Q7: Since $\lim_n T_n x$ exists for all x, uniform bounded principle tells us that there is some C so that $\sup_n \|T_n\| \leq C$. Since limits and T_n are linear, the definition of $Tx = \lim_n T_n x$ implies that T is linear. It is sufficient to show that T is continous. Let $\{x_m\}$ be a sequence converging to x. We compute that

$$\|Tx_m - Tx\| \leq \|(Tx_m - T_n x_m) + (T_n x_m - T_n x) + (T_n x - Tx)\| \leq \|Tx_m - T_n x_m\| + \|T_n x_m - T_n x\| + \|T_n x - Tx\| .$$

Uniform boundedness gives us $\|T_n x_m - T_n x\| \leq C \|x_m - x\|$, and the definition of T tells us that the other terms on the right hand side go to 0 for large n, m. Thus $Tx_m \to Tx$ as desired.