Q2: Let $\omega \in \Omega^k(V)$ such that ω is closed. Consider the k-form $f^{-1*}\omega \in \Omega^k(U)$. From the proprties of the exteriour derivative, we have that

$$d(f^{-1*}\omega) = f^{-1*}d(\omega) = 0$$

We get that $f^{-1*}\omega$ is a closed differential form. Hence there exists some $\eta \in \Omega^{k-1}(U)$ such that $d\eta = f^{-1*}\omega$. Pulling back by f^*, we then see that

$$d(f^*\eta) = f^*d\eta = f^* \circ f^{-1^*}\omega = (f^{-1} \circ f)^*\omega = \omega$$

Hence we have that ω is an exact form.