5.4.52a: The inclusion $M \subset N^0$ is clear since if $Tx = 0$ each f_i is certainly 0. Now suppose that $\varphi \in N^0$. Consider the mapping into C^{n+1} defined by $x \mapsto (\varphi(x), T(x))$. The image of this map must be an n dimensional subspace. By Hahn-Banach there exists a linear functional $g \in (\mathbb{C}^{n+1})^*$ that is 0 on the image of (φ, T) and nonzero on the remaining 1-d subspace. If $v_1 \ldots v_n$ is a basis for the image of (φ, T) and v_{n+1} is a basis for the subspace on which g is nonzer0, we have that

$$0 = g(\varphi(x), f_1(x), \ldots f_n(x)) = g(e_{n+1})\varphi(x) + \sum_i f_i(x) g(e_i) \implies \varphi(x) = -\sum_i \frac{g(e_i)}{g(e_{n+1}) f_i(x)}.$$

Thus φ is in the span. By exercise 23, we have that $M^* \cong (\mathfrak{X}/N)^*$.

5.4.52b: We have that the projection map $\pi_1 : \mathfrak{X} \to M$ is an isometric embedding by 5.4.23. Therefore so is $\pi_2 : M^* \to (\mathfrak{X}/N)^{**}$. Take $\varepsilon > 0$.