Q2a:

Suppose not, that is suppose that f is injective. Then we have that $D_1 f$ and $D_2 f$ are both not identically 0. $WLOG$ say that $D_1 f$ is nonzero on some open set A. Define $g(x, y) = (f(x, y), y)$. Then by properties of the differential,

$$Dg(x, y) = \begin{pmatrix} D_1 f(x, y) & D_2 f(x, y) \\ 0 & 1 \end{pmatrix}$$

We have that $DetDg = D_1 f(x, y)$ which is nonzero on A. Notice that our choice for g makes it injective. Thus by the inverse function theorem, there exists some open set A around the point (x_0, y_0) and open B around $g(x, y)$ such that $g^{-1} : B \to A$ exists and is both continous and differentiable. Consider distinct points $(\alpha, y_0), (\alpha, y_1) \in B$. Since g^{-1} is injective we have that there is some distinct (x_0, y_0) and (x_1, y_1) where $f(x_0, y_0) = f(x_1, y_1) = \alpha$. We obtain a contradiction and thus f is not injective.