

Exercise 1 Find

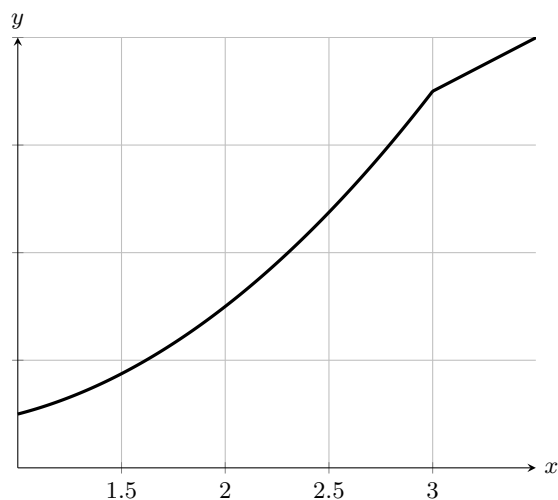
$$\lim_{x \rightarrow 3} f(x) = \boxed{7}.$$

where

$$f(x) = \begin{cases} x^2 - x + 1 & x \leq 3, \\ 2x + 1 & x > 3. \end{cases}$$

Hint: Both pieces of $f(x)$, $x^2 - x + 1$, for $x \leq 3$, and $2x + 1$, for $x > 3$, are continuous for all x . However, for the limit $\lim_{x \rightarrow 3} f(x)$ to exist, both the left-hand and the right-hand limits of $f(x)$ at 3 must exist and be equal.

Hint: Take a look at the graph of the function



Hint: Evaluating $\lim_{x \rightarrow 3^+} f(x)$ we see that it is equal to 7. This follows because, for $x > 3$, we are on the piece of $f(x)$ given by $2x + 1$ and the limit $\lim_{x \rightarrow 3} (2x + 1) = 2 \cdot \lim_{x \rightarrow 3} (x) + \lim_{x \rightarrow 3} (1) = 7$, certainly. On the other hand, evaluating $\lim_{x \rightarrow 3^-} f(x)$ we see it is equal to 7. This follows because, for $x \leq 3$, we are on the piece of $f(x)$ given by $x^2 - x + 1$ and the limit $\lim_{x \rightarrow 3} (x^2 - x + 1) = \left(\lim_{x \rightarrow 3} (x) \right)^2 - \lim_{x \rightarrow 3} (x) + \lim_{x \rightarrow 3} (1) = 7$, certainly. These are equal, so the limit exists is equal to 7.