Q1:
Suppose that $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable. Then, we can write $f(a+h) = f(a) + Df(h) + o(h)$, for some linear map $Df(a) \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)$. Consider the following expression.

$$
\begin{aligned}
&\lim_{h\to 0} f(a+h) - f(a) \\
&= \lim_{h\to 0} \|h\| \frac{f(a+h) - f(a) + Df(a)h}{\|h\|} - Df(a)h \\
&= \lim_{h\to 0} \|h\| \frac{o(h)}{\|h\|} - Df(a)h \text{ (by assumption)} \\
&= \lim_{h\to 0} \|h\| \lim_{h\to 0} \frac{o(h)}{\|h\|} - \lim_{h\to 0} Df(a)h \text{ (by properties of limits)} \\
&= 0 \\
&\implies \lim_{h\to 0} f(a+h) = f(a)
\end{aligned}
$$

It follows that f is continous at a. $\blacksquare$