

Q6: Consider the function $f(x) = \begin{cases} 6x(1-x) & x \in [0, 1] \\ 0 & x \notin [0, 1] \end{cases}$. Define $f_n(x) = nf(nx)$. We have that $f_n(x) = 0$ for $nx \geq 1$, therefore $\lim_{n \rightarrow \infty} f_n(x) = 0$. However,

$$\int_0^1 f_n(x) dx = \int_0^\infty f_n(x) dx = \int_0^\infty nf(nx) dx = \int_0^\infty f(y) dy = 1$$

And therefore, $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = 1$