

Q5i: Since any $\phi \in \text{Aut}(G)$ is determined by $\phi(\bar{1}) = a\bar{1}$, for some $a \in \mathbb{F}_p$ it is sufficient to determine which $a \in \mathbb{F}_p$ will give an invertible map. Note that from number theory, the elements in \mathbb{F}_p which have multiplicative inverses are the nonzero elements. Hence we can correspond every automorphism of G with an element \mathbb{F}_p^\times by identifying ϕ with $\phi(1)$. Hence

$$\text{Aut}(G) \cong F_p^\times$$

Q5ii: Similarly to 5i, an automorphism $\phi \in \text{Aut}(\mathbb{Z}/n\mathbb{Z})$ will be determined by $\phi(\bar{1}) = a\bar{1}$. Hence the set of all automorphisms can be corresponded to the set of all $a \in \mathbb{Z}/n\mathbb{Z}$ which have multiplicative inverses. We know from number theory this is exactly the set of all $a \in \mathbb{Z}/n\mathbb{Z}$ such that $\gcd(a, n) = 1$ i.e. the unit group of $\mathbb{Z}/n\mathbb{Z}$.