Continuity

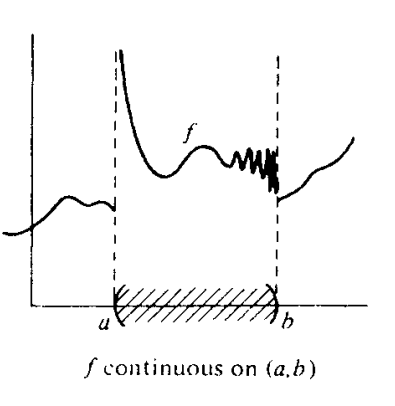
Def

Open interval

A function f is continuous at open interval iff

f is continuous at each point in .

Example



Closed interval

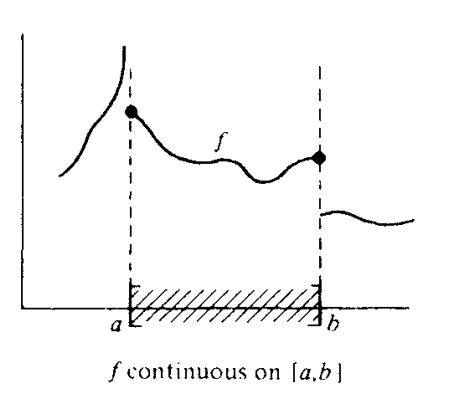
A function f is continuous at closed interval iff

f is continuous at open interval ,

f is continuous on the right of , and

f is continuous on the left of .

Example



Left open interval

A function f is continuous at left open interval iff

f is continuous at open interval , and

f is continous on the right of .

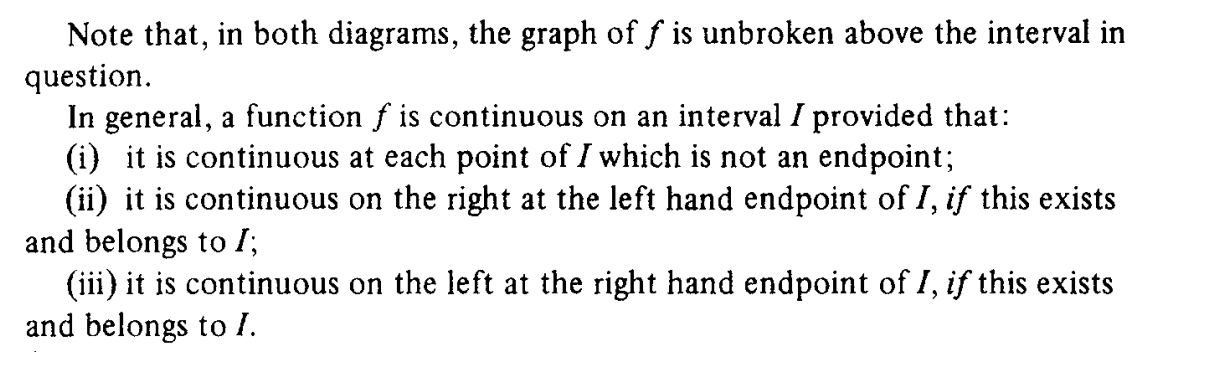
Right open interval

A function f is continuous at right open interval iff

f is continuous at open interval , and

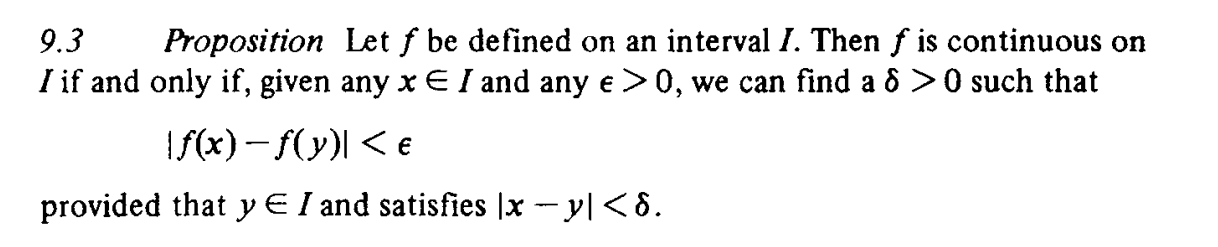
f is continuous on the left of .

Generalization for interval



Proposition

Epsilon-delta definition



Proof

1. Backward

Since there exists a real number which is very close to 0 but larger than 0.

(i.e. ) such that

one can imply that

=>

On the other hand, since there exist a real number such that

for all at the interval .

Now, use proof by contradiction of this subclaim.

The subclaim is, one of neccesity conditions of the claim

when for a small (i.e. )

is that is continuous at each point on the interval .

Suppose is NOT continuous at interval ,

then by definition of continuous function at interval, there must exist at least one point called such that is NOT continous at that point .

Therefore, there exist at least one point such that either one the following requirements satisfied (or more) by the definition of continuous function at one point.

1. does NOT exist
2. does NOT exist
3. !=

When one of the above requirement satisfied, one can find the fact that

neither does exist nor for a small number

(i.e. )

which completes the proof of first subclaim.

Notes:

does NOT exist

=> does NOT exist

=> does NOT exist

Similarly, when does NOT exist , then does NOT exist.

On the other hand, when != ,

then does NOT exist. Thus, does NOT exist.

Then, to prove second subclaim,

If is NOT true, then for is also NOT true.

If is NOT true, then it is NOT true that

for , .

Thus, it is also NOT true that

for

which completes the second subclaim.

1. Forward

is continuous on interval , and belongs to .

=> exists

When , then for , .

Since exists and when is at interval , one can imply that

for at interval

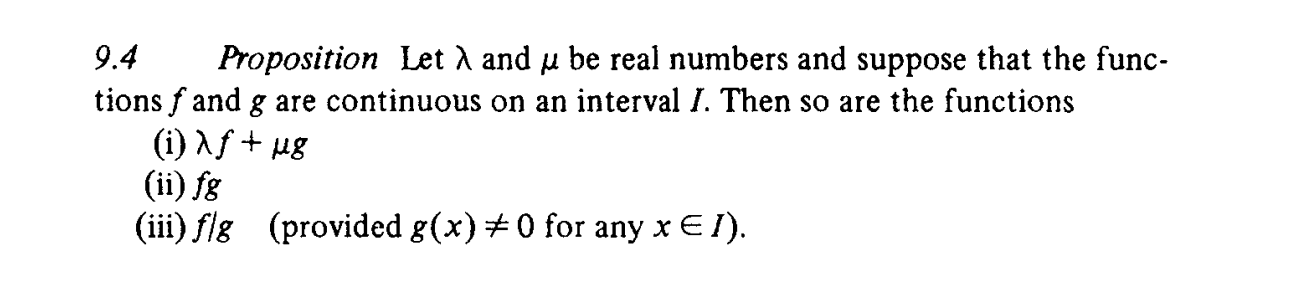
=>

Then there exist a real number which is very close to 0 but larger than 0.

(i.e.) such that

=>

Linear transformation



Proof

Since and are continuous on the interval , then

and is continuous at each point on the interval ,

Therefore, at each point on the interval , exists and so is .

Thus, for any scalar and ,

+

= +

=

does also exist.

Therefore,

is continuous on interval .

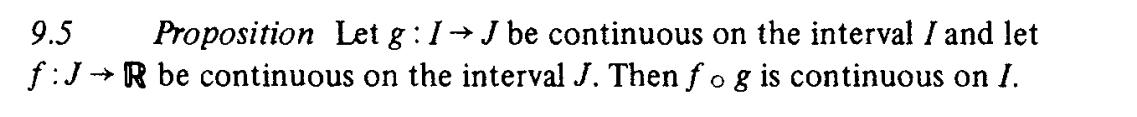
\*

=

does exist too. And so is .

Doing so for .

Composition function



Proof

Since is continuous on interval and is continuous on interval ,

then is continuous at each point on interval and so for at each point on interval .

Therefore,

For all point on interval , exists.

Now, let = .

Since is always on interval and is continuous at eacg point on interval , then does also exist.

which can imply that

=

=

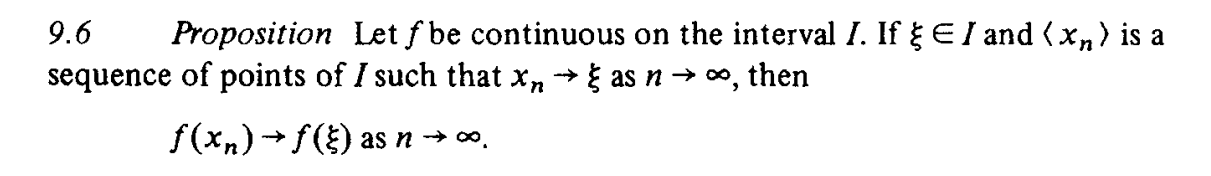
for all on interval intersects with .

does also exist.

Therefore,

is continuous.

Sequence



Proof

Since

(1) all points of sequence are on interval

(2) is continuous on interval .

then when ,

exists.

On the other hand, since on the interval .

does also exists.

By above claim, one has that

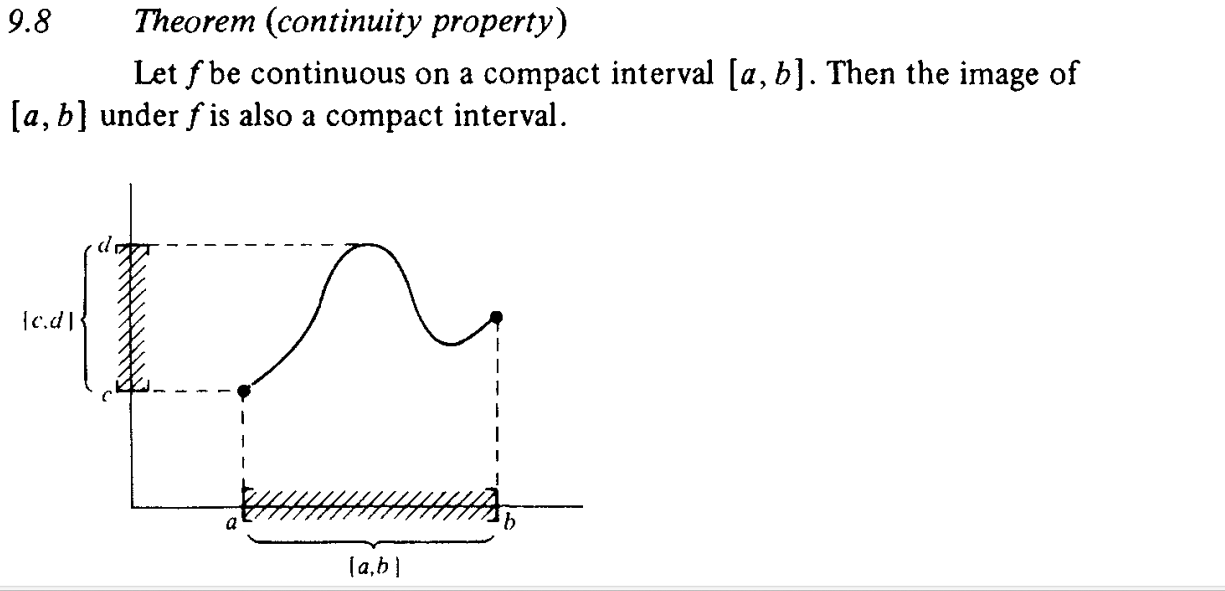
where

Thus,

as

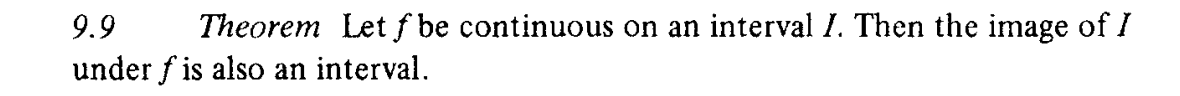
which completes the proof.

Continuity theorem



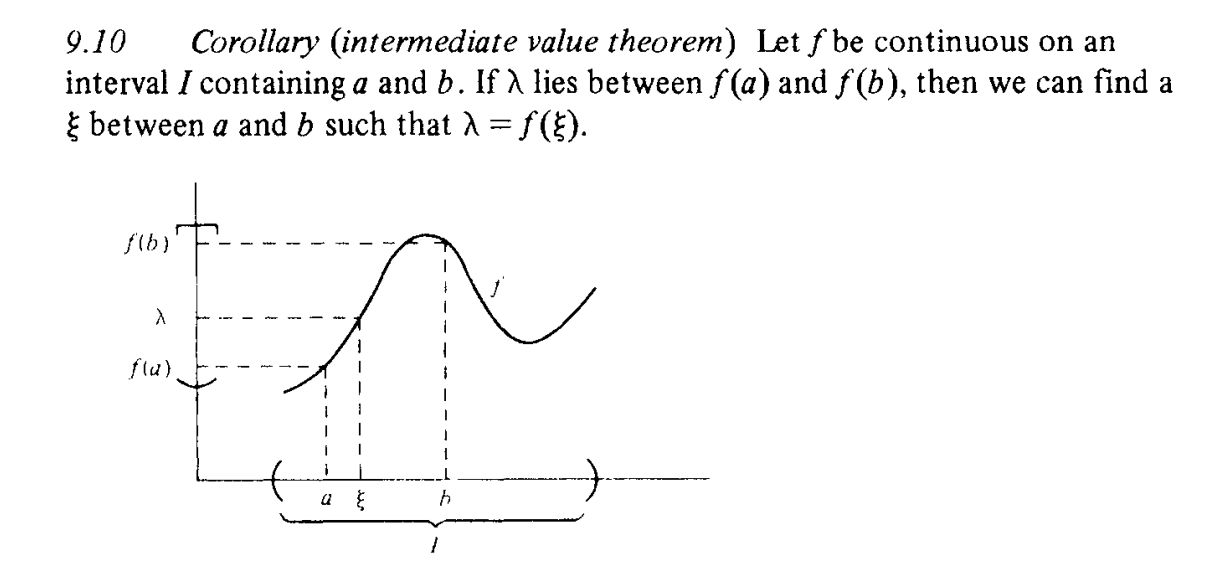
Proof

See section 9.8 in page 87 in cookbook.



Proof

See section 9.9 page 88 in cookbook.



Proof

Method 1:

See section 9.10 page 89 in cookbook.

It is a clumsy restatement of theroem 9.9.

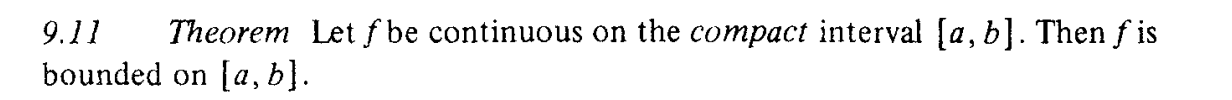
Given .

Since is continuous on interval ,

then for on interval , the corresponding point to the image of is also on the interval.

See the above figure for more detailed explanation.

Bounded function

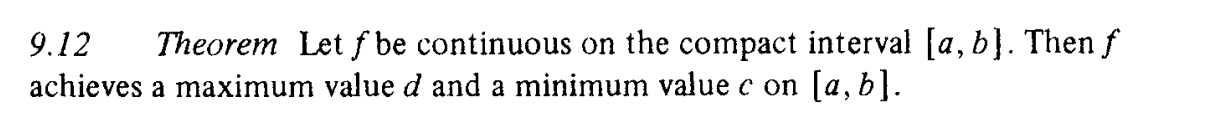


Proof

Proof by contradiction.

For more details, see section 9.11 page 88 in the cookbook.

Min value and Max value



Proof

For more details, see section 9.12 page 89 in the cookbook.