

Q2a: Let  $\{B_n\}$  be an disjoint sequence of sets in  $\mathcal{A}$ . We define

$$A_n = \bigcup_{i=1}^n B_i$$

We have that the  $A'_i$ s are an increasing sequence by construction. It is clear that each  $A_i$  belongs to  $\mathcal{A}$ , since each is the union of sets in  $\mathcal{A}$ . It is given that  $\mu$  is a finitely additive measure, so

$$\mu\left(\bigcup_{j=1}^n B_j\right) = \sum_{j=1}^n \mu(B_j)$$

By the construction of  $\{A_n\}$  it is also true that

$$\mu(A_n) = \mu\left(\bigcup_{j=1}^n B_n\right)$$

Taking the limit we see that

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{j=1}^n \mu(B_j) &= \lim_{n \rightarrow \infty} \mu\left(\bigcup_{j=1}^n B_j\right) \\ &= \lim_{n \rightarrow \infty} \mu(A_n) && \text{(by construction of } A_n) \\ &= \mu\left(\lim_{n \rightarrow \infty} \bigcup_{j=1}^n A_n\right) && \text{(by given measure continuity)} \\ &= \mu\left(\lim_{n \rightarrow \infty} \bigcup_{j=1}^n B_n\right) && \text{(by definition of } B_n) \end{aligned}$$

We get the desired equality and conclude that  $\mu$  is a premeasure.

Q2b: Let  $\{B_n\}$  be a sequence of disjoint sets in  $\mathcal{A}$ . Since  $\mu$  is finite, we have that downward measure continuity holds. We define a decreasing sequence of sets  $\{A_n\}$  by  $A_n = \bigcup_{i=n}^\infty B_i$ . Note that this sequence is contained in  $\mathcal{A}$  since it is the union of elements of  $\mathcal{A}$ . We note that by finiteness of  $\mu$  we have that

$$\sum_{i=1}^n \mu(B_i) = \mu\left(\bigcup_{i=1}^n B_i\right)$$

But also by the construction of  $\{A_n\}$  we get that

$$\sum_{i=1}^n \mu(B_i) = \sum_{i=1}^n \mu(A_i) - \mu(A_{i+1}) = \mu(A_1) - \mu(A_{n+1})$$

Taking the limit as  $n \rightarrow \infty$ , we get that

$$\begin{aligned} \sum_{i=1}^{\infty} \mu(B_i) &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \mu(B_i) \\ &= \mu(A_1) - \lim_{n \rightarrow \infty} \mu(A_{n+1}) \\ &= \mu(A_1) && \text{(by assumption of measure of nested sets)} \\ &= \mu\left(\bigcup_{i=1}^{\infty} B_i\right) \end{aligned}$$

We get the desired result and conclude that  $\mu$  is a premeasure on  $\mathcal{A}$