

Q4: Suppose that there is some inner automorphism ϕ on $GL(n, \mathbb{R})$ such that $\phi(g) = (g^{-1})^t = kgk^{-1}$ for some $k \in GL(n, \mathbb{R})$. Let g be an arbitrary member of $GL(n, \mathbb{R})$. From the properties of the determinant we get that

$$\det(\phi(g)) = \det((g^{-1})^t) = \det(g^{-1}) = \frac{1}{\det(g)}$$

But from the definition of ϕ we see that

$$\det(\phi(g)) = \det(kgk^{-1}) = \det(k) \cdot \det(g) \cdot \det(k^{-1}) = \det(k) \cdot \frac{1}{\det(k)} \cdot \det(g)$$

We have that $\det(g) = \frac{1}{\det(g)}$ which is untrue for invertible matrices with determinant not equal to 1.