Q2a: We will prove this via induction on e. For when $e = 1$, we have the simultaneous equations $x^{p-1} \equiv 1 \bmod (p)$ and $x \equiv i \bmod (p)$. For each i nonzero we have that $i^{p-1} \equiv 1 \bmod (P)$ by Fermats little Theorem. Now suppose that for e, there is some x_e such that $x_e^{p-1} \equiv 1 \bmod (p^e)$ and $x_e \equiv i \bmod (p)$ for some i. Take $x_{e+1} = x_e + p^e \cdot k$, for some $k \in \mathbb{Z}$. We have that

$$\begin{aligned}
f(x_{e+1}) &= f(x_e + kp^e) \\
&= (x_e + kp)^{p-1} - 1 \\
&= \sum_{j=0}^{n} \binom{p-1}{j} x_e^{p-i-j} (kp^e)^j - 1 \\
&= x_e^{p-1} + (p-1)x_e^{p-2} \cdot kp^e \cdots - 1 \\
&= x_e^{p-1} - 1 + (p-1)x_e^{p-2} \cdot kp^e \\
&= f(x_e) + f'(x_e)kp^e \bmod (p^{e+1})
\end{aligned}$$

By assumption, $f(x_e) \equiv 0 \bmod (p^e)$, we can write $f(x_e) = ap^e$ for some $a \in \mathbb{Z}$. Therefore, $ap^e + f'(x_e)kp^e \equiv 0 \bmod (p^{e+1})$ and so $q + f'(x_e)k \equiv 0 \bmod (p)$. Since $f'(x_e)$ is nonzero since $x_e \equiv i \not\equiv 0$, so we can take $k = (-q) \cdot (f'(x_e))^{-1}$. Inverses are unique hence x_{e+1} is unique.

Q2b: For each e_i we take $x_{e_i} \equiv -1 \bmod (p^{e_i})$. We can clearly see that $x_{e_i}^2 \equiv 1 \bmod (3^e)$ and $x_{e_i} \equiv 2 \bmod (3)$