Please review the law of total probability, conditional probability, and Bayes' rule before solving the first two problems.

1- An electronic fuse is produced by five production lines in a manufacturing operation. The fuses are costly, are quite reliable, and are shipped to suppliers in 100-unit lots. Because testing is destructive, most buyers of the fuses test only a small number of fuses before deciding to accept or reject lots of incoming fuses. All five production lines produce fuses at the same rate and normally produce only 2% defective fuses, which are dispersed randomly in the output. Unfortunately, production line 1 suffered mechanical difficulty and produced 5% defectives during the month of March. This situation became known to the manufacturer after the fuses had been shipped. A customer received a lot produced in March and tested three fuses. One failed. What is the probability that the lot was produced on line 1? What is the probability that the lot came from one of the four other lines? (Example 2.23 in the book)

2- A group of four components is known to contain two defectives. An inspector tests the components one at a time until the two defectives are located. Once he locates the two defectives, he stops testing. Let Y denote the number of the test on which the second defective is found. Draw the probability mass function of Y. (Problem 3.3 in the book)

3- If the mean, of a random variable Y , is denoted by μ and the variance by σ^2 then prove that:

$$\sigma^2 = E\left[Y^{\,2}\right] - \mu^2$$

4- Assume that the pure weight of a milk can is a random variable whose pdf takes a triangular shape symmetric around the value 100 gram. It is impossible to find a can whose weight is below 90 gram or above 110 gram. Sketch the pdf of the weight. Assume that we calibrated a can to be exactly 80 gram and named it CAL. Assume that we flip a fair coin and if the outcome is Head we select CAL and the weight Y is, therefore 80 gram. If the outcome is Tail we draw randomly any can from the product line and the weight Y will be the weight of the drawn can. Sketch the pdf of Y .

The following texts are recommended; Wackerly et al. (2001) is more thorough.

Wackerly, D. D., W. Mendenhall, et al. (2001). Mathematical statistics with applications. Pacific Grove, CA, Duxbury.

Hayter, A. J. (2007). Probability and statistics for engineers and scientists. Belmont, Calif., Thomson Brooks/Cole.