Chapter 5
Analytical Linkage Analysis

Loop Closure Equations for Four-Bar Linkages
ref : pp. 175-184

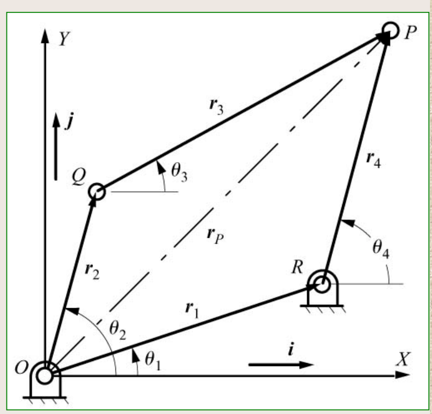
- We will represent each vector by a length r_i and an angle θ_i
- *All angles are measured counterclockwise from a line that remains parallel to the fixed x axis attached to the reference frame*

$$\mathbf{r}_p = \mathbf{r}_2 + \mathbf{r}_3 = \mathbf{r}_1 + \mathbf{r}_4$$

or

$$\boxed{\mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_4 - \mathbf{r}_1 = \mathbf{0}}$$

$$\begin{cases} r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 \cos\theta_1 = 0 \\ r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 - r_1 \sin\theta_1 = 0 \end{cases}$$

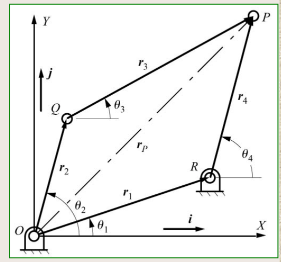


Position Analysis

$$\begin{cases} r_2\cos\theta_2 + r_3\cos\theta_3 - r_4\cos\theta_4 - r_1\cos\theta_1 = 0 \\ r_2\sin\theta_2 + r_3\sin\theta_3 - r_4\sin\theta_4 - r_1\sin\theta_1 = 0 \end{cases}$$

$$r_3\cos\theta_3 = -r_2\cos\theta_2 + r_4\cos\theta_4 + r_1\cos\theta_1 \qquad \text{(a)}$$

$$r_3\sin\theta_3 = -r_2\sin\theta_2 + r_4\sin\theta_4 + r_1\sin\theta_1 \qquad \text{(b)}$$



(a)^2+(b)^2 ➔

$$\begin{aligned} r_3^2 &= r_1^2 + r_2^2 + r_4^2 \\ &+ 2r_1r_4(\cos\theta_1\cos\theta_4 + \sin\theta_1\sin\theta_4) \\ &- 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) \\ &+ 2r_2r_4(\cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4) \qquad \text{(c)} \end{aligned}$$

Combining the coefficient of $\cos\theta_4$ and $\sin\theta_4$ yields

$$A\cos\theta_4 + B\sin\theta_4 + C = 0 \qquad \text{(d)}$$

where

$$\begin{aligned} A &= 2r_1r_4\cos\theta_1 - 2r_2r_4\cos\theta_2 \\ B &= 2r_1r_4\sin\theta_1 - 2r_2r_4\sin\theta_2 \\ C &= r_1^2 + r_2^2 + r_4^2 - r_3^2 - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) \end{aligned}$$

function of θ_2
(r_i and θ_1 are constants)

Position Analysis (cont)

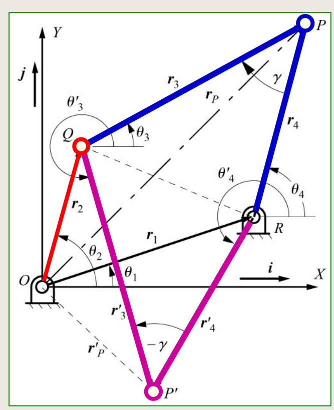
$$A\cos\theta_4 + B\sin\theta_4 + C = 0$$

$$\sin\theta_4 = \frac{2\tan\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)}, \quad \cos\theta_4 = \frac{1-\tan^2\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)}$$

$$(C-A)\tan^2\left(\frac{\theta_4}{2}\right) + 2B\tan\left(\frac{\theta_4}{2}\right) + (A+C) = 0$$

$$\tan\left(\frac{\theta_4}{2}\right) = \frac{-2B \pm \sqrt{4B^2 - 4(C-A)(C+A)}}{2(C-A)} = \frac{-B \pm \sqrt{B^2 - C^2 + A^2}}{(C-A)}$$

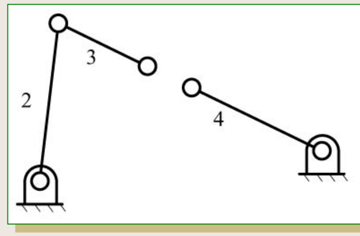
"±" corresponding to the two possible positions of the P for a given value of θ_2



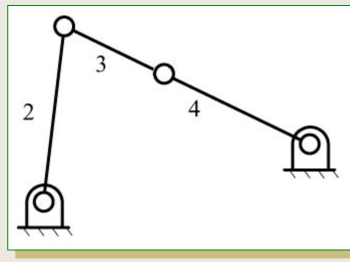
Position Analysis (cont)

$$\tan\left(\frac{\theta_4}{2}\right) = \frac{-B \pm \sqrt{B^2 - C^2 + A^2}}{(C - A)}$$

- Special cases:
  - ❑ No θ_4 can be found if B^2-C^2+A^2<0
  - ➔ the mechanism cannot be assembled in the position specified



  - ❑ One θ_4 can be found if B^2-C^2+A^2=0
  - ➔link 2 is in one of its limit positions



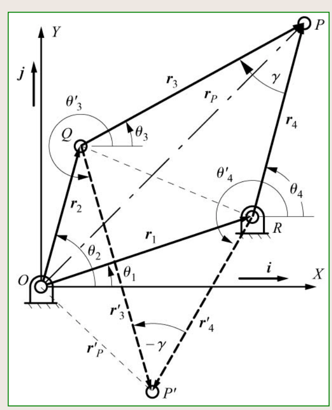
Position Analysis (cont)

$$r_3 \cos\theta_3 = -r_2 \cos\theta_2 + r_4 \cos\theta_4 + r_1 \cos\theta_1 \quad \text{(a)}$$
$$r_3 \sin\theta_3 = -r_2 \sin\theta_2 + r_4 \sin\theta_4 + r_1 \sin\theta_1 \quad \text{(b)}$$

- After θ_4 is known, dividing Eq. (b) be Eq. (a) and solving for θ_3 gives

$$\tan\theta_3 = \frac{-r_2 \sin\theta_2 + r_4 \sin\theta_4 + r_1 \sin\theta_1}{-r_2 \cos\theta_2 + r_4 \cos\theta_4 + r_1 \cos\theta_1}$$

- $\gamma \equiv \theta_4$-θ_3
- Note that the sign of γ provides a useful indicator as to which of the solution branches has been drawn, from the graphical point of view (p.178)
- In other words, the mechanism is kept in the same branch <u>if the sign of γ is not changed during the motion cycle</u>



Velocity Analysis

$$\begin{cases} r_2\cos\theta_2 + r_3\cos\theta_3 - r_4\cos\theta_4 - r_1\cos\theta_1 = 0 \\ r_2\sin\theta_2 + r_3\sin\theta_3 - r_4\sin\theta_4 - r_1\sin\theta_1 = 0 \end{cases} \quad \Longleftrightarrow \quad \mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_4 - \mathbf{r}_1 = \mathbf{0}$$

$$\begin{aligned} -r_2\dot\theta_2\sin\theta_2 - r_3\dot\theta_3\sin\theta_3 + r_4\dot\theta_4\sin\theta_4 + r_1\cancel{\dot\theta_1}\sin\theta_1 = 0 \\ r_2\dot\theta_2\cos\theta_2 + r_3\dot\theta_3\cos\theta_3 - r_4\dot\theta_4\cos\theta_4 - r_1\cancel{\dot\theta_1}\cos\theta_1 = 0 \end{aligned} \quad \Longleftrightarrow \quad \dot{\mathbf{r}}_2 + \dot{\mathbf{r}}_3 - \dot{\mathbf{r}}_4 - \dot{\mathbf{r}}_1 = \mathbf{0}$$

$$\begin{aligned} -r_3\dot\theta_3\sin\theta_3 + r_4\dot\theta_4\sin\theta_4 = r_2\dot\theta_2\sin\theta_2 \\ -r_3\dot\theta_3\cos\theta_3 + r_4\dot\theta_4\cos\theta_4 = r_2\dot\theta_2\cos\theta_2 \end{aligned}$$

$$\begin{bmatrix} -r_3\sin\theta_3 & r_4\sin\theta_4 \\ -r_3\cos\theta_3 & r_4\cos\theta_4 \end{bmatrix} \begin{bmatrix} \dot\theta_3 \\ \dot\theta_4 \end{bmatrix} = \begin{bmatrix} r_2\dot\theta_2\sin\theta_2 \\ r_2\dot\theta_2\cos\theta_2 \end{bmatrix}$$

$$\mathbf{A}_{n\times n}\,\mathbf{x}_{n\times 1} = \mathbf{b}_{n\times 1}$$

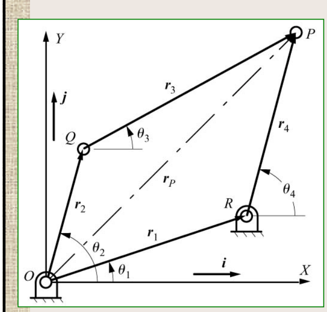
$d\theta_3/dt$ and $d\theta_4/dt$ can be determined by solving Ax=b

Velocity Analysis (cont.)

$$\mathbf{r}_P = \mathbf{r}_2 + \mathbf{r}_3 = \mathbf{r}_1 + \mathbf{r}_4$$

$$\begin{aligned} \mathbf{v}_P &= \dot{\mathbf{r}}_P = \dot{\mathbf{r}}_2 + \dot{\mathbf{r}}_3 \\ &= \frac{d}{dt}(r_2\cos\theta_2\mathbf{i} + r_2\sin\theta_2\mathbf{j}) + \frac{d}{dt}(r_3\cos\theta_3\mathbf{i} + r_3\sin\theta_3\mathbf{j}) \\ &= \left(-r_2\dot\theta_2\sin\theta_2 - r_3\dot\theta_3\sin\theta_3\right)\mathbf{i} + \left(r_2\dot\theta_2\cos\theta_2 + r_3\dot\theta_3\cos\theta_3\right)\mathbf{j} \end{aligned}$$

$$\begin{aligned} \mathbf{v}_P &= \dot{\mathbf{r}}_P = \cancel{\dot{\mathbf{r}}_1} + \dot{\mathbf{r}}_4 \\ &= 0 + \frac{d}{dt}(r_4\cos\theta_4\mathbf{i} + r_4\sin\theta_4\mathbf{j}) \\ &= \left(-r_4\dot\theta_4\sin\theta_4\right)\mathbf{i} + \left(r_4\dot\theta_4\cos\theta_4\right)\mathbf{j} \end{aligned}$$



$$\begin{aligned} \mathbf{r}_Q &= \mathbf{r}_2 \\ \mathbf{v}_Q &= \dot{\mathbf{r}}_Q = \dot{\mathbf{r}}_2 \\ &= \frac{d}{dt}(r_2\cos\theta_2\mathbf{i} + r_2\sin\theta_2\mathbf{j}) \\ &= \left(-r_2\dot\theta_2\sin\theta_2\right)\mathbf{i} + \left(r_2\dot\theta_2\cos\theta_2\right)\mathbf{j} \end{aligned}$$

Acceleration Analysis

$$\dot{\mathbf{r}}_2+\dot{\mathbf{r}}_3-\dot{\mathbf{r}}_4-\cancel{\dot{\mathbf{r}}_1}=\mathbf{0}$$

$$\dot{\mathbf{r}}_3-\dot{\mathbf{r}}_4=-\dot{\mathbf{r}}_2$$

$$\begin{aligned}
-r_3\dot{\theta}_3\sin\theta_3+r_4\dot{\theta}_4\sin\theta_4&=r_2\dot{\theta}_2\sin\theta_2\\
-r_3\dot{\theta}_3\cos\theta_3+r_4\dot{\theta}_4\cos\theta_4&=r_2\dot{\theta}_2\cos\theta_2
\end{aligned}$$

$$\begin{aligned}
-(r_3\ddot{\theta}_3\sin\theta_3+r_3\dot{\theta}_3^2\cos\theta_3)+(r_4\ddot{\theta}_4\sin\theta_4+r_4\dot{\theta}_4^2\cos\theta_4)&=(r_2\ddot{\theta}_2\sin\theta_2+r_2\dot{\theta}_2^2\cos\theta_2)\\
-(r_3\ddot{\theta}_3\cos\theta_3-r_3\dot{\theta}_3^2\sin\theta_3)+(r_4\ddot{\theta}_4\cos\theta_4-r_4\dot{\theta}_4^2\sin\theta_4)&=(r_2\ddot{\theta}_2\cos\theta_2-r_2\dot{\theta}_2^2\sin\theta_2)
\end{aligned}$$

$$\begin{aligned}
-r_3\ddot{\theta}_3\sin\theta_3+r_4\ddot{\theta}_4\sin\theta_4&=r_3\dot{\theta}_3^2\cos\theta_3-r_4\dot{\theta}_4^2\cos\theta_4+(r_2\ddot{\theta}_2\sin\theta_2+r_2\dot{\theta}_2^2\cos\theta_2)\\
-(r_3\ddot{\theta}_3\cos\theta_3)+r_4\ddot{\theta}_4\cos\theta_4&=-r_3\dot{\theta}_3^2\sin\theta_3+r_4\dot{\theta}_4^2\sin\theta_4+(r_2\ddot{\theta}_2\cos\theta_2-r_2\dot{\theta}_2^2\sin\theta_2)
\end{aligned}$$

$$\begin{bmatrix}-r_3\sin\theta_3 & r_4\sin\theta_4\\ -r_3\cos\theta_3 & r_4\cos\theta_4\end{bmatrix}\begin{bmatrix}\ddot{\theta}_3\\ \ddot{\theta}_4\end{bmatrix}=\begin{bmatrix}(r_2\ddot{\theta}_2\sin\theta_2+r_2\dot{\theta}_2^2\cos\theta_2)+r_3\dot{\theta}_3^2\cos\theta_3-r_4\dot{\theta}_4^2\cos\theta_4\\ (r_2\ddot{\theta}_2\cos\theta_2-r_2\dot{\theta}_2^2\sin\theta_2)-r_3\dot{\theta}_3^2\sin\theta_3+r_4\dot{\theta}_4^2\sin\theta_4\end{bmatrix}$$

$$\boxed{\mathbf{A}_{n\times n}\,\mathbf{x}_{n\times 1}=\mathbf{b}_{n\times 1}}$$

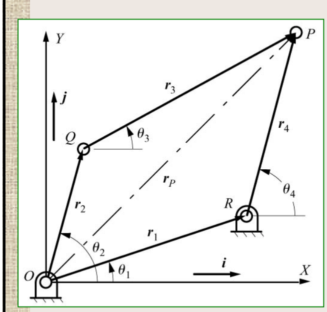
$d^2\theta_3/dt^2$ and $d^2\theta_4/dt^2$ can be determined by solving Ax=b

Acceleration Analysis (cont.)

$$\mathbf{r}_P=\mathbf{r}_2+\mathbf{r}_3=\mathbf{r}_1+\mathbf{r}_4$$

$$\begin{aligned}
\mathbf{v}_P&=\dot{\mathbf{r}}_P=\dot{\mathbf{r}}_2+\dot{\mathbf{r}}_3\\
&=\frac{d}{dt}(r_2\cos\theta_2\mathbf{i}+r_2\sin\theta_2\mathbf{j})+\frac{d}{dt}(r_3\cos\theta_3\mathbf{i}+r_3\sin\theta_3\mathbf{j})\\
&=\left(-r_2\dot{\theta}_2\sin\theta_2-r_3\dot{\theta}_3\sin\theta_3\right)\mathbf{i}+\left(r_2\dot{\theta}_2\cos\theta_2+r_3\dot{\theta}_3\cos\theta_3\right)\mathbf{j}
\end{aligned}$$

$$\begin{aligned}
\mathbf{v}_P&=\dot{\mathbf{r}}_P=\cancel{\dot{\mathbf{r}}_1}+\dot{\mathbf{r}}_4\\
&=0+\frac{d}{dt}(r_4\cos\theta_4\mathbf{i}+r_4\sin\theta_4\mathbf{j})\\
&=\left(-r_4\dot{\theta}_4\sin\theta_4\right)\mathbf{i}+\left(r_4\dot{\theta}_4\cos\theta_4\right)\mathbf{j}
\end{aligned}$$



$$\begin{aligned}
\mathbf{r}_Q&=\mathbf{r}_2\\
\mathbf{v}_Q&=\dot{\mathbf{r}}_Q=\dot{\mathbf{r}}_2\\
&=\frac{d}{dt}(r_2\cos\theta_2\mathbf{i}+r_2\sin\theta_2\mathbf{j})\\
&=\left(-r_2\dot{\theta}_2\sin\theta_2\right)\mathbf{i}+\left(r_2\dot{\theta}_2\cos\theta_2\right)\mathbf{j}
\end{aligned}$$

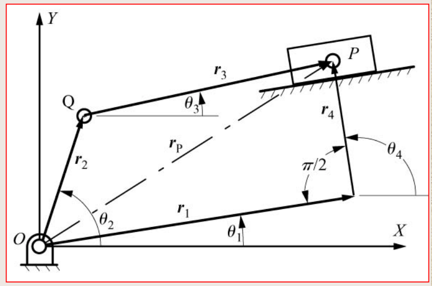
Loop Closure Equations for a Slider-Crank Mechanism

$$\mathbf{r}_p = \mathbf{r}_2 + \mathbf{r}_3 = \mathbf{r}_1 + \mathbf{r}_4$$

or

$$\boxed{\mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_4 - \mathbf{r}_1 = \mathbf{0}}$$

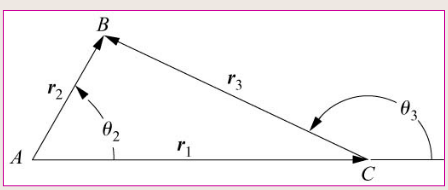
$$\begin{cases} r_2\cos\theta_2 + r_3\cos\theta_3 - r_4\cos\theta_4 - r_1\cos\theta_1 = 0 \\ r_2\sin\theta_2 + r_3\sin\theta_3 - r_4\sin\theta_4 - r_1\sin\theta_1 = 0 \end{cases}$$

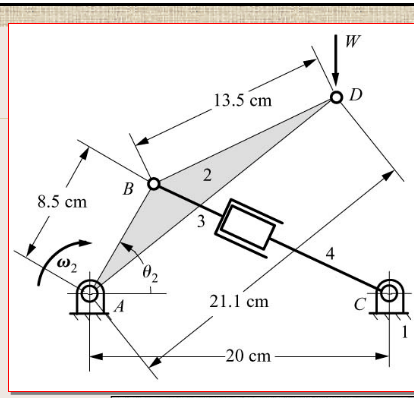


1. $\theta_4=\theta_1+\pi/2$, θ_1=const; r_2, r_3, and r_4 are also constants
2. There are two equations in terms of three independent variables, θ_2, θ_3, and r_1
3. Case 1: θ_2 is given, ➔ variables: r_1 and θ_3 (compressor)
4. Case 2: r_1 is given ➔ variables: θ_2 and θ_3 (Engine)

原則: 使向量迴路方程式中之每一個向量最多只含有一個變數

Loop Closure Equations for a Foot-Pump Mechanism



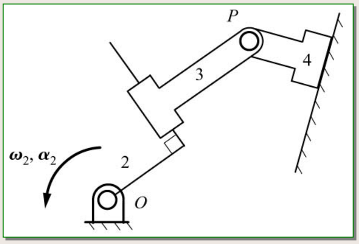


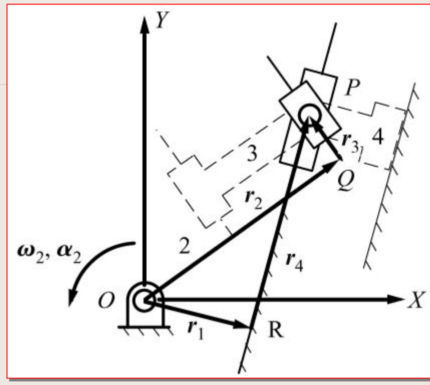
$$\boxed{\mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_1 = 0}$$

$$\begin{cases} r_2\cos\theta_2 + r_3\cos\theta_3 - r_1\cos\theta_1 = 0 \\ r_2\sin\theta_2 + r_3\sin\theta_3 - r_1\sin\theta_1 = 0 \end{cases}$$

1. $\mathbf{r}_1$ is a constant vector
2. There are two scalar equations in terms of three independent variables, θ_2, θ_3, and r_3
3. θ_2 is given, ➔ variables: r_3 and θ_3

Loop Closure Equations for an RPRP Mechanism



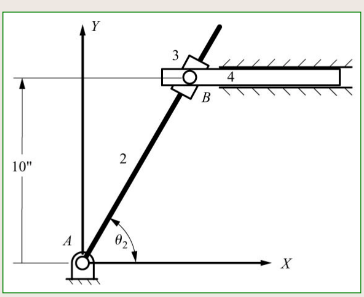


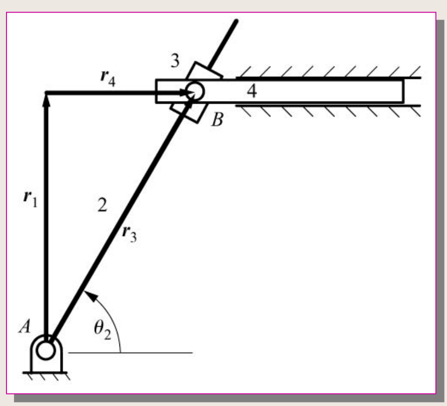
$$\mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_4 - \mathbf{r}_1 = \mathbf{0}$$

$$\begin{cases} r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 \cos\theta_1 = 0 \\ r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 - r_1 \sin\theta_1 = 0 \end{cases}$$

1. $\mathbf{r}_1$ *is a constant vector;* $\theta_4=\theta_1+\pi/2$, θ_4*=const;* $\theta_3=\theta_2+\pi/2$; r_2*=constant*
2. *There are two equations in terms of three independent variables,* θ_2, r_3, *and* r_4
3. θ_2 *is given,* ➔ *variables:* r_3 and r_4

Loop Closure Equations for an RPRP Mechanism

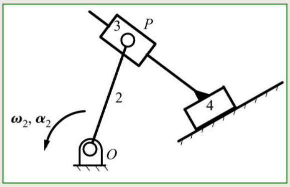


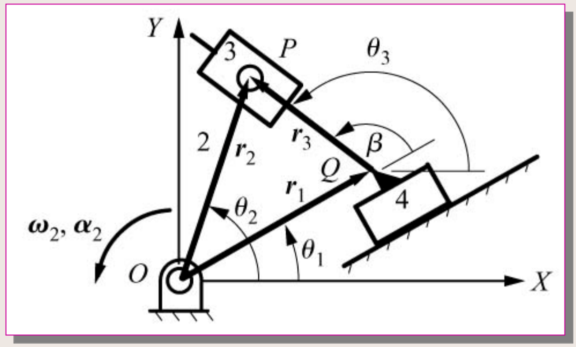


$$\mathbf{r}_1 + \mathbf{r}_4 - \mathbf{r}_3 = \mathbf{0}$$

原則: 使向量迴路方程式中之每一個向量最多只含有一個變數

Loop Closure Equations for an RRPP Mechanism

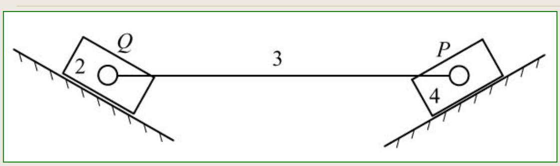


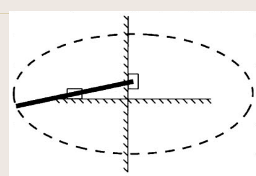


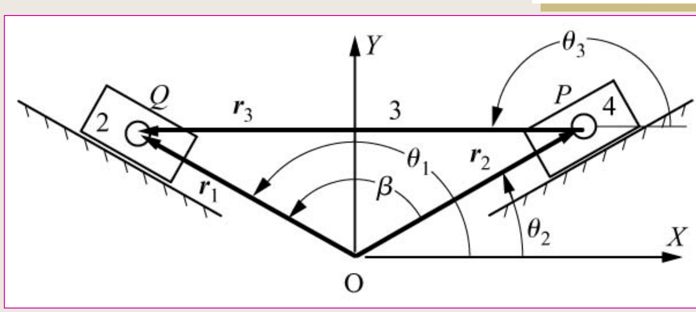
$$\mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_1 = \mathbf{0}$$

原則: 使向量迴路方程式中之每一個向量最多只含有一個變數

Loop Closure Equations for Elliptic Trammel

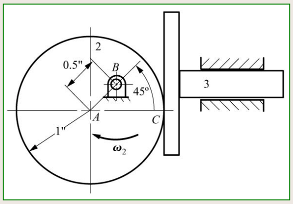


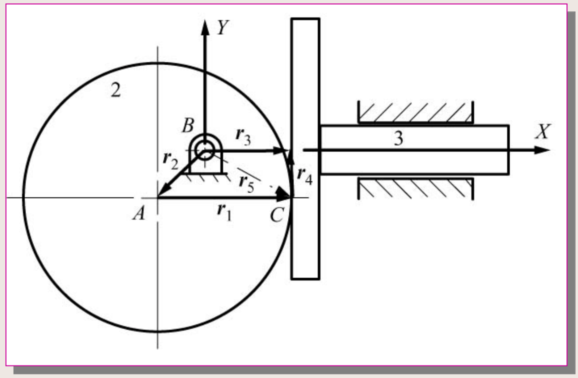




$$\mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_1 = \mathbf{0}$$

Loop Closure Equations for a Cam Mechanism





$$\mathbf{r}_2 + \mathbf{r}_1 + \mathbf{r}_4 - \mathbf{r}_3 = \mathbf{0}$$

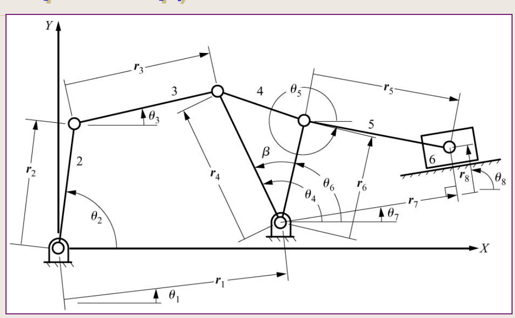
原則: 使向量迴路方程式中之每一個向量最多只含有一個變數

Vector Closure Conditions for Compound (Multiple-Loop) Mechanism

Euler's theory:

$U = J\text{-}N+1$

where
U : number of Independent loops
J : number of simple joints
N :number of links



1. $U=J\text{-}N+1=7\text{-}6+1=2$ ➔ two independent loops are needed
2. Vector loop equations:
   - **a.** $\mathbf{r}_2+\mathbf{r}_3-\mathbf{r}_4-\mathbf{r}_1=0$
   - **b.** $\mathbf{r}_6+\mathbf{r}_5-\mathbf{r}_8-\mathbf{r}_7=0$
3. $\mathbf{r}_1$ *and* $\mathbf{r}_8$ **are** *a constant vector;* $\theta_4=\theta_6+\beta$; $\theta_8=\theta_7+\pi/2$; r_2, r_3, r_4, r_5, *and* r_6 *are constants*
4. *There are four equations in terms of five independent variables*, θ_2, θ_3, θ_4, θ_5, and r_7
5. θ_2 *is given,* ➔ *variables:* θ_3, θ_4, θ_5, and r_7

Newton-Raphson method
One Equation in One Unknown

- Considering an nonlinear equation $\Phi(x)=0$, we have

$$\Phi_x \equiv \frac{d\Phi}{dx} \cong \frac{\Phi(x^{j+1})-\Phi(x^j)}{x^{j+1}-x^j}$$

- In order that $\Phi(x^{j+1})=0$, we must specify x^{j+1} to satisfy

$$\Phi_x = \frac{0-\Phi(x^j)}{x^{j+1}-x^j}$$

- Thus, we will approach a root of $\Phi(x^{j+1})=0$ by selecting

$$x^{j+1} = x^j - \frac{\Phi(x^j)}{\Phi_x}$$

Newton-Raphson method
One Equation in One Unknown (cont.)

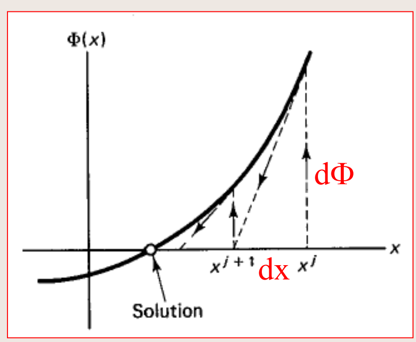
$$\Phi_x \equiv \frac{d\Phi}{dx} \cong \frac{\Phi(x^{j+1})-\Phi(x^j)}{x^{j+1}-x^j}$$

- If x_1^{j+1} are the root of $\Phi=0$, we have

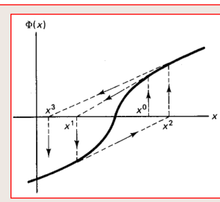
$$\Phi_x = \frac{0-\Phi(x^j)}{x^{j+1}-x^j}$$

- Then,

$$\boxed{x^{j+1} = x^j - \frac{\Phi(x^j)}{\Phi_x}}$$



- A case of failure



Newton-Raphson method
One Equation in One Unknown

```
function test
clc
x0=10
newton1(x0)

%-----------------------------
function y=newton1(x0)
x1=x0-fun(x0)/fq(x0);
n=1;
while (norm(x1-x0)>=1.0e-6)&(n<=1000)
    x0=x1;
    x1=x0-fun(x0)/fq(x0);
    n=n+1;
end
x1 % print solution
n % print num of iteration
```

```
%-----------------------------
function y=fun(x)
y = x*x - 4.0;

%-----------------------------
function y=fq(x)
y=2*x;
```

$$\boxed{x^{j+1} = x^j - \frac{\Phi(x^j)}{\Phi_x}}$$

Newton-Raphson method
2 Equations in 2 Unknowns

$$\begin{aligned} x_1^{j+1} &= x_1^j + \Delta x_1 \\ x_2^{j+1} &= x_2^j + \Delta x_2 \end{aligned} \Rightarrow \mathbf{x}^{j+1} = \mathbf{x}^j + \Delta\mathbf{x}$$

- Consider two nonlinear equations and Taylor theorem

$$\begin{aligned} \Phi_1(x_1, x_2) &= 0 \\ \Phi_2(x_1, x_2) &= 0 \end{aligned} \quad \text{Eq. (1)}$$

$$\Phi_1(x_1^{j+1}, x_2^{j+1}) = \Phi_1(x_1^j, x_2^j) + \Delta x_1 \frac{\partial \Phi_1}{\partial x_1} + \Delta x_2 \frac{\partial \Phi_1}{\partial x_2} + \cdots$$

$$\Phi_2(x_1^{j+1}, x_2^{j+1}) = \Phi_2(x_1^j, x_2^j) + \Delta x_1 \frac{\partial \Phi_2}{\partial x_1} + \Delta x_2 \frac{\partial \Phi_2}{\partial x_2} + \cdots$$

- If (x_1^{j+1}, x_2^{j+1}) are the roots Eq. (1), we get

$$\begin{aligned} 0 &= \Phi_1(x_1^j, x_2^j) + \Delta x_1 \frac{\partial \Phi_1}{\partial x_1} + \Delta x_2 \frac{\partial \Phi_1}{\partial x_2} \\ 0 &= \Phi_2(x_1^j, x_2^j) + \Delta x_1 \frac{\partial \Phi_2}{\partial x_1} + \Delta x_2 \frac{\partial \Phi_2}{\partial x_2} \end{aligned} \Rightarrow \begin{bmatrix} \frac{\partial \Phi_1}{\partial x_1} & \frac{\partial \Phi_1}{\partial x_2} \\ \frac{\partial \Phi_2}{\partial x_1} & \frac{\partial \Phi_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = -\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}$$

$$\boxed{\mathbf{\Phi}_{\mathbf{x}}(\mathbf{x}^j)\Delta\mathbf{x} = -\mathbf{\Phi}(\mathbf{x}^j)}$$

Newton-Raphson method
N Equations in N Unknown

- Similarly, for a set of *n* nonlinear equations with *n* unknowns, we can determine $\Delta\mathbf{x}$ by solving

$$\Phi_{\mathbf{x}}(\mathbf{x}^j)\Delta\mathbf{x} = -\Phi(\mathbf{x}^j)$$

$$\boxed{\mathbf{A}_{n\times n}\,\mathbf{x}_{n\times 1} = \mathbf{c}_{n\times 1}}$$

- Then, $\mathbf{x}^{j+1}$ can be evaluated from

$$\boxed{\mathbf{x}^{j+1} = \mathbf{x}^j + \Delta\mathbf{x}}$$

Example

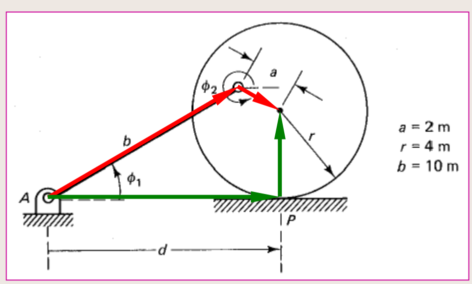
Apply the Newton-Raphson method to find Φ_2 and d when $\Phi_1 = 30°$.

$$\mathbf{b} + \mathbf{a} - \mathbf{r} - \mathbf{d} = \mathbf{0}$$

$$\Phi_1 \equiv b\cos\phi_1 + a\cos\phi_2 - d = 0$$
$$\Phi_2 \equiv b\sin\phi_1 + a\sin\phi_2 - r = 0$$

$$\mathbf{q} = [\phi_2, d, \phi_1]^T$$

$$\Phi_{\mathbf{q}} \equiv \left[\begin{array}{cc|c} -2\sin\phi_2 & -1 & -10\sin\phi_1 \\ 2\cos\phi_2 & 0 & 10\cos\phi_1 \end{array}\right]$$



Iteration number	$\phi_2^\dagger$	d
1	5.59 (326°)	10.0
2	5.77	10.43
3	5.76	10.3924

Initial guess x0