Q3i: Write

$$2 = (a + b\sqrt{-n})(c + d\sqrt{-n}).$$

Taking the norms of both sides, get that

$$4 = N(2) = N(a + b\sqrt{-n})N(c + d\sqrt{-n}) = (a^2 + nb^2)(c^2 + nd^2).$$

The terms on the right can either be both 2 or 1 and 4. Since $n > 3$ they can not both be 2. Therefore one must have norm 1 and another must have norm 4. So we have that $2 = \pm 1 \cdot \pm 2$. Hence 2 is irreducible. Next we will show that $\sqrt{-n}$ is irreducible. We write

$$\sqrt{-n} = (a + b\sqrt{-n})(c + d\sqrt{-n}).$$

Taking norms, we have that

$$n = N(\sqrt{-n}) = (a^2 + nb^2)(c^2 + nd^2).$$

If $\sqrt{-n}$ is not irreducible then both of the numbers on the right are not 1 or n. But this implies that $b, d = 0$. Thus $n = a^2c^2$. A contradiction. Finally we suppose that

$$1 + \sqrt{-n} = (a + b\sqrt{-n})(c + d\sqrt{-n}).$$

Taking norms, we see that

$$1 + n^2 = (a^2 + nb^2)(c^2 + nd^2) = a^2c^2 + n(a^2d^2 + b^2c^2) + b^2d^2n^2.$$

This yield $a^2c^2 = 1$, $b^2d^2 = 1$, and $a^2d^2 + b^2c^2 = 0$. This is impossible however, since this implies that $a^2d^2 = b^2c^2 = 0$ and since $\mathbb{Z}$ is an integral domain this implies that either a^2 or d^2 is 0, and b^2 or c^2 is 0, which will break the other equalities.

Q3ii: Suppose that $\sqrt{-n}$ and $1 + \sqrt{-n}$ are both prime. Then they will generate prime ideals, and so by Prop. 13 (p.255 dummite and foote), we have that $\mathbb{Z}[\sqrt{-n}]/(\sqrt{-n})$ and $Z[\sqrt{-n}]/(1 + \sqrt{-n})$ must be integral domains. In $\mathbb{Z}[\sqrt{-n}]/\sqrt{-n}$ we can write every element as $a + b\sqrt{-n}$. If $a \geq n$, then we can write $a = pn + r$ for some p, r. Since $pn \in (\sqrt{-n})$, we have that $a + b\sqrt{-n}$ is the same as r in the quotient ring. Thus this quotient ring is isomorphic to Z/nZ. Similarly, $Z[\sqrt{-n}]/(1 + \sqrt{-n}) \cong \mathbb{Z}/(n+1)\mathbb{Z}$. These rings can not both be integral domains, since one of $n, n+1$ is even, and hence can be factored into a product of 2 and another integer. These are both zero divisors.