Q3a: Suppose that V is a vector space whose inner product induces a norm. Let $x, y \in V$, consider the following chain of equalities:

$$\begin{aligned}
\|x+y\|^2 + \|x-y\|^2 &= \langle x+y, x+y\rangle + \langle x-y, x-y\rangle && \text{(by the defintion of the norm)} \\
&= \langle x,x\rangle + \langle x,y\rangle + \langle y,x\rangle + \langle y,y\rangle + \langle x,x\rangle - \langle x,y\rangle - \langle y,x\rangle + \langle y,y\rangle && \text{(by linearity)} \\
&= \langle x,x\rangle + \langle x,x\rangle + \langle y,y\rangle + \langle y,y\rangle \\
&= 2\|x\|^2 + 2\|y\|^2 && \text{(by definition of the norm)}
\end{aligned}$$

We obtain the desired equality, thus we are done.

Q3b: Suppose that $\|\cdot\| : V \to \mathbb{R}$ is a norm on V obeying the equality in $3a$. We claim that $\langle x, y\rangle := \left\|\frac{x+y}{2}\right\|^2 - \left\|\frac{x-y}{2}\right\|^2$ is an inner product on V uniquely determined by $\|\cdot\|$. We first show that $\langle x, y\rangle$ satisfies all the properties of an inner product. First, note that $\langle x, x\rangle = \left\|\frac{x+x}{2}\right\|^2 = \|x\|^2 \geq 0$, with equality holding iff $x = 0$, by the properties of the norm. Next, we get that $\langle x, y\rangle = \left\|\frac{x+y}{2}\right\|^2 - \left\|\frac{x-y}{2}\right\|^2 = \left\|\frac{y+x}{2}\right\|^2 - \left\|\frac{y-x}{2}\right\|^2 = \langle y, x\rangle$. We skip the proof of bilinearity. We now claim uniqueness. Suppose that $\|\cdot\|$ is a norm satisfying the parallelogram inequality, arising from two distinct inner products, $\langle\cdot,\cdot\rangle_1$ and $\langle\cdot,\cdot\rangle_2$, where $\langle\cdot,\cdot\rangle_1$ is defined as previously, and $\langle\cdot,\cdot\rangle_2 = \|x\|^2$ Then we have that for any $x, y \in V$

$$\begin{aligned}
\langle x, y\rangle_1 &= \left\|\frac{x+y}{2}\right\|^2 - \left\|\frac{x-y}{2}\right\|^2 \\
&= \frac{1}{4}[\|x+y\|^2 - \|x-y\|^2] \\
&= \frac{1}{4}[\langle x+y, x+y\rangle_2 - \langle x-y, x-y\rangle_2] \\
&= \frac{1}{4}[\langle x+x\rangle_2 + 2\langle x,y\rangle_2 + \langle y,y\rangle_2 - \langle x,x\rangle_2 + \langle x,y\rangle_2 - \langle y,y\rangle_2] \\
&= \frac{1}{4}4\langle x,y\rangle_2 \\
&= \langle x,y\rangle_2
\end{aligned}$$

Thus we have $\langle x, y\rangle_1 = \langle x, y\rangle_2$ and we conclude that any norm obeying the parallelogram equality arises from a unique inner product.