

Q6: Suppose that $x_n \rightarrow x$, and $Tx_n \rightarrow y$. Since $f \circ T$ is continuous, we have that $f(Tx_n) \rightarrow f(Tx)$. We also know that $f(Tx_n) \rightarrow f(y)$. Therefore $f(y) = f(Tx)$. Since continuous linear functions separate points, we have that $Tx = y$. Thus by the closed graph theorem, T is bounded.