Q2: It has been proven that an ideal I is maximal if and only if R/I is a field. Thus we will show that $\mathbb{R}[x]/(x^2+1)$ is a field. Note that any polynomial of degree greater than 2 will be equivalent to a linear polynomial, since we can subtract off a sufficiently large multiple of x^2+1. Elements of the ring $\mathbb{R}[x]/(x^2+1)$ will therefore take the form of $a+bx$. We claim that $\mathbb{R}[x]/(x^2+1) \cong \mathbb{C}$. If we take $a+bx, c+dx$ in this ring, we have that their sum will be

$$(a+bx)+(c+dx) = (a+c)+x(b+d).$$

We evaluate their product as

$$(a+bx)(c+dx) = ac+adx+cbx+dbx^2 = (ac-bd)+x(ad+cb).$$

We have an isomorphism between $\mathbb{R}[x]/(x^2+1)$ to $\mathbb{C}$ given by $\varphi(a+bx) = a+bi$. The structures of addition and multiplication will be preserved by our computations above.