Problem 11. *Shreya:*

Recall that for line bundle L,

$$\dim H^0(M, L \otimes X_M^{\otimes N}) - \dim H^1(M, L \otimes X_M^{\otimes N}) = \deg(L \otimes X_M^{\otimes N}) + (1 - g).$$

We know that $\deg(L \otimes X_M^{\otimes N}) = N$. The following inequality must then be true:

$$\dim H^0(M, L \otimes X_M^{\otimes N}) \geqslant N + (1 - g).$$

Selecting $N > g - 1$ will give us a nontrivial H^0 i.e. the existence of a nontrivial global section.