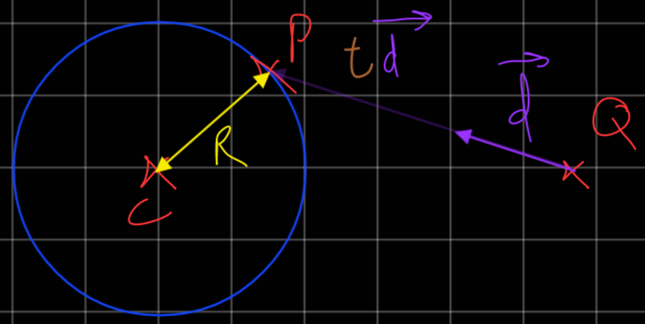

• = Real
• = Vector
• = Point
• = Unknown Variable

$$(C - P) \cdot (C - P) = R^2$$

$$Q + td$$

$$\big(C - (Q + td)\big) \cdot \big(C - (Q + td)\big) = R^2$$

$$(C - Q - td) \cdot (C - Q - td) = R^2$$

Define $\vec{v}$ as $C - Q$ for simplification

$$(V - td) \cdot (V - td) = R^2$$

Define $(V - td)$ as $\vec{W}$

$$(V - td) \cdot W = R^2$$

Property A
$(cU + V) \cdot W = c(U \cdot W) + (V \cdot W)$

$$(V - t d)\cdot W = -t(d\cdot W) + (V\cdot W)$$
$$= R^2$$

$$-t(d\cdot W) + (V\cdot W) = R^2$$

Replacing the first $\vec{W}$ as $(V - t d)$

$$\underbrace{-t(d\cdot(V - t d))}\ + V\cdot W = R^2$$

using property A on $d\cdot(V - t d)$

$$-t(d\cdot d) + (V\cdot d)$$
$$\Leftrightarrow -t\, d^2 + V\cdot d$$
$$-t(-t\, d^2 + V\cdot d) + V\cdot W = R^2$$

Replacing the last $\vec{W}$ as $(V - t d)$

$$-t(-t\, d^2 + V\cdot d) + \underbrace{V\cdot(V - t d)} = R^2$$

using property A on $V\cdot(V - t d)$

$$V\cdot(V - t d) = -t(d\cdot V) + (V\cdot V)$$
$$= -t(d\cdot V) + V^2$$

$$-t(-t\, d^2 + V\cdot d) - t(d\cdot V) + V^2 = R^2$$

$$-t(-t\, d^2 + 2V\cdot d) + V^2 = R^2$$

$$t^2 d^2 - 2t\, V \cdot d + V^2 = R^2$$

$$\Leftrightarrow \quad t^2 \underbrace{d^2} - t \underbrace{2V \cdot d} + \underbrace{V^2 - R^2} = 0$$

Polynomial of form $ax^2 + bx + c = 0$

$$a = d^2 \qquad b = -2V \cdot d \qquad c = V^2 - R^2$$

Solving for t:

$$\Delta = b^2 - 4(ac)$$

If $\Delta < 0$, the ray isn't touching the sphere

if $\Delta = 0$, the ray has 1 intersection

$$t = \frac{-b}{2a}$$

$$\Leftrightarrow \quad \frac{2V \cdot d}{2d^2} = \frac{V \cdot d}{d^2}$$

$$\boxed{t = \frac{(C - Q) \cdot d}{d \cdot d}}$$

if $\Delta > 0$, the ray has 2 intersections

$$t_1 = \frac{-b+\sqrt{\Delta}}{2a} \qquad \Delta = (-2V\cdot d)^2 - 4(d^2(V^2-R^2))$$

$$\boxed{t_1 = \frac{2V\cdot d+\sqrt{\Delta}}{2d\cdot d}}$$

$$t_2 = \frac{-b-\sqrt{\Delta}}{2a}$$

$$\boxed{t_2 = \frac{2V\cdot d-\sqrt{\Delta}}{2d\cdot d}}$$