5.4.47a: If $T_n \to T$ strongly, then for all n, $x \in \mathfrak{X}$, $\|T_n x\| < \infty$. Therefore $\sup_n \|T\| < \infty$ by the uniform boundedness principle. Now suppose that $T_n \to T$ weakly. That is for all $f \in \mathfrak{X}^*, x \in \mathfrak{X}$, we have $fT_n x \to fTx$. By uniform boundeness, we have that $\sup_n \|fT_n\| < \infty$. Since this holds for all f this implies that $\sup_n \|T_n\| < \infty$.

5.4.47b: Let $\langle x_\alpha \rangle$ be a net converging to x. We have that for all f, $f(x_\alpha) \to f(x)$. Therefore $\|f(x_\alpha)\| \to \|f(x)\|$. We also have that $\hat{x}_\alpha(f) \to \hat{x}(f)$ and the norms converge to the norm of $\|f(x_\alpha)\|$. Therefore $\sup_\alpha \|\hat{x}_\alpha(f)\| < \infty$. Therefore $\|\hat{x}\| = \|x\| < \infty$. Now for Weak * convergence, we have that for all x if $f_\alpha(x) \to f(x)$, then

$$\sup_\alpha \|f_\alpha(x)\| = \sup_\alpha \|\hat{x}(f_\alpha)\| < \infty.$$

Where the last inequality follows from convergence.