Note that $\mathbb{Z}[x,y] \cong \mathbb{Z}[x][y]$. This is like writing $p(x,y) = x^2 + 2xy + 1 = (x^2+1) + (2x)y$. Consider the ideal $I = (y)$. We know from prop. 12 of Ch 9.4 in Dummite and Foote, that if $f(x,y)$ is not reducible in $\mathbb{Z}[x][y]/(y)$ then it is not reducible in $\mathbb{Z}[x,y]$. Indeed, under the quotient mapping we have that $\tilde{f}(x) = x^2+1$. This is not irreducible since a polynomial of degree 2 is irreducible if for some α, $\tilde{f}(\alpha) = 0$. It is easy to see no such α exists in $\mathbb{Z}$.