Brief answers and hints to some of the past FPM exam questions.

- May 2013: solutions will be posted on Learn after the revision workshop in week 11.

- August 2013:

  1. Look at orders of groups, orders of elements, which groups are abelian.
  2. Many possible examples.
  3. (a) Use subgroup test, (b) similar to problem 4.6.
  4. (a) True, (b) False, (c) False
  5. Similar to Problem 40.
  6. Similar to Problem 26.
  7. (c) the map from (b)(ii) works here (d) False
  8. (a) (ii) Similar to Problem 6.19, (iii) False (b) (iii) True (iv) False.
  9. (a) Use the comparison test, (b) $p > 0$, (c) (i) True, (ii) False.
  10. (b) $1 + \frac{1}{2}x - \frac{1}{8}x^2$, (c) Similar to Problem 44.

- May 2014:

  1.
  2. (a) True, (b) False, (c) False.
  3. (b) Use the subgroup test.
  4. (a) True, (b) False , (c) True.
  5. See May 2013 exam, Problem 4b.
  6. Similar to Problem 33. The limit is $\sqrt{2}$.
  7.
  8. (a) (iii) Similar to Problem 6.19. (b)(ii) (A) True (B) False (C) True.
  9. See Problem 76.
  10. (a) There is a typo on the exam paper. The value of f at zero should be 1 (not 0). Answer is: $1 - \frac{x^2}{3!} + \frac{x^4}{5!}$. (b) $\frac{2^6}{7!}$. (c) Hint: write $\sin x = \sin x - \sin 0$ and use the Mean Value Theorem.

- August 2014:

  1. (a) True (b) False (c) False

2. (a) True (b) False (c) False

3. (b) 900 is the LCM of 25 and 36; use Lagrange's Theorem.

4. (a) False, (b) True, (c) False .

5. This is essentially based on the material in the enrichment section 2.5 which may have been covered in previous years. It is not on the current syllabus.

6. Radius of convergence is $R = 1$. For $|x| < 1$ the series converges absolutely. For $|x| > 1$ the series diverges. For $x = 1$ the series converges conditionally. For $x = -1$ the series diverges.

7. (a) 8, (c) $(n^8 + 4n^5 + 5n^4 + 2n^2 + 4n)/16$.

8. (b) $\mathbb{Z}_3$ works (c) (i) True, (ii) False, (iii) True, (iv) False.

9. (a) and (b) Use the Intermediate Value Theorem, (c) See Problem 78, (d) $f(x) = \frac{x^2}{1+x^2} \sin(1/x^2)$ for $x \neq 0$, $f(0) = 0$.

10. Similar to Problem 59.

- May 2015: all solutions have been posted on Learn.

- August 2015:

    1. (a) False (b) False (c) True

    2. (a) False (b) True (c) False

    3. Similar to Rules for Cosets (Thm 3.2.6).

    4. (a) True, (b) True, (c) False.

    5. This is essentially based on the material in the enrichment section 2.5 which may have been covered in previous years. It is not on the current syllabus.

    6. See Problem 31.

    7. (b) (i) 5 orbits: one containing 3 vertices, one containing 2 vertices, 3 containing one vertex. (c) (i) False (ii) True.

    8. (a) Similar to Problem 4.1. (b) (i) False (ii) True (iii) TRUE (c) Let G be the subgroup $\{e, h\}$ of $H = D_3$ (where h is a reflection) and let θ be the inclusion $G \subseteq H$.

    9. Similar to Problem 59.

    10. (a) Use the ratio test and the inequality $|\sin x| \leq |x|$, (b) See August 2013, Problem 9b, (c) (i) True, (ii) True, (iii) False.