Q2: We first claim that $\mathcal{N}(\mathcal{R})$ is an ideal. It is clear that if $r^n = 0$, then for any $a \in \mathcal{R}$ we have

$$(ar)^n = a^n r^n = 0.$$

Now suppose that $r, s \in \mathcal{N}(\mathcal{R})$ and $r^n = s^m = 0$. Then by the binomial expansion we compute that

$$(r+s)^{m+n} = \sum_{i=1}^{m+n} \binom{m+n}{i} r^{m+n-i} s^i = 0,$$

since until $i = m$, $r^{m+n-i} = 0$, and for $i > m$, $s^i = 0$. Thus $\mathcal{N}(\mathcal{R})$ is an ideal. Now let $r \in \mathcal{R}/\mathcal{N}(\mathcal{R})$ be a nilpotent element. Then we have that for some sufficiently large n,

$$r^n + \mathcal{N}(\mathcal{R}) = 0 + \mathcal{N}(\mathcal{R}).$$

Thus we have that $r^n \in \mathcal{N}(\mathcal{R})$. Hence for some k, $r^{n^k} = 0$ i.e. $r^{nk} = 0$ and so $r \in \mathcal{N}(\mathcal{R})$. Thus $\mathcal{N}(\mathcal{R})$ contains no nonzero nilpotent elements.