Q4: For $P > 3$, we employ the binomial formula to compute that

$$f(x) = \frac{\sum_{k=1}^{p} \binom{p}{k} x^{p-k}(-3)^k + 3^p}{x} = \sum_{k=1}^{p} \binom{p}{k} x^{p-k-1}(-3)^k = x^{p-1} + \cdots + p(-3)^{p-1}.$$

We see that p divides every coefficient except on the leading term, and p^2 does not divide the constant term. Thus by Eisenstiens criteron this polynomial is irreducible for $p > 3$. For $p = 3$, we can simply compute

$$f(x) = x^2 - 9x + 27.$$

If this polynomial splits, it must split into two linear factors or equivalently have two roots in $\mathbb{Z}$. The quadratic formula tells us that the roots are $x = \frac{1}{2}(9 \pm i3\sqrt{3})$, which are not in $\mathbb{Z}$. Thus this polynomial is irreducible.