☐PHYS115 ☐PHYS121 ☒PHYS123
☐PHYS116 ☐PHYS122 ☐PHYS124
Lab Cover Letter

Author (*You*) Wolf Mermelstein Signature: Wolf Mermelstein

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Lab Partner(s) Christopher Richner, Lily Kagy

Date Performed October 25, 2023 Date Submitted October 31, 2023

Lab (*such as #1: UNC*) #5: RKE

TA: Olivia Green

GRADE (*to be filled in by your TA*) See your TA for detailed feedback.
An '*x*' next to a subcategory means you need to improve this aspect of your work.

Paper Subtotals (points)

( ) ***General*** **(6)**
_____ Sig. figs.
_____ Units
_____ Clarity of Presentation
_____ Format

( ) ***Abstract*** **(4)**
_____ Quantity or principle
_____ How measurement was made
_____ Numerical Results
_____ Conclusion

( ) ***Intro & Theory*** **(9)**
_____ Basic principle
_____ Main equations to be used
_____ Apparatus
_____ What will be plotted
_____ Fitting parameters related

( ) ***Exp. Procedures*** **(15)**
_____ Description
_____ Stating and justifying uncertainties
_____ Data Record
_____ Quality of Lab Work

( ) ***Analysis & Error Analysis*** **(20)**
_____ Discussion
_____ Equations & Calculations
_____ Presentation inc. Graphs, Tables
_____ Results Reported & Reasonable
_____ Underlined items addressed

( ) ***Discussion & Conclusions*** **(6)**
_____ Numerical comparison of results
_____ Logical conclusions
_____ Discussion of pos. errors
_____ Suggestions to reduce errors

( ) **Paper Total (60 points)**
(30 points for CME or EPF)

( ) **Notebook (10 points)**
_____ Format (*proper style, following directions*)
_____ Apparatus (*brief description of equipment, including sketches*)
_____ Data (*including computer file names and manually recorded data*)
_____ Experimental Technique (*describing your procedures; stating & justifying uncerts.*)
_____ Analysis (*results and errors*)

( ) **Worksheet(s)/Fill-in-the-Blank-Report (30 points)** *if applicable*

( ) **Adjustments** – late submissions, improper procedures, etc. – or bonus points for exceptional work.

( ) Total Grade

Graded by __________ *(TA's initial)*

Rotational Kinetic Energy Lab

Wolf S. Mermelstein

October 31, 2023

Abstract

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Contents

1 Introduction

1.1 Moment of Inertia

Moment of inertia, often denoted by I, is a function of the specific geometry and mass distribution of an object. Moment of inertia is implicitly relative to the axis of rotation. Where R is the distance from the axis of rotation and M is the mass of the object, for a point mass the moment of inertia, I, is given by

$$I = MR^2 \tag{1}$$

The entire moment of inertia can be computed by thinking of a given object as a collection of tiny masses. As the masses' volumes shrink down to some small volume with some proportionately small mass, the can then be eventually said to be the differential dM. Integrating all the small point masses across the entire object implies that the entire moment of inertia of an object, I_{tot}, is given by

$$I = \sum_{k=1}^{\infty} \frac{m}{k} R^2 \tag{2}$$
$$= \int R^2 dM \tag{3}$$

This is conceptually helpful in understanding moment of inertia for arbitrary shapes, but is not practically useful for non simple (i.e. circles, squares, collections of discreet point masses) shapes, such as the mass-loaded, spoked wheel that we used in our experiment. As a result of such, it is often helpful to actually measure moment of inertia instead of attempting to compute it.

1.2 Conservation of Energy

The translational kinetic energy of an object in motion with mass M moving at speed v is given to be

$$K_T = \frac{1}{2} M v^2 \tag{4}$$

Since we know that

$$\frac{\theta}{2\pi} = \frac{s}{2\pi R}$$
$$s = \theta R \tag{5}$$

and

$$\frac{d}{dt} s = \frac{d}{dt} \theta R$$
$$v = \omega r \tag{6}$$

We can then derive from equation 4 that the rotational kinetic energy, K_R, is

$$\begin{aligned} K_R &= \frac{1}{2} M (\omega R)^2 \\ &= \frac{1}{2} (MR^2) \omega^2 \\ &= \frac{1}{2} I \omega^2 \end{aligned} \tag{7}$$

where I is defined to be the moment of inertia about the axis of rotation.

For the mass in figure **??** descends downwards due to gravity, it begins to lose its gravitational potential energy, U_W. The total energy of the system is internally conserved, however a small amount of energy is lost due to friction. So, where ΔU_W is the change in the gravitational potential energy of the counterweight, K_T is the translational kinetic energy of the counterweight, and K_R is the rotational kinetic energy of the counterweight, we state that

$$\Delta U_W + K_T + K_R = W_f \tag{8}$$

Which, using equations 7 and 4, implies that

$$\Delta U_W + (\frac{1}{2} M v^2) + (\frac{1}{2} I \omega^2) = W_f \tag{9}$$

ΔU_W should be negative, and K_T & K_R positive because the mass is falling, and, thus, losing gravitational kinetic energy, whilst simultaneously proportionately gaining kinetic energy.

Using the fact that gravitational potential energy for an object at height h of mass m in an environment where gravity can be approximated to g is given to be

$$U_G = (M \cdot g \cdot h) \quad (10)$$

Plugging this in, and renaming h be y, we get the final equation

$$-(M \cdot g \cdot y) + (\frac{1}{2}Mv^2) + (\frac{1}{2}I\omega^2) = W_f \quad (11)$$

1.3 Working Equation

Since for our specific experiment we used paperclips attached to the counterweight to cancel out friction, we can instead rewrite equation 11 to be

$$-(M \cdot g \cdot y) + (\frac{1}{2}Mv^2) + (\frac{1}{2}I\omega^2) = 0 \quad (12)$$

Carefully noting that we have discluded the energy of the moving paperclips, as it is negligible in comparison to the other energies of the system. And, to further simplify things, we will define y to be vertically positive, so as to make the equation into

$$-(M \cdot g \cdot y) + (\frac{1}{2}Mv^2) + (\frac{1}{2}I\omega^2) = 0 \quad (13)$$

Then, using relationship 6, we plug in $\frac{v}{r}$ for ω, resulting in the equation

$$-(M \cdot g \cdot y) + (\frac{1}{2}Mv^2) + (\frac{1}{2}I(\frac{v}{r})^2) = 0$$
$$= \frac{1}{2} \cdot v^2 \cdot (M + \frac{I}{r^2}) \quad (14)$$

Or, as will be used later for our computations, the equivalent equation in the form

$$gy = \frac{1}{2}(1 + \frac{I}{Mr^2}) \cdot v^2 \quad (15)$$

v is a value that is determined by our *Logger Pro*™software. It is computed with an advanced proprietary algorithm, but is similar to

$$v \approx \frac{\Delta s}{\Delta T} \quad (16)$$

Figure 1: Visual representation of k and r

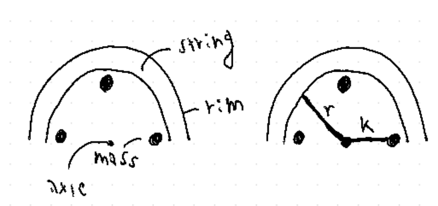
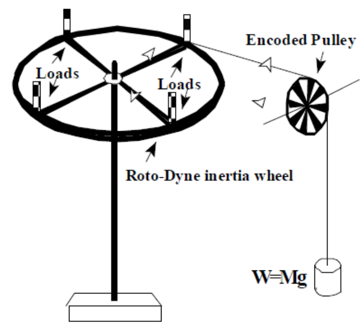


Figure 2: Roto-Dyne Inertia Wheel Apparatus [2]



2 Procedure

2.1 Taking Measurements

Before conducting our experiment, we took measurements of various parts of our setup. First we obtained d from the lab manual, twice the distance from the axle of the wheel to the string, and then we measured k, the distance from the axle of the wheel to the masses. We determined r to be half the diameter. For r we used the provided uncertainty of ± 0.002m, whereas for k we measured very carefully and chose the uncertainty to be 0.001m

$$d = 0.200 \pm 0.002 \text{ m}$$
$$r = 0.100 \pm 0.002 \text{ m}$$
$$k = 0.073 \pm 0.001 \text{ m}$$

We used a counterweight with a given mass of 0.06kg to provide a torque to spin our Roto-Dyne wheel, as can be seen in figure ??. To account for friction, we incrementally added paperclips to the bottom of the counterweight. We continued to add paperclips up until the mass would fall at a constant speed to counteract the force of friction, using *Logger Pro*™ software and an encoded pulley to monitor acceleration and velocity. Let M_c be the mass of the counterweight and M_p be the mass of the paperclips opposing friction, not used in our computations but still important to the experimental design.

$$\begin{aligned} M_p &= 0.0015 \pm 0.0001 \text{ kg} \\ M_c &= 0.06 \text{ kg} \end{aligned}$$

Also, we were provided with the mass of the Roto-Dyne wheel, M_R and the mass loads, M_L, of which there were 4.

$$\begin{aligned} M_R &= 1.5 \text{ kg} \\ M_L &= .225 \pm .002 \text{ kg} \end{aligned}$$

For the encoded pulley that we used to measure velocity and length of unrolled string it was given that the gaps between intervals of measurement, Δs, was

$$\Delta s = 0.015 \text{ m}$$

2.2 Estimating Moment of Inertia

Before measuring the moment of inertia, we decided to make a rough approximation. To do this, we used two different common models, the moment of inertia of a disc, $\frac{1}{2}Mr^2$, and the moment of inertia of a ring, Mr^2. These computations will not be accurate since the actual Rote-Dyne disc is neither a perfect disc nor a hoop. For our rough estimates, we took the radius to be r, and the mass to be

$$\begin{aligned} M_{tot} &= M_R + 4M_L \\ &= 2.4 \text{ kg} \end{aligned} \tag{17}$$

For the disc estimate, we got

$$\begin{aligned} I_{\text{disc}} &= \frac{1}{2}Mr^2 \\ &= 0.048 \text{ kg m}^2 \end{aligned} \tag{18}$$

For the hoop estimate, we got

$$\begin{aligned} I_{\text{hoop}} &= Mr^2 \\ &= 0.096 \text{ kg m}^2 \end{aligned} \tag{19}$$

To determine an overall estimate, I_{est}, we averaged I_{disc} and I_{hoop}, and then set the uncertainty to be that average. Assuming that the wheel is a hoop is an under-estimate, and assuming that the wheel is a disc is an over-estimate, so the actual moment of inertia should be somewhere between the two.

$$\begin{aligned} I_{\text{est}} &= \frac{I_{\text{disc}} + I_{\text{hoop}}}{2} \\ &= 0.072 \pm 0.072 \text{ kg m}^2 \end{aligned} \tag{20}$$

2.3 Monte Carlo Simulation

Before actually measuring the moment of inertia, we performed a Monte Carlo Simulation using our estimated I so that we could compare a graph of data of the estimated value to a graph of data for the actual value later on.

To perform this simulation, we began by creating a new Origin document, and arranged a table including a column for Δs, the overall displacement (vertical distance it has fallen) of the string, and ΔT_0, the time elapsed since dropping the counterweight to arrive at that overall displacement. The values for Δs were computed using the equation

$$y_i = i\Delta s \tag{21}$$

Which is simply stating that the total displacement of the rope is equal to the amount of a single displacement, a notch in the encoded pulley, times the number of increments, which is the total number of notches that passed by the laser at a given time point. To compute the ΔT_0 from the Δs we derived the following equation from equations 16 and 15.

First, we manipulated equation 15 by solving for v.

$$gy = \frac{1}{2}(1 + \frac{I}{Mr^2}) \cdot v^2$$
$$v^2 = \frac{\frac{1}{2}(1 + \frac{I}{Mr^2})}{gy}$$
$$v = \sqrt{\frac{\frac{1}{2}(1 + \frac{I}{Mr^2})}{gy}} \tag{22}$$

Next we solved equation 16 for t. For our use case here, we will allow ΔT to be ΔT_0.

$$v \approx \frac{\Delta s}{\Delta T}$$
$$\Delta T = \frac{\Delta s}{v}$$
$$\Delta T_0 = \frac{\Delta s}{v} \tag{23}$$

Then, we solved for ΔT_0 by plugging in v from equation 23 and Δs into equation 23.

$$\Delta T_0 = \frac{\Delta s}{v}$$
$$\Delta T_0 = \frac{\Delta s}{\sqrt{\frac{\frac{1}{2}(1+\frac{I}{Mr^2})}{gy}}}$$
$${\Delta T_0}^2 = \frac{\Delta s^2}{\frac{\frac{1}{2}(1+\frac{I}{Mr^2})}{gy}}$$
$${\Delta T_0}^2 = \frac{\Delta s^2(1 + \frac{I}{Mr^2})}{2gy}$$
$$\Delta T_0 = \Delta S\sqrt{\frac{(1 + \frac{I}{Mr^2})}{2gy}} \tag{24}$$

With a simple script we had *Origin Pro*™apply this equation to each row, utilizing that respective row's Δs value. Now, since this data is purely based on an estimated moment of inertia value, we applied Monte Carlo randomization. To do this, we shifted each $\Delta T0_i$ by some $\Delta T0_G i$ obtained from a Gaussian distribution G with the given mean 0 and σ equal to the estimated uncertainty for Δt, $\delta_{\Delta t}$, which was given to be 0.0002s, resulting with a column with values for δt_R. This can be expressed formulaically as

$$\delta T_R = \Delta T0 + \Delta t \cdot \mathrm{grnd}() \tag{25}$$

After running this computation, for the first three values of ΔT_r we got

Table 1: Samples of random Monte Carlo data generation

trial #	s
1	0.15409
2	0.10925
3	0.08891
1	0.15381
2	0.10883
3	0.08919
1	0.01540
2	0.10891
3	0.08891
1	0.15385
2	0.10918
3	0.08894

3 Results

For the actual results, we measured the velocity, acceleration, time (duration), and position of the falling counterweight mass using the encoded pulley and *Logger Pro*™. We then imported the data into *Origin Pro*™for analysis and to help us compile plots. To keep our data consistent, we decided to trim the first three rows and all rows after row 44. Data beyond that in either direction was problematic because of us abruptly setting up and stopping the counterweight from hitting the floor.

To visualize our data, we plotted v^2 against displacement for both the system with and without load masses, and did the same for our simulation.

Both plots and datatables can be found in the appendix, section 6.

4 Analysis

5 Conclusion

Acknowledgments

I would like to thank Christopher Richner and Lily Kagy, CWRU Department of Physics, for their help in obtaining the experimental data, collaborating on preparation of the figures, and checking calculations. Additionally, I would like to thank Olivia Green, CWRU Department of Physics, for helping facilitate our lab.

References

[1] Resnick Halliday and Walker. *Fundamentals of Physics*. Addison-Wesley Professional, 6 edition, 2023.

[2] D. Schultz. *General Physics I: Mechanics Lab Manual*. CWRU Bookstore, Spring 2004.

6 Appendix

Figure 3: Monte Carlo Simulation of Rotational Kinetic Energy Experiment plot

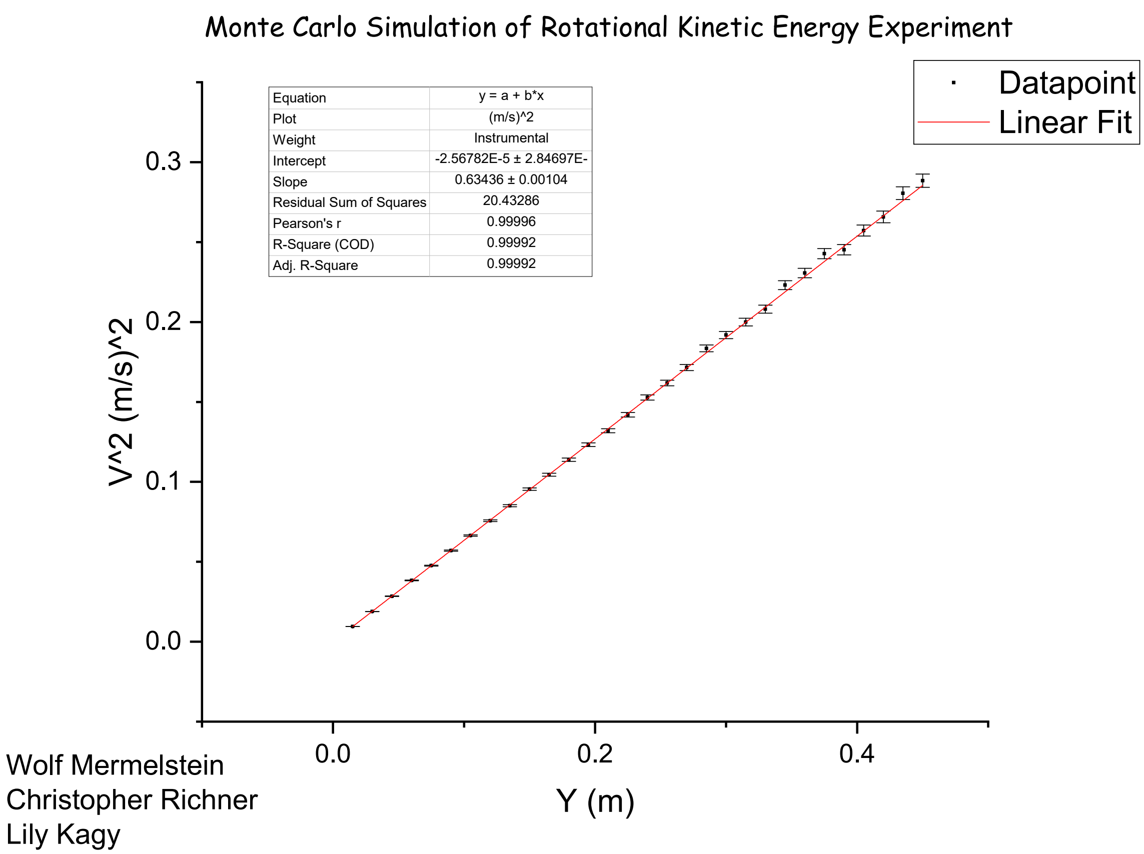


Figure 4: Without Masses v^2 vs sDist plot

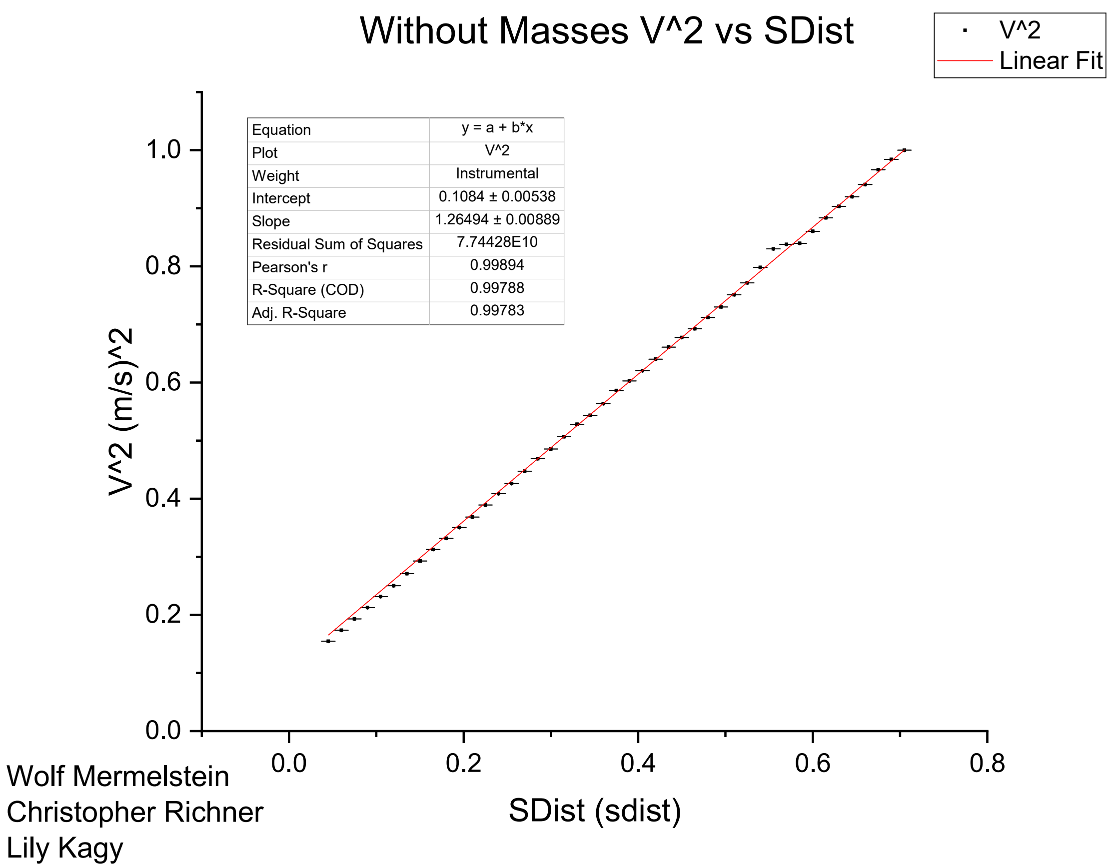


Figure 5: With Masses v^2 vs sDist plot

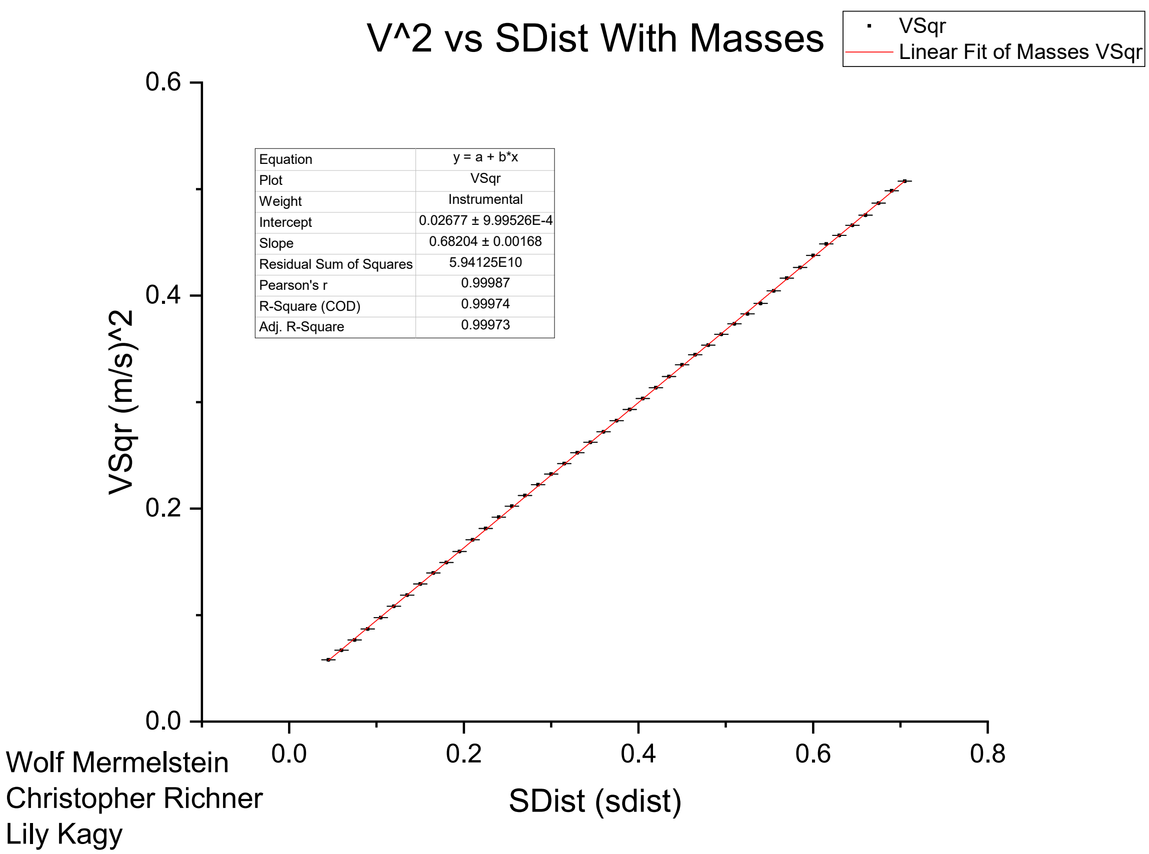


Table 2: Data for falling counterweight **without** mass loads in place

Time	STime	SDist	SVel	SAccel
t	stime	sdist	svel	saccel
s	s	m	m/s	m/s$\hat{2}$
0.2003832	0.3409468	0.045	0.240637469	0.2974361002
0.2421024	0.4010488	0.06	0.2587705469	0.3059740303
0.2727832	0.4571542	0.075	0.2768823709	0.3396616489
0.3099096	0.5096392	0.09	0.2950082819	0.3510465914
0.3375832	0.5590336	0.105	0.3125067287	0.3574728706
0.371984	0.6057826	0.12	0.3289729322	0.3469787101
0.3977972	0.6503324	0.135	0.3445762028	0.3535079613
0.4301136	0.6929332	0.15	0.3594867431	0.3465042602
0.4541972	0.7338524	0.165	0.3734799691	0.3374400019
0.4841948	0.7733118	0.18	0.3866063477	0.3278706007
0.506816	0.8114928	0.195	0.3997294453	0.3595444818
0.5350836	0.8484054	0.21	0.4130797945	0.3638046842
0.5563028	0.8841552	0.225	0.4258449397	0.3503333039
0.5829836	0.9188832	0.24	0.4381633461	0.3590888604
0.6031832	0.9526498	0.255	0.4497965385	0.3299465981
0.6285816	0.9856	0.27	0.4607194593	0.3330491259
0.6478832	1.0177834	0.285	0.4715773455	0.3417015924
0.6720832	1.0492332	0.3	0.4820423673	0.3238048217
0.6905832	1.0800322	0.315	0.492183323	0.3347201767
0.7137832	1.1101992	0.33	0.5023979233	0.3424834134
0.7315832	1.1397582	0.345	0.5121068853	0.3144374509
0.7539216	1.1687902	0.36	0.5216114938	0.3403302933
0.7710904	1.1972824	0.375	0.5315253346	0.355568289
0.792702	1.2252416	0.39	0.5413072074	0.3441565057
0.8092096	1.2527124	0.405	0.5507273167	0.3416705778
0.8302836	1.2797228	0.42	0.559880448	0.3360780896
0.8462136	1.3063022	0.435	0.5692255573	0.3671062769
0.8665272	1.3324336	0.45	0.5787253594	0.359973184
0.8820832	1.358147	0.465	0.5869743702	0.2816386499
0.9017832	1.383547	0.48	0.5946036744	0.3190939627
0.9168836	1.4086054	0.495	0.602973345	0.3489192067
0.9359832	1.4333056	0.51	0.6110845183	0.3078506391
0.9506832	1.4577022	0.525	0.6186489591	0.3122719003
0.9693164	1.481802	0.54	0.6266088147	0.3483025149
0.983612	1.5055832	0.555	0.6359267169	0.4353334838
1.0018836	1.5289834	0.57	0.6452122961	0.3582989806
1.0158992	1.5520834	0.585	0.6529222225	0.3092271113
1.0336832	1.5749334	0.6	0.661597301	0.4500795393
1.0474092	1.5974336	0.615	0.669666613	0.2671862678
1.0647832	1.6197336	0.63	0.6756628186	0.27059002
1.0782148	1.6418362	0.645	0.6826017458	0.3572933224
1.0952812	1.663686	0.66	0.6896131877	0.2844921273
1.1083832	1.6853406	0.675	0.69766979	0.4596086524
1.1251172	1.7066906	0.69	0.7060389196	0.32438475
1.1379832	1.7278332	0.705	0.7123894853	0.2763517435

Table 3: Data for falling counterweight **without** mass loads in place

Time	STime	SDist	SVel	SAccel
t	stime	sdist	svel	saccel
s	s	m	m/s	m/s$\hat{2}$
0.0942928	0.1784214	0.045	0.3931293508	0.6331416361
0.118666	0.2154714	0.06	0.4167682347	0.6429114748
0.13679	0.2505154	0.075	0.4395279097	0.6560084549
0.1593652	0.2838154	0.09	0.4611842509	0.6446726969
0.1763652	0.3156328	0.105	0.4813227422	0.6212064342
0.1974776	0.3461942	0.12	0.5002408825	0.6168350372
0.2134652	0.375645	0.135	0.5204425281	0.7550560866
0.2334652	0.403888	0.15	0.5411476689	0.7111579009
0.2485668	0.4311196	0.165	0.5590171297	0.6012483335
0.2675656	0.457576	0.18	0.5759222213	0.6767102446
0.2819648	0.4832344	0.195	0.5919181069	0.5701239701
0.3000652	0.5082738	0.21	0.6071150987	0.6437223597
0.3138984	0.5326654	0.225	0.6238645636	0.7296573993
0.3312004	0.5563802	0.24	0.6392812271	0.5705149372
0.3445048	0.5796034	0.255	0.6528166664	0.5951675979
0.361188	0.602345	0.27	0.668869986	0.8166345302
0.3740644	0.624472	0.285	0.6847467198	0.618420718
0.390102	0.6461654	0.3	0.6968292431	0.4955147032
0.4024884	0.6675292	0.315	0.7117831029	0.9044103837
0.417674	0.6883282	0.33	0.7267542228	0.53518959
0.4292784	0.7088134	0.345	0.737173249	0.4820351523
0.4445652	0.7290278	0.36	0.7508551143	0.8716399856
0.4560652	0.7487786	0.375	0.7655336953	0.6147383898
0.4705868	0.768221	0.39	0.7764021555	0.5032779268
0.4818648	0.7874214	0.405	0.7876149561	0.664697796
0.495882	0.8063156	0.42	0.8001386413	0.6609667181
0.5067812	0.8249194	0.435	0.8130346045	0.72541275
0.5206656	0.8432194	0.45	0.8231695853	0.3822354223
0.5311832	0.8613652	0.465	0.8321622217	0.6089180588
0.5446652	0.8792732	0.48	0.8437710657	0.6875800459
0.554982	0.8969236	0.495	0.8544283269	0.520014252
0.5680952	0.9143864	0.51	0.8665462466	0.8678410374
0.5782656	0.931549	0.525	0.8783503608	0.5077214323
0.5911116	0.948543	0.54	0.8933737653	1.260361948
0.6010652	0.965139	0.555	0.9110416678	0.8688140546
0.6135784	0.9814764	0.57	0.915255149	-0.3530060006
0.6232648	0.9979174	0.585	0.9163732773	0.4890230607
0.6353656	1.0142154	0.6	0.9275233967	0.8792576405
0.6447648	1.0302654	0.615	0.9399816885	0.673177482
0.6569652	1.046133	0.63	0.9503827681	0.6378058508
0.6663652	1.0618334	0.645	0.9590269548	0.4633363769
0.6780932	1.0774156	0.66	0.9699853551	0.9431916229
0.6870656	1.0927652	0.675	0.9830297701	0.7564507123
0.6985632	1.1079356	0.69	0.9919658955	0.4216494522
0.7075648	1.1230088	0.705	0.999952488	0.6380581699