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Delta Hedge Strategy based on the ObV theory Option Pricing

The option market starts to expand after Black, Merton and Scholes develop their option pricing theory. Today option market helps much for hedging purposes as well as for speculations. Most important assumption for Black-Scholes theory was normality of distribution of returns. Using ObV theory we overcome this assumption and run out with option pricing which possess power-law distribution of returns. In ObV option pricing we use t-Student distribution which very well can be fitted to commodity returns and not only. With a help of ObV theory we incorporate the well-know stylized fact that price returns from time to time behave more extremely than it would comes from standard Brownian motion theory.

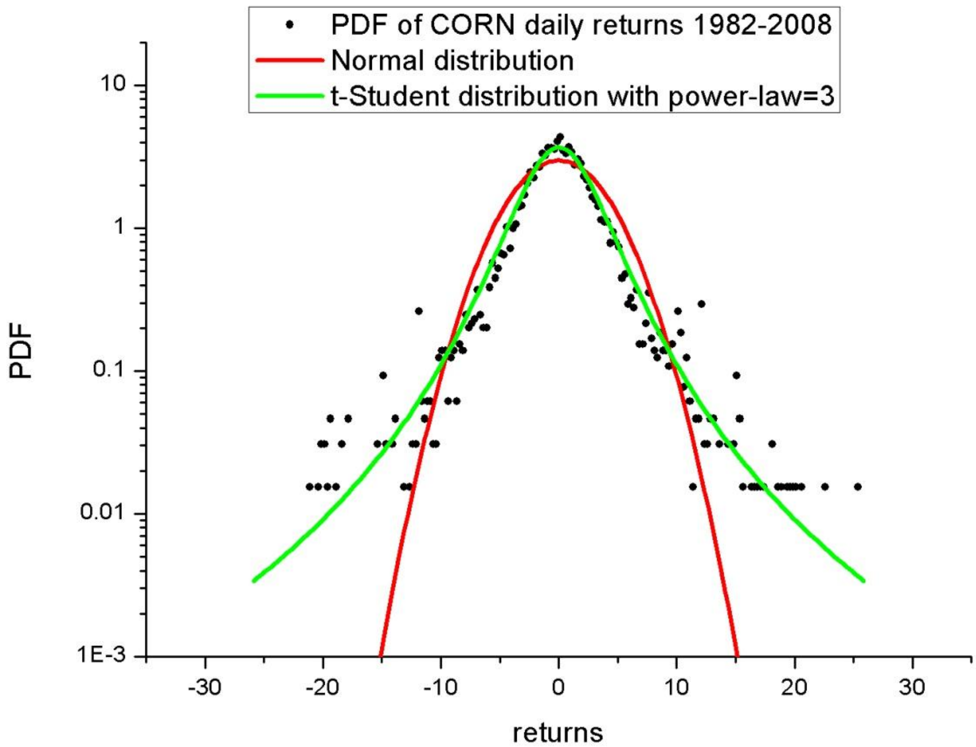

Fig. 1 Comparison between t-Student distribution and Normal distribution fitted to histogram of daily returns of Corn.

In the case of ObV option pricing as for Black-Scholes we can select a number of strategies to make the profit. Most popular strategy for options is Delta hedge Strategy. The delta hedge Strategy should make positive profit when the theory of option pricing precisely met the reality of options market, means better estimate the risk related to option. We have done back testing of Delta Hedge Strategy using ObV option pricing. In Fig. 2 we show the net profit of portfolio for agricultural commodity (Soybeans, Wheat, Corn, Soybean Meal, and Soybean Oil). In this back testing we try to simulate the situation of real trading when the trader before open of the market does not know the prices on open and send pending orders with limits which are filled or not with open prices. Of course this does not fully simulate the reality of trading so the next step is to establish the system working in real time which can fully simulate the trading in reality and after positive performance switch to trade with real money. In next sections we present aims, ObV model of option pricing, practical solution, last year backtesting and to do next.

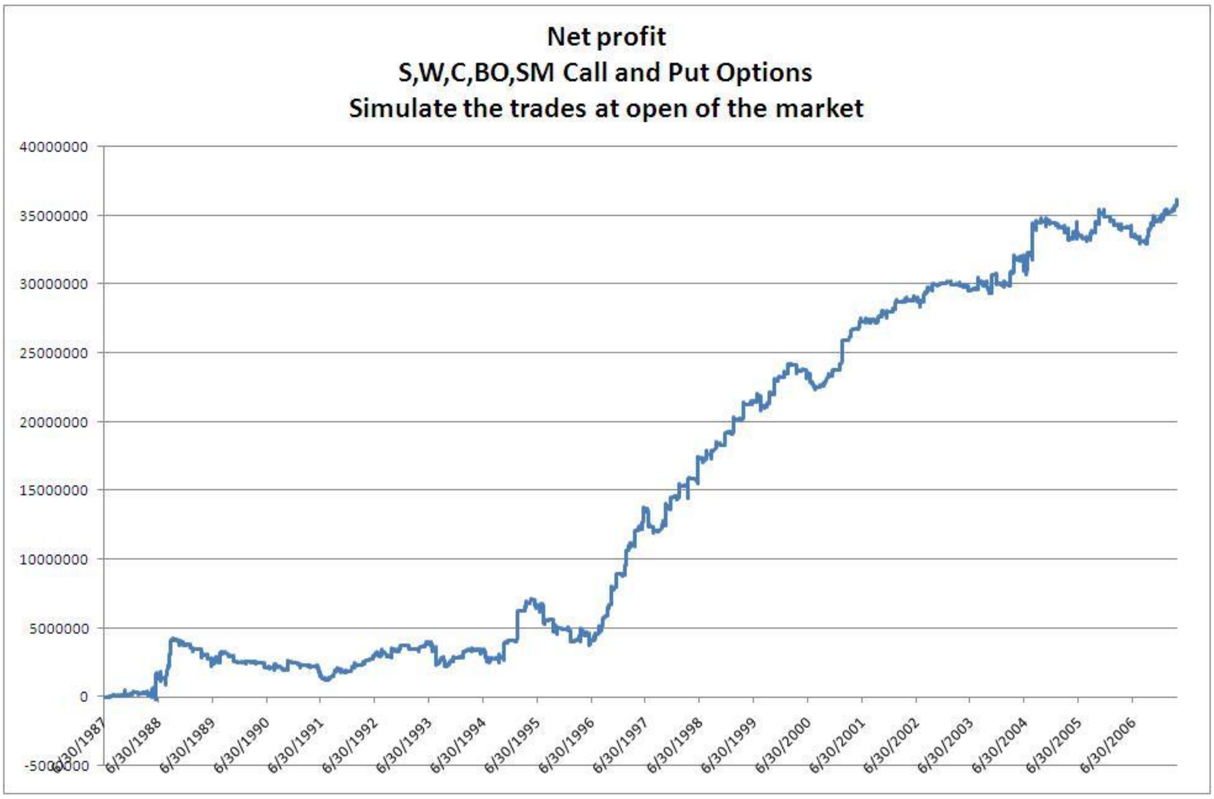

Fig. 2 The net profit of portfolio. Here we using Delta Hedge Strategy with ObV option pricing. Delta neutrality is satisfied only at opening on position on option.

Aims for real trading

Profit: 20% yearly

Risk: 10-15%

Yearly Sharp Ratio: 1-1.5

Description of Nongaussian Options Pricing based on ObV theory

Trading on options start to be more reliable from the moment of publishing Black, Scholes and Merton Model. This Model shows solution of risk pricing basing on Geometrical Brownian Motion. ObV options pricing is a generalization of this solution to stochastic motions with power-law distributions of increments. Especially t-Student distribution is used. The t-Student distribution is the power-law distribution, with input parameter 'degree of freedom' called here Power-Law: α. Power-Law can be from the range from 2 to infinity. When Power-Law goes to infinity, t-Student finishes as the Normal distribution, called also as Gaussian distribution and power-law distributions as nongaussian. Also ObV Option Pricing is attracted by Black-Scholes (B-S) in the case of infinity Power-Law.

Reassuming ObV option pricing is the generalization of B-S option pricing, with one additional parameter Power-Law: α, which shows the difference between ObV and B-S. One can say that distance between ObV and B-S is given by $1/(\alpha - 2)$.

$$P_\alpha(y) = \frac{\Gamma((\alpha+1)/2)}{\sqrt{\alpha\pi}\,\Gamma(\alpha/2)\left(1+\frac{y^2}{\alpha}\right)^{\frac{\alpha+1}{2}}}$$

Eq. 1 Formula for t-Student distribution.

Where Γ is Gamma function. Gamma function formula is as following

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$$

Eq. 2 Gamma function.

ObV options pricing assumes that logarithmic returns of basic instrument y is given by the following stochastic equation

$$dy = \mu dt + \sigma d\Omega$$

Eq. 3 Ito stochastic equation.

where Ω is generalization of Brownian Motion basing on t-Student distribution. Solution of above equation gives ObV option pricing.

In order to derive solution of above equation we use 'Objective Value Theory', which was published in Physica A[1].

Power-Law is automatically estimated basing on historical prices with the help of Maximum Likelihood Method. To calculate Power-Law are used only prices of basing instrument and is not fitted to the curve described by implied volatility.

Annualized standard deviation is estimated basing on AR model with closing prices (last price is substituted with current price) with the help of Singular Value Decomposition.

Pricing software resolve numerically integrals, which are derived from stochastic equations, so it is not Monte Carlo Method. Options Pricing can be very fast, even calculated in milliseconds. Here because of practically issues, pricing is slowed to be evaluated around second.

In Figs. 3-5 we present comparison of Black-Scholes, ObV and real market prices. Here we show the results on Implied Volatility versus strikes.

[1] K. Urbanowicz, P. Richmond and J.A. Hołyst, *Risk evaluation with enhanced covariance matrix,* Physica A: Statistical Mechanics and its Applications 384 (2), 468-474.

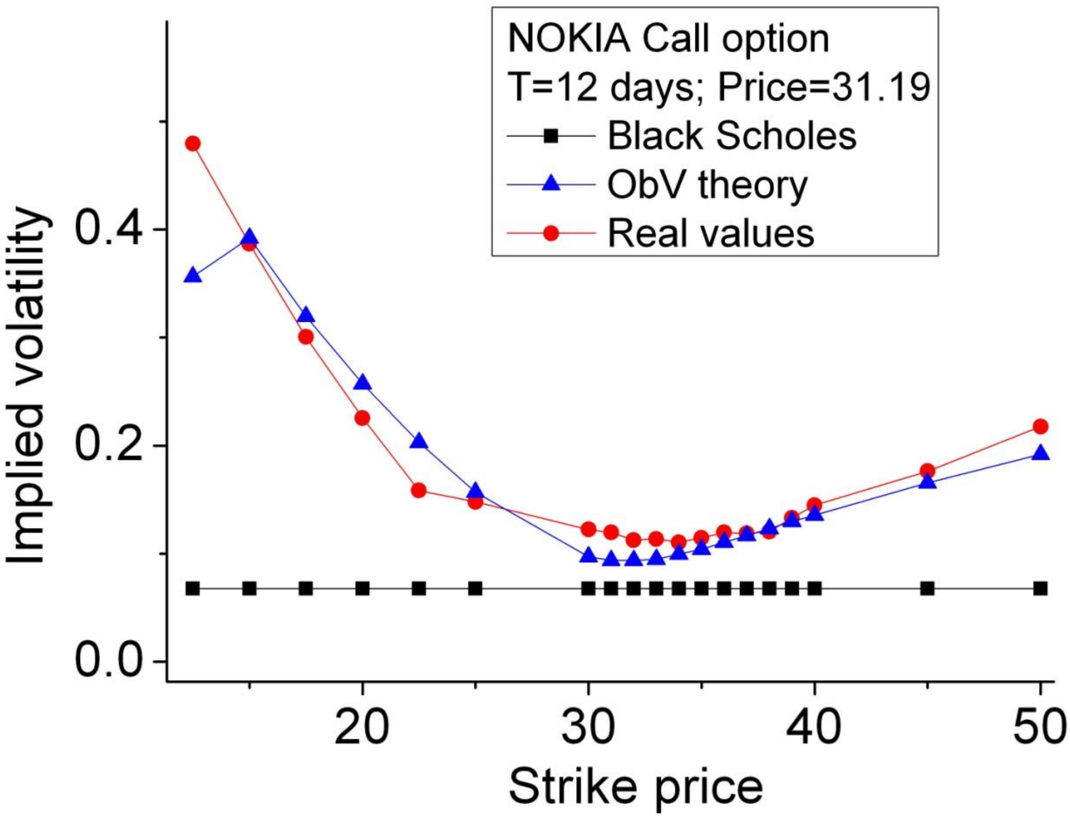

Fig. 3 Implied Volatility of Black Scholes, ObV model and real market implied volatility for NOKIA Call option. Source of data: CBOE site.

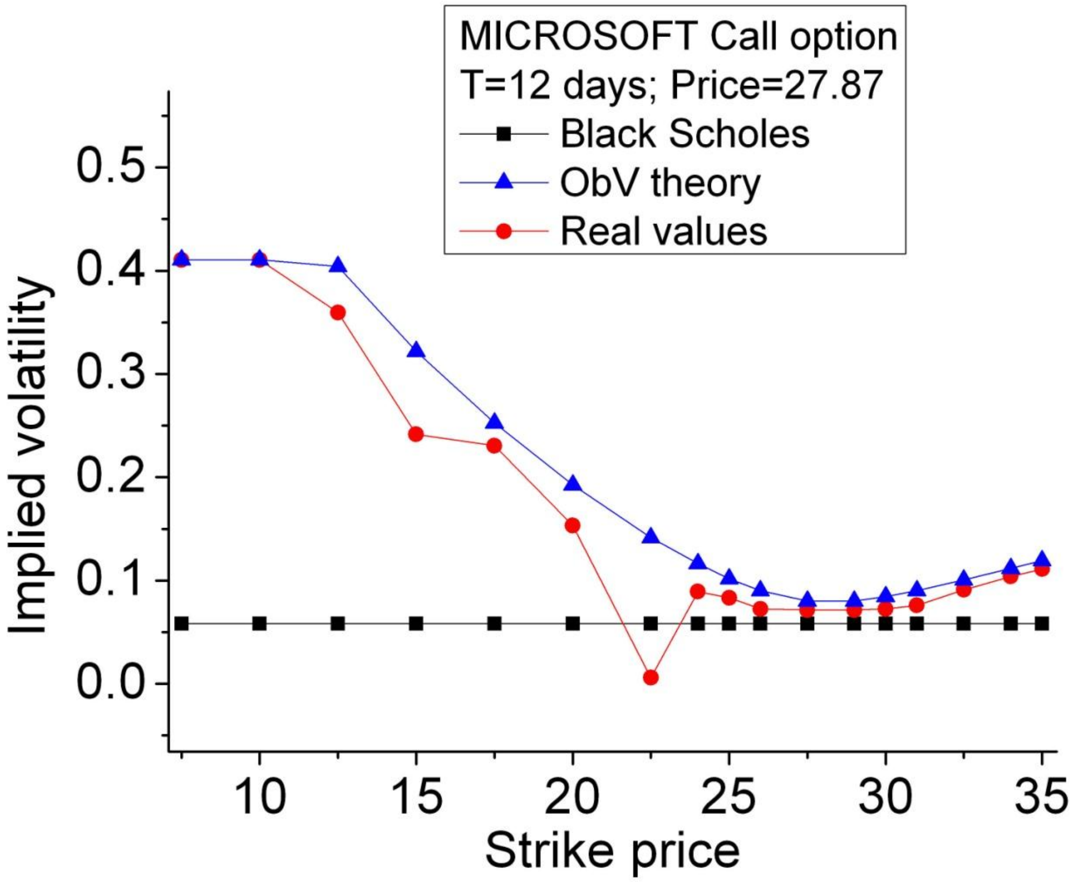

Fig. 4 Implied Volatility of Black Scholes, ObV model and real market implied volatility for MICROSOFT Call option. Source of data: CBOE site.

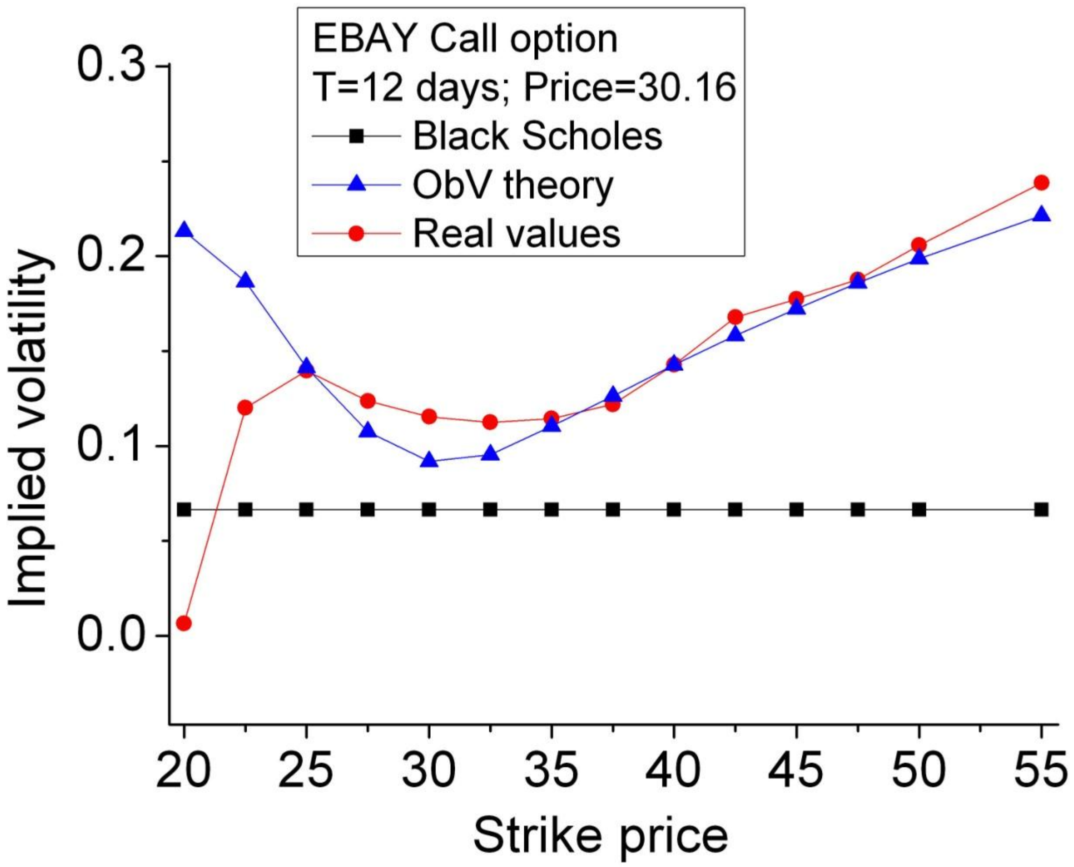

Fig. 5 Implied Volatility of Black Scholes, ObV model and real market implied volatility for EBAY Call option. Source of data: CBOE site.

Practical solution

Trading on options as well as its hedging is fully automated. We created software based on .NET solution which we test now. The results of on-line simulation of trading one can look at site http://www.wonabru.com/options. Now software is getting data from IqFeed data provider. It needs historical underlying prices to calculate ObV prices of options. One needs also current market options pricing to trade mispricing on options and hedge with underlying. We also ensure that spreads are tight in order to trade options or underlying only in time of large liquidity. In practical solution one should have .NET API from broker in order to stick to our current soft. It is good solution is to trade commodities like Crude Oil, Soybeans. Also indexes like E-mini S&P500 or E-mini NASDAQ100 or equities like Apple, Microsoft, JPMorgan etc (all followed instruments are here http://wonabru.com/options/plv/op.html). From my previous experiences FX options also is good solution but there are problems to have API.

Backtesting on last year historical daily data

Basing on IqFeed solution we can have historical daily settlement prices for current traded instruments (also for options). The reliable historical for future options prices are for last year. We have backtested from 20-10-2011 till 20-10-2012. Somehow the backtest will not fully support results of on-line trading because in backtest we traded once daily only. The backtest is done by the same software as for on-line simulating which we currently perform. The results of backtest are also in the same form as for simulation. The link is as follows: http://www.wonabru.com/options/backtest.