金融数学

Financial Mathematics

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目录

欢迎

在这里，我们同步课堂，总结每章的**重点**、**难点**，并发布**课后作业**。课后作业需在下次上课前交到老师信箱（明主 1036 门外邮箱柜右下角）。

我们这里主要以英文表述，有以下两个原因

1. 方便大家准备 SOA/CAS 的 Exam FM: Financial Mathematics考试；
2. 方便大家阅读相关英文文献。

此网站由授课老师高光远、助教程轶鹏、助教胡夏新管理，欢迎大家反馈意见到助教、微信群、或邮箱guangyuan.gao@ruc.edu.cn。

答疑

我定期把同学们的普遍疑问在这里解答，欢迎提问！

i 和 d 的关系（2020/09/16）

很多同学问课件上的这道题目。

问题：已知年实际利率为 5%。回答下述问题：

(1) 100 万元贷款在年末的利息是多少？100×5

(2) 如果在贷款起始日收取利息，应该收取多少利息？$100 \times i/(1+i) = 100 \times d$

(3) 年实际贴现率是多少？$d = i/(1+i)$

i 和 d 的区别可以理解为 i 是在**期末**付，d 是在**期初**付。$d = i \times v$，即期末 i 的**现值**是 d。

所以（1）是期末收的利息，（2）是期初收的利息。期初收的利息要比期末收的少，因为银行收到的这部分利息在这一年中还能产生利息，期初收的 d 到期末是 i。

贴现率 d 的另一种理解就是利息 i 的现值。

计算器（2020/09/10）

在课堂测验和期末考试，没有对计算器的严格要求，但至少需要科学计算器。大家不需要购买昂贵的可编程计算器，在这门课中，体现不出可编程计算器的优势。

建议的计算器是 SOA/CAS 要求的计算器。

最终成绩（2020/09/10）

1. 平时成绩占 40%，期末成绩占 60%。
2. 平时成绩主要根据课堂点名、课外作业的完成态度、随堂测试的准确度评定。

Chapter 1

Interest rate

1.1 Key concepts

Functions

- Accumulation function

$$a(t)$$

- Discount function

$$a^{-1}(t)$$

Interest rate

- Effective rate of interest/discount

$$i, d$$

- Simple interest

$$a(t) = 1 + it$$

- Compound interest

$$a(t) = (1 + i)^t$$

- Discount factor

$$v = (1+i)^{-1}$$

- Accumulation factor of t years

$$(1+i)^t$$

- Discount factor of t years

$$(1+i)^{-t}$$

- Nominal rate of interest/discount

$$i^{(m)}, d^{(m)}$$

- Force of interest

$$\delta$$

Values

- Accumulated value (future value)
- Present value

1.2 Key equations

Accumulation and discount

$$a(t) = (1+i)^t = (1-d)^{-t}$$

$$a^{-1}(t) = (1+i)^{-t} = (1-d)^t = v^t$$

Effective interest rate and discount rate

$$i = \frac{d}{1-d}$$

$$d = \frac{i}{1+i}$$

$$d = iv$$

$$v = 1 - d$$

$$i - d = id$$

Nominal interest rate and effective interest rate

$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^m$$

$$1 - d = \left(1 - \frac{d^{(m)}}{m}\right)^m$$

$$d^{(m)} = i^{(m)} \times \left(1 + \frac{i^{(m)}}{m}\right)^{-1}$$

Force of interest

$$\delta(t) = \frac{a'(t)}{a(t)}$$

$$a(t) = e^{\int_0^t \delta(s)ds}$$

$$\delta = \ln(1+i)$$

$$\delta = \lim_{m\to\infty} i^{(m)} = \lim_{m\to\infty} d^{(m)} = \ln(1+i)$$

$$d \le d^{(2)} \le d^{(3)} \le \cdots \le \delta \le \cdots \le i^{(3)} \le i^{(2)} \le i$$

Chapter 2

Level annuity

2.1 Key concepts

Annuity immediate

Annuity due

Deffered annuity

Perpetuity

m-thly payable annuity

Continuous payable annuity

2.2 Key equations

Homework

Week 2

Week 1

Problem 1

John invests X in a fund growing in accordance with the accumulation function implied by the *amount function*

$$A(t) = 4t^2 + 8t + 4.$$

Edna invests X in another fund growing in accordance with the accumulation function implied by the amount function

$$A(t) = 4t^2 + 2.$$

When does Edna's investment *exceed* John's?

Problem 2

What deposit made today will provide for a payment of \$1000 in 1 year and \$2000 in 3 years, if the effective rate of interest is 7.5%?

表 2.1: The cash flows of the three projects.

End of year	Project A	Project B	Project C
1	500	500	500
2	500	300	250
3	-175	-175	-175
4	100	150	200
5	0	200	200

Problem 3

Company X received the approval to start no more than two projects in the current calendar year. Three different projects were recommended, each of which requires an investment of 800 to be made at the beginning of the year.

The cash flows for each of the three projects are shown in Table 2.1:

The company uses an annual effective interest rate of 10% to discount its cash flows.

Determine which combination of projects the company should select.