Q6: By Gauss' Lemma, $x^2 - \sqrt{p}$ is irreducible in $\mathbb{Z}[\sqrt{p}][x]$ if and only if it is irreducible in $\mathbb{Q}[\sqrt{p}][x]$. If this polynomial is reducible, it must have two roots, and hence can be written as

$$x^2 - \sqrt{p} = (x - \alpha)(x - \beta).$$

This implies that $\alpha, \beta | -\sqrt{p}$. Since $\mathbb{Q}[\sqrt{p}]$ is a field, the only solutions that exist are $\alpha = \pm 1, \pm\sqrt{p}$. We can check that neither of these are a root of the polynomial in either $\mathbb{Q}[\sqrt{p}][x]$ or $\mathbb{Z}[\sqrt{p}][x]$. Therefore the polynomial is irreducible in $\mathbb{Z}[\sqrt{p}][x]$.