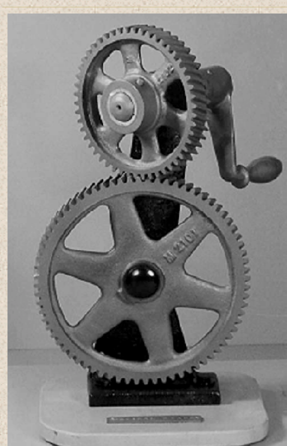
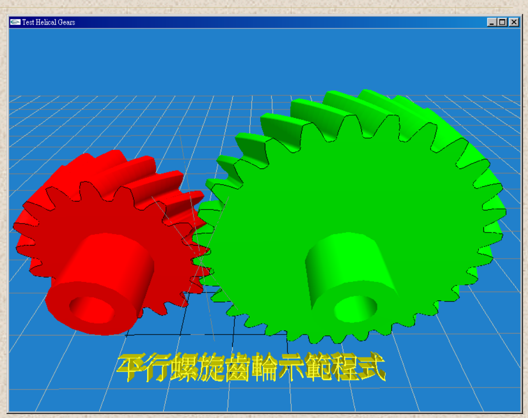
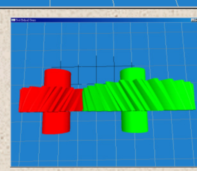
Chapter 11 Helical, Bevel, and Worm Gears

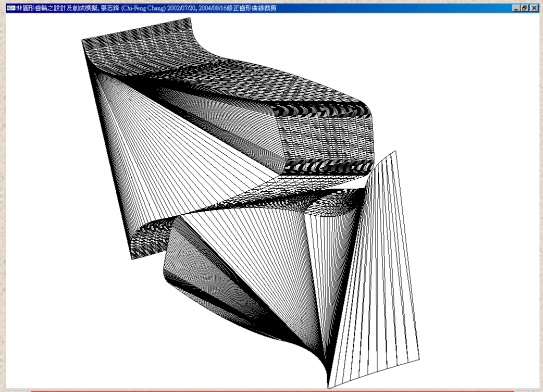
Helical Gears

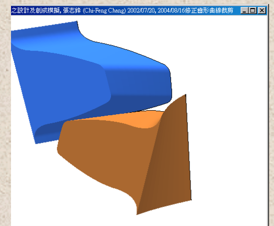


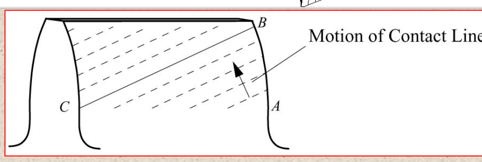




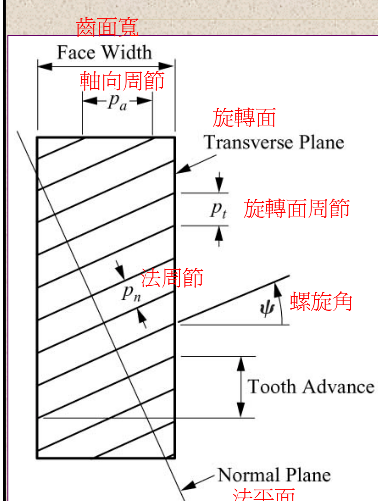
Motion of Contact Line







Helical Gear terminology



The properties in the normal plane:

P_n : normal diametral pitch (法平面徑節)

$m_n=1/P_n$ = the module for the hob (滾齒刀)

p_n : normal pitch (法平面周節)

$p_n P_n = \pi$

※ addendum $a = k\, m_n$

The properties in the transverse plane:

$P_t \equiv N/d_t$ = transverse diametral pitch (旋轉面徑節)

$m_t=1/P_t= d_t/N$

$p_t \equiv \pi\, d_t/N = \pi m_t$ =transverse circular pitch(旋轉面周節)

$p_t P_t = \pi$

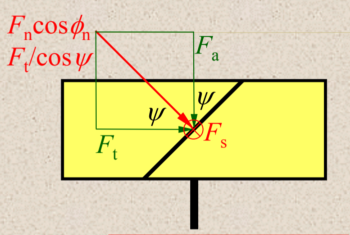
$$\boxed{p_n = p_t \cos\psi}$$

$$\boxed{\frac{\pi}{P_n} = \frac{\pi}{P_t}\cos\psi}$$

$$\boxed{P_t = P_n \cos\psi}$$

$$\boxed{m_n = m_t \cos\psi}$$

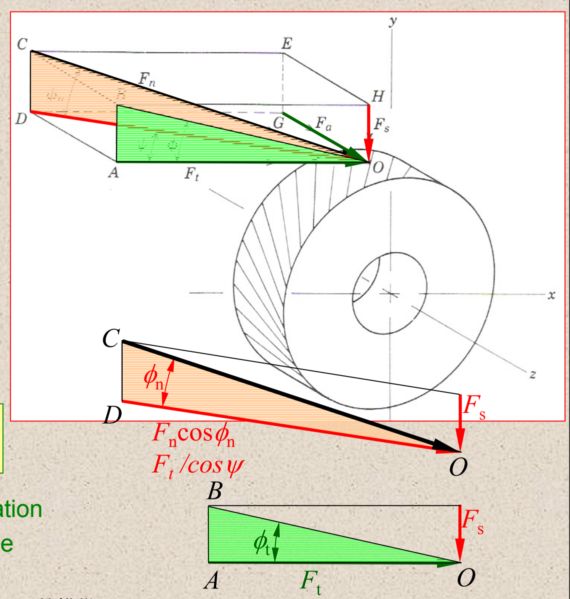
The relationship Among Pressure Angles ϕ_n and ϕ_t (P. 550)



$$F_n \Rightarrow \begin{cases} F_n\cos\phi_n \Rightarrow \begin{cases} F_t \\ F_a \end{cases} \\ F_s \end{cases}$$

$$\tan\phi_t = \frac{F_s}{F_t}$$

$$\tan\phi_n = \frac{F_s}{OD} = \frac{F_s}{F_t/\cos\psi} = \tan\phi_t\cos\psi$$

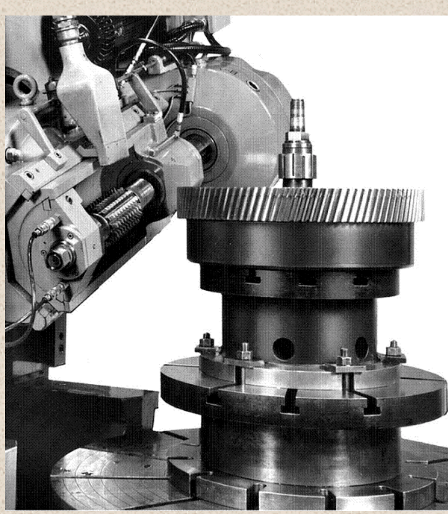


ϕ_t = pressure angle in plane of rotation

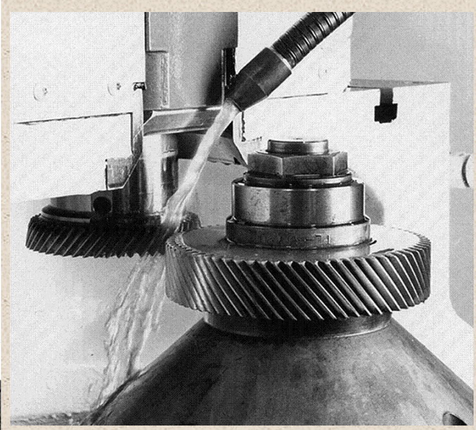
ϕ_n = pressure angle in normal plane

ψ = helix angle (螺旋角)

Helical Gear Manufacturing



Hobbing (滾齒法)



shapping(鉋齒法)

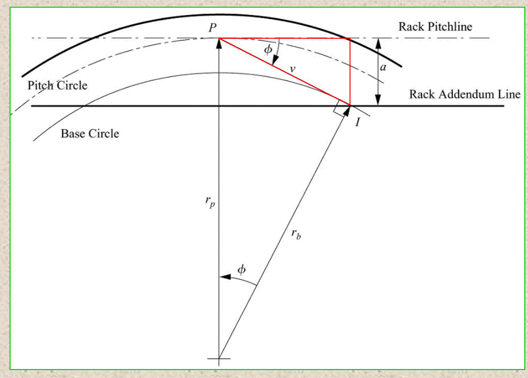
Minimum Tooth Number to Avoid Undercutting
pp. 501-503

$\boxed{\tan\phi_n = \tan\phi_t \cos\psi}$

$$r_p = \frac{a}{\sin^2\phi_t} = \frac{km_n}{\sin^2\phi_t} \qquad r_p = \frac{Nm_t}{2}$$

$$\frac{k(\cancel{m_t}\cos\psi)}{\sin^2\phi_t} = \frac{N\cancel{m_t}}{2}$$

$$N = \frac{2k\cos\psi}{\sin^2\phi_t}$$



Ex: ψ=30°, ϕ_n= 20° , standard gear k=1 ($a = m_n$)➔ ϕ_t= 22.796°, N_{min}=11.54=12T

Ex: ψ=45°, ϕ_n= 20° , standard gear k=1 ($a = m_n$)➔ ϕ_t= 22.796°, N_{min}=6.75=7T

Helical Gears with Parallel Shafts (pp. 503-509)
Velocity Ratio and Center Distance

$$p_n = p_t\cos\psi$$
$$\pi m_n = \pi m_t \cos\psi$$

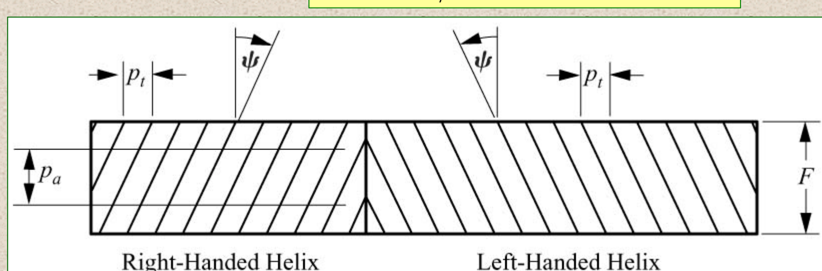
Velocity ratio:

$$\because v_{23} = r_{p_2}\omega_2 = r_{p_3}\omega_3$$

$$\therefore \frac{\omega_2}{\omega_3} = \frac{r_{p_3}}{r_{p_2}} = \frac{\frac{N_3 m_t}{2}}{\frac{N_2 m_t}{2}} = \frac{N_3}{N_2}$$

Center distance:

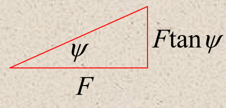
$$C = r_{p_2} + r_{p_3} = \frac{N_2 m_t}{2} + \frac{N_3 m_t}{2} = \frac{(N_2+N_3)m_t}{2}$$
$$= \frac{(N_2+N_3)m_n}{2\cos\psi}$$



Helical Gears with Parallel Shafts
-Minimum Face Width- (p. 504)

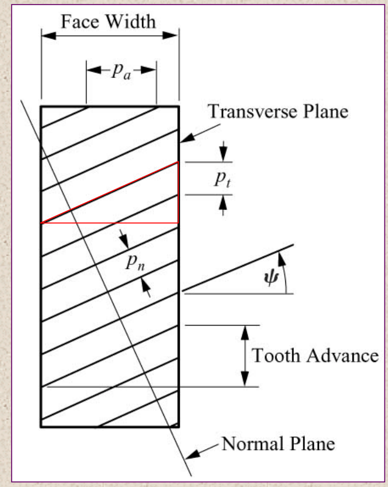
In order to provide overlapping action (i.e., the leading end of one tooth will come into contact before the trailing end of the adjacent tooth goes out of contact), the minimum face width should satisfy

$$F\tan\psi > p_t$$



The AGMA recommends that the limiting face width be increased by at least 15 percent, which results in the following equation:

$$F > 1.15\, p_t / \tan\psi$$



Helical Gears with Parallel Shafts
-Transverse Contact Ratio-

For spur gears, we have

$$m_c = \frac{\lambda}{p_b} = \frac{\lambda}{p_c \cos\phi}$$

where

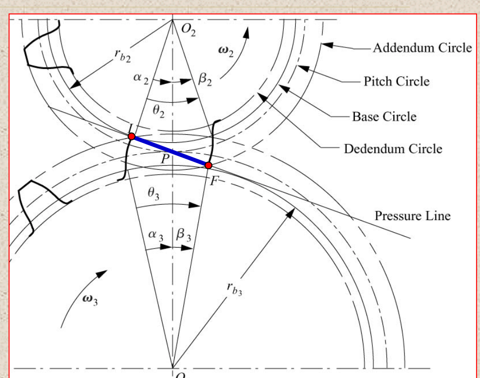
$$\lambda = IF = IP + PF = u + v$$

$$u = \sqrt{2r_{p_3}a_3 + a_3^2 + r_{p_3}^2\sin^2\phi} - r_{p_3}\sin\phi$$

$$v = \sqrt{2r_{p_2}a_2 + a_2^2 + r_{p_2}^2\sin^2\phi} - r_{p_2}\sin\phi$$

or

$$\lambda = \sum_{i=2}^{3}\left\{\sqrt{2r_{p_i}a_i + a_i^2 + r_{p_i}^2\sin^2\phi} - r_{p_i}\sin\phi\right\}$$



For helical gears, the transverse contact ratio is given as

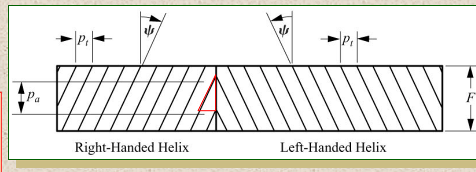
$$m_{c_t} = \frac{\lambda}{p_t \cos\phi_t} = \frac{P_t\lambda}{\pi\cos\phi_t} \qquad (11.16)$$

$$p_t P_t = \pi$$

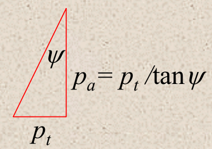
Helical Gears with Parallel Shafts
-Axial/Face Contact Ratio -

The axial ratio is the ratio of the face width of the gear to the axial pitch. That is,

$$m_{c_a} = \frac{F}{p_a} = \frac{F \tan\psi}{p_t} \qquad (11.17)$$



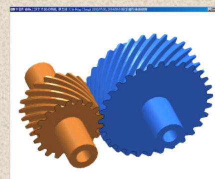
Notice that the axial contact ratio depends on the properties of a single gear. If the face widths of meshing gears are different, the smaller one is used in Eq. (11.17)



Then, the total contact ratio is

$$m_c = m_{ct} + m_{ca}$$

As in the case of spur gear, this ratio gives the average number of teeth in contact during the gear action.



Ex 11.1 Helical Gear Geometry

- Find the transverse diametral pitch, the pitch cylinder radii, and the axial, transverse, and total contact ratios.

Given: $p_n=4$, $\phi_n = 20°$ (cutter), $N_2=19$, $N_3=34$, $\psi=30°$, F=12 in.

(1) Find the transverse diametral pitch (旋轉面徑節)

$P_t = P_n \cos\psi = (4)\cos 30° = 3.464$ in ←ans(1)

(2) Find pitch cylinder radii (節圓柱之半徑=旋轉面之節圓半徑)

$r_{t2} = N_2/(2P_t) = 19/(2\times3.464) = 2.742$ in ←ans(2a)

$r_{t3} = N_3/(2P_t) = 34/(2\times3.464) = 4.908$ in ←ans(2b)

$P_t \equiv N/d_t = N/(2r_t) = P_n \cos\psi$ = transverse diameteral pitch (旋轉面徑節)
$m_t = 1/P_t = d_t/N$
$p_t \equiv \pi d_t/N = \pi m_t$ = transverse circular pitch(旋轉面周節)
$p_t P_t = \pi$

Ex 11.1 Helical Gear Geometry (cont)

(3) Find axial contact ratio (軸向之接觸比)

$p_t = \pi/P_t = 0.907$ in

$m_{ca} = F \tan\psi/p_t = 12 \tan 30°/0.907 = 7.6394$ ←ans(3a)

(4) Find transverse contact ratio (旋轉面之接觸比)

$a_2 = a_3 = m_n = 1/P_n = ¼ = 0.25$ in

$\tan\phi_n = \tan\phi_t \cos\psi$ ➔ ϕ_t=22.796°

$r_{p2} = r_{t2} = 2.742$ in

$r_{p3} = r_{t3} = 4.908$ in

∴ λ=1.1131 and m_{ct}= 0.3486←ans(3b)

(5) Find total contact ratio (旋轉面之接觸比)

$m_{ct} = m_{ca} + m_{ct}$= 8.9708 ←ans(3c)

$$m_{c_a} = \frac{F}{p_a} = \frac{F\tan\psi}{p_t} \qquad (11.17)$$

$$\lambda = \sum_{i=2}^{3}\left\{\sqrt{2r_{p_i}a_i + a_i^2 + r_{p_i}^2\sin^2\phi_t} - r_{p_i}\sin\phi_t\right\}$$

$$m_{c_t} = \frac{\lambda}{p_t\cos\phi_t} = \frac{\lambda P_t}{\pi\cos\phi_t} \qquad (11.16)$$

Example 11.2 Helical Gear Replacement of Spur Gears

$$r_p = \frac{Nm}{2} = \frac{N}{2P_d}$$

Problem:

It is desirable to replace an existing set of spur gears by helical gears, in order to eliminate a noise problem and to increase the capacity of a gear box.

Requirements: The center distance and angular velocity ratio are to remain the same. Use the same hob for spur gears to cut the new helical gears

Determine:

(1) the helix angle,

(2) outside diameters of the blanks, and

(3) the face width of the replacement gears.

Given: P_d=14, ϕ = 20° (origional spur gear), N_2=30, N_3=85 ψ=30 °, F=12 in.

Sol:

(1) original center distance

$$r_{p_2} = \frac{N_2}{2P_d} = \frac{30}{2(14)} = 1.071$$

$$r_{p_3} = \frac{N_3}{2P_d} = \frac{85}{2(14)} = 3.036$$

$$C = r_{p_2} + r_{p_2} = 1.071 + 3.036 = 4.286 \text{ in}$$

Example 11.2 (cont.)

$P_t = P_n \cos\psi$

(2) original angular velocity ratio

$$\therefore \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{85}{30} = \frac{17}{6}$$

(3) Tooth numbers of helical gears

$$C' = r_{p_2} + r_{p_3} = 4.286$$

$$\left(\frac{\omega_2}{\omega_3}\right)' = \frac{r_{p_3}}{r_{p_2}} = \frac{N_3}{N_2} = \frac{17}{6}$$

The pitch radii for helical gears must be the same as that of spur gears. That is, r_{p2}=1.071, r_{p3}=3.036

$$P_t = \left(\frac{N}{2r}\right)_{heli} = P_n \cos\psi = \left(\frac{N}{2r}\right)_{spur} \cos\psi$$

The tooth numbers on helical gears must be less than those on spur gears. Selecting N_2=24 yields N_3=68

P_t=24/(2×1.071)=11.199

$$\psi = \cos^{-1}\left(\frac{P_t}{P_n}\right) = 36.877°$$

Example 11.2 (cont.)

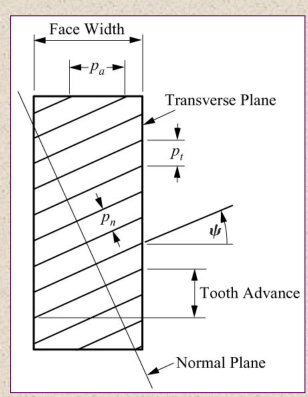
(4) the blank diameters of helical gears

$$D_{o_2} = 2(r_{p_2} + a_2) = 2(r_{p_2} + \frac{1}{P_n}) = 2.286 \text{ in}$$

$$D_{o_3} = 2(r_{p_3} + a_3) = 2(r_{p_3} + \frac{1}{P_n}) = 6.214 \text{ in}$$

(5) the face width of helical gears

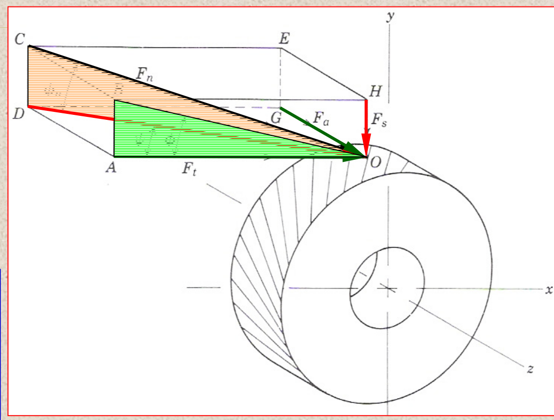
$$F \geq \frac{1.15\, p_t}{\tan\psi} = \frac{1.15\,\pi}{P_t \tan\psi} = 0.43 \text{ in}$$



Designing for Axial Force

When helical gears mesh, there will be an axial force F_a as shown in the figure. This axial force must be considered when designing the bearings.

Herringbone gears (人字齒輪) can be used for eliminating the axial forces



Crossed Helical Gears

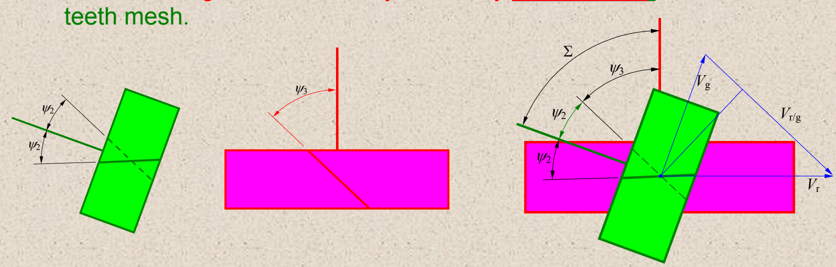
$$P_t = P_n \cos\psi$$

$$P_t \equiv N/d_t$$

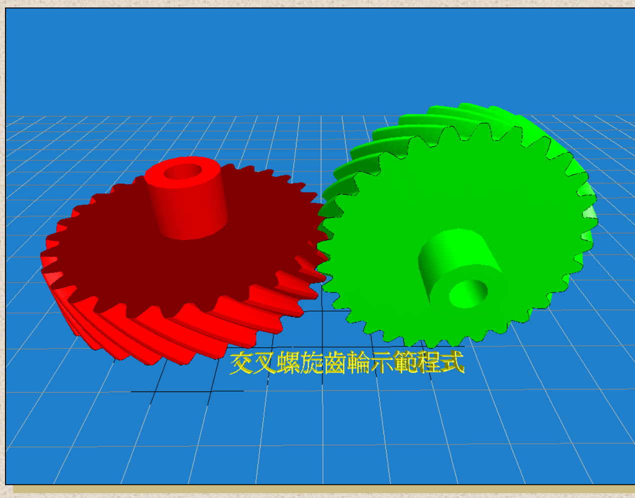
- The only requirement for crossed helical gears to mesh properly is that they have the same normal pitch.
- The velocity ratio for crossed helical gears:

$$R = \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{d_{t_3} P_{t_3}}{d_{t_2} P_{t_2}} = \frac{d_{t_3} P_n \cos\psi_3}{d_{t_2} P_n \cos\psi_2} = \frac{d_{t_3} \cos\psi_3}{d_{t_2} \cos\psi_2}$$

- Crossed helical gears are not used to transmit large amounts of power because the gears theoretically have only point contact where the teeth mesh.



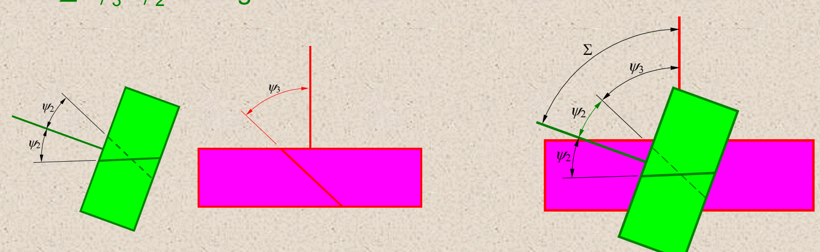
Demo



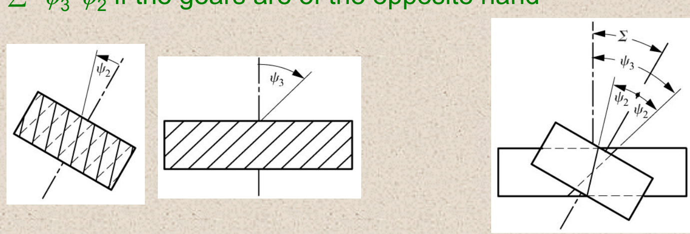
Helix angle ψ=45°, ϕ_n =20 °, m_n=2

Relationship Among Shaft angle and helix angles

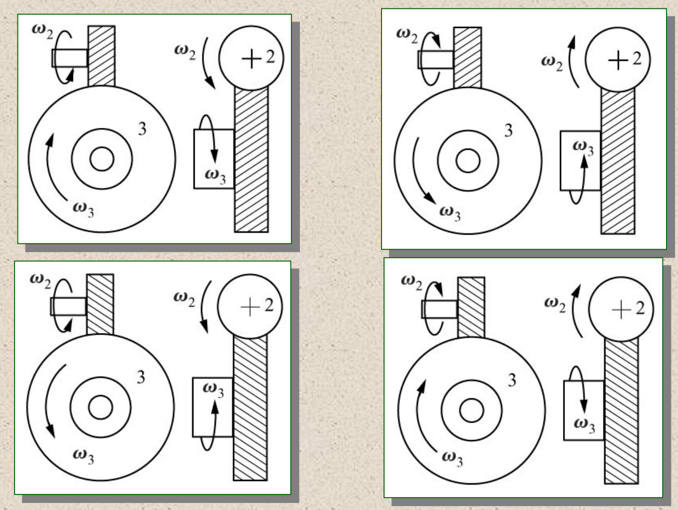
- $\Sigma = \psi_3 + \psi_2$ if the gears are of the same hand



- $\Sigma = \psi_3 - \psi_2$ if the gears are of the opposite hand



The Direction of the Angular Velocities for Crossed Helical Gears



$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2}$$

$$d_t = \frac{N}{P_t} = \frac{N}{P_n \cos \psi}$$

Ex 11.3 Crossed Helical Gear Geometry

- Determine:
  - the helix angle, number of teeth on the meshing gears, and center distance
- Given:
  - $\Sigma = 65°$, $\omega_2 : \omega_3 = 2:1$, $\psi_2 = 30°$, $N_2 = 70$, $P_n = 10$,
  - Both gears are right handed, and both are cut with the same hob

(1) The number of teeth on gear 3

$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{2}{1} \Rightarrow N_3 = 140$$

(2) The helix angle of gear 3

$$\sum = \psi_2 + \psi_3 \Rightarrow \psi_3 = 65 - 30 = 35°$$

(3) Center distance

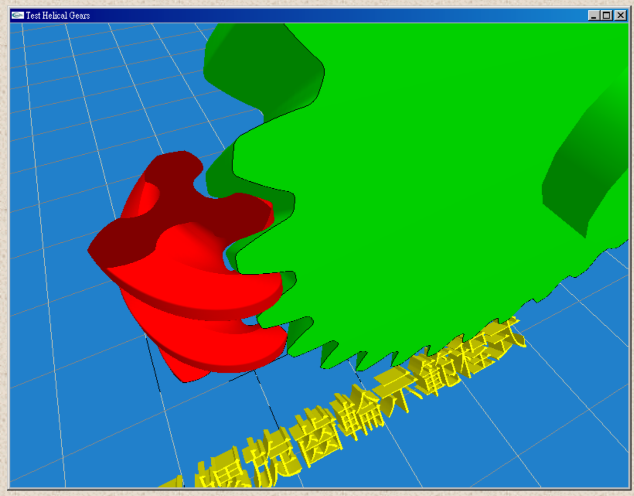
$$d_{t_2} = \frac{N_2}{P_n \cos \psi_2} = \frac{70}{10 \cos 30°} = 8.083 \text{ in}$$

$$d_{t_3} = \frac{N_3}{P_n \cos \psi_3} = \frac{140}{10 \cos 35°} = 17.091 \text{ in}$$

$$C = \frac{d_{t_2} + d_{t_3}}{2} = 12.587 \text{ in}$$

Special Case

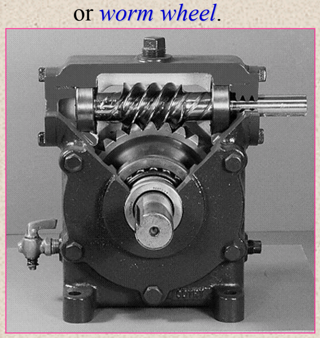
The Tooth of a Helical Gear Makes a Complete Revolution on the Pitch Cylinder

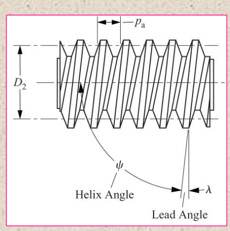


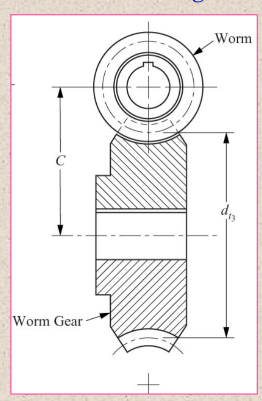
Helix angles $\psi_1 = 60°$, $\psi_2 = 30°$, $\Sigma = 90°$, $\phi_n = 20°$, $m_n = 2$

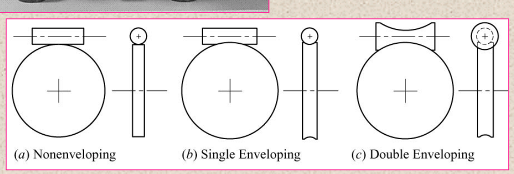
Worm Gears

If the tooth of a helical gear makes a complete revolution on the pitch cylinder, the resulting gear is called a *worm*, and the mating gear is called a *worm gear* or *worm wheel*.









$$\frac{\omega_w}{\omega_g} = \frac{N_g}{N_w}$$

Involute Bevel Gears

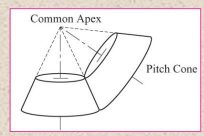
Velocity ratio:

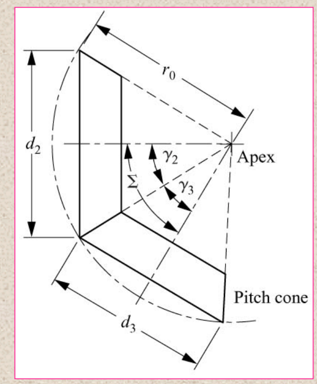
$$R = \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{d_3}{d_2} = \frac{r_3}{r_2}$$

Cone angles:

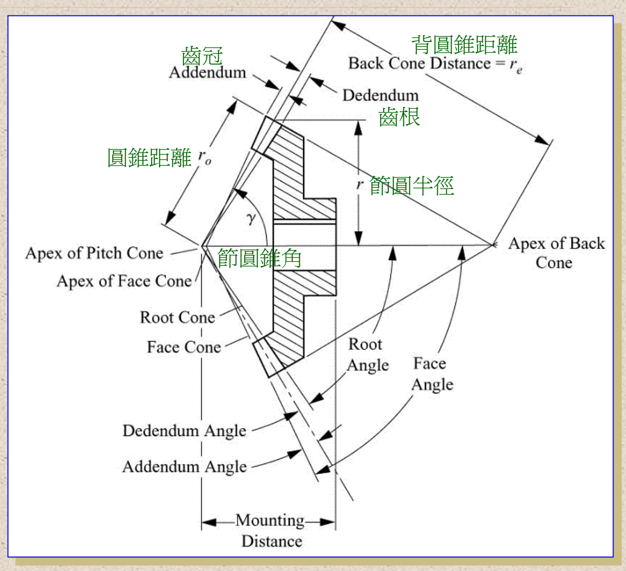
$$\tan \gamma_2 = \frac{\sin \Sigma}{(d_3 / d_2) + \cos \Sigma}$$

$$\tan \gamma_3 = \frac{\sin \Sigma}{(d_2 / d_3) + \cos \Sigma}$$





Nomenclature for bevel gears



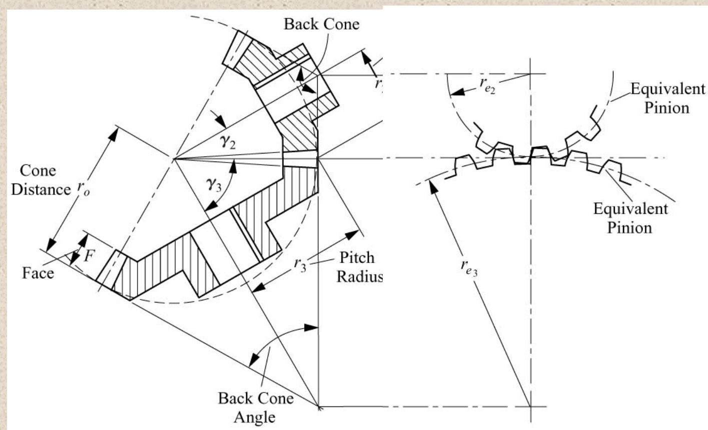
Notice that the apex of the face cone is not coincident with the apex of pitch cone

Approximation of Bevel Gears as Equivalent Spur Gears
Tredgold's Approximation

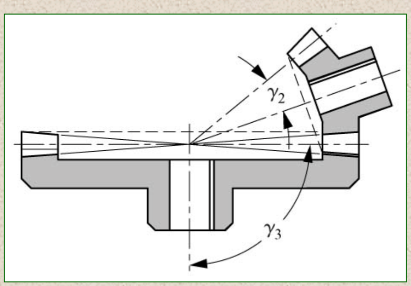
$$r_e = \frac{r}{\cos \lambda}$$

$$N_e = \frac{2\pi r_e}{p_c}$$

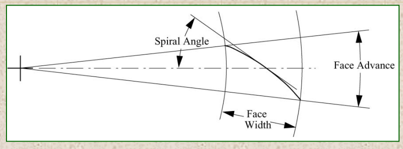
型成齒數

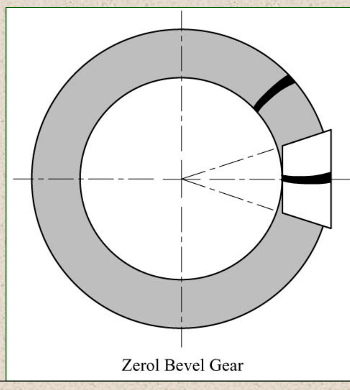


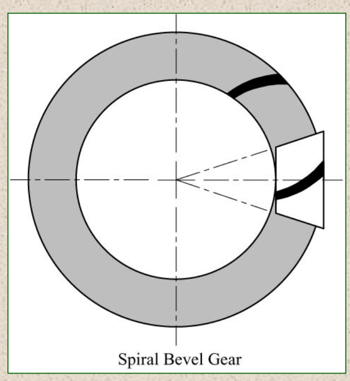
Crown bevel gears (γ=90°)
(冠狀斜齒輪)



Spiral bevel gears & zerol bevel gears
(蝸線斜齒輪 & 零角蝸線斜齒輪)







The spiral angle at the middle of the teeth = 0

Hypoid gears
(戟齒輪)