Q4a: Let $\varepsilon > 0$. Define $g_n = f_n - f$. Since $f_n \leq f_{n+1}$, we have that g_n is a decreasing sequence. g_n is also continuous, since it is the sum of continuous functions, with its limit being 0. Let $G_n = g_n^{pre}(-\infty, \varepsilon)$. This will be an open set in $[a, b]$ by continuity. We have that $G_n \subset G_{n+1}$. Therefore, the set $\{G_n\}$ forms an open cover of $[a, b]$. By compactness, there is a finite subcover. Let G_N be the largest such set, which contains every other element of the finite subcover. Therefore, if $n \geq N$, we have that $|g_n| < \varepsilon$ and so $|f_n(x) - f(x)| < \varepsilon$

4b: If g_n is an increasing convergent sequence bounded above by g_n, setting $g_n(x) = -f_n(x)$ and applying 4a shows that it is indeed uniformly convergent

4c: Not true, since we require that every open cover has a finite subcover.

4d: If $[a, b]$ is replaced with a compact metric space, this will also be true, since in our proof for 4a, we made no assumptions about the domain except that it is compact.