

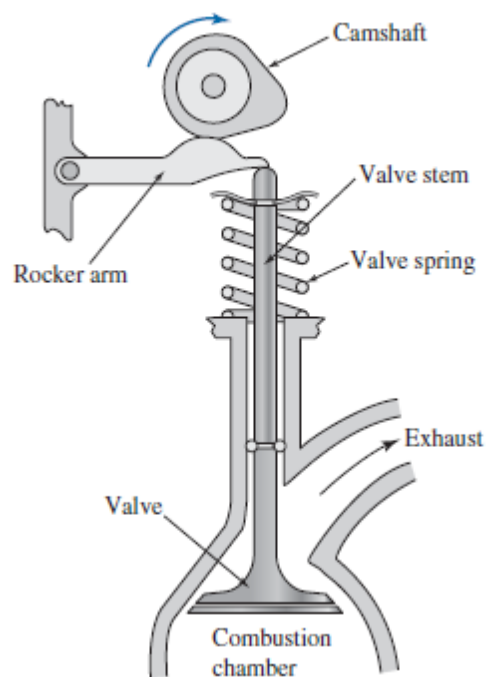
Cams

Introduction

A cam is a common mechanism element that drives a mating component known as a follower. The unique feature of a cam is that it can impart a very precise and distinctive motion to its follower that is sometimes impossible to achieve using other linkages. The cam is usually driven with a constant velocity however the profile of the cam controls the displacement, velocity and acceleration of the follower

Example:

The cam in engine valve train is mounted on an axle and driven by the engine. The axle of the cam rotates at high speeds and the follow arm (rocker arm) is in contact with the cam at all times and rocks between two positions that correspond to the open or closed valve. The motion is precisely synchronized.

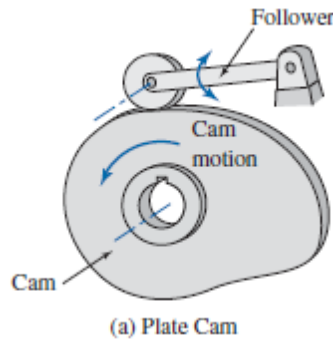


- To maintain full contact between the follower and the cam some mechanical means such as a spring may be utilized.

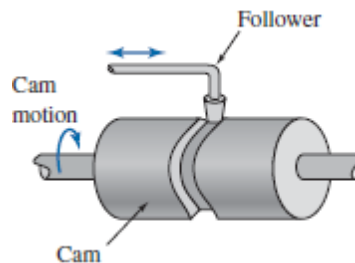
Types of cams

Cams can be classified into one of the following categories

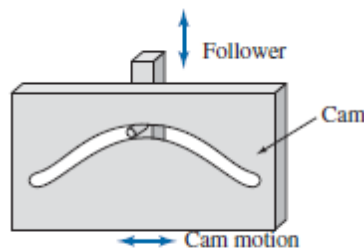
1. **Plate or disk cams:** they are formed on a disk or plate and the radial distance from the center of the disk is varied throughout the circumference of the cam, allowing a follower to ride on this outer edge.



Cylindrical or drum cam: This type of cam is formed on a cylinder. A groove is cut into the cylinder, with a varying location along the axis of rotation. Attaching a follower that rides in the groove gives the follower motion along the axis of rotation.



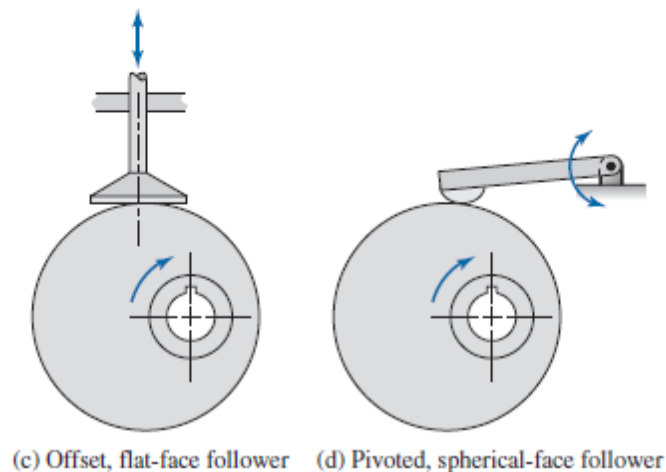
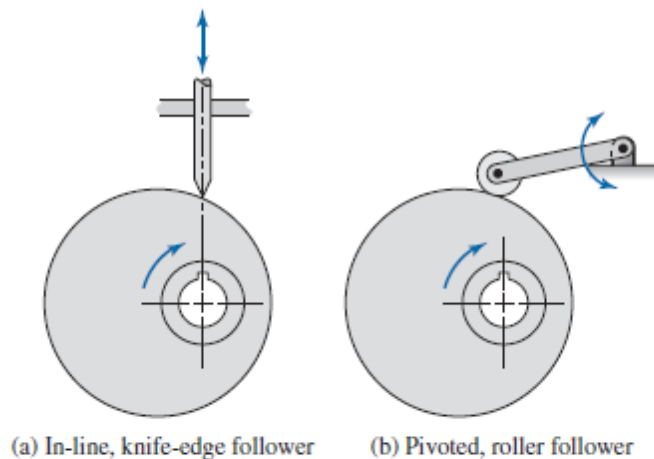
Linear cam: This type of cam is formed on a translated block. A groove is cut into the block with a distance that varies from the plane of translation. Attaching a follower that rides in the groove gives the follower motion perpendicular to the plane of translation.



Types of followers

Followers are classified based on their motion, shape and position

- Motion: translating or pivoting
- Position: For the translating followers distinction is made between inline motion where the line of translation passes through the center of rotation of the cam as opposed to an offset follower
- The shape of a follower can in general be classified as
 - Knife edge follower: The sharp edge produces high contact stresses and wears rapidly
 - Roller follower: friction and contact stresses are low however it can jam during steep cam displacement
 - Flat face follower: frictional forces are large and any follower deflection or misalignment can cause high surface stresses
 - Spherical faced follower: large frictional forces



Prescribed motion of the follower

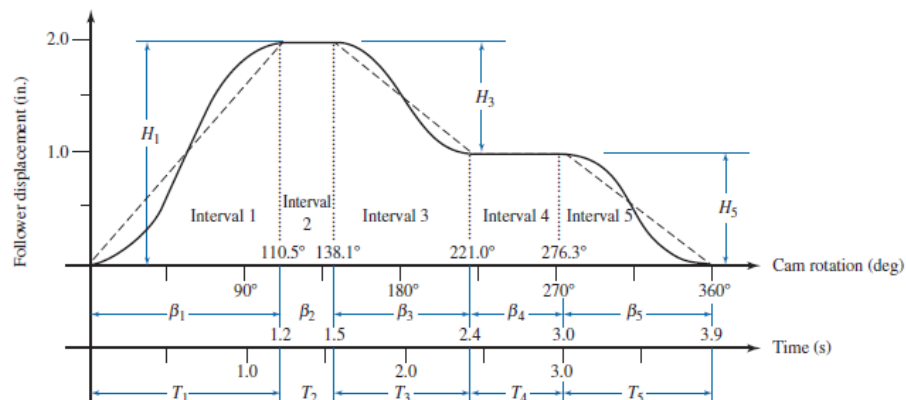
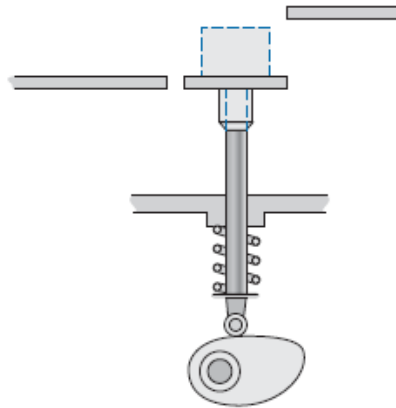
The motion of the follower is described by the required task and can be sketched on a displacement versus time or displacement versus angular displacement of the cam. For kinematic analysis the former is preferred, the latter if preferred for the design of the cam profile.

Cam rotation during an interval (i) of follower motion is typically expressed by the symbol β_i . Likewise, the time consumed during an interval is designated T_i . The amount of follower rise, or fall, during an interval is designated H_i . The period of cam rotation where there is no follower motion is termed dwell.

Example:

A cam is to be used for a platform that will repeatedly lift boxes from a lower conveyor to an upper conveyor. This machine is shown in the figure below. Plot a displacement diagram and determine the required constant speed of the cam when the follower motion sequence is as follows:

- Rise 2 in. in 1.2 s.
- Dwell for 0.3 s.
- Fall 1 in. in 0.9 s.
- Dwell 0.6 s.
- Fall 1 in. in 0.9 s.



Note:

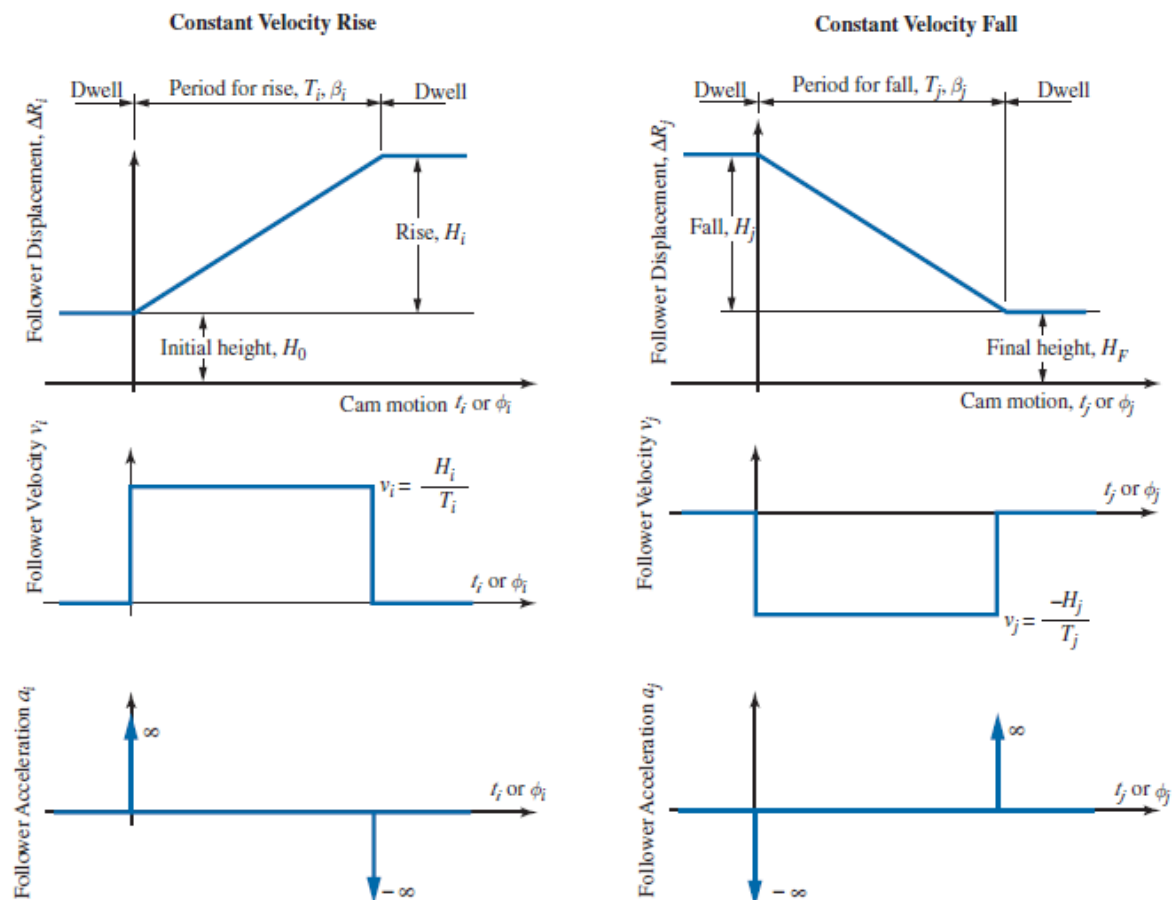
Different follower motion scheme can be prescribed during the rise and fall (constant velocity, constant acceleration....). From a dynamical point of view, larger accelerations impart larger inertial forces consequently high stresses, rapidly changing accelerations cause vibration and consequently noise.

Follower motion schemes

Constant velocity scheme

TABLE 9.1 Cam Follower Kinematics for Constant Velocity Motion

	Rise	Fall
Displacement:	$\Delta R_i = H_0 + \frac{H_i t_i}{T_i} = H_0 + \frac{H_i \phi_i}{\beta_i}$	$\Delta R_j = H_F + H_j \left(1 - \frac{t_j}{T_j}\right) = H_F + H_j \left(1 - \frac{\phi_j}{\beta_j}\right)$
Velocity:	$v_i = \frac{H_i}{T_i} = \frac{H_i \omega}{\beta_i}$	$v_j = \frac{-H_j}{T_j} = \frac{-H_j \omega}{\beta_j}$
Acceleration:	$a = 0$ (∞ at transitions)	$a = 0$ (∞ at transitions)

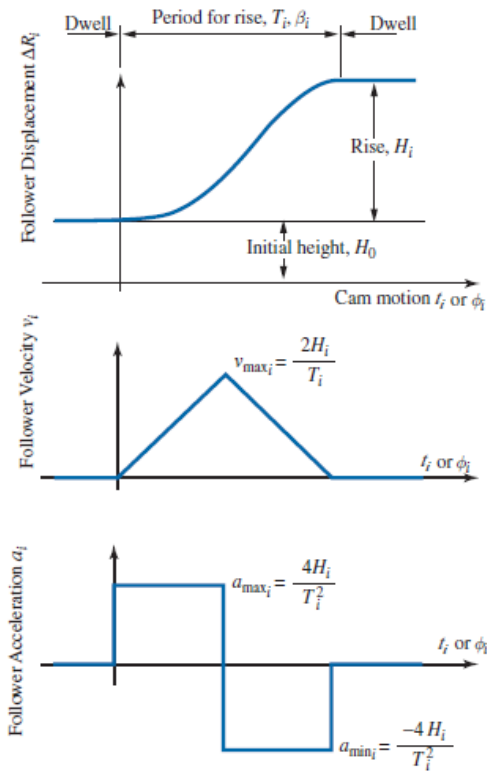


Constant acceleration scheme

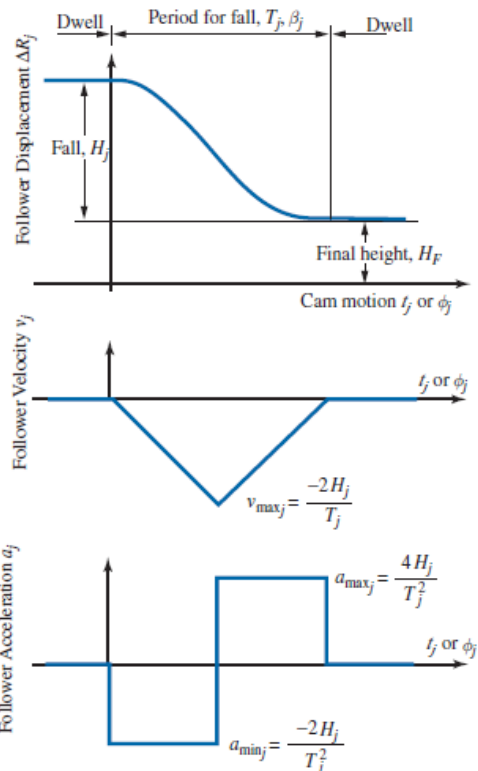
TABLE 9.2 Cam Follower Kinematics for Constant Acceleration Motion

	Rise	Fall
For $0 < t < 0.5 T$ ($0 < \phi < 0.5 \beta$):		
Displacement:	$\Delta R_i = H_0 + 2H_i \left(\frac{t_i}{T_i} \right)^2$ $= H_0 + 2H_i \left(\frac{\phi_i}{\beta_i} \right)^2$	$\Delta R_j = H_F + H_j - 2H_j \left(\frac{t_j}{T_j} \right)^2$ $= H_F + H_j - 2H_j \left(\frac{\phi_j}{\beta_j} \right)^2$
Velocity:	$v_i = \frac{4H_i t_i}{T_i^2} = \frac{4H_i \omega \phi_i}{\beta_i^2}$	$v_j = \frac{-4H_j t_j}{T_j^2} = \frac{-4H_j \omega \phi_j}{\beta_j^2}$
Acceleration:	$a_i = \frac{4H_i}{T_i^2} = \frac{4H_i \omega^2}{\beta_i^2}$	$a_j = \frac{-4H_j}{T_j^2} = \frac{-4H_j \omega^2}{\beta_j^2}$
For $0.5 T < t < T$ ($0.5 \beta < \phi < \beta$):		
Displacement:	$\Delta R_i = H_0 + H_i - 2H_i \left(1 - \frac{t_i}{T_i} \right)^2$ $= H_0 + H_i + 2H_i \left(1 - \frac{\phi_i}{\beta_i} \right)^2$	$\Delta R_j = H_F + 2H_j \left(1 - \frac{t_j}{T_j} \right)^2$ $= H_F + 2H_j \left(1 - \frac{\phi_j}{\beta_j} \right)^2$
Velocity:	$v_i = \frac{4H_i}{T_i} \left(1 - \frac{t_i}{T_i} \right) = \frac{4H_i \omega}{\beta_i} \left(1 - \frac{\phi_i}{\beta_i} \right)$	$v_j = \frac{-4H_j}{T_j} \left(1 - \frac{t_j}{T_j} \right) = \frac{-4H_j \omega}{\beta_j} \left(1 - \frac{\phi_j}{\beta_j} \right)$
Acceleration:	$a_i = \frac{-4H_i}{T_i^2} = \frac{-4H_i \omega^2}{\beta_i^2}$	$a_j = \frac{4H_j}{T_j^2} = \frac{4H_j \omega^2}{\beta_j^2}$

Constant Acceleration Rise

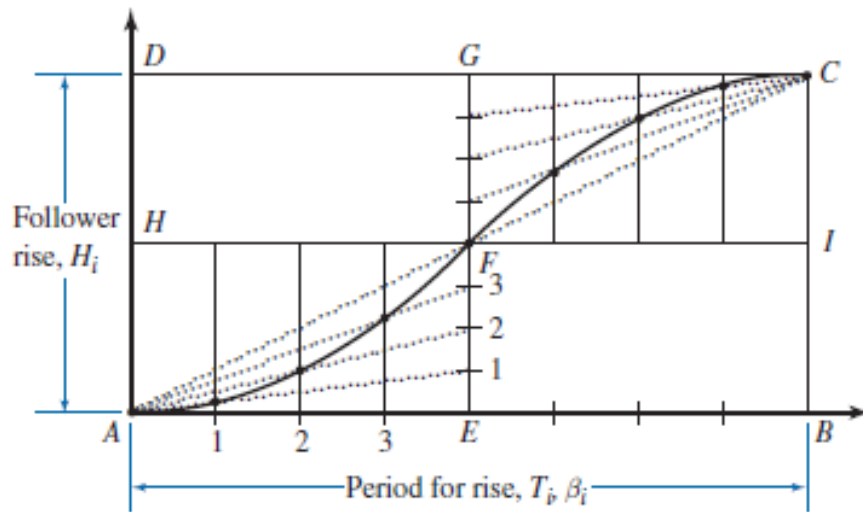


Constant Acceleration Fall



Graphical representation of the displacement

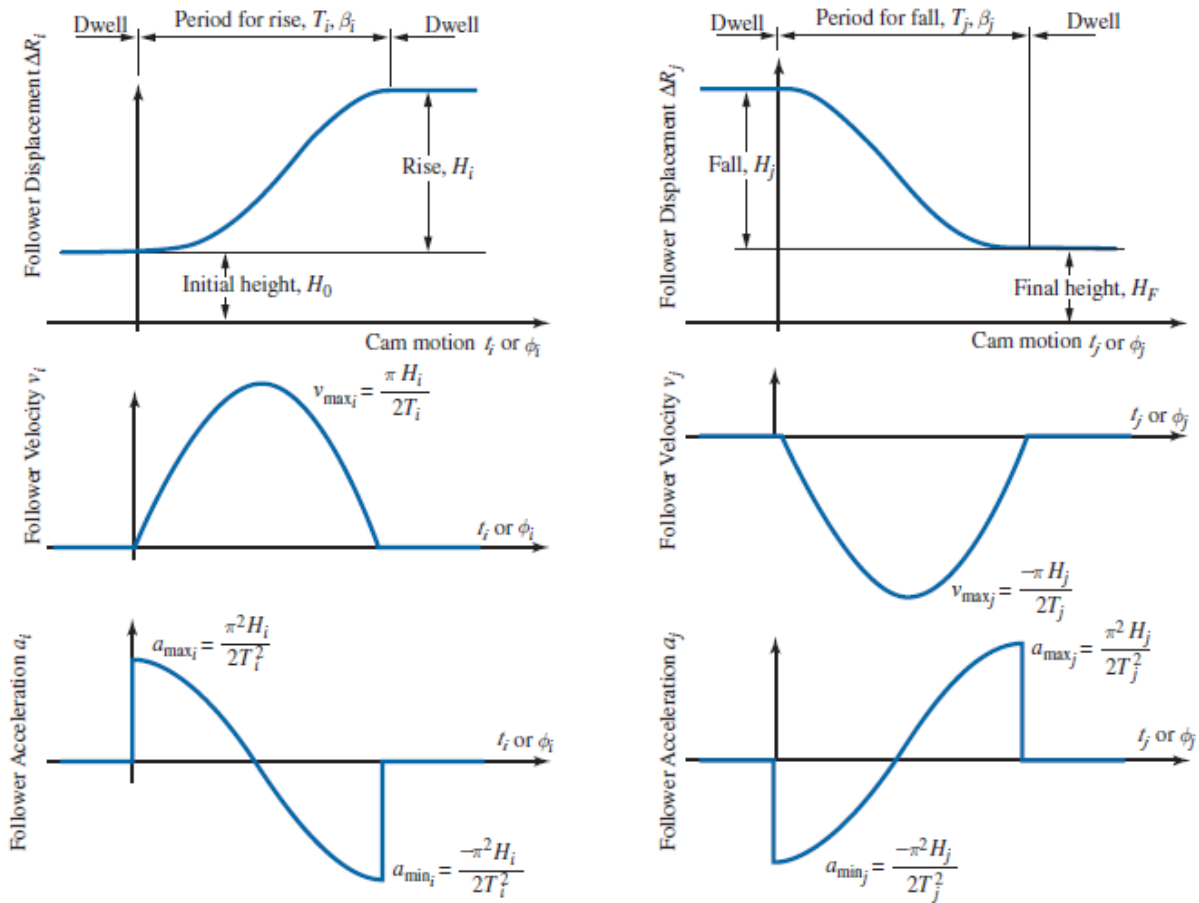
1. Divide the follower rise (or fall) sequence into two halves. AE represents the time period and EF the magnitude of rise for the first half of this motion scheme.
2. Divide both the horizontal and vertical axes of the quadrant AEFH into equal parts.
3. Construct vertical lines from the horizontal divisions.
4. Construct straight lines from corner A to the vertical divisions.
5. Draw a smooth curve through the points of intersection of the vertical lines and the lines drawn from corner A.
6. Repeat steps 2 through 5 for the remaining half of the curve as shown in quadrant FICG



Harmonic Motion

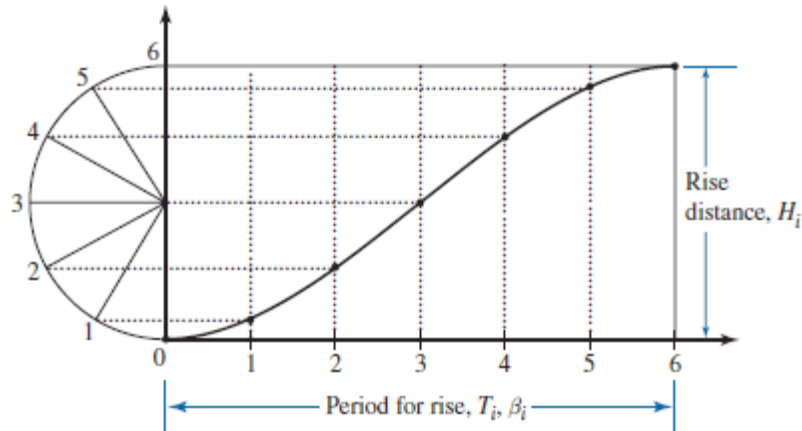
TABLE 9.3 Cam Follower Kinematics for Harmonic Motion

	Rise	Fall
Displacement:	$\Delta R_i = H_0 + \frac{H_i}{2} \left[1 - \cos \left(\frac{\pi t_i}{T_i} \right) \right]$ $= H_0 + \frac{H_i}{2} \left[1 - \cos \left(\frac{\pi \phi_i}{\beta_i} \right) \right]$	$\Delta R_j = H_F + \frac{H_j}{2} \left[1 + \cos \left(\frac{\pi t_j}{T_j} \right) \right]$ $= H_F + \frac{H_j}{2} \left[1 - \cos \left(\frac{\pi \phi_j}{\beta_j} \right) \right]$
Velocity:	$v_i = \frac{\pi H_i}{2 T_i} \left[\sin \left(\frac{\pi t_i}{T_i} \right) \right]$ $= \frac{\pi H_i \omega}{2 \beta_i} \left[\sin \left(\frac{\pi \phi_i}{\beta_i} \right) \right]$	$v_j = \frac{-\pi H_j}{2 T_j} \left[\sin \left(\frac{\pi t_j}{T_j} \right) \right]$ $= \frac{-\pi H_j \omega}{2 \beta_j} \left[\sin \left(\frac{\pi \phi_j}{\beta_j} \right) \right]$
Acceleration:	$a_i = \frac{\pi^2 H_i}{2 T_i^2} \left[\cos \left(\frac{\pi t_i}{T_i} \right) \right]$ $= \frac{\pi^2 H_i \omega^2}{2 \beta_i^2} \left[\cos \left(\frac{\pi \phi_i}{\beta_i} \right) \right]$	$a_j = \frac{-\pi^2 H_j}{2 T_j^2} \left[\cos \left(\frac{\pi t_j}{T_j} \right) \right]$ $= \frac{-\pi^2 H_j \omega^2}{2 \beta_j^2} \left[\cos \left(\frac{\pi \phi_j}{\beta_j} \right) \right]$



Graphical representation of the displacement

1. Construct a semicircle having a diameter equal to the amount of rise (or fall) desired.
2. Divide the rise time period into incremental divisions.
3. Divide the semicircle into the same number of equal divisions of the follower rise period.
4. Draw vertical lines from the divisions on the time axis.
5. Draw horizontal lines from the division points on the semicircle to the corresponding division lines on the time axis.
6. Draw a smooth curve through the points of intersection found in the previous step.

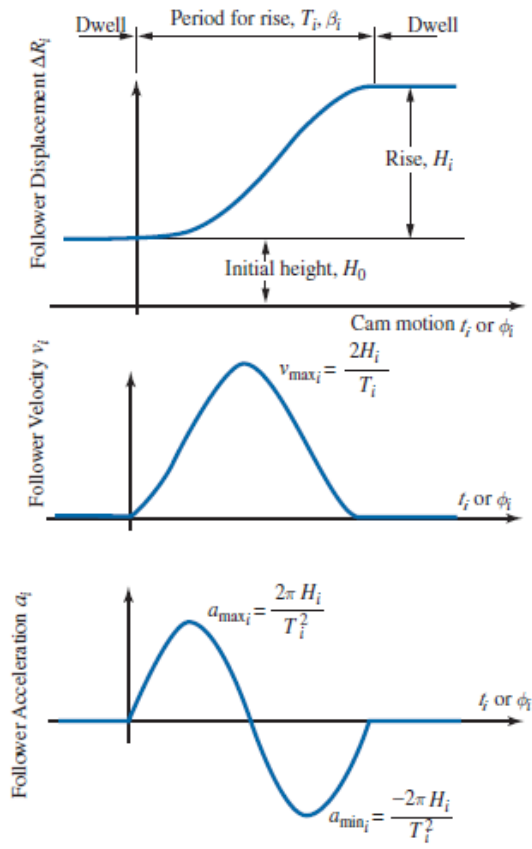


Cycloidal motion

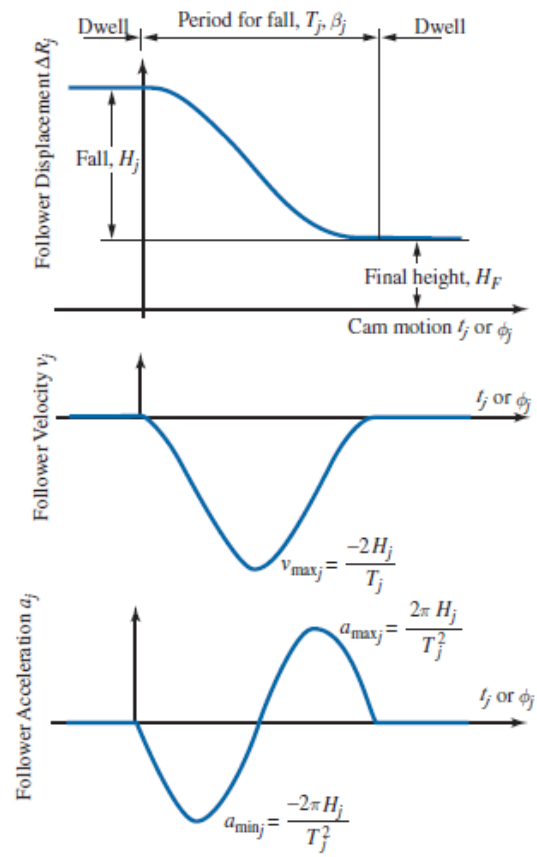
TABLE 9.4 Cam Follower Kinematics for Cycloidal Motion

	Rise	Fall
Displacement:	$\Delta R_i = H_0 + H_i \left[\frac{t_i}{T_i} - \frac{1}{2\pi} \sin \left(\frac{2\pi t_i}{T_i} \right) \right]$ $= H_0 + H_i \left[\frac{\phi_i}{\beta_i} - \frac{1}{2\pi} \sin \left(\frac{2\pi \phi_i}{\beta_i} \right) \right]$	$\Delta R_j = H_F + H_j \left[1 - \frac{t_j}{T_j} + \frac{1}{2\pi} \sin \left(\frac{2\pi t_j}{T_j} \right) \right]$ $= H_F + H_j \left[\frac{\phi_j}{\beta_j} - \frac{1}{2\pi} \sin \left(\frac{2\pi \phi_j}{\beta_j} \right) \right]$
Velocity:	$v_i = \frac{H_i}{T_i} \left[1 - \cos \left(\frac{2\pi t_i}{T_i} \right) \right]$ $= \frac{H_i \omega}{\beta_i} \left[1 - \cos \left(\frac{2\pi \phi_i}{\beta_i} \right) \right]$	$v_j = \frac{-H_j}{T_j} \left[1 - \cos \left(\frac{2\pi t_j}{T_j} \right) \right]$ $= \frac{-H_j \omega}{\beta_j} \left[1 - \cos \left(\frac{2\pi \phi_j}{\beta_j} \right) \right]$
Acceleration:	$a_i = \frac{2\pi H_i}{T_i^2} \left[\sin \left(\frac{2\pi t_i}{T_i} \right) \right]$ $= \frac{2\pi H_i \omega^2}{\beta_i^2} \left[\sin \left(\frac{2\pi \phi_i}{\beta_i} \right) \right]$	$a_j = \frac{-2\pi H_j}{T_j^2} \left[\sin \left(\frac{2\pi t_j}{T_j} \right) \right]$ $= \frac{-2\pi H_j \omega^2}{\beta_j^2} \left[\sin \left(\frac{2\pi \phi_j}{\beta_j} \right) \right]$

Cycloidal Rise



Cycloidal Fall



Graphical representation of the displacement

7. On a displacement diagram grid, draw a line from the beginning point of the rise (or fall) to the final point. This line is drawn from A to C
8. Extend the line drawn in the previous step and draw a circle, with radius $r = H/2\pi$ centered anywhere on that line.
9. Construct a vertical line through the center of the circle.
10. Divide the circle into an even number of parts.
11. Connect the circle division lines as shown
12. Mark the intersection points of the lines drawn in step 5 with the vertical line drawn in step 3.
13. Divide the time period into the same number of equal parts as the circle. Construct vertical lines from these division points.
14. Project the points identified in step 6 along a line parallel with the line constructed in step 1.
15. Mark intersection points of the lines constructed in step 8 with the vertical lines drawn in step 7, as shown
16. 10. Construct a smooth curve through the points identified in step 9.

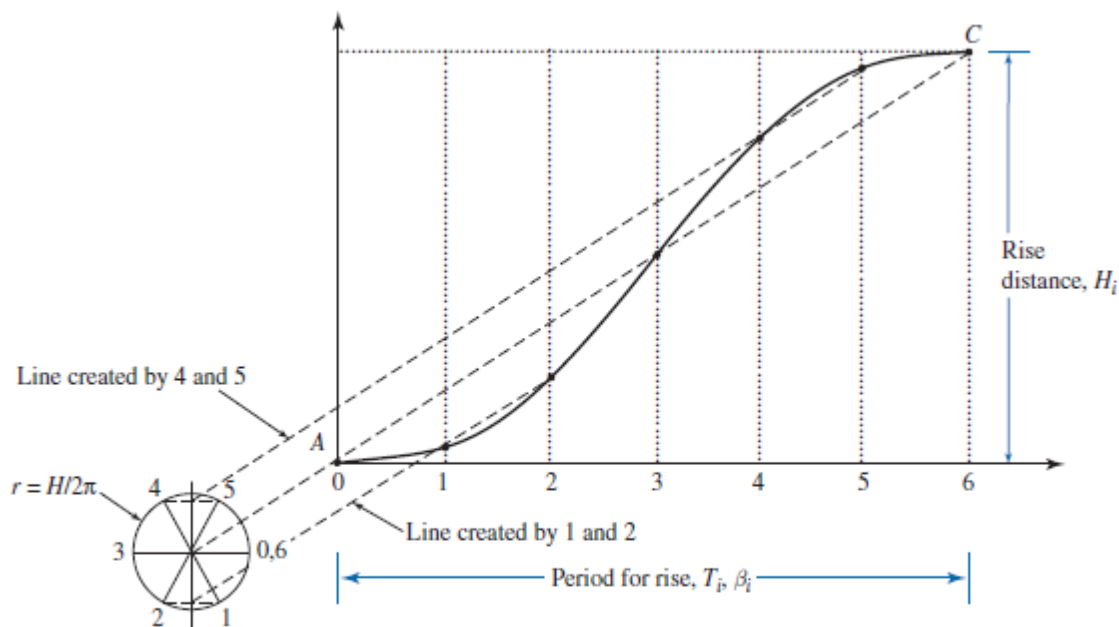
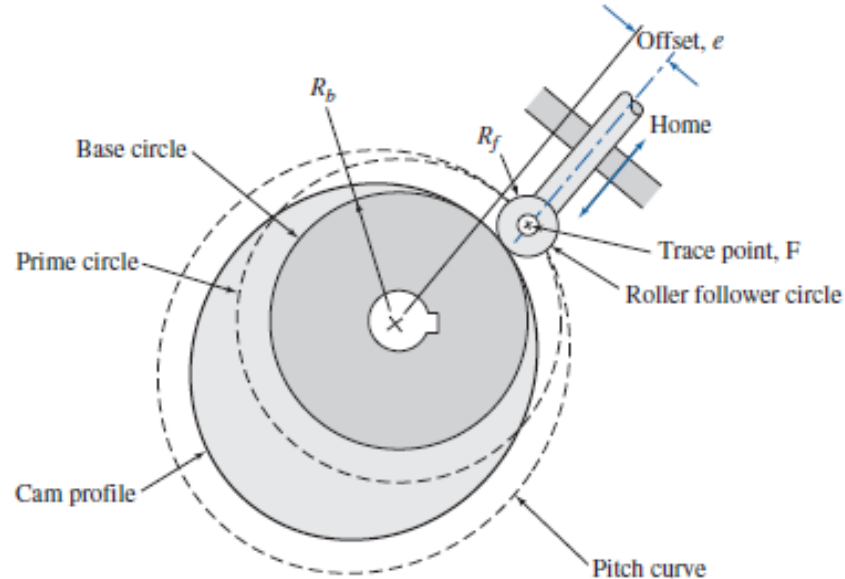


FIGURE 9.13 Construction of a cycloidal displacement diagram.

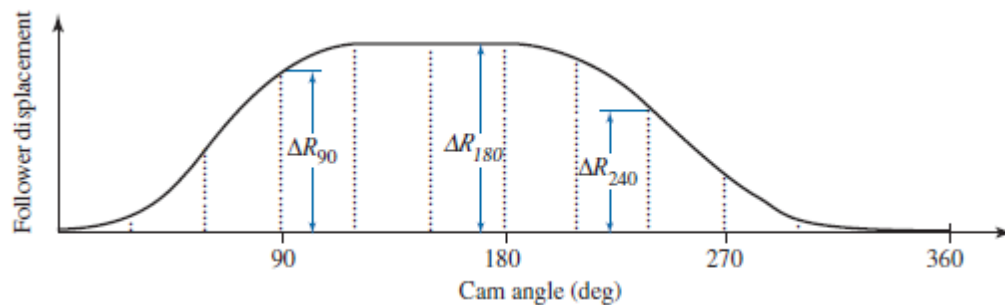
Disk cam profile

Definitions

1. The base circle is the smallest circle centered on the cam rotation axis and tangent to the cam surface.
2. The trace point serves as a reference to determine the effective location of the follower.
 - a. For a knife-edge follower, it is the point of cam and follower contact.
 - b. For a roller follower, the trace point is chosen at the center of the roller.
 - c. For a flat- or spherical-face follower, the trace point is chosen on the contact surface of the follower.
3. The home position of the cam is the orientation that corresponds to the 0° reference position on a displacement diagram.
4. The prime circle is a circle drawn through the trace point of the follower while the cam is at its home position.
5. The pitch curve is the path of the center of the follower.



The displacement diagram in the figure below will be used in the graphical representation of the cam profile for the different constructions that follow

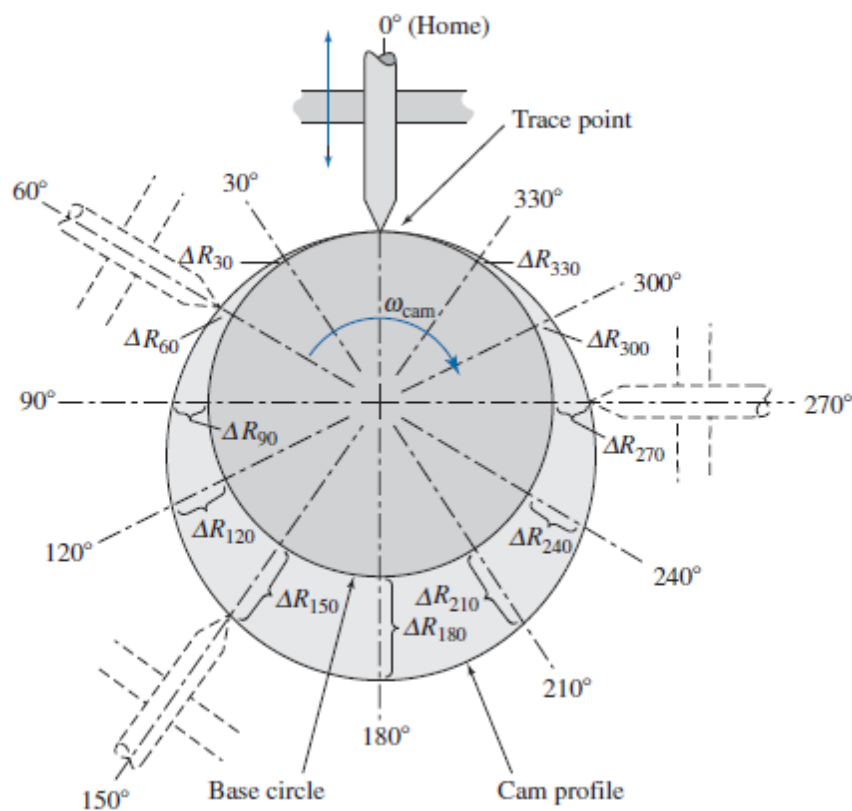


Design of a plate cam with an In-Line knife edge follower

Graphical design procedure

The following general procedure is used to graphically construct such a profile:

1. Draw the base circle of radius R_b . The size is typically a function of the spatial constraints of the application.
2. Draw the follower in the home position.
3. Draw radial lines from the center of the cam, corresponding to the cam angles identified on the displacement diagram. For construction purposes, the cam will remain stationary and the follower will be rotated in a direction opposite to the actual cam rotation.
4. Transfer the displacements from the displacement diagram to the radial lines. Measure these displacements from the base circle.
5. Draw a smooth curve through these prescribed displacements.
6. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall intervals.



Analytical design procedure

The profile of the cam can be constructed from the following x and y coordinates for clockwise rotation of the cam

$$\begin{cases} x = (R_b + \Delta R) \sin(\theta) \\ y = (R_b + \Delta R) \cos(\theta) \end{cases}$$

Where

R_b is the base circle radius

θ is the cam rotation angle measured against the direction of cam rotation from the home position

ΔR is the follower displacement at the cam angle θ

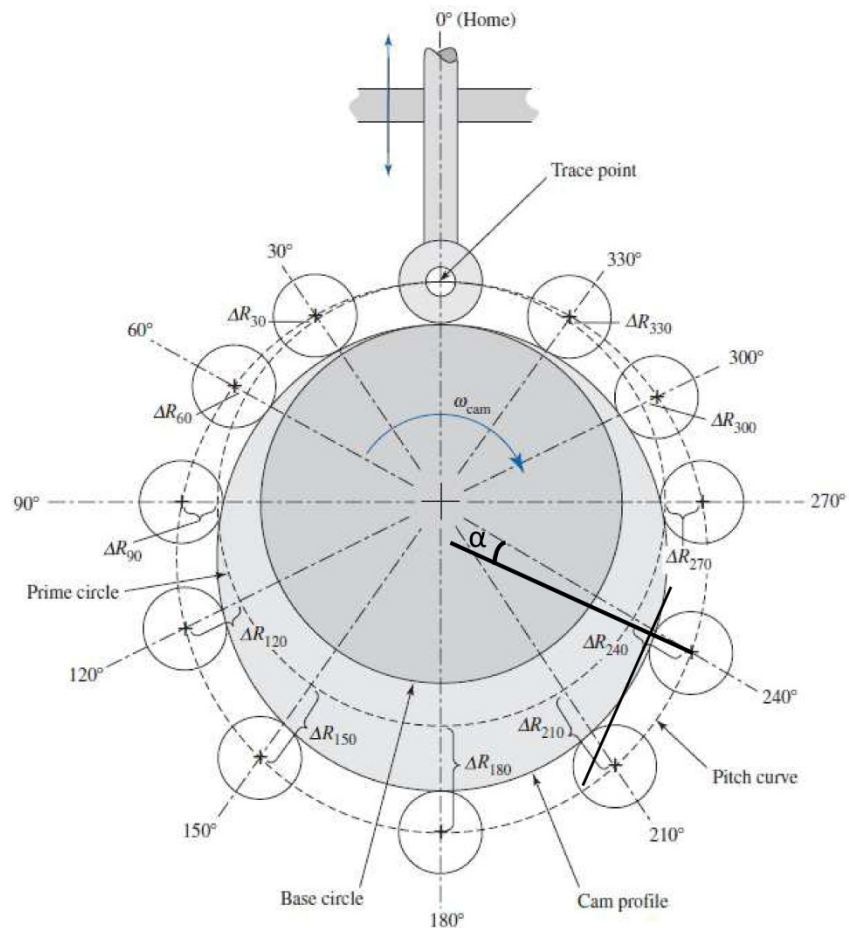
Note: A VBA program is available for this purpose

Design of a plate cam with an In-Line roller follower

Graphical design procedure

The following general procedure is used to graphically construct such a profile:

1. Draw the base circle of radius R_b . The size is typically a function of the spatial constraints of the application.
2. Draw the follower in the home position tangent to the base circle.
3. Draw radial lines from the center of the cam, corresponding to the cam angles identified on the displacement diagram. For construction purposes, the cam will remain stationary and the follower will be rotated in a direction opposite to the actual cam rotation.
4. Identify the trace point at the home position. For a roller follower, this is the point at the center of the roller.
5. Draw the prime circle through the trace point at its home position.
6. Transfer the displacements from the displacement diagram to the radial lines. Measure these displacements from the prime circle.
7. Draw the roller outline centered at the prescribed displacements identified in the previous step.
8. Draw a smooth curve tangent to the roller at these prescribed displacements.
9. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall intervals.



Analytical design procedure

The contact point of the roller and cam is in general slightly shifted from the line of action passing through the trace point and the axis of the cam. The angle changes from one cam angular position to another. The angular shift α is calculated as follow

$$\alpha = \tan^{-1} \left(\left(\frac{v}{\omega_{cam}} \right) \frac{1}{R_f + R_b + \Delta R} \right)$$

The profile of the cam can be constructed from the following x and y coordinates for clockwise rotation of the cam

$$\begin{cases} x = (R_f + R_b + \Delta R) \sin(\theta) - R_f \sin(\theta - \alpha) \\ y = (R_f + R_b + \Delta R) \cos(\theta) + R_f \cos(\theta - \alpha) \end{cases}$$

Where

R_b is the base circle radius

R_f is the radius of the follower

θ is the cam rotation angle measured against the direction of cam rotation from the home position

ΔR is the follower displacement at the cam angle θ

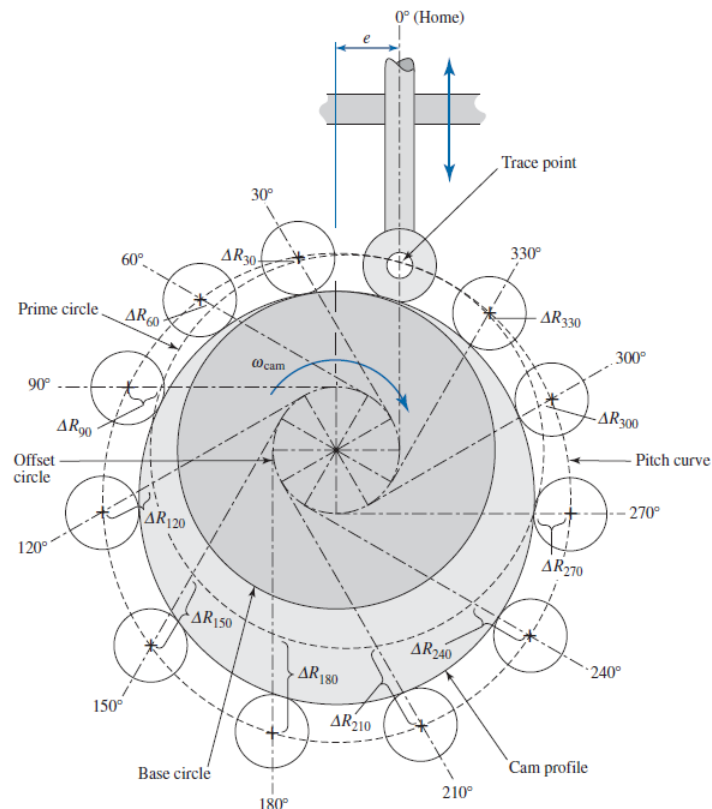
Note: A VBA program is available for this purpose

Design of a plate cam with an offset roller follower

Graphical design procedure

The following general procedure is used to graphically construct such a profile:

1. Draw the base circle of radius R_b . The size is typically a function of the spatial constraints of the application.
2. Draw the follower centerline in the home position.
3. Draw the prime circle, whose radius is equal to the sum of the base and roller follower radii
4. Draw the follower in the home position centered where the follower centerline intersects the prime circle.
5. Identify the trace point at the home position. For a roller follower, this is the point that is at the center of the roller.
6. Draw an offset circle of radius e , centered at the cam rotation axis. It will be tangent to the follower centerline.
7. Draw lines tangent to the offset circle, corresponding to the reference cam angles on the displacement diagram. For construction purposes, the cam will remain stationary and the follower will be rotated in a direction opposite to the actual cam rotation.
8. Transfer the displacements from the displacement diagram to the offset lines. Measure these displacements from the prime circle.
9. Draw the roller outline centered at the prescribed displacements identified in the previous step.
10. Draw a smooth curve tangent to the roller at these prescribed displacements.
11. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall intervals.



Analytical design procedure

The contact point of the roller and cam is in general slightly shifted from the line of action passing through the trace point and the axis of the cam. The angle changes from one cam angular position to another. The angular shift α is calculated as follow

$$\alpha = \tan^{-1} \left(\left(\frac{v}{\omega_{cam}} \right) \frac{R_f + R_b + \Delta R}{e^2 + (R_f + R_b + \Delta R)^2} - e \left(\frac{v}{\omega_{cam}} \right) \right)$$

The profile of the cam can be constructed from the following x and y coordinates for clockwise rotation of the cam

$$\begin{cases} x = -e \cos(\theta) + (R_f + R_b + \Delta R) \sin(\theta) - R_f \sin(\theta - \alpha) \\ y = -e \sin(\theta) + (R_f + R_b + \Delta R) \cos(\theta) + R_f \cos(\theta - \alpha) \end{cases}$$

Where

R_b is the base circle radius

R_f is the radius of the follower

θ is the cam rotation angle measured against the direction of cam rotation from the home position

ΔR is the follower displacement at the cam angle θ

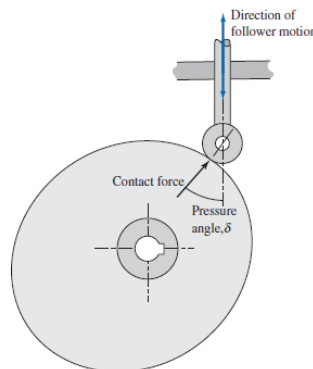
e is the offset

Note: A VBA program is available for this purpose

Pressure angle

The pressure angle is the angle between the center line of the follower and the line of action of the force pushing the follower (the perpendicular to the cam profile). For an offset roller follower this angle is equal to

$$\text{Pressure angle} = \sin^{-1} \left(\frac{e}{R_f + R_b + \Delta R} \right) + \alpha$$

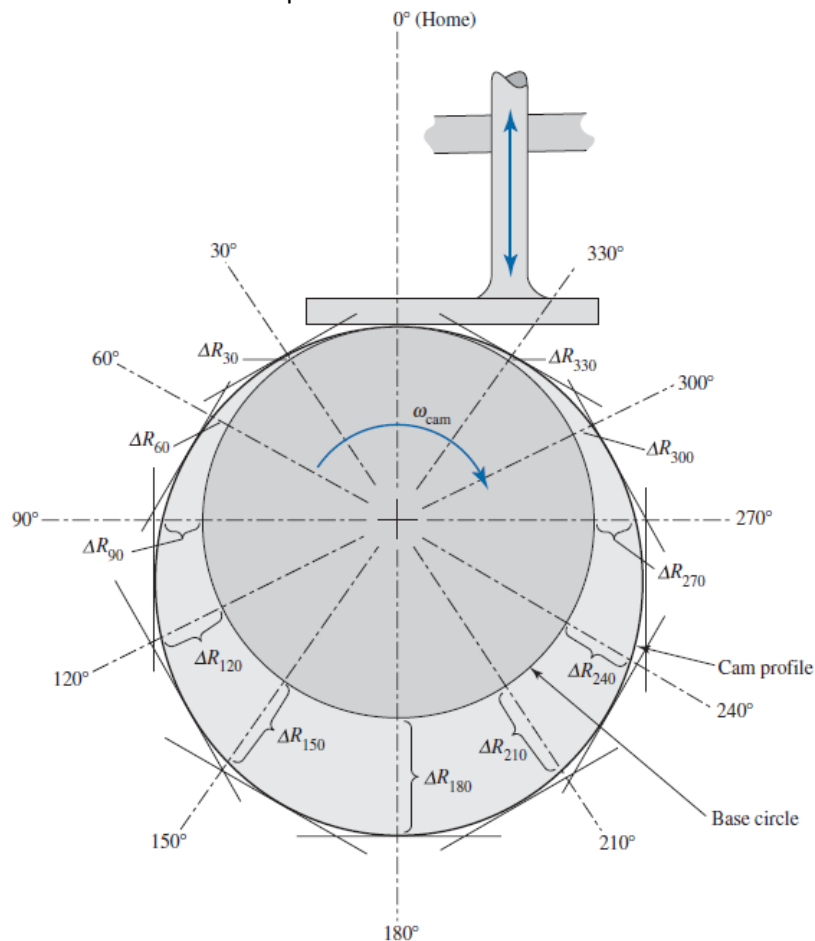


Design of a plate cam with a translating flat faced follower

Graphical design procedure

The following general procedure is used to graphically construct such a profile:

1. Draw the base circle of radius R_b . The size is typically a function of the spatial constraints of the application. Recall that for this type of follower, the base circle also serves as the prime circle.
2. Draw the follower in the home position, tangent to the base circle.
3. Draw radial lines from the center of the cam, corresponding to the cam angles on the displacement diagram. For construction purposes, the cam will remain stationary and the follower will be rotated in a direction opposite to the actual cam rotation.
4. Transfer the displacements from the displacement diagram to the radial lines, measured from the base circle.
5. Draw the flat-faced outline by constructing a line perpendicular to the radial lines at the prescribed displacements.
6. Draw a smooth curve tangent to the flat-faced outlines.
7. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall motions.



Analytical design procedure

The contact point of the flat faced follower can be in general slightly shifted from the line of action passing through the trace point and the axis of the cam. The angle changes from one cam angular position to another. The angular shift α is calculated as follow

$$\alpha = \tan^{-1} \left(\left(\frac{v}{\omega_{cam}} \right) \frac{1}{R_b + \Delta R} \right)$$

The profile of the cam can be constructed from the following x and y coordinates for clockwise rotation of the cam

$$\begin{cases} x = \left(\frac{R_b + \Delta R}{\cos(\alpha)} \right) \sin(\theta + \alpha) \\ y = \left(\frac{R_b + \Delta R}{\cos(\alpha)} \right) \cos(\theta + \alpha) \end{cases}$$

Where

R_b is the base circle radius

θ is the cam rotation angle measured against the direction of cam rotation from the home position

ΔR is the follower displacement at the cam angle θ

Note: A VBA program is available for this purpose

Design of a plate cam with a pivoted roller follower

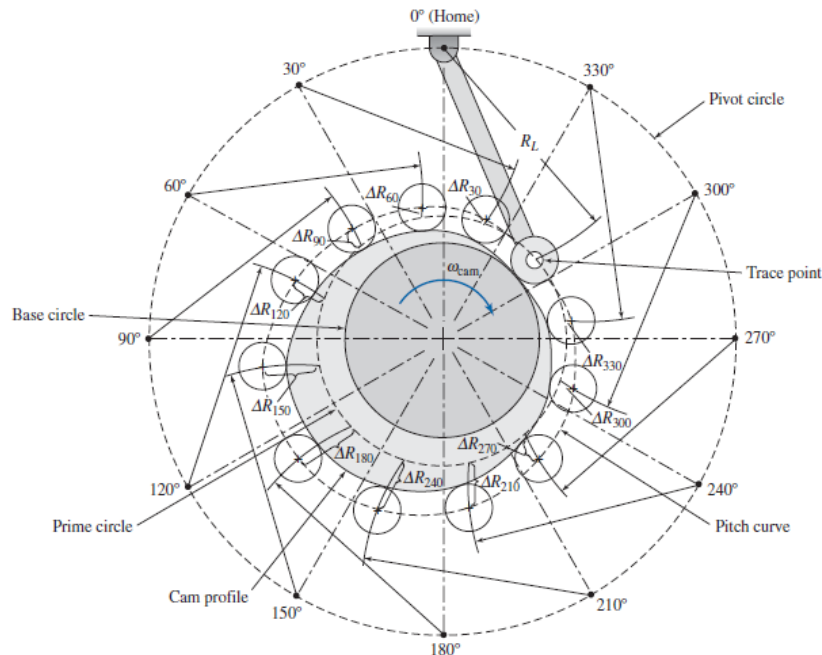
Graphical design procedure

The following general procedure is used to graphically construct such a profile:

1. Draw the base circle of radius R_b , where the size is a function of the spatial constraints of the application.
2. Draw the prime circle, whose radius is equal to the sum of the base and roller follower radii.
3. Draw the pivot circle of radius R_p . The distance between the pivot and the cam axis is also a function of the spatial constraints of the application.
4. Locate the home position of the pivot.
5. Draw an arc, centered at the home pivot, with a radius equal to the length of the pivoted follower link, R_L .
6. Draw the follower in the home position of radius R_f , centered where the arc drawn in step 5 intersects the prime circle.
7. Draw radial lines from the center of the cam to the pivot circle, corresponding to the cam angles on the displacement diagram. Recall that the follower is being rotated in a direction opposite to the cam rotation.
8. From each pivot point, draw an arc with a radius equal to the length of the follower arm, R_L , outward from the prime circle.
9. Transfer the displacements from the displacement diagram to the pivot arcs drawn in step 8. As mentioned, the prescribed displacements for a pivoted follower can be angular. The following equation can be used to convert from angular displacement of the follower link, $\Delta\theta_L$, to linear displacement of the roller center, ΔR_F .

$$\Delta R_F = R_L \sqrt{2 (1 - \cos(\Delta\theta_L))}$$

10. Draw the roller outline, centered at the prescribed displacements identified in the previous step.
11. Draw a smooth curve tangent to the roller at these prescribed displacements.
12. To accurately construct a profile consistent with the displacement diagram, it may be necessary to transfer additional intermediate points from the rise and fall motions.



Analytical design procedure

The contact point of the pivoted roller follower can in general be slightly shifted from the line of action passing through the trace point and the axis of the cam. The angle changes from one cam angular position to another. The angular shift α is calculated as follow

$$\alpha = \tan^{-1} \left(\left(\frac{v}{\omega_{cam}} \right) \frac{1}{R_b + R_f + \Delta R - \left(\frac{v}{\omega_{cam}} \right) \cos(\gamma)} \right)$$

where

$$\gamma = \cos^{-1} \left(\frac{R_L^2 + (R_f + R_b + \Delta R)^2 - R_p^2}{2 R_L (R_f + R_b + \Delta R)} \right)$$

The profile of the cam can be constructed from the following x and y coordinates for clockwise rotation of the cam

$$\begin{cases} x = (R_f + R_b + \Delta R) \cos(\theta) + R_f \sin(\theta - \alpha) \\ y = (R_f + R_b + \Delta R) \sin(\theta) + R_f \cos(\theta - \alpha) \end{cases}$$

Where

R_b is the base circle radius

θ is the cam rotation angle measured against the direction of cam rotation from the home position

ΔR is the follower displacement at the cam angle θ

R_f is the radius of the follower

R_L length of the follower pivot link

R_p distance between the cam center and the pivot location

Note: A VBA program is available for this purpose
