

Q5: We first claim that for any normal subgroups M, N that $M \cap N \trianglelefteq G$. Note if we take some $a \in M \cap N$, we have that for any $g \in G$, $gag^{-1} \in M$ and $gag^{-1} \in N$ since they are normal in G . Therefore $gag^{-1} \in M \cap N$. Hence $M \cap N \trianglelefteq G$. We now claim that $\langle M, N \rangle \trianglelefteq G$. Note that by definition, every element of $\langle M, N \rangle$ must be of the form mn for some $m, n \in M, N$. Then for any $g \in G$, we have that

$$gmng^{-1} = (gmg^{-1})(gng^{-1})$$

Since $gmg^{-1} \in M$ and $gng^{-1} \in N$, their product must also belong to $\langle M, N \rangle$.