# Two Sector New Keynesian Model with Weather Shocks and Land

## 1 Households

A representative household consumes a CES bundle of agricultural and non agricultural goods and supplies differentiated labor to the two sectors.

Habit adjusted consumption:

$$\tilde{c}_t = c_t - bc_{t-1}.$$

Preferences:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\tilde{c}_t^{1-\sigma_C}}{1-\sigma_C} - \chi \, h_{u,t}^{1+\sigma_H} \right],$$

where the labor aggregator with reallocation costs is

$$h_{u,t} = \left( h_{N,t}^{1+iota} + h_{A,t}^{1+iota} \right)^{\frac{1}{1+iota}}.$$

Marginal utility of consumption and stochastic discount factor:

$$u_{c,t} = \tilde{c}_t^{-\sigma_C}, \qquad m_t = \beta \frac{u_{c,t}}{u_{c,t-1}}.$$

Intratemporal optimality:

$$w_t^N \, u_{c,t} = u_t^N, \qquad w_t^A \, u_{c,t} = u_t^A,$$

with sector specific marginal disutilities of labor

$$u_t^N = e_t^h \, \chi \, h_{u,t}^{\sigma_H} \left( \frac{h_{N,t}}{h_{u,t}} \right)^{\iota}, \qquad u_t^A = e_t^h \, \chi \, h_{u,t}^{\sigma_H} \left( \frac{h_{A,t}}{h_{u,t}} \right)^{\iota}.$$

Nominal Euler equation (one period nominal bond):

$$m_{t+1} \frac{R_t}{\pi_{t+1}} = 1,$$

where $R_t$ is the gross nominal interest rate and $\pi_t$ is gross aggregate inflation.

## 2 Production

The economy has two producing sectors: non agriculture ($N$) and agriculture ($A$).

Production is linear in TFP and concave in labor:

$$y_t^N = e_t^z \, h_{N,t}^{1-\alpha}.$$

With real marginal cost $mc_t^N$, cost minimization implies:

$$w_t^N = mc_t^N (1-\alpha) \, p_t^N \frac{y_t^N}{h_{N,t}},$$

where $p_t^N$ is the relative price of non agricultural goods.

Agricultural sector with land and weather damage. Weather affects agricultural production through a damage function. Let $d_t$ denote the weather damage factor

$$d_t = (e_t^s)^{-\theta_1}.$$

Agricultural output:

$$y_t^A = (d_t \, \ell_{t-1})^{\omega} \left( e_t^z \kappa_A \, h_{A,t}^{1-\alpha} \right)^{1-\omega}.$$

With real marginal cost $mc_t^A$, labor demand in agriculture:

$$w_t^A = mc_t^A (1-\omega)(1-\alpha) \, p_t^A \, \frac{y_t^A}{h_{A,t}},$$

where $p_t^A$ is the relative price of agricultural goods.

## 3 Price setting and nominal rigidities

Prices are sticky in both sectors. We adopt Rotemberg type quadratic adjustment costs on inflation. For sector $j \in \{N, A\}$, let $\epsilon_j$ denote the elasticity of substitution and $\kappa_j$ the adjustment cost parameter.

Denote sectoral inflation by $\pi_{j,t}$ and aggregate inflation by $\pi_t$. The New Keynesian Phillips curves in each sector take the form

$$(1-\epsilon_j) \, p_t^j y_t^j + \epsilon_j \, mc_t^j y_t^j - \kappa_j \, p_t^j \pi_{j,t}(\pi_{j,t} - 1) \, y_t^j + \kappa_j \, \mathbb{E}_t \left[ m_{t+1} p_{t+1}^j \pi_{j,t+1}(\pi_{j,t+1} - 1) \, y_{t+1}^j \right] = 0,$$

for $j = N, A$.

## 4 Land, land costs, and damage

Land evolves according to

$$\ell_t = (1-\delta_L) \, d_t \, \ell_{t-1} + \phi_t,$$

where $\phi_t$ is land investment.

Land cost function:

$$\phi_t = \frac{\tau}{\psi} \, x_t^{\psi} \, d_t \, \ell_{t-1},$$

with $x_t$ the land investment intensity.

Shadow value of land $\varrho_t$ satisfies

$$\varrho_t = \mathbb{E}_t \left\{ m_{t+1} \left[ mc^A_{t+1} \frac{\omega\, p^A_{t+1} y^A_{t+1}}{\ell_t} + (1-\delta_L) d_{t+1} \varrho_{t+1} + \frac{\phi_{t+1}}{\ell_t} \right] \right\}.$$

FOC for land investment $x_t$:

$$p^N_t = \tau\, x_t^{\psi-1}\, \varrho_t\, d_t\, \ell_{t-1}.$$

# 5 Aggregation, demand, and prices

Let $n_t$ be the agricultural sector share, driven by a reallocation shock:

$$n_t = \bar{n}\, e^n_t.$$

Aggregate output:

$$y_t = (1-n_t)\, p^N_t y^N_t + n_t\, p^A_t y^A_t.$$

Aggregate hours:

$$h_t = (1-n_t) h_{N,t} + n_t h_{A,t}.$$

Non agricultural goods:

$$(1-n_t) y^N_t = (1-\varphi)\,(p^N_t)^{-\mu}\, c_t + n_t x_t + g_y\, \bar{y}^N\, e^g_t.$$

Agricultural goods:

$$n_t y^A_t = \varphi\,(p^A_t)^{-\mu}\, c_t.$$

## 5.1 Relative price index

The CES price index is

$$1 = (1-\varphi)(p^N_t)^{1-\mu} + \varphi (p^A_t)^{1-\mu}.$$

Inflation measures, let $P_t$ denote the aggregate price index. Gross inflation is

$$\pi_t = \frac{P_t}{P_{t-1}}.$$

Sectoral inflation rates are

$$\pi_{N,t} = \frac{P^N_t}{P^N_{t-1}}, \qquad \pi_{A,t} = \frac{P^A_t}{P^A_{t-1}}.$$

Gross Domestic Product:

$$GDP_t = y_t - n_t p^N_t x_t.$$

# 6 Monetary policy

Monetary policy follows a Taylor rule in logs around the steady state:

$$\log\left(\frac{R_t}{\bar{R}}\right) = \rho\, \log\left(\frac{R_{t-1}}{\bar{R}}\right) + (1-\rho)\left[\phi_y \log\left(\frac{GDP_t}{\overline{GDP}}\right) + \phi_\pi \log(\pi_t)\right],$$

where $\bar{R}$ and $\overline{GDP}$ denote steady state values.

# 7 Shock processes

All exogenous processes follow AR(1) dynamics:

$$\log e_t^z = \rho_z \log e_{t-1}^z + \sigma_z\, \eta_t^z,$$

$$\log e_t^h = \rho_h \log e_{t-1}^h + \sigma_h\, \eta_t^h,$$

$$\log e_t^g = \rho_g \log e_{t-1}^g + \sigma_g\, \eta_t^g,$$

$$\log e_t^n = \rho_n \log e_{t-1}^n + \sigma_n\, \eta_t^n,$$

$$\log e_t^s = \rho_s \log e_{t-1}^s + \sigma_s\, \eta_t^s.$$

All innovations $\eta_t^j$ are iid standard normal.