Q5: First note that since $\mathbb{F}_3$ is a field, constant numbers are irreducible. Next, we have that linear terms $x, (x-1), (x-2)$ are all irreducible, since they are all linear. This is also an exhaustive list of all degree 1 irreducible polynomials, since they each have a unique root. Note that for degree 2 and 3 polynomials, it is sufficient to check when they are monic, and their constant term is nonzero. The complete list of degree 2 monic polynomials over $\mathbb{F}_3$ are:

$$\begin{aligned} x^2+1 \\ x^2+2 \\ x^2+x+1 \\ x^2+x+2 \\ x^2+2x+1 \\ x^2+2x+2 \end{aligned}$$

Applying Prop. 9 and Prop 10, it is enough to evaluate each polynomial on $\mathbb{F}_3$, and the ones which do not vanish will therefore be irreducible. We see that the only such polynomials are

$$x^2+1, x^2+x+2, x^2+2x+2.$$

Finally, we will find the degree 3 irreducible polynomials. Similarly to the degree 2 case, we use Prop. 9 and Prop. 10. We can find such degree 3 polynomials by checking all degree 3 monic polynommials who do not vanish at 0. First note that the two simplest cases that are irreducible are

$$x(x-1)(x-2)+1 = x^3+2x+1, x(x-1)(x-2)+2 = x^3+2x+2.$$

These are irreducible by construction. They will never be 0. Checking the remaining degree polynomials, we have that the following are irreducible:

$$\begin{aligned} x^3+x^2+2 \\ x^3+x^2+x+2 \\ x^3+2x^2+x+1 \\ x^3+2x^2+2x+2 \\ x^3+x^2+2x+1 \\ x^3+2x^2+1 \end{aligned}$$

Thus by brute force these are all the irreducible polynomials over $\mathbb{F}_3$ of degree less than 3.