Q1: Suppose that μ is a measure on $\mathcal{P}(\mathbb{R})$, which only takes on values of 0 or 1. We first check the value of $\mu(\mathbb{R})$. If it is the case that it is 0, μ must clearly be 0 on any subset of $\mathbb{R}$. If it is 1, then we proceed. We write $\mathbb{R} = \bigcup_{n\in\mathbb{Z}}[n, n+1]$. Since μ can only take on values of 0 and 1, we have that $\mu([n, n+1]) = 0$ for either all but one n or some $n, n+1$. If it is nonzero for some consecutive intervals,$[n, n+1], [n+1, n+2]$, then we must have that

$$\mu([n, n+1] \cup [n+1, n+2]) = \mu([n, n+2]) = 1$$

This together with the fact that each interval has measure 1 implies that

$$\mu([n, n+1] \cap [n+1, n+2]) = \mu(\{n+1\}) = 1 \quad (*)$$

Since if the intersection were measure 0, we would have two disjoint sets with each measure 1. Hence this case leads to the conclusion that $\mu = \delta_x$ for $x = n+1$. Now consider the case that only one $[n, n+1]$ has a measure of 1. We call $E_1 = [n, n+1]$. We now define a sequence of sets $\{E_i\}$ in the following way. If $E_i = [a, b]$,

$$E_{i+1} = \begin{cases} [a, a+\frac{b-a}{2}], & \text{if } \mu([a, a+\frac{b-a}{2}]) = 1 \\ [a+\frac{b-a}{2}, b], & \text{if } \mu([a+\frac{b-a}{2}, b])\text{=}1 \end{cases}$$

If at some point, we have that $\mu([a+\frac{b-a}{2}, b]) = \mu([a, a+\frac{b-a}{2}]) = 1$ we terminate the sequence and using the same process as (*), we can conclude that $\mu = \delta_{a+\frac{b-a}{2}}$. Note that it will never be the case that both are of measure 0, since their union will be E_i, which defined recursively implies that each E_i is not measure 0. Furthermore, note that each $E_i \supset E_{i+1}$. Hence this is a decreasing sequence. We consider their intersection

$$A = \bigcap_i E_i$$

Note that downward measure continuity implies that $\mu(A) = 1$. Furthermore, from toplogy we know that the countable intersection of nested cauchy closed intervals in $\mathbb{R}$ is a singleton. Hence we have that $A = \{x\}$ for some point x. Therefore our measure μ is 0 on every set except those containing the point x. Namely, $\mu = \delta_x$.