Q2: If our differential form is given by

$$f(z)dz = \frac{1}{(z-3)(z^5-1)}dz.$$

To compute the integral along $|z|=4$, we can instead change to coordinates at ∞ and compute the residue. Changing coordiantes to ∞ by substituting $z \mapsto \frac{1}{z}$ gives us

$$f(\frac{1}{z})d\frac{1}{z} = \frac{-z^4}{(1-3z)(1-z^5)}dz.$$

Thus we compute that

$$\oint_{|z|=4} f(z)dz = \oint_{|z|=\frac{1}{4}} \frac{-z^4}{(1-3z)(1-z^5)}dz = 2\pi i Res\Big(-\frac{1}{z^2}f(\frac{1}{z}),0\Big) = 0,$$

since our function is holomorphic at ∞. To compute the same integral along $|z|=2$, we know that the sum of the residues including at infinty is 0. Therefore we just compute the residue at $z=3$ since this is not enclosed by $|z|=2$, and our final answer will be $-(2\pi i \cdot res(f,3))$. We compute the residue at 3 as

$$res(f,3) = \frac{1}{[(z-3)(z^5-1)]'}\Big|_{z=3} = \frac{1}{(6z^5-1-15z^3)}\Big|_{z=3} = \frac{1}{242}.$$

Therefore

$$\oint_{|z|=2} \frac{1}{(z-3)(z^5-1)}dz = -(2\pi i\frac{1}{242}) = -\frac{\pi i}{141}$$