Work, Energy, and Power

- Newton's Second Law
- Momentum
  - Linear momentum
  - Angular momentum

- Work
- Energy
  - Kinetic Energy
  - Potential Energy

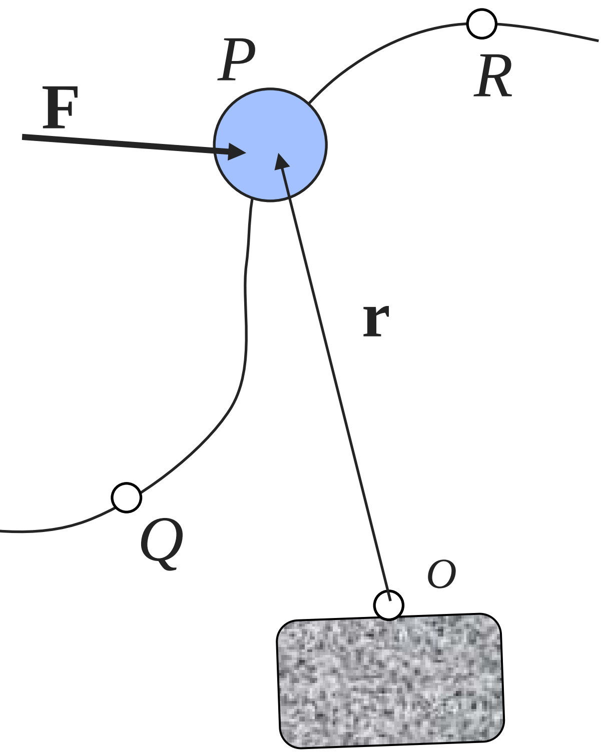
Work

mass, m

Work done by the force $\mathbf{F}$ on the particle P over the path from Q to R is given by:

$$W = \int_Q^R \mathbf{F} \cdot d\mathbf{r}$$

Note O is a point fixed in an *inertial* frame



$$\begin{aligned} dW &= \mathbf{F} \cdot d\mathbf{r} \\ &= m\ddot{\mathbf{r}} \cdot d\mathbf{r} \\ &= \tfrac{1}{2} m \, d\left(\dot{\mathbf{r}} \cdot \dot{\mathbf{r}}\right) \end{aligned}$$

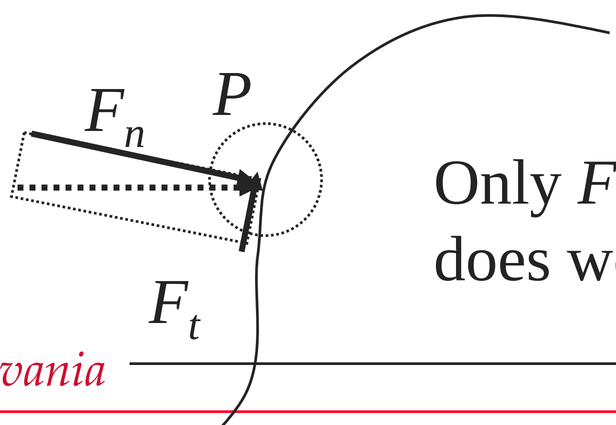
Recall

$$a = \frac{dv}{dt} \qquad adx = vdv$$

$$a = \frac{dv}{dx} v \qquad adx = \tfrac{1}{2} d\left(v.v\right)$$

The work done by $\mathbf{F}$ is equal to the change in the *kinetic energy* of the particle

$$W_{QR} = \int_Q^R \mathbf{F} \cdot d\mathbf{r} = \frac{1}{2} m\left(v_R^2 - v_Q^2\right)$$



Only F_t does work!

Mechanical Power

F acts on the particle

- Work done by **F**

$$dW = \mathbf{F} \cdot d\mathbf{r}$$

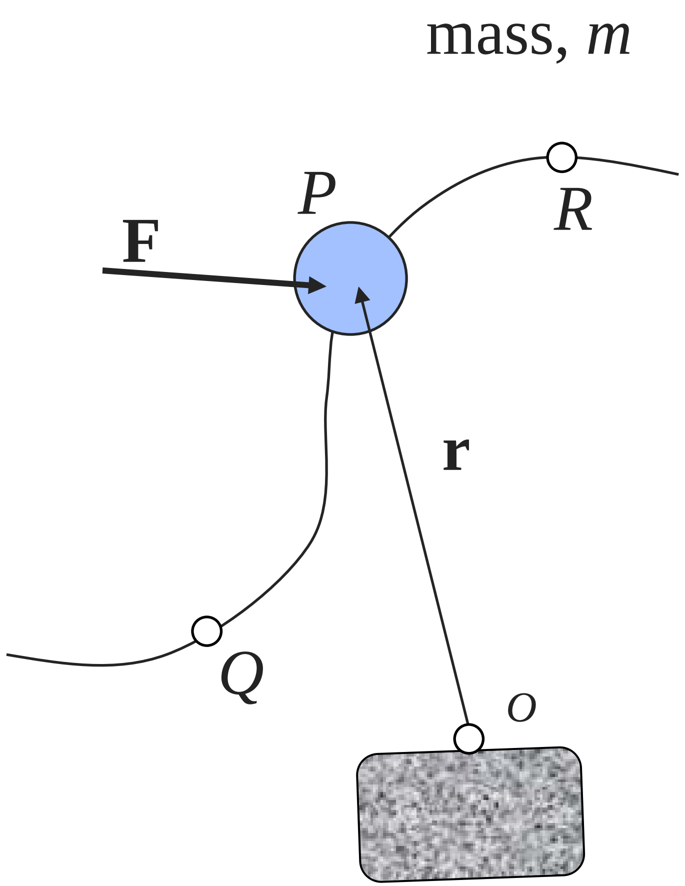
- Power developed by **F**

$$\frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$$
$$= \mathbf{F} \cdot \mathbf{v}_P$$

- Units

  Metric
  - Watts = Newton meter/second

  British
  - Lb ft/second
  - Horsepower
    - 1HP = 550 lb ft/sec
    - 1 HP = 746 W



Boeing 777-200

2 P&W turbofan engines providing ~ 74,000 lbs of thrust

Maximum take-off weight (MTOW) ~230,000 kg.

1 mile runway ~ 1600 m

What do you estimate the speed to be at the end of a 1 mile runway?

Estimated speed at the end of runway without drag = 96 m/s

But… take-off speed ~ 300 kmph (83.33 m/s)

What is the estimated drag force?

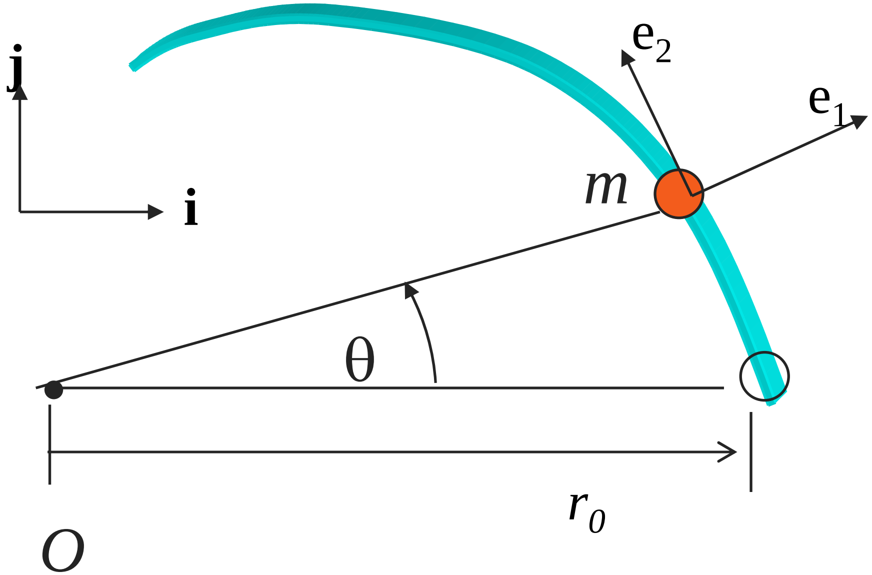
KE at the speed of 83.33 m/s = 798,610,000 Joules

Estimated average force through the length of the runway = 496 kN

Thrust = 2 × 329 kN

Drag force ~ 161 kN

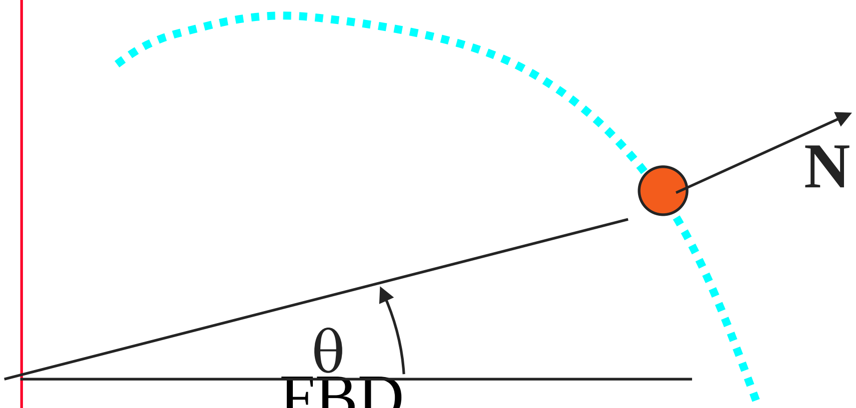
Example



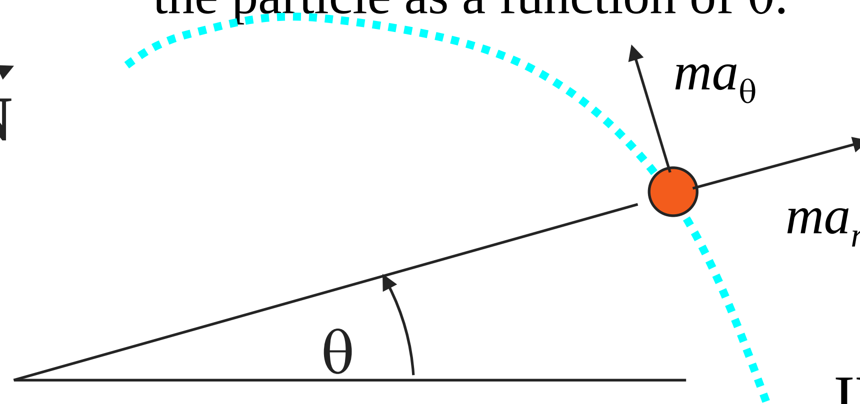
A particle of mass m slides along a horizontal frictionless track which is shaped like a logarithmic spiral:

$$r = r_0\, exp(-a\theta)$$

If the initial speed is v_0 when θ=0, find the speed of the particle and the magnitude of the track force acting on the particle as a function of θ.

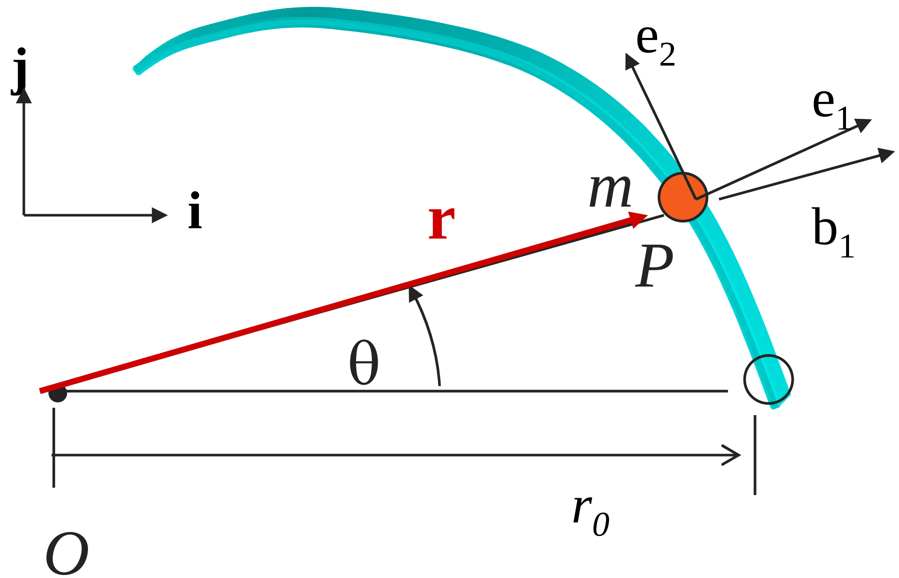


FBD



IRD

Example



Horizontal frictionless track which is shaped like a logarithmic spiral

$$r = r_0 \, exp(-a\theta)$$

The initial speed is v_0 when $\theta=0$

Find the speed of the particle and the magnitude of the track force acting on the particle.

$$\mathbf{r} = r\,\mathbf{b}_1$$

$$\mathbf{v}_P = r\dot{\theta}\,(-a\,\mathbf{b}_1 + \mathbf{b}_2)\ \boxed{= v\,\mathbf{e}_2}$$

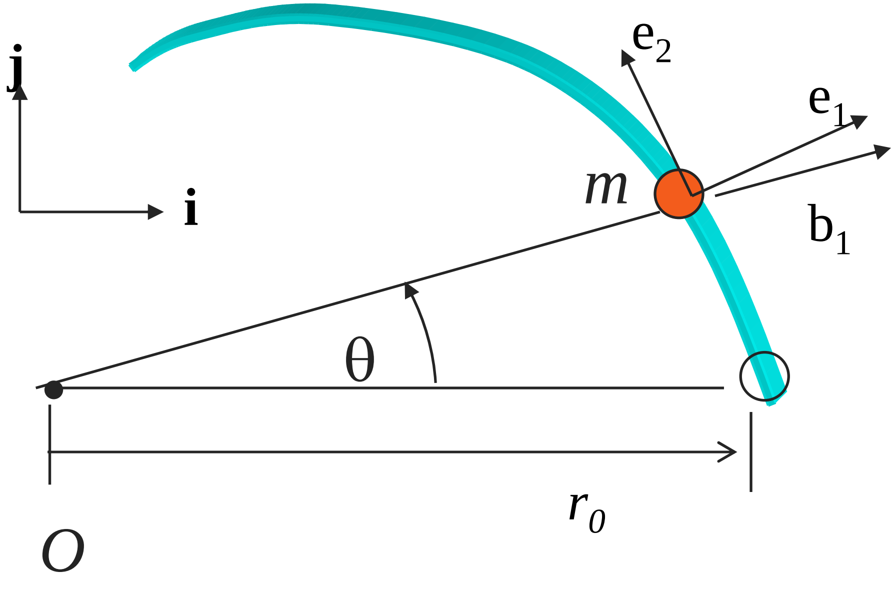
$$\mathbf{e}_1 = \frac{\mathbf{b}_1 + a\mathbf{b}_2}{\sqrt{1+a^2}} \quad \text{(unit normal)}$$

$$\mathbf{e}_2 = \frac{-a\mathbf{b}_1 + \mathbf{b}_2}{\sqrt{1+a^2}} \quad \text{(Same as } \mathbf{e}_t\text{)}$$

Track force = $N\,\mathbf{e}_1$

Force in the $\mathbf{e}_2$ direction?

Example



Since **N** is normal to the track, d**r** is tangential to the track.

$$W = \int_{r_0}^{r} \mathbf{N} \cdot d\mathbf{r} = \boxed{}$$

Therefore the speed of the particle is…

$$\boxed{}$$

$$\mathbf{e}_1 = \frac{\mathbf{b}_1 + a\mathbf{b}_2}{\sqrt{1+a^2}}$$

$$\mathbf{e}_2 = \frac{-a\mathbf{b}_1 + \mathbf{b}_2}{\sqrt{1+a^2}}$$

$$\mathbf{v}_P = v\,\mathbf{e}_2 \Rightarrow \mathbf{a}_P = \boxed{}$$

Newton's Laws

$$\frac{d}{dt}\left(m\ \mathbf{v}_P\right) = N\mathbf{e}_1$$

$$\frac{d}{dt}\left(r\mathbf{b}_1 \times m\ \mathbf{v}_P\right) = r\mathbf{b}_1 \times N\mathbf{e}_1$$

Conservative Force Field

Four equivalent definitions.

F is conservative

- **F** is a function only of the position of the particle and the work done by the force **F** on the particle *P* to get it from *Q* to *R* is independent of the path

- **F** is a function only of the position of the particle and the work done by the force **F** on the particle *P* is zero along *any* closed path

- There exists a scalar function ϕ ($\mathcal{PE}$) such that

$$dW = \mathbf{F} \cdot d\mathbf{r} = -d\phi$$

- There exists a scalar function ϕ ($\mathcal{PE}$) and a coordinate *s* such that*

$$F = \frac{-d\phi}{ds}$$

For multiple coordinates say *x* and *y*: $F_x = \dfrac{-\partial\phi}{\partial x}; \quad F_y = \dfrac{-\partial\phi}{\partial y}$

Conservation of Mechanical Energy

F is conservative

- There exists a scalar function ϕ such that

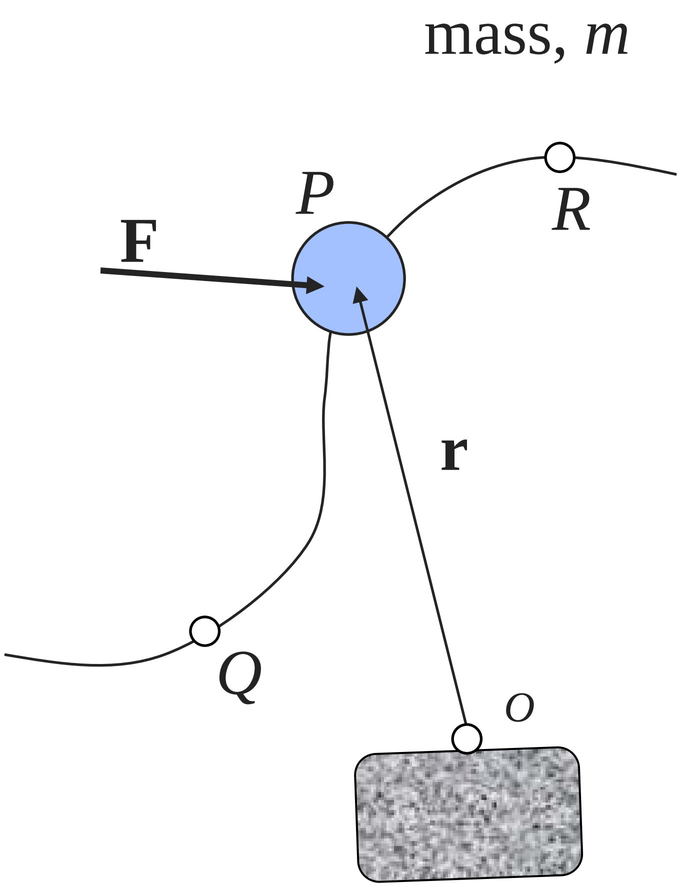
$$dW = \mathbf{F}\cdot d\mathbf{r} = -d\phi$$

- Work done by **F**

$$W_{QR} = \int_Q^R \mathbf{F}\cdot d\mathbf{r} = \frac{1}{2}m\left(v_R^2 - v_Q^2\right)$$
$$= \phi(Q) - \phi(R)$$

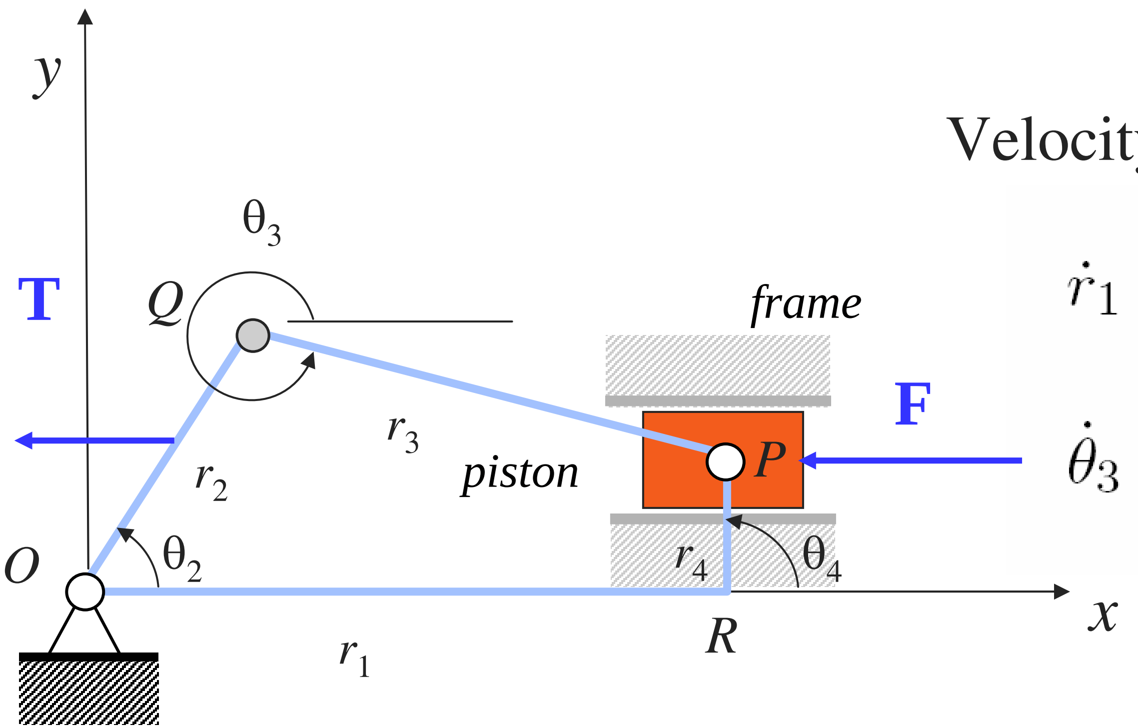
- Total energy is constant

$$\frac{1}{2}m\left(v_R^2\right) + \phi(R) = \frac{1}{2}m\left(v_Q^2\right) + \phi(Q)$$



Example

Find the magnitude of the force **T** which acts on the midpoint of the crank, given the force **F** acts on the piston. Neglect inertia, friction and gravity.



Velocity Equations (from before)

$$\dot{r}_1 = r_2 \frac{sin(\theta_3 - \theta_2)}{cos(\theta_3)} \dot{\theta}_2$$

$$\dot{\theta}_3 = -\frac{r_2 cos(\theta_2)}{r_3 cos(\theta_3)} \dot{\theta}_2$$