Q3a:

$$f(x,y) = \int_0^x g_1(t,0)dt + \int_0^y g_2(x,t)dt$$

Applying the fundamental theorem of calculus, we get that

$$D_2 f(x,y) = \frac{\partial}{\partial y}\int_0^x g_1(t,0)dt + \frac{\partial}{\partial y}\int_0^y g_2(x,t)dt = g_2(x,y)$$

3b:
For f to satisfy $D_2 f(x,y) = g_1(x,y)$, define f in the following way.

$$f(x,y) = \int_0^x g_1(t,y)dt + \int_0^y g_2(0,t)dt$$

by the FTC we will have that

$$D_1 f(x,y) = g_1(x,y)$$

3c:
Choose f_c in the following way

$$f_c(x,y) = \frac{1}{2}(x^2+y^2)$$

From single variable calculus, we have that $\frac{\partial f_c}{\partial x} = x$ and $\frac{\partial f_c}{\partial y} = y$
3d:
Choose f_d in the following way

$$f_d(x,y) = xy$$

From single variable calculus, we have that $\frac{\partial f_d}{\partial x} = y$ and $\frac{\partial f_d}{\partial y} = x$