

Q5: Let $H \leq G$, $|G| = p^\alpha m$ and $|H| = p^\beta n$. By Sylow's Theorem, there is a $Q \leq H$, which is a Sylow p subgroup of H . By Sylow's Theorem, there exists a p subgroup P of G , and there exists some $g \in G$ such that $Q \leq gPg^{-1}$. Since $Q \subset gPg^{-1} \cap H$, we have that $|gPg^{-1} \cap H| \geq |Q| = p^\beta$. But by Lagrange's theorem we also have that $p^\beta \leq |gPg^{-1} \cap H|$, so $p^\beta \leq |gPg^{-1} \cap H|$. This proves the result.