Since Cassini's oval is symmetric, it is sufficient to map the right half of it to the right half of the disk conformally. We first map $|z^2 - a^2| < r^2$ under $\zeta = z^2$, to get the circle $|\zeta - a^2| < r^2$. We can translate and scale this using

$$f(\zeta) = \frac{\frac{a^2}{r^2}\zeta}{\zeta + (a^2 + r^2)(\frac{a^2}{r^2} - 1)} = Z.$$

This maps to the unit disk. We now apply $w = Z^{\frac{1}{2}}$, on the branch $[-\pi, \pi]$. This will give the right half disk, and preserve the axis of symmetries. By the principle of symmetry, composing the above coformal mappings will give the desired result. For the other half of the