Q5: We have that the function $f(\frac{1}{z})$ is holomorphic on $\mathbb{C}$, and $\left|f(\frac{1}{z})\right| \geq \sqrt{|z|}$. Furthermore we have that $\frac{1}{f(\frac{1}{z})}$ is holomorphic, and so

$$\left|\frac{1}{f(\frac{1}{z})}\right| \leq \frac{1}{\sqrt{|z|}}.$$

If we write $\frac{1}{f(\frac{1}{z})} = \sum_{n=0}^{\infty} a_n z^n$, by cauchy's inequality we have that for all $n \geq 1$,

$$a_n \leq \frac{\sup_\theta \frac{1}{f(\frac{1}{re^{i\theta}})}}{r^n} \leq \frac{1}{r^{n-\frac{1}{2}}}.$$

Taking r sufficiently large tells us that $a_n = 0$ for $n \geq 1$. Therefore $f(z)$ is constant. However no constant can satisfy $c \geq \frac{1}{\sqrt{|z|}}$ since the righthand side is unbounded on $C \setminus \{0\}$. Therefore no such f can exist.