CS 495:
Data Visualization for Data Scientists

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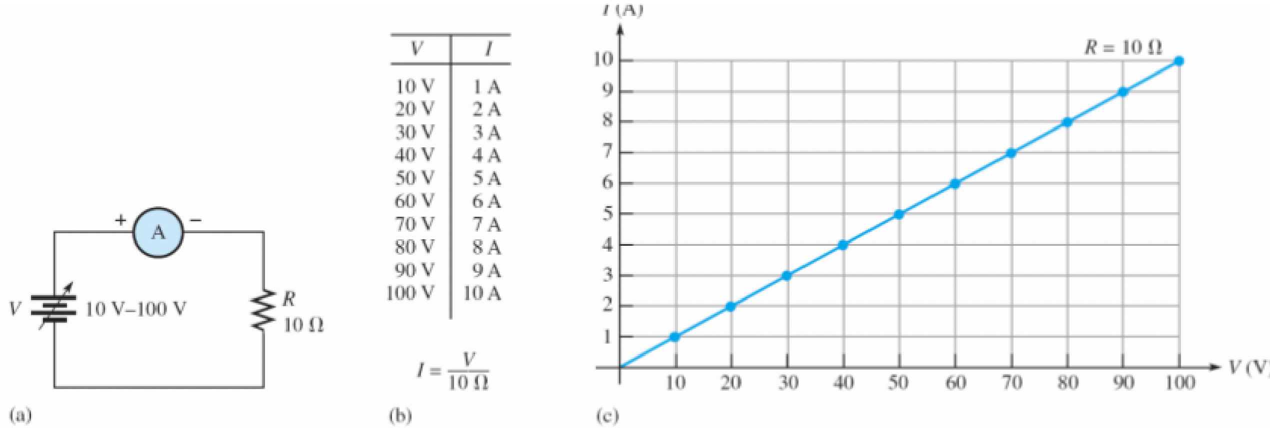
Prologue and Motivation

- "A picture is worth a thousand words" (English idiom).
- Recent research suggests that:
  "*Retina communicates to brain at 10 million bits per second. 40 words per second are read at 10 sec.; call it 1000 bits/sec. which is 1/10,000*"[1]

[1] https://www.edwardtufte.com/bboard/q-and-a-fetch-msg?msg_id=0002NC

Data Visualization for Exploration

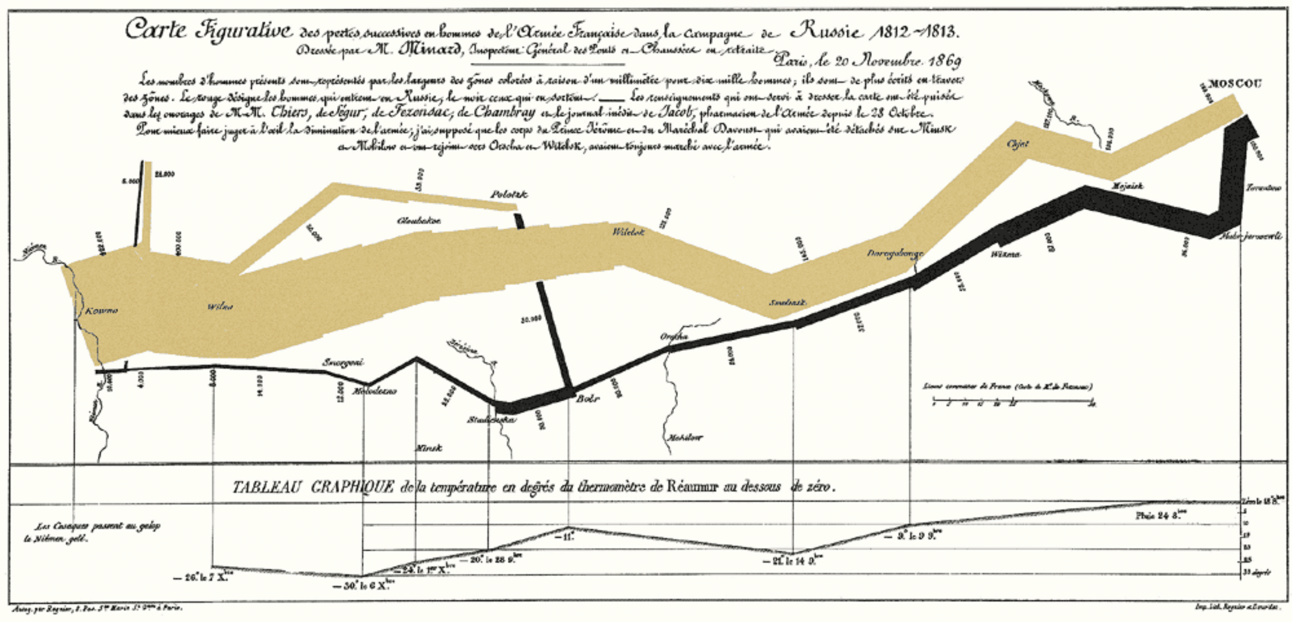
The discovery of the very classic Ohm's law



- Then, comes Statistics, Statistical Learning, Pattern Recognition, to formalize the observed relationship: model, regression, $p-$values, variance, confidence interval, etc.

Data Visualization for Illustration and Presentation

Invasion/retreat of French army to/from Russia:

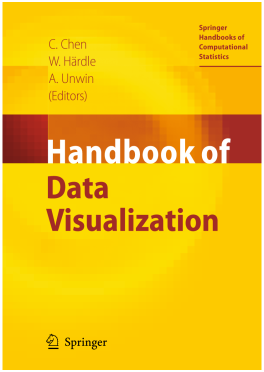


"Vivid historical content and brilliant design combine to make this one of the best statistical graphics ever" (Tufte, 2006, P. 122)

Course Objectives

- Data Visualization:
  - for exploring, extracting secrets, and understanding
    * build intuition and insight.
    * allow you getting the feeling of the patterns, secrets, hiding in data.
    * understand your data before any mathematical treatment.
  - for illustration, displaying, and conveying what has been explored.
- Linking to real life problems.
- Coding and scientific computing.
- We will emphasize on the foundations than the evolving technology.
- This course is just a very interesting voyage in high dimensions and hyperspace. Please, prepare your baggage, video cam, juice, and say cheese.

Texts, References, and Prerequisites



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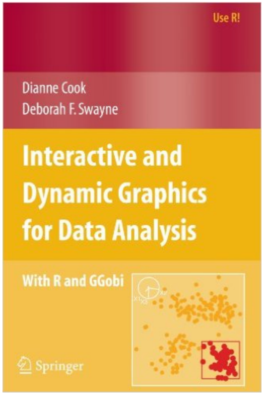

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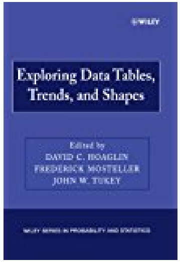

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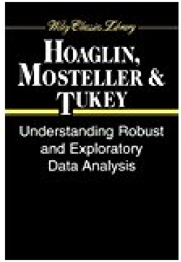

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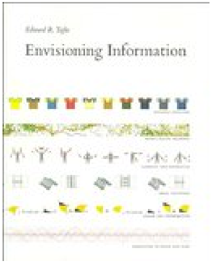

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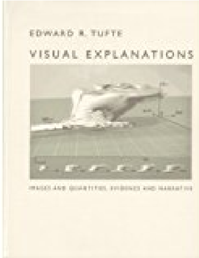

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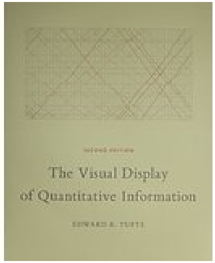

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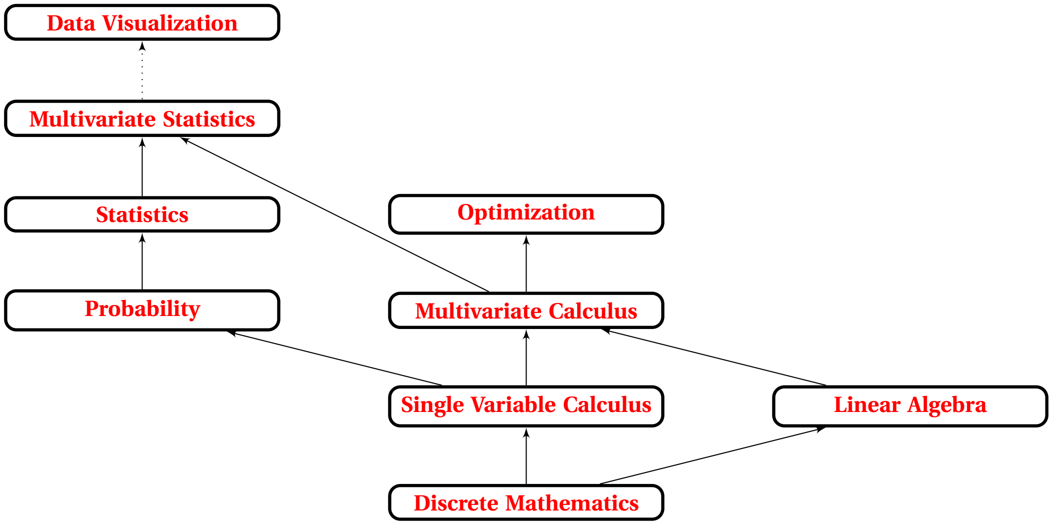

Data Visualization
Multivariate Statistics
Statistics
Optimization
Probability
Multivariate Calculus
Single Variable Calculus
Linear Algebra
Discrete Mathematics

Contents

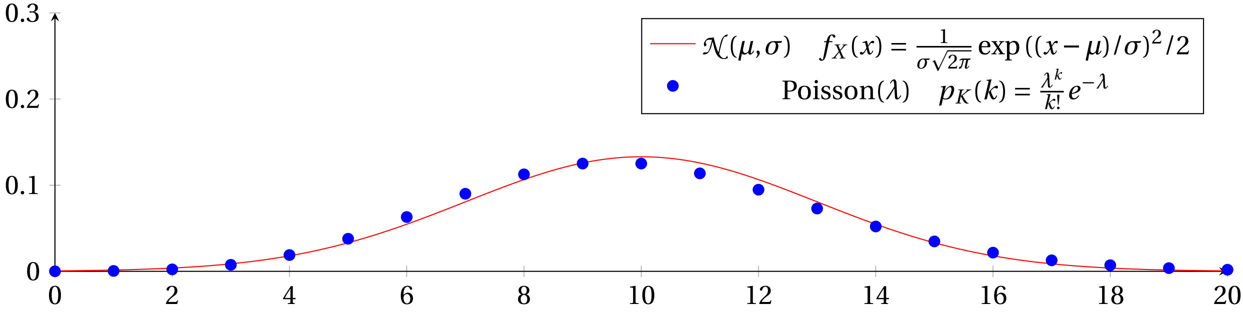
Part I

Exploration

Chapter 1

Some Necessary Probability and Statistics

1.1 Samples from Discrete and Continuous Distributions



- Here, $\mu = 10$, $\sigma = 3, \lambda = 10$ (how do you know from figure?)
- $P(X = x) = 0, P(K = k) \neq 0$.
- How samples look like?
- What about cluttering (observations overlaying each other).

1.2 Cumulative Distribution Function (CDF)

$$
\begin{aligned}
F(x) &= P(X \leq x) \\
&= \int_{-\infty}^{x} f(u)\ du = P(X < x) \qquad \text{(cont. var.)}
\end{aligned}
$$

Definition 1 **(F^{-1})** *: The p^{th} quantile is defined as, the value x_p of the r.v. that satisfies $F\left(x_p\right) = p$.*

- If F is monotonically (strictly) increasing, the pth quantile is unique (see figure).
- $F^{-1}(.5)$ is the median.
- $F^{-1}(.25)$ and $F^{-1}(.75)$is the lower and upper quartile.

Example 2 *Suppose*

$$
F(x) = x^2,\ 0 \leq x \leq 1,
$$

$$
\begin{aligned}
x_p^2 &= p, \\
x_p &= \sqrt{p}, \\
x_{.5} &= \sqrt{.5} = .707 \\
x_{.25} &= \sqrt{.25} = .5 \\
x_{.75} &= \sqrt{.75} = .866
\end{aligned}
$$

1.3 Normal Distribution

Corollary 3 *If $X \sim \mathcal{N}(\mu, \sigma)$ and $Z =\sim \mathcal{N}(0,1)$ (a standard normal), then*

$$P(Z < z) = \int_{-\infty}^{z} f_Z(u)\, du = \Phi(z)$$

$$\Phi(z) = 1 - \Phi(-z)$$

$$\frac{(X-\mu)}{\sigma} \sim Z$$

$$P(X < x) = P\left(\frac{X-\mu}{\sigma} < \frac{x-\mu}{\sigma}\right) = P\left(Z < \frac{x-\mu}{\sigma}\right) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

Example 4 *[σ and μ]:*

$$\begin{aligned}
P\left(\left|X-\mu\right| < \sigma\right) &= P\left(-\sigma < X-\mu < \sigma\right) \\
&= P\left(-1 < \frac{X-\mu}{\sigma} < 1\right) \\
&= P\left(-1 < Z < 1\right) \\
&= \Phi(1) - \Phi(-1) \\
&= .68 \\
P\left(\left|X-\mu\right| < 2\sigma\right) &= \Phi(2) - \Phi(-2) \\
&= .9545, \\
P\left(\left|X-\mu\right| < 3\sigma\right) &= \Phi(3) - \Phi(-3) \\
&= .9973 \qquad \text{(almost all the probability measure)}
\end{aligned}$$

1.4 Quantile Estimation, Outliers Cutoff, and Thick Tails

The ordered statistic $x_{(p=i.d)}$ is defined by interpolation as:

$$x_{(i.d)} = x_{(i)} + d(x_{(i+1)} - x_{(i)}) = (1-d)x_{(i)} + d\,x_{(i+1)} \qquad = \widehat{F}^{-1}(p) \qquad (p^{\text{th}} \text{ quantile})$$

$$x_{((n+1)/2)} = \begin{cases} x_{((n+1)/2)}, & n \text{ is odd.} \\ x_{(n/2+1/2)} = (1/2)(x_{(n/2)} + x_{(n/2+1)}) & n \text{ is even.} \end{cases} \qquad = \widehat{F}^{-1}(0.5) \qquad (\text{median: M})$$

$$x_{((1+(n+1)/2)/2)} = x_{((n+3)/4)} \qquad = \widehat{F}^{-1}(0.25) \qquad (\text{lower quartile: } Q_L)$$

$$x_{(((n+1)/2+n)/2)} = x_{((3n+1)/4)} \qquad = \widehat{F}^{-1}(0.75) \qquad (\text{upper quartile: } Q_U)$$

$$W_L = Q_L - 1.5(Q_U - Q_L) \qquad (\text{lower cutoff})$$

$$W_U = Q_U - 1.5(Q_U - Q_L) \qquad (\text{upper cutoff})$$

Example 5

34	*35*	*36*	*37*	*45*	*52*	*56*	*58*	*66*	*68*	*74*	*90*	*100*	*145*
1	2	3	4	5	6	7	8	9	10	11	12	13	14

Rank of M, QL, QU is 7.5, 4.25, 10.75

$M = 56 + 0.5(58 - 56) = 57$

$Q_L = 37 + 0.25(45 - 37) = 39$

$Q_U = 68 + 0.75(74 - 68) = 72.5$

$d_Q = 1.5(72.5 - 39) = 50.25$

$W_L = Q_L - d_Q = 39 - 50.25 = -11.25$

$W_U = Q_U + d_Q = 72.5 + 50.25 = 122.75$

Example 6 (meaning of quantile from $X \sim \mathcal{N}(\mu,\sigma)$) :

$$
\begin{aligned}
p = F(x_p) = P(X < x_p) &= \Phi\left(\frac{x_p - \mu}{\sigma}\right) \\
F^{-1}(p) = x_p &= \mu + \left(\Phi^{-1}(p)\right)\sigma \\
Q_L &= F^{-1}(0.25) = \mu - 0.6745\sigma \\
Q_U &= F^{-1}(0.75) = \mu + 0.6745\sigma \\
d_Q &= 1.349\sigma \\
W_L &= Q_L - 1.5d_Q = \mu - 2.698\sigma \\
W_U &= Q_U + 1.5d_Q = \mu + 2.698\sigma
\end{aligned}
$$

$$P(X < W_L) + P(X > W_U) = 2P(X < W_L) = 2\Phi\left(\frac{(\mu - 2.698\sigma) - \mu}{\sigma}\right) = 2\Phi(-2.698) = 0.00698$$

So, a sample (patch) of 1000 obs. will have almost 7 obs. outside the cutoffs.

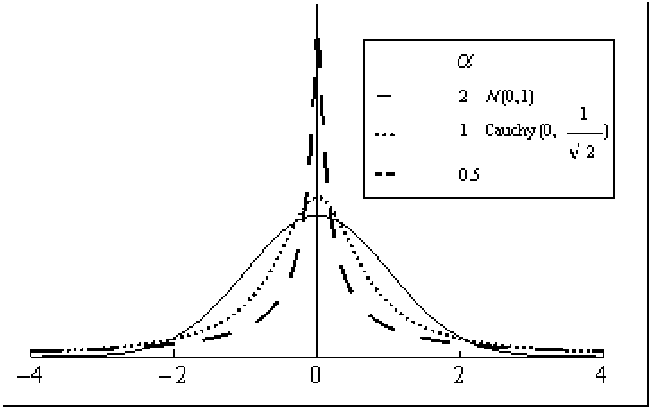
Definition 7 (Hoaglin et al. (2000)) :
Outlier *observation with different underlying behavior as compared with the bulk of the data which deserves more investigation. The cutoffs W_L and W_U will be arbitrarily used for outlier detection. Outliers could be:*

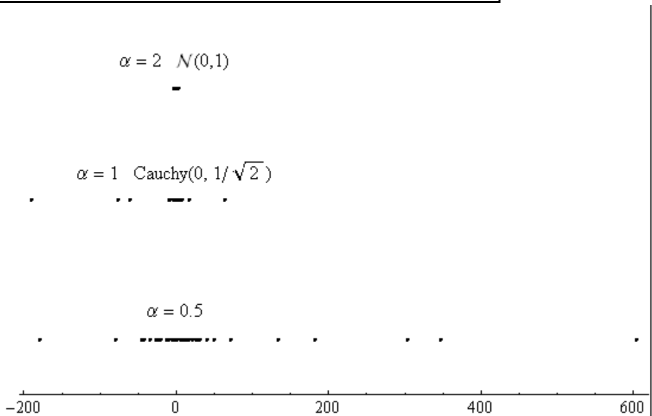
- *false value due to measurement error.*
- *right value due to thick tail.*

Resistance *insensitivity to misbehavior in data. A resistant method produces results that change only slightly when small part of the data is replaced by new numbers, possibly very different from the original ones.*

Robustness *insensitivity to departure from assumptions surrounding an underlying probabilistic model.*

Example 8 (Stable Distributions: thick tailed) :





1.5 Transformation and Log-scale

Chapter 2

History and Introduction

2.1 Evolution of Data Visualization

2.2 Types of Variable

Quantitative, where some measure is given as a value; e.g.,$X = 1, 3, -2.5$.

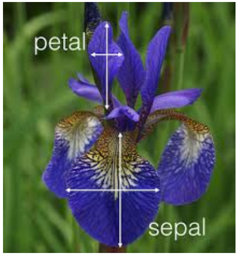
Qualitative (or Categorical), where no measures or metrics are associated; e.g., $X = Diseased, Nondiseased$.

Ordered Categorical; e.g., $X = small, medium, \ldots$. The variable $X \in \mathcal{G}$, a set of possible values.

Example 9 (`iris` dataset) *: (150 observations, by R. A. Fisher, the father of Statistics)*

Index	*SepalLength*	*SepalWidth*	*PetalLength*	*PetalWidth*	*Class*
1	5.1	3.5	1.4	0.2	*Iris-setosa*
2	4.9	3	1.4	0.2	*Iris-setosa*
3	4.7	3.2	1.3	0.2	*Iris-setosa*
4	4.6	3.1	1.5	0.2	*Iris-setosa*
5	5	3.6	1.4	0.2	*Iris-setosa*
6	5.4	3.9	1.7	0.4	*Iris-setosa*
7	4.6	3.4	1.4	0.3	*Iris-setosa*
8	5	3.4	1.5	0.2	*Iris-setosa*
9	4.4	2.9	1.4	0.2	*Iris-setosa*
10	4.9	3.1	1.5	0.1	*Iris-setosa*
⋮					
51	7	3.2	4.7	1.4	*Iris-versicolor*
52	6.4	3.2	4.5	1.5	*Iris-versicolor*
53	6.9	3.1	4.9	1.5	*Iris-versicolor*
54	5.5	2.3	4	1.3	*Iris-versicolor*
55	6.5	2.8	4.6	1.5	*Iris-versicolor*
56	5.7	2.8	4.5	1.3	*Iris-versicolor*
57	6.3	3.3	4.7	1.6	*Iris-versicolor*
58	4.9	2.4	3.3	1	*Iris-versicolor*
59	6.6	2.9	4.6	1.3	*Iris-versicolor*
60	5.2	2.7	3.9	1.4	*Iris-versicolor*
⋮					
101	6.3	3.3	6	2.5	*Iris-virginica*
102	5.8	2.7	5.1	1.9	*Iris-virginica*
103	7.1	3	5.9	2.1	*Iris-virginica*
104	6.3	2.9	5.6	1.8	*Iris-virginica*
105	6.5	3	5.8	2.2	*Iris-virginica*
106	7.6	3	6.6	2.1	*Iris-virginica*
107	4.9	2.5	4.5	1.7	*Iris-virginica*
108	7.3	2.9	6.3	1.8	*Iris-virginica*
109	6.7	2.5	5.8	1.8	*Iris-virginica*
110	7.2	3.6	6.1	2.5	*Iris-virginica*
⋮					

- *Knowing the physics of the problem helps understanding data.*
- *Iris is a genus of species of flowering plants with showy flowers. (In Arabic: Alsawsan).*
- *Iris is extensively grown as ornamental plant, medicine, drugs.*



Chapter 3

1-D charts

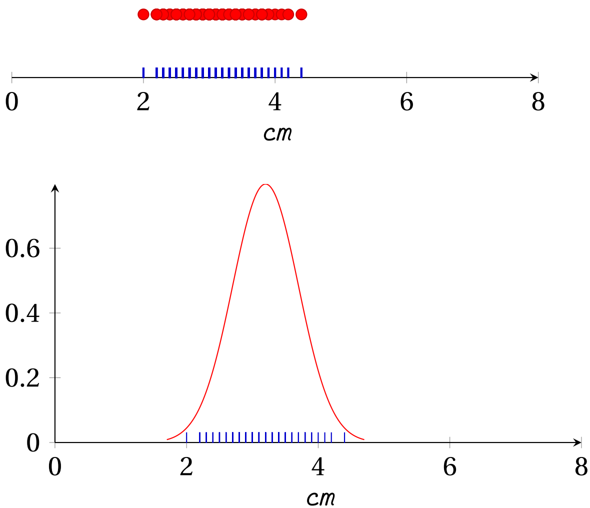
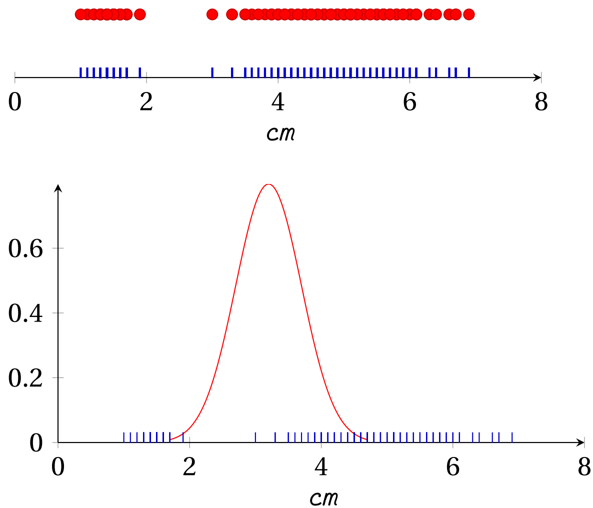
Per Hoaglin et al. (2000):

- How nearly symmetric the sample is?
- How spread out the numbers are?
- Whether a few values are far removed from the rest?
- Whether there are concentrations of data?
- Whether there are gaps in the data?

3.1 A Quantitative Variable

3.1.1 Rug Plot (the simplest ever)

Example 10 (`iris` **dataset**) .

SepalWidth	*PetalLength*
	

Hints for sense: Some observations clutter each other; standardize scale,
@@@ use Gaussian fit (blindly before visualization) then Gaussian mixture (after visualization). Then calculate the probability of having 2<X<3 and compare to no.obs/150 .

@@@ for the red normal distribution, calculate mu and sigma from data.

3.1.2 Stem-and-Leaf

@@@ needs revisiting and drawing
Invented by by John W. Tukey, (who also coined the word **bi**nary digi**t**).
It is a multi-functioning of the "data measure", i.e., the displaying element (here the digit) has more than one function (position and value).
Variations: e.g., adding rank left to each stem.
$L = \left[10 \times \log_{10} n\right]$ very good for $20 < n < 300$

3.1.3 Histograms: (for more details check St 121.)

$$I_{(c)} = \begin{cases} 1 & \text{if } c \text{ is } T \\ 0 & \text{if } c \text{ is } F \end{cases}, \qquad \text{(indicator function)}$$

$$I_{(c)} \sim Bernoulli\,(\Pr(c))\,.$$

For data $x_1, \ldots, x_n$ divide the data range T to K equal regions of equal width Δ (so that $K = T/\Delta$)

$$\begin{aligned} T_k &= [t_0 + \Delta k, t_0 + \Delta(k+1)[\\ &= [t_k, t_{k+1}[, \; k = 0, \ldots, K-1, \end{aligned}$$

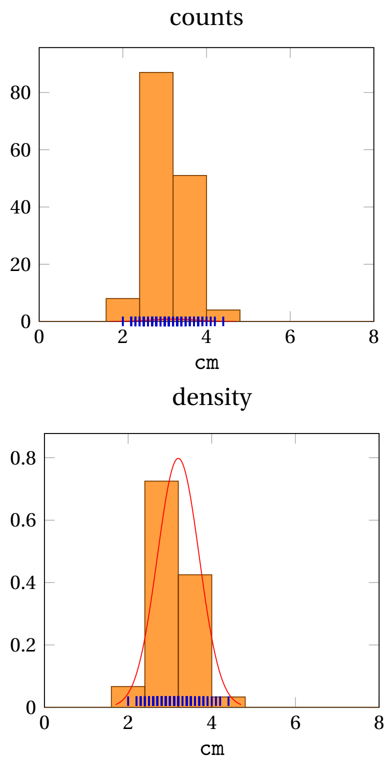
Notice: decreasing Δ increases K.
We have three versions of histogram:

$$N_k = \sum_{i=1}^{n} I_{(X_i \in T_k)}, \qquad \text{(counts)}$$

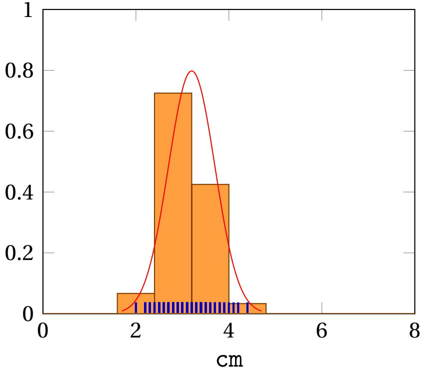
$$R_k = \frac{N_k}{n} \xrightarrow{p} \Pr(X \in T_k) \qquad \text{(relative counts)}$$

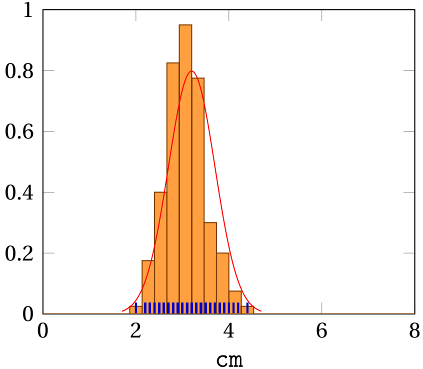
$$f_k = \frac{N_k}{\Delta n} \xrightarrow{p} \frac{\Pr(X \in T_k)}{\Delta} \approx \frac{f_X(t_k)\,\Delta}{\Delta} = f_X(t_k) \qquad \text{(density)}$$

SepalWidth

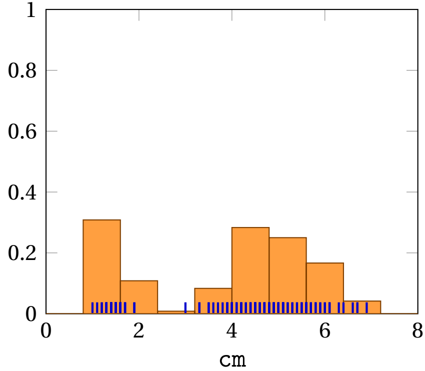


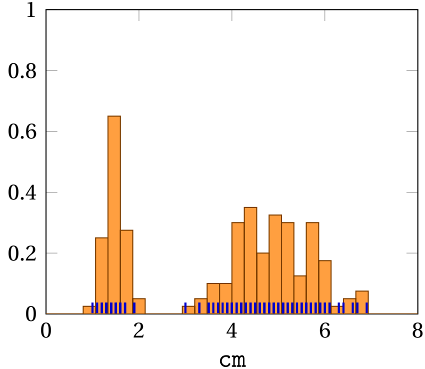
SepalWidth





PetalLength





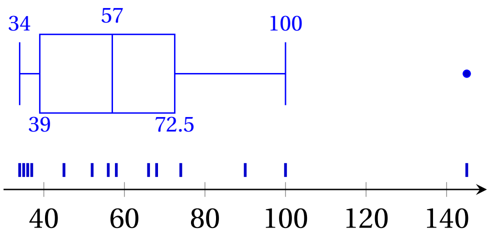
Hints for sense: bins = 10 vs. 30; unify X and Y scale for comparison;

3.1.4 Box Plot

back to example 11
To observe a glance: location, spread, skewness, tail length, and outlying data points.

lower whisker	lower quartile	median	upper quartile	upper whisker
$Q_L - 1.5 d_Q \le \min x_i = W_L$	Q_L	M	Q_U	$W_U = \max x_i \le Q_U + 1.5 d_Q$

Example 11 (Letter Values) .



Rank of M, QL, QU is 7.5, 4.25, 10.75

$M = 56 + 0.5(58 - 56) = 57$

$Q_L = 37 + 0.25(45 - 37) = 39$

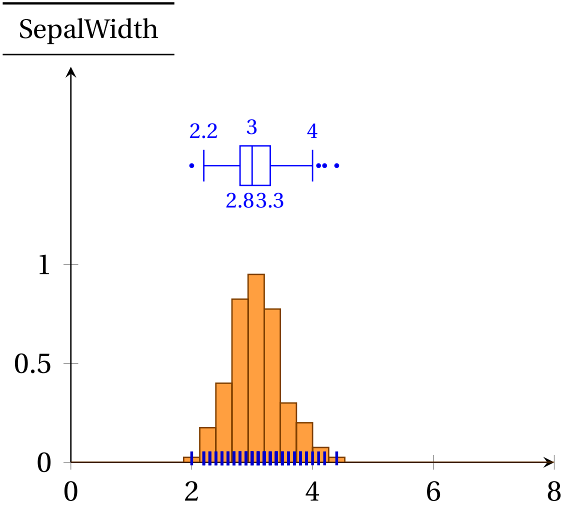
$Q_U = 68 + 0.75(74 - 68) = 72.5$

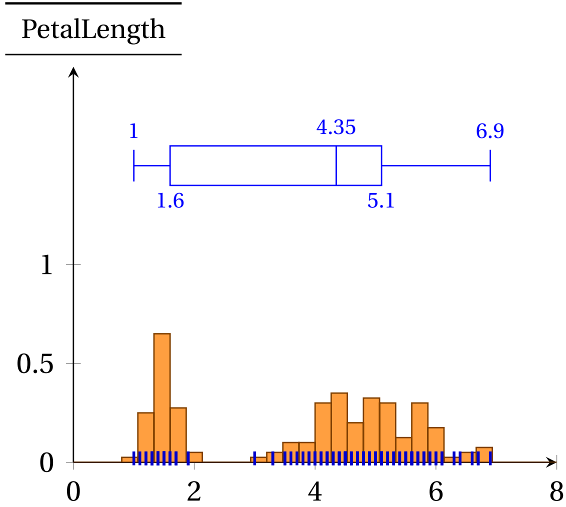
$d_Q = 1.5(72.5 - 39) = 50.25$

$39 - 50.25 = -11.25$

$72.5 + 50.25 = 122.75$

we could have defined a boxplot based on mean and variance => less resistant.
Why boxplot is not defined in terms of $W_L = \widehat{F}^{-1}(0.05)$
why boxplot is not defined in terms of mean and variance
for small patches $\frac{\#\text{ of obs.} n}{>} \Pr(X < W_L)$





Comparison

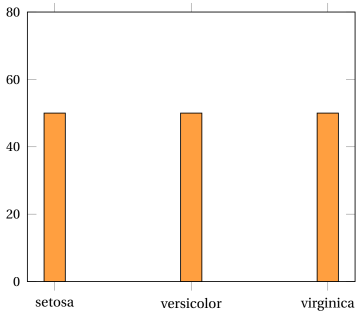
	Rug plot	**Histogram**	**Boxplot**	**Stem-and-leaf**
density	0 (clutter)	1	0 (region)	1
values	1	1	0 (region)	1
large N	0 (clutter)	1	1	0
resistance	0 (outliers)	0 (outliers)	1	0 (outliers)
discrete	0 (clutter)	1	1	1

Clutter could be alleviated by α-channel.

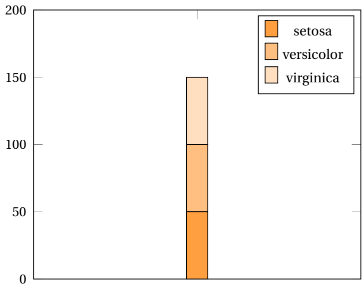
3.2 A Categorical Variable

Suppose we have only last column of table in Sec. 9; no numerical values. Only histogram-like charts: bar chart, stacked bar, pie chart, or any equivalent.

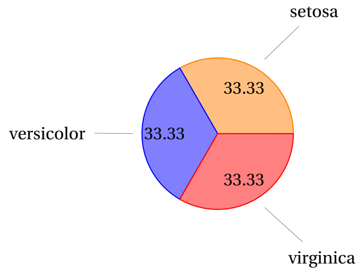
3.2.1 Bar chart



3.2.2 Stacked plot



3.2.3 Pie chart



- Bar chart is more professional and scientific; pie chart is more for illustration.
- More details can be put on the bar chart (including boxplot for each class, etc.)
- Bar chart and Stacked plot are utilized more for several patches.

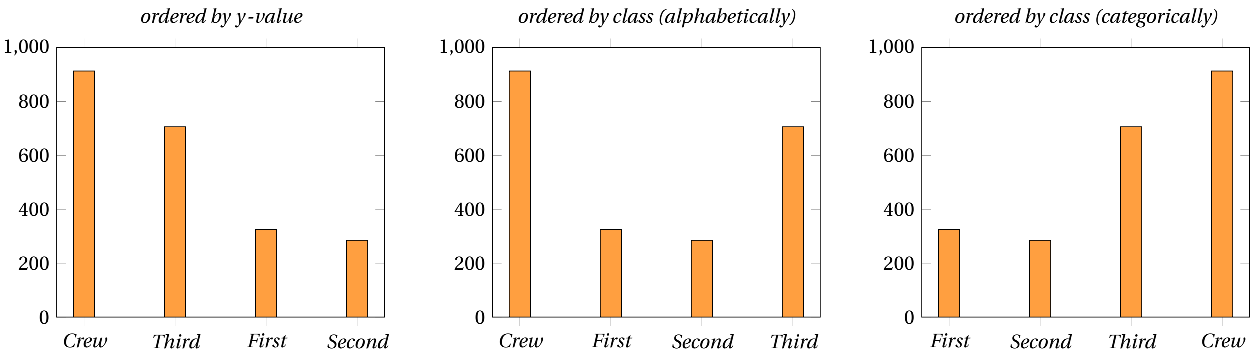
3.3 An Ordered Categorical Variable

Exactly as "bar chart" with ordered x-axis.

Example 12 (No. of Titanic passengers and crew) *: We can consider the variable (`passenger class`) as:*

- *categorical (as previous example) and order by y-value. (sorting will provide more information for the same ink).*
- *ordered categorical and order it alphabetically (nonsense in this example).*
- *ordered categorical and order it by class rank (makes sense here).*

Reproduced



Part II

The Art of Visual Display, Presentation, and Illustration

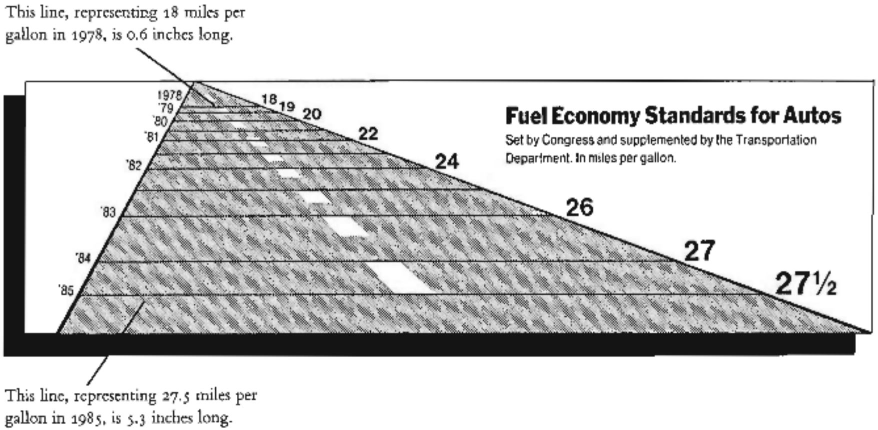
Chapter 4

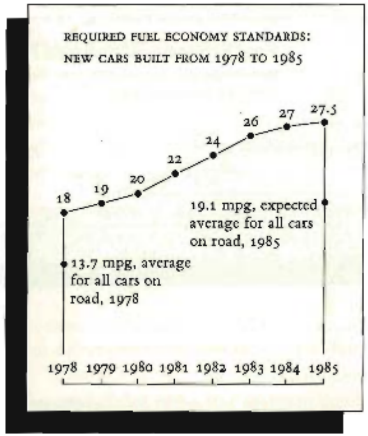
The Visual Display of Quantitative Information (Tufte, 2001)

@@@ data ink ratio before this example, because it has a problem in this ratio as well.

Example 13 (Lie Factor) *: (Tufte, 2001, P. 53) (The figure was published in New York Times, August 9, 1978)*

- *Three kinds of lies: lie, damn lie, and Statistics; also charts as well.*
- *Example of Statistics: Stock letters.*
- *Example of chart this one.*
- *Could have been decorated honestly like this one.*





$$\textbf{\textit{Lie Factor}} = \frac{\textit{size of effect shown in graphic}}{\textit{size of effect in data}} = \frac{(5.3-0.6)/0.6}{(27.5-18)/18} = \frac{7.83}{0.53} = 14.8$$

Part III

Applications

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