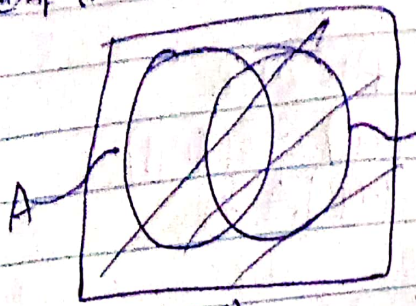
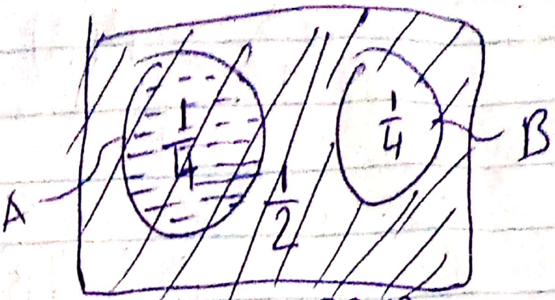
SEMINAR SHEET 2

1. (a) If $(A\cap B)=\emptyset$, then show that $P(A)\leq P(B')$





$P(A)\leq P(B')$

$P(A)=\frac{1}{4}$ $\quad P(B')=1-\frac{1}{4}=\frac{3}{4}$

$\frac{1}{4}\leq\frac{3}{4}$

$\therefore$ Hence shown

$P(A)=P(A\cap B)+P(A\cap B')$

$P(A)=0+P(A)\cdot P(B')$

$\frac{P(A)}{P(A)}=\frac{P(A)\cdot P(B')}{P(A)}$

$P(B')=1$

Area A is within Area of B'

(b) $P(A)=\frac{1}{2}$, $P(B)=\frac{3}{5}$, Show that $P(A\cup B)\geq\frac{3}{5}$ and $\frac{1}{10}\leq P(A\cap B)\leq\frac{3}{5}$

from; $B < A\cup B$

$P(B)\leq P(A\cup B)$

$\frac{3}{5}\leq P(A\cup B)$

$P(A\cup B)\geq\frac{3}{5}$

$\therefore$ Hence shown

$A\cap B < B$

$P(A\cap B)\leq P(B)$

$A\cup B < M$

$P(A\cup B)\leq P(M)$

$P(A\cup B)\leq 1$

$P(A)+P(B)-P(A\cap B)\leq 1$

$\frac{1}{2}+\frac{3}{5}-P(A\cap B)\leq 1$

$\frac{1}{2}+\frac{3}{5}-1\leq P(A\cap B)$

$\therefore \frac{1}{10}\leq P(A\cap B)\leq\frac{3}{5}$

$\cancel{B < A\cup B}$

$\cancel{P(B)\leq P(A\cup B)}$

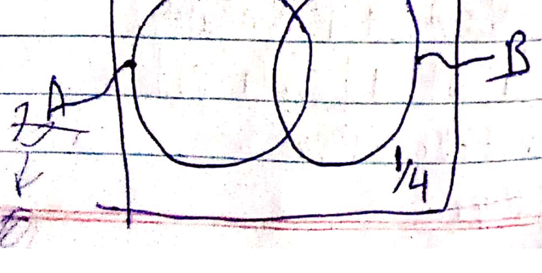
$\cancel{P(B)\leq 3/4}$

$B\geq A\cap B$

$P(B)\geq P(A\cap B)$

$\frac{3}{5}\geq P(A\cup B)$

(c) $P(A')=\frac{2}{3}$, $P(A)=\frac{1}{3}$, $P(A\cup B)=\frac{3}{4}$, $P(A\cup B)'=\frac{1}{4}$



$P(B\text{ only})=P(A')-P(A\cup B)'$

$P(B\text{ only})=\frac{2}{3}-\frac{1}{4}$

$P(B\text{ only})=5/12$

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(c) $P(B\text{ only}) \leq P(B)$

$\frac{5}{12} \leq P(B)$

$B < A \cup B$

$P(B) \leq P(A \cup B)$

$P(B) \leq \frac{3}{4}$

$$\frac{5}{12} \leq P(B) \leq \frac{3}{4}$$

(d) $P(A \cup B) = 1 - P(A')P(B')$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

~~$P(A \cup B) = P(A) + P(B)$~~ $-$ $P(A \cup B) = (1 - P(A')) + (1 - P(B')) - P(A \cap B)$

$P(A \cup B) = 2 - P(A') - P(B') - [(1 - P(A'))(1 - P(B'))]$

$P(A \cup B) = 2 - P(A') - P(B') - [1 - P(A') - P(B') + P(A')P(B')]$

$P(A \cup B) = 2 - P(A') - P(B') - 1 + P(A') + P(B') - P(A')P(B')$

$P(A \cup B) = 1 - P(A')P(B')$

$\therefore$ Hence shown

2. $P(A_1) = P$, $P(A_2) = q$ and $P(A_3) = r$

(i) Atleast one of the events occur $= 1 - P(A_1' \cap A_2' \cap A_3')$

$P(A_1') = 1 - P$, $P(A_2') = 1 - q$, $P(A_3') = 1 - r$

$1 - [(1 - P)(1 - q)(1 - r)]$

$1 - [(1 - q - P + Pq)(1 - r)]$

$1 - [1 - q - P + Pq - r + rq + Pr - Pqr]$

$1 - 1 + q + P - Pq + r - rq - Pr + Pqr$

$\therefore$ Atleast one of the events occur $= q + P + r - Pq - rq - Pr + Pqr$

(ii) Exactly two of the events occur

$P(A_1 \cap A_2 \cap A_3') + P(A_1 \cap A_2' \cap A_3) + P(A_1' \cap A_2 \cap A_3)$

$[Pq(1 - r)] + [Pr(1 - q)] + [qr(1 - P)]$

$Pq - Pqr + Pr - Pqr + qr - Pqr$

$= Pq + Pr + qr - 3Pqr$

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(iii) Atmost three of the events occur. $= 1$

PEA

3. $A = \frac{1}{2}$ $B = \frac{1}{3}$ $C = \frac{3}{4}$

Day 1 = 0

$P(A' \cap B' \cap C') = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{1}{4}\right) = \frac{1}{12}$

DAY 2 = 3

$P(A \cap B \cap C) = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{3}{4}\right) = \frac{1}{8}$

$\frac{1}{12} \times \frac{1}{8} = \frac{1}{96} \times 2 = \frac{1}{48}$

~~Day 1 = 3 = $\frac{1}{8}$~~ ~~Day 2 = 0~~

~~$\frac{1}{8} \times \frac{1}{12} = \frac{1}{96} \times$~~

Day 1; 1

$P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$\left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}\right)$

$\frac{1}{12} + \frac{1}{24} + \frac{1}{4} = \frac{2+1+6}{24} = \frac{9}{24} = \frac{3}{8}$

Day 2; 2.

$P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C)$

$\left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}\right)$

$\frac{1}{24} + \frac{1}{4} + \frac{1}{8} = \frac{1+6+3}{24} = \frac{10}{24} = \frac{5}{12}$

$\frac{3}{8} \times \frac{5}{12} = \frac{15}{96} \times 2 = \frac{15}{48}$

$\frac{1}{48} + \frac{15}{48} = \frac{16}{48} = \frac{1}{3}$

$\therefore$ The probability that the total number of attendances in two consecutive days is exactly three is $\frac{1}{3}$.

4. $P(A_i) = \frac{1}{i+1}$ $i = 1, 2, ----, n$

for $n = 1$

$\cancel{P(A_1)} = \frac{1}{\cancel{1+1}}$ $=$ $P(A_i') = 1 - \frac{1}{i+1}$

$P(A_i') = \frac{i+1-1}{i+1} = \frac{i}{i+1}$

for $i = 1$ $\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{8} \times \cdots\cdots , \frac{\cancel{n-2}}{\cancel{n-1}} \times \frac{\cancel{n-1}}{\cancel{n}} \times \frac{\cancel{n}}{n+1}$

$i = 2$ $\frac{2}{3}$

$i = 3$ $\frac{3}{}$

$\frac{1}{n+1}$

$\therefore$ Hence shown

5. Let E_1 = Chances that doctor A will diagnose a disease X correctly

E_2 = Patient will die

$P(E_1) = 0.6$

$P\left(\frac{E_2}{E_1'}\right) = 0.7$

$P\left(\frac{E_2}{E_1}\right) = 0.4$

$P\left(\frac{E_1}{E_2}\right) = ?$

$P\left(\frac{E_2}{E_1'}\right) = \frac{P(E_2 \cap E_1')}{P(E_1')}$

$0.7 \times 0.4 = P(E_2 \cap E_1')$

$P(E_2 \cap E_1') = 0.28$

$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_2 \cap E_1)}{P(E_1)}$

$0.4 \times 0.6 = P(E_2 \cap E_1)$

$P(E_2 \cap E_1) = 0.24$

$P(E_2) = P(E_2 \cap E_1') + P(E_2 \cap E_1)$

$P(E_2) = 0.28 + 0.24 = 0.52$

$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

$P\left(\frac{E_1}{E_2}\right) = \frac{0.24}{0.52} = \frac{6}{13}$

6. ~~Prior~~ ~~Likelihood~~

Let $E_1 \rightarrow$ factory machine A $A \rightarrow$ Defective bolts

$E_2 \rightarrow$ " " B

$E_3 \rightarrow$ " " C

Prior	Likelihood	Product
$P(E_1) = 0.25$	$P(A/E_1) = 0.05$	0.0125
$P(E_2) = 0.35$	$P(A/E_2) = 0.04$	0.014
$P(E_3) = 0.4$	$P(A/E_3) = 0.02$	0.008
		0.0345

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum P(E_i) \cdot P(A/E_i)}$$

$$P(E_1/A) = \frac{0.0125}{0.0345} = \frac{25}{69}$$

$$P(E_2/A) = \frac{0.014}{0.0345} = \frac{28}{69}$$

$$P(E_3/A) = \frac{0.008}{0.0345} = \frac{16}{69}$$

SEMINAR SHEET 3

1. It is geometric probability distribution

$P(X=x) = q^x p$

∴ It has one number of success.

2. (a) (i) Geometric probability distribution

$P(X=x) = q^x p$

(ii) Negative binomial probability distribution

$P(X=x) = \binom{x+r-1}{r-1} \cdot p^r \cdot q^x$

$x \rightarrow$ Number of failure

$r \rightarrow$ Number of success

$n \rightarrow$ Total numbers of trial $n = x + r$

3. Mean $= np = 5$

Variance $= npq = 3$ but $np = 5$

$\frac{5q}{5} = \frac{3}{5}$

$q = 3/5$ $n = 5/p$ but $p = 1 - q$ $p = 2/5$

$n = \frac{5}{2/5} = \frac{5}{1} \times \frac{5}{2} = \frac{25}{2} = 12.5 \simeq 13$

Binomial model $X \sim B(n, p)$

$X \sim B(13, 2/5)$.

Binomial probability distribution:

$\therefore P(X=x) = {}^{13}C_x \left(\frac{2}{5}\right)^x \left(\frac{3}{5}\right)^{13-x}$

4. It is binomial probability distribution

$P(X=x) = {}^nC_x p^x q^{n-x}$

but $p = 0.2$ $q = 1 - p = 0.8$ $n = 6$

$P(x \geq 4) = P(x=4) + P(x=5) + P(x=6)$

$P(x \geq 4) = {}^6C_4 (0.2)^4 (0.8)^2 + {}^6C_5 (0.2)^5 (0.8)^1 + {}^6C_6 (0.2)^6 (0.8)^0$

$P(x \geq 4) = 0.01536 + 0.001536 + 0.000064$

$\therefore P(x \geq 4) = 0.01696$

5 $P(x=1) = 0.4096$

$P(x=2) = 0.2048$

$P(x=1) = {}^5C_1 pq^4 = 0.4096$

$P(x=2) = {}^5C_2 p^2q^3 = 0.2048$

$5pq^4 = \frac{0.4096}{5}$

$\frac{10p^2q^3}{10} = \frac{0.2048}{10}$

$pq^4 = 0.08192 \text{ --- (i)}$

$p^2q^3 = 0.02048 \text{ --- (ii)}$

$p = \frac{0.08192}{q^4} \text{ --- (iii)}$

$\left(\frac{0.08192}{q^4}\right)^2 q^3 = 0.02048$

$\frac{0.0067108864}{q^8} \cdot q^3 = 0.02048$

$\frac{0.0067108864}{0.02048} = q^5$

$\sqrt[5]{0.32768} = \sqrt[5]{q^5}$

$q = 0.8$

but $p = 1 - q$

$\therefore p = 0.2$

6. $n=6$

$$9P(x=4) = P(x=2)$$

$$9\left({}^{6}C_{4}\, p^4 q^2\right) = {}^{6}C_{2}\, p^2 q^4$$

$$\therefore \frac{9(15p^4q^2)}{15} = \frac{15p^2q^4}{15}$$

$$9p^4q^2 = p^2q^4$$

$$9p^2 = q^2$$

$$\sqrt{(3p)^2} = \sqrt{(q)^2}$$

$$3p = 1-p$$

$$4p = 1$$

$$\therefore p = 1/4$$

7. It is poisson probability distribution

Mean $= \lambda = 20$

$$P(X<15) = P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)+P(x=7)+P(x=8)+P(x=9)+P(x=10)+P(x=11)+P(x=12)+P(x=13)+P(x=14)$$

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(X=x) = \frac{e^{-20}\, 20^x}{x!}$$

$P(x=0) = 2.061153622 \times 10^{-9}$

$P(x=1) = 4.122307245 \times 10^{-8}$

$P(x=2) = 4.122307245 \times 10^{-7}$

$P(x=3) = 2.74820483 \times 10^{-6}$

$P(x=4) = 1.374102415 \times 10^{-5}$

$P(x=5) = 5.49640966 \times 10^{-5}$

$P(x=6) = 1.832136553 \times 10^{-4}$

$P(x=7) = 5.234675867 \times 10^{-4}$

$P(x=8) = 1.308668967 \times 10^{-3}$

$P(x=9) = 2.908153259 \times 10^{-3}$

$P(x=10) = 5.816306518 \times 10^{-3}$

$P(x=11) = 0.010575102$

$P(x=12) = 0.017625171$

$P(x=13) = 0.027115648$

$P(x=14) = 0.03873664$

$\therefore P(X<15) = 0.104864279$

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8 It is binomial probability distribution

$p = 8/9$ $\quad q = 1/9$ $\quad n = 6$ $\quad x = 3$ $\quad \cancel{r = 3}$

$$P(X=3) = \binom{x+r-1}{r-1} p^r q^x$$

$$P(X=3) = \binom{3+3-1}{3-1}\left(\frac{8}{9}\right)^3\left(\frac{1}{9}\right)^3$$

$$P(X=3) = {}^5C_2\left(\frac{8}{9}\right)^3\left(\frac{1}{9}\right)^3$$

$$P(X=3) = {}^6C_3\left(\frac{8}{9}\right)^3\left(\frac{1}{9}\right)^3$$

$$P(X=3) = 0.019268366$$

$\therefore P(X=3) = \cancel{0.009634183287}\ 0.019268366$

9. It is binomial probability distribution

$q = 3/100 \quad p = \frac{97}{100} \quad n = 10$

(i) $P(x=6) = {}^{10}C_6\left(\frac{97}{100}\right)^6\left(\frac{3}{100}\right)^4$

$\therefore P(x=6) = 0.000141688538$

(ii) $P(x \geq 6) = P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$P(x \geq 6) = {}^{10}C_6\left(\frac{97}{100}\right)^6\left(\frac{3}{100}\right)^4 + {}^{10}C_7\left(\frac{97}{100}\right)^7\left(\frac{3}{100}\right)^3 + {}^{10}C_8\left(\frac{97}{100}\right)^8\left(\frac{3}{100}\right)^2 + {}^{10}C_9\left(\frac{97}{100}\right)^9\left(\frac{3}{100}\right)^1 +$
${}^{10}C_{10}\left(\frac{97}{100}\right)^{10}\left(\frac{3}{100}\right)^0$

$P(x \geq 6) = 0.000141688538 + 0.002617864417 + 0.031741606 + 0.228069317$
$+ 0.737424126$

$\therefore P(x \geq 6) = 0.999994602$

10 It is a binomial probability distribution

(i) $n = 10$

p (Probability of Correct answer) = 1/2

q [Probability of wrong answer] = 1/2

$$P(X=x) = {}^nC_x\, p^x q^{n-x}$$

$$P(x=x) = {}^{10}C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{10-x}$$

10 $P(x \geq 5) = 1 - P(x<5)$

but $P(x<5) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$P(x<5) = {}^{10}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10} + {}^{10}C_1 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^9 + {}^{10}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8 + {}^{10}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^7 + {}^{10}C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^6$

$P(x<5) = 0.0009765625 + 0.009765625 + 0.043945312 + 0.1171875 + 0.205078$

$P(x<15) = 0.376953124$

$P(x \geq 5) = 1 - 0.376953124$

$\therefore P(x \geq 5) = 0.623046875$

(ii) $P(X \geq 9) = P(x=9) + P(x=10)$

$P(x \geq 9) = {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right) + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$

$P(x \geq 9) = 0.009765625 + 0.0009765625$

$\therefore P(x \geq 9) =$

(iii)

X	0	1	2	3	4	5	6	7
$P(x=x)$	0.00098	0.00980	0.04395	0.11719	0.20508	0.24609	0.20508	
$F(x)$	0.00098	0.01078	0.05473	0.17192	0.377	0.62309		

$P(X \leq x) < 0.5$

$x = 4, 3, 2, 1, 0$ but the smallest is 0

$\therefore x = 0$

11 $n = 20$ $r = 3$ $x = 17$ $p = 0.25$ $q = 0.75$

$P(X=x) = \binom{x+r-1}{r-1} \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^x$

$P(x=17) = \binom{17+3-1}{3-1} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{17}$

$P(x=17) = \binom{19}{2} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{17}$

$P(x=17) = {}^{19}C_2 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{17}$

$\therefore P(x=17) = 0.020084342$

12 $p = 1/5$ $q = 4/5$

It is binomial probability distribution

$P(X=5) = {}^nC_x p^x q^{n-x}$

$n = 25$ $x = 5$

$P(X=5) = {}^{25}C_{\cancel{8}} \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^{25-5}$

$P(X=5) = {}^{25}C_5 \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^{20}$

$P(X=5) = 0.196015102$ is for that gets them before the twenty fifth questions.

$\cancel{P(X=x) = \binom{x+r-1}{r-1} p^r q^x}$

~~P(X~~ The probability that he gets them on the twenty-fifth questions it is negative binomial probability distribution.

$P(X=x) = \binom{x+r-1}{r-1} p^r q^x$

$n = 25$, $r = 5$, $x = 20$ $p = 1/5$ $q = 4/5$

$P(X=20) = \binom{20+5-1}{5-1} \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^{20}$

$P(X=20) = {}^{24}C_4 \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^{20}$

$P(X=20) = 0.03920302$

Probability $= 0.196015102 + 0.03920302$

$\therefore$ Probability $= 0.235218122$

14. It is geometric probability distribution

$P(X=x) = q^{x-1} p$

$P = 0.5$ $q = 0.5$ $x = 6$

$P(X=6) = (0.5)^{6-1} (0.5)$

$\cdot P(X=6) = 0.015625$

It is hypergeometric Probability

16 (i) $P(X=K) = \dfrac{{}^{12}C_3 \cdot {}^{8}C_2}{{}^{20}C_5} = \dfrac{6160}{15504} = \dfrac{389}{969}$

$N=20 \quad n=5 \quad K=3 \quad M=12$

$\therefore P(X=K) = \dfrac{389}{969}$

It is hypergeometric Proba

(iii) $P(X \geq K) = \dfrac{{}^{12}C_3 \cdot {}^{8}C_2}{{}^{20}C_5} + \dfrac{{}^{12}C_4 \cdot {}^{8}C_1}{{}^{20}C_5} + \dfrac{{}^{12}C_5 \cdot {}^{8}C_0}{{}^{20}C_5}$

$P(X \geq K) = \dfrac{389}{969} + \dfrac{165}{646} + \dfrac{33}{646}$

$\therefore P(X=x) = \dfrac{686}{969}$

$N=20, n=5, M=12, K=3$

17 It is hypergeometric probability distribution.

$$P(X=K) = \frac{\binom{M}{K}\binom{N-M}{n-K}}{\binom{N}{n}}$$

$K=3 \quad n=5 \quad N=12 \quad M=6$

$$P(X=3) = \frac{\binom{6}{3}\binom{12-6}{5-3}}{\binom{12}{5}}$$

$$P(X=3) = \frac{{}^{6}C_3 \cdot {}^{6}C_2}{{}^{12}C_5} = \frac{300}{792} = \frac{25}{66} = 0.378787878$$

$\therefore P(X=3) = \dfrac{25}{66}$

18

Tickets	number
0 0	0
0 1	3
1 0	2
1 1	0
	$n=5$

$p = 1/4 \quad q = 3/4$

$$\text{Probability} = C_x \cdot P_1^{x_1} \cdot P_2^{x_2} \cdot P_3^{x_3} \cdot P_4^{x_4}$$

$$\text{but } C_x = \frac{n!}{x_1!\, x_2!\, x_3!\, x_4!} = \frac{5!}{0!\, 3!\, 2!\, 0!} = 10$$

$$\text{Probability} = 10 \times \left(\tfrac{1}{4}\right)^0 \times \left(\tfrac{1}{4}\right)^3 \times \left(\tfrac{1}{4}\right)^2 \times \left(\tfrac{1}{4}\right)^0$$

$P_1 = 0.009765625$

Tickets	number
0 0	1
0 1	2
1 0	1
1 1	1
	5

$P = 1/4$

$$\text{Probability} = C_x \cdot P_1^{x_1} \cdot P_2^{x_2} \cdot P_3^{x_3} \cdot P_4^{x_4}$$

$$\text{but } C_x = \frac{n!}{x_1!\, x_2!\, x_3!\, x_4!} = \frac{5!}{1!\, 2!\, 1!\, 1!} = 60$$

$$\text{Probability} = 60 \times \left(\tfrac{1}{4}\right)^1 \times \left(\tfrac{1}{4}\right)^2 \times \left(\tfrac{1}{4}\right)^1 \times \left(\tfrac{1}{4}\right)^1$$

Probability $= 0.05859375$

$P_2 = 0.05859375$

18

Tickets	number
00	2
01	1
10	0
11	2
	5

$P = 1/4$

$$\text{Probability} = C_K \cdot P_1^{x_1} \cdot P_2^{x_2} \cdot P_3^{x_3} \cdot P_4^{x_4}$$

$$\text{but } C_K = \frac{n!}{x_1!\,x_2!\,x_3!\,x_4!} = \frac{5!}{2!\,0!\,2!\,1!} = 30$$

$$\text{Probability} = 30 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^1 \times \left(\frac{1}{4}\right)^0 \times \left(\frac{1}{4}\right)^2$$

$$\text{Probability} = 0.029296875$$

$$P_3 = 0.029296875$$

$$P = P_1 + P_2 + P_3$$

$$P = 0.009765625 + 0.05859375 + 0.029296875$$

$$\therefore \text{Probability} = 0.09765625$$

19. It is binomial probability distribution

$$P(X=x) = {}^nC_x\, p^x q^{n-x}$$

$$p = 1/3 \quad q = 2/3 \quad n = 5$$

$$P(x=4) = {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1$$

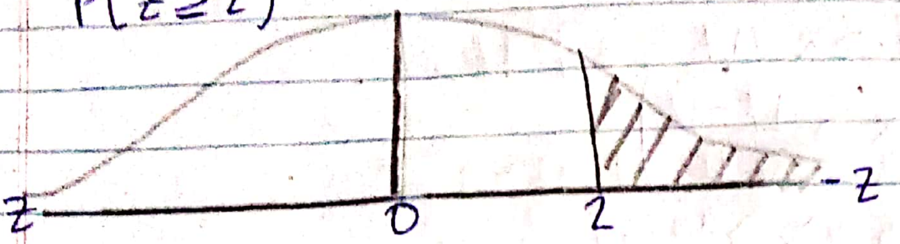
$$\therefore P(x=4) = 0.041152263$$

21. (a) Mean = 12   S.D = 4

(i) $X \geq 20$

$P\left(Z \geq \frac{20-12}{4}\right)$

$P(Z \geq 2)$

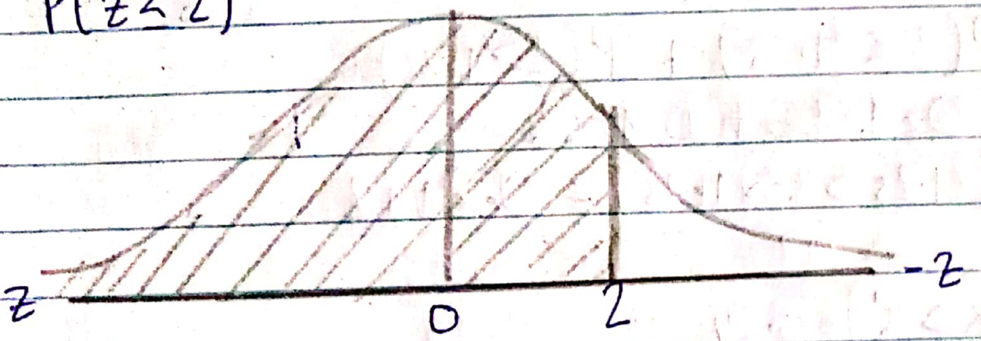


$\therefore P(Z \geq 2) = 0.0228$

(ii) $X \leq 20$

$P\left(Z \leq \frac{20-12}{4}\right)$

$P(Z \leq 2)$

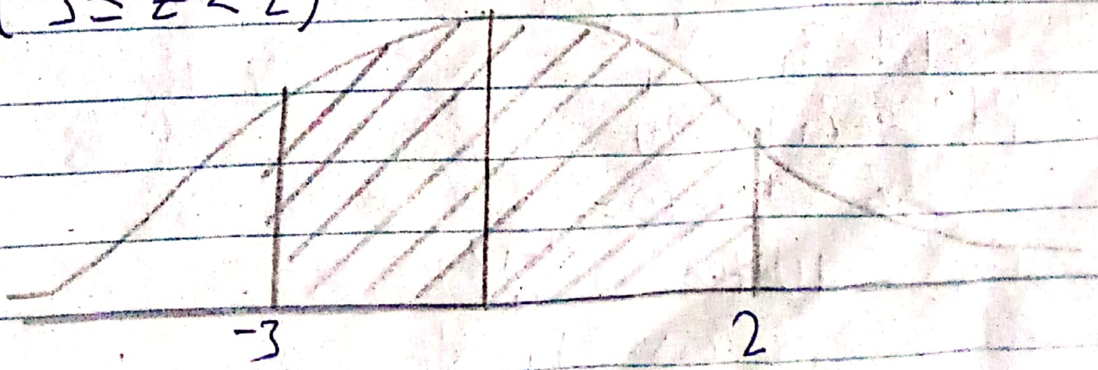


$\therefore P(Z \leq 2) = 0.9772$

(iii) $0 \leq X \leq 20$

$P\left(\frac{0-12}{4} \leq Z \leq \frac{20-12}{4}\right)$

$P(-3 \leq Z < 2)$



From normal table $P(Z \leq 2) - P(Z < -3) = 0.9772 - 0.0013 = \underline{0.9759}$

21 (i) (a)

$|x-12|>5$

$x-12>5 \qquad -x+12>5$

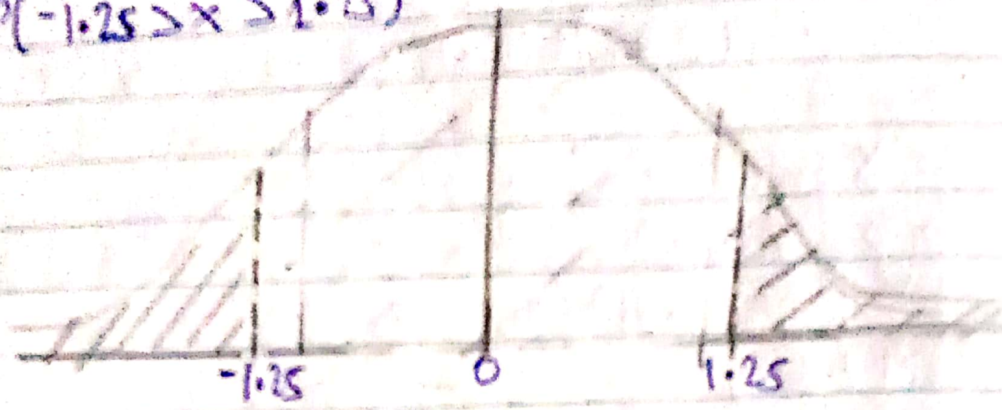
$x>5+12 \qquad 12-5>x$

$x>17 \qquad x<7$

$7>x>17$

$P\left(\frac{7-12}{4}>x>\frac{17-12}{4}\right)$

$P(-1.25>x>1.25)$



$= P(z<-1.25)+P(z>1.25)$

$= 0.1056+0.1056$

$\therefore P(-1.25>x>1.25)=0.2112$

(b) $P(x>x')=0.24$

$P\left(z>\frac{x'-12}{4}\right)=0.24$

let $z_1=\frac{x'-12}{4}$

$P(z>z_1)=0.24$

$z_1=+0.705$

$+0.705=\frac{x'-12}{4}$

$(+0.705\times4)+12=x'$

$x'=14.82$

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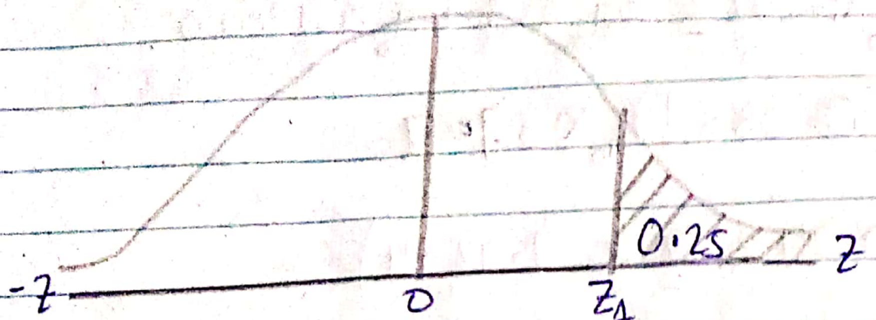
21 ⓒ find x_0' and x_1' $P(x_0' \le x \le x_1') = 0.5$ and $P(X \ge x_1') = 0.25$

$$P\left(Z \ge \frac{x_1' - 12}{4}\right) = 0.25$$



Let $Z_1 = \frac{x_1' - 12}{4}$

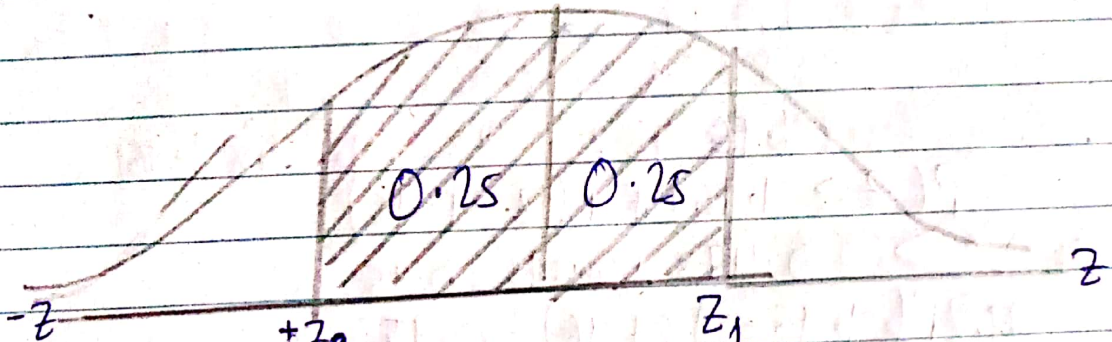
$P(Z \ge Z_1) = 0.25$

$Z_1 = 0.675$

$(0.675 \times 4) + 12 = x_1'$

$x_1' = 14.7$

$$P\left(\frac{x_0' - 12}{4} \le Z \le \frac{x_1' - 12}{4}\right) = 0.5$$



Let $Z_2 = \frac{x_0' - 12}{4}$

$P(Z_2 \le Z \le Z_1) = 0.5$

~~$P(Z_1)$~~ $P(Z \le Z_1) + P(Z \ge Z_2) = 0.5$

$P(Z \ge Z_2) = 0.5 - 0.25$

$P(Z \ge Z_2) = 0.25$

So $Z_2 = -Z_1 = -0.675$

$Z_2 = \frac{x_0' - 12}{4}$

$(-0.675 \times 4) + 12 = x_0'$

$x_0' = 9.3$

∴ The value of $x_0' = 9.3$ and $x_1' = 14.7$

22 (a) Mean = 4 Variance = 4 $S.D = \sqrt{Var}$

$S.D = \sqrt{4} = 2$

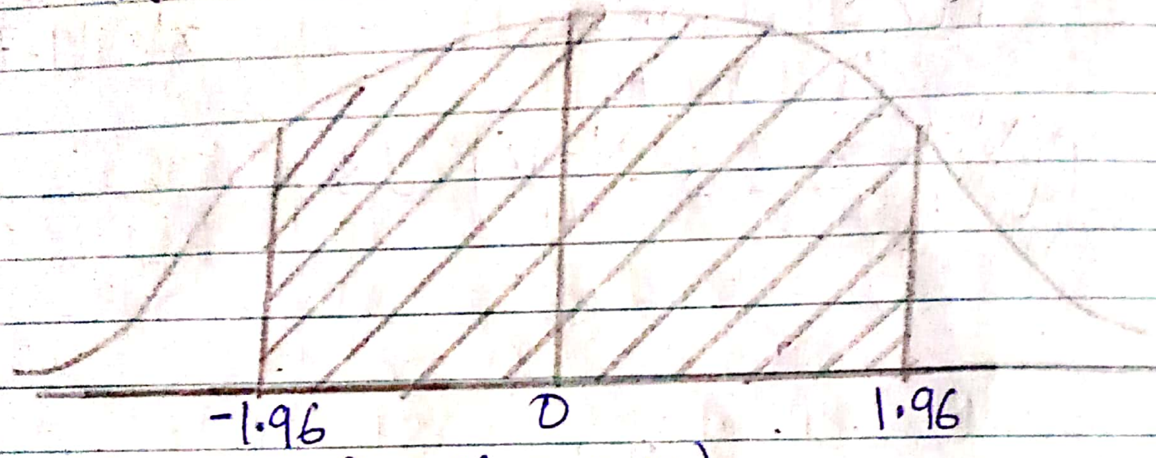
$1.202 \le x \le 83180000$

$\log_{10} 1.202 \le \log_{10} x \le \log_{10} 83180000$

$0.08 \le \log_{10} x \le 7.92$

$P\left(\frac{0.08-4}{2} \le Z \le \frac{7.92-4}{2}\right)$

$P(-1.96 \le Z \le 1.96)$

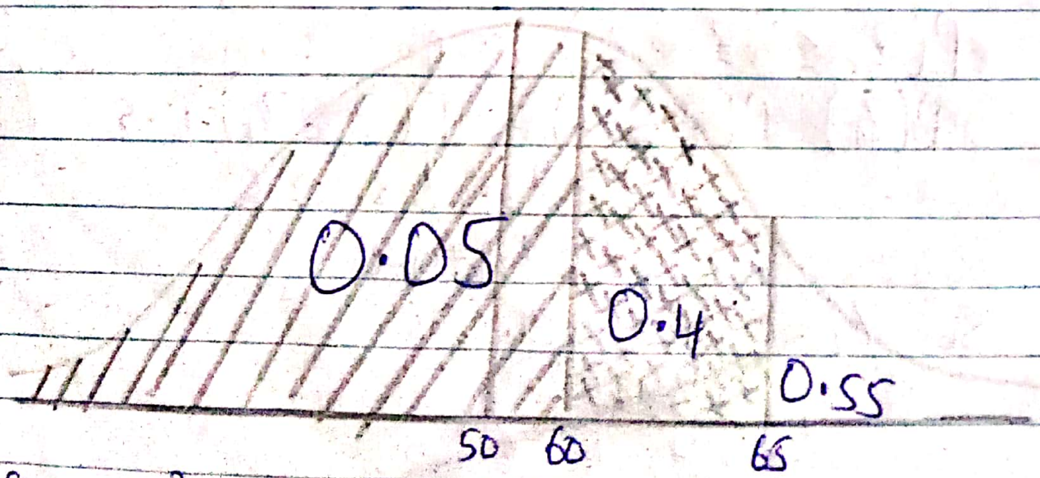


$P(Z \ge -1.96) + P(Z \le 1.96)$

$0.475 + 0.475 = 0.95$

$\therefore P(-1.96 \le Z \le 1.96) = 0.95$

(b)



$P(X \le 60) = 0.05$

$P(60 \le X \le 65) = 0.4$

$P(X \le 65) = 0.45$

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$$P\left(Z \le \frac{60-\mu}{\sigma}\right) = 0.05$$

Let $Z_1 = \frac{60-\mu}{\sigma}$

$$P(Z \le Z_1) = 0.05$$

$$Z_1 = 1.645$$

$$1.645\sigma + \mu = 60 \quad \text{-----} \quad (i)$$

$$P\left(Z \le \frac{65-\mu}{\sigma}\right) = 0.45$$

Let $Z_2 = \frac{65-\mu}{\sigma}$

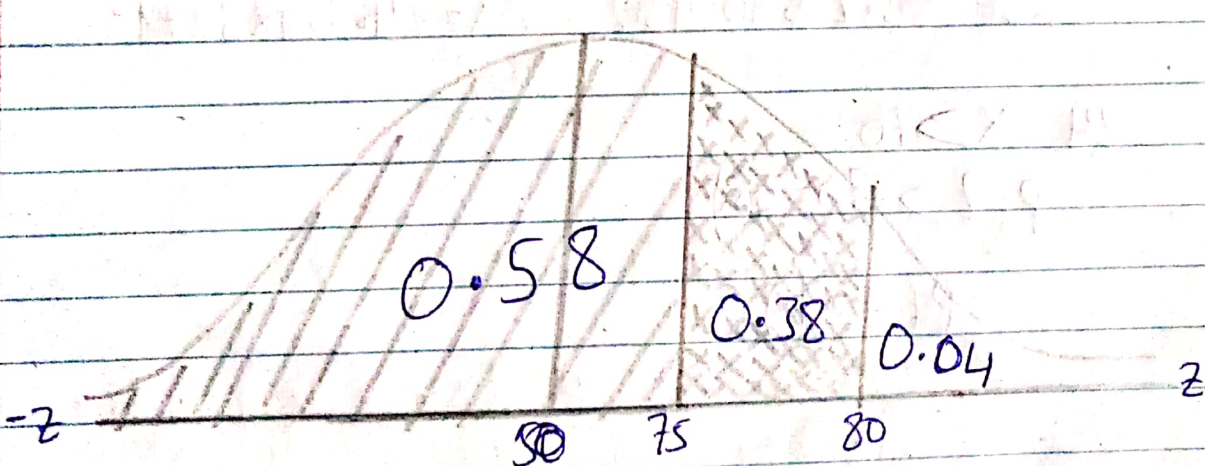
$$P(Z \le Z_2) = 0.45$$

$$\cancel{P(Z \ge} \; Z_2 = 0.125$$

$$0.125\sigma + \mu = 65 \quad \text{----} \quad (ii)$$

$\cancel{\sigma = 3.289}$ $\quad \mu = 65.411 \quad \sigma = {}^{-}3.289$

23.



$$P(X \le 75) = 0.58$$

$$P(75 \le X \le 80) = 0.38$$

$$P(X \ge 80) = 0.4$$

$$P\left(Z \le \frac{75-\mu}{\sigma}\right) = 0.58$$

Let $Z_1 = \frac{75-\mu}{\sigma}$

$$P(Z \le Z_1) = 0.58 - 0.5 = 0.08$$

23 $f(z_1) = 0.08$

$f(0.2) = 0.08 \qquad z_1 = 0.2$

$0.2\sigma + \mu = 75 \quad \ldots\ldots \quad (i)$

$$P\left(\frac{75-\mu}{\sigma} \leq Z \leq \frac{80-\mu}{\sigma}\right) = 0.38$$

$$P(z_1 \leq Z \leq z_2) = 0.38$$

$P(Z \geq 80) = 0.04$

$$P\left(Z \geq \frac{80-\mu}{\sigma}\right) = 0.5 - 0.04$$

Let $z_2 = \frac{80-\mu}{\sigma}$

$P(Z \geq z_2) = 0.46$

$f(z_2) = 0.46$

$f(1.75) = 0.46 \qquad z_2 = 1.75$

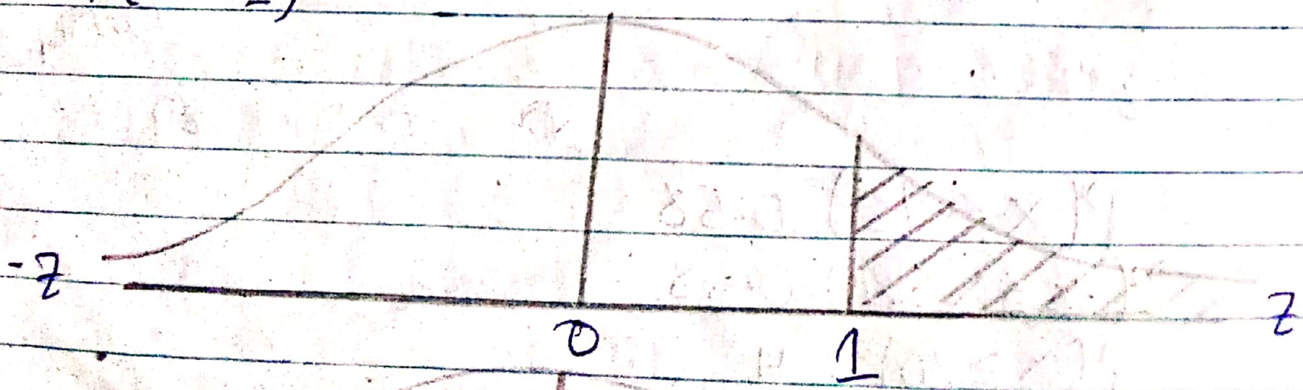
$1.75\sigma + \mu = 80 \quad \ldots \quad (ii)$

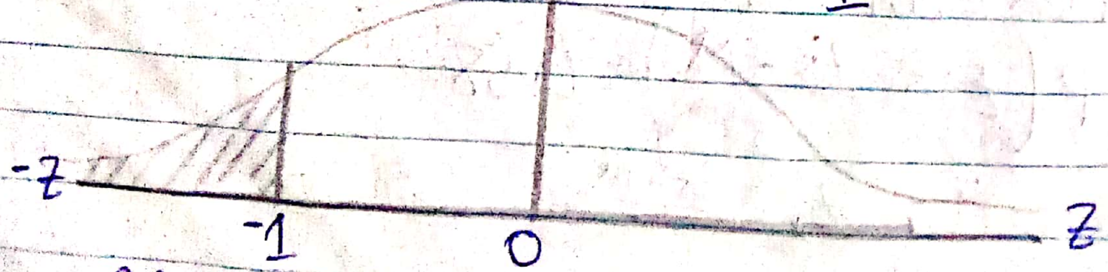
$\therefore \sigma = 3.225806452 \qquad \mu = 74.35483871$

24 $X \geq 70$

$$P\left(Z > \frac{70-65}{5}\right)$$

$P(Z > 1)$





$P(Z < -1) = 0.1587 = P(z > 1)$

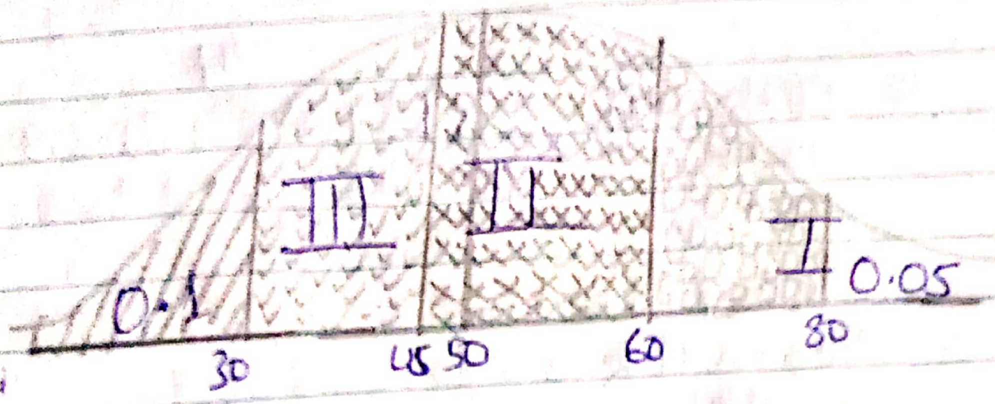
24 Binomial probability distribution

$P(X=x) = {}^nC_x\, p^x q^{n-x}$

$P(X=2) = {}^3C_2 (0.1587)^2 (0.8413)^1$

$\therefore P(X=2) = 0.06356612$

25.



$P(\cancel{x}<30) = 0.1$

$P\left(Z < \frac{30-\mu}{\sigma}\right) = 0.1$

let $Z_1 = \frac{30-\mu}{\sigma}$

$P(Z<Z_1) = 0.1$

$Z_1 = -1.2$

-0.09

-1.29

$+0.005$

-1.225

$-1.285\sigma + \mu = 30 \quad \cdots\cdots \quad (1)$

$P(Z \geq 80) = 0.05$

$P\left(Z \geq \frac{80-\mu}{\sigma}\right) = 0.05$

let $Z_2 = \frac{80-\mu}{\sigma}$

$P(Z \geq Z_2) = 0.05$

$Z_2 = 1.645 \qquad 1.645\sigma + \mu = 80$

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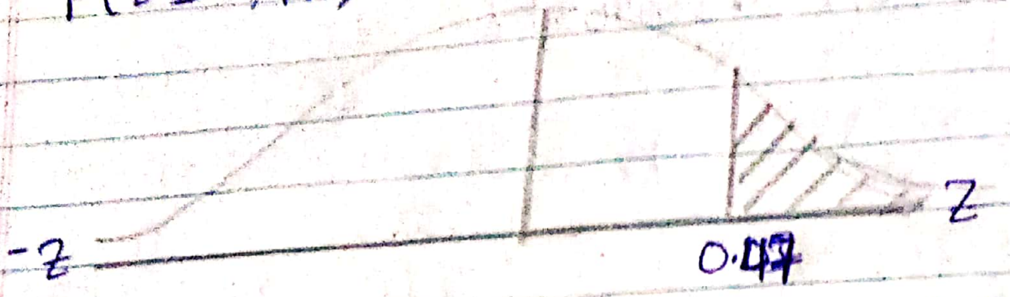
25 $\sigma = 17.06$ $\mu = 51.93$

Probability of first division;

$$P\left(Z \geq \frac{60-51.93}{17.06}\right)$$

$$P(Z \geq 0.47)$$



$P(Z \geq 0.47) = \cancel{0.31809} \times 100 = \cancel{31.809}$ $0.3192 \times 100 = 31.$

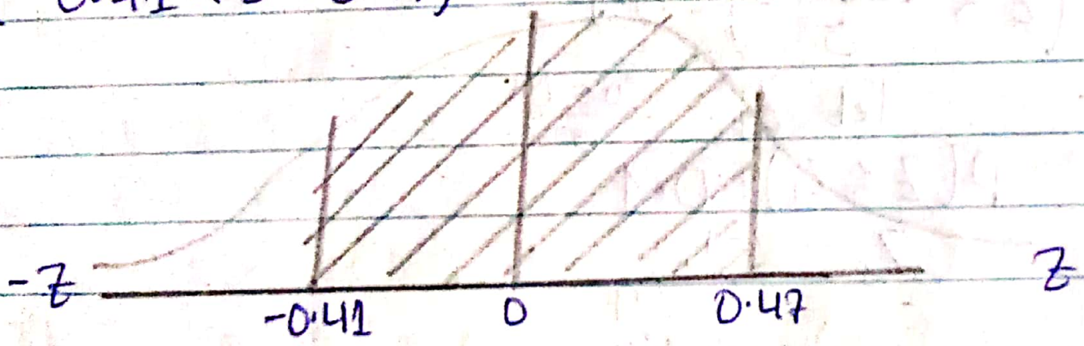
∴ ~~48.961~~% is ~~31.809% is for first division~~

∴ 31.92% is for first division.

Probability of second division;

$$P\left(\frac{45-51.93}{17.06} < Z < \frac{60-51.93}{17.06}\right)$$

$$P(-0.41 < Z < 0.47)$$



$P(-0.41 < Z < 0.47) = 0.6808 - 0.3409 = 0.3399 \times 100 = 33.99$

∴ 33.99% is for second division

Probability of third division:

$$P\left(\frac{30-51.93}{17.06} \leq Z \leq \frac{45-51.93}{17.06}\right)$$

$P(-1.29 \leq Z \leq -0.41) = 0.2424 \times 100 = 24.24\%$

∴ 24.24% for third division.

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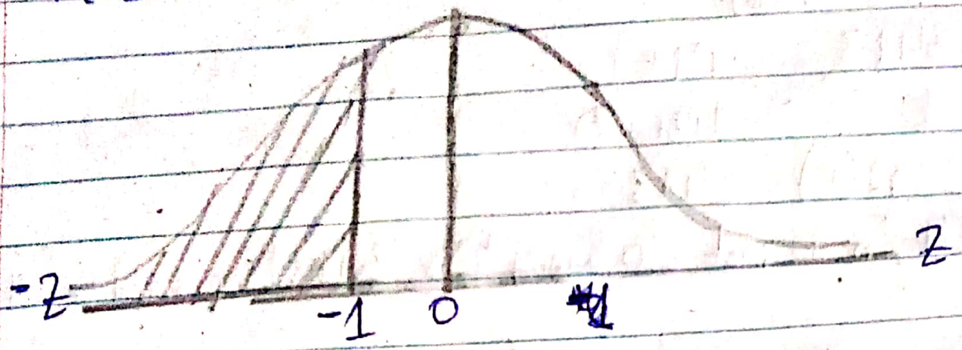
26 Mean = 1,000 σ = 200

(a) 10000 Lamps would fail

(i) In the first 800 burning hours?

$$P\left(Z < \frac{800-1000}{200}\right)$$

$$P(Z < -1)$$



$$P(Z < -1) = 0.1587$$

$$P = \frac{n(E)}{n(S)}$$

$$\frac{0.1587}{1} = \frac{n(E)}{10,000}$$

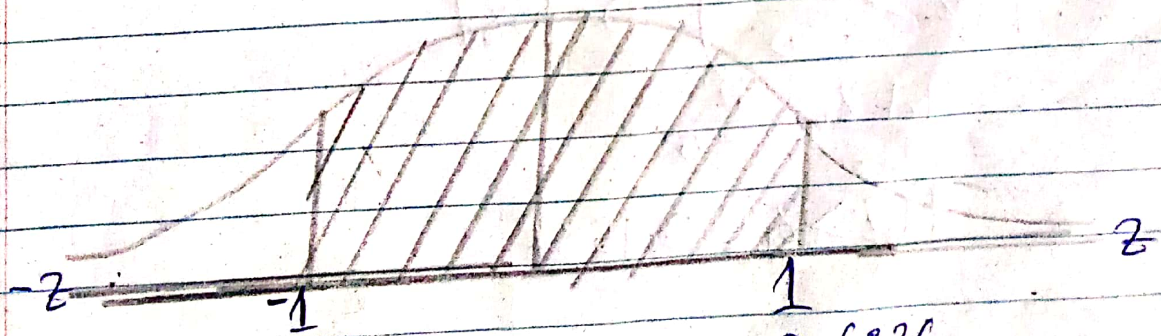
$$n(E) = 10,000 \times 0.1587 = 1587$$

∴ In the first burning hours 1587 lamps would fails.

(ii) Between 800 and 1200 burning hours?

$$P\left(\frac{800-1000}{200} < Z < \frac{1200-1000}{200}\right)$$

$$P(-1 < Z < 1)$$



$$P(-1 < Z < 1) = 0.8413 - 0.1587 = 0.6826$$

$$P(E) = \frac{n(E)}{n(S)} \qquad \frac{0.6826}{1} = \frac{n(E)}{10000} \qquad n(E) = 6826$$

∴ Between 800 and 1200 burning hours 6826 lamps would fail.

26 (b) ~~Lamp~~ 10000 Lamps would be still burning

(i) In the first 800 burning hours?

P(Lamps still burning) = 1 − P(Lamps would fail)

P(Lamps still burning) = 1 − 0.1587

P(Lamps still burning) = 0.8413

$$P(E) = \frac{n(E)}{n(S)}$$

$$\frac{0.8413}{1} = \frac{n(E)}{10{,}000}$$

$$n(E) = 8413$$

∴ ~~84~~ In the first 800 burning hours 8413 Lamps would be still burning.

(ii) Between 800 and 1200 burning hours?

P(Lamps still burning) = 1 − P(Lamp would fail)

P(Lamps still burning) = 1 − 0.6826

P(Lamps still burning) = 0.3174

$$P(E) = \frac{n(E)}{n(S)}$$

$$\frac{0.3174}{1} = \frac{n(E)}{10{,}000}$$

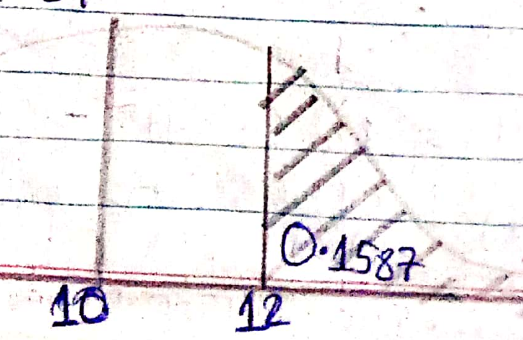
$$n(E) = 3174$$

∴ Between 800 and 1200 burning hours 3174 Lamps would be still burning.

28. $P\left(Z > \frac{12-10}{\sigma}\right) = 0.1587$

Let $Z_1 = \frac{12-10}{\sigma}$

$P(Z > Z_1) = 0.1587$



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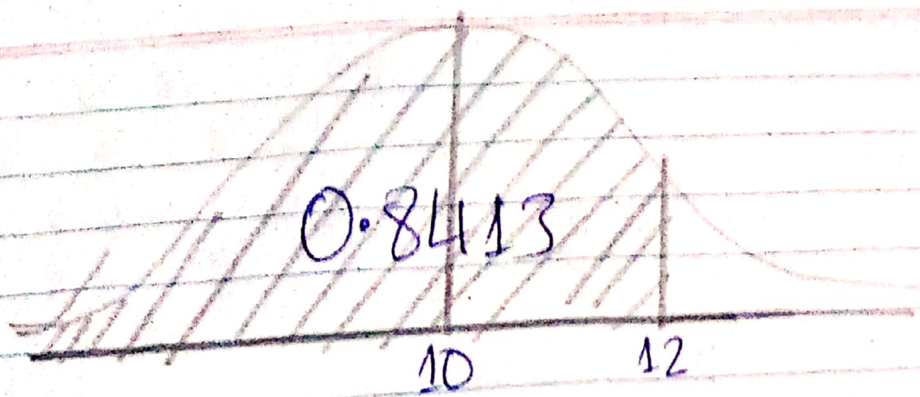
28



$P(Z \cancel{<} Z_1) = 0.8413$

$\varphi(Z_1) = 0.8413$

$\varphi(1) = 0.8413 \qquad Z_1 = 1$

$Z_1 = \frac{2}{\sigma}$

$1 = \frac{2}{\sigma}$

$\therefore \sigma = 2 \qquad \mu = 10$

$\cancel{P(\mu=10} \quad \cancel{P} \quad 9 \leq X \leq 11$

$P\left(\frac{\cancel{X}-\mu}{\sigma} \leq Z \leq \frac{\cancel{X}-\mu}{\sigma}\right)$

$P\left(\frac{9-10}{2} \leq Z \leq \frac{11-10}{2}\right)$

$P(-0.5 \leq Z \leq 0.5) = 2P(Z \geq -0.5)$

But $\varphi(-0.5) = 0.1587$

$P(-0.5 \leq Z \leq 0.5) = 2(0.1587)$

$\therefore P(9 \leq X \leq 11) = 0.3174$

15 It is geometric probability distribution

$p = \frac{1}{3}$ $q = \frac{2}{3}$

↑ Male ↑ female

$E(x) = q/p = \frac{2}{3} \div \frac{1}{3} = \underline{\underline{2}}$

$\therefore$ They should expect 2 female before the 1st male child is born.