

Q7: We have that if

$$w = z - \sqrt{z^2 - 1}$$

Therefore,

$$w - z = -\sqrt{z^2 - 1} \implies w^2 - 2zw + 1 = 0.$$

So we consider the space $X = \{(z, w) : w^2 - 2zw + 1 = 0\} \subset \mathbb{C}^2$. This takes on multiple values on the interval $[-1, 1]$. Hence we can construct a Riemann surface on which w is single valued by taking 2 copies of \mathbb{C} and identifying them on the interval $[-1, 1]$. We identify the point -1 with 1 and vice versa. This will ensure that w lifts to a single valued function.