

Exercise 1 Find

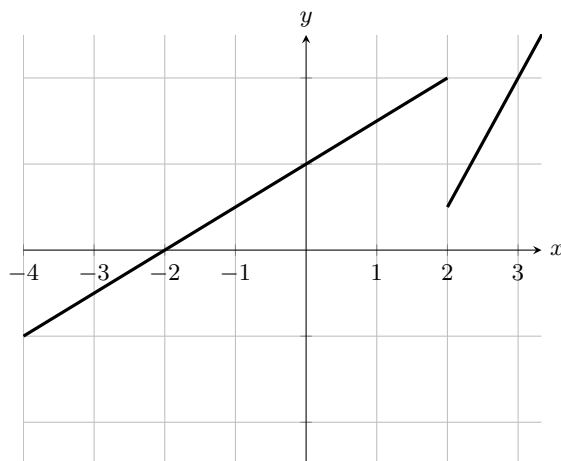
$$\lim_{x \rightarrow 2} f(x) = \boxed{DNE}.$$

where

$$f(x) = \begin{cases} x + 2 & x \leq 2, \\ 3x - 5 & x > 2. \end{cases}$$

Hint: Both pieces of $f(x)$, $x + 2$, for $x \leq 2$, and $3x - 5$, for $x > 2$, are continuous for all x . However, for the limit $\lim_{x \rightarrow 2} f(x)$ to exist, both the left-hand and the right-hand limits of $f(x)$ at 2 must exist and be equal.

Hint: Take a look at the graph of the function



Hint: Evaluating $\lim_{x \rightarrow 2^+} f(x)$ we see that it is equal to 1. This follows because, for $x > 2$, we are on the piece of $f(x)$ given by $3x - 5$ and the limit $\lim_{x \rightarrow 2} (3x - 5) = 3 \cdot \lim_{x \rightarrow 2} (x) - \lim_{x \rightarrow 2} (5) = 1$, certainly. On the other hand, evaluating $\lim_{x \rightarrow 2^-} f(x)$ we see it is equal to 4. This follows because, for $x \leq 2$, we are on the piece of $f(x)$ given by $x + 2$ and the limit $\lim_{x \rightarrow 2} (x + 2) = \lim_{x \rightarrow 2} (x) + \lim_{x \rightarrow 2} (2) = 4$, certainly. These are not equal, so $\lim_{x \rightarrow 2} f(x)$ does not exist.