Q2: Note that for any path from 0 to 1 in $\mathbb{C}$, we can write it as a sum of a closed path γ and one of 3 paths $\gamma_1, \gamma_2, \gamma_3$ which are paths from 0 to 1 which travel around i, 0 to 1 and $-i$ respectively. Using Cauchy's integral formula, we have that

$$\int_{\gamma+\gamma_i} \frac{dz}{1+z^2} = \int_{\gamma} \frac{dz}{1+z^2} + \int_{\gamma_i} \frac{dz}{1+z^2}.$$

We will compute each integral seperately. By Cauchy's Integral Formula, we have that

$$\int_{\gamma} \frac{dz}{1+z^2} = \frac{-i}{2}\int_{\gamma} \frac{dz}{z-(-i)} + \frac{i}{2}\int_{\gamma} \frac{dz}{z-i} = \frac{-i}{2}(2\pi i)w(\gamma,-i) + \frac{i}{2}(2\pi i)w(\gamma,i) = \pi w(\gamma,-i) - \pi w(\gamma,i)$$

We now compute the value of $\int_{\gamma_i} \frac{dz}{1+z^2}$. First for γ_1, we have that

$$\int_{\gamma_1} \frac{dz}{1+z^2} = \frac{-i}{2}\int_{\gamma_1} \frac{dz}{z-(-i)} + \frac{i}{2}\int_{\gamma_1} \frac{dz}{z-i} = \frac{-3\pi}{4}.$$

Since a complete path around i will have an integral of $-\pi$, and we add $\frac{\pi}{4}$ from integrating along $[0,1]$. Similarly, we have that

$$\int_{\gamma_3} \frac{dz}{1+z^2} = \frac{-3\pi}{4}.$$

Finally, if we integrate along γ_2, we will have

$$\int_{[0,1]} \frac{dz}{1+z^2} = \arctan(1) - \arctan(0) = \frac{\pi}{4}.$$

Thus,

$$\int_{\gamma+\gamma_i} \frac{dz}{1+z^2} = \pi(w(\gamma,-i) - w(\gamma,i)) + \begin{cases} \frac{-3\pi}{4} : i = 1,3 \\ \frac{\pi}{4} : i = 2 \end{cases}$$