BİLKENT UNIVERSITY **CTIS163 - DISCRETE MATHEMATICS** **2022 – 2023 Fall** **FINAL** **120 minutes – January 5, 2023**	**Name**	
	Surname	
	Student Id	

NOTICE TO THE STUDENTS

Read the instructions carefully listed below and sign the box:

1. An A4 size paper both sides written by the student can be used during the exam.
2. Textbooks, lecture notes, calculators with extensive memories, and any kind of computers are not permitted.
3. Cell phones and smart watches should be totally switched off (not in silent or flight modes).
4. Permitted material to be kept on your desks are; pencils, sharpeners, erasers (and in case you may need: water and tissues). **Pencil boxes are strictly forbidden**.
5. You are not allowed to **talk** to other students during the exam whatever the reason may be.
6. Disobeying the above rules will be severely penalized and a **disciplinary action** will be conducted.
7. Please **prepare your ID's (with photos)** on your desk for identity check.

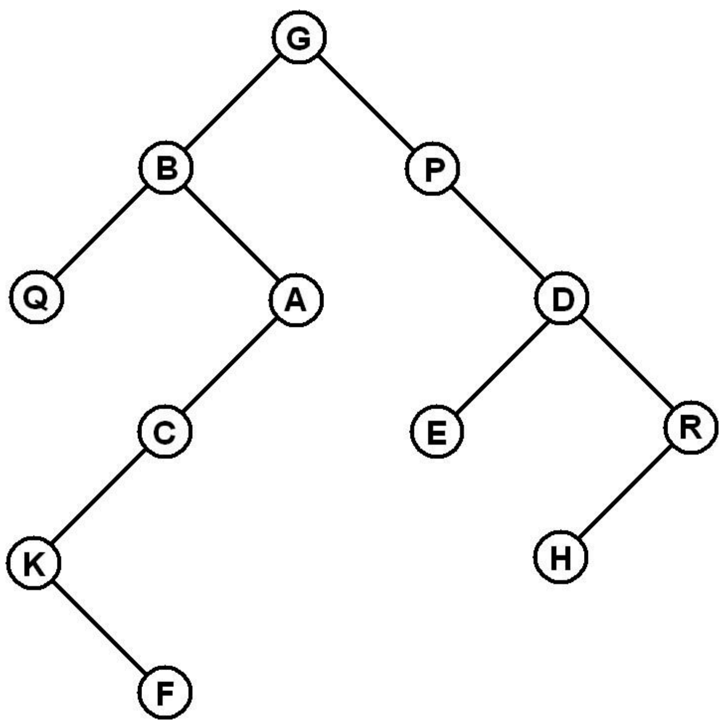
1/15	2/10	3/10	4/15	5/30	6/20	Total/100

Q1. (a) (10 pts) Draw the corresponding binary tree for the following tree traversals.

Preorder : G B Q A C K F P D E R H

Inorder : Q B K F C A G P E D H R

Solution:

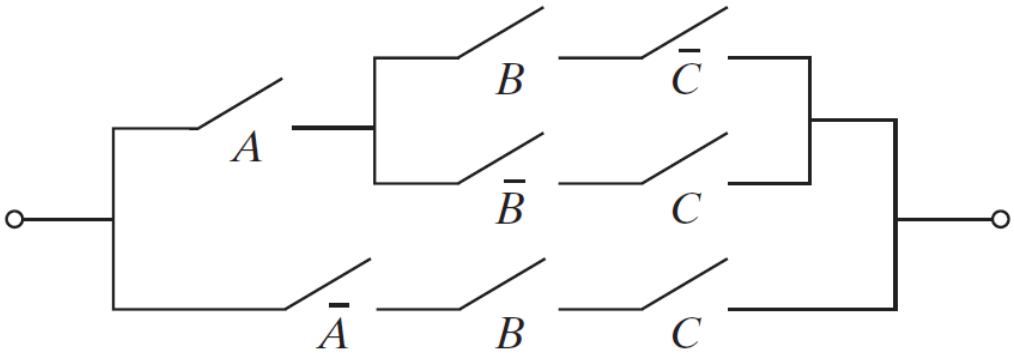


(b) (5 pts) Write the **Postorder** vertex listing of the binary tree in part (a).

Solution: Postorder: Q F K C A B E H R D P G

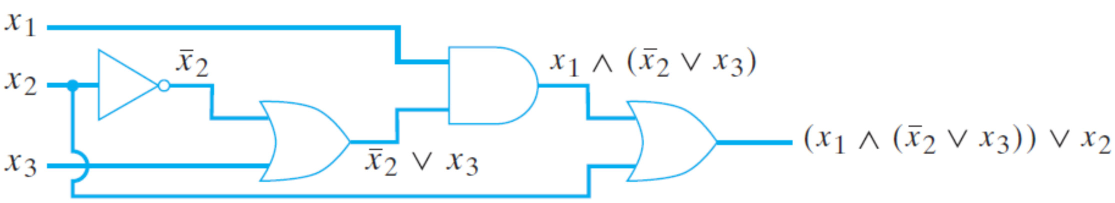
Q2. (10 pts) Represent $\{ A \wedge [(B \wedge \overline{C}) \vee (\overline{B} \wedge C)] \} \vee (\overline{A} \wedge B \wedge C)$ as switching circuit

Solution:



Q3. (10 pts) Draw combinatorial circuit corresponding to $(x_1 \wedge (\overline{x}_2 \vee x_3)) \vee x_2$.

Solution:



Q4. (a) (15 pts) Prove the following statement using the laws and properties of Boolean algebra.

$$\boldsymbol{xy' = 0} \quad \text{if and only if} \quad \boldsymbol{xy = x}$$

Solution: *If* $xy' = 0$ *then* $xy = x$

<u>*Proof:*</u>

x	$=$	$x1$	*Identity law*
	$=$	$x(y+y')$	*Complement law*
	$=$	$xy + xy'$	*Distributive law*
	$=$	$xy + 0$	*Given* $xy' = 0$
	$=$	xy	*Identity law*

If $xy = x$ *then* $xy' = 0$

<u>*Proof:*</u>

xy'	$=$	$(xy)y'$	*Given* $xy = x$
	$=$	$x(yy')$	*Associative law*
	$=$	$x0$	*Complement law*
	$=$	0	*Bound law*

Q5. Given the Boolean function $f(x,y,z) = (x'z + yy')(y + y')z + x'y + xy'$

(a) (15 pts) Find the **disjunctive** normal form (*sum of **minterms***) of the function by using algebraic methods only. *Don't use logic table.*

Solution: $f(x,y,z) = (x'z + \underbrace{yy'}_{0})\underbrace{(y + y')}_{1}z + x'y + xy' = (x'\underbrace{z)z}_{z} + x'y + xy'$

$f(x,y,z) = x'z + x'y + xy' = x(y + y')z + x'y(z + z') + xy'(z + z')$

$f(x,y,z) = x'yz + x'y'z + x'yz + x'yz' + xy'z + xy'z'$

$f(x,y,z) = x'yz + x'y'z + x'yz' + xy'z + xy'z'$

011 001 010 101 100

3 1 2 5 4

$f(x,y,z) = m_1 + m_2 + m_3 + m_4 + m_5 = \sum(1,2,3,4,5)$

(b) (15 pts) Find the **conjunctive** normal form (*product of **maxterms***) of the function by using algebraic methods only. *Don't use logic table.*

Solution: $f(x,y,z) = (x'z + \underbrace{yy'}_{0})\underbrace{(y + y')}_{1}z + x'y + xy' = (x'\underbrace{z)z}_{z} + x'y + xy'$

$f(x,y,z) = x'z + x'y + xy' = (x' + x'y + xy')(z + x'y + xy')$

$f(x,y,z) = (\underbrace{x' + x'}_{x'} + xy')(x' + y + xy')(z + x' + xy')(z + y + xy')$

$f(x,y,z) = (x' + xy')(x' + y + xy')(z + x' + xy')(z + y + xy')$

$f(x,y,z) = (x' + x)(x' + y')(x' + y + x)(x' + y + y')(z + x' + x)(z + x' + y')$

$(z + y + x)(z + y + y')$ **All red terms give 1. ∴ We can remove them**

$f(x,y,z) = (x' + y')(z + x' + y')(z + x + y) = (x' + y' + zz')(x' + y' + z)(x + y + z)$

$f(x,y,z) = (x' + y' + z)(x' + y' + z')(x' + y' + z)(x + y + z)$ **Blue terms are same**

$f(x,y,z) = (x' + y' + z)(x' + y' + z')(x + y + z)$

110 111 000

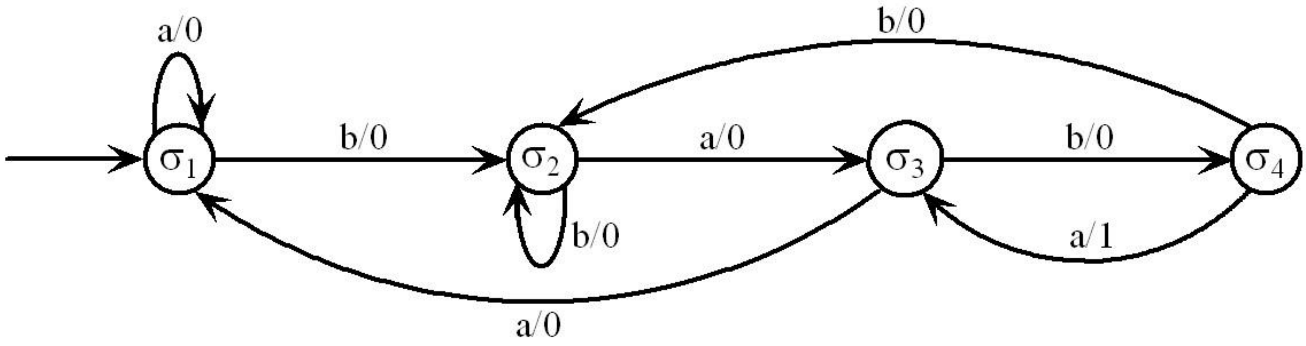
6 **7 0**

$f(x,y,z) = M_0 \cdot M_6 \cdot M_7 = \prod(0,6,7)$

Q6. Design a finite-state machine that outputs 1 whenever it sees baba in the input string; otherwise, outputs 0. (The machine receives strings over {a, b} as input.)

a. (10 pts) Draw the **state transition diagram** of the finite-state machine.

Solution:



b. (5 pts) Write down the sets *I*, *O*, *S*, the initial state and the table defining the next-state function (f) and output function (g) for the finite-state machine.

Solution: *$I = \{a, b\}$, $O = \{0,1\}$, $S = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ and the initial state is σ_1.*

	f		*g*	
S \ *I*	*a*	*b*	*a*	*b*
σ_1	σ_1	σ_2	0	0
σ_2	σ_3	σ_2	0	0
σ_3	σ_1	σ_4	0	0
σ_4	σ_3	σ_2	1	0