5.6.67: Claim: $|S_n x - Px| \to 0$ as $n \to \infty$. First if $x \in M$, then

$$S_n x = \frac{1}{n}\sum_{i=0}^{n-1} U^i x = \frac{1}{n}\sum_{i=0}^{n-1} x = x$$

and $Px = x$. Now, if $x = y - Uy$, We see that

$$\|S_n(x)\| = \frac{1}{n}\left\|\sum_{i=0}^{n-1} U^i y - \sum_{i=0}^{n-1} U^{i+1} y\right\| = \frac{1}{n}\|z - U^n z\| \leq \frac{1}{n}\|z\| + \|Uz\| = \frac{2\|z\|}{n}.$$

Thus we can make $\|S_n(x)\|$ as small as we wish if x is of the above form.