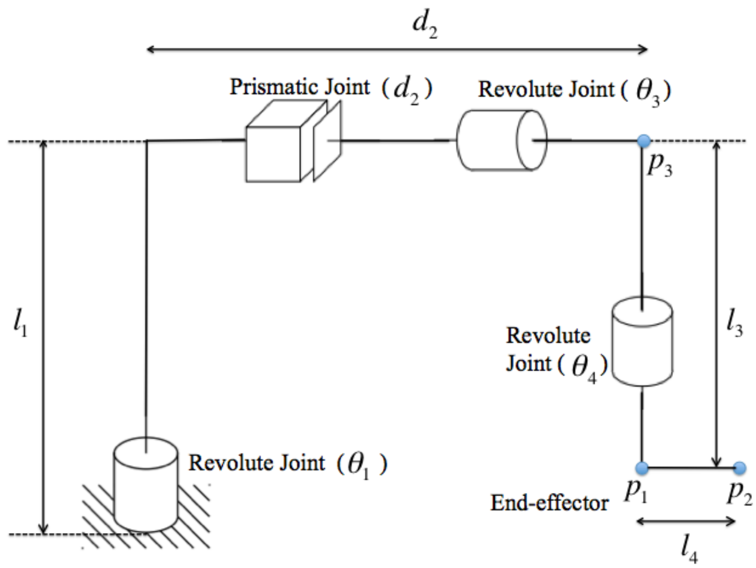
Additional Problems for Practice

1. Consider the RPRR (revolute-prismatic-revolute-revolute) robot manipulator shown in the figure below. Here, you can use indicated distance d_2 as the joint parameter for the prismatic joint. l_1, l_3, and l_4 are constant lengths. Consider the point p_1 to be the end-effector location. Find the Jacobian for this manipulator (find both the linear velocity Jacobian J_v and the angular velocity Jacobian J_ω).



2. Consider a revolute-revolute (RR) robot manipulator with the D-H parameters as shown below:

Link	a_i	α_i	d_i	θ_i
1	0	90^o	d_1	θ_1^*
2	l_2	0	0	θ_2^*

The end-effector is at the origin of the coordinate frame (x_2, y_2, z_2). The axis z_0 is along the direction of gravity. Find the Jacobian for this robot manipulator (find both the linear velocity Jacobian J_v and the angular velocity Jacobian J_ω).

3. Consider the robot manipulator in the previous question. Assume that the entire mass of each link is a point located at the end of the link.
   (a) Find the dynamics of the robot manipulator using the Euler-Lagrange method.
   (b) If the end-effector has a general force/torque on it (from the environment), write the dynamics including this force/torque.

4. Consider the robot manipulator in the previous question. Write the dynamics of the manipulator using the Newton-Euler method.

5. Consider the robot manipulator in the previous question.
   (a) Write a proportional-derivative (PD) controller (independent joint control) for each joint in the manipulator.
   (b) Write the inverse dynamics controller for the manipulator.

6. Consider a rigid body undergoing a purely rotational motion (no translation of the center of mass) with no external forces/torques acting on it. Denoting the angular velocity of the rigid body to be $\omega = [\omega_x, \omega_y, \omega_z]^T$ and considering the body-fixed coordinate frame to be aligned with the principal axes of the rigid body (i.e., a coordinate frame in which the inertia matrix is a diagonal matrix), the kinetic energy can be written in the form

$$K = \frac{1}{2}(I_{xx}\omega_x^2 + I_{yy}\omega_y^2 + I_{zz}\omega_z^2).$$

Consider the generalized coordinates (degrees of freedom) of this system to be the Euler angles ϕ, θ, ψ, corresponding to the rotation matrix $R = R_{z,\phi}R_{y,\theta}R_{z,\psi}$. Hence, define $q = [\phi, \theta, \psi]^T$.

   (a) By differentiating R with respect to time, write the angular velocity ω in the form $\omega = J(q)\dot{q}$ with $J(q)$ being a 3×3 matrix.

   (b) Using $\omega = J(q)\dot{q}$, write the kinetic energy as a function of q and $\dot{q}$.

   (c) Now, explain how you would obtain the dynamics of the rotating rigid body using the Euler-Lagrange method. *(Note: You do not need to actually derive the dynamics; just explain the technique).*

7. Consider a revolute-revolute (RR) manipulator shown below. The end effector of this manipulator is required to move along a wall as shown in the figure. Consider that a controller has been implemented for the manipulator to make its effective dynamics have a stiffness (in task space, i.e., in terms of end effector generalized position) of 50 N/m. Also, assume that the wall has a stiffness of 1000 N/m and that we want to exert a force of 25 N on the wall. Find the offset that we should choose for the virtual trajectory (i.e., how much inside the wall should we set the virtual trajectory?).