

Q4: Let  $\varepsilon > 0$ . Take the open cover  $\{B(\frac{\varepsilon}{3}, f)\}_{f \in \mathcal{K}}$ . By compactness there is a finite subcover  $\{B(\frac{\varepsilon}{3}, f_i)\}$  for some  $f_1, \dots, f_n$ . Since each  $f_i$  is defined on a compact set and continuous, they are all uniformly continuous. Thus there is some  $\delta_i$  such that  $|x - y| < \delta_i$  implies  $|f_i(x) - f_i(y)| < \frac{\varepsilon}{3}$ . Any  $g \in \mathcal{K}$  belongs to some open ball, so we have that for all  $x$ ,  $|g(x) - f(x)| < \frac{\varepsilon}{3}$ . Let  $\delta = \min\{\delta_1 \dots \delta_n\}$ . If we have  $|x - y| < \delta$  then

$$|g(x) - g(y)| < |g(x) - f_i(x)| + |f_i(x) - f_i(y)| + |f_i(y) - g(y)| < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon$$

Our choice of  $g$  is arbitrary, hence  $\mathcal{K}$  is an equicontinuous family of functions.