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Abstract:

It is well known that the finite-sample null distribution of the Jarque-Bera Lagrange Multiplier (LM) test for normality and its adjusted version (ALM) introduced by Urzua differ considerably from their asymptotic $\chi^2(2)$ limit. Here, we present results from Monte Carlo simulations using 10^7 replications which yield very precise numbers for the LM and ALM statistic over a wide range of critical values and sample sizes. Depending on the sample size and values of the statistic we get p values which significantly deviate from numbers previously published and used in hypothesis tests in many statistical software packages. The p values listed in this short Letter enable for the first time a precise implementation of the Jarque-Bera LM and ALM tests for finite samples.

1 Introduction

The Jarque-Bera (1980, 1987) Lagrange multiplier test is likely the most widely used procedure for testing normality of economic time series returns. The algorithm provides a joint test of the null hypothesis of normality in that the sample skewness b_1 equals zero and the sample kurtosis b_2 equals three. The null is rejected when the Lagrange multiplier statistic

$$LM = N\left(\frac{(b_1^{1/2})^2}{6} + \frac{(b_2 - 3)^2}{24}\right)$$
 (1)

exceeds some critical value, which is taken in the asymptotic limit from the $\chi^2(2)$ distribution. N is the sample size, $b_1^{1/2} = m_3/m_2^{3/2}$, $b_2 = m_4/m_2^2$ where m_i is the i-th central moment of the observations $m_i = \Sigma (x_j - \overline{x})^i/N$, and \overline{x} the sample mean.

Urzua (1996) modified the Jarque-Bera test replacing the asymptotic means and variances by their exact finite-sample values yielding

$$ALM = N \left(\frac{(b_1^{1/2})^2}{c_1} + \frac{(b_2 - c_2)^2}{c_3} \right) . {2}$$

Here the parameters $c_{1,2,3}$ are given by the expectation value and variances of the skewness and kurtosis

$$c_1 = var(b_1^{1/2}) = \frac{6(N-2)}{(N+1)(N+3)} ,$$

$$c_2 = E(b_2) = \frac{3(N-1)}{(N+1)} ,$$

$$c_3 = var(b_2) = \frac{24N(N-2)(N-3)}{(N+1)^2(N+3)(N+5)} .$$

Note, that the ALM has the same asymptotic distribution as the LM statistic.

The work of Urzua (1996) as well as the work by Deb and Sefton (1996) already warn about the incorrect use of the Jarque-Bera test in the case of small- and medium-sized samples. The authors performed Monte Carlo simulations and tabulated significance points for 5% and 10%, on a series of sample sizes ranging between 10 and 800. Deb and Sefton used 600'000 replications in their Monte Carlo simulations and Urzua used 10'000 replications and added results for the 1%, 15% and 20% significance points. Very recently Lawford (2004) developed an accurate response surface approximation for the 5% and 10% critical values of the Jarque-Bera test based on Monte Carlo simulations using 1 Million replications. The tables for the LM and ALM statistic values presented in these papers are restricted usually to a small set of parameters and the precision is in most cases limited to two digits. Furthermore, for small N we observe significant differences in comparison to previously published values. For some parameter settings the differences are so large, that this may result in inaccurate hypothesis tests or even more this may lead to situations with wrong decisions.

In this Letter we present tables with very precise values for both, the LM and ALM statistic. Since the slow convergence of the Monte Carlo simulation is well known we extend the simulations to 10 Million replications and enhance the mesh of p-values and sample sizes considerably.

The results have been used to implement R functions for the finite sample Jarque-Bera test and the distribution itself, using either the LM or ALM statistic. R (2004) is a powerful and widely used GPL-licensed statistical software environment based on the S language. In this sense our functions can also be called from the commercial S-Plus software package. The R functions are part of the Rmetrics software project, www.rmetrics.org. The software is GPL licensed and can be downloaded from the CRAN Server www.r-project.org.

2 Monte Carlo Simulation

We performed Monte Carlo simulations of the LM and ALM statistic using 10^7 replications. The results are summarized in Table 1 for both the LM and ALM statistic.

I M. m\NI	10	20	25	En	75	100	150	200	200	EOO	900	1000	1600	2400	10000
LM: p\N	10	40,000	35	74 704	75	100	150	200	300	500	800	1000	1600	2400 24.249	19,940
0.01% 0.05%	15.345 12.444	46.996 31.159	66.612 40.759	71.734 43.256	69.910 41.909	68.032 40.430	60.632 37.229	54.736 34.330	47.572 30.561	38.847 26.270	33.247 23.045	31.213 21.979	26.956 19.760	18.366	16.052
0.05%	10.995	24.970	31.969	33.753	32.738	31.840	29.547	27.551	24.830	21.812	19.521	18.736	17.150	16.083	14.397
0.10%	7.3004	13.471	16.414	17.281	17.305	16.959	16.257	15.638	14.669	13.583	12.726	12.366	11.762	11.384	10.792
1.00%	5.7029	9.7182	11.736	12.392	12.586	12.491	12.185		11.3580	10.778	10.299	10.117	9.8095	9.6084	9.3128
5.00%	2.5247	3.7954	4.5929	4.9757	5.2777	5.4300	5.5984	5.6758		5.8551	5.9103	5.9242	5.9569	5.9671	5.9857
10.00%	1.6232	2.3470	2.8814	3.1834	3.4862	3.6734	3.9041	4.0327	4.1891	4.3317		4.4568	4.5132	4.5424	4.5888
15.00%	1.2826	1.8230	2.2533	2.5094	2.7713	2.9390	3.1416	3.2580	3.4003	3.5312	3.6198	3.6507	3.7016	3.7309	3.7778
20.00%	1.1236	1.5623	1.9162	2.1278	2.3463	2.4865	2.6558	2.7559	2.8764	2.9882	3.0645	3.0909	3.1360	3.1611	3.2036
30.00%	0.9389	1.2516	1.4997	1.6466	1.7975	1.8944	2.0112	2.0807	2.1639		2.2962	2.3153	2.3460	2.3650	2.3968
40.00%	0.8077	1.0360	1.2115	1.3128	1.4165	1.4828	1.5619	1.6087	1.6649	1.7175	1.7547	1.7679	1.7889	1.8024	1.8248
50.00%	0.6950	0.8574	0.9771	1.0447	1.1126	1.1563	1.2076	1.2385	1.2752	1.3101	1.3338	1.3420	1.3568	1.3655	1.3808
60.00%	0.5885	0.6948	0.7699	0.8114	0.8529	0.8800	0.9105	0.9292	0.9518	0.9732	0.9882	0.9931	1.0024	1.0085	1.0181
70.00%	0.4801	0.5378	0.5769	0.5985	0.6202	0.6348	0.6508	0.6610	0.6730	0.6851	0.6940	0.6965	0.7018	0.7056	0.7108
80.00%	0.3618	0.3777	0.3896	0.3969	0.4046	0.4105	0.4168	0.4213	0.4267	0.4325	0.4368	0.4376	0.4402	0.4421	0.4451
85.00%	0.2950	0.2938	0.2958	0.2982	0.3010	0.3044	0.3071	0.3096	0.3130	0.3163	0.3189	0.3194	0.3209	0.3221	0.3245
90.00%	0.2192	0.2047	0.2002	0.1997	0.1997	0.2006	0.2016	0.2024	0.2040	0.2060	0.2071	0.2074	0.2081	0.2089	0.2106
95.00%	0.1272	0.1084	0.1022	0.1005	0.0995	0.0996	0.0992	0.0995	0.1000	0.1005	0.1010	0.1012	0.1013	0.1019	0.1024
99.00%	0.0304	0.0230	0.0208	0.0203	0.0198	0.0197	0.0196	0.0196	0.0197	0.0198	0.0197	0.0199	0.0020	0.0020	0.0020
99.50%	0.0156	0.0116	0.0104	0.0101	0.0099	0.0098	0.0098	0.0098	0.0099	0.0099	0.0098	0.0099	0.0099	0.0099	0.0100
99.90%	0.0032	0.0023	0.0021	0.0020	0.0020	0.0019	0.0019	0.0019	0.0020	0.0019	0.0020	0.0020	0.0020	0.0020	0.0020
99.95%	0.0016	0.0012	0.0010	0.0010	0.0010	0.0009	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
99.99%	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
A1 88\\	40		0.5		75	400	450	200	200	500	000	4000	4000	0.400	40000
ALM: p\N	10	20	35	50	75	100	150	200	300	500	800	1000	1600	2400	10000
0.01%	51.600	91.217	99.883	96.158	85.696	79.523	67.685	59.600	50.530	40.472	34.231	31.993	27.454	24.575	20.040
0.01% 0.05%	51.600 41.502	91.217 60.508	99.883 60.927	96.158 58.013	85.696 51.413	79.523 47.444	67.685 41.713	59.600 37.524	50.530 32.579	40.472 27.431	34.231 23.751	31.993 22.545	27.454 20.128	24.575 18.621	20.040 16.114
0.01% 0.05% 0.10%	51.600 41.502 36.538	91.217 60.508 48.399	99.883 60.927 47.780	96.158 58.013 45.318	85.696 51.413 40.266	79.523 47.444 37.414	67.685 41.713 33.106	59.600 37.524 30.158	50.530 32.579 26.498	40.472 27.431 22.785	34.231 23.751 20.147	31.993 22.545 19.244	27.454 20.128 17.474	24.575 18.621 16.314	20.040 16.114 14.457
0.01% 0.05% 0.10% 0.50%	51.600 41.502 36.538 23.831	91.217 60.508 48.399 25.963	99.883 60.927 47.780 24.569	96.158 58.013 45.318 23.229	85.696 51.413 40.266 21.334	79.523 47.444 37.414 19.986	67.685 41.713 33.106 18.285	59.600 37.524 30.158 17.156	50.530 32.579 26.498 15.689	40.472 27.431 22.785 14.211	34.231 23.751 20.147 13.129	31.993 22.545 19.244 12.694	27.454 20.128 17.474 11.971	24.575 18.621 16.314 11.525	20.040 16.114 14.457 10.827
0.01% 0.05% 0.10% 0.50% 1.00%	51.600 41.502 36.538 23.831 18.374	91.217 60.508 48.399 25.963 18.643	99.883 60.927 47.780 24.569 17.540	96.158 58.013 45.318 23.229 16.659	85.696 51.413 40.266 21.334 15.506	79.523 47.444 37.414 19.986 14.719	67.685 41.713 33.106 18.285 13.707	59.600 37.524 30.158 17.156 13.042	50.530 32.579 26.498 15.689 12.149	40.472 27.431 22.785 14.211 11.271	34.231 23.751 20.147 13.129 10.616	31.993 22.545 19.244 12.694 10.372	27.454 20.128 17.474 11.971 9.9667	24.575 18.621 16.314 11.525 9.7158	20.040 16.114 14.457 10.827 9.3386
0.01% 0.05% 0.10% 0.50% 1.00% 5.00%	51.600 41.502 36.538 23.831 18.374 7.4161	91.217 60.508 48.399 25.963 18.643 6.9317	99.883 60.927 47.780 24.569 17.540 6.6788	96.158 58.013 45.318 23.229 16.659 6.5533	85.696 51.413 40.266 21.334 15.506 6.4144	79.523 47.444 37.414 19.986 14.719 6.3192	67.685 41.713 33.106 18.285 13.707 6.2182	59.600 37.524 30.158 17.156 13.042 6.1493	50.530 32.579 26.498 15.689 12.149 6.0925	40.472 27.431 22.785 14.211 11.271 6.0497	34.231 23.751 20.147 13.129 10.616 6.0309	31.993 22.545 19.244 12.694 10.372 6.0218	27.454 20.128 17.474 11.971 9.9667 6.0182	24.575 18.621 16.314 11.525 9.7158 6.0077	20.040 16.114 14.457 10.827 9.3386 5.9961
0.01% 0.05% 0.10% 0.50% 1.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650	20.040 16.114 14.457 10.827 9.3386
0.01% 0.05% 0.10% 0.50% 1.00% 5.00%	51.600 41.502 36.538 23.831 18.374 7.4161	91.217 60.508 48.399 25.963 18.643 6.9317	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895	96.158 58.013 45.318 23.229 16.659 6.5533	85.696 51.413 40.266 21.334 15.506 6.4144	79.523 47.444 37.414 19.986 14.719 6.3192	67.685 41.713 33.106 18.285 13.707 6.2182	59.600 37.524 30.158 17.156 13.042 6.1493	50.530 32.579 26.498 15.689 12.149 6.0925	40.472 27.431 22.785 14.211 11.271 6.0497	34.231 23.751 20.147 13.129 10.616 6.0309	31.993 22.545 19.244 12.694 10.372 6.0218	27.454 20.128 17.474 11.971 9.9667 6.0182	24.575 18.621 16.314 11.525 9.7158 6.0077	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941
0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812
0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00% 20.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058
0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00% 20.00% 30.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980
0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00% 20.00% 40.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251
0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00% 20.00% 40.00% 50.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 1.3663	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808
0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00% 20.00% 30.00% 40.00% 60.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.8344	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 1.3663 1.0087	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179
0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00% 20.00% 40.00% 50.00% 60.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464 0.6406	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165 0.6065	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.8344 0.6100	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554 0.6735 0.4257	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 1.3663 1.0087 0.7054	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108
0.01% 0.05% 0.10% 0.50% 1.00% 1.00% 15.00% 20.00% 30.00% 40.00% 60.00% 70.00% 80.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464 0.6406 0.4376	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165 0.6065	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.8344 0.6100 0.4005	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.4022	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309 0.4064	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.4111	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538 0.4163	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625 0.4203	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554 0.6735 0.4257	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 0.4316	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963 0.4369	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014 0.4397	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 1.3663 1.0087 0.7054	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451
0.01% 0.05% 0.10% 0.50% 1.00% 1.00% 15.00% 20.00% 30.00% 40.00% 50.00% 60.00% 80.00% 85.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464 0.6406 0.4376 0.3344	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165 0.6065 0.4056	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.8344 0.6100 0.4005 0.2994	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.4022 0.2991	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.1604 0.8781 0.6309 0.4064 0.3005	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.4111	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538 0.4163 0.3058	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625 0.4203 0.3082	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554 0.6735 0.4257 0.3117	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 0.4316 0.3154	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359 0.3181	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963 0.4369 0.3187	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014 0.4397 0.3204	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 1.3663 1.0087 0.7054 0.4420 0.3219	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451 0.3243
0.01% 0.05% 0.10% 0.50% 1.00% 15.00% 20.00% 30.00% 40.00% 60.00% 80.00% 85.00% 90.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.0658 0.8464 0.6406 0.4376 0.3344 0.2284	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.0460 0.8165 0.6065 0.4056 0.3061 0.2060	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.6100 0.4005 0.2994 0.1992	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 0.8532 0.6183 0.4022 0.2991 0.1982	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 0.6309 0.4064 0.3005 0.1979	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.4111 0.3029 0.1991	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538 0.4163 0.3058 0.2000	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 0.9359 0.6625 0.4203 0.3082 0.2012	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.2003 1.6840 0.9554 0.6735 0.4257 0.3117 0.2029	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 0.4316 0.3154 0.2051	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 0.9888 0.6937 0.4359 0.3181 0.2065	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963 0.4369 0.3187 0.2071	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.0027 0.7014 0.4397 0.3204 0.2078	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 0.7054 0.7054 0.4420 0.3219 0.2087	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451 0.3243 0.2105
0.01% 0.05% 0.10% 0.50% 1.00% 10.00% 10.00% 20.00% 30.00% 40.00% 60.00% 80.00% 85.00% 90.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464 0.6406 0.4376 0.3344 0.2284	91.217 60.508 48.399 25.963 18.643 6.9317 2.7736 2.2164 1.6569 1.3130 0.0460 0.8165 0.4056 0.3061 0.2060 0.1044	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.0844 0.6100 0.4005 0.4005 0.2994 0.1992 0.0996	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.4022 0.2991 0.1982 0.0986	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309 0.4064 0.3005 0.1979	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.64116 0.4111 0.3029 0.1991	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538 0.4163 0.3058 0.2000 0.0982	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 0.9359 0.6625 0.4203 0.3082 0.2012 0.0985	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 2.9462 2.2003 1.6840 0.2846 0.6735 0.4257 0.3117 0.2029 0.0993	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 0.4316 0.3154 0.2051 0.1001	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359 0.3181 0.2065 0.1007	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963 0.4369 0.4369 0.3187 0.2071 0.1009	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014 0.4397 0.3204 0.2078 0.1013	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 1.0087 0.7054 0.4420 0.3219 0.2087 0.1017	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451 0.3243 0.2105 0.1023
0.01% 0.05% 0.10% 0.50% 1.00% 10.00% 15.00% 20.00% 30.00% 40.00% 60.00% 70.00% 80.00% 90.00% 99.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464 0.6406 0.4376 0.3344 0.2284 0.1177	91.217 60.508 48.399 25.963 18.643 6.9317 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165 0.4056 0.4056 0.4056 0.2060 0.1044 0.0213	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.0844 0.6100 0.4005 0.4005 0.2994 0.1992 0.0996 0.0199	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.4022 0.2991 0.1982 0.0986 0.0197	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309 0.4064 0.3005 0.1979 0.0980 0.0194	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.4111 0.3029 0.1991 0.0981	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538 0.4163 0.2000 0.0982 0.0193	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625 0.4203 0.2012 0.0985 0.0195	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 2.9462 2.2003 1.6840 1.2846 0.9554 0.4257 0.4257 0.3117 0.2029 0.0993 0.0195	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 0.4316 0.3154 0.2051 0.1001 0.0196	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359 0.3181 0.2065 0.1007 0.0196	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963 0.4369 0.3187 0.2071 0.1009 0.0198	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014 0.4397 0.2078 0.1013 0.0199	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 2.3694 1.8042 1.3663 1.0087 0.7054 0.4420 0.3219 0.2087 0.1017	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451 0.4213 0.2105 0.1023 0.0200
0.01% 0.05% 0.10% 0.50% 1.00% 10.00% 15.00% 20.00% 30.00% 40.00% 60.00% 70.00% 85.00% 95.00% 99.00%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464 0.64376 0.4376 0.3344 0.2284 0.1177 0.0242 0.0122	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.2164 1.6569 1.3130 1.0460 0.8165 0.4056 0.4056 0.4056 0.2060 0.1044 0.0213 0.0106	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.6100 0.4005 0.4005 0.02994 0.1992 0.0996 0.0199	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.4022 0.2991 0.1982 0.0986 0.0197 0.0098	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309 0.4064 0.3005 0.1979 0.0980 0.0194 0.0097	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.4111 0.3029 0.1991 0.0981 0.0994	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538 0.4163 0.3058 0.2000 0.0982 0.0193 0.0097	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625 0.4203 0.3082 0.2012 0.0985 0.0195 0.0097	50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 0.9554 0.6735 0.4257 0.3117 0.2029 0.0993 0.0195 0.0097	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 0.4315 0.3154 0.2051 0.1001 0.0098	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359 0.3181 0.2065 0.1007 0.0196	31.993 22.545 19.244 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963 0.3187 0.2071 0.1009 0.0198 0.0099	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014 0.4397 0.3204 0.2078 0.1013 0.0199	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 2.3694 1.8042 1.3663 1.0087 0.7054 0.4420 0.3219 0.2087 0.1017 0.0199	20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451 0.3243 0.2105 0.1023 0.0200 0.0100
0.01% 0.05% 0.10% 0.50% 1.00% 1.00% 15.00% 20.00% 30.00% 40.00% 60.00% 70.00% 85.00% 95.00% 99.00% 99.90%	51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464 0.6406 0.4376 0.3344 0.2284 0.1177 0.0242 0.0012 0.0025 0.0013	91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165 0.6065 0.4056 0.3061 0.2060 0.1044 0.0213 0.0106 0.0021 0.0021	99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.08444 0.6100 0.4005 0.2994 0.1992 0.0996 0.0199 0.0100 0.0020	96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.4022 0.2991 0.1982 0.0986 0.0197 0.0098 0.0020 0.0010	85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309 0.4064 0.3005 0.1979 0.0980 0.0194 0.0097	79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.4111 0.3029 0.1991 0.09981 0.01997 0.0019 0.0010	67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.6538 0.4163 0.3058 0.2000 0.0982 0.0193 0.0019 0.0019	59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625 0.4203 0.3082 0.2012 0.0985 0.0195 0.0019 0.0019	50.530 32.579 26.498 12.149 12.149 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554 0.6735 0.4257 0.3117 0.2029 0.0993 0.0195 0.0019	40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 0.4316 0.3154 0.2051 0.1001 0.0196 0.0098 0.0020 0.0010	34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359 0.3181 0.2065 0.1007 0.0196 0.0098 0.0020	31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963 0.4369 0.3187 0.2071 0.1009 0.0198 0.0099 0.0020 0.0010	27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 0.7014 0.4397 0.3204 0.2078 0.1013 0.0199 0.0020 0.0020	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 1.3663 0.7054 0.4420 0.3219 0.2087 0.1017 0.0199 0.0099 0.0020 0.0010	20.040 16.114 14.457 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 0.4151 0.3243 0.2105 0.1023 0.0200 0.0020

Table 1: Top: Significance points for the finite sample Jarque-Bera test. Bottom: Same values for the adjusted Jarque-Bera Test. The numbers are based on Monte Carlo simulations using 10^7 replications. Note, that the p values are listed in reverse order as 1-p. The three major levels, 1%, 5% and 10%, are written in bold face.

Figure 1 illustrates the results in a graph. The simulated p values and the deviations from the asymptotic $\chi^2(2)$ limit are shown. The curves belong to the same values of sample sizes N as listed in table 1.

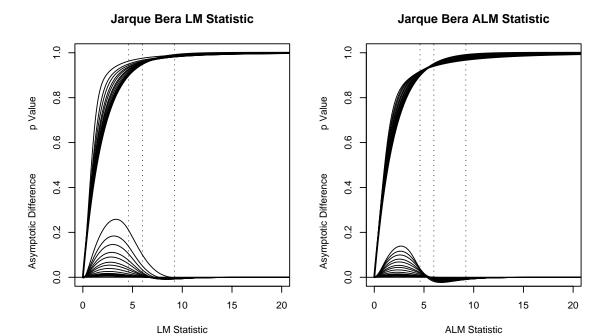


Figure 1: LM (left) and ALM (right) finite sample p values and their differences with respect to the asymptotic limit. The upper bundle of curves shows the p values. The lower bundle of curves measures the difference $p_N - p_\infty$ to the asymptotic limit. The graph clearly demonstrates that the adjusted Jarqua-Bera test outperforms the original version of the test. The three dotted vertical lines mark the 1% (99%), 5% (95%) and 10% (95%) levels in the asymptotic limit, respectively.

3 Response Surface and Hypothesis Test

To compute the LM and ALM statistic for a wide range of quantiles and sample sizes one usually approximates the response surface for a fixed value of p as a series in powers of 1 over N

$$q(p,N) = q(p,\infty) + \sum_{k=1}^{K} \beta_k N^{-k}$$
 (3)

Lawford (2004) has done this for the 5% and 10% quantile lines. He fitted his Monte Carlo data based on 1 Million replications for K=9. The regression coefficients β are listed in the aforementioned paper. We have done fits over a wide range of p-values. The results are shown in figure 2 in comparison with those obtained by Lawford. Note that Lawford's fit becomes less reliable for small lengths where the convergence of the Monte Carlo simulation slows down.

Another approach would be an Edgeworth (1917) expansion of the distribution in 1/N. Unfortunately, we found out that the expansion converges extremely slow. So we applied "Curve Fitting", as suggested by Rothenberg (1984), to approximate the response surface. Simple linear interpolation, 2-dimensional splines or connectionist function approximators are only three possibilities from many others. We followed the first approach fitting on logarithmic scales. The results are shown in Figure 3 for both the traditional Jarque-Bera test as well as its adjusted version.

We have implemented the Jarque-Bera test for finite samples into S functions using the statistical software packages R and SPlus, but it can be done very easily in any other software environment like Matlab, Eviews, or SAS among others. The underlying simulations with 10^7 replications were done with a separate C program using a multiplicative lagged Fibonacci random number generator with a lag of size 1279. The software allows to compute the distribution function and the quantile function for finite samples and the asymptotic limit either for the LM or ALM test version. These functions are used to derive the p values by the hypothesis test function.

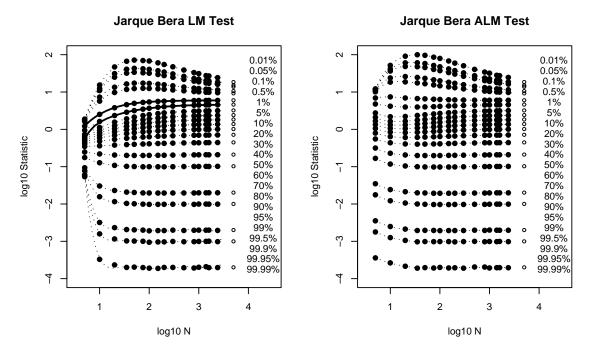


Figure 2: The figures show the LM (left) and ALM (right) statistic for a wide range of p values as a function of sample sizes. The dots show the results from the Monte Carlo simulations using 10^7 replications together with the asymptotic limit (marked by the open circles). The dotted lines are fitted series expansions of order K=6 in 1/N. The two thick lines in the left LM graph display the results of Lawford for the 5% and 10% levels.

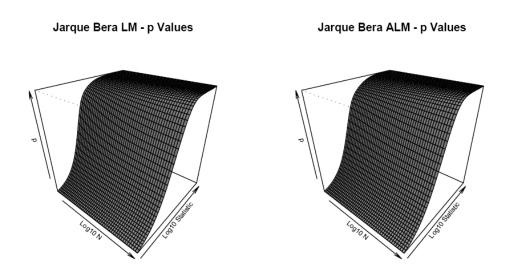


Figure 3: The figures show the LM (left) and ALM (right) surface of p values for a wide range of statistics (0.4 ... 100) and sample sizes (10 ... 10'000). Note, that the x- and y-axis are on logarithmic scales. The inputs consist of almost 2000 p-values ranging between 0.0001 and 0.9999.

4 Summary

This Letter tabulates precise p-values for the Jarque-Bera finite sample normality test. In addition to the original version of the Lagrange Multiplier test we have also computed finite sample p-values for its adjusted version formulated by Urzua (1996). In contrast to previous investigations the results were derived from a MC simulation with 10^7 replications. To our knowledge this is one of the largest simulations ever done in statistics. The outcome of the simulation are very precise values for finite samples which we have tabulated and can now be used for an improved hypothesis testing.

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