Q3: To show that $B \in \mathcal{T}^2(\mathcal{T}^k(V))$, we must show that it is 2-linear. Thus, for $T_1, T_2, T_3 \in \mathcal{T}^k(V)$ and $\alpha \in \mathbb{R}$, we evaluate $B(T_1 + \alpha T_2, T_3)$ and $B(T_1, T_2 + \alpha T_3)$. We see the following:

$$\begin{aligned}
B(T_1 + \alpha T_2, T_3) &= \sum_{i_1,\ldots,i_k=1}^{n} (T_1 + \alpha T_2)(v_{i_1} \ldots v_{i_k}) T_3(v_{i_1} \ldots v_{i_k}) \\
&= \sum_{i_1,\ldots,i_k=1}^{n} [T_1(v_{i_1} \ldots v_{i_k}) + \alpha T_2(v_{i_1} \ldots v_{i_k})] T_3(v_{i_1} \ldots v_{i_k}) \\
&= \sum_{i_1,\ldots,i_k=1}^{n} T_1(v_{i_1} \ldots v_{i_k}) T_3(v_{i_1} \ldots v_{i_k}) + \alpha T_2(v_{i_1} \ldots v_{i_k}) T_3(v_{i_1} \ldots v_{i_k}) \\
&= \sum_{i_1,\ldots,i_k=1}^{n} T_1(v_{i_1} \ldots v_{i_k}) T_3(v_{i_1} \ldots v_{i_k}) + \alpha \sum_{i_1,\ldots,i_k}^{n} T_2(v_{i_1} \ldots v_{i_k}) T_3(v_{i_1} \ldots v_{i_k}) \\
&= B(T_1, T_3) + \alpha B(T_2, T_3)
\end{aligned}$$

By almost exactly the same computation, we see that $B(T_1, T_2 + \alpha T_3) = B(T_1, T_2) + \alpha B(T_1, T_3)$. B is bilinear and hence belongs to $\mathcal{T}^2(\mathcal{T}^k(V))$

Q3b: We now wish to show that B is an inner product on $\mathcal{T}^k(V)$. We have shown above that B is bilinear. It remains to prove it is symmetric and positive definite. First, observe the following:

$$\begin{aligned}
B(T_1, T_2) &= \sum_{i_1,\ldots,i_k=1}^{n} T_1(v_{i_1} \ldots v_{i_k}) T_2(v_{i_1} \ldots v_{i_k}) \\
&= \sum_{i_1,\ldots,i_k=1}^{n} T_2(v_{i_1} \ldots v_{i_k}) T_1(v_{i_1} \ldots v_{i_k}) \\
&= B(T_2, T_1)
\end{aligned}$$

Hence B is symmetric. We will now show that for any $T \in \mathcal{T}^k(V)$, $B(T,T) \geq 0$ with equality holding if and only if $T = 0$. Observe:

$$\begin{aligned}
B(T, T) &= \sum_{i_1,\ldots,i_k=1}^{n} T(v_{i_1} \ldots v_{i_k}) T(v_{i_1} \ldots v_{i_k}) \\
&= \sum_{i_1,\ldots,i_k}^{n} [T(v_{i_1} \ldots v_{i_k})]^2 \geq 0
\end{aligned}$$

We note that equality holds if and only iff for each v_{i_j}, $T(v_{i_1} \ldots v_{i_k}) = 0$, meaning that on any k-tuple of basis vectors, $T = 0$. This is equivalent to saying that T is the 0-mapping.