Q1a: Using Hadamards Formula we compute:

$$\begin{aligned}
\frac{1}{R} &= \lim_{n\to\infty} \sup \sqrt[n]{|a_n|} \\
&= \lim_{n\to\infty} \sup \sqrt[n]{|q^{n^2}|} \\
&= \lim_{n\to\infty} \sup \sqrt[n]{|q|^{n^2}} \\
&= \lim_{n\to\infty} \sup |q|^n \\
&= 0 && \text{(since } |q| < 1 \text{ and hence its power approaches 0)}
\end{aligned}$$

We conclude that $R = \infty$ i.e. the power series converges everywhere

Q1b: Using Hadamards Formula, we compute:

$$\begin{aligned}
\frac{1}{R} &= \lim_{n\to\infty} \sup \sqrt[n]{|a_n|} \\
&= \lim_{n\to\infty} \sup \sqrt[n]{|n^p|} \\
&= \lim_{n\to\infty} \sup \sqrt[n]{|n|}^p \\
&= (\lim_{n\to\infty} \sup \sqrt[n]{|n|})^p \\
&= 1
\end{aligned}$$

Hence $R = 1$.

Q1c: Using Hadamards formula, we compute:

$$\frac{1}{R} = \lim_{n\to\infty} sup \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sup\{\sqrt[2n+1]{a^{2n+1}}, \sqrt[2n]{b^{2n}}\} = \lim_{n\to\infty} \sup\{a, b\} = \max\{a, b\}$$

Therefore we get that

$$R = \frac{1}{\max\{a, b\}}$$