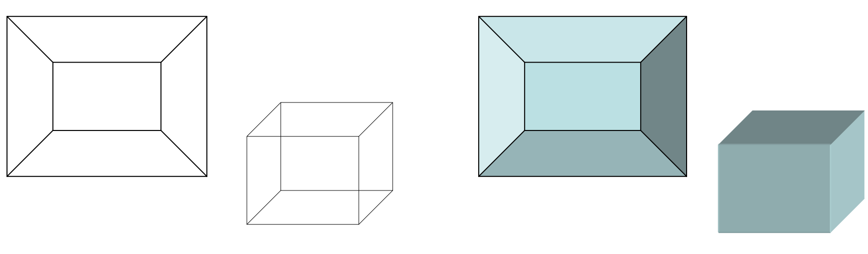
AML710 CAD LECTURE 30

Solid Modeling

Concepts

Evolution of Geometric Modeling

A wireframe representaion of an object is done using edges (lines curves) and vertices. Surface representation then is the logical evolution using faces (surfaces), edges and vertices. In this sequence of developments, the solid modeling uses topological information in addition to the geometrical information to represent the object unambiguously and completely.



Wireframe Model **Solid Model**

Advantages of Solid Models

Unlike wireframes and surface representations which contain only geometrical data, the solid model uses topological information in addition to the geometrical information to represent the object unambiguously and completely. Solid model results in accurate design, helps to further the goal of CAD/ CAM like CIM, Flexible manufacturing leading to better automation of the manufacturing process.

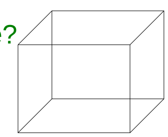
Geometry: The graphical information of dimension, length, angle, area and transformations

Topology: The invisible information about the connectivity, neighborhood, associatively etc

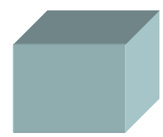
Is a solid model just a shaded image?

Three dimensional addressability?

Suitable for automation?



Wireframe Model



Solid Model

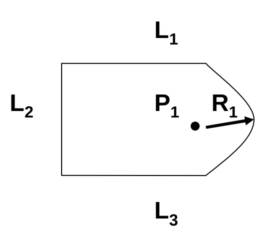
Geometry Vs Topology

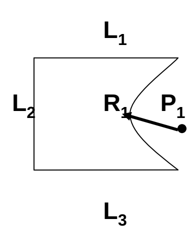
Geometry:

Metrics and dimensions of the solid object. Location of the object in a chosen coordinate system

Topology:

Combinatorial information like connectivity, associativity and neighborhood information. Invisible relationship information.





Same geometry and different Topology

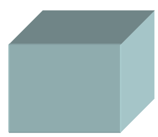
Manifold Vs Non-manifold

Two Manifold Representations:

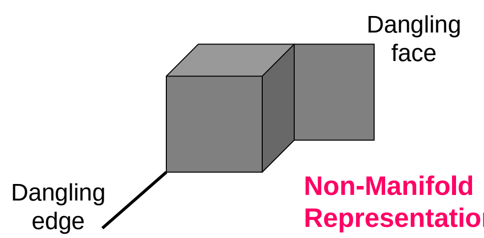
Two manifold objects are well bound, closed and homomorphic to a topological ball.

Non-manifold Representations:

When restrictions of closure and completeness are removed from the solid definition, wireframe entities can coexist with volume based solids.



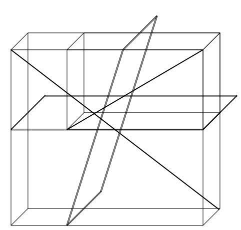
Two-Manifold Object



Non-Manifold Representation

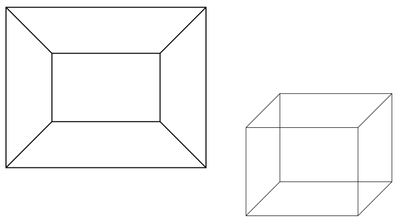
Disadvantages of Wireframe Models

- Ambiguity
- Subjective human interpretation
- Complex objects with many edges become confusing
- Lengthy and verbose to define
- Not possible to calculate Volume and Mass properties, NC tool path, cross sectioning etc

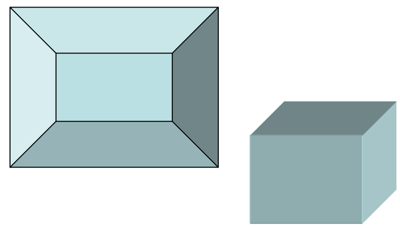


Definition of a Solid Model

A solid model of an object is a more complete representation than its surface (wireframe) model. It provides more topological information in addition to the geometrical information which helps to represent the solid unambiguously.



Wireframe Model

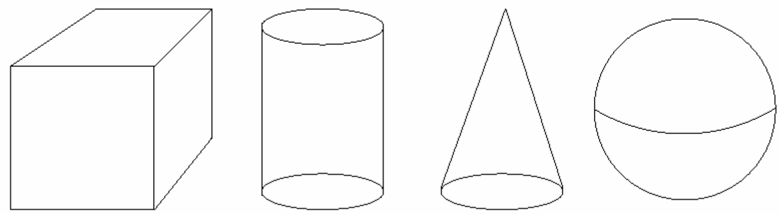


Solid Model

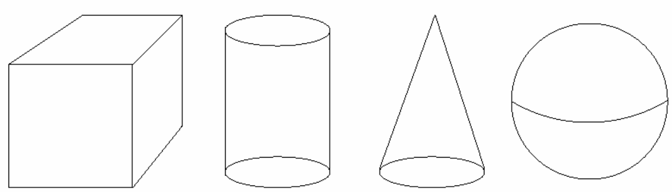
History of Geometric Modeling

Year	Modeler	Developer
1972	PAP, PADL-I, PADL2	Univ. of Rochester, Voelcker & Requicha
1973	Build-I Build-II	Braid's CAD Group in Cambridge, UK
1973	TIPS-I	Hokkaido University, Japan
1975	GLIDE-I	Eastman's Group in CMU, USA
1975	Euler Ops Winged Edge, B-rep	Baumgart, Stanford Univ., USA
1981	Romulus	Evans and Sutherland, 1st commercial

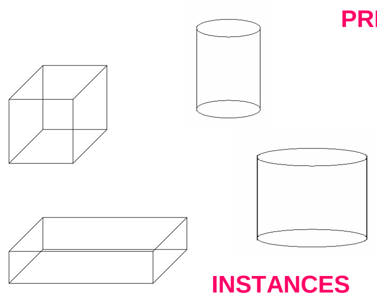
Geometric Modeling - Primitives



Primitives and Instances



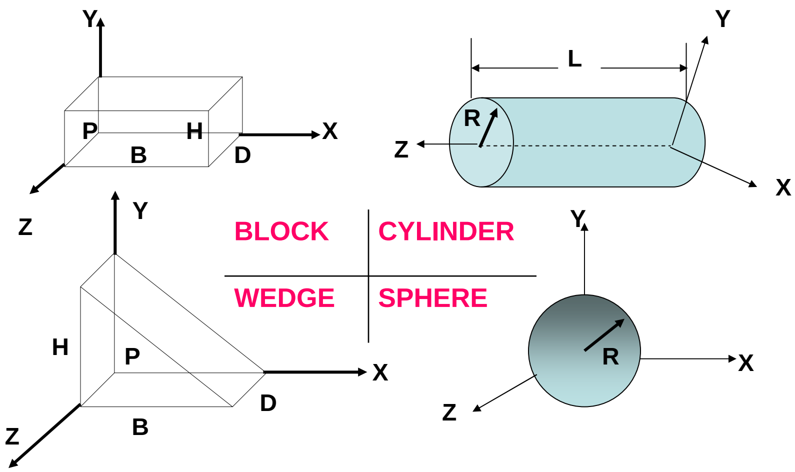
PRIMITIVES



INSTANCES

Instancing: An instance is a scaled / transformed replica of its original. The scaling may be uniform or differential. It is a method of keeping primitives in their basic minimum condition like unit cube and unit sphere etc.

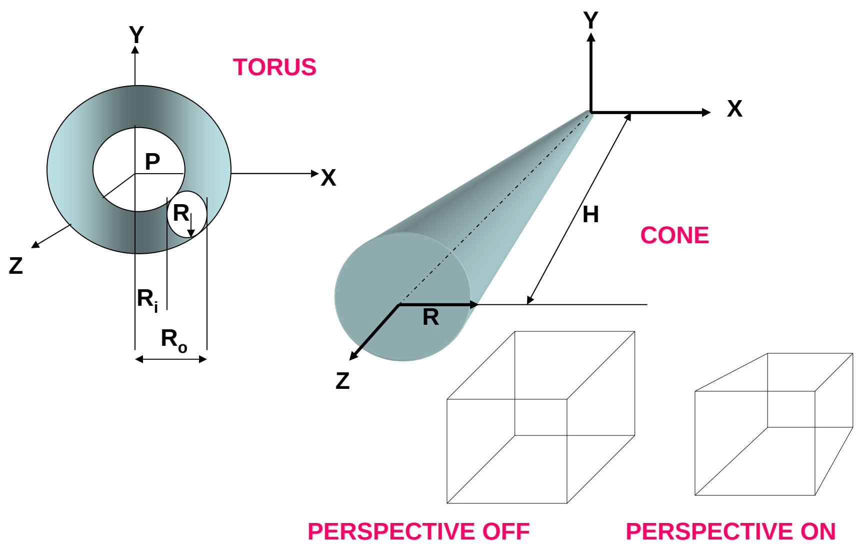
Solid Primitives: Ready starting objects



Origin of MCS: P   Orientation   Topology and shape

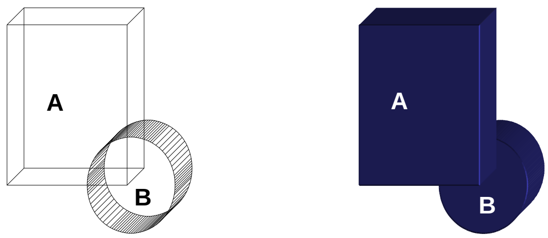
Dimension (size) L,B,H.D,R   Perspective on/off

Solid Primitives: Ready starting objects



Operation on Primitives

- A desired solid can be obtained by combining two or more solids
- When we use Boolean (set) operations the validity of the third (rusulting) solid is ensured

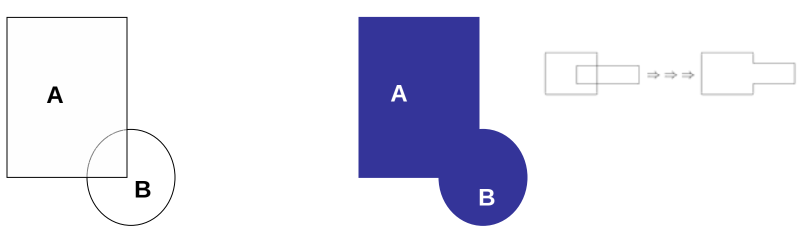


3 Dimensional

UNION: BLOCK $\cup$ CYLINDER $\mathbf{A \cup B}$

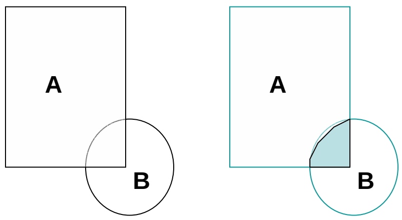
Operation on Primitives

- A desired solid can be obtained by combining two or more solids
- When we use Boolean (set) operations the validity of the third (rusulting) solid is ensured



UNION: BLOCK $\cup$ CYLINDER $\mathbf{A \cup B}$

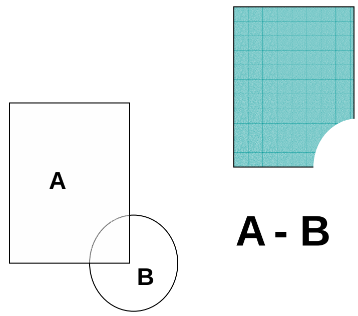
Operation on Primitives



INTERSECTION:

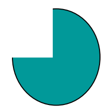
BLOCK $\cap$ CYLINDER

$A \cap B$



A - B

B - A

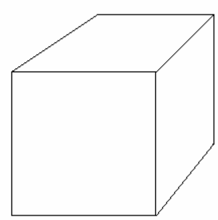


DIFFERENCE:

BLOCK - CYLINDER

Simple Polyhedron – Euler's Formula

- Euler's formula gives the relation amongst faces, edges and vertices of simple polyhedron.
- ***V-E+F=2.*** This formula tells us that there are only 5 regular polyhedra
- Consider polyhedra with every face having h edges, k edges emanating from each vertex and each edge sharing exactly 2 faces, then we can write an invariant for the solid as:
- ***hF=2E=kV*** Substituting this in Euler's formula we get



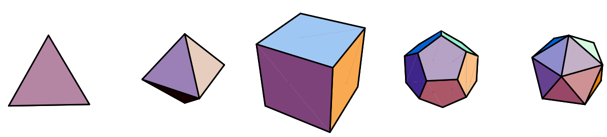
$$\frac{2E}{k}-E+\frac{2E}{h}=2 \Rightarrow \frac{1}{E}=\frac{1}{h}+\frac{1}{k}-\frac{1}{2}$$

Five Regular Polyhedra

$$\frac{2E}{k} - E + \frac{2E}{h} = 2 \qquad \Rightarrow \qquad 0 < \frac{1}{E} = \frac{1}{h} + \frac{1}{k} - \frac{1}{2}$$

- **We notice that for a polyhedron *h,k≥3***
- **What happens if h=k=4 ?**
- **If we take h=3, then 3≥k≥5**
- **Then by symmetry, if k=3 then 3≥h≥5**
- **Thus the possible polyhedra can be symbolically given as:**

(h,k,E)=(3,3,6), (3, 4, 12), (4,3,12), (5, 3, 30) and (3, 5, 30); tetrahedron, octahedron, cube, dodecahedron, icosahedron



Solid Representation

- The **solid representation** of an object is based on the idea that a physical object divides an **n-dimensional** space **E^n** into two regions: **interior** and **exterior** separated by the object **boundaries (two-manifold objects)**
- From this point of view any solid model is defined mathematically as a point set **S** in the three dimensional Euclidean space **E^3**

$$S = iS \bigcup bS$$

where iS is the interior point set and bS the boundary set.

The universal set is now defined as

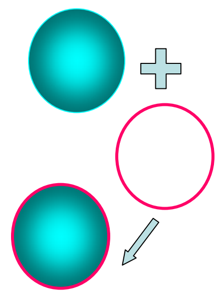
$$W = iS \bigcup bS \bigcup cS$$

Where cS is the complementary set of iS U bS

Interior, Exterior and Closure

- The following example will illustrate the concepts of interior, boundary, closure and exterior.
- Consider a unit sphere

- It's interior is $x^2 + y^2 + z^2 < 1$
- Its Boundary is $x^2 + y^2 + z^2 = 1$
- Solid sphere is Its closure $x^2 + y^2 + z^2 \leq 1$



- Whatever is outside of this closure is exterior

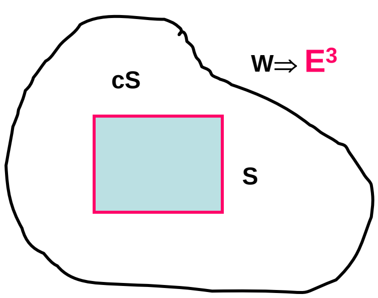
Geometric Closure

- The universal set is now defined as

$$W = iS \bigcup bS \bigcup cS$$

where cS is the complementary set of iS U bS

- The interior of the solid is geometrically closed by its boundaries $S = kS = iS \bigcup bS$

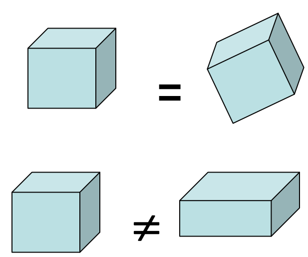


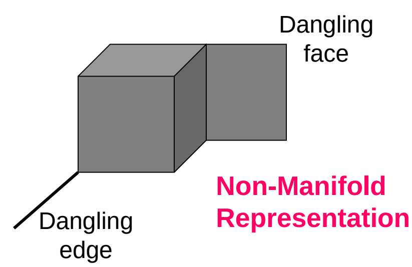
$$\boxed{iS} \bigcup \boxed{bS} = \boxed{S}$$

- **Wireframe models lack closure propery**
- **Curve in E^3 is one dimensional in parametric space**
- **A surface in E^3 is two dimensional in parametric space**

Properties of Solid Models

- **Rigidity:** Shape of the solid is invariant w.r.t location/orientation
- **Homogeneous 3-dimensionality**: The solid boundaries must be in contact with the interior. No isolated and dangling edges are permitted.





Properties of Solid Models

- **Finiteness and finite describability:** Size is finite and a finite amount of information can describe the solid model.
- **Closure under rigid motion and regularised Boolean operations**: Movement and Boolean operations should produce other valid solids.
- **Boundary determinism:** The boundary must contain the solid and hence determine the solid distinctly.
- **Any valid solid must be bounded, closed, regular and semi-analytic subsets of E^3**

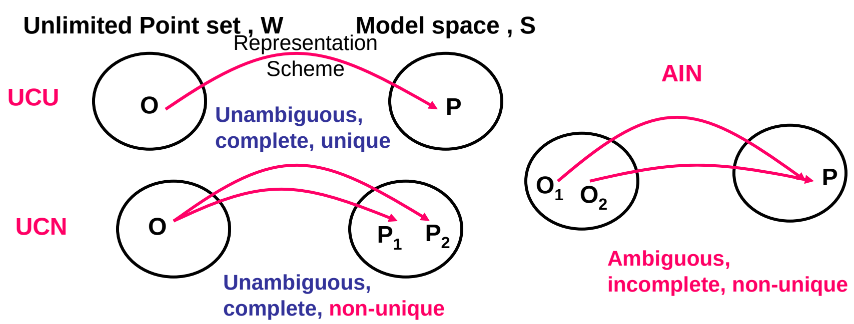
Properties of Solid Models

- The regularized point sets which represent the resulting solids from Boolean operations are called **r-sets (regularized sets**.
- **R-sets** can be imagined as curved polyhedra with well behaved boundaries.

Properties of Solid Modeling Schemes

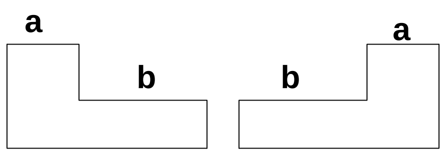
The schemes operate on r-sets or point sets to produce the desired solid model, which is automatically a valid solid

- For unambiguous, complete and unique representation the scheme maps points form point set (Euclidian) into a valid model to represent the physical object.

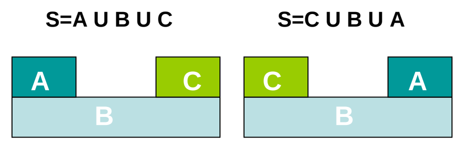


Common Features of Modeling Schemes

- **Domain**: The type of objects that can be represented or the geometric coverage.
- **Validity**: The resulting solid model when subjected to algorithms should succeed as valid solid.
- **Completeness** and **Unambiguousness**.
- **Uniqueness**: Positional and Permutational uniqueness.



Positionally non-unique



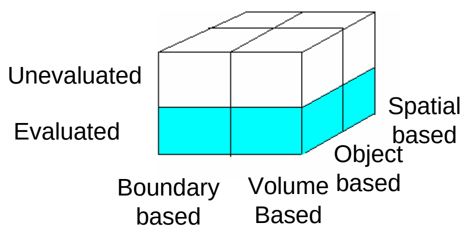
Permutationally non-unique

Desirable Properties of Modeling Schemes

- Conciseness, ease of creation and efficiency in application.
- **Conciseness means compact database to represent the object.**
- **Ease of operation implies user-friendliness. Most of the packages use CSG approach to create the solids**
- The representation must be usable for later operations like CAM / CNS etc

Classification of Solid Modeling

- Boundary based or Volume based
- Object based or spatially based
- Evaluated or unevaluated



		Boundary	Volume
Unevaluated	Spatial	Half space	Octree
Unevaluated	Object	Euler Ops	CSG
Evaluated	**Spatial**	**Boundary Cell Enum**	**Cell Enum**
Evaluated	**Object**	**Boundary Rep**	**Non-parametric**

Major Modeling Schemes

1. Half Spaces.
2. Boundary Representation (B-Rep)
3. Constructive Solid Geometry (CSG)
4. Sweeping
5. Analytical Solid Modeling (ASM)
6. Cell decomposition
7. Spatial Ennumeration
8. Octree encoding
9. Primitive Instancing

- The 3 most popular schemes: B-rep, CSG,Sweeping

Conversion Among Representations

Some representations can be converted to other representations.

- CSG to B-REP (but not B-rep to CSG)

2. Sweep
   - B-Rep
   - CSG
   - Cell Decomposition

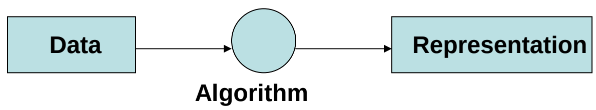
Some Solid Modelers in Practice

Modeler	Developer	Primary Scheme	User Input
CATIA	**IBM**	CSG	BREP+CSG
GEOMOD / I-DEAS	**SDRC/EDS**	BREP	BREP+CSG
PATRAN-G	**PDA ENGG.**	ASM	**HYPERPATCHES+CSG**
PADL-2	**CORNELL UNI.**	CSG	CSG
SOLIDESIGN	**COMPUTER VISION**	BREP	BREP+CSG
UNISOLIDS / UNIGRAPHICS	**McDONELL DOUGLAS**	CSG	BREP+CSG
PRO-E	**PARAMETRIC**	BREP	BREP+CSG
SOL. MOD. SYS	**INTERGRAPH**	BREP	BREP+CSG

Algorithms in Solid Modelers

- Three types of algorithms are in use in many solid modelers according to the kind of input and output

1. a; data -> representation

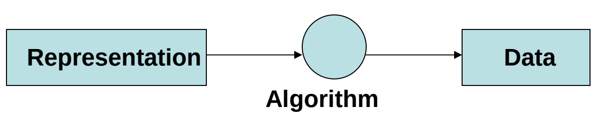


most common – build, maintain, manage representations

Algorithms in Solid Modelers

- Some algorithms operate on existing models to produce data.

2. a; representation -> data

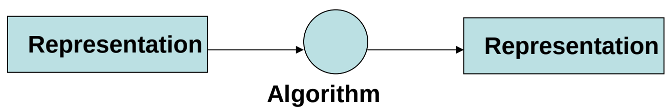


E.g.. Mass property calculation

Algorithms in Solid Modelers

- Some representations use the available representations to produce another representation.

3. a: representation -> representation



- E.g. Conversion from CSG to B-rep

Closure and Regularized set Operations

- The interior of the solid is geometrically closed by its boundaries

$$S = kiS = iS \bigcup bS$$

$$iS \bigcup bS = S$$

•If the closure of the interior of a given set yields that same given set then the set is regular

•Mathematically a given set is regular if and only if:

$$S = kiS = iS \bigcup bS$$

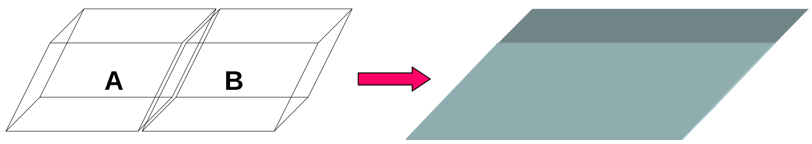
$$P \cup {*}Q = ki(P \cup Q); \quad P \cap {*}Q = ki(P \cap Q)$$

$$P - {*}Q = ki(P - Q); \quad c * Q = ki(cQ)$$

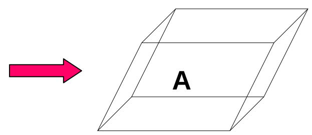
Regularized set Operations

- The interior of the solid is geometrically closed by its boundaries

$$A \cup^* B$$



$$A -^* B$$



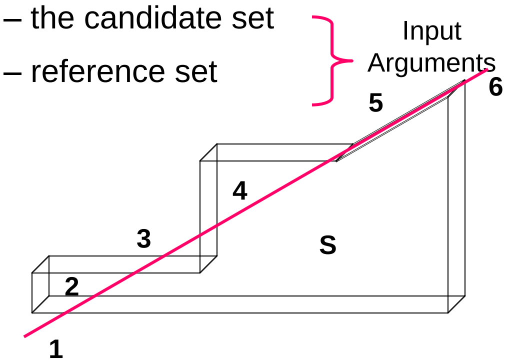
$$A \cap^* B = \varphi$$

Set Membership Classification

- The process by which various parts of a candidate set X (points, lines or solids) are assigned to (is tested) with a reference set iS, bS or cS is called set membership classification.

$$M[X, S] = (X \text{ in } S, X \text{ on } S, X \text{ out } S)$$

- X – the candidate set
- S – reference set



Line Segment	Test Result
1	**L out S**
2	**L in S**
3	**L out S**
4	**L in S**
5	**L on S**
6	**L out S**