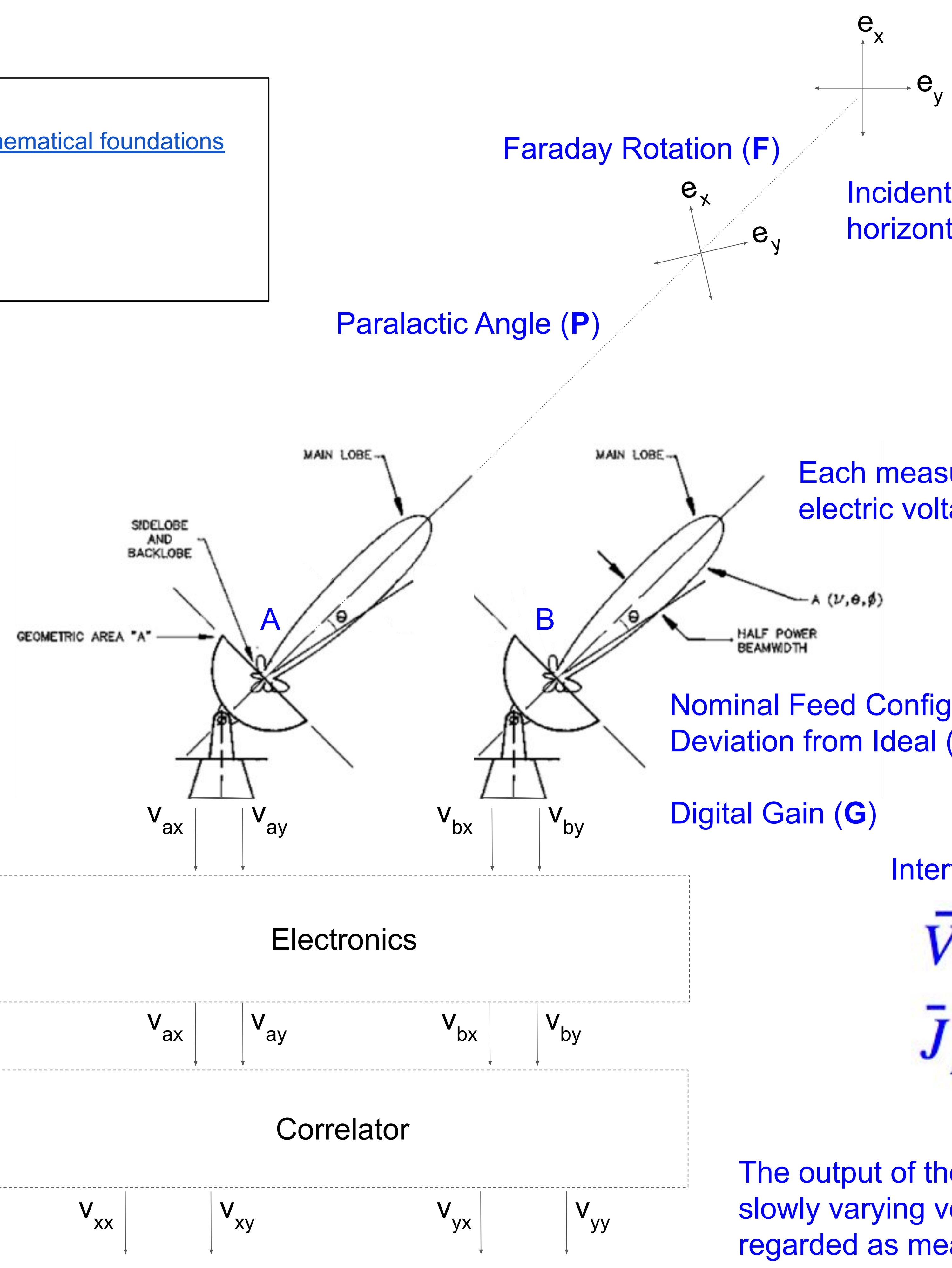
Basic Interferometry for Engineers

References
[Understanding radio polarimetry. I. Mathematical foundations]
[Interferometry Concepts]
[Advanced Interferometry & VLBI]
[Advanced Interferometry]
[A Brief Introduction to Interferometry]

An Interferometer Arm
Single baseline, 2 antennas, each with a receptor per polarisation connected to a correlator that cross-multiplies the input signals and time average the products.

Angular Resolution
Resolving power is the ability of an imaging device to separate points of an object that are located at a small angular distance. Angular resolution describes the ability of the radio telescope to distinguish small detail of an object and is limited by the size of the interference fringes.



Incident electromagnetic field has both a horizontal and a vertical component.

Each measured polarization is converted to an electric voltage by the receptor.

Nominal Feed Configuration (**C**)
Deviation from Ideal (**D**)

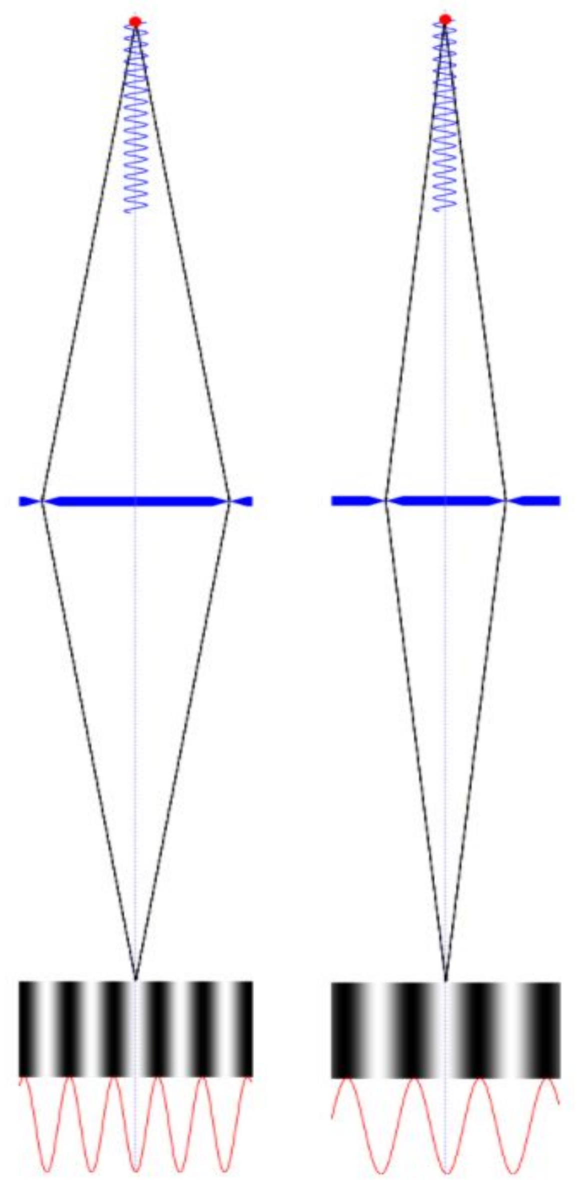
Digital Gain (**G**)

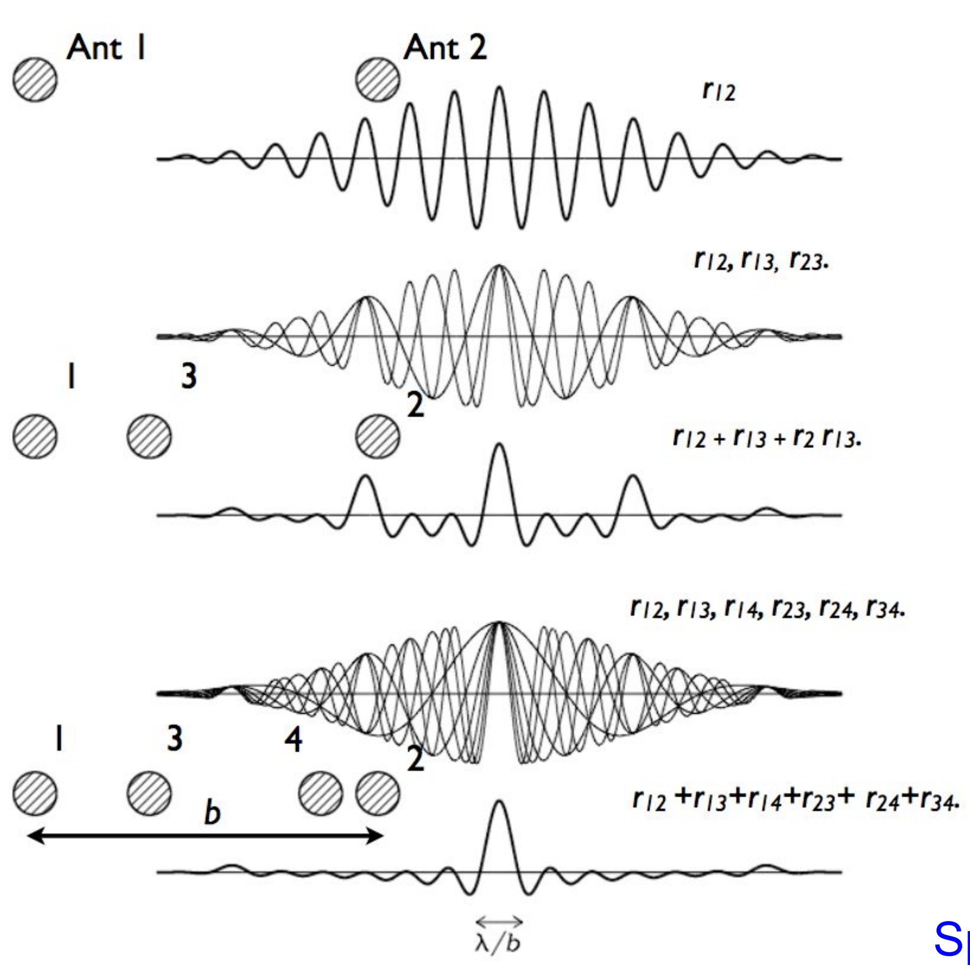
In the path between the source and the correlator, a number of intentional and spurious transformations occur.

Interferometer / **Measurement Equation**

$$\bar{V}_A = \bar{J}_A \bar{e}_A$$

$$\bar{J}_A = \bar{G}_A \bar{D}_A \bar{C}_A \bar{P}_A \bar{F}_A$$

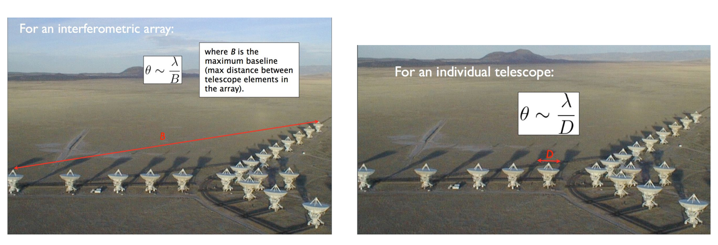


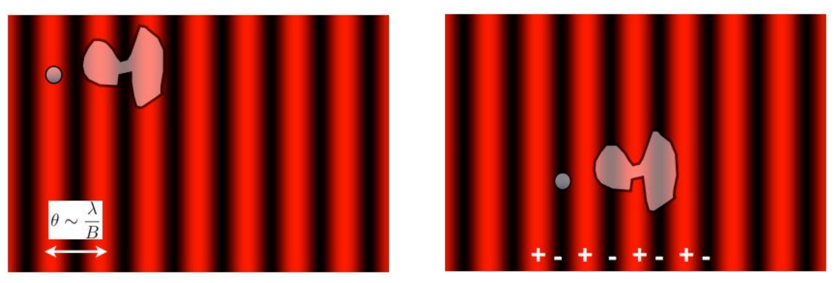


The output of the correlator is slowly varying voltage and can be regarded as measuring the real and imaginary parts of the complex interference fringe pattern, or complex visibilities.
Thus, **the correlator's basic output is a set of measurements of the complex visibility function**.
The visibility function is simply the cross-correlation measuring the coherence of the signal as a function of spacing between pairs of antennas.

$$v = <\begin{bmatrix} v_{AX}v^*_{BX} \\ v_{AX}v^*_{BY} \\ v_{AY}v^*_{BX} \\ v_{AY}v^*_{BY} \end{bmatrix}>$$

Stokes visibilities
Total intensity
$$I = \ <|v_{XX}|^2 + |v_{YY}|^2>$$
Linear polarisation
$$Q = \ <|v_{XX}|^2 - |v_{YY}|^2>$$
Orthogonal linear polarisation
$$U = \ 2 < Re|v_{XY}|>$$
circular polarisation
$$V = \ 2 < Im|v_{XY}|>$$





Spatial resolution is inversely proportional to baseline length.
Shorter baselines are required to image sources resolved by longer baselines.

Imaging

The sky brightness function is the Fourier transform of the visibility function: $I(l,m) = \iint S(u,v)V(u,v)e^{2\pi i(ul+vm)}dudv$ [Jy = 10^{-26} $WHz^{-1}m^{-2}$]
where V(u,v) is the spatial coherence or visibility function and I(l,m) is the sky intensity or brightness function. Consequently, a 2D Fourier transform of the measured visibility function will produce an image of the target object.

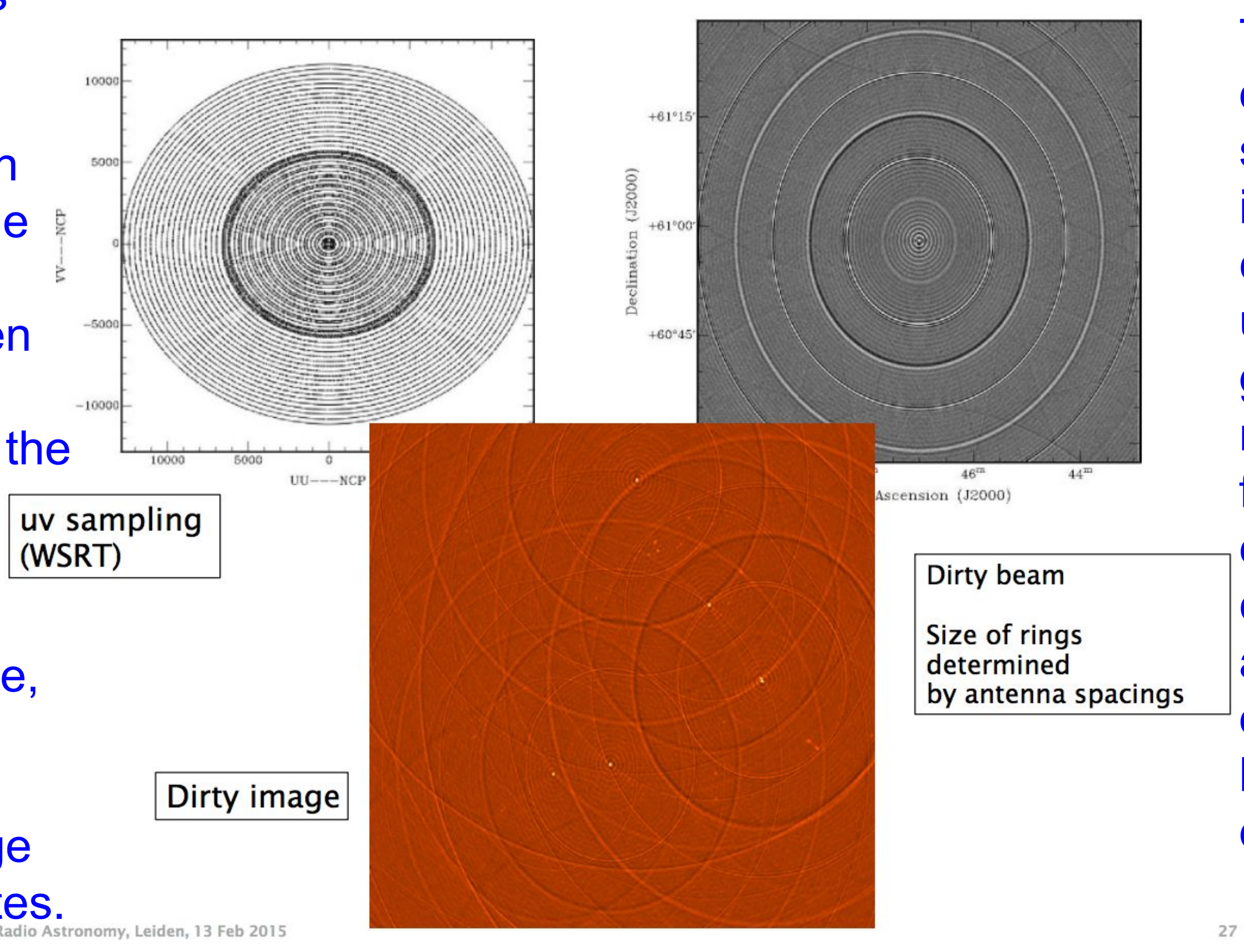
$$I^D = I * B$$
$$B(l,m) = \iint S(u,v)e^{2\pi i(ul+vm)}dudv$$

The data captured by a radio telescope is essentially a convolution of the source observed and the beam shape, which smooths (or blurs) the observed source.

However, the sky brightness cannot be retrieved directly, since the uv-plane is only sampled, S(u,v), and must be obtained through deconvolution.

Pairs of antennas record a visibility function in the uv-plane, which in turn, describes the distribution of baselines between antennas -- commonly called the baseline tracks.

For sources not located at the pole, the projected baseline length appears to change as the Earth rotates.



The finite number of baselines sampled, leads to incomplete coverage of the uv-plane, leaving gaps in the measured visibility function. This effect of sampling can be described as the convolution of the target brightness with the dirty beam B(l,m).

Deconvolution estimates the sky brightness function using a model of the source and interpolating across the gaps in the uv-coverage. The CLEAN algorithm assumes the source consists of a number of Gaussian components in an otherwise empty sky. It models these components as a series of delta functions, convolved with a beam the same dimension as the main lobe. This fitting and subtracting of emission is iterated until all the emission is subtracted from the original image.

The final image is formed by convolving the accumulated point source model with a Gaussian fitted to the mail lobe and adding the residual left after cleaning.