Q5:

"$\subset$"

Suppose that point $x \in BdA$. By definition, $x \in \overline{A} \setminus \text{int}A$. We want to show that $\overline{A} \subset [0,1]$. It suffices to show that $\text{ext}[0,1] \subset \text{ext}A$. Suppose that $y \in \text{ext}[0,1]$. Clearly, it must be that $y < 0$ or $y > 1$. Letting $\epsilon = min(|y-1|, |y|)$ taking $R = (y-\epsilon, y+\epsilon)$will ensure that the rectangle is disjoint from A. Hence, $y \in \text{ext}A$ and so $\text{ext}[0,1] \subset \text{ext}A$. Equivalently, $\overline{A} \subset [0,1]$ Since A is the union of open intervals, it is an open set as well, so $\text{int}A = A$. It follows that $\overline{A} \setminus \text{int}A \subset [0,1] \setminus A$, and so $x \in [0,1] \setminus A$. Therfore $\text{Bd}A \subset [0,1] \setminus A$

"$\supset$"

Suppose that $x \in [0,1] \setminus A$. Consider the open set $U = (x-\epsilon, x+\epsilon)$ for $\epsilon > 0$. By the density of the rational numbers in $\mathbb{R}$, there exists some rational number $r \in (x-\epsilon, x+\epsilon)$. By the definition of A, there must exist an i such that $r \in (a_i, b_i) \subset A$. From our choice of x we see that $U \cap \mathbb{R} \setminus A \neq \emptyset$ and $U \cap A \neq \emptyset$. This is exactly what it means to be in the boundary of A. Thus $\text{Bd}A \supset [0,1] \setminus A$ ■