5.5.59: For $f \in \mathcal{H}$, we define $\delta = \inf\{\|f - v\| : v \in K\}$. Let $\{v_n\}$ be a sequence in K so that $\|f - v_n\| \to \delta$. By the parrelelogram law we have that

$$2(\|v_n - f\|^2 + \|v_m - f\|^2) = \|v_n - v_m\|^2 + \|v_n + v_m - 2f\|^2 .$$

By conevexity we have that $\frac{1}{2}(v_n + v_m) \in K$ for all n, m, therefore we get that

$$\|y_n - y_m\|^2 = 2\|v_n - f\|^2 + 2\|v_m - f\|^2 - 4\left\|\frac{1}{2}(v_n + v_m) - f\right\|^2 \leq 2\|v_n - f\|^2 + 2\|v_m - f\|^2 - 4\delta^2.$$

Taking $n, m \to \infty$ we get that the sequence must be cauchy. Therefore it must converge since K is closed in $\mathcal{H}$. Let u be the limit. We claim u is unique. If not, let u_1, u_2 satisfy minimality. Then,

$$2\left\|f - \frac{1}{2}(u_1 + u_2)\right\| \leq \|f - u_1\| + \|f - u_2\| .$$

Minimality implies that this is an equality and so $u_1 = u_2$. We now claim that $\langle f - u, v - u\rangle \leq 0$ for all $v \in K$.