Q1: Let $\varepsilon > 0$. Define $A_\varepsilon = \{x : |f(x)| < \varepsilon\}$. Choose $\delta = \frac{\varepsilon}{1+\varepsilon}$. We cover $[0,1]$ with intervals of the form $(x-\frac{\delta}{2}, x+\frac{\delta}{2})$. By compactness, there exists finitely many $x_i$ corresponding to sets of the form $(x_i-\frac{\delta}{2}, x_i+\frac{\delta}{2})$ which cover $[0,1]$. It is sufficient to check that each set $(x_i-\frac{\delta}{2}, x_i+\frac{\delta}{2}) \cap A_\varepsilon$ is of measure 0 when mapped under $f$, since

$$f(\bigcup_{i=1}^{n} A_\varepsilon \cap (x_i - \frac{\delta}{2}, x_i + \frac{\delta}{2})) = \bigcup_{i=1}^{n} f(A_\varepsilon \cap (x_i - \frac{\delta}{2}, x_i + \frac{\delta}{2}))$$

We have that $f$ is differentiable on $(x_i-\frac{\delta}{2}, x_i+\frac{\delta}{2}) \cap [0,1]$ and continuous on $[x_i-\frac{\delta}{2}, x_i+\frac{\delta}{2}] \cap [0,1]$. Hence we can apply the mean value theorem and get that

$$\frac{|f(x_i + \frac{\delta}{2}) - f(x_i - \frac{\delta}{2})|}{\delta} \leq \varepsilon$$

For convinience we let $a = f(x_i+\frac{\delta}{2})$ and $b = f(x_i-\frac{\delta}{2})$. Consider the interval $X = (\min(a,b)-\frac{\delta}{2}, \max(a,b)+\frac{\delta}{2})$. We have that $X \supset f(A_\varepsilon \cap (x_i-\frac{\delta}{2}, x_i+\frac{\delta}{2}))$, since the image of connected intervals is a connected interval. We therefore get that

$$m^*(f(A_\varepsilon \cap (x_i - \frac{\delta}{2}, x_i + \frac{\delta}{2}))) \leq m^*(X) < \delta + \delta \cdot \varepsilon = \varepsilon$$

Therefore we have that $m^*(f(A_\varepsilon \cap (x_i-\frac{\delta}{2}, x_i+\frac{\delta}{2})))$ is of measure 0, and hence $A_\varepsilon$ is measure 0. We conclude that $f(\{x : |f'(x)| = 0\})$ is measure 0.