

Q4:

Suppose that  $T$  is a linear map from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ . It has been shown previously that for such map, there exists an  $M \in \mathbb{R}_+$  such that  $\|T(x)\| \leq M \|x\| \forall x \in \mathbb{R}^n$ . Take  $\epsilon > 0$ . Choose  $\delta = \frac{\epsilon}{M}$ . Then we see that.

$$\begin{aligned} \|x - y\| &< \frac{\epsilon}{M} \\ \implies M \|x - y\| &< \epsilon \\ \implies \|T(x) - T(y)\| &\leq M \|x - y\| < \epsilon \\ \implies \|T(x) - T(y)\| &< \epsilon \quad \blacksquare \end{aligned}$$