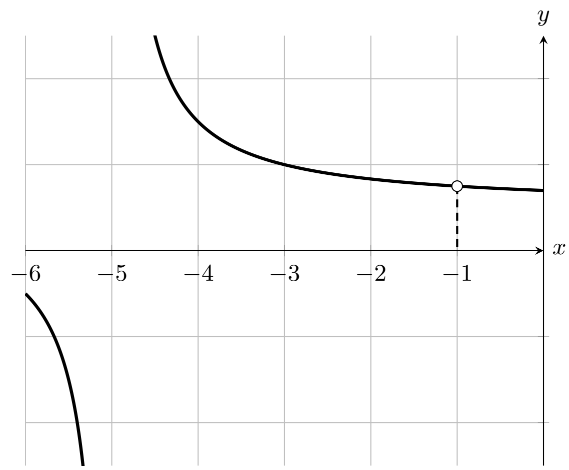
Exercise 1 *Find*

$$\lim_{x \to -1} \left(\frac{x^2 + 8x + 7}{x^2 + 6x + 5} \right) = \boxed{1.5}.$$

Hint: *This function is* <u>*not*</u> *continuous everywhere, but both the numerator and denominator are continuous everywhere as functions. Thus, if the limit of* $\frac{x^2+8x+7}{x^2+6x+5}$ *as* $x \to a$ *does not exist, then the denominator* $x^2 + 6x + 5$ *must be zero at* a*. Does* $x^2 + 6x + 5 = 0$ *when* $x = -1$*? Does* $x^2 + 8x + 7 = 0$ *at* $x = -1$ *as well?*

Hint: *Take a look at the graph of the function*



There is a removable discontinuity at $x = -1$*. This suggests something about the factorization of both polynomials* $x^2 + 8x + 7$ *and* $x^2 + 6x + 5$*. Recall that if both* $\lim_{x \to a} f(x)$ *and* $\lim_{x \to a} g(x)$ *exist, then, if* $\lim_{x \to a} g(x) \neq 0$ *, then* $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$*.*

Hint: *Notice that the quadratic equation tells us that* $x^2 + 8x + 7 = 0$ *has solutions* -4 ± 3 *and* $x^2 + 6x + 5 = 0$ *has solutions* -3 ± 2*. Thus,* $x^2 + 8x + 7 = (x+1)(x+7)$ *and* $x^2 + 6x + 5 = (x+1)(x+5)$*. Then for all* $x \neq -1$*,* $\frac{x^2+8x+7}{x^2+6x+5} = \frac{x+7}{x+5}$*, upon canceling the common factor of* $(x+1)$*. Since we are not asking what value* $\frac{x^2+8x+7}{x^2+6x+5}$ *takes at* $x = -1$*, but rather what value* $\frac{x^2+8x+7}{x^2+6x+5}$ <u>*approaches*</u> *as* $x \to -1$*, it suffices in every case to look at* $\lim_{x \to -1} \left(\frac{x+7}{x+5} \right)$*. We see that* $\lim_{x \to -1} (x+7) = 6$ *while* $\lim_{x \to -1} (x+5) = 4$*, and since* $\lim_{x \to -1} (x+5) \neq 0$*,* $\lim_{x \to -1} \left(\frac{x+7}{x+5} \right) = \frac{\lim_{x \to -1} (x+7)}{\lim_{x \to -1} (x+5)} = \frac{3}{2}$*. Or, in decimal form, 1.5.*