Q2: We will proceed by the contrapositive. Suppose that $\mu \not\perp \nu$. Then there exists some maximal sets E, F such that $\mu|_{E^c} \equiv 0$ and $\nu|_F^c \equiv 0$ but $E \cap F \neq \emptyset$ and $E \cap F$ is not measure 0. We compute that

$$\begin{aligned}
\|\alpha\mu - (1-\alpha)\nu\| &= \int_X \alpha\mu + (1-\alpha)\nu \\
&= \int_E \alpha\mu + (1-\alpha)\nu + \int_F \alpha\mu + (1-\alpha)\nu - \int_{E\cap F} \alpha\mu + (1-\alpha)\nu \\
&= \alpha + (1-\alpha) - \int_{E\cap F} \alpha\mu + (1-\alpha)\nu \\
&= 1 - \int_{E\cap F} \alpha\mu + (1-\alpha)\nu
\end{aligned}$$

Now we verify that $\int_{E\cap F} \alpha\mu + (1-\alpha)\nu > 0$. Note that by definition of E, F and since μ, ν are both positive on their intersection, we have that $\alpha\mu + (1 - \alpha\nu) > 0$ on a nonempty set. Hence we have that $\int_{E\cap F} \alpha\mu + (1-\alpha)\nu > 0$. Thus we conclude that if $\mu \not\perp \nu$, then $\|\alpha\mu - (1-\alpha)\nu\| \neq 1$.