Q1a: Suppose that $R(T)$ is closed. By contiuity we have that we have that $N(T)$. Therefore $E \backslash N(T) \cong R(T)$. T factors as an isomorphism composed with a projection, $\tilde{T} \circ \pi$. Therefore we have that

$$d(x, N(T)) \leq \|x + N(T)\| = \left\|\tilde{T}^{-1}\tilde{T}(x + N(T))\right\| \leq C \left\|\tilde{T}(x + N(T))\right\| = C \left\|T(x)\right\|.$$

Now suppose that for some C we have

$$d(x, N(T)) \leq C \left\|T(x)\right\|.$$

Let $\{x_n\}$ be a sequence such that , $T(x_n) \to y$. Then,

$$\|x_n - x_m\| \leq C \left\|T(x_n) - T(x_m)\right\| \to 0$$

as $n, m \to \infty$. Therefore by completeness $x_n \to x$ and so $T(x_n) \to T(x)$. Thus $R(T)$ is closed.

1b: Define the mapping

$$T : G \times L, (x, y) \mapsto x - y.$$

This has kernel $G \cap L$ so we apply $1a$ to conclude that $T(G, L)$ is closed. Equivalently we have that $G + L$ is closed.