5.4.50: Let $\{x_n\}, \{q_n\}$ be an enumeration of the countable dense set of B and $\mathbb{Q}$. Take $f_n \in \mathfrak{X}^*$ so that $f_n(x_n) = q_n$ and $\|f_n\| \leq 1$. Let $\varepsilon > 0$, define the set $V_{f_n,\varepsilon} = \{g \in \mathfrak{X}^* : \|f_n - g\| < \varepsilon\}$. We claim that $\{V_{f_n\,\varepsilon}\}$ is a covering of B^*. Let $\|g\| \leq q$. Then we have that for some $x \in B$, $\|g\|$ attains a maximum, since it is bounded. We have that

$$\|g(x) - f(x)\| \leq \|g(x_n) - g(x)\| + \|f(x_n) - f(x)\| + \|f(x_m) - f(x_n)\| + \|f(x_m) - f(x)\|\,.$$

If we take x_n, x_m with sufficient large so that $|x - x_m|, |x - x_n|, |x_n - x_m| < \frac{\varepsilon}{4}$, since the norms of each operator is 1 we get that

$$\|g(x) - f(x)\| < \varepsilon\,.$$

Thus we have a countable covering of B^* by basis elements. Therefore this space is second countable. Furthermore, since it is compact and T_1, it must be T_3 by basic topology. It therefore must be metrizable by elementary topology results.