

Q1a: Need to show whether  $f(x, y) = x_1y_2 - x_2y_1 + x_1y_1$  is an alternating tensor on  $\mathbb{R}^4$  or not. We can verify by computation that

$$f(x, x) = x_1x_2 - x_2x_1 + x_1x_1 \neq 0$$

This function is not alternatin and thus can not be an alternating tensor.

Q1b: We need to show whether  $g(x, y) = x_1y_3 - x_3y_2$  is an alternating tensor on  $\mathbb{R}^4$  or not. We see that

$$g(x, x) = x_1x_3x_3x_2 \neq 0$$

This is not an alternating tensor on  $\mathbb{R}^4$ .

Q1c: Need to show whether  $h(x, y) = (x_1)^3(y_2)^3 - (x_2)^3(y_1)^3$  is an alternating tensor on  $\mathbb{R}^4$  or not. We can verify that  $h$  is not even a 2 tensor on  $\mathbb{R}^4$  since

$$h(\lambda x, y) = (\lambda x_1)^3(y_2)^3 - (\lambda x_2)^3(y_1)^3 = \lambda^3 h(x, y)$$

This function fails homogeneity and thus is not a tensor, much less an alternating tensor.