Q6: It is sufficient to compute $\frac{1}{2\pi i}\int_\gamma \frac{zf'(z)}{f(z)-a}dz$ in a neighbourhood of a pole, and of a root. Then extend to all of the domain of integration by linearity. Furthermore, it is enough to check when $a = 0$, since we can always translate our closed curve γ. Taking a path γ enclosing some zero z_i with order k, we can write f locally as $f(z) = (z - z_i)^k g(z)$ for some holomorphic $g(z)$. We compute that

$$\frac{zf'(z)}{f(z)-a} = \frac{zk(z-z_i)^{k-1}g(z) + z(z-z_i)^k g'(z)}{(z-z_i)^k g(z)} = \frac{zkg(z) + z(z-z_i)g'(z)}{(z-z_i)g(z)}.$$

The residue theorem tells us that this integral over γ will be $z_i k$. Similarly, if we evaluate this at a pole, we will get $-mp_j$ where m is the order of the pole. Therefore, the entire integral will give us the sum of the poles and zeros counted with multiplicity.