Q4:
Since Df has rank n, assume $WLOG$ that the last n columns of Df are linearly independant. Define $F(x,y) = (x, f(x,y))$. This will be a function from $\mathbb{R}^{k+n}$ to $\mathbb{R}^{k+n}$. We have that $DF = \begin{bmatrix} I & 0 \\ \frac{\partial f}{\partial(x_1 \ldots x_k)} & \frac{\partial f}{\partial(x_{k+1} \ldots x_{k+n})} \end{bmatrix}$. At the point a, $detDF = \frac{\partial f}{\partial(x_{k+1} \ldots x_{k+n})} \neq 0$ since we assume that the columns of $\frac{\partial f}{\partial(x_{k+1} \ldots x_{k+n})}$ are linearly independant. Since the differential of F is invertible, we can apply the Inverse function theorem. So there must be an open neighbourhood $U \times V \ni a$ and an open neighbourhood $W_1 \times W_2 \ni (a_1, \ldots a_k, 0 \ldots 0)$ along with a $G : W_1 \times W_2 :\to U \times V$. We now show that for all $c \in W_2$, there is some $x \in U \times V$ with $f(x) = c$. First, note that $G(x,y) = (x, h(x,y))$ for some h. We see that

$$(a_1, \ldots a_k, c) = F(G(a_1, \ldots a_k, c)) = F(a_1, \ldots a_k, h(a_1, \ldots a_k, c)) = (a_1, \ldots a_k, f(a_1, \ldots a_k, h(a_1, \ldots a_k, c)))$$

. We see that $c = f(a_1, \ldots a_k, h(a_1, \ldots a_k, c))$ as desired.