

Q1a: Since  $a|b$ , we have for some  $k \in \mathbb{Z}$ ,  $ak = b$ , and since  $b|c$  for some  $l \in \mathbb{Z}$ ,  $lb = c$ . Therefore,  $c = lb = lka$  and so  $a|c$ .

Q1b: Since  $a|b$ , we have for some  $k \in \mathbb{Z}$ ,  $ak = b$ , and since  $c|d$  we have for some  $l \in \mathbb{Z}$ ,  $lc = d$ . Therefore, we have  $bd = (ka)(lc) = (kl)(ac)$  and so we conclude that  $ac|bd$ .

Q1c: Let  $m \neq 0$ . We note that  $a|c \iff ak = c$  for some  $k \in \mathbb{Z} \iff mak = bm \iff ma|mb$ .

Q1d: Since  $d|a$ , for some  $k$ ,  $dk = a$ . Since  $a \neq 0$ , we have that  $k \neq 0$ . So thus

$$|dk| = |a| \implies |k||d| = |a| \implies |d| \leq |a|$$

With equality holding when  $|k| = 1$