Q4: Let $(p, v) \in T_p \mathbb{R}^n$. We compute $D_{(p,v)}f$ as

$$\begin{aligned}
D_{(p,v)}f &= \frac{\partial f}{\partial x_1}(p) \cdot v_1 + \cdots + \frac{\partial f}{\partial x_n}(p) \cdot v_n \\
&= \langle (\frac{\partial f}{\partial x_1}(p), \ldots \frac{\partial f}{\partial x_n}(p)), (v_1, \ldots, v_n) \rangle && \text{(definition of inner product)} \\
&= \langle (p, \frac{\partial f}{\partial x_1}(p), \ldots \frac{\partial f}{\partial x_n}(p)), (p, v) \rangle && \text{(by definition of inner product on a tangent space)} \\
&= \langle (gradf)(p), (p, v) \rangle && \text{(by definition of grad f)}
\end{aligned}$$

By the Cauchy-Schwartz inequality, this quantity is maximized when (p, v) is collinear with $gradf(p)$, hence it represents the growth of the function in the direction of p.