

Q3a: Without loss of generality, $n < p$. If there existed a diffeomorphism $h : \mathbb{R}^n \rightarrow \mathbb{R}^p$ for $n \neq p$, then we have that

$$D(h \circ h^{-1}(a)) = I = h'(h^{-1}(a)) \circ h^{-1'}(a).$$

The identity matrix has rank p , but h' and $h^{-1'}$ both have rank n . We obtain a contradiction.

Q3b: Let U be an open neighborhood of 0 in \mathbb{H}^n . We can write $U = V \cap \mathbb{R}^n$ for some open $V \subset \mathbb{H}^n$. If there exists a diffeomorphism f from U to W for some open $W \subset \mathbb{R}^n$. We have that f extends to a diffeomorphism $\tilde{f} : V \rightarrow \mathbb{R}^n$. Since \tilde{f} is a diffeomorphism, we have that $\tilde{f}(V)$ is open. Furthermore, $\tilde{f}(V) \cap \tilde{f}(V \cap \mathbb{H}^{n^c})^c$ cannot be open, since \mathbb{H}^n is closed in \mathbb{R}^n , so its complement is open and thus any intersection with an open set is open. Pushing this open set by \tilde{f} gives us an open set, and taking the set difference will yield us a set that is not open. However, \tilde{f} is a diffeomorphism, $\tilde{f}(V) \cap \tilde{f}(V \cap \mathbb{H}^{n^c})^c = \tilde{f}(U) = f(U)$. Which contradicts the assumption that $f(U)$ is open.