Q1i: Consider the group $C_n$ for $n$ even. The subgroup $\langle\overline{2}\rangle$ is invariant under any automorphism, since previously we have characterized automorphisms of $C_n$ with multiplication by any number coprime to $n$.

Q1ii: Consider the group $\mathcal{Q}_8$ with normal subgroup $\langle i\rangle$. If we consider the map $\varphi$ defined by $\varphi(i) = j, \varphi(j) = k, \varphi(k) = i$. This will be an automorphism since it simply relabels the generators of the group. However

$$\varphi(\langle i\rangle) = \langle j\rangle.$$

Thus this subgroup can not be characteristic. We now give another example of a normal subgroup that is not characteristic. Consider the group $G = (\mathbb{Q}, +)$, with subgroup $H = \{\frac{n}{2} : n \in \mathbb{Z}\}$. Since $G$ is abelian we have that $H \trianglelefteq G$. Consider the map $\varphi : G \to G$ defined by $\varphi(x) = 2x$. This is an linear operation so it must preserve the additive structure of $\mathbb{Q}$. Furthermore it is invertible since it is just multiplication by a scalar. We see that $\varphi(H) = \mathbb{Z}$. Hence $H$ is not characteristic.