5.4.48a: Let $\{x_n\} \subset B$ be a sequence converging to some x. We claim that for any $f \in \mathfrak{X}^*$, $f(x_n) \to f(x)$. We have that $\|f(x_n)\| = \|\hat{x}_n(f)\| \leq 1$ by Theorem 5.8d. Therefore $\|\hat{x}(f)\| = \|f(x)\| \leq 1$. As desired.

5.4.48b: Let $\langle x_\alpha \rangle$ be a net in a bounded set E. Suppose that $f(x_\alpha) \to f(x)$. Then, we have that $\sup_\alpha \|f(x_\alpha)\| = \sup_\alpha \|\hat{x}_\alpha(f)\| = \sup_\alpha \|x_\alpha\| < \infty$.

5.4.48c: Let $\{f_n\}$ be a sequence in $F \subset \mathfrak{X}^*$, F bounded, that weak converges to some f in the weak closure. Then for all $\|x\| = 1$, we have that

$$\sup_n \|f_n(x)\| \leq C$$

for some C. Since $\|\cdot\|$ is continuous, we have that $\|f\| = \|\lim_{n\to\infty} f_n\| \leq C$

5.4.48.d: Let $\langle f_\alpha \rangle_{\alpha \in I}$ such that $\langle f_i - f_j \rangle_{(i,j)\in I^2} \to 0$. We have that for sufficiently large n, m, $\|f_n(x) - f_m(x)\| \to 0$. Therefore $\langle f_n(x) \rangle$ is a cauchy sequence. Hence it pointwise converges to some $f \in \mathfrak{X}^*$ by assignment 1 question 7.