# 1 Towards the lattice simulation

With the current status of the action (??) we could almost start to discretize the operators and fields, at least for the bosonic part this would not be a problem. For the fermions however this is not so straight forward. In order to include the fermionic contribution into the weight factor of the path integral like explained in section ?? one needs to integrate out the GRASSMANN variables to result into a PFAFFIAN or determinant of a fermionic operator. As presented in Appendix ?? this is only possible if the fermions appearing are of quadratic order. But in the fluctuation action (??) also quartic contributions of fermions appear, which have to be linearized with help of a HUBBARD-STATONOVICH transformation.[1]

## 1.1 Linearization of fermionic contributions

The only quartic contributions are coming from the $\eta$ fields and we can write these contributions to the action as

$$S^{\mathrm{F}}_{\mathrm{quart}}[\eta_i, \eta^i] = g \int \mathrm{d}t\mathrm{d}s \left[ -\frac{1}{z^2} \left(\eta^2\right)^2 + \left( \frac{i}{z^2} z_N \eta_i \left(\rho^{MN}\right)^i{}_j \eta^j \right)^2 \right] . \tag{1.1}$$

In the path integral representation the euclidean action contributes in an exponential $e^{-S_{\mathrm{E}}}$. By performing a naive HUBBARD-STATONOVICH transformation to this exponential we can reduce the four-fermion contributions to quadratic YUKAWA terms whereas we have to introduce 7 bosonic real auxiliary fields $\phi$ and $\phi^N$

$$\begin{aligned} &\exp\left\{ -g \int \mathrm{d}t\mathrm{d}s \left[ -\tfrac{1}{z^2} \left(\eta^2\right)^2 + \left( \tfrac{i}{z^2} z_N \eta_i \left(\rho^{MN}\right)^i{}_j \eta^j \right)^2 \right] \right\} \qquad (1.2) \\ &\quad \sim \int \mathcal{D}\phi \mathcal{D}\phi^{\mathcal{M}} \; \exp\left\{ -g \int \mathrm{d}t\mathrm{d}s \left[ \tfrac{1}{2}\phi^2 + \tfrac{\sqrt{2}}{z}\phi\eta^2 + \tfrac{1}{2}(\phi^M)^2 \right.\right. \\ &\qquad\qquad \left.\left. - i\tfrac{\sqrt{2}}{z^4}\phi_M \left( i\eta_i (\rho^{MN})^i{}_j \eta^j \right) z_N \right] \right\}. \end{aligned}$$

[1]See Appendix ?? for details