Q8a: By $Q4$ it is enough to show that the identity $id : S^1 \to S^1/\{z = -z\}$ is a diffeomorphism. Consider coordinate charts on S^1 given by stereographic projection, ψ_x, ψ_y. Let $\mathbb{R}P^1$ have the atlas induced by the quotient mapping of projection of S^1, with coordinate charts given as $\varphi_x(x, y) = sgn(x)y$ for $x \neq 0$ and $\psi_y(x, y) = sgn(y)x$ for $y \neq 0$. The inverses of the projections are given as $\varphi_x^{-1}(x) = (\sqrt{1 - x^2}, x)$. Similarly for $\varphi_y^{-1}(y)$. These are smooth on the domain since we exclude $x, y = \pm 1$. The inverses of $\psi_{x,y}$ are given as in $Q5$. We verify that the transition maps are smooth. Indeed, by question 5 they are smooth. Since we can identify S^1 with $\mathbb{R}P^1$, by question 7 they are diffeomorphic.

Q8b: Given the charts on $\mathbb{C}P^1$ as

$$U_1 = \{[x, y] : x \neq 0\}, U_2 = \{[x, y] : y \neq 0\},$$

with coordinate charts $\varphi_1([x, y]) = \frac{y}{x}$, $\varphi_2([x, y]) = \frac{x}{y}$. We note that when $x, y \neq 0$ we have that the transition map $\phi_1 \circ \phi_2^{-1} = \frac{1}{z}$. By question 7, the charts on the riemann sphere and on $\mathbb{C}P^1$ have the same transition map and hence induce the same equivalence relation on points in $\mathbb{C}$, with the equivalence relation defined as in $Q7$. Therefore the induced quotient space is the same and so $\mathbb{C}P^1$ is biholomorphic to $\S^2$.