Q5:
First notice that $|\det g'| = r > 0$. We can apply the COV Theorem to compute the value of the integral.

$$\begin{aligned}
\int_{T(A)} 1 &= \int_A 1|\det g'| \\
&= \int_0^{2\pi}\int_{b-a}^{b+a}\int_{-\sqrt{a^2-(r-b)^2}}^{\sqrt{a^2-(r-b)^2}} r \quad dzdrd\theta && \text{(by Fubini's Theorem)} \\
&= 2\int_0^{2\pi}\int_{b-a}^{b+a} r\sqrt{a^2-(r-b)^2} \quad drd\theta \\
&= 2\int_0^{2\pi}\int_{-a}^{a} (u+b)\sqrt{a^2-u^2} \quad dud\theta && \text{(substitution u = r-b)} \\
&= 2\int_0^{2\pi}\int_{-a}^{a} u\sqrt{a^2-u^2} \quad dud\theta + 2\int_0^{2\pi}\int_{-a}^{a} b\sqrt{a^2-u^2} \quad dud\theta \\
&= 2\int_0^{2\pi}\int_{-a}^{a} b\sqrt{a^2-u^2} \quad dud\theta && \text{(since first integral is of an odd function)} \\
&= 2b\int_0^{2\pi} \frac{\pi}{2}a^2 d\theta \\
&= 2\pi^2 a^2 b
\end{aligned}$$