Q3: Let σ have a disjoint cycle decomposition of

$$\sigma = \sigma_1 \dots \sigma_k.$$

We claim that if each σ_i is conjugate to its inverse, then so is σ. Let τ_i be the permutation which sends $\sigma_i \to \sigma_i^{-1}$ by conjugating. Note that the τ_i must be disjoint from eachother, since it can only act on the elements each σ_i acts on. We have that

$$\sigma^{-1} = \sigma_k^{-1} \dots \sigma_1^{-1} = (\tau_k \sigma_k \tau_k^{-1}) \dots (\tau_1 \sigma_1 \tau_1^{-1}) = (\tau_k \dots \tau_1)(\sigma_k \dots \sigma_1)(\tau_1^{-1} \dots \tau_k^{-1}) = (\tau_k \dots \tau_1)(\sigma)(\tau_k \dots \tau_1)^{-1}$$

Hence without loss of generality we assume that $\sigma = (a_1 a_2 \dots a_k)$. We know that $\sigma^{-1} = (a_1 a_k \dots a_2)$. Take $\tau = (a_2 a_k)(a_3 a_{k-1}) \dots$. We compute that

$$\tau\sigma = (a_1 a_k)(a_2 a_{k-1}) \cdots = \sigma^{-1}\tau,$$

As desired.