Q1a: Assume $f : A \to A$ has some fixed point b. Then we have that $(b, f(b)) = (b, b)$ will be in the graph of f. By definition, this will also belong to the diagonal. Conversely, assume that the graph of f intersects the diagonal of $A \times A$. Then for some point $b \in A$ we have that $(b, f(b)) = (b, b)$.

Q1b: We assume away the case in which $f(0) = 0$ and $f(1) = 1$. Define $g(x) := f(x) - x$, this will be continuous. We have that by our assumption, $g(0) > 0$ and $g(1) < 1$. Thus by the intermediate value theorem there is some x_0 where $g(x_0) = 0$ or equivalently, $f(x_0) = x_0$.

Q1c: No, consider the function $f(x) = x^2$. If we solve for $x^2 = x$ we see that $x = 1$ and $x = 0$ are both fixed points, yet they are not elements of the domain $(0, 1)$.

Q1d: No, consider the function $\chi_{\mathbb{Q}}$ on $(0, 1)$, defined to be 1 on every rational, and 0 else. This function will fix no points on $(0, 1)$.