Q4: Note that there are $(3^2-1)(3^2-3)$ invertible 2 by 2 matrices, since the first column has 3^2-1 choices (we exclude the 0 column), and the second column can not be a scalar multiple. So there are 3^2-3 different choices for the second column. We divide this product by 2 since half of the matrices have a determinant of 1. Thus $|SL(2,\mathbb{F}_3)| = 24 = 2^3 \cdot 3$. By Sylows theorem, the number of 3 subgroups must be either $1, 4, 7....$ We can check that the following subgroups are of order 3:

$$\left\langle \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix} \right\rangle = \left\{ \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}, e \right\}$$
$$\left\langle \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right\rangle = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, e \right\}$$
$$\left\langle \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix} \right\rangle = \left\{ \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}, e \right\}$$
$$\left\langle \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \right\rangle = \left\{ \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, e \right\}$$

We claim that there are no other subgroups of order 3 i.e. there are 4 Sylow 3 subgroups. If P is a Sylow $3-$group, note that $n_3(G) = \frac{|G|}{|N_G(P|}$. Furthermore, if a normalizes P then so does $2a$ since

$$(2a)P(2a)^{-1} = 4aPa^{-1} = aPa^{-1}.$$

We also know that each element of P normalizes P so $|N_G(P)| \geq 5$. This is a subgroup of G so be Lagranges Theorem, we have that $N_G(P) \geq 6$. Thus $n_3(G) \leq 4$. But we have produced 4 3 subgroups hence we conclude that the list we have produced is the complete characterization of 3 subgroups of G.