Q4: $\Longrightarrow$

Suppose that M is an orientable k-manifold. Let $p \in M$. Let μ_p be an orientation of M at p. By assumption there exists coordinate patches α, β such that there is an $\alpha : W_1 \to U$, and $\alpha(x) = p$ for some $x \in W_1$. Similarly, let $\beta : W_2 \to V$ be another coordinate chart around p, such that $p = \beta(y)$ for some $y \in W_2$. Therefore, we know that $\alpha_* \mu_x = \mu_p$ and $\beta_* \mu_y = \mu_p$. Thus we have that

$$\mu_y = (\beta \circ \beta^{-1})_* \mu_y = \beta_*^{-1} \mu_p = (\beta^{-1} \circ \alpha)_* \mu_x$$

Hence we have that the pushforward of the transition map $\beta^{-1} \circ \alpha$ pushes forward orientation and thus $\det(\beta^{-1} \circ \alpha) > 0$.

$\Longleftarrow$

Suppose that the tranistion map between each coordinate patch has a positive determinant differential. We know that we can find an orientation preserving map around each point in M, since we can precompose each coordinate patch with a orientation reversing linear map to flip the orientation back. Thus we will be assuming without loss of generality that there is an orientation preserving coordinate system around each point. Let $p \in M$ with corresponding coordinate patch $f_1 : W_1 \to U \ni p$, and let $q \in U$ such that $f_2 : W_2 \to U$ preserves orientation. Let x satisfy $f_1(x) = p$. Letting μ_x be an orientation of U_1, we have that $f_{1_*} \mu_x = \mu_p$. However, we have that

$$\mu_p = (f_2 \circ f_2^{-1} \circ f_1)_* \mu_x = f_{2_*} \circ (f_2^{-1} \circ f_1)_* \mu_x$$

We have that $(f_2^{-1} \circ f_1)_*$ is orientation preserving by assumption, so thus $\mu_p = \mu_q$, and so we conclude that M is orientable.