

Q4: Given $\mathcal{A} = \{(U_i, \varphi_i)\}, \mathcal{B} = \{(V_j, \psi_j)\}$ The following chain of equivalences hold:

$$\begin{aligned} id \text{ is a diffeomorphism} &\iff \forall i, j, \varphi_i \circ id \circ \psi_j^{-1} \in \mathcal{C}^\infty \\ &\iff \forall i, j, \varphi_i \circ \psi_j^{-1} \in \mathcal{C}^\infty \\ &\iff \mathcal{A}, \mathcal{B} \text{ lie in the same maximal atlas} \end{aligned}$$