Q2: If $v_1 \dots v_n$, and $I \in \underline{n}^n$, then $\det(v_I) = \omega_J(v_I)$ where $J = \{1, 2, \dots, n\}$. We know this is true since if I has any repeating elements, $\omega_J(v_I) = 0$, and if $I \in \underline{n}^n_a$. Let $\tau \in S_n$ be the unique permutation which satisfies $\tau(J) = I$, then

$$\omega_J(v_I) = (\sum_{\sigma \in S_n} (-1)^\sigma \varphi_J \circ \sigma^*)(v_I) = \sum_{\sigma \in S_n} (-1)^\sigma \varphi_I(v_{\sigma(I)}) = (-1)^\tau$$

This lines up with what we know about applying the determinant to the basis of a vector space. Now consider a collection of n vectors, $u_1, \dots, u_n$, where each $u_i = \sum_{j=1}^n a_{ij} v_j$ We compute $\det(u_1, \dots u_n)$ as follows:

$$\begin{aligned}
\det(u_1, \dots, u_n) &= \det(\sum_{j_1=1}^n a_{1j_1} v_{j_1}, \dots, \sum_{j_n=1}^n a_{n,j_n} v_{j_n}) \\
&= \sum_{j_1=1}^n a_{1j_1} \det(v_{j_1}, \sum_{j_2=1}^n a_{2j_2} v_{j_2}, \cdots \sum_{j_n=1}^n a_{n,j_n} v_{j_n}) \\
&= \sum_{j_1=1}^n a_{1j_1} (\sum_{j_2=1}^n a_{2j_2} \det(v_{j_1}, v_{j_2}, \dots u_n)) \\
&\vdots \\
&= \sum_{j_1=1}^n a_{1j_1} \sum_{j_2=1}^n a_{2j_2} \cdots \sum_{j_n=1}^n a_{nj_n} \det(v_{j_1}, \dots v_{j_n}) \\
&= \sum_{\sigma \in S_n} (-1)^\sigma \prod_{j=1}^n a_{i_j \sigma(j)} \\
&= \sum_{\sigma \in S_n} (-1)^\sigma \varphi_1 \otimes \cdots \otimes \varphi_n \circ \sigma^*(u_1, \dots u_n) \\
&= \omega_I(u_1 \dots u_n)
\end{aligned}$$