

Q6b: We will argue by contrapositive. Suppose that $f^{-1}(-\infty, a)$ is not open. Then there exists some $x \in f^{-1}(-\infty, a)$ such that for all $\varepsilon > 0$, there is some $y \in M_\varepsilon(x)$ with $y \notin f^{-1}(-\infty, a)$. Thus define $\{y_k\}$ to be a decreasing sequence of such y 's which satisfy the previous condition and converge to x . Then we have that $\lim_{k \rightarrow \infty} y_k = x$, but $\limsup_{k \rightarrow \infty} f(y_k) \geq f(x)$, since they are not in the preimage they must be greater than a . Now suppose that $f^{-1}(-\infty, a)$ is open. Then for each x , for all $\varepsilon > 0$, we have that the set $f^{-1}(-\infty, f(x) + \varepsilon)$ is open. Since x is in this set, there must be some open ball around it. For all y in this ball, we have that $f(y) < f(x) + \varepsilon$. Since this is true for all ε , if we take a sequence $\{x_k\}$ converging up to x , we have that $\limsup_{k \rightarrow \infty} f(x_k) \leq f(x)$, as desired.