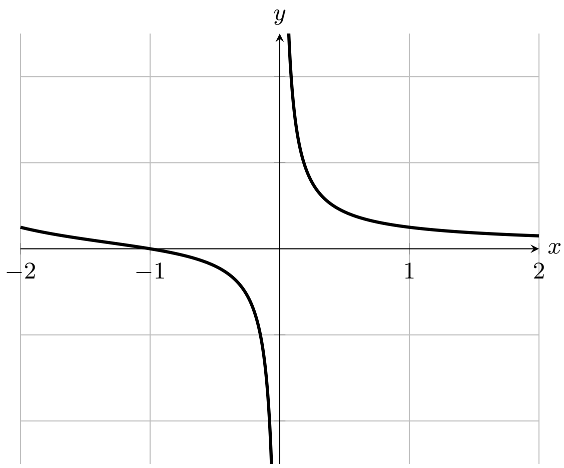
Exercise 1 *Find*

$$\lim_{x\to 0}\left(\frac{x+1}{x^2+3x}\right)=\boxed{DNE}.$$

Hint: *This function is* <u>*not*</u> *continuous everywhere, but both the numerator and denominator are continuous everywhere as functions. Thus, if the limit of* $\frac{x+1}{x^2+3x}$ *as* $x\to a$ *does not exist, then the denominator* x^2+3x *must be zero at* a.

Hint: *Take a look at the graph of the function*



Apply the limit law which says that, if both $\lim_{x\to a}f(x)$ *and* $\lim_{x\to a}g(x)$ *exist, then, if* $\lim_{x\to a}g(x)\neq 0$, *then* $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}$. *Observe what happens around (but not at)* $x=0$.

Hint: *On the one hand, for* $-3<x<0$, $x^2+3x<0$ *and for* $-1<x<0$, $x+1>0$; *hence, for* $-1<x<0$, $\frac{x+1}{x^2+3x}<0$; *conversely, for* $x>0$, $\frac{x+1}{x^2+3x}>0$. *On the other hand, for every number* a *satisfying* $-1<a<0$ *or* $a>0$, *the limit* $\lim_{x\to a}\left(\frac{x+1}{x^2+3x}\right)$ *exists because both the numerator and the denominator are continuous and nonzero for all* x *satisfying* $-1<x<0$ *or* $x>0$. *Applying several limit laws tells us that* $\lim_{x\to a}(x+1)=\lim_{x\to a}(x)+\lim_{x\to a}(1)=a+1$ *and* $\lim_{x\to a}\left(x^2+3x\right)=\left(\lim_{x\to a}(x)\right)^2+3\cdot\lim_{x\to a}(x)=a^2+3a$. *Hence, as both limits of the preceeding functions exist and* $a^2+3a\neq 0$ *for* $-1<a<0$ *or* $a>0$, $\lim_{x\to a}\left(\frac{x+1}{x^2+3x}\right)=\frac{\lim_{x\to a}(x+1)}{\lim_{x\to a}\left(x^2+3x\right)}=\frac{a+1}{a^2+3a}$.

Combining these two observations with the fact that for any $a<0$, *we can make the denominator arbitrarily close to* 0, *while the numerator becomes arbitrarily close to* 1, *we see that* $\lim_{x \to 0^-} \left(\frac{x+1}{x^2+3x} \right) = -\infty$. *Similarly, when* a *approaches* 0 *from the right,* $\lim_{x \to 0^+} \left(\frac{x+1}{x^2+3x} \right) = \infty$. *Since these are not equal, the limit does not exist.*