Q4: Since $\mathcal{R}$ is a euclidean domain, it is a UFC so we factor into irreducible polynomials $f(x) = p_1(x) \dots p_k(x)$. Since $\mathcal{R}$ is a PID it is enough to determine where ideals $I = (g(x))$ are mapped to under the quotient mapping, by the fourth isomorphism theorem. Any ideal containing $p(x)$ must be of the form $(p_{i_1}(x) \dots p_{i_m}(x))$. Therefore ideals of $\mathcal{R}/(f(x))$ are of the form $(p_{i_1}(x) \dots p_{i_m}(x))/f(x)$