Assignment 2 MAT 454

Q7: Using the addition formula from Q6, we get that

$$0 = \wp(u)(\wp'(v) + \wp'(u+p)) - \wp'(u)(\wp(v) - \wp(u+v)) - \wp(v)\wp'(u+v) - \wp'(v)\wp(u+v).$$

Now, subsituting the relationship

$$\wp'(u+v) = -\frac{\wp'(v) - \wp'(u)}{\wp(v) - \wp(u)}\wp(u+v) - \frac{\wp'(u)\wp(v) - \wp'(v)\wp(u)}{\wp(v) - \wp(u)},$$

Which must hold when the addition formula holds, we get that

$$4\wp(u+v)(\wp(u)-\wp(v))^{2} = -(\wp(u)+\wp(v))(\wp(u)-\wp(v))^{2} + (\wp'(u)-\wp'(v))^{2}$$

And rearranging this gives us

$$\wp(u+v) = -\wp(u) - \wp(v) + \frac{1}{4} \left(\frac{\wp'(u) - \wp'(v)}{\wp(u) - \wp(v)} \right)^2$$

As desired.