Q3: For $\lambda \in \Lambda^{n-k}(V)$, we claim that $\psi_k(\lambda)$ which assigns λ to $f_\lambda(\eta)$ defined by $f_\lambda(\eta) = \chi(\lambda \wedge \eta)$, $\eta \in \Lambda^k(V)$ is our desired choice free linear isomorphism. We will first show linearity of ψ_k. For $\alpha \in \mathbb{R}$ and $\lambda_1, \lambda_2 \in \Lambda^{n-k}(V)$, we compute

$$\begin{aligned}
\psi_k(\alpha\lambda_1 + \lambda_2) &= f_{\alpha\lambda_1+\lambda_2}(\eta) \\
&= \chi((\alpha\lambda_1 + \lambda_2) \wedge \eta) \\
&= \chi(\alpha\lambda_1 \wedge \eta + \lambda_2 \wedge \eta) && \text{(by linearity of } \wedge\text{)} \\
&= \alpha\chi(\lambda_1 \wedge \eta) + \chi(\lambda_2 \wedge \eta) && \text{(by linearity of } \chi\text{)} \\
&= \alpha f_{\lambda_1}(\eta) + f_{\lambda_2}(\eta) \\
&= \alpha\psi_k(\lambda_1) + \psi_k(\lambda_2)
\end{aligned}$$

Thus ψ_k is a linear mapping. It remains to show that it is a bijection between vector spaces. By the rank nullity theorem it is sufficient to show that ψ_k is either injective or surjective. Suppose that $\psi_k(\lambda) = 0$. This is the same as saying $f_\lambda(\eta) = 0$ for all η. Equivalently, $\chi(\lambda \wedge \eta) = 0$ for all η. Since χ is a linear isomorphism, it must be that $\lambda \wedge \eta = 0$ for all η. Therefore, we can conclude that $\lambda = 0$. Therefore, ψ_k has a trivial null space. Therefore it is injective and it follows that it is a linear isomorphism.