

Q3:

Define $A = \{(u, v) : 1 < u < \sqrt{2}, 1 < v < 2\}$. Let $g(u, v) = (\frac{u}{v}, vu)$. On the set A , g will be 1-1 and onto on $g(A)$. Note that $g(A) = B$ so we can apply the COV theorem to compute $\int_B f$. We have that $g' = \begin{bmatrix} \frac{1}{v} & v \\ \frac{-u}{v^2} & u \end{bmatrix}$. Therefore, $\det g' = \frac{2u}{v}$. This will always be nonzero in our domain, so by the COV theorem

$$\begin{aligned}
 \int_B f &= \int_A f \circ g |\det g'| \\
 &= \int_A u^5 v \cdot \frac{2u}{v} \\
 &= \int_1^4 \int_1^{\sqrt{2}} 2u^6 du dv && \text{(by Fubini's Theorem)} \\
 &= \int_1^4 \left[\frac{2u^7}{7} \right]_1^{\sqrt{2}} dv \\
 &= \frac{2}{7} \int_1^2 (8\sqrt{2} - 1) dv \\
 &= \frac{2}{7} (8\sqrt{2} - 1)
 \end{aligned}$$