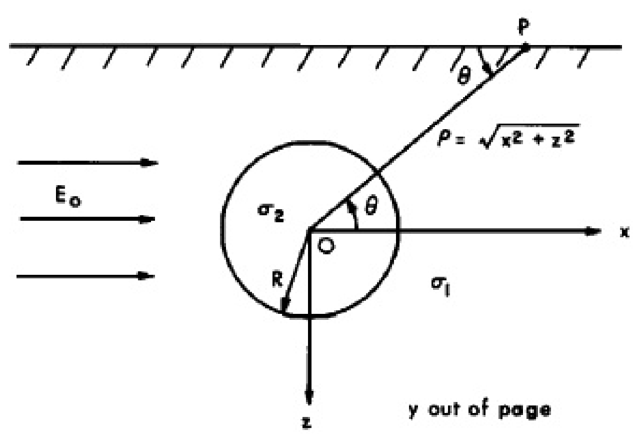
ANALYTICAL RESULTS AND PHYSICAL UNDERSTANDING

TEAM C

Uniform electric field illuminating a sphere in a uniform earth (analytic solution, reference)

Let consider a resistive uniform half-space, of conductivity $\sigma_1$enclosing a conductive sphere σ_2. Let assume a uniform, unidirectional static electric field E_0 going through this half-space.

FIGURE 1. Uniform electric field illuminating a sphere in a uniform earth



Maxwell equations

In this case, we need:

$\nabla \times E = 0 \quad$,so $\quad E = -\nabla V \quad (1)$

$J = \sigma E \quad (2)$

The primary field E_0 can then be expressed by:

$E^p{}_0 = -\frac{dV^p}{dx}$ (3) Assuming a primary potential null at the origin:

$$V^p = E_0 x = E_0 r cos\theta \quad (4)$$

As the primary potential respects $\nabla^2 V = 0$, as only a dependence in x direction, the anomalous or secondary field can be expressed as (using spherical coordinates):

$$V^s = (Ar + Br^{-2})cos\theta \quad (5)$$

If we assume finite values of the potential everywhere, we can divide the anomalous potential in two domain:

$$V^s{}_e = Br^{-2}cos\theta \quad \text{if } r > R \quad (6)$$

$$V^s{}_i = Arcos\theta \quad \text{if } r < R \quad (7)$$

The total external potential is then:

$$V_e = V^s{}_e + V^p = (-E_0 r + Br^{-2})cos\theta \quad (8)$$

On the surface of the sphere, both the normal current density and potential have to be continuous across the interface.

Using the continuity of current density, we got: $\sigma_1 E_e = \sigma_2 E_i \quad \sigma_1 \frac{dV_e}{dr} = \sigma_2 \frac{dV_e}{dr}$ (9)

$$2\sigma_1 BR^{-3} + \sigma_1 E_0 \quad (10)$$

Using the continuity of potential, we got:

$$V_e = V^s{}_i \quad (11)$$

$$-E_0 R + BR^{-2} = AR \quad (12)$$

From equations (10) and (12), we get:

$$A = -\frac{3\sigma_1}{\sigma_2 + 2\sigma_1} E_0$$

$$B = E_0 R^3 \frac{\sigma_2 - \sigma 1}{\sigma_2 + 2\sigma_1}$$

And the anomalous electric field is:

$$\mathbf{E_s} = -\nabla \mathbf{V^s{}_e} = \mathbf{E_0 R^3} \frac{\boldsymbol{\sigma_2 - \sigma_1}}{\boldsymbol{\sigma_2 - 2\sigma_1}} \frac{\mathbf{(2x^2 - y^2 - z^2)u_x + 3xyu_y + 3xzu_z}}{\mathbf{r^5}}$$

Continuity of current and charge accumulation.

Charges, Coulomb?s law and potentials.

Anomalous currents and electric fields

DC app for looking at currents, charges etc with a current source at the surface.

Analytic solution for a buried sphere in a uniform space