Scratch work

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Do not read if you are irascible; I apologize for the typos.
git clone https : // github.com / 43 d168f3e / Eternal - DEFLATION - Inflation.git
"git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git"

NOTES

HYPOTHESIS : If ,
employing the Einstein (or Einstein - Lovelock) eqs,
superluminal inflation / deflation exists,
then at time x4 =
0 (**before** the particles of the standard model exist)
a pair of universes with MASSES ± M is created
(i.e., universes are created in pairs).
Dark matter / dark energy may be
related to wave function overlap of the two universes

THE FOLLOWING SCRATCH IS CONSISTENT WITH
UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES ± M.

Note that we are ONLY looking for
superluminal inflation or deflation type solutions,
and NOT solutions that are even / odd functions of t
(like Cos[ν[j]'[0] * t], Sin[ν[j]'[0] * t],
Sech[ν[j]'[0] * t], Tanh[ν[j]'[0] * t], ...),
which also have ± M type eigenvalues,
or even solutions involving the well -
known special functions that might also have ± M type eigenvalues.

Bigger Bang:
Question: Are Universe (s) of masses ± M created in pairs at time x4 = 0
(before the particles of the standard model exist) ?
Are dark matter/dark energy related to wave function overlap of these two universes?

Ever wonder why there are 3 space dimensions and 1 time dimension?
Ever wonder if Nature loves the norm relation ||XY|| = ||X|| * ||Y||, for X, Y ∈ Nature?

Well, Hurwitz's theorem says that there are only 4 normed division algebras (||XY||=||X|| ||Y||) over the real numbers, which, up to isomorphism, are the 1-dim (over ℝ) real numbers ℝ, the 2-dim complex numbers,the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions. [This notebook's use of the split octonions is unrelated to the beautiful 1973 work of Murat Günaydin and his advisor Feza Gürsey, who used a split basis for octonions to study quark structure, linking them to symmetries like G_2. (Günaydin, M., & Gürsey, F. (1973). Quark structure and octonions. Journal of Mathematical Physics, 14(11), 1651–1667. DOI: 10.1063/1.1666240).]

The origin of this calculation has its roots in "A Remarkable Representation of the 3+2 de Sitter Group" by P.A.M.Dirac, J.Math.Phys.4,901–909 (1963).

Comment: The term "reduced Brauer–Weyl generators", below, refers to a specific set of irreducible generators for certain complex Clifford algebras, particularly in the context of mathematical physics and the study of Dirac spinors. These generators were introduced by E.A. Lord (E. A. Lord. "The Dirac spinor in six dimensions". Mathematical Proceedings of the Cambridge Philosophical Society, vol. 64, pp. 765–778, 1968) and are described as "reduced" because they form an irreducible representation of the Clifford algebra C_{2n} from $C_{2(n-1)}$.

Here, we employ Lord's idea of reduced Brauer–Weyl generators to extend Dirac's 4×4 gamma matrices into this notebook's real tau8 8×8 and tau16 16×16 matrices,
which are employed to write down the 'Dirac Equation for the Universe',
[bad name, I know; this has NOTHING to do with electrons/positrons. Remember, this is before the particles of the standard model exist; --just trying to get the reader comfortable with the formalism.]
These 'Euler-Lagrange equations for the Universe' are formulated in terms of a $\overline{O(4,4)}$ spinor Ψ16
(Ψ16 AKA 'WAVE FUNCTION OF the UNIVERSE,' which could possibly be named something more pompous).
Then this equation is used to [syncope, presyncope NEXT]
couple the WAVE FUNCTION OF the UNIVERSE to Gravity.

THIS WORK IS PARTIALLY BASED ON :

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 4, NUMBER 7 JUI

A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. DIRAC
Cambridge University, Cambridge, England and
*Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

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765

The Dirac spinor in six dimensions

BY E. A. LORD
Department of Mathematics, King's College, University of London

(*Received* 17 *November* 1966)

Identities satisfied by the generators of the Dirac algebra

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(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin-$\frac{1}{2}$ wave equation

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(Received 24 January 1984; accepted for publication 16 November 1984)

Here we use spacetime coordinates that are selected so that we are somewhat consistent with the xact libraries,
(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):
Cartesian coordinates:
x0 = hidden space (a small circle); x1, x2, x3 are the usual 3-space coords;
x4 = time coord,
and x5, x6, x7= superluminal deflating time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{SO(4, 4; \mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M8 , whose tangent bundle has
$\overline{SO(4, 4; \mathbb{R})}$ ≈ Spin(4,4;ℝ)
as iso group, and in which
3 of the 4 space dimensions superluminally INFLATE,
3 of the 4 time dimensions superluminally DEFLATE (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),
and the 4th space dim, x0, curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

In passing we remark that an analogous construction may be defined on an octonion space of one time dimension and seven space dimensions, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group
$SU(3) \otimes SU(2)_L \otimes U(1)_Y$;
The photon should remain massless after the Higgs field acquires a vacuum expectation value

(spontaneous symmetry breaking).

The Einstein - Lovelock vacuum field equations are
$A_a^b = 0$, where

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g}\sum_{k=1}^{m-1}\alpha_{(k)}g^{jl}\delta^{hh_1\cdots h_{2k}}_{jj_1\cdots j_{2k}}R^{j_1j_2}{}_{h_1h_2}\cdots R^{j_{2k-1}j_{2k}}{}_{h_{2k-1}h_{2k}} + \lambda\sqrt{g}\,g^{lh}, \quad (4.38)$$

where $\alpha_{(k)}$, λ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

and $m - 1 = \frac{8}{2} - 1 = 3$.
Citation : Tensors, Differential Forms, and Variational Principles (Dover Books on Mathematics), by David Lovelock and Hanno Rund

Let {w1, w2, w3, Λ} be pure numbers;
then by Eq. [4.38] the vacuum equations are
$0 == -\Lambda + H^{-2}$ w1 Lovelock1 + H^{-4} w2 Lovelock2 + H^{-6} w3 Lovelock3
in a hopefully obvious notation.
Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein - Lovelock tensors have different dimensions.
For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre - Hubble parameter. For now, let's pretend that they are independent.

NOTES :

Under the spacetime coordinate transformation

$x^k \mapsto \overline{x}^k\left(x^j\right)$

according to

$\Psi16\,(x) \mapsto \overline{\Psi16\,(\overline{x})} = S\,(x\,(\overline{x}))\,.\Psi16\,(x\,(\overline{x}))$,

i.e., $\overline{\Psi16^{(a)}\,(\overline{x})} = \left(S^{(a)}{}_{(b)}\;\Psi16^{(b)}\,(x\,(\overline{x}))\right)$

Therefore,

$$\frac{\partial}{\partial\overline{x}^k}\overline{\Psi16\,(\overline{x})} = \frac{\partial x^j}{\partial\overline{x}^k}\frac{\partial}{\partial x^j}S\,(x)\,.\Psi16\,(x) =$$

$$\frac{\partial x^j}{\partial\overline{x}^k}\left\{S\,(x)\,\boldsymbol{.}\,\frac{\partial}{\partial x^j}\Psi16\,(x) + \left[\frac{\partial}{\partial x^j}S\,(x)\right]\boldsymbol{.}\,\Psi16\,(x)\right\}$$

Identify the Octad connection : $\frac{\partial}{\partial x^j}S\,(x) =$

$$-S\,\boldsymbol{.}\,\Gamma_j\,(x) + \frac{\partial\overline{x}^k}{\partial x^j}\overline{\Gamma_k}\,(\overline{x})\,.\,S$$

hence

$$\frac{\partial}{\partial\overline{x}^k}\overline{\Psi16\,(\overline{x})} - \overline{\Gamma_k}\;\boldsymbol{.}\;\overline{\Psi16\,(\overline{x})} =$$

$$S\,(x\,(\overline{x}))\,.\,\frac{\partial x^j}{\partial\overline{x}^k}\left\{\frac{\partial}{\partial x^j}\Psi16\,(x\,(\overline{x})) - \Gamma_j\,.\Psi16\,(x\,(\overline{x}))\right\}$$

or

$$\left(1_{16\times16}\,\frac{\partial}{\partial\overline{x}^k} - \overline{\Gamma_k}\right)\boldsymbol{.}\;\overline{\Psi16\,(\overline{x})} =$$

$$S\,(x\,(\overline{x}))\,.\left[\frac{\partial x^j}{\partial\overline{x}^k}\left\{1_{16\times16}\,\frac{\partial}{\partial x^j} - \Gamma_j\right\}.\Psi16\,(x\,(\overline{x}))\right]$$

The connection transforms according to

$$\overline{\Gamma_k}\,(\overline{x}) = \frac{\partial\overline{x}^j}{\partial x^k}\left\{S\;\boldsymbol{.}\;\Gamma_j\,(x)\,.\,S^{-1} + S_{,j}\,.\,S^{-1}\right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2}\left[e_k^{(a)}\,\nabla_j\,e^k{}_{(b)}\right]\,SAB^{(b)}{}_{(a)}$$

Here, spin connection coefficients are $e_k^{(a)}\,\left(\nabla_j\,e^k{}_{(b)}\right)$

and

$$S = \exp\left(\frac{1}{2}\,\omega_{(A)\ (B)}\ (x)\ \mathrm{SAB}^{(A)\ (B)}\right)\ ,$$

where $\omega_{(B)\ (A)} = -\omega_{(A)\ (B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the Ψ16 field [A.K.A., wave function characterizing the Universe(s) of masses ±M];

TODO: prove **Universe(s) of masses ±M are created in pairs!**

K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility**: The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
  - $\nabla_\mu e^a{}_\nu = 0$
  - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

apologies, I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu}e^I_{\nu]} = \partial_{[\mu}e^I_{\nu]} + \omega^I_{[\mu J}e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu}e^I_\rho + \omega^I_{\mu J}\, e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistancy" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (neverthless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will neverthless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

? SOURCE terms

$TU^{\mu\nu}$ for $g_{\alpha\beta}$ come from "Universes' Wave Function Ψ16 Lagrangian"

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{ LagrangianΨ16} \right)$$

(hope that $TU^{\mu\nu} = \Lambda g^{\mu\nu}$, and

H = some function of M, where

Universe (s) of masses ± M created in pairs at time x4 = 0,

before the particles of the standard model exist) ;

WARNING:

Universes Ψ16 source $g_{\alpha\beta}$;

The Euler–Lagrange equations for Ψ16 must have ‘solutions’ such that

all off–diagonal terms of $TU^{\mu\nu}$ ARE ZERO .

The Ψ16 Lagrangian (please see below) =

$\sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg}[] =$

$$\sqrt{\text{Det}[g_{\mu\nu}]} * \left(\text{Transpose}[\Psi16].\sigma16.\text{Sum}\left[T16^{\alpha}[\alpha1-1].\left\{1_{16\times16} \frac{\partial}{\partial x^{\alpha1-1}} - \Gamma_{\alpha1-1}\right\}.\Psi16, \{\alpha1, 1, \text{Length}[X]\}\right] + \frac{\text{mASs}}{2} * \text{Transpose}[\Psi16].\sigma16.\Psi16 \right)$$

Next,

assume that Ψ16soln solves the Euler – Lagrange equations.

SOURCE terms

$TU^{\mu\nu}$ [evaluate terms after performing differentiation] (using LagrangianΨ16 ≡

$$\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\ \mathrm{Lg}[\,]\right)\Big|_{\Psi16=\text{solution-toEL-eqs}}:$$

$$\frac{1}{\kappa}\,\mathrm{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial g_{\mu\nu}}\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\ \mathrm{Lg}[\,]\right)$$

$$= \left(\mathrm{Lg}[\,]*\left[\frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial g_{\mu\nu}}\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\right)\right]+\frac{\partial}{\partial g_{\mu\nu}}\left(\mathrm{Lg}[\,]\right)\right)\Bigg|_{\Psi16=\Psi16\mathrm{soln}}$$

$$= \left(0?*\frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial g_{\mu\nu}}\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\right)+\frac{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial g_{\mu\nu}}\left(\mathrm{Lg}[\,]\right)\right)\Bigg|_{\Psi16=\Psi16\mathrm{soln}}$$

$$= \frac{\partial}{\partial g_{\mu\nu}}\Bigg(\mathrm{Transpose}[\Psi16].\sigma16.\ \mathrm{Sum}\Big[\Big(\mathrm{D}[\Psi16,\ X[\![\alpha1]\!]]+\Big(\Big(\frac{1}{2}\Big)\mathrm{connectionMatrix}\Big).\mathrm{Sum}[\omega[\![\alpha1,\ a,\ b]\!]*\mathrm{SAB}[\![a,\ b]\!],\ \{a,1,8\},\{b,1,8\}].\Psi16\Big),\ \{\alpha1,\ 1,\ \mathrm{Length}[X]\}\Big]+ \mathrm{mASs}*\mathrm{Transpose}[\Psi16].\mathrm{symm16}[\![j,\ 1]\!].\Psi16\Bigg)\Bigg|_{\Psi16=\Psi16\mathrm{soln}}$$

$$= \mathrm{Transpose}[\Psi16].\sigma16.\mathrm{Sum}\Big[\frac{\partial}{\partial g_{\mu\nu}}\left(\mathrm{T16}^{\alpha}[\alpha1-1]\right)\ .\ \Psi16_{\alpha1-1},\ \{\alpha1,\ 1,\ \mathrm{Length}[X]\}\Big]\Big|_{\Psi16=\text{solution-toEL-eqs}}$$

$$= \widetilde{\Psi16}\,.\sigma16.\ \mathrm{T16}^{A}\ .\ \Psi16_{,\alpha}\ \frac{\partial}{\partial g_{\mu\nu}}\left(g^{-1\,\alpha\beta}\ e^{B}_{\beta}\ \eta_{BA}\right) = \widetilde{\Psi16}\,.\sigma16.\ \mathrm{T16}^{A}\ .\ \Psi16_{,\alpha}\ \frac{\partial}{\partial g_{\mu\nu}}\left(e^{\alpha}{}_{(A)}\right)$$

NOTES :

Let g be a square matrix; we could use : $\frac{\partial}{\partial q}\left(g^{-1}\right) = -\,g^{-1}.\frac{\partial g}{\partial q}.\,g^{-1}$,

where q is a parameter $\left(\text{result from } \frac{\partial}{\partial q}\left(g.g^{-1}\right) = 0\right)$,

In this notebook,
g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] ,
g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}}\left(g^{-1}\right) = - g^{-1}.\frac{\partial g}{\partial g_{\mu\nu}}.g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In metric matrix g, we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate, for example.

This should be understood before reading further.

$$\text{Therefore } \frac{\partial}{\partial g_{\mu\nu}}\left(e^{\alpha}_{(A)}\right) = \frac{\partial}{\partial g_{\mu\nu}}\left(\text{Inverse}\left[e^{(A)}_{\alpha}\right]\right) =$$

$$- \text{Inverse}\left[e^{(A)}_{\alpha}\right].\frac{\partial e^{(A)}_{\alpha}}{\partial g_{\mu\nu}}.\text{Inverse}\left[e^{(A)}_{\alpha}\right] = - e^{\alpha}_{(A)}[\![\alpha 1, A1]\!] \frac{\partial e^{(A)}_{\alpha}[\![A1, \alpha 2]\!]}{\partial g_{\mu\nu}}.e^{\alpha}_{(A)}[\![\alpha 2, A2]\!]$$

$$\text{Consider } \frac{\partial}{\partial g_{\mu\nu}}(g_{\alpha\beta}) = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e^{(A)}_{\alpha}\, e^{B}_{\beta}\right) = \eta_{AB}\, e^{B}_{\beta} \frac{\partial}{\partial g_{\mu\nu}}\left(e^{(A)}_{\alpha}\right) + \eta_{AB}\, e^{B}_{\alpha} \frac{\partial}{\partial g_{\mu\nu}}\left(e^{A}_{\beta}\right);$$

$$e^{\beta}_{C} \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e^{(A)}_{\alpha}\, e^{B}_{\beta}\right) = \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}}\left(e^{(A)}_{\alpha}\right) + \eta_{AB}\, e^{B}_{\alpha}\, e^{\beta}_{C} \frac{\partial}{\partial g_{\mu\nu}}\left(e^{A}_{\beta}\right)$$

In metric matrix g, we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate.

Some of my calculations employ “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see
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http://www2.iap.fr/users/pitrou/
“FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb”
MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify

I copied and pasted too often; however, I am going to leave this here.

Universe sources for $g_{\alpha\beta}$;
be sure to append these to the Einstein and/or Einstein–Lovelock field equations.
WARNING: all off–diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous 'hacked solutions' for Ψ16 to be valid

Let T^{α} = {$T16^{\alpha}$[0], $T16^{\alpha}$[4]} and

$\sqrt{Det[g_{\mu\nu}]}$ * Lg[][mASs_, j_] =
$\sqrt{Det[g_{\mu\nu}]}$ * (Transpose[Ψ16].σ16.Sum[
T^{α}[α1 - 1].D[Ψ16, vars⟦α1⟧], {α1, 1, Length[vars]}] +
mASs * Transpose[Ψ16].symm16⟦134, 1⟧.Ψ16)

Let j = 134, σ16.(mass Matrix) = $T16^{\alpha}$[5].$T16^{\alpha}$[6].$T16^{\alpha}$[7]
and
usingLagrangianF16massive = (Transpose[Ψ16].σ16.
Sum[T^{α}[α1 - 1].D[Ψ16, vars⟦α1⟧], {α1, 1, Length[vars]}] +
mASs * Transpose[Ψ16].symm16⟦j = 134, 1⟧.Ψ16);
where it is assumed that Ψ16 solves the Euler - Lagrange equations, above; then usingLagrangianF16massive = 0, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{Det[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{Det[g_{\alpha\beta}]}\, Lg[] \right)$$

$$= \left(Lg[] * \left[\frac{1}{\sqrt{Det[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{Det[g_{\alpha\beta}]} \right) \right] + \frac{\partial}{\partial g_{\mu\nu}} (Lg[]) \right) \Big|_{Lg[] = usingLagrangianF16massive}$$

$$= \left(0 * \frac{1}{\sqrt{Det[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{Det[g_{\alpha\beta}]} \right) + \right.$$

$\sqrt{Det[g_{\alpha\beta}]}$ ∂ (Lg[]) | ...

remark: the term

$$\tilde{\Psi}16 .\sigma16. T16^{A} . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (e^{\alpha}_{A}) = \frac{\partial}{\partial g_{\mu\nu}} \tilde{\Psi}16 .\sigma16. T16^{A} . \Psi16_{,\alpha} e^{\alpha}_{A} =$$

$$\frac{\partial}{\partial g_{\mu\nu}} \Bigg(\text{Transpose}[\Psi16].\sigma16.$$

$$\text{Sum}\Big[T^{\alpha}\overbrace{[\alpha1 - 1].\text{D}[\Psi16, \text{vars}[\![\alpha1]\!]]}^{T16^{A}[A1-1].\Psi16_{,\alpha1}\ \eta_{A1\,B}\ e^{B}_{\beta}\ g^{-1\,\alpha1\,\beta}}, \{\alpha1, 1, \text{Length}[\text{vars}]\}\Big]$$

Incomplete theories	[show]
Scientists	[show]
	V • T • E

Frame fields [edit]

We use a set of vierbein or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab} e^a_\mu e^b_\nu = \eta_{\mu\nu}.$$

The vierbein defines a local rest frame, allowing the constant Gamma matrices to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a section of the frame bundle, and so defines a local trivialization of the frame bundle.

Spin connection [edit]

To write down the equation we also need the spin connection, also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e^\mu_a e^a_\nu = \delta^\mu{}_\nu.$$

The connection 1-form is then

$$\omega^\mu{}_{\nu a} := e^\mu_b \nabla_a e^b_\nu$$

where ∇_a is a covariant derivative, or equivalently a choice of connection on the frame bundle, most often taken to be the Levi-Civita connection.

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space T_pM to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

Raising and lowering indices is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract connection on a principal bundle, specifically on the frame bundle, which is defined on any smooth manifold, but which restricts to an *orthonormal* frame bundle on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu{}_{\nu\rho} = e_\rho^\alpha \omega^\mu{}_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2}(\omega_{\nu\rho})_\mu \sigma^{\nu\rho}\psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu R(M^{\nu\rho})v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $\mathrm{SO}(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

OTHER WORK THAT I USED :

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A Remarkable Representation of the 3 + 2 de Sitter Grou

P. A. M. DIRAC
Cambridge University, Cambridge, England and
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