Assignment 1 MAT 457

Q4: First we take note that $\mu(\bigcup_{i=1}^{\infty} E_n) \leq \sum_{i=1}^{\infty} E_i < \infty$ and clearly this implies that $\mu(E_1) < \infty$. Using insight gleaned from Q3b about $\mu(\limsup_n E_n)$ we evaluate that:

$$\mu(\limsup_n E_n) = \mu(\bigcap_{k \ge 1} \bigcup_{n \ge k} E_n)$$

$$= \lim_{k \to \infty} \mu(\bigcup_{n \ge k} E_n)$$
 (by measure continuity and assumption)
$$\leq \lim_{k \to \infty} \sum_{n = k}^{\infty} \mu(E_n)$$

$$= 0$$
 (since the sum converges, the tail must converge to 0)