Q4: We first map $\mathbb{C}\setminus[-1,1]$ to $\mathbb{C}\setminus[-\infty,0]$ using the conformal mapping $f_1(z)$ defined by

$$z \mapsto \frac{z+1}{z-1}.$$

Now we map $C\setminus[-\infty,0]$ to $\mathbb{C}\setminus\{z=x+iy : x<0, \arg(z)\in[\pi-\arccos(r), \pi+\arccos(r)]\}$, using the conformal mapping $f_2(z)$ defined by

$$z \mapsto z^{1-\frac{\arccos(r)}{\pi}}.$$

We choose the power of z such that the boundary of $C\setminus\{z=x+iy : x<0, \arg(z)\in[\pi-\arccos(r), \pi+\arccos(r)]\}$ gets sent to the boundary of the lense by our choice of f_4. We now apply $f_3(z)=-z$, which is conformal, to rotate the plane. We finally apply

$$f_4(z)=\frac{1-z}{1+z}$$

to transform this unbounded region into the compliment of the lense. Hence we take

$$f = f_4\circ f_3\circ f_2\circ f_1,$$

to be our conformal mapping of the given region