Q4: Consider $G = \mathcal{Q}_8$. As per A1Q4, we have classified all the subgroups of $\mathcal{Q}_8$. We know from properties of multiplication in $\mathcal{Q}_8$ that for any $g \in \mathcal{Q}_8$, $H \leq G$ that $gH = Hg$. For example, we can compute that

$$k\langle j\rangle = \{k, -i, -k, i\}$$
$$\langle j\rangle k = \{k, i, -k, -i\}$$

We see that the left and right cosets are equal, although we know that this group is not commutative. We claim that a subgroup is normal if and only if the left and right cosets coinside. We proceed with the forward implication. Suppose $H \trianglelefteq G$. We see that for all $g \in G$,

$$gHg^{-1} = H \implies gHg^{-1}g = Hg \implies gH = Hg$$

As desired. Now suppose that $H \leq G$, and $gH = Hg$. Applying g^{-1} to the right side we get that $gHg^{-1} = H$. Hence H is normal.