Exercise 1 *It is a well known fact that* $\lim_{x\to\infty}\left(\left(1+\frac{1}{x}\right)^x\right) = \lim_{x\to 0}\left((1+x)^{1/x}\right) = e$. *Find*

$$\lim_{x\to 0}\left(\frac{\ln(1+x)}{x}\right) = \boxed{1}.$$

Hint: *Recall that* $\lim_{x\to\infty}\left(\left(1+\frac{1}{x}\right)^x\right) = e$ *and, equivalently,* $\lim_{x\to 0}\left((1+x)^{1/x}\right) = e$. *Observe that we can rewrite this in a more illuminating way as* $\frac{\ln(1+x)}{x} = x^{-1}\ln(1+x)$. *Apply some properties of logarithms to this.*

Hint: *We know, from the properties of logarithms, that* $b\ln(a) = \ln(a^b)$ *when* $a > 0$. *Hence,* $x^{-1}\ln(1+x) = \ln((1+x)^{1/x})$ *for* $x > -1$ *(and, of course,* $x^{-1} = \frac{1}{x}$*). Hence,* $\ln((1+x)^{1/x}) = \frac{\ln(1+x)}{x}$ *when* $x \neq 0$. *So* $\lim_{x\to 0}\left(\ln((1+x)^{1/x})\right) = \lim_{x\to 0}\left(\frac{\ln(1+x)}{x}\right)$. *Recall that* $\ln(x)$ *is continuous everywhere besides* $x = 0$. *What can you say about* $\lim_{x\to 0}\ln((1+x)^{1/x})$*?*

Hint: *Let* $y = (1+x)^{1/x}$. *Then, since* $\lim_{x\to 0}\left((1+x)^{1/x}\right) = e$, *and* $\ln(x)$ *is continuous at* e, *it follows that* $\lim_{x\to 0}\left(\ln((1+x)^{1/x})\right) = \lim_{y\to e}(\ln(y)) = 1$ *and, hence, that* $\lim_{x\to 0}\left(\frac{\ln(1+x)}{x}\right) = 1$.