Q4: Note that when we refer to $\mathbb{R}^{k-1}$, we will be referring to $\mathbb{R}^{k-1}$ as a subset of $\mathbb{R}^k$. First note that since $U = U' \cap \mathbb{R}^k$, and $V = V' \cap \mathbb{R}^k$, it suffices to show that $\phi(U' \cap \mathbb{R}^{k-1}) = V' \cap \mathbb{R}^{k-1}$. First suppose that $x \in \phi(U\prime \cap \mathbb{R}^{k-1})$. Since ϕ is a diffeomorphism, there must exist some unique $y \in U' \cap \mathbb{R}^{k-1}$ such that $\phi(y) = x$. We can find some open set $B \cap \mathbb{R}^k$ with $y \in B \cap \mathbb{R}^k$ and $B \cap \mathbb{R}^k \subset U' \cap \mathbb{R}^k$. Hence since ϕ is a diffeomorphism, we have that $\phi(B \cap \mathbb{R}^{k-1}) \subset V\prime \cap \mathbb{R}^{k-1}$. Hence $x \in V' \cap \mathbb{R}^{k-1}$. Now suppose that $x \in V' \cap \mathbb{R}^{k-1}$. Then we have that x must belong to $V \cap \mathbb{R}^k$. There must therefore exist some $y \in U$ such that $\phi(y) = x$. We claim that such a y must belong to $U \cap \mathbb{R}^{k-1}$. Suppose not, then it must not belong to either U or $\mathbb{R}^{k-1}$. It must definitely belong to U, so suppose it was not an element of $\mathbb{R}^{k-1}$. If we take a sufficiently small open set C around y, such that C is disjoint from $\mathbb{R}^{k-1}$, Then the image of C under ϕ must also be disjoint from $\mathbb{R}^{k-1}$. However it contains y and $\phi(y) = x \in V' \cap \mathbb{R}^{k-1}$, a contradiction. We conclude that indeed $\phi(U \cap \mathbb{R}^{k-1}) = V \cap \mathbb{R}^{k-1}$.