MAT217 HW 5
Due Tues. Mar. 12, 2013

1. Let V and W be finite dimensional vector spaces and $\xi_V : V \to V^{**}$ and $\xi_W : W \to W^{**}$ be the isomorphisms
$$\xi_V(v) = eval_v \text{ and } \xi_W(w) = eval_w \ .$$
Show that if $T : V \to W$ is linear then $\xi_W^{-1} \circ (T^t)^t \circ \xi_V = T$.

2. Let V be finite-dimensional and C a basis for V^*. Show that there is a basis B of V such that $B^* = C$.

   Hint. Consider $C^* \subset V^{**}$, the dual basis to C.

3. (From Hoffman-Kunze) Let V be finite dimensional and define $\Psi : L(V, V) \to L(V^*, V^*)$ by $\Psi(T) = T^t$. Show that Ψ is an isomorphism.

4. Let $S \subset V$, a finite dimensional vector space. Show that if $\xi_V : V \to V^{**}$ is the map $\xi_V(v) = eval_v$, then $\xi_V^{-1}((S^\perp)^\perp) = \text{Span}(S)$.