Classical computation with a quantum computer

The Toffoli gate is universal for classical computation: Any program can be compiled into many Toffolis. If we show that a quantum computer can do a Toffoli, we therefore prove that quantum computers can solve any problem at least as efficiently as a classical one. Your task this week is to do just this, and do it with a real device!

The Toffoli has the effect

$$a,\ b,\ c\ \rightarrow\ a,\ b,\ (c + a \times b) \tag{1}$$

Where a, b and c are bit values and addition is taken mod 2.

Our quantum Toffoli could therefore be implemented by a unitary of the form

$$U:\ |\,a,\ b,\ c\,\rangle \rightarrow e^{i\theta_{a,b,c}}\,|\,a,\ b,\ (c + a \times b)\,\rangle \tag{2}$$

The standard quantum implementation of the Toffoli gate, which is sometimes used as a component of quantum algorithms, is that for which $\theta_{a,b,c} = 0\ \forall a, b, c$. However, since we are interested only in reproducing its effects on classical inputs, arbitrary values of the $\theta_{a,b,c}$ will be allowed for this exercise.

Your task is to find the best way possible to implement a Toffoli using a real device, namely *ibmqx4*. This can be accessed using IBM's Quantum Experience [1].

For testing of your design you can also use the simulator function of the Quantum Experience. You could also use the Quirk simulator[2], which was made by one of Google's quantum guys.

To determine the quality of your Toffoli, you'll need some measure of fidelity. We'll define one specifically for this job.

For this we will consider only inputs with $c = 1$. We can then interpret a and b as the two input bits of a NAND gate, and the resulting $1 + a \times b$ as its single output bit.

---

[1] https://quantumexperience.ng.bluemix.net/qx/editor
[2] http://algassert.com/quirk

Using the results from many runs on a given input a, b we can determine the probability of getting the correct result: $P(1 + a \times b|a, b)$. The fidelity is then the minimum of these for all possible inputs

$$F = \min_{a,b} P(1 + a \times b|a, b) \tag{3}$$

The skills you learnt in the tutorial programme could be used to help you design a Toffoli, or to understand circuits you may find elsewhere. For this you should know that the Q gate is a $\pi/4$ rotation around the Y axis. On the Quantum Experience this is the gate $u3(\pi/4, 0, 0)$ (make sure the 'Advanced' box is ticked to see this). On Quirk it is $Y^{1/8}$. The Q gate is closely related to the T gate that you may see around, which is a $\pi/4$ rotation around the Z axis.

You can design your Toffoli any way you like, but you'll need to use the gates available in the Quantum Experience. Whether you look up existing circuits for implementing Toffolis, or design your own, you'll need to tailor it to the needs of the *ibmqx4* device. So make sure to hand in an explanation of what you did and why, as well as the final circuit diagram. You should also list the $P(1 + a \times b|a, b)$ you got as results.

Whoever gets the Toffoli with the best fidelity will get an IBM Quantum Experience t-shirt!