

Q1i: We claim the element  $x^2y$  is in  $I \cap J$  but not in  $IJ$ . Note that it belongs to both the ideal  $I$  and  $J$ , since we can take  $y \cdot x^2$ , or  $x \cdot xy$  respectively. However, every element of  $IJ$  can be written as

$$\sum_i a_i \cdot x^3y$$

for some  $a_i \in \mathcal{R}$ . In this polynomial,  $x$  must have a degree of at least 3. Thus  $x^2y \notin IJ$ .

Q1ii: Consider the ideals  $I = (2), J = (4)$ . It is clear that  $I \cap J = J$ , since every multiple of 4 is also a multiple of 2. However, we have that  $IJ$  is all elements of the form

$$\sum_i a_i(2 \cdot 4) = 8 \cdot \sum_i a_i,$$

for  $a_i \in \mathbb{Z}$ . Thus we have that  $IJ = (8)$ .