Problem 3. *Jonathon: Show that* $F(v)$ *is harmonic.*

By definition:

$$\Delta F(v^*) = \sum_{k=1}^{n} \rho(v^*, v_k^*)\,[F(v^*) - F(v_k^*)] = \sum_{k=1}^{n} \rho(v^*, v_k^*) \int_{(v^*, v_k^*)} i \star df.$$

Since f harmonic, get

$$\begin{aligned}
\Delta F(v^*) &= \sum_{k=1}^{n} \rho(v^*, v_k^*) \int_{(v^*, v_k^*)} i \star df \\
&= i \sum_{k=1}^{n} \rho(v, v_k) \cdot \rho((v, v_k)^*) \int_{(v, v_k)^*} df && \text{(defn of } \star\text{)} \\
&= i \sum_{k=1}^{n} (f(v) - f(v_k)) \\
&= i \sum_{k=1}^{n} \left[\frac{1}{\sum_{i=1}^{n} \rho(v, v_k)} \sum_{l=1}^{n} \rho(v, v_l) f(v_l) \right] - f(v_k) \\
&= \frac{i}{\sum_{i=1}^{n} \rho(v, v_k)} \sum_{k=1}^{n} \left[\sum_{l=1}^{n} \rho(v, v_l) f(v_l) - \rho(v, v_l) f(v_l) \right] \\
&= 0
\end{aligned}$$