

Q5: Consider the orbits of two elements $x, y \in X$, $G \cdot x$ and $G \cdot y$. We have two cases, either $G \cdot x \cap G \cdot y \neq \emptyset$ or $G \cdot x \cap G \cdot y = \emptyset$. Consider the first case. For some $g, h \in G$ we have that

$$gx = hy$$

Applying the inverse of g , we get that

$$g^{-1}(gx) = (g^{-1}g)x = ex = x = g^{-1}hy$$

Now we see that

$$G \cdot x = \{ax : a \in G\} = \{ag^{-1}y : a \in G\} = G \cdot y$$

Hence if the orbits of two elements have nonempty intersection, they must be equal. If it is not the case, then their intersection is empty. We have shown the desired result.