1a: True: If each f_n has no discontinuities, then it must be continuous. By Chapter 4 Theorem 1(Pugh), the uniform limit of a sequence of continuous function is continuous as well. Hence continuity is preserved by uniform convergence.

1b: False: Let $A = \{q_1, \dots, q_{10} : q_i \in \mathbb{Q} \cap [0,1]\}$ consider $f_n(x) = \begin{cases} \frac{1}{n} & x \in A \\ 0 & \text{otherwise} \end{cases}$ We have that $f_n \rightrightarrows 0$, but 0 does not have any discontinuities.

1c: False: Take A and f_n as exactly the same as in 1b. Each f_n has at least 10 discontinuties, namely it has exactly 10. We have an identical conclusion to 1b.

1d: False: Take let $m \in \mathbb{N}$. Let $A = \{q_1, \dots, q_m : q_i \in \mathbb{Q} \cap [0,1]\}$. Take f_n as defined in 1b. Each f_n has finitely many discontinuities, and it uniformly converges to 0, which is continuous.

1e: False: Take $A = \{x \in [0,1] : \exists k \text{ such that } x = \frac{1}{k}\}$, and take $f_n(x)$ as defined in 1b. Clearly, each discontinuity is of the jump type, and occurs countably many times, yet the limit of $f_n(x)$ uniformly converges to 0, which has no jump discontinuities.

1f: False: Take $f_n(x) = \begin{cases} \frac{1}{n}\sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$ on the domain $[-1,1]$. This has an oscillating discontinuity at $x = 0$. We claim that $f_n \rightrightarrows 0$. Let $\varepsilon > 0$. Choose $N > \varepsilon$. Then if $n \geq N$,

$$\left\| \frac{1}{n} sin(\frac{1}{x}) - 0 \right\| < \left\| \frac{1}{n} \right\| < \varepsilon$$

Where the second inequality follows from $-1 \leq \sin(y) \leq 1$. The function 0 has no discontinuities of the oscillation type.

1g: False: Take the function from $1d$. This has no oscillating discontinuities.