Q6: Let γ be a parametrization of a circle with radius R. Cauchy's Integral formula tells us that

$$|f^{(n)}(z)| = \Big|\frac{n!}{2\pi i}\int_\gamma \frac{f(\zeta)}{(\zeta - z)^{n+1}}d\zeta\Big|.$$

Substituting for the bound on $f(z)$, we get that

$$|f^{(n)}(z)| \leq \Big|\frac{Mn!}{2\pi i}\Big| \cdot \int_\gamma \Big|\frac{1}{(\zeta - z)^{n+1}}d\zeta\Big|.$$

To maximize the integral, we can minimize the denominator. We do this by taking any z with $|z| = r$. Hence we have that

$$|f^{(n)}(z)| \leq \Big|\frac{Mn!}{2\pi i}\Big| \cdot \int_{[0,2\pi]} \Big|\frac{1}{(Re^{it} - re^{it})^n}\Big| \leq \Big|\frac{Mn!}{2\pi i}\Big| \cdot \Big|\frac{1}{(R-r)^n}\Big| \cdot \int_{[0,2\pi]} \Big|\frac{1}{Re^{it} - z}dt\Big|.$$

By Cauchy's integral formula,

$$\int_{[0,2\pi]} \Big|\frac{1}{Re^{it} - z}\Big| \leq |2\pi i|,$$

for some z. Hence

$$|f^{(n)}(z)| \leq \Big|\frac{Mn!}{(R-r)^n}\Big|$$