Q1a:

Since f is C^1, $f(3,-1,2)=0$ and $\frac{\partial f}{\partial y} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$ has nonzero determinant, the Implicit Function Theorem asserts that there must exist an open neighbourhood B of 3 and an open neighbourhood A of $(-1,2)$ with a continous differentiable function $g : B \to A$ such that $f(x, g_1(x), g_2(x)) = 0$.

Q1b:

By the Implicit Function Theorem, we can compute g' in the following way.

$$g' = -\left[\frac{\partial f}{\partial y}\right]^{-1} \cdot \frac{\partial f}{\partial x}$$

As given, we know that $\frac{\partial f}{\partial x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\frac{\partial f}{\partial y} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$. We compute $\frac{\partial f}{\partial y}^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$. Therefore

$$g'(3) = -\begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Q1c:

To be able to find a function that that gives us (y_1, y_2) in terms of x we would require that $\frac{\partial f}{\partial y}$ at that point be invertible, in order to guarantee the excistence of a function $g(x) = (g_1(x), g_2(x)) = (y_1, y_2)$. For instance, we can not solve for y_1 as a function of x and y_2 since the matrix $\frac{\partial f}{\partial x, y_2}$ has determinant zero and is not invertible.