Q1:

A1: We will determine the interiour, boundary and exteriour of A_1. First we consider the exteriour. Since A_1 is the closed unit ball, the exteriour must be its compliment in $\mathbb{R}^n$. Thus, ext $A_1 = \{ x \in \mathbb{R}^n : \|x\| > 1 \}$. Now we consider the boundary of A_1. If we take an open ball of radius ϵ around any x such that $\|x\| = 1$, this ball will intersect both A_1 and A_1^c, since it will contain at least one point with a norm greater than 1. It will also contain our chosen point, x which belongs to A_1. Now suppose that $x \in bdA_1$. then, consider then open ball with radius ϵ centered at x. This ball will contain points both in A_1 and A_1^c so long as $\|x\| = 1$. If we chose a point y with $\|y\| < 1$ then we could take ϵ to be $1 - \|y\|$. This would be fully contained in the set A_1. Thus, bd $A_1 = \{ x \in \mathbb{R}^n : \|x\| = 1\}$. Since the interiour, boundary and exteriour are disjoint from eachother and have union $\mathbb{R}^n$, the interiour of A_1 must be whatever is left over. So int $A_1 = \{ x \in \mathbb{R}^n : \|x\| < 1 \}$.

A2: We will first determine the exteriour of A_2. Suppose that $x \in extA_2$. Then there exists some $\epsilon > 0$ with $B_\epsilon(x)$ disjoint from A_2. x will never have norm 1, since the ball centered at it will always contain itself and so have nonempty intersection with A_2. Therefore $\|x\| > 1$ or $\|x\| < 1$. Now suppose that $\|x\| > 1$ or $\|x\| < 1$. Choose $\epsilon = \frac{|\|x\|-1|}{2}$. This ball by construction will contain no points with norm 1. Thus ext $A_2 = \{x \in \mathbb{R}^n : \|x\| > 1 \text{or} \, \|x\| < 1\}$. Now we will determine the interiour of A_2. We suppose that $x \in intA_2$. Then there is some $\epsilon > 0$ such that $B_\epsilon(x) \subset A_2$. However, every ball about a point with norm 1 will contain some other points with norm<1 or norm >1, by definition of the ball. Thus $\emptyset \supset intA_2$. Trivially, $\emptyset \subset A_2$. Thus we see that $intA_2 = \emptyset$. Since the interiour, boundary and exteriour are disjoint from eachother and have union of $\mathbb{R}^n$, the boundary of A_2 must be whatever is left over i.e. $bdA_2 = A_2$.

A3: We claim that $bdA_3 = \mathbb{R}^n$. Clearly, $bdA_3 \subset \mathbb{R}^n$. Now suppose that $x \in \mathbb{R}^n$. Let $\epsilon > 0$. Consider the ϵ cube around x, $C = (x_1 - \epsilon, x_1 + \epsilon) \times \ldots (x_n - \epsilon, x_n + \epsilon)$. By the density of rationals in $\mathbb{R}$, for each open interval $(x_i - \epsilon, x_i + \epsilon)$ we can find some rational $q_i \in (x_i - \epsilon, x_i + \epsilon)$, with $(q_1, \ldots q_n) \in A_3$. Similarly, we can find an irrational r_i with $r_i \in (x_i - \epsilon, x_i + \epsilon)$, and $(r_1, \ldots r_n) \notin A$. Our choice of x was arbitrary and so $bdA_3 = \mathbb{R}^n$. Since the union of the boundary, exteriour and interior is $\mathbb{R}^n$ and they are pairwise disjoint, it follows that $intA_3 = extA_3 = \emptyset$.