Q4a: By q2a, we have that $\gcd(1485, 1745) = 5$. Once again, by 2a, we know that $5 = (-47)\cdot 1485 + (40)\cdot 1745$. Thus we have a particular solution of $x_0 = -47 \cdot 3 = -141$ and $y_0 = 40 \cdot 3 = 120$. Thus by Theorem 1.13, the general solution takes the form of

$$x = -141 + \frac{1745n}{5} = -141 + 349n$$

$$y = 120 - \frac{1485n}{5} = 120 - 297n$$

for $n \in \mathbb{Z}$.

Q4b: We claim $a_1x_1 + \ldots a_nx_n = c$ if and only iff $gcd(a_1 \ldots a_n)|c$. We prove the forward implication. Suppose $(x_1, \ldots x_n)$ solves the equation. By definition of the gcd, $gcd(a_1 \ldots a_n)|a_i$ for each i. Then $gcd(a_1 \ldots a_n)|a_ix_i$ and so $gcd(a_1 \ldots a_n)|a_1x_1 + \cdots + a_nx_n = c$. We now show the reverse implication. Assume that $gcd(a_1 \ldots a_n)|c$. By 1.11, there exists $v_1 \ldots v_n$ with $gcd(a_1 \ldots a_n) = a_1v_1 + \cdots + a_nv_n$ Therefore for some $d \in \mathbb{Z}$ where $d \cdot gcd(a_1 \ldots a_n) = c$ i.e. $a_1 \cdot d \cdot v_1 + \ldots a_n \cdot d \cdot v_n = c$. Thus a solution exists.

Q4c: We want to find a solution to $2x + 3y + 5z = 1$. By above, a solution will exist since 2,3,5 are coprime. We first set $y = 1$. This reduces the equation to $2x + 5y = -2$. Now we can choose an even y to proceed with finding x. If we choose $y = 2$, then we have that $2x = -12$. This is solved by settings $x = -6$.