Q1: Let $M$ be a $k$ dimensonal submanifold of $\mathbb{R}^n$. Since $M$ is a subspace of $\mathbb{R}^n$, it inherits second countability and is hausdorff. Therefore it is sufficient to show that given two submanifold coordinate charts $(U_1, \varphi_1^{-1}), (U_2, \varphi_2^{-1})$ , the transition map $\varphi_2^{-1} \circ \varphi_1$ is a diffeomorphism. Define the maps $h_i : \mathbb{R}_x^k \times \mathbb{R}_y^{n-k} \to \mathbb{R}^n$ by

$$h_i(x, y) = \varphi_i(x) + (0, y).$$

Similarly as in A1, we have that $h_i$ is a diffeomorphism. Therefore the composition

$$h_2^{-1} \circ h_1$$

is a diffeomorphism of $\mathbb{R}^n$. If $\iota_x$ is the inclusion of $\mathbb{R}_x^k$ into $\mathbb{R}_{x,y}^n$ into the first $k$ coordinates and $\pi_x$ is the projection onto the $x$ coordinates, the composition,

$$\pi_x \circ (h_2^{-1} \circ h_1) \circ \iota_x = \pi_x(h_2^{-1} \circ (\varphi_1(x), 0)) = \pi_x(\varphi_2^{-1}(\varphi_1(x), 0)) = \varphi_2^{-1} \circ \varphi_1(x).$$

This is a diffeomorphism since it has rank $k$.