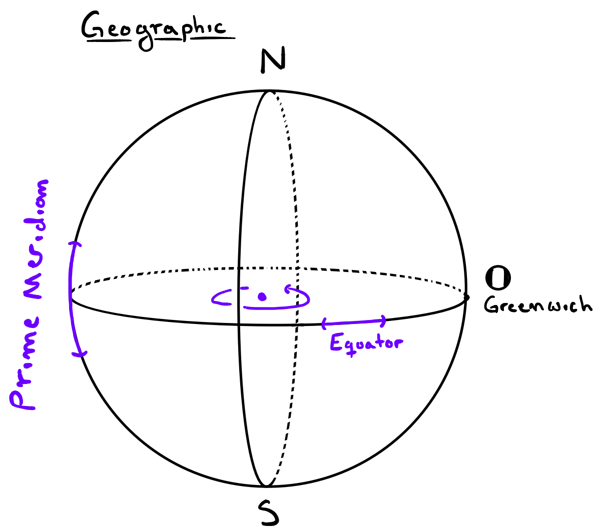
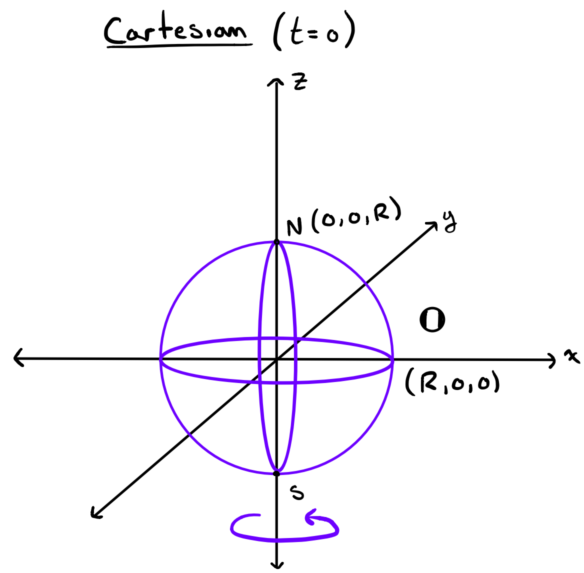
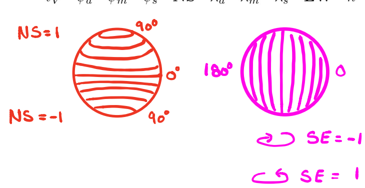
Model





$$t_V \quad \overbrace{\psi_d \quad \psi_m \quad \psi_s}^{\text{Latt}} \quad \mathbf{NS} \quad \overbrace{\lambda_d \quad \lambda_m \quad \lambda_s}^{\text{Long}} \quad \mathbf{EW} \quad h \qquad (8)$$



t_V is a real number given to an accuracy of 10^{-2} seconds ranging from 0 to 10^6. This is the time at which the `vehicle` is at the specified position.

ψ_d is an integer ranging from 0° (i.e., the Equator) to +90° (i.e., the North or South Pole).

ψ_m is an integer ranging from 0 to 59 minutes of degree.

ψ_s is a real number ranging from 0 to 59.9999 seconds of degree. It should be given to an accuracy of 10^{-2} (which corresponds to an accuracy of about a foot).

NS is an integer that is +1 North of the equator and −1 South of the equator.

λ_d is an integer ranging from 0° (i.e., the meridian of Greenwich) to 180 (i.e., 180 degrees east, or west, the *date line*).

λ_m is an integer ranging from 0 to 59 minutes of degree.

λ_s is a real number ranging from 0 to 59.9999 seconds of degree, given to the same accuracy as ψ_s.

EW is an integer that is +1 east of Greenwich and −1 west of Greenwich.

h is a real number giving the altitude in meters, to an accuracy of 1cm.

$\theta = \omega t$

$F = ma$

$d = rt$

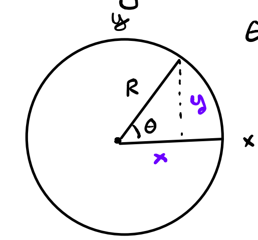
Exercise 1: Find a formula that describes the trajectory of the point **O** in Cartesian coordinates as a function of time.

Bird eye

$v = \frac{d}{t}$

$\theta = \omega \cdot t$

$= \frac{2\pi}{S} \cdot t$



$$\mathbf{O}_{car}(t) = \begin{bmatrix} R\cos\left(\frac{2\pi}{S}t\right) \\ R\sin\left(\frac{2\pi}{S}t\right) \\ 0 \end{bmatrix}$$

$\cos\theta = \frac{x}{R}$

$x = R \cdot \cos\theta$

Exercise 2: Write a program that converts angles from degrees, minutes, and seconds to radians, and vice versa. Make sure your program does what it's supposed to do.

For the following four exercises assume that t_V equals true Universal time and denote it by t.

* Convert Program

```python
import math
pi = math.pi

def deg_to_rad(deg, min, sec):
    rad = (deg + min / 60 + sec / 3600) / 180 * pi
    return rad

#I need to find a way to seperate the whole number
def rad_to_deg(rad):
    dummy = rad*180/math.pi
    deg = int(dummy)            #Just took the whole number
    min = int((dummy-int(dummy))*60)          #Take the left over decimals of the oringnal number and mult by 60
    sec = int(((dummy - int(dummy))*60-min)*60)
    return (deg, min, sec)
```

Floor ?

We can change this of course.

Python math.~ returns radians.

Exercise 3: Find a formula that converts position as given in (8) at time $t = 0$ into Cartesian coordinates.

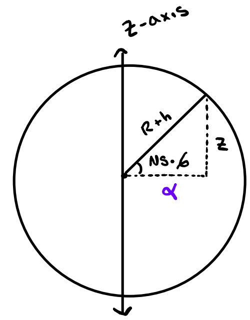
$$\underset{=0}{t_V} \quad \underbrace{\psi_d \quad \psi_m \quad \psi_s}_{\in [0, \frac{\pi}{2}]} \quad \underbrace{\mathbf{NS}}_{\in \{-1,1\}} \quad \underbrace{\lambda_d \quad \lambda_m \quad \lambda_s}_{\in [0, \pi]} \quad \underbrace{\mathbf{EW}}_{\in\{-1,1\}} \quad \underbrace{h}_{=h} \tag{8}$$

Need x (eqn 8), y (eqn 8), z (eqn 8)

① $\psi_d, \psi_m, \psi_s \rightsquigarrow \phi \in [0, \frac{\pi}{2}]$ (Convert to radians using * Convert program)

$\lambda_d, \lambda_m, \lambda_s \rightsquigarrow \theta \in [0, \pi]$ (Convert to radians using * Convert program)

② $\underline{z}$ $\quad \sin(NS \cdot \phi) = \frac{z}{R+h}$



$$\Longrightarrow$$

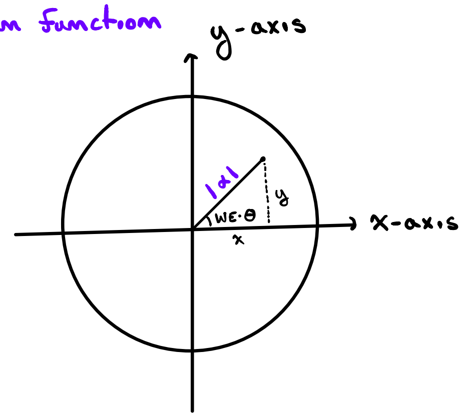
$$z(\phi, NS, h) = (R + h)\sin(NS \cdot \phi)$$

$$\alpha(\phi, NS, h) = (R + h) \cdot \cos(NS \cdot \phi)$$

we don't need this

$\Rightarrow$ cos is even function

$\underline{x \ \& \ y}$



$$\begin{aligned} x(\theta, WE, \alpha) &= |\alpha| \cdot \cos(WE \cdot \theta) \\ &= |(R+h) \cdot \cos(NS \cdot \phi)| \cos(WE \cdot \theta) \end{aligned}$$

$$\begin{aligned} y(\theta, WE, \alpha) &= |\alpha| \cdot \sin(WE \cdot \theta) \\ &= |(R+h) \cdot \cos(NS \cdot \phi)| \sin(WE \cdot \theta) \end{aligned}$$

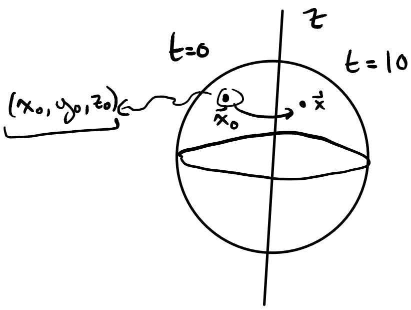
Thus Given we know ϕ & θ (using program from exercise 2) we have at $t=0$

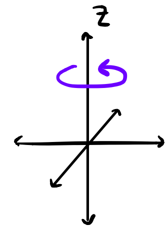
$$\text{Eqn}(8) := \vec{x}_0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{bmatrix} |\alpha| \cdot \cos(WE \cdot \theta) \\ |\alpha| \cdot \sin(WE \cdot \theta) \\ (R + h) \cdot \sin(NS \cdot \phi) \end{bmatrix}$$

Exercise 4: Find a formula that converts position and general time t as given in (8) into Cartesian coordinates.

We just use our formula from exercise 3 to find the Cartesian cordinates at $t=0$ & then use a rotation matrix to find position at time t.

Rotation Matrix: $$R(t) = \underbrace{\begin{bmatrix} \cos\left(\frac{2\pi}{S}t\right) & -\sin\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ \sin\left(\frac{2\pi}{S}\cdot t\right) & \cos\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Rotates around z-axis}}$$





$$\Longrightarrow \vec{X}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = R(t)\,\vec{x}_0$$

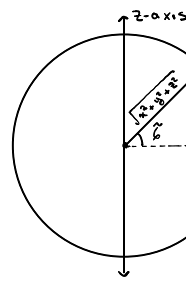
$$= \begin{bmatrix} \cos\left(\frac{2\pi}{S}t\right) & -\sin\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ \sin\left(\frac{2\pi}{S}\cdot t\right) & \cos\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} |\alpha|\cdot\cos(WE\cdot\theta) \\ |\alpha|\cdot\sin(WE\cdot\theta) \\ h\cdot\sin(NS\cdot\phi) \end{bmatrix}$$

$$\vec{X}(t) = \begin{array}{cc} \begin{bmatrix} |\alpha|\cos\left(\frac{2\pi}{S}\cdot t\right)\cos(WE\cdot\theta) - |\alpha|\sin\left(\frac{2\pi}{S}\cdot t\right)\sin(WE\cdot\theta) \\ |\alpha|\sin\left(\frac{2\pi}{S}\cdot t\right)\cos(WE\cdot\theta) + |\alpha|\cos\left(\frac{2\pi}{S}\cdot t\right)\sin(WE\cdot\theta) \\ (R+h)\cdot\sin(NS\cdot\phi) \end{bmatrix} & \begin{array}{c} x \\ y \\ z \end{array} \end{array}$$

Exercise 5: Find a formula that converts a position given in Cartesian coordinates at time $t = 0$ into a position of the form (8).

Cartesian $\rightsquigarrow$ Geographical

Given $[x, y, z]^T$ need to find: t_V ψ_d ψ_m ψ_s **NS** λ_d λ_m λ_s **EW** h



$\sin(\tilde{\zeta}) = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

$\tilde{\zeta} = \arcsin\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$\in [-\frac{\pi}{2}, \frac{\pi}{2}]$

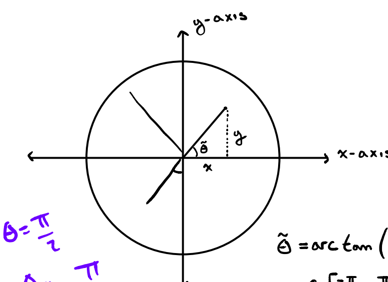
0, 90

0, $\frac{\pi}{2}$

- $t_V = 0$
- $h = \sqrt{x^2+y^2+z^2} - R$
- $NS = \begin{cases} 1 & z \geq 0 \\ -1 & z < 0 \end{cases}$
- $\psi_d, \psi_m, \psi_s = \left| \arcsin\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right) \right| \xrightarrow{\text{function}} [\psi_d, \psi_m, \psi_s]$

  * convert program

- $WE = \begin{cases} 1 & y \geq 0 \\ -1 & y < 0 \end{cases}$



$\tilde{\theta} = \arctan\left(\frac{y}{x}\right)$

$\in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$y < 0$ $\theta = \frac{\pi}{2}$

$y > 0$ $\theta = -\frac{\pi}{2}$

- $\lambda_d, \lambda_m, \lambda_s := \begin{cases} \frac{\pi}{2} & x = 0 \\ 0 & y = 0 \text{ \& } x > 0 \\ \pi & y = 0 \text{ \& } x < 0 \\ \arctan\left(\frac{y}{x}\right) & y > 0 \text{ \& } x > 0 \\ \left|\arctan\left(\frac{y}{x}\right)\right| & y < 0 \text{ \& } x > 0 \\ \pi + \arctan\left(\frac{y}{x}\right) & y > 0 \text{ \& } x < 0 \\ \pi - \arctan\left(\frac{y}{x}\right) & y < 0 \text{ \& } x < 0 \end{cases}$

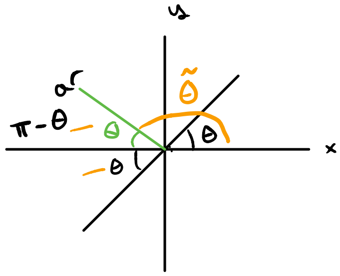
Not sure entirely how to deal w/ boundary cases.

unnessary?

$\xrightarrow{\text{function}} [\lambda_d, \lambda_m, \lambda_s]$

* convert program

y



Exercise 6: Find a formula that converts general time t and a position given in Cartesian coordinates into a position of the form (8).

Need to unrotate for t seconds

$$R^{-1}(t) = R(t)^T = \begin{bmatrix} \cos\left(\frac{2\pi}{S}t\right) & \sin\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ -\sin\left(\frac{2\pi}{S}\cdot t\right) & \cos\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Given $\vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$ we have $\underline{\vec{x}_0 = R^T(t)\, x(t)}$

$$\vec{x}_0 = \begin{bmatrix} \cos\left(\frac{2\pi}{S}t\right) & \sin\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ -\sin\left(\frac{2\pi}{S}\cdot t\right) & \cos\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

$$\vec{x}_0 = \begin{bmatrix} x(t)\cos\left(\frac{2\pi}{S}t\right) + y(t)\sin\left(\frac{2\pi}{S}\cdot t\right) \\ -x(t)\sin\left(\frac{2\pi}{S}\cdot t\right) + y(t)\cos\left(\frac{2\pi}{S}\cdot t\right) \\ z(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

Then we just apply formula from exercise 5 to get geographic coordinates.

Exercise 7: Find a formula that describes the trajectory of lamp post B12 in Cartesian coordinates as a function of time.

For example, according to my Magellan Trailblazer the street light labeled B12[6] in front of the South Window of my office (at time t) is located at:

$$\underbrace{t}_{\text{Assume } t=0??} \quad 40 \quad 45 \quad 55.0 \quad 1 \quad 111 \quad 50 \quad 58.0 \quad -1 \quad 1372.00 \qquad (9)$$ B12

i.e., at latitude 40° 45' 55" North, longitude 111° 50' 58" West, and an altitude of 1372 m.

× No, I don't think you assume this

⇒ Yes you can... (geographical doesn't change w/ time)

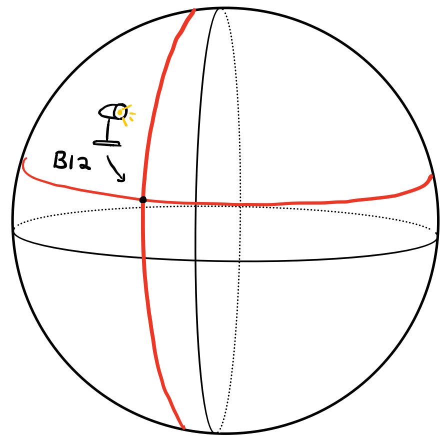
* Convert Program

Define $\phi = \phi(40, 45, 55)$

$$= \left(40 + \frac{45}{60} + \frac{55}{3600}\right) \cdot \frac{\pi}{180}$$

$$\theta = \theta(111, 50, 58.0)$$

$$= \left(111 + \frac{56}{60} + \frac{58}{3600}\right) \cdot \frac{\pi}{180}$$



⇒ We can use formula from exercise 4.

$$\vec{x}_{Lamp}(t+t_v) = \begin{bmatrix} |\alpha| \cos\left(\frac{2\pi}{S}(t+t_v)\right)\cos(WE\cdot\theta) - |\alpha|\sin\left(\frac{2\pi}{S}(t+t_v)\right)\sin(WE\cdot\theta) \\ |\alpha|\sin\left(\frac{2\pi}{S}\cdot(t+t_v)\right)\cos(WE\cdot\theta) + |\alpha|\cos\left(\frac{2\pi}{S}(t+t_v)\right)\sin(WE\cdot\theta) \\ (R+h)\cdot\sin(NS\cdot\phi) \end{bmatrix}$$

where $\alpha = (R+h)\cdot\cos(NS\cdot\phi)$

We can simplify by evaluating numerical values

Satellite

Satellite Program

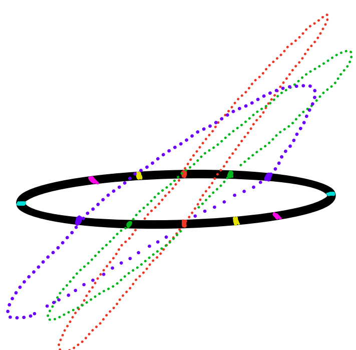
① Compute the position of Vehicle in Cartesian Coordinates

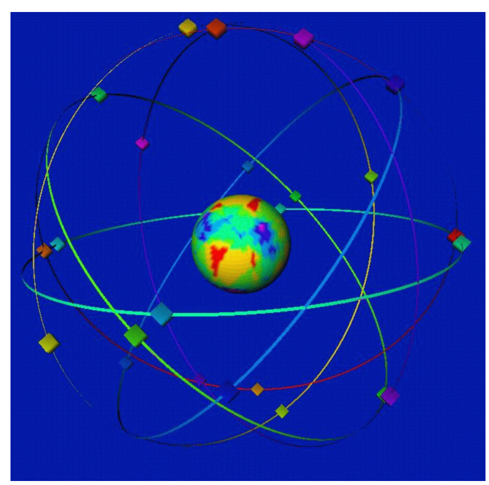
- (Bassically program exercise 4)

② • Compute data (i_s, t_s, x_s) (See if 15° above horizon)

$$\begin{cases} i_s: \text{Which satellite}, \in [0,23] \text{ (24 satellites)} \\ x_s: \text{Find cartesian coordinate of Satellite. (How ?)} \\ t_s: c(t_v - t_s) = \| x_v - x_s \| \end{cases}$$

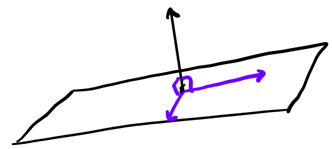
↳ Solve system of equations from satellites above horizon.





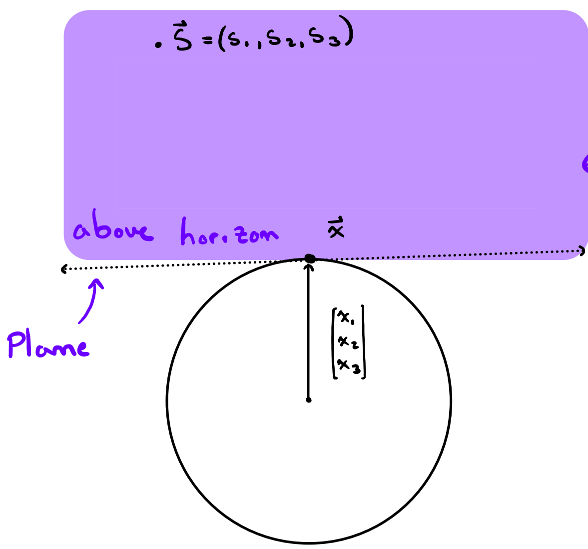
Exercise 8: Given a point $\mathbf{x}$ on earth and a point $\mathbf{s}$ in space, both in Cartesian coordinates, find a condition that tells you whether $\mathbf{s}$ as viewed from $\mathbf{x}$ is above the horizon.

Plane equation $\quad \vec{n} \cdot \overrightarrow{P_0P} = 0$

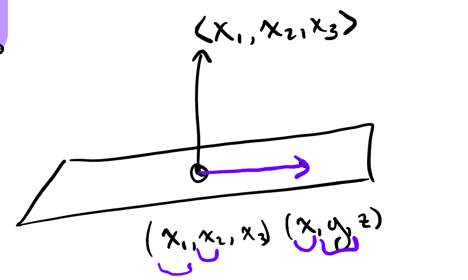


$$\langle x_1, x_2, x_3 \rangle \cdot \langle x - x_1, y - x_2, z - x_3 \rangle = 0$$

$$x_1 x + x_2 y + x_3 z = x_1^2 + x_2^2 + x_3^2$$



$$\underbrace{x_1 x + x_2 y + x_3 z \geq x_1^2 + x_2^2 + x_3^2}$$



$$\underbrace{\langle x - x_1, y - x_2, z - x_3 \rangle}$$

Thus $\vec{S} = (s_1, s_2, s_3)$ is in the horizon of point $\vec{x} = (x_1, x_2, x_3)$ if

$$\underline{x_1 \cdot s_1 + x_2 s_2 + x_3 s_3 \geq x_1^2 + x_2^3 + x_3^2}$$

or $\vec{S}$ must satisfy the inequality...

$$\underline{x_1 \cdot x + x_2 y + x_3 z \geq x_1^2 + x_2^3 + x_3^2}$$

Exercise 9: Discuss how to compute t_S and $\mathbf{x}_S$.

Well we are given x_v (position of Vechicle) and t_v (time of Vechicle location).

Cartesian

Want to find x_s and t_s where x_s is position and t_s is time such that

$$c|t_v - t_s| = \|x_v - x_s\|$$

Use equation (20)