Q1: First note that by the class equation, we have that

$$1 \neq |Z(G)|.$$

Since $|Z(G)| \mid p^2$, suppose that $|Z(G)| = p$. We therefore have that $|G/Z(G)| = p$. Hence, $G/Z(G) \cong C_p$. Taking some $x \notin Z(G)$, we can present $G/Z(G) = \{e, \overline{x}, \ldots \overline{x}^{p-1}\}$. Note that $|\overline{x}| = p$ therefore $x^p \in Z(G)$. We must also have that $|x| = p$ or p^2. If $|x| = p^2$, we are done since $G \cong C_{p^2}$. If we have that $|x| = p$, we write

$$G = \bigcup_{k=1}^{p} x^k Z(G).$$

Since $|Z(G)| = p$, we have that it must be cyclic. We can therefore write $Z(G) = \{e, z, z^2, \ldots, z^{p-1}\}$ for some $z \in Z(G)$. Furtherfore, we have that $G = \{x^i z^j : 0 \leq i, j \leq p\}$. If we take any $h, g \in G$, we can express them $g = x^a z^b, h = x^c z^d$. We compute that

$$gh = x^a z^b x^c z^d = x^{a+c} z^{c+d} = x^c z^d x^a z^b = hg$$

Hence we get that G is abelian.