Q3:

Since U open, U^c must be closed. it then follows from 2b that there exists some d such that $\|x-y\| \geq d$ for all $x \in U^c$ and $y \in C$. Now, we cover each $y \in C$ with a ball of radius $\frac{d}{2}$. This is an open cover of C, so we can take a finite subcover by compactness of C. Let D be the closure of this finite subcover. We first claim that D is a compact set. Note that D is the finite union of closed balls with radius $\frac{d}{2}$. It follows that D is closed and bounded and thus is compact by the Heine Borel Theorem. Note that as well, since every point in D is at most $\frac{d}{2}$ away from U^c, D must be disjoint with U^c and as such $D \subset U$. Finally it remains to show that $C \subset intD$. Suppose that $x \in C$. From our choice of D , there must exist an open ball centered at some point y, $B_{\frac{d}{2}}(y)$, such that x belongs to this open ball. Since this ball is open we can find some smaller open set U with $x \in U \subset B_{\frac{d}{2}}(y)$. This is exactly what it means to be in the interiour of D. Therefore, $C \subset intD$.