Q1a: Since $\mathcal{M}$ is an infinite σ-algebra, there exists an infinite sequence of nonempty sets $\{E_i\}$ belonging to $\mathcal{M}$ whose countable union and intersection belong to $\mathcal{M}$ and $E_i \in \mathcal{M}$ implies that $E_i^c \in \mathcal{M}$. We can generate a disjoint sequence of sets belonging to $\mathcal{M}$ in the following way. Define $F_1 = E_1$, and $F_n = \cup_{i=1}^n E_i \setminus \cup_{i=1}^{n-1} E_i$. We note that by construction, the F_i's are disjoint from one another, and that they belong to $\mathcal{M}$, since they are built using unions and compliments. Note as well that not all F_i's are empty, since we presume that $\mathcal{M}$ is infinite hence we can take distinct E_i. We have the desired result.

Q1b: We now claim that $\mathcal{M}$ is uncountable. It is sufficient to construct a subset of $\mathcal{M}$ that is uncountable, since we can find an injection from any such subset to $\mathcal{M}$. Let $\mathbb{P}(\mathbb{N})$ be the power set of the naturals. Now for each $\lambda \in \mathbb{P}(\mathbb{N})$, define

$$A_\lambda = \bigcup_{i \in \lambda} F_i$$

Each A_λ is a countable union of sets in $\mathcal{M}$, hence it belongs to $\mathcal{M}$. Furthermore if we consider $\{A_\lambda\}_{\lambda \in \mathbb{P}(\mathbb{N})}$, we can identify each A_λ with $\lambda \in \mathbb{P}(\mathbb{N})$ bijectively. It is a fact that $\mathbb{P}(\mathbb{N})$ is uncountable, hence $\{A_\lambda\}_{\lambda \in \mathbb{P}(\mathbb{N})}$ is our desired uncountable subset of $\mathcal{M}$.