Q6:
Define h as $h(t) = f(tx)$. This is differnetiable since it is the composition of differnetiable functions so from the chain rule we have that

$$h'(t) = f'(tx) \cdot x = \sum_{i=1}^{n} D_i f(tx) \cdot x_i$$

By the fundamental theorem of calculus

$$\int_0^1 h'(t)dt = f(x) = \int_0^1 \sum_{i=1}^{n} D_i f(tx) x_i dt$$

From the linearity of the integral we can rewrite it as

$$f(x) = \sum_{i=1}^{n} x_i \int_0^1 D_i f(tx) dt$$

Thus we choose each $g_i = \int_0^1 D_i f(tx)dt$. and so

$$f(x) = \sum_{i=1}^{n} x_i \cdot g_i$$