Automatic Addison

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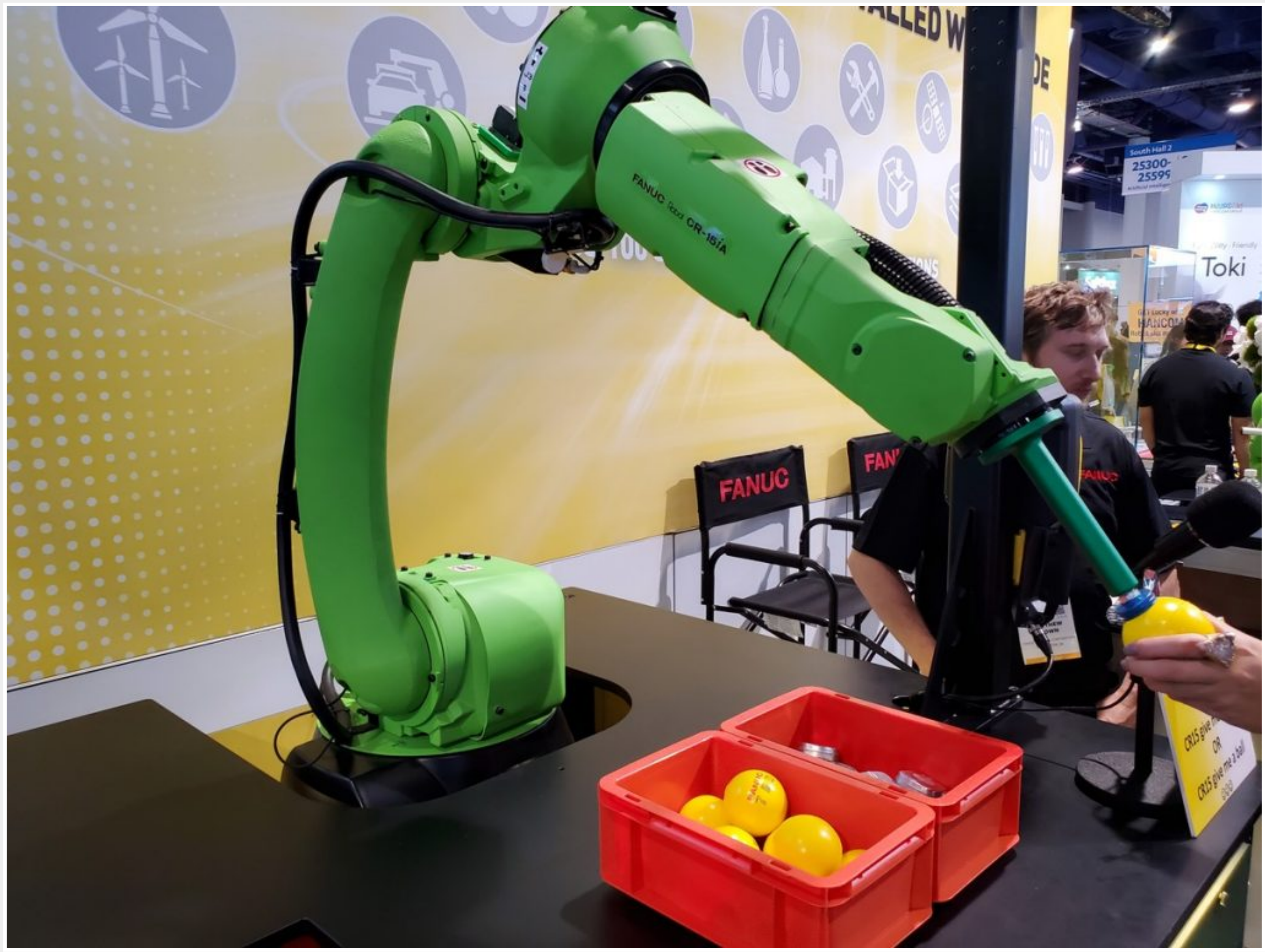
How to Find Denavit-Hartenberg Parameter Tables



In this section, we'll learn how to find the Denavit-Hartenberg Parameter table for robotic arms. This method is a **shortcut for finding homogeneous transformation matrices** and is commonly seen in documentation for industrial robots as well as in the research literature. The real-world example we'll consider in this tutorial is a SCARA robotic arm, like the one below.

Remember that homogeneous transformation matrices enable you to express the position and orientation of the end effector frame (e.g. robotics gripper, hand, vacuum suction cup, etc.) in terms of the base frame.

To calculate the homogeneous transformation matrix from the base frame to the end effector frame, the only values you need to have are the length of each link and the angle of each servo motor.

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The Three Steps

Here are the three steps for finding the Denavit-Hartenbeg parameter table and the homogeneous transformation matrices for a robotic manipulator:

1. Draw the kinematic diagram according to the four Denavit-Hartenberg rules.

2. Create the Denavit-Hartenberg parameter table.

- Number of Rows = Number of Frames – 1
- Number of Columns = 4: Two columns for rotation and two columns for displacement

3. Find the homogeneous transformation matrices (I'll cover how to do this step in my next post).

Definition of the Parameters

The Denavit-Hartenberg parameter tables consist of four variables:

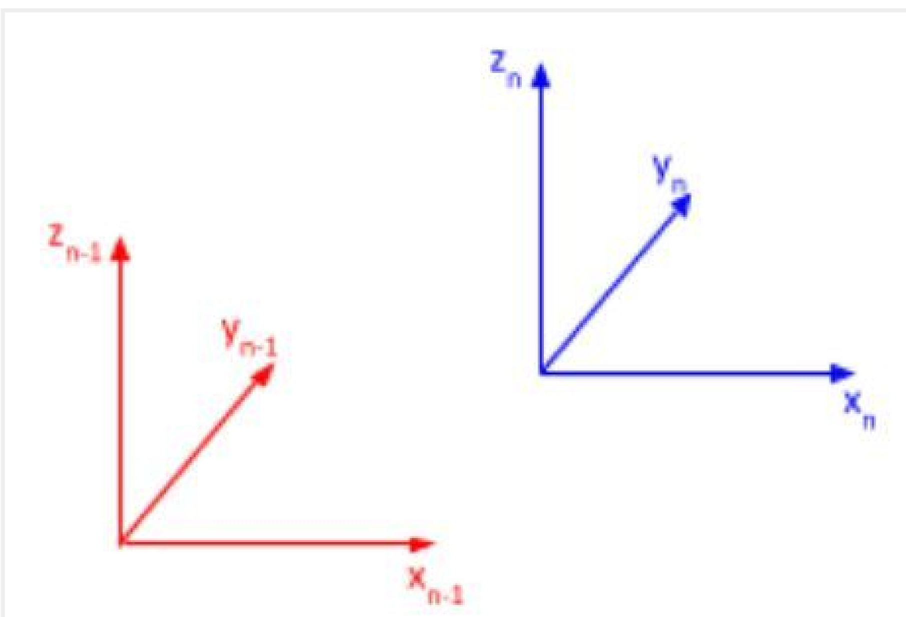
- The two variables used for rotation are **θ** and **α**.
- The two variables used for displacement are **r** and **d**.

Here is the D-H parameter table template for a robotic arm with four reference frames:

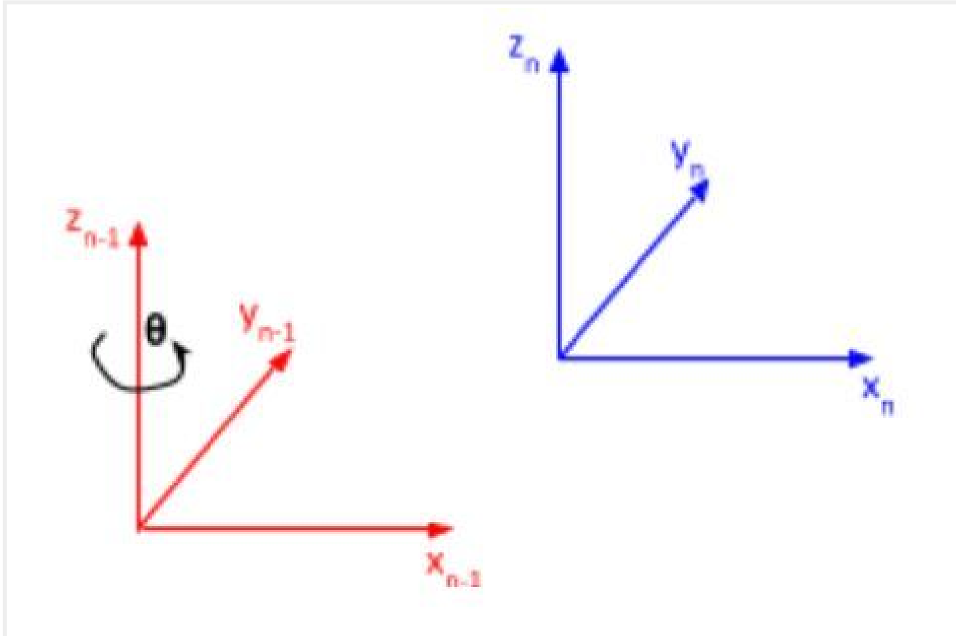
Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1				
2				
3				

Let's take a look at what these parameters mean by looking at two different frames. The n-1 frame is the frame before the n frame. We assume that both frames are connected by a link.

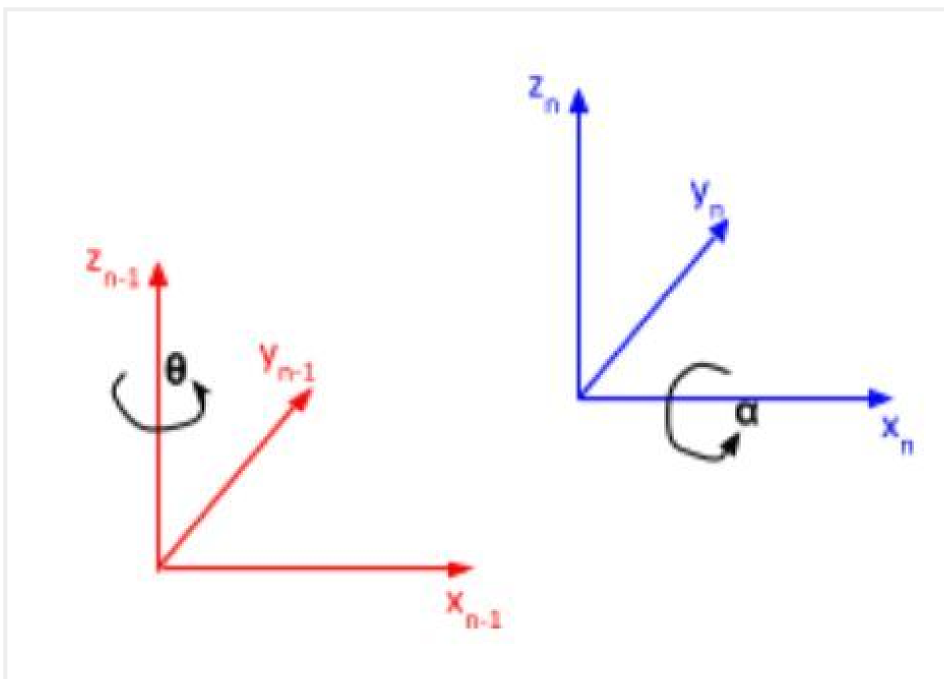
For example, if the n-1 frame is frame 2, the n frame is frame 3.



θ is the angle from x_{n-1} to x_n around z_{n-1}.

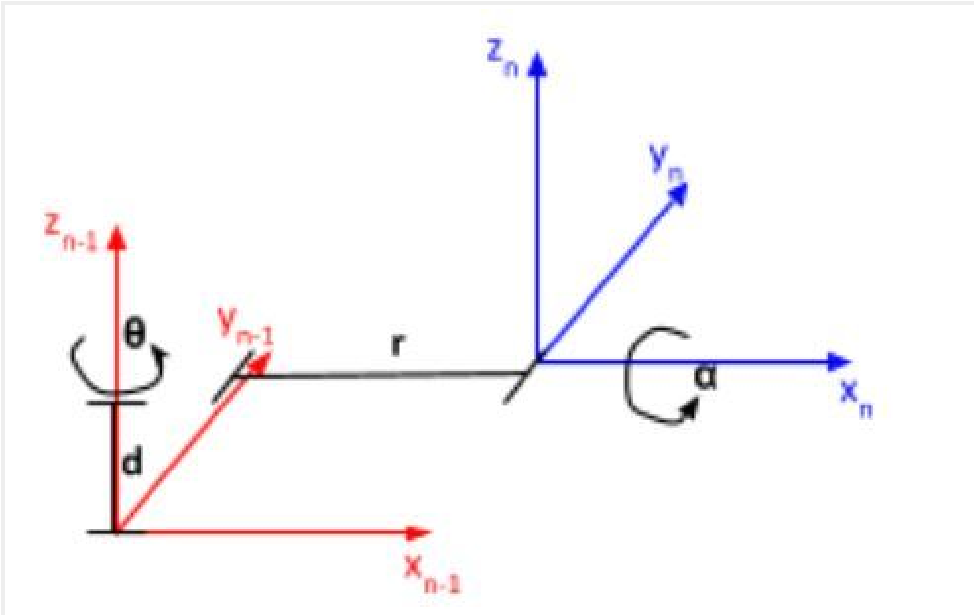


α is the angle from z_{n-1} to z_n around x_n.

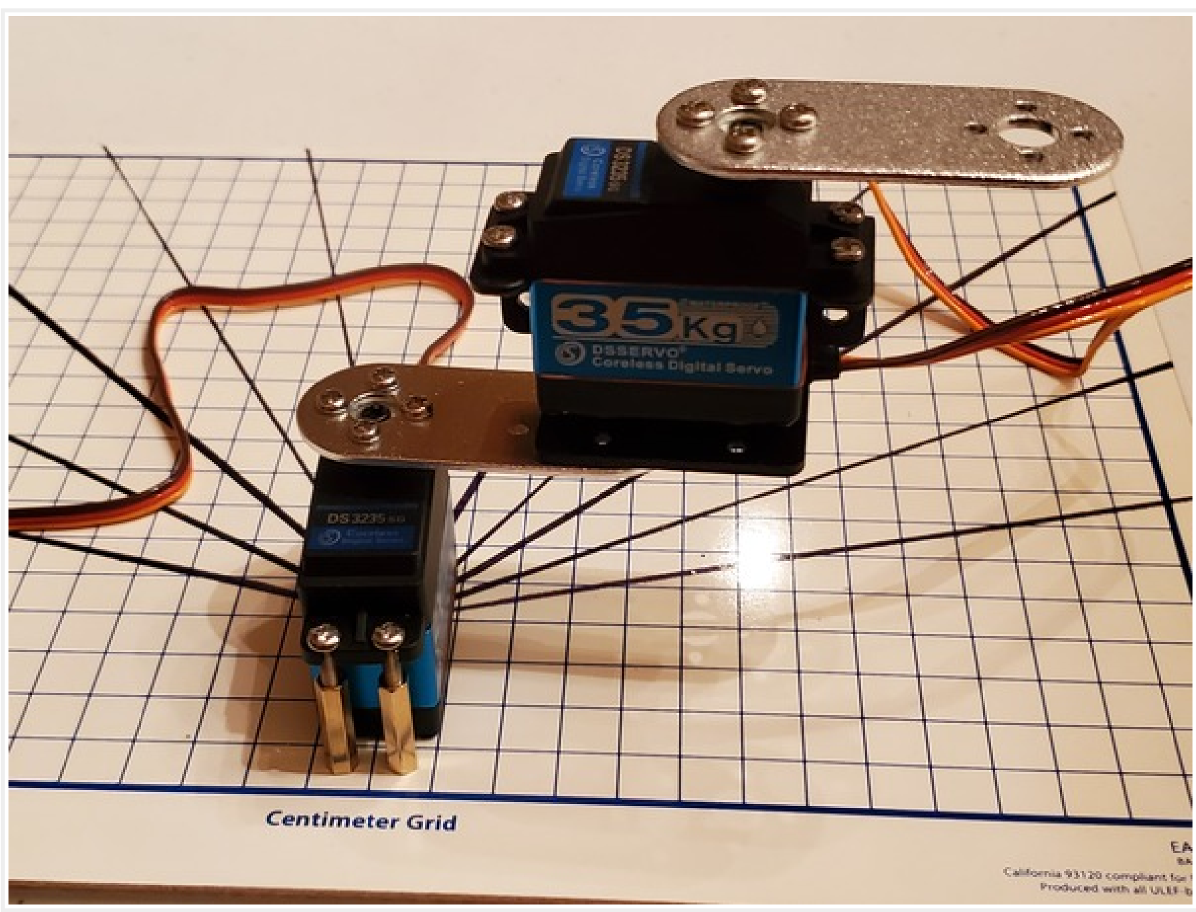


r (sometimes you'll see the letter 'a' instead of 'r') is the distance between the origin of the n-1 frame and the origin of the n frame along the x_n direction.

d is the distance from x_{n-1} to x_n along the z_{n-1} direction.



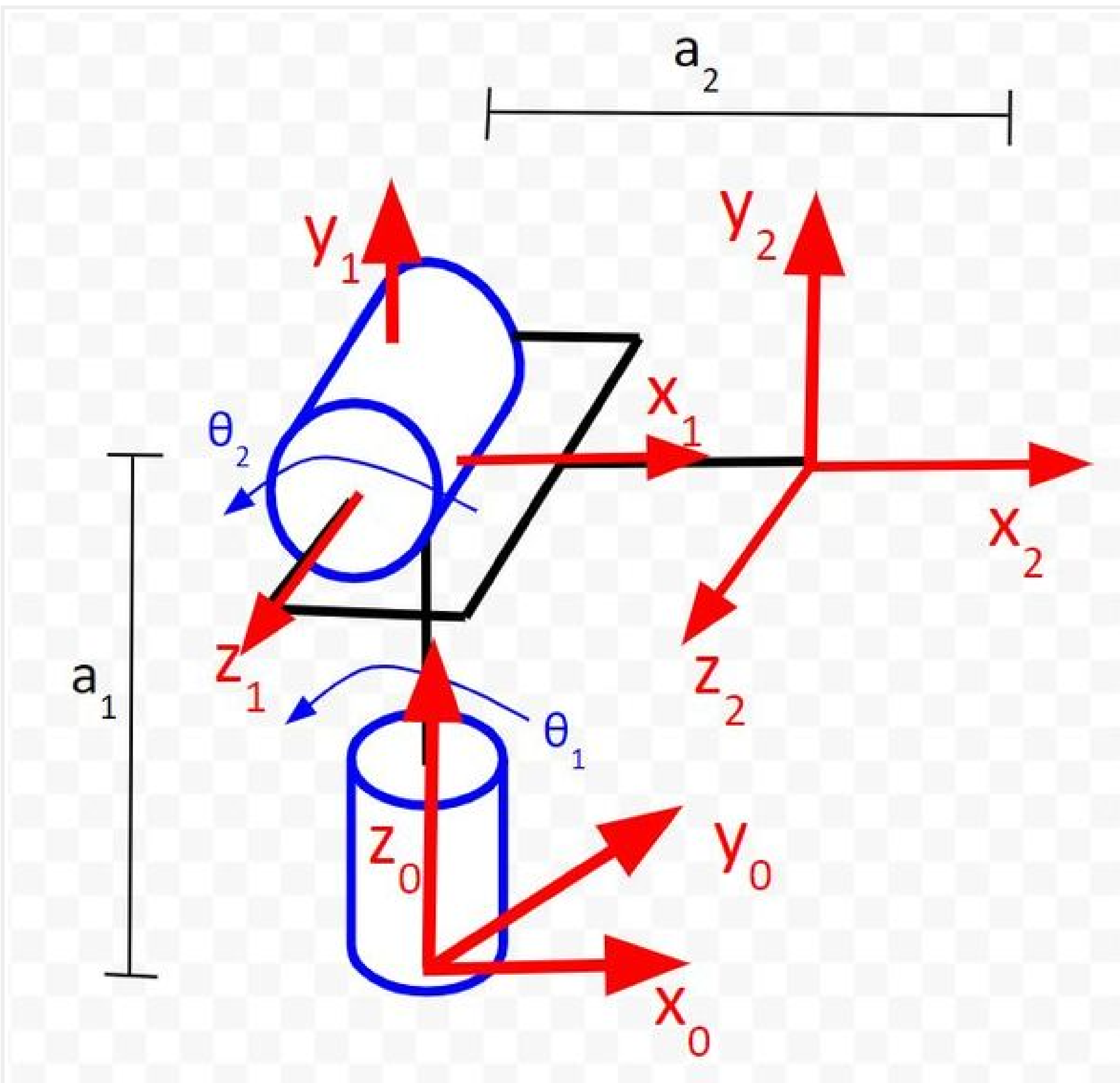
Example 1 – Two Degree of Freedom Robotic Arm



Let's take a look at an example so that we can walk through the process of creating and filling in Denavit-Hartenberg parameter tables.

Draw the Kinematic Diagram According to the Denavit-Hartenberg Rules

We'll start by drawing the kinematic diagram of a two degree of freedom robotic arm.



Create the Denavit-Hartenberg Parameter Table

Now, we need to find the Denavit-Hartenberg parameters. We have three coordinate frames here, so we need to have two rows in our D-H table (i.e. number of rows = number of frames – 1).

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1				
2				

Find θ

For the Joint 1 (Servo 0) row, we are going to focus on the relationship between frame 0 and frame 1. **θ is the angle from x_0 to x_1 around z_0.**

If you look at the diagram, x_0 and x_1 both point in the same direction. The axes are therefore aligned. When the robot is in motion, θ_1 will change (which will cause frame 1 to move relative to frame 0). The angle from x_0 to x_1 around z_0 will be θ_1, so let's put that in our table.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1			
2				

Now, let's look at the Joint 2 (Servo 1) row. We take a look at the rotation between frame 1 and frame 2. **θ is the angle from x_1 to x_2 around z_1.**

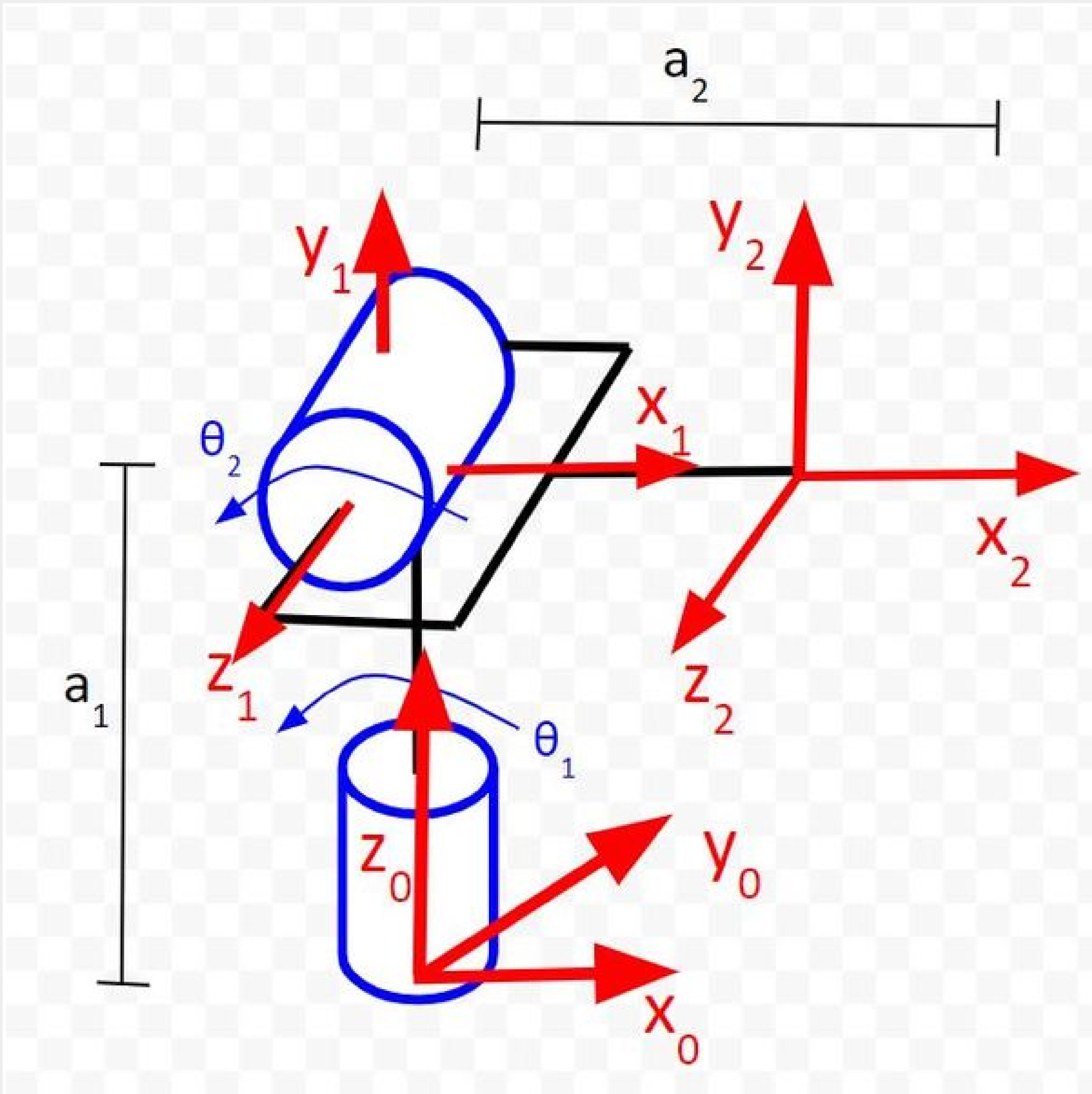
If you look at the diagram, x_1 and x_2 both point in the same direction. The axes are therefore aligned. When the robot is in motion, θ_2 will change (which will cause frame 2 to move). The angle from x_1 to x_2 around z_1 will be θ_2, so let's put that in the second row of our table.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1			
2	θ_2			

Find α

Let's start with the Joint 1 (Servo 0) row of the table.

α is the angle from z_0 to z_1 around x_1.



In the diagram above, you can see that this angle is 90 degrees, so we put 90 in the table.

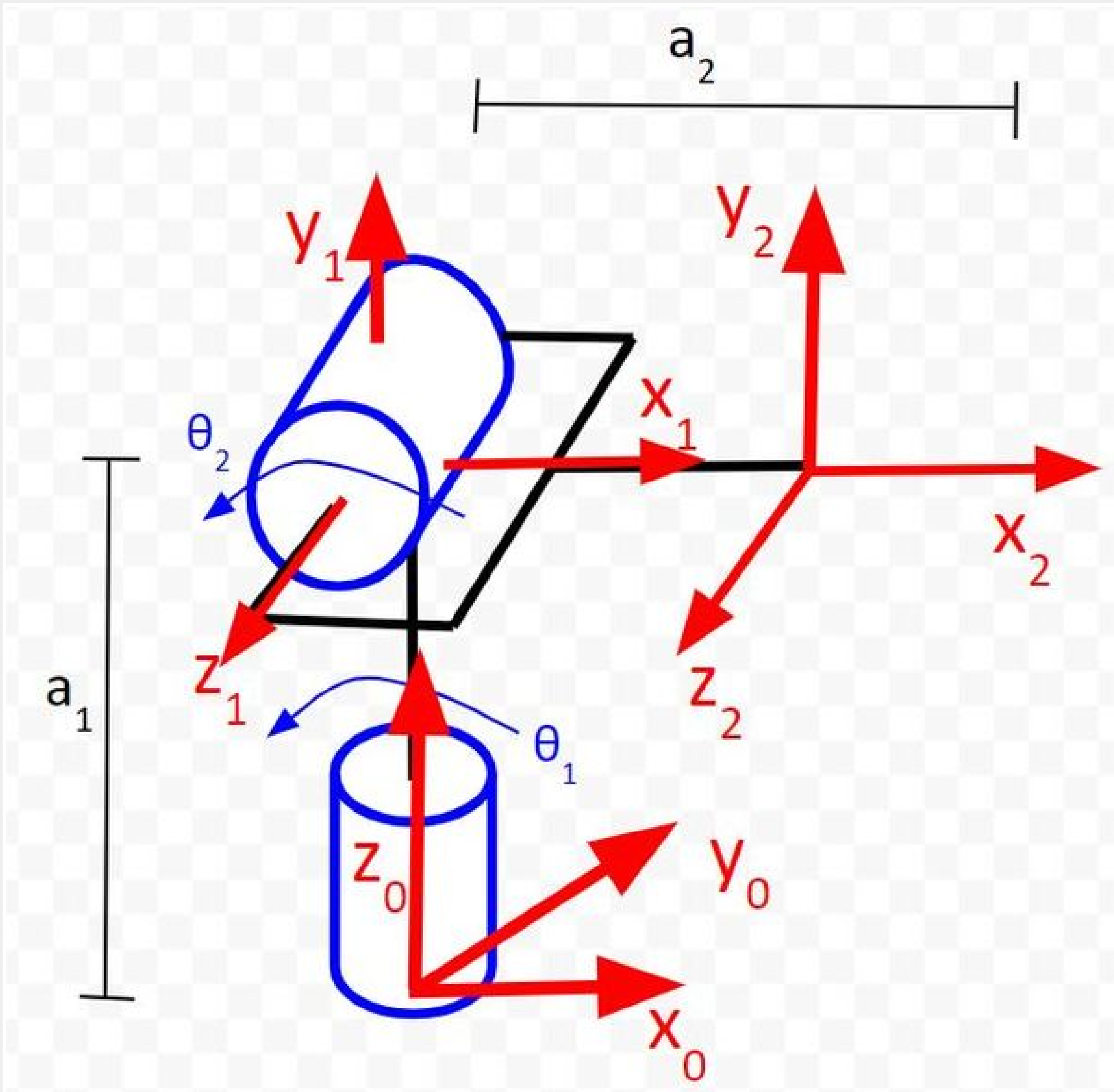
Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	90		
2	θ_2			

Let's go to the Joint 2 row of the table.

α is the angle from z_1 to z_2 around x_2. You can see that no matter what happens to θ_2, the angle from z_1 to z_2 will be 0 (since both axes point in the same direction). We put 0 degrees into the table.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	90		
2	θ_2	0		

Find r



Let's start with the Joint 1 row of the table.

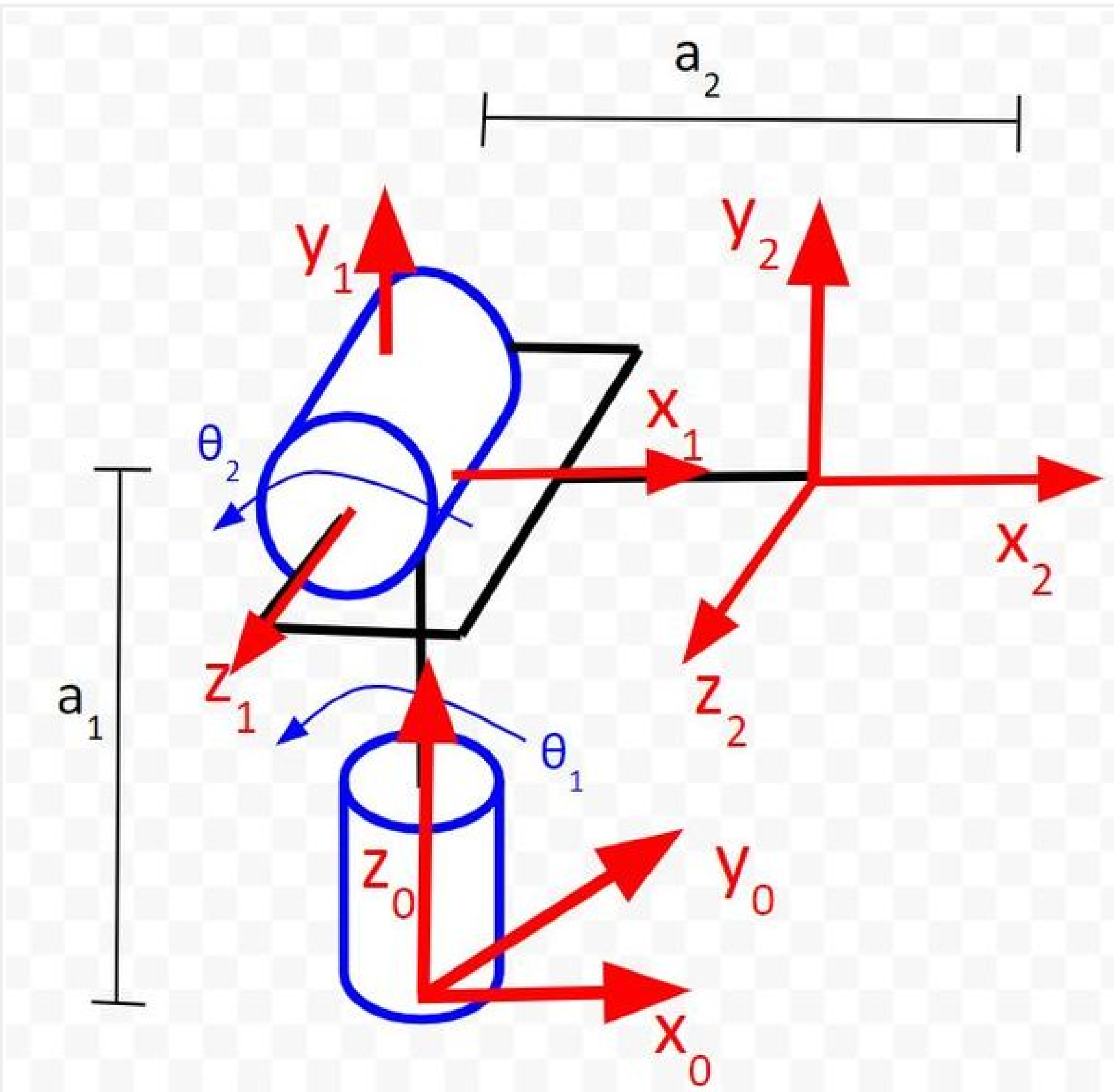
r is the distance between the origin of frame 0 and the origin of frame 1 along the x_1 direction. You can see in the diagram that this distance is 0.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	90	0	
2	θ_2	0		

Now let's look at the Joint 2 row. r is the distance between the origin of frame 1 and the origin of frame 2 along the x_2 direction. You can see in the diagram that this distance is a_2.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	90	0	
2	θ_2	0	a_2	

Find d



Now let's find d. We'll start on the first row of the table as usual.

d is the distance from x_0 to x_1 along the z_0 direction. You can see that this distance is a_1.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	90	0	a_1
2	θ_2	0	a_2	0

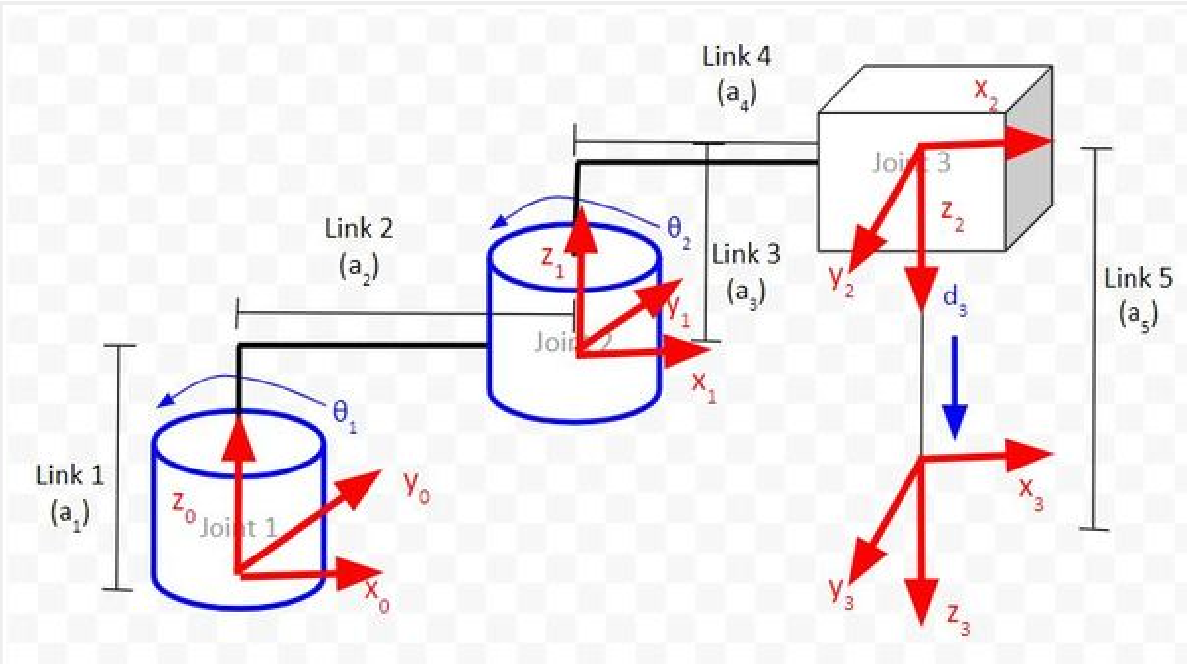
Now let's take a look at frame 1 to frame 2. d is the distance from x_1 to x_2 along the z_1 direction. You can see that this distance is 0.

Example 2 – SCARA Robot

Let's get some more practice filling in D-H parameter tables by looking at the SCARA robot.

Draw the Kinematic Diagram According to the Denavit-Hartenberg Rules

Here is the kinematic diagram using the D-H convention.

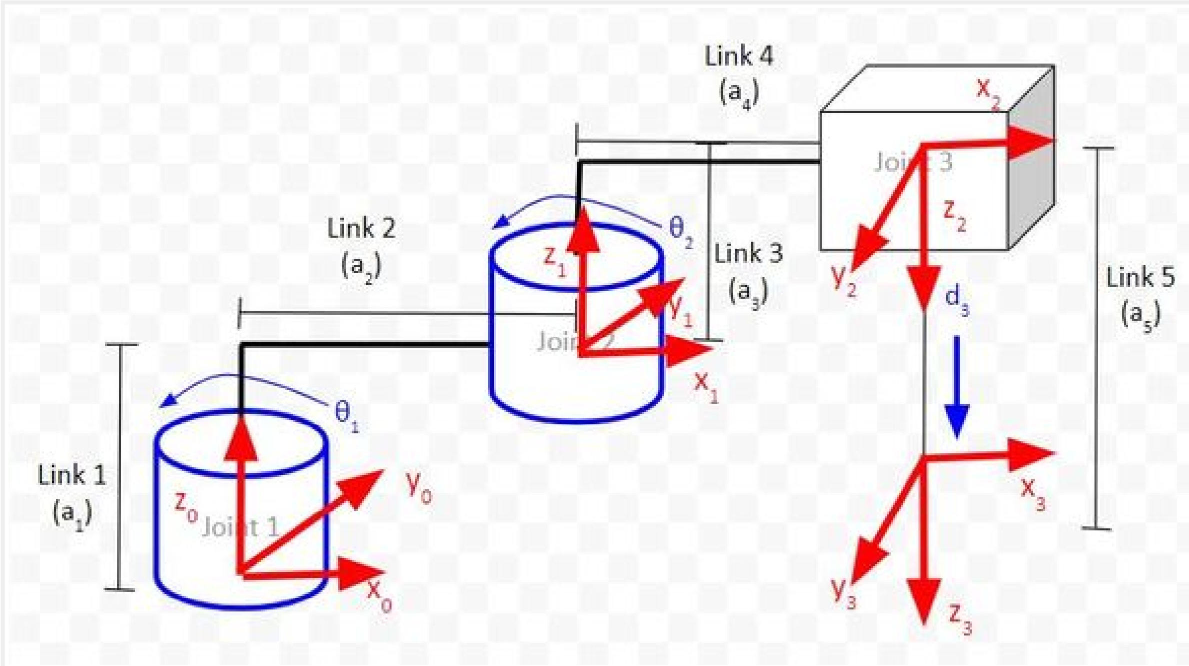


Create the Denavit-Hartenberg Parameter Table

Now, we need to find the Denavit-Hartenberg parameters. We have four coordinate frames here, so we need to have three rows in our D-H table.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1				
2				
3				

Find θ



For the Servo 0 row, we are going to focus on the relationship between frame 0 and frame 1. **θ is the angle from x_0 to x_1 around z_0.**

If you look at the diagram, x_0 and x_1 both point in the same direction. The axes are therefore aligned. When the robot is in motion, θ_1 will change (which will cause frame 1 to move). The angle from x_0 to x_1 around z_0 will be θ_1, so let's put that in our table.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1			
2				
3				

Now, let's look at the Servo 1 row. We take a look at the rotation between frame 1 and frame 2. **θ is the angle from x_1 to x_2 around z_1.**

If you look at the diagram, x_1 and x_2 both point in the same direction. The axes are therefore aligned. When the robot is in motion, θ_2 will change (which will cause frame 2 to move). The angle from x_1 to x_2 around z_1 will be θ_2, so let's put that in the second row of our table.

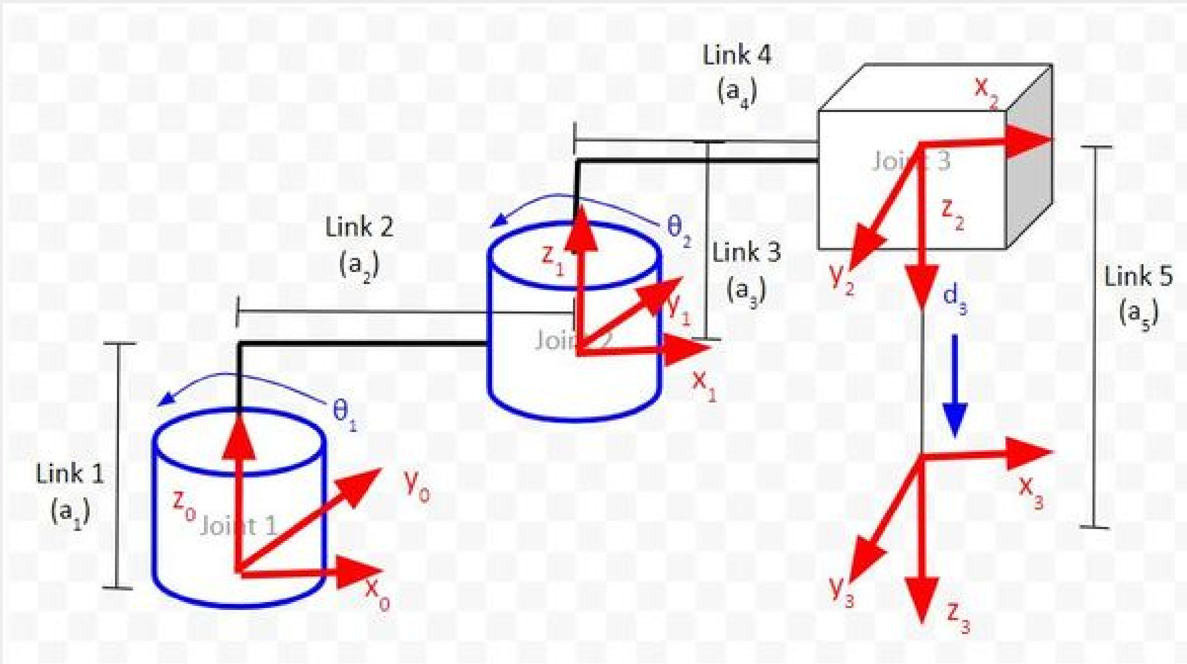
Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1			
2	θ_2			
3				

Now, let's look at the Servo 2 row. We take a look at the rotation between frame 2 and frame 3. **θ is the angle from x_2 to x_3 around z_2.**

If you look at the diagram, x_2 and x_3 both point in the same direction. The axes are therefore aligned. When the robot is in motion, there is only linear motion along z_2. The angle from x_2 to x_3 around z_2 will remain 0, so let's put that in the third row of our table.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1			
2	θ_2			
3	0			

Find α



Let's start with the Servo 0 row of the table.

α is the angle from z_0 to z_1 around x_1.

In the diagram above, you can see that this angle is 0 degrees, so we put 0 in the table.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0		
2	θ_2			
3	0			

Let's go to the Servo 1 row of the table.

α is the angle from z_1 to z_2 around x_2. In the diagram above, you can see that this angle is 180 degrees, so we put 180 in the table.

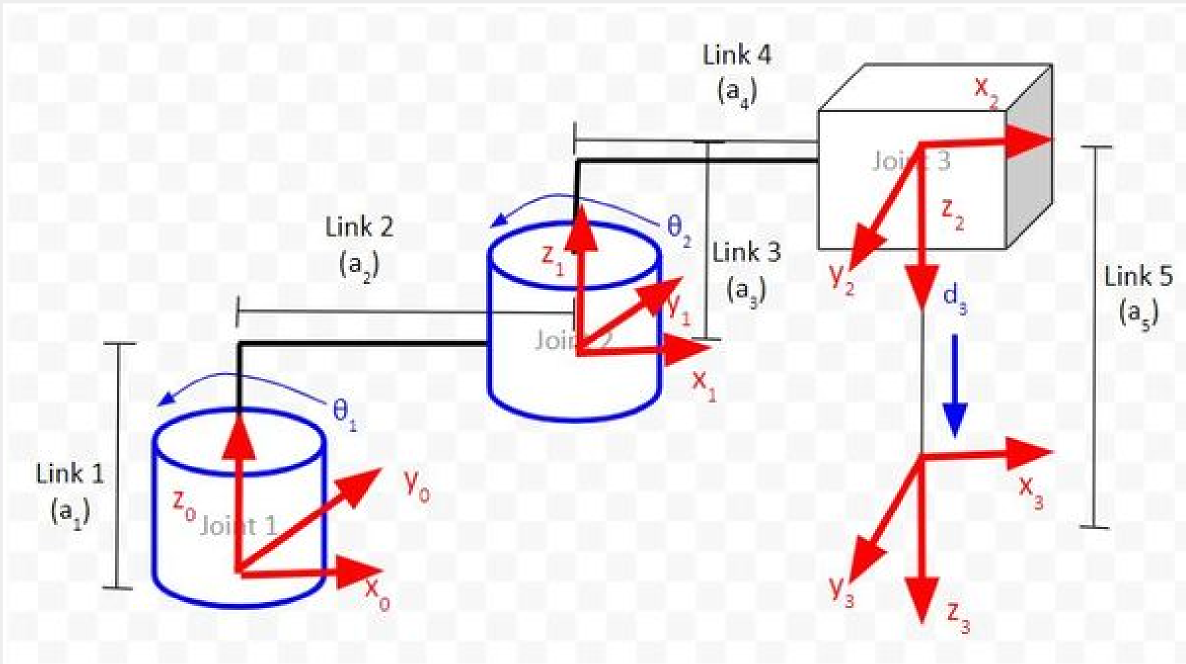
Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0		
2	θ_2	180		
3	0			

Let's go to the Servo 2 row of the table.

α is the angle from z_2 to z_3 around x_3. In the diagram above, you can see that this angle is 0 degrees, so we put 0 in the table.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0		
2	θ_2	180		
3	0	0		

Find r



Let’s start with the Servo 0 row of the table.

r is the distance between the origin of frame 0 and the origin of frame 1 along the x_1 direction. You can see in the diagram that this distance is a_2.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0	a_2	
2	θ_2	180		
3	0	0		

Now let’s look at the Servo 1 row. r is the distance between the origin of frame 1 and the origin of frame 2 along the x_2 direction. You can see in the diagram that this distance is a_4.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0	a_2	
2	θ_2	180	a_4	
3	0	0		

Now let's look at the Servo 2 row. r is the distance between the origin of frame 2 and the origin of frame 3 along the x_3 direction. You can see in the diagram that this distance is 0.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0	a_2	
2	θ_2	180	a_4	
3	0	0	0	

Find d

Now let's find d. We'll start on the first row of the table as usual.

d is the distance from x_0 to x_1 along the z_0 direction. You can see that this distance is a_1.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0	a_2	a_1
2	θ_2	180	a_4	
3	0	0	0	

Now let's take a look at frame 1 to frame 2. d is the distance from x_1 to x_2 along the z_1 direction. You can see that this distance is a_3.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0	a_2	a_1
2	θ_2	180	a_4	a_3
3	0	0	0	

Now let's take a look at frame 2 to frame 3. d is the distance from x_2 to x_3 along the z_2 direction. You can see that this distance is $a_5 + d_3$.

Joint i	θ_i (deg)	α_i (deg)	r_i (cm)	d_i (cm)
1	θ_1	0	a_2	a_1
2	θ_2	180	a_4	a_3
3	0	0	0	$a_5 + d_3$

References

Credit to Professor Angela Sodemann for teaching me this stuff. She is an excellent teacher (She runs a course on RoboGrok.com). On her YouTube channel, she provides some of the clearest explanations on robotics fundamentals you'll ever hear.

automaticaddison / August 26, 2020 / Robotics / manipulator