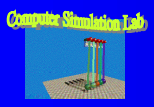
Chapter 4

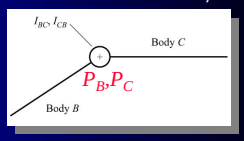
Instant Centers of Velocity



Definition of Instant Center (ref pp. 145-147)

If points P_C and P_B are fixed on different bodies, but they are coincident and have the same velocity at an instant, the location of the coincident point is called *the instant center of velocity* for the two bodies. That is,

$$\mathbf{v}_{P_B} = \mathbf{v}_{P_C} \quad \text{or} \quad \mathbf{v}_{P_B/P_C} = \mathbf{v}_{P_C/P_B} = 0$$

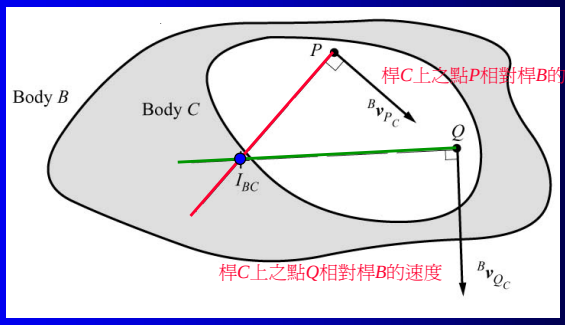


簡言之, 瞬心是兩桿件之一個重合點, 兩桿件在該點有相同之速度(即無相對速度)

- In general, there is only one instant center for two bodies.
- If more than one location is found to be an instant center for two bodies, then all locations are instant centers and the two bodies can be considered to be instantaneously fixed to each other.
- If no finite location can be found as an instant center of relative motion of two bodies, then the two bodies are translating w.r.t each other.
- Permanent instant center: points A and B are permanently coincident
- Instantaneous instant center: points A and B are momentarily coincident

Location of an Instant Center from the Directions of Two Velocities (ref pp. 147-148)

※Points *P* and *Q* are fixed in body C
Given: the directions of *relative velocities* $\mathbf{v}_P$ and $\mathbf{v}_Q$ (relative to body *B*)
Find: the instant center, I_{BC}, for bodies *B* and *C*



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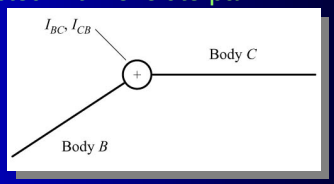
Primary Instant Centers (主要瞬心)

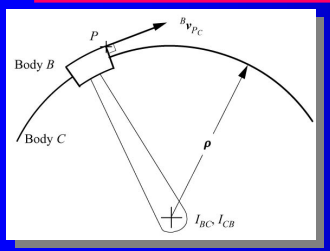
- All instant centers *which can be found merely by inspection* are called primary instant centers
- Case 1: for the two bodies connected with revolute pair

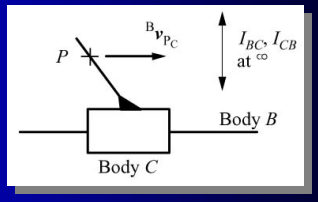
IC is located at the revolute joint



- Case 2: for the two bodies connected with sliding pair

IC is located at the center of curvature





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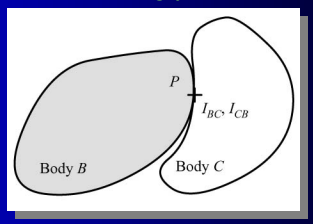
Primary Instant Centers-2

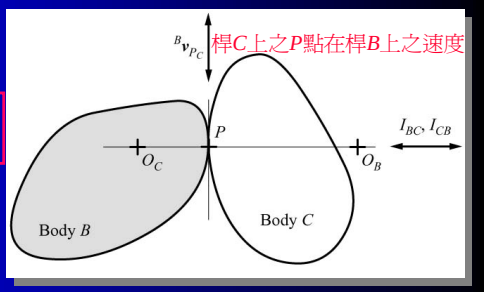
- Case 3: for the two bodies connected with rolling pair

IC is located at the point of contact



- Case 4: for the two bodies connected with cam pair

IC is located on the common normal, O_BO_C, through the contact point



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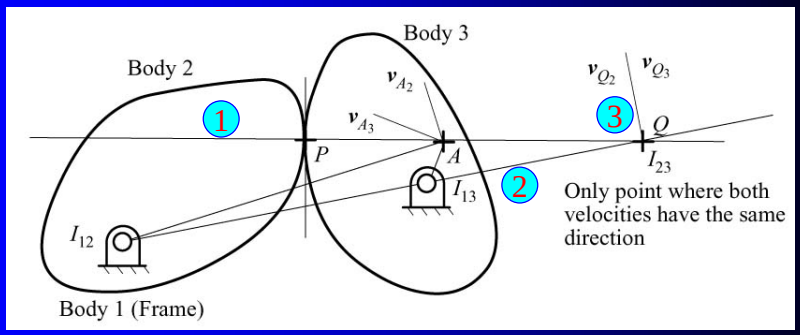
Example: The Instant Center Location Between Two Frame-Mounted Cams



Q is the only point where both velocities $\mathbf{v}_{Q2}$ and $\mathbf{v}_{Q3}$ have the same direction

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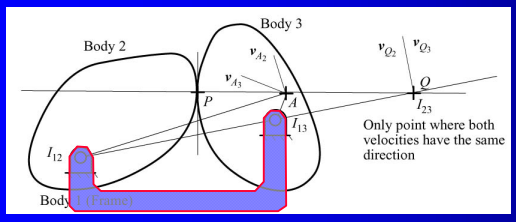
The Kennedy-Aronholdt Theorem
(三心定理)

- The total number of instant centers for *n* bodies is given by
  - *N*=*n*(*n*-1)/2
  - 類比: 兩桿之相對運動有一瞬心⇔兩點間有一連線
  - *Ex: For four-bar linkage, we have N=4(4-1)/2=6*
- If three bodies are in relative planar motion, there are three instant centers pertaining to the relative motion of pairs of those bodies. Those three instant centers are collinear.
  - 進行平面相對運動之三支桿件的三個瞬心恒在一直線上



Pf: 設若I_{23}不在I_{12} I_{13}之連線上, 例如在點A處, 則桿2和桿3在瞬心處之速度方向必不同, 即$\mathbf{v}_{A2} \neq \mathbf{v}_{A3}$, 亦即違反瞬心之定義, 故該三瞬心必在一直線上

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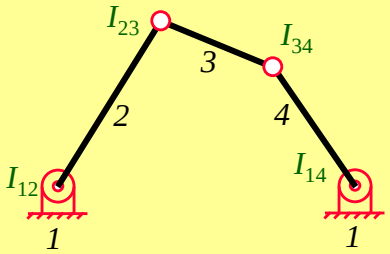
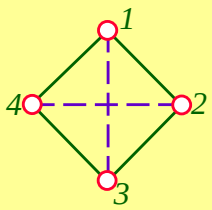
Example1 (p. 155)

- Find all of the instant centers
- Step 1: determine primary instant centers by inspection





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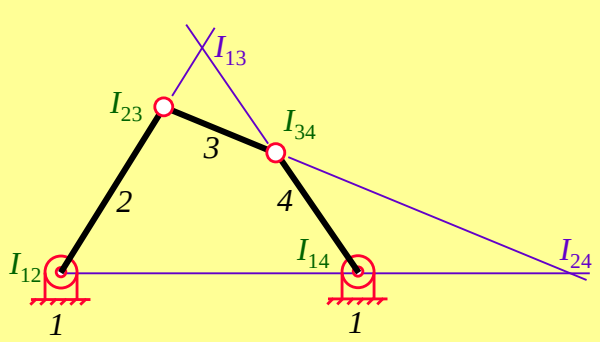
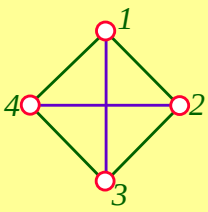
Example1 (cont.)

- Step 2: determine the other instant centers by using the Kennedy-Aronholdt theorem

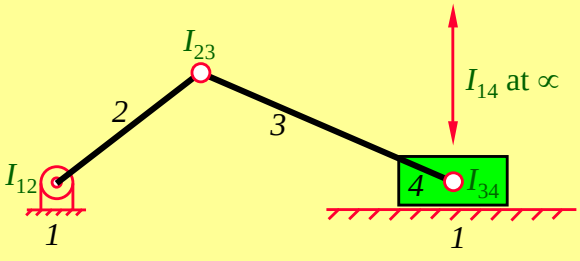
$13 \begin{cases} 12\text{-}23 \\ 14\text{-}34 \end{cases}$ $24 \begin{cases} 12\text{-}14 \\ 23\text{-}34 \end{cases}$

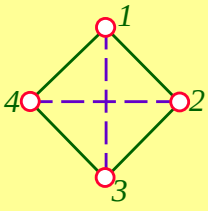




Example2 (p. 156)

- Find all of the instant centers
- Step 1: determine primary instant centers by inspection

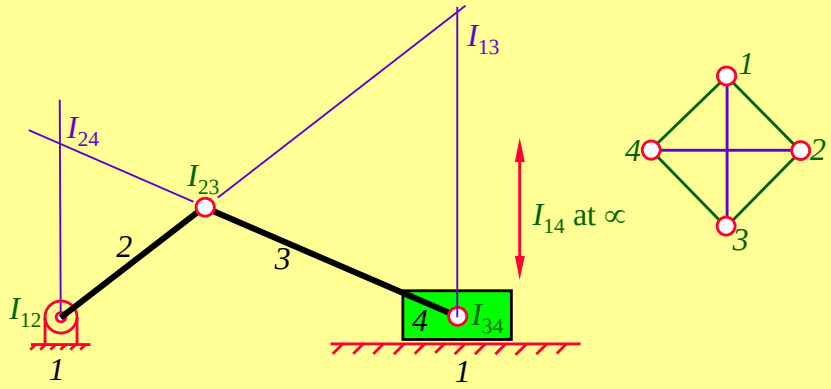




Example 2 (cont.)

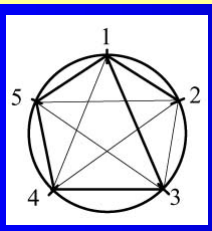
- Step 2: determine the other instant centers by using the Kennedy-Aronholdt theorem

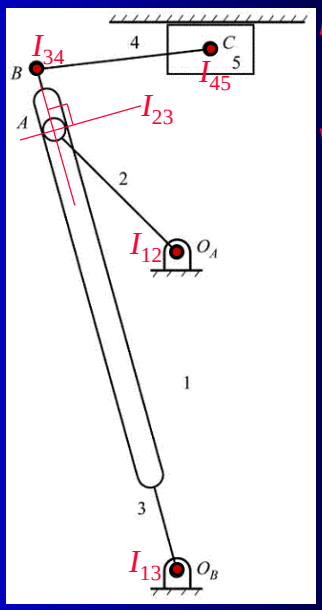
13 { 12-23, 14-34 } 24 { 12-14, 23-34 }



Example 4.4

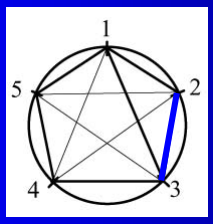
- Find: all instant centers for the Quick-Return Mechanism
- Procedure:
  - Determine primary instant centers: I_{12}, I_{13}, I_{34} , I_{45} and I_{15}
  - Applying Kennedy-Aronholdt theorem to determine the other instant centers

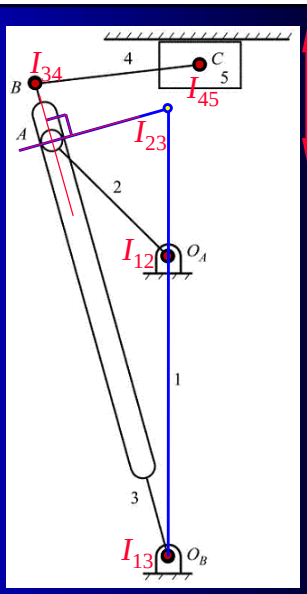




Example 4.4 (I_{23})

23 { 12-13





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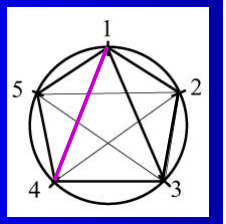
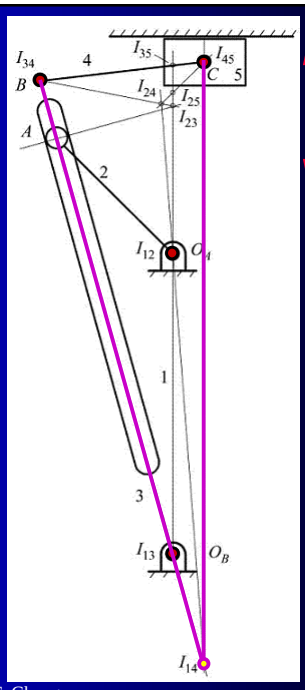
Example 4.4 (I_{14})

23 { 12-13

14 { 13-34, 15-45





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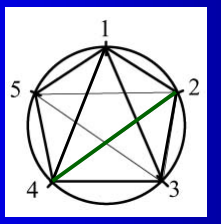
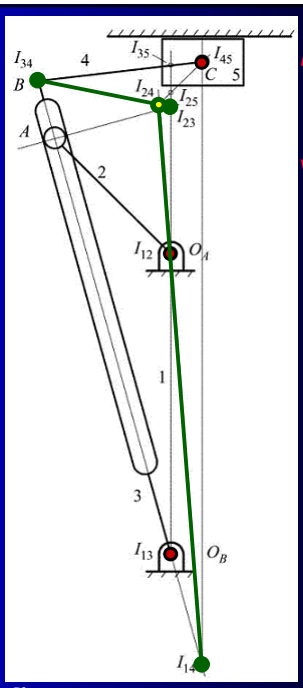
Example 4.4 (I_{24})

23 { 12-13

14 { 13-34
15-45

24 { 12-14
23-34





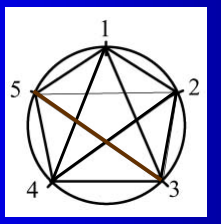
Example 4.4 (I_{35})

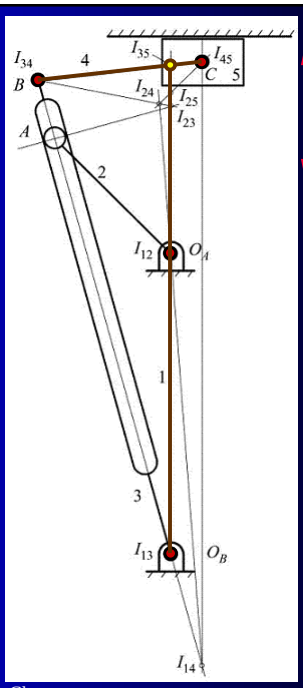
23 { 12-13

14 { 13-34
15-45

24 { 12-14
23-34

35 { 13-15
34-45





Example 4.4 (I_{25})

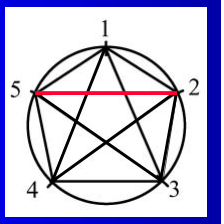
23 { 12-13

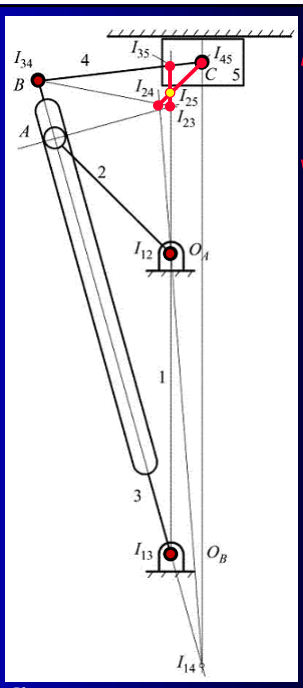
14 { 13-34
15-45

24 { 12-14
23-34

35 { 13-15
34-45

25 { 24-45
23-35





Example 4.4 (whole procedure)

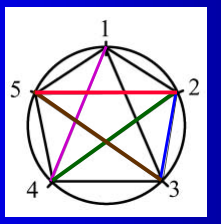
23 { 12-13

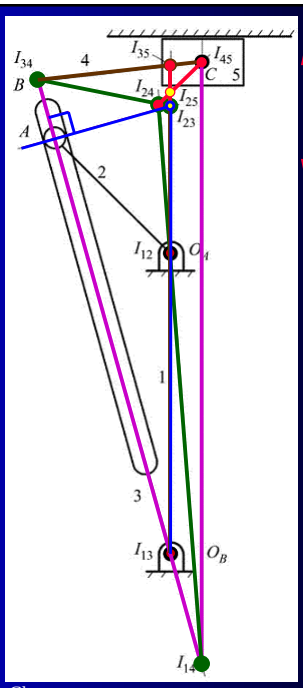
14 { 13-34
15-45

24 { 12-14
23-34

35 { 13-15
34-45

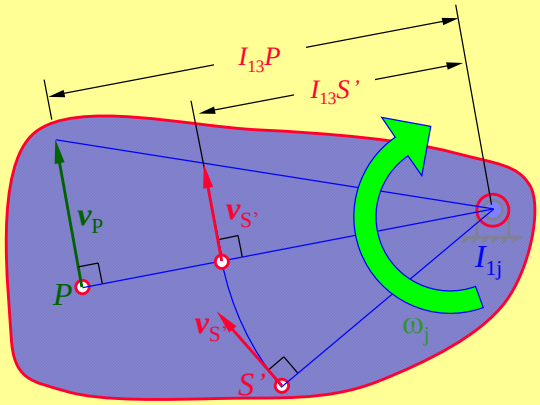
25 { 24-45
23-35





Rotating-Radius Method (pp. 156-157)

- Given:
  - the velocity of a point on link j ($\mathbf{v}_P$)
  - the instantaneous rotating center of link j (I_{1j})
- Find: the velocity of any other point on link j ($\mathbf{v}_{S'}$)

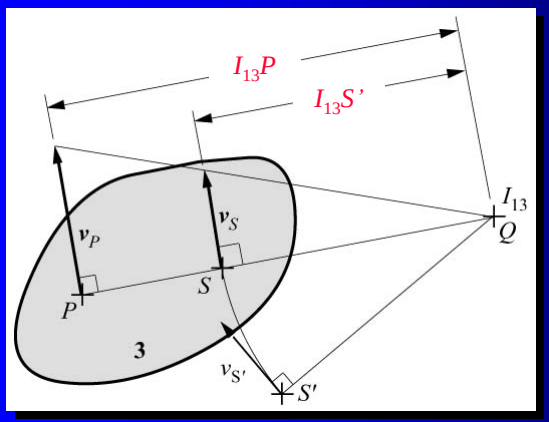


$$\because \frac{\mathbf{v}_{S'}}{\mathbf{v}_P} = \frac{\overline{I_{1j}S'}\omega_j}{\overline{I_{1j}P}\omega_j} = \frac{\overline{I_{1j}S'}}{\overline{I_{1j}P}}$$

$$\therefore \mathbf{v}_{S'} = \frac{\overline{I_{1j}S'}}{\overline{I_{1j}P}}\mathbf{v}_P$$

Rotating-Radius Method (cont.)

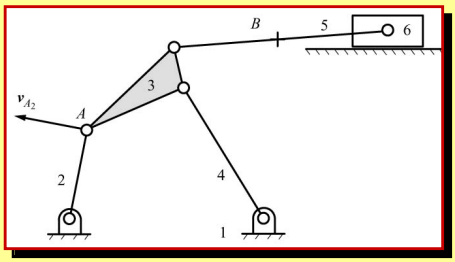
- Given:
  - the velocity of a point on link 3 ($\mathbf{v}_P$)
  - the instantaneous rotating center of link 3 (I_{13})
- Find: the velocity of any other point on link 3 ($\mathbf{v}_{S'}$)



$$\because \frac{\mathbf{v}_{S'}}{\mathbf{v}_P} = \frac{\overline{I_{13}S'}\omega_3}{\overline{I_{13}P}\omega_3} = \frac{\overline{I_{13}S'}}{\overline{I_{13}P}}$$

$$\therefore \mathbf{v}_{S'} = \frac{\overline{I_{13}S'}}{\overline{I_{13}P}}\mathbf{v}_P$$

Example 4.2 (Rotating-Radius Method, p.158)

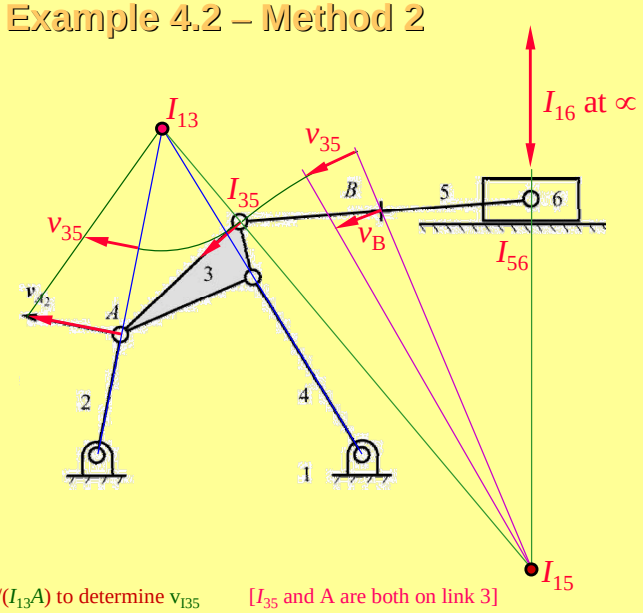


- Given: $\mathbf{v}_A$
- Find: $\mathbf{v}_B$
- Analysis:
  - Point A lies on link 2 and link 3
  - Point B lies on link 5
- **Method 1** (use I_{25}):
  - 1. Find I_{25} , I_{13} and I_{15}
  - 2. Use $v_{I25}/v_A = (I_{12}I_{25})/(I_{12}A)$ to determine v_{I25} [I_{25} and A are both on link 2]
  - 3. Use $v_B/v_{I25} = (I_{15}B)/(I_{15}I_{25})$ to determine v_B [I_{25} and B are both on link 5]
- **Method 2** (use I_{35})
  - 1. Find I_{35} , I_{13} and I_{15}
  - 2. Use $v_{I35}/v_A = (I_{13}I_{35})/(I_{13}A)$ to determine v_{I35} [I_{35} and A are both on link 3]
  - 3. Use $v_B/v_{I35} = (I_{15}B)/(I_{15}I_{35})$ to determine v_B [I_{35} and B are both on link 5]

Example 4.2 – Method 2

13 { 12-23, 14-34 }

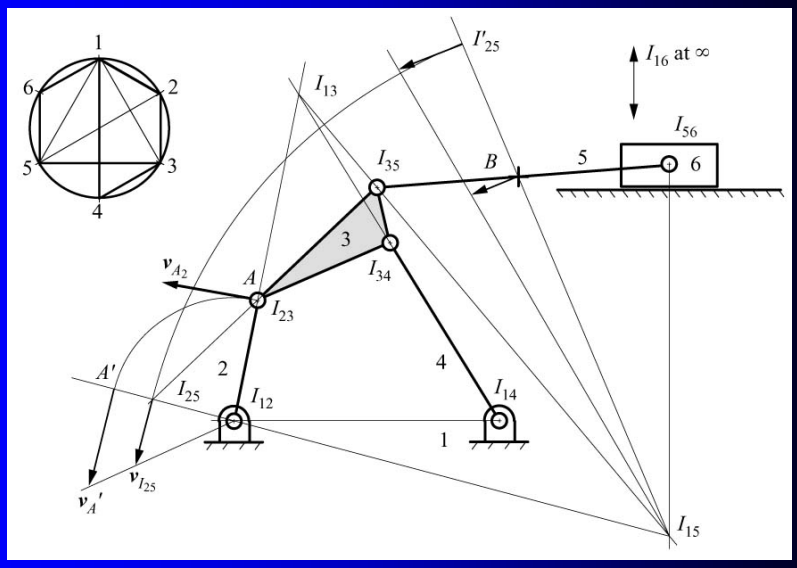
15 { 13-35, 16-56 }



Method 2 (use I_{35})

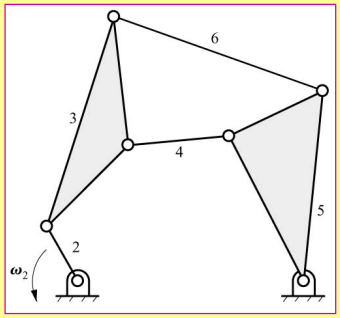
- 1. Find I_{35} , I_{13} and I_{15}
- 2. Use $v_{I35}/v_A = (I_{13}I_{35})/(I_{13}A)$ to determine v_{I35} [I_{35} and A are both on link 3]
- 3. Use $v_B/v_{I35} = (I_{15}B)/(I_{15}I_{35})$ to determine v_B [I_{35} and B are both on link 5]

Example 4.2 – Method 1



Example 4.3 (pp.159-160)

- Given: ω_2
- Find: ω_5
- Analysis:
  I_{25} lies on link 2 and link 5➔
  1. considering I_{25} as a point in link 2 yields
     $v_{I25} = (I_{12}I_{25})\,\omega_2$
  2. considering I_{25} as a point in link 5 yields
     $v_{I25} = (I_{15}I_{25})\,\omega_5$
- Procedures:
  1. Find I_{25} , I_{12} and I_{15}
  2. Use $v_{I25}= (I_{12}I_{25})\,\omega_2$ to determine v_{I25}
  3. $\omega_5=v_{I25}/(I_{15}I_{25})$

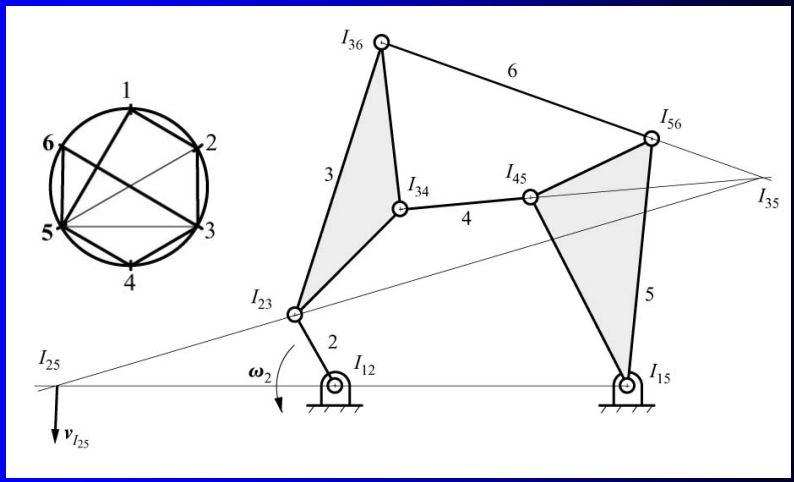


Example 4.3 (cont.)

25 { 12-15, 23-35 } 35 { 34-45, 35-56 }

$v_{I25} = (I_{12}I_{25})\omega_2$

$\omega_5 = v_{I25}/(I_{15}I_{25})$



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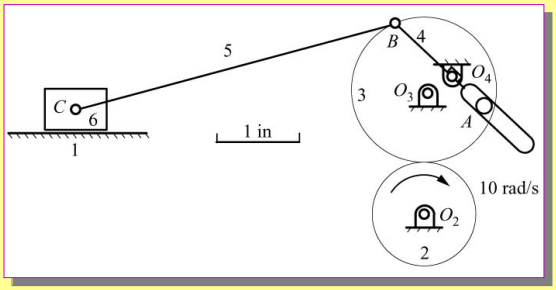
Example 4.6 (pp.163-164)

- Given: ω_2=10 rad/s, CW
- Find: v_C



- Procedures:
  1. Find I_{25} , I_{12} and I_{15}
  2. Use $v_{I25} = (I_{12}I_{25})\omega_2 = (I_{15}I_{25})\omega_5$ to determine ω_5
  3. $Vc = (I_{15}C)\ \omega_5$

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Example 4.6 (cont.)

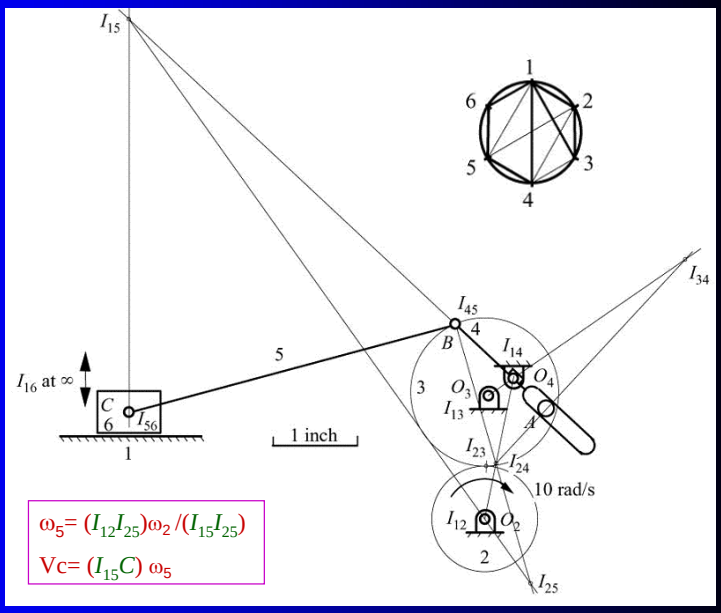
15 { 14-45, 15-56 }

34 { 13-14 }

24 { 12-14, 23-34 }

25 { 12-15, 24-45 }

$I_{12}I_{25} = 0.940"$
$I_{15}I_{25} = 7.261"$
$I_{15}C = 4.653"$



$\omega_5 = (I_{12}I_{25})\omega_2 / (I_{15}I_{25})$
$Vc = (I_{15}C)\,\omega_5$

End of chapter 4