Assignment 01 Optimization

Department of Computer Science Optimization

1. (Stewart, 2003, P. 53–58, Sec. 15.1, PP. 936–937) Match the function (a) with its graph (labeled A-F on next page) and (b) with its contour map (labeled I-VI). Give reasons for your choices.

$$z = \sin\sqrt{x^2 + y^2} \tag{1}$$

$$z = x^2 y^2 e^{-x^2 - y^2} (2)$$

$$z = \frac{1}{x^2 + 4y^2} \tag{3}$$

$$z = x^3 - 3xy^2 \tag{4}$$

$$z = \sin x \sin y \tag{5}$$

$$z = \sin^2 x + \frac{1}{4}y^2. {(6)}$$

2. Consider the function $\Phi(t) = \int_{g(t)^{h(t)} f(x,t) dx}$, then Leibniz' rule states:

$$\Phi'(t) = f(h(t), t)h'(t) - f(g(t), t)g'(t) + \int_{q(t)}^{h(t)} f'(x, t) dx,$$

where all derivatives are taken w.r.t. to t.

a) Apply Leibniz' rule to find the minima of the function

$$\Phi(t) = \int_{(t-1)}^{-(t-1)^2} (x-t) \ dx.$$

- b) Solve the problem without Leibniz' rule by direct integration then differentiation and conform the two answers.
- c) Provide geometrical interpretation to why Φ has a minimum.
- 3. Prove that the population median is the best decision under the absolute deviance loss. I.e. the population median is the solution to the following optimization problem

$$\underset{\widehat{X}}{\operatorname{minimize}} \operatorname{E}_X |X - \widehat{X}|.$$

4. Prove that the sample median is the best decision under the absolute deviance loss for a set of observations. I.e., the sample median is the solution to the following optimization problem

$$\underset{\widehat{X}}{\operatorname{minimize}} \sum_i |X_i - \widehat{X}|.$$

Bibliography

Stewart, J. (2003), Calculus, Belmont, CA: Thomson Brooks/Cole, 5th ed.

