Q2: We claim that $\mathbb{Z}/(m)\otimes_{\mathbb{Z}}\mathbb{Z}/(n)$ is isomorphic to $\mathbb{Z}/(m,n)$. Consider the following commutative diagram:

$$\begin{tikzcd} \mathbb{Z}/(m)\times\mathbb{Z}/(n) \arrow[r, "\otimes"] \arrow[d, "f(x,y)=xy"'] & \mathbb{Z}/(m)\otimes_{\mathbb{Z}}\mathbb{Z}/(n) \arrow[dl, "\tilde{f}"] \\ \mathbb{Z}/(m,n) & \end{tikzcd}$$

We have that by the universal property of $\otimes$, $f(x,y)=xy$ factors. We claim that $\tilde{f}$ is a group isomorphism. Note that it must be a homeomorphism. Note that $\tilde{f}$ is surjective since for $r\in\mathbb{Z}/(m,n)$ we can just take $r\otimes 1$ and see that

$$\tilde{f}(r\otimes 1)=r\tilde{f}(1\otimes 1)=r.$$

We now compute the cardinality of $\mathbb{Z}/(m)\otimes_{\mathbb{Z}}\mathbb{Z}/(n)$. If we take some element $a\otimes b=ab\otimes 1=1\otimes ab$. Using the division algorithm, we can write $ab=p_1m+r_1=p_2n+r_2$. So $a\otimes b=r_1\otimes 1=1\otimes r_2$. For this to be nontrivial we must have that $r_1,r_2<\gcd(n,m)$. Therefore the cardinalities of the groups are the same. Since $\tilde{f}$ is a surjection, it must also be a bijection.