Physics Lab

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Voltage Division Rule

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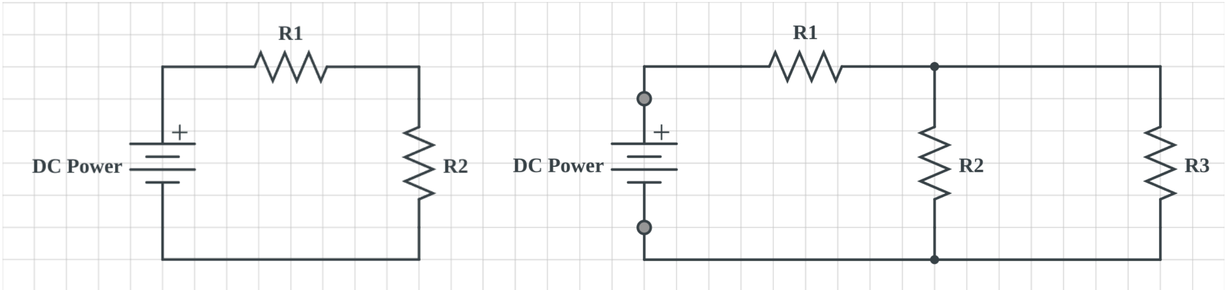
Overview

Here is book of Physics Lab experiment 1, Voltage Divider Rule overview

Experiment Info

- **Purpose >** Validation of voltage division law
- **Necessary Equipment >** Selective Resistance, DC Power, Digital Voltmeter, Wires

We have two types of resistors, series and parallel and the voltage division rule for them are :



for series resistors :

$$V_1 = \frac{V_S R_1}{R_1 + R_2}$$

$$V_2 = \frac{V_S R_2}{R_1 + R_2}$$

for parallel resistors :

final parallel resistor : $$(R_2 || R_3) = \frac{R_2 R_3}{R_2 + R_3}$$

$$V_1 = \frac{V_S R_1}{R_1 + (R_2 || R_3)} \qquad V_2 = V_3 = \frac{V_S (R_2 || R_3)}{R_1 + (R_2 || R_3)}$$

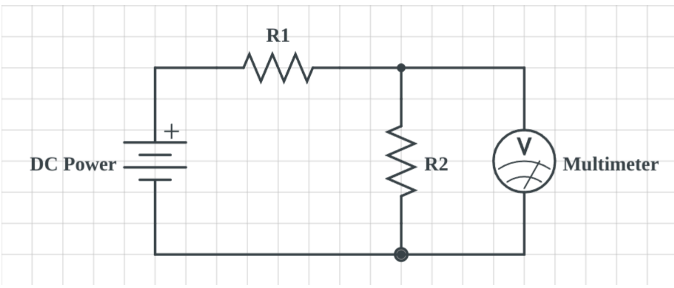
Division Rule on Series Resistors

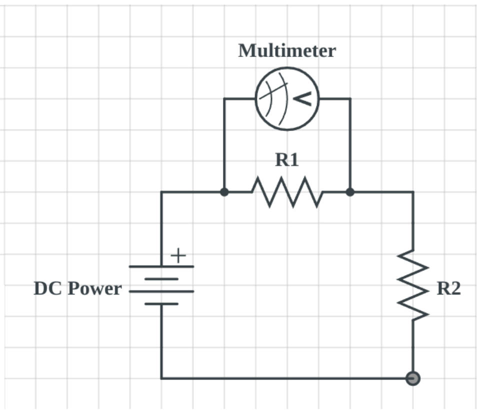
First we need to do this experiment on series resistors, R_1 is a selective and R_2 is a box resistor.

We conducted 3 experiments to validate Division Rule on series resistors.

Steps

1. for experiment ⅓, We set selective resistor on 7 so output will be $(10 - 7) * 100 = 300\Omega$ (because of using two black inputs, we used this conversion) but after connecting to multimeter we got $0.299k\Omega$. then we set box resistor on 3400Ω and multimeter showed us $3.410k\Omega$.
2. We set source DC power to $14V$.
3. Turn on DC power and enable output.
4. Connected multimeter to R_1 and written down its voltage, then did the same to R_2.
5. Calculated voltage of R_1 and R_2 via formula and written down.
6. for experiment ⅔, We only set DC power to $17.5V$ and did the previous steps again.
7. for last experiment, We set selective resistor on 6 so output will be $(10 - 6) * 100 = 400\Omega$ but after connecting to multimeter we got $0.399k\Omega$. then we set box resistor on 1500Ω and multimeter showed us $1.516k\Omega$. After that we did the previous steps again.





Next pages are experiments calculations

Division Rule on Series Resistors - Experiments Table

V_S	R_1	R_2	$V_{1_{measured}}$	$V_{2_{measured}}$	$V_{1_{real}}$	$V_{2_{real}}$
$14.0V$	$0.299k\Omega$	$3.410k\Omega$	$1.13V$	$12.93V$	$1.129V : \alpha_1$	$12.871V : \alpha_2$
$17.5V$	$0.299k\Omega$	$3.410k\Omega$	$1.43V$	$16.18V$	$1.411V : \beta_1$	$16.089V : \beta_2$
$17.5V$	$0.399k\Omega$	$1.516k\Omega$	$3.67V$	$13.91V$	$3.646V : \gamma_1$	$13.854V : \gamma_2$

Calculations of α, β and γ :

$$V_{\alpha_1} = \frac{14.0V \times 0.299k\Omega}{0.299k\Omega + 3.41k\Omega} \approx 1.129V \qquad V_{\alpha_2} = \frac{14.0V \times 3.410k\Omega}{0.299k\Omega + 3.41k\Omega} \approx 12.871V$$

$$V_{\beta_1} = \frac{17.5V \times 0.299k\Omega}{0.299k\Omega + 3.41k\Omega} \approx 1.411V \qquad V_{\beta_2} = \frac{17.5V \times 3.410k\Omega}{0.299k\Omega + 3.41k\Omega} \approx 16.089V$$

$$V_{\gamma_1} = \frac{17.5V \times 0.399k\Omega}{0.399k\Omega + 1.516k\Omega} \approx 3.646V \qquad V_{\gamma_2} = \frac{17.5V \times 1.516k\Omega}{0.399k\Omega + 1.516k\Omega} \approx 13.854V$$

Division Rule on Series Resistors - Error Calculations

Error calculations are divided into 3 categories, absolute, relative and systematic errors and my personal task is to calculate errors of **second experiment** :

from absolute formula : $Absolute\ Error = |real\ value - measured\ value|$

We get this absolute error for β_1 : $\varepsilon_{abs_{\beta_1}} = |1.411V - 1.43V| = 0.019$

Also we get this absolute error for β_2 : $\varepsilon_{abs_{\beta_2}} = |16.089V - 16.18V| = 0.091$

from relative formula : $Relative\ Error = \frac{|real\ value - measured\ value|}{real\ value}$

We get this absolute error for β_1 : $\varepsilon_{rel_{\beta_1}} = \frac{0.019V}{1.411V} \approx 0.016$

Also we get this absolute error for β_2 : $\varepsilon_{rel_{\beta_2}} = \frac{0.091V}{16.089V} \approx 0.006$

for systematic formula, We must do several mathematics operations :

1. Consider we have formula $V = \frac{V_S R_\alpha}{R_\alpha + R_\beta}$.
2. Take the ln from both sides.
3. Take the differential from both sides.
4. Convert differentials to delta (Δ).
5. Convert every $-$ to $+$.
6. accuracy of **Analog Device** is equal to the lowest shown range on the device.
7. Error of **Analog Device** is equal to the lowest shown range on device multiplied by 0.5.
8. accuracy of **Digital Device** is equal to 10^{-x} where x is float level on the screen.
9. Error of **Digital Device** is equal to 10^{-x} where x is float level on the screen.

Now we can calculate systematic error using main formula :

$$Step\ 1: \quad V = \frac{V_S R_1}{R_1 + R_2} \qquad V_2 = \frac{V_S R_2}{R_1 + R_2}$$

for V_1 :

$$Step\ 2: \quad ln(V_1) = ln(V_s) + ln(R_1) - ln(R_1 + R_2)$$

$$Step\ 3: \quad \frac{dV_1}{V_1} = \frac{dV_s}{V_s} + \frac{dR_1}{R_1} - \frac{d(R_1 + R_2)}{R_1 + R_2}$$

$$Step\ 4: \quad \frac{\Delta V_1}{V_1} = \frac{\Delta V_s}{V_s} + \frac{\Delta R_1}{R_1} - \frac{\Delta(R_1+R_2)}{R_1+R_2}$$

$$Step\ 5: \quad \frac{\Delta V_1}{V_1} = \frac{\Delta V_s}{V_s} + \frac{\Delta R_1}{R_1} + \frac{\Delta(R_1)+\Delta(R_2)}{R_1+R_2}$$

$$Step\ 6\sim: \quad \Delta V_S = 10^{-1} \qquad \Delta R_1 = 10^{-3} \qquad \Delta R_2 = 10^{-3}$$

$$Final: \quad \frac{\Delta V_1}{V_1} = \frac{10^{-1}V}{17.5V} + \frac{10^{-3}k\Omega}{0.299k\Omega} + \frac{10^{-3}k\Omega + 10^{-3}k\Omega}{0.299k\Omega + 3.410k\Omega} = 0.0096$$

$$\Rightarrow \varepsilon_{sys_{\beta_1}} = 0.0096$$

for V_2 :

$$Step\ 2: \quad ln(V_2) = ln(V_s) + ln(R_2) - ln(R_1 + R_2)$$

$$Step\ 3: \quad \frac{dV_2}{V_2} = \frac{dV_s}{V_s} + \frac{dR_2}{R_2} - \frac{d(R_1+R_2)}{R_1+R_2}$$

$$Step\ 4: \quad \frac{\Delta V_2}{V_2} = \frac{\Delta V_s}{V_s} + \frac{\Delta R_2}{R_2} - \frac{\Delta(R_1+R_2)}{R_1+R_2}$$

$$Step\ 5: \quad \frac{\Delta V_2}{V_2} = \frac{\Delta V_s}{V_s} + \frac{\Delta R_2}{R_2} + \frac{\Delta(R_1)+\Delta(R_2)}{R_1+R_2}$$

$$Step\ 6\sim: \quad \Delta V_S = 10^{-1} \qquad \Delta R_1 = 10^{-3} \qquad \Delta R_2 = 10^{-3}$$

$$Final: \quad \frac{\Delta V_2}{V_2} = \frac{10^{-1}V}{17.5V} + \frac{10^{-3}k\Omega}{3.410k\Omega} + \frac{10^{-3}k\Omega + 10^{-3}k\Omega}{0.299k\Omega + 3.410k\Omega} = 0.0065$$

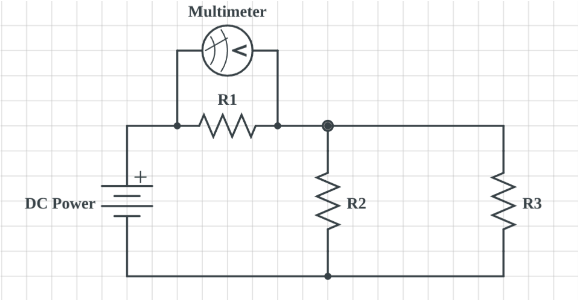
$$\Rightarrow \varepsilon_{sys_{\beta_2}} = 0.0065$$

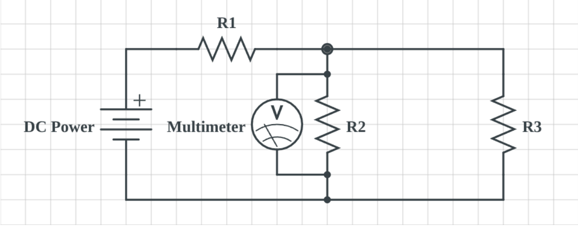
Division Rule on Parallel Resistors

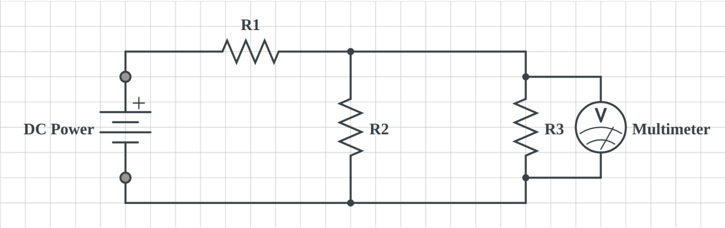
Next we need to do this experiment on parallel resistors, R_1 and R_2 are the same as the previous experiment and we add R_3 as a box resistor. We conducted 2 experiments to validate Division Rule on parallel resistors.

Steps

8. for experiment ½, We set selective resistor on 6 so output will be $(10 - 6) * 100 = 400\Omega$ (because of using two black inputs, we used this conversion) but after connecting to multimeter we got $0.399k\Omega$. then we set box resistor R_2 on 1500Ω and multimeter showed us $1.516k\Omega$. then we set R_3 on $2400k\Omega$ and multimeter showed us $2.420k\Omega$.
9. We set source DC power to $17.5V$ and turn on DC power and enable output.
10. Connected multimeter to R_1 and written down its voltage, then did the same to R_2 and R_3.
11. Calculated voltage of R_1, R_2 and R_3 via formula and written down.
12. for last experiment, We set selective resistor on 3 so output will be $(10 - 3) * 100 = 700\Omega$ but after connecting to multimeter we got $0.699k\Omega$. then we set box resistor R_2 on 1300Ω and multimeter showed us $1.365k\Omega$. After that we did the







Next pages are experiments calculations

Division Rule on Parallel Resistors - Experiments Table

V_S	R_1	R_2	R_3	$V_{1_{measured}}$	$V_{3_{measured}}$	$V_{1_{real}}$	$V_{3_{real}}$
$17.5V$	$0.399k\Omega$	$1.516k\Omega$	$2.400k\Omega$	$5.28V$	$12.24V$	$5.246V$: α_1	12.254V : α_3
$17.5V$	$0.699k\Omega$	$1.365k\Omega$	$2.400k\Omega$	$7.90V$	$9.69V$	$7.783V$: β_1	$9.717V$: β_3

Consider that we have these formulas :

$$V_1 = \frac{V_S R_1}{R_1 + (R_2||R_3)} \qquad V_2 = V_3 = \frac{V_S (R_2||R_3)}{R_1 + (R_2||R_3)} \qquad (R_2||R_3) = \frac{R_2 R_3}{R_2 + R_3}$$

Calculations of α and β :

$$V_{\alpha_1} = \frac{17.5V \times 0.399k\Omega}{0.399k\Omega + \frac{1.516k\Omega \times 2.400k\Omega}{1.516k\Omega + 2.400k\Omega}} \approx 5.246V \quad V_{\alpha_3} = \frac{17.5V \times \frac{1.516k\Omega \times 2.400k\Omega}{1.516k\Omega + 2.400k\Omega}}{0.399k\Omega + \frac{1.516k\Omega \times 2.400k\Omega}{1.516k\Omega + 2.400k\Omega}} \approx 12.254V$$

$$V_{\beta_1} = \frac{17.5V \times 0.699k\Omega}{0.699k\Omega + \frac{1.365k\Omega \times 2.400k\Omega}{1.365k\Omega + 2.400k\Omega}} \approx 7.783V \quad V_{\beta_3} = \frac{17.5V \times \frac{1.365k\Omega \times 2.400k\Omega}{1.365k\Omega + 2.400k\Omega}}{0.699k\Omega + \frac{1.365k\Omega \times 2.400k\Omega}{1.365k\Omega + 2.400k\Omega}} \approx 9.717V$$

Division Rule on Parallel Resistors - Error Calculations

Error calculations are divided into 3 categories, absolute, relative and systematic errors and my personal task is to calculate errors of **second experiment** :

from absolute formula : $Absolute\ Error = |real\ value - measured\ value|$

We get this absolute error for β_1 : $\varepsilon_{abs_{\beta_1}} = |7.783V - 7.90V| = 0.117$

Also we get this absolute error for β_3 : $\varepsilon_{abs_{\beta_3}} = |9.717V - 9.69V| = 0.027$

from relative formula : $Relative\ Error\ = \frac{|real\ value - measured\ value|}{real\ value}$

We get this absolute error for β_1 : $\varepsilon_{rel_{\beta_1}} = \frac{0.117V}{7.783V} \approx 0.015$

Also we get this absolute error for β_2 : $\varepsilon_{rel_{\beta_2}} = \frac{0.027V}{9.717V} \approx 0.003$

for systematic formula, We must do several mathematics operations :

1. Take the ln from both sides.
2. Take the differential from both sides.
3. Convert differentials to delta (Δ).
4. Convert every $-$ to $+$.
5. accuracy of **Analog Device** is equal to the lowest shown range on the device.
6. Error of **Analog Device** is equal to the lowest shown range on device multiplied by 0.5.
7. accuracy of **Digital Device** is equal to 10^{-x} where x is float level on the screen.
8. Error of **Digital Device** is equal to 10^{-x} where x is float level on the screen.

Now we can calculate systematic errors for both V_1 and V_3 :

for V_1 :

$$Step\ 1: \quad ln(V_1) = ln(V_s) + ln(R_1) - ln(R_1 + (R_2||R_3))$$

$$Step\ 2: \quad \frac{dV_1}{V_1} = \frac{dV_s}{V_s} + \frac{dR_1}{R_1} - \frac{d(R_1 + (R_2||R_3))}{R_1 + (R_2||R_3)}$$

$$Step\ 3: \quad \frac{\Delta V_1}{V_1} = \frac{\Delta V_s}{V_s} + \frac{\Delta R_1}{R_1} - \frac{\Delta(R_1 + (R_2||R_3))}{R_1 + (R_2||R_3)}$$

$$Step\ 4: \quad \frac{\Delta V_1}{V_1} = \frac{\Delta V_s}{V_s} + \frac{\Delta R_1}{R_1} + \frac{\Delta(R_1) + \Delta((R_2||R_3))}{R_1 + (R_2||R_3)}$$

$$Step\ 5\sim: \quad \Delta V_S = 10^{-1} \qquad \Delta R_1 = 10^{-3} \qquad \Delta(R_2||R_3) = 10^{-3}$$

$$Final: \quad \frac{\Delta V_2}{V_2} = \frac{10^{-1}V}{17.5V} + \frac{10^{-3}k\Omega}{0.699k\Omega} + \frac{10^{-3}k\Omega + 10^{-3}k\Omega}{0.699k\Omega + 0.873k\Omega} \approx 0.0084$$

$$\Rightarrow \varepsilon_{sys_{\beta_1}} \approx 0.0084$$

for V_3 :

$$Step\ 1.1: \ ln(V_3) = ln(V_s) + ln((R_2||R_3)) - ln(R_1 + (R_2||R_3))$$

$$Step\ 1.2: \ ln(V_3) = ln(V_s) + ln(R_2) + ln(R_3) - ln(R_2 + R_3) - ln(R_1 + (R_2||R_3))$$

$$Step\ 2: \quad \frac{dV_3}{V_3} = \frac{dV_s}{V_s} + \frac{dR_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{d(R_2+R_3)}{R_2+R_3} - \frac{d(R_1+(R_2||R_3))}{R_1+(R_2||R_3)}$$

$$Step\ 3: \quad \frac{\Delta V_3}{V_3} = \frac{\Delta V_s}{V_s} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta(R_2+R_3)}{R_2+R_3} - \frac{\Delta(R_1+(R_2||R_3))}{R_1+(R_2||R_3)}$$

$$Step\ 4: \quad \frac{\Delta V_3}{V_3} = \frac{\Delta V_s}{V_s} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} + \frac{\Delta(R_2)+\Delta(R_3)}{R_2+R_3} + \frac{\Delta(R_1)+\Delta((R_2||R_3))}{R_1+(R_2||R_3)}$$

$$Step\ 5\sim: \quad \Delta V_S = 10^{-1} \qquad \Delta R_1 = \Delta R_2 = \Delta R_3 = 10^{-3} \qquad \Delta(R_2||R_3) = 10^{-3}$$

$$Final:$$

$$\frac{\Delta V_3}{V_3} = \frac{10^{-1}V}{17.5V} + \frac{10^{-3}k\Omega}{1.365k\Omega} + \frac{10^{-3}k\Omega}{2.420k\Omega} + \frac{10^{-3}k\Omega + 10^{-3}k\Omega}{1.365k\Omega + 2.420k\Omega} + \frac{10^{-3}k\Omega + 10^{-3}k\Omega}{0.699k\Omega + 0.873k\Omega} \approx 0.0086$$

$$\Rightarrow \varepsilon_{sys_{\beta_3}} \approx 0.0086$$

Result

1. in parallel experiment table row No.2 in real we set parallel box resistor R_2 to $1.300k\Omega$, but after measuring via multimeter it showed us $1.365k\Omega$ and that's why we got $\varepsilon_{abs_{\beta_1}} = 0.117$.
2. If we measure resistors with the wrong multimeter, errors will be higher, in this experiment, the highest multimeter did not measured as well.
3. We proved the division rule for both parallel and series resistors.