3. Identity element $I = 1 + j0 = 1e^{j0}$
   For all $c \in \mathbb{C}$,

$$cI = c = Ic.$$

4. Inverse element
   For all $c_1 \in \mathbb{C}$, let inverse $c_2 \in \mathbb{C}$ be defined as $c_2 = \frac{1}{m_1} e^{-j\theta_1}$.

$$c_1 c_2 = m_1 \frac{1}{m_1} e^{j\theta_1} e^{-j\theta_1} = c_2 c_1 = 1e^{j0} = I$$

2-28 Quaternion $Q = q_o + iq_1 + jq_2 + kq_3 = (q_0, q_1, q_2, q_3)$
$R_{k,\theta} \to Q = (\cos\frac{\theta}{2}, n_x \sin\frac{\theta}{2}, n_y \sin\frac{\theta}{2}, n_z \sin\frac{\theta}{2})$
Now, $\|k\| = \sqrt{n_x^2 + n_y^2 + n_z^2} = 1$ because $k = [n_x n_y n_z]^T$ is a unit vector.

$$\begin{aligned} \|Q\| &= \sqrt{\cos^2\frac{\theta}{2} + (n_x^2 + n_y^2 + n_z^2)\sin^2\frac{\theta}{2}} \\ &= \sqrt{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}} \\ &= \sqrt{1} = 1 \end{aligned}$$

2-29 $Q = (q_0, q_1, q_2, q_3) = (\cos\frac{\theta}{2}, n_x \sin\frac{\theta}{2}, n_y \sin\frac{\theta}{2}, n_z \sin\frac{\theta}{2})$.
Find rotation matrix $R_{k,\theta} \Rightarrow$ find k, θ.

1. $\theta = \cos^{-1}(2q_0)$
2. $k = [n_x, n_y, n_z]^T = \left[\frac{q_1}{\sin\frac{\theta}{2}}, \frac{q_2}{\sin\frac{\theta}{2}}, \frac{q_3}{\sin\frac{\theta}{2}}\right]^T$
3. Substitute values for k, θ into

$$R_{k,\theta} = \begin{bmatrix} k_x^2 v_\theta + c_\theta & k_x k_y v_\theta - k_z s_\theta & k_x k_z v_\theta + k_y s_\theta \\ k_x k_y v_\theta + k_z s_\theta & k_y^2 v_\theta + c_\theta & k_y k_z v_\theta - k_x s_\theta \\ k_x k_z v_\theta - k_y s_\theta & k_y k_z v_\theta + k_x s_\theta & k_z^2 v_\theta + c_\theta \end{bmatrix}$$

   where v_θ =vers$\theta = 1 - c_\theta$.

2-30 Given R, find $Q = (q_0, q_1, q_2, q_3)$.

$$\theta = \cos^{-1}\left[\frac{Tr(R) - 1}{2}\right]$$

$$k = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \frac{1}{2\sin\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

If $\|k\| \neq 1$, then $k' = \frac{k}{\|k\|}$.
$q_0 = \cos\frac{\theta}{2}, q_1 = n_x \sin\frac{\theta}{2}, q_2 = n_y \sin\frac{\theta}{2}, q_3 = n_z \sin\frac{\theta}{2}$