Q4a:
$f(x,y,z) = x^y$ can be rewritten in the following way. $f(x,y,z) = x^y = e^{ylogx} = e^{\pi_2 log(\pi_1)}$. Note that this is only defined for $x > 0$.Thus according to the chain rule this function is differntiable. By the chain rule and the differential of the product function, we get that

$$\begin{aligned}
f'(x,y,z) &= e^{ylogx'}[y(log\pi_1)' + (logx)\pi_2{}']\\
&= e^{ylogx}[y\frac{1}{\pi(x,y,z)}\pi' + log(x)(0,1,0)]\\
&= e^{ylogx}[\frac{y}{x}(1,0,0) + (0, log(x), 0)]\\
&= x^y[(\frac{y}{x}, log(x), 0)]\\
&= (yx^{y-1}, log(x)x^y, 0)
\end{aligned}$$

4b: By Spivak Theorem 2.3(3), it follows that $f^{1'} = (yx^{y-1}, log(x)x^y, 0)$. It remains to determine $(f^2)'$. Since $f^2 = z$ we can rewrite it as $f^2 = 0\pi_1 + 0\pi_2 + \pi_3$. This is differentiable since it is the sum of differntiable functions. Thus we have

$$(f^2)' = \pi_3{}' = (0,0,1)$$

Applying Spivak theorem 2.3(3) again, we get that $f' = \begin{bmatrix} yx^{y-1} & log(x)x^y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q4c: We first note that we can rewrite $f = (x+y)^z$ as follows. $f = (x+y)^z = e^{zlog(x+y)} = e^{\pi_3(log(\pi_1+\pi_2))}$, for $x + y > 0$. Thus we compute f' using the chain rule and the differentials of linear maps:

$$\begin{aligned}
f' &= e^{z(log(x+y))'}(log(\pi_1+\pi_2)\pi_3' + \pi_3 log'(\pi_1+\pi_2))\\
&= e^{z(log(x+y))'}(log(x+y)(0,0,1) + z\frac{1}{\pi_1+\pi_2}(\pi_1' + \pi_2'))\\
&= e^{z(log(x+y))'}[(0,0,log(x+y)) + (\frac{z}{x+y}, \frac{z}{x+y}, 0)]\\
&= (x+y)^z[(\frac{z}{x+y}, \frac{z}{x+y}, log(x+y)]\\
&= (z(x+y)^{z-1}, z(x+y)^{z-1}, log(x+y)(x+y)^z)
\end{aligned}$$