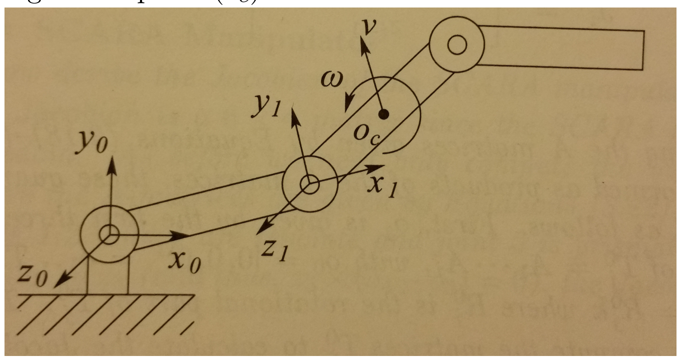
Solutions for Problems 4.16 and 4.17

4.16. We are given the three-link planar manipulator shown below and asked to find the Jacobian matrix corresponding to the point (o_c) at the center of link 2.



Denote a_1 to be the length of the first link and a_c to be the distance between the second joint and the point o_c. Denoting the origins (numerically written relative to the base frame, i.e., frame 0) of frames 0 and 1 by o_0 and o_1, we have $o_0 = [0,0,0]^T$ and $o_1 = [a_1c_1, a_1s_1, 0]^T$. We also have $o_c = [a_1c_1 + a_cc_{12}, a_1s_1 + a_cs_{12}, 0]^T$. Also, the z-axes of frames 0 and 1 are (when numerically written relative to the base frame, i.e., frame 0)we have $z_0 = [0,0,1]^T$ and $z_1 = [0,0,1]^T$. Since when writing the Jacobian matrix for the point o_c, we only have two revolute joints that are relevant (since the third joint will not kinematically influence the velocity of the point o_c) , the Jacobian matrix is (where the third column with all zeros corresponds to the third joint which is not kinematically relevant for the velocity of point o_c)

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} z_0 \times (o_c - o_0) & z_1 \times (o_c - o_1) & 0 \\ z_0 & z_1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_1s_1 - a_cs_{12} & -a_cs_{12} & 0 \\ a_1c_1 + a_cc_{12} & a_cc_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \tag{1}$$

4.17. We are given a three-link manipulator and asked to find the linear velocity Jacobian. Since all the three joints are revolute, the linear velocity Jacobian is

$$J_v = J_{11} = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \times (o_3 - o_2) \end{bmatrix} \tag{2}$$

Here, we have

$$\begin{aligned}
z_0 &= [0,0,1]^T \\
z_1 = z_2 &= [s_1, -c_1, 0]^T \\
o_0 &= [0,0,0]^T \\
o_1 &= [0,0,d_1]^T \\
o_2 &= [a_2c_1c_2\ ,\ a_2s_1c_2\ ,\ a_2s_2 + d_1]^T \\
o_3 &= [a_2c_1c_2 + a_3c_1c_{23}\ ,\ a_2s_1c_2 + a_3s_1c_{23}\ ,\ a_2s_2 + a_3s_{23} + d_1]^T
\end{aligned} \tag{3}$$

Hence, we get

$$J_{11} = \begin{bmatrix} -a_2s_1c_2 - a_3s_1c_{23} & -a_2s_2c_1 - a_3s_{23}c_1 & -a_3c_1s_{23} \\ a_2c_1c_2 + a_3c_1c_{23} & -a_2s_1s_2 - a_3s_1s_{23} & -a_3s_1s_{23} \\ 0 & a_2c_2 + a_3c_{23} & a_3c_{23} \end{bmatrix}$$

Computing the determinant of this matrix, we get

$$\det J_{11} = -a_2a_3s_3(a_2c_2 + a_3c_{23})$$