Solutions for Problem 5.19

5.19 Denoting the cubic polynomial by

$$q(t) = a_0 + a_1(t - t_0) + a_2(t - t_0)^2 + a_3(t - t_0)^3, \tag{1}$$

the given initial position (q_0), initial velocity (0 rad/sec), final position (q_1), and final velocity (1 rad/sec) at time ($t_0 + 2$) seconds give the conditions:

$$a_0 = q_0 \tag{2}$$

$$a_1 = 0 \tag{3}$$

$$a_0 + 2a_1 + 4a_2 + 8a_3 = q_1 \tag{4}$$

$$a_1 + 4a_2 + 12a_3 = 1. \tag{5}$$

Hence, we get

$$a_3 = \frac{1 - (q_1 - q_0)}{4} \tag{6}$$

$$a_2 = \frac{-2 + 3(q_1 - q_0)]}{4}. \tag{7}$$

Therefore, the cubic polynomial satisfying the given constraints is

$$q(t) = q_0 + \frac{-2 + 3(q_1 - q_0)]}{4}(t - t_0)^2 + \frac{1 - (q_1 - q_0)}{4}(t - t_0)^3. \tag{8}$$

The trajectory as a function of time for $q_0 = 0$ and $q_1 = 5$ is shown in the figure below.

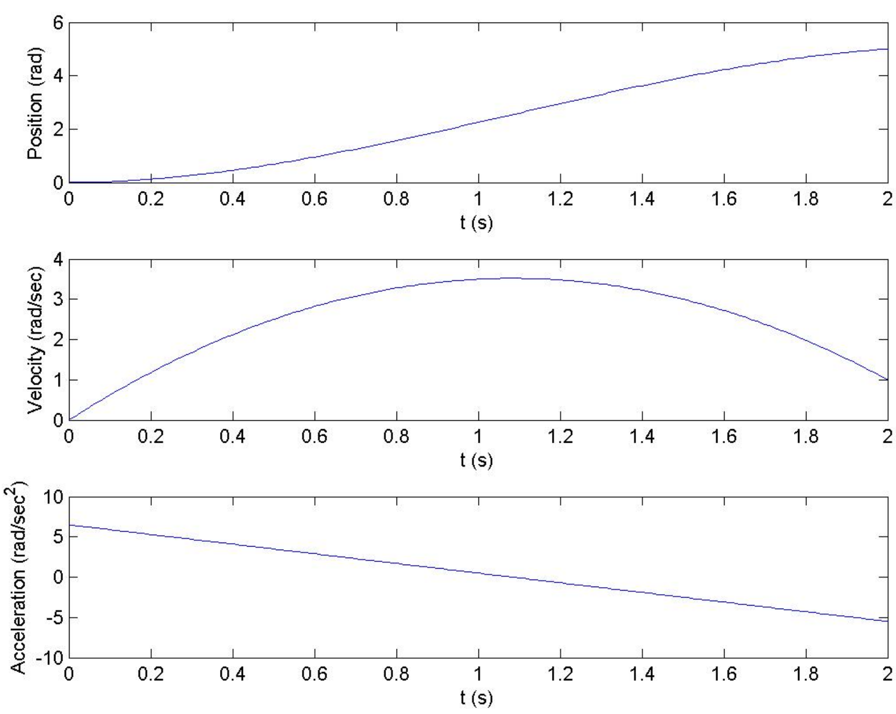

Figure 1: Cubic trajectory: position, velocity, and acceleration profiles.