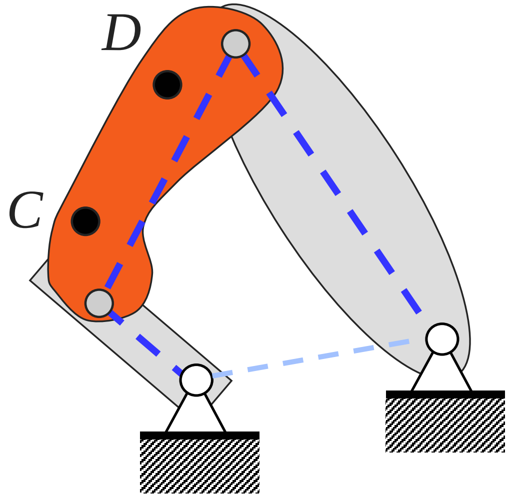
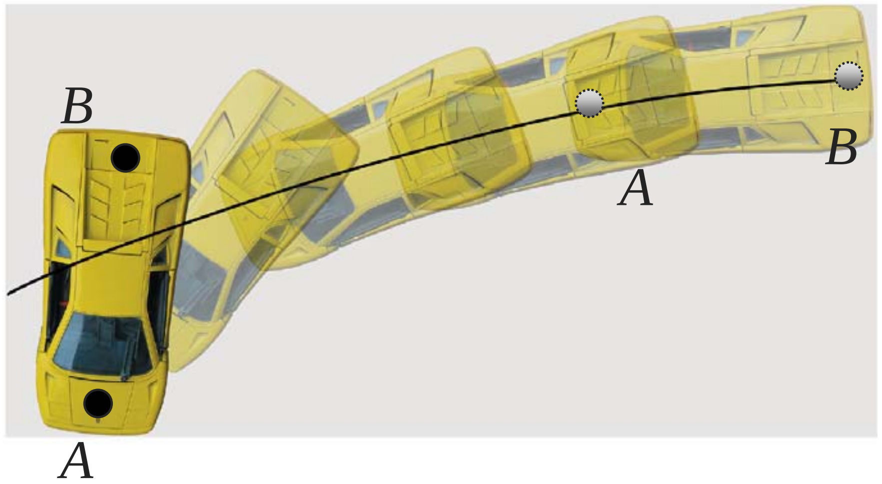
Kinematics of Planar Rigid Bodies

Chapter 6

Planar motion



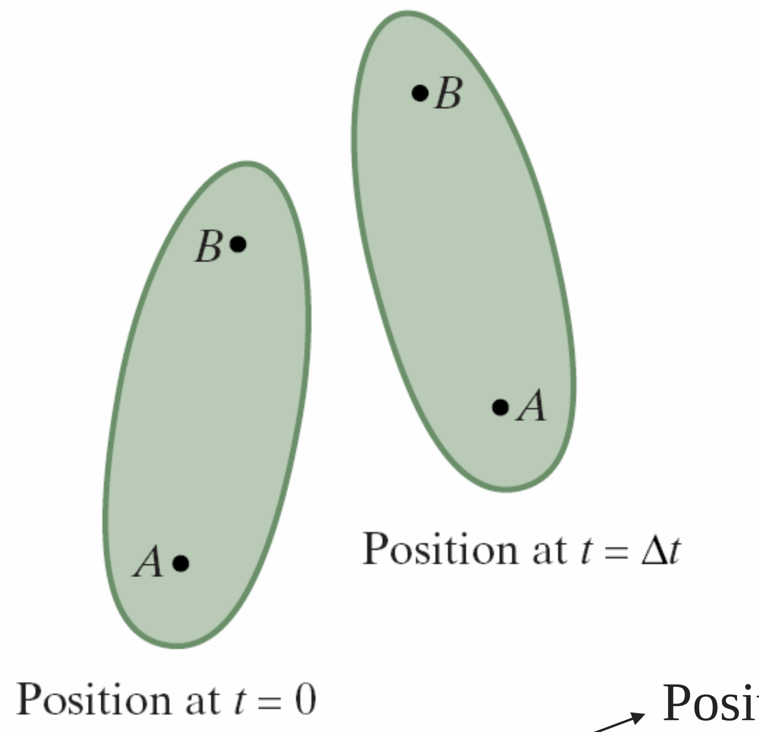


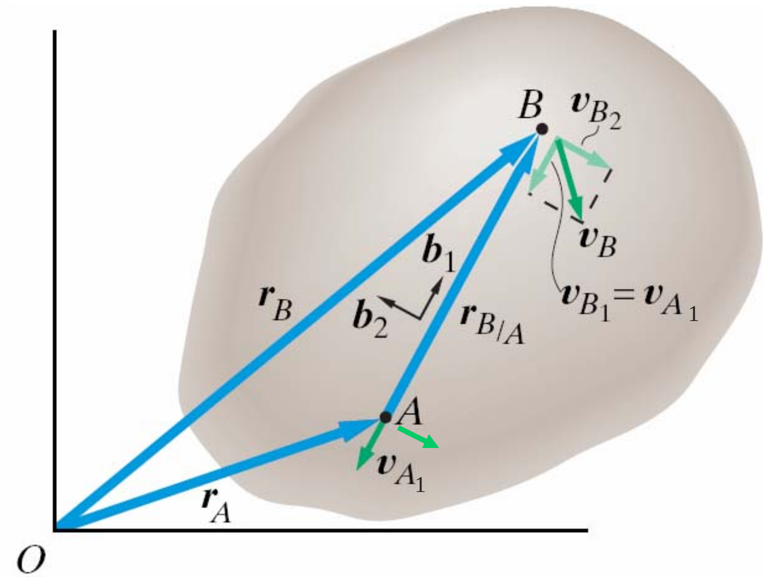
Does the coupler/car rotate? Translate?

Planar Rigid Body Motion

Fig 6.6 needs to be corrected

For any two points (say *A*, *B*) fixed to a rigid body





$$\mathbf{v}_{B/A} = \frac{\quad}{dt} = \frac{\quad}{dt} = \mathbf{v}_B - \mathbf{v}_A$$

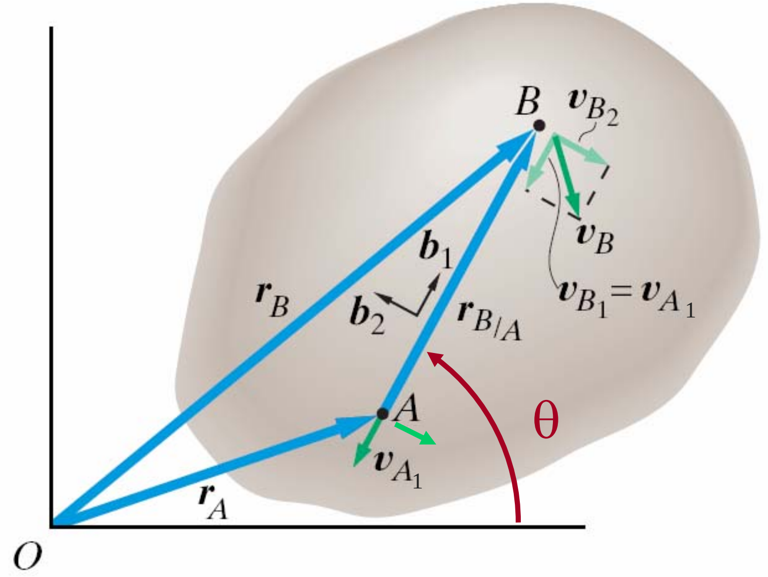
Rigid body constraint

Position of *A* and *B*: ________________

Velocity of *A* and *B*: ________________

Expression for $\mathbf{v}_B$ - $\mathbf{v}_A$

For any two points (say *A*, *B*) fixed to a rigid body



$$\mathbf{v}_{B/A} = \frac{d\mathbf{r}_{B/A}}{dt} = \frac{d(\mathbf{r}_B - \mathbf{r}_A)}{dt} = \mathbf{v}_B - \mathbf{v}_A$$

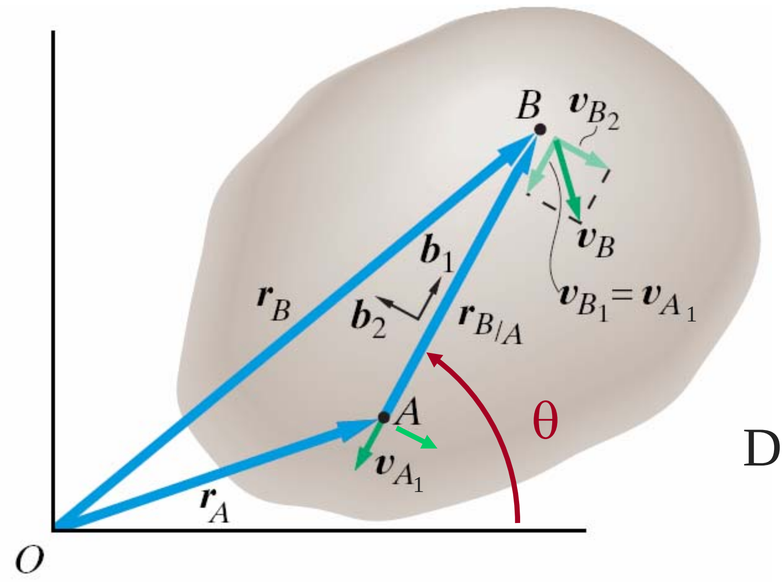
$$\mathbf{r}_{B/A} = r_{B/A}(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$

$$\frac{d}{dt}(\quad)$$

$$\frac{d}{dt}\mathbf{r}_{B/A} = \underbrace{r_{B/A}}_{\text{constant}}\left(\frac{d}{dt}\cos\theta\mathbf{i} + \frac{d}{dt}\sin\theta\mathbf{j}\right)$$

$$\mathbf{v}_{B/A} = r_{B/A}(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j})\dot{\theta}$$

Definition of Angular Velocity



$$\mathbf{v}_{B/A} = r_{B/A}\left(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j}\right)\dot{\theta}$$

Can rewrite as

$$\mathbf{v}_{B/A} = \dot{\theta}\mathbf{k} \times r_{B/A}\left(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}\right)$$

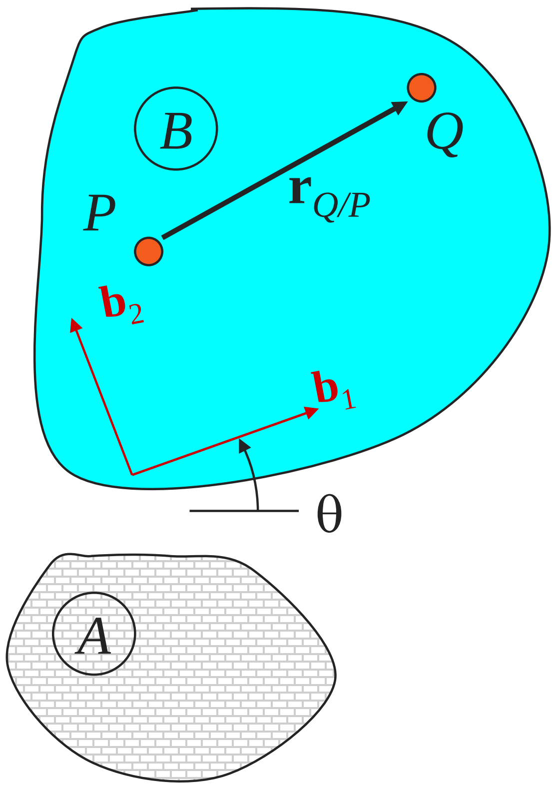
Define angular velocity, **w**

$$\omega = \dot{\theta}\mathbf{k}$$

So the relative velocity for points *A*, *B*

$$\mathbf{v}_{B/A} = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

Kinematics of Planar Rigid Bodies: Key Fact!



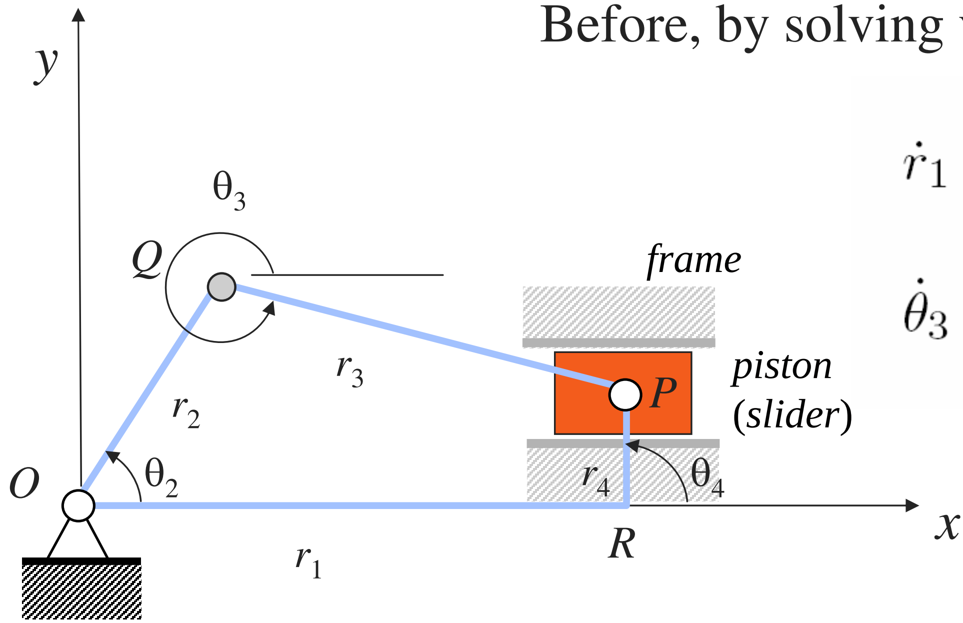
Relative velocity between *any* two points fixed on *any* rigid body, $\mathbf{v}_{Q/P}$

body, B

$$\mathbf{v}_{Q/P} = \frac{d\mathbf{r}_{Q/P}}{dt} = \omega^{B} \times \mathbf{r}_{Q/P}$$

angular velocity of the rigid body

Slider Crank Linkage Velocity Analysis



Before, by solving velocity equations

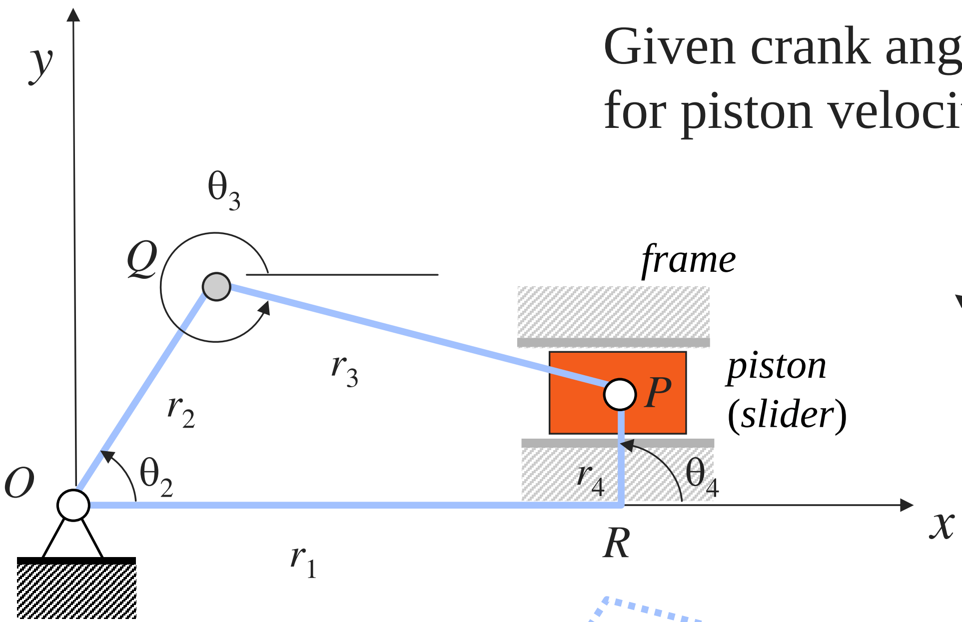
$$\dot{r}_1 = r_2 \frac{sin(\theta_3 - \theta_2)}{cos(\theta_3)} \dot{\theta}_2$$

$$\dot{\theta}_3 = -\frac{r_2 cos(\theta_2)}{r_3 cos(\theta_3)} \dot{\theta}_2$$

Alternative method: solve by writing vector equations representing rigid body constraints

Example

Given crank angular velocity, ω_2, solve for piston velocity, $\mathbf{v}_P$



$$\mathbf{v}_{Q/O} = \omega_2 \times \mathbf{r}_{Q/O}$$

$$\mathbf{v}_{P/Q} = \omega_3 \times \mathbf{r}_{P/Q}$$

Magnitude of ω_3 unknown but direction is known

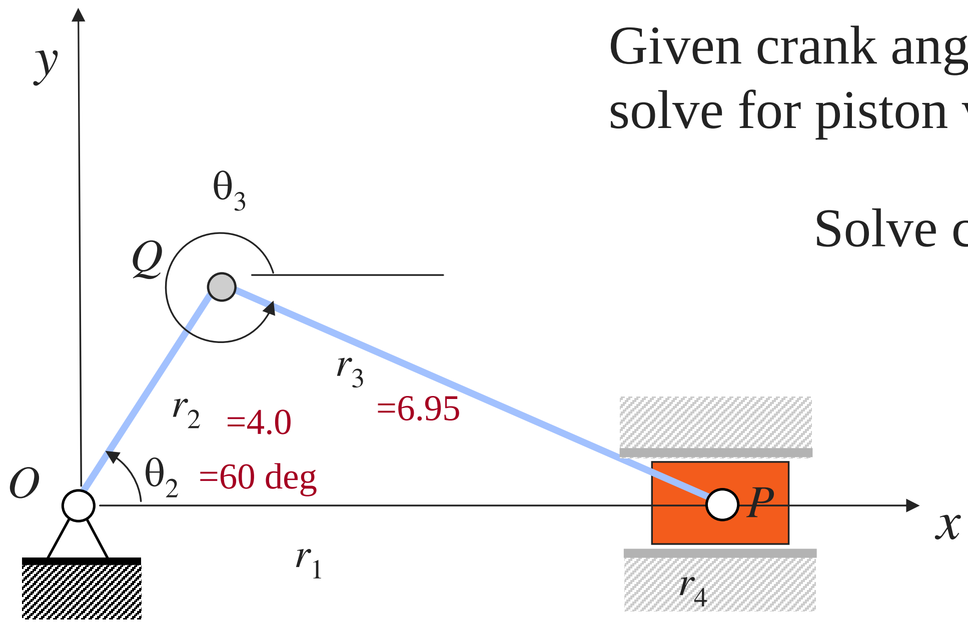
$\mathbf{v}_P$ in this direction

$$\mathbf{v}_P = \mathbf{v}_Q + \mathbf{v}_{P/Q}$$

Magnitude of ω_3, v_P unknown

Example

Given crank angular velocity, $\omega_2 = 1$ rad/s, solve for piston velocity, $\mathbf{v}_P$



Solve closure equations to get:

θ_3 = -30 deg

r_1 = 8.02

All dimensions in cm.

Examples: Transmissions

Gears

- Spur gears
- Helical gears
- Hypoid gears

Gear reductions

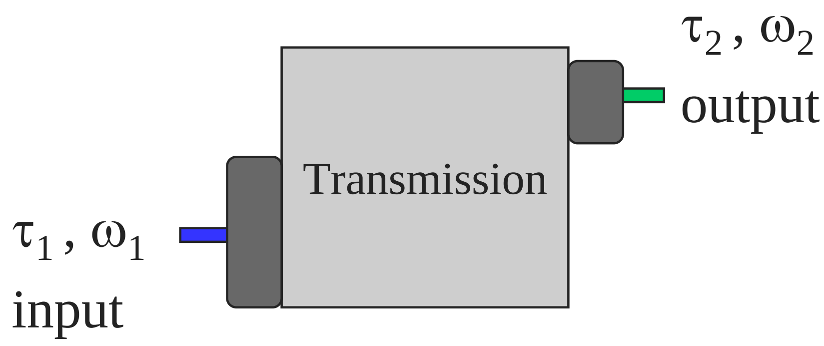
- Gear trains
- Worm
- Planetary
- Harmonic

Chain & Chain Drives

Need gears/transmissions to:

Decrease (increase) speeds

Increase (decrease) torques



Spur and Helical Gears

Spur gear

- Loud: Each time a gear tooth engages a tooth on the other gear, the teeth collide, and this impact makes a noise
- Wear and tear

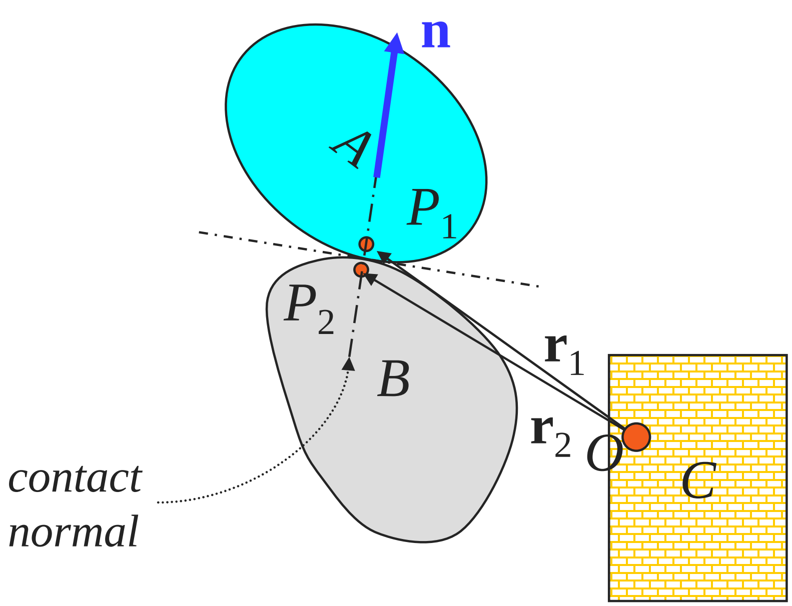
Helical gears

- Contact starts with point contact to line contact

Crossed helical gears

- Shaft angles need not be parallel

Rolling Contact



Contact points

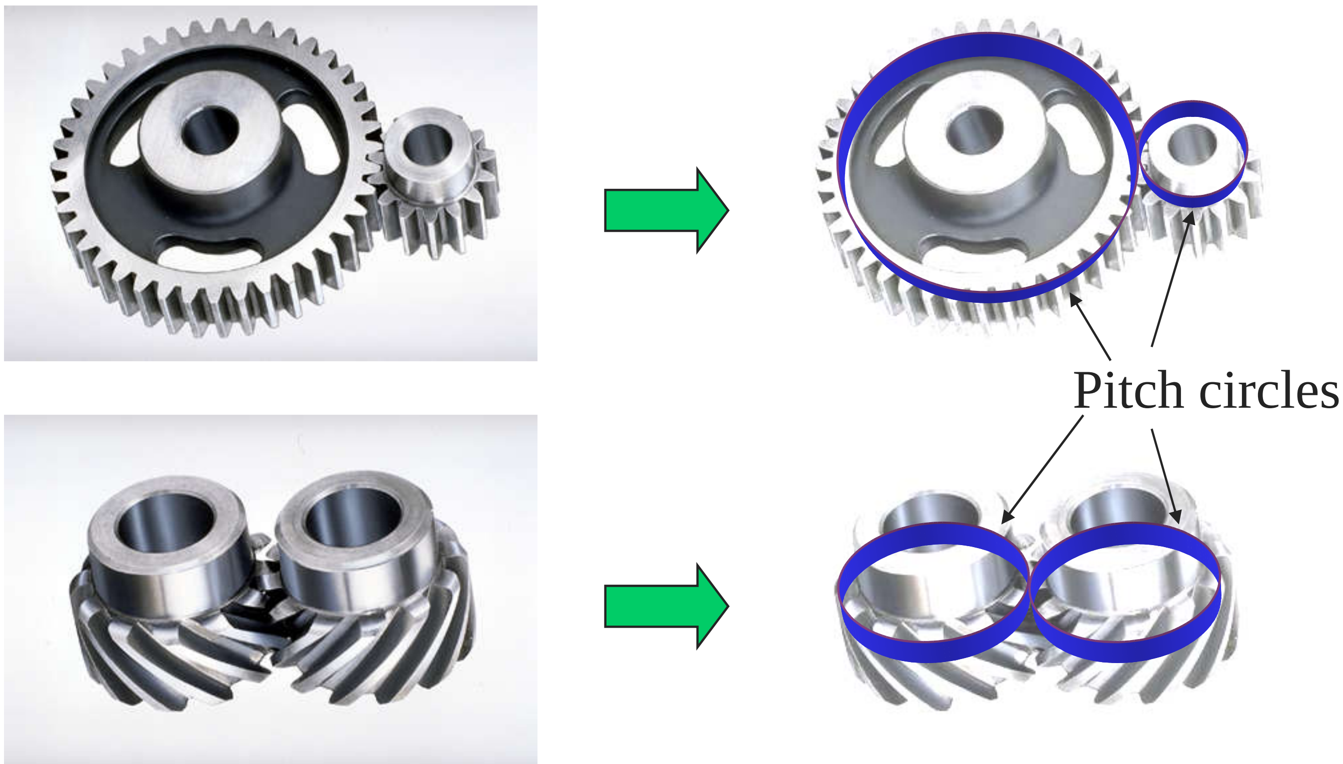
P_1 and P_2, coincident instantaneously

$$\mathbf{v}_{P_1} = \frac{d\mathbf{r}_1}{dt}$$

$$\mathbf{v}_{P_2} = \frac{d\mathbf{r}_2}{dt}$$

Body A rolls on *body B* ⇨ $\mathbf{v}_{P1} = \mathbf{v}_{P2}$

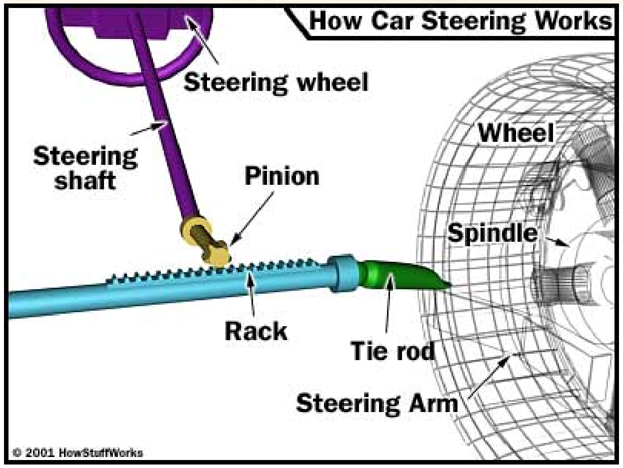
Modeling of Gears



The kinematics of rotation of a pair of meshing gears can be modeled as a rotation of the corresponding pitch circles.

Rack and Pinion

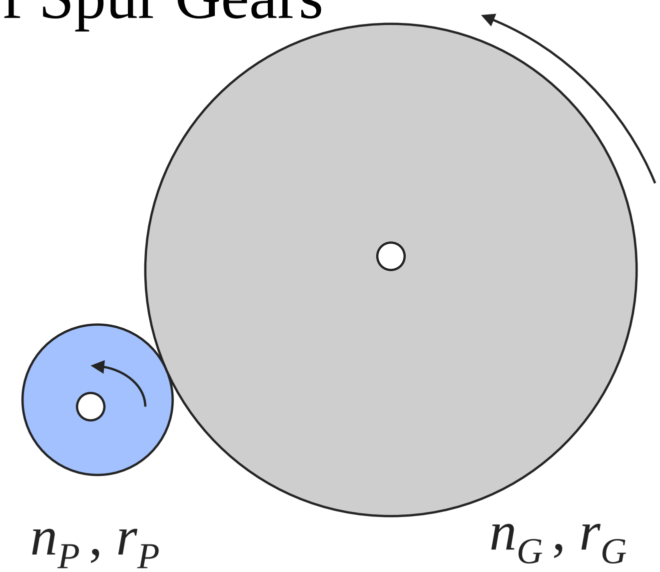
- Similar to a wheel on a ground with friction
  - But positive engagement
- Rack is a gear with infinite pitch circle radius
- Converts rotary motion to linear motion



- Linear speed
  - Proportional to pinion speed
  - $v = r_p \omega$

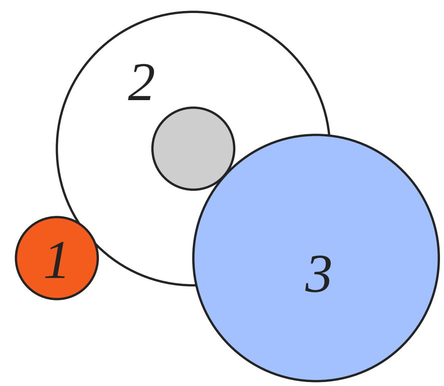
Analysis of Spur Gears

- Pinion, *P*
- Gear, *G*
- Number of teeth, *n*
- Radius, *r*
- Angular velocity, ω



$$\frac{-\omega_G}{\omega_P} = \frac{r_P}{r_G} = \frac{n_P}{n_G}$$

- The maximum reduction in a single stage is limited!
- To get higher reduction
  - Multiple stages
  - But…
    - lead to bulky package and weight
- Spur gears have high wear and tear and are noisy



Analysis of Planetary Gears

Simple Example

- Ring gear, *R*
- Sun gear, *S*
- Carrier arm, *C*
- Planet gear, *P*
- Frame, *F*

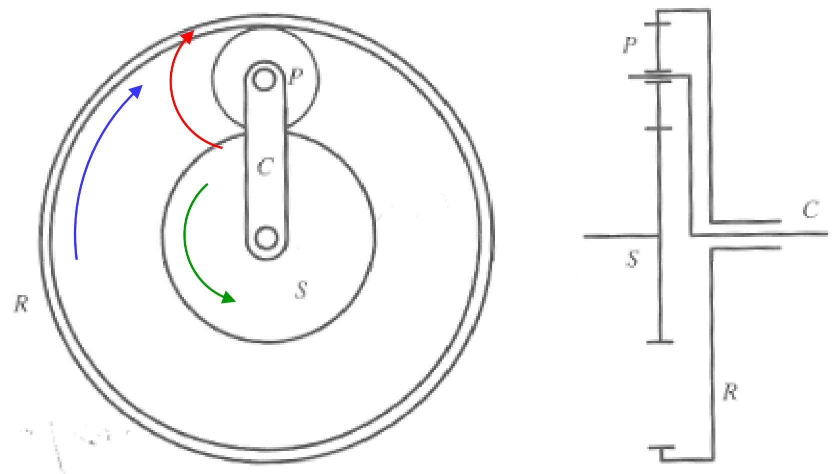


Figure 10.15 A simple planetary gear train.

[Waldron and Kinzel, 1999]

But suppose the ring gear is stationary and the carrier is not stationary

If carrier is stationary…

$$\frac{\omega_S}{\omega_P} = \frac{-r_P}{r_S} \qquad \frac{\omega_S}{\omega_R} = -\frac{r_R}{r_S}$$

Analysis of Planetary Gears

Simple Example

- Ring gear, R [stationary]
- Sun gear, S
- Carrier arm, C
- Planet gear, P
- Frame, F

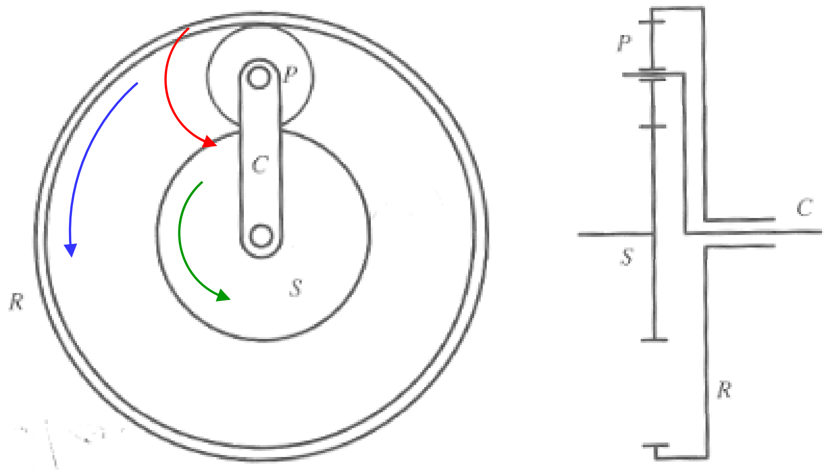


Figure 10.15 A simple planetary gear train.
[Waldron and Kinzel, 1999]

If $r_P = 2$, $r_S = 2$, $r_R = 6$, and $\omega_R = 0$:

$$\frac{\omega_S}{\omega_C} = 4$$

Assume positive counter clockwise directions

Planetary Gears

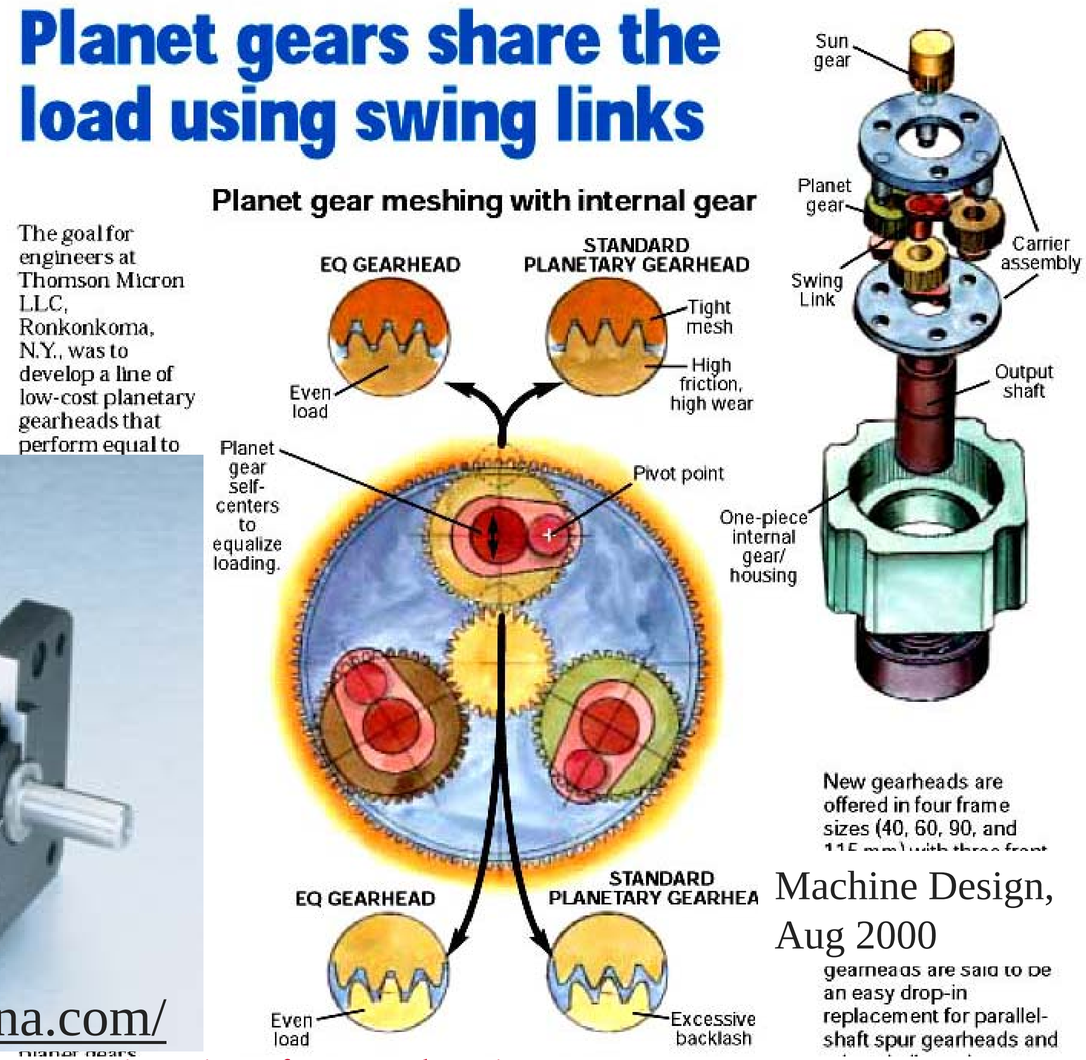
90-95% efficiency

30-50 Nm

3 lbs

20 arc min

http://www.apexdyna.com/



Planet gears share the load using swing links

The goal for engineers at Thomson Micron LLC, Ronkonkoma, N.Y., was to develop a line of low-cost planetary gearheads that perform equal to

New gearheads are offered in four frame sizes (40, 60, 90, and

gearheads are said to be an easy drop-in replacement for parallel-shaft spur gearheads and

Machine Design, Aug 2000