

Exercise 1 Find

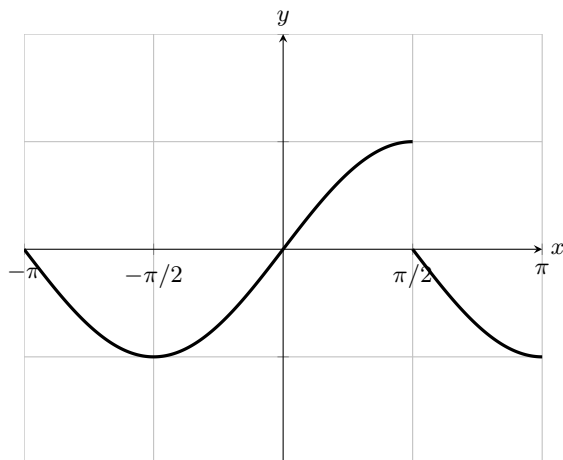
$$\lim_{x \rightarrow \pi/2} f(x) = \boxed{DNE}.$$

where

$$f(x) = \begin{cases} \sin(x) & x \leq \pi/2, \\ \cos(x) & x > \pi/2. \end{cases}$$

Hint: Both pieces of $f(x)$, $\sin(x)$, for $x \leq \pi/2$, and $\cos(x)$, for $x > \pi/2$ are continuous for all x . However, for the limit $\lim_{x \rightarrow \pi/2} f(x)$ to exist, both the left-hand and the right-hand limits of $f(x)$ at $\pi/2$ must exist and be equal.

Hint: Take a look at the graph of the function



Hint: Evaluating $\lim_{x \rightarrow \pi/2^+} f(x)$ we see that it is equal to $\cos(\pi/2) = 0$, which follows because for $x > \pi/2$, we are on the piece of $f(x)$ given by $\cos(x)$ and the limit $\lim_{x \rightarrow \pi/2} \cos(x) = \cos(\pi/2) = 0$, certainly, due to the continuity of $\cos(x)$. On the other hand, evaluating $\lim_{x \rightarrow \pi/2^-} f(x)$ we see it is equal to $\sin(\pi/2) = 1$, which follows because, for $x \leq \pi/2$, we are on the piece of $f(x)$ given by $\sin(x)$ and the limit $\lim_{x \rightarrow \pi/2} \sin(x) = \sin(\pi/2) = 1$, certainly, due to the continuity of $\sin(x)$. These are not equal, so $\lim_{x \rightarrow \pi/2} f(x)$ does not exist.