# Online Supplement: Robust Repositioning for Vehicle Sharing

## Appendix A: Proofs

### A.1. Proof of Lemma 1

Let $f(\mathbf{w_t}) = \sum_{i\in[N]} \bar{p}_{it}(d_{it} - w_{it}) + V_{t+1}\left(\mathbf{x_{t+1}}, \mathbf{d_{[t]}}\right)$ for given $\mathbf{d_{[t]}}$. For the first part, we show that under the condition $\bar{p}_{it} \geq \sum_{j\neq i} s_{ji(t+1)}\alpha_{ijt}$, the solution $w^*_{it} = d_{it} \wedge \left(x_{it} + \sum_{j\in[N]} r_{jit} - \sum_{j\in[N]} r_{ijt}\right)$ is optimal for the minimization problem in (2), which then confirms that $J_t(\mathbf{x_t}, \mathbf{r_t}, \mathbf{d_{[t]}}) = f(\mathbf{w_t}^*)$. Suppose, on the contrary, there exists an optimal solution $\mathbf{w'_t}$ to (2) with $w'_{it} < d_{it} \wedge \left(x_{it} + \sum_{j\in[N]} r_{jit} - \sum_{j\in[N]} r_{ijt}\right)$ for some $i \in [N]$. We denote the state in $t+1$ under $\mathbf{w'_t}$ as $\mathbf{x'_{t+1}}$. Let $\epsilon = d_{it} \wedge \left(x_{it} + \sum_{j\in[N]} r_{jit} - \sum_{j\in[N]} r_{ijt}\right) - w'_{it} > 0$, then the solution constructed by $w_{it} = w'_{it} + \epsilon$ and $w_{jt} = w'_{jt}$ for any $j \neq i$ is still a feasible solution to (2) with a cost $f(\mathbf{w_t}) = \sum_{i\in[N]} \bar{p}_{it}(d_{it} - w_{it}) + V_{t+1}\left(\mathbf{x_{t+1}}, \mathbf{d_{[t]}}\right) = \sum_{i\in[N]} \bar{p}_{it}(d_{it} - w'_{it}) - \bar{p}_{it}\epsilon + V_{t+1}\left(\mathbf{x_{t+1}}, \mathbf{d_{[t]}}\right)$. Observe here that $x_{i(t+1)} = x'_{i(t+1)} - \epsilon + \alpha_{iit}\epsilon$ and $x_{j(t+1)} = x'_{j(t+1)} + \alpha_{ijt}\epsilon$, $\forall j \neq i$. In period $t+1$ at state $\mathbf{x_{t+1}}$, it is always feasible to first reach the state $\mathbf{x'_{t+1}}$ by letting $r_{ji(t+1)} = \alpha_{ijt}\epsilon$ for $j \neq i$, and $r_{jk(t+1)} = 0$ for any $j \in [N], k \neq i$, at a cost $\sum_{j\neq i} s_{ji(t+1)}\alpha_{ijt}\epsilon$. After reaching to state $\mathbf{x'_{t+1}}$, one can reposition again according to the optimal policy at state $\mathbf{x'_{t+1}}$, which yields an optimal cost-to-go $V_{t+1}\left(\mathbf{x'_{t+1}}, \mathbf{d_{[t]}}\right)$. This two-step repositioning, however, may not be optimal at state $\mathbf{x_{t+1}}$. Thus, we must have $V_{t+1}\left(\mathbf{x_{t+1}}, \mathbf{d_{[t]}}\right) \leq \sum_{j\neq i} s_{ji(t+1)}\alpha_{ijt}\epsilon + V_{t+1}\left(\mathbf{x'_{t+1}}, \mathbf{d_{[t]}}\right)$. Consequently,

$$\begin{aligned}
f(\mathbf{w_t}) &\leq \sum_{i\in[N]} \bar{p}_{it}(d_{it} - w'_{it}) + V_{t+1}\left(\mathbf{x'_{t+1}}, \mathbf{d_{[t]}}\right) - \bar{p}_{it}\epsilon + \sum_{j\neq i} s_{ji(t+1)}\alpha_{ijt}\epsilon \\
&= f(\mathbf{w'_t}) + \sum_{j\neq i} s_{ji(t+1)}\alpha_{ijt}\epsilon - \bar{p}_{it}\epsilon \\
&\leq f(\mathbf{w'_t}),
\end{aligned}$$

where in the last inequality we have used the condition that $\bar{p}_{it} \geq \sum_{j\neq i} s_{ji(t+1)}\alpha_{ijt}$. Hence, $\mathbf{w_t}$ must also be optimal. This establishes that $w^*_{it} = d_{it} \wedge \left(x_{it} + \sum_{j\in[N]} r_{jit} - \sum_{j\in[N]} r_{ijt}\right)$ must be optimal.

For the second part, clearly $V_{T+1}(\mathbf{x_{T+1}})$ is convex. Suppose $V_{t+1}\left(\mathbf{x_{t+1}}, \mathbf{d_{[t]}}\right)$ is convex in $\mathbf{x_{t+1}}$ for $t \leq T$. The objective function in (2) is then jointly convex in $\mathbf{x_t}, \mathbf{r_t}, \mathbf{w_t}$ and the constraint set is a convex set. As a result, for any $\mathbf{d_{[t]}}$, $J_t\left(\mathbf{x_t}, \mathbf{r_t}, \mathbf{d_{[t]}}\right)$ is convex in $\mathbf{x_t}, \mathbf{r_t}$ (e.g., Proposition 2.2.15 in Simchi-Levi et al. 2005), and hence $\mathbb{E}_{\mathbb{P}}[J_t(\mathbf{x_t}, \mathbf{r_t}, \mathbf{d_{[t]}})]$ is also jointly convex in $\mathbf{x_t}, \mathbf{r_t}$ for any given $\mathbf{d_{[t-1]}}$. Thus, the objective function in (1) is jointly convex in $\mathbf{x_t}, \mathbf{r_t}$. As the constraint set in (1) is also convex, $V_t\left(\mathbf{x_t}, \mathbf{d_{[t-1]}}\right)$ is convex in $\mathbf{x_t}$. $\square$

### A.2. Proof of Proposition 1

For a 2-region system, the stochastic dynamic program (1) can then be simplified as:

$$V_t(x_t) = \min_{x_t - C \leq r_t \leq x_t} \left\{ s_{12t} r_t^+ + s_{21t} r_t^- + \mathbb{E}_{\mathbb{P}}[J_t(y_t, \mathbf{d_t})] \right\}$$

where

$$\begin{aligned}
J_t(y_t, \mathbf{d_t}) = &\min_{w_{1t}, w_{2t}} \left\{ \bar{p}_{1t}(d_{1t} - w_{1t}) + \bar{p}_{2t}(d_{2t} - w_{2t}) + V_{t+1}(x_{t+1}) \right\}, \\
\text{s.t.}\quad & x_{t+1} = y_t - \alpha_{12t} w_{1t} + \alpha_{21t} w_{2t}, \\
& w_{1t} \leq y_t \wedge d_{1t}, \\
& w_{2t} \leq (C - y_t) \wedge d_{2t},
\end{aligned}$$

and the terminal cost $V_{T+1}(x_{T+1}) = 0$.

To prove Proposition 1, we show the lemma below with the index $t$ dropped for the ease of presentation.

LEMMA 1. *If $y = x - r$ and the function $F(\cdot)$ is convex, then there exist $\underline{x}$ and $\overline{x}$ such that the optimal solution to $\min_{x-C\leq r\leq x}\{s_{12}r^+ + s_{21}r^- + F(y)\}$, denoted as $r^*(x)$, is given by*

$$r^*(x) = \begin{cases} x-\underline{x}, & x\in[0,\underline{x}),\\ 0, & x\in[\underline{x},\overline{x}],\\ x-\overline{x}, & x\in(\overline{x},C], \end{cases} \quad \text{and } y^*(x) = \begin{cases} \underline{x}, & x\in[0,\underline{x}),\\ x, & x\in[\underline{x},\overline{x}],\\ \overline{x}, & x\in(\overline{x},C]. \end{cases}$$

*where $\underline{x}$ and $\overline{x}$ are provided by the following two convex programs*

$$\underline{x} = \arg\min_{0\leq y\leq C}\{s_{21}y + F(y)\}, \quad \overline{x} = \arg\min_{0\leq y\leq C}\{-s_{12}y + F(y)\}.$$

*Proof of Lemma 1:* Since $F(y)$ is convex, $F(x-r)$ is submodular in $x$ and $r$ (see Theorem 2.3.6 in Simchi-Levi et al. 2005). Moreover, we see that $[x-C,x]$ is an increasing set in $x$. By Theorem 2.3.7 in Simchi-Levi et al. (2005), $r^*(x)$ is increasing in $x$. Note that $r^*(0)\leq 0$ and $r^*(C)\geq 0$. Define $\underline{x} = \inf\{x : r^*(x)\geq 0\}$ and $\overline{x} = \sup\{x : r^*(x)\leq 0\}$. By monotonicity of $r^*(x)$, it holds $0\leq \underline{x}\leq \overline{x}\leq C$. By definition, when $x\in[0,\underline{x})$ , $r^*(x) < 0$ and $y^*(x) > x$; when $x\in(\overline{x},C]$ , $r^*(x) > 0$ and $y^*(x) < x$; and when $x\in[\underline{x},\overline{x}]$, $r^*(x) = 0$ and $y^*(x) = x$.

Next, we show that when $x\in[0,\underline{x})$, $y^*(x) = \underline{x}$ and hence $r^*(x) = \underline{x} - x$. Note that when $x\in[0,\underline{x})$, the problem $\min_{x-C\leq r\leq x}\{s_{12}r^+ + s_{21}r^- + F(y)\}$ is equivalent to $\min_{\substack{0\leq y\leq C\\ y\geq x}} \; s_{12}(x-y)^+ + s_{21}(x-y)^- + F(y)$, where we have added the constraint $y\geq x$ and this will not affect the optimality since we know $y^*(x) > x$ for $x\in[0,\underline{x})$. Thus, we can further reduce the problem to

$$\min_{0\leq y\leq C} s_{21}(y-x) + F(y), \tag{1}$$

where under the dummy constraint $y\geq x$, $(x-y)^+ = 0$ and $(x-y)^- = y-x$. Observe that the solution to (1) does not depend on $x$; we denote this solution as $\underline{y}$. That is, $y^*(x) = \underline{y}$ when $x\in[0,\underline{x})$. However, we also know that $y^*(x) = x$, when $x\in[\underline{x},\overline{x}]$. Therefore, by continuity of $y^*(x)$, we must have $\lim_{x\uparrow\underline{x}} y^*(x) = \underline{y} = \underline{x} = y^*(\underline{x})$. The other part when $x\in(\overline{x},C]$ can be shown similarly. This completes the proof of Lemma 1. $\square$

By Lemma 1, $V_t(x_t)$ is convex in $x_t$ for any $t\in[T]$. As a result, $\mathbb{E}_{\mathbb{P}}[J_t(y,\mathbf{d_t})]$ is convex in $y$. Proposition 1 then follows directly from Lemma 1 by letting $F(y) = \mathbb{E}_{\mathbb{P}}[J_t(y,\mathbf{d_t})]$. $\square$

### A.3. Proof of Corollary 1

From Proposition 1, we know that $\underline{x}_t = y^*(s_{21t}) = \arg\min_{0\leq y\leq C}\{s_{21t}y + \mathbb{E}_{\mathbb{P}}[J_t(y,\mathbf{d_t})]\}$ and $\overline{x}_t = y^*(s_{12t}) = \arg\min_{0\leq y\leq C}\{-s_{12t}y + \mathbb{E}_{\mathbb{P}}[J_t(y,\mathbf{d_t})]\}$. Since $s_{21t}y + \mathbb{E}_{\mathbb{P}}[J_t(y,\mathbf{d_t})]$ is supermodular in $s_{21t}$ and $y$ (or submodular in $-s_{21t}$ and $y$), $\underline{x}_t = y^*(s_{21t})$ is decreasing in $s_{21t}$. Similarly, $-s_{12t}y + \mathbb{E}_{\mathbb{P}}[J_t(y,\mathbf{d_t})]$ is submodular in $s_{12t}$ and $y$, thus $\bar{x}_t = y^*(s_{12t})$ is increasing in $s_{12t}$. $\square$

### A.4. Proof of Corollary 2

When $T=1$, the optimization problem becomes $\min_{x-C\leq r\leq x}\{s_{12}r^+ + s_{21}r^- + \bar{p}_1\mathbb{E}_{\mathbb{P}}[(d_1-y)^+] + \bar{p}_2\mathbb{E}_{\mathbb{P}}[(d_2 - C+y)^+]\}$. One can solve the thresholds $\underline{x}$ and $\overline{x}$ as the following two newsvendor type problems: $\underline{x} = \arg\min_{0\leq y\leq C}\{s_{21}y + \bar{p}_1\mathbb{E}_{\mathbb{P}}[(d_1-y)^+] + \bar{p}_2\mathbb{E}_{\mathbb{P}}[(d_2-C+y)^+]\}$ and $\overline{x} = \arg\min_{0\leq y\leq C}\{-s_{12}y + \bar{p}_1\mathbb{E}_{\mathbb{P}}[(d_1-y)^+] + \bar{p}_2\mathbb{E}_{\mathbb{P}}[(d_2-C+y)^+]\}$. Note that by the first-order condition, $\underline{x}_0$ and $\overline{x}_0$ are respectively the unconstrained optimal solution to the above two problems. Since the above two objective functions are convex in $y$, we must have $\underline{x} = \underline{x}_0^+\wedge C$ and $\overline{x} = \overline{x}_0^+\wedge C$.

### A.5. Proof of Proposition 2

This proof follows similar procedures in Bertsimas et al. (2018). First, we show that $\mathbb{F} \subseteq \Pi_{\mathbf{d}}\mathbb{G}$. Suppose $\mathbb{P} \in \mathbb{F}$. We have $\mathbb{P}\left(\left(\mathbf{d}, \{(d_{it}-\mu_{it})^2, \forall i \in [N], t \in [T]\}, \left\{\left(\sum_{l=k}^{t} \mathbf{1}'(\mathbf{d}_l - \boldsymbol{\mu}_l)\right)^2, \forall k,t \in [T], k \le t\right)\right\} \in \bar{\mathcal{W}}\right) = 1$. We can construct a probability distribution $\mathbb{Q} \in \mathcal{P}_0\left(\mathbb{R}^{NT} \times \mathbb{R}^{NT} \times \mathbb{R}^{\frac{T(T+1)}{2}}\right)$ for $(\mathbf{d}, \mathbf{u}, \mathbf{v})$ by letting $\mathbb{P} = \Pi_{\mathbf{d}}\mathbb{Q}$, $u_{it} = (d_{it}-\mu_{it})^2$ for all $i \in [N], t \in [T]$ and $v_{kt} = \left(\sum_{l=k}^{t} \mathbf{1}'(\mathbf{d}_l - \boldsymbol{\mu}_l)\right)^2$ for all $k,t \in [T], k \le t$ $\mathbb{P}$-a.s. One can verify that $u_{it} \le \bar{u}_{it}$ and $v_{kt} \le \bar{v}_{kt}$ also hold, because $\underline{\mathbf{d}} \le \mathbf{d} \le \bar{\mathbf{d}}$. Therefore, $\mathbb{Q}\left((\mathbf{d}, \mathbf{u}, \mathbf{v}) \in \bar{\mathcal{W}}\right) = 1$. Moreover, since $\mathbb{E}_{\mathbb{Q}}(u_{it}) = \mathbb{E}_{\mathbb{P}}((d_{it}-\mu_{it})^2) \le \sigma_{it}^2$ and $\mathbb{E}_{\mathbb{Q}}(v_{kt}) = \mathbb{E}_{\mathbb{P}}\left(\left(\sum_{l=k}^{t} \mathbf{1}'(\mathbf{d}_l - \boldsymbol{\mu}_l)\right)^2\right) \le \gamma_{kt}^2$, we have $\mathbb{F} \subseteq \Pi_{\mathbf{d}}\mathbb{G}$.

Next, we show that $\Pi_{\mathbf{d}}\mathbb{G} \subseteq \mathbb{F}$. For any $\mathbb{Q} \in \mathbb{G}$ and $\mathbb{P}$ is the marginal distribution of $\mathbf{d}$ under any $\mathbb{Q}$, i.e., $\mathbb{P} \in \Pi_{\mathbf{d}}\mathbb{G}$, we have $\mathbb{E}_{\mathbb{P}}(\mathbf{d}) = \mathbb{E}_{\mathbb{Q}}(\mathbf{d}) = \boldsymbol{\mu}$. Also, because $\mathbb{Q}\left((\mathbf{d}, \mathbf{u}, \mathbf{v}) \in \bar{\mathcal{W}}\right) = 1$, we have $\mathbb{Q}\left(\mathbf{d} \in (\underline{\mathbf{d}}, \bar{\mathbf{d}})\right) = 1$, $\mathbb{Q}\left((d_{it}-\mu_{it})^2 \le u_{it}\right) = 1$ for all $i \in [N], t \in [T]$ and $\mathbb{Q}\left(\left(\sum_{l=k}^{t} \mathbf{1}'(\mathbf{d}_l - \boldsymbol{\mu}_l)\right)^2 \le v_{kt}\right) = 1$ for all $k,t \in [T], k \le t$. Hence, we have $\mathbb{P}\left(\mathbf{d} \in (\underline{\mathbf{d}}, \bar{\mathbf{d}})\right) = 1$, $\mathbb{E}_{\mathbb{P}}\left((d_{it}-\mu_{it})^2\right) = \mathbb{E}_{\mathbb{Q}}\left((d_{it}-\mu_{it})^2\right) \le \mathbb{E}_{\mathbb{Q}}(u_{it}) \le \sigma_{it}^2$ and $\mathbb{E}_{\mathbb{P}}\left(\left(\sum_{l=k}^{t} \mathbf{1}'(\mathbf{d}_l - \boldsymbol{\mu}_l)\right)^2\right) = \mathbb{E}_{\mathbb{Q}}\left(\left(\sum_{l=k}^{t} \mathbf{1}'(\mathbf{d}_l - \boldsymbol{\mu}_l)\right)^2\right) \le \mathbb{E}_{\mathbb{Q}}(v_{kt}) \le \gamma_{kt}^2$. Thus, $\mathbf{P} \in \mathbb{F}$. Consequently, we conclude that $\Pi_{\mathbf{d}}\mathbb{G} \subseteq \mathbb{F}$. $\square$

### A.6. Proof of Lemma 2

Given the first stage decision $\mathbf{r}$, we derive $\sup_{\mathbb{P}\in\mathbb{F}} \mathbb{E}_{\mathbb{P}}\left[\bar{p}_i\left(d_i - w_i(\mathbf{d})\right)\right]$. By Proposition 2, we have $\sup_{\mathbb{P}\in\mathbb{F}} \mathbb{E}_{\mathbb{P}}\left[\bar{p}_i\left(d_i - w_i(\mathbf{d})\right)\right] = \sup_{\mathbb{Q}\in\mathbb{G}} \mathbb{E}_{\mathbb{Q}}\left[\bar{p}_i\left(d_i - w_i(\mathbf{d})\right)\right]$, under ambiguity sets $\mathbb{F}$ and $\mathbb{G}$. The second stage worst-case cost can then be calculated by the following infinite dimensional linear program:

$$
\begin{aligned}
\max_{\mathbb{Q}\in\mathbb{G}} \quad & \int_{(\mathbf{d},\mathbf{u},v)\in\bar{\mathcal{W}}} \sum_{i\in[N]} \bar{p}_i\left(d_i - w_i(\mathbf{d})\right) d\mathbb{Q}\left(\mathbf{d},\mathbf{u},v\right) \\
\text{s.t.} \quad & \int_{(\mathbf{d},\mathbf{u},v)\in\bar{\mathcal{W}}} \mathbf{d} d\mathbb{Q}\left(\mathbf{d},\mathbf{u},v\right) = \boldsymbol{\mu} && (\cdots \text{ dual variable } \boldsymbol{\eta} \in \mathbb{R}^N) \\
& \int_{(\mathbf{d},\mathbf{u},v)\in\bar{\mathcal{W}}} u_i d\mathbb{Q}\left(\mathbf{d},\mathbf{u},v\right) \le \sigma_i^2, \forall i \in [N] && (\cdots \text{ dual variable } \theta_i \in \mathbb{R}) \\
& \int_{(\mathbf{d},\mathbf{u},v)\in\bar{\mathcal{W}}} v d\mathbb{Q}\left(\mathbf{d},\mathbf{u},v\right) \le \gamma^2 && (\cdots \text{ dual variable } \delta \in \mathbb{R}) \\
& \int_{(\mathbf{d},\mathbf{u},v)\in\bar{\mathcal{W}}} d\mathbb{Q}\left(\mathbf{d},\mathbf{u},v\right) = 1 && (\cdots \text{ dual variable } \lambda \in \mathbb{R})
\end{aligned}
$$

By the strong duality (Bertsimas et al. 2018), we have the dual formulation as

$$
\begin{aligned}
\min_{\boldsymbol{\theta}\ge\mathbf{0},\delta\ge 0,\lambda,\boldsymbol{\eta}} \quad & \lambda + \boldsymbol{\eta}'\boldsymbol{\mu} + \sum_{i\in[N]} \sigma_i^2\theta_i + \gamma^2\delta \\
\text{s.t.} \quad & \lambda + \boldsymbol{\eta}'\boldsymbol{d} + \sum_{i\in[N]} \theta_i u_i + \delta v \ge \sum_{i\in[N]} \bar{p}_i\left(d_i - w_i(\mathbf{d})\right), \forall (\mathbf{d}, \boldsymbol{u}, v) \in \bar{\mathcal{W}}
\end{aligned}
$$

By $w_i(\mathbf{d}) = d_i \wedge \left(x_i + \sum_{j\in[N]} r_{ji} - \sum_{j\in[N]} r_{ij}\right)$, we have $(d_i - w_i(\mathbf{d})) = \left(d_i - x_i - \sum_{j\in[N]} r_{ji} + \sum_{j\in[N]} r_{ij}\right)^+$. Combining the second stage worst-case cost with the first stage problem, we have the formulation (4). $\square$

### A.7. Proof of Proposition 3

Let $\mathcal{P}(N)$ be the power set of $[N]$. Since $\left(d_i - x_i - \sum_{j\in[N]} r_{ji} + \sum_{j\in[N]} r_{ij}\right)^+$ takes value either $d_i - x_i - \sum_{j\in[N]} r_{ji} + \sum_{j\in[N]} r_{ij}$ or 0, we are able to write the sum of piecewise linear functions as

$$
\sum_{i\in[N]} \bar{p}_i \left(d_i - x_i - \sum_{j\in[N]} r_{ji} + \sum_{j\in[N]} r_{ij}\right)^+ = \max_{S\in\mathcal{P}(N)} \sum_{i\in S} \bar{p}_i \left(d_i - x_i - \sum_{j\in[N]} r_{ji} + \sum_{j\in[N]} r_{ij}\right).
$$

Let $y_i = x_i + \sum_{j\in[N]} r_{ji} - \sum_{j\in[N]} r_{ij}$, the first constraint in problem (4) can then be written as $\lambda + \boldsymbol{\eta}'\mathbf{d} + \sum_{i\in[N]} \theta_i u_i + \delta v \geq \max_{S\in\mathcal{P}(N)} \sum_{i\in S} \bar{p}_i (d_i - y_i), \forall(\mathbf{d}, \boldsymbol{u}, v) \in \bar{\mathcal{W}}$. This is also equivalent to

$$\lambda + \boldsymbol{\eta}'\mathbf{d} + \sum_{i\in[N]} \theta_i u_i + \delta v \geq \sum_{i\in S} \bar{p}_i (d_i - y_i), \forall S \in \mathcal{P}(N), \forall(\mathbf{d}, \boldsymbol{u}, v) \in \bar{\mathcal{W}} \tag{2}$$

We reorganize the terms in the above constraint into: $\lambda + \sum_{i\in S} \bar{p}_i y_i \geq \max_{(\mathbf{d},\boldsymbol{u},v)\in\bar{\mathcal{W}}} \left\{ (\bar{\mathbf{p}}(S) - \boldsymbol{\eta})' \mathbf{d} - \sum_{i\in[N]} \theta_i u_i - \delta v \right\}, \forall S \in \mathcal{P}(N)$. The subproblem on the right-hand-side (RHS) is a SOCP. To see this, we can equivalently rewrite $\bar{\mathcal{W}}$ into a set of second-order conic constraints:

$$\bar{\mathcal{W}} \equiv \left\{ (\mathbf{d}, \mathbf{u}, \mathbf{v}) \in \mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R} \;\middle|\; \begin{array}{l} \underline{\mathbf{d}} \leq \mathbf{d} \leq \bar{\mathbf{d}} \\ \sqrt{(d_i - \mu_i)^2 + \left(\frac{u_i - 1}{2}\right)^2} \leq \frac{u_i + 1}{2}, \ \forall i \in [N] \\ \sqrt{(\mathbf{1}'(\mathbf{d} - \boldsymbol{\mu}))^2 + \left(\frac{v-1}{2}\right)^2} \leq \frac{v+1}{2} \end{array} \right\}.$$

Note that the subproblem $\max_{(\boldsymbol{d},\boldsymbol{u},\boldsymbol{v})\in\bar{\mathcal{W}}} \left\{ (\bar{\mathbf{p}}(S) - \boldsymbol{\eta})' \boldsymbol{d} - \sum_{i\in[N]} \theta_i u_i - \delta v \right\}$ is bounded and there exists an interior point in $\bar{\mathcal{W}}$. Let $\bar{\boldsymbol{\beta}}(S) \in \mathbb{R}^N$, $\underline{\boldsymbol{\beta}}(S) \in \mathbb{R}^N$, $\boldsymbol{\beta}(S) \in \mathbb{R}^N$, $\beta_0(S) \in \mathbb{R}$, $\boldsymbol{\rho}_i(S) \in \mathbb{R}^2$ and $\boldsymbol{\rho}_0(S) \in \mathbb{R}^2$ be the corresponding dual variables. By strong duality, the equivalent dual problem is also a SOCP:

$$\begin{aligned} \min_{\bar{\boldsymbol{\beta}}(S), \underline{\boldsymbol{\beta}}(S) \geq \mathbf{0}} \quad & \bar{\boldsymbol{\beta}}(S)'\bar{\mathbf{d}} - \underline{\boldsymbol{\beta}}(S)'\underline{\mathbf{d}} + \frac{1}{2}\mathbf{1}'\boldsymbol{\beta}(S) + \frac{1}{2}\beta_0(S) - \sum_{i\in[N]} \boldsymbol{\rho}_i(S)'\mathbf{b}_i - \boldsymbol{\rho}_0(S)\mathbf{b}_0 \\ \text{s.t.} \quad & \begin{pmatrix} \boldsymbol{\eta} - \bar{\mathbf{p}}(S) \\ \boldsymbol{\theta} \\ \delta \end{pmatrix} = \begin{pmatrix} \underline{\boldsymbol{\beta}}(S) - \bar{\boldsymbol{\beta}}(S) \\ \mathbf{0} \\ 0 \end{pmatrix} + \sum_{i\in[N]} \left(\mathbf{A}_i'\boldsymbol{\rho}_i(S) + \beta_i(S)\mathbf{c}_i\right) + \mathbf{A}_0'\boldsymbol{\rho}_0(S) + \beta_0\mathbf{c}_0 \\ & \|\boldsymbol{\rho}_i(S)\|_2 \leq \beta_i(S), \forall i \in [N] \\ & \|\boldsymbol{\rho}_0(S)\|_2 \leq \beta_0(S) \end{aligned}$$

where $\mathbf{A}_0 = \begin{bmatrix} \mathbf{1}' & \mathbf{0}' & 0 \\ \mathbf{0}' & \mathbf{0}' & \frac{1}{2} \end{bmatrix}$, $\mathbf{b}_0 = \begin{pmatrix} \mathbf{1}'\boldsymbol{\mu} \\ \frac{1}{2} \end{pmatrix}$, $\mathbf{c}_0 = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \frac{1}{2} \end{pmatrix}$, $\mathbf{A}_i = \begin{bmatrix} \mathbf{e}_i' & \mathbf{0}' & 0 \\ \mathbf{0} & \frac{1}{2}\mathbf{e}_i' & 0 \end{bmatrix}$, $\mathbf{b}_i = \begin{pmatrix} \mu_i \\ \frac{1}{2} \end{pmatrix}$, $\mathbf{c}_i = \begin{pmatrix} \mathbf{0} \\ \frac{1}{2}\mathbf{e}_i \\ 0 \end{pmatrix}, \forall i \in [N]$.

We are now able to replace the maximization problem in constraint (2) with the dual problem. Consequently, constraint (2) is equivalently saying that there exists a feasible solution $\left(\bar{\boldsymbol{\beta}}(S), \underline{\boldsymbol{\beta}}(S), \boldsymbol{\beta}(S), \beta_0(S), \boldsymbol{\rho}_i(S), \boldsymbol{\rho}_0(S)\right)$ satisfying the following constraints for all $S \in \mathcal{P}(N)$:

$$\begin{aligned} & \lambda + \sum_{i\in S} \bar{p}_i y_i \geq \bar{\boldsymbol{\beta}}(S)'\bar{\mathbf{d}} - \underline{\boldsymbol{\beta}}(S)'\underline{\mathbf{d}} + \frac{1}{2}\mathbf{1}'\boldsymbol{\beta}(S) + \frac{1}{2}\beta_0(S) - \sum_{i\in[N]} \boldsymbol{\rho}_i(S)'\mathbf{b}_i - \boldsymbol{\rho}_0(S)\mathbf{b}_0 \\ & \begin{pmatrix} \boldsymbol{\eta} - \bar{\mathbf{p}}(S) \\ \boldsymbol{\theta} \\ \delta \end{pmatrix} = \begin{pmatrix} \underline{\boldsymbol{\beta}}(S) - \bar{\boldsymbol{\beta}}(S) \\ \mathbf{0} \\ 0 \end{pmatrix} + \sum_{i\in[N]} \left(\mathbf{A}_i'\boldsymbol{\rho}_i(S) + \beta_i(S)\mathbf{c}_i\right) + \mathbf{A}_0'\boldsymbol{\rho}_0(S) + \beta_0\mathbf{c}_0 \\ & \|\boldsymbol{\rho}_i(S)\|_2 \leq \beta_i(S), \forall i \in [N] \\ & \|\boldsymbol{\rho}_0(S)\|_2 \leq \beta_0(S) \\ & \bar{\boldsymbol{\beta}}(S), \underline{\boldsymbol{\beta}}(S) \geq \mathbf{0} \end{aligned}$$

By replacing the first constraint in problem (4) with the above, we have the SOCP in the proposition. $\square$

### A.8. Proof of Proposition 4

Before proving Proposition 4, we first introduce several notations. Let $\hat{w}_i(\mathbf{d}) = d_i - w_i(\mathbf{d})$, $\hat{w}_i(\mathbf{d},\mathbf{u},v) = d_i - w_i(\mathbf{d},\mathbf{u},v)$, $y_i = x_i + \sum_{j\in[N]} r_{ji} - \sum_{j\in[N]} r_{ij}$,

$$\begin{aligned}\beta(\mathbf{y}) = \min_{\hat{w}_i(\cdot)} \sup_{\mathbb{P}\in\mathbb{F}} & \mathbb{E}_{\mathbb{P}}\left[\sum_{i\in[N]} \bar{p}_i \hat{w}_i(\mathbf{d})\right] \\ \text{s.t.} \quad & \hat{w}_i(\mathbf{d}) \geq (d_i - y_i)^+, \forall \mathbf{d}\in\mathcal{W}, i\in[N].\end{aligned} \tag{3}$$

and

$$\begin{aligned}\beta^{\text{ELDR}}(\mathbf{y}) = \min_{\hat{w}_i(\cdot)} \sup_{\mathbb{Q}\in\mathbb{G}} & \mathbb{E}_{\mathbb{Q}}\left[\sum_{i\in[N]} \bar{p}_i \hat{w}_i(\mathbf{d},\mathbf{u},v)\right] \\ \text{s.t.} \quad & \hat{w}_i(\mathbf{d},\mathbf{u},v) \geq (d_i - y_i)^+, \forall (\mathbf{d},\mathbf{u},v)\in\bar{\mathcal{W}}, i\in[N], \\ & \hat{\mathbf{w}}(\cdot) \in \bar{\mathcal{L}}^N(\mathbf{d},\mathbf{u},v).\end{aligned} \tag{4}$$

Here $\hat{w}_i(\mathbf{d})$ in $\beta(\mathbf{y})$ refers to any decision rule and $w_i(\mathbf{d},\mathbf{u},v)$ in $\beta^{\text{ELDR}}(\mathbf{y})$ is restricted to ELDR. Then,

$$Z^* = \min_{\sum_{j\in[N]} r_{ij}\leq x_i, r_{ij}\geq 0} \left\{\sum_{i,j\in[N]} s_{ij} r_{ij} + \beta(\mathbf{y})\right\}, \quad Z^{\text{ELDR}} = \min_{\sum_{j\in[N]} r_{ij}\leq x_i, r_{ij}\geq 0} \left\{\sum_{i,j\in[N]} s_{ij} r_{ij} + \beta^{\text{ELDR}}(\mathbf{y})\right\}.$$

In addition, for $i\in[N]$, let

$$\begin{aligned}\beta_i(y_i) = \min_{\hat{w}_i(\cdot)} \sup_{\mathbb{P}_i\in\mathbb{F}_i} & \mathbb{E}_{\mathbb{P}_i}\left[\bar{p}_i \hat{w}_i(d_i)\right] \\ \text{s.t.} \quad & \hat{w}_i(d_i) \geq (d_i - y_i)^+, \forall d_i\in\mathcal{W}_i,\end{aligned}$$

$$\text{where} \quad \mathbb{F}_i = \left\{ \mathbb{P}\in\mathcal{P}_0(\mathbb{R}) \;\middle|\; \begin{array}{l} \mathbb{E}_{\mathbb{P}}(d_i) = \mu_i \\ \mathbb{E}_{\mathbb{P}}\left((d_i-\mu_i)^2\right) \leq \sigma_i^2, \\ \mathbb{P}\left(d_i \in \left(\underline{d}_i, \bar{d}_i\right)\right) = 1 \end{array} \right\}, \quad \text{and} \quad \mathcal{W}_i = [\underline{d}_i, \bar{d}_i], \quad \text{and let}$$

$$\begin{aligned}\beta_i^{\text{ELDR}}(y_i) = \min_{\hat{w}_i(\cdot)} \sup_{\mathbb{Q}_i\in\mathbb{G}_i} & \mathbb{E}_{\mathbb{Q}_i}\left[\bar{p}_i \hat{w}_i(d_i,u_i)\right] \\ \text{s.t.} \quad & \hat{w}_i(d_i,u_i) \geq (d_i - y_i)^+, \forall (d_i,u_i)\in\bar{\mathcal{W}}_i, \\ & \hat{w}_i(\cdot) \in \bar{\mathcal{L}}(d_i,u_i),\end{aligned}$$

$$\text{where} \quad \mathbb{G}_i = \left\{ \mathbb{Q}\in\mathcal{P}_0\left(\mathbb{R}\times\mathbb{R}\right) \;\middle|\; \begin{array}{l} \mathbb{E}_{\mathbb{Q}}(d_i) = \mu_i \\ \mathbb{E}_{\mathbb{Q}}(u_i) \leq \sigma_i^2, \\ \mathbb{Q}\left((d_i,u_i)\in\bar{\mathcal{W}}_i\right) = 1 \end{array} \right\}, \quad \text{with} \quad \bar{\mathcal{W}}_i = \left\{(d,u) \middle|\; \underline{d}_i \leq d_i \leq \bar{d}_i, (d_i-\mu_i)^2 \leq u_i \leq \bar{u}_i\right\}.$$

To show that $Z^{\text{ELDR}} = Z^*$, it is then sufficient to show that $\beta^{\text{ELDR}}(\mathbf{y}) = \beta(\mathbf{y})$ for any $\mathbf{y}$. We establish this via the following two lemmas.

LEMMA 2. *If $\gamma \geq \sqrt{\sum_{i\in[N]}\sigma_i^2}$, then $\beta(\mathbf{y}) = \sum_{i\in[N]}\beta_i(y_i)$ for any $\mathbf{y}$.*

*Proof* First note that $\beta(\mathbf{y}) = \sup_{\mathbb{P}\in\mathbb{F}} \mathbb{E}_{\mathbb{P}}\left[\sum_{i\in[N]}\bar{p}_i(d_i-y_i)^+\right]$ and $\sum_{i\in[N]}\beta_i(y_i) = \sum_{i\in[N]}\sup_{\mathbb{P}_i\in\mathbb{F}_i}\mathbb{E}_{\mathbb{P}_i}\left[\bar{p}_i(d_i-y_i)^+\right]$. We first show that $\beta(\mathbf{y}) \leq \sum_{i\in[N]}\beta_i(y_i)$. Indeed, for any $\mathbb{P}\in\mathbb{F}$, let $\mathbb{P}_i$ be the marginal distribution for $d_i$. By the feasibility of $\mathbb{P}$, we have $\mathbb{E}_{\mathbb{P}_i}(d_i) = \mu_i$, $\mathbb{E}_{\mathbb{P}_i}\left((d_i-\mu_i)^2\right) \leq \sigma_i^2$, and $\mathbb{P}_i\left(d_i\in\left(\underline{d}_i,\bar{d}_i\right)\right) = 1$. That is, $\mathbb{P}_i\in\mathbb{F}_i$. In addition, $\mathbb{E}_{\mathbb{P}}\left[\sum_{i\in[N]}\bar{p}_i(d_i-y_i)^+\right] = \sum_{i\in[N]}\mathbb{E}_{\mathbb{P}_i}\left[\bar{p}_i(d_i-y_i)^+\right] \leq \sum_{i\in[N]}\sup_{\mathbb{P}_i\in\mathbb{F}_i}\mathbb{E}_{\mathbb{P}_i}\left[\bar{p}_i(d_i-y_i)^+\right]$. This shows that $\beta(\mathbf{y}) \leq \sum_{i\in[N]}\beta_i(y_i)$.

On the other hand, for any $\mathbb{P}_i \in \mathbb{F}_i$, we construct a joint distribution $\mathbb{P}$ for $(d_1, ..., d_N)$ as $\mathbb{P}(d_1 \le \omega_1, ..., d_N \le \omega_N) = \prod_{i\in[N]} \mathbb{P}_i(d_i \le \omega_i)$, $\forall \omega_i \in [\underline{d}_i, \bar{d}_i]$. In other word, given the marginal distributions of $d_i, i \in [N]$, we can find a joint distribution with the marginal distributions such that $d_i, i \in [N]$ are independent. To verify $\mathbb{P} \in \mathbb{F}$, note that $\mathbb{E}_{\mathbb{P}}(d_i) = \mu_i$, $\mathbb{E}_{\mathbb{P}}\left((d_i - \mu_i)^2\right) \le \sigma_i^2$, and $\mathbb{P}\left(d_i \in \left(\underline{d}_i, \bar{d}_i\right)\right) = 1$, by the feasibility of $\mathbb{P}_i$. By the independence of $\mathbb{P}_i$, and the assumption that $\gamma \ge \sqrt{\sum_{i\in[N]} \sigma_i^2}$, we have $\mathbb{E}_{\mathbb{P}}\left((\sum_{i\in[N]}(d_i - \mu_i))^2\right) = \sum_{i\in[N]} \mathbb{E}_{\mathbb{P}}\left((d_i - \mu_i)^2\right) = \sum_{i\in[N]} \sigma_i^2 \le \gamma^2$. Hence, $\mathbb{P} \in \mathbb{F}$, and it follows that $\sum_{i\in[N]} \mathbb{E}_{\mathbb{P}_i}[\bar{p}_i(d_i - y_i)^+] = \mathbb{E}_{\mathbb{P}}\left[\sum_{i\in[N]} \bar{p}_i(d_i - y_i)^+\right] \le \sup_{\mathbb{P}\in\mathbb{F}} \mathbb{E}_{\mathbb{P}}\left[\sum_{i\in[N]} \bar{p}_i(d_i - y_i)^+\right]$, i.e., $\sum_{i\in[N]} \beta_i(y_i) \le \beta(\mathbf{y})$. □

LEMMA 3. *For any $y_i$, $\beta_i(y_i) = \beta_i^{ELDR}(y_i)$.*

*Proof* Theorem 4 in Bertsimas et al. (2018) shows that ELDR can obtain the optimal objective value for linear DRO problem with only one second stage decision variable. Recall that $y_i$ is the only second stage decision variable in $\beta_i(y_i)$, ELDR can achieve the same optimal objective value. □

We now complete the proof of Proposition 4. Let $\hat{w}_i^*(d_i, u_i) = w_i^0 + w_i^1 d_i + w_i^2 u_i$ be the optimal solution in solving $\beta_i^{\text{ELDR}}(y_i)$. Let $\hat{w}_i^*(\mathbf{d}, \mathbf{u}, v) = w_i^0 + w_i^1 d_i + w_i^2 u_i$. Clearly, $\hat{w}_i^*(\mathbf{d}, \mathbf{u}, v) \in \bar{\mathcal{L}}(\mathbf{d}, \mathbf{u}, v)$ since the linear coefficients corresponding to the variables $d_j, u_j, j \ne i$ and $v$ are simply zero. In addition, for any $i \in [N]$, $\hat{w}_i^*(\mathbf{d}, \mathbf{u}, v) \ge (d_i - y_i)^+, \forall (\mathbf{d}, \mathbf{u}, v) \in \bar{\mathcal{W}}$ by the feasibility of $\hat{w}_i^*(d_i, u_i)$. Hence, $\hat{w}_i^*(\mathbf{d}, \mathbf{u}, v)$ is a feasible solution to (4). It follows that $\beta^{\text{ELDR}}(\mathbf{y}) \le \sup_{\mathbb{Q}\in\mathbb{G}} \mathbb{E}_{\mathbb{Q}}\left[\sum_{i\in[N]} \bar{p}_i \hat{w}_i^*(\mathbf{d}, \mathbf{u}, v)\right]$. For any $\mathbb{Q} \in \mathbb{G}$, let $\mathbb{Q}_i$ be the marginal distribution of $(d_i, u_i)$. It is easy to see that $\mathbb{Q}_i \in \mathbb{G}_i$. It then follows that $\mathbb{E}_{\mathbb{Q}}\left[\sum_{i\in[N]} \bar{p}_i \hat{w}_i^*(\mathbf{d}, \mathbf{u}, v)\right] = \sum_{i\in[N]} \mathbb{E}_{\mathbb{Q}_i}[\bar{p}_i \hat{w}_i^*(d_i, u_i)] \le \sum_{i\in[N]} \sup_{\mathbb{Q}_i\in\mathbb{G}_i} \mathbb{E}_{\mathbb{Q}_i}[\bar{p}_i \hat{w}_i^*(d_i, u_i)] = \sum_{i\in[N]} \beta_i^{\text{ELDR}}(y_i)$. Thus, $\beta^{\text{ELDR}}(\mathbf{y}) \le \sup_{\mathbb{Q}\in\mathbb{G}} \mathbb{E}_{\mathbb{Q}}\left[\sum_{i\in[N]} \bar{p}_i \hat{w}_i^*(\mathbf{d}, \mathbf{u}, v)\right] \le \sum_{i\in[N]} \beta_i^{\text{ELDR}}(y_i)$. By Lemma 2 and Lemma 3, we know that for $\gamma \ge \sqrt{\sum_{i\in[N]} \sigma_i^2}$, we have $\beta(\mathbf{y}) = \sum_{i\in[N]} \beta_i(y_i) = \sum_{i\in[N]} \beta_i^{\text{ELDR}}(y_i)$. Therefore, $\beta^{\text{ELDR}}(\mathbf{y}) \le \beta(\mathbf{y})$. On the other hand, for any $\hat{\mathbf{w}}(\cdot) \in \bar{\mathcal{L}}^N(\mathbf{d}, \mathbf{u}, v)$, it must also be a feasible solution to (3). Hence, $\beta^{\text{ELDR}}(\mathbf{y}) \ge \beta(\mathbf{y})$, which then implies $\beta^{\text{ELDR}}(\mathbf{y}) = \beta(\mathbf{y})$. Therefore, we have $Z^{\text{ELDR}} = Z^*$. □

### A.9. Proof of Robust Repositioning Policy in a 2-Region System

Consider a 2-region system with regions 1 and 2. For any period $t \in [T]$, we denote $r_{12t}$ as the vehicle repositioning from region 1 to 2 and $r_{21t}$ from region 2 to 1. Similar to Section 3.1, let $x_t$ be the available vehicles at region 1 at the beginning of period $t$. The rest of the variables follow from the previous discussions. Our robust model for the 2-region system can be formulated as below.

$$\begin{aligned} \min_{\substack{0\le r_{121}\le x_1\\ 0\le r_{211}\le C-x_1}} \quad & s_{121}r_{121} + s_{211}r_{211} + F(y_1) \\ \text{s.t.} \quad & y_1 = x_1 - r_{121} + r_{211} \end{aligned} \tag{5}$$

where $F(y_1)$ is directly from formulation (7) by setting $N = 2$.

By Lemma 3, we can write $F(y_1)$ as the optimal value to the following infinite dimensional linear program:

$$\begin{aligned} \min_{\substack{\boldsymbol{\theta},\boldsymbol{\delta}\ge\mathbf{0},\lambda,\boldsymbol{\eta}\\ \mathbf{x}_{t+1}^0,\mathbf{x}_{il(t+1)}^1,\mathbf{x}_{il(t+1)}^2,\mathbf{x}_{kl(t+1)}^3\\ \mathbf{r}_{t+1}^0,\mathbf{r}_{il(t+1)}^1,\mathbf{r}_{il(t+1)}^2,\mathbf{r}_{kl(t+1)}^3\\ \mathbf{w}_t^0,\mathbf{w}_{ilt}^1,\mathbf{w}_{ilt}^2,\mathbf{w}_{klt}^3}} \quad & \lambda + \boldsymbol{\eta}'\boldsymbol{\mu} + \sum_{\substack{i\in[2]\\ t\in[T]}} \sigma_i^2\theta_i + \sum_{\substack{k,t\in[T]\\ k\le t}} \gamma_{kt}^2\delta_{kt} \\ \text{s.t.} \quad & \text{Constraints in (7)} \\ & \lambda + \boldsymbol{\eta}'\boldsymbol{d} + \sum_{\substack{i\in[2]\\ t\in[T]}} \theta_{it}u_{it} + \sum_{\substack{k,t\in[T]\\ k\le t}} \delta_{kt}v_{kt} \ge \sum_{\substack{i\in[2]\\ t\in[T]}} \bar{p}_{it}\left(d_{it} - w_{it}(\cdot)\right) + \sum_{\substack{t\in[T-1]\\ i,j\in[2]}} s_{ij(t+1)}r_{ij(t+1)}(\cdot), \forall(\boldsymbol{d},\boldsymbol{u},\boldsymbol{v}) \in \bar{\mathcal{W}}. \end{aligned}$$

All constraints in the above program are linear and hence the constraint set is convex. In addition, the objective is linear, which is then jointly convex in all decision variables and $y_1$. As a result, $F(y_1)$ is convex in $y_1$ (see, for example, Proposition 2.2.15 in Simchi-Levi et al. 2005). Using Lemma 1 in the Appendix, we then have the following result on the structure of the robust repositioning policy.

Consider a 2-region system. For any $x_1$, let $r^*_{121}, r^*_{211}$ be the optimal solution to (5), $r_1^{\mathrm{R}}(x_1) = r^*_{121} - r^*_{211}$ and $y_1^{\mathrm{R}}(x_1) = x_1 - r_1^{\mathrm{R}}(x_1)$, then there exist $\underline{x}_1$ and $\overline{x}_1$ such that

$$r_1^{\mathrm{R}}(x_1) = \begin{cases} x_1 - \underline{x}_1, & x_1 \in [0, \underline{x}_1), \\ 0, & x_1 \in [\underline{x}_1, \overline{x}_1], \\ x_1 - \overline{x}_1, & x \in (\overline{x}_1, C], \end{cases} \quad \text{and} \quad y_1^{\mathrm{R}}(x_1) = \begin{cases} \underline{x}_1, & x \in [0, \underline{x}_1), \\ x_1, & x_1 \in [\underline{x}_1, \overline{x}_1], \\ \overline{x}_1, & x_1 \in (\overline{x}_1, C]. \end{cases}$$

where $\underline{x}_1$ and $\overline{x}_1$ are solutions to the following two convex programs

$$\underline{x}_1 = \arg\min_{0 \le y \le C} \{s_{211} y + F(y)\}, \quad \overline{x}_1 = \arg\min_{0 \le y \le C} \{-s_{121} y + F(y)\}.$$

The result confirms the desirable structural properties of our robust repositioning policy, even without the assumption of demand temporal independence.

## Appendix B: Data and Setup for Numerical Studies

### B.1. The MVP Formulation

The MVP is solved as the following linear program:

$$\begin{aligned} \min_{r_{ijt} \ge 0} \quad & \sum_{i \in [N]} \left( \sum_{j \in [N]} s_{ijt} r_{ijt} + \bar{p}_{it} \left( \mu_{it} - w_{it} \right) \right) \\ \text{s.t.} \quad & x_{i(t+1)} = x_{it} + \sum_{j \in [N]} \left( \alpha_{jit} w_{jt} + r_{jit} \right) - \sum_{j \in [N]} \left( \alpha_{ijt} w_{it} + r_{ijt} \right), \forall i \in [N], t \in [T] \\ & \sum_{j \in [N]} r_{ijt} \le x_{it}, \forall i \in [N], t \in [T] \\ & w_{it} \le \mu_{it} \wedge \left( x_{it} + \sum_{j \in [N]} r_{jit} - \sum_{j \in [N]} r_{ijt} \right), \forall i \in [N], t \in [T]. \end{aligned}$$

### B.2. Parameters for 2-Region System

The trip distribution, lost sales penalty per trip, and repositioning cost per trip are

$$(\alpha_{ijt}) = \begin{bmatrix} 0.87 & 0.13 \\ 0.72 & 0.28 \end{bmatrix}, (p_{ijt}) = \begin{bmatrix} 15.34 & 12.58 \\ 15.05 & 9.60 \end{bmatrix}, (s_{ijt}) = \begin{bmatrix} 13.81 & 11.33 \\ 13.55 & 8.63 \end{bmatrix}, \forall t \in [T].$$

### B.3. Sample Data of car2go San Diego

Here we provide 10 lines of sample vehicle status data that record the vehicle ID, address, GPS coordinates and time stamp. We then use the vehicle status data to track the movement of each vehicle and identify trips by checking the changes of locations as well as the trip durations.

### B.4. Region Clustering

Based on the inter-region travel intensities, we cluster the zip codes into 5 regions as below:

**Table 1 Sample Data of Vehicle Status in car2go San Diegon**

| Vehicle ID | Address | Coordinates | TimeGMT+8 |
|---|---|---|---|
| 6UK E956 | India St 2166, 92101 | [-117.16968,32.72708,0] | 23/3/14 19:41 |
| 6UK F014 | Collier Ave 3490, 92116 | [-117.11775,32.76548,0] | 23/3/14 19:41 |
| 6RF N765 | India St 3209, 92103 | [-117.17541,32.73672,0] | 23/3/14 19:41 |
| 6UK C946 | Park Blvd 4113, 92103 | [-117.14623,32.75251,0] | 23/3/14 19:41 |
| 6UK E958 | Mission Blvd 3822, 92109 | [-117.25317,32.7852,0] | 23/3/14 19:41 |
| 6UK F000 | Camino de la Reina 663, 92108 | [-117.15991,32.76641,0] | 23/3/14 19:41 |
| 6TK Z247 | Alvarado Rd 6329, 92120 | [-117.06368,32.77833,0] | 23/3/14 19:41 |
| 6UK E953 | Mission Center Ct 7820, 92108 | [-117.15555,32.77348,0] | 23/3/14 19:41 |
| 6RF N728 | Columbia St 2688, 92103 | [-117.17116,32.73238,0] | 23/3/14 19:41 |
| 6RF N708 | Sunset St 2638, 92110 | [-117.19451,32.75549,0] | 23/3/14 19:41 |

**Table 2 Region Clustering for car2go San Diego**

| Region | Zip Codes |
|---|---|
| 1 | 92101, 92102, 92134, 92132 |
| 2 | 92103, 92104, 92105 |
| 3 | 92108, 92110, 92116 |
| 4 | 92106, 92107 |
| 5 | 92109 |

### B.5. Ambiguity Sets for Section 5.3

We estimate the correlation coefficient $\rho_{ikjt}$ between the demand in region $i$ at time $k$ and the demand in region $j$ at time $t$. The ambiguity sets $\mathbb{F}_1$ and $\mathbb{F}_2$ are formulated as

$$\mathbb{F}_1 = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{NT}) \;\middle|\; \begin{array}{ll} \mathbb{E}_{\mathbb{P}}(\mathbf{d}) = \boldsymbol{\mu} & \\ \mathbb{E}_{\mathbb{P}}\left((d_{it}-\mu_{it})^2\right) \le \sigma_{it}^2, & \forall i \in [N], t \in [T] \\ \mathbb{E}_{\mathbb{P}}\left(\left(\sum_{l=k}^{t} \mathbf{1}'(\mathbf{d}_l - \boldsymbol{\mu}_l)\right)^2\right) \le \gamma_{kt}^2, & \forall k, t \in [T], k \le t \\ \mathbb{E}_{\mathbb{P}}\left((d_{ik}-\mu_{ik})(d_{jt}-\mu_{jt})\right) \le \phi_{ikjt}^2, & \forall (i,k,j,t) \in \{(i,k,j,t) | \rho_{ikjt} \ge 0.5\} \\ \mathbb{P}\left(\mathbf{d} \in (\underline{\mathbf{d}}, \bar{\mathbf{d}})\right) = 1 & \end{array} \right\},$$

and

$$\mathbb{F}_2 = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{NT}) \;\middle|\; \begin{array}{ll} \mathbb{E}_{\mathbb{P}}(\mathbf{d}) = \boldsymbol{\mu} & \\ \mathbb{E}_{\mathbb{P}}\left((d_{it}-\mu_{it})^2\right) \le \sigma_{it}^2, & \forall i \in [N], t \in [T] \\ \mathbb{E}_{\mathbb{P}}\left(\left(\sum_{l=k}^{t} \mathbf{1}'(\mathbf{d}_l - \boldsymbol{\mu}_l)\right)^2\right) \le \gamma_{kt}^2, & \forall k, t \in [T], k \le t \\ \mathbb{E}_{\mathbb{P}}\left((d_{ik}-\mu_{ik})(d_{jt}-\mu_{jt})\right) \le \phi_{ikjt}^2, & \forall i,j \in [N], k,t \in [T] \\ \mathbb{P}\left(\mathbf{d} \in (\underline{\mathbf{d}}, \bar{\mathbf{d}})\right) = 1 & \end{array} \right\}.$$

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