

## Integration of $\dot{R} = RS(\omega)$

**Definition of quaternion  $q$  in terms of Euler angles  $\theta_x, \theta_y, \theta_z$ :**

$$q_0 = \cos\left(\frac{\theta_x}{2}\right) * \cos\left(\frac{\theta_y}{2}\right) * \cos\left(\frac{\theta_z}{2}\right) + \sin\left(\frac{\theta_x}{2}\right) * \sin\left(\frac{\theta_y}{2}\right) * \sin\left(\frac{\theta_z}{2}\right) \quad (1)$$

$$q_1 = \sin\left(\frac{\theta_x}{2}\right) * \cos\left(\frac{\theta_y}{2}\right) * \cos\left(\frac{\theta_z}{2}\right) - \cos\left(\frac{\theta_x}{2}\right) * \sin\left(\frac{\theta_y}{2}\right) * \sin\left(\frac{\theta_z}{2}\right) \quad (2)$$

$$q_2 = \cos\left(\frac{\theta_x}{2}\right) * \sin\left(\frac{\theta_y}{2}\right) * \cos\left(\frac{\theta_z}{2}\right) + \sin\left(\frac{\theta_x}{2}\right) * \cos\left(\frac{\theta_y}{2}\right) * \sin\left(\frac{\theta_z}{2}\right) \quad (3)$$

$$q_3 = \cos\left(\frac{\theta_x}{2}\right) * \cos\left(\frac{\theta_y}{2}\right) * \sin\left(\frac{\theta_z}{2}\right) - \sin\left(\frac{\theta_x}{2}\right) * \sin\left(\frac{\theta_y}{2}\right) * \cos\left(\frac{\theta_z}{2}\right) \quad (4)$$

**Euler angles in terms of  $q$ :**

$$\theta_x = \text{atan2}(2 * (q_2 * q_3 + q_0 * q_1), q_0 * q_0 - q_1 * q_1 - q_2 * q_2 + q_3 * q_3) \quad (5)$$

$$\theta_y = \text{asin}(-2 * (q_1 * q_3 - q_0 * q_2)) \quad (6)$$

$$\theta_z = \text{atan2}(2 * (q_1 * q_2 + q_0 * q_3), (q_0 * q_0 + q_1 * q_1 - q_2 * q_2 - q_3 * q_3)) \quad (7)$$

$$(8)$$

**Rotation matrix in terms of  $q$ :**

$$R = \begin{bmatrix} q_0 * q_0 + q_1 * q_1 - q_2 * q_2 - q_3 * q_3 & 2 * (q_1 * q_2 - q_0 * q_3) & 2 * (q_1 * q_3 + q_0 * q_2) \\ 2 * (q_1 * q_2 + q_0 * q_3) & q_0 * q_0 - q_1 * q_1 + q_2 * q_2 - q_3 * q_3 & 2 * (q_2 * q_3 - q_0 * q_1) \\ 2 * (q_1 * q_3 - q_0 * q_2) & 2 * (q_2 * q_3 + q_0 * q_1) & q_0 * q_0 - q_1 * q_1 - q_2 * q_2 + q_3 * q_3 \end{bmatrix}$$

**Integration in terms of  $q$ :**

$$\dot{q} = \frac{1}{2} \bar{\omega} q \quad (9)$$

$$= \frac{1}{2} \begin{bmatrix} -\omega_x * q_1 - \omega_y * q_2 - \omega_z * q_3 \\ \omega_x * q_0 + \omega_z * q_2 - \omega_y * q_3 \\ \omega_y * q_0 - \omega_z * q_1 + \omega_x * q_3 \\ \omega_z * q_0 + \omega_y * q_1 - \omega_x * q_2 \end{bmatrix} \quad (10)$$

where  $\bar{\omega} = [0, w_x, w_y, w_z]^T$