Q4: Consider the identity mapping between $(\mathfrak{X}, \|\cdot\|_1)$ and $(\mathfrak{X}, \|\cdot\|_2)$. This map is bounded by assumption. So by Corr 5.11 (Folland) it follows that $T^{-1} \in \mathcal{L}(\mathcal{Y}, \mathfrak{X})$. Therefore T^{-1} is bounded i.e. there is some C so that $\|\cdot\|_2 \leq C \|\cdot\|_1$