Q6: Let $\alpha \in \mathbb{C}$, $|z| < 1$. Let $\sum_{k=0}^{\infty} a_n z^n = (1+z)^\alpha$. First, note that $1 = (1+0)^\alpha = a_0$. We claim that each a_n takes the form

$$a_n = \binom{\alpha}{n}$$

We note that differentiating n times, we get that

$$\frac{\partial^n}{\partial z^n}(1+z)^\alpha = (\alpha)\cdot(\alpha-1)\cdots(\alpha-n+1)(1+z)^\alpha = \frac{\partial^n}{\partial z^n}\sum_{k=0}^{\infty} a_n z^n = \sum_{k=n}^{\infty} n! a_n z^{n-k}$$

At $z = 0$ we have that

$$(\alpha)(\alpha-1)\cdots(\alpha-n+1) = n! a_n$$

Since every term in the power series vanishes after a_n, and we conclude that

$$a_n = \binom{\alpha}{n}$$

Therefore we have that

$$(1+z)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{n} z^n$$