Q1:

Since f is injective it has an inverse f^{-1} on $f(A)$. Therefore, for all $b \in f(A)$ there exists a unique $a \in A$ where $f(a) = b$. We now apply the inverse function theorem. For each $y \in f(A)$, there is an open set B_y, and some open set A_x around $f^{-1}(y) = x$ where there exists a C^1 inverse $f_x^{-1} : B_y \to A_x$. From the uniqueness of the inverse, we have that $\forall w \in B_y$. $f_x^{-1}(w) = f^{-1}(w)$. This is true for all $w \in F(A)$, so f^{-1} is continous and differentiable. We can write $f(A) = (f^{-1})^{-1}(A)$. Since f^{-1} is continous, and the pre image of open sets under continous maps is open, it follows that A is open. Similarly for any open $B \subset A$, $f(B) = (f^{-1})^{-1} = B$.