Q4: Take γ to be sufficiently large. Using Cauchy's Integral formula, we can write $g(z)$ as

$$g(z) = \frac{1}{2\pi i}\sum_{m=0}^{\infty} z^m \int_\gamma \frac{f(\zeta)}{\zeta^{m+1}} d\zeta - \frac{1}{2\pi i}\int_\gamma \frac{f(\zeta)}{\zeta - z}\cdot\frac{z^n}{\zeta^n}$$

We can write

$$\frac{1}{\zeta - z} = \frac{1}{\zeta}\sum_{m=0}^{\infty}\frac{z^m}{\zeta^m}.$$

We have that $g(z)$ becomes

$$g(z) = \frac{1}{2\pi i}\sum_{m=0}^{\infty} z^m \int_\gamma \frac{f(\zeta)}{\zeta^{m+1}} d\zeta - \frac{1}{2\pi i}\sum_{m=n}^{\infty} z^m \int_\gamma \frac{f(\zeta)}{\zeta^{m+1}} d\zeta = \frac{1}{2\pi i}\Big(\sum_{m=0}^{n-1} z^m \int_\gamma \frac{f(\zeta)}{\zeta^{m+1}} d\zeta\Big)$$

Which is clearly a polynomial of degree $n-1$. Using Cauchy's Integral Formula to construct a power series for f, we have that for $k \leq n-1$,

$$g^{(k)}(0) = \frac{k!}{2\pi i}\int_\gamma \frac{f(\zeta)}{\zeta^{k+1}} d\zeta = f^{(k)}(0).$$

As desired.