

Q3a: On the curve  $\bar{\gamma}$ , we have that  $\overline{f(\bar{z})}$  will be continuous since  $\overline{f(\bar{\gamma})} = \bar{f}$ . Since  $f$  is continuous for any  $(\varepsilon, \delta)$  pair satisfying the definition of continuity, we have that the same pair will satisfy for  $\overline{f(z)}$  since if  $|x - y| < \delta$  then  $|\overline{f(x)} - \overline{f(y)}| = |f(x) - f(y)| < \varepsilon$ . We now compute that

$$\begin{aligned}
 \int_{\bar{\gamma}} \overline{f(\bar{z})} dz &= \int_{[0,1]} \overline{f(\gamma(t))} \cdot \overline{\gamma'(t)}' dt \\
 &= \int_{[0,1]} \overline{f(\gamma(t))} \cdot \overline{\gamma'(t)} dt && \text{(since } \gamma \text{ is a function of real variable)} \\
 &= \int_{[0,1]} \operatorname{Re}(f(\gamma(t)) \cdot \gamma'(t)) dt - i \int_{[0,1]} \operatorname{Im}(f(\gamma(t)) \cdot \gamma'(t)) dt \\
 &= \overline{\int_{\gamma} f(z) dz} && \text{(by definition)}
 \end{aligned}$$

Q3b: Since integration is independent of parametrization, we can choose  $\gamma : [0, 2\pi] \rightarrow \mathbb{C}$  defined by  $\gamma(t) = e^{it}$ . We also note that  $\gamma'(t) = ie^{it}$ . Using 3a, we compute that

$$\begin{aligned}
 \overline{\int_{\gamma} f(z) dz} &= \int_{\bar{\gamma}} \overline{f(\bar{z})} dz \\
 &= \int_{[0,2\pi]} \overline{f(\gamma(t))} \cdot \overline{\gamma'(t)} dt \\
 &= \int_{[0,2\pi]} \overline{f(e^{it})} \cdot -ie^{-it} dt \\
 &= - \int_{[0,2\pi]} \overline{f(e^{it})} e^{-2it} \cdot ie^{it} dt \\
 &= - \int_{[0,2\pi]} \overline{f(\gamma(t))} \frac{1}{\gamma^2(t)} \cdot \gamma'(t) dt \\
 &= - \int_{\gamma} \frac{\overline{f(z)}}{z^2} dz
 \end{aligned}$$