

Seminar exercises

1. (a) If $(A \cap B) = \emptyset$, then show that $P(A) \leq P(B')$
(b) Let A and B be two events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{3}{5}$. Show that
$$P(A \cup B) \geq \frac{3}{5} \quad \text{and} \quad \frac{1}{10} \leq P(A \cap B) \leq \frac{3}{5}$$

(c) Given two events A and B. If the odds against A are 2 to 1 and those in favour of $(A \cup B)$ are 3 to 1. Show that $\frac{5}{12} \leq P(B) \leq \frac{3}{4}$
(d) If A and B are two independent events show that $p(A \cup B) = 1 - P(A')P(B')$
2. Let A_1, A_2 and A_3 , be three independent events for which $P(A_1) = p$, $P(A_2) = q$ and $P(A_3) = r$. Find the probability that (i) at least one of the events occurs, (ii) exactly two of the events occur, and (iii) at most three of the events occur.
3. A language class has only three students A, B, C and they independently attend the class. The probabilities of attendance of A, B and C on any given day are $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ respectively. Find the probability that the **total number of attendances in two consecutive days is exactly three.**
4. Suppose the events A_1, A_2, \dots, A_n are independent and that $P(A_i) = \frac{1}{i+1}, i = 1, 2, \dots, n$. Show that the probability that none of the **n events** occur is $\frac{1}{n+1}$.
5. The chances that doctor A **will diagnose a disease X correctly** is 60%. The chances that a **patient will die by his treatment after correct diagnosis** is 40% and the **chance of death by wrong diagnosis** is 70%. A **patient of doctor A, who had disease X died**. What is **the chance that his disease was diagnosed correctly?** (*Hint: use conditional probabilities*)
6. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5%, 4% and 2 percent are defective bolts. A bolt is drawn at random from the product and **is found to be defective**. What are the probabilities that it was manufactured by machines A, B and C? (*Hint: use Bayes' theorem*)