Q5: Let A be the set of all polynomials $p(x)$ on $[a, b]$ such that $p'(a) = 0$. We claim that A is a function algebra which vanishes nowhere and separates points. First note that this is indeed a function algebra, since it is closed under addition and scaling by the properties of differentiation. Similarly, by the product rule it is closed under multiplication. We now claim that A vanishes nowhere. Let $p \in [a, b]$. We will construct an $f \in A$ which does not vanish at p. Let $p \in [a, b]$. Define $f(x) = (x - a)^2 + c$ with constant c chosen so that $c \neq -(p - a)^2$. We see that $f'(a) = 0$ and f does not vanish at p, hence this function algebra is nowhere vanishing. We will now show that A separates points. Let $p_1, p_2 \in [a, b]$ be distinct points. We define $f(x) = (x - a)^2 - (p_1 - a)^2$. We see that f belongs to A and $f(p_1) = 0 \neq f(p_2)$. We have that A is a function algebra, which vanishes nowhere and separates points. Therefore, by the Stone-Weierstrass Theorem for each $\frac{\varepsilon}{2} > 0$ there exists some $q(x)$ where $q'(a) = 0$ and $|q(x) - f(x)| < \frac{\varepsilon}{2}$. Let $\varepsilon_0 = q(a) - f(a)$. Now let $p(x) = q(x) - \varepsilon_0$. We have that $p'(a) = 0, p(a) = f(a)$ and $|p(x) - f(x)| < \varepsilon$.