Problem 2. *Jack:* $\star \mathrm{d}f = -i\mathrm{d}f \iff f$ *discrete holomorphic.*

Suppose that $\star \mathrm{d}f = -i\mathrm{d}f$. The following equality is true:

$$\int_e \star \mathrm{d}f = \int_e -i\mathrm{d}f \tag{1}$$

We compute:

$$\int_e \star \mathrm{d}f = -\rho(e^*)\int_{e^*} \mathrm{d}f = -\rho(e^*)f(\partial e^*),$$

and

$$-i\int_e \mathrm{d}f = -if(\partial e).$$

These integrals are equal, so

$$if(\partial e) = \rho(e^*)f(\partial e^*) \iff i\frac{f(\partial e)}{l(e)} = \frac{f(\partial e^*)}{l(e^*)},$$

where we use the definition of $\rho(e^*)$. this is exactly the cauchy-riemann equation for discrete holomorphic functions. To see the reverse impliciation perform the steps of the proof backwards.