

# Sequences and Containers

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# Announcements

# Tree Recursion Review

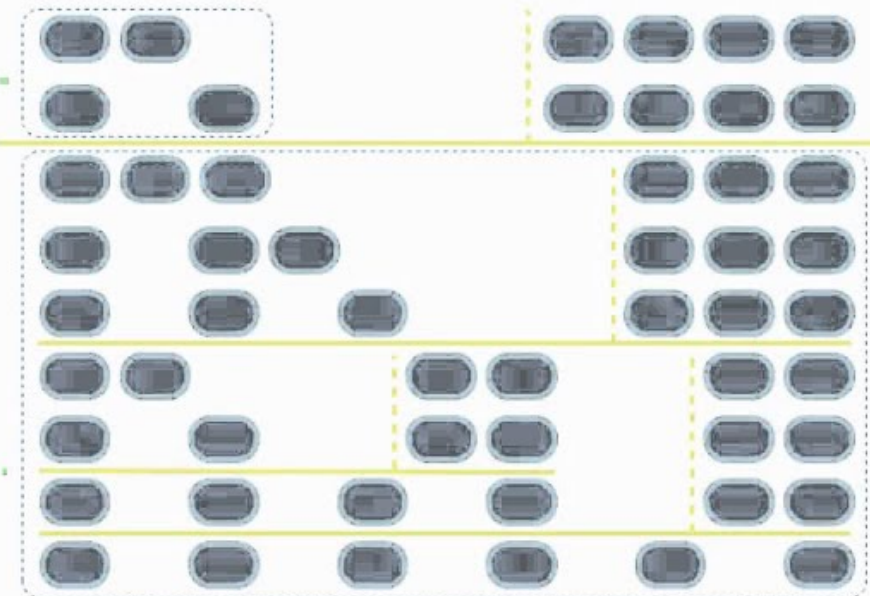
## Advice: Watch The Count Partitions Video Again!!!

### Counting Partitions

The number of partitions of a positive integer  $n$ , using parts up to size  $m$ , is the number of ways in which  $n$  can be expressed as the sum of positive integer parts up to  $m$  in increasing order.

```
count_partitions(6, 4)
```

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - `count_partitions(2, 4)`
  - `count_partitions(6, 3)`
- Tree recursion often involves exploring different choices.



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# Tree Recursion Exam Problem

## Spring 2023 Midterm 2 Question 5

**Definition.** When parking vehicles in a row, a motorcycle takes up 1 parking spot and a car takes up 2 adjacent parking spots. A string of length  $n$  can represent  $n$  adjacent parking spots using % for a motorcycle, <> for a car, and . for an empty spot.

For example: `'.%%.<><>'` (Thanks to the Berkeley Math Circle for introducing this question.)

Implement **count\_park**, which returns the number of ways that vehicles can be parked in  $n$  adjacent parking spots for positive integer  $n$ . Some or all spots can be empty.

```
def count_park(n):
    """Count the ways to park cars and motorcycles in n adjacent spots.
    >>> count_park(1) # '.' or '%'
    2
    >>> count_park(2) # '.. ', '.%', '%.', '%%', or '<>'
    5
    >>> count_park(4) # some examples: '<><>', '.%%', '%<>%', '%.<>'
    29
    """
    if n < 0:
        return 0
    elif n == 0:
        return 1
    else:
        return count_park(n-2) + count_park(n-1) + count_park(n-1)
```

# Lists

[ 'Demo' ]

Ranges

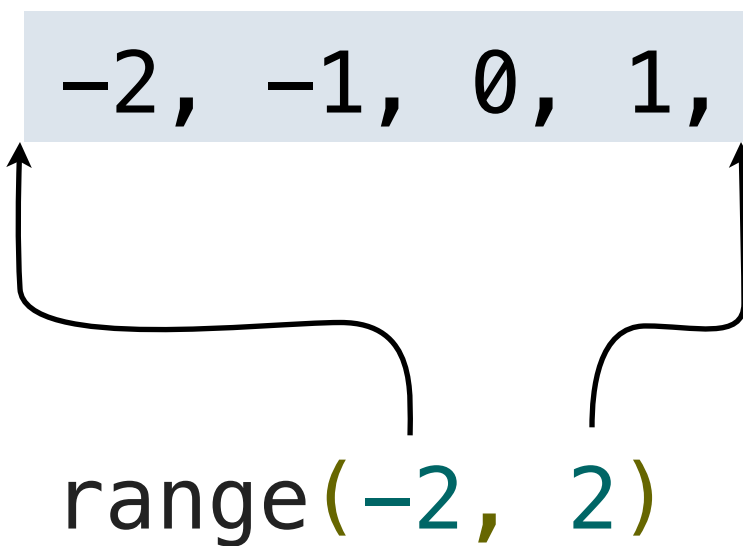


# The Range Type

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A range is a sequence of consecutive integers.\*

..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...



range(-2, 2)

**Length:** ending value - starting value

(Demo)

**Element selection:** starting value + index

```
>>> list(range(-2, 2))  
[-2, -1, 0, 1]
```

List constructor

```
>>> list(range(4))  
[0, 1, 2, 3]
```

Range with a 0 starting value

\* Ranges can actually represent more general integer sequences.

# List Comprehensions

# List Comprehensions

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```
[<map exp> for <name> in <iter exp> if <filter exp>]
```

Short version: 

```
[<map exp> for <name> in <iter exp>]
```

## Example: Two Lists

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Given these two related lists of the same length:

```
xs = range(-10, 11)
```

```
ys = [x*x - 2*x + 1 for x in xs]
```

Write a list comprehension that evaluates to:

A list of all the x values (from xs) for which the corresponding y (from ys) is below 10.

```
>>> list(xs)
```

```
[-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

```
>>> ys
```

```
[121, 100, 81, 64, 49, 36, 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
```

```
>>> xs_where_y_is_below_10
```

```
[-2, -1, 0, 1, 2, 3, 4]
```

Example: Promoted

## First in Line

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Implement **promoted**, which takes a sequence **s** and a one-argument function **f**. It returns a list with the same elements as **s**, but with all elements **e** for which **f(e)** is a true value ordered first. Among those placed first and those placed after, the order stays the same.

```
def promoted(s, f):  
    """Return a list with the same elements as s, but with all  
    elements e for which f(e) is a true value placed first.  
  
    >>> promoted(range(10), odd) # odds in front  
    [1, 3, 5, 7, 9, 0, 2, 4, 6, 8]  
    """  
    return [e for e in s if f(e)] + [e for e in s if not f(e)]
```

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# Lists, Slices, & Recursion

## A List is a First Element and the Rest of the List

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For any list `s`, the expression `s[1:]` is called a *slice* from index 1 to the end (or 1 onward)

- The value of `s[1:]` is a list whose length is one less than the length of `s`
- It contains all of the elements of `s` except `s[0]`
- Slicing `s` doesn't affect `s`

```
>>> s = [2, 3, 6, 4]
>>> s[1:]
[3, 6, 4]
>>> s
[2, 3, 6, 4]
```

In a list `s`, the first element is `s[0]` and the rest of the elements are `s[1:]`.



## Recursion Example: Sum

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Implement `sum_list`, which takes a list of numbers `s` and returns their sum. If a list is empty, the sum of its elements is 0.

```
def sum_list(s):  
    """Sum the elements of list s.  
  
    >>> sum([2, 4, 1, 3])  
    10  
    """  
  
    if len(s) == 0:  
        return 0  
  
    else:  
        return s[0] + sum_list(s[1:])
```

**Recursive idea:** The sum of the elements of a list is the result of adding the first element to the sum of the rest of the elements

## Recursion Example: Large Sums

**Definition:** A sublist of a list `s` is a list with some (or none or all) of the elements of `s`.

Implement **large**, which takes a list of positive numbers `s` and a non-negative number `n`.

It returns the sublist of `s` with the largest sum that is less than or equal to `n`.

You may call **sum\_list**, which takes a list and returns the sum of its elements.

```
def large(s, n):
    """Return the sublist of positive numbers s with the
    largest sum that is less than or equal to n.

    >>> large([4, 2, 5, 6, 7], 3)
    [2]
    >>> large([4, 2, 5, 6, 7], 8)
    [2, 6]
    >>> large([4, 2, 5, 6, 7], 19)
    [4, 2, 6, 7]
    >>> large([4, 2, 5, 6, 7], 20)
    [2, 5, 6, 7]
    """
    if s == []:
        return []
    elif s[0] > n:
        return large(s[1:], n)
    else:
        first = s[0]
        with_s0 = [first] + large(s[1:], n - first)
        without_s0 = large(s[1:], n)
        if sum_list(with_s0) > sum_list(without_s0):
            return with_s0
        else:
            return without_s0
```

# Building Lists Recursively

Add Consecutive

<https://cs61a.org/exam/su24/midterm/61a-su24-midterm.pdf#page=11>

## More Tree Recursion Practice

## Tree Recursion Exam Problem 2

<https://cs61a.org/exam/su22/midterm/61a-su22-midterm.pdf#page=10>