Q2: We will verify that ω satisfies the 3 conditions of being a measure. First note that the cardinality of $\emptyset$ is 0 by definition, so $\omega(\emptyset) = 0$. Next, let $A \subset B$. There exists an injection $\iota : A \to B$ via inclusion, which by definition means that $\omega(A) \leq \omega(B)$. Finally let $\{E_i\}$ be a collection of measurable sets. We have that

$$\omega(\bigcup_{k\geq 1} E_k) = \# \bigcup_{k\geq 1} E_k \leq \sum_{k\geq 1} \# E_k = \sum_{k\geq 1} \omega(E_k)$$

Where the inequality follows from cardinality of sets being at most the sum of the cardinalities, since the sets may not necessaily be disjoint. We now show that any $E \subset M$ is measurable. We see that for any $E \subset M$, and an arbitrary test set X,

$$\omega(X) = \#X = \#X \cap (E \cup E^c) = \#X \cap E + \#X \cap E^c = \omega(X \cap E) + \omega(X \cap E^c)$$

Where the third equality follows from the fact that E is disjoint with E^c.