## 1 Pre-processing

### 1.1 Matrix Transformation

First, all datasets are transformed into a matrix with each row containing one single rating and the columns of this matrix containing user IDs, item IDs, and ratings. Here, a user ID is the ID number of the person who rates the jokes and item ID is the ID number of the joke. To use Non-negative Matrix Factorization (NMF), all ratings must be non-negative. Thus, the ratings in the matrix are simply shifted to the positive direction by a value of 10. For Multilayer Perceptron model (MLP), the ratings are shifted back with the center being 0. Below is a summary of the matrix with an extra column of "nRated" which is just the number of jokes this person has rated for the purpose of organizing; only the first three columns of the matrix will be used for training. Notice that this matrix is a full matrix with no missing value.

1		uID	jID	rating	nRated
2	1:	1	1	2.18	74
3	2:	1	2	18.79	74
4	3:	1	3	0.34	74
5	4:	1	4	1.84	74
6	5:	1	5	2.48	74
7					
8	4136356:	73421	65	11.36	35
9	4136357:	73421	66	17.18	35
10	4136358:	73421	69	10.49	35
11	4136359:	73421	72	15.87	35
12	4136360:	73421	82	16.65	35

### 1.2 Cross Validation

Recommender systems, especially non-negative matrix factorization, should not encounter new user or item ID during prediction that the models have not yet encountered during training. Thus, to achieve the goal of predicting ratings of all 100 jokes for every pre-selected 300 users, we will have multiple

pairs of training and testing sets to guarantee mutual-exclusiveness. For instance, for a training set with a size of 30% of the total data, pseudocode of the operation is the following:

```
for i in allUserIDs:
    pair1_trainSet.append(30 random ratings from user i)
    if i is one of the 300 testing users:
        pair1_testSet.append(remaining 70 jokes from user i)
        pair2_trainSet.append(30 random ratings from pair1_testSet)
    pair2 testSet = ratings rated by the 300 users in pair1 trainSet
```

This algorithm follows three rules: training set in each pair contains only the specified percentage of the total dataset, the union of test sets from all pairs cover all of the joke IDs, and user IDs in the training set are guaranteed to not exist in the testing set for each pair. Thus, if there were only 10 joke IDs, for a user who is one of the 300 testing users, assignments of joke IDs of ratings would look like this:

```
trainSetJokeIDs testSetJokeIDs
pair1: [1,2,3] [4,5,6,7,8,9,10]
pair2: [4,5,9] [1,2,3]
```

where [1,2,3] are first randomly chosen from 1 to 10, and [4,5,9] are randomly chosen from [4,5,6,7,8,9,10]. Notice that we indeed only contain 30% of the dataset for training, the union of test sets from all pairs contains each testing joke ID once, and user IDs in the training set are guaranteed to not exist in the testing set for each pair.

For a training set with a size of higher than 50% of the total data, a similar strategy is used but with more pairs of training and testing sets. For instance for a training set with a size of 60% of the total data, 3 pairs of training and testing sets are required, and joke assignments to datasets would look like this for one of the user in the testing set (IDs during actual operations are randomly chosen; they are in numerical order for demonstration purpose):

```
trainSetJokeIDs testSetJokeIDs
pair1: [1,2,3,4,5,6] [8,9,10]
pair2: [1,2,3,8,9,10] [4,5,6,7]
pair3: [4,5,6,7,8,9] [1,2,3]
```

The same three rules are also followed.

# 2 Result

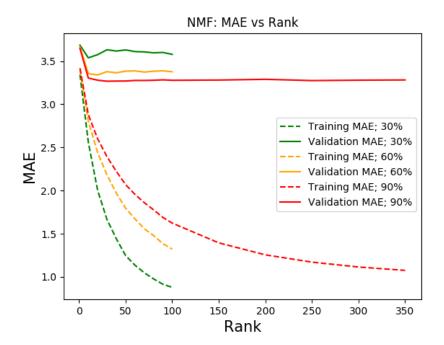
# 2.1 Non-negative Matrix Factorization

### 2.1.1 MAE

Training MAE Validation MAE

	Tre	oining S	izo		Training Size				
	Training Size				Ü				
Rank	30%	60%	90%	Rank	30%	60%	90%		
1	3.342	3.398	3.418	1	3.684	3.664	3.648		
10	2.559	2.802	2.879	10	3.537	3.354	3.303		
20	2.001	2.438	2.599	20	3.574	3.339	3.278		
30	1.665	2.181	2.392	30	3.631	3.377	3.266		
40	1.442	1.970	2.223	40	3.616	3.364	3.268		
50	1.246	1.796	2.068	50	3.629	3.383	3.269		
60	1.136	1.671	1.956	60	3.610	3.386	3.275		
70	1.047	1.558	1.860	70	3.606	3.372	3.275		
80	0.976	1.479	1.776	80	3.595	3.382	3.277		
90	0.914	1.385	1.688	90	3.600	3.387	3.282		
100	0.878	1.321	1.622	100	3.577	3.375	3.277		
150			1.393	150			3.280		
200			1.255	200			3.288		
250			1.170	250			3.274		
300			1.114	300			3.278		
350			1.075	350			3.281		

Note: The lowest validation MAE for each training size is underlined.



The MAE of the training set is calculated in the following way where n denotes the number of pairs,  $\hat{y}$  denotes vector containing the predicted values, and y denotes the vector containing the true values. The superscript indicates the pair index and the subscript indicates the item in the vector.

$$\frac{1}{n} \frac{1}{size(y)} \sum_{i=1}^{n} \sum_{j=1}^{size(y)} |\hat{y}_{j}^{(i)} - y_{j}^{(i)}|$$

Since MAE of the testing set is calculated after combining the predicted values together, the predicted values here are simply the  $300 \times 100$  ratings in the pre-selected testing set. Thus, the MAE is simply

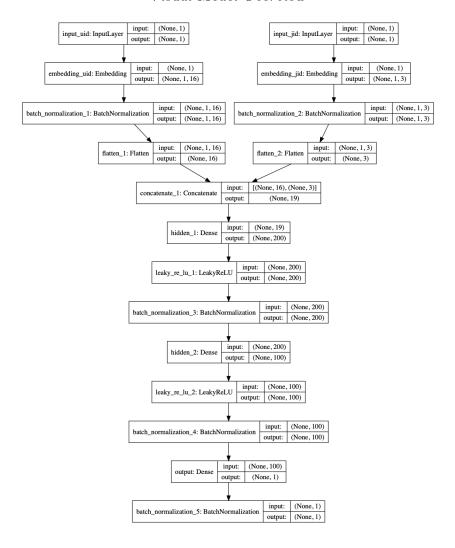
$$\frac{1}{size(y)} \sum_{j=1}^{size(y)} |\hat{y}_j - y_j|$$

.

### 2.2 Multilayer Perceptron

#### 2.2.1 Model Structure

#### Visual Model Overview



The loss function and the optimizer of the network are mean squared error and Adam with a learning rate of 0.1, and a batch size of 4096 is used for training.

The architecture of the layers follows the structure discussed by He et al in his paper published in 2017[1]. This model takes two scalar inputs: user ID and joke ID. Then, each scalar input is fed into a different embedding layer. The idea of embedding was first introduced in the field of Natural Language Processing by Bengio et al[2]. The purpose of an embedding layer is to reduce the high dimensionality of high cardinality discrete variable representation into a much smaller vector space. Here, the embedding layers convert each input into a vector of length 16 and 3 for user ID and joke ID which have a cardinality of 73,421 and 100 respectively, and the length of each vector is computed by taking the fourth root of the cardinality of the input. The way the IDs are converted into vectors is by assigning one vector of numerical values to every unique ID. The numerical values in these embedding vectors are initialized uniformly randomly and are updated during backpropagation based on the network's loss function in the same way as other layers' weights. Then, each of the vectors is normalized with Batch Normalization, and the normalized outputs are flattened and concatenated together as one single layer which is then fed to the hidden layers.

There are two hidden layers where the first layer contains 200 neurons and the second contains 100 neurons. Both of them are fully connected layers, have Leaky ReLU as their activations, and are normalized with Batch Normalization. The output layer has a size of one and a linear function as its activation, and its output is also normalized with Batch Normalization. Layer dropout is not used as it interferes with Batch Normalization which reduces performance when these two techniques are used together as discussed in [3].

## 2.2.2 MAE

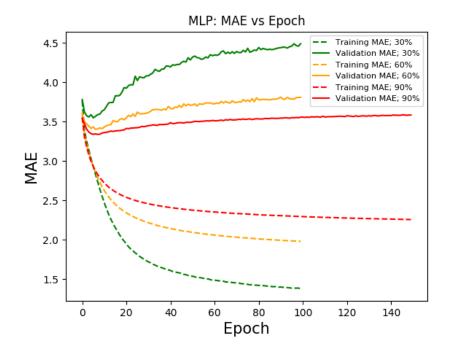
Training MAE

Training Size	$Training \ MAE$											
2         3.755         3.619         3.554         51         1.533         2.094         2.376         101         2.294           3         3.355         3.337         3.301         52         1.529         2.091         2.373         102         2.293           3         3.250         3.214         3.177         53         1.516         2.083         2.368         104         2.291           5         3.019         3.012         2.994         55         1.516         2.0283         2.366         105         2.290           6         2.917         2.928         2.854         2.876         57         1.496         2.072         2.361         107         2.288           8         2.729         2.789         2.829         58         1.499         2.068         2.359         108         2.287           9         2.635         2.729         2.788         59         1.481         2.064         2.356         109         2.286           10         2.547         2.675         6.0         1.481         2.064         2.356         109         2.286           11         2.463         2.626         2.721         61												
2         3, 3,250         3,214         3,177         53         1,517         2,086         2,371         103         2,292           4         3,132         3,109         3,073         54         1,516         2,086         2,371         103         2,292           5         3,019         3,012         2,994         55         1,516         2,083         106         105         2,291           6         2,917         2,928         2,930         56         1,506         2,076         2,363         106         2,289           7         2,823         2,854         2,876         57         1,496         2,072         2,361         107         2,288           8         2,729         2,788         59         1,499         2,068         2,359         108         2,2281           10         2,547         2,675         2,752         60         1,481         2,064         2,354         110         2,285           11         2,633         2,636         6,721         61         1,483         2,059         2,352         111         2,280           12         2,385         2,583         2,693         6         1,480										30%	60%	
3         3,250         3,214         3,177         53         1,516         2,083         2,368         104         2,291           5         3,019         3,012         2,994         55         1,516         2,076         2,366         105         6         2,291         2,928         2,930         56         1,506         2,076         2,363         106         2,229           7         2,823         2,854         2,876         57         1,496         2,072         2,361         107         2,289           8         2,729         2,789         2,883         59         1,490         2,068         2,359         109         2,286           10         2,547         2,675         2,752         60         1,481         2,064         2,356         109         2,286           11         2,463         2,626         2,721         61         1,481         2,064         2,354         110         2,285           12         2,365         2,583         2,693         62         1,480         2,056         2,350         111         2,284           12         2,365         2,528         2,311         1,264         2,492         1,41 <td></td>												
4         3.132         3.109         3.073         54         1.516         2.083         2.366         105         2.291           5         3.019         3.012         2.994         55         1.513         2.079         2.366         105         2.290           6         2.917         2.928         2.930         56         1.506         2.072         2.363         106         2.288           8         2.729         2.789         2.829         58         1.490         2.068         2.359         108         2.286           9         2.635         2.729         2.788         59         1.490         2.064         2.356         109         2.286           10         2.547         2.675         2.752         60         1.481         2.064         2.354         110         2.285           11         2.463         2.626         2.721         61         1.483         2.055         2.350         111         2.285           12         2.238         2.583         2.693         62         1.480         2.056         2.350         112         2.281           13         2.317         2.544         2.6693         63												
5         3.019         3.012         2.994         55         1.513         2.079         2.366         105         2.290           6         2.917         2.928         2.930         56         1.506         2.076         2.363         106         2.289           7         2.833         2.854         2.876         57         1.496         2.072         2.361         107         2.288           8         2.729         2.788         59         1.499         2.068         2.356         109         2.286           10         2.547         2.675         2.752         60         1.481         2.064         2.354         110         2.285           11         2.263         2.271         61         1.483         2.059         2.352         111         2.263         2.281           12         2.385         2.583         2.669         62         1.480         2.056         2.350         112         2.283           13         2.317         2.544         2.669         63         1.471         2.053         2.348         113         2.2281           14         2.250         2.478         2.677         65         1.468												
6         2.917         2.928         2.930         56         1.506         2.072         2.361         106         2.289           7         2.823         2.854         2.876         57         1.496         2.072         2.361         107         2.288           8         2.729         2.789         2.829         58         1.499         2.068         2.356         109         2.286           10         2.547         2.675         2.752         60         1.481         2.064         2.354         110         2.285           11         2.463         2.626         2.721         61         1.483         2.059         2.352         111         2.283           12         2.385         2.583         2.693         62         1.480         2.056         2.350         112         2.281           13         2.317         2.544         2.699         63         1.475         2.053         2.348         113         113         2.232         144         2.254         2.509         2.646         64         1.471         2.051         141         2.282           15         2.200         2.478         2.6292         65         1.468 <td></td>												
7         2,823         2,854         2,876         57         1,496         2,072         2,359         108         2,228           8         2,729         2,789         2,829         58         1,499         2,064         2,359         108         2,228           9         2,635         2,729         2,788         59         1,490         2,064         2,356         109         2,286           10         2,247         2,675         2,752         60         1,481         2,064         2,354         110         2,284           11         2,463         2,626         2,721         61         1,483         2,059         2,352         111         2,284           12         2,388         2,583         2,693         62         1,480         2,056         2,350         112         2,284           14         2,254         2,509         2,646         64         1,471         2,051         2,346         114         2,282           15         2,200         2,478         2,627         65         1,468         2,043         2,341         115         2,281           16         2,144         2,507         68         1,455												
8         2.729         2.789         2.829         58         1.499         2.068         2.356         109         2.286           10         2.547         2.675         2.752         60         1.481         2.064         2.356         109         2.286           11         2.663         2.626         2.721         61         1.483         2.059         2.352         111         2.283           12         2.385         2.583         2.693         62         1.480         2.056         2.350         112         2.283           13         2.317         2.544         2.669         63         1.475         2.053         2.348         113         2.282           14         2.254         2.509         2.66         64         1.477         2.051         2.346         114         2.282           15         2.200         2.478         2.627         65         1.468         2.049         2.344         115         2.281           16         2.148         2.450         2.609         66         1.462         2.040         2.332         116         2.280           17         2.102         2.474         2.592         67												
9         2.635         2.729         2.788         59         1.490         2.064         2.356         109         2.286           10         2.547         2.675         2.752         60         1.481         2.064         2.354         110         2.285           11         2.463         2.626         2.721         61         1.483         2.059         2.352         111         2.284           12         2.385         2.583         2.693         62         1.480         2.056         2.350         112         2.283           14         2.254         2.509         2.646         64         1.471         2.051         2.348         113         2.282           15         2.200         2.478         2.627         65         1.468         2.046         1.411         2.228           16         2.148         2.450         2.609         66         1.462         2.046         2.342         116         2.280           17         2.102         2.424         2.592         67         1.459         2.046         2.342         116         2.278           18         2.057         2.401         2.577         68         1.453 <td></td>												
10												
11         2.463         2.626         2.721         61         1.483         2.059         2.352         111         2.284           12         2.385         2.583         2.693         62         1.480         2.056         2.350         112         2.281           13         2.317         2.544         2.669         63         1.475         2.053         2.348         113         2.282           14         2.254         2.509         2.646         64         1.471         2.051         2.346         114         2.281           16         2.148         2.450         2.609         66         1.462         2.046         2.342         116         2.281           17         2.102         2.424         2.592         67         1.459         2.040         2.338         118         2.279           18         2.057         2.401         2.577         68         1.455         2.040         2.338         118         2.278           19         2.014         2.379         2.563         69         1.453         2.040         2.338         118         2.278           20         1.981         2.343         2.539         71 <td></td>												
12         2.385         2.583         2.693         62         1.480         2.056         2.350         112         2.283           13         2.317         2.544         2.669         63         1.475         2.053         2.348         113         2.282           15         2.200         2.478         2.627         65         1.468         2.049         2.344         115         2.281           16         2.148         2.450         2.609         66         1.462         2.046         2.342         116         2.280           17         2.102         2.424         2.592         67         1.459         2.043         2.340         117         2.279           18         2.057         2.401         2.577         68         1.455         2.040         2.338         118         2.278           19         2.014         2.379         2.563         69         1.453         2.036         2.335         119         2.278           20         1.981         2.360         2.551         70         1.450         2.035         2.331         121         2.277           21         1.940         2.343         2.539         71 <td></td>												
13         2.317         2.544         2.669         63         1.475         2.053         2.348         113         2.282           14         2.254         2.509         2.646         64         1.471         2.051         2.344         115         2.281           15         2.200         2.478         2.627         65         1.468         2.049         2.344         115         2.281           16         2.148         2.450         2.609         66         1.462         2.046         2.342         116         2.280           17         2.102         2.424         2.592         67         1.459         2.043         2.340         117         2.279           18         2.057         2.401         2.577         68         1.455         2.040         2.338         118         2.278           20         1.981         2.360         2.551         70         1.450         2.035         2.335         120         2.277           21         1.944         2.343         2.539         71         1.448         2.034         2.333         121         2.276           22         1.910         2.325         2.599         74 <td></td>												
14         2.254         2.599         2.646         64         1.471         2.051         2.346         114         2.282           15         2.200         2.478         2.627         65         1.462         2.046         2.344         115         1         2.281           16         2.148         2.450         2.609         66         1.462         2.046         2.342         116         2.280           17         2.102         2.424         2.592         67         1.459         2.043         2.340         117         2.279           18         2.057         2.401         2.577         68         1.455         2.040         2.338         118         2.278           19         2.014         2.379         2.563         69         1.453         2.036         2.336         119         2.278           20         1.981         2.360         2.551         70         1.450         2.034         2.336         119         2.277           21         1.944         2.343         2.539         71         1.448         2.034         2.331         121         2.275           22         1.910         2.325         2.528												
15         2,200         2,478         2,627         65         1,468         2,049         2,344         115         2,281           16         2,148         2,450         2,609         66         1,462         2,046         2,342         116         2,280           17         2,102         2,424         2,592         67         1,459         2,043         2,340         117         2,279           18         2,057         2,401         2,577         68         1,455         2,040         2,338         118         2,278           19         2,014         2,369         2,551         70         1,450         2,035         2,335         120         2,277           20         1,981         2,360         2,551         70         1,450         2,035         2,335         120         2,277           21         1,944         2,343         2,539         71         1,448         2,034         2,331         121         2,275           22         1,910         2,325         2,528         72         1,446         2,030         2,331         122         2,275           24         1,856         2,296         2,509         74 <td></td>												
16         2.148         2.450         2.609         66         1.462         2.046         2.342         116         2.280           17         2.102         2.424         2.592         67         1.459         2.040         2.338         118         2.278           19         2.014         2.379         2.563         69         1.453         2.036         2.336         119         2.278           20         1.981         2.360         2.551         70         1.450         2.035         2.335         120         2.277           21         1.944         2.343         2.539         71         1.448         2.034         2.333         121         2.276           22         1.910         2.325         2.528         72         1.446         2.030         2.331         122         2.275           23         1.880         2.310         2.518         73         1.440         2.027         2.330         123         2.275           24         1.856         2.296         2.509         74         1.436         2.026         2.328         124         2.274           25         1.832         2.281         2.500         75 <td></td>												
17         2.102         2.424         2.592         67         1.459         2.043         2.340         117         2.279           18         2.057         2.401         2.577         68         1.455         2.040         2.338         118         2.278           19         2.014         2.379         2.563         69         1.453         2.036         2.336         119         2.278           20         1.981         2.360         2.551         70         1.450         2.035         2.335         120         2.277           21         1.944         2.343         2.539         71         1.448         2.034         2.331         121         2.276           22         1.910         2.325         2.528         72         1.446         2.030         2.331         122         2.275           24         1.856         2.296         2.509         74         1.436         2.026         2.388         124         2.274           25         1.832         2.281         2.500         75         1.433         2.025         2.326         125         2.273           26         1.812         2.270         2.492         76 <td>15</td> <td>2.200</td> <td>2.478</td> <td>2.627</td> <td>65</td> <td>1.468</td> <td>2.049</td> <td>2.344</td> <td>115</td> <td></td> <td></td> <td>2.281</td>	15	2.200	2.478	2.627	65	1.468	2.049	2.344	115			2.281
18         2.057         2.401         2.577         68         1.455         2.040         2.338         118         2.278           19         2.014         2.379         2.563         69         1.453         2.036         2.336         119         2.278           20         1.981         2.360         2.551         70         1.448         2.035         2.335         120         2.277           21         1.944         2.343         2.539         71         1.448         2.034         2.333         121         2.276           22         1.910         2.325         2.528         72         1.446         2.030         2.331         122         2.275           23         1.880         2.310         2.518         73         1.446         2.026         2.328         124         2.275           24         1.856         2.296         2.509         74         1.436         2.025         2.328         124         2.273           25         1.832         2.281         2.500         75         1.433         2.025         2.325         126         2.272           27         1.789         2.246         2.476         78 <td></td> <td>2.148</td> <td>2.450</td> <td>2.609</td> <td></td> <td>1.462</td> <td>2.046</td> <td></td> <td></td> <td></td> <td></td> <td></td>		2.148	2.450	2.609		1.462	2.046					
19         2.014         2.379         2.563         69         1.453         2.036         2.336         119         2.278           20         1.981         2.360         2.551         70         1.450         2.035         2.335         120         2.277           21         1.944         2.343         2.331         121         2.276           22         1.910         2.325         2.528         72         1.446         2.030         2.331         122         2.275           23         1.880         2.310         2.518         73         1.440         2.027         2.330         123         2.275           24         1.856         2.296         2.509         74         1.436         2.026         2.328         124         2.275           24         1.852         2.291         2.500         75         1.433         2.025         2.326         125         2.273           26         1.812         2.270         2.492         76         1.433         2.025         2.326         125         2.271           27         1.789         2.246         2.476         78         1.426         2.018         2.322         128 <td>17</td> <td>2.102</td> <td>2.424</td> <td>2.592</td> <td></td> <td>1.459</td> <td>2.043</td> <td>2.340</td> <td></td> <td></td> <td></td> <td>2.279</td>	17	2.102	2.424	2.592		1.459	2.043	2.340				2.279
20         1.981         2.360         2.551         70         1.450         2.035         2.335         120         2.277           21         1.944         2.343         2.539         71         1.448         2.034         2.331         121         2.275           23         1.880         2.310         2.518         73         1.440         2.027         2.330         123         2.275           24         1.856         2.296         2.509         74         1.436         2.026         2.328         124         2.275           24         1.856         2.296         2.509         74         1.436         2.026         2.328         124         2.271           25         1.832         2.270         2.492         76         1.430         2.022         2.325         126         2.272           27         1.789         2.258         2.484         77         1.423         2.019         2.323         127         2.271           28         1.760         2.235         2.469         79         1.421         2.015         2.320         129         2.270           30         1.735         2.225         2.463         80 <td></td> <td>2.057</td> <td>2.401</td> <td>2.577</td> <td></td> <td>1.455</td> <td>2.040</td> <td></td> <td></td> <td></td> <td></td> <td></td>		2.057	2.401	2.577		1.455	2.040					
21         1.944         2.343         2.539         71         1.448         2.034         2.333         121         2.276           22         1.910         2.325         2.528         72         1.446         2.030         2.331         122         2.275           24         1.856         2.296         2.509         74         1.436         2.026         2.328         124         2.274           25         1.832         2.281         2.500         75         1.433         2.025         2.326         125         2.272           26         1.812         2.270         2.492         76         1.433         2.025         2.325         126         2.277           27         1.789         2.258         2.484         77         1.423         2.019         2.323         127         2.271           28         1.769         2.246         2.476         78         1.426         2.018         2.322         128         2.271           30         1.735         2.225         2.463         80         1.417         2.013         2.319         130         2.266           31         1.718         2.216         2.457         81 <td>19</td> <td>2.014</td> <td>2.379</td> <td>2.563</td> <td></td> <td>1.453</td> <td>2.036</td> <td></td> <td>119</td> <td>ll .</td> <td></td> <td></td>	19	2.014	2.379	2.563		1.453	2.036		119	ll .		
22         1.910         2.325         2.528         72         1.446         2.030         2.331         122         2.275           23         1.880         2.310         2.518         73         1.440         2.027         2.330         123         2.275           24         1.856         2.296         2.509         74         1.436         2.026         2.328         124         2.274           25         1.832         2.281         2.500         75         1.433         2.025         2.326         125         2.273           26         1.812         2.270         2.492         76         1.430         2.022         2.325         126         2.271           27         1.789         2.246         2.476         78         1.426         2.018         2.322         128         2.271           29         1.750         2.235         2.469         79         1.421         2.015         2.30         129         2.270           30         1.735         2.225         2.463         80         1.417         2.010         2.318         130         2.266           31         1.718         2.216         2.457         81	20	1.981	2.360	2.551	70	1.450	2.035	2.335	120			2.277
23         1.880         2.310         2.518         73         1.440         2.027         2.330         123         2.275           24         1.856         2.296         2.509         74         1.436         2.026         2.328         124         2.274           25         1.832         2.270         2.492         76         1.433         2.025         2.326         125         2.273           26         1.812         2.270         2.492         76         1.430         2.022         2.325         126         2.272           27         1.789         2.258         2.484         77         1.423         2.019         2.323         127         2.271           28         1.769         2.235         2.469         79         1.421         2.015         2.320         128         2.271           29         1.750         2.235         2.469         79         1.421         2.015         2.320         129         2.270           30         1.735         2.225         2.463         80         1.417         2.013         2.319         130         2.266           32         1.705         2.208         2.451         82 <td>21</td> <td>1.944</td> <td>2.343</td> <td>2.539</td> <td></td> <td>1.448</td> <td></td> <td></td> <td>121</td> <td>ll .</td> <td></td> <td></td>	21	1.944	2.343	2.539		1.448			121	ll .		
24         1.856         2.296         2.509         74         1.436         2.026         2.328         124         2.274           25         1.832         2.281         2.500         75         1.433         2.025         2.326         125         2.272           26         1.812         2.270         2.492         76         1.430         2.022         2.325         126         2.272           27         1.789         2.258         2.484         77         1.423         2.019         2.323         127         2.271           28         1.769         2.246         2.476         78         1.426         2.018         2.322         129         2.270           30         1.735         2.225         2.463         80         1.417         2.013         2.319         130         2.269           31         1.718         2.216         2.457         81         1.417         2.010         2.318         131         2.268           32         1.705         2.208         2.451         82         1.413         2.010         2.316         132         2.268           33         1.688         2.199         2.440         84 <td></td> <td>1.910</td> <td>2.325</td> <td>2.528</td> <td></td> <td>1.446</td> <td>2.030</td> <td></td> <td></td> <td></td> <td></td> <td>2.275</td>		1.910	2.325	2.528		1.446	2.030					2.275
25         1.832         2.281         2.500         75         1.433         2.025         2.326         125         2.273           26         1.812         2.270         2.492         76         1.430         2.022         2.325         126         2.272           27         1.789         2.246         2.476         78         1.423         2.019         2.323         127         2.271           28         1.769         2.246         2.476         78         1.426         2.018         2.322         128         2.271           29         1.750         2.235         2.469         79         1.421         2.015         2.320         129         2.270           30         1.735         2.225         2.469         79         1.421         2.015         2.319         130         2.269           31         1.718         2.216         2.457         81         1.417         2.010         2.318         131         2.268           32         1.705         2.208         2.451         82         1.413         2.010         2.315         133         2.266           32         1.675         2.191         2.440         84 <td>23</td> <td>1.880</td> <td>2.310</td> <td>2.518</td> <td>73</td> <td>1.440</td> <td>2.027</td> <td>2.330</td> <td>123</td> <td></td> <td></td> <td>2.275</td>	23	1.880	2.310	2.518	73	1.440	2.027	2.330	123			2.275
25         1.832         2.281         2.500         75         1.433         2.025         2.326         125         2.273           26         1.812         2.270         2.492         76         1.430         2.022         2.325         126         2.272           27         1.789         2.246         2.476         78         1.423         2.019         2.323         127         2.271           28         1.769         2.246         2.476         78         1.426         2.018         2.322         128         2.271           29         1.750         2.235         2.469         79         1.421         2.015         2.320         129         2.270           30         1.735         2.226         2.457         81         1.417         2.010         2.318         130         2.268           31         1.718         2.216         2.457         81         1.417         2.010         2.318         131         2.268           32         1.705         2.208         2.451         82         1.413         2.010         2.316         132         2.268           32         1.675         2.191         2.440         84 <td>24</td> <td>1.856</td> <td>2.296</td> <td>2.509</td> <td>74</td> <td>1.436</td> <td>2.026</td> <td>2.328</td> <td>124</td> <td>ll</td> <td></td> <td>2.274</td>	24	1.856	2.296	2.509	74	1.436	2.026	2.328	124	ll		2.274
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	25	1.832	2.281	2.500	75	1.433	2.025	2.326	125			
28         1.769         2.246         2.476         78         1.426         2.018         2.322         128         2.271           29         1.750         2.235         2.469         79         1.421         2.015         2.320         129         2.270           30         1.735         2.225         2.463         80         1.417         2.013         2.319         130         2.269           31         1.718         2.216         2.457         81         1.417         2.010         2.318         131         2.268           32         1.705         2.208         2.451         82         1.413         2.010         2.316         132         2.268           33         1.688         2.191         2.440         84         1.409         2.005         2.314         134         2.266           34         1.675         2.191         2.440         84         1.409         2.005         2.314         134         2.266           35         1.663         2.181         2.435         85         1.408         2.003         2.312         135         2.266           36         1.654         2.175         2.430         86 <td>26</td> <td>1.812</td> <td>2.270</td> <td>2.492</td> <td>76</td> <td>1.430</td> <td>2.022</td> <td>2.325</td> <td>126</td> <td>ll .</td> <td></td> <td>2.272</td>	26	1.812	2.270	2.492	76	1.430	2.022	2.325	126	ll .		2.272
29         1.750         2.235         2.469         79         1.421         2.015         2.320         129         2.270           30         1.735         2.225         2.463         80         1.417         2.013         2.319         130         2.269           31         1.718         2.216         2.457         81         1.417         2.010         2.318         131         2.268           32         1.705         2.208         2.451         82         1.413         2.010         2.316         132         2.268           33         1.688         2.199         2.445         83         1.415         2.007         2.315         133         2.267           34         1.675         2.191         2.440         84         1.409         2.005         2.314         134         2.266           35         1.663         2.181         2.435         85         1.408         2.002         2.310         136         2.266           36         1.654         2.175         2.430         86         1.409         2.002         2.310         136         2.265           37         1.643         2.168         2.425         87 <td></td> <td></td> <td></td> <td>2.484</td> <td></td> <td>1.423</td> <td>2.019</td> <td></td> <td></td> <td></td> <td></td> <td></td>				2.484		1.423	2.019					
30         1.735         2.225         2.463         80         1.417         2.013         2.319         130         2.269           31         1.718         2.216         2.457         81         1.417         2.010         2.318         131         2.268           32         1.705         2.208         2.451         82         1.413         2.010         2.316         132         2.268           33         1.688         2.199         2.445         83         1.415         2.007         2.315         133         2.267           34         1.675         2.191         2.440         84         1.409         2.005         2.314         134         2.266           35         1.663         2.181         2.435         85         1.408         2.003         2.312         135         2.266           36         1.654         2.175         2.430         86         1.409         2.002         2.310         136         2.266           37         1.643         2.168         2.425         87         1.401         2.000         2.309         137         2.265           38         1.631         2.161         2.421         88 <td>28</td> <td>1.769</td> <td>2.246</td> <td>2.476</td> <td>78</td> <td>1.426</td> <td>2.018</td> <td>2.322</td> <td>128</td> <td></td> <td></td> <td>2.271</td>	28	1.769	2.246	2.476	78	1.426	2.018	2.322	128			2.271
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	29	1.750	2.235	2.469	79	1.421	2.015	2.320	129			2.270
32         1.705         2.208         2.451         82         1.413         2.010         2.316         132         2.268           33         1.688         2.199         2.445         83         1.415         2.007         2.315         133         2.266           34         1.675         2.191         2.440         84         1.409         2.005         2.314         134         2.266           35         1.663         2.181         2.435         85         1.408         2.003         2.312         135         2.266           36         1.654         2.175         2.430         86         1.409         2.002         2.310         136         2.265           37         1.643         2.168         2.425         87         1.401         2.000         2.309         137         2.265           38         1.631         2.161         2.421         88         1.400         1.999         2.308         138         2.264           39         1.626         2.155         2.417         89         1.402         1.998         2.307         139         2.263           40         1.616         2.149         2.412         90 <td>30</td> <td>1.735</td> <td>2.225</td> <td>2.463</td> <td>80</td> <td>1.417</td> <td>2.013</td> <td>2.319</td> <td>130</td> <td></td> <td></td> <td>2.269</td>	30	1.735	2.225	2.463	80	1.417	2.013	2.319	130			2.269
33         1.688         2.199         2.445         83         1.415         2.007         2.315         133         2.267           34         1.675         2.191         2.440         84         1.409         2.005         2.314         134         2.266           35         1.663         2.181         2.435         85         1.408         2.003         2.312         135         2.266           36         1.654         2.175         2.430         86         1.409         2.002         2.310         136         2.265           37         1.643         2.161         2.421         88         1.401         2.000         2.309         137         2.265           38         1.631         2.161         2.421         88         1.400         1.999         2.308         138         2.264           39         1.626         2.155         2.417         89         1.402         1.998         2.307         139         2.263           40         1.616         2.149         2.412         90         1.392         1.995         2.306         140         2.263           41         1.607         2.144         2.408         91 <td>31</td> <td>1.718</td> <td>2.216</td> <td>2.457</td> <td>81</td> <td>1.417</td> <td>2.010</td> <td>2.318</td> <td></td> <td>ll .</td> <td></td> <td>2.268</td>	31	1.718	2.216	2.457	81	1.417	2.010	2.318		ll .		2.268
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	32	1.705	2.208	2.451	82	1.413	2.010	2.316				2.268
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	33	1.688	2.199	2.445	83	1.415	2.007	2.315	133	ll .		2.267
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	34	1.675		2.440	84	1.409	2.005	2.314	134			2.266
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	35	1.663	2.181	2.435	85	1.408	2.003	2.312	135			2.266
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	36	1.654	2.175	2.430	86	1.409	2.002	2.310	136	ll .		2.265
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	37	1.643	2.168	2.425	87	1.401	2.000	2.309	137			2.265
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	38	1.631	2.161	2.421	88	1.400	1.999	2.308	138	ll .		2.264
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	39	1.626	2.155	2.417	89	1.402	1.998	2.307	139			2.263
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	40	1.616	2.149	2.412	90	1.392	1.995	2.306	140			2.263
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	41	1.607	2.144	2.408	91	1.397	1.994	2.305	141	ll		2.262
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$												
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	43	1.589		2.401								
45     1.575     2.121     2.394     95     1.383     1.988     2.300     145     2.259       46     1.570     2.116     2.391     96     1.381     1.987     2.299     146     2.259       47     1.557     2.110     2.388     97     1.381     1.984     2.298     147     2.258       48     1.553     2.106     2.385     98     1.381     1.985     2.297     148     2.258       49     1.545     2.104     2.382     99     1.383     1.982     2.296     149     2.257												
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$												
47     1.557     2.110     2.388     97     1.381     1.984     2.298     147     2.258       48     1.553     2.106     2.385     98     1.381     1.985     2.297     148     2.258       49     1.545     2.104     2.382     99     1.383     1.982     2.296     149     2.257												
48     1.553     2.106     2.385     98     1.381     1.985     2.297     148     2.258       49     1.545     2.104     2.382     99     1.383     1.982     2.296     149     2.257												
49   1.545   2.104   2.382   99     1.383   1.982   2.296   149     2.257												
00    1.004   2.000   2.310    100    1.313   1.700   2.200    100        2.231	50	1.534	2.098	2.379	100	1.373	1.980	2.295	150			2.257

Validation MAE

Validation MAE											
		raining Si			Training Size			Training Size			
Epoch	30%	60%	90%	Epoch	30%	60%	90%	Epoch	30%	60%	90%
1	3.778	3.579	3.535	51	4.318	3.710	3.495	101			3.556
2	3.622	3.499	3.475	52	4.331	3.724	3.503	102			3.553
3	3.574	3.471	3.403	53	4.313	3.697	3.501	103			3.556
4	3.559	3.443	3.367	54	4.307	3.725	3.500	104			3.553
5	3.588	3.412	3.346	55	4.290	3.720	3.504	105			3.562
6	3.547	3.432	3.339	56	4.297	3.722	3.504	106			3.557
7	3.567	3.401	3.343	57	4.320	3.738	3.509	107			3.560
8	3.588	3.407	3.338	58	4.308	3.734	3.509	108			3.560
9	3.595	3.419	3.340	59	4.348	3.739	3.508	109			3.555
10	3.634	3.410	3.354	60	4.336	3.726	3.514	110			3.568
11	3.650	3.430	3.361	61	4.345	3.726	3.510	111			3.558
12	3.694	3.449	3.364	62	4.340	3.735	3.510	112			3.561
13	3.738	3.456	3.371	63	4.350	3.729	3.514	113			3.563
14	3.742	3.461	3.380	64	4.385	3.755	3.518	114			3.562
15	3.744	3.513	3.375	65	4.397	3.737	3.513	115			3.564
16	3.827	3.500	3.382	66	4.361	3.755	3.523	116			3.564
17	3.826	3.499	3.383	67	4.381	3.742	3.519	117			3.566
18	3.832	3.527	3.385	68	4.364	3.734	3.526	118			3.568
19	3.873	3.539	3.390	69	4.386	3.764	3.521	119			3.567
20	3.932	3.523	3.395	70	4.365	3.744	3.521	120			3.563
21	3.924	3.549	3.413	71	4.389	3.743	3.523	121			3.574
22	3.957	3.579	3.410	72	4.376	3.752	3.530	122			3.571
23	3.968	3.563	3.417	73	4.376	3.747	3.526	123			3.568
24	3.966	3.596	3.418	74	4.404	3.779	3.533	124			3.564
25	4.076	3.566	3.421	75	4.384	3.756	3.528	125			3.573
26	4.020	3.620	3.423	76	4.430	3.774	3.531	126			3.566
27	4.068	3.599	3.427	77	4.380	3.753	3.534	127			3.569
28	4.059	3.599	3.438	78	4.392	3.791	3.526	128			3.573
29	4.056	3.601	3.435	79	4.408	3.753	3.533	129			3.572
30	4.080	3.605	3.441	80	4.402	3.801	3.536	130			3.569
31	4.082	3.614	3.448	81	4.440	3.777	3.534	131			3.574
32	4.100	3.632	3.450	82	4.414	3.775	3.532	132			3.570
33	4.119	3.651	3.457	83	4.427	3.783	3.534	133			3.577
34	4.163	3.653	3.455	84	4.419	3.780	3.540	134			3.576
35	4.146	3.654	3.451	85	4.412	3.779	3.537	135			3.571
36	4.138	3.652	3.464	86	4.419	3.775	3.539	136			3.574
37	4.165	3.645	3.462	87	4.412	3.781	3.543	137			3.579
38	4.167	3.659	3.464	88	4.419	3.786	3.541	138			3.576
39	4.205	3.650	3.465	89	4.427	3.785	3.540	139			3.575
40	4.204	3.692	3.469	90	4.435	3.795	3.541	140			3.578
41	4.193	3.667	3.486	91	4.438	3.811	3.544	141			3.575
42	4.221	3.661	3.474	92	4.445	3.784	3.545	142			3.584
43	4.215	3.684	3.475	93	4.442	3.783	3.550	143			3.580
44	4.225	3.679	3.484	94	4.434	3.808	3.547	144			3.581
45	4.224	3.704	3.486	95	4.450	3.789	3.543	145			3.579
46	4.224	3.680	3.486	96	4.450	3.789	3.550	146			3.585
47	4.265	3.724	3.482	97	4.488	3.784	3.548	147			3.586
48	4.255	3.724	3.482	98	4.488	3.784	3.548	147			3.586
49	4.255	3.700	3.491	99	4.457	3.806	3.549	148			3.584
	4.310			100							3.584
50	4.282	3.706	3.490	100	4.487	3.808	3.555	150		l	3.383

Note: The lowest validation MAE for each training size is underlined.



For a pair of training and validation sets, the validation MAEs are computed after each epoch is finished, but the training MAEs are taken directly from keras.models.Model.fit() which is the mean of all MAEs calculated after each batch during an epoch of training. Although the sizes of the training set across different pairs are identical, the sizes of validation sets are not. Therefore, validation MAE of each pair is multiplied by a constant  $c_i$  before summation where  $c_i = size(val\_set_i)/30,000$  and i is the index of the pair, which is just a linear combination of constants  $c_i$  and the absolute error matrix and can be computed by finding

$$\begin{bmatrix} MAE_1 & MAE_2 & \dots & MAE_n \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & \dots & c_p \end{bmatrix} \times \begin{bmatrix} E_1^{(1)} & E_2^{(1)} & \dots & E_n^{(1)} \\ E_1^{(2)} & E_2^{(2)} & \dots & E_n^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ E_1^{(p)} & E_2^{(p)} & \dots & E_n^{(p)} \end{bmatrix}$$

where p denotes the total number of pairs, n denotes the total number of epochs,  $E_j^{(i)}$  denotes the absolute error of pair i at epoch j.

# 2.3 Ternary Comparison

Proportion of each ternary with each training size's optimal rank or epoch.

	Ternary Proportion					
Model	a	b	c			
NMF (90%)	55.300%	29.640%	15.060%			
NMF (60%)	54.523%	29.647%	15.830%			
NMF (30%)	51.253%	30.703%	18.043%			
MLP (90%)	54.813%	29.333%	15.853%			
MLP (60%)	53.760%	29.967%	16.273%			
MLP (30%)	52.373%	29.280%	18.347%			
Total Average	14.257%	11.180%	74.563%			
Uniformly Random	12.720%	13.560%	73.720%			
User Average	8.463%	12.283%	79.253%			

Note: a:  $MAE \in (-\infty, 3)$ ; b:  $MAE \in [3, 6)$ ; c:  $MAE \in [6, \infty)$ .

# References

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