Q1: Suppose that f is not injective at some points z_1, z_2. Then we have that

$$\Big| \int_{z_1}^{z_2} f'(z) - f'(0) dz \Big| = |f(z_2) - f(z_1) - f'(0)(z_2 - z_1)| = |f'(0)| \cdot |z_2 - z_1|.$$

We also have that

$$\Big| \int_{z_1}^{z_2} f'(z) - f'(0) dz \Big| \leq \Big| \int_{z_1}^{z_2} |f'(z) - f(0)| dz \Big| < \Big| \int_{z_1}^{z_2} |f'(0)| dz \Big| = |f'(0)| \cdot |z_1 - z_2|.$$

Putting these two facts together yields us the absurdity

$$|z_1 - z_2| < |z_1 - z_2|.$$

Thus f must be injective.