Problem 10. *Diba:*

For $F(z) = \int_a^z f dg$ we verify that it is not discrete analytic. Take a path $\{z_0, \ldots, z\}$ from a to z. Then

$$
\begin{aligned}
\frac{F(z+i+1) - F(z)}{i+1} &= \frac{1}{i+1}\left[\int_a^{z+i+1} f dg - \int_a^z f dg\right] \\
&= \frac{(f(z+i)+f(z))[g(z+i)-g(z)] + (f(z+i+1)+f(z+i))[g(z+i+1)-g(z+i)]}{2(i+1)} \\
&= 2z^3 + (3+i)z^2 + 4iz - 1 + i
\end{aligned}
$$

Similarly we compute:

$$
\begin{aligned}
\frac{F(z+1) - F(z+i)}{1-i} &= \frac{1}{1-i}\left[\int_a^{z+1} f dg - \int_a^{z+i} f dg\right] \\
&= \frac{(f(z+1)+f(z))(g(z+1)-g(z)) + (f(z+i)+f(z))(g(z+i)-g(z))}{2(1-i)} \\
&= \frac{1}{2}\left(4iz^3 + 4z + 1 + i\right)
\end{aligned}
$$

These are unequal.