

Q1: Suppose that the two sided ideal I is not 0. Then for $A \in J$ with rank k , we have that there exists $X, Y \in M_{n \times n}(\mathbb{F})$ so that

$$XAY = \begin{bmatrix} I_k & 0 \\ 0 & 0 \end{bmatrix}$$

with I_k the $k \times k$ identity matrix. So $XAY \in J$. We can apply permutation matrices so that the block I_k can be shifted diagonally. So therefore the sum of these diagonal matrices must be in I . We can rescale this matrix to get the identity. So $I \in J$. So $J = M_{n \times n}(\mathbb{F})$.