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GEOMETRICAL APPROACH FOR INDUSTRIAL ROBOT AXIS CALIBRATION USING LASER TRACKER

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Abstract

Absolute accuracy of industrial robots as well as good repeatability is very important technical characteristics in such tasks like machining operations and using the robot as a measuring system. The goal of work is to propose a geometrical approach for axis orientation parameters calibration of industrial robot based on laser tracker measurement. For the process of parameters identification using CPA-analysis, that allows determinate positions of robot joint axes.
This article demonstrate procedures that permit change first axes orientation of 6 DOF industrial robot kinematic for compensation of the non-coaxiality and non-perpendicularity of some axis solving direct kinematic. Our approach is based on conjoint rotation. Obtained functions approximated by polynoms.

Keywords direct kinematics; geometrical approach; conjoint rotation; industrial robot calibration; laser tracker; polynomial approximation

1. Introduction to the geometrical approach

Application of industrial robots for machining is an actual task, since it can highly increase technological flexibility and economic effectiveness in comparison with traditional CNC machining.

Robots need a high value of absolute accuracy for robotic based machining tasks performance. The tool center point (TCP) needs to be positioned and orientated without significant errors with respect to the workpiece, outside all its volume during off-line programming [1]. Improving of absolute accuracy is possible only for robots with good repeatability using structural and parametrical change of the mathematical model and identification of its parameters for the robot control system. In other words – using a calibration procedure.

At the present time for positioning and orientation tasks under precise control of modern industrial robots using analytical and numerical iterative methods. Analytical method [2] provides single-shot direct and inverse kinematic tasks solving. Using numerical iterative method [3, 4] for achieving desired precision direct and inverse kinematic tasks are solved in several iterations. Also, using numerical iterative methods, it is possible to use a big number of parameters. But if the number of parameters is increased then computation time, also increased. For analytical approach the number of parameters is limited. Since, if we add even one parameter then it is necessary to found new analytical solution of inverse kinematic task.

In both cases for industrial robot calibration it is necessary to provide identification of mathematical model to determine joint offsets, actual length of the links, misalignments between them and some other parameters. Depending on factors, causing inaccuracies, calibration parameters of industrial robot models are divided in 3 levels [5]:

- The first level is joint calibration
- The second is the calibration of geometrical parameters.
- The third is calibration of non-geometrical parameters.

The developed geometrical approach provides calibration procedures related to first and second level.

The first level calibration means accurate determination of joint coordinate values θ_i (i=1…m, where m is number of joints of manipulator) in accordance with the next formula:

$$\theta_i = k_{i1} n_i + k_{i2} \tag{1}$$

where n_i – is the pulse count from the encoder output, that represent link location, k_{i1} – the angle represented by each pulse, k_{i2} – zero offset in i joint.

Using this linear model the sensors of the robot are calibrated and installed to zero offset.

For obtaining greater accuracy, a more complex non-linear model with a harmonic component is used. It calibrates the eccentricity between gears in the gear box:

$$\theta_i = k_{i1} n_i + k_{i2} + k_{i3} \sin(k_{i4} n_i + k_{i5}) \tag{2}$$

where k_{i3} – harmonic amplitude, k_{i4} – represent eccentricity between gears and k_{i5} – phase shift.

That model depends of nonlinearity type, that occurs in joint.

We propose a new approach, that besides of this nonlinearity in joint coordinate θ_i, has additional nonlinear component $\delta\theta_i$, that depends of joint j current position (j=1…m, where m – is the number of joints). Joint j depends of the factor that is necessary to eliminate by calibration.

As a result, total value of joint θ_i' taking additional nonlinear component into account, can be expressed by:

$$\theta_i' = \theta_i + \delta\theta_i \tag{3}$$

where additional nonlinear component can be expressed by polinomial dependence with k orders in accordance with the formula:

$$\delta\theta_i = a_k \theta_j^{k} + a_{k-1} \theta_j^{k-1} + ... + a_1 \theta_j^{1} + a_0 \tag{4}$$

Polynomial coefficients $a_k...a_0$ represents values of geometrical dependence on second level calibration.

For the description of industrial robot geometrical parameters a standard DH representation is used [6] that transform joint space into Cartesian space describing the transition from previous link to next link in model form:

$$A_i^{i-1} = Trans(z_i, d_i) \cdot Rot(z_i, \theta_i) \cdot Trans(x_i, a_i) \cdot Rot(x_i, \alpha_i) \tag{5}$$

where $Trans(h_i, \chi_i)$ and $Rot(h_i, \chi_i)$ – homogeneous matrixes of translation and rotation respectively. h_i - rotation or translation axis (x_i, y_i or z_i) of frame i, χ_i - rotation angle or translation length (d_i, θ_i, a_i, α_i, γ_i, g_i) of frame i. Extended representation of this structure expressed as:

$$B_i^{i-1} = Trans(z_i, d_i) \cdot Rot(z_i, \theta_i) \cdot Trans(y_i, g_i) \cdot Trans(x_i, a_i) \cdot Rot(x_i, \alpha_i) \cdot Rot(y_i, \gamma_i) \tag{6}$$

Model (6) describes industrial robot geometry better, since it includes 2 additional homogeneous matrixes, that defines rotation γ_i and translation g_i along the y_i axis.

However, adding those 2 parameters significantly complicate analytical solution of the inverse kinematics. In this case it is reasonable to use numerical iterative solutions. The developed approach excludes the γ_i and g_i parameters from homogeneous matrix structure (6), and forces the robot to change projection angles of its movements in the process of identification, including in the first calibration level additional joint coordinates. The main goal of this approach is the calibration of geometrical inaccuracies using conjoint rotation.

It is reasonable to primarily apply this approach for serial 6 DOF industrial robot calibration of last 3 joints. Last 3 joint axes not intersect in one point because of assembly inaccuracies and production procedures of robot. This initial condition doesn't allow to find accurate analytical solution of inverse kinematics.

In the article M. Švaco, B. Šekoranja, F. Šuligoj and B. Jerbic [7] parameters approximation of industrial robot kinematic chain provide using the "black box" method. Expressions describing polynomial dependencies are obtained analytically. The method of CPA-analysis is used for parameters identification [8, 9], that allows to determinate the industrial robot joint axes position. Error evaluation of joint axes centers determination was investigated in [10].

2. Results of experimental research

Identification is performed using experimental setup presented at the figure 1. Measuring of robot trajectory points during identification were made by dynamic measurements. Nonlinear least squares method with Levenberg algorithm was used for processing of the obtained data [11].

Fig. 1. Experimental setup: 1 – industrial robot Gelios 20, 2 – laser tracker LTD800, 3 – reflector of laser tracker, 4 – 2 axis positioning table..

Analytical expressions were obtained using geometrical approach. For validation of those expressions 2 experiments using Gelios 20 industrial robot (payload 20 kg) were provided. An experimental data shows reducing of non-coaxiality and non-perpendicularity of the industrial robot wrist axes.

2.1. Non-perpendicularity compensation between 5th and 6th axes of manipulator

During identification of the industrial robot Gelios 20 geometrical parameters [10], after calibration of joint coordinate zero offsets the non-perpendicularity of the 5th and 6th axes was found. The angle between those axes is 89,912˚. This problem is related to production errors and assembly of manipulator and expressed by the angle γ_5. Ideally this angle must be equal to 0, but in this case it is equal to 0,088˚. If we recalculate this value into the length of the 5th axis it gives a 0,255 mm displacement. The error influence can be reduced using additional movement $\delta\theta_6$ by 6th joint when the 5th joint is moving. For that issue we should solve the geometrical task showed at the figure 2 (a, b).
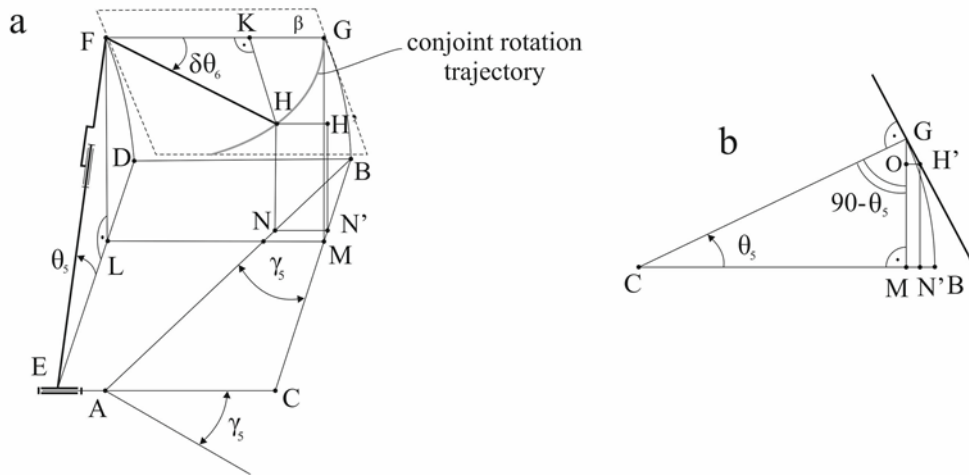
Fig. 2. (a) Conjoint rotation of 5th и 6th joints for reducing of non-perpendicularity; (b) projection of required points on plane CGB.

In this figure we can see the conjoint rotation trajectory of 5th и 6th joints: when the 5 axis moves on angle θ_5, the 6 axis moves on angle $\delta\theta_6$ in such a way that the H (TCP) point projection – point N lands up on the interval AB. So, the main question of this task is deflection of the projection CB of TCP the trajectory on the angle γ_5, for moving point N on projection AB. In the figure 2 (a) we also can see: plane β of 6th joint rotation if we elevate the 6th axis on the angle θ_5 and projection LM from interval FG if $\delta\theta_6 = 0$. In the figure 2 (b) we can see the plane CGB using for projection points E, F and L.

Using simple geometry we can obtain initial conditions and the relation between elements. First, BC=DE=EF=CG= d_6 - is the 6th link length and DB=LM=FG=FH= a_6 - is the displacement of TCP from the 6th joint rotation center. Second, points K, N and H' are H point projections to FG, AB lines and CGB plane, and N' and O points are H' point projections to CB and GM lines respectively. Third, all CGB plane parameters are EFD plane projections, so ΔEFL = ΔCGM.

For solving this task, first it is necessary to consider ΔGOH': the angle OGH' is equal to θ_5 angle (figure 2(b)), and $\sin(\theta_5)$ is equal to OH'/GH' (GH'=KH). Then from ΔFHK (figure 2(a)) we can determine KH as $a_6 \cdot \sin(\delta\theta_6)$. Now we should identify N'B as the difference between CB and (CM+MN'), where MN'=OH' and CM is equal to $d_6 \cdot \cos(\theta_5)$. After that, we should obtain NN'(NN'=HH'=KG). Where KG is the difference between intervals (FG-FK), and FK is equal to $a_6 \cdot \cos(\delta\theta_6)$. In the end, we can define the $tg(\gamma_5)$ as the ratio of interval NN' to N'B and obtain nonlinear equation:

$$d_6 \cdot tg(\gamma_5) - d_6 \cdot \cos(\theta_5) \cdot tg(\gamma_5) - a_6 \cdot \sin(\delta\theta_6) \cdot \sin(\theta_5) \cdot tg(\gamma_5) = a_6 - a_6 \cdot \cos(\delta\theta_6) \tag{7}$$

Solving that equation it is possible to obtain the θ_6 rotation angle value for conjoint rotation with the 5th axis for moving along γ_5 projection:

$$\delta\theta_6^{1,2} = -\left[\arcsin\left(\frac{-\sin(\theta_5) \cdot tg(\gamma_5) \cdot (P) \pm \sqrt{\sin^2(\theta_5) \cdot tg^2(\gamma_5) - (P)^2 + 1}}{\sin^2(\theta_5) \cdot tg^2(\gamma_5) + 1} \right) \right] \tag{8}$$

where

$$P = 1 - \frac{d_6 \cdot tg(\gamma_5) - d_6 \cdot tg(\gamma_5) \cdot \cos(\theta_5)}{a_6} \tag{9}$$

The equation (8) contains 2 solutions for $\delta\theta_6$ angle: one positive and one negative.

So that an angle value is identical and symmetrical in relation to θ_5 it is necessary to select for negative values θ_5 negative solutions $\delta\theta_6$ and for positive values θ_5 negative positive $\delta\theta_6$ respectively. The function graph before and after polynomial approximation is presented on the figure 4 in the section 3.

After reidentification, the angle value between 5th and 6th axes is equal to 89,993˚, reducing systematic error due to γ_5 angle up to 10 times

2.2. Non-coaxiality compensation between 4th and 6th axes of manipulator

In the same way as for non-perpendicularity in the identification process the non-coaxiality of the 4th and 6th joint axes was found. The distance between axes is about 0,342 mm. Influence of that error which could be reduced by means of 2 additional movements $\delta\theta_6$ 6th joint and $\delta\theta_5$ 5th joint when the 6th joint moves. For this propose it is necessary to solve the geometrical task presented on the figure 3(a,b).
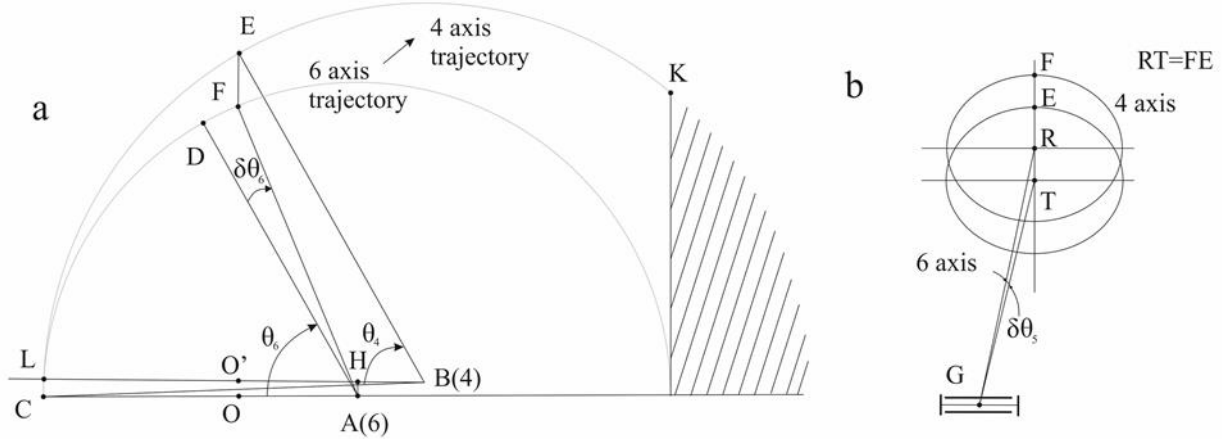

Fig. 3. (a) Conjoint rotation of 5th and 6th joints for non-coaxiality reducing; (b) rotation of the 5th joint for combining 4th and 6th joints trajectories.

The figure 3 illustrates 5th and 6th conjoint rotation. Apply the projection of 4th joint trajectory as the base trajectory. Points D and F belong to 6th joint trajectory with center at point A, point E belongs to 4th joint trajectory with center at point B. Point C - the common and first point of those 2 trajectories. Interval BL is parallel to AC, AH is perpendicular to LB. For decreasing influence of non-coaxiality it is necessary that the projection of 6th joint trajectory repeats the projection of 4th joint trajectory, i. e. point D locates in point E in each moment of time. For that reason we apply 2 additional rotations: $\delta\theta_5$ (figure 2 (b)) – at a distance of EF (EF=RT) and $\delta\theta_6$ (figure 2 (a)) – around point A.

Again, using simple geometry we can obtain initial conditions and the relation between elements. First, AC=AD=AF=l_6 - TCP displacement from point A, BC=BL=BE =l_4 - TCP displacement from point B, GR=GT=d_6 – length of 6th link and AB=Δ. Second, angle BCA equals to angle LBC. Third, $\theta_6 = \theta_4$.

For industrial robot Gelios 20 a special situation is typical: 2 circles have only one intersection point or the second point is located in a small distance. So it is not necessary to reinstall joint coordinate zero offset k_{62}. And solutions for $\delta\theta_6$ top and bottom parts of circle are symmetrical or almost symmetrical (when the small AH), so it is possible to use only one indicator. This indicator is equal to 1 when θ_6 is positive, and -1 when θ_6 is negative. In case when AH has big length it is necessary to use either a complex function, that should be represented using several indicators or change zero offset k_{62} to a case where points C, A and B will be located on one or "on almost one" line.

For solving set task it is necessary to determine an angle BCA and an angle EBL, which is equal to the difference between angles θ_4 and LBC. For projection determination of points E on BL we can use following expressions: $EO' = l_4 \cdot \sin(\theta_6)$ and $BO' = l_4 \cdot \cos(\theta_6)$. For OF and EO determination it is necessary to find out following: an angle ABC, projections BH and AH, length OA (BO'-BH) and angle CAF. Than FE can be expressed as a difference between EO and $indicator \cdot OF$. In the end, after all manipulations, we obtain nonlinear equations:

$$\delta\theta_5 = \arcsin(\frac{l_4 \cdot \sin(\theta_6) + \Delta \cdot \sin(Q) - indicator \cdot l_6 \cdot \sin(\arccos(\frac{l_4 \cdot \cos(\theta_6) - \Delta \cdot \cos(Q)}{l_6}))}{d_6}) \tag{10}$$

$$\delta\theta_6 = indicator \cdot (\arccos(\frac{l_4 \cdot \cos(\theta_4) - \Delta \cdot \cos(Q)}{l_6})) - \theta_4 \tag{11}$$

where

$$Q = \arccos(\frac{l_4^2 + \Delta^2 - l_6^2}{2 \cdot l_4 \cdot \Delta}) + \arccos(\frac{l_4^2 + l_6^2 - \Delta^2}{2 \cdot l_4 \cdot l_6}) \tag{12}$$

Graphs of these functions, before and after polynomial approximation, is presented on the figure 5.

After reidentifacation the value of non-coaxiality between 4th and 6th axes is equal to 0,142 mm, that is in 2 time lower, than the value, obtained before compensation.

Unfortunately, this approach is decreasing non-coaxiality using the elimination of displacement between axes; nevertheless, include additional error of orientation, which is caused by a small displacement of 5th axis. But, this error can be decreased by increasing the TCP distance from center of the robot's flange. Also, it is important to hold in mind, that a significant increase of backlash in the gear-box for this approach is totally unacceptable, so for represented technique it is appropriate to use modern approaches which can significantly reduce those values [12, 13, 14].

Represented geometrical approach calibrate only one orientation parameter of robot axis using conjoint rotation in comparison with different works [3, 4, 5, 7, 8, 9], where numerical iterative method is used for whole kinematic chain calibration. Including more calibration parameters we need to consistently solve similar tasks, represented in section 2. For applying geometrical approach it is necessary to develop different from represented in section 1 methods of solving inverse kinematics, that takes into account conjoint rotation.

3. Polynomial approximation and short function analysis

Represented functions takes much time of computation due to nonlinearity. Accordingly, polynomial dependencies will be suitable for control with fewer computations. Linear least square technique (OLS) was used for approximation to determine the vector $\boldsymbol{\omega}$ of the polynomial coefficients $\begin{bmatrix} a_k & a_{k-1} & \dots & a_0 \end{bmatrix}^T$.

In general the terms of OLS condition can be expressed by:

$$F(\mathrm{a}) = \sum_{p=1}^{N} (\mathrm{g}_p(\mathrm{a}))^2 = \sum_{p=1}^{N} (\delta\theta_{i,p} - a_k\theta_{j,p}^k - a_{k-1}\theta_{j,p}^{k-1} - \dots - a_1\theta_{j,p}^1 - a_0)^2 \to \min \tag{13}$$

where N - total amount of function value $\delta\theta_{i,p}$, and p - number of values. Then, applying methods of linear algebra, Jacobi matrix can be written as:

$$J = \begin{bmatrix} \partial g_1(\mathrm{a})\big/ a_1 & \dots & \partial g_1(\mathrm{a})\big/ a_k \\ \vdots & \ddots & \vdots \\ \partial g_N(\mathrm{a})\big/ a_1 & \dots & \partial g_N(\mathrm{a})\big/ a_k \end{bmatrix} \tag{14}$$

And found polynomial coefficients as:

$$\omega = (J^T J)^{-1} J^T \delta\theta \tag{15}$$

where $\boldsymbol{\delta\theta}$ represent vector of value $\begin{bmatrix} \delta\theta_{i,1} & \delta\theta_{i,2} & \dots & \delta\theta_{i,\mathrm{N}} \end{bmatrix}^T$ approximated by function.

To receive function for $\delta\theta_6^{1,2}$ and approximate its values, in case of non-perpendicularity compensation between 5th and 6th axes, we identify the next parameters using CPA-analysis: $\gamma_5 = 0,088\,°$, $d_6 = 166,0818$ mm, $a_6 = 31,704$ mm. The approximation was provided for polynoms with different order. We define, that for even-numbered order polynoms its even-numbered coefficients equal to 0. Third order polynom has a root-mean-square error about 0,414 mm. Computational error in the deviation γ_5 from nominal value using this polynom is about 0,002°. This orientation error gives us 10 µm if we recalculate it to length, which equals $2 \cdot d_6$. Fifth order polynom has a root-mean-square error is about 0,16 µm. Computational error in the deviation γ_5 from nominal value using this polynom is about 0,00004°. This orientation error gives us 0,146 µm if we recalculate it to length, which equals $2 \cdot d_6$. Approximating values of $\delta\theta_6^{1,2}$ function we receive equation of 5 axis compensation for joint coordinate θ_6 of Gelios 20 robot in terms of polynom:

$$\delta\theta_6 = 4,618 \cdot 10^{-12} \cdot \theta_5^5 - 1,2 \cdot 10^{-6} \cdot \theta_5^3 + 0,091 \cdot \theta_5^1 \tag{16}$$

This equation can be used for direct kinematic and compensated control. Dependency graph $\delta\theta_6^{1,2}$ from θ_5 is presented on the figure 4 by red line. Blue points represent the approximated function.
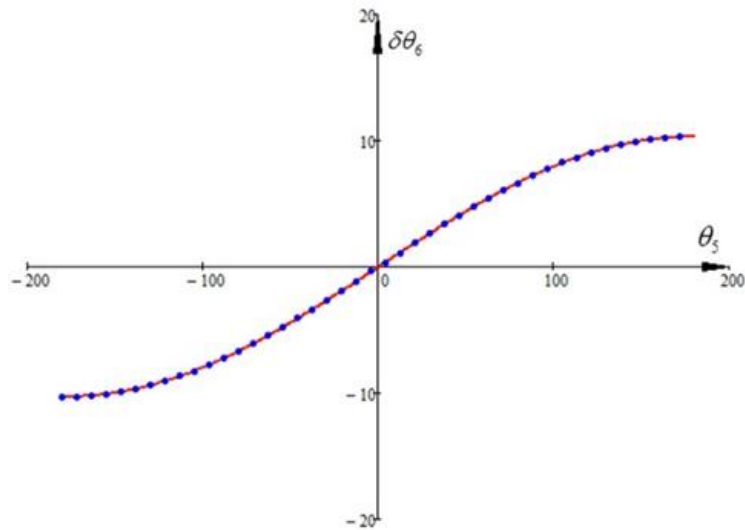
Fig. 4. Dependency graph   from . Red line – is function value. Blue points - is approximated function.

To receive functions for $\delta\theta_5$ and $\delta\theta_6$, and approximate their values, in case of non-coaxiality compensation between 4th and 6th axes, we identify next parameters using CPA-analysis: $\Delta = 0,3263$ mm, $d_6 = 166,0351$ mm, $l_4 = 32,5382$ mm and $l_6 = 32,212$ mm. There is a problem caused by point K (figure 3 (a)), that occurs during approximation of functions. This point is the frontier point, when the equations are solved. There is no 5 joint trajectory after that point, that can connect projections of 4th and 6th joints trajectory. The functions value which have no solutions are taken as 0, that the problem to be clearly visible on graphics (figure 5 (a)). These functions have suitable approximation only for θ_6 within the limits from -150º to +150º (figure 5 (b)). Outside those limits occurs loss of accuracy and it is necessary to extrapolate trajectory further. Inside those limits the accuracy of approximation is similar with accuracy for non-perpendicularity compensation.
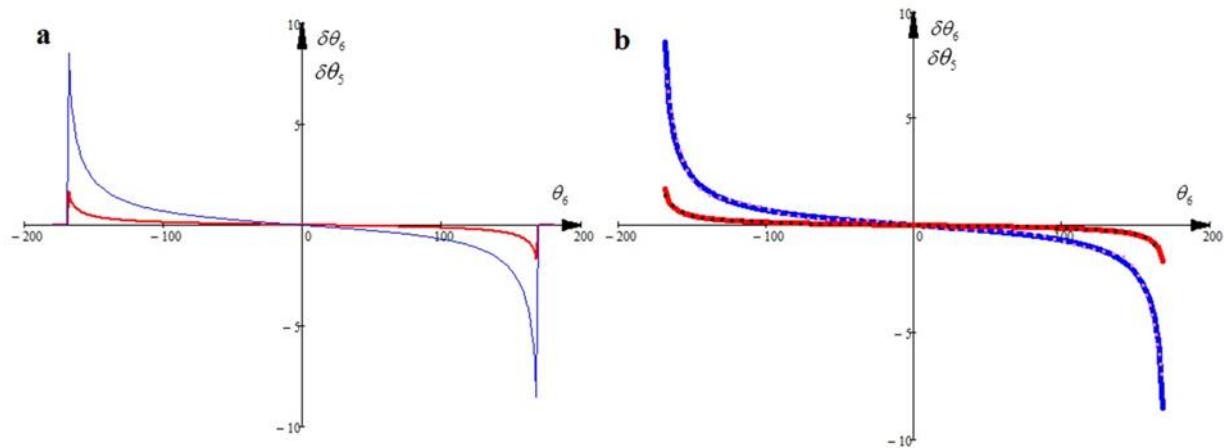

Fig. 5. (a) Dependency graph   from   - blue line, dependency graph   from   - red line; (b) graphs of the same dependencies, points showed the approximated functions.

In the case of non-perpendicularity compensation between 5th and 6th axes the problem of accuracy and extrapolation loss requirement is absent, due to the physical impossibility for the robot to reach extreme positions (close to ±180º).

Due to nonlinearity of dependencies represented in this article, the values of functions will be changed if we change distance from center of the 6 axis rotation to the TCP point. So, if the distance is large, than values of $\delta\theta_i$ will be minimal. Otherwise we will have maximal values. This approach cannot be applied, if the TCP point lies on the 6 axis. Error analysis of this approach depends on using CPA-analysis (because we use dynamical measurements). For joint center determination this kind of analysis is described in [10]. Error analysis for joint orientation determination will be described in future works.

4. Conclusion

In this article for increase accuracy of an industrial robot we propose a geometrical approach for axis orientation parameters calibration. For determination positions of robot joint axes was used CPA-analysis and laser tracker was also used as measurement equipment during the identification process. Applying conjoint rotation we represent 2 operations, that separate decrease essential errors. In one operation the value of non-perpendicularity between 5th and 6th axes was decreased by 0,081°. The angle between 5th and 6th axes equals 89,993° after compensation. It decreased the error in comparison with initial value by 10 times. In other operations the non-coaxiality between 4th and 6th joint axes was decreased from value 0,342 mm to 0,142 mm. The received dependences can be approximated by polynoms. That decreases computation time. In the case of non-perpendicularity compensation between 5th and 6th axes function approximation is absolutely successful. In the case of non-coaxiality compensation between 4th and 6th axes function approximation can be provided only with an additional limitation on 6th joint movement. Without limitation loss of accuracy occurs and it is necessary to extrapolate trajectory further on an unaccounted part of the circle. This approach can be also applied for orientation parameters calibration of positioning tables (figure 1).

Error analysis for joint orientation determination will be described in future works. Also, for the next applying of geometrical approach it is necessary to develop different from represented in section 1 methods of solving inverse kinematics, that takes into account conjoint rotation.

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5. References

[1] Yu. V. Poduraev, Mechatronics: fundamentals, methods, instrumentation, Engineering, Moscow, 2006.

[2] K. S. Fu, R.C. Gonzalez, C.S.G. Lee, Robotics: Control, Sensing, Vision, and Intelligence, McGraw-Hill Inc., New York, USA, 1987.

[3] Albert Nubiola, Ilian A. Bonev, Geometric approach to solving the inverse displacement problem of calibrated decoupled 6R serial robots. Transactions of the Canadian Society for Mechanical Engineering, 2014, Vol. 38, №1, pp. 31-44.

[4] N. Chen, G.A. Parker, Inverse kinematic solution to a calibrated Puma 560 industrial robot. Control Engineering Practice, Vol. 2, pp. 239-245.

[5] B. Mooring, Z. Roth, M. Driels, Fundamentals of manipulator calibration. John Wiley & Sons, INC. 1991.

[6] J. Denavit, R. Hartenberg, A kinematic notation for lower-pair mechanisms based on matrices. // Transactions of ASME – Journal of Applied Mechanics, 22(2), June 1955, pp. 215-221.

[7] M. Švaco, B. Šekoranja, F. Šuligoj, B. Jerbic, Calibration of an Industrial Robot using a Stereo Vision System, in: Annals of DAAAM for 2013 & Proceedings of the 24th International DAAAM Symposium, ISSN 1726-9679, ISBN 978-3-901509-85-8, Editor B[ranko] Katalinic, Published by DAAAM International, Vienna, Austria, 2013, pp. 459-463.

[8] H.W. Stone Kinematic, Modelling, Identification, and Control of Robotic Manipulators, Boston: Kluwer Academic Publishers, 1987. 225 p.

[9] M. Sclar, Geometric calibration of industrial manipulators by circle point analysis. // In Proceedings of Second Conference on Recent Advances in Robotics, Florida Atlantic University, May 1989, pp. 178-202.

[10] A. A. Vorotnikov, Yu. V. Poduraev, E.V. Romash, Estimation of error in determining the centers of rotation of links in a kinematic chain for industrial robot calibration techniques. Measurement Techniques, Vol. 58, No. 8, November, 2015, pp. 864-871.

[11] N. Chernov, Circular and Linear Regression Fitting Circles and Lines by Least Squares. CRC Press Taylor & Francis Group. 2011.

[12] Xiaolong Shen, Jiaying HU, Mingjun Zhang, Laixi Zhang. Experimental study on Backlash Compensation of CNC Machine Tool, Advanced Materials Research, Trans tech Publications, Switzerland, Vol 580, 2012, pp 419-422.

[13] Sven Gestegard Robertz, Lorenz Halt, Sameer Kelkar, Klas Nilsson, Anders Robertsson, Dominique Schar, Johannes Schiffer Precise robot motions using dual motor control, IEEE International Conference on Robotics and Automation, Anchorage, Alaska, USA. 2010, pp 5613 - 5620.

[14] Yu.V. Ilyukhin, Yu.V. Poduraev, A.V. Tatarintseva Nonlinear adaptive correction of continuous path speed of the tool for high , in: Annals of DAAAM for 2014 & Proceedings of the 25th International DAAAM Symposium, ISSN 1726-9679, ISBN 978-3-901509-85-8, Editor B[ranko] Katalinic, Published by DAAAM International, Vienna, Austria, 2014, pp. 994-1002.