Q1a: We first claim that for an open covering of $[a, b]$ by open balls, each ball will intersect at least one other. Suppose not, that is assume that (x, y) belongs to the open cover of $[a, b]$, and is disjoint from every other open cover. Then the point y belongs to some open set B belonging to the cover. By openness of B, there is some open ball U_y contained in B and containing y. This ball must intersect (x, y) which contradicts our assumption. We now prove the main result. Let ε be given. By equicontinuity of f_n, for some $\delta > 0$ we have an open covering of $[a, b]$ of the form $\mathcal{O} = \{B_{\frac{\delta}{2}}(x) = (x - \frac{\delta}{2}, x + \frac{\delta}{2}) : x \in [a, b]\}$. By compactness of $[a, b]$, there exists finitely many, and strictly increasing $x_1, \ldots, x_k$ where $B_{\frac{\delta}{2}}(x_1), \ldots, B_{\frac{\delta}{2}}(x_k)$ cover $[a, b]$. So p must belong to some $B_{\frac{\delta}{2}}(x_\alpha)$. By equicontinuinty of f_n, and by the boundedness at $f_n(p)$, we have that $f_n(x)$ will be bounded by $M + \varepsilon$ on $B_{\frac{\delta}{2}}(x_\alpha)$. Now take $x_0, x_1 \in B_{\frac{\delta}{2}}(x_\alpha)$ with $x_0 \in B_{\frac{\delta}{2}}(x_{\alpha-1})$ and $x_1 \in B_{\frac{\delta}{2}}(x_{\alpha+1})$. Repeating the same process as above we have that $|f_n(x)| \leq M + 2\varepsilon$ on $B_{\frac{\delta}{2}}(x_{\alpha-1}) \cup B_{\frac{\delta}{2}}(x_\alpha) \cup B_{\frac{\delta}{2}}(x_{\alpha+1})$. We repeat this process up to k times we have that $|f_n(x)| \leq M + k\varepsilon$. Therefore, f_n is uniformly bounded.

Q1b: Since being bounded at a point propagates to boundedness on entire domain, we can reformulate the Arzela-Ascoli in the following way. Let $(f_n) \subset C^o([a, b], \mathbb{R})$ be an equicontinuous sequence of function. Suppose that for some $p \in [a, b]$, $|f_n(p)| < M$ for some M, then (f_n) admits a uniformly convergent subsequence.

Q1c: This is true for (a, b). Since we can extend any continuous function on $[a, b]$ by $\tilde{f}(x) = f(x)$ for $x \in [a, b]$, and $\tilde{f}(a) = \lim_{x \to a_-} f(x)$ and $\tilde{f}(b) = \lim_{x \to b_+} f(x)$. The fact from 1a is true for $\tilde{f}$ so it will also be true for f. The result from $1a$ does not hold on either $\mathbb{Q}, \mathbb{R}, \mathbb{N}$ since they are not compact, or can not be extended to a compact set. Consider the family (fn) where $f_n(x) = x$. We have that on $\mathbb{Q}, \mathbb{N}, \mathbb{R}$ that $f_n(1)$ is bounded for all n, namely by 1 and (f_n) is equicontinuous. However, these functions are unbounded on $\mathbb{Q}, \mathbb{N}, \mathbb{R}$.