Q5: Let $f : C_{max} \to C_{int}$ by the identity function. It is clearly a bijection, since the sets C_{max} and C_{int} are identical as sets, only differing by the metric on them. We see f is also linear, since if $g, h \in C_{max}$,$\lambda \in \mathbb{R}$ we see $f(\lambda g + h) = \lambda g + h = \lambda f(g) + f(h)$. It remains to show that it is continous. Let $\varepsilon > 0$, and take any $\delta < \varepsilon$. We see that

$$max(|f(x) - g(x)|) < \delta \implies \int_0^1 |f(x) - g(x)| \leq \int_0^1 \delta = \delta < \varepsilon$$

Hence the identity mapping is continuous. We now claim that the inverse map $f^{-1} : C_{int} \to C_{max}$ is not continuous. Take $f = 0, g(x) = \frac{1}{(x+1)^n}$ for some $n \in \mathbb{N}$. Notice that f and g are both continuous on $[0, 1]$. Take $\varepsilon = 1$, and $\delta > 0$ We notice that $max(|f(x)-g(x)|) = 1$ regardless of our choice of n, with the maximum occuring at $x = 0$. Note as well that

$$\begin{aligned}\int_0^1 |0 - \frac{1}{(x+1)^n}|dx &= \int_0^1 \frac{1}{(x+1)^n}dx \\ &= \frac{2^n - 2}{(n-1)2^n} \\ &= \frac{(1 - 2^{n-1})}{(n-1)}\end{aligned}$$

We can make the quanitity $\frac{(1-2^{n-1})}{(n-1)} < \delta$ for sufficiently large n. Hence f^-1 is not continuous.