Q3: Let $C \subset \Omega$ be a compact subset. Since $\{\Omega_n\}$ form an open cover of Ω, there is a finite subcover indexed Ω_{n_i} that cover $\mathbb{C}$, say $M = \max\{n_i\}$. Hence Ω_M covers C, and so does $\{\Omega_N\}$ for $N \geq M$. Discarding the finitely many f_i's which are not defined on $\{\Omega_N\}$, we evaluate that on any closed $\gamma \subset C$,

$$\oint_\gamma f(z)dz = \lim_{\substack{n \geq M \\ n \to \infty}} \oint_\gamma f_n(z)dz = 0.$$

Therefore f is holomorphic on compact subsets. We now claim that $f' \to f'$ uniformly. Observe, on the same compact set C, taking γ to be a simple curve, we have that

$$f'(z) = \oint_\gamma \frac{f(\zeta)}{(\zeta - z)^2} d\zeta = \lim_{\substack{n \geq M \\ n \to \infty}} \oint_\gamma \frac{f_n(\zeta)}{(\zeta - z)^2} d\zeta = \lim_{\substack{n \geq M \\ n \to \infty}} f_n'(z).$$

This will uniformly converge since $\frac{1}{(\zeta - z)^2}$ is bounded on compact sets.