ANALYTICAL RESULTS AND PHYSICAL UNDERSTANDING OF DEMO: UNIFORM ELECTRIC FIELD ILLUMINATING A SPHERE IN A UNIFORM EARTH

Let consider a resistive uniform wholespace enclosing a sphere with conductivity σ_1 and radius R. The background has conductivity σ_0. We have uniform, unidirectional, static electric field E_0 going through this space.

Related Maxwell's equations:

$$\begin{aligned} \nabla \times E &= 0 \\ \nabla \times H &= J \\ J &= \sigma E. \end{aligned} \tag{0.1}$$

E_0 induces charge in the sphere which induces new E_s ? TODO: What's really happening?

1. Potentials

Total potential outside the sphere can be expressed as

$$V_t = -E_0 x \left(1 - \frac{\sigma_1 - \sigma_0}{\sigma_1 + 2\sigma_0} \frac{R^3}{r^3}\right) \tag{1.1}$$

and inside the sphere

$$V_t = -E_0 x \frac{3\sigma_0}{\sigma_1 + 2\sigma_0}. \tag{1.2}$$

The primary potential is related to the uniform electric field and the secondary is

2. Electric Fields

Since the first Maxwell's equation the electric field equals to the negative gradient of the potential:

$$E = -\nabla V \tag{2.1}$$

TODO: Why electric field only is in only x direction inside the sphere? Why there is discontinuity?

3. Current Densities

For current densities and electric field there is the following relation:

$$J = \sigma E. \tag{3.1}$$

The total current density is (always?) continuous, but as can be seen from the pictures of our demo, the primary and the secondary current densities can be highly discontinuous.

4. Charge Accumulation

Gauss's law of Maxwell's equation says that

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}, \tag{4.1}$$

where ρ is the total electric charge density. Charge is build on the surface of the sphere.

See EMGroupArchiveJuly2012.pdf and Ward and Hohmann for more details.