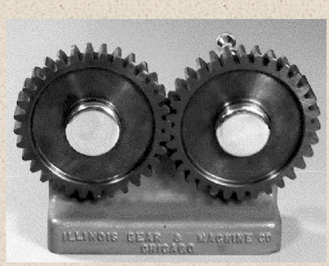
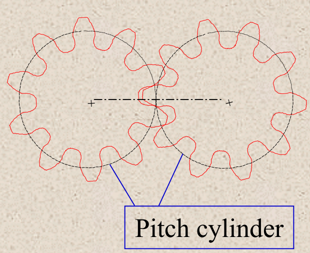
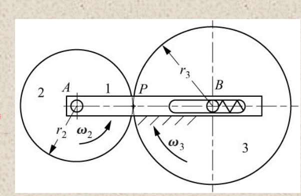
Chapter 10 Spur Gears

Spur Gears & Friction Disks
ref : pp. 458-460

- The motion of a pair of spur gears is equivalent to a pair of rolling cylinders called *pitch cylinder*







Friction disks

- When two gears of unequal size are meshed, the smaller gear is referred to as the ***pinion*** and the larger gear as the "***gear***" or "***wheel***"
- When the pitch cylinder radius becomes infinitely large, the teeth are located on a plane, and such a gear is called a ***rack*** (齒條)

Fundamental Law of Gearing

Condition for Constant-Velocity Ratio (pp. 460-461)

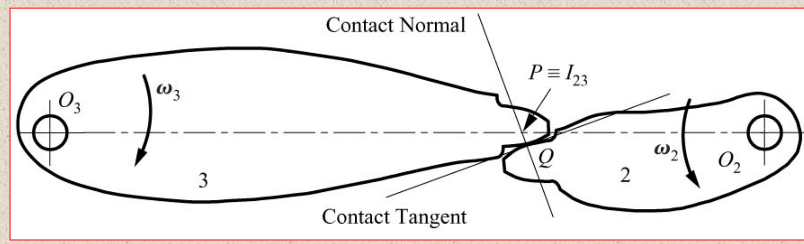
- Fundamental Law of Gearing:
  for conjugate profiles that yields constant-velocity ratio, the normal to the profiles at the point of contact always intersects the line of centers at the same point. That point is called the pitch point
  (兩齒輪若欲以等角速比傳動, 則通過接觸點之公法線恒交連心線於一個固定點-節點)

Pf:

$$V_P = \overline{O_3P}\omega_3 = \overline{O_2P}\omega_2 \rightarrow \frac{\omega_2}{\omega_3} = \frac{O_3P}{O_2P} = const$$

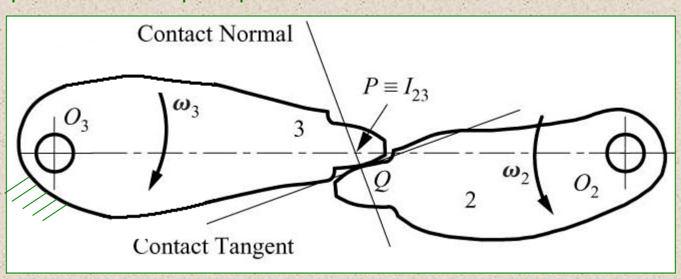
$$\overline{O_3P} + \overline{O_2P} = O_3O_2 = const$$

$\rightarrow$ P is a fixed point on the line O_3O_2



Sliding Velocity at Contact point Q

- *The rate of wear of the gear teeth depends on the sliding velocity*
- Consider the inverse motion that gear 3 is fixed ($V_{P2}=V_{P3}=V_{Q3}=0$), and gear 2 rotates with $(\omega_2 - \omega_3)$
- In this situation, pitch point P is the instantaneous rotating center of gear 2 and $V_{Q2}=(PQ)(\omega_2 - \omega_3)$
- Thus, the sliding velocity at contact point Q is
  - $V_s=V_{Q2}-V_{Q3}=(PQ)(\omega_2 - \omega_3)$
- The sliding velocity is proportional to the distance between the contact point Q and the pitch point P



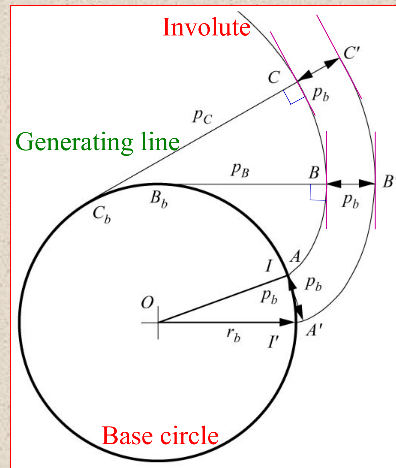
Involutes (漸開線)

Important properties of involute curves:

1.Generating line is always normal to the involute and tangent to the base circle

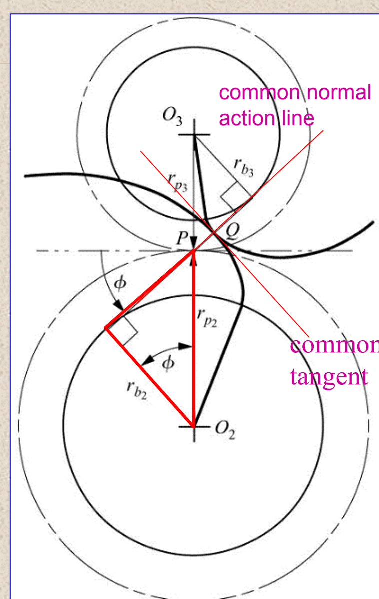
2. $\overline{B_bB} = \widehat{B_bI}$=radius of curvature at B

3. BB' = CC'= AA'=II'= P_b=base pitch(基節)



Spur Gears with Involute Profiles

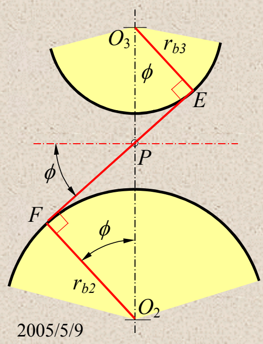
- Consider two involute profiles contact in an arbitrary position
- Since the generating line is always normal to the involute and tangent to the base circle, it always intersects the line of centers at the same point regardless of the motion of the profiles. Thus, involute profiles satisfy with the fundamental law of gearing

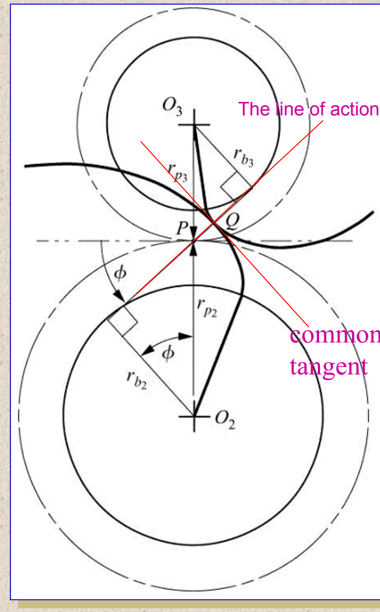


- r_b = the radius of the pitch circle r_b
- r_p = the radius of the pitch circle
- ϕ = pressure angle
- $r_b=r_p \cos\phi$

Properties of Spur Gears with Involute Profiles

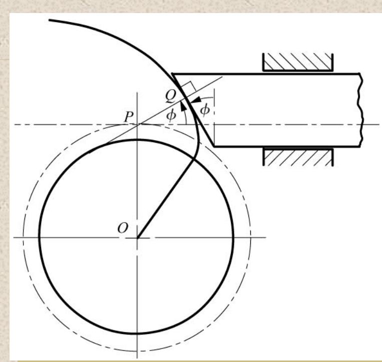
- The path of contact is rectilinear
  (接觸路徑爲一直線)
  - Since the normal at the point of contact, Q, is always the same line (the line of action), thus contact point Q simply moves along the action line
- The pressure angle is constant
  (壓力角爲常數--固定不變)
- Two involute profiles remain conjugate if their center distance is changed

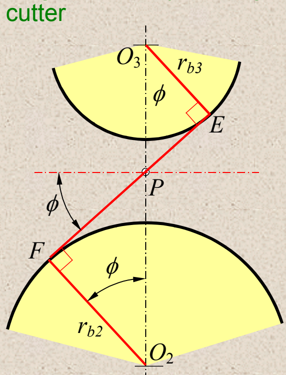


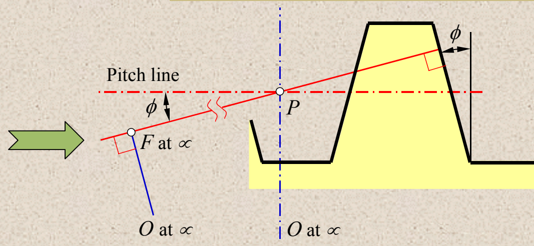


Properties of Spur Gears with Involute Profiles(cont)

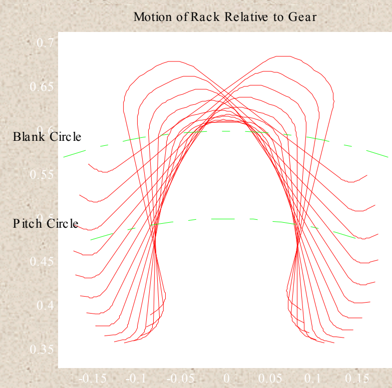
- The involute profile of a rack is a straight line and is perpendicular to the line of action
- A gear with involute tooth profiles can be generated by a straight-sided rack cutter

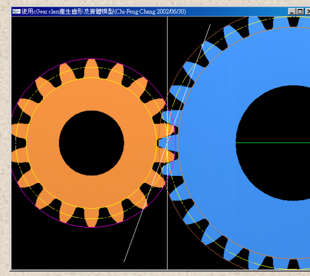


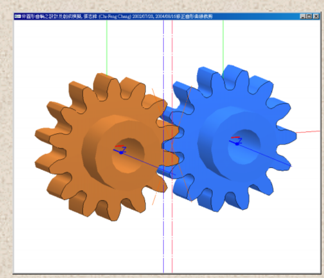




Spur Gear & Straight-sided Rack Cutter







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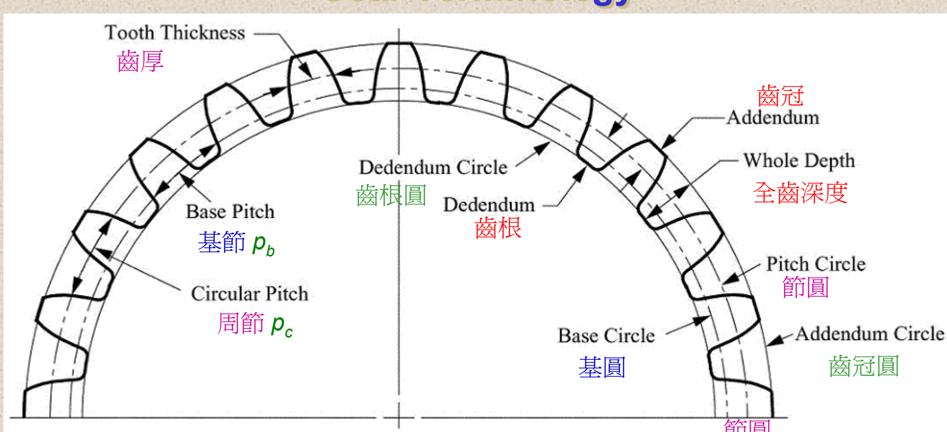
Gear Terminology



$m \equiv \dfrac{d_p}{N}$

$p_c = \dfrac{\pi d_p}{N}$

$p_b = \dfrac{\pi d_b}{N}$

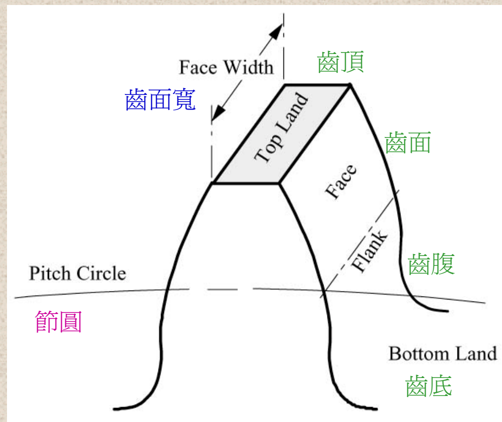
$r_b = r_c \cos\phi$

$p_c = \pi m$

$p_b = p_c \cos\phi$

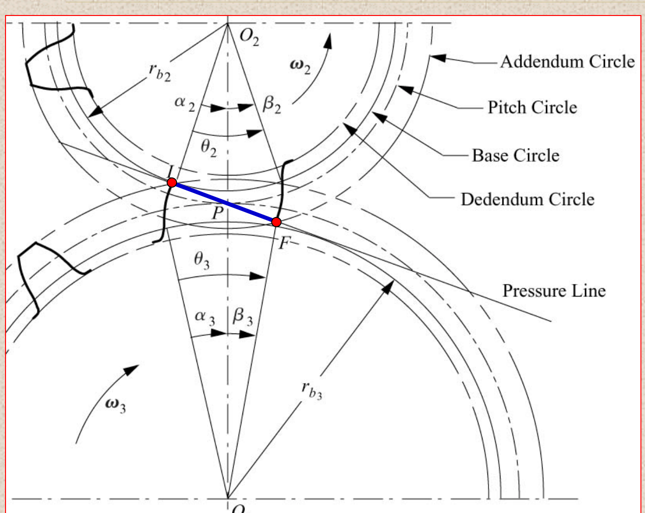
$d_b = 2r_b$
$d_p = 2r_c$
$P_d \equiv N/d_p$ (徑節)

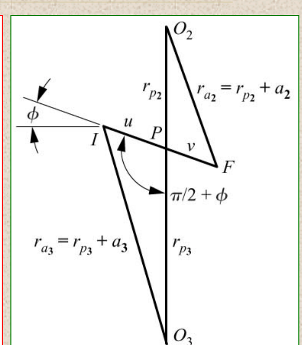
Gear Terminology (cont)



- If two gears are to mesh, it is necessary that their pitches and pressure angles be the same

The Length of Path of Contact
接觸路徑長度





The Length of Path of Contact (cont.)

$$u = \sqrt{r_{a_3}^2 - (r_{p_3}\cos\phi)^2} - r_{p_3}\sin\phi$$

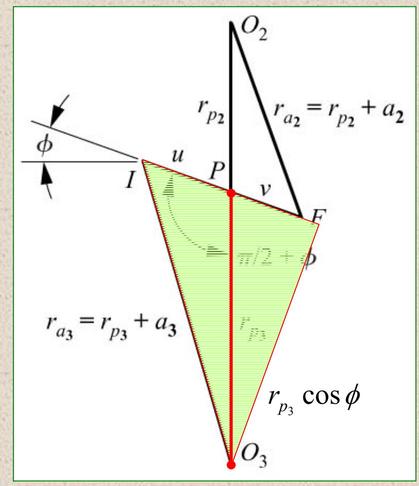
$$v = \sqrt{r_{a_2}^2 - (r_{p_2}\cos\phi)^2} - r_{p_2}\sin\phi$$

The length of the path of contact

$$\lambda = IF = IP + PF = u + v$$

$$\begin{aligned} u &= \sqrt{(r_{p_3} + a_3)^2 - (r_{p_3}\cos\phi)^2} - r_{p_3}\sin\phi \\ &= \sqrt{r_{p_3}^2 + 2r_{p_3}a_3 + a_3^2 - r_{p_3}^2\cos^2\phi} - r_{p_3}\sin\phi \\ &= \sqrt{2r_{p_3}a_3 + a_3^2 + r_{p_3}^2\sin^2\phi} - r_{p_3}\sin\phi \end{aligned}$$

$$\begin{aligned} v &= \sqrt{(r_{p_2} + a_2)^2 - (r_{p_2}\cos\phi)^2} - r_{p_2}\sin\phi \\ &= \sqrt{r_{p_2}^2 + 2r_{p_2}a_2 + a_2^2 - r_{p_2}^2\cos^2\phi} - r_{p_2}\sin\phi \\ &= \sqrt{2r_{p_2}a_2 + a_2^2 + r_{p_2}^2\sin^2\phi} - r_{p_2}\sin\phi \end{aligned}$$

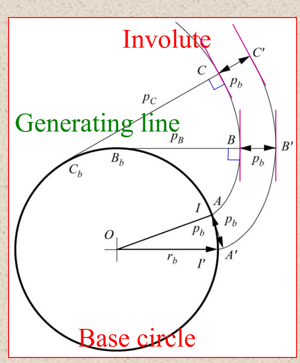


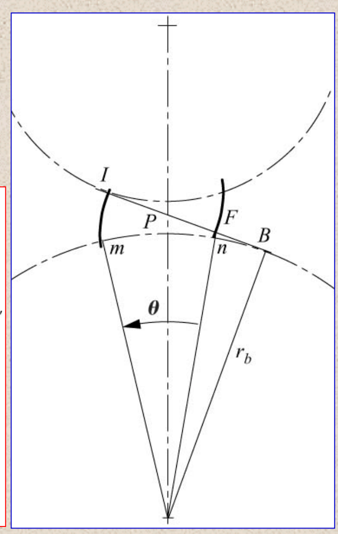
Contact Ratio 接觸比

The contact ratio m_c is defined as the average number of pairs of teeth which are in contact

m_c = length of path of contact / base pitch

$$m_c = \frac{\lambda}{p_b} = \frac{\lambda}{p_c\cos\phi}$$





Contact Angles
(Angle of Action 作用角)

Because the distance the contact point moves along the path of contact is equal to the curvilinear distance around the base circle, hence

The angle of approach (接近角)

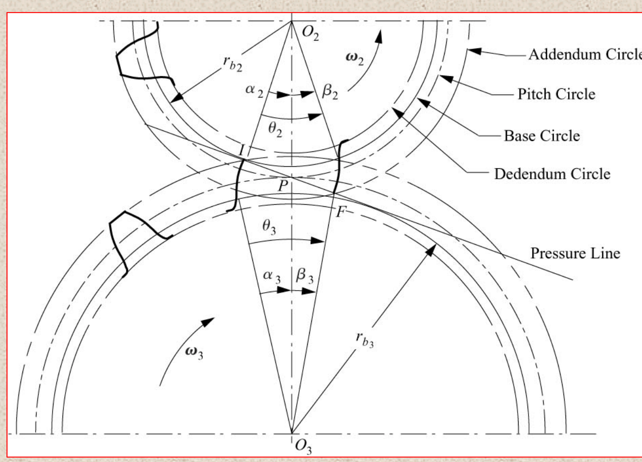
$$\alpha_2 = \frac{IP}{r_{b_2}}$$

The angle of recess (退遠角)

$$\beta_2 = \frac{PF}{r_{b_2}}$$

The angle of action (作用角)

$$\theta_2 = \alpha_2 + \beta_2 = \frac{IF}{r_{b_2}}$$



Example 10.1

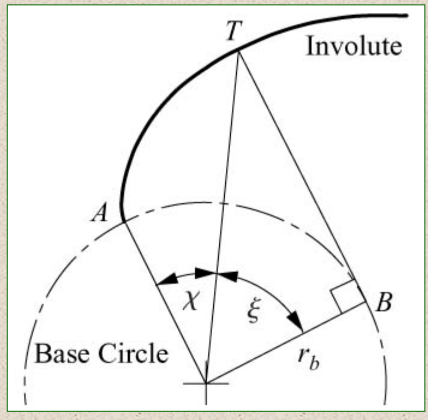
- Two gears are in mesh such that one gear (gear 2) has 20 teeth and the other (gear 3) has 30. The diametral pitch for each gear is 4, and the working pressure angle is 20°. Standard gears are involved in each case, and the addendum constant is 1. Determine the length of the contact line and the contact ratio.

$$\lambda = u + v = \sqrt{2r_{p_3}a_3 + a_3^2 + r_{p_3}^2 \sin^2\phi} - r_{p_3}\sin\phi + \sqrt{2r_{p_2}a_2 + a_2^2 + r_{p_2}^2 \sin^2\phi} - r_{p_2}\sin\phi$$

- N_2=20, N_3=30, ϕ=20 °, p_d=4
- Standard gears ➔ addendum a=1/p_d=1/4=0.25 in=a_2=a_3
- $p_d \equiv N/D$ ➔ $r_p = N/2p_d$
- $r_{p2}=N_2/2p_d$=20/(2)(4)=2.5 in
- $r_{p3}=N_3/2p_d$=30/(2)(4)=3.75
- Length of contact λ=1.185 in ←ANS
- $p_b=p_c\cos\phi=(\pi/p_d)\cos\phi$ =0.738 in
- Contact ratio $m_c= \lambda/p_b$=1.185/0.738=1.6052 ←ANS

Involutometry—Involute function

- $AB = BT = r_b \tan\xi$
- Involute function of ξ
  - $\mathrm{inv}(\xi) \equiv \chi = AB/r_b - \xi = \tan\xi - \xi$



Tooth Thickness *t* at Any Radius *r*

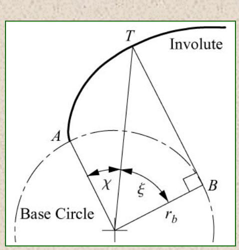
- From the figure we have

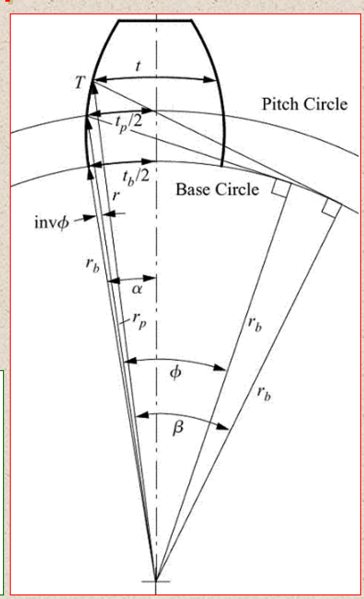
$$\frac{t}{2r} = \alpha - inv\beta \qquad \frac{t_p}{2r_b} = \alpha - inv\phi$$

$$\frac{t}{2r} - \frac{t_p}{2r_b} = -inv\beta + inv\phi$$

$$t = 2r\left(\frac{t_p}{2r_b} - inv\beta + inv\phi\right)$$

- $\cos\beta = r_b/r = r_p\cos\phi / r$





Example 10.2 (p. 472)

$$t = 2r\left(\frac{t_p}{2r_b} + inv\phi - inv\beta\right)$$

invξ = tan ξ - ξ

- Problem: Find the thickness at the addendum and base circles
- Given:
  - N=30, ϕ=20 °, p_d=4
  - Standard gear ➔ $t_p=p_c/2$, addendum a=1/p_d=1/4=0.25 in
- Sol:
  - $p_d \equiv N/D$ ➔ $r_p = N/2p_d = 30/(2)(4) = 3.75$ in
  - Radius of addendum circle $r_a = r_p+a = 3.75+0.25 = 4.0$ in
  - Radius of base circle $r_b = r_p\cos\phi = (3.75)\cos\phi = 3.524$ in
  - Circular tooth thickness $t_p = p_c/2 = (\pi/p_d)/2 = \pi/8 = 0.393$ in
- For the thickness at base circle, $r=r_b$, $\beta=0$ ➔
- $t_b=2r_b\{(t_p/2r_b)+inv\phi-inv\beta\}=2(3.524)\{(0.393/(2)(3.75)+inv20°-0\}$ =0.474 in ← ANS
- For the thickness at addendum circle, $r=r_a$, $\cos\beta=r_p\cos\phi/r_a$ ➔ β=28.241°
- $t_a=2r_a\{(t_p/2r_b)+inv\phi-inv\beta\}=2(4.0)\{(0.393/(2)(3.75)+inv20°-inv28.241°\}$ =0.184 in ← ANS

Example 10.3 (p. 473)

$$t = 2r\left(\frac{t_p}{2r_b} + inv\phi - inv\beta\right)$$

invξ = tan ξ - ξ

$\cos\beta = r_b/r = r_p\cos\phi / r$

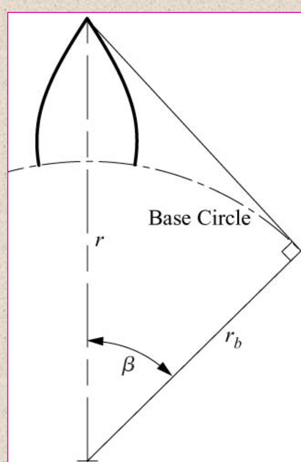
- Problem: For the gear in Example 10.2, determine the radius r that yields zero tooth thickness (t=0)
- Sol:

$$\because t = 2r\left(\frac{t_p}{2r_b} + inv\phi - inv\beta\right) = 0$$

$$\frac{t_p}{2r_b} + inv\phi - inv\beta = 0$$

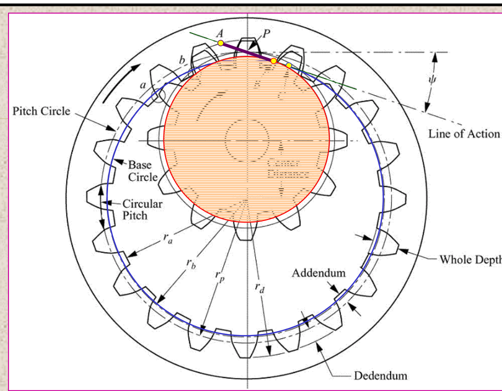
$$inv\beta \equiv \tan\beta - \beta = \frac{t_p}{2r_b} + inv\phi = \frac{0.393}{2(3.75)} + inv20^\circ = 0.0673$$

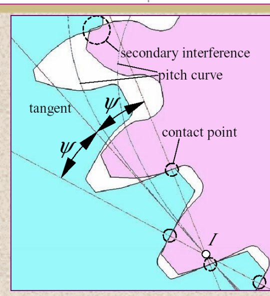
- Solving the nonlinear equation yields β=32.13°
- $r = r_b/\cos\beta$ =3.524/cos32.13° = 4.161 in ← ANS



Internal Gears

- Both the pinion and gear rotate in the same direction
- The advantages of an internal gear set are:
  - More compact than an external gear drive
  - Lower contact stresses (because a convex surface contacts a concave surface)
  - Lower relative sliding between teeth
  - Greater length of contact (note: there is no limit to the involute profile on the flank of the internal gear)
- There is a different type of interference called *fouling* or *secondary interference* (二次干涉)





Gear Manufacturing --Forming Gear teeth with Shaper--

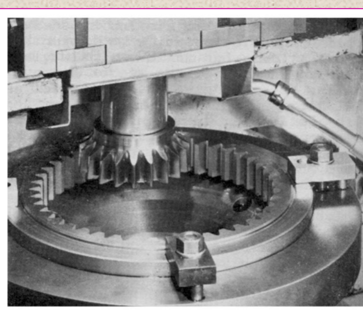
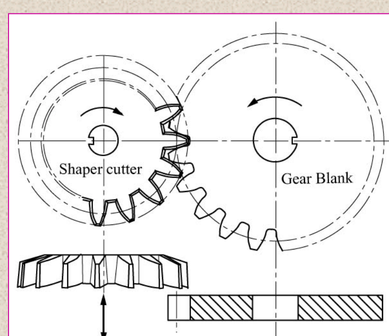
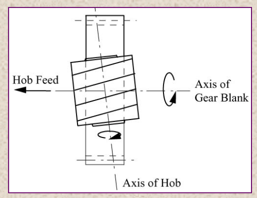
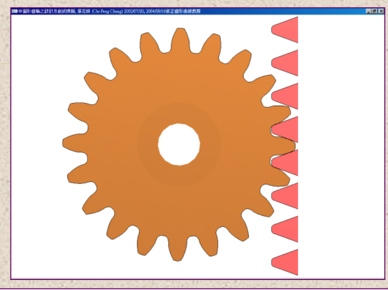


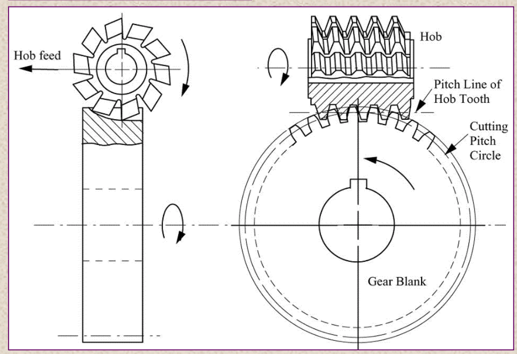
FIGURE 10.21 Shaping an internal gear. (Courtesy of Fellows Corporation, Springfield, Vermont.)



Gear Manufacturing
--Tooth Generation with a Hob--







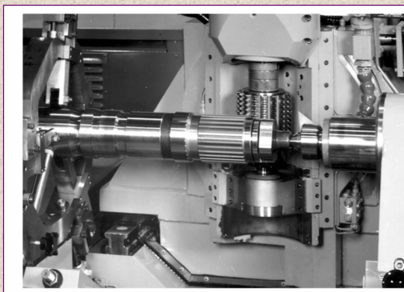
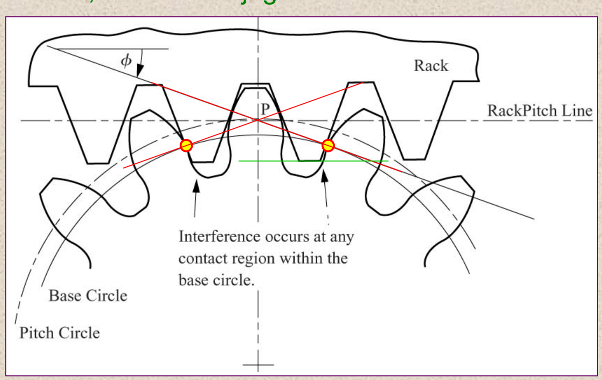
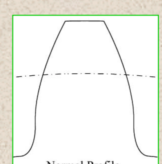


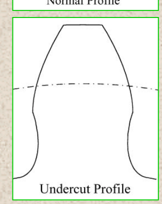
FIGURE 10.24 Hobbing spur gear teeth. (Courtesy of Bourn & Koch.)

Interference & Undercutting

- The cutter teeth will undercut the pinion teeth if the path of contact extends beyond the point of tangency (*interference point*) with the base circle of the pinion
- This is because the involute is not defined inside the base circle, and the conjugate action is lost







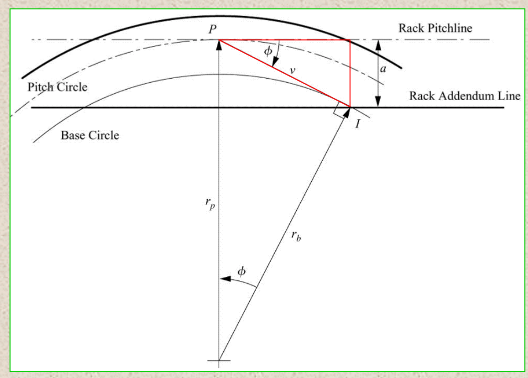
The Minimum Number of Teeth to Avoid Undercutting

- Interference will occur at all contact locations within the base circle
- Thus, there is a critical situation when the rack addendum line pass through the interference point *I*

$$v = \frac{a}{\sin\phi} = r_p \sin\phi$$

$$r_p = \frac{a}{\sin^2\phi} = \frac{km}{\sin^2\phi} \qquad r_p = \frac{Nm}{2}$$

$$N = \frac{2k}{\sin^2\phi}$$



- Ex: For standard gear, k=1
- $\phi=14.5°\Rightarrow N_{min}=2(1)/(\sin 14.5°)^2=31.9=32^T$
- $\phi=20°\Rightarrow N_{min}=2(1)/(\sin 20°)^2=17.1=18^T$

Another Condition to Avoid Undercutting

$k_u=R-R_b\cos\psi=R(1-\cos^2\psi)=R\sin^2\psi$

So, The condition to avoid undercutting can be written as

$k\leq R\sin^2\psi$