

Q4: Note that by definition of  $\cos(z)$ , we can write

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$$

We wish to find a  $z$  such that for all  $c \in \mathbb{C}$

$$\frac{e^{iz} + e^{-iz}}{2} = c$$

Take  $z = -i \log(-c - \sqrt{c^2 - 1})$ . We compute that:

$$\begin{aligned} \frac{e^{iz} + e^{-iz}}{2} &= \frac{e^{i(-i \log(-c - \sqrt{c^2 - 1}))} + e^{-i(-i \log(-c - \sqrt{c^2 - 1}))}}{2} \\ &= \frac{e^{\log(-c - \sqrt{c^2 - 1})} + e^{-\log(-c - \sqrt{c^2 - 1})}}{2} \\ &= \frac{\frac{-c - \sqrt{c^2 - 1}}{1} + \frac{1}{-c - \sqrt{c^2 - 1}}}{2} \\ &= \frac{1}{2} \cdot \frac{c^2 - 2c\sqrt{c^2 - 1} + c^2 - 1 + 1}{-c - \sqrt{c^2 - 1}} \\ &= \frac{1}{2} \cdot \frac{2c(-c - \sqrt{c^2 - 1})}{-c - \sqrt{c^2 - 1}} \\ &= c \end{aligned}$$

We conclude that  $\cos(z)$  is surjective onto the complex plane.