

# 1 Algebra & trigonometry

## Special angles

DEG	0°	30°	45°	60°	90°	120°	135°	150°	180°
RAD	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	DNE	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

For the resulting values of trig functions to be positive, it follows a quadrant with the order *all*, *sine*, *cosine*, *tangent* — starting at the top-right and moving anticlockwise.

## Cotangent & some reciprocal identities

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \sec \theta &= \frac{1}{\cos \theta} & &= \frac{\cos \theta}{\sin \theta} \end{aligned}$$

## Derivative of the inverse trig functions

$$\begin{aligned} \frac{d}{d\theta} \arcsin(\theta) &= \frac{1}{\sqrt{1-\theta^2}} & \frac{d}{d\theta} \arccos(\theta) &= -\frac{1}{\sqrt{1-\theta^2}} \\ \frac{d}{d\theta} \arctan(\theta) &= \frac{1}{1+\theta^2} & \frac{d}{d\theta} \operatorname{arccot}(\theta) &= -\frac{1}{1+\theta^2} \\ \frac{d}{d\theta} \operatorname{arcsec}(\theta) &= \frac{1}{\theta\sqrt{\theta^2-1}} & \frac{d}{d\theta} \operatorname{arccsc}(\theta) &= -\frac{1}{\theta\sqrt{\theta^2-1}} \end{aligned}$$

## Pythagorean identities

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta \end{aligned}$$

## Sum and difference formulae

$$\begin{aligned} \sin(\theta \pm \phi) &= \sin \theta \cos \phi \pm \cos \theta \sin \phi \\ \cos(\theta \pm \phi) &= \cos \theta \cos \phi \mp \sin \theta \sin \phi \\ \tan(\theta \pm \phi) &= \frac{\tan \theta \pm \tan \phi}{1 \mp \tan \theta \tan \phi} \end{aligned}$$

## Double-angle & half-angle formulae

These build upon the sum-difference formulae.

$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ \sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\ \cos \frac{\theta}{2} &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \tan \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \end{aligned}$$

## Logarithms & some of their properties

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\begin{aligned} \log_b b &= 1 & \log_b b^x &= x \\ b^{\log_b x} &= x & \log_b 1 &= 0 \end{aligned}$$

$$\begin{aligned} \log_b xy &= \log_b x + \log_b y & \log_b a^x &= x \log_b a \\ \log_b \frac{x}{y} &= \log_b x - \log_b y & \log_b \sqrt[x]{a} &= \frac{\log_b a}{x} \end{aligned}$$

## Inequalities, absolute values, & some of their properties

If  $a < b$  then  $a + c$  and  $a - c < b - c$

If  $a < b$  and  $c > 0$  then  $ac < bc$  and  $\frac{a}{c} < \frac{b}{c}$

If  $a < b$  and  $c < 0$  (i.e.  $c$  is negative) then  $ac > bc$  and  $\frac{a}{c} > \frac{b}{c}$

If  $b$  is positive, then:

$$\begin{aligned} |p| &= b & \Rightarrow & & p &= -b \text{ or } p = b \\ |p| &< b & \Rightarrow & & -b &< p < b \\ |p| &> b & \Rightarrow & & p &< -b \text{ or } p > b \end{aligned}$$

## 2 Limits

### Basics

**Definition** (rough).  $\lim_{x \rightarrow a} f(x) = L$  means that as we approach  $x$  closer to  $a$  *but not equal to*  $a$ , the value for  $f(x)$  can get arbitrarily closer to  $L$ .

Note that  $\lim_{x \rightarrow a} f(x) = L$  **iff**  $\lim_{x \rightarrow a^-} f(x) = L$  (LR limit) and  $\lim_{x \rightarrow a^+} f(x) = L$  (RH limit) are both true.

### Properties

Assuming that the limit exists, then all the following is true:

$$\lim_{x \rightarrow a} k = k \qquad \lim_{x \rightarrow a} x = a$$

$$\lim_{x \rightarrow a} f(x) = f(a) \text{ if the function } f \text{ is } \textit{continuous} \text{ at } x = a$$

$$\lim_{x \rightarrow a} (f(x) \pm g(x)) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} (k \cdot f(x)) = k \lim_{x \rightarrow a} f(x)$$

$$\lim_{x \rightarrow a} (f(x) \cdot g(x)) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \quad (g(a) \neq 0)$$

$$\lim_{x \rightarrow a} f(x)^n = \left( \lim_{x \rightarrow a} f(x) \right)^n$$

### Squeeze theorem

**Theorem.** If  $f(x) \leq g(x) \leq h(x)$  for all  $x$  near  $x = a$  (except possibly at  $x = a$ ), and

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L,$$

then

$$\lim_{x \rightarrow a} g(x) = L$$

### Some important-to-remember limits

$$\lim_{x \rightarrow 0} \frac{1}{x} = \text{DNE (1/0)}$$

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

$$\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} \sin \frac{1}{x} = \text{DNE (chaos)}$$

If  $\lim_{x \rightarrow a} f(x) = L$ ,  $\lim_{x \rightarrow L} g(x) = K$  s.t.  $g(L) = K$ , then

$$\lim_{x \rightarrow a} g(f(x)) = K.$$

### 3 Acknowledgements

The cheat sheet is partly inspired by (and copied from) Paul's Online Math Notes (<https://tutorial.math.lamar.edu/>); in particular his various cheat sheets.