Q4: We first claim that 1000^k has a remainder of $(-1)^k$ when divided by either 7 or 13. We see that

$$\begin{aligned}
1000^k &= (1001-1)^k \\
&= (7\cdot 11\cdot 13-1)^k \\
&= \sum_{i=0}^{k} \binom{k}{i} (7\cdot 11\cdot 13)^{k-i}\cdot(-1)^i \\
&= 7\cdot 13\cdot\left[\sum_{i=1}^{k-1} \binom{k}{i} (7\cdot 13)^{k-i-1}\cdot 11^{k-i}(-1)^i\right] + (-1)^k
\end{aligned}$$

Therefore, 1000^k has a remainder of $(-1)^k$ when divided by 7 or 13. Therefore, the divisibility rule for 7 or 13 will be as follows. If $a = \sum_{k=0}^{n} a_k \cdot 1000^k$, for $a_k \in 1, 2 \ldots 999$. Therefore the divisibility rule is a is divisible by 7 or 13 if $\sum_{k=1}^{n} a_k(-1)^k$ divides by 7 or 13, respectively.