

Q3: If $A \in I$ it will take the form of

$$A = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ a_{i1} & \cdots & a_{in} \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

for some i . The set of all matrices in this form will be closed under addition since matrix multiplication is done entry-wise. We compute right multiplication with an element of \mathcal{R} as

$$MA = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ a_{i1} & \cdots & a_{in} \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \cdot \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \vdots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ \sum_{j=1}^n a_{ij}b_{j1} & \cdots & \sum_{j=1}^n a_{ij}b_{jn} \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

Which is of the form that we desire. Hence this is a right ideal. We compute that the left multiplication will be

$$AM = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \vdots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix} \cdot \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ a_{i1} & \cdots & a_{in} \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} b_{i1}a_{i1} & \cdots & b_{in}a_{i1} \\ \vdots & \vdots & \vdots \\ b_{in}a_{i1} & \cdots & b_{in}a_{in} \end{bmatrix}$$

We note that this will not belong to the left ideal unless $n = 1$ which is just multiplication of real numbers.