Q7a: By the fundamental theorem of algebra, $P(z)$ factors fully over $\mathbb{C}$. We can write

$$P(z) = c(z - b_1)\dots(z - b_n)$$

We compute the derivative of $P'(z)$ as

$$P'(z) = \sum_{j=1}^{n} \prod_{i=1, j\neq i}^{n} (z_i - b_i)$$

Taking the quotient of these quantities we get

$$\frac{P'(z)}{P(z)} = \sum_{i=1}^{n} \frac{1}{(z_i - b_i)}$$

Q7b: Suppose that we had

$$0 = \sum_{k=1}^{n} \frac{1}{(z - b_k)}$$

Multuplying each $\frac{1}{z-b_k}$ term by $\frac{\overline{z}-\overline{b_k}}{\overline{z}-\overline{b_k}}$, we see that

$$0 = \sum_{k=1}^{n} \frac{\overline{z} - \overline{b_k}}{|z - b_k|^2} = \Big(\sum_{k=1}^{n} \frac{1}{|z - b_k|^2}\Big)\overline{z} - \sum_{k=1}^{n} \frac{\overline{b_k}}{|z - b_k|^2}$$

We conclude that

$$\Big(\sum_{k=1}^{n} \frac{1}{|z - b_k|^2}\Big)\overline{z} = \sum_{k=1}^{n} \frac{\overline{b_k}}{|z - b_k|^2}$$

Q7c: We claim that and z satisfying $P'(z) = 0$ is a convex linear combination of each b_k. Using the result from 7b and applying the complex conjugation, we get that

$$\Big(\sum_{k=1}^{n} \frac{1}{|z - b_k|^2}\Big)z = \sum_{k=1}^{n} \frac{b_k}{|z - b_k|^2}$$

Since $\sum_{k=1}^{n} \frac{1}{|z-b_k|^2}$ is nonzero we take no issue with writing the equation as

$$z = \frac{\sum_{k=1}^{n} \frac{b_k}{|z-b_k|^2}}{\Big(\sum_{k=1}^{n} \frac{1}{|z-b_k|^2}\Big)}$$

For all i denote $\frac{1}{|z-b_i|^2}$ as c_i. We see that

$$z = \frac{\sum_{k=1}^{n} bk \cdot c_k}{\sum_{k=1}^{n} c_k} = \frac{1}{\sum_{k=1}^{n} c_k} \sum_{k=1}^{n} b_k c_k$$

Evaluating for the sum of the coefficients on the $b_k's$ we get that

$$\frac{1}{\sum_{k=1}^{n} c_k} \sum_{k=1} c_k = 1$$

Since the coefficient on each b_k is positive, and they sum to 1, we can conclude that z is a convex linear combination of each b_i. Since the b_k's form a convex hull, we have that z must belong to the convex hull generated by $\{b_i\}$.