Appendix

A Grassmann Numbers

A.1 Grassmann Algebra

When we start considering a QFT including fermions, we have to conclude that our canonical quantisation rules for bosons no longer apply. Instead of a commutation relation

$$[\phi(x), \Pi(y)] = i\hbar\delta(x - y) \tag{A.1}$$

for bosonic fields, one has to introduce anticommutation relations for fermionic ones [1]

$$\{\psi(x), \pi(y)\} = i\hbar\delta(x - y), \tag{A.2}$$

which in the classical limit $\hbar \to 0$ is leading to objects that behave like anticommuting numbers

$$\psi\pi = -\pi\psi, \tag{A.3}$$

which seems rather strange at first. But the concept of anticommuting numbers, so called Grassmann variables, has proved quite useful in the path integral framework to represent the algebra of fermionic position and momentum observables as operators on a space of functions. In the following we want to define these objects and study their properties.

We can identify Grassmann numbers with elements of the exterior algebra $\Lambda^1(V)$ over a vector space V over a field K. According to the dimension of V the algebra has a finite collection of generators $\xi_1, \ldots, \xi_N$ with $N = \dim V \geq 1$ or infinitely many, if the vector space is infinite dimensional. With the exterior product there exists an associative connection on the algebra, which is linear in scalar multiplication and satisfies

$$\xi_i \wedge \xi_i = 0. \tag{A.4}$$

Thereby follows the demanded anticommutativity for the fermionic Grassmann variables

$$\xi_i \wedge \xi_j = -\xi_j \wedge \xi_i. \tag{A.5}$$

In general we can write the exterior algebra as a direct sum of subalgebras

$$\Lambda(V) = \bigoplus_{m=0}^{N} \Lambda^m(V), \tag{A.6}$$

where $\Lambda^m(V)$ is the quotient algebra of the tensor algebra $T^m(V)$ and the two-sided ideal $I^m(V)$ and $\Lambda^0(V) = K$. For an $a \in \Lambda^k(V)$ and $b \in \Lambda^l(V)$ we have a graded commutative exterior product

$$a \wedge b = (-1)^{kl} b \wedge a. \tag{A.7}$$

Therefore if we have an $a = \xi_i \wedge \xi_j$ and $b = \xi_m \wedge \xi_n$, then we get

$$a \wedge b = b \wedge a \tag{A.8}$$

and a and b behave like bosonic quantities. The highest alternating tensor one can create is

$$\xi_1 \wedge \xi_2 \wedge \ldots \wedge \xi_N \in \Lambda^N(V), \tag{A.9}$$

and with (A.4) it applies for all higher tensor spaces that $\Lambda^k(V) = 0,\ \forall\, k > N$. From now on we want to abbreviate the wedge notation simply by

$$\xi_i \xi_j \equiv \xi_i \wedge \xi_j. \tag{A.10}$$

So any analytic function of some real quantities $x_i \in \mathbb{R}$ and grassmanian generators $\xi_j \in \Lambda(V)$ can be represented by finitely many terms

$$f(x_1, \ldots, x_n, \xi_1, \ldots, \xi_N) = f_0 + f_1\, \xi_1 + \ldots + f_{12}\, \xi_1 \xi_2 + \ldots + f_{1 \ldots N}\, \xi_1 \cdots \xi_N, \tag{A.11}$$

where the coefficients are functions of the real quantities. In scope of this thesis we have to deal with vectors of four complex Grassmann variables and adapt our notation to the one used in [2]

$$\eta_i = \frac{1}{\sqrt{2}} \left(\xi_i^{\mathrm{R}} + i\, \xi_i^{\mathrm{I}} \right), \qquad \text{with} \quad \xi_i^{\mathrm{R}}, \xi_i^{\mathrm{I}} \in \Lambda^1(V), \quad i = (1, \ldots, 4). \tag{A.12}$$

We now define the Grassmann variables with an upper index as the complex conjugate of the ones with lower index $\eta^i \equiv (\eta_i)^\dagger = (\eta_i)^*$. Complex conjugation of two real Grassmann numbers is defined to include a change of position

$$(\xi_1 \xi_2)^* \equiv \xi_2^* \xi_1^* = -\xi_1 \xi_2. \tag{A.13}$$

They therefore behave like a formally purely imaginary quantity, whereas

$$(i \xi_1 \xi_2)^* = i \xi_1 \xi_2 \tag{A.14}$$

behaves like a formally real quantity.

A.2 Grassmann Analysis

Now we want to see how we can differentiate and integrate Grassmann variables. We define the derivative to be

$$\frac{\partial \xi_m}{\partial \xi_n} \equiv \delta_{mn}. \tag{A.15}$$

Following [1] and [3] we define the product rule to satisfy

$$\frac{\partial}{\partial \xi_n}\left(\xi_{m_1}\xi_{m_2}\cdots\xi_{m_r}\right) \equiv \delta_{m_1 n}\,\xi_{m_2}\cdots\xi_{m_r} - \delta_{m_2 n}\,\xi_{m_1}\xi_{m_3}\cdots\xi_{m_r} + \cdots + (-1)^{r-1}\delta_{m_r n}\,\xi_{m_1}\cdots\xi_{m_r}. \tag{A.16}$$

The tangent vectors are also elements of the exterior algebra and satisfy the same anticommutation rules

$$\frac{\partial}{\partial \xi_i}\frac{\partial}{\partial \xi_j} = -\frac{\partial}{\partial \xi_j}\frac{\partial}{\partial \xi_i}. \tag{A.17}$$

So if we perform a coordinate transformation $\xi_i = M_{ij}\,\theta_j$, an n-form is transforming according to the alternating properties of forms like

$$\frac{\partial}{\partial \xi_1}\cdots\frac{\partial}{\partial \xi_n} = \det\left(M^{-1}\right)\frac{\partial}{\partial \theta_1}\cdots\frac{\partial}{\partial \theta_n}, \tag{A.18}$$

where the matrix M is associated with the Jacobian

$$M_{ij} = \frac{\partial \xi_i}{\partial \theta_j}. \tag{A.19}$$

For an integral $\mathcal{I}[f]$ of a function $f(\xi)$ it holds true that

$$\frac{\partial}{\partial \xi}\mathcal{I}[f] = 0, \tag{A.20}$$

since the integral is independent of the integration variable. With (A.4) it is also true that

$$\frac{\partial}{\partial \xi}\frac{\partial}{\partial \xi} = 0 \tag{A.21}$$

and therefore one can identify the integration on Grassmann numbers with the differentiation up to some normalization constant C

$$\mathcal{I}[f] = \int \mathrm{d}\xi\; f(\xi) = C\frac{\partial}{\partial \xi}f(\xi). \tag{A.22}$$

So we end up with the following integration rules, if we define $C \equiv 1$:

$$\int \mathrm{d}\xi\;\xi = \frac{\partial}{\partial \xi}\xi = 1, \qquad \int \mathrm{d}\xi\; 1 = \frac{\partial}{\partial \xi}1 = 0. \tag{A.23}$$

We now want to perform an integration over the complex Grassmann variables η and $\eta^\dagger$. Therefore we want to investigate an integration over the function

$$e^{-a\eta\eta^\dagger}. \tag{A.24}$$

With

$$\eta\eta^\dagger = \frac{1}{2}\left(\xi_{\mathrm{R}} + i\,\xi_{\mathrm{I}}\right)\left(\xi_{\mathrm{R}} - i\,\xi_{\mathrm{I}}\right) = -i\,\xi_{\mathrm{R}}\xi_{\mathrm{I}} \tag{A.25}$$

and (A.18) we can transform the integral using that the determinant of the Jacobian is $\det J = i$ and find

$$\begin{aligned}\int \mathrm{d}\eta \mathrm{d}\eta^\dagger \; e^{-a\eta\eta^\dagger} &= i \int \mathrm{d}\xi_\mathrm{R} \mathrm{d}\xi_\mathrm{I} \; e^{ia\xi_\mathrm{R}\xi_\mathrm{I}} \\ &= i \int \mathrm{d}\xi_\mathrm{R} \mathrm{d}\xi_\mathrm{I} \; (1 + i\, a\xi_\mathrm{R}\xi_\mathrm{I}) \\ &= a \int \mathrm{d}\xi_\mathrm{R} \mathrm{d}\xi_\mathrm{I} \; \xi_\mathrm{I}\xi_\mathrm{R} = a\end{aligned} \tag{A.26}$$

We get the same result by integrating over two real Grassmann variables ξ_1 and ξ_2

$$\int \mathrm{d}\xi_1 \mathrm{d}\xi_2 \; e^{-a\xi_1\xi_2} = a. \tag{A.27}$$

By rewriting it with help of the antisymmetric matrix

$$A = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix} \tag{A.28}$$

we get

$$\int \mathrm{d}\xi_1 \mathrm{d}\xi_2 \; e^{-\frac{1}{2}\xi_i A_{ij}\xi_j} = \mathrm{Pf}\,(A)\,, \tag{A.29}$$

where $\mathrm{Pf}\,(A)$ is the Pfaffian of A, which is defined by the factorisation of the determinant in the following way

$$\mathrm{Pf}\,(A)^2 = \det(A) \tag{A.30}$$

This relation holds true for an even number of arbitrary many Grassmann generators. Its is therefore possible to integrate out Grassmann variables and be left with calculating a determinant or Pfaffian, which comes in quite essential when dealing with Grassmann integrals numerically.

A.3 Hubbard-Stratonovich Transformation

We have now seen how to deal with bilinear exponentials. But in many cases one also has to deal with higher forms than bilinears. In principle one can expand any function into finitely many terms, like we have seen earlier, and then perform the integration rules, which is but rather ugly and proved not very efficient in numerical simulations. But there is a method to transform an exponential with 4-forms into one with bilinears, by performing an integration over additional bosonic auxiliary fields. This is called a Hubbard-Stratonovich transformation. Assuming that we have a finite number of complex Grassmann variables η_i, we can see that the object $\eta^2 \equiv \eta^i \eta_i$ transforms like a formally real quantity

$$\left(\eta^i \eta_i\right)^* = \left(\eta_i\right)^* \left(\eta^i\right)^* = \eta^i \eta_i. \tag{A.31}$$

Therefore $\left(\eta^2\right)^2$ is a formally positive real value and we can apply the identity

$$e^{\frac{(\eta^2)^2}{4a}} = \sqrt{\frac{a}{\pi}} \int \mathrm{d}\phi \; e^{-a\phi^2 + \eta^2 \phi}, \tag{A.32}$$

which one can proof simply by square addition and performing a Gaussian integral. If we see this identity in the context of the path integral framework, we can say, that a new bosonic auxiliary field ϕ is introduced for the specific space time point, where the integration took place. Since we can do this transformation at every space time point, this leads to a whole path integral over ϕ. Now we also have to be aware of what happens if the exponent on the left side of (A.32) is negative

$$e^{-\frac{(\eta^2)^2}{4a}} = e^{\frac{(i\eta^2)^2}{4a}} = \sqrt{\frac{a}{\pi}} \int \mathrm{d}\phi \; e^{-a\phi^2 + i\eta^2 \phi}. \tag{A.33}$$

We can see that this introduces an imaginary term in the exponent and therefore a high oscillatory function in the integral. Analytically this term would not bother us, but since we want to perform a numerical calculation something like this often results into a numerical sign problem, discussed in section **??**.

B $\quad$ AdS$_5 \times$ S^5 spacetime

The $AdS_5 \times S^5$ space is in the central focus of the gauge/gravity duality and should be concerned more deeply. It is a direct product of five dimensional Anti-de Sitter (AdS) space and a five dimensional compact sphere. Both are maximally symmetric spaces and therefore inherit the isometry groups $SO(2,4)$ in case of AdS_5 and respectively $SO(6)$ for S^5 [4]. This is an important fact for the AdS/CFT correspondence since the direct product of these groups has the same amount of degrees of freedom as the superconformal group $SO(2,4) \times SO(6) = SU(2,2|4)$ as the undelying symmetry group of $\mathcal{N} = 4$ super Yang-Mills theory in four dimensional Minkowski space.

B.1 $\quad$ AdS$_5$ space

Since the construction of a sphere is rather simple, we focus on the Anti-de Sitter space. AdS_5 is a hyperboloid with constant negative curvature, that can be embedded in six dimensional Minkowski spacetime $X = (X^0, X^1, \ldots, X^5) \in \mathbb{R}^{4,2}$, with metric $\tilde{\eta} = \mathrm{diag}(-,+,+,+,+,-)$, so that

$$\mathrm{d}s^2 = -\left(\mathrm{d}X^0\right)^2 + \left(\mathrm{d}X^1\right)^2 + \cdots + \left(\mathrm{d}X^4\right)^2 - \left(\mathrm{d}X^5\right)^2 = \tilde{\eta}_{MN} \mathrm{d}X^M \mathrm{d}X^N, \tag{B.1}$$

where $M, N \in 0, \ldots, 5$. AdS_5 is then given by the hypersurface

$$\tilde{\eta}_{MN} X^M X^N = -\left(X^0\right)^2 + \left(X^1\right)^2 + \cdots + \left(X^4\right)^2 - \left(X^5\right)^2 = -R^2, \qquad \text{(B.2)}$$

where R is the radius of curvature of the AdS_5 space, see also Figure B.1

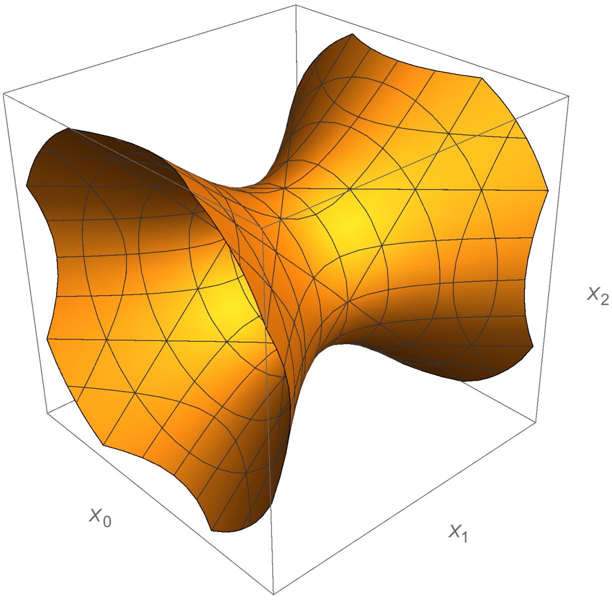

Figure B.1 Embedding of AdS_2 in $\mathbb{R}^3$ as a hypersurface given by the equation $-(X_0)^2 + (X_1)^2 - (X_3)^2 = -1$.

for an embedding of AdS_2 in $\mathbb{R}^3$. For large X^M the hyperboloid approaches the light-cone of the MINKOWSKI space $\mathbb{R}^{4,2}$, given by

$$\tilde{\eta}_{MN} X^M X^N = 0. \qquad \text{(B.3)}$$

We therefore can define the 'boundary' ∂AdS_5 of Anti-de Sitter space by the set of all lines on the light-cone (B.3) originating from $0 \in \mathbb{R}^{4,2}$. For a point $X \neq 0$ in AdS_5 close to the boundary and therefore satisfying (B.3) we can define (u, v) by

$$u = X^5 + X^4, \qquad v = X^5 - X^4, \qquad \text{(B.4)}$$

so we can rewrite (B.3) as

$$uv = \eta_{\mu\nu} X^\mu X^\nu, \qquad \text{(B.5)}$$

with $\mu, \nu \in 0, 1, 2, 3$ and $(\eta_{\mu\nu}) = \mathrm{diag}(-,+,+,+)$. Whenever $v \neq 0$ we can rescale the coordinates to set $v = 1$ and solve for u. Therefore one is left with a four dimensional MINKOWSKI space $\mathbb{R}^{3,1}$. Points with $v = 0$ are "points at infinity" added to four dimensional MINKOWSKI space. This makes ∂AdS_5 a conformal compactification of four dimensional MINKOWSKI space. According to Maldacena [5] the correspondence between a $\mathcal{N} = 4$ theory on $\mathbb{R}^{3,1}$ and Type IIB on $AdS_5 \times S^5$ therefore expresses a string theory on $AdS_5 \times S^5$ in terms of a theory on the boundary and thus is referred to as "holographic" [6].

B.2 Poincaré patch

Let us now introduce a parametrisation of the hyperboloid (B.2) by the following coordinates $x^\mu \in \mathbb{R}$, for $\mu \in 0,1,2,3$ and $z \in \mathbb{R}_+$. The parametrisation in these coordinates is given by

$$\begin{aligned} X^0 &= \frac{z}{2}\left(1+\frac{1}{z^2}\left(x_\mu x^\mu + R^2\right)\right), \\ X^i &= \frac{R}{z}x^i, \quad i \in 1,2,3, \\ X^4 &= \frac{z}{2}\left(1+\frac{1}{z^2}\left(x_\mu x^\mu - R^2\right)\right), \\ X^5 &= \frac{R}{z}x^0, \end{aligned} \tag{B.6}$$

with $x_\mu x^\mu = \eta_{\mu\nu}x^\mu x^\nu$ and $(\eta_{\mu\nu}) = \mathrm{diag}(-,+,+,+)$. These local coordinates are called Poincaré patch. The metric of AdS_5 in the Poincaré patch reads

$$\mathrm{d}s^2 = \frac{R^2}{z^2}\left(\mathrm{d}z^2 + \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu\right). \tag{B.7}$$

For the whole $AdS_5 \times S^5$ space in Poincaré patch we find

$$\mathrm{d}s^2 = \frac{R^2}{z^2}\left(\mathrm{d}z^2 + \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu\right) + \mathrm{d}u^I \mathrm{d}u^I, \tag{B.8}$$

where u^I $(I = 1,\dots,6)$ are coordinates on S^5 that satisfy $u^I u^I = R^2$. For $z \to 0$ we approach the boundary of $AdS_5 \times S^5$ and we can see from the metric that the contribution of the sphere becomes neglectable close to the boundary. If we include the infinite regime as $\partial(AdS_5 \times S^5)$ where $z = 0$ we can convince ourselves that the metric is conformally equivalent to 4d Minkowski space.

C SO(6) matrices

The matrices ρ^M_{ij} appearing in the action (**??**) are the off-diagonal blocks of $SO(6)$ Dirac matrices γ^M in chiral representation[1]

$$\gamma^M \equiv \begin{pmatrix} 0 & \rho^\dagger_M \\ \rho^M & 0 \end{pmatrix} = \begin{pmatrix} 0 & (\rho^M)^{ij} \\ (\rho^M)_{ij} & 0 \end{pmatrix}. \tag{C.1}$$

The ρ^M_{ij} shall all skew symmetric and we define the ones with the upper indices to satisfy $(\rho^M)^{ij} \equiv (\rho^M_{ij})^\dagger$. We can therefore state the following properties

$$\rho^M_{ij} = -\rho^M_{ji}, \qquad (\rho^M)^{ij} = -(\rho^M_{ij})^*, \tag{C.2}$$

[1]The upper or lower placement of the index M on the block matrices has no meaning and is only changed for the purpose of readability.

and we chose to use the following representation

$$\rho^1_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad \rho^2_{ij} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad \rho^3_{ij} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$\rho^4_{ij} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad \rho^5_{ij} = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}, \quad \rho^6_{ij} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{C.3}$$

From the Clifford algebra of the Dirac matrices $\{\gamma^M, \gamma^N\} = 2\delta^{MN}\mathbb{1}_8$ we can derive the relation

$$(\rho^M)^{il}(\rho^N)_{lj} + (\rho^N)^{il}(\rho^M)_{lj} = 2\delta^{MN}\delta^i_j. \tag{C.4}$$

The generators of the $SO(6)$ can be built by

$$\left(\rho^{MN}\right)^i{}_j = \frac{1}{2}\left[\left(\rho^M\right)^{il}\left(\rho^N\right)_{lj} - \left(\rho^N\right)^{il}\left(\rho^M\right)_{lj}\right]. \tag{C.5}$$

Further relations and identities are

$$\left(\rho^{MN}\right)^i{}_j = \left(\rho^{MN}\right)^{*j}_{i}\,, \qquad \left(\rho^{MN}\right)^i{}_j = \left(\rho^{NM}\right)_j{}^i\,, \tag{C.6}$$

$$(\rho^M)^{im}(\rho^M)^{kn} = 2\epsilon^{imkn}\,, \qquad (\rho^M)^{im}(\rho^M)_{nj} = 2\left(\delta^i_j\delta^m_n - \delta^i_n\delta^m_j\right)\,, \tag{C.7}$$

$$\begin{aligned} &\epsilon^{imkn}(\rho^M)_{mj}(\rho^L)_{nl} + \epsilon_{mjnl}(\rho^M)^{im}(\rho^L)^{kn} \\ &= (\rho^{\{M})^{ik}(\rho^{L\}})^{jl} + \delta^k_j(\rho^L)^{im}(\rho^M)_{ml} + \delta^i_l(\rho^M)^{km}(\rho^L)_{mj} \\ &\quad + \delta^{ML}\left(-4\delta^i_l\delta^k_j + 2\delta^i_j\delta^k_l\right)\,, \end{aligned} \tag{C.8}$$

$$-\,(\rho^{MN})^i{}_j(\rho^{ML})^k{}_l n_N n_L = -2(\rho^N)^{ik}(\rho^L)_{jl}n_N n_L - \delta^i_j\delta^k_l + 2\delta^i_l\delta^k_j\,. \tag{C.9}$$

D Discrete Fourier Transform

To perform a Fourier transform on the lattice one needs to discretize it to deal with a finite sequence of N complex numbers $x_0, x_1, \ldots, x_{N-1}$. We define the discrete Fourier transform X_k to be a vector in the base of roots of unit with components x_n as follows:

$$X_k = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x_n\, e^{-2\pi i k n/N} \qquad k \in \mathbb{Z}. \tag{D.1}$$

We can limit the domain of k to a finite set, because the exponential is periodic in k. In the following we want to stick to the domain $k \in$

$\left[-\frac{N}{2}+1,\ldots,\frac{N}{2}\right]$ and restrict us to even N. The inverse transform can be defined to be

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=-N/2+1}^{N/2} X_k \, e^{2\pi i n k/N}, \tag{D.2}$$

where due to periodicy n is in the domain $[0,\ldots,N-1]$ like defined in the beginning. If we now insert (D.1) into (D.2) we end up with

$$x_n = \frac{1}{N} \sum_{k=-N/2+1}^{N/2} \left(\sum_{m=0}^{N-1} x_m e^{-2\pi i k m/N} \right) e^{2\pi i k n/N} \tag{D.3}$$

$$= \frac{1}{N} \sum_{k',m=0}^{N-1} x_m e^{2\pi i k'(n-m)/N}, \tag{D.4}$$

where we shifted the summation for k' to be in the same domain as m. This is now only equal to x_n if the following important relation holds true

$$\frac{1}{N} \sum_{k'=0}^{N-1} e^{2\pi i k'(n-m)/N} = \delta_{m,n}. \tag{D.5}$$

We can prove this easily. For $n = m$ this is trivial and for $n \neq m$ we make use of the geometric series

$$\sum_{k'=0}^{N-1} \left(e^{2\pi i c/N}\right)^{k'} = \frac{1-e^{2\pi i c}}{1-e^{2\pi i c/N}} = 0, \qquad |c| = |n-m| \in [1,\ldots,N-1]. \tag{D.6}$$

Now we want to apply the Fourier transform on the two dimensional lattice used throughout this thesis. We defined the lattice to be

$$\Lambda = \{ n = (n_0, n_1) \, | n_0 = 1,\ldots,N_0 \, ; \; n_1 = 1,\ldots,N_1 \}, \tag{D.7}$$

with $N_0 = T$ and $N_1 = L$. Therefore the total number of lattice points is given by

$$|\Lambda| \equiv TL. \tag{D.8}$$

Following [7] we now want to calculate the discrete Fourier transform $\tilde{f}(p)$ of a function $f(n)$. Here for $f(n)$ we impose toroidal boundary conditions

$$f(n + \hat{\imath} N_i) = e^{2\pi i \theta_i} f(n), \tag{D.9}$$

where $\hat{\imath}$ is a unit vector in the i-direction and $\theta_i = 0$ corresponds to periodic and $\theta_i = 1/2$ to anti-periodic boundary conditions. The discrete momentum space corresponding to these boundary conditions is given by

$$\widetilde{\Lambda} = \left\{ p = (p_0, p_1) | \, p_i = \frac{2\pi}{aN_i}(k_i + \theta_i), \; k_i = -\frac{N_i}{2}+1,\ldots,\frac{N_i}{2} \right\}. \tag{D.10}$$

With (D.1) and (D.2) we can express the the Fourier transform as

$$\tilde{f}(p) = \frac{1}{\sqrt{|\Lambda|}} \sum_{n\in\Lambda} f(n) \, e^{-ip\cdot na} \tag{D.11}$$

and for the inverse transform we find

$$f(n) = \frac{1}{\sqrt{|\Lambda|}} \sum_{p \in \widetilde{\Lambda}} \tilde{f}(p)\, e^{ip \cdot na}. \tag{D.12}$$

Here again the important relations hold

$$\frac{1}{|\Lambda|} \sum_{n \in \Lambda} \exp(i(p - p') \cdot na) = \delta(p - p') \equiv \delta_{k_0, k_0'} \delta_{k_1, k_1'} \tag{D.13}$$

$$\frac{1}{|\Lambda|} \sum_{p \in \widetilde{\Lambda}} \exp(ip \cdot (n - n')a) = \delta(n - n') \equiv \delta_{n_0, n_0'} \delta_{n_1, n_1'}. \tag{D.14}$$

References

[1] P. Cartier, C. DeWitt-Morette, M. Ihl and C. Saemann, *"Supermanifolds: Application to supersymmetry"*, math-ph/0202026.

[2] S. Giombi, R. Ricci, R. Roiban, A. A. Tseytlin and C. Vergu, *"Quantum AdS(5) x S5 superstring in the AdS light-cone gauge"*, JHEP 1003, 003 (2010), arXiv:0912.5105.

[3] F. A. Berezin and A. A. Kirillov (eds.), *"Introduction to Superanalysis"*, 1 edition, Springer Netherlands (1987).

[4] M. Ammon and J. Erdmenger, *"Gauge/gravity duality"*, Cambridge Univ. Pr. (2015), Cambridge, UK.

[5] J. M. Maldacena, *"The Large N limit of superconformal field theories and supergravity"*, Int. J. Theor. Phys. 38, 1113 (1999), hep-th/9711200, [Adv. Theor. Math. Phys.2,231(1998)].

[6] E. Witten, *"Anti-de Sitter space and holography"*, Adv. Theor. Math. Phys. 2, 253 (1998), hep-th/9802150.

[7] C. Gattringer and C. Lang, *"Quantum Chromodynamics on the Lattice: An Introductory Presentation"*, Springer Berlin Heidelberg (2009).