

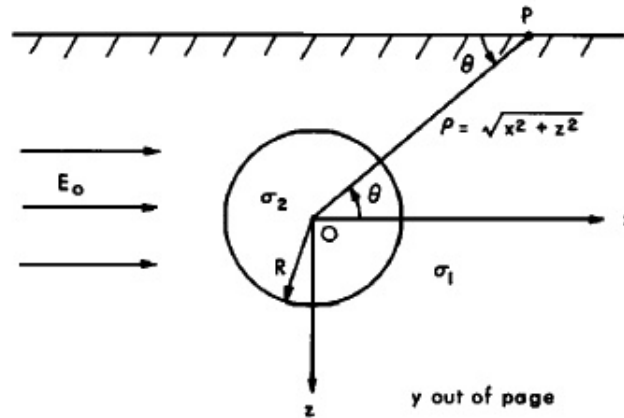
ANALYTICAL RESULTS AND PHYSICAL UNDERSTANDING

TEAM C

Uniform electric field illuminating a sphere in a uniform earth (analytic solution, reference)

Let consider a resistive uniform half-space, of conductivity σ_1 enclosing a conductive sphere σ_2 . Let assume a uniform, unidirectional static electric field E_0 going through this half-space.

FIGURE 1. Uniform electric field illuminating a sphere in a uniform earth



Maxwell equations

In this case, we need:

$$\nabla \times E = 0 \quad , \text{so} \quad E = -\nabla V \quad (1)$$

$$J = \sigma E \quad (2)$$

The primary field E_0 can then be expressed by:

$$E^p_0 = -\frac{dV^p}{dx} \quad (3) \text{ Assuming a primary potential null at the origin:}$$

$$V^p = E_0 x = E_0 r \cos \theta \quad (4)$$

As the primary potential respects $\nabla^2 V = 0$, as only a dependence in x direction, the anomalous or secondary field can be expressed as (using spherical coordinates):

$$V^s = (Ar + Br^{-2}) \cos \theta \quad (5)$$

If we assume finite values of the potential everywhere, we can divide the anomalous potential in two domain:

$$V^s_e = Br^{-2} \cos \theta \quad \text{if } r > R \quad (6)$$

$$V^s_i = Ar \cos \theta \quad \text{if } r < R \quad (7)$$

The total external potential is then:

$$V_e = V^s_e + V^p = (-E_0 r + Br^{-2}) \cos \theta \quad (8)$$

On the surface of the sphere, both the normal current density and potential have to be continuous across the interface.

$$\text{Using the continuity of current density, we got: } \sigma_1 E_e = \sigma_2 E_i \quad \sigma_1 \frac{dV_e}{dr} = \sigma_2 \frac{dV_i}{dr} \quad (9)$$

$$2\sigma_1 BR^{-3} + \sigma_1 E_0 \quad (10)$$

Using the continuity of potential, we got:

$$V_e = V^s_i \quad (11)$$

$$-E_0 R + BR^{-2} = AR \quad (12)$$

From equations (10) and (12), we get:

$$A = -\frac{3\sigma_1}{\sigma_2 + 2\sigma_1} E_0$$

$$B = E_0 R^3 \frac{\sigma_2 - \sigma_1}{\sigma_2 + 2\sigma_1}$$

And the anomalous electric field is:

$$\mathbf{E}_s = -\nabla V^s_e = \mathbf{E}_0 \mathbf{R}^3 \frac{\sigma_2 - \sigma_1}{\sigma_2 + 2\sigma_1} \frac{(2x^2 - y^2 - z^2)\mathbf{u}_x + 3xy\mathbf{u}_y + 3xz\mathbf{u}_z}{r^5}$$

Continuity of current and charge accumulation.

Charges, Coulomb's law and potentials.

Electric charge produces an electric potential; the Coulomb's electrostatic potential is

$$(0.1) \quad V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}.$$

Anomalous currents and electric fields

Anomalous current density is defined as

$$\mathbf{J}_a = \sigma_a \mathbf{E}.$$

Here \mathbf{E} is total electric field and σ_a is the difference between wholespace conductivity σ_1 and conductivity of the target σ_2 : $\sigma_a = \sigma_2 - \sigma_1$.

DC app for looking at currents, charges etc with a current source at the surface.

Analytic solution for a buried sphere in a uniform space