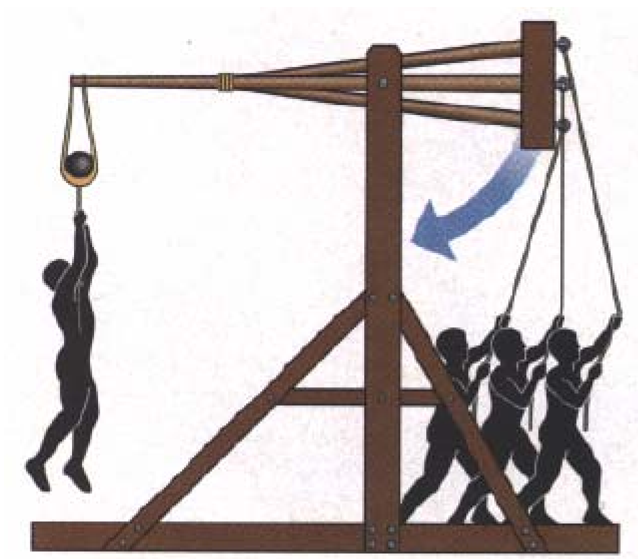
Rigid Body Kinematics and Kinetics

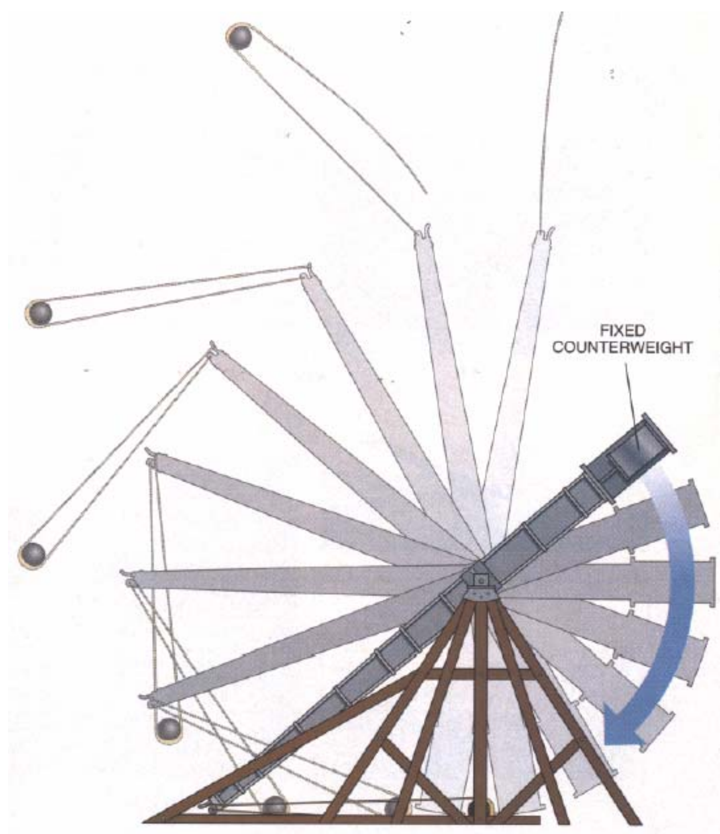
First consider the special case of rotation about a fixed point, O.

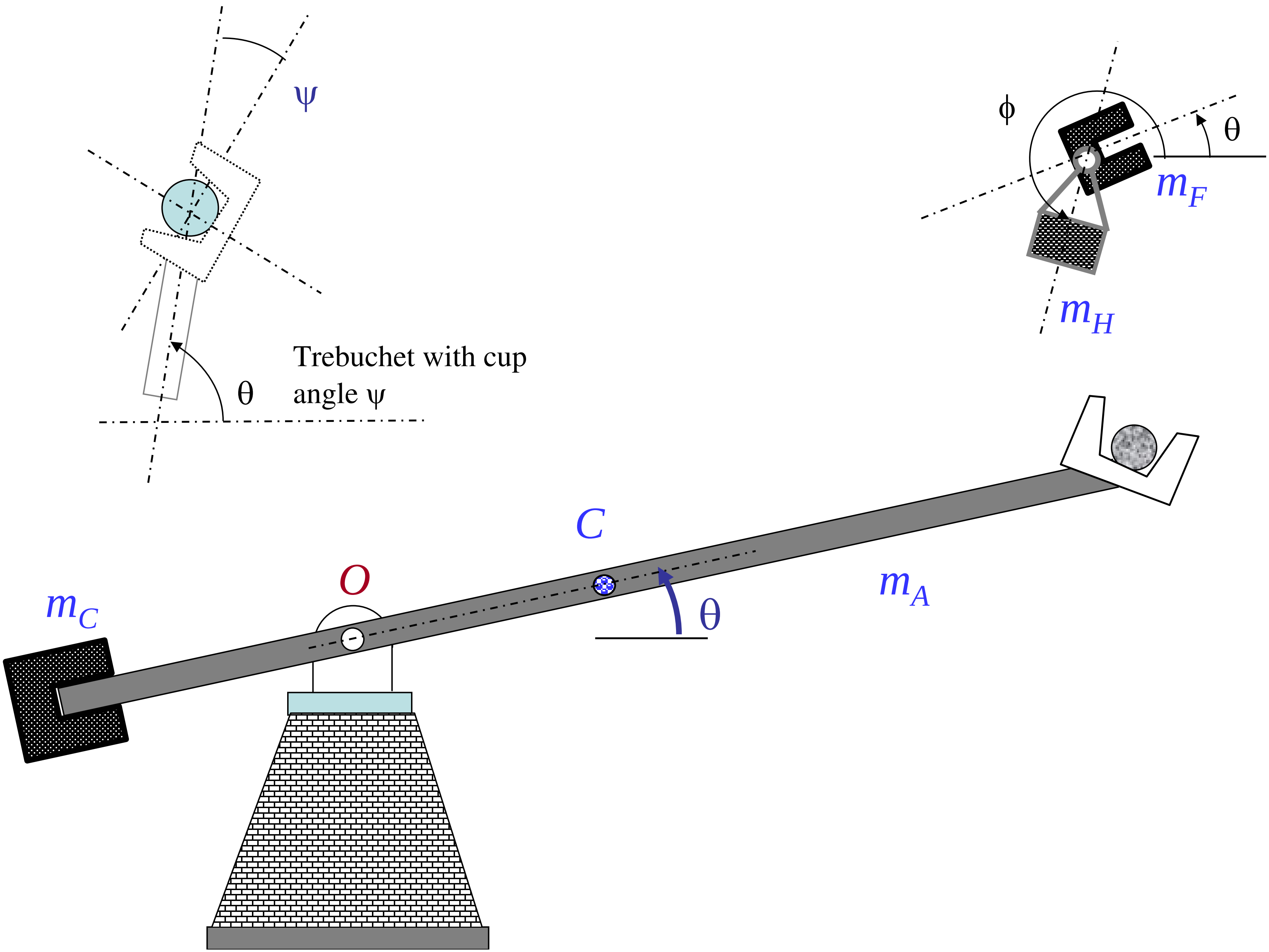
Project III

Design, Analysis and Prototyping of a Simplified Trebuchet

Skip ahead to Chapter 7.2

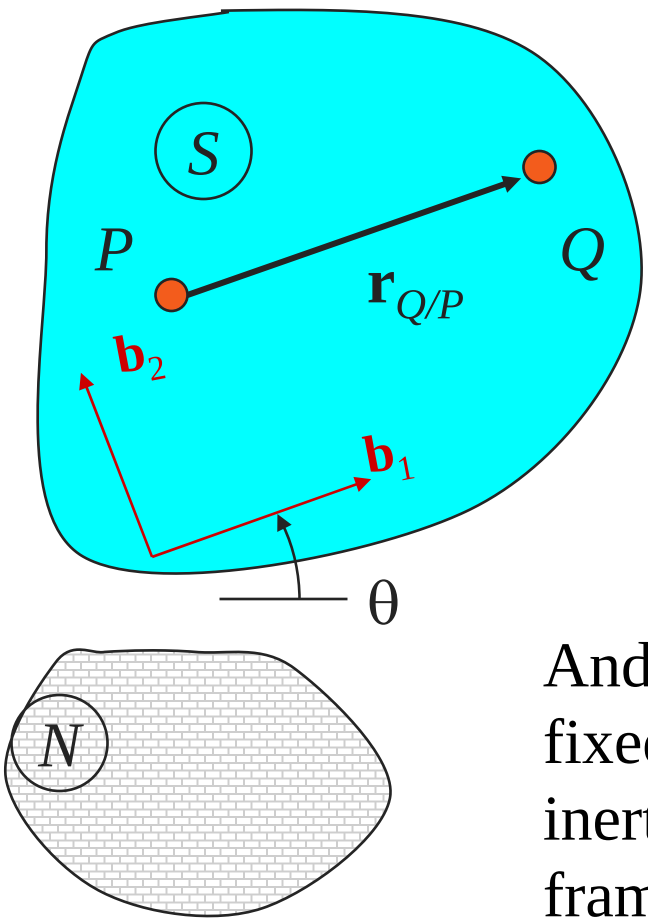






ψ
θ
Trebuchet with cup
angle ψ
ϕ
θ
m_F
m_H
C
O
m_C
m_A
θ

Kinematics of Planar Rigid Bodies



Relative velocity and acceleration between *any* two points fixed on *any* rigid body:

$$\mathbf{v}_Q - \mathbf{v}_P = \omega \times \mathbf{r}_{Q/P}$$

$$\mathbf{a}_Q - \mathbf{a}_P = \alpha \times \mathbf{r}_{Q/P} + \omega \times \left(\omega \times \mathbf{r}_{Q/P}\right)$$

And if *P* is fixed to the inertial frame *N*

$$\mathbf{v}_Q = \omega \times \mathbf{r}_{Q/P}$$

$$\mathbf{a}_Q = \alpha \times \mathbf{r}_{Q/P} + \omega \times \left(\omega \times \mathbf{r}_{Q/P}\right)$$

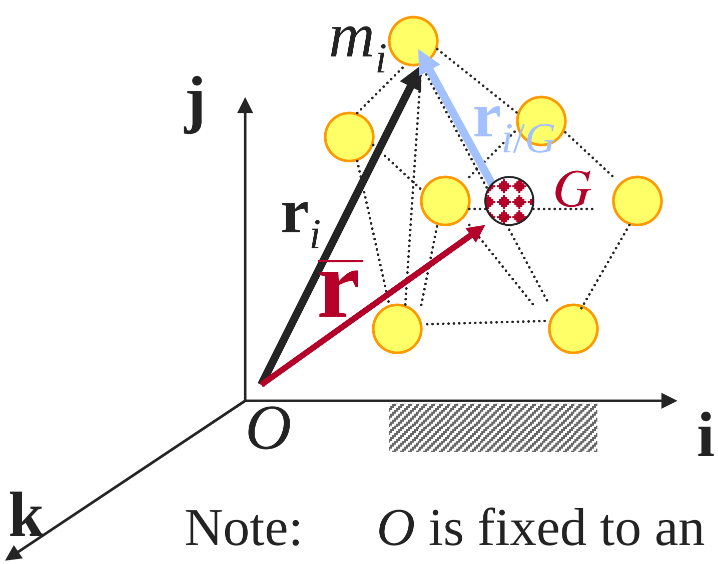
Recap: Rate of Change of Angular Momentum of a System of Particles about a fixed point, *O*

Angular Momentum of the *system* about *O*

$$\mathbf{H}_O = \sum_{i=1}^{N} \mathbf{r}_i \times m_i \, \mathbf{v}_i$$

Resultant moment of all external forces acting on the *system* about *O*

$$\mathbf{M}_O = \sum_{i=1}^{N} \mathbf{r}_i \times \mathbf{F}_i$$

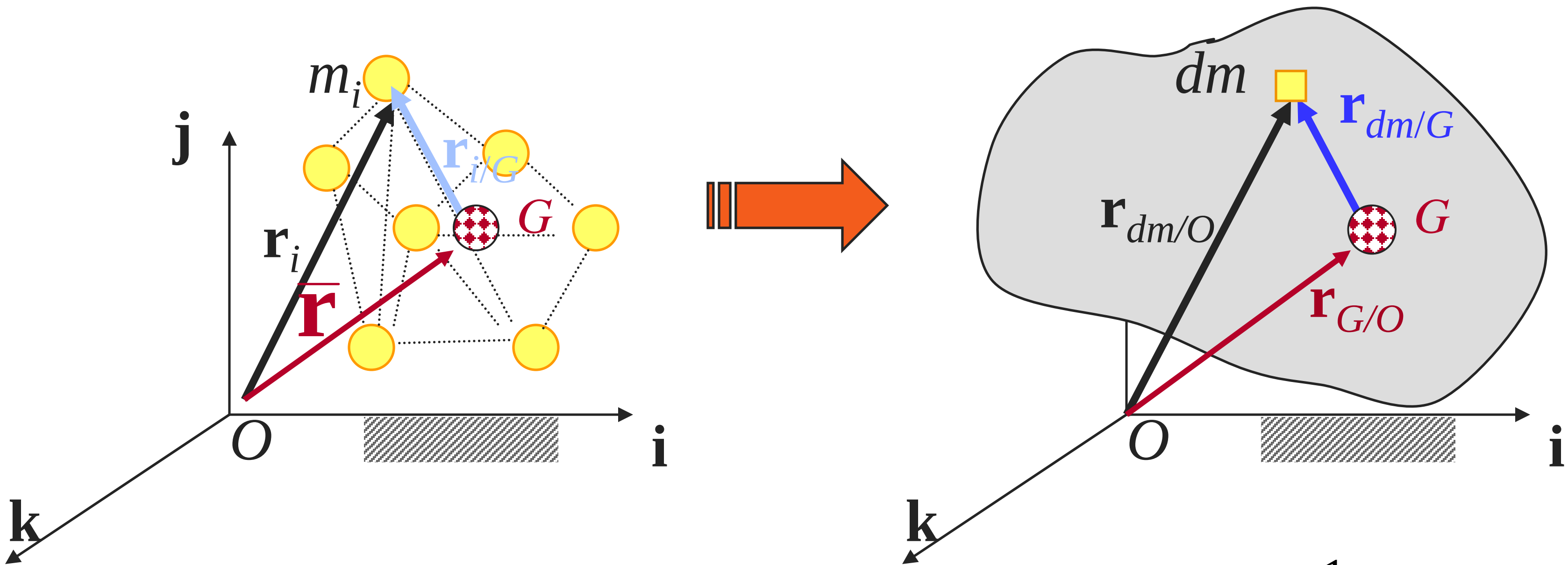


The rate of change of angular momentum of the *system* about *O* is equal to the resultant moment of all external forces acting on the *system* about *O*

$$\frac{d\,\mathbf{H}_O}{dt} = \mathbf{M}_O$$

Note: *O* is fixed to an inertial frame

Angular Momentum about a fixed point, *O*



Define $\bar{\mathbf{r}} = \frac{1}{m}\sum_i m_i \mathbf{r}_i$

Define $\mathbf{r}_{G/O} = \frac{1}{m}\int_{body} \mathbf{r}_{dm/O}\, dm$

Angular Momentum of the *system* about *O*

$$\mathbf{H}_O = \sum_{i=1}^{N} \mathbf{r}_i \times m_i\ \mathbf{v}_i$$

$$\mathbf{H}_O = \int_{body} \mathbf{r}_{dm/O} \times \mathbf{v}_{dm}\, dm$$

$$\boxed{\omega \times \mathbf{r}_{dm/O}}$$

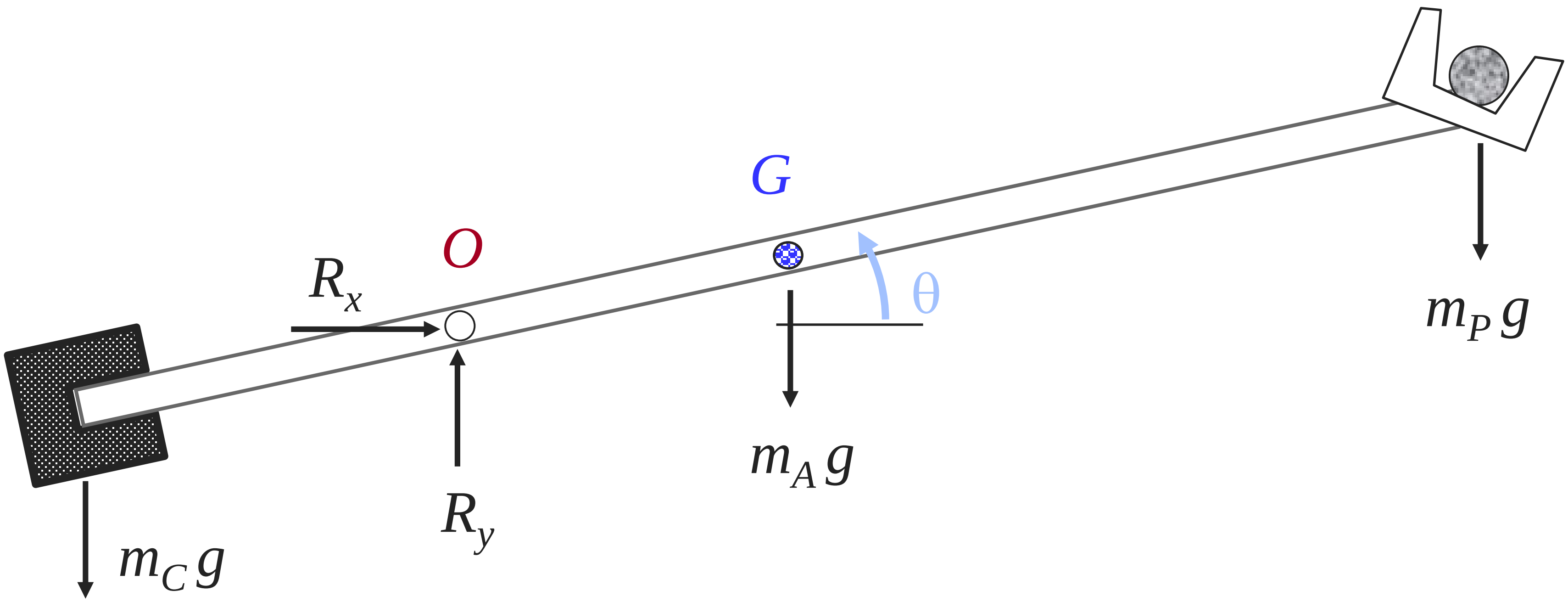
Mass Moment of Inertia about *O*

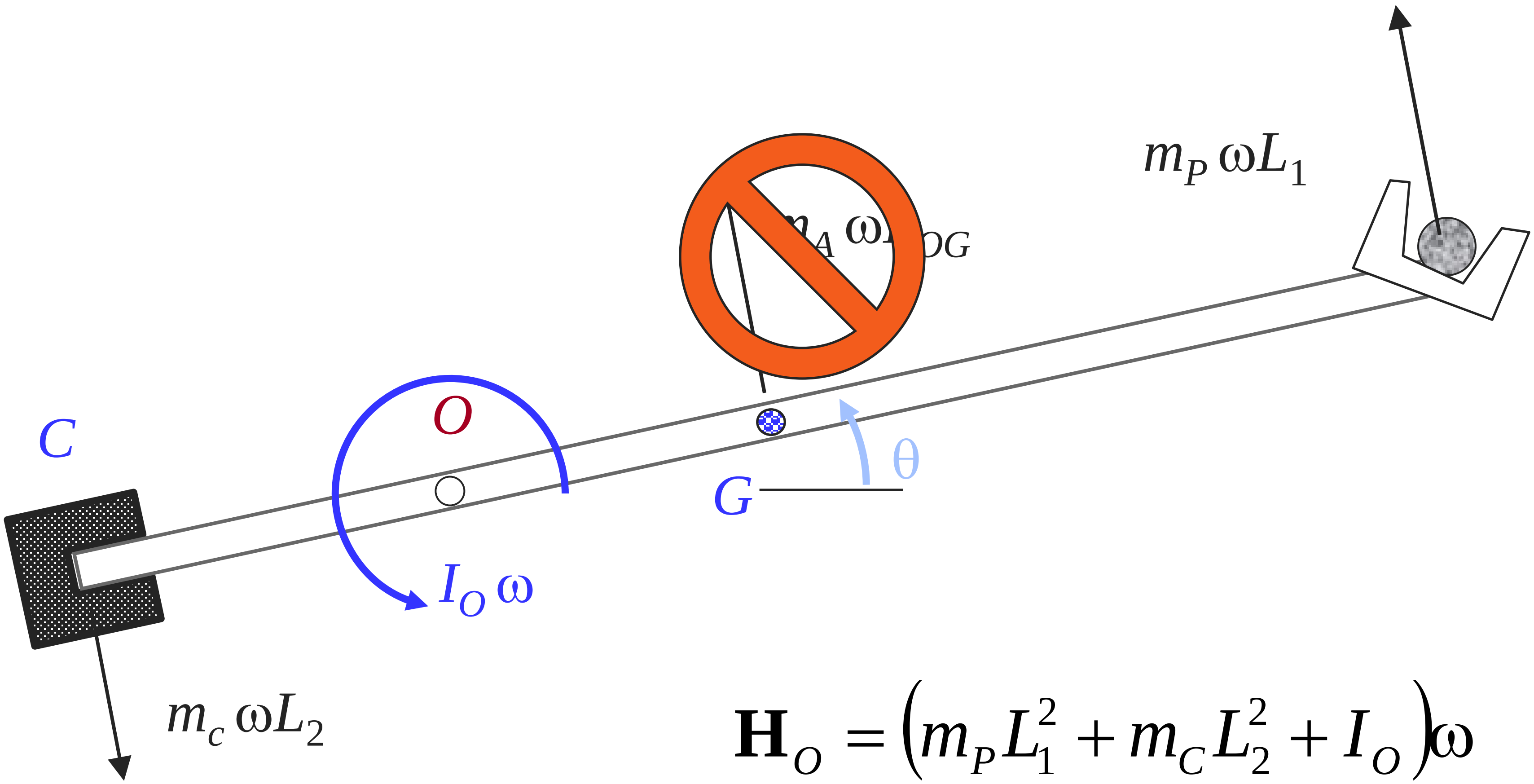
Definition

$$I_O = \int_{body} \left(r_{dm/O}\right)^2 dm$$

Why define it:

- Expression for angular momentum about *O*
  - $\mathbf{H}_O = \mathrm{I}_O\, \boldsymbol{\omega}$
- Expression for kinetic energy of a rigid body pivoted to a fixed point *O*
  - $KE = \frac{1}{2}\ \mathrm{I}_O\, \omega^2$


G
O
R_x
θ
$m_P\,g$
$m_A\,g$
R_y
$m_C\,g$



$$\mathbf{H}_O = \left(m_P L_1^2 + m_C L_2^2 + I_O\right)\omega$$

$$\sum \mathbf{M}_O = \dot{\mathbf{H}}_O$$
$$= \left(m_P L_1^2 + m_C L_2^2 + I_O\right)\dot{\omega}$$