Q1: Let M be a metric space such that for a set is compact if and only if it compact. We define (a_n) to be a cauchy sequence in M. We first claim that (a_n) is a bounded sequence. Recalling the definition of a Cauchy Sequence, we choose $\varepsilon = 1$. Then for some $N \in \mathbb{N}$, and for all $n, m \geq N$, we have that $|a_n - a_m| < 1$. Setting $d = max\{d(a_i, a_j) : 1 \leq i, j \leq N\}$ and taking $M = d + 1$, we see that $(a_n) \subset B_{M+1}(a_N)$ Thus, any cauchy sequence in this space is bounded. Let B be the closed ball containing (a_n). This is a closed and bounded set and hence is compact by assumption. Thus, for our sequence (a_n) there must exist some convergent subsequence (a_{n_k}). Let a be the limit of this subsequence. We claim that (a_n) converges to a. Let $\frac{\varepsilon}{2} > 0$. By convergence, there exists some $N \in \mathbb{N}$ such that for all $n \geq N$, $d(a_{n_k}, a) < \frac{\varepsilon}{2}$. Similarly, by Cauchy, for $\frac{\varepsilon}{2} > 0$ there is some $K \in \mathbb{N}$ such that for all $m, n \geq K$, $d(a_n, a_m) < \frac{\varepsilon}{2}$. If we take $L = \max(N, K)$ and $n_k, n > L$ we see that by the triangle inequality

$$d(a, a_n) \leq d(a_n, a_{n_k}) + d(a_{n_k}, a) < \varepsilon$$

And so our cauchy sequence converges. Thus this space M is complete.