Q7: We first note that the inequality

$$\frac{1-r}{1+r} \leq \frac{1-|z|^2}{|e^{i\theta}-z|^2} \leq \frac{1+r}{1-r}$$

holds since we have

$$\frac{1-r}{1+r} \leq 1-r^2 \leq \frac{1+r}{1-r}$$

. Multiplying by $u(re^{i\theta})$ and integrating, we get that

$$\int_0^{2\pi} u(re^{i\theta})\frac{1-r}{1+r}d\theta \leq \int_0^{2\pi} u(re^{i\theta})\frac{1-|z|^2}{|e^{i\theta}-z|^2}d\theta \leq \int_0^{2\pi} u(re^{i\theta})\frac{1+r}{1-r}.$$

Harmonicity of u and the mean value property imply that

$$2\pi\frac{1-r}{1+r}u(0) \leq 2\pi u(z) \leq 2\pi\frac{1+r}{1-r}u(0).$$

As desired.