

Q1i: If we have any two polynomials with coefficient 0 on x , adding them will clearly yield another polynomial with coefficient of 0 on x since polynomial addition is term by term. When multiplying two polynomials, the only way to get a nonzero coefficient on x over \mathbb{Q} is to multiply with a polynomial with another that has nonzero coefficient on x . Thus \mathcal{R} is a subring.

Q1ii: Note that we can factor the polynomial x^6 in two different ways:

$$x^6 = (x^2)(x^2)(x^2) = (x^3)(x^3).$$

We have two different ways to factor x^6 in \mathcal{R} thus it can not be a UFD.