Q2a: Solve $x' = 2x + t^2$. We guess that x takes the form of a quadratic, namely $x(t) = at^2 + bt + c$. We know that $x'(t) = 2at + b$. For equality to hold, we can choose constants $a = b = -\frac{1}{2}$, and $c = \frac{b}{2} = -\frac{1}{4}$. Therefore we have that

$$x' = -t + \frac{-1}{2} = 2(-\frac{1}{2}t^2 - \frac{-1}{2}t - \frac{1}{4}) + t^2 = 2x + t^2$$

Q2b: Solve $x' = 2x + e^{3t}$. We will guess that $x(t) = ce^(3t)$ for some constant c. We note that $x' = 3ce^{3t}$. Thus for equality to hold, we can take $c = 1$ and observe that

$$x' = 3e^{3t} = 2e^{3t} + e^{3t} = 2x + e^{3t}$$

Q2c: To solve $x' = 2x + te^{3t}$ we will proceed by guessing something of the form $x(t) = cte^{3t} - de^3t$. By simple computation we see that $x'(t) = 3ce^{3t} + 3cte^{3t} - 3de^{3t}$, and $2x + te^{3t} = 2cte^{3t} - 2de^{3t} + te^{3t}$. We have equality holding if we choose constants $c = d = 1$. Thus $x(t) = te^{3t} - e^{3t}$ solves.

Q2d: We guess a solution of the form $x(t) = c\sin(t) + 6d\cos(t) + e^{kt}$. We have that $x' = c \cdot cos(t) - d\sin(t) + ke^{kt}$ and $2x - cos(t) = 2c\sin(t) + (2d-1)cos(t) + 2e^{kt}$. We see that when $k = 2$, $c = -\frac{2}{5}$, $d = -\frac{2}{5}$. Therefore, $x(t) = -\frac{cos(t)}{5} - \frac{2\sin(t)}{5} + e^{2t}$ solves.

Q2e: To solve $x' = 2x + f(t)$, we guess a solution of the form $x(t) = te^{2t} + e^{2t}$. We see that $x' = e^{2t} + 2te^{3t} + 2e^{2t}$, and $2x + e^{2t} = 2te^{2t} + 2e^{2t} + e^{2t}$. These functions are equal, so our guess is correct.