Q3: Suppose that S is connected but S^o is not. Then we can write $S^o = A \sqcup B$ for some disjoint proper clopen subsets of S^o. Thus, we have that S^o is a closed set, and $S \setminus S^o$ is clopen. Clearly, $S \setminus S^o$ and S^o are disjoint so we can write $S = S^o \sqcup S \setminus S^o$. We obtain a contradiction, since S is assumed to be connected.