

Q6: Suppose not. That is, suppose that there exists an irrational number $q \in [0, 1]$ but with $q \notin A$. Since A is closed, it follows that $\mathbb{R} \setminus A$ is open. Therefore, there exists a $\epsilon > 0$ such that $(q - \epsilon, q + \epsilon) \subset \mathbb{R} \setminus A$. By the density of the rational numbers in \mathbb{R} , there exists some $r \in \mathbb{Q}$ with $r \in (q - \epsilon, q + \epsilon) \subset \mathbb{R} \setminus A$. Note that since $q \in [0, 1]$ and r is within ϵ of q , $q \in [0, 1]$. However, $q \in \mathbb{R} \setminus A \implies q \notin A$. We obtain a contradiction, since we assume that every rational between 0 and 1 is contained in A . ■