Q2a:
First we consider $h(t)$.

$$\begin{aligned}h(t) &= f(tx)\\ &= \|tx\|\, g(\frac{tx}{\|tx\|})\\ &= |t|\,\|x\|\, g(\frac{tx}{|t|\,\|x\|})\\ &= t\,\|x\|\, g(\frac{x}{\|x\|})\\ &= tf(x)\end{aligned}$$

Therefore, h is linear and so from single variable calculus we get that $h' = f(x)$
b:
First, if $g = 0$ then clearfly f is differntiable with $Df(x, y) = 0$ .If $g(x) \neq 0$ then suppose that $Df(0, 0)$ exists. We see that

$$\begin{aligned}&\lim_{h\to 0} \frac{\|f(h,0) - f(0,0) - Df(0,0)(h,0)\|}{\|h\|} = 0\\ &\implies Df(0,0)(h,0) = 0\end{aligned}$$

and

$$\begin{aligned}&\lim_{k\to 0} \frac{\|f(0,k) - f(0,0) - Df(0,0)(0,k)\|}{\|k\|} = 0\\ &\implies Df(0,0)(0,k) = 0\end{aligned}$$

Therefore, $Df(0, 0) = 0$.So if this is the differntial it must be the case that

$$\begin{aligned}&\lim_{x\to 0} \frac{f(x) - f(0) - Df(0,0)}{\|x\|}\\ &= \lim_{x\to 0} \frac{\|x\|\, g(\frac{x}{\|x\|})}{\|x\|}\\ &= \lim_{x\to 0} g(\frac{x}{\|x\|}) = 0\end{aligned}$$

However by the definition of g this limit does not exist unless $g = 0$. We obtain a contradiction and so $Df(0, 0)$ does not exist unless $g = 0$.