

Q1: Prove that $d(fg) = g \cdot df + f \cdot dg$. We compute that

$$\begin{aligned} d(fg) &= \sum_{i=1}^n \frac{\partial(f \cdot g)}{\partial x_i} dx_i \\ &= \sum_{i=1}^n \left[g \cdot \frac{\partial f}{\partial x_i} + f \cdot \frac{\partial g}{\partial x_i} \right] dx_i \\ &= g \cdot \sum_{i=1}^n \frac{\partial f}{\partial x_i} dx_i + f \cdot \sum_{i=1}^n \frac{\partial g}{\partial x_i} dx_i \\ &= g \cdot df + f \cdot dg \end{aligned}$$