

1. Suppose $C = C_1 \cup C_2$, where $C_1 \cap C_2 = \emptyset$. Then prove/disprove that: C is convex, C is not convex, or C can be, but not necessarily, convex.
2. Provide a conversion to/from the general affine combination $x = \sum_{i=1}^n \theta_i x_i$, $\sum_i \theta_i = 1$ and the α -combination studied in lectures:

$$x = \alpha_n x_n + (1 - \alpha_n) \left(\alpha_{n-1} x_{n-1} + (1 - \alpha_{n-1}) \left(\alpha_{n-2} x_{n-2} + (1 - \alpha_{n-2}) (\dots) \right) \right) \quad (\text{recursively})$$

$$= \left(((1 - \alpha_2)x_1 + \alpha_2 x_2)(1 - \alpha_3) + \alpha_3 x_3 \right) (1 - \alpha_4) + \alpha_4 x_4 + \dots \quad (\text{iteratively})$$

Hint: use the iterative expression; the recursive one is written just for clarification. The solution should be in the form:

$$\theta_i = \theta_i(\alpha_j, j = 1, \dots, n), i = 1, \dots, n$$

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