

Q1a: Consider the set of all piecewise linear functions with slope less than or equal to  $|2n|$ . They separate points and form a function algebra. Thus any continuous function on  $[0, 1]$  can be approximated by such function by Stone-Weierstrass Theorem. Thus for any  $f \in E_n$  there is some  $g$  such that  $|f - g| < \varepsilon$ . However we see that  $|g(x) - g(x_0)| > 2n|x - x_0| > n|x - x_0|$ , since  $n \geq 1$ . Therefore  $g \notin E_n$ , and so  $E_n$  is nowhere dense.

Q1b: Since  $\bigcup E_n$  is the set of all locally Lipschitz and hence somewhere differentiable functions, and is meager by 1a, it follows that the complement is the set of all nowhere differentiable functions, which must be a residual set.