

UNIVERSITY OF DAR ES SALAAM
COLLEGE OF SOCIAL SCIENCE
DEPARTMENT OF STATISTICS
ST122: LINEAR ALGEBRA WITH APPLICATIONS
SEMINAR QUESTIONS 3

1. Let $M = \begin{bmatrix} 9 & 0 & 0 \\ 1 & 6 & 0 \\ 1 & -1 & 3 \end{bmatrix}$

- (i) Find the eigenvalue of M .
- (ii) Determine whether M is positive definite or negative definite or semi-definite or indefinite. Provide a justification for your answer.
- (iii) Determine the eigenvectors corresponding to the largest eigenvalue.

2. Consider the following matrix over \mathbb{R}

$$A = \begin{bmatrix} 4 & 4 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

- (a) Find the characteristic equation for A . What are the eigenvalues of A ?
- (b) To each eigenvalue find the dimension of its corresponding eigenspace by finding a basis for the eigenspace. List a basis for each eigenspace.
- (c) Is A diagonalizable? Explain your answer. If it is finding an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

3.

- (a) What value(s) of k for which the quadratic form $5x_1^2 + x_2^2 + kx_3^2 + 4x_1x_2 - 2x_1x_3 - 2x_2x_3$ is positive definite.
- (b) In each part, determine whether the set of vectors is orthogonal or orthonormal with respect to the Euclidean inner product on \mathbb{R}^3 .
 - (i) $\left\{ \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right), \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right) \right\}$
 - (ii) $\left\{ \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right), \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \right), \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right) \right\}$

- (iii) $\left\{(1,0,0), \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), (0,0,1)\right\}$
 (iv) $\left\{\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right), \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)\right\}$

4.

(a) Express the following quadratic forms in matrix notation $\mathbf{x}^T \mathbf{A} \mathbf{x}$, where \mathbf{A} is symmetric matrix.

- (i) $2x^2 + 6xy - 5y^2$
 (ii) $x_1^2 + 7x_2^2 - 3x_3^2 + 4x_1x_2 - 2x_1x_3 + 8x_2x_3$
 (iii) $-7x_1x_3$
 (iv) $-x_1^2 - 2x_1x_2$

(b) For each of *i, ii, iii* and *iv* in question (a) determine whether \mathbf{A} is positive definite, negative definite or indefinite matrix.

5.

(a) Suppose that \mathbf{v} is an eigenvector for the matrix \mathbf{A} with eigenvalue 3. Let $\mathbf{B} = \mathbf{A}^2 - 2\mathbf{A} + 5\mathbf{I}$. Show that \mathbf{v} is an eigenvector for \mathbf{B} and determine its eigenvalue.

(b) Diagonalize the matrix

$$\mathbf{A} = \begin{bmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{bmatrix}$$

Using the information that the eigenvalues of \mathbf{A} are $\lambda = -3, 5$