

Q3:

Notice that we can rewrite $F(y) = \int_a^b f(x, y)dx = \int_a^b (\int_c^y D_2 f(x, y)dy + f(x, y)dx)$. We compute

$$\begin{aligned}\frac{\partial}{\partial y} F(y) &= \frac{\partial}{\partial y} \int_a^b \left(\int_c^y D_2 f(x, y)dy + f(x, y)dx \right) \\ &= \frac{\partial}{\partial y} \left[\int_a^b \int_c^y D_2 f(x, y)dydx + \int_a^b f(x, c)dx \right] && \text{(by linearity)} \\ &= \frac{\partial}{\partial y} \int_c^y \int_a^b D_2 f(x, y)dx dy + \frac{\partial}{\partial y} \int_a^b f(x, c)dx && \text{(by Fubini's Theorem)} \\ &= \frac{\partial}{\partial y} \int_c^y \int_a^b D_2 f(x, y)dx dy \\ &= \int_a^b D_2 f(x, y)dx && \text{(by FTC)}\end{aligned}$$