Q5a: Since M, N, are manifolds, for each $p \in M, q \in N$ there must exist open $U \ni p, V \ni q$ and a $g : U \to \mathbb{R}^{m-k}$, $f : V \to \mathbb{R}^{n-l}$ with $U \cap M = U \cap g^{-1}(\{0\})$ and $V \cap N = V \cap f^{-1}(\{0\})$ and $Dg(p)$ and $Df(q)$ have full rank. We define $h : U \times V \mapsto \mathbb{R}^{m+n-(k+l)}$ by $h(x, y) = (g(x), f(y))$. We can see that $(U \times V) \cap (M \times N) = (U \times V) \cap h^{-1}(\{0\})$, and we evaluate the differential of h at p, q as

$$Dh(p,q) = \begin{bmatrix} Dg(p) & 0 \\ 0 & Df(q) \end{bmatrix}$$

This will have a rank of $n + m - (k + l)$, and hence we conclude that $M \times N$ is a $k + l$ manifold.

Q5b: We first claim that $\partial M \times \partial N$ is a $k + l - 2$ manifold without boundary. First, note that from discussion in class, we know that ∂M, ∂N are $k - 1$ and $l - 1$ manifolds respectively. Hence by 5a, we have that their cartesian product will be a $(k - 1) + (l - 1) = k + l - 2$ manifold. Next, consider the manifold $M \setminus \partial M$. We have that every point in $M \setminus \partial M$ will have some coordinate chart with domain in the interiour of $\mathbb{R}^k_+$, and similarly for $N \setminus \partial N$ and $\mathbb{R}^l_+$. Hence we will have that $M \setminus \partial M$ will be a k manifold with boundary, and similarly $N \setminus \partial N$ will be an l manifold with boundary. Note that $M \setminus \partial M$ and $N \setminus \partial N$ have empty boundary, but are still manifolds with boundary. By $5a$, $M \setminus \partial M \times N \setminus \partial N$ is a $k + l$ manifold, and by construction is disjoint from $\partial M \times \partial N$.