Q4: We will show that the toplogists sin circle is path connected, yet not locally path connected. First suppose that we have two points x, y belonging to the circular subset of the toplogists sine circle. We simply take a path along the subset of the circle between these two points. If the points x belongs to the circular portion of our set, and y belongs to the sine curve, we choose a path along $y \mapsto \sin(\frac{1}{y})$ which is contiuous on $(0, 1)$. Since the sine curve has a common point with the circle segment, namely $(0, 0)$, choose a path from x to $0, 0$ along the circle. This path makes sense, since $(0, 1)$ is path connected, and $\sin(\frac{1}{x})$ is continuous on this set, hence the image of it is path connected as well, since we can compose and contious path with $\sin(\frac{1}{x})$ and obtain a new path Next case is when our points x, y belong to the sine curve. We simply travel along the continuous curve from x until we reach $(0, 0)$, and travel from y to $0, 0$ in the same fashion. We now claim that in some neighbourhood, the topoligists sin curve is not path connected. Suppose that it is.Let $B = M_{\frac{1}{2}}(0, 1)$ Then there exists a continuous path from $(\frac{2}{5\pi}, 1)$ to $(\frac{2}{9\pi}, 1)$ along the sin curve, which is contained in B intersected with the sin curve. Take the path $\gamma : [\frac{2}{5\pi}, \frac{2}{9\pi}] \to \mathbb{R}^2$ with $\gamma : x \mapsto (x, \sin(\frac{1}{x}))$. This is a path between the points, but it escapes the set B at the point $\frac{2}{7\pi}$, $\gamma(\frac{2}{7\pi}) = (\frac{2}{7\pi}, -1)$. Hence there is some neighbourhood in which the topologists sine circle is not path connected. So we conclude it is not locally path connected.