Q2: By a result from class, f bounded if and only if f is continuous. Hence $f^{-1}(\{0\})$ is closed by continuity. Note that it is a linear subspace by linearity of f. Conversely suppose that the kernel of f is closed. Suppose that f is unbounded. We have shown in lecture that there exists some element $v \in \mathfrak{X}$ such that $f(v) = 1$. Since f is unbounded there exists a sequence $\{x_n\}$ with $f(x_n) \geq n$. Define the sequence y_n by

$$y_n = v - \frac{x_n}{f(x_n)}.$$

We have that

$$f(y_n) = f(a) - \frac{f(x_n)}{f(x_n)} = 0.$$

Thus $\{y_n\} \subset \ker f$. Since the kernel is closed we have that $\lim_{n\to\infty} y_n = y$ is in the kernel. Thus $a \in \ker f$, a contradiction.