Q1: By Dummite and Foote proposition 7 (page 280), we have that P must be a maximal ideal. Apply Dummite and Foote Proposition 12 to get that $\mathcal{R}/P$ is a field. Clearly this means that $\mathcal{R}/P$ is a PID.