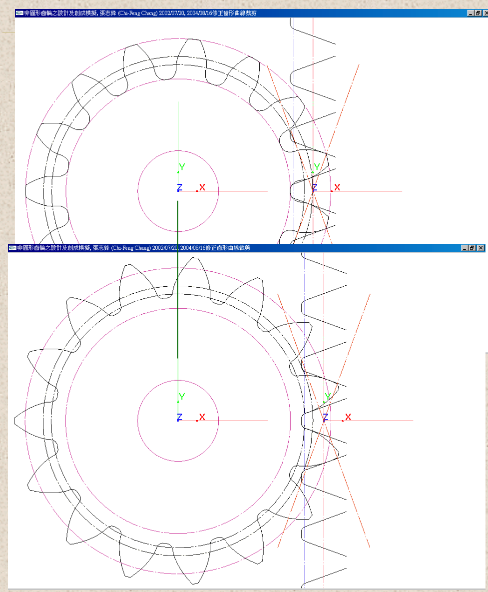
Chapter 10 Spur Gears-2

Nonstandard Spur Gears
ref: [Mabie & Reinholtz]

- There are two widely used systems for nonstandard gear:
  - Extended center distance system (加長中心距制)
  - Long and short addendum system (長短齒冠制)

- Long and short addendum system Advancing the cutter into the gear blank the same amount that it will be withdrawn from the pinion blank.
  - The working depth will be the same as if the gears were standard.
  - The center distance remains standard
  - The pressure angle remains standard

Standard and Nonstandard Gears

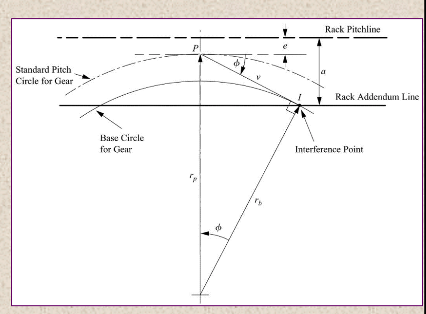


Standard Gear
Undercutting

Nonstandard Gear
No undercutting

Extended Center Distance System
(加長中心距制)

- In the *extended center distance system*, the cutter is withdrawn a certain amount from the blank so that the addendum of the basic rack passes through the interference point of the pinion when the pinion is being cut,
- When the pinion is mated with its gear, it will be found that
  - the center distance has been increased because of the decreased tooth space (齒間)
  - the pressure angle at which the gears operate increases

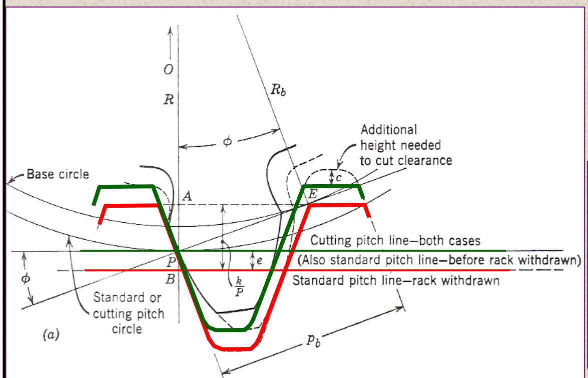


- The offset distance can be computed from

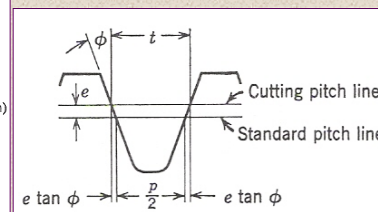
$$e = a + r_b \cos\phi - r_p = a + (r_p \cos\phi)\cos\phi - r_p$$
$$= a - r_p \sin^2\phi$$

Extended Center Distance System (cont)

- The tooth thickness is increased, but the tooth space is decreased and thus the center distance of mating gears has to be increased
  - (齒刀移位後, 小齒輪之標準節圓處的齒厚加大了, 但齒間卻縮小了, 故當此一小齒輪與其他標準齒輪嚙合時, 兩者之中心距必須比標準值大)



注意: 製作移位齒輪時, 齒條刀之切削節線仍在小齒輪原來之節圓上純滾動



Offset Distance e & The Tooth Thickness t at Cutting Pitch Circle

$$e = a - r_p \sin^2\phi$$

- The offset distance of the cutter can be computed from

$$e = a - r_p \sin^2\phi = m - \frac{mN}{2}\sin^2\phi$$

metric

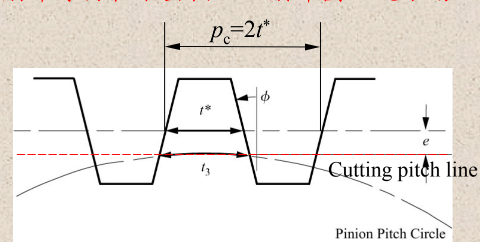
$$e = a - r_p \sin^2\phi = \frac{1}{P_d} - \frac{N}{2P_d}\sin^2\phi$$

U.S.

- The tooth thickness, *t* ,of the pinion on its cutting pitch circle can be determined from the tooth space of the rack on its cutting pitch line; i.e.,

注意: 製作移位齒輪時, 齒條刀之切削節線仍在小齒輪之切削節圓上純滾動

$$t = 2e\tan\phi + \frac{p_c}{2}$$



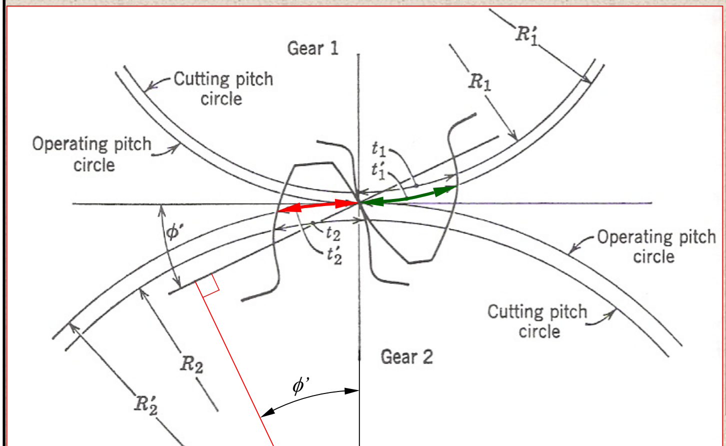
The Actual Meshing Pressure Angle ϕ' (ϕ_m)

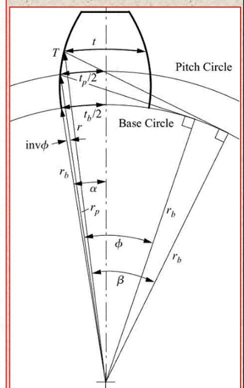
$$t = 2r\left(\frac{t_p}{2r_p} + inv\phi - inv\beta\right)$$

$$inv\xi = \tan\xi - \xi$$

$$t_1' = 2R_1'\left(\frac{t_1}{2R_1} + inv\phi - inv\phi'\right)$$

$$t_2' = 2R_2'\left(\frac{t_2}{2R_2} + inv\phi - inv\phi'\right)$$





The Actual Meshing Pressure Angle *(cont)*

$$t_1' = 2R_1'\left(\frac{t_1}{2R_1} + inv\phi - inv\phi'\right)$$

$$t_2' = 2R_2'\left(\frac{t_2}{2R_2} + inv\phi - inv\phi'\right)$$

$$\because t_1' + t_2' = \frac{2\pi R_1'}{N_1} = \frac{2\pi R_2'}{N_2}$$

$$\frac{R_1'}{R_2'} = \frac{N_1}{N_2} = \frac{R_1}{R_2}$$

$$\therefore \frac{2\pi R_2'}{N_2} = 2R_1'\left(\frac{t_1}{2R_1} + inv\phi - inv\phi'\right) + 2R_2'\left(\frac{t_2}{2R_2} + inv\phi - inv\phi'\right)$$

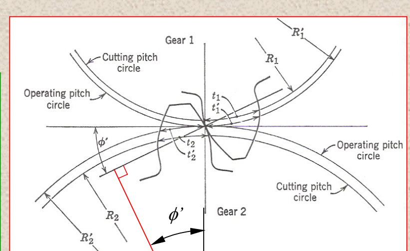
$$\frac{\pi}{N_2} = \frac{R_1'}{R_2'}\left(\frac{t_1}{2R_1} + inv\phi - inv\phi'\right) + \left(\frac{t_2}{2R_2} + inv\phi - inv\phi'\right)$$

$$\frac{\pi}{N_2} = \frac{R_1}{R_2}\left(\frac{t_1}{2R_1} + inv\phi - inv\phi'\right) + \left(\frac{t_2}{2R_2} + inv\phi - inv\phi'\right)$$

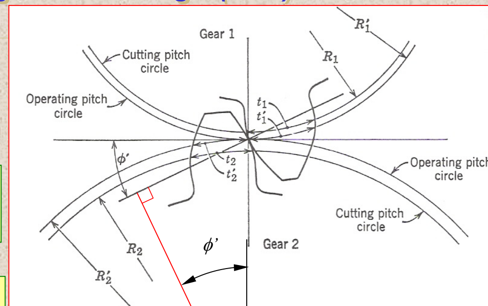
$$= \frac{t_1}{2R_2} + \frac{R_1}{R_2}(inv\phi - inv\phi') + \frac{t_2}{2R_2} + (inv\phi - inv\phi')$$

$$= \frac{t_1 + t_2}{2R_2} + \left(\frac{R_1}{R_2} + 1\right)(inv\phi - inv\phi')$$

$$= \frac{t_1 + t_2}{2R_2} + \left(\frac{N_1 + N_2}{N_2}\right)(inv\phi - inv\phi')$$



The Actual Meshing Pressure Angle *(cont)*



$$\frac{\pi}{N_2}=\frac{t_1+t_2}{2R_2}+\left(\frac{N_1+N_2}{N_2}\right)(inv\phi-inv\phi')$$

$$\frac{\pi}{N_2}-\frac{t_1+t_2}{2R_2}=\left(\frac{N_1+N_2}{N_2}\right)(inv\phi-inv\phi')$$

$$\left(\frac{N_2}{N_1+N_2}\right)\left(\frac{\pi}{N_2}-\frac{t_1+t_2}{2R_2}\right)=inv\phi-inv\phi'$$

$$inv\phi'=inv\phi-\left(\frac{\pi}{N_1+N_2}-\frac{N_2}{N_1+N_2}\frac{t_1+t_2}{2R_2}\right)$$

$$inv\phi'=inv\phi-\left(\frac{2\pi R_2-N_2(t_1+t_2)}{2R_2(N_1+N_2)}\right) \qquad (10.28)$$

Eq. (10.28) permits the computation of ϕ' if given

(a) tooth numbers: N_1, N_2
(b) cutting pitch radius R_2
(c) cutting pressure ϕ
(d) tooth thickness at cutting pitch radius: t_1, t_2

The center distance of mating gears = $R_1'+R_2'$

Example 10.4
Computing Nonstandard Gear Geometry

$$inv\phi'=inv\phi-\left(\frac{2\pi R_2-N_2(t_1+t_2)}{2R_2(N_1+N_2)}\right)$$

- A 13-tooth pinion with diametral pitch 6 and 20° cutting pressure angle is to mate with a 50-tooth gear. Find (1) *the center distance* and (2) *the meshing pressure angle* if the pinion is cut with a standard cutter offset so that the addendum line passes through the interference point

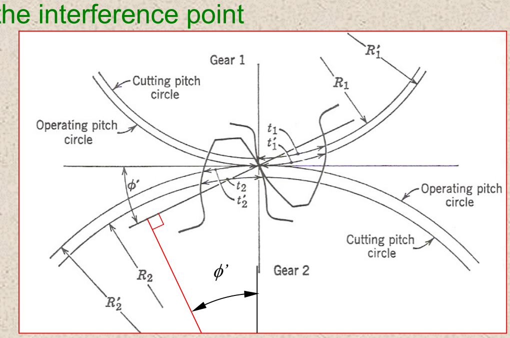
Given: $p_d=6$, $\phi=20°$, $N_1=13$, $N_2=50$.
pinion 1 is nonstandard and gear 2 is standard

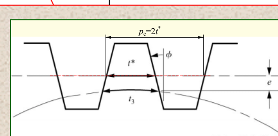
(1) Find t_1 and t_2

$$t_2=\frac{p_c}{2}=\frac{\pi}{2p_d}=0.2618$$

$$e=\frac{1}{p_d}-\frac{N_1}{2p_d}\sin^2\phi=\frac{1}{6}-\frac{13}{2(6)}\sin^2 20^\circ$$
$$=0.03994$$

$$t_1=2e\tan\phi+\frac{p_c}{2}=0.29087$$





Example 10.4 (cont)

(2) Find ϕ'

$$t_1 = 2e\tan\phi + \frac{p_c}{2} = 0.29087$$

$$t_2 = \frac{p_c}{2} = \frac{\pi}{2p_d} = 0.2618$$

$$r_2 = \frac{t_2 N_2}{\pi} = 4.1667$$

$$inv\phi' = inv\phi - \left(\frac{2\pi r_2 - N_2(t_1+t_2)}{2r_2(N_1+N_2)}\right)$$

$$inv\phi' = inv20^\circ - \frac{2\pi(4.1667) - 50(0.29087+0.2618)}{2(4.1667)(13+50)}$$
$$= 0.017673$$

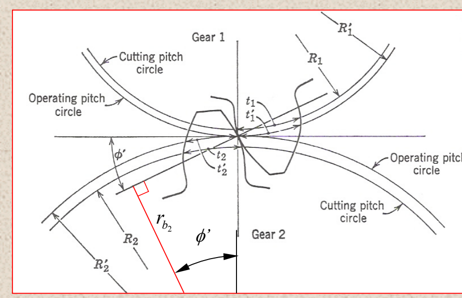
$$\phi' = 21.127^\circ$$

(3) Find the center distance

$$r_2' = \frac{r_{b_2}}{\cos\phi_m} = \frac{r_2\cos\phi}{\cos\phi_m} = 4.1975$$

$$r_1' = \frac{r_{b_1}}{\cos\phi_m} = \frac{r_1\cos\phi}{\cos\phi_m} = \frac{N_1}{2p_d}\frac{\cos\phi}{\cos\phi_m} = 1.0914$$

$$C - C' = r_2' + r_1' = 4.1975 + 1.0914 = 5.2889\ in$$



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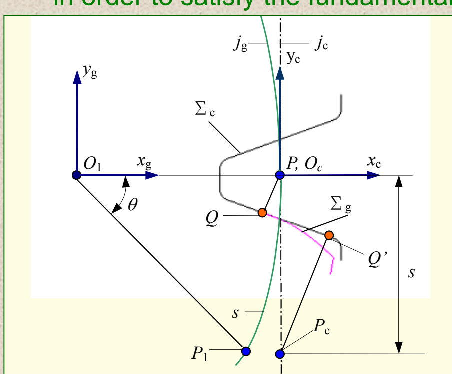
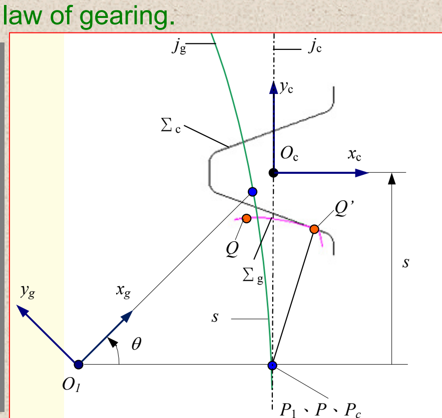
Equation of Meshing

- The normal vector **n** at a mesh point must pass through pitch point *P* (I) in order to satisfy the fundamental law of gearing.



Initial position
(note: *Q* is a meshing point because the normal at that point passing through pitch point *P*)



Point *Q'* becomes a meshing point when the normal passes *P*

Equation of Meshing (cont.)

- Thus, the following condition must be satisfied if point $Q(x_c, y_c)$ on the rack cutter is to be a meshing point

$\mathbf{v}_s \bullet \mathbf{n} = 0$

$v_s=(PQ)\omega_g$=sliding velocity
n=normal vector at Q

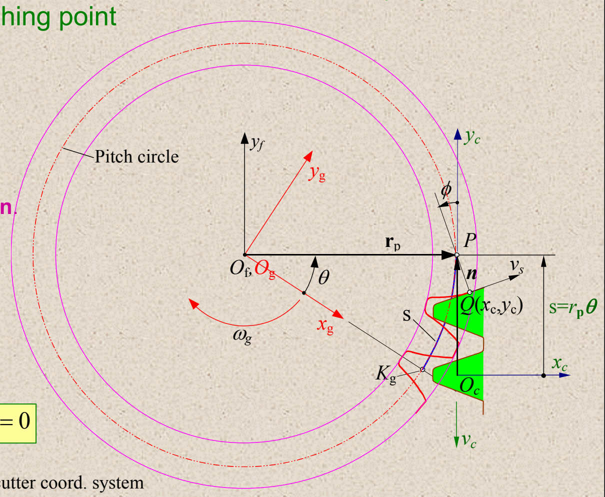
Since $\mathbf{v}_s \perp PQ$, we have $PQ \,//\, \mathbf{n}$.
That is,

$$\frac{P_y - y_c}{P_x - x_c} = \frac{n_y}{n_x}$$

$$P_x = 0$$
$$P_y = s = r_p\theta$$

$$-n_y x_c + n_x(y_c - r_p\theta) = 0$$

Cartesian coordinates of P in terms of rack cutter coord. system



Coordinates Transformation--Method 1: from cutter coord. O_c-x_cy_c to gear coord O_g-x_gy_g

$$-n_y x_c + n_x(y_c - r\theta) = 0$$

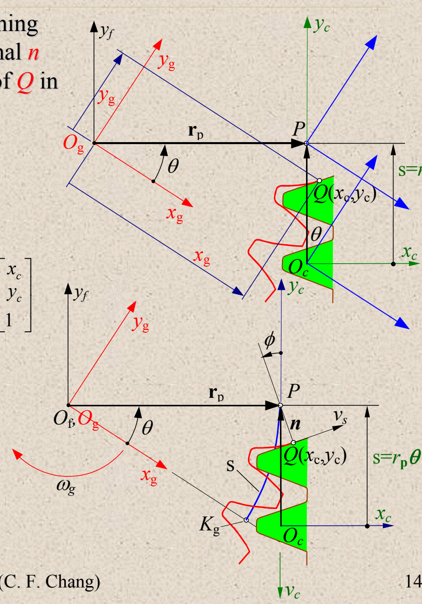
The values of θ and s can be determined if meshing point Q on the rack and the corresponding normal n are given. We can then determine the position of Q in gear coordinate system as follows

$$\begin{bmatrix} x_g \\ y_g \\ 1 \end{bmatrix} = T(\mathbf{r}_p)T(-\mathbf{s})R(\theta)\begin{bmatrix} x_c \\ y_c \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_g \\ y_g \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & r_p\cos\theta \\ 0 & 0 & r_p\sin\theta \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & s\sin\theta \\ 0 & 0 & -s\cos\theta \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_c \\ y_c \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_g \\ y_g \\ 1 \end{bmatrix} = \begin{bmatrix} x_c\cos\theta - y_c\sin\theta + s\sin\theta + r_p\cos\theta \\ x_c\sin\theta + y_c\cos\theta - s\cos\theta + r_p\sin\theta \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_g \\ y_g \\ 1 \end{bmatrix} = \begin{bmatrix} (x_c + r_p)\cos\theta - (y_c - s)\sin\theta \\ (x_c + r_p)\sin\theta + (y_c - s)\cos\theta \\ 1 \end{bmatrix}$$



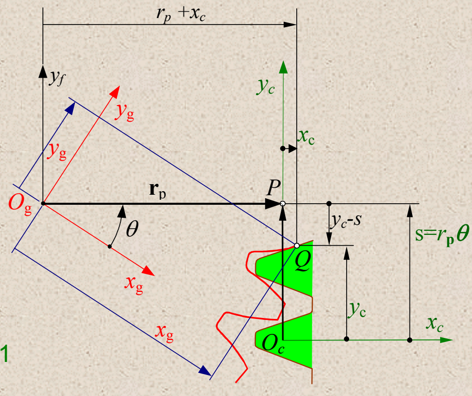
Coordinates Transformation--Method 2: from fixed coord. O_f-$x_f y_f$ to gear coord O_g-$x_g y_g$

- Let's expressed the position of meshing point Q in terms of fixed coordinates. That is
  - $X = x_c + r_p$
  - $Y = y_c - s$
- Then, we can determine the position of the meshing point by using the following coordinate transformation

$$\begin{bmatrix} x_g \\ y_g \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c + r_p \\ y_c - s \\ 1 \end{bmatrix}$$

- It can be seen that the result is the same as that obtained from method 1

$$\begin{bmatrix} x_g \\ y_g \\ 1 \end{bmatrix} = \begin{bmatrix} (x_c + r_p)\cos\theta - (y_c - s)\sin\theta \\ (x_c + r_p)\sin\theta + (y_c - s)\cos\theta \\ 1 \end{bmatrix}$$



Geometry of a Rack

$$\begin{bmatrix} x_g \\ y_g \\ 1 \end{bmatrix} = \begin{bmatrix} (x_c + r_p)\cos\theta - (y_c - s)\sin\theta \\ (x_c + r_p)\sin\theta + (y_c - s)\cos\theta \\ 1 \end{bmatrix}$$

$$\boxed{-n_y x_c + n_x (y_c - r\theta) = 0}$$

Taking section 1 as an example, we have

$$\mathbf{R}_c^{(1)} = \begin{bmatrix} x_c^{(1)} \\ y_c^{(1)} \end{bmatrix} = \begin{bmatrix} a_c \\ u^{(1)} \end{bmatrix}$$

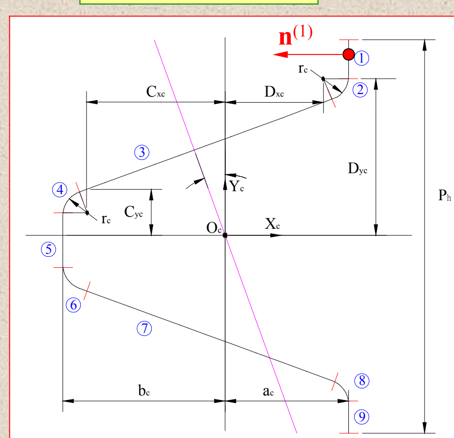
$$\mathbf{n}_c^{(1)} = \begin{bmatrix} n_{xc}^{(1)} \\ n_{yc}^{(1)} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\mathbf{u}^{(1)} = t^{(1)}\mathbf{A} + (1 - t^{(1)})\mathbf{B}$$

$$t^{(1)} = 1 \sim 0$$

$$\mathbf{A} = \frac{P_h}{2}$$

$$\mathbf{B} = \frac{P_h}{4} + a_c \tan\phi_c + r_c\left[(\sin\phi_c - 1)\tan\phi_c + \cos\phi_c\right]$$



Hence (x_c, y_c) and **n** can be determined by specifying the value of t. Then, (x_g, y_g) is determined by specifying

$$s = u^{(1)} \qquad \theta = \frac{s}{r_p}$$