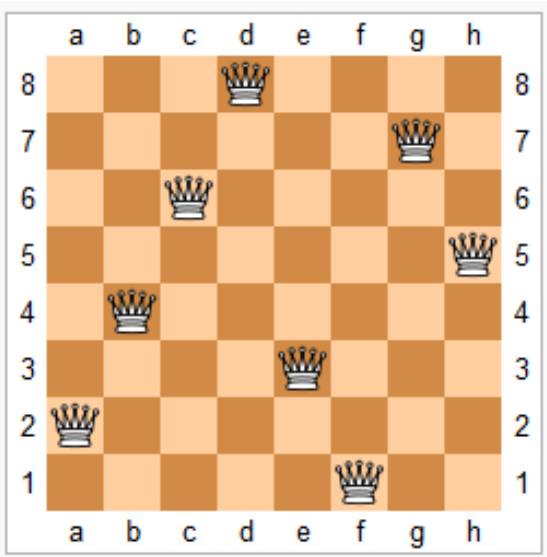
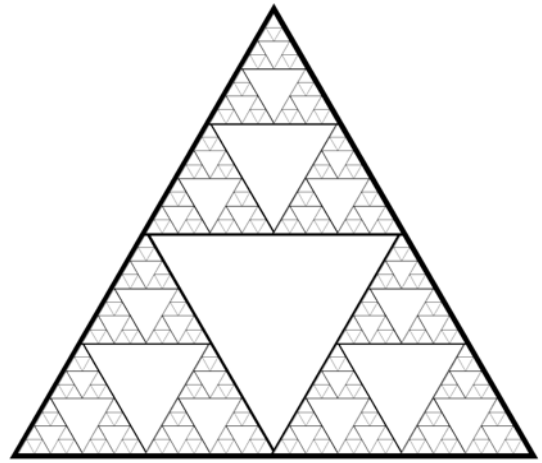
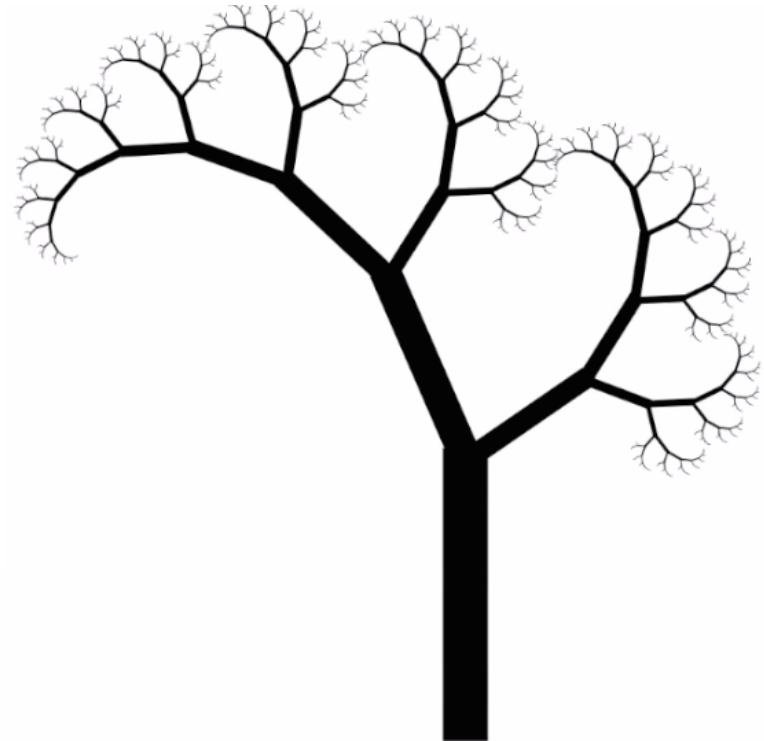
15-112
Fundamentals of Programming

Week 10 - Lecture 1:
More Advanced Recursion



One solution to the eight queens puzzle





March 22, 2016

Recursion vs Iteration

15-112 View

	Recursion	Iteration
Elegance	+	-
Performance	-	+
Debugability	-	+

Printing Call Stack

```
def fact(n, depth=0):
    print (“  ”*depth, “fact(”, n, “)”)
    if (n <= 1):
        result = 1
    else:
        result = n*fact(n-1, depth+1)
    print (“  ”*depth, “—>”, result)
    return result

fact(4)
```

Printing Call Stack

```
def fib(n, depth=0):
    print ("  "*depth, "fib(", n, ")")
    if (n < 2):
        result = 1
    else:
        result = fib(n-1, depth+1) + fib(n-2, depth+1)
    print ("  "*depth, "—>", result)
    return result

fib(4)
```

Printing Call Stack

```
def rangeSum(lo, hi, depth=0):
    print ("  "*depth, "rangeSum(", lo, hi ")")
    if (lo == hi):
        result = lo
    else:
        mid = (lo + hi)//2
        result = rangeSum(lo, mid) + rangeSum(mid+1, hi)
    print ("  "*depth, "—>", result)
    return result

print(rangeSum(10,15))
```

Memoization

```
def fib(n):
    if (n < 2):
        result = 1
    else:
        result = fib(n-1) + fib(n-2)
    return result

print(fib(6))
```

How many times is fib(2) computed? **5**

Memoization

```
fibResults = dict()

def fib(n):
    if (n in fibResults):
        return fibResults[n]
    if (n < 2):
        result = 1
    else:
        result = fib(n-1) + fib(n-2)
    fibResults[n] = result
    return result
```

Expanding the stack size and recursion limit

```
def rangeSum(lo, hi):
    if (lo > hi):
        return 0
    else:
        return lo + rangeSum(lo+1, hi)

print(rangeSum(1, 1234))
# RuntimeError: maximum recursion depth exceeded


print(callWithLargeStack(rangeSum(1, 123456)))
# Works
```

More Examples

Power set

Given a list, return a list of all the subsets of the list.

[1,2,3] -> [[], [1], [2], [3], [1,2], [2,3], [1,3], [1,2,3]]

Power set

Given a list, return a list of all the subsets of the list.

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All subsets = All subsets that do not contain 1 +

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All subsets that contain 1

Power set

Given a list, return a list of all the subsets of the list.

[1,2,3] -> [[], [1], [2], [3], [1,2], [2,3], [1,3], [1,2,3]]

[1] + subset that doesn't contain a 1

All subsets = All subsets that do not contain 1 +

All subsets that contain 1

Power set

Given a list, return a list of all the subsets of the list.

[1,2,3] -> [[], [1], [2], [3], [1,2], [2,3], [1,3], [1,2,3]]

```
def powerset(a):
    if (len(a) == 0):
        return [[]]
    else:
        allSubsets = [ ]
        for subset in powerset(a[1:]):
            allSubsets += [subset]
            allSubsets += [[a[0]] + subset]
        return allSubsets
```

Power set

Given a list, return a list of all the subsets of the list.

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Permutations

Given a list, return all permutations of the list.

[1,2,3] -> [[1,2,3], [2,1,3], [2,3,1], [1,3,2], [3,1,2], [3,2,1]]

Permutations

Given a list, return all permutations of the list.

[1,2,3] -> [[1,2,3], [2,1,3], [2,3,1], [1,3,2], [3,1,2], [3,2,1]]

[1,2,3], [2,1,3], [2,3,1]

Permutations

Given a list, return all permutations of the list.

[1,2,3] -> [[1,2,3], [2,1,3], [2,3,1], [1,3,2], [3,1,2], [3,2,1]]

[1,2,3], [2,1,3], [2,3,1] [1,3,2], [3,1,2], [3,2,1]

Permutations

<u>Permutations of [2,3,4]</u>

[2,3,4] → [1,2,3,4], [2,1,3,4], [2,3,1,4], [2,3,4,1]

[2,4,3] → [1,2,4,3], [2,1,4,3], [2,4,1,3], [2,4,3,1]

[1,2,3,4]

[3,2,4]

[3,4,2]

[4,2,3]

[4,3,2]

Permutations

Given a list, return all permutations of the list.

[1,2,3] -> [[1,2,3], [2,1,3], [2,3,1], [1,3,2], [3,1,2], [3,2,1]]

```
def permutations(a):
    if (len(a) == 0):
        return [[]]
    else:
        allPerms = [ ]
        for subPermutation in permutations(a[1:]):
            for i in range(len(subPermutation)+1):
                allPerms += [subPermutation[:i] + [a[0]] + subPermutation[i:]]
        return allPerms
```

Permutations

Given a list, return all permutations of the list.

[1,2,3] -> [[1,2,3], [2,1,3], [2,3,1], [1,3,2], [3,1,2], [3,2,1]]

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Permutations

Given a list, return all permutations of the list.

[1,2,3] -> [[1,2,3], [2,1,3], [2,3,1], [1,3,2], [3,1,2], [3,2,1]]

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    if (len(a) == 0):
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        allPerms = [ ]
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            for i in range(len(subPermutation)+1):
                allPerms += [subPermutation[:i] + [a[0]] + subPermutation[i:]]
        return allPerms
```

Print files in a directory

Name	Date Modified	Size	Kind
Folder1	Today, 10:11 PM	--	Folder
Folder2	Today, 10:12 PM	--	Folder
helloworld.py	Oct 7, 2014, 1:10 PM	812 bytes	Python
todo	Oct 3, 2014, 1:04 PM	1 KB	rich tex

Print files in a directory

Name	Date Modified	Size	Kind
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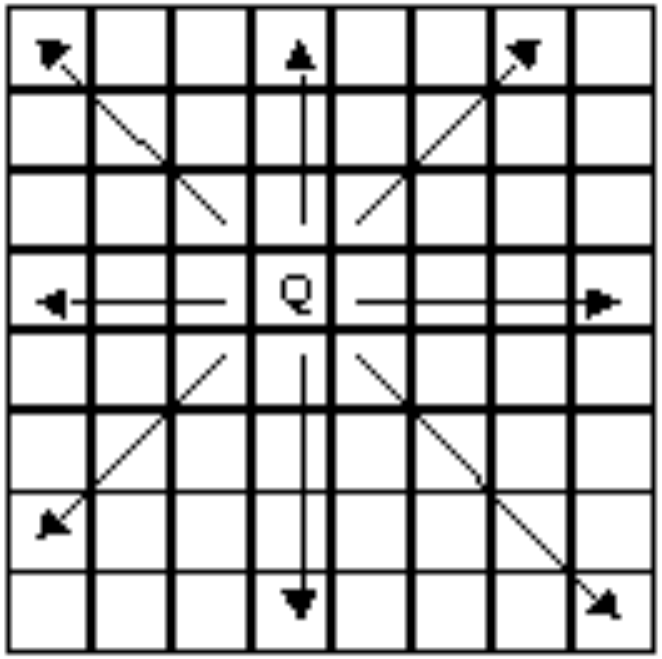
fooo.py	Oct 7, 2014, 1:10 PM	812 bytes	Python
SubFolder1	Today, 10:11 PM	--	Folder
foooo.py	Oct 7, 2014, 1:10 PM	812 bytes	Python
SubFolder2	Today, 10:12 PM	--	Folder
fooooo.py	Oct 7, 2014, 1:10 PM	812 bytes	Python
foooooo.py	Oct 7, 2014, 1:10 PM	812 bytes	Python
SubSubFolder1	Today, 10:13 PM	--	Folder
somePic	Today, 9:32 PM	56 KB	PNG ir
Folder2	Today, 10:12 PM	--	Folder
haha	Oct 3, 2014, 1:04 PM	1 KB	rich tex
helloworld.py	Oct 7, 2014, 1:10 PM	812 bytes	Python
todo	Oct 3, 2014, 1:04 PM	1 KB	rich tex

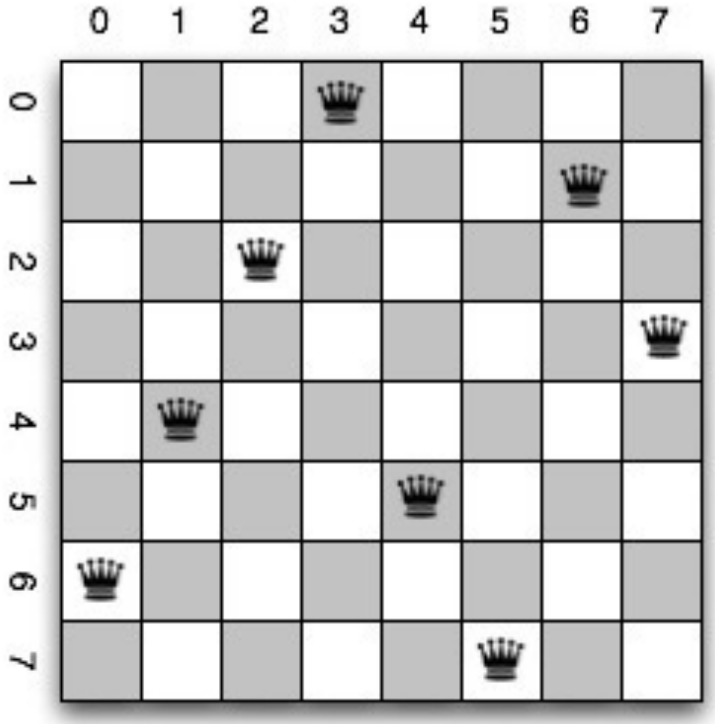
Print files in a directory

```
import os
def printFiles(path):
    if (os.path.isdir(path) == False):
        # base case:  not a folder, but a file, so print its path
        print(path)
    else:
        # recursive case: it's a folder
        for filename in os.listdir(path):
            printFiles(path + "/" + filename)
```

nQueens Problem

Place n queens on a n by n board so that no queen is attacking another queen.





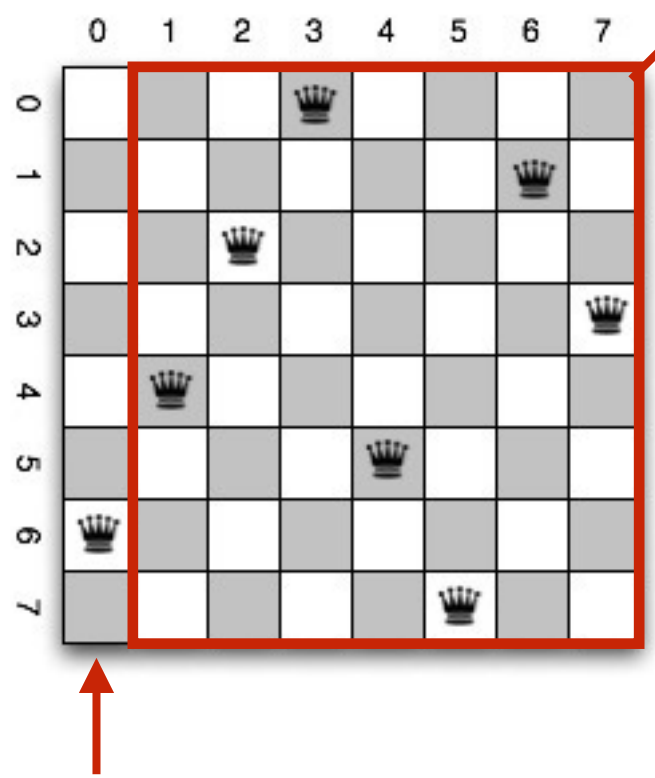
def solve(n): —> [6, 4, 2, 0, 5, 7, 1, 3]

list of rows

nQueens Problem

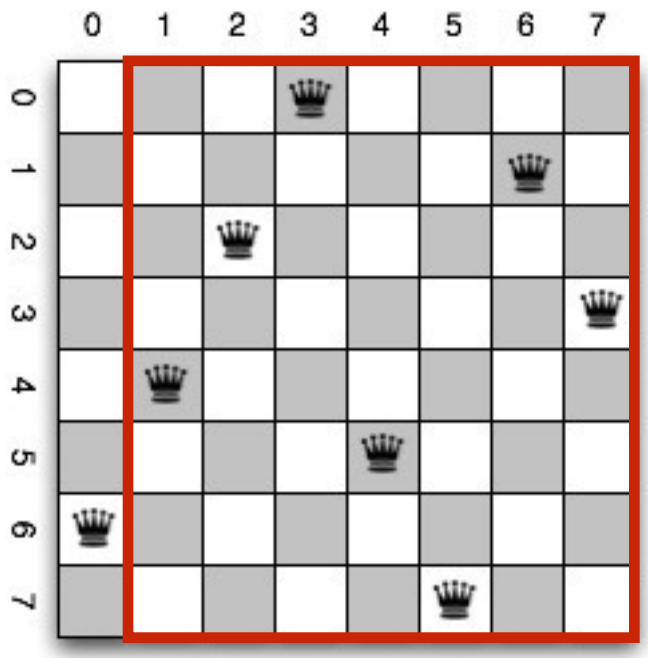
Place n queens on a n by n board so that no queen is attacking another queen.

n rows and n-1 columns



one queen has to be on first column

nQueens Problem



First attempt:

- try rows 0 to 7 for first queen
- for each try, recursively solve the red part

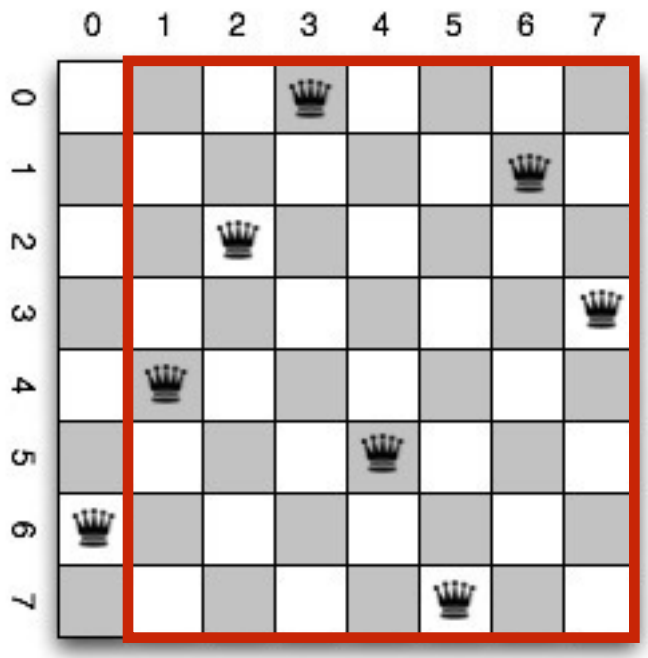
Problem:

Can't solve red part without taking into account first queen
First queen puts constraints on the solution to the red part

Need to be able to solve nQueens with added constraints.
Need to generalize our function:

```
def solve(n, m, constraints):
```

nQueens Problem



def solve(n, m, constraints):

n = number or rows

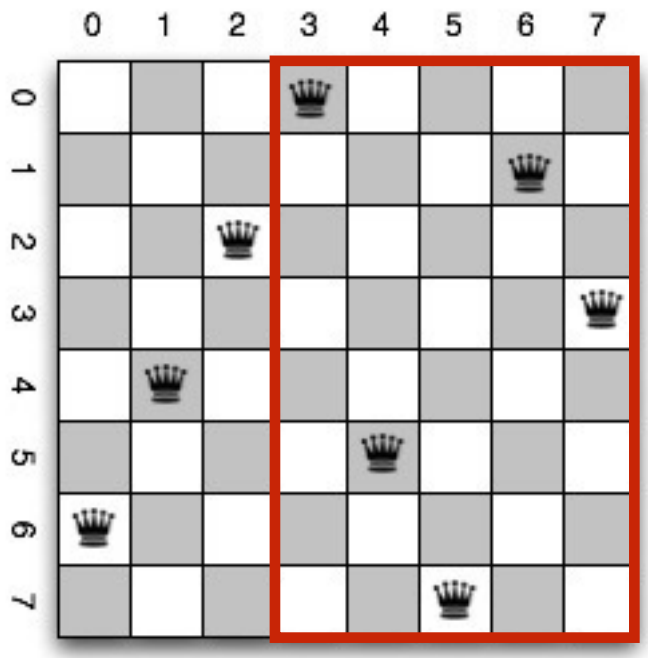
m = number or columns

constraints (in what form?)

list of rows

For the red part, we have the constraint [6]

nQueens Problem



def solve(n, m, constraints):

n = number or rows

m = number or columns

constraints (in what form?)

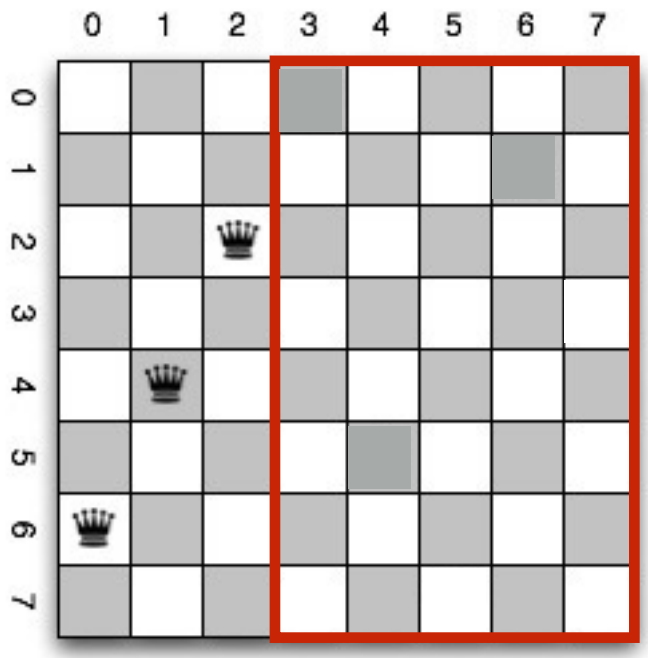
list of rows

For the red part, we have the constraint [6,4,2]

The constraint tells us which cells are <u>unusable</u> for the red part.

To solve original nQueens problem, call: solve(n, n, [])

nQueens Problem



[?,?,?,?,?]

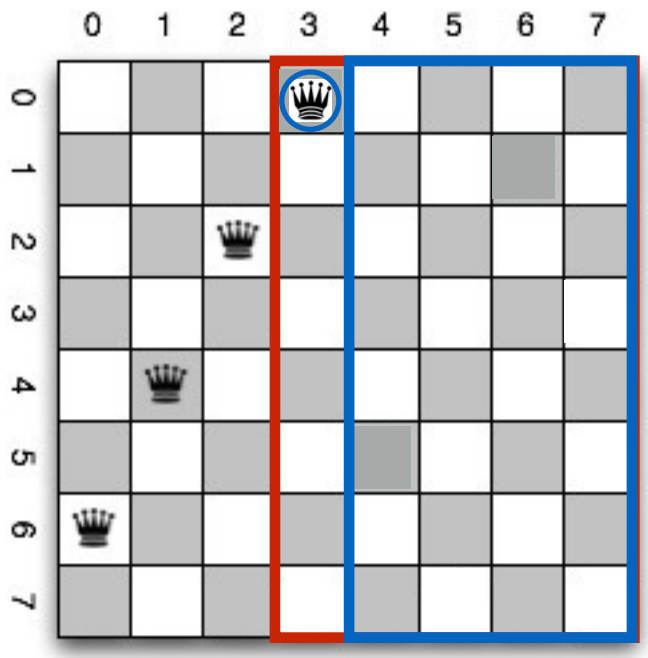
```
def solve(n, m, constraints):
```

n = 8

m = 5

constraints = [6,4,2]

nQueens Problem



[0,?,?,?,?]

```
def solve(n, m, constraints):
```

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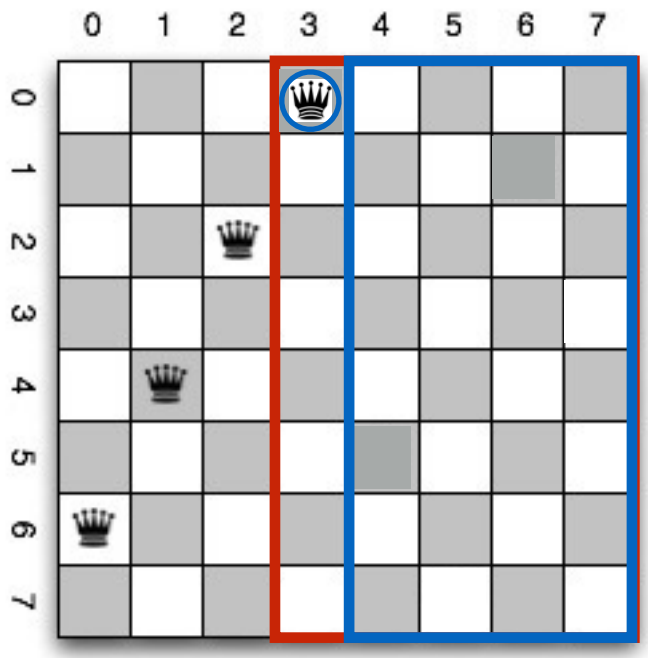
nQueens Problem



[0,?,?,?,?]

[5,7,1,3]

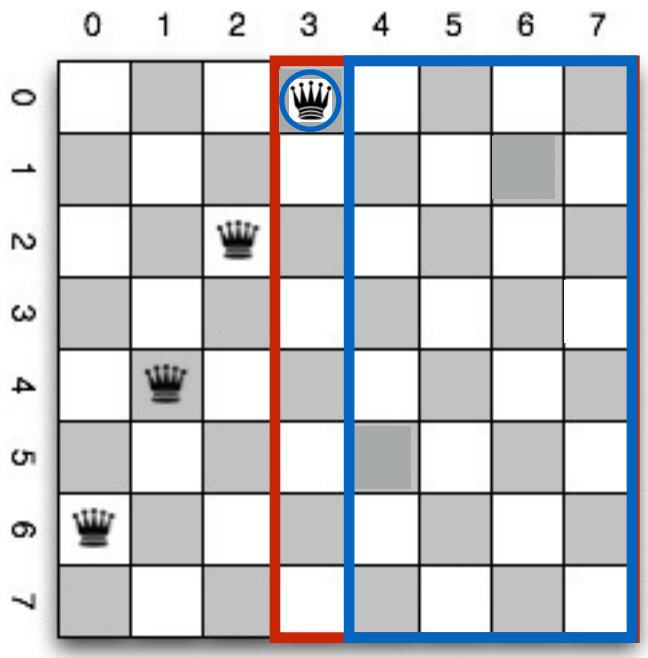
n = 8

m = 5

constraints = [6,4,2]

def solve(n, m, constraints):

nQueens Problem



[0,?,?,?,?]

[5,7,1,3] —> [0,5,7,1,3]

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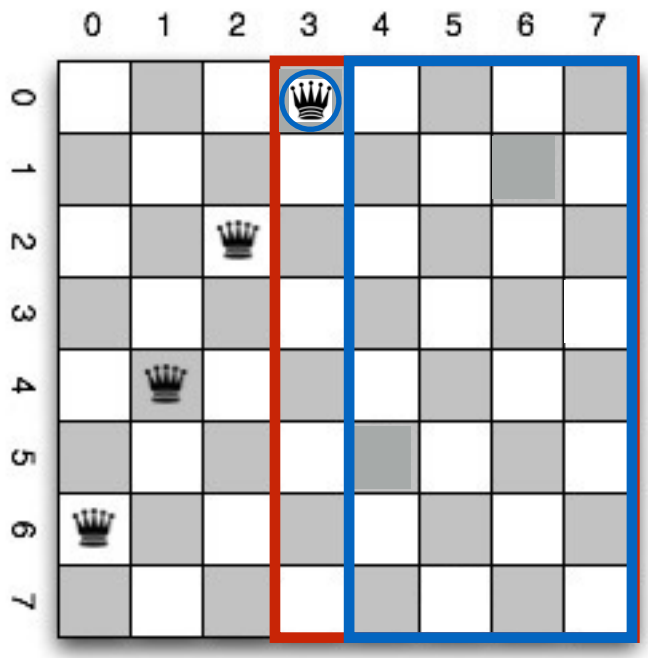


[0,?,?,?,?]

```
def solve(n, m, constraints):
```

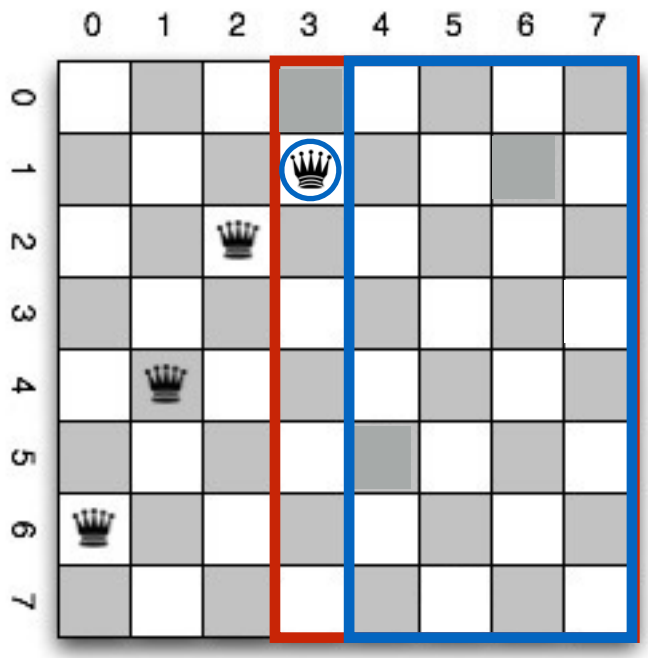
Suppose no solution

n = 8

m = 5

constraints = [6,4,2]

nQueens Problem



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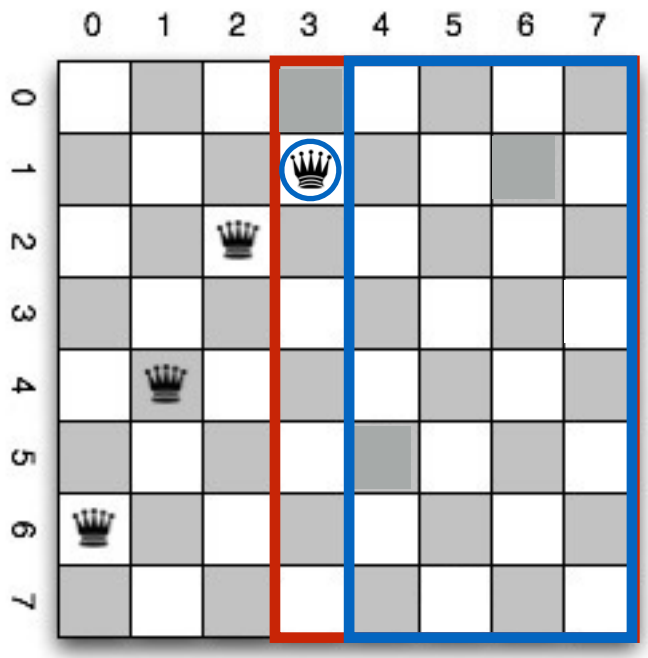


[0,?,?,?,?]

def solve(n, m, constraints):

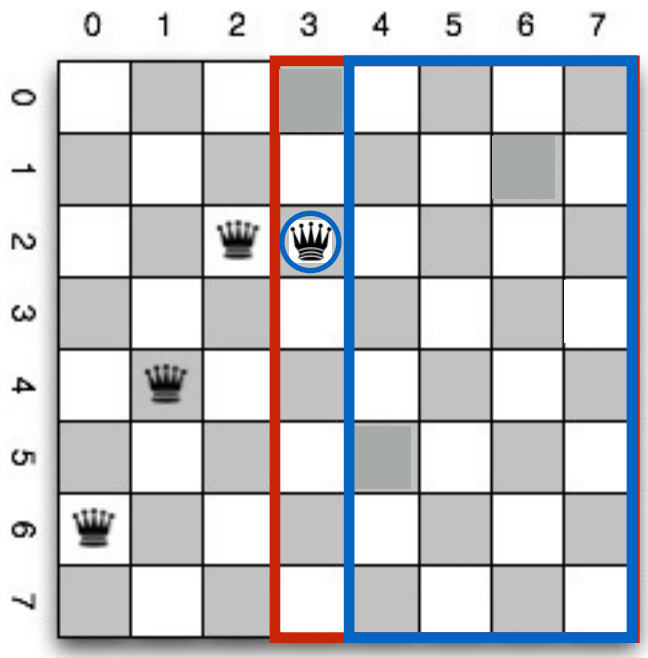
NOT LEGAL

n = 8

m = 5

constraints = [6,4,2]

nQueens Problem



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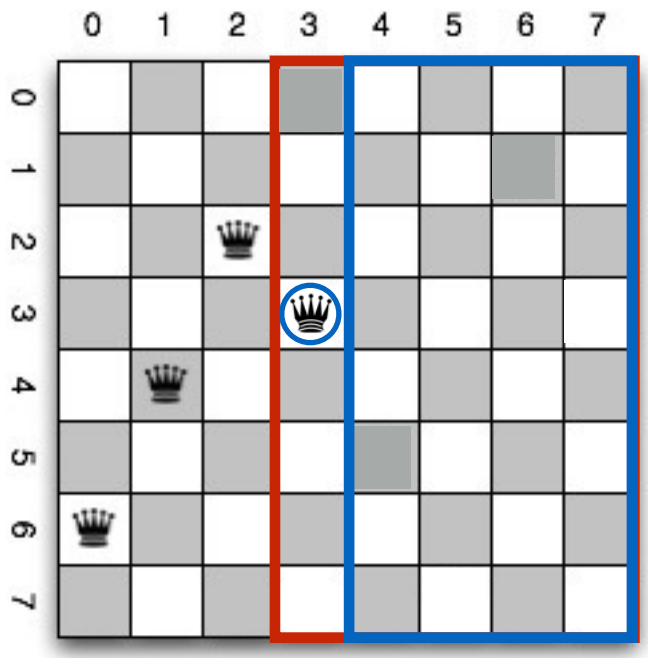

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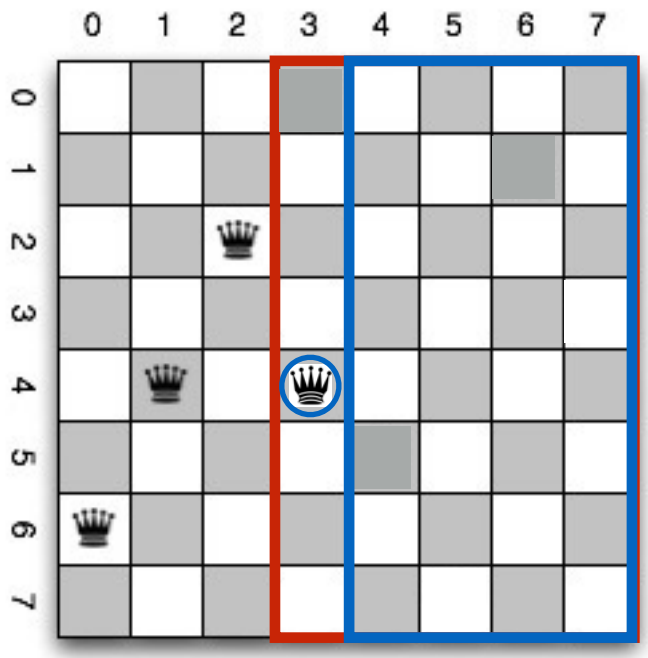

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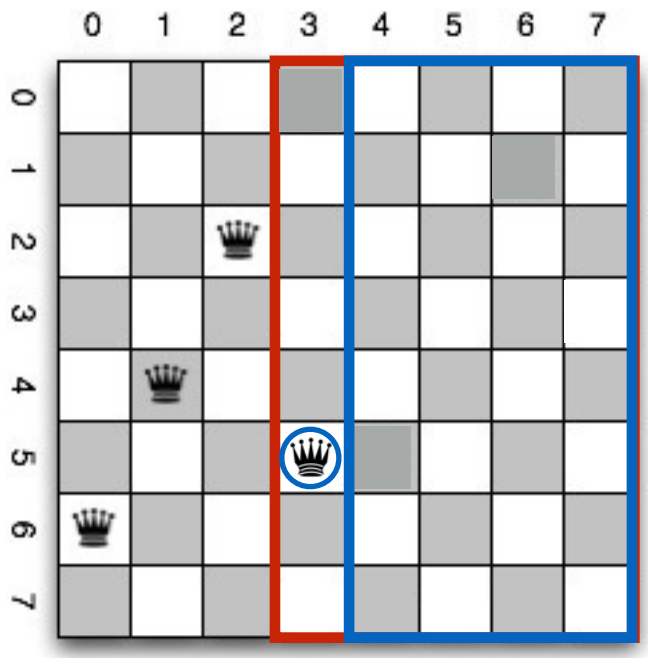

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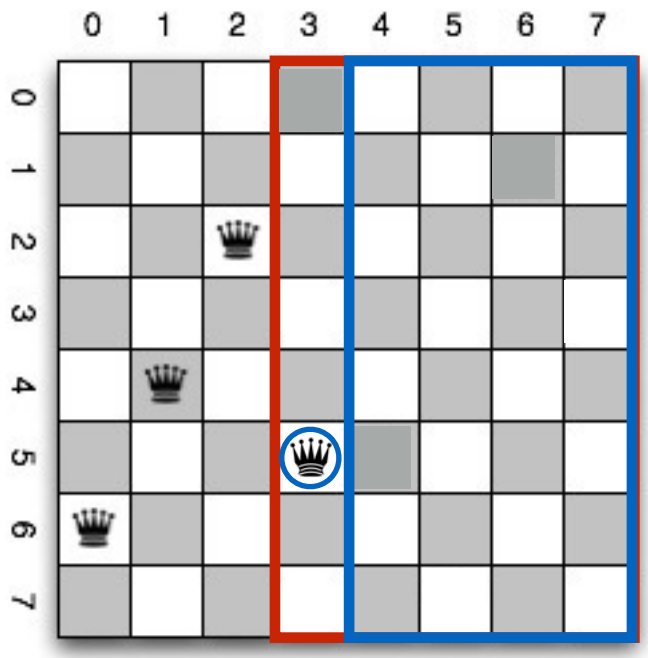

[0,?,?,?,?]

no solution

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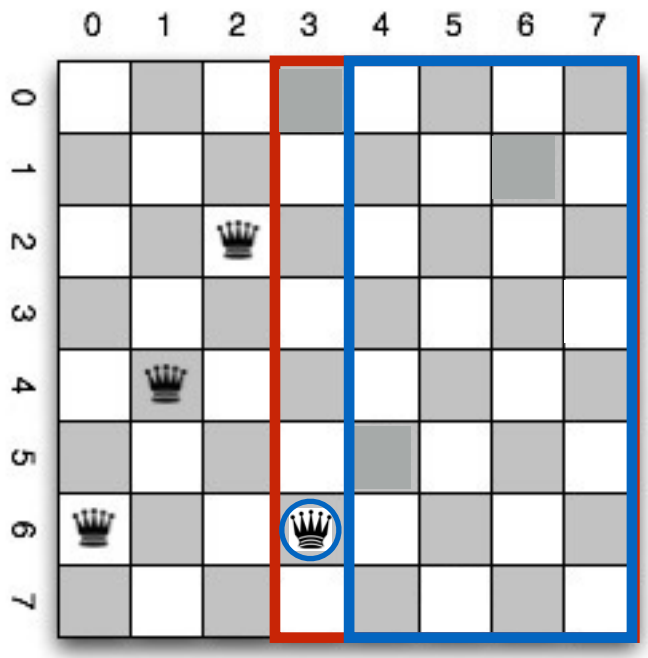
nQueens Problem



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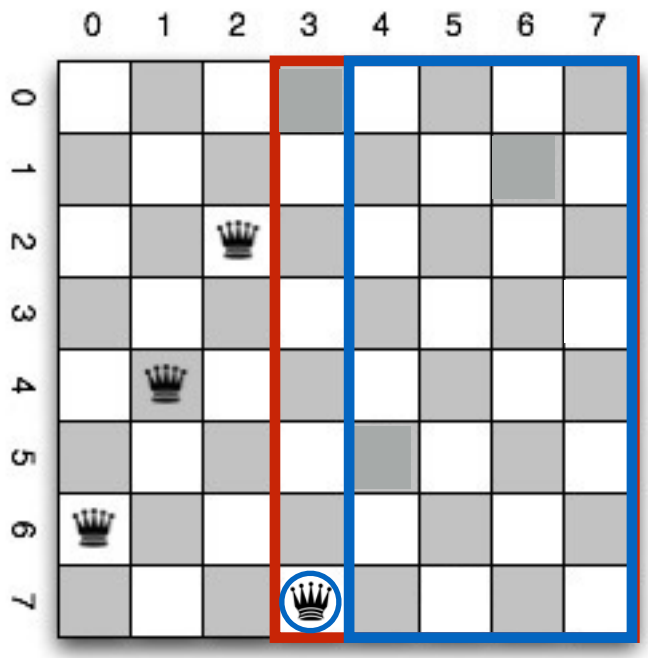

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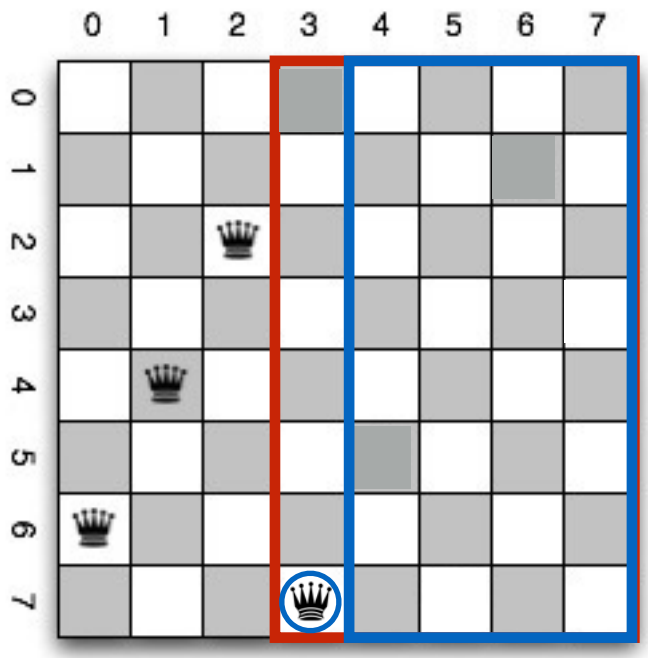

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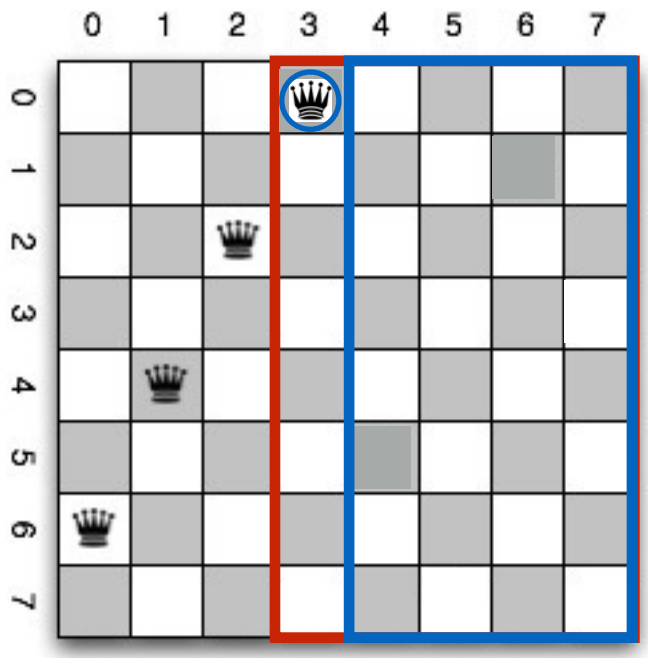


[0,?,?,?,?]

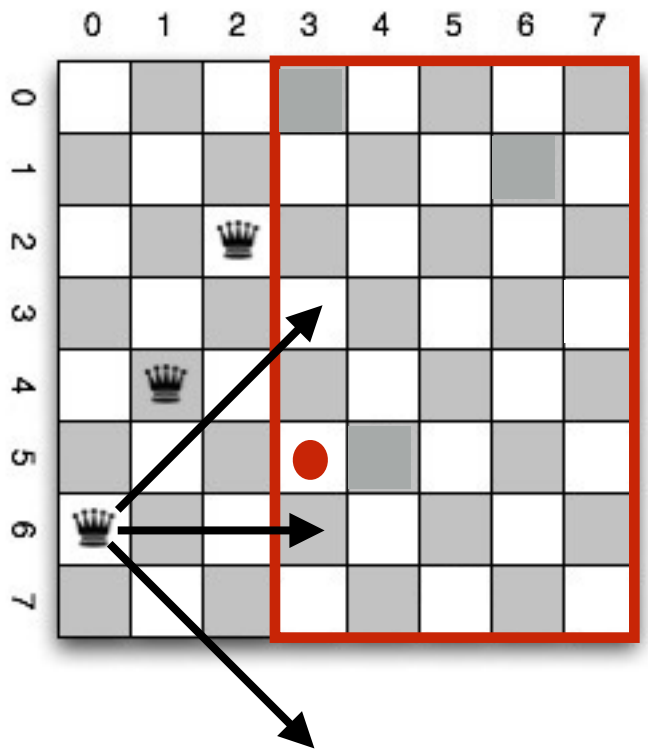
```
def solve(n, m, constraints):
    if(m == 0):
        return []
    for row in range(n):
        if (isLegal(row, constraints)):
            newConstraints = constraints + [row]
            result = solve(n, m-1, newConstraints)
            if (result != False):
                return [row] + result
    return False
```

n = 8

m = 5

constraints = [6,4,2]

nQueens Problem



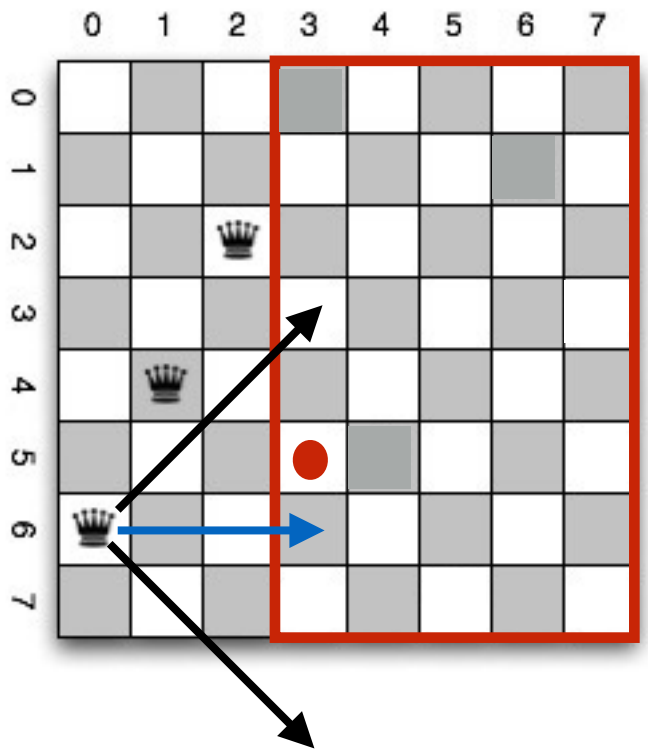

```
def isLegal(row, constraints):
    for ccol in range(len(constraints)):
        crow = constraints[ccol]
        shift = len(constraints) - ccol
        if ((row == crow) or
            (row == crow + shift) or
            (row == crow - shift)):
            return False
    return True
```

n = 8

m = 5

constraints = [6,4,2]

nQueens Problem

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            return False
    return True
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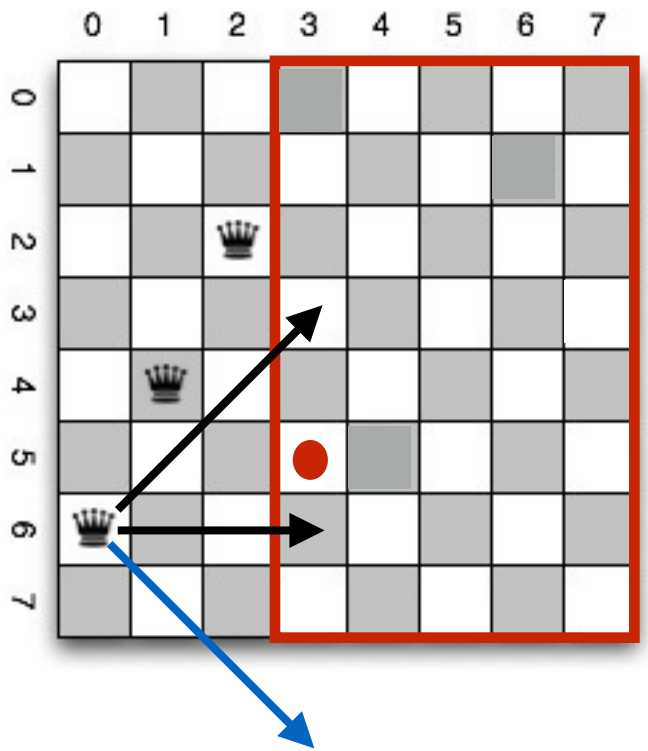
n = 8

m = 5

constraints = [6,4,2]

nQueens Problem



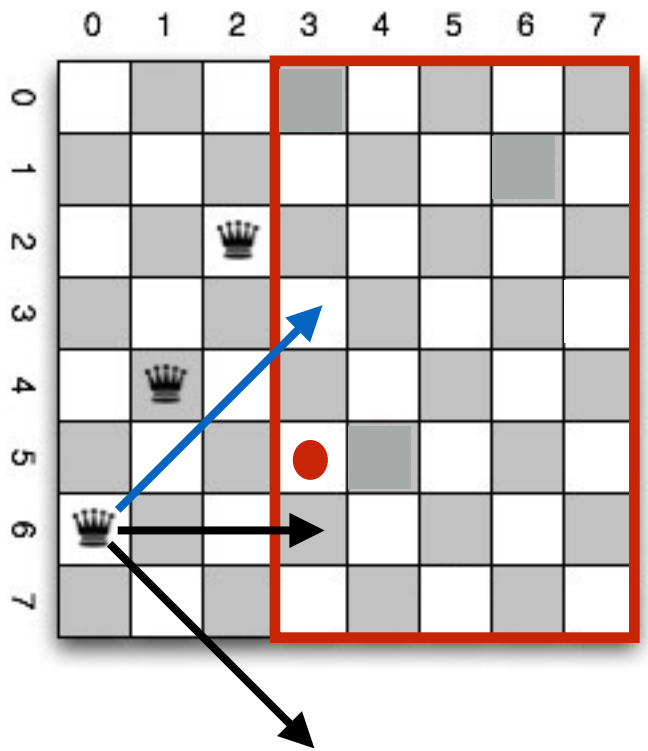

```
def isLegal(row, constraints):
    for ccol in range(len(constraints)):
        crow = constraints[ccol]
        shift = len(constraints) - ccol
        if ((row == crow) or
            (row == crow + shift) or
            (row == crow - shift)):
            return False
    return True
```

n = 8

m = 5

constraints = [6,4,2]

nQueens Problem



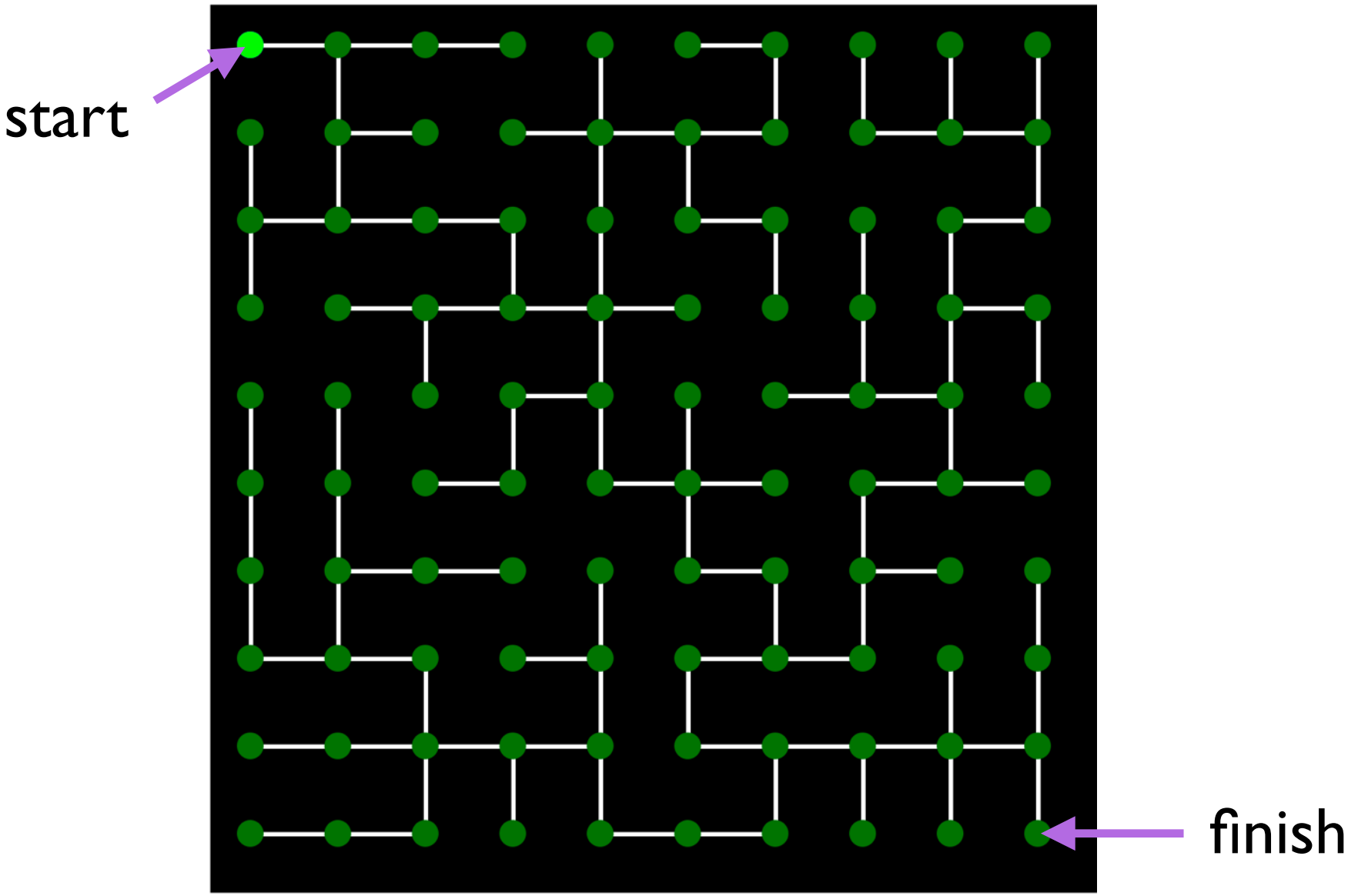

```
def isLegal(row, constraints):
    for ccol in range(len(constraints)):
        crow = constraints[ccol]
        shift = len(constraints) - ccol
        if ((row == crow) or
            (row == crow + shift) or
            (row == crow - shift)):
            return False
    return True
```

n = 8

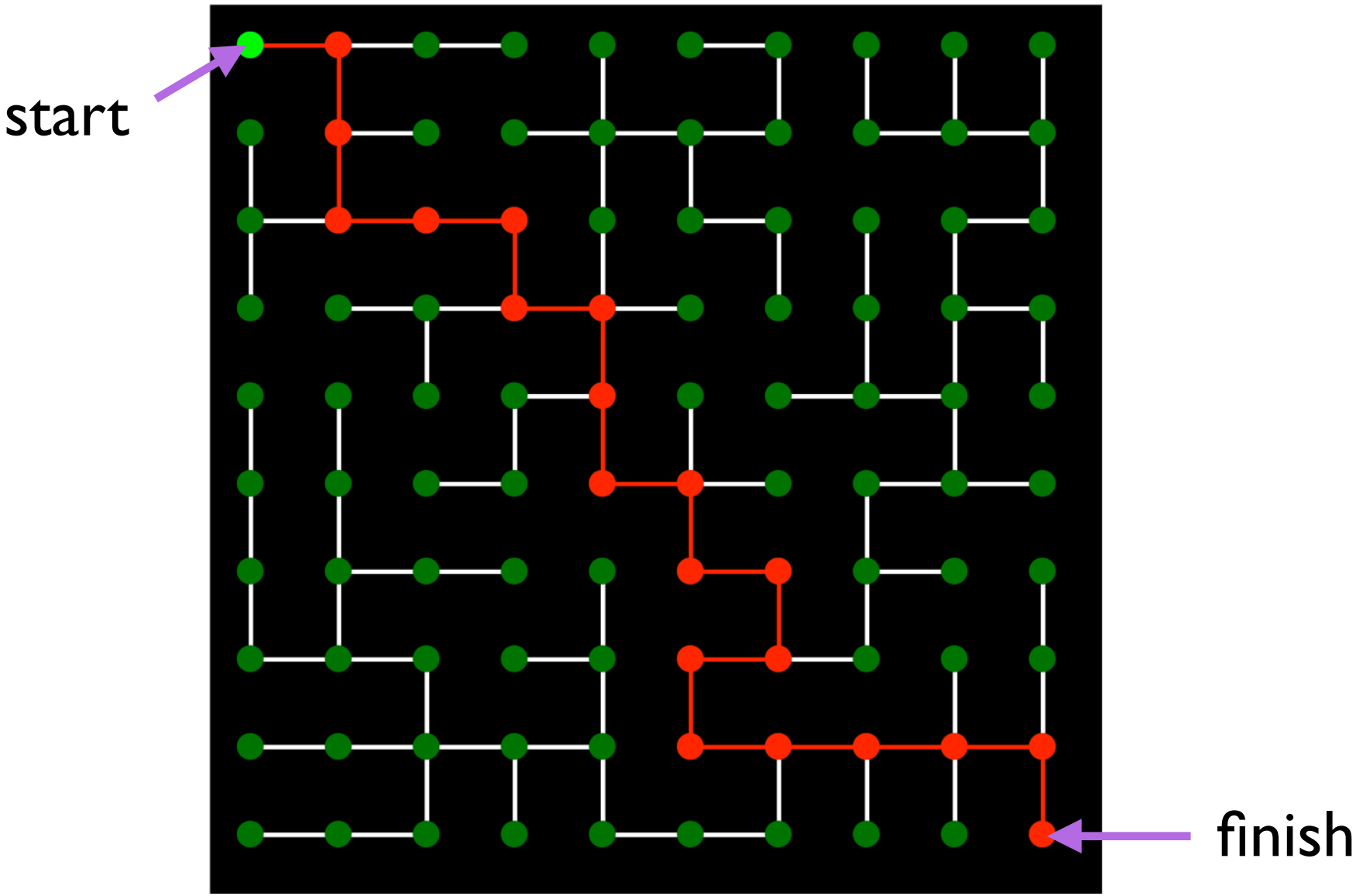
m = 5

constraints = [6,4,2]

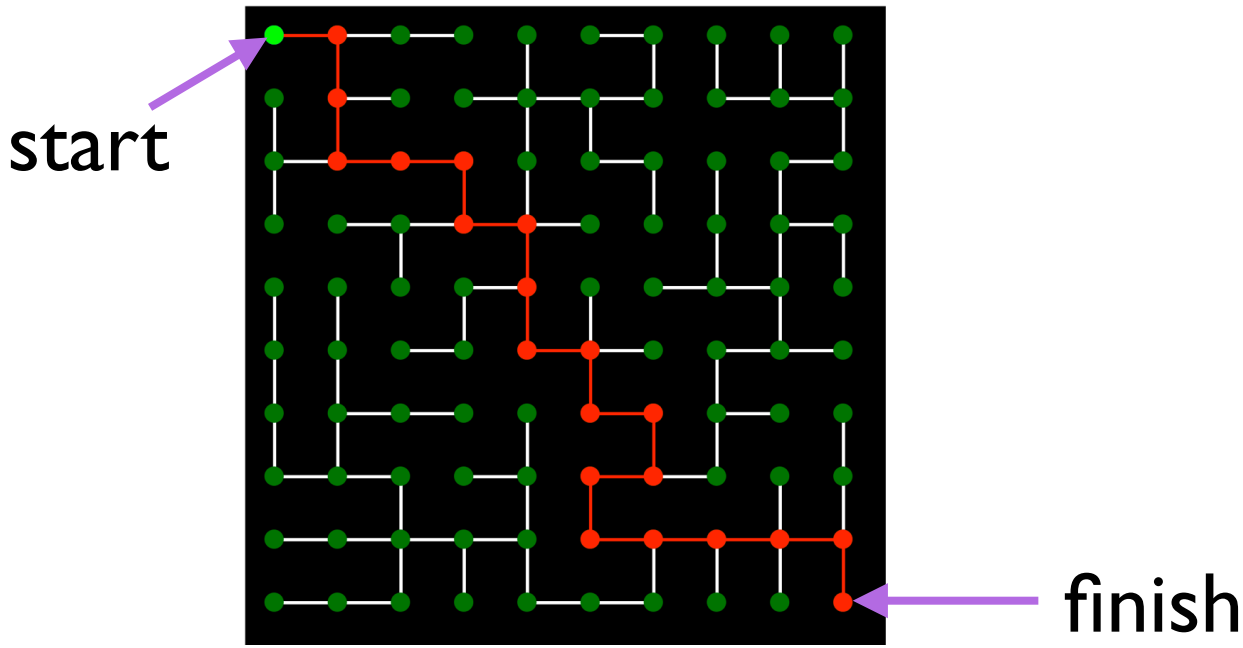
Solving a maze puzzle



Solving a maze puzzle



Solving a maze puzzle



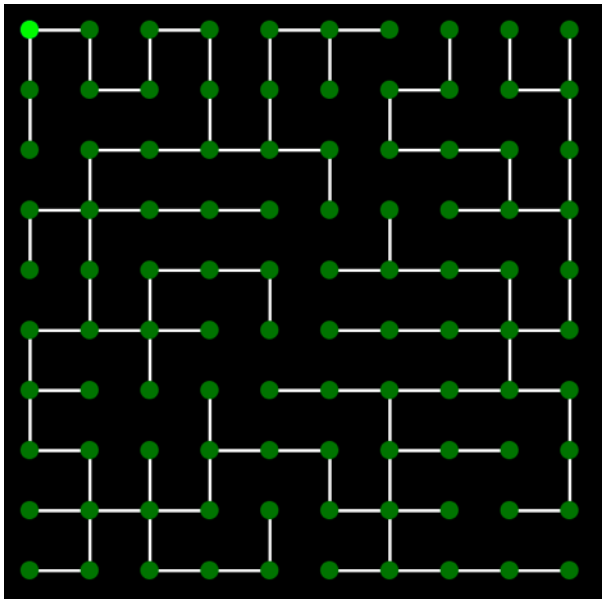
def isSolvable(maze, (rowStart, colStart), (rowEnd, colEnd)):
—> True or False

<u>Main Idea:</u>

if isSolvable(maze, (rowStart, colStart), (rowEnd, colEnd)),
then for some neighbor (rowN, colN) of (rowStart, colStart),
isSolvable(maze, (rowN, colN), (rowEnd, colEnd))

Solving a maze puzzle

```
def isSolvable(maze, (rowStart, colStart), (rowEnd, colEnd)):
    if ((rowStart, colStart) == (rowEnd, colEnd)):
        return True
                    U      D      R      L
    for dir in [(-1,0), (1,0), (0,1), (0,-1)]:
        newCell = (rowStart, colStart) + dir
        if (isLegal(maze, (rowStart, colStart), dir) and
            isSolvable(maze, newCell, (rowEnd, colEnd))):
            return True
    return False
```



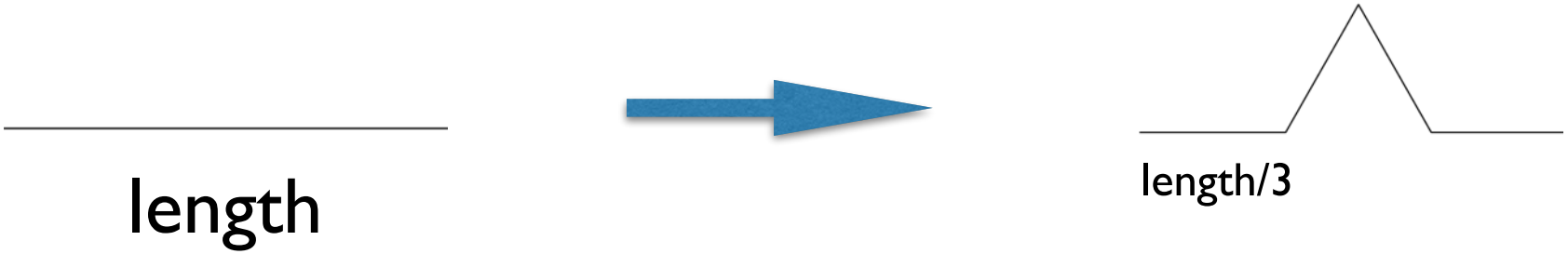
Where is the bug?

Fractals

Self similar image

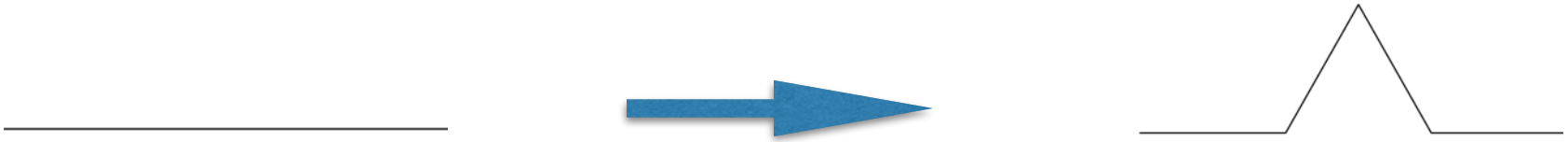
An image made up of smaller versions of itself.

A change rule:



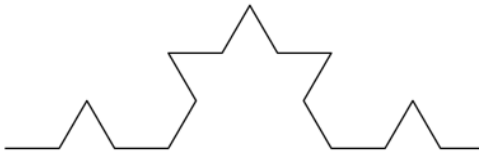
Fractals

n = 1

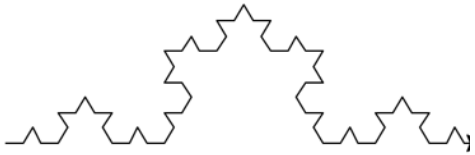


n = 2

n = 3



n = 4



```
def kN(length, n):
    if (n == 1):
        turtle.forward(length)
    else:
        kN(length/3.0, n-1)
        turtle.left(60)
        kN(length/3.0, n-1)
        turtle.right(120)
        kN(length/3.0, n-1)
        turtle.left(60)
        kN(length/3.0, n-1)
```