Q1a: Suppose that $p|a^k$. By Corr 2.2 $p|a$. Therefore, by prime factorization of a, we must have that $p^k|a^k$. This does not hold when p is composite. Consider when $p = 4, a = 2, k = 4$. Then $4|16$ but $4 \nmid 2$.

Q1b: It is easy to check that $132 = 2^2 \cdot 3 \cdot 11$, $400 = 2^4 \cdot 5^2$, and $1995 = 3 \cdot 5 \cdot 7 \cdot 19$. By the formula for the gcd of 2 numbers (pg. 23 jones and jones), we see $\gcd(132, 400) = 2^2 = 4$, $\gcd(132, 1995) = 3$, $\gcd(400, 1995) = 5$,$\gcd(132, 400, 1995) = \gcd(\gcd(132, 400), 1995) = \gcd(4, 1995) = 1$

Q1c: i) This is true. Given that $gcd(a, p^2) = p$, this means that for some $k \in \mathbb{Z}$, $kp = a$. Therefore,

$$\gcd(a^2, p^2) = \gcd(k^2p^2, p^2) = p^2 \gcd(k^2, 1) = p^2$$

ii) False. Consider $a = p$ and $b = p^3$, then we have that $\gcd(p, p^2) = p$, $\gcd(p^3, p^2) = p^2$, but $\gcd(p^4, p^4) = p^4$.

iii) This is true. Since $\gcd(a, p^2) = p$, there exists some $k \in \mathbb{Z}$, with k coprime to p such that $kp = a$. Similarly, there is some $l \in \mathbb{Z}$ with the the same proprty but $lp = b$. Therefore, the product kl is also coprime with p and p^2 by the contrapositive of lemma 2.1 b. Thus we have

$$\gcd(ab, p^4) = \gcd(klp^2, p^4) = p^2 \gcd(kl, p^2) = p^2$$

iv) False. Take $a = p^2 - p$. First we claim for p prime, $\gcd(p, p - 1) = 1$. Suppose that $k|p$ and $k|p - 1$. By Corr 1.4, $k|p - (p - 1) = 1$, so $k = 1$. Then we have that $\gcd(p^2 - p, p^2) = p \gcd(p - 1, p) = p$. We also have that $\gcd(a + p, p^2) = \gcd(p^2, p^2) = p^2$.