

Q2i: It is sufficient to show that for any  $a, b \in L$ , we have that  $ab^{-1} \in L$ . We write  $a = h_1k_1$  and  $b = h_2k_2$  and compute that

$$\begin{aligned}
 ab^{-1} &= h_1k_1k_2^{-1}h_2^{-1} \\
 &= h_1ek_1ek_2^{-1}h_2^{-1} \\
 &= h_1(h_2^{-1}h_2)k_1(h_2^{-1}h_2)k_2^{-1}h_2^{-1} \\
 &= (h_1h_2^{-1})(h_2k_1h_2^{-1})(h_2k_2^{-1}h_2^{-1}) \quad (\text{by generalized associativity})
 \end{aligned}$$

We have that  $h_1h_2^{-1} \in H$  and  $(h_2k_1h_2^{-1})(h_2k_2^{-1}h_2^{-1}) \in K$  since  $h_2 \in \text{Norm}_G(K)$ . We conclude that  $L$  is a subgroup

Q2ii: Let  $l = hk \in L$ . We evaluate that

$$\begin{aligned}
 Kl &= Khk \\
 &= hKk && (\text{since } h \text{ is in } \text{Norm}_G(K)) \\
 &= hK && (\text{since } kK = K) \\
 &= gkK && (\text{since } K = kK) \\
 &= lK
 \end{aligned}$$

Hence  $K$  is normal in  $L$ .