Q2: Since V a 3 dimensional vector space we have from basic linear algebra that V^* will also be 3 dimensional. It suffices to check that $\phi_{-1}, \phi_0, \phi_1$ either span V^* or are linearly independant. We will show linear independance. We will denote $p \in V$ as $p(x) = ax^2 + bx + c$. Suppose that for some scalars $\alpha_1, \alpha_2, \alpha_3$,

$$\alpha_1\phi_{-1}(p) + \alpha_2\phi_0(p) + \alpha_3\phi_1(p) = 0, \forall p \in V$$

Then we have that

$$\alpha_1(a - b + c) + \alpha_2(c) + \alpha_3(a + b + c) = 0$$

Re writing this expression get that

$$a(\alpha_1 + \alpha_4) + b(\alpha_3 - \alpha_3) + c(\alpha_2 + \alpha_3)$$

Since this is true for all polynomials, we, taking $b = 1 = a, c = 0$ we see that $\alpha_3 = 0$, taking $a = c = 0, b = 1$ gives us that $\alpha_1 = \alpha_3 = 0$. Finally, if we take $a = 1$, we see that $\alpha_2 = -\alpha_1 = 0$. Thus we conclude this is a linearly independant list, and so it is a basis of V^*. We now will find a basis $\beta = (p_{-1}, p_0, p_1)$ of V so that $\beta^* = \gamma$. In other words, for each ϕ_i, $\phi_i(p_j) = \delta_{ij}$. First consider p_{-1}. We require that $p_{-1}(-1) = 1, p_{-1}(0) = p_{-1}(1) = 0$. Choosing $p_{-1}(x) = \frac{1}{2}x^2 - \frac{1}{2}x$ will satisfy these properties. Setting $p_0(x) = -x^2 + 1$, we see that $p_0(-1) = p_0(1) = 0$ and $p_0(0) = 1$. Finally, setting $p_1(x) = \frac{1}{2}x^2 + \frac{1}{2}$ will give us the desired properties. Hence the basis $\beta = (\frac{1}{2}x^2 - \frac{1}{2}x, -x^2 + 1, \frac{1}{2}x^2 + \frac{1}{2}x)$ satisifes $\beta^* = \gamma$.