

Q4i: First consider any permutation $\sigma \in S_3$. Let $w \in W$. We find that

$$\sigma \cdot w = \sum_{i=1}^3 w_i e_{\sigma(i)}.$$

Since the coefficients do not change we have that $\sigma \cdot w \in W$. Now for any $\sum_i a_i \sigma_i$ we have that

$$\left(\sum_i a_i \sigma_i \right) \cdot w = \sum_i a_i \sigma_i \cdot w.$$

Since each $\sigma_i \cdot w \in W$, and W is a subspace the entire expression must also belong to W .

Q4ii: Note that the cardinality of \mathbb{F}_3^3 is 27 by a combinatorics argument. Similarly, W has a cardinality of 9. If there was a direct sum decomposition of \mathbb{F}_3^3 as $W \oplus V$ for some V , then V must contain 0 but no other vectors in W , so $|V| = 19$. Since V must be a subgroup of \mathbb{F}_3^3 we must have that $19|27$ which is absurd.