Q3: If $f \equiv 0$ then this is clearly true. Therefore we can assume that f is not identically 0. By the proof of Folland 6.10, we have that

$$\|f\|_q \leq \|f\|_p^{\frac{p}{q}} \|f\|_\infty^{1-\frac{p}{q}}.$$

Taking the lim sup as $q \to \infty$ we get that

$$\lim_{q\to\infty} \sup \|f\|_q \leq \lim_{q\to\infty} \sup \|f\|_p^{\frac{p}{q}} \|f\|_\infty^{1-\frac{p}{q}} = \|f\|_\infty.$$

Now choose M with $0 < M < \|f\|_\infty$. We define the set $E_M = \{x : |f(x)| \geq M\}$. Since

$$\|f\|_q^q = \int |f|^q d\mu,$$

monotonicity of the integral implies that

$$M^q \mu(E_M) \leq \|f\|_q^q$$

and so

$$M\mu(E_m)^{\frac{1}{q}} \leq \|f\|_q.$$

Therefore

$$M \lim_{q\to\infty} \inf \mu(E_M)^{\frac{1}{q}} \leq \lim_{q\to\infty} \inf \|f\|_q.$$

Taking the limit as $M \to \|f\|_\infty$ yields

$$\|f\|_\infty \leq \lim_{q\to\infty} \inf \|f\|_q.$$

Thus we have that

$$\lim_{q\to\infty} \sup \|f\|_q \leq \|f\|_\infty \leq \lim_{q\to\infty} \inf \|f\|_q.$$

Thus we have equality.