Q4a: Fix $z_0 \in \Omega$. Since Ω simply connected, for any $z \in \Omega$ one can define the curve $\gamma(t) : [0,1] \to \Omega$ so that $\gamma(0) = z_0$ and $\gamma(1) = z$. We define the function

$$g(z) = \int_\gamma \frac{f'(z)}{f(z)} dz - f(z_0).$$

This makes sense since $f(z) \neq 0$ for all z, and is well defined since on any other path δ satisfying the same endpoint conditions as γ, the holomorphicity of $\frac{f'(z)}{f(z)}$ implies that

$$\int_{\gamma-\delta} \frac{f'(z)}{f(z)} dz = 0 \implies \int_\gamma \frac{f'(z)}{f(z)} dz = \int_\delta \frac{f'(z)}{f(z)} dz.$$

We compute that

$$g(z) = \int_\gamma \frac{f'(z)}{f(z)} dz - f(z_0) = \int_{[0,1]} \frac{f'(\gamma(t))}{f(\gamma(t))} \cdot \gamma'(t) dt - f(z_0) = \log(f(\gamma(t)))\Big|_0^1 - f(z_0) = \log(f(z)).$$

This shows that $g(z)$ is our desired construction, and is independant of choice of base point. Finally, we have that $g(z)$ is holomorphic, since it is equal to a composition of holomorphic functions.

Q4b: We define the function $g(z) = \exp(\frac{1}{n}\log(f(z)))$. We have that $g(z)$ is a composition of holomorphic functions. We see that

$$\log(g(z)) = \frac{1}{n}\log(f(z)) \implies log(g(z)^n) = \log(f(z)) \implies g(z)^n = f(z),$$

As desired.