

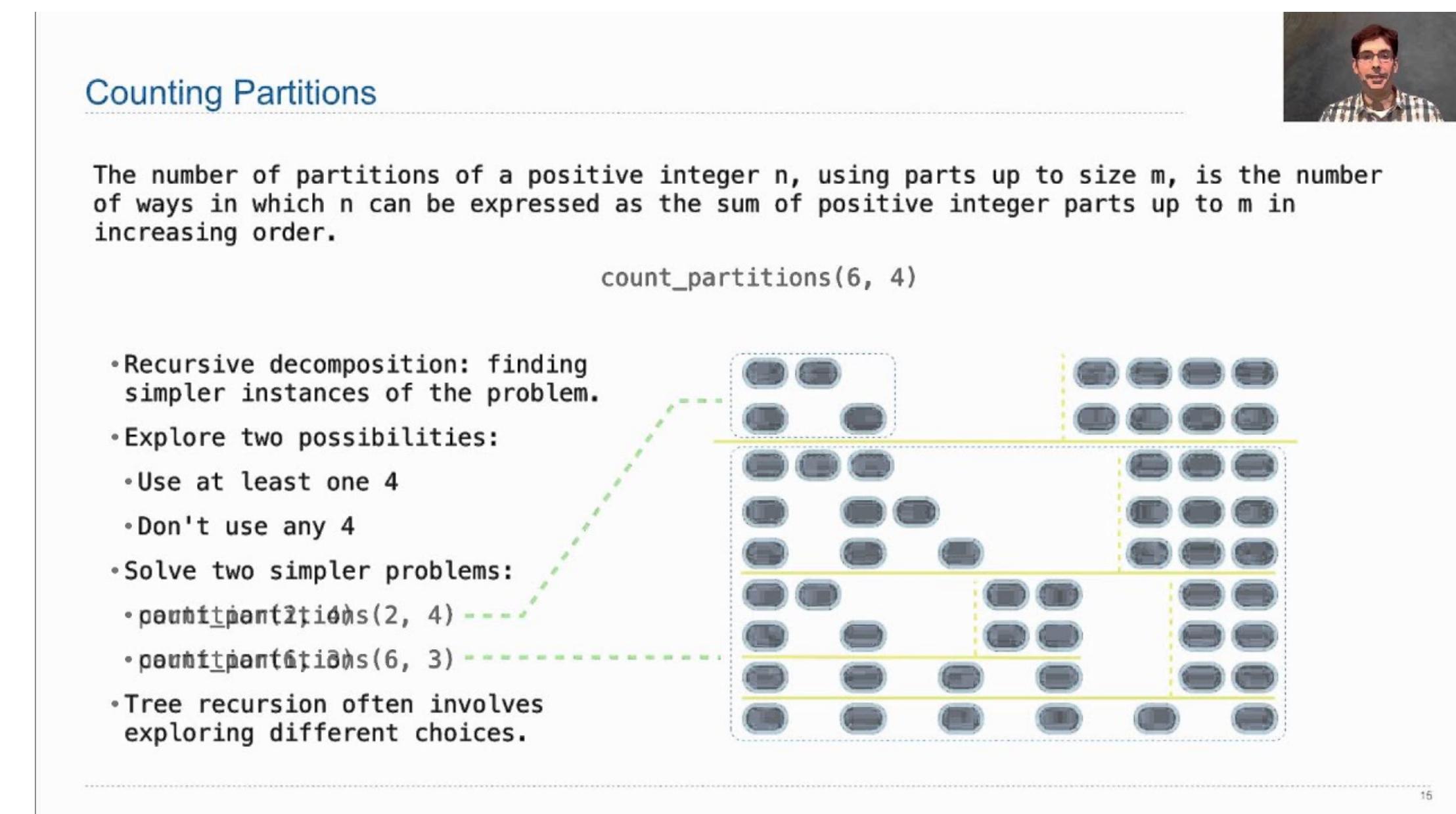
Sequences and Containers

Announcements

Tree Recursion Review

Advice: Watch The Count Partitions Video Again!!!

Counting Partitions



The number of partitions of a positive integer n , using parts up to size m , is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

count_partitions(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
 - Use at least one 4
 - Don't use any 4
- Solve two simpler problems:
 - `partitions(2, 4)`
 - `partitions(6, 3)`
- Tree recursion often involves exploring different choices.

<https://www.youtube.com/watch?v=DvgT4dnSMVM&list=PL6BsET-8jgYUUWPap4etQjZVwlWUeFxn0&index=4>

Tree Recursion Exam Problem

Spring 2023 Midterm 2 Question 5

Definition. When parking vehicles in a row, a motorcycle takes up 1 parking spot and a car takes up 2 adjacent parking spots. A string of length n can represent n adjacent parking spots using `%` for a motorcycle, `<>` for a car, and `.` for an empty spot.

For example: `'.%<>%<>'` (Thanks to the Berkeley Math Circle for introducing this question.)

Implement `count_park`, which returns the number of ways that vehicles can be parked in n adjacent parking spots for positive integer n . Some or all spots can be empty.

```
def count_park(n):
    """Count the ways to park cars and motorcycles in n adjacent spots.
    >>> count_park(1)  # '.' or '%'
    2
    >>> count_park(2)  # '...', '.%', '%.', '%%', or '<>'
    5
    >>> count_park(4)  # some examples: '<><>', '.%<>%', '%<>%', '%.<>'
    29
    .....
    if n < 0:
        return _____
    elif n == 0:
        return _____
    else:
        return _____
```

`count_park(n-2) + count_park(n-1) + count_park(n-1)`

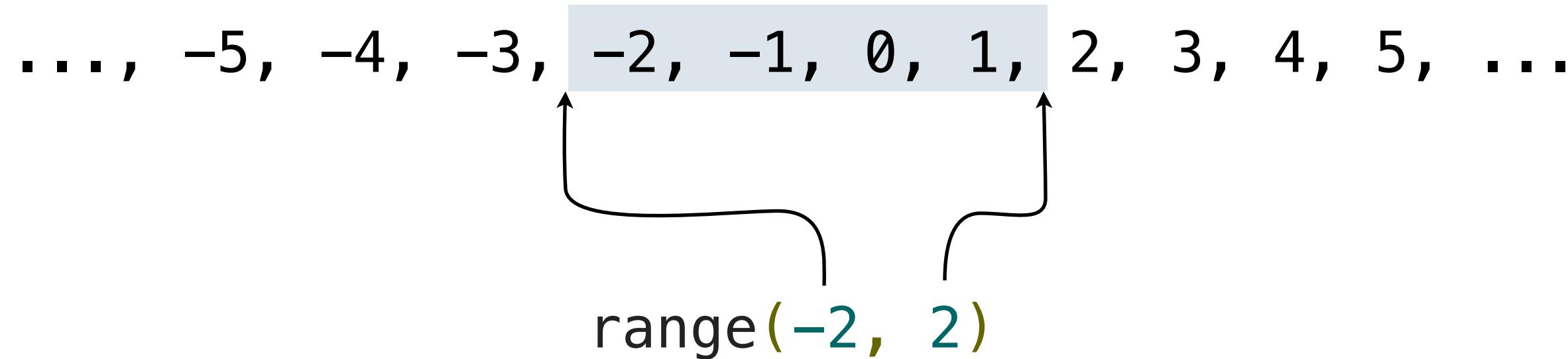
Lists

['Demo']

Ranges

The Range Type

A range is a sequence of consecutive integers.*



Length: ending value – starting value

(Demo)

Element selection: starting value + index

```
>>> list(range(-2, 2))  
[-2, -1, 0, 1]
```

List constructor

```
>>> list(range(4))  
[0, 1, 2, 3]
```

Range with a 0 starting value

* Ranges can actually represent more general integer sequences.

List Comprehensions

List Comprehensions

[<map exp> for <name> in <iter exp> if <filter exp>]

Short version: [<map exp> for <name> in <iter exp>]

Example: Two Lists

Given these two related lists of the same length:

```
xs = range(-10, 11)
```

```
ys = [x*x - 2*x + 1 for x in xs]
```

Write a list comprehension that evaluates to:

A list of all the x values (from xs) for which the corresponding y (from ys) is below 10.

```
>>> list(xs)
```

```
[-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

```
>>> ys
```

```
[121, 100, 81, 64, 49, 36, 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
```

```
>>> xs_where_y_is_below_10
```

```
[-2, -1, 0, 1, 2, 3, 4]
```

Example: Promoted

First in Line

Implement **promoted**, which takes a sequence **s** and a one-argument function **f**. It returns a list with the same elements as **s**, but with all elements **e** for which **f(e)** is a true value ordered first. Among those placed first and those placed after, the order stays the same.

```
def promoted(s, f):
    """Return a list with the same elements as s, but with all
    elements e for which f(e) is a true value placed first.

    >>> promoted(range(10), odd)  # odds in front
    [1, 3, 5, 7, 9, 0, 2, 4, 6, 8]
    """
    return [e for e in s if f(e)] + [e for e in s if not f(e)]
```

Lists, Slices, & Recursion

A List is a First Element and the Rest of the List

For any list `s`, the expression `s[1:]` is called a *slice* from index 1 to the end (or 1 onward)

- The value of `s[1:]` is a list whose length is one less than the length of `s`
- It contains all of the elements of `s` except `s[0]`
- Slicing `s` doesn't affect `s`

```
>>> s = [2, 3, 6, 4]
>>> s[1:]
[3, 6, 4]
>>> s
[2, 3, 6, 4]
```

In a list `s`, the first element is `s[0]` and the rest of the elements are `s[1:]`.

Recursion Example: Sum

Implement `sum_list`, which takes a list of numbers `s` and returns their sum. If a list is empty, the sum of its elements is 0.

```
def sum_list(s):
    """Sum the elements of list s.

    >>> sum([2, 4, 1, 3])
    10
    """

if len(s) == 0:
    return 0
else:
    return s[0] + sum_list(s[1:])
```

Recursive idea: The sum of the elements of a list is the result of adding the first element to the sum of the rest of the elements

Recursion Example: Large Sums

Definition: A sublist of a list s is a list with some (or none or all) of the elements of s .

Implement **large**, which takes a list of positive numbers s and a non-negative number n .

It returns the sublist of s with the largest sum that is less than or equal to n .

You may call **sum_list**, which takes a list and returns the sum of its elements.

```
def large(s, n):
    """Return the sublist of positive numbers s with the
    largest sum that is less than or equal to n.

    >>> large([4, 2, 5, 6, 7], 3)
    [2]
    >>> large([4, 2, 5, 6, 7], 8)
    [2, 6]
    >>> large([4, 2, 5, 6, 7], 19)
    [4, 2, 6, 7]
    >>> large([4, 2, 5, 6, 7], 20)
    [2, 5, 6, 7]
    .....

    if s == []:
        return []
    elif s[0] > n:
        return large(s[1:], n)
    else:
        first = s[0]
        with_s0 = _____ [first] + large(s[1:], n - first)
        without_s0 = _____ large(s[1:], n)
        if sum_list(with_s0) > sum_list(without_s0):
            return with_s0
        else:
            return without_s0
```

Building Lists Recursively

Add Consecutive

<https://cs61a.org/exam/su24/midterm/61a-su24-midterm.pdf#page=11>

More Tree Recursion Practice

Tree Recursion Exam Problem 2

<https://cs61a.org/exam/su22/midterm/61a-su22-midterm.pdf#page=10>