

Q4:

Given that  $f(x, y) = \int_x^x g_1(t, 0)dt + \int_0^y g_2(x, t)dt$ , we will compute  $D_1f$  and  $D_2f$ . First,  $D_1f$

$$\begin{aligned}
 D_1f &= D_1 \int_0^x g_1(t, 0)dt + D_1 \int_0^y g_2(x, t)dt \\
 &= g_1(x, 0) + D_1 \int_0^y g_2(x, t)dt && \text{(by FTC)} \\
 &= g_1(x, 0) + \int_0^y D_1g_2(x, t)dt && \text{(by the Leibniz Rule)} \\
 &= g_1(x, 0) + \int_0^y D_2g_1(x, t)dt && \text{(by assumption)} \\
 &= g_1(x, 0) + g_1(x, y) - g(x, 0) && \text{(by FTC)} \\
 &= g_1
 \end{aligned}$$

As desired. We now will compute  $D_2f$

$$\begin{aligned}
 D_2f &= D_2 \int_0^x g_1(t, 0)dt + D_2 \int_0^y g_2(x, t)dt \\
 &= D_2 \int_0^x g_1(t, 0)dt + g_2(x, y) && \text{(by FTC)} \\
 &= 0 + g_2(x, y) && \text{(since first integral constant in y)} \\
 &= g_2(x, y)
 \end{aligned}$$

As expected.