FE630 Portfolio Theory & Applications Final Project

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Objective

- Using a French Fama 3-factor model, build an investment strategy that maximizes portfolio return subject to a constraint of target beta and weekly portfolio rebalancing from March 2007 – November 2020.

- In addition, evaluate portfolio sensitivity to variations in both target beta as well as length of estimators for expected returns and covariance matrix under different market scenarios.

Assumptions

Investment Universe & Analysis Setup

- Investment Universe is composed of 12 ETFs (FXE, EWJ, GLD, QQQ, SPY, SHV, DBA, USO, XBI, ILF, EPP, FEZ)
- Market returns are represented by SPY ETF
- Analysis broken up into 3 market scenarios:
  - Pre-crisis (March 1, 2007 – March 31, 2008)
  - Crisis (April 1, 2008 –Sept 30, 2008)
  - Post-crisis (October 1, 2008 – June 30, 2020)

Objective Function & Constraints

$$\begin{cases} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda(\omega - \omega_p)^T \Sigma (\omega - \omega_p) \\ \sum_{i=1}^{n} \beta_i^m \omega_i = \beta_T^m \\ \sum_{i=1}^{n} \omega_i = 1, \ -2 \leq \omega_i \leq 2, \end{cases}$$

Target Beta

- Empirical calculation of individual asset betas for all 3 market scenarios yielded a range from 0 to 1.5
- Sequence used for testing target beta sensitivity: [0, 0.5, 1, 1.5]
- Lack of assets with significant inverse correlations; individual betas mostly stable through all 3 market scenarios (slight exception: GLD turned negative during crisis, possibly due to investor fears at the time)

Expected Returns & Covariance

- Term-structure of estimators is divided into three cases:
  - Short-term (<=40 days)
  - Long-term (>=120 days)
  - Medium-term

Notations

- CAPM Model is used to derive beta of each asset:

$$\rho_i = r_f + \beta_i^M(\rho_M - r_f) + \alpha_i$$

- ρ_i is the expected return of the asset
- r_f is the risk-free return
- ρ_M is the market return
- α_i is excess return

- For each market scenario, individual betas were calculated as follows:

$$\beta_i^M = \frac{cov(r_i, r_M)}{\sigma^2(r_M)}$$

- French Fama 3-Factor Model's coefficients are estimated by the linear regression of the time series data (ARIMA Model) based on the specified market scenario:

$$r_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{SMB} + b_i^v \rho_{HML} + \alpha_i$$

- ρ_{SMB} is the size factor
- ρ_{HML} is the value factor
- β_i^3, b_i^s, b_i^v are the coefficients to be estimated.

- $S_{60}^{90}(\beta_T^m = 0)$ is a description of the assumptions of a given portfolio strategy. In this case, we are using 90 days for estimation of expected returns, 60 days for estimation of covariance, and a target Beta of 0.

Workflow

Write a function to realize the optimization problem

$$\begin{cases} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda(\omega - \omega_p)^T \Sigma(\omega - \omega_p) \\ \sum_{i=1}^{n} \beta_i^m \omega_i = \beta_T^m \\ \sum_{i=1}^{n} \omega_i = 1, \ -2 \leq \omega_i \leq 2, \end{cases}$$

Apply FF model to calculate covariance matrix with input of its look back period and the end date

With the FF model:

Write a trend estimator to predict expected returns.

Every time pass in the value: back period, end date, ticker symbol and variance, of a single ETF. Then the estimator will provide the expected return of the ETF in the coming week.

Beginning on the first day of the target period, update the strategy every week

Record daily returns of the portfolio during the target period, plot its cumulative return versus the that of SPY

$$-1 + \prod_{t=0}^{T} (1 + r_t)$$

Trend Estimator:

When building or updating the portfoli strategy, for each ETF, first run linear regression on specific look back period to find the 3 coefficients.

Then fit ARIMA model on all the 3 factors, one time for each

$$\rho_i = r_f + \beta_i^3 \boxed{(\rho_M - r_f)} + b_i^s \boxed{\rho_{SMB}} + b_i^v \boxed{\rho_{HML}} + \alpha_i$$

coefficients

factors

Use the predictions of the 3 factors, and the coefficients we just get from the linear regression, to get the expected return for the coming week.

Do the process above for 12 times and get the rho array for the whole universe. Then use the updated rho series to build the strategy accordingly

Overview of Model Parameters

- All possible combinations of model parameters summarized below

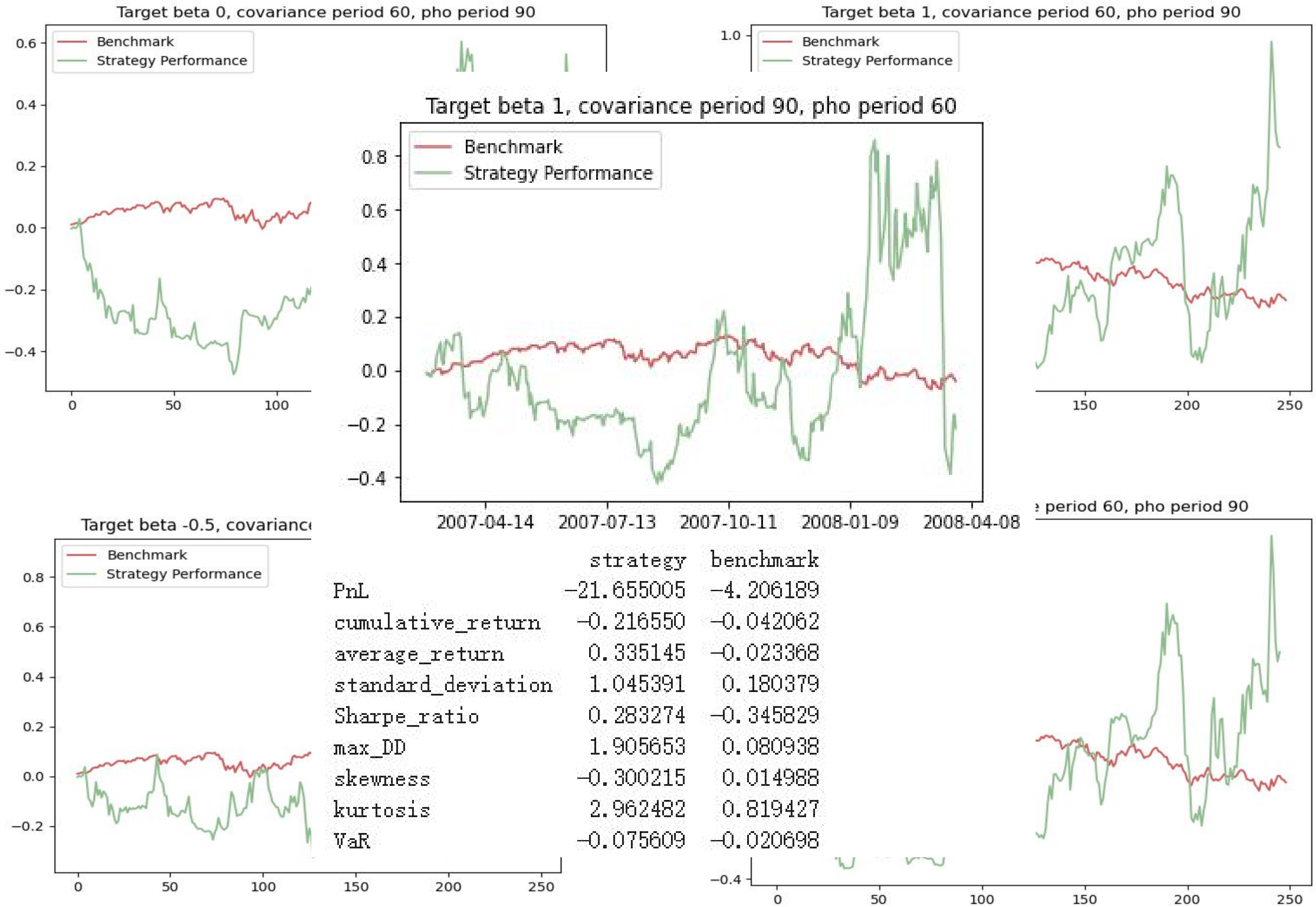
Market scenarios	Target Beta	Covariance estimators (days)	Rho estimators (days)
04/01/2007 – 03/31/2008	1.0	30	30
04/01/2008 – 12/31/2008	.5	60	60
1/1/2009 – 6/30/2020	0	90	90
04/01/2007 – 06/30/2020	-.5	120	120
	-1.0	150	150

Sensitivity on target beta:

- Market Scenario: 04/01/2007 – 03/31/2008

Target Beta Sensitivity	$S_{60}^{90}(\beta_T^m = 0)$	$S_{60}^{90}(\beta_T^m = 1)$	$S_{60}^{90}(\beta_T^m = -.5)$	$S_{60}^{90}(\beta_T^m = .5)$	SPY
Cum PnL	22.455	55.2541	47.6626	40.8289	-5.69
Mean Return	.659	.8808	.8256	.7834	-.0419
Max 10 Days DD	1.0557	.9859	.6519	.8856	.0889
Standard Deviation	.9506	.9364	.9271	.9383	.1839
Sharpe Ratio	.653	.8997	.8492	.7941	-.4363
Skewness	.0695	.4087	.1952	.4122	.0295
Kurtosis	2.0736	2.1974	2.9613	2.2943	.7669
VaR	-.1251	-.0913	-.0812	-.0969	-.0203

Some visualizations:



	strategy	benchmark
PnL	-21.655005	-4.206189
cumulative_return	-0.216550	-0.042062
average_return	0.335145	-0.023368
standard_deviation	1.045391	0.180379
Sharpe_ratio	0.283274	-0.345829
max_DD	1.905653	0.080938
skewness	-0.300215	0.014988
kurtosis	2.962482	0.819427
VaR	-0.075609	-0.020698

Sensitivity on look back period for covariance matrix :

- Market Scenario: 04/01/2007 – 03/31/2008

Sigma period sensitivity	$S_{30}^{60}(\beta_T^m = .5)$	$S_{60}^{60}(\beta_T^m = .5)$	$S_{90}^{60}(\beta_T^m = .5)$	$S_{120}^{60}(\beta_T^m = .5)$	SPY
Cum PnL	59.5887	-61. 8408	32.1388	-16.7347	-5.69
Mean Return	1.0512	-.343	.884	.4339	-.0419
Max 10 Days DD	.6537	1.061	.6663	1.0329	.0889
Standard Deviation	1.0731	1.1144	1.0943	1.1067	.1839
Sharpe Ratio	.944	-.3422	.7729	.3575	-.4363
Skewness	.1529	-.1269	.1873	-.0269	.0295
Kurtosis	2.4027	2.4955	3.4519	2.3049	.7669
VaR	-.1055	-.1152	-.0883	-.0933	-.0203

Sensitivity on look back period for expected return :

- Market Scenario: 04/01/2007 – 03/31/2008

Rho period sensitivity	$S_{90}^{30}(\beta_T^m = 1)$	$S_{90}^{60}(\beta_T^m = 1)$	$S_{90}^{90}(\beta_T^m = 1)$	$S_{90}^{120}(\beta_T^m = 1)$	SPY
Cum PnL	-1.4673	-21.655	2.5649	15.7183	-5.69
Mean Return	.5186	.3351	.4317	.5833	-.0419
Max 10 Days DD	.9322	1.9057	.7768	.6405	.0889
Standard Deviation	1.0337	1.0454	.9044	.9294	.1839
Sharpe Ratio	.4646	.2833	.435	.5864	-.4363
Skewness	.2158	-.3002	.3533	.0463	.0295
Kurtosis	2.243	2.9625	2.3995	3.5201	.7669
VaR	-.1039	-.0756	-.0858	-.0797	-.0203

Sensitivity on target beta:

- Market Scenario: 04/01/2008 – 12/31/2008

Target Beta sensitivity	$S_{60}^{90}(\beta_T^m = -.5)$	$S_{60}^{90}(\beta_T^m = 0)$	$S_{60}^{90}(\beta_T^m = .5)$	$S_{60}^{90}(\beta_T^m = 1)$	SPY
Cum PnL	-72.7936	-16.0105	13.6613	16.3497	-33.634
Mean Return	-.8655	.6402	1.0628	1.0489	-.4365
Max 10 Days DD	1.3751	1.1447	.7416	.8209	.3324
Standard Deviation	1.3066	1.2998	1.3165	1.28	.4538
Sharpe Ratio	-.6729	.482	.7969	.8089	-.9922
Skewness	-.0487	-.3134	-.281	-.405	.554
Kurtosis	.6593	2.5026	2.1897	1.8322	5.2836
VaR	-.1472	-.1358	-.1202	-.1157	-.0641

Sensitivity on look back period for covariance matrix :

- Market Scenario: 04/01/2008 – 12/31/2008

Sigma period sensitivity	$S_{30}^{90}(\beta_T^m = 1)$	$S_{60}^{90}(\beta_T^m = 1)$	$S_{90}^{90}(\beta_T^m = 1)$	$S_{120}^{90}(\beta_T^m = 1)$	SPY
Cum PnL	191.6674	16.3497	-11.1577	71.9459	-33.634
Mean Return	2.3114	1.0489	.6368	1.562	-.4365
Max 10 Days DD	.8226	.8209	.8159	1.223	.3324
Standard Deviation	1.3044	1.2798	1.2388	1.2675	.4538
Sharpe Ratio	1.7615	.8089	.503	1.2215	-.9922
Skewness	-.4331	-.405	-.4126	-.5817	.554
Kurtosis	1.7074	1.8322	2.3022	1.9342	5.2836
VaR	-.0999	-.1157	-.1088	-.1101	-.0641

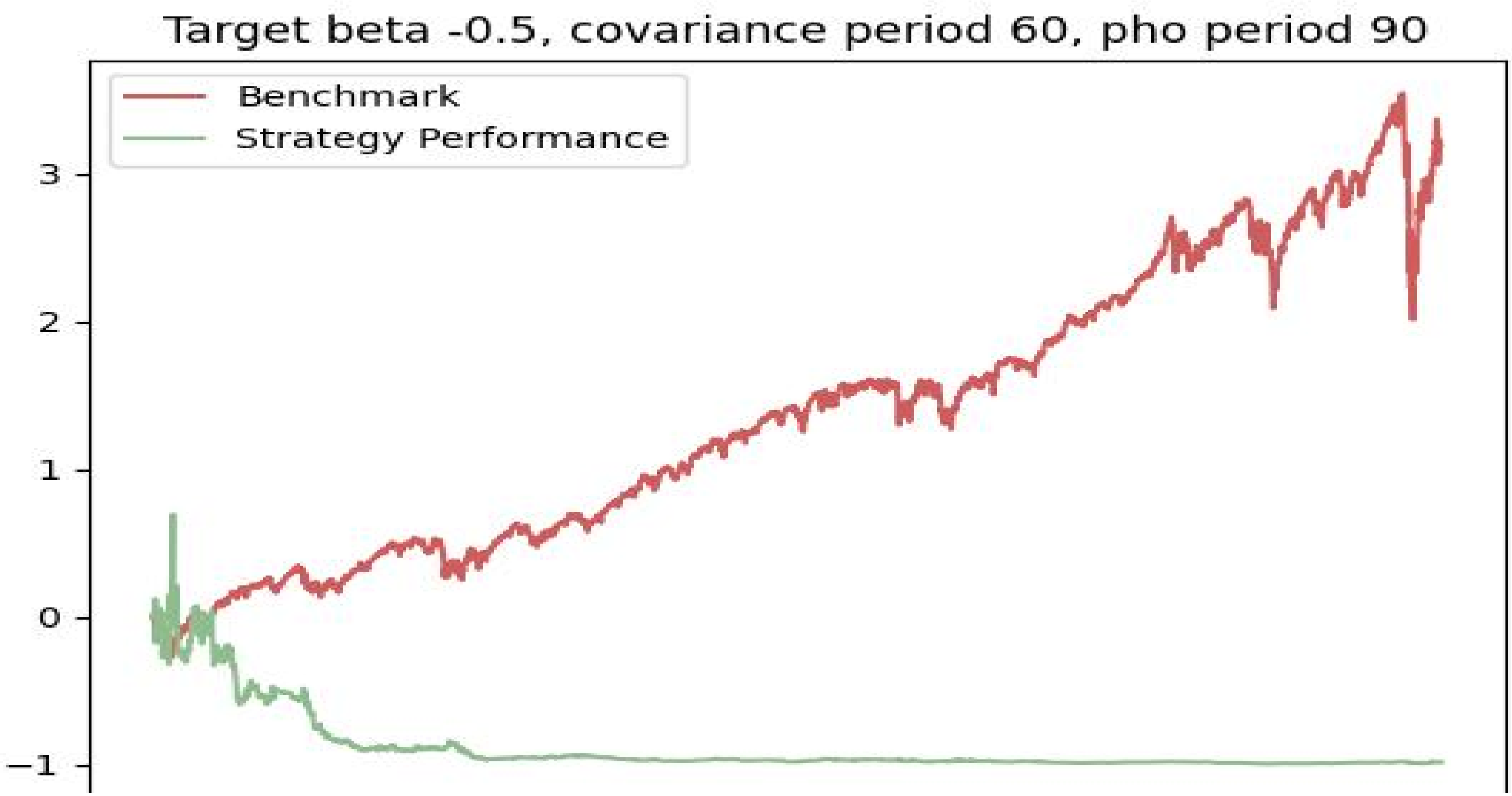
Sensitivity on look back period for expected return :

- Market Scenario: 04/01/2008 – 12/31/2008

Rho period sensitivity	$S_{60}^{30}(\beta_T^m = .5)$	$S_{60}^{60}(\beta_T^m = .5)$	$S_{60}^{90}(\beta_T^m = .5)$	$S_{60}^{120}(\beta_T^m = .5)$	SPY
Cum PnL	-96.2156	277.8341	13.6613	-2.0344	-33.634
Mean Return	-2.8985	2.8366	1.0628	.7809	-.4365
Max 10 Days DD	3.1765	.9542	.7416	.6934	.3324
Standard Deviation	1.6171	1.5099	1.3165	1.2732	.4538
Sharpe Ratio	-1.8009	1.8696	.7969	.6026	-.9922
Skewness	-.3743	1.6171	-.281	.1957	.554
Kurtosis	5.355	8.2819	2.1897	.8234	5.2836
VaR	-.1692	-.1042	-.1202	-.1142	-.0641

Performance Statistics

- Market Scenario: 1/1/2009 – 6/30/2020



Performance Statistics

Due to the complexity of our trend estimator, it takes too long to run an estimation so we have to leave this table for later work. The code is still running after 9 hours.

- 04/01/2007 – 06/30/2020

	$S_{30}^{60}(\beta_T^m) = .5$	$S_{60}^{90}(\beta_T^m) = .5$	$S_{60}^{90}(\beta_T^m) = 1$	$S_{60}^{90}(\beta_T^m) = -.5$	SPY
Cum PnL					
Mean Return					
Max 10 Days DD					
Standard Deviation					
Sharpe Ratio					
Skewness					
Kurtosis					
VaR					

Summary and Next Steps

Summary:

During normal periods, target beta should be close to the benchmark for best results. During times of crisis, the impact of beta is less clear; therefore, we do not recommend using target beta as an effective metric of portfolio performance during this time.

Changes in the covariance lookback period appears to have the greatest impact on PnL. Shorter covariance lookback periods appears to be optimal for short period portfolio strategy.

Investors can also choose which groups of look-back periods and beta to choose, based on their risk preference.

Next Steps:

Expanding the universe of asset such that the range of target betas is increased may provide greater insight into portfolio strategy.

Change the value of lambda in order to observe the impact on portfolio sensitivity analysis.

Try different models to predict factors which may lead to more efficient code.

Questions?