Q1a: Using the definition of the exteriour derivative on 0-forms, we compute that

$$df = \sum_{i=1}^{3} \frac{\partial f}{\partial x_i} dx_i = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz = \omega^1_{\text{grad f}}$$

We now will evaluate $d(\omega^1_F)$ using the definition of the exteriour derivative:

$$\begin{aligned}
d(\omega^1_F) &= \sum_{i=1}^{3} dx_i \wedge \frac{\partial \omega^1_F}{\partial x_i} \\
&= dx \wedge (\frac{\partial F_1}{\partial x} dx + \frac{\partial F_2}{\partial x} dy + \frac{\partial F_3}{\partial x} dz) + dy \wedge (\frac{\partial F_1}{\partial y} dx + \frac{\partial F_2}{\partial y} dy + \frac{\partial F_3}{\partial y} dz) + dz \wedge (\frac{\partial F_1}{\partial z} dx + \frac{\partial F_2}{\partial z} dy + \frac{\partial F_3}{\partial z} dz) \\
&= \frac{\partial F_2}{\partial x} dx \wedge dy + \frac{\partial F_3}{\partial x} dx \wedge dz + \frac{\partial F_1}{\partial y} dy \wedge dx + \frac{\partial F_3}{\partial y} dy \wedge dz + \frac{\partial F_1}{\partial z} dz \wedge dx + \frac{\partial F_2}{\partial z} dz \wedge dy \\
&= (\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}) dy \wedge dz + (\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}) dz \wedge dx + (\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}) dx \wedge dy \\
&= \omega^2_{\text{curl F}}
\end{aligned}$$

Finally we compute $d(\omega^2_F)$. Using the definition of d we see that

$$\begin{aligned}
d(\omega^2_F) &= \sum_{i=1}^{3} dx_i \wedge \frac{\partial \omega^2_F}{\partial x_i} \\
&= dx \wedge (\frac{\partial F_1}{\partial x} dy \wedge dz + \frac{\partial F_2}{\partial x} dz \wedge dx + \frac{\partial F_3}{\partial x} dx \wedge dy) + dy \wedge (\frac{\partial F_1}{\partial y} dy \wedge dz + \frac{\partial F_2}{\partial y} dx \wedge dz + \frac{\partial F_3}{\partial y} dx \wedge dy) \\
&+ dz \wedge (\frac{\partial F_1}{\partial z} dy \wedge dz + \frac{\partial F_2}{\partial z} dz \wedge dx + \frac{\partial F_3}{\partial z} dx \wedge dy) \\
&= \frac{\partial F_1}{\partial x} dx \wedge dy \wedge dz + \frac{\partial F_2}{\partial y} dx \wedge dy \wedge dz + \frac{\partial F_3}{\partial z} dx \wedge dy \wedge dz \wedge \\
&= (\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}) dx \wedge dy \wedge dz \\
&= (\text{div F}) dx \wedge dy \wedge dz
\end{aligned}$$

As desired.

Q1b: We will now show that gradient and curl fields are closed. Using the identity that $d^2 = 0$, we see that

$$0 = d^2 f = d(\omega^1_{\text{grad f}}) = \omega^2_{\text{grad curl f}}$$

Since $dy \wedge dz, dz \wedge dx, dx \wedge dy$ form a basis, by linear independence the coefficient functions must be 0 and we conclude the curl of a gradient field is 0. Now by a similar computation, we see that

$$0 = d^2 F = d(\omega^2_{\text{curl F}}) = (\text{div curl F}) dx \wedge dy \wedge dz$$

Hence this is a 0 3-form and we conclude that the divergence of a curl field is 0.