Q2: Let T be a surjective linear mapping from banach spaces E to F. Suppose that T has a right inverse S. We claim that $R(S)$ is the compliment of $N(T)$. First we show that

$$R(S) \cap N(T) = \{0\}.$$

Let $v \in R(S) \cap N(T)$. For some $u, Su = v$. We also have that $Tv = TSu = id_F u = 0$. Therefore $u = 0$ and so $v = 0$. We now claim that $E = R(S) + N(T)$. If $v \in E$, then $u = STv - v \in \ker T$. Therefore $v = STv - u$. Now suppose that $N(T)$ has a compliment in E. Let G be the compliment. We can write any $v \in E$ as $v = x + y$ for $x \in N(T), y \in G$. Define the right inverse S as $S(Tv) = x$. We see that $T(S(Tv)) = T(S(Tx)) = Tx$.