Ch. 5. Neural Network and Deep Learning

Yongjae Yoo, Ph. D.
Assistant Professor
Department of Artificial Intelligence,
Hanyang University ERICA



So Far,

- We have learnt how your computer understand the data and make decisions.
- (In the simplest ways.)
- Based on the data, the machine can identify
 - The trends of data
 - Which group the new input belongs to

What If

- The question goes "deep," i.e.,
 - Questions gets sophisticated multiple times we should ask.
 - Questions have many possibilities
 - New point (input) has many dimensions
 - New point (input) has totally new one.
- Then, the basic approach goes impossible.
- In most cases in real life, we have such issues.

Examples

- Coke vs. Pepsi
 - So many ingredients, answers diverge.
- Conditional/confounded plans
 - "It depends on whether my friends' opinions."
 - We eat 빵. Turkish eat Ekmek. Chinese eat Mian Bao.
 - What will you do after this class?
 - Etc.

Dealing With Many Questions

Nested If statements

```
x = 41

if x > 10:
    print("Above ten,")
    if x > 20:
        print("and also above 20!")
    else:
        print("but not above 20.")
```

With and/or/not (binary conditional operators)

Binary Conditional Operators

And, and OR

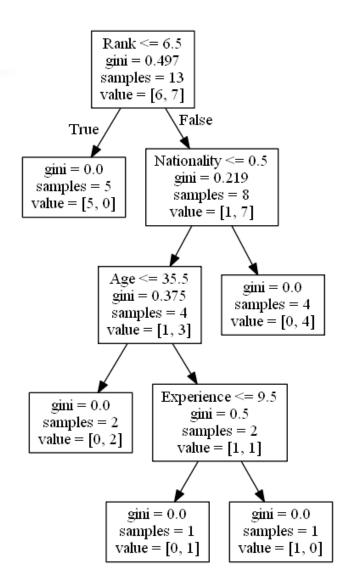
```
a = 200
b = 33
c = 500
if a > b and c > a:
    print("Both conditions are True")

a = 200
b = 33
c = 500
if a > b or a > c:
    print("At least one of the conditions is True")
```

Decision Tree

 Dealing with multiple conditions in a systematic way.

 A series of true-false questions, and the answers will be decided by the sequential inputs.



Decision Tree

- Example:
 - "I am trying to decide if I should go to a comedy show or not."
- Fortunately, I know the comedian's experience, age, nationality, rank, and my preference (Yay or Nay).

Age	Experience	Rank	Nationality	Go
36	10	9	UK	NO
42	12	4	USA	NO
23	4	6	N	NO
52	4	4	USA	NO
43	21	8	USA	YES
44	14	5	UK	NO
66	3	7	N	YES
35	14	9	UK	YES
52	13	7	N	YES
35	5	9	N	YES
24	3	5	USA	NO
18	3	7	UK	YES
45	9	9	UK	YES

Python Codes

• We will use a new tool, Pandas in this example.

```
import pandas

df = pandas.read_csv("data.csv")

print(df)
```

- Then, we need to convert the text data into numbers.
- We will use Pandas' map() method.

```
{'UK': 0, 'USA': 1, 'N': 2}
```

Python Codes

Mapping texts into numbers:

```
d = {'UK': 0, 'USA': 1, 'N': 2}
df['Nationality'] = df['Nationality'].map(d)
d = {'YES': 1, 'NO': 0}
df['Go'] = df['Go'].map(d)

print(df)
```

• Then,

Python Codes

• Then, we will extract "features" to make decisions

```
features =
['Age', 'Experience', 'Rank', 'Nationality']

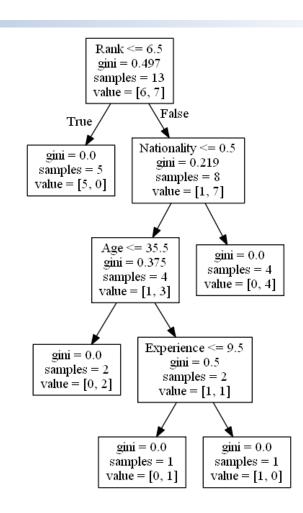
X = df[features]
y = df['Go']

print(X)
print(y)
```

Final Decision Tree Here:

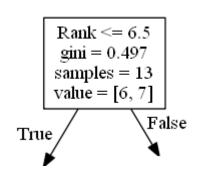
```
import pandas
from sklearn import tree
from sklearn.tree import DecisionTreeClassifier
import matplotlib.pyplot as plt
df = pandas.read csv("data.csv")
d = {'UK': 0, 'USA': 1, 'N': 2}
df['Nationality'] = df['Nationality'].map(d)
d = {'YES': 1, 'NO': 0}
df['Go'] = df['Go'].map(d)
features = ['Age', 'Experience', 'Rank', 'Nationality']
X = df[features]
y = df['Go']
dtree = DecisionTreeClassifier()
dtree = dtree.fit(X, y)
tree.plot tree(dtree, feature names=features)
```

Results



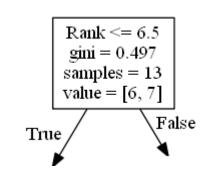
Explaining Each Level

- It shows the criteria of decision on each
- First, see "rank."
 - Rank <= 6.5 means that every comedian with a rank of 6.5 or lower will follow the True arrow (to the left), and the rest will follow the False arrow (to the right).
- Gini is the split quality between 0 0.5
 - Gini = 1 (x/n)² (y/n)²
 Where x is the number of positive answers("GO")
 , n is the number of samples, and y is the number of negative answers ("NO"), which gives us this calcula tion: 1 (7 / 13)²



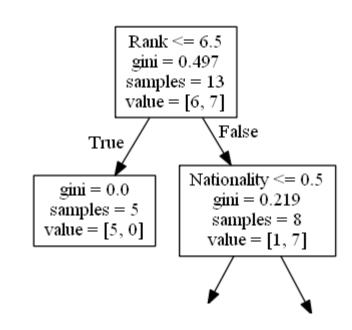
Explaining Each Level

- samples = 13 means that there are 13 comedians left at this point in the decision, which is all of them since this is the first step.
- value = [6, 7] means that of these 13 comedians, 6 will get a "NO", and 7 will get a "GO".



2nd Level

- True: five comedians will end here.
- (five samples, and five "NO"s.
- False:
 - Then the next way is checking nationality.
 - Among 8 samples, 1 No and 7 Yeses.
 - Gini would be 0.219.



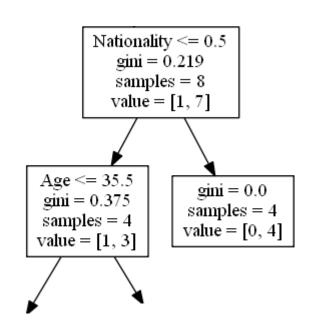
3rd Level

• True - 4 Comedians Continue:

- Age <= 35.5 means that comedians at the age of 35.5 or younger will follow the arrow to the left, and the rest will follow the arrow to the right.
- gini = 0.375 means that about 37.5% of the samples would go in one direction.
- samples = 4 means that there are 4 comedians left in this branch (4 comedians from the UK).
- value = [1, 3] means that of these 4 comedians, 1 will get a "NO" and 3 will get a "GO".

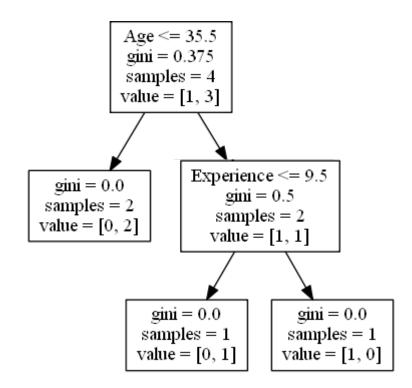
False - 4 Comedians End Here:

- gini = 0.0 means all of the samples got the same result.
- samples = 4 means that there are 4 comedians left in this branch (4 comedians not from the UK).
- value = [0, 4] means that of these 4 comedians, 0 will get a "NO" and 4 will get a "GO".



4th and 5th levels

- In 4th level, it checks age.
 - If younger than 35.5, 2 samples will end with "Yes."
 - Otherwise, it will check experience.
 If one's experience is shorter than 9.5 years, it ends up to "No."
 Otherwise, goes to "Yes."

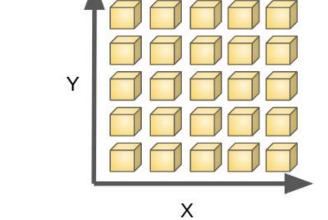


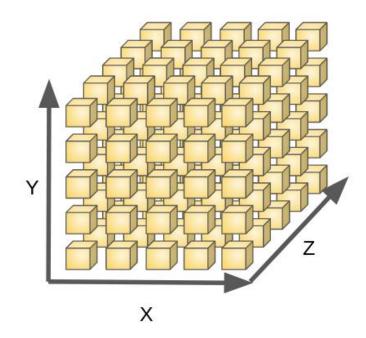
So… Is The Decision Tree Always Works?

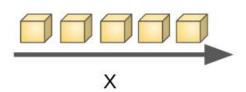
- Testing with different samples:
 - print(dtree.predict([[40, 10, 7, 1]]))
 - print(dtree.predict([[40, 10, 6, 1]]))
- You will see that the Decision Tree gives you different results, even you put in the same data when you try multiple times.
- That is because the Decision Tree is based on the probability of an outcome, and the answer will vary.

Curse of Dimensionality

- If we set multiple domain(s) variables,
- The possibility and the number of calculation explodes.
- In your cases of







Anyhow Let's Do This

Try the example.

• If you have any good data, you can try that with your data for your final report.

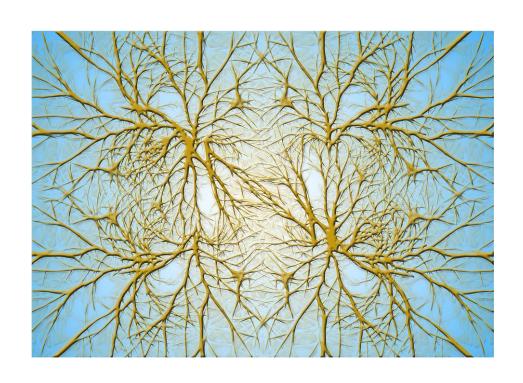
Dealing with Complexity

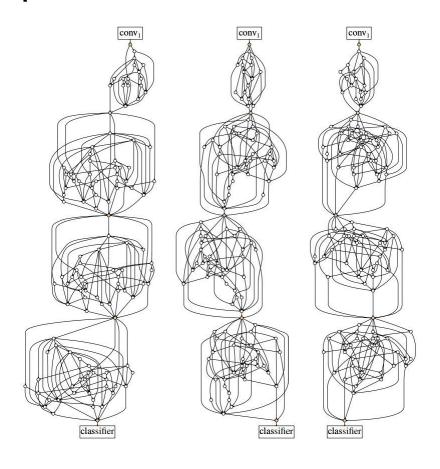
- Once again, computer can deal with yes/no questions.
- In other words, it can answer "data with linearity."
- In theory, we can deal with MANY linear combinations of que stions if we have enough memory. That is so-called Linear score function.

- To resolve this, we will let computer think as like we do.
- Neural network!

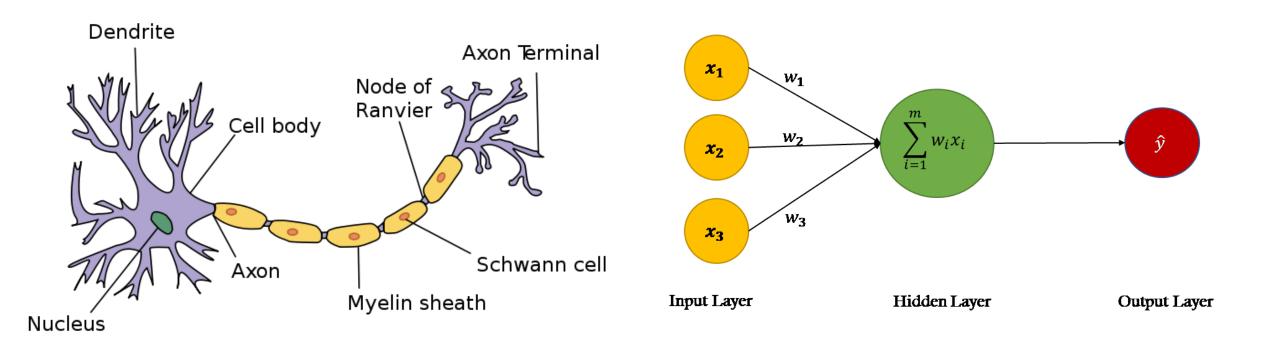
Analogy to (Physiologic) Neurons

• Physical neural networks vs. computational neural networks.



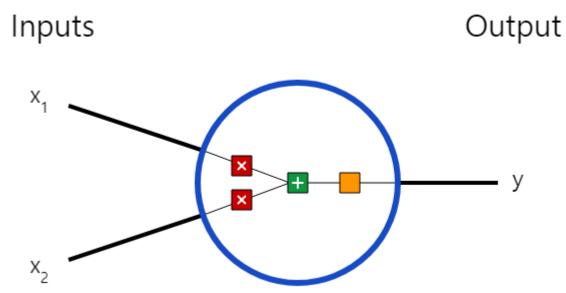


Looking into Single Neuron



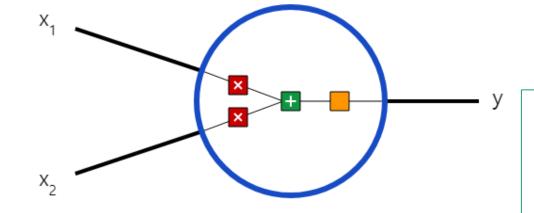
Neuron as The Basic Building Block

- A neuron takes inputs, does some math with them, and produces one output.
- 2-input neuron:



In a Neuron:

Inputs Output



3 things are happening here. First, each input is multiplied by a weight: ■

$$x_1
ightarrow x_1 * w_1$$

$$x_2
ightarrow x_2 * w_2$$

Next, all the weighted inputs are added together with a bias b:

$$(x_1*w_1)+(x_2*w_2)+b$$

Finally, the sum is passed through an activation function:

$$y = f(x_1 * w_1 + x_2 * w_2 + b)$$

Activation function

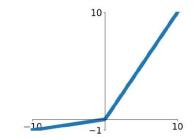
Works as a switch – it cuts out conditions we do not want.

$$f=W_2\max(0,W_1x)$$

Activation Functions

Sigmoid $\sigma(x) = \frac{1}{1 + e^{-x}}$ tanh tanh(x)ReLU $\max(0,x)$





Maxout

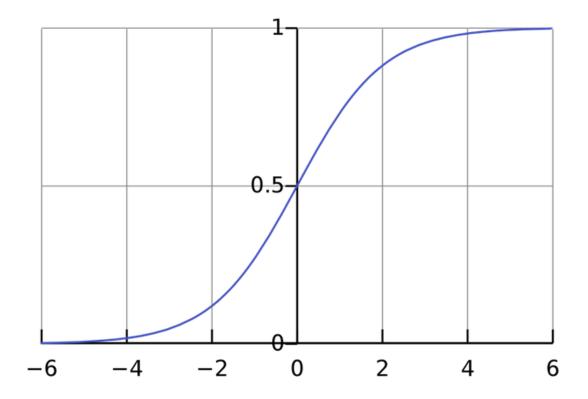
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

$$\begin{cases} x & x \ge 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$

Rectified Linear Unit (ReLU) often used as Default

Activation Function

- Consider a sigmoid function
 - Converts –inf to +inf to (0, 1)



Example

Assume we have a 2-input neuron that uses the sigmoid activation and has the following parameters:

$$w = [0, 1]$$
 $b = 4$

- w = [0, 1]: $w_1 = 0, w_1 = 1$.
- Now give an input x = [2, 3]. Applying a dot product:

$$(w \cdot x) + b = ((w_1 * x_1) + (w_2 * x_2)) + b$$

= $0 * 2 + 1 * 3 + 4$
= 7

$$y = f(w \cdot x + b) = f(7) = \boxed{0.999}$$

A Single Neuron's Output

- Gives you a single output here 0.999, as a "feedforward" result.
- Feedforward process means: passing inputs forward to get an output

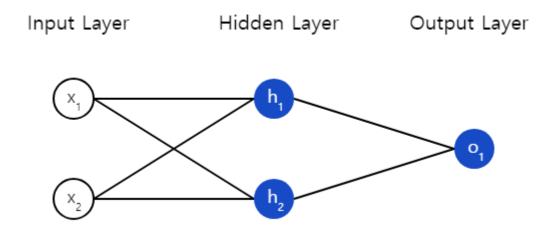
Now, Coding a Neuron Here:

Here just we implemented example converted to code:

```
import numpy as np
def sigmoid(x): # Our activation function: f(x) = 1 / (1 + e^{-(-x)})
  return 1 / (1 + np.exp(-x))
class Neuron:
  def init (self, weights, bias):
    self.weights = weights
    self.bias = bias
  def feedforward(self, inputs):
   # Weight inputs, add bias, then use the activation function
   total = np.dot(self.weights, inputs) + self.bias
    return sigmoid(total)
weights = np.array([0, 1]) # w1 = 0, w2 = 1
bias = 4 \# b = 4
n = Neuron(weights, bias)
x = np.array([2, 3]) # x1 = 2, x2 = 3
print(n.feedforward(x)) # 0.9990889488055994
```

So, What's The Difference?

Neural network can be concatenated, like:



- This network has 2 inputs, a hidden layer with 2 neurons h1 and h2, a
 nd an output layer with 1 neuron (o1).
- Notice that the inputs for o1 are the outputs from h1 and h2 that's what makes this a network.

Feedforwarding the Network

- A neural network can have any number of layers with any number of neurons in those layers.
- The basic idea stays the same: feed the input(s) forward through the ne urons in the network to get the output(s) at the end.

What happens if we pass in the input x = [2, 3]?

$$h_1 = h_2 = f(w \cdot x + b)$$

 $= f((0 * 2) + (1 * 3) + 0)$
 $= f(3)$
 $= 0.9526$
 $o_1 = f(w \cdot [h_1, h_2] + b)$
 $= f((0 * h_1) + (1 * h_2) + 0)$
 $= f(0.9526)$
 $= \boxed{0.7216}$

Let's Implement a 2-Layer Network

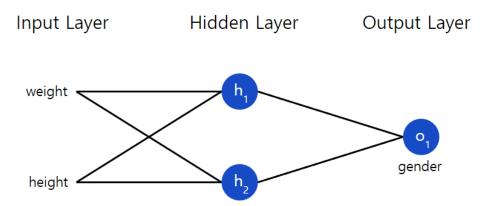
```
import numpy as np # ... code from previous section here
class OurNeuralNetwork:
  ''' A neural network with:
    - 2 inputs
    - a hidden layer with 2 neurons (h1, h2)
    - an output layer with 1 neuron (o1)
    - Each neuron has the same weights and bias:
      - w = [0, 1]
      - b = 0
   def init (self):
     weights = np.array([0, 1])
     bias = 0
     # The Neuron class here is from the previous section
     self.h1 = Neuron(weights, bias)
     self.h2 = Neuron(weights, bias)
     self.o1 = Neuron(weights, bias)
   def feedforward(self, x):
     out_h1 = self.h1.feedforward(x)
     out_h2 = self.h2.feedforward(x)
     # The inputs for o1 are the outputs from h1 and h2
     out o1 = self.o1.feedforward(np.array([out h1, out h2]))
     return out_o1 network = OurNeuralNetwork()
x = np.array([2, 3])
print(network.feedforward(x)) # 0.7216325609518421
```

So, More about Training

Assume we have the following measurements:

Name	Weight (lb)	Height (in)	Gender
Alice	133	65	F
Bob	160	72	М
Charlie	152	70	М
Diana	120	60	F

Let's train our network to predict someone's gender given the eir weight and height:



So, More about Training

• We'll represent Male with a 0 and Female with a 1, and we'll a lso shift the data to make it easier to use:

Name	Weight (minus 135)	Height (minus 66)	Gender
Alice	-2	-1	1
Bob	25	6	0
Charlie	17	4	0
Diana	-15	-6	1

I arbitrarily chose the shift amounts (135 and 66) to make the numbers look nice. Normally, you'd shift by the mean.

Understanding Loss

- Before we train our network, we first need a way to quantify how "good" it's doing so that it can try to do "better".
- That's what the loss is.

We'll use the **mean squared error** (MSE) loss:

 $ext{MSE} = rac{1}{n} \sum_{i=1}^n (y_{true} - y_{pred})^2$

- n is the number of samples, which is 4 (Alice, Bob, Charlie, Diana).
- y represents the variable being predicted, which is Gender.
- y_{true} is the *true* value of the variable (the "correct answer"). For example, y_{true} for Alice would be 1 (Female).
- ullet y_{pred} is the *predicted* value of the variable. It's whatever our network outputs.

Understanding Loss

- The Keys:
 - Better predictions = Lower loss.
 - Training a network = trying to minimize its loss.

Example

An Example Loss Calculation

Let's say our network always outputs 0 - in other words, it's confident all humans are Male \mathfrak{S} . What would our loss be?

Name	y_{true}	y_{pred}	$(y_{true}-y_{pred})^2$
Alice	1	0	1
Bob	0	0	0
Charlie	0	0	0
Diana	1	0	1

$$MSE = \frac{1}{4}(1+0+0+1) = \boxed{0.5}$$

Here MSE Loss Code

We can convert the procedure as code.

```
import numpy as np

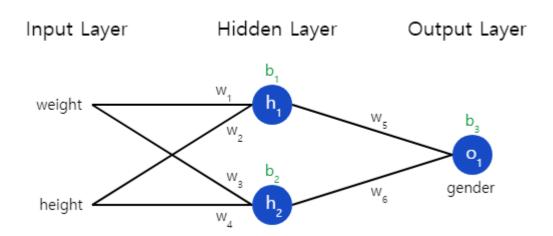
def mse_loss(y_true, y_pred):
    # y_true and y_pred are numpy arrays of the same length.
    return ((y_true - y_pred) ** 2).mean()

y_true = np.array([1, 0, 0, 1])
y_pred = np.array([0, 0, 0, 0])

print(mse_loss(y_true, y_pred)) # 0.5
```

Getting A Little Bit More Difficult

- So-called Backprop
- We now have a clear goal: minimize the loss of the neural network.
- We will skip the details of math here, but general way of updating the parameters here.



BackProp - Mathematics

To start, let's rewrite the partial derivative in terms of $\frac{\partial y_{pred}}{\partial w_1}$ instead:

$$rac{\partial L}{\partial w_1} = rac{\partial L}{\partial y_{pred}} * rac{\partial y_{pred}}{\partial w_1}$$

This works because of the Chain Rule.

We can calculate $rac{\partial L}{\partial y_{pred}}$ because we computed $L=(1-y_{pred})^2$ above:

$$rac{\partial L}{\partial y_{pred}} = rac{\partial (1-y_{pred})^2}{\partial y_{pred}} = \boxed{-2(1-y_{pred})}$$

Now, let's figure out what to do with $\frac{\partial y_{pred}}{\partial w_1}$. Just like before, let h_1, h_2, o_1 be the outputs of the neurons they represent. Then

$$y_{pred} = o_1 = f(w_5h_1 + w_6h_2 + b_3)$$

f is the sigmoid activation function, remember?

BackProp - Mathematics

Since w_1 only affects h_1 (not h_2), we can write

$$egin{aligned} rac{\partial y_{pred}}{\partial w_1} &= rac{\partial y_{pred}}{\partial h_1} * rac{\partial h_1}{\partial w_1} \ & \ rac{\partial y_{pred}}{\partial h_1} &= igg[w_5 * f'(w_5 h_1 + w_6 h_2 + b_3) igg] \end{aligned}$$

More Chain Rule.

We do the same thing for $\frac{\partial h_1}{\partial w_1}$:

$$h_1 = f(w_1x_1 + w_2x_2 + b_1)$$
 $rac{\partial h_1}{\partial w_1} = oxed{x_1 * f'(w_1x_1 + w_2x_2 + b_1)}$

You guessed it, Chain Rule.

 x_1 here is weight, and x_2 is height. This is the second time we've seen f'(x) (the derivate of the sigmoid function) now! Let's derive it:

$$f(x) = rac{1}{1+e^{-x}}$$
 $f'(x) = rac{e^{-x}}{(1+e^{-x})^2} = f(x)*(1-f(x))$

BackProp - Mathematics

We're done! We've managed to break down $\frac{\partial L}{\partial w_1}$ into several parts we can calculate:

$$oxed{rac{\partial L}{\partial w_1} = rac{\partial L}{\partial y_{pred}} * rac{\partial y_{pred}}{\partial h_1} * rac{\partial h_1}{\partial w_1}}$$

We will use this derived equation For actual calculation ©

This system of calculating partial derivatives by working backwards is known as **backpropagation**, or "backprop".

Training: Stochastic Gradient Descent

- We'll use an optimization algorithm called stochastic gradien t descent (SGD) that tells us how to change our weights and b iases to minimize loss.
- It's basically just this update equation:

$$w_1 \leftarrow w_1 - \eta rac{\partial L}{\partial w_1}$$

- η : learning rate
 - If $\frac{\partial L}{\partial w_1}$ is positive, w_1 will decrease, which makes L decrease.
 - If $\frac{\partial L}{\partial w_1}$ is negative, w_1 will increase, which makes L decrease.

Just Jumping into Calculations

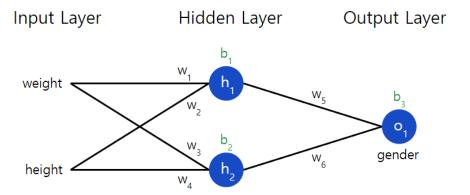
Choose a single item (stochastic) and try to learn from data.

- 1. Choose **one** sample from our dataset. This is what makes it *stochastic* gradient descent we only operate on one sample at a time.
- 2. Calculate all the partial derivatives of loss with respect to weights or biases (e.g. $\frac{\partial L}{\partial w_1}$, $\frac{\partial L}{\partial w_2}$, etc).
- 3. Use the update equation to update each weight and bias.
- 4. Go back to step 1.

Let's Make a Complete Implementation

It's *finally* time to implement a complete neural network:

Name	Weight (minus 135)	Height (minus 66)	Gender
Alice	-2	-1	1
Bob	25	6	0
Charlie	17	4	0
Diana	-15	-6	1



Codes (1)

```
import numpy as np
def sigmoid(x):
 # Sigmoid activation function: f(x) = 1 / (1 + e^{-x})
  return 1 / (1 + np.exp(-x))
def deriv_sigmoid(x):
 # Derivative of sigmoid: f'(x) = f(x) * (1 - f(x))
 fx = sigmoid(x)
  return fx * (1 - fx)
def mse_loss(y_true, y_pred):
 # y_true and y_pred are numpy arrays of the same length.
 return ((y_true - y_pred) ** 2).mean()
```

Codes (2)

```
class OurNeuralNetwork:
''' A neural network with:
- 2 inputs, a hidden layer with 2 neurons (h1, h2), an output layer with 1 neuron (o1)
,,,
def init (self):
 # Weights
 self.w1 = np.random.normal()
 self.w2 = np.random.normal()
 self.w3 = np.random.normal()
 self.w4 = np.random.normal()
 self.w5 = np.random.normal()
 self.w6 = np.random.normal()
 # Biases
 self.b1 = np.random.normal()
 self.b2 = np.random.normal()
 self.b3 = np.random.normal()
def feedforward(self, x):
 # x is a numpy array with 2 elements.
 h1 = sigmoid(self.w1 * x[0] + self.w2 * x[1] + self.b1)
 h2 = sigmoid(self.w3 * x[0] + self.w4 * x[1] + self.b2)
 o1 = sigmoid(self.w5 * h1 + self.w6 * h2 + self.b3)
 return o1
```

Codes (3)

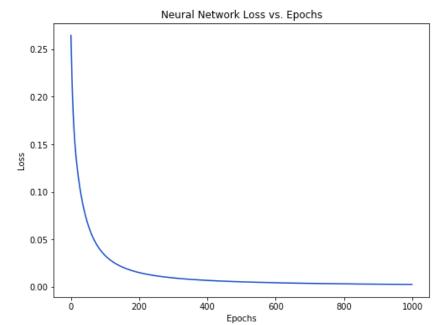
```
def train(self, data, all y trues):
  ''' - data is a (n \times 2) numpy array, n = \# of samples in the dataset.
  - all y trues is a numpy array with n elements.
  Elements in all y trues correspond to those in data. "
 learn rate = 0.1 epochs = 1000
 # number of times to loop through the entire dataset
 for epoch in range(epochs):
   for x, y true in zip(data, all y trues):
     # --- Do a feedforward (we'll need these values later)
     sum h1 = self.w1 * x[0] + self.w2 * x[1] + self.b1 h1 = sigmoid(sum h1)
     sum h2 = self.w3 * x[0] + self.w4 * x[1] + self.b2 h2 = sigmoid(sum h2)
     sum o1 = self.w5 * h1 + self.w6 * h2 + self.b3
     o1 = sigmoid(sum o1) y pred = o1
     # --- Calculate partial derivatives.
     # --- Naming: d L d w1 represents "partial L / partial w1"
     d L d ypred = -2 * (y true - y pred)
     # Neuron o1
     d ypred d w5 = h1 * deriv_sigmoid(sum_o1)
     d ypred d w6 = h2 * deriv sigmoid(sum o1)
     d ypred d b3 = deriv sigmoid(sum o1)
     d ypred d h1 = self.w5 * deriv sigmoid(sum o1)
     d ypred d h2 = self.w6 * deriv sigmoid(sum o1)
```

```
# Neuron h1
d h1 d w1 = x[0] * deriv sigmoid(sum h1)
d h1 d w2 = x[1] * deriv sigmoid(sum h1)
d h1 d b1 = deriv sigmoid(sum h1)
# Neuron h2
d_h2_dw3 = x[0] * deriv_sigmoid(sum_h2)
d_h2_dw4 = x[1] * deriv_sigmoid(sum_h2)
d h2 d b2 = deriv sigmoid(sum h2)
# --- Update weights and biases
# Neuron h1
self.w1 -= learn_rate * d_L_d_ypred * d_ypred_d_h1 * d_h1_d_w1
self.w2 -= learn rate * d L d ypred * d ypred d h1 * d h1 d w2
self.b1 -= learn rate * d L d ypred * d ypred d h1 * d h1 d b1
# Neuron h2
self.w3 -= learn rate * d L d ypred * d ypred d h2 * d h2 d w3
self.w4 -= learn rate * d L d ypred * d ypred d h2 * d h2 d w4
self.b2 -= learn rate * d L d ypred * d ypred d h2 * d h2 d b2
# Neuron o1
self.w5 -= learn_rate * d_L_d_ypred * d_ypred_d_w5
self.w6 -= learn rate * d L d ypred * d ypred d w6
self.b3 -= learn rate * d L d ypred * d ypred d b3
# --- Calculate total loss at the end of each epoch
if epoch % 10 == 0:
  y preds = np.apply along axis(self.feedforward, 1, data)
  loss = mse_loss(all y trues, y preds)
  print("Epoch %d loss: %.3f" % (epoch, loss))
```

Codes (4)

```
# Define dataset
data = np.array([ [-2, -1], # Alice [25, 6], # Bob [17, 4], # Charlie [-15, -6], # Diana ])
all_y_trues = np.array([ 1, # Alice 0, # Bob 0, # Charlie 1, # Diana ])
# Train our neural network!
network = OurNeuralNetwork()
network.train(data, all_y_trues)
```

Training decreases the loss



Making Actual Prediction

```
# Make some predictions
emily = np.array([-7, -3]) # 128 pounds, 63 inches
frank = np.array([20, 2]) # 155 pounds, 68 inches
print("Emily: %.3f" % network.feedforward(emily)) # 0.951 - F
print("Frank: %.3f" % network.feedforward(frank)) # 0.039 - M
```

More Resource

- Victor Zhou's blog page on easy introduction to NN
- https://victorzhou.com/blog/intro-to-neural-networks/#cod e-a-complete-neural-network
- https://replit.com/@vzhou842/An-Introduction-to-Neural-Ne tworks

Lab Session

- Follow the whole code and try to understand.
- Consider which of prediction you can make with this.
 - What is your major?

Dataset Treasure Island: Kaggle

• www.kaggle.com

