1. Prove that in the centered model, i.e., $\mathbf{X}_c = \mathbf{X}_1 - \frac{1}{n}\mathbf{J}\mathbf{X}_1$,

$$\mathbf{1}'\mathbf{X}_c = 0.$$

2. For the centered model, prove that

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{pmatrix} 1/n & \mathbf{0}' \\ \mathbf{0} & (\mathbf{X}_c'\mathbf{X}_c)^{-1} \end{pmatrix}$$

3. Prove that different linear transformations for two r.v. will preserve ρ.

4. Consider the previous homework regression problem: $Y = \sin X + \varepsilon$. For $N = 50$, use a regularized model with $p = 20$.

   • Use the large testing set to select the value of best λ.

   • Compare the best model to the best model obtained from the previous homework using nested stepwise regression.

   • For each λ, plot the value of each $\beta_i, i = 1, \ldots, p$.