

Q6: Let $|G| = 75$. Note that by Sylow's First Theorem, there exists a Sylow 5 group. Furthermore, $n_5(G) \equiv 1 \pmod{5}$. We claim that there is only 1 such subgroup. Suppose there was at least 6. Then we would have that there are 24 distinct elements in each group (excluding the identity). Since there are 6 subgroups, we would have at least $6 \cdot 24 > 75$ elements in the group. Thus $n_5(G) = 1$ and $P \triangleleft G$. Now let $Q \in \text{Syl}_3(G)$. Since $Q \cap P = \{e\}$, we can write $G = N \rtimes Q$. Since $|Q| = 3$, we have that $Q \cong \mathbb{Z}_3$. Therefore we have 3 possible $\varphi_i : Q \rightarrow \text{Aut}(P)$, with $\varphi_i(x) = x^i$. Since P is abelian, by a previous result, We have either $P = \mathbb{Z}_{25}$ or $P = \mathbb{Z}_5 \times \mathbb{Z}_5$. Thus we are done.