Q3: We begin by defining the relevant objects. Let $u_1, \ldots, u_m$ be a basis for $\mathbb{R}^m$. Let $v_1, \ldots, v_n$ be a basis for $\mathbb{R}^n$. Let $I = \{i_1 \ldots i_k\} \in \underline{n}^k_a$ and let $J' = \{j_1, \ldots, j_k\}$. Let $A = (a_{ij})$ where i ranges over $1, \ldots, n$ and j ranges over $1, \ldots, m$. We know that for some $\{c_J\}$, $L^*\omega_I = \sum_{J \in \underline{m}^k_a} c_J \omega_J$. If we apply both sides of the equation to $u_{J'}$. We see on the right that $\sum_{J \in \underline{m}^k_a} c_J \omega_J(v_{J'}) = c_{j'}$, since $\omega_J(v_I) = \delta_{IJ}$. The left side will be $L^*\omega_I(v_{J'})$. This quantity is equal to $c_{J'}$ so we will determine what it is.

$$
\begin{aligned}
L^*\omega_I(v'_J) &= \omega_I(L(v_{J'})) \\
&= \omega_I(L(u_{j_1}), \ldots, L(u_{j_k})) \\
&= \omega_I((a_{1j_1}v_1 + \cdots + a_{nj_1}v_n), \ldots, (a_{1j_k})v_1 + \cdots + a_{nj_k}v_n) \\
&= \sum_\sigma (-1)^\sigma \varphi_I \circ \sigma^*((a_{1j_1}v_1 + \cdots + a_{nj_1}v_n), \ldots, (a_{1j_k})v_1 + \cdots + a_{nj_k}v_n) \\
&= \sum_\sigma (-1)^\sigma \prod_{\alpha=1}^{k} a_{i_\alpha \sigma \alpha} \\
&= \det(B)
\end{aligned}
$$

Where $B = (a_{i_\alpha j_\beta})$ for α, β ranging over $1, \ldots k$. Therefore $c_{J'} = \det(B)$.