

Q3: For each $x \in C$, we can rewrite it as a base 3 decimal expansion whose entries are either 0 or 2. Denote this by $x = (x_n)$. Consider the mapping $f : C \rightarrow [0, 1]$ defined as

$$f((x_n)_{base3}) = (g(x_1), g(x_2) \dots)_{base2}$$

Where

$$g(x_i) = \begin{cases} 1 & x_i = 2 \\ 0 & x_i = 0 \end{cases}$$

We claim that f is a continuous surjective map. Suppose that $y \in [0, 1]$. Then it has some decimal expansion of the form $y = (a_n)$. In base 2 this will be a string of 0's and 1's. If we take $x \in C$ to have a 2 where ever there is a 1 in the decimal expansion of y , with the 0's remaining unchanged, we have that $f(x) = y$. We now claim that such an f is continuous. Notice that f can be represented as the composition of several continuous maps. First, we swap every 2 to a 1, then we convert from base 3 to base 2. Converting between number bases amounts to addition and multiplication, and thus is continuous. It remains to show that swapping 2's to 1's in a trinary number is a continuous function. Let h be the function on the Cantor set which changes a 2 to a 1. Since each element in the Cantor set has a trinary expansion with either a 2 or a 0, then $h(x) = \frac{x}{2}$. Thus h is division by a nonzero and hence is continuous.