Q2: First let α, β be the radii of the concentric circles, and assume that $\alpha < \beta$. If

$$f = \frac{az+b}{cz+d}$$

is a fractional linear transformation with the supposed properties, we can assume without loss of generality that the circles S_α, S_β are centered about 0, and $f(S_\alpha)$ and $f(S_\beta)$ will also be centered around 0, since we can always translate the images to 0 while f will remain a fractional linear transformation. Consider any pair of lines $l_1(t), l_2(t)$ which intersect at a right angle and pass through the origin. Note that they must also intersect at ∞. Since f is a conformal mapping and it takes lines to lines, we have that $f(l_1(t))$ and $f(l_2(t))$ must also intersect at a right angle at the origin. Therefore one of 2 cases must hold. Either $f(0) = 0, f(\infty) = \infty$, or $f(\infty) = 0$ and $f(0) = \infty$. Consider the first case. If we have that $f(0) = 0$, this implies that $b = 0$. Furthermore since $f^{-1}(\infty) = -\frac{d}{c} = \infty$, this implies that $c = 0$. Hence f takes the form

$$f(z) = \frac{az}{d} = \lambda z$$

for $\lambda = \frac{a}{d}$. We compute that

$$\frac{f(\beta)}{f(\alpha)} = \frac{\lambda\beta}{\lambda\alpha} = \frac{\beta}{\alpha}$$

Now consider the second case. If we have that $f(0) = \infty$, then it must be that $d = 0$. Since $f^{-1}(0) = -\frac{b}{a} = \infty$, this implies that $a = 0$. Hence f takes the form

$$f(z) = \frac{b}{cz} = \frac{\lambda}{z}.$$

We can evaluate that

$$\frac{f(\beta)}{f(\alpha)} = \frac{\frac{\lambda}{\beta}}{\frac{\lambda}{\alpha}} = \frac{\alpha}{\beta}$$

The ratio is conserved. Thus we are done.