

Q3: Let  $\varepsilon > 0$ . There exists some  $\delta > 0$  such that  $|x - y| < \delta$  implies  $|f_n(x) - f_n(y)| < \varepsilon$ . We write that  $|f(x) - f(y)| = |f_n(\frac{x}{n}) - f_n(\frac{y}{n})| < \varepsilon$ . Therefore, by continuity of  $f_n$ ,  $|\frac{x}{n} - \frac{y}{n}| = \frac{1}{n}|x - y| < \delta$ . Therefore,  $|f(x) - f(y)| < \varepsilon$  for all  $\varepsilon > 0$  and sufficiently large  $n$ . Therefore,  $f(x) = f(y)$ , and we conclude that  $f$  is a constant function.