Investment strategies - fit your desire risk level and the best plausible profit expectation.

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Abstract

Temporal linear correlations, which are not visible in average, can leads to changes in time series probability distribution function. In the case of stock market data the visible pivot point of the time series is zero. The unbalance of the gaussianity of left and right side of zero point can lead to some statistical forecasting of stochastic systems. Here we show the principle of objective value minimization which helps us to correct gaussianity estimation. The experimental testing of investments strategies based on this knowledge are presented. The positive efficiency of strategies can be as the proof of correctness of the objective value theory.

Key words: information entropy, stochastic processes, probability distribution, stock market data
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1 Introduction

Prediction the future is always most desire property of analysis, nevertheless which kind of analysis we perform. When we would like to earn some money on some investment, we try to catch the future in different ways. Sometime we use knowledge and experience, sometimes computers show the results for us, and many times we use intuition. In all the cases we can describe it and put it in some schemes which rule the prices on the market. However number of used schemes to forecast the future is large, one can say that too large to capture it in some reasonable way. Nevertheless there are some dominant ways and well-known knowledge which are used by many investors which generate some little predictable future. Sometimes the panic-like behavior increases the power of some sell/buy trigger and therefore enlarges its impact for self-realized prediction. Every market is made by buyers and sellers and prices

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are just the effect of their behavior, so established knowledge on forecasting is unstable. Usage of some prediction method by many investors will decrease its efficiency. On the other hand it generates the self made prophecy, but prophecy only for those who are the first, because the most of the people make a use of the prediction too late. This is the sense of minority games in which the only who are in minority wins, it is true also for stock market games. When we would like to speculate on the stock market, then we try to be on the minor site when we do invest and close the position when most people follow us. In speculation we have to be first so we take much of the risk, because on the beginning well-established triggers are ease to misunderstand. The well-known knowledge is overused make it many times useless or generates self made prediction which of course should not be employed by reasonable investor because it is too risky. There is for sure unpublished knowledge which makes money for big investors or others, which can be on the beginning stable (not altering in time or make too small profit to cover costs).

On the beginning we should tell what is known about stock market data for sure that is statistic and known property. Most of the people working on stock market are almost sure that the data are likely to be unpredictable, so that it is difficult to have universal in time and space method to collect money. We can say also that there are the ways of making the profit for very simple reason, because there are people that do it and got rich on stock market investments. From Physics point of view the data that we are dealing with are likely to be stochastic, not linearly correlated on scales larger than an hour. Randomness is dominant in such data but we think that there are deterministic component which can be reveal by some method but complex, nonlinear method. Lets go to the practice on investing, what most people know earn money.

1) Diversification.

The dominant way in optimization of portfolio is just diversification. Most of papers on the investment strategies are somehow related to this issue. Diversification is the simplest way to decrease the risk with having the profit on the same level. The assumption for efficient diversification is no correlation among investments. Sometimes it is difficult to manage if we for example are specialist in investments on banks only. On stock market many firms from the same branch are highly correlated, so investor have to look at stocks from very different branch to be consistent with effective diversification, or even should make money not only on stock market but also on FOREX or other. This is because it is known that highly develop Bessa shows much correlations in all of the stocks so the risk is high. Diversification is the only one investment strategy which can help investors without doing prediction. Rests of the methods are just prediction methods and prediction together with diversification.

2) Markowitz method. Putting together prediction and diversification with

simplest way of prediction and risk evaluation. The standard way of portfolio selection.

In late fifties last century Markowitz found the analytical solution of efficient portfolio selection. The standard Markowitz issue corresponds to simplest way of risk and profit evaluation. Risk is found to be variance of the past data and profit as the average from returns on the same region. In order to apply his procedure to another type of risk and profit evaluation one has to exchange in formulas below only m and D or C on correspond values. This is especially valid when we would like to optimize portfolio to forecast future values of m and D or C.

3) Fundamental analysis.

The Method of predictions can be selected on fundamentals and technical. Fundamental evaluation are based on knowledge about condition of firms or political or investor information about decision making process. Fundamental analysis is rather long-term, which uses the disproportion between the market value and the evaluation. There are twofold problems within:

a. Is it the evaluation properly made?

b. Market prices are not always showing the capital of a firm and future profit from the capital, but what investors thought about this.

The second case is much visible when we calculate the capital of a firm and compare to the capitalization on the stock market. It is well-known that the second is much higher than the first balance. The ratio of these two capitalizations is not universal and differs among stocks even in the same branch of industry. In this case investor uses some intuitive knowledge about the proper value.

4) Technical analysis.

The very beginning coefficients which may help in forecasting are moments and smoothing methods. These coefficients are based on the statistical property of price changes. Moments comes from the mathematics and are just first and second moment of time series so average and variance. In finance we use names of trends and volatility to show the same but volatility uses the second moment in many different ways but it is not as strictly defined as in mathematics. Trends are believed by investors to be much stronger than on beginning one can suppose. One can say that to know in which trend we are (bullish or bearish) is most important. Wisdom is to know the trend and follow it. The finest analysis about trend searching is Elliot Waves. Our expression about this analysis is that waves which shows you trend are not always visible. The correct investing structure in data can be seen from time to time and can be easily misunderstand by non specialist. In the case of volatility (in technical analysis is called moment) the established investing procedure is to buy when volatility is low and sell when is high. It can be understood as not to enter the market in the case of large risk and changes.

5) Capital Asset Pricing Model $--$ CAPM theory.

Many economists assume perfectly efficient market, market in which additional profit one can be gathered only when he take additional risk. This procedure one can turn round and say that if someone would like to look for more profitable stock then he should search for more risk in the case of efficient portfolio. This is an essence of CAPM theory [14] which go further and show more risky assets that is assets which are more correlated with a market index. Correlations with average increase the risk because it do not allowed for efficient diversification.

In the CAPM theory future profit is related to the covariance of the portfolio β_i with market index:

$$x_i - x_f = \nu_i + \beta_i(x_m - x_f) + \eta_i \tag{1}$$

where x_i,x_f and x_m is portfolio i, risk free and market return respectively. The linear trend ν_i and random noise η_i is selected to better fit the data. The relation profit=covariance could be explained in two ways:

1) Market portfolio is assumed to be most optimal portfolio one can get. Larger correlation of returns from portfolio x_i to the most optimal x_m should be awarded with higher profit in long time horizon.

2) The covariance β_i shows the level of decreased profit from efficient diversification. Higher risk connected to the efficient portfolio should collect more profit in longer time. The most important risk factor related to β_i is connected to collective fail of all investments in the portfolio, so that the risk of shortage of money would force to close the position in the most unprofitable way. In long time horizon investment the collective panic-like behavior which would force us to change the investment decision is related to the correlation β_i in average. One can say that the risk which can bear by individual investor can differ, so the risk should possess some deviations from the mean β_i. On the other hand covariance β_i is collected from the past, but one never knows if it will persist in the future. One more problem appears in CAPM modeling that is risk related to the correlation to the market β_i is not only one risk factor in efficient market, so one should look at some more correlations of the profit portfolio to existing fundamental analysis parameters [18].

The fundamental law of nature tells that free energy in the relaxation procedure stays in the minimum [10]. This is related to the limited sources of energy in nature. Nevertheless humans, because of desire, struggle with nature to possess more sources of the energy. Limited amount of energy as well as information, money etc. are then minimized by nature laws but we would like to have its unlimited sources. That is why we can say that the value which are minimized express the real value of the corresponding source such a value we call objective. In this paper we try to convince that the absolute value of some source demonstrate not always the real value, the value which are desired to maximize by humans, because what we would like to have more are express by its objective value. Stock market is thought to be stochastic [1–3] or mixture of deterministic and stochastic processes [4–8] but with prevailing part of the stochastic one. Regarding last we apply the principle of objective value minimization to the stock market data. We try to reveal the important thing of the stock market analysis, i.e. which stock is most desirable to invest bringing higher profit than can do average investors.

2 The principle of objective value minimization

Let us define x as the value of the studied variable. The information entropy for stochastic process is given by the formula [1,9]

$$H = -\int P(x) \ln P(x) dx. \tag{2}$$

It is common knowledge that in the equilibrium the entropy has its maximal value [1,2,10]. One can think about certain function $w(x)$ of x that we assign as the objective value. We would like to show some properties of this function. Let us create the functional which maximize the information entropy with constraints of the given average objective value [1]:

$$S = -\int P(x) \ln P(x) dx + \lambda \int w(x) P(x) dx + \beta \int P(x) dx = max. \tag{3}$$

The last factor in Eq. (3) gives the normalization of all probability to 1. We assume that for properly defined the objective value $w(x)$, one should have $w(0) = 0$. Resolving functional (3), i.e. calculate derivatives that should vanish

$$\frac{\partial S}{\partial P(x)} = 0, \tag{4}$$

we get the following formula for the probability distribution

$$P(x) = e^{-1+\beta} e^{\lambda w(x)}. \tag{5}$$

Table 1
Examples of objective values $w(x)$ and corresponding the probability distribution $P(x)$.

Objective value	$P(x)$	Probability distribution
$w(x) = \|x\|$	$P(x) \sim e^{-\|x\|}$	exponential
$w(x) = x^2$	$P(x) \sim e^{-x^2}$	gaussian
$w(x) = \ln\left(\frac{c+\|x\|}{c}\right)$	$P(x) \sim \left(\frac{c}{c+x}\right)^{-\lambda}$	power-law

The table 1 presents the examples of objective values $w(x)$ and corresponding to it the probability distribution $P(x)$. Let's change a little functional (3) in order to minimized objective value with the constraint of given value of the information entropy

$$S' = \int w(x)P(x)dx - \lambda \int P(x)\ln P(x)dx + \beta \int P(x)dx = min. \tag{6}$$

The above functional is minimized after calculating the the derivative that should vanish, because

$$\frac{\partial^2 S'}{\partial P(x)^2} > 0. \tag{7}$$

The minimization of the functional S' gives the same dependence $P(x)$ versus x as maximization of the functional S. Using the fact that probability distribution $P(x)$ include all the information about the stochastic process one can say that: the principle of maximum entropy express in the principle of minimum of the objective value.

Proposition In the equilibrium, for stochastic not correlated process, *the average objective value is minimized for constant entropy*

$$\langle w(x) \rangle = \sum_n w(x_n) = min, \tag{8}$$

where x_n is a realization of stochastic process in the step n. The proof can be made from the minimization of functional (6).

One can conclude from the above that objective value must be increasing function of absolute value $|x|$. The next conclusion is the following that having the probability density function (pdf) of certain absolute value one can calculate its objective value.

The objective value is a state function of a given system. One can say that it is a parameter of the system that is minimized. The energy in the canonical system known from statistical mechanics can serve as a objective value. Very fast one can conclude that distribution of energy is $P(E) \sim e^{-E}$ what agrees with statistical mechanics [10]. One can calculate from the above principle Maxwell-Bolzmann distribution of velocities. Minimization of the energy of the whole system which consist of particles without interaction can be regard as a minimization of the energy of every particles alone, so minimized should be $E \sim v^2$. The above gives the Maxwell-Boltzmann distribution $P(v) \sim e^{-v^2}$ [10].

3 Objective portfolio selection

Random Matrix Theory can be used to describe correlations eigenvalues spectrum of random data with the same probability distributions [2]. Correlations calculated from time series with different probability distributions will show lower values than in the case of the same Pdf. In correlation calculations the absolute value has its important impact so the smooth transormation of the data can lead to change the correlation. In financial interest is to relate correlations with risk but the aim of analysis should be the maximal correlation so the real risk. Let us imagine such a case that we are dealing with two time series transformed from one original that both possess different probability distribution. Let the first time series has Gauss distribution and second Couchy one. Correlation calculated between these two time series is apparent from one and can be evaluate as an integrate of the join part of these two probabilities. When we would like to calculate some property of two time series with different probability distributions we have to renormalized them to the distributions with the same properties. We use the Objective Value Theory to do this. Objective Value can be calculated from data by looking on the probability distribution function (Pdf). The ObV correspond to each data point representing its objectiveness, the value which in average is minimized in isothermal processes. ObV is extensive, so one can use the mean value as the reference value. Pdf renormalization process can be explained as follows. We calculate the objective value of each point and on this level we normalize average ObV of each time series to the mean ObV of all data. Next we calculate inverse function to objective value using only one set of parameters in Pdf description. Now we possess normalized all time series that the statistical properties calculated among these time series are find to be extreme (maximal or minimal). Apply the renormalization procedure to our example of time series with Couchy and other with Gauss distribution will give us transformation of these two different distributions to the universal one so the join part will give us unity. The validation of the theory will be calculations of correlation showing larger values in the case of renormalization procedure.

$$\tilde{x} = w^{-1}\left(\frac{w(x)}{\langle w(x)\rangle}\right) \cdot SD(x) \tag{9}$$

where SD(x) is standard deviation od x. The ObV as well as the normalization procedure is calculated for left and right side of Pdf separately.

The covariance matrix of normalized time series one can use to evaluate risk in standard portfolio optimization [**?**]. The ObV $\langle w(x)\rangle$ can be a good measure of risk related to the stock. The parameter $\langle w(x)\rangle$ for Gauss distribution is just variance of original data but for any other data described by power law is larger than variance and it depends from power exponent. Smaller power exponent benefit in much larger ObV than variance of original data. The extreme shortfall of prices will increase ObV with respect to variance. The Markowitz optimization problem can be replaced by the following functional:

$$w_p(x) + \lambda \cdot m_p + \beta(\int P(x)dx - 1) = min, \tag{10}$$

where $w_p(x)$ is measure for risk and m_p is expected profit from portfolio. Here we use our forecasting method by unbalance in gaussianity:

$$m_p = \sum_i p_i F_i \tag{11}$$

4 Statistical forecasting of stochastic systems

The existence of some method of forecasting of stock market data is rather controversial. Many scientists claim that there should not be any reasonable method because we are dealing with efficient market (CAPM theory). This is only the assumption, very reasonable assumption, but the strict proof one never saw. We would like to show here some, the same, very reasonable theory that could help to statistical forecasting of stochastic systems.

Let us consider time series of returns $\{x_i\}$, which was made as follows:

$$x_i = \ln \frac{P_i}{P_{i-1}}, \tag{12}$$

where P_i is the price at time i.

From the point of view of investors very important pivot point in returns time series is just zero. In all cases of visualization of changes in prices the

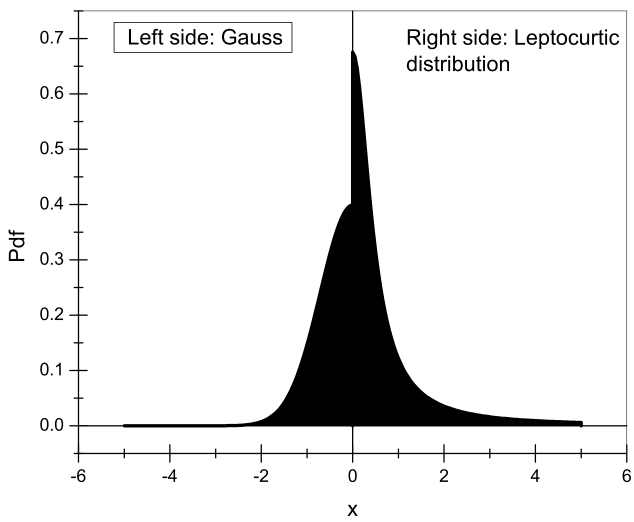

Fig. 1. Academic example of distribution behaviour with normal distribution on the left side and leptokurtic distribution on the right.

green color reflects growth and red falls of prices. That is why we can separate our return time series on bullish and bearish which reflects another type of subsystem responsible for action in investors' brain. We would like to convince that there should exist two separate parts which are acting separately only when in the stock market is hossa and second when is bessa. This is because we sell or buy so zero is the threshold in which we earn or loose. Let us then separate our time series to positive and negative values and put a notation as follows:

$$x_i^+ : x_i > 0 \text{ and } x_i^- : x_i < 0 \text{ for i=1,...,N} \tag{13}$$

All the time we would like to assign zero as the pivot point in both time series.

Let us consider that there exist some asymmetry between $\{x_i^-\}$ and $\{x_i^+\}$ probability distribution function – Pdf (see example on Fig. 1). This unbalance will lead us to do some forecasting basing on gaussianity of left and right Pdf (our meaning of gaussianity we explain in details in section 5).

Gaussianity in a simple word would mean a level of maturity of the distribution. Gaussianity for Gaussian distribution is maximal. This is well known from statistic that sum of many not correlated increments with finite variance will give us a Gaussian distribution, which further does not change in its figure after adding additional data. Gauss distribution is the attractor of convolutions of many distributions with finite second moment, that is why it is so ubiquitous. When we are dealing in stock market data with situation represented in Fig. 1 then our method says that we should BUY for three reasons:

1) If the distribution will persist then average of the right side is higher than average of the left side.

$$\left\langle \left| x_i^+ \right| \right\rangle > \left\langle \left| x_i^- \right| \right\rangle \qquad (14)$$

2) Gaussianity of the left side distribution G_{left} is larger than gaussianity of the right side G_{right}, so a parameter of unbalance

$$F = \frac{G_{left} - G_{right}}{\max(G_{left}, G_{right})} \qquad (15)$$

will show us a future. It can be understood twofold:

a. Maximal value of F is 1, so as minimal -1, so unbalance after approaching some large value should go back to zero because mean value is just this point.

b. Not stable leptokurtic distributions can be understood as generated by temporal correlations which are not visible in average. These temporal correlations will lead to extreme or zero values when correlate data are added, so if $x_i > 0$ then is more probable that $x_{i+1} > 0$. It generate a more frequent appearance of x_i^+ which gives shorter correlation lag in real time. On the end we will finished in higher gaussianity of right side distribution and positive trend.

3) Gauss is mature form of all distributions with finite second moment (what is always the case of stock market data). In other words gaussianity should increase in time when there are no external forces in the market. The increase of gaussianity can happen after more frequent appearance of not correlated data. In our example more frequent positive returns means statistically positive trend.

5 Gaussianity evaluation

For calculations of gaussianity we use fitted Pdf to histogram by the maximum likelihood principle. The distribution function, which we would like to fit to the data, should be the one which can emulate power-law distributions as well as Gaussian. We took the Student distribution which is well known as a power-law distribution with convergence to Gaussian for infinite power [2].

The Student function look as follows:

$$P_{St}(\nu, x) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)(1+x^2/\nu)^{(\nu+1)/2}} \tag{16}$$

where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$. Now we would like to fit our Student distribution to data. Let us note as $\tilde{x}_i$ 'clean' time series which correspond to Student distribution Eq. (16) best fitted to data, so we would like to minimize $\langle(\tilde{x}_i - x_i)^2\rangle$. In order to do so we use Maximum likelihood principle. We use equation on the conditional probability which we would like to maximize:

$$P(x_i|\tilde{x}_i) = \frac{P(\tilde{x}_i \text{ and } x_i)}{P(\tilde{x}_i)} = P(x_i), \tag{17}$$

where x_i and $\tilde{x}_i$ are independent from each other. The lack of dependence should be true because we are dealing with stochastic data. The method of maximum likelihood try to find maximum sum of logarithmus of probability given by Eq. (17), so to find ν we should resolve the following equality:

$$\frac{\partial \sum_i \ln(P_{St}(\nu, x_i))}{\partial \nu} = 0 \tag{18}$$

We solve this equation numerically with constraint that variance of the data should follow $\sigma^2 = \nu/(\nu-2)$ and mean equal zero. In the case of power finding one should preliminary normalized time series by division with average absolute value of x:

$$\tilde{x}_i = \frac{x_i}{\langle|x_i|\rangle}\; i = 1, ..., N \tag{19}$$

The objective value of all stocks one can present as

$$w(x) = \ln\left(1 + \frac{((x - \langle x\rangle)/\langle|x|\rangle)^2}{\nu}\right). \tag{20}$$

The gaussianity is calculated as the ratio of statistical Shannon entropy of returns distribution and objective value $w(x)$ as the consequence of objective value minimization theory:

$$G = \frac{-\int_{-\infty}^{\infty} P(x)\ln P(x)}{\langle w(x)\rangle}. \tag{21}$$

The gaussinianity can be further expand in order to release entropy and ObV evaluation from data. Complex calculations from data sometimes lead to large uncertainty. We can say that entropy as well as objective value (ObV) is calculated only from Pdf which are dependent from power exponent. Next we will show the approximation which relate gaussianity only from power exponent ν calculated from Student distribution. In the case of functional

$$w(x) - \lambda S(w(x)) + \beta \int P(x)dx = min, \tag{22}$$

we get probability distribution as follows

$$P(x) = exp\left(-1 + \frac{\beta}{\lambda} + \frac{1}{\lambda}w(x)\right) \tag{23}$$

Parameter β can be assigned from Pdf normalization and it is:

$$1 - \frac{\beta}{\lambda} = \ln \int exp\left(\frac{1}{\lambda}w(x)\right) dx \tag{24}$$

The entropy S in further calculations comes as:

$$S = -\int P(x)\ln P(x)dx = \frac{1}{\lambda}\langle w(x)\rangle - \ln \int exp\left(\frac{1}{\lambda}w(x)\right) \tag{25}$$

The parameter of our aim gaussianity implicate as the formula:

$$G = \frac{1}{\lambda} - \frac{\ln \int exp\left(w(x)/\lambda\right) dx}{\langle w(x)\rangle} \tag{26}$$

In the case when we fit to data Student distribution we have the following ObV:

$$w(x) = 0.5 \cdot (\nu + 1) \ln\left(1 + x^2/\nu\right), \tag{27}$$

where variance of x is one. In the simply approximation $w(x)$ can be replaced as follows

$$w(x) = (\nu + 1) \ln\left(1 + x/\nu\right) \tag{28}$$

Next we put Eq. (28) to the formula on gaussianity Eq. (26) we get

$$G = \ln\left(\nu \cdot (\nu + 2)\right) - \frac{1}{\nu + 1}, \tag{29}$$

where we put for parameter $\lambda = 1$. The forecasting procedure needs unbalance of left and right side of Pdf so it is

$$F = G_- - G_+ = \ln \frac{\nu_- \cdot (\nu_- + 2)}{\nu_+ \cdot (\nu_+ + 2)} + \frac{\nu_- - \nu_+}{(\nu_- + 1)(\nu_+ + 1)} \tag{30}$$

6 Investment strategy using only objective value

In order to omit entropy of Pdf calculations we can apply second method of forecasting which uses only objective value to look for the leptokurtic distributions. The same as in previous section the objective value can be presented very similar as in Eq. 20

$$w(x) = \ln \left(1 + \frac{((x - \langle x \rangle)/SD)^2}{\nu} \right), \tag{31}$$

where SD is the standard deviation of x. We divide w(x) for left side $w_-(x)$ and right side $w_+(x)$ of the value of x with the respect of $\langle x \rangle$. The inverse function calculated from normalized objective value is as follows

$$\tilde{x} = \tilde{w}^{-1}(w) = \sqrt{(e^{w/\langle w \rangle} - 1)\nu}. \tag{32}$$

Then one can create the normalized average value of left side R_- and right side R_+ of $\tilde{x}$ i.e.

$$R_{-/+} = \left\langle SD \cdot \tilde{w}^{-1} \left(\frac{w_{-/+}(x)}{\left\langle w_{-/+}(x) \right\rangle} \right) \right\rangle \tag{33}$$

The interpretation of $R = R_+ - R_-$ as the forecasting is that it is normalized mean return with respect of normalization using the objective value. If R is larger than zero then statistically on the right hand side of Pdf of x the distribution is more leptokurtic, so previous explanation in section 4 is valid also to forecasting value R.

7 Numerical results for stock market data

We calculate the objective value for stocks from New York Stock Exchange (NYSE), Warsaw Stock Exchange (WSE) and FOREX market. Results of calculated ν of different stocks from New York Stock Exchange in years 1999 and 2000 are presented in Table 2. In Figs 2,3 there are plotted the Pdf's with fitted t-Student distribution for Ford and Bank of America respectively.

We use the forecasting described in section 6 in investment strategy (IS1) as follows. In order to create create the portfolio with a help of R we do so that $p_i = R/\sum_j |p_j|$, where p_i is the amount of money invested in stock i (negative values are possible). In our approach we have to find optimal size of window N_{win} taken in calculation of objective value. Our optimization method is quite straightforward and it resembles a genetic algorithm. During the optimization process we change the selection probability for actual values of optimized parameters that is we increase the probability if the profit from portfolio is positive and we decrease in the opposite case. The optimization process is terminated when we reach a satisfactory mean value of a yearly profit from past data (here it is 30%).

We begin our algorithm by generating randomly chosen stocks in the initial portfolio. Then we randomly select a starting moment for our virtual investment. The next step is to optimize the parameters N_{win} and risk value r: $p_i = r \cdot R/\sum_j |p_j/r|$ using available data from the period prior to the selected starting point. Finally we invest in the portfolio but only if risk value r is positive. The procedure was repeated over 1000 times and at the end we calculated an average profit i.e. the efficiency of the method. At Fig. 4 we show a distribution of returns for our portfolio at Warsaw Stock Exchange. We have calculated recommendations for windows $27 - 65$ and $17 - 41$ days long for NYSE and WSE respectively. In the case of NYSE it was the period October 1999 - December 2000 and for WSE on the period of July 2002 - December 2003 (see Figs 5,6,7). For NYSE The annual return received in such a way after commissions subtracting is around 15.2% for 11 stocks (7.5% for 60 stocks) and for WSE it was 14.6% (the commission level has been set to 0.25%). To omit artificially large price changes that can be caused by such effects as stock splitting, extreme returns larger than 0.4 have been rejected. Looking at the results of investment strategy one can conclude that in the case of hossa (WSE) and bessa (NYSE) we generate positive high profit.

The second forecasting procedure (IS2) which use the gaussianity (see sec. 4) we applied to the FOREX market. FOREX market is global market of currency exchange, commodities as well as derivatives. In the FOREX there is possibility to sell or buy given currency rate, e.g. EUR to USD (euro to American dollar). The common amount of currency handled on the FOREX is lot (100000 of basing currency). The minimal value of lot in order to open position is 0.1. Nevertheless if investor open the position on one lot of currency it should possess on the account only 1% from one lot so 1000. The cost of opening the position is spread, so difference between ask and bit offer. Another cost of the investor is swap, which is constant cost subtracted at closing of each day. Here swap is the difference between costs of credit in one and second currency. Swap can be positive but mostly is negative in order to prevent riskless opportunities of making the profit. In this strategy IS2 we use parameter F in Eq. (15) to forecast future prices movement as follows:

$$R_{IS2}(N_f) = \sum_{i=1}^{N_f} F_i, \tag{34}$$

where F_i is F calculated on window of the size N_{win}^2 shifted i data back. The parameter N_{win}^2 is equal 20000 minutes. Strategy consist of two parameters $R_{IS2}(N_f)$. First one is slow changing parameter with $N_f = 5000$ minutes and second fast changing with $N_f = 90$ minutes. In our calculations we use 15 minuts high, low and close price that is time series is: $P_i^{IS2} = P_i^{high} + P_i^{low} + 2 \cdot P_i^{close}$. Time series is calculated as in Eq. (12). Investment strategy tells to buy if $R_{IS2}(5000) > R_{IS2}(90)$ and $R_{IS2}(5000) > 0.2$. In the opposite one should sell that is $R_{IS2}(5000) < R_{IS2}(90)$ and $R_{IS2}(5000) < -0.2$. After the opening in 2-6 weeks the position should be close. The trigger to close buy position is when $R_{IS2}(5000) > R_{IS2}(90)$ and $R_{IS2}(5000) < 0$. In the case of sell position it is respectively: $R_{IS2}(5000) < R_{IS2}(90)$ and $R_{IS2}(5000) > 0$. In Table 3 are presented profits of investment procedure IS2 using smallest value of investment 0.1 lot. The average profit from past investments are positive about 152 USD in two and half month (smallest deposit is about 100 USD, - one per cent of 0.1 lot), what is the experimental proof of our theory used in method IS2. One can see on the above that profit is high but it should be fitted to the risk aversion of the investor, because the price movements are sometimes against the method so it appears the risk of shortage of money on the account. Even if further method wins but it can be so that without us. Each investor should answer the question how much money it can invest to bear the risk which is inherit property of each stochastic system forecasting.

We placed the source files of method IS2 written in MetaQuotas Language 4 on the site http://www.urbanowicz.org.pl as well as on-line trades using this method and details of testing.

8 Conclusions

We would like to emphasize that correctness of the theory lays on the temporal correlations which are not visible in the average. These correlations lead to leptokurtic probability density distribution. With a help of our parameters we see these correlations in the unbalanced left and right side of the returns distribution. Such a correlation is more visible in FOREX when we use 15 minutes lags between the data. The theory based on the objective value help us to more appropriate compute the unbalance in gaussianity, what is most

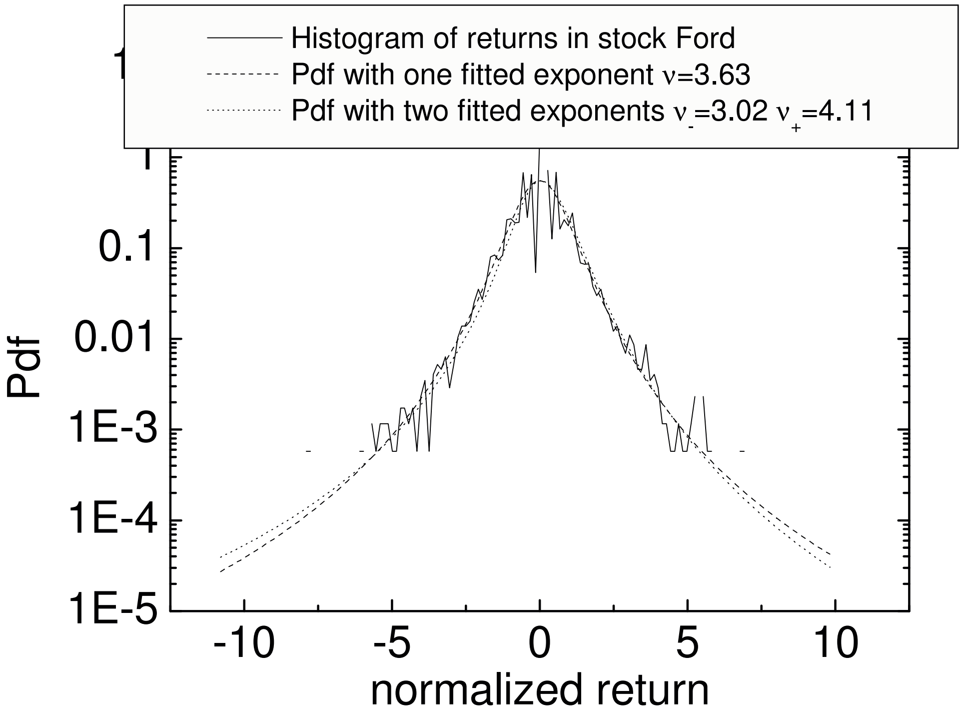

Fig. 2. Plot of the histogram of Ford stock counted in NYSE in years 1999 and 2000 and corresponding Student distributions with $\nu = 3.63$, $\nu_{-} = 3.02$ and $\nu_{+} = 4.11$.

important in our methods of stochastic systems forecasting. In the case of both investment methods we gain, so we conclude that the theory is correct. In the stock market data the possibility of cumulating of temporal correlations is plausible and can be explained with some sociological knowledge of collective behavior of investors. People when investing are incorporated in whole financial world of information transfer, so temporal correlation can appear and we reveal it with our parameters. Statistical forecasting can be possible of stock market behavior, because it is not purely random system.

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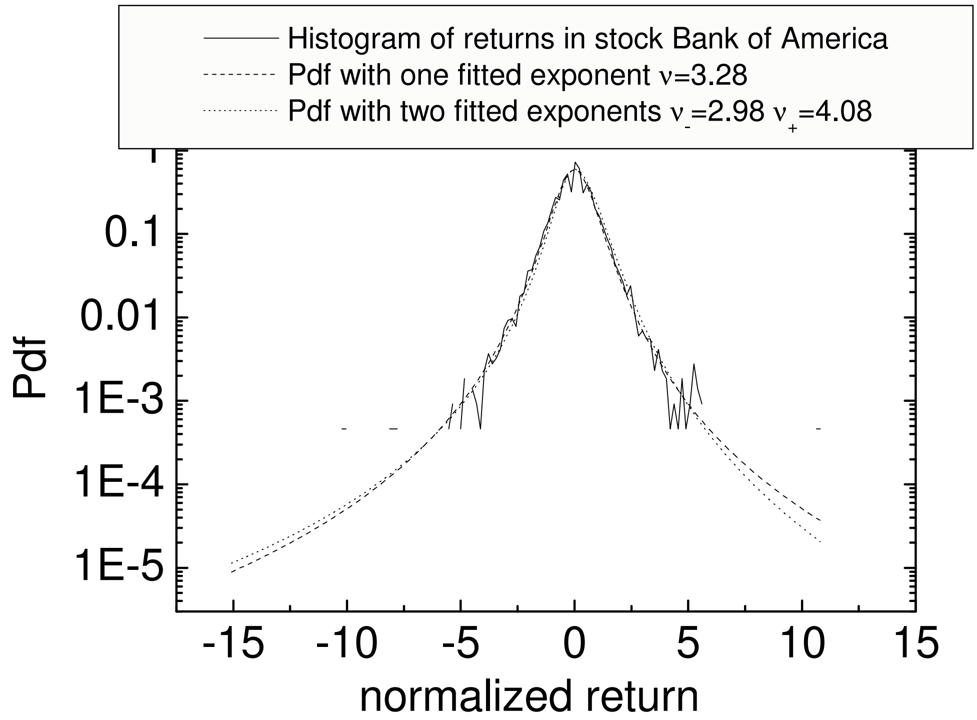

Fig. 3. Plot of the histogram of Bank of America stock counted in NYSE in years 1999 and 2000 and corresponding Student distributions with $\nu = 3.28$,$\nu_- = 2.98$ and $\nu_+ = 4.08$.

Table 2
Results of calculations of ν,ν_- and ν_+ parameter occurring in Student distribution for 11 stocks counted on NYSE.

Stock	ν	ν_-	ν_+
Apple	2.94	2.85	3.06
Bank of America	3.28	2.98	4.08
Boeing	3.8	4.07	3.24
Cisco	3.14	3.16	3.11
Compaq	3.05	2.67	3.99
Ford	3.63	3.02	4.11
General Electrics	3.52	3.76	3.34
General Motors	3.93	3.35	3.96
IBM	3.08	3.2	2.99
McDonalds	3.83	3.12	4.35
Texas Instruments	3.56	3.94	3.25

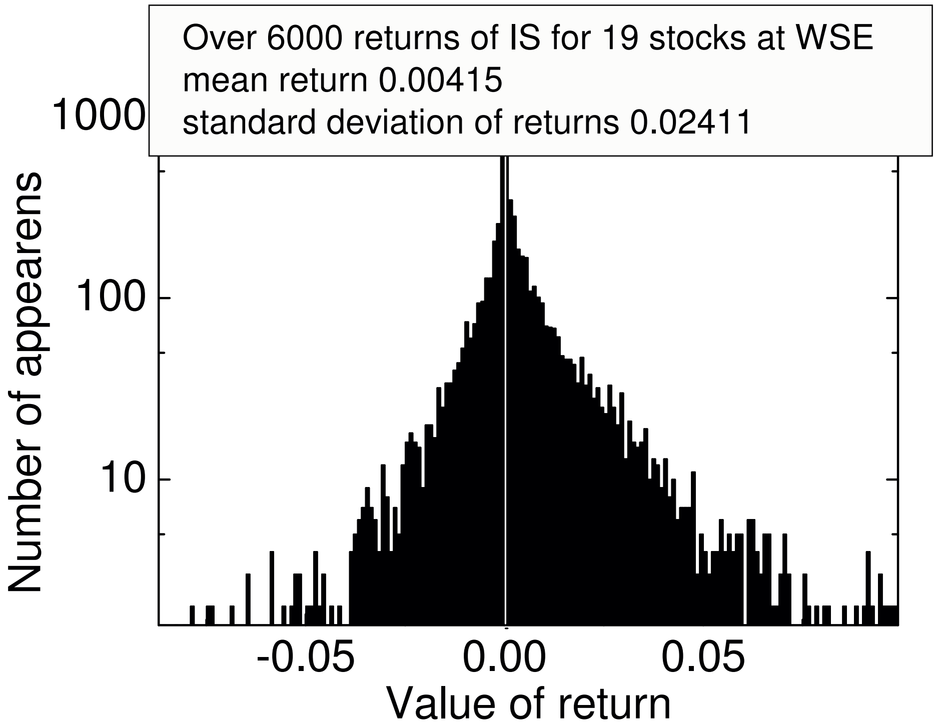

Fig. 4. Histogram of returns received by investment strategy – IS1. The mean return equals to 0.415% while the histogram dispersion is about 2.4%.

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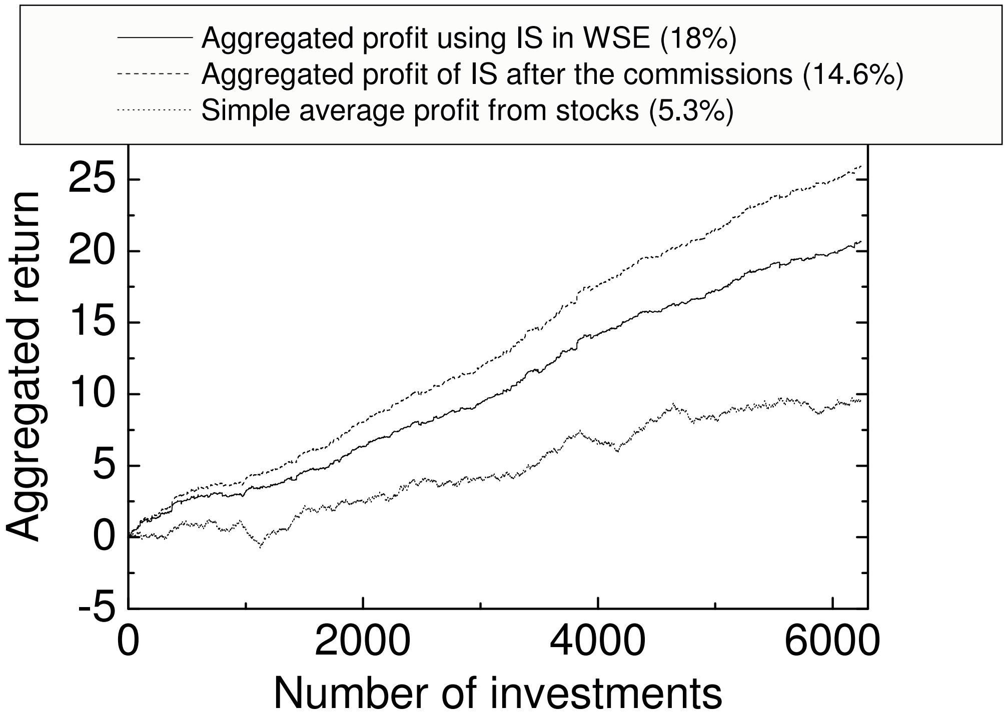

Fig. 5. The aggregated return for our investment strategy IS1 applied for 19 stocks from the Warsaw Stock Exchange. The return corresponds to the mean annual return 18% while the simple average return of 19 stocks was about 5.3% at the same time period.

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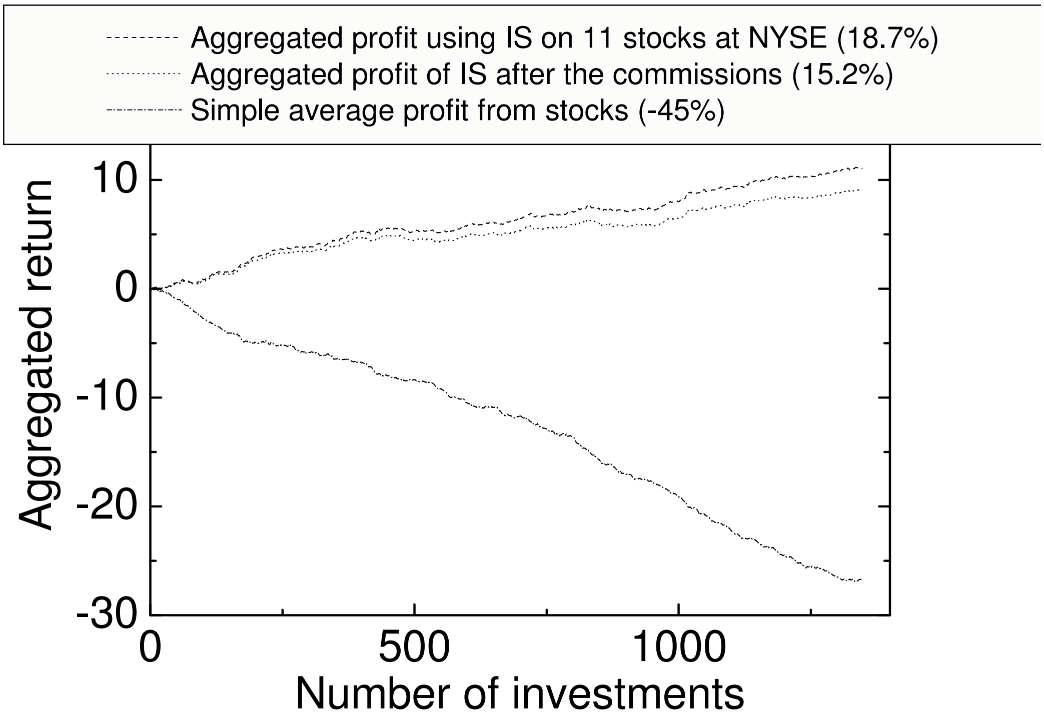

Fig. 6. The aggregated return for our investment strategy IS1 applied for 11 stocks from the New York Stock Exchange. The return corresponds to the mean annual return 18.7% while the simple average method of all 11 stocks was about -45% at the same time period.

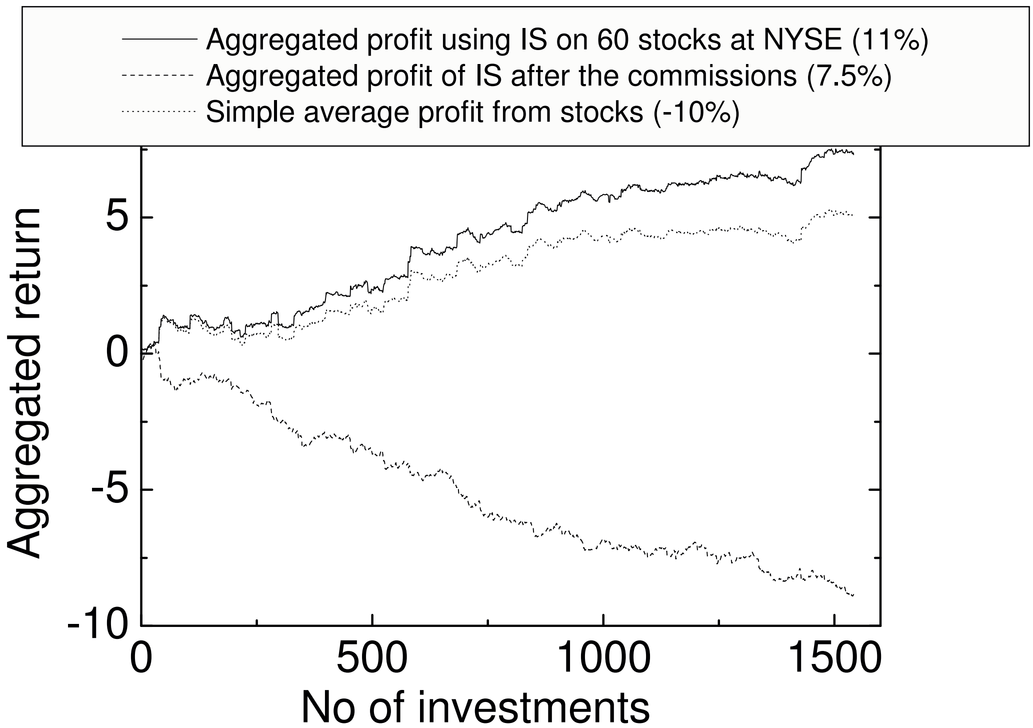

Fig. 7. The aggregated return for our investment strategy IS1 applied for 60 stocks from the New York Stock Exchange. The return corresponds to the mean annual return 11% while the simple average return of all 60 stocks was about -10% at the same time period.

Table 3
Results of testing the investment method IS2 on different currency rates in the FOREX market from 2.06.2006 to 18.08.2006.

Currency rate	size in lots	No of investments	Profit in USD
EURUSD	0.1	2	399.21
GBPUSD	0.1	3	677.27
USDCHF	0.1	1	86.31
CHFJPY	0.1	1	53.57
GBPCHF	0.1	1	-333.94
USDPLN	0.1	1	191.20
NZDUSD	0.1	1	173.14
GBPNZD	0.1	1	178.57
USDCAD	0.1	2	391.60
EURJPY	0.1	1	229.12
AUDJPY	0.1	1	393.70
EURCAD	0.1	1	176.61
AUDJPY	0.1	1	115.94
AUDNZD	0.1	1	127.22
AUDUSD	0.1	1	60.93
EURGBP	0.1	1	-74.03
EURCHF	0.1	1	-134.13
USDJPY	0.1	1	40.26