

Q6:

It has been shown that $\partial A = [0, 1] \setminus A$. We first claim that $\mathbb{Q} \cap [0, 1]$ is of measure 0. Given $\varepsilon > 0$ we consider a bijection $f : \mathbb{Q} \cap [0, 1] \rightarrow \mathbb{N}$. For each $r \in \mathbb{Q} \cap [0, 1]$ take the interval $(r - \frac{\varepsilon}{2^{f(r)+1}}, r + \frac{\varepsilon}{2^{f(r)+1}})$. This will cover $\mathbb{Q} \cap [0, 1]$. We compute $\sum_{r \in \mathbb{Q}} \frac{\varepsilon}{2^{f(r)}} < \varepsilon$, so $\mathbb{Q} \cap [0, 1]$ is measure 0. Therefore if $\sum_i (b_i - a_i) = l < 1$, then any cover of ∂A has length of at least $1 - l$ and thus is not of measure 0.