Dimensional synthesis of a leg mechanism

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Abstract. An eight bar leg mechanism dimensional synthesis is presented. The mathematical model regarding the synthesis is described and the results obtained after computation are verified with help of 2D mechanism simulation in Matlab. This mechanism, inspired from proposed solution of Theo Jansen, is integrated into the structure of a 2 DOF quadruped robot. With help of the kinematic synthesis method described, it is tried to determine new dimensions for the mechanism, based on a set of initial conditions. These are established by taking into account the movement of the end point of the leg mechanism, which enters in contact with the ground, during walking. An optimization process based on the results obtained can be conducted further in order to find a better solution for the leg mechanism.

1. Introduction

Designing mobile robots involves a number of problems related to creating systems that respect certain features motion. Synthesis of mechanisms refers to the design of an appropriate mechanism that corresponds to a prescribed movement or trajectories, of speeds and elements on the basis of several methods and techniques available in the scientific literature [1], [2], [3].

In determining the structure and geometry of a mechanism to carry out the laws of motion imposed by the theme, careful consideration should be taken regarding other factors such as the condition of existence of the crank, the possibility of transmitting forces in good dynamic conditions (by limiting the angle of transmission) so to avoid self-locking, acceptable dimension, economy, reliability [4].

In general, we can distinguish three types of requirements for kinematic synthesis of mechanisms by: generating of a function, generating of a movement or of a trajectory [3]. Kinematic synthesis aims at determining the mechanisms kinematic dimensions of composing elements, while the various conditions are imposed on the movement of points or elements. In the most common problems of synthesis are prescribed conditions on the positions of points or elements, so are used for writing the synthesis equations only the transmission functions of the positions. There are situations where it is required also to respect some requirements regarding the distribution of speed and / or acceleration and therefore will consider higher order transmission functions.

The synthesis of mechanisms using also resources and computational methods has been a research topic in the last years [5], [6], [7].

The aim of the paper consists in presenting a variant of dimensional synthesis for the eight-bar Jansen type of leg mechanism used in the structure of a quadruped robot. The mathematical model for the synthesis of the mechanism was computed in Mathcad environment and the results were verified using a simulating program developed inMatlab. This type of leg mechanism was considered for its advantages regarding the reduced number of DOF's which makes it easier to control, the scalable

design, the reduced impact on the ground during walking and because of less energy consumption [8],[9]. Analysis regarding the kinematics and kinetostatics of the leg mechanism were presented in [10]. Details about the design, simulation and construction of the quadruped robot based on the investigated leg mechanism could also be found in [11].

The synthesis of the kinematic chain assimilated to the leg of the walking robot, in addition to those mentioned earlier, must take into account the conditions of contact with the ground. It is therefore necessary motion analysis for the tibia part of the foot in contact with the ground and setting assumptions for the kinematic chain synthesis according to these conditions.

2. Dimensional synthesis

The dimensional synthesis of the kinematic chain presented in figure 1, involves determination of lengths values for component elements of the mechanism (l_j, with j= 0,...,7) using analytical equations with complex numbers based on an algorithm of synthesis. Constant length of the fixed element is selected initially.

In general, a synthesis problem can be reduced to generate functional dependencies between positional parameter of an element of the mechanism and particularly chosen parameter for the synthesis problem. This functional dependence represents the function which should be generated [7].

In the analysed synthesis problem, the positional parameter is the φ_1 angle, which corresponds to the driving element.

The end point F, situated at the contact with the ground describes during movement an irregularly ovoid trajectory depending on the positional parameters of the drive and driven elements, on the lengths of the elements and on the coordinates of the fixed joints (x_{O1}, y_{O1}).

The crank is considered as input element and realise a complete rotation during a walking cycle. The rocker elements O_2B and O_2E achieve a generating function dependent of φ_1 angle. In the right kinematic chain from figure 1 the kinematic joint E is considered in point E' having the angular position $\varphi_5 + \delta$ towards fixed joint O_2. The length of O_2E' element is identical with the one of the rocker O_2E ($l_5 = l_{5\_1}$). The fix element O_1O_2 is situated along O_1x axis in the negative way.
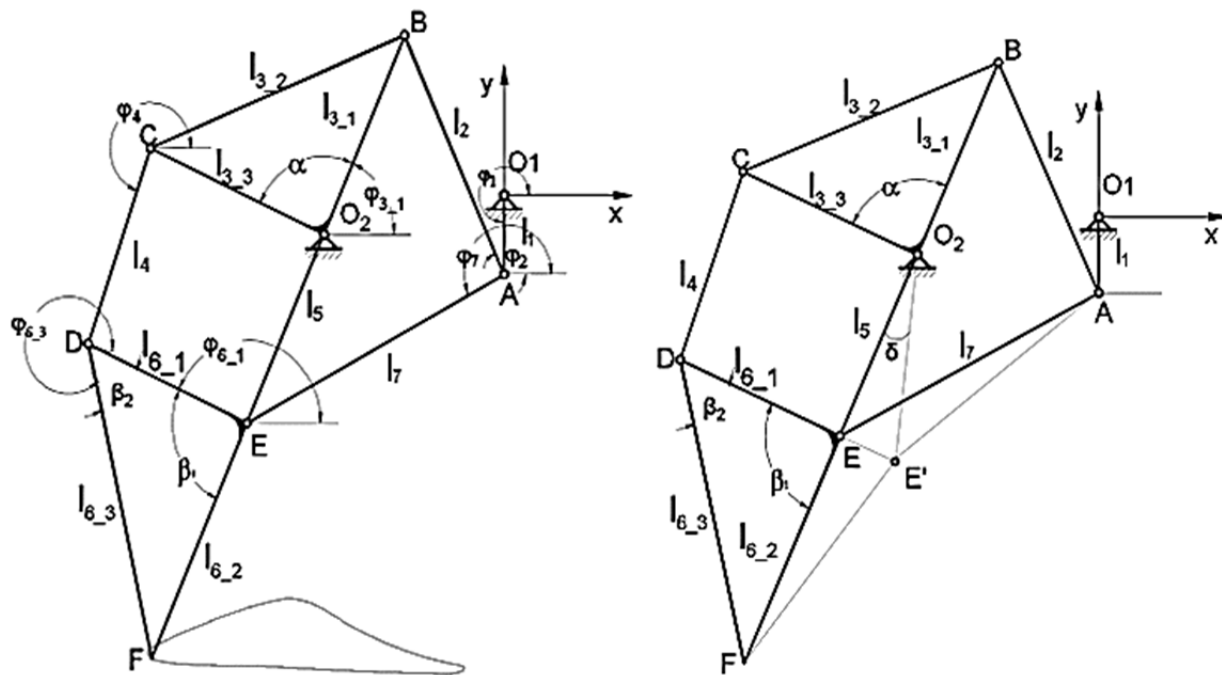

Figure 1. Scheme of the leg mechanism considered for synthesis.

The position of the point F, towards the coordinate system established in xO_1y can be written from analytical point of view in two ways.

The first way involves writing equation departing from point O_1, following points O_2, E and final point F:

$$F = -l_0 + l_5 \cdot e^{i\varphi_5} + l_{6\_2} \cdot e^{i(\varphi_{6\_1} + \beta_1)} \tag{1}$$

The position angle of the fixed element is considered equal to Π (180°). By rewriting the Equation 1 and adding the equation in complex conjugate form is obtained:

$$
\begin{aligned}
x_F + i \cdot y_F &= -l_0 + l_5 \cdot e^{i\varphi_5} + l_{6\_2} \cdot e^{i(\varphi_{6\_1} + \beta_1)} \\
x_F - i \cdot y_F &= -l_0 + l_5 \cdot e^{-i\varphi_5} + l_{6\_2} \cdot e^{-i(\varphi_{6\_1} + \beta_1)}
\end{aligned} \tag{2}
$$

By multiplying the above relations, it results:

$$
\begin{aligned}
x_F^2 + y_F^2 &= l_0^2 + l_5^2 + l_{6\_2}^2 - A + B \\
A &= 2l_0 l_5 \cos\varphi_5 + 2l_0 l_{6\_2} \cos(\varphi_{6\_1} + \beta_1) \\
B &= 2l_5 l_{6_2} \cos(\varphi_{6\_1} - \varphi_5 + \beta_1)
\end{aligned} \tag{3}
$$

Further, are considered 4 points (1,..., 4) from the trajectory described by the point F corresponding to the tibia part of the leg (figure 2). The chosen points correspond to four angular positions of the input element (φ_{1-1}, φ_{1-2}, φ_{1-3} respective φ_{1-4}). With the movement of point F along the path from point 1 to point 4 occurs also an angular variation of the positional parameter φ_1.

This angular variation of the angle of entry causes a variation of the angular position corresponding to the elements that compose the mechanism.

Figure 2. The position on the path curve of the four points considered.

This is considered:

- For point 1 (the starting point): $\varphi_1 = \varphi_{1-1} = \varphi_0$ (initial phase), $\varphi_{3\_1} = \varphi_{3\_1_0}$, $\varphi_5 = \varphi_{5_0}$, $\varphi_{6\_1} = \varphi_{6\_1_0}$;

- For point 2 (an intermediate point): $\varphi_1 = \varphi_{1-2} = \varphi_0 + \varphi_{12}$, $\varphi_{3\_1} = \varphi_{3\_1_0} + \varphi_{3\_12}$, $\varphi_5 = \varphi_{5_0} + \varphi_{512}$, $\varphi_{6\_1} = \varphi_{6\_1_0} = \varphi_{6\_1_{12}}$;

- For point 3 (another intermediate point): $\varphi_1 = \varphi_{1-3} = \varphi_0 + \varphi_{13}$, $\varphi_{3\_1} = \varphi_{3\_1_0} + \varphi_{3\_13}$, $\varphi_5 = \varphi_{5_0} + \varphi_{513}$, $\varphi_{6\_1} = \varphi_{6\_1_0} = \varphi_{6\_1_{13}}$;

- For point 4 (final point): $\varphi_1 = \varphi_{1-4} = \varphi_0 + \varphi_{14}$, $\varphi_{3\_1} = \varphi_{3\_1_0} + \varphi_{3\_14}$, $\varphi_5 = \varphi_{5_0} + \varphi_{514}$, $\varphi_{6\_1} = \varphi_{6\_1_0} = \varphi_{6\_1_{14}}$.

The transmission equation can be written as:

$$F_1\left(\varphi_1, \varphi_{6\_1}, \varphi_5\right) : l_0^2 - x_F^2 - y_F^2 + l_5^2 + l_{6\_2}^2 - A + B \tag{4}$$

From Equation 4, it can be seen that point F and its trajectory depends on parameter φ_1, on the geometry of the mechanism and on the lengths of the elements.

In the calculation algorithm for the synthesis described in detail in [11], conducted in the working environment Matlab, transmission equation (4) is written for four angular positions (φ_{1-1}, φ_{1-2}, φ_{1-3}, respective φ_{1-4}) resulting a system with 4 equations.

The input data for the transmission function $F_1(\varphi_1, \varphi_{6\_1}, \varphi_5)$ are the positional parameters, β_1 angle and the length of fixed element (l_0). The unknowns are the lengths l_5, l_{6_2}, φ_{5_0} respective angle $\varphi_{6\_1_0}$.

By rewriting the position equation of point F starting from joint O_1 to O_2, the joint C, D, E and final point F results:

$$F_2\left(\varphi_1, \varphi_{3\_1}, \varphi_{6\_1}\right): l_0^2 + x_F^2 + y_F^2 + l_4^2 + l_{3\_2}^2 + A_1 + B_1 + B_2 + C$$

$$A_1 = -2l_0 x_F - 2l_{3\_2} x_F \cos(\varphi_{3\_1} + \alpha) - 2l_{3\_2} y_F \sin(\varphi_{3\_1} + \alpha) + 2l_{6\_1} x_F \cos\varphi_{6\_1} + 2l_{6\_1} y_F \sin\varphi_{6\_1}$$

$$B_1 = 2l_0 l_{6\_1} x_F cos(\varphi_{6\_1} + \beta_1) - 2l_{6\_2} y_F \sin(\varphi_{6\_1} + \beta_1) - 2l_0 l_3 \cos(\varphi_{3\_1} + \alpha) \qquad (5)$$

$$B_2 = 2l_0 l_{6\_1} x_F \cos\varphi_{6\_1} - 2l_0 l_{6\_2} \cos(\varphi_{6\_1} + \beta_1)$$

$$C = -2l_{3\_2} l_{6\_1} \cos(\varphi_{3\_1} - \varphi_{6\_1} + \alpha) + 2l_{3\_2} l_{6\_2} \cos(\varphi_{3\_1} - \varphi_{6\_1} + \alpha - \beta_1) - 2l_{6\_1} l_{6\_2} \cos\beta_1$$

In this case the data consist of known angles $\varphi_{6\_1_0}$, $\varphi_{6\_2_0}$ respective the lengths l_0 and l_{6_2} and unknown are the lengths l_{3_2}, l_5, l_{6_1} respective angle $\varphi_{3\_1_0}$.

For the four bar mechanism O_1ABO_2, by writing the closed loop contour method in complex numbers:

$$l_1 e^{i\varphi_1} + l_2 e^{i\varphi_2} = -l_0 + l_{3\_1} e^{i\varphi_{3\_1}} \qquad (6)$$

The resulted transmission equation is:

$$G\left(\varphi_0, \varphi_{6\_1}, \varphi_5\right): l_0^2 - x_F^2 - y_F^2 + l_5^2 + l_{6\_2}^2 - D + E$$

$$D = 2l_0 l_5 cos\varphi_5 - 2l_0 l_{6\_2} cos(\varphi_{6_1} + \beta_1) \qquad (7)$$

$$E = 2l_5 l_{6_2} cos(\varphi_{6_1} - \varphi_5 + \beta_1)$$

Similarly, for the four bar mechanism O_1AEO_2 after closing the vector loop contour, and taking the projection of vector loop equation towards the two axes of the coordinate system and eliminating the parameter φ_7, gives the equation of transmission $H(\delta, l_{5\_1}, l_0, l_7)$.

The unknowns of this equation elements consist of lengths l_{5_1}, l_7, l_0 and respective on positional parameters δ and φ_{5_1}.

Synthesis algorithm for the analyzed leg mechanism implies passing through the following steps:

▪ Introduction of initial data (Cartesian coordinates of the four positions through which it describes the trajectory point F: (x_{F1}, y_{F1}), ..., (x_{F4}, y_{F4}), respective the angular positions of the mechanism elements corresponding to the four successive positions: $\varphi_{1-1, ..., }\varphi_{1-4}$);

▪ Writing the synthesis equations for:

- The polygonal contour loop O_1O_2EF which describes the trajectory F: $F_{11}\left(\varphi_1, \varphi_{6\_1}, \varphi_5\right)$, $F_{12}\left(\varphi_1, \varphi_{6\_1}, \varphi_5\right)$, $F_{13}\left(\varphi_1, \varphi_{6\_1}, \varphi_5\right)$, $F_{14}\left(\varphi_1, \varphi_{6\_1}, \varphi_5\right)$;

- The polygonal contour loop O_1O_2CDEF which describes the trajectory F: $F_{21}\left(\varphi_1, \varphi_{3\_1}, \varphi_{6\_1}\right)$, $F_{22}\left(\varphi_1, \varphi_{3\_1}, \varphi_{6\_1}\right)$, $F_{23}\left(\varphi_1, \varphi_{3\_1}, \varphi_{6\_1}\right)$, $F_{24}\left(\varphi_1, \varphi_{3\_1}, \varphi_{6\_1}\right)$;

- The four bar mechanism O_1ABO_2: $G_1\left(\varphi_0, l_{3\_1}, l_2, l_1\right)$, $G_2\left(\varphi_0, l_{3\_1}, l_2, l_1\right)$, $G_3\left(\varphi_0, l_{3\_1}, l_2, l_1\right)$, $G_4\left(\varphi_0, l_{3\_1}, l_2, l_1\right)$;

- The four bar mechanism O_1AEO_2: $H_1\left(\delta, l_{5\_1}, l_0, l_7\right)$, $H_2\left(\delta, l_{5\_1}, l_0, l_7\right)$, $H_3\left(\delta, l_{5\_1}, l_0, l_7\right)$, $H_4\left(\delta, l_{5\_1}, l_0, l_7\right)$.

- Determination of the sixteen unknown values (the lengths of the elements and angular positions associated to these) from the sixteen linear equation systems. The angles α and β_1 are determined from the triangles O_2CB respective DEF

The initial link lengths considered and the new link dimensions obtained after synthesis are presented in Table 1.

In figure 3 are comparedthe initial configuration of the leg mechanism with the one obtained after the synthesis process conducted.

The results regarding the shape of the trajectory described by the end point F were simulated in Matlab.

Table 1. The initial and the new link lengths considered.

| Elements	Initial lengths [mm]	Description	Synthesis link lengths [mm]
x_{B0}	-38	x coordinate of joint B_0	-39.4
y_{B0}	-7.5	y coordinate of joint B_0	-6.8
l_1	15	crank O_1A	15.47
l_2	50	connecting rod	51.74
$l_{3\ 1}$	41.5	rocker BO_2	42
$l_{3\ 2}$	40.1	element BC	41.3
l_4	39.3	element CD	42.2
l_5	39.4	rocker O_2E	40.6
$l_{6\ 1}$	36.7	element DE	38
$l_{6\ 2}$	49	element EF	52
l_7	61.9	connecting rod EA	63
$\alpha=\beta_1$	90°	angles of ternary elements	90°
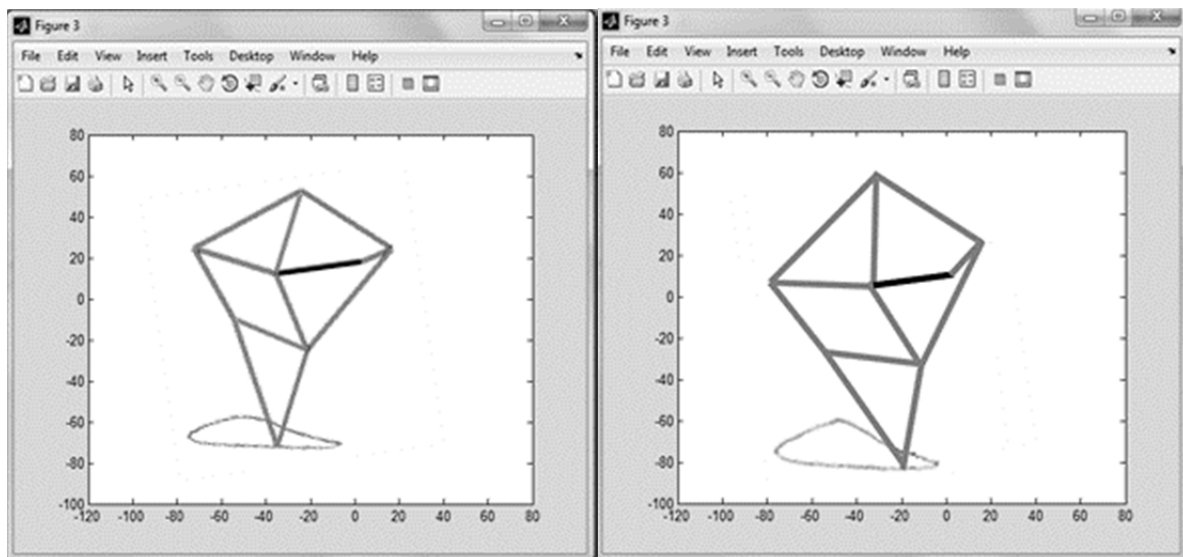
Figure 3. The comparison of the two leg mechanisms analyzed (in the left is the initial configuration and in the right is the configuration obtained after synthesis).

3. Conclusions

The advantage of the geometric model obtained figure 3 in the right, compared to the originally considered consists of a greater step heightlength that helps the robotic structure to overcome larger obstacles.

The problem identified for the analyzed configuration is that the control of the leg mechanism for walking in environments with obstacles is conditioned to the number of DOFs possessed. In this situation,the degrees of freedom (DOFs) are limited to one.Additional conditions are necessary for the synthesis in order to obtain a better trajectory that will ensure a stable walking. First is necessary to find a practical solution for increasing the number of DOFs at two, in order to improve the control of the leg during walking. It is taken into consideration the possibility to use another actuator that will be placed at the hip joint.

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