Trigonometria

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1 TRIGONOMETRIA PIANA

1.1 Limitazioni

$|\sin x| \leq 1$

$|\cos x| \leq 1$

1.2 Teorema di Pitagora

$\sin^2 x + \cos^2 x = 1$

1.3 Archi opposti

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

1.4 Archi complementari

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \frac{1}{\tan x}$

1.5 Archi supplementari

$\sin(\pi - x) = \sin x$

$\cos(\pi - x) = -\cos x$

$\tan(\pi - x) = -\tan x$

1.6 Formule di duplicazione

$\sin(2x) = 2\sin x \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x = \begin{cases} 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$

$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$

$\sin^2(x)\cos^2(x) = \frac{1}{4}\sin^2(2x) = \left(\frac{\sin(2x)}{2}\right)^2$

1.7 Formule parametriche

$\forall t = \frac{\tan x}{2}$

$\sin x = \frac{2t}{1 + t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$
$\tan x = \frac{2t}{1-t^2}$

1.8   Formule di addizione

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}$

1.9   Formule di prostaferesi

$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$
$cos x + \cos y = 2 \cos \frac{x+y}{2} \cos$
$\tan x + \tan y = \frac{\sin(x+y)}{\cos x \cos y}$

1.10   Formule di Werner

$\sin x + \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$
$\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$
$\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$

1.11   Valori per angoli particolari

x	$\cos x$	$\sin x$	$\tan x$	$\arctan x$
0	1	0	0	$\nexists$
$\frac{\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$\frac{\pi}{10}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{5+2\sqrt{5}}$
$\frac{\pi}{8}$	$\frac{2+\sqrt{2}}{2}$	$\frac{\sqrt{2}-\sqrt{2}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
$\frac{\pi}{5}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\sqrt{5-2\sqrt{5}}$	$\sqrt{10+2\sqrt{5}}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{3\pi}{10}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{5-2\sqrt{5}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
$\frac{3\pi}{8}$	$\frac{\sqrt{2}-\sqrt{2}}{2}$	$\frac{2+\sqrt{2}}{2}$	$\sqrt{2}+1$	$\sqrt{2}-1$
$\frac{2\pi}{5}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sqrt{25-10\sqrt{5}}$	$\frac{\sqrt{5}-1}{4}$
$\frac{5\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\sqrt{3}+2$	$2-\sqrt{3}$
$\frac{\pi}{2}$	0	1	$\nexists$	0
π	-1	0	0	$\nexists$
$\frac{3\pi}{2}$	0	-1	$\nexists$	0
2π	1	0	0	$\nexists$

2   TRIGONOMETRIA SFERICA