

Q2: The normal subgroups of S_4 are S_4 , A_4 , $\{e\}$ and $\{e, (12)(34), (13)(24), (14)(23)\}$. We claim that there are no other normal subgroups of S_4 . We know from the class equation that

$$24 = |Z(G)| + \sum_{g_i \in G} [G : C(g_i)] = 1 + 3 + 6 + 6 + 8$$

We can't have a subgroup of order 12 not equal to A_4 , since there's no way to get a divisor of 24 without adding 8, 3, 1, which already correspond to A_4 . The only other way to get a divisor of 24, is to have $3 + 1 = 4$, which corresponds to the subgroup $\{e, (12)(34), (13)(24), (14)(23)\}$.