Q1i: We write $f(z) = c(z-b_1)\dots(z-b_k)$. It follows that $f'(z) = c\sum_{i=1}^k \prod_{j\neq i}^k (z-b_j)$. When $f(z)$ iz nonzero the quotient becomes

$$\frac{f'(z)}{f(z)} = \frac{\sum_{i=1}^k \prod_{j\neq i}^k (z-b_j)}{(z-b_1)\dots(z-b_k)} = \sum_{i=1}^k \frac{1}{(z-b_i)}.$$

If $f(z) = 0$, and the zero is of degree 1, then the quotient will just be infinty. If z_0 is a zero of degree $k > 1$, then in the summation, the z_0 term will appear $k-1$ times.

Q1ii: We have that

$$0 = \sum_{i=1}^k \frac{1}{(z-b_k)} = 0 \implies 0 = \sum_{i=1}^k \frac{1}{(z-b_i)} \cdot \overline{\left(\frac{z-b_i}{z-b_i}\right)} \implies 0 = \sum_{i=1}^k \frac{\overline{z} - \overline{b_i}}{|z-b_i|^2}$$

Seperating the summands we get that

$$\overline{z} \cdot \sum_{i=1}^k \frac{1}{|z-b_i|^2} = \sum_{i=1}^k \frac{\overline{b_i}}{|z-b_i|^2},$$

as desired. If we have that $z = b_l$ for some i, then as we take $z \to b_l$ then the terms in the summation with index l will both approach the same value and hence they will not be counted.

Suppose that z satisfies $P'(z) = 0$. Then by conjugating the result from $Q1ii$, we have that z satisfies the expression

$$z \cdot \sum_{i=1}^k \frac{1}{|z-b_i|^2} = \sum_{i=1}^k \frac{b_i}{|z-b_i|^2}.$$

Thus we write

$$z = \frac{\sum_{i=1}^k \frac{b_i}{|z-b_i|^2}}{\sum_{i=1}^k \frac{1}{|z-b_i|^2}}.$$

The coefficients of the b_i are all positive since it is a sum of norms. Their sum will be

$$\frac{\sum_{i=1}^k \frac{1}{|z-b_i|^2}}{\sum_{i=1}^k \frac{1}{|z-b_i|^2}} = 1.$$

Therefore z is a convex linear combination of the $b_i's$ and so it belongs to the convex hull they generate.