Q1:

We first note that $1 - \overline{z}w$ and $1 - z\overline{w}$ are complex conjugates of each other, hence they have identical modulus. Using properties of the modulus we rewrite the lefthand side of the equality as such.

$$1 - \left| \frac{z-w}{1-z\overline{w}} \right|^2 = 1 - \frac{|z-w|^2}{|1-z\overline{w}|^2} = \frac{|1-z\overline{w}|^2 - |z-w|^2}{|1-z\overline{w}|^2}$$

Since the denominators are equal and nonzero, it is sufficient to show that the numerators of the given equality are indeed equal to each other. We compute that:

$$\begin{aligned} |1-z\overline{w}|^2 - |z-w|^2 &= (1-z\overline{w})(1-\overline{z}w) - (z-w)(\overline{z}-\overline{w}) \\ &= 1 - \overline{z}w - z\overline{w} + \overline{z}z\overline{w}w - \overline{z}z + z\overline{w} + \overline{z}w - \overline{w}w \\ &= 1 - |z|^2 - |w|^2 - |z|^2|w|^2 \\ &= (1-|z|^2)(1-|w|^2) \end{aligned}$$

As desired. This concludes the proof.