Q4: If H is a $p-$subgroup that contains N, then the fourth isomorphism theorem tells us that H/N will be a $p-$group of G/N. If $H \leq N$ is a $p-$group of G, then every element of H/N will be sent to eN by the projection mapping. Hence there can be at most $n_p(G)$ p groups in G/N. Therefore $n_p(G/N) \leq n_p(G)$. If $N = \{e\}$, then we have that $n_p(G) = n_p(G/N)$, so the inequality may not be strict.