5.5.62a: Suppose that $f \in L^2(\mu)$. For $\varepsilon > 0$, by Lusins theorem there is a compact set E_ε with $\mu(E_\varepsilon) < \varepsilon$ so that the function $f|_{E_\varepsilon}$ is continuous. Let f_ε be the continuous extension, which exists by Tietze extension theorem. Since f, f_ε agree on $[0,1] \setminus E_\varepsilon$, we see that $\int_{E_\varepsilon} |f - f_\varepsilon|^2 \to 0$ as $\varepsilon \to 0$ since $|f - f_\varepsilon|^2$ is integrable. Thus we are done.

b: Since the polynomials are dense in $C[0,1]$ by elementary real analysis results, we have that the polynomials are dense in $L^2(\mu)$.

c: Consider the set of polynomials with integer coefficients, P. This forms a function algebra, vanishes nowhere and seperates points. It is therefore dense in the set of all polynomials, so dense in $L^2(0,1)$. Furthermore, it is countable, since we can write P as the union of the set of polynomials with degree less than n, for all n. Each of these sets is countable since there is a bijection with $\mathbb{Z}^n$. Apply Gram-Schmidt Prodedure to P to get O, an orthonormal set of vectors. Since each $v \in O$ is a linear combination of vectors in P, we have that $span\{O\}$ is dense in $L^2([0,1])$. Therefore for each $f \in L^2([0,1])$, we have that we can write $f = \sum_{v_i \in O} a_i v_i$. Therefore $L^2([0,1])$ is separable.

d: We have that for all $n \in \mathbb{Z}$, $L^2([n, n+1])$ is seperable by applying the same proof from c to a translated interval. By 5.5.60 we have that $L^2(\mathbb{R})$ is seperable, since $\mathbb{R} = \bigcup_n [n, n+1]$, and each $L^2([n, n+1])$ is seperable.

e: By 5.5.61, since $\mathbb{R}^n = \mathbb{R} \otimes \mathbb{R} \cdots \otimes \mathbb{R}$, and $L^2(\mathbb{R})$ is seperable, so is $L^2(\mathbb{R}^n)$.