Q4: Given that a ring $\mathcal{R}$ has a euclidean division algorithm with the zero norm, we can write and a, b as

$$a = qb + r.$$

The definition of the division algorithm forces that $r = 0$. This means that we can write $a = qb$ for any a, b for some $q \in \mathcal{R}$. Taking $a = 1$ means that we can write $1 = qb$. Hence every element has an inverse. Thus $\mathcal{R}$ is a field.