${\bf Trigonometria}$

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1 Trigonometria piana

1.1 Limitazioni

$$|\sin x| \le 1$$
$$|\cos x| \le 1$$

Teorema di Pitagora

$$\sin^2 x + \cos^2 x = 1$$

Archi opposti 1.3

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

1.4 Archi complementari

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \frac{1}{\tan x}$$

Archi supplementari

$$\sin(\pi - x) = \sin x$$

$$\cos(\pi - x) = -\cos x$$

$$\tan(\pi - x) = -\tan x$$

Formule di duplicazione

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = \begin{cases} 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$$

$$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$$

$$\sin^2(x)\cos^2(x) = \frac{1}{4}\sin^2(2x) = \left(\frac{\sin(2x)}{2}\right)^2$$

1.7 Formule parametriche

$$\forall t = \frac{\tan x}{2}$$

$$\forall t = \frac{\tan x}{2}$$
$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$
$$\tan x = \frac{2t}{1-t^2}$$

1.8 Formule di addizione

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}$$

1.9 Formule di prostaferesi

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos$$

$$\tan x + \tan y = \frac{\sin(x+y)}{\cos x \cos y}$$

1.10 Formule di Werner

$$\sin x + \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x + y) + \cos(x - y))$$

$$\sin x \cos y = \frac{1}{2}(\sin(x + y) + \sin(x - y))$$

1.11 Valori per angoli particolari

x	$\cos x$	$\sin x$	$\tan x$	$\arctan x$
0	1	0	0	∄
$\frac{\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$\frac{\pi}{10}$	$ \frac{\sqrt{6}+\sqrt{2}}{4} $ $ \frac{\sqrt{10+2\sqrt{5}}}{4} $ $ \frac{2+\sqrt{2}}{2} $ $ \frac{\sqrt{3}}{2} $	$ \frac{\sqrt{6} - \sqrt{2}}{4} $ $ \frac{\sqrt{5} - 1}{4} $ $ \frac{\sqrt{2} - \sqrt{2}}{2} $ $ \frac{1}{2} $	$\frac{\sqrt{25-10\sqrt{5}}}{5}$ $\sqrt{2}-1$	$\sqrt{5+2\sqrt{5}}$
	$\frac{2+\sqrt{2}}{2}$	$\frac{\sqrt{2}-\sqrt{2}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
$\frac{\frac{\pi}{8}}{\frac{\pi}{6}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
$\frac{\pi}{5}$	$ \begin{array}{c} 2 \\ \sqrt{5}+1 \\ 4 \\ \hline \sqrt{2} \\ 2 \end{array} $	$\frac{\sqrt{10-2\sqrt{5}}}{4}$ $\frac{\sqrt{2}}{2}$	$\sqrt{5-2\sqrt{5}}$	$\sqrt{10 + 2\sqrt{5}}$
$\frac{\frac{\pi}{5}}{\frac{\pi}{4}}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{3\pi}{10}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$ $\frac{1}{2}$	$ \frac{\sqrt{5}+1}{4} $ $ \frac{\sqrt{3}}{2} $ $ \frac{2+\sqrt{2}}{2} $	$\sqrt{5+2\sqrt{5}}$	$\sqrt{5-2\sqrt{5}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
$\frac{3\pi}{8}$	$\frac{\sqrt{2}-\sqrt{2}}{2}$	$\frac{2+\sqrt{2}}{2}$	$\sqrt{2}+1$	$\sqrt{2}-1$
$\frac{2\pi}{5}$	$\frac{\frac{\sqrt{5}-1}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}}$	$ \frac{\sqrt{10+2\sqrt{5}}}{4} $ $ \frac{\sqrt{6}+\sqrt{2}}{4} $	$\sqrt{25 - 10\sqrt{5}}$	$ \frac{\frac{1}{\sqrt{3}}}{\sqrt{2}-1} $ $ \frac{\sqrt{5}-1}{4} $
$\frac{5\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\sqrt{3}+2$	$2-\sqrt{3}$
$\frac{\pi}{2}$	0	1	∄	0
π	-1	0	0	∄
$\frac{3\pi}{2}$	0	-1	∄	0
2π	1	0	0	∄

2 Trigonometria sferica