

NETAJI SUBHAS UNIVERSITY OF TECHNOLOGY



LAB FILE

Course: Artificial Intelligence in IoT

Course Code: EIECE02

Submitted By:

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2.	To train a linear neuron regression model to map data of the given nature using Stochastic Gradient Descent Algorithm.	
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Experiment-1

Aim: - Design an artificial neuron which takes a three-dimensional data as input and uses sigmoidal function as its activation function.

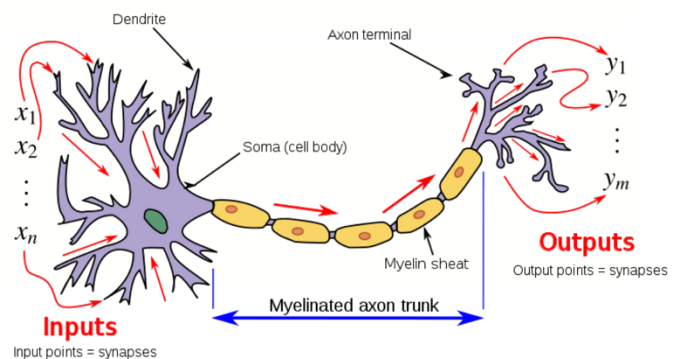
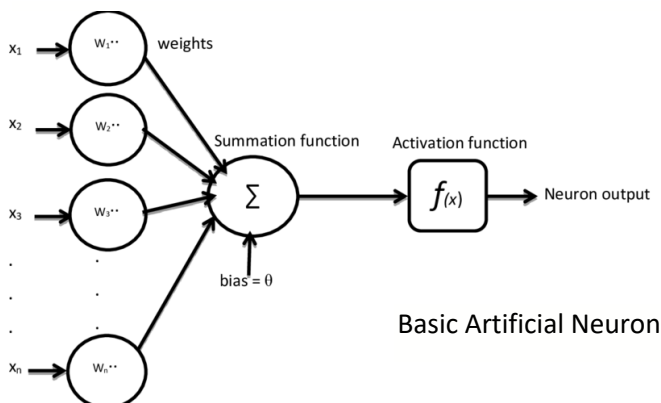
Apparatus: -

- Laptop Configuration
 - MacBook Air M1
 - 8GB RAM
 - 8-Core CPU
 - 7-Core GPU
- Libraries Used: -
 - TensorFlow 1.40
 - NumPy
 - PyLab
 - Matplotlib
- Coding Environment: - Python 3.7.

Theory: -

- **Artificial Neuron**

An artificial neuron is the basic unit of neural network



The basic element of Artificial Neuron

- i. A set of input signal. The input is a vector.

$$X = [x_1, x_2, x_3, \dots, x_n]^T.$$

where n is the number or the dimension of input signals. Inputs are also referred to as features. input are connected to neuron by a synaptic connection whose strength are represented by their weight. the weight vector W or, where W_i is a synaptic weight connecting i^{th} input to the neuron.

$$W = [w_1, w_2, w_3, \dots, w_n]^T.$$

The total synaptic input u to the neuron is given by the sum of the products of input and their corresponding connecting weight minus threshold The total synaptic input to the neuron

$$U = x_1 w_1 + x_2 w_2 + x_3 w_3 + \dots + x_n w_n + (-1)\theta$$

$$U = \sum_{i=1}^n x_i w_i - \theta$$

$$U = [x_1 \quad x_2 \quad x_3 \cdots \quad x_n] \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} - \theta$$

$$U = X^T W - \theta$$

The activation function f relates synaptic input to the activation function of neuron.
 $f(u)$ denotes activation function of the neuron
So, the output will be
$$y = f(u)$$

Python CODE: -

```
import tensorflow.compat.v1 as tf
tf.disable_v2_behavior()
#building computational graph
W=tf.Variable([2.5,-0.2,1.0],tf.float32)
b=tf.Variable([-0.5],tf.float32)
x=tf.placeholder(tf.float32)
#placeholder makes x to expect some input
#dot product of
u=tf.tensordot(W,x,axes=1)+b
y=0.8/1+tf.exp(-1.2*u)
#Evaluating computational graph
sess=tf.Session()
init=tf.global_variables_initializer()
#as name suggests
sess.run(init)
u,y=sess.run([u,y],{x:[0.8,2.0,-0.5]})
print(u,y)
```

OUTPUT: -

```
[0.6] [1.2867522]
```

Precautions: -

- Design an artificial neur
- on which takes a three-dimensional
- data as input and uses sigmoidal function
- n as its activation function

Experiment-2

Aim: - To train a linear neuron regression model to map data of the given nature using Stochastic Gradient Descent Algorithm.

$x = (x_1, x_2)$	y
(0.54, -0.95)	1.33
(0.27, 0.50)	0.45
(0.00, -0.55)	0.56
(-0.60, 0.52)	-1.66
(-0.66, -0.82)	-1.07
(0.37, 0.91)	0.30

Use Learning rate $\alpha = 0.01$ and number of iterations = 200.

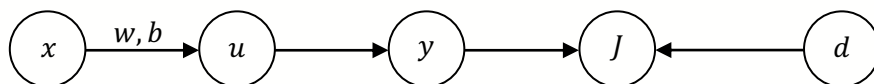
Plot the optimized best fit line and the graph of cost function vs epoch.

Apparatus: -

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Theory: -

Linear Regression: -



Representing a dependent (output) variables as a linear combination of independent (input) variables is known as linear regression.

The output of a linear neuron can be written as

$$y = w_1x_1 + w_2x_2 + w_3x_3 + \dots + w_nx_n + b$$

Where $x_1, x_2, x_3 \dots x_n$ are input features.

i.e., a Linear neuron performs linear regression and the weights and biases acts as regression coefficients.

In a given data set $\{x_p, d_p\}_{p=1}^P$

where input $x_p \in R^n$
& $d_p \in R$

Training a linear neuron finds a regression function

$$\varphi: R^n \rightarrow R$$

Given by Linear Mapping;

$$y = W^T X + b$$

The cost function $J(w, b)$, for regression is given as the square error b/w. neuron outputs, and the target.

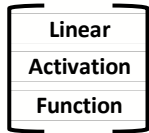
Given a training pattern $\{\bar{x}, d\}$, half squared error Cost function is given by: $-\frac{1}{2}(d - y)^2$

Where y is the neuron for the input pattern \bar{x} & $y = W^T X + b$

The $\frac{1}{2}$ in the cost function is introduced to simplify the learning process and data does not affect the optimization parameters (weights and biases)

$$J = \frac{1}{2}(d - y)^2$$

$$\# y = f(u) = u = W^T X + b$$



$$\# \frac{\partial J}{\partial w} = \frac{\partial J}{\partial y} \times \frac{\partial y}{\partial u} \times \frac{\partial u}{\partial w}$$

$$\# \frac{\partial J}{\partial y} = -(d - y)$$

$$\# \frac{\partial y}{\partial u} = 1$$

$$\# \frac{\partial u}{\partial w} = x$$

$$\Rightarrow \frac{\partial J}{\partial w} = -(d - y)\vec{x}$$

Now, we will update bias-

$$\begin{aligned} \# \frac{\partial J}{\partial w} &= \frac{\partial J}{\partial y} \times \frac{\partial y}{\partial u} \times \frac{\partial u}{\partial w} \\ &= -(d - y) \end{aligned}$$

#The Gradient Learning Equation

$$\# w = w - \alpha \nabla_w J$$

$$\# w = w + \alpha(d - y)\vec{x}$$

$$\# b = b - \alpha \nabla_b J$$

$$\# b = b + \alpha(d - y)$$

Learning Algorithm: -

for training patten $\{x_p, d_p\}_{p=1}^P$

1) Set learning Rate α

2) initialize (w, b)

3) Until the Convergence:

for each pattern in $\{x_p, d_p\}_{p=1}^P$

$$y = w^T x_p + b$$

$$w = w + \alpha(d_p - y_p)\vec{x}$$

$$b = b + \alpha(d_p - y_p)$$

Python CODE: -

```
import tensorflow as tf
import numpy as np
import pylab as plt
from mpl_toolkits.mplot3d import Axes3D

import os
if not os.path.isdir('figures'):
    os.makedirs('figures')

tf.logging.set_verbosity(tf.compat.v1.logging.ERROR)

no_iters = 200
lr = 0.01

SEED = 10
np.random.seed(SEED)

# generate training data
X = 2*np.random.rand(6, 2) - 1
Y = np.dot(X, [2.53, -0.47]) - 0.5 + np.random.rand(6)

print(X)
print(Y)
print(lr)

# Model parameters
w = tf.Variable(np.random.rand(2), dtype=tf.float32)
b = tf.Variable(0., dtype=tf.float32)

# Model input and output
x = tf.placeholder(tf.float32, [2])
d = tf.placeholder(tf.float32)

y = tf.tensordot(x, w, axes=1) + b
loss = tf.square(d - y) # sum of the squares
# optimizer
grad_w = -(d - y)*x
grad_b = -(d - y)
w_new = w.assign(w - lr*grad_w)
b_new = b.assign(b - lr*grad_b)

# initialize variables
sess = tf.Session()
sess.run(tf.global_variables_initializer())
# print initial weights and biases
w_, b_ = sess.run([w, b])
print('w: {}, b: {}'.format(w_, b_))
```

```

# training loop begins
err = []
idx = np.arange(len(X))
for i in range(no_iters):

    err_ = []
    np.random.shuffle(idx)
    X, Y = X[idx], Y[idx]
    for p in np.arange(len(X)):
        y_, loss_, w_, b_ = sess.run([y, loss, w_new, b_new], {x: X[p], d: Y[p]})

        if i == 0:
            print('iter: {}'.format(i+1))
            print('p: {}'.format(p+1))
            print('x: {}, d: {}'.format(X[p], Y[p]))
            print('y: {}'.format(y_))
            print('se: {}'.format(loss_))
            print('w: {}, b: {}'.format(w_, b_))

    err_.append(loss_)
    err.append(np.mean(err_))
    if i%10 == 0:
        print('iter: %d, mse: %g'%(i, err[i]))

# print final weights and error
w_, b_ = sess.run([w, b])
print('w: %s, b: %s'%(w_, b_))
print('mse: %.3f'%err[no_iters-1])

# plot learning curve
plt.figure(1)
plt.plot(range(no_iters), err)
plt.xlabel('epochs')
plt.ylabel('mean square error')
plt.savefig('./figures/2.1a_1.png')

# find the predicted values of inputs
pred = []
for p in np.arange(len(X)):
    pred.append(sess.run(y, {x:X[p]}))

# plot targets and predictions
fig = plt.figure(2)
ax = fig.gca(projection = '3d')
ax.scatter(X[:,0], X[:,1], Y, 'ro', label='targets')
ax.scatter(X[:,0], X[:,1], pred, 'b^', label='predicted')

X1 = np.arange(-1, 1, 0.1)
X2 = np.arange(-1, 1, 0.1)
X1,X2 = np.meshgrid(X1,X2)

```



```

Z = w_[0]*X1 + w_[1]*X2 + b_
ax.plot_surface(X1, X2, Z)

ax.set_zticks([-2, -1, 0, 1])
ax.set_xticks([-0.5, 0, 0.5])
ax.set_yticks([-0.5, 0, 0.5])
ax.set_xlabel('$x_1$')
ax.set_ylabel('$x_2$')
ax.set_zlabel('$y$')
plt.title('targets and predictions')
plt.legend()
plt.savefig('./figures/2.1a_2.png')

plt.show()

```

OUTPUT: -

```

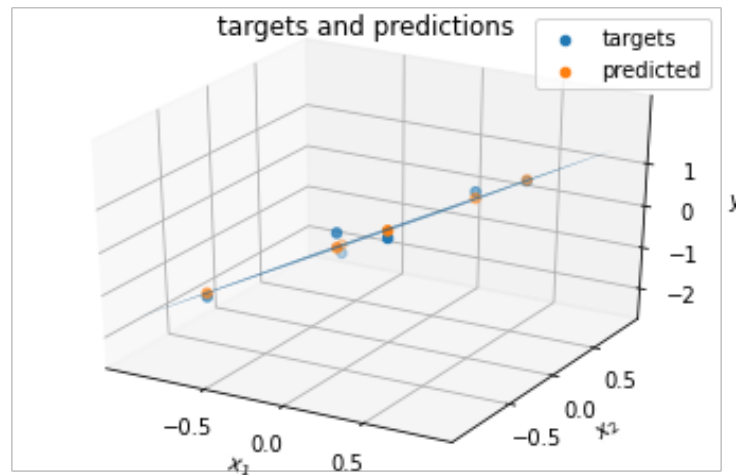
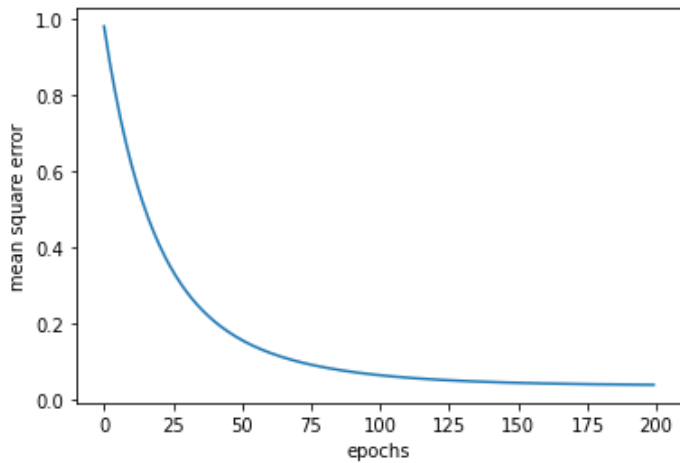
[[ 0.54264129 -0.9584961 ]
 [ 0.26729647  0.49760777]
 [-0.00298598 -0.55040671]
 [-0.60387427  0.52106142]
 [-0.66177833 -0.82332037]
 [ 0.37071964  0.90678669]]
[ 1.32732389  0.45457668  0.5637576 -1.66017471 -1.06558327  0.303607 ]
0.01
w: [0.91777414 0.71457577], b: 0.0
iter: 1
p: 1
x:[ 0.54264129 -0.9584961 ], d:1.3273238888598116
y: -0.1868959665298462
se: 2.2928619384765625
w: [0.92599094 0.70006204], b: 0.015142198652029037
iter: 1
p: 2
x:[-0.66177833 -0.82332037], d:-1.0655832748853409
y: -1.1740338802337646
se: 0.011761543340981007
w: [0.92527324 0.69916916], b: 0.01622670516371727
iter: 1
p: 3
x:[-0.00298598 -0.55040671], d:0.5637575971042242
y: -0.3713635206222534
se: 0.8744515180587769
w: [0.92524534 0.6940222 ], b: 0.025577915832400322
iter: 1
p: 4
x:[0.26729647 0.49760777], d:0.454576682526435
y: 0.6182435750961304
se: 0.02678685635328293
w: [0.92480785 0.69320774], b: 0.023941246792674065
iter: 1
p: 5
x:[-0.60387427  0.52106142], d:-1.6601747069539354
y: -0.1733226180076599
se: 2.2107293605804443
w: [0.9337866 0.6854603], b: 0.009072725661098957
iter: 1
p: 6

```

```

x:[0.37071964 0.90678669], d:0.30360700368844684
y: 0.9768120646476746
se: 0.4532049894332886
w: [0.93129086 0.6793558 ], b: 0.0023406757973134518
iter: 0, mse: 0.978299
iter: 10, mse: 0.613169
...
...
iter: 180, mse: 0.0383732
iter: 190, mse: 0.0376284
w: [ 2.0039272 -0.43821055], b: -0.013066508
mse: 0.037

```



Precautions: -

- Design an artificial neur
- on which takes a three-dimensional
- data as input and uses sigmoidal function
- n as its activation function

Experiment-3

Aim: - To train a discrete perceptron classification model to classify the following two dimensional model.

$x = (x_1, x_2)$	y
(1.0, 2.5)	B
(2.0, -1.0)	A
(1.5, 3.0)	B
(0.0, -1.5)	A
(-3.5, 1.0)	B
(2.5, 0.0)	A
(0.5, 1.5)	A
(0.0, -2.0)	A

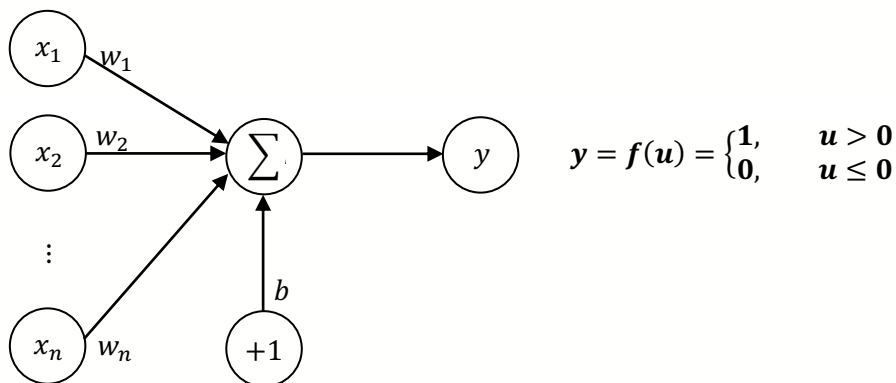
Plot the decision boundary and the graph of cost vs epoch.

Apparatus: -

- Laptop Configuration
 - MacBook Air M1
 - 8GB RAM
 - 8-Core CPU
 - 7-Core GPU
- Libraries Used: -
 - TensorFlow 1.40
 - NumPy
 - PyLab
 - Matplotlib
- Coding Environment: - Python 3.7.

Theory: -

Discrete Perceptron: - It's a neuron that has a threshold of unit step activation function.



Discrete Perceptron Classifier: - Classifies input patterns into two or more classes with a linear discriminant function.

Indicator Function $I()$: - It takes a value 1 when the condition given is true and value 0 when the condition is false.

$$I(x) = \begin{cases} 1, & x \text{ is true} \\ 0, & x \text{ is false} \end{cases}$$

Using an indicator function, the output of a discrete perceptron can be written as , $y = 1(u > 0)$ where $u = W^T X + b$

Learning Algorithm: -

Given \vec{p} training pairs $\{x_p, d_p\}_{p=1}^P$

where, $x_p \in \mathbb{R}^n$

is the n – dimensional input & $d_p \in (0, 1)$

Discrete Perceptron Learning Algorithm: - is a supervised scheme. It was proposed by Ministry in 1950 and its convergence can be proved.

However, because of non-differentiable character states of the activation, the discrete perceptron learning algorithm can't be derived from cost function.

Discrete perceptron learning algorithm finds a linear decision boundary in the feature space.

The change of weight is proportional to the difference (error) between the desired output 'd' & the perceptron output 'y'.

for training pattern (X, d) :

$$u = W^T X + b$$

$$y = 1(u > 0)$$

$$\delta = d - y$$

until the convergence:

$$w \leftarrow w + \alpha \delta \vec{x}$$

$$b \leftarrow b + \alpha \delta$$

Note that $\delta \in \{-1, 0, 1\}$

$$\alpha \in (0, 1)$$

Where $\alpha = 0.4$, Learning equation are referred to as simple perceptron rule

Python CODE: -

```
import tensorflow as tf
import numpy as np
import pylab as plt
from mpl_toolkits.mplot3d import Axes3D

import os
if not os.path.isdir('figures'):
    os.makedirs('figures')

tf.compat.v1.logging.set_verbosity(tf.compat.v1.logging.ERROR)

no_iters = 30
lr = 0.4
SEED = 10
np.random.seed(SEED)

# training data
x_train = np.array([[1.0, 2.5], [2.0, -1.0], [1.5, 3.0],
                    [0.0, -1.5], [-3.5, 1.0], [2.5, 0.0], [0.5, 1.5], [0.0, -2.0]])
y_train = np.array([1, 0, 1, 0, 1, 0, 0, 0])

print(x_train)
print(y_train)
print(lr)
# Model parameters
```

```

w = tf.Variable(np.random.rand(2), dtype=tf.float32)
b = tf.Variable(0., dtype=tf.float32)

# Model input and output
x = tf.placeholder(tf.float32)
d = tf.placeholder(tf.int32)

u = tf.tensordot(x,w, axes=1) + b
y = tf.where(tf.greater(u, 0), 1, 0)

delta = d - y
delta = tf.cast(delta, tf.float32)

w_new = w.assign(w + lr*delta*x)
b_new = b.assign(b + lr*delta)

# initialize the variables
init = tf.global_variables_initializer()
sess = tf.Session()
sess.run(init)

# print initial weights
w_, b_ = sess.run([w, b])
print('w: {}, b: {}'.format(w_, b_))

# training loop
err = []
idx = np.arange(len(x_train))
for i in range(no_iters):
    np.random.shuffle(idx)
    x_train, y_train = x_train[idx], y_train[idx]

    err_ = 0
    for p in np.arange(len(x_train)):
        u_, y_, w_, b_ = sess.run([u, y, w_new, b_new], {x: x_train[p], d: y_train[p]})
        err_ += y_ != y_train[p]

        if (i == 0):
            print('p: {}'.format(p+1))
            print('x: {}'.format(x_train[p]))
            print('d: {}'.format(y_train[p]))
            print('u: {}'.format(u_))
            print('y: {}'.format(y_))
            print('w: {}, b: {}'.format(w_, b_))

    err.append(err_)

    print('iter: {}, error: {}'.format(i+1, err[i]))
# print final weights

```

```

print('w: {}, b: {}'.format(w_, b_))

# plot the learning curves
plt.figure(2)
plt.plot(range(no_iters), err)
plt.xlabel('epochs')
plt.ylabel('classification error')
plt.yticks([0, 1, 2])
plt.savefig('./figures/3.1a_2.png')

# find predicctions
pred = []
for p in np.arange(len(x_train)):
    pred.append(sess.run(y, {x: x_train[p]}))
print(y_train, pred)

# plot the training data
plt.figure(1)
plt.plot(x_train[y_train==1,0], x_train[y_train==1,1], 'bx', label='class A')
plt.plot(x_train[y_train==0,0], x_train[y_train==0,1], 'ro', label='class B')
plt.xlabel('$x_1$')
plt.ylabel('$x_2$')
plt.title('training data')
plt.legend()
plt.savefig('./figures/3.1a_1.png')

# plot the decision boundary
x1 = np.arange(-4, 4, 0.1)
x2 = -(x1*w_[0] + b_)/w_[1]
plt.figure(3)
plt.plot(x_train[y_train==1,0], x_train[y_train==1,1], 'bx', label='class A')
plt.plot(x_train[y_train==0,0], x_train[y_train==0,1], 'ro', label='class B')
plt.plot(x1, x2, '-')
plt.xlabel('$x_1$')
plt.ylabel('$x_2$')
plt.title('decision boundary')
plt.legend()
plt.savefig('./figures/3.1a_3.png')
plt.show()

```

OUTPUT: -

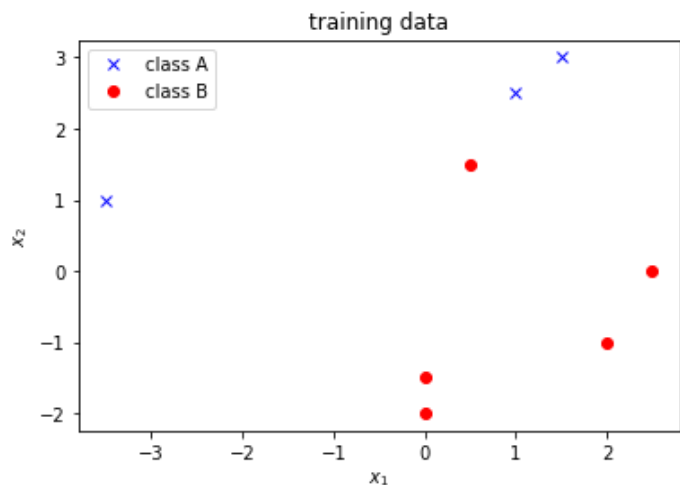
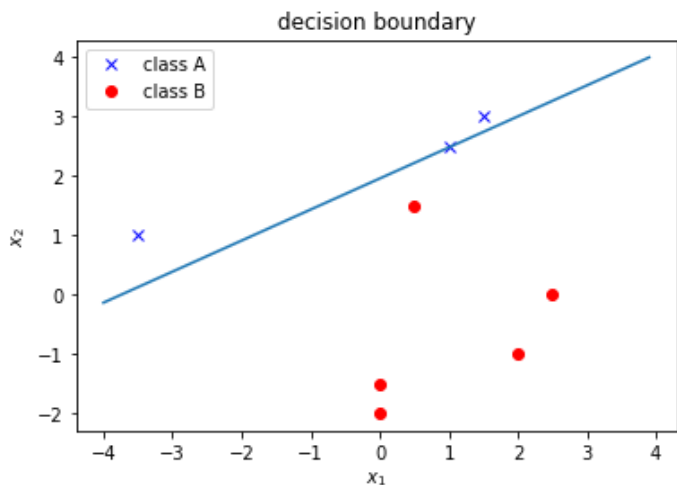
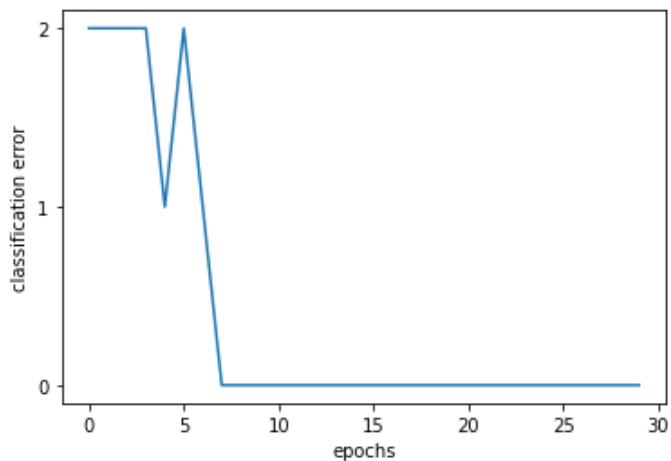
```

[[ 1.  2.5]
 [ 2. -1. ]
 [ 1.5  3. ]
 [ 0. -1.5]
 [-3.5  1. ]
 [ 2.5  0. ]
 [ 0.5  1.5]
 [ 0. -2. ]]
[1 0 1 0 1 0 0 0]

```

0.4

```
w: [0.77132064 0.02075195], b: 0.0
p: 1
x: [1.5 3. ]
d: 1
u: 1.2192368507385254
y: 1
w: [0.77132064 0.02075195], b: 0.0
p: 2
x: [ 0. -2.]
d: 0
u: -0.0415038987994194
:
:
:
w: [-0.22867936 0.02075195], b: -0.4000000059604645
p: 7
x: [ 2. -1.]
d: 0
u: -0.8781106472015381
y: 0
w: [-0.22867936 0.02075195], b: -0.4000000059604645
p: 8
x: [1. 2.5]
d: 1
u: -0.5767995119094849
y: 0
w: [0.17132065 1.020752 ], b: 0.0
iter: 1, error: 2
iter: 2, error: 2
:
:
:
iter: 29, error: 0
iter: 30, error: 0
w: [-0.4286793 0.8207519],
b: -1.6000000023841858
[1 1 0 1 0 0 0 0]
[1, 1, 0, 1, 0, 0, 0, 0]
```



Experiment-4

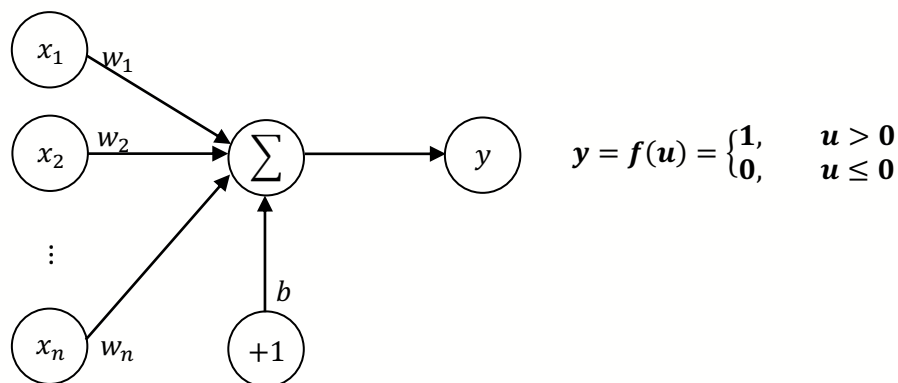
Aim: - To train a discr

Apparatus: -

- Laptop Configuration CPU, GPU, Core, clock etc.
- Laptop Configuration
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 - 8GB RAM
 - 8-Core CPU
 - 7-Core GPU
- Libraries Used: -
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Discrete perceptron learning algorithm finds a linear decision boundary in the feature space.

The change of weight is proportional to the difference (error) between the desired output 'd' & the perceptron output 'y'.

for training patten (X, d):

$$u = W^T X + b$$

$$y = 1(u > 0)$$

$$\delta = d - y$$

untill the convergence:

$$w \leftarrow w + \alpha \delta \vec{x}$$

$$b \leftarrow b + \alpha \delta$$

Note that $\delta = \{-1, 0, 1\}$

$$\alpha \in (0, 1)$$

Where $\alpha = 1.0$, Learning equation are referred to as simple perceptron rule

Python CODE: -

```
import tensorflow as tf
import numpy as np
import pylab as plt
import multiprocessing as mp

import os
if not os.path.isdir('figures'):
    print('creating the figures folder')
    os.makedirs('figures')

tf.compat.v1.logging.set_verbosity(tf.compat.v1.logging.ERROR)

no_iters = 30
SEED = 10
np.random.seed(SEED)

# training data
x_train = np.array([[1.0, 2.5], [2.0, -1.0], [1.5, 3.0],
                    [0.0, -1.5], [-3.5, 1.0], [2.5, 0.0], [0.5, 1.5], [0.0, -2.0]])
y_train = np.array([1, 0, 1, 0, 1, 0, 0, 0])

# Model parameters
w = tf.Variable(np.random.rand(2), dtype=tf.float32)
b = tf.Variable(0., dtype=tf.float32)
lr = tf.Variable(0.4, dtype=tf.float32)

# Model input and output
x = tf.placeholder(tf.float32)
d = tf.placeholder(tf.int32)

u = tf.tensordot(x, w, axes=1) + b
y = tf.where(tf.greater(u, 0), 1, 0)
```

```

delta = d - y
delta = tf.cast(delta, tf.float32)

w_new = w.assign(w + lr*delta*x)
b_new = b.assign(b + lr*delta)

# training loop
def my_train(rate):
    init = tf.global_variables_initializer()
    sess = tf.Session()
    sess.run(init) # reset values to wrong

    X, Y = x_train, y_train
    err = []
    idx = np.arange(len(X))
    for i in range(no_iters):
        np.random.shuffle(idx)
        X, Y = X[idx], Y[idx]
        err_ = 0
        for p in np.arange(len(X)):
            y_, w_, b_ = sess.run([y, w_new, b_new], {x: X[p], d: Y[p]})
            err_ += y_ != Y[p]

        err.append(err_)

    return err

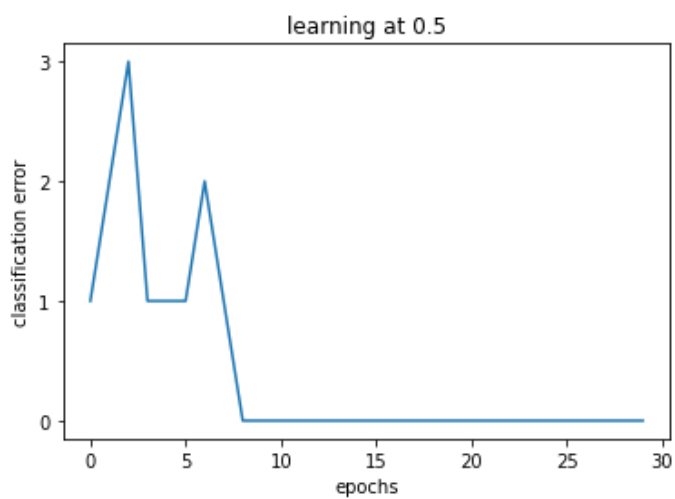
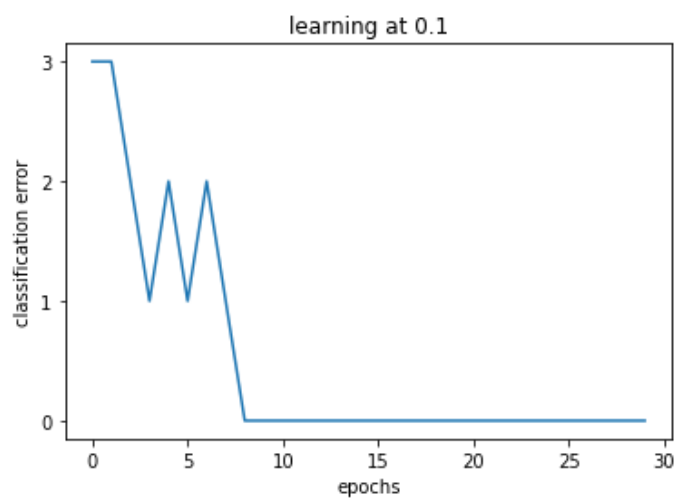
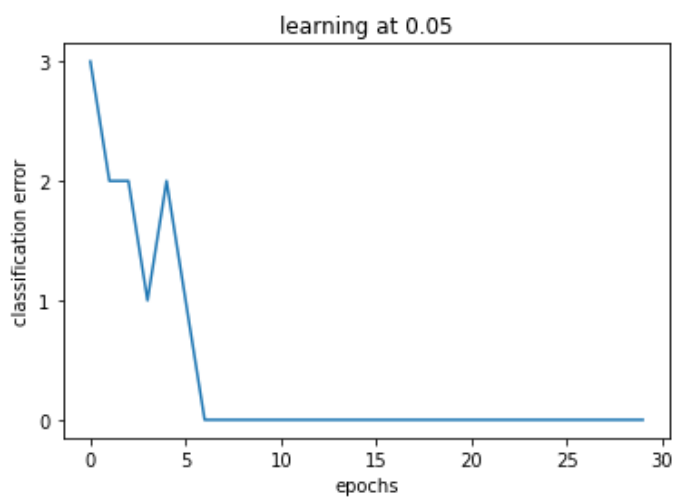
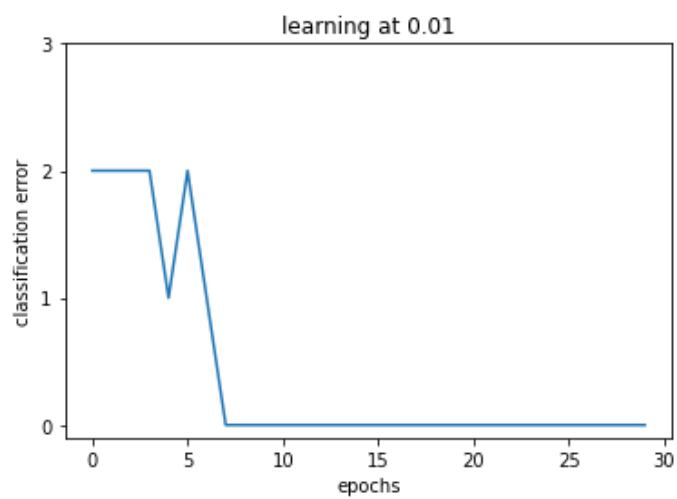
rates = [0.01, 0.05, 0.1, 0.5]

for i in range(len(rates)):
    cost = my_train(rates[i])
    plt.figure()
    plt.plot(range(no_iters), cost)
    plt.xlabel('epochs')
    plt.ylabel('classification error')
    plt.yticks([0, 1, 2, 3])
    plt.title('learning at {}'.format(rates[i]))
    plt.savefig('./figures/3.1b_{}.png'.format(rates[i]))

plt.show()

```

OUTPUT: -



Experiment-5

Aim: - 3.2

Apparatus: -

- Laptop Configuration CPU, GPU, Core, clock etc.
- Laptop Configuration
 - MacBook Air M1
 - 8GB RAM
 - 8-Core CPU
 - 7-Core GPU
- Libraries Used: -
 - TensorFlow 1.40
 - NumPy
 - PyLab
 - Matplotlib
- Coding Environment: - Python 3.7.

Theory: -

Logistic Regression Neuron: - A Logistic regression neuron performs a binary classification of input, i.e., it classifies inputs into two classes with levels 0 & 1.

The activation of logistic regression neuron gives the probability of the neuron, belongs to class one.

Given an input \vec{X} , the activation of neuron is-

$$f(u) = P(y = 1/\vec{x}) \\ = \frac{1}{1 + e^{-u}}$$

Activation function of neuron is given by sigmoidal or logistic function.

The output y of the neuron is not equal to the activation: -

Now,

$$P\left(y = \frac{0}{\vec{x}}\right) = 1 - P(y = 1/\vec{x}) \\ = 1 - f(u)$$

And,

$$y = 1(f(u) > 0.5)$$

Given a training pattern \vec{X}, \vec{b} where $\vec{X} \in \mathbb{R}^n$ and $\vec{b} \in (0, 1)$.

The cost function of classification is given by cross entropy.

$$J = -d \cdot \log(f(u)) - (1 - d) \cdot \log(1 - f(u))$$

The cost function J, is minimized using the gradient descent procedure.

Computing the gradient $\left(\frac{\partial J}{\partial u}\right)$: -

$$\frac{\partial J}{\partial u} = -\frac{\partial}{\partial u} [d \cdot \log(f(u)) - (1-d) \cdot \log(1-f(u))] \times \frac{\partial f(u)}{\partial u}$$

$$\frac{\partial J}{\partial u} = -\left[\frac{d}{f(u)} - \frac{(1-d)}{(1-f(u))} \right] \times f'(u)$$

$$\text{where, } f'(u) = f(u) \cdot [1 - f(u)]$$

$$\text{thus, } \frac{\partial J}{\partial u} = \left[\frac{d - f(u)}{f(u)[1 - f(u)]} \right] \times f(u)[1 - f(u)]$$

$$\frac{\partial J}{\partial u} = -[d - f(u)]$$

Now,

$$\nabla_w J = \frac{\partial J}{\partial u} \times \frac{\partial u}{\partial w} = -[d - f(u)] \vec{x}$$

$$\nabla_b J = \frac{\partial J}{\partial u} \times \frac{\partial u}{\partial b} = -[d - f(u)]$$

Thus, our weight updating equation will become-

$$w \leftarrow w + \alpha [d - f(u)] \vec{x}$$

$$b \leftarrow b + \alpha [d - f(u)]$$

Learning Algorithm: -

for a given input patten \vec{X}, \vec{d}

1) Set learning Rate α

2) initialize (w, b)

3) Repet until the Convergence

$$w \leftarrow w + \alpha [d - f(u)] \vec{x}$$

$$b \leftarrow b + \alpha [d - f(u)]$$

Python CODE: -

```
import tensorflow as tf
import numpy as np
import pylab as plt
from mpl_toolkits.mplot3d import Axes3D

import os
if not os.path.isdir('figures'):
    print('creating the figures folder')
    os.makedirs('figures')

tf.compat.v1.logging.set_verbosity(tf.compat.v1.logging.ERROR)

no_iters = 300
lr = 0.4
```

```

SEED = 10
np.random.seed(SEED)

# training data
x_train = np.array([[1.33, 0.72], [-1.55, -0.01], [0.62, -0.72],
                    [0.27, 0.11], [0.0, -0.17], [0.43, 1.2], [-0.97, 1.03], [0.23, 0.45]])
y_train = np.array([0, 1, 1, 1, 1, 0, 0, 0]).reshape(8,1)

print(x_train)
print(y_train)
print(lr)

# Model parameters
w = tf.Variable(np.random.rand(2,1), dtype=tf.float32)
b = tf.Variable(0., dtype=tf.float32)

# Model input and output
x = tf.placeholder(tf.float32, x_train.shape)
d = tf.placeholder(tf.int32, y_train.shape)

u = tf.matmul(x, w) + b
f_u = tf.sigmoid(u)
d_float = tf.cast(d, tf.float32)

loss = -tf.reduce_sum(d_float*tf.log(f_u) + (1-d_float)*tf.log(1-f_u))
class_err = tf.reduce_sum(tf.cast(tf.not_equal(f_u > 0.5, y_train), tf.int32))

grad_u = -(d_float - f_u)
grad_w = tf.matmul(tf.transpose(x), grad_u)
grad_b = tf.reduce_sum(grad_u)

w_new = w.assign(w - lr*grad_w)
b_new = b.assign(b - lr*grad_b)

# training loop
init = tf.global_variables_initializer()
sess = tf.Session()
sess.run(init) # reset values to wrong
w_, b_ = sess.run([w, b])
print('w: {}, b: {}'.format(w_, b_))

err = []
c_err = []
for i in range(no_iters):
    u, f_u, loss_, c_err_, w_, b_ = sess.run([u, f_u, loss, class_err, w_new, b_new],
        {x: x_train, d: y_train})

```

```

if (i == 0):
    print('u:{}'.format(u_))
    print('f_u:{}'.format(f_u_))
    print('y:{}'.format(f_u_ > 0.5))
    print('loss:{}'.format(loss_))
    print('error:{}'.format(c_err_))
    print('w: {}, b: {}'.format(w_, b_))

err.append(loss_)
c_err.append(c_err_)

if (i%10 == 0):
    print('iter: {}, err: {}, cost: {}'.format(i, c_err[i], err[i]))

# evaluate training accuracy
print('w: {}, b: {}'.format(w_, b_))

print(f_u_ > 0.5)

plt.figure(1)
plt.plot(x_train[y_train[:,0]==1,0], x_train[y_train[:,0]==1,1], 'bx', label='class A')
plt.plot(x_train[y_train[:,0]==0,0], x_train[y_train[:,0]==0,1], 'ro', label='class B')
plt.xlabel('$x_1$')
plt.ylabel('$x_2$')
plt.title('training data')
plt.legend()
plt.savefig('./figures/3.2_1.png')

plt.figure(2)
plt.plot(range(no_iters), err)
plt.xlabel('epochs')
plt.ylabel('cross-entropy')
plt.savefig('./figures/3.2_2.png')

plt.figure(3)
plt.plot(range(25), np.array(c_err)[:25])
plt.xlabel('epochs')
plt.ylabel('classification error')
plt.savefig('./figures/3.2_3.png')

x1 = np.arange(-2, 2, 0.1)
x2 = -(x1*w_[0] + b_)/w_[1]

plt.figure(4)
plt.plot(x_train[y_train[:,0]==1,0], x_train[y_train[:,0]==1,1], 'bx', label='class A')
plt.plot(x_train[y_train[:,0]==0,0], x_train[y_train[:,0]==0,1], 'ro', label='class B')
plt.plot(x1, x2, '-')
plt.xlabel('$x_1$')

```

```
plt.ylabel('$x_2$')
plt.title('decision boundary')
plt.legend()
plt.savefig('./figures/3.2_4.png')
```

```
plt.show()
```

OUTPUT: -

```
[[ 1.33  0.72]
 [-1.55 -0.01]
 [ 0.62 -0.72]
 [ 0.27  0.11]
 [ 0.   -0.17]
 [ 0.43  1.2 ]
 [-0.97  1.03]
 [ 0.23  0.45]]
[[0]
 [1]
 [1]
 [1]
 [1]
 [0]
 [0]
 [0]]
0.4
w: [[0.77132064]
     [0.02075195]], b: 0.0
u:[[ 1.040798 ]
   [-1.1957545 ]
   [ 0.4632774 ]
   [ 0.2105393 ]
   [-0.00352783]
   [ 0.3565702 ]
   [-0.7268065 ]
   [ 0.18674213]]
f_u:[[0.73900396]
     [0.2322313 ]
     [0.6137914 ]
     [0.5524413 ]
     [0.49911806]
     [0.5882099 ]
     [0.3258959 ]
     [0.54655033]]
y:[[ True]
   [False]
   [ True]
   [ True]
   [False]
   [ True]
   [False]
   [ True]]
loss:6.6521759033203125
error:5
w: [[ 0.021263 ]
     [-0.8357367]], b: -0.038896895945072174
iter: 0, err: 5, cost: 6.6521759033203125
iter: 10, err: 0, cost: 1.4870713949203491
:
:
:
```



```
iter: 280, err: 0, cost: 0.2432633638381958
iter: 290, err: 0, cost: 0.23725102841854095
w: [[ -1.2952098]
 [-12.666678 ]], b: 3.8386905193328857
[[False]
 [ True]
 [ True]
 [ True]
 [ True]
 [False]
 [False]
 [False]]
```

