

Q2: Clearly we have that C_{15} and $\langle e \rangle$ are both subgroups of C_{15} . By Lagrange's theorem, we have that any subgroup of C_{15} must have order of 3 or 5. Taking

$$H_1 = \langle 3 \rangle = \{0, 3, 6, 9, 12\}$$

and

$$H_2 = \langle 5 \rangle = \{0, 5, 10\}$$

We claim that these are the only subgroups of order 5 and 3 respectively. Note that since C_{15} is cyclic, every subgroup of it can be generated by 1 element. From A1Q4ii, we know that every nonidentity element of C_{15} which is coprime to 15 is a generator of C_{15} . Therefore the elements 3, 5, 6, 9, 10, 12 do not generate C_{15} . We can verify through simple computation that

$$\langle 3 \rangle = H_1$$

$$\langle 5 \rangle = H_2$$

$$\langle 6 \rangle = \{6, 12, 3, 9, 0\} = H_1$$

$$\langle 9 \rangle = \{9, 3, 12, 6, 0\} = H_1$$

$$\langle 10 \rangle = \{10, 5, 0\} = H_2$$

$$\langle 12 \rangle = \{12, 9, 6, 3, 0\} = H_1$$

We see that the only subgroups which are not $\{e\}$ and C_{15} are H_1, H_2 .