Q1a: It is clear that if $b = \partial c$, then $\partial b = \partial^2 c = 0$

Q1b: Suppose that there is a chain B such that $\partial B = b$. Consider the 1-form $\omega = \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$. Hence by Stoke's Theorem on Chains, we have that $\int_{\partial B} \omega = \int_B d\omega$. Thus we compute

$$\int_{\partial B} \omega = \int_b \omega = \int_{[0,1]} b^*\omega = \int_{[0,1]} 2\pi \sin^2(2\pi t)dt + 2\pi \cos^2(2\pi t)dt \int_{[0,1]} 2\pi dt = 2\pi$$

However we know that $d\omega = 0$ and so we have that $\int_B d\omega = 0$. We obtain a contradiction and conclude that b is not in the image of the boundary operator. We will now verify using Stoke's theorem that $\partial b = 0$:

$$0 = \int_b d\omega = \int_{\partial b} \omega$$

We see that the integral is 0 and conclude that $\partial b = 0$.