

Q2: We claim that $\mathbb{Z}/(m) \otimes_{\mathbb{Z}} \mathbb{Z}/(n) \cong \mathbb{Z}/(m,n)$. Consider the following commutative diagram:

$$\mathbb{Z}/(m) \times \mathbb{Z}/(n) \mathbb{Z}/(m) \otimes_{\mathbb{Z} \mathbb{Z}/(n)} \mathbb{Z}/(m,n) [\text{"} f(x,y) = xy \text{"}' , from = 1 - 1, to = 2 - 1] [\text{"} \otimes \text{"} , from = 1 - 1, to = 1 - 2] [\text{"} \tilde{f} \text{"} , from = 1 - 1, to = 1 - 2]$$