Q3: We first claim that $\mu_1 \leq \mu_2 \leq \mu_3$. Let $E \in \mathcal{M}$.Suppose $E_1 \dots E_n$ are disjoint with $\bigcup_{n=1}^N E_n = E$. Then if $\{F_i\}_{i=1}^\infty$ are a disjoint sequence with $\bigcup_{i=1}^\infty F_i = E$ we have that

$$\sum_{n=1}^N |\nu(E_n)| \leq \sum_{n=1}^\infty |\nu(F_n)|$$

Taking supremums yields that $\mu_1(E) \leq \mu_2(E)$. We now claim that $\mu_2(E) \leq \mu_3(E)$. Take any $E_1, E_2, \dots$ pairwise disjoint with $\bigcup_{n=1}^\infty E_n = E$. We have that

$$\sum_{n=1}^\infty |\nu(E_n)| = \sum_{n=1}^\infty |\int_{E_n} d\nu| = |\int_E d\nu| \leq \mu_3(E)$$

Hence we have $\mu_2(E) \leq \mu_3(E)$. Thus we have the chain of inequalities $\mu_1 \leq \mu_2 \leq \mu_3$. We now claim that $\mu_3(E) = |\nu|(E)$. By prop 3.13 from Folland, we have that

$$\mu_3(E) \leq \sup\Big\{\int_E |f| d|\nu| : |f| \leq 1, f \text{ measurable}\Big\} = \int_X \chi_E d|\nu| = |\nu|(E),$$

by approximating $|f|$ with simple functions. We can also check that

$$\begin{aligned}
|\nu|(E) &= \Big|\int_E 1 d|\nu|\Big| \\
&= \Big|\int_E \Big|\frac{d\nu}{d|\nu|}\Big| d|\nu|\Big| && \text{(by prop 3.13)} \\
&= \Big|\int_E \overline{\frac{d\nu}{d|\nu|}} \cdot \frac{d\nu}{d|\nu|} d|\nu|\Big| && \text{(by definition of modulus)} \\
&= \Big|\int_E \overline{\frac{d\nu}{d|\nu|}} d\nu\Big| \\
&\leq \mu_3(E) && \text{(since } |\overline{\tfrac{d\nu}{d|\nu|}}| = |\tfrac{d\nu}{d|\nu|}| = 1 \text{ a.e. by prop 3.13)}
\end{aligned}$$

Hence $|\nu|(E) = \mu_3(E)$. Finally we will show that $\mu_3 \leq \mu_1$, which will prove the result. Let $E \in \mathcal{M}$, f be any measurable function with $|f| \leq 1$, and let $\{E_n\}_{n=1}^N$ be any partition of E. Choose a simple function $\phi = \sum_{n=1}^N a_n \chi_{E_n}$ with $|a_n| \leq 1$ so that $\int_E |\phi - f| < \varepsilon$. It is sufficient to show that the result holds for ϕ. We have that

$$\Big|\int_E \phi d\nu\Big| = \Big|\sum_{n=1}^N \int_X a_n \chi_{E_n} d\nu\Big| \leq \sum_{n=1}^N |a_n| \Big|\int_{E_n} 1 d\nu\Big| = \sum_{n=1}^N |a_n||\nu(E_n)| \leq \sum_{n=1}^N |\nu(E_n)|$$

Taking supremums yields the desired inequality. Hence we have that

$$|\nu| \leq \mu_1 \leq \nu_2 \leq |\nu|$$

and conclude that

$$|\nu| = \mu_1 = \mu_2$$