Q3: The claim is false. Consider $G = S_4$, $K = A_4$. It is known that A_4 is normal in S_4 since it is the kernel of the homeomorphism $sgn : S_4 \to \{-1, 1\}$. Consider the subgroup $H = \{e, (12)(34)\}$. It is clear that H is a subgroup and $H < K$. Consider the permutation $\sigma = (23) \in S_4$. We compute that

$$\sigma H = \{\sigma, (1243)\}$$

but on the other hand

$$H\sigma = \{\sigma, (1342)\}$$

We have found $H < K \triangleleft G$ with H not normal in G. The claim is false.