Q4:
Suppose that f is integrable. Choose partition P such that $U(f,P) - L(f,P) < \varepsilon$. For any $S \in P$ we have that

$$M_S(|f|) - m_S(|f|) \leq M_S(f) - m_S(f)$$

Therefore

$$U(|f|,P) - L(|f|,P) = \sum_{S\in P}[M_S(|f|) - m_S(|f|)] \cdot vol(S) \leq \sum_{S\in P}[M_S(f) - m_S(f)] \cdot vol(S) < \varepsilon$$

hence $|f|$ is integrable. We know want to show that $|\int_A f| \leq \int_A |f|$. We have

$$\begin{aligned}
|\int_A f| &= |\inf U(f,P)| \\
&\leq \inf |U(f,P)| && \text{(from discussion in class)} \\
&= \inf |\sum_{S\in P} M_S(f) \cdot vol(S)| \\
&\leq \inf \sum_{S\in P} |M_S(f)| \cdot vol(S) \\
&\leq \inf \sum_{S\in P} M_S(|f|) \cdot vol(S) \\
&= \int_A |f|
\end{aligned}$$

As desired.