CSC17103 – KHAI THÁC DỮ LIỆU ĐỒ THỊ

HOMEWORK 03: Frequent Graph Pattern Mining

Problem 1. Hierarchical Networks

Calculate the degree exponent of the hierarchical network shown in figure below.

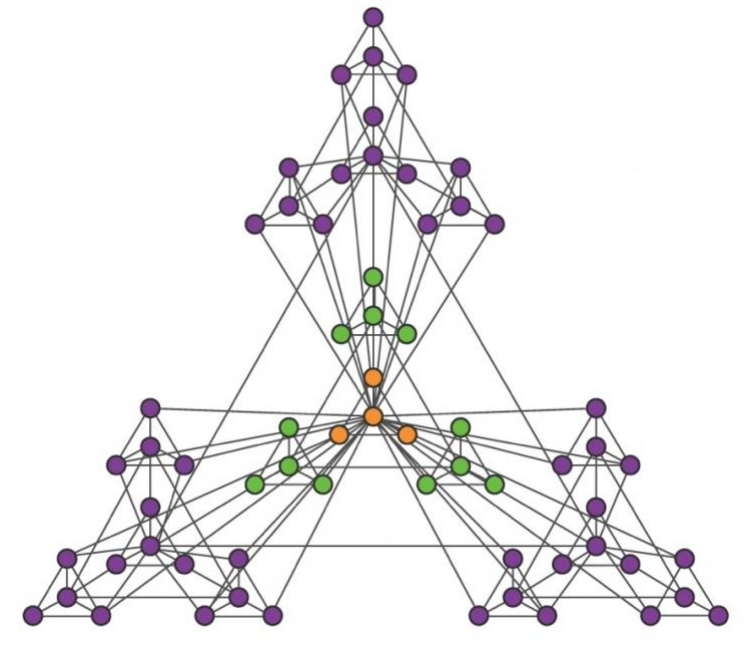

Figure 1. Hierarchical Networks

Problem 2. Communities on a Circle

Consider a one dimensional lattice with N nodes that form a circle, where each node connects to its two neighbors. Partition the line into n_c consecutive clusters of size $N_c=N/n_c$.

a) Calculate the modularity of the obtained partition.

b) According to the Maximum Modularity Hypothesis, the maximum of M_c corresponds to the best partition. Obtain the community size n_c corresponding to the best partition.

Problem 3. Modularity Resolution Limit

Consider a network consisting of a ring of n_c cliques, each clique having N_c nodes and $m(m–1)/2$ links. The neighboring cliques are connected by a single link (Figure 2). The network has an obvious community structure, each community corresponding to a clique.

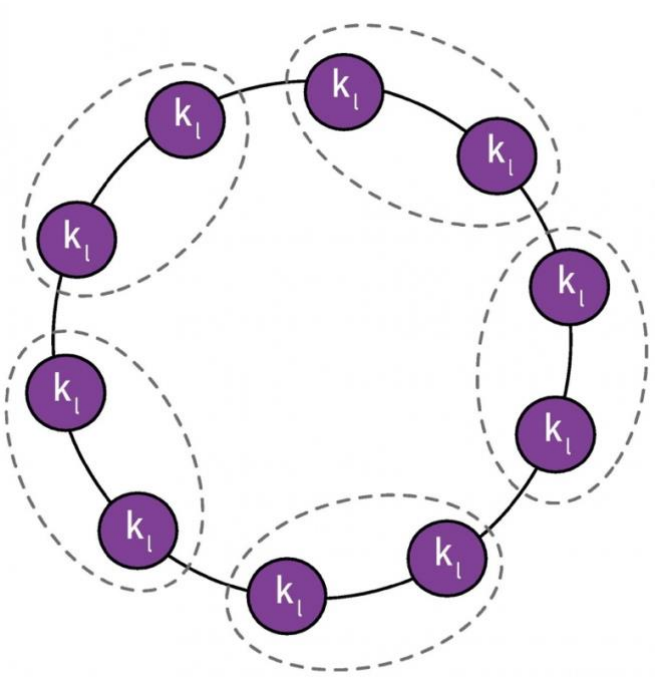

Figure 2. Modularity

a) Determine the modularity M_{single} of this natural partition, and the modularity M_{pairs} of the partition in which pairs of neighboring cliques are merged into a single community, as indicated by the dotted lines in Figure 2.

(b) Show that only for $n_c < 2L$ will the modularity maximum predict the intuitively correct community partition, where $L = \frac{n_c m(m-1)}{2} + n_c$.

(c) Discuss the consequences of violating the above inequality.

Problem 4. Modularity Maximum.

$$M = \sum_{c=1}^{n_c} \left[\frac{L_c}{L} - \left(\frac{k_c}{2L} \right)^2 \right]$$

Show that the maximum value of modularity M cannot exceed one.

Tài liệu tham khảo

[1] Network Science by Albert-László Barabási. (n.d.). BarabásiLab. http://networksciencebook.com