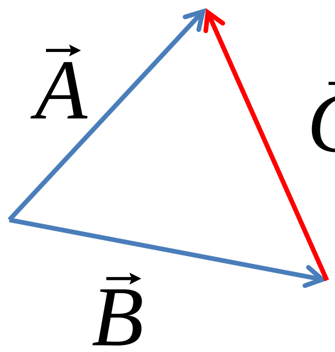
Institut supérieure des sciences appliquées et de technologie de Gafsa

TD 1 Électromagnétisme

Complément mathématique

Algèbre des vecteurs

1) Le vecteur différence:



$$\vec{C} = \vec{A} - \vec{B}$$

2) Le module de $\vec{C}$

$$\vec{C}^2 = \left(\vec{A} - \vec{B}\right)^2 = A^2 + B^2 - 2\vec{A} \bullet \vec{B}$$

$$C = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

3) Le P.V de deux vecteurs colinéaire est nul:

$$\vec{A}\times\vec{A}=\vec{0}$$

4) $$\left(\vec{A}\times\vec{B}\right)\bullet\left(\vec{C}\times\vec{D}\right)=\vec{A}\bullet\left[\vec{B}\times\left(\vec{C}\times\vec{D}\right)\right]$$

$$=\vec{A}\bullet\left[\vec{C}\bullet\left(\vec{B}\bullet\vec{D}\right)-\vec{D}\bullet\left(\vec{B}\bullet\vec{C}\right)\right]$$

$$=\left(\vec{A}\bullet\vec{C}\right)\bullet\left(\vec{B}\bullet\vec{D}\right)-\left(\vec{A}\bullet\vec{D}\right)\bullet\left(\vec{B}\bullet\vec{C}\right)$$

5) $$\left(\vec{A}\times\vec{B}\right)\times\vec{C}=-\vec{C}\times\left(\vec{A}\times\vec{B}\right)$$

$$=-\vec{A}\bullet\left(\vec{C}\bullet\vec{B}\right)+\vec{B}\bullet\left(\vec{C}\bullet\vec{A}\right)$$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{B}\bullet\left(\vec{A}\bullet\vec{C}\right)-\vec{C}\bullet\left(\vec{A}\bullet\vec{B}\right)$$

$$\Longrightarrow\quad\left(\vec{A}\times\vec{B}\right)\times\vec{C}\neq\vec{A}\times\left(\vec{B}\times\vec{C}\right)$$

Systèmes de coordonnées

1) Coordonnée cartésienne

$$\overrightarrow{OM} = x\hat{i} + y\hat{j} + z\hat{k}$$

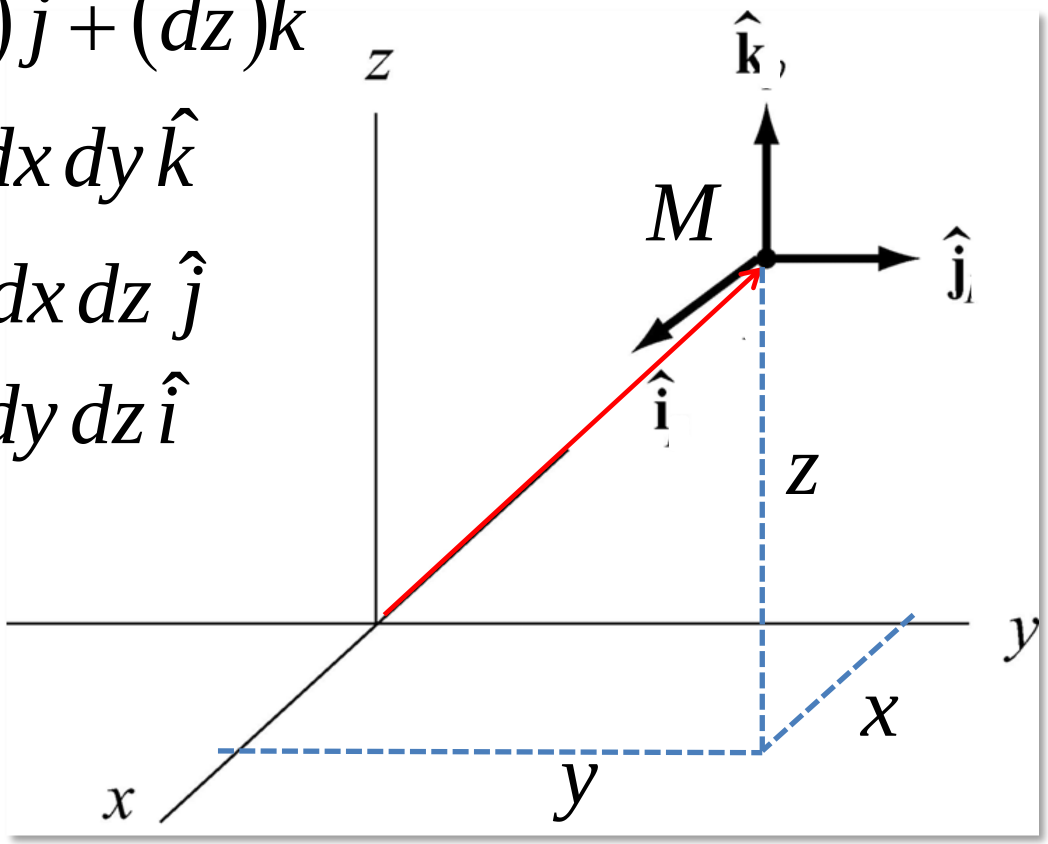
$$d\overrightarrow{OM} = d\vec{l}\,(dx)\hat{i} + (dy)\hat{j} + (dz)\hat{k}$$

$$d\vec{A}_1 = (dx)\hat{i} \times (dy)\hat{j} = dx\,dy\,\hat{k}$$

$$d\vec{A}_2 = (dz)\hat{k} \times (dx)\hat{i} = dx\,dz\,\hat{j}$$

$$d\vec{A}_3 = (dy)\hat{j} \times (dz)\hat{k} = dy\,dz\,\hat{i}$$

$$dV = dx\,dy\,dz$$



2) Coordonnée cylindrique

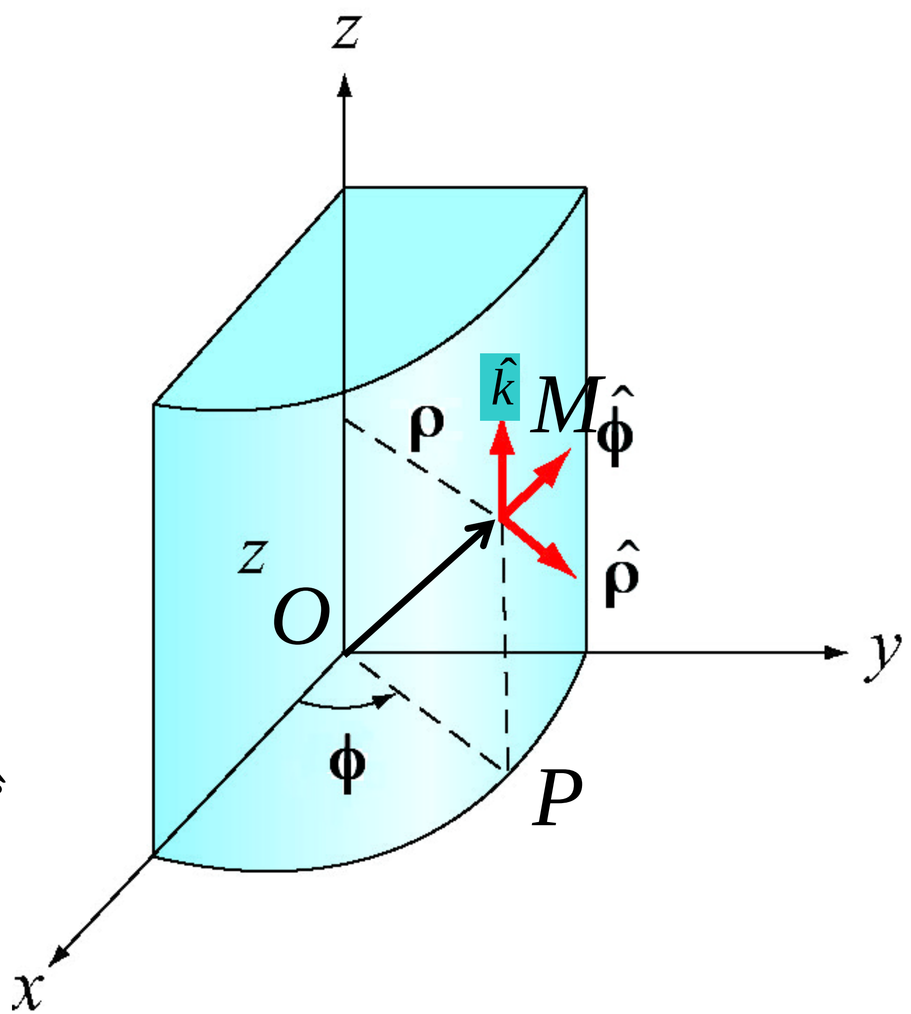
$$\vec{OM} = \rho\,\hat{\rho} + z\,\hat{k}$$

$$\hat{\rho} = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$$

$$\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$$

$$\begin{aligned} d\hat{\rho} &= d(\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) \\ &= -\sin\phi\, d\phi\,\hat{i} + \cos\phi\, d\phi\,\hat{j} \\ &= (d\phi)\hat{\phi} \end{aligned}$$

$$\begin{aligned} d\hat{\phi} &= d\left(-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}\right) \\ &= -\cos\phi\, d\phi\,\hat{i} - \sin\phi\, d\phi\,\hat{j} \\ &= -(d\phi)\,\hat{\rho} \end{aligned}$$



$$d\vec{OM} = d\left(\rho\,\hat{\rho} + z\,\hat{k}\right) = (d\rho)\hat{\rho} + \rho(d\hat{\rho}) + (dz)\hat{k} + z\underbrace{\left(d\hat{k}\right)}^{0}$$

D'où $d\vec{OM} = d\vec{l} = (d\rho)\hat{\rho} + (\rho d\phi)\hat{\phi} + (dz)\hat{k}$

Les surfaces sont données par les P.V suivants:

$$d\vec{A}_1 = (d\rho)\hat{\rho} \times (\rho d\phi)\hat{\phi} = \rho d\rho d\phi\,\hat{k}$$

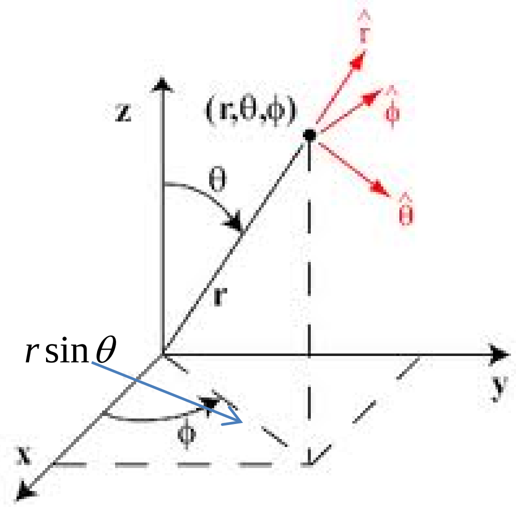
$$d\vec{A}_2 = (\rho d\phi)\hat{\phi} \times (dz)\hat{k} = \rho d\phi dz\,\hat{\rho}$$

$$d\vec{A}_3 = (dz)\hat{k} \times (d\rho)\hat{\rho} = d\rho dz\,\hat{\phi}$$

Le volume est donnée par le T.P.S

$$dV = (d\rho)\hat{\rho} \bullet \left[(\rho d\phi)\hat{\phi} \times (dz)\hat{k}\right] = \rho d\rho d\phi dz$$

3) Coordonnée sphérique



$$\vec{O}M = r\,\hat{r} = r\sin\theta\cos\phi\,\hat{i} + r\sin\theta\sin\phi\,\hat{j} + r\cos\theta\,\hat{k}$$

$$\hat{r} = \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}$$

$$\hat{\phi} = \frac{\hat{k}\times\hat{r}}{\sin\theta} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$$

$$\hat{\theta} = \hat{r}\times\hat{\phi} = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}$$

$$d\vec{OM} = d(r\,\hat{r}) = (dr)\hat{r} + r(d\hat{r})$$

$$= dr\,\hat{r} + r\,d\left(\sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}\right)$$

$$= dr\,\hat{r} + rd\theta\left(\overbrace{\cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}}^{\hat{\theta}}\right) +$$

$$r\sin\theta d\phi\left(\underbrace{-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}}_{\hat{\phi}}\right)$$

D’où on retrouve l’expression du déplacement élémentaire

$$d\vec{OM} = d\vec{l} = dr\,\hat{r} + rd\theta\,\hat{\theta} + r\sin\theta d\phi\,\hat{\phi}$$

$$d\vec{A}_{r\theta} = (dr\,\hat{r}) \times \left(r\,d\theta\,\hat{\theta}\right) = r dr d\theta\,\hat{\phi}$$

$$d\vec{A}_{\theta\phi} = \left(r\,d\theta\,\hat{\theta}\right) \times \left(r \sin\theta\,d\phi\,\hat{\phi}\right) = r^2 \sin\theta\,d\theta\,d\phi\,\hat{r}$$

$$d\vec{A}_{\phi r} = \left(r \sin\theta\,d\varphi\,\hat{\phi}\right) \times (dr\,\hat{r}) = r \sin\theta\,dr d\phi\,\hat{\theta}$$

$$dV = dr\,\hat{r} \bullet \left(r\,d\theta\,\hat{\theta} \times r \sin\theta\,d\phi\right) = r^2 dr \sin\theta\,d\theta\,d\phi$$