Q3: Let $c_r(t) = (r\cos(2\pi t), r\sin(2\pi t))$, defined on $[0,1]$. Let $c_r^*\omega = f dt$. We have that $f(0) = f(1)$ so by Q2 there exists some λ_r and g so that $dg = c^*\omega - \lambda_r dt$. We see that $c^{-1}(x,y) = \frac{1}{2\pi}\theta(x,y)$. If we apply the pullback of c_r^{-1} we see that

$$d(c_r^{-1^*} g) = c_r^{-1^*} c_r^*\omega - \lambda_r d(c^{-1^*} t) = (c \circ c^{-1})^*\omega - \frac{\lambda_r}{2\pi} d(\theta(x,y)) = \omega - \frac{\lambda_r}{2\pi} \cdot \frac{-ydx + xdy}{x^2 + y^2}$$

We know claim that such a λ_r is in fact unique. Suppose that there is distinct λ_1, λ_2 and g_1, g_2 where $dg_1 = \omega - \lambda_1\eta$ and $dg_2 = \omega - \lambda_2\eta$. We define the one form h as

$$h = g_1 - g_2 = (\lambda_2 - \lambda_1)\eta$$

Let c be any 1-chain, By Stoke's theorem, we compute the integral

$$\int_{\partial c} h = \int_c dh = 0$$

Therefore, we have that for all chains, $(\lambda_2 - \lambda_1)\eta = 0$. Therefore, $\lambda_2 - \lambda_1 = 0$. We conclude that λ is unique.