Assignment 8 MAT 357

Q1a: Since f is measurable, we have that $\mathcal{U}(f)$ and $\hat{\mathcal{U}}(f)$ are measurable. We know that the graph of f is $\hat{\mathcal{U}}(f) \setminus \mathcal{U}(f)$. Hence it is measurable. We can cover both $\mathcal{U}(f)$ and $\hat{\mathcal{U}}(f)$ with closed rectangles to obtain their measure. Therefore, their set difference is the zero set.

Q1b: No consider the following. If $C \subset \mathbb{R}$ is any non measurable set, Then the graph of the indicator function χ_C will not be measurable, but since it is contained in the graph of the constant function f = 1 it must be measure 0.

Q1e: Let Γ_f be the graph of f. Since $m_*\Gamma_f \leq m^*\Gamma_f = 0$. Hence the inner measure is 0