Q3: It is given that $|\varphi_n| \leq C$ and $|\varphi| \leq C$. We have that

$$\lim_{n\to\infty} \int |\varphi_n f_n - \varphi f| \leq \lim_{n\to\infty} \int |\varphi_n f_n - \varphi_n f| + \lim_{n\to\infty} \int |\varphi_n f - \varphi f| \leq C \lim_{n\to\infty} \int |f_n - f| + \lim_{n\to\infty} \int |\varphi_n f - \varphi f|$$

Notice that since $f_n \to f$ in $L^1$, we have that $\lim_{n\to\infty} \int |f_n - f| = 0$. Hence we have that

$$\lim_{n\to\infty} \int |\varphi_n f_n - \varphi f| \leq \lim_{n\to\infty} \int |\varphi_n f - \varphi f|$$

We have that

$$|\varphi_n f - \varphi f| \leq |\varphi_n f| + |\varphi f| \leq 2C|f|$$

Since $f \in L^1$, we have that $2C|f|$ is an integrable dominator of $|\varphi_n f - \varphi f|$. Hence we can apply the Dominating Convergence Theorem, and get that

$$\lim_{n\to\infty} \int |\varphi_n f - \varphi f| = \int \lim_{n\to\infty} |\varphi_n f - \varphi f| = 0$$

Which follows by a.e. convergence of $\varphi_n$ to $\varphi$. Therefore we have that $\lim_{n\to\infty} \int |\varphi_n f_n - \varphi f| = 0$, and therefore $\varphi_n f_n \to \varphi f$ almost everywhere. Since $|\varphi f| \leq C|f|$ it follows that $\varphi f \in L^1$.