

Q3a: Suppose *WLOG* that $f'(a) > 0$ for all $a \in \mathbb{R}$. Suppose not, that is suppose that $f'(a) > 0$ yet f not injective. Then there must exist some distinct a, b and $a < b \in \mathbb{R}$ where $f(a) = f(b)$. Then by the mean value theorem there must exist some $c \in (a, b)$ such that

$$f'(c) = \frac{f(a) - f(b)}{a - b}$$

By our assumption it must be that $f'(c) = 0$. We obtain a contradiction.

Q3b:

From Spivak Theorem 2-7 we have that

$$f'(x, y) = \begin{bmatrix} e^x \cos(y) & -e^x \sin(y) \\ e^x \sin(y) & e^x \cos(y) \end{bmatrix}$$

Computing the determinant, we have $\text{Det}(f'(x, y)) = e^{2x} \cos^2(y) + e^{2x} \sin^2(y) = e^{2x}$. This is always nonzero hence f' is invertible for all $(x, y) \in \mathbb{R}^2$. However the function f is not injective since for any choices of (x, y) , by the periodicity of \sin and \cos we will have

$$f(x, y) = (e^x \cos(y), e^x \sin(y)) = (e^x \cos(y + 2\pi), e^x \sin(y + 2\pi)) = f(x, y + 2\pi)$$