

Q1i: This is false, consider the groups $C_2 \times C_2$. We showed that this is not isomorphic to C_4 on assignment 4 Q3.

Q1ii: We define a map $\varphi : C_{10} \rightarrow C_2 \times C_5$ by $\varphi([x]_{10}) = ([x]_2, [x]_5)$. From the properties of mod n equivalence classes, we know that this is a homeomorphism. It remains to show that it is an injection between the two groups. Suppose that for some $x \in C_{10}$, $\varphi(x) = e$. This is equivalent to $([0]_2, [0]_5) = ([x]_2, [x]_5)$. By the chinese remainder theorem, we must have that $x = [0]_{10}$. Hence φ is an injection between 2 groups of the same size i.e. it is an isomorphism.