Q6: Note that for any $a, b \in \mathbb{C}$ the function

$$g(z) = \wp'(z) - a\wp(z) - b$$

is periodic, with period Γ. By Cartan iii§5 prop 5.1, as proven in lecture the number of zeros and periods is equal in any given parellelogram. Thus it is enough to show that there are 3 poles. Note that from the definition of $\wp'$, 0 will be a triple pole. Since the pole is of order 3, for any a, b, $g(z)$ will also have 3 poles and hence 3 zeros. We now claim that the sum of the zeros is equal to a period. Note that by prop 5.2 and class discussion, the sum of the roots for $\wp'(z) - a\wp(z) = b$, α_i and the poles β_i satisfy

$$\sum \alpha_i \equiv \sum \beta_i mod(\Gamma).$$

Since $\beta_i = 0$, the sum of the $\alpha_i' s$ will be congruent to 0 mod Γ and hence be a period. Now given u, v we wish to find a, b so that

$$g(u) = g(v) = 0.$$

Since the zeros must sum to 0 the last zero in the period parellelogram must be $-u - v$. So any a, b that make $g(u) = g(v) = 0$ will also make $g(-u - v) = 0$. We see that by solving the system of equations given by $g(u) = g(v) = 0$, we can take

$$a = \frac{\wp'(v) - \wp'(u)}{\wp(v) - \wp(u)}, b = \frac{\wp'(u)\wp(v) - \wp'(v)\wp(u)}{\wp(v) - \wp(u)}.$$

Therefore we get that

$$0 = \wp'(-u - v) - \frac{\wp'(v) - \wp'(u)}{\wp(v) - \wp(u)} \cdot \wp(-u - v) - \frac{\wp'(u)\wp(v) - \wp'(v)\wp(u)}{\wp(v) - \wp(u)}.$$

Multiplying across by $\wp(v) - \wp(u)$ gives us

$$\det \begin{pmatrix} \wp(u) & \wp'(u) & 1 \\ \wp(v) & \wp'(v) & 1 \\ \wp(-u - v) & \wp'(-u - v) & 1 \end{pmatrix} = 0$$