# EL5223 Homework 1 Solutions

1. (a) With $\theta_2$ measured relative to the horizontal direction (i.e., at $\theta_2 = 0$, the second link is parallel to the ground), we find the forward kinematics for the given robotic manipulator to be

$$x = D\cos(\theta_2) \tag{1}$$

$$y = d_1 - D\sin(\theta_2). \tag{2}$$

(b) To find the inverse kinematics, we need to solve for $(d_1, \theta_2)$ given $(x, y)$. Using equation (1), we get

$$\theta_2 = \text{acos}\Big(\frac{x}{D}\Big). \tag{3}$$

where acos denotes the inverse cosine. In general, we get two values for acos since $\cos(\theta_2) = \cos(-\theta_2)$. For either of these two values of $\theta_2$, we can find $d_1$ using (2) as:

$$d_1 = y + D\sin(\theta_2). \tag{4}$$

Hence, in general, we get two possible solutions for the inverse kinematics for this robotic manipulator. As discussed above, one solution would have the second link pointed downward (i.e., positive $\theta_2$ for this robotic manipulator) and the second solution would have the second link pointed upward (i.e., negative $\theta_2$ for this robotic manipulator).

(c) The velocity kinematics can be found directly from (1) and (2) as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & -D\sin(\theta_2) \\ 1 & -D\cos(\theta_2) \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{\theta}_2 \end{bmatrix}. \tag{5}$$

Hence, we can also write $\begin{bmatrix} \dot{d}_1 \\ \dot{\theta}_2 \end{bmatrix}$ in terms of $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$ as

$$\begin{bmatrix} \dot{d}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -\frac{\cos(\theta_2)}{\sin(\theta_2)} & 1 \\ -\frac{1}{D\sin(\theta_2)} & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}. \tag{6}$$