Q3: Suppose $z, w \in S^2$. We have that their distance on the sphere is given by

$$d(z,w)^2 = |z-w|^2 = (z_1-w_1)^2 + (z_2-w_2)^2 + (z_3-w_3)^2 = 2 - 2(z_1w_1 + z_2w_2 + z_3w_3).$$

Using the fact that for any vector $x \in \mathbb{C}$,

$$(x_1, x_2, x_3) = \Big(\frac{x+\overline{x}}{1+|x|^2}, \frac{x-\overline{x}}{i(1+|x|^2)}, \frac{|x|^2-1}{|x|^2+1}\Big),$$

therefore we compute that

$$\begin{aligned}
d(z,w)^2 &= 2 - 2\Big(\frac{(z+\overline{z})(w+\overline{w})}{(1+|z|^2)(1+|w|^2)} - \frac{(z-\overline{z})(w-\overline{w})}{(1+|z|^2)(1+|w|^2)} + \frac{(|z|^2-1)(|w|^2-1)}{(1+|z|^2)(1+|w|^2)}\Big) \\
&= 2 - 2\Big(\frac{z\overline{w} + \overline{z}w + z\overline{w} + \overline{z}w + |z|^2|w^2| - |z|^2 - |w|^2 + 1}{(1+|z|^2)(1+|w|^2)}\Big) \\
&= \frac{2(1+|z|^2+|w|^2+|z|^2|w|^2) - 2(z\overline{w} + \overline{z}w + z\overline{w} + \overline{z}w + |z|^2|w^2| - |z|^2 - |w|^2 + 1)}{(1+|z|^2)(1+|w|^2)} \\
&= \frac{4|w-z|}{(1+|z|^2)(1+|w|^2)}
\end{aligned}$$

Taking the square root gives the desired result. If we wish to compute $d(z,\infty)$, we write $\infty = (0,0,1)$. The same computations as above except with $w = (0,0,1)$ gives us

$$d(z,\infty)^2 = 2 - 2(\frac{|z|^2-1}{|z|^2+1}) = \frac{4}{|z|^2+1}.$$

We are done.