Q4: Suppose that $|G| = p^\alpha m$ for some α, and $p \nmid m$. By Sylows theorems, there exists a subgroup P with $|P| = p^\alpha$. There must exist some element x with $|x| > 1$. By Lagranges theorem, $|x| = p^\beta$, for some $1 \leq \beta \leq \alpha$. Hence we have that $x^{\beta^p} = e$. Therefore the element $x^{p^{\beta-1}}$ will have an order of p since

$$(x^{p^{\beta-1}})^p = x^{p \cdot p^{\beta-1}} = x^{p^\beta} = e.$$