Q7:
" $\Longrightarrow$ "
Suppose that C is compact. Let O be an open cover of C such that O is closed under union of pairs of its elements. Since C is compact, there exists some finite set of covers $U_1 \dots U_n$ where $C \subset \cup_{i=1}^{n} U_i$. O is closed under unions,so we have that $\cup_{i=1}^{n} U_i \in O$. Take $T = \cup_{i=1}^{n} U_i$ and the result follows.
" $\Longleftarrow$ "
Suppose that O is an open cover of the set C. We construct a new open cover O' in the following way. Let $O' = \{\text{finite unions of } u \in O\}$. O' is closed under pairwise union, since when we take the union of 2 sets, $u_1, u_2 \in O'$, there are some finitely many sets in O which have unions of u_1 and u_2, and their union will also be a finite union of sets in O. By assumption, there is some $T \in O'$ with $C \subset T$. Therefore, T is the finite union of some open sets in O. And so for any open cover O, we can find a finite subcover which also covers C. Thus C is compact. $\blacksquare$