Tree

[alias]

Free tree

[def]

A connected, undirected, acyclic graph.

[term]

1. parent: x is parent of y iff the edge (x,y) exists and the depth of x is smaller than the depth of y.
2. child: y is child of x means x is parent of y.
3. ancestor: x is an ancestor of y iff x is on a unique simple path from r to y.
4. descendent: y in a descendent of x means x is an ancestor of y.
5. proper ancestor: x is a proper ancestor of y iff x is an ancestor of y and
6. proper descendent: y is a proper ancestor of x iff y is descendent of x and
7. siblings: y and z are siblings iff they have same parent x.
8. leaf node (external node): node with no child.
9. Non-leaf node (internal node): it is opposite of the word leaf node.

More details of (3) to (6) can see my article named

“rooted\_tree”.

[identity]

If T is a tree and .

(1)

(2)

[pf]

We prove the identity (2)

1. For 1st part,

since it is connected.

1. For 2nd part,

since it is acyclic.

Let’s do more details with proof by contradiction.

Suppose that

then it must occurs:

there are at least two paths to connected from vertex v and u where .

Thus, this graph has at least 1 cycle.

It contradicts with the assumption.

1. From 1st and 2nd part, we can get

Hence proved.