Trace

Intro

Trace is defined as sum of diagonal entries.

Def

Given with size .

=

Def of terms related to trace

= 0 iff is trace orthogonal to .

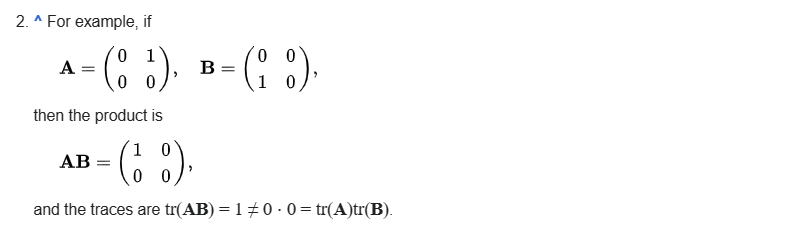
Myth

Non-communicativity

!=

Proof of myth

Non-communicativity



Property

Given:

Given matrix , with size and scalar .

Linear mapping (1)

= +

Linear mapping (2)

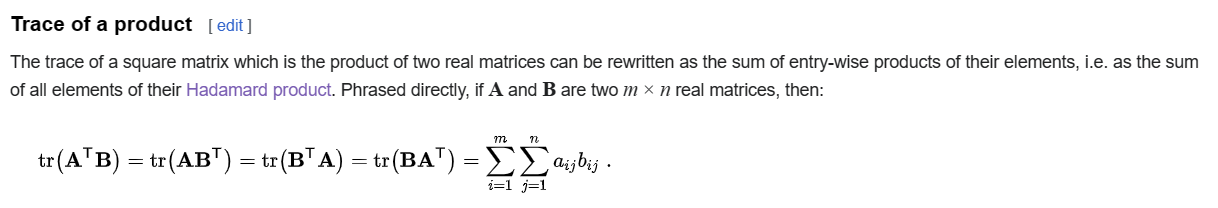
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Communicativity of product

=

Trace of a product

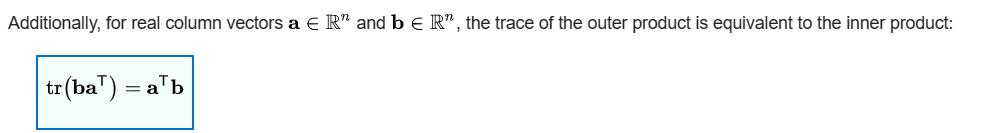
= ===



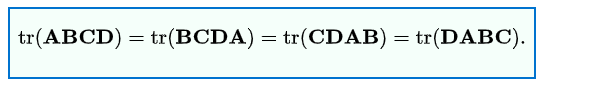
Trace of transpose for matrix

=

Trace of transpose for two vectors



Cyclic

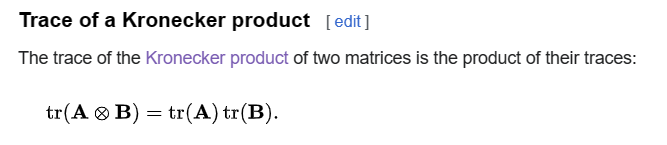


Trace as sum eigenvalue

The trace of is sum of which refers eigenvalue of A.

=

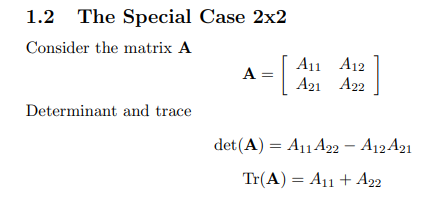
Trace of Kronecker product

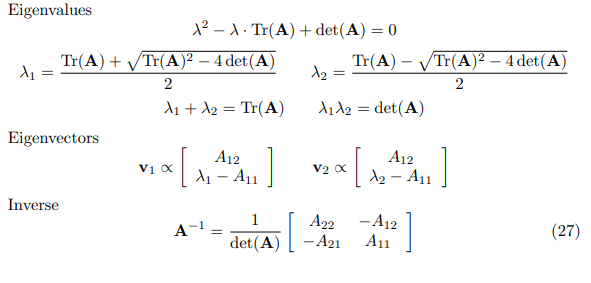


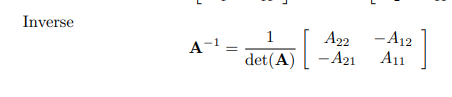
Communicativity of derivatives

=

Root and coefficients







Proof of property

Linear mapping (1)

=

= +

= +

Linear mapping (2)

=

=

=

Communicativity of product

=

where

=

if the fact is assumed = .

On the other hand,

=

where

=

if the fact is assumed = .

Because one only wants to evaluate the sum of diagonal entries,

One puts to , for matrix and . Getting:

1. =

=>

=

=

1. =

=>

=

=

Due to communicativity of multiplication bewteen numbers,

one has

=

Interchange to and to , one gets

=

=

Combining above equations, one has

=

=

=

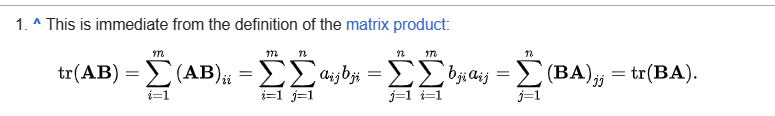
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which completes the proof.

For simpler proof, see Notes section in Wiki.

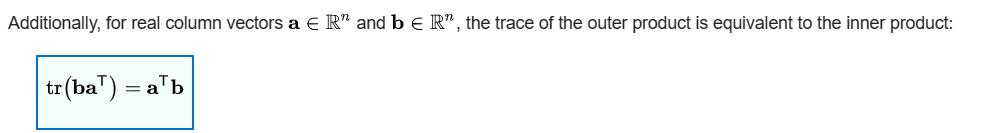


Trace of transpose for matrix

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Trace of transpose for two vectors



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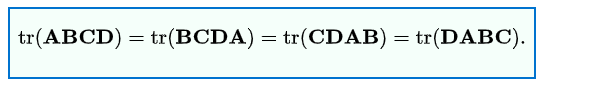
=

Cyclic

Since and has both size , it holds

=

Extending it to ,,, and . One has



which completes the proof.

Trace as sum eigenvalue

By definition of eigenvalue and eigenvector, one has

=

where

is an arbitrary matrix,

is a eigenvector of ,

is the eigenvalue of

One can imply property by definition, which is,

We can reduce to

=

=

Root and coefficients

By roots and coefficients,

Since are roots of

= =

Then simply use quadratic formula.

Special case

Identity matrix

=

Zero matrix

= 0

Idempotent matrix

=

Hermitian matrix

is always a real number.

Proof of special case

Identity matrix

=

=

=

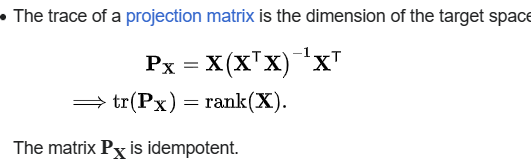
Zero matrix

=

=

=

Idempotent matrix



The second proof of claim:

=

=> =

Hermitian matrix

By definition of Hermitian, the diagonal entries are real numbers, thus

=

=

which is a real number.

End of proof.

Ref

[Trace (linear algebra) - Wikipedia](https://en.wikipedia.org/wiki/Trace_(linear_algebra))

[Web Cookbook](https://web.archive.org/web/20090521075124/http://www2.imm.dtu.dk/pubdb/views/edoc_download.php/3274/pdf/imm3274.pdf)