Exercise 1 *Find*

$$\lim_{x\to\infty}\left(2x-\sqrt{4x^2-x}\right)=\boxed{1/4}$$

Hint: *Recall that the conjugate of* $a+\sqrt{b}$ *is* $a-\sqrt{b}$ *(similarly, the conjugate of* $a-\sqrt{b}$ *is* $a+\sqrt{b}$*). Multiply through by the clever form of* 1*.*

Hint: *The clever form of* 1 *alluded to is simply the fraction containing the conjugate:* $1=\dfrac{2x+\sqrt{4x^2-x}}{2x+\sqrt{4x^2-x}}$*. After some algebra, you should see that the new numerator is* $\left(2x-\sqrt{4x^2-x}\right)\left(2x+\sqrt{4x^2-x}\right)=4x^2-4x^2+x=x$*. Similarly, the new denominator is* $2x+\sqrt{4x^2-x}$*. Hence,* $2x-\sqrt{4x^2-x}=\dfrac{x}{2x+\sqrt{4x^2-x}}$*. This is certainly a more a tractable form!*

Hint: *Multiplying both the numerator and the denominator by* 1 *over the highest power of* x *(i.e.,* $1=\frac{\frac{1}{x}}{\frac{1}{x}}$*) we obtain the expression* $\dfrac{x\frac{1}{x}}{2x\frac{1}{x}+\sqrt{4x^2-x}\frac{1}{x}}=\dfrac{1}{2+\sqrt{4x^2-x}\frac{1}{x}}$*. Of course,* $\sqrt{4x^2-x}\dfrac{1}{x}=\sqrt{\dfrac{4x^2-x}{x^2}}=\sqrt{4-\dfrac{1}{x}}$*. How exceedingly fortunate, since we have transformed our formally intractable problem into* $\displaystyle\lim_{x\to\infty}\left(\frac{1}{2+\sqrt{4-\frac{1}{x}}}\right)$ *and* $\displaystyle\lim_{x\to\infty}\left(\frac{1}{2+\sqrt{4-\frac{1}{x}}}\right)=\frac{1}{2+2}=\frac{1}{4}$*, hence,* $\displaystyle\lim_{x\to\infty}\left(2x-\sqrt{4x^2-x}\right)=\frac{1}{4}$*.*