

Q2: Take $\lambda = \int_{[0,1]} f dx$. We claim that $g(x) = \int_0^x f dt - (\int_{[0,1]} f dx)x$ will satisfy. We see that

$$dg = d\left(\int_{[0,x]} f dx\right) - d\left(x \int_{[0,1]} f dx\right) = f(x)dx - \int_{[0,1]} f dx$$

We see that $g(1) = \int_{[0,1]} f dt - \int_{[0,1]} f dx = 0$ and $g(0) = \int_{[0,0]} f dt - 0 = 0$. The uniqueness of λ follows from the uniqueness of $\int_{[0,1]} f dx$.