

Exercise 1 Find

$$\lim_{x \rightarrow 0} \left(\sin(x) \cos\left(\frac{1}{x^2}\right) \right) = \boxed{0}.$$

Hint: For $x \neq 0$, $-1 \leq \cos\left(\frac{1}{x^2}\right) \leq 1$. Therefore, multiplying the inequality by $\sin(x)$, we have that $-\sin(x) \leq \sin(x) \cos\left(\frac{1}{x^2}\right) \leq \sin(x)$. Apply the Squeeze Theorem to this inequality.

Hint: Since $\lim_{x \rightarrow 0} (-\sin(x)) = \lim_{x \rightarrow 0} (\sin(x)) = 0$, by the Squeeze Theorem, it follows that $\lim_{x \rightarrow 0} \left(\sin(x) \cos\left(\frac{1}{x^2}\right) \right) = 0$.
