Q1a: We claim that $\gamma = (\iota(v_1) \dots \iota(v_n))$ is a basis for V^{**} and that it is the dual basis of $(\phi_1 \dots \phi_n)$, the dual basis of V^* We will prove that γ is linearly independant and spans V^{**}. First suppose that for some scalar $\alpha_1, \dots, \alpha_n$ we have that

$$\alpha_1 \iota(v_1) + \dots \alpha_n \iota(v_n) = 0$$

From the definition of ι, we have

$$\alpha_1 \phi(v_1) + \dots \alpha_n \phi(v_n) = 0, \forall \phi \in V^*$$

Now write $\phi = \beta_1 \phi_q + \dots \beta_n \phi_n$ for scalars $\beta_1, \dots \beta_n$. We see that

$$\alpha_1(\beta_1 \phi_1(v_1) + \dots + \beta_n \phi_n(v_1)) + \dots \alpha_n(\beta_1 \phi_1(v_n) + \dots \beta_n \phi_n(v_n)) = 0$$

From the definition of the dual basis this gives us:

$$\alpha_1 \beta_1 + \dots + \alpha_n \beta_n = 0$$

Since this is true for every β_i, we must have that $\alpha_1 = \dots = \alpha_n = 0$. Hence γ is a linearly independant set. We now claim that it spans V^{**}. Now suppose that $\psi \in V^{**}$, and $\psi(\phi_i) = k_i$. Let $\phi = \beta_1 \phi_1 + \dots \beta_n \phi_n$. We see that

$$\begin{aligned}
\psi(\phi) &= \psi(\beta_1 \phi_1 + \dots + \beta_n \phi_n) \\
&= \beta_1 k_1 + \dots + \beta_n k_n \\
&= k_1 \phi(v_1) + \dots k_n \phi(v_n) \\
&= k_1 \iota_{v_1}(\phi) + \dots + k_n \iota_{v_n}(\phi)
\end{aligned}$$

Thus γ spans V^{**} and we conclude it is a basis. We now want to show that γ is dual to $(\phi_1, \dots, \phi_n)$. Notice that

$$\iota(v_i)(\phi_j) = \phi_j(v_i) = \delta_{ij}$$

We conclude that γ is indeed the dual of $(\phi_1, \dots \phi_n)$

Q1b: First, observe that $\iota(v)(\alpha\phi + \psi) = \alpha\phi + \psi(v) = \alpha\phi(v) + \phi(v) = \alpha\iota(v)(\phi) + \iota(v)(\psi)$, so ι is linear. Note that by 1b, we see that the image of ι is n dimensional, and the domain is as well n dimensional. Hence by the Rank-Nullity theorem we conclude it is a bijection . Thus it is a linear isomorphism between V and V^{**}