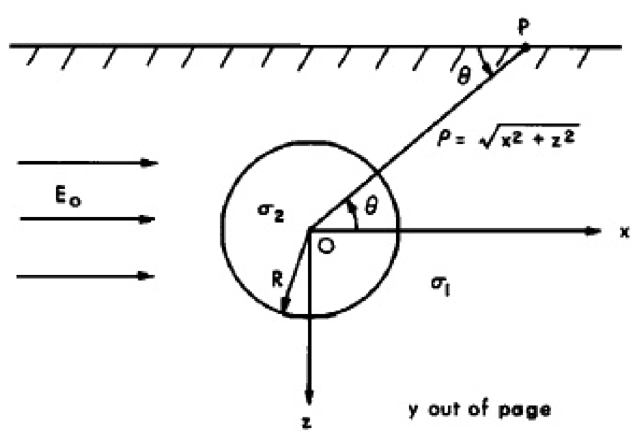
ANALYTICAL RESULTS AND PHYSICAL UNDERSTANDING

TEAM C

Uniform electric field illuminating a sphere in a uniform earth (analytic solution, reference)

Let consider a resistive uniform half-space, of conductivity $\sigma_1$enclosing a conductive sphere σ_2. Let assume a uniform, unidirectional static electric field E_0 going through this half-space.

FIGURE 1. Uniform electric field illuminating a sphere in a uniform earth



Maxwell equations

In this case, we need:

$$\nabla \times E = 0 \quad \text{,so} \quad E = -\nabla V \quad (1)$$

$J = \sigma E \quad (2)$

The primary field E_0 can then be expressed by:

$E^p{}_0 = -\frac{dV^p}{dx} \quad (3)$ Assuming a primary potential null at the origin:

$V^p = E_0 x = E_0 r cos\theta \quad (4)$

As the primary potential respects $\nabla^2 V = 0$, as only a dependence in x direction, the anomalous or secondary field can be expressed as (using spherical coordinates):

$V^s = (Ar + Br^{-2})cos\theta \quad (5)$

If we assume finite values of the potential everywhere, we can divide the anomalous potential in two domain:

$V^s{}_e = Br^{-2}cos\theta \quad \text{if } r > R \quad (6)$

$V^s{}_i = Arcos\theta \quad \text{if } r < R \quad (7)$

The total external potential is then:

$V_e = V^s{}_e + V^p = (-E_0 r + Br^{-2})cos\theta \quad (8)$

On the surface of the sphere, both the normal current density and potential have to be continuous across the interface.

Using the continuity of current density, we got: $\sigma_1 E_e = \sigma_2 E_i \quad \sigma_1 \frac{dV_e}{dr} = \sigma_2 \frac{dV_e}{dr} \quad (9)$

$2\sigma_1 BR^{-3} + \sigma_1 E_0 \quad (10)$

Using the continuity of potential, we got:

$V_e = V^s{}_i \quad (11)$

$-E_0 R + BR^{-2} = AR \quad (12)$

From equations (10) and (12), we get:

$A = -\frac{3\sigma_1}{\sigma_2+2\sigma_1} E_0 \quad (13)$

$B = E_0 R^3 \frac{\sigma_2-\sigma 1}{\sigma_2+2\sigma_1} \quad (14)$

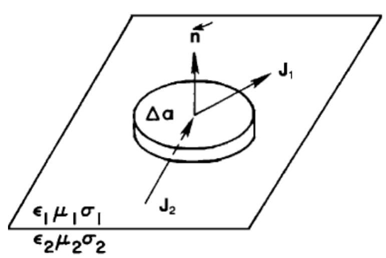
And the anomalous electric field is:

$$\mathbf{E_s} = -\nabla \mathbf{V^s}_\mathbf{e} = \mathbf{E_0 R^3} \frac{\boldsymbol{\sigma_2-\sigma_1}}{\boldsymbol{\sigma_2-2\sigma_1}} \frac{\mathbf{(2x^2-y^2-z^2)u_x+3xyu_y+3xzu_z}}{\mathbf{r^5}} \quad (15)$$

Continuity of current and charge accumulation

We assume to be here in a steady state with direct current. The current entering a cylinder through an interface as in figure 2 consists both in tangential and normal components.

FIGURE 2. Uniform electric field illuminating a sphere in a uniform earth



As the cylinder height is collapsed to zero, we can write the normal component as:
$I = J_1 \cdot \mathbf{n}\Delta a \quad (16)$ or as $I = J_2 \cdot \mathbf{n}\Delta a \quad (17)$

Note: Otherwise in steady state we would have an infinite built up of charges at the interface Then $J_2 \cdot \mathbf{n} = J_1 \cdot \mathbf{n} \quad (18)$

so $\mathbf{J_1}^{\mathbf{n}} = \mathbf{J_2}^{\mathbf{n}}$ (19)

Note: This is only true in a direct current case in a steady state. It appears to be satisfactory up to 10^5 Hz, as long as displacement currents can be considered negligeable.

Charges, Coulomb's law and potentials.

Electric charge produces an electric potential; the Coulomb's electrostatic potential is

$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$. (20)

Anomalous currents and electric fields

Anomalous current density is defined as $\mathbf{J}_a = \sigma_a \mathbf{E}$. (21)

Here $\mathbf{E}$ is total electric field and σ_a is the difference between wholespace conductivity σ_1 and conductivity of the target σ_2: $\sigma_a = \sigma_2 - \sigma_1$.

DC app for looking at currents, charges etc with a current source at the surface.

Analytic solution for a buried sphere in a uniform space