

Q5: Using the binomial theorem, we expand:

$$\begin{aligned}
 \left(1 + \frac{z}{n}\right)^n &= \sum_{k=0}^n \binom{n}{k} \cdot 1^{n-k} \frac{z^k}{n^k} \\
 &= \sum_{k=0}^n \frac{n(n-1) \cdots (n-k+1)(n-k)!}{k! \cdot (n-k)!} \frac{z^k}{n^k} \\
 &= \sum_{k=0}^n \frac{n(n-1) \cdots (n-k+1)}{n^k} \cdot \frac{z^k}{k!} \\
 &= 1 + z + \sum_{k=2}^n \frac{n(n-1) \cdots (n-k+1)}{n^k} \cdot \frac{z^k}{k!} \\
 &= 1 + z + \sum_{k=2}^n \left(1 - \frac{1}{n}\right) \cdots \left(1 - \frac{k-1}{n}\right) \frac{z^k}{k!}
 \end{aligned}$$

As desired. Taking the limits, we get that

$$\lim_{n \rightarrow \infty} \left(1 + \frac{z}{n}\right)^n = \lim_{n \rightarrow \infty} 1 + z + \sum_{k=2}^n \left(1 - \frac{1}{n}\right) \cdots \left(1 - \frac{k-1}{n}\right) \frac{z^k}{k!} = \sum_{k=0}^{\infty} \frac{z^k}{k!}$$

Since as n get arbitrarily large, the partial sums approach $\sum_{k=0}^{\infty} \frac{z^k}{k!}$.