

Q2: We claim $m^{\frac{1}{n}} \in \mathbb{Q}$ if and only if $m = k^n$ for some $k \in \mathbb{N}$. We will prove the forward implication first. Suppose that $m^{\frac{1}{n}} \in \mathbb{Q}$. Then for $a, b \in \mathbb{N}$ we have that $m = \frac{a^n}{b^n}$. Therefore, $b^n \cdot m = a^n$. By the fundamental theorem of arithmetic, a, b and m admit a unique prime factorization of the form $a = \prod_{i=1}^n p_i^{e_i}$, $b = \prod_{i=1}^k p_i^{f_i}$ and $m = \prod_{i=1}^k p_i^{g_i}$ for primes p_i , and $e_i, f_i, g_i \in \mathbb{N}$. We see that

$$\prod_{i=1}^k p_i^{g_i} \cdot \prod_{i=1}^k p_i^{ne_i} = \prod_{i=1}^k p_i^{g_i + ne_i} = \prod_{i=1}^k p_i^{nf_i}$$

We see that g_i must be a multiple of n , and we conclude that m is an n' th power. Now suppose that $m = k^n$ for some k . We have that $(k^n)^{\frac{1}{n}} = k$. Thus we are done.