

Q8: It is sufficient to show that for $\{(x_n, y_m)\} \rightarrow (0, 0)$ then $B(x_n, y_m) \rightarrow 0$. Consider the mapping $B_x(y) = B(x, y)$. We know that there must exist some C_x that satisfies $\|B_x(y)\| \leq C_x \|y\|$. By the uniform boundedness principle, we have that there is a maximal C such that $\|B_x(y)\| \leq C \|y\|$ for all x, y . Therefore we have that $\|B(x_n, y_m)\| \leq C \|B_{x_n}(y_m)\| \rightarrow 0$ as $y_m, x_n \rightarrow 0$. Therefore $B(x, y)$ is continuous.