

Exercise 1 Find

$$\lim_{x \rightarrow 0} \left(\frac{\cos^2(x) - 1}{\cos(x) - 1} \right) = \boxed{2}$$

Hint: Recall that for any two numbers, a and b , $a^2 - b^2 = (a - b)(a + b)$. Factor the numerator of the expression in this manner.

Hint: $\cos^2(x) - 1 = (\cos(x) - 1)(\cos(x) + 1)$ so $\frac{(\cos(x) - 1)(\cos(x) + 1)}{\cos(x) - 1} = \cos(x) + 1$ for $x \neq 2\pi n$ for any integer n (since, if $x = 2\pi n$, then $\cos(2\pi n) - 1 = 0$ in the denominator) and $\lim_{x \rightarrow 0} \left(\frac{\cos^2(x) - 1}{\cos(x) - 1} \right) = \lim_{x \rightarrow 0} (\cos(x) + 1)$ and this is $\lim_{x \rightarrow 0} (\cos(x)) + \lim_{x \rightarrow 0} (1) = 2$.
