

**Exercise 1** Find

$$\lim_{x \rightarrow 0} \left( x \sin \left( \frac{1}{x} \right) \right) = \boxed{0}.$$

**Hint:** Notice that  $-x \leq x \sin(\frac{1}{x}) \leq x$  for all  $x > 0$  (emphatically,  $x \neq 0$ ) and  $x \leq x \sin(\frac{1}{x}) \leq -x$  for all  $x < 0$  (again, emphatically,  $x \neq 0$ ). This can be restated as  $-|x| \leq x \sin(\frac{1}{x}) \leq |x|$  for  $x \neq 0$ . Our statement follows because  $-1 \leq \sin(\frac{1}{x}) \leq 1$  for all  $x \neq 0$ , hence, we obtained our inequality by multiplying by  $x$ . Apply the Squeeze Theorem to the inequality.

**Hint:** We see that  $\lim_{x \rightarrow 0} (-x) = \lim_{x \rightarrow 0} (x) = 0$ . It follows, by the Squeeze Theorem, that  $\lim_{x \rightarrow 0} \left( x \sin \left( \frac{1}{x} \right) \right) = 0$ .

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