

Q2: We claim that if $f : M \rightarrow N$ is continuous, and M is covering compact, then f is uniformly continuous. Since M is compact, it follows that its image under the continuous f is a compact subset of N . Now consider the following open cover of N . Let $\varepsilon > 0$, define $\mathcal{U} = \{N_{\frac{\varepsilon}{2}}(q) : q \in N\}$. We will denote each open cover corresponding to point q as U_q . Thus by continuity of f , we will have that $\mathcal{V} = \{f^{pre}(U_q) : q \in N\}$ will be an open cover of M . By covering compactness, we can extract a finite subcover corresponding to points q_1, \dots, q_n where U_{q_1}, \dots, U_{q_n} and $(f^{pre}(U_{q_1}), \dots, f^{pre}(U_{q_n}))$ are finite subcovers of N and M , respectively. Since we know that sequential compactness is equivalent to covering compactness, we can apply the lebesgue number lemma to our open cover of M . Hence there exists some $\lambda(\varepsilon) > 0$ with the property that for all $x \in M$, there exists some $f^{pre}(U_{q_x})$ with $M_\lambda(x) \subset f^{pre}(U_{q_x})$. We see that for any $\varepsilon > 0$, we choose $\delta = \lambda(\varepsilon)$. Then for any $x \in M$, we can find a $M_\lambda(x) \subset f^{pre}(U_{q_x})$. If we take any $y \in M_\lambda(x)$, we have that $d_M(x, y) < \delta$. Their image under f will belong to some U_{q_x} and thus $d_N(f(x), f(y)) < \varepsilon$ by the triangle inequality. This is exactly what it means for f to be uniformly continuous.