This is an example to show how to put a linear transformation into its matrix form (relative to two bases). Let us consider $V = \mathbb{R}^2, W = \mathbb{R}^3$, and $T : V \to W$ given by

$$T(x_1, x_2) = (x_1 - x_2, x_1, -x_2) \ .$$

Let's fix bases $B = \{(1,0),(1,1)\}$ of V and $C = \{(1,0,0),(0,1,1),(0,1,0)\}$ of W. We will now build $[T]_C^B$. Since V is 2-dimensional and W is 3-dimensional, this matrix will be 3×2.

To build the matrix $[T]_C^B$, we begin with the first basis vector in B, $(1,0)$ and take T of it:

$$T((1,0)) = (1,1,0) \ .$$

Now we express this in terms of the basis C for W:

$$(1,1,0) = 1 \cdot (1,0,0) + 0 \cdot (0,1,1) + 1 \cdot (0,1,0) \ .$$

This gives our first column for $[T]_C^B$:

$$\begin{pmatrix} 1 & ? \\ 0 & ? \\ 1 & ? \end{pmatrix} \ .$$

We continue with the next basis vector in B, $(1,1)$:

$$T((1,1)) = (0,1,-1) \ .$$

Express this in terms of the basis for W:

$$(0,1,-1) = 0 \cdot (1,0,0) + (-1) \cdot (0,1,1) + 2 \cdot (0,1,0) \ .$$

This gives the second column of the matrix and completes the exercise:

$$[T]_C^B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 2 \end{pmatrix} \ .$$

Now this was an example using the *column convention*. That is, the matrix is set up so that if we want to evaluate $[T(v)]_C$ for some $v \in V$ then we turn v into a column vector (relative to the basis B and write it as $[v]_B$, enter all the data into the columns of $[T]_C^B$, and then multiply $[T]_C^B$ by $[v]_B$ on the *right*. We could have done this using the row convention, where we do everything the same, except the data is entered into the rows, $[v]_B$ is expressed as a row vector and the multiplication by $[v]_B$ occurs on the left.