Q1a:

$$f(x,y) = \int_a^{x+y} g$$

. By the Fundamental Theorem of Calculus,

$$\frac{\partial f}{\partial x} = g(x+y)$$

and similarly

$$\frac{\partial f}{\partial y} = g(x+y)$$

1b:

$$f(x,y) = \int_y^x g$$

By properties of integration, we can rewrite f in the following way, for some $a \in (x,y)$

$$f(x,y) = \int_y^x g = \int_y^a g + \int_a^x g = -\int_a^y g + \int_a^x g$$

Applying FTC, we see that

$$\frac{\partial f}{\partial x} = g(x)$$

and

$$\frac{\partial f}{\partial y} = -g(y)$$

1c:

$$f(x,y) = \int_a^{xy} g$$

By applying both FTC and chain rule, we get

$$\frac{\partial f}{\partial x} = g(xy)y$$

and

$$\frac{\partial f}{\partial y} = g(xy)x$$

1d:

$$f(x,y) = \int_a^{\int_b^y g} g$$

By the FTC and chain rule,

$$\frac{\partial f}{\partial x} = 0$$

and

$$\frac{\partial f}{\partial y} = g(\int_b^y g) \cdot g(y)$$