GPU Programming

(in Cuda)

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NUCAR

Session 11

Outline

- Applications
  - Parallel Reduction
  - Prefix Sum (Scan)
  - Histogram
  - Convolution

Parallel Reduction

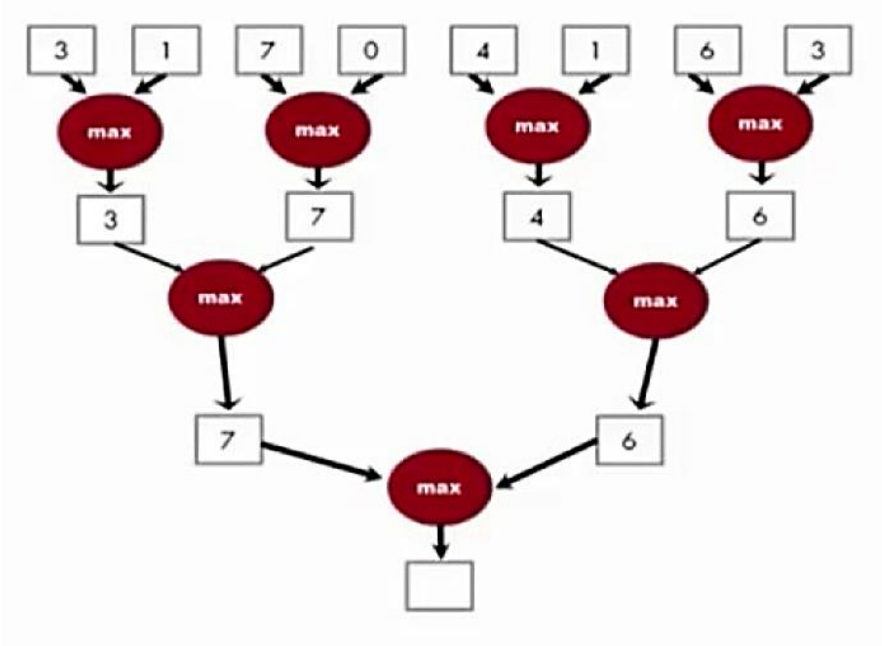
- A popular class of computation
- Goal: To master the concept of control divergence through reduction trees (how to be work efficient).

Parallel Reduction

- Example
- Calculate Max Value from vector:

Parallel Reduction

- Example
- Calculate Max Value from vector:
- A parallel reduction tree algorithm performs N-1 operations in log(N) steps



Parallel Reduction

- A commonly used strategy for processing large input data sets
  - There is no required order of processing elements in a data set (associative and commutative)
  - Partition the data set into smaller chunks
  - Have each thread to process a chunk
  - Use a reduction tree to summarize the results from each chunk into the final answer
- Google and Hadoop MapReduce frameworks are examples of this pattern
- We will focus on the reduction tree step for now.

Parallel Reduction

- Example: Handling privatization
  - Multiple threads write into an output location
  - Replicate the output location so that each thread has a private output location
  - Use a reduction tree to combine the values of private locations into the original output location

Parallel Reduction

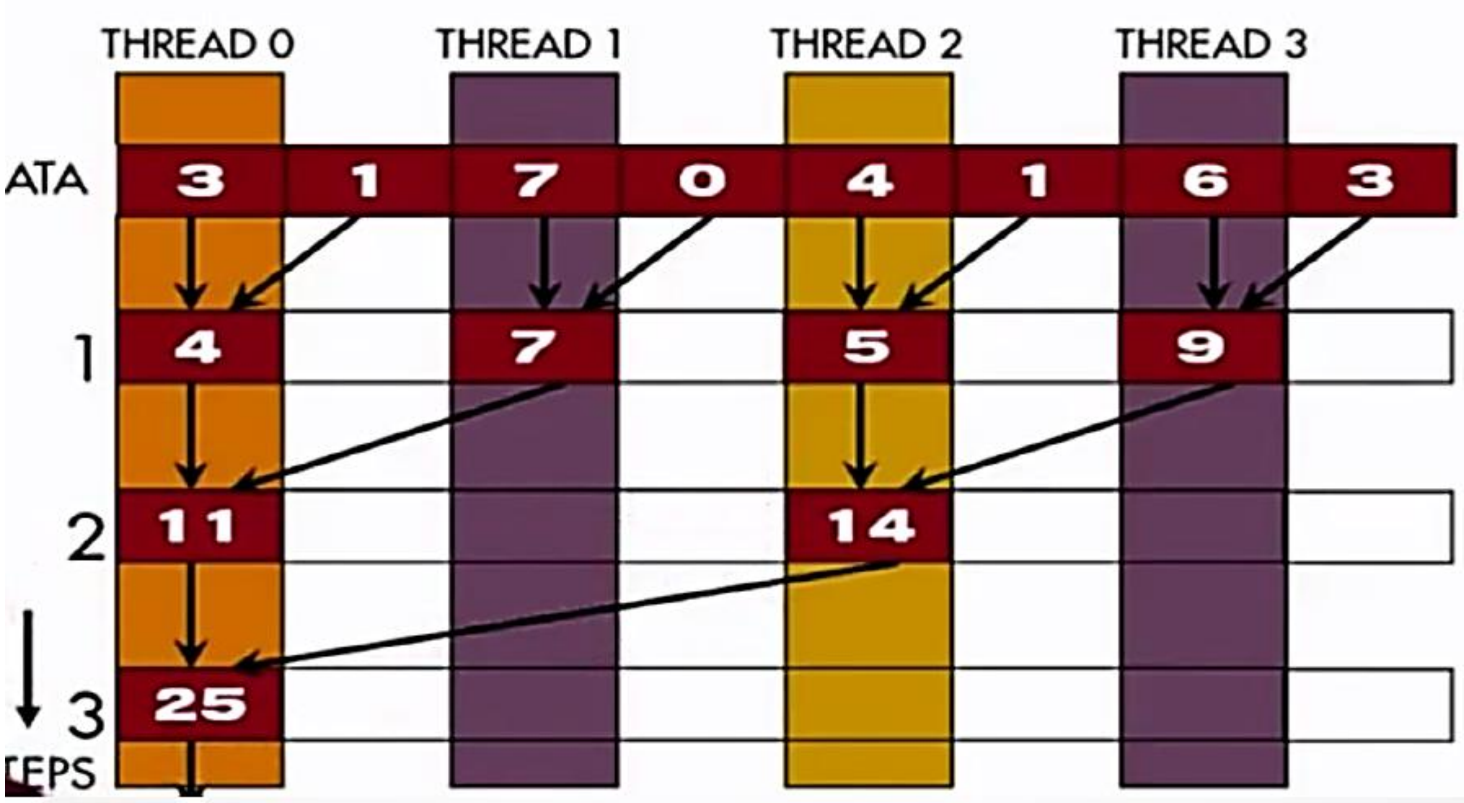
- Use: Summarize a set of input values into one value using a “reduction operation”
  - Max
  - Min
  - Sum
  - Product
- You need to initialize the result as an identity value for the reduction operation
  - Smallest possible value for max reduction
  - Largest possible value for min reduction
  - 0 for sum reduction
  - 1 for product reduction

Parallel Reduction

- For N input values, the reduction tree performs:
  - (1/2)N + (1/4)N + (1/8)N + … (1/N) = N-1 operations
  - In log(N) steps – 1 000 000 input values take 20 steps
    - Assuming that we have enough execution resources
  - Average parallelism (N-1)/log(N)
    - For N = 1 000 000 average parallelism is 50 000
    - However, peak resource requirement is 500 000!
- This is a work-efficient parallel algorithm
  - The amount of work done is comparable to sequential
  - Many parallel algorithms are not work efficient

Parallel Reduction

- An example



Parallel Reduction

- An example
  - Each thread block takes 2*BlockDim input elements
  - Each threads loads 2 elements into shared memory

```
__ shared __ float partialSum[2*BLOCK _ SIZE];

unsigned int t = threadIdx.x;
Unsigned int start = 2*blockIdx.x*blockDim.x;
partialSum[t] = input[start + t];
partialSum[blockDim+t] = input[start+ blockDim.x+t];
```

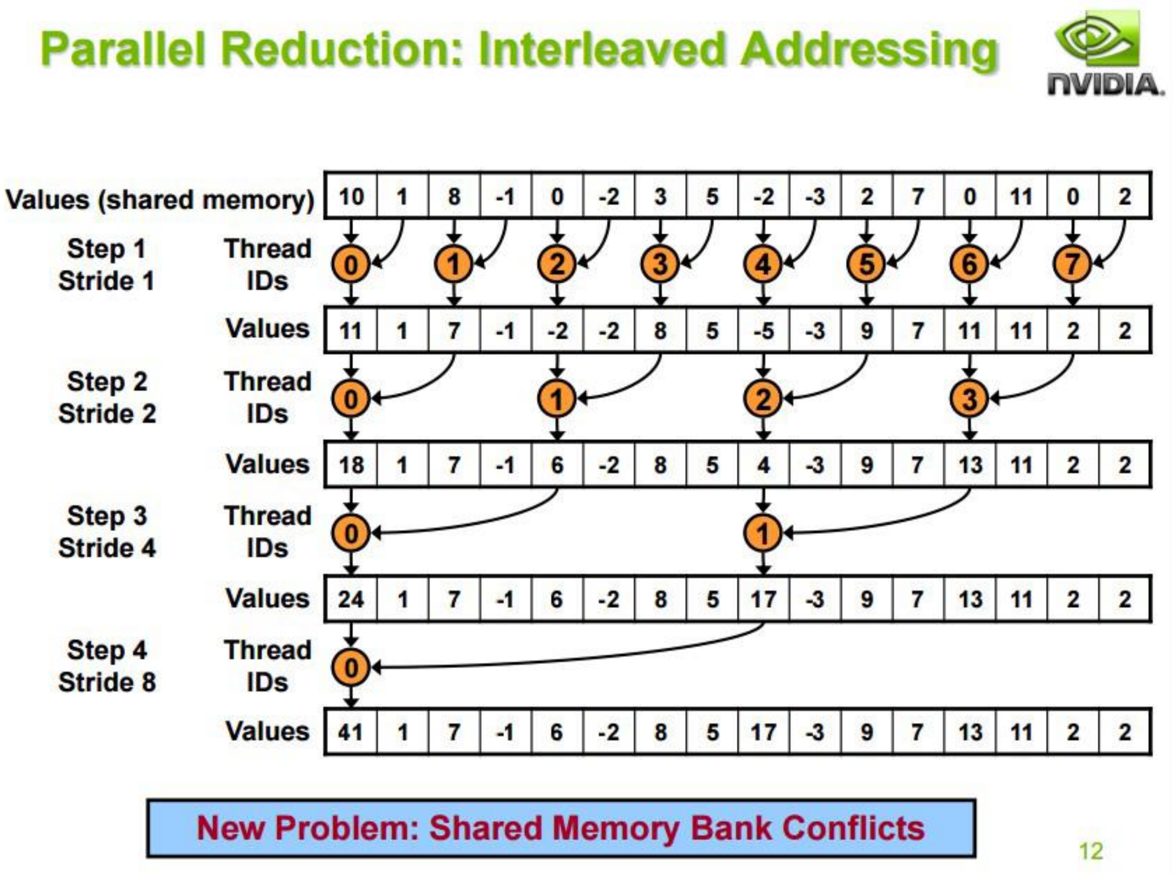
Parallel Reduction

```
for (unsigned int stride = 1;
    stride <= blockDim.x;  stride *= 2)
{
   __syncthreads();
  if (t % stride == 0)
     partialSum[2*t]+= partialSum[2*t+stride];
}
```

- Why do we need syncthreads()?

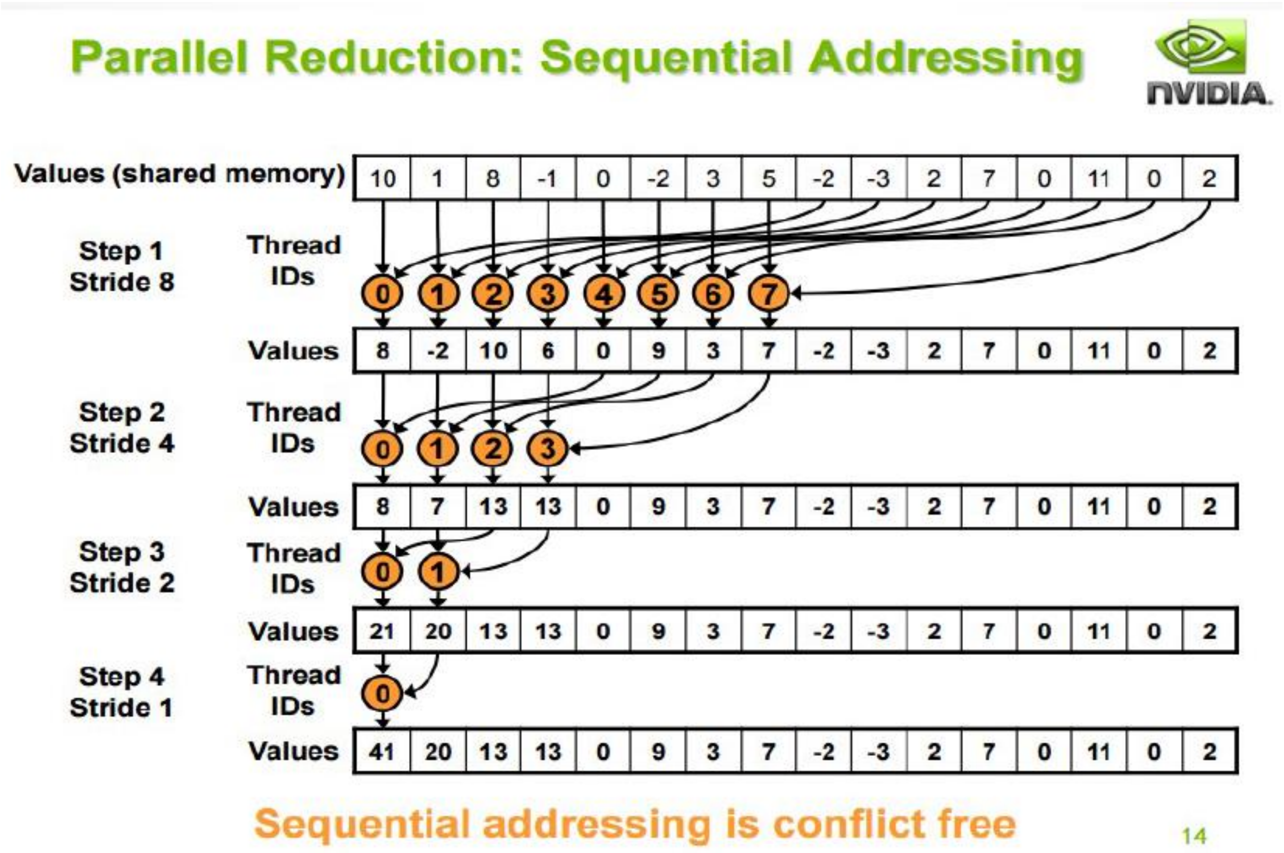
- Thread 0 in each thread block write the sum of the thread block in partialSum[0] into a vector indexed by the blockIdx.x
- There can be a large number of such sums if the original vector is very large
  - The host code may iterate and launch another kernel
- If there are only a small number of sums, the host can simply transfer the data back and add them together

Parallel Reduction



Parallel Reduction

- A better reduction?



Parallel Reduction

- There's a problem

```
for (unsigned int s=blockDim.x/2; s>0; s>>=1) {
    if (tid < s) {
        sdata[tid] += sdata[tid + s];
    }
    __syncthreads();
}
```

Parallel Reduction

- There's a problem
  - Idle threads!
  - Half of the threads are idle on first loop iteration!
  - This is wasteful….

```
for (unsigned int s=blockDim.x/2; s>0; s>>=1) {
    if (tid < s) {
        sdata[tid] += sdata[tid + s];
    }
    __syncthreads();
}
```

Parallel Reduction

- Halve the number of blocks, and replace single load:

```
// each thread loads one element from global to shared mem
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x*blockDim.x + threadIdx.x;
sdata[tid] = g_idata[i];
__syncthreads();
```

- With two loads and first add of the reduction:

```
// perform first level of reduction,
// reading from global memory, writing to shared memory
unsigned int tid = threadIdx.x;
unsigned int i = blockIdx.x*(blockDim.x*2) + threadIdx.x;
sdata[tid] = g_idata[i] + g_idata[i+blockDim.x];
__syncthreads();
```

Parallel Reduction

- Unrolling the last warp
  - As reduction proceeds, # “active” threads decreases
    - When s <= 32, we have only one warp left
  - Instructions are SIMD synchronous within a warp
  - That means when s <= 32:
    - We don’t need to __syncthreads()
    - We don’t need “if (tid < s)” because it doesn’t save any work
  - Lets unroll the last 6 iterations of the inner loop

Parallel Reduction

- Note: This saves useless work in all warps, not just the last one!
- Without unrolling, all warps execute every iteration of the for loop and if statement

```
__device__ void warpReduce(volatile int* sdata, int tid) {
    sdata[tid] += sdata[tid + 32];
    sdata[tid] += sdata[tid + 16];
    sdata[tid] += sdata[tid +  8];
    sdata[tid] += sdata[tid +  4];
    sdata[tid] += sdata[tid +  2];
    sdata[tid] += sdata[tid +  1];
}
```

IMPORTANT:
For this to be correct,
we must use the
"volatile" keyword!

```
// later…
for (unsigned int s=blockDim.x/2; s>32; s>>=1) {
    if (tid < s)
        sdata[tid] += sdata[tid + s];
    __syncthreads();
}

if (tid < 32) warpReduce(sdata, tid);
```

Parallel Reduction

```
template <unsigned int blockSize>
__device__ void warpReduce(volatile int *sdata, unsigned int tid) {
    if (blockSize >=  64) sdata[tid] += sdata[tid + 32];
    if (blockSize >=  32) sdata[tid] += sdata[tid + 16];
    if (blockSize >=  16) sdata[tid] += sdata[tid +  8];
    if (blockSize >=   8) sdata[tid] += sdata[tid +  4];
    if (blockSize >=   4) sdata[tid] += sdata[tid +  2];
    if (blockSize >=   2) sdata[tid] += sdata[tid +  1];
}
template <unsigned int blockSize>
__global__ void reduce6(int *g_idata, int *g_odata, unsigned int n) {
    extern __shared__ int sdata[];
    unsigned int tid = threadIdx.x;
    unsigned int i = blockIdx.x*(blockSize*2) + tid;
    unsigned int gridSize = blockSize*2*gridDim.x;
    sdata[tid] = 0;

    while (i < n) { sdata[tid] += g_idata[i] + g_idata[i+blockSize];  i += gridSize;  }
    __syncthreads();

    if (blockSize >= 512) { if (tid < 256) { sdata[tid] += sdata[tid + 256]; } __syncthreads(); }
    if (blockSize >= 256) { if (tid < 128) { sdata[tid] += sdata[tid + 128]; } __syncthreads(); }
    if (blockSize >= 128) { if (tid <   64) { sdata[tid] += sdata[tid +   64]; } __syncthreads(); }

    if (tid < 32) warpReduce(sdata, tid);
    if (tid == 0) g_odata[blockIdx.x] = sdata[0];
}
```

NVIDIA.

Final Optimized Kernel

Prefix Sum (Scan)

- Definition
  - Prefix sum, cumulative sum, inclusive scan, or simply scan of a sequence of numbers x0, x1, x2, ... Gives a second sequence of numbers y0, y1, y2, ..., the sums of prefixes (running totals) of the input sequence.
  - Any binary operation (not just the addition operation).
- Example
  - If (+) is addition, then scan on the set: [3 1 7 0 4 1 6 3]
    - Returns the set: [0 3 4 11 11 15 16 22]

Note: Exclusive scan: last input element is not included in the result

Prefix Sum (Scan)

- A Naïve inclusive parallel Scan
  - Assign one thread to calculate each y element
  - Have every thread add up all x elements needed for the y element

$$y0 = x0$$
$$y1 = x0 + x1$$
$$y2 = x0 + x1 + x2$$

Prefix Sum (Scan)

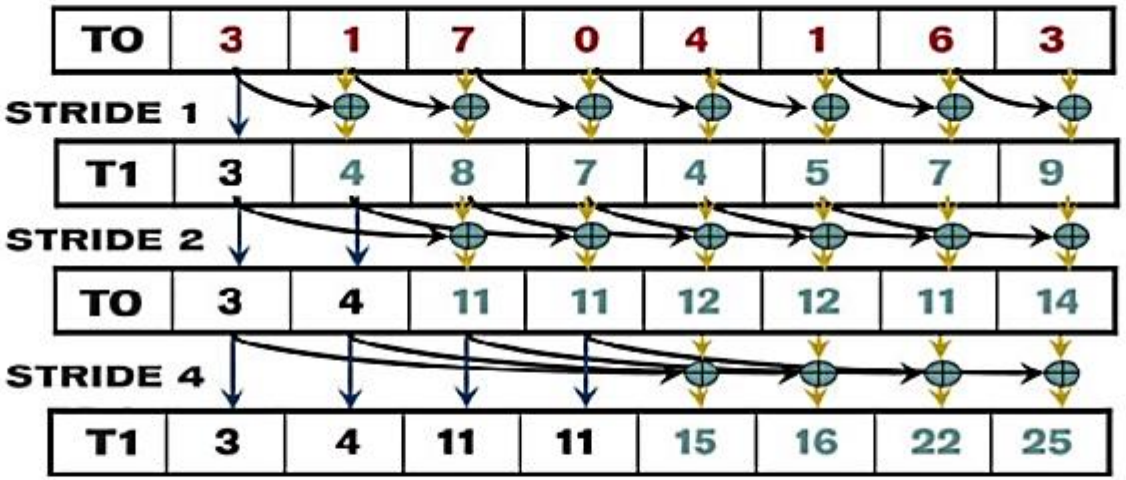
- A Naïve inclusive parallel Scan
  - Assign one thread to calculate each y element
  - Have every thread add up all x elements needed for the y element

$$y0 = x0$$
$$y1 = x0 + x1$$
$$y2 = x0 + x1 + x2$$

- Note: Parallel programming is easy as long as you do not care about performance

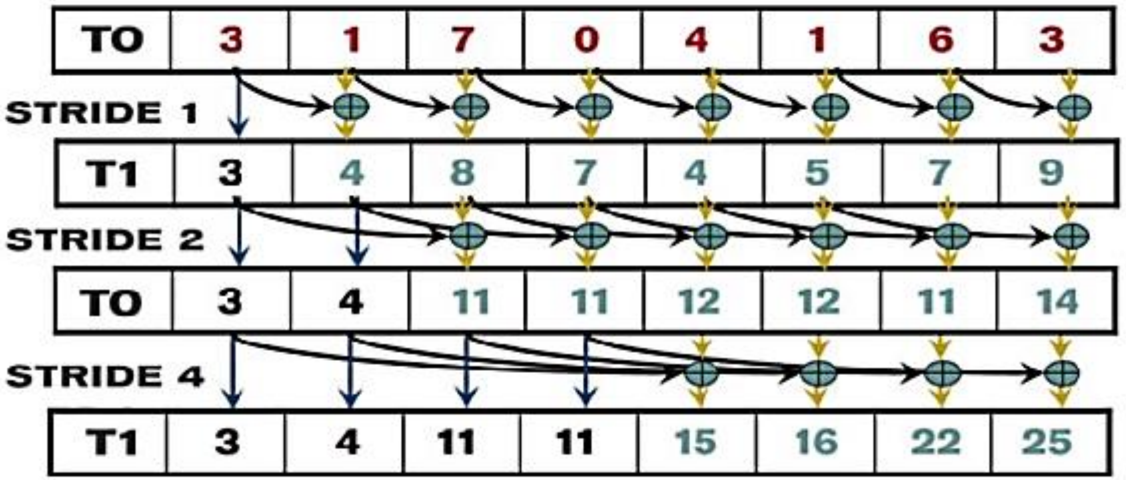
Prefix Sum (Scan)

- Compared to naïve version, a slight better version – Hillis Steele Scan.
- Iterate log(n) times: Threads stride to n: Add pairs of elements stride elements apart. Double stride at each iteration. (note must double buffer shared memory arrays)



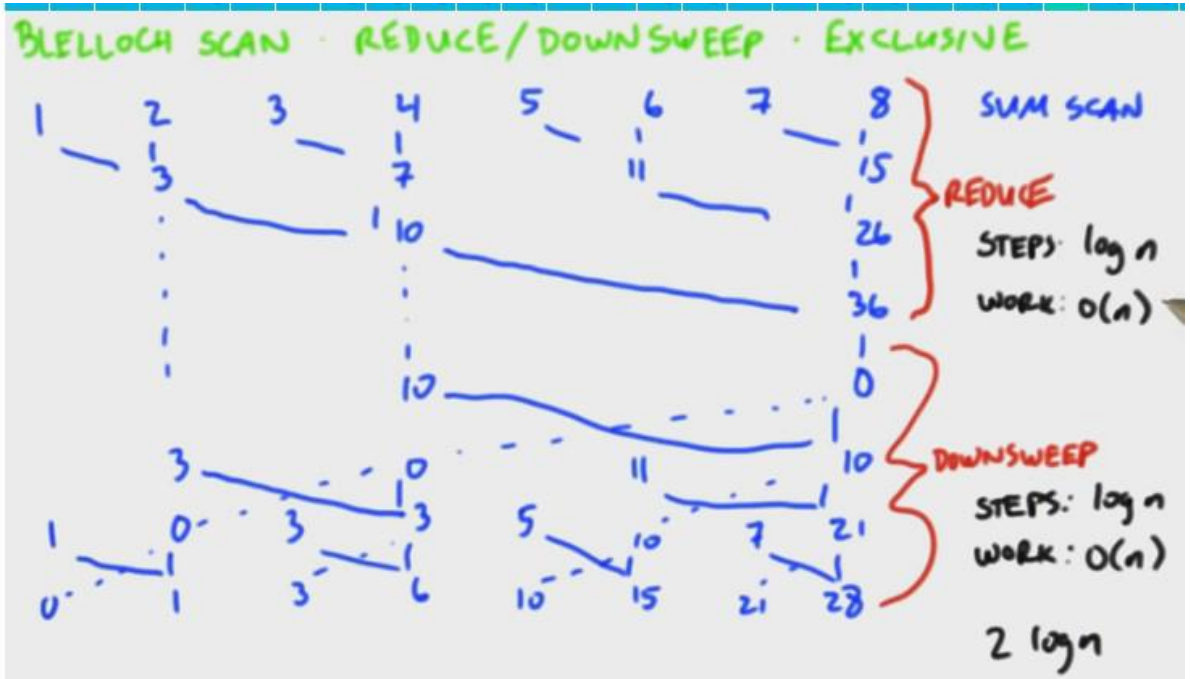
Prefix Sum (Scan)

- Compared to naïve version, a slight better version – Hillis Steele Scan.
- Iterate log(n) times: Threads stride to n: Add pairs of elements stride elements apart. Double stride at each iteration. (note must double buffer shared memory arrays)
- This scan algorithm is not that work efficient
  - Sequential scan algorithm does n-1 adds
  - How many does this one do?
  - What happens if the # of elements is 10^6?



Prefix Sum (Scan)

- Even better one: Blelloch Scan.
- Reduces in log(n), downsweeps in log(n).
- Total steps: 2 log(n)
- This scan algorithm is work efficient



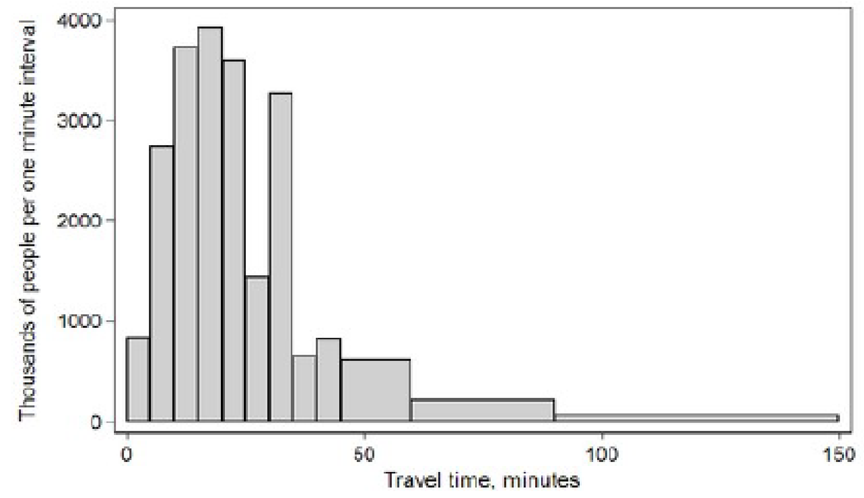
Prefix Sum (Scan)

- Useful in implementation of several parallel algorithms:
  - Radix sort
  - Quicksort
  - String comparison
  - Lexical analysis
  - Stream compaction
  - Polynomial evaluation
  - Solving recurrences
  - Tree operations
  - histograms

- Examples
  - Assigning camp slots
  - Assigning farmer market space
  - Allocating memory to parallel threads
  - Allocating memory buffer for communication channels

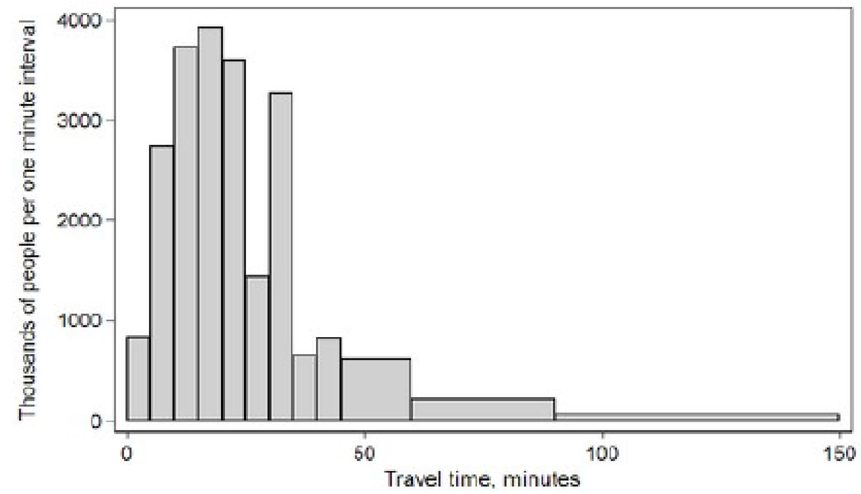
Histogram

- A histogram is a graphical representation of the distribution of numerical data.
- Bar Graph



Histogram

- A histogram is a graphical representation of the distribution of numerical data.
- Bar Graph
- Serial Algorithm




```
For (i=0; I < BIN_COUNT; i++)
   result[i]=0;
For (i=0; I < measurements.size(); i++)
   result[computeBin(measurements[i])]++;
```

Histogram

- Naïve implementation:

```
__global__ void naive_histo(int *d_bins, const int *d_in, const int BIN_COUNT)
{
    int myId = threadIdx.x + blockDim.x * blockIdx.x;
    int myItem = d_in[myId];
    int myBin = myItem % BIN_COUNT;
    d_bins[myBin]++;
}
```

Histogram

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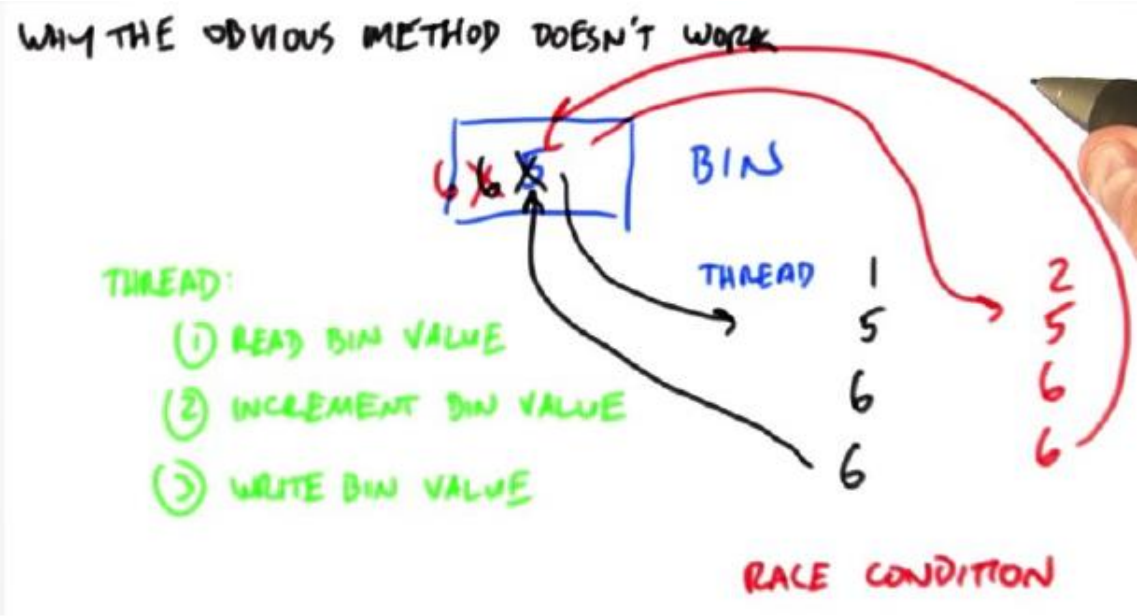
    d_bins[myBin]++;
}
```



Histogram

- Naïve implementation using atomic operations (Method 1):
  - This will avoid RAW hazards.

```
__global__ void simple_histo(int *d_bins, const int *c
{
    int myId = threadIdx.x + blockDim.x * blockIdx.x;
    int myItem = d_in[myId];
    int myBin = myItem % BIN_COUNT;
    atomicAdd(&(d_bins[myBin]), 1);
}
```

Histogram

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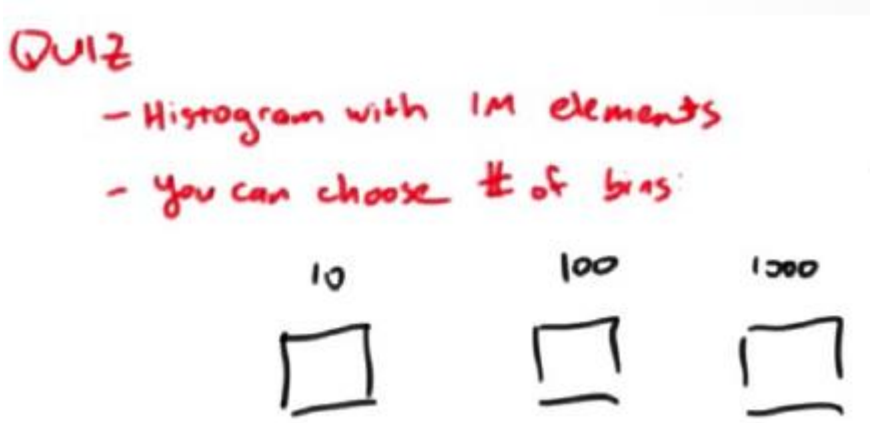
```
__global__ void simple_histo(int *d_bins, const int *
{
    int myId = threadIdx.x + blockDim.x * blockIdx.x;
    int myItem = d_in[myId];
    int myBin = myItem % BIN_COUNT;
    atomicAdd(&(d_bins[myBin]), 1);
}
```



Histogram

- Redefining the method. Local histogram + reduction (Method 2)

Histogram

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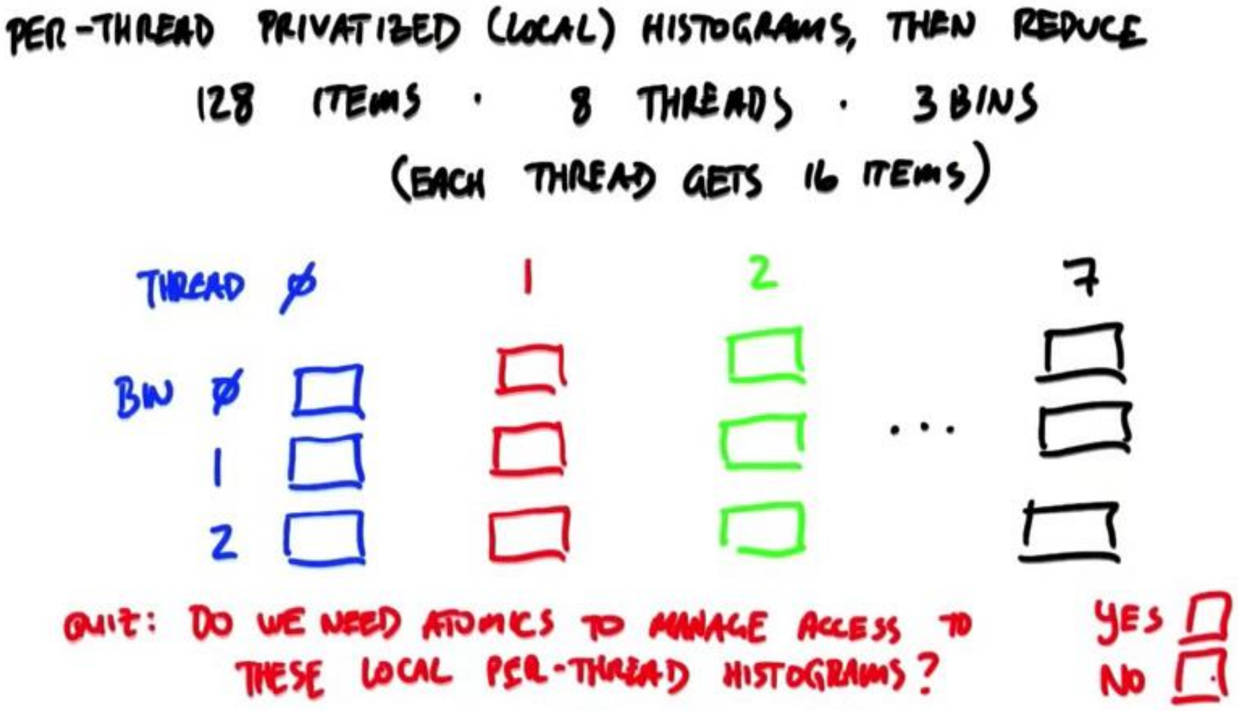
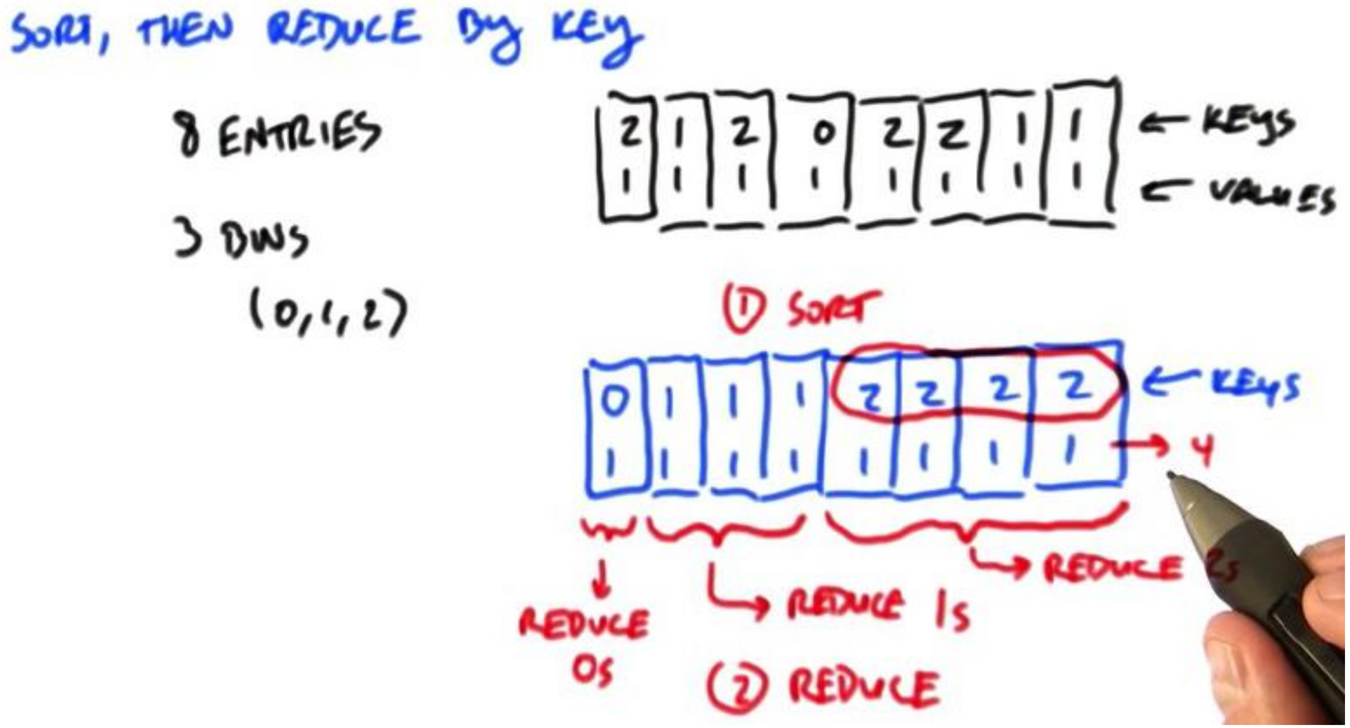


Histogram

- Redefining the method. Sort then reduce by key (Method 3)



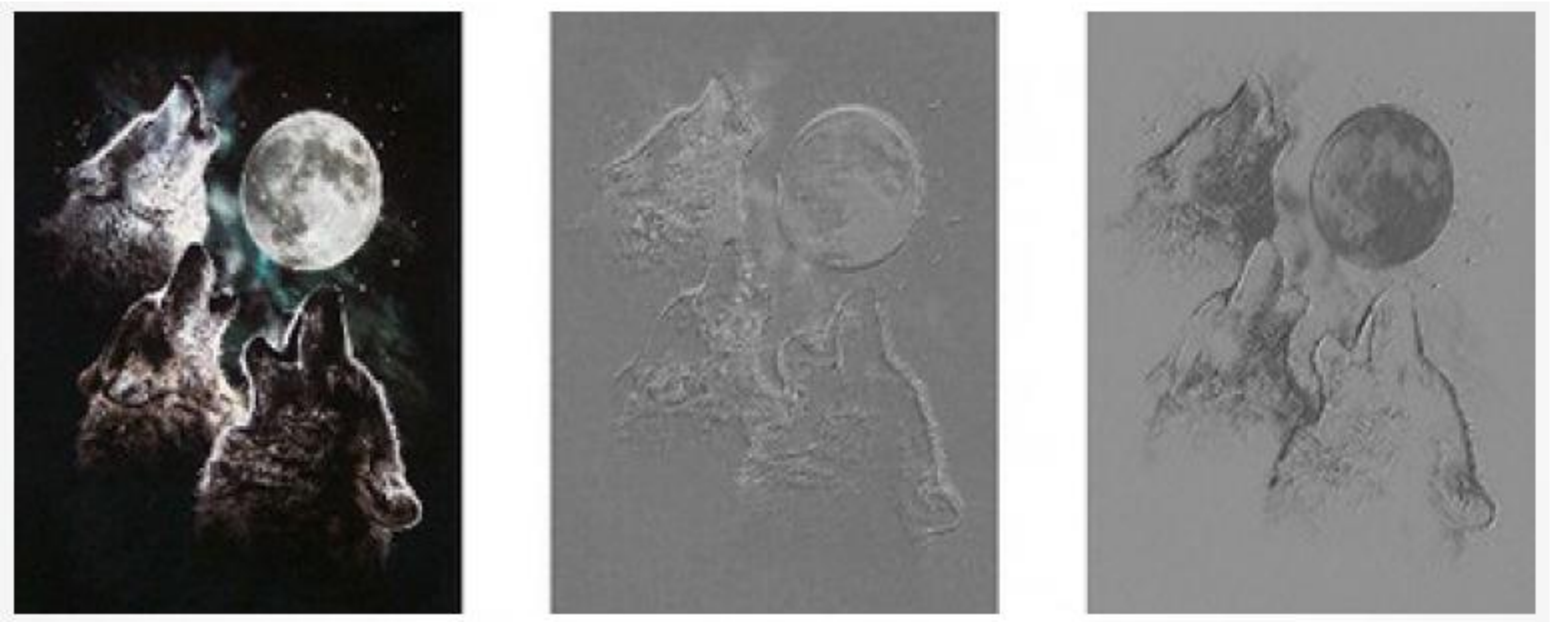
Histogram

- Final thoughts on histogram:
  - Using Atomic operations
  - Using per-thread histograms, and then reduce
  - Sort, then reduce by key
- Question
  - 256 threads, 8 bins. How many atomic adds are needed?
    - Atomic Technique.
    - Processing 16 elements per thread with local histogram, and then atomics.

Convolution

- Applications
  - A popular array operation that is used in various forms in signal processing, digital recording, image processing, video processing, and computer vision.
  - Convolution is often performed as a filter that transforms signals and pixels into more desirable values
    - Some filters smooth out the signal values so that one can see the big-picture trend
    - Others like Gaussian filters can be used to sharpen boundaries and edges of objects in images

Convolution

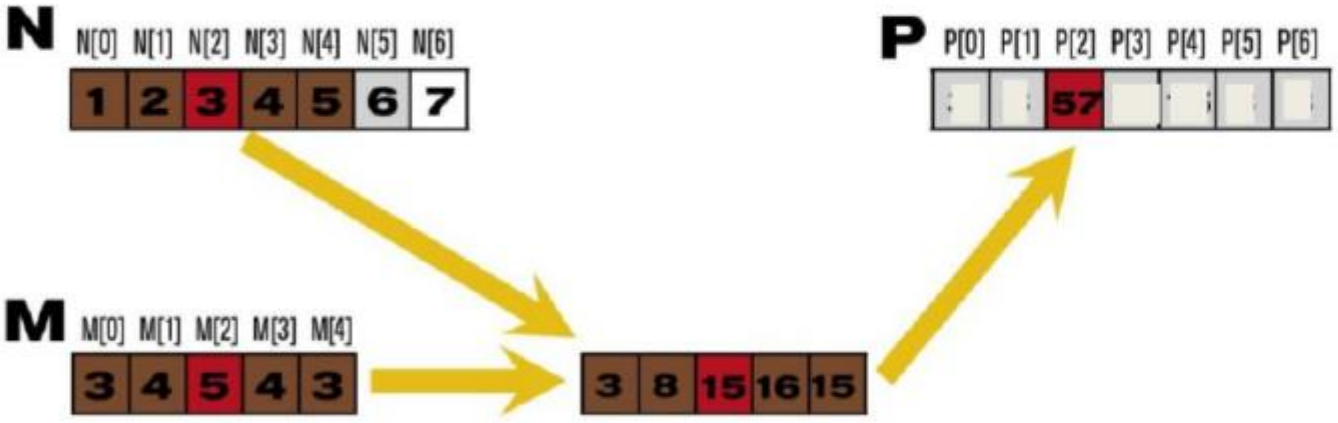


Convolution

- An array operation where each output data element is a weighted sum of a collection of neighboring input elements
- The weights used in the weighted sum calculation are defined by an input mask array, commonly referred to as the convolution kernel
  - We will refer to these mask arrays as convolution masks to avoid confusion.
  - The same convolution mask is typically used for all elements of the array.

Convolution

- 1D Convolution example
  - Commonly used for audio processing
    - Mask size is usually an odd number of elements for symmetry



Convolution

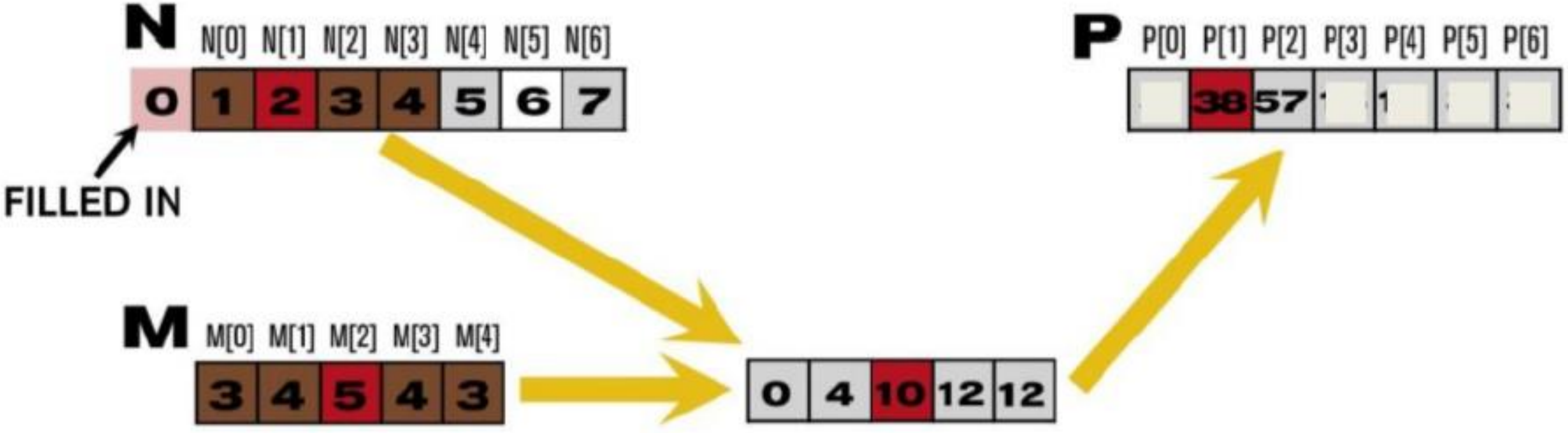
- Definition
  - For a causal discrete-time FIR filter of order N, each value of the output sequence is a weighted sum of the most recent input values:

$$y[n] = b_0 x[n] + b_1 x[n-1] + \cdots + b_N x[n-N]$$
$$= \sum_{i=0}^{N} b_i \cdot x[n-i],$$

  - Where:
    - X[n] is the input signal
    - Y[n] is the output signal,
    - N is the filter order, an Nth-order filter has (N+1) terms on the right-hand side
    - Bi is the value of the impulse response at the ith instance for 0<= I <= N of an Nth-order filter. If the filter is a direct form FIR filter then bi is also a coefficient of the filter.

Convolution

- Calculation of output elements near the boundaries (beginning and end) of the input array need to deal with “ghost” elements
    - Different policies (0, replicates of boundary values, etc.)



Convolution

- This kernel forces all elements outside the vector range to 0

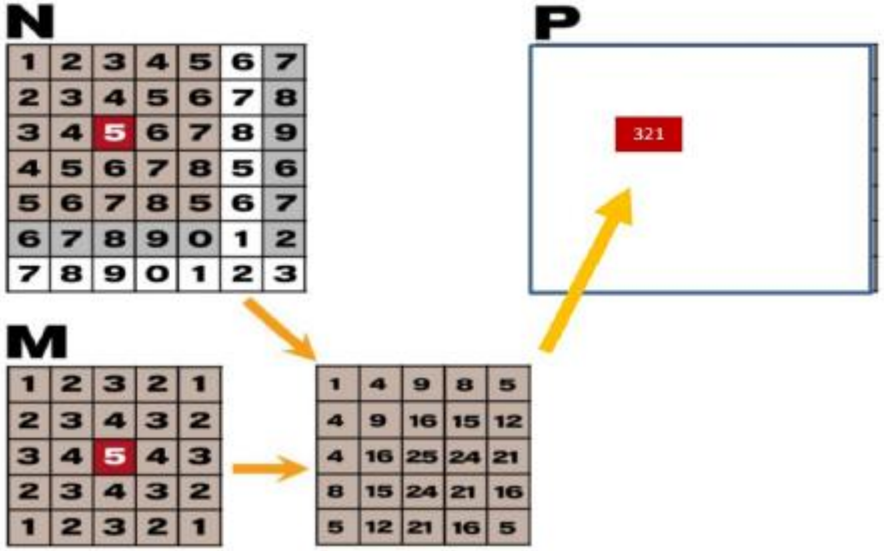
```
__global__ void convolution_1D_basic_kernel(float *N, float *M, float *P,
 int Mask_Width, int Width) {

 int i = blockIdx.x*blockDim.x + threadIdx.x;

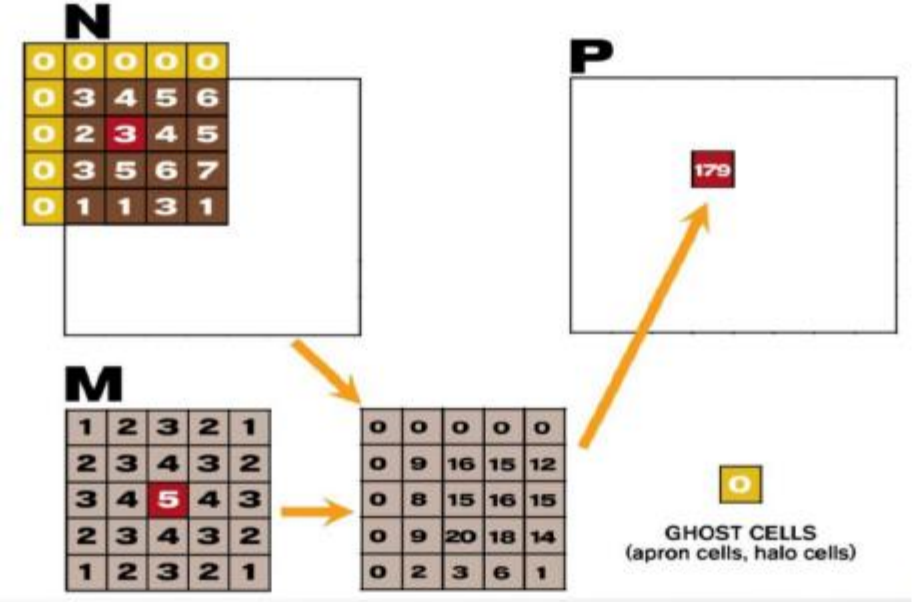
 float Pvalue = 0;
 int N_start_point = i - (Mask_Width/2);
 for (int j = 0; j < Mask_Width; j++) {
   if (N_start_point + j >= 0 && N_start_point + j < Width) {
     Pvalue += N[N_start_point + j]*M[j];
   }
 }
 P[i] = Pvalue;
}
```

Convolution

- 2D convolution

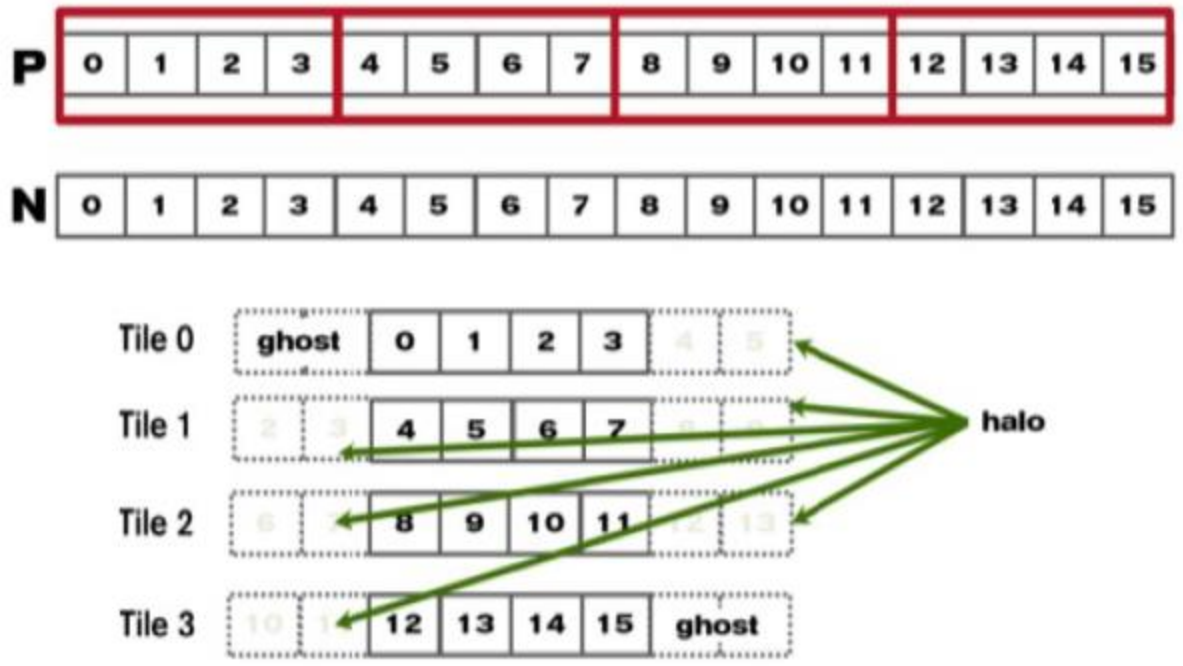


- 2D convolution – ghost cells



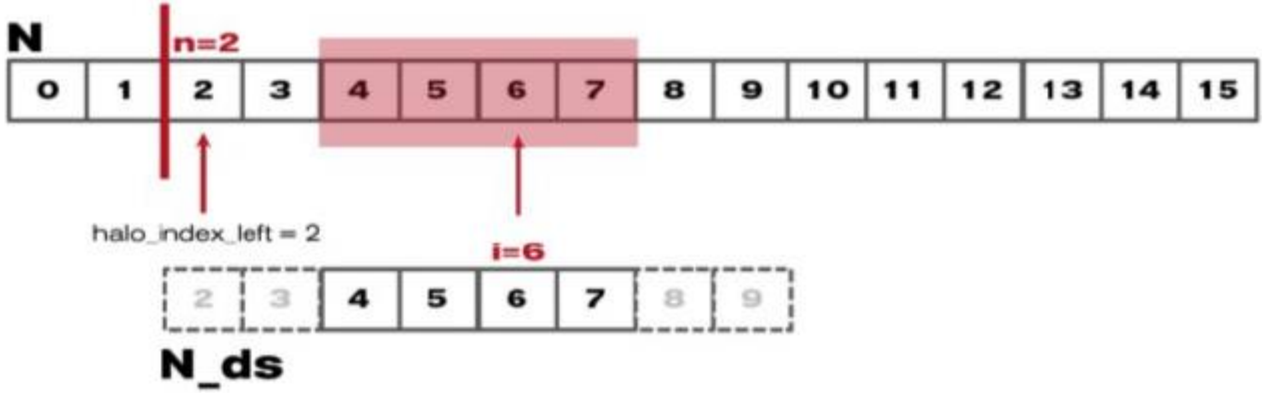
Convolution

- Tiled Convolution for 1D



Convolution

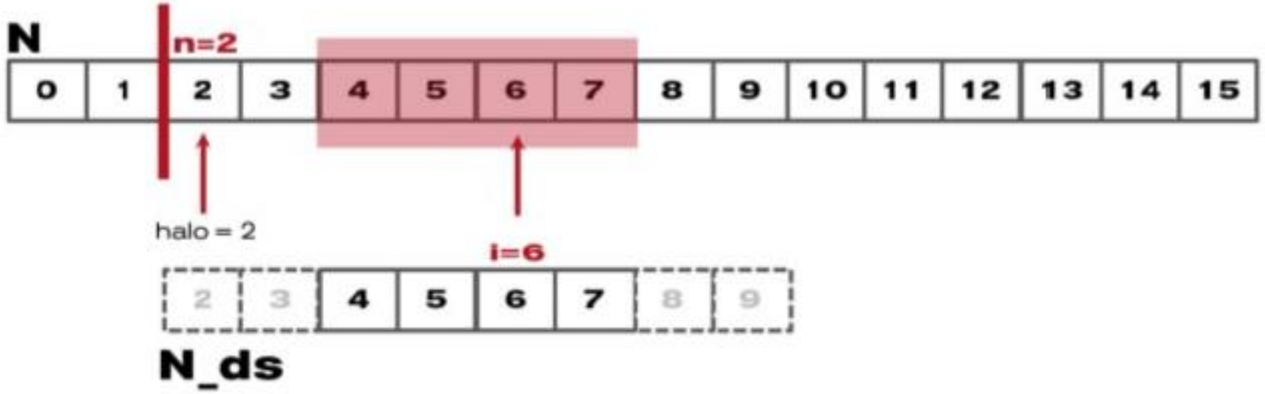
- Loading the left halo




```
int n = Mask_Width/2;
int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;
 if (threadIdx.x >= blockDim.x - n) {
  N_ds[threadIdx.x - (blockDim.x - n)] =
   (halo_index_left < 0) ? 0 : N[halo_index_left];
 }
```

Convolution

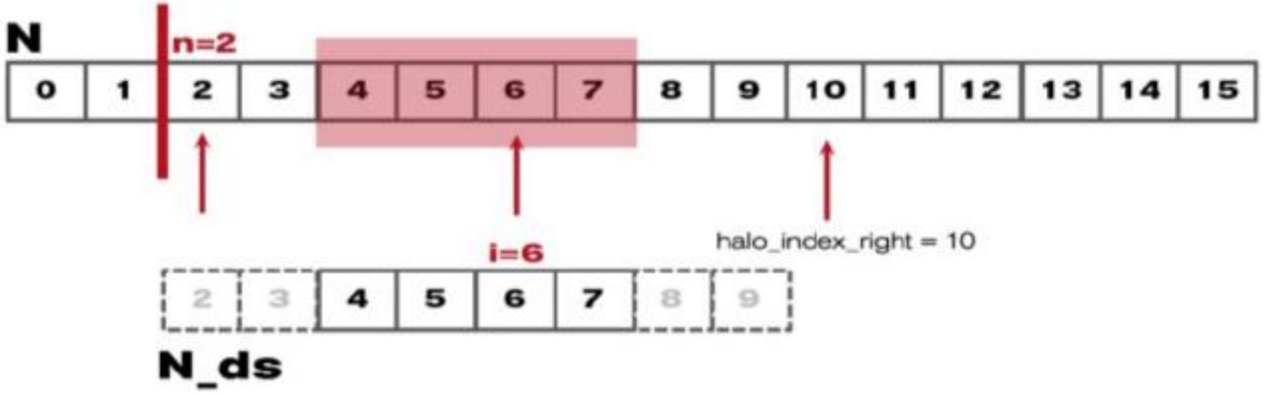
- Loading the internals




```
N_ds[n + threadIdx.x] = N[blockIdx.x*blockDim.x + threadIdx.x];
```

Convolution

- Loading the right halo




```
int halo_index_right = (blockIdx.x + 1)*blockDim.x + threadIdx.x;
if (threadIdx.x < n) {
  N_ds[n + blockDim.x + threadIdx.x] =
   (halo_index_right >= Width) ? 0 : N[halo_index_right];
}
```

Convolution

```
__global__ void convolution_1D_basic_kernel(float *N, const float __restrict__ *M,
          float *P, int Mask_Width, int Width) {

 int i = blockIdx.x*blockDim.x + threadIdx.x;
 __shared__ float  N_ds[TILE_SIZE + MAX_MASK_WIDTH - 1];

 int n = Mask_Width/2;

 int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;
 if (threadIdx.x >= blockDim.x - n) {
  N_ds[threadIdx.x - (blockDim.x - n)] =
   (halo_index_left < 0) ? 0 : N[halo_index_left];
 }

 N_ds[n + threadIdx.x] = N[blockIdx.x*blockDim.x + threadIdx.x];

 int halo_index_right = (blockIdx.x + 1)*blockDim.x + threadIdx.x;
 if (threadIdx.x < n) {
  N_ds[n + blockDim.x + threadIdx.x] =
   (halo_index_right >= Width) ? 0 : N[halo_index_right];
 }

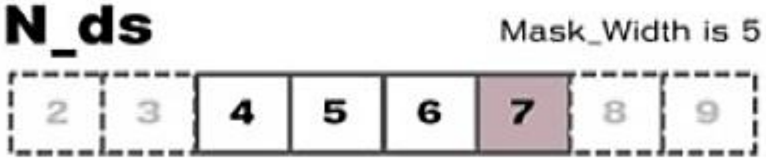
 __syncthreads();

 float Pvalue = 0;
 for(int j = 0; j < Mask_Width; j++) {
  Pvalue += N_ds[threadIdx.x + j]*M[j];
 }
 P[i] = Pvalue;

}
```

Convolution

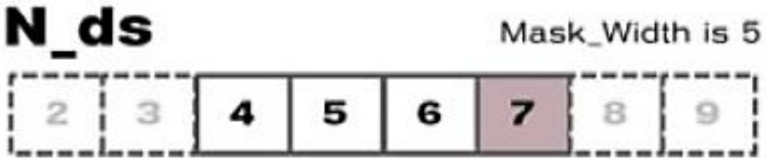
- Shared memory data reuse



Do we want to use Shared memory?

Convolution

- Shared memory data reuse



Element 2 is used by thread 4 (1X)
Element 3 is used by threads 4, 5 (2X)
Element 4 is used by threads 4, 5, 6 (3X)
Element 5 is used by threads 4, 5, 6, 7 (4X)
Element 6 is used by threads 4, 5, 6, 7 (4X)
Element 7 is used by threads 5, 6, 7 (3X)
Element 8 is used by threads 6, 7 (2X)
Element 9 is used by thread 7 (1X)

Do we want to use Shared memory?