

Q1: We first define $\Omega = \{(x, y) : a < x \leq y \leq b\}$. Using Fubini-Tonelli Theorem, we compute

$$\begin{aligned}
 \mu_F \times \mu_G(\Omega) &= \int_{(a,b]} \int_{(a,y]} dF(x) dG(y) \\
 &= \int_{(a,b]} [F(y) - F(a)] dG(y) \\
 &= \int_{(a,b]} \Delta F dG(y) + \int_{(a,b]} F_- dG(y) - \int_{(a,b]} F(a) dG(y) \quad (\text{since } F = \Delta F + F_- \text{ and linearity}) \\
 &= \sum_{x, \Delta F \neq 0} \int_x \Delta F dG(x) + \int_{(a,b]} F_- dG(x) - F(a)[G(b) - G(a)] \\
 &\quad (\Delta F \text{ nonzero on countable points}) \\
 &= \sum_{x, \Delta F \neq 0} \Delta F(x) \Delta G(x) + \int_{(a,b]} F_- dG(x) - F(a)[G(b) - G(a)] \quad (\text{using the definition of } dG)
 \end{aligned}$$

Similarly, we compute that

$$\begin{aligned}
 \mu_F \times \mu_G(\Omega) &= \int_{(a,b]} \int_{(x,b]} dG(y) dF(x) \\
 &= \int_{(a,b]} [G(b) - G(x)] dF(x) \\
 &\quad (\text{take decreasing sequence to } (x, b], \text{ apply downward measure cont.}) \\
 &= \int_{(a,b]} G(b) dF(x) - \int_{(a,b]} \Delta G(x) dF(x) - \int_{(a,b]} G_-(x) dF(x) \\
 &= G(b)[F(b) - F(a)] - \sum_{x, \Delta G(x) \neq 0} \int_x \Delta G(x) dF(x) - \int_{(a,b]} G_- dF(x) \\
 &= G(b)[F(b) - F(a)] - \sum_{x, \Delta G(x) \neq 0} \Delta F(x) \Delta G(x) - \int_{(a,b]} G_- dF(x)
 \end{aligned}$$

Taking the differences we get that

$$G(b)F(b) - G(a)F(a) = \int_{(a,b]} F_- dG + \int_{(a,b]} G_- dF + \sum_{a < x \leq b} \Delta G(x) \Delta F(x)$$

Since the points where either is $\Delta G(x)$ or $\Delta F(x)$ are zero will vanish in either of the equations we take the difference.