

Q1: First note that $|f_n - f| \leq |f_n| + |f| \leq 2|g|$. If we define $E_n(k) = \bigcup_{m=n}^{\infty} \{x : |f_m - f| \geq \frac{1}{k}\}$, and $A(k) = \{x : 2|g| \geq \frac{1}{k}\}$. We see that $A(k) \supset E_n(k)$. We have that for fixed k , $E_n(k)$ is a decreasing sequence. We wish to show that $\mu(E_n(k))$ is of finite measure. Applying Markov's inequality to $A(k)$, we get that

$$\mu(A(k)) \leq k \int_X 2|g|$$

Since $g \in L^1$ we have that $\mu(A(k)) < \infty$. Therefore we get that $\mu(E_1(k)) < \infty$. Since $\bigcap_{n=1}^{\infty} E_n(k) = \emptyset$, measure continuity implies that $\mu(E_n(k)) \rightarrow 0$ as $n \rightarrow \infty$. Given $\varepsilon > 0$, and $k \in \mathbb{N}$, choose n_k sufficiently large so that $\mu(E_{n_k}(k)) < \varepsilon 2^{-k}$. Let $E = \bigcap_{k=1}^{\infty} E_{n_k}(k)$. We get that $\mu(E) < \varepsilon$ and $|f_n(x) - f(x)| < \frac{1}{k}$ for $n > n_k$ and $x \notin E$. Therefore $f_n \rightarrow f$ uniformly on E^c .