

Q3: The inverse function theorem tells us that given a map $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$, if the differential of f at some point p is nonsingular, then there exists a neighborhood of $f(p)$ where f is locally invertible, and given that f is locally invertible, $Df^{-1}(p) = (Df(p))^{-1}$. A consequence of this theorem is that if f is of class C^r , then so will be f^{-1} . This makes sense since when we invert a matrix of continuous functions, we have that the inverted matrix will be a continuous function. Therefore if we have an ODE of the form $f(y) = y'$, then we will have a corresponding ODE in the neighborhood of a point $f^{-1}(y') = y$ will also have a solution, and since the original system is well posed, our new system will be as well.