Q3i: Suppose that $H \triangleleft^c K \triangleleft G$. Note that for any $g \in G$, we have that $K = gKg^{-1}$. Hence conjugation by an element of G preserves K i.e. it is an outer automorphism. Furthermore since H is characteristic, we have that conjugation by an element of G must also preserve it. Hence $gHg^{-1} = H$.

Q3ii: Consider the chain of groups $\{e, (12)(34)\} \triangleleft A_4 \triangleleft^c S_4$. We first claim that this is true. Since any automorphism of A_4 is a conjugation by an element of S_4, we have that $A_4 \triangleleft^c S_4$. We have previously shown that $\{e, (12)(34)\}$ is normal in A_4 but not normal in S_4 by A3Q3. Thus we are done.