Credit Analytics Bond RV Calculation Methodology

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Introduction

This document outlines the methodology used in Credit Analytics (release 1.4 and above) for the calculation of the bond curve-based relative value measures.

The Bond RV Measure Set

Classification of a given bond measure as an "RV" measure is somewhat arbitrary. In general, it is used (here) to refer to any of the measure that is in use for spotting relative value across bonds for a given issuer (or any similar category), and which is usually determined straight from a bond market measure (price/yield/spread to treasury). Specifically, it excludes such bond measures as DV01, loss PV, principal PV etc.

Following is the list of the RV measures - refer to the section below for a precise definition of these terms.

- Bond Basis
- Convexity

- Credit Basis
- Discount Margin
- Duration
- G Spread (Spread to the Government/Treasury Discount Curve)
- I Spread (Interpolated Spread to the Discount Curve)
- Option Adjusted Spread
- Par Asset Swap Spread
- Par Equivalent CDS Spread (PECS)
- Price
- Spread over Treasury (TSY) benchmark
- Yield
- Yield Basis
- Yield Spread
- Zero Discount Margin (ZDM)
- Zero (Z) Spread

Bond Analytical Measures Calculation

Definitions, Symbols, and Terminology

Bond Basis

Bond Basis to Exercise (B_E) is a bond RV metric capturing the basis in the yield space. It is defined as the difference between the yield to exercise computed from the market price and the yield to exercise computed from the theoretical price off of the risk-free discount curve.

Convexity

Convexity to Exercise (C_E) measures the rate of change of duration with yield. It is defined as the change in market duration on 1 basis point increase in yield.

Credit Basis

Credit Basis to Exercise (Φ_E) captures the adjustment needed to the input credit curve to account for the bond market price. It is defined as the parallel shift needed to be applied across the input credit curves quotes to make create the credit curve that produces the market price.

Credit Basis can be negative; given that the credit curve does not typically calibrate for negative hazard rates, the credit basis may not be calculable for market prices above a certain range.

Discount Margin

Discount Margin to Exercise (Δ_E) measures that spread earned above the reference rate. For fixed coupon bonds, it is computed as the difference between market yield and the initial implied discount rate to the bond’s frequency. For floaters, it is computed as the difference between market yield and the initial reference index rate.

Duration

Duration to Exercise (D_E) captures the relative rate of change of bond price with yield. It is defined as the fractional change of price as the market yield increases by 1 basis point.

G Spread

G Spread to Exercise (G_E) accounts for the Spread over the Government/Treasury Discount Curve. It is defined as the difference between the market yield to exercise of the bond and the rate calculated to the exercise date, implied from the specified discount curve constructed from the government debt instruments.

I Spread

I-Spread to Exercise (I_E) measures the spread over the specified Discount Curve interpolated to the exercise date. It is defined as the difference between the market yield to exercise of the bond and the rate interpolated to the exercise date, implied from the specified discount curve.

Option Adjusted Spread

Option adjusted to Exercise (O_E) spread captures the value of the optionality embedded in the bond. It is calculated identical to the Z-Spread (see Z-Spread for details).

Par Asset Swap Spread

Par asset swap spread to Exercise (P_E) estimates the spread implied by the price that a par floater would be expected to pay. It is defined as the difference between the market price and the theoretical price computed using the discount curve, computed in units of the bond PV01 (duration times price).

Par Spread

Par spread to Exercise (Ω_E) estimates the fair fixed coupon implied by the market price that an equivalent fixed coupon bond trading at par would pay. It is defined as the difference between the market price and par, computed in units of the bond PV01 (duration times price).

Par Equivalent CDS Spread (PECS)

The PECS to Exercise (Θ_E) measures the flat credit spread premium implied by the bond price. It is computed as the implied flat spread of the fictitious CDS needed to recover the market price of the bond.

Price

The theoretical exercise price of the bond can be computed from the bond cash flows, the discount curve and/or the credit curve and recovery using the methodology described below.

Spread over Treasury (TSY) benchmark

Treasury Spread to Exercise (S_{TSY}) accounts for the returns over the given benchmark bond. It is defined as the difference between the market yield to exercise of the bond and the yield to maturity of the specified benchmark treasury bond.

Yield

The yield to exercise (y_E) implied from the bond market price is calculated according to the equations shown below.

Yield Basis

Yield basis to Exercise is defined identically as the bond basis. See Bond Basis for details.

Yield Spread

Yield spread is defined identically as the bond basis. See Bond Basis for details.

Zero Discount Margin (ZDM)

Zero Discount Margin to Exercise (Ψ_E) estimates the excess spread over the reference index curve. It is a measure valid only floaters; it is defined as the extra coupon spread to be applied to the reference index rate curve so as to be able to recover the market price.

Zero (Z) Spread

Z Spread to Exercise (Z_S) captures the excess spread over the discount curve. The details of implying the zero-curve and the corresponding calculation of the Z Spread are described below.

Symbol	**Description**
B_E	Bond Basis to Exercise
C_E	Convexity to Exercise
Φ_E	Credit Basis to Exercise
Δ_E	Discount Margin to Exercise
D_E	Duration to Exercise
G_E	G-Spread to Exercise
I_E	I Spread to Exercise

O_E	Option Adjusted Spread to Exercise
P_E	Par Asset Swap Spread to Exercise
Ω_E	Par Spread to Exercise
Θ_E	Par Equivalent CDS Spread to Exercise
Ψ_E	Zero Discount Margin to Exercise
ε_i	The Full Period Coupon Rate between t_{i-1} and t_i
φ_E	Government Curve implied Rate to Exercise
$\Gamma_c(i-1,i)$	Coupon Day Count Fraction between t_{i-1} and t_i
$\Gamma_y(i-1,i)$	Yield Quote Day Count Fraction between t_{i-1} and t_i
δ_{IR}	Spread applied to the Interest Rate curve
d_c	Coupon Day Count Convention
d_{yc}	Yield Quote Day Count Convention
f_c	Coupon Frequency
f_y	Frequency for Yield Quote
t_i	Time at coupon flow # i
t_E	Exercise Date Time
y_E	Yield To Exercise
$C_f(t_i)$	Coupon Flow at Date Time t_i
$D_f(t_i)$	Discount Curve based Discount Factor at Date Time t_i
$D_f(\delta,t_i)$	δ Bumped Discount Curve based Discount Factor at Date Time t_i
$D_f(y_E, f_y, d_{yc}, t_j)$	Discount Factor at Date Time t_i given Yield To Exercise y_E, Quote Frequency f_y, Quote Day Count Convention d_{yc}

$D_f(z_s, f_y, d_{yc}, t_i)$	Discount Factor at date time t_i given the Z Spread z_S, the quote frequency f_y, Quote Day Count Convention d_{yc}
N_E	Notional at Exercise
N_j	Outstanding Notional at Date Time t_j
ΔN_j	Principal Notional Payout at Date Time t_j
$P_{Dirty}(IR_{Theo})$	Theoretical Dirty Price calculated from the input IR Curve
$P_{CR,Dirty}(IR_{Theo}, CR_{Theo})$	Theoretical Dirty Price calculated from the input IR and Credit Curves
$P_{Dirty}(\delta, IR_{Theo})$	Theoretical Dirty Price calculated from the input IR Curve with a spread adjustment
$P_{CR,Dirty}(\lambda_{CR}, IR_{Theo}, CR_{Theo})$	Theoretical Dirty Price calculated from the input IR Curve and Credit Curve, where the Credit Curve is created off of a flat spread λ_{CR}
$P_{CR,Dirty}(\delta_{CR}, IR_{Theo}, CR_{Theo})$	Theoretical Dirty Price calculated from the input IR Curve and Credit Curve, with a spread adjustment applied to the Credit Curve
R_E	Discount Curve implied Rate to Exercise
$S_P(t)$	Survival Probability at time t
S_{TSY}	Treasury Benchmark Spread to Exercise (done)
y_{BMK}	Yield of the Specified Treasury Benchmark
y_E	Yield to Exercise
$y_E(IR_{Theo})$	Theoretical Yield to exercise
$\{z_i\}$	Collection of the ordered nodes { z_i, z_i, …, z_i } that constitute the Zero Curve
z_i	Zero Rate to the Date Time t_i
z_S	Z Spread

Basic Measures

Equation (1): The Coupon Cash Flow of the bond at coupon date time t_i is given as

$$C_f(t_i) = \varepsilon_i \Gamma_c(i-1, i) d_c$$

Equation (2): The Discount Factor at date time t given the yield to exercise y_E, the quote frequency f_y, and the annualized quote day count based time fraction $\Gamma_y(i-1, i)$ is given as

$$D_f(y_E, f_y, d_{yc}, t) = \frac{1}{\left(1 + \frac{y_E}{f_y}\right)^{f_y \Gamma(0,t)}}$$

Equation (3): The Zero Rate z_i to a date time t_i is determined by the solution to z_i that computes the discount factor $D_f(t_i)$ given the quote frequency f_y, and the annualized quote day count based time fraction $\Gamma_y(i-1, i)$ is given as

$$D_f(t_i) = \frac{1}{\left(1 + \frac{z_i}{f_y}\right)^{f_y \Gamma(0,t)}}$$

Equation (4): The Discount Factor at date time t_i given the zero rate z_i, the Z Spread z_S, the quote frequency f_y, and the annualized quote day count based time fraction $\Gamma_y(i-1,i)$ is given as

$$D_f(z_s, f_y, d_{yc}, t_i) = \frac{1}{(1+\frac{z_i + z_S}{f_y})^{f_y \Gamma(0,t_i)}}$$

Equation (5): The Principal redeemed, amortized, or capitalized at time t_j is given as

$$\Delta N_j = N_j - N_{j-1}$$

Equation (6): The Dirty Price of the bond at exercise given an exercise yield y_E is given as

$$P_{Dirty}(y_E) = \sum_i C_f(t_i) D_f(y_E, f_y, d_{yc}, t_j) + \sum_j \Delta N_j D_f(y_E, f_y, d_{yc}, t_j) + N_E D_f(y_E, f_y, d_{yc}, t_E)$$

Equation (7): The Dirty Price of the bond at exercise given a Z spread (z_S) is given as

$$P_{Dirty}(z_S) = \sum_i C_f(t_i) D_f(z_S, f_y, d_{yc}, t_j) + \sum_j \Delta N_j D_f(z_S, f_y, d_{yc}, t_j) + N_E D_f(z_S, f_y, d_{yc}, t_E)$$

Equation (8): The Theoretical IR implied Dirty Price $P_{Dirty}(IR_{Theo})$ of the bond at exercise calculated using the discount factors from the input discount curve is given as

$$P_{Dirty}(IR_{Theo}) = \sum_i C_f(t_i) D_f(t_j) + \sum_j \Delta N_j D_f(t_j) + N_E D_f(t_E)$$

Equation (9): The IR implied Dirty Price $P_{Dirty}(\delta_{IR}, IR_{Theo})$ of the bond at exercise calculated using the discount factors from the input discount curve bumped by a rate δ_{IR} is given as

$$P_{Dirty}(\delta_{IR}, IR_{Theo}) = \sum_i C_f(t_i) D_f(\delta_{IR}, t_j) + \sum_j \Delta N_j D_f(\delta_{IR}, t_j) + N_E D_f(\delta_{IR}, t_E)$$

Equation (10): The Theoretical Credit implied Dirty Price $P_{CR,Dirty}(IR_{Theo}, CR_{Theo})$ of the bond at exercise calculated using the discount factors and the survival probabilities from the input discount curve and the credit curve respectively is given as

$$P_{CR,Dirty}(IR_{Theo}, CR_{Theo}) = \sum_i C_f(t_i) D_f(t_j) S_P(t_j) + \sum_j \Delta N_j D_f(t_j) + N_E D_f(t_E) S_P(t_E)$$

Equation (11): The Theoretical Credit implied Dirty Price $P_{CR,Dirty}(\delta_{CR}, IR_{Theo}, CR_{Theo})$ of the bond at exercise calculated using the discount factors and the survival probabilities from the input discount curve and the credit curve respectively, where the credit curve is bumped by a rate δ_{CR}, is given as

$$P_{CR,Dirty}(\delta_{CR}, IR_{Theo}, CR_{Theo}) = \sum_i C_f(t_i) D_f(t_j) S_P(\delta_{CR}, t_j) + \sum_j \Delta N_j D_f(t_j) + N_E D_f(t_E) S_P(\delta_{CR}, t_E)$$

Equation (12): The Credit Basis to Exercise Φ_E of the bond given the market price (P_{MKT}) is given as the solution of δ_{CR} in Equation (11):

$$P_{MKT} = \sum_i C_f(t_i) D_f(t_j) S_P(\delta_{CR}, t_j) + \sum_j \Delta N_j D_f(t_j) + N_E D_f(t_E) S_P(\delta_{CR}, t_E)$$

Equation (13): The Theoretical Credit implied Dirty Price of the bond at exercise $P_{CR,Dirty}(\lambda_{CR}, IR_{Theo}, CR_{Theo})$ is calculated using the discount factors and the survival probabilities from the input discount curve and the credit curve respectively, where the credit curve is created off of a flat spread λ_{CR}, is given as

$$P_{CR,Dirty}(\lambda_{CR}, IR_{Theo}, CR_{Theo}) = \sum_i C_f(t_i) D_f(t_j) S_P(\lambda_{CR}, t_j) + \sum_j \Delta N_j D_f(t_j) + N_E D_f(t_E) S_P(\lambda_{CR}, t_E)$$

Equation (14): The Par Equivalent CDS Spread to Exercise of the bond given the market price (P_{MKT}) is given as the solution of δ_{CR} in Equation (13):

$$P_{MKT} = \sum_i C_f(t_i) D_f(t_j) S_P(\lambda_{CR}, t_j) + \sum_j \Delta N_j D_f(t_j) + N_E D_f(t_E) S_P(\lambda_{CR}, t_E)$$

Equation (15): The Bond Spread to Treasury Benchmark at exercise S_{TSY} is computed from the Bond Yield to Exercise y_E and the given Treasury Benchmark Yield y_{BMK} as

$$S_{TSY} = y_E - y_{BMK}$$

Equation (16): The Bond I Spread to exercise I_E is computed from the Bond Yield to Exercise y_E and the Discount rate to Exercise implied from the input Interest Rate Curve R_E as

$$I_E = y_E - R_E$$

Equation (17): The Bond G Spread to exercise G_E is computed from the Bond Yield to Exercise y_E and the Discount rate to Exercise implied from the input Government Rate Curve φ_E as

$$G_E = y_E - \varphi_E$$

Equation (18): The Theoretical Yield to exercise $y_E(IR_{Theo})$ of the bond at exercise calculated using the discount factors from the input discount curve is given as the solution of y_E in Equation (6), where the dirty price P_{Dirty} is substituted by $P_{Dirty}(IR_{Theo})$ of Equation (8).

Equation (19): The Bond Basis at exercise B_E (also referred to as yield basis or as yield spread) is computed from the Bond Yield to Exercise y_E and the Bond Yield to Exercise y_E as

$$\mathrm{B}_E = y_E - y_E(IR_{Theo})$$

Equation (20): The Bond Duration to exercise D_E is computed as the fractional change in bond market price (P_{MKT}) to the change in the market yield (Y_{MKT}) as

$$D_E = \frac{1}{P_{MKT}} \frac{\Delta P_{MKT}}{\Delta Y_{MKT}}$$

Equation (21): The Bond Convexity to exercise C_E is computed as the change in bond market duration to exercise (D_E) to the change in the market yield (Y_{MKT}) as

$$C_E = \frac{\Delta D_E}{\Delta Y_{MKT}}$$

Equation (22): The Discount Margin to Exercise Δ_E of the bond given the market yield to exercise (y_E) is given as:

$$\Delta_E = y_E - R_E$$

Equation (23): The Par Asset Swap Spread to Exercise (P_E) of the bond given the market price (P_{MKT}) is given as:

$$\mathrm{P}_E = \frac{1}{P_{MKT}} \frac{P_{Dirty}(IR_{Theo}) - P_{MKT}}{D_E}$$

Equation (24): The Option Adjusted Spread to Exercise O_E is calculated identical to Z Spread, as a solution to z_S in Equation (7).