Q3a: First, WLOG suppose that $y = \alpha x$, that is assume that the line goes through 0. This clearly does not change the measure of the set, since measure is invariant under translation. Define the set $A_\alpha = \{(x, y) \in \mathbb{R}^2 : x \in [0, 1], y = \alpha x\}$. Since any line through 0 is a countable union of sets of similar for to A_α, except with different ranges for the value of x, it will be sufficient to show that A is the zero set. Let $\varepsilon > 0$, choose N sufficiently large so that $N > \frac{\alpha}{\varepsilon}$. We now take $\{R_i\}_{i=1}^N$ to be the collection of rectangles R_i where each $R_i = [\frac{i-1}{n}, \frac{i}{N}] \times [\frac{\alpha(i-1)}{n}, \frac{\alpha(i)}{N}]$. This will be a cover of A_α, and we can compute its measure as follows:

$$
\begin{aligned}
m^* A &\leq \sum_{i=1}^{N} |R_i| \\
&= \sum_{i=1}^{N} (\frac{i}{N} - \frac{i-1}{N})(\frac{\alpha i}{N} - \frac{\alpha(i-1)}{N}) \\
&= \sum_{i=1}^{N} \frac{\alpha}{N^2} \\
&= \frac{\alpha}{N} \\
&< \varepsilon
\end{aligned}
$$

Hence this set is the zero set, and so any line is the zero set.

Q3b: Let $P_i(a) = \{x \in \mathbb{R}^n : x_i = a\}$ be an $n-1$ hyerplane in $\mathbb{R}^n$. For the same reasoning as above, it is sufficient to consider when $a = 0$. Let $\varepsilon > 0$, we define $I_k = [\frac{-\varepsilon}{k^{n-1}2^{k+n}}, \frac{\varepsilon}{k^{n-1}2^{k+n}}]$, and $J_k = [-k, k]$. Then consider the set of boxes $\{B_k\}$ where B_k is the product of $n-1$ copies of J_k, and one I_k in the $i'th$ spot We compute the volume of this covering as

$$
\begin{aligned}
\sum_k |B_k| &= \sum_k 2^{n-1} k^{n-1} \cdot \frac{2\varepsilon}{k^{n-1} \cdot 2^{k+n}} \\
&= \sum_k \frac{\varepsilon}{2^k} \\
&= \varepsilon
\end{aligned}
$$

Hence this is the zero set. The same is true for any arbitrary $n-1$ hyperplane, since we can map any $P_i(0)$ to it by an isometry, which is linear and has a determinant of 1, so by Pugh Theorem 16 (page 397) any plane will also be the zero set.