Q3: By Egorov's Theorem, there must exist some set E_k such that $\mu(X \setminus E_k) < \frac{1}{k}$ and $f_n \to 0$ uniformly on E_k. We define the set

$$E = \bigcup_{k=1}^{\infty} E_k$$

Since $X \setminus E \subset X \setminus E_k$ for all k, we have that $\mu(X \setminus E) = 0$. Since the f_n's converge to 0 on E, for any $m \in \mathbb{N}$, there is some $n_m \in \mathbb{N}$ that satisfies

$$|f_n(x)| < \frac{1}{2^m}$$

for all $x \in X$ and $n \geq n_m$. We can assume that each $n_m \leq n_{m+1}$ since we can choose the minimum such n_m satisfying the above. We define $C_n = m$ for $n_m \leq n < n_{m+1}$. We have that C_n will approach ∞, since if it were not then it would be constant at some point and $|f_n(x)|$ would not approach 0. Now if we take $\varepsilon > 0$ and chose a m sufficiently large so that $m2^{-m} < \varepsilon$. Take $n \geq n_m$, and $l \geq m$ such that $n_l \leq n \leq n_{l+1}$. Therefore if $x \in E$, we get that

$$|C_n f_n(x)| = |l f_n(x)| \leq l2^{-l} \leq m2^{-m} < \varepsilon$$