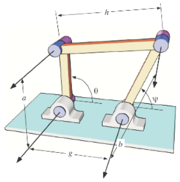
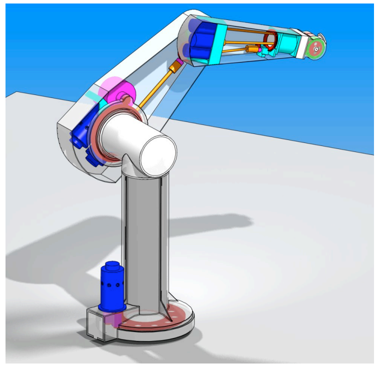


Elementary Robotics

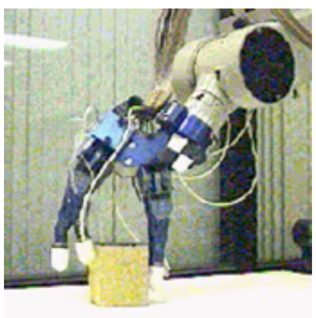
J. Michael McCarthy

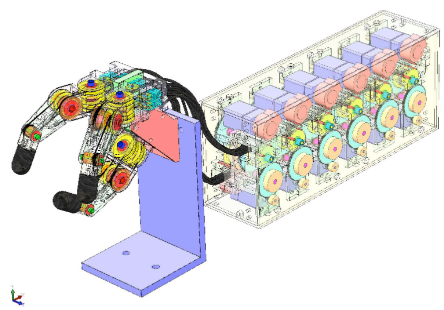
July 15, 2009

Examples of Robots

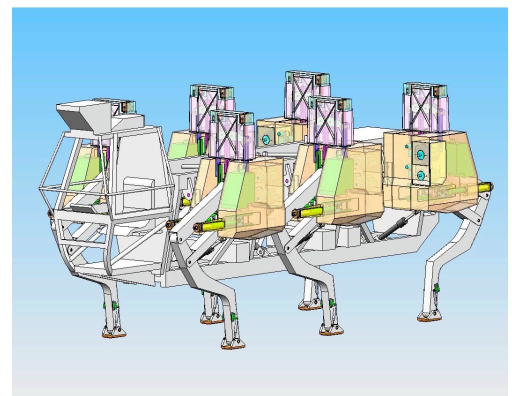


The PUMA robot developed Vic Scheinman in the late ‘70s was the first robot with an elbow.

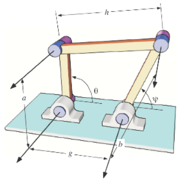




The Stanford-MIT hand designed by Ken Salisbury in the early ‘80s was the first robotic hand with articulated fingers.

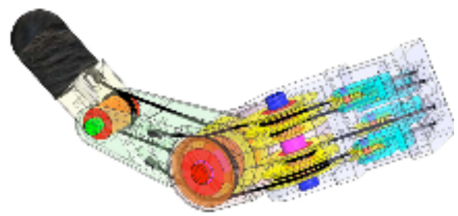


The Ohio State Adaptive Suspension Vehicle PUMA robot developed Ken Waldron was the first walking machine designed to carry a driver.

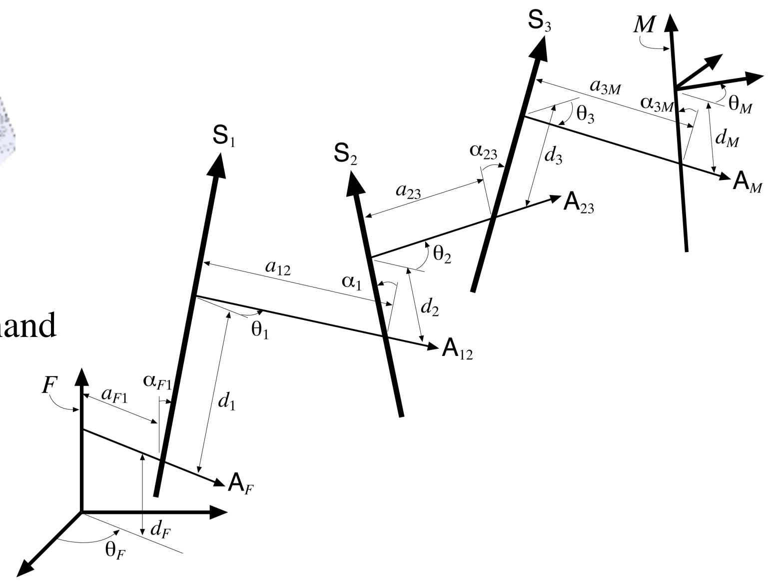


Serial Chains of Links

The Adept, Inc. SCARA robot



One finger of the Salisbury hand

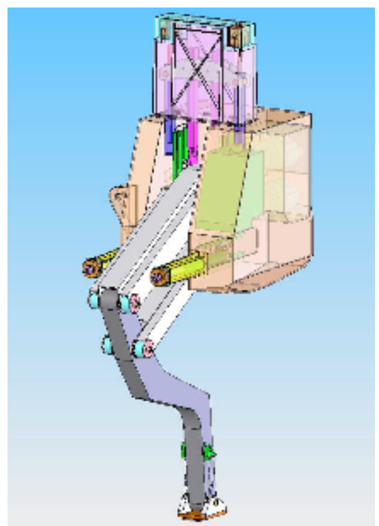


A **robot** is modeled as a sequence links connected by joints.

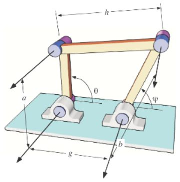
The joint axes are represented by lines in space denoted S_1, S_2, … S_6.

The common normal line A_{ij} between the two axes S_i and S_j define the *i*th link.

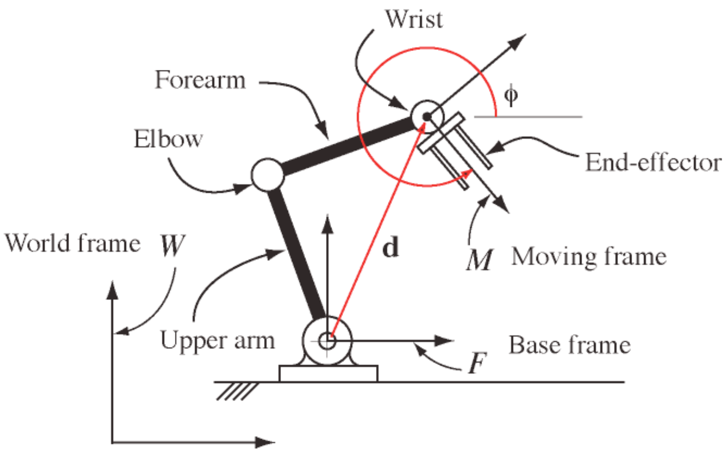
The Canadarm on the Space Shuttle



One leg of the ASV

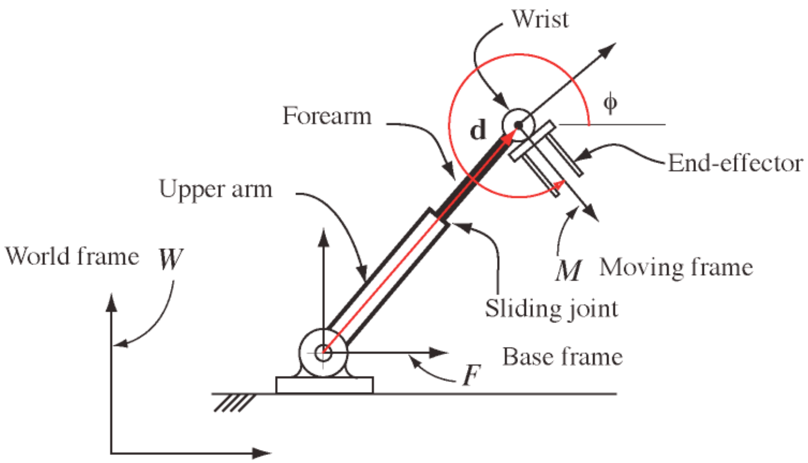


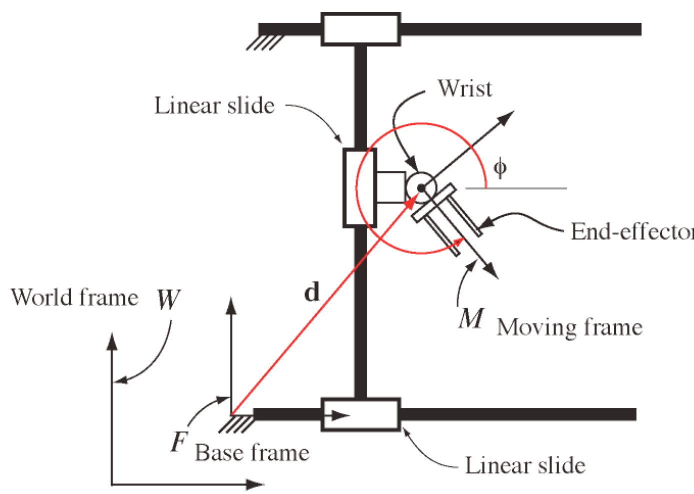
Planar Serial Chain Robots

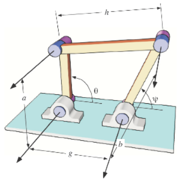


Planar robots come in three basic forms the RRR, RPR and PPR robots. In each case the purpose of the robot is to reliably position its moving frame *M* relative to its base frame *F*. This is how it positions the end-effector in the world frame.

This means the robot must precisely control the three parameters **d**=(x,y) and ϕ. The vector **d**=(x, y) defines the origin of *M* measured in *F*, and ϕ defines the angle measured from the *x*-axis of *F* to the *x*-axis of *M*.

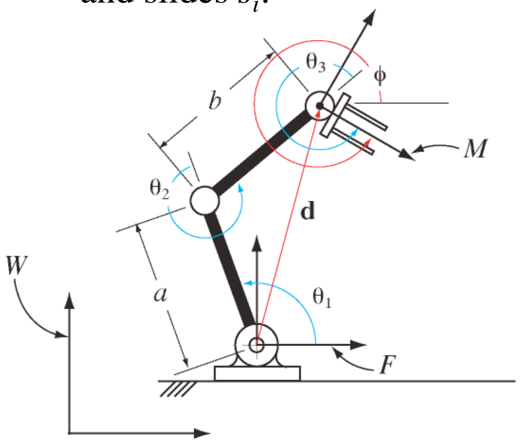






Forward Kinematics

The **forward kinematics equations** of serial chain define the location of the end-effector in terms of its joint parameters. In particular, they define the parameters **d** = (x, y) and ϕ in terms of the joint angles θ_i and slides s_i.

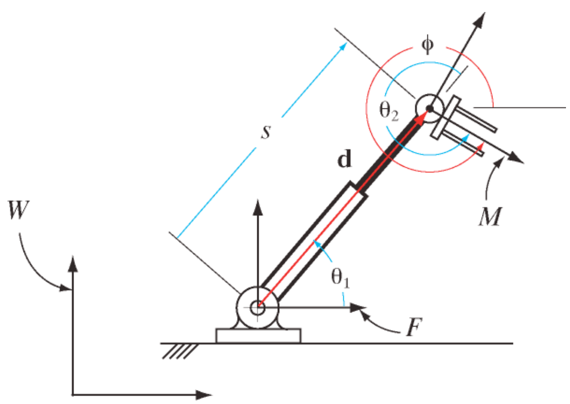


RRR:

$$\mathbf{d} = \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} a\cos\theta_1 \\ a\sin\theta_1 \end{Bmatrix} + \begin{Bmatrix} b\cos(\theta_1+\theta_2) \\ b\sin(\theta_1+\theta_2) \end{Bmatrix},$$
$$\phi = \theta_1 + \theta_2 + \theta_3.$$

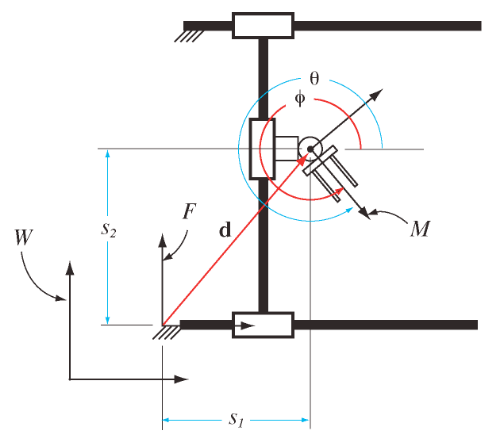
PPR:

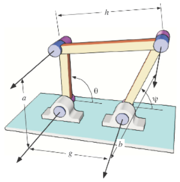
$$\mathbf{d} = \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} s_1 \\ s_2 \end{Bmatrix},$$
$$\phi = \theta.$$



RPR:

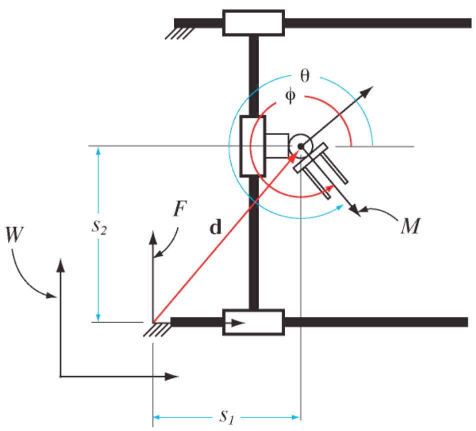
$$\mathbf{d} = \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} s\cos\theta_1 \\ s\sin\theta_1 \end{Bmatrix},$$
$$\phi = \theta_1 + \theta_2.$$





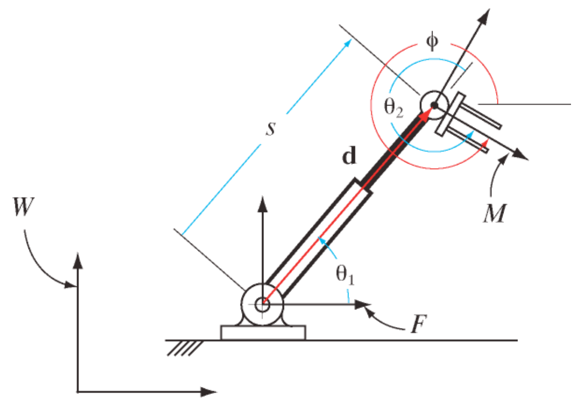
Inverse Kinematics

Inverse kinematics are the equations that compute the joint parameters of a serial chain that are needed to position the end-effector at a given location. This means the parameters **d** = (x, y) and ϕ are specified, and the equations define joint angles θ_i and slides s_i.



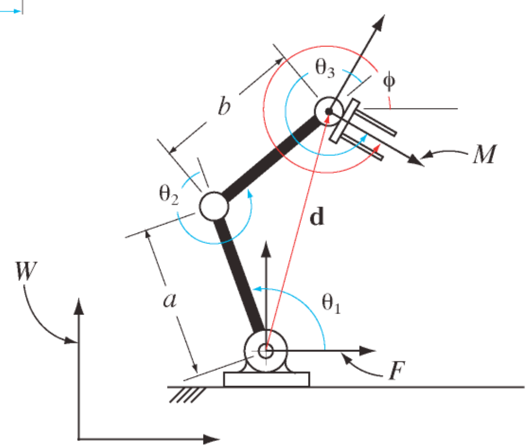
PPR:

$$s_1 = x,$$
$$s_2 = y,$$
$$\theta = \phi.$$



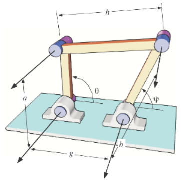
RPR:

$$s = \sqrt{x^2 + y^2},$$
$$\theta_1 = \arctan\left(\frac{y}{x}\right),$$
$$\theta_2 = \phi - \theta_1.$$

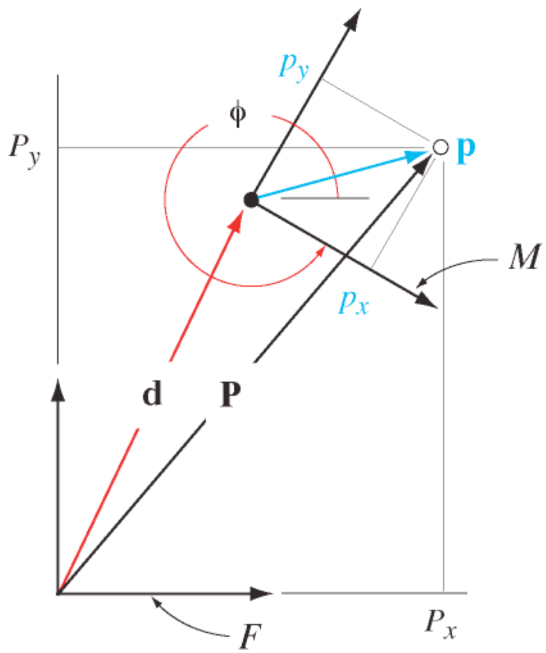


RRR:

$$d = \sqrt{x^2 + y^2}, \quad \eta = \arctan\left(\frac{y}{x}\right),$$
$$d^2 = a^2 + b^2 + 2ab\cos\theta_2,$$
$$\text{therefore,} \quad \theta_2 = \arccos\left(\frac{d^2 - a^2 - b^2}{2ab}\right),$$
$$\theta_1 = \eta - \arctan\left(\frac{b\sin\theta_2}{a + b\cos\theta_2}\right),$$
$$\theta_3 = \phi - \theta_1 - \theta_2.$$



Coordinate Transformations



The movement of one body relative to another is described mathematically by introducing ground frame F and a moving frame M attached to the body.

A point $\mathbf{p} = (p_x, p_y)$ in M coincides with a point $\mathbf{P} = (P_x, P_y)$ in F defined by the *coordinate transformation*:

$$\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \end{Bmatrix} + \begin{Bmatrix} x \\ y \end{Bmatrix},$$

or $$\mathbf{P} = [A(\phi)]\mathbf{p} + \mathbf{d}.$$

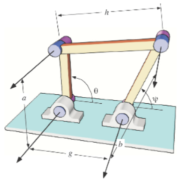
The 2x2 rotation matrix $[A(\phi)]$ and the 2x1 translation vector **d** define the position of the moving frame M relative to F.

As the body moves the parameters $\mathbf{d}(t)$ and $\phi(t)$ vary with time defining the trajectory $\mathbf{P}(t)$ of the point **p** moving in F.

$$\mathbf{P}(t) = [A(\phi(t))]\mathbf{p} + \mathbf{d}(t).$$

Notice that in this equation the coordinates **p** in M are constant. This is the point in the moving body that traces the trajectory $\mathbf{P}(t)$ in F.

A robot can be viewed as a device that is designed to control the parameters d and ϕ in order to position an end-effector frame M relative to the base frame F.



Summary

Robots are designed to position an end-effector within its workspace. *Position* means to locate the origin of the moving frame *M* at a specific point **d** in the base frame *F*, and to orient the moving frame at a specific angle ϕ ρεlative to the base frame.

Planar robots have the form of RRR, RPR and PPR serial chains. The planar RRR is often called a *SCARA robot* for Selective Compliant Articulated Robot for Assembly. The RPR is called a *cylindrical robot*, and the PPR is called a *Cartesian robot*.

The *forward* and *inverse kinematics equations* of the robot are the primary mathematical tools that are used to program its movement..