

Q2: Let  $G = D_8$ ,  $K = \{e, \sigma\rho, \sigma\rho, \rho^2\}$ ,  $H = \{e, \sigma\rho\}$ . We claim that  $H \triangleleft K \triangleleft G$ . Since  $[G : K] = 2$  we have that  $K \triangleleft G$ . Note that as well  $[K : H] = 2$ , so  $K \triangleleft H$ . We claim that  $H$  is not normal in  $G$ . We have that

$$\sigma H = \{\sigma, \rho\}$$

and

$$H\sigma = \{\sigma, \rho^3\}.$$

We have the desired result.