Dynamics of Particles

Beyond FBD=IRD
What else can we learn from Newton's 2^{nd} and 3^{rd} laws?

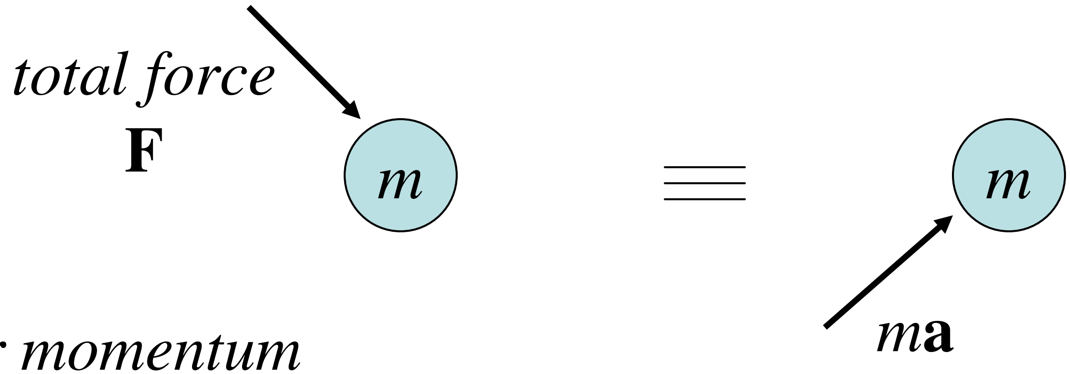
- Linear momentum and Linear impulse
- Angular momentum and Angular impulse
- Application
  - Impacts between particles

[TS, Chapter 3]

Linear Momentum

- Newton's 2nd Law

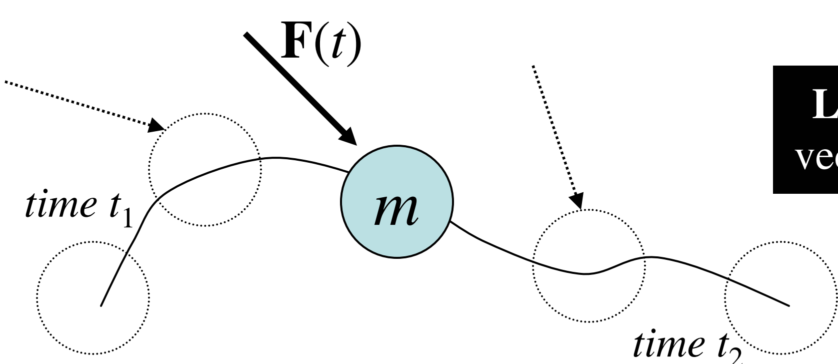
$$\mathbf{F} = m\mathbf{a}$$



linear momentum

$$\mathbf{F} = \frac{d\mathbf{L}}{dt} \qquad \mathbf{L} = m\,\mathbf{v}$$

- Linear Impulse



$$\mathbf{F} = m\mathbf{a} \quad \Rightarrow \quad \int_{t_1}^{t_2} \mathbf{F}\,dt = \int_{t_1}^{t_2} m\mathbf{a}\,dt$$

LI is a vector!!!

$$\boxed{LI_{1-2} = \int_{t_1}^{t_2} m\mathbf{a}\,dt = \mathbf{L}_2 - \mathbf{L}_1}$$

Angular Momentum

- Newton's 2nd Law

$$\mathbf{F} = m\mathbf{a}$$

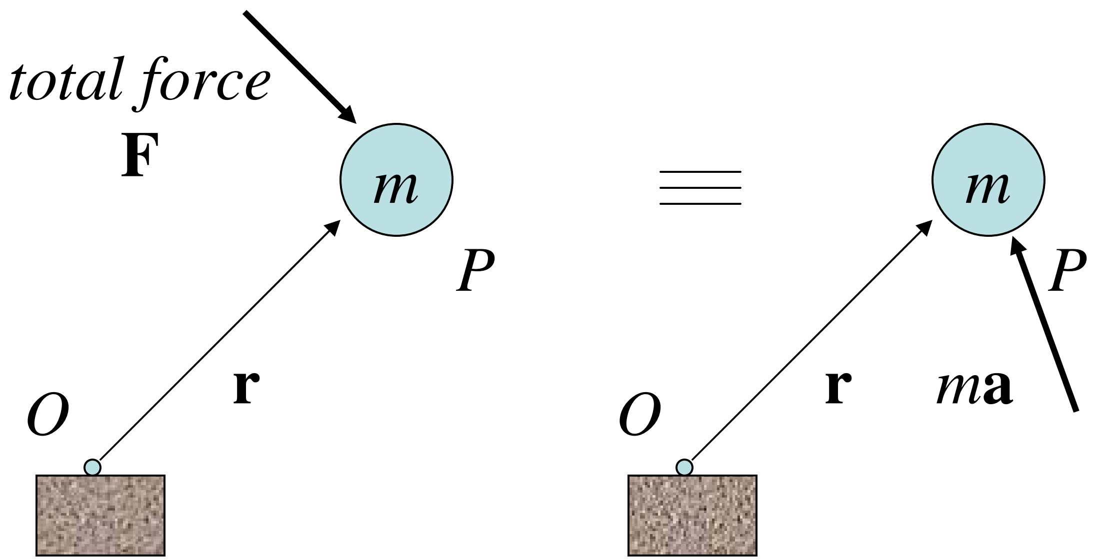
$$\mathbf{r} \times \mathbf{F} = \mathbf{r} \times m\mathbf{a}$$

Moment of the force (about O)

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Note really $\mathbf{r}_{P/O}$

?



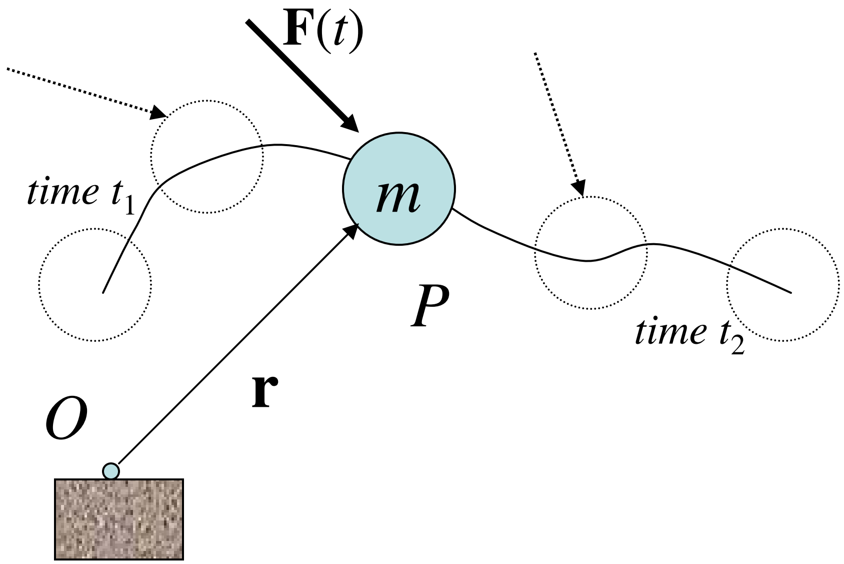
DEFINE

angular momentum

$$\mathbf{H}_O = \mathbf{r} \times m\,\mathbf{v}$$

$$\mathbf{M}_O = \frac{d\mathbf{H}_O}{dt}$$

Angular Impulse



$$LI_{1-2} = \int_{t_1}^{t_2} m\mathbf{a}dt = \mathbf{L}_2 - \mathbf{L}_1$$

$$AI_{O_{1-2}} = \int_{t_1}^{t_2} \mathbf{M}_O dt = \mathbf{H}_O(t_2) - \mathbf{H}_O(t_1)$$

Principles of Conservation of Momentum

- Conservation of Linear Momentum

$$LI_{1-2} = \int_{t_1}^{t_2} m\mathbf{a}dt = \mathbf{L}_2 - \mathbf{L}_1$$

If $\boldsymbol{LI} = 0$,
$\mathbf{L}_2 = \mathbf{L}_1$

- Conservation of Angular Momentum

$$AI_{O_{1-2}} = \int_{t_1}^{t_2} \mathbf{M}_O dt = \mathbf{H}_O(t_2) - \mathbf{H}_O(t_1)$$

If $\boldsymbol{AI}_O = 0$,
$\mathbf{H}_{O2} = \mathbf{H}_{O1}$

Example 1: Force/impulse associated with a swing

Golf club swing

Velocity ~ 150 ft/second (46 meters/second)

- Duration ~ 1 millisecond
- Mass = 1.62 oz. (0.046 kg.)
- Total linear impulse ~ 2.1 kg meters/second
- average force ~ 2100 Newtons (210 kg.)

Example 2: Force/Impulse

- mass of the ball = 0.145kg.
- mass of the bat = 0.9 kg.
- velocity of swing = 30 m/sec
- contact time < 1 msec
- fastball velocity ~ 100 mph (44.4 m/s)
- calculate average force, impulse

Impact

- Linear impulse = Change in Linear momentum
- Angular impulse = Change in Angular momentum
- Impact deals with finite impulses in very, very small time

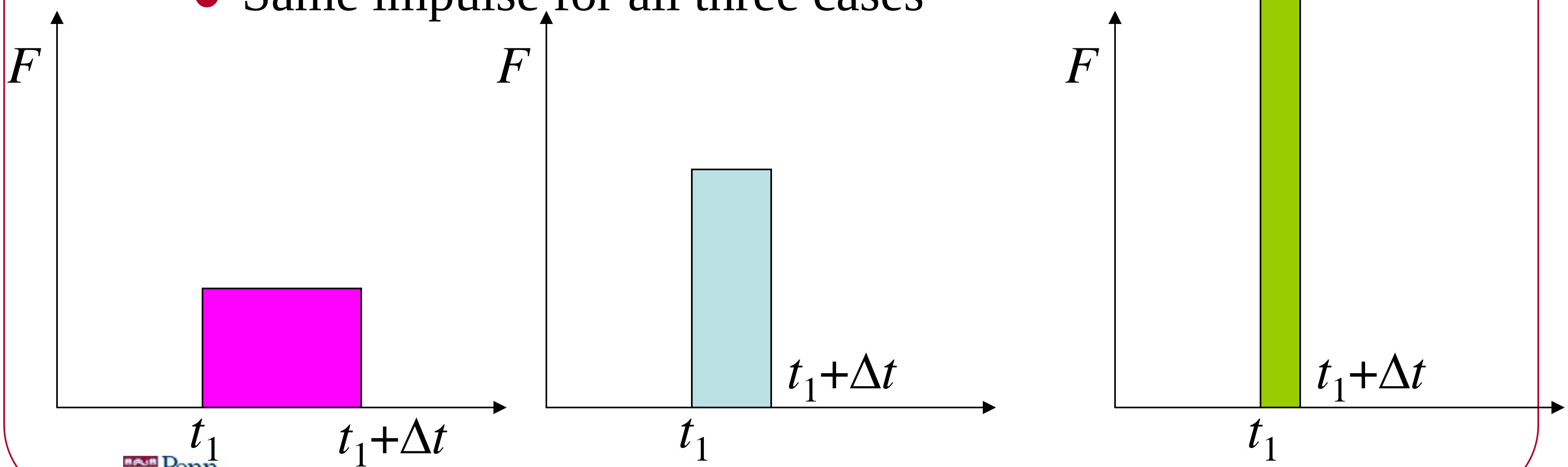
[TS, Chapter 3.7]

Linear Impulse with Small Time Intervals

- Large force, small time interval

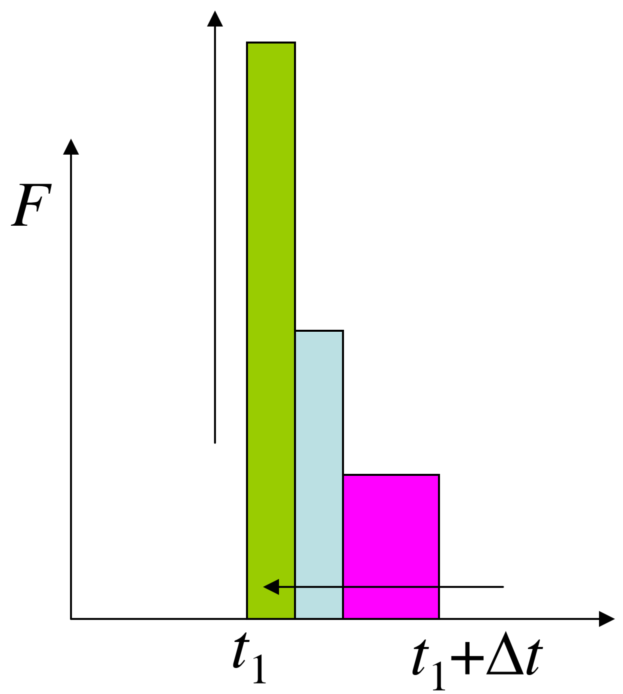
$$\boxed{LI_{t_1 - t_1+\Delta t} = \int_{t_1}^{t_1+\Delta t} \mathbf{F}\,dt = \mathbf{L}_{t_1+\Delta t} - \mathbf{L}_{t_1+\Delta t}}$$

- Impulse = area under the curve
  - Same impulse for all three cases



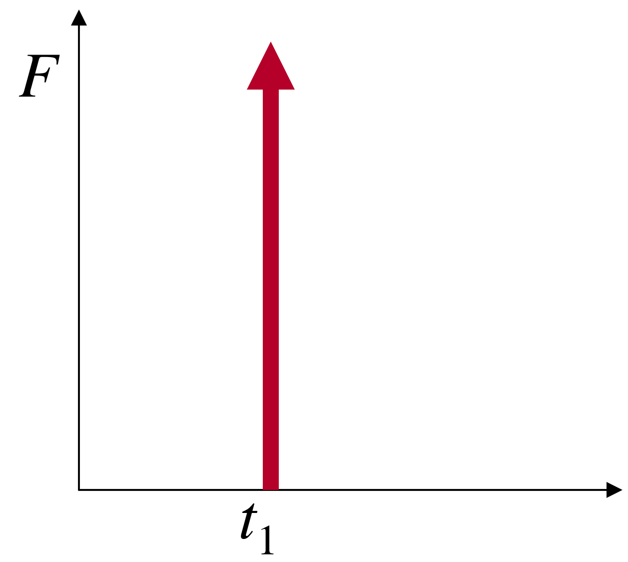
Linear Impulse with Infinitesimal time

- Large force, infinitesimal time interval
  - *lim* Δt tends to zero
- Impulse = area under the curve



3 key attributes

- Time interval is (almost) zero
- Force is (almost) infinitely large
- But,…impulse is finite



When is the "zero time" impulse model appropriate?

Not always…!

Are these impacts?

No for some calculations

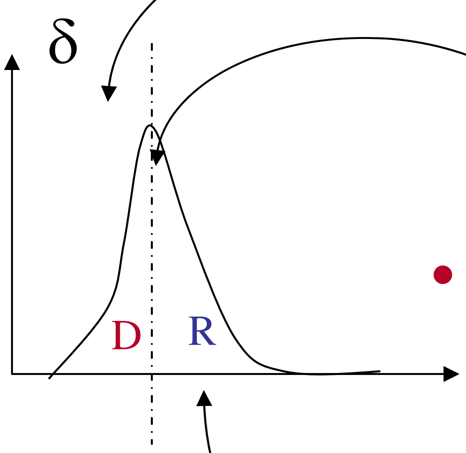
What if you have to predict/analyze the shape of the balloon?

Yes for some calculations

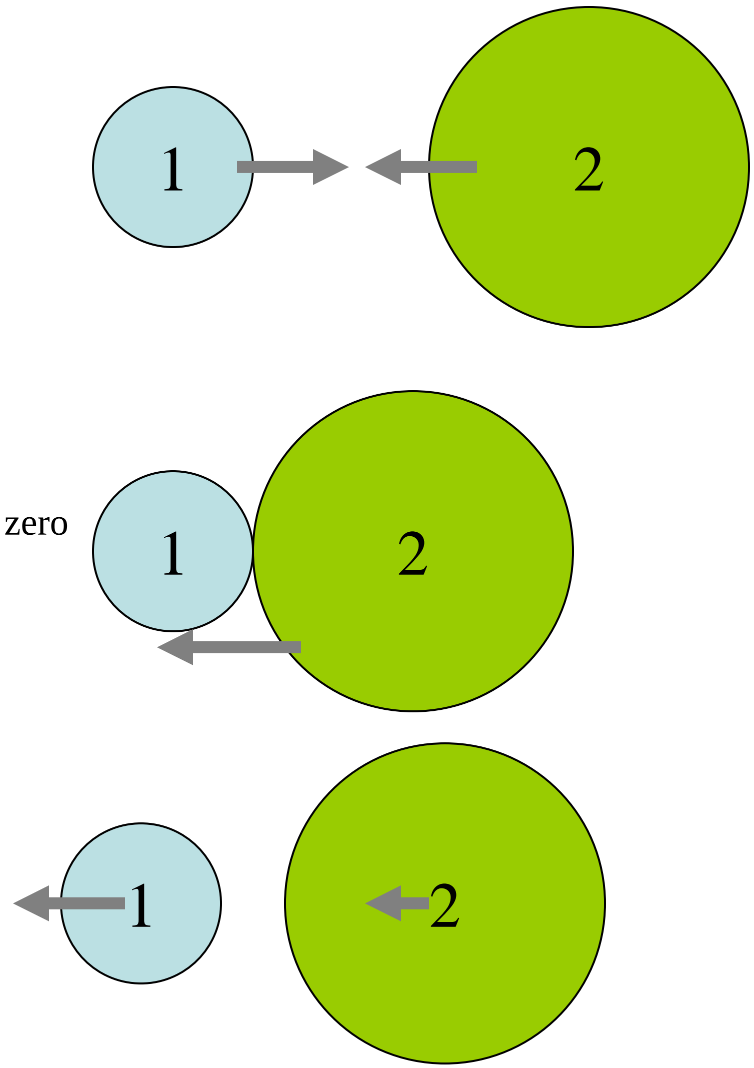
What if you have to estimate the maximum height of a bouncing basketball over successive bounces?

Reality

Collisions have three stages or states



- Deformation (compression)
  - Deformation increases
- State of maximum deformation
  - Rate of change of deformation is zero
- Restitution (expansion)
  - Deformation decreases

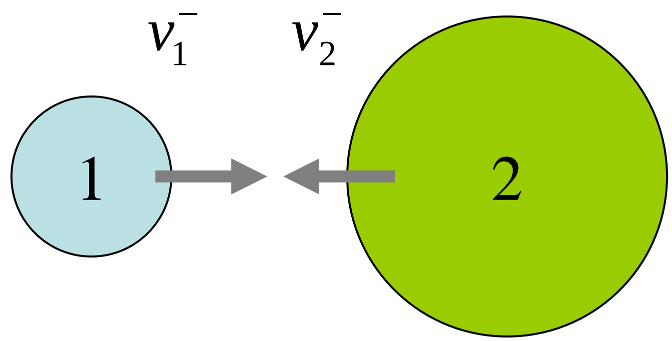


Collisions in 1 dimension

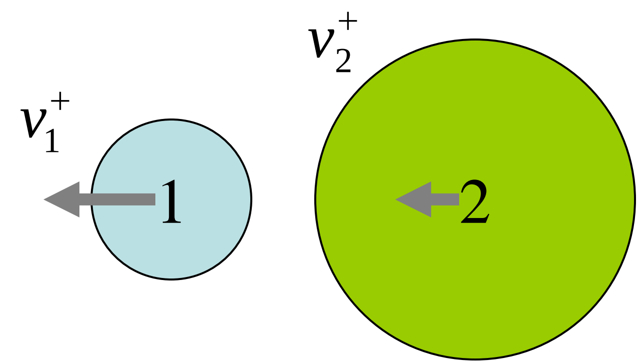
- Change in momentum
  - Let *LI* denote the linear impulse acting on 2

$$-LI = m_1 v_1^+ - m_1 v_1^-$$

$$LI = m_2 v_2^+ - m_2 v_2^-$$



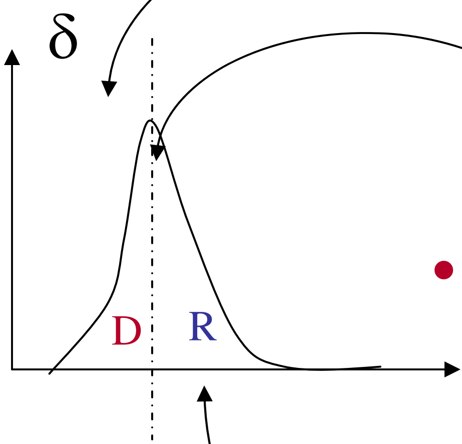
Note all quantities positive



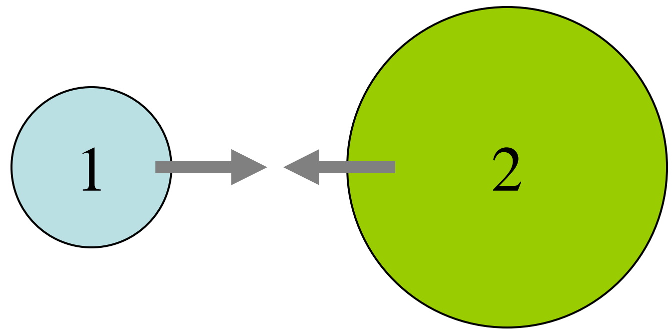
Adding

Reality

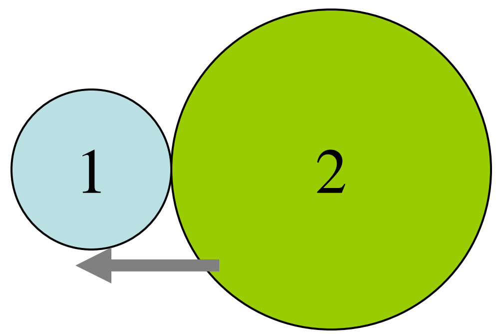
Collisions have three stages or states



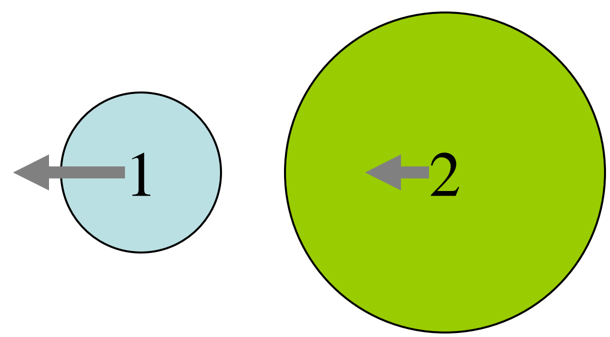
- Deformation (compression)
  - Deformation increases



- State of maximum deformation
  - Rate of change of deformation is zero



- Restitution (expansion)
  - Deformation decreases



How can we model impacts?

- Newton's laws
  - Accelerations are infinite if forces are infinite
- Conservation of energy
  - Only conserved for "elastic" collisions
  - Inelastic?
- Conservation of momentum
  - For each particle?
  - For the system of particles?
- Need an impact model
  - Poisson's model of restitution and compression (deformation) impulses
  - Newton's model of approach and separation velocities