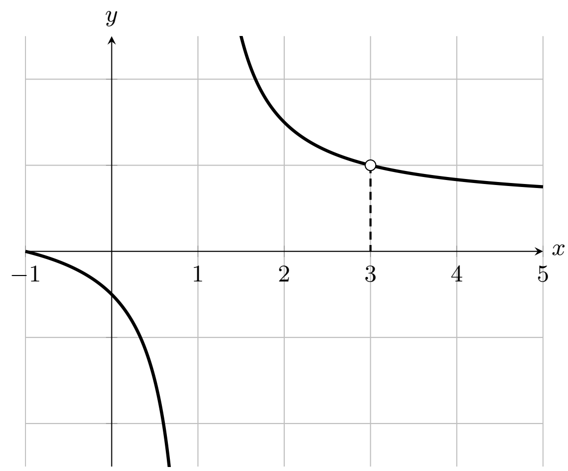
Exercise 1 *Find*

$$\lim_{x \to 3} \left(\frac{x^2 - 2x - 3}{x^2 - 4x + 3} \right) = \boxed{2}.$$

Hint: *This function is* <u>*not*</u> *continuous everywhere, but both the numerator and denominator are continuous everywhere as functions. Thus, if the limit of* $\frac{x^2 - 2x - 3}{x^2 - 4x + 3}$ *as* $x \to a$ *does not exist, then the denominator* $x^2 - 4x + 3$ *must be zero at* a*. Does* $x^2 - 4x + 3 = 0$ *when* $x = 3$*? Does* $x^2 - 2x - 3 = 0$ *at* $x = 3$ *as well?*

Hint: *Take a look at the graph of the function*



There is a removable discontinuity at $x = 3$*. This suggests something about the factorization of both polynomials* $x^2 - 2x - 3$ *and* $x^2 - 4x + 3$*. Recall that if both* $\lim_{x \to a} f(x)$ *and* $\lim_{x \to a} g(x)$ *exist, then, if* $\lim_{x \to a} g(x) \neq 0$ *, then* $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$.

Hint: *Notice that the quadratic equation tells us that* $x^2 - 2x - 3 = 0$ *has solutions* 1 ± 2 *and* $x^2 - 4x + 3 = 0$ *has solutions* 2 ± 1*. Thus,* $x^2 - 2x - 3 = (x - 3)(x + 1)$ *and* $x^2 - 4x + 3 = (x - 3)(x - 1)$*. Then for all* $x \neq 3$, $\frac{x^2 - 2x - 3}{x^2 - 4x + 3} = \frac{x + 1}{x - 1}$*, upon canceling the common factor of* $(x - 3)$*. Since we are not asking what value* $\frac{x^2 - 2x - 3}{x^2 - 4x + 3}$ *takes at* $x = 3$*, but rather what value* $\frac{x^2 - 2x - 3}{x^2 - 4x + 3}$ <u>*approaches*</u> *as* $x \to 3$*, it suffices in every case to look at* $\lim_{x \to 3} \left(\frac{x + 1}{x - 1} \right)$*. We see that* $\lim_{x \to 3} (x + 1) = 4$ *while* $\lim_{x \to 3} (x - 1) = 2$*, and since* $\lim_{x \to 3} (x - 1) \neq 0$, $\lim_{x \to 3} \left(\frac{x + 1}{x - 1} \right) = \frac{\lim_{x \to 3} (x + 1)}{\lim_{x \to 3} (x - 1)} = 2.$