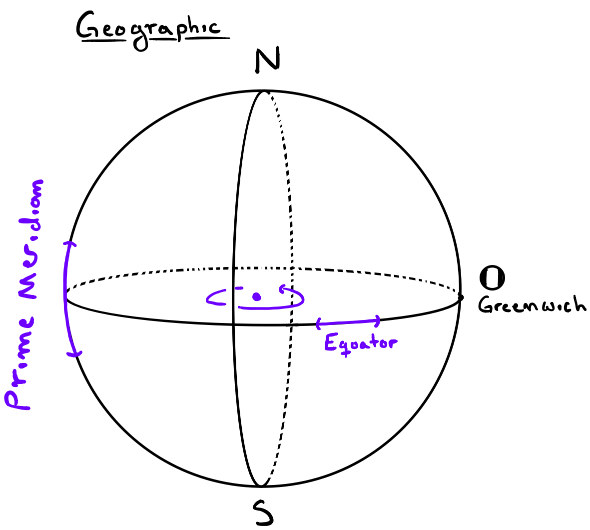
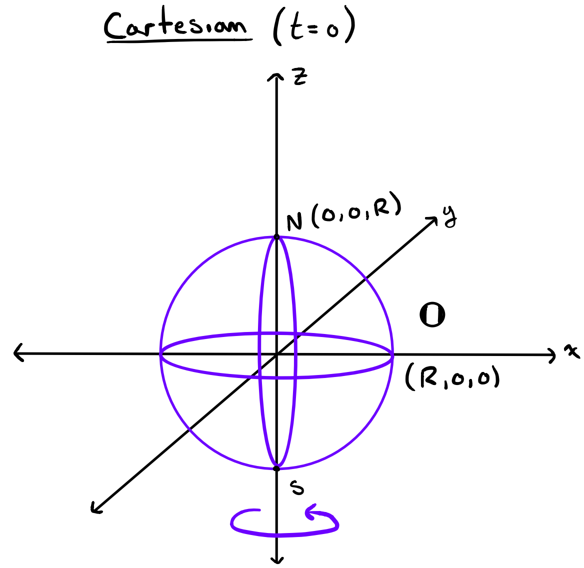
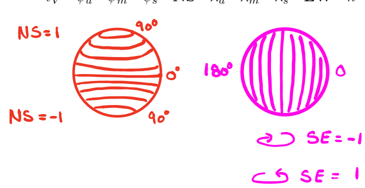
Model





$$t_V \quad \overbrace{\psi_d \quad \psi_m \quad \psi_s}^{\text{Latt}} \quad \mathbf{NS} \quad \overbrace{\lambda_d \quad \lambda_m \quad \lambda_s}^{\text{Long}} \quad \mathbf{EW} \quad h \tag{8}$$



- t_V is a real number given to an accuracy of 10^{-2} seconds ranging from 0 to 10^6. This is the time at which the `vehicle` is at the specified position.
- ψ_d is an integer ranging from 0° (i.e., the Equator) to +90° (i.e., the North or South Pole).
- ψ_m is an integer ranging from 0 to 59 minutes of degree.
- ψ_s is a real number ranging from 0 to 59.9999 seconds of degree. It should be given to an accuracy of 10^{-2} (which corresponds to an accuracy of about a foot).
- **NS** is an integer that is +1 North of the equator and −1 South of the equator.
- λ_d is an integer ranging from 0° (i.e., the meridian of Greenwich) to 180 (i.e., 180 degrees east, or west, the *date line*).
- λ_m is an integer ranging from 0 to 59 minutes of degree.
- λ_s is a real number ranging from 0 to 59.9999 seconds of degree, given to the same accuracy as ψ_s.
- **EW** is an integer that is +1 east of Greenwich and −1 west of Greenwich.
- h is a real number giving the altitude in meters, to an accuracy of 1cm.

$\theta = \omega t$

$F = ma$

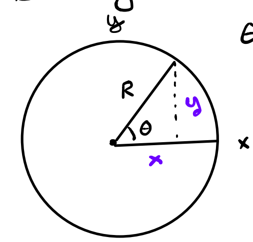
$d = rt$

Exercise 1: Find a formula that describes the trajectory of the point **O** in Cartesian coordinates as a function of time.

$$V = \frac{d}{t}$$

Bird eye

$$\mathbf{O}_{car}(t) = \begin{bmatrix} R\cos\left(\frac{2\pi}{S}t\right) \\ R\sin\left(\frac{2\pi}{S}t\right) \\ 0 \end{bmatrix}$$



$$\theta = \omega \cdot t = \frac{2\pi}{S} \cdot t$$

$$\cos\theta = \frac{x}{R}$$

$$x = R \cdot \cos\theta$$

Exercise 2: Write a program that converts angles from degrees, minutes, and seconds to radians, and vice versa. Make sure your program does what it's supposed to do.

For the following four exercises assume that t_V equals true Universal time and denote it by t.

```python
import math
pi = math.pi

def deg_to_rad(deg, min, sec):
    rad = (deg + min / 60 + sec / 3600) / 180 * pi
    return rad

# Need to find a way to separate the whole number.
def rad_to_deg(rad):
    dummy = rad * 180 / pi
    deg = floor(dummy) # Just use the whole number
    min = floor((dummy - floor(dummy)) * 60) # Multiply the decimals by 60
    sec = floor(((dummy - floor(dummy)) * 60 - min) * 60)
    return deg, min, sec

```

Exercise 3: Find a formula that converts position as given in (8) at time $t = 0$ into Cartesian coordinates.

$$\underset{=0}{t_V} \quad \underbrace{\psi_d \quad \psi_m \quad \psi_s}_{\in [0, \frac{\pi}{2}]} \quad \underbrace{\mathbf{NS}}_{\in\{-1,1\}} \quad \underbrace{\lambda_d \quad \lambda_m \quad \lambda_s}_{\in [0, \pi]} \quad \underbrace{\mathbf{EW}}_{\in\{-1,1\}} \quad \underbrace{h}_{=h} \tag{8}$$

Need x (eqn 8), y (eqn 8), z (eqn 8)

① $\psi_d, \psi_m, \psi_s \rightsquigarrow \varphi \in [0, \frac{\pi}{2}]$ (Convert to radians using * Convert program)

$\lambda_d, \lambda_m, \lambda_s \rightsquigarrow \theta \in [0, \pi]$ (Convert to radians using * Convert program)

② z

$$\sin(NS \cdot \varphi) = \frac{z}{R+h}$$

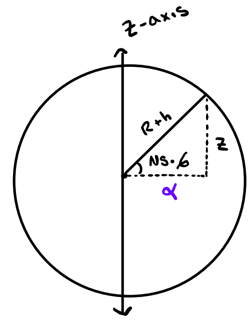
$$\Longrightarrow$$

$$z(\varphi, NS, h) = (R + h)\sin(NS \cdot \varphi)$$

$$\alpha(\varphi, NS, h) = (R + h) \cdot \cos(NS \cdot \varphi)$$

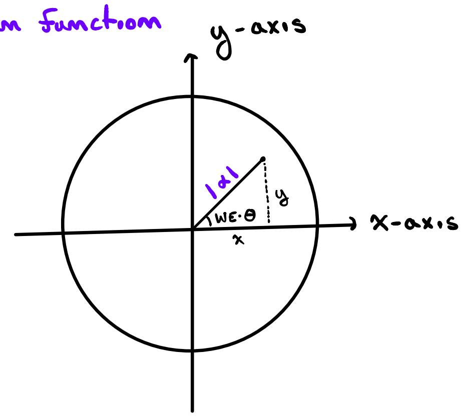
we don't need this

$\Rightarrow$ cos is even function



x & y

$$x(\theta, WE, \alpha) = |\alpha| \cdot \cos(WE \cdot \theta) = |(R+h) \cdot \cos(NS \cdot \varphi)| \cos(WE \cdot \theta)$$



$$y(\theta, WE, \alpha) = |\alpha| \cdot \sin(WE \cdot \theta) = |(R+h) \cdot \cos(NS \cdot \varphi)| \sin(WE \cdot \theta)$$

Thus Given we know φ & θ (using program from exercise 2) we have at $t=0$

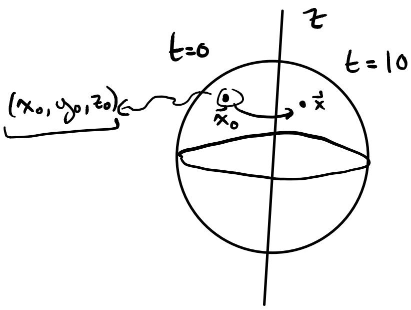
$$\text{Eqn}(8) := \vec{x}_0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} |\alpha| \cdot \cos(WE \cdot \theta) \\ |\alpha| \cdot \sin(WE \cdot \theta) \\ (R + h) \cdot \sin(NS \cdot \varphi) \end{bmatrix}$$

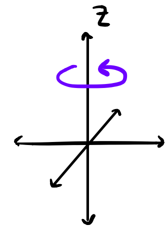
Exercise 4: Find a formula that converts position and general time t as given in (8) into Cartesian coordinates.

We just use our formula from exercise 3 to find the Cartesian coordinates at $t=0$ & then use a rotation matrix to find position at time t.

Rotation Matrix:

$$\underbrace{R(t) = \begin{bmatrix} \cos\left(\frac{2\pi}{S}t\right) & -\sin\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ \sin\left(\frac{2\pi}{S}\cdot t\right) & \cos\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Rotates around z-axis}}$$





$$\Longrightarrow \vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = R(t)\,\vec{x}_0$$

$$= \begin{bmatrix} \cos\left(\frac{2\pi}{S}t\right) & -\sin\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ \sin\left(\frac{2\pi}{S}\cdot t\right) & \cos\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} |\alpha|\cdot\cos(WE\cdot\theta) \\ |\alpha|\cdot\sin(WE\cdot\theta) \\ h\cdot\sin(NS\cdot\phi) \end{bmatrix}$$

$$\vec{x}(t) = \begin{array}{cl} \begin{bmatrix} |\alpha|\cos\left(\frac{2\pi}{S}\cdot t\right)\cos(WE\cdot\theta) - |\alpha|\sin\left(\frac{2\pi}{S}\cdot t\right)\sin(WE\cdot\theta) \\ |\alpha|\sin\left(\frac{2\pi}{S}t\right)\cos(WE\cdot\theta) + |\alpha|\cos\left(\frac{2\pi}{S}\cdot t\right)\sin(WE\cdot\theta) \\ (R+h)\cdot\sin(NS\cdot\phi) \end{bmatrix} & \begin{array}{l} x \\ y \\ z \end{array} \end{array}$$

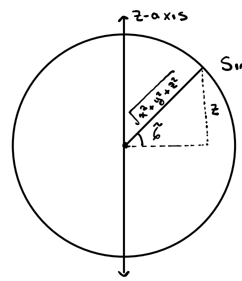
pizza

Exercise 5: Find a formula that converts a position given in Cartesian coordinates at time $t = 0$ into a position of the form (8).

Cartesian $\rightsquigarrow$ Geographical

Given $[x, y, z]^T$ need to find: t_V ψ_d ψ_m ψ_s **NS** λ_d λ_m λ_s **EW** h

- $t_V = 0$
- $h = \sqrt{x^2 + y^2 + z^2} - R$
- $NS = \begin{cases} 1 & z \geq 0 \\ -1 & z < 0 \end{cases}$



$\sin(\tilde{\zeta}) = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

$\tilde{\zeta} = \arcsin\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right)$

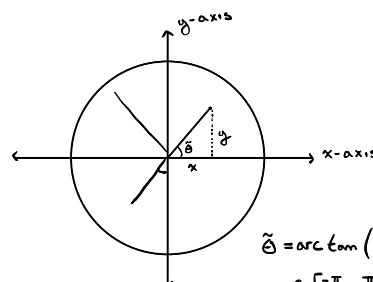
$\in [-\frac{\pi}{2}, \frac{\pi}{2}]$

0, 90

0, $\frac{\pi}{2}$

- $\psi_d, \psi_m, \psi_s = \left| \arcsin\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right) \right| \xrightarrow[\text{* convert program}]{\text{function}} [\psi_d, \psi_m, \psi_s]$

- $WE = \begin{cases} 1 & y \geq 0 \\ -1 & y < 0 \end{cases}$



$\tilde{\theta} = \arctan\left(\frac{y}{x}\right)$

$\in [-\frac{\pi}{2}, \frac{\pi}{2}]$

- $\lambda_d, \lambda_m, \lambda_s := \begin{cases} \frac{\pi}{2} & x = 0 \\ 0 & y = 0 \ \& \ x > 0 \\ \pi & y = 0 \ \& \ x < 0 \\ \left|\arctan\left(\frac{y}{x}\right)\right| & y \neq 0 \ \& \ x > 0 \\ \pi + \arctan\left(\frac{y}{x}\right) & y > 0 \ \& \ x < 0 \\ \pi - \arctan\left(\frac{y}{x}\right) & y < 0 \ \& \ x < 0 \end{cases}$ $\xrightarrow[\text{* convert program}]{\text{function}} [\lambda_d, \lambda_m, \lambda_s]$

Exercise 6: Find a formula that converts general time t and a position given in Cartesian coordinates into a position of the form (8).

Need to unrotate for t seconds

$$R^{-1}(t) = R(t)^T = \begin{bmatrix} \cos\left(\frac{2\pi}{S}t\right) & \sin\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ -\sin\left(\frac{2\pi}{S}\cdot t\right) & \cos\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Given $\vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$ we have $\vec{x}_0 = R^T(t)\,x(t)$

$$\vec{x}_0 = \begin{bmatrix} \cos\left(\frac{2\pi}{S}t\right) & \sin\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ -\sin\left(\frac{2\pi}{S}\cdot t\right) & \cos\left(\frac{2\pi}{S}\cdot t\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

$$\vec{x}_0 = \begin{bmatrix} x(t)\cos\left(\frac{2\pi}{S}t\right) + y(t)\sin\left(\frac{2\pi}{S}\cdot t\right) \\ -x(t)\sin\left(\frac{2\pi}{S}\cdot t\right) + y(t)\cos\left(\frac{2\pi}{S}\cdot t\right) \\ z(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

Then we just apply formula from exercise 5 to get geographic coordinates.

Exercise 7: Find a formula that describes the trajectory of lamp post B12 in Cartesian coordinates as a function of time.

For example, according to my Magellan Trailblazer the street light labeled B12[−6−] in front of the South Window of my office (at time t) is located at:

$$t \quad 40 \quad 45 \quad 55.0 \quad 1 \quad 111 \quad 50 \quad 58.0 \quad -1 \quad 1372.00 \qquad (9)$$

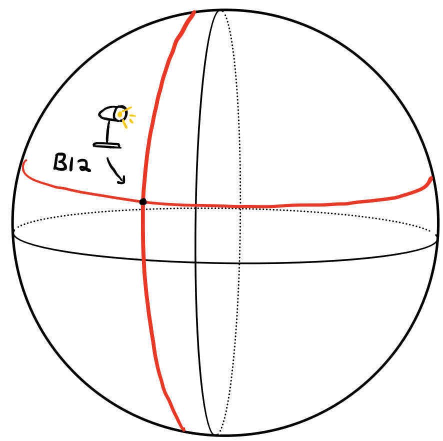
B12 

i.e., at latitude 40° 45' 55" North, longitude 111° 50' 58" West, and an altitude of 1372 m.

Assume $t=0$??

× No, I don't think you assume this

⇒ Yes you can... (geographical doesn't change w/ time)



* Convert Program

Define $\phi = \phi(40,45,55)$

$$= \left(40 + \frac{45}{60} + \frac{55}{3600}\right)\cdot\frac{\pi}{180}$$

$$\theta = \theta(111, 50, 58.0)$$

$$= \left(111 + \frac{56}{60} + \frac{58}{3600}\right)\cdot\frac{\pi}{180}$$

⇒ We can use formula from exercise 4.

$$\vec{x}_{Lamp}(t+t_v) = \begin{bmatrix} |\alpha| \cos\left(\frac{2\pi}{s}(t+t_v)\right)\cos(WE\cdot\theta) - |\alpha|\sin\left(\frac{2\pi}{s}(t+t_v)\right)\sin(WE\cdot\theta) \\ |\alpha|\sin\left(\frac{2\pi}{s}\cdot(t+t_v)\right)\cos(WE\cdot\theta) + |\alpha|\cos\left(\frac{2\pi}{s}(t+t_v)\right)\sin(WE\cdot\theta) \\ (R+h)\cdot\sin(NS\cdot\phi) \end{bmatrix}$$

where $\alpha = (R+h)\cdot\cos(NS\cdot\phi)$

We can simplify by evaluating numerical values

Satellite

Satellite Program

① Compute the position of Vehicle in Cartesian Coordinates

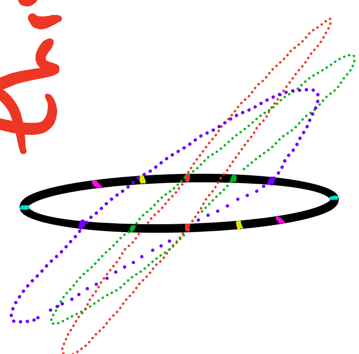
- (Bassically program exercise 4)

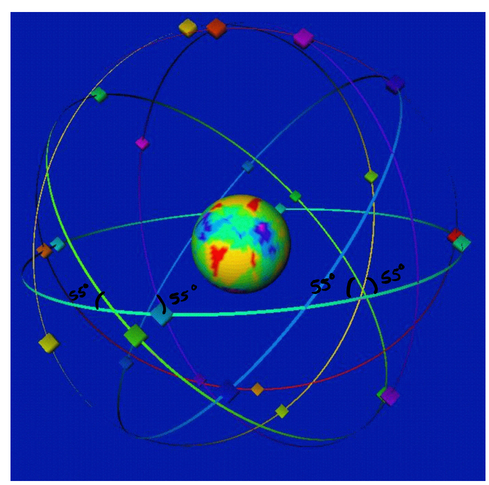
② • Compute data (i_s, t_s, x_s) (See if 15° above horizon)

$$\begin{cases} i_s: \text{Which satellite, } \in [0,23] \text{ (24 satellites)} \\ x_s: \text{Find cartesian coordinate of Satellite. (How ?)} \\ t_s: c(t_v - t_s) = \|x_v - x_s\| \end{cases}$$

↳ Solve system of equations from satellites above horizon.

Don't Need to Include this Page



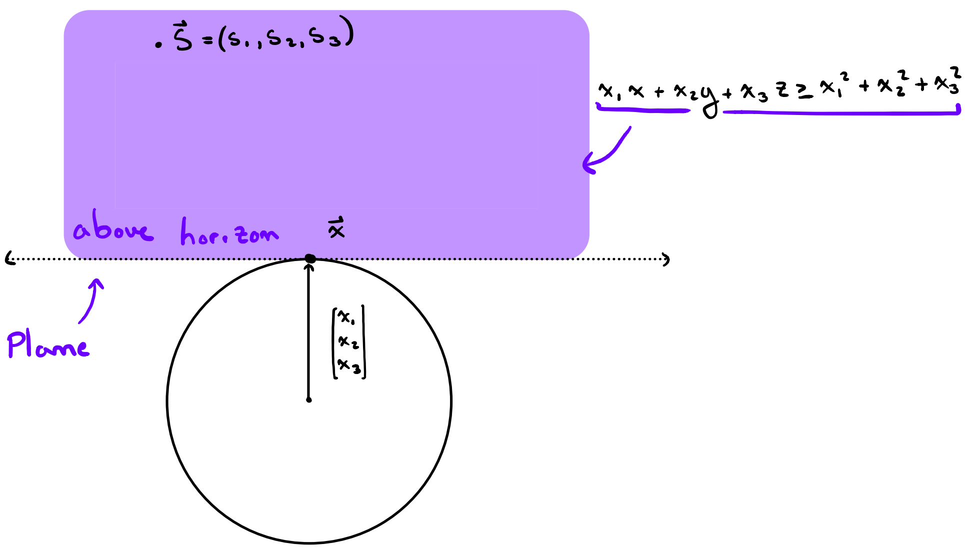


Exercise 8: Given a point **x** on earth and a point **s** in space, both in Cartesian coordinates, find a condition that tells you whether **s** as viewed from **x** is above the horizon.

Plane equation $\quad \vec{n} \cdot \overrightarrow{P_0 P} = 0$

$$\langle x_1, x_2, x_3 \rangle \cdot \langle x - x_1, y - x_2, z - x_3 \rangle = 0$$

$$x_1 x + x_2 y + x_3 z = x_1^2 + x_2^2 + x_3^2$$



Thus $\vec{S} = (s_1, s_2, s_3)$ is in the horizon of point $\vec{x} = (x_1, x_2, x_3)$ if

$$\underline{x_1 \cdot s_1 + x_2 s_2 + x_3 s_3 \geq x_1^2 + x_2^3 + x_3^2}$$

or $\vec{s}$ must satisfy the inequality...

$$\underline{x_1 \cdot x + x_2 y + x_3 z \geq x_1^2 + x_2^3 + x_3^2}$$

Exercise 9: Discuss how to compute t_S and $\mathbf{x}_S$.

Well we are given x_v (position of Vechicle) and t_v (time of Vechicle location).

Cartesian

Want to find x_s and t_s where x_s is position and t_s is time such that

$$c(t_v - t_s) = \| x_v - x_s \| \qquad (1)$$

Use equation (20) which describes the orbit of a Satellite

$$x_s(t) = (R+h)\left[\vec{u}\cos\left(\frac{2\pi t}{P} + \theta\right) + \vec{v}\sin\left(\frac{2\pi t}{P} + \theta\right)\right] \qquad (2)$$

Rearranging (1)

$$t_s = t_v - \frac{\| x_s(t_s) - x_v \|}{c}$$

$$t_s = t_v - \frac{1}{c}\cdot \left\| (R+h)\left[\vec{u}\cos\left(\frac{2\pi t_s}{P} + \theta\right) + \vec{v}\sin\left(\frac{2\pi t_s}{P} + \theta\right)\right] - x_v \right\|$$

This is now a Fixed point problem where,

$$t_{n+1} = t_v - \frac{1}{c}\| x_s(t_n) - x_v \|$$

We start w/ $t_0 = t_v$ and go until $c|t_v - t_s| < 10^{-2}$

1 cm of accuracy.

Exercise 10: Suppose you have data of the from (11) from 4 satellites. Write down a set of four equations whose solutions are the position of the vehicle in Cartesian coordinates, and t_V.

$i_s \quad t_s \quad X_s$

Provided $\quad i_0 \quad t_{s_0} \quad X_{s_0} \qquad$ Want $\quad t_v$ & X_v

$i_1 \quad t_{s_1} \quad X_{s_1}$

$i_2 \quad t_{s_2} \quad X_{s_2}$

$i_3 \quad t_{s_3} \quad X_{s_3}$

Equations:

$$\|x_v - x_{s_0}\| = c(t_v - t_{s_0})$$
$$\|x_v - x_{s_1}\| = c(t_v - t_{s_1})$$
$$\|x_v - x_{s_2}\| = c(t_v - t_{s_2})$$
$$\|x_v - x_{s_3}\| = c(t_v - t_{s_3})$$

Where $\|x_v - x_{s_i}\| = c(t_v - t_{s_i})$ can be writen as

$$\left[(x-\sigma_{i,1})^2 + (y-\sigma_{i,2})^2 + (z-\sigma_{i,3})^2\right]^{\frac{1}{2}} = c(t_v - t_{s_i}) \quad \text{for } i=0,1,2,3$$

We can of course eliminate t_v from equations by taking the difference of equations to maintain a level of accuracy. Then solve system of 3 equations for position. The solve for t_v.

Exercise 11: Suppose you have data of the form (11) from more than 4 satellites. Write down a least squares problem whose solution the position of the vehicle in Cartesian coordinates, and t_V.

Suppose we have data from $m+1$ satellites where $m+1 > 4$. Then we can create m equations in the form:

$$\|x_v - x_{s_0}\| - \|x_v - x_{s_i}\| + ct_{s_0} - ct_{s_i} = 0 \qquad \text{for } i = 1, 2, 3, \dots, m \tag{1}$$

Let $x_v = (x_1, x_2, x_3)$ and $x_{s_i} = (\sigma_{i,1}, \sigma_{i,2}, \sigma_{i,3})$ then we can write (1)

$$\left[(x_1-\sigma_{0,1})^2 + (x_2-\sigma_{0,2})^2 + (x_3-\sigma_{0,3})^2\right]^{\frac{1}{2}} - \left[(x_1-\sigma_{i,1})^2 + (x_2-\sigma_{i,2})^2 + (x_3-\sigma_{i,3})^2\right]^{\frac{1}{2}} - ct_{s_0} - ct_{s_i} = 0 \tag{2}$$

We will call equation (2) $F_i(x)$. Now define $f(x)$ as

$$f(x) = \sum_{i=1}^{m} F_i^2(x)$$

Now ideally $f(x_v) = 0$ would give position of satellite but this may not be possible to solve. So rather we minimize $f(x)$.

We set the gradient f to zero

$$\nabla f = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}\right]^T = \vec{0}$$

This will give us a system of equation

$$\frac{\partial f}{\partial x}(x, y, z) = 0$$
$$\frac{\partial f}{\partial y}(x, y, z) = 0$$
$$\frac{\partial f}{\partial z}(x, y, z) = 0$$

Which we can solve system using newtons method.

Once we solve $x_v = [x_1, x_2, x_3]^T$ we can then solve t_v

$$\|x_v - x_{s_0}\| = c(t_v - t_{s_0})$$

$$t_v = t_{s_0} + \frac{1}{c}\|x_v - x_{s_0}\|$$

Exercise 12: Find a formula for the *ground track* of satellite 1, i.e., the position in geographic coordinates directly underneath the satellite on the surface of the earth, as a function of time. Do you notice anything particular? What is the significance of the orbital period being exactly one half sidereal day.

The satellite having an orbital period of half a sidereal day is quite significant. It essentially completes its period twice as fast as earth completes it full rotation. From the perspective of an observer on earth it apears the satellite clompletes its full period in 1 sidereal day.

In fact from the perspective of earth, the satellites orbit pattern will look like a saddle or pringle chip as opposed to a perfect circle.

$$X_{trac,s_1}(t) = R\left[\cos\left(\frac{2\pi}{s}t+\theta\right), \sin\left(\frac{2\pi}{s}t+\theta\right), u_3\cos\left(\frac{2\pi}{p}t+\theta\right)+V_3\sin\left(\frac{2\pi}{p}t+\theta\right)\right]^T \qquad (1)$$

The equation from (1) describes the trajectory of our satellite in a cartesian coordinate system that is fixed to our geographic coordinate system. I.e C.C.S that rotates with earth.

Thus we can just use our equation from exercise 5 to find the geographic coordinates.

Concern: This works for a given t and then calculates geo-position. Not a function of time - Computationally expensive.

→ shouldn't be an issue.

Exercise 13: Find a precise description of Newton's method as it is applied to the nonlinear system obtained by processing data from 4 satellites, as derived in an earlier exercise. Your answer should include an explicit specification of the derivatives involved.

Given 4 satellites we can define the equation $F_i(x_v)$ where $x_v = (x_1, x_2, x_3)$ to be

$$F_i(x_v) = \|x_v - x_{s_0}\| - \|x_v - x_{s_i}\| - c(t_{s_0} - t_{s_i}) \qquad \text{for } i = 1, 2, 3$$

$$= \left[\sum_{k=1}^{3}(x_k - \sigma_{0,k})^2\right]^{\frac{1}{2}} - \left[\sum_{k=1}^{3}(x_k - \sigma_{i,k})^2\right]^{\frac{1}{2}} - c(t_{s_0} - t_{s_i})$$

Define,

$$F(\vec{x}) = [\, F_1(\vec{x}) \;\; F_2(\vec{x}) \;\; F_3(\vec{x}) \,]^T$$

Next we define the Jacobian of $F(x)$, we have the partial derivatives

- $$\frac{\partial F_i}{\partial x_j} = (x_j - \sigma_{0,j})\left[\sum_{k=1}^{3}(x_k - \sigma_{0,k})^2\right]^{-\frac{1}{2}} - (x_j - \sigma_{i,j})\left[\sum_{k=1}^{3}(x_k - \sigma_{i,k})^2\right]^{-\frac{1}{2}}$$

For $i = 1,2,3$ and $j = 1,2,3$. Thus

$$\nabla F = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial x_3} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial x_3} \\ \frac{\partial F_3}{\partial x_1} & \frac{\partial F_3}{\partial x_2} & \frac{\partial F_3}{\partial x_3} \end{bmatrix}$$

Now we use Newtons method:

We start with some $x^{(0)}$

We solve $\quad \nabla F(x^{(k)})\vec{s} = -F(x^{(k)})$

then define $\quad x^{(k+1)} = x^{(k)} + \vec{s}$

Repeat until $\quad \|s\| < 0.01$

In other words, until our error is less than 1 cm.

Exercise 14: Similarly, find Newton's method for the nonlinear system obtained from the least squares approach. Again, your answer should include an explicit specification of the derivatives involved.

We define $F_i(x_v)$ as in exercise 11.

$$F_i(x_v) = \left[\sum_{k=1}^{3}(x_k - \sigma_{0,k})^2\right]^{\frac{1}{2}} - \left[\sum_{k=1}^{3}(x_k - \sigma_{i,k})^2\right]^{\frac{1}{2}} - c(t_{s_0} - t_{s_i})$$

where $i = 1, 2, \ldots, m$ where $m > 3$

And

$$f(x) = \sum_{i=1}^{m} F_i^2(x)$$

Then we wish to solve $\nabla f(x) = 0$. This requires us to take the Jacobian of ∇f — this is the Hessian.

If we let

$$B = \nabla F = \left[\frac{\partial F_i}{\partial x_j}\right]_{\substack{i=1,2,\ldots,m \\ j=1,2,3}}$$

where

$$\frac{\partial F_i}{\partial x_j} = (x_j - \sigma_{0,j})\left[\sum_{k=1}^{3}(x_k - \sigma_{0,k})^2\right]^{-\frac{1}{2}} - (x_j - \sigma_{i,j})\left[\sum_{k=1}^{3}(x_k - \sigma_{1,k})^2\right]^{-\frac{1}{2}}$$

Then the Hessian is

$$H(x_v) \approx B^T B$$

$$= \begin{bmatrix} \sum_{i=1}^{3}\left(\frac{\partial F_i}{\partial x_1}\right)^2 & \sum_{i=1}^{3}\left(\frac{\partial F_i}{\partial x_1}\right)\left(\frac{\partial F_i}{\partial x_2}\right) & \sum_{i=1}^{3}\left(\frac{\partial F_i}{\partial x_1}\right)\left(\frac{\partial F_i}{\partial x_3}\right) \\ \sum_{i=1}^{3}\left(\frac{\partial F_i}{\partial x_1}\right)\left(\frac{\partial F_i}{\partial x_2}\right) & \sum_{i=1}^{3}\left(\frac{\partial F_i}{\partial x_2}\right)^2 & \sum_{i=1}^{3}\left(\frac{\partial F_i}{\partial x_2}\right)\left(\frac{\partial F_i}{\partial x_3}\right) \\ \sum_{i=1}^{3}\left(\frac{\partial F_i}{\partial x_1}\right)\left(\frac{\partial F_i}{\partial x_3}\right) & \sum_{i=1}^{3}\left(\frac{\partial F_i}{\partial x_2}\right)\left(\frac{\partial F_i}{\partial x_3}\right) & \sum_{i=1}^{3}\left(\frac{\partial F_i}{\partial x_3}\right)^2 \end{bmatrix}$$

Assume small residual we can neglect

Thus for some given $x^{(0)}$ solve

$$H(x^{(k)})\vec{s} = -\nabla f(x^{(k)})$$

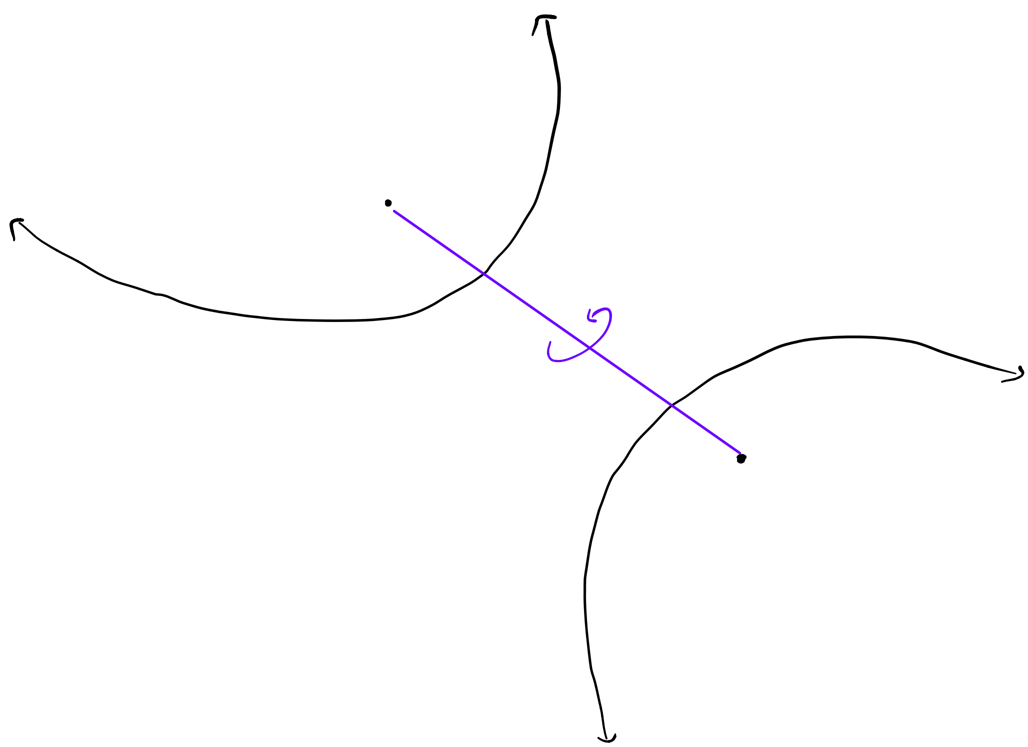
Then get

$$x^{(k+1)} = x^{(k)} + \vec{s}$$

Repeat until $\|\vec{s}\| < 0.01$

Exercise 15: Think about the number of solutions obtained by analyzing four satellite signals with an unknown vehicle time t_V. This is an open ended question that will not be graded!

Do not quite know yet...



Exercise 16: I gave an early draft of this assignment to my friend Meg Ikkal Anna Liszt^{-8-}. After muttering about the federal deficit she said that she has been talking to the Air Force (who operate GPS) for years. She does not understand why they are being so hard on themselves. She could save them billions of dollars because to determine position and altitude you only need three satellites, not four! Three satellites would give you three differences in signal run times, those would constitute three equations for the three components of position, once you know position you can compute true run time to the satellite, and from that you can compute the current time. She thinks that the Air Force is not implementing this approach because they don't want to pay her fee of 10% of the savings in launch costs of satellites alone. What do you think of this?

Suppose that we have info from only 3 satellites. Meg is proposing we can create three differences in signal runtime which would create our 3 equations with 3 unknowns.

This of course can be done but there is no gaurantee the system created will yield a unique solution.

It isn't possible in this case. We can only create 2 linearly independent difference equations. For example, if our 3 satellites are labeled 0, 1, 2 respectively, then the following difference equation are example of 2 that can generate all other difference equations.

$$F_1(x_v) = \|x_v - x_{s_0}\| - \|x_v - x_{s_1}\| - c(t_{s_0} - t_{s_1})$$

$$F_2(x_v) = \|x_v - x_{s_0}\| - \|x_v - x_{s_2}\| - c(t_{s_0} - t_{s_2})$$

Essentially, and 3 difference equations that Meg proposes will be a linearly dependent system. And therefore an underdetermined system that will not produce a unique result.

Exercise 17: After venting her frustration about the federal deficit Meg went to task with *me*. She said that "you academic types" like to be so "cumbersome". She thinks we don't use "common sense" because the very phrase isn't rooted in Latin or Greek. Why, she says, do I have to have integers **NS** and **EW** to indicate which hemisphere I'm on? Why, she says, don't I just make the degrees positive or negative? Indeed, why not?

There is some subtly to what Meg is proposing as an alternate.
If we just simply let degrees be negative or positive this doesn't account for the full representation we are actually using i.e. degrees AND minutes & seconds

This would be prone to giving funny/unexpected results in our program.