Chapter 2

Graphical Position, Velocity, and Acceleration Analysis for Mechanisms with Revolute Joints or Fixed Slides

Computer Simulation Lab

General Velocity Equation (ref pp. 98-100)

Velocity of a Point Moving on Moving Frame

In the course of Dynamics, we had shown that

1. For any vector, $\mathbf{r}$, rotating with angular velocity, ω, we have

$$\boxed{\dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}}$$

2. For any point, P, moving on the moving frame

$$\boxed{\mathbf{r}_p = \mathbf{r}_o + \boldsymbol{\rho}}$$

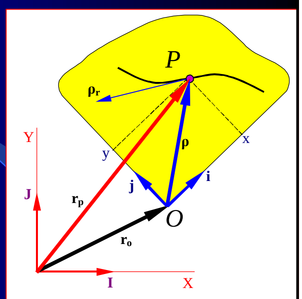
$$\boxed{\mathbf{v}_p = \dot{\mathbf{r}}_p = \dot{\mathbf{r}}_o + \dot{\boldsymbol{\rho}}}$$

$$\boxed{\therefore \mathbf{v}_p = \dot{\mathbf{r}}_o + \boldsymbol{\omega} \times \boldsymbol{\rho} + \dot{\boldsymbol{\rho}}_r}$$

$\dot{\boldsymbol{\rho}}_r$: P相對於動座標系之速度

$\boldsymbol{\omega} \times \boldsymbol{\rho}$: 動座標系旋轉所造成之速度, $\perp\rho$

$\dot{\mathbf{r}}_o$: 動座標系平移所造成之速度



$$\boldsymbol{\rho} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\dot{\boldsymbol{\rho}} = (\dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}) + (x\dot{\mathbf{i}} + y\dot{\mathbf{j}} + z\dot{\mathbf{k}})$$

$$= \dot{\boldsymbol{\rho}}_r + \{x(\boldsymbol{\omega} \times \mathbf{i}) + y(\boldsymbol{\omega} \times \mathbf{j}) + z(\boldsymbol{\omega} \times \mathbf{k})\}$$

$$= \dot{\boldsymbol{\rho}}_r + \boldsymbol{\omega} \times \{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}\}$$

$$= \dot{\boldsymbol{\rho}}_r + \boldsymbol{\omega} \times \boldsymbol{\rho}$$

$$\boxed{\dot{\boldsymbol{\rho}}_r \equiv \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}}$$

$$\boxed{\dot{\boldsymbol{\rho}} = \boldsymbol{\omega} \times \boldsymbol{\rho} + \dot{\boldsymbol{\rho}}_r}$$

General Acceleration Equation (ref. p. 101)

Acceleration of a Point Moving on Moving Frame

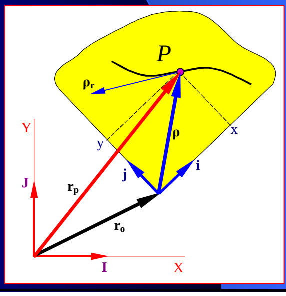
In the course of Dynamics, we also shown that

$$\mathbf{v}_p = \dot{\mathbf{r}}_o + \boldsymbol{\omega}\times\boldsymbol{\rho} + \dot{\boldsymbol{\rho}}_r$$

For any point, *P*, moving on the moving frame, the acceleration of point can be expressed as

$$\mathbf{a}_p = \ddot{\mathbf{r}}_o + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho}) + \dot{\boldsymbol{\omega}}\times\boldsymbol{\rho} + 2\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_r + \ddot{\boldsymbol{\rho}}_r$$

- $\ddot{\boldsymbol{\rho}}_r$：P在動座標系上運動之相對加速度
- $2\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_r$：動座標系旋轉所造成之科氏速度
- $\dot{\boldsymbol{\omega}}\times\boldsymbol{\rho}$：動座標系旋轉所造成之切線加速度
- $\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho})$：動座標系旋轉所造成之法向加速度
- $\ddot{\mathbf{r}}_o$：動座標系平移所造成之加速度



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Deviation of the Acceleration

$$\mathbf{v}_p = \dot{\mathbf{r}}_o + \boldsymbol{\omega}\times\boldsymbol{\rho} + \dot{\boldsymbol{\rho}}_r \qquad \dot{\boldsymbol{\rho}} = \boldsymbol{\omega}\times\boldsymbol{\rho} + \dot{\boldsymbol{\rho}}_r \qquad \dot{\boldsymbol{\rho}}_r \equiv \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}$$

$$\begin{aligned}
\mathbf{a}_p &= \ddot{\mathbf{r}}_o + \boldsymbol{\omega}\times\dot{\boldsymbol{\rho}} + \dot{\boldsymbol{\omega}}\times\boldsymbol{\rho} + \frac{d}{dt}(\dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}) \\
&= \ddot{\mathbf{r}}_o + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho} + \dot{\boldsymbol{\rho}}_r) + \dot{\boldsymbol{\omega}}\times\boldsymbol{\rho} + \{(\dot{x}\dot{\mathbf{i}} + \dot{y}\dot{\mathbf{j}} + \dot{z}\dot{\mathbf{k}}) + (\ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k})\} \\
&= \ddot{\mathbf{r}}_o + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho}) + \boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_r + \dot{\boldsymbol{\omega}}\times\boldsymbol{\rho} + \{\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_r + \ddot{\boldsymbol{\rho}}_r\} \\
&= \ddot{\mathbf{r}}_o + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho}) + \dot{\boldsymbol{\omega}}\times\boldsymbol{\rho} + 2\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_r + \ddot{\boldsymbol{\rho}}_r
\end{aligned}$$

- $\ddot{\boldsymbol{\rho}}_r$：P在動座標系上之相對加速度
- $2\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_r$：動座標系旋轉所造成之科氏速度
- $\dot{\boldsymbol{\omega}}\times\boldsymbol{\rho}$：動座標系旋轉所造成之切線加速度, $\perp\boldsymbol{\rho}$
- $\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho})$：動座標系旋轉所造成之法向加速度, 指向迴轉中心, $//\ \boldsymbol{\rho}$
- $\ddot{\mathbf{r}}_o$：動座標系平移所造成之加速度

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Velocity and Acceleration Relationships of Two Points Embedded in a Moving Lamina

- In the case under consideration, we may select point A as the reference point
- Since points A and B are embedded in a rigid body, we have

$$\dot{\boldsymbol{\rho}}_r = \ddot{\boldsymbol{\rho}}_r = 0$$

由於兩點皆固定在同一剛體上, 故由剛體上觀察時(相當於剛體靜止不動), A, B兩點之間並無任何相對運動

- Thus

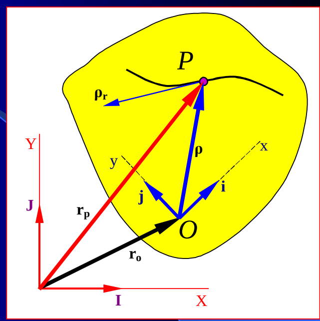
$$\mathbf{v}_p = \dot{\mathbf{r}}_o + \boldsymbol{\omega}\times\boldsymbol{\rho} + \cancel{\dot{\boldsymbol{\rho}}_r}$$

$$\mathbf{a}_p = \ddot{\mathbf{r}}_o + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{\rho}) + \dot{\boldsymbol{\omega}}\times\boldsymbol{\rho} + \cancel{2\boldsymbol{\omega}\times\dot{\boldsymbol{\rho}}_r} + \cancel{\ddot{\boldsymbol{\rho}}_r}$$

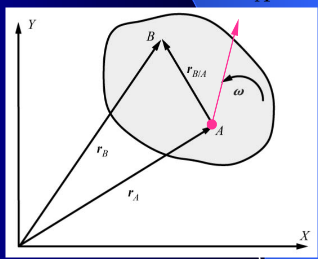
- Or

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}\times\mathbf{r}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}_{B/A}) + \dot{\boldsymbol{\omega}}\times\mathbf{r}_{B/A}$$



$\mathbf{v}_A$



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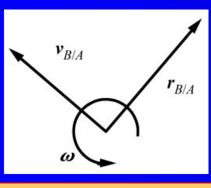
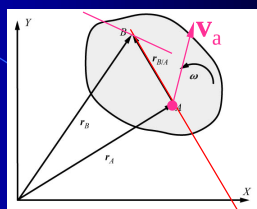
Basic Technique for Drawing Velocity Polygon

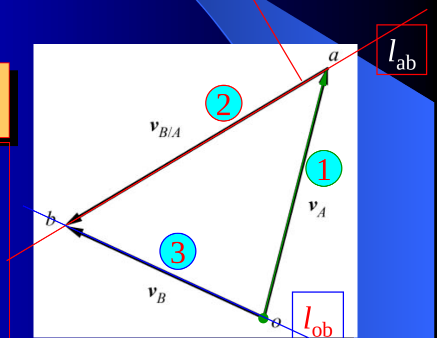
$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}\times\mathbf{r}_{B/A} = \mathbf{v}_A + \mathbf{v}_{B/A}$$



$|\mathbf{v}_{B/A}|=|\mathbf{r}_{B/A}|\omega$

$\mathbf{v}_{B/A} \perp \mathbf{r}_{B/A}$





Given: $\mathbf{v}_A$ and the direction of $\mathbf{v}_B$

Find: $\mathbf{v}_B$ and $\mathbf{v}_{B/A}$

Sol:

1. Choose a scale S_v
2. Draw $\mathbf{v}_A$
3. Draw line l_{ab} through a and $\perp AB$
4. Draw line l_{ob} through o and // $\mathbf{v}_B$ to intersect l_{ab} at b
5. $\mathbf{v}_B$ = (ob) S_v , $\mathbf{v}_{B/A}$ = (ab) S_v

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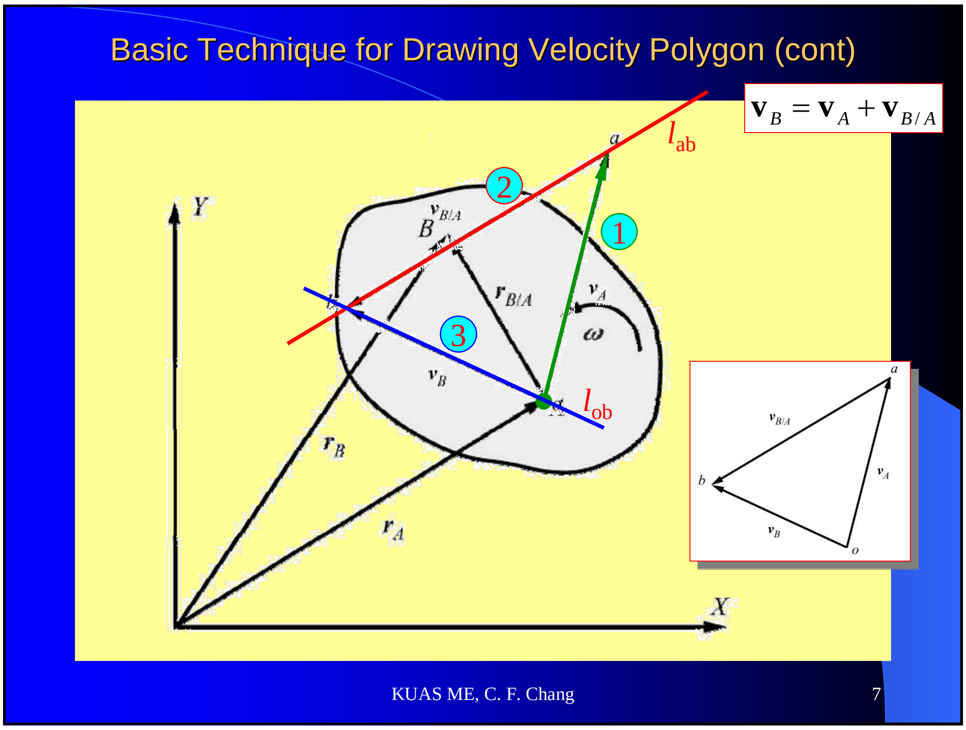
Basic Technique for Drawing Velocity Polygon (cont)
$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$
l_{ab}
l_{ob}
1
2
3
Y
X
B
$v_{B/A}$
$r_{B/A}$
v_A
ω
v_B
r_B
r_A
a
b
o
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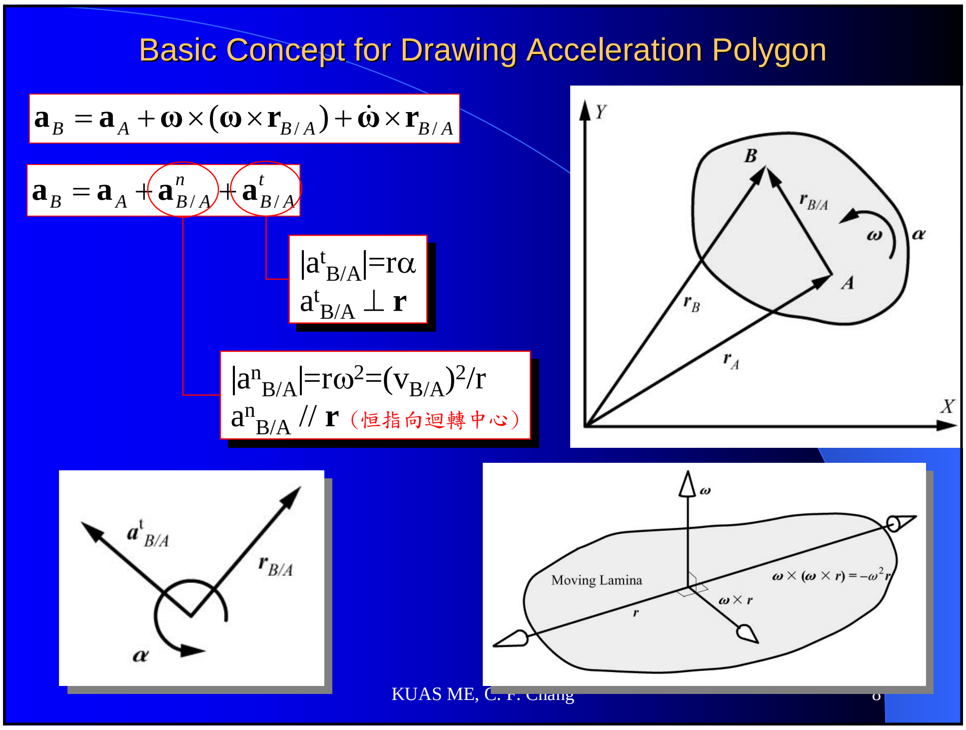
Basic Concept for Drawing Acceleration Polygon
$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{B/A}) + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{B/A}$
$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}^n_{B/A} + \mathbf{a}^t_{B/A}$
$|a^t_{B/A}| = r\alpha$
$a^t_{B/A} \perp \mathbf{r}$
$|a^n_{B/A}| = r\omega^2 = (v_{B/A})^2/r$
$a^n_{B/A} \,//\, \mathbf{r}$ （恒指向迴轉中心）
Y
X
B
A
$r_{B/A}$
ω
α
r_B
r_A
$a^t_{B/A}$
$r_{B/A}$
α
Moving Lamina
ω
$\omega \times (\omega \times r) = -\omega^2 r$
$\omega \times r$
r
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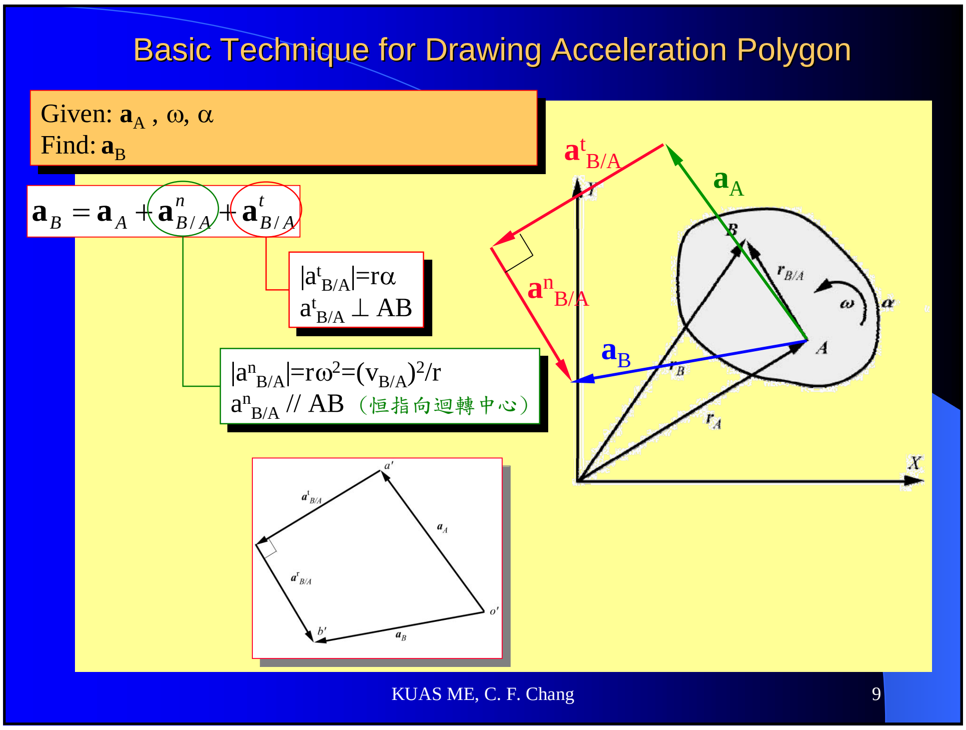
Basic Technique for Drawing Acceleration Polygon
Given: $\mathbf{a}_A$, ω, α
Find: $\mathbf{a}_B$
$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}^n_{B/A} + \mathbf{a}^t_{B/A}$
$|a^t_{B/A}|=r\alpha$
$a^t_{B/A} \perp AB$
$|a^n_{B/A}|=r\omega^2=(v_{B/A})^2/r$
$a^n_{B/A}$ // AB （恒指向迴轉中心）
$a^t_{B/A}$
a_A
$a^n_{B/A}$
a_B
Y
X
B
A
$r_{B/A}$
r_B
r_A
ω
α
a'
b'
o'
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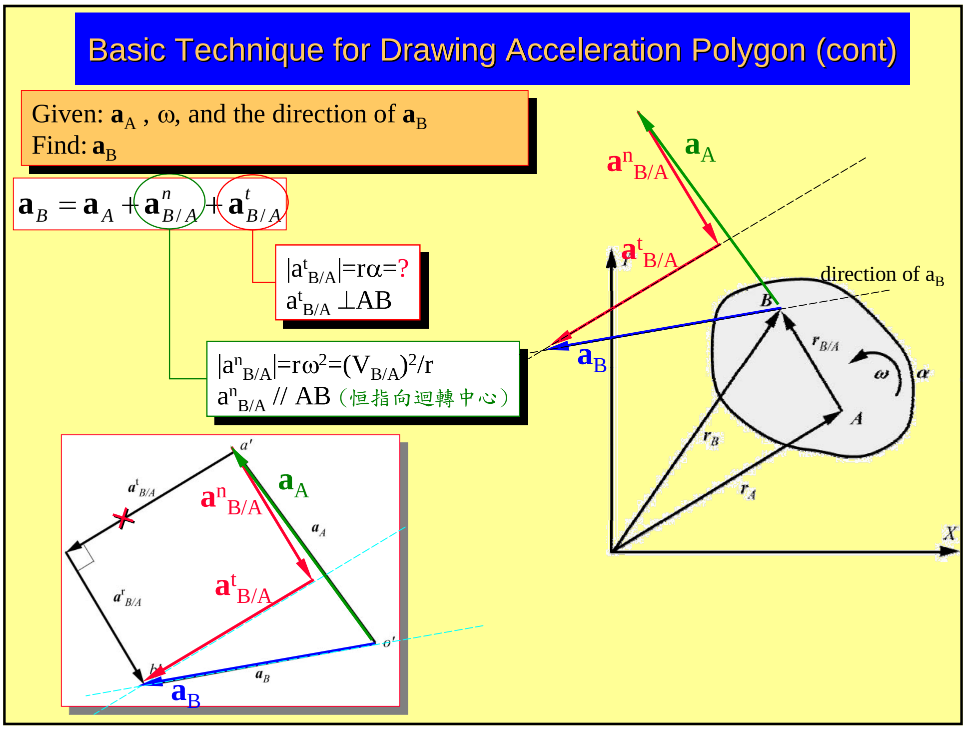
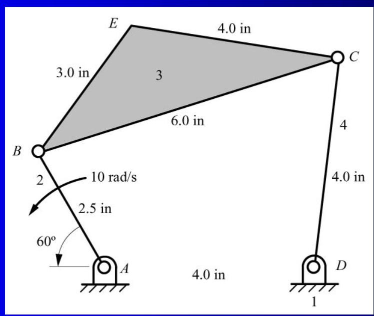

Basic Technique for Drawing Acceleration Polygon (cont)
Given: $\mathbf{a}_A$, ω, and the direction of $\mathbf{a}_B$
Find: $\mathbf{a}_B$
$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}^n_{B/A} + \mathbf{a}^t_{B/A}$
$|a^t_{B/A}|=r\alpha=?$
$a^t_{B/A} \perp AB$
$|a^n_{B/A}|=r\omega^2=(V_{B/A})^2/r$
$a^n_{B/A}$ // AB（恒指向迴轉中心）
$a^n_{B/A}$
a_A
$a^t_{B/A}$
direction of a_B
a_B
B
A
$r_{B/A}$
r_B
r_A
ω
α
Y
X
a'
o'
b'

Example 2-1 (pp. 67-74)

- Given:
  - Link *AB* is driven at a *constant angular velocity* of 10 rad/s CCW
  - Link *AB* is at 60° to the horizontal as shown in the figure
- Find:
  - angular positions,
  - angular velocities, and
  - angular accelerations of all members of the linkage



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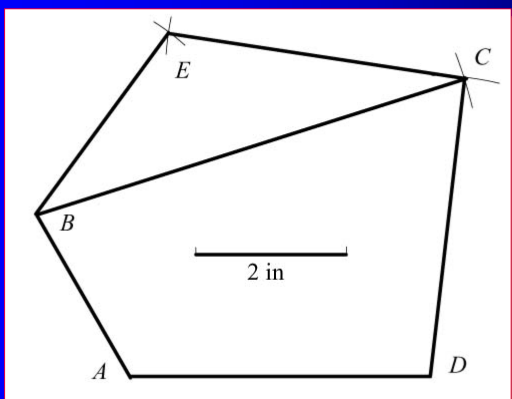
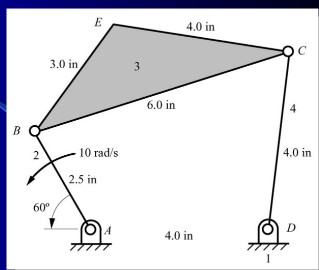
Example 2-1 : Position Analysis

Scale: *1-in on the drawing* corresponds to *2-in on the actual mechanism*





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Example 2-1 : Velocity Analysis

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$|\mathbf{v}_B| = |\mathbf{v}_{B/A}| = (AB)\omega_2 = (2.5\ in)(10\ rad/s) = 25\ in/s$
$\mathbf{v}_{B/A} \perp AB$

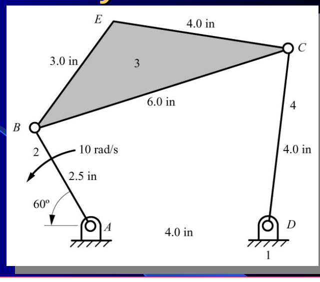
$|\mathbf{v}_{C/B}| = (BC)\omega_3 = (6.0\ in)(\omega_3\ rad/s) = ?$
$\mathbf{v}_{C/B} \perp BC$

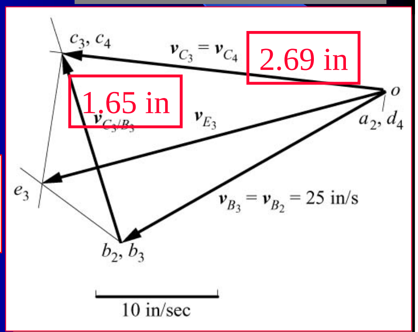
$|\mathbf{v}_C| = |\mathbf{v}_{C/D}| = (CD)\omega_4 = (4.0\ in)(\omega_4\ rad/s) = ?$
$\mathbf{v}_{C/D} \perp CD$

Scale: *1-in on the drawing* corresponds to *10-in/s on the actual velocity* ➔ $S_v = 10$

Measuring the lengths on the velocity polygon gives
$|\mathbf{v}_{C/B}| = (bc)S_v = (1.65)(10) = 16.5\ in/s$
$|\mathbf{v}_C| = (oc)S_v = (2.69)(10) = 26.9\ in/s$

$\omega_3 = |\mathbf{v}_{C/B}|/(BC) = 16.5/6.0 = 2.75\ rad/s$, *CCW* ← *ANS*
$\omega_4 = |\mathbf{v}_C|/(CD) = 26.9/4.0 = 6.73\ rad/s$, *CCW* ← *ANS*





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Example 2-1 : Acceleration Analysis

$\omega_3 = |\mathbf{v}_{C/B}|/(BC) = 16.5/6.0 = 2.75\ rad/s$
$\omega_4 = |\mathbf{v}_C|/(CD) = 26.9/4.0 = 6.73\ rad/s$

$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}^n_{C/B} + \mathbf{a}^t_{C/B}$

$(\mathbf{a}^n_C + \mathbf{a}^t_C) = (\mathbf{a}^n_B + \mathbf{a}^t_B) + (\mathbf{a}^n_{C/B} + \mathbf{a}^t_{C/B})$

$|\mathbf{a}^n_B| = (AB)(\omega_2)^2 = 2.5(10)^2 = 250\ in/s^2$, //AB
$|\mathbf{a}^t_B| = (AB)(\alpha_2) = 2.5(0) = 0\ in/s^2$, $\perp AB$

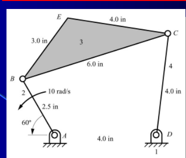
$|\mathbf{a}^n_{C/B}| = (BC)(\omega_3)^2 = (6.0)(2.75)^2 = 45.4\ in/s^2$, //BC
$|\mathbf{a}^t_{C/B}| = (BC)(\alpha_3) = (6.0)\ \alpha_3 = ?\ in/s^2$, $\perp BC$

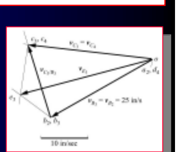
$|\mathbf{a}^n_C| = (CD)(\omega_4)^2 = (4.0)(6.73)^2 = 181.2\ in/s^2$, //CD
$|\mathbf{a}^t_C| = (CD)(\alpha_4) = (4.0)\ \alpha_4 = ?\ in/s^2$, $\perp CD$

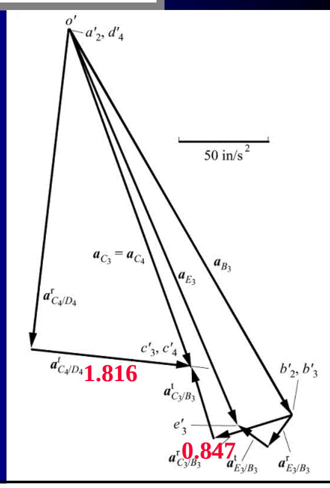
Scale: *1-in on the drawing* corresponds to *50-in/s²* *on the actual acceleration* ➔ $S_a = 50$

Measuring the lengths on the acceleration polygon yields
$|\mathbf{a}^t_{C/B}| = (bc)S_a = (0.847)(50) = 42.2\ in/s^2$
$|\mathbf{a}^t_C| = (oc)S_a = (1.816)(50) = 90.8\ in/s^2$

$\alpha_3 = |\mathbf{a}^t_{C/B}|/(BC) = 42.2/6.0 = 7.06\ rad/s^2$ ← *ANS*
$\alpha_4 = |\mathbf{a}^t_C|/(CD) = 90.8/4.0 = 22.7\ rad/s^2$ ← *ANS*







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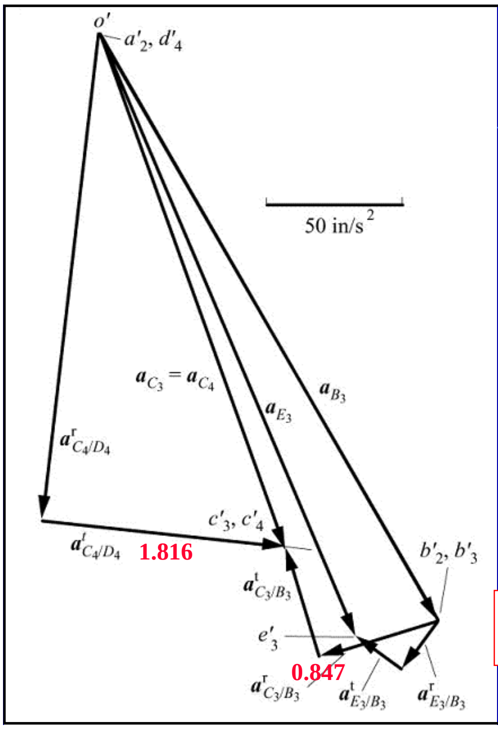

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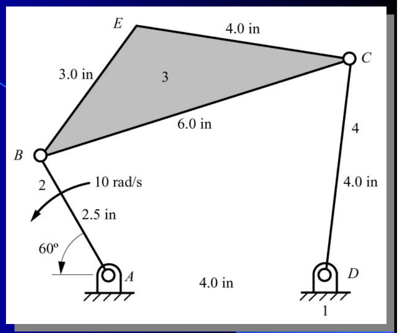

$$(\mathbf{a}_C^n + \mathbf{a}_C^t) = (\mathbf{a}_B^n + \mathbf{a}_B^t) + (\mathbf{a}_{C/B}^n + \mathbf{a}_{C/B}^t)$$

$|a^t_{C/B}|$ =(bc)S_a=(0.847)(50)=42.2 *in/s²*
$|a^t_C|$ =(oc)S_a=(1.816)(50)=90.8 *in/s²*

α_3=$|a^t_{C/B}|$/(BC) =42.2/6.0=7.06 *rad/s²* ← *ANS*
α_4=$|a^t_C|$/(CD) =90.8/4.0=22.7 *rad/s²* ← *ANS*

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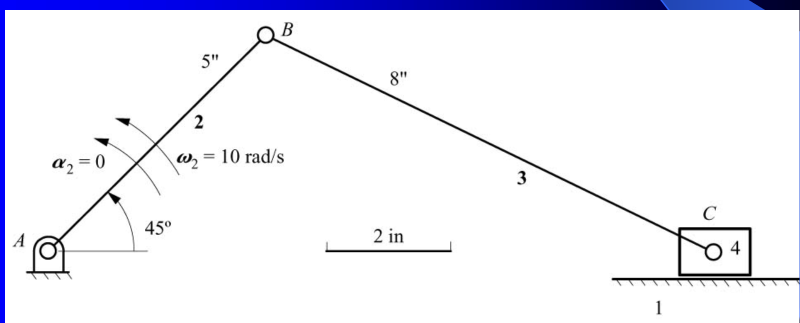
Example 2-2 (pp. 74-76)

- Given:
  - Link *AB* is driven at a *constant angular velocity* of 10 rad/s CCW
  - Link *AB* is at 45° to *AC* as shown in the figure
- Find:
  - angular positions,
  - angular velocities of link 3 (ω_3)
  - accelerations of the slider 4 (a_c)



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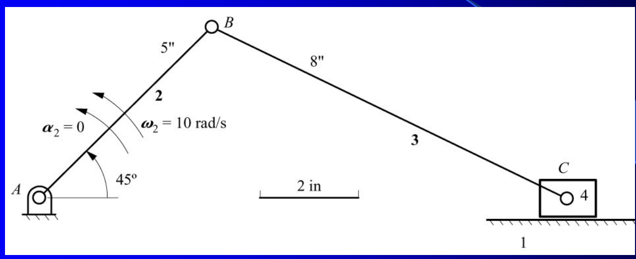
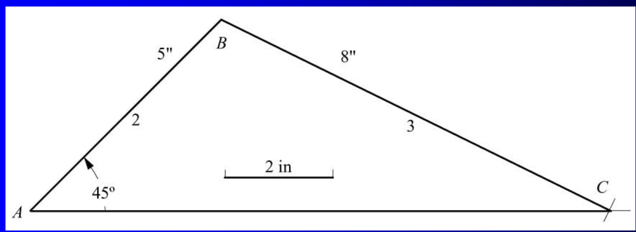
Example 2-2 : Position Analysis

Scale: *1-in on the drawing* corresponds to *2-in on the actual mechanism* ➔ $S_p=2$





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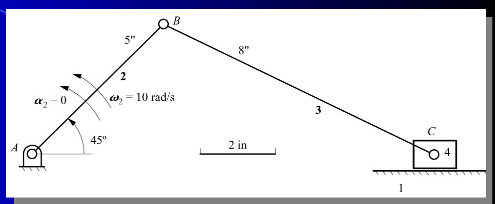
Example 2-2 : Velocity Analysis

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

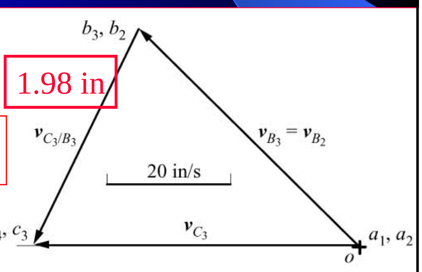
$|v_B|$=(AB)ω_2= (5 in)(10 rad/s) =5* *in/s*
$v_{B/A} \perp AB$

$|v_{C/B}|$=(BC)ω_3= (8.0 *in*)(ω_3 *rad/s*) =?
$v_{C/B} \perp BC$

$|v_C|$= ?
$v_{C/D}$ // (sliding direction, *AC*)



Scale: *1-in on the drawing* corresponds to *20-in/s on the actual velocity* ➔ $S_v=20$



Measuring the lengths on the velocity polygon gives
$|v_{C/B}|$ =(bc)S_v=(1.98)(20)=39.6 *in/s*

ω_3=$|v_{C/B}|$/(BC) =39.6/8.0=4.95 *rad/s, CW* ➔*ANS*

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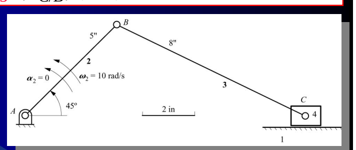
Example 2-2 : Acceleration Analysis

$\omega_3=|v_{C/B}|/(BC)=39.6/8.0=4.95$ *rad/s*



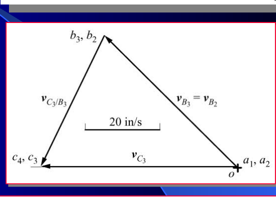
$$\mathbf{a}_C=\mathbf{a}_B+\mathbf{a}^n_{C/B}+\mathbf{a}^t_{C/B}$$

$$(\mathbf{a}^n_C+\mathbf{a}^t_C)=(\mathbf{a}^n_B+\mathbf{a}^t_B)+(\mathbf{a}^n_{C/B}+\mathbf{a}^t_{C/B})$$

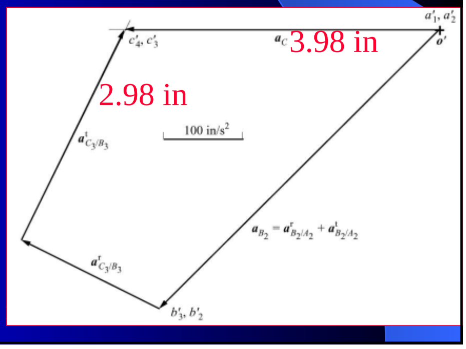
$|a^n_B|=(AB)(\omega_2)^2=5(10)^2=500$ *in/s²*, //*AB*
$|a^t_B|=(AB)(\alpha_2)=2.5(0)=0$ *in/s²*, ⊥ *AB*

$|a^n_{C/B}|=(BC)(\omega_3)^2=(8.0)(4.95)^2=196$ *in/s²*, //*BC*
$|a^t_{C/B}|=(BC)(\alpha_3)=(6.0)\ \alpha_3=?$ *in/s²*, ⊥ *BC*

$|a^n_C|=0$
$|a^t_C|=?$ *in/s²*, //sliding direction *AD*



Scale: *1-in on the drawing* corresponds to *100-in/s² on the actual acceleration* ➔ $S_a=100$



By measurement in figure, we get
$|a^t_{C/B}|=(bc)S_a=(2.98)(100)=298$ *in/s²*
$|a^t_C|=(oc)S_a=(3.98)(100)=398$ *in/s²* ←*ANS*

$\alpha_3=|a^t_{C/B}|/(BC)=298/8=37.3$ *rad/s²*

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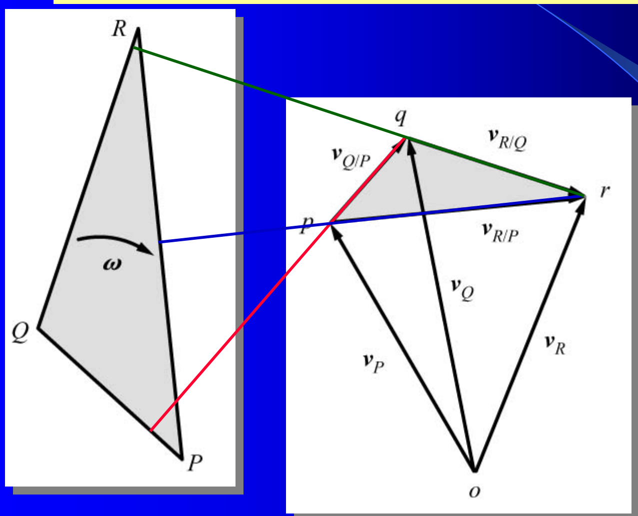
The Velocity Image Theorem

- Triangle ***pqr*** in the velocity polygon <u>is similar to</u> Link *PQR* and is rotated from it by 90° in the ω direction



$$\mathbf{v}_{Q/P}=\boldsymbol{\omega}\times\mathbf{r}_{Q/P}, \perp\mathbf{r}_{Q/P}$$

$$\mathbf{v}_{R/Q}=\boldsymbol{\omega}\times\mathbf{r}_{R/Q}, \perp\mathbf{r}_{R/Q}$$

$$\mathbf{v}_{R/P}=\boldsymbol{\omega}\times\mathbf{r}_{R/P}, \perp\mathbf{r}_{R/P}$$

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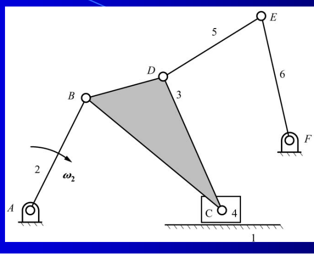
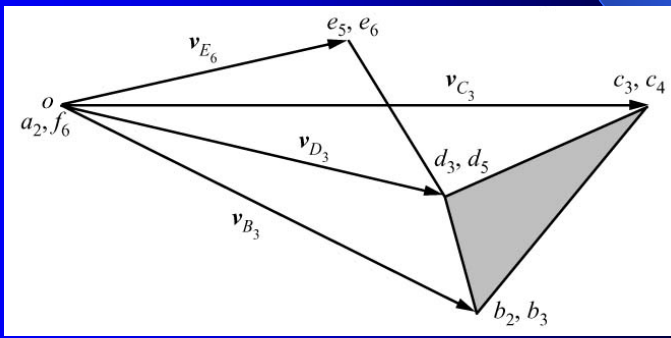
Example 2.3: Using Velocity Image Theorem





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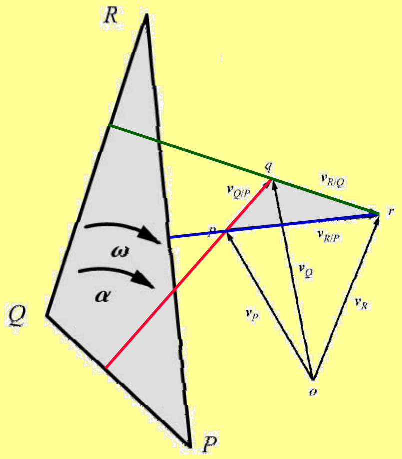
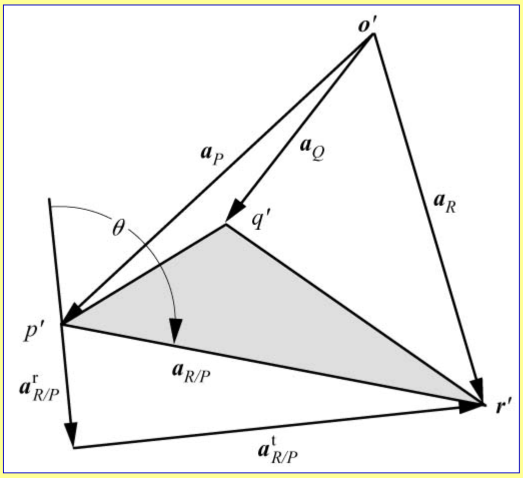
The Acceleration Image Theorem

- Triangle *p'q'r'* in the acceleration polygon <u>is similar to</u> Link *PQR*





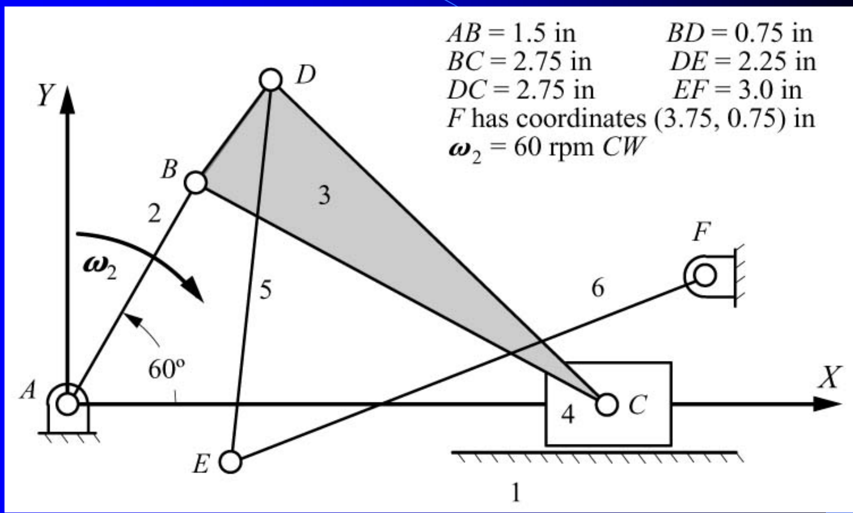
$$\left|\mathbf{a}_{R/P}\right| = \left|\mathbf{a}^n_{R/P} + \mathbf{a}^t_{R/P}\right| = \sqrt{(\overline{PR}\omega^2)^2 + (\overline{PR}\alpha)^2} = \overline{PR}\sqrt{\omega^4 + \alpha^2}$$

$$\left|\mathbf{a}_{Q/R}\right| = \overline{RQ}\sqrt{\omega^4 + \alpha^2}$$

$$\left|\mathbf{a}_{P/Q}\right| = \overline{QP}\sqrt{\omega^4 + \alpha^2}$$

$$\therefore \left|\mathbf{a}_{R/P}\right| : \left|\mathbf{a}_{Q/R}\right| : \left|\mathbf{a}_{P/Q}\right| = \overline{PR} : \overline{RQ} : \overline{QP}$$

Example 2.4 : Problem Statement (Fig. 2-16)



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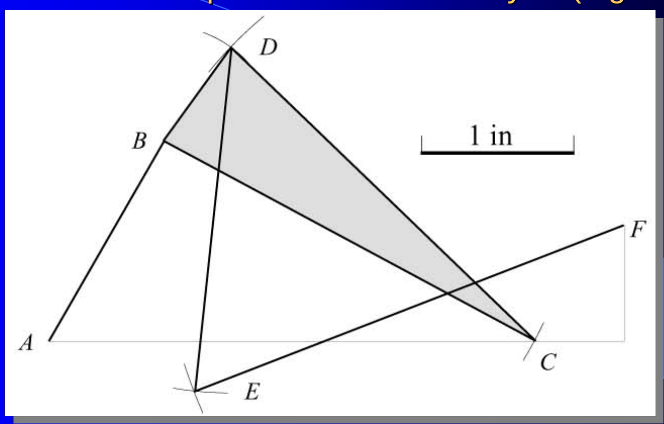
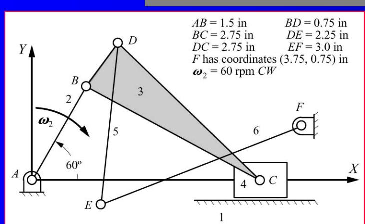
Solution to Example 2.4: Position Analysis (Fig2.17)





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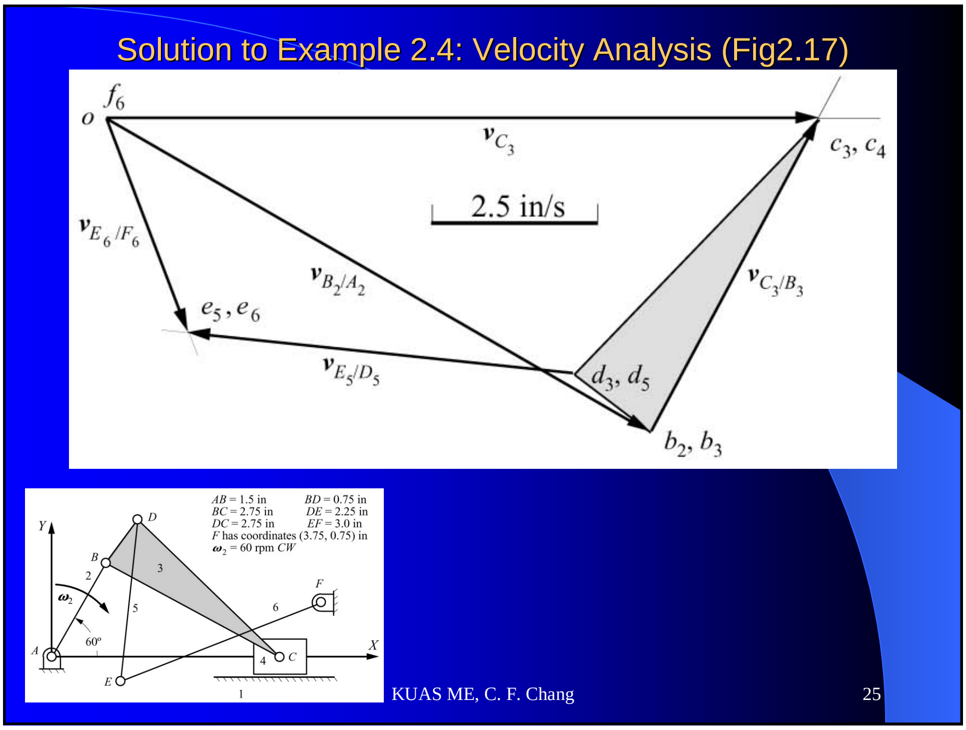
Solution to Example 2.4: Velocity Analysis (Fig2.17)
2.5 in/s
AB = 1.5 in BD = 0.75 in
BC = 2.75 in DE = 2.25 in
DC = 2.75 in EF = 3.0 in
F has coordinates (3.75, 0.75) in
ω2 = 60 rpm CW
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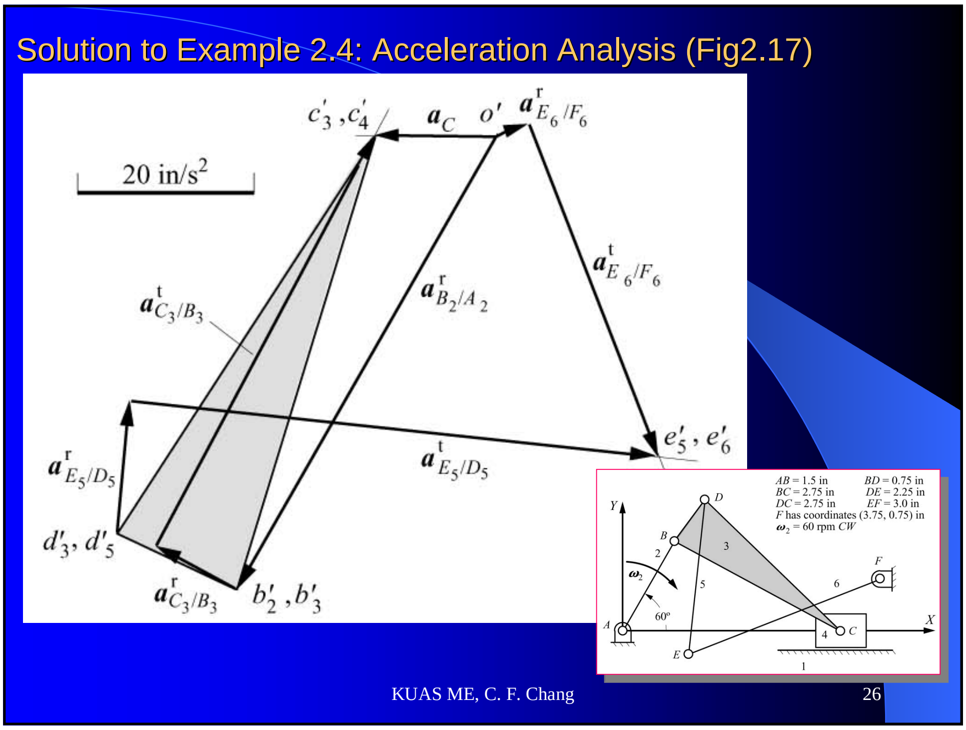
Solution to Example 2.4: Acceleration Analysis (Fig2.17)
20 in/s2
AB = 1.5 in BD = 0.75 in
BC = 2.75 in DE = 2.25 in
DC = 2.75 in EF = 3.0 in
F has coordinates (3.75, 0.75) in
ω2 = 60 rpm CW
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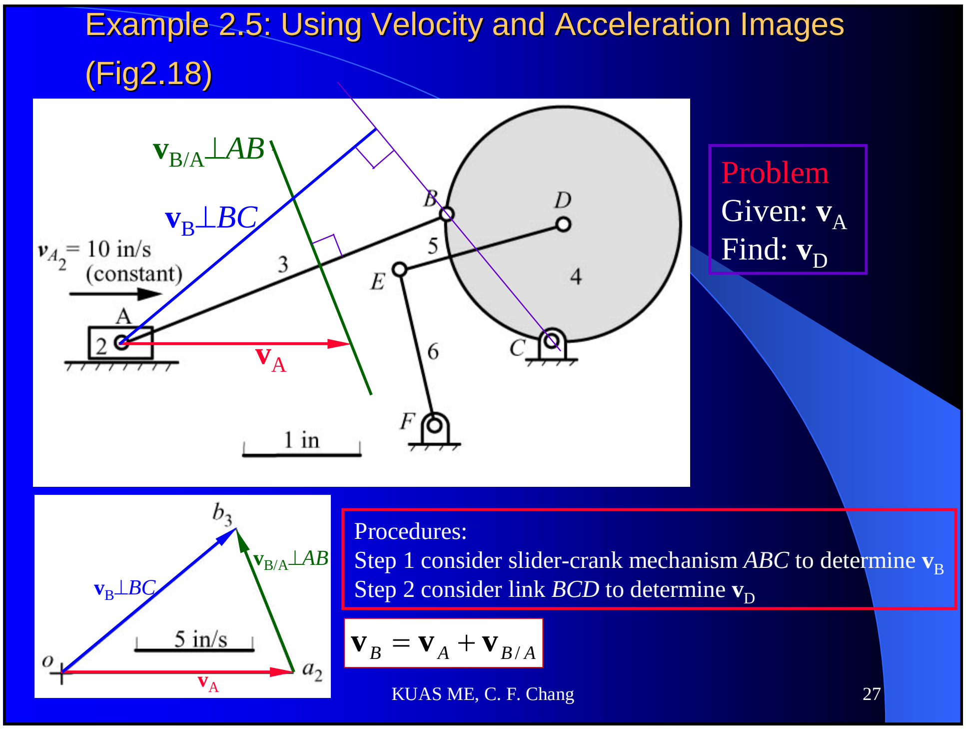
Example 2.5: Using Velocity and Acceleration Images (Fig2.18)
$\mathbf{v}_{B/A} \perp AB$
$\mathbf{v}_B \perp BC$
v_{A_2} = 10 in/s (constant)
A
2
3
B
D
5
4
E
C
6
F
$\mathbf{v}_A$
1 in
Problem
Given: $\mathbf{v}_A$
Find: $\mathbf{v}_D$
b_3
$\mathbf{v}_{B/A} \perp AB$
$\mathbf{v}_B \perp BC$
5 in/s
o
a_2
$\mathbf{v}_A$
Procedures:
Step 1 consider slider-crank mechanism *ABC* to determine $\mathbf{v}_B$
Step 2 consider link *BCD* to determine $\mathbf{v}_D$
$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$
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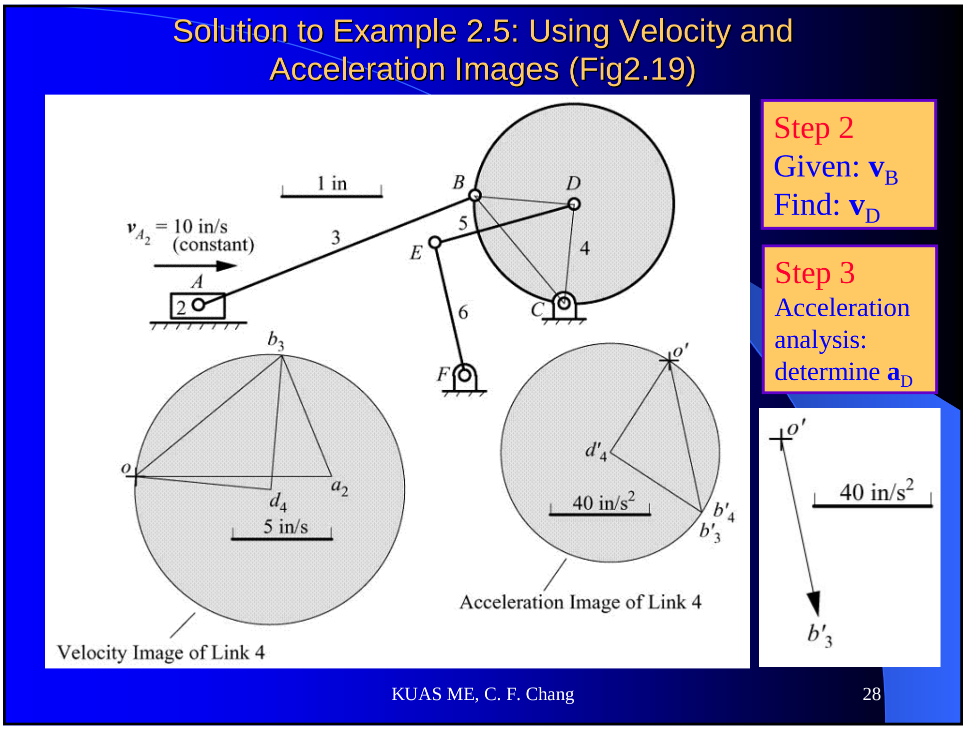
Solution to Example 2.5: Using Velocity and Acceleration Images (Fig2.19)
1 in
v_{A_2} = 10 in/s (constant)
A
2
3
B
D
5
4
E
C
6
F
b_3
o
a_2
d_4
5 in/s
Velocity Image of Link 4
o'
d'_4
40 in/s^2
b'_4
b'_3
Acceleration Image of Link 4
Step 2
Given: $\mathbf{v}_B$
Find: $\mathbf{v}_D$
Step 3
Acceleration analysis:
determine $\mathbf{a}_D$
o'
40 in/s^2
b'_3
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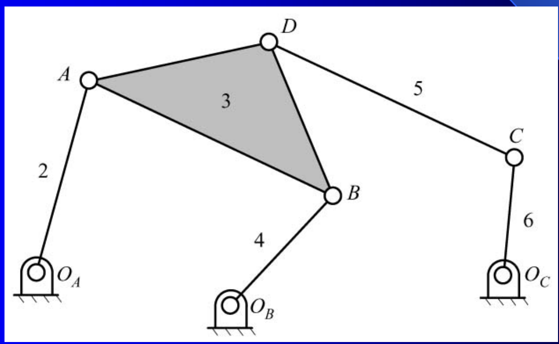
Solution by Inversion (Fig2.20)

- If the Stephenson linkage is driven by O_AA or O_BB, it can be analyzed by using the techniques of the preceding sections.
- However, this linkage cannot be solved graphically without the use of inversion techniques if it is driven by O_CC



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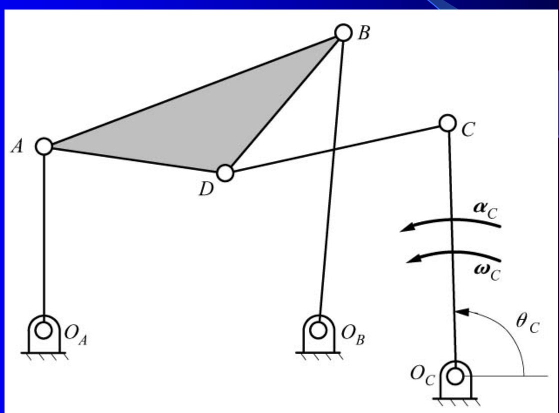
Example 2.6: Velocity Analysis by Inversion (Fig2.21)

- Find the angular velocities of all members of the linkage for the position in which θ_C=135°



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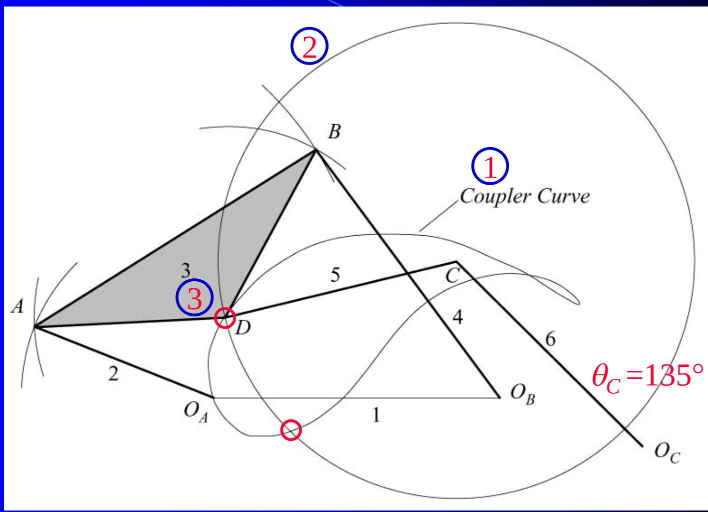
Position Solution for Example 2.6 (Fig2.22)



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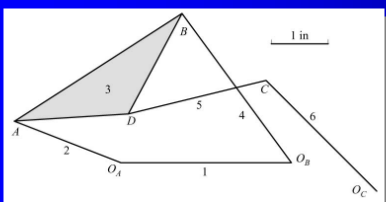
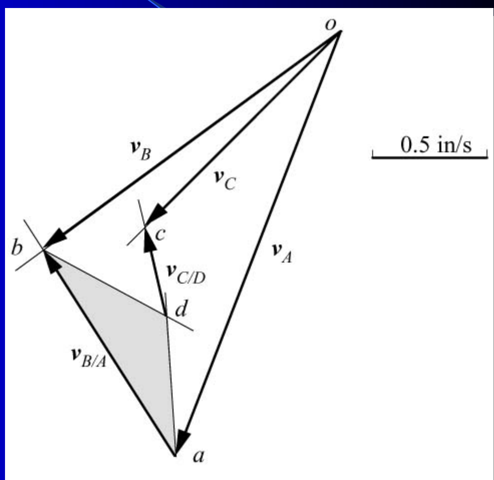
Velocity Solution for Example 2.6 (Fig2.23)

- Assume the angular velocity of link 2 to be Ω_2=1 ras/s
- By using the preceding techniques, the angular velocity of link 6 is found to be Ω_6=0.441 ras/s

- Since the specified angular velocity of link 6 is ω_6=10 ras/s, so the actual angular velocities of the other links should be multiplied by $\sigma=\omega_6/\ \Omega_6$=10/0.441=22.7; i.e., $\omega_i=\ \sigma\Omega_i$ (i=2, 3, 4, 5)





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