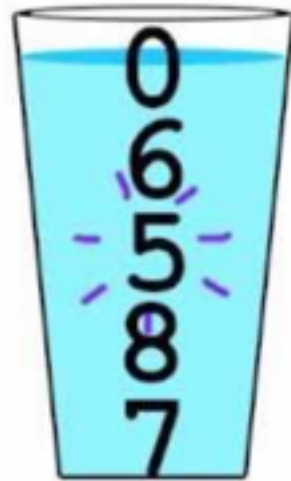


# 15-112

## Fundamentals of Programming

Week 5 - Lecture 1:  
Introduction to Efficiency. Searching. Sorting.



In a **bubble sort**,  
the “heaviest”  
item sinks to  
the bottom of the  
list while the  
“lightest” floats up  
to the top

# Principles of good programming

## Correctness

Your program does what it is supposed to.

Handles all cases (e.g. invalid user input).

## Maintainability

Readability, clarity of the code.

Reusability for yourself and others  
(proper use of functions/methods and *objects*)

➡ programs that are easy to handle and debug.

## Efficiency

In terms of running time and memory space used.

# Why care about efficiency?

multiplying two integers

sorting a list

factoring integers

computing Nash Equilibria of games

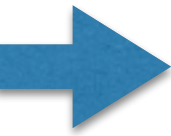
protein structure prediction

simulating quantum systems

building AI

proving theorems

# The Plan

- 
- > How to properly measure running time
  - > Searching a given list
    - Linear search
    - Binary search
  - > Big-Oh notation
  - > Sorting a given list
    - Selection sort
    - Bubble sort

# Motivating example: searching a list

Given a list of integers, and an integer, determine if the integer is in the list.

3	1	9	4	0	8	7	6	2	5
---	---	---	---	---	---	---	---	---	---

↑  
6

How many steps in the algorithm?

# Motivating example: searching a list

Given a list of integers, and an integer, determine if the integer is in the list.

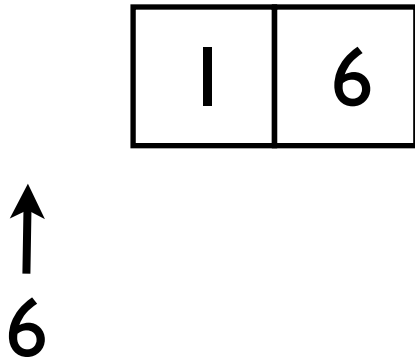
6	1	9	4	0	8	7	3	2	5
---	---	---	---	---	---	---	---	---	---

↑  
6

How many steps in the algorithm?

# Motivating example: searching a list

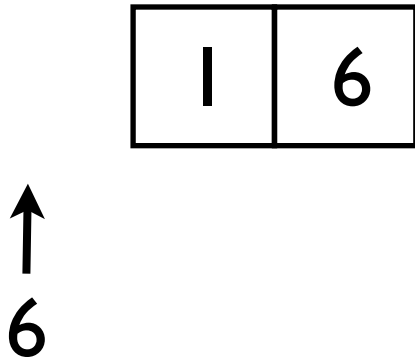
Given a list of integers, and an integer, determine if the integer is in the list.



How many steps in the algorithm?

# Motivating example: searching a list

Given a list of integers, and an integer, determine if the integer is in the list.



How many steps in the algorithm?

running time of an algorithm depends on:

- size of the list (size of input)
- the values in the input



# Measuring running time

running time of an algorithm depends on:

- size of the list (size of input)
- the values in the input

# Measuring running time

running time of an algorithm depends on:

- size of the list (size of input)
- the values in the input

size of the list:

Want to know running time with respect to any list size.

$N$  = list size

Measure running time as a function of  $N$ .

# Measuring running time

running time of an algorithm depends on:

- size of the list (size of input)
- the values in the input

the values in the input:

Measure running time with respect to *worst input*.

*worst input* = input that leads to most number of steps

# Measuring running time

## How to properly measure running time

- > Input length/size denoted by  $N$  (and sometimes by  $n$ )
  - for **lists**:  $N$  = number of elements
  - for **strings**:  $N$  = number of characters
  - for **ints**:  $N$  = number of digits
- > Running time is a function of  $N$ .
- > Look at worst-case scenario/input of length  $N$ .
- > Count algorithmic steps.
- > Ignore constant factors. (e.g.  $N^2 \approx 3N^2$ )  
(use **big-oh** notation)

# The Plan

> How to properly measure running time

 > Searching a given list

- Linear search

- Binary search

> Big-Oh notation

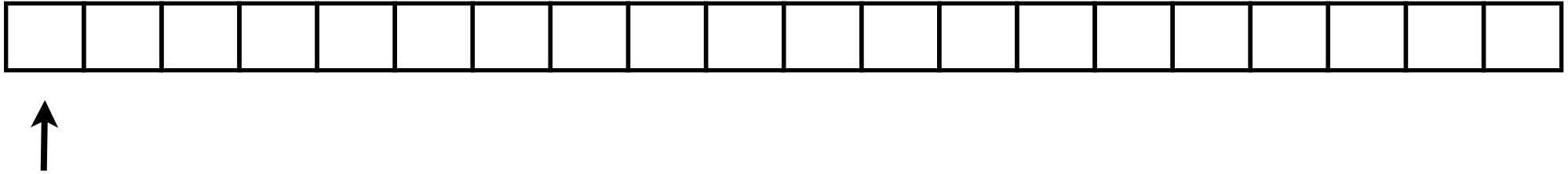
> Sorting a given list

- Selection sort

- Bubble sort

# Searching for an element in a list

Given a list of integers, and an integer, determine if the integer is in the list.



How many steps does this take?

$N$  steps

Can't do better (in the worst case)

This algorithm is called **Linear Search**.

# Searching for an element in a sorted list

Given a **sorted** list of integers, and an integer, determine if the integer is in the list.

0	1	2	4	5	5	6	8	9	9	50	60	99
---	---	---	---	---	---	---	---	---	---	----	----	----

↑  
50

running time:  $N$  steps

Can we do better?

How would you search for a name in a phonebook?

# Searching for an element in a sorted list

## Binary Search

0	1	2	4	5	5	6	8	9	9	50	60	99
---	---	---	---	---	---	---	---	---	---	----	----	----

↑  
50

Start in the middle



# Searching for an element in a sorted list

## Binary Search



↑  
50

Start in the middle

If (element > middle),  
can ignore left half of the list.

If (element < middle),  
can ignore right half of the list.

Repeat process on the remaining half.

# Searching for an element in a sorted list

## Binary Search



Start in the middle

If (element  $>$  middle),  
can ignore left half of the list.

If (element  $<$  middle),  
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Repeat process on the remaining half.

# Searching for an element in a sorted list

## Binary Search



Start in the middle

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Repeat process on the remaining half.

# Searching for an element in a sorted list

## Binary Search



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# Searching for an element in a sorted list

## Binary Search



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Repeat process on the remaining half.

# Searching for an element in a sorted list

## Binary Search



How many steps does this take (in the worst case)?

$$\sim \log_2 N$$

At each step we halve the list.

$$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \rightarrow \dots \rightarrow 1$$

After  $k$  steps:  $\frac{N}{2^k}$  elements left. When is this 1?

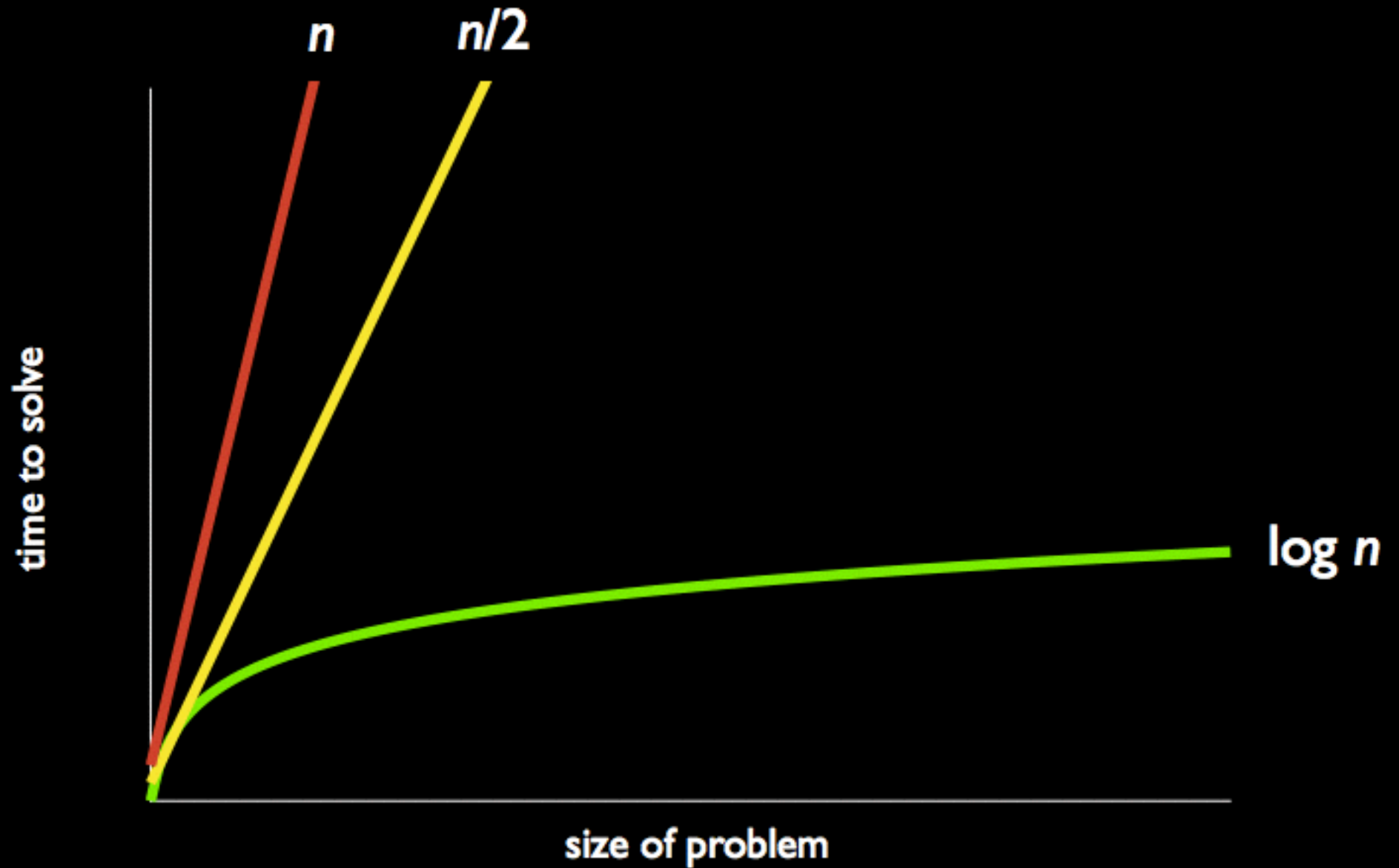
# N vs log N

How much better is log N compared to N ?

N	log N
2	1
8	3
128	7
1024	10
1,048,576	20
1,073,741,824	30
1,152,921,504,606,846,976	60

~ 1 quintillion

# $n$ vs $\log n$





# Linear search vs Binary search

## Linear Search

Takes  $\sim N$  steps.

Works for both sorted and unsorted lists.

## Binary Search

Takes  $\sim \log_2 N$  steps.

Works for only sorted lists.

# Linear search code

```
def linearSearch(L, target):  
    for index in range(len(L)):  
        if(L[index] == target):  
            return True  
    return False
```

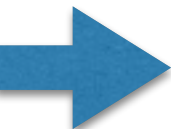
How many steps in the worst case?

# Binary search code

```
def binarySearch(L, target):  
    start = 0  
    end = len(L) - 1  
    while(start <= end):  
        middle = (start + end)//2  
        if(L[middle] == target):  
            return True  
        elif(L[middle] > target):  
            end = middle-1  
        else:  
            start = middle+1  
    return False
```

How many steps in the worst case?

# The Plan

- > How to properly measure running time
- > Searching a given list
  - Linear search
  - Binary search
-  > Big-Oh notation
- > Sorting a given list
  - Selection sort
  - Bubble sort

## The CS way to compare functions:

$$O(\cdot)$$

$$\leq$$

$$f(n) = O(g(n)) \quad \equiv \quad f(n) \text{ is } O(g(n))$$

means  $f(n) \leq g(n)$  , ignoring constant factors and small values of  $n$

## The CS way to compare functions:

$$O(\cdot)$$

$$\leq$$

$$10n + 25 = O(n) \quad \equiv \quad 10n + 25 \text{ is } O(n)$$

means  $10n + 25 \leq n$  , ignoring constant factors and small values of  $n$

# Big Oh Notation

A notation to ignore constant factors and small  $n$ .

$$2n \text{ is } O(n)$$

$$2 \log_2 n \text{ is } O(\log n)$$

$$3n \text{ is } O(n)$$

$$3 \log_2 n \text{ is } O(\log n)$$

$$1000n \text{ is } O(n)$$

$$1000 \log_2 n \text{ is } O(\log n)$$

$$0.00000001n \text{ is } O(n)$$

$$0.00000001 \log_2 n \text{ is } O(\log n)$$

$$n \text{ is } O(n^2)$$

$$\log_9 n \text{ is } O(\log n)$$

$$0.00000001n^2 \text{ is not } O(n)$$


$$n \log_7 n + 100 \text{ is not } O(n)$$

Running time of linear search is  $O(N)$

Running time of binary search is  $O(\log N)$

# Big Oh Notation

Why ignore constant factors and small  $n$ ?

- We want to capture the essence of an algorithm/problem.
- Technology independent. Language independent.
- Difference in Big Oh  a really fundamental difference.



# Big Oh Notation

Ignoring **constant factors** means  
ignoring **lower order additive terms**.

$$n^2 + 100n + 500 \text{ is } O(n^2)$$

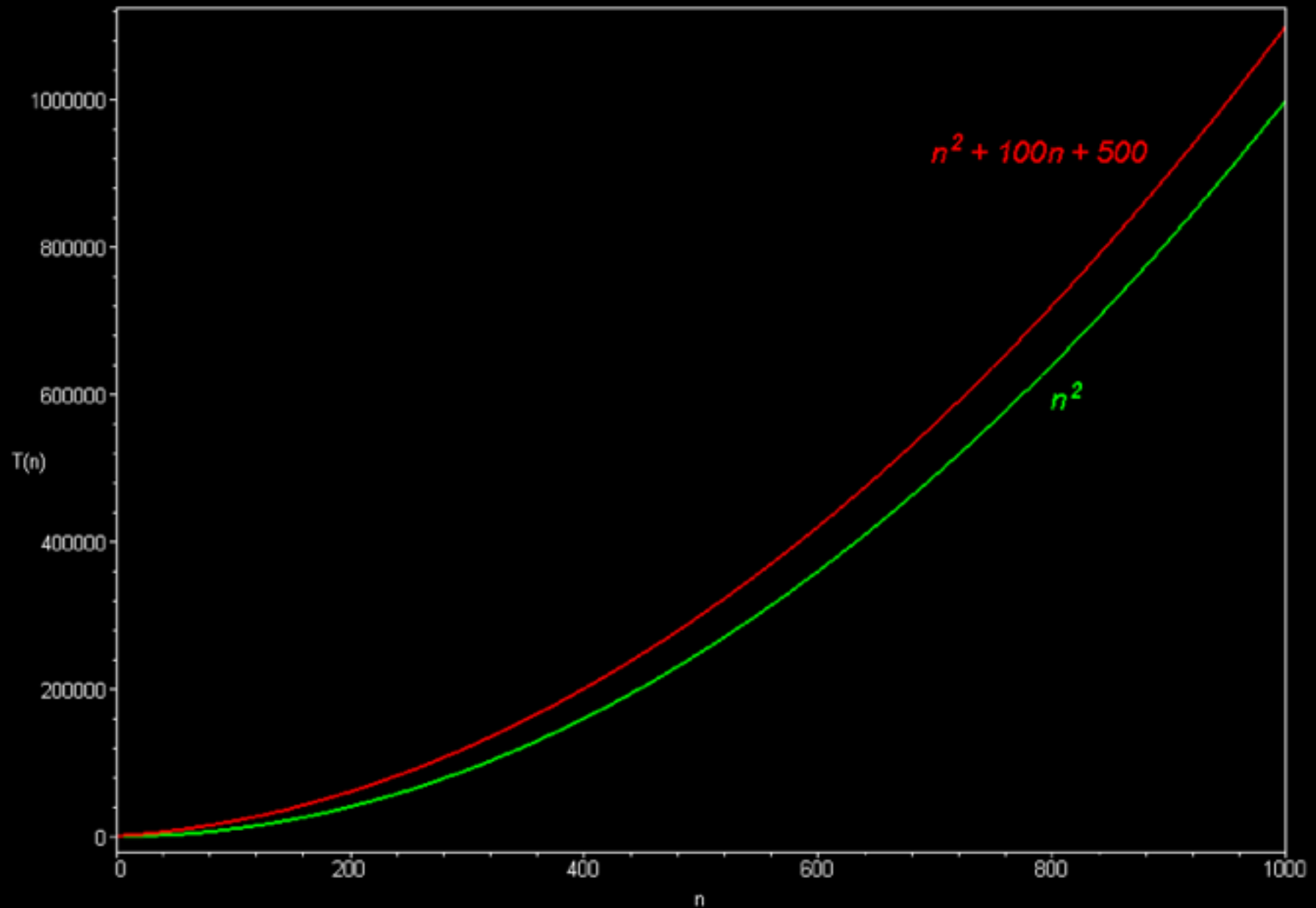
$$601n^2 = n^2 + 100n^2 + 500n^2 > n^2 + 100n + 500$$

**Also:**

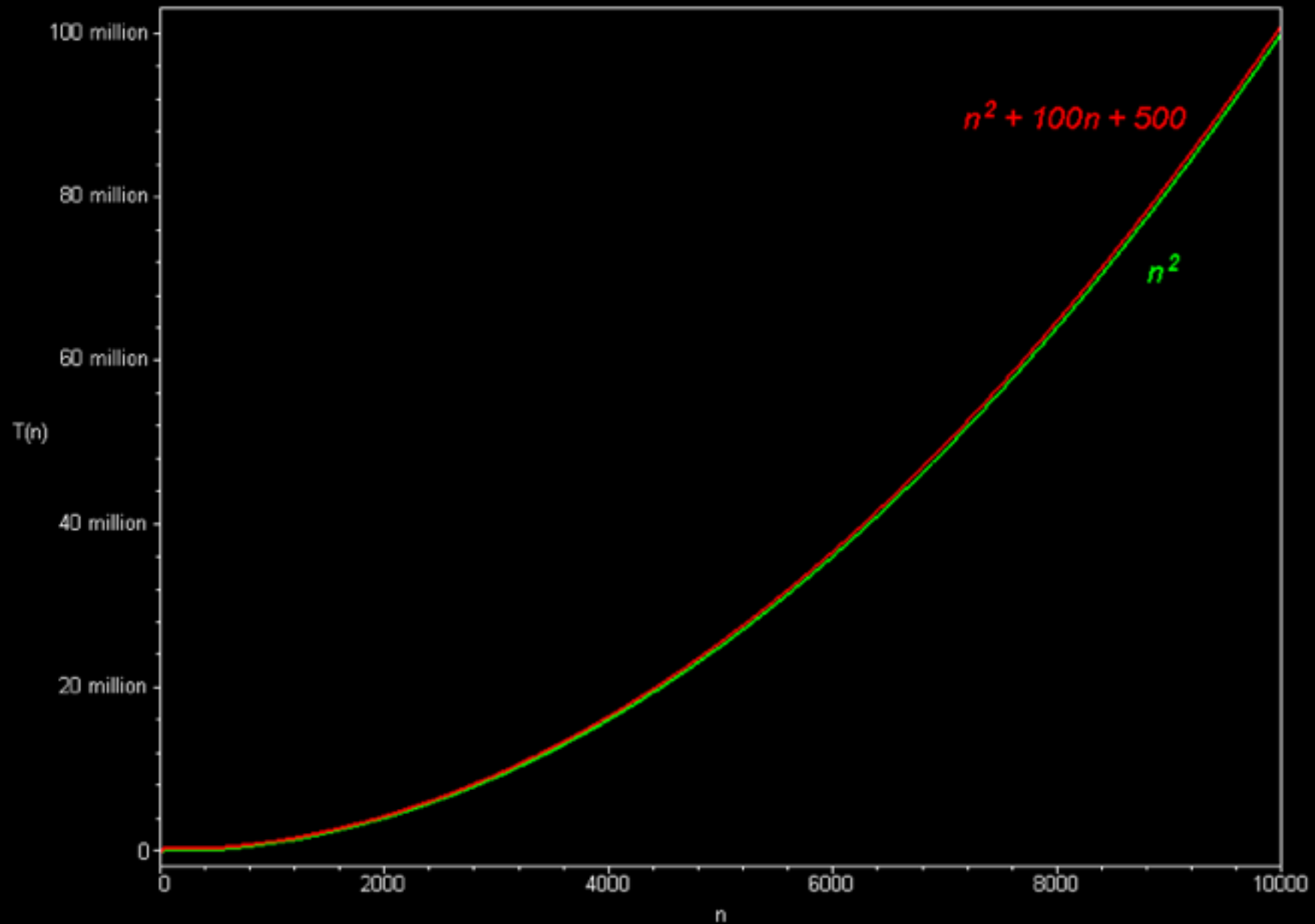
$$\frac{n^2 + 100n + 500}{n^2} = 1 + \frac{100n}{n^2} + \frac{500}{n^2} \longrightarrow 1$$

**Lower order terms don't matter!**

# Big Oh Notation



# Big Oh Notation



# Important Big Oh Classes

Again, not much interested in the difference between  $n$  and  $n/2$ .

We are very interested in the differences between

$$\log n \lll \sqrt{n} \ll n \ll n^2 \ll n^3 \lll 2^n$$

# Important Big Oh Classes

Common function families:

Constant:  $O(1)$

Logarithmic:  $O(\log n)$

Square-root:  $O(\sqrt{n}) = O(n^{0.5})$

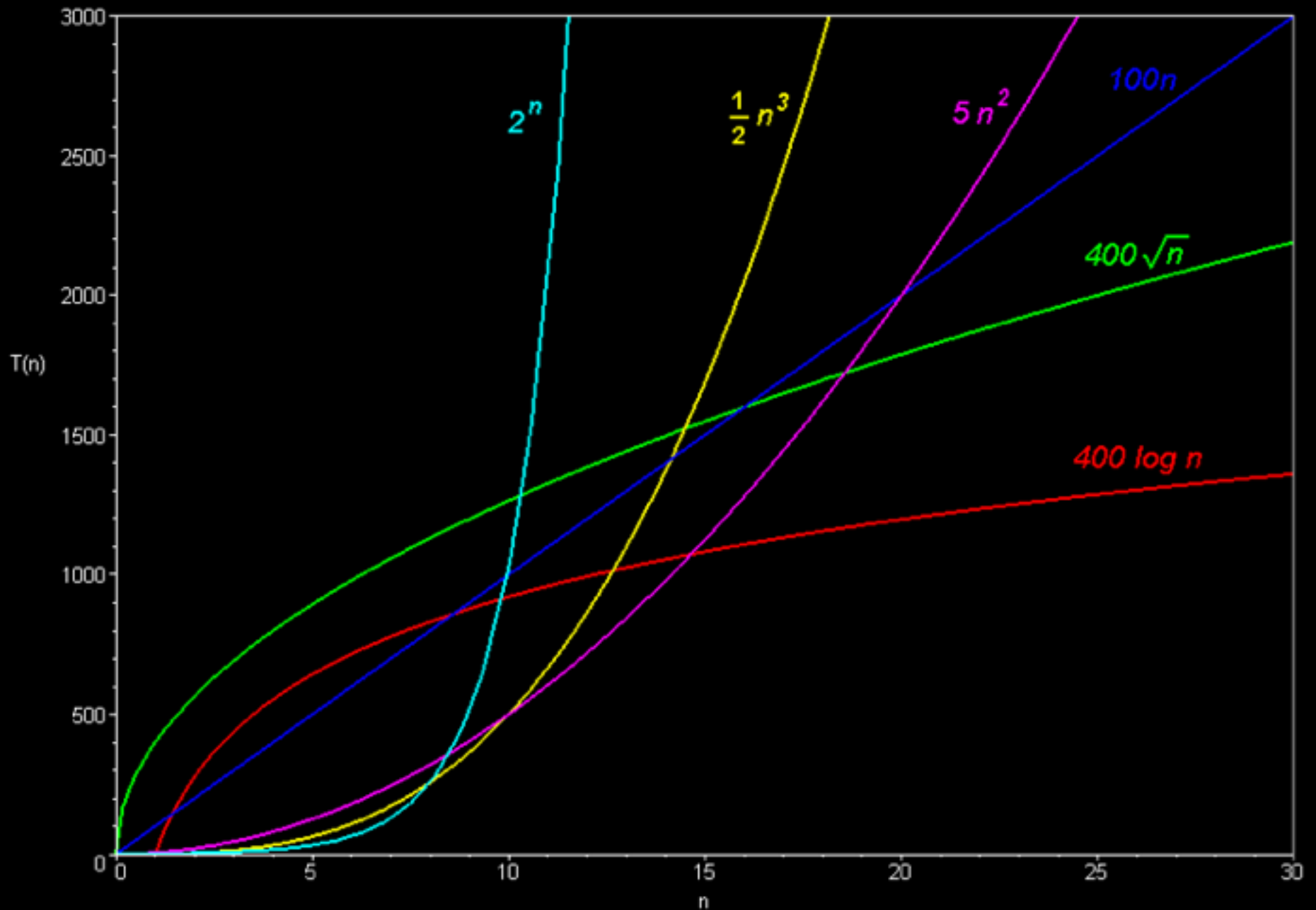
Linear:  $O(n)$

Loglinear:  $O(n \log n)$

Quadratic:  $O(n^2)$

Exponential:  $O(k^n)$

# Important Big Oh Classes



# Exponential running time

If your algorithm has exponential running time  
e.g.  $\sim 2^n$



No hope of being practical.

# n vs $2^n$

$2^n$	n
2	1
8	3
128	7
1024	10
1,048,576	20
1,073,741,824	30
1,152,921,504,606,846,976	60



# Exponential running time example

Given a list of integers, determine if there is a subset of the integers that sum to 0.

4	-3	-2	7	99	5	1
---	----	----	---	----	---	---

# Exponential running time example

Given a list of integers, determine if there is a subset of the integers that sum to 0.

4	-3	-2	7	99	5	1
---	----	----	---	----	---	---

## Exhaustive Search

Try every possible subset and see if it sums to 0.

Number of subsets is  $2^N$

So running time is at least  $2^N$



# The Plan

> How to properly measure running time

> Searching a given list

- Linear search

- Binary search

> Big-Oh notation

 > Sorting a given list

- Selection sort

- Bubble sort

1. Algorithm

2. Running time

3. Code

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

4	8	2	7	99	5	0
---	---	---	---	----	---	---

↑

## Selection Sort

Find the minimum element.

Put it on the left.

Repeat process on the remaining  $n-1$  elements.

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

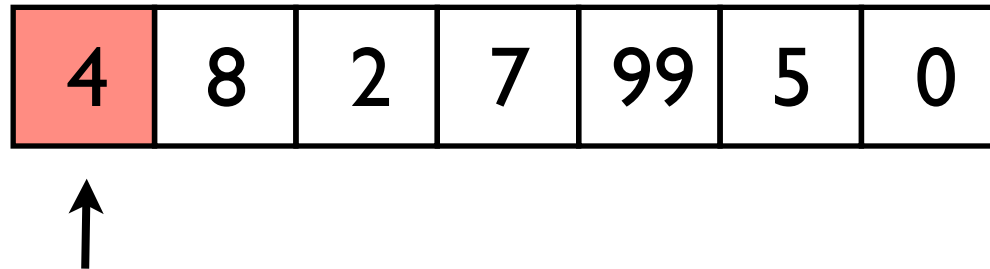
4	8	2	7	99	5	0
---	---	---	---	----	---	---



**Selection Sort**

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

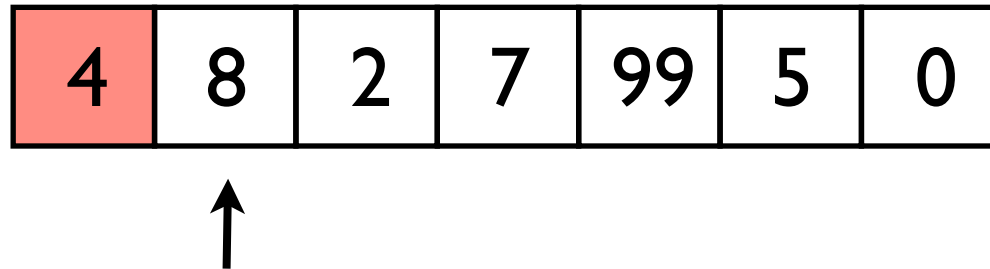


**Selection Sort**

Current min: 4

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

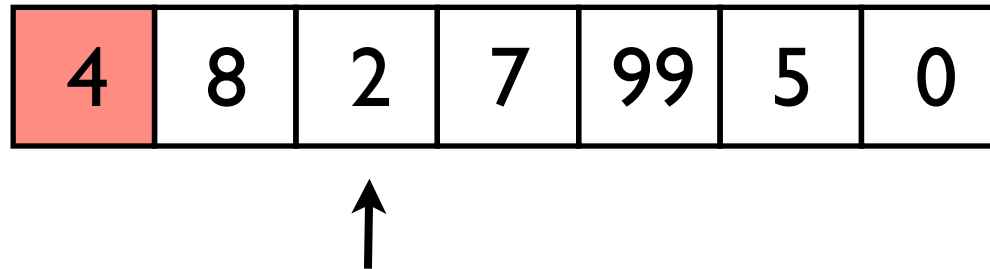


**Selection Sort**

Current min: 4

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).



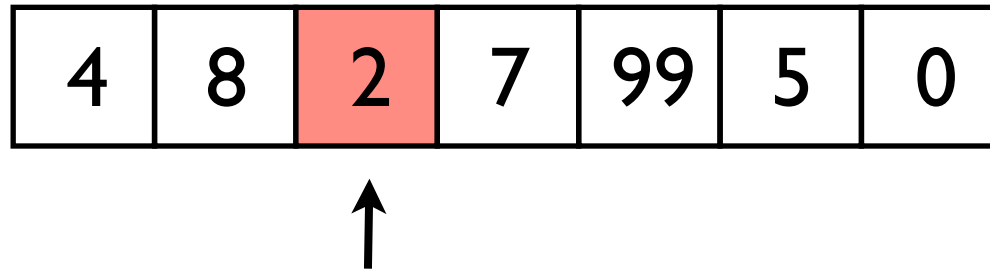
**Selection Sort**

Current min: 4



# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

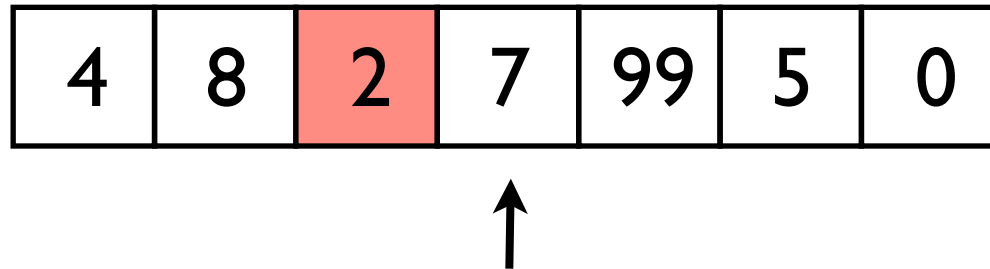


**Selection Sort**

Current min: 2

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

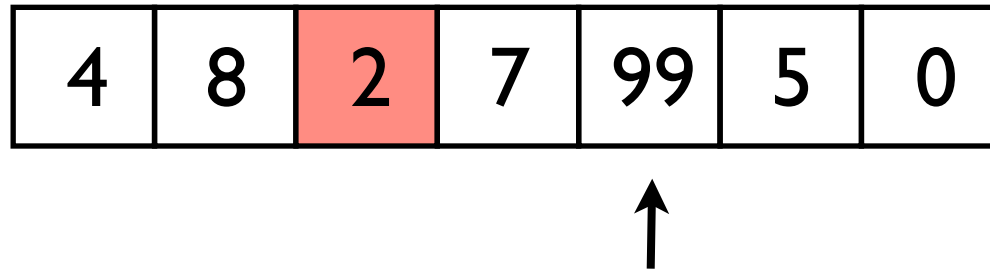


**Selection Sort**

Current min: 2

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

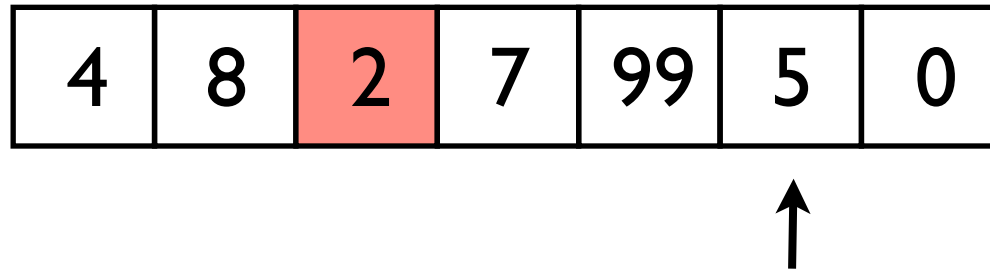


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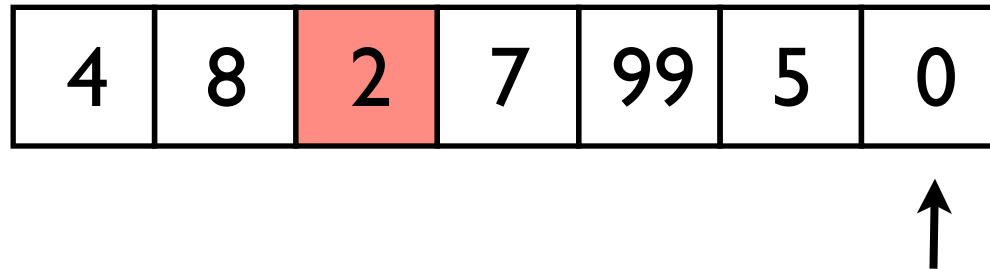


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Sort a given list of integers (from small to large).

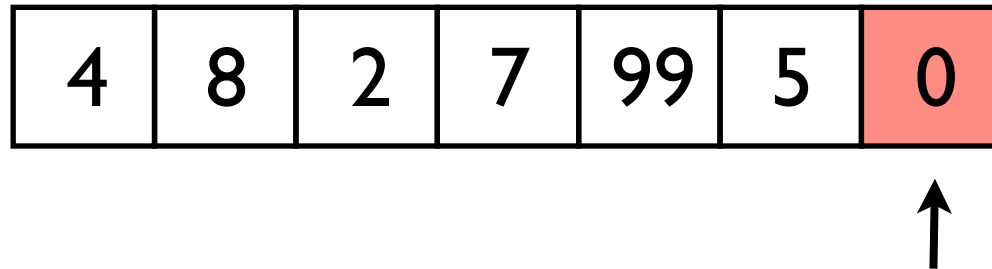


**Selection Sort**

Current min: 2

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

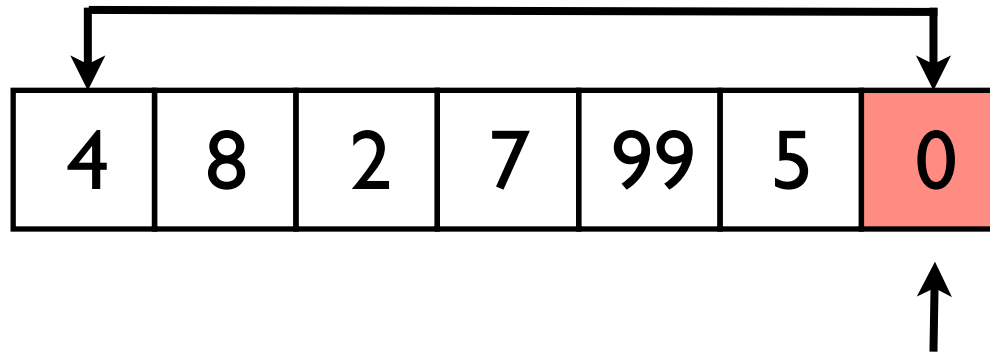


**Selection Sort**

Current min: 0

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

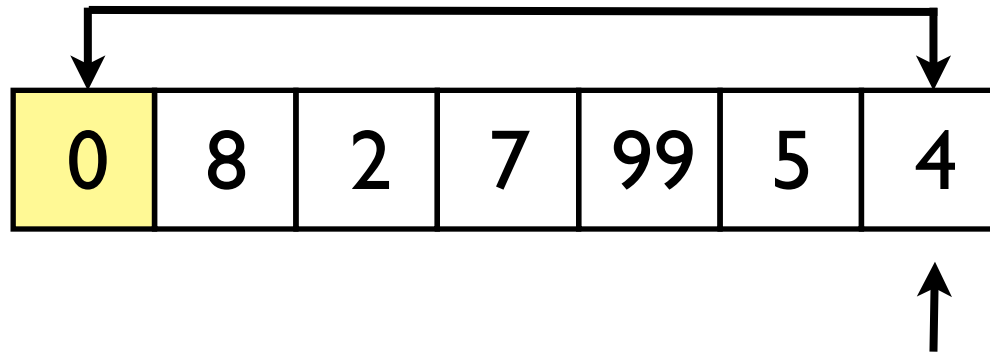


## Selection Sort

Swap current min with first element of the array

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).



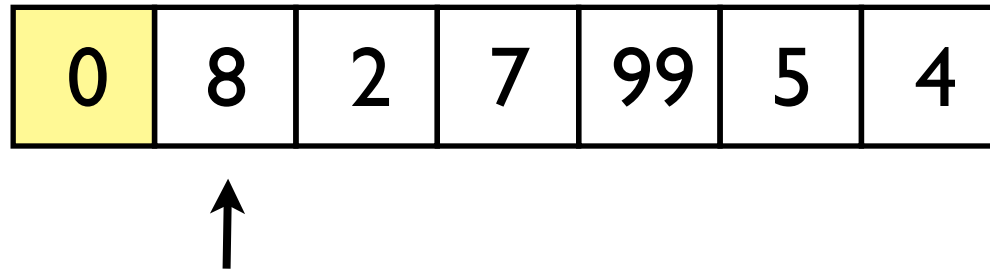
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# Selection Sort: Algorithm

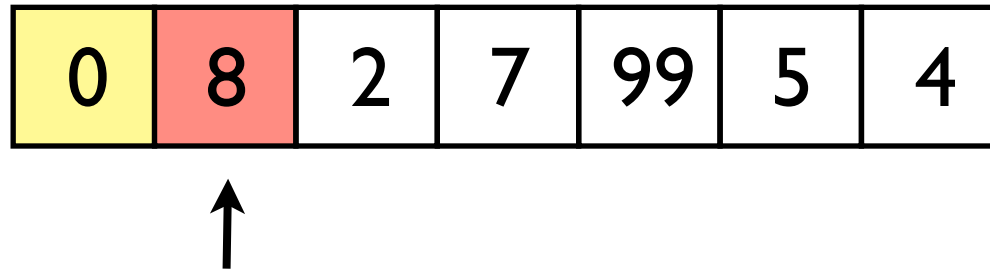
Sort a given list of integers (from small to large).



**Selection Sort**

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

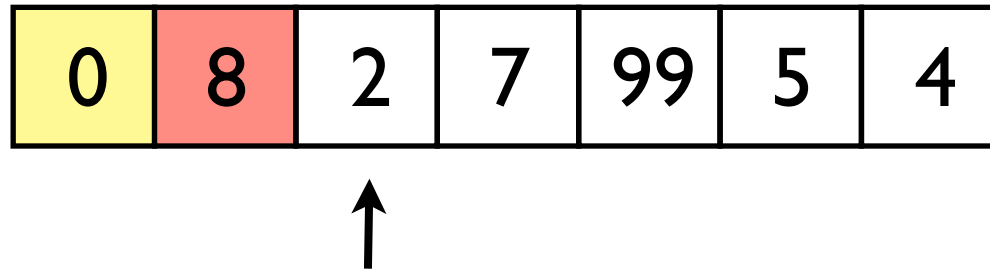


**Selection Sort**

Current min: 8

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

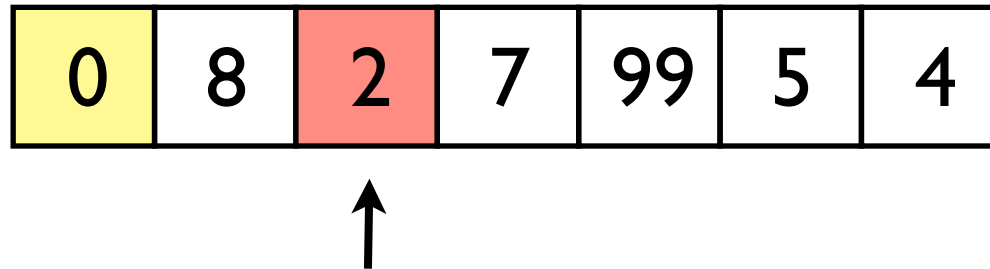


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Current min: 8

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

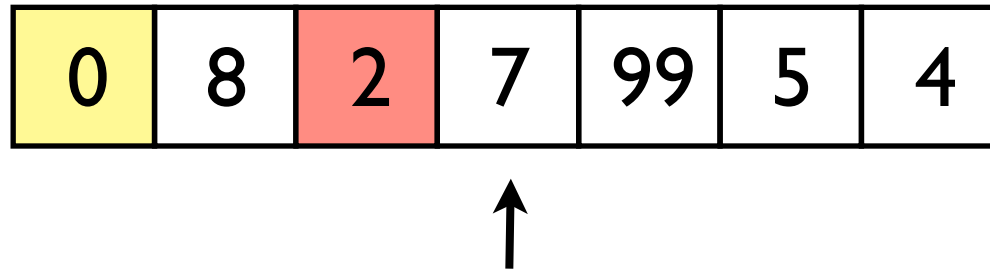


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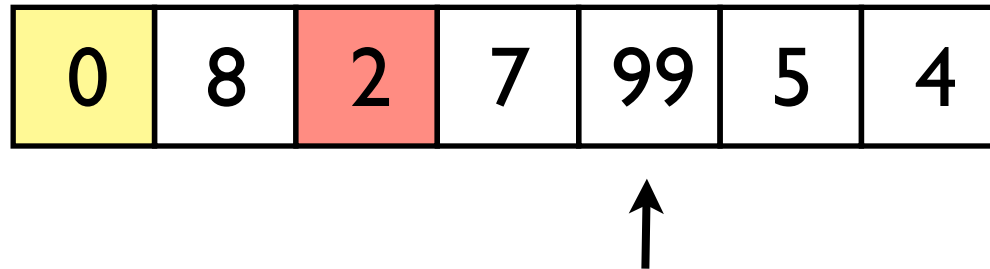


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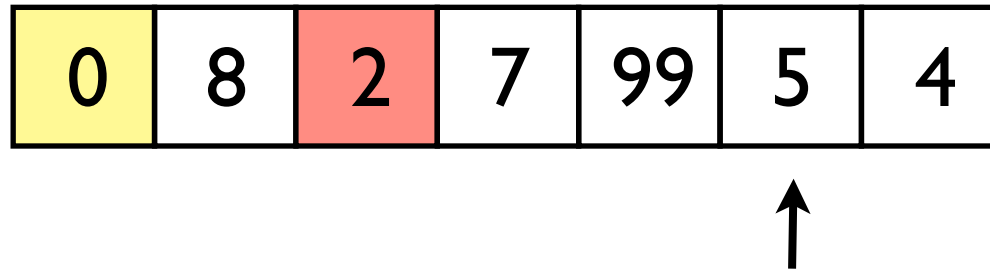


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# Selection Sort: Algorithm

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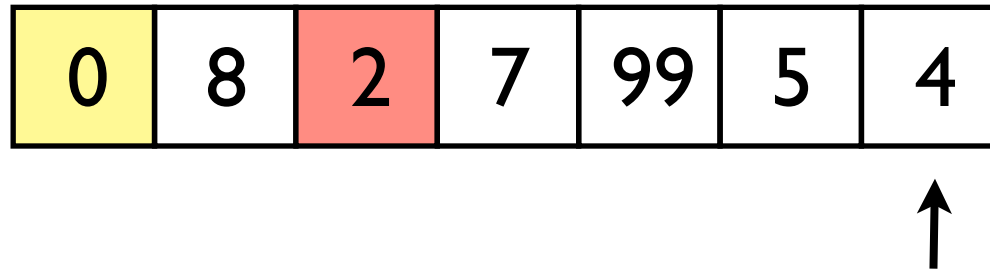


**Selection Sort**

Current min: 2

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).



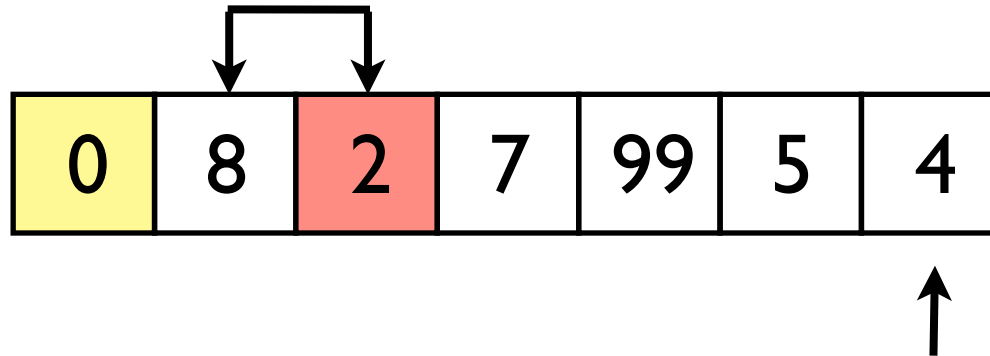
**Selection Sort**

Current min: 2



# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

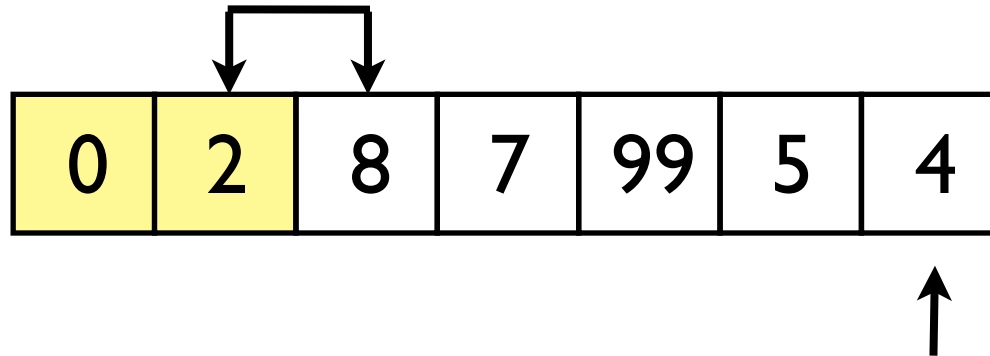


## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

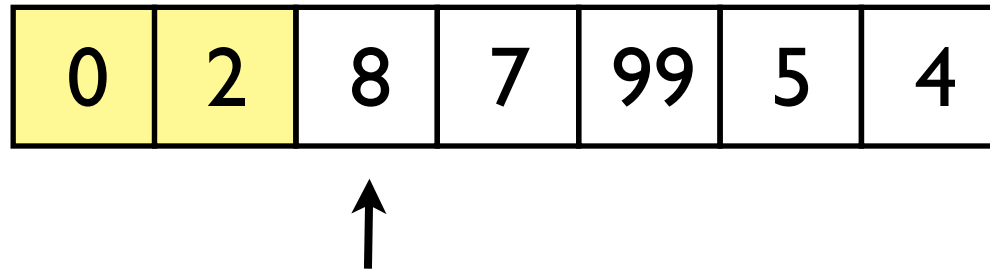


## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

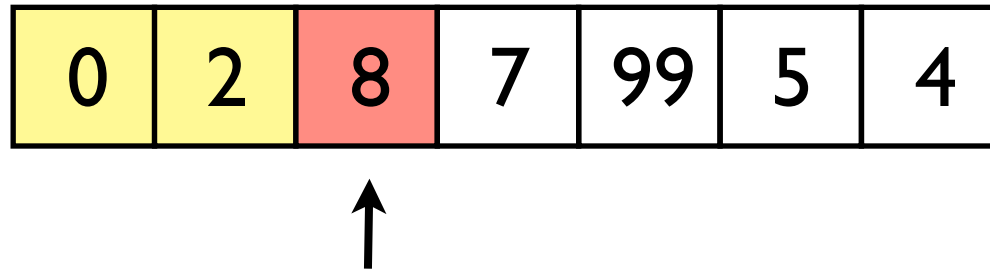
Sort a given list of integers (from small to large).



**Selection Sort**

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

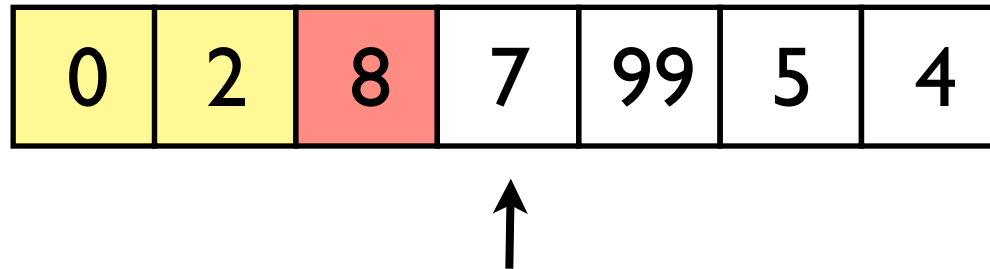


**Selection Sort**

Current min: 8

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

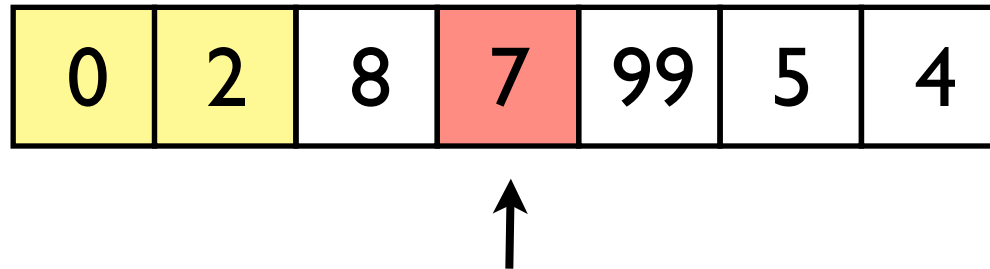


**Selection Sort**

Current min: 8

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

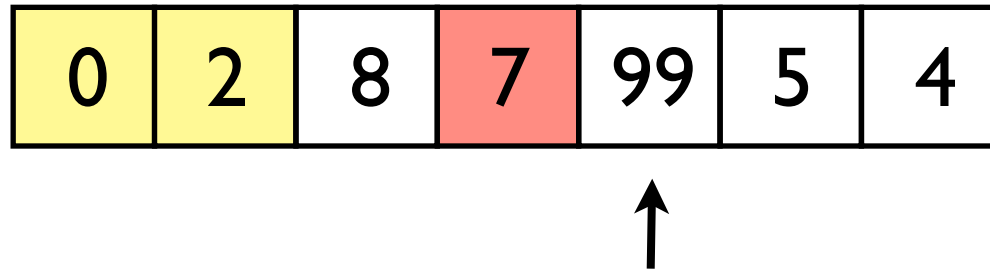


**Selection Sort**

Current min: 7

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

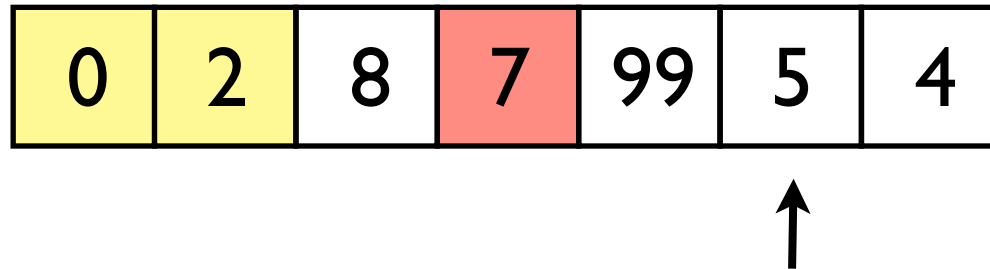


**Selection Sort**

Current min: 7

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).



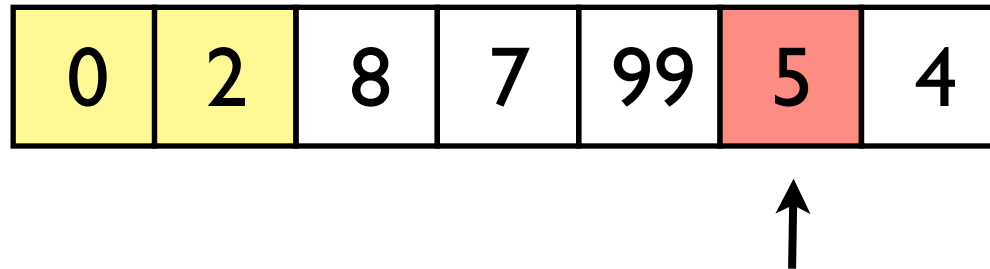
**Selection Sort**

Current min: 7



# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

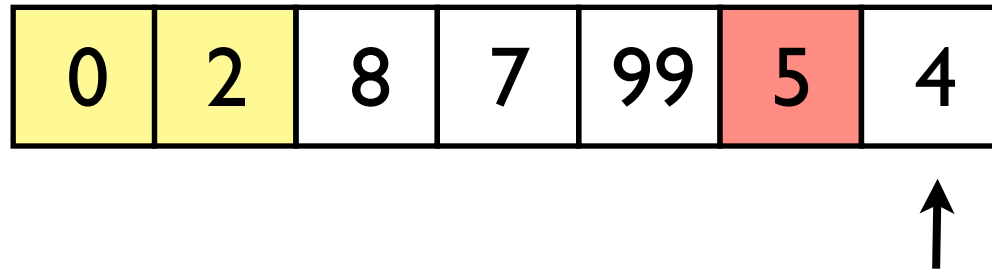


**Selection Sort**

Current min: 5

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

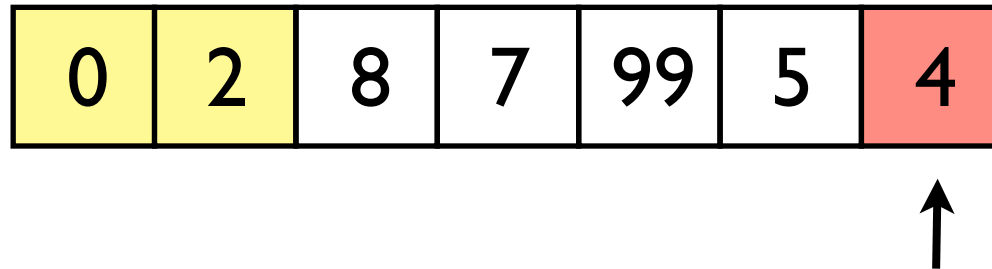


**Selection Sort**

Current min: 5

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

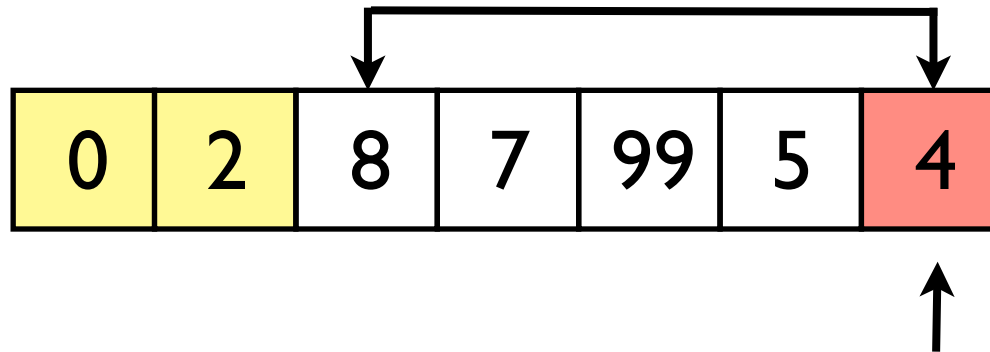


**Selection Sort**

Current min: 4

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

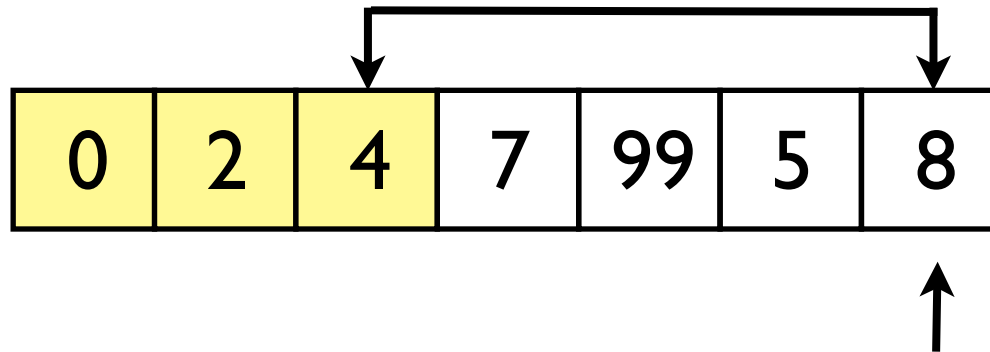


## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

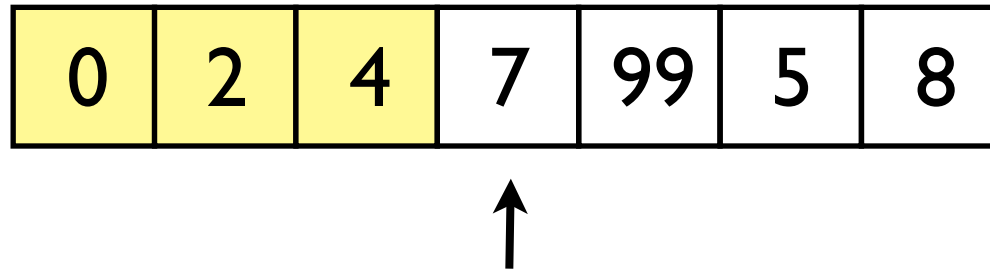


## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

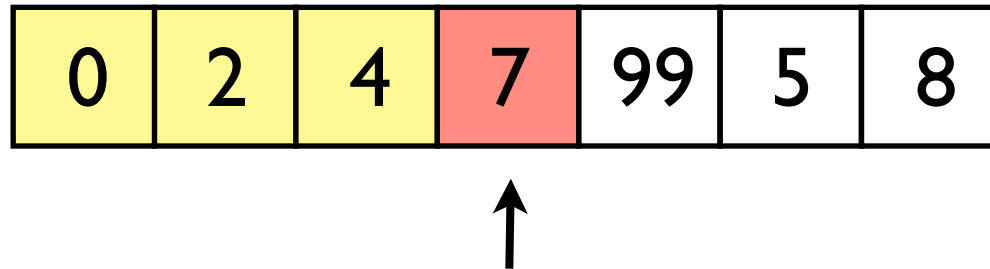
Sort a given list of integers (from small to large).



**Selection Sort**

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

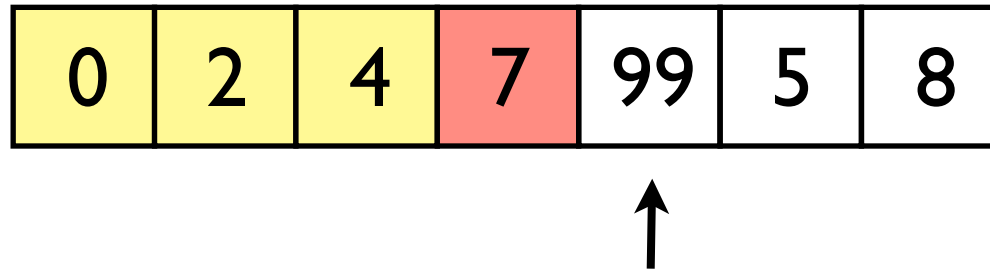


**Selection Sort**

Current min: 7

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).



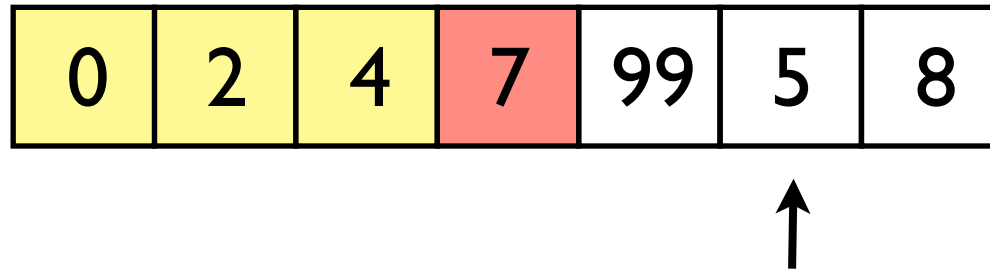
**Selection Sort**

Current min: 7



# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

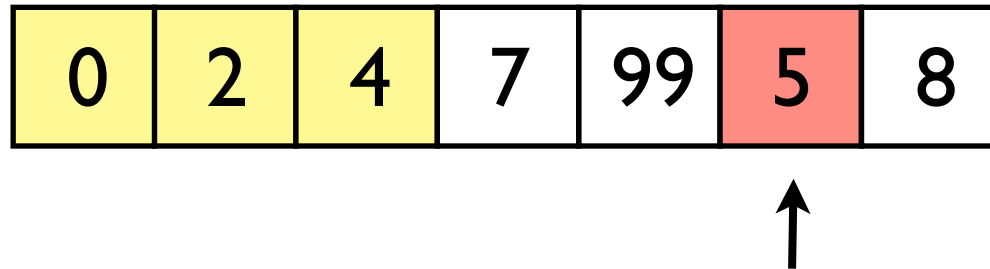


**Selection Sort**

Current min: 7

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

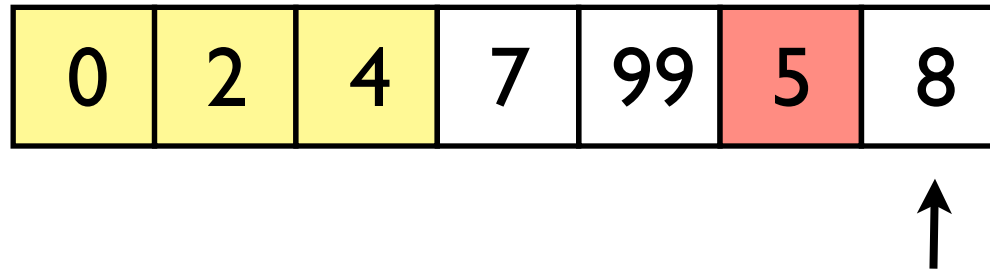


**Selection Sort**

Current min: 5

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

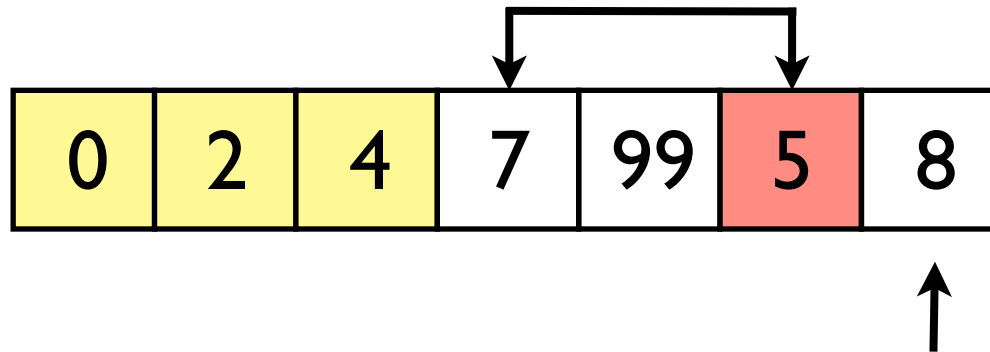


**Selection Sort**

Current min: 5

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

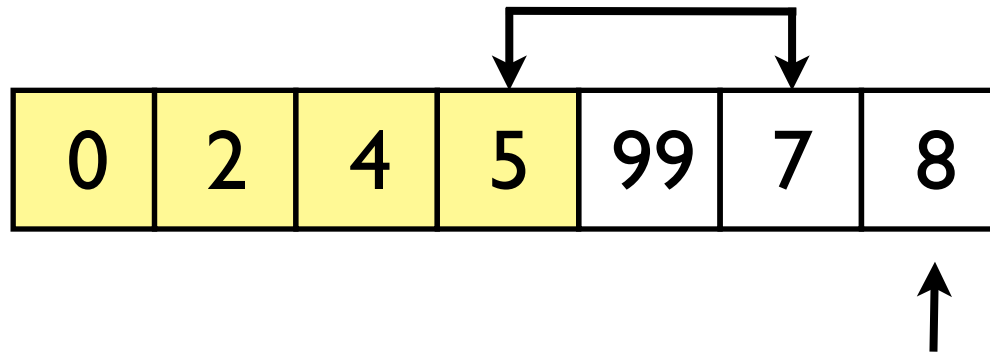


## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

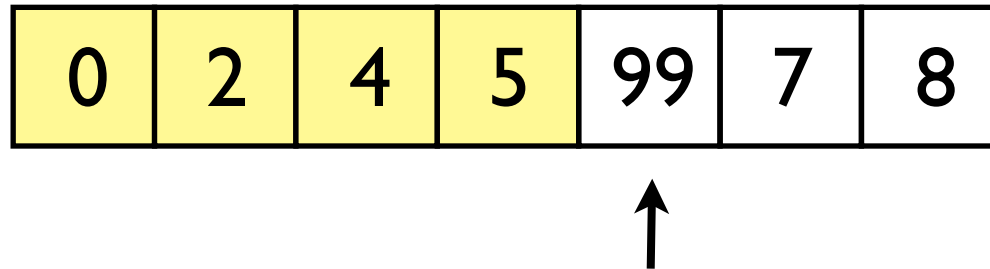


## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

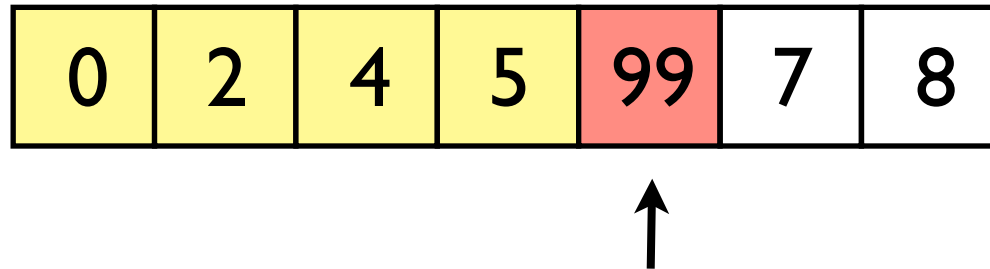
Sort a given list of integers (from small to large).



**Selection Sort**

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

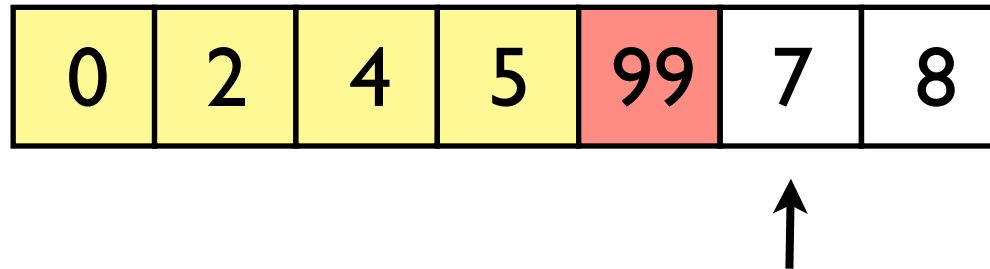


**Selection Sort**

Current min: 99

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).



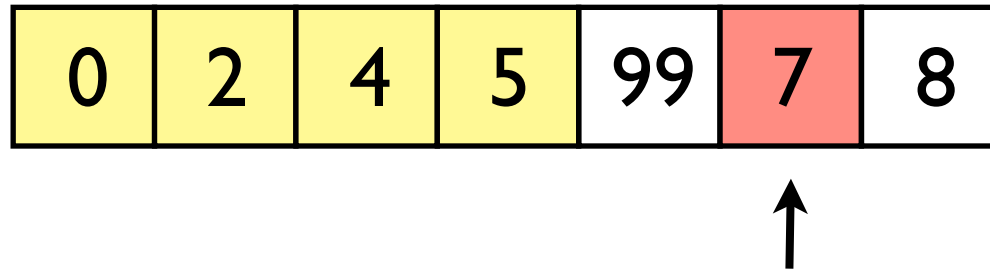
**Selection Sort**

Current min: 99



# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

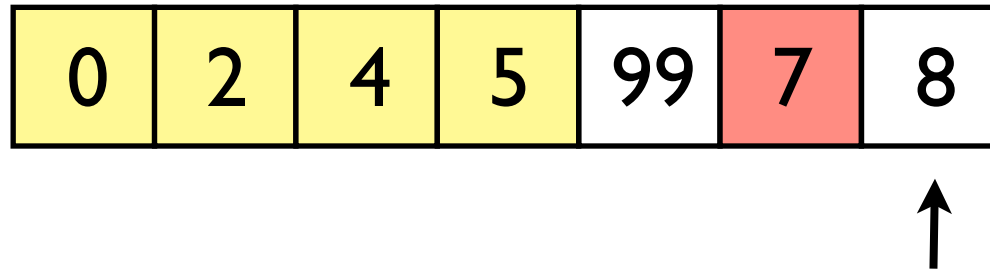


**Selection Sort**

Current min: 7

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

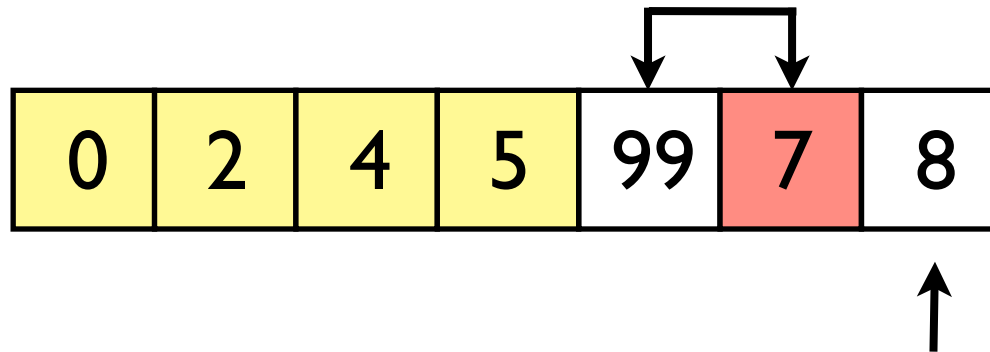


**Selection Sort**

Current min: 7

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

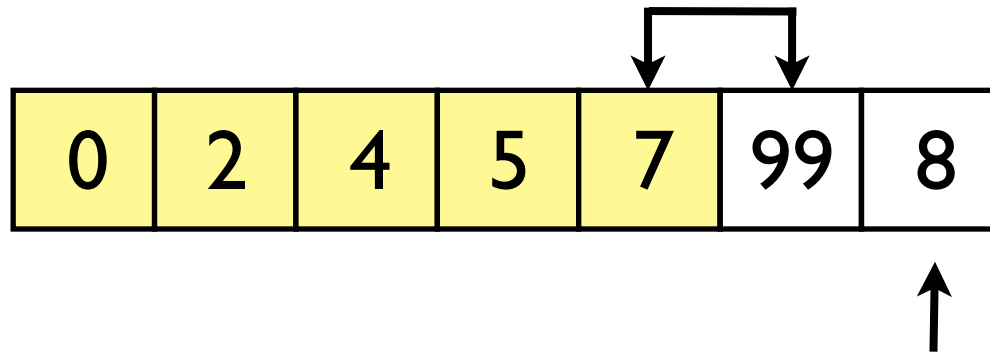


## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

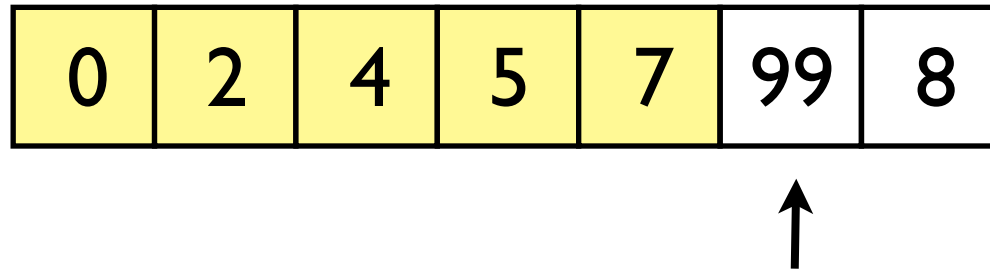


## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

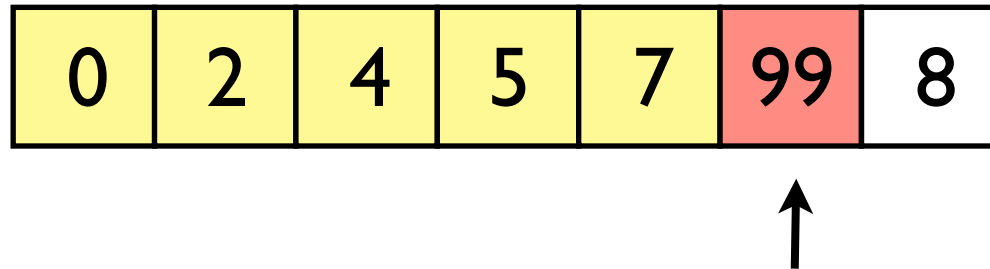
Sort a given list of integers (from small to large).



**Selection Sort**

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

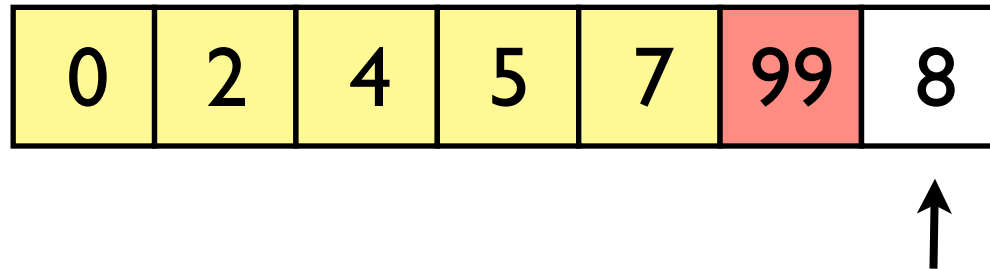


**Selection Sort**

Current min: 99

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

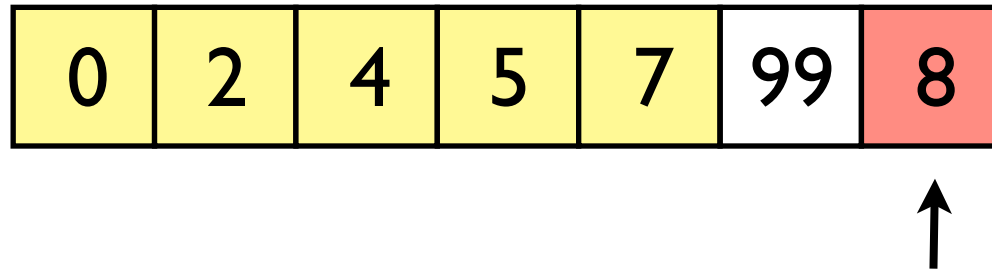


**Selection Sort**

Current min: 99

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).



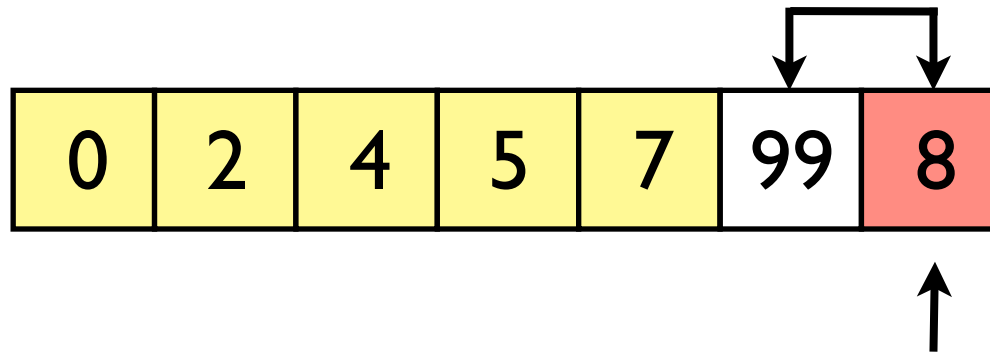
**Selection Sort**

Current min: 8



# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

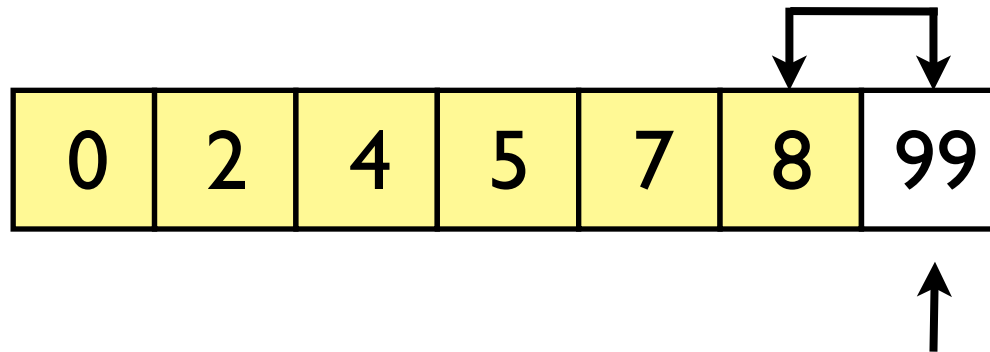


## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).



## Selection Sort

Swap current min with first element of unsorted part

# Selection Sort: Algorithm

Sort a given list of integers (from small to large).

0	2	4	5	7	8	99
---	---	---	---	---	---	----

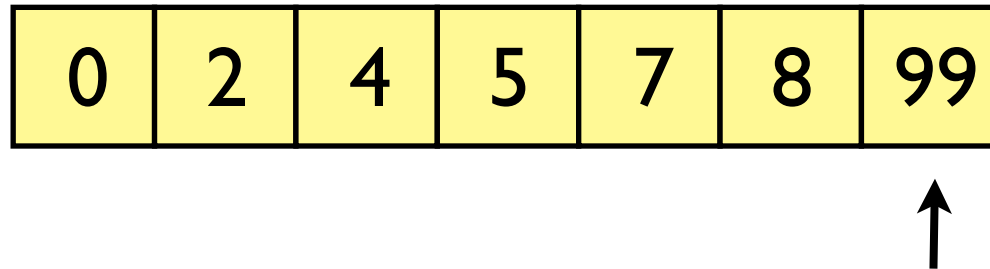


**Selection Sort**

**Done!**

# Selection Sort: Running Time

Sort a given list of integers (from small to large).



## Selection Sort

How many steps does this take (in the worst case)?

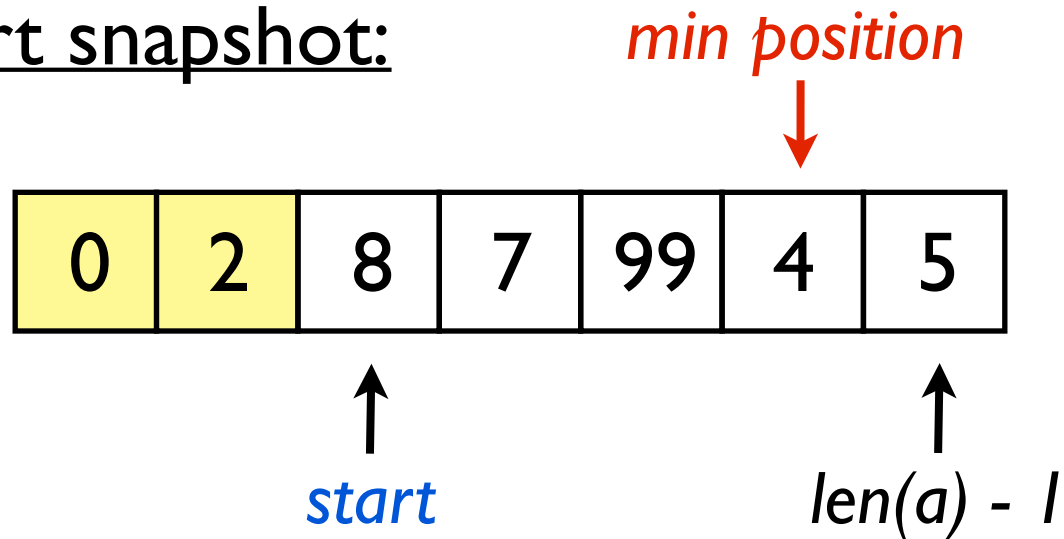
$$\sim N + (N - 1) + (N - 2) + \cdots + 1 = \frac{N^2}{2} + \frac{N}{2}$$

(As  $N$  increases, small terms lose significance.)

Running time is  $O(N^2)$ .

# Selection Sort: Code

Selection sort snapshot:



Find the *min position* from *start* to  $\text{len}(a) - 1$

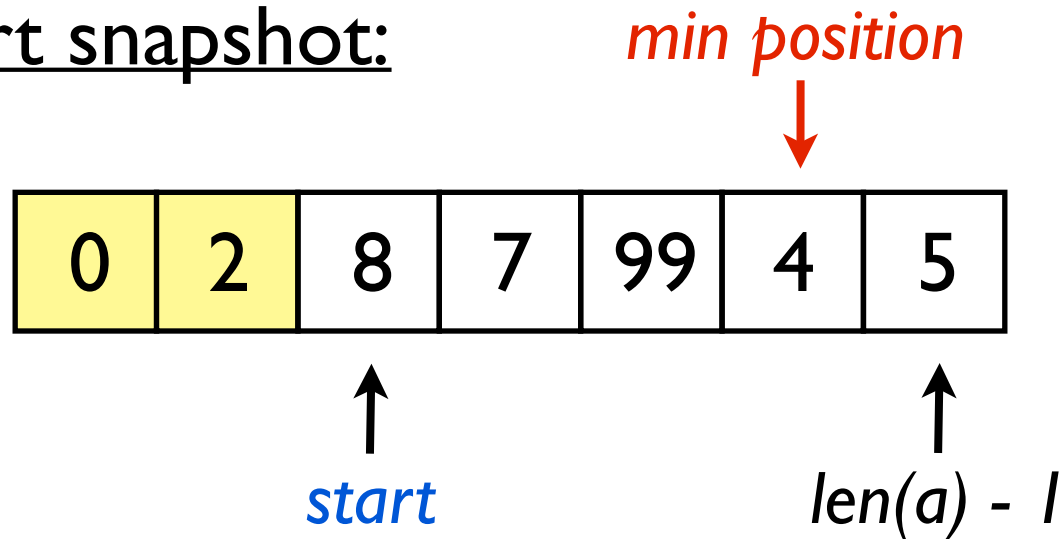
Swap elements in *min position* and *start*

Increment *start*

Repeat

# Selection Sort: Code

Selection sort snapshot:



for *start* = 0 to  $\text{len}(a) - 1$ :

Find the *min position* from *start* to  $\text{len}(a) - 1$

Swap elements in *min position* and *start*

# Selection Sort: Code

for *start* = 0 to  $\text{len}(a)-1$ :

Find the *min position* from *start* to  $\text{len}(a) - 1$

Swap elements in *min position* and *start*

0	2	8	7	99	4	5
---	---	---	---	----	---	---

**def** selectionSort(a):

for *start* in range(len(a)):

*currentMinIndex* = *start*

    for i in range(*start*, len(a)):

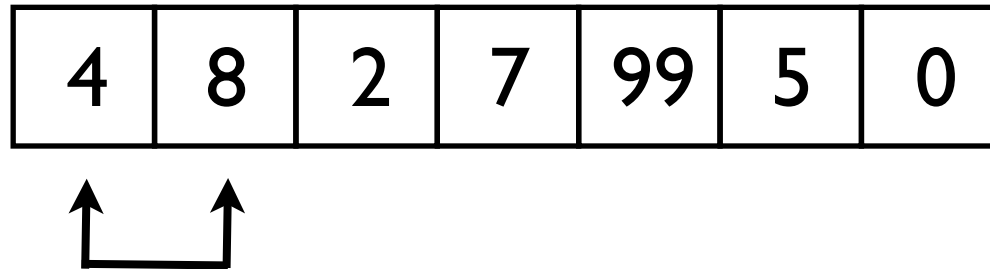
        if(a[i] < a[*currentMinIndex*]):

*currentMinIndex* = i

    (a[*currentMinIndex*], a[*start*]) = (a[*start*], a[*currentMinIndex*])

# Bubble Sort: Algorithm

Sort a given list of integers (from small to large).



## Bubble Sort

Compare each pair of adjacent items (left to right).

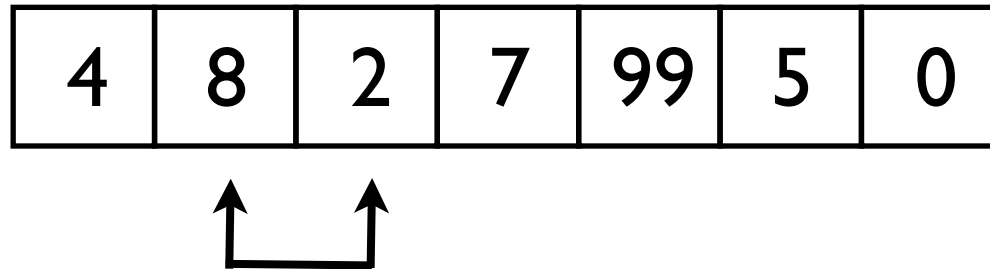
Swap them if they are in the wrong order.

Repeat until no more swaps are needed.



# Bubble Sort: Algorithm

Sort a given list of integers (from small to large).



## Bubble Sort

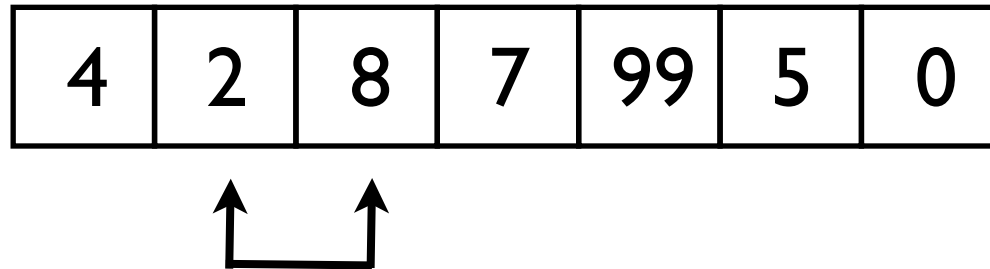
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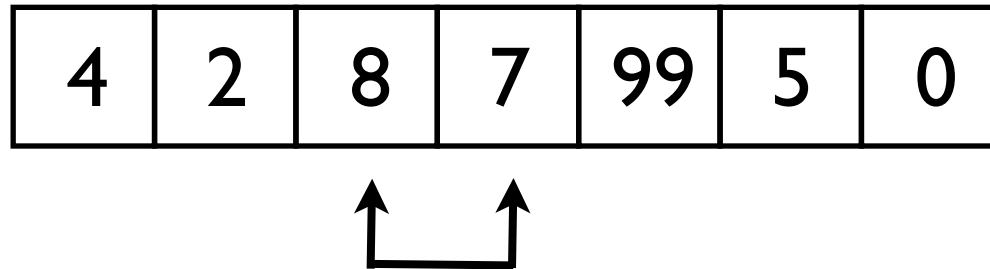
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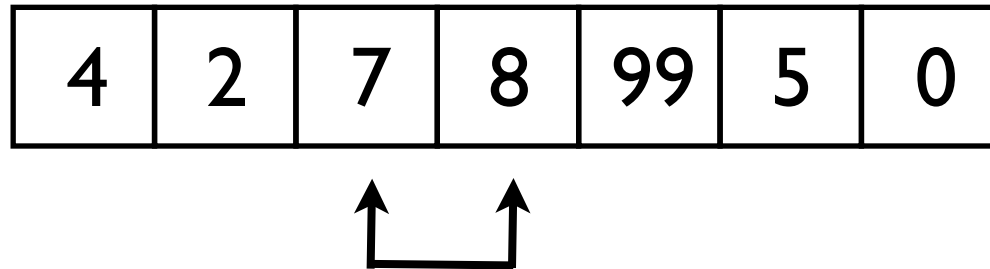
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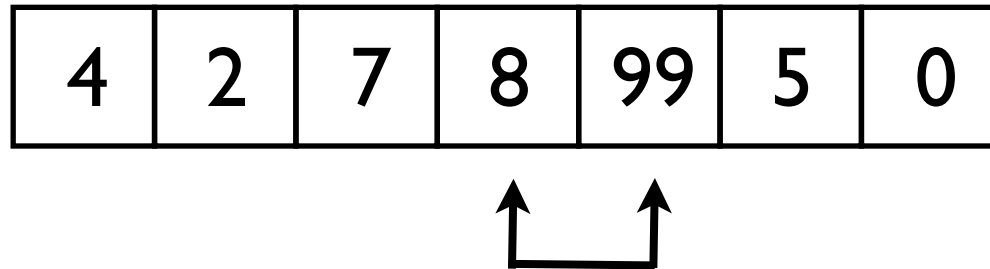
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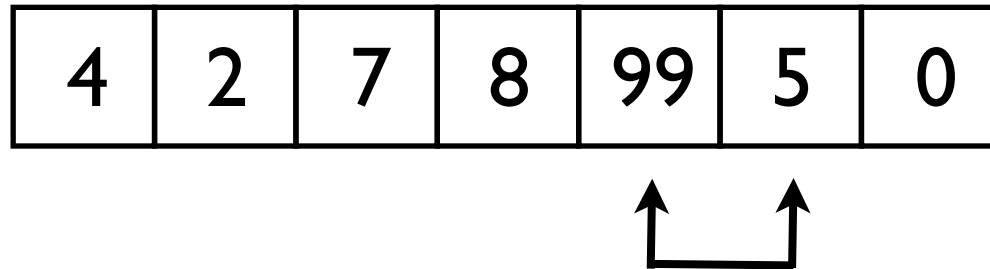
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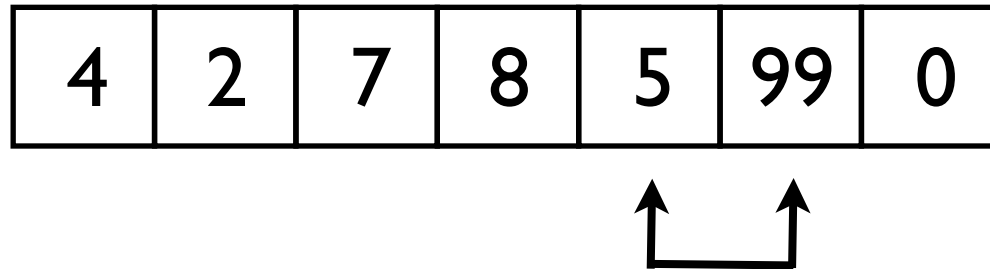
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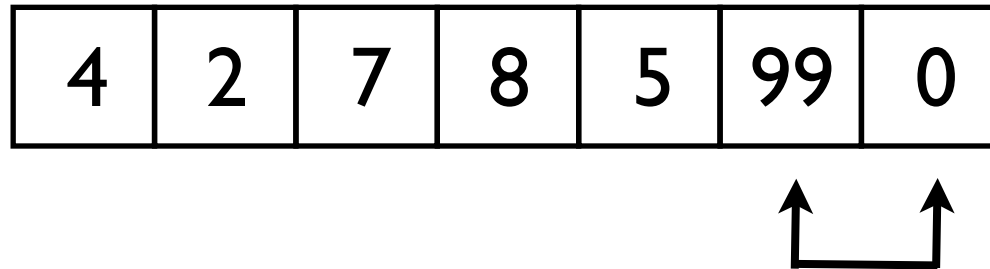
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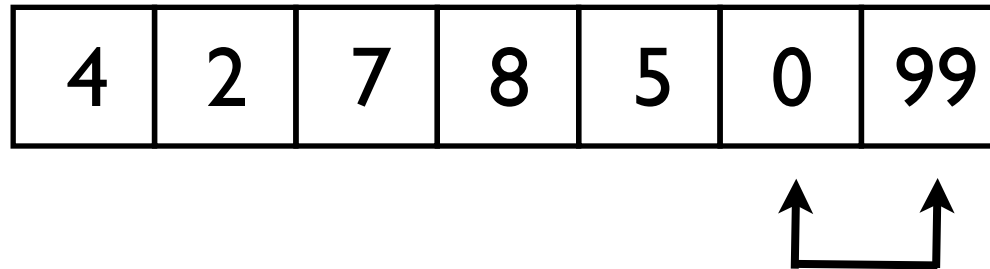
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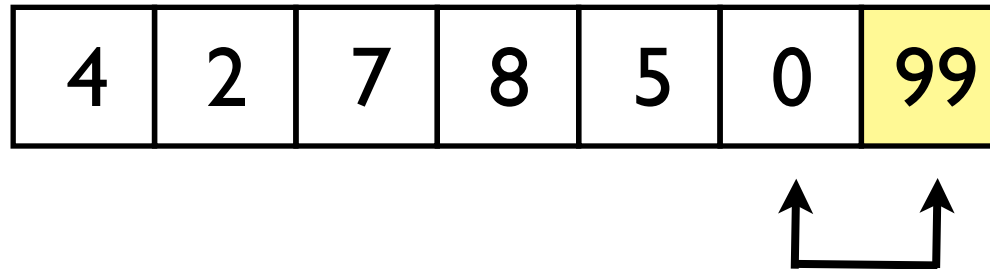
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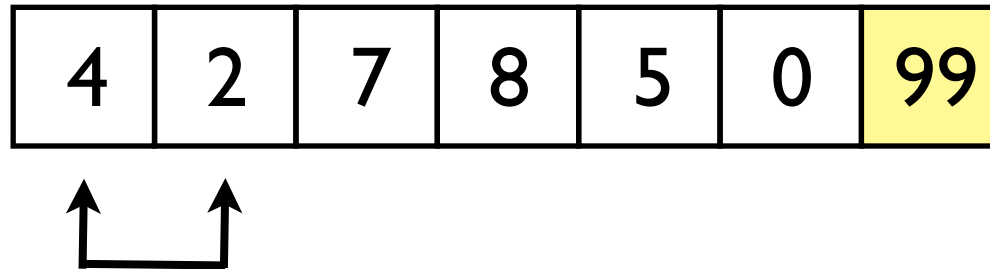
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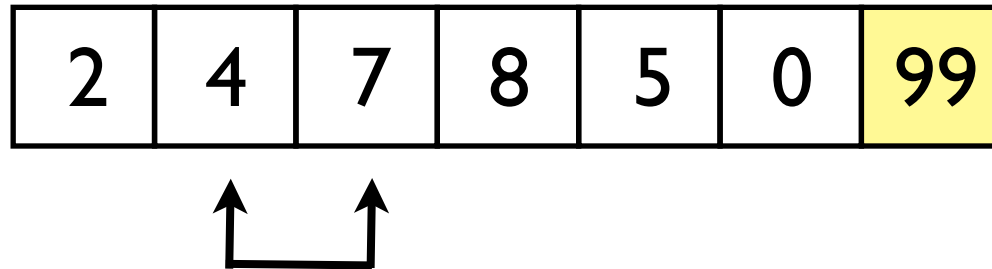
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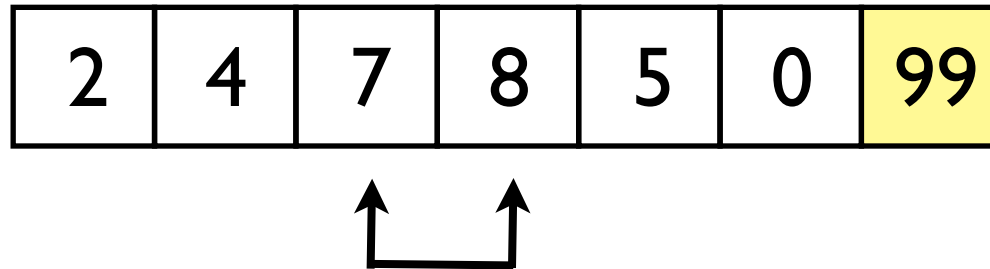
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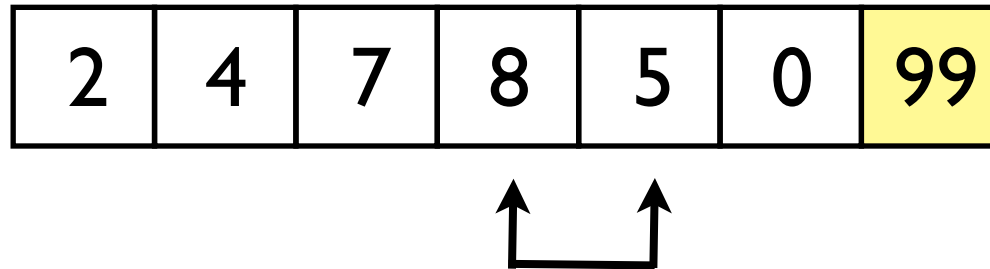
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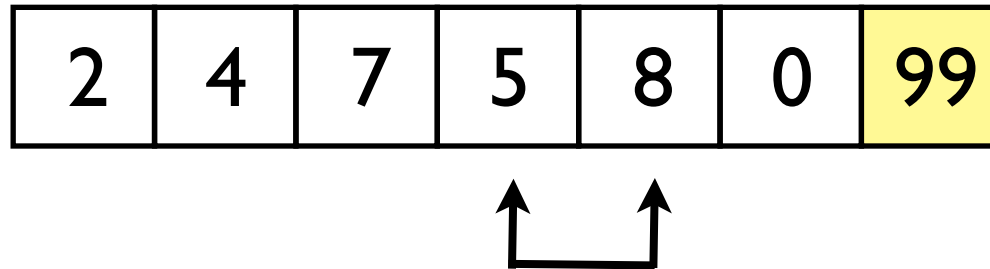
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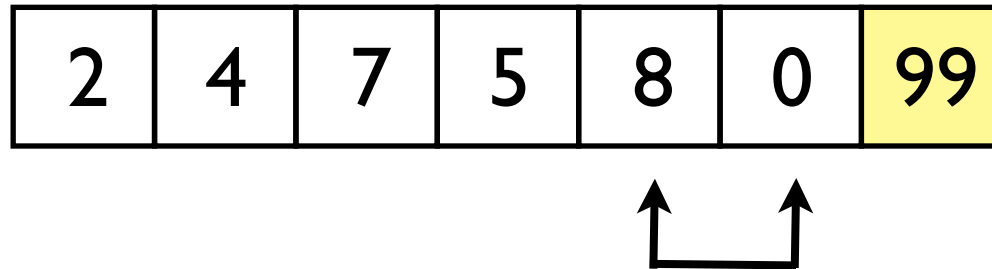
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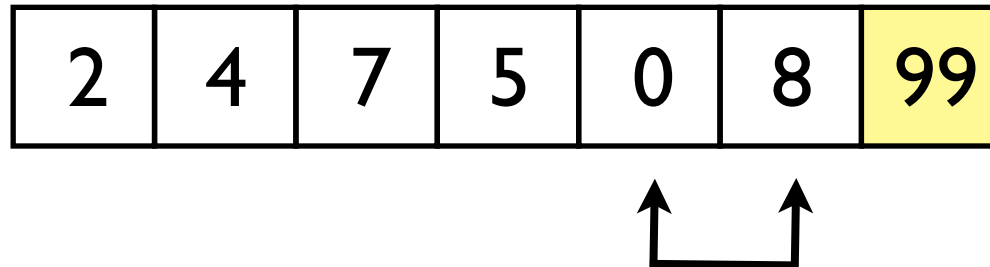
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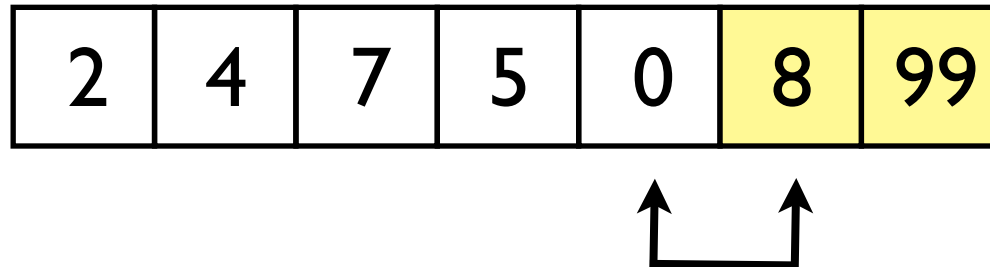
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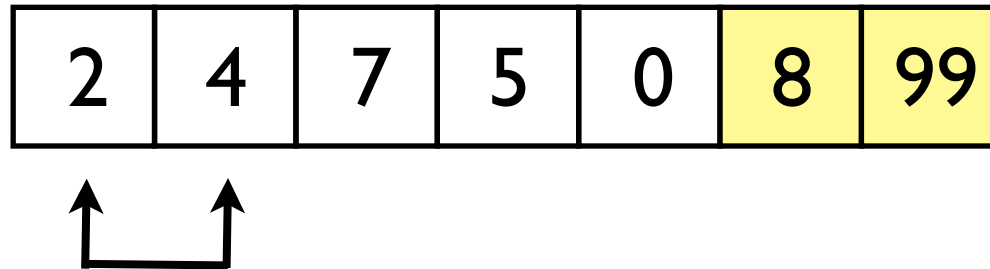
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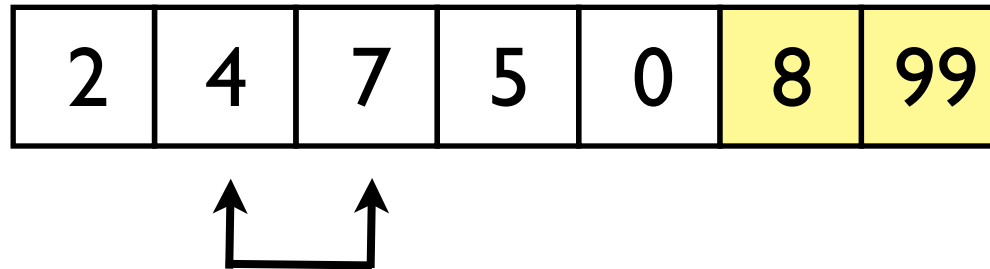
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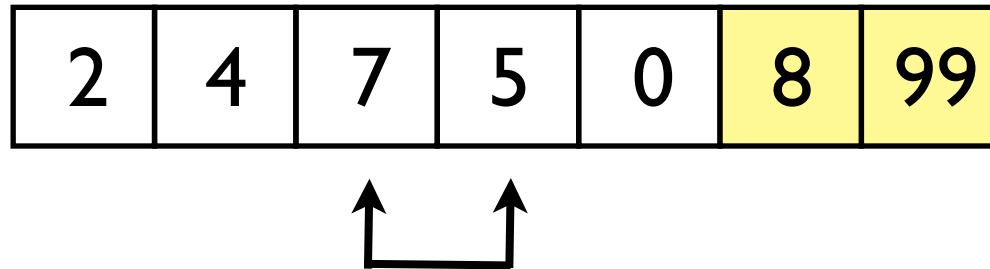
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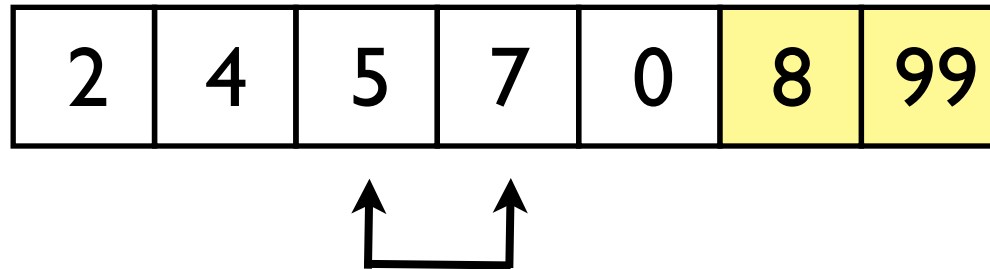
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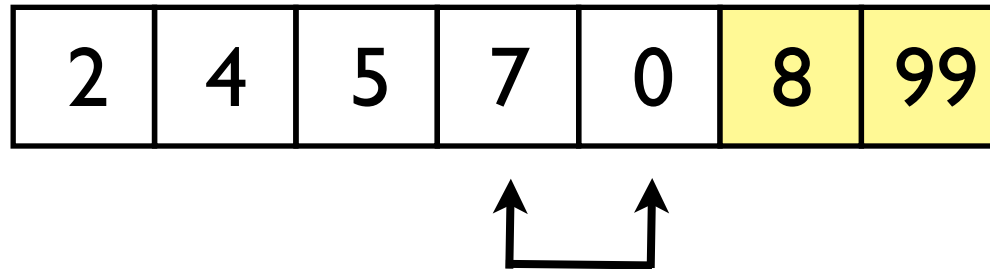
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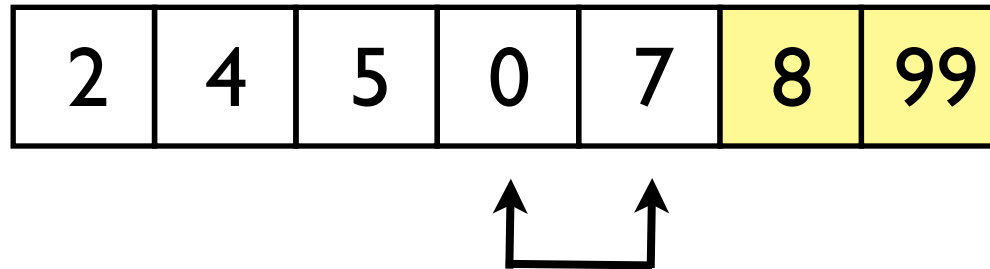
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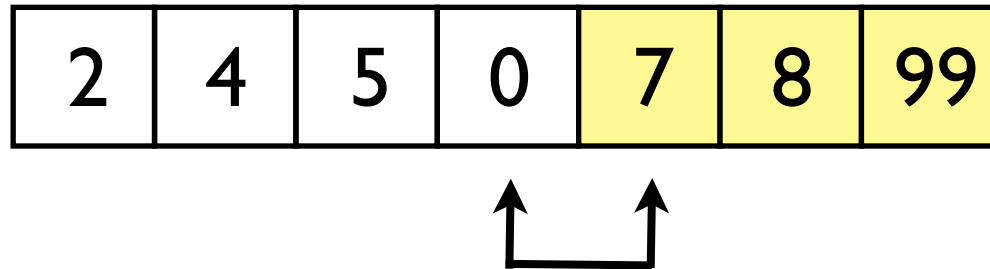
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## Bubble Sort

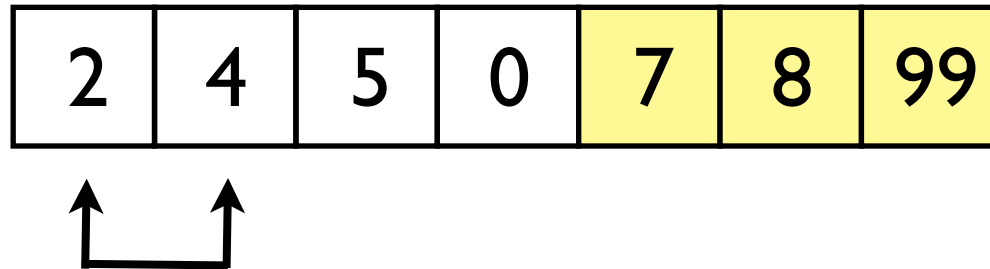
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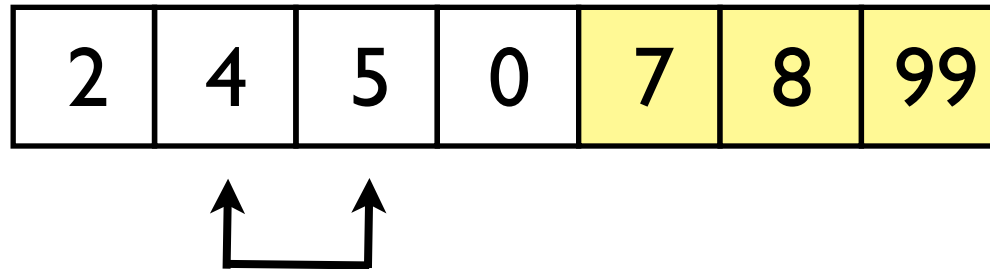
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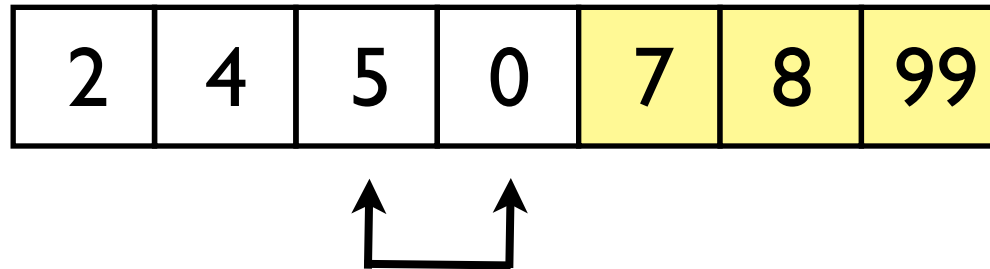
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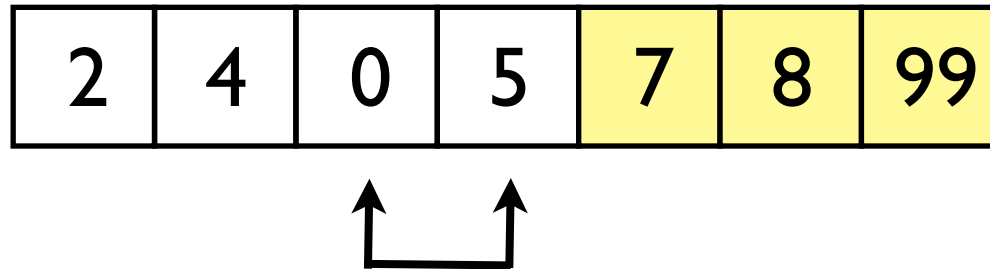
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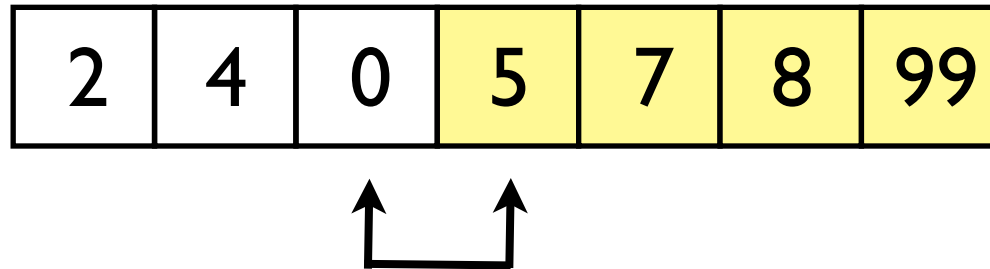
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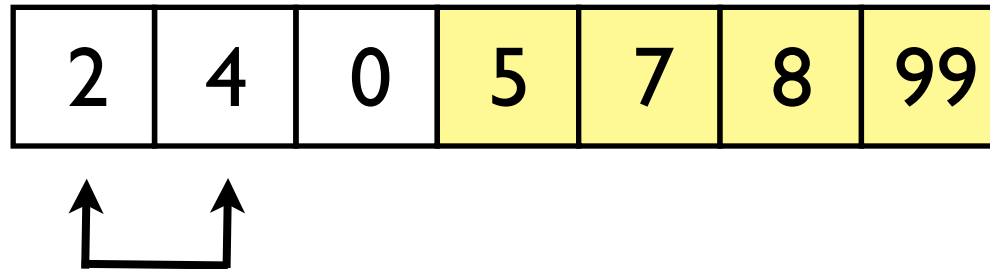
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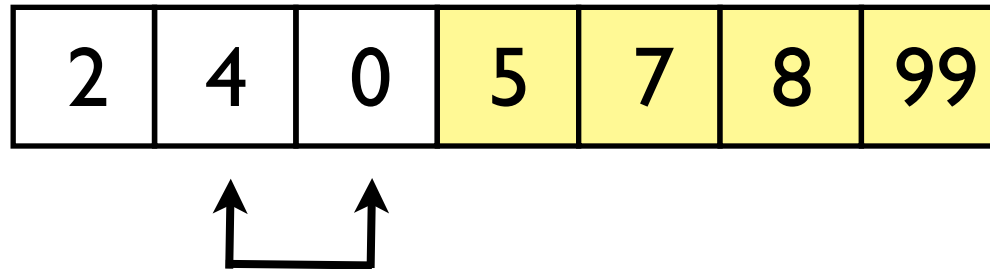
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# Bubble Sort: Algorithm

Sort a given list of integers (from small to large).



## Bubble Sort

Compare each pair of adjacent items (left to right).

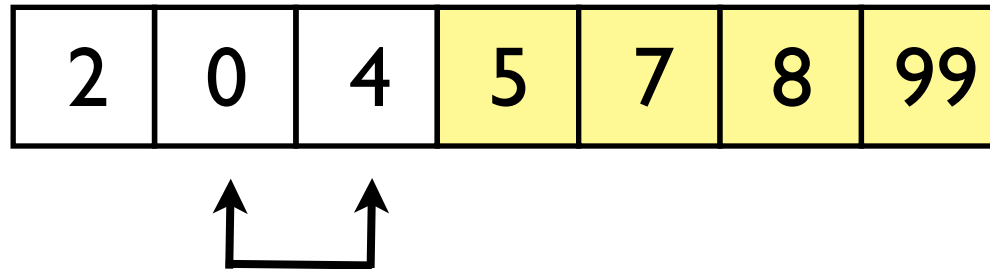
Swap them if they are in the wrong order.

Repeat until no more swaps are needed.



# Bubble Sort: Algorithm

Sort a given list of integers (from small to large).



## Bubble Sort

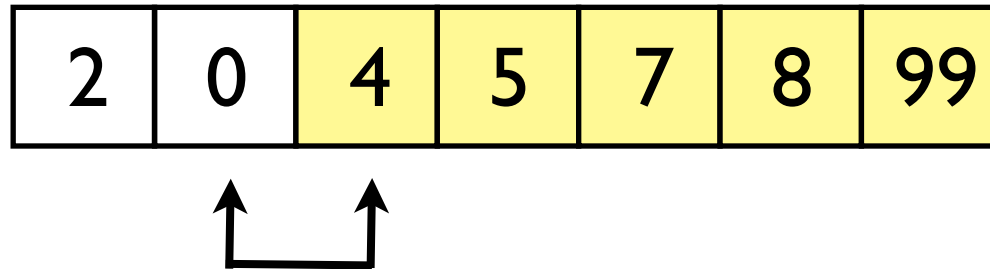
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Swap them if they are in the wrong order.

Repeat until no more swaps are needed.

# Bubble Sort: Algorithm

Sort a given list of integers (from small to large).



## Bubble Sort

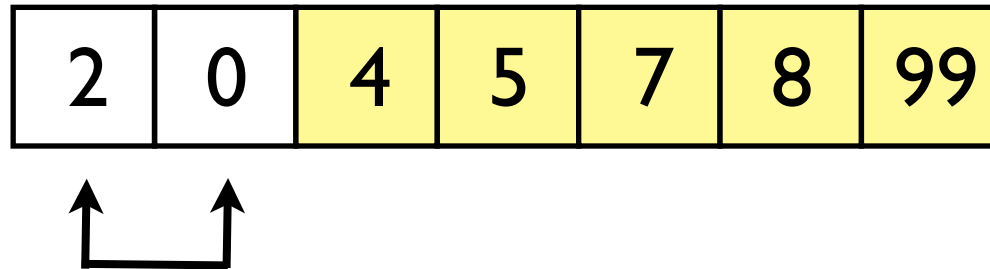
Compare each pair of adjacent items (left to right).

Swap them if they are in the wrong order.

Repeat until no more swaps are needed.

# Bubble Sort: Algorithm

Sort a given list of integers (from small to large).



## Bubble Sort

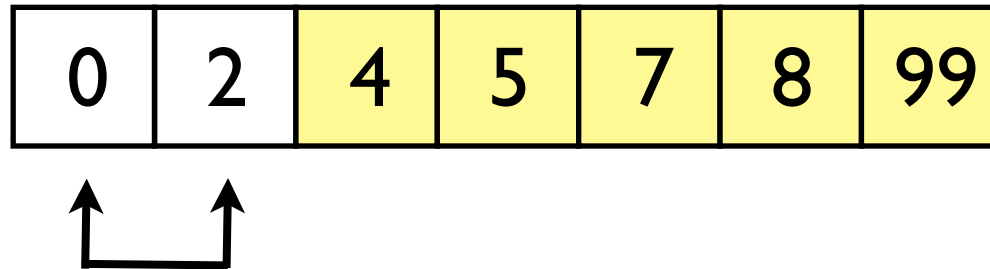
Compare each pair of adjacent items (left to right).

Swap them if they are in the wrong order.

Repeat until no more swaps are needed.

# Bubble Sort: Algorithm

Sort a given list of integers (from small to large).



## Bubble Sort

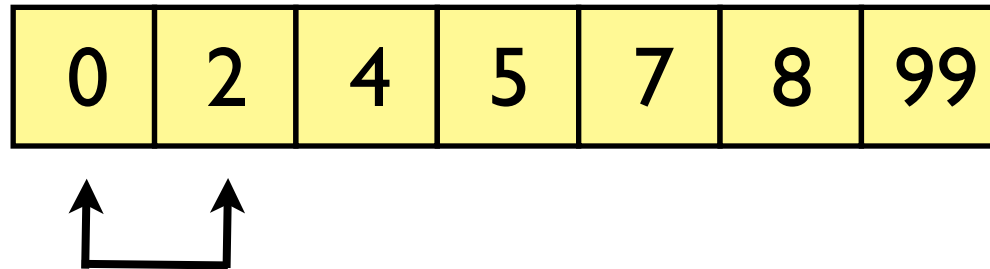
Compare each pair of adjacent items (left to right).

Swap them if they are in the wrong order.

Repeat until no more swaps are needed.

# Bubble Sort: Algorithm

Sort a given list of integers (from small to large).



## Bubble Sort

Compare each pair of adjacent items (left to right).

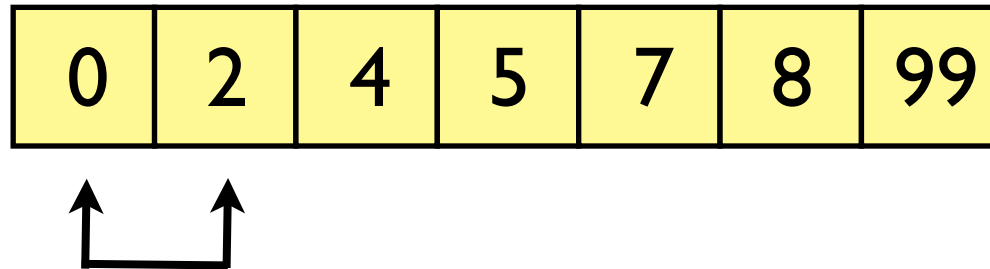
Swap them if they are in the wrong order.

Repeat until no more swaps are needed.

Large elements “bubble up”

# Bubble Sort: Running Time

Sort a given list of integers (from small to large).



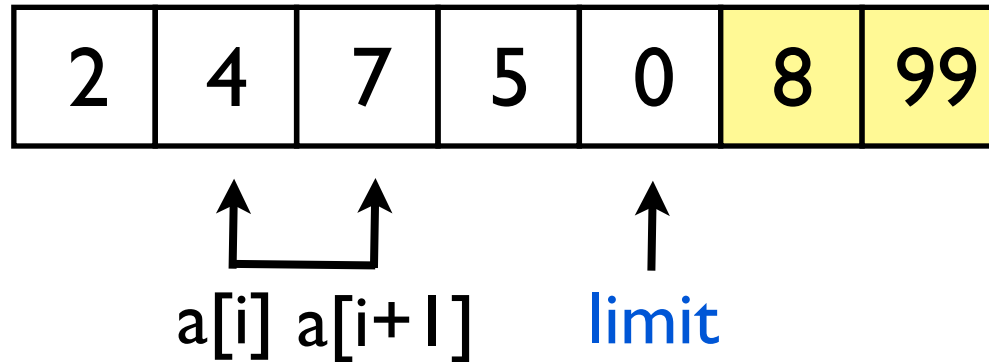
## Bubble Sort

How many steps does this take (in the worst case)?

$$O(N^2)$$

# Bubble Sort: Code

## Bubble sort snapshot



repeat until no more swaps:

for  $i = 0$  to **limit**:

if  $a[i] > a[i+1]$ , swap  $a[i]$  and  $a[i+1]$

decrement **limit**

# Bubble Sort: Code

repeat until no more swaps:

for  $i = 0$  to **limit**:

if  $a[i] > a[i+1]$ , swap  $a[i]$  and  $a[i+1]$

decrement **limit**

**def** bubbleSort(a):

swapped = True

**limit** = len(a)-1

**while**(swapped):

swapped = False

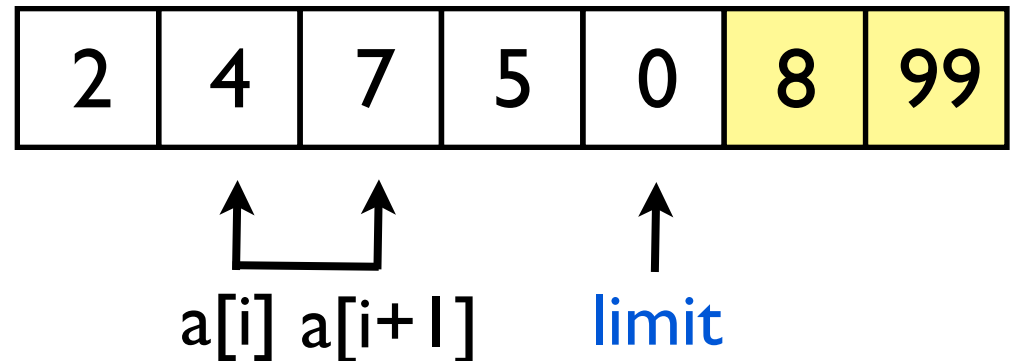
**for**  $i$  **in** range(limit):

**if**( $a[i] > a[i+1]$ ):

$(a[i], a[i+1]) = (a[i+1], a[i])$

swapped = True

**limit** -= 1





# Comparison: Selection Sort vs Bubble Sort

Worst case both take  $O(N^2)$  steps.

How about best case?

Selection sort:  $O(N^2)$

Bubble sort:  $O(N)$

If your list is close to being sorted,  
bubble sort can be better.

Is there a better way?

## Exercise

Write the code yourself:

linear search  
binary search  
selection sort  
bubble sort