BİLKENT UNIVERSITY
CTIS163 - DISCRETE MATHEMATICS
2022 – 2023 Spring
FINAL EXAM
150 minutes – June 15, 2023

Name	
Surname	
Student Id	

NOTICE TO THE STUDENTS

Read the instructions carefully listed below and sign the box:

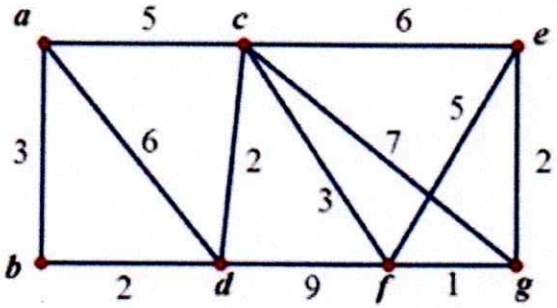
1. An A4 size paper both sides written by the student can be used during the exam.
2. Textbooks, lecture notes, calculators with extensive memories, and any kind of computers are not permitted.
3. Cell phones and smart watches should be totally switched off (not in silent or flight modes).
4. Permitted material to be kept on your desks are; pencils, sharpeners, erasers, and water.
5. You are not allowed to **talk** to other students during the exam whatever the reason may be.
6. Disobeying the above rules will be severely penalized and a **disciplinary action** will be conducted.
7. Please **prepare your ID's (with photos)** on your desk for identity check.

SIGNATURE	

Q1/10	Q2/10	Q3/20	Q4/10	Q5/10	Q6/20	Q7/10	Q8/10	Total/100

Q1. (10 pts) Find the **shortest path** from vertex **g** to vertex **a** by using **Dijkstra's** shortest-path algorithm for the weighted graph shown below. *Show your work by listing all the temporary and permanent labels for each vertex in each iteration of the algorithm, tabularly.*

- Write the shortest path as a vertex list. What is the **length** of your shortest path?



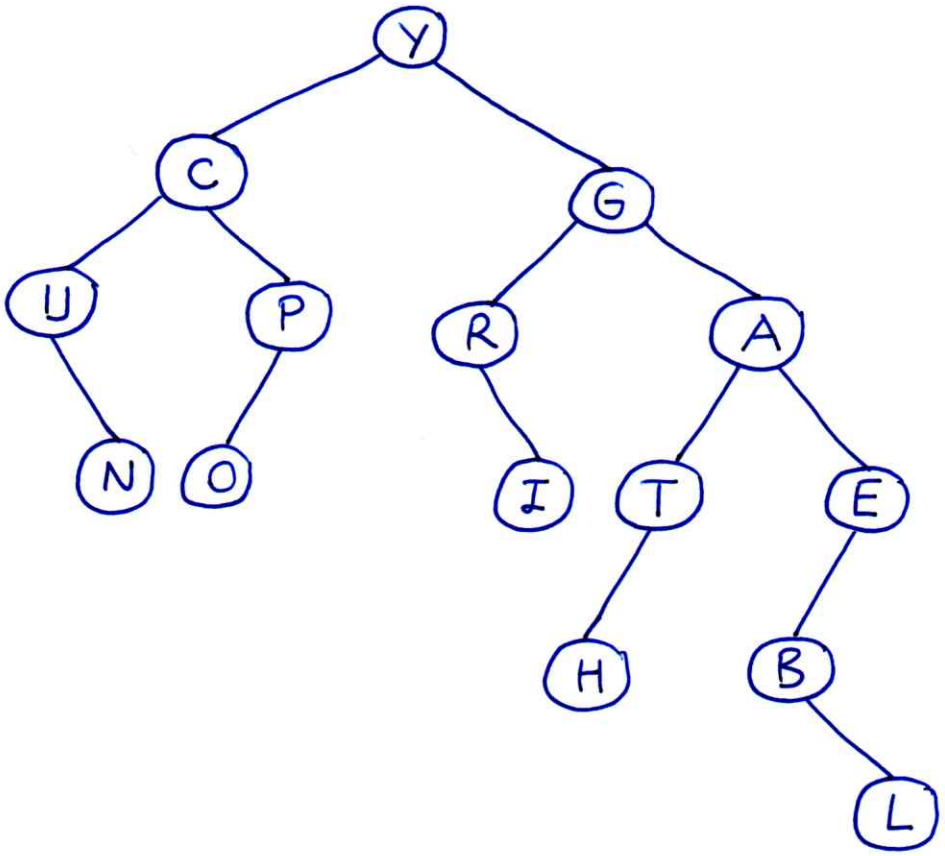
Iteration	a	b	c	d	e	f	g
0	∞	∞	∞	∞	∞	∞	0
1	∞	∞	7	∞	2	1	0
2	∞	∞	4	10	2	1	
3	∞	∞	4	10	2		
4	9	∞	4	6			
5	9	8		6			
6	9	8					
7	9						

Shortest path = (g, f, c, a) Length = 9

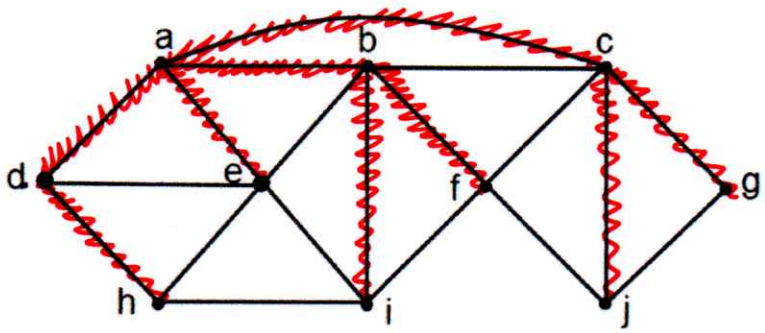
Q2. (10 pts) Assume that the *Preorder* and *Inorder* vertex orderings of a binary tree are given as follows. Draw the corresponding tree.

Preorder: **Y C U N P O G R I A T H E B L**

Inorder: **U N C O P Y R I G H T A B L E**



Q3. (a) (10 pts) Find and draw a **spanning tree** by using **Breadth-First** Search algorithm with the vertex ordering **a, b, c, d, e, f, g, h, i, j** for the following graph. *Explain your work. Write the edges added in each step.*

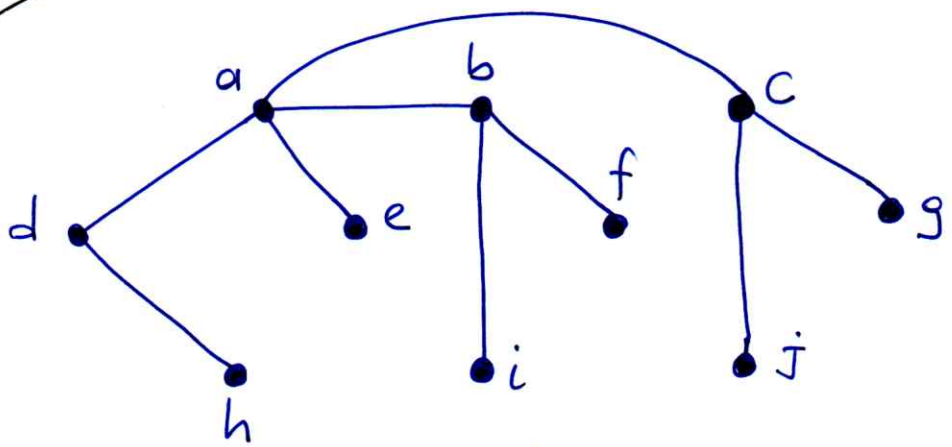


Level 0: a : add(a,b)
add(a,c)
add(a,d)
add(a,e)

Level 1: b : add(b,f)
add(b,i)
c : add(c,g)
add(c,j)
d : add(d,h)
e : None

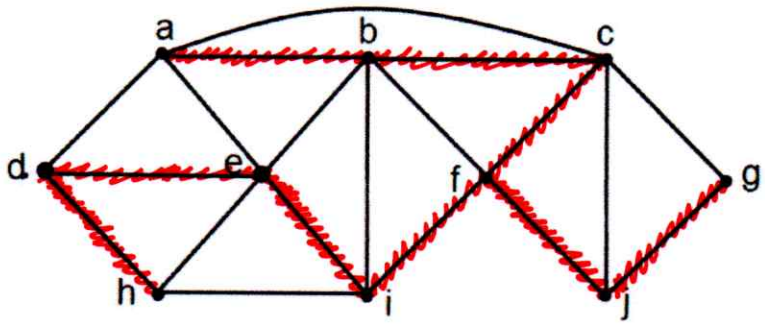
Level 2 : f : None
g : None
h : None
i : None
j : None

STOP

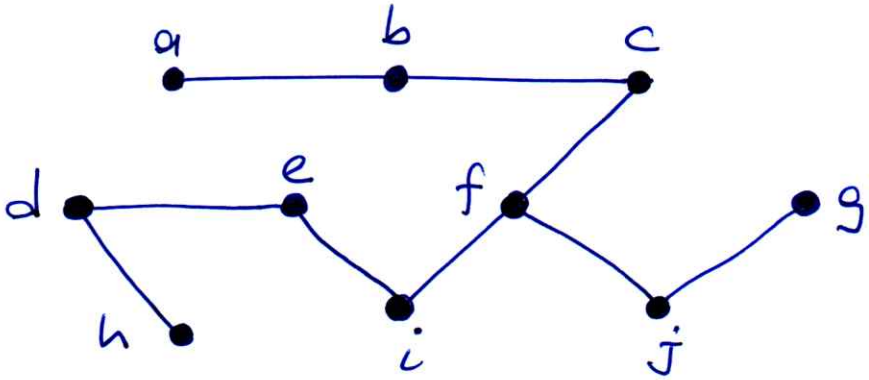


Spanning Tree

(b) (10 pts) Find and draw a **spanning tree** by using **Depth-First** Search algorithm with the vertex ordering **a, b, c, d, e, f, g, h, i, j** for the following graph. *Explain your work. Write the edges added in each step.*

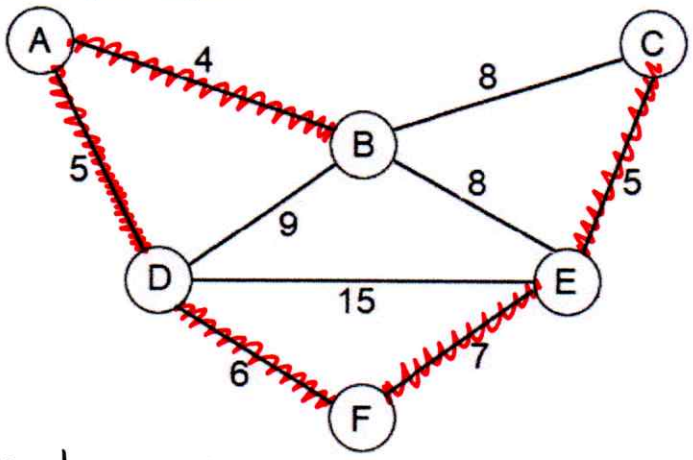


a : add(a, b)
b : add(b, c)
c : add(c, f)
f : add(f, i)
i : add(i, e)
e : add(e, d)
d : add(d, h)
h : None
Backtrack to d, e, i, f
f : add(f, j)
j : add(j, g)
g : None
Backtrack to j, f, c, b, a.
STOP



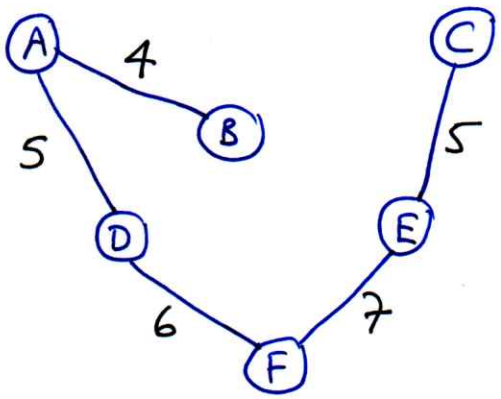
Spanning Tree

Q4. (10 pts) Find and **draw** a minimal spanning tree for the following graph by using **Prim's Algorithm**. Assume that the start vertex is **D**. *Show your work and write down the order of edges added to the minimal spanning tree in each iteration of the algorithm*. Write the **weight** of your minimal spanning tree.



Iteration	Edge	Weight
1	(D,A)	5 ← min
	(D,B)	9
	(D,E)	15
	(D,F)	6
2	(D,B)	9
	(D,E)	15
	(D,F)	6
	(A,B)	4 ← min
3	(D,E)	15
	(D,F)	6 ← min
	(B,C)	8
	(B,E)	8
4	(D,E)	15
	(B,C)	8
	(B,E)	8
	(F,E)	7 ← min
5	(B,C)	8
	(E,C)	5 ← min

STOP



Minimal Spanning Tree.

Weight = 27

Q5. (10 pts) Prove the following statement using the laws and properties of **Boolean algebra** (*write the name of the law used in each step*).

$$x = x \cdot x$$

$X = X \cdot 1$ Identity law

$= X \cdot (X + X')$ Complement law

$= X \cdot X + X \cdot X'$ Distributive law

$= X \cdot X + 0$ Complement law

$= X \cdot X$ Identity law.

Q6. Given the Boolean function $f(x, y, z) = x'y + xy' + (x + x')(yy' + x'z)z$.

(a) (10 pts) Find the **Disjunctive Normal Form** (*Sum of Products*) of the function by using algebraic methods. *Use the laws of Boolean Algebra.*

$f(x,y,z) = x'y + xy' + \underbrace{(x+x')}_{1}\underbrace{(yy')}_{0} + x'z)z$

$= x'y + xy' + x'z$ (SOP)

$= x'y(z+z') + xy'(z+z') + x'z(y+y')$

$= x'yz + x'yz' + xy'z + xy'z' + x'yz + x'y'z$

$011 \quad 010 \quad 101 \quad 100 \quad 001$

$m_3 \quad m_2 \quad m_5 \quad m_4 \quad m_1$

$= m_1 + m_2 + m_3 + m_4 + m_5$

$= \Sigma(1, 2, 3, 4, 5)$

$x \cdot 1 = x$

$x + 0 = x$

$x \cdot x = x$

$x + x' = 1$

(b) (10 pts) Find the **Conjunctive Normal Form** (*Products of Sum*) of the function by using algebraic methods. *Use the laws of Boolean Algebra.*

$f(x,y,z) = x'y + xy' + x'z$

$= (x' + xy' + x'z)(y + xy' + x'z)$ Distributive law

$= (x' + xy')(y + y' + x'z)(y + x + x'z)$

$= (x' + x)(x' + y')(x + y + x')(x + y + z)$

$= (x' + y')(x + y + z)$ (POS)

$= (x' + y' + zz')(x + y + z)$

$= (x' + y' + z)(x' + y' + z')(x + y + z)$

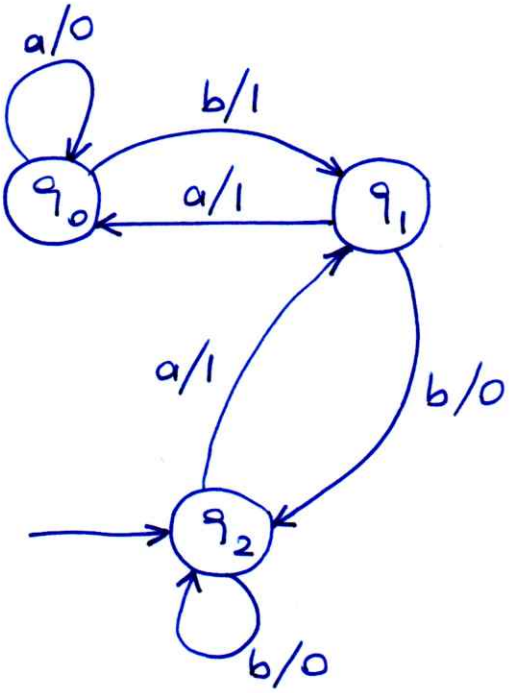
$1\ 1\ 0$ M_6 $\quad 1\ 1\ 1$ M_7 $\quad 0\ 0\ 0$ M_0

$= M_0 \cdot M_6 \cdot M_7$

$= \Pi(0,6,7)$

Q7. (10 pts) Draw the *state transition diagram* of the finite-state machine defined by the following table where $I = \{a, b\}$, $O = \{0, 1\}$, $S = \{q_0, q_1, q_2\}$, and the initial state is q_2.

	f		g	
S \ I	a	b	a	b
q_0	q_0	q_1	0	1
q_1	q_0	q_2	1	0
q_2	q_1	q_2	1	0



Q8. (10 pts) Design a finite state machine that outputs **1** whenever it sees **abc**; otherwise, outputs **0**. The input string is over {a, b, c}.