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**The Generalized AutoRegressive Conditional Heteroskedasticity  
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# The Generalized AutoRegressive Conditional Heteroskedasticity Parkinson Range (GARCH-PARK-R) Model for Forecasting Financial Volatility<sup>1</sup>

by

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## ABSTRACT

A new variant of the ARCH class of models for forecasting conditional variance, to be called the Generalized AutoRegressive Conditional Heteroskedasticity Parkinson Range (GARCH-PARK-R) Model, is proposed. The GARCH-PARK-R model, utilizing the extreme values, is a good alternative to the Realized Volatility that requires a large amount of intra-daily data, which remain relatively costly and are not readily available. Estimates of the GARCH-PARK-R model are derived using Quasi-Maximum Likelihood Estimation (QMLE). The results suggest that the GARCH-PARK-R model is a good middle ground between intra-daily models, such as the Realized Volatility and inter-daily models, such as the ARCH class. The forecasting performance of the models is evaluated using the daily Philippine Peso-U.S. Dollar exchange rate from January 1997 to December 2003.

*Key Words: Volatility, GARCH-PARK-R, QMLE*

## I. INTRODUCTION

Since the introduction of the seminal paper on the AutoRegressive Conditional Heteroskedasticity (ARCH) process of Robert Engle in 1982, researches on financial and time series econometrics have been dominated by the extensions of the ARCH process. One particular difficulty experienced in evaluating the various ARCH-type models is the fact that volatility is not directly measurable – the conditional variance is unobservable. The absence of such a benchmark that we can use to compare the forecasts of the various models makes it difficult to identify the good models from the bad ones.

Anderson and Bollerslev (1998) introduced the concept of “realized volatility” from which evaluation of the ARCH volatility models are to be made. Realized volatility models are calculated from high-frequency intra-daily data, rather than inter-daily data. In their seminal paper, Anderson and Bollerslev collected information on the DM-Dollar and Yen-Dollar spot exchange rates every five-minute interval, resulting in a total of 288 5-minute observations per day! The 288 observations

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<sup>1</sup> This paper was derived mainly from Chapter 4 of the M.S. (Statistics) thesis of the first author.

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were then used to compute the variance of the exchange rate of a particular day. Although volatility is an instantaneous phenomenon, the concept of realized volatility is by far the closest we have to a “model-free” measure of volatility.

Obviously, there is a trade-off when one is interested in estimating the conditional variance using realized volatility. While it may provide a model-free estimate of the unknown conditional variance, the data requirement (getting observation every 5 minutes, for instance) is simply enormous. In the case of the Philippines, the Philippine Stock Exchange (PSE) starts trading at 9:30 a.m. up to 12:00 noon, for a total of 150 minutes of trading time or 30 5-minute observations. Given the low market activity, it is highly probable that the price of a particular stock will not move during that 5-minute period. The same problem might be encountered in the foreign exchange market in the Philippine Dealing System (PDS). Data problem may hinder the use of realized volatility for emerging markets such as the Philippines.

An alternative approach to model volatility using intra-daily data is through the use of the range, the difference between the highest and lowest values for the day. The range is a popular measure of volatility in the area of quality control. The range is convenient to use, especially for researchers who do not have access to information on the trading floors of various markets, since major papers normally report the highest and lowest values of assets (stock prices, currencies, interest rates, etc.), together with the opening and closing prices.

This paper proposes the use of the Range, specifically the Parkinson Range, in estimating the conditional variance. The model will be called the Generalized Auto-Regressive Conditional Heteroskedasticity Parkinson Range (GARCH-PARK-R) model. This paper is organized as follows: section 1 serves as the introduction, section 2 discusses the ARCH process and its extensions. Section 3 introduces the concept of realized volatility. The GARCH-PARK-R model and the estimation procedure are discussed in section 4. Section 5 provides the empirical results and section 6 concludes.

## II. The ARCH Process and its Extensions

In this section, the ARCH process will be formally defined and some of its important properties discussed. Let  $\{u_t(\theta), t \in (\dots, -1, 0, 1, \dots)\}$  denote a discrete time stochastic process with the conditional mean and variance functions having parameterized by the finite dimensional vector  $\theta \in \Theta \subseteq \mathbb{R}^m$  and let  $\theta_0$  denote the true value of the parameter.

Let  $E[(\bullet) | I_{t-1}]$  or  $E_{t-1}(\bullet)$  denote the mathematical expectation conditioned on the information available at time  $(t-1)$ ,  $I_{t-1}$ .

**Definition 1.** In the relationship,  $u_t = Z_t \sigma_t$ , the stochastic process  $\{u_t(\theta), t \in (-\infty, \infty)\}$  follows an ARCH process if:

$$a. E(u_t(\theta_0) | I_{t-1}) = 0, \text{ for } t = 1, 2, \dots$$

- b.  $\text{Var}(u_t(\theta_0) | I_{t-1}) = \sigma_t^2(\theta_0)$  depends non-trivially on the sigma field generated by the past observations,  $\{u_{t-1}^2(\theta_0), u_{t-2}^2(\theta_0), \dots\}$ .

$\sigma_t^2(\theta_0) \equiv \sigma_t^2$  is the conditional variance of the process, conditioned on the information set  $I_{t-1}$ . The conditional variance is central to the ARCH process.

Letting  $Z_t(\theta_0) = u_t(\theta_0)/\sigma_t(\theta_0)$ ,  $t = 1, 2, \dots$  we have the standardized process  $\{Z_t(\theta_0) | t \in (-\infty, \infty)\}$  and it follows that,

$$\begin{aligned} \text{(i)} \quad E[Z_t(\theta_0) | I_{t-1}] &= 0 \quad \forall t \\ \text{(ii)} \quad \text{Var}[Z_t(\theta_0) | I_{t-1}] &= 1 \quad \forall t \end{aligned}$$

Thus, the conditional variance of  $Z_t(\cdot)$  is time invariant. Moreover, if we assume that the conditional distribution of  $Z_t(\cdot)$  is time invariant with a finite fourth moment, it follows from Jensen's inequality that,

$$E(u_t^4) = E(Z_t^4)E(\sigma_t^4) \geq E(Z_t^4) [E(\sigma_t^2)]^2 = E(Z_t^4) [E(u_t^2)]^2 \quad (1)$$

with the last equality holding only when the conditional variance is constant. Assuming that  $Z_t(\cdot)$  is normally distributed, it follows that the unconditional distribution of  $u_t$  is leptokurtic.

Engle (1982) has shown that for the first order ARCH process,

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$$

the unconditional variance and the fourth moment of this process are, respectively, given by,

$$\begin{aligned} E(u_t^2) &= \frac{\alpha_0}{1 - \alpha_1} \\ E(u_t^4) &= \left[ \frac{3\alpha_0^2}{(1 - \alpha_1)^2} \right] \left[ \frac{1 - \alpha_1^2}{1 - 3\alpha_1^2} \right] \end{aligned}$$

The condition for the variance to be finite is that  $\alpha_1 < 1$  and for the fourth moment,  $3\alpha_1^2 < 1$ .

Using the results in (1), it implies that  $E(u_t^4)/[E(u_t^2)]^2 \geq E(Z_t^4)$ . Thus, for the first order ARCH process,

$$\frac{E(u_t^4)}{(E(u_t^2))^2} = \frac{3(1 - \alpha_1^2)}{(1 - 3\alpha_1^2)} > 3$$

This result implies that the ARCH (1) process is a heavy-tailed distribution, that is, the process generates data with fatter tails than the normal distribution. This particular characteristic of the ARCH process is relevant in modeling time series

data especially financial time-series, like stock returns and asset prices, since these series tend to have thick-tailed distributions.

In general, the ARCH (q) process can be defined as,

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \cdots + \alpha_q u_{t-q}^2$$

For this model to be well defined and have a positive conditional variance almost surely, the parameters must satisfy  $\omega > 0$  and  $\alpha_1, \dots, \alpha_q \geq 0$ . Following the natural extension of the ARMA process as a parsimonious representation of a higher order AR process, Bollerslev (1986) extended the work of Engle to the Generalized ARCH or GARCH process. The GARCH (p,q) process is defined as,

$$\sigma_t^2 = \omega + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{i=1}^q \alpha_i u_{t-i}^2$$

$$\omega > 0, \alpha_i \geq 0, \beta_j \geq 0 \quad i = 1, \dots, q \quad j = 1, \dots, p$$

The conditional variance is a linear function of q lags of the squares of the error terms ( $u_t^2$ ) or the ARCH terms (also referred to as the “news” from the past) and p lags of the past values of the conditional variances ( $\sigma_t^2$ ) or the GARCH terms, and a constant  $\omega$ . The inequality restrictions are imposed to guarantee a positive conditional variance, almost surely.

The GARCH model of Bollerslev had been extended to account for an asymmetric response to a shock and this model is called an Exponential GARCH or EGARCH (Nelson, 1991). A model that accounts for the asymmetric effect of the “news” is the Threshold GARCH or TARCH model due independently to Zakoian (1994) and Glosten, Jaganathan and Runkle (1993). A GARCH model using the standard deviation was introduced independently by Taylor (1986) and Schwert (1989). These models are using the conditional standard deviation as a measure of volatility instead of the conditional variance. Such models were generalized by Ding *et al.* (1993) into the Power ARCH or PARCH model. A detailed discussion of the various models is provided in Mapa (2004).

### III. The Realized Volatility

This section provides a brief discussion of the concept of Realized Volatility. Let  $P_{n,t}$  denote the price of an asset (say US\$ 1 in Philippine Peso) at time  $n \geq 0$  at day t, where  $n = 1, 2, \dots, N$  and  $t = 1, 2, \dots, T$ . Note that when  $n=1$ ,  $P_t$  is simply the inter-daily price of the asset (normally recorded as the closing price). Let  $p_{n,t} = \log(P_{n,t})$ , denote the natural logarithm of the price of the asset. The observed discrete time series of continuously compounded returns with N observations per day is given by,

$$r_{n,t} = p_{n,t} - p_{n-1,t} \quad (2)$$

When  $n=1$ , we simply ignore the subscript  $n$  and  $r_t = p_t - p_{t-1} = \log(P_t) - \log(P_{t-1})$  where  $t = 2, \dots, T$ . In this case,  $r_t$  is the time series of daily return and is also the covariance-stationary series. In (6), we assume that:

- (a)  $E(r_{n,t}) = 0$
- (b)  $E(r_{n,t} r_{m,s}) = 0$  for  $n \neq m$  and  $t \neq s$
- (c)  $E(r_{n,t}^2 r_{m,s}^2) < \infty$  for  $n, m, s, t$

From (2), the continuously compounded daily return (Campbell, Lo, and Mackinlay, 1997 p.11) is given by,

$$r_t = \sum_{n=1}^N r_{n,t} \quad (3)$$

and the continuously compounded daily squared returns is,

$$\begin{aligned} r_t^2 &= \left( \sum_{n=1}^N r_{n,t} \right)^2 = \sum_{n=1}^N r_{n,t}^2 + \sum_{n=1}^N \sum_{m=1}^N r_{n,t} r_{m,t} \\ &= \sum_{n=1}^N r_{n,t}^2 + 2 \sum_{n=1}^N \sum_{m=n+1}^N r_{n,t} r_{m-n,t} \end{aligned} \quad (4)$$

Note that  $\sigma_t^2 = \text{Var}(r_t) = E(r_t^2)$  since  $E(r_t) = 0$ . From (4) and using assumption (b) of (2) above, we have,

$$\begin{aligned} \sigma_t^2 &= E(r_t^2) = E(s_t^2) \quad (5) \\ \text{where } s_t^2 &= \sum_{n=1}^N r_{n,t}^2 \end{aligned}$$

Thus, the sum of the intra-daily squared returns is an unbiased estimator of the daily population variance. The sum of the intra-daily squared returns is known as the **realized volatility** (also called the realized variance by Barndorff-Nielsen and Shephard (2002)). Given enough observations for a trading day, the realized volatility can be computed and is a model-free estimate of the conditional variance. The properties of the realized volatility are discussed in Anderson, Bollerslev, Diebold and Labys (1999). In particular, the authors found that the realized volatility is a consistent estimator of the daily population variance,  $\sigma_t^2$ .

#### IV. The GARCH-PARK-R Model

While the concept of realized volatility does provide a highly efficient way of estimating the unknown conditional variance, the problem of generating information on the price of an asset every five minutes or so is simply enormous. An alternative measure is to use extreme values, the highest and lowest prices of an asset, to produce two intra-daily observations. The range, the difference between the highest and lowest log prices, is a good proxy for volatility. The

range has the advantage of being available for researchers since high and low prices are available daily for a variety of financial time series.

Parkinson (1980) was the first to make use of the range in measuring volatility in the financial market. Parkinson developed the PARK daily volatility estimator based on the assumption that the intra-daily prices follow as Brownian motion. This study will make use of the PARK Range in modeling time-varying volatility. The model will be called the Generalized Auto-Regressive Conditional Heteroskedasticity Parkinson Range (GARCH-PARK-R) model.

Consider the covariance-stationary time series  $\{R_{pt}\}$  where,

$$R_{P_t} = \frac{\log(P_{(N),t}) - \log(P_{(1),t})}{\sqrt{4 \log(2)}} \quad t = 1, 2, \dots, T \quad (6)$$

$R_{Pt}$  is the PARK-Range of the asset at time  $t$ . Moreover, let  $R_{Pt} \geq 0$  for all  $t$  and that  $P(R_{Pt} < \delta | R_{Pt-1}, R_{Pt-2}, \dots) > 0$  for any  $\delta > 0$  and for all  $t$ . This condition states that the probability of observing zeros or near zeros in  $R_{Pt}$  is greater than zero.

Let,

$\mu_t = E[R_{Pt} | \mathcal{I}_{t-1}]$  be the conditional mean of the PARK range  
and

$\sigma_t^2 = \text{Var}[R_{Pt} | \mathcal{I}_{t-1}]$  be the conditional variance of the PARK range

The motivation behind the GARCH-PARK-R model is the Auto-Regressive Conditional Duration (ACD) model of Engle and Russel (1998) used to model observations that arrive at irregular intervals.

Let,

$$R_{P_t} = \mu_t \varepsilon_t \quad \text{where } \varepsilon_t | \mathcal{I}_{t-1} \sim iid(1, \phi_t^2) \quad \text{and}$$

$$\mu_t = \omega + \sum_{j=1}^q \alpha_j R_{P_{t-j}} + \sum_{j=1}^p \beta_j \mu_{t-j} \quad (7)$$

The model in (12) is known as the GARCH-PARK-R process of orders  $p$  and  $q$ . The mean and variance of the PARK range are given by,

$$\begin{aligned} (a) \quad E(R_{P_t}) &= \mu_t \\ (b) \quad \text{Var}(R_{P_t}) &= E(R_{P_t}^2) - [E(R_{P_t})]^2 \\ &= \mu_t^2 E(\varepsilon_t^2) - \mu_t^2 \\ &= \mu_t^2 (\phi_t^2 + 1) - \mu_t^2 = \mu_t^2 \phi_t^2 \end{aligned} \quad (8)$$

The GARCH-PARK-R model is similar to the Conditional Auto-Regressive Range (CARR) model suggested by Chou (2003). Two differences between this study and that of Chou's are to be clarified. First, this study uses the Parkinson range to

study volatility instead of the usual log range (Chou's measure). The Parkinson range has been found to be a better estimator of volatility (standard deviation). The second difference is the use of the data, this study makes use of the daily data while Chou's paper used weekly data. Weekly data might encounter problem in estimation due to the presence of aggregation effect and is the reason why the authors of this paper used the daily data.

For the density function of  $\varepsilon_t$  in (7), this study follows the suggestion of Engle and Russel (1998) and Engle and Gallo (2003) of using the gamma density,

$$f(\varepsilon_t | I_{t-1}) = \frac{\left(\frac{1}{\beta}\right)^\alpha}{\Gamma(\alpha)} \varepsilon_t^{\alpha-1} \exp\left\{-\frac{\varepsilon_t}{\beta}\right\} \quad (9)$$

Since  $E(\varepsilon_t) = \alpha\beta = 1$  (by assumption in (7)), it implies that  $\alpha = 1/\beta$ . Thus (9) now becomes,

$$\begin{aligned} f(\varepsilon_t | I_{t-1}) &= \frac{\alpha^\alpha}{\Gamma(\alpha)} \varepsilon_t^{\alpha-1} \exp\{-\alpha\varepsilon_t\} \\ \Rightarrow f(R_{P_t} | I_{t-1}) &= f\left(\frac{R_{P_t}}{\mu_t} | I_{t-1}\right) = \frac{1}{\Gamma(\alpha)} \alpha^\alpha \left(\frac{R_{P_t}}{\mu_t}\right)^{\alpha-1} \exp\left\{-\alpha\left(\frac{R_{P_t}}{\mu_t}\right)\right\} \\ &= \frac{1}{\Gamma(\alpha)} \alpha^\alpha (R_{P_t})^{\alpha-1} (\mu_t)^{-\alpha} \exp\left\{-\alpha\left(\frac{R_{P_t}}{\mu_t}\right)\right\} \\ &= \frac{\left(\frac{\alpha}{\mu_t}\right)^\alpha}{\Gamma(\alpha)} (R_{P_t})^{\alpha-1} \exp\left\{-R_{P_t}\left(\frac{\alpha}{\mu_t}\right)\right\} \end{aligned} \quad (10)$$

From (10), the conditional mean and variances of  $R_{P_t}$  are,

$$\begin{aligned} E(R_{P_t} | I_{t-1}) &= \frac{\alpha}{\alpha/\mu_t} = \mu_t \\ Var(R_{P_t} | I_{t-1}) &= \frac{\alpha}{\left(\frac{\alpha}{\mu_t}\right)^2} = \frac{(\mu_t)^2}{\alpha} \end{aligned}$$

The density function in (10) approaches the Gaussian density as  $\alpha$  increases. Moreover, the likelihood function is given by,

$$L = \prod_{t=1}^T \left[ \frac{1}{\Gamma(\alpha)} \alpha^\alpha (R_{P_t})^{\alpha-1} (\mu_t)^{-\alpha} \exp\left\{-\alpha\left(\frac{R_{P_t}}{\mu_t}\right)\right\} \right] \quad (11)$$



If the parameters of interest are only those that define  $\mu_t$  in (7), denoted by  $\mu_t(\underline{\theta})$ , then the log likelihood can be simplified into,

$$\begin{aligned}\log L &= \gamma - \alpha \sum_{t=1}^T \log(\mu_t) - \alpha \sum_{t=1}^T \left( \frac{R_{p_t}}{\mu_t} \right) \\ &= \gamma - \alpha \sum_{t=1}^T \left[ \log(\mu_t(\underline{\theta})) + \frac{R_{p_t}}{\mu_t(\underline{\theta})} \right]\end{aligned}\quad (12)$$

where  $\gamma = \gamma(\alpha, R_{p_t})$

Taking the derivative of the log likelihood function with respect to  $\underline{\theta}$ , we have,

$$\begin{aligned}\frac{\partial \log(L)}{\partial \underline{\theta}} &= -\alpha \sum_{t=1}^T \left[ \frac{1}{\mu_t(\underline{\theta})} \frac{\partial \mu_t(\underline{\theta})}{\partial \underline{\theta}} - \frac{R_{p_t}}{\mu_t^2(\underline{\theta})} \frac{\partial \mu_t(\underline{\theta})}{\partial \underline{\theta}} \right] = 0 \\ \Rightarrow \\ \sum_{t=1}^T \left[ \frac{R_{p_t}}{\mu_t^2(\underline{\theta})} - \frac{1}{\mu_t(\underline{\theta})} \right] \frac{\partial \mu_t(\underline{\theta})}{\partial \underline{\theta}} &= 0 \\ \Rightarrow \\ \sum_{t=1}^T \left[ \frac{R_{p_t} - \mu_t(\underline{\theta})}{\mu_t^2(\underline{\theta})} \right] \frac{\partial \mu_t(\underline{\theta})}{\partial \underline{\theta}} &= 0\end{aligned}\quad (13)$$

The parameter vector  $\underline{\theta}$  in (13) can be estimated numerically using some iterative algorithms such as the Marquardt or the BHHH.

An easier way of estimating the parameter vector  $\underline{\theta}$  is to apply the method of estimating the parameters of a GARCH (p,q) process. Recall that in the GARCH (p,q) process discussed in section 2,

$$u_t = \sigma_t Z_t \quad Z_t \sim N(0,1)$$

and

$$\sigma_t^2 = \omega + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{i=1}^q \alpha_i u_{t-i}^2$$

For the GARCH-PARK-R process let,

$$\begin{aligned}\sqrt{R_{p_t}} &= \sqrt{\mu_t} v_t & v_t &\sim iid(0,1) \\ E(\sqrt{R_{p_t}} | I_{t-1}) &= \sqrt{\mu_t} E(v_t) = 0 \\ \text{and } Var(\sqrt{R_{p_t}} | I_{t-1}) &= \mu_t E(v_t^2) = \mu_t \\ \text{with}\end{aligned}$$

$$\mu_t = \omega + \sum_{j=1}^q \alpha_j R_{p_{t-j}} + \sum_{j=1}^p \beta_j \mu_{t-j} \quad \text{given in (7)}$$

Thus, an analogous method of estimating the parameter vector  $\theta$  is to estimate the variance equation for the positive square root of the PARK R using GARCH (p,q) specification with zero in the mean specification. The Quasi-Maximum Likelihood estimators are consistent and distributed as Gaussian asymptotically even if the probability density function of the error is mis-specified following the results of Lee and Hansen and Lumsdaine for the GARCH (1,1) and Berkes *et al* (2003) for the GARCH (p,q) process.

## V. Empirical Results

This chapter discusses the results of forecasting conditional variance using the different ARCH and GARCH-PARK-R models. In this study, a total of 77 models were estimated: 68 ARCH-type models and 9 GARCH-PARK-R models. The model specifications are provided in Tables 1A and 1B below.

**Table 1A. Specification for ARCH-type Models \***

Model	Specification	Model	Specification
1	ARCH (1)	10	TARCH (1,1)
2	GARCH (1,1)	11	TARCH (1,2)
3	GARCH (1,2)	12	TARCH (2,1)
4	GARCH (2,1)	13	TARCH (2,2)
5	GARCH (2,2)	14	PARCH (1,1)
6	EGARCH (1,1)	15	PARCH (1,2)
7	EGARCH (1,2)	16	PARCH (2,1)
8	EGARCH (2,1)	17	PARCH (2,2)
9	EGARCH (2,2)		

\* The 17 models are estimated via the MLE using the Gaussian, Student's t and the Generalized Error Distribution and using the QMLE resulting to 68 models.

**Table 1B. Specification for GARCH PARK R Models\***

Model	Specification	Model	Specification
1	ARCH (1)	6	EGARCH (1,1)
2	GARCH (1,1)	7	EGARCH (1,2)
3	GARCH (1,2)	8	EGARCH (2,1)
4	GARCH (2,1)	9	EGARCH (2,2)
5	GARCH (2,2)		

\* The models are estimated using QMLE.

These models were estimated to fit the daily returns of the peso-dollar exchange rate from January 02, 1997 to December 05, 2003, a total of 1730 observations.

Following the approach of Hansen and Lunde (2001), the time series was divided into two sets, an estimation period and an evaluation period.

$$t = \underbrace{-T+1, \dots, 0}_{\text{estimation period}} \quad \underbrace{1, 2, \dots, n}_{\text{evaluation period}}$$

The parameters of the volatility models are estimated using the first T inter-daily observations and the estimates of the parameters are used to forecasts the remaining n periods. The estimation period made use of daily returns from January 02, 1997 to December 27, 2002, a total of 1493 observations.

In the evaluation period the daily volatility is estimated using the square of the Parkinson R, defined in (6). The square of the PARK R serves as the proxy for the unknown conditional variance. The evaluation period makes use of daily returns from January 02, 2003 to December 05, 2003, a total of 237 observations.

### Measures to Evaluate the Forecasting Performance

The main objective of building volatility models is to forecast future volatility. Given a number of competing models, there is a need to evaluate the forecasting performance of the models to segregate “good” models from “bad” ones. Let h denote the number of competing forecasting models. The  $j^{\text{th}}$  model provides a sequence of forecasts for the conditional variance,

$$\hat{\sigma}_{j,1}^2, \hat{\sigma}_{j,2}^2, \dots, \hat{\sigma}_{j,n}^2 \quad j = 1, 2, \dots, h$$

that will be compared to the square of the Parkinson range, the proxy of the intra-daily calculated volatility,

$$R_{P_1}^2, \dots, R_{P_n}^2$$

The forecast of  $j^{\text{th}}$  model leads to the observed loss,

$$L_{j,t}(\hat{\sigma}_{j,t}^2, R_{P_t}^2) \quad j = 1, 2, \dots, 77 \quad \text{and} \quad t = 1, 2, \dots, 237$$

In this study, five (5) different loss functions are used to evaluate the forecasting performance of the different models. The loss functions are based on the mean absolute deviations using the estimated conditional standard deviation (MAD1) and variance (MAD2), mean square error based on the conditional standard deviation (MSE1) and variance (MSE2) and a criterion equivalent to the  $R^2$  criterion using the regression equation,

$$\log(R_p^2) = a + b \log(\hat{\sigma}_t^2) + \varepsilon_t \quad t = 1, 2, \dots, 237$$

discussed in Engle and Patton (2001) and Taylor (1999).

The best over-all ARCH model is the TARCH (2,2) model with the Student's t as the underlying distribution. The second "best" model is the PARCH (2,2) model, also using the Student's t distribution. It is interesting to note that models using the Generalized Error Distribution performed relatively well using the five forecasting criteria, with 8 out of 17 models landing in the top 10 models. In general, the models with relatively superior forecasting performance, using the peso-dollar exchange rate, are those that accommodate the leverage effects such as the TARCH, PARCH and EGARCH.

The results also showed that volatility models that assumed the Gaussian distribution or those that used the QMLE performed worst compared to models that assumed the Student's t or Generalized Error distributions. Therefore, it is important to correctly specify the entire distribution and not only to focus on the specification of the volatility, even if it is the object of interest. A similar observation was made in the study of Hansen and Lunde (2001).

Using the five criteria discussed above, the top three GARCH-PARK-R models are GARCH (1,2), (2,1) and (1,1). As expected, the GARCH-PARK-R models performed better than most of the ARCH-type models. It should be noted that the proxy for the conditional variance in the evaluation period is the square of the Parkinson range. However, it is interesting to note that the forecasting performance of the "best" ARCH-type model, the TARCH (2,2) model with a student's t distribution, is relatively near the "best" GARCH-PARK-R model. The results somewhat provide an assurance that volatility models using inter-daily data can forecast the conditional variance rather well (at least using the Parkinson range).

## VI. Conclusion

This paper introduced a relatively simple, yet efficient, model to describe the variation in volatility of the peso-dollar exchange rate using intra-daily returns. The Generalized Auto-Regressive Conditional Heteroskedasticity Parkinson Range (GARCH-PARK-R) model can actually produce volatility estimates that are relatively superior to the ARCH class of models using inter-daily returns. The GARCH-PARK-R model is a good alternative to the so-called Realized Volatility that makes use of large quantity of intra-daily data, something that is difficult to obtain in emerging markets such as the Philippines.

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# Constructing a Coincident Index of Business Cycles Without Assuming a One-Factor Model\*

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## Abstract

The Stock–Watson coincident index and its subsequent extensions assume a static linear one-factor structure for the component indicators. Such assumption is restrictive in practice, however, with as few as four indicators. In fact, such assumption is unnecessary if one defines a coincident index as an estimate of latent monthly real GDP. This paper estimates VAR and factor models for latent monthly real GDP and other coincident indicators using the observable mixed-frequency series. For US data, Schwartz’s Bayesian information criterion selects a two-factor model. The smoothed estimate of latent monthly real GDP, or its common-factor component, is the proposed index.

*Keywords:* Mixed-frequency; VAR model; Multi-factor model; EM algorithm; Interpolation; Monthly real GDP

*JEL classification:* C32; C43; C53; E32

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# On the Sustainable Development Indices of Agriculture and Fisheries in Southeast Asia

by:

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## ABSTRACT

Agricultural production-related and macroeconomic data among Southeast Asian countries are used in the construction of indices on level of agricultural development. Models are constructed and used in the characterization of the dynamics of agricultural growth in Southeast Asia. Agricultural sustainability may still show from the models but some signs that it will soon fail also manifests.

**Keywords:** *cross-section time series, principal component analysis, sustainable agriculture*

## I. Introduction

The discourse on sustainability that gained so much attention globally this past two decades has so far reached several accomplishments. One important milestone is the creation of the Commission on Sustainable Development (CSD) under the United Nations. The multi-disciplinary nature of issues around sustainability would indeed require an institution to coordinate and integrate all the important learning from various fora that try to understand the problem and charting strategies towards attainment of solutions.

The CSD come up with the theme indicator framework that serves as the basis in identifying core indicators that can facilitate monitoring progress. The framework broadly grouped the indicators into social, environmental, economic and institutional themes. The social themes comprises of indicators that are also discussed when issues on poverty and vulnerability of the communities are considered. Much of the indicators under the social theme are also included in the Millennium Development Goals (MDG). This leaves the environmental and economic themes less pursued because the institutional theme would normally get significant share of the focus in technical assistance and capacity building intervention of international donors to the developing countries.

A possible reason for the dearth of work on the theme indicators under environmental and economic concerns is the difficulty of data generation on the identified indicators. There are surely bulk of data but whose nature is not necessarily aligned along the concerns of sustainability. There are quite a few indicators whose time series across various countries are available in some international databases like FAOSTAT of the UN-FAO. On the environmental theme, sub-issue of land, agriculture indicators like land usage and use of chemical inputs (fertilizers and pesticides) are available in FAOSTAT. For the oceans, seas and coast, annual catch data are also

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available in FAOSTAT. Air and Freshwater concerns though have scanty time series data that can facilitate quantitative monitoring. On the economic theme, data on agriculture and fishery production as well and international trade are also available from the said database.

(Anaedu and Engfeldt, 2002) developed the draft institutional framework for sustainable development as they believed that this is the key to the realization of the goals. The goals can be efficiently met once there is effective governance that will back up the programs and strategies that will fully implement Agenda 21.

This paper proposes a simple framework for agricultural sustainability that is backed up with time series data to allow empirical investigations on concerns related to monitoring. Specifically, indicators are summarized into indices that are linked into the proposed framework. Econometric models using the indices are developed along the provisions of the framework to illustrate the possible implementation strategy in monitoring agricultural sustainability.

## **II. On Sustainable Agriculture**

The promotion of sustainable agriculture while aiming for rural development is spelled out in Chapter 14 of Agenda 21. The eight session of UN Economic Council's Commission of Sustainable Development discussed the complementing issues on food security, poverty, agriculture sustainability and rural development. The session recommends the implementation of macro policy reform and trade liberalization that supports comprehensive rural development (improve access to land, poverty alleviation, rural employment, and reduction of rural emigration).

(de Vries, Van Keulen, Rabbinge, Luyten, 1995) explored a thesis on the biophysical limit of food production defined to have been reached when all land suitable for agriculture is cropped and the potential yield on each field is attained. They pointed out that the limit should be determined using sustainable agricultural practices in which the quality of soil and the non-agricultural environment either remain constant or improve, and limited natural resources are not overexploited. While expansion of irrigation systems can facilitate the expansion of crop areas, demand will necessarily increase due to population growth and demand for more affluent diet that contains more animal protein. They pointed out the potential problem in Asia since more than half of the global population will be in the region and sustainable agricultural practices would necessitate much larger area.

(Brown, et. al., 1987) proposed how agricultural sustainability can be realistically defined. Some trade-offs exist between land conservation and economic and social viability. While the carrying capacity concept is easily established among renewable resources, it is seriously flawed among humans. They agree that sustainability is a multidisciplinary issue covering the range of social, ecological and economic concerns. They continued that

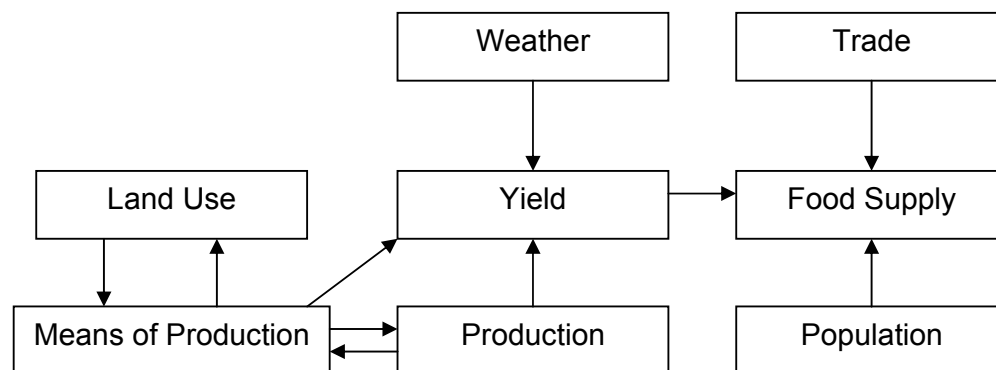
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sustainability and sustainable development vary across contexts and scales. Depending on the definition, it can either develop into a realistic goal or remain a utopian ideal. Different societies have different cultural expectations of sustainable development further complicating the process and setting the priorities of sustainability is value-laden and so requires a clear definition from the outset.

Monitoring agricultural sustainability in Southeast Asia is somewhat complicated considering that bulk of the producers are marginal and often has tenurial problems. A framework is proposed that will facilitate monitoring using the FAOSTAT database.

**Figure 1: Framework of Sustainable Agriculture**



Agricultural Sustainability is viewed to continuously support food supply (per capita fish for food, calories/day/capita, protein/day/capita, fat/day/capita, calorie/protein/fat from livestock) as it's ultimate goal. There are three determinants of food supply namely trade (animal/vegetable oil import/export, maize/milk/rice/meat import/export), population (total, rural, urban, agricultural population, non-agricultural population, total economically active population, total economically active population in agriculture) and yield (rice/corn/poultry yield, per capita fish production, per capita fish import/export, beef/chicken carcass weight/yield, milk yield, pig yield). Trade and population are exogenous while yield is affected by weather (southern oscillation index, a macro weather indicator), production (area of rice & maize, production of rice & maize, total fish production, fish import, fish export, fish processing, marine fish production, production of beef/pig/chicken, milk production, poultry production, agricultural production index, cereals production index), and means of production (maize seeds, fertilizer production and import, fertilizer consumption, agricultural requisite imports and exports, rice seeds). There is a complementation between production and means of production, similarly with means of production and land use (total area, agricultural area, irrigated area).

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While many other details may be lacking from the framework, the key elements included can help in the monitoring and assessment of the level of agricultural sustainability.

### III. Dimensions of Agricultural Production in SEA

The data on various food and agriculture indicators in Southeast Asian (SEA) countries were obtained from the FAOSTAT database (2004). The time series starts 1961 up to 2003 or 2002 for some indicators across the agricultural producing countries (Thailand, Cambodia, Laos, Vietnam, Myanmar, Indonesia, Malaysia and Philippines).

Cross-section time series data is pre-processed into indices that will measure/assess status of various components in the framework above. Principal component analysis is used in summarizing the indicators into fewer indices, scores of which are subsequently used in modeling.

There are 16 land use indicators and only one component suffice to explain 80% of total variation. The component whose score will be used in modeling is

*LANDUSE1 – aggregates all land use indicators with similar loadings for all indicators except for agricultural area in Myanmar (negligible loading).*

Of the 60 means of production indicators, three components are needed enough to explain 75% of the total variability in all indicators. The three components are defined as

*MEANS1-similar weights practically for all indicators, can be considered as an aggregate index.*

*MEANS2-more dominant loading for maize seed in Philippines and Thailand, fertilizer production and export in Myanmar, pesticide export in Myanmar were observed.*

*MEANS3-fertilizer import and consumption in Cambodia, pesticide import in Cambodia are dominating the loadings for this component.*

There are most number of indicators are available for production, totaling 148. Only two components however, are needed to account 74% of its total variability. The two indices are

*PROD1- similar weights for all indicators, aggregate index*

*PROD2-the loadings are complicated and does clearly help in dimension-reduction.*

There is various aggregates of the southern oscillation index used and two components would account for practically all of the total variability.

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*SOI1 – level of SOI*  
*SOI2- variability in SOI*

There are 76 trade indicators and it requires five components to explain 72% of the total variability. The five components of trade are

*TRADE1- aggregate index, loadings are similar for almost all indicators*  
*TRADE2-dominant loadings for import of rice/corn (cereals)*  
*TRADE3-dominant loadings for import of protein sources (maize, milk, meat)*  
*TRADE4-the loadings do not clearly indicate dimension-reduction.*  
*TRADE5- the loadings do not clearly indicate dimension-reduction*

The 56 population indicators can be summarized into a single component accounting for 94% of the total variability.

*POPN1-similar weights for all indicators*

The 72 yield indicators can be aggregated into two components accounting for 71% of the total variability.

*YIELD1- similar loadings for all indicators*  
*YIELD2-dominant loading for mostly the indicators on Vietnam*

The 64 food supply indicators can be summarized into two components accounting 82% of the total variability.

*FOOD1-similar loadings for all indicators*  
*FOOD2-loadings not clearly indicate dimension-reduction*

The plot of the time series indicates non-stationarity and the series seems to be drifting away in similar directions across countries. This is perhaps the reason why the component that assigns similar weights to practically all indicators would tend to account for bulk of the total variability of the indicators. There are also instances when the number of variables exceeded the number of observations, e.g., in production, a total of 148 indicators are used with 41 observations only. A detailed study on co-integration of the indicators or the use of sparse component analysis may help improve dimension-reducing characteristics of the components.

#### **IV. Dynamics of Agricultural Production in SEA**

The means of production are still efficiently used in Southeast Asia (SEA). Increased use of production inputs can still result to expansion of landuse that will perhaps improve the production opportunities in the region. The rate of land utilization however is much faster requiring higher intensity of production inputs. This is especially true in Thailand and Vietnam. If the production inputs are not carefully chosen, this can perhaps become a deterrent to agricultural sustainability.

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The means of production are efficiently converted into actual production in SEA. A 1% increase in the production inputs could mean 1.7% increase in production. The present system of resource allocation in the use of production inputs in production of various crops, animals and fishery products need to be preserved or enhanced further to foster a sustainable environment in the agriculture sector.

The yield index is not significantly affected by the weather index. This could mean any of two things. First, is that the weather indicator may not suffice to summarize the weather situation in SEA. Second, is that the continuous improvement of the irrigation systems may have mitigated the possible effects of weather fluctuations.

The present production structure is not very efficient in expanding the yield. A 1% increase in production level can only improve 0.66% in yield level. Though the means of production is efficiently converted into actual production, this is not positively contributing in expanding yield. The current production technology still needs further enhancement to ensure not just expansion of production, but of yield as well.

The present land use intensity is still sustainable since it still contributes in the expansion of yield. A 1% increase in land use index would mean a 1.84% increase in yield. For as long as SEA countries would be able to plan the productive allocation of land resources, yield can still improve in the coming years.

The population threshold is not yet reached in SEA since an increase in population could still motivate an increase in food supply. This is possibly approaching the limit as 1% increase in population index could mean a 0.71% increase in food supply. There is already a gap between the population growth and food supply increment that needs to be filled up. Fortunately, yield expansion can still serve as the buffer since an increase of 1% could cause an incremental 0.26% food supply. This is still a little short of the population-food supply gap and strategies to control population growth while expanding yield is necessary to ensure stable food supply in SEA. The improvement of trade relations of SEA within and outside the region is also contributing in the maintenance of a stable food supply, except when importation of cereals (rice and maize) dominates the trade patterns.

## **V. Conclusions and Recommendations**

The current dynamics of agricultural production in Southeast Asia (SEA) seems to indicate sustainability, some signs of failure are already beginning to manifest. Production inputs are still efficiently used but there is a need to expand yield further while population growth is being arrested to ensure a stable food supply in the region.

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The indexing system of various time series indicators across the SEA region is a good input in monitoring agricultural sustainability but methods that will enhance its dimension-reducing characteristics needs to be developed further.

## Appendix

The equations that provides empirical support to the framework using the principal components are as follows:

### *Landuse*

$$\text{LANDUSE1}_t = 0.1539 + 0.6037\text{MEANS1}_t + 0.1984\text{MEANS2}_t + a_t$$

(0.0001) (0.0001)

where  $a_t = 0.4384a_{t-1}$   
( $t=2.97$ )

$R^2=99.07\%$ ,  $DW=1.77$

### *Means of Production*

$$\text{MEANS1}_t = -0.2805 + 1.5457\text{LANDUSE1}_t + 0.4960\text{LANDUSE2}_t + a_t$$

(0.0001) (0.0003)

where  $a_t = 0.5161a_{t-1}$   
( $t=3.67$ )

$R^2=98.97\%$ ,  $DW=1.88$

### *Production*

$$\text{PROD1}_t = 0.4927 + 1.7012\text{MEAN1}_t + a_t$$

(0.0001)

where  $a_t = 0.3987a_{t-1}$   
( $t=2.68$ )

$R^2=99.08\%$ ,  $DW=1.4714$

### *Yield*

$$\text{YIELD1}_t = 0.0646 + 0.6571\text{PROD1}_t + 0.1948\text{PROD2}_t - 0.1094\text{MEAN3}_t + a_t$$

(0.0001) (0.0001) (0.0295)

where  $a_t = 0.6217a_{t-1}$   
( $t=4.76$ )

$R^2=99.66\%$ ,  $DW=1.4116$

YIELD not affected by SOI

$$\text{YIELD1}_t = 0.4548 + 1.8398\text{LANDUSE1}_t + a_t$$

(0.0001)

where  $a_t = 0.7641a_{t-1}$   
( $t=7.30$ )

$R^2=97.85\%$ ,  $DW=0.96$

### *Food Supply*

$$\text{FOOD1}_t = 0.1853 + 0.7144\text{POPN1}_t + 0.2611\text{YIELD1}_t + a_t$$

(0.0001) (0.0030)

where  $a_t = 0.6271a_{t-1}$   
( $t=4.90$ )

$R^2=99.53\%$ ,  $DW=1.56$

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$$\text{FOOD1}_t = 0.2493 + 1.1125\text{TRADE1}_t - 0.6061\text{TRADE2}_t + 0.2895\text{TRADE5}_t + a_t$$

(0.0001)
(0.0030)
(0.0216)

where  $a_t = 0.5906a_{t-1}$

(t=4.39)

$R^2 = 98.52\%$ ,  $DW = 1.53$

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# 1 Introduction

Since the seminal work by Stock and Watson (1989, 1991), it has been standard in the literature on business cycle indices to assume a static linear one-factor structure for coincident indicators, and use the estimated “common factor” as a coincident index; e.g., Kim and Yoo (1995), Diebold and Rudebusch (1996), Chauvet (1998), Kim and Nelson (1998), and Mariano and Murasawa (2003). This one-factor structure assumption is restrictive in practice. Indeed, Murasawa (2003) tests the autocovariance structure of the four US coincident indicators used in these works, and finds a strong evidence against the one-factor structure assumption.

This paper proposes a method for constructing a coincident index without assuming a one-factor model. The idea is simple. Many, if not all, will agree that if we observe real GDP promptly on monthly basis, then we do not need a coincident index. If so, then it suffices to predict the current monthly real GDP, which does not require a one-factor model. This paper thus relates index construction to interpolation of real GDP.

Mariano and Murasawa (2003) stress that a business cycle index must have an economic interpretation, because the “amplitude” of a cycle depends on the choice of an index. Figure 1 compares the composite index (CI) of coincident indicators, released by The Conference Board, and the Stock–Watson Experimental Coincident Index (XCI), released by themselves on their home pages, with seasonally-adjusted quarterly real GDP from 1979 to 1983, during which there are two peaks and two troughs. The XCI indicates that the trough in November 1982 is deeper than that in July 1980, whereas the CI indicates that the depth of the two are almost equal. In fact, real GDP is *higher* in the fourth quarter of 1982 than in the third quarter of 1980. Such inconsistency can arise because the levels of these indices have no economic interpretation. While Mariano and Murasawa (2003) include real GDP in the one-factor model to relate the common factor to monthly real GDP, this paper estimates monthly real GDP directly.

Specifically, this paper estimates VAR and factor models for latent monthly real GDP and other coincident indicators using the observable mixed-frequency series. The estimation procedure follows Mariano and Murasawa (2003), i.e., we derive a state-space model for the observable mixed-frequency series, and treat the mixed-frequency series as monthly series with missing observations. ML estimation of a linear Gaussian state-space model with missing observations is now standard. For US data, Schwartz’s Bayesian information criterion selects a two-factor model, in which the common factors jointly follow VAR(1) and the specific factors independently follow AR(1). The associated smoothed estimate of latent monthly real GDP, or its common-factor component, is the proposed index.

In practice, quasi-Newton methods may fail because both VAR and factor models may have too many unknown parameters. The EM algorithm is useful in such cases. Shumway and Stoffer (1982) derive the EM algorithm for ML estimation of a linear Gaussian state-space model, allowing for missing observations. As the EM algorithm slows down significantly near the (local) maximum, Watson and Engle (1983) suggest switching to a quasi-Newton method at some point, i.e., use the EM algorithm to find a good starting value for a quasi-Newton method. Unfortunately, the EM algorithm does not apply directly to factor models with mixed-frequency series; hence we use it only for VAR models in this paper.

In the literature on interpolation of real GDP, the best linear unbiased interpolation by Chow and Lin (1971) is most popular, but some authors use state-space models; e.g., Bernanke, Gertler, and Watson (1997), Cuche and Hess (1999, 2000), and Liu and Hall (2001). They consider univariate linear regression models for latent monthly real GDP, where for temporal aggregation, they do not take the log transformation. This paper takes the log transformation and considers multivariate