Q1i: We wish to solve $3x \equiv 5(\text{mod } 7)$. We see that $\gcd(3,7) = 1$. Since $1|5$, by theorem 3 this will have a solution. We can check and see that $[3(4)]_7 = [5]_7$. So $x = 4$ satisfies this equation. By theorem 3.7, we will have a general solution of the form $x = 4 + 7t$.

Q1ii: We see that $\gcd(12, 22) = 2$ and $2 \nmid 15$. Thus by theorem 3.7 this linear congruence has no solution.

Q1iii: We see that $\gcd(19, 50) = 1$ so a solution exists. Taking $x = 18$ we see that $19 \cdot 18 \equiv 342 \equiv 45\text{mod } 50$.

Q1iv: From 1c we know that $x = 19$ will solve. By lemma 3.9, we also have that $18x \equiv 42\text{mod } 50$ is equivalent to $9x \equiv 21\text{mod } 25$. This is solved by $x = 19$ as well.
Q1bi: Let $n = 4 \cdot 3 \cdot 5 = 60$. Let $c_1 = 15, c_2 = 20, c_3 = 12$. We want to find $d_i = x$ such that $c_i x \equiv 1\text{mod } (n_i)$. Take $d_1 = 3, d_2 = 2, d_3 = 3$. Thus by the CRT the solution will be $x = 1 \cdot 3 \cdot 15 + 2 \cdot 20 \cdot 2 + 3 \cdot 3 \cdot 12 = 53 mod 60$
Q1bii: Let $n = 7 \cdot 9 \cdot 4 = 252$. Let $c_1 = 36, c_2 = 28, c_3 = 63$. We want to find $d = x_i$ such that $c_i x \equiv 1 mod(n_i)$. Take $d_1 = 1, d_2 = 1, d_3 = 3$. Then the particular solution is

$$x_0 = 2 \cdot 1 \cdot 36 + 7 \cdot 1 \cdot 28 + 3 \cdot 3 \cdot 63 = 79 mod(252)$$

Q1c: Suppose the remainders of the given number x when divided by $3, 5, 7$ are a_1, a_2, a_3. We have that $n = 105$, $n_1 = 3, n_2 = 5, n_3 = 7$, $c_1 = 35, c_2 = 21, c_3 = 15$. We must find $x = d_i$ which solve $c_i x \equiv 1 mod(a_i)$. Take $d_1 = -1, d_2 = 1, d_3 = 1$. By the proof of theorem 3.10 the solution will be

$$x_0 = a_1 c_1 d_1 + a_2 d_2 c_2 + a_3 c_3 d_3 = -35a_1 + 21a_2 + 13a_3$$

This is in mod(105) so this formula will work for all numbers below 100.