Q3: First consider the polynomial $f(z) = z^4 + 3z^2 + 3$. Note that this is a quadratic in z^2, hence we can solve using the quadratic formula, yielding

$$z^2 = \frac{-3 \pm i\sqrt{3}}{2}.$$

Therefore there are 4 roots of $f(z)$, one in each quadrant. By Rouche's theorem it is enough to show that

$$|8z^3 + 8z| < |z^4 + 8z^3 + 3z^2 + 8z + 3|$$

on the right half plane. We can take a large semi circle so that it contains the two roots on the right half, and so that the degree 4 term dominates the degree 3 polynomial. We now claim that on the imaginary axis the inequality holds. We wish to show that

$$|-8iy^3 + 8iy| < |y^4 - 8iy^3 - 3y^2 + 8iy + 3|.$$

We know that

$$|y^4 - 8iy^3 - 3y^2 + 8iy + 3| = |y^4 - 3y^2 + 3| + |-8iy^3 + 8iy|.$$

Since the real part has a strictly positive modulus, the inequality must hold. Hence it follows that $P(z)$ has 2 roots in the right half plane.