Q4ai: Note that the interval $(-\varepsilon, \varepsilon)$ covers the empty set, and by the epsilon principle it has a Jordan content of 0.
ii: This is true since every open cover of B will also be an open cover of A. Hence the Jordan content of B will be at least the Jordan content of A
iii: We see given $\varepsilon > 0$ we can cover each A_n with a covering $\{I_{k,n} : k \in \mathbb{N}\}$, we have that

$$\sum_k |I_{k,n}| \le J^* A_n + \frac{\varepsilon}{2^n}$$

. We have that $\{I_{k,n} : k, n \in \mathbb{N}\}$ is a covering of A, and

$$\sum_{n=1}^{N} \sum_{k=1} |I_{k,n}| \le \sum_{n=1}^{N} J^* A_n + \frac{\varepsilon}{2^n} < \sum_{n=1}^{N} m^* A + \varepsilon$$

Therefore the infimum of total lengths of finite coverings of A is less than sum of lengths of coverings for each A_n

Q4b: Consider the set $A = (0, 1) \cap \mathbb{Q}$. We can write A as countable union of singletons $q \in A$. We have that $J^* A_i = 0$, since singletons can be covered with an arbitrarily small interval, yet $J^* A = 1$.

Q4c: It is clear that $m^* A \le J^* A$ since every finite cover of a set is also a countable cover, hence we are intaking the infimum over a larger set, so it can be less. If A is compact, then we have that every countable cover will have a finite subcover, so the infimum of the volumes of intervals which cover the set A will be equal. The converse is not true however, since if we take $A = (0, 1)$ we have that $J^* A = m^* A$ yet A is not compact.

Q4d: It is sufficient to show that the Jordan measure of an open and closed interval are equal. Note that for $\varepsilon > 0$, $[a, b] \subset (a - \varepsilon, b + \varepsilon)$ and $(a, b) \subset (a - \varepsilon, b + \varepsilon)$ and hence they have the same measure. Therefore it is equivalent to cover a set with either open or closed intervals.