

Q7:

It suffices to show that any increasing function is integrable. Let  $\varepsilon > 0$ . Choose partition  $P$  of  $[a, b]$  such that for each  $S \in P$ ,  $\text{vol}(S) < \frac{\varepsilon}{f(b) - f(a)}$ . If we let each  $S = [x_i, x_{i+1}]$  we compute that

$$\begin{aligned}
 & U(f, P) - L(f, P) \\
 &= \sum_{S \in P} [M_S(f) - m_S(f)] \cdot \text{vol}(S) \\
 &= \sum_{S \in P} [f(x_i) - f(x_{i-1})] \cdot \text{vol}(S) && \text{(since } f \text{ is increasing it attains sup and inf on boundary of } S) \\
 &= f(b) - f(a) \cdot \text{vol}(S) && \text{(by telescopic summation)} \\
 &< \varepsilon
 \end{aligned}$$

Since  $f$  is integrable the set of discontinuities must be of measure 0 by Spivak theorem 3-8.