

Solve the following problems.

- 8 marks**
1. Consider the following propositions then answer the following questions:
- $p$  : “you miss the practical exam”.
- $q$  : “you miss the final exam”.
- $r$  : “you will pass”.
- (a) Express in English the logical statement:  $(p \rightarrow \neg r) \wedge (q \rightarrow \neg r)$ .
- (b) Express in English the logical statement:  $(p \vee q) \rightarrow \neg r$ .
- (c) Are the two statements equivalent? (answer using truth table).
- (d) Are the two statements equivalent? (answer algebraically).
- 8 marks**
2. Suppose that the domain of  $x$  consists ONLY of -3, -1, 1, 3. Then for any predicate  $p(x)$  express the following using ONLY  $\wedge$ ,  $\vee$ ,  $\neg$ .
- (a)  $\exists x p(x)$ .
- (b)  $\forall x p(x)$ .
- (c)  $\forall x ((x \neq 1) \rightarrow p(x))$ .
- (d)  $\exists x ((x \geq 0) \wedge p(x))$ .
- 9 marks**
3. Consider the following statements:
- $$\exists x \forall y (y \neq 0 \rightarrow xy = 1).$$
- $$\forall x \exists y ((x + y = 2) \wedge (2x - y = 1)).$$
- (a) Express each statement in English.
- (b) Which statement is true and which is false? (why?)
- (c) For the false statement, give a counterexample.