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Speed Control of Jansen Linkage Mechanism for Exquisite Tasks

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Abstract. This paper reports a toe speed control approach to achieving complex gaits with the Jansen linkage mechanism. In order to achieve complex gaits, delicate control of the toe is required. Since, the Jansen linkage mechanism is a closed loop linkage mechanism, the trajectory of the toe is defined uniquely by the set of link lengths. Hence, by controlling the toe speed, the locomotion of the toe can be controlled arbitrarily in response to intended purposes of its gait pattern. In this paper, we proved that the norm of the toe speed bears a proportionate relationship to the angular velocity of the driving link in a Jansen mechanism based robot platform. Using this relationship as basis, we derived the angular trajectory that results in a constant toe speed in the robot platform. Numerical simulations were performed to demonstrate the efficacy and validity of the proposed approach.

Keywords: Legged robots, reconfigurable mechanisms, planar linkages, Jansen mechanism.

1. Introduction

Legged robots have dependably been a famous decision for roboticians due to their prevalence over conventional wheeled or followed automated stages on applications including mobility over harsh territories [1, 2, 3]. Bartsch et al in [4] introduced their endeavours in building up a six-legged, bioinspired, and vitality effective robot (SpaceClimber 1) for extraterrestrial surface investigation, especially for portability in lunar pits. Estremera et al in [5] expounded the improvement of crab and turning gaits for a hexapod robot, SILO-6, on a characteristic landscape containing uneven ground and additionally an application to humanitarian demining. Moro et al. [6] proposed an approach to directly map a range of gaits of a horse to a quadruped robot with an intention of generating a more life-like locomotion cycle. This work also presented the use of kinematic motion primitives in generating valid and stable walking, trotting, and galloping gaits that were tested on a compliant quadruped robot. In these works, the robots developed were generally effective in mimicking the gait cycles of their biological counterparts, but they suffered from high payload-to-machine-load ratio and high energy consumption. Several approaches were studied in developing energy efficient walking machines. In [7], a set of rules are put forward towards improving energy efficiency in statically stable walking robots by comparing two-legged, namely, mammal and insect, configurations on a hexapod robotic platform. In [8], an applied minimization criterion is presented for optimizing energy consumption in a hexapod robot over every half a locomotion cycle, especially while walking on uneven terrains. In [9], two significant energy efficient approaches are discussed towards determining optimal feet forces and joint torques for a six-legged robot.

Even though these works focused on the energy optimization problem, the robots experimented there in involved a series of links with multiple actuators to realize walking motion. Jansen [10], a Dutch kinetic artist, proposed an unconventional closed-kinematic-chain-based approach which requires actuation at only a single per leg through mapping internal cyclic motion into elliptical ones. Various aspects of the Jansen mechanism have been studied by a number of researchers. In [11] proposed an extension of the Theo Jansen mechanism by introducing an additional up-down motion in the linkage center for realizing new gait cycles with about ten times the height of the original for climbing over obstacles. Vector loop and simple geometric methods were used in conjunction with software tools such as ProEngineer and SAM for analyzing forward kinematics of the Theo Jansen mechanism in [12]. An attempt to optimize the leg geometry of the Theo Jansen mechanism using genetic algorithm is presented in [13]. The work explored the stability limits and tractive abilities while validating the kinematic and kinetic models through experiments with hardware prototypes. In [14], a preliminary dynamic analysis is performed using the superposition method with the intention of optimizing the Theo Jansen mechanism. In [15], we presented a dynamic analysis of a four legged Theo Jansen link mechanism using projection method that results in constraint force and equivalent Lagrange's equation of motion. In [16, 17], design of a novel reconfigurable Theo Jansen linkage is put forward and validated that produces a wide variety of gait cycles, opening new possibilities for innovative applications. The suggested mechanism switches from a pin-jointed Grübler kinematic chain to a 5-degree-of-freedom mechanism with slider joints during the reconfiguration process. An original design approach has been presented in [18] towards development of a trajectory generator that realizes a set of stable walking gaits, gait synchronization and transition for a reconfigurable Jansen platform.

This paper reports a toe speed control approach to achieving complex gaits with the Jansen linkage mechanism. In order to achieve complex gaits, delicate control of the toe is required. Since, the Jansen linkage mechanism is a closed loop linkage mechanism, the trajectory of the toe is defined uniquely by the set of link lengths. Hence, by controlling the toe speed, the locomotion of the toe can be controlled arbitrarily in response to intended purposes of its gait pattern. In this paper, we proved that the norm of the toe speed bears a proportionate relationship to the angular velocity of the driving link in a Jansen mechanism based robot platform. Using this relationship as basis, we derived the angular trajectory that results in a constant toe speed in the robot platform. Numerical simulations were performed to demonstrate the efficacy and validity of the proposed approach.

This paper is organized as follows: Section 2. derives the forward kinematics of the Jansen linkage mechanism based on Bilateration problem. In Section 3., the angular trajectory that results in a constant toe speed of the robot platform is derived. Section 4. presents the results of the numerical simulations and the validity of the proposed approach. Section 5. shows the application case that substantiates our results. Finally, concluding remarks are given in Section 6.

2. Forward Kinematics of a Jansen Linkage Mechanism

The Jansen linkage mechanism is an eight-link one degree-of-freedom planar linkage, that is, a Grübler kinematic chain, designed by the Dutch kinetic sculptor Theo Jansen during the 1990s for emulating a smooth and elegant walking motion [10]. This linkage, depicted in Fig. 2, has three independent loops and consists of six binary links, one ternary link, and a coupler link with seven revolute joints. Since one of the revolute joints involves there binary links, the topology of this kinematic chain does not correspond to any of the sixteen topologies of standard one degree-of-freedom eight-bar linkages [19, Appendix D].

In a Jansen linkage mechanism, according to the notation of Fig. 2, the centers of the revolute joints of the binary links define the line segments $\overline{P_1P_2}$, $\overline{P_1P_6}$, $\overline{P_2P_3}$, $\overline{P_3P_4}$, $\overline{P_3P_6}$, and $\overline{P_5P_7}$, those for the ternary link define the triangle $\triangle P_1P_4P_5$, and those for the coupler link with point P_8, the foot of a Jansen linkage mechanism, define the triangle $\triangle P_6P_7P_8$. The position analysis problem for this linkage corresponds to, given the dimensions of every link, the position of the revolute joint centers P_1 and P_2, and an angle θ for the input link, calculating all the feasible Cartesian locations of point P_8. To this end, instead of using joint angles through independent loop-closure equations [20], we will use squared distances and bilateration matrices to compute the corresponding values of P_8.

2.1. The bilateration matrix

The bilateration problem consists in finding the feasible locations of a point, say P_k, given its distances to two other points, say P_i and P_j, whose locations are known. Then, according to Fig. 1, the solution to this problem, in matrix form, can be expressed as:

$$\boldsymbol{p}_{i,k} = \boldsymbol{Z}_{i,j,k}\boldsymbol{p}_{i,j} \tag{1}$$

where $\boldsymbol{p}_{i,j} = \overrightarrow{P_iP_j}$ and

$$\boldsymbol{Z}_{i,j,k} = \frac{1}{2\,s_{i,j}}\begin{bmatrix} s_{i,j}+s_{i,k}-s_{j,k} & -4\,A_{i,j,k} \\ 4\,A_{i,j,k} & s_{i,j}+s_{i,k}-s_{j,k} \end{bmatrix}$$

is called a *bilateration matrix*, with $s_{i,j} = d_{i,j}^2 = \|\boldsymbol{p}_{i,j}\|^2$, the squared distance between P_i and P_j, and

$$A_{i,j,k} = \pm\frac{1}{4}\sqrt{\left(s_{i,j}+s_{i,k}+s_{j,k}\right)^2 - 2\left(s_{i,j}{}^2+s_{i,k}{}^2+s_{j,k}{}^2\right)}, \tag{2}$$

the oriented area of $\triangle P_iP_jP_k$ which is defined as positive if P_k is to the left of vector $\boldsymbol{p}_{i,j}$, and negative otherwise. It can be observed that the product of two bilateration matrices is commutative. Then, it is easy to prove that the set of bilateration matrices, *i.e.*, matrices of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, constitute a commutative group under the product and addition operations. Moreover, if $\boldsymbol{v} = \boldsymbol{Z}\boldsymbol{w}$, where $\boldsymbol{Z}$ is a bilateration matrix, then $\|\boldsymbol{v}\|^2 = \det(\boldsymbol{Z})\,\|\boldsymbol{w}\|^2$. The interested reader is addressed to [21] for a derivation of (1) and its properties.

It has been shown that by using bilateration matrices, the position analysis problem of linkages is greatly simplified –see, for instance, [22, 23]. This problem consists of finding the feasible assembly modes that a kinematic chain can adopt. An assembly mode is a possible relative transformation between the links of a kinematic chain or linkage. When an assignment of positions and orientations is made for all links with respect to a given reference frame, an assembly mode is called a configuration. Next, we present how to apply the bilateration method for solving the position analysis problem of a Jansen linkage mechanism.

2.2. Bilateration-based system of equations

First, according to the notation of Fig. 2, let us compute $\boldsymbol{p}_{1,3}$ from θ and the location of the revolute joint centers P_1 and P_2. That is,

$$\boldsymbol{p}_{1,3} = \boldsymbol{Z}_{1,2,3}\boldsymbol{p}_{1,2} \tag{3}$$

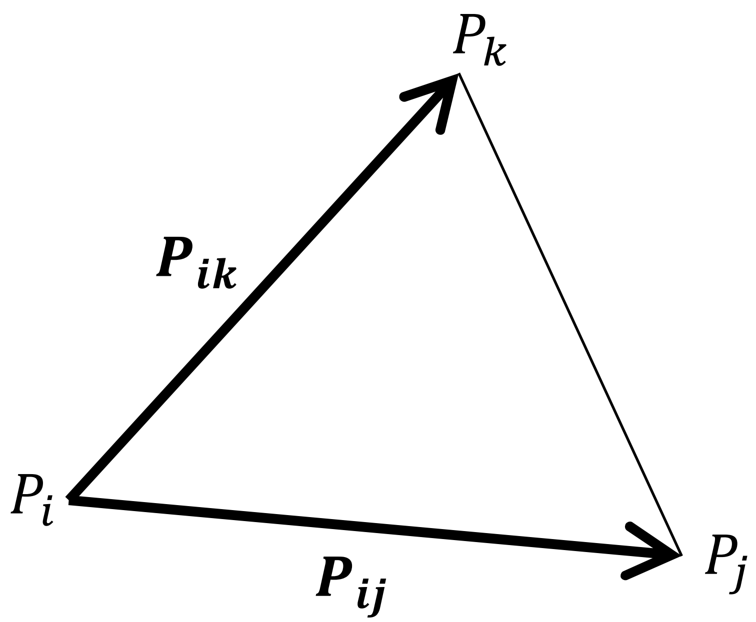

Figure 1: The bilateration problem.

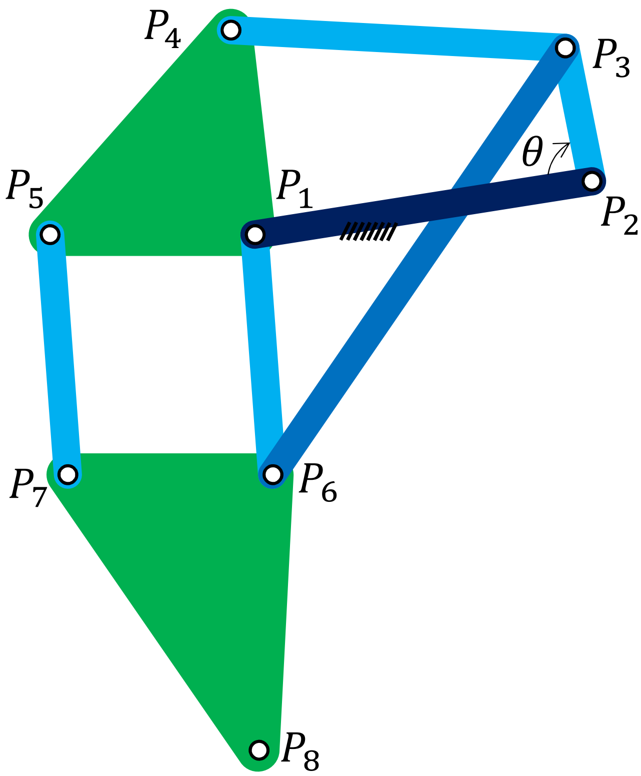

Figure 2: The Jansen linkage mechanism, a one degree-of-freedom planar linkage of eight links, seven revolute joints —one of them involving there binary links, and three independent loops.

with $s_{1,3} = s_{1,2} + s_{2,3} - 2\,d_{1,2}\,d_{2,3}\cos\theta$. Now, following a simple geometric constructive process from $\boldsymbol{p}_{1,3}$, we get

$$\boldsymbol{p}_{1,4} = \boldsymbol{Z}_{1,3,4}\boldsymbol{p}_{1,3}, \tag{4}$$
$$\boldsymbol{p}_{1,5} = \boldsymbol{Z}_{1,4,5}\boldsymbol{p}_{1,4} = \boldsymbol{Z}_{1,4,5}\boldsymbol{Z}_{1,3,4}\boldsymbol{p}_{1,3}, \text{ and} \tag{5}$$
$$\boldsymbol{p}_{1,6} = \boldsymbol{Z}_{1,3,6}\boldsymbol{p}_{1,3}. \tag{6}$$

Thus,

$$\boldsymbol{p}_{5,6} = -\boldsymbol{p}_{1,5} + \boldsymbol{p}_{1,6} = (-\boldsymbol{Z}_{1,4,5}\boldsymbol{Z}_{1,3,4} + \boldsymbol{Z}_{1,3,6})\boldsymbol{p}_{1,3}. \tag{7}$$

Then,

$$s_{5,6} = \det(-\boldsymbol{Z}_{1,4,5}\boldsymbol{Z}_{1,3,4} + \boldsymbol{Z}_{1,3,6})s_{1,3}. \tag{8}$$

Finally, from $\boldsymbol{p}_{5,6}$, we get

$$\boldsymbol{p}_{6,7} = -\boldsymbol{Z}_{5,6,7}\boldsymbol{p}_{5,6}, \tag{9}$$
$$\begin{aligned}\boldsymbol{p}_{6,8} &= \boldsymbol{Z}_{6,7,8}\boldsymbol{p}_{6,7}\\ &= -\boldsymbol{Z}_{6,7,8}\boldsymbol{Z}_{6,5,7}(-\boldsymbol{Z}_{1,4,5}\boldsymbol{Z}_{1,3,4} + \boldsymbol{Z}_{1,3,6})\,\boldsymbol{p}_{1,3}.\end{aligned} \tag{10}$$

Then,

$$\boldsymbol{p}_{1,8} = \boldsymbol{p}_{1,6} + \boldsymbol{p}_{6,8} = \left(\boldsymbol{Z}_{1,3,6} - \boldsymbol{Z}_{6,7,8}\boldsymbol{Z}_{6,5,7}(-\boldsymbol{Z}_{1,4,5}\boldsymbol{Z}_{1,3,4} + \boldsymbol{Z}_{1,3,6})\right)\boldsymbol{Z}_{1,2,3}\boldsymbol{p}_{1,2}. \tag{11}$$

Equation (11) defines the location of point P_8, the foot of a Jansen linkage mechanism. This equation depends on the set of link dimensions (S), the angle of the input link (θ), the orientation sign of the oriented areas $A_{1,2,3}$, $A_{1,3,4}$, $A_{1,3,6}$, and $A_{6,5,7}$, and the location of P_1 and P_2, the centers of the grounded revolute joints. For a given set of values for all these variables, a specific configuration of a Jansen linkage mechanism is determined, that is, the point P_8 is uniquely defined. We represent a configuration of a Jansen linkage mechanism as $(S, \theta, \eta, P_1, P_2)$ where $\eta = 0, \ldots, 15$ specifies the combination of signs for the areas $A_{1,2,3}$, $A_{1,3,4}$, $A_{1,3,6}$, and $A_{6,5,7}$. Thus, for example, $\eta = 10 = (1010)_2 \equiv + - +-$ implies that $A_{1,2,3} > 0$, $A_{1,3,4} < 0$, $A_{1,3,6} > 0$, and $A_{6,5,7} < 0$.

The ability of bilateration matrices to represent the solution of complex problems in a very compact form can be appreciated when comparing the solution for the position analysis of a Jansen linkage mechanism presented in [24] with the bilateration-based result of equation (11).

3. Trajectory Design for Constant Speed Control of Toes

This section presents the design approach to generating trajectory for achieving constant speed control of the toe based on the forward kinematics of the Jansen linkage mechanism. First, the derived forward kinematics of the Jansen linkage mechanism is analyzed, and relationship between the speed of the toe and the angular velocity of the driving angle is proven. Next, the angular velocity and trajectory of the driving link that is directly proportional to the speed of the toe are derived.

From (11), by solving Bilateration problem, the forward kinematics of the Jansen linkage mechanism can be obtain as follows:

$$\begin{aligned}\boldsymbol{p}_{1,8} &= \left(\boldsymbol{Z}_{1,3,6} - \boldsymbol{Z}_{6,7,8}\boldsymbol{Z}_{6,5,7}(-\boldsymbol{Z}_{1,4,5}\boldsymbol{Z}_{1,3,4} + \boldsymbol{Z}_{1,3,6})\right)\boldsymbol{Z}_{1,2,3}\boldsymbol{p}_{1,2}, \\ &= \boldsymbol{\zeta}(\theta)\boldsymbol{p}_{1,2}, \end{aligned} \tag{12}$$

where, θ represents the angle between the base link and the driving link from Fig. 2. And, by differentiating (12), the toe speed can be obtained as follows:

$$\dot{\boldsymbol{p}}_{1,8} = \dot{\boldsymbol{\zeta}}(\theta)\boldsymbol{p}_{1,2} + \boldsymbol{\zeta}(\theta)\dot{\boldsymbol{p}}_{1,2}.$$

Since $\dot{\boldsymbol{p}}_{1,2} = 0$ and $\dot{\boldsymbol{\zeta}}(\theta) = \frac{\partial}{\partial\theta}\boldsymbol{\zeta}(\theta)\dot{\theta}$,

$$\dot{\boldsymbol{p}}_{1,8} = \frac{\partial}{\partial\theta}\boldsymbol{\zeta}(\theta)\boldsymbol{p}_{1,2}\dot{\theta}. \tag{13}$$

Hence,

$$\left\|\dot{\boldsymbol{p}}_{1,8}\right\| = \left\|\frac{\partial}{\partial\theta}\boldsymbol{\zeta}(\theta)\boldsymbol{p}_{1,2}\right\|\left|\dot{\theta}\right|. \tag{14}$$

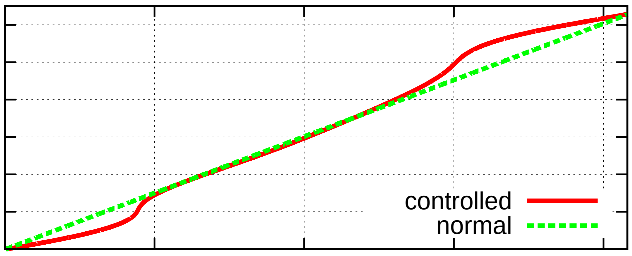

Figure 3: The time variation of the angle of the driving link.

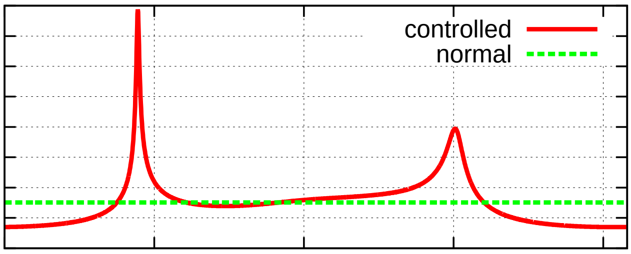

Figure 4: The time variation of the angular velocity of the driving link.

From (14), let a target speed of the toe is $\|\dot{\boldsymbol{r}}\|$ on a angle θ of the driving link, a target angular velocity of the driving link can be formulated as follows:

$$\dot{\theta} = \frac{\|\dot{\boldsymbol{r}}\|}{\left\|\frac{\partial}{\partial\theta}\zeta(\theta)\boldsymbol{p}_{1,2}\right\|}. \tag{15}$$

Finally, by integrating (15), we obtain the target driving link angle as follows:

$$\theta = \int \frac{\|\dot{\boldsymbol{r}}\|}{\left\|\frac{\partial}{\partial\theta}\zeta(\theta)\boldsymbol{p}_{1,2}\right\|}dt. \tag{16}$$

4. Numerical Simulation

This section verify that the constant speed of the toe can be realized by referencing (16) through numerical simulation. In the simulation, the target driving link angle from (16) is substituted into equation (12). Also, the target speed of the toe is set as $\|\dot{\boldsymbol{r}}\| = 4$ cm/s. Figs. 3-6 shows the simulation results, where the red line represents the results of the proposed method and the green line represents the results in the case of controlling the driving link with constant angular velocity, respectively.

From Fig. 5 and Fig. 6, it is confirmed that the norm of the velocity vector of the toe maintains the target velocity (4 cm/s) by utilizing the proposed method. Hence, the speed

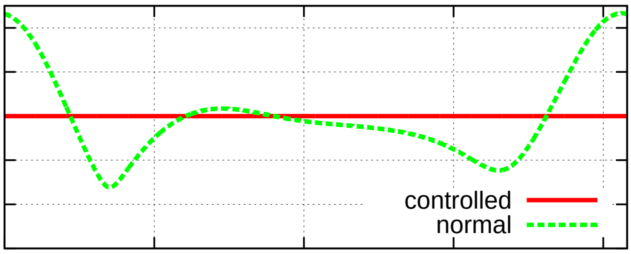

Figure 5: The time variation of the norm of the velocity vector of the toe.

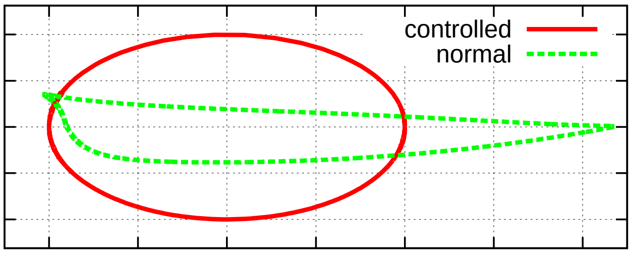

Figure 6: The trajectory of the norm of the velocity vector of the toe.

control of the toe can be realized by utilizing both the target angle shown in Fig. 3 and the target angular velocity shown in Fig. 4.

5. Application of Proposed Method

We have so far showed that the constant speed control of the toe can be realized by proposed method. This section applies the proposed method to derive the angular trajectory capable of achieving the speed control of the toe in horizontal direction. For example, a multi-legged robot manages the relationship between COG and the toe position appropriately to maintain stability. However, it is difficult for a Jansen mechanism based multi-legged robot to control its stability due to system complexity. With the proposed approach allowing for control of inputs speed of toes, the maintenance of system stability can be ensured.

Let the target velocity of the toe in the horizontal direction is r_x. Then, we obtain unknown target velocity in the vertical direction r_y because the velocity of a closed linkage mechanism is determined uniquely. Moreover, since it is difficult to obtain r_y without solving the forward kinematics, r_y should be handled as unknown parameter. Thus, since it is difficult to obtain the norm of the target velocity in this situation, (16) cannot be solved. In the application method, (13) is focused. In equation, we define

$$\frac{\partial}{\partial\theta}\boldsymbol{\zeta}(\theta)\boldsymbol{p}_{1,2} := \begin{bmatrix} p_x \\ p_y \end{bmatrix},$$

and obtain as follows:

$$\begin{bmatrix} r_x \\ r_y \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \end{bmatrix}\dot{\theta}. \tag{17}$$

From (17), since

$$r_x = p_x\dot{\theta},$$

the target driving link angle can be obtained as follows:

$$\theta = \int \frac{r_x}{p_x} dt. \tag{18}$$

By utilizing (18), in the case of the multi-legged robot as an example, since the velocity of the all supporting legs can be controlled arbitrarily, the walking control considering the stability of the robot can be realized.

6. Conclusion

This paper reported a design approach to using toe speed control towards achieving complex gaits with the Jansen linkage mechanism. It has been proven that the norm of the toe speed bears a proportionate relationship to the angular velocity of the driving link. Using this relationship as basis, we derived the angular trajectory that results in a constant toe speed in the robot platform. Numerical simulations were performed to demonstrate the efficacy and validity of the proposed approach. An application case demonstrating the use of the proposed approach has been presented that involves derivation of angular trajectory capable of speed control of the toe in horizontal direction. We have also showed that by utilizing the proposed method for the case of the multi-legged robot where the velocity of the all supporting legs can be controlled arbitrarily, the walking control ensuring system stability can be realized.

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