Q4: For $a, b \geq 0$, we consider
$$a - |a - b|$$
We wish to bound it above and below. First note that by the triangle inequality we have that
$$|a - b| \leq |a| + |b|$$
When we multiply by -1 and add a to get that
$$-b = a - |a| - |b| \leq a - |a - b|$$
We now wish to find an upper bound for
$$a - |a - b|$$
. If $a - b > 0$, then
$$a - |a - b| = a - (a - b) = b$$
If we have that $a - b < 0$, then
$$a - (-a + b) = 2a - b$$
Since $2a < 2b$ we know that $2a - b < b$ and hence we conclude that
$$-b \leq a - |a - b| \leq b$$
Replacing $a = f_n$, $b = f$, we get that
$$\int f_n - |f_n - f| \leq \int f$$
Applying the dominating convergence theorem, we have that
$$\lim_{n \to \infty} \int f_n - |f_n - f| = \int \lim_{n \to \infty} f_n - \int \lim_{n \to \infty} |f_n - f| = \int f$$