Q3: Note that we can write w as a composition of conformal mappings in the following way. We define $u = 1 - z, \xi = u^{\frac{1}{4}}, v = \frac{1-\xi}{1+\xi}$. We see that

$$w = v \circ \xi \circ u = \frac{1 - (1-z)^{\frac{1}{4}}}{1 + (1-z)^{\frac{1}{4}}}$$

We wish to find a z such that $Re(w(z)) = 0$. Note that since the FLT $w(\xi) = \frac{1+\xi}{1-\xi}$ acts on the region $\{x + iy : x \geq 0, x \geq y, y \geq -x\}$ onto the lens shaped region homomorhpically, it must take the boundary to the boundary. Therefore to find the height of the region, we solve for when

$$Re(w(a + ia)) = 0$$

We know that

$$Re(w(a + ia)) = 0 \iff \frac{1 - (a + ia)}{1 + (a + ia)} + \frac{1 - (a - ia)}{1 + (a - ia)} = 0.$$

Which implies that

$$(1 - a - ia)(1 + a - ia) + (1 - a + ia)(1 + a + ia) = 0.$$

Solving gives us that

$$a = \pm \frac{1}{\sqrt{2}}$$

We take the positive root since we are working over the right half of the complex plane. We therefore compute

$$w(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}) = -i(\sqrt{2} - 1),$$

and similarly

$$w(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}) = i(\sqrt{2} - 1)$$

Hence the height of this lense is $2\sqrt{2} - 2$.