Q4:
Define $A = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x \leq 1, 0 \leq y \leq 1 - x, 0 \leq z \leq 1 - x - y\}$. Choose

$$g(x, y, z) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

. Notice that $g(A) = T$, and g is invertible hence we can apply the COV theorem. Evaluate $\int_{g(A)} f$ as

$$\begin{aligned}
\int_{g(A)} f &= \int_A f \circ g |\det g'| \\
&= \int_A 4x \\
&= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 4x \quad dzdydx && \text{(by Fubini's Theorem)} \\
&= 4 \int_0^1 \int_0^{1-x} x - x^2 - xy \quad dydx \\
&= 4 \int_0^1 x(1-x) - x^2(1-x) - \frac{1}{2} x(1-x)^2 dx \\
&= \frac{1}{6}
\end{aligned}$$