

Q1: Bee flying problem:

Since the trains are approaching each other at $50m/s$, and they start at a distance of $100m$ apart, they will meet in 1 second according to basic kinematics. Since the bee travels at $100m/s$, it will move a total of $100m$ before the trains collide.

Q2: Derive Gauss' summation for first n integers:

We claim that the following formula holds

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

We proceed by induction. For $n = 1$ it is clear that $\sum_{i=1}^1 i = 1$. Suppose that the formula holds for n . We claim that this implies that the formula is true for $n + 1$. Observe:

$$\sum_{i=1}^{n+1} i = \sum_{i=1}^n i + (n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{n^2 + 3n + 2}{2} = \frac{(n+1)(n+2)}{2}$$

We conclude that the formula holds for all n by the principle of induction. Thus we see that Gauss' summation of integers from 1 to 100 was quite simple he only had to compute:

$$\sum_{i=1}^{100} i = \frac{100(100+1)}{2} = 5050$$

Q3: Suppose there is a tennis tournament with 128 players, how many matches are played in total?

We can compute this as a finite geometric sum.

$$\text{total played} = \sum_{i=1}^7 128 \frac{1}{2}^i = 64 \sum_{i=0}^7 \frac{1}{2}^{i-1} = 64 \left(\frac{1 - (\frac{1}{2})^7}{1 - \frac{1}{2}} \right) = 127$$

Alternatively, we could compute out the sum $64 + 32 + \dots + 1 = 127$.