

Q5:

Suppose that f is homogenous of degree m . Then we have that $f(tx) = t^m f(x)$. Taking the derivative with respect to t on both sides we see that

$$\begin{aligned}\frac{\partial f(tx)}{\partial t} &= f'(tx) \cdot \frac{\partial tx}{\partial t} \text{ (from chain rule)} \\ &= f'(tx) \cdot x \\ &= \sum_{i=1}^n D_i f(tx) \cdot x_i\end{aligned}$$

From simple differentiation in 1 dimension, $\frac{\partial(t^m f(x))}{\partial t} = mt^{m-1} f(x)$. Thus we get that

$$mt^{m-1} f(x) = \sum_{i=1}^n D_i f(tx) x_i$$

Choosing $t = 1$ gives the desired result.