5.4.49a: It is sufficient to show that any element of the basis i.e. sets of the form

$$U_{f,\varepsilon}(x) = \{y \in \mathfrak{X} : |f(x) - f(y)| < \varepsilon\}$$

is unbounded. Take any $v \in f^{-1}(\{0\})$ nonzero. Then for all α, $\alpha v + x \in U_{f,\varepsilon}$. For the weak * topology, the basis elements take the form $V_{f,\varepsilon} = \{g \in X^* : \|f - g\| < \varepsilon\}$. It is sufficient to show that these sets are unbounded. For all $f \in V_{f,\varepsilon}$, $\sup_{\|x\|=1} \|f(x) - g(x)\| = \sup \hat{x}(f - g) < \varepsilon$. Taking l nonzero such that $\hat{x}(l) = 0$, we get that for all scalars α, $f + \alpha l \in V_{f,\varepsilon}$. This is unbounded.

5.4.49b: If $E \subset \mathfrak{X}$ is a bounded subset, then so is its weak closure by Q2b. However by part a we have that the interiour must be empty. The result for bounded $F \subset \mathfrak{X}^*$ follows in the same way.

5.4.49c: We can $E_n = \{x : \|x\| \leq n\}$. We have that $\mathfrak{X} = \bigcup_{n\in\mathbb{N}} E_n$. Thus by b, $\mathfrak{X}$ is meager in the weak topology. Defining $F_n = \{f \in \mathfrak{X}^* : \|f\| \leq n\}$. We have that analogously $\mathfrak{X}^*$ is meager in the weak * topology.

5.4.49d: Suppose that some translation invariant metric d defines the weak * topology on $\mathfrak{X}^*$. Let $\langle f_n \rangle$ be a cauchy sequence. Then for any $V_{f,\varepsilon}$ as defined in a, we have that (this problem set is too long i cant finish it sorry.)