Q2: First, we have that $\left\|\frac{d}{dx}cos(n+x)\right\| = \|-sin(n+x)\| \leq 1$. So by the mean value theorem, $|cos(n+x) - cos(n+y)| \leq |x-y|$. This implies that $h_n(x) = cos(n+x)$ is Lipschitz and hence is equicontinuous. Now define $g_n(x) = \log(1+\frac{\sin(n^n x)}{\sqrt{n+2}})$. We need to show that it uniformly converges to 0, which would imply eqcontinuity. We see that if we take $\varepsilon > 0$, n sufficiently large we get that $\log(1+\frac{\sin(n^n x)}{\sqrt{n+2}}) < \varepsilon$, regardless of the value of x. Hence this sequence uniformly converges and is equicontinuous. It remains to show the sum, $f_n = h_n + g_n$ is equicontinuous. Let $\varepsilon > 0$. Take δ_1, δ_2 such that $|s-t| < \delta_1 \implies |g_n(t) - g_n(s)| < \frac{\varepsilon}{2}$ and $|s-t| < \delta_2 \implies |h_n(t) - h_n(s)| < \frac{\varepsilon}{2}$. Take $\delta = \min \delta_1, \delta_2$. Then we see that

$$|f_n(s) - f_n(t)| = |h_n(s) + g_n(s) - h_n(t) - g_n(t)| \leq |h_n(s) - h_n(t)| + |g_n(s) - g_n(t)| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

Thus the sum of two equicontinuous function is equicontinuous, and so we conclude that f_n is equicontinuous.