Time/Freq. Domain Motion Controller Design and Analysis

ME 597
Precision Motion Gen.
April 23, 2019

Lecture 7

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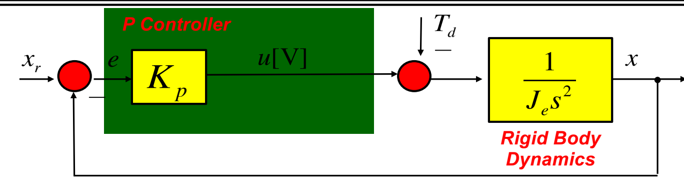

Position Dynamics $$G_x(s) = \frac{1}{J_e s^2}$$ $J_e = \text{Inertia[Volts/m/s}^2]$

P Controller $$C_v(s) = K_p$$

$K_p = \text{Proportional Gain[Volts/mm]}$

Open Loop Dynamics $$G_{ol} = \frac{x}{e} = \frac{K_p}{J_e s^2}$$

Closed Loop Dynamics $$G_{cl}(s) = \frac{K_p}{J_e s^2 + K_p}$$ $$G_{cl}(s) = \frac{K_p / J_e}{s^2 + K_p / J_e} \triangleq \frac{\omega_n^2}{s^2 + \omega_n^2}$$

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P Position Control for a Servo System

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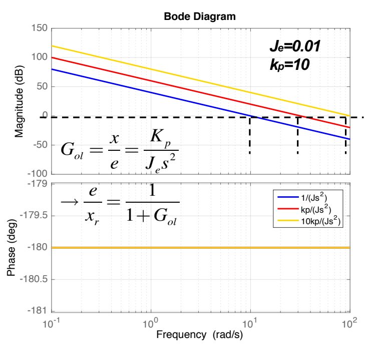
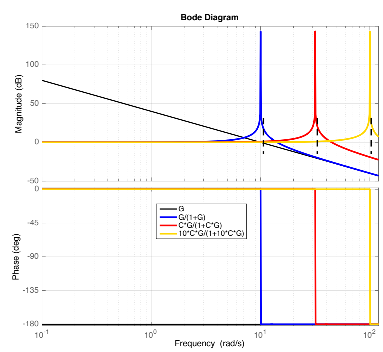
Closed Loop Dynamics

$$G_{cl}(s) = \frac{K_p / J_e}{s^2 + K_p / J_e} \triangleq \frac{\omega_n^2}{s^2 + \omega_n^2}$$

Proportional Feedback Introduces Spring Action $$\omega_n = \sqrt{\frac{K_p}{J_e}}$$





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Frequency Domain Analysis

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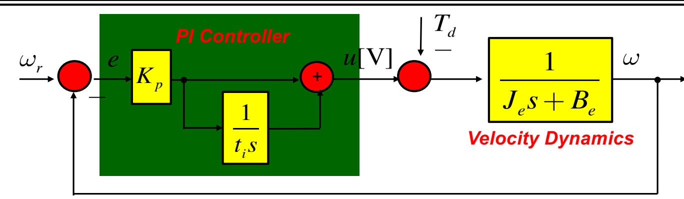

Velocity Dynamics $$G_v(s) = \frac{K_v}{\tau s + 1} = \frac{1}{J_e s + B_e}$$ $J_e = \text{Inertia[Volts/m/s}^2]$
$B_e = \text{Viscous Friction[Volts/m/s]}$

Inertia and Viscous Friction are Both Identified from Tests.

PI Velocity Controller $$C_v(s) = K_p\left(1 + \frac{1}{t_i s}\right) = \frac{K_p(t_i s + 1)}{t_i s}$$

$K_p = \text{Proportional Gain[Volts/mm/sec]}$
$t_i = \text{Integral Time Constant[sec]}$

It has a pole at the origin.*(s=0 [rad/sec])*

zero @s=$1/t_i$ [rad/sec]

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PI Velocity Control for a Servo System

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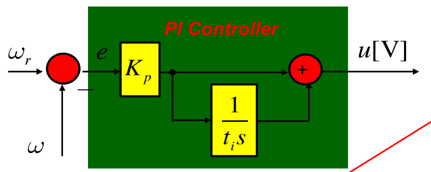
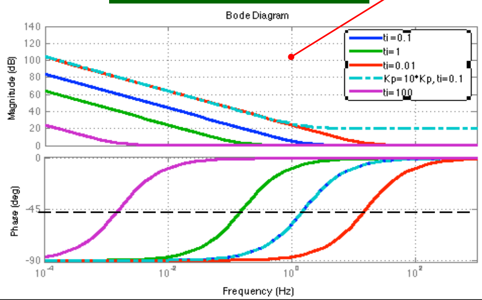


$$\frac{u(s)}{e(s)} = \underbrace{\frac{K_p(t_i s+1)}{t_i s}}_{C_v(s)}$$



Main Points to Consider

1. K_p In creases overall gain
2. Smaller integral time (t_i) constant introduces greater force in low frequency.

Tuning Rules: Use K_p to increase overall gain. Use t_i to increase the low frequency gain against disturbance

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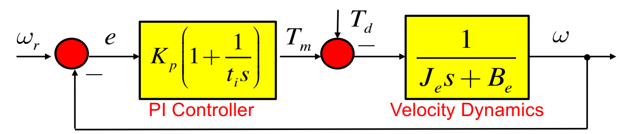
Closed Loop PI Control of the Motor Velocity



Closed Loop Dynamics

$$\omega = \omega_r \underbrace{\frac{GC}{1+GC}}_{\text{Tracking Dynamics}} - T_d \underbrace{\frac{G}{1+GC}}_{\text{Disturbance Dynamics}}$$

$$\omega = \frac{\frac{K_p}{J_e t_i}(t_i s+1)}{s^2 + \frac{K_p+B_e}{J_e}s + \frac{K_p}{J_e t_i}}\omega_r - \frac{\frac{1}{J_e}s}{s^2 + \frac{K_p+B_e}{J_e}s + \frac{K_p}{J_e t_i}}T_d$$

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Closed Loop Dynamics in 2nd Order System Form

$$\boxed{\omega = \frac{\omega_n^2(t_i s+1)}{s^2+2\zeta\omega_n s+\omega_n^2}\omega_r - \frac{1}{J_e}\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}T_d}$$

$$\begin{cases} \omega_n^2 = \dfrac{K_p}{J_e t_i} \\ 2\zeta\omega_n = \dfrac{K_p+B_e}{J_e} \end{cases}$$

Steady State Response for Step Velocity Command

$$e_{ss\,\text{STEP}} = \lim_{s\to 0} s\frac{s^2+2\zeta\omega_n s-\omega_n^2 t_i s}{s^2+2\zeta\omega_n s+\omega_n^2}\underbrace{\frac{\omega_r}{s}}_{\text{Cruse Velocity}} = 0$$

$$e_{ss\,\text{RAMP}} = \lim_{s\to 0} s\frac{s^2+2\zeta\omega_n s-\omega_n^2 t_i s}{s^2+2\zeta\omega_n s+\omega_n^2}\underbrace{\frac{A_r}{s^2}}_{\text{Acceleration}}$$

$$e_{ss\,\text{RAMP}} = \lim_{s\to 0}\frac{s+2\zeta\omega_n-\omega_n^2 t_i}{\omega_n^2}A_r = A_r(2\zeta/\omega_n - t_i) = \frac{t_i B_e}{K_p}A_r$$

*System gives steady state errors for ramp response. Errors are controlled by **K_p** & **t_i**.*

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Disturbance Response (Velocity change)

$$\omega = \frac{1}{J_e}\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}T_d$$

$$\omega_{ss\,\text{STEP}} = \lim_{s\to 0} s\frac{1/J_e s}{s^2+2\zeta\omega_n s+\omega_n^2}\frac{T_d}{s} = 0$$

Step Disturbances can not change the speed of the drive!

$$\omega_{ss\,\text{RAMP}} = \lim_{s\to 0} s\frac{1}{J_e}\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}\frac{T_d}{s^2} = \frac{T_d}{J_e\omega_n^2} = \frac{T_d t_i}{K_p}$$

Ramp Disturbances reduces the speed !

Some Comments: *For best disturbance rejection, we use high **K_p** and small integration time, **t_i**.*
*But, small **t_i** reduces damping of the system* $\zeta = \frac{(K_p+B_e)}{2\sqrt{J_e K_p}}\sqrt{t_i}$ *So, there is always a trade off in-between*

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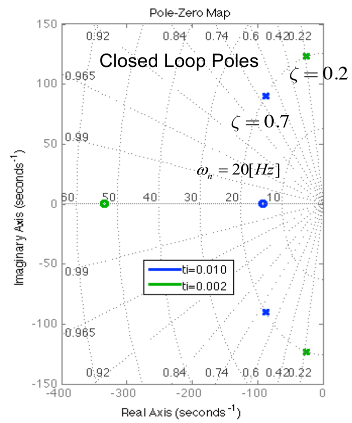
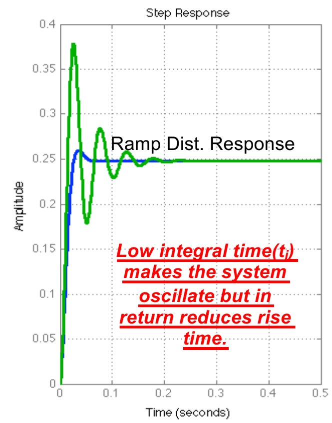
Je = 0.25536e-3, Be = 0.76467e-3;





ME 597 Precision Motion Gen.	Pole-Zero Map and Step Disturbance Response	Dr. Sencer Burak
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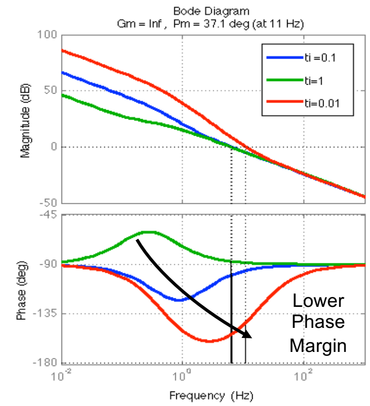
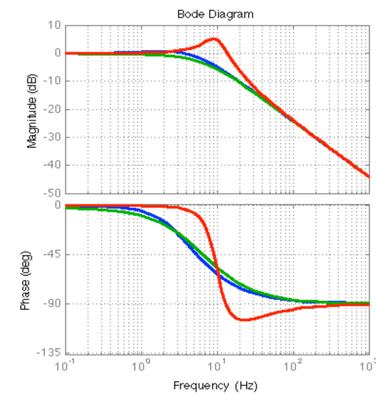
Open Loop Bode Plot **(GC)**

Closed Loop Bode**(GC/(1+GC))**





ME 597 Precision Motion Gen.	Open and Closed-loop Bode Plots of Velocity Control	Dr. Sencer Burak
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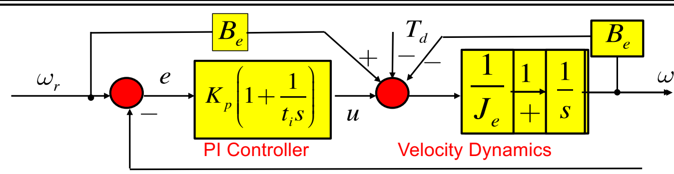

If we know the Viscous Friction (B_e) in the System, we can inject it as well. What happens if we do that???

$$\omega = \omega_r \underbrace{\frac{GC}{1+GC}}_{\text{Feedback}} + \omega_r \underbrace{\frac{B_e G}{1+GC}}_{\text{Feedforward}} - T_d \underbrace{\frac{G}{1+GC}}_{\text{Disturbance}}$$

$$\omega = \frac{\frac{K_p}{J_e t_i}(t_i s+1)}{s^2+\frac{K_p+B_e}{J_e}s+\frac{K_p}{J_e t_i}}\omega_r + \frac{\frac{B_e}{J_e}s}{s^2+\frac{K_p+B_e}{J_e}s+\frac{K_p}{J_e t_i}}\omega_r = \frac{\frac{K_p}{J_e t_i}(t_i s+1)+\frac{B_e}{J_e}s}{s^2+\frac{K_p+B_e}{J_e}s+\frac{K_p}{J_e t_i}}\omega_r$$

ME 597 Precision Motion Gen.	Velocity Dynamics(Feed-Forward)	Dr. Sencer Burak
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Tracking Error Dynamics with Friction FF

$$e = \frac{s^2+\frac{K_p+B_e}{J_e}s+\frac{K_p}{J_e t_i}-\frac{K_p t_i s}{J_e t_i}-\frac{K_p}{J_e t_i}-\frac{B_e}{J_e}s}{s^2+\frac{K_p+B_e}{J_e}s+\frac{K_p}{J_e t_i}}\omega_r = \frac{s^2+s\left(\frac{K_p+B_e}{J_e}-\frac{K_p}{J_e}-\frac{B_e}{J_e}\right)}{s^2+\frac{K_p+B_e}{J_e}s+\frac{K_p}{J_e t_i}}\omega_r$$

$$\boxed{=\frac{s^2}{s^2+\frac{K_p+B_e}{J_e}s+\frac{K_p}{J_e t_i}}\omega_r}$$

Final Value Theorem Gives:

$$\lim_{s\to 0} e = s\frac{s^2}{s^2+\frac{K_p+B_e}{J_e}s+\frac{K_p}{J_e t_i}}\frac{A_r}{s^2}=0$$

The system does not show any errors during ramp response!

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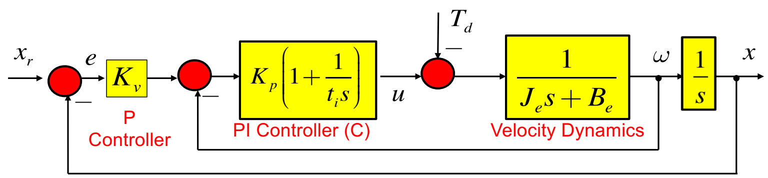

Kv is a simple proportional gain. Amplified position error is used to move the motor. If the error is zero, the motor does not move.

$$C(s)=\frac{K_p(t_i s+1)}{t_i s}, K_v \quad \text{and } G(s)=\frac{1}{J_e s^2+B_e s}$$

$$x=\frac{K_v GC}{1+K_v GC+sGC}x_r-\frac{G}{1+K_v GC+sGC}T_d$$

If higher Kv gain, the faster the motor moves to the target position, the smaller the error.

ME 597 Precision Motion Gen.	*P-PI Closed-loop Position Control*	Dr. Sencer Burak
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How do we tune the gains, Kv, Kp, ti ???

$$G_{tr}=\frac{x}{x_r}=\frac{K_v K_p(st_i+1)}{J_e t_i s^3+B_e t_i s^2+K_p s(t_i s+1)+K_p K_v(t_i s+1)}$$

$$G_d=\frac{x}{T_d}=\frac{st_i}{J_e t_i s^3+B_e t_i s^2+K_p s(t_i s+1)+K_p K_v(t_i s+1)}$$

$$\begin{aligned}J_e t_i s^3 &= J_e s^2(t_i s+1)-J_e s^2\\ &= J_e s^2(t_i s+1)-\frac{J_e}{t_i}s(t_i s+1)+\frac{J_e}{t_i^2}(t_i s+1)-\frac{J_e}{t_i^2}\end{aligned}$$

$$\begin{aligned}B_e t_i s^2 &= B_e s\ (t_i s+1)-B_e s\\ &= B_e s(t_i s+1)-\frac{B_e}{t_i}(t_i s+1)+\frac{B_e}{t_i}\end{aligned}$$

ME 597 Precision Motion Gen.	*Degree Reduction in P-PI Control*	Dr. Sencer Burak
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$$G_{tr}=\frac{x}{x_r}=\frac{K_v K_p}{J_e s^2+\left(K_p-\frac{J_e}{t_i}+B_e\right)s+\left(K_v K_p+\frac{J_e-B_e t_i}{t_i^2}\right)-\underbrace{\frac{\frac{J_e-B_e t_i}{t_i^2}}{t_i s+1}}_{\Gamma}}x_r$$

This equation can be separated into 2 different frequency regions.

$$\Gamma=\frac{\frac{J_e-B_e t_i}{t_i^2}}{t_i s+1}\begin{cases}\Gamma\approx\frac{J_e-B_e t_i}{t_i^2}, \text{ for } j\omega<\frac{1}{10t_i}\\ \Gamma\approx 0, \text{ for } j\omega>\frac{1}{10t_i}\end{cases}$$

ME 597 Precision Motion Gen.	*Degree Reduction in Freq. Domain*	Dr. Sencer Burak
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Let us design the Integral time constant as $\boxed{t_i=\frac{10}{\omega_p}}$ Position loop freq.

$$\frac{x}{x_r}\approx\frac{K_v K_p}{J_e s^2+\left(K_p-\frac{J_e}{t_i}+B_e\right)s+K_v K_p}x_r, \quad \text{for } j\omega<\frac{1}{10t_i}=\frac{\omega_p}{100}$$

$$\frac{x}{x_r}\approx\frac{K_v K_p}{J_e s^2+\left(K_p-\frac{J_e}{t_i}+B_e\right)s+\left(K_v K_p+\frac{J_e}{t_i^2}-\frac{B}{t_i}\right)}x_r, \quad \text{for } j\omega>\frac{10}{t_i}=\omega_p$$

$$\frac{K_v K_p}{J_e}=\omega_p^2$$

$$\left(\frac{K_v K_p}{J_e}+\frac{1}{t_i^2}-\frac{B_e}{J_e t_i}\right)=\omega_p^2 \text{ with } t_i=\frac{10}{\omega_p}\rightarrow\frac{K_v K_p}{J_e}-\frac{B_e}{J_e}\frac{\omega_p}{10}=\underbrace{\omega_p^2-\frac{\omega_p^2}{100}}_{0.99\omega_p^2}\Rightarrow\frac{K_v K_p}{J_e}-\omega_p\frac{B_e}{10J_e}\approx\omega_p^2$$

ME 597 Precision Motion Gen.	*2^{nd} Order Dynamics*	Dr. Sencer Burak
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Tuning of the P-Pi Control in terms of 2nd Dynamics

Generally the rigid body pole is at very low frequency

$$\frac{K_v K_p}{J_e} - \omega_p \frac{B_e}{10 J_e} \approx \omega_p \qquad \frac{B_e}{J_e} \cong 0.1 \ldots 10 \quad \text{and} \quad \underbrace{\omega_p}_{40\pi \sim 80\pi} \gg 1$$

$$\frac{K_v K_p}{J_e} \approx \omega_p \underbrace{(\omega_p + 1)}_{\approx \omega_p} \approx \omega_p^2 \qquad \boxed{\rightarrow \sqrt{\frac{K_v K_p}{J_e}} = \omega_p}$$

We got the natural freq.

Damping Ratio:

$$\frac{x}{x_r} \approx \frac{K_v K_p}{J_e s^2 + \underbrace{\left(K_p - \frac{J_e}{t_i} + B_e\right)}_{2\zeta\omega_p} s + \left(K_v K_p + \frac{J_e}{t_i^2} - \frac{B}{t_i}\right)} x_r$$

$$\boxed{2\zeta\omega_p = \frac{K_p}{J_e} - \frac{\omega_p}{10} + \frac{B_e}{J_e}} \qquad \boxed{\zeta = \frac{K_p + B_e}{2 J_e \omega_p} - 0.05} \qquad \boxed{\omega_p^2 = \frac{K_v K_p}{J_e}}$$

Example Tuning Study

Measured Inertia and friction for the linear motor system in our laboratory are given as ***Je=0.25536*10-3; Be = 0.76467*10-3.***
Design a P-PI controller for this system to achieve wn = 20[Hz] and a zeta = 0.7. Compute the controller parameters; K_v, K_p and t_i for the system.

First we start with the velocity controller parameters.

wn = 20*2*pi; zeta = .7;

ti = 10/wn; Kp = (zeta+0.05)*2*Je*wn-Be ;

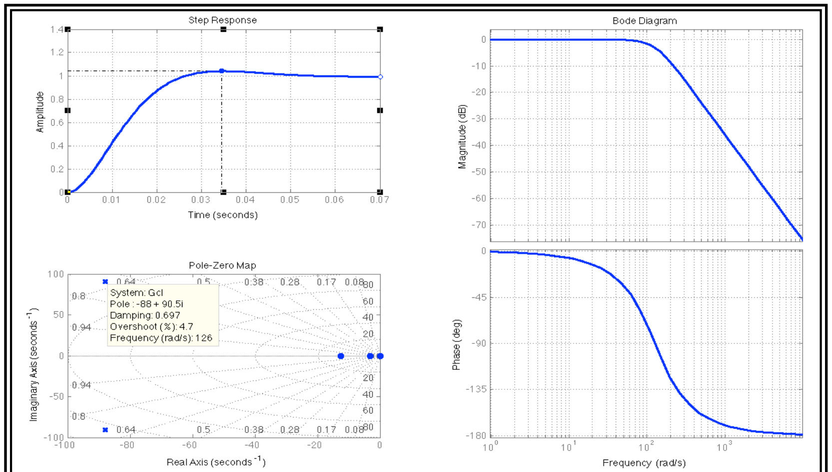
Then we get the Position Loop Gain

Kv = wn^2*Je/Kp;

Answer

Kp = .0474, ti= 0.0794, Kv = 85

Closed Loop Response of the Example



P-Pi Control Step and Ramp Disturbances

Disturbance TF:

$$G_d = \frac{x}{T_d} = \frac{s t_i}{J_e t_i s^3 + B_e t_i s^2 + K_p s(t_i s + 1) + K_p K_v (t_i s + 1)}$$

Step Disturbance Response

$$\lim_{s \to 0} s \frac{s t_i}{J_e t_i s^3 + B_e t_i s^2 + K_p s(t_i s + 1) + K_p K_v (t_i s + 1)} \frac{T_d}{s} = \mathbf{0}$$

Tracking TF

$$\frac{x}{x_r} \approx \frac{K_v K_p}{J_e s^2 + \left(K_p - \frac{J_e}{t_i} + B_e\right) s + K_v K_p} x_r = \frac{\omega_p^2}{s^2 + 2\zeta\omega_p s + \omega_p^2}$$

Step Position (x_r) Command Response

$$\lim_{s \to 0} s \frac{s^2 + 2\zeta\omega_p s}{s^2 + 2\zeta\omega_p s + \omega_p^2} \frac{x_r}{s} = \mathbf{0}$$

OK!

Ramp Position (V_r) Command Response

$$\lim_{s \to 0} s \frac{s^2 + 2\zeta\omega_p s}{s^2 + 2\zeta\omega_p s + \omega_p^2} \frac{V_r}{s^2} \neq \mathbf{0}$$

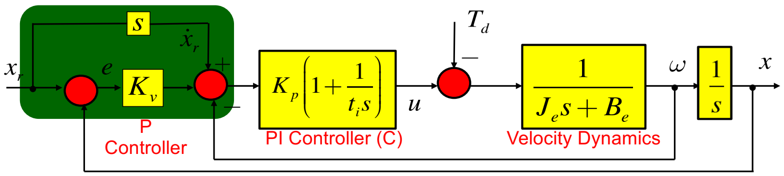
Something is wrong???

$$\lim_{s\to 0} s \frac{s^2+2\zeta\omega_p s}{s^2+2\zeta\omega_p s+\omega_p^2}\frac{V_r}{s^2} \neq 0$$

What is wrong?????

- We **knew** that the **velocity controller** has **zero** steady state error for a **step velocity** command.
- However, in P-PI control we can see that there is steady state error during constant velocity….why???

Answer : Velocity Feed-forward must be introduced to cancel out velocity dynamics.



Zero Error During Cruise Velocity

$$x = \frac{K_v GC}{1+K_v GC+sGC}x_r + \frac{sGC}{1+K_v GC+sGC}x_r$$

$$= \frac{(K_v+s)GC}{1+K_v GC+sGC} = \frac{K_v K_p(st_i+1)(1+\frac{s}{K_v})}{J_e t_i s^3+B_e t_i s^2+K_p s(t_i s+1)+K_p K_v(t_i s+1)}x_r$$

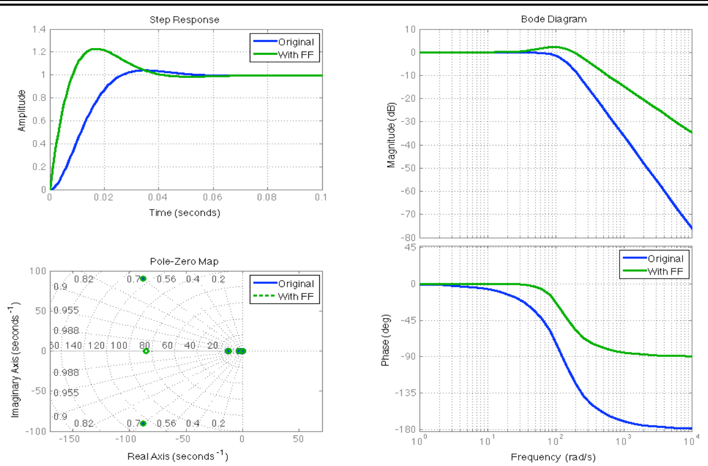
Final Value Theorem $\quad \lim_{s\to 0} s \frac{K_v K_p(st_i+1)(1+\frac{s}{K_v})}{J_e t_i s^3+B_e t_i s^2+K_p s(t_i s+1)+K_p K_v(t_i s+1)}\frac{V_r}{s^2}$

$$= \frac{J_e t_i s^3+B_e t_i s^2+K_p s(t_i s+1)-\frac{s}{K_v}K_p K_v(t_i s+1)}{J_e t_i s^3+B_e t_i s^2+K_p s(t_i s+1)+K_p K_v(t_i s+1)}\frac{V_r}{s} = \frac{J_e t_i s^2+B_e t_i s+K_p(t_i s+1)-\frac{1}{K_v}K_p K_v(t_i s+1)}{J_e t_i s^3+B_e t_i s^2+K_p s(t_i s+1)+K_p K_v(t_i s+1)}V_r$$

$$=0$$

By using the feed forward velocity, we can cancel velocity errors completely!

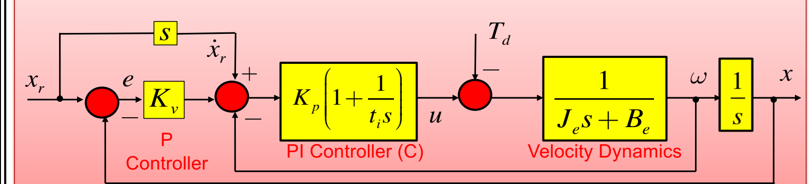
Ramp Response with Feed-Forward



P-Pi and P-Pi+FF

w_n = 20[Hz] and zeta = 0.7

Overall P-PI Control Structure For Drive Control with Velocity FeedForward

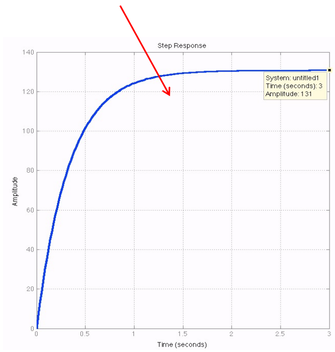


$$\frac{x}{x_r} \approx \frac{K_v K_p\left(1+\frac{s}{K_v}\right)}{J_e s^2+\left(K_p-\frac{J_e}{t_i}+B_e\right)s+K_v K_p}x_r$$

Now, system has a zero @ -Kv. It is not a simple 2nd order system anymore. Step "position" command demands infinite velocity.

Final Form of the P-Pi Controller for CNC Systems

The *step velocity* response of a drive system in our laboratory is measured. The input to the system is 0.1[V] torque command and measured velocity is in [mm/sec]. The response looks like this:



Do the following:

1. *Identify the inertia (Je) and viscous friction (Be) from measured experimental data.*
2. *Design a P-PI (No FF) position control with zeta=0.8 and wp = 40[Hz]. Compute gains.*
3. *Prepare a position trajectory with F=50[mm/sec]. Simulate the response of closed loop system in simulink. Plot errors, reference position, acceleration, actual position, acceleration.*
4. *Next, simulate the response using P-Pi+FF. Compare with step #3. What is different?*

Notes

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