

Exercise 1 Find

$$\lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{x} \right) = \boxed{3}.$$

Hint: Recall that $\lim_{x \rightarrow 0} \left(\frac{\sin(x)}{x} \right) = 1$. Multiplying $\frac{\sin(3x)}{x}$ by a clever form of 1, namely, $\frac{3}{3}$, gives us $3 \cdot \left(\frac{\sin(3x)}{3x} \right) = \frac{\sin(3x)}{x}$. Thus, $\lim_{x \rightarrow 0} \left(3 \cdot \left(\frac{\sin(3x)}{3x} \right) \right) = \lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{x} \right)$. What can you say about $\lim_{x \rightarrow 0} \left(3 \cdot \left(\frac{\sin(3x)}{3x} \right) \right)$?

Hint: Let $y = 3x$. Then since $\lim_{x \rightarrow 0} \left(3 \cdot \left(\frac{\sin(3x)}{3x} \right) \right) = \lim_{y \rightarrow 0} \left(3 \cdot \left(\frac{\sin(y)}{y} \right) \right)$, and we know that $\lim_{y \rightarrow 0} \left(\frac{\sin(y)}{y} \right) = 1$, applying the limit laws yields $\lim_{y \rightarrow 0} \left(3 \cdot \left(\frac{\sin(y)}{y} \right) \right) = 3 \cdot \lim_{y \rightarrow 0} \left(\frac{\sin(y)}{y} \right) = 3$.
