

Q4: We first verify that $d^2\omega = 0$. We compute that

$$\begin{aligned}
 d(d\omega) &= d\left(\sum_{i=1}^3 dx_i \wedge \frac{\partial \omega}{\partial x_i}\right) \\
 &= d(dx \wedge (ydx) + dy \wedge (xdx - zdz) + dz \wedge (-ydz)) \\
 &= d(-xdx \wedge dy - zdy \wedge dz) \\
 &= d(-xdx \wedge dy) - d(zdy \wedge dz) \\
 &= dx \wedge (-dx \wedge dy) - dz \wedge (dy \wedge dz) \\
 &= 0
 \end{aligned}$$

Unsurprisingly $d^2\omega = 0$. We now will verify that $d(\omega \wedge \eta) = d\omega \wedge \eta - \omega \wedge d\eta$. We first compute the left side.

$$\begin{aligned}
 d(\omega \wedge \eta) &= d((-xy^2z^2 - 3x)dx \wedge dy + (2x^2 + xz)dx \wedge dz + (6x - y^2z^3)dy \wedge dz) \\
 &= 6dx \wedge dy \wedge dz - (2x^2 + xz)dx \wedge dy \wedge dz - 2xy^2zdx \wedge dy \wedge dz \\
 &= (6 - 2x^2 - xz - 2xy^2z)dx \wedge dy \wedge dz
 \end{aligned}$$

We will now compute the right side:

$$\begin{aligned}
 d\omega \wedge \eta - \omega \wedge d\eta &= (-xdx \wedge dy - zdy \wedge dz) \wedge (xdx - yz^2dy + 2xdz) - (xydx + 3dy - yzdz) \wedge (2dx \wedge dz + 2yzdy \wedge dz) \\
 &= (-2x^2 - xz)dx \wedge dy \wedge dz - (2xy^2z - 6)dx \wedge dy \wedge dz \\
 &= (6 - 2x^2 - xz - 2xy^2z)dx \wedge dy \wedge dz
 \end{aligned}$$

Once again we are not surprised to see that indeed $d(\omega \wedge \eta) = d\omega \wedge \eta - \omega \wedge d\eta$.