

SEMINAR EXERCISES 3.

1. A thief has a bunch of n keys in which exactly one of which fits a lock. The thief tries to open the lock by trying the keys at random, what is the probability distribution involved in that attempt? Why?
2. In a precision bombing attack there is a 30% chance that anyone bomb will strike the target. What are the probability distributions involved in the following scenarios if:
 - a. One direct hit is required to destroy the target completely? Why?
 - b. Two direct hits are required to destroy the target completely? Why?
3. Describe the probability model from which the binomial distribution can be generated. Determine the binomial distribution for which the mean is 5 and variance 3
4. The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers chosen at random, four or more will suffer from the disease?
5. In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution.
6. With the usual notations, find p for a binomial random variable X if $n = 6$ and $9P(X = 4) = P(X = 2)$.
7. Red blood cell deficiency may be determined by examining a specimen of the blood under a microscope. Suppose a certain small fixed volume contains on the average 20 red cells for normal persons. Find the probability that a specimen from a normal person will contain less than 15 red cells.
8. During war 1 ship out of 9 was sunk on an average in making a certain voyage. What was the probability that exactly 3 out of a convoy of 6 ships would arrive safely?
9. In the long run 3, vessels out of every 100 are sunk. If 10 vessels are out, what is the probability that (i) exactly 6 will arrive safely, and (ii) at least 6 will arrive safely?
10. A student takes a true-false examination consisting of 10 questions. He is completely unprepared so he plans to guess each answer. The guesses are to be made at random. For example, he may toss a fair coin and use the outcome to determine his guess. (i) Compute the probability that he guesses correctly at least five times (ii) Compute the probability that he

guesses correctly at least 9 times. (iii) What is the smallest n that the probability of guessing at-least n correct answers is less than $\frac{1}{2}$

11. A scientist needs three diseased rabbits for an experiment. He has 20 rabbits available and inoculates them one at a time with a serum, quitting if and when he gets 3 positive reactions. If the probability is 0.25 that a rabbit can contract the disease from the serum, what is the probability that the scientist is able to get 3 diseased rabbits from 20?

Use: $\binom{x+r-1}{r-1} = \binom{-r}{x}$

12. A student has taken a 5-answer multiple choice examination orally. He continues to answer questions until he gets five correct answers. What is the probability that he gets them on or before the twenty-fifth question if he guesses at each answer?
13. In a series of independent trials with constant probability p of success in each trial, show that the number of successes in a fixed number n of independent trials follows a binomial distribution. Show further that the number of the trials required for a specified number r of successes follows a negative binomial distribution. Obtain the mean and the variance of this distribution
14. If the probability that a target is destroyed on anyone shot is 0.5, what is the probability that it would be destroyed on 6th attempt?
15. A couple decides to have children until they have a male child. What is the probability distribution of the number of children they would have? If the probability of a male child in their community is $\frac{1}{3}$, how many children are they expected to have before the first-place child is born?
16. A taxi cab company have 12 Ambassadors and 8 Fiats. If 5 of these taxi cabs are in the shop for repairs and Ambassador is as likely to be in for repairs as a fiat. what is the probability that (i) 3 of them are Ambassadors and 2 are fiats? (ii) at least 3 of them are Ambassadors? And (iii) all 5 of them are of the same make?
17. Find the probability that the income-tax official will catch 3 income-tax turns with illegitimate deductions, if he randomly selects 5 returns from among 12 returns of which 6 contain illegitimate deductions.

18. Four tickets marked 00,01,10, 11 respectively are placed in a bag. A ticket is drawn at random five times, being replaced each time. Find the probability that the sum of the numbers on the tickets thus drawn is 23. (Use: Multinomial Distribution Concept)
19. In a game of taking a chance, a contestant has to give correct answers to 4 out of 5 questions to win the contest. Questions are given with 3 answers each, out of which one is a correct answer. If a contestant answers the questions by selecting the answers at random, what is the probability that he will win the contest?
20. In a university examination of a particular year, 60% of the students failed when mean and s.d. of the marks were 50% and 5% respectively. The University decided to relax the conditions of passing by lowering the pass marks, to show its result 70%. Find the minimum marks for a student to pass, supposing the marks to be normally distributed and no change of the students' performance took place. Suppose there were 800 students, how many students passed in the both circumstances?
21. X is normally distributed and the mean of X is 12 and S.D. is 4. (a) Find out the probability of the following: (a) (i) $X \geq 20$ (ii) $X \leq 20$ (iii) $0 \leq X \leq 20$ (iv) $|x - 12| > 5$ (b) Find x' , when $P(X > x') = 0.24$ (c) Find x'_0 and x'_1 , when $P(x'_0 \leq X \leq x'_1) = .50$ and $P(X \geq x'_1) = 0.25$
22. (a) If $\log_{10} X$ is normally distributed with mean 4 and variance 4, find the probability of $1.202 \leq X \leq 83180000$. (b) Of a large group of men, 5% are under 60 inches in height and 40% are between 60 and 65 inches. Assuming a normal distribution, find the mean height and standard deviation.
23. If the skulls are classified as A, B and C according as the length-breadth index is under 75, between 75 and 80, or over 80, find approximately (assuming that the distribution is normal) the mean and standard deviation of a series in which A are 58%, B are 38% and C are 4%, being given that if

$$f(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-x^2/2} dx$$

Then $f(0.2) = 0.08$ and $f(1.75) = 0.46$

24. The marks obtained by a number of students for a certain subject are assumed to be approximately normally distributed with mean value 65 and with a standard deviation of 5. If 3 students are taken at random from this set what is the probability that exactly 2 of them will have marks over 70?
25. In an examination it is laid down that a student passes if he secures 30 per cent or more marks. He is placed in the first, second or third division according as he secures 60% or more marks, between 45% and 60% marks and marks between 30% and 45% respectively. He gets distinction in case he secures 80% or more marks. It is noticed from the result that 10% of the students failed in the examination, whereas 5% of them obtained distinction. Calculate the percentage of students placed in the first, second and third divisions. (Assume normal distribution of marks.)
26. The local authorities in a certain city install 10,000 electric lamps in the streets of the city. If these lamps have on average life of 1,000 burning hours with a standard deviation of 200 hours. Assuming normality, what number of lamps might be expected to fail (i) in the first 800 burning hours? (ii) between 800 and 1,200 burning hours? After what period of burning hours would you expect that (a) 10000 lamps would fail? (b) 10000 lamps would be still burning?
27. Explain how the following normal probability integrals are used in compute normal probabilities

$$f(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-x^2/2} dx \quad \text{and} \quad f(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-x^2/2} dx$$

28. Given that X is normally distributed with 10 and $P(X > 12) = 0.1587$. what is the probability that X will fall in the interval (9, 11)? Take $\varphi(1) = 0.8413$ and $\varphi(-.5) = 0.1587$ where

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-u^2/2} du$$