4a:
From the definition of $D_{e_i}f(a)$ we have that

$$D_{e_i}f(a) = \lim_{h\to 0} \frac{f(a+he_i) - f(a)}{h}$$
$$= \lim_{h\to 0} \frac{f(a_1, \dots a_i + h, a_{i+1} \dots a_n) - f(a)}{h}$$
$$= D_i f(a) \text{ by the definition of partial derivative}$$

4b:
Define the function h as $h(z,t) = f(a+ztx)$ for some points a and x. We have that

$$\frac{\partial h}{\partial z}(0,t) = \lim_{s\to 0} \frac{h(s,t) - h(0,t)}{s}$$
$$= \lim_{s\to 0} \frac{f(a+stx) - f(a)}{s}$$
$$= D_{tx}f(a)$$

Now define $g(k) = f(a+kx)$. Letting $k(t) = zt$ for some $z \in \mathbb{R}$, we have $g(zt) = f(a+ztx)$. Clearly $g = h$ and so $\frac{\partial g}{\partial z} = \frac{\partial h}{\partial z}$. So by the chain rule we have that

$$\frac{\partial g}{\partial z}(0,t)$$
$$= \frac{\partial g}{\partial k} \cdot \frac{\partial k}{\partial z}$$
$$= \lim_{m\to 0} \frac{g(m) - g(0)}{m} \cdot \frac{\partial(z\cdot t)}{\partial z}$$
$$= \lim_{m\to 0} \frac{f(a+mx) - f(a)}{m} \cdot t$$
$$= D_x f(a) t$$

Therefore we have that $D_{tx}f(a) = D_x f(a) \cdot t$
4c:
Let $z = a + kx$ for fixed $a, x \in \mathbb{R}^n$ and for $k \in \mathbb{R}$. The composition of differntiable functions is differntiable, so from chain rule we have that

$$\frac{\partial f}{\partial k}$$
$$= f'(z) \cdot \frac{\partial z}{\partial k}$$
$$= Df(z) \cdot x$$

Evaluating at $k = 0$ we have that $\frac{\partial f}{\partial k} = Df(a) \cdot x$. We now want to compute $\frac{\partial f}{\partial k}$ at $k = 0$ with limits.

$$\frac{\partial f}{\partial k} = \lim_{h\to 0} \frac{f(a+hx) - f(a)}{h}$$
$$= D_x f(a) \text{ (by definition)}$$

So we have that $D_x f(a) = Df(a) \cdot x$. By linearity,

$$D_{x+y}f(a) = Df(a)\cdot(x+y) = Df(a)\cdot x + Df(a)\cdot y = D_x f(a) + D_y f(a)$$