Q3: Suppose that there were a countable dense subset of l^∞. Let $\{\{x_n\}^k\}_{k\in\mathbb{N}}$ be an enumeration of the dense subset. Define the sequence $\{y_n\}$ by

$$y_i = \begin{cases} x_i^i + 1 \text{ if } |x_i^i| \leq 1 \\ 0 \text{ if } |x_i^i| > 1 \end{cases}$$

We have that $\{y_n\} \in l^\infty$ since each point in the sequence is either 0 or less than or equal to 2. By construction we have that $|y_i - x_i^k| \geq 1$ for all i, k. Hence $\sup_n |y_n - x_n^k| \geq 1$.