

Q2: Let $p(x) = x^3 + x + 1$. If p is reducible, it must split into a factors of degree 2 and 1. By prop.11 of section 9.4 (Dummit and Foote), p will be reducible if $p(-1)$ or $p(1)$ are 0. We see that clearly neither are 0. Thus $p(x)$ is irreducible over \mathbb{Q} .