DIGITAL SYSTEMS AND COMPUTER ORGANIZATION

Text Book:

Stephen Brown and Zvonko Vranesic, Fundamentals of Digital Logic with Verilog Design (3e), Tata McGraw Hill 2014

Logic Gates

- Digital circuits are hardware components that manipulate binary information.
- The circuits are implemented using transistors and interconnections in complex semiconductor devices called *integrated circuits*.
- Each basic circuit is referred to as a *logic gate*

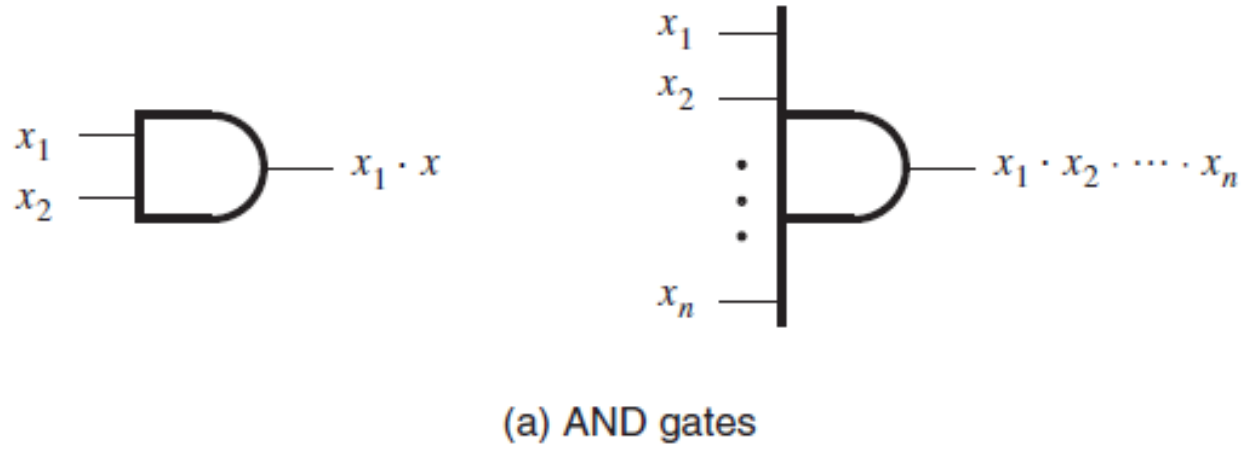
Truth Tables

x_1	x_2	$x_1 \cdot x_2$	$x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

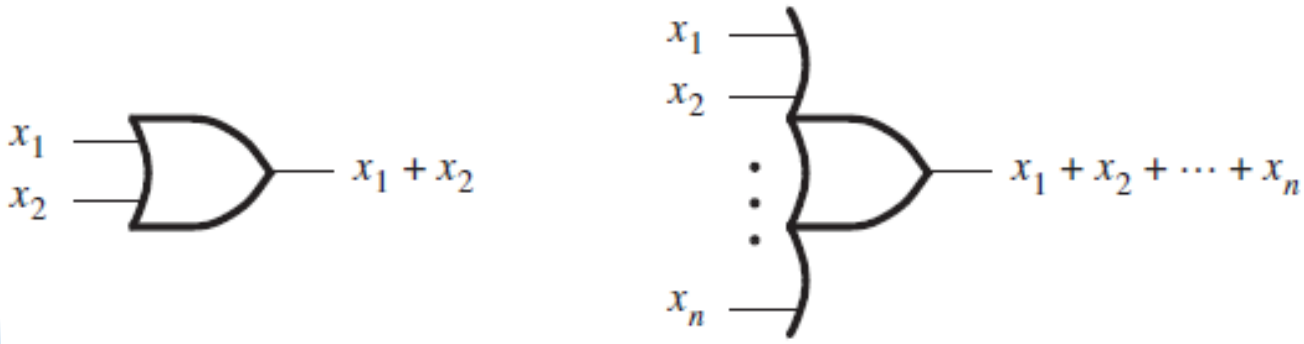
AND OR

(c) NOT gate

x	x’
0	1
1	0

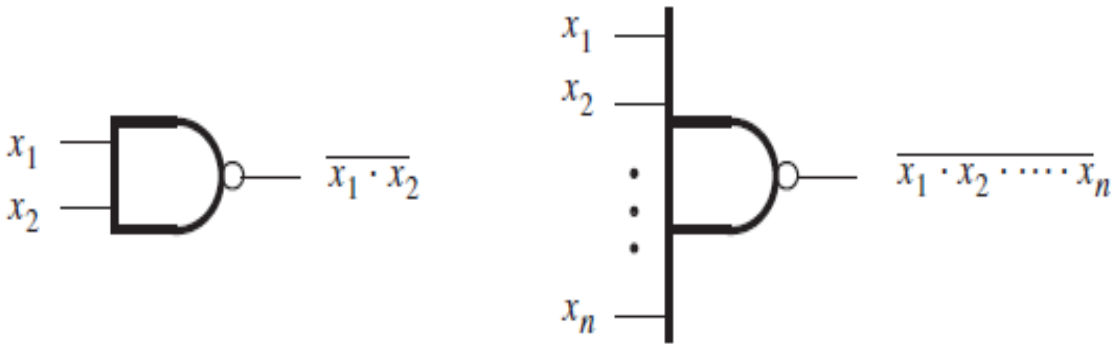


(a) AND gates

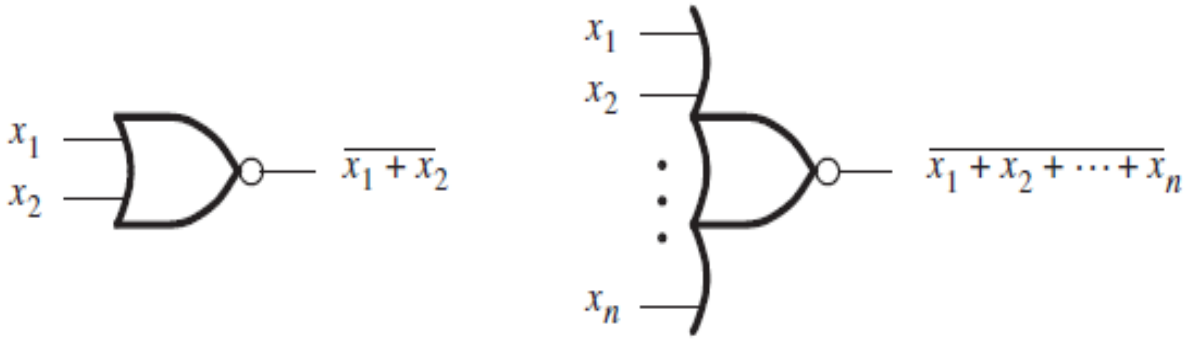


(b) OR gates

- AND gate gives *high* output if and only if all of its inputs are *high*
- OR gate output is LOW if and only if all of its inputs are LOW.



(a) NAND gates



(b) NOR gates

A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

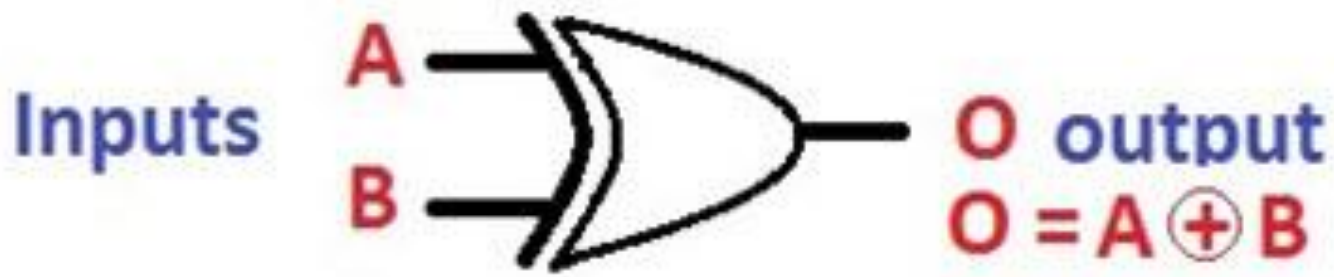
NAND

A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

NOR

- NAND gate gives LOW output if and only if all of its inputs are HIGH
- OR gate output is HIGH if and only if all of its inputs are LOW.

Exclusive OR (XOR)



Symbol

Inputs		Output
A	B	O
0	0	0
0	1	1
1	0	1
1	1	0

Truth table

XOR gate output is HIGH if it has odd number of HIGH inputs.

The complement operation can be applied to a single variable or to more complex operations.

For example, if

$$f(x1, x2) = x1 + x2$$

then the complement of f is

$$f'(x1, x2) = (x1 + x2)'$$

Precedence of Operations

- In the absence of parantheses, operation in a logic expression must be performed in the order: NOT, AND and OR.
- Given f=a.b+a’.b’

  first, a’ and b’ are calculated, then ab and a’b’ and finally ab+a’b’

Minterms

For a function of *n* variables, a product term in which each of the *n* variables appears once, either in uncomplemented or complemented form is called a *minterm*.
For a given row of the truth table, the minterm is formed by including *xi* if *xi* = 1 and by including *xi'* if *xi* = 0.

Row number	x_1	x_2	x_3	Minterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$
7	1	1	1	$m_7 = x_1x_2x_3$

Maxterms

- A sum term that contains all the variables in complemented or uncomplemented from is called a maxterm.

Row number	x_1	x_2	x_3	Maxterm
0	0	0	0	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

Sum-of-Products and Product-of-Sums Forms

If a function f is specified in the form of a truth table, then an expression that realizes f can be obtained by considering either the rows in the table for which $f = 1$, or by considering the rows for which $f = 0$.

Sum-of-Products Form

Any function f can be represented by a sum of minterms that correspond to the rows in the truth table for which $f = 1$.

Example:

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

$$f = \overline{x}_1\overline{x}_2 + \overline{x}_1 x_2 + x_1 x_2$$
$$= m_0 + m_1 + m_3$$

A logic expression consisting of product (AND) terms that are summed (ORed) is said to be in the *sum-of products* (*SOP*) form. If each product term is a minterm, then the expression is called a *canonical sum-of-products* for the function *f*

The *cost* of a logic circuit is the total number of gates plus the total number of inputs to all gates in the circuit.

The cost of the previous network is 13, because there are five gates and eight inputs to the gates.

The previous function f can also be specified as

$f(x1, x2, x3) = \sum (m1, m4, m5, m6)$

or even more simply as

$f(x1, x2, x3) = \sum m(1, 4, 5, 6)$

Product of Sums
Example:

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$= (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)$$

An alternative way of specifying our sample function is

$$f(x_1, x_2, x_3) = \Pi(M_0, M_2, M_3, M_7)$$

or more simply

$$f(x_1, x_2, x_3) = \Pi M(0, 2, 3, 7)$$

- A logic expression consisting of sum (OR) terms that are the factors of a logical product (AND) is said to be of the *product-of-sums* (*POS*) form.
- If each sum term is a maxterm, then the expression is called a *canonical product-of-sums* for the given function.