Q2a: We wish to solve the differential system of equations $X' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} X$. We first notice that the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 3$, with corresponding eigenvectors $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Therefore the general solution takes the form of

$$X(t) = \alpha e^{t} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta e^{3t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

for $\alpha, \beta \in \mathbb{R}$. This will correspond to the slope field 4.

Q2b: We wish to solve $X' = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} X$. We see that the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ has eigenvalues $\lambda_1 = 0$ corresponding to $v_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 7$ corresponding to $v_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ Therefore, the general solution will be of the form

$$X(t) = \alpha \cdot \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \beta e^{7t} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

for $\alpha, \beta \in \mathbb{R}$. This will correspond to slope field 2.

Q2c: To solve the system $X' = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} X$ we will compute the eigenvalues and the corresponding eigenvectors. We see that $\lambda_1 = 2$ corresponding to $v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = -1$ corresponding to $v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Therefore the solution to this ODE is

$$X(t) = \alpha e^{2t} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \beta e^{-1} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

for $\alpha, \beta \in \mathbb{R}$. This will correspond to slope field 1.

Q2d: To solve the system $X' = \begin{pmatrix} 1 & 2 \\ 3 & -3 \end{pmatrix} X$ we will compute the eigenvectors and eigenvalues of $\begin{pmatrix} 1 & 2 \\ 3 & -3 \end{pmatrix}$. We see that $\lambda_1 = -1 + \sqrt{10}$ coresponding to $v_1 = \begin{pmatrix} 2 + \sqrt{10} \\ 3 \end{pmatrix}$, and $\lambda_2 = -1\sqrt{10}$ with $v_2 = \begin{pmatrix} 2 - \sqrt{10} \\ 3 \end{pmatrix}$ Therefore the solution to this ODE will be

$$X(t) = \alpha e^{(-1+\sqrt{10})t} \cdot \begin{pmatrix} 2 + \sqrt{10} \\ 3 \end{pmatrix} + \beta e^{(-1-\sqrt{10})t} \cdot \begin{pmatrix} 2 - \sqrt{10} \\ 3 \end{pmatrix}$$

for $\alpha, \beta \in \mathbb{R}$. This will correspond to slope field 3.