Q3a, 7.12: First consider the case when $a = -3 \in Q_p$. We see by inspection that $-3 \in Q_2$ and $-3 \in Q_3$. So it is reasonable to assume that $p > 3$. By theorem 7.5, we have that

$$(\frac{-3}{p}) = (\frac{-1}{p})(\frac{3}{p})$$

If it is the case that $p =\equiv 1 \mod 4$, then $(\frac{-1}{p}) = 1$. By example 7.10, $(\frac{3}{p}) = 1$ when $p \equiv 1 \mod 12$. If if $p \equiv 3 \mod 4$, then we have that $(\frac{-1}{p}) = -1$. Once again by example 7.10, $(\frac{3}{p}) = -1$ for $p \equiv 7 \mod 12$. Therefore, $p \equiv 1 \mod 6$. A similar argument can be made for $a = 5$. We get that $(\frac{p}{5}) = 1$ if $p \equiv \pm 1 \mod 5$.For $a = 6$, since $6 \notin Q_3, Q_2$ we can assume that $p > 3$.If $p \equiv 1 \mod 4$, then $(\frac{6}{p}) = 1$ when $(\frac{p}{2}) = (\frac{p}{3})$. So, $p \equiv \pm 1 \mod 24$ or $p \equiv \pm 5 \mod 24$. When $a = 7$, $p = 2$, $p \equiv \pm 1 \mod 28, p \equiv \pm 3 \mod 28, p \equiv \pm 9 \mod 28$. When $a = 10$, we get that $p \equiv \pm 1, 3, 9, 13 \mod 40$. When $a = 169$, $p \neq 13$.

7.21: Notice that $-1 \in Q_n$ by theorem 7.15. This is only true iff $-1 \in Q_{p^e}$, for each $p^e || n$. Note that by thm 7.14, $-1 \in Q_{2^e} \iff e = 0$ or 1. If $p > 2, -1 \in Q_p \iff -1 \in Q_{p^e}$ by theorem 7.13. By cor. 7.7, this is true if and only if $p \equiv 1 \mod 4$. Therefore, $-1 \in Q_n$ if and only if 4 does not divide n or n is not divisible by a prime $p \equiv 3 \mod 4$.

7.22: We can observe that the quantities $\pm\sqrt{q}, \pm\sqrt{r}, \pm\sqrt{qr}$ are not integral. But at least one of $r, q, qr \equiv 1 \mod 8$. Therefore at least one belong to Q_{2^e} for all e, and so $(\frac{q}{r}) = 1$, and so $q \in Q_{r^e}$ for all e. Using the CRT, we want to show that for all prime p,p^e there exists some solution to $f(x) \equiv 0 \mod p^e$. By LQR, $(\frac{r}{q}) = (\frac{q}{r}) = 1$ and so $r \in Q_{p^e}$. By theorem 7.5, if $p \neq 2, p \neq q, p \neq r$ then $(\frac{qr}{p}) = (\frac{q}{p})(\frac{r}{p})$. So at minimum one of q, r, qr belong to Q_{p^e} for all e. So thus $h(x) \equiv 0 \mod n$ has a solution for all n.

7.11. We factor $219 = 3 \cdot 73$ so by thm 7.5,

$$(\frac{219}{383}) = (\frac{3}{383})(\frac{73}{383})$$

We use the LQR to evaluate

$$(\frac{3}{383}) = (-\frac{383}{3}) = -(\frac{2}{3}) = 1$$

Similarly by applying corr. 7.10, we get that

$$(\frac{73}{383}) = 1$$

And so

$$(\frac{219}{383}) = 1$$

and so $219 \in Q_{383}$

7.25 We factor $923 = 13 \cdot 71$ So $43 \equiv 4 \mod 13$ and $43 \equiv 4 \mod 13$ and $43 \in Q_{13}$. Hence by LGR we can write

$$(\frac{43}{71}) = -(\frac{71}{43}) = -(\frac{28}{43}) = -(\frac{7}{43}) = (\frac{43}{7}) = (\frac{1}{7}) = 1$$

And so $43 \in Q_{923}$