Design, Analysis and Prototyping of a Simplified Trebuchet

MEAM 211 Project III

March 31, 2007

1 Background

The trebuchet is a mechanical device derived from the catapult used for projecting artillery pieces going back to 300 B.C. See [1] for a historical account and a review of the many trebuchet designs including modern reconstructions. The annual *Punkin Chunkin* contest in Lewes, Delaware, brings many contestants who compete in building different pumpkin launchers [2]. Our goal is to build a mechanical device to launch a pumpkin approximately 75 - 100 feet and accurately knock down a designated target. The device must be strictly mechanical (pulleys, cables, pins) and activated by a simple release mechanism (for example, releasing a locking pin). It cannot have any external source of energy during the launch. However, you can manually add energy to initialize the trebuchet in a desired launch configuration.
We will divide the project into four phases.

Phase I In MEAM 211, we will analyze the simplest possible designs, all shown in Figure 1. You will develop a dynamic model of the configuration shown in the bottom right panel in Figure 1. We will think of different design modifications to improve the range of the device using strictly mechanical modifications. In particular, we will explore the use of a sling to improve the performance of the system.

Phase II In MEAM 211/247[1], you will build a physical prototype that will be able to launch a projectile with a range of approximately six feet. The better your Phase I model and the more accurate it is, the better your ability to predict the range of your trebuchet. You may need to revise your Phase I model in Phase II.

Phase III In MEAM 211, you will analyze the dynamics of a trebuchet with a sling and develop a dynamic model of the system that allows you to predict the range of the trebuchet. We will develop a computer simulation that will allow us simulate the system with different design parameters.

[1] All students, even those not registered in MEAM 247, will participate in Phase II.

Phase IV In MEAM 211/247[2], you will build a physical prototype of the trebuchet with a sling with the goal of maximizing the range of the trebuchet. You will use the Phase III model to predict the range of your trebuchet. However, you will be not be penalized if there are modeling errors that lead to discrepancies between the predicted and actual ranges. Instead, you will be expected to explain the reason for the discrepancies.

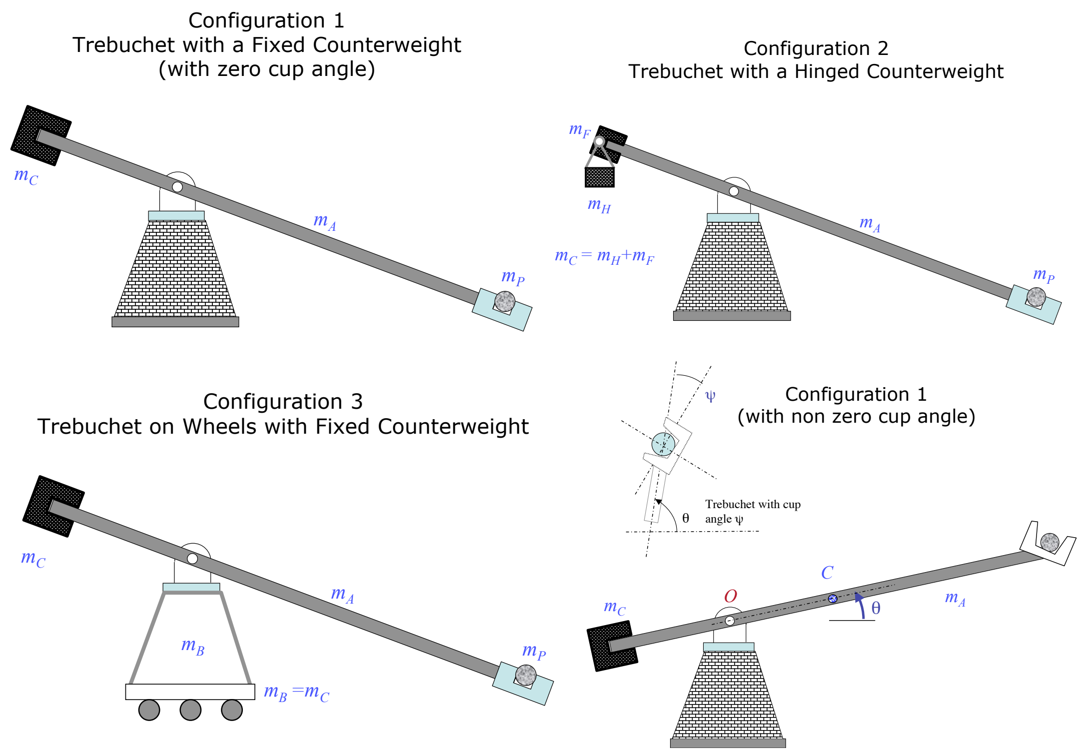

Figure 1: Three different configurations of a simple trebuchet with the projectile of mass m_P in a cup. The projectile is released when the normal force in the cup becomes zero. As shown in the bottom right panel, the cup can be fabricated so it forms an angle ψ with respect to the arm of the trebuchet. Configurations 2 and 3 have two degrees of freedom. Do either of these modifications improve the performance of the baseline configuration, Configuration 1?

[2]All students, even those not registered in MEAM 247, will participate in Phase IV.

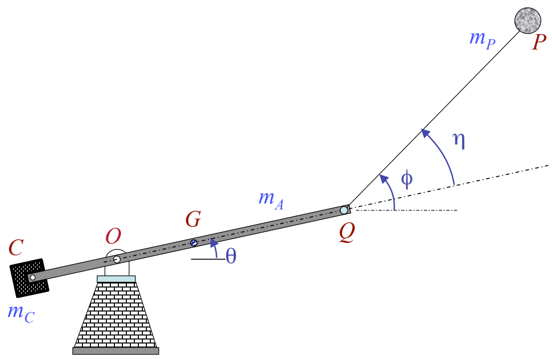

Figure 2: A trebuchet with the projectile in a sling (not shown).

2 Phase I

In Phase I, we will first a simple model of Configuration 1 (the trebuchet with a fixed counterweight) and gain some intuition to help with the design in Phase II.

- I.1 Derive equations of motion and expressions for the normal force on the projectile for each of the three configurations. See Section 3.
- I.2 Derive expressions that will allow you to calculate the range of the trebuchet. The parameters in Table 1 are for the prototype trebuchet demonstrated in class (Configuration 1 with non zero cup angle).
- I.3 Write a program that will allow you to change the parameters I_O, m_A, L_1, L_2, and ψ to determine their effect on the range of the trebuchet (Configuration 1 with non zero cup angle)?
- I.4 Assume the sum of the mass of the arm and counterweight must be less than 1750 grams and the length of the arm is at most 0.762 meters. The counterweight provided to you is approximately 1100 grams. Determine the trebuchet design parameters that will maximize your range. See Section 4.

3 Analysis

Introduction

In this section, a brief explanation of the process involved in deriving the required equations and the expression for the range - see Phase I.1, I.2 and I.3. A simple model for Configuration 1 along with the Free Body Diagram (FBD) and the Inertia Response Diagram (IRD) are shown in Figure 4.

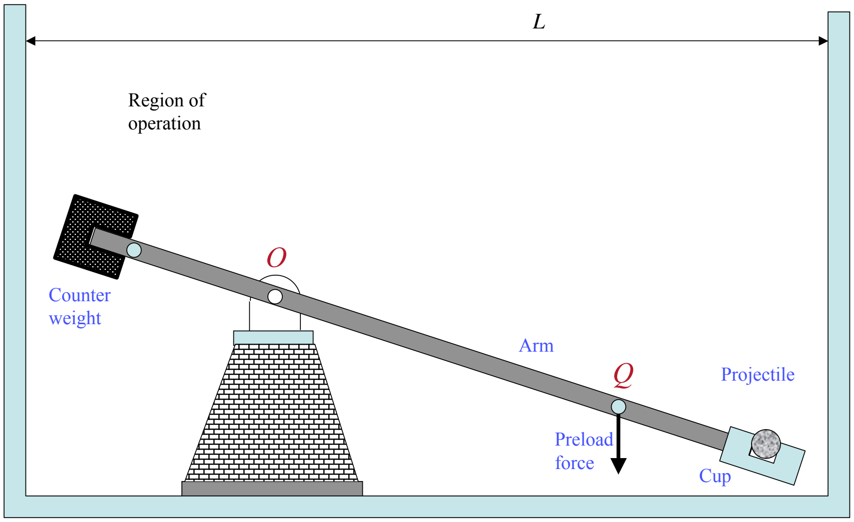

Figure 3: The projectile of the mass m_P is given. The working envelope of the trebuchet is limited to be in a bowl whose width can be at most L. No part of the trebuchet can go below the bowl during the operation of the trebuchet.

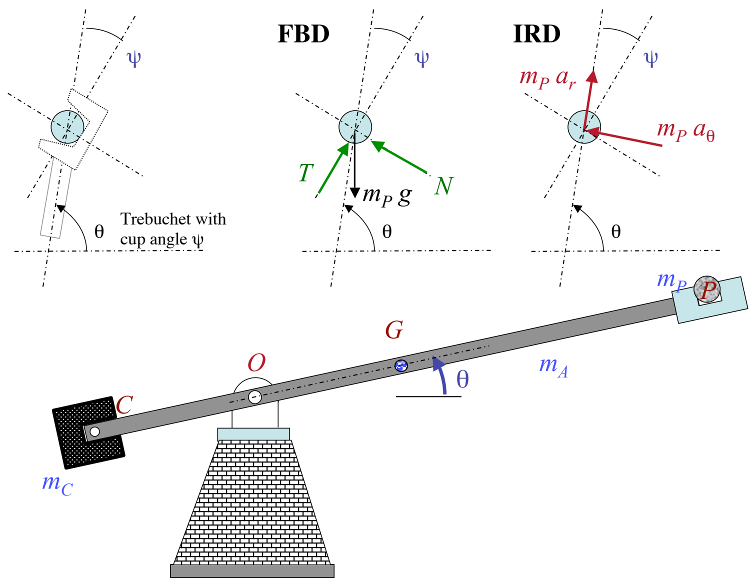

Figure 4: A simple trebuchet (Configuration 1) with the projectile of mass m_P in a cup. The projectile is released when the normal force in the cup becomes zero. The cup can be fabricated so it forms an angle ψ with respect to the arm of the trebuchet.

Symbol	Description
h	height of the pivot O above the base
L_1	length from the pivot to the center of the projectile, OP
L_2	length from the pivot to the center of the counterweight, OC
L_{OG}	length from the pivot to the center of mass of the arm OG
m_P	mass of the projectile
m_C	mass of the counterweight
m_A	mass of the arm (excluding the counterweight and the projectile)
I_O	mass moment of inertia of the arm about O (excluding the counterweight and the projectile)
m_t	the total mass of moving parts $m_t = m_P + m_C + m_A$
I_e	the effective mass moment of inertia of all moving parts about O $I_e = m_P L_1^2 + m_C L_2^2 + I_O$
θ_0	the angle from which the trebuchet is released
θ_s	the separation angle at which the normal force goes to zero
ψ	the angle made by the cup angle with the arm (see figure)
γ	a nondimensional parameter $\gamma = \frac{(m_C L_2 - m_P L_1 - m_A L_{OG}) L_1}{I_e}$

Table 1: Symbols used in the analysis

Assumptions

1. There is no friction in the system. No energy is lost either due to friction in the pin O, due to the projectile rubbing against the cup, or due to damping during the motion of the projectile.

2. Assume that the projectile does not move relative to the cup. Separation (and therefore relative motion between the projectile and the cup) occurs only when the normal force, N, between the projectile and the cup base goes to zero.

Observations

1. Energy is conserved. The total kinetic plus potential energy at the beginning must be equal to the total kinetic plus potential energy at the end.

2. Considering the FBD and the IRD for the projectile (as shown in Figure 4), we should be able to write an expression for the normal force, N, as a function of $\dot{\theta}$ and $\ddot{\theta}$.

3. We can consider the system consisting of the arm, the projectile and the counter weight as a single rigid body (before separation) and calculate the angular acceleration of the rigid body ($\ddot{\theta}$) at any configuration.

Model

Acceleration analysis for the projectile I first derived the radial and tangential components of acceleration for the projectile during the launch when it does not move relative to the arm.

$$\begin{aligned} a_r &= -L_1\dot{\theta}^2 & (1) \\ a_\theta &= L_1\ddot{\theta} & (2) \end{aligned}$$

Force balance for the projectile From the FBD and IRD, I get an expression for the normal force:

$$N = m_P\left[g\ cos\,(\theta-\psi) + L_1\left(\ddot{\theta}\ cos\ \psi - \dot{\theta}^2\ sin\ \psi\right)\right] \tag{3}$$

Force and moment balance for the system By analyzing moments about the pivot O acting on the system of the arm, the counterweight and the projectile, I was able to get an expression for the angular acceleration $\ddot{\theta}$:

$$\ddot{\theta} = \frac{[m_C L_2 - m_P L_1 - m_A L_{OG}]\ g\ cos\theta}{I_e} \tag{4}$$

where I_e is the effective inertia given by:

$$I_e = I_O + m_C L_2^2 + m_P L_1^2.$$

The term $[m_C L_2 - m_P L_1 - m_A L_{OG}]g\cos\theta$ is the net moment.

Conservation of energy Assume that the arm with the projectile is released at an initial angle θ_0 with an angular velocity 0. At any angle θ before separation, the total mechanical (potential plus kinetic) energy is the same as the mechanical energy at release. From this, I was able to obtain the expression:

$$\dot{\theta}^2 = \frac{2\ g\ (m_C L_2 - m_P L_1 - m_A L_{OG})(sin\ \theta - sin\ \theta_0)}{I_e}$$

Using the non dimensional parameter γ, this expression can be written more compactly as:

$$\dot{\theta}^2 = \frac{2\ g\ \gamma}{L_1}\ (sin\ \theta - sin\ \theta_0) \tag{5}$$

Calculation of separation angle Let θ_s be the separation angle. At that angle, the angular acceleration is given by Equation (4), and the angular velocity is given by Equation (5). Substituting both these expressions in Equation (3) gave me the following expression:

$$N = m_P\left[g\ cos(\theta_s - \psi) + \gamma g\ cos\theta_s\ cos\psi - 2\ \gamma\ g(sin\ \theta_s - sin\ \theta_0) sin\ \psi\right] \tag{6}$$

When separation occurs, the normal force goes to zero, which gives me the following equation.

$$cos(\theta_s - \psi) + \gamma cos\theta_s \; cos\psi - 2\,\gamma\,(sin\;\theta_s - sin\;\theta_0) sin\;\psi = 0 \tag{7}$$

which is linear in $cos(\theta_s)$ and $sin(\theta_s)$ for given θ_0, g and γ. We know how to solve such equations.

Determining the range of the projectile Establish a coordinate system at the pivot point, O. Given θ_0, g and γ you should be able to solve for the angle at which the projectile leaves the cup from Equation (7). Thus, the position at which the projectile is released is given by:

$$\begin{aligned} x_P(0) &= L_1 cos\theta_s \\ y_P(0) &= L_1 sin\theta_s \end{aligned}$$

Substitute θ_s into Equation (5) to get[3] $\dot{\theta}$. Calculate the velocity of the projectile when it is released at the separation time $t_s$as:

$$\mathbf{v}_P(t_s) = L_1 \dot{\theta} \mathbf{e}_\theta$$

If you know the initial position and the initial velocity you should be able to solve for the range.

Candidate design The design demonstrated in class was not optimized (see Table 2). For this poor choice of design parameters, I obtained a range of 2.52 meters.

4 Design optimization

You have three sets of choices to make.

1. The angle at which you release the trebuchet, θ_0. This translates to choosing the height of the pivot O and the length L_1. Why? If you use the given base, the height is fixed. Now choosing L_1 determines your choice for the angle of release θ_0.

2. The cup angle, ψ.

3. The lengths L_1 (if you have not already selected it) and L_2 and the mass of the counterweight m_C (if you choose not to use the counterweight provided), which will also affect the position of the center of mass, G, the length, L_{OG}, the mass m_A, and the inertia I_O. In reality, these parameters only enter into the range calculations through the parameter γ and the length L_1.

[3] When you solve for $\dot{\theta}$, make sure you chose the negative root from Equation (5).

You want to determine the best set of parameters to maximize your range. The simplest way to optimize the design is to write a program that will calculate the range for different values of these parameters, and run this program for different choices of parameters. Choose 10-12 values for each of these parameters. Run your program for each choice. If you choose n parameters to vary you will have n^{10} - n^{12} runs and results to choose the best design parameters from.

Parameter	Significance	Value
h	Height of the pivot O	80 cm.
m_P	mass of the projectile	45.69 grams
m_C	mass of the counterweight	1100 grams
m_R	mass of the rod (excluding the counterweight and the projectile)	525 grams
I_O	mass moment of inertia of the arm about O (excluding the counterweight and the projectile)	0.2660 kg. m^2
L_1	length (pivot to projectile)	87.63 cm.
L_2	length (pivot to counterweight)	49.53 cm.
L_{OG}	length (pivot to center of mass of arm)	22.2 cm.
θ_0	angle from which the trebuchet is released	-66 deg.
ψ	cup angle	10 deg.

Table 2: Examples of parameters for a trebuchet design without any optimization (Configuration 1)

5 Phase II

The goal of this phase of the project is to build a prototype and match the experimentally observed range with the range predicted by the model in Phase I. Since the focus of this part of the project is on modeling and prediction, the evaluation will be based primarily on accuracy of prediction and secondarily on range. Any discrepancies between the predicted and observed range must be carefully explained. You must conduct multiple experimental trials and provide an analysis of the variations from trial to trial.

Resources

You will be given two projectiles (a golf ball and a small rubber ball), a 1100 gram counterweight, a 5 inches $\times$ 32 inches sheet of acrylic for fabricating the arm, and a base for your projectile. However, you can also substitute these with materials that you have but with some exceptions. The total mass of your arm plus counterweight cannot exceed 1750 grams and the length of the arm can be at most 0.762 meters.

Note your arm must be designed so that the counterweights are easily mounted and removed.

6 Phase III

Materials in support for Phase III will be provided in class.

7 Phase IV

The focus of Phase IV is to design and build a trebuchet with a sling with the goal of maximizing the range of the trebuchet with a sling shown in Figure 2. You will use the model developed in Phase III to obtain a design for Phase IV. The resources available to you are limited to those in Phase II. No additional materials will be provided. Again, total mass of your arm plus counterweight cannot exceed 1750 grams and the length of the arm can be at most 0.762 meters. Your system must fit into a virtual bowl with $L = 2.0$ meters in Figure 3.

References

[1] P.E. Chevedden, L. Eigenbrod, V. Foley and W. Soedel *Scientific American*, July 1995.

[2] The Punkin Chunkin World Championship `http://www.punkinchunkin.com/`, November 2005.