

Q1: Let  $(g, e) \in G \times \{e\}$ . Let  $h = (h, h') \in P$ . We compute that

$$h(g, e)h^{-1} = (h, h')(g, e)(h^{-1}, h'^{-1}) = (hg, h')(h^{-1}, h'^{-1}) = (hgh^{-1}, e)$$

Which is clearly an element of  $G \times \{e\}$ . We now claim that  $G \times \{e\}$  is isomorphic to  $G'$ . Consider the map  $\phi : P \rightarrow G'$  defined by  $\phi(g, g') = g'$ . This will have a kernel of  $G \times \{e\}$  and is clearly surjective onto  $G'$ . Hence by the first isomorphism theorem, there exists an isomorphism from  $G \times G' / G \times \{e\}$  to  $G'$ .