

Q5: By question 21, it is sufficient to show that Lipschitz T sends zero sets to zero sets. Let L be the Lipschitz constant. Let Z be a zero set, let $\{I_k\}$ be a countable covering of Z by squares whose total volume is less than ε , which exists by lemma 16. The diameter of $I_k \cap Z \leq \text{diam } I_k$. By Lipschitz of T , $T(S_K \cap I_K)$ will have a diameter less than or equal to $LDiam I_k$. Thus we can find a square I'_k with diameter $LDiam I_k$ covering $T(Z \cap I_k)$. These squares cover $T(Z)$ and we see that

$$\sum_k |I'_k| \leq L^n \sum_k \text{diam}(I_k)^n \leq nL^n \sum_k |I_k| \leq nL^n \varepsilon$$

Since L, n both fixed, we have that $T(Z)$ is a measure 0 set.