

Q3: We claim that there exists some  $x \in \mathbb{R}$  satisfying  $E \cap E + x = \emptyset$ . Define the set  $A = \{x : E \cap E + x = \emptyset\}$ . It is sufficient to show that  $A$  is nowhere dense, i.e.  $A$  closed with empty interior. First we claim that  $A^c$  is open. If  $x \in A^c$ , then  $E \cap E + x$  is empty. If  $A^c$  were not open then for all  $\varepsilon > 0$ ,  $B_\varepsilon(x)$  is not contained in  $A^c$ . Since this holds for all  $\varepsilon$ , we have that  $x \notin A^c$ . A contradiction. Finally it remains to show that the interior of  $A$  is empty. If not then for some  $x \in A$  and  $\varepsilon > 0$ ,  $B_\varepsilon(x) \subset A$ . If  $y$  is such that  $y \in E, E + x$ , we have that  $y - x \in E$  and so  $(y - x - \varepsilon, y - x + \varepsilon) \subset E$ . Therefore  $m(E) > 0$ . A contradiction. Since translation is a homeomorphism, we apply the previous result, with our homeomorphism being translation.