Q1i: It is sufficient to show that for any $\alpha \in G$, $\alpha H = H\alpha$. First suppose that $\alpha \in G - H$. Since the index of H is 2, we know that we can obtain a partition of G as $G = H \sqcup \alpha H$. Similarly, we can obtain the partition of $G = H \sqcup H\alpha$. This implies that for $\alpha \in G - H$, $\alpha H = H\alpha$. Now suppose that $\alpha \in H$. Since H is a group which inherits its multiplication from G, we have that $\alpha H = H\alpha$. We have that $\alpha H = H\alpha$ for all $\alpha \in G$, hence we apply our result from assignment 2, question 4 and conclude that $H \trianglelefteq G$.

Q1ii: We write $n = ak$ for some positive integer a. Take $G = \mathbb{Z}/n\mathbb{Z}$, and $H = \langle [a] \rangle = \{[a], [2a], \ldots [ka]\}$. H is clearly a subgroup of G with order k, and since G is abelian, it follows that every subgroup is normal in G.