Q2:

WLOG suppose that $D_{12}f - D_{21}f > 0$ at some point (x, y). Since both functions are continuous, so must be their linear combination.Thus we know that in some open rectangle, B, around (x, y), $D_{12}f - D_{21}f > 0$ holds. Choose $A = [a, b] \times [c, d]$ such that it is contained in the interiour of B. By HW7Q3 it must be that $\int_A f(x, y)dxdy > \int_A 0 = 0$. We compute that

$$
\begin{aligned}
&\int_A D_{12}f - D_{21}f dxdy = \int_c^d \int_a^b D_{12}f - D_{21}f dxdy && \text{(by Fubini's Theorem)}\\
&= \int_c^d \int_a^b D_1D_2f(x, y)dxdy - \int_c^d \int_a^b D_2D_1f(x, y)dxdy && \text{(by linearity of the integral)}\\
&= \int_c^d \int_a^b D_1D_2f(x, y)dxdy - \int_a^b \int_c^d D_2D_1f(x, y)dydx && \text{(by Fubini's Theorem)}\\
&= \int_c^d [D_2f(a, y) - D_2f(b, y)]dy - \int_a^b [D_1f(x, c) - D_1f(x, d)]dx && \text{(by FTC)}\\
&= \int_c^d D_2f(a, y)dy - \int_c^d D_2f(b, y)dy - \int_a^b D_1f(x, c)dx + \int_a^b D_1f(x, d)dx && \text{(By linearity of the integral)}\\
&= f(a, d) - f(a, c) - [f(b, d) - f(b, c)] - [f(b, c) - f(a, c)] + f(b, d) - f(a, d) && \text{(by FTC)}\\
&= 0
\end{aligned}
$$

We get that $0 > 0$, a contradiction. Hence $D_{12}f = D_{21}f$ ■