Q3:

We know that $\frac{\partial(f,g)}{\partial(x,y,z)} = D(f,g)$ and so we have that

$$\frac{\partial(f,g)}{\partial(x,y,z)} = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \end{bmatrix}$$

Since the rank of $D(f,g)(p) = 2$ it must be that the dimension of the span of the columns is 2. Hence one of the column vectors is in the span of the other 2. Assume $WLOG$ that the first column is as such. So have that columns 2 and 3 must be linearly independant. From linear algebra it must be that $det\left(\begin{bmatrix} \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \end{bmatrix}\right) \neq 0$

By the Implicit Function Theorem, there exists an open neighbourhood $A \ni x_0$ and an open $B \ni (y_0, z_0)$ along with $(h,k) : A \to B$ with $(f,g)(x, h(x), k(x)) = 0$ for all $x \in A$. If we define $\gamma : A \to \mathbb{R}^3$ by $\gamma(x) = (x, h(x), k(x))$, this will solve both f and g near p.