Q2a: By Euclid's Algorithm, we compute $gcd(1745, 1485)$ as:

$$\begin{aligned}
1745 &= (1)1485 + 260 \\
1485 &= (5)260 + 185 \\
260 &= (1)185 + 75 \\
185 &= (2)75 + 35 \\
75 &= (2)35 + 5 \\
35 &= (7)5
\end{aligned}$$

So we conclude that $gcd(1745, 1485) = 5$. So working backwards, we see:

$$\begin{aligned}
5 &= 75 - 2(35) \\
&= 75 - 2(185 - 2 \cdot 75) \\
&= 5 \cdot (75) - 2 \cdot (185) \\
&= 5 \cdot (260 - 185) - 2 \cdot (185) \\
&= 5 \cdot (260) - 7 \cdot (185) \\
&= 5 \cdot (260) - 7 \cdot (1485 - 5 \cdot 260) \\
&= 40 \cdot (260) - 7 \cdot (1485) \\
&= 40(1745 - 1485) - 7 \cdot (1485) \\
&= -47 \cdot (1485) + 40 \cdot (1745)
\end{aligned}$$

We have written $5 = gcd(1745, 1485) = -47 \cdot 1485 + 40 \cdot 1745$

Q2b: By exercise 1.8, a number d is a divisor of a and b if and only if it is a divisor of $gcd(a, b)$. Therefore the set of all divisors of $a_1, a_2 \ldots a_k$ and $gcd(a_1, a_2) \ldots a_k$ are all the same, hence they share the game gcd.

Q2c: We will compute $\gcd(1092, 1155, 2002)$ and $\gcd(910, 780, 286, 195)$ using the 2b. We have

$$\gcd(1092, 1155, 2002) = \gcd(\gcd(1092, 1155), 2002)$$

So we evaluate

$$\begin{aligned}
1155 &= 1092 + 63 \\
1092 &= 17 \cdot 63 + 21 \\
63 &= 3 \cdot 21
\end{aligned}$$

Thus $gcd(1092, 1155) = 21$ Again, we compute $gcd(21, 2002)$

$$\begin{aligned}
2002 &= 95 \cdot 21 + 7 \\
21 &= 3 \cdot 7
\end{aligned}$$

Thus $\gcd(1092, 1155, 2002) = 7$ Now we will compute $\gcd(910, 780, 286, 195)$. We will iterate through using 2b to find the gcd of these numbers. First;

$$\begin{aligned}
910 &= 780 + 130 \\
780 &= 6 \cdot 130
\end{aligned}$$

And so $\gcd(910, 780) = 130$. Now we compute $\gcd(130, 286)$.

$$\begin{aligned}
286 &= 2 \cdot 130 + 26 \\
130 &= 5 \cdot 26
\end{aligned}$$

Hence we have that gcd$(130, 286) = 26$. Finally we wish to compute gcd$(26, 195)$

$$\begin{aligned} 195 &= 7 \cdot 26 + 13 \\ 26 &= 2 \cdot 13 \end{aligned}$$

So we have that gcd$(26, 195) = 13$ and we conclude by 2b that gcd$(910, 780, 286, 195) = 13$

Q2d: We proceed by induction. When $n = 2$, we have equality by Bezouts identity. Suppose that the formula holds for n. Then, for some $u_1, \ldots, u_n$, $gcd(a_1 \ldots a_n) = a_1u_1 + \cdots + a_nu_n$. Now consider gcd$(a_1, a_2 \ldots a_{n+1})$, By 2b, this is equal to gcd(gcd$(a_1, a_2), a_3, \ldots a_{n+1}))$. Therefore

$$\begin{aligned} &\gcd(\gcd(a_1, a_2), a_3, \ldots a_{n+1}) \\ &= \gcd(a_1, a_2)u_1 + \ldots u_k a_{k+1} && \text{(by induction hypothesis)} \\ &= (v_1a_1 + v_2a_2)u_1 + \cdots + u_k a_{k+1} && \text{(by bezouts identity)} \end{aligned}$$

As desired. We now apply this to gcd$(1092, 1155, 2002) = 7$. Using our derivation of the gcd, we have

$$\begin{aligned} 7 &= 2002 - 95(21) \\ &= 2002 - 95(1092 - 17 \cdot 63) \\ &= 2002 - 95 \cdot 1092 + (95)(17) \cdot 63 \\ &= 2002 - 95 \cdot 1092 + 1617 \cdot 63 \\ &= 2002 - 95 \cdot 1092 + 1617(1155 - 1092) \\ &= 2002 - 95 \cdot 1092 + 161 \cdot 1155 - 1617 \cdot 1092 \\ &= 2002 - 1712 \cdot 1092 + 1617 \cdot 1155 \end{aligned}$$