

Q5: We will identify all the subgroups of \mathcal{Q}_8 :

$$\begin{aligned}\langle 1 \rangle &= \{1\} \\ \langle -1 \rangle &= \{\pm 1\} \\ \langle i \rangle &= \{1, i, -1, -i\} \\ \langle j \rangle &= \{1, j, -1, -j\} \\ \langle k \rangle &= \{1, k, -1, -k\} \\ \langle i, j \rangle &= \{1, i, j, -1, -i, -j, k, -k\} = \mathcal{Q}_8\end{aligned}$$

There are no other subgroups of \mathcal{Q}_8 , since if we took any 2 of $\{i, j, k\}$ we could generate the entire group.