Q2a: Prove (2) $\iff$ (2′)
" $\implies$ "
Let $P(n)$ be the statement $1, \ldots n \in B$. P(1) is true since $1 \in B$. If $P(n)$ is true, then we have that $1, \ldots n \in B$, so $1 \ldots n, n+1 \in B$ and so $P(n+1)$ is true. $P(1)$ is true and $P(n) \implies P(n+1)$, so $P(n)$ is true for all $n \in \mathbb{N}$ by (2). Thus $1, \ldots n \in B$ for all $n \in N$, so $B = \mathbb{N}$.
" $\impliedby$ "
Given $P(1) \ldots P(n)$, take $B = \{n \in \mathbb{N} : P(1) \ldots P(n) true\}$. Then we have that $1 \in B$, since $P(1)$ is true, and if $1, \ldots n \in B$ then $P(1), \ldots P(n)$ is true so $P(n+1)$ is true and so $n+1 \in B$ hence $B = \mathbb{N}$. So $P(n)$ true for all $n \in N$.
Q2b:
Let $B = \{n \in \mathbb{N} : P(1) \ldots P(n) \text{ true}\}$. Since $P(1)$ is true by assumption, we have that $1 \in B$. Since $P(n) \implies P(n+1)$, by our assumption (2′) we know that if $1 \ldots n \in B$ then $n+1 \in B$, so $B = \mathbb{N}$. Thus $P(n)$ is true for all $n \in \mathbb{N}$.