Consider the $k+l$ form $\omega \wedge \eta$. By Stoke's Theorem, we have that

$$\int_M d(\omega \wedge \eta) = \int_{\partial M} \omega \wedge \eta = \int_{\emptyset} \omega \wedge \eta = 0$$

Using the properties of the exteriour derivative, we know that

$$d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^k \omega \wedge d\eta$$

Therefore

$$0 = \int_M d\omega \wedge \eta + (-1)^k \omega \wedge d\eta \implies (-1)^{k+1} \int_M d\omega \wedge \eta = \int_M \omega \wedge d\eta$$