1 Algebra & trigonometry

Special angles

DEG	0°	30°	45°	60°	90°	120°	135°	150°	180°
RAD	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	DNE	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

For the resulting values of trig functions to be positive, it follows a quadrant with the order *all, sine, cosine, tangent* — starting at the top-right and moving anticlockwise.

Cotangent & some reciprocal identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$
$$\sec\theta = \frac{1}{\cos\theta} \qquad = \frac{\cos\theta}{\sin\theta}$$

Derivative of the inverse trig functions

$$\frac{d}{d\theta}\arcsin(\theta) = \frac{1}{\sqrt{1-\theta^2}} \qquad \frac{d}{d\theta}\arccos(\theta) = -\frac{1}{\sqrt{1-\theta^2}}$$
$$\frac{d}{d\theta}\arctan(\theta) = \frac{1}{1+\theta^2} \qquad \frac{d}{d\theta}\operatorname{arccot}(\theta) = -\frac{1}{1+\theta^2}$$
$$\frac{d}{d\theta}\operatorname{arcsec}(\theta) = \frac{1}{\theta\sqrt{\theta^2-1}} \qquad \frac{d}{d\theta}\operatorname{arccsc}(\theta) = -\frac{1}{\theta\sqrt{\theta^2-1}}$$

Pythagorean identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Sum and difference formulae

$$\sin(\theta \pm \phi) = \sin a\cos b \pm \cos a\sin b$$
$$\cos(\theta \pm \phi) = \cos a\cos b \mp \sin a\sin b$$
$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

Double-angle & half-angle formulae

These build upon the sum-difference formulae.

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$
$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Logarithms & some of their properties

$$y = \log_b x \Leftrightarrow x = b^y$$

$$\log_b b = 1 \qquad \log_b b^x = x$$
$$b^{\log_b x} = x \qquad \log_b 1 = 0$$

$$\log_b xy = \log_b x + \log_b y \qquad \log_b a^x = x\log_b a$$
$$\log_b \frac{x}{y} = \log_b x - \log_b y \qquad \log_b \sqrt[x]{a} = \frac{\log_b a}{x}$$

Inequalities, absolute values, & some of their properties

If $a < b$ then $a + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ (i.e. c is negative) then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

If b is positive, then:

$$|p| = b \quad \Rightarrow \quad p = -b \text{ or } p = b$$
$$|p| < b \quad \Rightarrow \quad -b < p < b$$
$$|p| > b \quad \Rightarrow \quad p < -b \text{ or } p > b$$

2 Limits

Basics

Definition (rough)**.** $\lim_{x\to a} f(x) = L$ means that as we approach x closer to a *but not equal to* a, the value for $f(x)$ can get arbitrarily closer to L.

Note that $\lim_{x\to a} f(x) = L$ **iff** $\lim_{x\to a^-} f(x) = L$ (LR limit) and $\lim_{x\to a^+} f(x) = L$ (RH limit) are both true.

Properties

Assuming that the limit exists, then all the following is true:

$$\lim_{x\to a} k = k \qquad \lim_{x\to a} x = a$$

$\lim\limits_{x\to a} f(x) = f(a)$ if the function f is *continuous* at $x = a$

$$\begin{aligned}
\lim_{x\to a}(f(x) \pm g(x)) &= \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) \\
\lim_{x\to a}(k \cdot f(x)) &= k \lim_{x\to a} f(x) \\
\lim_{x\to a}(f(x) \cdot g(x)) &= \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) \\
\lim_{x\to a} \frac{f(x)}{g(x)} &= \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \quad (g(a) \neq 0) \\
\lim_{x\to a} f(x)^n &= \left(\lim_{x\to a} f(x)\right)^n
\end{aligned}$$

If $\lim\limits_{x\to a} f(x) = L$, $\lim\limits_{x\to L} g(x) = K$ s.t. $g(L) = K$, then

$$\lim_{x\to a} g(f(x)) = K.$$

Squeeze theorem

Theorem. If $f(x) \leq g(x) \leq h(x)$ for all x near $x = a$ (except possibly at $x = a$), and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L,$$

then

$$\lim_{x\to a} g(x) = L$$

Some important-to-remember limits

$$\begin{aligned}
\lim_{x\to 0} \frac{1}{x} &= \text{DNE } (1/0) & \lim_{x\to 0} \frac{1}{x^2} &= \infty \\
\lim_{x\to 0} \frac{\sin x}{x} &= 1 & \lim_{x\to 0} \frac{\cos x - 1}{x} &= 0 \\
\lim_{x\to 0} (1 + x)^{1/x} &= e & \lim_{x\to 0} \frac{e^x - 1}{x} &= 1 \\
\lim_{x\to 0} \sin \frac{1}{x} &= \text{DNE (chaos)}
\end{aligned}$$

3 Acknowledgements

The cheat sheet is partly inspired by (and copied from) Paul's Online Math Notes (`https://tutorial.math.lamar.edu/`); in particular his various cheat sheets.