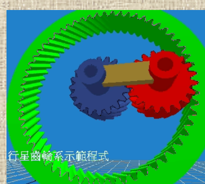
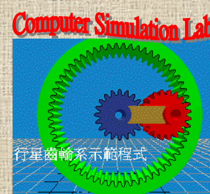
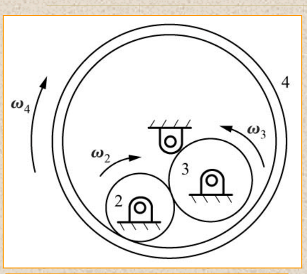
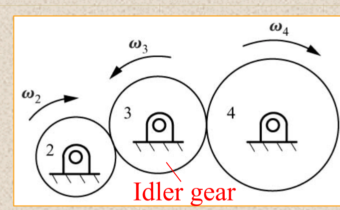
Chapter 12 Gear Trains

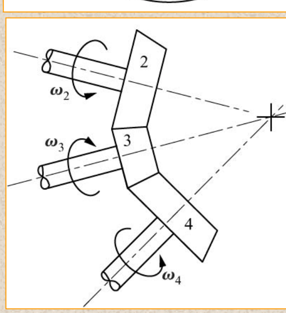




Simple Gear Trains (簡單輪系)





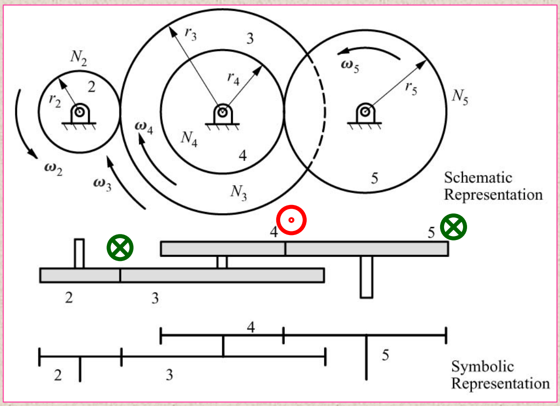


- Simple gear trains have only one gear on each shaft (各軸上皆只有一個齒輪)
- Velocity ratio, ω_i/ω_o, is a function of the numbers of teeth on the input and output gears only. That is, $\omega_i/\omega_o = N_o/N_i$. (簡單輪系之速度比僅與首尾兩輪之齒數比有關)
- Each gear must have the same normal pitch if the gears are to mesh properly (各齒輪之法周節必須相同)

Compound Gear Trains (複式齒輪系)

Compound gear trains are characterized by the presence of two or more gears attached to the same shaft (有些軸上有兩個以上的齒輪)

$$\frac{\omega_i}{\omega_o} = \pm \frac{\text{product of driven tooth numbers}}{\text{product of driver tooth numbers}}$$ = ± 從動輪之齒數乘積 / 驅動輪之齒數乘積



$$\frac{\omega_2}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_3}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_4}{\omega_5} = +\frac{N_3 N_5}{N_2 N_4}$$

Example 12.1
Analysis of a Compound Gear Train

$$r_p = \frac{Nm}{2} = \frac{N}{2P_d}$$ $$\frac{\omega_i}{\omega_o} = \pm \frac{\text{product of driven tooth numbers}}{\text{product of driver tooth numbers}}$$

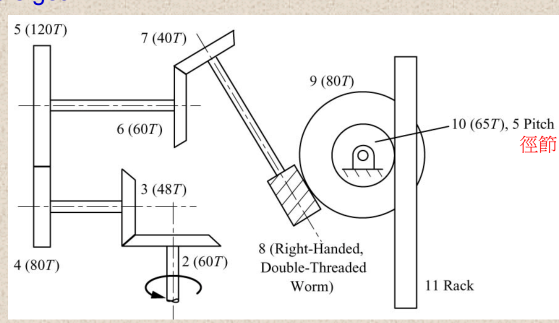
Problem:

Assume that the compound gear train in the figure has the tooth number given in parentheses. The angular velocity of gear 2 is 200 rpm in the direction shown.

Determine:

(1) the magnitude and direction of the angular velocity of gear 10,

(2) the velocity of the rack that is gear 11.



Sol:

(1) Determine $\omega_{10} = \omega_9$

$$\frac{\omega_2}{\omega_{10}} = \frac{\omega_2}{\omega_9} = +\frac{N_3 N_5 N_7 N_9}{N_2 N_4 N_6 N_8}$$

$$\frac{200}{\omega_{10}} = \frac{48}{60}\frac{120}{80}\frac{40}{60}\frac{80}{2} = 32$$

$$\omega_{10} = 200/32 = 6.25 \text{ rpm}$$

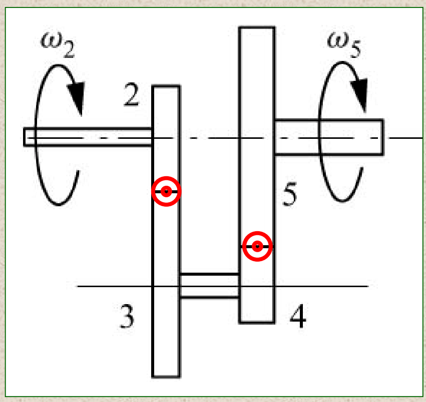
(2) Determine v_{11}

$v_{11} = r_{10}\omega_{10} = ((65/5) / 2) (6.25 (2\pi/60)) = 4.255$ in ←Ans

A Concentric Gear Reducer with a Two-Stage Reduction

$P_t = P_n \cos\psi$

$P_t = \dfrac{N}{2r}$



$r_2 + r_3 = r_4 + r_5$

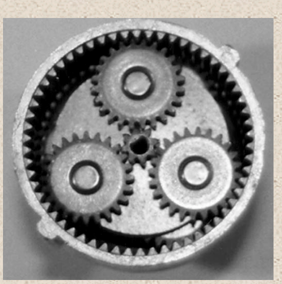
$$\frac{\omega_2}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_3}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_4}{\omega_5} = +\frac{N_3 N_5}{N_2 N_4}$$

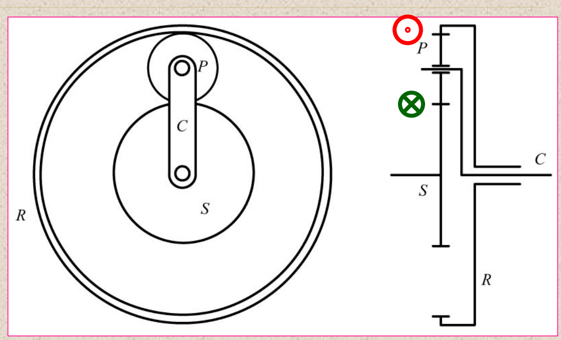
In concentric gear train, the input and output shafts are collinear

回歸齒輪系: 複式齒輪系之首尾兩輪的軸線重合 (節省空間)

Planetary Gear Trains / Epicyclic Gear Trains

行星齒輪系 / 周轉輪系





S: sun gear (太陽齒輪)

P: planet gear (行星齒輪)

C: carrier, arm (行星架, 旋臂)

R: internal ring gear (環齒輪)

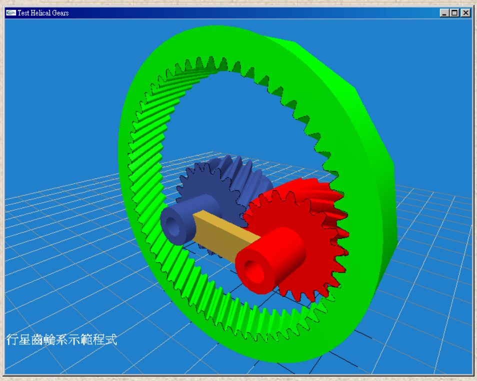
$$\frac{\omega_i - \omega_c}{\omega_j - \omega_c} = \pm\frac{N\ ...N_j}{N_i...N}$$

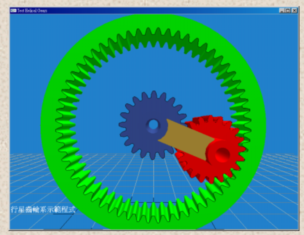
The ratio of velocities relative to the carrier

Gears i and j are gears that mesh with planet gear(s)

$$Ex: \frac{\omega_P - \omega_C}{\omega_S - \omega_C} = -\frac{N_C N_S}{N_P N_C} = -\frac{N_S}{N_P}$$

Demo





Spur gears N_s =20, N_p =20, N_R =60
Sun gear: $\omega_s = \omega_a(1+(N_r/ N_s)) = 4\omega_a$
Planet gear: $\omega_p= \omega_a(1-(N_r/ N_p)) = -2\omega_a$
Ring gear: $\omega_r= 0$
Driver: Carrier, Arm, ω_a

Sun gear: ψ_s =30°, N_s =20, ω_s =0
Planet gear: ψ_p = -30°, N_p =20, $\omega_p= \omega_a(1+(N_s/ N_p)) = 2\omega_a$
Ring gear: ψ_r= -30°, N_r=60, $\omega_r= \omega_a(1+(N_s/ N_r)) =1.333\ \omega_a$
Driver: Carrier, Arm, ω_a
Pressure angle: ϕ_n =20 °
Modulus: m_n=2

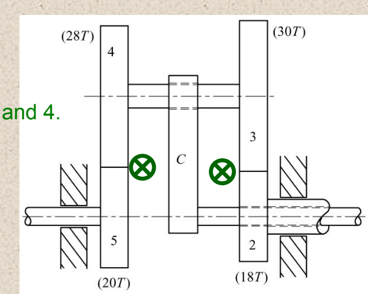
Ex: 12-4

Given:

ω_C= -150 rpm, ω_5= -50 rpm, and the tooth numbers are given in the figure

Find:

the angular velocity of gear 2



Sol:

Note that gears 2 and 5 mesh with planet gears 3 and 4.
Choosing i=2, j=5, we get

$$\frac{\omega_2-\omega_C}{\omega_5-\omega_C}=+\frac{N_5N_3}{N_2N_4}$$

$$\frac{\omega_2+150}{-50+150}=\frac{20}{18}\frac{30}{28}$$

ω_2 = -30.95 rpm ← ANS

Ex 12-5

Given:

ω_2= 60 rpm (ccw)

Find:

the angular velocity of gear 5

Sol:

Note that ω_7= 0 since gear 7 is a fixed ring gear

Besides, gears 2, 5, and 7 mesh with planet gears

(1) *i = 2, j= 5*

$$\frac{\omega_2-\omega_C}{\omega_5-\omega_C}=+\frac{N_5N_3}{N_2N_4} \rightarrow \frac{60-\omega_C}{\omega_5-\omega_C}=\frac{20}{18}\frac{30}{28}$$

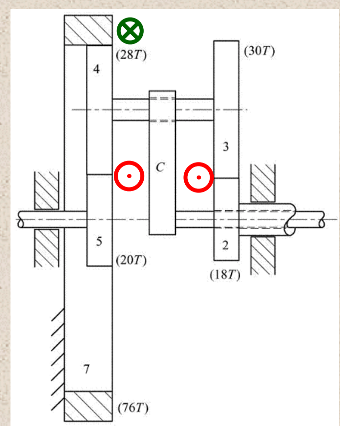
(2) *i = 2, j = 7*

$$\frac{\omega_2-\omega_C}{\omega_7-\omega_C}=-\frac{N_7N_3}{N_2N_4} \rightarrow \frac{60-\omega_C}{0-\omega_C}=-\frac{76}{18}\frac{30}{28}$$

$$\omega_C = 10.862 \text{ rpm}$$

(3) Solving (1) for ω_5 yields

$\omega_5 = 52.137$ rpm ← ANS



Ex 12-6

Given:

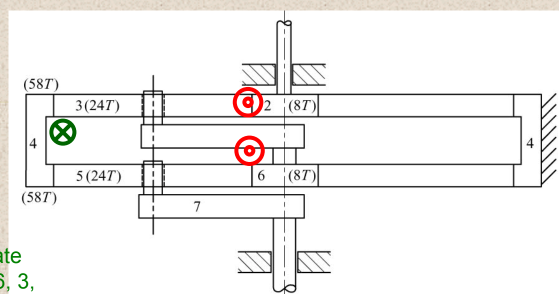
ω_2= 60 rpm (ccw), ω_4= 0

Find:

the velocity ratio ω_2 / ω_7

Sol:

There are two arms, so we separate the gears into two sets. Gears 2, 6, 3, and 4 are of set 1, while gears 6, 5, 4, and 7 are of set 2.



(1) For set 1

$$\frac{\omega_2-\omega_6}{\omega_4-\omega_6}=-\frac{N_4}{N_2} \rightarrow \frac{60-\omega_6}{0-\omega_6}=-\frac{58}{8} \rightarrow \omega_6=\frac{480}{66}=\frac{80}{11}$$

(2) For set 2

$$\frac{\omega_6-\omega_7}{\omega_4-\omega_7}=-\frac{N_4}{N_6} \rightarrow \frac{(80/11)-\omega_7}{0-\omega_7}=-\frac{58}{8} \rightarrow \omega_7 = 68.06 \text{ rpm} \leftarrow \text{ANS}$$

Ex 12-7

Given:

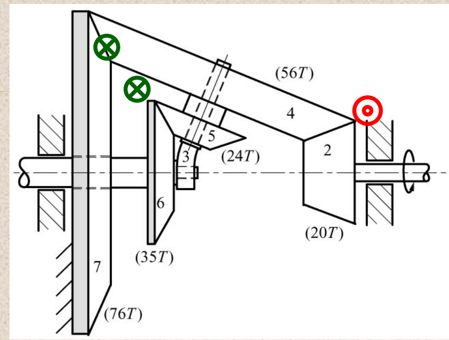
ω_2= 100 rpm (ccw)

Find:

the angular velocity of gear 6, ω_6

Sol:

1. $\omega_c = \omega_3$
2. ω_7= 0 (gear 7 is a fixed gear)
3. gears 2, 6, and 7 mesh with planet gears 4 and 5



(1) i = 2, j= 6

$$\frac{\omega_2-\omega_C}{\omega_6-\omega_C}=-\frac{N_6N_4}{N_2N_5} \rightarrow \frac{100-\omega_3}{\omega_6-\omega_3}=-\frac{35}{20}\frac{56}{24}$$

(2) i = 2, j = 7

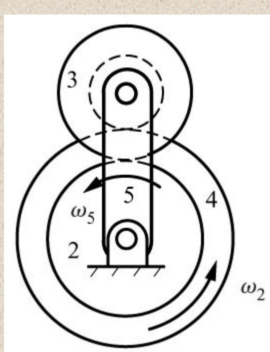
$$\frac{\omega_2-\omega_C}{\omega_7-\omega_C}=-\frac{N_7N_4}{N_2N_4}=-\frac{N_7}{N_2} \rightarrow \frac{100-\omega_3}{0-\omega_3}=-\frac{76}{20}$$

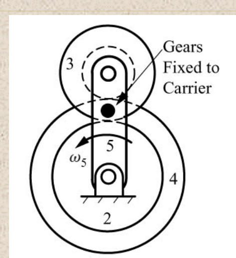
$$\omega_3 = 20.833 \text{ rpm}$$

(3) Solving (1) for ω_6 yields

$\omega_6 = 1.444$ rpm ← ANS

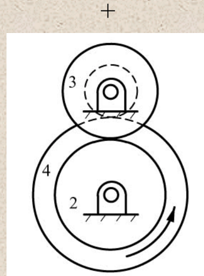
Analysis of Planetary Gear Trains Using Tabular Method





rotate with the arm
(gears locked to the arm)

+



relative to the arm
(carrier fixed)

$${}^5\omega_2 = \omega_2 - \omega_5$$

Ex 12.8

Given:

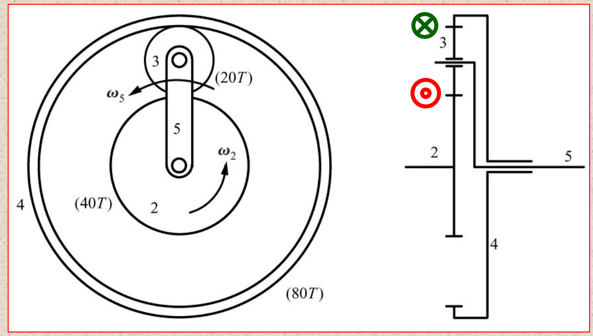
ω_2= 100 rpm (ccw)

ω_5= 200 rpm (ccw)

Find:

ω_3 and ω_4

Sol:



	Carrier 5	Gear 2	Gear 3	Gear 4
Gears locked	200	200	200	200
Carrier fixed	**0**	-100	$100(N_2/N_3)$ =+200	$200(N_3/N_4)$ =+50
Total	**200**	**100**	400	250

Ex 12.10

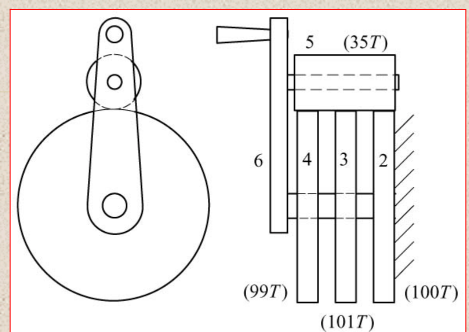
Given:

ω_6= 100 rpm (ccw)

Find:

ω_3 , ω_4, and ω_5

Sol:



	Carrier 6	Gear 2	Gear 3	Gear 4
Gears locked	100	100	100	100
Carrier fixed	**0**	-100	$-100(N_2/N_3)$ = -100/1.01	$-100(N_2/N_4)$ = -100/0.99
Total	**100**	**0**	100(101-1)/100 = 1	100(99-100)/99 = -100/99