Q4a: We claim that $x'(t) > 0$ for all $t \in \mathbb{R}$. First, notice that if $x'(t) = 0$ for some $t \in \mathbb{R}$, we have that by uniqueness $x \equiv a$ on all of $\mathbb{R}$. This can not be the case though, since $f(x(0)) = f(a) > 0$. Suppose now that $f(t_0) < 0$ at some point t_0. Then by taking a sufficiently large interval with endpoints t_0 and 0, the intermediate value theorem implies that at some point x_0, $f(x_0) = 0$. Once again, this implies that we have a constant solution which can not be the case

Q4b: We first take note that the function $x(t)$ can never be equal to 0 or 1, since this would imply it is a constant solution. Thus we have that $0 < x(t) < 1$. Since $x(t)$ is strictly increasing and continuous and bounded aboce at each point, by the monotone convergence theorem $\lim_{t\to\infty} x(t)$ converges to the supremum of $x(t)$. This will be 1. Simliarly, $\lim_{t\to-\infty} x(t)$ will converge to the infimum, which will be 0. We can evaluate that $\lim_{|t|\to\infty} x'(t) = 0$ in the following way.

$$\begin{aligned}
\lim_{t\to\infty} x' &= \lim_{|t|\to\infty} \lim_{h\to 0} \frac{x(t+h) - x(t)}{h} \\
&= \lim_{h\to 0} \lim_{|t|\to\infty} \frac{x(t+h) - x(t)}{h} \\
&= \lim_{h\to 0} \frac{0}{h} \\
&= 0
\end{aligned}$$

Q4c: Suppose that $y(t)$ is a solution of $x' = f(x)$, with initial value $y(0) >= b > a$. We know that y will share properties with x, namely those shown in a and b. Suppose that in some neighborhood of a point t_0, we have that $y(t_0) \leq x(t_0)$. The intermediate value theorem tells us that the functions x and y must be equal at some point. However, from our discussion in class we know that integreal curves can not cross eachother. Hence $y(t) > x(t) \forall t$.