

Q1a: Define the function $g(x, y) = y - f(x)$. This is clearly a C^∞ function, and $g^{-1}\{0\} = \Gamma_f$. To show that Γ_f is an n manifold it is enough to show that $\text{rank}(Dg) = n$. Notice that

$$Dg = \left[-\frac{\partial f}{\partial x} | I \right]$$

This will be an $(n) \times (n + m)$ matrix, and hence will have a rank of m , since the identity matrix will span a m dimensional space. Therefore we have that Γ_f is a manifold, if f is smooth.

Q1b: Consider the graph of the function $f(x) = x^{\frac{1}{3}}$. This will be a smooth manifold, since in every neighborhood, the function locally looks like \mathbb{R} , but $x^{\frac{1}{3}}$ is not C^1 since its derivative is not continuous at 0.