

1. Match the function (a) with its graph (labeled A-F on next page) and (b) with its contour map (labeled I-VI). Give reasons for your choices.

$$z = \sin \sqrt{x^2 + y^2} \quad (1)$$

$$z = x^2 y^2 e^{-x^2 - y^2} \quad (2)$$

$$z = \frac{1}{x^2 + 4y^2} \quad (3)$$

$$z = x^3 - 3xy^2 \quad (4)$$

$$z = \sin x \sin y \quad (5)$$

$$z = \sin^2 x + \frac{1}{4}y^2. \quad (6)$$

2. Consider the function $\Phi(t) = \int_{g(t)}^{h(t)} f(x, t) dx$, then Leibniz' rule states:

$$\Phi'(t) = f(h(t), t) h'(t) - f(g(t), t) g'(t) + \int_{g(t)}^{h(t)} f'(x, t) dx,$$

where all derivatives are taken w.r.t. to t .

- a) Apply Leibniz' rule to find the minima of the function

$$\Phi(t) = \int_{(t-1)}^{-(t-1)^2} (x-t) dx.$$

- b) Solve the problem without Leibniz' rule by direct integration then differentiation and conform the two answers.
c) Provide geometrical interpretation to why Φ has a minimum.

3. Prove that the population median is the best decision under the absolute deviance loss. I.e. the population median is the solution to the following optimization problem

$$\underset{\hat{X}}{\text{minimize}} E_X |X - \hat{X}|.$$

4. Prove that the sample median is the best decision under the absolute deviance loss for a set of observations. I.e., the sample median is the solution to the following optimization problem

$$\underset{\hat{X}}{\text{minimize}} \sum_i |X_i - \hat{X}|.$$

