Q1:
Since A is Jordan measurable, ∂A has measure 0. A is bounded, so ∂A must be as well. Since the boundary of a set is a closed set, it follows that ∂A is compact by Heine Borel Theorem. Since ∂A is of measure 0 for all $\varepsilon > 0$ there is some collection of open rectangles $\{U_\alpha\}$ which cover ∂A and $\sum_\alpha U_\alpha < \varepsilon$. Any compact set of measure 0 is also content 0 by Spivak Theorem 3-6. Therefore for any $\varepsilon > 0$, it suffices to choose a finite collection of rectangles $\{U_i\}_{i=1}^m$ with $\sum_{i=1}^m vol(U_i) < \varepsilon$. We define our set C as follows.

$$C = A \setminus ((\bigcup_{i=1}^m U_i) \cap intA)$$

Since $C \subset A$ it is bounded, and is the compliment of an open set it must be closed. Hence C is compact by the Heine Borel Theorem.

It remains to prove that the boundary of C will be of measure 0. We will find the boundary of C by determining the contents of $\overline{C} \setminus intC$. We note that

$$\begin{aligned}
\overline{C} \setminus intC &= C \setminus (A \setminus (\bigcup_{i=1}^m \overline{U}_i) \cap \overline{A}) \\
&= A \setminus ((\bigcup_{i=1}^m U_i) \cap intA) \setminus (A \setminus (\bigcup_{i=1}^m \overline{U}_i) \cap \overline{A} \\
&= \bigcup_{i=1}^m \partial U_i \cap C
\end{aligned}$$

Now, notice that $\bigcup_{i=1}^m \partial U_i \cap C \subset \bigcup_{i=1}^m \partial U_i$. Since each U_i is a rectangle, the boundary of each will be of measure 0. Hence the finite union over every boundary of the U_i's will also be measure 0. The subset of any measure 0 set is measure 0 so $\bigcup_{i=1}^m \partial U_i \cap C = \partial C$ is of measure 0 Moreover, it follows that $A \setminus C$ is bounded, since A boundned and $A \supset C$. Notice as well that $\partial A \setminus C = \partial A \cup \partial C$, which will be measure 0. Therefore, $\chi_{A\setminus C}$ is integrable. We now claim that $\int_{A\setminus C} \chi_{A\setminus C} < \varepsilon$. Since A is bounded, take a sufficiently large rectangle $D \supset A \setminus C$. We compute

$$\begin{aligned}
&\int_{A\setminus C} \chi_{A\setminus C} \\
&= \int_{(\bigcup_{i=1}^m U_i)\cap intA} \chi_{(\bigcup_{i=1}^m U_i)\cap intA} && \text{(using the definition of C)} \\
&\leq \int_{\bigcup_{i=1}^m U_i} \chi_{\bigcup_{i=1}^m U_i} && \text{(by A7Q3)} \\
&= \sum_{i=1}^m vol(U_i) && \text{(by defintion of volume)} \\
&< \varepsilon
\end{aligned}$$