

Q1:

First letting $A = (0, a) \times (0, \frac{\pi}{2}) \times (0, 2\pi)$ we see that $g(A) = V \setminus C$, for some content 0 set C . Thus by COV $\int_{g(A)} z = \int_A z \circ g \cdot |\det g'|$. We see that $z \circ g = r \sin \phi$ Computing g' get

$$g' = \begin{bmatrix} \cos \theta \cos \phi & -r \sin \phi \cos \theta & -r \cos \phi \sin \theta \\ \cos \phi \sin \theta & -r \sin \phi \sin \theta & r \cos \phi \cos \theta \\ \sin \phi & r \cos \phi & 0 \end{bmatrix}$$

We have that $|\det g'| = r^2 \cos \phi$. This will be nonzero on the domain of g , so we can apply COV. We evaluate:

$$\begin{aligned} \int_{g(A)} z &= \int_A z \circ g |\det g'| && \text{(by COV)} \\ &= \int_A r^3 \sin \phi \cos \phi \\ &= \int_0^a \int_0^{2\pi} \int_0^{\frac{\pi}{2}} r^3 \sin \phi \cos \phi \, d\phi d\theta dr && \text{(by Fubini's Theorem)} \\ &= \int_0^a \int_0^{2\pi} r^3 \frac{\sin^2 \phi}{2} \Big|_0^{\frac{\pi}{2}} d\theta dr \\ &= \int_0^a \int_0^{2\pi} \frac{r^3}{2} d\theta dr \\ &= \int_0^a \pi r^3 dr \\ &= \frac{\pi a^4}{4} \end{aligned}$$