Simplified Two-Sector Weather RBC Model

1 Households

A representative household consumes a CES bundle of agricultural and non-agricultural goods. Consumption with habits is defined as

$$\tilde{c}_t = c_t - bc_{t-1}.$$

Preferences are

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\tilde{c}_t^{1-\sigma_C}}{1-\sigma_C} - \chi \, h_{u,t}^{1+\sigma_H} \right],$$

where the labor aggregator, capturing reallocation costs, is

$$h_{u,t} = \left(h_{N,t}^{1+\iota} + h_{A,t}^{1+\iota} \right)^{\frac{1}{1+\iota}}.$$

The marginal utility of consumption and the stochastic discount factor are

$$u_{c,t} = \tilde{c}_t^{-\sigma_C}, \qquad m_t = \beta \frac{u_{c,t}}{u_{c,t-1}}.$$

Intratemporal optimality conditions:

$$w_t^N \, u_{c,t} = u_t^N, \qquad w_t^A \, u_{c,t} = u_t^A,$$

with

$$u_t^N = e_t^h \, \chi \, h_{u,t}^{\sigma_H} \left(\frac{h_{N,t}}{h_{u,t}} \right)^{\iota}, \qquad u_t^A = e_t^h \, \chi \, h_{u,t}^{\sigma_H} \left(\frac{h_{A,t}}{h_{u,t}} \right)^{\iota}.$$

Euler equation:

$$m_{t+1} r_{t+1} = 1.$$

2 Production

There are two sectors: non-agriculture and agriculture.

Non-agricultural production:

$$y_t^N = e_t^z \, h_{N,t}^{1-\alpha}.$$

Optimal labor demand:

$$w_t^N = (1-\alpha) \, p_t^N \, \frac{y_t^N}{h_{N,t}}.$$

Agricultural output is

$$y_t^A = (d_t\,\ell_{t-1})^{\omega}\left(e_t^z \kappa_A\, h_{A,t}^{1-\alpha}\right)^{1-\omega},$$

where the weather damage factor is

$$d_t = (e_t^s)^{-\theta_1}.$$

Labor demand:

$$w_t^A = (1-\omega)(1-\alpha)\,p_t^A\,\frac{y_t^A}{h_{A,t}}.$$

Land evolves according to

$$\ell_t = (1-\delta_L)\,d_t\,\ell_{t-1} + \phi_t,$$

where ϕ_t is land investment.

Land cost function:

$$\phi_t = \frac{\tau}{\psi}\,x_t^{\psi}\,d_t\,\ell_{t-1}.$$

Shadow value of land ϱ_t satisfies

$$\varrho_t = m_{t+1}\left[\frac{\omega\,p_{t+1}^A y_{t+1}^A}{\ell_t} + (1-\delta_L)d_{t+1}\varrho_{t+1} + \frac{\phi_{t+1}}{\ell_t}\right].$$

FOC for land investment x_t:

$$p_t^N = \tau\,x_t^{\psi-1}\,\varrho_t\,d_t\,\ell_{t-1}.$$

3 Aggregation and Prices

Let n_t be the agricultural sector share:

$$n_t = \bar{n}\,e_t^n.$$

Aggregate output:

$$y_t = (1-n_t)\,p_t^N y_t^N + n_t\,p_t^A y_t^A.$$

Aggregate hours:

$$h_t = (1-n_t)h_{N,t} + n_t h_{A,t}.$$

Consumption demand across sectors, with non-agricultural goods:

$$(1-n_t)y_t^N = (1-\varphi)\,(p_t^N)^{-\mu}\,c_t + n_t x_t + g_y\,\bar{y}^N\,e_t^g.$$

and agricultural goods:

$$n_t y_t^A = \varphi\,(p_t^A)^{-\mu}\,c_t.$$

Relative price index:

$$1 = (1-\varphi)(p_t^N)^{1-\mu} + \varphi(p_t^A)^{1-\mu}.$$

Gross Domestic Product:

$$GDP_t = y_t - n_t p_t^N x_t.$$

Shock processes:

All shocks follow AR(1) laws of motion:

$$\log e_t^z = \rho_z \log e_{t-1}^z + \sigma_z \, \eta_t^z,$$

$$\log e_t^h = \rho_h \log e_{t-1}^h + \sigma_h \, \eta_t^h,$$

$$\log e_t^g = \rho_g \log e_{t-1}^g + \sigma_g \, \eta_t^g,$$

$$\log e_t^n = \rho_n \log e_{t-1}^n + \sigma_n \, \eta_t^n,$$

$$\log e_t^s = \rho_s \log e_{t-1}^s + \sigma_s \, \eta_t^s.$$

All innovations η_t^j are iid standard normal.