5.5.64a: Show that for any x, $\|L_k x\| \to 0$ as $k \to \infty$. Let $x = \sum_n a_n u_n$. We have that

$$\|L_k x\|^2 = \left\| \sum_k a_k^2 \right\| \to 0,$$

since $\sum_{i=1}^{\infty} a_i^2 = \|a\| < \infty$, so the tail end converges to 0. However, in the norm toplogy, $\sup_k \|L_k\| = 1$ since for any L_k, we can always find a vector $x = u_{k+1}$ so $\|L_k x\| = 1$.

5.5.64b: For any linear functional f, we can write

$$f(R_k x) = \langle R_k x, y \rangle$$

for some $y = \sum_i b_i u_i$. If $x = \sum_i a_i u_i$, then we evaluate $f(R_k x)$ as

$$\langle \sum_n a_n u_{n+k}, \sum_n b_n u_n \rangle = \sum_{n=k} a_n b_{n+k}.$$

This converges to 0 since $\lim_{k\to\infty} b_{n+k} \to 0$. R_k does not converge to 0 in the strong operator topology since for any R_k, x, $\|R_k x\| = \|x\|$.

5.5.64c: We see that $\|R_k L_k x\| = \sum_{n=k} a_n^2$ which goes to 0 as $k \to \infty$, since $\|a\| < \infty$. However, $R_k L_k x = L_k \sum_1^{\infty} a_n u_{n+k} = \sum_{n=1}^{\infty} a_n u_n = x$.