Velocity Analysis

Introduction

Identifying the velocity of one or more points on a link is important in mechanism design. Often times a synchronous motion is required between two mechanisms such in the box packing system where boxes are fed at a certain rate and the packing mechanism must adapt to that rate. In other cases, like in the reciprocating motion of a slider inertial forces upon changes in direction must be minimized otherwise it could be damaging in the short and long rage.

Two common analyses types are usually used to identify the velocity of one or more points on a mechanism.

1. Relative velocity analysis
2. Instantaneous center method

The application of these methods can be completed analytically or graphically, in this chapter we use both methods to identify the velocity of one or more points on a link in planar motion.

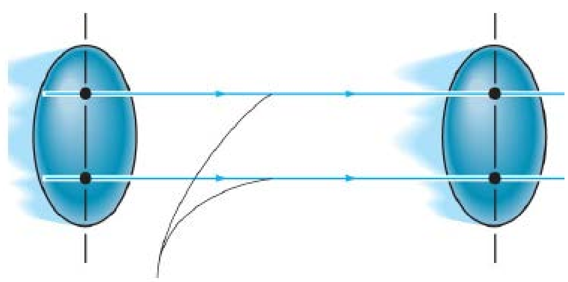
Planar motion

Planar motion can be divided into translation, rotation about a fixed axis or a combination of translation and rotation. Links experiencing motion in the third category are termed **floating links**.

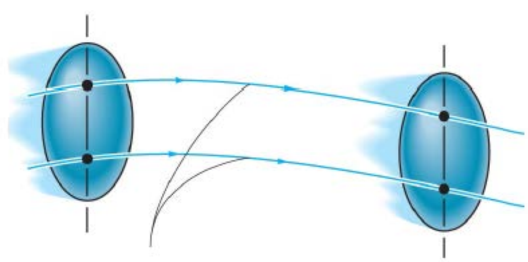
Translation

Every line segment in the body remains parallel to its original direction during the motion. Specifically, a body can undergo two types of translation:

- Rectilinear translation: All points follow parallel straight-line paths



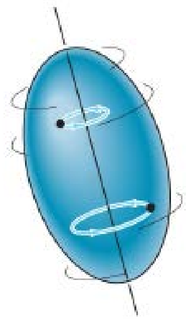
- Curvilinear translation: All points follow curved paths that are of the same shape and are equidistant from one another.



- **All points on the object have the same velocity**

Rotation about affixed axis

All the particles of the body, except those which lie on the axis of rotation move along circular paths

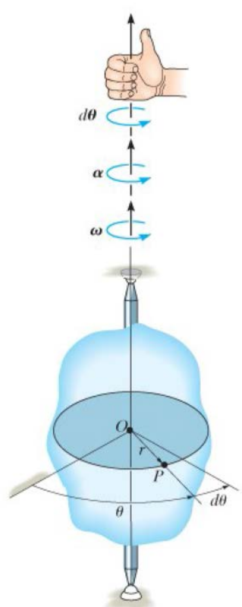


- The rotation motion of the object is measured by the time rate of change of the angular motion of a line connecting two points on the object, also known as angular velocity($\vec{\omega}$).

$$\vec{\omega} = \frac{d\theta}{dt}\ , direction\ of\ teh\ link\ rotation$$

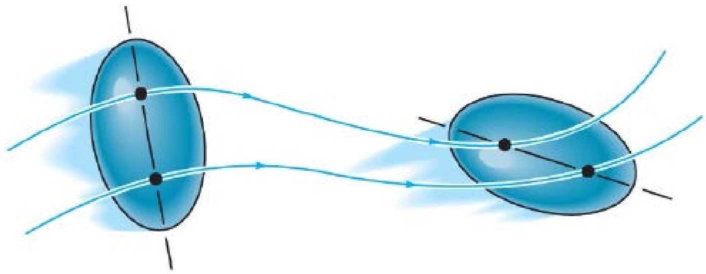
- The linear velocity of a point on the link is tangential to its circular path with a magnitude (speed) expressed in term of the distance from the axis of rotation and the angular velocity in radians per unit of time as

$$v = r\,\omega\ or\ \vec{V} = \vec{\omega} \times \vec{r}$$

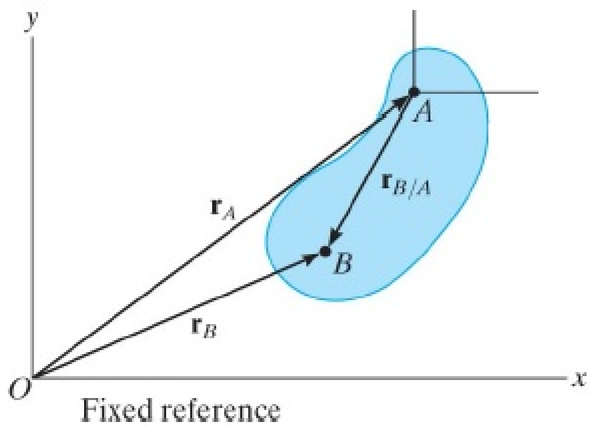


General plane motion

The body undergoes a combination of translation and rotation



Relative velocity



$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\frac{\overrightarrow{dr_B}}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_{B/A}}{dt}$$

$\frac{\overrightarrow{dr_B}}{dt}$: The absolute velocity of B

$\frac{d\vec{r}_A}{dt}$: The absolute velocity of A

$\frac{d\vec{r}_{B/A}}{dt}$: The relative velocity of B with reference to A

The relative velocity of B with reference to A is perpendicular to the line connecting A to B, this is true as long as A and B are on the same object since the distance from A to B is fixed and point B can only rotate with reference to A.

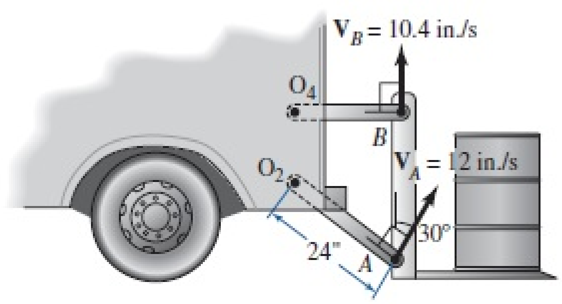
$$\frac{d\vec{r}_{B/A}}{dt} = \vec{\omega} \times \vec{r}_{B/A}$$

Note:

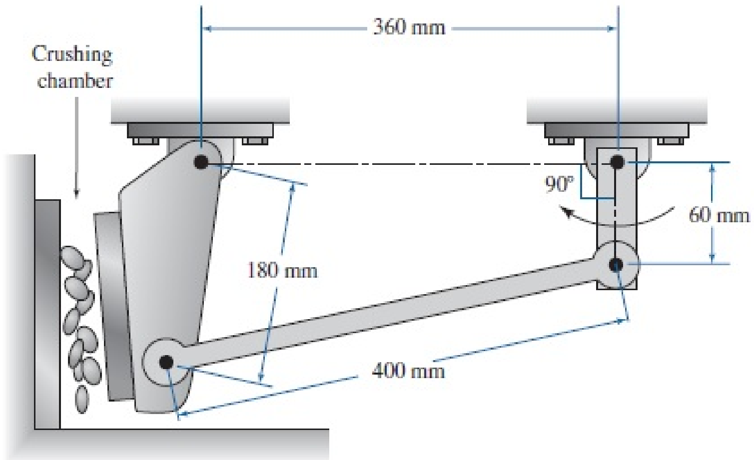
Since $\frac{d\vec{r}_{B/A}}{dt}$ is perpendicular to the line connecting A to B, the direction is known and the only unknown is the magnitude. If $\frac{d\vec{r}_A}{dt}$ is known in magnitude and direction and the direction of $\frac{\overrightarrow{dr_B}}{dt}$ is also known the previous equation can be solved graphically or analytically for the magnitudes of $\frac{\overrightarrow{dr_B}}{dt}$ and $\frac{d\vec{r}_{B/A}}{dt}$. More points on the link may be acquired to have as many unknown as equations.

Examples

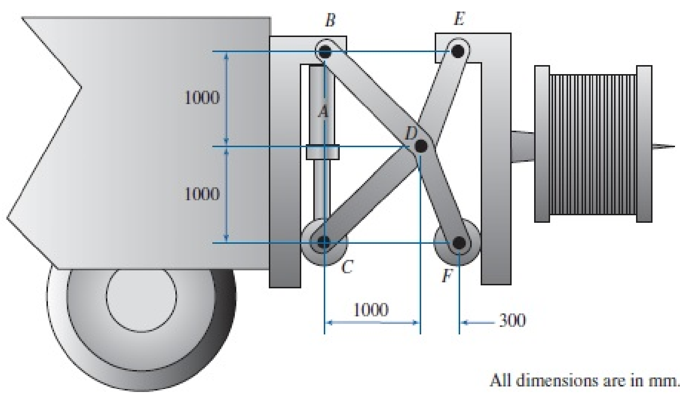
1. The figure below shows a cargo lift mechanism for a delivery truck. At this instant, point A has a velocity of 12 in./s in the direction shown, and point B has a velocity of 10.4 in./s, also in the direction shown. Determine the angular velocity of the lower link and the relative velocity of point B relative to point A.



2. The figure below shows a Rock crushing mechanism it is used in a machine where a large rock is placed in a vertical Hopper and falls into this crushing chamber. Properly sized aggregate which passes through a sieve, is discharged at the bottom. Rock not passing through the sieve is reintroduced into this crushing chamber. Determine the angular velocity of the crushing ram in the shown configuration as the 60 millimeter crank rotates 120 RPM clockwise.



3. The figure below illustrates a mechanism that extends reels of cable from a delivery truck. It is operated by a hydraulic cylinder at A. At this instant, the cylinder retracts at a rate of 5 mm/s. Determine the velocity of the top joint, point E.



Points on different links

When sliding occurs at the joints care must be taken so not to use the previous relation between two points on different links. The following relation must be used instead for points on different links.

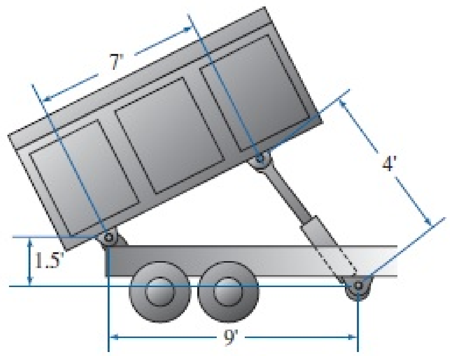
$$\frac{d\overrightarrow{r_B}}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_{B/A}}{dt}$$

$$\frac{d\vec{r}_{B/A}}{dt} = \vec{\omega} \times \vec{r}_{B/A} + \left(\frac{d\vec{r}_{B/A}}{dt}\right)'$$

$\left(\frac{d\vec{r}_{B/A}}{dt}\right)'$: is the velocity of B as seen by an observer on A rotating with the link that contains A.

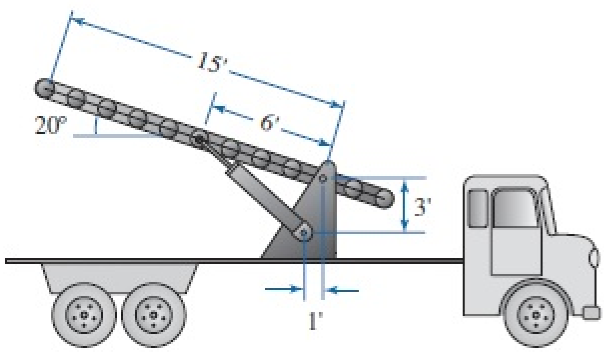
Example:

The figure below shows a mechanism that tips the bed of a dump truck. Determine the required speed of the hydraulic cylinder in order to tip the truck at a rate of 5 rad/min.



Example

The Figure illustrates a roofing material delivery truck conveyor. Heavy roofing materials can be transported on the conveyor to the roof. The conveyor is lifted into place by extending the hydraulic cylinder. At this instant, the cylinder is extending at a rate of 8 fpm (ft/min). Determine the rate that the conveyor is being lifted.

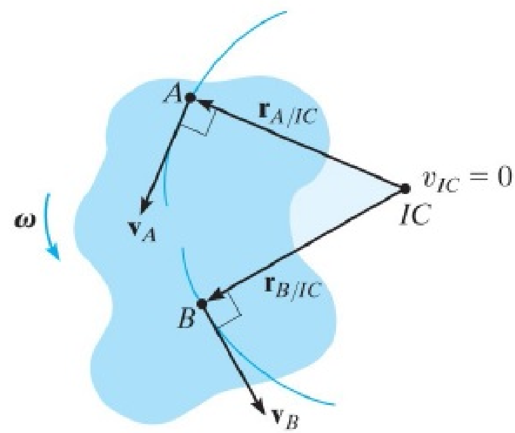


Instantaneous center of zero velocity

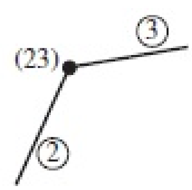
At any instant in time, an object can be viewed as rotating about an instantaneous center. This point is termed instantaneous center of zero velocity (IC) or instantaneous center of rotation. In terms of the instantaneous center of zero velocity, the velocity of any point on the object is perpendicular to the line connecting the (IC) to the point with a magnitude equal to the dis distance times the angular velocity of the object. The concept can be applied to the absolute as well as the relative motion between moving objects.

Few rules on how to identify the instantaneous center of zero velocity

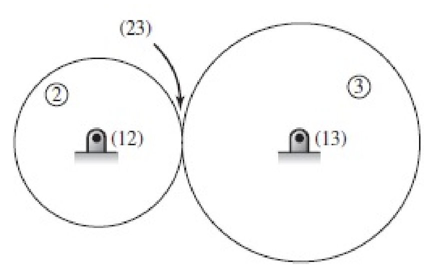
1. If you Know the direction of the velocity of two points on the object then the instantaneous center of zero velocity is at the intersection of the lines perpendicular to the velocity vectors



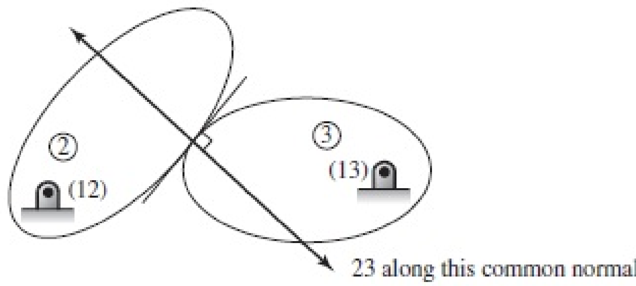
1. When two links are pin connected, the instantaneous center of rotation of the relative motion is the pin connection. In the figure (23) is the instantaneous center of rotation of the relative motion of 3 with reference to 2



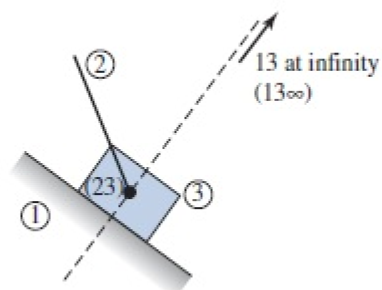
2. For two objects in rolling contact without slipping, the instantaneous center of the relative motion of one object with reference to the other is at the point of contact. In the following example (12) and (13) are the instantaneous centers of the absolute motion of object 2 and 3. (23) is the instantaneous center of rotation of the relative motion of 3 with reference to 2.



When sliding occurs at the contact point, the instantaneous center is somewhere along the line normal to the direction of the sliding contact



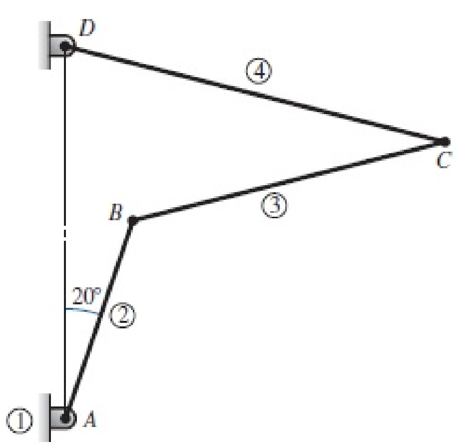
3. The instantaneous center of two links with straight line sliding is at infinity

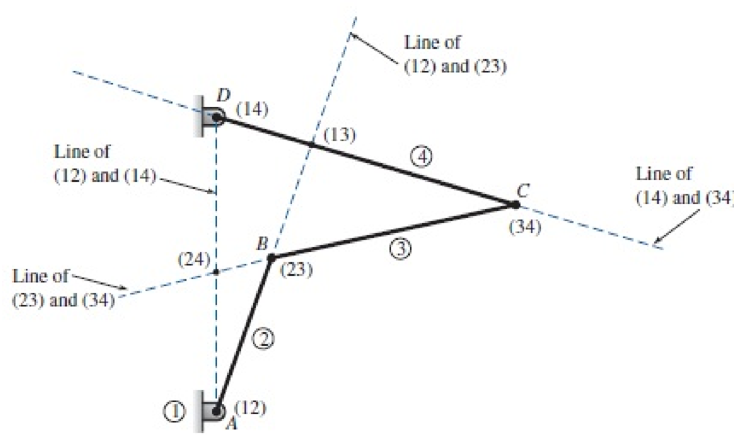


Kennedy's theorem:

Instant centers that cannot be found from the four rules for primary centers are located with the use of Kennedy's theorem. It states that "The three instant centers corresponding with any three bodies all lay on the same straight line"

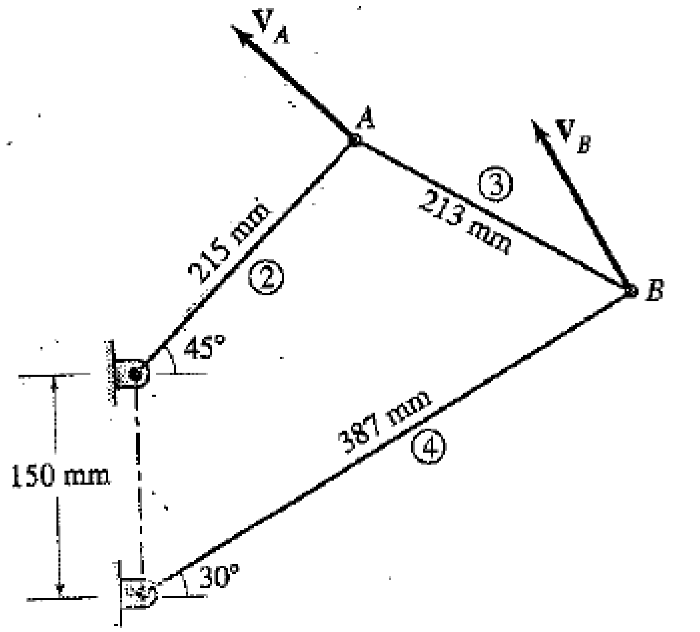
Example:

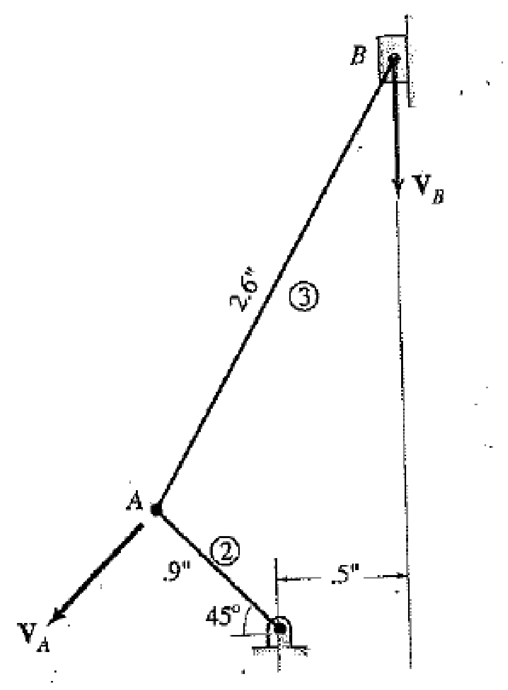


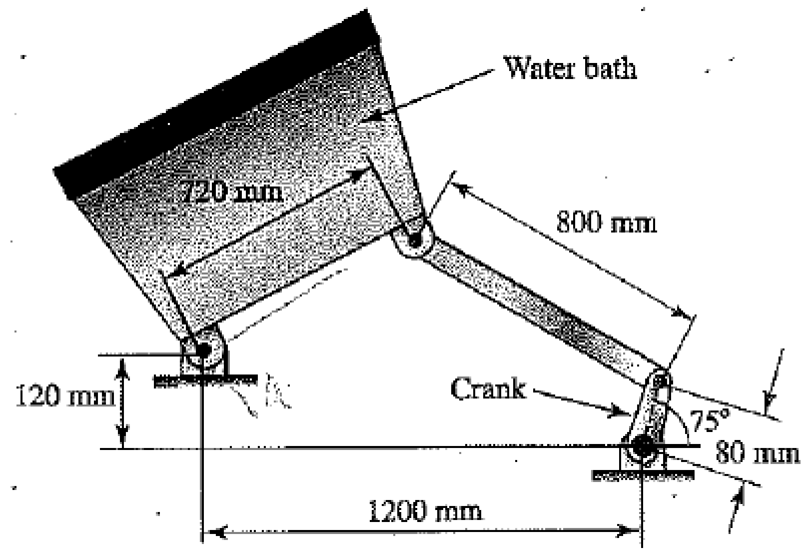


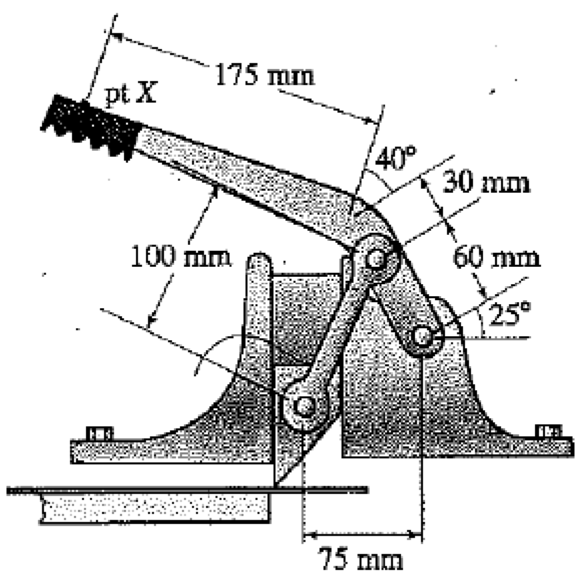
Example:

Identify the instantaneous centers of rotation of the following mechanism






Water bath
720 mm
800 mm
Crank
75°
80 mm
120 mm
1200 mm


175 mm
pt X
40°
30 mm
100 mm
60 mm
25°
75 mm

Finding the velocity of a link using the instantaneous center of rotation analysis

Consider the kinematic diagram shown in the figure below. Assume that the angular velocity of link 2 is known and we are asked to find the velocity of point C on link 3. The instantaneous center of rotation analysis consist of

1. Identify a link of known velocity (link 2)
2. Identify a link with the desired velocity (links 3)
3. Identify the frame the frame (link 1)
4. Identify the center of rotation of the link of known velocity with reference to the frame (12)
5. Identify the center of rotation of the link of desired velocity with reference to the link of known velocity (23)
6. Find the velocity of this point using the fact that both centers (12) and (23) are on link 2
7. Identify the center of rotation of the link with the desired velocity with reference to the frame (13) for link 3
8. The desired velocity can then be found using the fact that (13), (23) and the desired velocity are on the same object (link 3)