MEAM 211

Project 1^+

Only for students who have had prior Matlab experience

Department of Mechanical Engineering and Applied Mechanics

University of Pennsylvania

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1 Introduction

The goal of this project is to investigate the dynamics and guidance system for a cruise missile using the numerical integration capability built into MATLAB.

2 Background

The Raytheon Tomahawk Cruise Missile is approximately 20 feet (6.25 meters) long and 21 inches (0.52 meters) in diameter. It can be launched from aircraft, ships, or ground installations against land or naval targets. At launch it uses a 550-pound (250-kg) solid rocket booster to launch it into the air and get it up to the speed of several hundred miles per hour at an altitude of approximately 150 meters. The launch weight is approximately 3,200 pounds (1450 kg). The booster falls away once it has burned its fuel. The wings, tail fins and air inlet unfold, and the turbofan engine takes over. This engine weighs just 145 pounds (65 kg) and produces 600 pounds of thrust. The fuel load is 800 to 1,000 pounds (about 450 kg) of fuel at launch, or approximately 150 gallons (600 liters). The missile has a cruising speed of 550 mph. The guidance system uses internally stored computerized maps of its route to follow the contour of the terrain and also makes use of information from navigation satellites to adjust its course according to specified no-fly zones and the desired target. More information can be found online (see the references).

3 Dynamics

The dynamics of the cruise missile system are represented as a particle (point mass) subject to three distinct forces. The first force is the thrust due to the engine. The second force is drag which is due to the fact that the missile is moving through the air. Finally, acceleration due to gravity impacts the dynamics of the missile. Newton's second law gives us:

$$\mathbf{F}_T + \mathbf{F}_D + \mathbf{F}_G = m\mathbf{a}, \tag{1}$$

where m is the mass of the missile, and the three forces are

$$\begin{aligned}
\mathbf{F}_T &= F_{T,x}\mathbf{i} + F_{T,y}\mathbf{j}, \\
\mathbf{F}_D &= F_{D,x}\mathbf{i} + F_{D,y}\mathbf{j}, \\
\mathbf{F}_G &= F_{G,y}\mathbf{j},
\end{aligned}$$

where the subscripts T, D, and G signify thrust, drag, and the force due to gravity, respectively.

The engine thrust can be controlled to guide the system and can be treated as a control input. Because of the constraints on the engine, the engine thrust must be limited to a maximum value which we shall denote as T_{max}. If the guidance system (control system) requests a thrust that is greater than T_{max}, the thrust vector is scaled back in magnitude (but not in direction). This can be done by computing a unit vector in the same direction and then scaling it to the appropriate length, or mathematically,

$$\mathbf{F}_T = T_{max}\frac{\bar{\mathbf{F}}_T}{\|\bar{\mathbf{F}}_T\|}, \tag{2}$$

Here $\bar{\mathbf{F}}_T$ is the raw (unscaled) thrust vector which is scaled back to $\mathbf{F}_T$, a force within the thrust limit, T_{max}.

The drag on the missile is proportional to the square of the velocity. In reality this relationship is complicated and depends on the velocity of the missile and many other aerodynamic factors. For the purposes of this project, however, we will assume F_D is given by:

$$F_D = -\frac{1}{2}C_D\|\mathbf{v}\|\mathbf{v}.$$

where the drag coefficient, C_D, has the value 0.1. The drag force is in the opposite direction of the velocity vector. The force due to gravity is defined in the traditional manner, $\mathbf{F}_{G,y} = -mg\mathbf{j}$.

Note: There are many shortcomings of the simplified model presented here. For example, the point mass model is a gross simplification of the three-dimensional dynamic model that is generally used to simulate missiles. Also the constants that define the system play an important role in the simulation. You may feel the need to vary some of the constants in Section 2 based on your observations with the simulation and based on your own research on cruise missiles. If you do decide to change the model or the parameters, an explanation (see Section 7) must be provided to explain why the change was necessary and why the change is justifiable. For example, reducing the mass of the missile to a value below the mass of the engine is not justifiable. The drag coefficient cannot be reduced and the maximum thrust may not be increased.

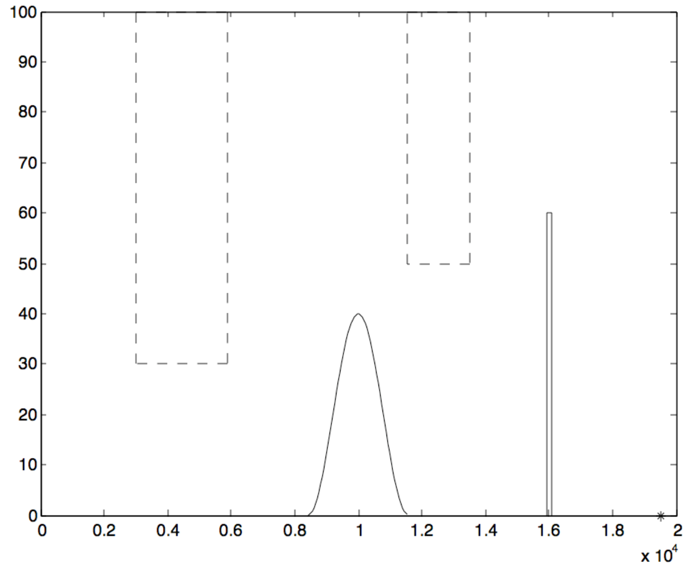

Figure 1: Simulated terrain profile for the missile. The intended target is denoted by a star (on the lower right hand corner). "No-fly zones" (because of radar exposure) and a hospital (which must be avoided at all cost) are also shown. The definition of this profile is given to you in the MATLAB script file ground_definition.m.

4 Control

The guidance system includes an estimator that estimates the position and velocity of the missile [1] based on on-board sensors and a controller that uses this information and information about the trajectory that needs to be followed to compute the thrust that the engine must generate. The state of the missile is given by the state vector:

$$X = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \tag{3}$$

We will assume that the estimator can accurately calculate the state, $X(t)$, at any time instant t.

The control system has available to it the desired trajectory, $X^{des}(t)$. In other words, it knows (has a look up table in its memory) the function:

$$X^{des}(t) = \begin{bmatrix} x^{des}(t) \\ y^{des}(t) \\ \dot{x}^{des}(t) \\ \dot{y}^{des}(t) \end{bmatrix} \tag{4}$$

[1] You are encouraged to look up internet based resources and find out how the position and velocity are sensed.

The controller has to calculate the discrepancy between the current state $X(t)$ and the desired state $X^{des}(t)$ and command corrective actions of the thruster.

The simplest control strategy that can be used in such a system is the so-called *proportional + derivative* control. The thrust is computed by adding two terms:

1. The proportional control term proportional to the error in position ; and

2. The derivative control term proportional to the error in the derivative of position (velocity).

Roughly speaking the proportional control term tries to minimize the error in the trajectory while the derivative control term allows the missile to "look ahead" to match its velocity to the desired velocity. The equations for computing the thrust are as follows. If the thrust vector has x and y components given by:

$$\mathbf{F}_T = F_{T,x}\mathbf{i} + F_{T,y}\mathbf{j},$$

the thrust vector is computed as:

$$F_{T,x} = K_{P,x}\left(x^{des}(t) - x(t)\right) + K_{V,x}\left(\dot{x}^{des}(t) - \dot{x}(t)\right) \tag{5}$$

$$F_{T,y} = K_{P,y}\left(y^{des}(t) - y(t)\right) + K_{V,y}\left(\dot{y}^{des}(t) - \dot{y}(t)\right) \tag{6}$$

If the constants, which are called *gains* in the control systems parlance, are chosen to be reasonable positive values, the thrust controller will allow the missile to follow the desired trajectory. Note that this thrust needs to be scaled according to Equation (2).

You will find that increasing the K_P values or the position gains will result in decreased errors but increased overshoots or oscillatory behavior. These overshoots can generally be successfully damped out by increasing the K_V values or the velocity gains.

5 The terrain description

You will be given one MATLAB script file that describes the terrain: `ground_definition.m`. It gives you a description of the relief of the surface over which the missile will be flying (see Fig. 1). The surface contains a hill and a hospital. Two no-fly zones are also depicted.

The parameters and initial conditions that resulted in the trajectory and thrust plots shown in Fig. 2 are presented in the following table.

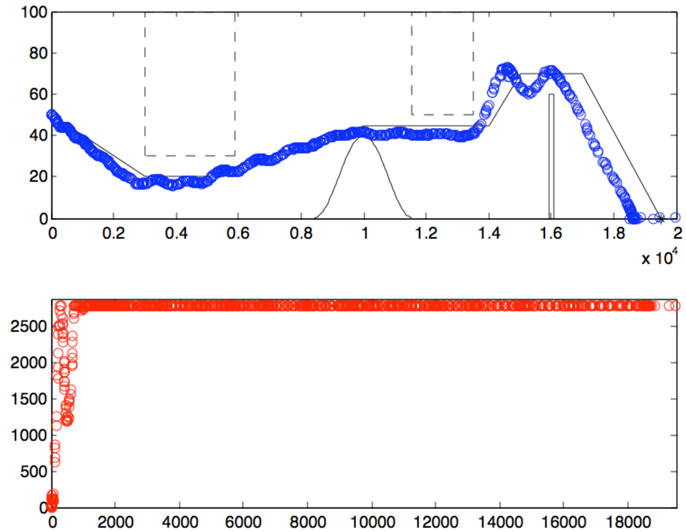

Figure 2: The trajectory as well as thrust are shown in the above plots. Note that the axes for the trajectory plot are meters vs. meters while for the thrust plot the axes are Newtons vs. meters. Also visible is the piecewise continuous trajectory defined as the desired trajectory.

Parameter	**Value**	**Units**
C_D	0.1	kg/m
g	9.81	m/s^2
m	150	kg
T_{max}	2770	N
Initial Condition	**Value**	**Units**
$x(0)$	0	m
$y(0)$	50	m
$\dot{x}(0)$	220	m/s
$\dot{y}(0)$	0	m/s

6 Project

The project consists of the following steps.

1. Write the equations of motion in state space notation to get it into the standard form:

$$\dot{X} = f(X, t),$$

   where $X = [x, y, \dot{x}, \dot{y}]^T$ is the state of the system.

2. Design a suitable trajectory $X^{des}(t) = [x^{des}(t), y^{des}(t), \dot{x}^{des}(t), \dot{y}^{des}(t)]^T$ for the missile. While you

will be given a trajectory to get you started, you will want to refine this trajectory to improve the performance of the missile.

3. Develop a MATLAB simulator to simulate $\dot{X} = f(X, t)$ at any time t and state Xwhich will involve calls to your controller to compute the thrust at any time t and state X.

4. Experiment with different choices of position gains $K_{P,x}$, $K_{P,y}$, $K_{V,x}$, and $K_{V,y}$, to see how well your controller performs.

7 Report

The report should follow the outline below.

1. The goal of the project (1-2 sentences);

2. The dynamics and control of the cruise missile (1 page maximum with equations);

3. The simulator (2 pages maximum, including a description of what each module does and the rationale underlying your code);

4. Analysis

   - Include all plots including a plot showing the terrain profile and the desired trajectory, a plot of the actual trajectory and the desired trajectory, and a plot of the speed as a function of time;
   - A brief description (1–2 paragraphs maximum) of the effects of changing the position and velocity gains; and
   - A discussion on the performance of your system (how will did it perform, did it hit the target without civilian casualties and no radar exposure?, why not?)

5. Discussion (1 page maximum, what did you learn from the project, what are the shortcomings of the modeling approach, what enhancements can you think of?)

6. Appendix: Source code with comments

You are encouraged to include background research on the dynamics of missiles in your write up.

References

[1] M, Brain, "How Cruise Missiles Work," http://science.howstuffworks.com/cruise-missile.htm, accessed Jan. 17, 2006.

[2] "Raytheon Product Data Sheet," http://www.raytheon.com/products/stellent/groups/public/documents/content/cms01_055764.pdf, Raytheon Company, accessed Jan. 17, 2006.

[3] J. Scott, "Missile Control Systems," http://www.aerospaceweb.org/question/weapons/q0158.shtml, accessed Jan. 17, 2006.