

Q2a: The orientation of M requires that our tangent vector is inward pointing. Hence we take the tangent vector to be $\xi = \left(\begin{pmatrix} x \\ 0 \\ z \end{pmatrix}, \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} \right)$.

Q2b: Using the chain $c : I \rightarrow \partial M, c(t) = (-\sin(2\pi t), 0, -\cos(2\pi t))$, we compute $\int_{\partial M} \omega$ as

$$\begin{aligned} \int_{\partial M} \omega &= \int_C \omega \\ &= \int_I c^* \omega \\ &= \int_{[0,1]} -3 \sin(2\pi t) \cdot -2\pi \sin(2\pi t) dt \\ &= 3\pi \end{aligned}$$

Q2c: We will compute $\int_M d\omega$ now

$$\begin{aligned} \int_M d\omega &= \int_{M-\partial M} d\omega \\ &= \int_{\alpha} \omega \\ &= \int_{u^2+v^2 < 1} \alpha^* \omega \\ &= \int_{u^2+v^2 < 1} 3du \wedge dv \\ &= 3\pi \end{aligned}$$