

# Applications: Scan/Histo/Conv

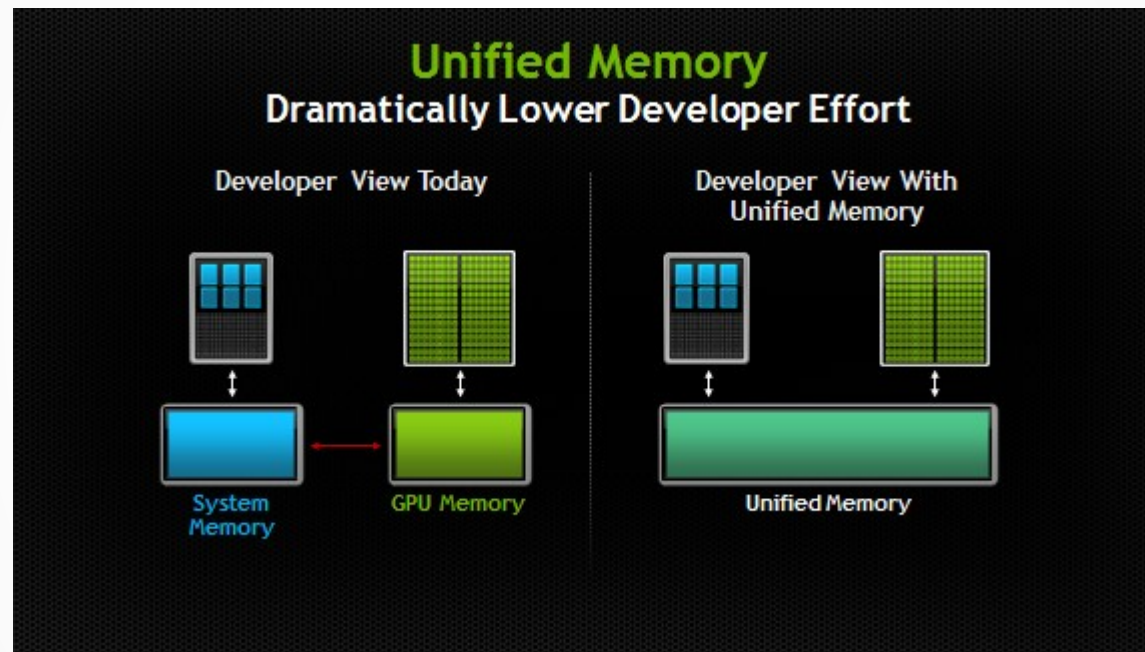
Leiming Yu  
yu.lei@husky.neu.edu



# Topics

- Prefix Sum
- Histogram
- Convolution

# Catch up

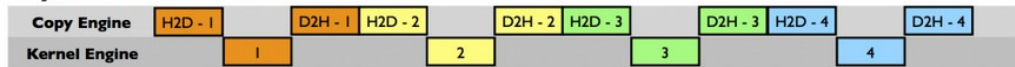


# Catch up

## Sequential Version



## Asynchronous Version 1

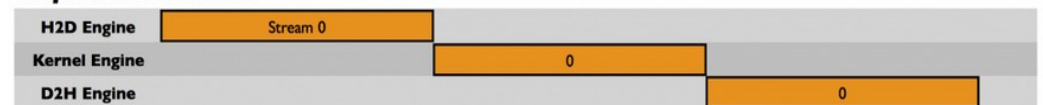


## Asynchronous Version 2



Time →

## Sequential Version



## Asynchronous Version 1



## Asynchronous Version 2



Time →

# Catch up

```
template <unsigned int blockSize>
__device__ void warpReduce(volatile int *sdata, unsigned int tid) {
    if (blockSize >= 64) sdata[tid] += sdata[tid + 32];
    if (blockSize >= 32) sdata[tid] += sdata[tid + 16];
    if (blockSize >= 16) sdata[tid] += sdata[tid + 8];
    if (blockSize >= 8) sdata[tid] += sdata[tid + 4];
    if (blockSize >= 4) sdata[tid] += sdata[tid + 2];
    if (blockSize >= 2) sdata[tid] += sdata[tid + 1];
}

template <unsigned int blockSize>
__global__ void reduce6(int *g_idata, int *g_odata, unsigned int n) {
    extern __shared__ int sdata[];
    unsigned int tid = threadIdx.x;
    unsigned int i = blockIdx.x*(blockSize*2) + tid;
    unsigned int gridSize = blockSize*2*gridDim.x;
    sdata[tid] = 0;

    while (i < n) { sdata[tid] += g_idata[i] + g_idata[i+blockSize]; i += gridSize; }
    __syncthreads();

    if (blockSize >= 512) { if (tid < 256) { sdata[tid] += sdata[tid + 256]; } __syncthreads(); }
    if (blockSize >= 256) { if (tid < 128) { sdata[tid] += sdata[tid + 128]; } __syncthreads(); }
    if (blockSize >= 128) { if (tid < 64) { sdata[tid] += sdata[tid + 64]; } __syncthreads(); }

    if (tid < 32) warpReduce(sdata, tid);
    if (tid == 0) g_odata[blockIdx.x] = sdata[0];
}
```



Final Optimized Kernel

# Parallel Prefix Sum (Scan)

What is prefix sum?

- Definition:

The all-prefix-sums operation takes a binary associative operator  $\oplus$  with identity  $I$ , and an array of  $n$  elements

$$[a_0, a_1, \dots, a_{n-1}]$$

and returns the ordered set

$$[I, a_0, (a_0 \oplus a_1), \dots, (a_0 \oplus a_1 \oplus \dots \oplus a_{n-2})].$$

- Example:

if  $\oplus$  is addition, then scan on the set

$$[3 \ 1 \ 7 \ 0 \ 4 \ 1 \ 6 \ 3]$$

returns the set

$$[0 \ 3 \ 4 \ 11 \ 11 \ 15 \ 16 \ 22]$$

Exclusive scan: last input element is not included in the result



# Parallel Prefix Sum (Scan)

Where it is applied?

- Useful in implementation of several parallel algorithms:

- radix sort
- quicksort
- String comparison
- Lexical analysis
- Stream compaction

- Polynomial evaluation
- Solving recurrences
- Tree operations
- Histograms
- Etc.

Assigning camp slots

Assigning farmer market space

Allocating memory to parallel threads

Allocating memory buffer for communication channels

# Parallel Prefix Sum (Scan)

## A Naïve Inclusive Parallel Scan

Assign one thread to calculate each  $y$  element

Have every thread to add up all  $x$  elements needed for the  $y$  element

$$y_0 = x_0$$

$$y_1 = x_0 + x_1$$

$$y_2 = x_0 + x_1 + x_2$$

“Parallel programming is easy as long as you do not care about performance.”



# Parallel Prefix Sum (Scan)

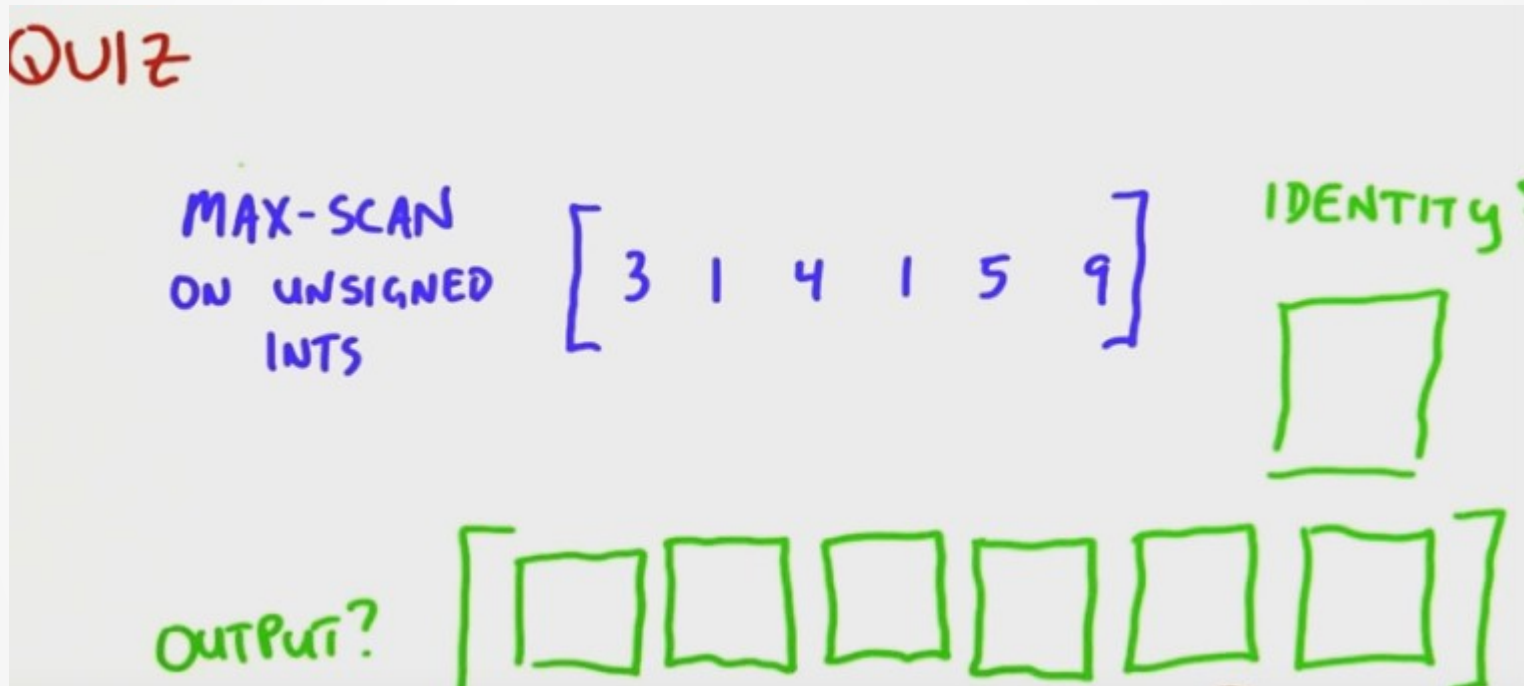
QUIZ

MAX-SCAN  
ON UNSIGNED  
INTS

$[3 \ 1 \ 4 \ 1 \ 5 \ 9]$

IDENTITY?

OUTPUT?

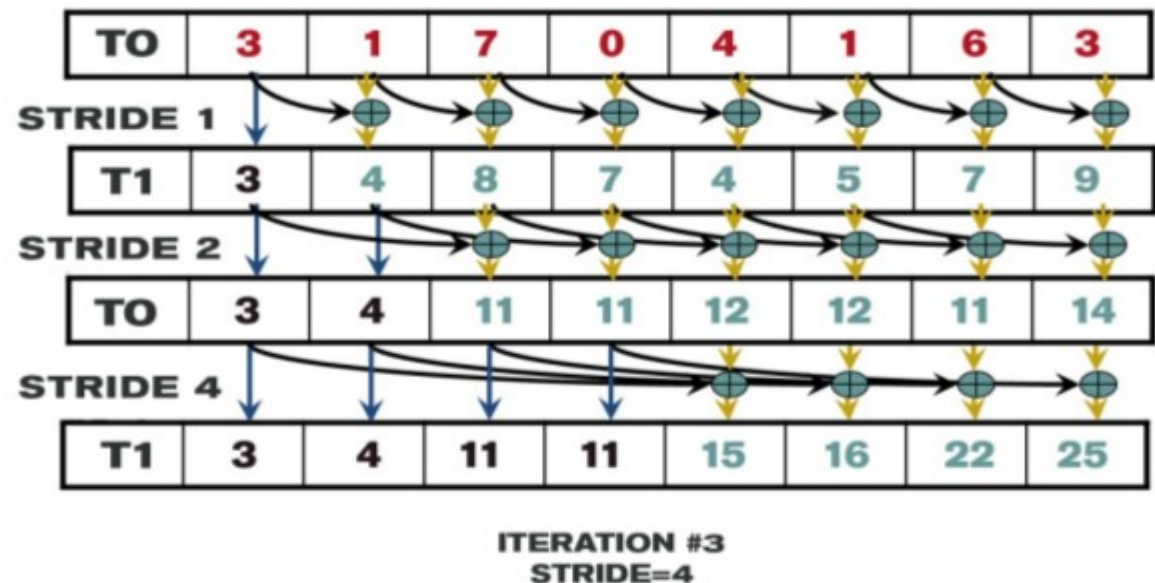


Be aware of inclusive and exclusive scan.

# Parallel Prefix Sum (Scan)

Compared to naïve version,  
a slight better version -  
**Hillis Steele Scan.**

1. ...
2. Iterate  $\log(n)$  times: Threads stride to  $n$ : Add pairs of elements stride elements apart. Double stride at each iteration. (note must double buffer shared memory arrays)
3. Write output from shared memory to device memory



- This scan algorithm is not that work efficient
  - Sequential scan algorithm does  $n-1$  adds
  - A factor of  $\log(n)$  might hurt: 20x more work for  $10^6$  elements!

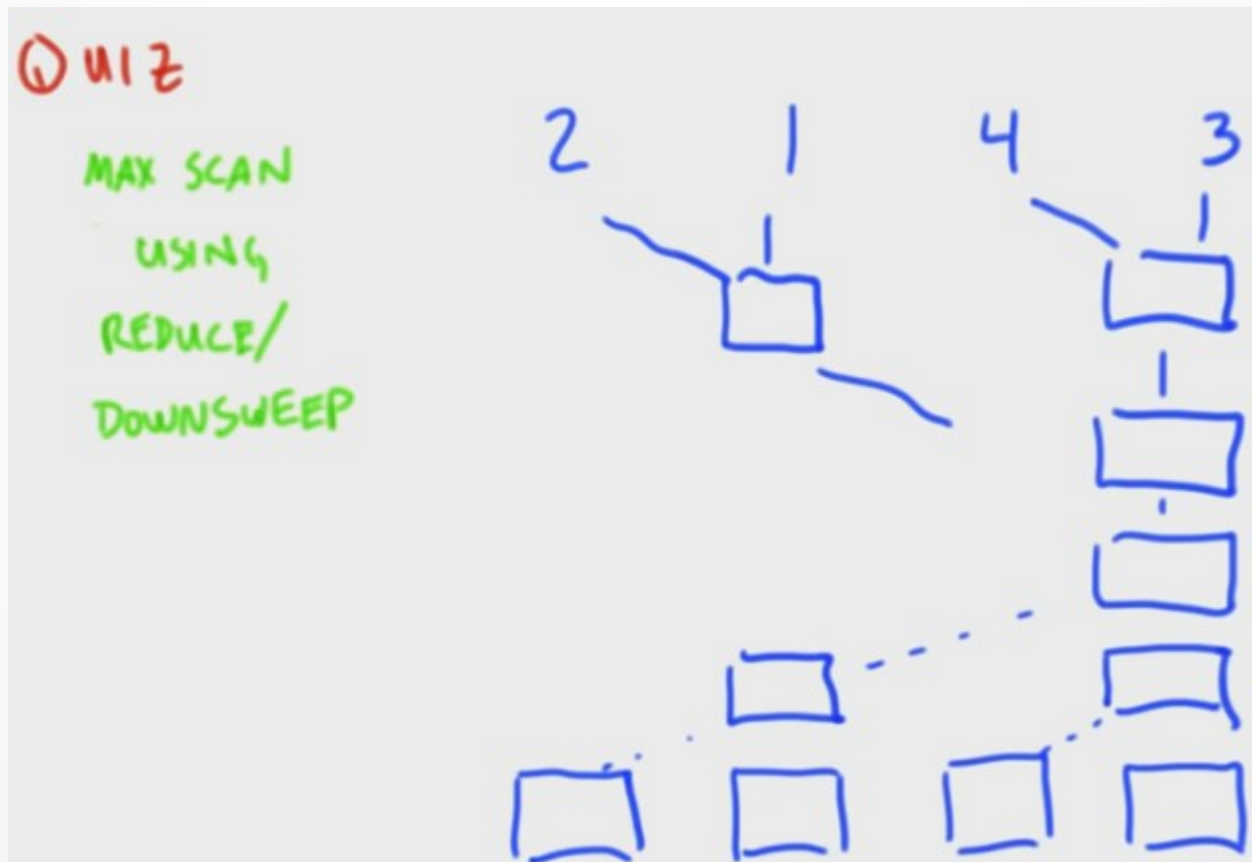
# Parallel Prefix Sum (Scan)

Work-efficient: **Blelloch Scan**

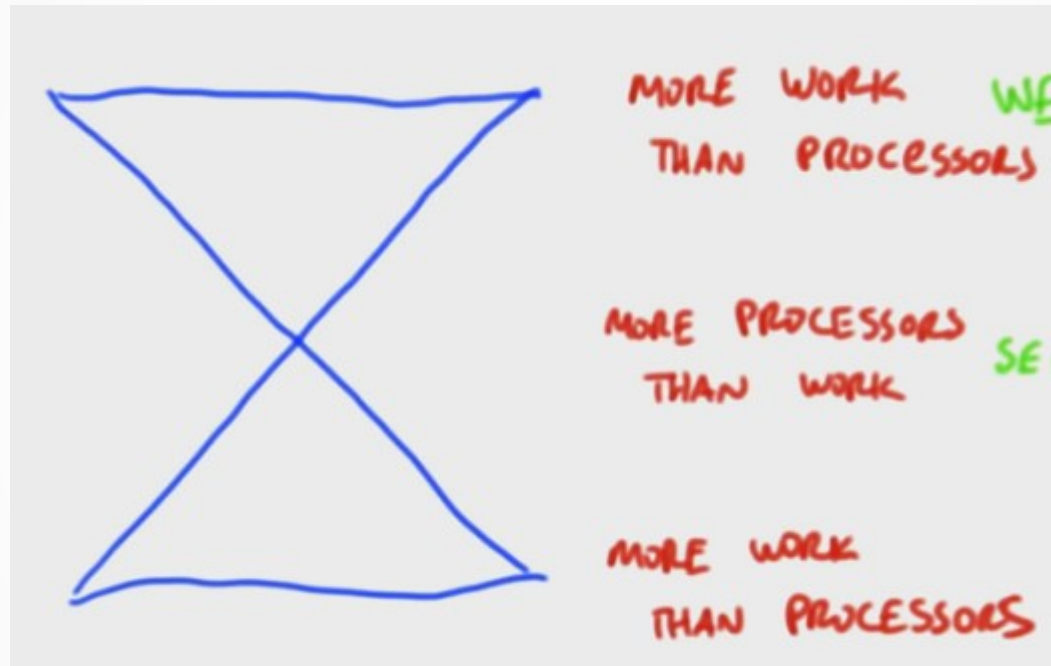


# Parallel Prefix Sum (Scan)

Work-efficient: **Blelloch Scan**



# Parallel Prefix Sum (Scan)



# Parallel Prefix Sum (Scan)

QUIZ

SERIAL

HILLIS  
STEELE

BLELLOCH

512 ELT. VECTOR  
512 PROCESSORS



1M ELT. VECTOR  
512 PROCESSORS



128 K ELT. VECTOR  
1 PROCESSOR

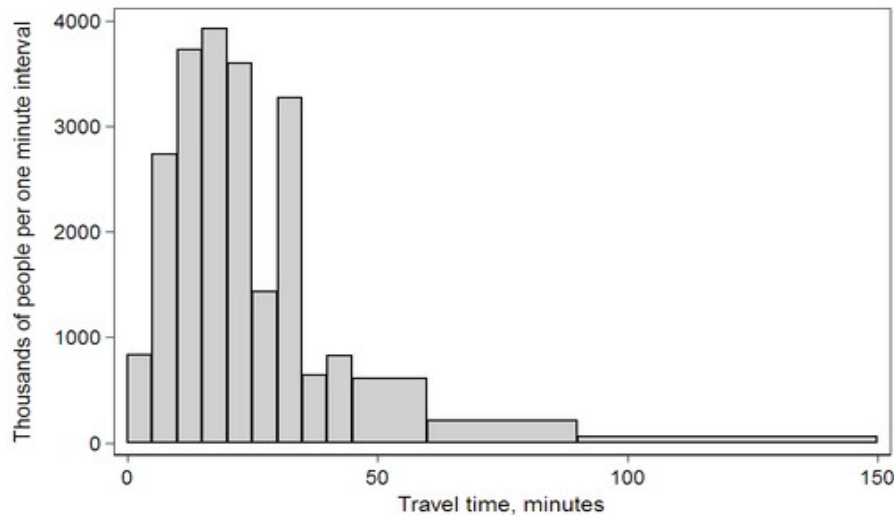




# Topics

- Prefix Sum
- **Histogram**
- Convolution

# Histogram



SERIAL ALGORITHM: HISTOGRAM

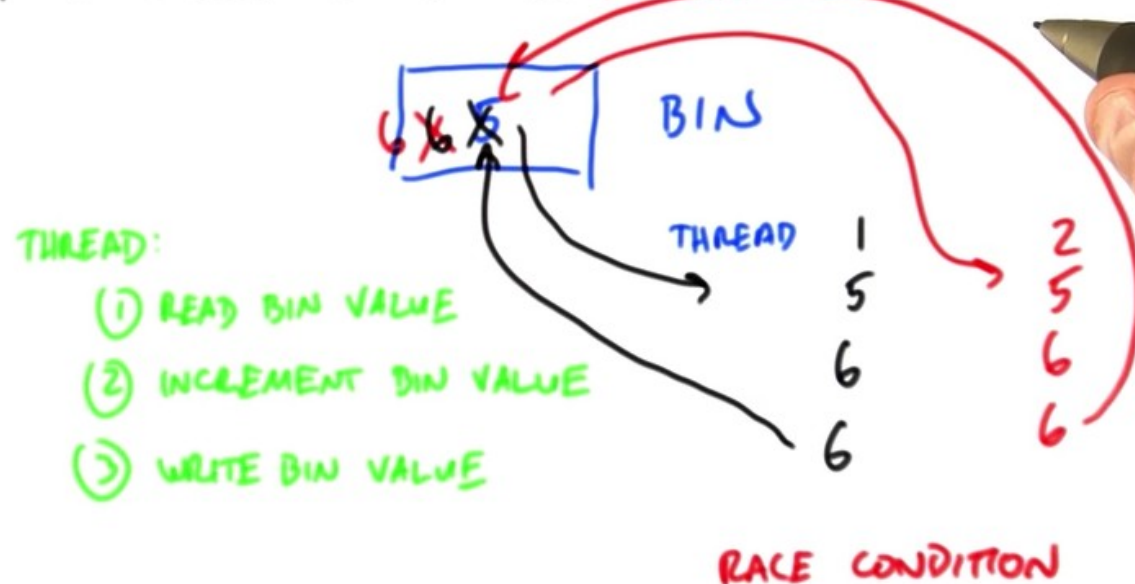
```
for (i=0; i < BIN_COUNT; i++)  
    result[i] = 0;
```

```
for (i=0; i < measurements.size(); i++)  
    result[computeBin(measurements[i])]++;
```

# Histogram

```
__global__ void naive_histo(int *d_bins, const int *d_in, const int BIN_COUNT)
{
    int myId = threadIdx.x + blockDim.x * blockIdx.x;
    int myItem = d_in[myId];
    int myBin = myItem % BIN_COUNT;
    d_bins[myBin]++;
}
```

WHY THE OBVIOUS METHOD DOESN'T WORK



# Histogram

Method 1 : Atomics

RAW hazard

```
__global__ void simple_histo(int *d_bins, const int *d_in)
{
    int myId = threadIdx.x + blockDim.x * blockIdx.x;
    int myItem = d_in[myId];
    int myBin = myItem % BIN_COUNT;
    atomicAdd(&(d_bins[myBin]), 1);
}
```

Quiz

- Histogram with 1M elements
- you can choose # of bins:



# Histogram

Method 2 : local histogram + reduction

PER-THREAD PRIVATIZED (LOCAL) HISTOGRAMS, THEN REDUCE  
128 ITEMS · 8 THREADS · 3 BINS  
(EACH THREAD GETS 16 ITEMS)

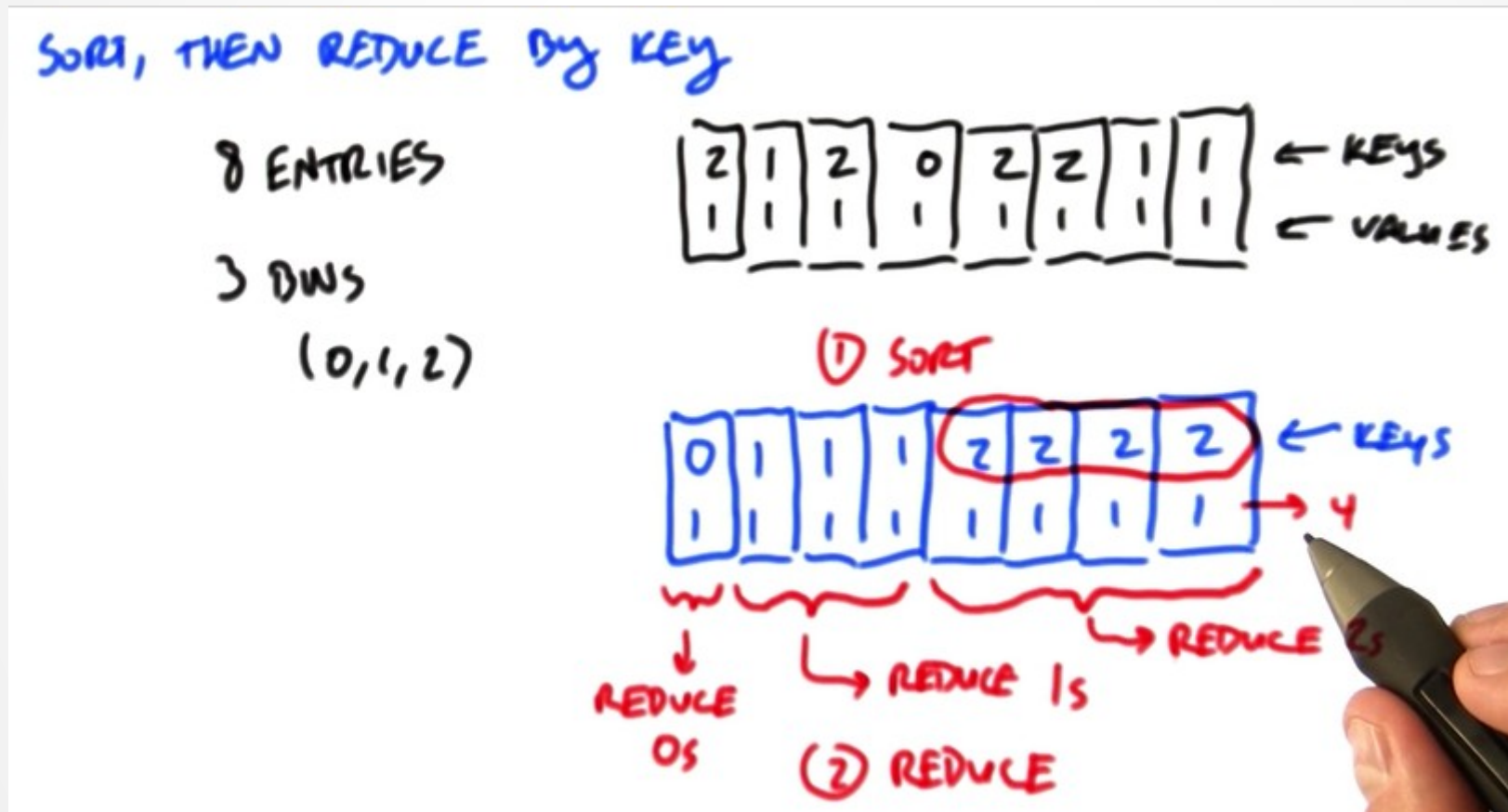


Q: DO WE NEED ATOMICS TO MANAGE ACCESS TO  
THESE LOCAL PER-THREAD HISTOGRAMS?

YES ☐  
NO ☐

# Histogram

Method 3 : sort then reduce by key





# Histogram

## FINAL THOUGHTS ON HISTOGRAM

- ATOMICS
- PER-THREAD HISTOGRAMS, THEN REDUCE (2)
- SORT, THEN REDUCE BY KEY (1)

256 THREADS, 8 BINS:

HOW MANY  
ATOMIC ADDS?

ATOMIC TECHNIQUE:



REDUCE TO 8-ELEMENT  
HISTOGRAM THEN  
ATOMICS



# Topics

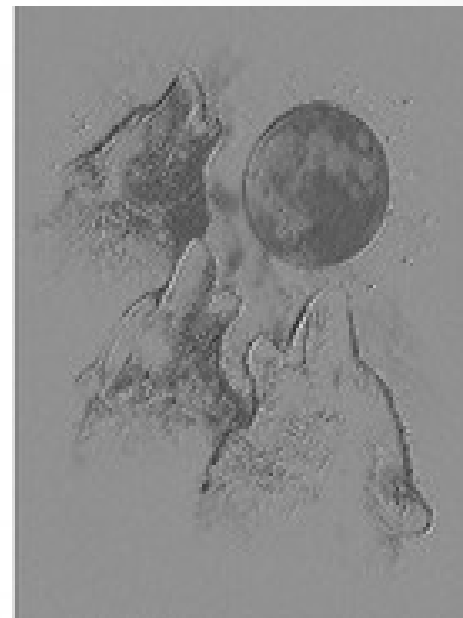
- Prefix Sum
- Histogram
- **Convolution**

# Convolution

## **Convolution Applications**

- A popular array operation that is used in various forms in signal processing, digital recording, image processing, video processing, and computer vision.
- Convolution is often performed as a filter that transforms signals and pixels into more desirable values.
  - Some filters smooth out the signal values so that one can see the big-picture trend
  - Others like Gaussian filters can be used to sharpen boundaries and edges of objects in images..

# Convolution



# Convolution

- An array operation where each output data element is a weighted sum of a collection of neighboring input elements
- The weights used in the weighted sum calculation are defined by an input mask array, commonly referred to as the *convolution kernel*
  - We will refer to these mask arrays as convolution masks to avoid confusion.
  - The same convolution mask is typically used for all elements of the array.

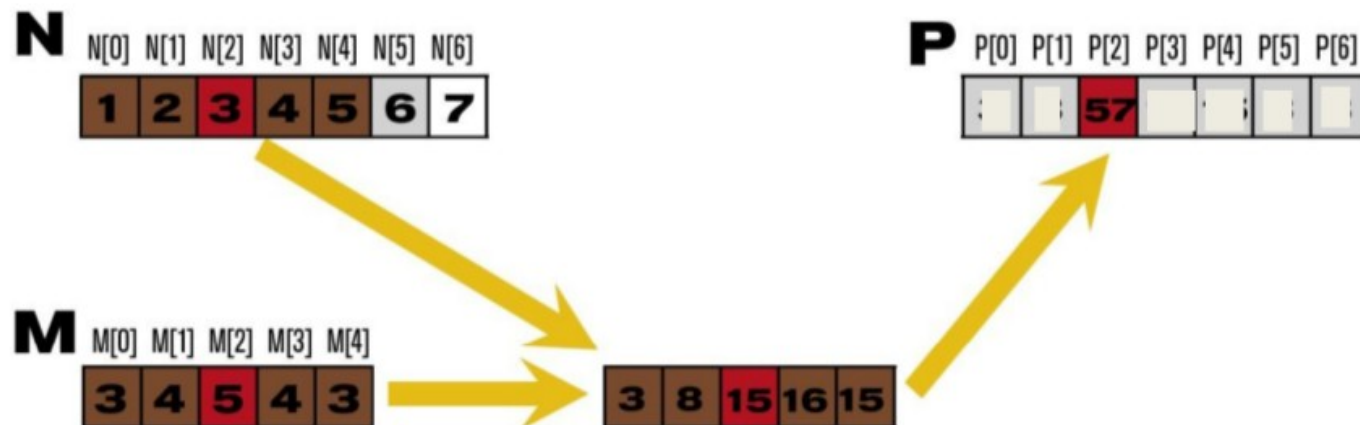
# Convolution

## 1D Convolution Example

Commonly used for audio processing

- Mask size is usually an odd number of elements for symmetry (5 in this example)

Calculation of P[2]





# Convolution

## Definition [\[ edit \]](#)

For a [causal discrete-time](#) FIR filter of order  $N$ , each value of the output sequence is a weighted sum of the most recent input values:

$$\begin{aligned} y[n] &= b_0x[n] + b_1x[n - 1] + \cdots + b_Nx[n - N] \\ &= \sum_{i=0}^N b_i \cdot x[n - i], \end{aligned}$$

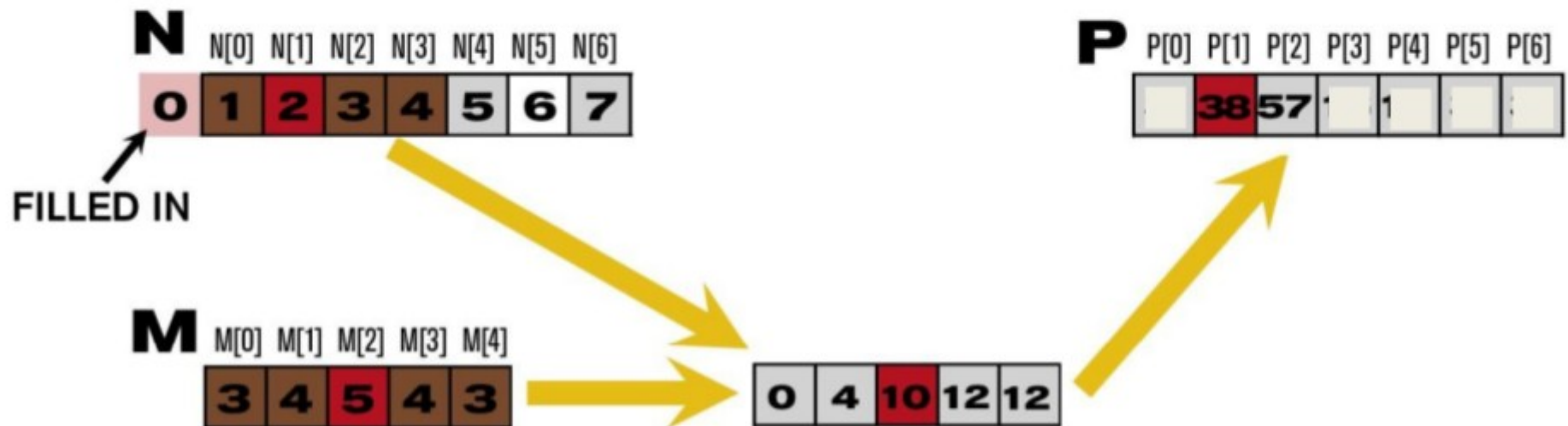
where:

- $x[n]$  is the input signal,
- $y[n]$  is the output signal,
- $N$  is the filter order; an  $N$ th-order filter has  $(N + 1)$  terms on the right-hand side
- $b_i$  is the value of the impulse response at the  $i$ 'th instant for  $0 \leq i \leq N$  of an  $N$ th-order FIR filter. If the filter is a direct form FIR filter then  $b_i$  is also a coefficient of the filter .

# Convolution

Calculation of output elements near the boundaries (beginning and end) of the input array need to deal with “ghost” elements

- Different policies (0, replicates of boundary values, etc.)



# Convolution

- This kernel forces all elements outside the image to 0

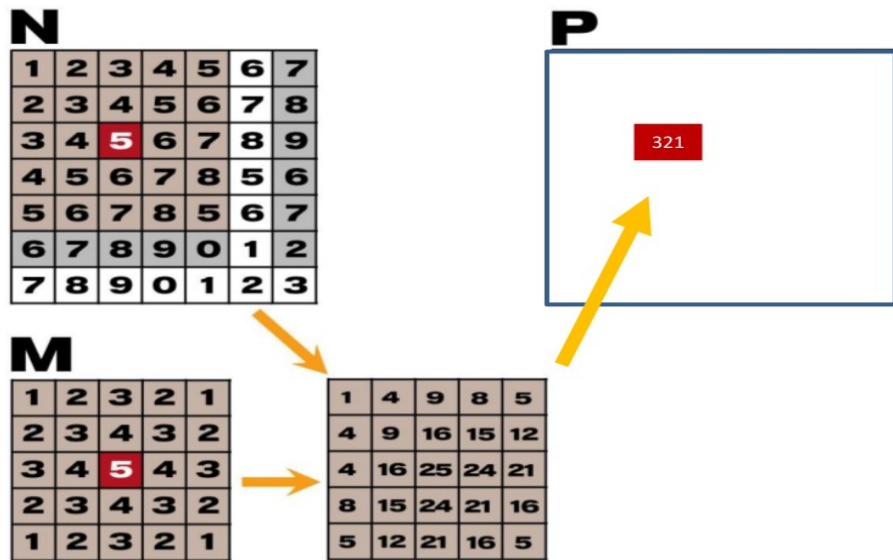
```
__global__ void convolution_1D_basic_kernel(float *N, float *M, float *P,
int Mask_Width, int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;

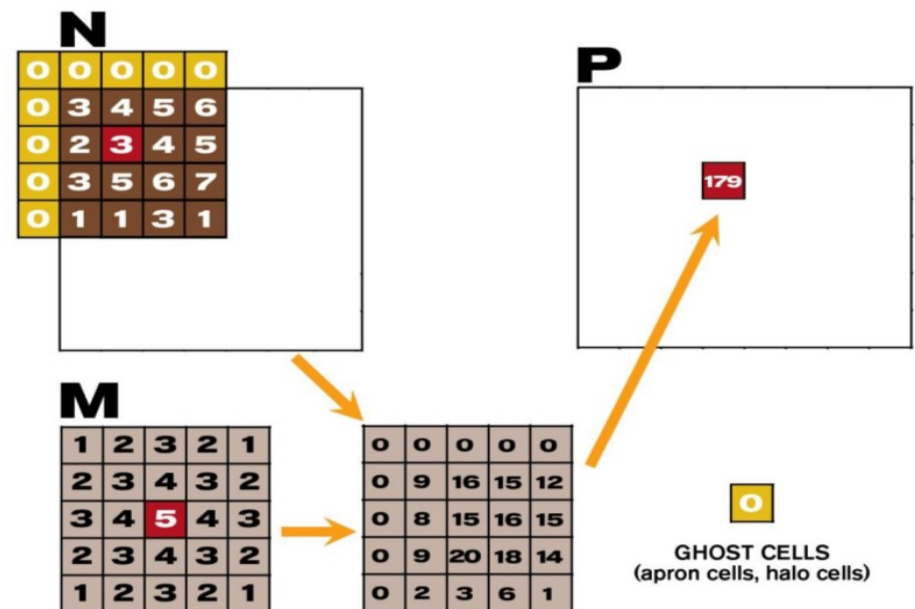
    float Pvalue = 0;
    int N_start_point = i - (Mask_Width/2);
    for (int j = 0; j < Mask_Width; j++) {
        if (N_start_point + j >= 0 && N_start_point + j < Width) {
            Pvalue += N[N_start_point + j]*M[j];
        }
    }
    P[i] = Pvalue;
}
```

# Convolution

## 2D Convolution



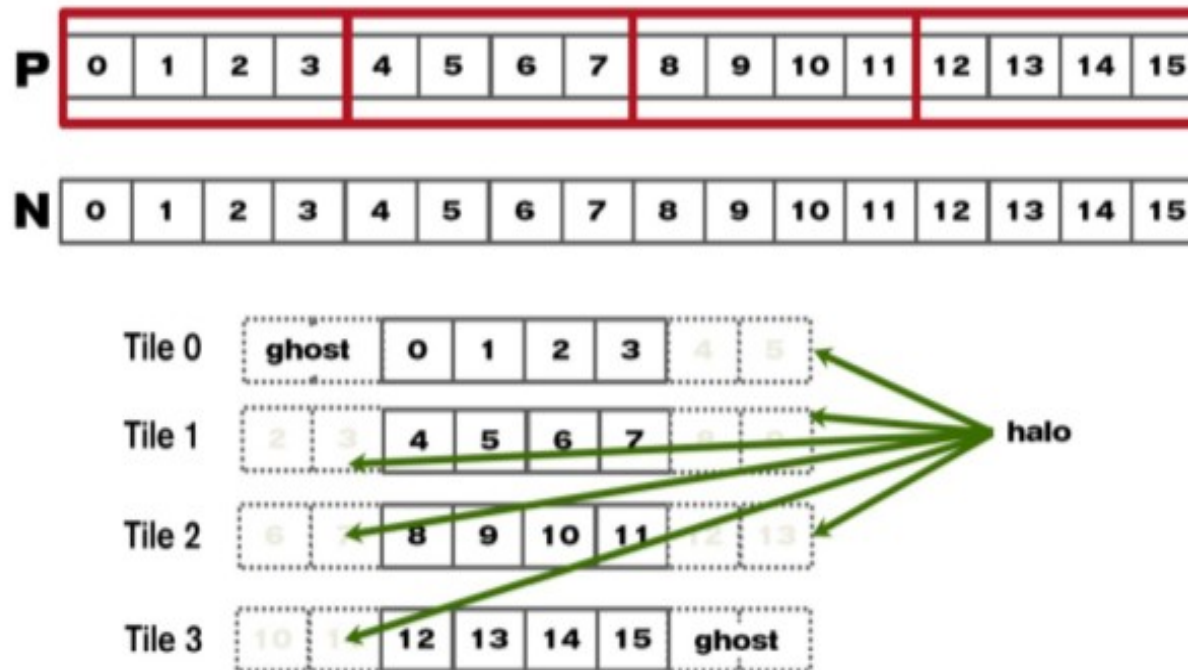
## 2D Convolution – Ghost Cells



# Convolution

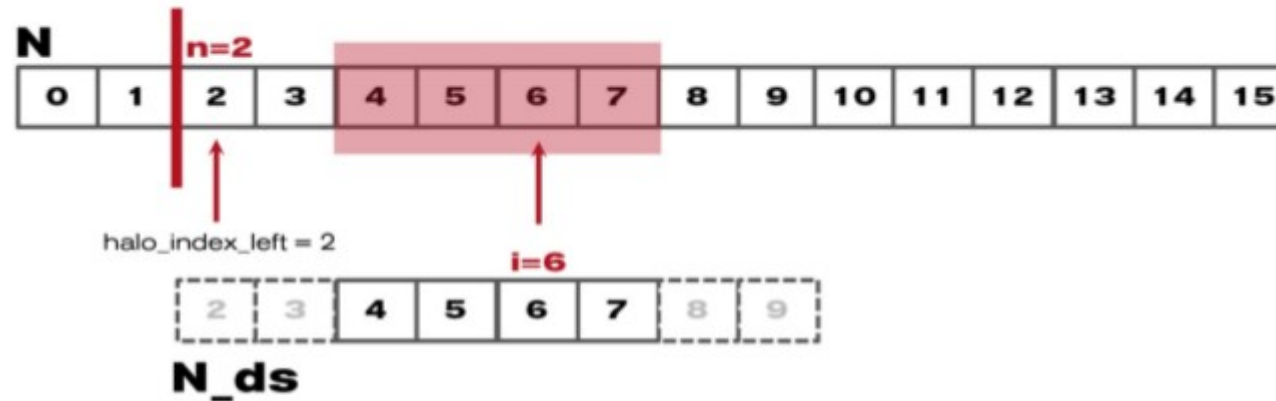
## Tiled Convolution

### Tiled 1D Convolution Basic Idea



# Convolution

## Loading the left halo

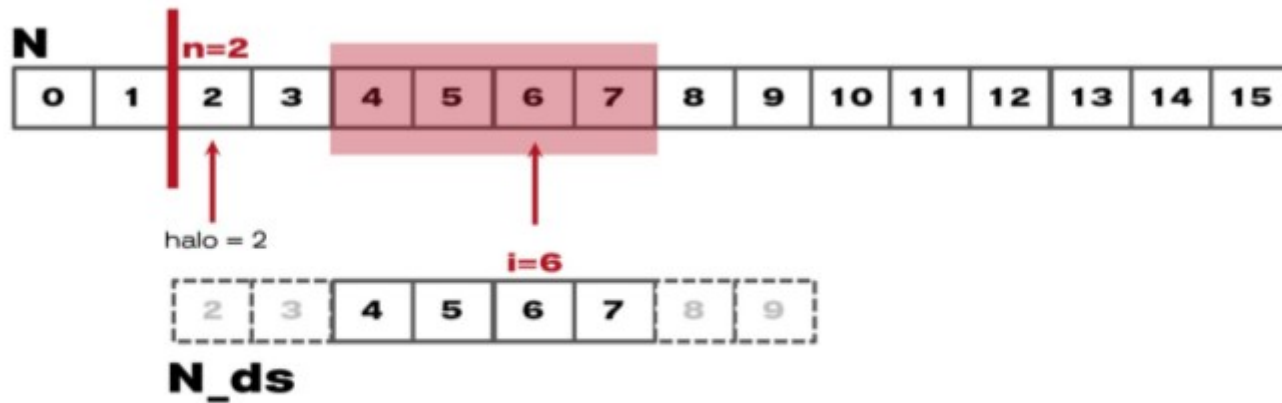


```
int n = Mask_Width/2;
int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;
if (threadIdx.x >= blockDim.x - n) {
    N_ds[threadIdx.x - (blockDim.x - n)] =
        (halo_index_left < 0) ? 0 : N[halo_index_left];
}
```



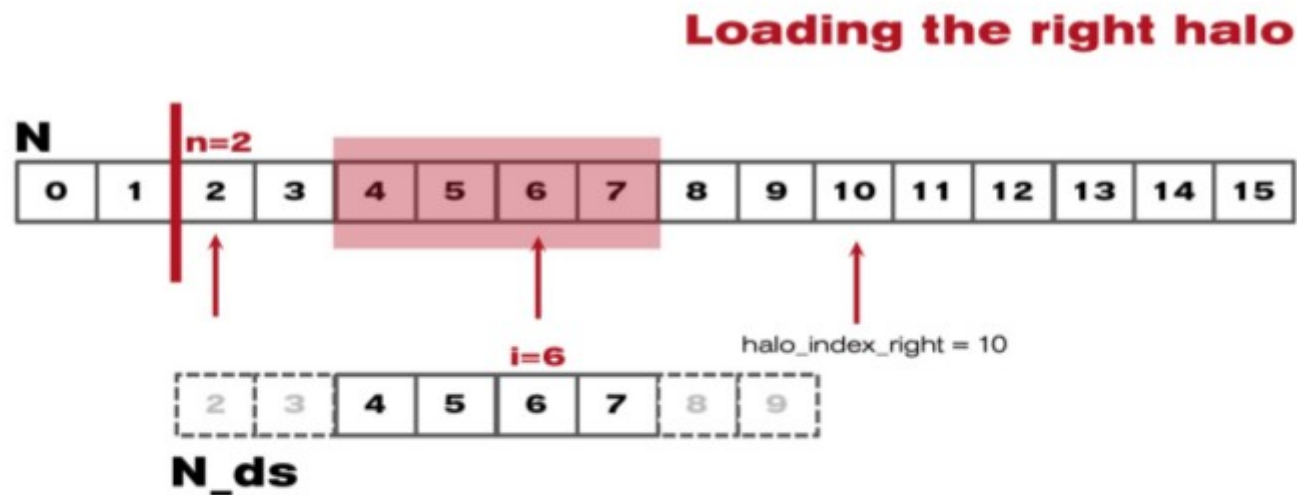
# Convolution

## Loading the internal elements



$N\_ds[n + threadIdx.x] = N[blockIdx.x * blockDim.x + threadIdx.x];$

# Convolution



```
int halo_index_right = (blockIdx.x + 1)*blockDim.x + threadIdx.x;
if (threadIdx.x < n) {
    N_ds[n + blockDim.x + threadIdx.x] =
        (halo_index_right >= Width) ? 0 : N[halo_index_right];
}
```

# Convolution

```
__global__ void convolution_1D_basic_kernel(float *N, const float __restrict__ *M,
                                           float *P, int Mask_Width, int Width) {

    int i = blockIdx.x*blockDim.x + threadIdx.x;
    __shared__ float N_ds[TILE_SIZE + MAX_MASK_WIDTH - 1];

    int n = Mask_Width/2;

    int halo_index_left = (blockIdx.x - 1)*blockDim.x + threadIdx.x;
    if (threadIdx.x >= blockDim.x - n) {
        N_ds[threadIdx.x - (blockDim.x - n)] =
            (halo_index_left < 0) ? 0 : N[halo_index_left];
    }

    N_ds[n + threadIdx.x] = N[blockIdx.x*blockDim.x + threadIdx.x];

    int halo_index_right = (blockIdx.x + 1)*blockDim.x + threadIdx.x;
    if (threadIdx.x < n) {
        N_ds[n + blockDim.x + threadIdx.x] =
            (halo_index_right >= Width) ? 0 : N[halo_index_right];
    }

    __syncthreads();

    float Pvalue = 0;
    for(int j = 0; j < Mask_Width; j++) {
        Pvalue += N_ds[threadIdx.x + j]*M[j];
    }
    P[i] = Pvalue;
}
```

# Convolution

## Shared Memory Data Reuse

**N\_ds**

Mask\_Width is 5



Element 2 is used by thread 4 (1X)

Element 3 is used by threads 4, 5 (2X)

Element 4 is used by threads 4, 5, 6 (3X)

Element 5 is used by threads 4, 5, 6, 7 (4X)

Element 6 is used by threads 4, 5, 6, 7 (4X)

Element 7 is used by threads 5, 6, 7 (3X)

Element 8 is used by threads 6, 7 (2X)

Element 9 is used by thread 7 (1X)

# References

- <http://www.eecs.umich.edu/courses/eecs570/hw/parprefix.pdf>
- [http://http.developer.nvidia.com/GPUGems3/gpugems3\\_ch39.html](http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html)
- Udacity : intro to parallel programming
- Coursera : heterogenous parallel programming