

Q2: Let $f(x) = e^x - e^2x$. We have that $f(-1) = \frac{1}{e} + e^2 > 0$ and $f(1) = e - e^2 < 0$. Thus by the intermediate value theorem, there is some x_0 with $f(x_0) = 0$. Thus $f(z) = e^z - e^2z$ has a real solution on $[-1, 1]$. We claim that it has only one complex root on the closed unit disk. Note that we have that

$$|e^z| < |-e^2z|$$

on S^1 , since $|e^2z| = e^2$ and $|e^z| = 1$, and $1 < e^2$. Thus $e^z - ze^2$ has only 1 zero on the closed disk by Rouchi's theorem.