Q5: We begin by proving the reverse implication. Suppose that f has a modulus of continuity, $\mu(y)$. Then by the definition of μ, we have that for all $\varepsilon > 0$, there exists some δ such that $|y| < \delta$ implies that $|\mu(y)| < \varepsilon$. Set $y = |s-t|$ for $s, t \in [a, b]$. Therefore, $|s-t| < \delta$. and by the assumption $|f(s) - f(t)| \leq \mu(|s-t|) < \varepsilon$. Therefore f is continuous, and by $Q2$ it is uniformly continuous. We now prove the forward implication. Suppose that f is uniformly continuous. Define $\mu(|s-t|) = \sup\{f(x) - f(y) : x, y \in M_{|s-t|}(a)\}$. By this definition of μ, we have that $|f(s) - f(t)| \leq \mu(|s-t|)$. Notice as well as $|s-t| \to 0$, we have that the supremum of the differences approaches 0 by continuity. This function is also strictly increasing, since as we increase $|s-t|$, the supremeum over all differences increases.