Q5: Let $\mathcal{A} = \{(\varphi_i, U_i^{\pm})\}$ where

$$\varphi_i(x) = (x_1, \dots \hat{x_i}, \dots x_{n+1}), U_i^{\pm} = \{x \in S^n : x_i > 0 \text{ if } U^+,\, x_i < 0 \text{ if } U^-\}.$$

We let $\mathcal{B} = \{(\psi_i, V_i)\}$ with

$$\psi_1(x) = \frac{(x_1, \dots, x_n)}{1 - x_{n+1}}, \psi_2 = \frac{(x_1, \dots, x_n)}{1 + x_{n+1}}, V_1 = S_n \setminus (0, 1), V_2 = S^n \setminus (0, -1).$$

It is readily checked that the inverse of φ_i on $\varphi_i(U_i^+)$ is given by

$$\varphi_i^{-1}(x) = (x_1, \dots, \overbrace{\sqrt{1 - \sum_{k \neq i} x_k^2}}^{\text{in the } i\text{'th slot}}, \dots x_{n+1}).$$

The inverse on $\varphi_i(U_i^-)$ is given by

$$\varphi_i^{-1}(x) = (x_1, \dots, \overbrace{-\sqrt{1 - \sum_{k \neq i} x_k^2}}^{\text{in the } i\text{'th slot}}, \dots x_{n+1}).$$

The stereographic projection will have inverse of

$$\psi_1^{-1}(x) = \Big(x - \frac{|x|^2 - 1}{|x|^2 + 1}x, \frac{|x|^2 - 1}{|x|^2 + 1}\Big), \psi_2^{-1}(x) = \Big(x - \frac{|x|^2 - 1}{|x|^2 + 1}x, -\frac{|x|^2 - 1}{|x|^2 + 1}\Big).$$

We see that on where they agree the homeomorphism,

$$\varphi_i \circ \psi_1^{-1}(x) = (x_1 - \frac{|x|^2 - 1}{|x|^2 + 1}x_1 \dots \overbrace{(x_i - \frac{|x|^2 - 1}{|x|^2 + 1}x_i)}^{\text{remove}}, \dots, \frac{|x|^2 - 1}{|x|^2 + 1}).$$

This is smooth on the domain of $\psi_1^{-1}(x)$. Similarly, the composition of homeomorphisms

$$\psi_1 \circ \varphi_i^{-1}(x) = \frac{(x_1, \dots \pm \sqrt{1 - \sum_{k \neq i} x_k^2}, \dots x_n)}{1 - x_{n+1}}$$

will be smooth on the domain of definition. Therefore, $\mathcal{A}, \mathcal{B}$ are contained in the same maximal atlas and so they are equivalent by question 4. Hi my name is alex. i am a redditor. my favorite subreddits are r/antiwork, r/sino, r/awwwww and r/politics. I am a moderator of r/antivaxxersdying and i run my own telegram channel based around sharing videos of white women getting attacked for saying racist slurs.