Q1a:
We define f in the following way.

$$f(x) = \begin{cases} \frac{3(-1)^n}{n} \text{ if } x \in [n+\frac{1}{3}, n+\frac{2}{3}] \text{for } n \in \mathbb{N} \\ 0 \text{ if } x < 1 \text{ or } x \in (n-\frac{1}{3}, n+\frac{1}{3}) \text{ for } n \in \mathbb{N} \end{cases}$$

It is readily available that the support of f is as desired, by construction. It suffices to check that $\int_{n+\frac{1}{3}}^{n+\frac{2}{3}} f = \frac{(-1)^n}{n}$.

$$\int_{n+\frac{1}{3}}^{n+\frac{2}{3}} f = \int_{n+\frac{1}{3}}^{n+\frac{2}{3}} \frac{3(-1)^n}{n} dx = (n+\frac{2}{3}-n-\frac{2}{3}) \cdot \frac{3(-1)^n}{n} = \frac{(-1)^n}{n}$$

1b:
Choose an open cover $\mathcal{U}$ of $\mathbb{R}$ in the following way. Each $A_n = [n+\frac{1}{3}, n+\frac{2}{3}]$ has some $U_n \in \mathcal{U}$ with $[n+\frac{1}{3}, n+\frac{2}{3}] \subset U_n$, and each U_n disjoint from one another. We cover the rest of $\mathbb{R}$ in any collection of open sets such that these sets will be disjoint from each A_n. Find a $PO1$ Φ subordinate to $\mathcal{U}$. We note that $\sum_{\phi \in \Phi} \int_{R \supset U} \varphi \cdot |f| = \sum_{i=1}^{\infty} \frac{1}{n}$. This is a divergent series, hence $\int_{\mathbb{R}} f$ is not integrable.
1c:
For each $A_n = [n+\frac{1}{3}, n+\frac{2}{3}]$ choose an open cover U_n which contains it disjoint from any other U_i. We take V_n to cover sets of the form $(n-\frac{1}{3}, n+\frac{1}{3})$ and we say that $V_0 = (-\infty, 1)$. Together, the set of all U_n and V_n cover all of $\mathbb{R}$. Consider the following rearrangement of the alternating harmonic series. We rearrange as

$$-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}\cdots = (-1+\frac{1}{2}+\frac{1}{4})-\frac{1}{3}\ldots$$

Where we rearrange each term to be in groups of 3 of the form $\frac{1}{2k-1} - \frac{1}{2(2k-1)} - \frac{1}{4k}$. From MAT157 this will absolutely converge. Choose a PO1 $\Phi = \{\phi_i\}$ such that each ϕ_{2i} corresponds to the $i'th$ term in our rearrangement of the series subordinate to an open set U_i, and each ϕ_{2k+1} is subordinate to a V_i. It follows that $\sum \int \phi_i |f|$ will converge, and $\sum \int \phi_i f = -\frac{1}{2}\log(2)$. We will now find a different rearrangement of our series. Consider the following rearrangement:

$$-1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}\cdots = (-1-\frac{1}{3}-\frac{1}{5})+(\frac{1}{2}+\frac{1}{4}\ldots)$$

Where the negative terms appear consecutively 3 times in a row, and the positive terms appear 7 times in a row. This sequence will absolutely converge, for simliar reason as the first. We also have that its limit will be $-(\log(2)+\log(\frac{3}{7}))$. Now consider another PO1, $\Psi = \{\psi_j\}$ where each ψ_i is chosen so that ψ_i will correspond to A_i. WLOG we may consider this specific PO1, since it is only defined on the support of the function f.We see that $\sum \int \psi_i f = -(\log(2)+\log(\frac{3}{7}))$