Q2a: Let $f(z)$ be a holomorphic funtion. Since every open set is the union of open balls, it is enough to show that f maps an open ball to an open set. Note that f must be analytic since it is holomorphic. Therefore the zeros of f' are isolated. Every open set either contains a zero of f', or it does not. If we take an open set U so that $f'(z) \neq 0$ on U, then the inverse function theorem tells us that $f|U$ is a bijection and hence is open when restricted to this set. Now take some open V such that it contains a zero of f', z_0. We have that $V \setminus \{z_0\}$ is open, and so $f(V \setminus \{z_0\})$ is open by the previous case. Continuity of f tells us that $f(z_0) \in int(f(V))$. Therefore $f(V)$ is open. Any open set can be written as the union of open sets either containing a zero of f' or not. Therefore f is an open mapping.

Q2b: Clearly $f(U) \subset V$. We show that $V \subset f(V)$. Suppose $x \in V$ such that $f^{-1}(\{x\}) = \emptyset$. Define $X = \overline{B_\varepsilon(x)}$. We have that $f^{-1}(X)$ is compact and nonempty. Take $\{K_n\}$ to be a family of nested decreasing compact sets, whose intersection is x. We have that for each n, $f^{-1}(K_n)$ is compact and the sequence $\{f^{-1}(K_n)\}$ is also a decreasing family of nested compact sets. From basic topology, the intersection $\bigcap_n f^{-1}(K_n)$ must be nonempty. It follows that $f^{-1}(\{x\})$ is nonempty. A contradiction. Therefore $f(U) = V$.

Q2c: Consider the mapping $f(z) : \mathbb{C} \to \mathbb{C}$ defined as $f(z) = |z|$. This will be a continuous but not holomorphic mapping. Note that $f(\mathbb{C}) = \mathbb{R}_{x \geq 0}$, which is not an open set.