Problem 1. *Jerry: Show* $d^2 = 0$.

Recall that the coboundary map $d : C^k(\Lambda) \to C^{k+1}(\Lambda)$ is defined as

$$d(\omega)(c) := \omega(\partial c).$$

$$d^2\omega(c) = \omega(\partial^2 c) = \int_{\partial^2 c} \omega$$

Since $\partial^2 v = 0$ for any vertex trivially, it is sufficient to show that $d^2 = 0$ on $0-\mathrm{forms}$. Let $f \in C^0(\Lambda)$, take $F \in \Gamma_2$ so that $\partial F = \sum_{i=1}^{n} v_i$. Then

$$\begin{aligned}
\int_F d^2 f &= \int_{\partial F} df && \text{(by defn)}\\
&= \int_{\partial^2 F} f && \text{(by defn)}\\
&= \int_{\partial \sum_{i=1}^{n} v_i} f && \text{(by defn)}\\
&= \int_{\sum_{i=1}^{n} (x_1^i - x_2^i)} f && \text{(setting } \partial v_i = x_1^i - x_2^i\text{)}\\
&= \sum_{i=1}^{n} f(x_1^i) - f(x_2^i) && \text{(by defn)}\\
&= 0 && \text{(since } x_2^i = x_1^{i+1} \text{ and } x_1^1 = x_2^n\text{)}
\end{aligned}$$

Thus on any face F $d^2(f)(F) = 0$. Therefore $d^2 f = 0$ for any given f.