Q5a: Let $f(z)$ have power series expansion of $\sum_{n=0}^{\infty} a_n z^n$. By Cauchy's inequality for $n > 0$, we have that

$$|a_n| \leq \frac{\sup_\theta |f(re^{i\theta})|}{|r|^n} < \frac{\sup_\theta 1 + |r|^{\frac{1}{2}}}{|r|^n}.$$

Since this holds for all $r$, we have that $|a_n| = 0$ except perhaps for $a_0$. The bound on $f(z)$ gives that $|a_0| < 1$. Hence $f(z) = a_0$ for some $|a_0| < 1$.

Q5b: In a similar flavour to $5a$, we have the inequality

$$|a_m| < \frac{1 + |r|^n}{|r|^n}.$$

Since this holds for all $r$, for $m > n$ we have that $|a_m| = 0$. Hence $f(z)$ has a finite power series expansion i.e. it is a polynomial.