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# Calibration of A 2-DOF Planar Parallel Robot: Home Position Identification and Experimental Verification

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**Abstract** - This paper presents the calibration of home position for a planar parallel robot. Two methods, geometric error iterative method and nonlinear fitting method are proposed to achieve joint angles at home position. Simulations and the experiments are performed based on these two methods. The simulation results prove the robustness of identification methods to the measurement noise. And the experimental results show the global average accuracy of the robot is improved from 0.99mm to less than 0.1mm.

**Index Terms** – Planar parallel robot; Calibration; Iterative method; Nonlinear regression

## I. INTRODUCTION

Although robot may have a high repeatability, calibration is needed when absolute accuracy is required. A planar parallel robot with parallelogram structures was developed for the application of MEMS device bonding [1]. In the volume manufacturing process, offline programming feature is expected, so the calibration of the robot should be performed.

The robot has been proved to have a high repeatability until 2.3  $\mu\text{m}$  [2], but the accuracy can't be guaranteed because absolute home position is not known accurately. The incremental encoders integrated in direct-driving motors just feedback incremental rotational angle from initial position. One photoelectric switch is used to find home position roughly and then zero pulse of the encoder near the switch determines the home position accurately. There are 64 zero pulse in the circle of encoder. The repeatability of home position has been also proved well by experiment. From the principle of home position determining, we know there must be an angle offset between designed and practical home position. To achieve high accuracy, the accurate home position should be identified by calibration using some high accuracy measuring system.

Many researches had been carried out calibration of parallel robot, most of them focused on the calibration of 6-DOF robot [2-6], some other papers present the calibration of 2 or 3 DOF parallel robot [7-9]. In general, these papers can be classified by the way of error measurement, using external sensors [5, 6, 8, 9] or internal sensors [2-4, 7]. Most of these papers didn't involve the accuracy of home position except [7], in which two iterative algorithms are proposed for auto-calibration of a planar parallel manipulator with sensor redundancy, but the experimental results didn't display the precision of robot after calibration. Some other literatures on

the calibration of serial robot are also valuable for reference on the geometric parameter estimation and error compensation. Three methodologies for parameter identification, nonlinear optimization, iterative linearization, and extended Kalman filter, are compared in the [10] according to the experimental results on a SCARA robot. In this paper, it is also mentioned that these methodologies calculate a robot model, which best fits the measured data, and isn't necessarily very consistent with the actual robot.

In this paper, two methods are proposed for the calibration of the planar robot by minimizing the residual between solution of nominal kinematic and practical measuring results. An iterative method was developed to find practical home position by the way of minimizing geometrically defined objective function. A nonlinear regression method was also used to find absolute home position. It finds a global optimal solution under total least square sense to minimize module of the position error vector. Based on this principle, the geometric parameters of robot were also identified. Simulation and experiment are carried out to verify the effectiveness of these two methods.

## II. THE CALIBRATION PROBLEM

### A. Description of the robot

The planar parallel robot with only five bars may have a uncontrollable rotational motion at the end point, which need to be constrained in the bonding application. In the design, the movable platform together with two adjacent links and a triangle part constitutes a parallelogram linked by four revolute joints. Triangle part, one proximal link and motor constitute another parallelogram linked by three revolute joints. These parallelogram linkages keep the movable platform in the end point parallel to the base whatever the motion of motors may be. As a result, the output of end effector is planar rigid translations of a part but not a point. So this novel robot can realize the full closed loop control or kinematic calibrate more easily by some external sensors such as planar encoder or laser interferometer. This kind of multi closed-loop structure also can improve the stiffness of robot in vertical direction and decrease error arising from every branch chains on the end point. To make full use of the feature of direct driving motor with big size outer rotor, two proximal links are replaced by rotor of motors. This is also help to reduce the mass of linkages and to improve the stiffness of

robot. The assembly of the robot with capability of 2-DOF pure translations is shown in fig.1.

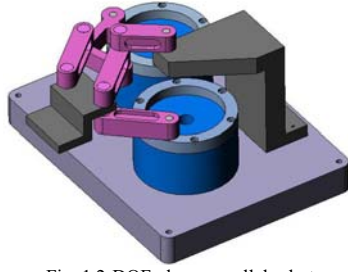


Fig. 1 2-DOF planar parallel robot

### B. The kinematic model

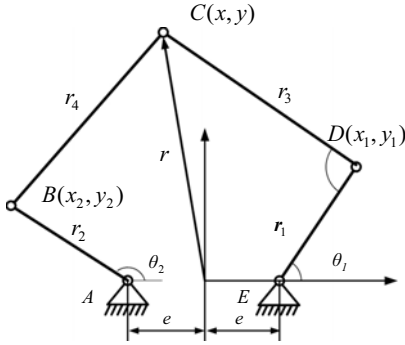


Fig. 2 Definitions of angles and lengths

Because the parallelogram structures don't influence the position of end effector, only five bar mechanism was considered in the kinematic analysis. The mechanism and definition of reference coordinate frame is shown in Fig. 2. From the figure, the following closed-chain kinematic equations in the form of vectors can be gotten,

$$\begin{cases} \vec{r}_1 + \vec{r}_3 + \vec{e} = \vec{r} \\ \vec{r}_2 + \vec{r}_4 - \vec{e} = \vec{r} \end{cases} \quad (1)$$

where  $|\vec{r}_1| = |\vec{r}_2| = l_1$ ,  $|\vec{r}_3| = |\vec{r}_4| = l_2$  and

$$\vec{r}_1 = (l_1 \cos \theta_1, l_1 \sin \theta_1)^T \quad \vec{r}_2 = (l_1 \cos \theta_2, l_1 \sin \theta_2)^T \quad \vec{e} = (e, 0)^T$$

According to the symmetrical configuration,  $r_3$  and  $r_4$  have the equal length. That is

$$(x - x_1)^2 + (y - y_1)^2 = l_2^2 \quad (2)$$

$$(x - x_2)^2 + (y - y_2)^2 = l_2^2 \quad (3)$$

Solving equations (2) and (3), yields forward kinematics:

$$\begin{cases} x = ky + w \\ y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{cases} \quad (4)$$

where

$$k = \frac{y_1 - y_2}{x_2 - x_1}, w = \frac{x_2^2 + y_2^2 - x_1^2 - y_1^2}{2(x_2 - x_1)}, a = k^2 + 1$$

$$b = 2k(w - x_1) - 2y_1, c = (w - x_1)^2 + y_1^2 - l_2^2$$

In equation (4), there are two different position solutions, which are corresponding to two assembly configurations. To select the assembly mode corresponding to the figure 2 and avoid singularity configuration, the sole solution can be got by following condition,

$$\begin{cases} \theta_2 > \alpha_{AC} \\ \theta_1 > \alpha_{EC} \end{cases} \quad (5)$$

where  $\alpha_{AC}$  and  $\alpha_{EC}$  are the direction angle of vector  $AC$  and  $EC$ , respectively. The constraint guarantee  $B, D$  always is outside of the triangle  $ACE$ , so singularity can be avoided.

Solve Eq. (1) yields inverse kinematics:

$$\begin{aligned} \theta_1 &= 2 \tan^{-1} \frac{-A + \sqrt{A^2 + B_1^2 - C_1^2}}{C_1 - B_1} \\ \theta_2 &= 2 \tan^{-1} \frac{-A - \sqrt{A^2 + B_2^2 - C_2^2}}{C_2 - B_2} \end{aligned} \quad (6)$$

where

$$\begin{aligned} A &= -2l_1 y, \quad B_1 = -2l_1(x - e), \quad B_2 = -2l_1(x + e) \\ C_1 &= x^2 + y^2 + e^2 + l_1^2 - l_2^2 - 2ex \\ C_2 &= x^2 + y^2 + e^2 + l_1^2 - l_2^2 + 2ex \end{aligned}$$

### C. Problem formulation

When the robot is asked to move to a desired pose  $P_d$ , the controller computes joint angles,  $Q_d$ , by means of inverse kinematics using nominal structural parameters  $L_n$ .

$$Q_d = I(P_d, L_n) \quad (7)$$

Where  $I(\cdot)$  is the inverse kinematics. Specially, we have.

$$Q_d = Q_0 + \Delta Q_d$$

Where  $Q_0$  is the vector  $(\theta_1, \theta_2)$  when robot is at the home position and  $\Delta Q_d$  is incremental rotational angle vector of the driving joints relative to home position.

However the robot will reach the actual pose  $P_a$ ,

$$P_a = F(Q_d, L_n + \Delta L) \quad (8)$$

Where  $F(\cdot)$  is the forward kinematics,  $\Delta L$  is the geometrical parameter errors. Here the forward kinematics may be different with model given in II.B, because there are some other geometric errors between links' orientation resulted from manufacturing and assemble. Here it is ignored because the mechanism is carefully manufactured and assembled. The objective of calibration is to reduce error,  $\Delta P$ , between desired pose and actual pose.

$$\Delta P = P_d - P_a \quad (9)$$

From analysis above, two parameters for kinematic model,  $Q_0$  and  $\Delta L$ , may cause the position error of end effector, which are also parameters needed to be identified. Although this paper focus on the home position identification, the influence of  $\Delta L$  on the estimation algorithms is also considered in the simulation and it is estimated according to the experiment data by the nonlinear regression method.

## III. IDENTIFICATION MEHTODS

#### A. Iterative geometric method

The basic idea of calibration this method is to minimize the position error  $\Delta P$ , as shown in fig.3. Assume the error vector  $\Delta P$  is small and the mapping between Cartesian space and joint space is thought as nearly linear in small range, the home position should be shifted from  $P_0$  to  $P_{0l}$  to compensate the error, based on analysis above. This procedure will be iterated to get a satisfied result. To minimize the position error in the whole workspace  $\Delta P$  is replaced by a global error vector  $E_G$ .

$$E_G = \frac{1}{n} \sum_{i=1}^n (P_{di} - P_{ai}) \quad (10)$$

where  $i$  is index number of measure points, the total number is  $n$ .

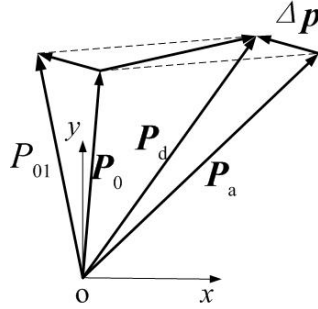


Fig. 3 The principle of geometric error iterative method

The process of this parameter estimate method can be depicted as follow:

- 1) Solve the inverse kinematic problems to get a set of joints' angles for specified poses in the workspace of the robot.
- 2) Measure the position of end effector by external sensors, then the vector from home position to practical position can be gotten. Therefore error between desired position and practical position is known.
- 3) Adjust home position, i.e. joint angle offset, by inverse kinematics according to global average position error  $E_G$ .
- 4) Calculate  $E_G$  by new home position, if error fall in the error tolerance then the result of 3) are thought as real home position, or else repeat 3) to update angle offsets again.

#### B. Nonlinear regression method

The second way is to find home position under total least squares sense by nonlinear regression, the objective function for the evaluation of regression analysis is,

$$\Delta P_i \rightarrow \min \quad i = 1, 2, \dots, n \quad (11)$$

Where  $n$  is the number of sample points measured.

The Gauss-Newton method was used for the parameter estimation and 95% confidence interval on the nonlinear least squares parameter estimates can be obtained also based on the residuals of estimation and Jacobian matrix at the parameters estimated. These statistical results are valuable to judge the validity of experimental results and validity of kinematic model. In principle, the other geometric parameters can be identified by this method also, which will be preformed by experimental results. The solve process of this method was

realized by mathematic software, MATLAB, which saves part time for programming.

### IV. SIMULATION STUDIES

#### A. Conditions for simulation studies

The common conditions for both simulations are:

- 1) Nominal geometric parameter:  
 $l_1 = 70 \text{ mm}$   $l_2 = 100 \text{ mm}$   $e = 25 \text{ mm}$   
 $\theta_1 = 45^\circ$   $\theta_2 = 135^\circ$
- 2) Number of sample points: 64
- 3) Three cases are studied: without noise and with noise, the mean of noise is zero and the standard deviation of noise is 0.01 mm and 0.001 mm respectively.
- 4) Assume true home position is  $\theta_1 = 40^\circ$   $\theta_2 = 140^\circ$
- 5) Error tolerance for iterative method is  $5e-4$  radian
- 6) For simulation, geometric error, it is:  
 $\Delta L = (\Delta e \quad \Delta l_1 \quad \Delta l_2) = (-0.03 \text{ mm} \quad 0.03 \text{ mm} \quad -0.03 \text{ mm})$

The position error in a measure point is evaluated by the module of resultant error along  $x$  and  $y$  direction,

$$\Delta P(x, y) = |P_d(x, y) - P_a(x, y)|$$

From the conditions set above,  $\Delta P(x, y)$  before calibration can be computed and the result is shown in fig. 4.

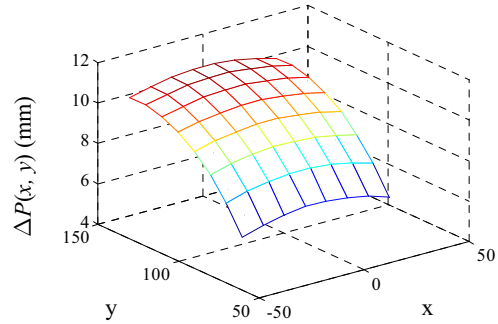


Fig. 4 Position error before calibration

#### B. Simulation results of geometric iterative method

TABLE I  
SIMULATION RESULTS FOR GEOMETRIC ITERATIVE METHOD (rad)

	True value	Geometric iteration (without $\Delta L$ )	Geometric iteration (without $\Delta L$ )	Geometric iteration (with $\Delta L$ )
$\sigma_{noise}$		0.01 mm	0.001 mm	0.01 mm
$\theta_{10}$	0.697778	0.697454	0.697735	0.698106
$\theta_{20}$	2.442222	2.441951	2.442144	2.442039

To verify the robustness of method, the noise was added in simulation experimental data computed by kinematic model, whose standard deviation is 0.01mm. The convergence process of  $E_G$  is shown in fig. 5, which is monotonously decrease from 0.81mm to 0.00021mm in 100 iterations. The resultant error after calibration is shown in fig. 6. Another two cases with 0.001mm measurement noise and with geometric error were also simulated. The results are shown in fig. 7 and fig. 8, respectively. All results are given in table I in detail. From

them, we can see that the noise and links' length error influence results slightly.

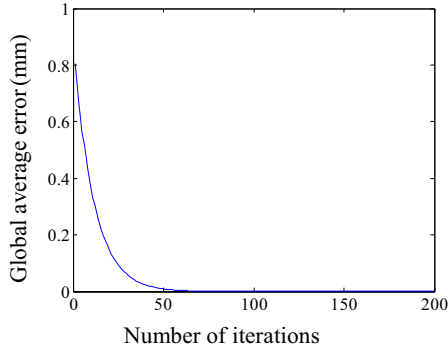


Fig. 5 Convergence process of  $|E_G|$  in geometric iterative method

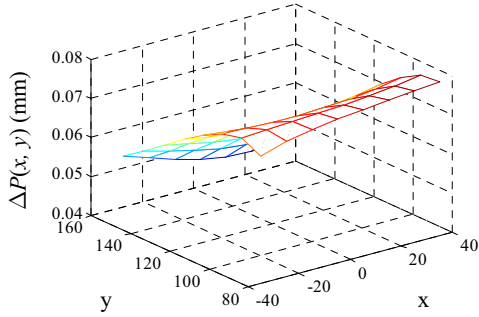


Fig. 6 Position error after calibration with 0.01mm measurement noise

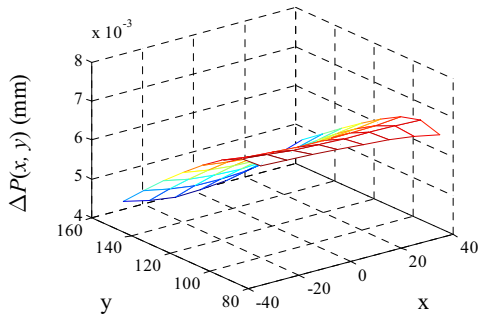


Fig. 7 after calibration with 0.001mm measurement noise

### C. Simulation results of nonlinear regression method

Three cases same with above are performed for nonlinear regression method also, calibration results in three cases are shown in fig. 9-11 and table II, where  $C_{i1}$  and  $C_{i2}$  denote 95% confidence interval of home position. These results show that this method is clearly better and robust compared with geometric iterative method. It is worth noting that the width of confidence width isn't sensitive to the noise but sensitive to  $\Delta L$ .

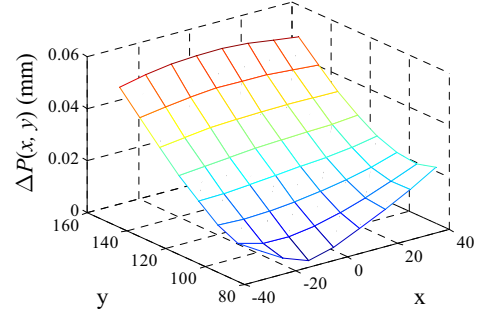


Fig.8 Position error after calibration with 0.01mm measurement noise and geometrical parameter errors

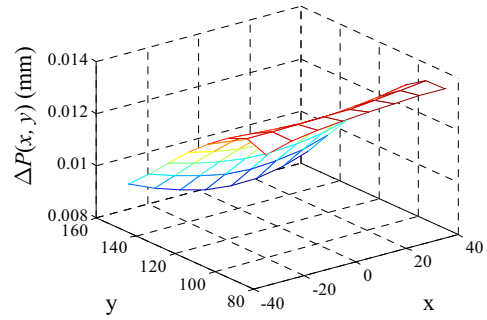


Fig. 9 Position error after calibration with 0.01mm measurement noise

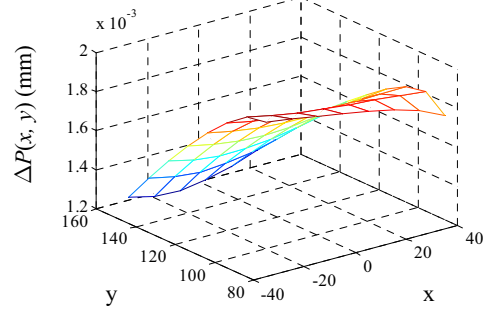


Fig. 10 Position error after calibration with 0.001mm measurement noise

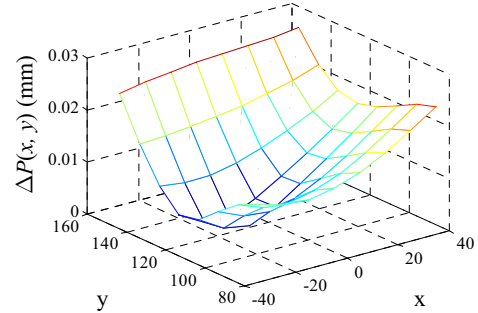


Fig. 11 Position error after calibration with 0.01mm measurement noise and geometrical parameter errors

The simulation results proved the validity and robustness of this method. Obviously the nonlinear regression is more

robust to the measurement noise and not so sensitive to  $\Delta L$  compared to iterative method. In general,  $\Delta L$  didn't worsen the calibration results too much in these two algorithms.

TABLE II  
SIMULATION RESULTS FOR NONLINEAR REGRESSION METHOD (rad)

	True value	Geometric iteration (without $\Delta L$ )	Geometric iteration (without $\Delta L$ )	Geometric iteration (with $\Delta L$ )
$\sigma_{noise}$		0.01 mm	0.001 mm	0.01 mm
$\theta_{10}$	0.697778	0.697963	0.697861	0.697278
$\theta_{20}$	2.442222	2.442462	2.442324	2.442068
$C_{i1}$		[0.696933, 0.698341]	[0.697711, 0.697818]	[0.696364, 0.699048]
$C_{i2}$		[2.441409, 2.442817]	[2.442146, 2.442253]	[2.441085, 2.443767]
Width of $C_i$		0.0086 deg	0.005 deg	0.15 deg

## V. EXPERIMENT SETUP AND RESULTS

### A. Experiment setup

Two laser interferometers, SP500 from SIOS GmbH, and capacitive Displacement measuring system, capaNCDT 620 from Micro Epsilon GmbH, are used to measure the position of the end effector. The experiment setup is shown in fig.12. According to the data measured by capacitive sensor, we know the displacement in vertical direction is neglectable.



Fig.12 Picture of experimental setup

A more important problem for the experiment setup is how to align measurement frame and base frame of robot. In our experiment, two mirrors are fixed on the base and holder on the end effector, which is precisely machined to guarantee the orthogonality of two mirrors. And the nonparallelism between two frames can be measured by collimator. In this way, rotational angles between two frames were reduced to arc seconds by adjusting mirror holder. In total, 49 sample points distributed in workspace evenly are measured in the experiment. So the whole calibration process can be summarized as:

- 1) Measurement of absolute position error according to the nominal home position in the design.
- 2) Identify the angles of driven angles at home position using the methods presented in the paper.
- 3) Verify the calibration results by experiment again.

The error  $\Delta P(x, y)$  before calibration is shown in fig.13 and the accuracy of robot is 1.66 mm (maximum) and 0.99 mm (average).

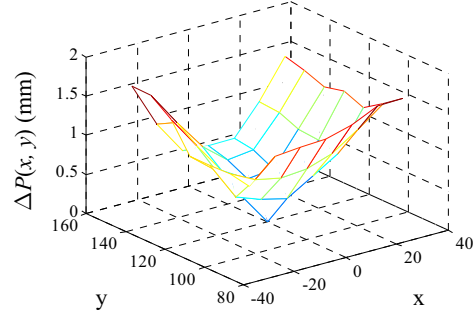


Fig. 13 The position error under nominal home position

### B. Experiment results of geometric iterative method

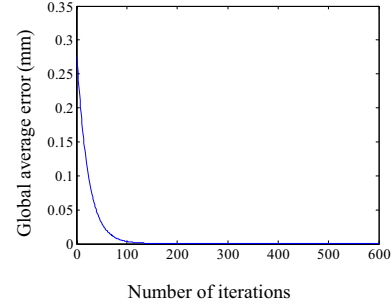


Fig. 14 Convergence process of  $|E_G|$  in geometric iterative method

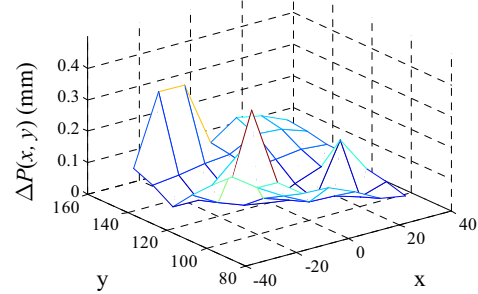


Fig. 15 Experimental result of geometric iterative method

TABLE III  
EXPERIMENTAL RESULTS OF PARAMETER ESTIMATION (mm, rad)

	Home position	Mean value	Maximum value	variance
Before calibration	0.785398 2.356194	0.993722	1.6614	0.1664
Geometric error iterative	0.730353 2.418533	0.1066	0.2809	0.0070
Nonlinear regression	0.736476 2.418244	0.084440	0.2918	0.005101
Nonlinear regression With links' length identification	0.734212 2.420469 24.889896 69.854834 99.864719	0.0561	0.3032	0.0051



From fig. 14, 15 and table III, it can be seen that the errors are small at most of measuring points. The position errors at several sub areas are still too large. The robot accuracy is improved to 0.2809 mm (maximum) and 0.1066 mm (average) after calibrated by this method.

### C. Experiment results of nonlinear regression method

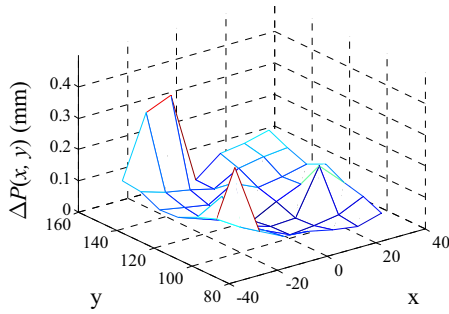


Fig. 16 Experimental result of nonlinear regression method without geometrical parameter errors

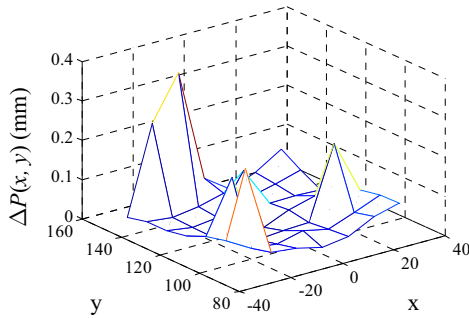


Fig. 17 Experimental result of nonlinear regression method with geometrical parameter errors

TABLE IV  
Confidence interval of PARAMETER ESTIMATION (mm, rad)

	Without length identification	With length identification
$\theta_1$	[0.734714, 0.738238]	[0.730867, 0.737557]
$\theta_2$	[2.416479, 2.420001]	[2.417167, 2.423772]
$\Delta e$	None	[24.428297, 25.351495]
$\Delta l_1$	None	[69.796439, 69.913229]
$\Delta l_2$	None	[99.340726, 100.388713]

From the calibration results (fig. 16, 17), the position error at most area of workspace is less than 0.1 mm. If higher accuracy is needed, we can divide the whole workspace into some subspaces and calibrate them respectively. Compare the results of cases with or without links' length identification (table III), the average error decrease when length error is also identified, while the peak value of error doesn't change distinctly. The confidence intervals of parameters estimated (table IV) are narrow, so the results are worth of believable. The position error is still big at some subareas after calibration; this may be caused by control issue or some other geometric errors, and need to be investigated in the future. The

accuracy was improved to 0.2918mm (maximum) and 0.084440mm (average) by this method.

### IV. CONCLUSIONS

This paper presented two procedures for the calibration of a novel planar parallel robot with parallelogram structures. According to the simulation and experiment results, the following conclusions can be drawn:

- 1) Effectiveness of both two methods was verified by simulation under condition and experiment. According to results, the nonlinear regression method is more robust against noise than the geometric error iterative method.
- 2) From the experimental results, the geometric iterative method achieves smaller peak value of the position error, whereas the nonlinear regression method achieves smaller mean value and variance of position error.
- 3) The absolute position accuracy of robot improved from 1.66mm (maximum) and 0.99mm (average) to 0.30mm (maximum) and 0.05mm (average).

These methods may also be applied to the calibration of other types of parallel robot by appropriate modifications.

### ACKNOWLEDGMENT

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