Q5: Let $f(z)$ be an analytic function with power series in an open neigborhood B of z_0 given by

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n.$$

We define

$$g(z) = c + \sum_{n=0}^{\infty} \frac{a_n}{n+1} (z - z_0)^{n+1}.$$

Note that from differentiation of power series, we have that

$$g'(z) = \sum_{n=0}^{\infty} \frac{a_n(n+1)}{(n+1)} (z - z_0)^n = f(z)$$

This fact implies that g is convergent on B as well. We now claim that such a $g(z)$ is unique, up to a constant term. Let $h(z)$ be another power series that satisfies $h'(z) = f(z)$. Then it must be that

$$(h(z) - g(z))' = 0,$$

i.e. $h(z) - g(z) = d$ for some $d \in \mathbb{C}$. Thus we are done.