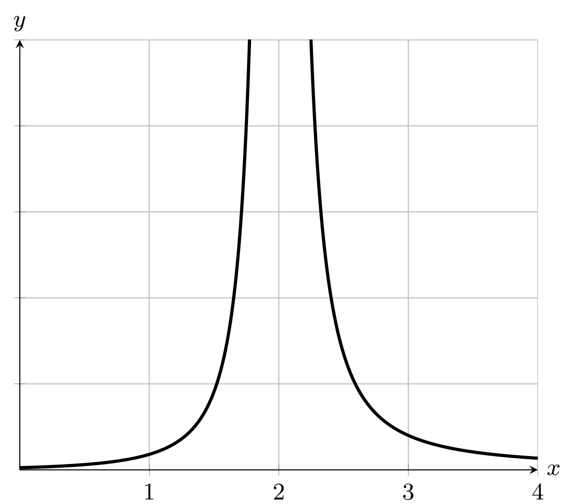
Exercise 1 *Find*

$$\lim_{x\to 2}\left(\frac{x^2+7x+10}{x^2-4x+4}\right)=\boxed{DNE}.$$

Hint: *This function is <u>not</u> continuous everywhere, but both the numerator and denominator are continuous everywhere as functions. Thus, if the limit of* $\frac{x^2+7x+10}{x^2-4x+4}$ *as* $x\to a$ *does not exist, then the denominator* x^2-4x+4 *must be zero at* a*. Does* $x^2-4x+4=0$ *when* $x=2$*? Does* $x^2+7x+10=0$ *at* $x=2$ *as well?*

Hint: *Take a look at the graph of the function*



Apply the limit law which says that, if both $\lim_{x\to a} f(x)$ *and* $\lim_{x\to a} g(x)$ *exist, then, if* $\lim_{x\to a} g(x)\neq 0$, $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$*. Observe what happens around (but not at)* $x=2$*.*

Hint: *First, notice that the quadratic equation gives only one solution to* $x^2-4x+4=0$*, namely* $x=2$*. Therefore,* $x^2-4x+4=(x-2)^2$*. Since the factorization of* $x^2+7x+10$ *does not contain two factors of* $(x-2)$*, there is a discontinuity at* $x=2$*. On the one hand, for* $x>-2$*,* $x^2+7x+10>0$ *and for all* x*,* $x^2-4x+4>0$ *(this latter result follows as the quadratic formula tells us that* $x^2-4x+4=0$ *only at* $x=2$*, so* $x^2-4x+4=(x-2)^2$*, and as we know, squared numbers are always positive); hence, for* $x>-2$*,* $\frac{x^2+7x+10}{x^2-4x+4}>0$*. On the other hand, for every number* a *satisfying* $-2<a<2$ *or* $a>2$*, the limit* $\lim_{x\to a}\frac{x^2+7x+10}{x^2-4x+4}$ *exists because both the numerator and the denominator are continuous and nonzero for all* x *satisfying* $-2<x<2$ *or* $x>2$*. Applying several limit laws tells us that* $\lim_{x\to a}\left(x^2+7x+10\right)=\left(\lim_{x\to a}(x)\right)^2+7\cdot\lim_{x\to a}(x)+$

$\lim_{x\to a}(10) = a^2 + 7a + 10$ *and* $\lim_{x\to a}\left(x^2 - 4x + 4\right) = \left(\lim_{x\to a}(x)\right)^2 - 4 \cdot \lim_{x\to a}(x) + \lim_{x\to a}(4) = a^2 - 4a + 4$. *Hence, as both limits of the preceeding functions exist and* $a^2 - 4a + 4 \neq 0$, *for* $-2 < a < 2$ *or* $a > 2$, $\lim_{x\to a}\left(\frac{x^2+7x+10}{x^2-4x+4}\right) = \frac{\lim_{x\to a}\left(x^2+7x+10\right)}{\lim_{x\to a}\left(x^2-4x+4\right)} = \frac{a^2+7a+10}{a^2-4a+4}$.

Combining these observations with the fact that the denominator becomes arbitrarily close to 0 *as* a *approaches* 2*, while the numerator approaches* 28*, we see that* $\lim_{x\to 2}\left(\frac{x^2+7x+10}{x^2-4x+4}\right) = \infty$, *because, from both sides of* $x = 2$, $\frac{x^2+7x+10}{x^2-4x+4}$ *approaches* ∞.