Q2:
By changing to polar coordinates we can rewrite define $V_1 = \{(r,\theta) : r \in (0,1), \theta \in (0,2\pi)\}$ and $V_2 = \{(r,\theta) : r > 1, \theta \in (0,2\pi)\}$. We see that $g(V_1) = U_1$ and $g(V_2) = U_2$. We know $|\det g'| = r$, and g injective on V_1 and V_2 so by the COV theorem we evaluate:

$$\begin{aligned}
\int_{g(V_1)} f &= \int_{V_1} f \circ g |\det g'| \\
&= \lim_{t\to 0} \int_t^1 \int_0^{2\pi} \frac{1}{r^2} r d\theta dr && \text{(by Fubini's Theorem and discussion in class)} \\
&= \lim_{t\to 0} 2\pi \int_0^1 \frac{1}{r} dr \\
&= \lim_{t\to 0} 2\pi [log(r)]\Big|_t^1 \\
&= -\infty
\end{aligned}$$

In other words, for some PO1, $\{\phi_i\}$, $\sum \int \phi_i f$ diverges so f is not integrable. We now evaluate $\int_{g(V_2)} f$ using the COV theorem.

$$\begin{aligned}
\int_{g(V_2)} f &= \int_{V_2} f \circ g |\det g'| \\
&= \lim_{t\to \infty} \int_1^t \int_0^{2\pi} \frac{1}{r^2} r d\theta dr && \text{(by Fubini's Theorem, and discussion in class)} \\
&= \lim_{t\to \infty} 2\pi \int_1^t \frac{1}{r} dr \\
&= \lim_{t\to \infty} 2\pi [\log(r)]\Big|_1^t \\
&= \infty
\end{aligned}$$

So, for some PO1 of V_2, $\sum \int \phi_i f$ diverges, so f is not integrable.