Q3a:
Let $P(n)$ denote $\sum_{k=1}^{n} k^3 = \left(\frac{(n)(n+1)}{2}\right)^2$. We proceed by induction on n. When $n = 1$,

$$\sum_{k=1}^{1} 1^3 = (\frac{(1)(2)}{2})^2 = 1$$

. Thus $P(1)$ is true. Now suppose the formula holds for n. We compute

$$\begin{aligned}
\sum_{k=1}^{n+1} k^3 &= \sum_{k=1}^{n} k^3 + (n+1)^3 \\
&= \left(\frac{(n)(n+1)}{2}\right)^2 + (n+1)^3 && \text{(by induction hypothesis)} \\
&= (n+1)^2 \left[\frac{n^2}{4} + (n+1)\right] \\
&= (n+1)^2 \left(\frac{n^2+4n+4}{4}\right) \\
&= (n+1)^2 \frac{(n+2)^2}{4} \\
&= \left(\frac{(n+1)(n+2)}{2}\right)^2
\end{aligned}$$

We have that $P(1)$ is true and $P(n) \implies P(n+1)$. By the principle of induction, $P(n)$ is true for all $n \in N$.
Q3b:
Let $P(n)$ denote $n \leq 2^n$. We proceed by induction on n. $P(1)$ corresponds to $1 \leq 2$. This is clearly true. Now Suppose that $P(n)$ is true, i.e. $n \leq 2^n$, Multiplying by 2, we have that $2n \leq 2^{n+1}$. For $n \geq 1$, $n + 1 \leq 2n$, so we will have $n + 1 \leq 2^{n+1}$. We have $P(1)$ true and $P(n) \implies P(n+1)$ so by the principle of induction $P(n)$ is true for all n.
Q3c:
Let $P(n)$ = "n has a binary representation". We proceed by strong induction on n. We see that $P(1) = 2^0 = 1$. Now suppose that $P(1) \ldots P(n)$ each have binary representations. We will consider 2 separate cases, one in which n is even and one in which n is odd. First, when n is even, we know that 2^0 is not included in our representation, since it is the only odd power of 2. Hence $n + 1$ is represented as $n + 2^0$. If n is odd, then $n + 1$ will be even and will be represented in the following way. $n + 1$ even implies $n + 1 = 2k$ for some $k < n$. By assumption k will have a binary representation, say $k = 2^{i_1} + \ldots 2^{i_j}$ for unique natural $i_1 \ldots i_j$. We have that $2k = 2(2^{i_1} + \ldots 2^{i_j}) = 2^{i_1+1} \ldots 2^{i_j+1} = n + 1$. Since We translate the list $i_1 \ldots i_j$ by 1 at each point, we preserve the uniqueness. Thus, $n + 1$ will have a binary representation. Since $P(1)$ is true, and $P(1) \ldots P(n) \implies P(n+1)$, $P(n)$ is true for all $n \in \mathbb{N}$ by the principle of Strong Induction.