

Q7: We first consider the sum

$$\sum_{k=0}^n \cos(kx + y) + i \sin(kx + y)$$

Using Eulers identity we can rewrite this as

$$\sum_{k=0}^n e^{i(kx+y)} = e^{iy} \sum_{k=0}^n e^{ikx} = e^{iy} \frac{1 - e^{i(n+1)x}}{1 - e^{ix}}$$

Further manipulating, we see that

$$\begin{aligned} e^{iy} \frac{1 - e^{i(n+1)x}}{1 - e^{ix}} &= e^{iy} \frac{(e^{\frac{ix(n+1)}{2}})(e^{\frac{-ix(n+1)}{2}} - e^{inx + \frac{ix(n+1)}{2}})}{e^{\frac{ix}{2}}(e^{\frac{-ix}{2}} - e^{\frac{ix}{2}})} \\ &= \frac{e^{\frac{ix}{2}} \cdot e^{i(\frac{nx}{2} + y)} \cdot (-2i \sin(\frac{n+1}{2}x))}{e^{\frac{ix}{2}} \cdot -2i \cdot \sin(\frac{x}{2})} \\ &= \frac{(\cos(\frac{n}{2}x + y) + i \sin(\frac{n}{2}x + y)) \cdot \sin(\frac{n+1}{2}x)}{\sin(\frac{x}{2})} \\ &= \frac{\cos(\frac{n}{2}x + y) \sin(\frac{n+1}{2}x)}{\sin(\frac{x}{2})} + i \frac{\sin(\frac{n}{2}x + y) \sin(\frac{n+1}{2}x)}{\sin(\frac{x}{2})} \end{aligned}$$

Since this is equal to the original quantity, the real and complex components must be equal and hence we achieve the desired equality.