

Q2i: Let  $g \in G$ . Suppose that  $a, b$  are both multiplicative inverses of  $g$ . Using the properties of group multiplication, we get that

$$bg = e \implies (bg)a = ea \implies b(ga) = a \implies b = a$$

Hence the multiplicative identity is unique.

Q2ii: Let  $g, h \in G$ . We let  $(gh)^{-1} = a$ . We see that

$$\begin{aligned} (gh)a &= e \\ g^{-1}(gh)a &= g^{-1}e && \text{(multiply left sides by } g^{-1}) \\ (g^{-1}g)(ha) &= g^{-1} && \text{(by associativity and identity)} \\ ha &= g^{-1} && \text{(by inverse)} \\ (h^{-1}h)a &= h^{-1}g^{-1} && \text{(multiply left sides by } h^{-1}) \\ a &= h^{-1}g^{-1} && \text{(by inverse)} \end{aligned}$$

Thus we see that  $a = (gh)^{-1} = h^{-1}g^{-1}$  as desired.

Q2iii: First, note that by multiplying  $gh = e$  by  $h^{-1}$  to the right, we get that  $g = h^{-1}$ . Similarly, when we multiply  $gh = e$  by  $g^{-1}$  to the left we get that  $h = g^{-1}$ . We can now verify that indeed

$$\begin{aligned} gh &= e \\ (hg)h &= he && \text{(multiply both sides by } h) \\ (hg)(hg) &= (he)g && \text{(multiply both sides by } g, \text{ associativity)} \\ (g^{-1}g)(h^{-1}h) &= hg \\ ee &= hg && \text{(by inverse)} \\ e &= hg && \text{(by identity)} \end{aligned}$$

Thus we have that  $gh = e = hg$  as desired.