Q2:
For each $x \in A$ we define $g_x : U_x \to \mathbb{R}$ to be the function which agrees with f on $U_x \cap A$. It will be C^∞. Note that $\{U_x\}$ is an open cover of A. Thus by $PO1$ there exists a countable collection $\Phi = \{\varphi_i\}$ where $Supp(\varphi_i)$ is contained in some U_x and, at any given point finitely many φ are nonzero, and $\sum_i \varphi_i(x) = 1$ for all x. We can define g as an extension of f in the following way. For $a \in A$ set

$$g(a) = \sum_{\varphi_x(a) \neq 0} \varphi_x(a) \cdot g_x(a) = \sum_{\varphi(a)_x \neq 0} \varphi_x(a) \cdot f(a) = f(a)$$

The above sum is well defined, since if $a \notin U_x$ for some x, $g_x(a)$ does not make sense and $\phi_x(a) = 0$, which would not be considered in our sum. On the interior of each U_x we will have that $\phi \cdot g_x$ is C^∞, on the exterior of U_x, we will have that $\phi \cdot g_x = 0$. On the boundary of U_x there will be some other $U_{x'}$ such that $\phi_{x'} \cdot g_x = 0$. By local finiteness, the above sum becomes finite and hence g will be C^∞, since it is finite sum and product of C^∞ functions. We see that g is defined on $\bigcup_{x \in A} U_x \supset A$. Hence ,we have extended f.