Q4a:
We define the set $A = \{a(x) - n(x)b(x) : n(x) \in \mathbb{R}[x]\}$, and $S = \{deg(s) : s \in S\}$. Note that both A and S are nonempty by construction. The set S admits 2 cases. Either $-\infty \in S$ or $-\infty \notin S$. In the first case, if $\infty \in S$ then there exists some $q(x) \in \mathbb{R}[x]$ with $a(x) - q(x)b(x) = 0$. We take $r = 0$, with $-\infty = deg(r(x)) < deg(b(x))$. We now consider $-\infty \notin S$. We have that $S \subset \mathbb{N} \cup \{0\}$. By the well ordering principle, there exists some minimum element, z of S. There must exist some $q \in \mathbb{R}[x]$ where $deg(a(x) - q(x)b(x)) = z$. Take $r(x) = a(x) - q(x)b(x) \iff a(x) = q(x)b(x) + r(x)$. We now claim that $deg(r(x)) < deg(b(x))$. Suppose not, that is assume that $deg(r(x)) \geq deg(b(x))$. Let $r(x)$ and $b(x)$ have the following form; $r(x) = \sum_{i=0}^{n} r_i x^i$ and $b(x) = \sum_{i=0}^{m} b_i x^i$, for $n \geq m$. We can rewrite $r(x)$ in the following way:

$$r(x) = \frac{r_n}{b_m} x^{n-m}(b_n x^m + \dots \frac{b_{n-m}}{r_n}) + \dots r_0$$

Now get that

$$a(x) = q(x)b(x) + r(x) = (q(x) + \frac{r_n}{b_n} x^{n-m})b(x) + r'(x)$$

for some $r'(x)$. This implies

$$a(x) = (q(x) + \frac{r_n}{b_m} x^{n-m})b(x) + [r(x) - \frac{r_n}{b_m} x^{n-m} b(x)]$$

The remainder term will have degree less than $r(x)$, contradicting that $r(x)$ has minimal degree. Thus $deg(r(x)) < deg(b(x))$
Q4b:
Suppose that

$$a(x) = b(x)q_1(x) + r_1(x) = b(x)q_2(x) + r_2(x)$$

Rewrite this as

$$b(x)(q_1(x) - q_2(x)) = (r_2(x) - r_1(x))$$

If $q_1(x) \neq q_2(x)$, then $deg(q_1(x) - q_2(x)) \geq 0$ and $deg(b(x)(q_1(x) - q_2(x))) \geq deg(b(x))$. Conversely, $deg(r_2(x) - r_1(x)) < deg(b(x))$, by 4a. We obtain a contradiction, since no number satisfies $x \geq deg(b(x))$ and $x < deg(b(x))$. Therefore, we have that $q_1(x) = q_2(x)$, which implies that $r_1(x) = r_2(x)$.