Github: github.com/3tz/Jester-Online-Jokes-Analysis

1 Pre-processing

1.1 Matrix Transformation

First, all datasets are transformed into a matrix with each row containing one single rating and the columns of this matrix containing user IDs, item IDs, and ratings. Here, a user ID is the ID number of the person who rates the jokes and item ID is the ID number of the joke. To use Non-negative Matrix Factorization (NMF), all ratings must be non-negative. Thus, the ratings in the matrix are simply shifted to the positive direction by a value of 10. For Multilayer Perceptron model (MLP), the ratings are shifted back with the center being 0. Below is a summary of the matrix with an extra column of "nRated" which is just the number of jokes this person has rated for the purpose of organizing; only the first three columns of the matrix will be used for training. Notice that this matrix is a full matrix with no missing value.

```
              uID jID rating nRated
      1:        1   1   2.18     74
      2:        1   2  18.79     74
      3:        1   3   0.34     74
      4:        1   4   1.84     74
      5:        1   5   2.48     74
     ---
4136356: 73421  65  11.36     35
4136357: 73421  66  17.18     35
4136358: 73421  69  10.49     35
4136359: 73421  72  15.87     35
4136360: 73421  82  16.65     35
```

1.2 Cross Validation

Recommender systems, especially non-negative matrix factorization, should not encounter new user or item ID during prediction that the models have not yet encountered during training. Thus, to achieve the goal of predicting ratings of all 100 jokes for every pre-selected 300 users, we will have multiple pairs of training and testing sets to guarantee mutual-exclusiveness. For instance, for a training set with a size of 30% of the total data, pseudocode of the operation is the following:

```
for i in allUserIDs:
  pair1_trainSet.append(30 random ratings from user i)
    if i is one of the 300 testing users:
      pair1_testSet.append(remaining 70 jokes from user i)
    pair2_trainSet.append(30 random ratings from pair1_testSet)
pair2_testSet = ratings rated by the 300 users in pair1_trainSet
```

This algorithm follows three rules: training set in each pair contains only the specified percentage of the total dataset, the union of test sets from all pairs cover all of the joke IDs, and user IDs in the training set are guaranteed to not exist in the testing set for each pair. Thus, if there were only 10 joke IDs, for a user who is one of the 300 testing users, assignments of joke IDs of ratings would look like this:

```
        trainSetJokeIDs  testSetJokeIDs
pair1:  [1,2,3]          [4,5,6,7,8,9,10]
pair2:  [4,5,9]          [1,2,3]
```

where `[1,2,3]` are first randomly chosen from 1 to 10, and `[4,5,9]` are randomly chosen from `[4,5,6,7,8,9,10]`. Notice that we indeed only contain 30% of the dataset for training, the union of test sets from all pairs contains each testing joke ID once, and user IDs in the training set are guaranteed to not exist in the testing set for each pair.

For a training set with a size of higher than 50% of the total data, a similar strategy is used but with more pairs of training and testing sets. For instance for a training set with a size of 60% of the total data, 3 pairs of training and testing sets are required, and joke assignments to datasets would look like this for one of the user in the testing set (IDs during actual operations are randomly chosen; they are in numerical order for demonstration purpose):

```
        trainSetJokeIDs  testSetJokeIDs
pair1:  [1,2,3,4,5,6]    [8,9,10]
pair2:  [1,2,3,8,9,10]   [4,5,6,7]
pair3:  [4,5,6,7,8,9]    [1,2,3]
```

The same three rules are also followed.

2 Result

2.1 Non-negative Matrix Factorization

2.1.1 MAE

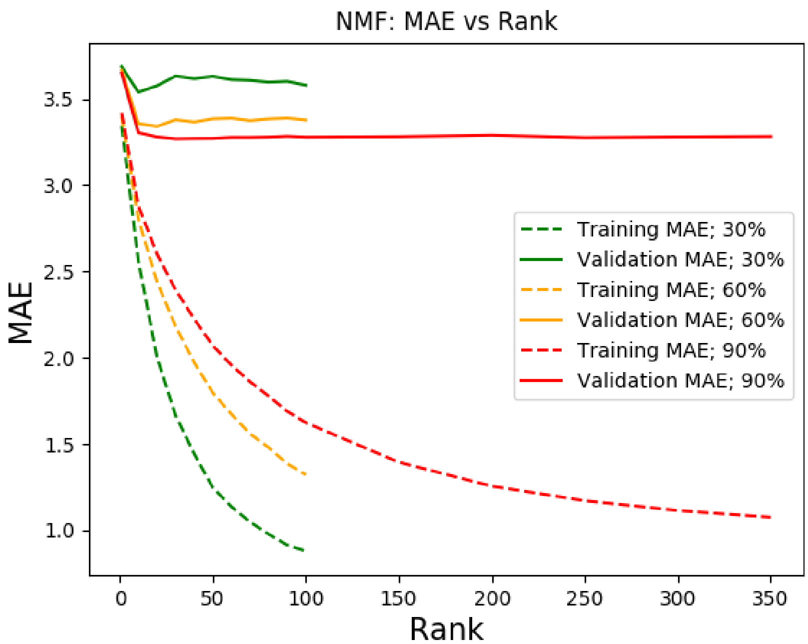
Training MAE

	Training Size		
Rank	30%	60%	90%
1	3.342	3.398	3.418
10	2.559	2.802	2.879
20	2.001	2.438	2.599
30	1.665	2.181	2.392
40	1.442	1.970	2.223
50	1.246	1.796	2.068
60	1.136	1.671	1.956
70	1.047	1.558	1.860
80	0.976	1.479	1.776
90	0.914	1.385	1.688
100	0.878	1.321	1.622
150			1.393
200			1.255
250			1.170
300			1.114
350			1.075

Validation MAE

	Training Size		
Rank	30%	60%	90%
1	3.684	3.664	3.648
10	**3.537**	3.354	3.303
20	3.574	**3.339**	3.278
30	3.631	3.377	**3.266**
40	3.616	3.364	3.268
50	3.629	3.383	3.269
60	3.610	3.386	3.275
70	3.606	3.372	3.275
80	3.595	3.382	3.277
90	3.600	3.387	3.282
100	3.577	3.375	3.277
150			3.280
200			3.288
250			3.274
300			3.278
350			3.281

Note: The lowest validation MAE for each training size is underlined.



The MAE of the training set is calculated in the following way where n denotes the number of pairs, $\hat{y}$ denotes vector containing the predicted values, and y denotes the vector containing the true values. The superscript indicates the pair index and the subscript indicates the item in the vector.

$$\frac{1}{n}\frac{1}{size(y)}\sum_{i=1}^{n}\sum_{j=1}^{size(y)}|\hat{y}_j^{(i)}-y_j^{(i)}|$$

Since MAE of the testing set is calculated after combining the predicted values together, the predicted values here are simply the 300×100 ratings in the pre-selected testing set. Thus, the MAE is simply

$$\frac{1}{size(y)}\sum_{j=1}^{size(y)}|\hat{y}_j-y_j|$$

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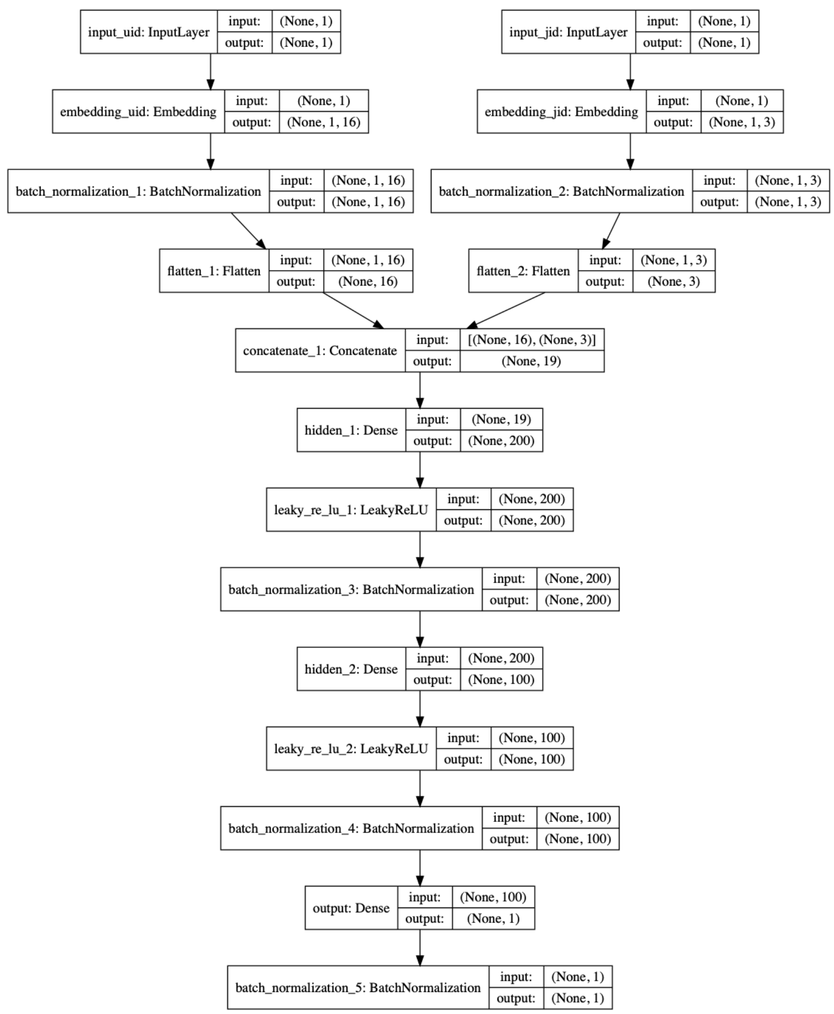
2.2 Multilayer Perceptron

2.2.1 Model Structure

Visual Model Overview



The loss function and the optimizer of the network are mean squared error and Adam with a learning rate of 0.1, and a batch size of 4096 is used for training.

The architecture of the layers follows the structure discussed by He *et al* in his paper published in 2017[1]. This model takes two scalar inputs: user ID and joke ID. Then, each scalar input is fed into a different embedding layer. The idea of embedding was first introduced in the field of Natural Language Processing by Bengio *et al*[2]. The purpose of an embedding layer is to reduce the high dimensionality of high cardinality discrete variable representation into a much smaller vector space. Here, the embedding layers convert each input into a vector of length 16 and 3 for user ID and joke ID which have a cardinality of $73,421$ and 100 respectively, and the length of each vector is computed by taking the fourth root of the cardinality of the input. The way the IDs are converted into vectors is by assigning one vector of numerical values to every unique ID. The numerical values in these embedding vectors are initialized uniformly randomly and are updated during backpropagation based on the network's loss function in the same way as other layers' weights. Then, each of the vectors is normalized with Batch Normalization, and the normalized outputs are flattened and concatenated together as one single layer which is then fed to the hidden layers.

There are two hidden layers where the first layer contains 200 neurons and the second contains 100 neurons. Both of them are fully connected layers, have Leaky ReLU as their activations, and are normalized with Batch Normalization. The output layer has a size of one and a linear function as its activation, and its output is also normalized with Batch Normalization. Layer dropout is not used as it interferes with Batch Normalization which reduces performance when these two techniques are used together as discussed in [3].

2.2.2 MAE

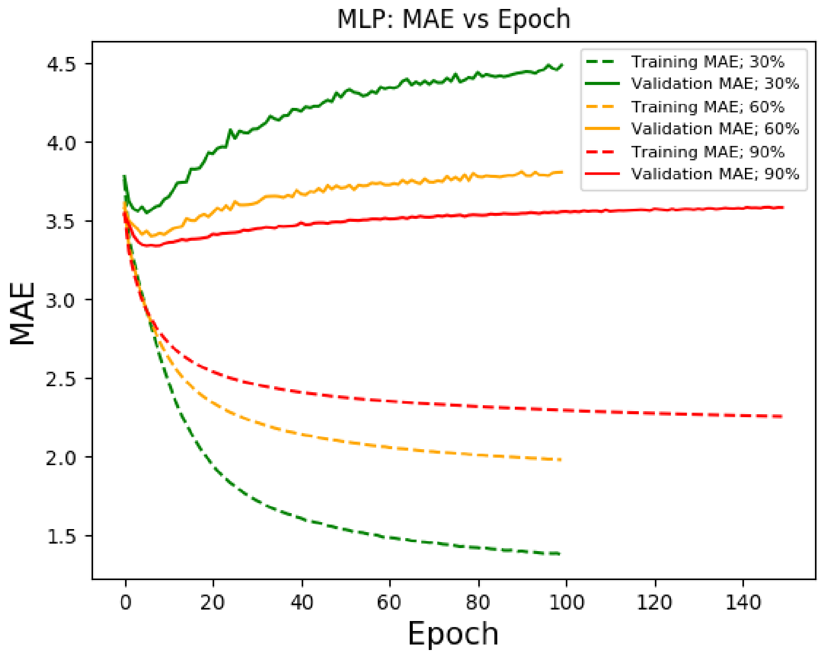
Training MAE

	Training Size		
Epoch	30%	60%	90%
1	3.755	3.619	3.554
2	3.385	3.337	3.301
3	3.250	3.214	3.177
4	3.132	3.109	3.073
5	3.019	3.012	2.994
6	2.917	2.928	2.930
7	2.823	2.854	2.876
8	2.729	2.789	2.829
9	2.635	2.729	2.788
10	2.547	2.675	2.752
11	2.463	2.626	2.721
12	2.385	2.583	2.693
13	2.317	2.544	2.669
14	2.254	2.509	2.646
15	2.200	2.478	2.627
16	2.148	2.450	2.609
17	2.102	2.424	2.592
18	2.057	2.401	2.577
19	2.014	2.379	2.563
20	1.981	2.360	2.551
21	1.944	2.343	2.539
22	1.910	2.325	2.528
23	1.880	2.310	2.518
24	1.856	2.296	2.509
25	1.832	2.281	2.500
26	1.812	2.270	2.492
27	1.789	2.258	2.484
28	1.769	2.246	2.476
29	1.750	2.235	2.469
30	1.735	2.225	2.463
31	1.718	2.216	2.457
32	1.705	2.208	2.451
33	1.688	2.199	2.445
34	1.675	2.191	2.440
35	1.663	2.181	2.435
36	1.654	2.175	2.430
37	1.643	2.168	2.425
38	1.631	2.161	2.421
39	1.626	2.155	2.417
40	1.616	2.149	2.412
41	1.607	2.144	2.408
42	1.594	2.137	2.405
43	1.589	2.133	2.401
44	1.580	2.127	2.397
45	1.575	2.121	2.394
46	1.570	2.116	2.391
47	1.557	2.110	2.388
48	1.553	2.106	2.385
49	1.545	2.104	2.382
50	1.534	2.098	2.379
51	1.533	2.094	2.376
52	1.529	2.091	2.373
53	1.517	2.086	2.371
54	1.516	2.083	2.368
55	1.513	2.079	2.366
56	1.506	2.076	2.363
57	1.496	2.072	2.361
58	1.499	2.068	2.359
59	1.490	2.064	2.356
60	1.481	2.064	2.354
61	1.483	2.059	2.352
62	1.480	2.056	2.350
63	1.475	2.053	2.348
64	1.471	2.051	2.346
65	1.468	2.049	2.344
66	1.462	2.046	2.342
67	1.459	2.043	2.340
68	1.455	2.040	2.338
69	1.453	2.036	2.336
70	1.450	2.035	2.335
71	1.448	2.034	2.333
72	1.446	2.030	2.331
73	1.440	2.027	2.330
74	1.436	2.026	2.328
75	1.433	2.025	2.326
76	1.430	2.022	2.325
77	1.423	2.019	2.323
78	1.426	2.018	2.322
79	1.421	2.015	2.320
80	1.417	2.013	2.319
81	1.417	2.010	2.318
82	1.413	2.010	2.316
83	1.415	2.007	2.315
84	1.409	2.005	2.314
85	1.408	2.003	2.312
86	1.409	2.002	2.310
87	1.401	2.000	2.309
88	1.400	1.999	2.308
89	1.402	1.998	2.307
90	1.392	1.995	2.306
91	1.397	1.994	2.305
92	1.391	1.993	2.303
93	1.390	1.990	2.302
94	1.387	1.989	2.301
95	1.383	1.988	2.300
96	1.381	1.987	2.299
97	1.381	1.984	2.298
98	1.381	1.985	2.297
99	1.383	1.982	2.296
100	1.373	1.980	2.295
101			2.294
102			2.293
103			2.292
104			2.291
105			2.290
106			2.289
107			2.288
108			2.287
109			2.286
110			2.285
111			2.284
112			2.283
113			2.282
114			2.282
115			2.281
116			2.280
117			2.279
118			2.278
119			2.278
120			2.277
121			2.276
122			2.275
123			2.275
124			2.274
125			2.273
126			2.272
127			2.271
128			2.271
129			2.270
130			2.269
131			2.268
132			2.268
133			2.267
134			2.266
135			2.266
136			2.265
137			2.265
138			2.264
139			2.263
140			2.263
141			2.262
142			2.261
143			2.261
144			2.260
145			2.259
146			2.259
147			2.258
148			2.258
149			2.257
150			2.257

Validation MAE

	Training Size				Training Size				Training Size		
Epoch	30%	60%	90%	Epoch	30%	60%	90%	Epoch	30%	60%	90%
1	3.778	3.579	3.535	51	4.318	3.710	3.495	101			3.556
2	3.622	3.499	3.475	52	4.331	3.724	3.503	102			3.553
3	3.574	3.471	3.403	53	4.313	3.697	3.501	103			3.556
4	3.559	3.443	3.367	54	4.307	3.725	3.500	104			3.553
5	3.588	3.412	3.346	55	4.290	3.720	3.504	105			3.562
6	**3.547**	3.432	3.339	56	4.297	3.722	3.504	106			3.557
7	3.567	**3.401**	3.343	57	4.320	3.738	3.509	107			3.560
8	3.588	3.407	**3.338**	58	4.308	3.734	3.509	108			3.560
9	3.595	3.419	3.340	59	4.348	3.739	3.508	109			3.555
10	3.634	3.410	3.354	60	4.336	3.726	3.514	110			3.568
11	3.650	3.430	3.361	61	4.345	3.726	3.510	111			3.558
12	3.694	3.449	3.364	62	4.340	3.735	3.510	112			3.561
13	3.738	3.456	3.371	63	4.350	3.729	3.514	113			3.563
14	3.742	3.461	3.380	64	4.385	3.755	3.518	114			3.562
15	3.744	3.513	3.375	65	4.397	3.737	3.513	115			3.564
16	3.827	3.500	3.382	66	4.361	3.755	3.523	116			3.564
17	3.826	3.499	3.383	67	4.381	3.742	3.519	117			3.566
18	3.832	3.527	3.385	68	4.364	3.734	3.526	118			3.568
19	3.873	3.539	3.390	69	4.386	3.764	3.521	119			3.567
20	3.932	3.523	3.395	70	4.365	3.744	3.521	120			3.563
21	3.924	3.549	3.413	71	4.389	3.743	3.523	121			3.574
22	3.957	3.579	3.410	72	4.376	3.752	3.530	122			3.571
23	3.968	3.563	3.417	73	4.376	3.747	3.526	123			3.568
24	3.966	3.596	3.418	74	4.404	3.779	3.533	124			3.564
25	4.076	3.566	3.421	75	4.384	3.756	3.528	125			3.573
26	4.020	3.620	3.423	76	4.430	3.774	3.531	126			3.566
27	4.068	3.599	3.427	77	4.380	3.753	3.534	127			3.569
28	4.059	3.599	3.438	78	4.392	3.791	3.526	128			3.573
29	4.056	3.601	3.435	79	4.408	3.753	3.533	129			3.572
30	4.080	3.605	3.441	80	4.402	3.801	3.536	130			3.569
31	4.082	3.614	3.448	81	4.440	3.777	3.534	131			3.574
32	4.100	3.632	3.450	82	4.414	3.775	3.532	132			3.570
33	4.119	3.651	3.457	83	4.427	3.783	3.534	133			3.577
34	4.163	3.653	3.455	84	4.419	3.780	3.540	134			3.576
35	4.146	3.654	3.451	85	4.412	3.779	3.537	135			3.571
36	4.138	3.652	3.464	86	4.419	3.775	3.539	136			3.574
37	4.165	3.645	3.462	87	4.412	3.781	3.543	137			3.579
38	4.167	3.659	3.464	88	4.419	3.786	3.541	138			3.576
39	4.205	3.650	3.465	89	4.427	3.785	3.540	139			3.575
40	4.204	3.692	3.469	90	4.435	3.795	3.541	140			3.578
41	4.193	3.667	3.486	91	4.438	3.811	3.544	141			3.575
42	4.221	3.661	3.474	92	4.445	3.784	3.545	142			3.584
43	4.215	3.684	3.475	93	4.442	3.783	3.550	143			3.580
44	4.225	3.679	3.484	94	4.434	3.808	3.547	144			3.581
45	4.224	3.704	3.486	95	4.450	3.789	3.543	145			3.579
46	4.250	3.680	3.486	96	4.458	3.789	3.550	146			3.585
47	4.265	3.724	3.482	97	4.488	3.784	3.548	147			3.586
48	4.255	3.700	3.489	98	4.465	3.803	3.552	148			3.579
49	4.310	3.721	3.491	99	4.457	3.806	3.549	149			3.584
50	4.282	3.706	3.490	100	4.487	3.808	3.555	150			3.583

Note: The lowest validation MAE for each training size is underlined.



For a pair of training and validation sets, the validation MAEs are computed after each epoch is finished, but the training MAEs are taken directly from `keras.models.Model.fit()` which is the mean of all MAEs calculated after each batch during an epoch of training. Although the sizes of the training set across different pairs are identical, the sizes of validation sets are not. Therefore, validation MAE of each pair is multiplied by a constant c_i before summation where $c_i = size(val\_set_i)/30,000$ and i is the index of the pair, which is just a linear combination of constants c_i and the absolute error matrix and can be computed by finding

$$\begin{bmatrix} MAE_1 & MAE_2 & \dots & MAE_n \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & \dots & c_p \end{bmatrix} \times \begin{bmatrix} E_1^{(1)} & E_2^{(1)} & \dots & E_n^{(1)} \\ E_1^{(2)} & E_2^{(2)} & \dots & E_n^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ E_1^{(p)} & E_2^{(p)} & \dots & E_n^{(p)} \end{bmatrix}$$

where p denotes the total number of pairs, n denotes the total number of epochs, $E_j^{(i)}$ denotes the absolute error of pair i at epoch j.

2.3 Ternary Comparison

Proportion of each ternary with each training size's optimal rank or epoch.

	Ternary Proportion		
Model	a	b	c
NMF (90%)	55.300%	29.640%	15.060%
NMF (60%)	54.523%	29.647%	15.830%
NMF (30%)	51.253%	30.703%	18.043%
MLP (90%)	54.813%	29.333%	15.853%
MLP (60%)	53.760%	29.967%	16.273%
MLP (30%)	52.373%	29.280%	18.347%
Total Average	14.257%	11.180%	74.563%
Uniformly Random	12.720%	13.560%	73.720%
User Average	8.463%	12.283%	79.253%

Note: a: $MAE \in (-\infty, 3)$; b: $MAE \in [3, 6)$; c: $MAE \in [6, \infty)$.

References

[1] Xiangnan He, Lizi Liao, Hanwang Zhang, Liqiang Nie, Xia Hu, Tat-Seng Chua. Neural Collaborative Filtering. *arXiv preprint arXiv:1708.05031*
[2] Yoshua Bengio, Rejean Ducharme, Pascal Vincent. A Neural Probabilistic Language Model (2001).
[3] Xiang Li, Shuo Chen, Xiaolin Hu, Jian Yang. Understanding the Disharmony between Dropout and Batch Normalization by Variance Shift. *arXiv preprint arXiv:1801.05134*