Exercise 1 *Find*

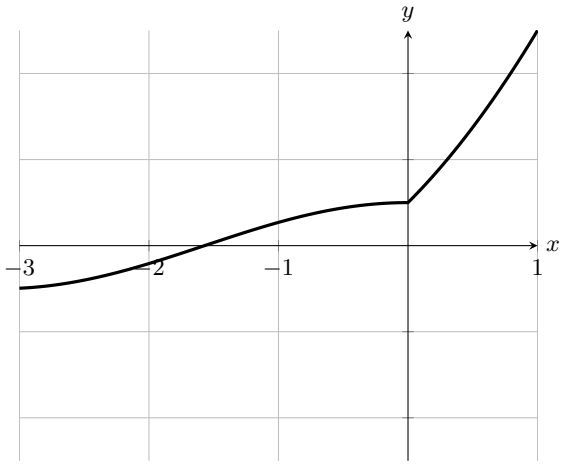
$$\lim_{x\to 0} f(x) = \boxed{1}.$$

where

$$f(x) = \begin{cases} \cos(x) & x \leq 0, \\ x^2 + 3x + 1 & x > 0. \end{cases}$$

Hint: *Both pieces of $f(x)$, $\cos(x)$, for $x \leq 0$, and $x^2 + 3x + 1$, for $x > 0$, are continuous for all x. However, for the limit $\lim_{x\to 0} f(x)$ to exist, both the left-hand and the right-hand limits of $f(x)$ at 0 must exist and be equal.*

Hint: *Take a look at the graph of the function*



Hint: *Evaluating $\lim_{x\to 0^+} f(x)$ we see that it is 1. This follows because for $x > 0$, we are on the piece of $f(x)$ given by $x^2 + 3x + 1$ and the limit $\lim_{x\to 0} \left(x^2 + 3x + 1\right) = \left(\lim_{x\to 0}(x)\right)^2 + 3 \cdot \lim_{x\to 0}(x) + \lim_{x\to 0}(1) = 1$, certainly. On the other hand, evaluating $\lim_{x\to 0^-} f(x)$ we see it is equal to 1. This follows because, for $x \leq 0$, we are on the piece of $f(x)$ given by $\cos(x)$ and the limit $\lim_{x\to 0} \cos(x) = 1$, certainly. These are equal, so the limit exists and is equal to 1.*