Exercise 1 *Let $f(x) = 3 - x$. Let $\varepsilon > 0$ be given. Verify that*

$$\lim_{x \to 5} f(x) = -2$$

by finding a suitable choice for $\delta > 0$ such that if $|x - 5| < \delta$, then $|f(x) - (-2)| < \varepsilon$. Choose $\delta = \boxed{\varepsilon}$.

Hint: *Observe that $|f(x) - (-2)| = |3 - x - (-2)|$ and $|3 - x - (-2)| = |5 - x|$. How should we choose $\delta > 0$ such that if $0 < |x - 5| < \delta$, then $|5 - x| < \varepsilon$?*

Hint: *Saying that $|5 - x| < \varepsilon$ is equivalent to saying that the quantity $(5 - x)$ satisfies $-\varepsilon < (5 - x) < \varepsilon$. Multiplying by a negative number reverses the direction of the inequalities (if you don't remember this, convince yourself of it).*

Therefore, upon multiplication of both sides of $-\varepsilon < (5 - x)$ by -1, we have $(-1 \cdot (-\varepsilon)) > (-1 \cdot (5 - x))$, which is simply $\varepsilon > (x - 5)$. Similarly, the statement $(5 - x) < \varepsilon$, upon multiplication of both sides by -1 becomes $(-1 \cdot (5 - x)) > (-1 \cdot (\varepsilon))$ which is simply $(x - 5) > -\varepsilon$.

Hint: *Combining the statements that $\varepsilon > (x - 5)$ and $(x - 5) > -\varepsilon$, we see they say that $-\varepsilon < (x - 5) < \varepsilon$. Thus, if $-\varepsilon < (5 - x) < \varepsilon$, then $-\varepsilon < (x - 5) < \varepsilon$, and if $-\varepsilon < (x - 5) < \varepsilon$, then $-\varepsilon < (5 - x) < \varepsilon$. This makes sense, because, in fact, $|x - 5| = |5 - x|$ for any real number x.*

Thus, we can recast the question as follows. How should we choose $\delta > 0$ such that if $0 < |x - 5| < \delta$, then $|x - 5| < \varepsilon$?

Hint: *Choose $\delta = \varepsilon$. We see that this choice gives us the desired result. Namely, if $0 < |x - 5| < \varepsilon$, then $|5 - x| < \varepsilon$. This follows because, from what we have shown, if $|x - 5| < \varepsilon$, then $|5 - x| < \varepsilon$.*