Q2: We first show that $H \times H'$ is normal in $G \times G'$ for normal H, H'. Let $(h, h') \in H \times H'$. Take any $(g, g') \in G \times G'$. We compute that

$$(g, g')(h, h')(g^{-1}, {g'}^{-1}) = (gh, g'h')(g^{-1}, {g'}^{-1}) = (ghg^{-1}, g'h'{g'}^{-1})$$

Since $ghg^{-1} \in H$ and $g'h'{g'}^{-1} \in H'$, the pairing must belong to the product $H \times H'$. Hence we conclude that $H \times H' \trianglelefteq G \times G'$. We now claim that $(G \times G')/(H \times H') \cong (G/H) \times (G'/H')$. We define a map

$$\varphi : (G \times G')/(H \times H') \to (G/H) \times (G'/H')$$

$$(g, g')(H \times H') \mapsto (gH, g'H')$$

We claim that φ is an isomorphism. We verify that

$$\varphi((g, g')(k, k')) = (gkH, g'k'H') = (gH, g'H)(kH, k'H') = \varphi(g, g')\varphi(k, k')$$

It remains to show that φ is a bijection. It is clearly onto, since if we take a pairing $(gH, g'H)$ we can take $a = (g, g')(H \times H')$ so that $\phi(a) = (gH, g'H')$. We claim that φ is injective. Note that if we have for some $(g, g')(H \times H')$, $\phi((g, g')H \times H') = (eH, eH')$, this implies that $gH = eH$ and $g'H' = eH'$. We can deduce that $(g, g') = (e, e)$. We conclude that ϕ is an isomorphism and hence the two groups must be isomorphic.