

Exercise 1 Find

$$\lim_{x \rightarrow 0} \left(\frac{\sin(5x)}{8x} \right) = \boxed{5/8}.$$

Hint: Recall that $\lim_{x \rightarrow 0} \left(\frac{\sin(x)}{x} \right) = 1$. Multiplying $\frac{\sin(5x)}{8x}$ by a clever form of 1, namely, $\frac{5}{5}$, gives us $\frac{5}{8} \cdot \left(\frac{\sin(5x)}{5x} \right) = \frac{\sin(5x)}{8x}$. Thus, $\lim_{x \rightarrow 0} \left(\frac{5}{8} \cdot \left(\frac{\sin(5x)}{5x} \right) \right) = \lim_{x \rightarrow 0} \left(\frac{\sin(5x)}{8x} \right)$. What can you say about $\lim_{x \rightarrow 0} \left(\frac{5}{8} \cdot \left(\frac{\sin(5x)}{5x} \right) \right)$?

Hint: Let $y = 5x$. Then since $\lim_{x \rightarrow 0} \left(\frac{5}{8} \cdot \left(\frac{\sin(5x)}{5x} \right) \right) = \lim_{y \rightarrow 0} \left(\frac{5}{8} \cdot \left(\frac{\sin(y)}{y} \right) \right)$, and we know that $\lim_{y \rightarrow 0} \left(\frac{\sin(y)}{y} \right) = 1$, applying the limit laws yields $\lim_{y \rightarrow 0} \left(\frac{5}{8} \cdot \left(\frac{\sin(y)}{y} \right) \right) = \frac{5}{8} \cdot \lim_{y \rightarrow 0} \left(\frac{\sin(y)}{y} \right) = \frac{5}{8}$.
