

Scratch work of Author

ideas

Scratch work

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Do not read if you are irascible; I apologize for typos.

git clone <https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git>

“git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git”

Ever wonder why there are 3 space dimensions and 1 time dimension?

Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in \text{Nature}$?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\| = \|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over \mathbb{R}) real numbers \mathbb{R} , the 2-dim complex numbers, the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in “A Remarkable Representation of the 3+2 de Sitter Group” by P.A.M. Dirac, J.Math.Phys.4,901–909 (1963).

Here, Dirac's gamma matrices have been extended into this paper's real τ_8 8×8 and $\tau_{16} = \tau$ 16×16 matrices, in order to simply incorporate and couple the **WAVE FUNCTION OF the UNIVERSE** to Gravity via a term resembling

ψ^\dagger (mass matrix for universe) ψ :

(mass matrix for universe) $\sim (\tau_0 \tau_1 \tau_2 \tau_3)(\tau_5 \tau_6 \tau_7)$,

[$\tau_5 \tau_6 \tau_7$ breaks “**x5, x6, x7**” Lorentz invariance].

Here we use **spacetime coordinates** that are selected so that we are somewhat consistent with the **xact libraries**,

(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):

Cartesian coordinates:

x_0 = hidden space (a small circle); **x1, x2, x3** are the usual 3-space coords;

x4 = time coord,

and **x5, x6, x7** = superluminal **deflating** time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{\text{SO}}(4, 4; \mathbb{R})$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M_8 , whose tangent bundle has

$$\overline{\text{SO}}(4, 4; \mathbb{R}) \approx \text{Spin}(4, 4; \mathbb{R})$$

as iso group, and in which

3 of the 4 space dimensions **superluminally INFLATE**,

3 of the 4 time dimensions **superluminally DEFLATE** (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),

and the 4th space dim, x_0 , curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

TODO: prove that Universe(s) of masses $\pm M$ are created in pairs!

In passing we remark that an analogous construction may be defined on an octonion space of **one time dimension and seven space dimensions**, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group

$$\text{SU}(3) \otimes \text{SU}(2)_L \otimes U(1)_Y;$$

The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

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The Einstein - Lovelock vacuum field equations are

$A_a^b = 0$, where

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n h completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta_{jj_1 \dots j_{2k}}^{hh_1 \dots h_{2k}} R^{j_1 j_2}_{h_1 h_2} \dots R^{j_{2k-1} j_{2k}}_{h_{2k-1} h_{2k}} + \lambda \sqrt{g} g^{lh},$$

where $\alpha_{(k)}$, λ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

and $m-1 = \frac{8}{2} - 1 = 3$.

Citation: [Tensors, Differential Forms, and Variational Principles \(Dover Books on Mathematics\)](#), by David Lovelock and Hanno Rund

Let $\{w_1, w_2, w_3, \Lambda\}$ be pure numbers;

then by Eq. [4.38] the vacuum equations are

$0 = -\Lambda + H^{-2} w_1 \text{ Lovelock1} + H^{-4} w_2 \text{ Lovelock2} + H^{-6} w_3 \text{ Lovelock3}$ in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein - Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre - Hubble parameter. For now, let's pretend that they are independent.

NOTES:

Under the spacetime coordinate transformation

$$x^k \mapsto \bar{x}^k (x^j)$$

according to

$$\Psi_{16}(x) \mapsto \overline{\Psi_{16}(\bar{x})} = S(x(\bar{x})) \cdot \Psi_{16}(x(\bar{x})),$$

$$\text{i.e., } \overline{\Psi_{16}^{(a)}(\overline{x})} = \left(S_{(b)}^{(a)} \Psi_{16}^{(b)} \right) (x(\overline{x}))$$

Therefore,

$$\begin{aligned} \frac{\partial}{\partial \overline{x}^k} \overline{\Psi_{16}(\overline{x})} &= \frac{\partial x^j}{\partial \overline{x}^k} \frac{\partial}{\partial x^j} S(x) \cdot \Psi_{16}(x) = \\ &= \frac{\partial x^j}{\partial \overline{x}^k} \left\{ S(x) \cdot \frac{\partial}{\partial x^j} \Psi_{16}(x) + \left[\frac{\partial}{\partial x^j} S(x) \right] \cdot \Psi_{16}(x) \right\} \end{aligned}$$

Identify the Octad connection :

$$\frac{\partial}{\partial x^j} S(x) =$$

$$- S \cdot \Gamma_j(x) + \frac{\partial \overline{x}^k}{\partial x^j} \overline{\Gamma_k}(\overline{x}) \cdot S$$

hence

$$\begin{aligned} \frac{\partial}{\partial \overline{x}^k} \overline{\Psi_{16}(\overline{x})} - \overline{\Gamma_k} \cdot \overline{\Psi_{16}(\overline{x})} &= \\ S(x(\overline{x})) \cdot \frac{\partial x^j}{\partial \overline{x}^k} \left\{ \frac{\partial}{\partial x^j} \Psi_{16}(x(\overline{x})) - \Gamma_j \cdot \Psi_{16}(x(\overline{x})) \right\} \\ \text{or} \\ \left(1_{16 \times 16} \frac{\partial}{\partial \overline{x}^k} - \overline{\Gamma_k} \right) \cdot \overline{\Psi_{16}(\overline{x})} &= \\ S(x(\overline{x})) \cdot \left[\frac{\partial x^j}{\partial \overline{x}^k} \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^j} - \Gamma_j \right\} \cdot \Psi_{16}(x(\overline{x})) \right] \end{aligned}$$

The connection transforms according to

$$\overline{\Gamma_k}(\overline{x}) = \frac{\partial \overline{x}^j}{\partial x^k} \left\{ S \cdot \Gamma_j(x) \cdot S^{-1} + S_{,j} \cdot S^{-1} \right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2} \left[e_k^{(a)} \nabla_j e_{(b)}^k \right] \text{SAB}^{(b)}_{(a)}$$

Here, spin connection coefficients are $e_k^{(a)} (\nabla_j e_{(b)}^k)$

and

$$\mathbf{S} = \exp \left(\frac{1}{2} \omega_{(a)(b)}(x) \text{SAB}^{(a)(b)} \right),$$

In the spinor Lagrangian $\text{Lg}[]$, below, \mathbf{M} is the mass of the Ψ_{16} field [A.K.A., wave function characterizing the Universe(s) of masses $\pm \mathbf{M}$];

TODO: prove **Universe(s) of masses $\pm M$ are created in pairs!**

K is used to track spin coefficients; $K == 1$; set $K \rightarrow 1$ to employ [total] covariant derivative of spinors; put $K \rightarrow 0$ to ignore

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
 - $\nabla_\mu e^a_\nu = 0$
 - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a_{b\nu}$, and e^a_ν is the tetrad component.

In[*]:= I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^{\omega}_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

SOURCE terms $TU^{\mu\nu}$ for $g_{\alpha\beta}$
 (hope that $TU^{\mu\nu} = \Lambda g^{\mu\nu}$) ;

Universes Ψ_{16} source $g_{\alpha\beta}$;

WARNING: all off-diagonal terms MUST BE ZERO in order for the ‘solutions,’ TODO: find them, to be valid

$Lg[] = \text{LagrangianF16massive}[3 \text{ HM}, 134]$ (please see previous notebook)

$\sqrt{\text{Det}[g_{\mu\nu}]} * \text{LagrangianF16massive}[mASs_ , j_] =$

$\sqrt{\text{Det}[g_{\mu\nu}]} * \left(\text{Transpose}[\Psi_{16}] . \sigma_{16} .$

$\text{Sum}\left[T_{16}^\alpha[\alpha_{1-1}] . \left\{ 1_{16 \times 16} \frac{\partial}{\partial X^{\alpha_{1-1}}} - \Gamma_{\alpha_{1-1}} \right\} . \Psi_{16}, \{\alpha_{1-1}, 1, \text{Length}[X]\}\right] +$

$\frac{mASs}{2} * \text{Transpose}[\Psi_{16}] . \text{symm16}[[134, 1]] . \Psi_{16} \right)$

Let $j = 134$, $\sigma_{16} . (\text{mass Matrix}) = T_{16}^\alpha[5] . T_{16}^\alpha[6] . T_{16}^\alpha[7]$

and

$\text{usingLagrangianF16massive} = (\text{Transpose}[\Psi_{16}] . \sigma_{16} .$

$\text{Sum}[T^\alpha[\alpha_{1-1}] . D[\Psi_{16}, \text{vars}[\alpha_{1-1}]], \{\alpha_{1-1}, 1, \text{Length}[\text{vars}]\}] +$

$mASs * \text{Transpose}[\Psi_{16}] . \text{symm16}[[j = 134, 1]] . \Psi_{16}) ;$

where it is assumed that Ψ_{16} solves the Euler - Lagrange equations, above; then $\text{usingLagrangianF16massive} = 0$, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms
after performing differentiation]
($\text{usingLagrangianF16massive} \equiv$
 $Lg[] |_{\Psi_{16}=\text{solution-toEL-eqs}}$) :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{LagrangianF16massive} \right)$$

$$= \left(\text{LagrangianF16massive} * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \frac{\partial}{\partial g_{\mu\nu}} \left(\text{LagrangianF16massive} \right) \right)$$

| $\text{LagrangianF16massive} = \text{usingLagrangianF16massive}$

$$\begin{aligned}
&= \left(0? * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \right. \\
&\quad \left. \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\text{LagrangianF16massive} \right) \right) \\
&\quad \left| \text{LagrangianF16massive} = \text{usingLagrangianF16massive} \right. \\
&= \frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\Psi16] . \sigma16 . \right. \\
&\quad \text{Sum} \left[\left(D[\Psi16, X[[\alpha1]]] + \left(\left(\frac{1}{2} \right) \text{connectionMatrix} \right) . \right. \right. \\
&\quad \left. \left. \text{Sum}[\omega[[\alpha1, a, b]] * \text{SAB}[[a, b]], \{a, 1, 8\}, \{b, 1, 8\}] . \Psi16 \right) , \right. \\
&\quad \left. \left. \{ \alpha1, 1, \text{Length}[X] \} \right] + \text{mASs} * \text{Transpose}[\Psi16] . \text{symm16}[[j, 1]] . \right. \\
&\quad \left. \Psi16 \right) \left| \text{LagrangianF16massive} = \text{usingLagrangianF16massive} \right. \\
&= \text{Transpose}[\Psi16] . \sigma16 . \text{Sum} \left[\frac{\partial}{\partial g_{\mu\nu}} \left(T16^\alpha[\alpha1 - 1] \right) . \Psi16_{\alpha1-1} , \right. \\
&\quad \left. \{ \alpha1, 1, \text{Length}[X] \} \right] \left| \Psi16 = \text{solution-toEL-eqs} \right. \\
&= \widetilde{\Psi16} . \sigma16 . T16^A . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1\alpha\beta} e_\beta^B \eta_{BA} \right) = \\
&\quad \widetilde{\Psi16} . \sigma16 . T16^A . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(e_{(A)}^\alpha \right)
\end{aligned}$$

NOTES :

Let g be a square matrix; we could use : $\frac{\partial}{\partial q} (g^{-1}) = - g^{-1} . \frac{\partial g}{\partial q} . g^{-1} ,$

where q is a parameter $\left(\text{result from } \frac{\partial}{\partial q} (g . g^{-1}) = 0 \right) ,$

In this notebook, g is usually some 8×8

matrix [with unconstrained elements; call them $g_{\mu\nu}$] ,

g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}} (g^{-1}) = - g^{-1} . \frac{\partial g}{\partial g_{\mu\nu}} . g^{-1} ;$

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In `metric` matrix `g`, we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$, and then differentiate, for example.

This should be understood before reading further.

$$\begin{aligned} \text{Therefore } \frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha) &= \\ \frac{\partial}{\partial g_{\mu\nu}} (\text{Inverse}[e_\alpha^{(A)}]) &= -\text{Inverse}[e_\alpha^{(A)}] \cdot \frac{\partial e_\alpha^{(A)}}{\partial g_{\mu\nu}} \cdot \text{Inverse}[e_\alpha^{(A)}] = \\ &- e_{(A)}^\alpha [[\alpha 1, A1]] \frac{\partial e_\alpha^{(A)} [[A1, \alpha 2]]}{\partial g_{\mu\nu}} \cdot e_{(A)}^\alpha [[\alpha 2, A2]] \end{aligned}$$

$$\begin{aligned} \text{Consider } \frac{\partial}{\partial g_{\mu\nu}} (g_{\alpha\beta}) &= \\ \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^B) &= \eta_{AB} e_\beta^B \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^B \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^A); \\ e_C^\beta \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} &= \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^B) = \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^B e_C^\beta \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^A) \end{aligned}$$

In metric matrix `g`, we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$, and then differentiate.

Some of my calculations employ “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see Cyril Pitrou

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<http://www2.iap.fr/users/pitrou/>

“FriedmannLemaîtreMetric_CoordinatesApproach_xCoba.nb”

`MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify`

Universe sources for $g_{\alpha\beta}$;

be sure to append these to the Einstein and/or Einstein-Lovelock field equations.

WARNING: all off-diagonal terms of all source

terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous 'hacked solutions' for Ψ_{16} to be valid

Let $T^\alpha = \{T_{16}^\alpha[0], T_{16}^\alpha[4]\}$ and

$$\begin{aligned} & \sqrt{\text{Det}[g_{\mu\nu}]} * \text{LagrangianF16massive}[mASs_ , j_] = \\ & \sqrt{\text{Det}[g_{\mu\nu}]} * (\text{Transpose}[\Psi_{16}] . \sigma_{16} . \\ & \quad \text{Sum}[T^\alpha[\alpha_1 - 1] . D[\Psi_{16}, \text{vars}[\alpha_1]], \{\alpha_1, 1, \text{Length}[\text{vars}]\}] + \\ & \quad mASs * \text{Transpose}[\Psi_{16}] . \text{symm}_{16}[[134, 1]] . \Psi_{16}) \end{aligned}$$

Let $j = 134$, $\sigma_{16} . (\text{massMatrix}) = T_{16}^\alpha[5] . T_{16}^\alpha[6] . T_{16}^\alpha[7]$

and

$$\begin{aligned} & \text{usingLagrangianF16massive} = (\text{Transpose}[\Psi_{16}] . \sigma_{16} . \\ & \quad \text{Sum}[T^\alpha[\alpha_1 - 1] . D[\Psi_{16}, \text{vars}[\alpha_1]], \{\alpha_1, 1, \text{Length}[\text{vars}]\}] + \\ & \quad mASs * \text{Transpose}[\Psi_{16}] . \text{symm}_{16}[[j = 134, 1]] . \Psi_{16}) ; \end{aligned}$$

where it is assumed that Ψ_{16} solves the Euler - Lagrange equations, above; then $\text{usingLagrangianF16massive} = 0$, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\begin{aligned} \frac{1}{x} TU^{\mu\nu} & \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{LagrangianF16massive} \right) \\ & = \left(\text{LagrangianF16massive} * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \right. \\ & \quad \left. \frac{\partial}{\partial g_{\mu\nu}} (\text{LagrangianF16massive}) \right) \\ & \quad | \text{LagrangianF16massive} = \text{usingLagrangianF16massive} \\ & = \left(0 * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \right. \\ & \quad \left. \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\text{LagrangianF16massive}) \right) \\ & \quad | \text{LagrangianF16massive} = \text{usingLagrangianF16massive} \\ & \quad (\end{aligned}$$

$$T_{16}^\alpha[A_1 - 1] . \Psi_{16, \alpha_1} \eta_{A_1 B} e_\beta^B g^{-1 \alpha_1 \beta}$$

remark:

the

term

$$\tilde{\Psi_{16}} . \sigma_{16} . T_{16}^A . \Psi_{16, \alpha} \frac{\partial}{\partial g_{\mu\nu}} (e_A^\alpha) = \frac{\partial}{\partial g_{\mu\nu}} \tilde{\Psi_{16}} . \sigma_{16} . T_{16}^A . \Psi_{16, \alpha} e_A^\alpha =$$

$$\frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\Psi_{16}] . \sigma_{16} .$$

$$\text{Sum} \left[T^{\alpha}[\alpha_1 - 1] . D \left[\underbrace{\Psi_{16, \alpha_1} \eta_{A_1 B} e_{\beta}^B g^{-1 \alpha_1 \beta}}_{\text{}} \right], \{\alpha_1, 1, \text{Length}[\text{vars}]\} \right]$$

Frame fields [\[edit\]](#)

We use a set of [vierbein](#) or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab}e_\mu^a e_\nu^b = \eta_{\mu\nu}.$$

The vierbein defines a local rest [frame](#), allowing the constant [Gamma matrices](#) to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a [section](#) of the [frame bundle](#), and so defines a local trivialization of the frame bundle.

Spin connection [\[edit\]](#)

To write down the equation we also need the [spin connection](#), also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e_a^\mu e_\nu^\mu = \delta^\mu_\nu.$$

The connection 1-form is then

$$\omega^\mu_{\nu a} := e_b^\mu \nabla_a e_\nu^b$$

where ∇_a is a [covariant derivative](#), or equivalently a choice of [connection](#) on the frame bundle, most often taken to be the [Levi-Civita connection](#).

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space $T_p M$ to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

[Raising and lowering indices](#) is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract [connection on a principal bundle](#), specifically on the [frame bundle](#), which is defined on any smooth manifold, but which restricts to an [orthonormal frame bundle](#) on pseudo-Riemannian manifolds.

[Incomplete theories](#) [\[show\]](#)
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Covariant derivative for fields in a representation of the Lorentz group [\[edit\]](#)

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu{}_{\nu\rho} = e_\rho^\alpha \omega^\mu{}_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2}(\omega_{\nu\rho})_\mu \sigma^{\nu\rho} \psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu R(M^{\nu\rho})v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $\text{SO}(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

Scratch work

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Do not read if you are irascible; I apologize for typos.

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Since this is the age of the Xtreme, let’s go with the 8-dim octonions. Actually

the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

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 [$\tau_5 \tau_6 \tau_7$ breaks “**x5, x6, x7**” Lorentz invariance].

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x4 = time coord,
 and **x5, x6, x7** = superluminal **deflating** time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{\text{SO}}(4, 4; \mathbb{R})$ (left-spinor, right-spinor and vector), which are equivalent [Cartan’s triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M_8 , whose tangent bundle has $\overline{\text{SO}}(4, 4; \mathbb{R}) \approx \text{Spin}(4, 4; \mathbb{R})$ as iso group, and in which 3 of the 4 space dimensions **superluminally INFLATE**, 3 of the 4 time dimensions **superluminally DEFLATE** (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting), and the 4th space dim, x_0 , curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

TODO: prove that Universe(s) of masses $\pm M$ are created in pairs!

In passing we remark that an analogous construction may be defined on an octonion space of **one time dimension and seven space dimensions**, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group $\text{SU}(3) \otimes \text{SU}(2)_L \otimes U(1)_Y$;

The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

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The Einstein - Lovelock vacuum field equations are

$A_a^b = 0$, where

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta_{jj_1 \dots j_{2k}}^{hh_1 \dots h_{2k}} R^{j_1 j_2}_{h_1 h_2} \dots R^{j_{2k-1} j_{2k}}_{h_{2k-1} h_{2k}} + \lambda \sqrt{g} g^{lh},$$

where $\alpha_{(k)}$, λ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

and $m-1 = \frac{8}{2} - 1 = 3$.

Citation: *Tensors, Differential Forms, and Variational Principles* (Dover Books on Mathematics), by David Lovelock and Hanno Rund

Let $\{w_1, w_2, w_3, \Lambda\}$ be pure numbers;

then by Eq. [4.38] the vacuum equations are

$0 = -\Lambda + H^{-2} w_1 \text{ Lovelock}_1 + H^{-4} w_2 \text{ Lovelock}_2 + H^{-6} w_3 \text{ Lovelock}_3$ in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein - Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre - Hubble parameter. For now, let's pretend that they are independent.

NOTES:

Under the spacetime coordinate transformation

$$x^k \mapsto \bar{x}^k (x^j)$$

according to

$$\Psi_{16}(x) \mapsto \overline{\Psi_{16}(\bar{x})} = S(x(\bar{x})) \cdot \Psi_{16}(x(\bar{x})),$$

$$\text{i.e., } \overline{\Psi_{16}^{(a)}(\overline{x})} = \left(S_{(b)}^{(a)} \Psi_{16}^{(b)} \right) (x(\overline{x}))$$

Therefore,

$$\begin{aligned} \frac{\partial}{\partial \overline{x}^k} \overline{\Psi_{16}(\overline{x})} &= \frac{\partial x^j}{\partial \overline{x}^k} \frac{\partial}{\partial x^j} S(x) \cdot \Psi_{16}(x) = \\ &= \frac{\partial x^j}{\partial \overline{x}^k} \left\{ S(x) \cdot \frac{\partial}{\partial x^j} \Psi_{16}(x) + \left[\frac{\partial}{\partial x^j} S(x) \right] \cdot \Psi_{16}(x) \right\} \end{aligned}$$

Identify the Octad connection :

$$\frac{\partial}{\partial x^j} S(x) =$$

$$- S \cdot \Gamma_j(x) + \frac{\partial \overline{x}^k}{\partial x^j} \overline{\Gamma_k(\overline{x})} \cdot S$$

hence

$$\begin{aligned} \frac{\partial}{\partial \overline{x}^k} \overline{\Psi_{16}(\overline{x})} - \overline{\Gamma_k} \cdot \overline{\Psi_{16}(\overline{x})} &= \\ S(x(\overline{x})) \cdot \frac{\partial x^j}{\partial \overline{x}^k} \left\{ \frac{\partial}{\partial x^j} \Psi_{16}(x(\overline{x})) - \Gamma_j \cdot \Psi_{16}(x(\overline{x})) \right\} \\ \text{or} \\ \left(1_{16 \times 16} \frac{\partial}{\partial \overline{x}^k} - \overline{\Gamma_k} \right) \cdot \overline{\Psi_{16}(\overline{x})} &= \\ S(x(\overline{x})) \cdot \left[\frac{\partial x^j}{\partial \overline{x}^k} \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^j} - \Gamma_j \right\} \cdot \Psi_{16}(x(\overline{x})) \right] \end{aligned}$$

The connection transforms according to

$$\overline{\Gamma_k}(\overline{x}) = \frac{\partial \overline{x}^j}{\partial x^k} \left\{ S \cdot \Gamma_j(x) \cdot S^{-1} + S_{,j} \cdot S^{-1} \right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2} \left[e_k^{(a)} \nabla_j e_{(b)}^k \right] \text{SAB}^{(b)}_{(a)}$$

Here, spin connection coefficients are $e_k^{(a)} (\nabla_j e_{(b)}^k)$

and

$$\mathbf{S} = \exp \left(\frac{1}{2} \omega_{(a)(b)}(x) \text{SAB}^{(a)(b)} \right),$$

In the spinor Lagrangian $\text{Lg}[]$, below, \mathbf{M} is the mass of the Ψ_{16} field [A.K.A., wave function characterizing the Universe(s) of masses $\pm \mathbf{M}$];

TODO: prove **Universe(s) of masses $\pm M$ are created in pairs!**

K is used to track spin coefficients; $K == 1$; set $K \rightarrow 1$ to employ [total] covariant derivative of spinors; put $K \rightarrow 0$ to ignore

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
 - $\nabla_\mu e^a_\nu = 0$
 - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a_{b\nu}$, and e^a_ν is the tetrad component.

In[*]:= I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

SOURCE terms $TU^{\mu\nu}$ for $g_{\alpha\beta}$
 (hope that $TU^{\mu\nu} = \Lambda g^{\mu\nu}$) ;

Universes Ψ_{16} source $g_{\alpha\beta}$;

WARNING: all off-diagonal terms MUST BE ZERO in order for the ‘solutions,’ TODO: find them, to be valid

`Lg[] = LagrangianF16massive[3 HM, 134] (please see previous notebook)`

`$\sqrt{\text{Det}[g_{\mu\nu}]} * \text{LagrangianF16massive}[\text{mASs_}, j_]$ =`

$$\sqrt{\text{Det}[g_{\mu\nu}]} * \left(\text{Transpose}[\Psi_{16}] . \sigma_{16} . \right. \\ \left. \text{Sum} \left[T_{16}^{\alpha}[\alpha_{1-1}] . \left\{ 1_{16 \times 16} \frac{\partial}{\partial X^{\alpha_{1-1}}} - \Gamma_{\alpha_{1-1}} \right\} . \Psi_{16}, \{\alpha_{1-1}, 1, \text{Length}[X]\} \right] + \right. \\ \left. \frac{\text{mASs}}{2} * \text{Transpose}[\Psi_{16}] . \text{symm16}[[134, 1]] . \Psi_{16} \right)$$

`Let j = 134, $\sigma_{16} . (\text{mass Matrix}) = T_{16}^{\alpha}[5] . T_{16}^{\alpha}[6] . T_{16}^{\alpha}[7]$`

`and`

`$\text{usingLagrangianF16massive} = (\text{Transpose}[\Psi_{16}] . \sigma_{16} .$`

`$\text{Sum}[T^{\alpha}[\alpha_{1-1}] . D[\Psi_{16}, \text{vars}[\alpha_{1-1}]], \{\alpha_{1-1}, 1, \text{Length}[\text{vars}]\}] +$`

`$\text{mASs} * \text{Transpose}[\Psi_{16}] . \text{symm16}[[j = 134, 1]] . \Psi_{16}) ;$`

`where it is assumed that Ψ_{16} solves the Euler - Lagrange equations, above; then $\text{usingLagrangianF16massive} = 0$, as shown below.`

SOURCE terms $TU^{\mu\nu}$ [evaluate terms
after performing differentiation]
($\text{usingLagrangianF16massive} \equiv$
 $\text{Lg[]} |_{\Psi_{16}=\text{solution-toEL-eqs}}$) :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{LagrangianF16massive} \right)$$

$$= \left(\text{LagrangianF16massive} * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \right. \\ \left. \frac{\partial}{\partial g_{\mu\nu}} \left(\text{LagrangianF16massive} \right) \right)$$

| $\text{LagrangianF16massive} = \text{usingLagrangianF16massive}$

$$\begin{aligned}
&= \left(0? * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \right. \\
&\quad \left. \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\text{LagrangianF16massive} \right) \right) \\
&\quad \left| \text{LagrangianF16massive} = \text{usingLagrangianF16massive} \right. \\
&= \frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\Psi16] . \sigma16 . \right. \\
&\quad \text{Sum} \left[\left(D[\Psi16, X[[\alpha1]]] + \left(\left(\frac{1}{2} \right) \text{covariantDiffMatrix} \right) . \right. \right. \\
&\quad \left. \left. \text{Sum}[\omega[[\alpha1, a, b]] * \text{SAB}[[a, b]], \{a, 1, 8\}, \{b, 1, 8\}] . \Psi16 \right) , \right. \\
&\quad \left. \{ \alpha1, 1, \text{Length}[X] \} \right] + \text{mASs} * \text{Transpose}[\Psi16] . \text{symm16}[[j, 1]] . \\
&\quad \left. \Psi16 \right) \left| \text{LagrangianF16massive} = \text{usingLagrangianF16massive} \right. \\
&= \text{Transpose}[\Psi16] . \sigma16 . \text{Sum} \left[\frac{\partial}{\partial g_{\mu\nu}} \left(T16^\alpha[\alpha1 - 1] \right) . \Psi16_{\alpha1-1} , \right. \\
&\quad \left. \{ \alpha1, 1, \text{Length}[X] \} \right] \left| \Psi16 = \text{solution-toEL-eqs} \right. \\
&= \widetilde{\Psi16} . \sigma16 . T16^A . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1\alpha\beta} e_\beta^B \eta_{BA} \right) = \\
&\quad \widetilde{\Psi16} . \sigma16 . T16^A . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(e_{(A)}^\alpha \right)
\end{aligned}$$

NOTES :

Let g be a square matrix; we could use : $\frac{\partial}{\partial q} (g^{-1}) = - g^{-1} . \frac{\partial g}{\partial q} . g^{-1} ,$

where q is a parameter $\left(\text{result from } \frac{\partial}{\partial q} (g . g^{-1}) = 0 \right) ,$

In this notebook, g is usually some 8×8

matrix [with unconstrained elements; call them $g_{\mu\nu}$] ,
 g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}} (g^{-1}) = - g^{-1} . \frac{\partial g}{\partial g_{\mu\nu}} . g^{-1} ;$

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In **metric** matrix g , we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$, and then differentiate, for example.

This should be understood before reading further .

$$\begin{aligned} \text{Therefore } \frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha) &= \\ \frac{\partial}{\partial g_{\mu\nu}} (\text{Inverse}[e_\alpha^{(A)}]) &= -\text{Inverse}[e_\alpha^{(A)}] \cdot \frac{\partial e_\alpha^{(A)}}{\partial g_{\mu\nu}} \cdot \text{Inverse}[e_\alpha^{(A)}] = \\ &- e_{(A)}^\alpha [[\alpha 1, A1]] \frac{\partial e_\alpha^{(A)} [[A1, \alpha 2]]}{\partial g_{\mu\nu}} \cdot e_{(A)}^\alpha [[\alpha 2, A2]] \end{aligned}$$

$$\begin{aligned} \text{Consider } \frac{\partial}{\partial g_{\mu\nu}} (g_{\alpha\beta}) &= \\ \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^B) &= \eta_{AB} e_\beta^B \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^B \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^A); \\ e_C^\beta \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} &= \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^B) = \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^B e_C^\beta \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^A) \end{aligned}$$

In metric matrix g , we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$, and then differentiate.

Some of my calculations employ “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see Cyril Pitrou

Institut d’Astrophysique de Paris (France)

pitrou@iap.fr

<http://www2.iap.fr/users/pitrou/>

“FriedmannLemaîtreMetric_CoordinatesApproach_xCoba.nb”

MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify

Universe sources for $g_{\alpha\beta}$;

be sure to append these to the Einstein and/or Einstein-Love-
lock field equations.

WARNING: all off-diagonal terms of all source

**terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in
order for the previous 'hacked solutions' for Ψ_{16} to
be valid**

Let $T^\alpha = \{T_{16}^\alpha[0], T_{16}^\alpha[4]\}$ and

$$\begin{aligned} & \sqrt{\text{Det}[g_{\mu\nu}]} * \text{LagrangianF16massive}[mASs_ , j_] = \\ & \sqrt{\text{Det}[g_{\mu\nu}]} * (\text{Transpose}[\Psi_{16}] . \sigma_{16} . \\ & \quad \text{Sum}[T^\alpha[\alpha_1 - 1] . D[\Psi_{16}, \text{vars}[\alpha_1]], \{\alpha_1, 1, \text{Length}[\text{vars}]\}] + \\ & \quad mASs * \text{Transpose}[\Psi_{16}] . \text{symm16}[[134, 1]] . \Psi_{16}) \end{aligned}$$

Let $j = 134$, $\sigma_{16} . (\text{mass Matrix}) = T_{16}^\alpha[5] . T_{16}^\alpha[6] . T_{16}^\alpha[7]$

and

$$\begin{aligned} & \text{usingLagrangianF16massive} = (\text{Transpose}[\Psi_{16}] . \sigma_{16} . \\ & \quad \text{Sum}[T^\alpha[\alpha_1 - 1] . D[\Psi_{16}, \text{vars}[\alpha_1]], \{\alpha_1, 1, \text{Length}[\text{vars}]\}] + \\ & \quad mASs * \text{Transpose}[\Psi_{16}] . \text{symm16}[[j = 134, 1]] . \Psi_{16}) ; \end{aligned}$$

where it is assumed that Ψ_{16} solves the Euler - Lagrange equations,
above; then $\text{usingLagrangianF16massive} = 0$, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms
after performing differentiation] :

$$\begin{aligned} \frac{1}{x} TU^{\mu\nu} & \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{LagrangianF16massive} \right) \\ & = \left(\text{LagrangianF16massive} * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \right. \\ & \quad \left. \frac{\partial}{\partial g_{\mu\nu}} \left(\text{LagrangianF16massive} \right) \right) \\ & \quad | \text{LagrangianF16massive} = \text{usingLagrangianF16massive} \\ & = \left(0 * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \right. \\ & \quad \left. \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\text{LagrangianF16massive} \right) \right) \\ & \quad | \text{LagrangianF16massive} = \text{usingLagrangianF16massive} \\ & \quad (\end{aligned}$$

$$T_{16}^A[A1-1] . \Psi_{16, \alpha_1} \eta_{A1B} e_B^{\alpha_1} g^{-1 \alpha_1 \beta}$$

remark: the term

$$\tilde{\Psi}_{16} . \sigma_{16} . T_{16}^A . \Psi_{16, \alpha} \frac{\partial}{\partial g_{\mu\nu}} (e_A^\alpha) = \frac{\partial}{\partial g_{\mu\nu}} \tilde{\Psi}_{16} . \sigma_{16} . T_{16}^A . \Psi_{16, \alpha} e_A^\alpha =$$

$$\frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\Psi_{16}] . \sigma_{16} . \right. \\ \left. \text{Sum} \left[T^\alpha[\alpha_1 - 1] . D[\Psi_{16}, \text{vars}[\alpha_1]], \{\alpha_1, 1, \text{Length}[\text{vars}]\} \right] \right.$$

Begin

```
In[*]:= Print["CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]
CopyRight (C) 2022, Patrick L. Nash, under the General Public License.

In[*]:= Print["For simplicity, metric  g_{\alpha\beta} =  g_{\alpha\beta}(x0,x1,x2,x3,x4,x5,x6,x7) =  g_{\alpha\beta}(x0,x4) = diag{
      g_{00}(x0),g_{11}(x0,x4),g_{11}(x0,x4),g_{11}(x0,x4),-1, g_{77}(x0,x4),g_{77}(x0,x4),g_{77}(x0,x4) },"]
For simplicity, metric  g_{\alpha\beta} =  g_{\alpha\beta}(x0,x1,x2,x3,x4,x5,x6,x7) =  g_{\alpha\beta}(x0,x4) = diag{
      g_{00}(x0),g_{11}(x0,x4),g_{11}(x0,x4),g_{11}(x0,x4),-1, g_{77}(x0,x4),g_{77}(x0,x4),g_{77}(x0,x4) },

In[*]:= Print["where g_{77}(x0,x4) = g_{11}(x0,-x4) "]
where g_{77}(x0,x4) = g_{11}(x0,-x4)

In[*]:= Unprotect[dir];

In[*]:= dir = FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]];
Protect[dir];
If[$Failed == SetDirectory[dir], {CreateDirectory[dir], SetDirectory[dir]}];

In[*]:= names = FileNameSplit[NotebookFileName[]]
Out[*]:=
{ , Users, nsh, Documents, 8-dim, 2025-11-04,
  2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix 3.nb}

In[*]:= name = StringReplace[names[[-1]], "nb" -> "mx"]
Out[*]:=
2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix 3.mx
```

```
In[ ]:= header = StringReplace[names[[-1]], ".nb" → "-"]
Out[ ]:=
2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix 3-

In[ ]:= DIM8 = 8;
In[ ]:= Protect[DIM8, M, K, H]
Out[ ]:=
{DIM8, M, K, H}

Coordinates :
{hidden space,  $\underbrace{\hspace{1cm}}$  3 - space,
time,  $\underbrace{\hspace{3cm}}$  superluminal deflating time}
{ $x_0$ ,  $\overbrace{x_1, x_2, x_3}$ ,
 $x_4$ ,  $\overbrace{x_5, x_6, x_7}$ } :
```

```
In[ ]:= X = {x0, x1, x2, x3, x4, x5, x6, x7};
Protect[X];
Protect[x0, x1, x2, x3, x4, x5, x6, x7];
```

```
In[ ]:= sX0 = And @@ Thread[X > 0]
Out[ ]:=
x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 && x7 > 0

In[ ]:= ssX = H > 0 && sX0 && 6 H x0 > 0 && 2 la[x4] > 0 && Cot[6 H x0] > 0 && Sin[6 H x0] > 0
Out[ ]:=
H > 0 && x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 &&
x7 > 0 && 6 H x0[] > 0 && 2 la[x4[]] > 0 && Cot[6 H x0[]] > 0 && Sin[6 H x0[]] > 0
```

For Maple_2025 :

```
In[ ]:= sreplacex = Z[#] → ToExpression["Z" <> ToString[#]] & /@ Range[0, 15]
Out[ ]:=
{Z[0] → Z0, Z[1] → Z1, Z[2] → Z2, Z[3] → Z3, Z[4] → Z4, Z[5] → Z5, Z[6] → Z6, Z[7] → Z7, Z[8] → Z8,
Z[9] → Z9, Z[10] → Z10, Z[11] → Z11, Z[12] → Z12, Z[13] → Z13, Z[14] → Z14, Z[15] → Z15}
```

hacks (a.k.a., lies):

In[]:=

```

constraintTrig = Sin[6 * H * x0] > 0 && Cos[6 * H * x0] > 0 && Csc[6 * H * x0] > 0 &&
  Sec[6 * H * x0] > 0 && Tan[6 * H * x0] > 0 && Cot[6 * H * x0] > 0 && Sin[z] > 0 &&  $\sqrt{\text{Sin}[z]}$  > 0 &&
  Sin[z]3/2 > 0 && Sin[z]1/2 > 0 && Sin[z]-3/2 > 0 && Sin[z]-1/2 > 0 && Cot[z] > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 &&
  Cot[z]3/2 > 0 && Cot[z]1/2 > 0 && Tan[z] > 0 && Sec[z] > 0 &&  $\sqrt{\text{Csc}[z]}$  > 0 && Csc[z] > 0

```

Out[]:=

```

Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
  Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 &&  $\sqrt{\text{Sin}[z]}$  > 0 && Sin[z]3/2 > 0 &&
 $\sqrt{\text{Sin}[z]}$  > 0 &&  $\frac{1}{\text{Sin}[z]^{3/2}}$  > 0 &&  $\frac{1}{\sqrt{\text{Sin}[z]}}$  > 0 && Cot[z] > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 &&
  Cot[z]3/2 > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 && Tan[z] > 0 && Sec[z] > 0 &&  $\sqrt{\text{Csc}[z]}$  > 0 && Csc[z] > 0

```

more hacks (damn lies):

In[]:=

```

(*&&K2-M2>0&& $\sqrt{K^2-M^2}$ >0&&e2 H  $\sqrt{K^2-M^2}$  x4>0&&2 H  $\sqrt{K^2-M^2}$  x4>0&&
  2 H  $\sqrt{1-M^2}$  x4>0&& $\sqrt{1-M^2}$ >0&&e2 H  $\sqrt{1-M^2}$  x4>0&&2 H  $\sqrt{1-M^2}$  x4>0*)

```

In[]:=

```

constraintX =
  x0 > 0 && x4 > 0 && H > 0 && 6 * H * x0 > 0 && 3 * H * x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
  z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 && ea4[H x4] > 0

```

Out[]:=

```

x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
  z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 && ea4[H x4] > 0

```

statistics :

In[]:=

```

constraintVars = constraintX && constraintTrig

```

Out[]:=

```

x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
  Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
  ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
  Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 &&  $\sqrt{\text{Sin}[z]}$  > 0 && Sin[z]3/2 > 0 &&
 $\sqrt{\text{Sin}[z]}$  > 0 &&  $\frac{1}{\text{Sin}[z]^{3/2}}$  > 0 &&  $\frac{1}{\sqrt{\text{Sin}[z]}}$  > 0 && Cot[z] > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 &&
  Cot[z]3/2 > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 && Tan[z] > 0 && Sec[z] > 0 &&  $\sqrt{\text{Csc}[z]}$  > 0 && Csc[z] > 0

```



```
In[*]:= subsDefects =
{
 $\sqrt{e^{2 a4 [H x4]}} \rightarrow e^{a4 [H x4]}$ ,  $\sqrt{e^{-2 a4 [H x4]}} \rightarrow e^{-a4 [H x4]}$ ,  $\sqrt{e^{2 a4 [H x4]} \text{Sin}[6 H x4]^{1/3}} \rightarrow e^{a4 [H x4]} \text{Sin}[6 H x4]^{1/6}$ ,
 $\frac{1}{\sqrt{e^{2 a4 [H x4]} \text{Sin}[6 H x4]^{1/3}}} \rightarrow \frac{1}{e^{a4 [H x4]} \text{Sin}[6 H x4]^{1/6}}$ ,
 $\frac{1}{\sqrt{e^{2 a4 [H x4]} \text{Sin}[6 H x0]^{1/6}}} \rightarrow \frac{1}{e^{a4 [H x4]} \text{Sin}[6 H x0]^{1/6}}$ ,
 $\frac{1}{\sqrt{e^{-2 a4 [H x4]} \text{Sin}[6 H x0]^{1/6}}} \rightarrow \frac{1}{e^{-a4 [H x4]} \text{Sin}[6 H x0]^{1/6}}$ ,  $\sqrt{e^{2 a4 [H x4]} \text{Sin}[6 H x0]^{1/6}} \rightarrow$ 
 $e^{a4 [H x4]} \text{Sin}[6 H x0]^{1/6}$ ,  $\sqrt{e^{-2 a4 [H x4]} \text{Sin}[6 H x0]^{1/6}} \rightarrow e^{-a4 [H x4]} \text{Sin}[6 H x0]^{1/6}$ 
}
```

```
Out[*]=
{
 $\sqrt{e^{2 a4 [H x4]}} \rightarrow e^{a4 [H x4]}$ ,  $\sqrt{e^{-2 a4 [H x4]}} \rightarrow e^{-a4 [H x4]}$ ,  $\sqrt{e^{2 a4 [H x4]} \text{Sin}[6 H x4]^{1/3}} \rightarrow e^{a4 [H x4]} \text{Sin}[6 H x4]^{1/6}$ ,
 $\frac{1}{\sqrt{e^{2 a4 [H x4]} \text{Sin}[6 H x4]^{1/3}}} \rightarrow \frac{e^{-a4 [H x4]}}{\text{Sin}[6 H x4]^{1/6}}$ ,  $\frac{1}{\sqrt{e^{2 a4 [H x4]} \text{Sin}[6 H x0]^{1/6}}} \rightarrow \frac{e^{-a4 [H x4]}}{\text{Sin}[6 H x0]^{1/6}}$ ,
 $\frac{1}{\sqrt{e^{-2 a4 [H x4]} \text{Sin}[6 H x0]^{1/6}}} \rightarrow \frac{e^{a4 [H x4]}}{\text{Sin}[6 H x0]^{1/6}}$ ,  $\sqrt{e^{2 a4 [H x4]} \text{Sin}[6 H x0]^{1/6}} \rightarrow e^{a4 [H x4]} \text{Sin}[6 H x0]^{1/6}$ ,
 $\sqrt{e^{-2 a4 [H x4]} \text{Sin}[6 H x0]^{1/6}} \rightarrow e^{-a4 [H x4]} \text{Sin}[6 H x0]^{1/6}$ 
}
```

future coordinate transformation :

```
In[*]:= 6 H x0 == z && H x4 == t
szt = Solve[%, {z, t}] [[1]]
sx0x4 = Solve[%%, {x0, x4}] [[1]]
Protect[sx0x4, szt];
```

```
Out[*]=
6 H x0 == z && H x4 == t
```

```
Out[*]=
{z → 6 H x0, t → H x4}
```

```
Out[*]=
{
 $x0 \rightarrow \frac{z}{6 H}$ ,  $x4 \rightarrow \frac{t}{H}$ 
}
```

```
In[*]:= sx0x4
```

```
Out[*]=
{
 $x0 \rightarrow \frac{z}{6 H}$ ,  $x4 \rightarrow \frac{t}{H}$ 
}
```

```
In[*]:= (*sa4={a4→((A4[3 H #2])&)}*)
```

```
In[*]:= Protect[sx0x4, szt]
```

```
Out[*]=
{}
```

S0 (4, 4) Minkowski Lorentz metric η_{4488} : 4 + 4 spacetime;
8 × 8 dimensional :

```
In[*]:= (η4488 = ArrayFlatten[{{IdentityMatrix[4], 0}, {0, -IdentityMatrix[4]}}]) // MatrixForm
Out[*]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```

$SO(4, 4)$ Spinor Lorentz metric σ ;

τ^A are analogous to the Dirac gamma matrices :
all true :

$$\text{Table}\left[\left\{A, B, \text{FullSimplify}\left[\frac{1}{2} (\tau[A] \cdot \bar{\tau}[B] + \tau[B] \cdot \bar{\tau}[A]) = \eta_{4488}[[A + 1, B + 1]] * \text{ID8}\right]\right\}, \{A, 0, 7\}, \{B, 0, 7\}\right]$$

Type - 1 and type - 2 employ σ (the usual notational abuse),
since $\sigma = \sigma^{-1}$: 4 + 4 spacetime,

8 × 8 dimensional and 16 × 16 dimensional :

indices. We define the reduced Brauer-Weyl generators $\{\bar{\tau}^{A'}, \tau^{A'}\}$ of the generators of the two real 8 × 8 inequivalent irreducible representations of $SO(4, 4)$ (see eqs. (2.6) and (2.7)) by demanding that the tau matrices satisfy (the tilde denotes transpose)

$$(2.3) \quad \tilde{\tau}^{A'} \sigma = \sigma \bar{\tau}^{A'}$$

and

$$(2.4) \quad \tau^{A'} \bar{\tau}^{B'} + \tau^{B'} \bar{\tau}^{A'} = 2IG^{A'B'} = \bar{\tau}^{A'} \tau^{B'} + \bar{\tau}^{B'} \tau^{A'},$$

where I denotes the 8 × 8 unit matrix. Denoting the matrix elements of $\tau^{A'}$ by $\tau^{A'a}_b$, we may write eq. (3) as

$$(2.5) \quad \bar{\tau}^{A'}_{ab} = \tau^{A'}_{ba},$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix}$$

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \rightarrow SO(4,4)$ is given by

$$(2.8) \quad 8L^{A'}_{B'}(g) = \text{tr} \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}_{C'} G_{A'B'} L^{B'}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}_{B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

```
In[*]:= (σ = ArrayFlatten[{{ConstantArray[0, {4, 4}], IdentityMatrix[4]},
{IdentityMatrix[4], ConstantArray[0, {4, 4}]}}]) // MatrixForm
```

```
Out[*] // MatrixForm =
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

below :

```
In[*]:= σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

below :

```
In[*]:= Table[T16^A[A1] = ArrayFlatten[{{0,  $\bar{\tau}$ [A1]}, { $\tau$ [A1], 0}},
      {A1, 0, 7}];
```

below :

```
In[*]:=  $\sigma$ 16.T16^A[#] == -Transpose[ $\sigma$ 16.T16^A[#]] & /@ Range[0, 7]
```

```
Out[*]= {True, True, True, True, True, True, True, True}
```

below :

```
In[*]:= Table[{A1, B1}, FullSimplify[
      ExpandAll[ $\frac{1}{2}$  ( T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1] ) ==
       $\eta$ 4488[[A1 + 1, B1 + 1]] * ID16 ]],
      {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

```
Out[*]//MatrixForm=
```

$\begin{pmatrix} \{0, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{1, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{2, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{3, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{4, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{5, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{6, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{7, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 7\} \\ \text{True} \end{pmatrix}$

defs; some Symbols; metric

?todo: NEED DefScalarFunction[#]; & /@ (f16[#]&/@Range[0,15]), and allow xact to compute?

Introduce the wave function, Ψ_{16} , for a Universe:

```
In[*]:=  $\Psi_{16} = f_{16}[\#][x_0, x_4] \& /@ \text{Range}[0, 15]$ 
```

```
Out[*]:=
```

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

```
In[*]:=
```

```
Clear[sf16Aa];
sf $\psi$ 16Aa = f16[#] → ToExpression[
  "((Z[" <> ToString[#] <> "][6*H*#1, H*#2]  $\frac{1}{\text{Sin}[6*H*#1]^{1/2}}$  )&)" ] & /@ Range[0, 15]
```

```
Out[*]:=
```

```
{f16[0] →  $\left( \frac{Z[0][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ , f16[1] →  $\left( \frac{Z[1][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ ,
 f16[2] →  $\left( \frac{Z[2][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ , f16[3] →  $\left( \frac{Z[3][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ ,
 f16[4] →  $\left( \frac{Z[4][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ , f16[5] →  $\left( \frac{Z[5][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ ,
 f16[6] →  $\left( \frac{Z[6][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ , f16[7] →  $\left( \frac{Z[7][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ ,
 f16[8] →  $\left( \frac{Z[8][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ , f16[9] →  $\left( \frac{Z[9][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ ,
 f16[10] →  $\left( \frac{Z[10][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ , f16[11] →  $\left( \frac{Z[11][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ ,
 f16[12] →  $\left( \frac{Z[12][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ , f16[13] →  $\left( \frac{Z[13][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ ,
 f16[14] →  $\left( \frac{Z[14][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ , f16[15] →  $\left( \frac{Z[15][6H\#1, H\#2]}{\sqrt{\text{Sin}[6H\#1]}} \& \right)$ }
```

0 (4, 4) related :

```
In[*]:= Needs["Notation`"]
```

```
In[*]:= Symbolize[  $T_{16}^A$  ]
```

```
In[*]:= Symbolize[  $T_{16}^\alpha$  ]
```

NEED DefScalarFunction[#]; & /@ Flatten[Universe]

```
In[*]:= Universe = ToExpression["U" <> ToString[#]] & /@ Range[0, 7]
```

```
In[*]:= Symbolize[  $F_a^A$  ]
```

```
In[*]:= Symbolize[  $F_A^a$  ]
```

```
In[*]:= Symbolize[gtrye $^{(A)}_\alpha$ ]
```

```
In[ ]:= Symbolize[gtryeα(A)]
```

SPACETIME METRIC

```
In[ ]:= (*einsteinMetric=Array[g[#1-1,#2-1]&,{8,8}];*)
```

SPACETIME METRIC:

```
In[ ]:= g4488 = Array[g[#1 - 1] [#2 - 1] [x0, x4] &, {8, 8}];
```

SPACETIME METRIC values:

MatrixMetric44

$6 H x0 = z \ \&\& \ H x4 = t$

$$\left\{ \left\{ a4''[t] \rightarrow 0, a4'[t] \rightarrow \frac{2}{3} (-1 + M) \right\}, \left\{ a4''[t] \rightarrow 0, a4'[t] \rightarrow \frac{2 (1 + M)}{3} \right\} \right\}$$

```
In[ ]:= (*β3=Exp[2 H x4 Sqrt[K2-M2]];*)
```

```
(*β3=Exp[2 * a4[3*H*x4]];*)
```

```
β3 = Exp[2 * a4[H * x4]] (* /. {a4 -> (((K1 * (2 (1+M))/3 + K2 * (2/3) (-1+M))) #) &}) *)
```

```
β1 = Sin[6 * H * x0]1/3;
```

```
β2 = Cot[6 * H * x0]2;
```

```
Out[ ]:=
```

$e^{2 a4 [H x4]}$

```
In[ ]:= MatrixForm[MatrixMetric44 =
  { {β2, 0, 0, 0, 0, 0, 0, 0}, {0, β1 β3, 0, 0, 0, 0, 0, 0}, {0, 0, β1 β3, 0, 0, 0, 0, 0},
    {0, 0, 0, β1 β3, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -β1/β3, 0, 0}, {0, 0,
    0, 0, 0, -β1/β3, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -β1/β3} } // FullSimplify[#, constraintVars] &]
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \cot[6 H x0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2 a4 [H x4]} \sin[6 H x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2 a4 [H x4]} \sin[6 H x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2 a4 [H x4]} \sin[6 H x0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -e^{-2 a4 [H x4]} \sin[6 H x0]^{1/3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= (*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"] [6*H*#1,3*H*#2] 1/Sin[6*H*#1]^(1/2) )&)" ]&/@
Range[0,15]*)
```

```
In[ ]:= (*Clear[sf16Aa];
sfψ16Aa=
f16[#]→ToExpression["((Z["<>ToString[#]<>"] [6*H*#1,3*H*#2] )&)" ]&/@Range[0,15]*)
```

```
In[*]:= (*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression["(Z["<>ToString[#]<>"] [6*H*#1,H*#2]  $\frac{1}{\sin[6*H*#1]^{1/2}}$  )&)" ]&/@
Range[0,15]*)
```

```
In[*]:= (*Inactive[(q)&] Evaluate[Activate[Evaluate[q]]] *)
```

```
In[*]:= Clear[makeSubstitution];
makeSubstitution[] := Module[{met, mat, t, f, p},
  met = Flatten[g4488][[All, 0]];
  mat = Flatten[MatrixMetric44];
  t = Thread[met → mat];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]};
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[(q) &];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]};
  Return[p];
]
```

```
In[*]:= makeSubstitution[a_, b_] := Module[{t, f, p},
  t = Thread[a → b];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]};
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[(q) &];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]};
  Return[p];
]
```

```
In[*]:= ssgm4488 = makeSubstitution[]
```

```
Out[*]=
```

```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
g[1][1] → (e^{2 a4 [H #2]} Sin[6 H #1]^{1/3} &), g[1][2] → (0 &), g[1][3] → (0 &),
g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
g[2][1] → (0 &), g[2][2] → (e^{2 a4 [H #2]} Sin[6 H #1]^{1/3} &), g[2][3] → (0 &), g[2][4] → (0 &),
g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
g[3][2] → (0 &), g[3][3] → (e^{2 a4 [H #2]} Sin[6 H #1]^{1/3} &), g[3][4] → (0 &), g[3][5] → (0 &),
g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
g[5][5] → (-e^{-2 a4 [H #2]} Sin[6 H #1]^{1/3} &), g[5][6] → (0 &), g[5][7] → (0 &),
g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
g[6][5] → (0 &), g[6][6] → (-e^{-2 a4 [H #2]} Sin[6 H #1]^{1/3} &), g[6][7] → (0 &),
g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^{-2 a4 [H #2]} Sin[6 H #1]^{1/3} &) }
```

```
In[*]:= Symbolize[E_{\alpha}^{(A)}]
```

```
In[*]:= Symbolize[e_{\alpha}^{(A)}]
```

```
In[*]:= e_{\alpha}^{(A)} = Array[E_{\alpha}^{(A)}[{#1 - 1, #2 - 1}] &, {8, 8}]
```

```
Out[*]=
```

```
{ {E_{\alpha}^{(A)}[0, 0], E_{\alpha}^{(A)}[0, 1], E_{\alpha}^{(A)}[0, 2], E_{\alpha}^{(A)}[0, 3], E_{\alpha}^{(A)}[0, 4], E_{\alpha}^{(A)}[0, 5], E_{\alpha}^{(A)}[0, 6], E_{\alpha}^{(A)}[0, 7]},
{E_{\alpha}^{(A)}[1, 0], E_{\alpha}^{(A)}[1, 1], E_{\alpha}^{(A)}[1, 2], E_{\alpha}^{(A)}[1, 3], E_{\alpha}^{(A)}[1, 4], E_{\alpha}^{(A)}[1, 5], E_{\alpha}^{(A)}[1, 6], E_{\alpha}^{(A)}[1, 7]},
{E_{\alpha}^{(A)}[2, 0], E_{\alpha}^{(A)}[2, 1], E_{\alpha}^{(A)}[2, 2], E_{\alpha}^{(A)}[2, 3], E_{\alpha}^{(A)}[2, 4], E_{\alpha}^{(A)}[2, 5], E_{\alpha}^{(A)}[2, 6], E_{\alpha}^{(A)}[2, 7]},
{E_{\alpha}^{(A)}[3, 0], E_{\alpha}^{(A)}[3, 1], E_{\alpha}^{(A)}[3, 2], E_{\alpha}^{(A)}[3, 3], E_{\alpha}^{(A)}[3, 4], E_{\alpha}^{(A)}[3, 5], E_{\alpha}^{(A)}[3, 6], E_{\alpha}^{(A)}[3, 7]},
{E_{\alpha}^{(A)}[4, 0], E_{\alpha}^{(A)}[4, 1], E_{\alpha}^{(A)}[4, 2], E_{\alpha}^{(A)}[4, 3], E_{\alpha}^{(A)}[4, 4], E_{\alpha}^{(A)}[4, 5], E_{\alpha}^{(A)}[4, 6], E_{\alpha}^{(A)}[4, 7]},
{E_{\alpha}^{(A)}[5, 0], E_{\alpha}^{(A)}[5, 1], E_{\alpha}^{(A)}[5, 2], E_{\alpha}^{(A)}[5, 3], E_{\alpha}^{(A)}[5, 4], E_{\alpha}^{(A)}[5, 5], E_{\alpha}^{(A)}[5, 6], E_{\alpha}^{(A)}[5, 7]},
{E_{\alpha}^{(A)}[6, 0], E_{\alpha}^{(A)}[6, 1], E_{\alpha}^{(A)}[6, 2], E_{\alpha}^{(A)}[6, 3], E_{\alpha}^{(A)}[6, 4], E_{\alpha}^{(A)}[6, 5], E_{\alpha}^{(A)}[6, 6], E_{\alpha}^{(A)}[6, 7]},
{E_{\alpha}^{(A)}[7, 0], E_{\alpha}^{(A)}[7, 1], E_{\alpha}^{(A)}[7, 2], E_{\alpha}^{(A)}[7, 3], E_{\alpha}^{(A)}[7, 4], E_{\alpha}^{(A)}[7, 5], E_{\alpha}^{(A)}[7, 6], E_{\alpha}^{(A)}[7, 7]} }
```

```
In[*]:= (*Symbolize[se_{\alpha}^{(A)}]*)
```

```
In[*]:= Symbolize[sge_{\alpha}^{(A)}]
```

```
In[*]:= Symbolize[sgtrye_{\alpha}^{(A)}]
```

```
In[*]:= Symbolize[E_{(A)}^{\alpha}]
```

```
In[*]:= Symbolize[sge_{(A)}^{\alpha}]
```

```
In[*]:= Symbolize[sgtrye_{(A)}^{\alpha}]
```

```
In[*]:= Symbolize[e_{(A)}^{\alpha}]
```



```

In[*]:= e(A)α = Array[E(A)α [#1 - 1, #2 - 1] &, {8, 8}]
Out[*]=
{ {E(A)α [0, 0], E(A)α [0, 1], E(A)α [0, 2], E(A)α [0, 3], E(A)α [0, 4], E(A)α [0, 5], E(A)α [0, 6], E(A)α [0, 7]},
  {E(A)α [1, 0], E(A)α [1, 1], E(A)α [1, 2], E(A)α [1, 3], E(A)α [1, 4], E(A)α [1, 5], E(A)α [1, 6], E(A)α [1, 7]},
  {E(A)α [2, 0], E(A)α [2, 1], E(A)α [2, 2], E(A)α [2, 3], E(A)α [2, 4], E(A)α [2, 5], E(A)α [2, 6], E(A)α [2, 7]},
  {E(A)α [3, 0], E(A)α [3, 1], E(A)α [3, 2], E(A)α [3, 3], E(A)α [3, 4], E(A)α [3, 5], E(A)α [3, 6], E(A)α [3, 7]},
  {E(A)α [4, 0], E(A)α [4, 1], E(A)α [4, 2], E(A)α [4, 3], E(A)α [4, 4], E(A)α [4, 5], E(A)α [4, 6], E(A)α [4, 7]},
  {E(A)α [5, 0], E(A)α [5, 1], E(A)α [5, 2], E(A)α [5, 3], E(A)α [5, 4], E(A)α [5, 5], E(A)α [5, 6], E(A)α [5, 7]},
  {E(A)α [6, 0], E(A)α [6, 1], E(A)α [6, 2], E(A)α [6, 3], E(A)α [6, 4], E(A)α [6, 5], E(A)α [6, 6], E(A)α [6, 7]},
  {E(A)α [7, 0], E(A)α [7, 1], E(A)α [7, 2], E(A)α [7, 3], E(A)α [7, 4], E(A)α [7, 5], E(A)α [7, 6], E(A)α [7, 7]} }

In[*]:= (*gtryeα(A) = (eα(A) /. sgtryeα(A)) *)
In[*]:= (*gtrye(A)α = (e(A)α /. sgtrye(A)α) *)
In[*]:= (*Symbolize[Γα βκ] *)
In[*]:= (*preSpinConnection=Array[Γα βκ [##] &, {8, 8, 8}]; *)
In[*]:= (*Symbolize[EaA] *)
In[*]:= (*Symbolize[EAa] *)
In[*]:= (*FaA FAa *)

In[*]:= (*Do[FaA = EaA [h] = Table[(u[[h]].σ.(τ[B])), {B, 1, 8}], {h, 1, Length[u]}];
Do[EAa [h] = FullSimplify[Inverse[EaA [h]]], {h, 1, Length[u]}];
Table[FAa = EAa [h] == (u[[h]].σ.u[[h]])
Transpose[Table[FullSimplify[ExpandAll[η8[B, B] τ[B].u[[h]]],
{B, 1, 8}]], {h, 1, Length[u]}] *)

In[*]:= (*Table[FullSimplify[ExpandAll[Transpose[EaA [h]].η8.EaA [h]]] -
(u[[h]].σ.u[[h]) σ == Zero, {h, 1, Length[u]}] *)
In[*]:= (*iη8 = FullSimplify[Inverse[η8]]; *)
In[*]:= (*Clear[ωμ (b)(a)]; *) Clear[ω]; Symbolize[ωμ (b)(a)]
In[*]:= ω = Array[ωμ (b)(a) [##] &, {8, 8, 8}];
Protect[ω]
Out[*]=
{ω}

```

constants

```

In[*]:= (*ArrayFlatten[{IdentityMatrix[4], 0}, {0, -IdentityMatrix[4]}] *)

```

```

In[ ]:= ID4 = IdentityMatrix[4];
        ID8 = IdentityMatrix[8];

In[ ]:=  $\eta$ 4488 // MatrixForm

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


In[ ]:=  $\eta$ 4488[[#, #]] & /@ Range[8]

Out[ ]:=
{1, 1, 1, 1, -1, -1, -1, -1}

In[ ]:=
Zero4 = ConstantArray[0, {4, 4}]; Zero8 = ConstantArray[0, {8, 8}];
mid = ExpandAll[-1 * ID4];
 $\epsilon$ 3 = Array[Signature[{{##}}] &, {3, 3, 3}]
 $\epsilon$ 4 = Array[Signature[{{##}}] &, {4, 4, 4, 4}];
MId = ExpandAll[-1 * ID8];
simp = {Zero4  $\rightarrow$  0, ID4  $\rightarrow$  1, mid  $\rightarrow$  -1};
Simp = {Zero8  $\rightarrow$  ZERO, ID8  $\rightarrow$  ONE, MId  $\rightarrow$  MONE};

Out[ ]:=
{{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}},
 {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}}, {{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}

In[ ]:= Zero16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{ $\eta$ 8, 0}, {0, - $\eta$ 8}}];
Id16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * Id16];
Simp16 = {Zero16  $\rightarrow$  ZERO16, Id16  $\rightarrow$  ONE16, MId16  $\rightarrow$  MONE16};

In[ ]:= ZERO16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{ $\eta$ 4488, 0}, {0, - $\eta$ 4488}}];
ID16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * ID16];
Simp16 = {ZERO16  $\rightarrow$  ZERO16, ID16  $\rightarrow$  ONE16, MId16  $\rightarrow$  MONE16};

In[ ]:=  $\omega$ s = Flatten[
  Table[ToExpression[" $\omega$ " <> ToString[A1] <> ToString[B1]], {A1, 1, 7}, {B1, A1 + 1, 8}]
Length[ $\omega$ s]

Out[ ]:=
{ $\omega$ 12,  $\omega$ 13,  $\omega$ 14,  $\omega$ 15,  $\omega$ 16,  $\omega$ 17,  $\omega$ 18,  $\omega$ 23,  $\omega$ 24,  $\omega$ 25,  $\omega$ 26,  $\omega$ 27,  $\omega$ 28,
  $\omega$ 34,  $\omega$ 35,  $\omega$ 36,  $\omega$ 37,  $\omega$ 38,  $\omega$ 45,  $\omega$ 46,  $\omega$ 47,  $\omega$ 48,  $\omega$ 56,  $\omega$ 57,  $\omega$ 58,  $\omega$ 67,  $\omega$ 68,  $\omega$ 78}

Out[ ]:=
28

```

```

In[*]:=
 $\sigma.\sigma - \text{ID8} == \text{Zero8}$ 
Tr[ $\sigma$ ] == 0

Out[*]:=
True

Out[*]:=
True

```

some function definitions :

```

In[*]:= eextract[a_] := Extract[a, {#}] & /@ Range[0, Length[a]]

In[*]:= imes[a_, b_] := {a, b}

In[*]:= imes[a_, b_, c_] := {a, b, c}

In[*]:= iid[a_, b_] := a

In[*]:= iid[a_, b_, c_] := {a, b}

In[*]:= iid[a_List] := a[[1]]

In[*]:= qid[a_] := a

In[*]:= skelx[x_, matQ_ : True] := Module[{t = {666}},
  If[matQ == True, t = Block[{r},
    r = x;
    Table[Block[{q}, q = SameQ[0, r[[j, i]]];
      If[True == q, Style[0, Blue], Style[Length[r[[j, i]]], Red]]],
      {j, 1, Length[r]}, {i, 1, Length[r[[1]]}]]];
  If[matQ == False, t = Block[{r},
    r = x;
    Table[Block[{q}, q = SameQ[0, r[[j]]];
      If[True == q, Style[0, {RGBColor -> {0, 1/3, 0}, Bold}], Style[Length[r[[j]]], Red]]],
      {j, 1, Length[r]}]]];
  MatrixForm[t, TableAlignments -> Left]]

In[*]:= makeSym[size_, fn_] :=
  Module[{rtmp}, rtmp = Table[fn[i, j], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[rtmp, {{2}, {1}}]}]]

In[*]:= makeAntiSym[size_, fn_] :=
  Module[{rtmp}, rtmp = Table[If[i == j, 0, fn[i, j]], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[-rtmp, {{2}, {1}}]}]]

```

```

In[*]:= Block[{MX}, MX = makeAntiSym[8, Subscript[ωω, ##] &]]
Out[*]=
{ {0, -ωω2,1, -ωω3,1, -ωω4,1, -ωω5,1, -ωω6,1, -ωω7,1, -ωω8,1},
  {ωω2,1, 0, -ωω3,2, -ωω4,2, -ωω5,2, -ωω6,2, -ωω7,2, -ωω8,2},
  {ωω3,1, ωω3,2, 0, -ωω4,3, -ωω5,3, -ωω6,3, -ωω7,3, -ωω8,3},
  {ωω4,1, ωω4,2, ωω4,3, 0, -ωω5,4, -ωω6,4, -ωω7,4, -ωω8,4},
  {ωω5,1, ωω5,2, ωω5,3, ωω5,4, 0, -ωω6,5, -ωω7,5, -ωω8,5},
  {ωω6,1, ωω6,2, ωω6,3, ωω6,4, ωω6,5, 0, -ωω7,6, -ωω8,6},
  {ωω7,1, ωω7,2, ωω7,3, ωω7,4, ωω7,5, ωω7,6, 0, -ωω8,7},
  {ωω8,1, ωω8,2, ωω8,3, ωω8,4, ωω8,5, ωω8,6, ωω8,7, 0} }

```

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2} \left(\frac{\partial g_{\mu\alpha}}{\partial x^\beta} + \frac{\partial g_{\mu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\mu} \right),$$

$$\Gamma^\mu_{\alpha\beta} = g^{\mu\nu} \Gamma_{\nu\alpha\beta},$$

$$R^\mu_{\nu\alpha\beta} = \frac{\partial \Gamma^\mu_{\nu\beta}}{\partial x^\alpha} - \frac{\partial \Gamma^\mu_{\nu\alpha}}{\partial x^\beta} + \Gamma^\mu_{\rho\alpha} \Gamma^\rho_{\nu\beta} - \Gamma^\mu_{\rho\beta} \Gamma^\rho_{\nu\alpha}.$$

$$R_{\mu\nu} = R^\alpha_{\mu\alpha\nu}, \quad R = g^{\mu\nu} R_{\mu\nu},$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

```

In[*]:= (*Christoffel, Einstein tensor, ...*)
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, r, Ricci, Rier, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    r = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
      (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}],
      {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
    Ricci = ParallelTable[FullSimplify[D[r[[μ, ν, β]], X[[α]]] - D[r[[μ, ν, α]], X[[β]]] +
      Sum[r[[μ, s, α]] * r[[s, ν, β]] - r[[μ, s, β]] * r[[s, ν, α]], {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}];
    Rier = ParallelTable[FullSimplify[Sum[Ricci[[α, μ, α, ν]], {α, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}];
    RS = FullSimplify[Tr[ginv.Rier], ass];
    G = ParallelTable[FullSimplify[Rier[[α]] - 1/2 g[[α]] RS, ass], {α, 1, Length[Rier]}];];
  Print[t];
  Print[Now];
  Return[{ginv, r, Ricci, Rier, RS, G}]

```

```

In[*]:= gtry = MatrixMetric44

```

```

Out[*]:= {{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^(2 a4 [H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0},
  {0, 0, e^(2 a4 [H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^(2 a4 [H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^(-2 a4 [H x4]) Sin[6 H x0]^(1/3), 0, 0},
  {0, 0, 0, 0, 0, 0, -e^(-2 a4 [H x4]) Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -e^(-2 a4 [H x4]) Sin[6 H x0]^(1/3)}}

```

```

In[*]:= Protect[gtry]

```

```

Out[*]:= {gtry}

```

```

In[*]:= Unprotect[ginv, r, Ricci, Rier, RS, EinsteinG]

```

```

Out[*]:= {}

```

```

In[*]:= result = {ginv, r, Ricci, Rier, RS, EinsteinG} = rt[gtry];

```

Sat 29 Nov 2025 05:16:31 GMT-8

{3.38879, Null}

Sat 29 Nov 2025 05:16:35 GMT-8

```

In[*]:= Protect[ginv, r, Ricci, Rier, RS, EinsteinG]

```

```

Out[*]:= {ginv, r, Ricci, Rier, RS, EinsteinG}

```

verify $\mathcal{G}_{\mu\nu|\alpha} = 0$:

```
In[ ]:= Simplify[Table[D[gtry[[j, k]], X[[i]]] -
  Sum[gtry[[k, s]] × r[[s, i, j]], {s, 1, DIM8}] - Sum[gtry[[j, s]] × r[[s, i, k]], {s, 1, DIM8}],
  {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] // Flatten // Union
```

```
Out[ ]:=
```

```
{0}
```

```
In[ ]:= Table[g[j][k] →
  If[j == k, ToExpression["(g[" <> ToString[j] <> "]" <> ToString[k] <> "]"[#1,#2])&"],
  (0 &)], {j, 0, 7}, {k, 0, 7}];
sg = % // Flatten
```

```
Out[ ]:=
```

```
{g[0][0] → (g[0][0][#1, #2] &), g[0][1] → (0 &), g[0][2] → (0 &),
g[0][3] → (0 &), g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &),
g[0][7] → (0 &), g[1][0] → (0 &), g[1][1] → (g[1][1][#1, #2] &), g[1][2] → (0 &),
g[1][3] → (0 &), g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &),
g[1][7] → (0 &), g[2][0] → (0 &), g[2][1] → (0 &), g[2][2] → (g[2][2][#1, #2] &),
g[2][3] → (0 &), g[2][4] → (0 &), g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &),
g[3][0] → (0 &), g[3][1] → (0 &), g[3][2] → (0 &), g[3][3] → (g[3][3][#1, #2] &),
g[3][4] → (0 &), g[3][5] → (0 &), g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &),
g[4][1] → (0 &), g[4][2] → (0 &), g[4][3] → (0 &), g[4][4] → (g[4][4][#1, #2] &),
g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &), g[5][0] → (0 &), g[5][1] → (0 &),
g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &), g[5][5] → (g[5][5][#1, #2] &),
g[5][6] → (0 &), g[5][7] → (0 &), g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &),
g[6][3] → (0 &), g[6][4] → (0 &), g[6][5] → (0 &), g[6][6] → (g[6][6][#1, #2] &),
g[6][7] → (0 &), g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &),
g[7][4] → (0 &), g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (g[7][7][#1, #2] &) }
```

```
In[ ]:= g[7][7][x0, x4] /. ssgm4488
% /. sx0x4
```

```
Out[ ]:=
```

```
 $-e^{-2 a_4 [H x^4]} \sin[6 H x_0]^{1/3}$ 
```

```
Out[ ]:=
```

```
 $-e^{-2 a_4 [t]} \sin[z]^{1/3}$ 
```

```
In[ ]:= MatrixMetric44
```

```
Out[ ]:=
```

```
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^{2 a_4 [H x^4]} Sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0},
{0, 0, e^{2 a_4 [H x^4]} Sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0}, {0, 0, 0, e^{2 a_4 [H x^4]} Sin[6 H x0]^{1/3}, 0, 0, 0, 0},
{0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^{-2 a_4 [H x^4]} Sin[6 H x0]^{1/3}, 0, 0},
{0, 0, 0, 0, 0, 0, -e^{-2 a_4 [H x^4]} Sin[6 H x0]^{1/3}, 0}, {0, 0, 0, 0, 0, 0, 0, -e^{-2 a_4 [H x^4]} Sin[6 H x0]^{1/3}}}
```

```
In[*]:= ssgm4488[x0, x4] /. sx0x4
```

```
Out[*]=
```

```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
  g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
  g[1][1] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
  g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
  g[2][1] → (0 &), g[2][2] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
  g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
  g[3][2] → (0 &), g[3][3] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
  g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
  g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
  g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
  g[5][5] → (-e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
  g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
  g[6][5] → (0 &), g[6][6] → (-e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
  g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
  g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &)} {z/6 H, t/H}
```

```
In[*]:= (*ssgGzt={G[0][0]→((Cot[#1]^2)&),G[0][1]→((0)&),G[0][2]→((0)&),
  G[0][3]→((0)&),G[0][4]→((0)&),G[0][5]→((0)&),G[0][6]→((0)&),G[0][7]→((0)&),
  G[1][0]→((0)&),G[1][1]→((e^(2*A4[#2]) Sin[#1]^(1/3))&),G[1][2]→((0)&),G[1][3]→((0)&),
  G[1][4]→((0)&),G[1][5]→((0)&),G[1][6]→((0)&),G[1][7]→((0)&),G[2][0]→((0)&),
  G[2][1]→((0)&),G[2][2]→((e^(2*A4[#2]) Sin[#1]^(1/3))&),G[2][3]→((0)&),G[2][4]→((0)&),
  G[2][5]→((0)&),G[2][6]→((0)&),G[2][7]→((0)&),G[3][0]→((0)&),G[3][1]→((0)&),
  G[3][2]→((0)&),G[3][3]→((e^(2*A4[#2]) Sin[#1]^(1/3))&),G[3][4]→((0)&),G[3][5]→((0)&),
  G[3][6]→((0)&),G[3][7]→((0)&),G[4][0]→((0)&),G[4][1]→((0)&),G[4][2]→((0)&),
  G[4][3]→((0)&),G[4][4]→((-1)&),G[4][5]→((0)&),G[4][6]→((0)&),G[4][7]→((0)&),
  G[5][0]→((0)&),G[5][1]→((0)&),G[5][2]→((0)&),G[5][3]→((0)&),G[5][4]→((0)&),
  G[5][5]→((-e^(2*A4[#2]) Sin[#1]^(1/3))&),G[5][6]→((0)&),G[5][7]→((0)&),
  G[6][0]→((0)&),G[6][1]→((0)&),G[6][2]→((0)&),G[6][3]→((0)&),G[6][4]→((0)&),
  G[6][5]→((0)&),G[6][6]→((-e^(2*A4[#2]) Sin[#1]^(1/3))&),G[6][7]→((0)&),
  G[7][0]→((0)&),G[7][1]→((0)&),G[7][2]→((0)&),G[7][3]→((0)&),G[7][4]→((0)&),
  G[7][5]→((0)&),G[7][6]→((0)&),G[7][7]→((-e^(2*A4[#2]) Sin[#1]^(1/3))&)}*)
```

```
In[*]:= (*G[#][#][z,t]/.ssgGzt&/@Range[0,7]*)
```

```
In[*]:= detgg = Det[g4488 /. sg] // FullSimplify[#, constraintVars] &
```

```
Out[*]=
```

```
g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
  g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]
```

```
In[ ]:= g4488 /. sg // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} g[0][0][x0, x4] & 0 & 0 & 0 & 0 & 0 \\ 0 & g[1][1][x0, x4] & 0 & 0 & 0 & 0 \\ 0 & 0 & g[2][2][x0, x4] & 0 & 0 & 0 \\ 0 & 0 & 0 & g[3][3][x0, x4] & 0 & 0 \\ 0 & 0 & 0 & 0 & g[4][4][x0, x4] & 0 \\ 0 & 0 & 0 & 0 & 0 & g[5][5][x0, x4] \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= MatrixMetric44.η4488
```

```
(*eAa=*)FullSimplify[√%, constraintVars]
```

```
Out[ ]:=
```

$$\begin{aligned} &\{ \{ \text{Cot}[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, e^{2 a4 [H x4]} \text{Sin}[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0 \}, \\ &\{ 0, 0, e^{2 a4 [H x4]} \text{Sin}[6 H x0]^{1/3}, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, e^{2 a4 [H x4]} \text{Sin}[6 H x0]^{1/3}, 0, 0, 0, 0 \}, \\ &\{ 0, 0, 0, 0, 1, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, e^{-2 a4 [H x4]} \text{Sin}[6 H x0]^{1/3}, 0, 0 \}, \\ &\{ 0, 0, 0, 0, 0, 0, e^{-2 a4 [H x4]} \text{Sin}[6 H x0]^{1/3}, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, e^{-2 a4 [H x4]} \text{Sin}[6 H x0]^{1/3} \} \end{aligned}$$

```
Out[ ]:=
```

$$\begin{aligned} &\{ \{ \text{Cot}[6 H x0], 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, \sqrt{e^{2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}, 0, 0, 0, 0, 0, 0 \}, \\ &\{ 0, 0, \sqrt{e^{2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, \sqrt{e^{2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}, 0, 0, 0, 0 \}, \\ &\{ 0, 0, 0, 0, 1, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, \sqrt{e^{-2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}, 0, 0 \}, \\ &\{ 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}, 0 \}, \\ &\{ 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6} \} \end{aligned}$$

eAa

```
In[ ]:= g4488.η4488 /. sg
```

```
(eAa = FullSimplify[√%, constraintVars]) // MatrixForm
```

```
Out[ ]:=
```

$$\begin{aligned} &\{ \{ g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0 \}, \\ &\{ 0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0 \}, \\ &\{ 0, 0, 0, 0, -g[4][4][x0, x4], 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, -g[5][5][x0, x4], 0, 0 \}, \\ &\{ 0, 0, 0, 0, 0, 0, -g[6][6][x0, x4], 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, -g[7][7][x0, x4] \} \end{aligned}$$

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \sqrt{g[0][0][x0, x4]} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{g[1][1][x0, x4]} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{g[2][2][x0, x4]} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{g[3][3][x0, x4]} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{-g[4][4][x0, x4]} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{-g[5][5][x0, x4]} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


```
In[ ]:= eAa
```

```
ssgm4488
```

```
(eAa04 = %% /. % // FullSimplify[#, constraintVars] &) // MatrixForm
```

```
Out[ ]:=
```

```
{ {  $\sqrt{g[0][0][x0, x4]}$ , 0, 0, 0, 0, 0, 0, 0 }, { 0,  $\sqrt{g[1][1][x0, x4]}$ , 0, 0, 0, 0, 0, 0 },
  { 0, 0,  $\sqrt{g[2][2][x0, x4]}$ , 0, 0, 0, 0, 0 }, { 0, 0, 0,  $\sqrt{g[3][3][x0, x4]}$ , 0, 0, 0, 0 },
  { 0, 0, 0, 0,  $\sqrt{-g[4][4][x0, x4]}$ , 0, 0, 0 }, { 0, 0, 0, 0, 0,  $\sqrt{-g[5][5][x0, x4]}$ , 0, 0 },
  { 0, 0, 0, 0, 0, 0,  $\sqrt{-g[6][6][x0, x4]}$ , 0 }, { 0, 0, 0, 0, 0, 0, 0,  $\sqrt{-g[7][7][x0, x4]}$  } }
```

```
Out[ ]:=
```

```
{ g[0][0] → (Cot[6 H #1]2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
  g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
  g[1][1] → ( $e^{2 a4 [H \#2]} \text{Sin}[6 H \#1]^{1/3}$  &), g[1][2] → (0 &), g[1][3] → (0 &),
  g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
  g[2][1] → (0 &), g[2][2] → ( $e^{2 a4 [H \#2]} \text{Sin}[6 H \#1]^{1/3}$  &), g[2][3] → (0 &), g[2][4] → (0 &),
  g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
  g[3][2] → (0 &), g[3][3] → ( $e^{2 a4 [H \#2]} \text{Sin}[6 H \#1]^{1/3}$  &), g[3][4] → (0 &), g[3][5] → (0 &),
  g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
  g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
  g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
  g[5][5] → ( $-e^{-2 a4 [H \#2]} \text{Sin}[6 H \#1]^{1/3}$  &), g[5][6] → (0 &), g[5][7] → (0 &),
  g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
  g[6][5] → (0 &), g[6][6] → ( $-e^{-2 a4 [H \#2]} \text{Sin}[6 H \#1]^{1/3}$  &), g[6][7] → (0 &),
  g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
  g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → ( $-e^{-2 a4 [H \#2]} \text{Sin}[6 H \#1]^{1/3}$  &) }
```

```
Out[ ]//MatrixForm=
```

```
( Cot[6 H x0] 0 0 0 0
  0  $\sqrt{e^{2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}$  0 0 0
  0 0  $\sqrt{e^{2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}$  0 0
  0 0 0  $\sqrt{e^{2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}$  0
  0 0 0 0 1
  0 0 0 0 0  $\sqrt{e^{-2 a4}}$ 
  0 0 0 0 0 0 )
```

OCTAD $e_{\alpha}^{(A)} \leftarrow \text{Lorentz}$: $e_{\alpha} \leftarrow \text{spacetime}$:

$$\blacksquare e_{\alpha}^{(A)} = e_{\alpha}^{(A)} = eAa$$

In[*]:= $(\text{sge}_{\alpha}^{(A)} = \text{Thread}[\text{Flatten}[e_{\alpha}^{(A)}] \rightarrow \text{Flatten}[eAa]]) (*//\text{Column}*)$

Out[*]=

$$\{E_{\alpha}^{(A)}[0, 0] \rightarrow \sqrt{g[0][0][x0, x4]}, E_{\alpha}^{(A)}[0, 1] \rightarrow 0, E_{\alpha}^{(A)}[0, 2] \rightarrow 0, E_{\alpha}^{(A)}[0, 3] \rightarrow 0, E_{\alpha}^{(A)}[0, 4] \rightarrow 0, \\ E_{\alpha}^{(A)}[0, 5] \rightarrow 0, E_{\alpha}^{(A)}[0, 6] \rightarrow 0, E_{\alpha}^{(A)}[0, 7] \rightarrow 0, E_{\alpha}^{(A)}[1, 0] \rightarrow 0, E_{\alpha}^{(A)}[1, 1] \rightarrow \sqrt{g[1][1][x0, x4]}, \\ E_{\alpha}^{(A)}[1, 2] \rightarrow 0, E_{\alpha}^{(A)}[1, 3] \rightarrow 0, E_{\alpha}^{(A)}[1, 4] \rightarrow 0, E_{\alpha}^{(A)}[1, 5] \rightarrow 0, E_{\alpha}^{(A)}[1, 6] \rightarrow 0, E_{\alpha}^{(A)}[1, 7] \rightarrow 0, \\ E_{\alpha}^{(A)}[2, 0] \rightarrow 0, E_{\alpha}^{(A)}[2, 1] \rightarrow 0, E_{\alpha}^{(A)}[2, 2] \rightarrow \sqrt{g[2][2][x0, x4]}, E_{\alpha}^{(A)}[2, 3] \rightarrow 0, E_{\alpha}^{(A)}[2, 4] \rightarrow 0, \\ E_{\alpha}^{(A)}[2, 5] \rightarrow 0, E_{\alpha}^{(A)}[2, 6] \rightarrow 0, E_{\alpha}^{(A)}[2, 7] \rightarrow 0, E_{\alpha}^{(A)}[3, 0] \rightarrow 0, E_{\alpha}^{(A)}[3, 1] \rightarrow 0, E_{\alpha}^{(A)}[3, 2] \rightarrow 0, \\ E_{\alpha}^{(A)}[3, 3] \rightarrow \sqrt{g[3][3][x0, x4]}, E_{\alpha}^{(A)}[3, 4] \rightarrow 0, E_{\alpha}^{(A)}[3, 5] \rightarrow 0, E_{\alpha}^{(A)}[3, 6] \rightarrow 0, E_{\alpha}^{(A)}[3, 7] \rightarrow 0, \\ E_{\alpha}^{(A)}[4, 0] \rightarrow 0, E_{\alpha}^{(A)}[4, 1] \rightarrow 0, E_{\alpha}^{(A)}[4, 2] \rightarrow 0, E_{\alpha}^{(A)}[4, 3] \rightarrow 0, E_{\alpha}^{(A)}[4, 4] \rightarrow \sqrt{-g[4][4][x0, x4]}, \\ E_{\alpha}^{(A)}[4, 5] \rightarrow 0, E_{\alpha}^{(A)}[4, 6] \rightarrow 0, E_{\alpha}^{(A)}[4, 7] \rightarrow 0, E_{\alpha}^{(A)}[5, 0] \rightarrow 0, E_{\alpha}^{(A)}[5, 1] \rightarrow 0, E_{\alpha}^{(A)}[5, 2] \rightarrow 0, \\ E_{\alpha}^{(A)}[5, 3] \rightarrow 0, E_{\alpha}^{(A)}[5, 4] \rightarrow 0, E_{\alpha}^{(A)}[5, 5] \rightarrow \sqrt{-g[5][5][x0, x4]}, E_{\alpha}^{(A)}[5, 6] \rightarrow 0, E_{\alpha}^{(A)}[5, 7] \rightarrow 0, \\ E_{\alpha}^{(A)}[6, 0] \rightarrow 0, E_{\alpha}^{(A)}[6, 1] \rightarrow 0, E_{\alpha}^{(A)}[6, 2] \rightarrow 0, E_{\alpha}^{(A)}[6, 3] \rightarrow 0, E_{\alpha}^{(A)}[6, 4] \rightarrow 0, E_{\alpha}^{(A)}[6, 5] \rightarrow 0, \\ E_{\alpha}^{(A)}[6, 6] \rightarrow \sqrt{-g[6][6][x0, x4]}, E_{\alpha}^{(A)}[6, 7] \rightarrow 0, E_{\alpha}^{(A)}[7, 0] \rightarrow 0, E_{\alpha}^{(A)}[7, 1] \rightarrow 0, E_{\alpha}^{(A)}[7, 2] \rightarrow 0, \\ E_{\alpha}^{(A)}[7, 3] \rightarrow 0, E_{\alpha}^{(A)}[7, 4] \rightarrow 0, E_{\alpha}^{(A)}[7, 5] \rightarrow 0, E_{\alpha}^{(A)}[7, 6] \rightarrow 0, E_{\alpha}^{(A)}[7, 7] \rightarrow \sqrt{-g[7][7][x0, x4]}\}$$

In[*]:= $(*\text{Symbolize}[\text{sge}_{\alpha}^{(A)}] *)$

In[*]:= $(*\text{Symbolize}[\text{sgtrye}_{\alpha}^{(A)}] *)$

In[*]:= $(\text{sgtrye}_{\alpha}^{(A)} = \text{Thread}[\text{Flatten}[e_{\alpha}^{(A)}] \rightarrow \text{Flatten}[eAa04]]) (*//\text{Column}*)$

Out[*]=

$$\{E_{\alpha}^{(A)}[0, 0] \rightarrow \text{Cot}[6 H x0], E_{\alpha}^{(A)}[0, 1] \rightarrow 0, E_{\alpha}^{(A)}[0, 2] \rightarrow 0, E_{\alpha}^{(A)}[0, 3] \rightarrow 0, \\ E_{\alpha}^{(A)}[0, 4] \rightarrow 0, E_{\alpha}^{(A)}[0, 5] \rightarrow 0, E_{\alpha}^{(A)}[0, 6] \rightarrow 0, E_{\alpha}^{(A)}[0, 7] \rightarrow 0, E_{\alpha}^{(A)}[1, 0] \rightarrow 0, \\ E_{\alpha}^{(A)}[1, 1] \rightarrow \sqrt{e^{2 a4[H x4]} \text{Sin}[6 H x0]^{1/6}}, E_{\alpha}^{(A)}[1, 2] \rightarrow 0, E_{\alpha}^{(A)}[1, 3] \rightarrow 0, E_{\alpha}^{(A)}[1, 4] \rightarrow 0, \\ E_{\alpha}^{(A)}[1, 5] \rightarrow 0, E_{\alpha}^{(A)}[1, 6] \rightarrow 0, E_{\alpha}^{(A)}[1, 7] \rightarrow 0, E_{\alpha}^{(A)}[2, 0] \rightarrow 0, E_{\alpha}^{(A)}[2, 1] \rightarrow 0, \\ E_{\alpha}^{(A)}[2, 2] \rightarrow \sqrt{e^{2 a4[H x4]} \text{Sin}[6 H x0]^{1/6}}, E_{\alpha}^{(A)}[2, 3] \rightarrow 0, E_{\alpha}^{(A)}[2, 4] \rightarrow 0, E_{\alpha}^{(A)}[2, 5] \rightarrow 0, \\ E_{\alpha}^{(A)}[2, 6] \rightarrow 0, E_{\alpha}^{(A)}[2, 7] \rightarrow 0, E_{\alpha}^{(A)}[3, 0] \rightarrow 0, E_{\alpha}^{(A)}[3, 1] \rightarrow 0, E_{\alpha}^{(A)}[3, 2] \rightarrow 0, \\ E_{\alpha}^{(A)}[3, 3] \rightarrow \sqrt{e^{2 a4[H x4]} \text{Sin}[6 H x0]^{1/6}}, E_{\alpha}^{(A)}[3, 4] \rightarrow 0, E_{\alpha}^{(A)}[3, 5] \rightarrow 0, E_{\alpha}^{(A)}[3, 6] \rightarrow 0, \\ E_{\alpha}^{(A)}[3, 7] \rightarrow 0, E_{\alpha}^{(A)}[4, 0] \rightarrow 0, E_{\alpha}^{(A)}[4, 1] \rightarrow 0, E_{\alpha}^{(A)}[4, 2] \rightarrow 0, E_{\alpha}^{(A)}[4, 3] \rightarrow 0, \\ E_{\alpha}^{(A)}[4, 4] \rightarrow 1, E_{\alpha}^{(A)}[4, 5] \rightarrow 0, E_{\alpha}^{(A)}[4, 6] \rightarrow 0, E_{\alpha}^{(A)}[4, 7] \rightarrow 0, E_{\alpha}^{(A)}[5, 0] \rightarrow 0, E_{\alpha}^{(A)}[5, 1] \rightarrow 0, \\ E_{\alpha}^{(A)}[5, 2] \rightarrow 0, E_{\alpha}^{(A)}[5, 3] \rightarrow 0, E_{\alpha}^{(A)}[5, 4] \rightarrow 0, E_{\alpha}^{(A)}[5, 5] \rightarrow \sqrt{e^{-2 a4[H x4]} \text{Sin}[6 H x0]^{1/6}}, \\ E_{\alpha}^{(A)}[5, 6] \rightarrow 0, E_{\alpha}^{(A)}[5, 7] \rightarrow 0, E_{\alpha}^{(A)}[6, 0] \rightarrow 0, E_{\alpha}^{(A)}[6, 1] \rightarrow 0, E_{\alpha}^{(A)}[6, 2] \rightarrow 0, \\ E_{\alpha}^{(A)}[6, 3] \rightarrow 0, E_{\alpha}^{(A)}[6, 4] \rightarrow 0, E_{\alpha}^{(A)}[6, 5] \rightarrow 0, E_{\alpha}^{(A)}[6, 6] \rightarrow \sqrt{e^{-2 a4[H x4]} \text{Sin}[6 H x0]^{1/6}}, \\ E_{\alpha}^{(A)}[6, 7] \rightarrow 0, E_{\alpha}^{(A)}[7, 0] \rightarrow 0, E_{\alpha}^{(A)}[7, 1] \rightarrow 0, E_{\alpha}^{(A)}[7, 2] \rightarrow 0, E_{\alpha}^{(A)}[7, 3] \rightarrow 0, \\ E_{\alpha}^{(A)}[7, 4] \rightarrow 0, E_{\alpha}^{(A)}[7, 5] \rightarrow 0, E_{\alpha}^{(A)}[7, 6] \rightarrow 0, E_{\alpha}^{(A)}[7, 7] \rightarrow \sqrt{e^{-2 a4[H x4]} \text{Sin}[6 H x0]^{1/6}}\}$$

```
In[*]:= (sgtrye(A)α = Thread[Flatten[e(A)α] → Flatten[Inverse[e(A)(A) /. sgtrye(A)(A)]]]) (*//Column*)
```

```
Out[*]=
```

$$\left\{ \begin{aligned} &E_{(A)}^{\alpha}[0, 0] \rightarrow \tan[6 H x 0], E_{(A)}^{\alpha}[0, 1] \rightarrow 0, E_{(A)}^{\alpha}[0, 2] \rightarrow 0, E_{(A)}^{\alpha}[0, 3] \rightarrow 0, \\ &E_{(A)}^{\alpha}[0, 4] \rightarrow 0, E_{(A)}^{\alpha}[0, 5] \rightarrow 0, E_{(A)}^{\alpha}[0, 6] \rightarrow 0, E_{(A)}^{\alpha}[0, 7] \rightarrow 0, E_{(A)}^{\alpha}[1, 0] \rightarrow 0, \\ &E_{(A)}^{\alpha}[1, 1] \rightarrow \frac{1}{\sqrt{e^{2 a 4[H x 4]}} \sin[6 H x 0]^{1/6}}, E_{(A)}^{\alpha}[1, 2] \rightarrow 0, E_{(A)}^{\alpha}[1, 3] \rightarrow 0, E_{(A)}^{\alpha}[1, 4] \rightarrow 0, \\ &E_{(A)}^{\alpha}[1, 5] \rightarrow 0, E_{(A)}^{\alpha}[1, 6] \rightarrow 0, E_{(A)}^{\alpha}[1, 7] \rightarrow 0, E_{(A)}^{\alpha}[2, 0] \rightarrow 0, E_{(A)}^{\alpha}[2, 1] \rightarrow 0, \\ &E_{(A)}^{\alpha}[2, 2] \rightarrow \frac{1}{\sqrt{e^{2 a 4[H x 4]}} \sin[6 H x 0]^{1/6}}, E_{(A)}^{\alpha}[2, 3] \rightarrow 0, E_{(A)}^{\alpha}[2, 4] \rightarrow 0, E_{(A)}^{\alpha}[2, 5] \rightarrow 0, \\ &E_{(A)}^{\alpha}[2, 6] \rightarrow 0, E_{(A)}^{\alpha}[2, 7] \rightarrow 0, E_{(A)}^{\alpha}[3, 0] \rightarrow 0, E_{(A)}^{\alpha}[3, 1] \rightarrow 0, E_{(A)}^{\alpha}[3, 2] \rightarrow 0, \\ &E_{(A)}^{\alpha}[3, 3] \rightarrow \frac{1}{\sqrt{e^{2 a 4[H x 4]}} \sin[6 H x 0]^{1/6}}, E_{(A)}^{\alpha}[3, 4] \rightarrow 0, E_{(A)}^{\alpha}[3, 5] \rightarrow 0, E_{(A)}^{\alpha}[3, 6] \rightarrow 0, \\ &E_{(A)}^{\alpha}[3, 7] \rightarrow 0, E_{(A)}^{\alpha}[4, 0] \rightarrow 0, E_{(A)}^{\alpha}[4, 1] \rightarrow 0, E_{(A)}^{\alpha}[4, 2] \rightarrow 0, E_{(A)}^{\alpha}[4, 3] \rightarrow 0, \\ &E_{(A)}^{\alpha}[4, 4] \rightarrow 1, E_{(A)}^{\alpha}[4, 5] \rightarrow 0, E_{(A)}^{\alpha}[4, 6] \rightarrow 0, E_{(A)}^{\alpha}[4, 7] \rightarrow 0, E_{(A)}^{\alpha}[5, 0] \rightarrow 0, E_{(A)}^{\alpha}[5, 1] \rightarrow 0, \\ &E_{(A)}^{\alpha}[5, 2] \rightarrow 0, E_{(A)}^{\alpha}[5, 3] \rightarrow 0, E_{(A)}^{\alpha}[5, 4] \rightarrow 0, E_{(A)}^{\alpha}[5, 5] \rightarrow \frac{1}{\sqrt{e^{-2 a 4[H x 4]}} \sin[6 H x 0]^{1/6}}, \\ &E_{(A)}^{\alpha}[5, 6] \rightarrow 0, E_{(A)}^{\alpha}[5, 7] \rightarrow 0, E_{(A)}^{\alpha}[6, 0] \rightarrow 0, E_{(A)}^{\alpha}[6, 1] \rightarrow 0, E_{(A)}^{\alpha}[6, 2] \rightarrow 0, \\ &E_{(A)}^{\alpha}[6, 3] \rightarrow 0, E_{(A)}^{\alpha}[6, 4] \rightarrow 0, E_{(A)}^{\alpha}[6, 5] \rightarrow 0, E_{(A)}^{\alpha}[6, 6] \rightarrow \frac{1}{\sqrt{e^{-2 a 4[H x 4]}} \sin[6 H x 0]^{1/6}}, \\ &E_{(A)}^{\alpha}[6, 7] \rightarrow 0, E_{(A)}^{\alpha}[7, 0] \rightarrow 0, E_{(A)}^{\alpha}[7, 1] \rightarrow 0, E_{(A)}^{\alpha}[7, 2] \rightarrow 0, E_{(A)}^{\alpha}[7, 3] \rightarrow 0, \\ &E_{(A)}^{\alpha}[7, 4] \rightarrow 0, E_{(A)}^{\alpha}[7, 5] \rightarrow 0, E_{(A)}^{\alpha}[7, 6] \rightarrow 0, E_{(A)}^{\alpha}[7, 7] \rightarrow \frac{1}{\sqrt{e^{-2 a 4[H x 4]}} \sin[6 H x 0]^{1/6}} \end{aligned} \right\}$$

```
In[*]:= gtrye(A)(A) = (e(A)(A) /. sgtrye(A)(A)) /. subsDefects
```

```
Out[*]=
```

$$\left\{ \begin{aligned} &\{\cot[6 H x 0], 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{a 4[H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, e^{a 4[H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a 4[H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-a 4[H x 4]} \sin[6 H x 0]^{1/6}, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, e^{-a 4[H x 4]} \sin[6 H x 0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, 0, e^{-a 4[H x 4]} \sin[6 H x 0]^{1/6}\} \end{aligned} \right\}$$

```
In[*]:= gtrye(A)α = (e(A)α /. sgtrye(A)α) /. subsDefects
```

```
Out[*]=
```

$$\left\{ \begin{aligned} &\{\tan[6 H x 0], 0, 0, 0, 0, 0, 0, 0\}, \left\{0, \frac{e^{-a 4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0\right\}, \\ &\left\{0, 0, \frac{e^{-a 4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{e^{-a 4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0\right\}, \\ &\{0, 0, 0, 0, 1, 0, 0, 0\}, \left\{0, 0, 0, 0, 0, \frac{e^{a 4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0\right\}, \\ &\left\{0, 0, 0, 0, 0, 0, \frac{e^{a 4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{e^{a 4[H x 4]}}{\sin[6 H x 0]^{1/6}}\right\} \end{aligned} \right\}$$

```

In[ ] :=  $\mathbf{e}_\alpha^{(A)}$ 
% /. sgtrye $_\alpha^{(A)}$  // MatrixForm

Out[ ] =
{ {  $E_\alpha^{(A)}[0, 0]$ ,  $E_\alpha^{(A)}[0, 1]$ ,  $E_\alpha^{(A)}[0, 2]$ ,  $E_\alpha^{(A)}[0, 3]$ ,  $E_\alpha^{(A)}[0, 4]$ ,  $E_\alpha^{(A)}[0, 5]$ ,  $E_\alpha^{(A)}[0, 6]$ ,  $E_\alpha^{(A)}[0, 7]$  },
  {  $E_\alpha^{(A)}[1, 0]$ ,  $E_\alpha^{(A)}[1, 1]$ ,  $E_\alpha^{(A)}[1, 2]$ ,  $E_\alpha^{(A)}[1, 3]$ ,  $E_\alpha^{(A)}[1, 4]$ ,  $E_\alpha^{(A)}[1, 5]$ ,  $E_\alpha^{(A)}[1, 6]$ ,  $E_\alpha^{(A)}[1, 7]$  },
  {  $E_\alpha^{(A)}[2, 0]$ ,  $E_\alpha^{(A)}[2, 1]$ ,  $E_\alpha^{(A)}[2, 2]$ ,  $E_\alpha^{(A)}[2, 3]$ ,  $E_\alpha^{(A)}[2, 4]$ ,  $E_\alpha^{(A)}[2, 5]$ ,  $E_\alpha^{(A)}[2, 6]$ ,  $E_\alpha^{(A)}[2, 7]$  },
  {  $E_\alpha^{(A)}[3, 0]$ ,  $E_\alpha^{(A)}[3, 1]$ ,  $E_\alpha^{(A)}[3, 2]$ ,  $E_\alpha^{(A)}[3, 3]$ ,  $E_\alpha^{(A)}[3, 4]$ ,  $E_\alpha^{(A)}[3, 5]$ ,  $E_\alpha^{(A)}[3, 6]$ ,  $E_\alpha^{(A)}[3, 7]$  },
  {  $E_\alpha^{(A)}[4, 0]$ ,  $E_\alpha^{(A)}[4, 1]$ ,  $E_\alpha^{(A)}[4, 2]$ ,  $E_\alpha^{(A)}[4, 3]$ ,  $E_\alpha^{(A)}[4, 4]$ ,  $E_\alpha^{(A)}[4, 5]$ ,  $E_\alpha^{(A)}[4, 6]$ ,  $E_\alpha^{(A)}[4, 7]$  },
  {  $E_\alpha^{(A)}[5, 0]$ ,  $E_\alpha^{(A)}[5, 1]$ ,  $E_\alpha^{(A)}[5, 2]$ ,  $E_\alpha^{(A)}[5, 3]$ ,  $E_\alpha^{(A)}[5, 4]$ ,  $E_\alpha^{(A)}[5, 5]$ ,  $E_\alpha^{(A)}[5, 6]$ ,  $E_\alpha^{(A)}[5, 7]$  },
  {  $E_\alpha^{(A)}[6, 0]$ ,  $E_\alpha^{(A)}[6, 1]$ ,  $E_\alpha^{(A)}[6, 2]$ ,  $E_\alpha^{(A)}[6, 3]$ ,  $E_\alpha^{(A)}[6, 4]$ ,  $E_\alpha^{(A)}[6, 5]$ ,  $E_\alpha^{(A)}[6, 6]$ ,  $E_\alpha^{(A)}[6, 7]$  },
  {  $E_\alpha^{(A)}[7, 0]$ ,  $E_\alpha^{(A)}[7, 1]$ ,  $E_\alpha^{(A)}[7, 2]$ ,  $E_\alpha^{(A)}[7, 3]$ ,  $E_\alpha^{(A)}[7, 4]$ ,  $E_\alpha^{(A)}[7, 5]$ ,  $E_\alpha^{(A)}[7, 6]$ ,  $E_\alpha^{(A)}[7, 7]$  } }

Out[ ] // MatrixForm =

$$\begin{pmatrix} \text{Cot}[6 H x 0] & 0 & 0 & 0 & 0 \\ 0 & \sqrt{e^{2 a 4 [H x 4]}} \text{Sin}[6 H x 0]^{1/6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{e^{2 a 4 [H x 4]}} \text{Sin}[6 H x 0]^{1/6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{e^{2 a 4 [H x 4]}} \text{Sin}[6 H x 0]^{1/6} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \sqrt{e^{-2 a 4}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


In[ ] :=  $\mathbf{ene} = \text{FullSimplify}[\text{Transpose}[\mathbf{e}_\alpha^{(A)}] . \eta 4488 . \mathbf{e}_\alpha^{(A)}, \text{constraintVars}];$ 
% /. sge $_\alpha^{(A)}$ 
%% /. sgtrye $_\alpha^{(A)}$ 
% = gtry // FullSimplify[#, constraintVars] &

Out[ ] =
{ { g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0 }, { 0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0 },
  { 0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0 }, { 0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0 },
  { 0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0 }, { 0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0 },
  { 0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0 }, { 0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4] } }

Out[ ] =
{ {  $\text{Cot}[6 H x 0]^2$ , 0, 0, 0, 0, 0, 0, 0 }, { 0,  $e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0, 0, 0, 0, 0, 0 },
  { 0, 0,  $e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0, 0, 0, 0, 0 }, { 0, 0, 0,  $e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0, 0, 0, 0 },
  { 0, 0, 0, 0, -1, 0, 0, 0 }, { 0, 0, 0, 0, 0,  $-e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0, 0 },
  { 0, 0, 0, 0, 0, 0,  $-e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0 }, { 0, 0, 0, 0, 0, 0, 0,  $-e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$  } }

Out[ ] =
True

In[ ] := gtry

Out[ ] =
{ {  $\text{Cot}[6 H x 0]^2$ , 0, 0, 0, 0, 0, 0, 0 }, { 0,  $e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0, 0, 0, 0, 0, 0 },
  { 0, 0,  $e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0, 0, 0, 0, 0 }, { 0, 0, 0,  $e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0, 0, 0, 0 },
  { 0, 0, 0, 0, -1, 0, 0, 0 }, { 0, 0, 0, 0, 0,  $-e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0, 0 },
  { 0, 0, 0, 0, 0, 0,  $-e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$ , 0 }, { 0, 0, 0, 0, 0, 0, 0,  $-e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3}$  } }

In[ ] := (*FullSimplify[Transpose[ $\mathbf{e}_\alpha^{(A)}$ ] .  $\eta 4488 . \mathbf{e}_\alpha^{(A)}, \text{constraintVars}$ ]-MatrixMetric44*)

```

```
In[ ]:= ssgm4488
```

```
Out[ ]:=
```

$$\begin{aligned} &\{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ &g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ &g[1][1] \rightarrow (e^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ &g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ &g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ &g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ &g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ &g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ &g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ &g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ &g[5][5] \rightarrow (-e^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ &g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ &g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ &g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ &g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&)\} \end{aligned}$$

```
In[ ]:= MatrixMetric44 // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \text{Cot}[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a4[Hx4]} \text{Sin}[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a4[Hx4]} \text{Sin}[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a4[Hx4]} \text{Sin}[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -e^{-2a4[Hx4]} \text{Sin}[6Hx0]^{1/3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= eα(A)
```

```
Out[ ]:=
```

$$\begin{aligned} &\{E_{\alpha}^{(A)}[0, 0], E_{\alpha}^{(A)}[0, 1], E_{\alpha}^{(A)}[0, 2], E_{\alpha}^{(A)}[0, 3], E_{\alpha}^{(A)}[0, 4], E_{\alpha}^{(A)}[0, 5], E_{\alpha}^{(A)}[0, 6], E_{\alpha}^{(A)}[0, 7], \\ &E_{\alpha}^{(A)}[1, 0], E_{\alpha}^{(A)}[1, 1], E_{\alpha}^{(A)}[1, 2], E_{\alpha}^{(A)}[1, 3], E_{\alpha}^{(A)}[1, 4], E_{\alpha}^{(A)}[1, 5], E_{\alpha}^{(A)}[1, 6], E_{\alpha}^{(A)}[1, 7], \\ &E_{\alpha}^{(A)}[2, 0], E_{\alpha}^{(A)}[2, 1], E_{\alpha}^{(A)}[2, 2], E_{\alpha}^{(A)}[2, 3], E_{\alpha}^{(A)}[2, 4], E_{\alpha}^{(A)}[2, 5], E_{\alpha}^{(A)}[2, 6], E_{\alpha}^{(A)}[2, 7], \\ &E_{\alpha}^{(A)}[3, 0], E_{\alpha}^{(A)}[3, 1], E_{\alpha}^{(A)}[3, 2], E_{\alpha}^{(A)}[3, 3], E_{\alpha}^{(A)}[3, 4], E_{\alpha}^{(A)}[3, 5], E_{\alpha}^{(A)}[3, 6], E_{\alpha}^{(A)}[3, 7], \\ &E_{\alpha}^{(A)}[4, 0], E_{\alpha}^{(A)}[4, 1], E_{\alpha}^{(A)}[4, 2], E_{\alpha}^{(A)}[4, 3], E_{\alpha}^{(A)}[4, 4], E_{\alpha}^{(A)}[4, 5], E_{\alpha}^{(A)}[4, 6], E_{\alpha}^{(A)}[4, 7], \\ &E_{\alpha}^{(A)}[5, 0], E_{\alpha}^{(A)}[5, 1], E_{\alpha}^{(A)}[5, 2], E_{\alpha}^{(A)}[5, 3], E_{\alpha}^{(A)}[5, 4], E_{\alpha}^{(A)}[5, 5], E_{\alpha}^{(A)}[5, 6], E_{\alpha}^{(A)}[5, 7], \\ &E_{\alpha}^{(A)}[6, 0], E_{\alpha}^{(A)}[6, 1], E_{\alpha}^{(A)}[6, 2], E_{\alpha}^{(A)}[6, 3], E_{\alpha}^{(A)}[6, 4], E_{\alpha}^{(A)}[6, 5], E_{\alpha}^{(A)}[6, 6], E_{\alpha}^{(A)}[6, 7], \\ &E_{\alpha}^{(A)}[7, 0], E_{\alpha}^{(A)}[7, 1], E_{\alpha}^{(A)}[7, 2], E_{\alpha}^{(A)}[7, 3], E_{\alpha}^{(A)}[7, 4], E_{\alpha}^{(A)}[7, 5], E_{\alpha}^{(A)}[7, 6], E_{\alpha}^{(A)}[7, 7]\} \end{aligned}$$

```
In[ ]:= Block[{s, r}, s = FullSimplify[eα(A) /. sgtryeα(A), constraintVars];
r = FullSimplify[Transpose[s].η4488.s, constraintVars] - MatrixMetric44;
FullSimplify[r, constraintVars]
```

```
Out[ ]:=
```

$$\begin{aligned} &\{\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\} \end{aligned}$$

```
In[*]:= (* (eα(A) = eAa) // MatrixForm *)
```

```
In[*]:= (* eα(A)
Inverse[%] *)
```

```
In[*]:= (* (eα(A) = Inverse[eα(A) /. sge_<Subsuperscript>_<and>_<LeftParenthesis>_<RightParenthesis>]) //
MatrixForm *)
```

```
In[*]:= e(A)α
```

```
Out[*]:=
```

```
{ {E(A)α [0, 0], E(A)α [0, 1], E(A)α [0, 2], E(A)α [0, 3], E(A)α [0, 4], E(A)α [0, 5], E(A)α [0, 6], E(A)α [0, 7]},
{E(A)α [1, 0], E(A)α [1, 1], E(A)α [1, 2], E(A)α [1, 3], E(A)α [1, 4], E(A)α [1, 5], E(A)α [1, 6], E(A)α [1, 7]},
{E(A)α [2, 0], E(A)α [2, 1], E(A)α [2, 2], E(A)α [2, 3], E(A)α [2, 4], E(A)α [2, 5], E(A)α [2, 6], E(A)α [2, 7]},
{E(A)α [3, 0], E(A)α [3, 1], E(A)α [3, 2], E(A)α [3, 3], E(A)α [3, 4], E(A)α [3, 5], E(A)α [3, 6], E(A)α [3, 7]},
{E(A)α [4, 0], E(A)α [4, 1], E(A)α [4, 2], E(A)α [4, 3], E(A)α [4, 4], E(A)α [4, 5], E(A)α [4, 6], E(A)α [4, 7]},
{E(A)α [5, 0], E(A)α [5, 1], E(A)α [5, 2], E(A)α [5, 3], E(A)α [5, 4], E(A)α [5, 5], E(A)α [5, 6], E(A)α [5, 7]},
{E(A)α [6, 0], E(A)α [6, 1], E(A)α [6, 2], E(A)α [6, 3], E(A)α [6, 4], E(A)α [6, 5], E(A)α [6, 6], E(A)α [6, 7]},
{E(A)α [7, 0], E(A)α [7, 1], E(A)α [7, 2], E(A)α [7, 3], E(A)α [7, 4], E(A)α [7, 5], E(A)α [7, 6], E(A)α [7, 7]} }
```

```
In[*]:= (sge(A)α = Thread[Flatten[e(A)α] → Flatten[Inverse[eα(A) /. sgeα(A)]]) (* // Column *)
```

```
Out[*]:=
```

```
{E(A)α [0, 0] →  $\frac{1}{\sqrt{g[0][0][\times 0, \times 4]}}$ , E(A)α [0, 1] → 0, E(A)α [0, 2] → 0, E(A)α [0, 3] → 0, E(A)α [0, 4] → 0,
E(A)α [0, 5] → 0, E(A)α [0, 6] → 0, E(A)α [0, 7] → 0, E(A)α [1, 0] → 0, E(A)α [1, 1] →  $\frac{1}{\sqrt{g[1][1][\times 0, \times 4]}}$ ,
E(A)α [1, 2] → 0, E(A)α [1, 3] → 0, E(A)α [1, 4] → 0, E(A)α [1, 5] → 0, E(A)α [1, 6] → 0, E(A)α [1, 7] → 0,
E(A)α [2, 0] → 0, E(A)α [2, 1] → 0, E(A)α [2, 2] →  $\frac{1}{\sqrt{g[2][2][\times 0, \times 4]}}$ , E(A)α [2, 3] → 0, E(A)α [2, 4] → 0,
E(A)α [2, 5] → 0, E(A)α [2, 6] → 0, E(A)α [2, 7] → 0, E(A)α [3, 0] → 0, E(A)α [3, 1] → 0, E(A)α [3, 2] → 0,
E(A)α [3, 3] →  $\frac{1}{\sqrt{g[3][3][\times 0, \times 4]}}$ , E(A)α [3, 4] → 0, E(A)α [3, 5] → 0, E(A)α [3, 6] → 0, E(A)α [3, 7] → 0,
E(A)α [4, 0] → 0, E(A)α [4, 1] → 0, E(A)α [4, 2] → 0, E(A)α [4, 3] → 0, E(A)α [4, 4] →  $\frac{1}{\sqrt{-g[4][4][\times 0, \times 4]}}$ ,
E(A)α [4, 5] → 0, E(A)α [4, 6] → 0, E(A)α [4, 7] → 0, E(A)α [5, 0] → 0, E(A)α [5, 1] → 0, E(A)α [5, 2] → 0,
E(A)α [5, 3] → 0, E(A)α [5, 4] → 0, E(A)α [5, 5] →  $\frac{1}{\sqrt{-g[5][5][\times 0, \times 4]}}$ , E(A)α [5, 6] → 0, E(A)α [5, 7] → 0,
E(A)α [6, 0] → 0, E(A)α [6, 1] → 0, E(A)α [6, 2] → 0, E(A)α [6, 3] → 0, E(A)α [6, 4] → 0, E(A)α [6, 5] → 0,
E(A)α [6, 6] →  $\frac{1}{\sqrt{-g[6][6][\times 0, \times 4]}}$ , E(A)α [6, 7] → 0, E(A)α [7, 0] → 0, E(A)α [7, 1] → 0, E(A)α [7, 2] → 0,
E(A)α [7, 3] → 0, E(A)α [7, 4] → 0, E(A)α [7, 5] → 0, E(A)α [7, 6] → 0, E(A)α [7, 7] →  $\frac{1}{\sqrt{-g[7][7][\times 0, \times 4]}}$  }
```

```
In[*]:= (* Symbolize[sge(A)α] *)
```

```

In[*]:= (*Symbolize[sgtryeα(A)])*)
In[*]:= (*/.sge(A)α
        %/.sgtrye(A)α*)
In[*]:= (*FullSimplify[Transpose[e(A)α].η4488.e(A)α,constraintVars]-MatrixMetric44*)

```

For Spin (4, 4); τ tau; T16; OCTAD : Nash; Introduce the wave function, Ψ16, for this Universe::

O(4,4); evalues, evects of σ

```

In[*]:= {evalues, evects} = Eigensystem[σ]
Out[*]:=
{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
  {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
  {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}

In[*]:= (u = ExpandAll[ $\frac{1}{\sqrt{2}}$  evects]) // MatrixForm
Out[*] // MatrixForm =

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$


In[*]:= Table[u[[h]].σ.u[[h]], {h, 1, Length[u]}]
Out[*]:=
{-1, -1, -1, -1, 1, 1, 1, 1}

In[*]:= Table[(Transpose[u] [[h]]).σ.u[[h]], {h, 1, Length[u]}]
Out[*]:=
{-1, -1, -1, -1, 1, 1, 1, 1}

In[*]:= Table[(Transpose[u] [[A]]) == -σ.u[[A]], {A, 1, 4}]
Table[(Transpose[u] [[A]]) == σ.u[[A]], {A, 5, Length[u]}]
Out[*]:=
{True, True, True, True}
Out[*]:=
{True, True, True, True}

```

$$Ax = \left(\frac{\text{KroneckerProduct}[dX, X] - \text{KroneckerProduct}[X, dX]}{2} \right) \cdot \sigma;$$

```

Au =  $\left( \frac{\text{KroneckerProduct}[dU, U] - \text{KroneckerProduct}[U, dU]}{2} \right) . \sigma;$ 
Lsquared =  $\dot{x}^A \dot{x}_A = dX . \sigma . dX +$ 
 $X . \sigma . X \left( \frac{dU . \sigma . dU}{U . \sigma . U} - \left( \frac{U . \sigma . dU}{U . \sigma . U} \right)^2 \right) + \frac{2}{U . \sigma . U} \text{Tr}[\bar{\tau}^A . A x . \tau_A . Au]$ 
Timing[Simplify[ExpandAll[dY.η8.dY - Lsquared]]]
{10.733, 0}

```

```

In[ ]:= FullSimplify[Sqrt[Flatten[MatrixMetric44.η4488]], constraintVars]
Out[ ]:=
{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0, 0, 0,  $\sqrt{e^{2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}$ , 0, 0, 0, 0, 0, 0, 0,
 $\sqrt{e^{2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}$ , 0, 0, 0, 0, 0, 0, 0, 0,  $\sqrt{e^{2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}$ , 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,  $\sqrt{e^{-2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}$ , 0, 0, 0, 0, 0, 0, 0, 0,
 $\sqrt{e^{-2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}$ , 0, 0, 0, 0, 0, 0, 0, 0,  $\sqrt{e^{-2 a4 [H x4]}} \text{Sin}[6 H x0]^{1/6}$ }

```

SO(4), γ; M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+,+,+,+,-,-,-}, {x0, x1, x2, x3, x4, x5, x6, x7}

```

In[ ]:= Qa[h_, p_, q_] := Signature[{h, p, q, 4}];
Qb[h_, p_, q_] := ID4[[p, 4]] * ID4[[q, h]] - ID4[[p, h]] * ID4[[q, 4]];
SelfDualAntiSymmetric[h_, p_, q_] := Qa[h, p, q] - Qb[h, p, q];
AntiSelfDualAntiSymmetric[h_, p_, q_] := (Qa[h, p, q] + Qb[h, p, q]);

In[ ]:= Qa[1, 2, 3]
Out[ ]:=
1

In[ ]:= ε4[[1, 2, 3, 4]]
Out[ ]:=
1

In[ ]:= Do[s4by4[h] = Table[Table[SelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
{h, 1, 3}]

In[ ]:= Do[
t4by4[h] = Table[Table[AntiSelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
{h, 1, 3}]

In[ ]:= (*Protect[{s4by4[1], s4by4[2], s4by4[3], t4by4[1], t4by4[2], t4by4[3]}] *)

In[ ]:= Protect[s4by4, t4by4]
Out[ ]:=
{s4by4, t4by4}

```



```
In[*]:= {s4by4[#] // MatrixForm, MatrixForm[t4by4[#]]} & /@ Range[3]
```

```
Out[*]=
```

$$\left\{ \left\{ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right\}, \right. \\ \left. \left\{ \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\}, \left\{ \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right\} \right\}$$

```
In[*]:= Table[{{J, K}, MatrixForm[s4by4[J].t4by4[K]]}, {J, 1, 3}, {K, 1, 3}]
```

```
Out[*]=
```

$$\left\{ \left\{ \left\{ \{1, 1\}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}, \left\{ \{1, 2\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right\}, \left\{ \{1, 3\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \right\} \right\}, \right. \\ \left\{ \left\{ \{2, 1\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \right\}, \left\{ \{2, 2\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}, \left\{ \{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right\} \right\}, \\ \left\{ \left\{ \{3, 1\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{3, 3\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\} \right\} \right\}$$

```
In[*]:= Do[st[J, K] = s4by4[J].t4by4[K], {J, 1, 3}, {K, 1, 3}]
```

```
In[*]:= Table[{{J, K}, MatrixForm[st[J, K]]}, {J, 1, 3}, {K, 1, 3}]
```

```
Out[*]=
```

$$\left\{ \left\{ \left\{ \{1, 1\}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}, \left\{ \{1, 2\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right\}, \left\{ \{1, 3\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \right\} \right\}, \right. \\ \left\{ \left\{ \{2, 1\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \right\}, \left\{ \{2, 2\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}, \left\{ \{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right\} \right\}, \\ \left\{ \left\{ \{3, 1\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{3, 3\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\} \right\} \right\}$$

```
In[*]:= Protect[st]
```

```
Out[*]=
```

```
{st}
```

```

In[ ]:= Table[{J, K}, MatrixForm[st[J, K] - Transpose[st[J, K]]], {J, 1, 3}, {K, 1, 3}]
Out[ ]=

$$\left\{ \left\{ \left\{ 1, 1 \right\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \left\{ 1, 2 \right\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \left\{ 1, 3 \right\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\} \right\},$$


$$\left\{ \left\{ \left\{ 2, 1 \right\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \left\{ 2, 2 \right\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \left\{ 2, 3 \right\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\} \right\},$$


$$\left\{ \left\{ \left\{ 3, 1 \right\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \left\{ 3, 2 \right\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \left\{ 3, 3 \right\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\} \right\}$$

In[ ]:= Permutations[Range[3], 2]
Out[ ]=
{{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}
In[ ]:= KroneckerProduct[Range[3], Range[3]]
Out[ ]=
{{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}
In[ ]:= Table[{J, K}, {J, 1, 3}, {K, 1, 3}]
st[##] & /@%
st[##] & @@ %%
Out[ ]=
{{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}}
Out[ ]=
{st[{{1, 1}, {1, 2}, {1, 3}}], st[{{2, 1}, {2, 2}, {2, 3}}], st[{{3, 1}, {3, 2}, {3, 3}}]}
Out[ ]=
st[{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}]

```

```
In[ ]:= Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
Length[%]
```

```
Out[ ]:=
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

```
Out[ ]:=
16
```

```
In[ ]:= Unprotect[γ];
γ = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
```

```
Out[ ]:=
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

```
In[ ]:= Length[γ]
```

```
Out[ ]:=
16
```

```

In[ ]:= Protect[γ]
Out[ ]=
{γ}

In[ ]:= γ[[1]]
Out[ ]=
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}

In[ ]:= γ[[-1]]
Out[ ]=
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}

```

Self Dual Anti-Symmetric

```

In[ ]:= Table[ $\left(\frac{1}{2} \sum_{j1=1}^4 \left(\sum_{j2=1}^4 \epsilon4[p, q, j1, j2] \times s4by4[h][j1, j2]\right)\right) - s4by4[h][p, q],$ 
{h, 1, 3}, {q, 4}, {p, 4}]
Out[ ]=
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}

```

Anti SelfDual Anti-Symmetric

```

In[ ]:= Table[ $\left(\frac{1}{2} \sum_{j1=1}^4 \left(\sum_{j2=1}^4 \epsilon4[p, q, j1, j2] \times t4by4[h][j1, j2]\right)\right) + t4by4[h][p, q],$ 
{h, 1, 3}, {q, 4}, {p, 4}]
Out[ ]=
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}

```

SO(4,4), Spin(4,4) = $\overline{SO(4,4)}$, τ ; if using xact: M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+,+,+,-,-,-,-}, {x0, x1, x2, x3, x4, x5, x6, x7}

```

In[ ]:= τ == τ
Out[ ]=
False

In[ ]:= τ
Head[%]
Out[ ]=
τ
Out[ ]=
OverBar

```

```

In[ ]:=  $\overline{\tau}$ 
Head[%]
Out[ ]=  $\overline{\tau}$ 
Out[ ]=
ParsedBoxWrapper

In[ ]:=  $\overline{\tau}$ 
Head[%]
Out[ ]=  $\overline{\tau}$ 
Out[ ]=
OverBar

In[ ]:= Symbol
Out[ ]=
Symbol

In[ ]:=  $\overline{\tau} === \overline{\tau}$ 
Out[ ]=
True

In[ ]:=  $\overline{\tau} === \overline{\tau}$ 
Out[ ]=
False

In[ ]:=  $\eta_{4488}$  // MatrixForm
Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


```

```

In[ ]:=  $\eta_{4488} = \text{DiagonalMatrix}[\text{RotateRight}[\text{evalues}, 4]]$ 

```

```

Out[ ]=
True

```

```

In[ ]:= DiagonalMatrix[RotateRight[evalues, 3]] // MatrixForm

```

```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


```

these also yield correct reps:

```

ζ[0] = ID8;
Table[ζ[7 - h] = ArrayFlatten[{{0, -t4by4[h]}, {t4by4[h], 0}}, {h, 1, 3}];
Table[ζ[h] = ArrayFlatten[{{0, s4by4[h]}, {s4by4[h], 0}}, {h, 1, 3}];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm

```

```

ζ[0] = ID8;
Table[ζ[h] = ArrayFlatten[{{0, t4by4[h]}, {t4by4[h], 0}}, {h, 1, 3}];
Table[ζ[7 - h] = ArrayFlatten[{{0, -s4by4[h]}, {s4by4[h], 0}}, {h, 1, 3}];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm

```

```

ζ[0] = ID8;
Table[ζ[h] = ArrayFlatten[{{0, t4by4[h]}, {t4by4[h], 0}}, {h, 1, 3}];
Table[ζ[7 - h] = ArrayFlatten[{{0, s4by4[h]}, {-s4by4[h], 0}}, {h, 1, 3}];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm

```

```

In[*]:= ζ[0] = ID8;
Table[
  ζ[7 - h] = ArrayFlatten[{{0, t4by4[h]}, {-t4by4[h], 0}}, {h, 1, 3}];
Table[ζ[h] = ArrayFlatten[{{0, s4by4[h]}, {s4by4[h], 0}}, {h, 1, 3}];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm

```

```

Out[*]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


```

```

In[*]:= σ = ζ[1].ζ[2].ζ[3]

```

```

Out[*]=
True

```

```

In[*]:= Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\zeta[A] \cdot \zeta[B] + \zeta[B] \cdot \zeta[A]) = -\eta 4488 [A + 1, B + 1] * ID8$ ]]},
  {A, 1, 7}, {B, 1, 7}]

```

```

Out[*]=
{{{1, 1, True}, {1, 2, True}, {1, 3, True}, {1, 4, True},
  {1, 5, True}, {1, 6, True}, {1, 7, True}}, {{2, 1, True}, {2, 2, True},
  {2, 3, True}, {2, 4, True}, {2, 5, True}, {2, 6, True}, {2, 7, True}},
{{3, 1, True}, {3, 2, True}, {3, 3, True}, {3, 4, True}, {3, 5, True}, {3, 6, True},
  {3, 7, True}}, {{4, 1, True}, {4, 2, True}, {4, 3, True}, {4, 4, True},
  {4, 5, True}, {4, 6, True}, {4, 7, True}}, {{5, 1, True}, {5, 2, True},
  {5, 3, True}, {5, 4, True}, {5, 5, True}, {5, 6, True}, {5, 7, True}},
{{6, 1, True}, {6, 2, True}, {6, 3, True}, {6, 4, True}, {6, 5, True},
  {6, 6, True}, {6, 7, True}}, {{7, 1, True}, {7, 2, True},
  {7, 3, True}, {7, 4, True}, {7, 5, True}, {7, 6, True}, {7, 7, True}}}

```

```
In[ ]:=  $\eta$ 4488 // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[ ]:= Table[{A, - $\eta$ 4488[[A + 1, A + 1]] *  $\xi$ [A] == Transpose[ $\xi$ [A]]}, {A, 1, 7}]
```

```
Out[ ]:=
```

```
{ {1, True}, {2, True}, {3, True}, {4, True}, {5, True}, {6, True}, {7, True} }
```

```
In[ ]:= ( $\sigma$  ==  $\xi$ [1]. $\xi$ [2]. $\xi$ [3])
```

```
Out[ ]:=
```

```
True
```

```
In[ ]:=  $\sigma$ . $\xi$ [1]. $\xi$ [2]. $\xi$ [3] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[ ]:=  $\tau$ [0] = ID8;
Do[
  { $\tau$ [A] = FullSimplify[ExpandAll[( $\xi$ [A])]}},
  {A, 1, 7}]
(*Do[
  { $\tau$ [A] = FullSimplify[ExpandAll[-( $\sigma$ . $\xi$ [1].
     $\xi$ [2]. $\xi$ [3]. $\xi$ [A])]}}, {A, 1, 7}]*)
```

```
In[ ]:= ( $\sigma$  ==  $\tau$ [1]. $\tau$ [2]. $\tau$ [3])
```

```
Out[ ]:=
```

```
True
```

```
In[ ]:=  $\overline{\tau}[0] = \text{ID8};$   
Do[{ $\overline{\tau}[A] = \text{FullSimplify}[\text{ExpandAll}[\text{Transpose}[\tau[A]] \cdot \sigma]]$ }, {A, 1, 7}]
```

```
In[ ]:=  $\sigma \cdot \overline{\tau}[\#] = \text{Transpose}[\sigma \cdot \tau[\#]] \& /@ \text{Range}[0, 7]$ 
```

```
Out[ ]:= {True, True, True, True, True, True, True, True}
```

```
In[ ]:= Table[{A, B, FullSimplify[  
ExpandAll[ $\frac{1}{2} (\tau[A] \cdot \overline{\tau}[B] + \tau[B] \cdot \overline{\tau}[A]) = \eta 4488 [A + 1, B + 1] * \text{ID8}$ ]]},  
{A, 0, 7}, {B, 0, 7}] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 1 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 2 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 3 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 4 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 5 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 6 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 7 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 7 \\ \text{True} \end{pmatrix} \end{pmatrix}$$

```
In[ ]:=  $\tau[7] // \text{MatrixForm}$ 
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


```
In[*]:= (Ω = σ.τ[7]) // MatrixForm
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[*]:= Ω == τ[4].τ[5].τ[6]
```

```
Out[*]=
```

True

```
In[*]:= τ[5].τ[6].τ[7] // MatrixForm
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

```
In[*]:= (σΩ = σ.Ω) // MatrixForm
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[*]:= Inverse[σΩ] // MatrixForm
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[*]:= τ[1].τ[2].τ[3].τ[4].τ[5].τ[6].τ[7] == τ[0] == ID8
```

```
Out[*]=
```

True

```
In[*]:= (σ == τ[1].τ[2].τ[3])
```

```
Out[*]=
```

True

```
In[*]:= (σ == τ[4].τ[5].τ[6].τ[7])
```

```
Out[*]=
True
```

```
In[*]:= τ[1].τ[2].τ[3].τ[#] == -Transpose[τ[1].τ[2].τ[3].τ[#]] & /@ Range[0, 7]
```

```
Out[*]=
{False, True, True, True, True, True, True, True}
```

```
In[*]:= σ.τ[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
```

```
Out[*]=
{True, True, True, True, True, True, True, True}
```

```
In[*]:= τ[5].τ[6].τ[7].τ[#] == Transpose[τ[5].τ[6].τ[7].τ[#]] & /@
Range[0, 7]
```

```
Out[*]=
{False, True, True, True, False, True, True, True}
```

```
In[*]:= τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```

```
Out[*]=
{True, True, True, True, True, False, False, False}
```

```
In[*]:= τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```

```
Out[*]=
{True, False, False, False, False, True, True, True}
```

```
In[*]:= τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```

```
Out[*]=
{True, True, True, True, True, False, False, False}
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}. \quad ; \quad 0 \ (4, 4) : \text{ SAB } ;$$

```
covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]
```

Need these in order to define the Lagrangian for the "universe," later :

```
In[*]:= (σ16 = T16^A[0].T16^A[1].T16^A[2].T16^A[3]) // MatrixForm
```

```
Out[*]//MatrixForm=
T16^A[0].T16^A[1].T16^A[2].T16^A[3]
```

```
In[ ]:= Table[T16^A[A1] = ArrayFlatten[{{0,  $\overline{\tau}$ [A1]}, { $\tau$ [A1], 0}}],
             {A1, 0, 7}];
```

```
In[ ]:= (T16^A[8] = T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]) // MatrixForm
```

```
Out[ ]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```

```
In[ ]:= T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#] ==
        -Transpose[T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#]] & /@ Range[0, 8]
```

```
Out[ ]:=
{True, True, True, True, True, True, True, True, False}
```

```
In[ ]:= T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#] ==
        Transpose[T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#]] & /@ Range[0, 8]
```

```
Out[ ]:=
{True, True, True, True, True, True, True, True, True}
```

```
In[ ]:= T16^A[8] ==  $\sigma$ 16.T16^A[4].T16^A[5].T16^A[6].T16^A[7]
```

```
Out[ ]:=
True
```

```
In[ ]:= T16^A[#] == Transpose[T16^A[#]] & /@ Range[0, 8]
```

```
Out[ ]:=
{True, True, True, True, False, False, False, False, True}
```

```
In[ ]:= T16^A[#] == -Transpose[T16^A[#]] & /@ Range[0, 8]
```

```
Out[ ]:=
{False, False, False, False, True, True, True, True, False}
```

```
In[ ]:=  $\sigma$ 16.T16^A[#] == -Transpose[ $\sigma$ 16.T16^A[#]] & /@ Range[0, 7]
```

```
Out[ ]:=
{True, True, True, True, True, True, True, True}
```

```
In[*]:= (covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]) // MatrixForm
```

[illegible]

```
In[*]:= σ16.covariantDiffMatrix == Transpose[σ16.covariantDiffMatrix]
```

```
Out[•]= True
```

```
In[•]:= ID16 // MatrixForm
```

[illegible]

```
In[*]:= (σ16(*=T16^A[0].T16^A[1].T16^A[2].T16^A[3]*)) // MatrixForm
```

```
Out[*]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

```
In[*]:= σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

```
Out[*]=
True
```

later, after defining $e_{(A)}^\alpha$:

```
In[*]:= (*Table[T16^α[α1-1]=
Sum[(e_{(A)}^α [[α1,A1]] ) T16^A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

```
In[*]:= (*(T16^α[8]=T16^α[0].T16^α[1].T16^α[2].T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7])//
MatrixForm*)
```

```
In[*]:= (*eLpairs*)
```

```
In[*]:= (*Ω16a=Array[0&,{16,16}];
{1+#[[1]],1+#[[2]]}&/@eLpairs
(Ω16a[[#[[1]],#[[2]]]=1)&/@%*)
```

```
In[*]:= (*Ω16a//MatrixForm*)
```

```
In[*]:= (*(Ω16=σ16.(Ω16a+Transpose[Ω16a]))//MatrixForm*)
```

```
In[*]:= T16^A[0].T16^A[4] // MatrixForm
T16^A[0].T16^A[4].σ16
(Ω16 = %) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[•] =

[illegible]

`Out[•]//MatrixForm=`

[illegible]

$$In[\bullet] := \text{Det}[\Omega_{16}]$$

Out[•] =

1

```
In[ ]:=  $\sigma_{16}.\Omega_{16}$  // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:=  $\sigma_{16}.\mathbf{T16}^A[\#] == -\text{Transpose}[\sigma_{16}.\mathbf{T16}^A[\#]] \& /@$   
 $\text{Range}[0, 7]$ 
```

```
Out[ ]:=  
{True, True, True, True, True, True, True, True}
```

```
In[ ]:=  $\sigma_{16}.\mathbf{T16}^A[\#] == -\text{Transpose}[\sigma_{16}.\mathbf{T16}^A[\#]] \& /@ \text{Range}[0, 7]$ 
```

```
Out[ ]:=  
{True, True, True, True, True, True, True, True}
```

```
In[ ]:=  $\sigma_{16}.\mathbf{T16}^A[\#] == -\text{Transpose}[\sigma_{16}.\mathbf{T16}^A[\#]] \& /@ \text{Range}[0, 7]$ 
```

```
Out[ ]:=  
{True, True, True, True, True, True, True, True}
```

```
In[*]:= Table[{A1, B1}, FullSimplify[
  ExpandAll[ $\frac{1}{2}$  (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) ==
   $\eta_{4488}$ [A1 + 1, B1 + 1] * ID16]]],
  {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

```
Out[*] // MatrixForm =

$$\begin{pmatrix} \begin{pmatrix} \{0, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{1, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{2, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{3, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{4, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{5, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{6, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{7, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 7\} \\ \text{True} \end{pmatrix} \end{pmatrix}$$

```

```
In[*]:=  $\Psi$ 16upper = Take[ $\Psi$ 16, 8]
 $\Psi$ 16lower = Take[ $\Psi$ 16, -8]
 $\Psi$ 16 = Flatten[{ $\Psi$ 16upper,  $\Psi$ 16lower}]
```

```
Out[*] = {f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
  f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4]}
```

```
Out[*] = {f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
  f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

```
Out[*] = True
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
Table[T16^A[A1] = ArrayFlatten[{{0,  $\bar{\tau}$ [A1]}, { $\tau$ [A1], 0}}],
  {A1, 0, 7}]
 $\sigma$ 16 = ArrayFlatten[{{- $\sigma$ , 0}, {0,  $\sigma$ }}]
 $\begin{pmatrix} 0 & \bar{\tau}[A1] \\ \tau[A1] & 0 \end{pmatrix} \begin{pmatrix} \Psi16upper \\ \Psi16lower \end{pmatrix} = \begin{pmatrix} \bar{\tau}[A1].\Psi16lower \\ \tau[A1].\Psi16upper \end{pmatrix}$ 
```


$$\sigma_{16} \cdot \begin{pmatrix} 0 & \tau[A1] \\ \tau[A1] & 0 \end{pmatrix} \begin{pmatrix} \Psi_{16\text{upper}} \\ \Psi_{16\text{lower}} \end{pmatrix} = \begin{pmatrix} -\sigma \tau[A1] \cdot \Psi_{16\text{lower}} \\ \sigma \tau[A1] \cdot \Psi_{16\text{upper}} \end{pmatrix}$$

$$\Psi_{16} \cdot \sigma_{16} \cdot \begin{pmatrix} 0 & \tau[A1] \\ \tau[A1] & 0 \end{pmatrix} \cdot \Psi_{16} = \begin{pmatrix} -\Psi_{16\text{upper}} \cdot \sigma \tau[A1] \cdot \Psi_{16\text{lower}} \\ \Psi_{16\text{lower}} \cdot \sigma \tau[A1] \cdot \Psi_{16\text{upper}} \end{pmatrix}$$

```
In[*]:= (*σ16.T16^[#]&/@Range[0,7]*)
```

```
In[*]:= (*
(T16^A[8]=FullSimplify[
T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]])//MatrixForm*)
```

```
In[*]:= η4488
```

```
Out[*]=
```

```
{ {1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0} }
```

```
In[*]:= η99 = DiagonalMatrix[{1, 1, 1, 1, -1, -1, -1, -1, 1}]
```

```
Out[*]=
```

```
{ {1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1} }
```

```
In[*]:= (*Table[{A1,B1,
FullSimplify[ExpandAll[1/2 ( T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]) == η99[[A1,B1]]*ID16]]},
{A1,0, 8},{B1,0, 8}]]//MatrixForm*)
```

```
In[*]:= (*Table[1/4 (T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]), {A1, 8},{B1,8}]**)
```

```
In[*]:= (Pl = 1/2 (ID16 - T16^A[8])) // MatrixForm
```

```
Out[*]//MatrixForm=
```

```
{ {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1} }
```

```
In[ ]:= 
$$\left( \mathbf{P}_R = \frac{1}{2} (\mathbf{ID16} + \mathbf{T16}^A[8]) \right) // \text{MatrixForm}$$

```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[ ]:= {ID16 == P_L + P_R, P_L.P_L == P_L, P_L.P_R == P_R.P_L == ZERO16}
```

```
Out[ ]=
```

{True, True, True}

```
In[ ]:=
```

```
SAB = Table[
  
$$\frac{1}{4} (\mathbf{T16}^A[A1] . \mathbf{T16}^A[B1] - \mathbf{T16}^A[B1] . \mathbf{T16}^A[A1]), \{A1, 0, 7\}, \{B1, 0, 7\}];$$

```

```
In[ ]:= SAB = Table[
$$\frac{1}{4} (\mathbf{T16}^A[A1] . \mathbf{T16}^A[B1] - \mathbf{T16}^A[B1] . \mathbf{T16}^A[A1]),$$

  {A1, 0, 7}, {B1, 0, 7}];
```

```
In[ ]:= ParallelTable[FullSimplify[σ16.SAB[[A1, B1]] == -Transpose[σ16.SAB[[A1, B1]]],
  {A1, 1, 8}, {B1, 1, 8}] // Flatten // Union
```

```
Out[ ]=
```

{True}

```
In[ ]:= ParallelTable[FullSimplify[SAB[[A1, B1]].SAB[[A2, B2]] - SAB[[A2, B2]].SAB[[A1, B1]] ==
  - (η4488[[A1, A2]] × SAB[[B1, B2]] - η4488[[A1, B2]] × SAB[[B1, A2]] -
    η4488[[B1, A2]] × SAB[[A1, B2]] + η4488[[B1, B2]] × SAB[[A1, A2]])],
  {A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}] // Flatten // Union
```

```
Out[ ]=
```

{True}

```
In[*]:= ParallelTable[FullSimplify[SAB[[A1, B1]].T16^A[B2 - 1] - T16^A[B2 - 1].SAB[[A1, B1]] ==
  (-η4488[[B2, A1]] T16^A[B1 - 1] + η4488[[B2, B1]] T16^A[A1 - 1])],
  {A1, 1, 8}, {B1, 1, 8}, {B2, 1, 8}] // Flatten // Union
```

```
Out[*]:=
{True}
```

```
In[*]:= (*ParallelTable[{ {A1,B1},FullSimplify[σ16.SAB[[A1,B1]]== -Transpose[σ16.SAB[[A1,B1]]] }},
  {A1,1, 8},{B1,1,8}]*)
```

```
In[*]:= (*ParallelTable[{ {A1,B1,A2,B2},
  FullSimplify[SAB[[A1,B1]].SAB[[A2,B2]] - SAB[[A2,B2]].SAB[[A1,B1]] == - (η4488[[A1,A2]] SAB[[B1,B2]] -
  η4488[[A1,B2]] SAB[[B1,A2]] - η4488[[B1,A2]] SAB[[A1,B2]] + η4488[[B1,B2]] SAB[[A1,A2]]) ]},
  {A1,1, 7},{B1,A1+1,8},{A2,1, 7},{B2,A2+1,8}]*)
```

```
In[*]:= (*ParallelTable[{ {A1,B1,B2},FullSimplify[SAB[[A1,B1]].T16^A[B2-1] - T16^A[B2-1].SAB[[A1,B1]] ==
  (-η4488[[B2,A1]] T16^A[B1-1] + η4488[[B2,B1]] T16^A[A1-1]) ]}, {A1,1, 8},{B1,1,8},{B2,1,8}]*)
```

```
In[*]:= Do[SAB[A1][B1] =  $\frac{1}{4} (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1])$ ,
  {A1, 0, 7}, {B1, 0, 7}];
```

```
In[*]:= SAB[-1 + 1][-1 + 2] == SAB[[1, 2]]
```

```
Out[*]:=
True
```

```
In[•]:= σ16.SAB[[1, 2]] // MatrixForm
```

Out[•]//MatrixForm=

[illegible]

```
In[•]:= σ16.sAB[1][2] // MatrixForm
```

Out[•]//MatrixForm=

[illegible]

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3, 3)$ Lie algebra (using the physics convention where generators are Hermitian) are **given by the formula:** ⓘ

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. ⓘ

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The $\mathfrak{so}(4, 4)$ Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature $(4, 4)$. ⓘ

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) ⓘ

Step 2: State the general commutation relation

The commutation relations for the generators of the $\mathfrak{so}(p, q)$ Lie algebra are a generalization of the $\mathfrak{so}(n)$ algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. ⓘ

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the $\mathfrak{so}(4, 4)$ Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a $(4, 4)$ signature (four plus signs and four minus signs, or vice versa). ⓘ

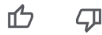
(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). ⓘ

what are the commutation relation for the generators of $so(4,4)$?

The commutation relations for the generators $J_{\mu\nu}$ of the $so(4,4)$ Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: \varnothing

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature $(4,4)$ (or some other convention, the structure constants are the same as for $so(8)$ in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



→ Why is the metric signature significant for $so(4,4)$ commutation relations?

Why is the metric signature significant for $so(4,4)$ commutation relations?

The metric signature is significant for the $so(p, q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). \varnothing

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$
 The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4,4)$ are distinct from $so(8)$ (which has a $(8,0)$ signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like $(4,4)$ or the more common $(1,3)$ Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

$$\ln[\bullet] := \eta_{4488}$$

Out[•] =

$$\begin{aligned} & \{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 0, 0, -1, 0\}, \{0, 0, 0, 0, 0, 0, 0, -1\} \end{aligned}$$

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

$$\ln[\bullet] := \text{SAB}[6, 2] \cdot \text{SAB}[6, 7] - \text{SAB}[6, 7] \cdot \text{SAB}[6, 2] = \eta_{4488}[6, 6] \times \text{SAB}[7, 2]$$

Out[•] =

True

$$In[\bullet] := \{\{6, 2, 6, 7\}, \text{False}\}$$

Out[•] =

```
{{6, 2, 6, 7}, False}
```

$$\ln[\bullet] := \text{SAB}[[1, 2]] \cdot \text{SAB}[[1, 3]] - \text{SAB}[[1, 3]] \cdot \text{SAB}[[1, 2]] == \eta_{4488}[[1, 1]] \times \text{SAB}[[2, 3]]$$

Out[•]=

False

$$In[\bullet] := \mathbf{sAB}[0][0]$$

Out[•]=

[illegible]

```
In[*]:= SAB[0][1]
```

```
Out[*]=
```

```
{ {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {-1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0} }
```

```
In[*]:= Table[Transpose[T16^A[8].SAB[A1][B1]] == T16^A[8].SAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
```

```
Out[*]=
```

```
{{True, False, False, False, True, True, True, True},
 {False, True, False, False, True, True, True, True},
 {False, False, True, False, True, True, True, True},
 {False, False, False, True, True, True, True, True},
 {True, True, True, True, True, False, False, False},
 {True, True, True, True, False, True, False, False},
 {True, True, True, True, False, False, True, False},
 {True, True, True, True, False, False, False, True}}
```

```
In[*]:= Table[Transpose[T16^A[7].SAB[A1][B1]] == T16^A[7].SAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
```

```
Out[*]=
```

```
{{True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {False, False, False, False, True, True, True, False},
 {False, False, False, False, True, True, True, False},
 {False, False, False, False, True, True, True, False},
 {True, True, True, True, False, False, False, True}}
```



```
In[*]:= Table[Transpose[σ16.sAB[A1][B1]] == σ16.sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
```

```
Out[*]:=
```

```
{{True, False, False, False, False, False, False, False},
 {False, True, False, False, False, False, False, False},
 {False, False, True, False, False, False, False, False},
 {False, False, False, True, False, False, False, False},
 {False, False, False, False, True, False, False, False},
 {False, False, False, False, False, True, False, False},
 {False, False, False, False, False, False, True, False},
 {False, False, False, False, False, False, False, True}}
```

```
In[*]:= Table[Transpose[σ16.SAB[[A1, B1]]] == σ16.SAB[[A1, B1]], {A1, 1, 8}, {B1, 1, 8}]
```

```
Out[*]:=
```

```
{{True, False, False, False, False, False, False, False},
 {False, True, False, False, False, False, False, False},
 {False, False, True, False, False, False, False, False},
 {False, False, False, True, False, False, False, False},
 {False, False, False, False, True, False, False, False},
 {False, False, False, False, False, True, False, False},
 {False, False, False, False, False, False, True, False},
 {False, False, False, False, False, False, False, True}}
```

```
In[*]:= SAB[[1, 2]][[1]]
```

```
Out[*]:=
```

```
{0, 0, 0, 0, 0, 0, 0, 0,  $\frac{1}{2}$ , 0, 0, 0, 0, 0, 0, 0}
```

```
In[*]:= SAB[[1, 2]][[2]]
```

```
Out[*]:=
```

```
{0, 0, 0, 0, 0, 0, 0, 0,  $\frac{1}{2}$ , 0, 0, 0, 0, 0, 0, 0}
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
In[*]:= SAB1 = Table[Take[SAB[[A1, B1]][[C1]], 8], {A1, 8}, {B1, 8}, {C1, 1, 8}];
```

```
In[*]:= SAB2 = Table[Take[SAB[[A1, B1]][[C1]], -8], {A1, 8}, {B1, 8}, {C1, 9, 16}];
```

```
In[*]:= SAB[[1, 2]] == ArrayFlatten[{{SAB1[[1, 2]], 0}, {0, SAB2[[1, 2]]}}
```

```
Out[*]:=
```

```
True
```

```
In[*]:= (*Table[
  SAB[[A1,B1]]== ArrayFlatten[{{SAB1[[A1,B1]],0},{0,SAB2[[A1,B1]]}},
  {A1,0, 7},{B1,0,7}]*)
```

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \rightarrow SO(4,4)$ is given by

$$(2.8) \quad 8L^{A'}_{B'}(g) = \text{tr} \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}_{C'} G_{A'B'} L^{B'}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}_{B'} \tau^{B'} = \tilde{D}^{(2)} \bar{\tau}^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

In[*]:= SAB2[[1, 2]]

Out[*]=

$$\begin{aligned} & \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \right\}, \left\{ 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0 \right\}, \right. \\ & \left\{ 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0 \right\}, \left\{ 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0 \right\}, \\ & \left. \left\{ 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \end{aligned}$$

In[*]:= \tau[0]

Out[*]=

$$\begin{aligned} & \{ \{ 1, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 1, 0, 0, 0, 0, 0, 0, 0 \}, \\ & \{ 0, 0, 1, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \}, \\ & \{ 0, 0, 0, 0, 0, 1, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 1, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 1, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 1 \} \} \end{aligned}$$

```

In[*]:=
(*Block[{b,A,Fa(μ),F(μ)a,ψ,Ψ},
  b=Array[A,{8}];
  ψ[1]=Array[Ψ[1],{8}];
  ψ[2]=Array[Ψ[2],{8}];
  Fa(μ)=Table[ $\left(\frac{1}{\sqrt{\psi[1].\sigma.\psi[1]}}\psi[1].\sigma.(\overline{\tau}[\mu L])\right)$ ,{μL,1,8}];
  F(μ)a=Transpose[Table[ $\left(\frac{1}{\sqrt{\psi[1].\sigma.\psi[1]}}\eta_{4488}[\mu L,\mu L]\tau[\mu L].\psi[1]\right)$ ,{μL,1,8}]];
  b===FullSimplify[Fa(μ).ψ[2]/.Thread[ψ[2]→F(μ)a.b]]*)

```

```

In[*]:=
(*Block[{b,A,B,q,F(μ)a,ψ,φ,Ψ},
  b=Array[A,{8}];
  ψ[1]=Array[Ψ[1],{8}];
  φ[1]=Array[Ψ[2],{8}];
  q=ParallelTable[FullSimplify[
     $\left(\frac{1}{\sqrt{\psi[1].\sigma.\psi[1]}}\psi[1].\sigma.\frac{1}{2}(\overline{\tau}[A1].\tau[B1]-\overline{\tau}[B1].\tau[A1]).\varphi[1]\right)$ ],{A1,1,8},{B1,1,8}];
  B=FullSimplify[ $\left(\frac{1}{\sqrt{\psi[1].\sigma.\psi[1]}}\text{ParallelSum}[\eta_{4488}[\mu L,\mu L]\tau[\mu L]b[\mu L],\{\mu L,1,8\}].\psi[1]\right)$ ];
  FullSimplify[q/.Thread[φ[1]→B]]*)

```

```

In[*]:=
(*Block[{b,A,B,q,F(μ)a,ψ,φ,Ψ},
  b=Array[A,{8}];
  ψ[1]=Array[Ψ[1],{8}];
  φ[1]=Array[Ψ[2],{8}];
  q=ParallelTable[FullSimplify[(ψ[1].σ.(τ̄[A1].τ[B1]).φ[1])],{A1,1,8},{B1,1,8}];
  B=FullSimplify[(ParallelSum[η4488[μL,μL]τ[μL]b[μL],{μL,1,8}].ψ[1])];
  FullSimplify[ $\frac{1}{\psi[1].\sigma.\psi[1]}$ q/.Thread[φ[1]→B]]*)

```

$T16^\alpha$

In[*]:= **ssgm4488**

Out[*]=

$\{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&),$
 $g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&),$
 $g[1][1] \rightarrow (e^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&),$
 $g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&),$
 $g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&),$
 $g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&),$
 $g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&),$
 $g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&),$
 $g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&),$
 $g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&),$
 $g[5][5] \rightarrow (-e^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&),$
 $g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&),$
 $g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&),$
 $g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&),$
 $g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&)\}$

In[*]:= $e_{(A)}^\alpha /. \text{sge}_{(A)}^\alpha$

Out[*]=

$\left\{ \left\{ \frac{1}{\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \right.$
 $\left\{ 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0 \right\},$
 $\left\{ 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\},$
 $\left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}} \right\} \right\}$

In[*]:=

Table[T16 $^\alpha$ [$\alpha 1 - 1$] =
Sum[($(e_{(A)}^\alpha /. \text{sge}_{(A)}^\alpha)$)[$\alpha 1, A1$]] T16 A [$A1 - 1$], { $A1, 1, 8$ }},
{ $\alpha 1, 1, 8$ }}]; (* /. ssgm4488 *)

In[*]:=

(T16 $^\alpha$ [8] = T16 $^\alpha$ [0] . T16 $^\alpha$ [1] . T16 $^\alpha$ [2] . T16 $^\alpha$ [3] . T16 $^\alpha$ [4] . T16 $^\alpha$ [5] . T16 $^\alpha$ [6] . T16 $^\alpha$ [7]);

In[*]:=

(*T16 $^\alpha$ [8]==T16 A [8]*Sec[6 H x0]*)

CHECK

Table[T16 $^\alpha$ [$\alpha 1 - 1$] = Sum[($e_{(A)}^\alpha$)[$\alpha 1, A1$]] T16 A [$A1 - 1$], { $A1, 1, 8$ }},

$\{\alpha_1, 1, 8\}$]; and . . . :

```
In[*]:= (*Table[T16α[α1-1]=
Sum[(eα(A)[[α1,A1]] T16A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

```
In[*]:= (*(T16α[8]=T16α[0].T16α[1].T16α[2].T16α[3].T16α[4].T16α[5].T16α[6].T16α[7])//
MatrixForm*)
```

```
In[*]:= Table[{ {A1, B1}, FullSimplify[
ExpandAll[ $\frac{1}{2}$  (T16A[A1].T16A[B1] + T16A[B1].T16A[A1]) ==
η4488[[A1 + 1, B1 + 1]] * ID16]]},
{A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[*]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} \{0, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{1, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{2, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{3, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{4, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{5, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{6, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{7, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 7\} \\ \text{True} \end{pmatrix} \end{pmatrix}$$

```
In[*]:= Table[{ $\alpha 1$ ,  $\beta 1$ }, FullSimplify[
   $\frac{1}{2}$  FullSimplify[(T16 $^\alpha$ [ $\alpha 1$ ].T16 $^\alpha$ [ $\beta 1$ ] + T16 $^\alpha$ [ $\beta 1$ ].T16 $^\alpha$ [ $\alpha 1$ )] /.
    ssgm4488), constraintVars] ==
  Inverse[MatrixMetric44][[ $\alpha 1 + 1$ ,  $\beta 1 + 1$ ] * ID16]]},
  {{ $\alpha 1$ , 0, 7}, { $\beta 1$ , 0, 7}}] // MatrixForm
```

```
Out[*] // MatrixForm =
```

$$\begin{pmatrix} \begin{pmatrix} \{0, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{0, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{1, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{2, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{3, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{4, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{5, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{5, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{6, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{6, 7\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{7, 0\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 4\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 5\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 6\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{7, 7\} \\ \text{True} \end{pmatrix} \end{pmatrix}$$

```
In[*]:= MatrixMetric44 // MatrixForm
```

```
Out[*] // MatrixForm =
```

$$\begin{pmatrix} \text{Cot}[6 H x 0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[*]:= ((e $^\alpha_{(A)}$  /. sgtrye $^\alpha_{(A)}$ ).MatrixMetric44.(e $^\alpha_{(A)}$  /. sgtrye $^\alpha_{(A)}$ )) // MatrixForm
```

```
Out[*] // MatrixForm =
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[*]:= Transpose[(e(A)α /. sgtrye(A)α)] . η4488 . (e(A)α /. sgtrye(A)α) - MatrixMetric44 //
FullSimplify[#, constraintVars] &
```

```
Out[*]:= {{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[*]:= Transpose[(eα(A) /. sgtryeα(A))] . MatrixMetric44 . (eα(A) /. sgtryeα(A)) - η4488 //
(*Full*)Simplify[#, constraintVars] &
```

```
Out[*]:= {{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[*]:= Inverse[(eα(A) /. sgtryeα(A))] == (e(A)α /. sgtrye(A)α)
```

```
Out[*]:= True
```

```
In[*]:= Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]
```

```
Out[*]:= {{0, 0}, {0, 0}}
```

BELOW : Lagrangian Ψ_{16} =

$$\kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi_{16}] . \sigma_{16} . T_{16}^A[A_1] .$$

$$\Psi_{16}|_{\alpha} g^{-1 \alpha \beta} \eta_{A_1 B_1} E_{\beta}^{B_1} + \text{mass term}$$

```
In[*]:= e(A)α /. sgtrye(A)α
```

```
Out[*]:= {{Cot[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0}, {0, sqrt[e2 a4 [H x 4] Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0, 0},
{0, 0, sqrt[e2 a4 [H x 4] Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, 0, sqrt[e2 a4 [H x 4] Sin[6 H x 0]1/6, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, sqrt[e-2 a4 [H x 4] Sin[6 H x 0]1/6, 0, 0, 0},
{0, 0, 0, 0, 0, 0, sqrt[e-2 a4 [H x 4] Sin[6 H x 0]1/6, 0, 0},
{0, 0, 0, 0, 0, 0, 0, sqrt[e-2 a4 [H x 4] Sin[6 H x 0]1/6, 0, 0}}}
```

In[*]:= $\mathbf{e}_{(A)}^\alpha /. \text{sgtrye}_{(A)}^\alpha$

Out[*]=

$$\left\{ \left\{ \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/6}}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ \left\{ 0, 0, \frac{1}{\sqrt{e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/6}}}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{e^{2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/6}}}, 0, 0, 0, 0 \right\}, \\ \left\{ 0, 0, 0, 0, 1, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/6}}}, 0, 0 \right\}, \\ \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/6}}}, 0 \right\}, \\ \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2 a 4 [H x 4]} \text{Sin}[6 H x 0]^{1/6}}} \right\} \right\}$$

In[*]:= $\mathbf{e}_{(A)}^\alpha /. \text{sge}_{(A)}^\alpha$

Out[*]=

$$\left\{ \left\{ \frac{1}{\sqrt{g[0][0][x 0, x 4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{g[1][1][x 0, x 4]}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ \left\{ 0, 0, \frac{1}{\sqrt{g[2][2][x 0, x 4]}}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{g[3][3][x 0, x 4]}}, 0, 0, 0, 0 \right\}, \\ \left\{ 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x 0, x 4]}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x 0, x 4]}}, 0, 0 \right\}, \\ \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x 0, x 4]}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x 0, x 4]}} \right\} \right\}$$

In[*]:=

```
Clear[con];
con[g_, ass_ : {}] := Module[{t, rT, ginv},
  ginv = FullSimplify[Inverse[g], ass];
  rT = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
    (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}],
    {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
  Return[{ginv, rT}]
```


In[^{*}]:= **ssgm4488**

Out[^{*}]=

$$\begin{aligned} &\{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ &g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ &g[1][1] \rightarrow (e^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ &g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ &g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ &g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ &g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ &g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ &g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ &g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ &g[5][5] \rightarrow (-e^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ &g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ &g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ &g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ &g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&)\} \end{aligned}$$

In[^{*}]:= **g4488**

g4488 /. sg

Out[^{*}]=

$$\begin{aligned} &\{\{g[0][0][x0, x4], g[0][1][x0, x4], g[0][2][x0, x4], g[0][3][x0, x4], \\ &g[0][4][x0, x4], g[0][5][x0, x4], g[0][6][x0, x4], g[0][7][x0, x4]\}, \\ &\{g[1][0][x0, x4], g[1][1][x0, x4], g[1][2][x0, x4], g[1][3][x0, x4], \\ &g[1][4][x0, x4], g[1][5][x0, x4], g[1][6][x0, x4], g[1][7][x0, x4]\}, \\ &\{g[2][0][x0, x4], g[2][1][x0, x4], g[2][2][x0, x4], g[2][3][x0, x4], \\ &g[2][4][x0, x4], g[2][5][x0, x4], g[2][6][x0, x4], g[2][7][x0, x4]\}, \\ &\{g[3][0][x0, x4], g[3][1][x0, x4], g[3][2][x0, x4], g[3][3][x0, x4], \\ &g[3][4][x0, x4], g[3][5][x0, x4], g[3][6][x0, x4], g[3][7][x0, x4]\}, \\ &\{g[4][0][x0, x4], g[4][1][x0, x4], g[4][2][x0, x4], g[4][3][x0, x4], \\ &g[4][4][x0, x4], g[4][5][x0, x4], g[4][6][x0, x4], g[4][7][x0, x4]\}, \\ &\{g[5][0][x0, x4], g[5][1][x0, x4], g[5][2][x0, x4], g[5][3][x0, x4], \\ &g[5][4][x0, x4], g[5][5][x0, x4], g[5][6][x0, x4], g[5][7][x0, x4]\}, \\ &\{g[6][0][x0, x4], g[6][1][x0, x4], g[6][2][x0, x4], g[6][3][x0, x4], \\ &g[6][4][x0, x4], g[6][5][x0, x4], g[6][6][x0, x4], g[6][7][x0, x4]\}, \\ &\{g[7][0][x0, x4], g[7][1][x0, x4], g[7][2][x0, x4], g[7][3][x0, x4], \\ &g[7][4][x0, x4], g[7][5][x0, x4], g[7][6][x0, x4], g[7][7][x0, x4]\} \end{aligned}$$

Out[^{*}]=

$$\begin{aligned} &\{\{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \{0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0\}, \{0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0\}, \{0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0\}, \{0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]\} \end{aligned}$$

```

In[ ]:= MatrixForm[#] & /@Block[{eAα, eαA, (*ginv,r,gg,*)colVecs, rowVecs, ab, ba, r},
  (*gg=g4488/.sg;*)
  (*{ginv,r}=con[gg];*)
  eαA = e(A)α /. sge(A)α;
  eAα = eα(A) /. sgeα(A);
  rowVecs = Table[eAα[[b, All]], {b, 1, Length[eAα]};
  colVecs = Table[eαA[[All, b]], {b, 1, Length[eαA]};
  ab = rowVecs.colVecs // FullSimplify[#, constraintVars] &;
  ba = Transpose[colVecs].Transpose[rowVecs] // FullSimplify[#, constraintVars] &;
  (*r={{ab},{ba}};*)
  r = {ab, ba};
  r]

```

Out[]:=

$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

metric compatibility condition for the Octad

$$\nabla_{\mu} e_{\nu}^I = \partial_{\mu} e_{\nu}^I - \Gamma_{\mu\nu}^{\rho} e_{\rho}^I + \omega_{\mu J}^I e_{\nu}^J = 0$$

```

In[ ]:= Clear[spinCoefficients];
spinCoefficients[eIv_] := Module[{evI, w, r, s},
  If[Dimensions[eIv] ≠ {8, 8}, Return[{}]];
  If[Det[eIv] == 0, Return[{}]];
  evI = FullSimplify[Inverse[eIv], constraintVars] /. subsDefects;
  w = Table[-Table[
    FullSimplify[Sum[D[eIv[[I1, v1]], X[[μ1]]] * evI[[v1, Jprime]], {v1, 1, Length[evI]}] -
    Sum[ eIv[[I1, ρ]] * Γ[ρ, μ1, v1] * evI[[v1, Jprime]], {v1, 1, Length[evI]},
    {ρ, 1, Length[X]}], constraintVars] /. subsDefects,
    {I1, 1, Length[eIv]}, {Jprime, 1, Length[evI]}, {μ1, 1, Length[X]}];
  Assert[Dimensions[w] == {8, 8, 8}];
  Return[w];
]

```

```

In[ ]:= spinCoeffs = spinCoefficients[gtrye(A)α]
Out[ ]=
{ {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, -ea4[H x4] H Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0}, {ea4[H x4] H Sin[6 H x0]1/6, 0,
    0, 0, ea4[H x4] H Sin[6 H x0]1/6 a4'[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, ea4[H x4] H Sin[6 H x0]1/6 a4'[H x4], 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, -ea4[H x4] H Sin[6 H x0]1/6, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ea4[H x4] H Sin[6 H x0]1/6, 0, 0, 0, ea4[H x4] H Sin[6 H x0]1/6 a4'[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, ea4[H x4] H Sin[6 H x0]1/6 a4'[H x4], 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, -ea4[H x4] H Sin[6 H x0]1/6, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {ea4[H x4] H Sin[6 H x0]1/6, 0, 0, 0, ea4[H x4] H Sin[6 H x0]1/6 a4'[H x4],
    0, 0, 0}, {0, 0, 0, ea4[H x4] H Sin[6 H x0]1/6 a4'[H x4], 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, e-a4[H x4] H Sin[6 H x0]1/6, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, e-a4[H x4] H Sin[6 H x0]1/6 a4'[H x4], 0, 0},
  {e-a4[H x4] H Sin[6 H x0]1/6, 0, 0, 0, -e-a4[H x4] H Sin[6 H x0]1/6 a4'[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0, e-a4[H x4] H Sin[6 H x0]1/6, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, e-a4[H x4] H Sin[6 H x0]1/6 a4'[H x4]}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
  {e-a4[H x4] H Sin[6 H x0]1/6, 0, 0, 0, -e-a4[H x4] H Sin[6 H x0]1/6 a4'[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, e-a4[H x4] H Sin[6 H x0]1/6, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, e-a4[H x4] H Sin[6 H x0]1/6 a4'[H x4]}},
  {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, e-a4[H x4] H Sin[6 H x0]1/6, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, e-a4[H x4] H Sin[6 H x0]1/6 a4'[H x4]}}

```



```

In[*]:= (*Γ[All,5,All]*)
In[*]:= (*Γ[All,1,All]*)
In[*]:= (*spinCoeffs[1]*)
In[*]:= (*spinCoeffs[5]*)
In[*]:= (*{X[1],X[5]}*)
In[*]:= (*spinCoeffs[1][1][1]*)
In[*]:= (*ωμIJ[1][1][1]*)
In[*]:= (*Block[{μ=1},FullSimplify[-(D[gtrye(A)α,X[μ]].gtryeα(A))-(gtrye(A)α.Γ[All,μ,All].gtryeα(A)),
  constraintVars]/.subsDefects]
  Dimensions[%]*)
In[*]:= (*Block[{μ=1},FullSimplify[
  -(D[gtrye(A)α,X[μ]].Transpose[gtryeα(A)])-(gtrye(A)α.Γ[All,μ,All].Transpose[gtryeα(A)]),
  constraintVars]/.subsDefects]*)
In[*]:= (*Block[{μ=1},
  FullSimplify[-(Sum[D[gtrye(A)α[All,v],X[μ]].gtryeα(A)[v,All],{v,1,8}])-(
    gtrye(A)α.Γ[All,μ,All].gtryeα(A)),constraintVars]/.subsDefects]
  Dimensions[%]*)
In[*]:= (*Table[{A1,1,8},{B1,1,8}]*
In[*]:= (*Block[{μ=1},
  Table[FullSimplify[-(Sum[D[gtrye(A)α[A1,v],X[μ]].gtryeα(A)[v,B1],{v,1,8}])-(
    Sum[gtrye(A)α[A1,ρ].Γ[ρ,μ,v].gtryeα(A)[v,B1],{ρ,1,8},{v,1,8}]),
    constraintVars]/.subsDefects,{A1,1,8},{B1,1,8}]]
  Dimensions[%]*)
In[*]:= Table[ ( ωμIJ[μ] - spinCoeffs[μ] ) /. subsDefects , {μ, 1, Length[X]}] // Flatten // Union
Out[*]:=
{0}

```

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:

- $\nabla_\mu e^a_\nu = 0$

- Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a_{b\nu}$, and e^a_ν is the tetrad component.

◦ Metric compatibility : The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame . The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is : ◦ V

He "

v=0

- Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a_{b\nu}$, and e^a_ν is the tetrad component .

No it is not the torsionless condition which is:

$$T_{\mu\nu}^I = D_{[\mu}^{\omega} e_{\nu]}^I = \partial_{[\mu} e_{\nu]}^I + \omega_{[\mu J}^I e_{\nu]}^J = 0$$

This postulate says:

$$\nabla_{\mu} e_{\nu}^I = \partial_{\mu} e_{\nu}^I - \Gamma_{\mu\nu}^{\rho} e_{\rho}^I + \omega_{\mu J}^I e_{\nu}^J = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D_{\mu}^{\omega} \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

```
In[*]:= Block[{eAa, eaA, t, ginv, r, gg},
  gg = g4488 /. sg;
  {ginv, r} = con[gg];
  eaA = e_{(A)}^{\alpha} /. sge_{(A)}^{\alpha}; eaA = e_{\alpha}^{(A)} /. sge_{\alpha}^{(A)};
  t = Table[Block[{v, V, contra, covari},
    V = eAa[[b, All]];
    v = eaA[[All, b]];
    contra = Table[D[v[[k]], X[[\mu]]] + Sum[v[[i]] \times r[[k, i, \mu]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before \omega terms*)
    covari = Table[D[V[[k]], X[[\mu]]] - Sum[V[[i]] \times r[[i, k, \mu]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before \omega terms*)
    {b - 1, X[[\mu]], {{contra}, {covari}}}], {b, 1, Length[eaA]}, {\mu, 1, DIM8}];
  t]
FullSimplify[#, /. ssgm4488, constraintVars] & /@ %
```

Out[*]=

$$\left\{ \left\{ \left\{ 0, x0, \left\{ \left\{ \left\{ 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \right\} \right\}, \right. \right. \\ \left. \left\{ \left\{ 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \right\} \right\} \right\},$$

$$\begin{aligned}
& \{0, x1, \left\{ \left\{ \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\}, \right. \right. \\
& \quad \left. \left\{ \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{0, x2, \left\{ \left\{ \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[2][2][x0, x4]}, 0, 0, 0, 0, 0 \right\}, \right. \right. \\
& \quad \left. \left\{ \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{0, x3, \left\{ \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[3][3][x0, x4]}, 0, 0, 0, 0 \right\}, \right. \right. \\
& \quad \left. \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{0, x4, \left\{ \left\{ \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0 \right\}, \right. \right. \\
& \quad \left. \left\{ \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{0, x5, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[5][5][x0, x4]}, 0, 0 \right\}, \right. \right. \\
& \quad \left. \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0 \right\} \right\} \right\}, \\
& \{0, x6, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[6][6][x0, x4]}, 0 \right\}, \right. \right. \\
& \quad \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0 \right\} \right\} \right\}, \\
& \{0, x7, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]} \right\}, \right. \right. \\
& \quad \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}} \right\} \right\} \right\}, \\
& \{1, x0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 1, x1, \right. \right. \\
& \quad \left. \left\{ \left\{ \left\{ -\frac{g[1][1]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, \right. \right. \right. \\
& \quad \left. \left. 0, 0, 0 \right\}, \left\{ \left\{ -\frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{1, x2, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \{1, x3, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \{1, x4, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \{1, x5, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \{1, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{2, x2, \\
& \quad \left\{ \left\{ \left\{ -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, \right. \right. \\
& \quad \left. \left. 0, 0, 0 \right\} \right\}, \left\{ \left\{ -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{2, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{3, x3, \\
& \quad \left\{ \left\{ \left\{ -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, \right. \right. \\
& \quad \left. \left. 0, 0, 0 \right\} \right\}, \left\{ \left\{ -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x0, \left\{ \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \\
& \quad \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{4, x1, \left\{ \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x0, x4]}{2 g[1][1][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{4, x2, \left\{ \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]}{2 g[2][2][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \{4, x3, \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2 g[3][3][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, 0 \right\} \right\}, \\
& \{4, x4, \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \right. \right. \\
& \quad \left. \frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. -\frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \\
& \{4, x5, \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[5][5][x0, x4]}, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0 \right\} \right\}, \\
& \{4, x6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[6][6][x0, x4]}, 0 \right\} \right\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0 \right\} \right\}, \\
& \{4, x7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]} \right\} \right\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]} \right\} \right\}, \left\{ \{5, x0, \right. \\
& \left. \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]}, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ \left\{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}} - \frac{g[5][5]^{(1,0)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]}, \right. \right. \\
& \quad \left. \left. 0, 0 \right\} \right\} \right\}, \{5, x1, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \right\} \}, \\
& \{5, x2, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \right\} \}, \\
& \{5, x3, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \right\} \}, \{5, x4, \\
& \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]}, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ \left\{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}} - \frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]}, \right. \right. \\
& \quad \left. \left. 0, 0 \right\} \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \{0, 0\}\}\}\}, \{5, x5, \left\{\left\{-\frac{g[5][5]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[5][5][x0, x4]}}, \right.\right. \\
& \left.0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0\right\}\}, \\
& \left\{-\frac{g[5][5]^{(1,0)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]}, 0, 0, 0, \right. \\
& \left. -\frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]}, 0, 0, 0\right\}\}\}\}, \\
& \{5, x6, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x7, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \left\{\left\{6, x0, \right.\right. \\
& \left.\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0\right\}\}, \right. \\
& \left.\left\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]}, \right.\right. \\
& \left.0\right\}\}\}\}, \{6, x1, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x2, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x3, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \left\{6, x4, \right. \\
& \left.\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0\right\}\}, \right. \\
& \left.\left\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]}, \right.\right. \\
& \left.0\right\}\}\}\}, \{6, x5, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \left\{6, x6, \right. \\
& \left.\left\{\left\{-\frac{g[6][6]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[6][6][x0, x4]}}, \right.\right. \\
& \left.0, 0, 0\right\}\}, \left\{-\frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]}, 0, \right. \\
& \left.0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]}, 0, 0, 0\right\}\}\}\}, \\
& \{6, x7, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \left\{7, x0, \right. \\
& \left.\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]}, \right.\right. \\
& \left.\left\{0, 0, 0, 0, 0, 0, 0, \right.\right. \\
& \left. -\frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 g[7][7][x0, x4]} \right\}\}\}\}\}, \\
& \{7, x1, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x2, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x3, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \left\{7, x4, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right\} \right\}, \right. \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \\
& \left. \left. - \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]} \right\} \right\} \}, \\
& \{7, x5, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x6, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{7, x7, \\
& \left\{ \left\{ \left\{ -\frac{g[7][7]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4] \sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]}{2g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}}, \right. \right. \\
& 0, 0, 0 \}, \left\{ \left\{ -\frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]}, 0, 0, \right. \right. \\
& \left. \left. 0, -\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]}, 0, 0, 0 \right\} \right\} \} \} \}
\end{aligned}$$

Out[8]=

$$\begin{aligned}
& \left\{ \left\{ 0, x0, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \right. \\
& \left\{ 0, x1, \{\{0, H, 0, 0, 0, 0, 0, 0\}\}, \{\{0, e^{2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 0, x2, \{\{0, 0, H, 0, 0, 0, 0, 0\}\}, \{\{0, 0, e^{2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 0, x3, \{\{0, 0, 0, H, 0, 0, 0, 0\}\}, \{\{0, 0, 0, e^{2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 0, x4, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 0, x5, \{\{0, 0, 0, 0, 0, 0, H, 0\}\}, \{\{0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0\}\}\}, \right. \\
& \left\{ 0, x6, \{\{0, 0, 0, 0, 0, 0, 0, H\}\}, \{\{0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0\}\}\}, \right. \\
& \left\{ 0, x7, \{\{0, 0, 0, 0, 0, 0, 0, H\}\}, \{\{0, 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} H \sin[6Hx0]^{1/3}\}\}\}, \right. \\
& \left\{ 1, x0, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x1, \left\{ \left\{ -\sqrt{e^{2a4[Hx4]}} H \sec[6Hx0] \sin[6Hx0]^{7/6}, \right. \right. \right. \\
& \left. \left. 0, 0, 0, \sqrt{e^{2a4[Hx4]}} H \sin[6Hx0]^{1/6} a4'[Hx4], 0, 0, 0 \right\}, \right. \\
& \left. \left\{ \left\{ -\frac{\sqrt{e^{2a4[Hx4]}} H \cos[6Hx0]}{\sin[6Hx0]^{5/6}}, 0, 0, 0, -\sqrt{e^{2a4[Hx4]}} H \sin[6Hx0]^{1/6} a4'[Hx4], 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 1, x2, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x3, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x4, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x5, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x6, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x7, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 2, x0, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 2, x1, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 2, x2, \left\{ \left\{ -\sqrt{e^{2a4[Hx4]}} H \sec[6Hx0] \sin[6Hx0]^{7/6}, \right. \right. \right. \\
& \left. \left. 0, 0, 0, \sqrt{e^{2a4[Hx4]}} H \sin[6Hx0]^{1/6} a4'[Hx4], 0, 0, 0 \right\}, \right. \\
& \left. \left\{ \left\{ -\frac{\sqrt{e^{2a4[Hx4]}} H \cos[6Hx0]}{\sin[6Hx0]^{5/6}}, 0, 0, 0, -\sqrt{e^{2a4[Hx4]}} H \sin[6Hx0]^{1/6} a4'[Hx4], 0, 0, 0 \right\} \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \{2, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ \{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x3, \left\{ \left\{ \left\{ -\sqrt{e^{2a4[Hx4]}} H \operatorname{Sec}[6Hx0] \operatorname{Sin}[6Hx0]^{7/6}, \right. \right. \\
& \quad \left. \left. 0, 0, 0, \sqrt{e^{2a4[Hx4]}} H \operatorname{Sin}[6Hx0]^{1/6} a4'[Hx4], 0, 0, 0 \right\} \right\}, \\
& \quad \left\{ \left\{ -\frac{\sqrt{e^{2a4[Hx4]}} H \operatorname{Cos}[6Hx0]}{\operatorname{Sin}[6Hx0]^{5/6}}, 0, 0, 0, -\sqrt{e^{2a4[Hx4]}} H \operatorname{Sin}[6Hx0]^{1/6} a4'[Hx4], 0, 0, 0 \right\} \right\} \}, \\
& \{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ \{4, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x1, \{\{\{0, Ha4'[Hx4], 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \left\{ \{0, -e^{2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4], 0, 0, 0, 0, 0, 0\} \right\} \}, \\
& \{4, x2, \{\{\{0, 0, Ha4'[Hx4], 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \left\{ \{0, 0, -e^{2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4], 0, 0, 0, 0, 0\} \right\} \}, \\
& \{4, x3, \{\{\{0, 0, 0, Ha4'[Hx4], 0, 0, 0, 0, 0\}\}, \\
& \quad \left\{ \{0, 0, 0, -e^{2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4], 0, 0, 0, 0\} \right\} \}, \\
& \{4, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x5, \{\{\{0, 0, 0, 0, 0, 0, -Ha4'[Hx4], 0, 0\}\}, \\
& \quad \left\{ \{0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4], 0, 0\} \right\} \}, \\
& \{4, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, -Ha4'[Hx4], 0\}\}, \\
& \quad \left\{ \{0, 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4], 0\} \right\} \}, \\
& \{4, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, -Ha4'[Hx4]\}\}, \\
& \quad \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4]\} \right\} \}, \\
& \{ \{5, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x5, \left\{ \left\{ \left\{ \sqrt{e^{-2a4[Hx4]}} H \operatorname{Sec}[6Hx0] \operatorname{Sin}[6Hx0]^{7/6}, \right. \right. \\
& \quad \left. \left. 0, 0, 0, \sqrt{e^{-2a4[Hx4]}} H \operatorname{Sin}[6Hx0]^{1/6} a4'[Hx4], 0, 0, 0 \right\} \right\}, \\
& \quad \left\{ \left\{ -\frac{\sqrt{e^{-2a4[Hx4]}} H \operatorname{Cos}[6Hx0]}{\operatorname{Sin}[6Hx0]^{5/6}}, 0, 0, 0, \sqrt{e^{-2a4[Hx4]}} H \operatorname{Sin}[6Hx0]^{1/6} a4'[Hx4], 0, 0, 0 \right\} \right\} \}, \\
& \{5, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},
\end{aligned}$$

$$\begin{aligned} & \{ \{6, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\ & \{6, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{6, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{6, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{6, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{6, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{6, x6, \left\{ \left\{ \left\{ \sqrt{e^{-2 a4 [H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, \right. \right. \\ & \quad \left. \left. 0, 0, 0, \sqrt{e^{-2 a4 [H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4' [H x4], 0, 0, 0 \right\} \right\}, \\ & \quad \left\{ \left\{ -\frac{\sqrt{e^{-2 a4 [H x4]}} H \operatorname{Cos}[6 H x0]}{\operatorname{Sin}[6 H x0]^{5/6}}, 0, 0, 0, \sqrt{e^{-2 a4 [H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4' [H x4], 0, 0, 0 \right\} \right\} \right\}, \\ & \{6, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{ \{7, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{7, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{7, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{7, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{7, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{7, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{7, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{7, x7, \left\{ \left\{ \left\{ \sqrt{e^{-2 a4 [H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, \right. \right. \\ & \quad \left. \left. 0, 0, 0, \sqrt{e^{-2 a4 [H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4' [H x4], 0, 0, 0 \right\} \right\}, \\ & \quad \left\{ \left\{ -\frac{\sqrt{e^{-2 a4 [H x4]}} H \operatorname{Cos}[6 H x0]}{\operatorname{Sin}[6 H x0]^{5/6}}, 0, 0, 0, \sqrt{e^{-2 a4 [H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4' [H x4], 0, 0, 0 \right\} \right\} \right\} \end{aligned}$$

```
In[*]:= (*Block[{eαA, eαA, t, ginv, r, gg},
  gg=g4488/.sg;
  {ginv, r}=con[gg];
  eαA=e(A)α/.sge(A)α;
  eAα=eα(A)/.sgeα(A);
  t=Table[Block[{v, V, d}, v=eαA[[All, b]];
    d=Table[D[v[[k]], X[[μ]]]+Sum[v[[i]] r[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    {b-1, X[[μ], d]}, {b, 1, Length[eαA]}, {μ, 1, DIM8}]; t]
FullSimplify[#, /. ssgm4488, constraintVars]&/@**)
```

```
in[*]:= (*Block[{eA $\alpha$ ,eA $\alpha$ ,t,ginv,r,gg},
  gg=g4488/.sg;
  {ginv,r}=con[gg];
  eA $\alpha$ =e $_{(A)}^{\alpha}$ /.sge $_{(A)}^{\alpha}$ ;
  eA $\alpha$ =e $_{\alpha}^{(A)}$ /.sge $_{\alpha}^{(A)}$ ;
  t=Table[Block[{v,V,d},v=eA $\alpha$ [[b,All]]];
    d=Table[D[v[[k]],X[[ $\mu$ ]]]-Sum[v[[i]]r[[k,i, $\mu$ ]],{i,1,DIM8}],{k,1,DIM8}];
    {b-1,X[[ $\mu$ ]],d}],{b,1,Length[eA $\alpha$ ]},{ $\mu$ ,1,DIM8}];t]
FullSimplify[#/.ssgm4488,constraintVars]&/@%*]
```

$$In[\bullet] := \text{Symbolize}[\omega_{\mu}^{(a)}]^{(b)}$$
$$In[\bullet] := \omega = \text{Array}\left[\omega_{\mu}^{(a)} \left[\begin{smallmatrix} \text{###} \\ (b) \end{smallmatrix}\right] \&, \{8, 8, 8\}\right];$$

In[*]:= $\omega[[1]$

Out[*]=

$$\left\{ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 1], \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 2], \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 3], \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 4], \right. \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 5], \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 6], \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 7], \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 8] \right\}, \\ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 1], \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 2], \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 3], \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 4], \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 5], \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 6], \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 7], \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 8] \right\}, \\ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 3, 1], \omega_{\mu}^{(a)}{}_{(b)} [1, 3, 2], \omega_{\mu}^{(a)}{}_{(b)} [1, 3, 3], \omega_{\mu}^{(a)}{}_{(b)} [1, 3, 4], \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 3, 5], \omega_{\mu}^{(a)}{}_{(b)} [1, 3, 6], \omega_{\mu}^{(a)}{}_{(b)} [1, 3, 7], \omega_{\mu}^{(a)}{}_{(b)} [1, 3, 8] \right\}, \\ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 4, 1], \omega_{\mu}^{(a)}{}_{(b)} [1, 4, 2], \omega_{\mu}^{(a)}{}_{(b)} [1, 4, 3], \omega_{\mu}^{(a)}{}_{(b)} [1, 4, 4], \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 4, 5], \omega_{\mu}^{(a)}{}_{(b)} [1, 4, 6], \omega_{\mu}^{(a)}{}_{(b)} [1, 4, 7], \omega_{\mu}^{(a)}{}_{(b)} [1, 4, 8] \right\}, \\ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 5, 1], \omega_{\mu}^{(a)}{}_{(b)} [1, 5, 2], \omega_{\mu}^{(a)}{}_{(b)} [1, 5, 3], \omega_{\mu}^{(a)}{}_{(b)} [1, 5, 4], \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 5, 5], \omega_{\mu}^{(a)}{}_{(b)} [1, 5, 6], \omega_{\mu}^{(a)}{}_{(b)} [1, 5, 7], \omega_{\mu}^{(a)}{}_{(b)} [1, 5, 8] \right\}, \\ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 6, 1], \omega_{\mu}^{(a)}{}_{(b)} [1, 6, 2], \omega_{\mu}^{(a)}{}_{(b)} [1, 6, 3], \omega_{\mu}^{(a)}{}_{(b)} [1, 6, 4], \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 6, 5], \omega_{\mu}^{(a)}{}_{(b)} [1, 6, 6], \omega_{\mu}^{(a)}{}_{(b)} [1, 6, 7], \omega_{\mu}^{(a)}{}_{(b)} [1, 6, 8] \right\}, \\ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 7, 1], \omega_{\mu}^{(a)}{}_{(b)} [1, 7, 2], \omega_{\mu}^{(a)}{}_{(b)} [1, 7, 3], \omega_{\mu}^{(a)}{}_{(b)} [1, 7, 4], \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 7, 5], \omega_{\mu}^{(a)}{}_{(b)} [1, 7, 6], \omega_{\mu}^{(a)}{}_{(b)} [1, 7, 7], \omega_{\mu}^{(a)}{}_{(b)} [1, 7, 8] \right\}, \\ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 8, 1], \omega_{\mu}^{(a)}{}_{(b)} [1, 8, 2], \omega_{\mu}^{(a)}{}_{(b)} [1, 8, 3], \omega_{\mu}^{(a)}{}_{(b)} [1, 8, 4], \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 8, 5], \omega_{\mu}^{(a)}{}_{(b)} [1, 8, 6], \omega_{\mu}^{(a)}{}_{(b)} [1, 8, 7], \omega_{\mu}^{(a)}{}_{(b)} [1, 8, 8] \right\} \}$$

In[*]:= $\text{sw} = \text{Block}[\{\text{eA}\alpha, \text{e}\alpha A, \mathbf{t}, \mathbf{V}, \mathbf{g} \text{inv}, \mathbf{r}, \mathbf{g}\mathbf{g}\},$

$\mathbf{g}\mathbf{g} = \mathbf{g4488} /. \mathbf{sg};$

$\{\mathbf{g} \text{inv}, \mathbf{r}\} = \text{con}[\mathbf{g}\mathbf{g}];$

$\text{e}\alpha A = \mathbf{e}_{(A)}^{\alpha} /. \mathbf{sge}_{(A)}^{\alpha};$

$\text{eA}\alpha = \mathbf{e}_{\alpha}^{(A)} /. \mathbf{sge}_{\alpha}^{(A)};$

$\text{Table}[\omega[\mu, \mathbf{a}, \mathbf{b}] \rightarrow \text{Block}[\{\mathbf{v}, \mathbf{d}\}, \mathbf{v} = \text{e}\alpha A[\text{All}, \mathbf{b}];$

$\mathbf{d} = \text{Sum}[\{\mathbf{D}[\mathbf{v}[\mathbf{k}], \mathbf{X}[\mu]]\} + \text{Sum}[\mathbf{v}[\mathbf{i}] \times \mathbf{r}[\mathbf{k}, \mathbf{i}, \mu], \{\mathbf{i}, 1, \text{DIM8}\}]] * \text{eA}\alpha[\mathbf{a}, \mathbf{k}], \{\mathbf{k}, 1, \text{DIM8}\}];$
 $\mathbf{d}], \{\mu, 1, \text{DIM8}\}, \{\mathbf{a}, 1, \text{Length}[\text{e}\alpha A]\}, \{\mathbf{b}, 1, \text{Length}[\text{e}\alpha A]\}]]$

Out[*]=

$$\left\{ \left\{ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 3] \rightarrow 0, \right. \right. \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 4] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 5] \rightarrow \frac{\mathbf{g}[0][0]^{(0,1)}[\times 0, \times 4]}{2 \sqrt{\mathbf{g}[0][0][\times 0, \times 4]} \sqrt{-\mathbf{g}[4][4][\times 0, \times 4]}}, \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 1, 8] \rightarrow 0 \right\}, \\ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 4] \rightarrow 0, \right. \\ \left. \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [1, 2, 8] \rightarrow 0 \right\},$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [1, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [1, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [1, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 5, 1] \rightarrow -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 5, 2] \rightarrow 0, \right. \\
& \omega_{\mu(b)}^{(a)} [1, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 5] \rightarrow \\
& \left. \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [1, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 3] \rightarrow 0, \right. \\
& \omega_{\mu(b)}^{(a)} [1, 6, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 6] \rightarrow \\
& \left. \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, \right. \\
& \omega_{\mu(b)}^{(a)} [1, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 8] \rightarrow 0 \left. \right\}, \left\{ \omega_{\mu(b)}^{(a)} [1, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 2] \rightarrow 0, \right. \\
& \omega_{\mu(b)}^{(a)} [1, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 6] \rightarrow 0, \\
& \omega_{\mu(b)}^{(a)} [1, 7, 7] \rightarrow \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \\
& \left. \sqrt{-g[6][6][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 4] \rightarrow 0, \right. \\
& \omega_{\mu(b)}^{(a)} [1, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 8] \rightarrow \\
& \left. \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 2] \rightarrow -\frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, \right. \\
& \omega_{\mu(b)}^{(a)} [2, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 5] \rightarrow 0, \\
& \omega_{\mu(b)}^{(a)} [2, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 8] \rightarrow 0 \left. \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 2, 1] \rightarrow \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, \right. \\
& \omega_{\mu(b)}^{(a)} [2, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 4] \rightarrow 0, \\
& \omega_{\mu(b)}^{(a)} [2, 2, 5] \rightarrow \frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} \sqrt{-g[4][4][x0, x4]}},
\end{aligned}$$

[illegible]

$$\begin{aligned}
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [4, 8, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [4, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [4, 8, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [4, 8, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [4, 8, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [4, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [4, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [4, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [5, 1, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 1, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 1, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 1, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 1, 5] \rightarrow -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 1, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 1, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [5, 2, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 2, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 2, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 2, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 2, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 2, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 2, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [5, 3, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 3, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 3, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 3, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 3, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 3, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 3, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [5, 4, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 4, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 4, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 4, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 4, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 4, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 4, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [5, 5, 1] \rightarrow \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, \omega_{\mu}^{(a)}{}_{(b)} [5, 5, 2] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 5, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 5, 4] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 5, 5] \rightarrow \right. \\
& \left. \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 5, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 5, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [5, 6, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 6, 3] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 6, 4] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 6, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 6, 6] \rightarrow \right. \\
& \left. \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 6, 8] \rightarrow 0 \right\}, \left\{ \omega_{\mu}^{(a)}{}_{(b)} [5, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 7, 2] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 7, 4] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 7, 6] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 7, 7] \rightarrow \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[6][6][x0, x4]}, \omega_{\mu}^{(a)}{}_{(b)} [5, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [5, 8, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 8, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 8, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [5, 8, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [5, 8, 8] \rightarrow \right. \\
& \left. \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [6, 1, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 1, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 1, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [6, 1, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 1, 6] \rightarrow -\frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[5][5][x0, x4]}}, \\
& \quad \left. \omega_{\mu}^{(a)}{}_{(b)} [6, 1, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [6, 2, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 2, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 2, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 2, 4] \rightarrow 0, \right. \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [6, 2, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 2, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 2, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 2, 8] \rightarrow 0 \left. \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [6, 3, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 3, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 3, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 3, 4] \rightarrow 0, \right. \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [6, 3, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 3, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 3, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 3, 8] \rightarrow 0 \left. \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [6, 4, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 4, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 4, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 4, 4] \rightarrow 0, \right. \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [6, 4, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 4, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 4, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 4, 8] \rightarrow 0 \left. \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [6, 5, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 5, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 5, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 5, 4] \rightarrow 0, \right. \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [6, 5, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 5, 6] \rightarrow -\frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[5][5][x0, x4]}}, \\
& \quad \left. \omega_{\mu}^{(a)}{}_{(b)} [6, 5, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [6, 6, 1] \rightarrow \frac{g[5][5]^{(1,0)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[5][5][x0, x4]}, \right. \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [6, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 6, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 6, 4] \rightarrow 0, \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [6, 6, 5] \rightarrow \frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[5][5][x0, x4]}, \\
& \quad \left. \omega_{\mu}^{(a)}{}_{(b)} [6, 6, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [6, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 7, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 7, 4] \rightarrow 0, \right. \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [6, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 7, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 7, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 7, 8] \rightarrow 0 \left. \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [6, 8, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 8, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 8, 4] \rightarrow 0, \right. \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [6, 8, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [6, 8, 8] \rightarrow 0 \left. \right\}, \\
& \left\{ \left\{ \omega_{\mu}^{(a)}{}_{(b)} [7, 1, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 1, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 1, 3] \rightarrow 0, \right. \right. \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [7, 1, 4] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 1, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 1, 6] \rightarrow 0, \\
& \quad \omega_{\mu}^{(a)}{}_{(b)} [7, 1, 7] \rightarrow -\frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[6][6][x0, x4]}}, \left. \omega_{\mu}^{(a)}{}_{(b)} [7, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [7, 2, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 2, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 2, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)}{}_{(b)} [7, 2, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 2, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 2, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [7, 2, 8] \rightarrow 0 \right\},
\end{aligned}$$

$$\begin{aligned}
& \omega_{\mu}^{(a)}{}_{(b)} [8, 5, 8] \rightarrow -\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}} \}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [8, 6, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 6, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 6, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [8, 6, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 6, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [8, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 7, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 7, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [8, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 7, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 7, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)}{}_{(b)} [8, 8, 1] \rightarrow \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]}, \right. \\
& \omega_{\mu}^{(a)}{}_{(b)} [8, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 8, 3] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 8, 4] \rightarrow 0, \\
& \omega_{\mu}^{(a)}{}_{(b)} [8, 8, 5] \rightarrow \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}, \\
& \left. \omega_{\mu}^{(a)}{}_{(b)} [8, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)}{}_{(b)} [8, 8, 8] \rightarrow 0 \right\} \}
\end{aligned}$$

```

In[*]:= (*sw=Block[{eAa,eaA,t,V,ginv,r,gg},
  gg=g4488/.sg;
  {ginv,r}=con[gg];
  eaA=e(A)α/.sge(A)α;
  eAa=eα(A)/.sgeα(A);
  Table[ ωμ(a)(b) [μ,a,b]→Block[{v,d},v=eaA[[All,b];
    d=Sum[{D[v[[k]],X[[μ]]]+Sum[ v[[i]] r[[k,i,μ]],{i,1,DIM8}]}*eAa[[a,k]],{k,1,DIM8}];
    d],{μ,1,DIM8},{a,1,Length[eaA]},{b,1,Length[eAa]}]]*)

```

```

In[*]:= swf = Flatten[sw];

```

```

In[*]:= (*ωμ(a)(b)*)
ω;
ωmat = % /. swf
Dimensions[%]

```

```

Out[*]:=

```

$$\begin{aligned}
& \left\{ \left\{ \left\{ 0, 0, 0, 0, \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \right. \right. \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left. \left\{ -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, \right. \right. \\
& \left. \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \right. \\
& \left. 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \right. \\
& \left. \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, 0, \right. \\
& \left. \left. \right\} \right\}
\end{aligned}$$

$$\begin{aligned}
& 0\}, \left\{0, 0, 0, 0, 0, 0, 0, \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}g[6][6][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[6][6][x0, x4]}, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \right. \\
& \left. \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{0, -\frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{\frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, \right. \\
& \left. \frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \left\{0, -\frac{g[1][1]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[1][1][x0, x4]}g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0\right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{0, 0, -\frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \left\{\frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, \right. \\
& \left. \frac{g[2][2]^{(0,1)}[x0, x4]}{2\sqrt{g[2][2][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[2][2][x0, x4]}g[4][4][x0, x4]}, 0, 0, 0, 0, 0\right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{0, 0, 0, -\frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \left\{\frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[3][3][x0, x4]}}, \right. \\
& 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2\sqrt{g[3][3][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\}, \\
& \left\{0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[3][3][x0, x4]}g[4][4][x0, x4]}, 0, 0, 0, 0\right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, \right. \\
& \left. \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, 0, \right. \\
& 0, 0\}, \left\{ 0, 0, 0, 0, 0, 0, \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[5][5][x0, x4]}, 0, 0\right\}, \left\{ 0, 0, 0, 0, 0, 0, \right. \\
& \left. \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \sqrt{-g[6][6][x0, x4]}, 0\right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[7][7][x0, x4]} \right\}, \left\{ \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \right. \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \\
& \left\{ \frac{g[5][5]^{(1,0)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[5][5][x0, x4]}, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[5][5][x0, x4]}, \right. \\
& 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \right. \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[6][6][x0, x4]}, 0, 0, 0, \right. \\
& \left. \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[6][6][x0, x4]}, 0, 0, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[7][7][x0, x4]}}, \right\}, \right. \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}}, \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]}, \right.
\end{aligned}$$

$$0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}, 0, 0, 0\}}\}$$

Out[*]=

{8, 8, 8}

```
In[*]:= swgtry = Block[{eAα, eaA, t, V},
  eaA = e(A)α /. sgtrye(A)α;
  eAα = eα(A) /. sgtryeα(A);
  Table[(*V=eAα[[a,All]];*) ωμ(a)(b) [μ, a, b] → Block[{v, d}, v = eaA[[All, b]];
    d = Sum[{D[v[[k]], X[[μ]]] + Sum[v[[i]] × r[[k, i, μ]], {i, 1, DIM8}]} * eAα[[a, k]], {k, 1, DIM8}];
    FullSimplify[d, constraintVars] /. {√e2 a4 [H x4] → ea4 [H x4], √e-2 a4 [H x4] → e-a4 [H x4]}},
    {μ, 1, DIM8}, {a, 1, Length[eAα]}, {b, 1, Length[eaA]}]]
```

Out[*]=

```
{ { { ωμ(a)(b) [1, 1, 1] → 0, ωμ(a)(b) [1, 1, 2] → 0, ωμ(a)(b) [1, 1, 3] → 0, ωμ(a)(b) [1, 1, 4] → 0,
  ωμ(a)(b) [1, 1, 5] → 0, ωμ(a)(b) [1, 1, 6] → 0, ωμ(a)(b) [1, 1, 7] → 0, ωμ(a)(b) [1, 1, 8] → 0},
  { ωμ(a)(b) [1, 2, 1] → 0, ωμ(a)(b) [1, 2, 2] → 0, ωμ(a)(b) [1, 2, 3] → 0, ωμ(a)(b) [1, 2, 4] → 0,
  ωμ(a)(b) [1, 2, 5] → 0, ωμ(a)(b) [1, 2, 6] → 0, ωμ(a)(b) [1, 2, 7] → 0, ωμ(a)(b) [1, 2, 8] → 0},
  { ωμ(a)(b) [1, 3, 1] → 0, ωμ(a)(b) [1, 3, 2] → 0, ωμ(a)(b) [1, 3, 3] → 0, ωμ(a)(b) [1, 3, 4] → 0,
  ωμ(a)(b) [1, 3, 5] → 0, ωμ(a)(b) [1, 3, 6] → 0, ωμ(a)(b) [1, 3, 7] → 0, ωμ(a)(b) [1, 3, 8] → 0},
  { ωμ(a)(b) [1, 4, 1] → 0, ωμ(a)(b) [1, 4, 2] → 0, ωμ(a)(b) [1, 4, 3] → 0, ωμ(a)(b) [1, 4, 4] → 0,
  ωμ(a)(b) [1, 4, 5] → 0, ωμ(a)(b) [1, 4, 6] → 0, ωμ(a)(b) [1, 4, 7] → 0, ωμ(a)(b) [1, 4, 8] → 0},
  { ωμ(a)(b) [1, 5, 1] → 0, ωμ(a)(b) [1, 5, 2] → 0, ωμ(a)(b) [1, 5, 3] → 0, ωμ(a)(b) [1, 5, 4] → 0,
  ωμ(a)(b) [1, 5, 5] → 0, ωμ(a)(b) [1, 5, 6] → 0, ωμ(a)(b) [1, 5, 7] → 0, ωμ(a)(b) [1, 5, 8] → 0},
  { ωμ(a)(b) [1, 6, 1] → 0, ωμ(a)(b) [1, 6, 2] → 0, ωμ(a)(b) [1, 6, 3] → 0, ωμ(a)(b) [1, 6, 4] → 0,
  ωμ(a)(b) [1, 6, 5] → 0, ωμ(a)(b) [1, 6, 6] → 0, ωμ(a)(b) [1, 6, 7] → 0, ωμ(a)(b) [1, 6, 8] → 0},
  { ωμ(a)(b) [1, 7, 1] → 0, ωμ(a)(b) [1, 7, 2] → 0, ωμ(a)(b) [1, 7, 3] → 0, ωμ(a)(b) [1, 7, 4] → 0,
  ωμ(a)(b) [1, 7, 5] → 0, ωμ(a)(b) [1, 7, 6] → 0, ωμ(a)(b) [1, 7, 7] → 0, ωμ(a)(b) [1, 7, 8] → 0},
  { ωμ(a)(b) [1, 8, 1] → 0, ωμ(a)(b) [1, 8, 2] → 0, ωμ(a)(b) [1, 8, 3] → 0, ωμ(a)(b) [1, 8, 4] → 0,
  ωμ(a)(b) [1, 8, 5] → 0, ωμ(a)(b) [1, 8, 6] → 0, ωμ(a)(b) [1, 8, 7] → 0, ωμ(a)(b) [1, 8, 8] → 0}},
  { { ωμ(a)(b) [2, 1, 1] → 0, ωμ(a)(b) [2, 1, 2] → -ea4 [H x4] H Sin[6 H x0]1/6, ωμ(a)(b) [2, 1, 3] → 0,
  ωμ(a)(b) [2, 1, 4] → 0, ωμ(a)(b) [2, 1, 5] → 0, ωμ(a)(b) [2, 1, 6] → 0, ωμ(a)(b) [2, 1, 7] → 0,
  ωμ(a)(b) [2, 1, 8] → 0}, { ωμ(a)(b) [2, 2, 1] → ea4 [H x4] H Sin[6 H x0]1/6, ωμ(a)(b) [2, 2, 2] → 0,
  ωμ(a)(b) [2, 2, 3] → 0, ωμ(a)(b) [2, 2, 4] → 0, ωμ(a)(b) [2, 2, 5] → ea4 [H x4] H Sin[6 H x0]1/6 a4' [H x4],
```


[illegible]

[illegible]

[illegible]

$$\begin{aligned}
& \omega_{\mu}^{(a)}(b) [8, 4, 5] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 4, 6] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 4, 7] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 4, 8] \rightarrow 0 \}, \\
& \{ \omega_{\mu}^{(a)}(b) [8, 5, 1] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 5, 2] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 5, 3] \rightarrow 0, \\
& \omega_{\mu}^{(a)}(b) [8, 5, 4] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 5, 5] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 5, 6] \rightarrow 0, \\
& \omega_{\mu}^{(a)}(b) [8, 5, 7] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 5, 8] \rightarrow e^{-a^4 [H \times 4]} H \sin[6 H \times 0]^{1/6} a^4 [H \times 4] \}, \\
& \{ \omega_{\mu}^{(a)}(b) [8, 6, 1] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 6, 3] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 6, 4] \rightarrow 0, \\
& \omega_{\mu}^{(a)}(b) [8, 6, 5] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 6, 6] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 6, 8] \rightarrow 0 \}, \\
& \{ \omega_{\mu}^{(a)}(b) [8, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 7, 2] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 7, 4] \rightarrow 0, \\
& \omega_{\mu}^{(a)}(b) [8, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 7, 6] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 7, 7] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 7, 8] \rightarrow 0 \}, \\
& \{ \omega_{\mu}^{(a)}(b) [8, 8, 1] \rightarrow e^{-a^4 [H \times 4]} H \sin[6 H \times 0]^{1/6}, \omega_{\mu}^{(a)}(b) [8, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 8, 3] \rightarrow 0, \\
& \omega_{\mu}^{(a)}(b) [8, 8, 4] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 8, 5] \rightarrow -e^{-a^4 [H \times 4]} H \sin[6 H \times 0]^{1/6} a^4 [H \times 4], \\
& \omega_{\mu}^{(a)}(b) [8, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 8, 8] \rightarrow 0 \} \} \}
\end{aligned}$$

In[*]:= **swfgtry = Flatten[swgtry];**

```

In[5]:= (*ωμ(a)(b)*)
ω;
ωω = % /. swfgtry // FullSimplify[#, constraintVars] &
Dimensions[%]

```

[illegible]

```
In[ ]:= g4488 /. sg
```

```
% /. ssgm4488
```

```
Out[ ]=
```

```
{ {g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
  {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
  {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
  {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]} }
```

```
Out[ ]=
```

```
{ {Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^(2 a4 [H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0},
  {0, 0, e^(2 a4 [H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^(2 a4 [H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^(-2 a4 [H x4]) Sin[6 H x0]^(1/3), 0, 0},
  {0, 0, 0, 0, 0, 0, -e^(-2 a4 [H x4]) Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -e^(-2 a4 [H x4]) Sin[6 H x0]^(1/3)} }
```

check metric compatibility condition for the Octad

```
In[ ]:= Block[{eAα, eaA, t, ginv, Γ, gg(*, ωω*)},
```

```
(*ωω=ωμ(a)(b) /. swf;*)
```

```
gg = g4488 /. sg /. ssgm4488;
```

```
{ginv, Γ} = con[gg];
```

```
eaA = e(A)α /. sge(A)α; eAα = eα(A) /. sgeα(A);
```

```
t = Table[Block[{v, V, contra, covari},
```

```
V = eAα[[b, All]];
```

```
v = eaA[[All, b]];
```

```
contra =
```

```
Table[D[V[[k]], X[[μ]]] + Sum[v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
```

```
(*no ω terms*)
```

```
covari = Table[D[V[[k]], X[[μ]]] - Sum[V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}] +
```

```
Sum[eAα[[J, All]] [[k]] × ωmat[[μ, b, J]], {J, 1, DIM8}] //
```

```
FullSimplify[#, constraintVars && e^(2 a4 [H x4]) > 0 && e^(a4 [H x4]) > 0 &&
```

```
e^(-2 a4 [H x4]) > 0 && e^(-a4 [H x4]) > 0] &, {k, 1, DIM8}]; (*ω terms*)
```

```
(*{b-1, X[[μ]], {{contra}, {covari}}}, {b, 1, Length[eaA]}, {μ, 1, DIM8}]; *)
```

```
{b-1, X[[μ]], covari}], {b, 1, Length[eaA]}, {μ, 1, DIM8}];
```

```
t]
```

```
FullSimplify[#, /. ssgm4488,
```

```
constraintVars && e^(2 a4 [H x4]) > 0 && e^(a4 [H x4]) > 0 && e^(-2 a4 [H x4]) > 0 && e^(-a4 [H x4]) > 0] & /@ %
```

```
Out[ ]=
```

```
{ { {0, x0, {
  {
    {
      {
        g[0][0]^(1,0)[x0, x4] + 24 H Csc[12 H x0] g[0][0][x0, x4],
        2 Sqrt[g[0][0][x0, x4]]
      },
      {
        g[0][0]^(0,1)[x0, x4],
        2 Sqrt[g[0][0][x0, x4]]
      },
      {
        -g[1][1]^(1,0)[x0, x4] + 2 e^(2 a4 [H x4]) H Sec[6 H x0] Sin[6 H x0]^(4/3) g[0][0][x0, x4],
        2 Sqrt[g[0][0][x0, x4]]
      }
    }
  }
}, {0, x1,
```

$$\begin{aligned}
& \{0, 0, 0, 0, 0, 0\}, \{0, x2, \{0, 0, \\
& \frac{-g[2][2]^{(1,0)}[x0, x4] + 2 e^{2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, \\
& 0, 0, 0, 0, 0\}, \{0, x3, \{0, 0, 0, \\
& \frac{-g[3][3]^{(1,0)}[x0, x4] + 2 e^{2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, \\
& 0, 0, 0, 0\}, \\
& \{0, x4, \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\}, \\
& \{0, x5, \{0, 0, 0, 0, 0, \\
& \frac{-g[5][5]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, \\
& 0, 0\}, \{0, x6, \{0, 0, 0, 0, 0, 0, \\
& \frac{-g[6][6]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, \\
& 0\}, \{0, x7, \{0, 0, 0, 0, 0, 0, 0, \\
& \frac{-g[7][7]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4]}{2 \sqrt{g[0][0][x0, x4]}} \} \} \}, \\
& \left\{ \{1, x0, \{0, \frac{g[1][1]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, \right. \\
& 0, 0, 0, 0, 0, 0\}, \\
& \{1, x1, \left\{ \frac{g[1][1]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, \right. \\
& 0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0 \} \}, \\
& \{1, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{1, x4, \{0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0\}, \\
& \{1, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{1, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{1, x7, \{0, 0, 0, 0, 0, 0, 0, 0\} \}, \\
& \left\{ \{2, x0, \{0, 0, \frac{g[2][2]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, \right.
\end{aligned}$$

$$\begin{aligned}
& \{0, 0, 0, 0, 0\}, \{2, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x2, \left\{ \frac{g[2][2]^{(1,0)}[x0, x4] - 2 H \text{Cot}[6 H x0] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, \right. \\
& \quad \left. 0, \frac{g[2][2]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0\right\}\}, \\
& \{2, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x4, \left\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0\right\}\}, \\
& \{2, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \left\{\left\{3, x0, \right. \right. \\
& \quad \left. \left\{0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4] - 2 H \text{Cot}[6 H x0] g[3][3][x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\right\}\right\}, \\
& \{3, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x3, \left\{ \frac{g[3][3]^{(1,0)}[x0, x4] - 2 H \text{Cot}[6 H x0] g[3][3][x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, \right. \\
& \quad \left. 0, \frac{g[3][3]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[3][3][x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, 0\right\}\}, \\
& \{3, x4, \left\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[3][3][x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\right\}\}, \\
& \{3, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{\left\{4, x0, \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}\right\}, \left\{4, \right. \\
& \quad x1, \left\{0, \frac{g[1][1]^{(0,1)}[x0, x4] + 6 e^{2 a4[H x4]} H \text{Sin}[6 H x0]^{1/3} a4'[H x4] g[4][4][x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}}, \right. \\
& \quad \left. 0, 0, 0, 0, 0, 0\right\}\}, \left\{4, x2, \right. \\
& \quad \left\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4] + 6 e^{2 a4[H x4]} H \text{Sin}[6 H x0]^{1/3} a4'[H x4] g[4][4][x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}}, \right. \\
& \quad \left. 0, 0, 0, 0, 0\right\}\}, \left\{4, x3, \left\{0, 0, 0, \right. \right. \\
& \quad \left. \frac{g[3][3]^{(0,1)}[x0, x4] + 6 e^{2 a4[H x4]} H \text{Sin}[6 H x0]^{1/3} a4'[H x4] g[4][4][x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}}, \right. \\
& \quad \left. 0, 0, 0, 0\right\}\}, \\
& \left\{4, x4, \left\{-\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}\right\}, \left\{4, \right.
\end{aligned}$$

$$\begin{aligned}
& \times 5, \left\{ 0, 0, 0, 0, 0, \frac{1}{2} \left(-6 e^{-2 a_4 [H x_4]} H \sin[6 H x_0]^{1/3} a_4' [H x_4] - \frac{g[5][5]^{(0,1)} [x_0, x_4]}{g[4][4][x_0, x_4]} \right) \right. \\
& \quad \left. \sqrt{-g[4][4][x_0, x_4]}, 0, 0 \right\}, \{4, x_6, \\
& \quad \left\{ 0, 0, 0, 0, 0, \frac{1}{2} \left(-6 e^{-2 a_4 [H x_4]} H \sin[6 H x_0]^{1/3} a_4' [H x_4] - \frac{g[6][6]^{(0,1)} [x_0, x_4]}{g[4][4][x_0, x_4]} \right) \right. \\
& \quad \left. \sqrt{-g[4][4][x_0, x_4]}, 0 \right\}, \{4, x_7, \{0, 0, 0, 0, 0, 0, 0, \\
& \quad \frac{1}{2} \left(-6 e^{-2 a_4 [H x_4]} H \sin[6 H x_0]^{1/3} a_4' [H x_4] - \frac{g[7][7]^{(0,1)} [x_0, x_4]}{g[4][4][x_0, x_4]} \right) \\
& \quad \sqrt{-g[4][4][x_0, x_4]} \} \}, \{ \{5, x_0, \\
& \quad \{0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)} [x_0, x_4] - 2 H \cot[6 H x_0] g[5][5][x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}}, 0, 0 \} \}, \\
& \quad \{5, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{5, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \\
& \quad \{5, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \\
& \quad \{5, x_4, \{0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)} [x_0, x_4] + 6 H a_4' [H x_4] g[5][5][x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}}, 0, 0 \} \}, \\
& \quad \{5, x_5, \left\{ -\frac{g[5][5]^{(1,0)} [x_0, x_4] - 2 H \cot[6 H x_0] g[5][5][x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}}, 0, 0, \right. \\
& \quad \left. 0, -\frac{g[5][5]^{(0,1)} [x_0, x_4] + 6 H a_4' [H x_4] g[5][5][x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}}, 0, 0, 0 \} \}, \\
& \quad \{5, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{5, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{ \{6, x_0, \\
& \quad \{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)} [x_0, x_4] - 2 H \cot[6 H x_0] g[6][6][x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]}}, 0 \} \}, \\
& \quad \{6, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{6, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \\
& \quad \{6, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \\
& \quad \{6, x_4, \{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)} [x_0, x_4] + 6 H a_4' [H x_4] g[6][6][x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]}}, 0 \} \}, \\
& \quad \{6, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \\
& \quad \{6, x_6, \left\{ -\frac{g[6][6]^{(1,0)} [x_0, x_4] - 2 H \cot[6 H x_0] g[6][6][x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]}}, 0, 0, \right. \\
& \quad \left. 0, -\frac{g[6][6]^{(0,1)} [x_0, x_4] + 6 H a_4' [H x_4] g[6][6][x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]}}, 0, 0, 0 \} \}, \\
& \quad \{6, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{ \{7, x_0, \\
& \quad \{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)} [x_0, x_4] - 2 H \cot[6 H x_0] g[7][7][x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]}}, 0 \} \}, \\
& \quad \{7, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{7, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \},
\end{aligned}$$

$$\begin{aligned}
& \{7, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{7, x4, \left\{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}}\right\}\right\}, \\
& \{7, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{7, x7, \left\{-\frac{g[7][7]^{(1,0)}[x0, x4] - 2 H \text{Cot}[6 H x0] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}}\right\}, 0, 0, \right. \\
& \left. 0, -\frac{g[7][7]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}}\right\}\} \}
\end{aligned}$$

Out[*]=

```

{{{0, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x7, {0, 0, 0, 0, 0, 0, 0, 0}}, {{1, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x7, {0, 0, 0, 0, 0, 0, 0, 0}}, {{2, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x7, {0, 0, 0, 0, 0, 0, 0, 0}}, {{3, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x7, {0, 0, 0, 0, 0, 0, 0, 0}}, {{4, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x7, {0, 0, 0, 0, 0, 0, 0, 0}}, {{5, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x7, {0, 0, 0, 0, 0, 0, 0, 0}}, {{6, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x7, {0, 0, 0, 0, 0, 0, 0, 0}}, {{7, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}}

```

```

In[*]:= Block[{eAα, eaA, t, ginv, Γ, gg(*, ωω*)},
  (*ωω=ωμ(a)(b) /. Sωf; *)
  gg = g4488 /. sg;
  {ginv, Γ} = con[gg];
  eaA = e(A)α /. sge(A)α; eAα = eα(A) /. sgeα(A);
  t = Table[Block[{v, V, contra, covari},
    V = eAα[[b, All]];
    v = eaA[[All, b]];
    contra =
      Table[D[V[[k]], X[[μ]]] + Sum[v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*no ω terms*)
    covari = Table[D[V[[k]], X[[μ]]] - Sum[V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}] +
      Sum[eAα[[J, All]] [[k]] × ωmat[[μ, b, J]], {J, 1, DIM8}],
    {k, 1, DIM8}]; (*ω terms*)
    {b - 1, X[[μ]], {{contra}, {covari}}}], {b, 1, Length[eaA]}, {μ, 1, DIM8}];
  t]

```

FullSimplify[#, /. ssgm4488,

constraintVars && e^{2 a4[H x4]} > 0 && e^{a4[H x4]} > 0 && e^{-2 a4[H x4]} > 0 && e^{-a4[H x4]} > 0] & /@ %

Out[*]=

$$\left\{\left\{\left\{0, x0, \left\{\left\{0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0\right\}\right\}\right\},
\right.$$

$$\begin{aligned}
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{0, x1, \{\{\{0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x2, \{\{\{0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[2][2][x0, x4]}, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x3, \{\{\{0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[3][3][x0, x4]}, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x4, \{\{\{0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x5, \{\{\{0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[5][5][x0, x4]}, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x6, \{\{\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[6][6][x0, x4]}, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]}\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x1, \{\{\{-\frac{g[1][1]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, \\
& \quad -\frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},
\end{aligned}$$

$$\begin{aligned}
& \{2, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x2, \left\{\left\{\left\{-\frac{g[2][2]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0\right\}\right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\right\}, \\
& \{2, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x3, \left\{\left\{\left\{-\frac{g[3][3]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0\right\}\right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\right\}, \\
& \{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x0, \left\{\left\{\left\{\frac{g[0][0]^{(0,1)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0\right\}\right\}, \right. \\
& \quad \left. \left\{\left\{0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]} - \right. \right. \right. \\
& \quad \left. \left. \left. \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]}\right) \right. \right. \right. \\
& \quad \left. \left. \left. g[4][4][x0, x4], 0, 0, 0\right\}\right\}\right\}, \\
& \{4, x1, \left\{\left\{\left\{0, \frac{g[1][1]^{(0,1)}[x0, x4]}{2 g[1][1][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}\right\}, \right. \\
& \quad \left. \left\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\right\}\right\}, \\
& \{4, x2, \left\{\left\{\left\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]}{2 g[2][2][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0\right\}\right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x3, \{\{\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2 g[3][3][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x4, \{\{\{-\frac{g[4][4]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \\
& \frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0\}\}, \\
& \{\{0, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]} - \\
& \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \\
& g[4][4][x0, x4], 0, 0, 0\}\}\}\}, \\
& \{4, x5, \{\{\{0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[5][5][x0, x4]}, 0, 0\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x6, \{\{\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[6][6][x0, x4]}, 0\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{5, x0, \\
& \{\{\{0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]}, \\
& 0, 0\}\}, \{\{0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}} - \\
& \frac{g[5][5]^{(1,0)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]} - \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \right. \\
& \left. \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) g[5][5][x0, x4], 0, 0\}\}\}\}, \\
& \{5, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{5, x4,
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]}, \right. \right. \right. \\
& \quad \left. \left. 0, 0 \right\} \right\}, \left\{ \left\{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}} - \right. \right. \\
& \quad \left. \left. \frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]} - \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \right. \right. \right. \\
& \quad \left. \left. \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) g[5][5][x0, x4], 0, 0 \right\} \right\} \Big\}, \\
& \{5, x5, \left\{ \left\{ -\frac{g[5][5]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. -\frac{g[5][5]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0 \right\} \right\}, \\
& \quad \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \Big\}, \{5, x6, \\
& \quad \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\}, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \Big\}, \\
& \{5, x7, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\}, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \Big\}, \\
& \left\{ \{6, x0, \left\{ \left\{ 0, 0, 0, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \frac{g[6][6]^{(1,0)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0 \right\} \right\}, \right. \\
& \quad \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \right. \right. \\
& \quad \left. \left. \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]} - \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \right. \right. \right. \\
& \quad \left. \left. \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) g[6][6][x0, x4], 0 \right\} \right\} \Big\}, \\
& \{6, x1, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\}, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \Big\}, \\
& \{6, x2, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\}, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \Big\}, \\
& \{6, x3, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\}, \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \Big\}, \\
& \{6, x4, \left\{ \left\{ 0, 0, 0, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0 \right\} \right\}, \\
& \quad \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \right. \right. \\
& \quad \left. \left. \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]} - \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) g[6][6][x0, x4], 0\}}\}}\}, \\
& \{6, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x6, \{\{\{-\frac{g[6][6]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0, \\
& 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{6, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{\{7, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, \\
& \frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]}\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}} - \\
& \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 g[7][7][x0, x4]} - \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \right. \\
& \left. \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) g[7][7][x0, x4]\}}\}}\}, \\
& \{7, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, \\
& \frac{g[7][7]^{(0,1)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]}\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}} - \\
& \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 g[7][7][x0, x4]} - \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \right. \\
& \left. \frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) g[7][7][x0, x4]\}}\}}\}, \\
& \{7, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x7, \{\{\{-\frac{g[7][7]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[7][7][x0, x4]}}, 0,
\end{aligned}$$

$$0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}}, 0, 0, 0\}}, \\ \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\}$$

Out[8]=

$$\left\{ \left\{ \{0, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\right\}, \right. \\ \{0, x1, \{\{\{0, H, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{0, x2, \{\{\{0, 0, H, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{0, x3, \{\{\{0, 0, 0, H, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{0, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{0, x5, \{\{\{0, 0, 0, 0, 0, 0, H, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{0, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, H\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{0, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, H\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \left\{ \{1, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\right\}, \\ \left\{ 1, x1, \left\{ \left\{ \left[-\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \right. \\ \left. \left. \left. 3 \sqrt{e^{2 a4[H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\} \right\}, \\ \{1, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \left\{ \{2, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\right\}, \\ \{2, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \left\{ 2, x2, \left\{ \left\{ \left[-\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \right. \\ \left. \left. \left. 3 \sqrt{e^{2 a4[H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\} \right\}, \\ \{2, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \left\{ \{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\right\}, \\ \{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \left\{ 3, x3, \left\{ \left\{ \left[-\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \right. \\ \left. \left. \left. 3 \sqrt{e^{2 a4[H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\} \right\}, \\ \{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \end{array}$$

$$\begin{aligned}
& \{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x1, \{\{\{0, 3 H a4'[H x4], 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x2, \{\{\{0, 0, 3 H a4'[H x4], 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x3, \{\{\{0, 0, 0, 3 H a4'[H x4], 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x5, \{\{\{0, 0, 0, 0, 0, 0, -3 H a4'[H x4], 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, -3 H a4'[H x4], 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{4, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, -3 H a4'[H x4]\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x5, \{\{\{\sqrt{e^{-2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, \\
& \quad 3 \sqrt{e^{-2 a4[H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x6, \{\{\{\sqrt{e^{-2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, \\
& \quad 3 \sqrt{e^{-2 a4[H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x7, \{\{\{\sqrt{e^{-2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, \\
& \quad 3 \sqrt{e^{-2 a4[H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\}
\end{aligned}$$

```

In[*]:= (*ωΨ=
Table[Sum[ExpandAll[(ω(a)μ(b) [μ,a,b]/.swf)Simplify[(Transpose[Ψ16].σ16.SAB[[a,b].Ψ16)]]],
{a,1,8},{b,1,8}],{μ,1,8}]*)

In[*]:= (*ωΨ=Table[Sum[ExpandAll[ωmat[[μ,a,b]]Simplify[(Transpose[Ψ16].σ16.SAB[[a,b].Ψ16)]]],
{a,1,8},{b,1,8}],{μ,1,8}]*)

In[*]:= (*jωΨ=ParallelTable[{j,base16[[j,2]],Table[
{X[[μ]],Simplify[(Transpose[Ψ16].σ16.(ZZZ0 *T16α[0]+T16α[4]* ZZZ4 ).base16[[j,1]].(Sum[
ωmat[[μ,a,b]]SAB[[a,b]],{a,1,8},{b,1,8}])Ψ16)]]},{μ,1,8}]],{j,1,Length[base16]}]*)

In[*]:= (*jωΨ=ParallelTable[{j,base16[[j,2]],Table[Sum[ExpandAll[(ω(a)μ(b) [μ,a,b]/.swf)Simplify[
(Transpose[Ψ16].σ16.(ZZZ0 *T16α[0]+T16α[4]* ZZZ4 ).base16[[j,1]].SAB[[a,b].Ψ16)]]],
{a,1,8},{b,1,8}],{μ,1,8}]],{j,1,Length[base16]}]*)

In[*]:= (*jωΨ[[92]]*)

In[*]:= (*jωΨ[[92]]/.ssgm4488//FullSimplify[#,constraintVars]&*)

In[*]:= SAB[[1]][[2]]

Out[*]=
{{0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{-1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0}}

In[*]:= (ω(a)μ(b) [8, 8, 1] /. Flatten[sugtry])

Out[*]=
e-a4[H x4] H Sin[6 H x0]1/6

```

In[*]:= SAB[[1, 2]]. Ψ 16

Out[*]=

$$\left\{ \frac{1}{2} f_{16}[7][x_0, x_4], \frac{1}{2} f_{16}[6][x_0, x_4], -\frac{1}{2} f_{16}[5][x_0, x_4], -\frac{1}{2} f_{16}[4][x_0, x_4], \right. \\ \frac{1}{2} f_{16}[3][x_0, x_4], \frac{1}{2} f_{16}[2][x_0, x_4], -\frac{1}{2} f_{16}[1][x_0, x_4], -\frac{1}{2} f_{16}[0][x_0, x_4], \\ -\frac{1}{2} f_{16}[15][x_0, x_4], -\frac{1}{2} f_{16}[14][x_0, x_4], \frac{1}{2} f_{16}[13][x_0, x_4], \frac{1}{2} f_{16}[12][x_0, x_4], \\ \left. -\frac{1}{2} f_{16}[11][x_0, x_4], -\frac{1}{2} f_{16}[10][x_0, x_4], \frac{1}{2} f_{16}[9][x_0, x_4], \frac{1}{2} f_{16}[8][x_0, x_4] \right\}$$

In[*]:= T16^A[0]

Out[*]=

```
{ {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0} }
```

In[*]:= SAB = Table $\left[\frac{1}{4} \left(T_{16}^A[A_1] \cdot T_{16}^A[B_1] - T_{16}^A[B_1] \cdot T_{16}^A[A_1] \right), \right.$
 $\left. \{A_1, 0, 7\}, \{B_1, 0, 7\} \right];$

In[*]:= Ψ 16

Out[*]=

```
{ f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
  f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
  f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
  f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4] }
```

```
In[*]:= Sum[ExpandAll[( $\omega_{\mu}^{(a)}(b)$ )[4, a, b] /. swfgtry] Simplify[(SAB[a, b])], {a, 1, 8}, {b, 1, 8}].
```

```
 $\Psi$ 16 // MatrixForm
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} -e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[5][x 0, x 4] \\ e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[4][x 0, x 4] \\ -e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[7][x 0, x 4] \\ e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[6][x 0, x 4] \\ -e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[1][x 0, x 4] \\ e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[0][x 0, x 4] \\ -e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[3][x 0, x 4] \\ e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[2][x 0, x 4] \\ e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[13][x 0, x 4] \\ -e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[12][x 0, x 4] \\ e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[15][x 0, x 4] \\ -e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[14][x 0, x 4] \\ e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[9][x 0, x 4] \\ -e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[8][x 0, x 4] \\ e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[11][x 0, x 4] \\ -e^{a^4[H \times 4]} H \sin[6 H x 0]^{1/6} f_{16}[10][x 0, x 4] \end{pmatrix}$$

```
In[*]:= (*Table[Block[{eA $\alpha$ , e $\alpha$ A, t, V}, eA $\alpha$ =e $\alpha_{(A)}$  /. sgtrye $\alpha_{(A)}$ ;
eA $\alpha$ =e $\alpha_{(A)}$  /. sgtrye $\alpha_{(A)}$ ;
V=eA $\alpha$ [[a, All]];
t=Table[Block[{v, d}, v=eA $\alpha$ [[All, b]];
d=D[v[[k]], X[[j]]]+Sum[v[[i]] r[[k, i, j]], {i, 1, DIM8}];
d.V], {k, 1, DIM8}];
t], {j, 1, DIM8}, {a, 1, Length[eA $\alpha$ ]}, {b, 1, Length[e $\alpha$ A]}] *)
```

```
In[*]:= Clear[rt];
rt[g_, ass_: {}] := Module[{t, r, RicciR, Rier, RS, G, ginv},
Print[Now];
t = AbsoluteTiming[
ginv = FullSimplify[Inverse[g], ass];
r = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
(D[g[[s, j]], X[[k]]]+D[g[[s, k]], X[[j]]]-D[g[[j, k]], X[[s]]]), {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
RicciR = ParallelTable[FullSimplify[D[r[[ $\mu$ , v,  $\beta$ ]], X[[ $\alpha$ ]]-D[r[[ $\mu$ , v,  $\alpha$ ]], X[[ $\beta$ ]]]+
Sum[r[[ $\mu$ , s,  $\alpha$ ]]*r[[s, v,  $\beta$ ]]-r[[ $\mu$ , s,  $\beta$ ]]*r[[s, v,  $\alpha$ ]], {s, 1, DIM8}], ass],
{ $\mu$ , 1, DIM8}, {v, 1, DIM8}, { $\alpha$ , 1, DIM8}, { $\beta$ , 1, DIM8}];
Rier = ParallelTable[FullSimplify[Sum[RicciR[[ $\alpha$ ,  $\mu$ ,  $\alpha$ , v]], { $\alpha$ , 1, DIM8}], ass],
{ $\mu$ , 1, DIM8}, {v, 1, DIM8}];
RS = FullSimplify[Tr[ginv.Rier], ass];
G = ParallelTable[FullSimplify[Rier[[ $\alpha$ ]]- $\frac{1}{2}$  g[[ $\alpha$ ]] RS, ass], { $\alpha$ , 1, Length[Rier]}];];
Print[t];
Print[Now];
Return[{ginv, r, RicciR, Rier, RS, G}]
```

verify $g_{\mu\nu}|_{\alpha} = 0 :$

```
In[ ]:= Simplify[Table[D[gtry[[j, k]], X[[i]]] -
    Sum[gtry[[k, s]] x r[[s, i, j]], {s, 1, DIM8}] - Sum[gtry[[j, s]] x r[[s, i, k]], {s, 1, DIM8}],
    {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] // Flatten // Union
```

```
Out[ ]:=
{0}
```

■ CHECK

```
In[ ]:= MatrixMetric44 // MatrixForm
```

```
Out[ ]//MatrixForm=
( Cot[6 H x0]^2      0      0      0      0
  0      e^2 a4[H x4] Sin[6 H x0]^(1/3)  0      0      0
  0      0      e^2 a4[H x4] Sin[6 H x0]^(1/3)  0      0
  0      0      0      e^2 a4[H x4] Sin[6 H x0]^(1/3)  0
  0      0      0      0      -1
  0      0      0      0      0      -e^-2 a4[H x4] S
  0      0      0      0      0      0
  0      0      0      0      0      0 )
```

```
In[ ]:= (e_alpha^alpha . MatrixMetric44 . e_alpha^alpha /. sgtrye_alpha^alpha) // MatrixForm
```

```
Out[ ]//MatrixForm=
( 1 0 0 0 0 0 0 0
  0 1 0 0 0 0 0 0
  0 0 1 0 0 0 0 0
  0 0 0 1 0 0 0 0
  0 0 0 0 -1 0 0 0
  0 0 0 0 0 -1 0 0
  0 0 0 0 0 0 -1 0
  0 0 0 0 0 0 0 -1 )
```

```
In[ ]:= Transpose[e_alpha^alpha] . eta4488 . e_alpha^alpha - MatrixMetric44 /. sgtrye_alpha^alpha // FullSimplify[#, constraintVars] &
```

```
Out[ ]:=
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[ ]:= Transpose[e_alpha^alpha] . MatrixMetric44 . e_alpha^alpha - eta4488 /. sgtrye_alpha^alpha // FullSimplify[#, constraintVars] &
```

```
Out[ ]:=
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[ ]:= Inverse[e_alpha^alpha /. sgtrye_alpha^alpha] == e_alpha^alpha /. sgtrye_alpha^alpha // FullSimplify[#, constraintVars] &
```

```
Out[ ]:=
True
```

```
In[ ]:= Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]
```

```
Out[ ]:=
{{0, 0}, {0, 0}}
```

```

In[*]:= (*Inverse[Transpose[e(A)α].MatrixMetric44.e(A)α]==
        eα(A).Inverse[MatrixMetric44].Transpose[eα(A)]]//FullSimplify[#,constraintVars]&*)
In[*]:= (*eα(A).Inverse[MatrixMetric44].Transpose[eα(A)]]//FullSimplify[#,constraintVars]&
        %-Inverse[η4488]]//Union[Flatten[#]]&*)
In[*]:= (*Transpose[eα(A)]==eα(A)]]//FullSimplify[#,constraintVars]&*)
In[*]:= (* (FullSimplify[eα(A).Inverse[MatrixMetric44].Transpose[eα(A)]-Inverse[η4488] ,
        constraintVars]]//Union[Flatten[#]]&)]//FullSimplify[#,constraintVars]&*)
In[*]:= (* (FullSimplify[Transpose[eα(A)].Inverse[MatrixMetric44].eα(A),constraintVars]-
        Inverse[η4488]]//Union[Flatten[#]]&)]//FullSimplify[#,constraintVars]&*)
In[*]:= (* (FullSimplify[Transpose[eα(A)].ginv.eα(A),constraintVars]-Inverse[η4488]]//
        Union[Flatten[#]]&)]//FullSimplify[#,constraintVars]&*)
In[*]:= (E(A)α = Table[Sum[eα(A)[[A1, α1]] ginv[[α1, β1]] × η4488[[A1, B1]] /. sgtryeα(A) ,
        {α1, 1, 8}, {A1, 1, 8}], {β1, 1, 8}, {B1, 1, 8}] //
        FullSimplify[#, constraintVars] &)] // Column

```

```

Out[*]:= {Tan[6 H x0], 0, 0, 0, 0, 0, 0, 0}
        {0,  $\frac{e^{-34[x4]}}{\sin[6 H x0]^{1/6}}$ , 0, 0, 0, 0, 0, 0}
        {0, 0,  $\frac{e^{-34[x4]}}{\sin[6 H x0]^{1/6}}$ , 0, 0, 0, 0, 0}
        {0, 0, 0,  $\frac{e^{-34[x4]}}{\sin[6 H x0]^{1/6}}$ , 0, 0, 0, 0}
        {0, 0, 0, 0, 1, 0, 0, 0}
        {0, 0, 0, 0, 0,  $\frac{e^{34[x4]}}{\sin[6 H x0]^{1/6}}$ , 0, 0}
        {0, 0, 0, 0, 0, 0,  $\frac{e^{34[x4]}}{\sin[6 H x0]^{1/6}}$ , 0}
        {0, 0, 0, 0, 0, 0, 0,  $\frac{e^{34[x4]}}{\sin[6 H x0]^{1/6}}$ }

```

$$\begin{aligned}
 \text{Lagrangian}\Psi16 = & \\
 & \kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi16].\sigma16.T16^A[A1]. \\
 & \Psi16_{,\alpha} g^{-1\ \alpha\beta} \eta_{A1\ B1} E_{\beta}^{B1} + \text{mass term}
 \end{aligned}$$

```
In[*]:= ass = constraintVars
```

```
Out[*]:=
```

```
x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 &&  $\sqrt{\text{Sin}[z]}$  > 0 && Sin[z]3/2 > 0 &&
 $\sqrt{\text{Sin}[z]}$  > 0 &&  $\frac{1}{\text{Sin}[z]^{3/2}}$  > 0 &&  $\frac{1}{\sqrt{\text{Sin}[z]}}$  > 0 && Cot[z] > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 &&
Cot[z]3/2 > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 && Tan[z] > 0 && Sec[z] > 0 &&  $\sqrt{\text{Csc}[z]}$  > 0 && Csc[z] > 0
```

```
In[*]:= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
```

```
Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 &&  $\sqrt{\text{Sin}[z]}$  > 0 && Sin[z]3/2 > 0 &&
 $\sqrt{\text{Sin}[z]}$  > 0 &&  $\frac{1}{\text{Sin}[z]^{3/2}}$  > 0 &&  $\frac{1}{\sqrt{\text{Sin}[z]}}$  > 0 && Cot[z] > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 &&
Cot[z]3/2 > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 && Tan[z] > 0 && Sec[z] > 0 &&  $\sqrt{\text{Csc}[z]}$  > 0 && Csc[z] > 0
```

```
Out[*]:=
```

```
x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 &&  $\sqrt{\text{Sin}[z]}$  > 0 && Sin[z]3/2 > 0 &&
 $\sqrt{\text{Sin}[z]}$  > 0 &&  $\frac{1}{\text{Sin}[z]^{3/2}}$  > 0 &&  $\frac{1}{\sqrt{\text{Sin}[z]}}$  > 0 && Cot[z] > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 &&
Cot[z]3/2 > 0 &&  $\sqrt{\text{Cot}[z]}$  > 0 && Tan[z] > 0 && Sec[z] > 0 &&  $\sqrt{\text{Csc}[z]}$  > 0 && Csc[z] > 0
```

```
In[*]:= ginv = FullSimplify[Inverse[gtry], ass]
```

```
Out[*]:=
```

```
True
```

```
In[*]:= ginv === Transpose[ginv]
```

```
Out[*]:=
```

```
True
```

verify metric covariant derivative again:

```
In[*]:= Simplify[Table[D[gtry[[j, k]], X[[i]]] -
Sum[gtry[[k, s]] × r[[s, i, j]], {s, 1, DIM8}] - Sum[gtry[[j, s]] × r[[s, i, k]], {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] // Flatten // Union
```

```
Out[*]:=
```

```
{0}
```

```
In[*]:= RS
```

```
Out[*]:=
```

```
6 H2 (-7 + a4'[H x4]2)
```



```
In[ ]:= EinsteinG
skelx[%]
```

```
Out[ ]=
{{-3 H^2 Cot[6 H x0]^2 (-5 + a4' [H x4]^2), 0, 0, 0, 0, 0, 0, 0},
{0, -e^{2 a4 [H x4]} H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 - a4'' [H x4]), 0, 0, 0, 0, 0, 0},
{0, 0, -e^{2 a4 [H x4]} H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 - a4'' [H x4]), 0, 0, 0, 0, 0},
{0, 0, 0, -e^{2 a4 [H x4]} H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 - a4'' [H x4]), 0, 0, 0, 0},
{0, 0, 0, 0, -3 H^2 (7 + a4' [H x4]^2), 0, 0, 0},
{0, 0, 0, 0, 0, e^{-2 a4 [H x4]} H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 + a4'' [H x4]), 0, 0},
{0, 0, 0, 0, 0, 0, e^{-2 a4 [H x4]} H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 + a4'' [H x4]), 0},
{0, 0, 0, 0, 0, 0, 0, e^{-2 a4 [H x4]} H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 + a4'' [H x4])}}
```

```
Out[ ]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

```

```
Union[Table[
Sum[τ[A].bas64[[k, 1]].σ.ϕ[A]×η8[A, A], {A, 1, 8}] - ID8 Tr[bas64[[k, 1]].σ] +
4 eg[[k]]  $\frac{\text{bas64}[[k, 1]] - \text{Transpose}[\text{bas64}[[k, 1]]]}{2}$ .σ, {k, 1, 64}]]]
```

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}
```

BASIS of 16 × 16 matrices :

```
In[ ]:= 16 * 16
```

```
Out[ ]=
256
```

```
In[ ]:= 2^8
```

```
Out[ ]=
256
```

```
In[ ]:= Sum[Binomial[8, p], {p, 0, 8}]
```

```
Out[ ]=
256
```

```
In[ ]:= Id16 == ID16 == Dot@@ (T16^A[#] & /@ Range[0, 8])
```

```
Out[ ]=
True
```

```
In[*]:= Clear[t160]; t160 = {{Id16, {Range[0, 8]}}}; Length[t160]  
Clear[t16A];  
t16A = {};  
Do[AppendTo[t16A, {T16^A[j], {j}}], {j, 0, 7}];  
Length[t16A]  
Clear[t16AB];  
t16AB = {};  
Do[AppendTo[t16AB, {T16^A[j].T16^A[k], {j, k}}], {j, 0, 6}, {k, j + 1, 7}];  
Length[t16AB]  
Clear[t16ABC];  
t16ABC = {};  
Do[AppendTo[t16ABC, {T16^A[j].T16^A[k].T16^A[h], {j, k, h}}],  
{j, 0, 5}, {k, j + 1, 6}, {h, k + 1, 7}];  
Length[t16ABC]  
Clear[t16ABCD];  
t16ABCD = {};  
Do[AppendTo[t16ABCD, {T16^A[j].T16^A[k].T16^A[h].T16^A[i], {j, k, h, i}}],  
{j, 0, 4}, {k, j + 1, 5}, {h, k + 1, 6}, {i, h + 1, 7}];  
Length[t16ABCD]  
Clear[t16ABCDE];  
t16ABCDE = {};  
Do[AppendTo[t16ABCDE, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l], {j, k, h, i, l}}],  
{j, 0, 3}, {k, j + 1, 4}, {h, k + 1, 5}, {i, h + 1, 6}, {l, i + 1, 7}];  
Length[t16ABCDE]  
Clear[t16ABCDEF];  
t16ABCDEF = {};  
Do[AppendTo[t16ABCDEF, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q], {j, k, h, i, l,  
q}}, {j, 0, 2}, {k, j + 1, 3}, {h, k + 1, 4}, {i, h + 1, 5}, {l, i + 1, 6}, {q, l + 1, 7}];  
Length[t16ABCDEF]  
Clear[t16ABCDEFGF];  
t16ABCDEFGF = {};  
Do[AppendTo[t16ABCDEFGF,  
{T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].T16^A[r], {j, k, h, i, l, q, r}}],  
{j, 0, 1}, {k, j + 1, 2}, {h, k + 1, 3}, {i, h + 1, 4}, {l, i + 1, 5}, {q, l + 1, 6}, {r, q + 1, 7}];  
Length[t16ABCDEFGF]  
Clear[t16ABCDEFGFH];  
t16ABCDEFGFH = {};  
Do[AppendTo[t16ABCDEFGFH, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].  
T16^A[q].T16^A[r].T16^A[s], {j, k, h, i, l, q, r, s}}], {j, 0, 1}, {k, j + 1, 1},  
{h, k + 1, 2}, {i, h + 1, 3}, {l, i + 1, 4}, {q, l + 1, 5}, {r, q + 1, 6}, {s, r + 1, 7}];  
Length[t16ABCDEFGFH]  
% + % + % + % + % + % + % + % + % + % + % + % + % + % + %
```

Out[•] =

Out[•] =

8

Out[•] =

28

$$Out[\bullet] =$$

56

$Out[\bullet] =$

70

Out[•] =

56

Out[•] =

28

Out[•] =

8

Out[•] =

1

$Out[\bullet] =$

256

```
In[*]:= (*base16=Flatten[  
        {t160,t16A,t16AB,t16ABC,t16ABCD,t16ABCDE,t16ABCDEF,t16ABCDEFGF,t16ABCDEFGH},1];  
Length[base16]*)
```

```
In[*]:= base16 = Flatten[
    {t16A, t16AB, t16ABC, t16ABCD, t16ABCDE, t16ABCDEF, t16ABCDEF, t16ABCDEF, t160}, 1];
Length[base16]
```

Out[•] =

256

```
In[•]:= Length[Union[base16[All, 1]]]
```

Out[•] =

256

```
In[*]:= dups = Block[{s}, s = base16[All, 1]; Select[s, MemberQ[s, -#] &]; Length[dups]
```

Out[•] =

0

```
In[6]:= dups = Select[base16, MemberQ[base16[[All, 1]], -#[[1]]] &]; Length[dups]
```

Out[•] =

0

```
In[*]:= BASE16 = use16 = Complement[base16, dups]; Length[use16]
```

Out[•] =

256

$$\ln[*]:= (* (T16^A[8]=T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]) //MatrixForm*)$$

```
In[•]:= MatrixForm[T16^A[#]] & /@ Range[0, 7]
```

Out[•] =

[illegible]

[illegible]

[illegible]

$$\left(\begin{array}{cccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad \left. \vphantom{\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}} \right\}$$

```
In[•] := (*BASE16[[-1]]*)
```

```
ln[*]:= (*BASE16[[2]]
          %[[1]]==BASE16[[1]] [[1]]*)
```

```
In[*]:= 16 * (16 + 1) / 2
         16 * (16 - 1) / 2
         % + %%
```

```
Out[*]=
136
```

```
Out[*]=
120
```

```
Out[*]=
256
```

```
In[*]:= Clear[symm16];
         symm16 = {};
         Do[If[base16[[k, 1]] == Transpose[base16[[k, 1]]],
             AppendTo[symm16, {base16[[k, 1]], {{k}, base16[[k, 2]]}}], {k, 1, Length[base16]}];
         Length[symm16]
         symm16[[]]
```

```
Out[*]=
136
```

```
Out[*]=
{{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

```
In[*]:= dupssymm16 = Select[symm16, MemberQ[symm16[All, 1], -#[[1]] &]; Length[dupssymm16]
```

```
Out[*]=
0
```

```
In[ ]:= #[[2]] & /@ symm16
```

```
Out[ ]=
```

```
{{{1}, {0}}, {{2}, {1}}, {{3}, {2}}, {{4}, {3}}, {{12}, {0, 4}}, {{13}, {0, 5}},
{{14}, {0, 6}}, {{15}, {0, 7}}, {{18}, {1, 4}}, {{19}, {1, 5}}, {{20}, {1, 6}},
{{21}, {1, 7}}, {{23}, {2, 4}}, {{24}, {2, 5}}, {{25}, {2, 6}}, {{26}, {2, 7}},
{{27}, {3, 4}}, {{28}, {3, 5}}, {{29}, {3, 6}}, {{30}, {3, 7}}, {{39}, {0, 1, 4}},
{{40}, {0, 1, 5}}, {{41}, {0, 1, 6}}, {{42}, {0, 1, 7}}, {{44}, {0, 2, 4}}, {{45}, {0, 2, 5}},
{{46}, {0, 2, 6}}, {{47}, {0, 2, 7}}, {{48}, {0, 3, 4}}, {{49}, {0, 3, 5}}, {{50}, {0, 3, 6}},
{{51}, {0, 3, 7}}, {{59}, {1, 2, 4}}, {{60}, {1, 2, 5}}, {{61}, {1, 2, 6}}, {{62}, {1, 2, 7}},
{{63}, {1, 3, 4}}, {{64}, {1, 3, 5}}, {{65}, {1, 3, 6}}, {{66}, {1, 3, 7}}, {{73}, {2, 3, 4}},
{{74}, {2, 3, 5}}, {{75}, {2, 3, 6}}, {{76}, {2, 3, 7}}, {{89}, {4, 5, 6}}, {{90}, {4, 5, 7}},
{{91}, {4, 6, 7}}, {{92}, {5, 6, 7}}, {{93}, {0, 1, 2, 3}}, {{102}, {0, 1, 4, 5}},
{{103}, {0, 1, 4, 6}}, {{104}, {0, 1, 4, 7}}, {{105}, {0, 1, 5, 6}}, {{106}, {0, 1, 5, 7}},
{{107}, {0, 1, 6, 7}}, {{112}, {0, 2, 4, 5}}, {{113}, {0, 2, 4, 6}}, {{114}, {0, 2, 4, 7}},
{{115}, {0, 2, 5, 6}}, {{116}, {0, 2, 5, 7}}, {{117}, {0, 2, 6, 7}}, {{118}, {0, 3, 4, 5}},
{{119}, {0, 3, 4, 6}}, {{120}, {0, 3, 4, 7}}, {{121}, {0, 3, 5, 6}}, {{122}, {0, 3, 5, 7}},
{{123}, {0, 3, 6, 7}}, {{132}, {1, 2, 4, 5}}, {{133}, {1, 2, 4, 6}}, {{134}, {1, 2, 4, 7}},
{{135}, {1, 2, 5, 6}}, {{136}, {1, 2, 5, 7}}, {{137}, {1, 2, 6, 7}}, {{138}, {1, 3, 4, 5}},
{{139}, {1, 3, 4, 6}}, {{140}, {1, 3, 4, 7}}, {{141}, {1, 3, 5, 6}}, {{142}, {1, 3, 5, 7}},
{{143}, {1, 3, 6, 7}}, {{148}, {2, 3, 4, 5}}, {{149}, {2, 3, 4, 6}}, {{150}, {2, 3, 4, 7}},
{{151}, {2, 3, 5, 6}}, {{152}, {2, 3, 5, 7}}, {{153}, {2, 3, 6, 7}}, {{162}, {4, 5, 6, 7}},
{{167}, {0, 1, 2, 4, 5}}, {{168}, {0, 1, 2, 4, 6}}, {{169}, {0, 1, 2, 4, 7}},
{{170}, {0, 1, 2, 5, 6}}, {{171}, {0, 1, 2, 5, 7}}, {{172}, {0, 1, 2, 6, 7}},
{{173}, {0, 1, 3, 4, 5}}, {{174}, {0, 1, 3, 4, 6}}, {{175}, {0, 1, 3, 4, 7}},
{{176}, {0, 1, 3, 5, 6}}, {{177}, {0, 1, 3, 5, 7}}, {{178}, {0, 1, 3, 6, 7}},
{{183}, {0, 2, 3, 4, 5}}, {{184}, {0, 2, 3, 4, 6}}, {{185}, {0, 2, 3, 4, 7}},
{{186}, {0, 2, 3, 5, 6}}, {{187}, {0, 2, 3, 5, 7}}, {{188}, {0, 2, 3, 6, 7}},
{{197}, {0, 4, 5, 6, 7}}, {{198}, {1, 2, 3, 4, 5}}, {{199}, {1, 2, 3, 4, 6}},
{{200}, {1, 2, 3, 4, 7}}, {{201}, {1, 2, 3, 5, 6}}, {{202}, {1, 2, 3, 5, 7}},
{{203}, {1, 2, 3, 6, 7}}, {{212}, {1, 4, 5, 6, 7}}, {{217}, {2, 4, 5, 6, 7}},
{{218}, {3, 4, 5, 6, 7}}, {{225}, {0, 1, 2, 4, 5, 6}}, {{226}, {0, 1, 2, 4, 5, 7}},
{{227}, {0, 1, 2, 4, 6, 7}}, {{228}, {0, 1, 2, 5, 6, 7}}, {{229}, {0, 1, 3, 4, 5, 6}},
{{230}, {0, 1, 3, 4, 5, 7}}, {{231}, {0, 1, 3, 4, 6, 7}}, {{232}, {0, 1, 3, 5, 6, 7}},
{{234}, {0, 2, 3, 4, 5, 6}}, {{235}, {0, 2, 3, 4, 5, 7}}, {{236}, {0, 2, 3, 4, 6, 7}},
{{237}, {0, 2, 3, 5, 6, 7}}, {{240}, {1, 2, 3, 4, 5, 6}}, {{241}, {1, 2, 3, 4, 5, 7}},
{{242}, {1, 2, 3, 4, 6, 7}}, {{243}, {1, 2, 3, 5, 6, 7}}, {{247}, {0, 1, 2, 3, 4, 5, 6}},
{{248}, {0, 1, 2, 3, 4, 5, 7}}, {{249}, {0, 1, 2, 3, 4, 6, 7}}, {{250}, {0, 1, 2, 3, 5, 6, 7}},
{{255}, {0, 1, 2, 3, 4, 5, 6, 7}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

```
In[ ]:=  $\sigma_{16} = T_{16}^A[0] \cdot T_{16}^A[1] \cdot T_{16}^A[2] \cdot T_{16}^A[3]$ 
```

```
Out[ ]=
```

```
True
```

```
In[ ]:=  $\sigma_{16} = \text{base16}[[93]][1]$ 
```

```
Out[ ]=
```

```
True
```

```

In[ ]:= Select[symm16, #[[2]][[1]][[1]] == 93 & → "Index"]
Out[ ]:=
{49}

In[ ]:=  $\sigma$ 16 == symm16[[49]][[1]]
Out[ ]:=
True

In[ ]:= symm16[[134]]
Out[ ]:=
{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{250}, {0, 1, 2, 3, 5, 6, 7}}

In[ ]:= Select[symm16, #[[2]][[1]][[1]] == 250 & → "Index"]
Out[ ]:=
{134}

In[ ]:= T16SuperscriptA
Out[ ]:=
T16A

In[ ]:=  $\sigma$ 16.T16A[5].T16A[6].T16A[7] == symm16[[134]][[1]]
Out[ ]:=
True

In[ ]:= T16A[8] == base16[[255]][[1]]
Out[ ]:=
True

In[ ]:= Select[symm16, #[[2]][[1]][[1]] == 255 & → "Index"]
Out[ ]:=
{135}

```


BASIS of 8×8 matrices :

```
In[*]:= tA = {}; Do[AppendTo[tA, {τ[j], {j}}], {j, 1, 7}]
Length[tA]
tAB = {}; Do[AppendTo[tAB, {τ[j].τ[k], {j, k}}], {j, 1, 6}, {k, j + 1, 7}]
Length[tAB]
tABC = {};
Do[AppendTo[tABC, {τ[j].τ[k].τ[h], {j, k, h}}], {j, 1, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[tABC]
```

```
Out[*]=
7
```

```
Out[*]=
21
```

```
Out[*]=
35
```

```
In[*]:= tA[[1]]
```

```
Out[*]=
{{{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}}, {1}}
```

```
In[*]:= tA[[#]] [[1]] == -Transpose[tA[[#]] [[1]]] & /@ Range[Length[tA]]
```

```
Out[*]=
{True, True, True, False, False, False, False}
```

```
In[*]:= bas64 = Flatten[{tA, tAB, tABC, {{Id, {64}}}}, 1]; Length[bas64]
```

```
Out[*]=
64
```

```
In[*]:= determineIF8AntiSymmetric[m_] := Module[{t, tt, sum, res, r},
  t = m.τ[#] & /@ Range[0, 7];
  tt = Transpose[#] & /@ t;
  sum = FullSimplify[t[[#]] + tt[[#]] & /@ Range[Length[t]]];
  res = Union[Flatten[#]] & /@ sum;
  If[Length[#] > 1, Return[False]] & /@ res;
  r = Union[Flatten[res]];
  Return[r == {0}];
]
```

```
In[*]:= determineIF8AntiSymmetric[Zero8]
```

```
Out[*]=
True
```

```
In[*]:= determineIF8AntiSymmetric[ID8]
```

```
Out[*]=
False
```



```

In[*]:= 32 × 65
Out[*]=
2080

In[*]:= (*Length[Flatten[countTrace,1]]*)
In[*]:= (*{Length[countTrace],Length[countTrace[[1]],Length[countTrace[[1,1]]]}*)
In[*]:= (*{Length[countTrace],Length[countTrace[[2]],Length[countTrace[[2,1]]]}*)
In[*]:= (*{Length[countTrace],Length[countTrace[[64]],Length[countTrace[[64,1]]]}*)
In[*]:= (*{Length[Flatten[countTrace,1]][All,3]],Count[Flatten[countTrace,1]][All,3],0,
Count[Flatten[countTrace,1]][All,3],1,Count[Flatten[countTrace,1]][All,3],-1]}*)
In[*]:= 32 × 63
Out[*]=
2016

In[*]:= anti = {};
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],
AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}], {k, 1, 64}];
Length[anti]
anti[[28]]

Out[*]=
28

Out[*]=
{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}

In[*]:= anti[[28, 1]]
Out[*]=
{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}

In[*]:= anti[[28, 2]]
Out[*]=
{{63}, {5, 6, 7}}

In[*]:= symm = {};
Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
AppendTo[symm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}], {k, 1, 64}];
Length[symm]
symm[[28]]

Out[*]=
35

Out[*]=
{{{ -1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
{0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, {{52}, {2, 5, 7}}}
```

Spinor Lagrangian

```
In[•]:= usegT16 = Table[FullSimplify[(T16α[α1 - 1]), constraintVars], {α1, 1, Length[X]}]
```

Out[•] =

[illegible]

$$\begin{aligned}
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, -\frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, -\frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{-\frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0\right\}, \right. \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[2][2][x0, x4]}}\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0\right\},
\end{aligned}$$

$$\begin{aligned} & \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, -\frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{\frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, -\frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0\right\}, \right. \\ & \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[3][3][x0, x4]}}\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \\ & \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \\ & \left.\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\right\}, \right. \\ & \left.\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0\right\}, \right. \end{aligned}$$

$$\begin{aligned}
& \left\{0, 0, 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, -\frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{-\frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, -\frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0\right\}, \right. \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, -\frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0\right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, -\frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{\frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, -\frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0\right\}, \right. \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}} \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, -\frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ -\frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0 \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[7][7][x0, x4]}}\right\}, \\
& \left\{-\frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, -\frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, -\frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, -\frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}, 0\right\}, \\
& \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}\right\}
\end{aligned}$$

```

In[ ]:= useT16 = Table[FullSimplify[(T16α[α1 - 1] /. ssgm4488), constraintVars] /.
  {
    Sqrt[e-2 a4[H x4]] -> e-a4[H x4], Sqrt[e2 a4[H x4] Sin[6 H x0]1/3] -> ea4[H x4] Sin[6 H x0]1/6,
    1 / (Sqrt[e2 a4[H x4] Sin[6 H x0]1/3]) -> 1 / (ea4[H x4] Sin[6 H x0]1/6)}, {α1, 1, Length[X]}] /.
  {
    1 / (Sqrt[e-2 a4[H x4] Sin[6 H x0]1/3]) -> 1 / (ea4[H x4] Sin[6 H x0]1/6), Sqrt[e2 a4[H x4]] -> ea4[H x4]}

```

```

Out[ ]:= {
  {{0, 0, 0, 0, 0, 0, 0, 0, 0, Tan[6 H x0], 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, Tan[6 H x0], 0, 0, 0, 0, 0, 0, 0},

```

$$\begin{aligned} & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Tan}[6 H x 0], 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Tan}[6 H x 0], 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Tan}[6 H x 0], 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Tan}[6 H x 0], 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Tan}[6 H x 0], 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Tan}[6 H x 0]\}, \\ & \{\text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0\}, \\ & \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}\}, \right. \\ & \quad \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}, 0\}, \\ & \quad \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4 [H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0\right\}, \\ & \quad \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4 [H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0\right\}, \\ & \quad \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0\}, \\ & \quad \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \\ & \quad \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4 [H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0\right\}, \\ & \quad \left\{0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4 [H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \quad \left\{0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4 [H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \quad \left\{0, 0, 0, 0, 0, 0, \frac{e^{-a^4 [H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \quad \left\{0, 0, 0, 0, 0, -e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \quad \left\{0, 0, 0, 0, -e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \quad \left\{0, 0, 0, \frac{e^{-a^4 [H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \quad \left\{0, 0, \frac{e^{-a^4 [H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \\ & \quad \{0, -e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \quad \{-e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \\ & \left\{ \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4 [H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0\right\}, \right. \\ & \quad \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}\}, \\ & \quad \left.\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{-a^4 [H x 4]} \text{Csc}[6 H x 0]^{1/6}, 0, 0, 0\right\} \right\}, \end{aligned}$$

$$\begin{aligned}
& \{-e^{-a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, -e^{-a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{-1, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0 \right\}, \right. \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, -e^{a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, -e^{a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \{-e^{a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, -e^{a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4[H \times 4]}}{\operatorname{Sin}[6 H x 0]^{1/6}}, 0 \right\}, \right. \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -e^{a^4[H \times 4]} \operatorname{Csc}[6 H x 0]^{1/6}, 0\} \right\},
\end{aligned}$$

[illegible]

$$\left\{0, 0, 0, 0, 0, \frac{e^{-a^4[H x^4]}}{\sin[6 H x^0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, \frac{e^{-a^4[H x^4]}}{\sin[6 H x^0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, \frac{e^{-a^4[H x^4]}}{\sin[6 H x^0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0, 0\right\}\}$$

```
In[*]:= Dimensions[useT16]
```

```
Out[*]=
{8, 16, 16}
```

```
In[*]:= Det[MatrixMetric44]
```

```
Out[*]=
Cos[6 H x0]^2
```

```
In[*]:= useDSQRT = Cos[6 H x0]
```

```
Out[*]=
Cos[6 H x0]
```

```
In[*]:= wmat /. sg /. ssgm4488 /. subsDefects;
```

```
(% /. {Sqrt[e^2 a4[H x4]] -> e^a4[H x4], Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) -> e^a4[H x4] H Sin[6 H x0]^(1/6)}) -
  spinCoeffs // FullSimplify[#, constraintVars] & /. subsDefects /.
  {Sqrt[e^2 a4[H x4]] -> e^a4[H x4], Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) -> e^a4[H x4] H Sin[6 H x0]^(1/6)} //
  FullSimplify[#, constraintVars] & /. subsDefects;
% // Flatten // Union
% // ExpandAll /. subsDefects /.
  {Sqrt[e^2 a4[H x4]] -> e^a4[H x4], Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) -> e^a4[H x4] H Sin[6 H x0]^(1/6)} /. subsDefects
% /. {a4[H x4] -> pi^4, Sin[6 H x0] -> e^12}
```

```
Out[*]=
{0, e^-a4[H x4] (-1 + e^a4[H x4] Sqrt[e^-2 a4[H x4]]) H Sin[6 H x0]^(1/6),
  -((-e^a4[H x4] + Sqrt[e^2 a4[H x4]]) H Sin[6 H x0]^(1/6)), (-e^a4[H x4] + Sqrt[e^2 a4[H x4]]) H Sin[6 H x0]^(1/6),
  e^-a4[H x4] (1 - e^a4[H x4] Sqrt[e^-2 a4[H x4]]) H Sin[6 H x0]^(1/6) a4'[H x4],
  e^-a4[H x4] (-1 + e^a4[H x4] Sqrt[e^-2 a4[H x4]]) H Sin[6 H x0]^(1/6) a4'[H x4],
  (-e^a4[H x4] + Sqrt[e^2 a4[H x4]]) H Sin[6 H x0]^(1/6) a4'[H x4]}
```

```
Out[*]=
{0, -e^-a4[H x4] H Sin[6 H x0]^(1/6) + Sqrt[e^-2 a4[H x4]] H Sin[6 H x0]^(1/6),
  e^a4[H x4] H Sin[6 H x0]^(1/6) - Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6),
  -e^a4[H x4] H Sin[6 H x0]^(1/6) + Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6),
  e^-a4[H x4] H Sin[6 H x0]^(1/6) a4'[H x4] - Sqrt[e^-2 a4[H x4]] H Sin[6 H x0]^(1/6) a4'[H x4],
  -e^-a4[H x4] H Sin[6 H x0]^(1/6) a4'[H x4] + Sqrt[e^-2 a4[H x4]] H Sin[6 H x0]^(1/6) a4'[H x4],
  -e^a4[H x4] H Sin[6 H x0]^(1/6) a4'[H x4] + Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) a4'[H x4]}
```

```
Out[*]=
{0, 0, 0, 0, 0, 0, 0}
```

```

In[*]:= Protect[QQQ1, QQQ2, QQQ3, QQQ4]

Out[*]=
{QQQ1, QQQ2, QQQ3, QQQ4}

In[*]:= (matTest = (QQQ1 * ID16 + QQQ2 * covariantDiffMatrix)) // MatrixForm

Out[*] // MatrixForm =

$$\begin{pmatrix} \text{QQQ1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \text{QQQ1} & 0 & 0 & 0 & 0 & 0 & 0 & -\text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \text{QQQ1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\text{QQQ2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{QQQ1} & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \text{QQQ1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\text{QQQ2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \text{QQQ1} & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ2} \\ 0 & -\text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ1} & 0 & 0 & 0 & 0 & 0 & -\text{QQQ2} & 0 \\ \text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{QQQ2} \end{pmatrix}$$


```

```

In[*]:= Clear[Lg];
Lg[] :=

$$\sqrt{\text{detgg}} * \left( \text{Transpose}[\Psi16] . \sigma16 . \text{Sum} \left[ \text{FullSimplify}[(T16^a[\alpha1 - 1] /. \text{sg}), \text{constraintVars}] . \right. \right. \\ \left. \left( D[\Psi16, X[\alpha1]] + \left( \frac{K}{2} /. \{K \rightarrow 1\} \right) * \text{Sum}[\omega \text{mat}[\alpha1, a, b] * \text{SAB}[a, b] . \text{matTest} . \Psi16, \right. \right. \\ \left. \left. \{a, 1, 8\}, \{b, 1, 8\}] \right), \{\alpha1, 1, \text{Length}[X]\} \right] + \\ \left. (H * M) * \text{Transpose}[\Psi16] . \sigma16 . \Psi16 \right) // \text{Simplify}[\#, \text{constraintVars}] \&$$


```

```

In[*]:=  $\omega \mu \text{IJ}[1]$ 

Out[*]=
{{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}}

```

```

In[*]:= Clear[La];
La[] := useDSQRT *

$$\left( \text{Transpose}[\Psi16] . \sigma16 . \text{Sum} \left[ \text{useT16}[\alpha1] . \left( D[\Psi16, X[\alpha1]] + \left( \frac{K}{2} /. \{K \rightarrow 1\} \right) * \text{Sum}[\omega \mu \text{IJ}[\alpha1][A1, \right. \right. \right. \\ \left. \left. \left. B1] * \text{SAB}[A1, B1], \{A1, 1, 8\}, \{B1, 1, 8\}] . \text{matTest} . \Psi16 \right), \{\alpha1, 1, \text{Length}[X]\} \right] + \right. \\ \left. (H * M) * \text{Transpose}[\Psi16] . \sigma16 . \Psi16 \right) // \text{Simplify}[\#, \text{constraintVars}] \&$$


```

```

In[*]:= testLa = La[]

Out[*]=

$$-\frac{1}{2} e^{-2 a^4 [H \times 4]} \text{Cos}[6 H \times 0]$$


```


$$\begin{aligned}
& (4 e^{2 a_4 [H x_4]} H M f_{16}[2][x_0, x_4] \times f_{16}[6][x_0, x_4] + 6 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[3][x_0, x_4] \times \\
& f_{16}[6][x_0, x_4] - 6 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[2][x_0, x_4] \times f_{16}[7][x_0, x_4] + \\
& 4 e^{2 a_4 [H x_4]} H M f_{16}[3][x_0, x_4] \times f_{16}[7][x_0, x_4] - 4 e^{2 a_4 [H x_4]} H M f_{16}[8][x_0, x_4] \times \\
& f_{16}[12][x_0, x_4] - 6 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[9][x_0, x_4] \times f_{16}[12][x_0, x_4] + \\
& 6 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[8][x_0, x_4] \times f_{16}[13][x_0, x_4] - 4 e^{2 a_4 [H x_4]} H M f_{16}[9][x_0, x_4] \times \\
& f_{16}[13][x_0, x_4] - 4 e^{2 a_4 [H x_4]} H M f_{16}[10][x_0, x_4] \times f_{16}[14][x_0, x_4] + \\
& 6 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[11][x_0, x_4] \times f_{16}[14][x_0, x_4] - \\
& 6 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[10][x_0, x_4] \times f_{16}[15][x_0, x_4] - \\
& 4 e^{2 a_4 [H x_4]} H M f_{16}[11][x_0, x_4] \times f_{16}[15][x_0, x_4] + \\
& 3 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[0][x_0, x_4]^2 a_4' [H x_4] + (1 + 2 e^{2 a_4 [H x_4]}) H Q Q Q 2 f_{16}[1][x_0, x_4]^2 a_4' [H x_4] + \\
& H Q Q Q 2 f_{16}[2][x_0, x_4]^2 a_4' [H x_4] + 2 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[2][x_0, x_4]^2 a_4' [H x_4] + \\
& 2 H Q Q Q 2 f_{16}[3][x_0, x_4]^2 a_4' [H x_4] + e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[3][x_0, x_4]^2 a_4' [H x_4] + \\
& 3 H Q Q Q 2 f_{16}[4][x_0, x_4]^2 a_4' [H x_4] + 2 H Q Q Q 2 f_{16}[5][x_0, x_4]^2 a_4' [H x_4] + \\
& e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[5][x_0, x_4]^2 a_4' [H x_4] + 2 H Q Q Q 2 f_{16}[6][x_0, x_4]^2 a_4' [H x_4] + \\
& e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[6][x_0, x_4]^2 a_4' [H x_4] + H Q Q Q 2 f_{16}[7][x_0, x_4]^2 a_4' [H x_4] + \\
& 2 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[7][x_0, x_4]^2 a_4' [H x_4] - 3 H Q Q Q 2 f_{16}[8][x_0, x_4]^2 a_4' [H x_4] - \\
& 2 H Q Q Q 2 f_{16}[9][x_0, x_4]^2 a_4' [H x_4] - e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[9][x_0, x_4]^2 a_4' [H x_4] - \\
& 2 H Q Q Q 2 f_{16}[10][x_0, x_4]^2 a_4' [H x_4] - e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[10][x_0, x_4]^2 a_4' [H x_4] - \\
& H Q Q Q 2 f_{16}[11][x_0, x_4]^2 a_4' [H x_4] - 2 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[11][x_0, x_4]^2 a_4' [H x_4] + \\
& 2 H Q Q Q 1 f_{16}[5][x_0, x_4] \times f_{16}[12][x_0, x_4] a_4' [H x_4] - \\
& 2 e^{2 a_4 [H x_4]} H Q Q Q 1 f_{16}[5][x_0, x_4] \times f_{16}[12][x_0, x_4] a_4' [H x_4] - \\
& 3 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[12][x_0, x_4]^2 a_4' [H x_4] - \\
& 2 H Q Q Q 1 f_{16}[4][x_0, x_4] \times f_{16}[13][x_0, x_4] a_4' [H x_4] + \\
& 2 e^{2 a_4 [H x_4]} H Q Q Q 1 f_{16}[4][x_0, x_4] \times f_{16}[13][x_0, x_4] a_4' [H x_4] - \\
& H Q Q Q 2 f_{16}[13][x_0, x_4]^2 a_4' [H x_4] - 2 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[13][x_0, x_4]^2 a_4' [H x_4] - \\
& H Q Q Q 2 f_{16}[14][x_0, x_4]^2 a_4' [H x_4] - 2 e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[14][x_0, x_4]^2 a_4' [H x_4] - \\
& 2 H Q Q Q 2 f_{16}[15][x_0, x_4]^2 a_4' [H x_4] - e^{2 a_4 [H x_4]} H Q Q Q 2 f_{16}[15][x_0, x_4]^2 a_4' [H x_4] - \\
& 2 e^{2 a_4 [H x_4]} f_{16}[9][x_0, x_4] f_{16}[0]^{(0,1)} [x_0, x_4] + 2 e^{2 a_4 [H x_4]} f_{16}[8][x_0, x_4] f_{16}[1]^{(0,1)} [x_0, x_4] + \\
& 2 e^{2 a_4 [H x_4]} f_{16}[11][x_0, x_4] f_{16}[2]^{(0,1)} [x_0, x_4] - 2 e^{2 a_4 [H x_4]} f_{16}[10][x_0, x_4] \\
& f_{16}[3]^{(0,1)} [x_0, x_4] + 2 e^{2 a_4 [H x_4]} f_{16}[13][x_0, x_4] f_{16}[4]^{(0,1)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} f_{16}[12][x_0, x_4] f_{16}[5]^{(0,1)} [x_0, x_4] - 2 e^{2 a_4 [H x_4]} f_{16}[15][x_0, x_4] \\
& f_{16}[6]^{(0,1)} [x_0, x_4] + 2 e^{2 a_4 [H x_4]} f_{16}[14][x_0, x_4] f_{16}[7]^{(0,1)} [x_0, x_4] + \\
& 2 e^{2 a_4 [H x_4]} f_{16}[3][x_0, x_4] f_{16}[10]^{(0,1)} [x_0, x_4] - 2 e^{2 a_4 [H x_4]} f_{16}[2][x_0, x_4] \\
& f_{16}[11]^{(0,1)} [x_0, x_4] + 2 e^{2 a_4 [H x_4]} f_{16}[5][x_0, x_4] f_{16}[12]^{(0,1)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} f_{16}[4][x_0, x_4] f_{16}[13]^{(0,1)} [x_0, x_4] - 2 e^{2 a_4 [H x_4]} f_{16}[7][x_0, x_4] \\
& f_{16}[14]^{(0,1)} [x_0, x_4] + 2 e^{2 a_4 [H x_4]} f_{16}[6][x_0, x_4] f_{16}[15]^{(0,1)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[12][x_0, x_4] f_{16}[0]^{(1,0)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[13][x_0, x_4] f_{16}[1]^{(1,0)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[14][x_0, x_4] f_{16}[2]^{(1,0)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[15][x_0, x_4] f_{16}[3]^{(1,0)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[8][x_0, x_4] f_{16}[4]^{(1,0)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[9][x_0, x_4] f_{16}[5]^{(1,0)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[10][x_0, x_4] f_{16}[6]^{(1,0)} [x_0, x_4] - \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[11][x_0, x_4] f_{16}[7]^{(1,0)} [x_0, x_4] + \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[4][x_0, x_4] f_{16}[8]^{(1,0)} [x_0, x_4] + \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[5][x_0, x_4] f_{16}[9]^{(1,0)} [x_0, x_4] + \\
& 2 e^{2 a_4 [H x_4]} \tan[6 H x_0] f_{16}[6][x_0, x_4] f_{16}[10]^{(1,0)} [x_0, x_4] +
\end{aligned}$$

$$\begin{aligned}
& 2 e^{2 a^4 [H x^4]} \operatorname{Tan}[6 H x^0] f_{16}[7][x^0, x^4] f_{16}[11]^{(1,0)}[x^0, x^4] + \\
& 2 f_{16}[0][x^0, x^4] \left(2 e^{2 a^4 [H x^4]} H M f_{16}[4][x^0, x^4] + 3 e^{2 a^4 [H x^4]} H Q Q Q 2 f_{16}[5][x^0, x^4] - \right. \\
& \quad H Q Q Q 1 f_{16}[9][x^0, x^4] a_4'[H x^4] + e^{2 a^4 [H x^4]} H Q Q Q 1 f_{16}[9][x^0, x^4] a_4'[H x^4] + \\
& \quad \left. e^{2 a^4 [H x^4]} f_{16}[9]^{(0,1)}[x^0, x^4] + e^{2 a^4 [H x^4]} \operatorname{Tan}[6 H x^0] f_{16}[12]^{(1,0)}[x^0, x^4] \right) - \\
& 2 f_{16}[1][x^0, x^4] \left(3 e^{2 a^4 [H x^4]} H Q Q Q 2 f_{16}[4][x^0, x^4] - 2 e^{2 a^4 [H x^4]} H M f_{16}[5][x^0, x^4] - \right. \\
& \quad H Q Q Q 1 f_{16}[8][x^0, x^4] a_4'[H x^4] + e^{2 a^4 [H x^4]} H Q Q Q 1 f_{16}[8][x^0, x^4] a_4'[H x^4] + \\
& \quad \left. e^{2 a^4 [H x^4]} f_{16}[8]^{(0,1)}[x^0, x^4] - e^{2 a^4 [H x^4]} \operatorname{Tan}[6 H x^0] f_{16}[13]^{(1,0)}[x^0, x^4] \right) + \\
& 2 e^{2 a^4 [H x^4]} \operatorname{Tan}[6 H x^0] f_{16}[2][x^0, x^4] f_{16}[14]^{(1,0)}[x^0, x^4] + \\
& 2 e^{2 a^4 [H x^4]} \operatorname{Tan}[6 H x^0] f_{16}[3][x^0, x^4] f_{16}[15]^{(1,0)}[x^0, x^4]
\end{aligned}$$

```
In[ ]:= (*D[testLa,a4[H x4]]//Flatten//Union*)
```

```
D[testLa, a4'[H x4]] // (*Full*)Simplify[#, constraintVars] & // Flatten // Union
```

```
Out[ ]=
```

$$\begin{aligned}
& -\frac{1}{2} e^{-2 a^4 [H x^4]} H \operatorname{Cos}[6 H x^0] \\
& \left(3 e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[0][x^0, x^4]^2 + (1 + 2 e^{2 a^4 [H x^4]}) Q Q Q 2 f_{16}[1][x^0, x^4]^2 + Q Q Q 2 f_{16}[2][x^0, x^4]^2 + \right. \\
& \quad 2 e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[2][x^0, x^4]^2 + 2 Q Q Q 2 f_{16}[3][x^0, x^4]^2 + e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[3][x^0, x^4]^2 + \\
& \quad 3 Q Q Q 2 f_{16}[4][x^0, x^4]^2 + 2 Q Q Q 2 f_{16}[5][x^0, x^4]^2 + e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[5][x^0, x^4]^2 + \\
& \quad 2 Q Q Q 2 f_{16}[6][x^0, x^4]^2 + e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[6][x^0, x^4]^2 + Q Q Q 2 f_{16}[7][x^0, x^4]^2 + \\
& \quad 2 e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[7][x^0, x^4]^2 - 2 (-1 + e^{2 a^4 [H x^4]}) Q Q Q 1 f_{16}[1][x^0, x^4] \times f_{16}[8][x^0, x^4] - \\
& \quad 3 Q Q Q 2 f_{16}[8][x^0, x^4]^2 + 2 (-1 + e^{2 a^4 [H x^4]}) Q Q Q 1 f_{16}[0][x^0, x^4] \times f_{16}[9][x^0, x^4] - \\
& \quad 2 Q Q Q 2 f_{16}[9][x^0, x^4]^2 - e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[9][x^0, x^4]^2 - 2 Q Q Q 2 f_{16}[10][x^0, x^4]^2 - \\
& \quad e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[10][x^0, x^4]^2 - Q Q Q 2 f_{16}[11][x^0, x^4]^2 - \\
& \quad 2 e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[11][x^0, x^4]^2 + 2 Q Q Q 1 f_{16}[5][x^0, x^4] \times f_{16}[12][x^0, x^4] - \\
& \quad 2 e^{2 a^4 [H x^4]} Q Q Q 1 f_{16}[5][x^0, x^4] \times f_{16}[12][x^0, x^4] - 3 e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[12][x^0, x^4]^2 - \\
& \quad 2 Q Q Q 1 f_{16}[4][x^0, x^4] \times f_{16}[13][x^0, x^4] + 2 e^{2 a^4 [H x^4]} Q Q Q 1 f_{16}[4][x^0, x^4] \times f_{16}[13][x^0, x^4] - \\
& \quad Q Q Q 2 f_{16}[13][x^0, x^4]^2 - 2 e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[13][x^0, x^4]^2 - Q Q Q 2 f_{16}[14][x^0, x^4]^2 - \\
& \quad \left. 2 e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[14][x^0, x^4]^2 - 2 Q Q Q 2 f_{16}[15][x^0, x^4]^2 - e^{2 a^4 [H x^4]} Q Q Q 2 f_{16}[15][x^0, x^4]^2 \right)
\end{aligned}$$

```
In[ ]:= testLg = Lg[];
```

```
In[ ]:= testLgm = testL /. sg /. ssgm4488 // (*Full*)Simplify[#, constraintVars] & /. subsDefects
```

```
Out[ ]=
```

```
testL
```

```
In[ ]:= D[testLgm, a4[H x4]]
```

```
D[testLgm, a4'[H x4]] // (*Full*)Simplify[#, constraintVars] &
```

```
Out[ ]=
```

```
0
```

```
Out[ ]=
```

```
0
```

In[*]:=

```

Clear[Lj];
Lj[j_] := useDSQRT * (((
  Transpose[Ψ16].σ16.useT16[[1]].D[Ψ16, X[[1]]] +

  Transpose[Ψ16].σ16.useT16[[5]].D[Ψ16, X[[5]]] +
  (K0/2) * Sum[(*useω[[1,A1,B1]]*) ωμIJ[1][[A1, B1]] × Transpose[Ψ16].σ16.
    useT16[[1]].SAB[[A1, B1]].(base16[[j, 1]].Ψ16, {A1, 1, 8}, {B1, 1, 8})] +

  Transpose[Ψ16].σ16.useT16[[5]].D[Ψ16, X[[5]]] + (K4/2) * Sum[(*useω[[5,A1,B1]]*)
    ωμIJ[5][[A1, B1]] × Transpose[Ψ16].σ16.useT16[[5]].SAB[[A1, B1]].(base16[[j, 1]].Ψ16,
    {A1, 1, 8}, {B1, 1, 8})] +

  (H*M) * Transpose[Ψ16].σ16.Ψ16) // Simplify[#, constraintVars] &

```

In[*]:= Lj[92]

Out[*]=

```

Cos[6 H x0] (-2 H M f16[2][x0, x4] × f16[6][x0, x4] -
  2 H M f16[3][x0, x4] × f16[7][x0, x4] + 2 H M f16[8][x0, x4] × f16[12][x0, x4] +
  2 H M f16[9][x0, x4] × f16[13][x0, x4] + 2 H M f16[10][x0, x4] × f16[14][x0, x4] +
  2 H M f16[11][x0, x4] × f16[15][x0, x4] + 2 f16[9][x0, x4] f16[0](0,1)[x0, x4] -
  2 f16[8][x0, x4] f16[1](0,1)[x0, x4] - 2 f16[11][x0, x4] f16[2](0,1)[x0, x4] +
  2 f16[10][x0, x4] f16[3](0,1)[x0, x4] - 2 f16[13][x0, x4] f16[4](0,1)[x0, x4] +
  2 f16[12][x0, x4] f16[5](0,1)[x0, x4] + 2 f16[15][x0, x4] f16[6](0,1)[x0, x4] -
  2 f16[14][x0, x4] f16[7](0,1)[x0, x4] - 2 f16[3][x0, x4] f16[10](0,1)[x0, x4] +
  2 f16[2][x0, x4] f16[11](0,1)[x0, x4] - 2 f16[5][x0, x4] f16[12](0,1)[x0, x4] +
  2 f16[4][x0, x4] f16[13](0,1)[x0, x4] + 2 f16[7][x0, x4] f16[14](0,1)[x0, x4] -
  2 f16[6][x0, x4] f16[15](0,1)[x0, x4] + Tan[6 H x0] f16[12][x0, x4] f16[0](1,0)[x0, x4] +
  Tan[6 H x0] f16[13][x0, x4] f16[1](1,0)[x0, x4] +
  Tan[6 H x0] f16[14][x0, x4] f16[2](1,0)[x0, x4] + Tan[6 H x0] f16[15][x0, x4]
  f16[3](1,0)[x0, x4] + Tan[6 H x0] f16[8][x0, x4] f16[4](1,0)[x0, x4] +
  Tan[6 H x0] f16[9][x0, x4] f16[5](1,0)[x0, x4] + Tan[6 H x0] f16[10][x0, x4]
  f16[6](1,0)[x0, x4] + Tan[6 H x0] f16[11][x0, x4] f16[7](1,0)[x0, x4] -
  Tan[6 H x0] f16[4][x0, x4] f16[8](1,0)[x0, x4] - Tan[6 H x0] f16[5][x0, x4]
  f16[9](1,0)[x0, x4] - Tan[6 H x0] f16[6][x0, x4] f16[10](1,0)[x0, x4] -
  Tan[6 H x0] f16[7][x0, x4] f16[11](1,0)[x0, x4] - f16[0][x0, x4]
  (2 H M f16[4][x0, x4] + 2 f16[9](0,1)[x0, x4] + Tan[6 H x0] f16[12](1,0)[x0, x4]) - f16[1][x0,
  x4] (2 H M f16[5][x0, x4] - 2 f16[8](0,1)[x0, x4] + Tan[6 H x0] f16[13](1,0)[x0, x4]) -
  Tan[6 H x0] f16[2][x0, x4] f16[14](1,0)[x0, x4] -
  Tan[6 H x0] f16[3][x0, x4] f16[15](1,0)[x0, x4])

```

```

In[*]:= detgg
Out[*]=
g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]

In[*]:= Clear[eL];
eL[Lagrangian_Symbol, detsqrt_] := Module[{L, t},
  L = Lagrangian[];
  t = Table[FullSimplify[ $\frac{1}{\text{detsqrt}}$  (D[L, f16[k][x0, x4]] - D[D[L, f16[k](1,0)[x0, x4]], x0] -
    D[D[L, f16[k](0,1)[x0, x4]], x4]), constraintVars], {k, 0, 15}];
  Return[t /. subsDefects];
]

```

eLa

```

In[*]:= eLa = eL[La, useDSQRT]
Out[*]=
{H (-2 M f16[4][x0, x4] - 3 QQQ2 f16[5][x0, x4] - 6 f16[12][x0, x4] +
  (-3 QQQ2 f16[0][x0, x4] + (-1 + e-2 a4[H x4]) QQQ1 f16[9][x0, x4]) a4'[H x4]) -
  2 (f16[9](0,1)[x0, x4] + Tan[6 H x0] f16[12](1,0)[x0, x4]),
H (3 QQQ2 f16[4][x0, x4] - 2 M f16[5][x0, x4] - 6 f16[13][x0, x4] +
  (-2 QQQ2 f16[1][x0, x4] + QQQ1 f16[8][x0, x4]) a4'[H x4] -
  e-2 a4[H x4] (QQQ2 f16[1][x0, x4] + QQQ1 f16[8][x0, x4]) a4'[H x4]) +
  2 (f16[8](0,1)[x0, x4] - Tan[6 H x0] f16[13](1,0)[x0, x4]),
H (-2 M f16[6][x0, x4] + 3 QQQ2 f16[7][x0, x4] - 6 f16[14][x0, x4] -
  e-2 a4[H x4] (1 + 2 e2 a4[H x4]) QQQ2 f16[2][x0, x4] a4'[H x4]) +
  2 (f16[11](0,1)[x0, x4] - Tan[6 H x0] f16[14](1,0)[x0, x4]),
H (-3 QQQ2 f16[6][x0, x4] - 2 M f16[7][x0, x4] - 6 f16[15][x0, x4] -
  e-2 a4[H x4] (2 + e2 a4[H x4]) QQQ2 f16[3][x0, x4] a4'[H x4]) -
  2 (f16[10](0,1)[x0, x4] + Tan[6 H x0] f16[15](1,0)[x0, x4]),
H (-2 M f16[0][x0, x4] + 3 QQQ2 f16[1][x0, x4] - 6 f16[8][x0, x4] -
  e-2 a4[H x4] (3 QQQ2 f16[4][x0, x4] + (-1 + e2 a4[H x4]) QQQ1 f16[13][x0, x4]) a4'[H x4]) +
  2 (f16[13](0,1)[x0, x4] - Tan[6 H x0] f16[8](1,0)[x0, x4]),
H (-3 QQQ2 f16[0][x0, x4] - 2 M f16[1][x0, x4] - 6 f16[9][x0, x4] - e-2 a4[H x4]
  ((2 + e2 a4[H x4]) QQQ2 f16[5][x0, x4] - (-1 + e2 a4[H x4]) QQQ1 f16[12][x0, x4]) a4'[H x4]) -
  2 (f16[12](0,1)[x0, x4] + Tan[6 H x0] f16[9](1,0)[x0, x4]),
H (-2 M f16[2][x0, x4] - 3 QQQ2 f16[3][x0, x4] - 6 f16[10][x0, x4] -
  e-2 a4[H x4] (2 + e2 a4[H x4]) QQQ2 f16[6][x0, x4] a4'[H x4]) -
  2 (f16[15](0,1)[x0, x4] + Tan[6 H x0] f16[10](1,0)[x0, x4]),
H (3 QQQ2 f16[2][x0, x4] - 2 M f16[3][x0, x4] - 6 f16[11][x0, x4] -
  e-2 a4[H x4] (1 + 2 e2 a4[H x4]) QQQ2 f16[7][x0, x4] a4'[H x4]) +
  2 (f16[14](0,1)[x0, x4] - Tan[6 H x0] f16[11](1,0)[x0, x4]),

```

$$\begin{aligned}
& H \left(6 f_{16}[4][x_0, x_4] + 2 M f_{16}[12][x_0, x_4] - 3 QQQ2 f_{16}[13][x_0, x_4] + \right. \\
& \quad \left. e^{-2 a^4 [H x_4]} \left((-1 + e^{2 a^4 [H x_4]}) QQQ1 f_{16}[1][x_0, x_4] + 3 QQQ2 f_{16}[8][x_0, x_4] \right) a_4' [H x_4] \right) - \\
& \quad 2 f_{16}[1]^{(0,1)} [x_0, x_4] + 2 \tan[6 H x_0] f_{16}[4]^{(1,0)} [x_0, x_4], \\
& H \left(6 f_{16}[5][x_0, x_4] + 3 QQQ2 f_{16}[12][x_0, x_4] + 2 M f_{16}[13][x_0, x_4] + \right. \\
& \quad \left. (-QQQ1 f_{16}[0][x_0, x_4] + QQQ2 f_{16}[9][x_0, x_4]) a_4' [H x_4] + \right. \\
& \quad \left. e^{-2 a^4 [H x_4]} (QQQ1 f_{16}[0][x_0, x_4] + 2 QQQ2 f_{16}[9][x_0, x_4]) a_4' [H x_4] \right) + \\
& \quad 2 \left(f_{16}[0]^{(0,1)} [x_0, x_4] + \tan[6 H x_0] f_{16}[5]^{(1,0)} [x_0, x_4] \right), \\
& H \left(6 f_{16}[6][x_0, x_4] + 2 M f_{16}[14][x_0, x_4] + 3 QQQ2 f_{16}[15][x_0, x_4] + \right. \\
& \quad \left. e^{-2 a^4 [H x_4]} \left(2 + e^{2 a^4 [H x_4]} \right) QQQ2 f_{16}[10][x_0, x_4] a_4' [H x_4] \right) + \\
& \quad 2 \left(f_{16}[3]^{(0,1)} [x_0, x_4] + \tan[6 H x_0] f_{16}[6]^{(1,0)} [x_0, x_4] \right), \\
& H \left(6 f_{16}[7][x_0, x_4] - 3 QQQ2 f_{16}[14][x_0, x_4] + 2 M f_{16}[15][x_0, x_4] + \right. \\
& \quad \left. e^{-2 a^4 [H x_4]} \left(1 + 2 e^{2 a^4 [H x_4]} \right) QQQ2 f_{16}[11][x_0, x_4] a_4' [H x_4] \right) - \\
& \quad 2 f_{16}[2]^{(0,1)} [x_0, x_4] + 2 \tan[6 H x_0] f_{16}[7]^{(1,0)} [x_0, x_4], \\
& H \left(6 f_{16}[0][x_0, x_4] + 2 M f_{16}[8][x_0, x_4] + 3 QQQ2 f_{16}[9][x_0, x_4] + \right. \\
& \quad \left. \left((QQQ1 - e^{-2 a^4 [H x_4]} QQQ1) f_{16}[5][x_0, x_4] + 3 QQQ2 f_{16}[12][x_0, x_4] \right) a_4' [H x_4] \right) + \\
& \quad 2 \left(f_{16}[5]^{(0,1)} [x_0, x_4] + \tan[6 H x_0] f_{16}[0]^{(1,0)} [x_0, x_4] \right), \\
& H \left(6 f_{16}[1][x_0, x_4] - 3 QQQ2 f_{16}[8][x_0, x_4] + 2 M f_{16}[9][x_0, x_4] + \right. \\
& \quad \left. e^{-2 a^4 [H x_4]} (QQQ1 f_{16}[4][x_0, x_4] + QQQ2 f_{16}[13][x_0, x_4]) a_4' [H x_4] + \right. \\
& \quad \left. (-QQQ1 f_{16}[4][x_0, x_4] + 2 QQQ2 f_{16}[13][x_0, x_4]) a_4' [H x_4] \right) - \\
& \quad 2 f_{16}[4]^{(0,1)} [x_0, x_4] + 2 \tan[6 H x_0] f_{16}[1]^{(1,0)} [x_0, x_4], \\
& H \left(6 f_{16}[2][x_0, x_4] + 2 M f_{16}[10][x_0, x_4] - 3 QQQ2 f_{16}[11][x_0, x_4] + \right. \\
& \quad \left. e^{-2 a^4 [H x_4]} \left(1 + 2 e^{2 a^4 [H x_4]} \right) QQQ2 f_{16}[14][x_0, x_4] a_4' [H x_4] \right) - \\
& \quad 2 f_{16}[7]^{(0,1)} [x_0, x_4] + 2 \tan[6 H x_0] f_{16}[2]^{(1,0)} [x_0, x_4], \\
& H \left(6 f_{16}[3][x_0, x_4] + 3 QQQ2 f_{16}[10][x_0, x_4] + 2 M f_{16}[11][x_0, x_4] + \right. \\
& \quad \left. e^{-2 a^4 [H x_4]} \left(2 + e^{2 a^4 [H x_4]} \right) QQQ2 f_{16}[15][x_0, x_4] a_4' [H x_4] \right) + \\
& \quad \left. 2 \left(f_{16}[6]^{(0,1)} [x_0, x_4] + \tan[6 H x_0] f_{16}[3]^{(1,0)} [x_0, x_4] \right) \right\}
\end{aligned}$$

```
In[ ] := DumpSave[ToString[header <> "eLa.mx"], eLa];
```

```
In[ ] := Print[ToString[header <> "eLa.mx"]]
```

```
2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix 3-eLa.mx
```

```
"2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix 3-eLa.mx"
```

```
(*Get["2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix  
3-eLa.mx"]*)
```

eLazt

```
In[ ] := time1 = Now
AbsoluteTiming[
  eLazt =  $\frac{2 \sqrt{\sin[z]}}{6 * H}$  * eLa /. sfψ16Aa /. sx0x4 // FullSimplify[#, constraintVars] &]
Now - time1
```

Out[*]=

Sat 29 Nov 2025 05:17:37 GMT-8

Out[*]=

$$\begin{aligned}
& \{25.4518, \\
& \left\{ \frac{1}{3} \left(-2 M Z[4][z, t] - 3 Q Q Q 2 Z[5][z, t] + \left(-3 Q Q Q 2 Z[0][z, t] + \left(-1 + e^{-2 a^4[t]} \right) Q Q Q 1 Z[9][z, t] \right) \right. \right. \\
& \quad \left. \left. a_4'[t] - 2 \left(Z[9]^{(0,1)}[z, t] + 6 \tan[z] Z[12]^{(1,0)}[z, t] \right) \right) \right\}, \\
& \frac{1}{3} \left(3 Q Q Q 2 Z[4][z, t] - 2 M Z[5][z, t] - e^{-a^4[t]} \left(Q Q Q 2 \left(3 \cosh[a_4[t]] + \sinh[a_4[t]] \right) Z[1][z, t] - \right. \right. \\
& \quad \left. \left. 2 Q Q Q 1 \sinh[a_4[t]] Z[8][z, t] \right) a_4'[t] + 2 \left(Z[8]^{(0,1)}[z, t] - 6 \tan[z] Z[13]^{(1,0)}[z, t] \right) \right), \\
& \frac{1}{3} \left(-2 M Z[6][z, t] + 3 Q Q Q 2 Z[7][z, t] - e^{-2 a^4[t]} \left(1 + 2 e^{2 a^4[t]} \right) Q Q Q 2 Z[2][z, t] a_4'[t] + \right. \\
& \quad \left. 2 \left(Z[11]^{(0,1)}[z, t] - 6 \tan[z] Z[14]^{(1,0)}[z, t] \right) \right), \\
& \frac{1}{3} \left(-3 Q Q Q 2 Z[6][z, t] - e^{-2 a^4[t]} \left(2 + e^{2 a^4[t]} \right) Q Q Q 2 Z[3][z, t] a_4'[t] - \right. \\
& \quad \left. 2 \left(M Z[7][z, t] + Z[10]^{(0,1)}[z, t] + 6 \tan[z] Z[15]^{(1,0)}[z, t] \right) \right), \frac{1}{3} \left(-2 M Z[0][z, t] + \right. \\
& \quad \left. 3 Q Q Q 2 Z[1][z, t] - e^{-2 a^4[t]} \left(3 Q Q Q 2 Z[4][z, t] + \left(-1 + e^{2 a^4[t]} \right) Q Q Q 1 Z[13][z, t] \right) a_4'[t] + \right. \\
& \quad \left. 2 \left(Z[13]^{(0,1)}[z, t] - 6 \tan[z] Z[8]^{(1,0)}[z, t] \right) \right), \frac{1}{3} \left(-3 Q Q Q 2 Z[0][z, t] + \right. \\
& \quad \left. e^{-a^4[t]} \left(Q Q Q 2 \left(-3 \cosh[a_4[t]] + \sinh[a_4[t]] \right) Z[5][z, t] + 2 Q Q Q 1 \sinh[a_4[t]] Z[12][z, t] \right) \right. \\
& \quad \left. a_4'[t] - 2 \left(M Z[1][z, t] + Z[12]^{(0,1)}[z, t] + 6 \tan[z] Z[9]^{(1,0)}[z, t] \right) \right), \\
& \frac{1}{3} \left(-2 M Z[2][z, t] - 3 Q Q Q 2 Z[3][z, t] - e^{-2 a^4[t]} \left(2 + e^{2 a^4[t]} \right) Q Q Q 2 Z[6][z, t] a_4'[t] - \right. \\
& \quad \left. 2 \left(Z[15]^{(0,1)}[z, t] + 6 \tan[z] Z[10]^{(1,0)}[z, t] \right) \right), \\
& \frac{1}{3} \left(3 Q Q Q 2 Z[2][z, t] - 2 M Z[3][z, t] - e^{-2 a^4[t]} \left(1 + 2 e^{2 a^4[t]} \right) Q Q Q 2 Z[7][z, t] a_4'[t] + \right. \\
& \quad \left. 2 \left(Z[14]^{(0,1)}[z, t] - 6 \tan[z] Z[11]^{(1,0)}[z, t] \right) \right), \frac{1}{3} \left(2 M Z[12][z, t] - \right. \\
& \quad \left. 3 Q Q Q 2 Z[13][z, t] + e^{-2 a^4[t]} \left(\left(-1 + e^{2 a^4[t]} \right) Q Q Q 1 Z[1][z, t] + 3 Q Q Q 2 Z[8][z, t] \right) a_4'[t] - \right. \\
& \quad \left. 2 \left(Z[1]^{(0,1)}[z, t] - 6 \tan[z] Z[4]^{(1,0)}[z, t] \right) \right), Q Q Q 2 Z[12][z, t] - \\
& \frac{1}{3} e^{-a^4[t]} \left(2 Q Q Q 1 \sinh[a_4[t]] Z[0][z, t] + Q Q Q 2 \left(-3 \cosh[a_4[t]] + \sinh[a_4[t]] \right) Z[9][z, t] \right) \\
& \quad a_4'[t] + \frac{2}{3} \left(M Z[13][z, t] + Z[0]^{(0,1)}[z, t] + 6 \tan[z] Z[5]^{(1,0)}[z, t] \right), \\
& \frac{1}{3} \left(2 M Z[14][z, t] + 3 Q Q Q 2 Z[15][z, t] + e^{-2 a^4[t]} \left(2 + e^{2 a^4[t]} \right) Q Q Q 2 Z[10][z, t] a_4'[t] + \right. \\
& \quad \left. 2 \left(Z[3]^{(0,1)}[z, t] + 6 \tan[z] Z[6]^{(1,0)}[z, t] \right) \right), \\
& \frac{1}{3} \left(-3 Q Q Q 2 Z[14][z, t] + 2 M Z[15][z, t] + \left(2 + e^{-2 a^4[t]} \right) Q Q Q 2 Z[11][z, t] a_4'[t] - \right. \\
& \quad \left. 2 \left(Z[2]^{(0,1)}[z, t] - 6 \tan[z] Z[7]^{(1,0)}[z, t] \right) \right), \frac{1}{3} \left(2 M Z[8][z, t] + 3 Q Q Q 2 Z[9][z, t] + \right. \\
& \quad \left(2 e^{-a^4[t]} Q Q Q 1 \sinh[a_4[t]] Z[5][z, t] + 3 Q Q Q 2 Z[12][z, t] \right) a_4'[t] + \\
& \quad \left. 2 \left(Z[5]^{(0,1)}[z, t] + 6 \tan[z] Z[0]^{(1,0)}[z, t] \right) \right), \frac{1}{3} \left(-3 Q Q Q 2 Z[8][z, t] + 2 M Z[9][z, t] + \right. \\
& \quad \left. e^{-a^4[t]} \left(-2 Q Q Q 1 \sinh[a_4[t]] Z[4][z, t] + Q Q Q 2 \left(3 \cosh[a_4[t]] + \sinh[a_4[t]] \right) Z[13][z, t] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& a4'[t] - 2 \left(Z[4]^{(0,1)}[z, t] - 6 \tan[z] Z[1]^{(1,0)}[z, t] \right), \\
& \frac{1}{3} \left(2 M Z[10][z, t] - 3 QQQ2 Z[11][z, t] + \left(2 + e^{-2 a4[t]} \right) QQQ2 Z[14][z, t] a4'[t] - \right. \\
& \quad \left. 2 \left(Z[7]^{(0,1)}[z, t] - 6 \tan[z] Z[2]^{(1,0)}[z, t] \right) \right), \\
& \frac{1}{3} \left(3 QQQ2 Z[10][z, t] + 2 M Z[11][z, t] + e^{-2 a4[t]} \left(2 + e^{2 a4[t]} \right) QQQ2 Z[15][z, t] a4'[t] + \right. \\
& \quad \left. 2 \left(Z[6]^{(0,1)}[z, t] + 6 \tan[z] Z[3]^{(1,0)}[z, t] \right) \right) \} \}
\end{aligned}$$

Out[*]=

25.455877 s

```
In[*]:= DumpSave[ToString[header <> "eLazt.mx"], eLazt];
```

```
In[*]:= Print[ToString[header <> "eLazt.mx"]]
```

2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix 3-eLazt.mx

"2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix 3-eLazt.mx"

```
(*Get["2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix
3-eLazt.mx"]*)
```

Explore eLazt

```
In[*]:= resolveWithRemainder[xy_, wrt_List] := Module[{b, remainder},
```

```
  b = D[xy, #] & /@wrt;
```

```
  remainder = xy - b.wrt // Expand;
```

```
  Return[{Style[b, Red], Style[remainder, Blue]}];
```

```
]
```

```
In[*]:= resolveResolve[xx_] := Module[{a, b, c, c0, d, dop, f, g, h, j, k, l, m, n, o, p, q, r, s},
```

```
  a = Flatten[xx];
```

```
  b = a[[1];
```

```
  c0 = Drop[a, 1];
```

```
  c = c0 * (Prime[#] & /@Range[Length[a] - 1]);
```

```
  (*ensure d[[j]][1] that is 'Times'*)
```

```
  d = eextract[#] & /@c;
```

```
  dop = Flatten[Take[#, 1] & /@d]; (*Print["dop: ",dop];*)
```

```
  f = Table[If[dop[[j]] === Times, Drop[d[[j]], 1], d[[j]], {j, 1, Length[d]}];
```

```
  g = Take[#, 1] & /@f; (*Print["g: ",g];*)
```

```
  h = #[[1] & /@g;
```

```
  j = If[# > 0, {1}, {-1}] & /@h;
```

```
  k = Take[#, -1] & /@f; (*Print["k: ",k];*)
```

```
  l = eextract[#] & /@Flatten[k]; (*Print["l: ",l];*)
```

```
  m = Drop[#, -2] & /@l; (*Print["m",m];*)
```

```
  n = eextract[#] & /@Flatten[m];
```

```
  o = Drop[#, 1] & /@n;
```

```
  p = Table[If[j == 3, (*Print["t",eextract[o[[j]][1]]];*)
```

```
    Take[Flatten[eextract[o[[j]][1]], -1], o[[j]], {j, 1, Length[o]}];
```

```
  q = Transpose[{j, p}];
```

```
  Return[{b, q}];]
```

Q1 v Q2

In[*]:= eLazt

Out[*]=

$$\begin{aligned}
& \left\{ \frac{1}{3} \left(-2 M Z[4][z, t] - 3 QQQ2 Z[5][z, t] + \right. \right. \\
& \quad \left(-3 QQQ2 Z[0][z, t] + (-1 + e^{-2 a^4[t]}) QQQ1 Z[9][z, t] \right) a4'[t] - \\
& \quad 2 \left(Z[9]^{(0,1)}[z, t] + 6 \tan[z] Z[12]^{(1,0)}[z, t] \right), \frac{1}{3} \left(3 QQQ2 Z[4][z, t] - 2 M Z[5][z, t] - \right. \\
& \quad e^{-a^4[t]} (QQQ2 (3 \cosh[a^4[t]] + \sinh[a^4[t]]) Z[1][z, t] - 2 QQQ1 \sinh[a^4[t]] Z[8][z, t]) \\
& \quad a4'[t] + 2 \left(Z[8]^{(0,1)}[z, t] - 6 \tan[z] Z[13]^{(1,0)}[z, t] \right), \\
& \quad \frac{1}{3} \left(-2 M Z[6][z, t] + 3 QQQ2 Z[7][z, t] - e^{-2 a^4[t]} (1 + 2 e^{2 a^4[t]}) QQQ2 Z[2][z, t] a4'[t] + \right. \\
& \quad \left. 2 \left(Z[11]^{(0,1)}[z, t] - 6 \tan[z] Z[14]^{(1,0)}[z, t] \right), \right. \\
& \quad \frac{1}{3} \left(-3 QQQ2 Z[6][z, t] - e^{-2 a^4[t]} (2 + e^{2 a^4[t]}) QQQ2 Z[3][z, t] a4'[t] - \right. \\
& \quad 2 \left(M Z[7][z, t] + Z[10]^{(0,1)}[z, t] + 6 \tan[z] Z[15]^{(1,0)}[z, t] \right), \frac{1}{3} \left(-2 M Z[0][z, t] + \right. \\
& \quad 3 QQQ2 Z[1][z, t] - e^{-2 a^4[t]} (3 QQQ2 Z[4][z, t] + (-1 + e^{2 a^4[t]}) QQQ1 Z[13][z, t]) a4'[t] + \\
& \quad 2 \left(Z[13]^{(0,1)}[z, t] - 6 \tan[z] Z[8]^{(1,0)}[z, t] \right), \frac{1}{3} \left(-3 QQQ2 Z[0][z, t] + \right. \\
& \quad e^{-a^4[t]} (QQQ2 (-3 \cosh[a^4[t]] + \sinh[a^4[t]]) Z[5][z, t] + 2 QQQ1 \sinh[a^4[t]] Z[12][z, t]) \\
& \quad a4'[t] - 2 \left(M Z[1][z, t] + Z[12]^{(0,1)}[z, t] + 6 \tan[z] Z[9]^{(1,0)}[z, t] \right), \\
& \quad \frac{1}{3} \left(-2 M Z[2][z, t] - 3 QQQ2 Z[3][z, t] - e^{-2 a^4[t]} (2 + e^{2 a^4[t]}) QQQ2 Z[6][z, t] a4'[t] - \right. \\
& \quad \left. 2 \left(Z[15]^{(0,1)}[z, t] + 6 \tan[z] Z[10]^{(1,0)}[z, t] \right), \right. \\
& \quad \frac{1}{3} \left(3 QQQ2 Z[2][z, t] - 2 M Z[3][z, t] - e^{-2 a^4[t]} (1 + 2 e^{2 a^4[t]}) QQQ2 Z[7][z, t] a4'[t] + \right. \\
& \quad 2 \left(Z[14]^{(0,1)}[z, t] - 6 \tan[z] Z[11]^{(1,0)}[z, t] \right), \frac{1}{3} \left(2 M Z[12][z, t] - \right. \\
& \quad 3 QQQ2 Z[13][z, t] + e^{-2 a^4[t]} ((-1 + e^{2 a^4[t]}) QQQ1 Z[1][z, t] + 3 QQQ2 Z[8][z, t]) a4'[t] - \\
& \quad 2 \left(Z[1]^{(0,1)}[z, t] - 6 \tan[z] Z[4]^{(1,0)}[z, t] \right), QQQ2 Z[12][z, t] - \\
& \quad \frac{1}{3} e^{-a^4[t]} (2 QQQ1 \sinh[a^4[t]] Z[0][z, t] + QQQ2 (-3 \cosh[a^4[t]] + \sinh[a^4[t]]) Z[9][z, t]) \\
& \quad a4'[t] + \frac{2}{3} \left(M Z[13][z, t] + Z[0]^{(0,1)}[z, t] + 6 \tan[z] Z[5]^{(1,0)}[z, t] \right), \\
& \quad \frac{1}{3} \left(2 M Z[14][z, t] + 3 QQQ2 Z[15][z, t] + e^{-2 a^4[t]} (2 + e^{2 a^4[t]}) QQQ2 Z[10][z, t] a4'[t] + \right. \\
& \quad \left. 2 \left(Z[3]^{(0,1)}[z, t] + 6 \tan[z] Z[6]^{(1,0)}[z, t] \right), \right. \\
& \quad \frac{1}{3} \left(-3 QQQ2 Z[14][z, t] + 2 M Z[15][z, t] + (2 + e^{-2 a^4[t]}) QQQ2 Z[11][z, t] a4'[t] - \right. \\
& \quad \left. 2 \left(Z[2]^{(0,1)}[z, t] - 6 \tan[z] Z[7]^{(1,0)}[z, t] \right), \right. \\
& \quad \frac{1}{3} \left(2 M Z[8][z, t] + 3 QQQ2 Z[9][z, t] + (2 e^{-a^4[t]}) QQQ1 \sinh[a^4[t]] Z[5][z, t] + 3 QQQ2 Z[12][z, t] \right) \\
& \quad a4'[t] + 2 \left(Z[5]^{(0,1)}[z, t] + 6 \tan[z] Z[0]^{(1,0)}[z, t] \right),
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3} \left(-3 \text{QQQ2} Z[8][z, t] + 2 M Z[9][z, t] + e^{-a4[t]} \right. \\
& \quad \left(-2 \text{QQQ1} \text{Sinh}[a4[t]] Z[4][z, t] + \text{QQQ2} (3 \text{Cosh}[a4[t]] + \text{Sinh}[a4[t]]) Z[13][z, t] \right) a4'[t] - \\
& \quad \left. 2 \left(Z[4]^{(0,1)}[z, t] - 6 \text{Tan}[z] Z[1]^{(1,0)}[z, t] \right) \right), \\
& \frac{1}{3} \left(2 M Z[10][z, t] - 3 \text{QQQ2} Z[11][z, t] + \left(2 + e^{-2 a4[t]} \right) \text{QQQ2} Z[14][z, t] a4'[t] - \right. \\
& \quad \left. 2 \left(Z[7]^{(0,1)}[z, t] - 6 \text{Tan}[z] Z[2]^{(1,0)}[z, t] \right) \right), \\
& \frac{1}{3} \left(3 \text{QQQ2} Z[10][z, t] + 2 M Z[11][z, t] + e^{-2 a4[t]} \left(2 + e^{2 a4[t]} \right) \text{QQQ2} Z[15][z, t] a4'[t] + \right. \\
& \quad \left. 2 \left(Z[6]^{(0,1)}[z, t] + 6 \text{Tan}[z] Z[3]^{(1,0)}[z, t] \right) \right) \}
\end{aligned}$$

In[]:= **(elazt1 = elazt /. {QQQ1 → 1, QQQ2 → 0} // ExpandAll) // Column**

Out[]:=

$$\begin{aligned}
& -\frac{2}{3} M Z[4][z, t] - \frac{1}{3} Z[9][z, t] a4'[t] + \\
& \quad \frac{1}{3} e^{-2 a4[t]} Z[9][z, t] a4'[t] - \frac{2}{3} Z[9]^{(0,1)}[z, t] - 4 \text{Tan}[z] Z[12]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[5][z, t] + \frac{2}{3} e^{-a4[t]} \text{Sinh}[a4[t]] Z[8][z, t] a4'[t] + \\
& \quad \frac{2}{3} Z[8]^{(0,1)}[z, t] - 4 \text{Tan}[z] Z[13]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[6][z, t] + \frac{2}{3} Z[11]^{(0,1)}[z, t] - 4 \text{Tan}[z] Z[14]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[7][z, t] - \frac{2}{3} Z[10]^{(0,1)}[z, t] - 4 \text{Tan}[z] Z[15]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[0][z, t] - \frac{1}{3} Z[13][z, t] a4'[t] + \\
& \quad \frac{1}{3} e^{-2 a4[t]} Z[13][z, t] a4'[t] + \frac{2}{3} Z[13]^{(0,1)}[z, t] - 4 \text{Tan}[z] Z[8]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[1][z, t] + \frac{2}{3} e^{-a4[t]} \text{Sinh}[a4[t]] Z[12][z, t] a4'[t] - \\
& \quad \frac{2}{3} Z[12]^{(0,1)}[z, t] - 4 \text{Tan}[z] Z[9]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[2][z, t] - \frac{2}{3} Z[15]^{(0,1)}[z, t] - 4 \text{Tan}[z] Z[10]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[3][z, t] + \frac{2}{3} Z[14]^{(0,1)}[z, t] - 4 \text{Tan}[z] Z[11]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[12][z, t] + \frac{1}{3} Z[1][z, t] a4'[t] - \\
& \quad \frac{1}{3} e^{-2 a4[t]} Z[1][z, t] a4'[t] - \frac{2}{3} Z[1]^{(0,1)}[z, t] + 4 \text{Tan}[z] Z[4]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[13][z, t] - \frac{2}{3} e^{-a4[t]} \text{Sinh}[a4[t]] Z[0][z, t] a4'[t] + \\
& \quad \frac{2}{3} Z[0]^{(0,1)}[z, t] + 4 \text{Tan}[z] Z[5]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[14][z, t] + \frac{2}{3} Z[3]^{(0,1)}[z, t] + 4 \text{Tan}[z] Z[6]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[15][z, t] - \frac{2}{3} Z[2]^{(0,1)}[z, t] + 4 \text{Tan}[z] Z[7]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[8][z, t] + \frac{2}{3} e^{-a4[t]} \text{Sinh}[a4[t]] Z[5][z, t] a4'[t] + \frac{2}{3} Z[5]^{(0,1)}[z, t] + 4 \text{Tan}[z] Z[0]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[9][z, t] - \frac{2}{3} e^{-a4[t]} \text{Sinh}[a4[t]] Z[4][z, t] a4'[t] - \frac{2}{3} Z[4]^{(0,1)}[z, t] + 4 \text{Tan}[z] Z[1]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[10][z, t] - \frac{2}{3} Z[7]^{(0,1)}[z, t] + 4 \text{Tan}[z] Z[2]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[11][z, t] + \frac{2}{3} Z[6]^{(0,1)}[z, t] + 4 \text{Tan}[z] Z[3]^{(1,0)}[z, t]
\end{aligned}$$

In[]:= **{{3, 6, 11, 14}, {2, 7, 10, 15}}**

Out[]:=

{{3, 6, 11, 14}, {2, 7, 10, 15}}

In[]:= **{{1, 4, 9, 12}, {0, 5, 8, 13}}**

Out[]:=

{{1, 4, 9, 12}, {0, 5, 8, 13}}

```

In[*]:= eLazt1Couplings = {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
Out[*]=
{{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}

In[*]:= Clear[jetvars1eLazt1, jetvars2eLazt1]

In[*]:= (*Clear[#]&/@
  (ToExpression["jetvars"<>ToString[#]<>"eLazt1"]&/@Range[Length[eLazt1Couplings]])*

In[*]:= jetvars =
  ToExpression["jetvars" <> ToString[#] <> "eLazt1"] & /@ Range[Length[eLazt1Couplings]]
Out[*]=
{jetvars1eLazt1, jetvars2eLazt1, jetvars3eLazt1, jetvars4eLazt1}

In[*]:= jetvarsVals =
  Table[Flatten[{Z[#][z, t], D[Z[#][z, t], z], D[Z[#][z, t], t]} & /@ eLazt1Couplings[[j]],
    {j, 1, Length[eLazt1Couplings]}]

Out[*]=
{{Z[0][z, t], Z[0](1,0)[z, t], Z[0](0,1)[z, t], Z[5][z, t], Z[5](1,0)[z, t], Z[5](0,1)[z, t],
  Z[8][z, t], Z[8](1,0)[z, t], Z[8](0,1)[z, t], Z[13][z, t], Z[13](1,0)[z, t], Z[13](0,1)[z, t]},
{Z[1][z, t], Z[1](1,0)[z, t], Z[1](0,1)[z, t], Z[4][z, t], Z[4](1,0)[z, t], Z[4](0,1)[z, t],
  Z[9][z, t], Z[9](1,0)[z, t], Z[9](0,1)[z, t], Z[12][z, t], Z[12](1,0)[z, t], Z[12](0,1)[z, t]},
{Z[2][z, t], Z[2](1,0)[z, t], Z[2](0,1)[z, t], Z[7][z, t], Z[7](1,0)[z, t],
  Z[7](0,1)[z, t], Z[10][z, t], Z[10](1,0)[z, t], Z[10](0,1)[z, t], Z[15][z, t],
  Z[15](1,0)[z, t], Z[15](0,1)[z, t]}, {Z[3][z, t], Z[3](1,0)[z, t], Z[3](0,1)[z, t],
  Z[6][z, t], Z[6](1,0)[z, t], Z[6](0,1)[z, t], Z[11][z, t], Z[11](1,0)[z, t],
  Z[11](0,1)[z, t], Z[14][z, t], Z[14](1,0)[z, t], Z[14](0,1)[z, t]}}

In[*]:= Thread[jetvars =
  Table[Flatten[{Z[#][z, t], D[Z[#][z, t], z], D[Z[#][z, t], t]} & /@ eLazt1Couplings[[j]],
    {j, 1, Length[eLazt1Couplings]}]]

Out[*]=
{{Z[0][z, t], Z[1][z, t], Z[2][z, t], Z[3][z, t]},
{Z[0](1,0)[z, t], Z[1](1,0)[z, t], Z[2](1,0)[z, t], Z[3](1,0)[z, t]},
{Z[0](0,1)[z, t], Z[1](0,1)[z, t], Z[2](0,1)[z, t], Z[3](0,1)[z, t]},
{Z[5][z, t], Z[4][z, t], Z[7][z, t], Z[6][z, t]},
{Z[5](1,0)[z, t], Z[4](1,0)[z, t], Z[7](1,0)[z, t], Z[6](1,0)[z, t]},
{Z[5](0,1)[z, t], Z[4](0,1)[z, t], Z[7](0,1)[z, t], Z[6](0,1)[z, t]},
{Z[8][z, t], Z[9][z, t], Z[10][z, t], Z[11][z, t]},
{Z[8](1,0)[z, t], Z[9](1,0)[z, t], Z[10](1,0)[z, t], Z[11](1,0)[z, t]},
{Z[8](0,1)[z, t], Z[9](0,1)[z, t], Z[10](0,1)[z, t], Z[11](0,1)[z, t]},
{Z[13][z, t], Z[12][z, t], Z[15][z, t], Z[14][z, t]},
{Z[13](1,0)[z, t], Z[12](1,0)[z, t], Z[15](1,0)[z, t], Z[14](1,0)[z, t]},
{Z[13](0,1)[z, t], Z[12](0,1)[z, t], Z[15](0,1)[z, t], Z[14](0,1)[z, t]}}

```

```

In[*]:= (eLazt1ResolveWithRemainder = resolveWithRemainder[#, {M, a4'[t], Tan[z]}] & /@ eLazt1) //
Column

Out[*]=
{
  {-2/3 Z[4][z, t], -1/3 Z[9][z, t] + 1/3 e^{-2 a4[t]} Z[9][z, t], -4 Z[12]^{(1,0)}[z, t]}, -2/3 Z[9]^{(0,1)}[z, t]}
  {-2/3 Z[5][z, t], 2/3 e^{-a4[t]} Sinh[a4[t]] Z[8][z, t], -4 Z[13]^{(1,0)}[z, t]}, 2/3 Z[8]^{(0,1)}[z, t]}
  {-2/3 Z[6][z, t], 0, -4 Z[14]^{(1,0)}[z, t]}, 2/3 Z[11]^{(0,1)}[z, t]}
  {-2/3 Z[7][z, t], 0, -4 Z[15]^{(1,0)}[z, t]}, -2/3 Z[10]^{(0,1)}[z, t]}
  {-2/3 Z[0][z, t], -1/3 Z[13][z, t] + 1/3 e^{-2 a4[t]} Z[13][z, t], -4 Z[8]^{(1,0)}[z, t]}, 2/3 Z[13]^{(0,1)}[z, t]}
  {-2/3 Z[1][z, t], 2/3 e^{-a4[t]} Sinh[a4[t]] Z[12][z, t], -4 Z[9]^{(1,0)}[z, t]}, -2/3 Z[12]^{(0,1)}[z, t]}
  {-2/3 Z[2][z, t], 0, -4 Z[10]^{(1,0)}[z, t]}, -2/3 Z[15]^{(0,1)}[z, t]}
  {-2/3 Z[3][z, t], 0, -4 Z[11]^{(1,0)}[z, t]}, 2/3 Z[14]^{(0,1)}[z, t]}
  {2/3 Z[12][z, t], 1/3 Z[1][z, t] - 1/3 e^{-2 a4[t]} Z[1][z, t], 4 Z[4]^{(1,0)}[z, t]}, -2/3 Z[1]^{(0,1)}[z, t]}
  {2/3 Z[13][z, t], -2/3 e^{-a4[t]} Sinh[a4[t]] Z[0][z, t], 4 Z[5]^{(1,0)}[z, t]}, 2/3 Z[0]^{(0,1)}[z, t]}
  {2/3 Z[14][z, t], 0, 4 Z[6]^{(1,0)}[z, t]}, 2/3 Z[3]^{(0,1)}[z, t]}
  {2/3 Z[15][z, t], 0, 4 Z[7]^{(1,0)}[z, t]}, -2/3 Z[2]^{(0,1)}[z, t]}
  {2/3 Z[8][z, t], 2/3 e^{-a4[t]} Sinh[a4[t]] Z[5][z, t], 4 Z[0]^{(1,0)}[z, t]}, 2/3 Z[5]^{(0,1)}[z, t]}
  {2/3 Z[9][z, t], -2/3 e^{-a4[t]} Sinh[a4[t]] Z[4][z, t], 4 Z[1]^{(1,0)}[z, t]}, -2/3 Z[4]^{(0,1)}[z, t]}
  {2/3 Z[10][z, t], 0, 4 Z[2]^{(1,0)}[z, t]}, -2/3 Z[7]^{(0,1)}[z, t]}
  {2/3 Z[11][z, t], 0, 4 Z[3]^{(1,0)}[z, t]}, 2/3 Z[6]^{(0,1)}[z, t]}
}

In[*]:= (*resolveResolve[#]&/@eLazt1ResolveWithRemainder[[1]]*)

In[*]:= (*resolveWithRemainder[#]&/@Transpose[{ }]*)

In[*]:= Solve[Thread[0 == Flatten[{eLazt1[[1]], eLazt1[[6]], eLazt1[[9]], eLazt1[[14]]}],
  {Z[1]^{(0,1)}[z, t], Z[4]^{(0,1)}[z, t], Z[9]^{(0,1)}[z, t], Z[12]^{(0,1)}[z, t]}] [[-1]]

Out[*]=
{
  Z[1]^{(0,1)}[z, t] -> 1/2 e^{-2 a4[t]} (2 e^{2 a4[t]} M Z[12][z, t] - Z[1][z, t] a4'[t] +
    e^{2 a4[t]} Z[1][z, t] a4'[t] + 12 e^{2 a4[t]} Tan[z] Z[4]^{(1,0)}[z, t]), Z[4]^{(0,1)}[z, t] ->
    e^{-a4[t]} (e^{a4[t]} M Z[9][z, t] - Sinh[a4[t]] Z[4][z, t] a4'[t] + 6 e^{a4[t]} Tan[z] Z[1]^{(1,0)}[z, t]),
  Z[9]^{(0,1)}[z, t] -> -1/2 e^{-2 a4[t]} (2 e^{2 a4[t]} M Z[4][z, t] - Z[9][z, t] a4'[t] +
    e^{2 a4[t]} Z[9][z, t] a4'[t] + 12 e^{2 a4[t]} Tan[z] Z[12]^{(1,0)}[z, t]), Z[12]^{(0,1)}[z, t] ->
    -e^{-a4[t]} (e^{a4[t]} M Z[1][z, t] - Sinh[a4[t]] Z[12][z, t] a4'[t] + 6 e^{a4[t]} Tan[z] Z[9]^{(1,0)}[z, t])
}

In[*]:= Solve[eLazt1[[1]] == 0, Z[9]^{(0,1)}[z, t]] [[1]]
eLazt1[[14]] /. % // FullSimplify[#, constraintVars] &

Out[*]=
{
  Z[9]^{(0,1)}[z, t] -> -1/2 e^{-2 a4[t]} (2 e^{2 a4[t]} M Z[4][z, t] -
    Z[9][z, t] a4'[t] + e^{2 a4[t]} Z[9][z, t] a4'[t] + 12 e^{2 a4[t]} Tan[z] Z[12]^{(1,0)}[z, t])
}

Out[*]=
2/3 (M Z[9][z, t] - e^{-a4[t]} Sinh[a4[t]] Z[4][z, t] a4'[t] - Z[4]^{(0,1)}[z, t] + 6 Tan[z] Z[1]^{(1,0)}[z, t])

```

```

In[*]:= (*ToExpression["Z["<>ToString[#]<>"] [z,t]"&/@eLazt1Couplings[[1]]*)

In[*]:= (*vars1eLazt1=Z[#] [z,t]&/@eLazt1Couplings[[1]]*)

In[*]:= (*jetvars1eLazt1=
  {Z[#] [z,t],D[Z[#] [z,t],z],D[Z[#] [z,t],t]}&/@eLazt1Couplings[[1]]//Flatten*)

In[*]:= (*ToExpression["Z["<>ToString[#]<>"] [z,t]"&/@eLazt1Couplings[[2]]*)

In[*]:= (*vars2eLazt1=Z[#] [z,t]&/@eLazt1Couplings[[2]]*)

In[*]:= (*jetvars2eLazt1=
  {Z[#] [z,t],D[Z[#] [z,t],z],D[Z[#] [z,t],t]}&/@eLazt1Couplings[[2]]//Flatten*)

In[*]:= jetvarsVals[[1]]
Out[*]:=
{Z[0] [z, t], Z[0] (1,0) [z, t], Z[0] (0,1) [z, t], Z[5] [z, t], Z[5] (1,0) [z, t], Z[5] (0,1) [z, t],
  Z[8] [z, t], Z[8] (1,0) [z, t], Z[8] (0,1) [z, t], Z[13] [z, t], Z[13] (1,0) [z, t], Z[13] (0,1) [z, t]}

In[*]:= eLazt1;
(*Table[tab1[p]=D[eLazt1[[j]],#]&/@jetvarsVals[[p]][[k]],{j,1,Length[eLazt1]]}
  Dimensions[%]*)
(*tab1=Table[D[eLazt1[[j]],jetvars1eLazt1[[k]],
  {k,1,Length[jetvars1eLazt1]],{j,1,Length[eLazt1]]}*)
Table[tab[p]=Table[D[eLazt1[[j]],jetvarsVals[[p]][[k]],{j,1,Length[eLazt1]],
  {k,1,Length[jetvarsVals[[p]]]}],{p,1,Length[jetvarsVals]}]
Dimensions[tab[1]]
complements = Table[Select[tab[p], # == {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0} & -> "Index"],
  {p, 1, Length[jetvarsVals]}]
Table[useEqsRows[p] = Complement[Range[16], complements[[p]], {p, 1, Length[jetvarsVals]}]
Table[useEqs[p] = eLazt1[[#]] & /@ useEqsRows[p], {p, 1, Length[jetvarsVals]}]
eLazt1Couplings

(*notEq1=Select[tab1,#=={0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}&"Index"]
eqZ2=Sort[eLazt1[[#]]&/@notEq1]
eqZ1=Sort[Complement[eLazt1,eqZ2]]*)
(*Select[eLgzt,#->"Index"!MemberQ[notEq1]&]*)

Out[*]:=
{{ {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, - $\frac{2M}{3}$ , 0, 0,  $\frac{2}{3} e^{-a4[t]} \sinh[a4[t]] a4'[t]$ , 0,  $\frac{2}{3}$ , 0, -4 Tan[z], 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {- $\frac{2M}{3}$ , 0, 0, 0, 0, 0, 0, 0, -4 Tan[z], 0, - $\frac{1}{3} a4'[t] + \frac{1}{3} e^{-2 a4[t]} a4'[t]$ , 0,  $\frac{2}{3}$ },
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```

$$\begin{aligned}
& \left\{ -\frac{2}{3} e^{-a4[t]} \sinh[a4[t]] a4'[t], 0, \frac{2}{3}, 0, 4 \tan[z], 0, 0, 0, 0, \frac{2M}{3}, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 4 \tan[z], 0, \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] a4'[t], 0, \frac{2}{3}, \frac{2M}{3}, 0, 0, 0, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \left\{ 0, 0, 0, -\frac{2M}{3}, 0, 0, -\frac{1}{3} a4'[t] + \frac{1}{3} e^{-2a4[t]} a4'[t], 0, -\frac{2}{3}, 0, -4 \tan[z], 0 \right\}, \right. \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left. \left\{ -\frac{2M}{3}, 0, 0, 0, 0, 0, 0, -4 \tan[z], 0, \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] a4'[t], 0, -\frac{2}{3} \right\}, \right. \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{a4'[t]}{3} - \frac{1}{3} e^{-2a4[t]} a4'[t], 0, -\frac{2}{3}, 0, 4 \tan[z], 0, 0, 0, 0, \frac{2M}{3}, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 4 \tan[z], 0, -\frac{2}{3} e^{-a4[t]} \sinh[a4[t]] a4'[t], 0, -\frac{2}{3}, \frac{2M}{3}, 0, 0, 0, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \right. \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, 0, 0, -\frac{2M}{3}, 0, 0, 0, 0, -\frac{2}{3}, 0, -4 \tan[z], 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left. \left\{ -\frac{2M}{3}, 0, 0, 0, 0, 0, 0, -4 \tan[z], 0, 0, 0, -\frac{2}{3} \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \right. \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, 0, -\frac{2}{3}, 0, 4 \tan[z], 0, 0, 0, 0, \frac{2M}{3}, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 4 \tan[z], 0, 0, 0, -\frac{2}{3}, \frac{2M}{3}, 0, 0, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \right. \\
& \left\{ 0, 0, 0, -\frac{2M}{3}, 0, 0, 0, 0, \frac{2}{3}, 0, -4 \tan[z], 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ -\frac{2M}{3}, 0, 0, 0, 0, 0, 0, -4 \tan[z], 0, 0, 0, \frac{2}{3} \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, \frac{2}{3}, 0, 4 \tan[z], 0, 0, 0, 0, \frac{2M}{3}, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, 4 \tan[z], 0, 0, 0, \frac{2}{3}, \frac{2M}{3}, 0, 0, 0, 0, 0 \right\} \right\}
\end{aligned}$$

Out[*]=

{16, 12}

Out[*]=

{{1, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15, 16}, {2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 15, 16},
{1, 2, 3, 5, 6, 8, 9, 10, 11, 13, 14, 16}, {1, 2, 4, 5, 6, 7, 9, 10, 12, 13, 14, 15}}

Out[*]=

{{2, 5, 10, 13}, {1, 6, 9, 14}, {4, 7, 12, 15}, {3, 8, 11, 16}}

Out[*]=

$$\left\{ \left\{ -\frac{2}{3} M Z[5][z, t] + \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[8][z, t] a4'[t] + \right. \right.$$

$$\frac{2}{3} Z[8]^{(0,1)}[z, t] - 4 \tan[z] Z[13]^{(1,0)}[z, t], -\frac{2}{3} M Z[0][z, t] - \frac{1}{3} Z[13][z, t] a4'[t] +$$

$$\frac{1}{3} e^{-2 a4[t]} Z[13][z, t] a4'[t] + \frac{2}{3} Z[13]^{(0,1)}[z, t] - 4 \tan[z] Z[8]^{(1,0)}[z, t],$$

$$\frac{2}{3} M Z[13][z, t] - \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[0][z, t] a4'[t] +$$

$$\frac{2}{3} Z[0]^{(0,1)}[z, t] + 4 \tan[z] Z[5]^{(1,0)}[z, t], \frac{2}{3} M Z[8][z, t] +$$

$$\frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] + \frac{2}{3} Z[5]^{(0,1)}[z, t] + 4 \tan[z] Z[0]^{(1,0)}[z, t] \Big\},$$

$$\left\{ -\frac{2}{3} M Z[4][z, t] - \frac{1}{3} Z[9][z, t] a4'[t] + \frac{1}{3} e^{-2 a4[t]} Z[9][z, t] a4'[t] - \frac{2}{3} Z[9]^{(0,1)}[z, t] - \right.$$

$$4 \tan[z] Z[12]^{(1,0)}[z, t], -\frac{2}{3} M Z[1][z, t] + \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[12][z, t] a4'[t] -$$

$$\frac{2}{3} Z[12]^{(0,1)}[z, t] - 4 \tan[z] Z[9]^{(1,0)}[z, t], \frac{2}{3} M Z[12][z, t] + \frac{1}{3} Z[1][z, t] a4'[t] -$$

$$\frac{1}{3} e^{-2 a4[t]} Z[1][z, t] a4'[t] - \frac{2}{3} Z[1]^{(0,1)}[z, t] + 4 \tan[z] Z[4]^{(1,0)}[z, t], \frac{2}{3} M Z[9][z, t] -$$

$$\frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[4][z, t] a4'[t] - \frac{2}{3} Z[4]^{(0,1)}[z, t] + 4 \tan[z] Z[1]^{(1,0)}[z, t] \Big\},$$

$$\left\{ -\frac{2}{3} M Z[7][z, t] - \frac{2}{3} Z[10]^{(0,1)}[z, t] - 4 \tan[z] Z[15]^{(1,0)}[z, t], \right.$$

$$-\frac{2}{3} M Z[2][z, t] - \frac{2}{3} Z[15]^{(0,1)}[z, t] - 4 \tan[z] Z[10]^{(1,0)}[z, t],$$

$$\frac{2}{3} M Z[15][z, t] - \frac{2}{3} Z[2]^{(0,1)}[z, t] + 4 \tan[z] Z[7]^{(1,0)}[z, t],$$

$$\frac{2}{3} M Z[10][z, t] - \frac{2}{3} Z[7]^{(0,1)}[z, t] + 4 \tan[z] Z[2]^{(1,0)}[z, t] \Big\},$$

$$\left\{ -\frac{2}{3} M Z[6][z, t] + \frac{2}{3} Z[11]^{(0,1)}[z, t] - 4 \tan[z] Z[14]^{(1,0)}[z, t], \right.$$

$$-\frac{2}{3} M Z[3][z, t] + \frac{2}{3} Z[14]^{(0,1)}[z, t] - 4 \tan[z] Z[11]^{(1,0)}[z, t],$$

$$\frac{2}{3} M Z[14][z, t] + \frac{2}{3} Z[3]^{(0,1)}[z, t] + 4 \tan[z] Z[6]^{(1,0)}[z, t],$$

$$\frac{2}{3} M Z[11][z, t] + \frac{2}{3} Z[6]^{(0,1)}[z, t] + 4 \tan[z] Z[3]^{(1,0)}[z, t] \Big\} \Big\}$$

Out[*]=

{ {0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14} }

In[*]:= {eLazt1Couplings[#, useEqs[#]] & /@ Range[4]}

Out[*]=

$$\left\{ \left\{ \{0, 5, 8, 13\}, \left\{ -\frac{2}{3} M Z[5][z, t] + \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[8][z, t] a4'[t] + \right. \right. \\ \frac{2}{3} Z[8]^{(0,1)}[z, t] - 4 \tan[z] Z[13]^{(1,0)}[z, t], -\frac{2}{3} M Z[0][z, t] - \frac{1}{3} Z[13][z, t] a4'[t] + \\ \frac{1}{3} e^{-2 a4[t]} Z[13][z, t] a4'[t] + \frac{2}{3} Z[13]^{(0,1)}[z, t] - 4 \tan[z] Z[8]^{(1,0)}[z, t], \\ \frac{2}{3} M Z[13][z, t] - \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[0][z, t] a4'[t] + \frac{2}{3} Z[0]^{(0,1)}[z, t] + \\ 4 \tan[z] Z[5]^{(1,0)}[z, t], \frac{2}{3} M Z[8][z, t] + \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] + \\ \left. \left. \frac{2}{3} Z[5]^{(0,1)}[z, t] + 4 \tan[z] Z[0]^{(1,0)}[z, t] \right\} \right\}, \\ \left\{ \{1, 4, 9, 12\}, \left\{ -\frac{2}{3} M Z[4][z, t] - \frac{1}{3} Z[9][z, t] a4'[t] + \frac{1}{3} e^{-2 a4[t]} Z[9][z, t] a4'[t] - \right. \right. \\ \frac{2}{3} Z[9]^{(0,1)}[z, t] - 4 \tan[z] Z[12]^{(1,0)}[z, t], -\frac{2}{3} M Z[1][z, t] + \\ \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[12][z, t] a4'[t] - \frac{2}{3} Z[12]^{(0,1)}[z, t] - 4 \tan[z] Z[9]^{(1,0)}[z, t], \\ \frac{2}{3} M Z[12][z, t] + \frac{1}{3} Z[1][z, t] a4'[t] - \frac{1}{3} e^{-2 a4[t]} Z[1][z, t] a4'[t] - \\ \frac{2}{3} Z[1]^{(0,1)}[z, t] + 4 \tan[z] Z[4]^{(1,0)}[z, t], \frac{2}{3} M Z[9][z, t] - \\ \left. \left. \frac{2}{3} e^{-a4[t]} \sinh[a4[t]] Z[4][z, t] a4'[t] - \frac{2}{3} Z[4]^{(0,1)}[z, t] + 4 \tan[z] Z[1]^{(1,0)}[z, t] \right\} \right\}, \\ \left\{ \{2, 7, 10, 15\}, \left\{ -\frac{2}{3} M Z[7][z, t] - \frac{2}{3} Z[10]^{(0,1)}[z, t] - 4 \tan[z] Z[15]^{(1,0)}[z, t], \right. \right. \\ -\frac{2}{3} M Z[2][z, t] - \frac{2}{3} Z[15]^{(0,1)}[z, t] - 4 \tan[z] Z[10]^{(1,0)}[z, t], \\ \frac{2}{3} M Z[15][z, t] - \frac{2}{3} Z[2]^{(0,1)}[z, t] + 4 \tan[z] Z[7]^{(1,0)}[z, t], \\ \left. \left. \frac{2}{3} M Z[10][z, t] - \frac{2}{3} Z[7]^{(0,1)}[z, t] + 4 \tan[z] Z[2]^{(1,0)}[z, t] \right\} \right\}, \\ \left\{ \{3, 6, 11, 14\}, \left\{ -\frac{2}{3} M Z[6][z, t] + \frac{2}{3} Z[11]^{(0,1)}[z, t] - 4 \tan[z] Z[14]^{(1,0)}[z, t], \right. \right. \\ -\frac{2}{3} M Z[3][z, t] + \frac{2}{3} Z[14]^{(0,1)}[z, t] - 4 \tan[z] Z[11]^{(1,0)}[z, t], \\ \frac{2}{3} M Z[14][z, t] + \frac{2}{3} Z[3]^{(0,1)}[z, t] + 4 \tan[z] Z[6]^{(1,0)}[z, t], \\ \left. \left. \frac{2}{3} M Z[11][z, t] + \frac{2}{3} Z[6]^{(0,1)}[z, t] + 4 \tan[z] Z[3]^{(1,0)}[z, t] \right\} \right\} \right\}$$

```

In[ ]:= (eLazt1Resolved = Table[{eLazt1Couplings[p],
    Solve[And@@ Thread[0 == useEqs[p]], Table[Z[eLazt1Couplings[p][k]](0,1)[z, t],
        {k, 1, Length[eLazt1Couplings[p]]}]]][1]], {p, 1, Length[jetvarsVals]}) // Column

Out[ ]:=
{ {0, 5, 8, 13}, {Z[0](0,1)[z, t] →
    -e-a4[t] (ea4[t] M Z[13][z, t] - Sinh[a4[t]] Z[0][z, t] a4'[t] + 6 ea4[t] Tan[z] Z[5](1,0)[z, t]),
    Z[5](0,1)[z, t] → -e-a4[t] (ea4[t] M Z[8][z, t] + Sinh[a4[t]] Z[5][z, t] a4'[t] +
    6 ea4[t] Tan[z] Z[0](1,0)[z, t]), Z[8](0,1)[z, t] →
    e-a4[t] (ea4[t] M Z[5][z, t] - Sinh[a4[t]] Z[8][z, t] a4'[t] + 6 ea4[t] Tan[z] Z[13](1,0)[z, t]),
    Z[13](0,1)[z, t] →  $\frac{1}{2}$  e-2 a4[t] (2 e2 a4[t] M Z[0][z, t] - Z[13][z, t] a4'[t] +
    e2 a4[t] Z[13][z, t] a4'[t] + 12 e2 a4[t] Tan[z] Z[8](1,0)[z, t])} }
{ {1, 4, 9, 12},
    {Z[1](0,1)[z, t] →  $\frac{1}{2}$  e-2 a4[t] (2 e2 a4[t] M Z[12][z, t] - Z[1][z, t] a4'[t] + e2 a4[t] Z[1][z, t] a4'[t] +
    12 e2 a4[t] Tan[z] Z[4](1,0)[z, t]), Z[4](0,1)[z, t] →
    e-a4[t] (ea4[t] M Z[9][z, t] - Sinh[a4[t]] Z[4][z, t] a4'[t] + 6 ea4[t] Tan[z] Z[1](1,0)[z, t]),
    Z[9](0,1)[z, t] → - $\frac{1}{2}$  e-2 a4[t] (2 e2 a4[t] M Z[4][z, t] - Z[9][z, t] a4'[t] +
    e2 a4[t] Z[9][z, t] a4'[t] + 12 e2 a4[t] Tan[z] Z[12](1,0)[z, t]), Z[12](0,1)[z, t] →
    -e-a4[t] (ea4[t] M Z[1][z, t] - Sinh[a4[t]] Z[12][z, t] a4'[t] + 6 ea4[t] Tan[z] Z[9](1,0)[z, t])} }
{ {2, 7, 10, 15}, {Z[2](0,1)[z, t] → M Z[15][z, t] + 6 Tan[z] Z[7](1,0)[z, t],
    Z[7](0,1)[z, t] → M Z[10][z, t] + 6 Tan[z] Z[2](1,0)[z, t],
    Z[10](0,1)[z, t] → -M Z[7][z, t] - 6 Tan[z] Z[15](1,0)[z, t],
    Z[15](0,1)[z, t] → -M Z[2][z, t] - 6 Tan[z] Z[10](1,0)[z, t]} }
{ {3, 6, 11, 14}, {Z[3](0,1)[z, t] → -M Z[14][z, t] - 6 Tan[z] Z[6](1,0)[z, t],
    Z[6](0,1)[z, t] → -M Z[11][z, t] - 6 Tan[z] Z[3](1,0)[z, t],
    Z[11](0,1)[z, t] → M Z[6][z, t] + 6 Tan[z] Z[14](1,0)[z, t],
    Z[14](0,1)[z, t] → M Z[3][z, t] + 6 Tan[z] Z[11](1,0)[z, t]} }

```



```

In[ ]:= eLazt1Resolved[[1]][2] // ExpandAll
ToString[FullForm[#]] & /@%;
StringReplace[#, "Rule" -> "Equal"] & /@%;
eLazt1Resolved1eqs = ToExpression[#] & /@%
DSolve[%,
  Table[Z[eLazt1Couplings[[1]][k]][z, t], {k, 1, Length[eLazt1Couplings[[1]]}], {z, t}]

Out[ ]:=

$$\begin{aligned}
& \{Z[0]^{(0,1)}[z, t] \rightarrow -M Z[13][z, t] + e^{-a4[t]} \sinh[a4[t]] Z[0][z, t] a4'[t] - 6 \tan[z] Z[5]^{(1,0)}[z, t], \\
& Z[5]^{(0,1)}[z, t] \rightarrow -M Z[8][z, t] - e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] - 6 \tan[z] Z[0]^{(1,0)}[z, t], \\
& Z[8]^{(0,1)}[z, t] \rightarrow M Z[5][z, t] - e^{-a4[t]} \sinh[a4[t]] Z[8][z, t] a4'[t] + 6 \tan[z] Z[13]^{(1,0)}[z, t], \\
& Z[13]^{(0,1)}[z, t] \rightarrow \\
& \quad M Z[0][z, t] + \frac{1}{2} Z[13][z, t] a4'[t] - \frac{1}{2} e^{-2 a4[t]} Z[13][z, t] a4'[t] + 6 \tan[z] Z[8]^{(1,0)}[z, t]\}
\end{aligned}$$


Out[ ]:=

$$\begin{aligned}
& \{Z[0]^{(0,1)}[z, t] == -M Z[13][z, t] + e^{-a4[t]} \sinh[a4[t]] Z[0][z, t] a4'[t] - 6 \tan[z] Z[5]^{(1,0)}[z, t], \\
& Z[5]^{(0,1)}[z, t] == -M Z[8][z, t] - e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] - 6 \tan[z] Z[0]^{(1,0)}[z, t], \\
& Z[8]^{(0,1)}[z, t] == M Z[5][z, t] - e^{-a4[t]} \sinh[a4[t]] Z[8][z, t] a4'[t] + 6 \tan[z] Z[13]^{(1,0)}[z, t], \\
& Z[13]^{(0,1)}[z, t] == \\
& \quad M Z[0][z, t] + \frac{1}{2} Z[13][z, t] a4'[t] - \frac{1}{2} e^{-2 a4[t]} Z[13][z, t] a4'[t] + 6 \tan[z] Z[8]^{(1,0)}[z, t]\}
\end{aligned}$$


Out[ ]:=
DSolve[ $\{Z[0]^{(0,1)}[z, t] ==$ 
 $-M Z[13][z, t] + e^{-a4[t]} \sinh[a4[t]] Z[0][z, t] a4'[t] - 6 \tan[z] Z[5]^{(1,0)}[z, t],$ 
 $Z[5]^{(0,1)}[z, t] == -M Z[8][z, t] - e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] - 6 \tan[z] Z[0]^{(1,0)}[z, t],$ 
 $Z[8]^{(0,1)}[z, t] == M Z[5][z, t] - e^{-a4[t]} \sinh[a4[t]] Z[8][z, t] a4'[t] + 6 \tan[z] Z[13]^{(1,0)}[z, t],$ 
 $Z[13]^{(0,1)}[z, t] ==$ 
 $M Z[0][z, t] + \frac{1}{2} Z[13][z, t] a4'[t] - \frac{1}{2} e^{-2 a4[t]} Z[13][z, t] a4'[t] + 6 \tan[z] Z[8]^{(1,0)}[z, t]\},$ 
 $\{Z[0][z, t], Z[5][z, t], Z[8][z, t], Z[13][z, t]\}, \{z, t\}]$ 

In[ ]:= eLazt1Resolved1eqs /. sreplaceZ // FullForm
Table[Z[eLazt1Couplings[[1]][k]][z, t], {k, 1, Length[eLazt1Couplings[[1]]]} /. sreplaceZ

Out[ ]//FullForm=
List[Equal[Derivative[0, 1][Z0][z, t], Plus[Times[-1, M, Z13[z, t]],
Times[Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z0[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z5][z, t]]],
Equal[Derivative[0, 1][Z5][z, t], Plus[Times[-1, M, Z8[z, t]],
Times[-1, Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z5[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z0][z, t]]],
Equal[Derivative[0, 1][Z8][z, t], Plus[Times[M, Z5[z, t]],
Times[-1, Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z8[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z13][z, t]]], Equal[Derivative[0, 1][Z13][z, t],
Plus[Times[M, Z0[z, t]], Times[Rational[1, 2], Z13[z, t], Derivative[1][a4][t]],
Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]], Z13[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z8][z, t]]]]]

Out[ ]:=
{Z0[z, t], Z5[z, t], Z8[z, t], Z13[z, t]}

```

```

In[ ]:= eLazt1Resolved[[2]][2] // ExpandAll;
ToString[FullForm[#]] & /@%;
StringReplace[#, "Rule" → "Equal"] & /@%;
eLazt1Resolved2eqs = ToExpression[#] & /@%;
% /. sreplaceZ // FullForm
Table[Z[eLazt1Couplings[[2]][k]][z, t], {k, 1, Length[eLazt1Couplings[[2]]]}] /. sreplaceZ
Out[ ]//FullForm=
List[Equal[Derivative[0, 1][Z1][z, t],
  Plus[Times[M, Z12[z, t]], Times[Rational[1, 2], Z1[z, t], Derivative[1][a4][t]],
    Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]], Z1[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][Z4][z, t]]],
  Equal[Derivative[0, 1][Z4][z, t], Plus[Times[M, Z9[z, t]],
    Times[-1, Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z4[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][Z1][z, t]]], Equal[Derivative[0, 1][Z9][z, t],
    Plus[Times[-1, M, Z4[z, t]], Times[Rational[-1, 2], Z9[z, t], Derivative[1][a4][t]],
    Times[Rational[1, 2], Power[E, Times[-2, a4[t]]], Z9[z, t], Derivative[1][a4][t]],
    Times[-6, Tan[z], Derivative[1, 0][Z12][z, t]]],
  Equal[Derivative[0, 1][Z12][z, t], Plus[Times[-1, M, Z1[z, t]],
    Times[Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z12[z, t], Derivative[1][a4][t]],
    Times[-6, Tan[z], Derivative[1, 0][Z9][z, t]]]]
Out[ ]:=
{Z1[z, t], Z4[z, t], Z9[z, t], Z12[z, t]}

In[ ]:= eLazt1Resolved[[3]][2] // ExpandAll;
ToString[FullForm[#]] & /@%;
StringReplace[#, "Rule" → "Equal"] & /@%;
eLazt1Resolved3eqs = ToExpression[#] & /@%;
% /. sreplaceZ // FullForm
Table[Z[eLazt1Couplings[[3]][k]][z, t], {k, 1, Length[eLazt1Couplings[[3]]]}] /. sreplaceZ
Out[ ]//FullForm=
List[Equal[Derivative[0, 1][Z2][z, t],
  Plus[Times[M, Z15[z, t]], Times[6, Tan[z], Derivative[1, 0][Z7][z, t]]],
  Equal[Derivative[0, 1][Z7][z, t], Plus[Times[M, Z10[z, t]],
    Times[6, Tan[z], Derivative[1, 0][Z2][z, t]]], Equal[Derivative[0, 1][Z10][z, t],
    Plus[Times[-1, M, Z7[z, t]], Times[-6, Tan[z], Derivative[1, 0][Z15][z, t]]],
  Equal[Derivative[0, 1][Z15][z, t],
    Plus[Times[-1, M, Z2[z, t]], Times[-6, Tan[z], Derivative[1, 0][Z10][z, t]]]]
Out[ ]:=
{Z2[z, t], Z7[z, t], Z10[z, t], Z15[z, t]}

```

```

In[*]:= eLazt1Resolved[[4]][2] // ExpandAll;
ToString[FullForm[#]] & /@%;
StringReplace[#, "Rule" → "Equal"] & /@%;
eLazt1Resolved4eqs = ToExpression[#] & /@%;
% /. sreplaceZ // FullForm
Table[Z[eLazt1Couplings[[4]][k]][z, t], {k, 1, Length[eLazt1Couplings[[4]]}] /. sreplaceZ
Out[*] // FullForm=
List[Equal[Derivative[0, 1][Z3][z, t],
  Plus[Times[-1, M, Z14[z, t]], Times[-6, Tan[z], Derivative[1, 0][Z6][z, t]]],
  Equal[Derivative[0, 1][Z6][z, t], Plus[Times[-1, M, Z11[z, t]],
    Times[-6, Tan[z], Derivative[1, 0][Z3][z, t]]], Equal[Derivative[0, 1][Z11][z, t],
    Plus[Times[M, Z6[z, t]], Times[6, Tan[z], Derivative[1, 0][Z14][z, t]]],
  Equal[Derivative[0, 1][Z14][z, t],
    Plus[Times[M, Z3[z, t]], Times[6, Tan[z], Derivative[1, 0][Z11][z, t]]]]
Out[*]=
{Z3[z, t], Z6[z, t], Z11[z, t], Z14[z, t]}

In[*]:= (*tab[1]
  Select[%,#=={0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}&→"Index"]
  useEqsRows[1]=Complement[Range[16],%]
  useEqs[1]=eLazt1[[#]]&/@useEqsRows[1]
  eLazt1Couplings[[1]*)

In[*]:= (*Solve[Thread[0==eqZ1],D[vars1eLazt1,t]][[-1]]//ExpandAll
  ToString[FullForm[#]]&/@%;
  StringReplace[#, "Rule"→ "Equal"]&/@%;
  (sdt11=ToExpression[#]&/@%) //Column
  eLazt1Couplings[[1]*)

In[*]:= (*Solve[Thread[0==eqZ2],D[vars2eLazt1,t]][[-1]]//ExpandAll
  ToString[FullForm[#]]&/@%;
  StringReplace[#, "Rule"→ "Equal"]&/@%;
  (sdt12=ToExpression[#]&/@%) //Column
  eLazt1Couplings[[2]*)

```

eLazt2

```
In[*]:= (eLazt2 = eLazt /. {QQQ1 → 0, QQQ2 → 1} // ExpandAll) // Column
```

```
Out[*]=
```

$$\begin{aligned}
& -\frac{2}{3} M Z[4][z, t] - Z[5][z, t] - Z[0][z, t] a4'[t] - \frac{2}{3} Z[9]^{(0,1)}[z, t] - 4 \tan[z] Z[12]^{(1,0)}[z, t] \\
& Z[4][z, t] - \frac{2}{3} M Z[5][z, t] - e^{-a4[t]} \cosh[a4[t]] Z[1][z, t] a4'[t] - \\
& \frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[1][z, t] a4'[t] + \frac{2}{3} Z[8]^{(0,1)}[z, t] - 4 \tan[z] Z[13]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[6][z, t] + Z[7][z, t] - \frac{2}{3} Z[2][z, t] a4'[t] - \\
& \frac{1}{3} e^{-2 a4[t]} Z[2][z, t] a4'[t] + \frac{2}{3} Z[11]^{(0,1)}[z, t] - 4 \tan[z] Z[14]^{(1,0)}[z, t] \\
& -Z[6][z, t] - \frac{2}{3} M Z[7][z, t] - \frac{1}{3} Z[3][z, t] a4'[t] - \\
& \frac{2}{3} e^{-2 a4[t]} Z[3][z, t] a4'[t] - \frac{2}{3} Z[10]^{(0,1)}[z, t] - 4 \tan[z] Z[15]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[0][z, t] + Z[1][z, t] - e^{-2 a4[t]} Z[4][z, t] a4'[t] + \\
& \frac{2}{3} Z[13]^{(0,1)}[z, t] - 4 \tan[z] Z[8]^{(1,0)}[z, t] \\
& -Z[0][z, t] - \frac{2}{3} M Z[1][z, t] - e^{-a4[t]} \cosh[a4[t]] Z[5][z, t] a4'[t] + \\
& \frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] - \frac{2}{3} Z[12]^{(0,1)}[z, t] - 4 \tan[z] Z[9]^{(1,0)}[z, t] \\
& -\frac{2}{3} M Z[2][z, t] - Z[3][z, t] - \frac{1}{3} Z[6][z, t] a4'[t] - \\
& \frac{2}{3} e^{-2 a4[t]} Z[6][z, t] a4'[t] - \frac{2}{3} Z[15]^{(0,1)}[z, t] - 4 \tan[z] Z[10]^{(1,0)}[z, t] \\
& Z[2][z, t] - \frac{2}{3} M Z[3][z, t] - \frac{2}{3} Z[7][z, t] a4'[t] - \\
& \frac{1}{3} e^{-2 a4[t]} Z[7][z, t] a4'[t] + \frac{2}{3} Z[14]^{(0,1)}[z, t] - 4 \tan[z] Z[11]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[12][z, t] - Z[13][z, t] + \\
& e^{-2 a4[t]} Z[8][z, t] a4'[t] - \frac{2}{3} Z[1]^{(0,1)}[z, t] + 4 \tan[z] Z[4]^{(1,0)}[z, t] \\
& Z[12][z, t] + \frac{2}{3} M Z[13][z, t] + e^{-a4[t]} \cosh[a4[t]] Z[9][z, t] a4'[t] - \\
& \frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[9][z, t] a4'[t] + \frac{2}{3} Z[0]^{(0,1)}[z, t] + 4 \tan[z] Z[5]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[14][z, t] + Z[15][z, t] + \frac{1}{3} Z[10][z, t] a4'[t] + \\
& \frac{2}{3} e^{-2 a4[t]} Z[10][z, t] a4'[t] + \frac{2}{3} Z[3]^{(0,1)}[z, t] + 4 \tan[z] Z[6]^{(1,0)}[z, t] \\
& -Z[14][z, t] + \frac{2}{3} M Z[15][z, t] + \frac{2}{3} Z[11][z, t] a4'[t] + \\
& \frac{1}{3} e^{-2 a4[t]} Z[11][z, t] a4'[t] - \frac{2}{3} Z[2]^{(0,1)}[z, t] + 4 \tan[z] Z[7]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[8][z, t] + Z[9][z, t] + Z[12][z, t] a4'[t] + \frac{2}{3} Z[5]^{(0,1)}[z, t] + 4 \tan[z] Z[0]^{(1,0)}[z, t] \\
& -Z[8][z, t] + \frac{2}{3} M Z[9][z, t] + e^{-a4[t]} \cosh[a4[t]] Z[13][z, t] a4'[t] + \\
& \frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[13][z, t] a4'[t] - \frac{2}{3} Z[4]^{(0,1)}[z, t] + 4 \tan[z] Z[1]^{(1,0)}[z, t] \\
& \frac{2}{3} M Z[10][z, t] - Z[11][z, t] + \frac{2}{3} Z[14][z, t] a4'[t] + \\
& \frac{1}{3} e^{-2 a4[t]} Z[14][z, t] a4'[t] - \frac{2}{3} Z[7]^{(0,1)}[z, t] + 4 \tan[z] Z[2]^{(1,0)}[z, t] \\
& Z[10][z, t] + \frac{2}{3} M Z[11][z, t] + \frac{1}{3} Z[15][z, t] a4'[t] + \\
& \frac{2}{3} e^{-2 a4[t]} Z[15][z, t] a4'[t] + \frac{2}{3} Z[6]^{(0,1)}[z, t] + 4 \tan[z] Z[3]^{(1,0)}[z, t]
\end{aligned}$$

```

In[*]:= Flatten[{{1, 4, 9, 12, 0, 5, 8, 13}}] // Sort
Complement[Range[0, 15], %]
eLazt2Couplings = Flatten[{{%, %}}, 1]

Out[*]=
{0, 1, 4, 5, 8, 9, 12, 13}

Out[*]=
{2, 3, 6, 7, 10, 11, 14, 15}

Out[*]=
{{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}}

In[*]:= eLazt2Couplings[[1]]

Out[*]=
{0, 1, 4, 5, 8, 9, 12, 13}

In[*]:= jetvarsValsOf2 =
Table[Flatten[{Z[#][z, t], D[Z[#][z, t], z], D[Z[#][z, t], t]} & /@ eLazt2Couplings[[j]],
{j, 1, Length[eLazt2Couplings]}]

Out[*]=
{{Z[0][z, t], Z[0](1,0)[z, t], Z[0](0,1)[z, t], Z[1][z, t],
Z[1](1,0)[z, t], Z[1](0,1)[z, t], Z[4][z, t], Z[4](1,0)[z, t], Z[4](0,1)[z, t],
Z[5][z, t], Z[5](1,0)[z, t], Z[5](0,1)[z, t], Z[8][z, t], Z[8](1,0)[z, t],
Z[8](0,1)[z, t], Z[9][z, t], Z[9](1,0)[z, t], Z[9](0,1)[z, t], Z[12][z, t],
Z[12](1,0)[z, t], Z[12](0,1)[z, t], Z[13][z, t], Z[13](1,0)[z, t], Z[13](0,1)[z, t]},
{Z[2][z, t], Z[2](1,0)[z, t], Z[2](0,1)[z, t], Z[3][z, t], Z[3](1,0)[z, t],
Z[3](0,1)[z, t], Z[6][z, t], Z[6](1,0)[z, t], Z[6](0,1)[z, t], Z[7][z, t],
Z[7](1,0)[z, t], Z[7](0,1)[z, t], Z[10][z, t], Z[10](1,0)[z, t], Z[10](0,1)[z, t],
Z[11][z, t], Z[11](1,0)[z, t], Z[11](0,1)[z, t], Z[14][z, t], Z[14](1,0)[z, t],
Z[14](0,1)[z, t], Z[15][z, t], Z[15](1,0)[z, t], Z[15](0,1)[z, t]}}

In[*]:= Length[jetvarsValsOf2]

Out[*]=
2

```

```

In[*]:= eLazt2;
(*Table[tabl[p]=D[eLazt2[[j]],#]&/@jetvarsValsOf2[[p]][[k]],{j,1,Length[eLazt2]]}
Dimensions[%]*)
(*tabl=Table[D[eLazt2[[j]],jetvars1eLazt1[[k]],
{k,1,Length[jetvars1eLazt1]],{j,1,Length[eLazt2]]}*)
Table[tabOf2[p] = Table[D[eLazt2[[j]], jetvarsValsOf2[[p]][[k]], {j, 1, Length[eLazt2]],
{k, 1, Length[jetvarsValsOf2[[p]]]], {p, 1, Length[jetvarsValsOf2]]};
Dimensions[tabOf2[1]]
complements = Table[Select[tabOf2[p],
# == {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0} & → "Index"],
{p, 1, Length[jetvarsValsOf2]}]
Table[useEqsRows[p] = Complement[Range[16], complements[[p]],
{p, 1, Length[jetvarsValsOf2]}]
Table[useEqs[p] = eLazt2[[#]] & /@ useEqsRows[p], {p, 1, Length[jetvarsValsOf2]}]
eLazt2Couplings
(*notEq1=Select[tabl,#=={0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}&→"Index"]
eqZ2=Sort[eLazt2[[#]]&/@notEq1]
eqZ1=Sort[Complement[eLazt2,eqZ2]]*)
(*Select[eLgzt,#→"Index"!MemberQ[notEq1]&]*)

Out[*]=
{16, 24}

Out[*]=
{{3, 4, 7, 8, 11, 12, 15, 16}, {1, 2, 5, 6, 9, 10, 13, 14}}

Out[*]=
{{1, 2, 5, 6, 9, 10, 13, 14}, {3, 4, 7, 8, 11, 12, 15, 16}}

Out[*]=

$$\left\{ \left\{ -\frac{2}{3} M Z[4][z, t] - Z[5][z, t] - Z[0][z, t] a4'[t] - \frac{2}{3} Z[9]^{(0,1)}[z, t] - 4 \tan[z] Z[12]^{(1,0)}[z, t], \right. \right.$$


$$Z[4][z, t] - \frac{2}{3} M Z[5][z, t] - e^{-a4[t]} \cosh[a4[t]] Z[1][z, t] a4'[t] -$$


$$\frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[1][z, t] a4'[t] + \frac{2}{3} Z[8]^{(0,1)}[z, t] - 4 \tan[z] Z[13]^{(1,0)}[z, t],$$


$$-\frac{2}{3} M Z[0][z, t] + Z[1][z, t] - e^{-2 a4[t]} Z[4][z, t] a4'[t] + \frac{2}{3} Z[13]^{(0,1)}[z, t] -$$


$$4 \tan[z] Z[8]^{(1,0)}[z, t], -Z[0][z, t] - \frac{2}{3} M Z[1][z, t] - e^{-a4[t]} \cosh[a4[t]] Z[5][z, t] a4'[t] +$$


$$\frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] - \frac{2}{3} Z[12]^{(0,1)}[z, t] - 4 \tan[z] Z[9]^{(1,0)}[z, t],$$


$$\frac{2}{3} M Z[12][z, t] - Z[13][z, t] + e^{-2 a4[t]} Z[8][z, t] a4'[t] - \frac{2}{3} Z[1]^{(0,1)}[z, t] +$$


$$4 \tan[z] Z[4]^{(1,0)}[z, t], Z[12][z, t] + \frac{2}{3} M Z[13][z, t] + e^{-a4[t]} \cosh[a4[t]] Z[9][z, t] a4'[t] -$$


$$\frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[9][z, t] a4'[t] + \frac{2}{3} Z[0]^{(0,1)}[z, t] + 4 \tan[z] Z[5]^{(1,0)}[z, t],$$


$$\frac{2}{3} M Z[8][z, t] + Z[9][z, t] + Z[12][z, t] a4'[t] + \frac{2}{3} Z[5]^{(0,1)}[z, t] + 4 \tan[z] Z[0]^{(1,0)}[z, t],$$


```

$$\begin{aligned}
& -Z[8][z, t] + \frac{2}{3} MZ[9][z, t] + e^{-a4[t]} \cosh[a4[t]] Z[13][z, t] a4'[t] + \\
& \frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[13][z, t] a4'[t] - \frac{2}{3} Z[4]^{(0,1)}[z, t] + 4 \tan[z] Z[1]^{(1,0)}[z, t] \Big\}, \\
& \Big\{ -\frac{2}{3} MZ[6][z, t] + Z[7][z, t] - \frac{2}{3} Z[2][z, t] a4'[t] - \frac{1}{3} e^{-2a4[t]} Z[2][z, t] a4'[t] + \\
& \frac{2}{3} Z[11]^{(0,1)}[z, t] - 4 \tan[z] Z[14]^{(1,0)}[z, t], \\
& -Z[6][z, t] - \frac{2}{3} MZ[7][z, t] - \frac{1}{3} Z[3][z, t] a4'[t] - \frac{2}{3} e^{-2a4[t]} Z[3][z, t] a4'[t] - \\
& \frac{2}{3} Z[10]^{(0,1)}[z, t] - 4 \tan[z] Z[15]^{(1,0)}[z, t], \\
& -\frac{2}{3} MZ[2][z, t] - Z[3][z, t] - \frac{1}{3} Z[6][z, t] a4'[t] - \frac{2}{3} e^{-2a4[t]} Z[6][z, t] a4'[t] - \\
& \frac{2}{3} Z[15]^{(0,1)}[z, t] - 4 \tan[z] Z[10]^{(1,0)}[z, t], \\
& Z[2][z, t] - \frac{2}{3} MZ[3][z, t] - \frac{2}{3} Z[7][z, t] a4'[t] - \frac{1}{3} e^{-2a4[t]} Z[7][z, t] a4'[t] + \\
& \frac{2}{3} Z[14]^{(0,1)}[z, t] - 4 \tan[z] Z[11]^{(1,0)}[z, t], \\
& \frac{2}{3} MZ[14][z, t] + Z[15][z, t] + \frac{1}{3} Z[10][z, t] a4'[t] + \frac{2}{3} e^{-2a4[t]} Z[10][z, t] a4'[t] + \\
& \frac{2}{3} Z[3]^{(0,1)}[z, t] + 4 \tan[z] Z[6]^{(1,0)}[z, t], -Z[14][z, t] + \frac{2}{3} MZ[15][z, t] + \\
& \frac{2}{3} Z[11][z, t] a4'[t] + \frac{1}{3} e^{-2a4[t]} Z[11][z, t] a4'[t] - \frac{2}{3} Z[2]^{(0,1)}[z, t] + \\
& 4 \tan[z] Z[7]^{(1,0)}[z, t], \frac{2}{3} MZ[10][z, t] - Z[11][z, t] + \frac{2}{3} Z[14][z, t] a4'[t] + \\
& \frac{1}{3} e^{-2a4[t]} Z[14][z, t] a4'[t] - \frac{2}{3} Z[7]^{(0,1)}[z, t] + 4 \tan[z] Z[2]^{(1,0)}[z, t], \\
& Z[10][z, t] + \frac{2}{3} MZ[11][z, t] + \frac{1}{3} Z[15][z, t] a4'[t] + \frac{2}{3} e^{-2a4[t]} Z[15][z, t] a4'[t] + \\
& \frac{2}{3} Z[6]^{(0,1)}[z, t] + 4 \tan[z] Z[3]^{(1,0)}[z, t] \Big\} \Big\}
\end{aligned}$$

Out[] =

{{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}}

```

In[ ] := (*eLazt1Resolved[[2]][[2]]//ExpandAll;
ToString[FullForm[#]]&/@%;
StringReplace[#, "Rule" -> "Equal"]&/@%;
eLazt1Resolved2eqs=ToExpression[#]&/@%;
%/.sreplaceZ//FullForm
Table[Z[eLazt1Couplings[[2]][[k]]][z,t],{k,1,Length[eLazt1Couplings[[2]]]}/.sreplaceZ*)

```

```

In[*]:= Table[Table[Z[eLazt2Couplings[[p]][k]](0,1)[z, t], {k, 1, Length[eLazt2Couplings[[p]]}],
  {p, 1, Length[jetvarsValsOf2]}] // Column

Out[*]:=
{Z[0](0,1)[z, t], Z[1](0,1)[z, t], Z[4](0,1)[z, t], Z[5](0,1)[z, t],
  Z[8](0,1)[z, t], Z[9](0,1)[z, t], Z[12](0,1)[z, t], Z[13](0,1)[z, t]}
{Z[2](0,1)[z, t], Z[3](0,1)[z, t], Z[6](0,1)[z, t], Z[7](0,1)[z, t],
  Z[10](0,1)[z, t], Z[11](0,1)[z, t], Z[14](0,1)[z, t], Z[15](0,1)[z, t]}

In[*]:= Clear[eLazt2Resolved];
(eLazt2Resolved = Table[{eLazt2Couplings[[p]],
  Block[{r, r1, r2, r3, r3maple, var, varmaple},
    r = Block[{s}, s = Solve[And @@ Thread[0 == useEqs[p]], Table[
      Z[eLazt2Couplings[[p]][k]](0,1)[z, t], {k, 1, Length[eLazt2Couplings[[p]]}]]][1];
    s // ExpandAll];
    r1 = ToString[FullForm[#]] & /@ r;
    r2 = StringReplace[#, "Rule" → "Equal"] & /@ r1;
    r3 = ToExpression[#] & /@ r2;
    r3maple = FullForm[r3] /. sreplaceZ;
    var = Table[Z[eLazt2Couplings[[p]][k]][z, t], {k, 1, Length[eLazt2Couplings[[p]]}];
    varmaple = FullForm[var] /. sreplaceZ; (*Print[{var, varmaple}];*)
    {{var, r3}, r, {varmaple, r3maple}}}], {p, 1, Length[jetvarsValsOf2]}]) // Column

Out[*]:=
{{0, 1, 4, 5, 8, 9, 12, 13}, {{Z[0][z, t], Z[1][z, t],
  Z[4][z, t], Z[5][z, t], Z[8][z, t], Z[9][z, t], Z[12][z, t], Z[13][z, t]},
  {Z[0](0,1)[z, t] = - $\frac{3}{2}$  Z[12][z, t] - M Z[13][z, t] -  $\frac{3}{2}$  e-a4[t] Cosh[a4[t]] Z[9][z, t] a4'[t] +
 $\frac{1}{2}$  e-a4[t] Sinh[a4[t]] Z[9][z, t] a4'[t] - 6 Tan[z] Z[5](1,0)[z, t], Z[1](0,1)[z, t] =
M Z[12][z, t] -  $\frac{3}{2}$  Z[13][z, t] +  $\frac{3}{2}$  e-2 a4[t] Z[8][z, t] a4'[t] + 6 Tan[z] Z[4](1,0)[z, t],
Z[4](0,1)[z, t] = - $\frac{3}{2}$  Z[8][z, t] + M Z[9][z, t] +  $\frac{3}{2}$  e-a4[t] Cosh[a4[t]] Z[13][z, t] a4'[t] +
 $\frac{1}{2}$  e-a4[t] Sinh[a4[t]] Z[13][z, t] a4'[t] + 6 Tan[z] Z[1](1,0)[z, t],
Z[5](0,1)[z, t] = -M Z[8][z, t] -  $\frac{3}{2}$  Z[9][z, t] -  $\frac{3}{2}$  Z[12][z, t] a4'[t] - 6 Tan[z] Z[0](1,0)[z, t],
Z[8](0,1)[z, t] = - $\frac{3}{2}$  Z[4][z, t] + M Z[5][z, t] +  $\frac{3}{2}$  e-a4[t] Cosh[a4[t]] Z[1][z, t] a4'[t] +
 $\frac{1}{2}$  e-a4[t] Sinh[a4[t]] Z[1][z, t] a4'[t] + 6 Tan[z] Z[13](1,0)[z, t],
Z[9](0,1)[z, t] = -M Z[4][z, t] -  $\frac{3}{2}$  Z[5][z, t] -  $\frac{3}{2}$  Z[0][z, t] a4'[t] - 6 Tan[z] Z[12](1,0)[z, t],
Z[12](0,1)[z, t] = - $\frac{3}{2}$  Z[0][z, t] - M Z[1][z, t] -  $\frac{3}{2}$  e-a4[t] Cosh[a4[t]] Z[5][z, t] a4'[t] +
 $\frac{1}{2}$  e-a4[t] Sinh[a4[t]] Z[5][z, t] a4'[t] - 6 Tan[z] Z[9](1,0)[z, t], Z[13](0,1)[z, t] =
M Z[0][z, t] -  $\frac{3}{2}$  Z[1][z, t] +  $\frac{3}{2}$  e-2 a4[t] Z[4][z, t] a4'[t] + 6 Tan[z] Z[8](1,0)[z, t]}},
  {Z[0](0,1)[z, t] → - $\frac{3}{2}$  Z[12][z, t] - M Z[13][z, t] -  $\frac{3}{2}$  e-a4[t] Cosh[a4[t]] Z[9][z, t] a4'[t] +
 $\frac{1}{2}$  e-a4[t] Sinh[a4[t]] Z[9][z, t] a4'[t] - 6 Tan[z] Z[5](1,0)[z, t], Z[1](0,1)[z, t] →
M Z[12][z, t] -  $\frac{3}{2}$  Z[13][z, t] +  $\frac{3}{2}$  e-2 a4[t] Z[8][z, t] a4'[t] + 6 Tan[z] Z[4](1,0)[z, t],
Z[4](0,1)[z, t] → - $\frac{3}{2}$  Z[8][z, t] + M Z[9][z, t] +  $\frac{3}{2}$  e-a4[t] Cosh[a4[t]] Z[13][z, t] a4'[t] +
 $\frac{1}{2}$  e-a4[t] Sinh[a4[t]] Z[13][z, t] a4'[t] + 6 Tan[z] Z[1](1,0)[z, t],
Z[5](0,1)[z, t] → -M Z[8][z, t] -  $\frac{3}{2}$  Z[9][z, t] -  $\frac{3}{2}$  Z[12][z, t] a4'[t] - 6 Tan[z] Z[0](1,0)[z, t],
Z[8](0,1)[z, t] → - $\frac{3}{2}$  Z[4][z, t] + M Z[5][z, t] +  $\frac{3}{2}$  e-a4[t] Cosh[a4[t]] Z[1][z, t] a4'[t] +

```



```

    1/2 e-a4[t] Sinh[a4[t]] Z[1][z, t] a4'[t] + 6 Tan[z] Z[13](1,0)[z, t],
    Z[9](0,1)[z, t] → -M Z[4][z, t] - 3/2 Z[5][z, t] - 3/2 Z[0][z, t] a4'[t] - 6 Tan[z] Z[12](1,0)[z, t],
    Z[12](0,1)[z, t] → -3/2 Z[0][z, t] - M Z[1][z, t] - 3/2 e-a4[t] Cosh[a4[t]] Z[5][z, t] a4'[t] +
    1/2 e-a4[t] Sinh[a4[t]] Z[5][z, t] a4'[t] - 6 Tan[z] Z[9](1,0)[z, t], Z[13](0,1)[z, t] →
    M Z[0][z, t] - 3/2 Z[1][z, t] + 3/2 e-2 a4[t] Z[4][z, t] a4'[t] + 6 Tan[z] Z[8](1,0)[z, t]},
    {List[Z0[z, t], Z1[z, t], Z4[z, t], Z5[z, t], Z8[z, t], Z9[z, t], Z12[z, t], Z13[z, t]],
    List[Equal[Derivative[0, 1][Z0][z, t],
    Plus[Times[Rational[-3, 2], Z12[z, t]], Times[-1, M, Z13[z, t]], Times[Rational[-3, 2],
    Power[E, Times[-1, a4[t]]], Cosh[a4[t]], Z9[z, t], Derivative[1][a4][t]],
    Times[Rational[1, 2], Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z9[z, t],
    Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][Z5][z, t]]],
    Equal[Derivative[0, 1][Z1][z, t], Plus[Times[M, Z12[z, t]],
    Times[Rational[-3, 2], Z13[z, t]], Times[Rational[3, 2], Power[E, Times[-2, a4[t]]],
    Z8[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][Z4][z, t]]],
    Equal[Derivative[0, 1][Z4][z, t], Plus[Times[Rational[-3, 2], Z8[z, t]],
    Times[M, Z9[z, t]], Times[Rational[3, 2], Power[E, Times[-1, a4[t]]],
    Cosh[a4[t]], Z13[z, t], Derivative[1][a4][t]], Times[Rational[1, 2],
    Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z13[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][Z1][z, t]]], Equal[Derivative[0, 1][Z5][z, t],
    Plus[Times[-1, M, Z8[z, t]], Times[Rational[-3, 2], Z9[z, t]], Times[Rational[-3, 2],
    Z12[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][Z0][z, t]]],
    Equal[Derivative[0, 1][Z8][z, t], Plus[Times[Rational[-3, 2], Z4[z, t]],
    Times[M, Z5[z, t]], Times[Rational[3, 2], Power[E, Times[-1, a4[t]]],
    Cosh[a4[t]], Z1[z, t], Derivative[1][a4][t]], Times[Rational[1, 2],
    Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z1[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][Z13][z, t]]], Equal[Derivative[0, 1][Z9][z, t],
    Plus[Times[-1, M, Z4[z, t]], Times[Rational[-3, 2], Z5[z, t]], Times[Rational[-3, 2],
    Z0[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][Z12][z, t]]],
    Equal[Derivative[0, 1][Z12][z, t], Plus[Times[Rational[-3, 2], Z0[z, t]],
    Times[-1, M, Z1[z, t]], Times[Rational[-3, 2], Power[E, Times[-1, a4[t]]],
    Cosh[a4[t]], Z5[z, t], Derivative[1][a4][t]], Times[Rational[1, 2],
    Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z5[z, t], Derivative[1][a4][t]],
    Times[-6, Tan[z], Derivative[1, 0][Z9][z, t]]], Equal[Derivative[0, 1][Z13][z, t],
    Plus[Times[M, Z0[z, t]], Times[Rational[-3, 2], Z1[z, t]],
    Times[Rational[3, 2], Power[E, Times[-2, a4[t]]], Z4[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][Z8][z, t]]]]]}},
    {{2, 3, 6, 7, 10, 11, 14, 15},
    {{Z[2][z, t], Z[3][z, t], Z[6][z, t], Z[7][z, t], Z[10][z, t], Z[11][z, t],
    Z[14][z, t], Z[15][z, t]}, {Z[2](0,1)[z, t] = -3/2 Z[14][z, t] + M Z[15][z, t] +
    Z[11][z, t] a4'[t] + 1/2 e-2 a4[t] Z[11][z, t] a4'[t] + 6 Tan[z] Z[7](1,0)[z, t],
    Z[3](0,1)[z, t] = -M Z[14][z, t] - 3/2 Z[15][z, t] - 1/2 Z[10][z, t] a4'[t] -
    e-2 a4[t] Z[10][z, t] a4'[t] - 6 Tan[z] Z[6](1,0)[z, t], Z[6](0,1)[z, t] = -3/2 Z[10][z, t] -
    M Z[11][z, t] - 1/2 Z[15][z, t] a4'[t] - e-2 a4[t] Z[15][z, t] a4'[t] - 6 Tan[z] Z[3](1,0)[z, t],
    Z[7](0,1)[z, t] = M Z[10][z, t] - 3/2 Z[11][z, t] + Z[14][z, t] a4'[t] +
    1/2 e-2 a4[t] Z[14][z, t] a4'[t] + 6 Tan[z] Z[2](1,0)[z, t],

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Z[10]^(0,1)[z,t] == -3/2 Z[6][z,t] - M Z[7][z,t] - 1/2 Z[3][z,t] a4'[t] -
  e^-2 a4[t] Z[3][z,t] a4'[t] - 6 Tan[z] Z[15]^(1,0)[z,t], Z[11]^(0,1)[z,t] == M Z[6][z,t] -
  3/2 Z[7][z,t] + Z[2][z,t] a4'[t] + 1/2 e^-2 a4[t] Z[2][z,t] a4'[t] + 6 Tan[z] Z[14]^(1,0)[z,t],
Z[14]^(0,1)[z,t] == -3/2 Z[2][z,t] + M Z[3][z,t] + Z[7][z,t] a4'[t] +
  1/2 e^-2 a4[t] Z[7][z,t] a4'[t] + 6 Tan[z] Z[11]^(1,0)[z,t],
Z[15]^(0,1)[z,t] == -M Z[2][z,t] - 3/2 Z[3][z,t] - 1/2 Z[6][z,t] a4'[t] -
  e^-2 a4[t] Z[6][z,t] a4'[t] - 6 Tan[z] Z[10]^(1,0)[z,t]},
{Z[2]^(0,1)[z,t] -> -3/2 Z[14][z,t] + M Z[15][z,t] + Z[11][z,t] a4'[t] +
  1/2 e^-2 a4[t] Z[11][z,t] a4'[t] + 6 Tan[z] Z[7]^(1,0)[z,t],
Z[3]^(0,1)[z,t] -> -M Z[14][z,t] - 3/2 Z[15][z,t] - 1/2 Z[10][z,t] a4'[t] -
  e^-2 a4[t] Z[10][z,t] a4'[t] - 6 Tan[z] Z[6]^(1,0)[z,t],
Z[6]^(0,1)[z,t] -> -3/2 Z[10][z,t] - M Z[11][z,t] - 1/2 Z[15][z,t] a4'[t] -
  e^-2 a4[t] Z[15][z,t] a4'[t] - 6 Tan[z] Z[3]^(1,0)[z,t],
Z[7]^(0,1)[z,t] -> M Z[10][z,t] - 3/2 Z[11][z,t] + Z[14][z,t] a4'[t] +
  1/2 e^-2 a4[t] Z[14][z,t] a4'[t] + 6 Tan[z] Z[2]^(1,0)[z,t],
Z[10]^(0,1)[z,t] -> -3/2 Z[6][z,t] - M Z[7][z,t] - 1/2 Z[3][z,t] a4'[t] -
  e^-2 a4[t] Z[3][z,t] a4'[t] - 6 Tan[z] Z[15]^(1,0)[z,t],
Z[11]^(0,1)[z,t] -> M Z[6][z,t] - 3/2 Z[7][z,t] + Z[2][z,t] a4'[t] +
  1/2 e^-2 a4[t] Z[2][z,t] a4'[t] + 6 Tan[z] Z[14]^(1,0)[z,t],
Z[14]^(0,1)[z,t] -> -3/2 Z[2][z,t] + M Z[3][z,t] + Z[7][z,t] a4'[t] +
  1/2 e^-2 a4[t] Z[7][z,t] a4'[t] + 6 Tan[z] Z[11]^(1,0)[z,t],
Z[15]^(0,1)[z,t] -> -M Z[2][z,t] - 3/2 Z[3][z,t] - 1/2 Z[6][z,t] a4'[t] -
  e^-2 a4[t] Z[6][z,t] a4'[t] - 6 Tan[z] Z[10]^(1,0)[z,t]},
List[Z2[z,t], Z3[z,t], Z6[z,t], Z7[z,t], Z10[z,t], Z11[z,t], Z14[z,t], Z15[z,t]],
List[Equal[Derivative[0,1][Z2][z,t], Plus[Times[Rational[-3,2], Z14[z,t]],
  Times[M, Z15[z,t]], Times[Z11[z,t], Derivative[1][a4][t]],
  Times[Rational[1,2], Power[E, Times[-2, a4[t]]], Z11[z,t], Derivative[1][a4][t]],
  Times[6, Tan[z], Derivative[1,0][Z7][z,t]]], Equal[Derivative[0,1][Z3][z,t],
  Plus[Times[-1, M, Z14[z,t]], Times[Rational[-3,2], Z15[z,t]],
  Times[Rational[-1,2], Z10[z,t], Derivative[1][a4][t]],
  Times[-1, Power[E, Times[-2, a4[t]]], Z10[z,t], Derivative[1][a4][t]],
  Times[-6, Tan[z], Derivative[1,0][Z6][z,t]]],
Equal[Derivative[0,1][Z6][z,t], Plus[Times[Rational[-3,2], Z10[z,t]],
  Times[-1, M, Z11[z,t]], Times[Rational[-1,2], Z15[z,t], Derivative[1][a4][t]],
  Times[-1, Power[E, Times[-2, a4[t]]], Z15[z,t], Derivative[1][a4][t]],
  Times[-6, Tan[z], Derivative[1,0][Z3][z,t]]],
Equal[Derivative[0,1][Z7][z,t], Plus[Times[M, Z10[z,t]],
  Times[Rational[-3,2], Z11[z,t]], Times[Z14[z,t], Derivative[1][a4][t]],
  Times[Rational[1,2], Power[E, Times[-2, a4[t]]], Z14[z,t], Derivative[1][a4][t]],
  Times[6, Tan[z], Derivative[1,0][Z2][z,t]]],
Equal[Derivative[0,1][Z10][z,t], Plus[Times[Rational[-3,2], Z6[z,t]],
  Times[-1, M, Z7[z,t]], Times[Rational[-1,2], Z3[z,t], Derivative[1][a4][t]],
  Times[-1, Power[E, Times[-2, a4[t]]], Z3[z,t], Derivative[1][a4][t]],
  Times[-6, Tan[z], Derivative[1,0][Z15][z,t]]],

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Times[-6, Tan[z], Derivative[1, 0][Z10][z, t]]],
Equal[Derivative[0, 1][Z11][z, t], Plus[Times[M, Z6[z, t]],
Times[Rational[-3, 2], Z7[z, t]], Times[Z2[z, t], Derivative[1][a4][t]],
Times[Rational[1, 2], Power[E, Times[-2, a4[t]]], Z2[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z14][z, t]]],
Equal[Derivative[0, 1][Z14][z, t], Plus[Times[Rational[-3, 2], Z2[z, t]],
Times[M, Z3[z, t]], Times[Z7[z, t], Derivative[1][a4][t]],
Times[Rational[1, 2], Power[E, Times[-2, a4[t]]], Z7[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z11][z, t]]], Equal[Derivative[0, 1][Z15][z, t],
Plus[Times[-1, M, Z2[z, t]], Times[Rational[-3, 2], Z3[z, t]],
Times[Rational[-1, 2], Z6[z, t], Derivative[1][a4][t]],
Times[-1, Power[E, Times[-2, a4[t]]], Z6[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z10][z, t]]]]]]]]}

```

In[*]:= Dimensions[eLazt2Resolved]

Out[*]=

{2, 2}

In[*]:= eLazt2Resolved[[1]]

Out[*]=

$$\begin{aligned}
& \left\{ \{0, 1, 4, 5, 8, 9, 12, 13\}, \left\{ \left\{ Z[0][z, t], Z[1][z, t], \right. \right. \right. \\
& \quad Z[4][z, t], Z[5][z, t], Z[8][z, t], Z[9][z, t], Z[12][z, t], Z[13][z, t] \}, \\
& \quad \left\{ Z[0]^{(0,1)}[z, t] = -\frac{3}{2} Z[12][z, t] - M Z[13][z, t] - \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[9][z, t] a4'[t] + \right. \\
& \quad \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[9][z, t] a4'[t] - 6 \tan[z] Z[5]^{(1,0)}[z, t], Z[1]^{(0,1)}[z, t] = \\
& \quad M Z[12][z, t] - \frac{3}{2} Z[13][z, t] + \frac{3}{2} e^{-2 a4[t]} Z[8][z, t] a4'[t] + 6 \tan[z] Z[4]^{(1,0)}[z, t], \\
& \quad Z[4]^{(0,1)}[z, t] = -\frac{3}{2} Z[8][z, t] + M Z[9][z, t] + \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[13][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[13][z, t] a4'[t] + 6 \tan[z] Z[1]^{(1,0)}[z, t], Z[5]^{(0,1)}[z, t] = \\
& \quad -M Z[8][z, t] - \frac{3}{2} Z[9][z, t] - \frac{3}{2} Z[12][z, t] a4'[t] - 6 \tan[z] Z[0]^{(1,0)}[z, t], \\
& \quad Z[8]^{(0,1)}[z, t] = -\frac{3}{2} Z[4][z, t] + M Z[5][z, t] + \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[1][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[1][z, t] a4'[t] + 6 \tan[z] Z[13]^{(1,0)}[z, t], Z[9]^{(0,1)}[z, t] = \\
& \quad -M Z[4][z, t] - \frac{3}{2} Z[5][z, t] - \frac{3}{2} Z[0][z, t] a4'[t] - 6 \tan[z] Z[12]^{(1,0)}[z, t], \\
& \quad Z[12]^{(0,1)}[z, t] = -\frac{3}{2} Z[0][z, t] - M Z[1][z, t] - \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[5][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] - 6 \tan[z] Z[9]^{(1,0)}[z, t], Z[13]^{(0,1)}[z, t] = \\
& \quad M Z[0][z, t] - \frac{3}{2} Z[1][z, t] + \frac{3}{2} e^{-2 a4[t]} Z[4][z, t] a4'[t] + 6 \tan[z] Z[8]^{(1,0)}[z, t] \left. \right\} \}, \\
& \left\{ Z[0]^{(0,1)}[z, t] \rightarrow -\frac{3}{2} Z[12][z, t] - M Z[13][z, t] - \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[9][z, t] a4'[t] + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[9][z, t] a4'[t] - 6 \tan[z] Z[5]^{(1,0)}[z, t], Z[1]^{(0,1)}[z, t] \rightarrow \\
& M Z[12][z, t] - \frac{3}{2} Z[13][z, t] + \frac{3}{2} e^{-2 a4[t]} Z[8][z, t] a4'[t] + 6 \tan[z] Z[4]^{(1,0)}[z, t], \\
& Z[4]^{(0,1)}[z, t] \rightarrow -\frac{3}{2} Z[8][z, t] + M Z[9][z, t] + \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[13][z, t] a4'[t] + \\
& \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[13][z, t] a4'[t] + 6 \tan[z] Z[1]^{(1,0)}[z, t], \\
& Z[5]^{(0,1)}[z, t] \rightarrow -M Z[8][z, t] - \frac{3}{2} Z[9][z, t] - \frac{3}{2} Z[12][z, t] a4'[t] - 6 \tan[z] Z[0]^{(1,0)}[z, t], \\
& Z[8]^{(0,1)}[z, t] \rightarrow -\frac{3}{2} Z[4][z, t] + M Z[5][z, t] + \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[1][z, t] a4'[t] + \\
& \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[1][z, t] a4'[t] + 6 \tan[z] Z[13]^{(1,0)}[z, t], \\
& Z[9]^{(0,1)}[z, t] \rightarrow -M Z[4][z, t] - \frac{3}{2} Z[5][z, t] - \frac{3}{2} Z[0][z, t] a4'[t] - 6 \tan[z] Z[12]^{(1,0)}[z, t], \\
& Z[12]^{(0,1)}[z, t] \rightarrow -\frac{3}{2} Z[0][z, t] - M Z[1][z, t] - \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[5][z, t] a4'[t] + \\
& \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] - 6 \tan[z] Z[9]^{(1,0)}[z, t], Z[13]^{(0,1)}[z, t] \rightarrow \\
& M Z[0][z, t] - \frac{3}{2} Z[1][z, t] + \frac{3}{2} e^{-2 a4[t]} Z[4][z, t] a4'[t] + 6 \tan[z] Z[8]^{(1,0)}[z, t] \}, \\
& \{ \text{List}[Z0[z, t], Z1[z, t], Z4[z, t], Z5[z, t], Z8[z, t], Z9[z, t], Z12[z, t], Z13[z, t]], \\
& \text{List}[\text{Equal}[\text{Derivative}[0, 1][Z0][z, t], \\
& \quad \text{Plus}[\text{Times}[\text{Rational}[-3, 2], Z12[z, t]], \text{Times}[-1, M, Z13[z, t]], \text{Times}[\text{Rational}[-3, 2], \\
& \quad \text{Power}[E, \text{Times}[-1, a4[t]]], \cosh[a4[t]], Z9[z, t], \text{Derivative}[1][a4][t]], \\
& \quad \text{Times}[\text{Rational}[1, 2], \text{Power}[E, \text{Times}[-1, a4[t]]], \sinh[a4[t]], Z9[z, t], \\
& \quad \text{Derivative}[1][a4][t]], \text{Times}[-6, \tan[z], \text{Derivative}[1, 0][Z5][z, t]]], \\
& \text{Equal}[\text{Derivative}[0, 1][Z1][z, t], \text{Plus}[\text{Times}[M, Z12[z, t]], \\
& \quad \text{Times}[\text{Rational}[-3, 2], Z13[z, t]], \text{Times}[\text{Rational}[3, 2], \text{Power}[E, \text{Times}[-2, a4[t]]], \\
& \quad Z8[z, t], \text{Derivative}[1][a4][t]], \text{Times}[6, \tan[z], \text{Derivative}[1, 0][Z4][z, t]]], \\
& \text{Equal}[\text{Derivative}[0, 1][Z4][z, t], \text{Plus}[\text{Times}[\text{Rational}[-3, 2], Z8[z, t]], \\
& \quad \text{Times}[M, Z9[z, t]], \text{Times}[\text{Rational}[3, 2], \text{Power}[E, \text{Times}[-1, a4[t]]], \\
& \quad \cosh[a4[t]], Z13[z, t], \text{Derivative}[1][a4][t]], \text{Times}[\text{Rational}[1, 2], \\
& \quad \text{Power}[E, \text{Times}[-1, a4[t]]], \sinh[a4[t]], Z13[z, t], \text{Derivative}[1][a4][t]], \\
& \quad \text{Times}[6, \tan[z], \text{Derivative}[1, 0][Z1][z, t]]], \text{Equal}[\text{Derivative}[0, 1][Z5][z, t], \\
& \quad \text{Plus}[\text{Times}[-1, M, Z8[z, t]], \text{Times}[\text{Rational}[-3, 2], Z9[z, t]], \text{Times}[\text{Rational}[-3, 2], \\
& \quad Z12[z, t], \text{Derivative}[1][a4][t]], \text{Times}[-6, \tan[z], \text{Derivative}[1, 0][Z0][z, t]]], \\
& \text{Equal}[\text{Derivative}[0, 1][Z8][z, t], \text{Plus}[\text{Times}[\text{Rational}[-3, 2], Z4[z, t]], \\
& \quad \text{Times}[M, Z5[z, t]], \text{Times}[\text{Rational}[3, 2], \text{Power}[E, \text{Times}[-1, a4[t]]], \\
& \quad \cosh[a4[t]], Z1[z, t], \text{Derivative}[1][a4][t]], \text{Times}[\text{Rational}[1, 2], \\
& \quad \text{Power}[E, \text{Times}[-1, a4[t]]], \sinh[a4[t]], Z1[z, t], \text{Derivative}[1][a4][t]], \\
& \quad \text{Times}[6, \tan[z], \text{Derivative}[1, 0][Z13][z, t]]], \text{Equal}[\text{Derivative}[0, 1][Z9][z, t], \\
& \quad \text{Plus}[\text{Times}[-1, M, Z4[z, t]], \text{Times}[\text{Rational}[-3, 2], Z5[z, t]], \text{Times}[\text{Rational}[-3, 2], \\
& \quad Z0[z, t], \text{Derivative}[1][a4][t]], \text{Times}[-6, \tan[z], \text{Derivative}[1, 0][Z12][z, t]]], \\
& \text{Equal}[\text{Derivative}[0, 1][Z12][z, t], \text{Plus}[\text{Times}[\text{Rational}[-3, 2], Z0[z, t]], \\
& \quad \text{Times}[-1, M, Z1[z, t]], \text{Times}[\text{Rational}[-3, 2], \text{Power}[E, \text{Times}[-1, a4[t]]],
\end{aligned}$$

```

Cosh[a4[t]], Z5[z, t], Derivative[1][a4][t]], Times[Rational[1, 2],
Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z5[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z9][z, t]]], Equal[Derivative[0, 1][Z13][z, t],
Plus[Times[M, Z0[z, t]], Times[Rational[-3, 2], Z1[z, t]],
Times[Rational[3, 2], Power[E, Times[-2, a4[t]]], Z4[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z8][z, t]]]]]]}}

```

```

In[8]:= eLazt2Resolved[[1]][[2]][[1]][[1]]
eLazt2Resolved[[1]][[2]][[1]][[2]]
DSolve[%, %, {z, t}]

```

```
Out[8]=
```

```

{Z[0][z, t], Z[1][z, t], Z[4][z, t], Z[5][z, t],
Z[8][z, t], Z[9][z, t], Z[12][z, t], Z[13][z, t]}

```

```
Out[8]=
```

$$\begin{aligned}
& \{Z[0]^{(0,1)}[z, t] = -\frac{3}{2} Z[12][z, t] - M Z[13][z, t] - \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[9][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[9][z, t] a4'[t] - 6 \tan[z] Z[5]^{(1,0)}[z, t], Z[1]^{(0,1)}[z, t] = \\
& \quad M Z[12][z, t] - \frac{3}{2} Z[13][z, t] + \frac{3}{2} e^{-2 a4[t]} Z[8][z, t] a4'[t] + 6 \tan[z] Z[4]^{(1,0)}[z, t], \\
& \quad Z[4]^{(0,1)}[z, t] = -\frac{3}{2} Z[8][z, t] + M Z[9][z, t] + \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[13][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[13][z, t] a4'[t] + 6 \tan[z] Z[1]^{(1,0)}[z, t], \\
& \quad Z[5]^{(0,1)}[z, t] = -M Z[8][z, t] - \frac{3}{2} Z[9][z, t] - \frac{3}{2} Z[12][z, t] a4'[t] - 6 \tan[z] Z[0]^{(1,0)}[z, t], \\
& \quad Z[8]^{(0,1)}[z, t] = -\frac{3}{2} Z[4][z, t] + M Z[5][z, t] + \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[1][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[1][z, t] a4'[t] + 6 \tan[z] Z[13]^{(1,0)}[z, t], \\
& \quad Z[9]^{(0,1)}[z, t] = -M Z[4][z, t] - \frac{3}{2} Z[5][z, t] - \frac{3}{2} Z[0][z, t] a4'[t] - 6 \tan[z] Z[12]^{(1,0)}[z, t], \\
& \quad Z[12]^{(0,1)}[z, t] = -\frac{3}{2} Z[0][z, t] - M Z[1][z, t] - \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[5][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] - 6 \tan[z] Z[9]^{(1,0)}[z, t], Z[13]^{(0,1)}[z, t] = \\
& \quad M Z[0][z, t] - \frac{3}{2} Z[1][z, t] + \frac{3}{2} e^{-2 a4[t]} Z[4][z, t] a4'[t] + 6 \tan[z] Z[8]^{(1,0)}[z, t]\}
\end{aligned}$$

Out[*]=

$$\begin{aligned}
 \text{DSolve}\left[\left\{Z[0]^{(0,1)}[z, t] = -\frac{3}{2} Z[12][z, t] - M Z[13][z, t] - \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[9][z, t] a4'[t] + \right. \right. \\
 \left. \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[9][z, t] a4'[t] - 6 \tan[z] Z[5]^{(1,0)}[z, t], Z[1]^{(0,1)}[z, t] = \right. \\
 \left. M Z[12][z, t] - \frac{3}{2} Z[13][z, t] + \frac{3}{2} e^{-2 a4[t]} Z[8][z, t] a4'[t] + 6 \tan[z] Z[4]^{(1,0)}[z, t], \right. \\
 Z[4]^{(0,1)}[z, t] = -\frac{3}{2} Z[8][z, t] + M Z[9][z, t] + \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[13][z, t] a4'[t] + \\
 \left. \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[13][z, t] a4'[t] + 6 \tan[z] Z[1]^{(1,0)}[z, t], \right. \\
 Z[5]^{(0,1)}[z, t] = -M Z[8][z, t] - \frac{3}{2} Z[9][z, t] - \frac{3}{2} Z[12][z, t] a4'[t] - 6 \tan[z] Z[0]^{(1,0)}[z, t], \\
 Z[8]^{(0,1)}[z, t] = -\frac{3}{2} Z[4][z, t] + M Z[5][z, t] + \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[1][z, t] a4'[t] + \\
 \left. \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[1][z, t] a4'[t] + 6 \tan[z] Z[13]^{(1,0)}[z, t], \right. \\
 Z[9]^{(0,1)}[z, t] = -M Z[4][z, t] - \frac{3}{2} Z[5][z, t] - \frac{3}{2} Z[0][z, t] a4'[t] - 6 \tan[z] Z[12]^{(1,0)}[z, t], \\
 Z[12]^{(0,1)}[z, t] = -\frac{3}{2} Z[0][z, t] - M Z[1][z, t] - \frac{3}{2} e^{-a4[t]} \cosh[a4[t]] Z[5][z, t] a4'[t] + \\
 \left. \frac{1}{2} e^{-a4[t]} \sinh[a4[t]] Z[5][z, t] a4'[t] - 6 \tan[z] Z[9]^{(1,0)}[z, t], Z[13]^{(0,1)}[z, t] = \right. \\
 \left. M Z[0][z, t] - \frac{3}{2} Z[1][z, t] + \frac{3}{2} e^{-2 a4[t]} Z[4][z, t] a4'[t] + 6 \tan[z] Z[8]^{(1,0)}[z, t]\right\}, \\
 \{Z[0][z, t], Z[1][z, t], Z[4][z, t], Z[5][z, t], Z[8][z, t], \\
 Z[9][z, t], Z[12][z, t], Z[13][z, t]\}, \{z, t\}]
 \end{aligned}$$

In[*]:= eLazt2Resolved[[2]][[2]][[1]][[1]]

eLazt2Resolved[[2]][[2]][[1]][[2]]

DSolve[%, %, {z, t}]

Out[*]=

$$\{Z[2][z, t], Z[3][z, t], Z[6][z, t], Z[7][z, t], \\
 Z[10][z, t], Z[11][z, t], Z[14][z, t], Z[15][z, t]\}$$

Out[8]=

$$\begin{aligned}
& \left\{ Z[2]^{(0,1)}[z, t] = -\frac{3}{2} Z[14][z, t] + M Z[15][z, t] + \right. \\
& \quad Z[11][z, t] a4'[t] + \frac{1}{2} e^{-2 a4[t]} Z[11][z, t] a4'[t] + 6 \tan[z] Z[7]^{(1,0)}[z, t], \\
& Z[3]^{(0,1)}[z, t] = -M Z[14][z, t] - \frac{3}{2} Z[15][z, t] - \frac{1}{2} Z[10][z, t] a4'[t] - \\
& \quad e^{-2 a4[t]} Z[10][z, t] a4'[t] - 6 \tan[z] Z[6]^{(1,0)}[z, t], \\
& Z[6]^{(0,1)}[z, t] = -\frac{3}{2} Z[10][z, t] - M Z[11][z, t] - \frac{1}{2} Z[15][z, t] a4'[t] - \\
& \quad e^{-2 a4[t]} Z[15][z, t] a4'[t] - 6 \tan[z] Z[3]^{(1,0)}[z, t], \\
& Z[7]^{(0,1)}[z, t] = M Z[10][z, t] - \frac{3}{2} Z[11][z, t] + Z[14][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-2 a4[t]} Z[14][z, t] a4'[t] + 6 \tan[z] Z[2]^{(1,0)}[z, t], \\
& Z[10]^{(0,1)}[z, t] = -\frac{3}{2} Z[6][z, t] - M Z[7][z, t] - \frac{1}{2} Z[3][z, t] a4'[t] - \\
& \quad e^{-2 a4[t]} Z[3][z, t] a4'[t] - 6 \tan[z] Z[15]^{(1,0)}[z, t], \\
& Z[11]^{(0,1)}[z, t] = M Z[6][z, t] - \frac{3}{2} Z[7][z, t] + Z[2][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-2 a4[t]} Z[2][z, t] a4'[t] + 6 \tan[z] Z[14]^{(1,0)}[z, t], \\
& Z[14]^{(0,1)}[z, t] = -\frac{3}{2} Z[2][z, t] + M Z[3][z, t] + Z[7][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-2 a4[t]} Z[7][z, t] a4'[t] + 6 \tan[z] Z[11]^{(1,0)}[z, t], \\
& Z[15]^{(0,1)}[z, t] = -M Z[2][z, t] - \frac{3}{2} Z[3][z, t] - \frac{1}{2} Z[6][z, t] a4'[t] - \\
& \quad e^{-2 a4[t]} Z[6][z, t] a4'[t] - 6 \tan[z] Z[10]^{(1,0)}[z, t] \left. \right\}
\end{aligned}$$

Out[8] =

$$\begin{aligned}
& \text{DSolve}\left[\left\{Z[2]^{(0,1)}[z, t] = -\frac{3}{2} Z[14][z, t] + M Z[15][z, t] + \right. \right. \\
& \quad Z[11][z, t] a4'[t] + \frac{1}{2} e^{-2 a4[t]} Z[11][z, t] a4'[t] + 6 \text{Tan}[z] Z[7]^{(1,0)}[z, t], \\
& \quad Z[3]^{(0,1)}[z, t] = -M Z[14][z, t] - \frac{3}{2} Z[15][z, t] - \frac{1}{2} Z[10][z, t] a4'[t] - \\
& \quad e^{-2 a4[t]} Z[10][z, t] a4'[t] - 6 \text{Tan}[z] Z[6]^{(1,0)}[z, t], \\
& \quad Z[6]^{(0,1)}[z, t] = -\frac{3}{2} Z[10][z, t] - M Z[11][z, t] - \frac{1}{2} Z[15][z, t] a4'[t] - \\
& \quad e^{-2 a4[t]} Z[15][z, t] a4'[t] - 6 \text{Tan}[z] Z[3]^{(1,0)}[z, t], \\
& \quad Z[7]^{(0,1)}[z, t] = M Z[10][z, t] - \frac{3}{2} Z[11][z, t] + Z[14][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-2 a4[t]} Z[14][z, t] a4'[t] + 6 \text{Tan}[z] Z[2]^{(1,0)}[z, t], \\
& \quad Z[10]^{(0,1)}[z, t] = -\frac{3}{2} Z[6][z, t] - M Z[7][z, t] - \frac{1}{2} Z[3][z, t] a4'[t] - \\
& \quad e^{-2 a4[t]} Z[3][z, t] a4'[t] - 6 \text{Tan}[z] Z[15]^{(1,0)}[z, t], \\
& \quad Z[11]^{(0,1)}[z, t] = M Z[6][z, t] - \frac{3}{2} Z[7][z, t] + Z[2][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-2 a4[t]} Z[2][z, t] a4'[t] + 6 \text{Tan}[z] Z[14]^{(1,0)}[z, t], \\
& \quad Z[14]^{(0,1)}[z, t] = -\frac{3}{2} Z[2][z, t] + M Z[3][z, t] + Z[7][z, t] a4'[t] + \\
& \quad \frac{1}{2} e^{-2 a4[t]} Z[7][z, t] a4'[t] + 6 \text{Tan}[z] Z[11]^{(1,0)}[z, t], \\
& \quad Z[15]^{(0,1)}[z, t] = -M Z[2][z, t] - \frac{3}{2} Z[3][z, t] - \frac{1}{2} Z[6][z, t] a4'[t] - \\
& \quad e^{-2 a4[t]} Z[6][z, t] a4'[t] - 6 \text{Tan}[z] Z[10]^{(1,0)}[z, t]\}, \\
& \{Z[2][z, t], Z[3][z, t], Z[6][z, t], Z[7][z, t], Z[10][z, t], \\
& \quad Z[11][z, t], Z[14][z, t], Z[15][z, t]\}, \{z, t\}]
\end{aligned}$$


```
In[ ]:= eLazt2Resolved[[1]][2][3]
```

```
Out[ ]=
```

```
{List[Z0[z, t], Z1[z, t], Z4[z, t], Z5[z, t], Z8[z, t], Z9[z, t], Z12[z, t], Z13[z, t]],
List[Equal[Derivative[0, 1][Z0][z, t],
Plus[Times[Rational[-3, 2], Z12[z, t]], Times[-1, M, Z13[z, t]], Times[Rational[-3, 2],
Power[E, Times[-1, a4[t]]], Cosh[a4[t]], Z9[z, t], Derivative[1][a4][t]],
Times[Rational[1, 2], Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z9[z, t],
Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][Z5][z, t]]], Equal[
Derivative[0, 1][Z1][z, t], Plus[Times[M, Z12[z, t]], Times[Rational[-3, 2], Z13[z, t]],
Times[Rational[3, 2], Power[E, Times[-2, a4[t]]], Z8[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z4][z, t]]], Equal[Derivative[0, 1][Z4][z, t],
Plus[Times[Rational[-3, 2], Z8[z, t]], Times[M, Z9[z, t]], Times[Rational[3, 2],
Power[E, Times[-1, a4[t]]], Cosh[a4[t]], Z13[z, t], Derivative[1][a4][t]],
Times[Rational[1, 2], Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z13[z, t],
Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][Z1][z, t]]],
Equal[Derivative[0, 1][Z5][z, t], Plus[Times[-1, M, Z8[z, t]],
Times[Rational[-3, 2], Z9[z, t]], Times[Rational[-3, 2], Z12[z, t],
Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][Z0][z, t]]],
Equal[Derivative[0, 1][Z8][z, t], Plus[Times[Rational[-3, 2], Z4[z, t]],
Times[M, Z5[z, t]], Times[Rational[3, 2], Power[E, Times[-1, a4[t]]],
Cosh[a4[t]], Z1[z, t], Derivative[1][a4][t]], Times[Rational[1, 2],
Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z1[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z13][z, t]]], Equal[Derivative[0, 1][Z9][z, t],
Plus[Times[-1, M, Z4[z, t]], Times[Rational[-3, 2], Z5[z, t]],
Times[Rational[-3, 2], Z0[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z12][z, t]]],
Equal[Derivative[0, 1][Z12][z, t], Plus[Times[Rational[-3, 2], Z0[z, t]],
Times[-1, M, Z1[z, t]], Times[Rational[-3, 2], Power[E, Times[-1, a4[t]]],
Cosh[a4[t]], Z5[z, t], Derivative[1][a4][t]], Times[Rational[1, 2],
Power[E, Times[-1, a4[t]]], Sinh[a4[t]], Z5[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z9][z, t]]], Equal[Derivative[0, 1][Z13][z, t],
Plus[Times[M, Z0[z, t]], Times[Rational[-3, 2], Z1[z, t]],
Times[Rational[3, 2], Power[E, Times[-2, a4[t]]], Z4[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z8][z, t]]]]]
```

```
In[*]:= eLazt2Resolved[[2]][[3]]
```

```
Out[*]=
```

```
{List[Z2[z, t], Z3[z, t], Z6[z, t], Z7[z, t], Z10[z, t], Z11[z, t], Z14[z, t], Z15[z, t]],
List[Equal[Derivative[0, 1][Z2][z, t], Plus[Times[Rational[-3, 2], Z14[z, t]],
Times[M, Z15[z, t]], Times[Z11[z, t], Derivative[1][a4][t]],
Times[Rational[1, 2], Power[E, Times[-2, a4[t]]], Z11[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z7][z, t]]], Equal[Derivative[0, 1][Z3][z, t],
Plus[Times[-1, M, Z14[z, t]], Times[Rational[-3, 2], Z15[z, t]],
Times[Rational[-1, 2], Z10[z, t], Derivative[1][a4][t]],
Times[-1, Power[E, Times[-2, a4[t]]], Z10[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z6][z, t]]],
Equal[Derivative[0, 1][Z6][z, t], Plus[Times[Rational[-3, 2], Z10[z, t]],
Times[-1, M, Z11[z, t]], Times[Rational[-1, 2], Z15[z, t], Derivative[1][a4][t]],
Times[-1, Power[E, Times[-2, a4[t]]], Z15[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z3][z, t]]],
Equal[Derivative[0, 1][Z7][z, t], Plus[Times[M, Z10[z, t]],
Times[Rational[-3, 2], Z11[z, t]], Times[Z14[z, t], Derivative[1][a4][t]],
Times[Rational[1, 2], Power[E, Times[-2, a4[t]]], Z14[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z2][z, t]]],
Equal[Derivative[0, 1][Z10][z, t], Plus[Times[Rational[-3, 2], Z6[z, t]],
Times[-1, M, Z7[z, t]], Times[Rational[-1, 2], Z3[z, t], Derivative[1][a4][t]],
Times[-1, Power[E, Times[-2, a4[t]]], Z3[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z15][z, t]]],
Equal[Derivative[0, 1][Z11][z, t], Plus[Times[M, Z6[z, t]],
Times[Rational[-3, 2], Z7[z, t]], Times[Z2[z, t], Derivative[1][a4][t]],
Times[Rational[1, 2], Power[E, Times[-2, a4[t]]], Z2[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z14][z, t]]],
Equal[Derivative[0, 1][Z14][z, t], Plus[Times[Rational[-3, 2], Z2[z, t]],
Times[M, Z3[z, t]], Times[Z7[z, t], Derivative[1][a4][t]],
Times[Rational[1, 2], Power[E, Times[-2, a4[t]]], Z7[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][Z11][z, t]]], Equal[Derivative[0, 1][Z15][z, t],
Plus[Times[-1, M, Z2[z, t]], Times[Rational[-3, 2], Z3[z, t]],
Times[Rational[-1, 2], Z6[z, t], Derivative[1][a4][t]],
Times[-1, Power[E, Times[-2, a4[t]]], Z6[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][Z10][z, t]]]]]}
```

```
In[*]:= (*vars1eLazt2=ToExpression["Z["<>ToString[#]<>"] [z,t]"&/@eLazt2Couplings[[1]]*)
```

```
In[*]:= (*Z[#] [z,t]&/@eLazt2Couplings[[1]]*)
```

```
In[*]:= (*jetvars1eLazt2=
{Z[#] [z,t],D[Z[#] [z,t],z],D[Z[#] [z,t],t]}&/@eLazt2Couplings[[1]]//Flatten*)
```

```
In[*]:= (*vars2eLazt2=ToExpression["Z["<>ToString[#]<>"] [z,t]"&/@eLazt2Couplings[[2]]*)
```

```
In[*]:= (*Z[#] [z,t]&/@eLazt2Couplings[[2]]*)
```

```

In[*]:= (*jetvars2eLazt2=
        {Z[#][z,t],D[Z[#][z,t],z],D[Z[#][z,t],t]}&/@eLazt2Couplings[[2]]//Flatten*)

In[*]:= (*eLazt2;
        D[eLazt2,#]&/@jetvars1eLazt2
        Dimensions[%]
        (*tab1=Table[D[eLazt2[[j]],jetvars1eLazt2[[k]],
            {k,1,Length[jetvars1eLazt2]},{j,1,Length[eLazt2]}]*)
        tab1=Table[D[eLazt2[[j]],jetvars1eLazt2[[k]],
            {j,1,Length[eLazt2]},{k,1,Length[jetvars1eLazt2]}]
        Dimensions[tab1]
        notEq1=Select[tab1,##={0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}&→"Index"]
        eqZ2=Sort[eLazt2[[#]]&/@notEq1]
        eqZ1=Sort[Complement[eLazt2,eqZ2]]
        (*Select[eLgzt,##→"Index"!MemberQ[notEq1]&]*)*)

```

```

In[*]:= (*Solve[Thread[0==eqZ1],D[vars1eLazt2,t]] [[-1]]//ExpandAll
        ToString[FullForm[#]]&/@%;
        StringReplace[#, "Rule"→ "Equal"]&/@%;
        (sdt1=ToExpression[#]&/@%)//Column
        eLazt2Couplings[[1]*)

```

```

In[*]:= (*Solve[Thread[0==eqZ2],D[vars2eLazt2,t]] [[-1]]//ExpandAll
        ToString[FullForm[#]]&/@%;
        StringReplace[#, "Rule"→ "Equal"]&/@%;
        (sdt2=ToExpression[#]&/@%)//Column
        eLazt2Couplings[[2]*)

```

```

In[*]:= vars1 = Z[#][z,t] &/@ couplings[[1]]
        vars0z1 = vars1[[All,0]]
        jetvarsZ1 = {Z[#][z,t],D[Z[#][z,t],z],D[Z[#][z,t],t]} &/@ couplings[[1]] // Flatten
        varsz1 = vars1 /. sreplacZ (*FOR MAPLE*)
        coupledEQS1 = Sort[eLgzt[[# + 1]] &/@ couplings[[1]]] WTF!!!!!!!!!!!!!!!!!!!!!!
        FullForm[varsz1] (*FOR MAPLE*)
        FullForm[coupledEQS1] (*FOR MAPLE*)

```

```

In[*]:= (eLazt2ResolveWithRemainder = resolveWithRemainder[#, {M, a4'[t], Tan[z]}] & /@ eLazt2) //
Column

Out[*]=

$$\left\{ \left\{ -\frac{2}{3} Z[4][z, t], -Z[0][z, t], -4 Z[12]^{(1,0)}[z, t] \right\}, -Z[5][z, t] - \frac{2}{3} Z[9]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ -\frac{2}{3} Z[5][z, t], -e^{-a4[t]} \cosh[a4[t]] Z[1][z, t] - \frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[1][z, t], \right. \right.$$


$$\left. -4 Z[13]^{(1,0)}[z, t] \right\}, Z[4][z, t] + \frac{2}{3} Z[8]^{(0,1)}[z, t] \}$$


$$\left\{ \left\{ -\frac{2}{3} Z[6][z, t], -\frac{2}{3} Z[2][z, t] - \frac{1}{3} e^{-2 a4[t]} Z[2][z, t], -4 Z[14]^{(1,0)}[z, t] \right\}, \right.$$


$$\left. Z[7][z, t] + \frac{2}{3} Z[11]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ -\frac{2}{3} Z[7][z, t], -\frac{1}{3} Z[3][z, t] - \frac{2}{3} e^{-2 a4[t]} Z[3][z, t], -4 Z[15]^{(1,0)}[z, t] \right\}, \right.$$


$$\left. -Z[6][z, t] - \frac{2}{3} Z[10]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ -\frac{2}{3} Z[0][z, t], -e^{-2 a4[t]} Z[4][z, t], -4 Z[8]^{(1,0)}[z, t] \right\}, Z[1][z, t] + \frac{2}{3} Z[13]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ -\frac{2}{3} Z[1][z, t], -e^{-a4[t]} \cosh[a4[t]] Z[5][z, t] + \frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[5][z, t], \right. \right.$$


$$\left. -4 Z[9]^{(1,0)}[z, t] \right\}, -Z[0][z, t] - \frac{2}{3} Z[12]^{(0,1)}[z, t] \}$$


$$\left\{ \left\{ -\frac{2}{3} Z[2][z, t], -\frac{1}{3} Z[6][z, t] - \frac{2}{3} e^{-2 a4[t]} Z[6][z, t], -4 Z[10]^{(1,0)}[z, t] \right\}, \right.$$


$$\left. -Z[3][z, t] - \frac{2}{3} Z[15]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ -\frac{2}{3} Z[3][z, t], -\frac{2}{3} Z[7][z, t] - \frac{1}{3} e^{-2 a4[t]} Z[7][z, t], -4 Z[11]^{(1,0)}[z, t] \right\}, \right.$$


$$\left. Z[2][z, t] + \frac{2}{3} Z[14]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ \frac{2}{3} Z[12][z, t], e^{-2 a4[t]} Z[8][z, t], 4 Z[4]^{(1,0)}[z, t] \right\}, -Z[13][z, t] - \frac{2}{3} Z[1]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ \frac{2}{3} Z[13][z, t], e^{-a4[t]} \cosh[a4[t]] Z[9][z, t] - \frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[9][z, t], \right. \right.$$


$$\left. 4 Z[5]^{(1,0)}[z, t] \right\}, Z[12][z, t] + \frac{2}{3} Z[0]^{(0,1)}[z, t] \}$$


$$\left\{ \left\{ \frac{2}{3} Z[14][z, t], \frac{1}{3} Z[10][z, t] + \frac{2}{3} e^{-2 a4[t]} Z[10][z, t], 4 Z[6]^{(1,0)}[z, t] \right\}, \right.$$


$$\left. Z[15][z, t] + \frac{2}{3} Z[3]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ \frac{2}{3} Z[15][z, t], \frac{2}{3} Z[11][z, t] + \frac{1}{3} e^{-2 a4[t]} Z[11][z, t], 4 Z[7]^{(1,0)}[z, t] \right\}, \right.$$


$$\left. -Z[14][z, t] - \frac{2}{3} Z[2]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ \frac{2}{3} Z[8][z, t], Z[12][z, t], 4 Z[0]^{(1,0)}[z, t] \right\}, Z[9][z, t] + \frac{2}{3} Z[5]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ \frac{2}{3} Z[9][z, t], e^{-a4[t]} \cosh[a4[t]] Z[13][z, t] + \frac{1}{3} e^{-a4[t]} \sinh[a4[t]] Z[13][z, t], \right. \right.$$


$$\left. 4 Z[1]^{(1,0)}[z, t] \right\}, -Z[8][z, t] - \frac{2}{3} Z[4]^{(0,1)}[z, t] \}$$


$$\left\{ \left\{ \frac{2}{3} Z[10][z, t], \frac{2}{3} Z[14][z, t] + \frac{1}{3} e^{-2 a4[t]} Z[14][z, t], 4 Z[2]^{(1,0)}[z, t] \right\}, \right.$$


$$\left. -Z[11][z, t] - \frac{2}{3} Z[7]^{(0,1)}[z, t] \right\}$$


$$\left\{ \left\{ \frac{2}{3} Z[11][z, t], \frac{1}{3} Z[15][z, t] + \frac{2}{3} e^{-2 a4[t]} Z[15][z, t], 4 Z[3]^{(1,0)}[z, t] \right\}, \right.$$


$$\left. Z[10][z, t] + \frac{2}{3} Z[6]^{(0,1)}[z, t] \right\}$$


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