Q2a: It is known that the 2 form $xdx + ydy + zdz$ vanishes everywhere on $\mathcal{S}^2$. Hence by wedging with dx, we see that

$$(xdx + ydy + zdz) \wedge dx = 0 \implies ydy \wedge dx + zdz \wedge dx = 0 \implies ydx \wedge dy = zdx \wedge dz$$

Similarly, when we wedge with dy, we see that

$$(xdx + ydy + zdz) \wedge dy = 0 \implies xdx \wedge dy + zdz \wedge dy = 0 \implies xdx \wedge dy = zdy \wedge dz$$

Lastly, when we wedge with dz we get

$$(xdx + ydy + zdz) \wedge dz = 0 \implies xdx \wedge dz + ydy \wedge dz = 0 \implies xdz \wedge dx = ydx \wedge dz$$

Q2b: We can write $x^2 + y^2 = 1 - z^2$ on $\mathcal{S}^2$, and since $x, y \neq 0$ division by these quantities makes sense. Hence, we see that

$$\begin{aligned}
\omega &= \frac{x(1-z^2)}{x^2+y^2}dy \wedge dz + \frac{y(1-z^2)}{x^2+y^2}dz \wedge dx + \frac{z(1-z^2)}{x^2+y^2}dx \wedge dy \\
&= \Big[\frac{x}{x^2+y^2}dy \wedge dz + \frac{y}{x^2+y^2}dz \wedge dx + \frac{z}{x^2+y^2}dx \wedge dy\Big] - \frac{z}{x^2+y^2}\Big[xzdy \wedge dz + yzdz \wedge dx + z^2 dx \wedge dy\Big] \\
&= \Big[\frac{x}{x^2+y^2}dy \wedge dz + \frac{y}{x^2+y^2}dz \wedge dx + \frac{z}{x^2+y^2}dx \wedge dy\Big] - \frac{z}{x^2+y^2}\Big[x^2 dx \wedge dy + y^2 dx \wedge dy + z^2 dx \wedge dy\Big] \\
&\hphantom{=} \qquad \text{(by 2a)} \\
&= \Big[\frac{x}{x^2+y^2}dy \wedge dz - \frac{y}{x^2+y^2}dx \wedge dz + \frac{z}{x^2+y^2}dx \wedge dy\Big] - \frac{z}{x^2+y^2}[(x^2+y^2+z^2)dx \wedge dy] \\
&= \Big[\frac{x}{x^2+y^2}dy \wedge dz - \frac{y}{x^2+y^2}dx \wedge dz + \frac{z}{x^2+y^2}dx \wedge dy\Big] - \frac{z}{x^2+y^2}dx \wedge dy \qquad \text{(since on } \mathcal{S}^2\text{)} \\
&= \Big(\frac{xdy}{x^2+y^2} - \frac{ydx}{x^2+y^2}\Big) \wedge dz
\end{aligned}$$

As desired.

Q2c: Suppose we have a spherical bread being sliced into n slices. Each slice will have a height of $h = \frac{2}{n}$. We wish to integrate $\left(\frac{xdy-ydx}{x^2+y^2}\right) \wedge dz$ on the chain whose image is given by $A = \{\}$