

Q1a: We claim that if $G = \mathbb{Z}$ and $H = 3\mathbb{Z}$, then $H \leq G$. Suppose that $a, b \in H$. We can write them in the form $a = 3n$ and $b = 3m$. It is sufficient to verify that $ab^{-1} \in H$. A simple computation verifies that indeed,

$$ab^{-1} = a + (-b) = 3n + (-3m) = 3(n - m) \in H$$

We have shown the desired result.

Q1b: Similarly to 1a, we will show that for any $q, r \in H$, $qr^{-1} \in H$.

$$qr^{-1} = q \cdot \frac{1}{r} = \frac{q}{r}$$

Since both r is positive and rational, its reciprocal is positive and rational as well, and positive rationals are closed under multiplication. Hence $H \leq G$.

Q1c: Suppose that H was a subgroup of D_6 containing every reflection. There are 3 possible reflections that can be put on a triangle. Additionally, H must contain e to be considered a subgroup. Therefore $|H| = 4$. This is not possible however, since $|D_6| = 6$ and 4 does not divide 6. Hence by Lagrange's theorem, this cannot be a subgroup.