Problem 5. *Gurman: Show d is an anti derivation*

Let $\alpha \in C^k(\Lambda), \beta \in C^l(\Lambda)$. If $k, l > 1$ then $d(\alpha \wedge \beta) = 0$. First check when $k = l = 0$. Then:

$$\int_{(e_1,e_2)} d(\alpha \wedge \beta) = \int_{\{e_1,e_2\}} \alpha \wedge \beta = \alpha(e_2)\beta(e_2) - \alpha(e_1)\beta(e_1).$$

On the other hand:

$$\int_{(e_1,e_2)} \alpha \wedge d\beta + \beta \wedge d\alpha = avg_{e_1,e_2}\alpha\,[\beta(e_2) - \beta(e_1)] + avg_{e_1,e_2}\beta\,[\alpha(e_2) - \alpha(e_1)] = \alpha(e_2)\beta(e_2) - \alpha(e_1)\beta(e_1).$$

Thus d satisfies the anti derivation property on $0 - \text{forms}$. Now verify on $1 - \text{forms}$ and $0 - \text{forms}$. Compute that:

$$\int_F d(\alpha \wedge \beta) = \int_{\partial F} \alpha \wedge \beta = \sum_{e \in \partial F} avg_e \alpha \int_e \beta.$$

On the other hand;

$$\int_F d\alpha \wedge \beta + \int_F \alpha \wedge d\beta = \frac{1}{4} \sum_{e.e' \in \partial F} \int_e d\alpha \int_{e'} \beta - \int_{e'} d\alpha \int_e \beta + avg_F \alpha \int_F d\beta = \sum_{e \in \partial F} avg_e \alpha \int_e \beta,$$

where the last equality follows from cancelling the summands with opposite signs and adding like terms of which come in pairs.