Q3: We know that the angle function $\theta$ is defined as follows: $\theta(x,y) = \begin{cases} \arctan(\frac{y}{x}) & x>0, y>0 \\ \pi + \arctan(\frac{y}{x}) & x<0 \\ 2\pi + \arctan\frac{y}{x} & x>0, y<0 \\ \frac{\pi}{2} & x=0, y>0 \\ \frac{3\pi}{2} & x=0, y<0 \end{cases}$ We see that $\theta(x,y)$ is a continiously differentiable function of $x, y$ on $\mathbb{R}^2\{0\}$. We compute $d\theta$ as

$$\begin{aligned} d\theta &= dx \wedge \frac{\partial\theta}{\partial x} + dy \wedge \frac{\partial\theta}{\partial y} \\ &= \frac{\frac{-y}{x^2}}{1+(\frac{y}{x})^2}dx + \frac{\frac{1}{x}}{1+(\frac{y}{x})^2}dy \\ &= \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy \end{aligned}$$

This is well defined on the domain of $\theta$.