Q6: Suppose that in some neighbourhood U_{z_0} of $z_0 \in I$, f has a power series given by

$$f(z) = \sum_{n=0}^{\infty} a_n(z - z_0)^n$$

with a radius of convergence of r. We claim that $f(z)$ absolutely converges on a ball in $\mathbb{C}$ given by $|z - z_0| < r$. We compute that

$$|\sum_{n=0}^{\infty} a_n(z - z_0)^n| \leq \sum_{n=0}^{\infty} |a_n| \cdot |(z - z_0)|^n < \infty$$

We now claim that the coefficients of the power series are the same regardless of which z_0 you choose. Take $z_0, z_1 \in I$ such that the neighborhoods containing them where the power series of f converges are not disjoint. Take $z_2 \in I$ in this intersection. There must be some power series expansion. By analytic continuation, it must agree with the power series of f at z_0 and z_1. Hence the power series is the same along I, and the neighborhood around I where it converges. If we take the unions of each $B_r(z_0)$ we have a connected open set containing I on which f is analytic.