Problem 4. *Marco: Show every class in* H^k *has a unique harmonic representative.*

Let $\alpha \in \ker d$. By hodge decomposition, we can uniquely write:

$$\alpha = \alpha_d + \alpha_{d'} + \alpha_\Delta.$$

Since $d\alpha = 0$, $\implies d\alpha_{d'} = 0$. By uniqueness of decomposition $\alpha_{d'} = 0$. Therefore we can write

$$\alpha = \alpha_d + \alpha_\Delta = df + \omega.$$

Suppose there was some form η so that $\omega - \eta \in \operatorname{image} d$. Harmonic functions form a subspace so $\omega - \eta \in \ker \Delta$ as well. By the hodge decomposition uniqueness $\omega - \eta = 0$.