

NOTES

HYPOTHESIS : If ,
employing the Einstein eqs (or Einstein - Lovelock eqs) ,
superluminal inflation / deflation exists,
then at time $x_4 = 0$ (before the particles of the standard model exist)
a pair of universes with $MASSES \pm M$ is created
(i.e., universes are created in pairs)
Dark matter / dark energy may be
related to wave function overlap of these two universes

THE FOLLOWING SCRATCH IS CONSISTENT WITH
UNIVERSES THAT ARE CREATED IN PAIRS, WITH $MASSES \pm M$.

Note that we are ONLY looking for
superluminal inflation or deflation type solutions ,
and NOT solutions that are even / odd functions of t
(like $\cos[\nu[j]^\theta * t]$, $\sin[\nu[j]^\theta * t]$,
 $\text{Sech}[\nu[j]^\theta * t]$, $\tanh[\nu[j]^\theta * t]$, ...),
which also have $\pm M$ type eigenvalues ,
or even solutions involving the
well -
known special functions that might also have $\pm M$ type parameters.

Bigger Bang:

Question: Are Universe (s) of masses $\pm M$ created in pairs at time $x_4 = 0$
(before the particles of the standard model exist) ?

Scratch work

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Do not read if you are irascible; I apologize for typos and duplications and

git clone <https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git>

“git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git”

Ever wonder why there are 3 space dimensions and 1 time dimension?

Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in \text{Nature}$?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\| = \|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over \mathbb{R}) real numbers \mathbb{R} , the 2-dim complex numbers, the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in "A Remarkable Representation of the 3+2 de Sitter Group" by P.A.M.Dirac, J.Math.Phys.4,901–909 (1963).

Comment: The term "reduced Brauer–Weyl generators", below, refers to a specific set of irreducible generators for certain complex Clifford algebras, particularly in the context of mathematical physics and the study of Dirac spinors. These generators were introduced by **E.A. Lord** (E. A. Lord. "The Dirac spinor in six dimensions". Mathematical Proceedings of the Cambridge Philosophical Society, vol. 64, pp. 765–778, 1968) and are described as "reduced" because they form an irreducible representation of the Clifford algebra C_{2n} from $C_{2(n-1)}$.

Here, we employ Lord's idea of reduced Brauer–Weyl generators to extend Dirac's 4×4 gamma matrices into this notebook's real tau8 8×8 and tau16 16×16 matrices,

which are employed to write down the 'Dirac Equation for the Universe',

[remember, this is before the particles of the standard model exist]

formulated in terms of a $O(4, 4)$ spinor Ψ_6

(Ψ_6 AKA '**WAVE FUNCTION OF the UNIVERSE**', which could possibly be named something more pompous).

Then this equation is used to

[WARNING: syncope, presyncope AHEAD]

couple the **WAVE FUNCTION OF the UNIVERSE** to Gravity.

THIS WORK IS PARTIALLY BASED ON :

JOURNAL OF MATHEMATICAL PHYSICS

VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. DIRAC

*Cambridge University, Cambridge, England and
Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

Proc. Camb. Phil. Soc. (1968), **64**, 765
 PCPS 64–96
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765

The Dirac spinor in six dimensions

By E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

JOURNAL OF MATHEMATICAL PHYSICS **51**, 042501 (2010)

Second gravity

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IL NUOVO CIMENTO

VOL. 105 B, N. 1

Gennaio 1990

On the Structure of the Split Octonion Algebra.

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(ricevuto il 24 Luglio 1989)

Identities satisfied by the generators of the Dirac algebra

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(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin- $\frac{1}{2}$ wave equation

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(Received 24 January 1984; accepted for publication 16 November 1984)

Here we use [spacetime coordinates](#) that are selected so that we are somewhat consistent with the [exact libraries](#),

(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):

[Cartesian coordinates:](#)

x_0 = hidden space (a small circle); x_1, x_2, x_3 are the usual 3-space coords;

x_4 = time coord,

and x_5, x_6, x_7 = superluminal [deflating](#) time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{SO(4, 4; \mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M8 , whose tangent bundle has

$\overline{SO(4, 4; \mathbb{R})} \approx \text{Spin}(4,4;\mathbb{R})$

as iso group, and in which

3 of the 4 space dimensions [superluminally INFLATE](#),

3 of the 4 time dimensions [superluminally DEFLATE](#) (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),

and the 4th space dim, x_0 , curls up into a ring ([hence a particle whose wave function penetrates this ring acquires a mass contribution](#)).

In passing we remark that an analogous construction may be defined on an octonion space of [one time dimension and seven space dimensions](#), if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

[Unsolved problem 1](#) (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group

$SU(3) \otimes SU(2)_L \otimes U(1)$;

The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

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The Einstein – Lovelock vacuum field equations are

$$A_a^b = 0, \text{ where}$$

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta_{jj_1 \dots j_{2k}}^{hh_1 \dots h_{2k}} R^{j_1 j_2}_{\quad h_1 h_2} \dots R^{j_{2k-1} j_{2k}}_{\quad h_{2k-1} h_{2k}} + \lambda \sqrt{g} g^{lh}, \quad (4.3)$$

where $\alpha_{(k)}, \lambda$ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

$$\text{and } m-1 = \frac{n}{2} - 1 = 3.$$

Citation: Tensors, Differential Forms,
and Variational Principles (Dover Books on Mathematics),
by David Lovelock and Hanno Rund

Let $\{w1, w2, w3, \Delta\}$ be pure numbers;

then by Eq. [4.38] the vacuum equations are

$$0 = -\Delta + H^{-2} w1 \text{ Lovelock1} + H^{-4} w2 \text{ Lovelock2} + H^{-6} w3 \text{ Lovelock3}$$

in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein – Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre – Hubble parameter. For now, let's pretend that they are independent.

NOTES:

Under the spacetime coordinate transformation

$$x^k \mapsto \bar{x}^k (x^j)$$

according to

$$\Psi^{16}(x) \mapsto \bar{\Psi}^{16}(\bar{x}) = S(x(\bar{x})) \cdot \Psi^{16}(x(\bar{x})),$$

$$\text{i.e., } \overline{\Psi 16^{(a)}(\bar{x})} = \left(S_{(b)}^{(a)} \Psi 16^{(b)}(x) \right)$$

Therefore,

$$\frac{\partial}{\partial \bar{x}^k} \overline{\Psi 16(\bar{x})} = \frac{\partial x^j}{\partial \bar{x}^k} \frac{\partial}{\partial x^j} S(x) \cdot \Psi 16(x) = \\ \frac{\partial x^j}{\partial \bar{x}^k} \left\{ S(x) \cdot \frac{\partial}{\partial x^j} \Psi 16(x) + \left[\frac{\partial}{\partial x^j} S(x) \right] \cdot \Psi 16(x) \right\}$$

$$\text{Identify the Octad connection : } \frac{\partial}{\partial x^j} S(x) =$$

$$- S \cdot \Gamma_j(x) + \frac{\partial \bar{x}^k}{\partial x^j} \bar{\Gamma}_k(\bar{x}) \cdot S$$

hence

$$\frac{\partial}{\partial \bar{x}^k} \overline{\Psi 16(\bar{x})} = \bar{\Gamma}_k \cdot \overline{\Psi 16(\bar{x})} = \\ S(x(\bar{x})) \cdot \frac{\partial x^j}{\partial \bar{x}^k} \left\{ \frac{\partial}{\partial x^j} \Psi 16(x(\bar{x})) - \Gamma_j \cdot \Psi 16(x(\bar{x})) \right\}$$

or

$$\left(1_{16 \times 16} \frac{\partial}{\partial \bar{x}^k} - \bar{\Gamma}_k \right) \cdot \overline{\Psi 16(\bar{x})} = \\ S(x(\bar{x})) \cdot \left[\frac{\partial x^j}{\partial \bar{x}^k} \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^j} - \Gamma_j \right\} \cdot \Psi 16(x(\bar{x})) \right]$$

The connection transforms according to

$$\bar{\Gamma}_k(\bar{x}) = \frac{\partial \bar{x}^j}{\partial x^k} \left\{ S \cdot \Gamma_j(x) \cdot S^{-1} + S_{,j} \cdot S^{-1} \right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2} [e_k^{(a)} \nabla_j e_{(b)}^k] \text{SAB}^{(b)(a)}$$

Here, spin connection coefficients are $e_k^{(a)} (\nabla_j e_{(b)}^k)$

and

$$S = \exp \left(\frac{1}{2} \omega_{(A)(B)}(x) \text{SAB}^{(A)(B)} \right),$$

where $\omega_{(B)(A)} = -\omega_{(A)(B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the $\Psi 16$ field [A.K.A., wave function characterizing the Universe(s) of masses $\pm M$];

TODO: prove Universe(s) of masses $\pm M$ are created in pairs!

K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:

- $\nabla_\mu e^a{}_\nu = 0$
- Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

apologies, I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

? SOURCE terms
 $T^{\mu\nu}$ for $g_{\alpha\beta}$ come from
 "Universes' Wave Function
 Ψ^{16} Lagrangian"

$$\frac{1}{\kappa} \text{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lagrangian}\Psi16)$$

(hope that $\text{TU}^{\mu\nu} = \Lambda g^{\mu\nu}$, and \times

$H = \text{some function of } M, \text{ where } \times$

Universe (s) of masses $\pm M$ created in pairs at time $x4 = 0$,

before the particles of the standard model exist) ;

WARNING:

Universes $\Psi16$ source $g_{\alpha\beta}$;

The Euler–Lagrange equations for $\Psi16$ must have ‘solutions’ such that

all off-diagonal terms of $\text{TU}^{\mu\nu}$ ARE ZERO .

The $\Psi16$ Lagrangian (please see below) =

$$\begin{aligned} \sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg[]} = & \\ \sqrt{\text{Det}[g_{\mu\nu}]} * \left(\text{Transpose}[\Psi16].\circ16.\text{Sum} \left[T16^\alpha [\alpha1 - 1] . \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^{\alpha1-1}} - \Gamma_{\alpha1-1} \right\} . \Psi16, \right. \right. & \\ \left. \left. \{\alpha1, 1, \text{Length}[X]\} \right] + \frac{\text{mASs}}{2} * \text{Transpose}[\Psi16].\circ16.\Psi16 \right) & \end{aligned}$$

Next,

assume that $\Psi16\text{soln}$ solves the Euler – Lagrange equations.

SOURCE terms

$\text{TU}^{\mu\nu}$ [evaluate terms after performing differentiation] (using $\text{Lagrangian}\Psi16 \equiv$
 $\left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lg[]} \right) |_{\Psi16=\text{solution-toEL-eqs}}$) :

$$\frac{1}{\kappa} \text{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lg[]})$$

$$\begin{aligned}
&= \left(\mathcal{L}g[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(-\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \frac{\partial}{\partial g_{\mu\nu}} (\mathcal{L}g[]) \right) \quad \boxed{\Psi16=\Psi16soln} \\
&= \left(0 ? * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(-\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\mathcal{L}g[]) \right) \quad \boxed{\Psi16=\Psi16soln} \\
&= \frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\Psi16].\sigma16. \right. \\
&\quad \left. \text{Sum} \left[\left(D[\Psi16, X[\alpha1]] + \left(\frac{1}{2} \right) \text{connectionMatrix} \right). \text{Sum}[\omega[\alpha1, a, b] * \text{SAB}[a, b], \right. \right. \\
&\quad \left. \left. \{a, 1, 8\}, \{b, 1, 8\}] . \Psi16 \right), \{\alpha1, 1, \text{Length}[X]\} \right] + \\
&\quad \left. \text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[j, 1].\Psi16 \right) \quad \boxed{\Psi16=\Psi16soln} \\
&= \text{Transpose}[\Psi16].\sigma16.\text{Sum} \left[\frac{\partial}{\partial g_{\mu\nu}} (\text{T16}^\alpha[\alpha1 - 1]) . \Psi16_{\alpha1-1}, \right. \\
&\quad \left. \{\alpha1, 1, \text{Length}[X]\} \right] \quad \boxed{\Psi16=solution_toEL_eqs} \\
&= \underset{\sim}{\Psi16} . \sigma16. \text{T16}^A . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1\alpha\beta} e_\beta^B \eta_{BA} \right) = \underset{\sim}{\Psi16} . \sigma16. \text{T16}^A . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha)
\end{aligned}$$

NOTES :

Let g be a square matrix; we could use: $\frac{\partial}{\partial q} (g^{-1}) = -g^{-1} \cdot \frac{\partial g}{\partial q} \cdot g^{-1}$,

where q is a parameter (result from $\frac{\partial}{\partial q} (g \cdot g^{-1}) = 0$),

In this notebook,

g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] ,
 g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}} (g^{-1}) = -g^{-1} \cdot \frac{\partial g}{\partial g_{\mu\nu}} \cdot g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In metric matrix g ,

we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate, for example.

This should be understood before reading further.

$$\text{Therefore } \frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha) = \frac{\partial}{\partial g_{\mu\nu}} (\text{Inverse}[e_\alpha^{(A)}]) = -\text{Inverse}[e_\alpha^{(A)}] \cdot \frac{\partial e_\alpha^{(A)}}{\partial g_{\mu\nu}} \cdot \text{Inverse}[e_\alpha^{(A)}] = -e_{(A)}^\alpha [[\alpha 1, A1]] \frac{\partial e_\alpha^{(A)} [[A1, \alpha 2]]}{\partial g_{\mu\nu}} \cdot e_{(A)}^\alpha [[\alpha 2, A2]]$$

$$\text{Consider } \frac{\partial}{\partial g_{\mu\nu}} (g_{\alpha\beta}) = \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^{(B)}) = \eta_{AB} e_\beta^{(B)} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^{(A)} \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^{(B)}) ; \\ e_C^\beta \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} = \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^{(B)}) = \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^{(A)} e_C^\beta \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^{(B)})$$

In metric matrix g , we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate.

Some of my calculations employ “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see
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“FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb”

`MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify`

I copied and pasted too often; however, I am going to leave this here.

Universe sources for $g_{\alpha\beta}$;
be sure to append these to the Einstein and/or Einstein–Lovelock field equations.

WARNING: all off-diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous ‘hacked solutions’ for $\Psi16$ to be valid

Let $T^\alpha = \{T16^\alpha[0], T16^\alpha[4]\}$ and

```
 $\sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg}[][\text{mASs}_-, j_-] =$ 
 $\sqrt{\text{Det}[g_{\mu\nu}]} * (\text{Transpose}[\Psi16].\sigma16.\text{Sum}[$ 
 $T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\}] +$ 
 $\text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[134, 1].\Psi16)$ 
```

Let $j = 134$, $\sigma16.(\text{mass Matrix}) = T16^\alpha[5].T16^\alpha[6].T16^\alpha[7]$

and

```
usingLagrangianF16massive = (Transpose[\Psi16].\sigma16.
 $\text{Sum}[T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\}] +$ 
 $\text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[j = 134, 1].\Psi16);$ 
```

where it is assumed that $\Psi16$ solves the Euler – Lagrange equations, above; then $\text{usingLagrangianF16massive} = 0$, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lg}[] \right)$$

$$= \left(\text{Lg}[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \right.$$

remark: the term

$$\underset{\partial g_{\mu\nu}}{\overset{\sim}{\partial}} \Psi^{16} \cdot \sigma^{16} \cdot T^{16^A} \cdot \Psi^{16}_{,\alpha} \underset{\partial g_{\mu\nu}}{\overset{\sim}{\partial}} (e_A^\alpha) = \underset{\partial g_{\mu\nu}}{\overset{\sim}{\partial}} \Psi^{16} \cdot \sigma^{16} \cdot T^{16^A} \cdot \Psi^{16}_{,\alpha} e_A^\alpha =$$

$$\frac{\partial}{\partial g_{\mu\nu}} \left(\begin{array}{l} \text{Transpose}[\Psi^{16}] \cdot \sigma^{16} \cdot \\ \\ T^{16^A}[A1-1] \cdot \Psi^{16}_{,\alpha1} \eta_{A1B} e_B^\beta g^{-1} \alpha1 \beta \\ \text{Sum} \left[T^\alpha[\alpha1 - 1] \cdot D[\Psi^{16}, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\} \right] \end{array} \right)$$

Frame fields [edit]

We use a set of [vierbein](#) or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab} e_\mu^a e_\nu^b = \eta_{\mu\nu}.$$

The vierbein defines a local rest [frame](#), allowing the constant [Gamma matrices](#) to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a [section](#) of the [frame bundle](#), and so defines a local trivialization of the frame bundle.

Incomplete theories	[show]
Scientists	v • t • e

Spin connection [edit]

To write down the equation we also need the [spin connection](#), also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e_a^\mu e_\nu^a = \delta_\nu^\mu.$$

The connection 1-form is then

$$\omega^\mu{}_{\nu a} := e_b^\mu \nabla_a e_\nu^b$$

where ∇_a is a [covariant derivative](#), or equivalently a choice of [connection](#) on the frame bundle, most often taken to be the [Levi-Civita connection](#).

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space $T_p M$ to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

[Raising and lowering indices](#) is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract [connection on a principal bundle](#), specifically on the [frame bundle](#), which is defined on any smooth manifold, but which restricts to an [orthonormal frame bundle](#) on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu_{\nu\rho} = e_\rho^\alpha \omega^\mu_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2} (\omega_{\nu\rho})_\mu \sigma^{\nu\rho} \psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu R(M^{\nu\rho}) v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $SO(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

OTHER WORK THAT I USED:

JOURNAL OF MATHEMATICAL PHYSICS

VOLUME 4, NUMBER 7

A Remarkable Representation of the $3 + 2$ de Sitter Group

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*Cambridge University, Cambridge, England and
Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

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The Dirac spinor in six dimensions

By E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

Identities satisfied by the generators of the Dirac algebra

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(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin- $\frac{1}{2}$ wave equation

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Begin

```
In[2]:= Print[
  "CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]
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In[3]:= Print["Please cite this work, and this web page, if you use it"]
Please cite this work, and this web page, if you use it

In[4]:= Print["For simplicity, metric gαβ= gαβ(x0,x1,x2,x3,x4,x5,x6,x7)=
           gαβ(x0,x4) = diag{ g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1,
           g77(x0,x4),g77(x0,x4),g77(x0,x4) },"]
For simplicity, metric gαβ= gαβ(x0,x1,x2,x3,x4,x5,x6,x7)= gαβ(x0,x4) = diag{
           g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4) } ,
In[5]:= Print["where g77(x0,x4) = g11(x0,-x4) "]
where g77(x0,x4) = g11(x0,-x4)
ConvertMapleToMathematicaV2

In[6]:= FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]]
mapleToMathematicaV2PATH =
  FileNameJoin[{%, "ConvertMapleToMathematicaV2.m"}]

Out[6]= /Users/nsh/Documents/8-dim/2026-01-04-Complex

Out[7]= /Users/nsh/Documents/8-dim/2026-01-04-Complex/ConvertMapleToMathematicaV2.m

In[8]:= Get[mapleToMathematicaV2PATH] (*Loads new maple to mathematica parser*)
ConvertMapleToMathematicaV2 loaded successfully!
BUT, WARNING: DO NOT USE IF YOU WANT A CORRECT RESULT!
Load the Maple output strings.

In[9]:= (*SetDirectory[NotebookDirectory[]];*)

In[10]:= (*maplestringEQ1=Get["string-seq1.txt"]
maplestringEQ2=Get["thinkpad_stringEQ2.txt"]*)
```

```
In[11]:= NotebookFileName[]
Out[11]= /Users/nsh/Documents/8-dim/2026-01-04-Complex/Pair-Crtn-Univ-same_E-L-eqs-alt-
approach.nb

In[12]:= Unprotect[dir];
Protect[dir];
If[$Failed == SetDirectory[dir], {.CreateDirectory[dir], SetDirectory[dir]}];

In[16]:= (*FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]]*)

In[17]:= names = FileNameSplit[NotebookFileName[]]
Out[17]= {, Users, nsh, Documents, 8-dim, 2026-01-04-Complex,
Pair-Crtn-Univ-same_E-L-eqs-alt-approach.nb}

In[18]:= name = StringReplace[names[[-1]], "nb" → "mx"]
Out[18]= Pair-Crtn-Univ-same_E-L-eqs-alt-approach.mx

In[19]:= header = StringReplace[names[[-1]], ".nb" → "-"]
Out[19]= Pair-Crtn-Univ-same_E-L-eqs-alt-approach-

In[20]:= SetOptions[Simplify, TimeConstraint → 1]
Out[20]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, 
TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[21]:= SetOptions[FullSimplify, TimeConstraint → 3]
Out[21]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, 
TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}

In[22]:= Options[Simplify]
Out[22]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, 
TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[23]:= Options[FullSimplify]
Out[23]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, 
TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}
```

```
In[24]:= {Off[General::spell], Off[General::spell1]};
```

```
In[25]:= DIM8 = 8;
```

```
In[26]:= Protect[DIM8, M, K, H]
```

```
Out[26]= {DIM8, M, K, H}
```

Coordinates :

$\left\{ \underbrace{\text{hidden space}}, \underbrace{\text{3-space}}, \underbrace{\text{time}}_{\text{superluminal deflating time}} \right\}$

$\{x_0, \overline{x_1, x_2, x_3}, x_4, \overline{x_5, x_6, x_7}\} :$

```
In[27]:= X = {x0, x1, x2, x3, x4, x5, x6, x7};
```

```
Protect[X];
```

```
Protect[x0, x1, x2, x3, x4, x5, x6, x7];
```

```
In[30]:= sX0 = And @@ Thread[X > 0]
```

```
Out[30]= x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 && x7 > 0
```

```
In[31]:= ssX = H > 0 && sX0 && 6 H x0 > 0 && 2 la[x4] > 0 && Cot[6 H x0] > 0 && Sin[6 H x0] > 0
```

```
Out[31]= H > 0 && x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 &&
x7 > 0 && 6 H x0[] > 0 && 2 la[x4[]] > 0 && Cot[6 H x0[]] > 0 && Sin[6 H x0[]] > 0
```

For Maple_2025:

```
In[32]:= sreplaceZ = Z[#] → ToExpression["Z" <> ToString[#]] & /@ Range[0, 15]
```

```
Out[32]= {Z[0] → Z0, Z[1] → Z1, Z[2] → Z2, Z[3] → Z3, Z[4] → Z4,
Z[5] → Z5, Z[6] → Z6, Z[7] → Z7, Z[8] → Z8, Z[9] → Z9, Z[10] → Z10,
Z[11] → Z11, Z[12] → Z12, Z[13] → Z13, Z[14] → Z14, Z[15] → Z15}
```

```
In[33]:= sreplacenZ = nZ[#] → ToExpression["nZ" <> ToString[#]] & /@ Range[0, 15]
```

```
Out[33]= {nZ[0] → nZ0, nZ[1] → nZ1, nZ[2] → nZ2, nZ[3] → nZ3, nZ[4] → nZ4,
nZ[5] → nZ5, nZ[6] → nZ6, nZ[7] → nZ7, nZ[8] → nZ8, nZ[9] → nZ9, nZ[10] → nZ10,
nZ[11] → nZ11, nZ[12] → nZ12, nZ[13] → nZ13, nZ[14] → nZ14, nZ[15] → nZ15}
```

hacks (a.k.a., lies):

```
In[34]:= constraintTrig = Sin[6 * H * x0] > 0 && Cos[6 * H * x0] > 0 && Csc[6 * H * x0] > 0 &&
Sec[6 * H * x0] > 0 && Tan[6 * H * x0] > 0 && Cot[6 * H * x0] > 0 && Sin[z] > 0 &&
Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 && Sin[z]^(1/2) > 0 && Sin[z]^(−3/2) > 0 &&
Sin[z]^(−1/2) > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 && Cot[z]^(3/2) > 0 &&
Cot[z]^(1/2) > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

```
Out[34]= Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]^(3/2) > 0 && 1/Sin[z] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]^(3/2) > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

more hacks (damn lies):

```
In[35]:= (*&&K^2-M^2>0&&Sqrt[K^2-M^2]>0&&e^(H Sqrt[K^2-M^2])^4>0&&2 H Sqrt[K^2-M^2]^4>0&&
2 H Sqrt[1-M^2]^4>0&&Sqrt[1-M^2]>0&&e^(H Sqrt[1-M^2])^4>0&&2 H Sqrt[1-M^2]^4>0*)
```

```
In[36]:= constraintX = x0 > 0 && x4 > 0 && H > 0 && 6 * H * x0 > 0 &&
3 * H * x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 && t > 0 && M > 0 &&
K > 0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0 && e^(2 a4[H x4]) > 0 && e^(a4[H x4]) > 0
```

```
Out[36]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 &&
t > 0 && M > 0 && K > 0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0 && e^(2 a4[H x4]) > 0 && e^(a4[H x4]) > 0
```

statistics:

```
In[37]:= constraintVars = constraintX && constraintTrig
```

```
Out[37]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0 && e^(2 a4[H x4]) > 0 &&
e^(a4[H x4]) > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]^(3/2) > 0 && 1/Sin[z] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]^(3/2) > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

```
In[38]:= subsDefects = {  $\sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}$ ,  $\sqrt{e^{-2 a4[H x4]}} \rightarrow e^{-a4[H x4]}$ ,
 $\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}} \rightarrow e^{a4[H x4]} \sin[6 H x4]^{1/6}$ ,
 $\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}}} \rightarrow \frac{1}{e^{a4[H x4]} \sin[6 H x4]^{1/6}}$ ,
 $\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{1}{e^{a4[H x4]} \sin[6 H x0]^{1/6}}$ ,
 $\frac{1}{\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{1}{e^{-a4[H x4]} \sin[6 H x0]^{1/6}}$ ,
 $\sqrt{e^{2 a4[H x4]}} \sin[6 H x0]^{1/6} \rightarrow e^{a4[H x4]} \sin[6 H x0]^{1/6}$ ,
 $\sqrt{e^{-2 a4[H x4]}} \sin[6 H x0]^{1/6} \rightarrow e^{-a4[H x4]} \sin[6 H x0]^{1/6}$  }
```

```
Out[38]= {  $\sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}$ ,  $\sqrt{e^{-2 a4[H x4]}} \rightarrow e^{-a4[H x4]}$ ,
 $\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}} \rightarrow e^{a4[H x4]} \sin[6 H x4]^{1/6}$ ,
 $\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}}} \rightarrow \frac{e^{-a4[H x4]}}{\sin[6 H x4]^{1/6}}$ ,  $\frac{1}{\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{e^{-a4[H x4]}}{\sin[6 H x0]^{1/6}}$ ,
 $\frac{1}{\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{e^{a4[H x4]}}{\sin[6 H x0]^{1/6}}$ ,
 $\sqrt{e^{2 a4[H x4]}} \sin[6 H x0]^{1/6} \rightarrow e^{a4[H x4]} \sin[6 H x0]^{1/6}$ ,
 $\sqrt{e^{-2 a4[H x4]}} \sin[6 H x0]^{1/6} \rightarrow e^{-a4[H x4]} \sin[6 H x0]^{1/6}$  }
```

future coordinate transformation :

```
In[39]:= 6 H x0 == z && H x4 == t
szt = Solve[%, {z, t}] [[1]]
sx0x4 = Solve[%%, {x0, x4}] [[1]]
Protect[sx0x4, szt];
```

```
Out[39]= 6 H x0 == z && H x4 == t
```

```
Out[40]= {z → 6 H x0, t → H x4}
```

```
Out[41]= {x0 →  $\frac{z}{6 H}$ , x4 →  $\frac{t}{H}$ }
```

```
In[43]:= sx0x4
```

```
Out[43]= {x0 →  $\frac{z}{6 H}$ , x4 →  $\frac{t}{H}$ }
```

```
In[44]:= (*sa4={a4→((A4[3 H #2])&)}*)
```

```
In[45]:= Protect[sx0x4, szt]
Out[45]= {}

SO(4, 4) Minkowski Lorentz metric  $\eta_{4488}$  : 4+4 spacetime; 8×8 dimensional :

In[46]:= ( $\eta_{4488}$  = ArrayFlatten[
  {{IdentityMatrix[4], 0}, {0, -IdentityMatrix[4]}}]) // MatrixForm
Out[46]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


SO(4, 4) Spinor Lorentz metric  $\sigma$ ;
 $\tau^A$  are analogous to the Dirac gamma matrices :
all true : Table[
```

$$\left\{ A, B, \text{FullSimplify}\left[\frac{1}{2} (\tau[A].\bar{\tau}[B] + \tau[B].\bar{\tau}[A]) == \eta_{4488}[[A+1, B+1]*ID8]\right], \{A, 0, 7\}, \{B, 0, 7\} \right]$$

Type - 1 and type - 2 employ σ (the usual notational abuse),
since $\sigma = \sigma^{-1}$: 4+4 spacetime, 8×8 dimensional and 16×16 dimensional :

indices. We define the reduced Brauer-Weyl generators $\{\tilde{\tau}^{A'}, \tau^{A'}\}$ of the
generators of the two real 8×8 inequivalent irreducible representations of
 $\overline{SO}(4, 4)$ (see eqs. (2.6) and (2.7)) by demanding that the tau matrices satisfy (the
tilde denotes transpose)

$$(2.3) \quad \tilde{\tau}^{A'} \sigma = \sigma \bar{\tau}^{A'}$$

and

$$(2.4) \quad \tau^{A'} \bar{\tau}^{B'} + \tau^{B'} \bar{\tau}^{A'} = 2IG^{A'B'} = \bar{\tau}^{A'} \tau^{B'} + \bar{\tau}^{B'} \tau^{A'},$$

where I denotes the 8×8 unit matrix. Denoting the matrix elements of $\tau^{A'}$ by
 $\tau^{A'a}_b$, we may write eq. (3) as

$$(2.5) \quad \bar{\tau}^{A'}_{ab} = \tau^{A'}_{ba},$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix}$$

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4, 4)}$ and $L \in SO(4, 4)$. The canonical 2-1 homomorphism $\overline{SO(4, 4)} \rightarrow SO(4, 4)$ is given by

$$(2.8) \quad 8L^{A'}{}_{B'}(g) = \text{tr } \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4, 4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}{}_{B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

```
In[47]:= (σ = ArrayFlatten[{{ConstantArray[0, {4, 4}], IdentityMatrix[4]}, {IdentityMatrix[4], ConstantArray[0, {4, 4}]}}]) // MatrixForm
```

Out[47]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

below:

```
In[48]:= σ16 == ArrayFlatten[{{{-σ, 0}, {0, σ}}}]
```

below:

```
In[=]:= Table[T16A[A1] = ArrayFlatten[{{{0, τ[A1]}, {τ[A1], 0}}}], {A1, 0, 7}];
```

below:

```
In[=]:= σ16.T16A[#] = -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

```
Out[=]= {True, True, True, True, True, True, True}
```

below:

```
In[=]:= Table[{A1, B1},  
FullSimplify[ExpandAll[ $\frac{1}{2} (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) =$   
 $\eta 4488 [A1+1, B1+1] * ID16$ ]], {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

```
Out[=]//MatrixForm=
```

({0, 0})	({0, 1})	({0, 2})	({0, 3})	({0, 4})	({0, 5})	({0, 6})	({0, 7})
(True)							
({1, 0})	({1, 1})	({1, 2})	({1, 3})	({1, 4})	({1, 5})	({1, 6})	({1, 7})
(True)							
({2, 0})	({2, 1})	({2, 2})	({2, 3})	({2, 4})	({2, 5})	({2, 6})	({2, 7})
(True)							
({3, 0})	({3, 1})	({3, 2})	({3, 3})	({3, 4})	({3, 5})	({3, 6})	({3, 7})
(True)							
({4, 0})	({4, 1})	({4, 2})	({4, 3})	({4, 4})	({4, 5})	({4, 6})	({4, 7})
(True)							
({5, 0})	({5, 1})	({5, 2})	({5, 3})	({5, 4})	({5, 5})	({5, 6})	({5, 7})
(True)							
({6, 0})	({6, 1})	({6, 2})	({6, 3})	({6, 4})	({6, 5})	({6, 6})	({6, 7})
(True)							
({7, 0})	({7, 1})	({7, 2})	({7, 3})	({7, 4})	({7, 5})	({7, 6})	({7, 7})
(True)							

defs; some Symbols; metric

$$\left\{ \left\{ a4[t] \rightarrow \frac{M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} + \right. \right.$$

$$\left. \left. c_1 + \frac{1}{2} \text{ProductLog}\left[-e^{-\frac{2 M t (c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1}-2 c_1}\right]\right\} \right\}$$

$$\text{DSolve}\left[\theta == -M (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7]) + (Q1 - e^{-2 a4[t]} Q1) a4'[t], a4[t], t\right]$$

$$M c[j] = (j + 1)^{\text{st}} \text{ Energy Eigenvalue for Transpose[cayZ].}\Psi16, j = 0, \dots, 7$$

`ProductLog[z]`

gives the principal solution for w in $z = we^w$.

if z and w are any complex numbers, then

$$we^w = z$$

holds if and only if

$$w = W_k(z) \text{ for some integer } k.$$

When dealing with real numbers only, the two branches W_0 and W_{-1} suffice: for real numbers x and y the equation

$$ye^y = x$$

can be solved for y only if $x \geq \frac{-1}{e}$; yields $y = W_0(x)$ if $x \geq 0$ and the two values $y = W_0(x)$ and $y = W_{-1}(x)$ if $\frac{-1}{e} \leq x < 0$.

?todo: NEED `DefScalarFunction[#]; & /@ (f16[#]&/@Range[0,15])`, and allow `xact` to compute?

Introduce the wave function, $\Psi16$, for a Universe:

In[48]:= `\Psi16 = f16[#][x0, x4] & /@ Range[0, 15]`

Out[48]=

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

```
In[49]:= processRawSets[rawSets_] :=
Module[{s1, s2, J, ret, L = Range[Length[rawSets]], r = {}},
While[Length[L] > 1,
s1 = Union[rawSets[[L[[1]]]]];
L = Drop[L, 1];
J = L[[1]];
While[J ≤ 16,
If[MemberQ[L, J],
s2 = Union[rawSets[[J]]];
If[Intersection[s1, s2] ≠ {}, s1 = Union[Flatten[{s1, s2}]];
L = Complement[L, {J}];, ##&[]];
##&[]];
J++];
AppendTo[r, s1];
];
ret = Sort[r, #1[[1]] < #2[[1]] &];
Return[ret];
]

In[50]:= rawSets\[Psi]16 = Cases[
#, h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] → n,
Infinity,
Heads → True
] & /@ \Psi16

Out[50]= {{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}, {15} }

In[51]:= processRawSets[rawSets\[Psi]16]
Out[51]= {{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14} }

In[52]:= (*\Psi new16=F16[#] [x0,x4]&/@Range[0,15]*)
```

```
In[53]:= Clear[sf16Aa];
sfψ16Aa = f16[#] → ToExpression[
  "((Z[" <> ToString[#] <> "] [6*H##1,H##2])&)"] & /@ Range[0, 15]
```

```
Out[53]= {f16[0] → (Z[0][6 H##1, H##2] &), f16[1] → (Z[1][6 H##1, H##2] &),
f16[2] → (Z[2][6 H##1, H##2] &), f16[3] → (Z[3][6 H##1, H##2] &),
f16[4] → (Z[4][6 H##1, H##2] &), f16[5] → (Z[5][6 H##1, H##2] &),
f16[6] → (Z[6][6 H##1, H##2] &), f16[7] → (Z[7][6 H##1, H##2] &),
f16[8] → (Z[8][6 H##1, H##2] &), f16[9] → (Z[9][6 H##1, H##2] &),
f16[10] → (Z[10][6 H##1, H##2] &), f16[11] → (Z[11][6 H##1, H##2] &),
f16[12] → (Z[12][6 H##1, H##2] &), f16[13] → (Z[13][6 H##1, H##2] &),
f16[14] → (Z[14][6 H##1, H##2] &), f16[15] → (Z[15][6 H##1, H##2] &)}
```

```
In[54]:= (*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression[
  "((Z["<>ToString[#]<>"] [6*H##1,H##2] 1/Sin[6*H##1]^(1/2))&)"]&/@Range[0,15]*)
```

```
In[55]:= Clear[snewf16Aa];
snewf16Aa = f16[#] & /@ Range[0, 15]
```

```
Out[55]= {f16[0] &gt; (nZ[0][6 H#1, H#2] &), f16[1] &gt; (nZ[1][6 H#1, H#2] &),
f16[2] &gt; (nZ[2][6 H#1, H#2] &), f16[3] &gt; (nZ[3][6 H#1, H#2] &),
f16[4] &gt; (nZ[4][6 H#1, H#2] &), f16[5] &gt; (nZ[5][6 H#1, H#2] &),
f16[6] &gt; (nZ[6][6 H#1, H#2] &), f16[7] &gt; (nZ[7][6 H#1, H#2] &),
f16[8] &gt; (nZ[8][6 H#1, H#2] &), f16[9] &gt; (nZ[9][6 H#1, H#2] &),
f16[10] &gt; (nZ[10][6 H#1, H#2] &), f16[11] &gt; (nZ[11][6 H#1, H#2] &),
f16[12] &gt; (nZ[12][6 H#1, H#2] &), f16[13] &gt; (nZ[13][6 H#1, H#2] &),
f16[14] &gt; (nZ[14][6 H#1, H#2] &), f16[15] &gt; (nZ[15][6 H#1, H#2] &)}
```

0 (4, 4) related:

```
In[56]:= Needs["Notation`"]
```

```
In[57]:= Symbolize[σga1 A]
```

```
In[58]:= Symbolize[σga2 A]
```

```
In[59]:= Symbolize[τ]
```

```
In[60]:= Symbolize[T16A]
```

```
In[61]:= Symbolize[T16α]
```

```
In[62]:= Symbolize[ua]
```

```
In[63]:= Symbolize[Jy]
```

```
In[64]:= Symbolize[ FAa ]
```

```
In[65]:= Symbolize[ FaA ]
```

```
In[66]:= Symbolize[ σ22 ]
```

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

```
In[67]:= Symbolize[ gαβ ]
```

note below: g_{AB} is really $g_{(A)(B)}$:: Minkowski – Lorentz metric

```
In[68]:= Symbolize[ gAB ]
```

note below: G_{AB} is really $G_{(A)(B)}$:: Minkowski – Lorentz

```
In[69]:= (*Symbolize[ GAB ]*)
```

```
In[70]:= Symbolize[ S44αβ ]
```

$$\Sigma^{\alpha\beta} = \xi S^{\alpha\beta} u,$$

```
In[71]:= Symbolize[ Σαβ ]
```

```
In[72]:= (*Symbolize[ Σαβ ]*)
```

$$\frac{1}{2} \Sigma^{jk} \Sigma_{jk} = \frac{1}{4} \partial_a u^a u^b \partial_b - \frac{1}{4} \partial_a u^b u^b \partial_a,$$

```
In[73]:= Symbolize[ Σ2 ]
```

NEED DefScalarFunction[#]; & /@ Flatten[Universe]

```
In[74]:= Universe = ToExpression["U" <> ToString[#]] & /@ Range[0, 7]
```

```
In[74]:= Symbolize[ FAa ]
```

```
In[75]:= Symbolize[ FaA ]
```

```
In[76]:= Symbolize[gtrye(A)α ]
```

```
In[77]:= Symbolize[gtryeα(A) ]
```

SPACETIME METRIC

```
In[78]:= (*einsteinsteinMetric=Array[g[#1-1,#2-1]&,{8,8}]*)
```

SPACETIME METRIC:

```
In[79]:= g4488 = Array[g[#1 - 1] [#2 - 1] [x0, x4] &, {8, 8}];
```

SPACETIME METRIC values:

MatrixMetric44

$$\begin{aligned} 6 H x0 = z \&& H x4 = t \\ \left\{ \left\{ a4''[t] \rightarrow 0, a4'[t] \rightarrow \frac{2}{3} (-1 + M) \right\}, \left\{ a4''[t] \rightarrow 0, a4'[t] \rightarrow \frac{2 (1 + M)}{3} \right\} \right\} \end{aligned}$$

```
In[80]:= (*β3=Exp[2 H x4 √(K²-M²)];*)
(*β3=Exp[2 * a4[3*H*x4]];*)
β3 = Exp[2 * a4[H * x4]] (* /. {a4 → ((K1 * 2^(1+M)/3 + K2 * 2/3 (-1+M)) #) &} *)
β1 = Sin[6 * H * x0]^(1/3);
β2 = Cot[6 * H * x0]^2;
```

```
Out[80]= e^2 a4[H x4]
```

```
In[83]:= MatrixForm[
  MatrixMetric44 = {{β2, 0, 0, 0, 0, 0, 0, 0}, {0, β1 β3, 0, 0, 0, 0, 0, 0}, {0, 0, β1 β3, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -β1/β3, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -β1/β3}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}} // FullSimplify[#, constraintVars] &]
```

```
Out[83]//MatrixForm=
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[84]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression[
"((Z["<>ToString[#]<>"] [6*H*#1,3*H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@Range[0,15]*)
```

```
In[85]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"] [6*H#1,3*H#2])&)"]&/@Range[0,15]*)

In[86]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression[
"((Z["<>ToString[#]<>"] [6*H#1,H#2] 1/Sin[6*H#1]^(1/2))&)"]&/@Range[0,15]*)

In[87]:= (*Inactivate[((q)&)]Evaluate[Activate[Evaluate[q]]]*)

In[88]:= Clear[makeSubstitution];
makeSubstitution[] := Module[{met, mat, t, f, p},
  met = Flatten[g4488][[All, 0]];
  mat = Flatten[MatrixMetric44];
  t = Thread[met → mat];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]

In[90]:= makeSubstitution[a_, b_] := Module[{t, f, p},
  t = Thread[a → b];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]
```

```
In[91]:= ssgm4488 = makeSubstitution[]
```

Out[91]=

$$\begin{aligned} \{g[0][0] \rightarrow (\text{Cot}[6 H \#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] \rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&), \\ g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&), \\ g[2][2] \rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&), \\ g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&), \\ g[3][3] \rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&)\} \end{aligned}$$

```
In[92]:= Symbolize[Eα(A)]
```

```
In[93]:= Symbolize[eα(A)]
```

```
In[94]:= eα(A) = Array[Eα(A) [##1 - 1, ##2 - 1] \&, {8, 8}]
```

Out[94]=

$$\begin{aligned} &\{\{E_{\alpha}^{(A)}[0, 0], E_{\alpha}^{(A)}[0, 1], E_{\alpha}^{(A)}[0, 2], E_{\alpha}^{(A)}[0, 3], \\ &E_{\alpha}^{(A)}[0, 4], E_{\alpha}^{(A)}[0, 5], E_{\alpha}^{(A)}[0, 6], E_{\alpha}^{(A)}[0, 7]\}, \{E_{\alpha}^{(A)}[1, 0], E_{\alpha}^{(A)}[1, 1], \\ &E_{\alpha}^{(A)}[1, 2], E_{\alpha}^{(A)}[1, 3], E_{\alpha}^{(A)}[1, 4], E_{\alpha}^{(A)}[1, 5], E_{\alpha}^{(A)}[1, 6], E_{\alpha}^{(A)}[1, 7]\}, \\ &\{E_{\alpha}^{(A)}[2, 0], E_{\alpha}^{(A)}[2, 1], E_{\alpha}^{(A)}[2, 2], E_{\alpha}^{(A)}[2, 3], E_{\alpha}^{(A)}[2, 4], E_{\alpha}^{(A)}[2, 5], \\ &E_{\alpha}^{(A)}[2, 6], E_{\alpha}^{(A)}[2, 7]\}, \{E_{\alpha}^{(A)}[3, 0], E_{\alpha}^{(A)}[3, 1], E_{\alpha}^{(A)}[3, 2], E_{\alpha}^{(A)}[3, 3], \\ &E_{\alpha}^{(A)}[3, 4], E_{\alpha}^{(A)}[3, 5], E_{\alpha}^{(A)}[3, 6], E_{\alpha}^{(A)}[3, 7]\}, \{E_{\alpha}^{(A)}[4, 0], E_{\alpha}^{(A)}[4, 1], \\ &E_{\alpha}^{(A)}[4, 2], E_{\alpha}^{(A)}[4, 3], E_{\alpha}^{(A)}[4, 4], E_{\alpha}^{(A)}[4, 5], E_{\alpha}^{(A)}[4, 6], E_{\alpha}^{(A)}[4, 7]\}, \\ &\{E_{\alpha}^{(A)}[5, 0], E_{\alpha}^{(A)}[5, 1], E_{\alpha}^{(A)}[5, 2], E_{\alpha}^{(A)}[5, 3], E_{\alpha}^{(A)}[5, 4], E_{\alpha}^{(A)}[5, 5], \\ &E_{\alpha}^{(A)}[5, 6], E_{\alpha}^{(A)}[5, 7]\}, \{E_{\alpha}^{(A)}[6, 0], E_{\alpha}^{(A)}[6, 1], E_{\alpha}^{(A)}[6, 2], E_{\alpha}^{(A)}[6, 3], \\ &E_{\alpha}^{(A)}[6, 4], E_{\alpha}^{(A)}[6, 5], E_{\alpha}^{(A)}[6, 6], E_{\alpha}^{(A)}[6, 7]\}, \{E_{\alpha}^{(A)}[7, 0], E_{\alpha}^{(A)}[7, 1], \\ &E_{\alpha}^{(A)}[7, 2], E_{\alpha}^{(A)}[7, 3], E_{\alpha}^{(A)}[7, 4], E_{\alpha}^{(A)}[7, 5], E_{\alpha}^{(A)}[7, 6], E_{\alpha}^{(A)}[7, 7]\}\} \end{aligned}$$

```
In[95]:= (*Symbolize[seα(A)]*)
```

```
In[96]:= Symbolize[sgeα(A)]
```

```
In[97]:= Symbolize[sgtryeα(A)]
```

```
In[98]:= Symbolize[E(A)α]
```

```
In[99]:= Symbolize[sgeα(A)]

In[100]:= Symbolize[sgtryeα(A)]

In[101]:= Symbolize[eα(A)]

In[102]:= eα(A) = Array[Eα(A) [##1 - 1, ##2 - 1] &, {8, 8}]

Out[102]= { {Eα(A) [0, 0], Eα(A) [0, 1], Eα(A) [0, 2], Eα(A) [0, 3], Eα(A) [0, 4], Eα(A) [0, 5], Eα(A) [0, 6], Eα(A) [0, 7]}, {Eα(A) [1, 0], Eα(A) [1, 1], Eα(A) [1, 2], Eα(A) [1, 3], Eα(A) [1, 4], Eα(A) [1, 5], Eα(A) [1, 6], Eα(A) [1, 7]}, {Eα(A) [2, 0], Eα(A) [2, 1], Eα(A) [2, 2], Eα(A) [2, 3], Eα(A) [2, 4], Eα(A) [2, 5], Eα(A) [2, 6], Eα(A) [2, 7]}, {Eα(A) [3, 0], Eα(A) [3, 1], Eα(A) [3, 2], Eα(A) [3, 3], Eα(A) [3, 4], Eα(A) [3, 5], Eα(A) [3, 6], Eα(A) [3, 7]}, {Eα(A) [4, 0], Eα(A) [4, 1], Eα(A) [4, 2], Eα(A) [4, 3], Eα(A) [4, 4], Eα(A) [4, 5], Eα(A) [4, 6], Eα(A) [4, 7]}, {Eα(A) [5, 0], Eα(A) [5, 1], Eα(A) [5, 2], Eα(A) [5, 3], Eα(A) [5, 4], Eα(A) [5, 5], Eα(A) [5, 6], Eα(A) [5, 7]}, {Eα(A) [6, 0], Eα(A) [6, 1], Eα(A) [6, 2], Eα(A) [6, 3], Eα(A) [6, 4], Eα(A) [6, 5], Eα(A) [6, 6], Eα(A) [6, 7]}, {Eα(A) [7, 0], Eα(A) [7, 1], Eα(A) [7, 2], Eα(A) [7, 3], Eα(A) [7, 4], Eα(A) [7, 5], Eα(A) [7, 6], Eα(A) [7, 7]}}

In[103]:= (*gtryeα(A) = (eα(A) /. sgtryeα(A))*)

In[104]:= (*gtryeα(A) = (eα(A) /. sgtryeα(A))*)

In[105]:= (*Symbolize[Rκα β]*)

In[106]:= (*preSpinConnection=Array[Rκα β[##]&,{8,8,8}];*)

In[107]:= (*Symbolize[EAa]*)

In[108]:= (*Symbolize[EaA]*)

In[109]:= (*FAa FaA*)
```

In[110]:=

```
(*Do[ FAa=EAa[h]=Table[(u[[h]].σ.(τ[B])),{B,1,8}],{h,1,Length[u]}];
Do[EaA[h]=FullSimplify[Inverse[EAa[h]]],{h,1,Length[u]}];
Table[ FaA=EaA[h]==(u[[h]].σ.u[[h]]);
Transpose[Table[FullSimplify[ExpandAll[η8[[B,B]]τ[B].u[[h]]]],
{B,1,8}]],{h,1,Length[u]}]*)
```

In[111]:=

```
(*Table[
FullSimplify[ExpandAll[Transpose[EAa[h]].η8.EAa[h]]]-(u[[h]].σ.u[[h]])σ==Zero,
{h,1,Length[u]}]*)
```

In[112]:=

```
(*η88=FullSimplify[Inverse[η8]];*)
```

In[113]:=

```
(*Clear[ω(a)μ(b);*)Clear[ω];Symbolize[ω(a)μ(b)]
```

In[114]:=

```
ω=Array[ω(a)μ(b)[[##]&,{8,8,8}];
Protect[ω]
```

Out[115]=

```
{ω}
```

constants

```
← ERROR : 08similarityTransformation has 1 st index that transforms as  $\frac{\partial}{\partial \Psi}$  ,
not as Ψs
```

In[116]:=

```
(* 08similarityTransformation has 1st index that transforms as  $\frac{\partial}{\partial \Psi}$  ,
not as Ψ *)
```

```
08similarityTransformation has 1 st index that transforms as  $\frac{\partial}{\partial \Psi}$  , not as Ψ :
```

In[117]:=

```
(*ArrayFlatten[{{IdentityMatrix[4],0},{0,-IdentityMatrix[4]}}]*)
```

In[118]:=

```
ID4=IdentityMatrix[4];
ID8=IdentityMatrix[8];
```

```

In[120]:=  $\eta4488 // \text{MatrixForm}$ 
Out[120]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


In[121]:=  $\eta4488[[\#, \#]] \& /@ \text{Range}[8]$ 
Out[121]= {1, 1, 1, 1, -1, -1, -1, -1}

In[122]:= 

Zero4 = ConstantArray[0, {4, 4}]; Zero8 = ConstantArray[0, {8, 8}];
mid = ExpandAll[-1 * ID4];
e3 = Array[Signature[{##}] &, {3, 3, 3}]
e4 = Array[Signature[{##}] &, {4, 4, 4, 4}];
MId = ExpandAll[-1 * ID8];
simp = {Zero4 \[Rule] 0, ID4 \[Rule] 1, mid \[Rule] -1};
Simp = {Zero8 \[Rule] ZERO, ID8 \[Rule] ONE, MId \[Rule] MONE};

Out[124]= {{{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}}, {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}}}, {{{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}}
```

In[129]:=

```

Zero16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{{\eta8, 0}, {0, -η8}}}];
Id16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * Id16];
Simp16 = {Zero16 \[Rule] ZERO16, Id16 \[Rule] ONE16, MId16 \[Rule] MONE16};
```

In[133]:=

```

ZERO16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{{\eta4488, 0}, {0, -η4488}}}];
ID16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * ID16];
Simp16 = {ZERO16 \[Rule] ZERO16, ID16 \[Rule] ONE16, MId16 \[Rule] MONE16};
```

```
In[137]:= ws = Flatten[Table[ToExpression["ω" <> ToString[A1] <> ToString[B1]], {A1, 1, 7}, {B1, A1 + 1, 8}]];
Length[ws]

Out[137]= {ω12, ω13, ω14, ω15, ω16, ω17, ω18, ω23, ω24, ω25, ω26, ω27, ω28, ω34, ω35, ω36, ω37, ω38, ω45, ω46, ω47, ω48, ω56, ω57, ω58, ω67, ω68, ω78}

Out[138]= 28

In[139]:= σ.σ - ID8 == Zero8
Tr[σ] == 0

Out[139]= True

Out[140]= True
```

some function definitions:

```
In[141]:= Clear[rawSets];
rawSets[l_, f_, o_] := Module[{t, h},
  t = Table[Cases[
    l[[j]],
    h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], f] & n + o,
    Infinity,
    Heads → True
  ], {j, 1, Length[l]}];
  Return[t];
]

In[143]:= (*rawSetsel16=rawSets[el16,"f16",0]*)

In[144]:= (*couplings=showCoupledEquations[rawSetsel16]
%//Flatten//Sort
%==Range[0,15]*)
```

```

In[145]:= Clear[dualRawSets];
dualRawSets[l_, f_, o_, h_, oh_] := Module[{r, r1, r2},
  r1 = rawSets[l, f, o];
  r2 = rawSets[l, h, oh];
  r = Union[Flatten[{r1, r2}, 1]] // Drop[#, 1] &;
  Return[r];
]

In[147]:= (* Helper to perform one pass of merging *)
MergeSetsStep[currentSets_List] :=
  Module[{n = Length[currentSets], i, j, merged = currentSets},
    (* We look for the first pair (i, j) that intersects *)
    Catch[
      For[i = 1, i < Length[merged], i++,
        For[j = i + 1, j < Length[merged], j++,
          If[Intersection[merged[[i]], merged[[j]]] != {}, {},
            (* Found intersection: Merge and Throw to restart/finish this step *)
            merged = Delete[merged, {{i}, {j}}];
            AppendTo[merged, Sort[Union[currentSets[[i]], currentSets[[j]]]]];
            Throw[Sort[merged]]; (* Sort for canonical form *)
          ];
        ];
      ];
      (* If no intersection found, return original sorted *)
      Sort[merged]
    ];
  ];
]

In[148]:= showCoupledEquations[items_List] :=
  FixedPoint[MergeSetsStep, items];

```

```
In[149]:= blockPartition[mat_]:=
Module[{{(*blocks,block11,block12,block21,block22,*)ret},
blocks = Partition[mat, {8, 8}];
(*block11=blocks[[1,1]] ;*)(*Top-left block*)
(*block12=blocks[[1,2]] ;*)(*Top-right block*)
(*block21=blocks[[2,1]] ;*)(*Bottom-left block*)
(*block22=blocks[[2,2]] ;*)(*Bottom-right block*)
ret = ArrayFlatten[
    {{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}] === mat;
If[ret, Return[
    {{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}, ## &[]];
Return[Null];]

In[150]:= eextract[a_]:=Extract[a, {#}] & /@ Range[0, Length[a]]

In[151]:= Clear[iimes]

In[152]:= iimes[a_]:= {a}

In[153]:= iimes[a_, b_]:= {a, b}

In[154]:= iimes[a_, b_, c_]:= {a, b, c}

In[155]:= iimes[a_, b_, c_, d_]:= {a, b, c, d}

In[156]:= iimes[a_, b_, c_, d_, f_]:= {a, b, c, d, f}

In[157]:= iimes[a_, b_, c_, d_, f_, h_]:= {a, b, c, d, f, h}

In[158]:= iimes[a_, b_, c_, d_, f_, h_, j_]:= {a, b, c, d, f, h, j}

In[159]:= times[a__]:= Flatten[{Flatten[#] & /@ a}]

In[160]:= (*times[a_,b_,c_,d_]:= {a,b} *)
```

In[161]:=

$$(*\text{imes}[a_-, b_-, c_-, d_-][0] := \{a, d\}*)$$

In[162]:=

$$(*\text{imes}[a_-, b_-][0]^{\wedge} := \{a, b\}*)$$

In[163]:=

$$\text{id}[a_-, b_-] := a$$

In[164]:=

$$\text{id}[a_-, b_-, c_-] := \{a, b\}$$

In[165]:=

$$\text{id}[a_List] := a[[1]]$$

In[166]:=

$$\text{qid}[a__] := a$$

In[167]:=

$$\text{Clear}[der]; der[a__] [c_-][b__] := c$$

In[168]:=

$$\text{Clear}[der3]; der3[a__] [c_-][b__] := \{c, a, b\}$$

In[169]:=

$$(*der[a_-, b_-] [c_-] := c*)$$

In[170]:=

$$(*derz[a_-, b_-] := a*)$$

In[171]:=

$$(*dert[a_-, b_-] := b*)$$

In[172]:=

$$(*derivative[a__] [c_-] := \text{FullForm}[c] /. \{\text{Derivative} \rightarrow \text{der}\}*)$$

In[173]:=

$$\{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]\}$$

$$\text{FullForm}[\#] \& /@ \%$$

$$(*\text{ToString}[\#] \& /@ \%*)$$

$$\% /. \{\text{Derivative} \rightarrow \text{der}\}$$

Out[173]=

$$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$$

Out[174]=

$$\{Z[0][z, t], \text{Derivative}[1, 0][Z[0]][z, t], \text{Derivative}[0, 1][Z[0]][z, t]\}$$

Out[175]=

$$\{Z[0][z, t], Z[0], Z[0]\}$$

In[176]:=

```

{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
% * (Prime[#] & /@ Range[Length[%]])
Plus @@ %
FullForm[%] (* #]&/@%*)
% /. {Plus -> qid, Times -> qid, Derivative -> der}
List[%]
Partition[%, 2]
eextract[#] & /@ %
(*eextract[#]&/@%*)
(*#__List[0]&/@%*)
#\[2]& /@ %
#\[3]& /@ %%

```

Out[176]=

$$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$$

Out[177]=

$$\{2 Z[0][z, t], 3 Z[0]^{(1,0)}[z, t], 5 Z[0]^{(0,1)}[z, t]\}$$

Out[178]=

$$2 Z[0][z, t] + 5 Z[0]^{(0,1)}[z, t] + 3 Z[0]^{(1,0)}[z, t]$$

Out[179]/.FullForm=

```

Plus[Times[2, Z[0][z, t]], Times[5, Derivative[0, 1][Z[0]][z, t]],
Times[3, Derivative[1, 0][Z[0]][z, t]]]

```

Out[180]=

$$\text{Sequence}[2, Z[0][z, t], 5, Z[0], 3, Z[0]]$$

Out[181]=

$$\{2, Z[0][z, t], 5, Z[0], 3, Z[0]\}$$

Out[182]=

$$\{\{2, Z[0][z, t]\}, \{5, Z[0]\}, \{3, Z[0]\}\}$$

Out[183]=

$$\{\{\text{List}, 2, Z[0][z, t]\}, \{\text{List}, 5, Z[0]\}, \{\text{List}, 3, Z[0]\}\}$$

Out[184]=

$$\{2, 5, 3\}$$

Out[185]=

$$\{Z[0][z, t], Z[0], Z[0]\}$$

Sequence[Subscript[expr, 1], Subscript[expr, 2], ...]

represents a sequence of arguments to be spliced automatically into any function.

```

In[186]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#+ & /@ %]
(*ToString[#+&/@%*])
% /. {Derivative -> der3}
Out[186]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[187]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[188]= {Z[0][z, t], List[Z[0], 1, 0, z, t], List[Z[0], 0, 1, z, t]}

In[189]:= helpf[x__] := Length[{x}]
In[190]:= {helpf[x, y, z], helpf[]}
Out[190]= {3, helpf[]}

In[191]:= f[##] & @@ X
Out[191]= f[x0, x1, x2, x3, x4, x5, x6, x7]

In[192]:= StringExtract["a--bbb--ccc--ddd", "--" -> 3]
Out[192]= ccc

In[193]:= StringExtract["a bbb cccc aa d", -1]
Out[193]= d

In[194]:= StringRiffle[{{"a", "b", "c"}, {"d", "e", "f"}}, "\n", "\t"]
Out[194]= a      b      c
          d      e      f

In[195]:= {"", "z, t"}
Out[195]= {, z, t}

```



```
In[202]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
(*InputForm[TextString[#]]&/@%*)
StringExtract[#, "[" → All] & /@ %
StringExtract[#, "]" → All] & /@ %
StringExtract[#, "[" → 1] & /@ %%%
StringExtract[#, "]" → 1] & /@ %%%%
StringExtract[#, "Z[" → 1] & /@ %%%%%

Out[202]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[203]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[204]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[205]= {{Z, 0}, z, t}}, {Derivative, 1, 0}, Z, 0]], z, t}},
{Derivative, 0, 1}, Z, 0]], z, t}]

Out[206]= {{Z[0], [z, t, ], {Derivative[1, 0, [Z[0, , [z, t, ],
{Derivative[0, 1, [Z[0, , [z, t, ]]

Out[207]= {Z, Derivative, Derivative}

Out[208]= {Z[0, Derivative[1, 0, Derivative[0, 1]

Out[209]= {, Derivative[1, 0][, Derivative[0, 1][}

In[210]:= (*TemplateApply[StringTemplate["Z `["` then `b`"],
<|"a"→1234,"b"→5678|>]*)

```

```
In[211]:= skelx[x_, matQ_ : True] := Module[{t = {666}}, 
  If[matQ == True, t = Block[{r}, 
    r = x; 
    Table[Block[{q}, q = SameQ[0, r[[j, i]]]; 
      If[True == q, Style[0, Blue], Style[Length[r[[j, i]]], Red]], 
      {j, 1, Length[r]}, {i, 1, Length[r[[1]]]}]]; 
    If[matQ == False, t = Block[{r}, 
      r = x; 
      Table[Block[{q}, q = SameQ[0, r[[j]]]; 
        If[True == q, Style[0, {RGBColor -> {0, 1/3, 0}, Bold}], 
        Style[Length[r[[j]]], Red]], {j, 1, Length[r]}]]; 
      MatrixForm[t, TableAlignments -> Left]]]
```

```
In[212]:= makeSym[size_, fn_] := 
Module[{rtmp}, rtmp = Table[fn[i, j], {i, 1, size}, {j, 1, i}]; 
MapThread[Join, {rtmp, Rest /@ Flatten[rtmp, {{2}, {1}}]}]]
```

```
In[213]:= makeAntiSym[size_, fn_] := 
Module[{rtmp}, rtmp = Table[If[i == j, 0, fn[i, j]], {i, 1, size}, {j, 1, i}]; 
MapThread[Join, {rtmp, Rest /@ Flatten[-rtmp, {{2}, {1}}]}]]
```

```
In[214]:= Block[{MX}, MX = makeAntiSym[8, Subscript[\omega, ##] &]
```

```
Out[214]= {{0, -\omega_{2,1}, -\omega_{3,1}, -\omega_{4,1}, -\omega_{5,1}, -\omega_{6,1}, -\omega_{7,1}, -\omega_{8,1}}, 
{\omega_{2,1}, 0, -\omega_{3,2}, -\omega_{4,2}, -\omega_{5,2}, -\omega_{6,2}, -\omega_{7,2}, -\omega_{8,2}}, 
{\omega_{3,1}, \omega_{3,2}, 0, -\omega_{4,3}, -\omega_{5,3}, -\omega_{6,3}, -\omega_{7,3}, -\omega_{8,3}}, 
{\omega_{4,1}, \omega_{4,2}, \omega_{4,3}, 0, -\omega_{5,4}, -\omega_{6,4}, -\omega_{7,4}, -\omega_{8,4}}, 
{\omega_{5,1}, \omega_{5,2}, \omega_{5,3}, \omega_{5,4}, 0, -\omega_{6,5}, -\omega_{7,5}, -\omega_{8,5}}, 
{\omega_{6,1}, \omega_{6,2}, \omega_{6,3}, \omega_{6,4}, \omega_{6,5}, 0, -\omega_{7,6}, -\omega_{8,6}}, 
{\omega_{7,1}, \omega_{7,2}, \omega_{7,3}, \omega_{7,4}, \omega_{7,5}, \omega_{7,6}, 0, -\omega_{8,7}}, 
{\omega_{8,1}, \omega_{8,2}, \omega_{8,3}, \omega_{8,4}, \omega_{8,5}, \omega_{8,6}, \omega_{8,7}, 0}}
```

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2} \left(\frac{\partial g_{\mu\alpha}}{\partial x^\beta} + \frac{\partial g_{\mu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\mu} \right),$$

$$\Gamma^\mu{}_{\alpha\beta} = g^{\mu\nu} \Gamma_\nu{}^{}_{\alpha\beta},$$

$$R^\mu{}_{\nu\alpha\beta} = \frac{\partial \Gamma^\mu{}_{\nu\beta}}{\partial x^\alpha} - \frac{\partial \Gamma^\mu{}_{\nu\alpha}}{\partial x^\beta} + \Gamma^\mu{}_{\rho\alpha} \Gamma^\rho{}_{\nu\beta} - \Gamma^\mu{}_{\rho\beta} \Gamma^\rho{}_{\nu\alpha}.$$

$$R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}, \quad R = g^{\mu\nu}R_{\mu\nu},$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

In[215]:=

```
(*Christoffel, Einstein tensor, ...*)
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, rΓ, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    rΓ = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
      (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8} *
      {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
    RicciΓ =
      ParallelTable[FullSimplify[D[rΓ[[μ, ν, β]], X[[α]]] - D[rΓ[[μ, ν, α]], X[[β]]] +
        Sum[rΓ[[μ, s, α]] * rΓ[[s, ν, β]] - rΓ[[μ, s, β]] * rΓ[[s, ν, α]],
        {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}];
    RieΓ = ParallelTable[FullSimplify[Sum[RicciΓ[[α, μ, α, ν]], {α, 1, DIM8}],
      ass], {μ, 1, DIM8}, {ν, 1, DIM8}];
    RS = FullSimplify[Tr[ginv.RieΓ], ass];
    G = ParallelTable[
      FullSimplify[rieΓ[[α]] - 1/2 g[[α]] RS, ass], {α, 1, Length[RieΓ]}];
    Print[t];
    Print[Now];
    Return[{ginv, rΓ, RicciΓ, RieΓ, RS, G}]
  ]]
```

If ‘paste’ Maple MathML to ToExpression[MathMLForm[]] fails, then try to edit this :

In[217]:=

```
Clear[mapleztStringToMathematica];
mapleztStringToMathematica[
  input_String, toExpression_ : False, debug_ : False] :=
Module[{prepare, post, custom, use = input, save = input, linenumber = 0},
(*use=ToString[InputForm[ToString[input]]];*)(*Print[use];*)
  prepare[in_String] := Module[{s = in}, linenumber++;
  s = in;
  (*If[s==ToString[InputForm[ToString[s]]]],
```

```
(*###&[]*) Print[s],Print[ToUpperCase[s]]];*)
Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,z)" >
  "D[Z[" <> ToString[j] <> "][z,t],{z,2}]]"], {j, 0, 15}];
Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,t)" >
  "D[D[Z[" <> ToString[j] <> "][z,t],z],t]"], {j, 0, 15}];
Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,z)" >
  "D[D[Z[" <> ToString[j] <> "][z,t],t],z]"], {j, 0, 15}];
Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,t)" >
  "D[Z[" <> ToString[j] <> "][z,t],{t,2}]]"], {j, 0, 15}];

Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t)" >
  "D[Z[" <> ToString[j] <> "][z,t],t]"], {j, 0, 15}];
Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z)" >
  "D[Z[" <> ToString[j] <> "][z,t],z]"], {j, 0, 15}];

s = StringReplace[s, "tan(z)" >> "Tan[z]"];
s = StringReplace[s, "cot(z)" >> "Cot[z]"];
s = StringReplace[s, "sin(z)" >> "Sin[z]"];
s = StringReplace[s, "cos(z)" >> "Cos[z]"];
s = StringReplace[s, "sec(z)" >> "Sec[z]"];
s = StringReplace[s, "csc(z)" >> "Csc[z]"];

s = StringReplace[s, "=" >> "->"];

(*Do[s=StringReplace[s,
  "Z"<>ToString[j]<>"(z,t)">>"Z["<>ToString[j]<>"[z,t]"],{j,0,15}];*)
(*s=StringReplace[s,"diff(a4(t),t)">>"D[a4[t],t]"];*)
(*s=StringReplace[s,"exp(-2*a4(t))">>"Exp[-2a4[t]]"];*)
(*s=StringReplace[s,"a4(t)">>"a4[t]"];*)

Return[s];
];
post[in_String] := Module[{s = in}, linenumber++;
s = in;
s = StringReplace[s, "a4(t)" >> "a4[t]"];
Do[s = StringReplace[s, "Z" <> ToString[j] <> "(z,t)" >
  "Z[" <> ToString[j] <> "][z,t]"], {j, 0, 15}];
Return[s];
];
custom[in_String] := Module[{s = in}, linenumber++;

```

```

s = in;

s = StringReplace[s, "diff(a4(t),t)" → "D[a4[t],t]"];
s = StringReplace[s, "exp(-2*a4(t))" → "Exp[-2a4[t]]"];

Return[s];
];
use = prepare[use];
use = custom[use];
Return[post[use]];
(*If[toExpression,Return[ToExpression[s/.{z→#1,t→#2}],##&[]];*)

]

```

Mathematica Lexer and Parser for Maple - like Syntax Supports : Basic arithmetic, function calls, lists {}, equations =, and specific Maple keywords .

```

In[219]:= (*---LEXER---*) (*Helper:Check if character is a digit*)
IsDigit[char_] := StringMatchQ[char, RegularExpression["\\d"]];

(*Helper:Check if character is a letter*)
IsAlpha[char_] := StringMatchQ[char, RegularExpression"[a-zA-Z_"]];

(*Helper:Check if character is whitespace*)
IsSpace[char_] := StringMatchQ[char, RegularExpression["\\s"]];

(*Main Lexer Function*)
GetTokens[str_String] :=
Module[{chars, len, i, char, tokens = {}, token, numStr, idStr},
chars = Characters[str];
len = Length[chars];
i = 1;
While[i ≤ len, char = chars[[i]];
Which[(*Skip Whitespace*) IsSpace[char],
i++, (*Numbers*) IsDigit[char], numStr = char;
i++;
While[i ≤ len && IsDigit[chars[[i]]], numStr = numStr <> chars[[i]];
i++];
AppendTo[tokens, {"NUMBER", ToExpression[numStr]}],
(*Identifiers and Keywords*) IsAlpha[char], idStr = char;
i++];

```

```

While[i <= len && (IsAlpha[chars[[i]]] || IsDigit[chars[[i]]]),
  idStr = idStr <> chars[[i]];
  i++];
(*Check for specific keywords if needed,
otherwise IDENTIFIER*)AppendTo[tokens, {"IDENTIFIER", idStr}],
(*Operators and Punctuation*)char == "+", AppendTo[tokens, {"PLUS", "+"}];
i++, char == "-", AppendTo[tokens, {"MINUS", "-"}];
i++, char == "*", AppendTo[tokens, {"STAR", "*"}];
i++, char == "/", AppendTo[tokens, {"SLASH", "/"}];
i++, char == "^", AppendTo[tokens, {"CARET", "^"}];
i++, char == "(", AppendTo[tokens, {"LPAREN", "("}];
i++, char == ")", AppendTo[tokens, {"RPAREN", ")"}];
i++, char == "{", AppendTo[tokens, {"LBRACE", "{"}];
i++, char == "}", AppendTo[tokens, {"RBRACE", "}" }];
i++, char == "=", AppendTo[tokens, {"EQUALS", "="}];
i++, char == ",", AppendTo[tokens, {"COMMA", ","}];
i++, char == "$", AppendTo[tokens, {"DOLLAR", "$"}];
i++, (*Ellipsis... or Dot.*)char == ".", If[i + 2 <= len && chars[[i + 1]] ==
".." && chars[[i + 2]] == "..", AppendTo[tokens, {"ELLIPSIS", "..."}];
i += 3, AppendTo[tokens, {"DOT", "."}];
i++], True, Print["Unknown character: ", char];
i++]];
AppendTo[tokens, {"EOF", "EOF"}];
tokens];

```

In[223]:=

```

(*---PARSER---*)
(*Grammar:Expression→Equation Equation→AddExp["=" AddExp] AddExp→
  MulExp {("+"|"-") MulExp} MulExp→PowerExp {("*"|"/") PowerExp} PowerExp→
    Primary["^" PowerExp] (Right associative) Primary→
      NUMBER|IDENTIFIER|"(" Expression ")"|IDENTIFIER "(" ArgList ")"|
      "{" ArgList "}" ArgList→Expression {"," Expression}|Empty*)
ParseTokens[tokens_List] := Module[{pos = 1, currentToken, eat,
  peek, parseExpression, parseEquation, parseAddExp,
  parseMulExp, parsePowerExp, parseUnaryExp, parsePrimary,
  parseArgs, parseList}, currentToken := tokens[[pos]];
  peek[] := tokens[[pos]];
  eat[type_] := If[currentToken[[1]] == type, pos++;
    True, Print["Error: Expected ", type, " but found ", currentToken[[1]]];
    False];
  (*Entry point*)parseExpression[] := parseEquation[];
  (*Equation:a=b*)
```

```

parseEquation[] := Module[{left, right}, left = parseAddExp[];
  If[currentToken[1] == "EQUALS", eat["EQUALS"];
    right = parseAddExp[];
    {"Equation", left, right}, left]];
parseAddExp[] := Module[{node, right, op}, node = parseMulExp[];
  While[MemberQ>{"PLUS", "MINUS"}, currentToken[1]], op = currentToken[2];
  eat[currentToken[1]];
  right = parseMulExp[];
  node = {"BinaryOp", op, node, right}];
  node];
parseMulExp[] := Module[{node, right, op}, node = parsePowerExp[];
  While[True,
    If[MemberQ>{"STAR", "SLASH"}, currentToken[1]], op = currentToken[2];
    eat[currentToken[1]];
    right = parsePowerExp[];
    node = {"BinaryOp", op, node, right}, (*Implicit Multiplication*)
    If[MemberQ>{"IDENTIFIER", "NUMBER", "LPAREN", "LBRACE", "ELLIPSIS"},
      currentToken[1]], right = parsePowerExp[];
    node = {"BinaryOp", "*", node, right},
    Break[] (*Not an operator or start of expression*)]];
  node];
parsePowerExp[] := Module[{node, right}, node = parseUnaryExp[];
  If[currentToken[1] == "CARET", eat["CARET"];
    right = parsePowerExp[];
    (*Right associative recursion*) {"BinaryOp", "^", node, right}, node]];
parseUnaryExp[] := Module[{op, node},
  If[MemberQ>{"PLUS", "MINUS"}, currentToken[1]], op = currentToken[2];
  eat[currentToken[1]];
  node = parseUnaryExp[];
  {"UnaryOp", op, node}, parsePrimary[]];
parsePrimary[] := Module[{token, node, name, args}, token = currentToken;
  Switch[token[1], "NUMBER", eat["NUMBER"];
    {"Number", token[2]}, "IDENTIFIER", eat["IDENTIFIER"];
    name = token[2];
    (*Check if it's a function call:IDENTIFIER followed by LPAREN*)
    If[currentToken[1] == "LPAREN", eat["LPAREN"];
      args = parseArgs[];
      eat["RPAREN"];
      {"Call", name, args}, {"Identifier", name}], "LPAREN", eat["LPAREN"]];
    node = parseExpression[]];

```

```

eat["RPAREN"];
node, "LBRACE", parseList[], "ELLIPSIS", eat["ELLIPSIS"];
{"Identifier", "..."},
"EOF", Print["Error: Unexpected EOF in Primary"];
{"Error", "EOF"}, (*Do NOT eat EOF*)_,
Print["Error: Unexpected token in Primary: ", token];
eat[token[[1]]];
(*Advance to avoid infinite loop*) {"Error", token}]];
(*Parse List:{a,b,c}*)parseList[] := Module[{elements}, eat["LBRACE"]];
elements = parseArgs[];
eat["RBRACE"];
 {"List", elements}];
(*Parse Argument List:expr,expr,...*)
parseArgs[] := Module[{args = {}, arg}, If[currentToken[[1]] != "RPAREN" &&
currentToken[[1]] != "RBRACE", arg = parseExpression[];
AppendTo[args, arg];
While[currentToken[[1]] == "COMMA", eat["COMMA"];
arg = parseExpression[];
AppendTo[args, arg]];];
args];
parseExpression[]];

```

X

In[224]:=

```

(*---CONVERTER---*)
(*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunction[funcName_String] :=
Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate",
"DESol", "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan",
"Tan", "sin", "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc",
"Csc", "csch", "Csch", "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh",
"coth", "Coth", "sech", "Sech", "det", "Det", "transpose", "Transpose",
"sphericalbessel", "SphericalBesselJ", "lambertw", "ProductLog",
"gamma", "Gamma", "airy", "AiryAi", "besselJ", "BesselJ", "besselY",
"BesselY", "hypergeometricU", "HypergeometricU", "zeta", "Zeta",
"erf", "Erf", "erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNode[nodeType_, nodeData_, childResults_] :=
Module[{funcName, cleanFuncName, mathFunc, argsStr},
Switch[nodeType, "Number", ToString[nodeData],
```

```

"Identifier", (*Handle special identifiers like _Y→Y*)
If[StringLength[nodeData] > 0 && StringTake[nodeData, 1] == "_",
  StringDrop[nodeData, 1], nodeData], "BinaryOp",
"(" <> childResults[[1]] <> " " <> nodeData <> " " <> childResults[[2]] <> ")",
"UnaryOp", nodeData <> "(" <> childResults[[1]] <> ")", "Equation",
childResults[[1]] <> " == " <> childResults[[2]], "List", "{" <>
  StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
(*Handle special identifiers like _Y→Y for function names too*)
cleanFuncName = If[StringLength[funcName] > 0 &&
  StringTake[funcName, 1] == "_", StringDrop[funcName, 1], funcName];
mathFunc = MapleToMathematicaFunction[cleanFuncName];
(*Special handling for DSolve-requires 3 arguments*)
If[cleanFuncName == "DESol" || mathFunc == "DSolve",
  (*DSolve[equations,functions,variables]*)]
(*Ensure we have exactly 3 arguments,pad with empty lists if needed*)
argsStr = StringRiffle[Join[childResults,
  Table["{}", {i, 1, Max[0, 3 - Length[childResults]]}]], ", "];
mathFunc <> "[" <> argsStr <> "]", 
(*Default function call*)argsStr = StringRiffle[childResults, ", "];
mathFunc <> "[" <> argsStr <> "]"], _, "Error"]];

(*Convert AST to Mathematica String-Non-
recursive iterative version using post-order traversal*)
ToMathematicaString[ast_] :=
Module[{stack, outputStack, currentItem, node,.nodeType, children,
  childCount, childResults, i, result, nodeData}, (*Use two
  stacks:one for traversal,one for results*)stack = {{ast, False}};
(*{node,visited}* )outputStack = {};
(*Post-order traversal:process children before parents*)
While[Length[stack] > 0, currentItem = Last[stack];
  node = currentItem[[1]];
  nodeType = node[[1]];
  If[currentItem[[2]],
    (*Node already visited,process it*)stack = Most[stack];
    (*Determine children and extract data*)
    {children, nodeData} = Switch[nodeType, "Number", {{}, node[[2]]},
      "Identifier", {{}, node[[2]]}, "BinaryOp", {{node[[3]], node[[4]]}, node[[2]]},
      "UnaryOp", {{node[[3]]}, node[[2]]}, "Equation", {{node[[2]], node[[3]]}}, ""},
      "List", {node[[2]], ""}, "Call", {node[[3]], node[[2]]}, _, {{}, ""}}];
    (*Pop child results from output stack*)childCount = Length[children];
    stack = {stack, childCount}];]
  
```

```

childResults = {};
If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
  outputStack = Most[outputStack];, {i, 1, childCount}]];
(*Process this node*)
result = ProcessASTNode[nodeType, nodeData, childResults];
AppendTo[outputStack, result];, (*Node not visited,
mark as visited and push children*) stack[[Length[stack]]] = {node, True};
(*Push children onto stack
in reverse order (so they process left-to-right)*)
children = Switch[nodeType, "Number", {}, "Identifier", {},
  "BinaryOp", {node[[3]], node[[4]]}, "UnaryOp", {node[[3]]}, "Equation",
  {node[[2]], node[[3]]}, "List", node[[2]], "Call", node[[3]], _, {}];
Do[AppendTo[stack, {children[[i]], False}];,
{i, Length[children], 1, -1}];];
(*Return the final result*)
If[Length[outputStack] > 0, Last[outputStack], "Error"]];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematica[inputStr_String] :=
Module[{tokens, ast}, tokens = GetTokens[inputStr];
ast = ParseTokens[tokens];
ToMathematicaString[ast]];

In[228]:= (*---CONVERTER---*)
(*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunctionOLD[funcName_String] :=
Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate",
"DESol", "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan",
"Tan", "sin", "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc",
"Csc", "csch", "Csch", "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh",
"coth", "Coth", "sech", "Sech", "det", "Det", "transpose", "Transpose",
"sphericalbessel", "SphericalBesselJ", "lambertw", "ProductLog",
"gamma", "Gamma", "airy", "AiryAi", "besselJ", "BesselJ", "besselY",
"BesselY", "hypergeometricU", "HypergeometricU", "zeta", "Zeta",
"erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNodeOLD[nodeType_, nodeData_, childResults_] :=
Module[{funcName, cleanFuncName, mathFunc, argsStr},
Switch[nodeType, "Number", ToString[nodeData],
"Identifier", (*Handle special identifiers like _Y→Y*)]

```

```

If[StringLength[nodeData] > 0 && StringTake[nodeData, 1] == "_",
  StringDrop[nodeData, 1], nodeData], "BinaryOp",
("" <> childResults[[1]] <> " " <> nodeData <> " " <> childResults[[2]] <> ")",
"UnaryOp", nodeData <> "(" <> childResults[[1]] <> ")", "Equation",
childResults[[1]] <> " == " <> childResults[[2]], "List", "{" <>
  StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
(*Handle special identifiers like _Y→Y for function names too*)
cleanFuncName = If[StringLength[funcName] > 0 &&
  StringTake[funcName, 1] == "_", StringDrop[funcName, 1], funcName];
mathFunc = MapleToMathematicaFunction[cleanFuncName];
argsStr = StringRiffle[childResults, ", "];
mathFunc <> "[" <> argsStr <> "]", _, "Error"]];

(*Convert AST to Mathematica String-Non-
recursive iterative version using post-order traversal*)
ToMathematicaStringOLD[ast_] :=
Module[{stack, outputStack, currentItem, node,.nodeType, children,
  childCount, childResults, i, result, nodeData}, (*Use two
  stacks:one for traversal,one for results*)stack = {{ast, False}};
(*{node,visited}*)
outputStack = {};
(*Post-order traversal:process children before parents*)
While[Length[stack] > 0, currentItem = Last[stack];
  node = currentItem[[1]];
  nodeType = node[[1]];
  If[currentItem[[2]],
    (*Node already visited,process it*)stack = Most[stack];
    (*Determine children and extract data*)
    {children, nodeData} = Switch[nodeType, "Number", {}, node[[2]],
      "Identifier", {}, node[[2]], "BinaryOp", {{node[[3]], node[[4]]}, node[[2]]},
      "UnaryOp", {{node[[3]]}, node[[2]]}, "Equation", {{node[[2]], node[[3]]}}, ""},
      "List", {node[[2]], ""}, "Call", {node[[3]], node[[2]]}, _, {}, ""}];
    (*Pop child results from output stack*)childCount = Length[children];
    childResults = {};
    If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
      outputStack = Most[outputStack];, {i, 1, childCount}]];
    (*Process this node*)
    result = ProcessASTNode[nodeType, nodeData, childResults];
    AppendTo[outputStack, result];, (*Node not visited,
    mark as visited and push children*)stack[[Length[stack]]] = {node, True};
    (*Push children onto stack
  
```

```

    in reverse order (so they process left-to-right)*)
children = Switch[nodeType, "Number", {}, "Identifier", {},
  "BinaryOp", {node[[3]], node[[4]]}, "UnaryOp", {node[[3]]}, "Equation",
  {node[[2]], node[[3]]}, "List", node[[2]], "Call", node[[3]], _, {}];
Do[AppendTo[stack, {children[[i]], False}]]|,
{i, Length[children], 1, -1}];];
(*Return the final result*)
If[Length[outputStack] > 0, Last[outputStack], "Error"]];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematicaOLD[inputStr_String] :=
Module[{tokens, ast}, tokens = GetTokens[inputStr];
ast = ParseTokens[tokens];
ToMathematicaString[ast]];

In[232]:= sta = "{nZ10(z, t) = (c2C7*sin(C2Q1*t) +
  c2C8*cos(C2Q1*t))*(c2C5*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) +
  c2C6*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ11(z, t) = (c2C3*sin(C2Q1*t) +
  c2C4*cos(C2Q1*t))*(c2C1*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) +
  c2C2*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ8(z, t) = (((-C2Q1*c2C2*c2C3 +
  M*c2C6*c2C8)*cos(C2Q1*t) + sin(C2Q1*t)*(C2Q1*c2C2*c2C4 +
  M*c2C6*c2C7))*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) + ((C2Q1*c2C1*c2C3 -
  M*c2C5*c2C8)*cos(C2Q1*t) - sin(C2Q1*t)*(C2Q1*c2C1*c2C4 +
  M*c2C5*c2C7))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) + (-cos(M*t)*c2C10 +
  sin(M*t)*c2C9)*sqrt(-C2Q1^2 + M^2))/sqrt(-C2Q1^2 + M^2),
nZ9(z, t) = (((C2Q1*c2C6*c2C7 + M*c2C2*c2C4)*cos(C2Q1*t) +
  sin(C2Q1*t)*(-C2Q1*c2C6*c2C8 + M*c2C2*c2C3))*sin(z)^(-sqrt(-C2Q1^2
+ M^2)/3) + sqrt(-C2Q1^2 + M^2)*(cos(M*t)*c2C9 +
  sin(M*t)*c2C10)*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) + (-C2Q1*c2C5*c2C7 -
  M*c2C1*c2C4)*cos(C2Q1*t) - sin(C2Q1*t)*(-C2Q1*c2C5*c2C8 +
  M*c2C1*c2C3))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6)/sqrt(-C2Q1^2 + M^2)}";

```

In[233]:=

ConvertMapleToMathematica [sta]

Out[233]=

```

{nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5
* (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) + (c2C6 * (Sin[z]
^ (-(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))))), nZ11[z, t] == (((c2C3
* Sin[(C2Q1 * t)]) + (c2C4 * Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z] ^
(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) + (c2C2 * (Sin[z] ^ (-(Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))] / 6))))), nZ8[z, t] == ((((((((-(C2Q1) * c2C2) * c2C3)
+ ((M * c2C6) * c2C8)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((C2Q1 *
c2C2) * c2C4) + ((M * c2C6) * c2C7)))) * (Sin[z] ^ (-(Sqrt[((-(C2Q1) ^
2) + (M ^ 2))] / 6))) + (((((C2Q1 * c2C1) * c2C3) - ((M * c2C5) * c2C8))
* Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C1) * c2C4) + ((M *
c2C5) * c2C7)))) * (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6)))) + (((-(Cos[(M * t)]) * c2C10) + (Sin[(M * t)] * c2C9)) * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]) / Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]), nZ9[z, t] ==
((((((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)])
+ (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) *
(Sin[z] ^ (-(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 3))) + ((Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))] * ((Cos[(M * t)] * c2C9) + (Sin[(M * t)] * c2C10))) *
(Sin[z] ^ (-(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6)))) + ((((-(C2Q1) *
c2C5) * c2C7) - ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)])) - (Sin[(C2Q1 *
t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z] ^
(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) / Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))}

```

In[234]:=

```

1) + ((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*
c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt((-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1^2
+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*
c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt((-C2Q1^2+M^2)))-sin(z)^(1/6*
sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*
t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*sin(M*t)-C2Q1*Int(-cos(z)*(((-C:
2Q1*c2C6*c2C8+M*c2C2*c2C3)*cos(C2Q1*t)-sin(C2Q1*t)*(C2Q1*c2C6*c2C7+M*
c2C2*c2C4))*sin(z)^(-1/6*sqrt((-C2Q1^2+M^2)-1))+((-C2Q1*c2C5*c2C8+M*c2C1
*c2C3)*cos(C2Q1*t)-sin(C2Q1*t)*(C2Q1*c2C5*c2C7+M*c2C1*c2C4))*sin(z)^(1
/6*sqrt((-C2Q1^2+M^2)-1)),z))/M,nZ9(z,t)=1/6*((6*M*c2C9-Int(-sin(M*t)*(((-C:
sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt((-C2Q1^2+M
^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*
c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt((-C2Q1^2+M^2)-1)),z)-6*sqrt(-C:
2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C:
2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt((-C2Q1^2+M^2))-sin(z)^(1/
6*sqrt((-C2Q1^2+M^2)))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(
C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*cos(M*t)+(6*M*c2C10+Int(-c:
os(M*t)*((-C2Q1^2+M^2)*Int(-cos(z)*((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(
C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt((-C:
2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C:
2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt((-C2Q1^2+M^2)-1)),z)-6*
sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*
(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt((-C2Q1^2+M^2))-sin(z)^(1/
6*sqrt((-C2Q1^2+M^2)))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(
C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*sin(M*t)+M*Int(-cos(z)*((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*
c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt((-C2Q1^2+M^2)-1))+((C2Q1*c2C5*c2C7+
M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(
z)^(1/6*sqrt((-C2Q1^2+M^2)-1)),z))/M}";
```

In[235]:=

ConvertMapleToMathematica[sti]

Out[235]=

```

{nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5 *
(Sin[z]^(1/6) * Sqrt[((-C2Q1)^2 + (M^2))])) + (c2C6 * (Sin[z]^
((-1)/6) * Sqrt[((-C2Q1)^2 + (M^2))]))), nZ11[z, t] == (((c2C3
* Sin[(C2Q1 * t)]) + (c2C4 * Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z]^(1/
6) * Sqrt[((-C2Q1)^2 + (M^2))])) + (c2C2 * (Sin[z]^((-1)/6) *
Sqrt[((-C2Q1)^2 + (M^2)))))), nZ8[z, t] == (((1/6) * (((((6
* Sqrt[((-C2Q1)^2 + (M^2))]) * c2C6) * ((c2C7 * Sin[(C2Q1 * t)])
+ (c2C8 * Cos[(C2Q1 * t)]))) * (Sin[z]^((-1)/6) * Sqrt[((-C2Q1)^
2 + (M^2))])) - (((6 * Sqrt[((-C2Q1)^2 + (M^2))]) * c2C5) *
((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)]))) * (Sin[z]^((1/6) * Sqrt[((-C2Q1)^2 + (M^2))])))
```

```

/ 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])))) + ((((-(6) * M) * c2C10) -
Integrate[(-(Cos[(M * t)]) * (((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) *
((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)])]
+ (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))) + ((((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) +
(Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))), z]) - ((6 *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * ((((((C2Q1 * c2C6) * c2C7) + ((M *
c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) *
c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))])) - ((Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) *
((((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) +
(Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))), t]) * Cos[(M * t)]) + (((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) * *
((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * ((((((C2Q1 * c2C6) *
c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * *
(((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((1 / 6) *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))) + ((((((C2Q1 * c2C5) * c2C7) +
((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) *
c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))]] - 1))), z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * *
((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))), t]) * Sin[(M * t)]) - (C2Q1 * Integrate[(-(Cos[z]) * ((((((-(C2Q1) *
c2C6) * c2C8) + ((M * c2C2) * c2C3)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] *
(((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)))) * (Sin[z] ^ (((1 / 6) *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))) + ((((((-(C2Q1) * c2C5) *
c2C8) + ((M * c2C1) * c2C3)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] * *
(((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)))) * (Sin[z] ^ (((1 / 6) *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))), z])) / M), nZ9[z, t] == (((1 /
6) * (((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) * (((((-(C2Q1) ^
2) + (M ^ 2)) * Integrate[(-(Cos[z]) * ((((((C2Q1 * c2C6) * c2C7) + ((M *
c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) *
c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))]] - 1))) + ((((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) *
c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) +
((M * c2C1) * c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) +
(M ^ 2))]] - 1))), z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * *
((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) +

```

$$\begin{aligned}
& (\text{Sin}[(\text{C2Q1} * \text{t})] * (((-(\text{C2Q1}) * \text{c2C6}) * \text{c2C8}) + ((\text{M} * \text{c2C2}) * \text{c2C3}))) * \\
& (\text{Sin}[z] ^ ((-1) / 6) * \text{Sqrt}[((-(\text{C2Q1}) ^ 2) + (\text{M} ^ 2))])) - ((\text{Sin}[z] ^ \\
& ((1 / 6) * \text{Sqrt}[((-(\text{C2Q1}) ^ 2) + (\text{M} ^ 2))])) * (((((\text{C2Q1} * \text{c2C5}) * \text{c2C7}) + \\
& ((\text{M} * \text{c2C1}) * \text{c2C4})) * \text{Cos}[(\text{C2Q1} * \text{t})]) + (\text{Sin}[(\text{C2Q1} * \text{t})] * (((-(\text{C2Q1}) * \\
& \text{c2C5}) * \text{c2C8}) + ((\text{M} * \text{c2C1}) * \text{c2C3}))))))), \text{t})] * \text{Cos}[(\text{M} * \text{t})]) + (((6 \\
& * \text{M}) * \text{c2C10}) + \text{Integrate}[(-(\text{Cos}[(\text{M} * \text{t})])) * (((-(\text{C2Q1}) ^ 2) + (\text{M} ^ 2)) \\
& * \text{Integrate}[(-(\text{Cos}[z]) * (((((\text{C2Q1} * \text{c2C6}) * \text{c2C7}) + ((\text{M} * \text{c2C2}) * \\
& \text{c2C4})) * \text{Cos}[(\text{C2Q1} * \text{t})]) + (\text{Sin}[(\text{C2Q1} * \text{t})] * (((-(\text{C2Q1}) * \text{c2C6}) * \text{c2C8}) \\
& + ((\text{M} * \text{c2C2}) * \text{c2C3})))) * (\text{Sin}[z] ^ ((-1) / 6) * \text{Sqrt}[((-(\text{C2Q1}) ^ 2) + \\
& (\text{M} ^ 2))]) - 1))) + (((((\text{C2Q1} * \text{c2C5}) * \text{c2C7}) + ((\text{M} * \text{c2C1}) * \text{c2C4})) * \\
& \text{Cos}[(\text{C2Q1} * \text{t})]) + (\text{Sin}[(\text{C2Q1} * \text{t})] * (((-(\text{C2Q1}) * \text{c2C5}) * \text{c2C8}) + ((\text{M} * \\
& \text{c2C1}) * \text{c2C3})))) * (\text{Sin}[z] ^ ((1 / 6) * \text{Sqrt}[((-(\text{C2Q1}) ^ 2) + (\text{M} ^ 2))]) - \\
& 1))), z]) - ((6 * \text{Sqrt}[((-(\text{C2Q1}) ^ 2) + (\text{M} ^ 2))]) * (((((\text{C2Q1} * \text{c2C6}) \\
& * \text{c2C7}) + ((\text{M} * \text{c2C2}) * \text{c2C4})) * \text{Cos}[(\text{C2Q1} * \text{t})]) + (\text{Sin}[(\text{C2Q1} * \text{t})] * \\
& (((-(\text{C2Q1}) * \text{c2C6}) * \text{c2C8}) + ((\text{M} * \text{c2C2}) * \text{c2C3}))) * (\text{Sin}[z] ^ ((-1) / 6) \\
& * \text{Sqrt}[((-(\text{C2Q1}) ^ 2) + (\text{M} ^ 2))])) - ((\text{Sin}[z] ^ ((1 / 6) * \text{Sqrt}[((-(\text{C2Q1}) \\
& ^ 2) + (\text{M} ^ 2))]))) * (((((\text{C2Q1} * \text{c2C5}) * \text{c2C7}) + ((\text{M} * \text{c2C1}) * \text{c2C4})) * \\
& \text{Cos}[(\text{C2Q1} * \text{t})]) + (\text{Sin}[(\text{C2Q1} * \text{t})] * (((-(\text{C2Q1}) * \text{c2C5}) * \text{c2C8}) + ((\text{M} * \\
& \text{c2C1}) * \text{c2C3}))))))), \text{t})] * \text{Sin}[(\text{M} * \text{t})]) + (\text{M} * \text{Integrate}[(-(\text{Cos}[z]) \\
& * (((((\text{C2Q1} * \text{c2C6}) * \text{c2C7}) + ((\text{M} * \text{c2C2}) * \text{c2C4})) * \text{Cos}[(\text{C2Q1} * \text{t})]) \\
& + (\text{Sin}[(\text{C2Q1} * \text{t})] * (((-(\text{C2Q1}) * \text{c2C6}) * \text{c2C8}) + ((\text{M} * \text{c2C2}) * \text{c2C3})))) \\
& * (\text{Sin}[z] ^ ((-1) / 6) * \text{Sqrt}[((-(\text{C2Q1}) ^ 2) + (\text{M} ^ 2))]) - 1))) + \\
& (((((\text{C2Q1} * \text{c2C5}) * \text{c2C7}) + ((\text{M} * \text{c2C1}) * \text{c2C4})) * \text{Cos}[(\text{C2Q1} * \text{t})]) + \\
& (\text{Sin}[(\text{C2Q1} * \text{t})] * (((-(\text{C2Q1}) * \text{c2C5}) * \text{c2C8}) + ((\text{M} * \text{c2C1}) * \text{c2C3}))) * \\
& (\text{Sin}[z] ^ ((1 / 6) * \text{Sqrt}[((-(\text{C2Q1}) ^ 2) + (\text{M} ^ 2))]) - 1))), z])) / \text{M})
\end{aligned}$$

gtry and Γ and ...

In[236]:=

```
gtry = MatrixMetric44
```

Out[236]=

$$\begin{aligned}
& \{\{\text{Cot}[6 \text{H} x 0]^2, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2 a4[\text{H} x 4]} \text{Sin}[6 \text{H} x 0]^{1/3}, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, e^{2 a4[\text{H} x 4]} \text{Sin}[6 \text{H} x 0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, e^{2 a4[\text{H} x 4]} \text{Sin}[6 \text{H} x 0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, -e^{-2 a4[\text{H} x 4]} \text{Sin}[6 \text{H} x 0]^{1/3}, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, -e^{-2 a4[\text{H} x 4]} \text{Sin}[6 \text{H} x 0]^{1/3}, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, -e^{-2 a4[\text{H} x 4]} \text{Sin}[6 \text{H} x 0]^{1/3}\}
\end{aligned}$$

In[237]:=

```
Protect[gtry]
```

Out[237]=

```
{gtry}
```

```
In[238]:= Unprotect[ginv, \[Gamma], Ricci\[Gamma], Rie\[Gamma], RS, EinsteinG]
Out[238]= {}

In[239]:= result = {ginv, \[Gamma], Ricci\[Gamma], Rie\[Gamma], RS, EinsteinG} = rt[gtry];
Tue 3 Feb 2026 15:34:36 GMT-8
{3.63541, Null}
Tue 3 Feb 2026 15:34:40 GMT-8

In[240]:= Protect[ginv, \[Gamma], Ricci\[Gamma], Rie\[Gamma], RS, EinsteinG]
Out[240]= {ginv, \[Gamma], Ricci\[Gamma], Rie\[Gamma], RS, EinsteinG}

verify g\mu\nu|\alpha = 0 :

In[241]:= Simplify[
Table[D[gtry[[j, k]], X[[i]]] - Sum[gtry[[k, s]] \times \[Gamma][[s, i, j]], {s, 1, DIM8}] -
Sum[gtry[[j, s]] \times \[Gamma][[s, i, k]], {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Simplify[
Table[D[gtry[[j, k]], X[[i]]] - Sum[gtry[[k, s]] \times \[Gamma][[s, i, j]], {s, 1, DIM8}] -
Sum[gtry[[j, s]] \times \[Gamma][[s, i, k]], {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union

```
Out[241]= {0}
```

```
In[242]:= 
Table[g[j][k] → If[j == k,
  ToExpression["((g[" <> ToString[j] <> "] [" <> ToString[k] <> "] [#1, #2]) &) "],
  (0 &)], {j, 0, 7}, {k, 0, 7}];
sg = % // Flatten

Out[243]=
{g[0][0] → (g[0][0][#1, #2] &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &),
g[1][0] → (0 &), g[1][1] → (g[1][1][#1, #2] &), g[1][2] → (0 &), g[1][3] → (0 &),
g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &),
g[2][0] → (0 &), g[2][1] → (0 &), g[2][2] → (g[2][2][#1, #2] &), g[2][3] → (0 &),
g[2][4] → (0 &), g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &),
g[3][0] → (0 &), g[3][1] → (0 &), g[3][2] → (0 &), g[3][3] → (g[3][3][#1, #2] &),
g[3][4] → (0 &), g[3][5] → (0 &), g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &),
g[4][1] → (0 &), g[4][2] → (0 &), g[4][3] → (0 &), g[4][4] → (g[4][4][#1, #2] &),
g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &), g[5][0] → (0 &), g[5][1] → (0 &),
g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &), g[5][5] → (g[5][5][#1, #2] &),
g[5][6] → (0 &), g[5][7] → (0 &), g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &),
g[6][3] → (0 &), g[6][4] → (0 &), g[6][5] → (0 &), g[6][6] → (g[6][6][#1, #2] &),
g[6][7] → (0 &), g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &),
g[7][4] → (0 &), g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (g[7][7][#1, #2] &)}

In[244]:= 
g[7][7][x0, x4] /. ssgm4488
% /. sx0x4

Out[244]=
- $e^{-2 a4[H x4]} \sin[6 H x0]^{1/3}$ 

Out[245]=
- $e^{-2 a4[t]} \sin[z]^{1/3}$ 

In[246]:= 
MatrixMetric44

Out[246]=
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0,  $e^{2 a4[H x4]} \sin[6 H x0]^{1/3}$ , 0, 0, 0, 0, 0, 0}, {0, 0,  $e^{2 a4[H x4]} \sin[6 H x0]^{1/3}$ , 0, 0, 0, 0, 0}, {0, 0, 0,  $e^{2 a4[H x4]} \sin[6 H x0]^{1/3}$ , 0, 0, 0, 0}, {0, 0, 0, 0, - $e^{-2 a4[H x4]} \sin[6 H x0]^{1/3}$ , 0, 0}, {0, 0, 0, 0, 0, - $e^{-2 a4[H x4]} \sin[6 H x0]^{1/3}$ , 0}, {0, 0, 0, 0, 0, 0, - $e^{-2 a4[H x4]} \sin[6 H x0]^{1/3}$ }}
```

In[247]:=

ssgm4488[x0, x4] /. sx0x4

Out[247]=

$$\{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&),$$

$$g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&),$$

$$g[1][1] \rightarrow (e^{2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&),$$

$$g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&),$$

$$g[2][2] \rightarrow (e^{2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&),$$

$$g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&),$$

$$g[3][3] \rightarrow (e^{2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&),$$

$$g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&),$$

$$g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&),$$

$$g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&),$$

$$g[5][5] \rightarrow (-e^{-2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&),$$

$$g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&),$$

$$g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&),$$

$$g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&),$$

$$g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2A4[H\#2]} \sin[6H\#1]^{1/3} \&) \} \left[\frac{z}{6H}, \frac{t}{H} \right]$$

In[248]:=

$$(*ssgGzt=\{G[0][0]\rightarrow((\text{Cot}[\#1]^2)\&), G[0][1]\rightarrow((0)\&), G[0][2]\rightarrow((0)\&),$$

$$G[0][3]\rightarrow((0)\&), G[0][4]\rightarrow((0)\&), G[0][5]\rightarrow((0)\&), G[0][6]\rightarrow((0)\&),$$

$$G[0][7]\rightarrow((0)\&), G[1][0]\rightarrow((0)\&), G[1][1]\rightarrow((e^{2A4[\#2]} \sin[\#1]^{1/3})\&),$$

$$G[1][2]\rightarrow((0)\&), G[1][3]\rightarrow((0)\&), G[1][4]\rightarrow((0)\&), G[1][5]\rightarrow((0)\&),$$

$$G[1][6]\rightarrow((0)\&), G[1][7]\rightarrow((0)\&), G[2][0]\rightarrow((0)\&), G[2][1]\rightarrow((0)\&),$$

$$G[2][2]\rightarrow((e^{2A4[\#2]} \sin[\#1]^{1/3})\&), G[2][3]\rightarrow((0)\&), G[2][4]\rightarrow((0)\&),$$

$$G[2][5]\rightarrow((0)\&), G[2][6]\rightarrow((0)\&), G[2][7]\rightarrow((0)\&), G[3][0]\rightarrow((0)\&),$$

$$G[3][1]\rightarrow((0)\&), G[3][2]\rightarrow((0)\&), G[3][3]\rightarrow((e^{2A4[\#2]} \sin[\#1]^{1/3})\&),$$

$$G[3][4]\rightarrow((0)\&), G[3][5]\rightarrow((0)\&), G[3][6]\rightarrow((0)\&), G[3][7]\rightarrow((0)\&),$$

$$G[4][0]\rightarrow((0)\&), G[4][1]\rightarrow((0)\&), G[4][2]\rightarrow((0)\&), G[4][3]\rightarrow((0)\&),$$

$$G[4][4]\rightarrow((-1)\&), G[4][5]\rightarrow((0)\&), G[4][6]\rightarrow((0)\&), G[4][7]\rightarrow((0)\&),$$

$$G[5][0]\rightarrow((0)\&), G[5][1]\rightarrow((0)\&), G[5][2]\rightarrow((0)\&), G[5][3]\rightarrow((0)\&),$$

$$G[5][4]\rightarrow((0)\&), G[5][5]\rightarrow((-e^{-2A4[\#2]} \sin[\#1]^{1/3})\&),$$

$$G[5][6]\rightarrow((0)\&), G[5][7]\rightarrow((0)\&), G[6][0]\rightarrow((0)\&), G[6][1]\rightarrow((0)\&),$$

$$G[6][2]\rightarrow((0)\&), G[6][3]\rightarrow((0)\&), G[6][4]\rightarrow((0)\&), G[6][5]\rightarrow((0)\&),$$

$$G[6][6]\rightarrow((-e^{-2A4[\#2]} \sin[\#1]^{1/3})\&), G[6][7]\rightarrow((0)\&), G[7][0]\rightarrow((0)\&),$$

$$G[7][1]\rightarrow((0)\&), G[7][2]\rightarrow((0)\&), G[7][3]\rightarrow((0)\&), G[7][4]\rightarrow((0)\&),$$

$$G[7][5]\rightarrow((0)\&), G[7][6]\rightarrow((0)\&), G[7][7]\rightarrow((-e^{-2A4[\#2]} \sin[\#1]^{1/3})\&)\}*)$$

In[249]:=

(*G[#[]#[]z,t]/.ssgGzt&/@Range[0,7]*)

```
In[250]:= detgg = Det[g4488 /. sg] // FullSimplify[#, constraintVars] &
Out[250]= g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]

In[251]:= g4488 /. sg // MatrixForm
Out[251]//MatrixForm=

$$\begin{pmatrix} g[0][0][x0, x4] & 0 & 0 & 0 & 0 \\ 0 & g[1][1][x0, x4] & 0 & 0 & 0 \\ 0 & 0 & g[2][2][x0, x4] & 0 & 0 \\ 0 & 0 & 0 & g[3][3][x0, x4] & 0 \\ 0 & 0 & 0 & 0 & g[4][4][x0, x4] \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


In[252]:= MatrixMetric44.η4488
(*eAa=*) FullSimplify[#, constraintVars]
Out[252]= {{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

Out[253]= {{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0}, {0, Sqrt[e^2 a4[H x4]] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {0, 0, Sqrt[e^2 a4[H x4]] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, Sqrt[e^2 a4[H x4]] Sin[6 H x0]^(1/6), 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, Sqrt[e^-2 a4[H x4]] Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, Sqrt[e^-2 a4[H x4]] Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, Sqrt[e^-2 a4[H x4]] Sin[6 H x0]^(1/6)}}

eAa
```

In[254]:=

```
g4488.η4488 /. sg
( eAa = FullSimplify[  $\sqrt{\%}$ , constraintVars] ) // MatrixForm
```

Out[254]=

```
{ {g[0][0][x0, x4], 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0}, {0, 0, 0, 0, -g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, -g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, -g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, -g[7][7][x0, x4]} }
```

Out[255]//MatrixForm=

$$\begin{pmatrix} \sqrt{g[0][0][x0, x4]} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{g[1][1][x0, x4]} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{g[2][2][x0, x4]} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{g[3][3][x0, x4]} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{-g[4][4][x0, x4]} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[256]:=

eAa

ssgm4488

(eAa04 = %% /. % // FullSimplify[#, constraintVars] &) // MatrixForm

Out[256]=

$$\left\{ \left\{ \sqrt{g[0][0][x_0, x_4]}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \sqrt{g[1][1][x_0, x_4]}, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, \sqrt{g[2][2][x_0, x_4]}, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \sqrt{g[3][3][x_0, x_4]}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, \sqrt{-g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \sqrt{-g[5][5][x_0, x_4]}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, \sqrt{-g[6][6][x_0, x_4]}, 0, 0, 0, 0, 0, \sqrt{-g[7][7][x_0, x_4]} \right\} \right\}$$

Out[257]=

$$\begin{aligned} g[0][0] &\rightarrow (\text{Cot}[6 H \#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] &\rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] &\rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&), \\ g[1][5] &\rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&), \\ g[2][2] &\rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&), \\ g[2][6] &\rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&), \\ g[3][3] &\rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] &\rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] &\rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] &\rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] &\rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] &\rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] &\rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] &\rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] &\rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&) \end{aligned}$$

Out[258]/MatrixForm=

$\text{Cot}[6 H x_0]$	0	0	0	0	0
0	$\sqrt{e^{2 a4[H \#4]}} \sin[6 H x_0]^{1/6}$	0	0	0	0
0	0	$\sqrt{e^{2 a4[H \#4]}} \sin[6 H x_0]^{1/6}$	0	0	0
0	0	0	$\sqrt{e^{2 a4[H \#4]}} \sin[6 H x_0]^{1/6}$	0	0
0	0	0	0	0	1
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

OCTAD $e_{\alpha \leftarrow \text{spacetime}}^{(A) \leftarrow \text{Lorentz}}$:

$$e_{\alpha}^{(A)} = e_{\alpha}^{(A)} = eAa$$

In[259]:=

$$(sge_{\alpha}^{(A)} = \text{Thread}[\text{Flatten}[e_{\alpha}^{(A)}] \rightarrow \text{Flatten}[eAa]]) (*//Column*)$$

Out[259]=

$$\begin{aligned} & \{E_{\alpha}^{(A)}[0, 0] \rightarrow \sqrt{g[0][0][x0, x4]}, E_{\alpha}^{(A)}[0, 1] \rightarrow 0, E_{\alpha}^{(A)}[0, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[0, 3] \rightarrow 0, E_{\alpha}^{(A)}[0, 4] \rightarrow 0, E_{\alpha}^{(A)}[0, 5] \rightarrow 0, E_{\alpha}^{(A)}[0, 6] \rightarrow 0, E_{\alpha}^{(A)}[0, 7] \rightarrow 0, \\ & E_{\alpha}^{(A)}[1, 0] \rightarrow 0, E_{\alpha}^{(A)}[1, 1] \rightarrow \sqrt{g[1][1][x0, x4]}, E_{\alpha}^{(A)}[1, 2] \rightarrow 0, E_{\alpha}^{(A)}[1, 3] \rightarrow 0, \\ & E_{\alpha}^{(A)}[1, 4] \rightarrow 0, E_{\alpha}^{(A)}[1, 5] \rightarrow 0, E_{\alpha}^{(A)}[1, 6] \rightarrow 0, E_{\alpha}^{(A)}[1, 7] \rightarrow 0, E_{\alpha}^{(A)}[2, 0] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 1] \rightarrow 0, E_{\alpha}^{(A)}[2, 2] \rightarrow \sqrt{g[2][2][x0, x4]}, E_{\alpha}^{(A)}[2, 3] \rightarrow 0, E_{\alpha}^{(A)}[2, 4] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 5] \rightarrow 0, E_{\alpha}^{(A)}[2, 6] \rightarrow 0, E_{\alpha}^{(A)}[2, 7] \rightarrow 0, E_{\alpha}^{(A)}[3, 0] \rightarrow 0, E_{\alpha}^{(A)}[3, 1] \rightarrow 0, \\ & E_{\alpha}^{(A)}[3, 2] \rightarrow 0, E_{\alpha}^{(A)}[3, 3] \rightarrow \sqrt{g[3][3][x0, x4]}, E_{\alpha}^{(A)}[3, 4] \rightarrow 0, E_{\alpha}^{(A)}[3, 5] \rightarrow 0, \\ & E_{\alpha}^{(A)}[3, 6] \rightarrow 0, E_{\alpha}^{(A)}[3, 7] \rightarrow 0, E_{\alpha}^{(A)}[4, 0] \rightarrow 0, E_{\alpha}^{(A)}[4, 1] \rightarrow 0, E_{\alpha}^{(A)}[4, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[4, 3] \rightarrow 0, E_{\alpha}^{(A)}[4, 4] \rightarrow \sqrt{-g[4][4][x0, x4]}, E_{\alpha}^{(A)}[4, 5] \rightarrow 0, \\ & E_{\alpha}^{(A)}[4, 6] \rightarrow 0, E_{\alpha}^{(A)}[4, 7] \rightarrow 0, E_{\alpha}^{(A)}[5, 0] \rightarrow 0, E_{\alpha}^{(A)}[5, 1] \rightarrow 0, E_{\alpha}^{(A)}[5, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[5, 3] \rightarrow 0, E_{\alpha}^{(A)}[5, 4] \rightarrow 0, E_{\alpha}^{(A)}[5, 5] \rightarrow \sqrt{-g[5][5][x0, x4]}, \\ & E_{\alpha}^{(A)}[5, 6] \rightarrow 0, E_{\alpha}^{(A)}[5, 7] \rightarrow 0, E_{\alpha}^{(A)}[6, 0] \rightarrow 0, E_{\alpha}^{(A)}[6, 1] \rightarrow 0, E_{\alpha}^{(A)}[6, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[6, 3] \rightarrow 0, E_{\alpha}^{(A)}[6, 4] \rightarrow 0, E_{\alpha}^{(A)}[6, 5] \rightarrow 0, E_{\alpha}^{(A)}[6, 6] \rightarrow \sqrt{-g[6][6][x0, x4]}, \\ & E_{\alpha}^{(A)}[6, 7] \rightarrow 0, E_{\alpha}^{(A)}[7, 0] \rightarrow 0, E_{\alpha}^{(A)}[7, 1] \rightarrow 0, E_{\alpha}^{(A)}[7, 2] \rightarrow 0, E_{\alpha}^{(A)}[7, 3] \rightarrow 0, \\ & E_{\alpha}^{(A)}[7, 4] \rightarrow 0, E_{\alpha}^{(A)}[7, 5] \rightarrow 0, E_{\alpha}^{(A)}[7, 6] \rightarrow 0, E_{\alpha}^{(A)}[7, 7] \rightarrow \sqrt{-g[7][7][x0, x4]} \} \end{aligned}$$

In[260]:=

$$(*\text{Symbolize}[sge_{\alpha}^{(A)}]*)$$

In[261]:=

$$(*\text{Symbolize}[sgtrye_{\alpha}^{(A)}]*)$$

In[262]:=

$$(\text{sgtrye}_{\alpha}^{(A)} = \text{Thread}[\text{Flatten}[e_{\alpha}^{(A)}] \rightarrow \text{Flatten}[eAa04]]) (*//Column*)$$

Out[262]=

$$\left\{ E_{\alpha}^{(A)} [0, 0] \rightarrow \text{Cot}[6 H x 0], E_{\alpha}^{(A)} [0, 1] \rightarrow 0, E_{\alpha}^{(A)} [0, 2] \rightarrow 0, E_{\alpha}^{(A)} [0, 3] \rightarrow 0, \right. \\ E_{\alpha}^{(A)} [0, 4] \rightarrow 0, E_{\alpha}^{(A)} [0, 5] \rightarrow 0, E_{\alpha}^{(A)} [0, 6] \rightarrow 0, E_{\alpha}^{(A)} [0, 7] \rightarrow 0, E_{\alpha}^{(A)} [1, 0] \rightarrow 0, \\ E_{\alpha}^{(A)} [1, 1] \rightarrow \sqrt{e^{2 a4[H \times 4]}} \sin[6 H x 0]^{1/6}, E_{\alpha}^{(A)} [1, 2] \rightarrow 0, E_{\alpha}^{(A)} [1, 3] \rightarrow 0, E_{\alpha}^{(A)} [1, 4] \rightarrow 0, \\ E_{\alpha}^{(A)} [1, 5] \rightarrow 0, E_{\alpha}^{(A)} [1, 6] \rightarrow 0, E_{\alpha}^{(A)} [1, 7] \rightarrow 0, E_{\alpha}^{(A)} [2, 0] \rightarrow 0, E_{\alpha}^{(A)} [2, 1] \rightarrow 0, \\ E_{\alpha}^{(A)} [2, 2] \rightarrow \sqrt{e^{2 a4[H \times 4]}} \sin[6 H x 0]^{1/6}, E_{\alpha}^{(A)} [2, 3] \rightarrow 0, E_{\alpha}^{(A)} [2, 4] \rightarrow 0, E_{\alpha}^{(A)} [2, 5] \rightarrow 0, \\ E_{\alpha}^{(A)} [2, 6] \rightarrow 0, E_{\alpha}^{(A)} [2, 7] \rightarrow 0, E_{\alpha}^{(A)} [3, 0] \rightarrow 0, E_{\alpha}^{(A)} [3, 1] \rightarrow 0, E_{\alpha}^{(A)} [3, 2] \rightarrow 0, \\ E_{\alpha}^{(A)} [3, 3] \rightarrow \sqrt{e^{2 a4[H \times 4]}} \sin[6 H x 0]^{1/6}, E_{\alpha}^{(A)} [3, 4] \rightarrow 0, E_{\alpha}^{(A)} [3, 5] \rightarrow 0, E_{\alpha}^{(A)} [3, 6] \rightarrow 0, \\ E_{\alpha}^{(A)} [3, 7] \rightarrow 0, E_{\alpha}^{(A)} [4, 0] \rightarrow 0, E_{\alpha}^{(A)} [4, 1] \rightarrow 0, E_{\alpha}^{(A)} [4, 2] \rightarrow 0, E_{\alpha}^{(A)} [4, 3] \rightarrow 0, E_{\alpha}^{(A)} [4, 4] \rightarrow 1, \\ E_{\alpha}^{(A)} [4, 5] \rightarrow 0, E_{\alpha}^{(A)} [4, 6] \rightarrow 0, E_{\alpha}^{(A)} [4, 7] \rightarrow 0, E_{\alpha}^{(A)} [5, 0] \rightarrow 0, E_{\alpha}^{(A)} [5, 1] \rightarrow 0, \\ E_{\alpha}^{(A)} [5, 2] \rightarrow 0, E_{\alpha}^{(A)} [5, 3] \rightarrow 0, E_{\alpha}^{(A)} [5, 4] \rightarrow 0, E_{\alpha}^{(A)} [5, 5] \rightarrow \sqrt{e^{-2 a4[H \times 4]}} \sin[6 H x 0]^{1/6}, \\ E_{\alpha}^{(A)} [5, 6] \rightarrow 0, E_{\alpha}^{(A)} [5, 7] \rightarrow 0, E_{\alpha}^{(A)} [6, 0] \rightarrow 0, E_{\alpha}^{(A)} [6, 1] \rightarrow 0, E_{\alpha}^{(A)} [6, 2] \rightarrow 0, \\ E_{\alpha}^{(A)} [6, 3] \rightarrow 0, E_{\alpha}^{(A)} [6, 4] \rightarrow 0, E_{\alpha}^{(A)} [6, 5] \rightarrow 0, E_{\alpha}^{(A)} [6, 6] \rightarrow \sqrt{e^{-2 a4[H \times 4]}} \sin[6 H x 0]^{1/6}, \\ E_{\alpha}^{(A)} [6, 7] \rightarrow 0, E_{\alpha}^{(A)} [7, 0] \rightarrow 0, E_{\alpha}^{(A)} [7, 1] \rightarrow 0, E_{\alpha}^{(A)} [7, 2] \rightarrow 0, E_{\alpha}^{(A)} [7, 3] \rightarrow 0, \\ \left. E_{\alpha}^{(A)} [7, 4] \rightarrow 0, E_{\alpha}^{(A)} [7, 5] \rightarrow 0, E_{\alpha}^{(A)} [7, 6] \rightarrow 0, E_{\alpha}^{(A)} [7, 7] \rightarrow \sqrt{e^{-2 a4[H \times 4]}} \sin[6 H x 0]^{1/6} \right\}$$

In[263]:=

```
(sgtrye $_{(A)}^{\alpha}$  = Thread[Flatten[e $_{(A)}^{\alpha}$ ]  $\rightarrow$  Flatten[Inverse[e $_{\alpha}^{(A)}$  /. sgtrye $_{\alpha}^{(A)}$ ]]])  
(*//Column*)
```

Out[263]=

$$\left\{ \begin{array}{l} E_{(A)}^{\alpha}[0, 0] \rightarrow \tan[6 H x 0], E_{(A)}^{\alpha}[0, 1] \rightarrow 0, E_{(A)}^{\alpha}[0, 2] \rightarrow 0, E_{(A)}^{\alpha}[0, 3] \rightarrow 0, \\ E_{(A)}^{\alpha}[0, 4] \rightarrow 0, E_{(A)}^{\alpha}[0, 5] \rightarrow 0, E_{(A)}^{\alpha}[0, 6] \rightarrow 0, E_{(A)}^{\alpha}[0, 7] \rightarrow 0, E_{(A)}^{\alpha}[1, 0] \rightarrow 0, \\ E_{(A)}^{\alpha}[1, 1] \rightarrow \frac{1}{\sqrt{e^{2 a 4 [H x 4]} \sin[6 H x 0]^{1/6}}}, E_{(A)}^{\alpha}[1, 2] \rightarrow 0, E_{(A)}^{\alpha}[1, 3] \rightarrow 0, E_{(A)}^{\alpha}[1, 4] \rightarrow 0, \\ E_{(A)}^{\alpha}[1, 5] \rightarrow 0, E_{(A)}^{\alpha}[1, 6] \rightarrow 0, E_{(A)}^{\alpha}[1, 7] \rightarrow 0, E_{(A)}^{\alpha}[2, 0] \rightarrow 0, E_{(A)}^{\alpha}[2, 1] \rightarrow 0, \\ E_{(A)}^{\alpha}[2, 2] \rightarrow \frac{1}{\sqrt{e^{2 a 4 [H x 4]} \sin[6 H x 0]^{1/6}}}, E_{(A)}^{\alpha}[2, 3] \rightarrow 0, E_{(A)}^{\alpha}[2, 4] \rightarrow 0, E_{(A)}^{\alpha}[2, 5] \rightarrow 0, \\ E_{(A)}^{\alpha}[2, 6] \rightarrow 0, E_{(A)}^{\alpha}[2, 7] \rightarrow 0, E_{(A)}^{\alpha}[3, 0] \rightarrow 0, E_{(A)}^{\alpha}[3, 1] \rightarrow 0, E_{(A)}^{\alpha}[3, 2] \rightarrow 0, \\ E_{(A)}^{\alpha}[3, 3] \rightarrow \frac{1}{\sqrt{e^{2 a 4 [H x 4]} \sin[6 H x 0]^{1/6}}}, E_{(A)}^{\alpha}[3, 4] \rightarrow 0, E_{(A)}^{\alpha}[3, 5] \rightarrow 0, E_{(A)}^{\alpha}[3, 6] \rightarrow 0, \\ E_{(A)}^{\alpha}[3, 7] \rightarrow 0, E_{(A)}^{\alpha}[4, 0] \rightarrow 0, E_{(A)}^{\alpha}[4, 1] \rightarrow 0, E_{(A)}^{\alpha}[4, 2] \rightarrow 0, E_{(A)}^{\alpha}[4, 3] \rightarrow 0, E_{(A)}^{\alpha}[4, 4] \rightarrow 1, \\ E_{(A)}^{\alpha}[4, 5] \rightarrow 0, E_{(A)}^{\alpha}[4, 6] \rightarrow 0, E_{(A)}^{\alpha}[4, 7] \rightarrow 0, E_{(A)}^{\alpha}[5, 0] \rightarrow 0, E_{(A)}^{\alpha}[5, 1] \rightarrow 0, \\ E_{(A)}^{\alpha}[5, 2] \rightarrow 0, E_{(A)}^{\alpha}[5, 3] \rightarrow 0, E_{(A)}^{\alpha}[5, 4] \rightarrow 0, E_{(A)}^{\alpha}[5, 5] \rightarrow \frac{1}{\sqrt{e^{-2 a 4 [H x 4]} \sin[6 H x 0]^{1/6}}}, \\ E_{(A)}^{\alpha}[5, 6] \rightarrow 0, E_{(A)}^{\alpha}[5, 7] \rightarrow 0, E_{(A)}^{\alpha}[6, 0] \rightarrow 0, E_{(A)}^{\alpha}[6, 1] \rightarrow 0, E_{(A)}^{\alpha}[6, 2] \rightarrow 0, \\ E_{(A)}^{\alpha}[6, 3] \rightarrow 0, E_{(A)}^{\alpha}[6, 4] \rightarrow 0, E_{(A)}^{\alpha}[6, 5] \rightarrow 0, E_{(A)}^{\alpha}[6, 6] \rightarrow \frac{1}{\sqrt{e^{-2 a 4 [H x 4]} \sin[6 H x 0]^{1/6}}}, \\ E_{(A)}^{\alpha}[6, 7] \rightarrow 0, E_{(A)}^{\alpha}[7, 0] \rightarrow 0, E_{(A)}^{\alpha}[7, 1] \rightarrow 0, E_{(A)}^{\alpha}[7, 2] \rightarrow 0, E_{(A)}^{\alpha}[7, 3] \rightarrow 0, \\ E_{(A)}^{\alpha}[7, 4] \rightarrow 0, E_{(A)}^{\alpha}[7, 5] \rightarrow 0, E_{(A)}^{\alpha}[7, 6] \rightarrow 0, E_{(A)}^{\alpha}[7, 7] \rightarrow \frac{1}{\sqrt{e^{-2 a 4 [H x 4]} \sin[6 H x 0]^{1/6}}} \end{array} \right\}$$

In[264]:=

```
gtrye $_{\alpha}^{(A)} = (e_{\alpha}^{(A)} /. sgtrye_{\alpha}^{(A)}) /. subsDefects$ 
```

Out[264]=

$$\left\{ \begin{array}{l} \{\cot[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, e^{a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, e^{-a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0\}, \\ \{0, 0, 0, 0, 0, 0, 0, e^{-a 4 [H x 4]} \sin[6 H x 0]^{1/6}\} \end{array} \right\}$$

In[265]:=

$$\text{gtrye}_{(A)}^{\alpha} = (\mathbf{e}_{(A)}^{\alpha} / . \text{sgtrye}_{(A)}^{\alpha}) / . \text{subsDefects}$$

Out[265]=

$$\begin{aligned} & \left\{ \{ \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ 0, \frac{e^{-a4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{e^{-a4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{e^{-a4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, \frac{e^{a4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[H x 4]}}{\sin[6 H x 0]^{1/6}} \right\} \right\} \end{aligned}$$

In[266]:=

$$\begin{aligned} & \mathbf{e}_{\alpha}^{(A)} \\ & \% / . \text{sgtrye}_{\alpha}^{(A)} // \text{MatrixForm} \end{aligned}$$

Out[266]=

$$\begin{aligned} & \left\{ \{ E_{\alpha}^{(A)}[0, 0], E_{\alpha}^{(A)}[0, 1], E_{\alpha}^{(A)}[0, 2], E_{\alpha}^{(A)}[0, 3], \right. \\ & E_{\alpha}^{(A)}[0, 4], E_{\alpha}^{(A)}[0, 5], E_{\alpha}^{(A)}[0, 6], E_{\alpha}^{(A)}[0, 7] \}, \{ E_{\alpha}^{(A)}[1, 0], E_{\alpha}^{(A)}[1, 1], \right. \\ & E_{\alpha}^{(A)}[1, 2], E_{\alpha}^{(A)}[1, 3], E_{\alpha}^{(A)}[1, 4], E_{\alpha}^{(A)}[1, 5], E_{\alpha}^{(A)}[1, 6], E_{\alpha}^{(A)}[1, 7] \}, \\ & \left. \{ E_{\alpha}^{(A)}[2, 0], E_{\alpha}^{(A)}[2, 1], E_{\alpha}^{(A)}[2, 2], E_{\alpha}^{(A)}[2, 3], E_{\alpha}^{(A)}[2, 4], E_{\alpha}^{(A)}[2, 5], \right. \\ & E_{\alpha}^{(A)}[2, 6], E_{\alpha}^{(A)}[2, 7] \}, \{ E_{\alpha}^{(A)}[3, 0], E_{\alpha}^{(A)}[3, 1], E_{\alpha}^{(A)}[3, 2], E_{\alpha}^{(A)}[3, 3], \right. \\ & E_{\alpha}^{(A)}[3, 4], E_{\alpha}^{(A)}[3, 5], E_{\alpha}^{(A)}[3, 6], E_{\alpha}^{(A)}[3, 7] \}, \{ E_{\alpha}^{(A)}[4, 0], E_{\alpha}^{(A)}[4, 1], \right. \\ & E_{\alpha}^{(A)}[4, 2], E_{\alpha}^{(A)}[4, 3], E_{\alpha}^{(A)}[4, 4], E_{\alpha}^{(A)}[4, 5], E_{\alpha}^{(A)}[4, 6], E_{\alpha}^{(A)}[4, 7] \}, \\ & \left. \{ E_{\alpha}^{(A)}[5, 0], E_{\alpha}^{(A)}[5, 1], E_{\alpha}^{(A)}[5, 2], E_{\alpha}^{(A)}[5, 3], E_{\alpha}^{(A)}[5, 4], E_{\alpha}^{(A)}[5, 5], \right. \\ & E_{\alpha}^{(A)}[5, 6], E_{\alpha}^{(A)}[5, 7] \}, \{ E_{\alpha}^{(A)}[6, 0], E_{\alpha}^{(A)}[6, 1], E_{\alpha}^{(A)}[6, 2], E_{\alpha}^{(A)}[6, 3], \right. \\ & E_{\alpha}^{(A)}[6, 4], E_{\alpha}^{(A)}[6, 5], E_{\alpha}^{(A)}[6, 6], E_{\alpha}^{(A)}[6, 7] \}, \{ E_{\alpha}^{(A)}[7, 0], E_{\alpha}^{(A)}[7, 1], \right. \\ & E_{\alpha}^{(A)}[7, 2], E_{\alpha}^{(A)}[7, 3], E_{\alpha}^{(A)}[7, 4], E_{\alpha}^{(A)}[7, 5], E_{\alpha}^{(A)}[7, 6], E_{\alpha}^{(A)}[7, 7] \} \} \end{aligned}$$

Out[267]//MatrixForm=

$$\begin{array}{cccccc} \text{Cot}[6 H x 0] & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{e^{2 a4[H x 4]}} \sin[6 H x 0]^{1/6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{e^{2 a4[H x 4]}} \sin[6 H x 0]^{1/6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{e^{2 a4[H x 4]}} \sin[6 H x 0]^{1/6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

In[268]:=

```
eηe = FullSimplify[Transpose[e(A)α] . η4488.e(A)α, constraintVars];
% /. sge(A)α
%% /. sgtrye(A)α
% == gtry // FullSimplify[#, constraintVars] &
```

Out[269]=

```
{ {g[0][0][x0, x4], 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]} }
```

Out[270]=

```
{ {Cot[6 H x0]2, 0, 0, 0, 0, 0, 0}, {0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0}, {0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0}, {0, 0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0}, {0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0, 0}, {0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0}, {0, 0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3} }
```

Out[271]=

```
True
```

In[272]:=

```
gtry
```

Out[272]=

```
{ {Cot[6 H x0]2, 0, 0, 0, 0, 0, 0}, {0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0}, {0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0}, {0, 0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0}, {0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0, 0}, {0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0}, {0, 0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3} }
```

In[273]:=

```
(*FullSimplify[Transpose[e(A)α] . η4488.e(A)α, constraintVars]-MatrixMetric44*)
```

In[274]:=

ssgm4488

Out[274]=

$$\{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&), \\ g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&), \\ g[2][2] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&), \\ g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&), \\ g[3][3] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&)\}$$

In[275]:=

MatrixMetric44 // MatrixForm

Out[275]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a^4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a^4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a^4[Hx4]} \sin[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[276]:=

$$\mathbf{e}_\alpha^{(A)}$$

Out[276]=

$$\left\{ \begin{array}{l} \{E_\alpha^{(A)}[0, 0], E_\alpha^{(A)}[0, 1], E_\alpha^{(A)}[0, 2], E_\alpha^{(A)}[0, 3], \\ E_\alpha^{(A)}[0, 4], E_\alpha^{(A)}[0, 5], E_\alpha^{(A)}[0, 6], E_\alpha^{(A)}[0, 7]\}, \{E_\alpha^{(A)}[1, 0], E_\alpha^{(A)}[1, 1], \\ E_\alpha^{(A)}[1, 2], E_\alpha^{(A)}[1, 3], E_\alpha^{(A)}[1, 4], E_\alpha^{(A)}[1, 5], E_\alpha^{(A)}[1, 6], E_\alpha^{(A)}[1, 7]\}, \\ \{E_\alpha^{(A)}[2, 0], E_\alpha^{(A)}[2, 1], E_\alpha^{(A)}[2, 2], E_\alpha^{(A)}[2, 3], E_\alpha^{(A)}[2, 4], E_\alpha^{(A)}[2, 5], \\ E_\alpha^{(A)}[2, 6], E_\alpha^{(A)}[2, 7]\}, \{E_\alpha^{(A)}[3, 0], E_\alpha^{(A)}[3, 1], E_\alpha^{(A)}[3, 2], E_\alpha^{(A)}[3, 3], \\ E_\alpha^{(A)}[3, 4], E_\alpha^{(A)}[3, 5], E_\alpha^{(A)}[3, 6], E_\alpha^{(A)}[3, 7]\}, \{E_\alpha^{(A)}[4, 0], E_\alpha^{(A)}[4, 1], \\ E_\alpha^{(A)}[4, 2], E_\alpha^{(A)}[4, 3], E_\alpha^{(A)}[4, 4], E_\alpha^{(A)}[4, 5], E_\alpha^{(A)}[4, 6], E_\alpha^{(A)}[4, 7]\}, \\ \{E_\alpha^{(A)}[5, 0], E_\alpha^{(A)}[5, 1], E_\alpha^{(A)}[5, 2], E_\alpha^{(A)}[5, 3], E_\alpha^{(A)}[5, 4], E_\alpha^{(A)}[5, 5], \\ E_\alpha^{(A)}[5, 6], E_\alpha^{(A)}[5, 7]\}, \{E_\alpha^{(A)}[6, 0], E_\alpha^{(A)}[6, 1], E_\alpha^{(A)}[6, 2], E_\alpha^{(A)}[6, 3], \\ E_\alpha^{(A)}[6, 4], E_\alpha^{(A)}[6, 5], E_\alpha^{(A)}[6, 6], E_\alpha^{(A)}[6, 7]\}, \{E_\alpha^{(A)}[7, 0], E_\alpha^{(A)}[7, 1], \\ E_\alpha^{(A)}[7, 2], E_\alpha^{(A)}[7, 3], E_\alpha^{(A)}[7, 4], E_\alpha^{(A)}[7, 5], E_\alpha^{(A)}[7, 6], E_\alpha^{(A)}[7, 7]\} \end{array} \right\}$$

In[277]:=

```
Block[{s, r}, s = FullSimplify[e_\alpha^{(A)} /. sgtrye_\alpha^{(A)}, constraintVars];
r = FullSimplify[Transpose[s].\eta4488.s, constraintVars] - MatrixMetric44;
FullSimplify[r, constraintVars]]
```

Out[277]=

$$\left\{ \begin{array}{l} \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \end{array} \right.$$

In[278]:=

```
(* (e_\alpha^{(A)}=eAa) //MatrixForm*)
```

In[279]:=

```
(*e_\alpha^{(A)}
Inverse[%]*)
```

In[280]:=

```
(* (e_\alpha^{(A)}=Inverse[
e_\alpha^{(A)} /. sge_Subsuperscript_\alpha_and_LeftParenthesis_A_RightParenthesis]) //
MatrixForm*)
```

```
In[281]:= eα(A)
Out[281]= { {Eα(A) [0, 0], Eα(A) [0, 1], Eα(A) [0, 2], Eα(A) [0, 3], Eα(A) [0, 4], Eα(A) [0, 5], Eα(A) [0, 6], Eα(A) [0, 7]}, {Eα(A) [1, 0], Eα(A) [1, 1], Eα(A) [1, 2], Eα(A) [1, 3], Eα(A) [1, 4], Eα(A) [1, 5], Eα(A) [1, 6], Eα(A) [1, 7]}, {Eα(A) [2, 0], Eα(A) [2, 1], Eα(A) [2, 2], Eα(A) [2, 3], Eα(A) [2, 4], Eα(A) [2, 5], Eα(A) [2, 6], Eα(A) [2, 7]}, {Eα(A) [3, 0], Eα(A) [3, 1], Eα(A) [3, 2], Eα(A) [3, 3], Eα(A) [3, 4], Eα(A) [3, 5], Eα(A) [3, 6], Eα(A) [3, 7]}, {Eα(A) [4, 0], Eα(A) [4, 1], Eα(A) [4, 2], Eα(A) [4, 3], Eα(A) [4, 4], Eα(A) [4, 5], Eα(A) [4, 6], Eα(A) [4, 7]}, {Eα(A) [5, 0], Eα(A) [5, 1], Eα(A) [5, 2], Eα(A) [5, 3], Eα(A) [5, 4], Eα(A) [5, 5], Eα(A) [5, 6], Eα(A) [5, 7]}, {Eα(A) [6, 0], Eα(A) [6, 1], Eα(A) [6, 2], Eα(A) [6, 3], Eα(A) [6, 4], Eα(A) [6, 5], Eα(A) [6, 6], Eα(A) [6, 7]}, {Eα(A) [7, 0], Eα(A) [7, 1], Eα(A) [7, 2], Eα(A) [7, 3], Eα(A) [7, 4], Eα(A) [7, 5], Eα(A) [7, 6], Eα(A) [7, 7]} }
```

In[282]:=

$$(\text{sge}_{(A)}^{\alpha} = \text{Thread}[\text{Flatten}[e_{(A)}^{\alpha}] \rightarrow \text{Flatten}[\text{Inverse}[e_{\alpha}^{(A)} / . \text{sge}_{\alpha}^{(A)}]]]) (*//Column*)$$

Out[282]=

$$\left\{ \begin{array}{l} E_{(A)}^{\alpha} [0, 0] \rightarrow \frac{1}{\sqrt{g[0][0][x0, x4]}}, E_{(A)}^{\alpha} [0, 1] \rightarrow 0, E_{(A)}^{\alpha} [0, 2] \rightarrow 0, \\ E_{(A)}^{\alpha} [0, 3] \rightarrow 0, E_{(A)}^{\alpha} [0, 4] \rightarrow 0, E_{(A)}^{\alpha} [0, 5] \rightarrow 0, E_{(A)}^{\alpha} [0, 6] \rightarrow 0, E_{(A)}^{\alpha} [0, 7] \rightarrow 0, \\ E_{(A)}^{\alpha} [1, 0] \rightarrow 0, E_{(A)}^{\alpha} [1, 1] \rightarrow \frac{1}{\sqrt{g[1][1][x0, x4]}}, E_{(A)}^{\alpha} [1, 2] \rightarrow 0, E_{(A)}^{\alpha} [1, 3] \rightarrow 0, \\ E_{(A)}^{\alpha} [1, 4] \rightarrow 0, E_{(A)}^{\alpha} [1, 5] \rightarrow 0, E_{(A)}^{\alpha} [1, 6] \rightarrow 0, E_{(A)}^{\alpha} [1, 7] \rightarrow 0, E_{(A)}^{\alpha} [2, 0] \rightarrow 0, \\ E_{(A)}^{\alpha} [2, 1] \rightarrow 0, E_{(A)}^{\alpha} [2, 2] \rightarrow \frac{1}{\sqrt{g[2][2][x0, x4]}}, E_{(A)}^{\alpha} [2, 3] \rightarrow 0, E_{(A)}^{\alpha} [2, 4] \rightarrow 0, \\ E_{(A)}^{\alpha} [2, 5] \rightarrow 0, E_{(A)}^{\alpha} [2, 6] \rightarrow 0, E_{(A)}^{\alpha} [2, 7] \rightarrow 0, E_{(A)}^{\alpha} [3, 0] \rightarrow 0, E_{(A)}^{\alpha} [3, 1] \rightarrow 0, \\ E_{(A)}^{\alpha} [3, 2] \rightarrow 0, E_{(A)}^{\alpha} [3, 3] \rightarrow \frac{1}{\sqrt{g[3][3][x0, x4]}}, E_{(A)}^{\alpha} [3, 4] \rightarrow 0, E_{(A)}^{\alpha} [3, 5] \rightarrow 0, \\ E_{(A)}^{\alpha} [3, 6] \rightarrow 0, E_{(A)}^{\alpha} [3, 7] \rightarrow 0, E_{(A)}^{\alpha} [4, 0] \rightarrow 0, E_{(A)}^{\alpha} [4, 1] \rightarrow 0, E_{(A)}^{\alpha} [4, 2] \rightarrow 0, \\ E_{(A)}^{\alpha} [4, 3] \rightarrow 0, E_{(A)}^{\alpha} [4, 4] \rightarrow \frac{1}{\sqrt{-g[4][4][x0, x4]}}, E_{(A)}^{\alpha} [4, 5] \rightarrow 0, \\ E_{(A)}^{\alpha} [4, 6] \rightarrow 0, E_{(A)}^{\alpha} [4, 7] \rightarrow 0, E_{(A)}^{\alpha} [5, 0] \rightarrow 0, E_{(A)}^{\alpha} [5, 1] \rightarrow 0, E_{(A)}^{\alpha} [5, 2] \rightarrow 0, \\ E_{(A)}^{\alpha} [5, 3] \rightarrow 0, E_{(A)}^{\alpha} [5, 4] \rightarrow 0, E_{(A)}^{\alpha} [5, 5] \rightarrow \frac{1}{\sqrt{-g[5][5][x0, x4]}}, \\ E_{(A)}^{\alpha} [5, 6] \rightarrow 0, E_{(A)}^{\alpha} [5, 7] \rightarrow 0, E_{(A)}^{\alpha} [6, 0] \rightarrow 0, E_{(A)}^{\alpha} [6, 1] \rightarrow 0, E_{(A)}^{\alpha} [6, 2] \rightarrow 0, \\ E_{(A)}^{\alpha} [6, 3] \rightarrow 0, E_{(A)}^{\alpha} [6, 4] \rightarrow 0, E_{(A)}^{\alpha} [6, 5] \rightarrow 0, E_{(A)}^{\alpha} [6, 6] \rightarrow \frac{1}{\sqrt{-g[6][6][x0, x4]}}, \\ E_{(A)}^{\alpha} [6, 7] \rightarrow 0, E_{(A)}^{\alpha} [7, 0] \rightarrow 0, E_{(A)}^{\alpha} [7, 1] \rightarrow 0, E_{(A)}^{\alpha} [7, 2] \rightarrow 0, E_{(A)}^{\alpha} [7, 3] \rightarrow 0, \\ E_{(A)}^{\alpha} [7, 4] \rightarrow 0, E_{(A)}^{\alpha} [7, 5] \rightarrow 0, E_{(A)}^{\alpha} [7, 6] \rightarrow 0, E_{(A)}^{\alpha} [7, 7] \rightarrow \frac{1}{\sqrt{-g[7][7][x0, x4]}} \end{array} \right\}$$

In[283]:=

$$(*\text{Symbolize}[\text{sge}_{(A)}^{\alpha}]*)$$

In[284]:=

$$(*\text{Symbolize}[\text{sgtrye}_{(A)}^{\alpha}]*)$$

In[285]:=

$$(*%/. \text{sge}_{\alpha}^{(A)} \\ %%/. \text{sgtrye}_{\alpha}^{(A)} *)$$

In[286]:=

$$(*\text{FullSimplify}[\text{Transpose}[e_{\alpha}^{(A)}].\eta4488.e_{\alpha}^{(A)}, \text{constraintVars}] - \text{MatrixMetric44}*)$$

For Spin (4, 4); τ tau; T16; OCTAD : Nash; Introduce the wave function, Ψ_{16} , for this Universe:

O(4,4); values, evecs of σ

In[287]:=

```
{values, evecs} = Eigensystem[\sigma]
```

Out[287]=

```
{ {-1, -1, -1, -1, 1, 1, 1, 1}, {0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
{0, -1, 0, 0, 1, 0, 0}, {-1, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
{0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0} }}
```

In[288]:=

$$\left(u = \text{ExpandAll} \left[\frac{1}{\sqrt{2}} \text{evecs} \right] \right) // \text{MatrixForm}$$

Out[288]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

In[289]:=

```
Table[u[[h]].\sigma.u[[h]], {h, 1, Length[u]}]
```

Out[289]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[290]:=

```
Table[(Transpose[u][[h]]).\sigma.u[[h]], {h, 1, Length[u]}]
```

Out[290]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[291]:=

```
Table[(Transpose[u][[A]]) == -\sigma.u[[A]], {A, 1, 4}]
```

```
Table[(Transpose[u][[A]]) == \sigma.u[[A]], {A, 5, Length[u]}]
```

Out[291]=

```
{True, True, True, True}
```

Out[292]=

```
{True, True, True, True}
```

$$\begin{aligned} Ax &= \left(\frac{\text{KroneckerProduct}[dX, X] - \text{KroneckerProduct}[X, dX]}{2} \right) . \sigma; \\ Au &= \left(\frac{\text{KroneckerProduct}[dU, U] - \text{KroneckerProduct}[U, dU]}{2} \right) . \sigma; \end{aligned}$$

Lsquared =

$$\dot{x}^A \dot{x}_A = dX.\sigma.dX + X.\sigma.X \left(\frac{dU.\sigma.dU}{U.\sigma.U} - \left(\frac{U.\sigma.dU}{U.\sigma.U} \right)^2 \right) + \frac{2}{U.\sigma.U} \operatorname{Tr}[\bar{\tau}^A.Ax.\tau_A.Au]$$

```
Timing[Simplify[ExpandAll[dY.η8.dY - Lsquared]]]
```

{10.733, 0}

In[293]:=

```
FullSimplify[Sqrt[Flatten[MatrixMetric44.η4488]], constraintVars]
```

Out[293]=

$$\left\{ \text{Cot}[6 H x_0], 0, 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{2 a4[H x 4]}} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, \right. \\ \left. \sqrt{e^{2 a4[H x 4]}} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{2 a4[H x 4]}} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0, \right. \\ \left. 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x 4]}} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, \right. \\ \left. 0, 0, \sqrt{e^{-2 a4[H x 4]}} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x 4]}} \sin[6 H x_0]^{1/6} \right\}$$

$$SO(4), \quad \gamma; M8, \{0, 1, 2, 3, 4, 5, 6, 7\}, \{+, +, +, -, -, -, -\}, \{x0, x1, x2, x3, x4, x5, x6, x7\}$$

In[294]:=

Under the involutive automorphism $\gamma^{AB} \rightarrow -\tilde{\gamma}^{AB}$ of $so(3,3)$, the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra $so(4)$, which is the Lie algebra of $\overline{SO(4)}$, the maximal compact subgroup of $\overline{SL(4,\mathbb{R})}$. The subalgebra $so(4) \cong su(2) + su(2)$ may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of $so(4)$ whereby $\tau \in so(4)$ is mapped into its dual, ${}^*\tau$. The even subspace under $*$ of $so(4)$ corresponds to self-dual tensors, and, say, the first $su(2)$ in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second $su(2)$ in the direct sum. A basis for $so(4)$ may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by ($h = 1, 2, 3$),

$$2s^h = (\gamma^{23}, \gamma^{31}, \gamma^{21}), \quad (4)$$

and

$$2t^h = (\gamma^{45}, \gamma^{64}, \gamma^{65}). \quad (5)$$

Out[294]=

Under the involutive automorphism $\gamma^{AB} \rightarrow -\tilde{\gamma}^{AB}$ of $\text{so}(3,3)$, the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra $\text{so}(4)$, which is the Lie algebra of $\overline{\text{SO}(4)}$, the maximal compact subgroup of $\overline{\text{SL}(4,\mathbb{R})}$. The subalgebra $\text{so}(4) \cong \text{su}(2) + \text{su}(2)$ may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of $\text{so}(4)$ whereby $\tau \in \text{so}(4)$ is mapped into its dual, ${}^*\tau$. The even subspace under * of $\text{so}(4)$ corresponds to self-dual tensors, and, say, the first $\text{su}(2)$ in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second $\text{su}(2)$ in the direct sum. A basis for $\text{so}(4)$ may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by ($h = 1, 2, 3$),

$$2s^h = (\gamma^{23}, \gamma^{31}, \gamma^{21}), \quad (4)$$

and

$$2t^h = (\gamma^{45}, \gamma^{64}, \gamma^{65}). \quad (5)$$

In[295]:=

```
Qa[h_, p_, q_] := Signature[{h, p, q, 4}];
Qb[h_, p_, q_] := ID4[p, 4] * ID4[q, h] - ID4[p, h] * ID4[q, 4];
SelfDualAntiSymmetric[h_, p_, q_] := Qa[h, p, q] - Qb[h, p, q];
AntiSelfDualAntiSymmetric[h_, p_, q_] := (Qa[h, p, q] + Qb[h, p, q]);
```

In[299]:=

```
Qa[1, 2, 3]
```

Out[299]=

```
1
```

In[300]:=

```
 $\epsilon 4[1, 2, 3, 4]$ 
```

Out[300]=

```
1
```

In[301]:=

```
Do[s4by4[h] = Table[Table[SelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
{h, 1, 3}]
```

```

In[302]:= 
Do[
  t4by4[h] = Table[Table[AntiSelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
  {h, 1, 3}]

In[303]:= (*Protect[{s4by4[1],s4by4[2],s4by4[3],t4by4[1],t4by4[2],t4by4[3]}]*)

In[304]:= Protect[s4by4, t4by4]
Out[304]= {s4by4, t4by4}

In[305]:= {s4by4[#] // MatrixForm, MatrixForm[t4by4[#]]} & /@ Range[3]
Out[305]=
{ {{ 0 0 0 1
   0 0 1 0
   0 -1 0 0
  -1 0 0 0 }, { 0 0 0 -1
   0 0 1 0
   0 -1 0 0
   1 0 0 0 } },
{ {{ 0 0 -1 0
   0 0 0 1
   1 0 0 0
  0 -1 0 0 }, { 0 0 -1 0
   0 0 0 -1
   1 0 0 0
   0 1 0 0 } },
{ {{ 0 1 0 0
   -1 0 0 0
   0 0 0 1
   0 0 1 0 }, { 0 1 0 0
   -1 0 0 0
   0 0 0 1
   0 0 -1 0 } },
{ {{ 0 0 1 0
   0 0 0 -1
   1 0 0 0
   0 1 0 0 }, { 0 0 0 1
   0 0 1 0
   0 1 0 0
   -1 0 0 0 } } }

In[306]:= Table[{{J, K}, MatrixForm[s4by4[J].t4by4[K]]}, {J, 1, 3}, {K, 1, 3}]
Out[306]=
{ {{ {1, 1}, { 1 0 0 0
   0 -1 0 0
   0 0 -1 0
   0 0 0 1 } },
{ {1, 2}, { 0 1 0 0
   1 0 0 0
   0 0 0 -1
   0 0 -1 0 } },
{ {1, 3}, { 0 0 1 0
   0 0 0 -1
   1 0 0 0
   0 -1 0 0 } } },
{ {{ {2, 1}, { 0 1 0 0
   1 0 0 0
   0 0 0 -1
   0 0 -1 0 } },
{ {2, 2}, { -1 0 0 0
   0 1 0 0
   0 0 -1 0
   0 0 0 1 } },
{ {2, 3}, { 0 0 0 1
   0 0 1 0
   0 1 0 0
   1 0 0 0 } } },
{ {{ {3, 1}, { 0 0 1 0
   0 0 0 1
   1 0 0 0
   0 1 0 0 } },
{ {3, 2}, { 0 0 0 -1
   0 0 1 0
   0 1 0 0
  -1 0 0 0 } },
{ {3, 3}, { -1 0 0 0
   0 -1 0 0
   0 0 1 0
   0 0 0 1 } } } }

```

```
In[308]:= Table[{{J, K}, MatrixForm[st[J, K]]}, {J, 1, 3}, {K, 1, 3}]
Out[308]=
{{{{1, 1}, {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}}, {{1, 2}, {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}}, {{1, 3}, {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}}}}, {{{2, 1}, {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}}}, {{2, 2}, {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}}, {{2, 3}, {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}}}}, {{{3, 1}, {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}, {{3, 2}, {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}}, {{3, 3}, {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}}}}
```

```
In[309]:= Protect[st]
Out[309]= {st}
```

```
In[310]:= Table[{{J, K}, MatrixForm[st[J, K] - Transpose[st[J, K]]]}, {J, 1, 3}, {K, 1, 3}]
Out[310]=
{{{{1, 1}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{1, 2}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{1, 3}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}, {{{2, 1}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{2, 2}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{2, 3}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}, {{{3, 1}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{3, 2}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{3, 3}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}}}
```

```
In[311]:= Permutations[Range[3], 2]
Out[311]= {{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}
```

```
In[312]:= KroneckerProduct[Range[3], Range[3]]
Out[312]= {{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}
```

```

In[313]:= Table[{J, K}, {J, 1, 3}, {K, 1, 3}]
st[##] & /@%
st[##] & @@%%
Out[313]= {{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}}

Out[314]= {st[{{1, 1}, {1, 2}, {1, 3}}], st[{{2, 1}, {2, 2}, {2, 3}}], st[{{3, 1}, {3, 2}, {3, 3}}]}

Out[315]= st[{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}]

In[316]:= Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}], 1]
Length[%]
Out[316]= {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
  {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
  {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
  {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
  {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
  {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
  {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
  {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
  {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
  {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
  {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
  {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
  {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
  {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
  {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
  {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

```

Out[317]=

16

```
In[318]:= Unprotect[γ];
γ = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
Out[319]= {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
{{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
{{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
{{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

In[320]:= Length[γ]
Out[320]= 16

In[321]:= Protect[γ]
Out[321]= {γ}

In[322]:= γ[[1]]
Out[322]= {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}

In[323]:= γ[[-1]]
Out[323]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Self Dual Anti-Symmetric

```
In[324]:= Table[ $\left(\frac{1}{2} \sum_{j_1=1}^4 \left(\sum_{j_2=1}^4 \epsilon 4[p, q, j_1, j_2] \times s4by4[h][j_1, j_2]\right)\right) - s4by4[h][p, q],$   

 $\{h, 1, 3\}, \{q, 4\}, \{p, 4\}\]$ 
```

Out[324]=
 $\{\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\}$

Anti SelfDual Anti-Symmetric

```
In[325]:= Table[ $\left(\frac{1}{2} \sum_{j_1=1}^4 \left(\sum_{j_2=1}^4 \epsilon 4[p, q, j_1, j_2] \times t4by4[h][j_1, j_2]\right)\right) + t4by4[h][p, q],$   

 $\{h, 1, 3\}, \{q, 4\}, \{p, 4\}\]$ 
```

Out[325]=
 $\{\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\}$

$SO(4,4), Spin(4,4) = \overline{SO(4,4)}, \tau$; if using xact: M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+, +, +, -, -, -}, {x0, x1, x2, x3, x4, x5, x6, x7}

```
In[326]:=  $\overline{\tau} == \overline{\tau}$ 
```

Out[326]= False

```
In[327]:=  $\overline{\tau}$ 
```

Head[%]

Out[327]= $\overline{\tau}$

Out[328]= Symbol

```
In[329]:=  $\overline{\tau}$ 
```

Head[%]

Out[329]= $\overline{\tau}$

Out[330]= ParsedBoxWrapper

```
In[331]:=  $\overline{\tau}$ 
Head[%]
Out[331]=  $\overline{\tau}$ 

Out[332]= Symbol

In[333]:= Symbol
Out[333]= Symbol

In[334]:=  $\overline{\tau} === \overline{\tau}$ 
Out[334]= True

In[335]:=  $\overline{\tau} === \overline{\tau}$ 
Out[335]= False

In[336]:=  $\eta4488 // \text{MatrixForm}$ 
Out[336]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


In[337]:=  $\eta4488 = \text{DiagonalMatrix}[\text{RotateRight}[\text{values}, 4]]$ 
Out[337]= True

In[338]:=  $\text{DiagonalMatrix}[\text{RotateRight}[\text{values}, 3]] // \text{MatrixForm}$ 
Out[338]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```

In[339]:=

```

 $\tau[0] = \text{ID8};$ 
Table[
   $\tau[7-h] = \text{ArrayFlatten}[\{\{0, t4by4[h]\}, \{-t4by4[h], 0\}\}], \{h, 1, 3\}];$ 
Table[ $\tau[h] = \text{ArrayFlatten}[\{\{0, s4by4[h]\}, \{s4by4[h], 0\}\}], \{h, 1, 3\}];$ 
( $\tau[7] = \tau[1].\tau[2].\tau[3].\tau[4].\tau[5].\tau[6]\}) // \text{MatrixForm}$ 

```

Out[342]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[343]:=

 $\sigma = \tau[1].\tau[2].\tau[3]$

Out[343]=

True

In[344]:=

```

Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\tau[A].\tau[B] + \tau[B].\tau[A]) = -\eta 4488 [A+1, B+1] * \text{ID8}] ]}],
{A, 1, 7}, {B, 1, 7}]$ 
```

Out[344]=

```

{{{1, 1, True}, {1, 2, True}, {1, 3, True},
  {1, 4, True}, {1, 5, True}, {1, 6, True}, {1, 7, True}},
 {{2, 1, True}, {2, 2, True}, {2, 3, True}, {2, 4, True}, {2, 5, True},
  {2, 6, True}, {2, 7, True}}, {{3, 1, True}, {3, 2, True},
  {3, 3, True}, {3, 4, True}, {3, 5, True}, {3, 6, True}, {3, 7, True}},
 {{4, 1, True}, {4, 2, True}, {4, 3, True}, {4, 4, True}, {4, 5, True},
  {4, 6, True}, {4, 7, True}}, {{5, 1, True}, {5, 2, True},
  {5, 3, True}, {5, 4, True}, {5, 5, True}, {5, 6, True}, {5, 7, True}},
 {{6, 1, True}, {6, 2, True}, {6, 3, True}, {6, 4, True}, {6, 5, True},
  {6, 6, True}, {6, 7, True}}, {{7, 1, True}, {7, 2, True},
  {7, 3, True}, {7, 4, True}, {7, 5, True}, {7, 6, True}, {7, 7, True}}}

```

In[345]:=

 $\eta4488 // \text{MatrixForm}$

Out[345]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[346]:=

Table[{A, - $\eta4488[A + 1, A + 1] * \tau[A]$ == Transpose[\tau[A]]}, {A, 1, 7}]

Out[346]=

{ {1, True}, {2, True}, {3, True}, {4, True}, {5, True}, {6, True}, {7, True} }

In[347]:=

 $(\sigma == \tau[1].\tau[2].\tau[3])$

Out[347]=

True

In[348]:=

 $\sigma.\tau[1].\tau[2].\tau[3] // \text{MatrixForm}$

Out[348]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[349]:=

```
(* $\tau[0]$ =ID8;
Do[{\tau[A] = FullSimplify[ExpandAll[(\tau[A])]]}, {A, 1, 7}]*)
(*Do[{\tau[A] = FullSimplify[ExpandAll[-(\sigma.\tau[1].\tau[2].\tau[3].\tau[A])]]}, {A, 1, 7}]*)
```

In[350]:=

 $(\sigma == \tau[1].\tau[2].\tau[3])$

Out[350]=

True

In[351]:=

(*Symbolize[\tau]*)

In[352]:=

```
 $\tau[0] = \text{ID8};$ 
Do[{ $\tau[A] = \text{FullSimplify}[\text{ExpandAll}[(\sigma.\text{Transpose}[\tau[A]).\sigma])]$ } , {A, 1, 7}]
```

In[354]:=

```
 $\sigma.\tau[\#] = \text{Transpose}[\sigma.\tau[\#]] \& /@ \text{Range}[0, 7]$ 
```

Out[354]=

```
{True, True, True, True, True, True, True, True}
```

In[355]:=

```
Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\tau[A].\tau[B] + \tau[B].\tau[A]) = \eta 4488 [A+1, B+1] * \text{ID8}$ ]}],
  {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

Out[355]//MatrixForm=

$$\begin{array}{cccccccc} \left(\begin{array}{c} 0 \\ 0 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 0 \\ 1 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 0 \\ 2 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 0 \\ 3 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 0 \\ 4 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 0 \\ 5 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 0 \\ 6 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 0 \\ 7 \\ \text{True} \end{array} \right) \\ \left(\begin{array}{c} 1 \\ 0 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 1 \\ 1 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 1 \\ 2 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 1 \\ 3 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 1 \\ 4 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 1 \\ 5 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 1 \\ 6 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 1 \\ 7 \\ \text{True} \end{array} \right) \\ \left(\begin{array}{c} 2 \\ 0 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 2 \\ 1 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 2 \\ 2 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 2 \\ 3 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 2 \\ 4 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 2 \\ 5 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 2 \\ 6 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 2 \\ 7 \\ \text{True} \end{array} \right) \\ \left(\begin{array}{c} 3 \\ 0 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 3 \\ 1 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 3 \\ 2 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 3 \\ 3 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 3 \\ 4 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 3 \\ 5 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 3 \\ 6 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 3 \\ 7 \\ \text{True} \end{array} \right) \\ \left(\begin{array}{c} 4 \\ 0 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 4 \\ 1 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 4 \\ 2 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 4 \\ 3 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 4 \\ 4 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 4 \\ 5 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 4 \\ 6 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 4 \\ 7 \\ \text{True} \end{array} \right) \\ \left(\begin{array}{c} 5 \\ 0 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 5 \\ 1 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 5 \\ 2 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 5 \\ 3 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 5 \\ 4 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 5 \\ 5 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 5 \\ 6 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 5 \\ 7 \\ \text{True} \end{array} \right) \\ \left(\begin{array}{c} 6 \\ 0 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 6 \\ 1 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 6 \\ 2 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 6 \\ 3 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 6 \\ 4 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 6 \\ 5 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 6 \\ 6 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 6 \\ 7 \\ \text{True} \end{array} \right) \\ \left(\begin{array}{c} 7 \\ 0 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 7 \\ 1 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 7 \\ 2 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 7 \\ 3 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 7 \\ 4 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 7 \\ 5 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 7 \\ 6 \\ \text{True} \end{array} \right) & \left(\begin{array}{c} 7 \\ 7 \\ \text{True} \end{array} \right) \end{array}$$

In[356]:=

 $\tau[7] // \text{MatrixForm}$

Out[356]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In[357]:=

 $(\Omega = \sigma \cdot \tau[7]) // \text{MatrixForm}$

Out[357]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[358]:=

 $\Omega == \tau[4] \cdot \tau[5] \cdot \tau[6]$

Out[358]=

True

In[359]:=

 $\tau[5] \cdot \tau[6] \cdot \tau[7] // \text{MatrixForm}$

Out[359]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

In[360]:=

 $(\sigma\Omega = \sigma \cdot \Omega) // \text{MatrixForm}$

Out[360]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

In[361]:= **Inverse**[$\sigma\Omega$] // MatrixForm

Out[361]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[362]:= $\tau[1].\tau[2].\tau[3].\tau[4].\tau[5].\tau[6].\tau[7] == \tau[0] == \text{ID8}$

Out[362]= True

In[363]:=

($\sigma == \tau[1].\tau[2].\tau[3]$)

Out[363]= True

In[364]:=

($\sigma == \tau[4].\tau[5].\tau[6].\tau[7]$)

Out[364]= True

In[365]:=

$\tau[1].\tau[2].\tau[3].\tau[\#] == -\text{Transpose}[\tau[1].\tau[2].\tau[3].\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[365]= {False, True, True, True, True, True, True, True}

In[366]:= $\sigma.\bar{\tau}[\#] == \text{Transpose}[\sigma.\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[366]= {True, True, True, True, True, True, True, True}

In[367]:= $\tau[5].\tau[6].\tau[7].\bar{\tau}[\#] == \text{Transpose}[\tau[5].\tau[6].\tau[7].\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[367]= {False, True, True, True, False, True, True, True}

In[368]:=

$\tau[4].\tau[\#] == \text{Transpose}[\tau[4].\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[368]= {True, True, True, True, True, False, False, False}

```
In[369]:=  $\tau[4].\bar{\tau}[\#] == \text{Transpose}[\tau[4].\tau[\#]] \& /@ \text{Range}[0, 7]$ 
Out[369]= {True, False, False, False, True, True, True}
```

```
In[370]:=  $\tau[4].\tau[\#] == \text{Transpose}[\tau[4].\tau[\#]] \& /@ \text{Range}[0, 7]$ 
Out[370]= {True, True, True, True, True, False, False}
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}. \quad ; \quad 0(4, 4) : \text{SAB} ;$$

$$\begin{aligned} \text{covariantDiffMatrix} &= T16^A[5].T16^A[6].T16^A[7] \\ S &= \begin{pmatrix} a & b \\ c & d \end{pmatrix}; \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix} \cdot \begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix} &= \begin{pmatrix} b.\tau^A & a.\bar{\tau}^A \\ d.\tau^A & c.\bar{\tau}^A \end{pmatrix} \cdot \begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix} \\ &= \begin{pmatrix} b.\tau^A.\tilde{a} + a.\bar{\tau}^A.\tilde{b} & b.\tau^A.\tilde{c} + a.\bar{\tau}^A.\tilde{d} \\ d.\tau^A.\tilde{a} + c.\bar{\tau}^A.\tilde{b} & d.\tau^A.\tilde{c} + c.\bar{\tau}^A.\tilde{d} \end{pmatrix} \end{aligned}$$

Need these in order to define the Lagrangian for the "universe," later :

```
In[371]:=  $(\sigma16 = T16^A[0].T16^A[1].T16^A[2].T16^A[3]) // \text{MatrixForm}$ 
Out[371]//MatrixForm=
 $T16^A[0].T16^A[1].T16^A[2].T16^A[3]$ 
```

```
In[372]:=  $\text{Table}[T16^A[A1] = \text{ArrayFlatten}[\{\{0, \bar{\tau}[A1]\}, \{\tau[A1], 0\}\}], \{A1, 0, 7\}]$ 
```

In[373]:=

```
(T16A[8] = T16A[0].T16A[1].T16A[2].T16A[3].T16A[4].T16A[5].T16A[6].T16A[7]) //  
MatrixForm
```

Out[373]//MatrixForm=

In[374]:=

$\sigma16 // \text{MatrixForm}$

Out[374]//MatrixForm=

In[375]:=

```
T16A[0].T16A[1].T16A[2].T16A[3].T16A[#] ==
-Transpose[T16A[0].T16A[1].T16A[2].T16A[3].T16A[#]] & /@ Range[0, 8]
```

Out[375]=

{True, True, True, True, True, True, True, True, False}

In[382]:=

```
σ16.covariantDiffMatrix = Transpose[σ16.covariantDiffMatrix]
```

Out[382]=

True

In[383]:=

ID16 // MatrixForm

Out[383]//MatrixForm=

In[384]:=

```
(σ16(*=T16A[0].T16A[1].T16A[2].T16A[3]*)) // MatrixForm
```

Out[384]//MatrixForm=

In[385]:=

```
 $\sigma_{16} = \text{ArrayFlatten}[\{\{-\sigma, 0\}, \{0, \sigma\}\}]$ 
```

Out[385]=

True

later, after defining $e_{(A)}^\alpha$:

In[386]:=

$$(*\text{Table}[\text{T16}^\alpha[\alpha 1-1]=\text{Sum}[(\text{e}_{(A)}^\alpha[[\alpha 1,A1]] \text{ T16}^A[A1-1],\{A1,1,8\}],\{\alpha 1,1,8\}];*)$$

In[387]:=

$$(*(\text{T16}^\alpha[8]=\text{T16}^\alpha[0].\text{T16}^\alpha[1].\text{T16}^\alpha[2].\text{T16}^\alpha[3].\text{T16}^\alpha[4].\text{T16}^\alpha[5].\text{T16}^\alpha[6].\text{T16}^\alpha[7])//\text{MatrixForm}*)$$

In[388]:=

$$(*\text{eLpairs}*)$$

In[389]:=

$$(*\Omega 16a=\text{Array}[0\&,\{16,16\}];\\ \{1+\#[[1]],1+\#[[2]]\}\&/@eLpairs\\ (\Omega 16a[[\#[[1]],\#[[2]]]]=1)\&/@%*)$$

In[390]:=

$$(*\Omega 16a//\text{MatrixForm}*)$$

In[391]:=

$$(*(\Omega 16=\sigma 16.\text{ (}\Omega 16a+\text{Transpose}[\Omega 16a]\text{)})//\text{MatrixForm}*)$$

In[392]:=

$$\begin{aligned} &\text{T16}^A[0].\text{T16}^A[4] // \text{MatrixForm} \\ &\text{T16}^A[0].\text{T16}^A[4].\sigma 16 \\ &(\Omega 16 = %) // \text{MatrixForm} \end{aligned}$$

Out[392]//MatrixForm=

$$\left(\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Out[393]=

Out[394]//MatrixForm=

In[395]:=

Det[Ω16]

Out[395]=

1

In[396]:=

$\sigma16.\Omega16 // \text{MatrixForm}$

Out[396]//MatrixForm=

0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0

In[397]:=

```
 $\sigma16.T16^A[\#] == -\text{Transpose}[\sigma16.T16^A[\#]] \& /@ \text{Range}[0, 7]$ 
```

Out[397]=

{True, True, True, True, True, True, True, True}

In[398]:=

```
σ16.T16A[#] == -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

Out[398]=

{True, True, True, True, True, True, True, True}

In[399]:=

$\sigma16.T16^A[\#] == -\text{Transpose}[\sigma16.T16^A[\#]] \& /@ \text{Range}[0, 7]$

Out[399]=

{True, True, True, True, True, True, True, True}

In[400]:=

```
 $\sigma16.\text{newT16}^A[\#] == -\text{Transpose}[\sigma16.\text{newT16}^A[\#]] \& /@ \text{Range}[0, 7]$ 
```

Out[400]=

In[401]:=

```
newσ16.newT16A[#] == -Transpose[newσ16.newT16A[#]] & /@ Range[0, 7]
```

Out[401]=

$$\left\{ \begin{aligned} \text{new}\sigma16.\text{newT16}^A[0] &= -(\text{new}\sigma16.\text{newT16}^A[0])^T, & \text{new}\sigma16.\text{newT16}^A[1] &= -(\text{new}\sigma16.\text{newT16}^A[1])^T, \\ \text{new}\sigma16.\text{newT16}^A[2] &= -(\text{new}\sigma16.\text{newT16}^A[2])^T, \\ \text{new}\sigma16.\text{newT16}^A[3] &= -(\text{new}\sigma16.\text{newT16}^A[3])^T, \\ \text{new}\sigma16.\text{newT16}^A[4] &= -(\text{new}\sigma16.\text{newT16}^A[4])^T, \\ \text{new}\sigma16.\text{newT16}^A[5] &= -(\text{new}\sigma16.\text{newT16}^A[5])^T, \\ \text{new}\sigma16.\text{newT16}^A[6] &= -(\text{new}\sigma16.\text{newT16}^A[6])^T, \\ \text{new}\sigma16.\text{newT16}^A[7] &= -(\text{new}\sigma16.\text{newT16}^A[7])^T \end{aligned} \right\}$$

In[402]:=

```
Table[{A1, B1},
  FullSimplify[ExpandAll[ $\frac{1}{2} (\text{T16}^A[\text{A1}] \cdot \text{T16}^A[\text{B1}] + \text{T16}^A[\text{B1}] \cdot \text{T16}^A[\text{A1}]) =$ 
     $\eta 4488 [\text{A1} + 1, \text{B1} + 1] * \text{ID16}] ]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm$ 
```

Out[402]//MatrixForm=

$\begin{pmatrix} \{0, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{1, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{2, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{3, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{4, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{5, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{6, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{7, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 7\} \\ \text{True} \end{pmatrix}$

In[403]:=

```
Table[{A1, B1}, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\text{newT16}^A[\text{A1}] \cdot \text{newT16}^A[\text{B1}] + \text{newT16}^A[\text{B1}] \cdot \text{newT16}^A[\text{A1}]) =$ 
     $\eta 4488 [\text{A1} + 1, \text{B1} + 1] * \text{ID16}] ]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm$ 
```

Out[403]//MatrixForm=

(... 1 ...)

Size in memory: 0.5 MB [Show more](#) [Show all](#) [Iconize](#) [Store full expression in notebook](#) [Settings](#)

In[404]:=

```
 $\Psi16upper = \text{Take}[\Psi16, 8]$ 
 $\Psi16lower = \text{Take}[\Psi16, -8]$ 
 $\Psi16 = \text{Flatten}[\{\Psi16upper, \Psi16lower\}]$ 
```

Out[404]=

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4]}
```

Out[405]=

```
{f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

Out[406]=

```
True
```

$$t^A = \begin{pmatrix} 0 & \vec{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
Table[T16^A[A1] = ArrayFlatten[{{0, τ[A1]}, {τ[A1], 0}}], {A1, 0, 7}]
σ16 = ArrayFlatten[{{{-σ, 0}, {0, σ}}}]
(0  τ[A1]) (Ψ16upper) = (τ[A1].Ψ16lower)
τ[A1]  0
σ16.(0  τ[A1]) (Ψ16lower) = (-σ τ[A1].Ψ16lower)
τ[A1]  0
Ψ16.σ16.(0  τ[A1]).Ψ16 = (-Ψ16upper.σ τ[A1] .Ψ16lower)
Ψ16lower.σ τ[A1] Ψ16upper
```

In[407]:= (*σ16.T16^A[#]&/@Range[0,7]*)

In[408]:=

```
(*  
(T16^A[8]=FullSimplify[T16^A[0].T16^A[1].T16^A[2].  
T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]])//MatrixForm*)
```

In[409]:=

η4488

Out[409]=

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1} }
```

In[410]:=

η99 = DiagonalMatrix[{1, 1, 1, 1, -1, -1, -1, 1}]

Out[410]=

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 1} }
```

In[411]:=

```
(*Table[{A1,B1,FullSimplify[
ExpandAll[1/2 ( T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]) == η99[[A1,B1]]*ID16]]},{A1,0, 8},{B1,0, 8}]//MatrixForm*)
```

In[412]:=

```
(*Table[1/4 (T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]), {A1, 8}, {B1, 8}]*)
```

In[413]:=

$$\left(P_L = \frac{1}{2} (ID16 - T16^A[8]) \right) // MatrixForm$$

Out[413]//MatrixForm=

In[414]:=

$$\left(P_R = \frac{1}{2} (ID16 + T16^A[8]) \right) // MatrixForm$$

Out[414]//MatrixForm=

In[415]:=

{ID16 == P_L + P_B, P_L . P_L == P_L , P_L . P_B == P_B , P_L == ZERO16}

Out[415]=

{True, True, True}

```
SAB = Table[ $\frac{1}{4} (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1])$ , {A1, 0, 7}, {B1, 0, 7}];
```

In[416]:=

```
SAB = Table[ $\frac{1}{4} (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1])$ , {A1, 0, 7}, {B1, 0, 7}];
```

In[417]:=

```
ParallelTable[
  FullSimplify[SAB[[A1, B1]] == -Transpose[SAB[[A1, B1]]]],
  {A1, 1, 8}, {B1, 1, 8}] // Flatten // Union
```

Out[417]=

```
{True}
```

In[418]:=

```
ParallelTable[
  FullSimplify[SAB[[A1, B1]].SAB[[A2, B2]] - SAB[[A2, B2]].SAB[[A1, B1]] ==
  - (η4488[[A1, A2]] × SAB[[B1, B2]] - η4488[[A1, B2]] × SAB[[B1, A2]] -
  η4488[[B1, A2]] × SAB[[A1, B2]] + η4488[[B1, B2]] × SAB[[A1, A2]])], {A1, 1, 7},
  {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}] // Flatten // Union
```

Out[418]=

```
{True}
```

In[419]:=

```
ParallelTable[
  FullSimplify[SAB[[A1, B1]].T16^A[B2 - 1] - T16^A[B2 - 1].SAB[[A1, B1]] ==
  (-η4488[[B2, A1]] T16^A[B1 - 1] + η4488[[B2, B1]] T16^A[A1 - 1])],
  {A1, 1, 8}, {B1, 1, 8}, {B2, 1, 8}] // Flatten // Union
```

Out[419]=

```
{True}
```

In[420]:=

```
(*ParallelTable[
 {{A1, B1}, FullSimplify[SAB[[A1, B1]] == -Transpose[SAB[[A1, B1]]]]},
 {A1, 1, 8}, {B1, 1, 8}]*)
```

In[421]:=

```
(*ParallelTable[
 {{A1, B1, A2, B2}, FullSimplify[SAB[[A1, B1]].SAB[[A2, B2]] - SAB[[A2, B2]].SAB[[A1, B1]] ==
  - (η4488[[A1, A2]] SAB[[B1, B2]] - η4488[[A1, B2]] SAB[[B1, A2]] -
  η4488[[B1, A2]] SAB[[A1, B2]] + η4488[[B1, B2]] SAB[[A1, A2]])]}, {A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}]*)
```

In[422]:=

```
(*ParallelTable[
{{A1,B1,B2},FullSimplify[SAB[A1,B1].T16^A[B2-1]-T16^A[B2-1].SAB[A1,B1]==
(-η4488[B2,A1]T16^A[B1-1]+η4488[B2,B1]T16^A[A1-1])]},{{A1,1,8},{B1,1,8},{B2,1,8}]*)
```

In[423]:=

```
Do[sAB[A1][B1]=1/4 (T16^A[A1].T16^A[B1]-T16^A[B1].T16^A[A1]),
{A1,0,7},{B1,0,7}];
```

In[424]:=

```
Dimensions[ArrayFlatten[{{0, ID8}, {ID8, 0}}]]
```

Out[424]=

```
{16, 16}
```

In[425]:=

```
sAB[-1+1][-1+2]==SAB[1,2]
```

Out[425]=

```
True
```

In[426]:=

```
σ16.SAB[1,2]//MatrixForm
```

Out[426]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

In[427]:=

s16.sAB[1][2] // MatrixForm

Out[427]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3, 3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula: \otimes

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. \otimes

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4). 🔗

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) 🔗

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. 🔗

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa). 🔗

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). 🔗

what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the *so*(4, 4) Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: 🔗

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for *so*(8) in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



↳ Why is the metric signature significant for \$so(4,4)\$ commutation relations?

Why is the metric signature significant for \$so(4,4)\$ commutation relations?

The metric signature is significant for the $so(p, q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). ☰

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$
The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4, 4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[428]:=

η4488

Out[428]=

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1} }
```

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

In[429]:=

$$\text{SAB}[6, 2] \cdot \text{SAB}[6, 7] - \text{SAB}[6, 7] \cdot \text{SAB}[6, 2] = \eta4488[6, 6] \times \text{SAB}[7, 2]$$

Out[429]=

True

In[430]:=

$$\{ \{6, 2, 6, 7\}, \text{False} \}$$

Out[430]=

$$\{ \{6, 2, 6, 7\}, \text{False} \}$$

In[431]:=

$$\text{SAB}[1, 2] \cdot \text{SAB}[1, 3] - \text{SAB}[1, 3] \cdot \text{SAB}[1, 2] = \eta4488[1, 1] \times \text{SAB}[2, 3]$$

Out[431]=

False

In[433]:=

sAB[0][1]

Out[433]=

$$\left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right.$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0 \right\},$$

$$\left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0 \right\} \right\}$$

```
In[434]:= Table[Transpose[T16^A[8].sAB[A1][B1]] == T16^A[8].sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[434]= {{True, False, False, False, True, True, True}, {False, True, False, False, True, True, True}, {False, False, True, False, True, True, True}, {False, False, False, True, True, True, True}, {True, True, True, True, False, False, False}, {True, True, True, True, False, True, False}, {True, True, True, True, False, False, True}, {True, True, True, True, False, False, False}}
In[435]:= Table[Transpose[T16^A[7].sAB[A1][B1]] == T16^A[7].sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[435]= {{True, True, True, True, False, False, True}, {True, True, True, True, False, False, False}, {True, True, True, True, False, False, True}, {True, True, True, True, False, False, False}, {False, False, False, False, True, True, False}, {False, False, False, False, True, True, True}, {False, False, False, False, True, True, False}, {True, True, True, True, False, False, True}}
In[436]:= Table[Transpose[\sigma16.sAB[A1][B1]] == \sigma16.sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[436]= {{True, False, False, False, False, False, False}, {False, True, False, False, False, False, False}, {False, False, True, False, False, False, False}, {False, False, False, True, False, False, False}, {False, False, False, False, True, False, False}, {False, False, False, False, False, True, False}, {False, False, False, False, False, False, True}}
```

```
In[437]:= Table[Transpose[\sigma16.SAB[A1, B1]] == \sigma16.SAB[A1, B1], {A1, 1, 8}, {B1, 1, 8}]
Out[437]= {{True, False, False, False, False, False, False, False}, {False, True, False, False, False, False, False, False}, {False, False, True, False, False, False, False, False}, {False, False, False, True, False, False, False, False}, {False, False, False, False, True, False, False, False}, {False, False, False, False, False, True, False, False}, {False, False, False, False, False, False, True, False}, {False, False, False, False, False, False, False, True}}
```

```
In[438]:= SAB[1, 2][1]
Out[438]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

$$\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}$$

```
In[439]:= SAB[1, 2][2]
Out[439]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

$$\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
In[440]:= SAB1 = Table[Take[SAB[A1, B1][C1], 8], {A1, 8}, {B1, 8}, {C1, 1, 8}];
In[441]:= SAB2 = Table[Take[SAB[A1, B1][C1], -8], {A1, 8}, {B1, 8}, {C1, 9, 16}];
In[442]:= SAB[1, 2] == ArrayFlatten[{{SAB1[1, 2], 0}, {0, SAB2[1, 2]}}]
Out[442]= True
```

```
In[443]:= (*Table[SAB[A1,B1]== ArrayFlatten[{{SAB1[A1,B1],0},{0,SAB2[A1,B1]}]], {A1,0, 7},{B1,0,7}]*)
```

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \rightarrow SO(4,4)$ is given by

$$(2.8) \quad 8L^{A'}_{\ B'}(g) = \text{tr} \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}_{\ C'} G_{A'B'} L^{B'}_{\ D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}_{\ B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}_{\ B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

In[444]:=

SAB2[[1, 2]]

Out[444]=

$$\left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \right\}, \right. \\ \left. \left\{ 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0 \right\}, \left\{ 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}$$

In[445]:=

τ[θ]

Out[445]=

$$\{ \{ 1, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 1, 0, 0, 0, 0, 0, 0 \}, \\ \{ 0, 0, 1, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 1, 0, 0, 0 \}, \\ \{ 0, 0, 0, 0, 0, 1, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 1, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 1 \} \}$$

In[446]:=

```
(*Block[{b,A,F_a^(μ),F_(μ)^a,ψ,Ψ},
b=Array[A,{8}];
ψ[1]=Array[Ψ[1],{8}];
ψ[2]=Array[Ψ[2],{8}];
F_a^(μ)=Table[(1/(Sqrt[ψ[1].σ.ψ[1]])ψ[1].σ.(τ[μL])),{μL,1,8}];
F_(μ)^a=Transpose[Table[(1/(Sqrt[ψ[1].σ.ψ[1]])η4488[[μL,μL]τ[μL].ψ[1]]),{μL,1,8}]];
b==FullSimplify[F_a^(μ).ψ[2]/.Thread[ψ[2]→F_(μ)^a.b]]]*)
```

In[447]:=

```
(*Block[{b,A,B,q,Fa(μ),ψ,φ,Ψ},
b=Array[A,{8}];
ψ[1]=Array[Ψ[1],{8}];
φ[1]=Array[Ψ[2],{8}];
q=ParallelTable[
  FullSimplify[(1/(ψ[1].σ.ψ[1])ψ[1].σ.(1/2(τ[A1].τ[B1]-τ[B1].τ[A1]).φ[1])],
  {A1,1,8},{B1,1,8}];
B=FullSimplify[
  (1/(ψ[1].σ.ψ[1]) ParallelSum[η4488[[μL,μL]τ[μL]b[[μL],{μL,1,8}].ψ[1]]];
FullSimplify[q/.Thread[φ[1]→B]]*)]
```

In[448]:=

```
(*Block[{b,A,B,q,Fa(μ),ψ,φ,Ψ},
b=Array[A,{8}];
ψ[1]=Array[Ψ[1],{8}];
φ[1]=Array[Ψ[2],{8}];
q=ParallelTable[
  FullSimplify[(ψ[1].σ.(τ[A1].τ[B1]).φ[1]),{A1,1,8},{B1,1,8}];
B=FullSimplify[( ParallelSum[η4488[[μL,μL]τ[μL]b[[μL],{μL,1,8}].ψ[1]]];
FullSimplify[(1/(ψ[1].σ.ψ[1])q/.Thread[φ[1]→B])]]*)]
```

T16^α

In[449]:=

ssgm4488

Out[449]=

$$\begin{aligned} \{ & g[0][0] \rightarrow (\text{Cot}[6 H \#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&), \\ & g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&), \\ & g[2][2] \rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&), \\ & g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&), \\ & g[3][3] \rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&) \} \end{aligned}$$

In[450]:=

e^α_(A) / . sge^α_(A)

Out[450]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}} \right\} \right\} \end{aligned}$$

In[451]:=

```
Table[T16α[α1 - 1] = Sum[(eα(A) / . sgeα(A)) [[α1, A1]]] T16A[A1 - 1], {A1, 1, 8}], {α1, 1, 8}]; (* /. ssgm4488 *)
```

In[452]:=

```
(T16α[8] = T16α[0].T16α[1].T16α[2].T16α[3].T16α[4].T16α[5].T16α[6].T16α[7]);
```

In[453]:=

```
(*T16α[8]==T16A[8]*Sec[6 H x0]*)
```

CHECK

```
Table[T16α[α1 - 1] = Sum[(eα(A) [[α1, A1]] ) T16A[A1 - 1], {A1, 1, 8}], {α1, 1, 8}];  
and • • • :
```

In[454]:=

```
(*Table[T16α[α1-1]= Sum[(eα(A) [[α1,A1]] ) T16A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

In[455]:=

```
(* (T16α[8]=T16α[0].T16α[1].T16α[2].T16α[3].T16α[4].T16α[5].T16α[6].T16α[7])//  
MatrixForm*)
```

In[456]:=

```
Table[{ {A1, B1},  
FullSimplify[ExpandAll[ $\frac{1}{2}$  ( T16A[A1].T16A[B1] + T16A[B1].T16A[A1] ) ==  
η4488[[A1+1, B1+1]*ID16]] }}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[456]//MatrixForm=

```
(({0, 0}) ({0, 1}) ({0, 2}) ({0, 3}) ({0, 4}) ({0, 5}) ({0, 6}) ({0, 7})  
True ({0, 1}) ({0, 2}) ({0, 3}) ({0, 4}) ({0, 5}) ({0, 6}) ({0, 7})  
({1, 0}) ({1, 1}) ({1, 2}) ({1, 3}) ({1, 4}) ({1, 5}) ({1, 6}) ({1, 7})  
True ({1, 1}) ({1, 2}) ({1, 3}) ({1, 4}) ({1, 5}) ({1, 6}) ({1, 7})  
({2, 0}) ({2, 1}) ({2, 2}) ({2, 3}) ({2, 4}) ({2, 5}) ({2, 6}) ({2, 7})  
True ({2, 1}) ({2, 2}) ({2, 3}) ({2, 4}) ({2, 5}) ({2, 6}) ({2, 7})  
({3, 0}) ({3, 1}) ({3, 2}) ({3, 3}) ({3, 4}) ({3, 5}) ({3, 6}) ({3, 7})  
True ({3, 1}) ({3, 2}) ({3, 3}) ({3, 4}) ({3, 5}) ({3, 6}) ({3, 7})  
({4, 0}) ({4, 1}) ({4, 2}) ({4, 3}) ({4, 4}) ({4, 5}) ({4, 6}) ({4, 7})  
True ({4, 1}) ({4, 2}) ({4, 3}) ({4, 4}) ({4, 5}) ({4, 6}) ({4, 7})  
({5, 0}) ({5, 1}) ({5, 2}) ({5, 3}) ({5, 4}) ({5, 5}) ({5, 6}) ({5, 7})  
True ({5, 1}) ({5, 2}) ({5, 3}) ({5, 4}) ({5, 5}) ({5, 6}) ({5, 7})  
({6, 0}) ({6, 1}) ({6, 2}) ({6, 3}) ({6, 4}) ({6, 5}) ({6, 6}) ({6, 7})  
True ({6, 1}) ({6, 2}) ({6, 3}) ({6, 4}) ({6, 5}) ({6, 6}) ({6, 7})  
({7, 0}) ({7, 1}) ({7, 2}) ({7, 3}) ({7, 4}) ({7, 5}) ({7, 6}) ({7, 7})  
True ({7, 1}) ({7, 2}) ({7, 3}) ({7, 4}) ({7, 5}) ({7, 6}) ({7, 7})
```

In[457]:=

```
Table[{ $\{\alpha_1, \beta_1\}$ , FullSimplify[
   $\frac{1}{2} \text{FullSimplify}[((T16^\alpha[\alpha_1].T16^\alpha[\beta_1] + T16^\alpha[\beta_1].T16^\alpha[\alpha_1]) / . \text{ssgm4488}),$ 
  constraintVars] == Inverse[MatrixMetric44][ $\alpha_1 + 1, \beta_1 + 1$ ] * ID16]},  

  { $\alpha_1, 0, 7$ }, { $\beta_1, 0, 7$ }]] // MatrixForm
```

Out[457]//MatrixForm=

$$\begin{pmatrix} \{0, 0\} & \{0, 1\} & \{0, 2\} & \{0, 3\} & \{0, 4\} & \{0, 5\} & \{0, 6\} & \{0, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{1, 0\} & \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} & \{1, 5\} & \{1, 6\} & \{1, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{2, 0\} & \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} & \{2, 5\} & \{2, 6\} & \{2, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{3, 0\} & \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} & \{3, 5\} & \{3, 6\} & \{3, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{4, 0\} & \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} & \{4, 5\} & \{4, 6\} & \{4, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{5, 0\} & \{5, 1\} & \{5, 2\} & \{5, 3\} & \{5, 4\} & \{5, 5\} & \{5, 6\} & \{5, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{6, 0\} & \{6, 1\} & \{6, 2\} & \{6, 3\} & \{6, 4\} & \{6, 5\} & \{6, 6\} & \{6, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{7, 0\} & \{7, 1\} & \{7, 2\} & \{7, 3\} & \{7, 4\} & \{7, 5\} & \{7, 6\} & \{7, 7\} \\ \text{True} & \text{True} \end{pmatrix}$$

In[458]:=

MatrixMetric44 // MatrixForm

Out[458]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6 H x \theta]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2 a 4 [H x 4]} \sin[6 H x \theta]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2 a 4 [H x 4]} \sin[6 H x \theta]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2 a 4 [H x 4]} \sin[6 H x \theta]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[459]:=

```
((e $_{(A)}^{\alpha} / . \text{sgtrye}_{(A)}^{\alpha}) \cdot \text{MatrixMetric44.}(e_{(A)}^{\alpha} / . \text{sgtrye}_{(A)}^{\alpha})) // \text{MatrixForm}$ 
```

Out[459]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[460]:= Transpose[(eα(A) /. sgtryeα(A))].η4488.(eα(A) /. sgtryeα(A)) - MatrixMetric44 // FullSimplify[#, constraintVars] &

Out[460]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[461]:= Transpose[(e(A)α /. sgtrye(A)α)].MatrixMetric44.(e(A)α /. sgtrye(A)α) - η4488 // (*Full*)Simplify[#, constraintVars] &

Out[461]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[462]:= Inverse[(e(A)α /. sgtrye(A)α)] == (eα(A) /. sgtryeα(A))

Out[462]= True
```

```
In[463]:= Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]

Out[463]= {{0, 0}, {0, 0}}
```

BELOW : Lagrangian₁₆ = $\kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi_{16}] . \sigma_{16} . T_{16}^A[A_1] . \Psi_{16} \Big|_{\alpha} g^{-1} \Big|_{\beta} \eta_{A_1 B_1} E_{\beta}^{B_1} + \text{mass term}$

```
In[464]:= eα(A) /. sgtryeα(A)

Out[464]= {{Cot[6 H x 0], 0, 0, 0, 0, 0, 0, 0}, {0,  $\sqrt{e^{2 a4[H x 4]}}$  Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0,  $\sqrt{e^{2 a4[H x 4]}}$  Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, 0,  $\sqrt{e^{2 a4[H x 4]}}$  Sin[6 H x 0]1/6, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0,  $\sqrt{e^{-2 a4[H x 4]}}$  Sin[6 H x 0]1/6, 0, 0}, {0, 0, 0, 0, 0, 0,  $\sqrt{e^{-2 a4[H x 4]}}$  Sin[6 H x 0]1/6, 0}, {0, 0, 0, 0, 0, 0, 0,  $\sqrt{e^{-2 a4[H x 4]}}$  Sin[6 H x 0]1/6}}
```

In[465]:=

$$\mathbf{e}_{(A)}^{\alpha} / . \text{sgttrye}_{(A)}^{\alpha}$$

Out[465]=

$$\begin{aligned} & \left\{ \{ \tan[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ 0, \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0, 0, 0, 0 \right\}, \{0, 0, 0, 0, 1, 0, 0, 0, 0 \}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}} \right\} \right\} \end{aligned}$$

In[466]:=

$$\mathbf{e}_{(A)}^{\alpha} / . \text{sge}_{(A)}^{\alpha}$$

Out[466]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}} \right\} \right\} \end{aligned}$$

In[467]:=

```

Clear[con];
con[g_, ass_] := Module[{t, rΓ, ginv},
  ginv = FullSimplify[Inverse[g], ass];
  rΓ = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
    (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}], {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
  Return[{ginv, rΓ}]];

```

In[469]:=

ssgm4488

Out[469]=

$$\begin{aligned} & \{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&), \\ & g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&), \\ & g[2][2] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&), \\ & g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&), \\ & g[3][3] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&)\} \end{aligned}$$

In[470]:=

g4488**g4488 /. sg**

Out[470]=

$$\begin{aligned} & \{\{g[0][0][x0, x4], g[0][1][x0, x4], g[0][2][x0, x4], g[0][3][x0, x4], \\ & g[0][4][x0, x4], g[0][5][x0, x4], g[0][6][x0, x4], g[0][7][x0, x4]\}, \\ & \{g[1][0][x0, x4], g[1][1][x0, x4], g[1][2][x0, x4], g[1][3][x0, x4], \\ & g[1][4][x0, x4], g[1][5][x0, x4], g[1][6][x0, x4], g[1][7][x0, x4]\}, \\ & \{g[2][0][x0, x4], g[2][1][x0, x4], g[2][2][x0, x4], g[2][3][x0, x4], \\ & g[2][4][x0, x4], g[2][5][x0, x4], g[2][6][x0, x4], g[2][7][x0, x4]\}, \\ & \{g[3][0][x0, x4], g[3][1][x0, x4], g[3][2][x0, x4], g[3][3][x0, x4], \\ & g[3][4][x0, x4], g[3][5][x0, x4], g[3][6][x0, x4], g[3][7][x0, x4]\}, \\ & \{g[4][0][x0, x4], g[4][1][x0, x4], g[4][2][x0, x4], g[4][3][x0, x4], \\ & g[4][4][x0, x4], g[4][5][x0, x4], g[4][6][x0, x4], g[4][7][x0, x4]\}, \\ & \{g[5][0][x0, x4], g[5][1][x0, x4], g[5][2][x0, x4], g[5][3][x0, x4], \\ & g[5][4][x0, x4], g[5][5][x0, x4], g[5][6][x0, x4], g[5][7][x0, x4]\}, \\ & \{g[6][0][x0, x4], g[6][1][x0, x4], g[6][2][x0, x4], g[6][3][x0, x4], \\ & g[6][4][x0, x4], g[6][5][x0, x4], g[6][6][x0, x4], g[6][7][x0, x4]\}, \\ & \{g[7][0][x0, x4], g[7][1][x0, x4], g[7][2][x0, x4], g[7][3][x0, x4], \\ & g[7][4][x0, x4], g[7][5][x0, x4], g[7][6][x0, x4], g[7][7][x0, x4]\}\} \end{aligned}$$

Out[471]=

$$\begin{aligned} & \{\{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \{0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0\}, \{0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0\}, \{0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0\}, \{0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]\}\} \end{aligned}$$

In[472]:=

```

MatrixForm[#] & /@ Block[{eAα, eαA, (*ginv,Γ,gg,*) colVecs, rowVecs, ab, ba, r},
  (*gg=g4488/.sg;*)
  (*{ginv,Γ}=con[gg];*)
  eαA = eα(A) /. sgeα(A);
  eAα = e(A)α /. sge(A)α;
  rowVecs = Table[eAα[[b]], All], {b, 1, Length[eAα]}];
  colVecs = Table[eαA[[All, b]], {b, 1, Length[eαA]}];
  ab = rowVecs.colVecs // FullSimplify[#, constraintVars] &;
  ba = Transpose[colVecs].Transpose[rowVecs] //
    FullSimplify[#, constraintVars] &;
  (*r={{ab},{ba}};*)
  r = {ab, ba};
  r]

```

Out[472]=

$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \right\}$$

metric compatibility condition for the Octad

$$\nabla_\mu e_\nu^I = \partial_\mu e_\nu^I - \Gamma_{\mu\nu}^\rho e_\rho^I + \omega_{\mu J}^I e_\nu^J = 0$$

In[473]:=

```
Clear[spinCoefficients];
spinCoefficients[eIv_] := Module[{evI, w, r, s},
  If[Dimensions[eIv] != {8, 8}, Return[]];
  If[Det[eIv] == 0, Return[]];
  evI = FullSimplify[Inverse[eIv], constraintVars] /. subsDefects;
  w = Table[
    -Table[FullSimplify[Sum[D[eIv[[I1, v1]], X[[μ1]]] * evI[[v1, Jprime]], {v1, 1,
      Length[evI]]] - Sum[ eIv[[I1, ρ]] * r[ρ, μ1, v1] * evI[[v1, Jprime]],
      {v1, 1, Length[evI]}, {ρ, 1, Length[X]}],
    constraintVars] /. subsDefects, {I1, 1, Length[eIv]},
    {Jprime, 1, Length[evI]}], {μ1, 1, Length[X]}];
  Assert[Dimensions[w] == {8, 8, 8}];
  Return[w];
]
```


In[476]:=

```
Table[ $\omega_{\mu IJ}[\mu] = \text{FullSimplify}\left[-\left(\text{D}\left[\text{gtrye}_{\alpha}^{(A)}, X[\mu]\right].\text{gtrye}_{(A)}^{\alpha}\right) - \left(\text{gtrye}_{\alpha}^{(A)}.r[\text{All}, \mu, \text{All}].\text{gtrye}_{(A)}^{\alpha}\right)\right],$ 
constraintVars] /. subsDefects, {\mu, 1, Length[X]}]
```

Out[476]=

```

In[477]:= (*gtrye(A)α.gtrye(A)α*)

In[478]:= (*gtryeα(A).gtrye(A)α*)

In[479]:= (*gtryeα(A)*)

In[480]:= (*Γ
Dimensions[%]*)

In[481]:= (*Table[Γ[[All,μ,All]],{μ,1,Length[X]]]*)
```

In[482]:= (*Γ[[All,5,All]]*)

In[483]:= (*Γ[[All,1,All]]*)

In[484]:= (*spinCoeffs[[1]]*)

In[485]:= (*spinCoeffs[[5]]*)

In[486]:= (*{X[[1]],X[[5]]}*)

In[487]:= (*spinCoeffs[[1]][[1]][[1]]*)

In[488]:= (*ωμIJ[1][[1]][[1]]*)

In[489]:= (*Block[{μ=1},
FullSimplify[-((D[gtrye_α^(A),X[[μ]]].gtrye_(A)^α)-(gtrye_α^(A).Γ[[All,μ,All]].gtrye_(A)^α)),
constraintVars]/.subsDefects]
Dimensions[%]*)

In[490]:= (*Block[{μ=1},
FullSimplify[-((D[gtrye_α^(A),X[[μ]]].Transpose[gtrye_(A)^α])-(gtrye_α^(A).Γ[[All,μ,All]].
Transpose[gtrye_(A)^α])),constraintVars]/.subsDefects]*)

In[491]:= (*Block[{μ=1},FullSimplify[
-((Sum[D[gtrye_α^(A),All,v],X[[μ]]]*gtrye_(A)^α[[v,All]],,{v,1,8}])-(
gtrye_α^(A).Γ[[All,μ,All]].gtrye_(A)^α)],constraintVars]/.subsDefects]
Dimensions[%]*)

In[492]:= (*Table[,{A1,1,8},{B1,1,8}]*)

```
In[493]:= (*Block[{μ=1},Table[FullSimplify[
  -((Sum[D[gtrye_α^(A)[A1,ν],X[μ]]*gtrye_α^(A)[ν,B1],{ν,1,8}])-(
    Sum[gtrye_α^(A)[A1,ρ]*Γ[ρ,μ,ν]*gtrye_α^(A)[ν,B1],{ρ,1,8},
     {ν,1,8}]),constraintVars]/.subsDefects,{A1,1,8},{B1,1,8}]]]
Dimensions[%]*)]
```

In[494]:=

```
Table[(ωμIJ[μ] - spinCoeffs[μ]) /. subsDefects, {μ, 1, Length[X]}] // 
Flatten // Union
```

Out[494]=

{0}

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
 - $\nabla_\mu e^a{}_\nu = 0$
 - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.
 - Metric compatibility : The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame . The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is :
- He "
- v=0
- Here, V_ν is the covariant derivative with respect to the connection $F^\mu{}_{\nu b}$, and $e_a{}^\nu$ is the tetrad component .

No it is not the torsionless condition which is:

$$T_{\mu\nu}^I = D_{[\mu}^\omega e_{\nu]}^I = \partial_{[\mu} e_{\nu]}^I + \omega_{[\mu J}^I e_{\nu]}^J = 0$$

This postulate says:

$$\nabla_\mu e_\nu^I = \partial_\mu e_\nu^I - \Gamma_{\mu\nu}^\rho e_\rho^I + \omega_{\mu J}^I e_\nu^J = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D_\mu^\omega \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

```
In[495]:= Block[{eAα, eαA, t, ginv, Γ, gg},
  gg = g4488 /. sg;
  {ginv, Γ} = con[gg];
  eAα = eα(A) /. sgeα(A); eαA = e(A)α /. sge(A)α;
  t = Table[ Block[{v, V, contra, covari},
    V = eAα[[b, All]];
    v = eαA[[All, b]];
    contra =
      Table[D[v[[k]], X[[μ]]] + Sum[v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
      (*before ω terms*)
    covari =
      Table[D[V[[k]], X[[μ]]] - Sum[V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
      (*before ω terms*)
    {b - 1, X[[μ]], {{contra}, {covari}}}], {b, 1, Length[eAα]}, {μ, 1, DIM8}];
  t]
  FullSimplify[#, ssym4488, constraintVars] & /@ %

```

Out[495]=

$$\left\{ \left\{ 0, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, 0, 0, 0 \right\}$$

$$\begin{aligned}
& \left\{ \left\{ 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x1, \left\{ \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x2, \left\{ \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[2][2][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x3, \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[3][3][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x4, \left\{ \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x5, \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[5][5][x0, x4]}, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0 \right\} \right\}, \\
& \left\{ 0, x6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[6][6][x0, x4]}, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0 \right\} \right\}, \\
& \left\{ 0, x7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]} \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}} \right\} \right\}, \\
& \left\{ 1, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \quad \left. \left\{ 1, x1, \left\{ \left\{ -\frac{g[1][1]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4]\sqrt{g[1][1][x0, x4]}} \right\} \right\}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 0, 0, 0, - \frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0 \} \}, \\
& \left\{ \left\{ - \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, - \frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0 \} \right\} \right\}, \\
& \{1, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x2, \left\{ \left\{ - \frac{g[2][2]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[2][2][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, - \frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ - \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, - \frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0 \right\} \right\}, \\
& \{2, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x3, \left\{ \left\{ - \frac{g[3][3]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[3][3][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, - \frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ - \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, - \frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, 0 \right\} \right\}, \\
& \{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 4, x_0, \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. - \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}} - \frac{g[4][4]^{(1,0)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x_1, \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x_0, x_4]}{2 g[1][1][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, \frac{g[1][1]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x_2, \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 g[2][2][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x_3, \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4]}{2 g[3][3][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x_4, \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. - \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}} - \frac{g[4][4]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x_5, \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0 \right\} \right\}, \\
& \left\{ 4, x_6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& - \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]}, 0 \} \} \}, \\
& \{6, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0 \} \}, \{\{0, 0, 0, \\
& \quad 0, 0, 0, - \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]}, 0 \} \} \}, \\
& \{6, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x6, \{\{\{- \frac{g[6][6]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0, \\
& \quad 0, 0, - \frac{g[6][6]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0\} \}, \\
& \quad \left\{ \left\{ - \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. - \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]}, 0, 0, 0 \right\} \right\} \}, \\
& \{6, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]}, 0, 0, 0, 0, \\
& \quad 0, 0, 0, - \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 g[7][7][x0, x4]}, 0 \} \} \}, \\
& \{7, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \frac{g[7][7]^{(0,1)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]}, 0, 0, 0, 0, \\
& \quad 0, 0, 0, - \frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 g[7][7][x0, x4]}, 0 \} \} \}, \\
& \{7, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},
\end{aligned}$$

$$\begin{aligned} & \left\{ 7, x_7, \left\{ \left\{ -\frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}, 0, \right. \right. \right. \\ & \quad \left. \left. \left. 0, 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\ & \left\{ \left\{ -\frac{g[7][7]^{(1,0)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2 g[7][7][x_0, x_4]}, 0, 0, 0, \right. \right. \\ & \quad \left. \left. \left. -\frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2 g[7][7][x_0, x_4]}, 0, 0, 0 \right\} \right\} \right\} \end{aligned}$$

Out[496]=

$$\begin{aligned} & \left\{ \left\{ 0, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 0, x_1, \left\{ \left\{ 0, H, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, e^{2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 0, x_2, \left\{ \left\{ 0, 0, H, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, e^{2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 0, x_3, \left\{ \left\{ 0, 0, 0, H, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, e^{2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 0, x_4, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 0, x_5, \left\{ \left\{ 0, 0, 0, 0, 0, H, 0, 0, 0 \right\} \right\}, \right. \\ & \quad \left. \left\{ 0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0, 0 \right\} \right\} \right\}, \left\{ 0, x_6, \right. \\ & \quad \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, H, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0 \right\} \right\} \right\}, \left\{ 0, \right. \\ & \quad \left. x_7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, H \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} \right\} \right\} \right\}, \\ & \left\{ 1, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 1, x_1, \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \sec[6 H x0] \sin[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \right. \\ & \quad \left. \left. \left. \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \left\{ -\frac{\sqrt{e^{2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, \right. \\ & \quad \left. 0, 0, 0, -\sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 1, x_2, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 1, x_3, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 1, x_4, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 1, x_5, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 1, x_6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 1, x_7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 2, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 2, x_1, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 2, x_2, \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \sec[6 H x0] \sin[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \right. \\ & \quad \left. \left. \left. \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \left\{ -\frac{\sqrt{e^{2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, \right. \\ & \quad \left. 0, 0, 0, -\sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 2, x_3, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ & \left\{ 2, x_4, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \end{aligned}$$

$$\begin{aligned}
& \{2, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\}, \\
& \{3, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x_3, \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \sin[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \left\{ \left\{ -\frac{\sqrt{e^{2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, \right. \right. \\
& \quad \left. \left. 0, 0, 0, -\sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \{3, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x_1, \{\{\{0, H a4'[H x4], 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, -e^{2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x_2, \{\{\{0, 0, H a4'[H x4], 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, -e^{2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x_3, \{\{\{0, 0, 0, H a4'[H x4], 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, -e^{2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x_5, \{\{\{0, 0, 0, 0, 0, 0, -H a4'[H x4], 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0, 0\}\}\}\}, \\
& \{4, x_6, \{\{\{0, 0, 0, 0, 0, 0, -H a4'[H x4], 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0\}\}\}\}, \\
& \{4, x_7, \{\{\{0, 0, 0, 0, 0, 0, -H a4'[H x4]\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4]\}\}\}\}, \\
& \{5, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x_5, \left\{ \left\{ \sqrt{e^{-2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \sin[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \left\{ \left\{ -\frac{\sqrt{e^{-2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, \right. \right. \\
& \quad \left. \left. 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \{5, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},
\end{aligned}$$

$$\begin{aligned}
& \{5, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\}, \\
& \{6, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_6, \left\{ \left\{ \sqrt{e^{-2 a4[Hx4]}} H \operatorname{Sec}[6 H x_0] \sin[6 H x_0]^{7/6}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. \sqrt{e^{-2 a4[Hx4]}} H \sin[6 H x_0]^{1/6} a4'[H x_4], 0, 0, 0 \right\} \right\}, \left\{ \left\{ -\frac{\sqrt{e^{-2 a4[Hx4]}} H \cos[6 H x_0]}{\sin[6 H x_0]^{5/6}}, \right. \right. \\
& \quad \left. \left. 0, 0, 0, \sqrt{e^{-2 a4[Hx4]}} H \sin[6 H x_0]^{1/6} a4'[H x_4], 0, 0, 0 \right\} \right\}, \\
& \{6, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_7, \left\{ \left\{ \sqrt{e^{-2 a4[Hx4]}} H \operatorname{Sec}[6 H x_0] \sin[6 H x_0]^{7/6}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. \sqrt{e^{-2 a4[Hx4]}} H \sin[6 H x_0]^{1/6} a4'[H x_4], 0, 0, 0 \right\} \right\} \}
\end{aligned}$$

In[497]:=

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(*Block[{eAα,eαA,t,ginv,Γ,gg},
gg=g4488/.sg;
{ginv,Γ}=con[gg];
eαA=eα(A) /.sgeα(A);
eAα=e(A)α /.sge(A)α;
t=Table[ Block[{v,V,d},v=eαA[[All,b]]];
d=Table[D[v[[k]],X[[μ]] ]+Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}],{k,1,DIM8}];
{b-1,X[[μ]],d}],{b,1,Length[eαA]}, {μ,1,DIM8}];t]
FullSimplify[#/ssgm4488,constraintVars]&/@%*)

```

```
In[498]:= (*Block[{eAα,eαA,t,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=eα(A) /.sg eα(A);
  eAα=e(A)α /.sg e(A)α;
  t=Table[ Block[{v,V,d},v=eAα[[b,All]];
    d=Table[D[v[[k]],X[[μ]]]-Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}],{k,1,DIM8}];
    {b-1,X[[μ]],d}],{b,1,Length[eαA]},{μ,1,DIM8}];t]
  FullSimplify[#/ssgm4488,constraintVars]&/@%*)

In[498]:= Symbolize[ω(a)μ(b)]

In[498]:= ω = Array[ω(a)μ(b) & , {8, 8, 8}];

In[499]:= ω[[1]]

Out[499]= {{ { ω(a)μ(b) [1, 1, 1], ω(a)μ(b) [1, 1, 2], ω(a)μ(b) [1, 1, 3], ω(a)μ(b) [1, 1, 4],
  ω(a)μ(b) [1, 1, 5], ω(a)μ(b) [1, 1, 6], ω(a)μ(b) [1, 1, 7], ω(a)μ(b) [1, 1, 8] },
  { ω(a)μ(b) [1, 2, 1], ω(a)μ(b) [1, 2, 2], ω(a)μ(b) [1, 2, 3], ω(a)μ(b) [1, 2, 4],
  ω(a)μ(b) [1, 2, 5], ω(a)μ(b) [1, 2, 6], ω(a)μ(b) [1, 2, 7], ω(a)μ(b) [1, 2, 8] },
  { ω(a)μ(b) [1, 3, 1], ω(a)μ(b) [1, 3, 2], ω(a)μ(b) [1, 3, 3], ω(a)μ(b) [1, 3, 4],
  ω(a)μ(b) [1, 3, 5], ω(a)μ(b) [1, 3, 6], ω(a)μ(b) [1, 3, 7], ω(a)μ(b) [1, 3, 8] },
  { ω(a)μ(b) [1, 4, 1], ω(a)μ(b) [1, 4, 2], ω(a)μ(b) [1, 4, 3], ω(a)μ(b) [1, 4, 4],
  ω(a)μ(b) [1, 4, 5], ω(a)μ(b) [1, 4, 6], ω(a)μ(b) [1, 4, 7], ω(a)μ(b) [1, 4, 8] },
  { ω(a)μ(b) [1, 5, 1], ω(a)μ(b) [1, 5, 2], ω(a)μ(b) [1, 5, 3], ω(a)μ(b) [1, 5, 4],
  ω(a)μ(b) [1, 5, 5], ω(a)μ(b) [1, 5, 6], ω(a)μ(b) [1, 5, 7], ω(a)μ(b) [1, 5, 8] },
  { ω(a)μ(b) [1, 6, 1], ω(a)μ(b) [1, 6, 2], ω(a)μ(b) [1, 6, 3], ω(a)μ(b) [1, 6, 4],
  ω(a)μ(b) [1, 6, 5], ω(a)μ(b) [1, 6, 6], ω(a)μ(b) [1, 6, 7], ω(a)μ(b) [1, 6, 8] },
  { ω(a)μ(b) [1, 7, 1], ω(a)μ(b) [1, 7, 2], ω(a)μ(b) [1, 7, 3], ω(a)μ(b) [1, 7, 4],
  ω(a)μ(b) [1, 7, 5], ω(a)μ(b) [1, 7, 6], ω(a)μ(b) [1, 7, 7], ω(a)μ(b) [1, 7, 8] },
  { ω(a)μ(b) [1, 8, 1], ω(a)μ(b) [1, 8, 2], ω(a)μ(b) [1, 8, 3], ω(a)μ(b) [1, 8, 4],
  ω(a)μ(b) [1, 8, 5], ω(a)μ(b) [1, 8, 6], ω(a)μ(b) [1, 8, 7], ω(a)μ(b) [1, 8, 8] } }
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In[500]:=

```

sw = Block[{eAα, eAα, t, V, ginv, Γ, gg},
  gg = g4488 /. sg;
  {ginv, Γ} = con[gg];
  eAα = eα(A) /. sgeα(A);
  eAα = e(A)α /. sge(A)α;
  Table[w[[μ, a, b]] → Block[{v, d}, v = eAα[[All, b]];
    d = Sum[(D[v[[k]], X[[μ]]] + Sum[v[[i]] × r[[k, i, μ]]], {i, 1, DIM8})] * eAα[[a, k]],
    {k, 1, DIM8}];
    d], {μ, 1, DIM8}, {a, 1, Length[eAα]}, {b, 1, Length[eAα]}]

```

Out[500]=

$$\begin{aligned}
& \left\{ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 1, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 1, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 1, 5] \rightarrow \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \\ \omega_{\mu(b)}^{(a)} [1, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 1, 8] \rightarrow 0 \end{array} \right\}, \right. \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 5, 1] \rightarrow -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [1, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 5, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 5, 5] \rightarrow \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \\ \sqrt{-g[4][4][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [1, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 5, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 6, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 6, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 6, 5] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 6, 6] \rightarrow \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \\ \sqrt{-g[5][5][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [1, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 6, 8] \rightarrow 0 \end{array} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [1, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 7, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 7, 7] \rightarrow \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[6][6][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 7] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 8, 8] \rightarrow \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [2, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 2] \rightarrow -\frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[1][1][x0, x4]}}, \right. \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 5] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 8] \rightarrow 0 \right\}, \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [2, 2, 1] \rightarrow \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[1][1][x0, x4]}}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 2, 5] \rightarrow \frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 8] \rightarrow 0 \right\}, \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [2, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 8] \rightarrow 0 \right\}, \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [2, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 8] \rightarrow 0 \right\}, \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [2, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 2] \rightarrow -\frac{g[1][1]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[1][1][x0, x4]}g[4][4][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 5] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 8] \rightarrow 0 \right\}, \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [2, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 6, 4] \rightarrow 0, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [3, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [4, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 1, 3] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 1, 4] \rightarrow -\frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[3][3][x0, x4]}}, \omega_{\mu(b)}^{(a)} [4, 1, 5] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 1, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [4, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [4, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [4, 4, 1] \rightarrow \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[3][3][x0, x4]}}, \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 4, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 4, 5] \rightarrow \frac{g[3][3]^{(0,1)}[x0, x4]}{2\sqrt{g[3][3][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 4, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [4, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 5, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 5, 3] \rightarrow 0, \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 5, 4] \rightarrow -\frac{g[3][3]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[3][3][x0, x4]}g[4][4][x0, x4]}, \omega_{\mu(b)}^{(a)} [4, 5, 5] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 5, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [4, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [4, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left\{ \omega_{\mu(b)}^{(a)} [4, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 8] \rightarrow 0 \right\} \right\}, \\
& \quad \left\{ \left\{ \omega_{\mu(b)}^{(a)} [5, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 1, 5] \rightarrow -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 8] \rightarrow 0 \right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [5, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [5, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [5, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [5, 5, 1] \rightarrow \frac{g[4][4]^{(1,0)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[4][4][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [5, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 5, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 5, 5] \rightarrow \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]} \right) \\ \sqrt{-g[4][4][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [5, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 5, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [5, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 6, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 6, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 6, 5] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 6, 6] \rightarrow \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}g[5][5][x0, x4]} \right) \\ \sqrt{-g[5][5][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [5, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 6, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [5, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 7, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 7, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 7, 6] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 7, 7] \rightarrow \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}g[6][6][x0, x4]} \right) \\ \sqrt{-g[6][6][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [5, 7, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [5, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 7] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [5, 8, 8] \rightarrow \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}g[7][7][x0, x4]} \right) \\ \sqrt{-g[7][7][x0, x4]} \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [6, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 4] \rightarrow 0, \end{array} \right\}
\end{aligned}$$

$$\begin{aligned}
& \omega_{\mu(b)}^{(a)} [6, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 6] \rightarrow -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[5][5][x0, x4]}}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 6] \rightarrow -\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 6, 1] \rightarrow \frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[5][5][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 6, 5] \rightarrow \frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}g[5][5][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [7, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 3] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [7, 1, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 6] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [7, 1, 7] \rightarrow -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[6][6][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [7, 1, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [7, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [7, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 8] \rightarrow 0 \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [7, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 5, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 5, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 5, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 5, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 5, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 5, 7] \rightarrow -\frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[6][6][x0, x4]}}, \omega_{\mu(b)}^{(a)} [7, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 7, 1] \rightarrow \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[6][6][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 7, 5] \rightarrow \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[6][6][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 7, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 7, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [8, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [8, 1, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 7] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [8, 1, 8] \rightarrow -\frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[7][7][x0, x4]}} \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [8, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [8, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [8, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [8, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [8, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 4, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [8, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 4, 8] \rightarrow 0 \right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [8, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 5, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 5, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 5, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 5, 7] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 5, 8] \rightarrow -\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}} \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 7, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 7, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 8, 1] \rightarrow \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 8, 5] \rightarrow \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 8, 8] \rightarrow 0 \right\} \}
\end{aligned}$$

In[501]:=

```
(*sw=Block[{eAα,eαA,t,V,ginv,Γ,gg},
gg=g4488/.sg;
{ginv,Γ}=con[gg];
eαA=eα(A)/.sgeα(A);
eAα=eα(A)/.sgeα(A);
Table[ ω(a)μ(b) [μ,a,b]→Block[{v,d},v=eαA[[All,b]];
d=Sum[
(D[v[[k]],X[[μ]]]+Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}])*eAα[[a,k]],{k,1,DIM8}];
d],{μ,1,DIM8},{a,1,Length[eαA]},{b,1,Length[eαA]}]*)
```

In[502]:=

swf = Flatten[sw];

In[503]:=

```
(*ω(a)μ(b) *)
ω;
ωmat=% /. swf
Dimensions[%]
```

Out[504]=

$$\left\{ \left\{ 0, 0, 0, 0, \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \right.$$

$$\begin{aligned}
& \left\{ -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, \right. \\
& \left. \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]}, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[5][5][x0, x4]}, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[6][6][x0, x4]}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \\
& \left. \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ 0, -\frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, \right. \\
& \left. \frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, -\frac{g[1][1]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, \right. \\
& \left. \frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ 0, 0, 0, -\frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0 \right\}, \right. \\
& \quad \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \quad \left\{ \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, \right. \\
& \quad \left. \frac{g[3][3]^{(0,1)}[x0, x4]}{2\sqrt{g[3][3][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \\
& \quad \left\{ 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[3][3][x0, x4]}g[4][4][x0, x4]}, 0, 0, 0, 0 \right\}, \\
& \quad \left. \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\}, \\
& \quad \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \\
& \quad \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \quad \left\{ \frac{g[4][4]^{(1,0)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[4][4][x0, x4]}, 0, 0, 0, \right. \\
& \quad \left. \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[4][4][x0, x4]}, 0, 0, 0 \right\}, \\
& \quad \left\{ 0, 0, 0, 0, 0, 0, \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}g[5][5][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[5][5][x0, x4]}, 0, 0 \right\}, \\
& \quad \left\{ 0, 0, 0, 0, 0, 0, 0, \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}g[6][6][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[6][6][x0, x4]}, 0 \right\}, \\
& \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}g[7][7][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \quad \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \\
& \quad \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \quad \left\{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \\
& \quad \left\{ \frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[5][5][x0, x4]}, 0, 0, \right.
\end{aligned}$$

$$\begin{aligned}
& 0, \frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 \sqrt{-g[4][4][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0, 0 \}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} \sqrt{-g[6][6][x_0, x_4]}}, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[6][6]^{(1,0)}[x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}{2 \sqrt{g[0][0][x_0, x_4]} g[6][6][x_0, x_4]}, 0, 0, 0, \right. \\
& \left. \frac{g[6][6]^{(0,1)}[x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}{2 \sqrt{-g[4][4][x_0, x_4]} g[6][6][x_0, x_4]}, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} \sqrt{-g[7][7][x_0, x_4]}} \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}} \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{g[7][7]^{(1,0)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2 \sqrt{g[0][0][x_0, x_4]} g[7][7][x_0, x_4]}, 0, 0, 0, \right. \\
& \left. \frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2 \sqrt{-g[4][4][x_0, x_4]} g[7][7][x_0, x_4]}, 0, 0, 0 \right\} \}
\end{aligned}$$

Out[505]=

{8, 8, 8}

In[506]:=

```

swgtry = Block[{eAα, eαA, t, V},
  eαA = eα(A) /. sgtryeα(A);
  eAα = e(A)α /. sgtrye(A)α;
  Table[(*V=eAα[[a,All]];*) ω(a)μ(b) [μ, a, b] → Block[{v, d}, v = eαA[[All, b]];
    d = Sum[(D[v[[k]], X[[μ]]] + Sum[v[[i]]×Γ[[k, i, μ]], {i, 1, DIM8}]) * eAα[[a, k]],
      {k, 1, DIM8}];
    FullSimplify[d, constraintVars] /. {Sqrt[e2 a4[H x4]] → ea4[H x4], Sqrt[e-2 a4[H x4]] →
      e-a4[H x4]}], {μ, 1, DIM8}, {a, 1, Length[eAα]}, {b, 1, Length[eαA]}]]

```

Out[506]=

$$\left\{ \left\{ \omega_{μ(b)}^{(a)} [1, 1, 1] \rightarrow 0, \omega_{μ(b)}^{(a)} [1, 1, 2] \rightarrow 0, \omega_{μ(b)}^{(a)} [1, 1, 3] \rightarrow 0, \omega_{μ(b)}^{(a)} [1, 1, 4] \rightarrow 0, \right. \right.$$

$$\left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 5, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 6] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 8] \rightarrow e^{-a4[H \times 4]} H \sin[6H \times 0]^{1/6} a4'[H \times 4], \\ \omega_{\mu(b)}^{(a)} [8, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 8] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 8] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 8, 1] \rightarrow e^{-a4[H \times 4]} H \sin[6H \times 0]^{1/6}, \quad \omega_{\mu(b)}^{(a)} [8, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 8, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 5] \rightarrow -e^{-a4[H \times 4]} H \sin[6H \times 0]^{1/6} a4'[H \times 4], \\ \omega_{\mu(b)}^{(a)} [8, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 8] \rightarrow 0 \end{array} \right\} \right\}$$

In[507]:=

swfgtry = Flatten[swgtry];

In[508]:=

```
(* $\omega_{\mu(b)}^{(a)}$ *)
ω;
ωω = % /. swfgtry // FullSimplify[#, constraintVars] &
Dimensions[%]
```



```
In[511]:= g4488 /. sg
% /. ssgm4488

Out[511]= {{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}

Out[512]= {{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

check metric compatibility condition for the Octad

In[5]:= Block[{eAα, eαA, t, ginv, Γ, gg(*, ωω*)},
(*ωω=ω^(a)_(b) /. swf;*)
gg = g4488 /. sg /. ssgm4488;
{ginv, Γ} = con[gg];
eαA = e^(α)_(A) /. sge^(α)_(A); eAα = e^(A)_(α) /. sge^(A)_(α);
t = Table[Block[{v, V, contra, covari},
V = eAα[[b, All]];
v = eαA[[All, b]];
contra =
Table[D[v[[k]], X[[μ]]] + Sum[v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
(*no ω terms*)
covari = Table[D[V[[k]], X[[μ]]] - Sum[V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}] +
Sum[eAα[[J, All]][[k]] × ωmat[[μ, b, J]], {J, 1, DIM8}] //*
FullSimplify[#, constraintVars && e^2 a4[H x4] > 0 && e^a4[H x4] > 0 &&
e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0] &, {k, 1, DIM8}]; (*ω terms*)
(*{b-1, X[[μ]], {{contra}, {covari}}}, {b, 1, Length[eαA]}, {μ, 1, DIM8}];*)
{b - 1, X[[μ]], covari}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];

t]
FullSimplify[#, ssgm4488,
constraintVars && e^2 a4[H x4] > 0 && e^a4[H x4] > 0 && e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0] & /@ %
Out[5]= {{0, x0, {g[0][0]^(1,0)[x0, x4] + 24 H Csc[12 H x0] g[0][0][x0, x4], 2 √g[0][0][x0, x4]}}}
```

$$\begin{aligned}
& \left. 0, 0, 0, \frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \Big\}, \\
& \{0, x1, \{0, \{-g[1][1]^{(1,0)}[x0, x4] + 2e^{2a4[Hx4]} H \operatorname{Sec}[6Hx0] \operatorname{Sin}[6Hx0]^{4/3} \\
& \quad g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0, 0\}\}, \\
& \{0, x2, \{0, 0, \{-g[2][2]^{(1,0)}[x0, x4] + 2e^{2a4[Hx4]} H \operatorname{Sec}[6Hx0] \operatorname{Sin}[6Hx0]^{4/3} \\
& \quad g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0, 0\}\}, \\
& \{0, x3, \{0, 0, 0, \{-g[3][3]^{(1,0)}[x0, x4] + 2e^{2a4[Hx4]} H \operatorname{Sec}[6Hx0] \\
& \quad \operatorname{Sin}[6Hx0]^{4/3} g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0\}\}, \\
& \{0, x4, \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \Big\}, \\
& \{0, x5, \{0, 0, 0, 0, 0, \{-g[5][5]^{(1,0)}[x0, x4] - 2e^{-2a4[Hx4]} H \operatorname{Sec}[6Hx0] \\
& \quad \operatorname{Sin}[6Hx0]^{4/3} g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0, 0\}\}, \\
& \{0, x6, \{0, 0, 0, 0, 0, 0, \{-g[6][6]^{(1,0)}[x0, x4] - 2e^{-2a4[Hx4]} H \operatorname{Sec}[6Hx0] \\
& \quad \operatorname{Sin}[6Hx0]^{4/3} g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0\}\}, \\
& \{0, x7, \{0, 0, 0, 0, 0, 0, 0, \{-g[7][7]^{(1,0)}[x0, x4] - 2e^{-2a4[Hx4]} H \\
& \quad \operatorname{Sec}[6Hx0] \operatorname{Sin}[6Hx0]^{4/3} g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]})\}\}, \\
& \left\{ \left\{ 1, x0, \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4] - 2H \operatorname{Cot}[6Hx0] g[1][1][x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 1, x1, \left\{ \frac{g[1][1]^{(1,0)}[x0, x4] - 2H \operatorname{Cot}[6Hx0] g[1][1][x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, \right. \right. \\
& \quad \left. \left. 0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6Ha4'[Hx4] g[1][1][x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0 \right\} \right\}, \\
& \{1, x2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{1, x4, \\
& \quad \left. \left. \left. 0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6Ha4'[Hx4] g[1][1][x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \{1, x5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{1, x6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{1, x7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \left\{ \left\{ 2, x0, \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4] - 2H \operatorname{Cot}[6Hx0] g[2][2][x0, x4]}{2\sqrt{g[2][2][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, 0, 0 \right\} \right\}, \{2, x1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 2, x_2, \left\{ \frac{g[2][2]^{(1,0)}[x_0, x_4] - 2 H \operatorname{Cot}[6 H x_0] g[2][2][x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, \right. \right. \\
& \quad \left. \left. 0, \frac{g[2][2]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[2][2][x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \{2, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_4, \\
& \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[2][2][x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0, 0, 0 \right\} \}, \\
& \{2, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \left\{ \{3, x_0, \right. \\
& \quad \left. \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x_0, x_4] - 2 H \operatorname{Cot}[6 H x_0] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, 0 \right\} \right\}, \\
& \{3, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{ 3, x_3, \left\{ \frac{g[3][3]^{(1,0)}[x_0, x_4] - 2 H \operatorname{Cot}[6 H x_0] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}} \right., 0, 0, 0, \right. \\
& \quad \left. \left. \frac{g[3][3]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \{3, x_4, \\
& \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, 0 \right\} \}, \\
& \{3, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{ \{4, x_0, \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \{4, x_1, \\
& \{0, (g[1][1]^{(0,1)}[x_0, x_4] + 6 e^{2 a4[H x_4]} H \operatorname{Sin}[6 H x_0]^{1/3} a4'[H x_4] g[4][4][x_0, x_4]) / \\
& (2 \sqrt{-g[4][4][x_0, x_4]}), 0, 0, 0, 0, 0, 0\}, \{4, x_2, \{0, 0, \\
& (g[2][2]^{(0,1)}[x_0, x_4] + 6 e^{2 a4[H x_4]} H \operatorname{Sin}[6 H x_0]^{1/3} a4'[H x_4] g[4][4][x_0, x_4]) / \\
& (2 \sqrt{-g[4][4][x_0, x_4]}), 0, 0, 0, 0, 0\}, \{4, x_3, \{0, 0, 0, \\
& (g[3][3]^{(0,1)}[x_0, x_4] + 6 e^{2 a4[H x_4]} H \operatorname{Sin}[6 H x_0]^{1/3} a4'[H x_4] g[4][4][x_0, x_4]) / \\
& (2 \sqrt{-g[4][4][x_0, x_4]}), 0, 0, 0, 0, 0\}, \\
& \left\{ 4, x_4, \left\{ -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \{4, x_5, \\
& \{0, 0, 0, 0, 0, 0, \frac{1}{2} \left(-6 e^{-2 a4[H x_4]} H \operatorname{Sin}[6 H x_0]^{1/3} a4'[H x_4] - \frac{g[5][5]^{(0,1)}[x_0, x_4]}{g[4][4][x_0, x_4]} \right) \}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-g[4][4][x0, x4]}, 0, 0 \} \}, \{ 4, x6, \{ 0, 0, 0, 0, 0, 0, 0, \\
& \frac{1}{2} \left(-6 e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4] - \frac{g[6][6]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]}, 0 \} \}, \{ 4, x7, \{ 0, 0, 0, 0, 0, 0, 0, 0, \\
& \frac{1}{2} \left(-6 e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4] - \frac{g[7][7]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]} \right) \}, \{ \{ 5, x0, \\
& \{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0 \} \}, \\
& \{ 5, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 5, x2, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 5, x3, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 5, x4, \\
& \{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0 \} \}, \\
& \{ 5, x5, \{ -\frac{g[5][5]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0, \\
& 0, -\frac{g[5][5]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0 \} \}, \\
& \{ 5, x6, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 5, x7, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \{ \{ 6, x0, \\
& \{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0 \} \}, \\
& \{ 6, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 6, x2, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 6, x3, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 6, x4, \\
& \{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0 \} \}, \\
& \{ 6, x5, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 6, x6, \{ -\frac{g[6][6]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0, 0, \\
& 0, -\frac{g[6][6]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0 \} \}, \\
& \{ 6, x7, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \{ \{ 7, x0, \\
& \{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} \} \}, \\
& \{ 7, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 7, x2, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \},
\end{aligned}$$

```

{7, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {7, x4,
  {0, 0, 0, 0, 0, 0, 0, 0, -g[7][7]^(0,1)[x0, x4] + 6 H a4' [H x4] g[7][7][x0, x4]
   2 sqrt[-g[7][7][x0, x4]]}},
{7, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {7, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}},
{7, x7, {-g[7][7]^(1,0)[x0, x4] - 2 H Cot[6 H x0] g[7][7][x0, x4]
  2 sqrt[-g[7][7][x0, x4]], 0, 0,
  0, -g[7][7]^(0,1)[x0, x4] + 6 H a4' [H x4] g[7][7][x0, x4]
  2 sqrt[-g[7][7][x0, x4]], 0, 0, 0}}}
Out[*]=
{{{0, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{1, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{2, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {2, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {2, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {2, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {2, x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {2, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {2, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {2, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{3, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {3, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {3, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {3, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {3, x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {3, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {3, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {3, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{4, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {4, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {4, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {4, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {4, x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {4, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {4, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {4, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{5, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {5, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {5, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {5, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {5, x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {5, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {5, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {5, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{6, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {6, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {6, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {6, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {6, x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {6, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {6, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {6, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{7, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {7, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {7, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {7, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {7, x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {7, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {7, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {7, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}
In[*]:= Block[{eA $\alpha$ , e $\alpha$ A, t, ginv,  $\Gamma$ , gg(*,  $\omega$  $\omega$ *)},
  (* $\omega$  $\omega$ = $\omega$  $_{\mu}^{(a)}$  $_{(b)}^{(a)}$ /.Swf;*)
  gg = g4488 /. sg;
  {ginv,  $\Gamma$ } = con[gg];
  e $\alpha$ A = e $_{(A)}^{\alpha}$  /. sge $_{(A)}^{\alpha}$ ; eA $\alpha$  = e $_{\alpha}^{(A)}$  /. sge $_{\alpha}^{(A)}$ ;
  t = Table[ Block[{v, V, contra, covari},
    V = eA $\alpha$ [[b, All]];
    v = e $\alpha$ A[[All, b]];
    contra =
      Table[D[v[[k]], X[[ $\mu$ ]]] + Sum[v[[i]] $\times$  $\Gamma$ [[k, i,  $\mu$ ]], {i, 1, DIM8}], {k, 1, DIM8}];
      (*no  $\omega$  terms*)
    covari = Table[D[V[[k]], X[[ $\mu$ ]]] - Sum[V[[i]] $\times$  $\Gamma$ [[i, k,  $\mu$ ]], {i, 1, DIM8}] +
      Sum[eA $\alpha$ [[J, All]][[k]] $\times$ omat[[ $\mu$ , b, J]], {J, 1, DIM8}],
      {k, 1, DIM8}];
      (* $\omega$  terms*)
    {b-1, X[[ $\mu$ ]], {{contra}, {covari}}}], {b, 1, Length[e $\alpha$ A]}, { $\mu$ , 1, DIM8}]];

```

$t]$

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FullSimplify[#, ssym4488,
  constraintVars &&  $e^{2 a4[x4]} > 0 \&\& e^{a4[x4]} > 0 \&\& e^{-2 a4[x4]} > 0 \&\& e^{-a4[x4]} > 0$ ] & /@ %
Out[#]=
{{{{0, x0, {{0, 0, 0, 0, -g[0][0]^(0,1)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[4][4][x0, x4]), 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0}}}, {0, x1, {{0, g[1][1]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[1][1][x0, x4]), 0, 0, 0, 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x2, {{0, 0, g[2][2]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[2][2][x0, x4]), 0, 0, 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x3, {{0, 0, 0, g[3][3]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[3][3][x0, x4]), 0, 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x4, {{0, 0, 0, 0, g[4][4]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[4][4][x0, x4]), 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x5, {{0, 0, 0, 0, 0, g[5][5]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[5][5][x0, x4]), 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x6, {{0, 0, 0, 0, 0, 0, g[6][6]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[6][6][x0, x4]), 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x7, {{0, 0, 0, 0, 0, 0, 0, g[7][7]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[7][7][x0, x4])}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x0, {{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x1, {{-g[1][1]^(1,0)[x0, x4]/(2 g[0][0][x0, x4] Sqrt[g[1][1][x0, x4]]), 0, 0, 0, 0, 0, 0, 0}}, {0, 0, -g[1][1]^(0,1)[x0, x4]/(2 Sqrt[g[1][1][x0, x4]] g[4][4][x0, x4]), 0, 0, 0, 0}}}}}

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{ { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { 1, x2,
  { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } ,
{ 1, x3, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } ,
{ 1, x4, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } ,
{ 1, x5, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } ,
{ 1, x6, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } ,
{ 1, x7, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ { 2, x0, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x1, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x2, { { { - $\frac{g[2][2]^{(1,0)}[x0,x4]}{2 g[0][0][x0,x4] \sqrt{g[2][2][x0,x4]}}$ , 0,
  0, 0, - $\frac{g[2][2]^{(0,1)}[x0,x4]}{2 \sqrt{g[2][2][x0,x4]} g[4][4][x0,x4]}$ , 0, 0, 0 } } } } ,
{ { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } , { 2, x3,
  { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x4, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x5, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x6, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x7, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } } ,
{ { 3, x0, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x1, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x2, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x3, { { { - $\frac{g[3][3]^{(1,0)}[x0,x4]}{2 g[0][0][x0,x4] \sqrt{g[3][3][x0,x4]}}$ , 0,
  0, 0, - $\frac{g[3][3]^{(0,1)}[x0,x4]}{2 \sqrt{g[3][3][x0,x4]} g[4][4][x0,x4]}$ , 0, 0, 0 } } } } ,
{ { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } , { 3, x4,
  { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x5, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x6, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x7, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } } ,
{ { 4, x0, { { {  $\frac{g[0][0]^{(0,1)}[x0,x4]}{2 g[0][0][x0,x4] \sqrt{-g[4][4][x0,x4]}}$ , 0, 0, 0,
   $\frac{g[4][4]^{(1,0)}[x0,x4]}{2 (-g[4][4][x0,x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0,x4]}{2 \sqrt{-g[4][4][x0,x4]} g[4][4][x0,x4]}$ , 0, 0, 0 } } } } ,

```

$$\begin{aligned}
& \left\{ \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}} - \right. \right. \\
& \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]} - \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \right. \right. \\
& \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]} \right) g[4][4][x0, x4], 0, 0, 0 \right\} \} \}, \\
& \left\{ 4, x1, \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x0, x4]}{2g[1][1][x0, x4]\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x2, \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]}{2g[2][2][x0, x4]\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x3, \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2g[3][3][x0, x4]\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x4, \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4]\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \left. \left. \left. \frac{g[4][4]^{(0,1)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}} - \right. \right. \\
& \left. \left. \frac{g[4][4]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]} - \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \right. \right. \right. \\
& \left. \left. \left. \frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]} \right) g[4][4][x0, x4], 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x5, \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[5][5][x0, x4]}, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[6][6][x0, x4]}, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 4, x_7, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[4][4][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\} \right\}, \\
& \left\{ 5, x_0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2(-g[5][5][x_0, x_4])^{3/2}} + \right. \right. \right. \right. \\
& \quad \left. \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \quad -\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]}} - \frac{g[5][5]^{(1,0)}[x_0, x_4]\sqrt{-g[5][5][x_0, x_4]}}{2g[5][5][x_0, x_4]} - \\
& \quad \left(\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2(-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]} \right) \\
& \quad \left. \left. \left. \left. g[5][5][x_0, x_4], 0, 0 \right\} \right\} \right\}, \\
& \left\{ 5, x_1, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 5, x_2, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 5, x_3, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 5, x_4, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2(-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]}} - \right. \\
& \quad \left. \frac{g[5][5]^{(0,1)}[x_0, x_4]\sqrt{-g[5][5][x_0, x_4]}}{2g[5][5][x_0, x_4]} - \left(\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2(-g[5][5][x_0, x_4])^{3/2}} + \right. \right. \\
& \quad \left. \left. \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]} \right) g[5][5][x_0, x_4], 0, 0 \right\} \right\}, \\
& \left\{ 5, x_5, \left\{ \left\{ -\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2g[0][0][x_0, x_4]\sqrt{-g[5][5][x_0, x_4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2g[4][4][x_0, x_4]\sqrt{-g[5][5][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \quad \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 5, x_6, \right. \\
& \quad \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 5, x_7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 6, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}
\end{aligned}$$

$$\begin{aligned}
& \frac{g[7][7]^{(1,0)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2 g[7][7][x_0, x_4]} - \left(\frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 (-g[7][7][x_0, x_4])^{3/2}} + \right. \\
& \left. \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right) g[7][7][x_0, x_4] \} \}, \\
& \{7, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 (-g[7][7][x_0, x_4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]}} - \right. \\
& \left. \frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2 g[7][7][x_0, x_4]} - \right. \\
& \left. \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right) g[7][7][x_0, x_4] \} \}, \\
& \{7, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_7, \{\{\{-\frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}, 0, \\
& 0, 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}, 0, 0, 0\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\} \}
\end{aligned}$$

Out[=]

$$\begin{aligned}
& \{\{\{0, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_1, \{\{\{0, H, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_2, \{\{\{0, 0, H, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_3, \{\{\{0, 0, 0, H, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_5, \{\{\{0, 0, 0, 0, 0, H, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_6, \{\{\{0, 0, 0, 0, 0, 0, H, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, H\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x_1, \{\{\{-\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{2 a4[H x4]}} \right. \\
& \left. H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},
\end{aligned}$$

$$\left\{ 5, x_5, \left\{ \left\{ \sqrt{e^{-2 a4[Hx4]}} H \operatorname{Sec}[6 H x_0] \operatorname{Sin}[6 H x_0]^{7/6}, 0, 0, 0, 0, 3 \sqrt{e^{-2 a4[Hx4]}} H \operatorname{Sin}[6 H x_0]^{1/6} a4'[Hx4], 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \right\},$$

$$\left\{ 5, x_6, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 5, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 6, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 6, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 6, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 6, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 6, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 6, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 6, x_6, \left\{ \left\{ \sqrt{e^{-2 a4[Hx4]}} H \operatorname{Sec}[6 H x_0] \operatorname{Sin}[6 H x_0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{-2 a4[Hx4]}} H \operatorname{Sin}[6 H x_0]^{1/6} a4'[Hx4], 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \right\},$$

$$\left\{ 6, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 7, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 7, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 7, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 7, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 7, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 7, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 7, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\},$$

$$\left\{ 7, x_7, \left\{ \left\{ \sqrt{e^{-2 a4[Hx4]}} H \operatorname{Sec}[6 H x_0] \operatorname{Sin}[6 H x_0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{-2 a4[Hx4]}} H \operatorname{Sin}[6 H x_0]^{1/6} a4'[Hx4], 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \right\}$$

```
In[513]:= (*wPsi=Table[Sum[ExpandAll[
  (wμ^(a)_(b) [μ,a,b]/.swf)Simplify[(Transpose[Ψ16].σ16.SAB[a,b].Ψ16)], 
  {a,1,8},{b,1,8}],{μ,1,8}]*]
```

```
In[514]:= (*wPsi=Table[
  Sum[ExpandAll[(ωmat[μ,a,b])Simplify[(Transpose[Ψ16].σ16.SAB[a,b].Ψ16)]], 
  {a,1,8},{b,1,8}],{μ,1,8}]*)
```

```
In[515]:= (*jwPsi=ParallelTable[{j,base16[[j,2]],
  Table[{X[μ]},Simplify[(Transpose[Ψ16].σ16.(ZZZ0*T16^α[0]+T16^α[4]*ZZZ4)). 
  base16[[j,1]].(Sum[ωmat[μ,a,b]SAB[a,b],{a,1,8},{b,1,8}]]Ψ16)]}, 
  {μ,1,8}]},{j,1,Length[base16]]}*)
```

```
In[516]:= (*jωΨ=ParallelTable[{j,base16[[j,2]],

Table[Sum[ExpandAll[(wμ(a)(b) [μ,a,b]/.swf)Simplify[(Transpose[Ψ16].σ16.

(ZZZ0 *T16α[0]+T16α[4]* ZZZ4 ).base16[[j,1]].SAB[a,b].Ψ16)],

{a,1,8},{b,1,8}],{μ,1,8}]},{j,1,Length[base16]}]*)

In[517]:= (*jωΨ[[92]]*)

In[518]:= (*jωΨ[[92]]/.ssgm4488//FullSimplify[#,constraintVars]&*)
```

In[519]:=

SAB[[1]][[2]]

Out[519]=

$$\begin{aligned} & \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0 \right\} \end{aligned}$$

In[520]:=

$$\left(\omega_{\mu}^{(a)}_{(b)} [8, 8, 1] /. \text{Flatten}[swgtry] \right)$$

Out[520]=

$$e^{-a4[H \times 4]} H \sin[6 H \times 0]^{1/6}$$

In[521]:=

SAB[1, 2].¶16

Out[521]=

$$\left\{ \frac{1}{2} f_{16}[7][x_0, x_4], \frac{1}{2} f_{16}[6][x_0, x_4], -\frac{1}{2} f_{16}[5][x_0, x_4], -\frac{1}{2} f_{16}[4][x_0, x_4], \right.$$

$$\frac{1}{2} f_{16}[3][x_0, x_4], \frac{1}{2} f_{16}[2][x_0, x_4], -\frac{1}{2} f_{16}[1][x_0, x_4], -\frac{1}{2} f_{16}[0][x_0, x_4],$$

$$-\frac{1}{2} f_{16}[15][x_0, x_4], -\frac{1}{2} f_{16}[14][x_0, x_4], \frac{1}{2} f_{16}[13][x_0, x_4], \frac{1}{2} f_{16}[12][x_0, x_4],$$

$$\left. -\frac{1}{2} f_{16}[11][x_0, x_4], -\frac{1}{2} f_{16}[10][x_0, x_4], \frac{1}{2} f_{16}[9][x_0, x_4], \frac{1}{2} f_{16}[8][x_0, x_4] \right\}$$

In[522]:=

T16^A[0]

Out[522]=

$$\begin{aligned} & \{ \{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \end{aligned}$$

In[523]:=

SAB =

$$\text{Table}\left[\frac{1}{4} (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]), \{A1, 0, 7\}, \{B1, 0, 7\}\right];$$

Out[523]=

¶16

Out[523]=

$$\begin{aligned} & \{f_{16}[0][x_0, x_4], f_{16}[1][x_0, x_4], f_{16}[2][x_0, x_4], f_{16}[3][x_0, x_4], \\ & f_{16}[4][x_0, x_4], f_{16}[5][x_0, x_4], f_{16}[6][x_0, x_4], f_{16}[7][x_0, x_4], \\ & f_{16}[8][x_0, x_4], f_{16}[9][x_0, x_4], f_{16}[10][x_0, x_4], f_{16}[11][x_0, x_4], \\ & f_{16}[12][x_0, x_4], f_{16}[13][x_0, x_4], f_{16}[14][x_0, x_4], f_{16}[15][x_0, x_4]\} \end{aligned}$$

In[524]:=

$$\text{Sum}\left[\text{ExpandAll}\left[\left(\omega_{\mu}^{(a)}_{(b)} [4, a, b] /. \text{swfgtry}\right) \text{Simplify}[(SAB[a, b])]\right], \{a, 1, 8\}, \{b, 1, 8\}\right]. \Psi16 // \text{MatrixForm}$$

Out[524]/MatrixForm=

$$\begin{aligned} & -e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[5][x0, x4] \\ & e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[4][x0, x4] \\ & -e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[7][x0, x4] \\ & e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[6][x0, x4] \\ & -e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[1][x0, x4] \\ & e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[0][x0, x4] \\ & -e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[3][x0, x4] \\ & e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[2][x0, x4] \\ & e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[13][x0, x4] \\ & -e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[12][x0, x4] \\ & e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[15][x0, x4] \\ & -e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[14][x0, x4] \\ & e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[9][x0, x4] \\ & -e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[8][x0, x4] \\ & e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[11][x0, x4] \\ & -e^{a4[H x^4]} H \sin[6 H x0]^{1/6} f16[10][x0, x4] \end{aligned}$$

In[525]:=

```
(*Table[Block[{eA\alpha,e\alpha A,t,V},e\alpha A=e^\alpha_{(A)}/.sgtry e^\alpha_{(A)};
eA\alpha=e_\alpha^{(A)}/.sgtry e_\alpha^{(A)};
V=eA\alpha[[a,All]];
t=Table[Block[{v,d},v=e\alpha A[[All,b]];
d=D[v[[k]],X[[j]]]+Sum[v[[i]]\Gamma[[k,i,j]],{i,1,DIM8}];
d.V],{k,1,DIM8}];
t],{j,1,DIM8},{a,1,Length[e\alpha A]},{b,1,Length[e\alpha A}]*)
```

In[526]:=

```
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, \Gamma, Ricci\Gamma, Rie\Gamma, RS, G, ginv},
Print[Now];
t = AbsoluteTiming[
ginv = FullSimplify[Inverse[g], ass];
\Gamma = FullSimplify[Table[(1/2)*Sum[(ginv[[i, s]) *
(D[g[[s, j], X[[k]]] + D[g[[s, k], X[[j]]] - D[g[[j, k], X[[s]]]), {s, 1, DIM8
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
Ricci\Gamma = ParallelTable[
FullSimplify[D[\Gamma[[\mu, \nu, \beta]], X[[\alpha]]] - D[\Gamma[[\mu, \nu, \alpha]], X[[\beta]]] + Sum[
\Gamma[[\mu, s, \alpha]] \times \Gamma[[s, \nu, \beta]] - \Gamma[[\mu, s, \beta]] \times \Gamma[[s, \nu, \alpha]], {s, 1, DIM8}], ass],
{\mu, 1, DIM8}, {\nu, 1, DIM8}, {\alpha, 1, DIM8}, {\beta, 1, DIM8}]];
Rie\Gamma =
```

```

ParallelTable[ FullSimplify[Sum[ Ricci $\Gamma$ [\mathbf{\alpha}, \mathbf{\mu}, \mathbf{\alpha}, \mathbf{\nu}], {\mathbf{\alpha}, 1, DIM8}], ass], 
  {\mathbf{\mu}, 1, DIM8}, {\mathbf{\nu}, 1, DIM8}] ;
RS = FullSimplify[Tr[ ginv.Rie $\Gamma$ ], ass];
G = ParallelTable[
  FullSimplify[Rie $\Gamma$ [\mathbf{\alpha}] -  $\frac{1}{2}$  g[\mathbf{\alpha}] RS, ass], {\mathbf{\alpha}, 1, Length[Rie $\Gamma$ ]}];
Print[t];
Print[Now];
Return[{ginv, \mathbf{\Gamma}, Ricci $\Gamma$ , Rie $\Gamma$ , RS, G}]

```

verify $g_{\mu\nu|\alpha} = 0$:

In[526]:=

```

Simplify[
  Table[D[gtry[j, k], X[i]] - Sum[gtry[k, s] \times \mathbf{\Gamma}[s, i, j], {s, 1, DIM8}] - 
    Sum[gtry[j, s] \times \mathbf{\Gamma}[s, i, k], {s, 1, DIM8}],
  {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union

```

Out[526]=

{0}

CHECK

In[527]:=

MatrixMetric44 // MatrixForm

Out[527]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6 H x \theta]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2 a^4 [H x 4]} \sin[6 H x \theta]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2 a^4 [H x 4]} \sin[6 H x \theta]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2 a^4 [H x 4]} \sin[6 H x \theta]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 - e^{-2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[528]:=

$(e_{(A)}^\alpha . \text{MatrixMetric44}.e_{(A)}^\alpha / . \text{sgtry} e_{(A)}^\alpha) // \text{MatrixForm}$

Out[528]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[529]:= Transpose[e(A)].η4488.e(A) - MatrixMetric44 /. sgtrye(A) //>
FullSimplify[#, constraintVars] &

Out[529]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}};

In[530]:= Transpose[e(A)].MatrixMetric44.e(A) - η4488 /. sgtrye(A) //>
FullSimplify[#, constraintVars] &

Out[530]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}};

In[531]:= Inverse[e(A) /. sgtrye(A)] == e(A) /. sgtrye(A) //> FullSimplify[#, constraintVars] &

Out[531]= True

In[532]:= Block[{a, b, c, d, m}, m = {{a, b}, {c, d}};
Inverse[Transpose[m]] - Transpose[Inverse[m]]]

Out[532]= {{0, 0}, {0, 0}};

In[533]:= (*Inverse[Transpose[e(A)].MatrixMetric44.e(A)] ==
e(A).Inverse[MatrixMetric44].Transpose[e(A)] //>
FullSimplify[#, constraintVars]&*)

In[534]:= (*e(A).Inverse[MatrixMetric44].Transpose[e(A)] //>
FullSimplify[#, constraintVars]& &
%-Inverse[η4488]//Union[Flatten[#]]&*)

In[535]:= (*Transpose[e(A)] == e(A) //> FullSimplify[#, constraintVars]&*)

In[536]:= (* (FullSimplify[e(A).Inverse[MatrixMetric44].Transpose[e(A)] -
Inverse[η4488], constraintVars] //>
Union[Flatten[#]]&) //> FullSimplify[#, constraintVars]&*)
```

In[537]:=

```
(* FullSimplify[Transpose[e(A)].Inverse[MatrixMetric44].e(A),
  constraintVars]-Inverse[η4488]//Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)
```

In[538]:=

```
(* FullSimplify[Transpose[e(A)].ginv.e(A),constraintVars]-Inverse[η4488]//Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)
```

In[539]:=

$$(E_{(A)}^{\alpha} = \text{Table}\left[\sum e_{\alpha}^{(A)} [A1, \alpha1] \text{ginv}[\alpha1, \beta1] \times \eta4488 [A1, B1] / . \text{sgtry} e_{\alpha}^{(A)}, \{\alpha1, 1, 8\}, \{A1, 1, 8\}, \{\beta1, 1, 8\}, \{B1, 1, 8\}\right] // \text{FullSimplify}[\#, \text{constraintVars}] \&) // \text{Column}$$

Out[539]=

$$\begin{aligned} &\{\tan[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0\} \\ &\left\{0, \frac{e^{-a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0\right\} \\ &\left\{0, 0, \frac{e^{-a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0\right\} \\ &\left\{0, 0, 0, \frac{e^{-a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0\right\} \\ &\{0, 0, 0, 0, 1, 0, 0, 0, 0\} \\ &\left\{0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0\right\} \\ &\left\{0, 0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}, 0\right\} \\ &\left\{0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}\right\} \end{aligned}$$
Lagrangian Ψ 16 =
$$\kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi16].\sigma16.\text{T16}^A[A1].\Psi16_{,\alpha} g^{-1}{}^{\alpha\beta} \eta_{A1 B1} E_{\beta}^{B1} + \text{mass term}$$

In[540]:=

ass = constraintVars

Out[540]=

$$\begin{aligned} &x0 > 0 \&& x4 > 0 \&& H > 0 \&& 6Hx0 > 0 \&& 3Hx4 > 0 \&& a4[x4] > 0 \&& A4[t] > 0 \&& Q > 0 \&& \\ &z > 0 \&& t > 0 \&& M > 0 \&& K > 0 \&& e^{-2a4[Hx4]} > 0 \&& e^{-a4[Hx4]} > 0 \&& e^{2a4[Hx4]} > 0 \&& \\ &e^{a4[Hx4]} > 0 \&& \sin[6Hx0] > 0 \&& \cos[6Hx0] > 0 \&& \csc[6Hx0] > 0 \&& \sec[6Hx0] > 0 \&& \\ &\tan[6Hx0] > 0 \&& \cot[6Hx0] > 0 \&& \sin[z] > 0 \&& \sqrt{\sin[z]} > 0 \&& \sin[z]^{3/2} > 0 \&& \\ &\sqrt{\sin[z]} > 0 \&& \frac{1}{\sin[z]^{3/2}} > 0 \&& \frac{1}{\sqrt{\sin[z]}} > 0 \&& \cot[z] > 0 \&& \sqrt{\cot[z]} > 0 \&& \\ &\cot[z]^{3/2} > 0 \&& \sqrt{\cot[z]} > 0 \&& \tan[z] > 0 \&& \sec[z] > 0 \&& \sqrt{\csc[z]} > 0 \&& \csc[z] > 0 \end{aligned}$$

```
In[540]:= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]3/2 > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]3/2 > 0 && 1/Sqrt[Sin[z]] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]3/2 > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

```
Out[540]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]3/2 > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]3/2 > 0 && 1/Sqrt[Sin[z]] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]3/2 > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

```
In[541]:= ginv = FullSimplify[Inverse[gtry], ass]
```

```
Out[541]= True
```

```
In[542]:= ginv === Transpose[ginv]
```

```
Out[542]= True
```

verify metric covariant derivative again:

```
In[543]:=
```

```
Simplify[
  Table[D[gtry[[j, k]], X[[i]]] - Sum[gtry[[k, s]] \[Cross] \[Gamma][[s, i, j]], {s, 1, DIM8}] -
    Sum[gtry[[j, s]] \[Cross] \[Gamma][[s, i, k]], {s, 1, DIM8}],
  {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union
```

```
Out[543]=
```

```
{}
```

```
In[544]:=
```

```
RS
```

```
Out[544]=
```

$$6 H^2 (-7 + a4' [H x4]^2)$$

In[545]:=

```
EinsteinG
skelx[%]
```

Out[545]=

$$\left\{ \begin{aligned} & \left\{ -3 H^2 \operatorname{Cot}[6 H x_0]^2 (-5 + a4'[H x_4]^2), 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, -e^{2 a4[H x_4]} H^2 \sin[6 H x_0]^{1/3} (-15 + 3 a4'[H x_4]^2 - a4''[H x_4]), 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, -e^{2 a4[H x_4]} H^2 \sin[6 H x_0]^{1/3} (-15 + 3 a4'[H x_4]^2 - a4''[H x_4]), 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, -e^{2 a4[H x_4]} H^2 \sin[6 H x_0]^{1/3} (-15 + 3 a4'[H x_4]^2 - a4''[H x_4]), 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, -3 H^2 (7 + a4'[H x_4]^2), 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, e^{-2 a4[H x_4]} H^2 \sin[6 H x_0]^{1/3} (-15 + 3 a4'[H x_4]^2 + a4''[H x_4]), 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, e^{-2 a4[H x_4]} H^2 \sin[6 H x_0]^{1/3} (-15 + 3 a4'[H x_4]^2 + a4''[H x_4]), 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, e^{-2 a4[H x_4]} H^2 \sin[6 H x_0]^{1/3} (-15 + 3 a4'[H x_4]^2 + a4''[H x_4]) \right\} \end{aligned} \right\}$$

Out[546]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{pmatrix}$$

$$\text{Union}\left[\text{Table}\left[\begin{aligned} & \text{Sum}[\tau[A].\text{bas64}[k, 1].\sigma.\overline{\tau}[A] \times \eta8[A, A], \{A, 1, 8\}] - \text{ID8}\text{Tr}[\text{bas64}[k, 1].\sigma] + \\ & \frac{\text{bas64}[k, 1] - \text{Transpose}[\text{bas64}[k, 1]]}{2}.\sigma, \{k, 1, 64\}\right]\right]$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}$$

BASIS of 16×16 matrices :

In[547]:=

16 * 16

Out[547]=

256

In[548]:=

 2^8

Out[548]=

256

In[549]:=

Sum[Binomial[8, p], {p, 0, 8}]

Out[549]=

256

```
In[550]:= Id16 == ID16 == Dot @@ (T16A[#] & /@ Range[0, 8])
Out[550]= True

In[551]:= Clear[t160]; t160 = {{Id16, {Range[0, 8]}}, Length[t160]
Clear[t16A];
t16A = {};
Do[AppendTo[t16A, {T16A[j], {j}}], {j, 0, 7}];
Length[t16A]
Clear[t16AB];
t16AB = {};
Do[AppendTo[t16AB, {T16A[j].T16A[k], {j, k}}], {j, 0, 6}, {k, j + 1, 7}];
Length[t16AB]
Clear[t16ABC];
t16ABC = {};
Do[AppendTo[t16ABC, {T16A[j].T16A[k].T16A[h], {j, k, h}}], {j, 0, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[t16ABC]
Clear[t16ABCD];
t16ABCD = {};
Do[AppendTo[t16ABCD, {T16A[j].T16A[k].T16A[h].T16A[i], {j, k, h, i}}], {j, 0, 4}, {k, j + 1, 5}, {h, k + 1, 6}, {i, h + 1, 7}];
Length[t16ABCD]
Clear[t16ABCDE];
t16ABCDE = {};
Do[AppendTo[t16ABCDE, {T16A[j].T16A[k].T16A[h].T16A[i].T16A[l], {j, k, h, i, l}}], {j, 0, 3}, {k, j + 1, 4}, {h, k + 1, 5}, {i, h + 1, 6}, {l, i + 1, 7}];
Length[t16ABCDE]
Clear[t16ABCDEF];
t16ABCDEF = {};
Do[AppendTo[t16ABCDEF, {T16A[j].T16A[k].T16A[h].T16A[i].T16A[l].T16A[q], {j, k, h, i, l, q}}], {j, 0, 2}, {k, j + 1, 3}, {h, k + 1, 4}, {i, h + 1, 5}, {l, i + 1, 6}, {q, l + 1, 7}];
Length[t16ABCDEF]
Clear[t16ABCDEFG];
t16ABCDEFG = {};
Do[AppendTo[t16ABCDEFG, {T16A[j].T16A[k].T16A[h].T16A[i].T16A[l].T16A[q].T16A[r], {j, k, h, i, l, q, r}}], {j, 0, 1}, {k, j + 1, 2}, {q, l + 1, 7}];
```

```

{h, k + 1, 3}, {i, h + 1, 4}, {l, i + 1, 5}, {q, l + 1, 6}, {r, q + 1, 7}];

Length[t16ABCDEFG]
Clear[t16ABCDEFGH];
t16ABCDEFGH = {};
Do[AppendTo[t16ABCDEFGH, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].
T16^A[r].T16^A[s], {j, k, h, i, l, q, r, s}}], {j, 0, 1}, {k, j + 1, 1}, {h,
k + 1, 2}, {i, h + 1, 3}, {l, i + 1, 4}, {q, l + 1, 5}, {r, q + 1, 6}, {s, r + 1, 7}];

Length[t16ABCDEFGH]
% + %% + %%% + %%%% + %%%%% + %%%%%% + %%%%%%% + %%%%%%%%

```

```

Out[551]=
1

Out[552]=
8

Out[553]=
28

Out[554]=
56

Out[555]=
70

Out[556]=
56

Out[557]=
28

Out[558]=
8

Out[559]=
1

Out[560]=
256

```

```

In[561]:= (*base16=Flatten[{t160,t16A,t16AB,t16ABC,
t16ABCD,t16ABCDE,t16ABCDEF,t16ABCDEFG,t16ABCDEFGH},1];
Length[base16]*)

```

```

In[562]:= base16 = Flatten[{t16A, t16AB, t16ABC, t16ABCD,
t16ABCDE, t16ABCDEF, t16ABCDEFG, t16ABCDEFGH, t160}, 1];
Length[base16]

Out[562]=
256

```

```

In[563]:= Length[Union[base16[[All, 1]]]]
Out[563]= 256

In[564]:= Tr[#.#/16] & /@ base16[[All, 1]] // Flatten // Union
Out[564]= {-1, 1}

In[565]:= positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]
Out[565]= {1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40,
41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74,
75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115,
116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139,
140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171,
172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198,
199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231,
232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[566]:= negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]
Out[566]= {5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54,
55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86,
87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126,
127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160,
161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194,
195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219,
220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[567]:= {Length[positiveTrMM], Length[negativeTrMM]}
Out[567]= {136, 120}

In[568]:= dups = Block[{s}, s = base16[[All, 1]]; Select[s, MemberQ[s, -#] &]]; Length[dups]
Out[568]= 0

In[569]:= dups = Select[base16, MemberQ[base16[[All, 1]], -#[[1]]] &]; Length[dups]
Out[569]= 0

```


In[573]:=

(*BASE16[-1]*)

In[574]:=

(*BASE16[-2])

```
%[[1]]==BASE16[-1][[1]]*)
```

```

In[575]:= 16 * (16 + 1) / 2
16 * (16 - 1) / 2
% + %%

Out[575]= 136

Out[576]= 120

Out[577]= 256

In[578]:= Clear[antisymm16];
antisymm16 = {};
Do[If[base16[[k, 1]] === -Transpose[base16[[k, 1]]], AppendTo[antisymm16,
{base16[[k, 1]], {{k}, base16[[k, 2]]}}], {k, 1, Length[base16]}];
Length[antisymm16]
antisymm16[[%]]

Out[579]= 120

Out[580]= {{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}, {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {{254}, {1, 2, 3, 4, 5, 6, 7}}}

```

In[581]:=

```
Clear[symmm16];
symmm16 = {};
Do[If[base16[[k, 1]] === Transpose[base16[[k, 1]]], AppendTo[symmm16,
    {base16[[k, 1]], {{k}, base16[[k, 2]]}}], {k, 1, Length[base16]}];
Length[symmm16]
symmm16[[%]]
```

Out[582]=

136

Out[583]=

```
{ {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
  {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
  {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
  {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
  {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
  {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, 
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, 
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, 
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, 
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, 
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, 
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, 
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, 
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}}
```

In[584]:=

```
dupssymmm16 = Select[symmm16, MemberQ[symmm16[[All, 1]], -#[[1]]] &];
Length[dupssymmm16]
```

Out[584]=

0

```
In[585]:= #[[2]] & /@ symm16
Out[585]= {{}, {{1}, {0}}, {{2}, {1}}, {{3}, {2}}, {{4}, {3}}, {{12}, {0, 4}}, {{13}, {0, 5}}, {{14}, {0, 6}}, {{15}, {0, 7}}, {{18}, {1, 4}}, {{19}, {1, 5}}, {{20}, {1, 6}}, {{21}, {1, 7}}, {{23}, {2, 4}}, {{24}, {2, 5}}, {{25}, {2, 6}}, {{26}, {2, 7}}, {{27}, {3, 4}}, {{28}, {3, 5}}, {{29}, {3, 6}}, {{30}, {3, 7}}, {{39}, {0, 1, 4}}, {{40}, {0, 1, 5}}, {{41}, {0, 1, 6}}, {{42}, {0, 1, 7}}, {{44}, {0, 2, 4}}, {{45}, {0, 2, 5}}, {{46}, {0, 2, 6}}, {{47}, {0, 2, 7}}, {{48}, {0, 3, 4}}, {{49}, {0, 3, 5}}, {{50}, {0, 3, 6}}, {{51}, {0, 3, 7}}, {{59}, {1, 2, 4}}, {{60}, {1, 2, 5}}, {{61}, {1, 2, 6}}, {{62}, {1, 2, 7}}, {{63}, {1, 3, 4}}, {{64}, {1, 3, 5}}, {{65}, {1, 3, 6}}, {{66}, {1, 3, 7}}, {{73}, {2, 3, 4}}, {{74}, {2, 3, 5}}, {{75}, {2, 3, 6}}, {{76}, {2, 3, 7}}, {{89}, {4, 5, 6}}, {{90}, {4, 5, 7}}, {{91}, {4, 6, 7}}, {{92}, {5, 6, 7}}, {{93}, {0, 1, 2, 3}}, {{102}, {0, 1, 4, 5}}, {{103}, {0, 1, 4, 6}}, {{104}, {0, 1, 4, 7}}, {{105}, {0, 1, 5, 6}}, {{106}, {0, 1, 5, 7}}, {{107}, {0, 1, 6, 7}}, {{112}, {0, 2, 4, 5}}, {{113}, {0, 2, 4, 6}}, {{114}, {0, 2, 4, 7}}, {{115}, {0, 2, 5, 6}}, {{116}, {0, 2, 5, 7}}, {{117}, {0, 2, 6, 7}}, {{118}, {0, 3, 4, 5}}, {{119}, {0, 3, 4, 6}}, {{120}, {0, 3, 4, 7}}, {{121}, {0, 3, 5, 6}}, {{122}, {0, 3, 5, 7}}, {{123}, {0, 3, 6, 7}}, {{132}, {1, 2, 4, 5}}, {{133}, {1, 2, 4, 6}}, {{134}, {1, 2, 4, 7}}, {{135}, {1, 2, 5, 6}}, {{136}, {1, 2, 5, 7}}, {{137}, {1, 2, 6, 7}}, {{138}, {1, 3, 4, 5}}, {{139}, {1, 3, 4, 6}}, {{140}, {1, 3, 4, 7}}, {{141}, {1, 3, 5, 6}}, {{142}, {1, 3, 5, 7}}, {{143}, {1, 3, 6, 7}}, {{148}, {2, 3, 4, 5}}, {{149}, {2, 3, 4, 6}}, {{150}, {2, 3, 4, 7}}, {{151}, {2, 3, 5, 6}}, {{152}, {2, 3, 5, 7}}, {{153}, {2, 3, 6, 7}}, {{162}, {4, 5, 6, 7}}, {{167}, {0, 1, 2, 4, 5}}, {{168}, {0, 1, 2, 4, 6}}, {{169}, {0, 1, 2, 4, 7}}, {{170}, {0, 1, 2, 5, 6}}, {{171}, {0, 1, 2, 5, 7}}, {{172}, {0, 1, 2, 6, 7}}, {{173}, {0, 1, 3, 4, 5}}, {{174}, {0, 1, 3, 4, 6}}, {{175}, {0, 1, 3, 4, 7}}, {{176}, {0, 1, 3, 5, 6}}, {{177}, {0, 1, 3, 5, 7}}, {{178}, {0, 1, 3, 6, 7}}, {{183}, {0, 2, 3, 4, 5}}, {{184}, {0, 2, 3, 4, 6}}, {{185}, {0, 2, 3, 4, 7}}, {{186}, {0, 2, 3, 5, 6}}, {{187}, {0, 2, 3, 5, 7}}, {{188}, {0, 2, 3, 6, 7}}, {{197}, {0, 4, 5, 6, 7}}, {{198}, {1, 2, 3, 4, 5}}, {{199}, {1, 2, 3, 4, 6}}, {{200}, {1, 2, 3, 4, 7}}, {{201}, {1, 2, 3, 5, 6}}, {{202}, {1, 2, 3, 5, 7}}, {{203}, {1, 2, 3, 6, 7}}, {{212}, {1, 4, 5, 6, 7}}, {{217}, {2, 4, 5, 6, 7}}, {{218}, {3, 4, 5, 6, 7}}, {{225}, {0, 1, 2, 4, 5, 6}}, {{226}, {0, 1, 2, 4, 5, 7}}, {{227}, {0, 1, 2, 4, 6, 7}}, {{228}, {0, 1, 2, 5, 6, 7}}, {{229}, {0, 1, 3, 4, 5, 6}}, {{230}, {0, 1, 3, 4, 5, 7}}, {{231}, {0, 1, 3, 4, 6, 7}}, {{232}, {0, 1, 3, 5, 6, 7}}, {{234}, {0, 2, 3, 4, 5, 6}}, {{235}, {0, 2, 3, 4, 5, 7}}, {{236}, {0, 2, 3, 4, 6, 7}}, {{237}, {0, 2, 3, 5, 6, 7}}, {{240}, {1, 2, 3, 4, 5, 6}}, {{241}, {1, 2, 3, 4, 5, 7}}, {{242}, {1, 2, 3, 4, 6, 7}}, {{243}, {1, 2, 3, 5, 6, 7}}, {{247}, {0, 1, 2, 3, 4, 5, 6}}, {{248}, {0, 1, 2, 3, 4, 5, 7}}, {{249}, {0, 1, 2, 3, 4, 6, 7}}, {{250}, {0, 1, 2, 3, 5, 6, 7}}, {{255}, {0, 1, 2, 3, 4, 5, 6, 7}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```



```
In[593]:= σ16.T16A[5].T16A[6].T16A[7] == symm16[[134]][1]
```

```
Out[593]= True
```

```
In[594]:= T16A[8] == base16[[255]][1]
```

```
Out[594]= True
```

```
In[595]:= Select[symm16, #[2][1][1] == 255 & → "Index"]
```

```
Out[595]= {135}
```

similarly:

```
In[596]:= coupled1 = {4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13} // Union
```

```
Out[596]= {0, 1, 4, 5, 8, 9, 12, 13}
```

```
In[597]:= coupled2 = Complement[Range[0, 15], coupled1]
```

```
Out[597]= {2, 3, 6, 7, 10, 11, 14, 15}
```

```
In[598]:= coupled = {coupled1, coupled2}
```

```
Out[598]= {{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}}
```

```
In[599]:= Block[{m, vars, eqs, eqs1, eqs2, s, Φ16},
Φ16 = f[#] & /@ Range[16];
m = Array[qb[#1, #2] &, {16, 16}];
vars = Flatten[m];
eqs1 = And @@
```

```
Thread[m.Φ16 == Flatten[{Φ16[[# + 1]] & /@ coupled1, Φ16[[# + 1]] & /@ coupled2}]];
eqs2 = And @@ Thread[m.Transpose[m] == Flatten[ID16]];
eqs = eqs1 && eqs2;
```

```
s = Solve[eqs, vars] // FullSimplify;
s
```

```
]
```

```
Out[599]= {}
```


In[601]:=

`linTrans // MatrixForm`

Out[601]//MatrixForm=

In[602]:=

```
linTrans.Transpose[linTrans] // FullSimplify // MatrixForm
```

Out[602]//MatrixForm=

```
In[603]:= linTrans.¶16 // Column
Union[%[[1]]]

Out[603]= f16[0] [x0, x4]
f16[1] [x0, x4]
f16[8] [x0, x4]
f16[9] [x0, x4]
f16[2] [x0, x4]
f16[3] [x0, x4]
f16[10] [x0, x4]
f16[11] [x0, x4]
f16[4] [x0, x4]
f16[5] [x0, x4]
f16[12] [x0, x4]
f16[13] [x0, x4]
f16[6] [x0, x4]
f16[7] [x0, x4]
f16[14] [x0, x4]
f16[15] [x0, x4]

Out[604]= {f16[0] [x0, x4], f16[1] [x0, x4], f16[2] [x0, x4], f16[3] [x0, x4],
f16[4] [x0, x4], f16[5] [x0, x4], f16[6] [x0, x4], f16[7] [x0, x4],
f16[8] [x0, x4], f16[9] [x0, x4], f16[10] [x0, x4], f16[11] [x0, x4],
f16[12] [x0, x4], f16[13] [x0, x4], f16[14] [x0, x4], f16[15] [x0, x4]}

In[6]:= positiveTrMM = Select[base16, Tr[#[[1]] . #[[1]]] > 0 & -> "Index"]

Out[6]= {1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59,
60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116,
117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151,
152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198,
199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242,
243, 247, 248, 249, 250, 255, 256}

In[7]:= negativeTrMM = Select[base16, Tr[#[[1]] . #[[1]]] < 0 & -> "Index"]

Out[7]= {5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77,
78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128,
129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182,
189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221,
222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[8]:= {Length[positiveTrMM], Length[negativeTrMM]}

Out[8]= {136, 120}
```

```
In[605]:= positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]
Out[605]= {1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40,
41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74,
75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115,
116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139,
140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171,
172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198,
199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231,
232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[606]:= negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]
Out[606]= {5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54,
55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86,
87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126,
127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160,
161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194,
195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219,
220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[607]:= {Length[positiveTrMM], Length[negativeTrMM]}
Out[607]= {136, 120}
```

```
In[608]:= base16[[#] & /@ positiveTrMM;
{#[[2]], Tr[Transpose[#[[1]].linTrans]]} & /@%
Select[% , #[[2]] ≠ 0 &]
linTransSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];
Out[609]= {{{0}, 0}, {{1}, 0}, {{2}, 0}, {{3}, 0}, {{0, 4}, 0}, {{0, 5}, 0}, {{0, 6}, 0},
{{0, 7}, -4}, {{1, 4}, 0}, {{1, 5}, 0}, {{1, 6}, 0}, {{1, 7}, 0}, {{2, 4}, 0},
{{2, 5}, 0}, {{2, 6}, 0}, {{2, 7}, 0}, {{3, 4}, -4}, {{3, 5}, 0}, {{3, 6}, 0},
{{3, 7}, 0}, {{0, 1, 4}, 0}, {{0, 1, 5}, 0}, {{0, 1, 6}, 0}, {{0, 1, 7}, 0},
{{0, 2, 4}, 0}, {{0, 2, 5}, 0}, {{0, 2, 6}, 0}, {{0, 2, 7}, 0}, {{0, 3, 4}, 0},
{{0, 3, 5}, 0}, {{0, 3, 6}, 0}, {{0, 3, 7}, 0}, {{1, 2, 4}, 0}, {{1, 2, 5}, 0},
{{1, 2, 6}, 0}, {{1, 2, 7}, 0}, {{1, 3, 4}, 0}, {{1, 3, 5}, 0}, {{1, 3, 6}, 0},
{{1, 3, 7}, 0}, {{2, 3, 4}, 0}, {{2, 3, 5}, 0}, {{2, 3, 6}, 0}, {{2, 3, 7}, 0},
{{4, 5, 6}, -4}, {{4, 5, 7}, 0}, {{4, 6, 7}, 0}, {{5, 6, 7}, 0}, {{0, 1, 2, 3}, 0},
{{0, 1, 4, 5}, 0}, {{0, 1, 4, 6}, 0}, {{0, 1, 4, 7}, 0}, {{0, 1, 5, 6}, 0},
{{0, 1, 5, 7}, 0}, {{0, 1, 6, 7}, 0}, {{0, 2, 4, 5}, 0}, {{0, 2, 4, 6}, 0},
{{0, 2, 4, 7}, 0}, {{0, 2, 5, 6}, 0}, {{0, 2, 5, 7}, 0}, {{0, 2, 6, 7}, 0},
{{0, 3, 4, 5}, 0}, {{0, 3, 4, 6}, 0}, {{0, 3, 4, 7}, 4}, {{0, 3, 5, 6}, 0},
{{0, 3, 5, 7}, 0}, {{0, 3, 6, 7}, 0}, {{1, 2, 4, 5}, 0}, {{1, 2, 4, 6}, 0},
{{1, 2, 4, 7}, 0}, {{1, 2, 5, 6}, 0}, {{1, 2, 5, 7}, 0}, {{1, 2, 6, 7}, -4},
{{1, 3, 4, 5}, 0}, {{1, 3, 4, 6}, 0}, {{1, 3, 4, 7}, 0}, {{1, 3, 5, 6}, 0},
{{1, 3, 5, 7}, 0}, {{1, 3, 6, 7}, 0}, {{2, 3, 4, 5}, 0}, {{2, 3, 4, 6}, 0},
{{2, 3, 4, 7}, 0}, {{2, 3, 5, 6}, 0}, {{2, 3, 5, 7}, 0}, {{2, 3, 6, 7}, 0},
{{4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5}, 4}, {{0, 1, 2, 4, 6}, 0}, {{0, 1, 2, 4, 7}, 0},
{{0, 1, 2, 5, 6}, 0}, {{0, 1, 2, 5, 7}, 0}, {{0, 1, 2, 6, 7}, 0}, {{0, 1, 3, 4, 5}, 0},
{{0, 1, 3, 4, 6}, 0}, {{0, 1, 3, 4, 7}, 0}, {{0, 1, 3, 5, 6}, 0}, {{0, 1, 3, 5, 7}, 0},
{{0, 1, 3, 6, 7}, 0}, {{0, 2, 3, 4, 5}, 0}, {{0, 2, 3, 4, 6}, 0}, {{0, 2, 3, 4, 7}, 0},
{{0, 2, 3, 5, 6}, 0}, {{0, 2, 3, 5, 7}, 0}, {{0, 2, 3, 6, 7}, 0}, {{0, 4, 5, 6, 7}, 4},
{{1, 2, 3, 4, 5}, 0}, {{1, 2, 3, 4, 6}, 0}, {{1, 2, 3, 4, 7}, 0}, {{1, 2, 3, 5, 6}, 0},
{{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 6, 7}, 0}, {{1, 4, 5, 6, 7}, 0}, {{2, 4, 5, 6, 7}, 0},
{{3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6}, 0}, {{0, 1, 2, 4, 5, 7}, 0},
{{0, 1, 2, 4, 6, 7}, 0}, {{0, 1, 2, 5, 6, 7}, 0}, {{0, 1, 3, 4, 5, 6}, 0},
{{0, 1, 3, 4, 5, 7}, 0}, {{0, 1, 3, 4, 6, 7}, 0}, {{0, 1, 3, 5, 6, 7}, 0},
{{0, 2, 3, 4, 5, 6}, 0}, {{0, 2, 3, 4, 5, 7}, 0}, {{0, 2, 3, 4, 6, 7}, 0},
{{0, 2, 3, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6}, 0}, {{1, 2, 3, 4, 5, 7}, 0},
{{1, 2, 3, 4, 6, 7}, -4}, {{1, 2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6}, 0},
{{0, 1, 2, 3, 4, 5, 7}, 0}, {{0, 1, 2, 3, 4, 6, 7}, 0}, {{0, 1, 2, 3, 5, 6, 7}, 0},
{{0, 1, 2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6, 8}, 4}}
Out[610]= {{0, 7}, -4}, {3, 4}, -4}, {4, 5, 6}, -4}, {0, 3, 4, 7}, 4},
{1, 2, 6, 7}, -4}, {0, 1, 2, 4, 5}, 4}, {0, 4, 5, 6, 7}, 4},
{1, 2, 3, 5, 7}, -4}, {1, 2, 3, 4, 6, 7}, -4}, {{0, 1, 2, 3, 4, 5, 6, 7}, 4}}
```



```
In[613]:= base16[[#]] & /@ negativeTrMM;
{#[[2]], Tr[Transpose[#[[1]].linTrans]]} & /@%
Select[% , #[[2]] ≠ 0 &]
linTransAntiSymmetric =
Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];
Out[614]=
{{{4}, 0}, {{5}, 0}, {{6}, 0}, {{7}, 0}, {{0, 1}, 0}, {{0, 2}, 0}, {{0, 3}, 0},
{{1, 2}, 0}, {{1, 3}, 0}, {{2, 3}, 0}, {{4, 5}, 0}, {{4, 6}, 0}, {{4, 7}, 0},
{{5, 6}, 0}, {{5, 7}, 0}, {{6, 7}, 0}, {{0, 1, 2}, 0}, {{0, 1, 3}, 0},
{{0, 2, 3}, 0}, {{0, 4, 5}, 0}, {{0, 4, 6}, 0}, {{0, 4, 7}, 0}, {{0, 5, 6}, 0},
{{0, 5, 7}, 0}, {{0, 6, 7}, 0}, {{1, 2, 3}, 0}, {{1, 4, 5}, 0}, {{1, 4, 6}, 0},
{{1, 4, 7}, 0}, {{1, 5, 6}, 0}, {{1, 5, 7}, 0}, {{1, 6, 7}, 0}, {{2, 4, 5}, 0},
{{2, 4, 6}, 0}, {{2, 4, 7}, 0}, {{2, 5, 6}, 0}, {{2, 5, 7}, 0}, {{2, 6, 7}, 0},
{{3, 4, 5}, 0}, {{3, 4, 6}, 0}, {{3, 4, 7}, 0}, {{3, 5, 6}, -4}, {{3, 5, 7}, 0},
{{3, 6, 7}, 0}, {{0, 1, 2, 4}, 0}, {{0, 1, 2, 5}, 0}, {{0, 1, 2, 6}, -4},
{{0, 1, 2, 7}, 0}, {{0, 1, 3, 4}, 0}, {{0, 1, 3, 5}, 0}, {{0, 1, 3, 6}, 0},
{{0, 1, 3, 7}, 0}, {{0, 2, 3, 4}, 0}, {{0, 2, 3, 5}, 0}, {{0, 2, 3, 6}, 0},
{{0, 2, 3, 7}, 0}, {{0, 4, 5, 6}, 0}, {{0, 4, 5, 7}, 0}, {{0, 4, 6, 7}, 0},
{{0, 5, 6, 7}, 0}, {{1, 2, 3, 4}, 0}, {{1, 2, 3, 5}, 0}, {{1, 2, 3, 6}, 0},
{{1, 2, 3, 7}, 0}, {{1, 4, 5, 6}, 0}, {{1, 4, 5, 7}, 0}, {{1, 4, 6, 7}, 0},
{{1, 5, 6, 7}, 0}, {{2, 4, 5, 6}, 0}, {{2, 4, 5, 7}, 0}, {{2, 4, 6, 7}, 0},
{{2, 5, 6, 7}, 0}, {{3, 4, 5, 6}, 0}, {{3, 4, 5, 7}, 0}, {{3, 4, 6, 7}, 0},
{{3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4}, 0}, {{0, 1, 2, 3, 5}, -4}, {{0, 1, 2, 3, 6}, 0},
{{0, 1, 2, 3, 7}, 0}, {{0, 1, 4, 5, 6}, 0}, {{0, 1, 4, 5, 7}, 0},
{{0, 1, 4, 6, 7}, 0}, {{0, 1, 5, 6, 7}, 0}, {{0, 2, 4, 5, 6}, 0}, {{0, 2, 4, 5, 7}, 0},
{{0, 2, 4, 6, 7}, 0}, {{0, 2, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6}, 0}, {{0, 3, 4, 5, 7}, 0},
{{0, 3, 4, 6, 7}, 0}, {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 6}, 0}, {{1, 2, 4, 5, 7}, 4},
{{1, 2, 4, 6, 7}, 0}, {{1, 2, 5, 6, 7}, 0}, {{1, 3, 4, 5, 6}, 0}, {{1, 3, 4, 5, 7}, 0},
{{1, 3, 4, 6, 7}, 0}, {{1, 3, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6}, 0}, {{2, 3, 4, 5, 7}, 0},
{{2, 3, 4, 6, 7}, 0}, {{2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5}, 0},
{{0, 1, 2, 3, 4, 6}, -4}, {{0, 1, 2, 3, 4, 7}, 0}, {{0, 1, 2, 3, 5, 6}, 0},
{{0, 1, 2, 3, 5, 7}, 0}, {{0, 1, 2, 3, 6, 7}, 0}, {{0, 1, 4, 5, 6, 7}, 0},
{{0, 2, 4, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6, 7}, 0}, {{1, 2, 4, 5, 6, 7}, 0},
{{1, 3, 4, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6, 7}, 0},
{{0, 1, 3, 4, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6, 7}, 0}}
Out[615]=
{{{3, 5, 6}, -4}, {{0, 1, 2, 6}, -4}, {{0, 1, 2, 3, 5}, -4},
{{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 7}, 4}, {{0, 1, 2, 3, 4, 6}, -4}}
In[617]:= {Length[linTransSymmetric], Length[linTransAntiSymmetric]}
Out[617]=
{10, 6}
```

BASIS of 8×8 matrices :

In[618]:=

```
tA = {} ; Do[AppendTo[tA, {τ[j], {j}}], {j, 1, 7}]
Length[tA]
tAB = {};
Do[AppendTo[tAB, {τ[j].τ[k], {j, k}}], {j, 1, 6}, {k, j + 1, 7}]
Length[tAB]
tABC = {};
Do[AppendTo[tABC, {τ[j].τ[k].τ[h], {j, k, h}}],
{j, 1, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[tABC]
```

Out[619]=

7

Out[621]=

21

Out[623]=

35

In[624]:=

tA[[1]]

Out[624]=

```
{ {{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
{0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0},
{0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0} }, {1} }
```

In[625]:=

tA[[#]] [[1]] == -Transpose[tA[[#]] [[1]]] & /@ Range[Length[tA]]

Out[625]=

{True, True, True, False, False, False}

In[626]:=

bas64 = Flatten[{tA, tAB, tABC, {{Id, {64}}}}, 1]; Length[bas64]

Out[626]=

64

```
In[627]:= determineIF8AntiSymmetric[m_] := Module[{t, tt, sum, res, r},
  t = m.τ[#] & /@ Range[0, 7];
  tt = Transpose[#, & /@ t];
  sum = FullSimplify[t[[#]] + tt[[#]]] & /@ Range[Length[t]];
  res = Union[Flatten[#, 1] & /@ sum];
  If[Length[#] > 1, Return[False]] & /@ res;
  r = Union[Flatten[res]];
  Return[r == {0}];
]
```

```
In[628]:= determineIF8AntiSymmetric[Zero8]
```

Out[628]=

```
In[629]:= determineTE8AntiSymmetric[ID81]
```

Out[629]=

```
find8AntiSymmetric =
  If[determineIF8AntiSymmetric[#[[1]], #[[2]], ##]&1 &/@ bas64
```

Out[630]=

In[631]:=

```

anti = {};
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],
    AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[anti]
anti[[28]]

```

Out[632]=

28

Out[633]=

```

\{\{0, 1, 0, 0, 0, 0, 0, 0, 0\}, \{-1, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, -1, 0, 0, 0, 0, 0\},
\{0, 0, 1, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0, 0\},
\{0, 0, 0, 0, 0, 0, 0, 1\}, \{0, 0, 0, 0, 0, 0, 0, -1, 0\}\}, \{\{63\}, \{5, 6, 7\}\}\}

```

In[634]:=

```

symm = {};
Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
  AppendTo[symm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[symm]
symm[[%]]

```

Out[635]=

35

Out[636]=

```

{{{{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1},
{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}}, {{59}, {3, 6, 7}}}}

```

In[637]:=

```

η64 = DiagonalMatrix[
Table[FullSimplify[ $\frac{1}{8} \text{Tr}[\text{bas64}[A, 1].\text{bas64}[A, 1]]$ ], {A, 64}]];
Tr[η64]

```

Out[637]=

$$7 + \frac{1}{8} \text{Tr}[\text{Id}.\text{Id}]$$

In[638]:=

```

(* (* (* (* (* (* (countTrace=
Table[{A, B, FullSimplify[ $\frac{1}{8} \text{Tr}[\text{bas64}[A, 1].\text{bas64}[B, 1]]$ ] }, {A, 1, 64}, {B, A, 64}]) // MatrixForm*) *) *) *)

```

In[639]:=

(***)

In[640]:=

32 × 65

Out[640]=

2080

In[641]:=

(*Length[Flatten[countTrace, 1]]*)

In[642]:=

(*{Length[countTrace], Length[countTrace[[1]]], Length[countTrace[[1, 1]]]}*)

In[643]:=

(*{Length[countTrace], Length[countTrace[[2]]], Length[countTrace[[2, 1]]]}*)

In[644]:=

(*{Length[countTrace], Length[countTrace[[64]]], Length[countTrace[[64, 1]]]}*)

```
In[645]:= (*{Length[Flatten[countTrace,1][All,3]],  
Count[Flatten[countTrace,1][All,3],0],  
Count[Flatten[countTrace,1][All,3],1],  
Count[Flatten[countTrace,1][All,3],-1]}*)
```

```
In[646]:= 32 × 63
```

```
Out[646]= 2016
```

```
In[647]:=
```

```
anti = {};  
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],  
AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}], {k, 1, 64}];  
Length[anti]  
anti[[28]]
```

```
Out[648]= 28
```

```
Out[649]= {{ {0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},  
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},  
{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}}
```

```
In[650]:= anti[[28, 1]]
```

```
Out[650]= {{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},  
{0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0},  
{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}
```

```
In[651]:= anti[[28, 2]]
```

```
Out[651]= {{63}, {5, 6, 7}}
```

In[652]:=

```

symm = {};
Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
AppendTo[symm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[symm]
symm[[28]]

```

Out[653]=

35

Out[654]=

```

{{{-1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
{0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 1}}, {{52}, {2, 5, 7}}}

```

For the split orthogonal group Spin(4,4) and its associated Lie algebra, there exist three fundamentally equivalent 8-dimensional representations:

1. The vector representation V
2. The type-1 spinor representation S+ (or S1), positive chirality
3. The type-2 spinor representation S- (or S2), negative chirality

These representations are related by outer automorphisms of Spin(4,4). All three representations have the same dimension: $\dim(V) = \dim(S1) = \dim(S2) = 8$

This is unique to dimension 8 and forms the basis of triality: Cartan, E. (1925). La th' eorie des spineurs. Hermann, Paris.

metric-preserving transformations; find automorphism $\sigma g \longleftrightarrow \sigma g^A_A$

$\sigma.g == \text{Transpose}[\sigma g^{-1}].g$

$\sigma == \text{Transpose}[\sigma g^{-1}].g.\sigma g^{-1}$

$(\sigma g)^A_A \equiv \sigma g^A_A$

$(\sigma g^{-1})^A_a \equiv \sigma g^A_a ; (\text{Transpose}[\sigma g^{-1}])^A_a \equiv \sigma g^A_a$

$\sigma_{ab} == \sigma g^A_a g_{AB} \sigma g^B_b$

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$N = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[655]:=

$$\text{Clear}[\sigma g_{1A}^a]; \sigma g_{1A}^a = \frac{1}{\sqrt{2}} (\eta 4488 + \sigma)$$

Out[655]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \\ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

In[656]:=

$$\sigma g_{2A}^a = \frac{1}{\sqrt{2}} \{ \{ 1, 0, 0, 0, 1, 0, 0, 0 \}, \{ 0, 0, 1, 0, 0, 0, 1, 0 \}, \\ \{ 0, 1, 0, 0, 0, 1, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, 1 \}, \\ \{ 1, 0, 0, 0, -1, 0, 0, 0 \}, \{ 0, 0, 1, 0, 0, 0, -1, 0 \}, \\ \{ 0, 1, 0, 0, 0, -1, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, -1 \} \}$$

Out[656]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \\ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \\ \left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

```

In[657]:= (s1s2 = σg1Aa.Transpose[σg2Aa] ) // MatrixForm
Out[657]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


In[658]:= (* (s1s2=Inverse[σg1Aa.Transpose[σg2Aa]])//MatrixForm*)

In[659]:= s1s2.s1s2 === ID8
Out[659]= True

In[660]:= (Thread[Π16[[1 ;; 8] → s1s2.Π16[[9 ;; 16]]]) // Column
%[[1]][All, 1, 0]
%[[1]][All, 2] /. {x0 → #1, x4 → #2}
ToExpression["((" <> ToString[#] <> ")&)"] & /@ %
forceDimensionReductionA = Thread[%%% → %]
Out[660]=
f16[0][x0, x4] → f16[8][x0, x4]
f16[1][x0, x4] → f16[10][x0, x4]
f16[2][x0, x4] → f16[9][x0, x4]
f16[3][x0, x4] → f16[11][x0, x4]
f16[4][x0, x4] → f16[12][x0, x4]
f16[5][x0, x4] → f16[14][x0, x4]
f16[6][x0, x4] → f16[13][x0, x4]
f16[7][x0, x4] → f16[15][x0, x4]

Out[661]=
{f16[0], f16[1], f16[2], f16[3], f16[4], f16[5], f16[6], f16[7]}

Out[662]=
{f16[8][#1, #2], f16[10][#1, #2], f16[9][#1, #2], f16[11][#1, #2],
 f16[12][#1, #2], f16[14][#1, #2], f16[13][#1, #2], f16[15][#1, #2]}

Out[663]=
{f16[8][#1, #2] &, f16[10][#1, #2] &, f16[9][#1, #2] &, f16[11][#1, #2] &,
 f16[12][#1, #2] &, f16[14][#1, #2] &, f16[13][#1, #2] &, f16[15][#1, #2] &}

Out[664]=
{f16[0] → (f16[8][#1, #2] &), f16[1] → (f16[10][#1, #2] &),
 f16[2] → (f16[9][#1, #2] &), f16[3] → (f16[11][#1, #2] &),
 f16[4] → (f16[12][#1, #2] &), f16[5] → (f16[14][#1, #2] &),
 f16[6] → (f16[13][#1, #2] &), f16[7] → (f16[15][#1, #2] &)}
```

```
In[665]:= (Thread[#[9 ;; 16] → s1s2.#[1 ;; 8]]) // Column
%[[1]] [[All, 1, 0]]
%[[1]] [[All, 2]] /. {x0 → #1, x4 → #2}
ToExpression["((" <> ToString[#] <> ")") &] & /@ %
forceDimensionReduction = Thread[%% → %]

Out[665]= f16[8][x0, x4] → f16[0][x0, x4]
f16[9][x0, x4] → f16[2][x0, x4]
f16[10][x0, x4] → f16[1][x0, x4]
f16[11][x0, x4] → f16[3][x0, x4]
f16[12][x0, x4] → f16[4][x0, x4]
f16[13][x0, x4] → f16[6][x0, x4]
f16[14][x0, x4] → f16[5][x0, x4]
f16[15][x0, x4] → f16[7][x0, x4]

Out[666]= {f16[8], f16[9], f16[10], f16[11], f16[12], f16[13], f16[14], f16[15]}

Out[667]= {f16[0][#1, #2], f16[2][#1, #2], f16[1][#1, #2], f16[3][#1, #2],
f16[4][#1, #2], f16[6][#1, #2], f16[5][#1, #2], f16[7][#1, #2]}

Out[668]= {f16[0][#1, #2] &, f16[2][#1, #2] &, f16[1][#1, #2] &, f16[3][#1, #2] &,
f16[4][#1, #2] &, f16[6][#1, #2] &, f16[5][#1, #2] &, f16[7][#1, #2] &}

Out[669]= {f16[8] → (f16[0][#1, #2] &), f16[9] → (f16[2][#1, #2] &),
f16[10] → (f16[1][#1, #2] &), f16[11] → (f16[3][#1, #2] &),
f16[12] → (f16[4][#1, #2] &), f16[13] → (f16[6][#1, #2] &),
f16[14] → (f16[5][#1, #2] &), f16[15] → (f16[7][#1, #2] &)}

In[670]:= (*σga2A == -σga1A & /@ bas64[[All, 1]]*)

In[671]:= σga2A.Transpose[σga2A] === ID8
```

$\sigma g_{2A}^a \cdot \text{Transpose}[\sigma g_{2A}^a] == ID8$

```
Out[671]= True

In[672]:= σga1A.Transpose[σga1A] === ID8
```

$\sigma g_{1A}^a \cdot \text{Transpose}[\sigma g_{1A}^a] == ID8$

```
Out[672]= True
```

metric - preserving transformation:

In[673]:=

$$\sigma g_{1A}^a \cdot \sigma.\text{Transpose}[g_{1A}^a] == \eta 4488$$

Out[673]=

True

metric - preserving transformation:

In[674]:=

$$\sigma g_{2A}^a \cdot \sigma.\text{Transpose}[g_{2A}^a] == \eta 4488$$

Out[674]=

True

In[675]:=

$$Mc = \frac{1}{\sqrt{2}} (\eta 4488 + \sigma)$$

Out[675]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \\ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

In[676]:=

$$Nc = \frac{1}{\sqrt{2}} \{ \{1, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 0\}, \\ \{0, 1, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 1\}, \\ \{1, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, -1, 0\}, \\ \{0, 1, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, -1\}\}$$

Out[676]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \\ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \\ \left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

In[677]:=

$$(* (s1s2=Mc.\text{Transpose}[Nc]) // \text{MatrixForm} *)$$

In[678]:= **Nc.Transpose[Nc] // MatrixForm**

Out[678]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[679]:= **Mc.Transpose[Mc] // MatrixForm**

Out[679]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[680]:= **Mc.σ.Transpose[Mc] === η4488**

Out[680]=

True

In[681]:= **Nc.σ.Transpose[Nc] === η4488**

Out[681]=

True

In[682]:= **(Thread[Π16[[9 ;; 16] → s1s2.Π16[[1 ;; 8]]]) // Column**

Out[682]=

$$\begin{aligned} f16[8][x_0, x_4] &\rightarrow f16[0][x_0, x_4] \\ f16[9][x_0, x_4] &\rightarrow f16[2][x_0, x_4] \\ f16[10][x_0, x_4] &\rightarrow f16[1][x_0, x_4] \\ f16[11][x_0, x_4] &\rightarrow f16[3][x_0, x_4] \\ f16[12][x_0, x_4] &\rightarrow f16[4][x_0, x_4] \\ f16[13][x_0, x_4] &\rightarrow f16[6][x_0, x_4] \\ f16[14][x_0, x_4] &\rightarrow f16[5][x_0, x_4] \\ f16[15][x_0, x_4] &\rightarrow f16[7][x_0, x_4] \end{aligned}$$

```
In[683]:= f8[#] [x0, x4] & /@ Range[0, 7] // Column
(s1s2.%[[1]]) // Column

Out[683]= f8[0] [x0, x4]
f8[1] [x0, x4]
f8[2] [x0, x4]
f8[3] [x0, x4]
f8[4] [x0, x4]
f8[5] [x0, x4]
f8[6] [x0, x4]
f8[7] [x0, x4]

Out[684]= f8[0] [x0, x4]
f8[2] [x0, x4]
f8[1] [x0, x4]
f8[3] [x0, x4]
f8[4] [x0, x4]
f8[6] [x0, x4]
f8[5] [x0, x4]
f8[7] [x0, x4]
```

Killing Vector Fields

```
In[685]:= 
$$\begin{aligned} & \left[ -DG\left(\left[ \left[ "vector", M8, [ ] \right], \left[ \left[ [6], \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right], \left[ [7], \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} (x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right]] \right], _DG\left(\left[ \left[ "vector", \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. M8, [ ] \right], \left[ [3], \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} (x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right], \left[ [4], \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} (x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right]] \right], _DG\left(\left[ \left[ "vector", \right. \right. \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \left. \left. \left. M8, [ ] \right], \left[ [3], \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} \right. \right. \right. \right. \right. \right. \right. \right]$$

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1/3 (cos(Hx0) - 1)1/6 (2 cos(Hx0) + 1)1/3 cos(Hx0)1/3 ]],  

]), _DG
-  $\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0)$   

+ 1)1/6 (-1 + 2 cos(2 Hx0))1/3 (cos(Hx0) - 1)1  

/6 (2 cos(Hx0) + 1)1/3 cos(Hx0)1/3 ]]]],  

-DG
-  $\frac{1}{\sin(6Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1$   

/6 (cos(Hx0) + 1)1/6 (2 cos(Hx0) - 1)1/3 (2 cos(Hx0)  

+ 1)1/3 (-1 + 2 cos(2 Hx0))1/3 ], [ [ [8],  

-  $\frac{1}{\sin(6Hx0)^{1/3}} (x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1$   

/6 (cos(Hx0) + 1)1/6 (2 cos(Hx0) - 1)1/3 (2 cos(Hx0)  

+ 1)1/3 (-1 + 2 cos(2 Hx0))1/3 ]]]], _DG
M8, []], [ [ [6], -  $\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0)$   

- 1)1/3 (cos(Hx0) + 1)1/6 (-1 + 2 cos(2 Hx0))1  

/3 (cos(Hx0) - 1)1/6 (2 cos(Hx0) + 1)1/3 cos(Hx0)1/3 ]]]]  

), _DG
-  $\frac{1}{\sin(6Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1$   

/6 (cos(Hx0) + 1)1/6 (2 cos(Hx0) - 1)1/3 (2 cos(Hx0)  

+ 1)1/3 (-1 + 2 cos(2 Hx0))1/3 ], [ [ [8],  

-  $\frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1$   

/6 (cos(Hx0) + 1)1/6 (2 cos(Hx0) - 1)1/3 (2 cos(Hx0)  

+ 1)1/3 (-1 + 2 cos(2 Hx0))1/3 ]]]], _DG
M8, []], [ [ [7], -  $\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0)$   

- 1)1/3 (cos(Hx0) + 1)1/6 (-1 + 2 cos(2 Hx0))1  

/3 (cos(Hx0) - 1)1/6 (2 cos(Hx0) + 1)1/3 cos(Hx0)1/3 ]]]]  

), _DG
-  $\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0)$   

+ 1)1/6 (-1 + 2 cos(2 Hx0))1/3 (cos(Hx0) - 1)1  

/6 (2 cos(Hx0) + 1)1/3 cos(Hx0)1/3 ]]]],  

-DG
-  $\frac{1}{\sin(6Hx0)^{1/3}} (x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1$   

/6 (cos(Hx0) + 1)1/6 (2 cos(Hx0) - 1)1/3 (2 cos(Hx0)  

+ 1)1/3 (-1 + 2 cos(2 Hx0))1/3 ] ]]
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$$\begin{aligned}
& - \frac{1}{\sin(6Hx0)^{1/3}} (xI \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) ]]]], \_DG \left( \left[ \begin{array}{l} \text{"vector",} \\ M8, [ ] \end{array} \right], \left[ \begin{array}{l} [2], \\ \frac{1}{\sin(6Hx0)^{1/3}} (x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \end{array} \right], \left[ \begin{array}{l} [4], \\ - \frac{1}{\sin(6Hx0)^{1/3}} (xI \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) ]]]], \_DG \left( \left[ \begin{array}{l} \text{"vector",} \\ M8, [ ] \end{array} \right], \left[ \begin{array}{l} [2], \\ \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} \\
& (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} \\
& /3 (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) \end{array} \right] \right] \right]
\end{aligned}$$


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Out[685]=

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$$\begin{aligned}
& \left[ \_DG \left( \left[ \begin{array}{l} \text{"vector",} M8, [ ] \end{array} \right], \left[ \begin{array}{l} [6], \\ - \frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \end{array} \right], \left[ \begin{array}{l} [7], \\ \frac{1}{\sin(6Hx0)^{1/3}} (x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) ]]]], \_DG \left( \left[ \begin{array}{l} \text{"vector",} \\ M8, [ ] \end{array} \right], \left[ \begin{array}{l} [3], \\ \frac{1}{\sin(6Hx0)^{1/3}} (x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \end{array} \right], \left[ \begin{array}{l} [4], \\ - \frac{1}{\sin(6Hx0)^{1/3}} (x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) ]]]], \_DG \left( \left[ \begin{array}{l} \text{"vector",} \\ M8, [ ] \end{array} \right], \left[ \begin{array}{l} [3], \\ \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} \\
& /3 (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} \\
& /3 (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) \end{array} \right] \right] \right] \right]
\end{aligned}$$


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$$\begin{aligned}
& + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/3} (\cos(Hx0) - 1)^1 \\
& ^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) ]]]], \\
& _DG \left( \left[ ["vector", M8, [ ]], \left[ [6], \right. \right. \right. \\
& - \frac{1}{\sin(6 Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \\
& ^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2 Hx0))^{1/3}) ], [8], \\
& \left. \left. \left. \frac{1}{\sin(6 Hx0)^{1/3}} (x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \\
& ^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2 Hx0))^{1/3}) ]]], _DG \left( \left[ ["vector", \right. \right. \\
& M8, [ ]], \left[ [6], - \frac{1}{\sin(6 Hx0)^{1/3}} ((2 \cos(Hx0) \right. \\
& - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^1 \\
& ^{1/3} (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) ]]] \\
& ), _DG \left( \left[ ["vector", M8, [ ]], \left[ [7], \right. \right. \right. \\
& - \frac{1}{\sin(6 Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \\
& ^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2 Hx0))^{1/3}) ], [8], \\
& \left. \left. \left. \frac{1}{\sin(6 Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \\
& ^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2 Hx0))^{1/3}) ]]], _DG \left( \left[ ["vector", \right. \right. \\
& M8, [ ]], \left[ [7], - \frac{1}{\sin(6 Hx0)^{1/3}} ((2 \cos(Hx0) \right. \\
& - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^1 \\
& ^{1/3} (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) ]]] \\
& ), _DG \left( \left[ ["vector", M8, [ ]], \left[ [8], \right. \right. \right. \\
& - \frac{1}{\sin(6 Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) \\
& + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/3}) (\cos(Hx0) - 1)^1 \\
& ^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) ]]], \\
& _DG \left( \left[ ["vector", M8, [ ]], \left[ [2], \right. \right. \right. \\
& \frac{1}{\sin(6 Hx0)^{1/3}} (x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \\
& ^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2 Hx0))^{1/3}) ], [3], \\
& \left. \left. \left. - \frac{1}{\sin(6 Hx0)^{1/3}} (xI \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \\
& ^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0)
\end{aligned}$$

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```

+ 1)1/3 (-1 + 2 cos(2 Hxθ))1/3)`]]]), _DG(`["vector",
M8, []], [[2],

$$\frac{1}{\sin(6Hx\theta)^{1/3}}(x^3 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2 Hx\theta))^{1/3})], [[4],

$$-\frac{1}{\sin(6Hx\theta)^{1/3}}(x I \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2 Hx\theta))^{1/3})]]], _DG(`["vector",
M8, []], [[2], 
$$\frac{1}{\sin(6Hx\theta)^{1/3}}((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2 Hx\theta))^{1/3} / \sin(6Hx\theta)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})]]]
]]]$$$$$$

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In[686]:= (*ToExpression[MathMLForm[{"http://www.w3.org/TR/MathML","math"}()]]*)

In[687]:= (*ToExpression[MathMLForm[\square]]*)

In[688]:= "[_DG([[\"vector\", M8, []], [[[6], -x^6 * cos(H*x\theta)^{1/3} * (cos(H*x\theta) - 1)^{1/6} * (cos(H*x\theta) + 1)^{1/6} * (2 * cos(H*x\theta) - 1)^{1/3} * (2 * cos(H*x\theta) + 1)^{1/3} * (-1 + 2 * cos(2 * H*x\theta))^{1/3} / sin(6 * H*x\theta)^{1/3}], [[7], x^5 * cos(H*x\theta)^{1/3} * (cos(H*x\theta) - 1)^{1/6} * (cos(H*x\theta) + 1)^{1/6} * (2 * cos(H*x\theta) - 1)^{1/3} * (2 * cos(H*x\theta) + 1)^{1/3} * (-1 + 2 * cos(2 * H*x\theta))^{1/3} / sin(6 * H*x\theta)^{1/3}]]], _DG(`["vector", M8, []], [[[3], x^3 * cos(H*x\theta)^{1/3} * (cos(H*x\theta) - 1)^{1/6} * (cos(H*x\theta) + 1)^{1/6} * (2 * cos(H*x\theta) - 1)^{1/3} * (2 * cos(H*x\theta) + 1)^{1/3} * (-1 + 2 * cos(2 * H*x\theta))^{1/3} / sin(6 * H*x\theta)^{1/3}]]], _DG(`["vector", M8, []], [[[3], (2 * cos(H*x\theta) - 1)^{1/3} * (cos(H*x\theta) + 1)^{1/6} * (cos(H*x\theta) - 1)^{1/6} * (2 * cos(H*x\theta) - 1)^{1/3} * (2 * cos(H*x\theta) + 1)^{1/3} * (-1 + 2 * cos(2 * H*x\theta))^{1/3} / sin(6 * H*x\theta)^{1/3}]]], _DG(`["vector", M8, []], [[[4], -x^2 * cos(H*x\theta)^{1/3} * (cos(H*x\theta) - 1)^{1/6} * (cos(H*x\theta) + 1)^{1/6} * (2 * cos(H*x\theta) - 1)^{1/3} * (2 * cos(H*x\theta) + 1)^{1/3} * (-1 + 2 * cos(2 * H*x\theta))^{1/3} / sin(6 * H*x\theta)^{1/3}]]], _DG(`["vector", M8, []], [[[4], (2 * cos(H*x\theta) - 1)^{1/3} * (cos(H*x\theta) + 1)^{1/6} * (cos(H*x\theta) - 1)^{1/6} * (2 * cos(H*x\theta) - 1)^{1/3} * (2 * cos(H*x\theta) + 1)^{1/3} * (-1 + 2 * cos(2 * H*x\theta))^{1/3} * cos(H*x\theta)^{1/3} / sin(6 * H*x\theta)^{1/3}]]], _DG(`["vector", M8, []], [[[6], -x^7 * cos(H*x\theta)^{1/3} * (cos(H*x\theta) - 1)^{1/6} * (cos(H*x\theta) + 1)^{1/6} * (2 * cos(H*x\theta) - 1)^{1/3} * (2 * cos(H*x\theta) + 1)^{1/3} * (-1 + 2 * cos(2 * H*x\theta))^{1/3} / sin(6 * H*x\theta)^{1/3}], [[8], x^5 * cos(H*x\theta)^{1/3} * (cos(H*x\theta) - 1)^{1/6} * (cos(H*x\theta) + 1)^{1/6} * (2 * cos(H*x\theta) - 1)^{1/3} * (2 * cos(H*x\theta) + 1)^{1/3} * (-1 + 2 * cos(2 * H*x\theta))^{1/3} / sin(6 * H*x\theta)^{1/3}]]]

```

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector", M8,
[]], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector", M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector", M8,
[]], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector", M8,
[]], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])

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Out[688]=

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[_DG([["vector", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector", M8,
[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]])

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2*cos(2*H*x0)^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0)
- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[{"vector": M8}], [[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[{"vector": M8}], [[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[{"vector": M8}], [[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], [[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[{"vector": M8}], [[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[{"vector": M8}], [[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], [[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[{"vector": M8}], [[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[{"vector": M8}], [[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[{"vector": M8}], [[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[{"vector": M8}], [[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], [[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[{"vector": M8}], [[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]])

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In[689]:=

killingVectors =

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"[ _DG([[["vector\"", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[["vector\"", M8,
[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[["vector\"", M8,
[]], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)],
[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[["vector\"", M8,
[]], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[["vector\"", M8,
[]], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)],
[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[["vector\"", M8,
[]], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)],
[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[["vector\"", M8,
[]], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) -

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1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], 
[[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\", M8,
[]], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], 
[[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\", M8,
[]], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])"
Out[689]=
[_DG([[\"vector\", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], 
[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\", M8,
[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], 
_DG([[\"vector\", M8, []], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)], 
[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\",
M8, []], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]), 
_DG([[\"vector\", M8, []], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\",
M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], 
_DG([[\"vector\", M8, []], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\",
M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], 
_DG([[\"vector\", M8, []], [[[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], 
[[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], 
[[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]

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- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector", M8, []], [[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector",
M8, []], [[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector",
M8, []], [[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/3)*(2*cos(H*x0) - 1)^(1/6)*(cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([["vector",
M8, []], [[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([["vector",
M8, []], [[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])

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In[690]:=

$$\begin{aligned}
& -DG \left(\left[\left["vector", M8, [] \right], \left[\left[6 \right], \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \\
& \quad \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \\
& \quad \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right] \right], \left[\left[7 \right], \right. \\
& \quad \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} (x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \right. \\
& \quad \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \\
& \quad \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right] \right] \right], _DG \left(\left[\left["vector", \right. \right. \right. \\
& \quad \left. \left. \left. M8, [] \right], \left[\left[3 \right], \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} (x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \right. \\
& \quad \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \\
& \quad \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right] \right] \right]
\end{aligned}$$

```


$$\begin{aligned}
& \left. \frac{1}{\sin(6Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right], [8], \\
& \frac{1}{\sin(6Hx0)^{1/3}} (x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right], [9]
\end{aligned}$$


```

$$\begin{aligned}
& M8, [], \left[[6], -\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) \right. \\
& \left. - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \\
& \left. /^3 (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right]] \\
&), _DG \left(\left[["vector", M8, []], \left[[7], \right. \right. \\
& \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right] \right], \left[[8], \right. \\
& \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right] \right]], _DG \left(\left[["vector", \right. \right. \\
& M8, [], \left[[7], -\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) \right. \\
& \left. - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \\
& \left. /^3 (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right]] \\
&), _DG \left(\left[["vector", M8, []], \left[[8], \right. \right. \\
& \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) \right. \right. \\
& \left. \left. - 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (2 \cos(Hx0) + 1)^1 \right. \right. \\
& \left. \left. /^3 (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right]], _DG \left(\left[["vector", \right. \right. \\
& M8, [], \left[[2], \right. \right. \\
& \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}} (x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^3 (\cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) \right] \right]
\end{aligned}$$

```


$$\begin{aligned}
& \left. \frac{1}{\sin(6Hx0)^{1/3}} (xI \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \\
& \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \\
& \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right], \left[ [3], \right. \\
& \left. - \frac{1}{\sin(6Hx0)^{1/3}} (xI \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \\
& \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \\
& \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right]]], \_DG \left( \left[ ["vector", \right. \right. \\
& M8, [ ]], \left[ \left[ [2], \right. \right. \\
& \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} (x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right], \left[ [4], \right. \right. \\
& \left. \left. - \frac{1}{\sin(6Hx0)^{1/3}} (xI \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right]]], \_DG \left( \left[ ["vector", \right. \right. \\
& M8, [ ]], \left[ \left[ [2], \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^3 (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} \right. \right. \\
& \left. \left. /^3 (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \right] \right]
\end{aligned}$$


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Out[690]=

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$$-DG\left(\left["vector", M8, [ ]\right], \left[\left[ [6], \right.\right.$$


$$\left.-\frac{1}{\sin(6Hx\theta)^{1/3}}(x\theta \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3}\right)\right], \left[[7], \right.$$


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$$\begin{aligned}
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /^6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]), _DG\left(\left[["vector",\right. \right. \\
& M8, []], \left[\left[[3], \right. \right. \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x3 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /^6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], \left[[4], \right. \\
& \left. \left. - \frac{1}{\sin(6Hx\theta)^{1/3}} (x2 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \right. \right. \\
& /^6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]), _DG\left(\left[["vector",\right. \right. \\
& M8, []], \left[\left[[3], \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^1 \right. \right. \\
& /^3 (\cos(Hx\theta) - 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^1 \\
& \left. \left. /^3 (2 \cos(Hx\theta) + 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3})] \right], _DG\left(\left[["vector",\right. \right. \\
& \left. \left. M8, []], \left[\left[[4], \right. \right. \right. \\
& \left. \left. \left. \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) \right. \right. \right. \\
& \left. \left. \left. - 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} (2 \cos(Hx\theta) + 1)^1 \right. \right. \right. \\
& \left. \left. \left. /^3 (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3})] \right], _DG\left(\left[["vector",\right. \right. \\
& M8, []], \left[\left[[6], \right. \right. \right. \\
& \left. \left. \left. - \frac{1}{\sin(6Hx\theta)^{1/3}} (x7 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \right. \right. \right. \\
& \left. \left. \left. /^6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \right. \right. \right. \\
& \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})] \right] \right]
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3}), [[8], \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]], _DG\left(\left[["vector",\right. \right. \\
& M8, []], \left[[6], -\frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) \\
& - 1)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} \\
& /^3 (2 \cos(Hx\theta) + 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3})] \right] \\
&), _DG\left(\left[["vector", M8, []], \left[[7], \right. \right. \\
& -\frac{1}{\sin(6Hx\theta)^{1/3}} (x7 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /^6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [[8], \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x6 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /^6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]], _DG\left(\left[["vector",\right. \right. \\
& M8, []], \left[[7], -\frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) \\
& - 1)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} \\
& /^3 (2 \cos(Hx\theta) + 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3})] \right] \\
&), _DG\left(\left[["vector", M8, []], \left[[8], \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{\sin(6Hx\theta)^{1/3}} ((2\cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) \\
& - 1)^{1/6} (-1 + 2\cos(2Hx\theta))^{1/3} (2\cos(Hx\theta) + 1)^1 \\
& /^3 (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3})]])], _DG\left(\left[\begin{array}{l} \text{"vector"}, \\ M8, [], \left[\left[[2], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \frac{1}{\sin(6Hx\theta)^{1/3}} (x2\cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \right. \right. \right. \right. \right. \\
& /^6 (\cos(Hx\theta) + 1)^{1/6} (2\cos(Hx\theta) - 1)^{1/3} (2\cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2\cos(2Hx\theta))^{1/3}) \right], \left[[3], \right. \left. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. - \frac{1}{\sin(6Hx\theta)^{1/3}} (x1\cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \right. \right. \right. \right. \right. \\
& /^6 (\cos(Hx\theta) + 1)^{1/6} (2\cos(Hx\theta) - 1)^{1/3} (2\cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2\cos(2Hx\theta))^{1/3}) \right]]], _DG\left(\left[\begin{array}{l} \text{"vector"}, \\ M8, [], \left[\left[[2], \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \frac{1}{\sin(6Hx\theta)^{1/3}} (x3\cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \right. \right. \right. \right. \right. \\
& /^6 (\cos(Hx\theta) + 1)^{1/6} (2\cos(Hx\theta) - 1)^{1/3} (2\cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2\cos(2Hx\theta))^{1/3}) \right], \left[[4], \right. \left. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. - \frac{1}{\sin(6Hx\theta)^{1/3}} (x1\cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \right. \right. \right. \right. \right. \\
& /^6 (\cos(Hx\theta) + 1)^{1/6} (2\cos(Hx\theta) - 1)^{1/3} (2\cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2\cos(2Hx\theta))^{1/3}) \right]]], _DG\left(\left[\begin{array}{l} \text{"vector"}, \\ M8, [], \left[\left[[2], \frac{1}{\sin(6Hx\theta)^{1/3}} ((2\cos(Hx\theta) - 1)^1 \right. \right. \right. \right. \right. \\
& /^3 (\cos(Hx\theta) - 1)^{1/6} (-1 + 2\cos(2Hx\theta))^1 \\
& /^3 (2\cos(Hx\theta) + 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3}) \right] \right]
\end{aligned}$$

13

In[691]:=

```

ConvertMapleToMathematicaV2[
" -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3), , x5*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)"]

```

Out[691]=

$$-\frac{1}{\text{Sin}[6 \text{H} x \theta]^{1/3}} x^6 (-1 + \text{Cos}[\text{H} x \theta])^{1/6} \text{Cos}[\text{H} x \theta]^{1/3} (1 + \text{Cos}[\text{H} x \theta])^{1/6} \\ (-1 + 2 \text{Cos}[\text{H} x \theta])^{1/3} (1 + 2 \text{Cos}[\text{H} x \theta])^{1/3} (-1 + 2 \text{Cos}[2 \text{H} x \theta])^{1/3}$$

In[692]:=

```

ConvertMapleToMathematica[
" x3*cos(H*x0)^(1/3)*(cos(H*x0)-1)^(1/6)*(cos(H*x0)+1)^(1/6)*(2*cos(H*x0)-1)^(1/3)*(2*cos(H*x0)+1)^(1/3)*(-1+2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3), -x2*cos(H*x0)^(1/3)*(cos(H*x0)-1)^(1/6)*(cos(H*x0)+1)^(1/6)*(2*cos(H*x0)-1)^(1/3)*(2*cos(H*x0)+1)^(1/3)*(-1+2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)"]

```

Out[692]=

$$((((((x3 * (\cos[(H * x0)] \wedge (1 / 3))) * ((\cos[(H * x0)] - 1) \wedge (1 / 6))) * ((\cos[(H * x0)] + 1) \wedge (1 / 6))) * (((2 * \cos[(H * x0)]) - 1) \wedge (1 / 3))) * (((2 * \cos[(H * x0)]) + 1) \wedge (1 / 3))) * ((-(1) + (2 * \cos[((2 * H) * x0)])) \wedge (1 / 3))) / (\sin[((6 * H) * x0)] \wedge (1 / 3)))$$

In[693]:=

ConvertMapleToMathematica

```
"(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)"]
```

Out[693]=

$$((((((2 * \cos[(H * x\theta)]) - 1) ^ (1 / 3)) * ((\cos[(H * x\theta)] - 1) ^ (1 / 6))) * ((-1 + (2 * \cos[((2 * H) * x\theta)])) ^ (1 / 3))) * (((2 * \cos[(H * x\theta)] + 1) ^ (1 / 3))) * ((\cos[(H * x\theta)] + 1) ^ (1 / 6))) * (\cos[(H * x\theta)] ^ (1 / 3))) / (\sin[((6 * H) * x\theta)] ^ (1 / 3)))$$

In[694]:=

M8 > $KV := \text{KillingVectors}(g);$

$$KV := \left[-\frac{x6 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x5}}{\sin(6 Hx0)^1 \wedge} \right.$$

$$+ \frac{x5 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x6}}{\sin(6 Hx0)^1 \wedge},$$

$$\frac{x3 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x2}}{\sin(6 Hx0)^1 \wedge}$$

$$- \frac{x2 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x3}}{\sin(6 Hx0)^1 \wedge},$$

$$\frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x2}}{\sin(6 Hx0)^1 \wedge}$$

$$\frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x3}}{\sin(6 Hx0)^1 \wedge},$$

$$- \frac{x7 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x5}}{\sin(6 Hx0)^1 \wedge}$$

$$+ \frac{x5 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x7}}{\sin(6 Hx0)^1 \wedge},$$

$$- \frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x5}}{\sin(6 Hx0)^1 \wedge},$$

$$- \frac{x7 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x6}}{\sin(6 Hx0)^1 \wedge}$$

$$+ \frac{x6 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x7}}{\sin(6 Hx0)^1 \wedge},$$

$$- \frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x6}}{\sin(6 Hx0)^1 \wedge},$$

$$- \frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x7}}{\sin(6 Hx0)^1 \wedge},$$

$$\frac{x2 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x1}}{\sin(6 Hx0)^1 \wedge}$$

Out[694]=

In[695]:=

$$\begin{aligned}
& - \frac{xI \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2Hx0))^{1/\beta} D_{x2}}{\sin(6Hx0)^{1/\beta}}, \\
& \frac{x^3 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2Hx0))^{1/\beta} D_{x1}}{\sin(6Hx0)^{1/\beta}}, \\
& - \frac{xI \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2Hx0))^{1/\beta} D_{x3}}{\sin(6Hx0)^{1/\beta}}, \\
& \left. \frac{(2 \cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (-1 + 2 \cos(2Hx0))^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} \cos(Hx0)^{1/\beta} D_{x1}}{\sin(6Hx0)^{1/\beta}} \right]
\end{aligned}$$

Out[605]=

$$\begin{aligned}
& - \frac{xI \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2Hx0))^1 \wedge D_{x2}}{\sin(6Hx0)^1 \wedge} \\
& x3 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2Hx0))^1 \wedge D_{x1} \\
& - \frac{xI \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2Hx0))^1 \wedge D_{x3}}{\sin(6Hx0)^1 \wedge} \\
& \left. \frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x1}}{\sin(6Hx0)^1 \wedge} \right]
\end{aligned}$$

In[696]:=

$$\begin{aligned}
& \frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x2}}{\sin(6 Hx0)^{1/6}}, \\
& \frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x3}}{\sin(6 Hx0)^{1/6}}, \\
& - \frac{x^7 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x5}}{\sin(6 Hx0)^{1/6}}, \\
& + \frac{x^5 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x7}}{\sin(6 Hx0)^{1/6}}, \\
& - \frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x5}}{\sin(6 Hx0)^{1/6}}, \\
& - \frac{x^7 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x6}}{\sin(6 Hx0)^{1/6}}, \\
& + \frac{x^6 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x7}}{\sin(6 Hx0)^{1/6}}, \\
& - \frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x6}}{\sin(6 Hx0)^{1/6}}, \\
& - \frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x7}}{\sin(6 Hx0)^{1/6}}, \\
& x^2 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x1}}{\sin(6 Hx0)^{1/6}}, \\
& - \frac{x^1 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x2}}{\sin(6 Hx0)^{1/6}}, \\
& x^3 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x1}}{\sin(6 Hx0)^{1/6}}, \\
& - \frac{x^1 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x3}}{\sin(6 Hx0)^{1/6}}, \\
& (2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x1}]
\end{aligned}$$

8 >

Out[696]=

8 >

4×4 Dirac γ matrices:

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta], \quad (22)$$

$$S^{\alpha\beta} = -\frac{1}{2}\gamma^{\alpha\beta}. \quad (23)$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \quad (24)$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^\alpha \epsilon = -\epsilon \gamma^\alpha, \quad (25)$$

$$\tilde{S}^{\alpha\beta} \epsilon = -\epsilon S^{\alpha\beta}, \quad (26)$$

$$[S^{\alpha\beta}, \gamma_\mu] = \delta_\mu^\alpha \gamma^\beta - \delta_\mu^\beta \gamma^\alpha, \quad (27)$$

$$[S^{\alpha\beta}, S^{\mu\nu}] = g^{\alpha\mu} S^{\beta\nu} - g^{\alpha\nu} S^{\beta\mu} - g^{\beta\mu} S^{\alpha\nu} + g^{\beta\nu} S^{\alpha\mu}, \quad (28)$$

and

$$\gamma^5 S^{\alpha\beta} = \frac{1}{2} g^{\alpha\mu} g^{\beta\nu} \epsilon_{\mu\nu\lambda\sigma} S^{\lambda\sigma}. \quad (29)$$

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

In[697]:=

(*Symbolize[g_{αβ}]*)

In[698]:=

η4488

Dimensions[%]

g_{αβ} = η4488[[2 ;; 5, 2 ;; 5]]

Out[698]=

{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1} }

Out[699]=

{8, 8}

Out[700]=

{ {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1} }

In[701]:=

Protect[g_{αβ}]

Out[701]=

{g_{Subscript αβ}}

In[702]:=

g3 = DiagonalMatrix[{1, 1, -1}]

Out[702]=

{ {1, 0, 0}, {0, 1, 0}, {0, 0, -1} }

In[703]:=

Do[Do[Gr[h, k] = t4by4[h].s4by4[k] * (-g3[[h, h]] * g3[[k, k]]), {k, 1, 3}], {h, 1, 3}]

In[704]:=

Gr[1, 1] // MatrixForm

Out[704]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In[705]:=

 $\gamma = \{\text{Gr}[1, 1], \text{Gr}[1, 2], \text{Gr}[1, 3], \text{t4by4}[2]\};$ ✖ Set: Symbol γ is Protected. [i](#)

In[706]:=

Table[FullSimplify[\mathbf{\gamma[h].\gamma[k]} == g_{\alpha\beta}[h, k] * ID4], {h, 1, 4}]

Out[706]=

{False, False, False, True}

$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\gamma_0 g^{\alpha\beta}, \quad (4)$$

where

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

and γ_0 denotes the 4×4 unit matrix. The $\Sigma^{\alpha\beta}$ of Eq. (1)

In[707]:=

$$\text{Table}\left[\left\{\{h, k\}, \text{FullSimplify}\left[\frac{\gamma[h].\gamma[k] + \gamma[k].\gamma[h]}{2}\right] == g_{\alpha\beta}[h, k] \text{ID4}\right\}, \{h, 1, 4\}, \{k, 1, 4\}\right] // \text{MatrixForm}$$

Out[707]//MatrixForm=

$$\begin{pmatrix} (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) \\ (\text{False}) & (\text{True}) & (\text{True}) & (\text{False}) \\ (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) \\ (\text{True}) & (\text{False}) & (\text{True}) & (\text{False}) \\ (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) \\ (\text{True}) & (\text{True}) & (\text{False}) & (\text{False}) \\ (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) \\ (\text{False}) & (\text{False}) & (\text{False}) & (\text{True}) \end{pmatrix}$$

In[708]:=

$$\text{Table}\left[\left\{\{h, k\}, \text{FullSimplify}\left[\frac{\gamma[h] \cdot \gamma[k] + \gamma[k] \cdot \gamma[h]}{2}\right] == \eta 4488 [1+h, 1+k] \text{ID4}\right\}, \{h, 1, 4\}, \{k, 1, 4\}\right] // \text{MatrixForm}$$

Out[708]//MatrixForm=

$$\begin{pmatrix} (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) \\ \text{False} & \text{True} & \text{True} & \text{False} \\ (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) \\ \text{True} & \text{False} & \text{True} & \text{False} \\ (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) \\ \text{True} & \text{True} & \text{False} & \text{False} \\ (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) \\ \text{False} & \text{False} & \text{False} & \text{True} \end{pmatrix}$$

$$\gamma^5 = -\gamma^1 \gamma^2 \gamma^3 \gamma^4$$

In[709]:=

$$-\gamma[1] \cdot \gamma[2] \cdot \gamma[3] \cdot \gamma[4]$$

Out[709]=

$$\{\{0, 0, 0, 1\}, \{0, 0, -1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\}$$

In[710]:=

$$\text{AppendTo}[\gamma, (-\gamma[1] \cdot \gamma[2] \cdot \gamma[3] \cdot \gamma[4])]$$

Set: Symbol γ is Protected. [i](#)

Out[710]=

$$\begin{aligned} &\{\{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-1, 0, 0, 0\}\}, \\ &\quad \{\{0, 0, -1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\}, \\ &\quad \{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\}, \\ &\quad \{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{1, 0, 0, 0\}\}, \\ &\quad \{\{0, 0, -1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\}, \\ &\quad \{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 1, 0\}\}, \\ &\quad \{\{1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\}, \\ &\quad \{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\}, \\ &\quad \{\{0, 0, 1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\}, \\ &\quad \{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, -1, 0\}\}, \\ &\quad \{\{-1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\}, \\ &\quad \{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{1, 0, 0, 0\}\}, \\ &\quad \{\{0, 0, 1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\}, \\ &\quad \{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\}, \\ &\quad \{\{-1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}, \\ &\quad \{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}, \\ &\quad \{\{0, 0, 0, 1\}, \{0, 0, -1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\} \} \end{aligned}$$

In[711]:=

$$\text{Protect}[\gamma]$$

Out[711]=

$$\{\}$$

```
In[712]:= γ[[5]]
Out[712]= {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}
```

```
In[713]:= γ[[5]] === -t4by4[3]
Out[713]= False
```

```
In[714]:= Table[
  {{h, k}, FullSimplify[γ[h].γ[k] + γ[k].γ[h]] === 2 * η4488[[1 + h, 1 + k] ID4],
  {h, 1, 5}, {k, 1, 5}] // MatrixForm}
```

```
Out[714]//MatrixForm=
{{{{1, 1}, {{1, 2}, {{1, 3}, {{1, 4}, {{1, 5}, {False, True, True, False, False}}}, {False, False, False, False, False}}}, {False, False, False, False, False}}, {{{2, 1}, {{2, 2}, {{2, 3}, {{2, 4}, {{2, 5}, {True, False, True, False, False}}}, {False, False, False, False, False}}}, {True, False, True, False, False}}, {{{3, 1}, {{3, 2}, {{3, 3}, {{3, 4}, {{3, 5}, {True, True, False, False, False}}}, {False, False, False, False, False}}}, {True, True, False, False, False}}, {{{4, 1}, {{4, 2}, {{4, 3}, {{4, 4}, {{4, 5}, {True, True, True, False, False}}}, {False, False, True, False, False}}}, {False, False, False, True, False}}, {{{5, 1}, {{5, 2}, {{5, 3}, {{5, 4}, {{5, 5}, {True, True, True, True, False}}}, {False, False, True, True, False}}}, {False, False, False, True, True}}}}
```

$$\gamma^\alpha = \gamma^{\alpha 6}, \quad (19)$$

$$\gamma^5 = \gamma^{56}, \quad (20)$$

$$\gamma^\alpha \gamma^5 = \gamma^{\alpha 5}, \quad (21)$$

$$\epsilon = \gamma^{64}.$$

```
In[715]:= -γ[[4]] // MatrixForm
```

```
Out[715]//MatrixForm=
{{0, 0, 0, 1}, {0, 0, -1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}
```

```
In[716]:= se = {ε → t4by4[2]}
Out[716]= {ε → {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}
```

```
In[717]:= Transpose[ε /. se] === (-ε /. se)
Out[717]= True

In[718]:= ParallelTable[{{A1},
  FullSimplify[(ε /. se).γ[A1] === Transpose[(ε /. se).γ[A1]]]}, {A1, 1, 5}]
Out[718]= {{1, True}, {2, True}, {3, True}, {4, False}, {5, True}}

In[719]:= γ[4] === t4by4[2]
Out[719]= False

In[720]:= t4by4[1]
Out[720]= {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}}



$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta]. \quad (3)$$


In[721]:= S44αβ = -1/4 Table[FullSimplify[γ[h].γ[k] - γ[k].γ[h]], {h, 1, 4}, {k, 1, 4}];
In[722]:= Protect[S44αβ]
Out[722]= {S44αβ}

In[723]:= η44 = DiagonalMatrix[{1, 1, 1, -1}]
Out[723]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}


In[724]:= Protect[η44]
Out[724]= {η44}
```

S44 $\alpha\beta$ commutation relations; misc :

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta], \quad (22)$$

$$S^{\alpha\beta} = -\frac{1}{2}\gamma^{\alpha\beta}. \quad (23)$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \quad (24)$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^\alpha \epsilon = -\epsilon \gamma^\alpha, \quad (25)$$

$$\tilde{S}^{\alpha\beta} \epsilon = -\epsilon S^{\alpha\beta}, \quad (26)$$

$$[S^{\alpha\beta}, \gamma_\mu] = \delta_\mu^\alpha \gamma^\beta - \delta_\mu^\beta \gamma^\alpha, \quad (27)$$

$$[S^{\alpha\beta}, S^{\mu\nu}] = g^{\alpha\mu} S^{\beta\nu} - g^{\alpha\nu} S^{\beta\mu} - g^{\beta\mu} S^{\alpha\nu} + g^{\beta\nu} S^{\alpha\mu}, \quad (28)$$

and

$$\gamma^5 S^{\alpha\beta} = \frac{1}{2} g^{\alpha\mu} g^{\beta\nu} \epsilon_{\mu\nu\lambda\sigma} S^{\lambda\sigma}. \quad (29)$$

$$S44\alpha\beta = -\frac{1}{4} \text{Table}[\text{FullSimplify}[\gamma[h].\gamma[k] - \gamma[k].\gamma[h]], \{h, 1, 4\}, \{k, 1, 4\}];$$

In[725]:=

```
(*SAB=
Table[1/4 (T16^A[A1].T16^A[B1]-T16^A[B1].T16^A[A1]), {A1,0, 7},{B1,0, 7}];*)
```

In[726]:=

```
(*ParallelTable[
 FullSimplify[\sigma16.SAB[A1,B1]==-Transpose[\sigma16.SAB[A1,B1]]],
 {A1,1, 8},{B1,1,8}]/.Flatten//Union*)
```

In[727]:=

```
(*ParallelTable[
 {{A1,B1}},FullSimplify[SAB[A1,B1]==-Transpose[SAB[A1,B1]]]}, {A1,1, 8},{B1,1,8}]*)
```

In[728]:=

```
ParallelTable[
 FullSimplify[S44\alpha\beta[A1, B1].S44\alpha\beta[A2, B2] - S44\alpha\beta[A2, B2].S44\alpha\beta[A1, B1] ==
 (\eta44[A1, A2]\times S44\alpha\beta[B1, B2] - \eta44[A1, B2]\times S44\alpha\beta[B1, A2] -
 \eta44[B1, A2]\times S44\alpha\beta[A1, B2] + \eta44[B1, B2]\times S44\alpha\beta[A1, A2])], {A1, 1, 3},
 {B1, A1 + 1, 4}, {A2, 1, 3}, {B2, A2 + 1, 4}] // Flatten // Union
```

Out[728]=

```
{False, True}
```

In[729]:=

```
ParallelTable[{{A1, B1, A2, B2},  

  FullSimplify[S44αβ[A1, B1].S44αβ[A2, B2] - S44αβ[A2, B2].S44αβ[A1, B1] ==  

    (η44[A1, A2] × S44αβ[B1, B2] - η44[A1, B2] × S44αβ[B1, A2] -  

     η44[B1, A2] × S44αβ[A1, B2] + η44[B1, B2] × S44αβ[A1, A2])]},  

  {A1, 1, 4}, {B1, 1, 4}, {A2, 1, 4}, {B2, 1, 4}] // MatrixForm
```

Out[729]//MatrixForm=

({1, 1, 1, 1})	({1, 1, 1, 2})	({1, 1, 1, 3})	({1, 1, 1, 4})	({1, 2, 1, 1})	({1, True})
({1, 1, 2, 1})	({1, 1, 2, 2})	({1, 1, 2, 3})	({1, 1, 2, 4})	({1, 2, 2, 1})	({1, True})
({1, 1, 3, 1})	({1, 1, 3, 2})	({1, 1, 3, 3})	({1, 1, 3, 4})	({1, 2, 3, 1})	({1, False})
({1, 1, 4, 1})	({1, 1, 4, 2})	({1, 1, 4, 3})	({1, 1, 4, 4})	({1, 2, 4, 1})	({1, True})
({2, 1, 1, 1})	({2, 1, 1, 2})	({2, 1, 1, 3})	({2, 1, 1, 4})	({2, 2, 1, 1})	({2, True})
({2, 1, 2, 1})	({2, 1, 2, 2})	({2, 1, 2, 3})	({2, 1, 2, 4})	({2, 2, 2, 1})	({2, True})
({2, 1, 3, 1})	({2, 1, 3, 2})	({2, 1, 3, 3})	({2, 1, 3, 4})	({2, 2, 3, 1})	({2, True})
({2, 1, 4, 1})	({2, 1, 4, 2})	({2, 1, 4, 3})	({2, 1, 4, 4})	({2, 2, 4, 1})	({2, True})
({3, 1, 1, 1})	({3, 1, 1, 2})	({3, 1, 1, 3})	({3, 1, 1, 4})	({3, 2, 1, 1})	({3, True})
({3, 1, 2, 1})	({3, 1, 2, 2})	({3, 1, 2, 3})	({3, 1, 2, 4})	({3, 2, 2, 1})	({3, False})
({3, 1, 3, 1})	({3, 1, 3, 2})	({3, 1, 3, 3})	({3, 1, 3, 4})	({3, 2, 3, 1})	({3, False})
({3, 1, 4, 1})	({3, 1, 4, 2})	({3, 1, 4, 3})	({3, 1, 4, 4})	({3, 2, 4, 1})	({3, True})
({4, 1, 1, 1})	({4, 1, 1, 2})	({4, 1, 1, 3})	({4, 1, 1, 4})	({4, 2, 1, 1})	({4, True})
({4, 1, 2, 1})	({4, 1, 2, 2})	({4, 1, 2, 3})	({4, 1, 2, 4})	({4, 2, 2, 1})	({4, True})
({4, 1, 3, 1})	({4, 1, 3, 2})	({4, 1, 3, 3})	({4, 1, 3, 4})	({4, 2, 3, 1})	({4, True})
({4, 1, 4, 1})	({4, 1, 4, 2})	({4, 1, 4, 3})	({4, 1, 4, 4})	({4, 2, 4, 1})	({4, False})

In[730]:=

```
ParallelTable[FullSimplify[S44αβ[A1, B1].γ[B2] - γ[B2].S44αβ[A1, B1] ==  

  (η44[B2, A1] × γ[B1] - η44[B2, B1] × γ[A1])],  

  {A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // Flatten // Union
```

Out[730]=

{False, True}

In[731]:=

```
ParallelTable[
 {{A1, B1, B2}, FullSimplify[S44 $\alpha\beta$ [A1, B1]. $\gamma$ [B2] -  $\gamma$ [B2].S44 $\alpha\beta$ [A1, B1] ==
  ( $\eta$ 44[B2, A1] $\times$  $\gamma$ [B1] -  $\eta$ 44[B2, B1] $\times$  $\gamma$ [A1])]}, {A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // MatrixForm
```

Out[731]//MatrixForm=

$$\begin{pmatrix} \{1, 1, 1\} & \text{True} \\ \{1, 1, 2\} & \text{True} \\ \{1, 1, 3\} & \text{True} \\ \{1, 1, 4\} & \text{True} \\ \{2, 1, 1\} & \text{False} \\ \{2, 1, 2\} & \text{False} \\ \{2, 1, 3\} & \text{True} \\ \{2, 1, 4\} & \text{True} \\ \{3, 1, 1\} & \text{False} \\ \{3, 1, 2\} & \text{True} \\ \{3, 1, 3\} & \text{False} \\ \{3, 1, 4\} & \text{True} \\ \{4, 1, 1\} & \text{False} \\ \{4, 1, 2\} & \text{True} \\ \{4, 1, 3\} & \text{True} \\ \{4, 1, 4\} & \text{False} \end{pmatrix} \begin{pmatrix} \{1, 2, 1\} & \text{False} \\ \{1, 2, 2\} & \text{False} \\ \{1, 2, 3\} & \text{True} \\ \{1, 2, 4\} & \text{True} \\ \{2, 2, 1\} & \text{True} \\ \{2, 2, 2\} & \text{True} \\ \{2, 2, 3\} & \text{True} \\ \{2, 2, 4\} & \text{True} \\ \{3, 2, 1\} & \text{True} \\ \{3, 2, 2\} & \text{False} \\ \{3, 2, 3\} & \text{False} \\ \{3, 2, 4\} & \text{True} \\ \{4, 2, 1\} & \text{True} \\ \{4, 2, 2\} & \text{False} \\ \{4, 2, 3\} & \text{True} \\ \{4, 2, 4\} & \text{False} \end{pmatrix} \begin{pmatrix} \{1, 3, 1\} & \text{False} \\ \{1, 3, 2\} & \text{True} \\ \{1, 3, 3\} & \text{False} \\ \{1, 3, 4\} & \text{True} \\ \{2, 3, 1\} & \text{True} \\ \{2, 3, 2\} & \text{False} \\ \{2, 3, 3\} & \text{False} \\ \{2, 3, 4\} & \text{True} \\ \{3, 3, 1\} & \text{True} \\ \{3, 3, 2\} & \text{True} \\ \{3, 3, 3\} & \text{True} \\ \{3, 3, 4\} & \text{True} \\ \{4, 3, 1\} & \text{True} \\ \{4, 3, 2\} & \text{True} \\ \{4, 3, 3\} & \text{False} \\ \{4, 3, 4\} & \text{False} \end{pmatrix} \begin{pmatrix} \{1, 4, 1\} & \text{False} \\ \{1, 4, 2\} & \text{True} \\ \{1, 4, 3\} & \text{True} \\ \{1, 4, 4\} & \text{False} \\ \{2, 4, 1\} & \text{True} \\ \{2, 4, 2\} & \text{False} \\ \{2, 4, 3\} & \text{True} \\ \{2, 4, 4\} & \text{False} \\ \{3, 4, 1\} & \text{True} \\ \{3, 4, 2\} & \text{True} \\ \{3, 4, 3\} & \text{False} \\ \{3, 4, 4\} & \text{False} \\ \{4, 4, 1\} & \text{True} \\ \{4, 4, 2\} & \text{True} \\ \{4, 4, 3\} & \text{True} \\ \{4, 4, 4\} & \text{True} \end{pmatrix}$$

In[732]:=

```
ParallelTable[{{A1, B1}, FullSimplify[( $\epsilon$  /. se).S44 $\alpha\beta$ [A1, B1] ==
 Transpose[( $\epsilon$  /. se).S44 $\alpha\beta$ [A1, B1]]]}, {A1, 1, 3}, {B1, A1 + 1, 4}]
```

Out[732]=

```
{{{{1, 2}, True}, {{1, 3}, True}, {{1, 4}, True}}, {{{2, 3}, True}, {{2, 4}, True}}, {{{3, 4}, True}}}
```

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3, 3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula: \circlearrowleft

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. \circlearrowleft

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The $\mathfrak{so}(4,4)$ Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature $(4, 4)$. \circlearrowleft

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) \circlearrowleft

Step 2: State the general commutation relation

The commutation relations for the generators of the $\mathfrak{so}(p,q)$ Lie algebra are a generalization of the $\mathfrak{so}(n)$ algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. \circlearrowleft

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the $\mathfrak{so}(4,4)$ Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a $(4,4)$ signature (four plus signs and four minus signs, or vice versa). \circlearrowleft

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). \circlearrowleft

what are the commutation relation for the generators of $so(4,4)$?

The commutation relations for the generators $J_{\mu\nu}$ of the $so(4,4)$ Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$ are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: ⚡

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for $so(8)$ in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



↳ Why is the metric signature significant for $so(4,4)$ commutation relations?

Why is the metric signature significant for $so(4,4)$ commutation relations?

The metric signature is significant for the $so(p,q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). ⚡

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p,q)$ is given by:
$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$
The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4,4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[733]:=

η4488

Out[733]=

$$\{\{1, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, 1, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, -1\}\}$$

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

In[734]:=

```
(*Table[SAB[[A1,B1]]== ArrayFlatten[{{SAB1[[A1,B1]],0},{0,SAB2[[A1,B1]]}}]],{A1,0,7},{B1,0,7}]*)
```

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \rightarrow SO(4,4)$ is given by

$$(2.8) \quad 8L^{A'}{}_{B'}(g) = \text{tr } \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}{}_{B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

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A Remarkable Representation of the $3 + 2$ de Sitter Group

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Identities satisfied by the generators of the Dirac algebra

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In[735]:=

```
Unprotect[\gammaABs];
\gammaABs = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}], 1]
```

Out[736]=

```
\{\{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-1, 0, 0, 0\}\},
 \{\{0, 0, -1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\},
 \{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\},
 \{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{1, 0, 0, 0\}\},
 \{\{0, 0, -1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\},
 \{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 1, 0\}\},
 \{\{1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\},
 \{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\},
 \{\{0, 0, 1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\},
 \{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, -1, 0\}\},
 \{\{-1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\},
 \{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{1, 0, 0, 0\}\},
 \{\{0, 0, 1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\},
 \{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\},
 \{\{-1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\},
 \{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}}
```

In[737]:=

Length[\gammaABs]

Out[737]=

16

In[738]:=

Protect[\gammaABs]

Out[738]=

{\gammaABs}

```
In[739]:=  $\gamma \text{ABS}[1]$ 
Out[739]=  $\{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-1, 0, 0, 0\}\}$ 
```

```
In[740]:=  $\gamma \text{ABS}[-1]$ 
Out[740]=  $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$ 
```

Recall Self Dual Anti-Symmetric

```
In[741]:= Table[ $\left(\frac{1}{2} \sum_{j1=1}^4 \left(\sum_{j2=1}^4 \epsilon 4[p, q, j1, j2] \times s4by4[h][j1, j2]\right)\right) - s4by4[h][p, q],$   

 $\{h, 1, 3\}, \{q, 4\}, \{p, 4\}\right]$ 
Out[741]=  $\{\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$   

 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$   

 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\}$ 
```

Recall Anti SelfDual Anti-Symmetric

```
In[742]:= Table[ $\left(\frac{1}{2} \sum_{j1=1}^4 \left(\sum_{j2=1}^4 \epsilon 4[p, q, j1, j2] \times t4by4[h][j1, j2]\right)\right) + t4by4[h][p, q],$   

 $\{h, 1, 3\}, \{q, 4\}, \{p, 4\}\right]$ 
Out[742]=  $\{\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$   

 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$   

 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\}$ 
```

BASIS of 4×4 matrices :

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The Dirac spinor in six dimensions

BY E. A. LORD

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(Received 17 November 1966)

3. Decomposition of the basic spinor representations. We have already stated that the basic spinor representations of $SO(n, c)$, for n even, are reducible. We shall consider here the reduction of the basic 8-component spinor representation of $SO(6, c)$ into constituent 4-component representations. The procedure is easily generalized to apply to any even-dimensional space. An irreducible representation of the Clifford algebra C_6 can be constructed from a given irreducible representation of C_5 . The $\gamma_\mu (\mu = 1, \dots, 5)$ of an irreducible representation of C_5 satisfy (2.5), so that the quantities $e_\alpha (\alpha = 1, \dots, 6)$ defined by

$$\left. \begin{aligned} e_\mu &= (-i) \begin{pmatrix} & \gamma_\mu B \\ -B^{-1}\gamma_\mu & \end{pmatrix} \quad (\mu = 1, \dots, 5), \\ e_6 &= \begin{pmatrix} & B \\ B^{-1} & \end{pmatrix}, \end{aligned} \right\} \quad (3.1)$$

satisfy

$$\frac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \dots, 6).$$

For the moment the matrix B is completely arbitrary. The e_α therefore give an irreducible representation of C_6 . They can be written more concisely in terms of the sets of matrices a_α and \bar{a}_α , defined as follows:

$$\left. \begin{aligned} a_\mu &= \gamma_\mu B, & \bar{a}_\mu &= -B^{-1}\gamma_\mu \quad (\mu = 1, \dots, 5), \\ a_6 &= iB, & \bar{a}_6 &= iB^{-1}. \end{aligned} \right\} \quad (3.2)$$

The generator e_α of C_6 can then be written

$$e_\alpha = (-i) \begin{pmatrix} & a_\alpha \\ \bar{a}_\alpha & \end{pmatrix} \quad (\alpha = 1, \dots, 6). \quad (3.3)$$

With $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1e_2e_3e_4e_5e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

and $e_{\alpha\beta} = \frac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2} \begin{pmatrix} a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha & \\ \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha & \end{pmatrix}.$

This latter expression shows that the infinitesimal generators of the 8-component spinor representation of $SO(6, c)$ is the direct sum of two 4-component representations. The basic spinor can be written

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix},$$

where ϕ and χ are four component spinors which transform according to representations with infinitesimal generators

$$\left. \begin{aligned} G_{\alpha\beta}^{(1)} &= -\frac{1}{4}(a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha) \\ \text{and} \quad G_{\alpha\beta}^{(2)} &= -\frac{1}{4}(\bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha), \end{aligned} \right\} \quad (3.4)$$

$$\underbrace{\text{sy}}_1, \underbrace{\text{sy}}_4, \underbrace{\text{asy}}_6, \underbrace{\text{sy}}_4, \underbrace{\text{sy}}_1$$

```
In[743]:= Plus @@ {1, 4, 6, 4, 1}
Out[743]= 16

{sy, sy, asy, sy, sy}
  1   4   6   4   1

In[744]:= Plus @@ {1, 4, 6, 4, 1}
Out[744]= 16

In[745]:= MatrixForm[If[# == 2, i, 1] * PauliMatrix[#]] & /@ Range[3]
Out[745]= { ( 0  1 ), ( 0  1 ), ( 1  0 ) }
           1  0      -1  0      0  -1

In[746]:= σ22 =
  Flatten[{ { IdentityMatrix[2]}, If[# == 2, i, 1] * PauliMatrix[#] & /@ Range[3]}, 1]
Out[746]= { {{1, 0}, {0, 1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}} }

In[747]:= σ22 = Flatten[
  {{-IdentityMatrix[2]}, If[# == 2, i, 1] * PauliMatrix[#] & /@ Range[3]}, 1]
Out[747]= { {{-1, 0}, {0, -1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}} }

In[748]:= -IdentityMatrix[2] === Dot @@ (σ22[[#]] & /@ Range[Length[σ22]])
Out[748]= True

In[749]:= Table[ { {A, B}, FullSimplify[ 1/2 Tr[ 1/2 ( σ22[[A]].σ22[[B]] + σ22[[B]].σ22[[A]] ) ] ] }, {A, 1, 4}, {B, 1, 4} ] // MatrixForm
Out[749]//MatrixForm=

$$\begin{pmatrix} \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$


```

```
In[750]:=  $\eta_{2244} = \text{DiagonalMatrix}[\{-1, 1, -1, 1\}]$ 
Out[750]=  $\{\{-1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\}$ 
```

```
In[751]:=  $\text{Table}\left[\left\{\{A, B\}, \text{FullSimplify}\left[\text{ExpandAll}\left[\frac{1}{2} (\sigma_{22}[A] \cdot \overline{\sigma_{22}[B]} + \sigma_{22}[B] \cdot \overline{\sigma_{22}[A]}) = \eta_{2244}[A, B] * \text{IdentityMatrix}[2]\right]\right]\right\}, \{A, 1, 4\}, \{B, 1, 4\}\right] // \text{MatrixForm}$ 
```

Out[751]//MatrixForm=

$$\begin{pmatrix} \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} \\ \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} \\ \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} \\ \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} \end{pmatrix}$$

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The Dirac spinor in six dimensions

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$$\left. \begin{aligned} e_\mu &= (-i) \begin{pmatrix} \gamma_\mu B \\ -B^{-1} \gamma_\mu \end{pmatrix} \quad (\mu = 1, \dots, 5), \\ e_6 &= \begin{pmatrix} B \\ B^{-1} \end{pmatrix}, \end{aligned} \right\}$$

$$\frac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \dots, 6).$$

$$\left. \begin{array}{l} \alpha_\mu = \gamma_\mu B, \quad \bar{a}_\mu = -B^{-1}\gamma_\mu \quad (\mu = 1, \dots 5), \\ a_6 = iB, \quad \bar{a}_6 = iB^{-1}. \end{array} \right\}$$

generator e_α of C_6 can then be written

$$e_\alpha = (-i) \begin{pmatrix} & a_\alpha \\ \bar{a}_\alpha & \end{pmatrix} \quad (\alpha = 1, \dots 6).$$

$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1e_2e_3e_4e_5e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$e_{\alpha\beta} = \frac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2} \begin{pmatrix} a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha & \\ \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha & \end{pmatrix}.$$

In[752]:=

```
Do[yyy[j] = ArrayFlatten[{{0, σ22[[j]]}, {-σ22[[j]], 0}}], {j, 4}];
yyy[5] = ArrayFlatten[{{0, IdentityMatrix[2]}, {IdentityMatrix[2], 0}}]
```

Out[752]=

```
{ {0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0} }
```

In[753]:=

```
yyy[5] = {{0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0}}
```

Out[753]=

```
{ {0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0} }
```

In[754]:=

```
(yyy[6] = Dot @@ (yyy[#[ ] & /@ Range[5])) // MatrixForm
```

Out[754]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

In[755]:=

```
MatrixForm[yyy[#[ ] ] & /@ Range[6]]
```

Out[755]=

$$\left\{ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \right\}$$

In[756]:=

```
Do[xxx[j] = ArrayFlatten[{{0, σ22[[j]]}, {σ22[[j]], 0}}], {j, 4}]
```

```
In[757]:= MatrixForm[xxx[#]] & /@ Range[4]
Out[757]=
{
$$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}}$$
}

In[758]:= xxx[5] = Dot @@ (xxx[#] & /@ Range[4])
Out[758]=
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}

In[759]:= IdentityMatrix[4]
Out[759]=
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}

In[760]:=
```

```
txA0 = {{IdentityMatrix[4], {0}}};
Length[txA0]
txA = {} ; Do[AppendTo[txA, {xxx[j], {j}}], {j, 1, 4}]
Length[txA]
txAB = {};
Do[AppendTo[txAB, {xxx[j].xxx[k], {j, k}}], {j, 1, 3}, {k, j + 1, 4}]
Length[txAB]
txABC = {};
Do[AppendTo[txABC, {xxx[j].xxx[k].xxx[h], {j, k, h}}],
{j, 1, 2}, {k, j + 1, 3}, {h, k + 1, 4}];
Length[txABC]
txABCD = {{xxx[1].xxx[2].xxx[3].xxx[4], {1, 2, 3, 4}}};
Length[txABCD]
```

```
Out[761]=
1
Out[763]=
4
Out[765]=
6
Out[767]=
4
Out[769]=
1
```

In[770]:=

```
base4by4 = Flatten[{txA0, txA, txA B, txA BC, txA BC D}, 1]; Length[base4by4]
```

Out[770]=

16

In[771]:=

```

Table[{{A, B}, Block[{r}, r = FullSimplify[ $\frac{1}{4}$  Tr[
 $\frac{1}{2}$  (base4by4[[A, 1]].base4by4[[B, 1]] + base4by4[[B, 1]].base4by4[[A, 1]])]];
If[r == 0, Style[r, Red], Style[r, Darker[Green]]]], {A, 1, Length[base4by4]}, {B, 1, Length[base4by4]}] // MatrixForm

```

Out[771]//MatrixForm=

```
In[772]:= base4by4[[16]][[1]]
Out[772]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}

In[773]:= anti4by4 = {};
Do[If[base4by4[[k, 1]] == -Transpose[base4by4[[k, 1]]], AppendTo[anti4by4,
    {base4by4[[k, 1]], {{k}, base4by4[[k, 2]]}}] ], {k, 1, Length[base4by4]}];
Length[anti4by4]
anti4by4[[%]]

Out[774]= 6

Out[775]= {{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}, {{14}, {1, 3, 4}}}}
```

```
In[776]:= symm4by4 = {};
Do[If[base4by4[[k, 1]] == Transpose[base4by4[[k, 1]]], AppendTo[symm4by4,
    {base4by4[[k, 1]], {{k}, base4by4[[k, 2]]}}] ], {k, 1, Length[base4by4]}];
Length[symm4by4]
symm4by4[[%]]

Out[777]= 10

Out[778]= {{{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}, {{16}, {1, 2, 3, 4}}}}
```

```
In[779]:= η16 =
DiagonalMatrix[Table[FullSimplify[ $\frac{1}{4}$  Tr[symm4by4[[A, 1]].symm4by4[[A, 1]]],
    {A, Length[symm4by4]}]]];
Tr[η16]

Out[779]= 10
```

O(4,4); values, evecs of σ

```
In[780]:= {values, evecs} = Eigensystem[σ]
Out[780]= {{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0}, {0, -1, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1}, {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}}
```

In[781]:= $\left(\text{u} = \text{ExpandAll} \left[\frac{1}{\sqrt{2}} \text{evecs} \right] \right) // \text{MatrixForm}$

Out[781]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

In[782]:= $\text{Table}[\text{u}[[\text{h}]] \cdot \sigma \cdot \text{u}[[\text{h}]], \{\text{h}, 1, \text{Length}[\text{u}]\}]$

Out[782]= $\{-1, -1, -1, -1, 1, 1, 1, 1\}$

In[783]:= $\text{Table}[(\text{Transpose}[\text{u}][[\text{h}]]). \sigma. \text{u}[[\text{h}]], \{\text{h}, 1, \text{Length}[\text{u}]\}]$

Out[783]= $\{-1, -1, -1, -1, 1, 1, 1, 1\}$

In[784]:= $\text{Table}[(\text{Transpose}[\text{u}][[\text{A}]] == -\sigma. \text{u}[[\text{A}]], \{\text{A}, 1, 4\}]$
 $\text{Table}[(\text{Transpose}[\text{u}][[\text{A}]] == \sigma. \text{u}[[\text{A}]], \{\text{A}, 5, \text{Length}[\text{u}]\})]$

Out[784]= $\{\text{True}, \text{True}, \text{True}, \text{True}\}$

Out[785]= $\{\text{True}, \text{True}, \text{True}, \text{True}\}$

In[786]:=

hUSE = 8;

In[787]:= $\text{Protect}[\text{hUSE}]$

Out[787]= $\{\text{hUSE}\}$

In[788]:= $\text{unit} = \text{u}[[\text{hUSE}]]$

Out[788]= $\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$

```
In[789]:= Protect[unit]
Out[789]= {unit}

In[790]:= (Transpose[unit].σ.unit)
Out[790]= 1

In[791]:= (Transpose[unit].σ.(τ[0]))
Out[791]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}

In[792]:= myid[MX_] := Sum[τ[A].MX.τ[A] η4488[A, A], {A, 1, 8}]
Myid[MX_] := Sum[τ[A].MX.τ[A] × η448[A, A], {A, 1, 8}]

Union[Table[
  Sum[τ[A].bas64[k, 1].σ.τ[A] × η8[A, A], {A, 1, 8}] - IdTr[bas64[k, 1].σ] +
  4 eg[k] bas64[k, 1] - Transpose[bas64[k, 1]] .σ, {k, 1, 64}],]

{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}, {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}]

In[794]:= τ[0]
Out[794]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}


In[795]:= (Transpose[unit].σ.(τ[0]))
Out[795]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
```

In[796]:=

$$F_A^a = \text{Transpose}[\eta 4488 [\# + 1, \# + 1] * (\tau [\#].\text{unit}) \& /@ \text{Range}[0, 7]]$$

Out[796]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \\ & \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \end{aligned}$$

In[797]:=

$$F_a^A = \text{Transpose}[\text{unit}.\sigma.(\bar{\tau}[\#]) \& /@ \text{Range}[0, 7]]$$

Out[797]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \\ & \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \\ & \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\} \end{aligned}$$

In[798]:=

$$F_A^a.F_a^A === \text{ID8}$$

Out[798]=

True

pretend that the X are Minkowski Coordinates:

```
In[799]:= Block[{U, u = U[#] & /@ Range[8], fx = Fa.X, eq, sx, su, ret},
  eq = And @@ Thread[u == fx];
  sx = Solve[eq, X][[-1]];
  su = Solve[eq, u][[-1]];
  ret = {sx, su};
  ret]

Out[799]=
{ {x0 -> U[1]/Sqrt[2] + U[5]/Sqrt[2], x1 -> -U[4]/Sqrt[2] - U[8]/Sqrt[2], x2 -> U[3]/Sqrt[2] + U[7]/Sqrt[2],
  x3 -> -U[2]/Sqrt[2] - U[6]/Sqrt[2], x4 -> 1/2 (Sqrt[2] U[2] - Sqrt[2] U[6]), x5 -> 1/2 (-Sqrt[2] U[3] + Sqrt[2] U[7]),
  x6 -> 1/2 (-Sqrt[2] U[4] + Sqrt[2] U[8]), x7 -> 1/2 (Sqrt[2] U[1] - Sqrt[2] U[5])},
  {U[1] -> 1/2 (Sqrt[2] x0 + Sqrt[2] x7), U[2] -> 1/2 (-Sqrt[2] x3 + Sqrt[2] x4), U[3] -> 1/2 (Sqrt[2] x2 - Sqrt[2] x5),
  U[4] -> 1/2 (-Sqrt[2] x1 - Sqrt[2] x6), U[5] -> 1/2 (Sqrt[2] x0 - Sqrt[2] x7), U[6] -> 1/2 (-Sqrt[2] x3 - Sqrt[2] x4),
  U[7] -> 1/2 (Sqrt[2] x2 + Sqrt[2] x5), U[8] -> 1/2 (-Sqrt[2] x1 + Sqrt[2] x6)}}

In[800]:= (sgtrye(A)α = Thread[Flatten[e(A)α] → Flatten[eAa04]]) (* // Column *)
```

```
In[800]:= subsDefects

Out[800]=
{ √e2 a4[H x4] → ea4[H x4], √e-2 a4[H x4] → e-a4[H x4],
  √e2 a4[H x4] Sin[6 H x4]1/3 → ea4[H x4] Sin[6 H x4]1/6,
  1/√e2 a4[H x4] Sin[6 H x4]1/3 → e-a4[H x4]/Sin[6 H x4]1/6, 1/√e2 a4[H x4] Sin[6 H x0]1/6 → e-a4[H x4]/Sin[6 H x0]1/6,
  1/√e-2 a4[H x4] Sin[6 H x0]1/6 → ea4[H x4]/Sin[6 H x0]1/6,
  √e2 a4[H x4] Sin[6 H x0]1/6 → ea4[H x4] Sin[6 H x0]1/6,
  √e-2 a4[H x4] Sin[6 H x0]1/6 → e-a4[H x4] Sin[6 H x0]1/6 }
```

```
In[801]:= Thread[X → 0]

Out[801]=
{x0 → 0, x1 → 0, x2 → 0, x3 → 0, x4 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Stop pretending:

In[802]:=

```
FaA.e(A)α.X /. sgtrye(A)α // FullSimplify[#, constraintVars] &;
frameOctadX = # /. {Sqrt[e2 a4[H x4]] → ea4[H x4], Sqrt[e-2 a4[H x4]] → e-a4[H x4]} & /@ %
% /. {x1 → 0, x2 → 0, x3 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Out[803]=

$$\left\{ \frac{x_0 \operatorname{Cot}[6 H x_0] + e^{-a4[H x4]} x_7 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \frac{x_4 - e^{a4[H x4]} x_3 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \right. \\ \frac{(e^{a4[H x4]} x_2 - e^{-a4[H x4]} x_5) \sin[6 H x_0]^{1/6}}{\sqrt{2}}, -\frac{(e^{a4[H x4]} x_1 + e^{-a4[H x4]} x_6) \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ \frac{x_0 \operatorname{Cot}[6 H x_0] - e^{-a4[H x4]} x_7 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, -\frac{x_4 + e^{a4[H x4]} x_3 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ \left. \frac{(e^{a4[H x4]} x_2 + e^{-a4[H x4]} x_5) \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \frac{(-e^{a4[H x4]} x_1 + e^{-a4[H x4]} x_6) \sin[6 H x_0]^{1/6}}{\sqrt{2}} \right\}$$

Out[804]=

$$\left\{ \frac{x_0 \operatorname{Cot}[6 H x_0]}{\sqrt{2}}, \frac{x_4}{\sqrt{2}}, 0, 0, \frac{x_0 \operatorname{Cot}[6 H x_0]}{\sqrt{2}}, -\frac{x_4}{\sqrt{2}}, 0, 0 \right\}$$

In[805]:=

```
FaA.eAα.X - frameOctadX // FullSimplify[#, constraintVars] &
```

Out[805]=

$$\left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\ \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \right. \\ \left. \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\ \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}.eA_α. \\ \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} - \frac{x_0 \operatorname{Cot}[6 H x_0] + e^{-a4[H x4]} x_7 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \right. \\ \left. \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\ \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \right. \\ \left. \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\ \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \right.$$

$$\begin{aligned}
& eA\alpha \cdot \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} + \frac{-x_4 + e^{a4[H \times 4]} x_3 \sin[6H \times 0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \cdot eA\alpha. \\
& \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} + \frac{e^{-a4[H \times 4]} (-e^{2a4[H \times 4]} x_2 + x_5) \sin[6H \times 0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \cdot eA\alpha. \\
& \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} + \frac{(e^{a4[H \times 4]} x_1 + e^{-a4[H \times 4]} x_6) \sin[6H \times 0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \cdot eA\alpha. \\
& \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} + \frac{-x_0 \cot[6H \times 0] + e^{-a4[H \times 4]} x_7 \sin[6H \times 0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \cdot eA\alpha.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \\
& eA\alpha. \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{x4 + e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. eA\alpha. \\
& \{x0, x1, x2, x3, x4, x5, x6, x7\} - \frac{(e^{a4[H x4]} x2 + e^{-a4[H x4]} x5) \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. eA\alpha. \\
& \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{e^{-a4[H x4]} (e^{2 a4[H x4]} x1 - x6) \sin[6 H x0]^{1/6}}{\sqrt{2}}
\end{aligned}$$

In[806]:=

 $\{ \{ F_A^a \}, \{ eA\alpha \}, \{ X \} \} // Column$

Out[806]=

$$\begin{aligned}
& \left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\
& \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \right. \\
& \left. \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\
& \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}
\end{aligned}$$

{eA\alpha}

{ {x0, x1, x2, x3, x4, x5, x6, x7} }

In[807]:=

```

{x2u, u2x} = Block[{U, u = U[#] & /@ Range[8],
  fx = frameOctadX, eq, sx, su, ret, altEQ, sCot, skot},
  altEQ = And @@ {Cot[6 H x0] == kot, Sin[6 H x0] == sin, a4[H x4] == A44};
  sCot = Solve[altEQ, {Cot[6 H x0], Sin[6 H x0], a4[H x4]}][[1]];
  skot = Solve[altEQ, {kot, sin, A44}][[1]];
  eq = And @@ Thread[u == (fx /. sCot)];
  sx = Solve[eq, X][[-1]];
  su = Solve[eq, u][[-1]];
  ret = {sx /. skot // FullSimplify, su /. skot // ExpandAll};
  ret]

```

Out[807]=

$$\left\{ \begin{array}{l} x_0 \rightarrow \frac{\tan[6Hx_0](U[1] + U[5])}{\sqrt{2}}, x_1 \rightarrow -\frac{e^{-a4[Hx4]}(U[4] + U[8])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, \\ x_2 \rightarrow \frac{e^{-a4[Hx4]}(U[3] + U[7])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, x_3 \rightarrow -\frac{e^{-a4[Hx4]}(U[2] + U[6])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, x_4 \rightarrow \frac{U[2] - U[6]}{\sqrt{2}}, \\ x_5 \rightarrow \frac{e^{a4[Hx4]}(-U[3] + U[7])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, x_6 \rightarrow \frac{e^{a4[Hx4]}(-U[4] + U[8])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, x_7 \rightarrow \frac{e^{a4[Hx4]}(U[1] - U[5])}{\sqrt{2}\sin[6Hx_0]^{1/6}} \}, \\ \{ U[1] \rightarrow \frac{x_0 \cot[6Hx_0]}{\sqrt{2}} + \frac{e^{-a4[Hx4]}x_7 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, U[2] \rightarrow \frac{x_4}{\sqrt{2}} - \frac{e^{a4[Hx4]}x_3 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[3] \rightarrow \frac{e^{a4[Hx4]}x_2 \sin[6Hx_0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]}x_5 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[4] \rightarrow -\frac{e^{a4[Hx4]}x_1 \sin[6Hx_0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]}x_6 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[5] \rightarrow \frac{x_0 \cot[6Hx_0]}{\sqrt{2}} - \frac{e^{-a4[Hx4]}x_7 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[6] \rightarrow -\frac{x_4}{\sqrt{2}} - \frac{e^{a4[Hx4]}x_3 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[7] \rightarrow \frac{e^{a4[Hx4]}x_2 \sin[6Hx_0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]}x_5 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[8] \rightarrow -\frac{e^{a4[Hx4]}x_1 \sin[6Hx_0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]}x_6 \sin[6Hx_0]^{1/6}}{\sqrt{2}} \} \}$$

In[808]:=

u2x

Out[808]=

$$\begin{aligned} U[1] &\rightarrow \frac{x_0 \operatorname{Cot}[6 H x_0]}{\sqrt{2}} + \frac{e^{-a4[H x4]} x_7 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \quad U[2] \rightarrow \frac{x_4}{\sqrt{2}} - \frac{e^{a4[H x4]} x_3 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[3] &\rightarrow \frac{e^{a4[H x4]} x_2 \sin[6 H x_0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x_5 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[4] &\rightarrow -\frac{e^{a4[H x4]} x_1 \sin[6 H x_0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x_6 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[5] &\rightarrow \frac{x_0 \operatorname{Cot}[6 H x_0]}{\sqrt{2}} - \frac{e^{-a4[H x4]} x_7 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \quad U[6] \rightarrow -\frac{x_4}{\sqrt{2}} - \frac{e^{a4[H x4]} x_3 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[7] &\rightarrow \frac{e^{a4[H x4]} x_2 \sin[6 H x_0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x_5 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[8] &\rightarrow -\frac{e^{a4[H x4]} x_1 \sin[6 H x_0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x_6 \sin[6 H x_0]^{1/6}}{\sqrt{2}} \} \end{aligned}$$

In[809]:=

```
{sumu2x, diffu2x} = Block[{U, u, a1, a2, a3 = {}, aa4 = {}},
(*Clear[U,u,a1,a2,a3,a4];*)
u = U[#] & /@ Range[8]; Print[u];
a1 = u /. u2x; Print[a1];
a2 = Transpose[{{Take[a1, 4]}, {Take[a1, -4]} }]; Print[a2];
a3 = ExpandAll[FullSimplify[\frac{#[1] + #[2]}{2}]] & /@ a2;
aa4 = ExpandAll[FullSimplify[\frac{#[1] - #[2]}{2}]] & /@ a2;
{a3, aa4}]
```

$$\begin{aligned}
& \{U[1], U[2], U[3], U[4], U[5], U[6], U[7], U[8]\} \\
& \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \\
& \quad \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad - \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad \left. - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\} \\
& \left\{ \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \right. \\
& \quad \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad \left. \left. - \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, - \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \right. \right. \\
& \quad \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad \left. \left. - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\} \right\} \\
\end{aligned}$$

Out[809]=

$$\left\{ \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}}, - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \right. \\
\left. \left. - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\}, \left\{ \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \right. \\
\left. \left. \frac{x4 - \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, - \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\} \right\}$$

split octonions; evals, evecs of σ

```

# Type - 1 spinor basis (constructed from Clifford algebra considerations)
# These create a maximal isotropic subspace with respect to the split form
self.type1_spinor_matrix = sqrt2_inv*np.array([[1, 1, 0, 0, 0, 0, 0, 0], # s1_0[1, -1, 0, 0, 0, 0, 0, 0], # s1_
1[0, 0, 1, 1, 0, 0, 0, 0], # s1_2[0, 0, 1, -1, 0, 0, 0, 0], # s1_3[0, 0, 0, 1, 1, 0, 0], # s1_4[0, 0, 0, 1, -1, 0, 0], # s1_
5[0, 0, 0, 0, 0, 1, 1], # s1_6[0, 0, 0, 0, 0, 1, -1], # s1_7])

```

Type - 2 spinor basis (the complementary spinor representation)

```
# Related to Type - 1 by another triality automorphism
self.type2_spinor_basis_matrix = sqrt2_inv*np.array([[1, 0, 1, 0, 0, 0, 0, 0], # s2_0[1, 0, -1, 0, 0, 0, 0, 0], # s2_
1[0, 1, 0, 1, 0, 0, 0, 0], # s2_2[0, 1, 0, -1, 0, 0, 0, 0], # s2_3[0, 0, 0, 1, 0, 1, 0], # s2_4[0, 0, 0, 1, 0, -1, 0], # s2_5[0, 0, 0, 0, 1, 0, 1], # s2_6[0, 0, 0, 0, 1, 0, -1], # s2_7])
```

In[810]:=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

Out[810]=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

In[811]:=

4 Transformation Matrices

We provide explicit 8×8 matrices that transform coordinates between the three bases.

4.1 Vector to Type-1 Spinor

The transformation matrix from vector basis to type-1 spinor basis is:

$$M_{V \rightarrow S_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (11)$$

4.2 Vector to Type-2 Spinor

The transformation matrix from vector basis to type-2 spinor basis is:

$$M_{V \rightarrow S_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (12)$$

Out[811]=

4 Transformation Matrices

We provide explicit 8×8 matrices that transform coordinates between the three bases.

4.1 Vector to Type-1 Spinor

The transformation matrix from vector basis to type-1 spinor basis is:

$$M_{V \rightarrow S_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (11)$$

4.2 Vector to Type-2 Spinor

The transformation matrix from vector basis to type-2 spinor basis is:

$$M_{V \rightarrow S_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (12)$$

In[812]:=

```
(type1SpinorBasisMatrix =
 {{1, 1, 0, 0, 0, 0, 0, 0, 0}, {1, -1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 1, 0, 0, 0, 0, 0}, {0, 0, 1, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 1, 0, 0, 0}, {0, 0, 0, 0, 1, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 1}, {0, 0, 0, 0, 0, 0, 1, -1}}) // MatrixForm
```

Out[812]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{pmatrix}$$

In[813]:=

```
(type2SpinorBasisMatrix =
 {{1, 0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, -1, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 1, 0, 0, 0, 0, 0}, {0, 1, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 1, 0}, {0, 0, 0, 0, 1, 0, -1, 0},
 {0, 0, 0, 0, 0, 1, 0, 1}, {0, 0, 0, 0, 0, 1, 0, -1}}) // MatrixForm
```

Out[813]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix}$$

In[814]:=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

Out[814]=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

In[815]:=

$$\mathbf{MS1S2} = \text{type1SpinorBasisMatrix}.\text{Transpose}[\text{type2SpinorBasisMatrix}] // \text{MatrixForm}$$

Out[815]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 \end{pmatrix}$$

In[816]:=

$$\mathbf{g}_{AB} = \eta 4488$$

Out[816]=

$$\{\{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 0, 0, -1\}\}$$

In[817]:=

$$\{\mathbf{evals}_{AB}, \mathbf{evecs}_{AB}\} = \text{Eigensystem}[\mathbf{g}_{AB}]$$

Out[817]=

$$\{\{-1, -1, -1, -1, 1, 1, 1, 1\}, \{\{0, 0, 0, 0, 0, 0, 0, 1\}, \{0, 0, 0, 0, 0, 0, 1, 0\}, \{0, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\}, \{1, 0, 0, 0, 0, 0, 0, 0\}\}\}$$

In[818]:=

$$\{\mathbf{evals}, \mathbf{evecs}\} = \text{Eigensystem}[\sigma]$$

Out[818]=

$$\{\{-1, -1, -1, -1, 1, 1, 1, 1\}, \{\{0, 0, 0, -1, 0, 0, 0, 1\}, \{0, 0, -1, 0, 0, 0, 1, 0\}, \{0, -1, 0, 0, 0, 1, 0, 0\}, \{-1, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 1\}, \{0, 0, 1, 0, 0, 0, 1, 0\}, \{0, 1, 0, 0, 0, 1, 0, 0\}, \{1, 0, 0, 0, 0, 1, 0, 0\}\}\}$$

In[819]:=

$$\left(\mathbf{u} = \text{ExpandAll} \left[\frac{1}{\sqrt{2}} \mathbf{evecs} \right] \right) // \text{MatrixForm}$$

Out[819]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

```
In[820]:= Table[u[[h]].σ.u[[h]], {h, 1, Length[u]}]
Out[820]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[821]:= Table[(Transpose[u][[h]]).σ.u[[h]], {h, 1, Length[u]}]
Out[821]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[822]:= Table[(Transpose[u][[A]] == -σ.u[[A]], {A, 1, 4}]
Table[(Transpose[u][[A]] == σ.u[[A]], {A, 5, Length[u]}]
Out[822]= {True, True, True, True}

Out[823]= {True, True, True, True}

In[824]:= hUSE = 8;
*** Set: Symbol hUSE is Protected. i

In[825]:= Protect[hUSE]
Out[825]= {}

In[826]:= unit = u[[hUSE]]
*** Set: Symbol unit is Protected. i
Out[826]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}

In[827]:= Protect[unit]
Out[827]= {}

In[828]:= (Transpose[unit].σ.unit)
Out[828]= 1

In[829]:= (Transpose[unit].σ.(τ[θ]))
Out[829]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
```

In[830]:=

```

Do[E^A_a[h] = Table[(u[[h]].\sigma.(\overline{\tau}[B])), {B, 0, 7}], {h, 1, Length[u]}];

$$\tilde{g}^A_a = \frac{1}{\sqrt{u}\sigma u} \sigma^c \sigma_b \tilde{\tau}^{ab}{}_{ac}$$

Do[E^a_A[h] = FullSimplify[Inverse[E^A_a[h]]], {h, 1, Length[u]}];

$$\tilde{g}^a_b = \frac{1}{\sqrt{u}\sigma u} \tau^a{}_b u^b$$

Table[E^a_A[h] === (u[[h]].\sigma.u[[h]])
Transpose[Table[FullSimplify[ExpandAll[\eta4488[B, B] \times \tau[B - 1].u[[h]]]],
{B, 1, 8}]], {h, 1, Length[u]}]

```

Out[830]=

$$\mathfrak{F}_a^A = \frac{1}{\sqrt{\tilde{u}\sigma u}} u^c \sigma_{cb} \bar{\tau}^{Ab}{}_a$$

Out[831]=

$$\mathfrak{F}_A^a = \frac{1}{\sqrt{\tilde{u}\sigma u}} \tau_A{}^a{}_b u^b$$

Out[832]=

{True, True, True, True, True, True, True, True}

In[833]:=

```
(*Do[E^A_a[h]=Table[(u[[h]].\sigma.\tau[B]),{B,0,7}],{h,1,Length[u]}];
Do[E^a_A[h]=FullSimplify[Inverse[E^A_a[h]]],{h,1,Length[u]}];
Table[E^a_A[h]==(u[[h]].\sigma.u[[h]])
    Transpose[Table[FullSimplify[ExpandAll[\eta4488[[B,B]\tau[B-1].u[[h]]]],
{B,1,8}]],{h,1,Length[u]}]*]
```

In[834]:=

```

myid[MX_] := Sum[ $\bar{\tau}[A].MX.\tau[A] \eta4488[A, A]$ , {A, 1, 8}]
Myid[MX_]:=Sum[ $\tau[A].MX.\bar{\tau}[A] \times \eta4488[A, A]$ , {A, 1, 8}]

```

```

Union[Table[
  Sum[τ[A].bas64[[k, 1]].σ.τ[A] × η8[[A, A]], {A, 1, 8}] - IdTr[bas64[[k, 1]].σ] +
  4 eg[[k]] 
$$\frac{\text{bas64}[[k, 1]] - \text{Transpose}[\text{bas64}[[k, 1]]]}{2} \cdot \sigma, \{k, 1, 64\}\Big]$$

```

```
{{{0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0},  
{0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0}}}
```

In[836]:=

10

Out[836]=

```

{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}

```

```
In[837]:= (Transpose[unit].σ. (τ[0]))
```

```
Out[837]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
```

```
In[838]:= FAa = Transpose[η4488[[# + 1, # + 1] * (τ[#].unit) & /@ Range[0, 7]]]
```

```
Out[838]= { {1/√2, 0, 0, 0, 0, 0, 0, 1/√2}, {0, 0, 0, -1/√2, 1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, 0, 0, 0}, {0, -1/√2, 0, 0, 0, 0, -1/√2, 0}, {1/√2, 0, 0, 0, 0, 0, 0, -1/√2}, {0, 0, 0, -1/√2, -1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, 0, 0, 0}, {0, -1/√2, 0, 0, 0, 0, 1/√2, 0} }
```

```
In[839]:= FAa = Transpose[unit].σ. (τ[#]) & /@ Range[0, 7]
```

```
Out[839]= { {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}, {0, 0, 0, -1/√2, 0, 0, 0, -1/√2}, {0, 0, 1/√2, 0, 0, 0, 0, 0}, {0, -1/√2, 0, 0, 0, 0, -1/√2, 0}, {1/√2, 0, 0, 0, 0, 0, 0, 1/√2}, {0, 0, 0, -1/√2, -1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, 0, 0, 0}, {0, -1/√2, 0, 0, 0, 0, 1/√2, 0} }
```

```
In[840]:= FAa.FAa === ID8
```

```
Out[840]= True
```

```
gtrye(A)α = (e(A)α /. sgtrye(A)α) /. subsDefects
```

```
gtryeα(A) = (eα(A) /. sgtryeα(A)) /. subsDefects
```

```
(sgtryeα(A) = Thread[Flatten[eα(A)] → Flatten[Inverse[e(A)α /. sgtrye(A)α]]])
```

```
(*//Column*)
```

```
gtryeα(A) = (eα(A) /. sgtryeα(A)) /. subsDefects
```

Out[8]:=

$$\begin{aligned} & \left\{ \{\tan[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0 \right\}, \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}} \right\} \right\} \end{aligned}$$

pretend that the X are Minkowski Coordinates:

In[841]:=

```
Block[{U, u = U[#] & /@ Range[8], fx = Fa.X, eq, sx, su, ret},
eq = And @@ Thread[u == fx];
sx = Solve[eq, X][[-1]];
su = Solve[eq, u][[-1]];
ret = {sx, su};
ret]
```

Out[841]:=

$$\begin{aligned} & \left\{ x0 \rightarrow \frac{U[1]}{\sqrt{2}} + \frac{U[5]}{\sqrt{2}}, x1 \rightarrow -\frac{U[4]}{\sqrt{2}} - \frac{U[8]}{\sqrt{2}}, x2 \rightarrow \frac{U[3]}{\sqrt{2}} + \frac{U[7]}{\sqrt{2}}, \right. \\ & x3 \rightarrow -\frac{U[2]}{\sqrt{2}} - \frac{U[6]}{\sqrt{2}}, x4 \rightarrow \frac{1}{2} (\sqrt{2} U[2] - \sqrt{2} U[6]), x5 \rightarrow \frac{1}{2} (-\sqrt{2} U[3] + \sqrt{2} U[7]), \\ & x6 \rightarrow \frac{1}{2} (-\sqrt{2} U[4] + \sqrt{2} U[8]), x7 \rightarrow \frac{1}{2} (\sqrt{2} U[1] - \sqrt{2} U[5]) \Big\}, \\ & \left\{ U[1] \rightarrow \frac{1}{2} (\sqrt{2} x0 + \sqrt{2} x7), U[2] \rightarrow \frac{1}{2} (-\sqrt{2} x3 + \sqrt{2} x4), U[3] \rightarrow \frac{1}{2} (\sqrt{2} x2 - \sqrt{2} x5), \right. \\ & U[4] \rightarrow \frac{1}{2} (-\sqrt{2} x1 - \sqrt{2} x6), U[5] \rightarrow \frac{1}{2} (\sqrt{2} x0 - \sqrt{2} x7), U[6] \rightarrow \frac{1}{2} (-\sqrt{2} x3 - \sqrt{2} x4), \\ & \left. U[7] \rightarrow \frac{1}{2} (\sqrt{2} x2 + \sqrt{2} x5), U[8] \rightarrow \frac{1}{2} (-\sqrt{2} x1 + \sqrt{2} x6) \right\} \end{aligned}$$

In[842]:=

```
(sgtrye(A)α = Thread[Flatten[e(A)α] → Flatten[eAa04]]) (* //Column*)
```

In[842]:=

subsDefects

Out[842]=

$$\left\{ \begin{aligned} \sqrt{e^{2a4[Hx4]}} &\rightarrow e^{a4[Hx4]}, \quad \sqrt{e^{-2a4[Hx4]}} \rightarrow e^{-a4[Hx4]}, \\ \sqrt{e^{2a4[Hx4]}} \sin[6Hx4]^{1/3} &\rightarrow e^{a4[Hx4]} \sin[6Hx4]^{1/6}, \\ \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx4]^{1/3}} &\rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx4]^{1/6}}, \quad \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, \\ \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}} &\rightarrow \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, \\ \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6} &\rightarrow e^{a4[Hx4]} \sin[6Hx0]^{1/6}, \\ \sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6} &\rightarrow e^{-a4[Hx4]} \sin[6Hx0]^{1/6} \end{aligned} \right\}$$

In[843]:=

Thread[X → 0]

Out[843]=

$$\{x0 \rightarrow 0, x1 \rightarrow 0, x2 \rightarrow 0, x3 \rightarrow 0, x4 \rightarrow 0, x5 \rightarrow 0, x6 \rightarrow 0, x7 \rightarrow 0\}$$

Stop pretending:

$$(sgtrye_{(A)}^{\alpha} = Thread[Flatten[e_{(A)}^{\alpha}] \rightarrow Flatten[Inverse[e_{\alpha}^{(A)} /. sgtrye_{\alpha}^{(A)}]]])$$

(*//Column*)

$$gtrye_{(A)}^{\alpha} = (e_{(A)}^{\alpha} /. sgtrye_{(A)}^{\alpha}) /. subsDefects$$

$$\begin{aligned} &\left\{ \{ \tan[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}} \right\} \right\} \end{aligned}$$

In[844]:=

$$e_{\alpha}^{(A)} = eA\alpha = gtrye_{\alpha}^{(A)}$$

Out[844]=

$$\begin{aligned} &\left\{ \{ \cot[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6} \right\} \right\} \end{aligned}$$

$$In[844]:= gtrye_{\alpha}^{(A)} = (e_{\alpha}^{(A)} /. sgtrye_{\alpha}^{(A)}) /. subsDefects$$

In[845]:= $\text{gtrye}_{(A)}^{\alpha} = (\mathbf{e}_{(A)}^{\alpha} /. \text{sgtrye}_{(A)}^{\alpha}) /. \text{subsDefects}$

In[845]:= $\{\text{gtrye}_{\alpha}^{(A)}, \text{gtrye}_{(A)}^{\alpha}\} // \text{MatrixForm}$

Out[845]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6Hx0] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ e^{a4[Hx4]} \sin[6Hx0]^{1/6} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ e^{a4[Hx4]} \sin[6Hx0]^{1/6} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ e^{a4[Hx4]} \sin[6Hx0]^{1/6} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \tan[6Hx0] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In[846]:=

```
FAa.eα(A).X /. sgtryeα(A) // FullSimplify[#, constraintVars] &;
frameOctadX = # /. {Sqrt[e2 a4[Hx4]] → ea4[Hx4], Sqrt[e-2 a4[Hx4]] → e-a4[Hx4]} & /@ %
% /. {x1 → 0, x2 → 0, x3 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Out[847]=

$$\left\{ \frac{x0 \cot[6Hx0] + e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{x4 - e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right.$$

$$\frac{e^{-a4[Hx4]} (e^{2 a4[Hx4]} x2 - x5) \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{-e^{-a4[Hx4]} (e^{2 a4[Hx4]} x1 + x6) \sin[6Hx0]^{1/6}}{\sqrt{2}},$$

$$\frac{x0 \cot[6Hx0] - e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{x4 + e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}},$$

$$\left. \frac{e^{-a4[Hx4]} (e^{2 a4[Hx4]} x2 + x5) \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{e^{-a4[Hx4]} (-e^{2 a4[Hx4]} x1 + x6) \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\}$$

Out[848]=

$$\left\{ \frac{x0 \cot[6Hx0]}{\sqrt{2}}, \frac{x4}{\sqrt{2}}, 0, 0, \frac{x0 \cot[6Hx0]}{\sqrt{2}}, -\frac{x4}{\sqrt{2}}, 0, 0 \right\}$$

In[849]:=

```
FAa.eAα.X - frameOctadX // FullSimplify[#, constraintVars] &
```

Out[849]=

$$\{0, 0, 0, 0, 0, 0, 0, 0\}$$

```
In[850]:= eAα
Out[850]=
{{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0}, {0, e^a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {0, 0, e^a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {0, 0, 0, e^a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e^-a4[H x4] Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, e^-a4[H x4] Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, e^-a4[H x4] Sin[6 H x0]^(1/6)}}

In[851]:= {{F_A^a}, {eAα}, {X}} // Column
Out[851]=
{{{{1/√2, 0, 0, 0, 0, 0, 0, 1/√2}, {0, 0, 0, -1/√2, 1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, -1/√2, 0, 0}, {0, -1/√2, 0, 0, 0, 0, -1/√2, 0}, {1/√2, 0, 0, 0, 0, 0, -1/√2}, {0, 0, 0, -1/√2, -1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, 0, 0, 1/√2}}, {{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0}, {0, e^a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {0, 0, e^a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e^-a4[H x4] Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, e^-a4[H x4] Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, e^-a4[H x4] Sin[6 H x0]^(1/6)}}, {x0, x1, x2, x3, x4, x5, x6, x7}}}
```

split octonion multiplication constants:

$$m_{ab}^c = \mathbb{F}_a^A \tau_A{}_b{}^c$$

$$m_{AB}^C = \mathbb{F}_c^C \tau_A{}_b{}^c \mathbb{F}_B^b$$

```
In[852]:= (xw)w = xw^2
Out[852]= (xw)w = xw^2
```

In[853]:=

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

Out[853]=

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

In[854]:=

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex structure J_u . If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on \mathbf{O} . However, because S^6 is connected, we see that the orientation of \mathbf{O} induced by the natural orientation of \mathbf{O}_u as a complex vector space does not depend on u . We refer to this orientation as the natural orientation of \mathbf{O} .

Out[854]=

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In[855]:=

Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

so J_u is an isometry for each $u \in S$. Moreover, it follows that \mathbf{O}_u is endowed with a natural Hermitian structure with respect to the inner product $\langle \cdot, \cdot \rangle$. We denote the group of complex linear transformations of \mathbf{O}_u by $GL(\mathbf{O}_u)$ and the special unitary transformations of \mathbf{O}_u with its Hermitian metric by $SU(\mathbf{O}_u)$.

Out[855]=

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$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

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J_u

In[856]:=

```
Table[FullSimplify[
Sum[\psi[A1] \times J[B1] gAB[[A1, A1]] E^A_a[hUSE][[C]][c] \tau[A1 - 1][[c]][b] E^a_A[hUSE][[b]][B1],
{c, 1, 8}, {b, 1, 8}, {A1, 1, 8}, {B1, 1, 8}]], {C, 1, 8}]
```

Out[856]=

$$\begin{aligned}
& \{J[1] \times \psi[1] - J[2] \times \psi[2] - J[3] \times \psi[3] - J[4] \times \psi[4] + J[5] \times \psi[5] + \\
& J[6] \times \psi[6] + J[7] \times \psi[7] + J[8] \times \psi[8], J[2] \times \psi[1] + J[1] \times \psi[2] - \\
& J[4] \times \psi[3] + J[3] \times \psi[4] + J[6] \times \psi[5] - J[5] \times \psi[6] - J[8] \times \psi[7] + J[7] \times \psi[8], \\
& J[3] \times \psi[1] + J[4] \times \psi[2] + J[1] \times \psi[3] - J[2] \times \psi[4] + J[7] \times \psi[5] + J[8] \times \psi[6] - \\
& J[5] \times \psi[7] - J[6] \times \psi[8], J[4] \times \psi[1] - J[3] \times \psi[2] + J[2] \times \psi[3] + \\
& J[1] \times \psi[4] + J[8] \times \psi[5] - J[7] \times \psi[6] + J[6] \times \psi[7] - J[5] \times \psi[8], \\
& J[5] \times \psi[1] + J[6] \times \psi[2] + J[7] \times \psi[3] + J[8] \times \psi[4] + J[1] \times \psi[5] - J[2] \times \psi[6] - \\
& J[3] \times \psi[7] - J[4] \times \psi[8], J[6] \times \psi[1] - J[5] \times \psi[2] + J[8] \times \psi[3] - \\
& J[7] \times \psi[4] + J[2] \times \psi[5] + J[1] \times \psi[6] + J[4] \times \psi[7] - J[3] \times \psi[8], \\
& J[7] \times \psi[1] - J[8] \times \psi[2] - J[5] \times \psi[3] + J[6] \times \psi[4] + J[3] \times \psi[5] - J[4] \times \psi[6] + \\
& J[1] \times \psi[7] + J[2] \times \psi[8], J[8] \times \psi[1] + J[7] \times \psi[2] - J[6] \times \psi[3] - \\
& J[5] \times \psi[4] + J[4] \times \psi[5] + J[3] \times \psi[6] - J[2] \times \psi[7] + J[1] \times \psi[8]\}
\end{aligned}$$

In[857]:=

Table

```
FullSimplify[Sum[gAB[[B1, B1]] E^A_a[hUSE][[C]][[c]] \[Tau][[B1 - 1]][[c]][[b]] E^a_A[hUSE][[b]][[B1]], {c, 1, 8}, {b, 1, 8}]], {C, 1, 8}, {A1, 1, 8}, {B1, 1, 8}]
```

Out[857]=


```

In[860]:= EA = Array[eA, 8]
Out[860]= {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[861]:= e(A)α
Out[861]= {{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0}, {0, ea4[H x4] Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, ea4[H x4] Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, 0, ea4[H x4] Sin[6 H x0]1/6, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e-a4[H x4] Sin[6 H x0]1/6, 0, 0}, {0, 0, 0, 0, 0, 0, e-a4[H x4] Sin[6 H x0]1/6, 0}, {0, 0, 0, 0, 0, 0, 0, e-a4[H x4] Sin[6 H x0]1/6}}

In[862]:= h = hUSE;
B = 1; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 2; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 3; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 4; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 5; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 6; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 7; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 8;
FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]

Out[863]= {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

Out[864]= {eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}

Out[865]= {eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}

Out[866]= {eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]}

Out[867]= {eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]}

Out[868]= {eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]}

Out[869]= {eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]}

Out[870]= {eA[8], eA[7], -eA[6], -eA[5], -eA[4], -eA[3], eA[2], eA[1]}

```

In[871]:=

```
B = 1;
Grid[
{EA, FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]}, Frame → All]
```

Out[871]=

eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]

In[872]:=

```
gridx = Grid[Partition[Flatten[{{{x}, EA}, Table[
({{x}, FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]} /.
{x → ToExpression["eA[" <> ToString[B] <> "]"]}),
{B, 1, 8}]}], 9], Frame → All]
```

Out[872]=

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

In[873]:=

$\epsilon_A =$	$\epsilon_B =$	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8
ϵ_1		$-\epsilon_8$	ϵ_3	$-\epsilon_2$	$-\epsilon_5$	ϵ_4	$-\epsilon_7$	ϵ_6	ϵ_1
ϵ_2		$-\epsilon_3$	$-\epsilon_8$	ϵ_1	$-\epsilon_6$	ϵ_7	ϵ_4	$-\epsilon_5$	ϵ_2
ϵ_3		ϵ_2	$-\epsilon_1$	$-\epsilon_8$	$-\epsilon_7$	$-\epsilon_6$	ϵ_5	ϵ_4	ϵ_3
ϵ_4		ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4
ϵ_5		$-\epsilon_4$	$-\epsilon_7$	ϵ_6	$-\epsilon_1$	ϵ_8	ϵ_3	$-\epsilon_2$	ϵ_5
ϵ_6		ϵ_7	$-\epsilon_4$	$-\epsilon_5$	$-\epsilon_2$	$-\epsilon_3$	ϵ_8	ϵ_1	ϵ_6
ϵ_7		$-\epsilon_6$	ϵ_5	$-\epsilon_4$	$-\epsilon_3$	ϵ_2	$-\epsilon_1$	ϵ_8	ϵ_7
ϵ_8		ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8

Out[873]=

$\epsilon_A =$	$\epsilon_B =$	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8
ϵ_1		$-\epsilon_8$	ϵ_3	$-\epsilon_2$	$-\epsilon_5$	ϵ_4	$-\epsilon_7$	ϵ_6	ϵ_1
ϵ_2		$-\epsilon_3$	$-\epsilon_8$	ϵ_1	$-\epsilon_6$	ϵ_7	ϵ_4	$-\epsilon_5$	ϵ_2
ϵ_3		ϵ_2	$-\epsilon_1$	$-\epsilon_8$	$-\epsilon_7$	$-\epsilon_6$	ϵ_5	ϵ_4	ϵ_3
ϵ_4		ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4
ϵ_5		$-\epsilon_4$	$-\epsilon_7$	ϵ_6	$-\epsilon_1$	ϵ_8	ϵ_3	$-\epsilon_2$	ϵ_5
ϵ_6		ϵ_7	$-\epsilon_4$	$-\epsilon_5$	$-\epsilon_2$	$-\epsilon_3$	ϵ_8	ϵ_1	ϵ_6
ϵ_7		$-\epsilon_6$	ϵ_5	$-\epsilon_4$	$-\epsilon_3$	ϵ_2	$-\epsilon_1$	ϵ_8	ϵ_7
ϵ_8		ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8

1	i	j	k	ℓ	li	lj	lk
i	-1	k	$-j$	$-li$	ℓ	$-lk$	lj
j	$-k$	-1	i	$-lj$	lk	ℓ	$-li$
k	j	$-i$	-1	$-lk$	$-lj$	li	ℓ
ℓ	li	lj	ℓk	1	i	j	k
li	$-\ell$	$-\ell k$	lj	$-i$	1	k	$-j$
lj	ℓk	$-\ell$	$-li$	$-j$	$-k$	1	i
lk	$-\ell j$	li	$-\ell$	$-k$	j	$-i$	1

In[874]:=

```
(τABC = Table[ExpandAll[gAB[[B, B]] E^A_a[hUSE].τ[B - 1].E^a_A[hUSE]], {B, 1, 8}]);  
(τABC[[#]] // MatrixForm) & /@ Range[8]
```

Out[874]=

$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[875]:=

EA

Out[875]=

{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[876]:=

EA.τABC[[1]]

Out[876]=

{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[877]:=

EA.τABC[[2]]

Out[877]=

{eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}

In[878]:=

EA.τABC[[3]]

Out[878]=

{eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}

```
In[879]:= EA.τABC[[4]]
Out[879]= {eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]}

In[880]:= EA.τABC[[5]]
Out[880]= {eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]}

In[881]:= EA.τABC[[6]]
Out[881]= {eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]}

In[882]:= EA.τABC[[7]]
Out[882]= {eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]}

In[883]:= TeXForm[gridx]
Out[883]//TeXForm=
\begin{array}{cccccccccc}
x & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \text{eA}(4) & \\
& \text{eA}(5) & \text{eA}(6) & \text{eA}(7) & \text{eA}(8) & \\
\text{eA}(1) & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \\
& \text{eA}(4) & \text{eA}(5) & \text{eA}(6) & \text{eA}(7) & \\
& \text{eA}(8) & & & & \\
\text{eA}(2) & \text{eA}(2) & -\text{eA}(1) & -\text{eA}(4) & \\
& \text{eA}(3) & -\text{eA}(6) & \text{eA}(5) & \text{eA}(8) & \\
& -\text{eA}(7) & & & & \\
\text{eA}(3) & \text{eA}(3) & -\text{eA}(4) & -\text{eA}(1) & \\
& -\text{eA}(2) & -\text{eA}(7) & -\text{eA}(8) & \text{eA}(5) & \\
& \text{eA}(6) & & & & \\
\text{eA}(4) & \text{eA}(4) & -\text{eA}(3) & \text{eA}(2) & \\
& -\text{eA}(1) & -\text{eA}(8) & \text{eA}(7) & -\text{eA}(6) & \\
& \text{eA}(5) & & & & \\
\text{eA}(5) & \text{eA}(5) & -\text{eA}(6) & \text{eA}(7) & \\
& \text{eA}(8) & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \\
& \text{eA}(4) & & & & \\
\text{eA}(6) & \text{eA}(6) & -\text{eA}(5) & \text{eA}(8) & \\
& -\text{eA}(7) & -\text{eA}(2) & \text{eA}(1) & -\text{eA}(4) & \\
& \text{eA}(3) & & & & \\
\text{eA}(7) & \text{eA}(7) & -\text{eA}(8) & -\text{eA}(5) & \\
& \text{eA}(6) & -\text{eA}(3) & \text{eA}(4) & \text{eA}(1) & \\
& -\text{eA}(2) & & & & \\
\text{eA}(8) & \text{eA}(8) & -\text{eA}(7) & -\text{eA}(6) & \\
& -\text{eA}(5) & -\text{eA}(4) & -\text{eA}(3) & \text{eA}(2) & \\
& \text{eA}(1) & & & & \\
& & & & &
\end{array}
```

```
In[884]:= (*StringReplace[ToString[TeXForm[gridx]], "\text{eA}" \rightarrow "\epsilon"]*)

In[885]:= Grid[Partition[
  Flatten[{{{x}, EA}, Table[({{x}, Table[Sum[FullSimplify[ExpandAll[gAB[[B, B]
    EA[[C1]] E^A_a[hUSE][[C1, c1]] \[Tau][B - 1][[c1, d1]] E^a_A[hUSE][[d1, B1]]]], ,
    {C1, 1, 8}, {c1, 1, 8}, {d1, 1, 8}], {B1, 1, 8}}] /. ,
    {x \[Rule] ToExpression["eA" \[LessThan> ToString[B] \[LessThan>] ""]}], ,
    {B, 1, 8}]}]}, 9], Frame \[Rule] All]

Out[885]=
```

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]


```
In[886]:= gridx

Out[886]=
```

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

Complex Structure

J_u

```
In[887]:= J[#] \[Rule] 0 & /@ Range[0, 7]

Out[887]= {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 0, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}

In[888]:= sJu = {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 1, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}

Out[888]= {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 1, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}
```

```
for 1 :
i
```

In[889]:=

```
Block[{ψ, J, t, r, sJu},
sJu =
{J[0] → 0, J[1] → 0, J[2] → 0, J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0};
t = Table[FullSimplify[Sum[ψ[A1 - 1] × J[B1 - 1] gAB[[A1, A1]],
E^A_a[hUSE][[C]][c] τ[A1 - 1][[c]][b] E^a_A[hUSE][[b]][B1]],
{c, 1, 8}, {b, 1, 8}, {A1, 1, 8}, {B1, 1, 8}]], {C, 1, 8}];
r = t /. sJu
]
```

Out[889]=

{ψ[4], -ψ[5], -ψ[6], -ψ[7], ψ[0], -ψ[1], -ψ[2], -ψ[3]}

In[890]:=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \tilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elements

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

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$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix} \quad (10)$$

of the 16-component spinor representation of $\overline{\text{SO}(4,4;\mathbb{R})}$. We see that, as is, in fact, well known from the general theory, the 16-component spinor representation of $\text{SO}(4,4;\mathbb{R})$ is the direct sum of two (inequivalent) real 8×8 irreducible spinor representations $D_{(1)}=D_{(1)}(g)$ and $D_{(2)}=D_{(2)}(g)$ of $\text{SO}(4,4;\mathbb{R}) \ni g$ that are generated by $D_{(1)}^{AB}$ and $D_{(2)}^{AB}$, respectively. This we record as

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A \quad (11)$$

and

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A. \quad (12)$$

Out[890]=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \widetilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elements

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

042501-7 Second gravity

J. Math. Phys. 51, 042501 (2010)

$$\tau^A t^B - t^B \tau^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix} \quad (10)$$

of the 16-component spinor representation of $\overline{\text{SO}(4,4;\mathbb{R})}$. We see that, as is, in fact, well known from the general theory, the 16-component spinor representation of $\text{SO}(4,4;\mathbb{R})$ is the direct sum of two (inequivalent) real 8×8 irreducible spinor representations $D_{(1)}=D_{(1)}(g)$ and $D_{(2)}=D_{(2)}(g)$ of $\text{SO}(4,4;\mathbb{R}) \ni g$ that are generated by $D_{(1)}^{AB}$ and $D_{(2)}^{AB}$, respectively. This we record as

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A \quad (11)$$

and

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A. \quad (12)$$

In[891]:=

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$$\widetilde{D_{(1)}}^{AB} \sigma = -\sigma D_{(1)}^{AB} \Rightarrow \tilde{D}_{(1)} \sigma = \sigma D_{(1)}^{-1}, \quad (18)$$

$$\widetilde{D_{(2)}}^{AB} \sigma = -\sigma D_{(2)}^{AB} \Rightarrow \tilde{D}_{(2)} \sigma = \sigma D_{(2)}^{-1}, \quad (19)$$

$$L^A{}_C G_{AB} L^B{}_D = G_{CD} = \{\tilde{L} G L\}_{CD}, \quad (20)$$

$$L^A{}_B \bar{\tau}^B = D_{(1)}^{-1} \bar{\tau}^A D_{(2)}, \quad (21)$$

$$L^A{}_B \tau^B = D_{(2)}^{-1} \tau^A D_{(1)}, \quad (22)$$

The canonical 2-1 homomorphism $\overline{\mathrm{SO}(4,4;\mathbb{R})} \rightarrow \mathrm{SO}(4,4;\mathbb{R}): g \mapsto L(g)$ is given by

$$8L^A{}_B = \mathrm{tr}(D_{(1)}^{-1} \bar{\tau}^A D_{(2)} \tau^B) G_{CB}, \quad (23)$$

where tr denotes the trace. Note that $D_{(1)}(g(\omega)) = D_{(2)}(g(\omega))$ when $\omega_{A8} = 0$, i.e., when one restricts $\overline{\mathrm{SO}(4,4;\mathbb{R})}$ to

$$g = \begin{pmatrix} \exp\left(\frac{1}{4}\omega_{AB} D_{(1)}^{AB}\right) & 0 \\ 0 & \exp\left(\frac{1}{4}\omega_{AB} D_{(2)}^{AB}\right) \end{pmatrix} \quad \text{and} \quad \omega_{A8} = 0 \quad (24)$$

[one of the real forms of spin (7, C)].

Out[891]=

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$$\widetilde{D_{(1)}}^{AB}\sigma = -\sigma D_{(1)}^{AB} \Rightarrow \tilde{D}_{(1)}\sigma = \sigma D_{(1)}^{-1}, \quad (18)$$

$$\widetilde{D_{(2)}}^{AB}\sigma = -\sigma D_{(2)}^{AB} \Rightarrow \tilde{D}_{(2)}\sigma = \sigma D_{(2)}^{-1}, \quad (19)$$

$$L^A{}_C G_{AB} L^B{}_D = G_{CD} = \{\tilde{L}GL\}_{CD}, \quad (20)$$

$$L^A{}_B \bar{\tau}^B = D_{(1)}^{-1} \bar{\tau}^A D_{(2)}, \quad (21)$$

$$L^A{}_B \tau^B = D_{(2)}^{-1} \tau^A D_{(1)}, \quad (22)$$

The canonical 2-1 homomorphism $\overline{\mathrm{SO}(4,4;\mathbb{R})} \rightarrow \mathrm{SO}(4,4;\mathbb{R}): g \mapsto L(g)$ is given by

$$8L^A{}_B = \mathrm{tr}(D_{(1)}^{-1} \bar{\tau}^A D_{(2)} \tau^B) G_{CB}, \quad (23)$$

where tr denotes the trace. Note that $D_{(1)}(g(\omega)) = D_{(2)}(g(\omega))$ when $\omega_{A8}=0$, i.e., when one restricts $\overline{\mathrm{SO}(4,4;\mathbb{R})}$ to

$$\overline{\mathrm{SO}(3,4;\mathbb{R})} = \{g \in \overline{\mathrm{SO}(4,4;\mathbb{R})} \mid ,$$

$$g = \begin{pmatrix} \exp\left(\frac{1}{4}\omega_{AB} D_{(1)}^{AB}\right) & 0 \\ 0 & \exp\left(\frac{1}{4}\omega_{AB} D_{(2)}^{AB}\right) \end{pmatrix} \quad \text{and} \quad \omega_{A8}=0 \} \quad (24)$$

[one of the real forms of spin (7, C)].

In[892]:=

```
Block[{\psi, J, t, r, sJu},
sJu = {J[0] → 0, J[1] → 0, J[2] → 0,
       J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0};
r = {\psi[4], -ψ[5], -ψ[6], -ψ[7], ψ[0], -ψ[1], -ψ[2], -ψ[3]};
ψ = τ[A].τ[B].ψ16[[1;;8]];
]
```

Out[892]=

```
τ[A].{f16[7][x0, x4], f16[6][x0, x4], -f16[5][x0, x4], -f16[4][x0, x4],
      f16[3][x0, x4], f16[2][x0, x4], -f16[1][x0, x4], -f16[0][x0, x4]}

τ[A1].τ[B1] → D-1(1).τ[A1].τ[B1].D(1);
τ[A1].τ[B1] → D-1(2).τ[A1].τ[B1].D(2);
σ.τ[A1].τ[B1] = Transpose[τ[A1]].σ.τ[B1] =
      Transpose[τ[A1]].Transpose[σ.τ[B1]] = Transpose[σ.τ[B1].τ[A1]];
σ.τ[A1].τ[B1] = Transpose[τ[B1].τ[A1]].Transpose[σ];
σ.τ[A1].τ[B1].ψ1 = Transpose[τ[B1].τ[A1]].Transpose[σ].ψ1;
Transpose[ψ2].σ ↔ Transpose[σ].ψ1 ? ;
σ.τ[A1].τ[B1].ψ1 ↔ Transpose[τ[B1].τ[A1]].Transpose[ψ2].σ;
```

```

 $\sigma \cdot \bar{\tau}[A1] \cdot \tau[B1] \cdot \psi_1 \leftrightarrow \text{Transpose}[\psi_2 \cdot \bar{\tau}[B1] \cdot \tau[A1]] \cdot \sigma;$ 
B1 = A1
 $\sigma \cdot \psi_1 \leftrightarrow \text{Transpose}[\psi_2] \cdot \sigma^{-1}$  ;
 $\psi_1 \leftrightarrow \text{Transpose}[\psi_2] \cdot \sigma^{-1}$  ;
 $\text{Transpose}[\psi_2] \leftrightarrow \sigma \cdot \psi_1$  ;
In[893]:= {ψ1[4], -ψ1[5], -ψ1[6], -ψ1[7], ψ1[0], -ψ1[1], -ψ1[2], -ψ1[3]} /. {ψ1[#] → };
In[893]:= (*Block[{ψ1,ψ2,σψ,sσψ,r,sJu},
σψ=σ.(ψ2[#]&/@Range[0,7]);
sσψ=(ψ1[#]→σψ[[#+1]]&/@Range[0,7]);
r={ψ1[4],-ψ1[5],-ψ1[6],-ψ1[7],ψ1[0],-ψ1[1],-ψ1[2],-ψ1[3]};
(*ψ=τ[A].τ[B].Π16[1;;8]*)
r/.sσψ
]*)
```

In[894]:= Block[{ψ1, ψ2, σψ, sσψ, r},
 $\sigma\psi = \sigma \cdot (\psi_2[\#] & /@ Range[0, 7]);$
 $s\sigma\psi = (\psi_1[\#] \rightarrow \sigma\psi[[\# + 1]] & /@ Range[0, 7]);$
 $r = \{\psi_1[4], -\psi_1[5], -\psi_1[6], -\psi_1[7], \psi_1[0], -\psi_1[1], -\psi_1[2], -\psi_1[3]\};$
 $(*ψ=τ[A].τ[B].Π16[1;;8]*)$
 $r /. s\sigma\psi$
]

Out[894]= {ψ2[0], -ψ2[1], -ψ2[2], -ψ2[3], ψ2[4], -ψ2[5], -ψ2[6], -ψ2[7]}

for $\frac{1}{i}$:

```
In[895]:=  $\Psi_{16cccc} = \text{Block}[\{\psi_1, \psi_2, \sigma\psi, s\sigma\psi, s\psi_1, s\psi_2, r\},$ 
 $\sigma\psi = \sigma.(\Psi_{16}[9 ; ; 16]);$ 
 $s\sigma\psi = (\psi_1[\#] \rightarrow \sigma\psi[\#+1] \& /@ \text{Range}[0, 7]);$ 
 $s\psi_1 = \text{Thread}[(\psi_1[\#] \& /@ \text{Range}[0, 7]) \rightarrow \Psi_{16}[1 ; ; 8]];$ 
 $s\psi_2 = \text{Thread}[(\psi_1[\#] \& /@ \text{Range}[0, 7]) \rightarrow \Psi_{16}[9 ; ; 16]];$ 
 $r = \{\psi_1[4], -\psi_1[5], -\psi_1[6], -\psi_1[7], \psi_1[0], -\psi_1[1], -\psi_1[2], -\psi_1[3]\};$ 
 $(* \psi = \tau[A].\tau[B] .*)$ 
 $\text{Flatten}[\{r /. s\psi_1, r /. s\psi_2\}]$ 
]
```

```
Out[895]= {f16[4][x0, x4], -f16[5][x0, x4], -f16[6][x0, x4], -f16[7][x0, x4],
f16[0][x0, x4], -f16[1][x0, x4], -f16[2][x0, x4], -f16[3][x0, x4],
f16[12][x0, x4], -f16[13][x0, x4], -f16[14][x0, x4], -f16[15][x0, x4],
f16[8][x0, x4], -f16[9][x0, x4], -f16[10][x0, x4], -f16[11][x0, x4]}
```

```
In[896]:=  $\text{ca}\Psi_{16cccc} = \text{CoefficientArrays}[\Psi_{16cccc} - \Psi_{16}, \Psi_{16}]$ 
```

```
Out[896]= {SparseArray[ Specified elements: 12 Dimensions: {16}], SparseArray[ Specified elements: 8 Dimensions: {16, 16}]}
```

```
In[897]:=  $(\text{ca}\Psi_{16cccc}^2 = \text{Normal}[\text{ca}\Psi_{16cccc}[[2]]]) // \text{MatrixForm}$ 
```

```
Out[897]//MatrixForm= 
$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

```
In[898]:=  $\text{ca}\Psi_{16cccc} = \text{CoefficientArrays}[\Psi_{16cccc} - \Psi_{16}, \Psi_{16}]$ 
```

```
Out[898]= {SparseArray[ Specified elements: 0 Dimensions: {16}], SparseArray[ Specified elements: 32 Dimensions: {16, 16}]}
```

In[899]:=

```
(ca甫cc甫2 = Normal[ca甫cc甫[[2]]]) // MatrixForm
```

Out[899]//MatrixForm=

In[900]:=

`ca\Pcc\P2` ==> Transpose [`ca\Pcc\P2`]

Out[900]=

True

In[901]:=

```
ca\Pcc\P2.ca\Pcc\P2 // MatrixForm
```

Out[901]/MatrixForm=

In[902]:=

$$\begin{aligned} & \text{ca}\Psi\text{cc}\Psi 2.\Psi 16\text{cccc} - \Psi 16 + \Psi 16\text{cccc} \\ & \text{ca}\Psi\text{cc}\Psi 2.\Psi 16 - \Psi 16\text{cccc} + \Psi 16 \end{aligned}$$

Out[902]=

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

Out[903]=

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

for $\frac{1}{i}$:

In[904]:=

$$\text{jjcc} = \text{ca}\Psi\text{cc}\Psi 2 + \text{ID16};$$

In[905]:=

$$\text{jjcc}.\Psi 16 === \Psi 16\text{cccc}$$

Out[905]=

True

In[906]:=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

Out[906]=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

In[907]:=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

Out[907]=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

$$F_A^a = \text{Transpose}[\eta 4488[\#, 1, \# + 1] * (\tau[\#].\text{unit}) \& /@ \text{Range}[0, 7]]$$

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \\ & \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \end{aligned}$$

$$F_a^A = \text{Transpose}[\text{unit}].\sigma.(\tau[\#]) \& /@ \text{Range}[0, 7]$$

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \end{aligned}$$

$$\left\{ \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\}$$

In[908]:=

 F_a^A

Out[908]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\}$$

In[909]:=

unit

Out[909]=

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

In[910]:=

 $(F_a^A.\text{unit})$

Out[910]=

$$\{1, 0, 0, 0, 0, 0, 0, 0\}$$

In[911]:=

 $(F_a^A[8].\text{unit})$

Out[911]=

$$0$$

In[912]:=

unit.F_A^a

Out[912]=

$$\{1, 0, 0, 0, 0, 0, 0, 0\}$$

In[913]:=

unit.unit // FullSimplify

Out[913]=

$$1$$

In[914]:=

unit. σ .unit // FullSimplify

Out[914]=

$$1$$

```
In[915]:= realProjection8 = KroneckerProduct[unit, unit]
Out[915]=

$$\left\{ \left\{ \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \right.$$


$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \right\}$$


In[916]:= realProjection8.realProjection8 === realProjection8
Out[916]=
True

In[917]:= realProjection16 = ArrayFlatten[{{realProjection8, 0}, {0, realProjection8}}]
Out[917]=

$$\left\{ \left\{ \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right.$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\left\{ \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\}$$


In[918]:= realProjection16.realProjection16 === realProjection16
Out[918]=
True
```

In[919]:=

We denote $\langle x, 1 \rangle$ by $\text{Re } x$ and $(x - \text{Re } x)$ by $\text{Im } x$.

Out[919]=

We denote $\langle x, 1 \rangle$ by $\text{Re } x$ and $(x - \text{Re } x)$ by $\text{Im } x$.

In[920]:=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

Out[920]=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

In[921]:=

```
¶16cc = 2 realProjection16.¶16 - ¶16 // FullSimplify
```

Out[921]=

$$\begin{aligned} & \{ f16[4][x_0, x_4], -f16[1][x_0, x_4], -f16[2][x_0, x_4], -f16[3][x_0, x_4], \\ & f16[0][x_0, x_4], -f16[5][x_0, x_4], -f16[6][x_0, x_4], -f16[7][x_0, x_4], \\ & f16[12][x_0, x_4], -f16[9][x_0, x_4], -f16[10][x_0, x_4], -f16[11][x_0, x_4], \\ & f16[8][x_0, x_4], -f16[13][x_0, x_4], -f16[14][x_0, x_4], -f16[15][x_0, x_4] \} \end{aligned}$$

In[922]:=

```
¶16ccA =  

2 (Flatten[{unit.¶16[[1;;8]] * (unit.F_A^a), unit.¶16[[9;;16]] * (unit.F_A^a)}]) -  

¶16 // FullSimplify
```

Out[922]=

$$\begin{aligned} & \{ (-1 + \sqrt{2}) f16[0][x_0, x_4] + \sqrt{2} f16[4][x_0, x_4], -f16[1][x_0, x_4], -f16[2][x_0, x_4], \\ & -f16[3][x_0, x_4], -f16[4][x_0, x_4], -f16[5][x_0, x_4], -f16[6][x_0, x_4], \\ & -f16[7][x_0, x_4], (-1 + \sqrt{2}) f16[8][x_0, x_4] + \sqrt{2} f16[12][x_0, x_4], \\ & -f16[9][x_0, x_4], -f16[10][x_0, x_4], -f16[11][x_0, x_4], -f16[12][x_0, x_4], \\ & -f16[13][x_0, x_4], -f16[14][x_0, x_4], -f16[15][x_0, x_4] \} \end{aligned}$$

In[923]:=

```
¶16cc - ¶16ccA // FullSimplify
```

Out[923]=

$$\begin{aligned} & \{ -((-1 + \sqrt{2}) (f16[0][x_0, x_4] + f16[4][x_0, x_4])), \\ & 0, 0, 0, f16[0][x_0, x_4] + f16[4][x_0, x_4], 0, 0, 0, \\ & -((-1 + \sqrt{2}) (f16[8][x_0, x_4] + f16[12][x_0, x_4])), \\ & 0, 0, 0, f16[8][x_0, x_4] + f16[12][x_0, x_4], 0, 0, 0 \} \end{aligned}$$

In[924]:=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \tilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elements

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

Out[924]=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \tilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elements

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

```
Table[{A, B, FullSimplify[
  ExpandAll[1/2 (τ[B].τ[A] + τ[A].τ[B]) == η4488[[A+1, B+1]*ID8]]}, {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

```
Table[{A, B, FullSimplify[
  ExpandAll[1/2 (τ[A].τ[B] + τ[B].τ[A]) == η4488[[A+1, B+1]*ID8]]}, {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

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In[]:=

SUBMANIFOLDS AND SPECIAL STRUCTURES ON THE OCTONIANS

ROBERT L. BRYANT

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

Classically, this observation was used to define an almost complex structure on S^6 as follows: If $u \in S^6$, then R_u preserves the 2-plane spanned by 1 and u and therefore preserves its orthogonal 6-plane, which may be identified with $T_u S^6 \subseteq \text{Im } \mathbf{O}$ after translation to the origin. Thus R_u induces a complex

structure on $T_u S^6$ for each $u \in S^6$. This almost complex structure is not integrable (even locally) to a complex structure (see below). In [Ca], Calabi noticed that for *any* oriented $M^6 \subseteq \text{Im } \mathbf{O}$, $R_{N(x)}$ induces a complex structure on $T_x M^6$ (where $N(x)$ is the oriented unit normal). Thus every oriented $M^6 \subseteq \text{Im } \mathbf{O}$ inherits an almost complex structure. Moreover, M^6 inherits a metric from $\text{Im } \mathbf{O}$, so we actually have a $U(3)$ -structure on M^6 . (Calabi calls these structures “almost Hermitian.” He also proves that such M^6 possess a canonical $SU(3)$ -substructure but we will not need this.) Calabi shows that the second fundamental form Π of M decomposes with respect to the $U(3)$ -structure into a piece $\Pi^{1,1}$ of type (1,1) and a piece $\Pi^{0,2}$ of type (0,2). He then shows that the almost complex structure of M is integrable if and only if $\Pi^{1,1} = 0$ and that the canonical 2-form of the $U(3)$ -structure, say Ω , is closed if and only if $\Pi^{0,2} = 0$ and $\text{tr}_1 \Pi^{1,1} = 0$. From this it follows that the $U(3)$ -structure on M^6 is Kähler if and only if $\Pi \equiv 0$, so that M^6 is a hyperplane (or a union of pieces of hyperplanes). Calabi then constructs nontrivial examples of $M^6 \subseteq \text{Im } \mathbf{O}$ for which the almost complex structure is integrable. His examples are of the form $S \times \mathbf{R}^4 \subseteq \text{Im } \mathbf{O}$, where $S \subseteq \mathbf{R}^3$ is a minimal surface, $\mathbf{R}^3 \subseteq \text{Im } \mathbf{O}$ is an *associative* 3-plane, and $\mathbf{R}^4 = (\mathbf{R}^3)^\perp$. Calabi leaves open the problem of determining whether or not there are nontrivial $M^6 \subseteq \text{Im } \mathbf{O}$ for which the canonical 2-form is closed.

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex structure J_u . If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on \mathbf{O} . However, because S^6 is connected, we see that the orientation of \mathbf{O} induced by the natural orientation of \mathbf{O}_u as a complex vector space does not depend on u . We refer to this orientation as the natural orientation of \mathbf{O} .

Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

so J_u is an isometry for each $u \in S$. Moreover, it follows that \mathbf{O}_u is endowed with a natural Hermitian structure with respect to the inner product $\langle \cdot, \cdot \rangle$. We denote the group of complex linear transformations of \mathbf{O}_u by $GL(\mathbf{O}_u)$ and the special unitary transformations of \mathbf{O}_u with its Hermitian metric by $SU(\mathbf{O}_u)$.

We let $\text{Spin}(7) \subseteq SO(8)$ denote the subgroup generated by the set $\{J_u \mid u \in S^6\} \subseteq SO(8)$. It is known (see [12]) that $\text{Spin}(7)$ is a connected, simply connected, compact Lie group of real dimension 21. Its center is $\{\pm I_8\} \simeq \mathbb{Z}/2$ and $\text{Spin}(7)/\{\pm I_8\}$ is isomorphic to $SO(7)$, a simple group. We want to make explicit the structure equations of $\text{Spin}(7)$ as a subgroup of $SO(8)$ in such a way that its relationship with the complex structures J_u is made clear.

In[1]:=

Let $u \in S^6$ be an imaginary unit which is orthogonal to $\epsilon \in \mathbf{O}$. For each $\lambda \in \mathbb{R}$, $(\cos \lambda \epsilon + \sin \lambda u)$ is an imaginary unit. Hence $J_\epsilon \circ J_{(\cos \lambda \epsilon + \sin \lambda u)} = -\cos \lambda I + \sin \lambda J_\epsilon \circ J_u$ is an element of $\text{Spin}(7)$. We easily compute that $J_\epsilon \circ J_u + J_u \circ J_\epsilon = 0$ by using (1.10). Thus $(J_\epsilon \circ J_u)^2 = J_\epsilon \circ J_u \circ J_\epsilon \circ J_u = -J_\epsilon^2 \circ J_u^2 = -I$. It follows that

$$(1.16) \quad \exp(\lambda J_\epsilon \circ J_u) = \cos \lambda I + \sin \lambda J_\epsilon \circ J_u.$$

Thus, if $\text{spin}(7) \subseteq so(8)$ is the Lie algebra of $\text{Spin}(7)$, we see that $J_\epsilon \circ J_u \in \text{spin}(7)$ for all $u \in S^6$ with $\langle u, \epsilon \rangle = 0$. Since $\text{spin}(7)$ is a vector space, we see that $L \subseteq \text{spin}(7)$ where

$$(1.17) \quad L = \{J_\epsilon \circ J_w \mid w \in \text{Im } \mathbf{O}, \langle \epsilon, w \rangle = 0\}.$$

Note that $\dim_{\mathbb{R}} L = 6$.

Transpose[a]. σ16.b == Transpose[J.a]. σ16.J.b = a.Transpose[J]. σ16.J.b

want ψcc.σ16.?Ψ16 = Ψ16.J.σ16.?Ψ16 ≠ 0

need 1 irrep for {i, u}

need 1 irrep for {i, u} _perp

1 == x0 or x4; u == x4 or x0

x0 has periodic fns; use x4

In[925]:=

t16A[All, 2]

Out[925]=

{ {0}, {1}, {2}, {3}, {4}, {5}, {6}, {7} }

In[926]:=

T16^A[4].T16^A[4] === -Id16

Out[926]=

True

In[927]:=

(*JT4=T16^A[4] *) (* complex structure on S⁶ *)

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

J.J === -ID16

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex

matrix multiplication is associative

(split) octonion multiplication is NOT associative

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

$$\text{Transpose}[J].\sigma_{16}.J == \sigma_{16}$$

In[928]:=

```
Clear[possibleComplexStructureIndex2];
(*Clear[possibleComplexStructureMatrix2];*)
possibleComplexStructureIndex2[j_] := Module[{m, r, jsj, jj},
  J = base16[[j]][1];
  jj = (FullSimplify[J.J] == -ID16);
  jsj = FullSimplify[Transpose[J].\sigma16.J] == \sigma16;
  Return[{jj, jsj}];]
```

In[930]:=

```
possibleComplexStructureIndex2[#] & /@ {12, 97, 100, 109, 127, 225, 230, 236}
```

Out[930]=

```
{ {False, False}, {True, False}, {True, False}, {True, False},
  {True, False}, {False, False}, {False, False}, {False, False} }
```

In[931]:=

```

Clear[possibleComplexStructureIndex];
Clear[possibleComplexStructureMatrix];
possibleComplexStructureIndex = (*Parallel*)Table[
  Block[{J, r, m},
    Clear[J, r, m];
    m = r = {};
    J = base16[[j]][1];
    If[(J.J === -ID16) &&
      (FullSimplify[Transpose[J].σ16.J] === σ16), r = base16[[j]][2];
     m = σ16.J;, ##&[]];
    (*If[!(m===-Transpose[m]),r=base16[[j]][2],r={}]*)
    r]
  , {j, 1, Length[base16]}] // Union // Drop[#, 1] &

```

Out[933]=

```

{{4}, {5}, {6}, {7}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {4, 5}, {4, 6},
{4, 7}, {5, 6}, {5, 7}, {6, 7}, {0, 1, 2, 3, 4}, {0, 1, 2, 3, 5}, {0, 1, 2, 3, 6},
{0, 1, 2, 3, 7}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7}, {0, 1, 5, 6, 7},
{0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 2, 5, 6, 7}, {0, 3, 4, 5, 6},
{0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {0, 3, 5, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7},
{1, 2, 4, 6, 7}, {1, 2, 5, 6, 7}, {1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7},
{1, 3, 5, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7}, {2, 3, 4, 6, 7}, {2, 3, 5, 6, 7},
{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 6}, {0, 1, 2, 3, 4, 7}, {0, 1, 2, 3, 5, 6},
{0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}, {0, 1, 4, 5, 6, 7}, {0, 2, 4, 5, 6, 7},
{0, 3, 4, 5, 6, 7}, {1, 2, 4, 5, 6, 7}, {1, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 7}}

```

In[934]:=

```
Length[possibleComplexStructureIndex]
```

Out[934]=

```
56
```

In[935]:=

```
sixersJ = If[Length[#] ≥ 6, #, ##&[]] & /@ possibleComplexStructureIndex
```

Out[935]=

```

{{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 6}, {0, 1, 2, 3, 4, 7}, {0, 1, 2, 3, 5, 6},
{0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}, {0, 1, 4, 5, 6, 7}, {0, 2, 4, 5, 6, 7},
{0, 3, 4, 5, 6, 7}, {1, 2, 4, 5, 6, 7}, {1, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 7}}

```

In[936]:=

```

Clear[possibleT4ComplexStructureIndex];
Clear[possibleT4ComplexStructureMatrix];
possibleComplexT4StructureIndex = (*Parallel*)Table[
  Block[{J, a1, a2, b1, b2, c1, c2,
    c3, c3ori, c3s, c2ed, found, r, m, debug = False},

```

```

Clear[J, a1, a2, b1, b2, c1, c2, c3, c3ori, c3s, c2ed, found, r, m];
c3s = found = r = {};
If[FullSimplify[T16^A[4].base16[[j]][1] === base16[[j]][1].T16^A[4]],
J = T16^A[4].base16[[j]][1] // FullSimplify;
a1 = J.J // FullSimplify;
If[(a1 === -ID16),
a2 = Transpose[J].σ16.J // FullSimplify;
If[(a2 === σ16),
m = σ16.J;
c1 = Append[base16[[j]][2], 4] // Sort; (* for J = T16^A[4]*#[1]*)
c3ori = c3 = Select[base16, #[[2]] == c1 & → "Index"];
(*Print["c1= ",c1];*)
If[c3 === {},
c3s = StringReplace[ToString[c1], "4, 4" → "256"] //
ToExpression // Sort // Drop[#, -1] &;
c3 = Select[base16, #[[2]] == c3s & → "Index"],
##&[]];
If[debug, Print["Append[base16[[j]][[2]],4]//Sort=c1= ",
c1, "Select[base16,#[[2]]==c1&]= ", c3ori,
";StringReplace[c1,'4, 4'->'256']= ", c3s, ";c3= ", c3], ##&[]];
If[! (c3 === {}),
c2 = c3[[1]];
(*Print["c2= ",c2];*)
If[c2 > 0 && c2 ≤ Length[base16], found = base16[[c2]], ##&[]];
If[! (found === {}) && (J === found[[1]] || J === -found[[1]])
(&&! (m=====Transpose[m])*),
r = found[[2]];
(*Print[base16[[j]][2],"";Tr[J.J]/16=",Tr[a1]/16,
";c1=",c1,";c2=Select[base16,#[[2]]==c1&= ",c2,
";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
Tr[found[[1]].found[[1]]/16];*)
,
##&[]];
, ##&[], ##&[], ##&[], ##&[], ##&[]];
r]
, {j, 1, Length[base16]}] // Union // Drop[#, 1] &

```

```

Out[938]=
{{4}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {5, 6}, {5, 7},
{6, 7}, {0, 1, 2, 3, 4}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7},
{0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 3, 4, 5, 6},
{0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7}, {1, 2, 4, 6, 7},
{1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7},
{2, 3, 4, 6, 7}, {0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}
```

In[939]:=

```
JT4 = T16^A[4] ;(* complex structure on S^6 *)
Jcomplement <→( ^q[[7]]*Cos[Q7] + ^q[[6]]*Sin[Q7] Cos[Q8] + ^q[[5]]*Sin[Q7] Sin[Q8])
```

In[940]:=

```
basisProjections16 =
Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@
Range[16];
```

In[941]:=

```
basisProjections8 =
Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@
Range[8];
```

In[942]:=

```
JcomplementProjection8 =
(basisProjections8[[7]] + basisProjections8[[6]] + basisProjections8[[5]]);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8
```

Out[943]=

```
True
```

In[944]:=

```
JcomplementProjection8 = (basisProjections8[[7]] +
basisProjections8[[6]] + basisProjections8[[5]] + basisProjections8[[4]]);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8
```

Out[945]=

```
True
```

In[946]:=

```
(*JcomplementProjection8=
(basisProjections8[[7]]*Cos[Q7]+basisProjections8[[6]]*Sin[Q7]Cos[Q8]+
basisProjections8[[5]]*Sin[Q7]Sin[Q8]);*)
```

In[947]:=

```
JcomplementProjection8.JcomplementProjection8 // FullSimplify // MatrixForm
JcomplementProjection8 // MatrixForm
%% - % :
```


In[950]:=

```
(parallel04 =
  Block[{x = Table[KroneckerDelta[1, j], {j, 16}]], KroneckerProduct[x, x]] +
  Block[{x = Table[KroneckerDelta[5, j], {j, 16}]],
    KroneckerProduct[x, x]]) // MatrixForm
```

Out[950]//MatrixForm=

```
In[951]:= (projection04 = ID16 - Block[{x = Table[KroneckerDelta[1, j], {j, 16}]}, 
    KroneckerProduct[x, x]] - Block[{x = Table[KroneckerDelta[5, j], {j, 16}]}, 
    KroneckerProduct[x, x]]) // MatrixForm

Out[951]//MatrixForm= 

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


In[952]:= projection04.parallel04 === Zero16

Out[952]= True

{{0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}
```

In[953]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 5, 6}; % === -T16^A[4].T16^A[7].T16^A[8]

Out[954]= True

In[955]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 5, 7}; % === T16^A[4].T16^A[6].T16^A[8]

Out[956]= True

In[957]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 6, 7}; % === -T16^A[4].T16^A[5].T16^A[8]

Out[958]= True

```
In[959]:= sixers = If[Length[#] ≥ 6, #, ## &[]] & /@ possibleComplexT4StructureIndex
sixersIndices = Table[Select[base16, #[[2]] === sixers[[j]] & → "Index"], {j, 1, Length[sixers]}] // Flatten
base16[[#][[1]] & /@ sixersIndices;
Jcomplement = (%[[1]] * Cos[Q7] + %[[2]] * Sin[Q7] Cos[Q8] + %[[3]] * Sin[Q7] Sin[Q8])
Dimensions[Jcomplement]
FullSimplify[Jcomplement.Jcomplement] === -ID16

Out[959]= {{0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}
```

```
Out[960]= {222, 223, 224}

Out[962]= {{0, 0, -Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, -Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0}, {Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7], 0, 0, 0, 0, -Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -Cos[Q7], 0, 0, 0, 0, -Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0, 0}, {Cos[Q7], 0, 0, 0, 0, 0, Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, Cos[Q7], Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -Cos[Q7], 0, Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8], 0, Cos[Q7], 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, -Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7], -Cos[Q7], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8], 0, 0, 0, 0, -Cos[Q7]}, {0, 0, 0, 0, 0, 0, 0, -Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], 0, 0, 0, 0, Cos[Q7], 0}, {0, 0, 0, 0, 0, 0, 0, 0, Cos[Q7], 0, 0, 0, 0, Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}, {0, 0, 0, 0, 0, 0, 0, -Cos[Q7], 0, 0, 0, 0, -Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7]}, {0, 0, 0, 0, 0, 0, 0, 0, -Cos[Q7], -Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8], 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, Cos[Q7], 0, -Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], 0, 0}}
```

```
Out[963]= {16, 16}

Out[964]= True

In[965]:= realProjection8 = KroneckerProduct[unit, unit]

Out[965]= { {1/2, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {1/2, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0} }
```

```
In[966]:= realProjection8.realProjection8 === realProjection8
Out[966]= True

In[967]:= (*KroneckerProduct[unit,Transpose[unit]]*)

In[968]:= realProjection16 = ArrayFlatten[{{realProjection8, 0}, {0, realProjection8}}]
Out[968]=
{ { 1 , 0 , 0 , 0 , 1 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 1 , 0 , 0 , 0 , 1 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 },
{ 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 } }
```

In[969]:= realProjection16.realProjection16 === realProjection16

Out[969]= True

In[970]:= We denote $\langle x, 1 \rangle$ by $\text{Re } x$ and $(x - \text{Re } x)$ by $\text{Im } x$.

Out[970]= We denote $\langle x, 1 \rangle$ by $\text{Re } x$ and $(x - \text{Re } x)$ by $\text{Im } x$.

In[971]:= $\bar{x} = 2\langle x, 1 \rangle - x.$

Out[971]= $\bar{x} = 2\langle x, 1 \rangle - x.$

```

In[972]:= unit.unit
Out[972]= 1

In[973]:= unit.σ.unit
Out[973]= 1

In[974]:= T16A[4].T16A[4] === -ID16
Out[974]= True

In[975]:= T4cc = T16A[4] - (realProjection16.T16A[4]) // FullSimplify
Out[975]= { {0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, -1/2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1/2, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0} }

```

In[976]:=

T4cc.T4cc

Out[976]=

$$\left\{ \left\{ -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right.$$

$$\left\{ 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\{0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\{0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0 \right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0 \right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1\},$$

$$\left. \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1\} \right\}$$

In[977]:=

T16^A[7].T16^A[7] === -ID16

Out[977]=

True

In[978]:=

```
T7cc = T16A[7] - (realProjection16.T16A[7]) // FullSimplify
```

Out[978]=

$$\begin{aligned} & \left\{ \left(0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right) \right\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1\}, \\ & \left\{ -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \{0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \left\{ \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \{0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \end{aligned}$$

In[979]:=

T7CC, T7CC

Out[979]=

$$\begin{aligned}
& \left\{ 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], -\cos[Q7]^2 + \frac{1}{4} (-3 + \cos[2 Q8]) \sin[Q7]^2, \right. \\
& \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \\
& \left. \frac{1}{2} \cos[Q8]^2 \sin[Q7]^2, \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \right. \\
& -\cos[Q7]^2 - \frac{1}{4} (3 + \cos[2 Q8]) \sin[Q7]^2, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \\
& \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \frac{1}{2} \sin[Q7]^2 \sin[Q8]^2, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{\cos[Q7]^2}{2}, \right. \\
& \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], 0, \frac{1}{4} (-3 + \cos[2 Q7]), \\
& \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q8]^2 \sin[Q7]^2, \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \right. \\
& 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], -\cos[Q7]^2 + \frac{1}{4} (-3 + \cos[2 Q8]) \sin[Q7]^2, \\
& \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \frac{1}{2} \sin[Q7]^2 \sin[Q8]^2, \right. \\
& 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \\
& -\cos[Q7]^2 - \frac{1}{4} (3 + \cos[2 Q8]) \sin[Q7]^2, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \frac{1}{4} (-3 + \cos[2 Q7]), \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \\
& 0, \frac{\cos[Q7]^2}{2}, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8] \}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \right. \\
& -\cos[Q7]^2 + \frac{1}{4} (-3 + \cos[2 Q8]) \sin[Q7]^2, \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], 0, \\
& \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q8]^2 \sin[Q7]^2, \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8] \}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8] \right\},
\end{aligned}$$

$$\begin{aligned}
& -\cos[Q7]^2 - \frac{1}{4} (3 + \cos[2 Q8]) \sin[Q7]^2, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \\
& \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \frac{1}{2} \sin[Q7]^2 \sin[Q8]^2 \}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& 0, \frac{\cos[Q7]^2}{2}, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], 0, \\
& \frac{1}{4} (-3 + \cos[2 Q7]), \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8] \}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q8]^2 \sin[Q7]^2, \right. \\
& \left. \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \right. \\
& \left. -\cos[Q7]^2 + \frac{1}{4} (-3 + \cos[2 Q8]) \sin[Q7]^2, \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8] \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \right. \\
& \left. \frac{1}{2} \sin[Q7]^2 \sin[Q8]^2, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \right. \\
& \left. \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], -\cos[Q7]^2 - \frac{1}{4} (3 + \cos[2 Q8]) \sin[Q7]^2 \right\}
\end{aligned}$$

In[983]:=

T16^A[4].Jcomplement // MatrixForm

Out[983]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\cos[Q7] & 0 & 0 & 0 & 0 \\
0 & \cos[Q7] & 0 & 0 & 0 \\
0 & 0 & \cos[Q7] & 0 & -\sin[Q7] \sin[Q8] \\
0 & 0 & 0 & \cos[Q7] & \cos[Q8] \sin[Q7] \\
0 & 0 & -\sin[Q7] \sin[Q8] & \cos[Q8] \sin[Q7] & -\cos[Q7] \\
0 & 0 & -\cos[Q8] \sin[Q7] & -\sin[Q7] \sin[Q8] & 0 \\
\sin[Q7] \sin[Q8] & \cos[Q8] \sin[Q7] & 0 & 0 & 0 \\
-\cos[Q8] \sin[Q7] & \sin[Q7] \sin[Q8] & 0 & 0 & 0
\end{pmatrix}$$

```
In[984]:= T16A[4].Jcomplement === Jcomplement.T16A[4]
FullSimplify[Jcomplement.Jcomplement] === -ID16
T16A[4].T16A[4] === -ID16

Out[984]= True

Out[985]= True

Out[986]= True

In[987]:= unit3 = {Cos[Q7], Sin[Q7] Cos[Q8], Sin[Q7] Sin[Q8]}

Out[987]= {Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}

In[988]:= unit3.unit3 // FullSimplify

Out[988]= 1
```

```
In[989]:= unit3
sixers
Complement[{0, 1, 2, 3, 5, 6, 7}, #] & /@%
eqsRQQ7Q8 = Thread[Subscript[q, #][1]] & /@%) == RQ * unit3
sRQQ7Q8 = Solve[And @@%, {RQ, Q7, Q8}] // FullSimplify
seqsRQQ7Q8 = %% /. {Equal  $\rightarrow$  Rule}

Out[989]= {Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}

Out[990]= {{0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7} }

Out[991]= {{7}, {6}, {5} }

Out[992]= {q7 == RQ Cos[Q7], q6 == RQ Cos[Q8] Sin[Q7], q5 == RQ Sin[Q7] Sin[Q8]}

Out[993]= 
$$\left\{ \begin{array}{l} \text{RQ} \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, \quad Q7 \rightarrow 2\pi c_1 - \frac{i}{2} \operatorname{Log} \left[ \frac{-\frac{i}{2} \sqrt{q_5^2 + q_6^2} - q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, \\ Q8 \rightarrow \operatorname{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \end{array} \right\},$$


$$Q7 \rightarrow 2\pi c_1 - \frac{i}{2} \operatorname{Log} \left[ \frac{\frac{i}{2} \left( \sqrt{q_5^2 + q_6^2} + i q_7 \right)}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, \quad Q8 \rightarrow 2\pi c_2 - \frac{i}{2} \operatorname{Log} \left[ \frac{-\frac{i}{2} q_5 - q_6}{\sqrt{q_5^2 + q_6^2}} \right] \text{ if } c_2 \in \mathbb{Z},$$


$$\left\{ \begin{array}{l} \text{RQ} \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, \quad Q7 \rightarrow 2\pi c_1 - \frac{i}{2} \operatorname{Log} \left[ \frac{-\frac{i}{2} \sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, \\ Q8 \rightarrow 2\pi c_2 - \frac{i}{2} \operatorname{Log} \left[ \frac{-\frac{i}{2} q_5 - q_6}{\sqrt{q_5^2 + q_6^2}} \right] \text{ if } c_2 \in \mathbb{Z} \end{array} \right\},$$


$$Q7 \rightarrow 2\pi c_1 - \frac{i}{2} \operatorname{Log} \left[ \frac{\frac{i}{2} \sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, \quad Q8 \rightarrow \operatorname{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \} }$$


Out[994]= {q7  $\rightarrow$  RQ Cos[Q7], q6  $\rightarrow$  RQ Cos[Q8] Sin[Q7], q5  $\rightarrow$  RQ Sin[Q7] Sin[Q8]}
```

```
In[995]:= FullSimplify[(\sqrt{q_6^2 + q_5^2} /. seqsRQQ7Q8), Q7 > 0 && Sin[Q7] > 0 && RQ > 0]
% / q_7 /. seqsRQQ7Q8
Solve[\frac{\sqrt{q_6^2 + q_5^2}}{q_7} == FullSimplify[% , Q7 > 0 && Sin[Q7] > 0 && RQ > 0], Q7]
sQ7 = {Q7 \rightarrow ArcTan[q_7, \sqrt{q_5^2 + q_6^2}]}

Out[995]= RQ Sin[Q7]

Out[996]= Tan[Q7]

Out[997]= {Q7 \rightarrow ArcTan[\frac{\sqrt{q_5^2 + q_6^2}}{q_7} + \pi c_1 | if | c_1 \in \mathbb{Z}]}

Out[998]= {Q7 \rightarrow ArcTan[q_7, \sqrt{q_5^2 + q_6^2}]}

In[999]:= (*sQ7=*) Solve[\frac{\sqrt{q_6^2 + q_5^2}}{q_7} == FullSimplify[\left(\frac{\sqrt{q_6^2 + q_5^2}}{q_7} /. seqsRQQ7Q8\right),
Q7 > 0 && Sin[Q7] > 0 && RQ > 0], Q7] // FullSimplify
Out[999]= {Q7 \rightarrow ArcCot[\frac{q_7}{\sqrt{q_5^2 + q_6^2}} + \pi c_1 | if | c_1 \in \mathbb{Z}]}
```

```
In[1000]:= eqsRQQ7Q8[[2 ;; 3]]
Transpose[{{%[[All, 1], %[[All, 2]]]}]
%[[2]][[1]] / %[[1]][[1]] == %[[2]][[2]] / %[[1]][[2]] // FullSimplify
sQ8 = Solve[% , Q8][[1]] // FullSimplify
sQ8 = {Q8 → ArcTan[q6, q5]}
sRQQ7Q8 /. % // FullSimplify

Out[1000]= {q6 == RQ Cos[Q8] Sin[Q7], q5 == RQ Sin[Q7] Sin[Q8]}

Out[1001]= {{q6, RQ Cos[Q8] Sin[Q7]}, {q5, RQ Sin[Q7] Sin[Q8]}}
```

Out[1002]= $\frac{q_5}{q_6} == \tan[Q8]$

Out[1003]= $\left\{ Q8 \rightarrow \text{ArcTan}\left[\frac{q_5}{q_6}\right] + \pi c_1 \text{ if } c_1 \in \mathbb{Z} \right\}$

Out[1004]= {Q8 → ArcTan[q6, q5]}

Out[1005]= $\left\{ \begin{array}{l} RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - \frac{i}{2} \log\left[\frac{-\frac{i}{2} \sqrt{q_5^2 + q_6^2} - q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}, \\ \text{ArcTan}[q_6, q_5] \rightarrow \text{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \end{array} \right\},$

$\left\{ RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - \frac{i}{2} \log\left[\frac{\frac{i}{2} \left(\sqrt{q_5^2 + q_6^2} + i q_7\right)}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}, \right.$

$\left. \text{ArcTan}[q_6, q_5] \rightarrow 2\pi c_2 - \frac{i}{2} \log\left[\frac{-\frac{i}{2} q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z} \right\},$

$\left\{ RQ \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - \frac{i}{2} \log\left[\frac{-\frac{i}{2} \sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}, \right.$

$\left. \text{ArcTan}[q_6, q_5] \rightarrow 2\pi c_2 - \frac{i}{2} \log\left[\frac{-\frac{i}{2} q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z} \right\},$

$\left\{ RQ \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - \frac{i}{2} \log\left[\frac{\frac{i}{2} \sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}, \right.$

$\left. \text{ArcTan}[q_6, q_5] \rightarrow \text{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \right\}$

```

In[1006]:= sQ7Q8 = Flatten[{sQ7, sQ8}]
Out[1006]= {Q7 → ArcTan[q7, √q52 + q62], Q8 → ArcTan[q6, q5]}

In[1007]:= Block[{a, b, c},
  {a, b, c} * unit3]
Out[1007]= {a Cos[Q7], b Cos[Q8] Sin[Q7], c Sin[Q7] Sin[Q8]}

In[1008]:= base16[[#][1].base16[[#][1]] === -ID16 & /@ {222, 223, 224}
Out[1008]= {True, True, True}

In[1009]:= (*sixersT4=%*{Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}
{Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}.
{Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}/.FullSimplify*)

In[1010]:= Block[{J = T16A[4], m = σ16.J},
  {J.J === -ID16, σ16 === Transpose[J].σ16.J, !(m === -Transpose[m])} ]
Out[1010]= {True, True, False}

```

In[1011]:=

```
(*Clear[possibleComplexStructureIndex];
Clear[possibleComplexStructureMatrix];
possibleComplexStructureIndex=Select[base16,Block[{J,a0,a1,a2,b1,b2,r},
Clear[J,a0,a1,a2,b1,b2,r];
a0=#[[1]];
J=T16^A[4]*a0//FullSimplify;
a1=J.J//FullSimplify;
b1=(a1===-ID16);
a2=Transpose[J].σ16.J//FullSimplify;
b2=(a2===σ16);
r=b1&&b2;
If[r,Block[{c1,c2,c3},
c1=Append[#[[2]],4]//Sort; (* for J = T16^A[4]*#[[1]]*)
c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
Print[#[[2]],";",Tr[a1]/16,";c1=",c1,";c2=",
c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
Tr[base16[[c2]][[1]].base16[[c2]][[1]]/16],##&[]];
r
]&→"Index"]
possibleComplexStructureMatrix=base16[[#][[2]]&/@%*)
```

In[1012]:=

$$(*\text{Tr}[(T16^A[3].T16^A[4]).(T16^A[3].T16^A[4])]/16*)$$

In[1013]:=

$$(*\text{base16}[[5]][[2]]*)$$

In[1014]:=

$$(*\text{Select}[\text{base16}, \#[[2]] \in \{1, 2, 3, 4, 5, 6\} \& \rightarrow "Index"]*)$$

In[1015]:=

$$(*\text{base16}[[240]][[2]]
\text{Tr}[\text{base16}[[240]][[1]].\text{base16}[[240]][[1]]/16]*)$$

In[1016]:=

```
(*(*Parallel*)Do[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
  Clear[J,a0,a1,a2,b1,b2,r];
  a0=base16[[j]][1];
  J=T16^A[4]*a0//FullSimplify;
  a1=J.J//FullSimplify;
  If[! (a1===-ID16),Continue[],##&[]];
  (*b1=(a1===-ID16);*)
  a2=Transpose[J].σ16.J//FullSimplify;
  If[! (a2==σ16),Continue[],##&[]];
  (*b2=(a2==σ16);*)
  (*r=b1&&b2;*)

  c1=Append[base16[[j]][2],4]//Sort; (* for J = T16^A[4]*#[1]*)
  c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
  Print[base16[[j]][2],";",Tr[a1]/16,";c1=",c1,
  ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
  Tr[base16[[c2]][1].base16[[c2]][1]]/16];
  {J,base16[[c2]][1]}];
  ,{j,1,Length[base16}]*)
```

In[1017]:=

```
(*ParallelDo[
  Block[{r,t,a1},
  t=Tr[base16[[j]][1].base16[[k]][1]/16];
  r=t^2==KroneckerDelta[j,k];
  a1=If[r,##&[],Print[{{j,k},t,{base16[[j]][2],base16[[k]][2]}]]];
  ],{j,1,Length[base16}],{k,1,Length[base16}]*)
```

In[1018]:=

```
(*(*Parallel*)Table[Catch[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
  Clear[J,a0,a1,a2,b1,b2,r];
  a0=base16[[j]][1];
  J=T16^A[4]*a0//FullSimplify;
  a1=J.J//FullSimplify;
  If[! (a1===-ID16),Throw[Null],##&[]];
  (*b1=(a1===-ID16);*)
  a2=Transpose[J].σ16.J//FullSimplify;
  If[! (a2==σ16),Throw[Null],##&[]];
  (*b2=(a2==σ16);*)
  (*r=b1&&b2;*)

  c1=Append[base16[[j]][2],4]//Sort; (* for J = T16^A[4]*#[1]*)
  c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
  Print[base16[[j]][2],";",Tr[a1]/16,";c1=",c1,
  ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
  Tr[base16[[c2]][1].base16[[c2]][1]]/16];
  {{J},{base16[[c2]][1]},{J==base16[[c2]][1]||J===-base16[[c2]][1]}}]];
  ,{j,1,Length[base16]/128}]*)
```

In[1019]:=

```
(*(*Parallel*)Table[
  Block[{J,a1,a2,b1,b2,c1,c2,c3,r},
    Clear[J,a1,a2,b1,b2,r];
    r={};
    J=T16^A[4].base16[[j]][1]//FullSimplify;
    a1=J.J//FullSimplify;
    If[(a1===-ID16),
      a2=Transpose[J].σ16.J//FullSimplify;
      If[(a2==σ16),
        c1=Append[base16[[j]][2],4]//Sort; (* for J = T16^A[4]*#[1]*)
        c3=Select[base16,#[2]==c1&→"Index"];
        If[!(c3=={}),
          c2=c3[[1]];
          Print[base16[[j]][2],";Tr[J.J]/16=",Tr[a1]/16,";c1=",c1,
            ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
            Tr[base16[[c2]][1].base16[[c2]][1]]/16];
          r={{J},{base16[[c2]][1]},{J==base16[[c2]][1]||J===-base16[[c2]][1]}};
          ,##&[],##&[],##&[]];
        ];
      ];
    ];
  r]
,{j,1,Length[base16}]]//Union*)
```

In[1020]:=

```
(*possibleComplexStructureIndex=
Select[base16,Block[{J=T16^A[4]*#[1],a1,a2,b1,b2,r},
  a1=J.J;
  a2=Transpose[J].σ16.J;
  b1=a1===-ID16;
  b2=a2==σ16;
  r=b1&&b2;
  (*If[r,Print[a1]];*)
  r
]→"Index"]
possibleComplexStructureMatrix=base16[[#][2]&/@%*)
```

In[1021]:=

base16[[201]][2]

Out[1021]=

{1, 2, 3, 5, 6}

In[1022]:=

$$\text{T16}^A[4].\text{base16}[[201][1]] // \text{MatrixForm}$$

Out[1022]//MatrixForm=

$$\left(\begin{array}{cccccccccccccccc} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

In[1023]:=

$$\text{T16}^A[1].\text{T16}^A[2].\text{T16}^A[3].\text{T16}^A[4].\text{T16}^A[5].\text{T16}^A[6] // \text{MatrixForm}$$

Out[1023]//MatrixForm=

$$\left(\begin{array}{cccccccccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{array} \right)$$

In[1024]:=

```
(*possibleComplexStructureIndex=
Select[base16,Block[{J=T16^A[4]*#[[1]],a1=J.J,a2=Transpose[J].σ16.J,
b1=a1==ID16,b2=a2==σ16},b1&&b2]&→"Index"]
possibleComplexStructureMatrix=base16[[#][2]]&/@%*)
```

In[1025]:=

```
possibleComplexStructureIndex =
Select[base16, Block[{J = T16^A[4] * #\[1], m = σ16.J}, J.J === -ID16 &&
σ16 === Transpose[J].σ16.J(*&&! (m === -Transpose[m]) *)] & → "Index"]
possibleComplexStructureMatrix = base16[[#][2]] & /@ %
```

Out[1025]=

{4, 178, 187, 201}

Out[1026]=

{{3}, {0, 1, 3, 6, 7}, {0, 2, 3, 5, 7}, {1, 2, 3, 5, 6}}

In[1027]:=

Select[base16, #\[2] == {1, 2, 3, 4, 5, 6} & → "Index"]

Out[1027]=

{240}

In[1028]:=

base16[[240]][2]

Out[1028]=

{1, 2, 3, 4, 5, 6}

In[1029]:=

base16[[240]][1] // MatrixForm

Out[1029]//MatrixForm=

$$\left(\begin{array}{cccccccccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

In[1030]:=

```
(*possibleComplexStructureIndex=
Select[base16,Block[{J=#\[1],m=σ16.#\[1]},J.J===-ID16&&
σ16==Transpose[J].σ16.J&&! (m===-Transpose[m]) ]&→"Index"]
possibleComplexStructureMatrix=base16[[#][2]]&/@%*)
```

```
findComplexStucture[] := Module[{list, filter(*, innerProduct*)},
(*innerProduct[a_, b_] := FullSimplify[Transpose[a].σ16.b];*)
(*filter[J_][a_, b_] := J.J == -ID16 && innerProduct[a, b] == innerProduct[J.a, J.b];*)
```

```

filter[J_]:=J . J === -ID16 && σ16 === Transpose[J] . σ16 . J;
Return[list];
]

In[1031]:= Ω16 === T16^A[1].T16^A[2].T16^A[3].T16^A[4]
Out[1031]= True

```

Spinor Lagrangian

```

In[1032]:= usegT16 =
  Table[FullSimplify[(T16^α[α1 - 1]), constraintVars], {α1, 1, Length[X]}];

In[1033]:= useT16 = Table[FullSimplify[(T16^α[α1 - 1] /. ssgm4488), constraintVars] /.
  {
    √e^-2 a4[H x4] → e^-a4[H x4], √e^2 a4[H x4] Sin[6 H x0]^(1/3) → e^a4[H x4] Sin[6 H x0]^(1/6),
    1/√e^2 a4[H x4] Sin[6 H x0]^(1/3) → 1/e^a4[H x4] Sin[6 H x0]^(1/6)
  }, {α1, 1, Length[X]}] /.
  {
    1/√e^-2 a4[H x4] Sin[6 H x0]^(1/3) → 1/e^a4[H x4] Sin[6 H x0]^(1/6), √e^2 a4[H x4] → e^a4[H x4]
  };

In[1034]:= Dimensions[useT16]
Out[1034]= {8, 16, 16}

In[1035]:= Det[MatrixMetric44]
Out[1035]= Cos[6 H x0]^2

In[1036]:= useDSQRT = Cos[6 H x0]
Out[1036]= Cos[6 H x0]

```

```

In[1037]:= 
wmat /. sg /. ssgm4488 /. subsDefects;

$$\left( \frac{\sqrt{e^{2 a4[H x4]}}}{\sqrt{e^{2 a4[H x4]}}} \rightarrow e^{a4[H x4]}, \right.$$


$$\left. \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} \rightarrow e^{a4[H x4]} H \sin[6 H x0]^{1/6} \right) - spinCoeffs //$$

```

```

FullSimplify[#, constraintVars] & /. subsDefects /.

$$\left\{ \sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}, \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} \rightarrow e^{a4[H x4]} H \sin[6 H x0]^{1/6} \right\} //$$

```

```

FullSimplify[#, constraintVars] & /. subsDefects;
% // Flatten // Union
% // ExpandAll /. subsDefects /.

$$\left\{ \sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}, \right.$$


$$\left. \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} \rightarrow e^{a4[H x4]} H \sin[6 H x0]^{1/6} \right\} /. subsDefects
% /. {a4[H x4] \rightarrow \pi^4, Sin[6 H x0] \rightarrow e^12}

Out[1039]=

$$\left\{ 0, e^{-a4[H x4]} \left( -1 + e^{a4[H x4]} \sqrt{e^{-2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6}, \right.$$


$$- \left( \left( -e^{a4[H x4]} + \sqrt{e^{2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6}, \left( -e^{a4[H x4]} + \sqrt{e^{2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6}, \right.$$


$$e^{-a4[H x4]} \left( 1 - e^{a4[H x4]} \sqrt{e^{-2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6} a4'[H x4],$$


$$e^{-a4[H x4]} \left( -1 + e^{a4[H x4]} \sqrt{e^{-2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6} a4'[H x4],$$


$$\left. \left( -e^{a4[H x4]} + \sqrt{e^{2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6} a4'[H x4] \right\}$$


Out[1040]=

$$\left\{ 0, -e^{-a4[H x4]} H \sin[6 H x0]^{1/6} + \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6}, \right.$$


$$e^{a4[H x4]} H \sin[6 H x0]^{1/6} - \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6},$$


$$-e^{a4[H x4]} H \sin[6 H x0]^{1/6} + \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6},$$


$$e^{-a4[H x4]} H \sin[6 H x0]^{1/6} a4'[H x4] - \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4],$$


$$-e^{-a4[H x4]} H \sin[6 H x0]^{1/6} a4'[H x4] + \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4],$$


$$\left. -e^{a4[H x4]} H \sin[6 H x0]^{1/6} a4'[H x4] + \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4] \right\}$$


Out[1041]=
{0, 0, 0, 0, 0, 0, 0}

In[1042]=
Q2 = 0; Protect[Q1, Q2]

Out[1042]=
{Q1, Q2}$$

```

In[1043]:=

```

Clear[Lg];
Lg[] :=  $\sqrt{\det gg} \cdot \left( \text{Transpose}[\Psi_{16}] \cdot \sigma_{16} \cdot \right.$ 
 $\text{Sum}\left[ \text{FullSimplify}\left[ (\text{T16}^\alpha [\alpha_1 - 1] / . \text{sg}), \text{constraintVars} \right] \cdot \left( D[\Psi_{16}, X[\alpha_1]] + \right. \right.$ 
 $\left. \left( \frac{Q_1}{2} \right) \cdot \text{Sum}[\omega_{\text{mat}}[\alpha_1, a, b] * \text{SAB}[a, b] \cdot \Psi_{16}, \{a, 1, 8\}, \{b, 1, 8\}] \right),$ 
 $\left. \left. \{\alpha_1, 1, \text{Length}[X]\} \right] + (H * M) * \text{Transpose}[\Psi_{16}] \cdot \sigma_{16} \cdot \Psi_{16} \right) //$ 
Simplify[#, constraintVars] &

```

In[1045]:=

 $\omega_{\mu IJ}[1]$

Out[1045]=

```

{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}

```

In[1046]:=

```

Clear[La];
La[] :=
useDSQRT *  $\left( \text{Transpose}[\Psi_{16}] \cdot \sigma_{16} \cdot \text{Sum}\left[ \text{useT16}[\alpha_1] \cdot \left( D[\Psi_{16}, X[\alpha_1]] + \left( \frac{Q_1}{2} \right) \cdot \right. \right.$ 
 $\left. \left. \text{Sum}[\omega_{\mu IJ}[\alpha_1][A_1, B_1] * \text{SAB}[A_1, B_1], \{A_1, 1, 8\}, \{B_1, 1, 8\}] \cdot \Psi_{16} \right), \right.$ 
 $\left. \left. \{\alpha_1, 1, \text{Length}[X]\} \right] + (H * M) * \text{Transpose}[\Psi_{16}] \cdot \sigma_{16} \cdot \Psi_{16} \right) //$ 
Simplify[#, constraintVars] &

```

In[1048]:=

(*testLa=La[]*)

In[1049]:=

(*testLg=Lg[];*)

In[1050]:=

```

(*D[testLa,a4[H x4]]//Flatten//Union*)
(*D[testLa,a4'[H x4]]//(*Full*)Simplify[#,constraintVars]&//Flatten//Union*)

```

In[1051]:=

```

(*testLgm=
testL/.sg/.ssgm4488//(*Full*)Simplify[#,constraintVars]&/.subsDefects*)

```

In[1052]:=

```

(*D[testLgm,a4[H x4]]
D[testLgm,a4'[H x4]]//(*Full*)Simplify[#,constraintVars]&*)

```

In[1053]:=

```

Clear[Lj];
Lj[j_] := useDSQRT * ((((
Transpose[\Psi16].\sigma16.useT16[1].D[\Psi16, X[1]] +
Transpose[\Psi16].\sigma16.useT16[5].D[\Psi16, X[5]] + (K0/2) * Sum[
(*use\omega[1,A1,B1]*) \omega\mu IJ[1][A1, B1] \times Transpose[\Psi16].\sigma16.useT16[1].SAB[A1, B1].(base16[j, 1]).\Psi16, {A1, 1, 8}, {B1, 1, 8}] +
Transpose[\Psi16].\sigma16.useT16[5].D[\Psi16, X[5]] + (K4/2) * Sum[
(*use\omega[5,A1,B1]*) \omega\mu IJ[5][A1, B1] \times Transpose[\Psi16].\sigma16.useT16[5].SAB[A1, B1].(base16[j, 1]).\Psi16, {A1, 1, 8}, {B1, 1, 8}] +
(H * M) * Transpose[\Psi16].\sigma16.\Psi16) // Simplify[#, constraintVars] &

```

In[1055]:=

(*Lj[92]*)

In[1056]:=

detgg

Out[1056]=

$$g[0][0][x_0, x_4] \times g[1][1][x_0, x_4] \times g[2][2][x_0, x_4] \times g[3][3][x_0, x_4] \times g[4][4][x_0, x_4] \times g[5][5][x_0, x_4] \times g[6][6][x_0, x_4] \times g[7][7][x_0, x_4]$$

In[1057]:=

Clear[eL];

```

eL[Lagrangian_Symbol, detsqrt_] := Module[{L, t},
  L = Lagrangian[];
  t = Table[FullSimplify[
    
$$\frac{1}{detsqrt} (D[L, f16[k][x_0, x_4]] - D[D[L, f16[k]^{(1,0)}[x_0, x_4]], x_0] - D[D[L, f16[k]^{(0,1)}[x_0, x_4]], x_4]), constraintVars], {k, 0, 15}];
  Return[t /. subsDefects];
]$$

```

eLa

```
In[1059]:= (*Get[
"2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLa.
mx"] ;*)

In[1060]:= eLa = eL[La, useDSQRT];
```

```
In[1061]:= eLa
Out[1061]= { -2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[9][x0, x4] a4'[H x4] -
 2 (H M f16[4][x0, x4] + 3 H f16[12][x0, x4] +
 f16[9](0,1)[x0, x4] + Tan[6 H x0] f16[12](1,0)[x0, x4]),
 2 (-H M f16[5][x0, x4] - 3 H f16[13][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[8][x0, x4] a4'[H x4] + f16[8](0,1)[x0, x4] - Tan[6 H x0] f16[13](1,0)[x0, x4]),
 -2 (H M f16[6][x0, x4] + 3 H f16[14][x0, x4] - f16[11](0,1)[x0, x4] +
 Tan[6 H x0] f16[14](1,0)[x0, x4]),
 -2 (H M f16[7][x0, x4] + 3 H f16[15][x0, x4] + f16[10](0,1)[x0, x4] +
 Tan[6 H x0] f16[15](1,0)[x0, x4]), -2 H M f16[0][x0, x4] -
 6 H f16[8][x0, x4] - 2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[13][x0, x4] a4'[H x4] +
 2 f16[13](0,1)[x0, x4] - 2 Tan[6 H x0] f16[8](1,0)[x0, x4],
 -2 H M f16[1][x0, x4] - 6 H f16[9][x0, x4] +
 e-2 a4[H x4] (-1 + e2 a4[H x4]) H Q1 f16[12][x0, x4] a4'[H x4] -
 2 (f16[12](0,1)[x0, x4] + Tan[6 H x0] f16[9](1,0)[x0, x4]),
 -2 (H M f16[2][x0, x4] + 3 H f16[10][x0, x4] +
 f16[15](0,1)[x0, x4] + Tan[6 H x0] f16[10](1,0)[x0, x4]),
 -2 (H M f16[3][x0, x4] + 3 H f16[11][x0, x4] - f16[14](0,1)[x0, x4] +
 Tan[6 H x0] f16[11](1,0)[x0, x4]),
 2 (3 H f16[4][x0, x4] + H M f16[12][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[1][x0, x4] a4'[H x4] - f16[1](0,1)[x0, x4] + Tan[6 H x0] f16[4](1,0)[x0, x4]),
 2 (3 H f16[5][x0, x4] + H M f16[13][x0, x4] - e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[0][x0, x4] a4'[H x4] + f16[0](0,1)[x0, x4] + Tan[6 H x0] f16[5](1,0)[x0, x4]),
 2 (3 H f16[6][x0, x4] + H M f16[14][x0, x4] + f16[3](0,1)[x0, x4] +
 Tan[6 H x0] f16[6](1,0)[x0, x4]), 2 (3 H f16[7][x0, x4] +
 H M f16[15][x0, x4] - f16[2](0,1)[x0, x4] + Tan[6 H x0] f16[7](1,0)[x0, x4]),
 2 (3 H f16[0][x0, x4] + H M f16[8][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[5][x0, x4] a4'[H x4] + f16[5](0,1)[x0, x4] + Tan[6 H x0] f16[0](1,0)[x0, x4]),
 6 H f16[1][x0, x4] + 2 H M f16[9][x0, x4] - 2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[4][x0, x4] a4'[H x4] - 2 f16[4](0,1)[x0, x4] + 2 Tan[6 H x0] f16[1](1,0)[x0, x4],
 2 (3 H f16[2][x0, x4] + H M f16[10][x0, x4] - f16[7](0,1)[x0, x4] +
 Tan[6 H x0] f16[2](1,0)[x0, x4]), 2 (3 H f16[3][x0, x4] +
 H M f16[11][x0, x4] + f16[6](0,1)[x0, x4] + Tan[6 H x0] f16[3](1,0)[x0, x4]) }
```

```
In[1062]:= Length[eLa]
```

```
Out[1062]= 16
```

```
In[1063]:= DumpSave[ToString[header <> "eLa.mx"], eLa];
```

```
In[1064]:= Print[ToString[header <> "eLa.mx"]]
Pair-Crtn-Univ-same_E-L-eqs-alt-approach-eLa.mx

In[1065]:= (*Cases[
  eLa,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
  Infinity,
  Heads → True
]*)

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE:

In[1066]:= eLaRawSets = Cases[
  #,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
  Infinity,
  Heads → True
] & /@ eLa

Out[1066]= {{9, 4, 12, 9, 12}, {5, 13, 8, 8, 13}, {6, 14, 11, 14},
{7, 15, 10, 15}, {0, 8, 13, 13, 8}, {1, 9, 12, 12, 9}, {2, 10, 15, 10},
{3, 11, 14, 11}, {4, 12, 1, 1, 4}, {5, 13, 0, 0, 5}, {6, 14, 3, 6},
{7, 15, 2, 7}, {0, 8, 5, 5, 0}, {1, 9, 4, 4, 1}, {2, 10, 7, 2}, {3, 11, 6, 3}]

In[1067]:= Dimensions[eLaRawSets]
Out[1067]= {16}

In[1068]:= rawSetsel16 = rawSets[eLa, "f16", 0]
% === eLaRawSets
Out[1068]= {{9, 4, 12, 9, 12}, {5, 13, 8, 8, 13}, {6, 14, 11, 14},
{7, 15, 10, 15}, {0, 8, 13, 13, 8}, {1, 9, 12, 12, 9}, {2, 10, 15, 10},
{3, 11, 14, 11}, {4, 12, 1, 1, 4}, {5, 13, 0, 0, 5}, {6, 14, 3, 6},
{7, 15, 2, 7}, {0, 8, 5, 5, 0}, {1, 9, 4, 4, 1}, {2, 10, 7, 2}, {3, 11, 6, 3}]

Out[1069]= True
```

In[1070]:=

```
eLaCouplings = showCoupledEquations[eLaRawSets]
% // Flatten // Sort
% === Range[0, 15]
```

Out[1070]=

```
{ {0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14} }
```

Out[1071]=

```
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
```

Out[1072]=

```
True
```

In[1073]:=

```
eLaLaztCouplings = eLaCouplings
```

Out[1073]=

```
{ {0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14} }
```

In[1074]:=

```
(*processRawSets[]:=Module[{s1,s2,J,L=Range[Length[eLaRawSets]],r={}},
While[Length[L]>1,
  s1=Union[eLaRawSets[[L[[1]]]]];
  L=Drop[L,1];
  J=L[[1]];
  While[J≤16,
    If[MemberQ[L,J],
      s2=Union[eLaRawSets[[J]]];
      If[Intersection[s1,s2]≠{},s1=Union[Flatten[{s1,s2}]];
       L=Complement[L,{J}];,##&[]];
      ##&[]];
    J++]];
  AppendTo[r,s1];
];
Return[r];
]*)
```

eLaLazt

In[1075]:=

```
(*Get[
"2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLaLazt
.mx"];*)
```

In[1076]:=

```

time1 = Now
AbsoluteTiming[
  eLazt =  $\frac{1}{2 \cdot H} * eLa /. sf\psi16Aa /. sx0x4 // FullSimplify[\#, constraintVars] &$ 
Now - time1

```

Out[1076]=

Tue 3 Feb 2026 15:35:31 GMT-8

Out[1077]=

$$\begin{aligned}
& \{5.68581, \{-M Z[4][z, t] - 3 Z[12][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] - \\
& \quad Z[9]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t], -M Z[5][z, t] - 3 Z[13][z, t] + \\
& \quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + Z[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t], \\
& \quad -M Z[6][z, t] - 3 Z[14][z, t] + Z[11]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t], \\
& \quad -M Z[7][z, t] - 3 Z[15][z, t] - Z[10]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t], \\
& \quad -M Z[0][z, t] - 3 Z[8][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + \\
& \quad Z[13]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t], -M Z[1][z, t] - 3 Z[9][z, t] + \\
& \quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] - Z[12]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t], \\
& \quad -M Z[2][z, t] - 3 Z[10][z, t] - Z[15]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t], \\
& \quad -M Z[3][z, t] - 3 Z[11][z, t] + Z[14]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t], \\
& \quad 3 Z[4][z, t] + M Z[12][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] - \\
& \quad Z[1]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t], 3 Z[5][z, t] + M Z[13][z, t] - \\
& \quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] + Z[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t], \\
& \quad 3 Z[6][z, t] + M Z[14][z, t] + Z[3]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t], \\
& \quad 3 Z[7][z, t] + M Z[15][z, t] - Z[2]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t], \\
& \quad 3 Z[0][z, t] + M Z[8][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] + \\
& \quad Z[5]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t], 3 Z[1][z, t] + M Z[9][z, t] - \\
& \quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] - Z[4]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t], \\
& \quad 3 Z[2][z, t] + M Z[10][z, t] - Z[7]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t], \\
& \quad 3 Z[3][z, t] + M Z[11][z, t] + Z[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t]\}\}
\end{aligned}$$

Out[1078]=

5.688943 s

In[1079]:=

DumpSave[ToString[header <> "eLazt.mx"], eLazt];

In[1080]:=

```

Print[ToString[header <> "eLazt.mx"]]
Pair-Crtn-Univ-same_E-L-eqs-alt-approach-eLazt.mx

```

In[1081]:=

varZ = Z[#] & /@ Range[0, 15]

Out[1081]=

$$\{Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6], Z[7], \\
Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]\}$$

```

In[1082]:= varZzt = #[z, t] & /@ varZ
Out[1082]= {Z[0][z, t], Z[1][z, t], Z[2][z, t], Z[3][z, t], Z[4][z, t],
Z[5][z, t], Z[6][z, t], Z[7][z, t], Z[8][z, t], Z[9][z, t], Z[10][z, t],
Z[11][z, t], Z[12][z, t], Z[13][z, t], Z[14][z, t], Z[15][z, t]}

In[1083]:= DzvarZ = D[varZzt, z]
Out[1083]= {Z[0]^(1,0)[z, t], Z[1]^(1,0)[z, t], Z[2]^(1,0)[z, t], Z[3]^(1,0)[z, t],
Z[4]^(1,0)[z, t], Z[5]^(1,0)[z, t], Z[6]^(1,0)[z, t], Z[7]^(1,0)[z, t],
Z[8]^(1,0)[z, t], Z[9]^(1,0)[z, t], Z[10]^(1,0)[z, t], Z[11]^(1,0)[z, t],
Z[12]^(1,0)[z, t], Z[13]^(1,0)[z, t], Z[14]^(1,0)[z, t], Z[15]^(1,0)[z, t]}

In[1084]:= DtvarZ = D[varZzt, t]
Out[1084]= {Z[0]^(0,1)[z, t], Z[1]^(0,1)[z, t], Z[2]^(0,1)[z, t], Z[3]^(0,1)[z, t],
Z[4]^(0,1)[z, t], Z[5]^(0,1)[z, t], Z[6]^(0,1)[z, t], Z[7]^(0,1)[z, t],
Z[8]^(0,1)[z, t], Z[9]^(0,1)[z, t], Z[10]^(0,1)[z, t], Z[11]^(0,1)[z, t],
Z[12]^(0,1)[z, t], Z[13]^(0,1)[z, t], Z[14]^(0,1)[z, t], Z[15]^(0,1)[z, t]}

```

```
In[1085]:= (DtvarZsubs = Solve[And @@ Thread[θ == eLazt], DtvarZ][[1]] // FullSimplify[#, constraintVars] &) // Column
Out[1085]= Z[0]^(0,1)[z, t] → -3 Z[5][z, t] - M Z[13][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] - 6 Tan[z] Z[5]^(1,0)[z, t]
Z[1]^(0,1)[z, t] → 3 Z[4][z, t] + M Z[12][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] + 6 Tan[z] Z[4]^(1,0)[z, t]
Z[2]^(0,1)[z, t] → 3 Z[7][z, t] + M Z[15][z, t] + 6 Tan[z] Z[7]^(1,0)[z, t]
Z[3]^(0,1)[z, t] → -3 Z[6][z, t] - M Z[14][z, t] - 6 Tan[z] Z[6]^(1,0)[z, t]
Z[4]^(0,1)[z, t] → 3 Z[1][z, t] + M Z[9][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] + 6 Tan[z] Z[1]^(1,0)[z, t]
Z[5]^(0,1)[z, t] → -3 Z[0][z, t] - M Z[8][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] - 6 Tan[z] Z[0]^(1,0)[z, t]
Z[6]^(0,1)[z, t] → -3 Z[3][z, t] - M Z[11][z, t] - 6 Tan[z] Z[3]^(1,0)[z, t]
Z[7]^(0,1)[z, t] → 3 Z[2][z, t] + M Z[10][z, t] + 6 Tan[z] Z[2]^(1,0)[z, t]
Z[8]^(0,1)[z, t] → M Z[5][z, t] + 3 Z[13][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + 6 Tan[z] Z[13]^(1,0)[z, t]
Z[9]^(0,1)[z, t] → -M Z[4][z, t] - 3 Z[12][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] - 6 Tan[z] Z[12]^(1,0)[z, t]
Z[10]^(0,1)[z, t] → -M Z[7][z, t] - 3 (Z[15][z, t] + 2 Tan[z] Z[15]^(1,0)[z, t])
Z[11]^(0,1)[z, t] → M Z[6][z, t] + 3 Z[14][z, t] + 6 Tan[z] Z[14]^(1,0)[z, t]
Z[12]^(0,1)[z, t] → -M Z[1][z, t] - 3 Z[9][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] - 6 Tan[z] Z[9]^(1,0)[z, t]
Z[13]^(0,1)[z, t] → M Z[0][z, t] + 3 Z[8][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] + 6 Tan[z] Z[8]^(1,0)[z, t]
Z[14]^(0,1)[z, t] → M Z[3][z, t] + 3 Z[11][z, t] + 6 Tan[z] Z[11]^(1,0)[z, t]
Z[15]^(0,1)[z, t] → -M Z[2][z, t] - 3 (Z[10][z, t] + 2 Tan[z] Z[10]^(1,0)[z, t])
```

In[1086]:=

```

ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(DtvarZEQS = ToExpression[#] & /@ %) // Column

```

Out[1088]=

$$\begin{aligned}
Z[0]^{(0,1)}[z, t] &= -3 Z[5][z, t] - M Z[13][z, t] + \\
&\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t] \\
Z[1]^{(0,1)}[z, t] &= 3 Z[4][z, t] + M Z[12][z, t] + \\
&\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t] \\
Z[2]^{(0,1)}[z, t] &= 3 Z[7][z, t] + M Z[15][z, t] + 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t] \\
Z[3]^{(0,1)}[z, t] &= -3 Z[6][z, t] - M Z[14][z, t] - 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t] \\
Z[4]^{(0,1)}[z, t] &= 3 Z[1][z, t] + M Z[9][z, t] - \\
&\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t] \\
Z[5]^{(0,1)}[z, t] &= -3 Z[0][z, t] - M Z[8][z, t] - \\
&\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t] \\
Z[6]^{(0,1)}[z, t] &= -3 Z[3][z, t] - M Z[11][z, t] - 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t] \\
Z[7]^{(0,1)}[z, t] &= 3 Z[2][z, t] + M Z[10][z, t] + 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t] \\
Z[8]^{(0,1)}[z, t] &= M Z[5][z, t] + 3 Z[13][z, t] - \\
&\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t] \\
Z[9]^{(0,1)}[z, t] &= -M Z[4][z, t] - 3 Z[12][z, t] - \\
&\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t] \\
Z[10]^{(0,1)}[z, t] &= -M Z[7][z, t] - 3 (Z[15][z, t] + 2 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t]) \\
Z[11]^{(0,1)}[z, t] &= M Z[6][z, t] + 3 Z[14][z, t] + 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t] \\
Z[12]^{(0,1)}[z, t] &= -M Z[1][z, t] - 3 Z[9][z, t] + \\
&\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t] \\
Z[13]^{(0,1)}[z, t] &= M Z[0][z, t] + 3 Z[8][z, t] + \\
&\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t] \\
Z[14]^{(0,1)}[z, t] &= M Z[3][z, t] + 3 Z[11][z, t] + 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t] \\
Z[15]^{(0,1)}[z, t] &= -M Z[2][z, t] - 3 (Z[10][z, t] + 2 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t])
\end{aligned}$$

In[1089]:=

```

ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(DtvarZrelations = ToExpression[#] & /@ %) // Column

```

Out[1091]=

$$\begin{aligned}
& 3 Z[5][z, t] + M Z[13][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] + \\
& Z[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t] \\
& -3 Z[4][z, t] - M Z[12][z, t] - \\
& e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] + Z[1]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t] \\
& -3 Z[7][z, t] - M Z[15][z, t] + Z[2]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t] \\
& 3 Z[6][z, t] + M Z[14][z, t] + Z[3]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t] \\
& -3 Z[1][z, t] - M Z[9][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] + \\
& Z[4]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t] \\
& 3 Z[0][z, t] + M Z[8][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] + \\
& Z[5]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t] \\
& 3 Z[3][z, t] + M Z[11][z, t] + Z[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t] \\
& -3 Z[2][z, t] - M Z[10][z, t] + Z[7]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t] \\
& -M Z[5][z, t] - 3 Z[13][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + \\
& Z[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t] \\
& M Z[4][z, t] + 3 Z[12][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] + \\
& Z[9]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t] \\
& M Z[7][z, t] + Z[10]^{(0,1)}[z, t] + 3 (Z[15][z, t] + 2 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t]) \\
& -M Z[6][z, t] - 3 Z[14][z, t] + Z[11]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t] \\
& M Z[1][z, t] + 3 Z[9][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] + \\
& Z[12]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t] \\
& -M Z[0][z, t] - 3 Z[8][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + \\
& Z[13]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t] \\
& -M Z[3][z, t] - 3 Z[11][z, t] + Z[14]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t] \\
& M Z[2][z, t] + Z[15]^{(0,1)}[z, t] + 3 (Z[10][z, t] + 2 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t])
\end{aligned}$$

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE:

In[1092]:=

```
eLaztCouplings = eLaCouplings
```

Out[1092]=

```
{ {0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14} }
```

In[1093]:=

```
(*{{5,13,0,8,5,5,0,5,13,8},{4,12,1,1,9,4},{7,15,2,2,10,7},{6,14,3,3,11,6}}
eLaztCouplings=Union[#]&/@%
(*eLaztCouplings=Union[Flatten[#]]&/@%*)
Union[Flatten[%]]==Range[0,15]*)
```

SEPARATE VARIABLES THAT ARE COUPLED TO EACH OTHER (reorder):

In[1094]:=

```

Thread[(Z[#] & /@ Flatten[eLastCouplings]) → (yZ[#] & /@ Range[0, 15])]
% // Sort
sZt0yZ
% - %%

```

Out[1094]=

```

{Z[0] → yZ[0], Z[5] → yZ[1], Z[8] → yZ[2], Z[13] → yZ[3], Z[1] → yZ[4], Z[4] → yZ[5],
 Z[9] → yZ[6], Z[12] → yZ[7], Z[2] → yZ[8], Z[7] → yZ[9], Z[10] → yZ[10],
 Z[15] → yZ[11], Z[3] → yZ[12], Z[6] → yZ[13], Z[11] → yZ[14], Z[14] → yZ[15]}

```

Out[1095]=

```

{Z[0] → yZ[0], Z[1] → yZ[4], Z[2] → yZ[8], Z[3] → yZ[12], Z[4] → yZ[5], Z[5] → yZ[1],
 Z[6] → yZ[13], Z[7] → yZ[9], Z[8] → yZ[2], Z[9] → yZ[6], Z[10] → yZ[10],
 Z[11] → yZ[14], Z[12] → yZ[7], Z[13] → yZ[3], Z[14] → yZ[15], Z[15] → yZ[11]}

```

Out[1096]=

```
sZt0yZ
```

Out[1097]=

```

{sZt0yZ - (Z[0] → yZ[0]), sZt0yZ - (Z[1] → yZ[4]),
 sZt0yZ - (Z[2] → yZ[8]), sZt0yZ - (Z[3] → yZ[12]), sZt0yZ - (Z[4] → yZ[5]),
 sZt0yZ - (Z[5] → yZ[1]), sZt0yZ - (Z[6] → yZ[13]), sZt0yZ - (Z[7] → yZ[9]),
 sZt0yZ - (Z[8] → yZ[2]), sZt0yZ - (Z[9] → yZ[6]), sZt0yZ - (Z[10] → yZ[10]),
 sZt0yZ - (Z[11] → yZ[14]), sZt0yZ - (Z[12] → yZ[7]), sZt0yZ - (Z[13] → yZ[3]),
 sZt0yZ - (Z[14] → yZ[15]), sZt0yZ - (Z[15] → yZ[11])}

```

```
In[1098]:= (*Thread[(Z[#]&/@eLaztCouplings[[1]])>(yZ[#]&/@Range[0,3])]*
 Thread[(Z[#]&/@eLaztCouplings[[2]])>(yZ[#]&/@Range[4,7])]
 Thread[(Z[#]&/@eLaztCouplings[[3]])>(yZ[#]&/@Range[8,11])]
 Thread[(Z[#]&/@eLaztCouplings[[4]])>(yZ[#]&/@Range[12,15])]*)
sZt0yZ = Thread[(Z[#]&/@Flatten[eLaztCouplings])>(yZ[#]&/@Range[0,15])]
(*Union[Flatten[{%,%%,%%%,%%%%}]]*)
ZyZEQS = sZt0yZ /. {Rule -> Equal}
ZyZforCaEQS = -sZt0yZ /. {Rule -> Subtract}
(*ToString[FullForm[#]]&/@sZt0yZ;
StringReplace[#, "Rule" -> "Equal"]&/@%;*)
(*(ZyZEQS=ToExpression[#]&/@%)*)
syZtoZ = Solve[And @@ ZyZEQS, (yZ[#]&/@Range[0,15])][[-1]]
Out[1098]= {Z[0] -> yZ[0], Z[5] -> yZ[1], Z[8] -> yZ[2], Z[13] -> yZ[3], Z[1] -> yZ[4], Z[4] -> yZ[5],
Z[9] -> yZ[6], Z[12] -> yZ[7], Z[2] -> yZ[8], Z[7] -> yZ[9], Z[10] -> yZ[10],
Z[15] -> yZ[11], Z[3] -> yZ[12], Z[6] -> yZ[13], Z[11] -> yZ[14], Z[14] -> yZ[15]}
Out[1099]= {Z[0] == yZ[0], Z[5] == yZ[1], Z[8] == yZ[2], Z[13] == yZ[3], Z[1] == yZ[4], Z[4] == yZ[5],
Z[9] == yZ[6], Z[12] == yZ[7], Z[2] == yZ[8], Z[7] == yZ[9], Z[10] == yZ[10],
Z[15] == yZ[11], Z[3] == yZ[12], Z[6] == yZ[13], Z[11] == yZ[14], Z[14] == yZ[15]}
Out[1100]= {yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1], yZ[5] - Z[4],
yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}
Out[1101]= {yZ[0] -> Z[0], yZ[1] -> Z[5], yZ[2] -> Z[8], yZ[3] -> Z[13], yZ[4] -> Z[1], yZ[5] -> Z[4],
yZ[6] -> Z[9], yZ[7] -> Z[12], yZ[8] -> Z[2], yZ[9] -> Z[7], yZ[10] -> Z[10],
yZ[11] -> Z[15], yZ[12] -> Z[3], yZ[13] -> Z[6], yZ[14] -> Z[11], yZ[15] -> Z[14]}
Out[1102]= (*Thread[(Z[#]&/@Flatten[eLaztCouplings])>(yZ[#]&/@Range[0,15])]
%//Sort
sZt0yZ
%-%%*)
Out[1103]= ToString[FullForm[#]] & /@ sZt0yZ;
StringReplace[#, "Rule" -> "Subtract"] & /@ %;
(ZyZforCaEQS = -ToExpression[#]& /@ %)
Out[1105]= {yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1], yZ[5] - Z[4],
yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}
```

```
In[1106]:= cayZ = CoefficientArrays[ZyZforCaEQS, (yZ[#] & /@ Range[0, 15])]

Out[1106]= {SparseArray[ Specified elements: 16 Dimensions: {16}], SparseArray[ Specified elements: 16 Dimensions: {16, 16}]}
```



```
In[1107]:= caZ = CoefficientArrays[ZyZforCaEQS, (Z[#] & /@ Range[0, 15])]

Out[1107]= {SparseArray[ Specified elements: 16 Dimensions: {16}], SparseArray[ Specified elements: 16 Dimensions: {16, 16}]}
```



```
In[1108]:= (cayZ2 = cayZ[[2]] // Normal) // MatrixForm
```



```
Out[1108]//MatrixForm= 
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```

IDENTIFY ORTHOGONAL SIMILARITY TRANSFORMATION FROM SET OF VARIABLES Z[j] TO SET OF VARIABLES yZ[j] :

In[1109]:=

```
cayZ2.(yZ[#] & /@ Range[0, 15])
(*   cayZ2.yZ === Z : *)
% /. syZtoZ
(*   inverse: yZ === Transpose[cayZ2].Z *)
Transpose[cayZ2].% /. sZt0YZ
```

Out[1109]=

```
{yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}
```

Out[1110]=

```
{Z[0], Z[5], Z[8], Z[13], Z[1], Z[4], Z[9],
Z[12], Z[2], Z[7], Z[10], Z[15], Z[3], Z[6], Z[11], Z[14]}
```

Out[1111]=

```
{yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}
```

ORTHOGONAL O(16) SIMILARITY TRANSFORMATION :

In[1112]:=

```
cayZ2.Transpose[cayZ2] === ID16
```

Out[1112]=

```
True
```

BUT NOT ORTHOGONAL O(8,8) SIMILARITY TRANSFORMATION :

In[1113]:=

```
cayZ2.σ16.Transpose[cayZ2] === σ16
```

Out[1113]=

```
True
```

Also, Not a Direct Sum :

In[1114]:=

```
cayZ2 === ArrayFlatten[{{cayZ2[[1;;8, 1;;8]], cayZ2[[1;;8, 9;;16]]},
{cayZ2[[9;;16, 1;;8]], cayZ2[[9;;16, 9;;16]]}}]
```

Out[1114]=

```
True
```

In[1115]:=

```
cayZ2 === ArrayFlatten[{{cayZ2[[1;;8, 1;;8]], {0, cayZ2[[9;;16, 9;;16]]}}]
```

Out[1115]=

```
True
```

TRANSFORM Euler Lagrange equations to yZ[j] :

```
In[1116]:= (DtYzRelations = Transpose[cayZ2].DtvarZrelations /. sZt0yZ // FullSimplify) // Column
Out[1116]=
```

$$\begin{aligned}
& 3 yZ[1][z, t] + M yZ[3][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[0][z, t] a4'[t] + \\
& yZ[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[1]^{(1,0)}[z, t] \\
& -3 yZ[5][z, t] - M yZ[7][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[4][z, t] a4'[t] + \\
& yZ[4]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[5]^{(1,0)}[z, t] \\
& -3 yZ[9][z, t] - M yZ[11][z, t] + yZ[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[9]^{(1,0)}[z, t] \\
& 3 yZ[13][z, t] + M yZ[15][z, t] + yZ[12]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[13]^{(1,0)}[z, t] \\
& -3 yZ[4][z, t] - M yZ[6][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[5][z, t] a4'[t] + \\
& yZ[5]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[4]^{(1,0)}[z, t] \\
& 3 yZ[0][z, t] + M yZ[2][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[1][z, t] a4'[t] + \\
& yZ[1]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[0]^{(1,0)}[z, t] \\
& 3 yZ[12][z, t] + M yZ[14][z, t] + yZ[13]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[12]^{(1,0)}[z, t] \\
& -3 yZ[8][z, t] - M yZ[10][z, t] + yZ[9]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[8]^{(1,0)}[z, t] \\
& -M yZ[1][z, t] - 3 yZ[3][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[2][z, t] a4'[t] + \\
& yZ[2]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[3]^{(1,0)}[z, t] \\
& M yZ[5][z, t] + 3 yZ[7][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[6][z, t] a4'[t] + \\
& yZ[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[7]^{(1,0)}[z, t] \\
& M yZ[9][z, t] + 3 yZ[11][z, t] + yZ[10]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[11]^{(1,0)}[z, t] \\
& -M yZ[13][z, t] - 3 yZ[15][z, t] + yZ[14]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[15]^{(1,0)}[z, t] \\
& M yZ[4][z, t] + 3 yZ[6][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[7][z, t] a4'[t] + \\
& yZ[7]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[6]^{(1,0)}[z, t] \\
& -M yZ[0][z, t] - 3 yZ[2][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[3][z, t] a4'[t] + \\
& yZ[3]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[2]^{(1,0)}[z, t] \\
& -M yZ[12][z, t] - 3 yZ[14][z, t] + yZ[15]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[14]^{(1,0)}[z, t] \\
& M yZ[8][z, t] + 3 yZ[10][z, t] + yZ[11]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[10]^{(1,0)}[z, t]
\end{aligned}$$

CHECK THAT $yZ[j]$ ARE IN PROPER ORDER (4 things, 4 at a time):

```
In[1117]:= (DtyZsubs = Solve[And @@ Thread[0 == DtyZrelations], 
  (D[(yZ[#][z, t] & /@ Range[0, 15]), t])[[1]] // 
  FullSimplify[#, constraintVars] &) // Column
Out[1117]=
yZ[0]^(0,1)[z, t] → -3 yZ[1][z, t] - M yZ[3][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - 6 Tan[z] yZ[1]^(1,0)[z, t]
yZ[1]^(0,1)[z, t] → -3 yZ[0][z, t] - M yZ[2][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t]
yZ[2]^(0,1)[z, t] → M yZ[1][z, t] + 3 yZ[3][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] + 6 Tan[z] yZ[3]^(1,0)[z, t]
yZ[3]^(0,1)[z, t] → M yZ[0][z, t] + 3 yZ[2][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]
yZ[4]^(0,1)[z, t] → 3 yZ[5][z, t] + M yZ[7][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] + 6 Tan[z] yZ[5]^(1,0)[z, t]
yZ[5]^(0,1)[z, t] → 3 yZ[4][z, t] + M yZ[6][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t]
yZ[6]^(0,1)[z, t] → -M yZ[5][z, t] - 3 yZ[7][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - 6 Tan[z] yZ[7]^(1,0)[z, t]
yZ[7]^(0,1)[z, t] → -M yZ[4][z, t] - 3 yZ[6][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]
yZ[8]^(0,1)[z, t] → 3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t]
yZ[9]^(0,1)[z, t] → 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t]
yZ[10]^(0,1)[z, t] → -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t])
yZ[11]^(0,1)[z, t] → -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])
yZ[12]^(0,1)[z, t] → -3 yZ[13][z, t] - M yZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t]
yZ[13]^(0,1)[z, t] → -3 yZ[12][z, t] - M yZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t]
yZ[14]^(0,1)[z, t] → M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t]
yZ[15]^(0,1)[z, t] → M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]
```

```

In[1118]:= (caZ2 = caZ[[2]] // Normal) // MatrixForm
Out[1118]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


In[1119]:= sx0x4
Out[1119]=  $\left\{ x_0 \rightarrow \frac{z}{6H}, x_4 \rightarrow \frac{t}{H} \right\}$ 

In[1120]:= (*DtvarZEQS*)
In[1121]:= (*DtyZsubs*)
In[1122]:= (* (DtvarZsubs=Solve[And@@Thread[0==eLast],DtvarZ][[1]]//FullSimplify[#,constraintVars]&)//Column*)
In[1123]:= (* (DtyZsubs=
Solve[And@@Thread[0==DtyZrelations],(D[(yZ[#][z,t]&/@Range[0,15]),t])][[1]]//FullSimplify[#,constraintVars]&)//Column*)
In[1124]:= (*ToString[FullForm[#]]&/@DtyZsubs;
StringReplace[#, "Rule" \[Rule] "Equal"]&/@%;
(DtvaryZEQS=ToExpression[#]&/@%)//Column*)
DtvaryZEQS = DtyZsubs /. {Rule \[Rule] Equal};
zeroDtyZeqs = DtyZsubs /. {Rule \[Rule] Subtract};
coupledDtyZeqs = Partition[DtvaryZEQS, 4]
```

```

Out[1125]=
{3 yZ[1] [z, t] + M yZ[3] [z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[0] [z, t] a4'[t] +
yZ[0]^(0,1) [z, t] + 6 Tan[z] yZ[1]^(1,0) [z, t], 3 yZ[0] [z, t] + M yZ[2] [z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[1] [z, t] a4'[t] + yZ[1]^(0,1) [z, t] + 6 Tan[z] yZ[0]^(1,0) [z, t],
-M yZ[1] [z, t] - 3 yZ[3] [z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[2] [z, t] a4'[t] +
yZ[2]^(0,1) [z, t] - 6 Tan[z] yZ[3]^(1,0) [z, t], -M yZ[0] [z, t] - 3 yZ[2] [z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[3] [z, t] a4'[t] + yZ[3]^(0,1) [z, t] - 6 Tan[z] yZ[2]^(1,0) [z, t],
-3 yZ[5] [z, t] - M yZ[7] [z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[4] [z, t] a4'[t] +
yZ[4]^(0,1) [z, t] - 6 Tan[z] yZ[5]^(1,0) [z, t], -3 yZ[4] [z, t] - M yZ[6] [z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[5] [z, t] a4'[t] + yZ[5]^(0,1) [z, t] - 6 Tan[z] yZ[4]^(1,0) [z, t],
M yZ[5] [z, t] + 3 yZ[7] [z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[6] [z, t] a4'[t] +
yZ[6]^(0,1) [z, t] + 6 Tan[z] yZ[7]^(1,0) [z, t], M yZ[4] [z, t] + 3 yZ[6] [z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[7] [z, t] a4'[t] + yZ[7]^(0,1) [z, t] + 6 Tan[z] yZ[6]^(1,0) [z, t],
-3 yZ[9] [z, t] - M yZ[11] [z, t] + yZ[8]^(0,1) [z, t] - 6 Tan[z] yZ[9]^(1,0) [z, t],
-3 yZ[8] [z, t] - M yZ[10] [z, t] + yZ[9]^(0,1) [z, t] - 6 Tan[z] yZ[8]^(1,0) [z, t],
M yZ[9] [z, t] + yZ[10]^(0,1) [z, t] + 3 (yZ[11] [z, t] + 2 Tan[z] yZ[11]^(1,0) [z, t]),
M yZ[8] [z, t] + yZ[11]^(0,1) [z, t] + 3 (yZ[10] [z, t] + 2 Tan[z] yZ[10]^(1,0) [z, t]),
3 yZ[13] [z, t] + M yZ[15] [z, t] + yZ[12]^(0,1) [z, t] + 6 Tan[z] yZ[13]^(1,0) [z, t],
3 yZ[12] [z, t] + M yZ[14] [z, t] + yZ[13]^(0,1) [z, t] + 6 Tan[z] yZ[12]^(1,0) [z, t],
-M yZ[13] [z, t] - 3 yZ[15] [z, t] + yZ[14]^(0,1) [z, t] - 6 Tan[z] yZ[15]^(1,0) [z, t],
-M yZ[12] [z, t] - 3 yZ[14] [z, t] + yZ[15]^(0,1) [z, t] - 6 Tan[z] yZ[14]^(1,0) [z, t]}

```

```

Out[1126]=
{ {yZ[0]^(0,1)[z, t] == -3 yZ[1][z, t] - M yZ[3][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - 6 Tan[z] yZ[1]^(1,0)[z, t],  

  yZ[1]^(0,1)[z, t] == -3 yZ[0][z, t] - M yZ[2][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t],  

  yZ[2]^(0,1)[z, t] == M yZ[1][z, t] + 3 yZ[3][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] + 6 Tan[z] yZ[3]^(1,0)[z, t],  

  yZ[3]^(0,1)[z, t] == M yZ[0][z, t] + 3 yZ[2][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]},  

{yZ[4]^(0,1)[z, t] == 3 yZ[5][z, t] + M yZ[7][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] + 6 Tan[z] yZ[5]^(1,0)[z, t],  

  yZ[5]^(0,1)[z, t] == 3 yZ[4][z, t] + M yZ[6][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t],  

  yZ[6]^(0,1)[z, t] == -M yZ[5][z, t] - 3 yZ[7][z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - 6 Tan[z] yZ[7]^(1,0)[z, t],  

  yZ[7]^(0,1)[z, t] == -M yZ[4][z, t] - 3 yZ[6][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]},  

{yZ[8]^(0,1)[z, t] == 3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t],  

  yZ[9]^(0,1)[z, t] == 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t],  

  yZ[10]^(0,1)[z, t] == -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),  

  yZ[11]^(0,1)[z, t] == -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])},  

{yZ[12]^(0,1)[z, t] == -3 yZ[13][z, t] - M yZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t],  

  yZ[13]^(0,1)[z, t] == -3 yZ[12][z, t] - M yZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t],  

  yZ[14]^(0,1)[z, t] == M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t],  

  yZ[15]^(0,1)[z, t] == M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]}}

```

```

In[1127]:= (*ToString[FullForm[#]]&/@Take[coupled yZe qs,2]
StringReplace[#, "Equal" → "Subtract"]&/@%;
(ToExpression[#]&/@%)
#/.{Equal→Subtract}&/@Take[coupled yZe qs,2]
%-%%*)

```

```

In[1128]:= Clear[yZ, yZvar];
yZvar = (yZ[#][z, t] & /@ Range[0, 15])

Out[1129]= {yZ[0][z, t], yZ[1][z, t], yZ[2][z, t], yZ[3][z, t], yZ[4][z, t],
yZ[5][z, t], yZ[6][z, t], yZ[7][z, t], yZ[8][z, t], yZ[9][z, t], yZ[10][z, t],
yZ[11][z, t], yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t]}

```

```
In[1130]:= DSolve[coupledYZeqs[[1]], yZvar[[1 ;; 4]], {z, t}]
Out[1130]= DSolve[{yZ[0]^(0,1)[z, t] == -3 yZ[1][z, t] - M yZ[3][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - 6 Tan[z] yZ[1]^(1,0)[z, t],
yZ[1]^(0,1)[z, t] == -3 yZ[0][z, t] - M yZ[2][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t],
yZ[2]^(0,1)[z, t] == M yZ[1][z, t] + 3 yZ[3][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] + 6 Tan[z] yZ[3]^(1,0)[z, t],
yZ[3]^(0,1)[z, t] == M yZ[0][z, t] + 3 yZ[2][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]},
{yZ[0][z, t], yZ[1][z, t], yZ[2][z, t], yZ[3][z, t]}, {z, t}]

In[1131]:= DSolve[coupledYZeqs[[2]], yZvar[[5 ;; 8]], {z, t}]
Out[1131]= DSolve[{yZ[4]^(0,1)[z, t] == 3 yZ[5][z, t] + M yZ[7][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] + 6 Tan[z] yZ[5]^(1,0)[z, t],
yZ[5]^(0,1)[z, t] == 3 yZ[4][z, t] + M yZ[6][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t],
yZ[6]^(0,1)[z, t] == -M yZ[5][z, t] - 3 yZ[7][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - 6 Tan[z] yZ[7]^(1,0)[z, t],
yZ[7]^(0,1)[z, t] == -M yZ[4][z, t] - 3 yZ[6][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]},
{yZ[4][z, t], yZ[5][z, t], yZ[6][z, t], yZ[7][z, t]}, {z, t}]

In[1132]:= DSolve[coupledYZeqs[[3]], yZvar[[9 ;; 12]], {z, t}]
Out[1132]= DSolve[{yZ[8]^(0,1)[z, t] == 3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t],
yZ[9]^(0,1)[z, t] == 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t],
yZ[10]^(0,1)[z, t] == -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),
yZ[11]^(0,1)[z, t] == -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])},
{yZ[8][z, t], yZ[9][z, t], yZ[10][z, t], yZ[11][z, t]}, {z, t}]

In[1133]:= DSolve[coupledYZeqs[[4]], yZvar[[13 ;; 16]], {z, t}]
Out[1133]= DSolve[{yZ[12]^(0,1)[z, t] == -3 yZ[13][z, t] - M yZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t],
yZ[13]^(0,1)[z, t] == -3 yZ[12][z, t] - M yZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t],
yZ[14]^(0,1)[z, t] == M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t],
yZ[15]^(0,1)[z, t] == M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]},
{yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t]}, {z, t}]
```

$$\begin{aligned}
eq4 := & [3 yZ13(z, t) + MyZ15(z, t) + D_2(yZ12)(z, t) + 6 \tan(z) I \\
& = 0, 3 yZ12(z, t) + MyZ14(z, t) + D_2(yZ13)(z, t) + 6 \tan(z) I \\
& = 0, MyZ13(z, t) + 3 yZ15(z, t) + 6 \tan(z) D_1(yZ15)(z, t) = I \\
& t), MyZ12(z, t) + 3 yZ14(z, t) + 6 \tan(z) D_1(yZ14)(z, t) = D_2(I
\end{aligned}$$

TRY MAPLE: MUST use ` List[Equal[Derivative[0,1][yZ8][z,t],Pl...`

```

In[1134]:= syZzt = yZ[#] → ToExpression["((yZ" <> ToString[#] <> "[#1,#2])&)"] & /@ Range[0, 15]
maplexyZeqs = FullForm[coupledyeZeqs /. % // FullSimplify]
Out[1134]= {yZ[0] → (yZ0[#1, #2] &), yZ[1] → (yZ1[#1, #2] &),
yZ[2] → (yZ2[#1, #2] &), yZ[3] → (yZ3[#1, #2] &), yZ[4] → (yZ4[#1, #2] &),
yZ[5] → (yZ5[#1, #2] &), yZ[6] → (yZ6[#1, #2] &), yZ[7] → (yZ7[#1, #2] &),
yZ[8] → (yZ8[#1, #2] &), yZ[9] → (yZ9[#1, #2] &), yZ[10] → (yZ10[#1, #2] &),
yZ[11] → (yZ11[#1, #2] &), yZ[12] → (yZ12[#1, #2] &),
yZ[13] → (yZ13[#1, #2] &), yZ[14] → (yZ14[#1, #2] &), yZ[15] → (yZ15[#1, #2] &)}

```

```

Out[1135]//FullForm=
List[List[
  Equal[Plus[Times[3, yZ1[z, t]], Times[M, yZ3[z, t]], Derivative[0, 1][yZ0][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ1][z, t]]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ0[z, t], Derivative[1][a4][t]]],
  Equal[Plus[Times[3, yZ0[z, t]], Times[M, yZ2[z, t]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ1[z, t], Derivative[1][a4][t]],
    Derivative[0, 1][yZ1][z, t], Times[6, Tan[z], Derivative[1, 0][yZ0][z, t]]], 0],
  Equal[Plus[Times[M, yZ1[z, t]], Times[3, yZ3[z, t]],

    Times[6, Tan[z], Derivative[1, 0][yZ3][z, t]]],
  Plus[Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ2[z, t],
    Derivative[1][a4][t]], Derivative[0, 1][yZ2][z, t]]],
  Equal[Plus[Times[M, yZ0[z, t]], Times[3, yZ2[z, t]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ3[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][yZ2][z, t]]], Derivative[0, 1][yZ3][z, t]]],
  List[Equal[Plus[Times[3, yZ5[z, t]], Times[M, yZ7[z, t]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ4[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][yZ5][z, t]]], Derivative[0, 1][yZ4][z, t]],
  Equal[Plus[Times[3, yZ4[z, t]], Times[M, yZ6[z, t]],

    Times[6, Tan[z], Derivative[1, 0][yZ4][z, t]]],
  Plus[Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ5[z, t],
    Derivative[1][a4][t]], Derivative[0, 1][yZ5][z, t]]],
  Equal[Plus[Times[M, yZ5[z, t]], Times[3, yZ7[z, t]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ6[z, t], Derivative[1][a4][t]],
    Derivative[0, 1][yZ6][z, t], Times[6, Tan[z], Derivative[1, 0][yZ7][z, t]]], 0],
  Equal[Plus[Times[M, yZ4[z, t]], Times[3, yZ6[z, t]], Derivative[0, 1][yZ7][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ6][z, t]]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ7[z, t], Derivative[1][a4][t]]],
  List[Equal[Plus[Times[M, yZ11[z, t]], Times[3, yZ9[z, t]],

    Times[6, Tan[z], Derivative[1, 0][yZ9][z, t]]], Derivative[0, 1][yZ8][z, t]],
  Equal[Plus[Times[M, yZ10[z, t]], Times[3, yZ8[z, t]],

    Times[6, Tan[z], Derivative[1, 0][yZ8][z, t]]], Derivative[0, 1][yZ9][z, t]],
  Equal[Plus[Times[3, yZ11[z, t]], Times[M, yZ9[z, t]], Derivative[0, 1][yZ10][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ11][z, t]]], 0],
  Equal[Plus[Times[3, yZ10[z, t]], Times[M, yZ8[z, t]], Derivative[0, 1][yZ11][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ10][z, t]]], 0]],
  List[Equal[Plus[Times[3, yZ13[z, t]], Times[M, yZ15[z, t]],

    Derivative[0, 1][yZ12][z, t], Times[6, Tan[z], Derivative[1, 0][yZ13][z, t]]], 0],
  Equal[Plus[Times[3, yZ12[z, t]], Times[M, yZ14[z, t]],

    Derivative[0, 1][yZ13][z, t], Times[6, Tan[z], Derivative[1, 0][yZ12][z, t]]], 0],
  Equal[Plus[Times[M, yZ13[z, t]], Times[3, yZ15[z, t]],

    Times[6, Tan[z], Derivative[1, 0][yZ15][z, t]]], Derivative[0, 1][yZ14][z, t]],
  Equal[Plus[Times[M, yZ12[z, t]], Times[3, yZ14[z, t]],

    Times[6, Tan[z], Derivative[1, 0][yZ14][z, t]]], Derivative[0, 1][yZ15][z, t]]]
]

```

```
In[1136]:= mapleyZvars =
FullForm[yZ[#] [z, t] & /@ Range[4 * (# - 1), 4 * (# - 1) + 3] /. syZzt] & /@ Range[4]
Out[1136]= {List[yZ0[z, t], yZ1[z, t], yZ2[z, t], yZ3[z, t]], 
List[yZ4[z, t], yZ5[z, t], yZ6[z, t], yZ7[z, t]], 
List[yZ8[z, t], yZ9[z, t], yZ10[z, t], yZ11[z, t]], 
List[yZ12[z, t], yZ13[z, t], yZ14[z, t], yZ15[z, t]]}
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

```
“maple-output.py”:
import textwrap
filename = "maple-output.txt"
maple_long_string = "paste maple string here"
# Wraps the text and joins with newlines
with open (filename, "w") as f:
    f.write ("\n" . join (textwrap . wrap (maple_long_string, width = 80)))
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

"Solution" by Maple

```
In[1174]:=  $\Psi 16;$ 
% /. sfy16Aa /. sx0x4;
% /. szt0yz;
Thread[% == %%];
Solve[And @@ %, yZ[#][z, t] & /@ Range[0, 15]] [[1]]
% /. szt /. {Rule -> Equal};
Solve[And @@ %, f16[#][x0, x4] & /@ Range[0, 15]] [[1]];
% /. {Rule -> Equal}

Out[1178]= {yZ[0][z, t] -> f16[0][x0, x4], yZ[1][z, t] -> f16[5][x0, x4],
yZ[2][z, t] -> f16[8][x0, x4], yZ[3][z, t] -> f16[13][x0, x4],
yZ[4][z, t] -> f16[1][x0, x4], yZ[5][z, t] -> f16[4][x0, x4],
yZ[6][z, t] -> f16[9][x0, x4], yZ[7][z, t] -> f16[12][x0, x4],
yZ[8][z, t] -> f16[2][x0, x4], yZ[9][z, t] -> f16[7][x0, x4],
yZ[10][z, t] -> f16[10][x0, x4], yZ[11][z, t] -> f16[15][x0, x4],
yZ[12][z, t] -> f16[3][x0, x4], yZ[13][z, t] -> f16[6][x0, x4],
yZ[14][z, t] -> f16[11][x0, x4], yZ[15][z, t] -> f16[14][x0, x4]}

Out[1181]= {f16[0][x0, x4] == yZ[0][6 H x0, H x4], f16[1][x0, x4] == yZ[4][6 H x0, H x4],
f16[2][x0, x4] == yZ[8][6 H x0, H x4], f16[3][x0, x4] == yZ[12][6 H x0, H x4],
f16[4][x0, x4] == yZ[5][6 H x0, H x4], f16[5][x0, x4] == yZ[1][6 H x0, H x4],
f16[6][x0, x4] == yZ[13][6 H x0, H x4], f16[7][x0, x4] == yZ[9][6 H x0, H x4],
f16[8][x0, x4] == yZ[2][6 H x0, H x4], f16[9][x0, x4] == yZ[6][6 H x0, H x4],
f16[10][x0, x4] == yZ[10][6 H x0, H x4], f16[11][x0, x4] == yZ[14][6 H x0, H x4],
f16[12][x0, x4] == yZ[7][6 H x0, H x4], f16[13][x0, x4] == yZ[3][6 H x0, H x4],
f16[14][x0, x4] == yZ[15][6 H x0, H x4], f16[15][x0, x4] == yZ[11][6 H x0, H x4]}
```

Load the Maple output strings. Get is a lexer and parer and ... , so use ReadString

```
In[1]:= SetDirectory[NotebookDirectory[]];
```

$$\begin{aligned} seq3 := \left\{ yZ10(z, t) = \frac{1}{\sqrt{\sin(z)}} \left((c37 \sin(\sqrt{M^2 - 36 C3 - 9} t) \right. \right. \\ \left. \left. + c38 \cos(\sqrt{M^2 - 36 C3 - 9} t) \right) \left(c36 \sin(z) - \frac{\sqrt{1 + 4 C3}}{2} \right. \right. \\ \left. \left. + c35 \sin(z) \frac{\sqrt{1 + 4 C3}}{2} \right) \right), yZ11(z, t) \end{aligned}$$

$$-\cos(\sqrt{M^2 - 36 C4 - 9} t) c43 \left(c42 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}}$$

$$+ c41 \sin(z)^{\frac{\sqrt{1+4C4}}{2}} \right) \sqrt{M^2 - 36 C4 - 9} \Big)$$

$$+ \sqrt{1+4C4} \left(c47 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right.$$

$$+ c48 \cos(\sqrt{M^2 - 36 C4 - 9} t) \Big) \left(c45 \sin(z)^{\frac{\sqrt{1+4C4}}{2}}$$

$$- c46 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}} \Big) \Big) \Big), yZ13(z, t) =$$

$$-\frac{1}{\sqrt{\sin(z)} M} \left(\left(c46 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}}$$

$$+ c45 \sin(z)^{\frac{\sqrt{1+4C4}}{2}} \right) \left(\sin(\sqrt{M^2 - 36 C4 - 9} t) c48$$

$$- \cos(\sqrt{M^2 - 36 C4 - 9} t) c47 \right) \sqrt{M^2 - 36 C4 - 9}$$

$$+ 3 \sqrt{1+4C4} \left(c41 \sin(z)^{\frac{\sqrt{1+4C4}}{2}}$$

$$- c42 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}} \right) \left(c43 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right.$$

$$+ c44 \cos(\sqrt{M^2 - 36 C4 - 9} t) \Big) \Big), yZ14(z, t)$$

$$= \frac{1}{\sqrt{\sin(z)}} \left(\left(c46 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}}$$

$$+ c45 \sin(z)^{\frac{\sqrt{1+4C4}}{2}} \right) \left(c47 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right.$$

$$+ c48 \cos(\sqrt{M^2 - 36 C4 - 9} t) \Big) \Big), yZ15(z, t)$$

$$= \frac{1}{\sqrt{\sin(z)}} \left(\left(c42 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}}$$

$$\left. \left. + c41 \sin(z) \frac{\sqrt{1+4C4}}{2} \right) \left(c43 \sin(\sqrt{M^2 - 36C4 - 9} t) \right. \right. \\ \left. \left. + c44 \cos(\sqrt{M^2 - 36C4 - 9} t) \right) \right)$$

In[1139]:=

```
maplestringEQ3 =
"{{yZ10(z,t)=((c37 sin(sqrt(M^2-36 C3-9) t)+c38 cos(sqrt(M^2-36 C3-9) t)) (c36 (
sin(z))^(-(sqrt(1+4 C3))/2)+c35 (sin(z))^((sqrt(1+4 C3))/2)))/(sqrt(sin(z))) ,
yZ11(z,t)=((c33 sin(sqrt(M^2-36 C3-9) t)+c34 cos(sqrt(M^2-36 C3-9) t)) (c32
(sin(z))^(-(sqrt(1+4 C3))/2)+c31 (sin(z))^((sqrt(1+4 C3))/2)))/(sqrt(sin(z)))
),yZ8(z,t)=-(3 (((cos(sqrt(M^2-36 C3-9) t) c33-sin(sqrt(M^2-36 C3-9) t) c34
(c32 (sin(z))^(-(sqrt(1+4 C3))/2)+c31 (sin(z))^((sqrt(1+4 C3))/2)) sqrt(M^
2-36 C3-9))/3+(c37 sin(sqrt(M^2-36 C3-9) t)+c38 cos(sqrt(M^2-36 C3-9) t))
sqrt(1+4 C3) (c35 (sin(z))^((sqrt(1+4 C3))/2)-c36 (sin(z))^(-(sqrt(1+4 C3))/
2)))/(sqrt(sin(z)) M),yZ9(z,t)=-(c36 (sin(z))^(-(sqrt(1+4 C3))/2)+c35 (sin
(z))^((sqrt(1+4 C3))/2)) (cos(sqrt(M^2-36 C3-9) t) c37-sin(sqrt(M^2-36 C3-9)
t) c38) sqrt(M^2-36 C3-9)+3 sqrt(1+4 C3) (c31 (sin(z))^((sqrt(1+4 C3))/2)-
c32 (sin(z))^(-(sqrt(1+4 C3))/2)) (c33 sin(sqrt(M^2-36 C3-9) t)+c34 cos(sqrt
(M^2-36 C3-9) t)))/(sqrt(sin(z)) M)} }";
```

In[1137]:=

```
maplestringEQ4 =
"{{yZ12(z, t) = -3*((sin(sqrt(M^2 - 36*C4 - 9)*t)*c44 - cos(sqrt(M^2 - 36*C4
- 9)*t)*c43)*(c42*sin(z)^(-sqrt(1 + 4*C4)/2) + c41*sin(z)^(sqrt(1 +
4*C4)/2))*sqrt(M^2 - 36*C4 - 9))/3 + sqrt(1 + 4*C4)*(c47*sin(sqrt(M^2 -
36*C4 - 9)*t) + c48*cos(sqrt(M^2 - 36*C4 - 9)*t))*(c45*sin(z)^(sqrt(1
+ 4*C4)/2) - c46*sin(z)^(-sqrt(1 + 4*C4)/2)))/(sqrt(sin(z))*M),
yZ13(z, t) = -(c46*sin(z)^(-sqrt(1 + 4*C4)/2) + c45*sin(z)^(sqrt(1 +
4*C4)/2))*(sin(sqrt(M^2 - 36*C4 - 9)*t)*c48 - cos(sqrt(M^2 - 36*C4 -
9)*t)*c47)*sqrt(M^2 - 36*C4 - 9) + 3*sqrt(1 + 4*C4)*(c41*sin(z)^(sqrt(1
+ 4*C4)/2) - c42*sin(z)^(-sqrt(1 + 4*C4)/2))*(c43*sin(sqrt(M^2 -
36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4 - 9)*t)))/(sqrt(sin(z))*M),
yZ14(z, t) = (c46*sin(z)^(-sqrt(1 + 4*C4)/2) + c45*sin(z)^(sqrt(1
+ 4*C4)/2))*(c47*sin(sqrt(M^2 - 36*C4 - 9)*t) + c48*cos(sqrt(M^2 -
36*C4 - 9)*t))/sqrt(sin(z)), yZ15(z, t) = (c42*sin(z)^(-sqrt(1
+ 4*C4)/2) + c41*sin(z)^(sqrt(1 + 4*C4)/2))*(c43*sin(sqrt(M^2 -
36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4 - 9)*t))/sqrt(sin(z))}}";
```

```
In[1140]:= solvedEQ3 = ConvertMapleToMathematicaV2[ReleaseHold[maplestringEQ3]]
Out[1140]=
{YZ10[z, t] ==  $\frac{1}{\sqrt{\sin[z]}} \left( C35[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} + C36[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right)$ 
 $\left( c38 \cos[\sqrt{-9 - 36C3 + M^2} t] + c37 \sin[\sqrt{-9 - 36C3 + M^2} t] \right),$ 
YZ11[z, t] ==  $\frac{1}{\sqrt{\sin[z]}} \left( C31[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} + C32[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right)$ 
 $\left( c34 \cos[\sqrt{-9 - 36C3 + M^2} t] + c33 \sin[\sqrt{-9 - 36C3 + M^2} t] \right),$ 
YZ8[z, t] ==
 $- \frac{1}{M \sqrt{\sin[z]}} 3 \left( \frac{1}{3} \sqrt{-9 - 36C3 + M^2} \left( C31[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} + C32[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right)$ 
 $\left( c33 \cos[\sqrt{-9 - 36C3 + M^2} t] - c34 \sin[\sqrt{-9 - 36C3 + M^2} t] \right) +$ 
 $\sqrt{1 + 4C3} \left( C35[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} - C36[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right)$ 
 $\left( c38 \cos[\sqrt{-9 - 36C3 + M^2} t] + c37 \sin[\sqrt{-9 - 36C3 + M^2} t] \right),$ 
YZ9[z, t] ==
 $\frac{1}{M \sqrt{\sin[z]}} \left( -3 \sqrt{1 + 4C3} \left( C31[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} - C32[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right)$ 
 $\left( c34 \cos[\sqrt{-9 - 36C3 + M^2} t] + c33 \sin[\sqrt{-9 - 36C3 + M^2} t] \right) -$ 
 $\sqrt{-9 - 36C3 + M^2} \left( C35[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} + C36[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right)$ 
 $\left( c37 \cos[\sqrt{-9 - 36C3 + M^2} t] - c38 \sin[\sqrt{-9 - 36C3 + M^2} t] \right) \right) \}$ 
```



```
time1 = Now
If[FindFile["solvedEQ2.mx"] === $Failed,
 AbsoluteTiming[solvedEQ2 = ConvertMapleToMathematicaV2[maplestringEQ2]];
 DumpSave[ToString[header <> "solvedEQ2.mx"], solvedEQ2];
 Print[ToString[header <> "solvedEQ2.mx"]];
 , Get[ "solvedEQ2.mx"]]
Now - time1
```