Q6i: If $G = \mathbb{Z}/12\,\mathbb{Z}$, we claim that the only subgroups of order 2 and 3 are $H_1 = \{0, 6\}$ and $H_2 = \{0, 4, 8\}$ respectively. We claim that these are the only subgroups of order 2 and 3 respectively. First note that in any order 2 subgroup of G, we must have that $e = 0$ belongs to the subgroup, and some other element a satisfying $a = a^{-1}$. Hence we have that

$$12 = a + a^{-1} = a + a \implies a = 6$$

So a subgroup of order 2 must be $\{0, 6\}$. Now consider a subgroup of order 3. It must clearly contain the identity, and two other elements that are inverses of eachother. If we have that $H = \{0, a, b\}$, we know that $a^{-1} = b$, but also that $a + a = b$. This implies that

$$12 = a + b = a + a + a = 3a \implies a = 4$$

Therefore the only subgroups of G with order 2 and 3 are H_1 and H_2.

Q6ii: Sicne 2 does not divide 45, we have that there does not exist a subgroup of order 2 in $G = \mathbb{Z}/45\,\mathbb{Z}$. We claim that the only subgroup of G with order 3 is $H = \{0, 15, 30\}$. The proof that this is the only subgroup of order 3 follows the same procedure as in Q6ii. We conclude that H is the only subgroup of G.

Q6iii: Let $G = D_{12}$, We claim that the only subgroups of order 2 are $H_1 = \{e, \rho^3\}$, $H_2 = \{e, \sigma\}$ and $H_3 = \{e, \sigma\rho^3\}$. It is clear that these are the only such subgroups, since they must be generated by an element with order 2. It is clear that these are the only subgroups of order 2 since they are generated by the only elements of D_{12} that have an order of 2. We now claim that the only subgroup of order 3, is $H_4 = \{e, \rho^2, \rho^4\}$. We know that σ can not belong to a subgroup with odd order, since $|\sigma| = 2$. Hence it must be be generated by only rotations. Clearly the only rotation satisfying $a^3 = e$ is $a = \rho^2$.

Q6iv: Let $G = D_{10}$. We know by lagranges theorem that there does not exist a subgroup of order 3, since 3 does not divide 10. Thus there may only exist subgroups of order 2. We claim that the only subgroups of order 2 in D_{10} is $H_1 = \{e, \sigma\}$. We require that a subgroup of order 2 must be generated by an element with order 2. Clearly σ satisfies this. No other element of D_{10} can satisfy this, since a pentagon has an odd number of sides, hence no rotation or rotation and reflection can be applied successively to obtain the identity transformation.