Scratch work of Author

NOTES

HYPOTHESIS : If , employing the Einstein eqs,
superluminal inflation / deflation exists,
then at time x4 =
0 (**before** the particles of the standard model exist)
a pair of universes with MASSES ± M is created
(i.e., universes are created in pairs)

THE FOLLOWING SCRATCH IS CONSISTENT WITH
UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES ± M.

Note that we are ONLY looking for
superluminal inflation or deflation type solutions,
and NOT solutions that are even / odd functions of t ×
(like Cos[ν[j]'[0] * t], Sin[ν[j]'[0] * t],
Sech[ν[j]'[0] * t], Tanh[ν[j]'[0] * t], ...),
which also have ± M type eigenvalues,
or even solutions involving the well -
known special functions that might also have ± M type eigenvalues.

Bigger Bang:
Question: Are Universe (s) of masses ± M created in pairs at time x4 = 0 (before the particles of the standard model exist) ?

Scratch work

Patrick L. Nash, Ph.D.

Professor, UTSA Physics and Astronomy, Retired (UTSA)
Patrick299Nash at gmail ...
Do not read if you are irascible; I apologize for typos.
git clone https://github.com/43d168f3e/Eternal-DEFLATION-

Inflation.git
“git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git”

Ever wonder why there are 3 space dimensions and 1 time dimension?
Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in$ Nature?

Well, Hurwitz’s theorem says that there are only 4 normed division algebras ($\|XY\|=\|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over $\mathbb{R}$) real numbers $\mathbb{R}$,the 2-dim complex numbers,the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let’s go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in “A Remarkable Representation of the 3+2 de Sitter Group” by P.A.M.Dirac, J.Math.-Phys.4,901–909 (1963).

Comment: The term “reduced Brauer–Weyl generators”, below, refers to a specific set of irreducible generators for certain complex Clifford algebras, particularly in the context of mathematical physics and the study of Dirac spinors. These generators were introduced by **E.A. Lord** (E. A. Lord. “The Dirac spinor in six dimensions”. Mathematical Proceedings of the Cambridge Philosophical Society, vol. 64, pp. 765–778, 1968) and are described as “reduced” because they form an irreducible representation of the Clifford algebra C_{2n} from $C_{2(n-1)}$.

Here, we employ Lord's idea of reduced Brauer–Weyl generators to extend Dirac's 4×4 gamma matrices into this notebook's real tau8 8×8 and tau16 16×16 matrices,
which are employed to write down the 'Dirac Equation for the Universe',
[remember, this is before the particles of the standard model exist]
formulated in terms of a $\overline{O(4,4)}$ spinor Ψ16
(Ψ16 AKA 'WAVE FUNCTION OF the UNIVERSE,' which could possibly be named something more pompous).
Then this equation is used to
[WARNING: syncope, presyncope AHEAD]
couple the WAVE FUNCTION OF the UNIVERSE to Gravity.

THIS WORK IS PARTIALLY BASED ON :

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. Dirac
Cambridge University, Cambridge, England and
*Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

Proc. Camb. Phil. Soc. (1968), **64**, 765 765
PCPS 64–96
Printed in Great Britain

The Dirac spinor in six dimensions

By E. A. Lord
Department of Mathematics, King's College, University of London

(*Received* 17 *November* 1966)

Identities satisfied by the generators of the Dirac algebra

Patrick L. Nash
M. S. 474, NASA Langley Research Center, Hampton, Virginia 23665
(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin-$\frac{1}{2}$ wave equation

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(Received 24 January 1984; accepted for publication 16 November 1984)

Here we use spacetime coordinates that are selected so that we are somewhat consistent with the
xact libraries,
(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):
Cartesian coordinates:
x0 = hidden space (a small circle); x1, x2, x3 are the usual 3-space coords;
x4 = time coord,
and x5, x6, x7= superluminal deflating time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{SO(4,4;\mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M8 , whose tangent bundle has
$\overline{SO(4,4;\mathbb{R})} \approx \mathrm{Spin}(4,4;\mathbb{R})$
as iso group, and in which
3 of the 4 space dimensions superluminally INFLATE,
3 of the 4 time dimensions superluminally DEFLATE (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),
and the 4th space dim, x0, curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

In passing we remark that an analogous construction may be defined on an octonion space of one time dimension and seven space dimensions, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group
$\mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes U(1)_Y$;
The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

Please cite this work and this web page if you use it.

The Einstein - Lovelock vacuum field equations are
$A_a^b = 0$, where

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has bee completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g}\sum_{k=1}^{m-1}\alpha_{(k)}g^{jl}\delta^{hh_1\cdots h_{2k}}_{jj_1\cdots j_{2k}}R^{j_1j_2}{}_{h_1h_2}\cdots R^{j_{2k-1}j_{2k}}{}_{h_{2k-1}h_{2k}} + \lambda\sqrt{g}\,g^{lh}, \quad (4.3$$

where $\alpha_{(k)}$, λ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

and $m - 1 = \frac{8}{2} - 1 = 3$.

Citation: Tensors, Differential Forms, and Variational Principles (Dover Books on Mathematics), by David Lovelock and Hanno Rund

Let {w1, w2, w3, Λ} be pure numbers;
then by Eq. [4.38] the vacuum equations are
$0 == -\Lambda + H^{-2}\ \mathrm{w1\ Lovelock1} + H^{-4}\ \mathrm{w2\ Lovelock2} + H^{-6}\ \mathrm{w3\ Lovelock3}$
in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein - Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre - Hubble parameter. For now, let's pretend that they are independent.

NOTES :

Under the spacetime coordinate transformation

$x^k \mapsto \overline{x}^k\left(x^j\right)$

according to

$$\Psi 16\ (x)\ \mapsto \overline{\Psi 16\,(\overline{x})} =$$

$$S\,(\,x\,(\overline{x})\,)\,.\Psi 16\,(\,x\,(\overline{x})\,)\,,\ \text{i.e.,}$$

$$\overline{\Psi 16^{(a)}}\,(\overline{x}) = \left(\,S^{(a)}_{\ (b)}\,\Psi 16^{(b)}\,(\,x\,(\overline{x})\,)\,\right)$$

Therefore,

$$\frac{\partial}{\partial \overline{x}^k}\,\overline{\Psi 16\,(\overline{x})} =$$

$$\frac{\partial x^j}{\partial \overline{x}^k}\,\frac{\partial}{\partial x^j}\,S\,(x)\,.\Psi 16\,(x) = \frac{\partial x^j}{\partial \overline{x}^k}\,\left\{\,S\,(x)\,.\,\frac{\partial}{\partial x^j}\,\Psi 16\,(x) + \left[\frac{\partial}{\partial x^j}\,S\,(x)\right].\Psi 16\,(x)\,\right\}$$

Identify the Octad connection : $\frac{\partial}{\partial x^j}\,S\,(x) =$

$$-\,S\,.\,\Gamma_j\,(\,x\,) + \frac{\partial \overline{x}^k}{\partial x^j}\,\overline{\Gamma_k}\,(\overline{x})\,.\,S$$

hence

$$\frac{\partial}{\partial \overline{x}^k}\,\overline{\Psi 16\,(\overline{x})} - \overline{\Gamma_k}\,.\,\overline{\Psi 16\,(\overline{x})} = S\,(\,x\,(\overline{x})\,)\,.\,\frac{\partial x^j}{\partial \overline{x}^k}\,\left\{\,\frac{\partial}{\partial x^j}\,\Psi 16\,(x\,(\overline{x})\,) - \Gamma_j\,.\Psi 16\,(x\,(\overline{x})\,)\,\right\}$$

or

$$\left(1_{16\times 16}\,\frac{\partial}{\partial \overline{x}^k} - \overline{\Gamma_k}\right).\,\overline{\Psi 16\,(\overline{x})} =$$

$$S\,(x\,(\overline{x}))\,.\left[\frac{\partial x^j}{\partial \overline{x}^k}\left\{1_{16\times16}\frac{\partial}{\partial x^j}-\Gamma_j\right\}.\Psi16\,(x\,(\overline{x}))\right]$$

The connection transforms according to

$$\overline{\Gamma_k}\,(\overline{x}) = \frac{\partial \overline{x}^j}{\partial x^k}\left\{S\,.\,\Gamma_j\,(x)\,.\,S^{-1} + S_{,j}\,.\,S^{-1}\right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2}\left[e_k^{(a)}\,\nabla_j\,e^k_{(b)}\right]\,SAB^{(b)}{}_{(a)}$$

Here, spin connection coefficients are $e_k^{(a)}\,(\nabla_j\,e^k_{(b)})$

and

$$\mathbf{S = exp}\left(\frac{1}{2}\,\omega_{(A)\,(B)}\,(x)\;SAB^{(A)\,(B)}\right),$$

where $\omega_{(B)\,(A)} = -\omega_{(A)\,(B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the Ψ16 field [A.K.A., wave function characterizing the Universe(s) of masses ±M];

TODO: prove **Universe(s) of masses ±M are created in pairs!**

K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility**: The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
  - $\nabla_\mu e^a{}_\nu = 0$
  - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

apologies, I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J}\, e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistancy" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (neverthless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will neverthless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called ‘Tetrad Postulate’ and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

? SOURCE terms $TU^{\mu\nu}$ for $g_{\alpha\beta}$ come from "Universes' Wave Function Ψ16 Lagrangian"

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv$$

$$\frac{1}{\sqrt{Det[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{Det[g_{\alpha\beta}]} \text{ Lagrangian}\Psi16 \right)$$

(hope that $TU^{\mu\nu} = \Lambda g^{\mu\nu}$, and

H = some function of M, where

Universe (s) of masses ±

M created in pairs at time x4 = 0,

before the particles

of the standard model exist);

WARNING:

Universes Ψ16 source $g_{\alpha\beta}$;

The Euler–Lagrange equations for Ψ16 must have ‘solutions’ such that

all off–diagonal terms of $TU^{\mu\nu}$ ARE ZERO .

The Ψ16 Lagrangian (please see below) =

$\sqrt{Det[g_{\mu\nu}]}$ * Lg[] =

$$\sqrt{Det[g_{\mu\nu}]} * \Big(\text{Transpose}[\Psi16].\sigma16.$$

$$\text{Sum}\Big[T16^{\alpha}[\alpha1-1].\Big\{ 1_{16\times16} \frac{\partial}{\partial x^{\alpha1-1}} - \Gamma_{\alpha1-1} \Big\}.$$

$$\Psi16, \{\alpha1, 1, \text{Length}[X]\} \Big] +$$

$$\frac{\text{mASs}}{2} * \text{Transpose}[\Psi 16].\sigma 16.\Psi 16 \Big)$$

Next,

assume that Ψ16soln solves the Euler - Lagrange equations.

SOURCE terms

$TU^{\mu\nu}$ [evaluate terms after performing differentiation] (using LagrangianΨ16 ≡ $\left(\sqrt{\text{Det}[g_{\alpha\beta}]}\ \text{Lg}[\,] \right) \big|_{\Psi 16=\text{solution-toEL-eqs}}$) :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial\, g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]}\ \text{Lg}[\,] \right)$$

$$= \left(\text{Lg}[\,] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial\, g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \frac{\partial}{\partial\, g_{\mu\nu}} (\text{Lg}[\,]) \right) \Bigg|_{\Psi 16=\Psi 16\text{soln}}$$

$$= \left(0\,? * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial\, g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial\, g_{\mu\nu}} (\text{Lg}[\,]) \right) \Bigg|_{\Psi 16=\Psi 16\text{soln}}$$

$$= \frac{\partial}{\partial g_{\mu\nu}} \Big($$

Transpose[Ψ16].
σ16.Sum[(D[Ψ16, X⟦α1⟧] +
((1/2) connectionMatrix).
Sum[ω⟦α1, a, b⟧ * SAB⟦a, b⟧,
{a, 1, 8}, {b, 1, 8}].Ψ16),
{α1, 1, Length[X]}] +
mASs * Transpose[Ψ16].symm16⟦j, 1⟧.
Ψ16) |Ψ16=Ψ16soln

= Transpose[Ψ16].σ16.
Sum[$\frac{\partial}{\partial g_{\mu\nu}}$ (T16^α[α1 - 1]) . Ψ16_{α1-1} ,
{α1, 1, Length[X]}] |Ψ16=solution-toEL-eqs

$$= \widetilde{\Psi 16}\,.\sigma 16.\,T16^{A}\,.\,\Psi 16_{,\alpha}$$

$$\frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1\,\alpha\beta}\; e^{B}_{\beta}\, \eta_{BA} \right) =$$

$$\widetilde{\Psi 16}\,.\sigma 16.\,T16^{A}\,.\,\Psi 16_{,\alpha}\;\; \frac{\partial}{\partial g_{\mu\nu}} \left(e^{\alpha}_{(A)} \right)$$

NOTES :

Let g be a square matrix;

we could use : $\frac{\partial}{\partial q}\left(g^{-1} \right) = - g^{-1} . \frac{\partial g}{\partial q} . g^{-1}$,

where q is a parameter

$\left(\text{result from } \frac{\partial}{\partial q}\left(g.g^{-1} \right) = 0\right)$,

In this notebook, g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] , g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}}\left(g^{-1} \right) = - g^{-1} . \frac{\partial g}{\partial g_{\mu\nu}} . g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ;

note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In metric matrix g,

we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate, for example.

This should be understood before reading further.

Therefore $\frac{\partial}{\partial g_{\mu\nu}}\left(e^{\alpha}_{(A)}\right) =$

$\frac{\partial}{\partial g_{\mu\nu}}\left(\text{Inverse}\left[e_{\alpha}^{(A)}\right]\right) =$

$$-\,\mathrm{Inverse}\left[e_{\alpha}^{(A)}\right].\frac{\partial e_{\alpha}^{(A)}}{\partial g_{\mu\nu}}.\mathrm{Inverse}\left[e_{\alpha}^{(A)}\right]=$$

$$-\,e^{\alpha}_{(A)}[\![\alpha 1, A1]\!]\,\frac{\partial e_{\alpha}^{(A)}[\![A1,\alpha 2]\!]}{\partial g_{\mu\nu}}.e^{\alpha}_{(A)}[\![\alpha 2, A2]\!]$$

$$\text{Consider}\ \frac{\partial}{\partial g_{\mu\nu}}(g_{\alpha\beta}) = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e_{\alpha}^{(A)}\ e_{\beta}^{B}\right) =$$

$$\eta_{AB}\ e_{\beta}^{B}\,\frac{\partial}{\partial g_{\mu\nu}}\left(e_{\alpha}^{(A)}\right) + \eta_{AB}\, e_{\alpha}^{B}\,\frac{\partial}{\partial g_{\mu\nu}}\left(e_{\beta}^{A}\right);$$

$$e_{C}^{\beta}\,\frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e_{\alpha}^{(A)}\ e_{\beta}^{B}\right) =$$

$$\eta_{AC}\,\frac{\partial}{\partial g_{\mu\nu}}\left(e_{\alpha}^{(A)}\right) + \eta_{AB}\, e_{\alpha}^{B}\, e_{C}^{\beta}\,\frac{\partial}{\partial g_{\mu\nu}}\left(e_{\beta}^{A}\right)$$

In metric matrix g,

we must replace element $g_{\mu\nu}$ with $\dfrac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate.

Some of my calculations employ “http://www.xact.es/download-/xAct_1.2.0.tgz” ; also, see
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Institut d’Astrophysique de Paris (France)
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http://www2.iap.fr/users/pitrou/
"FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb"
MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify

I copied and pasted too often; however, I am going to leave this here.

Universe sources for $g_{\alpha\beta}$;
be sure to append these to the Einstein and/or Einstein–Lovelock field equations.
WARNING: all off–diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous 'hacked solutions' for Ψ16 to be valid

```
Let Tα = {T16α[0], T16α[4]} and
√Det[gμν] * Lg[][mASs_, j_] =
  √Det[gμν] * ( Transpose[Ψ16].σ16.Sum[
       Tα[α1 - 1].D[ Ψ16, vars⟦α1⟧], {α1, 1, Length[vars]}] +
     mASs * Transpose[Ψ16].symm16⟦134, 1⟧.Ψ16 )

Let j = 134, σ16.(mass Matrix) = T16α[5].T16α[6].T16α[7]
and
usingLagrangianF16massive = (Transpose[Ψ16].σ16.
      Sum[Tα[α1 - 1].D[ Ψ16, vars⟦α1⟧], {α1, 1, Length[vars]}] +
     mASs * Transpose[Ψ16].symm16⟦j = 134, 1⟧.Ψ16);
where it is assumed that Ψ16 solves the Euler - Lagrange equations,
above; then usingLagrangianF16massive = 0, as shown below.
```

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa}\,TU^{\mu\nu} \equiv \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial g_{\mu\nu}}\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\;\mathrm{Lg}[]\right)$$

$$= \left(\mathrm{Lg}[] * \left[\frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial g_{\mu\nu}}\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\right)\right] + \frac{\partial}{\partial g_{\mu\nu}}\left(\mathrm{Lg}[]\right)\right)\Bigg|_{\mathrm{Lg}[]=\mathrm{usingLagrangianF16massive}}$$

$$= \left(0 * \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial g_{\mu\nu}}\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\right) + \frac{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial g_{\mu\nu}}\left(\mathrm{Lg}[]\right)\right)\Bigg|_{\mathrm{Lg}[]=\mathrm{usingLagrangianF16massive}}$$

remark: the term

$$\tilde{\Psi}16 . \sigma 16 . T16^{A} . \Psi 16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (e_{A}^{\alpha}) = \frac{\partial}{\partial g_{\mu\nu}} \tilde{\Psi}16 . \sigma 16 . T16^{A} . \Psi 16_{,\alpha} e_{A}^{\alpha} =$$

$$\frac{\partial}{\partial g_{\mu\nu}} \Big(\texttt{Transpose}[\Psi 16].\sigma 16.$$

$$\texttt{Sum}\Big[T^{\alpha}\overbrace{[\alpha 1 - 1].D[\Psi 16, \texttt{vars}[[\alpha 1]]]}^{T16^{A}[A1-1].\Psi 16_{,\alpha 1}\, \eta_{A1\,B}\, e_{\beta}^{B}\, g^{-1\,\alpha 1\,\beta}}, \{\alpha 1, 1, \texttt{Length}[\texttt{vars}]\}\Big]$$

Incomplete theories	[show]
Scientists	[show]

V • T • E

Frame fields [edit]

We use a set of vierbein or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab} e^a_\mu e^b_\nu = \eta_{\mu\nu}.$$

The vierbein defines a local rest frame, allowing the constant Gamma matrices to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a section of the frame bundle, and so defines a local trivialization of the frame bundle.

Spin connection [edit]

To write down the equation we also need the spin connection, also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e^\mu_a e^a_\nu = \delta^\mu{}_\nu.$$

The connection 1-form is then

$$\omega^\mu{}_{\nu a} := e^\mu_b \nabla_a e^b_\nu$$

where ∇_a is a covariant derivative, or equivalently a choice of connection on the frame bundle, most often taken to be the Levi-Civita connection.

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space T_pM to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

Raising and lowering indices is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract connection on a principal bundle, specifically on the frame bundle, which is defined on any smooth manifold, but which restricts to an *orthonormal* frame bundle on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu{}_{\nu\rho} = e_\rho^\alpha \omega^\mu{}_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2}(\omega_{\nu\rho})_\mu \sigma^{\nu\rho} \psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu R(M^{\nu\rho}) v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $\mathrm{SO}(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

OTHER WORK THAT I USED :

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Grou

P. A. M. DIRAC
Cambridge University, Cambridge, England and
*Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

Proc. Camb. Phil. Soc. (1968), **64**, 765 765
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The Dirac spinor in six dimensions

BY E. A. LORD
Department of Mathematics, King's College, University of London

(*Received* 17 *November* 1966)

Identities satisfied by the generators of the Dirac algebra

Patrick L. Nash
M. S. 474, NASA Langley Research Center, Hampton, Virginia 23665

(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin-$\frac{1}{2}$ wave equation

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(Received 24 January 1984; accepted for publication 16 November 1984)

Begin

In[∘]:=
```
Print["CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]
```

CopyRight (C) 2022, Patrick L. Nash, under the General Public License.

In[∘]:=
```
Print["Please  cite  this  work,  and this web page, if you use it"]
```

Please cite this work, and this web page, if you use it

In[∘]:=
```
Print["For simplicity, metric  gαβ=  gαβ(x0,x1,x2,x3,x4,x5,x6,x7)= gαβ(x0,x4) = diag{
   g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4) },"]
```

For simplicity, metric $g_{\alpha\beta}$= $g_{\alpha\beta}$(x0,x1,x2,x3,x4,x5,x6,x7)= $g_{\alpha\beta}$(x0,x4) = diag{
g_{00}(x0),g_{11}(x0,x4),g_{11}(x0,x4),g_{11}(x0,x4),-1, g_{77}(x0,x4),g_{77}(x0,x4),g_{77}(x0,x4) },

In[∘]:=
```
Print["where g77(x0,x4) = g11(x0,-x4) "]
```

where g_{77}(x0,x4) = g_{11}(x0,-x4)

In[∘]:=
```
Unprotect[dir];
```

In[∘]:=
```
dir = FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]];
Protect[dir];
If[$Failed == SetDirectory[dir], {CreateDirectory[dir], SetDirectory[dir]}];
```

In[∘]:=
```
names = FileNameSplit[NotebookFileName[]]
```

Out[∘]=

{C:, Users, nsh, Documents, 8-dim, 2025-12-13-Partitioned-Determinant,
Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT.nb}

In[∘]:= name = StringReplace[names[[-1]], "nb" → "mx"]

Out[∘]=

Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT.mx

In[∘]:= header = StringReplace[names[[-1]], ".nb" → "-"]

Out[∘]=

Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT-

In[∘]:= DIM8 = 8;

In[∘]:= Protect[DIM8, M, K, H]

Out[∘]=

{DIM8, M, K, H}

Coordinates :

$$\Big\{\underbrace{\text{hidden space}}, \ \underbrace{3-\text{space}}, \ \underbrace{\text{time}} \ \underbrace{\text{superluminal deflating time}}\Big\}$$

$$\{x0,\ \overbrace{x1,\ x2,\ x3},\ x4,\ \ \overbrace{x5,\ x6,\ x7}\}\ :$$

```
X = {x0, x1, x2, x3, x4, x5, x6, x7};
Protect[X];
Protect[x0, x1, x2, x3, x4, x5, x6, x7];
```

In[∘]:= sX0 = And @@ Thread[X > 0]

Out[∘]=

x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 && x7 > 0

In[∘]:= ssX = H > 0 && sX0 && 6 H x0 > 0 && 2 la[x4] > 0 && Cot[6 H x0] > 0 && Sin[6 H x0] > 0

Out[∘]=

H > 0 && x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 &&
x7 > 0 && 6 H x0[] > 0 && 2 la[x4[]] > 0 && Cot[6 H x0[]] > 0 && Sin[6 H x0[]] > 0

For Maple_2025 :

In[∘]:= sreplaceZ = Z[#] → ToExpression["Z" <> ToString[#]] & /@ Range[0, 15]

Out[∘]=

{Z[0] → Z0, Z[1] → Z1, Z[2] → Z2, Z[3] → Z3, Z[4] → Z4, Z[5] → Z5, Z[6] → Z6, Z[7] → Z7, Z[8] → Z8,
Z[9] → Z9, Z[10] → Z10, Z[11] → Z11, Z[12] → Z12, Z[13] → Z13, Z[14] → Z14, Z[15] → Z15}

In[∘]:= sreplacenZ = nZ[#] → ToExpression["nZ" <> ToString[#]] & /@ Range[0, 15]

Out[∘]=

{nZ[0] → nZ0, nZ[1] → nZ1, nZ[2] → nZ2, nZ[3] → nZ3, nZ[4] → nZ4,
nZ[5] → nZ5, nZ[6] → nZ6, nZ[7] → nZ7, nZ[8] → nZ8, nZ[9] → nZ9, nZ[10] → nZ10,
nZ[11] → nZ11, nZ[12] → nZ12, nZ[13] → nZ13, nZ[14] → nZ14, nZ[15] → nZ15}

hacks (a.k.a., lies):

In[ ]:= constraintTrig = Sin[6 * H * x0] > 0 && Cos[6 * H * x0] > 0 && Csc[6 * H * x0] > 0 && Sec[6 * H * x0] > 0 && Tan[6 * H * x0] > 0 && Cot[6 * H * x0] > 0 && Sin[z] > 0 && $\sqrt{\text{Sin}[z]}$ > 0 && $\text{Sin}[z]^{3/2}$ > 0 && $\text{Sin}[z]^{1/2}$ > 0 && $\text{Sin}[z]^{-3/2}$ > 0 && $\text{Sin}[z]^{-1/2}$ > 0 && Cot[z] > 0 && $\sqrt{\text{Cot}[z]}$ > 0 && $\text{Cot}[z]^{3/2}$ > 0 && $\text{Cot}[z]^{1/2}$ > 0 && Tan[z] > 0 && Sec[z] > 0 && $\sqrt{\text{Csc}[z]}$ > 0 && Csc[z] > 0

Out[ ]= Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 && Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && $\sqrt{\text{Sin}[z]}$ > 0 && $\text{Sin}[z]^{3/2}$ > 0 && $\sqrt{\text{Sin}[z]}$ > 0 && $\frac{1}{\text{Sin}[z]^{3/2}}$ > 0 && $\frac{1}{\sqrt{\text{Sin}[z]}}$ > 0 && Cot[z] > 0 && $\sqrt{\text{Cot}[z]}$ > 0 && $\text{Cot}[z]^{3/2}$ > 0 && $\sqrt{\text{Cot}[z]}$ > 0 && Tan[z] > 0 && Sec[z] > 0 && $\sqrt{\text{Csc}[z]}$ > 0 && Csc[z] > 0

more hacks (damn lies):

In[ ]:= (*&&K^2-M^2>0&& $\sqrt{K^2-M^2}$>0&&$e^{2\ H\ \sqrt{K^2-M^2}\ x4}$>0&&2 H $\sqrt{K^2-M^2}$ x4>0&& 2 H $\sqrt{1-M^2}$ x4>0&& $\sqrt{1-M^2}$>0&&$e^{2\ H\ \sqrt{1-M^2}\ x4}$>0&&2 H $\sqrt{1-M^2}$ x4>0*)

In[ ]:= constraintX = x0 > 0 && x4 > 0 && H > 0 && 6 * H * x0 > 0 && 3 * H * x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && $e^{-2\ a4[H\ x4]}$ > 0 && $e^{-a4[H\ x4]}$ > 0 && $e^{2\ a4[H\ x4]}$ > 0 && $e^{a4[H\ x4]}$ > 0

Out[ ]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && $e^{-2\ a4[H\ x4]}$ > 0 && $e^{-a4[H\ x4]}$ > 0 && $e^{2\ a4[H\ x4]}$ > 0 && $e^{a4[H\ x4]}$ > 0

statistics :

In[ ]:= constraintVars = constraintX && constraintTrig

Out[ ]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && $e^{-2\ a4[H\ x4]}$ > 0 && $e^{-a4[H\ x4]}$ > 0 && $e^{2\ a4[H\ x4]}$ > 0 && $e^{a4[H\ x4]}$ > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 && Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && $\sqrt{\text{Sin}[z]}$ > 0 && $\text{Sin}[z]^{3/2}$ > 0 && $\sqrt{\text{Sin}[z]}$ > 0 && $\frac{1}{\text{Sin}[z]^{3/2}}$ > 0 && $\frac{1}{\sqrt{\text{Sin}[z]}}$ > 0 && Cot[z] > 0 && $\sqrt{\text{Cot}[z]}$ > 0 && $\text{Cot}[z]^{3/2}$ > 0 && $\sqrt{\text{Cot}[z]}$ > 0 && Tan[z] > 0 && Sec[z] > 0 && $\sqrt{\text{Csc}[z]}$ > 0 && Csc[z] > 0

In[∘]:= subsDefects =

$$\left\{\sqrt{e^{2\,a4[H\,x4]}}\to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}}\to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}}\to\frac{1}{e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6}},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{1}{e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{1}{e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\right\}$$

Out[∘]=

$$\left\{\sqrt{e^{2\,a4[H\,x4]}}\to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}}\to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}}\to\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x4]^{1/6}},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{e^{a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\right\}$$

future coordinate transformation :

In[∘]:=
```
6 H x0 == z && H x4 == t
szt = Solve[%, {z, t}][[1]]
sx0x4 = Solve[%%, {x0, x4}][[1]]
Protect[sx0x4, szt];
```

Out[∘]=

6 H x0 == z && H x4 == t

Out[∘]=

{z → 6 H x0, t → H x4}

Out[∘]=

$$\left\{x0\to\frac{z}{6\,H},\ x4\to\frac{t}{H}\right\}$$

In[∘]:= sx0x4

Out[∘]=

$$\left\{x0\to\frac{z}{6\,H},\ x4\to\frac{t}{H}\right\}$$

In[∘]:= (*sa4={a4→((A4[3 H #2])&)}*)

In[∘]:= Protect[sx0x4, szt]

Out[∘]=

{}

$\overline{\text{SO (4, 4)}}$ Minkowski Lorentz metric

η4488 : 4 + 4 spacetime; 8 × 8 dimensional :

In[ ]:=
```
(η4488 = ArrayFlatten[{{IdentityMatrix[4], 0}, {0, -IdentityMatrix[4]}}]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$\overline{\text{SO}(4,4)}$ Spinor Lorentz metric σ;
τ^A are analogous to
the Dirac gamma matrices : all true : Table$\left[\left\{A, B, \text{FullSimplify}\left[\frac{1}{2}(\tau[A].\overline{\tau}[B] + \tau[B].\overline{\tau}[A]) == \eta4488[[A+1, B+1]] * \text{ID8}\right]\right\}, \{A, 0, 7\}, \{B, 0, 7\}\right]$

Type - 1 and type - 2 employ σ (the ususal notational abuse), since $\sigma == \sigma^{-1}$: 4 + 4 spacetime, 8 × 8 dimensional and 16 × 16 dimensional :

indices. We define the reduced Brauer-Weyl generators $\{\bar{\tau}^{A'}, \tau^{A'}\}$ of the generators of the two real 8×8 inequivalent irreducible representations of $\overline{SO(4,4)}$ (see eqs. (2.6) and (2.7)) by demanding that the tau matrices satisfy (the tilde denotes transpose)

$$\tilde{\tau}^{A'}\sigma = \sigma\bar{\tau}^{A'} \tag{2.3}$$

and

$$\tau^{A'}\bar{\tau}^{B'} + \tau^{B'}\bar{\tau}^{A'} = 2IG^{A'B'} = \bar{\tau}^{A'}\tau^{B'} + \bar{\tau}^{B'}\tau^{A'}, \tag{2.4}$$

where I denotes the 8×8 unit matrix. Denoting the matrix elements of $\tau^{A'}$ by $\tau^{A'a}{}_b$, we may write eq. (3) as

$$\bar{\tau}^{A'}{}_{ab} = \tau^{A'}{}_{ba}, \tag{2.5}$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A\tau^B - \bar{\tau}^B\tau^A & 0 \\ 0 & \tau^A\bar{\tau}^B - \tau^B\bar{\tau}^A \end{pmatrix} = 4\begin{pmatrix} D_{(1)}{}^{AB} & 0 \\ 0 & D_{(2)}{}^{AB} \end{pmatrix}$$

$$4D_{(1)}{}^{AB} = \bar{\tau}^A\tau^B - \bar{\tau}^B\tau^A$$

$$4D_{(2)}{}^{AB} = \tau^A\bar{\tau}^B - \tau^B\bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \to SO(4,4)$ is given by

(2.8) $$8L^{A'}{}_{B'}(g) = \mathrm{tr}\, \widetilde{D}^{(2)}(g)\, \tau^{A'}\, D^{(1)}(g)\, \overline{\tau}_{B'}\,.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

(2.9) $$\widetilde{D}^{(1)}\sigma = \sigma D^{(1)-1}\,,$$

(2.10) $$\sigma^{-1}\widetilde{D}^{(2)} = D^{(2)-1}\sigma^{-1}\,,$$

(2.11) $$L^{A'}{}_{C'}\, G_{A'B'}\, L^{B'}{}_{D'} = G_{C'D'}\,,$$

(2.12) $$L^{A'}{}_{B'}\, \tau^{B'} = \widetilde{D}^{(2)}\, \tau^{A'}\, D^{(1)}\,,$$

(2.13) $$L^{A'}{}_{B'}\, \overline{\tau}^{B'} = D^{(1)-1}\, \overline{\tau}^{A'}\, \widetilde{D}^{(2)-1}\,.$$

In[∘]:=
```
(σ = ArrayFlatten[{{ConstantArray[0, {4, 4}], IdentityMatrix[4]},
    {IdentityMatrix[4], ConstantArray[0, {4, 4}]}}]) // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

below :

In[∘]:=
```
σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

below :

In[∘]:=
```
Table[T16^A[A1] = ArrayFlatten[
    {{0, τ̄[A1]}, {τ[A1], 0}}], {A1, 0, 7}];
```

below :

In[∘]:=
```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@
  Range[0, 7]
```

Out[ ]=

{True, True, True, True, True, True, True, True}

below :

In[ ]:=

```
Table[{{A1, B1}, FullSimplify[
    ExpandAll[1/2 (T16^A[A1].T16^A[B1] +
        T16^A[B1].T16^A[A1]) ==
      η4488[[A1 + 1, B1 + 1]] * ID16]]},
  {A1, 0, 7}, {B1, 0, 7}] //
 MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
\begin{pmatrix}\{0,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,3\}\\ \text{True}\end{pmatrix} & \Big(\begin{matrix}\{0,\\ \text{Tru}\end{matrix}\\
\begin{pmatrix}\{1,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,3\}\\ \text{True}\end{pmatrix} & \Big(\begin{matrix}\{1,\\ \text{Tru}\end{matrix}\\
\begin{pmatrix}\{2,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,3\}\\ \text{True}\end{pmatrix} & \Big(\begin{matrix}\{2,\\ \text{Tru}\end{matrix}\\
\begin{pmatrix}\{3,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,3\}\\ \text{True}\end{pmatrix} & \Big(\begin{matrix}\{3,\\ \text{Tru}\end{matrix}\\
\begin{pmatrix}\{4,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,3\}\\ \text{True}\end{pmatrix} & \Big(\begin{matrix}\{4,\\ \text{Tru}\end{matrix}\\
\begin{pmatrix}\{5,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,3\}\\ \text{True}\end{pmatrix} & \Big(\begin{matrix}\{5,\\ \text{Tru}\end{matrix}\\
\begin{pmatrix}\{6,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,3\}\\ \text{True}\end{pmatrix} & \Big(\begin{matrix}\{6,\\ \text{Tru}\end{matrix}\\
\begin{pmatrix}\{7,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,3\}\\ \text{True}\end{pmatrix} & \Big(\begin{matrix}\{7,\\ \text{Tru}\end{matrix}
\end{pmatrix}$$

defs; some Symbols; metric

$$\left\{\left\{\text{a4}[t] \to \frac{M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{\text{Q1}} + \mathbb{c}_1 + \frac{1}{2}\,\text{ProductLog}\left[-e^{-\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{\text{Q1}}-2\,\mathbb{c}_1}\right]\right\}\right\}$$

$$\text{DSolve}\Big[0 == -M\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7]) + \left(\text{Q1} - e^{-2\,\text{a4}[t]}\,\text{Q1}\right)\,\text{a4}'[t],\ \text{a4}[t],\ t\Big]$$

$M\,c[j]$ = $(j+1)^{st}$ Energy Eigenvalue for Transpose[cayZ].Ψ16 , $j = 0, \ldots, 7$

ProductLog[z]
gives the principal solution for w in $z = we^w$.

if z and w are any complex numbers, then

$$we^w = z$$

holds if and only if

$$w = W_k(z) \quad \text{for some integer } k.$$

When dealing with real numbers only, the two branches W_0 and W_{-1} suffice: for real numbers x and y the equation

$$ye^y = x$$

can be solved for y only if $x \geq \frac{-1}{e}$; yields $y = W_0(x)$ if $x \geq 0$ and the two values $y = W_0(x)$ and $y = W_{-1}(x)$ if $\frac{-1}{e} \leq x < 0$.

$$-\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} - 2\,\mathbb{c}_1 ==$$

-1

$$\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} + 2\,\mathbb{c}_1 ==$$

$$1,\ ,\ t = 0 \quad \Rightarrow \mathbb{c}_1 = \frac{1}{2}$$

$$\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} + 1 == 1$$

$$\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} ==$$

0 $\Rightarrow$ sum Energy Eigenvalues =

$$\sum_{j=0}^{7} M\,c[j] = 0 \overset{?}{\Rightarrow} \text{if } +M \text{ then } -M$$

?todo: NEED DefScalarFunction[#]; & /@ (f16[#]&/@Range[0,15]), and allow xact to compute?

Introduce the wave function, Ψ16, for a Universe:

In[◦]:=
```
Ψ16 = f16[#][x0, x4] & /@ Range[0, 15]
```

Out[◦]=
```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

In[◦]:=
```
(*Ψnew16=F16[#][x0,x4]&/@Range[0,15]*)
```

In[◦]:=
```
Clear[sf16Aa];
sfψ16Aa = f16[#] → ToExpression[
    "((Z[" <> ToString[#] <> "][6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"] & /@ Range[0, 15]
```

Out[◦]=

$$\left\{f16[0]\to\left(\frac{Z[0][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),\ f16[1]\to\left(\frac{Z[1][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),\right.$$

$$f16[2]\to\left(\frac{Z[2][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),\ f16[3]\to\left(\frac{Z[3][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),$$

$$f16[4]\to\left(\frac{Z[4][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),\ f16[5]\to\left(\frac{Z[5][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),$$

$$f16[6]\to\left(\frac{Z[6][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),\ f16[7]\to\left(\frac{Z[7][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),$$

$$f16[8]\to\left(\frac{Z[8][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),\ f16[9]\to\left(\frac{Z[9][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),$$

$$f16[10]\to\left(\frac{Z[10][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),\ f16[11]\to\left(\frac{Z[11][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),$$

$$f16[12]\to\left(\frac{Z[12][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),\ f16[13]\to\left(\frac{Z[13][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),$$

$$\left.f16[14]\to\left(\frac{Z[14][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right),\ f16[15]\to\left(\frac{Z[15][6H\#1,H\#2]}{\sqrt{Sin[6H\#1]}}\&\right)\right\}$$

In[•]:=
```
Clear[snewfψ16Aa];
snewfψ16Aa = f16[#] → ToExpression[
    "((nZ[" <> ToString[#] <> "][6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"] & /@ Range[0, 15]
```

Out[•]=

$$\left\{f16[0]\to\left(\frac{nZ[0][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[1]\to\left(\frac{nZ[1][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[2]\to\left(\frac{nZ[2][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[3]\to\left(\frac{nZ[3][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[4]\to\left(\frac{nZ[4][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[5]\to\left(\frac{nZ[5][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[6]\to\left(\frac{nZ[6][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[7]\to\left(\frac{nZ[7][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[8]\to\left(\frac{nZ[8][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[9]\to\left(\frac{nZ[9][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[10]\to\left(\frac{nZ[10][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[11]\to\left(\frac{nZ[11][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[12]\to\left(\frac{nZ[12][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[13]\to\left(\frac{nZ[13][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[14]\to\left(\frac{nZ[14][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\ f16[15]\to\left(\frac{nZ[15][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right)\right\}$$

O(4, 4) related :

In[•]:= `Needs["Notation`"]`

In[•]:= Symbolize[$\overline{\tau}$]

In[•]:= Symbolize[$\overline{\text{new}\tau}$]

In[•]:= Symbolize[$T16^{A}$]

In[•]:= Symbolize[$T16^{\alpha}$]

In[•]:= Symbolize[$newT16^{A}$]

In[•]:= Symbolize[$newT16^{\alpha}$]

NEED DefScalarFunction[#]; & /@ Flatten[Universe]

In[•]:= Universe = ToExpression["U" <> ToString[#]] & /@ Range[0, 7]

In[•]:= Symbolize[Ŧ_{a}^{A}]

In[•]:= Symbolize[Ŧ_{A}^{a}]

In[∘]:= Symbolize[$gtrye_{\alpha}^{(A)}$]

In[∘]:= Symbolize[$gtrye^{\alpha}_{(A)}$]

SPACETIME METRIC

```
In[∘]:= (*einsteinMetric=Array[g[#1-1,#2-1]&,{8,8}]*)
```

SPACETIME METRIC:

```
In[∘]:= g4488 = Array[g[#1 - 1][#2 - 1][x0, x4] &, {8, 8}];
```

SPACETIME METRIC values:

MatrixMetric44

$$6\,H\,x0 == z\ \&\&\ H\,x4 == t$$

$$\left\{\left\{a4''[t] \to 0,\ a4'[t] \to \frac{2}{3}(-1+M)\right\},\ \left\{a4''[t] \to 0,\ a4'[t] \to \frac{2\,(1+M)}{3}\right\}\right\}$$

In[∘]:= (*β3=Exp[2 H x4 $\sqrt{K^2-M^2}$];*)

(*β3=Exp[2 * a4[3*H*x4]];*)

β3 = Exp[2 * a4[H * x4]] (*/.{a4→(((K1*$\frac{2\ (1+M)}{3}$+K2*$\frac{2}{3}$ (-1+M))#)&)}*)

β1 = Sin[6 * H * x0]$^{\frac{1}{3}}$;

β2 = Cot[6 * H * x0]^2;

Out[∘]=

$$e^{2\,a4[H\,x4]}$$

In[∘]:= MatrixForm[MatrixMetric44 = {{β2, 0, 0, 0, 0, 0, 0, 0}, {0, β1 β3, 0, 0, 0, 0, 0, 0}, {0, 0, β1 β3, 0, 0, 0, 0, 0}, {0, 0, 0, β1 β3, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, $-\frac{β1}{β3}$, 0, 0}, {0, 0, 0, 0, 0, 0, $-\frac{β1}{β3}$, 0}, {0, 0, 0, 0, 0, 0, 0, $-\frac{β1}{β3}$}} // FullSimplify[#, constraintVars] &]

Out[∘]//MatrixForm=

$$\begin{pmatrix} Cot[6\,H\,x0]^2 & 0 & 0 & 0 & 0 & \\ 0 & e^{2\,a4[H\,x4]}\,Sin[6\,H\,x0]^{1/3} & 0 & 0 & 0 & \\ 0 & 0 & e^{2\,a4[H\,x4]}\,Sin[6\,H\,x0]^{1/3} & 0 & 0 & \\ 0 & 0 & 0 & e^{2\,a4[H\,x4]}\,Sin[6\,H\,x0]^{1/3} & 0 & \\ 0 & 0 & 0 & 0 & -1 & \\ 0 & 0 & 0 & 0 & 0 & -e^{-2\,a4[H\,x4]}\,S \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{pmatrix}$$

In[ ]:=
```
(*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"][6*H*#1,3*H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@
  Range[0,15]*)
```

In[ ]:=
```
(*Clear[sf16Aa];
sfψ16Aa=
 f16[#]→ToExpression["((Z["<>ToString[#]<>"][6*H*#1,3*H*#2])&)"]&/@Range[0,15]*)
```

In[ ]:=
```
(*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"][6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@
  Range[0,15]*)
```

In[ ]:=
```
(*Inactivate[((q)&)]Evaluate[Activate[Evaluate[q]]]*)
```

In[ ]:=
```
Clear[makeSubstitution];
makeSubstitution[] := Module[{met, mat, t, f, p},
  met = Flatten[g4488]〚All, 0〛;
  mat = Flatten[MatrixMetric44];
  t = Thread[met → mat];
  f = Table[FullForm[t〚j〛 /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f〚j〛〚1〛〚2〛;
     expr = Inactivate[((q) &)];
     s = f〚j〛〚1〛〚1〛 → expr;
     r = Activate[s // Evaluate] // Evaluate;
     r], {j, 1, Length[f]}];
  Return[p];
 ]
```

In[ ]:=
```
makeSubstitution[a_, b_] := Module[{t, f, p},
  t = Thread[a → b];
  f = Table[FullForm[t〚j〛 /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f〚j〛〚1〛〚2〛;
     expr = Inactivate[((q) &)];
     s = f〚j〛〚1〛〚1〛 → expr;
     r = Activate[s // Evaluate] // Evaluate;
     r], {j, 1, Length[f]}];
  Return[p];
 ]
```

In[∘]:= ssgm4488 = makeSubstitution[]

Out[∘]=

$\{g[0][0] \to (\mathrm{Cot}[6\,H\,\#1]^2\,\&),\ g[0][1] \to (0\,\&),\ g[0][2] \to (0\,\&),\ g[0][3] \to (0\,\&),$
$g[0][4] \to (0\,\&),\ g[0][5] \to (0\,\&),\ g[0][6] \to (0\,\&),\ g[0][7] \to (0\,\&),\ g[1][0] \to (0\,\&),$
$g[1][1] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[1][2] \to (0\,\&),\ g[1][3] \to (0\,\&),$
$g[1][4] \to (0\,\&),\ g[1][5] \to (0\,\&),\ g[1][6] \to (0\,\&),\ g[1][7] \to (0\,\&),\ g[2][0] \to (0\,\&),$
$g[2][1] \to (0\,\&),\ g[2][2] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[2][3] \to (0\,\&),\ g[2][4] \to (0\,\&),$
$g[2][5] \to (0\,\&),\ g[2][6] \to (0\,\&),\ g[2][7] \to (0\,\&),\ g[3][0] \to (0\,\&),\ g[3][1] \to (0\,\&),$
$g[3][2] \to (0\,\&),\ g[3][3] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[3][4] \to (0\,\&),\ g[3][5] \to (0\,\&),$
$g[3][6] \to (0\,\&),\ g[3][7] \to (0\,\&),\ g[4][0] \to (0\,\&),\ g[4][1] \to (0\,\&),\ g[4][2] \to (0\,\&),$
$g[4][3] \to (0\,\&),\ g[4][4] \to (-1\,\&),\ g[4][5] \to (0\,\&),\ g[4][6] \to (0\,\&),\ g[4][7] \to (0\,\&),$
$g[5][0] \to (0\,\&),\ g[5][1] \to (0\,\&),\ g[5][2] \to (0\,\&),\ g[5][3] \to (0\,\&),\ g[5][4] \to (0\,\&),$
$g[5][5] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[5][6] \to (0\,\&),\ g[5][7] \to (0\,\&),$
$g[6][0] \to (0\,\&),\ g[6][1] \to (0\,\&),\ g[6][2] \to (0\,\&),\ g[6][3] \to (0\,\&),\ g[6][4] \to (0\,\&),$
$g[6][5] \to (0\,\&),\ g[6][6] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[6][7] \to (0\,\&),$
$g[7][0] \to (0\,\&),\ g[7][1] \to (0\,\&),\ g[7][2] \to (0\,\&),\ g[7][3] \to (0\,\&),\ g[7][4] \to (0\,\&),$
$g[7][5] \to (0\,\&),\ g[7][6] \to (0\,\&),\ g[7][7] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&)\}$

In[∘]:= Symbolize$[E_\alpha^{(A)}]$

In[∘]:= Symbolize$[e_\alpha^{(A)}]$

In[∘]:= $e_\alpha^{(A)}$ = Array$[E_\alpha^{(A)}[\#1-1, \#2-1]\,\&, \{8, 8\}]$

Out[∘]=

$\{\{E_\alpha^{(A)}[0,0], E_\alpha^{(A)}[0,1], E_\alpha^{(A)}[0,2], E_\alpha^{(A)}[0,3], E_\alpha^{(A)}[0,4], E_\alpha^{(A)}[0,5], E_\alpha^{(A)}[0,6], E_\alpha^{(A)}[0,7]\},$
$\{E_\alpha^{(A)}[1,0], E_\alpha^{(A)}[1,1], E_\alpha^{(A)}[1,2], E_\alpha^{(A)}[1,3], E_\alpha^{(A)}[1,4], E_\alpha^{(A)}[1,5], E_\alpha^{(A)}[1,6], E_\alpha^{(A)}[1,7]\},$
$\{E_\alpha^{(A)}[2,0], E_\alpha^{(A)}[2,1], E_\alpha^{(A)}[2,2], E_\alpha^{(A)}[2,3], E_\alpha^{(A)}[2,4], E_\alpha^{(A)}[2,5], E_\alpha^{(A)}[2,6], E_\alpha^{(A)}[2,7]\},$
$\{E_\alpha^{(A)}[3,0], E_\alpha^{(A)}[3,1], E_\alpha^{(A)}[3,2], E_\alpha^{(A)}[3,3], E_\alpha^{(A)}[3,4], E_\alpha^{(A)}[3,5], E_\alpha^{(A)}[3,6], E_\alpha^{(A)}[3,7]\},$
$\{E_\alpha^{(A)}[4,0], E_\alpha^{(A)}[4,1], E_\alpha^{(A)}[4,2], E_\alpha^{(A)}[4,3], E_\alpha^{(A)}[4,4], E_\alpha^{(A)}[4,5], E_\alpha^{(A)}[4,6], E_\alpha^{(A)}[4,7]\},$
$\{E_\alpha^{(A)}[5,0], E_\alpha^{(A)}[5,1], E_\alpha^{(A)}[5,2], E_\alpha^{(A)}[5,3], E_\alpha^{(A)}[5,4], E_\alpha^{(A)}[5,5], E_\alpha^{(A)}[5,6], E_\alpha^{(A)}[5,7]\},$
$\{E_\alpha^{(A)}[6,0], E_\alpha^{(A)}[6,1], E_\alpha^{(A)}[6,2], E_\alpha^{(A)}[6,3], E_\alpha^{(A)}[6,4], E_\alpha^{(A)}[6,5], E_\alpha^{(A)}[6,6], E_\alpha^{(A)}[6,7]\},$
$\{E_\alpha^{(A)}[7,0], E_\alpha^{(A)}[7,1], E_\alpha^{(A)}[7,2], E_\alpha^{(A)}[7,3], E_\alpha^{(A)}[7,4], E_\alpha^{(A)}[7,5], E_\alpha^{(A)}[7,6], E_\alpha^{(A)}[7,7]\}\}$

In[∘]:= (*Symbolize$[se_\alpha^{(A)}]$*)

In[∘]:= Symbolize$[sge_\alpha^{(A)}]$

In[∘]:= Symbolize$[sgtrye_\alpha^{(A)}]$

In[∘]:= Symbolize$[E_{(A)}^\alpha]$

In[∘]:= Symbolize$[sge_{(A)}^\alpha]$

In[∘]:= Symbolize$[sgtrye_{(A)}^\alpha]$

In[∘]:= Symbolize$[e_{(A)}^\alpha]$

```
In[∘]:= e^α_(A) = Array[E^α_(A)[#1 - 1, #2 - 1] &, {8, 8}]
Out[∘]=
{{E^α_(A)[0, 0], E^α_(A)[0, 1], E^α_(A)[0, 2], E^α_(A)[0, 3], E^α_(A)[0, 4], E^α_(A)[0, 5], E^α_(A)[0, 6], E^α_(A)[0, 7]},
 {E^α_(A)[1, 0], E^α_(A)[1, 1], E^α_(A)[1, 2], E^α_(A)[1, 3], E^α_(A)[1, 4], E^α_(A)[1, 5], E^α_(A)[1, 6], E^α_(A)[1, 7]},
 {E^α_(A)[2, 0], E^α_(A)[2, 1], E^α_(A)[2, 2], E^α_(A)[2, 3], E^α_(A)[2, 4], E^α_(A)[2, 5], E^α_(A)[2, 6], E^α_(A)[2, 7]},
 {E^α_(A)[3, 0], E^α_(A)[3, 1], E^α_(A)[3, 2], E^α_(A)[3, 3], E^α_(A)[3, 4], E^α_(A)[3, 5], E^α_(A)[3, 6], E^α_(A)[3, 7]},
 {E^α_(A)[4, 0], E^α_(A)[4, 1], E^α_(A)[4, 2], E^α_(A)[4, 3], E^α_(A)[4, 4], E^α_(A)[4, 5], E^α_(A)[4, 6], E^α_(A)[4, 7]},
 {E^α_(A)[5, 0], E^α_(A)[5, 1], E^α_(A)[5, 2], E^α_(A)[5, 3], E^α_(A)[5, 4], E^α_(A)[5, 5], E^α_(A)[5, 6], E^α_(A)[5, 7]},
 {E^α_(A)[6, 0], E^α_(A)[6, 1], E^α_(A)[6, 2], E^α_(A)[6, 3], E^α_(A)[6, 4], E^α_(A)[6, 5], E^α_(A)[6, 6], E^α_(A)[6, 7]},
 {E^α_(A)[7, 0], E^α_(A)[7, 1], E^α_(A)[7, 2], E^α_(A)[7, 3], E^α_(A)[7, 4], E^α_(A)[7, 5], E^α_(A)[7, 6], E^α_(A)[7, 7]}}
In[∘]:= (*gtrye_α^(A) = (e_α^(A) /.sgtrye_α^(A))*)
In[∘]:= (*gtrye^α_(A) = (e^α_(A) /.sgtrye^α_(A))*)
In[∘]:= (*Symbolize[Γ^κ_α β]*)
In[∘]:= (*preSpinConnection=Array[Γ^κ_α β[##]&,{8,8,8}];*)
In[∘]:= (*Symbolize[ E^A_a ]*)
In[∘]:= (*Symbolize[ E^a_A ]*)
In[∘]:= (* Ƒ^A_a Ƒ^a_A *)
In[∘]:= (*Do[ Ƒ^A_a =E^A_a[h]=Table[(u⟦h⟧.σ.(τ̅[B])),{B,1,8}],{h,1,Length[u]}];
        Do[E^a_A[h]=FullSimplify[Inverse[E^A_a[h]]],{h,1,Length[u]}];
        Table[ Ƒ^a_A =E^a_A[h]== (u⟦h⟧.σ.u⟦h⟧)
            Transpose[Table[FullSimplify[ExpandAll[ η8⟦B,B⟧τ[B].u⟦h⟧ ]],
              {B,1,8}]],{h,1,Length[u]}]*)
In[∘]:= (*Table[FullSimplify[ExpandAll[Transpose[E^A_a[h]].η8.E^A_a[h]]]-
           (u⟦h⟧.σ.u⟦h⟧)σ== Zero,{h,1,Length[u]}]*)
In[∘]:= (*iη88=FullSimplify[Inverse[η8]];*)
In[∘]:= (*Clear[ω_μ^(a)_(b)];*)Clear[ω]; Symbolize[ω_μ^(a)_(b)]
In[∘]:= ω = Array[ω_μ^(a)_(b)[##] &, {8, 8, 8}];
        Protect[ω]
Out[∘]=
{ω}
```

constants

In[ ]:=
```
similarityTransformation00 =
  {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
   {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
   {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
   {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

← ERROR : O8similarityTransformation has 1 st index that transforms as $\frac{\partial}{\partial \Psi}$, not as Ψs

In[ ]:=
```
(* O8similarityTransformation has 1st
 index that transforms as ∂/∂Ψ , not as Ψ *)
```

O8similarityTransformation has 1 st index that transforms as $\frac{\partial}{\partial \Psi}$, not as Ψ :

In[∘]:=
```
O8similarityTransformation = Transpose[similarityTransformation00]
Transpose[O8similarityTransformation].O8similarityTransformation === ID16
O8similarityTransformation.σ16.Transpose[O8similarityTransformation] === σ16
(*O8similarityTransformation.σ16.Transpose[O8similarityTransformation]-σ16//MatrixForm*)
```

Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

Out[∘]=
```
False
```

Out[∘]=
```
False
```

In[∘]:=
```
σ16 === ArrayFlatten[{{-σ, 0}, {0, σ}}]
σ16 // MatrixForm
O8similarityTransformation.σ16.Transpose[O8similarityTransformation] // MatrixForm
```

Out[∘]=
```
False
```

Out[∘]//MatrixForm=
```
σ16
```

Out[∘]//MatrixForm=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}.σ16.
  {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
   {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
   {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
   {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[∘]:=
```
(*ArrayFlatten[{{IdentityMatrix[4],0},{0,-IdentityMatrix[4]}}]*)
```

In[∘]:=
```
ID4 = IdentityMatrix[4];
ID8 = IdentityMatrix[8];
```

In[∘]:=
```
η4488 // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
η4488[[#, #]] & /@ Range[8]
```
Out[∘]=
```
{1, 1, 1, 1, -1, -1, -1, -1}
```

In[∘]:=
```
Zero4 = ConstantArray[0, {4, 4}]; Zero8 = ConstantArray[0, {8, 8}];
mid = ExpandAll[-1 * ID4];
ϵ3 = Array[Signature[{##}] &, {3, 3, 3}]
ϵ4 = Array[Signature[{##}] &, {4, 4, 4, 4}];
MId = ExpandAll[-1 * ID8];
simp = {Zero4 → 0, ID4 → 1, mid → -1};
Simp = {Zero8 → ZERO, ID8 → ONE, MId → MONE};
```
Out[∘]=
```
{{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}},
 {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}}, {{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}
```

In[∘]:=
```
Zero16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η8, 0}, {0, -η8}}];
Id16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * Id16];
Simp16 = {Zero16 → ZERO16, Id16 → ONE16, MId16 → MONE16};
```

In[∘]:=
```
ZERO16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η4488, 0}, {0, -η4488}}];
ID16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * ID16];
Simp16 = {ZERO16 → ZERO16, ID16 → ONE16, MId16 → MONE16};
```

In[∘]:=
```
ωs = Flatten[
   Table[ToExpression["ω" <> ToString[A1] <> ToString[B1]], {A1, 1, 7}, {B1, A1 + 1, 8}]]
Length[ωs]
```
Out[∘]=
```
{ω12, ω13, ω14, ω15, ω16, ω17, ω18, ω23, ω24, ω25, ω26, ω27, ω28,
 ω34, ω35, ω36, ω37, ω38, ω45, ω46, ω47, ω48, ω56, ω57, ω58, ω67, ω68, ω78}
```
Out[∘]=
```
28
```

In[ ]:=
```
σ.σ - ID8 == Zero8
Tr[σ] == 0
```
Out[ ]=
```
True
```
Out[ ]=
```
True
```

some function definitions :

In[ ]:=
```
blockPartition[mat_] := Module[{(*blocks,block11,block12,block21,block22,*)ret},
  blocks = Partition[mat, {8, 8}];
  (*block11=blocks[[1,1]] ;*) (*Top-left block*)
  (*block12=blocks[[1,2]] ;*) (*Top-right block*)
  (*block21=blocks[[2,1]] ;*) (*Bottom-left block*)
  (*block22=blocks[[2,2]] ;*) (*Bottom-right block*)
  ret =
   ArrayFlatten[{{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}] === mat;
  If[ret, Return[{{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}], ## &[]];
  Return[Null];]
```
In[ ]:=
```
eextract[a_] := Extract[a, {#}] & /@ Range[0, Length[a]]
```
In[ ]:=
```
Clear[imes]
```
In[ ]:=
```
imes[a_] := {a}
```
In[ ]:=
```
imes[a_, b_] := {a, b}
```
In[ ]:=
```
imes[a_, b_, c_] := {a, b, c}
```
In[ ]:=
```
imes[a_, b_, c_, d_] := {a, b, c, d}
```
In[ ]:=
```
imes[a_, b_, c_, d_, f_] := {a, b, c, d, f}
```
In[ ]:=
```
imes[a_, b_, c_, d_, f_, h_] := {a, b, c, d, f, h}
```
In[ ]:=
```
imes[a_, b_, c_, d_, f_, h_, j_] := {a, b, c, d, f, h, j}
```
In[ ]:=
```
times[a__] := Flatten[{Flatten[#] & /@ a}]
```
In[ ]:=
```
(*times[a_,b_,c_,d_]:={a,b}*)
```
In[ ]:=
```
(*imes[a_,b_,c_,d_][0]:={a,d}*)
```
In[ ]:=
```
(*imes[a_,b_][0]^:={a,b}*)
```

In[ ]:=
```
iid[a_, b_] := a
```

In[ ]:=
```
iid[a_, b_, c_] := {a, b}
```

In[ ]:=
```
iid[a_List] := a⟦1⟧
```

In[ ]:=
```
qid[a__] := a
```

In[ ]:=
```
Clear[der]; der[a__] [c_][b__] := c
```

In[ ]:=
```
Clear[der3]; der3[a__] [c_][b__] := {c, a, b}
```

In[ ]:=
```
(*der[a_, b_] [c_]:= c*)
```

In[ ]:=
```
(*derz[a_, b_] := a*)
```

In[ ]:=
```
(*dert[a_, b_] := b*)
```

In[ ]:=
```
(*derivative[a__] [c_]:= FullForm[c]/.{Derivative→der}*)
```

In[ ]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
(*ToString[#]&/@%*)
% /. {Derivative → der}
```

Out[ ]=

$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$

Out[ ]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[ ]=
```
{Z[0][z, t], Z[0], Z[0]}
```

In[∘]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
%* (Prime[#] & /@ Range[Length[%]])
Plus @@ %
FullForm[%]  (* #]&/@%*)
% /. {Plus → qid, Times → qid, Derivative → der}
List[%]
Partition[%, 2]
eextract[#] & /@ %
(*eextract[#]&/@%*)
(*#__List[[0]]&/@%*)
#[[2]] & /@ %
#[[3]] & /@ %%
```

Out[∘]=
```
{Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}
```

Out[∘]=
```
{2 Z[0][z, t], 3 Z[0]^(1,0)[z, t], 5 Z[0]^(0,1)[z, t]}
```

Out[∘]=
```
2 Z[0][z, t] + 5 Z[0]^(0,1)[z, t] + 3 Z[0]^(1,0)[z, t]
```

Out[∘]//FullForm=
```
Plus[Times[2, Z[0][z, t]], Times[5, Derivative[0, 1][Z[0]][z, t]],
 Times[3, Derivative[1, 0][Z[0]][z, t]]]
```

Out[∘]=
```
Sequence[2, Z[0][z, t], 5, Z[0], 3, Z[0]]
```

Out[∘]=
```
{2, Z[0][z, t], 5, Z[0], 3, Z[0]}
```

Out[∘]=
```
{{2, Z[0][z, t]}, {5, Z[0]}, {3, Z[0]}}
```

Out[∘]=
```
{{List, 2, Z[0][z, t]}, {List, 5, Z[0]}, {List, 3, Z[0]}}
```

Out[∘]=
```
{2, 5, 3}
```

Out[∘]=
```
{Z[0][z, t], Z[0], Z[0]}
```

Sequence[Subscript[expr, 1], Subscript[expr, 2], …] represents a sequence of arguments to be spliced automatically into any function .

In[∘]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
(*ToString[#]&/@%*)
% /. {Derivative → der3}
```

Out[∘]=
```
{Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}
```

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{Z[0][z, t], List[Z[0], 1, 0, z, t], List[Z[0], 0, 1, z, t]}
```

In[ ]:=
```
helpf[x__] := Length[{x}]
```

In[ ]:=
```
{helpf[x, y, z], helpf[]}
```

Out[ ]=
```
{3, helpf[]}
```

In[ ]:=
```
f[##] & @@ X
```

Out[ ]=
```
f[x0, x1, x2, x3, x4, x5, x6, x7]
```

In[ ]:=
```
StringExtract["a--bbb--ccc--dddd", "--" → 3]
```

Out[ ]=
```
ccc
```

In[ ]:=
```
StringExtract["a bbb   cccc aa    d", -1]
```

Out[ ]=
```
d
```

In[ ]:=
```
StringRiffle[{{"a", "b", "c"}, {"d", "e", "f"}}, "\n", "\t"]
```

Out[ ]=
```
a	b	c
d	e	f
```

In[ ]:=
```
{"", "z, t"}
```

Out[ ]=
```
{, z, t}
```

```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
StringSplit[#, "]"] & /@ %
StringExtract[#, "[" → All] & /@ %
StringExtract[#,
   "\!\(\*StyleBox[\"
      \",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"{\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*
      StyleBox[\",\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"
      \",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"z\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*
      StyleBox[\",\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"
      \",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"t\",ShowStringCharacters->True,NumberMarks->True]\)" →
    All] & /@ %
InputForm[TextString[#]] & /@ %
```

In[∘]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
StringSplit[#, "]"] & /@ %
StringExtract[#, "[" → All] & /@ %
(*StringExtract[#," {, z, t"→All]&/@%*)
InputForm[TextString[#]] & /@ %
```

Out[∘]=
$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{{Z[0, [z, t}, {Derivative[1, 0, [Z[0, , [z, t}, {Derivative[0, 1, [Z[0, , [z, t}}
```

Out[∘]=
```
{{{Z, 0}, {, z, t}}, {{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}},
 {{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}}
```

Out[∘]=
```
{"{{Z, 0}, {, z, t}}", "{{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}",
 "{{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}"}
```

In[∘]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
(*InputForm[TextString[#]]&/@%*)
StringExtract[#, "[" → All] & /@ %
StringExtract[#, "]" → All] & /@ %%
StringExtract[#, "[" → 1] & /@ %%%
StringExtract[#, "]" → 1] & /@ %%%%
StringExtract[#, "Z[" → 1] & /@ %%%%%
```

Out[∘]=
$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{{Z, 0], z, t]}, {Derivative, 1, 0], Z, 0]], z, t]}, {Derivative, 0, 1], Z, 0]], z, t]}}
```

Out[∘]=
```
{{Z[0, [z, t, }, {Derivative[1, 0, [Z[0, , [z, t, }, {Derivative[0, 1, [Z[0, , [z, t, }}
```

Out[∘]=
```
{Z, Derivative, Derivative}
```

Out[∘]=
```
{Z[0, Derivative[1, 0, Derivative[0, 1}
```

Out[∘]=
```
{, Derivative[1, 0][, Derivative[0, 1][}
```

In[∘]:=
```
(*TemplateApply[StringTemplate["Z `[` then `b`"],
 <|"a"→1234,"b"→5678|>]*)
```

In[∘]:=
```
skelx[x_, matQ_ : True] := Module[{t = {666}},
  If[matQ == True, t = Block[{r},
     r = x;
     Table[Block[{q}, q = SameQ[0, r[[j, i]]];
       If[True == q, Style[0, Blue], Style[Length[r[[j, i]]], Red]]],
      {j, 1, Length[r]}, {i, 1, Length[r[[-1]]]}]]];
  If[matQ == False, t = Block[{r},
     r = x;
     Table[Block[{q}, q = SameQ[0, r[[j]]];
       If[True == q, Style[0, {RGBColor → {0, 1 / 3, 0}, Bold}], Style[Length[r[[j]]], Red]]],
      {j, 1, Length[r]}]]];
  MatrixForm[t, TableAlignments → Left]]
```

In[∘]:=
```
makeSym[size_, fn_] :=
 Module[{rtmp}, rtmp = Table[fn[i, j], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[rtmp, {{2}, {1}}]}]]
```

In[∘]:=
```
makeAntiSym[size_, fn_] :=
 Module[{rtmp}, rtmp = Table[If[i == j, 0, fn[i, j]], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[-rtmp, {{2}, {1}}]}]]
```

In[∘]:=
```
Block[{MX}, MX = makeAntiSym[8, Subscript[ωω, ##] &]]
```

Out[∘]=

$$\{\{0, -\omega\omega_{2,1}, -\omega\omega_{3,1}, -\omega\omega_{4,1}, -\omega\omega_{5,1}, -\omega\omega_{6,1}, -\omega\omega_{7,1}, -\omega\omega_{8,1}\},$$
$$\{\omega\omega_{2,1}, 0, -\omega\omega_{3,2}, -\omega\omega_{4,2}, -\omega\omega_{5,2}, -\omega\omega_{6,2}, -\omega\omega_{7,2}, -\omega\omega_{8,2}\},$$
$$\{\omega\omega_{3,1}, \omega\omega_{3,2}, 0, -\omega\omega_{4,3}, -\omega\omega_{5,3}, -\omega\omega_{6,3}, -\omega\omega_{7,3}, -\omega\omega_{8,3}\},$$
$$\{\omega\omega_{4,1}, \omega\omega_{4,2}, \omega\omega_{4,3}, 0, -\omega\omega_{5,4}, -\omega\omega_{6,4}, -\omega\omega_{7,4}, -\omega\omega_{8,4}\},$$
$$\{\omega\omega_{5,1}, \omega\omega_{5,2}, \omega\omega_{5,3}, \omega\omega_{5,4}, 0, -\omega\omega_{6,5}, -\omega\omega_{7,5}, -\omega\omega_{8,5}\},$$
$$\{\omega\omega_{6,1}, \omega\omega_{6,2}, \omega\omega_{6,3}, \omega\omega_{6,4}, \omega\omega_{6,5}, 0, -\omega\omega_{7,6}, -\omega\omega_{8,6}\},$$
$$\{\omega\omega_{7,1}, \omega\omega_{7,2}, \omega\omega_{7,3}, \omega\omega_{7,4}, \omega\omega_{7,5}, \omega\omega_{7,6}, 0, -\omega\omega_{8,7}\},$$
$$\{\omega\omega_{8,1}, \omega\omega_{8,2}, \omega\omega_{8,3}, \omega\omega_{8,4}, \omega\omega_{8,5}, \omega\omega_{8,6}, \omega\omega_{8,7}, 0\}\}$$

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(\frac{\partial g_{\mu\alpha}}{\partial x^{\beta}} + \frac{\partial g_{\mu\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\mu}}\right),$$

$$\Gamma^{\mu}{}_{\alpha\beta} = g^{\mu\nu}\Gamma_{\nu\alpha\beta},$$

$$R^{\mu}{}_{\nu\alpha\beta} = \frac{\partial \Gamma^{\mu}{}_{\nu\beta}}{\partial x^{\alpha}} - \frac{\partial \Gamma^{\mu}{}_{\nu\alpha}}{\partial x^{\beta}} + \Gamma^{\mu}{}_{\rho\alpha}\Gamma^{\rho}{}_{\nu\beta} - \Gamma^{\mu}{}_{\rho\beta}\Gamma^{\rho}{}_{\nu\alpha}.$$

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}, \qquad R = g^{\mu\nu}R_{\mu\nu},$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

In[ ]:=
```
(*Christoffel, Einstein tensor, ...*)
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, ΓΓ, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    ΓΓ = FullSimplify[Table[(1 / 2) * Sum[(ginv[[i, s]]) *
          (D[g[[s, j]], X[[k]] ] + D[g[[s, k]], X[[j]] ] - D[g[[j, k]], X[[s]] ]), {s, 1, DIM8}],
       {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] , ass];
    RicciΓ = ParallelTable[FullSimplify[D[ΓΓ[[μ, ν, β]], X[[α]] ] - D[ΓΓ[[μ, ν, α]], X[[β]] ] +
         Sum[ΓΓ[[μ, s, α]] × ΓΓ[[s, ν, β]] - ΓΓ[[μ, s, β]] × ΓΓ[[s, ν, α]], {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}] ;
    RieΓ = ParallelTable[ FullSimplify[Sum[ RicciΓ[[α, μ, α, ν]], {α, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}] ;
    RS = FullSimplify[Tr[ ginv.RieΓ] , ass];
    G = ParallelTable[FullSimplify[RieΓ[[α]] - 1/2 g[[α]] RS, ass], {α, 1, Length[RieΓ]}];];
  Print[t];
  Print[Now];
  Return[{ginv, ΓΓ, RicciΓ, RieΓ, RS, G}]]
```

If 'paste' Maple MathML to ToExpression[MathMLForm[]] fails, then try to edit this :

In[ ]:=
```
Clear[mapleztStringToMathematica];
mapleztStringToMathematica[input_String, toExpression_ : False, debug_ : False] :=
 Module[{prepare, post, custom, use = input, save = input, linenumber = 0},
  (*use=ToString[InputForm[TextString[input]]];*) (*Print[use];*)
  prepare[in_String] := Module[{s = in}, linenumber ++;
    s = in;
    (*If[s==ToString[InputForm[TextString[s]]],
      (*##&[]*)Print[s],Print[ToUpperCase[s]]];*)
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,z)" →
        "D[Z[" <> ToString[j] <> "][z,t],{z,2}]"], {j, 0, 15}];
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,t)" →
        "D[D[Z[" <> ToString[j] <> "][z,t],z],t]"], {j, 0, 15}];
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,z)" →
        "D[D[Z[" <> ToString[j] <> "][z,t],t],z]"], {j, 0, 15}];
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,t)" →
        "D[Z[" <> ToString[j] <> "][z,t],{t,2}]"], {j, 0, 15}];

    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t)" →
        "D[Z[" <> ToString[j] <> "][z,t],t]"], {j, 0, 15}];
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z)" →
        "D[Z[" <> ToString[j] <> "][z,t],z]"], {j, 0, 15}];
```

```
    s = StringReplace[s, "tan(z)" → "Tan[z]"];
    s = StringReplace[s, "cot(z)" → "Cot[z]"];
    s = StringReplace[s, "sin(z)" → "Sin[z]"];
    s = StringReplace[s, "cos(z)" → "Cos[z]"];
    s = StringReplace[s, "sec(z)" → "Sec[z]"];
    s = StringReplace[s, "csc(z)" → "Csc[z]"];

    s = StringReplace[s, "=" → "->"];

    (*Do[s=StringReplace[s,"Z"<>ToString[j]<>"(z,t)"→"Z["<>ToString[j]<>"[z,t]"],
      {j,0,15}];*)
    (*s=StringReplace[s,"diff(a4(t),t)"→"D[a4[t],t]"];*)
    (*s=StringReplace[s,"exp(-2*a4(t))"→"Exp[-2a4[t]]"];*)
    (*s=StringReplace[s,"a4(t))"→"a4[t]"];*)

    Return[s];
   ];
  post[in_String] := Module[{s = in}, linenumber++;
    s = in;
    s = StringReplace[s, "a4(t))" → "a4[t]"];
    Do[s = StringReplace[s, "Z" <> ToString[j] <> "(z,t)" → "Z[" <> ToString[j] <> "][z,t]"],
     {j, 0, 15}];
    Return[s];
   ];
  custom[in_String] := Module[{s = in}, linenumber++;
    s = in;

    s = StringReplace[s, "diff(a4(t),t)" → "D[a4[t],t]"];
    s = StringReplace[s, "exp(-2*a4(t))" → "Exp[-2a4[t]]"];

    Return[s];
   ];
  use = prepare[use];
  use = custom[use];
  Return[post[use]];
  (*If[toExpression,Return[ToExpression[s/.{z→#1,t→#2}],##&[]];*)
 ]
```

Mathematica Lexer and Parser for Maple - like Syntax Supports : Basic arithmetic, function calls, lists {}, equations =, and specific Maple keywords .

In[•]:=
```
(*---LEXER---*)(*Helper:Check if character is a digit*)
IsDigit[char_] := StringMatchQ[char, RegularExpression["\\d"]];

(*Helper:Check if character is a letter*)
IsAlpha[char_] := StringMatchQ[char, RegularExpression["[a-zA-Z_]"]];

(*Helper:Check if character is whitespace*)
IsSpace[char_] := StringMatchQ[char, RegularExpression["\\s"]];

(*Main Lexer Function*)
GetTokens[str_String] :=
  Module[{chars, len, i, char, tokens = {}, token, numStr, idStr}, chars = Characters[str];
   len = Length[chars];
   i = 1;
   While[i ≤ len, char = chars⟦i⟧;
    Which[(*Skip Whitespace*)IsSpace[char], i++, (*Numbers*)IsDigit[char], numStr = char;
     i++;
     While[i ≤ len && IsDigit[chars⟦i⟧], numStr = numStr <> chars⟦i⟧;
      i++;];
     AppendTo[tokens, {"NUMBER", ToExpression[numStr]}],
     (*Identifiers and Keywords*)IsAlpha[char], idStr = char;
     i++;
     While[i ≤ len && (IsAlpha[chars⟦i⟧] || IsDigit[chars⟦i⟧]), idStr = idStr <> chars⟦i⟧;
      i++;];
     (*Check for specific keywords if needed,
     otherwise IDENTIFIER*)AppendTo[tokens, {"IDENTIFIER", idStr}],
     (*Operators and Punctuation*)char == "+", AppendTo[tokens, {"PLUS", "+"}];
     i++, char == "-", AppendTo[tokens, {"MINUS", "-"}];
     i++, char == "*", AppendTo[tokens, {"STAR", "*"}];
     i++, char == "/", AppendTo[tokens, {"SLASH", "/"}];
     i++, char == "^", AppendTo[tokens, {"CARET", "^"}];
     i++, char == "(", AppendTo[tokens, {"LPAREN", "("}];
     i++, char == ")", AppendTo[tokens, {"RPAREN", ")"}];
     i++, char == "{", AppendTo[tokens, {"LBRACE", "{"}];
     i++, char == "}", AppendTo[tokens, {"RBRACE", "}"}];
     i++, char == "=", AppendTo[tokens, {"EQUALS", "="}];
     i++, char == ",", AppendTo[tokens, {"COMMA", ","}];
     i++, char == "$", AppendTo[tokens, {"DOLLAR", "$"}];
     i++, (*Ellipsis... or Dot.*)char == ".", If[i + 2 ≤ len &&
       chars⟦i + 1⟧ == "." && chars⟦i + 2⟧ == ".", AppendTo[tokens, {"ELLIPSIS", "..."}];
      i += 3, AppendTo[tokens, {"DOT", "."}];
      i++], True, Print["Unknown character: ", char];
     i++];];
   AppendTo[tokens, {"EOF", "EOF"}];
   tokens];
```

In[•]:=
```
(*---PARSER---*)(*Grammar:Expression→
 Equation Equation→AddExp["=" AddExp] AddExp→MulExp {("+"|"-") MulExp} MulExp→
```

```
    PowerExp {("*"|"/") PowerExp} PowerExp→Primary["^" PowerExp] (Right associative)
       Primary→NUMBER|IDENTIFIER|"(" Expression ")"|IDENTIFIER "(" ArgList ")"|
        "{" ArgList "}" ArgList→Expression {"," Expression}|Empty*)
ParseTokens[tokens_List] := Module[{pos = 1, currentToken, eat, peek, parseExpression,
    parseEquation, parseAddExp, parseMulExp, parsePowerExp, parseUnaryExp,
    parsePrimary, parseArgs, parseList}, currentToken := tokens⟦pos⟧;
   peek[] := tokens⟦pos⟧;
   eat[type_] := If[currentToken⟦1⟧ == type, pos++;
     True, Print["Error: Expected ", type, " but found ", currentToken⟦1⟧];
     False];
   (*Entry point*)parseExpression[] := parseEquation[];
   (*Equation:a=b*)parseEquation[] := Module[{left, right}, left = parseAddExp[];
     If[currentToken⟦1⟧ == "EQUALS", eat["EQUALS"];
      right = parseAddExp[];
      {"Equation", left, right}, left]];
   parseAddExp[] := Module[{node, right, op}, node = parseMulExp[];
     While[MemberQ[{"PLUS", "MINUS"}, currentToken⟦1⟧], op = currentToken⟦2⟧;
      eat[currentToken⟦1⟧];
      right = parseMulExp[];
      node = {"BinaryOp", op, node, right};];
     node];
   parseMulExp[] := Module[{node, right, op}, node = parsePowerExp[];
     While[True, If[MemberQ[{"STAR", "SLASH"}, currentToken⟦1⟧], op = currentToken⟦2⟧;
       eat[currentToken⟦1⟧];
       right = parsePowerExp[];
       node = {"BinaryOp", op, node, right}, (*Implicit Multiplication*)
       If[MemberQ[{"IDENTIFIER", "NUMBER", "LPAREN", "LBRACE", "ELLIPSIS"},
         currentToken⟦1⟧], right = parsePowerExp[];
        node = {"BinaryOp", "*", node, right},
        Break[] (*Not an operator or start of expression*)]]];
     node];
   parsePowerExp[] := Module[{node, right}, node = parseUnaryExp[];
     If[currentToken⟦1⟧ == "CARET", eat["CARET"];
      right = parsePowerExp[];
      (*Right associative recursion*){"BinaryOp", "^", node, right}, node]];
   parseUnaryExp[] := Module[{op, node},
     If[MemberQ[{"PLUS", "MINUS"}, currentToken⟦1⟧], op = currentToken⟦2⟧;
      eat[currentToken⟦1⟧];
      node = parseUnaryExp[];
      {"UnaryOp", op, node}, parsePrimary[]]];
   parsePrimary[] := Module[{token, node, name, args}, token = currentToken;
     Switch[token⟦1⟧, "NUMBER", eat["NUMBER"];
      {"Number", token⟦2⟧}, "IDENTIFIER", eat["IDENTIFIER"];
      name = token⟦2⟧;
      (*Check if it's a function call:IDENTIFIER followed by LPAREN*)
      If[currentToken⟦1⟧ == "LPAREN", eat["LPAREN"];
       args = parseArgs[];
```

```
        eat["RPAREN"];
        {"Call", name, args}, {"Identifier", name}], "LPAREN", eat["LPAREN"];
       node = parseExpression[];
       eat["RPAREN"];
       node, "LBRACE", parseList[], "ELLIPSIS", eat["ELLIPSIS"];
       {"Identifier", "..."}, "EOF", Print["Error: Unexpected EOF in Primary"];
       {"Error", "EOF"}, (*Do NOT eat EOF*)
       _, Print["Error: Unexpected token in Primary: ", token];
       eat[token⟦1⟧];
       (*Advance to avoid infinite loop*){"Error", token}]];
   (*Parse List:{a,b,c}*)parseList[] := Module[{elements}, eat["LBRACE"];
     elements = parseArgs[];
     eat["RBRACE"];
     {"List", elements}];
   (*Parse Argument List:expr,expr,...*)parseArgs[] := Module[{args = {}, arg},
     If[currentToken⟦1⟧ ≠ "RPAREN" && currentToken⟦1⟧ ≠ "RBRACE", arg = parseExpression[];
      AppendTo[args, arg];
      While[currentToken⟦1⟧ == "COMMA", eat["COMMA"];
       arg = parseExpression[];
       AppendTo[args, arg];];];
     args];
   parseExpression[]];
```

X

```
In[•]:= (*---CONVERTER---*)(*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunction[funcName_String] :=
  Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate", "DESol",
   "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan", "Tan", "sin",
   "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc", "Csc", "csch", "Csch",
   "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh", "coth", "Coth", "sech", "Sech",
   "det", "Det", "transpose", "Transpose", "sphericalbessel", "SphericalBesselJ",
   "lambertw", "ProductLog", "gamma", "Gamma", "airy", "AiryAi", "besselJ",
   "BesselJ", "besselY", "BesselY", "hypergeometricU", "HypergeometricU", "zeta",
   "Zeta", "erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNode[nodeType_, nodeData_, childResults_] :=
  Module[{funcName, cleanFuncName, mathFunc, argsStr},
   Switch[nodeType, "Number", ToString[nodeData], "Identifier",
    (*Handle special identifiers like _Y→Y*)If[StringLength[nodeData] > 0 &&
      StringTake[nodeData, 1] == "_", StringDrop[nodeData, 1], nodeData], "BinaryOp",
    "(" <> childResults⟦1⟧ <> " " <> nodeData <> " " <> childResults⟦2⟧ <> ")",
    "UnaryOp", nodeData <> "(" <> childResults⟦1⟧ <> ")", "Equation",
    childResults⟦1⟧ <> " == " <> childResults⟦2⟧, "List",
    "{" <> StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
    (*Handle special identifiers like _Y→Y for function names too*)
    cleanFuncName = If[StringLength[funcName] > 0 && StringTake[funcName, 1] == "_",
```

```
        StringDrop[funcName, 1], funcName];
      mathFunc = MapleToMathematicaFunction[cleanFuncName];
      (*Special handling for DSolve-requires 3 arguments*)
      If[cleanFuncName == "DESol" || mathFunc == "DSolve",
       (*DSolve[equations,functions,variables]*)(*Ensure we have exactly 3 arguments,
       pad with empty lists if needed*)argsStr = StringRiffle[
         Join[childResults, Table["{}", {i, 1, Max[0, 3 - Length[childResults]]}]], ", "];
       mathFunc <> "[" <> argsStr <> "]",
       (*Default function call*)argsStr = StringRiffle[childResults, ", "];
       mathFunc <> "[" <> argsStr <> "]"], _, "Error"]];

(*Convert AST to Mathematica String-Non-
 recursive iterative version using post-order traversal*)
ToMathematicaString[ast_] := Module[{stack, outputStack, currentItem,
     node, nodeType, children, childCount, childResults, i, result, nodeData},
    (*Use two stacks:one for traversal,one for results*)stack = {{ast, False}};
    (*{node,visited}*)outputStack = {};
    (*Post-order traversal:process children before parents*)
    While[Length[stack] > 0, currentItem = Last[stack];
     node = currentItem⟦1⟧;
     nodeType = node⟦1⟧;
     If[currentItem⟦2⟧, (*Node already visited,process it*)stack = Most[stack];
      (*Determine children and extract data*)
      {children, nodeData} = Switch[nodeType, "Number", {{}, node⟦2⟧},
        "Identifier", {{}, node⟦2⟧}, "BinaryOp", {{node⟦3⟧, node⟦4⟧}, node⟦2⟧},
        "UnaryOp", {{node⟦3⟧}, node⟦2⟧}, "Equation", {{node⟦2⟧, node⟦3⟧}, ""},
        "List", {node⟦2⟧, ""}, "Call", {node⟦3⟧, node⟦2⟧}, _, {{}, ""}];
      (*Pop child results from output stack*)childCount = Length[children];
      childResults = {};
      If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
         outputStack = Most[outputStack];, {i, 1, childCount}];];
      (*Process this node*)result = ProcessASTNode[nodeType, nodeData, childResults];
      AppendTo[outputStack, result];, (*Node not visited,
      mark as visited and push children*)stack⟦Length[stack]⟧ = {node, True};
      (*Push children onto stack in reverse order (so they process left-to-right)*)
      children = Switch[nodeType, "Number", {}, "Identifier", {},
        "BinaryOp", {node⟦3⟧, node⟦4⟧}, "UnaryOp", {node⟦3⟧}, "Equation",
        {node⟦2⟧, node⟦3⟧}, "List", node⟦2⟧, "Call", node⟦3⟧, _, {}];
      Do[AppendTo[stack, {children⟦i⟧, False}];, {i, Length[children], 1, -1}];];];
    (*Return the final result*)If[Length[outputStack] > 0, Last[outputStack], "Error"]];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematica[inputStr_String] :=
  Module[{tokens, ast}, tokens = GetTokens[inputStr];
   ast = ParseTokens[tokens];
   ToMathematicaString[ast]];
```

```
In[ ]:= (*---CONVERTER---*)(*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunctionOLD[funcName_String] :=
```

```
  Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate", "DESol",
   "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan", "Tan", "sin",
   "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc", "Csc", "csch", "Csch",
   "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh", "coth", "Coth", "sech", "Sech",
   "det", "Det", "transpose", "Transpose", "sphericalbessel", "SphericalBesselJ",
   "lambertw", "ProductLog", "gamma", "Gamma", "airy", "AiryAi", "besselJ",
   "BesselJ", "besselY", "BesselY", "hypergeometricU", "HypergeometricU", "zeta",
   "Zeta", "erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNodeOLD[nodeType_, nodeData_, childResults_] :=
  Module[{funcName, cleanFuncName, mathFunc, argsStr},
   Switch[nodeType, "Number", ToString[nodeData], "Identifier",
    (*Handle special identifiers like _Y→Y*)If[StringLength[nodeData] > 0 &&
      StringTake[nodeData, 1] == "_", StringDrop[nodeData, 1], nodeData], "BinaryOp",
    "(" <> childResults⟦1⟧ <> " " <> nodeData <> " " <> childResults⟦2⟧ <> ")",
    "UnaryOp", nodeData <> "(" <> childResults⟦1⟧ <> ")", "Equation",
    childResults⟦1⟧ <> " == " <> childResults⟦2⟧, "List",
    "{" <> StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
    (*Handle special identifiers like _Y→Y for function names too*)
    cleanFuncName = If[StringLength[funcName] > 0 && StringTake[funcName, 1] == "_",
      StringDrop[funcName, 1], funcName];
    mathFunc = MapleToMathematicaFunction[cleanFuncName];
    argsStr = StringRiffle[childResults, ", "];
    mathFunc <> "[" <> argsStr <> "]", _, "Error"]];

(*Convert AST to Mathematica String-Non-
 recursive iterative version using post-order traversal*)
ToMathematicaStringOLD[ast_] := Module[{stack, outputStack, currentItem,
    node, nodeType, children, childCount, childResults, i, result, nodeData},
   (*Use two stacks:one for traversal,one for results*)stack = {{ast, False}};
   (*{node,visited}*)outputStack = {};
   (*Post-order traversal:process children before parents*)
   While[Length[stack] > 0, currentItem = Last[stack];
    node = currentItem⟦1⟧;
    nodeType = node⟦1⟧;
    If[currentItem⟦2⟧, (*Node already visited,process it*)stack = Most[stack];
     (*Determine children and extract data*)
     {children, nodeData} = Switch[nodeType, "Number", {{}, node⟦2⟧},
       "Identifier", {{}, node⟦2⟧}, "BinaryOp", {{node⟦3⟧, node⟦4⟧}, node⟦2⟧},
       "UnaryOp", {{node⟦3⟧}, node⟦2⟧}, "Equation", {{node⟦2⟧, node⟦3⟧}, ""},
       "List", {node⟦2⟧, ""}, "Call", {node⟦3⟧, node⟦2⟧}, _, {{}, ""}];
     (*Pop child results from output stack*)childCount = Length[children];
     childResults = {};
     If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
        outputStack = Most[outputStack];, {i, 1, childCount}];];
     (*Process this node*)result = ProcessASTNode[nodeType, nodeData, childResults];
```

```
      AppendTo[outputStack, result];, (*Node not visited,
      mark as visited and push children*)stack⟦Length[stack]⟧ = {node, True};
      (*Push children onto stack in reverse order (so they process left-to-right)*)
      children = Switch[nodeType, "Number", {}, "Identifier", {},
        "BinaryOp", {node⟦3⟧, node⟦4⟧}, "UnaryOp", {node⟦3⟧}, "Equation",
        {node⟦2⟧, node⟦3⟧}, "List", node⟦2⟧, "Call", node⟦3⟧, _, {}];
      Do[AppendTo[stack, {children⟦i⟧, False}];, {i, Length[children], 1, -1}];];];
    (*Return the final result*)If[Length[outputStack] > 0, Last[outputStack], "Error"]];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematicaOLD[inputStr_String] :=
  Module[{tokens, ast}, tokens = GetTokens[inputStr];
   ast = ParseTokens[tokens];
   ToMathematicaString[ast]];
```

In[∘]:=
```
sta = "{nZ10(z, t) = (c2C7*sin(C2Q1*t) + c2C8*cos(C2Q1*t))*(c2C5*sin(z)^(sqrt(-C2Q1^2
    + M^2)/6) + c2C6*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ11(z, t) =
    (c2C3*sin(C2Q1*t) + c2C4*cos(C2Q1*t))*(c2C1*sin(z)^(sqrt(-C2Q1^2
    + M^2)/6) + c2C2*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ8(z, t) =
    (((-C2Q1*c2C2*c2C3 + M*c2C6*c2C8)*cos(C2Q1*t) + sin(C2Q1*t)*(C2Q1*c2C2*c2C4
    + M*c2C6*c2C7))*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) + ((C2Q1*c2C1*c2C3
    - M*c2C5*c2C8)*cos(C2Q1*t) - sin(C2Q1*t)*(C2Q1*c2C1*c2C4 +
    M*c2C5*c2C7))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) + (-cos(M*t)*c2C10 +
    sin(M*t)*c2C9)*sqrt(-C2Q1^2 + M^2))/sqrt(-C2Q1^2 + M^2), nZ9(z, t) =
    (((C2Q1*c2C6*c2C7 + M*c2C2*c2C4)*cos(C2Q1*t) + sin(C2Q1*t)*(-C2Q1*c2C6*c2C8
    + M*c2C2*c2C3))*sin(z)^(-sqrt(-C2Q1^2 + M^2)/3) + sqrt(-C2Q1^2 +
    M^2)*(cos(M*t)*c2C9 + sin(M*t)*c2C10)*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) +
    (-C2Q1*c2C5*c2C7 - M*c2C1*c2C4)*cos(C2Q1*t) - sin(C2Q1*t)*(-C2Q1*c2C5*c2C8
    + M*c2C1*c2C3))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6)/sqrt(-C2Q1^2 + M^2)}";
```

In[ ]:= **ConvertMapleToMathematica[sta]**

Out[ ]=

```
{nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5 * (Sin[z]
  ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) + (c2C6 * (Sin[z] ^ (-(Sqrt[((-(C2Q1)
  ^ 2) + (M ^ 2))]) / 6))))), nZ11[z, t] == (((c2C3 * Sin[(C2Q1 * t)]) + (c2C4 *
  Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6)))
  + (c2C2 * (Sin[z] ^ (-(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) / 6))))), nZ8[z, t] ==
  ((((((((-(C2Q1) * c2C2) * c2C3) + ((M * c2C6) * c2C8)) * Cos[(C2Q1 * t)]) +
  (Sin[(C2Q1 * t)] * (((C2Q1 * c2C2) * c2C4) + ((M * c2C6) * c2C7)))) * (Sin[z] ^
  (-(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) / 6))) + (((((C2Q1 * c2C1) * c2C3) - ((M *
  c2C5) * c2C8)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C1) * c2C4)
  + ((M * c2C5) * c2C7)))) * (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6)))) +
  (((-(Cos[(M * t)]) * c2C10) + (Sin[(M * t)] * c2C9)) * Sqrt[((-(C2Q1) ^ 2) + (M ^
  2))]))) / Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]), nZ9[z, t] == ((((((((((C2Q1 * c2C6) *
  c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1)
  * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (-(Sqrt[((-(C2Q1) ^ 2) +
  (M ^ 2))]) / 3))) + ((Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] * ((Cos[(M * t)] * c2C9)
  + (Sin[(M * t)] * c2C10))) * (Sin[z] ^ (-(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) /
  6)))) + ((((-(C2Q1) * c2C5) * c2C7) - ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)])) -
  (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z]
  ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) / Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])}
```

```
In[•]:= sti =
  "{nZ10(z,t)=(c2C7*sin(C2Q1*t)+c2C8*cos(C2Q1*t))*(c2C5*sin(z)^(1/6*sqrt(-C2Q1^2+M^2))+
    c2C6*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))),nZ11(z,t)=(c2C3*sin(C2Q1*t)+c2C4*cos(C2Q1*t)
    )*(c2C1*sin(z)^(1/6*sqrt(-C2Q1^2+M^2))+c2C2*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))),nZ8(z
    ,t)=1/6*(6*sqrt(-C2Q1^2+M^2)*c2C6*(c2C7*sin(C2Q1*t)+c2C8*cos(C2Q1*t))*sin(z)^(-1/6
    *sqrt(-C2Q1^2+M^2))-6*sqrt(-C2Q1^2+M^2)*c2C5*(c2C7*sin(C2Q1*t)+c2C8*cos(C2Q1*t))*
    sin(z)^(1/6*sqrt(-C2Q1^2+M^2))+(-6*M*c2C10-Int(-cos(M*t)*((-C2Q1^2+M^2)*Int(-cos(z
    )*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*
    c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)
    +sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6
    *sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*
    c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M
    ^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1
    *c2C3)))),t))*cos(M*t)+(6*M*c2C9-Int(-sin(M*t)*((-C2Q1^2+M^2)*Int(-cos(z)*(((C2Q1*
    c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(
    z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t
    )*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1
    ^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*
    c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M^2))*((C2Q1*
    c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3)))),t)
    )*sin(M*t)-C2Q1*Int(-cos(z)*(((-C2Q1*c2C6*c2C8+M*c2C2*c2C3)*cos(C2Q1*t)-sin(C2Q1*t
    )*(C2Q1*c2C6*c2C7+M*c2C2*c2C4))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((-C2Q1*c2C5*
    c2C8+M*c2C1*c2C3)*cos(C2Q1*t)-sin(C2Q1*t)*(C2Q1*c2C5*c2C7+M*c2C1*c2C4))*sin(z)^(1/
    6*sqrt(-C2Q1^2+M^2)-1)),z))/M,nZ9(z,t)=1/6*((6*M*c2C9-Int(-sin(M*t)*((-C2Q1^2+M^2)
    *Int(-cos(z)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*
    c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)
    *cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M
    ^2)-1)),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1
    *t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*
    sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*
    c2C5*c2C8+M*c2C1*c2C3)))),t))*cos(M*t)+(6*M*c2C10+Int(-cos(M*t)*((-C2Q1^2+M^2)*Int(-cc
    *c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(
    C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1
    )),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q
     2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8
    +M*c2C1*c2C3)))),t))*sin(M*t)+M*Int(-cos(z)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(
    C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-
    1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*
    c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z))/M}";

In[•]:= ConvertMapleToMathematica[sti]

Out[•]=
  {nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5 * (Sin[z]
    ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))) + (c2C6 * (Sin[z] ^ ((-(1) / 6)
    * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))))), nZ11[z, t] == (((c2C3 * Sin[(C2Q1 * t)])
    + (c2C4 * Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^
    2) + (M ^ 2))]))) + (c2C2 * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
    2))]))))), nZ8[z, t] == (((1 / 6) * (((((((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])
```

```
* c2C6) * ((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)]))) * (Sin[z] ^
((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))) - ((((6 * Sqrt[((-(C2Q1) ^ 2)
+ (M ^ 2))]) * c2C5) * ((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)]))) *
(Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])))) + ((((-(6) * M) * c2C10) -
Integrate[(-(Cos[(M * t)]) * ((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) *
((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1
* t)] * (((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1)
/ 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))) + (((((C2Q1 * c2C5) * c2C7) +
((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5)
* c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) +
(M ^ 2))]) - 1))))), z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * ((((((C2Q1
* c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] *
(((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((-(1) / 6) *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) +
(M ^ 2))])) * ((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)])
+ (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))))))),
t]) * Cos[(M * t)])) + ((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) *
((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * ((((((C2Q1 * c2C6) * c2C7)
+ ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) *
c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]) - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) *
Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1)
* c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))),
z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * ((((((C2Q1 * c2C6) * c2C7) +
((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6)
* c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))]))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) *
((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1
* t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))))))), t]) * Sin[(M
* t)])) - (C2Q1 * Integrate[(-(Cos[z]) * ((((((-(C2Q1) * c2C6) * c2C8) + ((M *
c2C2) * c2C3)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C6) * c2C7)
+ ((M * c2C2) * c2C4)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]) - 1))) + ((((((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)) * Cos[(C2Q1
* t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))), z]))) / M),
nZ9[z, t] == (((1 / 6) * (((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) *
((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * ((((((C2Q1 * c2C6) * c2C7)
+ ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) *
c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]) - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) *
Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1)
* c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))),
z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * ((((((C2Q1 * c2C6) * c2C7) +
((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6)
* c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))]))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) *
((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1
```

```
* t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))))))), t]) * Cos[(M
* t)]) + ((((6 * M) * c2C10) + Integrate[(-(Cos[(M * t)]) * ((((-(C2Q1) ^ 2)
+ (M ^ 2)) * Integrate[(-(Cos[z]) * ((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]) - 1))) + ((((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))
* (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))), z]) - ((6 *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * ((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) * (((((C2Q1
* c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)]
* (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))))), t]) * Sin[(M *
t)])) + (M * Integrate[(-(Cos[z]) * ((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]) - 1))) + ((((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))
* (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))), z]))) / M)}
```

gtry and Γ and ...

In[∘]:= `gtry = MatrixMetric44`

Out[∘]=

$$\{\{\text{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \{0, 0, e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}, 0, 0\}, \{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}\}\}$$

In[∘]:= `Protect[gtry]`

Out[∘]=

{gtry}

In[∘]:= `Unprotect[ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG]`

Out[∘]=

{}

In[∘]:= `result = {ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG} = rt[gtry];`

Mon 15 Dec 2025 12:05:25 GMT-8

{5.16043, Null}

Mon 15 Dec 2025 12:05:31 GMT-8

In[∘]:= `Protect[ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG]`

Out[∘]=

{ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG}

verify $g_{\mu\nu|\alpha} = 0$:

In[ ]:=
```
Simplify[Table[D[gtry[[j, k]], X[[i]] ] -
    Sum[ gtry[[k, s]] × Γ[[s, i, j]], {s, 1, DIM8}] - Sum[ gtry[[j, s]] × Γ[[s, i, k]], {s, 1, DIM8}],
   {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Out[ ]=
```
{0}
```

In[ ]:=
```
Table[g[j][k] →
   If[j == k, ToExpression["((g[" <> ToString[j] <> "][" <> ToString[k] <> "][#1,#2])&)"],
    (0 &)], {j, 0, 7}, {k, 0, 7}];
sg = % // Flatten
```

Out[ ]=
```
{g[0][0] → (g[0][0][#1, #2] &), g[0][1] → (0 &), g[0][2] → (0 &),
 g[0][3] → (0 &), g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &),
 g[0][7] → (0 &), g[1][0] → (0 &), g[1][1] → (g[1][1][#1, #2] &), g[1][2] → (0 &),
 g[1][3] → (0 &), g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &),
 g[1][7] → (0 &), g[2][0] → (0 &), g[2][1] → (0 &), g[2][2] → (g[2][2][#1, #2] &),
 g[2][3] → (0 &), g[2][4] → (0 &), g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &),
 g[3][0] → (0 &), g[3][1] → (0 &), g[3][2] → (0 &), g[3][3] → (g[3][3][#1, #2] &),
 g[3][4] → (0 &), g[3][5] → (0 &), g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &),
 g[4][1] → (0 &), g[4][2] → (0 &), g[4][3] → (0 &), g[4][4] → (g[4][4][#1, #2] &),
 g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &), g[5][0] → (0 &), g[5][1] → (0 &),
 g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &), g[5][5] → (g[5][5][#1, #2] &),
 g[5][6] → (0 &), g[5][7] → (0 &), g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &),
 g[6][3] → (0 &), g[6][4] → (0 &), g[6][5] → (0 &), g[6][6] → (g[6][6][#1, #2] &),
 g[6][7] → (0 &), g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &),
 g[7][4] → (0 &), g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (g[7][7][#1, #2] &)}
```

In[ ]:=
```
g[7][7][x0, x4] /. ssgm4488
% /. sx0x4
```

Out[ ]=
$$-e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}$$

Out[ ]=
$$-e^{-2\,a4[t]}\,\mathrm{Sin}[z]^{1/3}$$

In[ ]:=
```
MatrixMetric44
```

Out[ ]=
$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

```
In[∘]:= ssgm4488[x0, x4] /. sx0x4

Out[∘]=
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
  g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
  g[1][1] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
  g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
  g[2][1] → (0 &), g[2][2] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
  g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
  g[3][2] → (0 &), g[3][3] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
  g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
  g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
  g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
  g[5][5] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
  g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
  g[6][5] → (0 &), g[6][6] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
  g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
  g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &)}[z/(6 H), t/H]

In[∘]:= (*ssgGzt={G[0][0]→((Cot[#1]^2)&),G[0][1]→((0)&),G[0][2]→((0)&),
  G[0][3]→((0)&),G[0][4]→((0)&),G[0][5]→((0)&),G[0][6]→((0)&),G[0][7]→((0)&),
  G[1][0]→((0)&),G[1][1]→((ⅇ^(2*A4[#2]) Sin[#1]^(1/3))&),G[1][2]→((0)&),G[1][3]→((0)&),
  G[1][4]→((0)&),G[1][5]→((0)&),G[1][6]→((0)&),G[1][7]→((0)&),G[2][0]→((0)&),
  G[2][1]→((0)&),G[2][2]→((ⅇ^(2*A4[#2]) Sin[#1]^(1/3))&),G[2][3]→((0)&),G[2][4]→((0)&),
  G[2][5]→((0)&),G[2][6]→((0)&),G[2][7]→((0)&),G[3][0]→((0)&),G[3][1]→((0)&),
  G[3][2]→((0)&),G[3][3]→((ⅇ^(2*A4[#2]) Sin[#1]^(1/3))&),G[3][4]→((0)&),G[3][5]→((0)&),
  G[3][6]→((0)&),G[3][7]→((0)&),G[4][0]→((0)&),G[4][1]→((0)&),G[4][2]→((0)&),
  G[4][3]→((0)&),G[4][4]→((-1)&),G[4][5]→((0)&),G[4][6]→((0)&),G[4][7]→((0)&),
  G[5][0]→((0)&),G[5][1]→((0)&),G[5][2]→((0)&),G[5][3]→((0)&),G[5][4]→((0)&),
  G[5][5]→((-ⅇ^(-2*A4[#2]) Sin[#1]^(1/3))&),G[5][6]→((0)&),G[5][7]→((0)&),
  G[6][0]→((0)&),G[6][1]→((0)&),G[6][2]→((0)&),G[6][3]→((0)&),G[6][4]→((0)&),
  G[6][5]→((0)&),G[6][6]→((-ⅇ^(-2*A4[#2]) Sin[#1]^(1/3))&),G[6][7]→((0)&),
  G[7][0]→((0)&),G[7][1]→((0)&),G[7][2]→((0)&),G[7][3]→((0)&),G[7][4]→((0)&),
  G[7][5]→((0)&),G[7][6]→((0)&),G[7][7]→((-ⅇ^(-2*A4[#2]) Sin[#1]^(1/3))&)}*)

In[∘]:= (*G[#][#][z,t]/.ssgGzt&/@Range[0,7]*)

In[∘]:= detgg = Det[g4488 /. sg] // FullSimplify[#, constraintVars] &

Out[∘]=
g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
 g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]
```

In[∘]:= g4488 /. sg // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} g[0][0][x0, x4] & 0 & 0 & 0 & 0 & 0 \\ 0 & g[1][1][x0, x4] & 0 & 0 & 0 & 0 \\ 0 & 0 & g[2][2][x0, x4] & 0 & 0 & 0 \\ 0 & 0 & 0 & g[3][3][x0, x4] & 0 & 0 \\ 0 & 0 & 0 & 0 & g[4][4][x0, x4] & 0 \\ 0 & 0 & 0 & 0 & 0 & g[5][5][x0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[∘]:= MatrixMetric44.η4488

(*eAa=*)FullSimplify[$\sqrt{\%}$, constraintVars]

Out[∘]=

$\{\{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2 a4[H x4]} Sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \{0, 0, e^{2 a4[H x4]} Sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2 a4[H x4]} Sin[6 H x0]^{1/3}, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-2 a4[H x4]} Sin[6 H x0]^{1/3}, 0, 0\}, \{0, 0, 0, 0, 0, 0, e^{-2 a4[H x4]} Sin[6 H x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, e^{-2 a4[H x4]} Sin[6 H x0]^{1/3}\}\}$

Out[∘]=

$\{\{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0\}, \{0, \sqrt{e^{2 a4[H x4]}} Sin[6 H x0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \{0, 0, \sqrt{e^{2 a4[H x4]}} Sin[6 H x0]^{1/6}, 0, 0, 0, 0, 0\}, \{0, 0, 0, \sqrt{e^{2 a4[H x4]}} Sin[6 H x0]^{1/6}, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} Sin[6 H x0]^{1/6}, 0, 0\}, \{0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} Sin[6 H x0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} Sin[6 H x0]^{1/6}\}\}$

eAa

In[∘]:= g4488.η4488 /. sg

(**eAa** = FullSimplify[$\sqrt{\%}$, constraintVars]) // MatrixForm

Out[∘]=

{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, -g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, -g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, -g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, -g[7][7][x0, x4]}}

Out[∘]//MatrixForm=

$$\begin{pmatrix} \sqrt{g[0][0][x0, x4]} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{g[1][1][x0, x4]} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{g[2][2][x0, x4]} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{g[3][3][x0, x4]} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{-g[4][4][x0, x4]} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[∘]:=
```
eAa
ssgm4488
(eAa04 = %% /. % // FullSimplify[#, constraintVars] &) // MatrixForm
```

Out[∘]=

$$\{\{\sqrt{g[0][0][x0, x4]}, 0, 0, 0, 0, 0, 0, 0\}, \{0, \sqrt{g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0\}, \{0, 0, \sqrt{g[2][2][x0, x4]}, 0, 0, 0, 0, 0\}, \{0, 0, 0, \sqrt{g[3][3][x0, x4]}, 0, 0, 0, 0\}, \{0, 0, 0, 0, \sqrt{-g[4][4][x0, x4]}, 0, 0, 0\}, \{0, 0, 0, 0, 0, \sqrt{-g[5][5][x0, x4]}, 0, 0\}, \{0, 0, 0, 0, 0, 0, \sqrt{-g[6][6][x0, x4]}, 0\}, \{0, 0, 0, 0, 0, 0, 0, \sqrt{-g[7][7][x0, x4]}\}\}$$

Out[∘]=

$$\{g[0][0] \to (\mathrm{Cot}[6\,H\,\#1]^2\,\&),\ g[0][1] \to (0\,\&),\ g[0][2] \to (0\,\&),\ g[0][3] \to (0\,\&),\ g[0][4] \to (0\,\&),\ g[0][5] \to (0\,\&),\ g[0][6] \to (0\,\&),\ g[0][7] \to (0\,\&),\ g[1][0] \to (0\,\&),\ g[1][1] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[1][2] \to (0\,\&),\ g[1][3] \to (0\,\&),\ g[1][4] \to (0\,\&),\ g[1][5] \to (0\,\&),\ g[1][6] \to (0\,\&),\ g[1][7] \to (0\,\&),\ g[2][0] \to (0\,\&),\ g[2][1] \to (0\,\&),\ g[2][2] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[2][3] \to (0\,\&),\ g[2][4] \to (0\,\&),\ g[2][5] \to (0\,\&),\ g[2][6] \to (0\,\&),\ g[2][7] \to (0\,\&),\ g[3][0] \to (0\,\&),\ g[3][1] \to (0\,\&),\ g[3][2] \to (0\,\&),\ g[3][3] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[3][4] \to (0\,\&),\ g[3][5] \to (0\,\&),\ g[3][6] \to (0\,\&),\ g[3][7] \to (0\,\&),\ g[4][0] \to (0\,\&),\ g[4][1] \to (0\,\&),\ g[4][2] \to (0\,\&),\ g[4][3] \to (0\,\&),\ g[4][4] \to (-1\,\&),\ g[4][5] \to (0\,\&),\ g[4][6] \to (0\,\&),\ g[4][7] \to (0\,\&),\ g[5][0] \to (0\,\&),\ g[5][1] \to (0\,\&),\ g[5][2] \to (0\,\&),\ g[5][3] \to (0\,\&),\ g[5][4] \to (0\,\&),\ g[5][5] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[5][6] \to (0\,\&),\ g[5][7] \to (0\,\&),\ g[6][0] \to (0\,\&),\ g[6][1] \to (0\,\&),\ g[6][2] \to (0\,\&),\ g[6][3] \to (0\,\&),\ g[6][4] \to (0\,\&),\ g[6][5] \to (0\,\&),\ g[6][6] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[6][7] \to (0\,\&),\ g[7][0] \to (0\,\&),\ g[7][1] \to (0\,\&),\ g[7][2] \to (0\,\&),\ g[7][3] \to (0\,\&),\ g[7][4] \to (0\,\&),\ g[7][5] \to (0\,\&),\ g[7][6] \to (0\,\&),\ g[7][7] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&)\}$$

Out[∘]//MatrixForm=

$$\left(\begin{array}{cccccc}
\mathrm{Cot}[6\,H\,x0] & 0 & 0 & 0 & 0 & \\
0 & \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & 0 & 0 & \\
0 & 0 & \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & 0 & \\
0 & 0 & 0 & \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & \\
0 & 0 & 0 & 0 & 1 & \\
0 & 0 & 0 & 0 & 0 & \sqrt{e^{-2\,a4}} \\
0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 &
\end{array}\right.$$

OCTAD $e_{\alpha\leftarrow\text{spacetime}}^{(A)\longleftarrow\text{Lorentz}}$:

- $\mathbf{e_\alpha^{(A)}}$ **=** $\mathbf{e_\alpha^{(A)}}$ **= eAa**

In[∘]:=
```
(sge_α^(A) = Thread[Flatten[e_α^(A)] → Flatten[eAa]]) (*//Column*)
```

Out[∘]=
```
{E_α^(A)[0, 0] → Sqrt[g[0][0][x0, x4]], E_α^(A)[0, 1] → 0, E_α^(A)[0, 2] → 0, E_α^(A)[0, 3] → 0, E_α^(A)[0, 4] → 0,
 E_α^(A)[0, 5] → 0, E_α^(A)[0, 6] → 0, E_α^(A)[0, 7] → 0, E_α^(A)[1, 0] → 0, E_α^(A)[1, 1] → Sqrt[g[1][1][x0, x4]],
 E_α^(A)[1, 2] → 0, E_α^(A)[1, 3] → 0, E_α^(A)[1, 4] → 0, E_α^(A)[1, 5] → 0, E_α^(A)[1, 6] → 0, E_α^(A)[1, 7] → 0,
 E_α^(A)[2, 0] → 0, E_α^(A)[2, 1] → 0, E_α^(A)[2, 2] → Sqrt[g[2][2][x0, x4]], E_α^(A)[2, 3] → 0, E_α^(A)[2, 4] → 0,
 E_α^(A)[2, 5] → 0, E_α^(A)[2, 6] → 0, E_α^(A)[2, 7] → 0, E_α^(A)[3, 0] → 0, E_α^(A)[3, 1] → 0, E_α^(A)[3, 2] → 0,
 E_α^(A)[3, 3] → Sqrt[g[3][3][x0, x4]], E_α^(A)[3, 4] → 0, E_α^(A)[3, 5] → 0, E_α^(A)[3, 6] → 0, E_α^(A)[3, 7] → 0,
 E_α^(A)[4, 0] → 0, E_α^(A)[4, 1] → 0, E_α^(A)[4, 2] → 0, E_α^(A)[4, 3] → 0, E_α^(A)[4, 4] → Sqrt[-g[4][4][x0, x4]],
 E_α^(A)[4, 5] → 0, E_α^(A)[4, 6] → 0, E_α^(A)[4, 7] → 0, E_α^(A)[5, 0] → 0, E_α^(A)[5, 1] → 0, E_α^(A)[5, 2] → 0,
 E_α^(A)[5, 3] → 0, E_α^(A)[5, 4] → 0, E_α^(A)[5, 5] → Sqrt[-g[5][5][x0, x4]], E_α^(A)[5, 6] → 0, E_α^(A)[5, 7] → 0,
 E_α^(A)[6, 0] → 0, E_α^(A)[6, 1] → 0, E_α^(A)[6, 2] → 0, E_α^(A)[6, 3] → 0, E_α^(A)[6, 4] → 0, E_α^(A)[6, 5] → 0,
 E_α^(A)[6, 6] → Sqrt[-g[6][6][x0, x4]], E_α^(A)[6, 7] → 0, E_α^(A)[7, 0] → 0, E_α^(A)[7, 1] → 0, E_α^(A)[7, 2] → 0,
 E_α^(A)[7, 3] → 0, E_α^(A)[7, 4] → 0, E_α^(A)[7, 5] → 0, E_α^(A)[7, 6] → 0, E_α^(A)[7, 7] → Sqrt[-g[7][7][x0, x4]]}
```

In[∘]:=
```
(*Symbolize[sge_α^(A)]*)
```

In[∘]:=
```
(*Symbolize[sgtrye_α^(A)]*)
```

In[∘]:=
```
(sgtrye_α^(A) = Thread[Flatten[e_α^(A)] → Flatten[eAa04]]) (*//Column*)
```

Out[∘]=
```
{E_α^(A)[0, 0] → Cot[6 H x0], E_α^(A)[0, 1] → 0, E_α^(A)[0, 2] → 0, E_α^(A)[0, 3] → 0,
 E_α^(A)[0, 4] → 0, E_α^(A)[0, 5] → 0, E_α^(A)[0, 6] → 0, E_α^(A)[0, 7] → 0, E_α^(A)[1, 0] → 0,
 E_α^(A)[1, 1] → Sqrt[ⅇ^(2 a4[H x4])] Sin[6 H x0]^(1/6), E_α^(A)[1, 2] → 0, E_α^(A)[1, 3] → 0, E_α^(A)[1, 4] → 0,
 E_α^(A)[1, 5] → 0, E_α^(A)[1, 6] → 0, E_α^(A)[1, 7] → 0, E_α^(A)[2, 0] → 0, E_α^(A)[2, 1] → 0,
 E_α^(A)[2, 2] → Sqrt[ⅇ^(2 a4[H x4])] Sin[6 H x0]^(1/6), E_α^(A)[2, 3] → 0, E_α^(A)[2, 4] → 0, E_α^(A)[2, 5] → 0,
 E_α^(A)[2, 6] → 0, E_α^(A)[2, 7] → 0, E_α^(A)[3, 0] → 0, E_α^(A)[3, 1] → 0, E_α^(A)[3, 2] → 0,
 E_α^(A)[3, 3] → Sqrt[ⅇ^(2 a4[H x4])] Sin[6 H x0]^(1/6), E_α^(A)[3, 4] → 0, E_α^(A)[3, 5] → 0, E_α^(A)[3, 6] → 0,
 E_α^(A)[3, 7] → 0, E_α^(A)[4, 0] → 0, E_α^(A)[4, 1] → 0, E_α^(A)[4, 2] → 0, E_α^(A)[4, 3] → 0,
 E_α^(A)[4, 4] → 1, E_α^(A)[4, 5] → 0, E_α^(A)[4, 6] → 0, E_α^(A)[4, 7] → 0, E_α^(A)[5, 0] → 0, E_α^(A)[5, 1] → 0,
 E_α^(A)[5, 2] → 0, E_α^(A)[5, 3] → 0, E_α^(A)[5, 4] → 0, E_α^(A)[5, 5] → Sqrt[ⅇ^(-2 a4[H x4])] Sin[6 H x0]^(1/6),
 E_α^(A)[5, 6] → 0, E_α^(A)[5, 7] → 0, E_α^(A)[6, 0] → 0, E_α^(A)[6, 1] → 0, E_α^(A)[6, 2] → 0,
 E_α^(A)[6, 3] → 0, E_α^(A)[6, 4] → 0, E_α^(A)[6, 5] → 0, E_α^(A)[6, 6] → Sqrt[ⅇ^(-2 a4[H x4])] Sin[6 H x0]^(1/6),
 E_α^(A)[6, 7] → 0, E_α^(A)[7, 0] → 0, E_α^(A)[7, 1] → 0, E_α^(A)[7, 2] → 0, E_α^(A)[7, 3] → 0,
 E_α^(A)[7, 4] → 0, E_α^(A)[7, 5] → 0, E_α^(A)[7, 6] → 0, E_α^(A)[7, 7] → Sqrt[ⅇ^(-2 a4[H x4])] Sin[6 H x0]^(1/6)}
```

In[ ]:= $\left(\text{sgtrye}^{\alpha}_{(A)} = \text{Thread}\left[\text{Flatten}\left[e^{\alpha}_{(A)}\right] \to \text{Flatten}\left[\text{Inverse}\left[e^{(A)}_{\alpha} \text{ /. sgtrye}^{(A)}_{\alpha}\right]\right]\right]\right)$ (*//Column*)

Out[ ]=

$$\Big\{E^{\alpha}_{(A)}[0,0]\to \text{Tan}[6\,H\,x0],\ E^{\alpha}_{(A)}[0,1]\to 0,\ E^{\alpha}_{(A)}[0,2]\to 0,\ E^{\alpha}_{(A)}[0,3]\to 0,\ E^{\alpha}_{(A)}[0,4]\to 0,\ E^{\alpha}_{(A)}[0,5]\to 0,\ E^{\alpha}_{(A)}[0,6]\to 0,\ E^{\alpha}_{(A)}[0,7]\to 0,\ E^{\alpha}_{(A)}[1,0]\to 0,\ E^{\alpha}_{(A)}[1,1]\to \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[1,2]\to 0,\ E^{\alpha}_{(A)}[1,3]\to 0,\ E^{\alpha}_{(A)}[1,4]\to 0,\ E^{\alpha}_{(A)}[1,5]\to 0,\ E^{\alpha}_{(A)}[1,6]\to 0,\ E^{\alpha}_{(A)}[1,7]\to 0,\ E^{\alpha}_{(A)}[2,0]\to 0,\ E^{\alpha}_{(A)}[2,1]\to 0,\ E^{\alpha}_{(A)}[2,2]\to \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[2,3]\to 0,\ E^{\alpha}_{(A)}[2,4]\to 0,\ E^{\alpha}_{(A)}[2,5]\to 0,\ E^{\alpha}_{(A)}[2,6]\to 0,\ E^{\alpha}_{(A)}[2,7]\to 0,\ E^{\alpha}_{(A)}[3,0]\to 0,\ E^{\alpha}_{(A)}[3,1]\to 0,\ E^{\alpha}_{(A)}[3,2]\to 0,\ E^{\alpha}_{(A)}[3,3]\to \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[3,4]\to 0,\ E^{\alpha}_{(A)}[3,5]\to 0,\ E^{\alpha}_{(A)}[3,6]\to 0,\ E^{\alpha}_{(A)}[3,7]\to 0,\ E^{\alpha}_{(A)}[4,0]\to 0,\ E^{\alpha}_{(A)}[4,1]\to 0,\ E^{\alpha}_{(A)}[4,2]\to 0,\ E^{\alpha}_{(A)}[4,3]\to 0,\ E^{\alpha}_{(A)}[4,4]\to 1,\ E^{\alpha}_{(A)}[4,5]\to 0,\ E^{\alpha}_{(A)}[4,6]\to 0,\ E^{\alpha}_{(A)}[4,7]\to 0,\ E^{\alpha}_{(A)}[5,0]\to 0,\ E^{\alpha}_{(A)}[5,1]\to 0,\ E^{\alpha}_{(A)}[5,2]\to 0,\ E^{\alpha}_{(A)}[5,3]\to 0,\ E^{\alpha}_{(A)}[5,4]\to 0,\ E^{\alpha}_{(A)}[5,5]\to \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[5,6]\to 0,\ E^{\alpha}_{(A)}[5,7]\to 0,\ E^{\alpha}_{(A)}[6,0]\to 0,\ E^{\alpha}_{(A)}[6,1]\to 0,\ E^{\alpha}_{(A)}[6,2]\to 0,\ E^{\alpha}_{(A)}[6,3]\to 0,\ E^{\alpha}_{(A)}[6,4]\to 0,\ E^{\alpha}_{(A)}[6,5]\to 0,\ E^{\alpha}_{(A)}[6,6]\to \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[6,7]\to 0,\ E^{\alpha}_{(A)}[7,0]\to 0,\ E^{\alpha}_{(A)}[7,1]\to 0,\ E^{\alpha}_{(A)}[7,2]\to 0,\ E^{\alpha}_{(A)}[7,3]\to 0,\ E^{\alpha}_{(A)}[7,4]\to 0,\ E^{\alpha}_{(A)}[7,5]\to 0,\ E^{\alpha}_{(A)}[7,6]\to 0,\ E^{\alpha}_{(A)}[7,7]\to \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}}\Big\}$$

In[ ]:= $\text{gtrye}^{(A)}_{\alpha} = \left(e^{(A)}_{\alpha} \text{ /. sgtrye}^{(A)}_{\alpha}\right)$ /. subsDefects

Out[ ]=

$$\Big\{\{\text{Cot}[6\,H\,x0],0,0,0,0,0,0,0\},\ \{0,e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0,0\},\ \{0,0,e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0\},\ \{0,0,0,e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0,0,0,0\},\ \{0,0,0,0,1,0,0,0\},\ \{0,0,0,0,0,e^{-a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0,0\},\ \{0,0,0,0,0,0,e^{-a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0\},\ \{0,0,0,0,0,0,0,e^{-a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6}\}\Big\}$$

In[ ]:= $\text{gtrye}^{\alpha}_{(A)} = \left(e^{\alpha}_{(A)} \text{ /. sgtrye}^{\alpha}_{(A)}\right)$ /. subsDefects

Out[ ]=

$$\Big\{\{\text{Tan}[6\,H\,x0],0,0,0,0,0,0,0\},\ \Big\{0,\frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0,0\Big\},\ \Big\{0,0,\frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0\Big\},\ \Big\{0,0,0,\frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0\Big\},\ \{0,0,0,0,1,0,0,0\},\ \Big\{0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0\Big\},\ \Big\{0,0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0\Big\},\ \Big\{0,0,0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}}\Big\}\Big\}$$

In[•]:=
$$e_\alpha^{(A)}$$
$$\%\ /.\ sgtrye_\alpha^{(A)}\ //\ \mathrm{MatrixForm}$$

Out[•]=
$$\{\{E_\alpha^{(A)}[0,0], E_\alpha^{(A)}[0,1], E_\alpha^{(A)}[0,2], E_\alpha^{(A)}[0,3], E_\alpha^{(A)}[0,4], E_\alpha^{(A)}[0,5], E_\alpha^{(A)}[0,6], E_\alpha^{(A)}[0,7]\},$$
$$\{E_\alpha^{(A)}[1,0], E_\alpha^{(A)}[1,1], E_\alpha^{(A)}[1,2], E_\alpha^{(A)}[1,3], E_\alpha^{(A)}[1,4], E_\alpha^{(A)}[1,5], E_\alpha^{(A)}[1,6], E_\alpha^{(A)}[1,7]\},$$
$$\{E_\alpha^{(A)}[2,0], E_\alpha^{(A)}[2,1], E_\alpha^{(A)}[2,2], E_\alpha^{(A)}[2,3], E_\alpha^{(A)}[2,4], E_\alpha^{(A)}[2,5], E_\alpha^{(A)}[2,6], E_\alpha^{(A)}[2,7]\},$$
$$\{E_\alpha^{(A)}[3,0], E_\alpha^{(A)}[3,1], E_\alpha^{(A)}[3,2], E_\alpha^{(A)}[3,3], E_\alpha^{(A)}[3,4], E_\alpha^{(A)}[3,5], E_\alpha^{(A)}[3,6], E_\alpha^{(A)}[3,7]\},$$
$$\{E_\alpha^{(A)}[4,0], E_\alpha^{(A)}[4,1], E_\alpha^{(A)}[4,2], E_\alpha^{(A)}[4,3], E_\alpha^{(A)}[4,4], E_\alpha^{(A)}[4,5], E_\alpha^{(A)}[4,6], E_\alpha^{(A)}[4,7]\},$$
$$\{E_\alpha^{(A)}[5,0], E_\alpha^{(A)}[5,1], E_\alpha^{(A)}[5,2], E_\alpha^{(A)}[5,3], E_\alpha^{(A)}[5,4], E_\alpha^{(A)}[5,5], E_\alpha^{(A)}[5,6], E_\alpha^{(A)}[5,7]\},$$
$$\{E_\alpha^{(A)}[6,0], E_\alpha^{(A)}[6,1], E_\alpha^{(A)}[6,2], E_\alpha^{(A)}[6,3], E_\alpha^{(A)}[6,4], E_\alpha^{(A)}[6,5], E_\alpha^{(A)}[6,6], E_\alpha^{(A)}[6,7]\},$$
$$\{E_\alpha^{(A)}[7,0], E_\alpha^{(A)}[7,1], E_\alpha^{(A)}[7,2], E_\alpha^{(A)}[7,3], E_\alpha^{(A)}[7,4], E_\alpha^{(A)}[7,5], E_\alpha^{(A)}[7,6], E_\alpha^{(A)}[7,7]\}\}$$

Out[•]//MatrixForm=
$$\left(\begin{array}{cccccc}
\mathrm{Cot}[6\,H\,x0] & 0 & 0 & 0 & 0 & \\
0 & \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & 0 & 0 & \\
0 & 0 & \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & 0 & \\
0 & 0 & 0 & \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & \\
0 & 0 & 0 & 0 & 1 & \\
0 & 0 & 0 & 0 & 0 & \sqrt{e^{-2\,a4}} \\
0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 &
\end{array}\right.$$

In[•]:=
$$e\eta e = \mathrm{FullSimplify}\left[\mathrm{Transpose}\left[e_\alpha^{(A)}\right].\eta 4488.e_\alpha^{(A)},\ \mathrm{constraintVars}\right];$$
$$\%\ /.\ sge_\alpha^{(A)}$$
$$\%\%\ /.\ sgtrye_\alpha^{(A)}$$
$$\% == gtry\ //\ \mathrm{FullSimplify}[\#,\ \mathrm{constraintVars}]\ \&$$

Out[•]=
```
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}
```

Out[•]=
$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

Out[•]=
True

In[•]:=
gtry

Out[•]=
$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

In[•]:=
$$(*\mathrm{FullSimplify}\left[\mathrm{Transpose}\left[e_\alpha^{(A)}\right].\eta 4488.e_\alpha^{(A)},\mathrm{constraintVars}\right]-\mathrm{MatrixMetric44}*)$$

In[∘]:= **ssgm4488**

Out[∘]=

```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
 g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
 g[1][1] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
 g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
 g[2][1] → (0 &), g[2][2] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
 g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
 g[3][2] → (0 &), g[3][3] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
 g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
 g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
 g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
 g[5][5] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
 g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
 g[6][5] → (0 &), g[6][6] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
 g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
 g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &)}
```

In[∘]:= **MatrixMetric44 // MatrixForm**

Out[∘]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0]^2 & 0 & 0 & 0 & 0 & \\ 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 & \\ 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & \\ 0 & 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & \\ 0 & 0 & 0 & 0 & -1 & \\ 0 & 0 & 0 & 0 & 0 & -e^{-2\,a4[H\,x4]}\,\mathrm{S} \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{pmatrix}$$

In[∘]:= $\mathbf{e}_\alpha^{(\mathbf{A})}$

Out[∘]=

$$\{\{E_\alpha^{(A)}[0,0], E_\alpha^{(A)}[0,1], E_\alpha^{(A)}[0,2], E_\alpha^{(A)}[0,3], E_\alpha^{(A)}[0,4], E_\alpha^{(A)}[0,5], E_\alpha^{(A)}[0,6], E_\alpha^{(A)}[0,7]\},$$
$$\{E_\alpha^{(A)}[1,0], E_\alpha^{(A)}[1,1], E_\alpha^{(A)}[1,2], E_\alpha^{(A)}[1,3], E_\alpha^{(A)}[1,4], E_\alpha^{(A)}[1,5], E_\alpha^{(A)}[1,6], E_\alpha^{(A)}[1,7]\},$$
$$\{E_\alpha^{(A)}[2,0], E_\alpha^{(A)}[2,1], E_\alpha^{(A)}[2,2], E_\alpha^{(A)}[2,3], E_\alpha^{(A)}[2,4], E_\alpha^{(A)}[2,5], E_\alpha^{(A)}[2,6], E_\alpha^{(A)}[2,7]\},$$
$$\{E_\alpha^{(A)}[3,0], E_\alpha^{(A)}[3,1], E_\alpha^{(A)}[3,2], E_\alpha^{(A)}[3,3], E_\alpha^{(A)}[3,4], E_\alpha^{(A)}[3,5], E_\alpha^{(A)}[3,6], E_\alpha^{(A)}[3,7]\},$$
$$\{E_\alpha^{(A)}[4,0], E_\alpha^{(A)}[4,1], E_\alpha^{(A)}[4,2], E_\alpha^{(A)}[4,3], E_\alpha^{(A)}[4,4], E_\alpha^{(A)}[4,5], E_\alpha^{(A)}[4,6], E_\alpha^{(A)}[4,7]\},$$
$$\{E_\alpha^{(A)}[5,0], E_\alpha^{(A)}[5,1], E_\alpha^{(A)}[5,2], E_\alpha^{(A)}[5,3], E_\alpha^{(A)}[5,4], E_\alpha^{(A)}[5,5], E_\alpha^{(A)}[5,6], E_\alpha^{(A)}[5,7]\},$$
$$\{E_\alpha^{(A)}[6,0], E_\alpha^{(A)}[6,1], E_\alpha^{(A)}[6,2], E_\alpha^{(A)}[6,3], E_\alpha^{(A)}[6,4], E_\alpha^{(A)}[6,5], E_\alpha^{(A)}[6,6], E_\alpha^{(A)}[6,7]\},$$
$$\{E_\alpha^{(A)}[7,0], E_\alpha^{(A)}[7,1], E_\alpha^{(A)}[7,2], E_\alpha^{(A)}[7,3], E_\alpha^{(A)}[7,4], E_\alpha^{(A)}[7,5], E_\alpha^{(A)}[7,6], E_\alpha^{(A)}[7,7]\}\}$$

In[∘]:=

```
Block[{s, r}, s = FullSimplify[e_α^(A) /. sgtrye_α^(A), constraintVars];
 r = FullSimplify[Transpose[s].η4488.s, constraintVars] - MatrixMetric44;
 FullSimplify[r, constraintVars]]
```

Out[∘]=

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[ ]:= (*$\left(e_{\alpha}^{(A)}\right.$=eAa)//MatrixForm*)

In[ ]:= (*$e_{\alpha}^{(A)}$
Inverse[%]*)

In[ ]:= (*$\left(e^{\alpha}_{(A)}\right.$=Inverse[$e_{\alpha}^{(A)}$/.sge_Subsuperscript_α_and_LeftParenthesis_A_RightParenthesis])//MatrixForm*)

In[ ]:= $e^{\alpha}_{(A)}$

Out[ ]=

$$\{\{E^{\alpha}_{(A)}[0,0], E^{\alpha}_{(A)}[0,1], E^{\alpha}_{(A)}[0,2], E^{\alpha}_{(A)}[0,3], E^{\alpha}_{(A)}[0,4], E^{\alpha}_{(A)}[0,5], E^{\alpha}_{(A)}[0,6], E^{\alpha}_{(A)}[0,7]\},$$
$$\{E^{\alpha}_{(A)}[1,0], E^{\alpha}_{(A)}[1,1], E^{\alpha}_{(A)}[1,2], E^{\alpha}_{(A)}[1,3], E^{\alpha}_{(A)}[1,4], E^{\alpha}_{(A)}[1,5], E^{\alpha}_{(A)}[1,6], E^{\alpha}_{(A)}[1,7]\},$$
$$\{E^{\alpha}_{(A)}[2,0], E^{\alpha}_{(A)}[2,1], E^{\alpha}_{(A)}[2,2], E^{\alpha}_{(A)}[2,3], E^{\alpha}_{(A)}[2,4], E^{\alpha}_{(A)}[2,5], E^{\alpha}_{(A)}[2,6], E^{\alpha}_{(A)}[2,7]\},$$
$$\{E^{\alpha}_{(A)}[3,0], E^{\alpha}_{(A)}[3,1], E^{\alpha}_{(A)}[3,2], E^{\alpha}_{(A)}[3,3], E^{\alpha}_{(A)}[3,4], E^{\alpha}_{(A)}[3,5], E^{\alpha}_{(A)}[3,6], E^{\alpha}_{(A)}[3,7]\},$$
$$\{E^{\alpha}_{(A)}[4,0], E^{\alpha}_{(A)}[4,1], E^{\alpha}_{(A)}[4,2], E^{\alpha}_{(A)}[4,3], E^{\alpha}_{(A)}[4,4], E^{\alpha}_{(A)}[4,5], E^{\alpha}_{(A)}[4,6], E^{\alpha}_{(A)}[4,7]\},$$
$$\{E^{\alpha}_{(A)}[5,0], E^{\alpha}_{(A)}[5,1], E^{\alpha}_{(A)}[5,2], E^{\alpha}_{(A)}[5,3], E^{\alpha}_{(A)}[5,4], E^{\alpha}_{(A)}[5,5], E^{\alpha}_{(A)}[5,6], E^{\alpha}_{(A)}[5,7]\},$$
$$\{E^{\alpha}_{(A)}[6,0], E^{\alpha}_{(A)}[6,1], E^{\alpha}_{(A)}[6,2], E^{\alpha}_{(A)}[6,3], E^{\alpha}_{(A)}[6,4], E^{\alpha}_{(A)}[6,5], E^{\alpha}_{(A)}[6,6], E^{\alpha}_{(A)}[6,7]\},$$
$$\{E^{\alpha}_{(A)}[7,0], E^{\alpha}_{(A)}[7,1], E^{\alpha}_{(A)}[7,2], E^{\alpha}_{(A)}[7,3], E^{\alpha}_{(A)}[7,4], E^{\alpha}_{(A)}[7,5], E^{\alpha}_{(A)}[7,6], E^{\alpha}_{(A)}[7,7]\}\}$$

In[ ]:= ($sge^{\alpha}_{(A)}$ = Thread[Flatten[$e^{\alpha}_{(A)}$] → Flatten[Inverse[$e_{\alpha}^{(A)}$ /. $sge_{\alpha}^{(A)}$]]]) (*//Column*)

Out[ ]=

$$\{E^{\alpha}_{(A)}[0,0]\to\frac{1}{\sqrt{g[0][0][x0,x4]}}, E^{\alpha}_{(A)}[0,1]\to 0, E^{\alpha}_{(A)}[0,2]\to 0, E^{\alpha}_{(A)}[0,3]\to 0, E^{\alpha}_{(A)}[0,4]\to 0,$$
$$E^{\alpha}_{(A)}[0,5]\to 0, E^{\alpha}_{(A)}[0,6]\to 0, E^{\alpha}_{(A)}[0,7]\to 0, E^{\alpha}_{(A)}[1,0]\to 0, E^{\alpha}_{(A)}[1,1]\to\frac{1}{\sqrt{g[1][1][x0,x4]}},$$
$$E^{\alpha}_{(A)}[1,2]\to 0, E^{\alpha}_{(A)}[1,3]\to 0, E^{\alpha}_{(A)}[1,4]\to 0, E^{\alpha}_{(A)}[1,5]\to 0, E^{\alpha}_{(A)}[1,6]\to 0, E^{\alpha}_{(A)}[1,7]\to 0,$$
$$E^{\alpha}_{(A)}[2,0]\to 0, E^{\alpha}_{(A)}[2,1]\to 0, E^{\alpha}_{(A)}[2,2]\to\frac{1}{\sqrt{g[2][2][x0,x4]}}, E^{\alpha}_{(A)}[2,3]\to 0, E^{\alpha}_{(A)}[2,4]\to 0,$$
$$E^{\alpha}_{(A)}[2,5]\to 0, E^{\alpha}_{(A)}[2,6]\to 0, E^{\alpha}_{(A)}[2,7]\to 0, E^{\alpha}_{(A)}[3,0]\to 0, E^{\alpha}_{(A)}[3,1]\to 0, E^{\alpha}_{(A)}[3,2]\to 0,$$
$$E^{\alpha}_{(A)}[3,3]\to\frac{1}{\sqrt{g[3][3][x0,x4]}}, E^{\alpha}_{(A)}[3,4]\to 0, E^{\alpha}_{(A)}[3,5]\to 0, E^{\alpha}_{(A)}[3,6]\to 0, E^{\alpha}_{(A)}[3,7]\to 0,$$
$$E^{\alpha}_{(A)}[4,0]\to 0, E^{\alpha}_{(A)}[4,1]\to 0, E^{\alpha}_{(A)}[4,2]\to 0, E^{\alpha}_{(A)}[4,3]\to 0, E^{\alpha}_{(A)}[4,4]\to\frac{1}{\sqrt{-g[4][4][x0,x4]}},$$
$$E^{\alpha}_{(A)}[4,5]\to 0, E^{\alpha}_{(A)}[4,6]\to 0, E^{\alpha}_{(A)}[4,7]\to 0, E^{\alpha}_{(A)}[5,0]\to 0, E^{\alpha}_{(A)}[5,1]\to 0, E^{\alpha}_{(A)}[5,2]\to 0,$$
$$E^{\alpha}_{(A)}[5,3]\to 0, E^{\alpha}_{(A)}[5,4]\to 0, E^{\alpha}_{(A)}[5,5]\to\frac{1}{\sqrt{-g[5][5][x0,x4]}}, E^{\alpha}_{(A)}[5,6]\to 0, E^{\alpha}_{(A)}[5,7]\to 0,$$
$$E^{\alpha}_{(A)}[6,0]\to 0, E^{\alpha}_{(A)}[6,1]\to 0, E^{\alpha}_{(A)}[6,2]\to 0, E^{\alpha}_{(A)}[6,3]\to 0, E^{\alpha}_{(A)}[6,4]\to 0, E^{\alpha}_{(A)}[6,5]\to 0,$$
$$E^{\alpha}_{(A)}[6,6]\to\frac{1}{\sqrt{-g[6][6][x0,x4]}}, E^{\alpha}_{(A)}[6,7]\to 0, E^{\alpha}_{(A)}[7,0]\to 0, E^{\alpha}_{(A)}[7,1]\to 0, E^{\alpha}_{(A)}[7,2]\to 0,$$
$$E^{\alpha}_{(A)}[7,3]\to 0, E^{\alpha}_{(A)}[7,4]\to 0, E^{\alpha}_{(A)}[7,5]\to 0, E^{\alpha}_{(A)}[7,6]\to 0, E^{\alpha}_{(A)}[7,7]\to\frac{1}{\sqrt{-g[7][7][x0,x4]}}\}$$

In[ ]:= (*Symbolize[$sge^{\alpha}_{(A)}$]*)

In[∘]:= (*Symbolize[$sgtrye^{\alpha}_{(A)}$]*)

In[∘]:= (*%/.$sge^{(A)}_{\alpha}$
%%/.$sgtrye^{(A)}_{\alpha}$*)

In[∘]:= (*FullSimplify[Transpose[$e^{(A)}_{\alpha}$].η4488.$e^{(A)}_{\alpha}$,constraintVars]-MatrixMetric44*)

For Spin (4, 4) ; τ tau; T16; OCTAD : Nash; Introduce the wave function, Ψ16, for this Universe::

O(4,4); evalues, evecs of σ

In[∘]:= {evalues, evecs} = Eigensystem[σ]

Out[∘]=

```
{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
  {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
  {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}
```

In[∘]:= $\left(u = \text{ExpandAll}\left[\frac{1}{\sqrt{2}}\ \text{evecs}\right]\right)$ // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

In[∘]:= Table[u〚h〛.σ.u〚h〛, {h, 1, Length[u]}]

Out[∘]=

{-1, -1, -1, -1, 1, 1, 1, 1}

In[∘]:= Table[(Transpose[u]〚h〛).σ.u〚h〛, {h, 1, Length[u]}]

Out[∘]=

{-1, -1, -1, -1, 1, 1, 1, 1}

In[∘]:= Table[(Transpose[u]〚A〛) == -σ.u〚A〛, {A, 1, 4}]
Table[(Transpose[u]〚A〛) == σ.u〚A〛, {A, 5, Length[u]}]

Out[∘]=

{True, True, True, True}

Out[∘]=

{True, True, True, True}

Ax = $\left(\frac{1}{2}\right.$ (KroneckerProduct[dX, X] -

```
KroneckerProduct[X, dX])).σ;
Au = (1/2 (KroneckerProduct[dU, U] -
       KroneckerProduct[U, dU])).σ;
```

$$Lsquared = \dot{x}^{A}\,\dot{x}_{A} = dX.\sigma.dX + X.\sigma.X\left(\frac{dU.\sigma.dU}{U.\sigma.U} - \left(\frac{U.\sigma.dU}{U.\sigma.U}\right)^{2}\right) + \frac{2}{U.\sigma.U}\,Tr\left[\tau^{A}.Ax.\tau_{A}.Au\right]$$

```
Timing[
 Simplify[ExpandAll[dY.η8.dY - Lsquared]]]
{10.733, 0}
```

In[∘]:=
```
FullSimplify[Sqrt[Flatten[MatrixMetric44.η4488]], constraintVars]
```

Out[∘]=

$$\{Cot[6\,H\,x0],\,0,0,0,0,0,0,0,0,\,\sqrt{e^{2\,a4[H\,x4]}}\,Sin[6\,H\,x0]^{1/6},\,0,0,0,0,0,0,0,0,\,\sqrt{e^{2\,a4[H\,x4]}}\,Sin[6\,H\,x0]^{1/6},\,0,0,0,0,0,0,0,0,\,\sqrt{e^{2\,a4[H\,x4]}}\,Sin[6\,H\,x0]^{1/6},\,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,\,\sqrt{e^{-2\,a4[H\,x4]}}\,Sin[6\,H\,x0]^{1/6},\,0,0,0,0,0,0,0,0,\,\sqrt{e^{-2\,a4[H\,x4]}}\,Sin[6\,H\,x0]^{1/6},\,0,0,0,0,0,0,0,0,\,\sqrt{e^{-2\,a4[H\,x4]}}\,Sin[6\,H\,x0]^{1/6}\}$$

SO(4), γ; M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+,+,+,+,-,-,-,-}, {x0, x1, x2, x3, x4, x5, x6, x7}

In[∘]:=
```
Qa[h_, p_, q_] := Signature[{h, p, q, 4}];
Qb[h_, p_, q_] := ID4⟦p, 4⟧ * ID4⟦q, h⟧ - ID4⟦p, h⟧ * ID4⟦q, 4⟧;
SelfDualAntiSymmetric[h_, p_, q_] := Qa[h, p, q] - Qb[h, p, q] ;
AntiSelfDualAntiSymmetric[h_, p_, q_] := (Qa[h, p, q] + Qb[h, p, q] );
```

In[∘]:=
```
Qa[1, 2, 3]
```

Out[∘]=
```
1
```

In[∘]:=
```
ϵ4⟦1, 2, 3, 4⟧
```

Out[∘]=
```
1
```

In[∘]:=
```
Do[s4by4[h] = Table[Table[SelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
 {h, 1, 3} ]
```

In[ ]:=
```
Do[
 t4by4[h] = Table[Table[AntiSelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
 {h, 1, 3} ]
```

In[ ]:=
```
(*Protect[{s4by4[1],s4by4[2],s4by4[3],t4by4[1],t4by4[2],t4by4[3]}]*)
```

In[ ]:=
```
Protect[s4by4, t4by4]
```

Out[ ]=
```
{s4by4, t4by4}
```

In[ ]:=
```
{s4by4[#] // MatrixForm, MatrixForm[t4by4[#]]} & /@ Range[3]
```

Out[ ]=

$$\left\{\left\{\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}\right\},\right.$$
$$\left.\left\{\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\right\}, \left\{\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\right\}\right\}$$

In[ ]:=
```
Table[{{J, K}, MatrixForm[s4by4[J].t4by4[K]]}, {J, 1, 3}, {K, 1, 3}]
```

Out[ ]=

$$\left\{\left\{\left\{\{1, 1\}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}, \left\{\{1, 2\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\right\}, \left\{\{1, 3\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}\right\}\right\},\right.$$
$$\left\{\left\{\{2, 1\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\right\}, \left\{\{2, 2\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}, \left\{\{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}\right\}\right\},$$
$$\left.\left\{\left\{\{3, 1\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\right\}, \left\{\{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}\right\}, \left\{\{3, 3\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}\right\}\right\}$$

In[ ]:=
```
Do[st[J, K] = s4by4[J].t4by4[K], {J, 1, 3}, {K, 1, 3}]
```

In[ ]:=
```
Table[{{J, K}, MatrixForm[st[J, K]]}, {J, 1, 3}, {K, 1, 3}]
```

Out[ ]=

$$\left\{\left\{\left\{\{1, 1\}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}, \left\{\{1, 2\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\right\}, \left\{\{1, 3\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}\right\}\right\},\right.$$
$$\left\{\left\{\{2, 1\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\right\}, \left\{\{2, 2\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}, \left\{\{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}\right\}\right\},$$
$$\left.\left\{\left\{\{3, 1\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\right\}, \left\{\{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}\right\}, \left\{\{3, 3\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}\right\}\right\}$$

In[∘]:=
```
Protect[st]
```
Out[∘]=
```
{st}
```
In[∘]:=
```
Table[{{J, K}, MatrixForm[st[J, K] - Transpose[st[J, K]]]}, {J, 1, 3}, {K, 1, 3}]
```
Out[∘]=

$$\left\{\left\{\left\{\{1,1\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{1,2\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{1,3\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\}\right\},\right.$$

$$\left\{\left\{\{2,1\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{2,2\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{2,3\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\}\right\},$$

$$\left.\left\{\left\{\{3,1\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{3,2\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{3,3\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\}\right\}\right\}$$

In[∘]:=
```
Permutations[Range[3], 2]
```
Out[∘]=
```
{{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}
```
In[∘]:=
```
KroneckerProduct[Range[3], Range[3]]
```
Out[∘]=
```
{{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}
```
In[∘]:=
```
Table[{J, K}, {J, 1, 3}, {K, 1, 3}]
st[##] & /@ %
st[##] & @@ %%
```
Out[∘]=
```
{{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}}
```
Out[∘]=
```
{st[{{1, 1}, {1, 2}, {1, 3}}], st[{{2, 1}, {2, 2}, {2, 3}}], st[{{3, 1}, {3, 2}, {3, 3}}]}
```
Out[∘]=
```
st[{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}]
```

In[ ]:=
```
Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
Length[%]
```

Out[ ]=
```
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

Out[ ]=
```
16
```

In[ ]:=
```
Unprotect[γ];
γ = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
   Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
```

Out[ ]=
```
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

In[ ]:=
```
Length[γ]
```

Out[ ]=
```
16
```

In[∘]:= Protect[γ]

Out[∘]=

{γ}

In[∘]:= γ⟦1⟧

Out[∘]=

{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}

In[∘]:= γ⟦-1⟧

Out[∘]=

{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}

Self Dual Anti-Symmetric

In[∘]:= Table$\left[\left(\frac{1}{2}\sum_{j1=1}^{4}\left(\sum_{j2=1}^{4}\text{ϵ4}[[p,q,j1,j2]]\times\text{s4by4}[h][[j1,j2]]\right)\right)-\text{s4by4}[h][[p,q]],\right.$

$\left.\{h,1,3\},\{q,4\},\{p,4\}\right]$

Out[∘]=

{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}

Anti SelfDual Anti-Symmetric

In[∘]:= Table$\left[\left(\frac{1}{2}\sum_{j1=1}^{4}\left(\sum_{j2=1}^{4}\text{ϵ4}[[p,q,j1,j2]]\times\text{t4by4}[h][[j1,j2]]\right)\right)+\text{t4by4}[h][[p,q]],\right.$

$\left.\{h,1,3\},\{q,4\},\{p,4\}\right]$

Out[∘]=

{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}

SO(4,4), Spin(4,4) = $\overline{\text{SO}(4, 4)}$, τ ; if using xact: M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+,+,+,+,-,-,-,-}, {x0, x1, x2, x3, x4, x5, x6, x7}

In[∘]:= $\overline{\tau}$ === $\overline{\tau}$

Out[∘]=

False

In[∘]:= $\overline{\tau}$

Head[%]

Out[∘]=

$\overline{\tau}$

Out[∘]=

Symbol

In[ ]:=
```
τ̅
Head[%]
```
Out[ ]=
```
τ̅
```
Out[ ]=
```
ParsedBoxWrapper
```
In[ ]:=
```
τ̅
Head[%]
```
Out[ ]=
```
τ̅
```
Out[ ]=
```
Symbol
```
In[ ]:=
```
Symbol
```
Out[ ]=
```
Symbol
```
In[ ]:=
```
τ̅ === τ̅
```
Out[ ]=
```
True
```
In[ ]:=
```
τ̅ === τ̅
```
Out[ ]=
```
False
```
In[ ]:=
```
η4488 // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[ ]:=
```
η4488 == DiagonalMatrix[RotateRight[evalues, 4]]
```
Out[ ]=
```
True
```
In[ ]:=
```
DiagonalMatrix[RotateRight[evalues, 3]] // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

these also yield correct reps:

ζ[0] = ID8;
Table[ζ[7 - h] = ArrayFlatten[{{0, -t4by4[h]}, {t4by4[h], 0}}], {h, 1, 3}];
Table[ζ[h] = ArrayFlatten[{{0, s4by4[h]}, {s4by4[h], 0}}], {h, 1, 3}];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm

ζ[0] = ID8;
Table[ζ[h] = ArrayFlatten[{{0, t4by4[h]}, {t4by4[h], 0}}], {h, 1, 3}];
Table[ζ[7 - h] = ArrayFlatten[{{0, -s4by4[h]}, {s4by4[h], 0}}], {h, 1, 3}];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm

ζ[0] = ID8;
Table[ζ[h] = ArrayFlatten[{{0, t4by4[h]}, {t4by4[h], 0}}], {h, 1, 3}];
Table[ζ[7 - h] = ArrayFlatten[{{0, s4by4[h]}, {-s4by4[h], 0}}], {h, 1, 3}];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm

In[∘]:=
```
ξ[0] = ID8;
Table[
  ξ[7 - h] = ArrayFlatten[{{0, t4by4[h]}, {-t4by4[h], 0}}], {h, 1, 3}];
Table[ξ[h] = ArrayFlatten[{{0, s4by4[h]}, {s4by4[h], 0}}], {h, 1, 3}];
(ξ[7] = ξ[1].ξ[2].ξ[3].ξ[4].ξ[5].ξ[6]) // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[∘]:=
```
σ == ξ[1].ξ[2].ξ[3]
```

Out[∘]=

True

In[•]:=
```
Table[{A, B, FullSimplify[
   ExpandAll[1/2 (ξ[A].ξ[B] + ξ[B].ξ[A]) == -η4488[[A + 1, B + 1]] * ID8]]},
  {A, 1, 7}, {B, 1, 7}]
```

Out[•]=
```
{{{1, 1, True}, {1, 2, True}, {1, 3, True}, {1, 4, True},
  {1, 5, True}, {1, 6, True}, {1, 7, True}}, {{2, 1, True}, {2, 2, True},
  {2, 3, True}, {2, 4, True}, {2, 5, True}, {2, 6, True}, {2, 7, True}},
 {{3, 1, True}, {3, 2, True}, {3, 3, True}, {3, 4, True}, {3, 5, True}, {3, 6, True},
  {3, 7, True}}, {{4, 1, True}, {4, 2, True}, {4, 3, True}, {4, 4, True},
  {4, 5, True}, {4, 6, True}, {4, 7, True}}, {{5, 1, True}, {5, 2, True},
  {5, 3, True}, {5, 4, True}, {5, 5, True}, {5, 6, True}, {5, 7, True}},
 {{6, 1, True}, {6, 2, True}, {6, 3, True}, {6, 4, True}, {6, 5, True},
  {6, 6, True}, {6, 7, True}}, {{7, 1, True}, {7, 2, True},
  {7, 3, True}, {7, 4, True}, {7, 5, True}, {7, 6, True}, {7, 7, True}}}
```

In[•]:=
```
η4488 // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[•]:=
```
Table[{A, -η4488[[A + 1, A + 1]] * ξ[A] == Transpose[ξ[A]]} , {A, 1, 7}]
```

Out[•]=
```
{{1, True}, {2, True}, {3, True}, {4, True}, {5, True}, {6, True}, {7, True}}
```

In[•]:=
```
(σ == ξ[1].ξ[2].ξ[3])
```

Out[•]=
```
True
```

In[•]:=
```
σ.ξ[1].ξ[2].ξ[3] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[•]:=
```
τ[0] = ID8;
Do[
 {τ[A] = FullSimplify[ExpandAll[(ξ[A])]]} ,
 {A, 1, 7}]
(*Do[
 {τ[A] = FullSimplify[ExpandAll[-(σ.ξ[1].
        ξ[2].ξ[3].ξ[A])]]} , {A, 1, 7}]*)
```

In[•]:=
```
(σ == τ[1].τ[2].τ[3])
```

Out[•]=
```
True
```

In[•]:=
```
(*Symbolize[τ̅]*)
```

In[•]:=
```
τ̅[0] = ID8;
Do[{τ̅[A] = FullSimplify[ExpandAll[
      (σ.Transpose[τ[A]].σ)]]} , {A, 1, 7}]
```

In[•]:=
```
σ.τ̅[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
```

Out[•]=
```
{True, True, True, True, True, True, True, True}
```

In[∘]:=
$$\text{Table}\left[\left\{A, B, \text{FullSimplify}\left[\text{ExpandAll}\left[\frac{1}{2}\,(\tau[A].\overline{\tau}[B] + \tau[B].\overline{\tau}[A]) == \eta 4488[\![A+1, B+1]\!] * \text{ID8}\right]\right]\right\}, \{A, 0, 7\}, \{B, 0, 7\}\right] \,//\, \text{MatrixForm}$$

Out[∘]//MatrixForm=
$$\begin{pmatrix}
\begin{pmatrix}0\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\7\\\text{True}\end{pmatrix} \\
\begin{pmatrix}1\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\7\\\text{True}\end{pmatrix} \\
\begin{pmatrix}2\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\7\\\text{True}\end{pmatrix} \\
\begin{pmatrix}3\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\7\\\text{True}\end{pmatrix} \\
\begin{pmatrix}4\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\7\\\text{True}\end{pmatrix} \\
\begin{pmatrix}5\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\7\\\text{True}\end{pmatrix} \\
\begin{pmatrix}6\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\7\\\text{True}\end{pmatrix} \\
\begin{pmatrix}7\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\7\\\text{True}\end{pmatrix}
\end{pmatrix}$$

In[∘]:= $\tau[7]$ // MatrixForm

Out[∘]//MatrixForm=
$$\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

In[∘]:= $(\Omega = \sigma.\tau[7])$ // MatrixForm

Out[∘]//MatrixForm=
$$\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0
\end{pmatrix}$$

In[∘]:= Ω == τ[4].τ[5].τ[6]

Out[∘]=

True

In[∘]:= τ[5].τ[6].τ[7] // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

In[∘]:= (σΩ = σ.Ω) // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[∘]:= Inverse[σΩ] // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[∘]:= τ[1].τ[2].τ[3].τ[4].τ[5].τ[6].τ[7] == τ[0] == ID8

Out[∘]=

True

In[∘]:= (σ == τ[1].τ[2].τ[3])

Out[∘]=

True

In[∘]:= (σ == τ[4].τ[5].τ[6].τ[7])

Out[∘]=

True

In[∘]:= τ[1].τ[2].τ[3].τ[#] == -Transpose[τ[1].τ[2].τ[3].τ[#]] & /@ Range[0, 7]

Out[∘]=

{False, True, True, True, True, True, True, True}

In[ ]:=
```
σ.τ̅[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
```

Out[ ]=
```
{True, True, True, True, True, True, True, True}
```

In[ ]:=
```
τ[5].τ[6].τ[7].τ̅[#] == Transpose[τ[5].τ[6].τ[7].τ[#]] & /@
 Range[0, 7]
```

Out[ ]=
```
{False, True, True, True, False, True, True, True}
```

In[ ]:=
```
τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```

Out[ ]=
```
{True, True, True, True, True, False, False, False}
```

In[ ]:=
```
τ[4].τ̅[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```

Out[ ]=
```
{True, False, False, False, False, True, True, True}
```

In[ ]:=
```
τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```

Out[ ]=
```
{True, True, True, True, True, False, False, False}
```

$$t^A = \begin{pmatrix} 0 & \overline{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$; O (4, 4) : SAB ;

covariantDiffMatrix = $T16^A[5].T16^A[6].T16^A[7]$

$$S = \begin{pmatrix} a & b \\ c & d \end{pmatrix};$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}.\begin{pmatrix} 0 & \overline{\tau}^A \\ \tau^A & 0 \end{pmatrix}.\begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix} = \begin{pmatrix} b.\tau^A & a.\overline{\tau}^A \\ d.\tau^A & c.\overline{\tau}^A \end{pmatrix}.\begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix}$$

$$= \begin{pmatrix} b.\tau^A.\tilde{a} + a.\overline{\tau}^A.\tilde{b} & b.\tau^A.\tilde{c} + a.\overline{\tau}^A.\tilde{d} \\ d.\tau^A.\tilde{a} + c.\overline{\tau}^A.\tilde{b} & d.\tau^A.\tilde{c} + c.\overline{\tau}^A.\tilde{d} \end{pmatrix}$$

In[∘]:=
```
O8similarityTransformation // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

In[∘]:=
```
blocks = Partition[O8similarityTransformation, {8, 8}];
```

In[∘]:=
```
(*To access the individual blocks:*)
block11 = blocks⟦1, 1⟧ (*Top-left block*)
block12 = blocks⟦1, 2⟧ (*Top-right block*)
block21 = blocks⟦2, 1⟧ (*Bottom-left block*)
block22 = blocks⟦2, 2⟧ (*Bottom-right block*)
```

Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

Out[∘]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}}
```

Out[∘]=
```
{{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

Out[∘]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[∘]:=
```
ArrayFlatten[{{blocks⟦1, 1⟧, blocks⟦1, 2⟧}, {blocks⟦2, 1⟧, blocks⟦2, 2⟧}}] ===
 O8similarityTransformation
```

Out[∘]=
```
True
```

In[∘]:=

$$\begin{pmatrix} \text{blocks}⟦1,1⟧ & \text{blocks}⟦1,2⟧ \\ \text{blocks}⟦2,1⟧ & \text{blocks}⟦2,2⟧ \end{pmatrix} . \begin{pmatrix} 0 & \overline{\tau}^{A} \\ \tau^{A} & 0 \end{pmatrix} . \begin{pmatrix} \text{block}\tilde{\text{s}}⟦1,1⟧ & \text{block}\tilde{\text{s}}⟦2,1⟧ \\ \text{block}\tilde{\text{s}}⟦1,2⟧ & \text{block}\tilde{\text{s}}⟦2,2⟧ \end{pmatrix} =$$

$$\begin{pmatrix} \text{blocks}⟦1,2⟧ . \tau^{A} & \text{blocks}⟦1,1⟧ . \overline{\tau}^{A} \\ \text{blocks}⟦2,2⟧ . \tau^{A} & \text{blocks}⟦2,1⟧ . \overline{\tau}^{A} \end{pmatrix} .$$

$$\begin{pmatrix} \text{blocks}\tilde{}[[1,1]] & \text{blocks}\tilde{}[[2,1]] \\ \text{blocks}\tilde{}[[1,2]] & \text{blocks}\tilde{}[[2,2]] \end{pmatrix}$$

$$= \left(\begin{array}{l} \text{blocks}[[1,2]].\tau^{A}.\text{blocks}\tilde{}[[1,1]] + \text{blocks}[[1,1]]. \\ \text{blocks}[[2,2]].\tau^{A}.\text{blocks}\tilde{}[[1,1]] + \text{blocks}[[2,1]]. \end{array} \right.$$

In[ ]:=
```
Table[
 newT16^A[A1] === ArrayFlatten[{{blocks[[1, 2]].τ[A1].Transpose[blocks[[1, 1]]] + blocks[[1, 1]].
      τ̄[A1].Transpose[blocks[[1, 2]]], blocks[[1, 2]].τ[A1].Transpose[blocks[[2, 1]]] +
     blocks[[1, 1]].τ̄[A1].Transpose[blocks[[2, 2]]]},
    {blocks[[2, 2]].τ[A1].Transpose[blocks[[1, 1]]] + blocks[[2, 1]].τ̄[A1].
      Transpose[blocks[[1, 2]]], blocks[[2, 2]].τ[A1].Transpose[blocks[[2, 1]]] +
     blocks[[2, 1]].τ̄[A1].Transpose[blocks[[2, 2]]]}}], {A1, 0, 7}]
```

Out[ ]=
```
{False, False, False, False, False, False, False, False}
```

In[ ]:=
```
MatrixForm[newT16^A[#]] & /@ Range[0, 8]
```

Out[ ]=
```
{newT16^A[0], newT16^A[1], newT16^A[2], newT16^A[3],
 newT16^A[4], newT16^A[5], newT16^A[6], newT16^A[7], newT16^A[8]}
```

In[ ]:=
```
(*Example 16x16 matrix*)(*matrix=Table[RandomInteger[{1,100}],{16},{16}];*)

(*Partition the matrix into 8x8 blocks*)
(*blocks=Partition[matrix,{8,8}];*)

(*The result is a 2x2 list of 8x8 matrices*)
(*blocks*)
```

In[ ]:=
```
(*TakeList[O8similarityTransformation,{8,8}]*)
blocks = TakeList[O8similarityTransformation, {8, 8}, {8, 8}]
ArrayFlatten[%] === O8similarityTransformation
```

Out[ ]=

{{{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}}}

Out[•]=

True

In[•]:=
```
Length[blocks]
```

Out[•]=

2

In[•]:=
```
(*O8similarityTransformation==ArrayFlatten[{{Take[],Take[]},{Take[],Take[]}}]*)
```

Take [*list*, seq_1, seq_2]

gives a nested list in which elements specified by seq_i are taken at level i in *list*. effectively extracts a submatrix from *list*.

In[•]:=
```
Take[]
```

Take: Take called with 0 arguments; 1 or more arguments are expected.

Out[•]=

Take[]

Need these in order to define the Lagrangian for the "universe," later :

In[•]:=
```
(σ16 = T16^A[0].T16^A[1].T16^A[2].T16^A[3]) // MatrixForm
```

Out[•]//MatrixForm=

$T16^A[0].T16^A[1].T16^A[2].T16^A[3]$

In[•]:=
```
Table[T16^A[A1] = ArrayFlatten[{{0, τ̄[A1]}, {τ[A1], 0}}],
  {A1, 0, 7}];
```

In[∘]:=
```
(T16^A[8] = T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]) // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

In[∘]:=
```
Table[newT16^A[A1] = O8similarityTransformation. T16^A[A1].
    Transpose[O8similarityTransformation], {A1, 0, 8}];
```

In[∘]:=
```
Transpose[Ψ16]. σ16 ===
 Transpose[Transpose[O8similarityTransformation].O8similarityTransformation.Ψ16]. σ16 ===
 Transpose[O8similarityTransformation.Ψ16].O8similarityTransformation. σ16
```

Out[∘]=
```
True
```

In[∘]:=
```
Transpose[Ψ16]. σ16 .Transpose[O8similarityTransformation] ===
 Transpose[Transpose[O8similarityTransformation].O8similarityTransformation.Ψ16]. σ16.
  Transpose[O8similarityTransformation] === Transpose[O8similarityTransformation.Ψ16].
  O8similarityTransformation. σ16.Transpose[O8similarityTransformation]
```

Out[∘]=
```
True
```

In[∘]:=
```
(newσ16 = O8similarityTransformation. σ16.Transpose[O8similarityTransformation]) //
 MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0
\end{pmatrix}$$

In[∘]:=
```
σ16 // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0
\end{pmatrix}$$

In[∘]:=
```
T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#] ==
    -Transpose[T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#]] & /@ Range[0, 8]
```

Out[∘]=
```
{True, True, True, True, True, True, True, True, False}
```

In[∘]:=
```
T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#] ==
    Transpose[T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#]] & /@ Range[0, 8]
```

Out[∘]=
```
{True, True, True, True, True, True, True, True, True}
```

In[∘]:=
```
T16^A[8] == σ16.T16^A[4].T16^A[5].T16^A[6].T16^A[7]
```

Out[∘]=
```
True
```

In[∘]:=
```
T16^A[#] == Transpose[T16^A[#]] & /@ Range[0, 8]
```

Out[∘]=
```
{True, True, True, True, False, False, False, False, True}
```

In[∘]:=
```
T16^A[#] == -Transpose[T16^A[#]] & /@ Range[0, 8]
```

Out[∘]=
```
{False, False, False, False, True, True, True, True, False}
```

In[∘]:=
```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
```

Out[∘]=
```
{True, True, True, True, True, True, True, True}
```

In[∘]:=
```
(covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]) // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

In[∘]:=
```
σ16.covariantDiffMatrix == Transpose[σ16.covariantDiffMatrix]
```

Out[∘]=
```
True
```

In[∘]:=
```
ID16 // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

In[∘]:=
```
(σ16(*=T16^A[0].T16^A[1].T16^A[2].T16^A[3]*)) // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}$$

In[∘]:=
```
σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

Out[∘]=

True

later, after defining $e^{\alpha}_{(A)}$:

In[∘]:=
```
(*Table[T16^α[α1-1] =
    Sum[(e^α_(A)[[α1,A1]]) T16^A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

In[∘]:=
```
(*(T16^α[8]=T16^α[0].T16^α[1].T16^α[2].T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7])//
 MatrixForm*)
```

In[∘]:=
```
(*eLpairs*)
```

In[∘]:=
```
(*Ω16a=Array[0&,{16,16}];
 {1+#[[1]],1+#[[2]]}&/@eLpairs
 (Ω16a[[#[[1]],#[[2]]]]=1)&/@%*)
```

In[∘]:=
```
(*Ω16a//MatrixForm*)
```

In[∘]:=
```
(*(Ω16=σ16.(Ω16a+Transpose[Ω16a]))//MatrixForm*)
```

In[ ]:=
```
T16^A[0].T16^A[4] // MatrixForm
T16^A[0].T16^A[4].σ16
(Ω16 = %) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0
\end{pmatrix}$$

Out[ ]=

```
{{0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}}
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0
\end{pmatrix}$$

In[ ]:=
```
Det[Ω16]
```

Out[ ]=

1

In[∘]:= σ16.Ω16 // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

In[∘]:= σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]

Out[∘]=

{True, True, True, True, True, True, True, True}

In[∘]:= σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]

Out[∘]=

{True, True, True, True, True, True, True, True}

In[∘]:= σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]

Out[∘]=

{True, True, True, True, True, True, True, True}

In[∘]:= σ16.newT16^A[#] == -Transpose[σ16.newT16^A[#]] & /@ Range[0, 7]

Out[∘]=

{False, False, False, True, False, True, True, False}

In[∘]:= newσ16.newT16^A[#] == -Transpose[newσ16.newT16^A[#]] & /@ Range[0, 7]

Out[∘]=

{True, True, True, True, True, True, True, True}

In[ ]:=
```
Table[{{A1, B1}, FullSimplify[
    ExpandAll[1/2 (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) ==
      η4488[[A1 + 1, B1 + 1]] * ID16]]},
  {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix}\{0,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{1,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{2,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{3,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{4,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{5,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{6,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{7,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,7\}\\ \text{True}\end{pmatrix} \end{pmatrix}$$

In[ ]:=
```
Table[{{A1, B1}, FullSimplify[ExpandAll[
    1/2 (newT16^A[A1].newT16^A[B1] + newT16^A[B1].newT16^A[A1]) ==
     η4488[[A1 + 1, B1 + 1]] * ID16]]},
  {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix}\{0,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{1,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{2,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{3,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{4,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{5,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{6,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{7,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,7\}\\ \text{True}\end{pmatrix} \end{pmatrix}$$

```
In[∘]:= Ψ16upper = Take[Ψ16, 8]
        Ψ16lower = Take[Ψ16, -8]
        Ψ16 == Flatten[{Ψ16upper, Ψ16lower}]
Out[∘]=
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4]}
Out[∘]=
{f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
Out[∘]=
True
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
Table[T16^A[A1] = ArrayFlatten[
    {{0, τ̄[A1]}, {τ[A1], 0}}], {A1, 0, 7}]
σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

$$\begin{pmatrix} 0 & \bar{\tau}[A1] \\ \tau[A1] & 0 \end{pmatrix} \begin{pmatrix} \Psi16upper \\ \Psi16lower \end{pmatrix} = \begin{pmatrix} \bar{\tau}[A1].\Psi16lower \\ \tau[A1].\Psi16upper \end{pmatrix}$$

$$\sigma16.\begin{pmatrix} 0 & \bar{\tau}[A1] \\ \tau[A1] & 0 \end{pmatrix} \begin{pmatrix} \Psi16upper \\ \Psi16lower \end{pmatrix} = \begin{pmatrix} -\sigma\,\bar{\tau}[A1].\Psi16lower \\ \sigma\,\tau[A1].\Psi16upper \end{pmatrix}$$

$$\Psi16.\sigma16.\begin{pmatrix} 0 & \bar{\tau}[A1] \\ \tau[A1] & 0 \end{pmatrix}.\Psi16 = \begin{pmatrix} -\Psi16upper.\sigma\,\bar{\tau}[A1]\,.\Psi16lower \\ \Psi16lower.\sigma\,\tau[A1]\,\Psi16upper \end{pmatrix}$$

```
In[∘]:= (*σ16.T16^A[#]&/@Range[0,7]*)
```

```
In[•]:= (*
(T16^A[8]=FullSimplify[
    T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]])//MatrixForm*)

In[•]:= η4488
Out[•]=
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}

In[•]:= η99 = DiagonalMatrix[{1, 1, 1, 1, -1, -1, -1, -1, 1}]
Out[•]=
{{1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1}}

In[•]:= (*Table[{A1,B1,
   FullSimplify[ExpandAll[1/2 ( T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1])== η99[[A1,B1]]*ID16]]},
  {A1,0, 8},{B1,0, 8}]//MatrixForm*)

In[•]:= (*Table[1/4 (T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]), {A1, 8},{B1,8}]*)

In[•]:= (P_L = 1/2 (ID16 - T16^A[8])) // MatrixForm
Out[•]//MatrixForm=
```

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

In[∘]:=
$$\left(P_R = \frac{1}{2}\left(\text{ID16} + \text{T16}^A[8]\right)\right) \text{ // MatrixForm}$$

Out[∘]//MatrixForm=
$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

In[∘]:= $\{\text{ID16} == P_L + P_R,\ P_L.P_L == P_L,\ P_L.P_R == P_R.P_L == \text{ZERO16}\}$

Out[∘]=
```
{True, True, True}
```

SAB = Table[
$$\frac{1}{4}\left(\text{T16}^A[\text{A1}].\text{T16}^A[\text{B1}] - \text{T16}^A[\text{B1}].\text{T16}^A[\text{A1}]\right),\ \{\text{A1}, 0, 7\}, \{\text{B1}, 0, 7\}];$$

In[∘]:=
$$\text{SAB} = \text{Table}\left[\frac{1}{4}\left(\text{T16}^A[\text{A1}].\text{T16}^A[\text{B1}] - \text{T16}^A[\text{B1}].\text{T16}^A[\text{A1}]\right),\ \{\text{A1}, 0, 7\}, \{\text{B1}, 0, 7\}\right];$$

In[∘]:=
```
ParallelTable[FullSimplify[σ16.SAB[[A1, B1]] === -Transpose[σ16.SAB[[A1, B1]]]],
    {A1, 1, 8}, {B1, 1, 8}] // Flatten // Union
```

Out[∘]=
```
{True}
```

In[ ]:=
```
ParallelTable[{{A1, B1}, FullSimplify[SAB⟦A1, B1⟧ === -Transpose[SAB⟦A1, B1⟧]]},
 {A1, 1, 8}, {B1, 1, 8}]
```

Out[ ]=
```
{{{{1, 1}, True}, {{1, 2}, True}, {{1, 3}, True}, {{1, 4}, True},
  {{1, 5}, False}, {{1, 6}, False}, {{1, 7}, False}, {{1, 8}, False}},
 {{{2, 1}, True}, {{2, 2}, True}, {{2, 3}, True}, {{2, 4}, True},
  {{2, 5}, False}, {{2, 6}, False}, {{2, 7}, False}, {{2, 8}, False}},
 {{{3, 1}, True}, {{3, 2}, True}, {{3, 3}, True}, {{3, 4}, True},
  {{3, 5}, False}, {{3, 6}, False}, {{3, 7}, False}, {{3, 8}, False}},
 {{{4, 1}, True}, {{4, 2}, True}, {{4, 3}, True}, {{4, 4}, True},
  {{4, 5}, False}, {{4, 6}, False}, {{4, 7}, False}, {{4, 8}, False}},
 {{{5, 1}, False}, {{5, 2}, False}, {{5, 3}, False}, {{5, 4}, False},
  {{5, 5}, True}, {{5, 6}, True}, {{5, 7}, True}, {{5, 8}, True}},
 {{{6, 1}, False}, {{6, 2}, False}, {{6, 3}, False}, {{6, 4}, False},
  {{6, 5}, True}, {{6, 6}, True}, {{6, 7}, True}, {{6, 8}, True}},
 {{{7, 1}, False}, {{7, 2}, False}, {{7, 3}, False}, {{7, 4}, False},
  {{7, 5}, True}, {{7, 6}, True}, {{7, 7}, True}, {{7, 8}, True}},
 {{{8, 1}, False}, {{8, 2}, False}, {{8, 3}, False}, {{8, 4}, False},
  {{8, 5}, True}, {{8, 6}, True}, {{8, 7}, True}, {{8, 8}, True}}}
```

In[ ]:=
```
forO44Similarity =
 ParallelTable[If[FullSimplify[SAB⟦A1, B1⟧ === -Transpose[SAB⟦A1, B1⟧]],
    {A1, B1}, ## &[]], {A1, 1, 7}, {B1, A1 + 1, 8}] // Flatten[#, 1] &
Length[%]
```

Out[ ]=
```
{{1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {5, 6}, {5, 7}, {5, 8}, {6, 7}, {6, 8}, {7, 8}}
```

Out[ ]=
```
12
```

In[ ]:=
```
SAB⟦##⟧ & @@ forO44Similarity⟦1⟧
MatrixExp[%]
```

Out[ ]=

$$\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{-\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0\right\}\right\}$$

Out[ ]=

```
{{Cos[1/2], 0, 0, 0, 0, 0, 0, Sin[1/2], 0, 0, 0, 0, 0, 0, 0, 0},
 {0, Cos[1/2], 0, 0, 0, 0, Sin[1/2], 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, Cos[1/2], 0, 0, -Sin[1/2], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, Cos[1/2], -Sin[1/2], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, Sin[1/2], Cos[1/2], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, Sin[1/2], 0, 0, Cos[1/2], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -Sin[1/2], 0, 0, 0, 0, Cos[1/2], 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {-Sin[1/2], 0, 0, 0, 0, 0, 0, Cos[1/2], 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, Cos[1/2], 0, 0, 0, 0, 0, 0, -Sin[1/2]},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[1/2], 0, 0, 0, 0, -Sin[1/2], 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[1/2], 0, 0, Sin[1/2], 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[1/2], Sin[1/2], 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -Sin[1/2], Cos[1/2], 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -Sin[1/2], 0, 0, Cos[1/2], 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, Sin[1/2], 0, 0, 0, 0, Cos[1/2], 0},
 {0, 0, 0, 0, 0, 0, 0, 0, Sin[1/2], 0, 0, 0, 0, 0, 0, Cos[1/2]}}
```

In[ ]:=

```
ParallelTable[FullSimplify[SAB[[A1, B1]].SAB[[A2, B2]] - SAB[[A2, B2]].SAB[[A1, B1]] ==
    -(η4488[[A1, A2]] × SAB[[B1, B2]] - η4488[[A1, B2]] × SAB[[B1, A2]] -
      η4488[[B1, A2]] × SAB[[A1, B2]] + η4488[[B1, B2]] × SAB[[A1, A2]])],
  {A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}] // Flatten // Union
```

Out[ ]=

```
{True}
```

In[ ]:=

```
ParallelTable[FullSimplify[SAB[[A1, B1]].T16^A[B2 - 1] - T16^A[B2 - 1].SAB[[A1, B1]] ==
    (-η4488[[B2, A1]] T16^A[B1 - 1] + η4488[[B2, B1]] T16^A[A1 - 1])],
  {A1, 1, 8}, {B1, 1, 8}, {B2, 1, 8}] // Flatten // Union
```

Out[ ]=

```
{True}
```

In[∘]:=
```
(*ParallelTable[{{A1,B1},FullSimplify[σ16.SAB〚A1,B1〛==-Transpose[σ16.SAB〚A1,B1〛]]},
 {A1,1, 8},{B1,1,8}]*)
```

In[∘]:=
```
(*ParallelTable[{{A1,B1,A2,B2},
  FullSimplify[SAB〚A1,B1〛.SAB〚A2,B2〛 -SAB〚A2,B2〛.SAB〚A1,B1〛==-(η4488〚A1,A2〛SAB〚B1,B2〛-
       η4488〚A1,B2〛SAB〚B1,A2〛-η4488〚B1,A2〛SAB〚A1,B2〛+η4488〚B1,B2〛SAB〚A1,A2〛)]},
 {A1,1, 7},{B1,A1+1,8},{A2,1, 7},{B2,A2+1,8}]*)
```

In[∘]:=
```
(*ParallelTable[{{A1,B1,B2},FullSimplify[SAB〚A1,B1〛.T16^A[B2-1] -T16^A[B2-1].SAB〚A1,B1〛==
     (-η4488〚B2,A1〛T16^A[B1-1]+η4488〚B2,B1〛T16^A[A1-1])]}, {A1,1, 8},{B1,1,8},{B2,1,8}]*)
```

In[∘]:=
```
Do[sAB[A1][B1] = 1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]),
  {A1, 0, 7}, {B1, 0, 7}];
```

In[∘]:=
```
O8similarityTransformation
```

Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[∘]:=
```
O8similarityTransformation.Ψ16 // Column
```

Out[∘]=
```
f16[0][x0, x4]
f16[1][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
f16[2][x0, x4]
f16[3][x0, x4]
f16[6][x0, x4]
f16[7][x0, x4]
f16[10][x0, x4]
f16[11][x0, x4]
f16[14][x0, x4]
f16[15][x0, x4]
```

In[∘]:=
```
scale16 = {1, 1, -1, -1, 1, 1, -1, -1, 1, 1, -1, -1, 1, 1, -1, -1};
Length[scale16]
```

Out[∘]=
```
16
```

In[•]:=
```
scale16mat = DiagonalMatrix[scale16]
```

Out[•]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}
```

In[•]:=
```
O8similarityTransformation.(scale16 * Ψ16) // Column
```

Out[•]=
```
f16[0][x0, x4]
f16[1][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
-f16[2][x0, x4]
-f16[3][x0, x4]
-f16[6][x0, x4]
-f16[7][x0, x4]
-f16[10][x0, x4]
-f16[11][x0, x4]
-f16[14][x0, x4]
-f16[15][x0, x4]
```

In[•]:=
```
Dimensions[ArrayFlatten[{{0, ID8}, {ID8, 0}}]]
```

Out[•]=
```
{16, 16}
```

In[•]:=
```
(try1616 =
   Table[Block[{m = O8similarityTransformation.(scale16 * Ψ16), d}, d = D[m[[j]], #] & /@ Ψ16;
     d], {j, 1, 16}]) // MatrixForm
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1
\end{pmatrix}
$$

In[ ]:=
```
try1616.Transpose[try1616] === ID16
```
Out[ ]=
```
True
```
In[ ]:=
```
(try1616.Ψ16) // Column
```
Out[ ]=
```
f16[0][x0, x4]
f16[1][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
-f16[2][x0, x4]
-f16[3][x0, x4]
-f16[6][x0, x4]
-f16[7][x0, x4]
-f16[10][x0, x4]
-f16[11][x0, x4]
-f16[14][x0, x4]
-f16[15][x0, x4]
```
In[ ]:=
```
(sqrta = ArrayFlatten[{{0, ID8}, {ID8, 0}}]) // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \end{pmatrix}$$

In[ ]:= {sJordanDecomposition, jJordanDecomposition} = JordanDecomposition[sqrta]

Out[ ]=

```
{{{0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {-1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}},
 {{-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}}
```

In[ ]:= sJordanDecomposition.jJordanDecomposition.Inverse[sJordanDecomposition] === sqrta

Out[ ]=

True

In[∘]:=
```
(sqrtb = sJordanDecomposition.√jJordanDecomposition.Inverse[sJordanDecomposition]) //
  MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
\frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} \\
\frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2}
\end{pmatrix}$$

In[∘]:=
```
O44similarityTransformation00 =
  Table[Block[{m = O8similarityTransformation.(scale16 * Ψ16), d}, d = D[m[[j]], #] & /@ Ψ16;
    d], {j, 1, 16}]
```

Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}
```

In[∘]:=
```
(O44similarityTransformation = sqrtb.O44similarityTransformation00.Inverse[sqrtb]) //
  MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} \\
\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} \\
-\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} \\
\frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2}
\end{pmatrix}$$

In[∘]:=
```
(*O44similarityTransformation=ArrayFlatten[{{0,ID8},{ID8,0}}].
  Table[Block[{m=O8similarityTransformation.(scale16*Ψ16),d},d=D[m[[j]],#]&/@Ψ16;
    d],{j,1,16}]*)
```

In[∘]:=
```
O44similarityTransformation.Transpose[O44similarityTransformation] === Id16
```

Out[∘]=

False

In[∘]:=
```
(O44similarityTransformation = sqrtb.O44similarityTransformation00.Transpose[sqrtb]);
O44similarityTransformation.Transpose[O44similarityTransformation] === Id16
```

Out[∘]=

False

In[∘]:=
```
(O44similarityTransformation = sqrtb.O44similarityTransformation00.sqrtb);
O44similarityTransformation.Transpose[O44similarityTransformation] === Id16
```

Out[∘]=

False

In[∘]:=
```
SAB[[1, 2]] // MatrixForm
O8similarityTransformation. SAB[[1, 2]].
  Transpose[O8similarityTransformation] // MatrixForm
O44similarityTransformation. SAB[[1, 2]].Transpose[O44similarityTransformation] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0\\
0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

In[ ]:= **sAB[-1 + 1][-1 + 2] == SAB⟦1, 2⟧**

Out[ ]=

True

In[ ]:= **σ16.SAB⟦1, 2⟧ // MatrixForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{1}{2}&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&\frac{1}{2}&0&0&0
\end{pmatrix}$$

In[ ]:=
```
σ16.sAB[1][2] // MatrixForm
```

Out[ ]//MatrixForm=

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula:

$$[J_{ab}, J_{cd}] = i(\eta_{ac} J_{bd} - \eta_{ad} J_{bc} - \eta_{bc} J_{ad} + \eta_{bd} J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+,+,+,-,-,-)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \mathbf{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties.

The final answer is $[\mathbf{J_{ab}}, \mathbf{J_{cd}}] = \mathbf{i}(\boldsymbol{\eta}_{\mathbf{ac}} \mathbf{J_{bd}} - \boldsymbol{\eta}_{\mathbf{ad}} \mathbf{J_{bc}} - \boldsymbol{\eta}_{\mathbf{bc}} \mathbf{J_{ad}} + \boldsymbol{\eta}_{\mathbf{bd}} \mathbf{J_{ac}})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4).

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \ldots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention)

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$.

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa).

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not).

what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the $so(4,4)$ Lie algebra, where $\mu, \nu \in \{0, 1, \ldots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the standard relations for orthogonal Lie algebras, given by a generalization of the angular momentum commutation relations:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for $so(8)$ in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).

↳ Why is the metric signature significant for $so(4,4)$ commutation relations?

Why is the metric signature significant for $so(4,4)$ commutation relations?

The metric signature is significant for the $so(p,q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable).

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p,q)$ is given by:
  $[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$
  The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4,4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
  - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[ ]:=
```
η4488
```
Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
```

orthogonal Lie algebra $so(p,q)$ is given by:
$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$

In[ ]:=
```
SAB⟦6, 2⟧.SAB⟦6, 7⟧ - SAB⟦6, 7⟧.SAB⟦6, 2⟧ == η4488⟦6, 6⟧ × SAB⟦7, 2⟧
```
Out[ ]=
```
True
```
In[ ]:=
```
{{6, 2, 6, 7}, False}
```
Out[ ]=
```
{{6, 2, 6, 7}, False}
```
In[ ]:=
```
SAB⟦1, 2⟧.SAB⟦1, 3⟧ - SAB⟦1, 3⟧.SAB⟦1, 2⟧ == η4488⟦1, 1⟧ × SAB⟦2, 3⟧
```
Out[ ]=
```
False
```

In[ ]:= **sAB[0][0]**

Out[ ]=

```
{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

In[ ]:= **sAB[0][1]**

Out[ ]=

```
{{0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {-1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0}}
```

In[ ]:= **Table[Transpose[T16^A[8].sAB[A1][B1]] == T16^A[8].sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]**

Out[ ]=

```
{{True, False, False, False, True, True, True, True},
 {False, True, False, False, True, True, True, True},
 {False, False, True, False, True, True, True, True},
 {False, False, False, True, True, True, True, True},
 {True, True, True, True, True, False, False, False},
 {True, True, True, True, False, True, False, False},
 {True, True, True, True, False, False, True, False},
 {True, True, True, True, False, False, False, True}}
```

```
In[ ]:= Table[Transpose[T16^A[7].sAB[A1][B1]] == T16^A[7].sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[ ]=
{{True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {False, False, False, False, True, True, True, False},
 {False, False, False, False, True, True, True, False},
 {False, False, False, False, True, True, True, False},
 {True, True, True, True, False, False, False, True}}
In[ ]:= Table[Transpose[σ16.sAB[A1][B1]] == σ16.sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[ ]=
{{True, False, False, False, False, False, False, False},
 {False, True, False, False, False, False, False, False},
 {False, False, True, False, False, False, False, False},
 {False, False, False, True, False, False, False, False},
 {False, False, False, False, True, False, False, False},
 {False, False, False, False, False, True, False, False},
 {False, False, False, False, False, False, True, False},
 {False, False, False, False, False, False, False, True}}
In[ ]:= Table[Transpose[σ16.SAB[[A1, B1]]] == σ16.SAB[[A1, B1]], {A1, 1, 8}, {B1, 1, 8}]
Out[ ]=
{{True, False, False, False, False, False, False, False},
 {False, True, False, False, False, False, False, False},
 {False, False, True, False, False, False, False, False},
 {False, False, False, True, False, False, False, False},
 {False, False, False, False, True, False, False, False},
 {False, False, False, False, False, True, False, False},
 {False, False, False, False, False, False, True, False},
 {False, False, False, False, False, False, False, True}}
In[ ]:= SAB[[1, 2]][[1]]
Out[ ]=
```

$$\left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0\right\}$$

```
In[ ]:= SAB[[1, 2]][[2]]
Out[ ]=
```

$$\left\{0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}$$

$$t^A = \begin{pmatrix} 0 & \overline{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
In[ ]:= SAB1 = Table[Take[SAB[[A1, B1]][[C1]], 8],  {A1, 8}, {B1, 8}, {C1, 1, 8}];
In[ ]:= SAB2 = Table[Take[SAB[[A1, B1]][[C1]], -8],  {A1, 8}, {B1, 8}, {C1, 9, 16}];
```

In[∘]:=
```
SAB⟦1, 2⟧ == ArrayFlatten[{{SAB1⟦1, 2⟧, 0}, {0, SAB2⟦1, 2⟧}}]
```
Out[∘]=
```
True
```

In[∘]:=
```
(*Table[
 SAB⟦A1,B1⟧== ArrayFlatten[{{SAB1⟦A1,B1⟧,0},{0,SAB2⟦A1,B1⟧}}],
 {A1,0, 7},{B1,0,7}]*)
```

$$4D_{(1)}{}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}{}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A .$$

Let $g \in \overline{SO}(4,4)$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO}(4,4) \to SO(4,4)$ is given by

(2.8) $$8L^{A'}{}_{B'}(g) = \mathrm{tr}\, \widetilde{D}^{(2)}(g)\, \tau^{A'} D^{(1)}(g)\, \bar{\tau}_{B'} .$$

As usual tr denotes the trace. Under the action of $\overline{SO}(4,4)$

(2.9) $$\widetilde{D}^{(1)} \sigma = \sigma D^{(1)-1} ,$$

(2.10) $$\sigma^{-1} \widetilde{D}^{(2)} = D^{(2)-1} \sigma^{-1} ,$$

(2.11) $$L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'} ,$$

(2.12) $$L^{A'}{}_{B'} \tau^{B'} = \widetilde{D}^{(2)} \tau^{A'} D^{(1)} ,$$

(2.13) $$L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \widetilde{D}^{(2)-1} .$$

In[∘]:=
```
SAB2⟦1, 2⟧
```
Out[∘]=

$$\left\{\left\{0,0,0,0,0,0,0,-\frac{1}{2}\right\},\left\{0,0,0,0,0,0,-\frac{1}{2},0\right\},\left\{0,0,0,0,0,\frac{1}{2},0,0\right\},\left\{0,0,0,0,\frac{1}{2},0,0,0\right\},\left\{0,0,0,-\frac{1}{2},0,0,0,0\right\},\left\{0,0,-\frac{1}{2},0,0,0,0,0\right\},\left\{0,\frac{1}{2},0,0,0,0,0,0\right\},\left\{\frac{1}{2},0,0,0,0,0,0,0\right\}\right\}$$

In[∘]:=
```
τ[0]
```

Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[∘]:=
```
(*Block[{b,A,F_a^(μ),F^a_(μ),ψ,Ψ},
 b=Array[A,{8}];
 ψ[1]=Array[Ψ[1],{8}];
 ψ[2]=Array[Ψ[2],{8}];
 F_a^(μ)=Table[(1/Sqrt[ψ[1].σ.ψ[1]] ψ[1].σ.(τbar[μL])),{μL,1,8}];
 F^a_(μ)= Transpose[Table[(1/Sqrt[ψ[1].σ.ψ[1]] η4488[[μL,μL]] τ[μL].ψ[1]),{μL,1,8}]];
 b===FullSimplify[F_a^(μ).ψ[2]/.Thread[ψ[2]→F^a_(μ).b]]]*)
```

In[∘]:=
```
(*Block[{b,A,B,q,F^a_(μ),ψ,φ,Ψ},
 b=Array[A,{8}];
 ψ[1]=Array[Ψ[1],{8}];
 φ[1]=Array[Ψ[2],{8}];
 q=ParallelTable[FullSimplify[
     (1/Sqrt[ψ[1].σ.ψ[1]] ψ[1].σ.1/2 (τbar[A1].τ[B1]-τbar[B1].τ[A1]).φ[1])],{A1,1,8},{B1,1,8}];
 B=FullSimplify[(1/Sqrt[ψ[1].σ.ψ[1]]  ParallelSum[η4488[[μL,μL]] τ[μL] b[[μL]],{μL,1,8}].ψ[1])];
 FullSimplify[q/.Thread[φ[1]→B]]]*)
```

In[∘]:=
```
(*Block[{b,A,B,q,F^a_(μ),ψ,φ,Ψ},
 b=Array[A,{8}];
 ψ[1]=Array[Ψ[1],{8}];
 φ[1]=Array[Ψ[2],{8}];
 q=ParallelTable[FullSimplify[(ψ[1].σ.(τbar[A1].τ[B1]).φ[1])],{A1,1,8},{B1,1,8}];
 B=FullSimplify[( ParallelSum[η4488[[μL,μL]] τ[μL] b[[μL]],{μL,1,8}].ψ[1])];
 FullSimplify[1/(ψ[1].σ.ψ[1]) q/.Thread[φ[1]→B]]]*)
```

$T16^{\alpha}$

In[ ]:= ssgm4488

Out[ ]=

$$\{g[0][0] \to (\mathrm{Cot}[6 H \#1]^2 \&), g[0][1] \to (0 \&), g[0][2] \to (0 \&), g[0][3] \to (0 \&), g[0][4] \to (0 \&), g[0][5] \to (0 \&), g[0][6] \to (0 \&), g[0][7] \to (0 \&), g[1][0] \to (0 \&), g[1][1] \to (e^{2 a4[H \#2]} \mathrm{Sin}[6 H \#1]^{1/3} \&), g[1][2] \to (0 \&), g[1][3] \to (0 \&), g[1][4] \to (0 \&), g[1][5] \to (0 \&), g[1][6] \to (0 \&), g[1][7] \to (0 \&), g[2][0] \to (0 \&), g[2][1] \to (0 \&), g[2][2] \to (e^{2 a4[H \#2]} \mathrm{Sin}[6 H \#1]^{1/3} \&), g[2][3] \to (0 \&), g[2][4] \to (0 \&), g[2][5] \to (0 \&), g[2][6] \to (0 \&), g[2][7] \to (0 \&), g[3][0] \to (0 \&), g[3][1] \to (0 \&), g[3][2] \to (0 \&), g[3][3] \to (e^{2 a4[H \#2]} \mathrm{Sin}[6 H \#1]^{1/3} \&), g[3][4] \to (0 \&), g[3][5] \to (0 \&), g[3][6] \to (0 \&), g[3][7] \to (0 \&), g[4][0] \to (0 \&), g[4][1] \to (0 \&), g[4][2] \to (0 \&), g[4][3] \to (0 \&), g[4][4] \to (-1 \&), g[4][5] \to (0 \&), g[4][6] \to (0 \&), g[4][7] \to (0 \&), g[5][0] \to (0 \&), g[5][1] \to (0 \&), g[5][2] \to (0 \&), g[5][3] \to (0 \&), g[5][4] \to (0 \&), g[5][5] \to (-e^{-2 a4[H \#2]} \mathrm{Sin}[6 H \#1]^{1/3} \&), g[5][6] \to (0 \&), g[5][7] \to (0 \&), g[6][0] \to (0 \&), g[6][1] \to (0 \&), g[6][2] \to (0 \&), g[6][3] \to (0 \&), g[6][4] \to (0 \&), g[6][5] \to (0 \&), g[6][6] \to (-e^{-2 a4[H \#2]} \mathrm{Sin}[6 H \#1]^{1/3} \&), g[6][7] \to (0 \&), g[7][0] \to (0 \&), g[7][1] \to (0 \&), g[7][2] \to (0 \&), g[7][3] \to (0 \&), g[7][4] \to (0 \&), g[7][5] \to (0 \&), g[7][6] \to (0 \&), g[7][7] \to (-e^{-2 a4[H \#2]} \mathrm{Sin}[6 H \#1]^{1/3} \&)\}$$

In[ ]:= $e^{\alpha}_{(A)}$ /. $sge^{\alpha}_{(A)}$

Out[ ]=

$$\left\{\left\{\frac{1}{\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\right\}, \left\{0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}\right\}\right\}$$

In[ ]:= Table[$T16^{\alpha}$[α1 - 1] = Sum[(($e^{\alpha}_{(A)}$ /. $sge^{\alpha}_{(A)}$)⟦α1, A1⟧) $T16^{A}$[A1 - 1], {A1, 1, 8}], {α1, 1, 8}];(*/.ssgm4488*)

In[ ]:= ($T16^{\alpha}$[8] = $T16^{\alpha}$[0].$T16^{\alpha}$[1].$T16^{\alpha}$[2].$T16^{\alpha}$[3].$T16^{\alpha}$[4].$T16^{\alpha}$[5].$T16^{\alpha}$[6].$T16^{\alpha}$[7]);

In[ ]:= Table[new$T16^{\alpha}$[A1] = O8similarityTransformation.$T16^{\alpha}$[A1].Transpose[O8similarityTransformation], {A1, 0, 8}];

In[ ]:= (*$T16^{\alpha}$[8]==$T16^{A}$[8]*Sec[6 H x0]*)

CHECK

Table$\left[T16^{\alpha}[\alpha 1-1] = \text{Sum}\left[\left(e^{\alpha}_{(A)}[\![\alpha 1, A1]\!]\right) T16^{A}[A1-1], \{A1, 1, 8\}\right], \{\alpha 1, 1, 8\}\right]$; and ● ● ● :

In[∘]:=
```
(*Table[T16^α[α1-1]=
    Sum[(e^α_(A)[[α1,A1]] ) T16^A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

In[∘]:=
```
(*(T16^α[8]=T16^α[0].T16^α[1].T16^α[2].T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7])//
 MatrixForm*)
```

In[∘]:=
```
Table[{{A1, B1}, FullSimplify[
    ExpandAll[1/2 (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) ==
      η4488[[A1 + 1, B1 + 1]] * ID16]]},
  {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
\begin{pmatrix}\{0,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{1,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{2,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{3,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{4,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{5,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{6,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{7,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,7\}\\ \text{True}\end{pmatrix}
\end{pmatrix}$$

In[ ]:=
```
Table[{{α1, β1}, FullSimplify[
    1/2 FullSimplify[((T16^α[α1].T16^α[β1] + T16^α[β1].T16^α[α1]) /.
         ssgm4488), constraintVars] ==
     Inverse[MatrixMetric44][[α1 + 1, β1 + 1]] * ID16]},
  {α1, 0, 7}, {β1, 0, 7}] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
\begin{pmatrix}\{0,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{1,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{2,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{3,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{4,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{5,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{6,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,7\}\\ \text{True}\end{pmatrix} \\
\begin{pmatrix}\{7,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,7\}\\ \text{True}\end{pmatrix}
\end{pmatrix}$$

In[∘]:=
```
Table[{{α1, β1}, FullSimplify[
   1/2 FullSimplify[((newT16^α[α1].newT16^α[β1] + newT16^α[β1].
          newT16^α[α1]) /. ssgm4488), constraintVars] ==
    Inverse[MatrixMetric44][[α1 + 1, β1 + 1]] * ID16]},
  {α1, 0, 7}, {β1, 0, 7}] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix}\{0,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{0,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{1,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{1,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{2,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{2,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{3,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{3,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{4,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{4,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{5,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{5,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{6,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{6,7\}\\ \text{True}\end{pmatrix} \\ \begin{pmatrix}\{7,0\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,1\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,2\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,3\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,4\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,5\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,6\}\\ \text{True}\end{pmatrix} & \begin{pmatrix}\{7,7\}\\ \text{True}\end{pmatrix} \end{pmatrix}$$

In[∘]:= `MatrixMetric44 // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6\,H\,x0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -e^{-2\,a4[H\,x4]}\,\text{S} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[∘]:= $((e^{\alpha}_{(A)}\ /.\ \text{sgtrye}^{\alpha}_{(A)}).\text{MatrixMetric44}.(e^{\alpha}_{(A)}\ /.\ \text{sgtrye}^{\alpha}_{(A)}))$ // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
Transpose[(e_α^(A) /. sgtrye_α^(A))].η4488.(e_α^(A) /. sgtrye_α^(A)) - MatrixMetric44 //
  FullSimplify[#, constraintVars] &
```

Out[∘]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[∘]:=
```
Transpose[(e^α_(A) /. sgtrye^α_(A))].MatrixMetric44.(e^α_(A) /. sgtrye^α_(A)) - η4488 //
  (*Full*)Simplify[#, constraintVars] &
```

Out[∘]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[∘]:=
```
Inverse[(e^α_(A) /. sgtrye^α_(A))] == (e_α^(A) /. sgtrye_α^(A))
```

Out[∘]=
```
True
```

In[∘]:=
```
Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]
```

Out[∘]=
```
{{0, 0}, {0, 0}}
```

BELOW : LagrangianΨ16 = $\kappa\sqrt{\mathrm{Det}[g_{\mu\nu}]}$ Transpose[Ψ16].σ16.T16^{A}[A1]. $\Psi16_{|\alpha}\, g^{-1\ \alpha\beta}\, \eta_{A1\,B1}\, E^{B1}_{\beta}$ + mass term

In[∘]:=
```
e_α^(A) /. sgtrye_α^(A)
```

Out[∘]=

$$\left\{\{\mathrm{Cot}[6\,H\,x0], 0, 0, 0, 0, 0, 0, 0\}, \left\{0, \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}, 0, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, 0, \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}, 0, 0, 0, 0\right\},$$
$$\{0, 0, 0, 0, 1, 0, 0, 0\}, \left\{0, 0, 0, 0, 0, \sqrt{e^{-2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, \sqrt{e^{-2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}\right\}\right\}$$

In[∘]:= $e^{\alpha}_{(A)}$ /. $sgtrye^{\alpha}_{(A)}$

Out[∘]=

$$\left\{\{\text{Tan}[6\,H\,x0], 0, 0, 0, 0, 0, 0, 0\}, \left\{0, \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}}, 0, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, 0, \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}}, 0, 0, 0, 0\right\},$$
$$\{0, 0, 0, 0, 1, 0, 0, 0\}, \left\{0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}}, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}}, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}}\right\}\right\}$$

In[∘]:= $e^{\alpha}_{(A)}$ /. $sge^{\alpha}_{(A)}$

Out[∘]=

$$\left\{\left\{\frac{1}{\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}\right\}\right\}$$

In[∘]:=
```
Clear[con];
con[g_, ass_ : {}] := Module[{t, ΓΓ, ginv},
  ginv = FullSimplify[Inverse[g], ass];
  ΓΓ = FullSimplify[Table[(1 / 2) * Sum[(ginv⟦i, s⟧) *
        (D[g⟦s, j⟧, X⟦k⟧ ] + D[g⟦s, k⟧, X⟦j⟧ ] - D[g⟦j, k⟧, X⟦s⟧ ]), {s, 1, DIM8}],
     {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] , ass];
  Return[{ginv, ΓΓ}]]
```

In[∘]:= ssgm4488

Out[∘]=

```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
 g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
 g[1][1] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
 g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
 g[2][1] → (0 &), g[2][2] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
 g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
 g[3][2] → (0 &), g[3][3] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
 g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
 g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
 g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
 g[5][5] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
 g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
 g[6][5] → (0 &), g[6][6] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
 g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
 g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &)}
```

In[∘]:= g4488

g4488 /. sg

Out[∘]=

```
{{g[0][0][x0, x4], g[0][1][x0, x4], g[0][2][x0, x4], g[0][3][x0, x4],
  g[0][4][x0, x4], g[0][5][x0, x4], g[0][6][x0, x4], g[0][7][x0, x4]},
 {g[1][0][x0, x4], g[1][1][x0, x4], g[1][2][x0, x4], g[1][3][x0, x4],
  g[1][4][x0, x4], g[1][5][x0, x4], g[1][6][x0, x4], g[1][7][x0, x4]},
 {g[2][0][x0, x4], g[2][1][x0, x4], g[2][2][x0, x4], g[2][3][x0, x4],
  g[2][4][x0, x4], g[2][5][x0, x4], g[2][6][x0, x4], g[2][7][x0, x4]},
 {g[3][0][x0, x4], g[3][1][x0, x4], g[3][2][x0, x4], g[3][3][x0, x4],
  g[3][4][x0, x4], g[3][5][x0, x4], g[3][6][x0, x4], g[3][7][x0, x4]},
 {g[4][0][x0, x4], g[4][1][x0, x4], g[4][2][x0, x4], g[4][3][x0, x4],
  g[4][4][x0, x4], g[4][5][x0, x4], g[4][6][x0, x4], g[4][7][x0, x4]},
 {g[5][0][x0, x4], g[5][1][x0, x4], g[5][2][x0, x4], g[5][3][x0, x4],
  g[5][4][x0, x4], g[5][5][x0, x4], g[5][6][x0, x4], g[5][7][x0, x4]},
 {g[6][0][x0, x4], g[6][1][x0, x4], g[6][2][x0, x4], g[6][3][x0, x4],
  g[6][4][x0, x4], g[6][5][x0, x4], g[6][6][x0, x4], g[6][7][x0, x4]},
 {g[7][0][x0, x4], g[7][1][x0, x4], g[7][2][x0, x4], g[7][3][x0, x4],
  g[7][4][x0, x4], g[7][5][x0, x4], g[7][6][x0, x4], g[7][7][x0, x4]}}
```

Out[∘]=

```
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}
```

In[ ]:=
```
MatrixForm[#] & /@ Block[{eAα, eαA, (*ginv,Γ,gg,*)colVecs, rowVecs, ab, ba, r},
  (*gg=g4488/.sg;*)
  (*{ginv,Γ}=con[gg];*)
  eαA = e^α_(A) /. sge^α_(A);
  eAα = e_α^(A) /. sge_α^(A);
  rowVecs = Table[eAα[[b, All]], {b, 1, Length[eAα]}];
  colVecs = Table[eαA[[All, b]], {b, 1, Length[eαA]}];
  ab = rowVecs.colVecs // FullSimplify[#, constraintVars] &;
  ba = Transpose[colVecs].Transpose[rowVecs] // FullSimplify[#, constraintVars] &;
  (*r={{ab},{ba}};*)
  r = {ab, ba};
  r]
```

Out[ ]=

$$\left\{\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}\right\}$$

metric compatibility condition for the Octad

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

In[ ]:=
```
Clear[spinCoefficients];
spinCoefficients[eIν_] := Module[{evI, w, r, s},
  If[Dimensions[eIν] ≠ {8, 8}, Return[{}]];
  If[Det[eIν] == 0, Return[{}]];
  evI = FullSimplify[Inverse[eIν], constraintVars] /. subsDefects;
  w = Table[-Table[
       FullSimplify[Sum[D[eIν[[I1, ν1]], X[[μ1]]] * evI[[ν1, Jprime]], {ν1, 1, Length[evI]}] -
          Sum[ eIν[[I1, ρ]] * Γ[[ρ, μ1, ν1]] * evI[[ν1, Jprime]], {ν1, 1, Length[evI]},
            {ρ, 1, Length[X]}], constraintVars] /. subsDefects,
       {I1, 1, Length[eIν]}, {Jprime, 1, Length[evI]}], {μ1, 1, Length[X]}];
  Assert[Dimensions[w] == {8, 8, 8}];
  Return[w];
 ]
```

In[∘]:= spinCoeffs = spinCoefficients$\left[\text{gtrye}_{\alpha}^{(A)}\right]$

Out[∘]=

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, -e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0,
   0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, -e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, -e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4],
   0, 0, 0}, {0, 0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0},
  {e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6)},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4]},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}}
```

In[ ]:=

```
Table[
 ωμIJ[μ] = FullSimplify[-((D[gtrye_α^(A), X[[μ]]].gtrye^α_(A)) - (gtrye_α^(A).Γ[[All, μ, All]].gtrye^α_(A))),
    constraintVars] /. subsDefects, {μ, 1, Length[X]}]
```

Out[ ]=

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, -e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0,
   0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, -e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, -e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4],
   0, 0, 0}, {0, 0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0},
  {e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6)},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4]},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}}
```

In[ ]:=

```
(*gtrye^α_(A).gtrye_α^(A)*)
```

In[ ]:=

```
(*gtrye_α^(A).gtrye^α_(A)*)
```

In[ ]:=

```
(*gtrye_α^(A)*)
```

In[ ]:=

```
(*Γ
 Dimensions[%]*)
```

In[ ]:=

```
(*Table[Γ[[All,μ,All]],{μ,1,Length[X]}]*)
```

```
In[∘]:= (*Γ⟦All,5,All⟧*)

In[∘]:= (*Γ⟦All,1,All⟧*)

In[∘]:= (*spinCoeffs⟦1⟧*)

In[∘]:= (*spinCoeffs⟦5⟧*)

In[∘]:= (*{X⟦1⟧,X⟦5⟧}*)

In[∘]:= (*spinCoeffs⟦1⟧⟦1⟧⟦1⟧*)

In[∘]:= (*ωμIJ[1]⟦1⟧⟦1⟧*)

In[∘]:= (*Block[{μ=1},FullSimplify[-((D[gtrye_α^(A),X⟦μ⟧].gtrye^α_(A))-(gtrye_α^(A).Γ⟦All,μ,All⟧.gtrye^α_(A))),
          constraintVars]/.subsDefects]
        Dimensions[%]*)

In[∘]:= (*Block[{μ=1},FullSimplify[
          -((D[gtrye_α^(A),X⟦μ⟧].Transpose[gtrye^α_(A)])-(gtrye_α^(A).Γ⟦All,μ,All⟧.Transpose[gtrye^α_(A)])),
          constraintVars]/.subsDefects]*)

In[∘]:= (*Block[{μ=1},
         FullSimplify[-((Sum[    D[gtrye_α^(A)⟦All,ν⟧,X⟦μ⟧]*gtrye^α_(A)⟦ν,All⟧     ,{ν,1,8}]   )-
              (gtrye_α^(A).Γ⟦All,μ,All⟧.gtrye^α_(A))),constraintVars]/.subsDefects]
        Dimensions[%]*)

In[∘]:= (*Table[,{A1,1,8},{B1,1,8}]*)

In[∘]:= (*Block[{μ=1},
         Table[FullSimplify[-((Sum[    D[gtrye_α^(A)⟦A1,ν⟧,X⟦μ⟧]*gtrye^α_(A)⟦ν,B1⟧     ,{ν,1,8}]   )-
              (Sum[    gtrye_α^(A)⟦A1,ρ⟧*Γ⟦ρ,μ,ν⟧*gtrye^α_(A)⟦ν,B1⟧    ,{ρ,1,8},{ν,1,8}]   )),
            constraintVars]/.subsDefects,{A1,1,8},{B1,1,8}]]
        Dimensions[%]*)

In[∘]:= Table[ ( ωμIJ[μ] - spinCoeffs⟦μ⟧ ) /. subsDefects , {μ, 1, Length[X]}] // Flatten // Union

Out[∘]= {0}
```

- **Metric compatibility**: The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
    - $\nabla_\mu e^a{}_\nu = 0$
    - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

○ Metric compatibility : The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame . The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is : ○ ∇

не "
v=0
• Here, Vu is the covariant derivative with respect to the connection F" bv, and ea v is
the tetrad component .

No it is not the torsionless condition which is:

$$T^{I}_{\mu\nu} = D^{\omega}_{[\mu}e^{I}_{\nu]} = \partial_{[\mu}e^{I}_{\nu]} + \omega^{I}_{[\mu J}e^{J}_{\nu]} = 0$$

This postulate says:

$$\nabla_{\mu}e^{I}_{\nu} = \partial_{\mu}e^{I}_{\nu} - \Gamma^{\rho}_{\mu\nu}e^{I}_{\rho} + \omega^{I}_{\mu J}\, e^{J}_{\nu} = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistancy" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^{\omega}_{\mu}\eta_{IJ} = 0$ (neverthless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will neverthless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called ‘Tetrad Postulate’ and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

In[∘]:=
```
Block[{eAα, eαA, t, ginv, Γ, gg},
 gg = g4488 /. sg;
 {ginv, Γ} = con[gg];
 eαA = e^α_(A) /. sge^α_(A); eAα = e_α^(A) /. sge_α^(A);
 t = Table[ Block[{v, V, contra, covari},
    V = eAα[[b, All]];
    v = eαA[[All, b]];
    contra = Table[D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before ω terms*)
    covari = Table[D[V[[k]], X[[μ]] ] - Sum[ V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before ω terms*)
    {b - 1, X[[μ]], {{contra}, {covari}}}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
 t]
FullSimplify[# /. ssgm4488, constraintVars] & /@ %
```

Out[∘]=

$$\left\{\left\{\left\{0,\ x0,\ \left\{\left\{\left\{0,\ 0,\ 0,\ 0,\ -\frac{g[0][0]^{(0,1)}[x0,\ x4]}{2\,\sqrt{g[0][0][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0\right\}\right\},\right.\right.\right.\right.$$

$$\left\{\left\{0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{0, x1, \left\{\left\{\left\{0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\ g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0\right\}\right\},\right.\right.$$

$$\left\{\left\{0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{0, x2, \left\{\left\{\left\{0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\ g[2][2][x0, x4]}, 0, 0, 0, 0, 0\right\}\right\},\right.\right.$$

$$\left\{\left\{0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{0, x3, \left\{\left\{\left\{0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\ g[3][3][x0, x4]}, 0, 0, 0, 0\right\}\right\},\right.\right.$$

$$\left\{\left\{0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{0, x4, \left\{\left\{\left\{0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\ g[4][4][x0, x4]}, 0, 0, 0\right\}\right\},\right.\right.$$

$$\left\{\left\{0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{0, x5, \left\{\left\{\left\{0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\ g[5][5][x0, x4]}, 0, 0\right\}\right\},\right.\right.$$

$$\left\{\left\{0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{0, x6, \left\{\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\ g[6][6][x0, x4]}, 0\right\}\right\},\right.\right.$$

$$\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0\right\}\right\}\right\}\right\},$$

$$\left\{0, x7, \left\{\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\ g[7][7][x0, x4]}\right\}\right\},\right.\right.$$

$$\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}\right\}\right\}\right\}\right\}\right\},$$

$$\left\{\{1, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \left\{1, x1,\right.\right.$$

$$\left\{\left\{\left\{-\frac{g[1][1]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\ \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}\ g[4][4][x0, x4]},\right.\right.\right.$$

$$0, 0, 0\Big\}\Big\}, \left\{\left\{-\frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0\right\}\right\}\Big\}\Big\},$$

{1, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{1, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\{1, x4, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{1, x5, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{1, x6, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{1, x7, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\}\Big\},$$
$$\Big\{\{2, x0, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{2, x1, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\}, \Big\{2, x2,$$
$$\Big\{\Big\{\Big\{-\frac{g[2][2]^{(1,0)}[x0, x4]}{2\,g[0][0][x0, x4]\,\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4]}{2\,\sqrt{g[2][2][x0, x4]}\,g[4][4][x0, x4]},$$
$$0, 0, 0\Big\}\Big\}, \Big\{\Big\{-\frac{g[2][2]^{(1,0)}[x0, x4]}{2\,\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4]}{2\,\sqrt{g[2][2][x0, x4]}}, 0, 0, 0\Big\}\Big\}\Big\}\Big\},$$
$$\{2, x3, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{2, x4, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{2, x5, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{2, x6, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{2, x7, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\}\Big\},$$
$$\Big\{\{3, x0, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{3, x1, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{3, x2, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\}, \Big\{3, x3,$$
$$\Big\{\Big\{\Big\{-\frac{g[3][3]^{(1,0)}[x0, x4]}{2\,g[0][0][x0, x4]\,\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]}{2\,\sqrt{g[3][3][x0, x4]}\,g[4][4][x0, x4]},$$
$$0, 0, 0\Big\}\Big\}, \Big\{\Big\{-\frac{g[3][3]^{(1,0)}[x0, x4]}{2\,\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]}{2\,\sqrt{g[3][3][x0, x4]}}, 0, 0, 0\Big\}\Big\}\Big\}\Big\},$$
$$\{3, x4, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{3, x5, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{3, x6, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\},$$
$$\{3, x7, \{\{\{0,0,0,0,0,0,0,0\}\}, \{\{0,0,0,0,0,0,0,0\}\}\}\}\Big\},$$
$$\Big\{\Big\{4, x0, \Big\{\Big\{\Big\{\frac{g[0][0]^{(0,1)}[x0, x4]}{2\,g[0][0][x0, x4]\,\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0,$$
$$\frac{g[4][4]^{(1,0)}[x0, x4]}{2\,(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2\,\sqrt{-g[4][4][x0, x4]}\,g[4][4][x0, x4]}, 0, 0, 0\Big\}\Big\},$$
$$\Big\{\Big\{\frac{g[0][0]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0,$$
$$-\frac{g[4][4]^{(1,0)}[x0, x4]}{2\,\sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(1,0)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0\Big\}\Big\}\Big\}\Big\},$$
$$\Big\{4, x1, \Big\{\Big\{\Big\{0, \frac{g[1][1]^{(0,1)}[x0, x4]}{2\,g[1][1][x0, x4]\,\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0\Big\}\Big\},$$
$$\Big\{\Big\{0, \frac{g[1][1]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{4,\ x2,\ \Big\{\Big\{\Big\{0,\ 0,\ \frac{g[2][2]^{(0,1)}[x0,\ x4]}{2\,g[2][2][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ \frac{g[2][2]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]},\ 0,\ 0,\ 0,\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{4,\ x3,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ \frac{g[3][3]^{(0,1)}[x0,\ x4]}{2\,g[3][3][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ \frac{g[3][3]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]},\ 0,\ 0,\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{4,\ x4,\ \Big\{\Big\{\Big\{-\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\,g[0][0][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0,$$

$$\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\,(-g[4][4][x0,\ x4])^{3/2}}+\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\,\sqrt{-g[4][4][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{-\frac{g[4][4]^{(1,0)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]},\ 0,\ 0,\ 0,$$

$$-\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\,\sqrt{-g[4][4][x0,\ x4]}}-\frac{g[4][4]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]},\ 0,\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{4,\ x5,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\,\sqrt{-g[4][4][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]},\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{4,\ x6,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\,\sqrt{-g[4][4][x0,\ x4]}\ g[6][6][x0,\ x4]},\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]},\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{4,\ x7,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\,\sqrt{-g[4][4][x0,\ x4]}\ g[7][7][x0,\ x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[7][7]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]}\Big\}\Big\}\Big\}\Big\}\Big\},\ \Big\{\Big\{5,\ x0,$$

$$\Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\,(-g[5][5][x0,\ x4])^{3/2}}+\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\,\sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\,\sqrt{-g[5][5][x0,\ x4]}}-\frac{g[5][5]^{(1,0)}[x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}}{2\,g[5][5][x0,\ x4]},$$

$$0,\ 0\Big\}\Big\}\Big\}\Big\},\ \{5,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{5,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{5,\ x3,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\ \Big\{5,\ x4,$$

$$\Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\,(-g[5][5][x0,\ x4])^{3/2}}+\frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\,\sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0\Big\}\Big\},$$

$$\left\{\left\{0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}} - \frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]},\right.\right.$$

$$\left.\left.0, 0\right\}\right\}\right\}\right\}, \left\{5, x5, \left\{\left\{\left\{-\frac{g[5][5]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\sqrt{-g[5][5][x0, x4]}},\right.\right.\right.\right.$$

$$\left.0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2\, g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0\right\}\right\},$$

$$\left\{\left\{-\frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]}, 0, 0, 0,\right.\right.$$

$$\left.\left.\left.\left.-\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]}, 0, 0, 0\right\}\right\}\right\}\right\},$$

{5, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}}, {{6, x0,

$$\left\{\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\,(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}\; g[6][6][x0, x4]}, 0\right\}\right\},\right.$$

$$\left\{\left\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]},\right.\right.$$

0}}}}, {6, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {6, x4,

$$\left\{\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2\,(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}\; g[6][6][x0, x4]}, 0\right\}\right\},\right.$$

$$\left\{\left\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]},\right.\right.$$

0}}}}, {6, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {6, x6,

$$\left\{\left\{\left\{-\frac{g[6][6]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2\, g[4][4][x0, x4]\sqrt{-g[6][6][x0, x4]}},\right.\right.\right.$$

$$\left.0, 0, 0\right\}\right\}, \left\{\left\{-\frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]}, 0,\right.\right.$$

$$\left.\left.\left.\left.0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]}, 0, 0, 0\right\}\right\}\right\}\right\},$$

{6, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}}, {{7, x0,

$$\left\{\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}\; g[7][7][x0, x4]}\right\}\right\},\right.$$

$$\left\{\left\{0, 0, 0, 0, 0, 0, 0,\right.\right.$$

$$\left.\left.\left.\left.-\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2\, g[7][7][x0, x4]}\right\}\right\}\right\}\right\},$$

{7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

```
{7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {7, x4,
 {{{0, 0, 0, 0, 0, 0, 0, g[7][7]^(0,1)[x0, x4]/(2 (-g[7][7][x0, x4])^(3/2)) + g[7][7]^(0,1)[x0, x4]/(2 Sqrt[-g[7][7][x0, x4]] g[7][7][x0, x4])}},
  {{0, 0, 0, 0, 0, 0, 0,
    -g[7][7]^(0,1)[x0, x4]/(2 Sqrt[-g[7][7][x0, x4]]) - g[7][7]^(0,1)[x0, x4] Sqrt[-g[7][7][x0, x4]]/(2 g[7][7][x0, x4])}}}},
{7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {7, x7,
 {{{-g[7][7]^(1,0)[x0, x4]/(2 g[0][0][x0, x4] Sqrt[-g[7][7][x0, x4]]), 0, 0, 0, -g[7][7]^(0,1)[x0, x4]/(2 g[4][4][x0, x4] Sqrt[-g[7][7][x0, x4]]),
    0, 0, 0}}, {{-g[7][7]^(1,0)[x0, x4] Sqrt[-g[7][7][x0, x4]]/(2 g[7][7][x0, x4]), 0, 0,
    0, -g[7][7]^(0,1)[x0, x4] Sqrt[-g[7][7][x0, x4]]/(2 g[7][7][x0, x4]), 0, 0, 0}}}}}}
```

Out[∘]=

```
{{{0, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {0, x1, {{{0, H, 0, 0, 0, 0, 0, 0}}, {{0, e^(2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}}}},
  {0, x2, {{{0, 0, H, 0, 0, 0, 0, 0}}, {{0, 0, e^(2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}}}},
  {0, x3, {{{0, 0, 0, H, 0, 0, 0, 0}}, {{0, 0, 0, e^(2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0, 0, 0}}}},
  {0, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {0, x5, {{{0, 0, 0, 0, 0, H, 0, 0}}, {{0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0}}}},
  {0, x6, {{{0, 0, 0, 0, 0, 0, H, 0}}, {{0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3), 0}}}},
  {0, x7, {{{0, 0, 0, 0, 0, 0, 0, H}}, {{0, 0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3)}}}}},
 {{1, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {1, x1, {{{-Sqrt[e^(2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
      0, 0, 0, Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}},
    {{-Sqrt[e^(2 a4[H x4])] H Cos[6 H x0]/Sin[6 H x0]^(5/6), 0, 0, 0, -Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}}},
  {1, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {1, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {1, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {1, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {1, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {1, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{2, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x2, {{{-Sqrt[e^(2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
      0, 0, 0, Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}},
```

$$\left\{\left\{-\frac{\sqrt{e^{2\,a4[H\,x4]}}\;H\,Cos[6\,H\,x0]}{Sin[6\,H\,x0]^{5/6}},\,0,\,0,\,0,\,-\sqrt{e^{2\,a4[H\,x4]}}\;H\,Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4],\,0,\,0,\,0\right\}\right\}\right\},$$

{2, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{3, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\left\{3,\,x3,\,\left\{\left\{\left\{-\sqrt{e^{2\,a4[H\,x4]}}\;H\,Sec[6\,H\,x0]\,Sin[6\,H\,x0]^{7/6},\right.\right.\right.\right.$$
$$\left.\left.0,\,0,\,0,\,\sqrt{e^{2\,a4[H\,x4]}}\;H\,Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4],\,0,\,0,\,0\right\}\right\},$$
$$\left.\left.\left\{\left\{-\frac{\sqrt{e^{2\,a4[H\,x4]}}\;H\,Cos[6\,H\,x0]}{Sin[6\,H\,x0]^{5/6}},\,0,\,0,\,0,\,-\sqrt{e^{2\,a4[H\,x4]}}\;H\,Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4],\,0,\,0,\,0\right\}\right\}\right\}\right\},$$

{3, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{4, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\left\{4,\,x1,\,\left\{\{\{0,\,H\,a4'[H\,x4],\,0,\,0,\,0,\,0,\,0,\,0\}\},\right.\right.$$
$$\left.\left.\left\{\left\{0,\,-e^{2\,a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/3}\,a4'[H\,x4],\,0,\,0,\,0,\,0,\,0,\,0\right\}\right\}\right\}\right\},$$
$$\left\{4,\,x2,\,\left\{\{\{0,\,0,\,H\,a4'[H\,x4],\,0,\,0,\,0,\,0,\,0\}\},\right.\right.$$
$$\left.\left.\left\{\left\{0,\,0,\,-e^{2\,a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/3}\,a4'[H\,x4],\,0,\,0,\,0,\,0,\,0\right\}\right\}\right\}\right\},$$
$$\left\{4,\,x3,\,\left\{\{\{0,\,0,\,0,\,H\,a4'[H\,x4],\,0,\,0,\,0,\,0\}\},\right.\right.$$
$$\left.\left.\left\{\left\{0,\,0,\,0,\,-e^{2\,a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/3}\,a4'[H\,x4],\,0,\,0,\,0,\,0\right\}\right\}\right\}\right\},$$

{4, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\left\{4,\,x5,\,\left\{\{\{0,\,0,\,0,\,0,\,0,\,-H\,a4'[H\,x4],\,0,\,0\}\},\right.\right.$$
$$\left.\left.\left\{\left\{0,\,0,\,0,\,0,\,0,\,-e^{-2\,a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/3}\,a4'[H\,x4],\,0,\,0\right\}\right\}\right\}\right\},$$
$$\left\{4,\,x6,\,\left\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,-H\,a4'[H\,x4],\,0\}\},\right.\right.$$
$$\left.\left.\left\{\left\{0,\,0,\,0,\,0,\,0,\,0,\,-e^{-2\,a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/3}\,a4'[H\,x4],\,0\right\}\right\}\right\}\right\},$$
$$\left\{4,\,x7,\,\left\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,-H\,a4'[H\,x4]\}\},\right.\right.$$
$$\left.\left.\left.\left\{\left\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,-e^{-2\,a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/3}\,a4'[H\,x4]\right\}\right\}\right\}\right\}\right\},$$

{{5, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\left\{5,\,x5,\,\left\{\left\{\left\{\sqrt{e^{-2\,a4[H\,x4]}}\;H\,Sec[6\,H\,x0]\,Sin[6\,H\,x0]^{7/6},\right.\right.\right.\right.$$
$$\left.\left.0,\,0,\,0,\,\sqrt{e^{-2\,a4[H\,x4]}}\;H\,Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4],\,0,\,0,\,0\right\}\right\},$$
$$\left.\left.\left\{\left\{-\frac{\sqrt{e^{-2\,a4[H\,x4]}}\;H\,Cos[6\,H\,x0]}{Sin[6\,H\,x0]^{5/6}},\,0,\,0,\,0,\,\sqrt{e^{-2\,a4[H\,x4]}}\;H\,Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4],\,0,\,0,\,0\right\}\right\}\right\}\right\},$$

{5, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{6, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\Big\{6,\ x6,\ \Big\{\Big\{\Big\{\sqrt{e^{-2\,a4[H\,x4]}}\ H\,Sec[6\,H\,x0]\ Sin[6\,H\,x0]^{7/6},\ 0,\ 0,\ 0,\ \sqrt{e^{-2\,a4[H\,x4]}}\ H\,Sin[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\Big\}\Big\},$$
$$\Big\{\Big\{-\frac{\sqrt{e^{-2\,a4[H\,x4]}}\ H\,Cos[6\,H\,x0]}{Sin[6\,H\,x0]^{5/6}},\ 0,\ 0,\ 0,\ \sqrt{e^{-2\,a4[H\,x4]}}\ H\,Sin[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

{6, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{7, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\Big\{7,\ x7,\ \Big\{\Big\{\Big\{\sqrt{e^{-2\,a4[H\,x4]}}\ H\,Sec[6\,H\,x0]\ Sin[6\,H\,x0]^{7/6},\ 0,\ 0,\ 0,\ \sqrt{e^{-2\,a4[H\,x4]}}\ H\,Sin[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\Big\}\Big\},$$
$$\Big\{\Big\{-\frac{\sqrt{e^{-2\,a4[H\,x4]}}\ H\,Cos[6\,H\,x0]}{Sin[6\,H\,x0]^{5/6}},\ 0,\ 0,\ 0,\ \sqrt{e^{-2\,a4[H\,x4]}}\ H\,Sin[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\Big\}\Big\}\Big\}\Big\}\Big\}\Big\}$$

In[ ]:=
```
(*Block[{eAα,eαA,t,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=e^α_(A)/.sge^α_(A);
  eAα=e^(A)_α/.sge^(A)_α;
  t=Table[ Block[{v,V,d},v=eαA〚All,b〛;
     d=Table[D[v〚k〛,X〚μ〛 ]+Sum[ v〚i〛 Γ〚k,i,μ〛,{i,1,DIM8}],{k,1,DIM8}];
     {b-1,X〚μ〛,d}],{b,1,Length[eαA]},{μ,1,DIM8}];t]
 FullSimplify[#/.ssgm4488,constraintVars]&/@%*)
```

In[ ]:=
```
(*Block[{eAα,eαA,t,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=e^α_(A)/.sge^α_(A);
  eAα=e^(A)_α/.sge^(A)_α;
  t=Table[ Block[{v,V,d},v=eAα〚b,All〛;
     d=Table[D[v〚k〛,X〚μ〛 ]-Sum[ v〚i〛 Γ〚k,i,μ〛,{i,1,DIM8}],{k,1,DIM8}];
     {b-1,X〚μ〛,d}],{b,1,Length[eαA]},{μ,1,DIM8}];t]
 FullSimplify[#/.ssgm4488,constraintVars]&/@%*)
```

In[ ]:= Symbolize[$\omega_{\mu}{}^{(a)}{}_{(b)}$]

In[ ]:= ω = Array[$\omega_{\mu}{}^{(a)}{}_{(b)}$[##] &, {8, 8, 8}];

In[ ]:= ω[[1]]

Out[ ]=

$\{\{\omega_{\mu}{}^{(a)}{}_{(b)}[1,1,1], \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,2], \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,3], \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,4], \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,5], \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,6], \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,7], \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,8]\},$

$\{\omega_{\mu}{}^{(a)}{}_{(b)}[1,2,1], \omega_{\mu}{}^{(a)}{}_{(b)}[1,2,2], \omega_{\mu}{}^{(a)}{}_{(b)}[1,2,3], \omega_{\mu}{}^{(a)}{}_{(b)}[1,2,4], \omega_{\mu}{}^{(a)}{}_{(b)}[1,2,5], \omega_{\mu}{}^{(a)}{}_{(b)}[1,2,6], \omega_{\mu}{}^{(a)}{}_{(b)}[1,2,7], \omega_{\mu}{}^{(a)}{}_{(b)}[1,2,8]\},$

$\{\omega_{\mu}{}^{(a)}{}_{(b)}[1,3,1], \omega_{\mu}{}^{(a)}{}_{(b)}[1,3,2], \omega_{\mu}{}^{(a)}{}_{(b)}[1,3,3], \omega_{\mu}{}^{(a)}{}_{(b)}[1,3,4], \omega_{\mu}{}^{(a)}{}_{(b)}[1,3,5], \omega_{\mu}{}^{(a)}{}_{(b)}[1,3,6], \omega_{\mu}{}^{(a)}{}_{(b)}[1,3,7], \omega_{\mu}{}^{(a)}{}_{(b)}[1,3,8]\},$

$\{\omega_{\mu}{}^{(a)}{}_{(b)}[1,4,1], \omega_{\mu}{}^{(a)}{}_{(b)}[1,4,2], \omega_{\mu}{}^{(a)}{}_{(b)}[1,4,3], \omega_{\mu}{}^{(a)}{}_{(b)}[1,4,4], \omega_{\mu}{}^{(a)}{}_{(b)}[1,4,5], \omega_{\mu}{}^{(a)}{}_{(b)}[1,4,6], \omega_{\mu}{}^{(a)}{}_{(b)}[1,4,7], \omega_{\mu}{}^{(a)}{}_{(b)}[1,4,8]\},$

$\{\omega_{\mu}{}^{(a)}{}_{(b)}[1,5,1], \omega_{\mu}{}^{(a)}{}_{(b)}[1,5,2], \omega_{\mu}{}^{(a)}{}_{(b)}[1,5,3], \omega_{\mu}{}^{(a)}{}_{(b)}[1,5,4], \omega_{\mu}{}^{(a)}{}_{(b)}[1,5,5], \omega_{\mu}{}^{(a)}{}_{(b)}[1,5,6], \omega_{\mu}{}^{(a)}{}_{(b)}[1,5,7], \omega_{\mu}{}^{(a)}{}_{(b)}[1,5,8]\},$

$\{\omega_{\mu}{}^{(a)}{}_{(b)}[1,6,1], \omega_{\mu}{}^{(a)}{}_{(b)}[1,6,2], \omega_{\mu}{}^{(a)}{}_{(b)}[1,6,3], \omega_{\mu}{}^{(a)}{}_{(b)}[1,6,4], \omega_{\mu}{}^{(a)}{}_{(b)}[1,6,5], \omega_{\mu}{}^{(a)}{}_{(b)}[1,6,6], \omega_{\mu}{}^{(a)}{}_{(b)}[1,6,7], \omega_{\mu}{}^{(a)}{}_{(b)}[1,6,8]\},$

$\{\omega_{\mu}{}^{(a)}{}_{(b)}[1,7,1], \omega_{\mu}{}^{(a)}{}_{(b)}[1,7,2], \omega_{\mu}{}^{(a)}{}_{(b)}[1,7,3], \omega_{\mu}{}^{(a)}{}_{(b)}[1,7,4], \omega_{\mu}{}^{(a)}{}_{(b)}[1,7,5], \omega_{\mu}{}^{(a)}{}_{(b)}[1,7,6], \omega_{\mu}{}^{(a)}{}_{(b)}[1,7,7], \omega_{\mu}{}^{(a)}{}_{(b)}[1,7,8]\},$

$\{\omega_{\mu}{}^{(a)}{}_{(b)}[1,8,1], \omega_{\mu}{}^{(a)}{}_{(b)}[1,8,2], \omega_{\mu}{}^{(a)}{}_{(b)}[1,8,3], \omega_{\mu}{}^{(a)}{}_{(b)}[1,8,4], \omega_{\mu}{}^{(a)}{}_{(b)}[1,8,5], \omega_{\mu}{}^{(a)}{}_{(b)}[1,8,6], \omega_{\mu}{}^{(a)}{}_{(b)}[1,8,7], \omega_{\mu}{}^{(a)}{}_{(b)}[1,8,8]\}\}$

In[ ]:=
```
sω = Block[{eAα, eαA, t, V, ginv, Γ, gg},
  gg = g4488 /. sg;
  {ginv, Γ} = con[gg];
  eαA = e^α_(A) /. sge^α_(A);
  eAα = e_α^(A) /. sge_α^(A);
  Table[ω[[μ, a, b]] → Block[{v, d}, v = eαA[[All, b]];
     d = Sum[(D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}]) * eAα[[a, k]], {k, 1, DIM8}];
     d], {μ, 1, DIM8}, {a, 1, Length[eαA]}, {b, 1, Length[eαA]}]]
```

Out[ ]=

$\{\{\{\omega_{\mu}{}^{(a)}{}_{(b)}[1,1,1] \to 0, \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,2] \to 0, \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,3] \to 0,$

$\omega_{\mu}{}^{(a)}{}_{(b)}[1,1,4] \to 0, \omega_{\mu}{}^{(a)}{}_{(b)}[1,1,5] \to \frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[4][4][x0, x4]}},$

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 1, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 1, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 1, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [1, 2, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 2, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 2, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 2, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 2, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 2, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 2, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 2, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [1, 3, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 3, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 3, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 3, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 3, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 3, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 3, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 3, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [1, 4, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 4, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 4, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 4, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 4, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 4, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 4, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 4, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [1, 5, 1] → $-\frac{g[0][0]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ \sqrt{g[0][0][x0,\ x4]}\ g[4][4][x0,\ x4]}$, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 5, 2] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 5, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 5, 4] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 5, 5] →

$\left(\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ (-g[4][4][x0,\ x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[4][4][x0,\ x4]}\right)\ \sqrt{-g[4][4][x0,\ x4]}$,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 5, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 5, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 5, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [1, 6, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 6, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 6, 3] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 6, 4] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 6, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 6, 6] →

$\left(\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ (-g[5][5][x0,\ x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]}\right)\ \sqrt{-g[5][5][x0,\ x4]}$,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 6, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 6, 8] → 0}, {$\omega_{\mu\ (b)}^{\ (a)}$ [1, 7, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 7, 2] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 7, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 7, 4] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 7, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 7, 6] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 7, 7] → $\left(\frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ (-g[6][6][x0,\ x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[6][6][x0,\ x4]}\ g[6][6][x0,\ x4]}\right)$

$\sqrt{-g[6][6][x0,\ x4]}$, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 7, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [1, 8, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 8, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 8, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 8, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [1, 8, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 8, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 8, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [1, 8, 8] →

$\left(\frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ (-g[7][7][x0,\ x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[7][7][x0,\ x4]}\ g[7][7][x0,\ x4]}\right)\ \sqrt{-g[7][7][x0,\ x4]}$}},

{{$\omega_{\mu\ (b)}^{\ (a)}$ [2, 1, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [2, 1, 2] → $-\frac{g[1][1]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}\ \sqrt{g[1][1][x0,\ x4]}}$,

$\omega_{\mu\ (b)}^{\ (a)}$ [2, 1, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [2, 1, 4] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [2, 1, 5] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [2, 1, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [2, 1, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [2, 1, 8] → 0},

$$\left\{\omega_{\mu\ (b)}^{(a)}[2,2,1]\to\frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[1][1][x0,x4]}},\right.$$
$$\omega_{\mu\ (b)}^{(a)}[2,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,2,4]\to 0,$$
$$\omega_{\mu\ (b)}^{(a)}[2,2,5]\to\frac{g[1][1]^{(0,1)}[x0,x4]}{2\sqrt{g[1][1][x0,x4]}\sqrt{-g[4][4][x0,x4]}},$$
$$\left.\omega_{\mu\ (b)}^{(a)}[2,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,2,8]\to 0\right\},$$
$$\left\{\omega_{\mu\ (b)}^{(a)}[2,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,3,4]\to 0,\right.$$
$$\left.\omega_{\mu\ (b)}^{(a)}[2,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,3,8]\to 0\right\},$$
$$\left\{\omega_{\mu\ (b)}^{(a)}[2,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,4,4]\to 0,\right.$$
$$\left.\omega_{\mu\ (b)}^{(a)}[2,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,4,8]\to 0\right\},$$
$$\left\{\omega_{\mu\ (b)}^{(a)}[2,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,5,2]\to -\frac{g[1][1]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[1][1][x0,x4]}\,g[4][4][x0,x4]},\right.$$
$$\omega_{\mu\ (b)}^{(a)}[2,5,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,5,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,5,5]\to 0,$$
$$\left.\omega_{\mu\ (b)}^{(a)}[2,5,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,5,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,5,8]\to 0\right\},$$
$$\left\{\omega_{\mu\ (b)}^{(a)}[2,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,6,4]\to 0,\right.$$
$$\left.\omega_{\mu\ (b)}^{(a)}[2,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,6,8]\to 0\right\},$$
$$\left\{\omega_{\mu\ (b)}^{(a)}[2,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,7,4]\to 0,\right.$$
$$\left.\omega_{\mu\ (b)}^{(a)}[2,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,7,8]\to 0\right\},$$
$$\left\{\omega_{\mu\ (b)}^{(a)}[2,8,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,8,4]\to 0,\right.$$
$$\left.\omega_{\mu\ (b)}^{(a)}[2,8,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,8,8]\to 0\right\}\Big\},$$
$$\Big\{\left\{\omega_{\mu\ (b)}^{(a)}[3,1,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,1,2]\to 0,\right.$$
$$\omega_{\mu\ (b)}^{(a)}[3,1,3]\to -\frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[2][2][x0,x4]}},\ \omega_{\mu\ (b)}^{(a)}[3,1,4]\to 0,$$
$$\left.\omega_{\mu\ (b)}^{(a)}[3,1,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,1,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,1,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,1,8]\to 0\right\},$$
$$\left\{\omega_{\mu\ (b)}^{(a)}[3,2,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,2,4]\to 0,\right.$$
$$\left.\omega_{\mu\ (b)}^{(a)}[3,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,2,8]\to 0\right\},$$
$$\left\{\omega_{\mu\ (b)}^{(a)}[3,3,1]\to\frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[2][2][x0,x4]}},\right.$$
$$\omega_{\mu\ (b)}^{(a)}[3,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,3,4]\to 0,$$
$$\omega_{\mu\ (b)}^{(a)}[3,3,5]\to\frac{g[2][2]^{(0,1)}[x0,x4]}{2\sqrt{g[2][2][x0,x4]}\sqrt{-g[4][4][x0,x4]}},$$

$\omega_{\mu\ (b)}^{\ (a)}[3,3,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,3,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,3,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{\ (a)}[3,4,1]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,4,2]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,4,3]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,4,4]\to 0,$

$\omega_{\mu\ (b)}^{\ (a)}[3,4,5]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,4,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,4,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,4,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{\ (a)}[3,5,1]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,5,2]\to 0,$

$$\omega_{\mu\ (b)}^{\ (a)}[3,5,3]\to -\frac{g[2][2]^{(0,1)}[x0,x4]\ \sqrt{-g[4][4][x0,x4]}}{2\ \sqrt{g[2][2][x0,x4]}\ g[4][4][x0,x4]},\ \omega_{\mu\ (b)}^{\ (a)}[3,5,4]\to 0,$$

$\omega_{\mu\ (b)}^{\ (a)}[3,5,5]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,5,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,5,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,5,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{\ (a)}[3,6,1]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,6,2]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,6,3]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,6,4]\to 0,$

$\omega_{\mu\ (b)}^{\ (a)}[3,6,5]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,6,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,6,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,6,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{\ (a)}[3,7,1]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,7,2]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,7,3]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,7,4]\to 0,$

$\omega_{\mu\ (b)}^{\ (a)}[3,7,5]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,7,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,7,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,7,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{\ (a)}[3,8,1]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,8,2]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,8,3]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,8,4]\to 0,$

$\omega_{\mu\ (b)}^{\ (a)}[3,8,5]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,8,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,8,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[3,8,8]\to 0\}\},$

$\{\{\omega_{\mu\ (b)}^{\ (a)}[4,1,1]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,1,2]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,1,3]\to 0,$

$$\omega_{\mu\ (b)}^{\ (a)}[4,1,4]\to -\frac{g[3][3]^{(1,0)}[x0,x4]}{2\ \sqrt{g[0][0][x0,x4]}\ \sqrt{g[3][3][x0,x4]}},\ \omega_{\mu\ (b)}^{\ (a)}[4,1,5]\to 0,$$

$\omega_{\mu\ (b)}^{\ (a)}[4,1,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,1,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,1,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{\ (a)}[4,2,1]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,2,2]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,2,3]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,2,4]\to 0,$

$\omega_{\mu\ (b)}^{\ (a)}[4,2,5]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,2,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,2,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,2,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{\ (a)}[4,3,1]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,3,2]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,3,3]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,3,4]\to 0,$

$\omega_{\mu\ (b)}^{\ (a)}[4,3,5]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,3,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,3,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,3,8]\to 0\},$

$$\{\omega_{\mu\ (b)}^{\ (a)}[4,4,1]\to \frac{g[3][3]^{(1,0)}[x0,x4]}{2\ \sqrt{g[0][0][x0,x4]}\ \sqrt{g[3][3][x0,x4]}},$$

$\omega_{\mu\ (b)}^{\ (a)}[4,4,2]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,4,3]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,4,4]\to 0,$

$$\omega_{\mu\ (b)}^{\ (a)}[4,4,5]\to \frac{g[3][3]^{(0,1)}[x0,x4]}{2\ \sqrt{g[3][3][x0,x4]}\ \sqrt{-g[4][4][x0,x4]}},\ \omega_{\mu\ (b)}^{\ (a)}[4,4,6]\to 0,$$

$\omega_{\mu\ (b)}^{\ (a)}[4,4,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,4,8]\to 0\},\ \{\omega_{\mu\ (b)}^{\ (a)}[4,5,1]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,5,2]\to 0,$

$$\omega_{\mu\ (b)}^{\ (a)}[4,5,3]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,5,4]\to -\frac{g[3][3]^{(0,1)}[x0,x4]\ \sqrt{-g[4][4][x0,x4]}}{2\ \sqrt{g[3][3][x0,x4]}\ g[4][4][x0,x4]},$$

$\omega_{\mu\ (b)}^{\ (a)}[4,5,5]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,5,6]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,5,7]\to 0,\ \omega_{\mu\ (b)}^{\ (a)}[4,5,8]\to 0\},$

$$\left\{\omega_{\mu\,(b)}^{(a)}[4,6,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,6,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,6,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,6,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,6,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,6,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,6,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,6,8]\to 0\right\},$$

$$\left\{\omega_{\mu\,(b)}^{(a)}[4,7,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,7,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,7,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,7,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,7,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,7,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,7,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,7,8]\to 0\right\},$$

$$\left\{\omega_{\mu\,(b)}^{(a)}[4,8,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,8,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,8,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,8,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,8,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,8,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,8,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[4,8,8]\to 0\right\}\Big\},$$

$$\Big\{\left\{\omega_{\mu\,(b)}^{(a)}[5,1,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,1,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,1,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,1,4]\to 0,\right.$$

$$\omega_{\mu\,(b)}^{(a)}[5,1,5]\to -\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[4][4][x0,x4]}},$$

$$\left.\omega_{\mu\,(b)}^{(a)}[5,1,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,1,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,1,8]\to 0\right\},$$

$$\left\{\omega_{\mu\,(b)}^{(a)}[5,2,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,2,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,2,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,2,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,2,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,2,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,2,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,2,8]\to 0\right\},$$

$$\left\{\omega_{\mu\,(b)}^{(a)}[5,3,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,3,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,3,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,3,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,3,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,3,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,3,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,3,8]\to 0\right\},$$

$$\left\{\omega_{\mu\,(b)}^{(a)}[5,4,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,4,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,4,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,4,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,4,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,4,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,4,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,4,8]\to 0\right\},$$

$$\left\{\omega_{\mu\,(b)}^{(a)}[5,5,1]\to \frac{g[4][4]^{(1,0)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[4][4][x0,x4]},\ \omega_{\mu\,(b)}^{(a)}[5,5,2]\to 0,\right.$$

$$\omega_{\mu\,(b)}^{(a)}[5,5,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,5,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,5,5]\to$$

$$\left(\frac{g[4][4]^{(0,1)}[x0,x4]}{2\,(-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(0,1)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}\,g[4][4][x0,x4]}\right)\sqrt{-g[4][4][x0,x4]},$$

$$\left.\omega_{\mu\,(b)}^{(a)}[5,5,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,5,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,5,8]\to 0\right\},$$

$$\left\{\omega_{\mu\,(b)}^{(a)}[5,6,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,6,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,6,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,6,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,6,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,6,6]\to\right.$$

$$\left(\frac{g[5][5]^{(0,1)}[x0,x4]}{2\,(-g[5][5][x0,x4])^{3/2}}+\frac{g[5][5]^{(0,1)}[x0,x4]}{2\sqrt{-g[5][5][x0,x4]}\,g[5][5][x0,x4]}\right)\sqrt{-g[5][5][x0,x4]},$$

$$\left.\omega_{\mu\,(b)}^{(a)}[5,6,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,6,8]\to 0\right\},\ \left\{\omega_{\mu\,(b)}^{(a)}[5,7,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,7,2]\to 0,\right.$$

$$\omega_{\mu\,(b)}^{(a)}[5,7,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,7,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,7,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,7,6]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[5,7,7]\to \left(\frac{g[6][6]^{(0,1)}[x0,x4]}{2\,(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(0,1)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}\,g[6][6][x0,x4]}\right)$$

$\sqrt{-g[6][6][x0, x4]}$, $\omega_{\mu\ (b)}^{\ (a)}$ [5, 7, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [5, 8, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [5, 8, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [5, 8, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [5, 8, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [5, 8, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [5, 8, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [5, 8, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [5, 8, 8] →

$\left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\,\sqrt{-g[7][7][x0, x4]}\;g[7][7][x0, x4]}\right)\sqrt{-g[7][7][x0, x4]}$}},

{{$\omega_{\mu\ (b)}^{\ (a)}$ [6, 1, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 1, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 1, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 1, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 1, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 1, 6] → $-\frac{g[5][5]^{(1,0)}[x0, x4]}{2\,\sqrt{g[0][0][x0, x4]}\;\sqrt{-g[5][5][x0, x4]}}$,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 1, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 1, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [6, 2, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 2, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 2, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 2, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 2, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 2, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 2, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 2, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [6, 3, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 3, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 3, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 3, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 3, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 3, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 3, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 3, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [6, 4, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 4, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 4, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 4, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 4, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 4, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 4, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 4, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [6, 5, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 5, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 5, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 5, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 5, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 5, 6] → $-\frac{g[5][5]^{(0,1)}[x0, x4]\;\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]\;\sqrt{-g[5][5][x0, x4]}}$,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 5, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 5, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [6, 6, 1] → $\frac{g[5][5]^{(1,0)}[x0, x4]\;\sqrt{-g[5][5][x0, x4]}}{2\,\sqrt{g[0][0][x0, x4]}\;g[5][5][x0, x4]}$,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 6, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 6, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 6, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 6, 5] → $\frac{g[5][5]^{(0,1)}[x0, x4]\;\sqrt{-g[5][5][x0, x4]}}{2\,\sqrt{-g[4][4][x0, x4]}\;g[5][5][x0, x4]}$,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 6, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 6, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 6, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [6, 7, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 7, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 7, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 7, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 7, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 7, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 7, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 7, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [6, 8, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 8, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 8, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 8, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [6, 8, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 8, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 8, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [6, 8, 8] → 0}},

{{$\omega_{\mu\ (b)}^{\ (a)}$ [7, 1, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 1, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 1, 3] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 1, 4] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 1, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 1, 6] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 1, 7] → $-\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[6][6][x0, x4]}}$, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 1, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [7, 2, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 2, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 2, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 2, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 2, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 2, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 2, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 2, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [7, 3, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 3, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 3, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 3, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 3, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 3, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 3, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 3, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [7, 4, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 4, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 4, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 4, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 4, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 4, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 4, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 4, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [7, 5, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 5, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 5, 3] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 5, 4] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 5, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 5, 6] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 5, 7] → $-\frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]\sqrt{-g[6][6][x0, x4]}}$, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 5, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [7, 6, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 6, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 6, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 6, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 6, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 6, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 6, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 6, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [7, 7, 1] → $\frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}\,g[6][6][x0, x4]}$,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 7, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 7, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 7, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 7, 5] → $\frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}\,g[6][6][x0, x4]}$,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 7, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 7, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 7, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [7, 8, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 8, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 8, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 8, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [7, 8, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 8, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 8, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [7, 8, 8] → 0}},

{{$\omega_{\mu\ (b)}^{\ (a)}$ [8, 1, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 1, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 1, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 1, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [8, 1, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 1, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 1, 7] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [8, 1, 8] → $-\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[7][7][x0, x4]}}$},

{$\omega_{\mu\ (b)}^{\ (a)}$ [8, 2, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 2, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 2, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 2, 4] → 0,

$\omega_{\mu\ (b)}^{\ (a)}$ [8, 2, 5] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 2, 6] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 2, 7] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 2, 8] → 0},

{$\omega_{\mu\ (b)}^{\ (a)}$ [8, 3, 1] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 3, 2] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 3, 3] → 0, $\omega_{\mu\ (b)}^{\ (a)}$ [8, 3, 4] → 0,

$\omega_{\mu\ (b)}^{(a)}[8,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[8,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[8,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,7]\to 0,$

$$\omega_{\mu\ (b)}^{(a)}[8,5,8]\to -\frac{g[7][7]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]\sqrt{-g[7][7][x0,x4]}}\},$$

$\{\omega_{\mu\ (b)}^{(a)}[8,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[8,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,8]\to 0\},$

$$\{\omega_{\mu\ (b)}^{(a)}[8,8,1]\to \frac{g[7][7]^{(1,0)}[x0,x4]\sqrt{-g[7][7][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[7][7][x0,x4]},$$

$\omega_{\mu\ (b)}^{(a)}[8,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,4]\to 0,$

$$\omega_{\mu\ (b)}^{(a)}[8,8,5]\to \frac{g[7][7]^{(0,1)}[x0,x4]\sqrt{-g[7][7][x0,x4]}}{2\sqrt{-g[4][4][x0,x4]}\,g[7][7][x0,x4]},$$

$\omega_{\mu\ (b)}^{(a)}[8,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,8]\to 0\}\}\}$

In[∘]:=
```
(*sω=Block[{eAα,eαA,t,V,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=e^α_(A)/.sge^α_(A);
  eAα=e^(A)_α/.sge^(A)_α;
  Table[ω^(a)_μ (b)[μ,a,b]→Block[{v,d},v=eαA[[All,b]];
    d=Sum[(D[v[[k]],X[[μ]] ]+Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}])*eAα[[a,k]],{k,1,DIM8}];
    d],{μ,1,DIM8},{a,1,Length[eαA]},{b,1,Length[eαA]}]]*)
```

In[∘]:=
```
sωf = Flatten[sω];
```

In[∘]:=
```
(*ω^(a)_μ (b)*)
ω;
ωmat = % /. sωf
Dimensions[%]
```

Out[∘]=

$$\{\{\{0,0,0,0,\frac{g[0][0]^{(0,1)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[4][4][x0,x4]}},0,0,0\},$$

$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$

$$\Big\{-\frac{g[0][0]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\;g[4][4][x0,x4]},0,0,0,$$

$$\left(\frac{g[4][4]^{(1,0)}[x0,x4]}{2\,(-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}\;g[4][4][x0,x4]}\right)\sqrt{-g[4][4][x0,x4]},$$

$$0,0,0\Big\},\Big\{0,0,0,0,0,$$

$$\left(\frac{g[5][5]^{(1,0)}[x0,x4]}{2\,(-g[5][5][x0,x4])^{3/2}}+\frac{g[5][5]^{(1,0)}[x0,x4]}{2\sqrt{-g[5][5][x0,x4]}\;g[5][5][x0,x4]}\right)\sqrt{-g[5][5][x0,x4]},0,$$

$$0\Big\},\Big\{0,0,0,0,0,0,\left(\frac{g[6][6]^{(1,0)}[x0,x4]}{2\,(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(1,0)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}\;g[6][6][x0,x4]}\right)$$

$$\sqrt{-g[6][6][x0,x4]},0\Big\},\Big\{0,0,0,0,0,0,0,$$

$$\left(\frac{g[7][7]^{(1,0)}[x0,x4]}{2\,(-g[7][7][x0,x4])^{3/2}}+\frac{g[7][7]^{(1,0)}[x0,x4]}{2\sqrt{-g[7][7][x0,x4]}\;g[7][7][x0,x4]}\right)\sqrt{-g[7][7][x0,x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0,-\frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\;\sqrt{g[1][1][x0,x4]}},0,0,0,0,0,0\Big\},$$

$$\Big\{\frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\;\sqrt{g[1][1][x0,x4]}},0,0,0,$$

$$\frac{g[1][1]^{(0,1)}[x0,x4]}{2\sqrt{g[1][1][x0,x4]}\;\sqrt{-g[4][4][x0,x4]}},0,0,0\Big\},\{0,0,0,0,0,0,0,0\},$$

$$\{0,0,0,0,0,0,0,0\},\Big\{0,-\frac{g[1][1]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[1][1][x0,x4]}\;g[4][4][x0,x4]},0,0,0,0,0,0\Big\},$$

$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\Big\},$$

$$\Big\{\Big\{0,0,-\frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\;\sqrt{g[2][2][x0,x4]}},0,0,0,0,0\Big\},$$

$$\{0,0,0,0,0,0,0,0\},\Big\{\frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\;\sqrt{g[2][2][x0,x4]}},0,0,0,$$

$$\frac{g[2][2]^{(0,1)}[x0,x4]}{2\sqrt{g[2][2][x0,x4]}\;\sqrt{-g[4][4][x0,x4]}},0,0,0\Big\},\{0,0,0,0,0,0,0,0\},$$

$$\Big\{0,0,-\frac{g[2][2]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[2][2][x0,x4]}\;g[4][4][x0,x4]},0,0,0,0,0\Big\},$$

$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\Big\},$$

$$\Big\{\Big\{0,0,0,-\frac{g[3][3]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\;\sqrt{g[3][3][x0,x4]}},0,0,0,0\Big\},\{0,0,0,0,0,0,0,0\},$$

$$\{0,0,0,0,0,0,0,0\},\Big\{\frac{g[3][3]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\;\sqrt{g[3][3][x0,x4]}},$$

$$0,0,0,\frac{g[3][3]^{(0,1)}[x0,x4]}{2\sqrt{g[3][3][x0,x4]}\;\sqrt{-g[4][4][x0,x4]}},0,0,0\Big\},$$

$$\left\{0,0,0,-\frac{g[3][3]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[3][3][x0,x4]}\,g[4][4][x0,x4]},0,0,0,0\right\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\Big\},$$
$$\Big\{\left\{0,0,0,0,-\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[4][4][x0,x4]}},0,0,0\right\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\left\{\frac{g[4][4]^{(1,0)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[4][4][x0,x4]},0,0,0,\right.$$
$$\left(\frac{g[4][4]^{(0,1)}[x0,x4]}{2(-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(0,1)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}\,g[4][4][x0,x4]}\right)\sqrt{-g[4][4][x0,x4]},0,$$
$$\left.0,0\right\},\left\{0,0,0,0,0,\left(\frac{g[5][5]^{(0,1)}[x0,x4]}{2(-g[5][5][x0,x4])^{3/2}}+\frac{g[5][5]^{(0,1)}[x0,x4]}{2\sqrt{-g[5][5][x0,x4]}\,g[5][5][x0,x4]}\right)\right.$$
$$\left.\sqrt{-g[5][5][x0,x4]},0,0\right\},\Big\{0,0,0,0,0,0,$$
$$\left.\left(\frac{g[6][6]^{(0,1)}[x0,x4]}{2(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(0,1)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}\,g[6][6][x0,x4]}\right)\sqrt{-g[6][6][x0,x4]},0\right\},$$
$$\left\{0,0,0,0,0,0,0,\left(\frac{g[7][7]^{(0,1)}[x0,x4]}{2(-g[7][7][x0,x4])^{3/2}}+\frac{g[7][7]^{(0,1)}[x0,x4]}{2\sqrt{-g[7][7][x0,x4]}\,g[7][7][x0,x4]}\right)\right.$$
$$\left.\sqrt{-g[7][7][x0,x4]}\right\}\Big\},\Big\{\left\{0,0,0,0,0,-\frac{g[5][5]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[5][5][x0,x4]}},0,0\right\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\left\{0,0,0,0,0,-\frac{g[5][5]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]\sqrt{-g[5][5][x0,x4]}},0,0\right\},$$
$$\left\{\frac{g[5][5]^{(1,0)}[x0,x4]\sqrt{-g[5][5][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[5][5][x0,x4]},0,0,0,\frac{g[5][5]^{(0,1)}[x0,x4]\sqrt{-g[5][5][x0,x4]}}{2\sqrt{-g[4][4][x0,x4]}\,g[5][5][x0,x4]},\right.$$
$$\left.0,0,0\right\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\Big\},$$
$$\Big\{\left\{0,0,0,0,0,0,-\frac{g[6][6]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[6][6][x0,x4]}},0\right\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\left\{0,0,0,0,0,0,-\frac{g[6][6]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]\sqrt{-g[6][6][x0,x4]}},0\right\},$$
$$\{0,0,0,0,0,0,0,0\},\left\{\frac{g[6][6]^{(1,0)}[x0,x4]\sqrt{-g[6][6][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[6][6][x0,x4]},0,0,0,\right.$$
$$\left.\frac{g[6][6]^{(0,1)}[x0,x4]\sqrt{-g[6][6][x0,x4]}}{2\sqrt{-g[4][4][x0,x4]}\,g[6][6][x0,x4]},0,0,0\right\},\{0,0,0,0,0,0,0,0\}\Big\},$$
$$\Big\{\left\{0,0,0,0,0,0,0,-\frac{g[7][7]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[7][7][x0,x4]}}\right\},$$

```
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, - (g[7][7]^(0,1)[x0, x4] Sqrt[-g[4][4][x0, x4]]) / (2 g[4][4][x0, x4] Sqrt[-g[7][7][x0, x4]])}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {(g[7][7]^(1,0)[x0, x4] Sqrt[-g[7][7][x0, x4]]) / (2 Sqrt[g[0][0][x0, x4]] g[7][7][x0, x4]),
 0, 0, 0, (g[7][7]^(0,1)[x0, x4] Sqrt[-g[7][7][x0, x4]]) / (2 Sqrt[-g[4][4][x0, x4]] g[7][7][x0, x4]), 0, 0, 0}}}
```

Out[∘]=

```
{8, 8, 8}
```

In[∘]:=

```
sωgtry = Block[{eAα, eαA, t, V},
  eαA = e^α_(A) /. sgtrye^α_(A);
  eAα = e^(A)_α /. sgtrye^(A)_α;
  Table[(*V=eAα[[a,All]];*) ω^(a)_μ (b) [μ, a, b] → Block[{v, d}, v = eαA[[All, b]];
     d = Sum[(D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}]) * eAα[[a, k]], {k, 1, DIM8}];
     FullSimplify[d, constraintVars] /. {Sqrt[e^(2 a4[H x4])] → e^(a4[H x4]), Sqrt[e^(-2 a4[H x4])] → e^(-a4[H x4])}],
   {μ, 1, DIM8}, {a, 1, Length[eAα]}, {b, 1, Length[eαA]}]]
```

Out[∘]=

```
{{{ω^(a)_μ (b) [1, 1, 1] → 0, ω^(a)_μ (b) [1, 1, 2] → 0, ω^(a)_μ (b) [1, 1, 3] → 0, ω^(a)_μ (b) [1, 1, 4] → 0,
   ω^(a)_μ (b) [1, 1, 5] → 0, ω^(a)_μ (b) [1, 1, 6] → 0, ω^(a)_μ (b) [1, 1, 7] → 0, ω^(a)_μ (b) [1, 1, 8] → 0},
  {ω^(a)_μ (b) [1, 2, 1] → 0, ω^(a)_μ (b) [1, 2, 2] → 0, ω^(a)_μ (b) [1, 2, 3] → 0, ω^(a)_μ (b) [1, 2, 4] → 0,
   ω^(a)_μ (b) [1, 2, 5] → 0, ω^(a)_μ (b) [1, 2, 6] → 0, ω^(a)_μ (b) [1, 2, 7] → 0, ω^(a)_μ (b) [1, 2, 8] → 0},
  {ω^(a)_μ (b) [1, 3, 1] → 0, ω^(a)_μ (b) [1, 3, 2] → 0, ω^(a)_μ (b) [1, 3, 3] → 0, ω^(a)_μ (b) [1, 3, 4] → 0,
   ω^(a)_μ (b) [1, 3, 5] → 0, ω^(a)_μ (b) [1, 3, 6] → 0, ω^(a)_μ (b) [1, 3, 7] → 0, ω^(a)_μ (b) [1, 3, 8] → 0},
  {ω^(a)_μ (b) [1, 4, 1] → 0, ω^(a)_μ (b) [1, 4, 2] → 0, ω^(a)_μ (b) [1, 4, 3] → 0, ω^(a)_μ (b) [1, 4, 4] → 0,
   ω^(a)_μ (b) [1, 4, 5] → 0, ω^(a)_μ (b) [1, 4, 6] → 0, ω^(a)_μ (b) [1, 4, 7] → 0, ω^(a)_μ (b) [1, 4, 8] → 0},
  {ω^(a)_μ (b) [1, 5, 1] → 0, ω^(a)_μ (b) [1, 5, 2] → 0, ω^(a)_μ (b) [1, 5, 3] → 0, ω^(a)_μ (b) [1, 5, 4] → 0,
   ω^(a)_μ (b) [1, 5, 5] → 0, ω^(a)_μ (b) [1, 5, 6] → 0, ω^(a)_μ (b) [1, 5, 7] → 0, ω^(a)_μ (b) [1, 5, 8] → 0},
  {ω^(a)_μ (b) [1, 6, 1] → 0, ω^(a)_μ (b) [1, 6, 2] → 0, ω^(a)_μ (b) [1, 6, 3] → 0, ω^(a)_μ (b) [1, 6, 4] → 0,
   ω^(a)_μ (b) [1, 6, 5] → 0, ω^(a)_μ (b) [1, 6, 6] → 0, ω^(a)_μ (b) [1, 6, 7] → 0, ω^(a)_μ (b) [1, 6, 8] → 0},
  {ω^(a)_μ (b) [1, 7, 1] → 0, ω^(a)_μ (b) [1, 7, 2] → 0, ω^(a)_μ (b) [1, 7, 3] → 0, ω^(a)_μ (b) [1, 7, 4] → 0,
   ω^(a)_μ (b) [1, 7, 5] → 0, ω^(a)_μ (b) [1, 7, 6] → 0, ω^(a)_μ (b) [1, 7, 7] → 0, ω^(a)_μ (b) [1, 7, 8] → 0},
  {ω^(a)_μ (b) [1, 8, 1] → 0, ω^(a)_μ (b) [1, 8, 2] → 0, ω^(a)_μ (b) [1, 8, 3] → 0, ω^(a)_μ (b) [1, 8, 4] → 0,
   ω^(a)_μ (b) [1, 8, 5] → 0, ω^(a)_μ (b) [1, 8, 6] → 0, ω^(a)_μ (b) [1, 8, 7] → 0, ω^(a)_μ (b) [1, 8, 8] → 0}},
```

$\{\{\omega^{(a)}_{\mu\ (b)}[2,1,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,1,2]\to -e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\ (b)}[2,1,3]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,1,4]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,1,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,1,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,1,7]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,1,8]\to 0\},\ \{\omega^{(a)}_{\mu\ (b)}[2,2,1]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\ (b)}[2,2,2]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,2,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,2,4]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,2,5]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$

$\omega^{(a)}_{\mu\ (b)}[2,2,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,2,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,2,8]\to 0\},$

$\{\omega^{(a)}_{\mu\ (b)}[2,3,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,3,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,8]\to 0\},$

$\{\omega^{(a)}_{\mu\ (b)}[2,4,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,4,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,8]\to 0\},$

$\{\omega^{(a)}_{\mu\ (b)}[2,5,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,2]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$

$\omega^{(a)}_{\mu\ (b)}[2,5,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,4]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,5]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,5,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,8]\to 0\},$

$\{\omega^{(a)}_{\mu\ (b)}[2,6,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,6,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,8]\to 0\},$

$\{\omega^{(a)}_{\mu\ (b)}[2,7,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,7,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,8]\to 0\},$

$\{\omega^{(a)}_{\mu\ (b)}[2,8,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,8,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,8]\to 0\}\},$

$\{\{\omega^{(a)}_{\mu\ (b)}[3,1,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,1,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,1,3]\to -e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},$

$\omega^{(a)}_{\mu\ (b)}[3,1,4]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,1,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,1,6]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[3,1,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,1,8]\to 0\},$

$\{\omega^{(a)}_{\mu\ (b)}[3,2,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[3,2,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,8]\to 0\},$

$\{\omega^{(a)}_{\mu\ (b)}[3,3,1]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\ (b)}[3,3,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,3,3]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[3,3,4]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,3,5]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$

$\omega^{(a)}_{\mu\ (b)}[3,3,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,3,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,3,8]\to 0\},$

$\{\omega^{(a)}_{\mu\ (b)}[3,4,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,4,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,4,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,4,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[3,4,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,4,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,4,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,4,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[3,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,5,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,5,3]\to e^{a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4],$
$\omega_{\mu\ (b)}^{(a)}[3,5,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,5,6]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,5,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,5,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[3,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[3,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[3,8,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,8,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,8]\to 0\}\},$
$\{\{\omega_{\mu\ (b)}^{(a)}[4,1,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,1,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,1,3]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,1,4]\to -e^{a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/6},\ \omega_{\mu\ (b)}^{(a)}[4,1,5]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,1,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,1,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,1,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,2,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,4,1]\to e^{a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/6},\ \omega_{\mu\ (b)}^{(a)}[4,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,4,3]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,4,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,4,5]\to e^{a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ \omega_{\mu\ (b)}^{(a)}[4,4,6]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,4,8]\to 0\},\ \{\omega_{\mu\ (b)}^{(a)}[4,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,2]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,5,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,4]\to e^{a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4],$
$\omega_{\mu\ (b)}^{(a)}[4,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,8,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,8,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,8]\to 0\}\},$
$\{\{\omega_{\mu\ (b)}^{(a)}[5,1,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,1,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[5,2,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[5,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[5,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[5,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[5,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[5,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[5,8,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,8,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,8,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,8,8]\to 0\}\},$

$\{\{\omega_{\mu\ (b)}^{(a)}[6,1,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,1,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,1,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,1,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[6,1,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,1,6]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},$

$\omega_{\mu\ (b)}^{(a)}[6,1,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,1,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[6,2,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,2,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[6,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,2,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[6,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,3,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[6,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,3,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[6,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,4,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[6,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,4,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[6,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,5,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,5,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,5,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[6,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,5,6]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ \omega_{\mu\ (b)}^{(a)}[6,5,7]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[6,5,8]\to 0\},\ \{\omega_{\mu\ (b)}^{(a)}[6,6,1]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega_{\mu\ (b)}^{(a)}[6,6,2]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[6,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,6,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[6,6,5]\to -e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$

$\omega_{\mu\ (b)}^{(a)}$ [6, 6, 6] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 6, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 6, 8] → 0},

{$\omega_{\mu\ (b)}^{(a)}$ [6, 7, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 7, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 7, 3] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 7, 4] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [6, 7, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 7, 6] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 7, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 7, 8] → 0},

{$\omega_{\mu\ (b)}^{(a)}$ [6, 8, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 8, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 8, 3] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 8, 4] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [6, 8, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 8, 6] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 8, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [6, 8, 8] → 0}},

{{$\omega_{\mu\ (b)}^{(a)}$ [7, 1, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 1, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 1, 3] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 1, 4] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 1, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 1, 6] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 1, 7] → $e^{-a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$, $\omega_{\mu\ (b)}^{(a)}$ [7, 1, 8] → 0},

{$\omega_{\mu\ (b)}^{(a)}$ [7, 2, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 2, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 2, 3] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 2, 4] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 2, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 2, 6] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 2, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 2, 8] → 0},

{$\omega_{\mu\ (b)}^{(a)}$ [7, 3, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 3, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 3, 3] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 3, 4] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 3, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 3, 6] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 3, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 3, 8] → 0},

{$\omega_{\mu\ (b)}^{(a)}$ [7, 4, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 4, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 4, 3] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 4, 4] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 4, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 4, 6] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 4, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 4, 8] → 0},

{$\omega_{\mu\ (b)}^{(a)}$ [7, 5, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 5, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 5, 3] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 5, 4] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 5, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 5, 6] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 5, 7] → $e^{-a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$ a4′[H x4], $\omega_{\mu\ (b)}^{(a)}$ [7, 5, 8] → 0},

{$\omega_{\mu\ (b)}^{(a)}$ [7, 6, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 6, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 6, 3] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 6, 4] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 6, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 6, 6] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 6, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 6, 8] → 0},

{$\omega_{\mu\ (b)}^{(a)}$ [7, 7, 1] → $e^{-a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$, $\omega_{\mu\ (b)}^{(a)}$ [7, 7, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 7, 3] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 7, 4] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 7, 5] → $-e^{-a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$ a4′[H x4],

$\omega_{\mu\ (b)}^{(a)}$ [7, 7, 6] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 7, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 7, 8] → 0},

{$\omega_{\mu\ (b)}^{(a)}$ [7, 8, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 8, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 8, 3] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 8, 4] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [7, 8, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 8, 6] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 8, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [7, 8, 8] → 0}},

{{$\omega_{\mu\ (b)}^{(a)}$ [8, 1, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [8, 1, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [8, 1, 3] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [8, 1, 4] → 0, $\omega_{\mu\ (b)}^{(a)}$ [8, 1, 5] → 0, $\omega_{\mu\ (b)}^{(a)}$ [8, 1, 6] → 0,

$\omega_{\mu\ (b)}^{(a)}$ [8, 1, 7] → 0, $\omega_{\mu\ (b)}^{(a)}$ [8, 1, 8] → $e^{-a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$},

{$\omega_{\mu\ (b)}^{(a)}$ [8, 2, 1] → 0, $\omega_{\mu\ (b)}^{(a)}$ [8, 2, 2] → 0, $\omega_{\mu\ (b)}^{(a)}$ [8, 2, 3] → 0, $\omega_{\mu\ (b)}^{(a)}$ [8, 2, 4] → 0,

$\omega_{\mu\ (b)}^{(a)}[8,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,2,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[8,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[8,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[8,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,3]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,5,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,6]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,5,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,8]\to e^{-a4[H\,x4]}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\ a4'[H\,x4]\},$

$\{\omega_{\mu\ (b)}^{(a)}[8,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[8,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,8]\to 0\},$

$\{\omega_{\mu\ (b)}^{(a)}[8,8,1]\to e^{-a4[H\,x4]}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega_{\mu\ (b)}^{(a)}[8,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,3]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[8,8,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,5]\to -e^{-a4[H\,x4]}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\ a4'[H\,x4],$

$\omega_{\mu\ (b)}^{(a)}[8,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,8]\to 0\}\}\}$

```
In[]:= sωfgtry = Flatten[sωgtry];
```

In[◦]:=
```
(*ω_μ^(a)_(b)*)
ω;
ωω = % /. sωfgtry // FullSimplify[#, constraintVars] &
Dimensions[%]
```

Out[◦]=
```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, -ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0,
   0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, -ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, -ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4],
   0, 0, 0}, {0, 0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0},
  {ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6)},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4]},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}}
```

Out[◦]=
```
{8, 8, 8}
```

In[•]:=
```
g4488 /. sg
% /. ssgm4488
```

Out[•]=
```
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}
```

Out[•]=

$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

check metric compatibility condition for the Octad

In[•]:=
```
Block[{eAα, eαA, t, ginv, Γ, gg(*,ωω*)},
  (*ωω=ω_μ^(a)_(b) /.sωf;*)
  gg = g4488 /. sg /. ssgm4488;
  {ginv, Γ} = con[gg];
  eαA = e^α_(A) /. sge^α_(A);
  eAα = e_α^(A) /. sge_α^(A);
  t = Table[ Block[{v, V, contra, covari},
      V = eAα[[b, All]];
      v = eαA[[All, b]];
      contra = Table[
         D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]],
           {i, 1, DIM8}], {k, 1, DIM8}];
      (*no ω terms*)
      covari = Table[
         D[V[[k]], X[[μ]] ] - Sum[ V[[i]] × Γ[[i, k, μ]],
            {i, 1, DIM8}] + Sum[ eAα[[J, All]][[k]] ×
```

```
ωmat[[μ, b, J]], {J, 1, DIM8}] //
FullSimplify[#, constraintVars &&
    e^(2 a4[H x4]) > 0 && e^(a4[H x4]) > 0 &&
    e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0] &,
  {k, 1, DIM8}]; (*ω terms*)
 (*{b-1,X[[μ]],{{contra},{covari}}}],
   {b,1,Length[eαA]},{μ,1,DIM8}];*)
 {b - 1, X[[μ]], covari}],
 {b, 1, Length[eαA]}, {μ, 1, DIM8}];
t]
FullSimplify[# /. ssgm4488,
  constraintVars && e^(2 a4[H x4]) > 0 && e^(a4[H x4]) >
   0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0] & /@ %
```

Out[ ]=

$$\left\{\left\{\left\{0,\ x0,\ \left\{\left(g[0][0]^{(1,0)}[x0,\ x4] + 24\,H\,\text{Csc}[12\,H\,x0]\,g[0][0][x0,\ x4]\right) / \left(2\sqrt{g[0][0][x0,\ x4]}\right),\ 0,\ 0,\ 0,\ \frac{g[0][0]^{(0,1)}[x0,\ x4]}{2\sqrt{g[0][0][x0,\ x4]}},\ 0,\ 0,\ 0\right\}\right\},\right.\right.$$

$$\left\{0,\ x1,\ \left\{0,\ \left(-g[1][1]^{(1,0)}[x0,\ x4] + 2\,e^{2\,a4[H\,x4]}\,H\,\text{Sec}[6\,H\,x0]\,\text{Sin}[6\,H\,x0]^{4/3}\,g[0][0][x0,\ x4]\right) /\right.\right.$$

$$\left(2\sqrt{g[0][0][x0, x4]}\right), 0, 0, 0, 0, 0, 0\}\},$$

$$\{0, x2, \{0, 0, \left(-g[2][2]^{(1,0)}[x0, x4] + 2\, e^{2\, a4[H\, x4]}\, H\, Sec[6\, H\, x0]\, Sin[6\, H\, x0]^{4/3}\, g[0][0][x0, x4]\right) / \left(2\sqrt{g[0][0][x0, x4]}\right), 0, 0, 0, 0, 0\}\},$$

$$\{0, x3, \{0, 0, 0, \left(-g[3][3]^{(1,0)}[x0, x4] + 2\, e^{2\, a4[H\, x4]}\, H\, Sec[6\, H\, x0]\, Sin[6\, H\, x0]^{4/3}\, g[0][0][x0, x4]\right) / \left(2\sqrt{g[0][0][x0, x4]}\right), 0, 0, 0, 0\}\},$$

$$\{0, x4, \left\{\frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0\right\}\},$$

$$\{0, x5, \{0, 0, 0, 0, 0, \left(-g[5][5]^{(1,0)}[x0, x4] - 2\, e^{-2\, a4[H\, x4]}\, H\, Sec[6\, H\, x0]\, Sin[6\, H\, x0]^{4/3}\, g[0][0][x0, x4]\right) / \left(2\sqrt{g[0][0][x0, x4]}\right), 0, 0\}\},$$

$$\{0, x6, \{0, 0, 0, 0, 0, 0, \left(-g[6][6]^{(1,0)}[x0, x4] - 2\, e^{-2\, a4[H\, x4]}\, H\, Sec[6\, H\, x0]\, Sin[6\, H\, x0]^{4/3}\, g[0][0][x0, x4]\right) / \left(2\sqrt{g[0][0][x0, x4]}\right), 0\}\},$$

$$\{0, x7, \{0, 0, 0, 0, 0, 0, 0, \left(-g[7][7]^{(1,0)}[x0, x4] - 2\, e^{-2\, a4[H\, x4]}\, H\, Sec[6\, H\, x0]\, Sin[6\, H\, x0]^{4/3}\, g[0][0][x0, x4]\right) \Big/ \left(2\sqrt{g[0][0][x0, x4]}\right)\}\}\},$$

$$\{\{1, x0, \{0, \left(g[1][1]^{(1,0)}[x0, x4] - 2\, H\, Cot[6\, H\, x0]\, g[1][1][x0, x4]\right) \Big/ \left(2\sqrt{g[1][1][x0, x4]}\right), 0, 0, 0, 0, 0, 0\}\},$$

$$\{1, x1, \{\left(g[1][1]^{(1,0)}[x0, x4] - 2\, H\, Cot[6\, H\, x0]\, g[1][1][x0, x4]\right) \Big/ \left(2\sqrt{g[1][1][x0, x4]}\right), 0, 0, 0, \left(g[1][1]^{(0,1)}[x0, x4] - 6\, H\, a4'[H\, x4]\, g[1][1][x0, x4]\right) \Big/ \left(2\sqrt{g[1][1][x0, x4]}\right), 0, 0, 0\}\},$$

$$\{1, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{1, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{1, x4, \{0, \left(g[1][1]^{(0,1)}[x0, x4] - 6\, H\, a4'[H\, x4]\, g[1][1][x0, x4]\right) \Big/ \left(2\sqrt{g[1][1][x0, x4]}\right), 0, 0, 0, 0, 0, 0\}\},$$

$$\{1, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{1, x6,$$

```
   {0, 0, 0, 0, 0, 0, 0, 0}},
  {1, x7, {0, 0, 0, 0, 0, 0, 0, 0}}},
 {{2, x0, {0, 0, (g[2][2]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[2][2][x0, x4]) /
      (2 Sqrt[g[2][2][x0, x4]]), 0, 0, 0, 0, 0}},
  {2, x1, {0, 0, 0, 0, 0, 0, 0, 0}},
  {2, x2, {(g[2][2]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[2][2][x0, x4]) /
      (2 Sqrt[g[2][2][x0, x4]]), 0, 0,
    0, (g[2][2]^(0,1)[x0, x4] -
        6 H a4'[H x4] g[2][2][x0, x4]) /
      (2 Sqrt[g[2][2][x0, x4]]), 0, 0, 0}},
  {2, x3, {0, 0, 0, 0, 0, 0, 0, 0}},
  {2, x4,
   {0, 0, (g[2][2]^(0,1)[x0, x4] -
        6 H a4'[H x4] g[2][2][x0, x4]) /
      (2 Sqrt[g[2][2][x0, x4]]), 0, 0, 0, 0, 0}},
  {2, x5, {0, 0, 0, 0, 0, 0, 0, 0}},
  {2, x6, {0, 0, 0, 0, 0, 0, 0, 0}},
  {2, x7, {0, 0, 0, 0, 0, 0, 0, 0}}},
 {{3, x0, {0, 0, 0, (g[3][3]^(1,0)[x0, x4] -
```

$$2\,H\,\text{Cot}[6\,H\,x0]\,g[3][3][x0,x4]\Big)\Big/$$
$$\Big(2\sqrt{g[3][3][x0,x4]}\Big),0,0,0,0\}\},$$
$$\{3,x1,\{0,0,0,0,0,0,0,0\}\},$$
$$\{3,x2,\{0,0,0,0,0,0,0,0\}\},$$
$$\Big\{3,x3,\Big\{\Big(g[3][3]^{(1,0)}[x0,x4]-$$
$$2\,H\,\text{Cot}[6\,H\,x0]\,g[3][3][x0,x4]\Big)\Big/$$
$$\Big(2\sqrt{g[3][3][x0,x4]}\Big),0,0,$$
$$0,\Big(g[3][3]^{(0,1)}[x0,x4]-$$
$$6\,H\,a4'[H\,x4]\,g[3][3][x0,x4]\Big)\Big/$$
$$\Big(2\sqrt{g[3][3][x0,x4]}\Big),0,0,0\Big\}\Big\},$$
$$\Big\{3,x4,\Big\{0,0,0,\Big(g[3][3]^{(0,1)}[x0,x4]-$$
$$6\,H\,a4'[H\,x4]\,g[3][3][x0,x4]\Big)\Big/$$
$$\Big(2\sqrt{g[3][3][x0,x4]}\Big),0,0,0,0\Big\}\Big\},$$
$$\{3,x5,\{0,0,0,0,0,0,0,0\}\},$$
$$\{3,x6,\{0,0,0,0,0,0,0,0\}\},$$
$$\{3,x7,\{0,0,0,0,0,0,0,0\}\}\Big\},$$
$$\Big\{\Big\{4,x0,\Big\{\frac{g[0][0]^{(0,1)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}},0,0,$$
$$0,-\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}},0,0,0\Big\}\Big\},$$
$$\Big\{4,x1,\Big\{0,\Big(g[1][1]^{(0,1)}[x0,x4]+$$
$$6\,e^{2\,a4[H\,x4]}\,H\,\text{Sin}[6\,H\,x0]^{1/3}$$

$$\text{a4}'[\text{H x4}]\ \text{g}[4][4][\text{x0, x4}]\Big) \Big/$$
$$\left(2\sqrt{-\text{g}[4][4][\text{x0, x4}]}\right),\ 0,$$
$$0,\ 0,\ 0,\ 0,\ 0\}\},$$
$$\Big\{4,\ \text{x2},\ \Big\{0,\ 0,\ \Big(\text{g}[2][2]^{(0,1)}[\text{x0, x4}] +$$
$$6\ e^{2\ \text{a4}[\text{H x4}]}\ \text{H Sin}[6\ \text{H x0}]^{1/3}$$
$$\text{a4}'[\text{H x4}]\ \text{g}[4][4][\text{x0, x4}]\Big) \Big/$$
$$\left(2\sqrt{-\text{g}[4][4][\text{x0, x4}]}\right),\ 0,\ 0,\ 0,\ 0,\ 0\Big\}\Big\},$$
$$\Big\{4,\ \text{x3},\ \Big\{0,\ 0,\ 0,\ \Big(\text{g}[3][3]^{(0,1)}[\text{x0, x4}] +$$
$$6\ e^{2\ \text{a4}[\text{H x4}]}\ \text{H Sin}[6\ \text{H x0}]^{1/3}$$
$$\text{a4}'[\text{H x4}]\ \text{g}[4][4][\text{x0, x4}]\Big) \Big/$$
$$\left(2\sqrt{-\text{g}[4][4][\text{x0, x4}]}\right),\ 0,\ 0,\ 0,\ 0\Big\}\Big\},$$
$$\Big\{4,\ \text{x4},\ \Big\{-\frac{\text{g}[4][4]^{(1,0)}[\text{x0, x4}]}{2\sqrt{-\text{g}[4][4][\text{x0, x4}]}},\ 0,\ 0,$$
$$0,\ -\frac{\text{g}[4][4]^{(0,1)}[\text{x0, x4}]}{2\sqrt{-\text{g}[4][4][\text{x0, x4}]}},\ 0,\ 0,\ 0\Big\}\Big\},$$
$$\Big\{4,\ \text{x5},\ \Big\{0,\ 0,\ 0,\ 0,\ 0,$$
$$\frac{1}{2}\left(-6\ e^{-2\ \text{a4}[\text{H x4}]}\ \text{H Sin}[6\ \text{H x0}]^{1/3}\ \text{a4}'[\text{H x4}] -\right.$$
$$\left.\frac{\text{g}[5][5]^{(0,1)}[\text{x0, x4}]}{\text{g}[4][4][\text{x0, x4}]}\right)$$
$$\sqrt{-\text{g}[4][4][\text{x0, x4}]},\ 0,\ 0\Big\}\Big\},$$

$$\Big\{4,\ x6,\ \Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2}\left(-6\ e^{-2\ a4[H\ x4]}\ H\ Sin[6\ H\ x0]^{1/3}\ a4'[H\ x4] - \frac{g[6][6]^{(0,1)}[x0,\ x4]}{g[4][4][x0,\ x4]}\right)\sqrt{-g[4][4][x0,\ x4]},\ 0\Big\}\Big\},$$

$$\Big\{4,\ x7,\ \Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2}\left(-6\ e^{-2\ a4[H\ x4]}\ H\ Sin[6\ H\ x0]^{1/3}\ a4'[H\ x4] - \frac{g[7][7]^{(0,1)}[x0,\ x4]}{g[4][4][x0,\ x4]}\right)\sqrt{-g[4][4][x0,\ x4]}\Big\}\Big\}\Big\},\ \Big\{\Big\{5,\ x0,$$

$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ -\Big(\Big(g[5][5]^{(1,0)}[x0,\ x4] - 2\ H\ Cot[6\ H\ x0]\ g[5][5][x0,\ x4]\Big)\Big/\Big(2\sqrt{-g[5][5][x0,\ x4]}\Big)\Big),\ 0,\ 0\Big\}\Big\},$$

$$\{5,\ x1,\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},$$

$$\{5,\ x2,\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},$$

$$\{5,\ x3,\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},$$

$$\Big\{5,\ x4,\ \Big\{0,\ 0,\ 0,\ 0,\ 0,\ -\Big(\Big(g[5][5]^{(0,1)}[x0,\ x4] + 6\ H\ a4'[H\ x4]\ g[5][5][x0,\ x4]\Big)\Big/\Big(2\sqrt{-g[5][5][x0,\ x4]}\Big)\Big),\ 0,\ 0\Big\}\Big\},$$

```
{5, x5, {-((g[5][5]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[5][5][x0, x4]) /
      (2 Sqrt[-g[5][5][x0, x4]])), 0,
   0, 0, -((g[5][5]^(0,1)[x0, x4] +
        6 H a4'[H x4] g[5][5][x0, x4]) /
      (2 Sqrt[-g[5][5][x0, x4]])), 0, 0, 0}},
 {5, x6, {0, 0, 0, 0, 0, 0, 0, 0}},
 {5, x7,
  {0, 0, 0, 0, 0, 0, 0, 0}}},
{{6, x0, {0, 0, 0, 0, 0, 0,
   -((g[6][6]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[6][6][x0, x4]) /
      (2 Sqrt[-g[6][6][x0, x4]])), 0}},
 {6, x1, {0, 0, 0, 0, 0, 0, 0, 0}},
 {6, x2,
  {0, 0, 0, 0, 0, 0, 0, 0}},
 {6, x3, {0, 0, 0, 0, 0, 0, 0, 0}},
 {6, x4,
  {0, 0, 0, 0, 0, 0, -((g[6][6]^(0,1)[x0, x4] +
        6 H a4'[H x4] g[6][6][x0, x4]) /
      (2 Sqrt[-g[6][6][x0, x4]])), 0}},
 {6, x5, {0, 0, 0, 0, 0, 0, 0, 0}},
```

```
{6, x6,
  {-((g[6][6]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[6][6][x0, x4]) /
      (2 Sqrt[-g[6][6][x0, x4]])), 0,
   0, 0, -((g[6][6]^(0,1)[x0, x4] +
        6 H a4'[H x4] g[6][6][x0, x4]) /
      (2 Sqrt[-g[6][6][x0, x4]])), 0, 0, 0}},
 {6, x7, {0, 0, 0, 0, 0, 0, 0, 0}}},
{{7, x0, {0, 0, 0, 0, 0, 0, 0,
   -((g[7][7]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[7][7][x0, x4]) /
      (2 Sqrt[-g[7][7][x0, x4]]))}},
 {7, x1, {0, 0, 0, 0, 0, 0, 0, 0}},
 {7, x2,
  {0, 0, 0, 0, 0, 0, 0, 0}},
 {7, x3, {0, 0, 0, 0, 0, 0, 0, 0}},
 {7, x4, {0, 0, 0, 0, 0, 0,
   0, -((g[7][7]^(0,1)[x0, x4] +
        6 H a4'[H x4] g[7][7][x0, x4]) /
      (2 Sqrt[-g[7][7][x0, x4]]))}},
 {7, x5, {0, 0, 0, 0, 0, 0, 0, 0}},
 {7, x6,
```

```
   {0, 0, 0, 0, 0, 0, 0, 0}},
  {7, x7, {-((g[7][7]^(1,0)[x0, x4] -
         2 H Cot[6 H x0] g[7][7][x0, x4]) /
       (2 Sqrt[-g[7][7][x0, x4]])), 0,
    0, 0, -((g[7][7]^(0,1)[x0, x4] +
         6 H a4′[H x4] g[7][7][x0, x4]) /
       (2 Sqrt[-g[7][7][x0, x4]])), 0, 0, 0}}}}
```

Out[∘]=

{{{0, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{2, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{3, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{4, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{5, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{6, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{7, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}}

In[∘]:=
```
Block[{eAα, eαA, t, ginv, Γ, gg(*,ωω*)},
  (*ωω=ω_μ^(a)_(b) /.sωf;*)
  gg = g4488 /. sg;
  {ginv, Γ} = con[gg];
  eαA = e^α_(A) /. sge^α_(A);
  eAα = e^(A)_α /. sge^(A)_α;
  t = Table[Block[{v, V, contra, covari},
      V = eAα⟦b, All⟧;
      v = eαA⟦All, b⟧;
      contra = Table[
        D[v⟦k⟧, X⟦μ⟧] + Sum[v⟦i⟧ × Γ⟦k, i, μ⟧,
          {i, 1, DIM8}], {k, 1, DIM8}];
      (*no ω terms*)
      covari = Table[D[V⟦k⟧, X⟦μ⟧] -
         Sum[V⟦i⟧ × Γ⟦i, k, μ⟧, {i, 1, DIM8}] +
         Sum[eAα⟦J, All⟧⟦k⟧ × ωmat⟦μ, b, J⟧,
```

```
              {J, 1, DIM8}],
            {k, 1, DIM8}]; (*ω terms*)
          {b - 1, X⟦μ⟧, {{contra}, {covari}}}],
        {b, 1, Length[eαA]}, {μ, 1, DIM8}];
   t]
  FullSimplify[# /. ssgm4488,
      constraintVars && ⅇ^(2 a4[H x4]) > 0 && ⅇ^(a4[H x4]) >
        0 && ⅇ^(-2 a4[H x4]) > 0 && ⅇ^(-a4[H x4]) > 0] & /@ %
```

Out[ ]=

$$\Big\{\Big\{\Big\{0,\ x0,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ -\frac{g[0][0]^{(0,1)}[x0,\ x4]}{2\sqrt{g[0][0][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},\ \Big\{0,\ x1,\ \Big\{\Big\{\Big\{0,\ \frac{g[1][1]^{(1,0)}[x0,\ x4]}{2\sqrt{g[0][0][x0,\ x4]}\ g[1][1][x0,\ x4]},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x2,\ \Big\{\Big\{\Big\{0,\ 0,\ \frac{g[2][2]^{(1,0)}[x0,\ x4]}{2\sqrt{g[0][0][x0,\ x4]}\ g[2][2][x0,\ x4]},$$

$$0,\ 0,\ 0,\ 0,\ 0\}\}\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\Big\{0,\ x3,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ \frac{g[3][3]^{(1,0)}[x0,\ x4]}{2\sqrt{g[0][0][x0,\ x4]}\ g[3][3][x0,\ x4]},\ 0,$$

$$0,\ 0,\ 0\Big\}\Big\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x4,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ \frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\sqrt{g[0][0][x0,\ x4]}\ g[4][4][x0,\ x4]},$$

$$0,\ 0,\ 0\Big\}\Big\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x5,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\sqrt{g[0][0][x0,\ x4]}\ g[5][5][x0,\ x4]},$$

$$0,\ 0\Big\}\Big\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x6,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\sqrt{g[0][0][x0,\ x4]}\ g[6][6][x0,\ x4]},$$

$$0\Big\}\Big\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x7,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$$

$$\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\ g[7][7][x0, x4]}\Big\}\Big\},$$

{{0, 0, 0, 0, 0, 0, 0, 0}}}}},

{{1, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},

{{0, 0, 0, 0, 0, 0, 0, 0}}}}, {1, x1,

$$\Big\{\Big\{\Big\{-\frac{g[1][1]^{(1,0)}[x0, x4]}{2\,g[0][0][x0, x4]\sqrt{g[1][1][x0, x4]}},$$

0, 0, 0,

$$-\frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}\ g[4][4][x0, x4]},$$

0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{1, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},

{{0, 0, 0, 0, 0, 0, 0, 0}}}},

{1, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},

{{0, 0, 0, 0, 0, 0, 0, 0}}}},

{1, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},

{{0, 0, 0, 0, 0, 0, 0, 0}}}},

{1, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},

{{0, 0, 0, 0, 0, 0, 0, 0}}}},

{1, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},

{{0, 0, 0, 0, 0, 0, 0, 0}}}},

{1, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},

{{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{2, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}, {2, x2,

$$\left\{\left\{\left\{-\frac{g[2][2]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\, \sqrt{g[2][2][x0, x4]}},\right.\right.\right.$$

0, 0, 0,

$$-\frac{g[2][2]^{(0,1)}[x0, x4]}{2\, \sqrt{g[2][2][x0, x4]}\, g[4][4][x0, x4]},$$

0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}},

$$\Big\{\{3,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{3,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{3,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\ \Big\{3,\ x3,$$

$$\Big\{\Big\{\Big\{-\frac{g[3][3]^{(1,0)}[x0,\ x4]}{2\ g[0][0][x0,\ x4]\ \sqrt{g[3][3][x0,\ x4]}},\ 0,\ 0,\ 0,\ -\frac{g[3][3]^{(0,1)}[x0,\ x4]}{2\ \sqrt{g[3][3][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0\Big\}\Big\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\{3,\ x4,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{3,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{3,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{3,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\Big\},\ \Big\{\Big\{4,\ x0,$$

$$\Bigg\{\Bigg\{\Bigg\{\frac{g[0][0]^{(0,1)}[x0,\ x4]}{2\ g[0][0][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},$$

$$0,\ 0,\ 0,\ \frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ (-g[4][4][x0,\ x4])^{3/2}} +$$

$$\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[4][4][x0,\ x4]},$$

$$0,\ 0,\ 0\Bigg\}\Bigg\},$$

$$\Bigg\{\Bigg\{0,\ 0,\ 0,\ 0,\ -\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}} -$$

$$\frac{g[4][4]^{(1,0)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ g[4][4][x0,\ x4]}$$

$$-\left(\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ (-g[4][4][x0,\ x4])^{3/2}} + g[4][4]^{(1,0)}[x0,\ x4] \Big/ \left(2\ \sqrt{-g[4][4][x0,\ x4]}\ g[4][4][x0,\ x4]\right)\right)$$

$$g[4][4][x0,\ x4],\ 0,\ 0,\ 0\Bigg\}\Bigg\}\Bigg\}\Bigg\},\ \Bigg\{4,\ x1,$$

$$\Bigg\{\Bigg\{\Bigg\{0,\ \frac{g[1][1]^{(0,1)}[x0,\ x4]}{2\ g[1][1][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},$$

$$0,\ 0,\ 0,\ 0,\ 0,\ 0\Bigg\}\Bigg\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\Big\{4, x2, \Big\{\Big\{\Big\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]}{2\, g[2][2][x0, x4]\, \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0\Big\}\Big\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\Big\}\Big\},$$

$$\Big\{4, x3, \Big\{\Big\{\Big\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2\, g[3][3][x0, x4]\, \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0\Big\}\Big\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\Big\}\Big\}, \Big\{4, x4,$$

$$\Big\{\Big\{\Big\{-\frac{g[4][4]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\, \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \frac{g[4][4]^{(0,1)}[x0, x4]}{2\, (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\, \sqrt{-g[4][4][x0, x4]}\, g[4][4][x0, x4]}, 0, 0, 0\Big\}\Big\},$$

$$\Big\{\Big\{0, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x0, x4]}{2\, \sqrt{-g[4][4][x0, x4]}} -$$

$$\frac{g[4][4]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ g[4][4][x0,\ x4]}$$

$$-\left(\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\ (-g[4][4][x0,\ x4])^{3/2}}+\right.$$
$$g[4][4]^{(0,1)}[x0,\ x4]\ \Big/$$
$$\left(2\ \sqrt{-g[4][4][x0,\ x4]}\right.$$
$$\left.\left.g[4][4][x0,\ x4]\right)\right)$$

$$\left.\left.\left.\left.g[4][4][x0,\ x4],\ 0,\ 0,\ 0\right\}\right\}\right\}\right\},$$

$$\left\{4,\ x5,\ \left\{\left\{\left\{0,\ 0,\ 0,\ 0,\ 0,\right.\right.\right.\right.$$
$$\frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[5][5][x0,\ x4]},$$
$$\left.\left.0,\ 0\right\}\right\},$$
$$\left.\left.\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\right\}\right\},$$

$$\left\{4,\ x6,\ \left\{\left\{\left\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\right.\right.\right.\right.$$
$$\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[6][6][x0,\ x4]},$$
$$\left.\left.0\right\}\right\},$$
$$\left.\left.\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\right\}\right\},$$

$$\left\{4,\ x7,\ \left\{\left\{\left\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\right.\right.\right.\right.$$

$$\frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[7][7][x0,\ x4]}\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\}\Big\},$$

$$\Big\{\Big\{5,\ x0,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,$$

$$\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ (-g[5][5][x0,\ x4])^{3/2}} +$$

$$\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]},$$

$$0,\ 0\Big\}\Big\},\ \Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,$$

$$-\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}} -$$

$$\frac{g[5][5]^{(1,0)}[x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}}{2\ g[5][5][x0,\ x4]}$$

$$-\left(\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ (-g[5][5][x0,\ x4])^{3/2}} + g[5][5]^{(1,0)}[x0,\ x4] \Big/ \left(2\ \sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]\right)\right)$$

$$g[5][5][x0,\ x4],\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\{5,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},$$

{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}, {5, x4,

$$\left\{\left\{\left\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ (-g[5][5][x0,\ x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0\right\}\right\},\ \left\{\left\{0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}} - \frac{g[5][5]^{(0,1)}[x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}}{2\ g[5][5][x0,\ x4]} - \left(\frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ (-g[5][5][x0,\ x4])^{3/2}} + g[5][5]^{(0,1)}[x0,\ x4] \Big/ \left(2\ \sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]\right)\right) g[5][5][x0,\ x4],\ 0,\ 0\right\}\right\}\right\},\ \{5,\ x5,$$

$$\left\{\left\{\left\{-\frac{g[5][5]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\, \sqrt{-g[5][5][x0, x4]}},\right.\right.\right.$$
$$0, 0, 0,$$
$$-\frac{g[5][5]^{(0,1)}[x0, x4]}{2\, g[4][4][x0, x4]\, \sqrt{-g[5][5][x0, x4]}},$$
$$\left.\left.0, 0, 0\right\}\right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$
$$\{5, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$
$$\{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\},$$
$$\left\{\left\{6, x0, \left\{\left\{\left\{0, 0, 0, 0, 0, 0,\right.\right.\right.\right.\right.$$
$$\frac{g[6][6]^{(1,0)}[x0, x4]}{2\, (-g[6][6][x0, x4])^{3/2}} +$$
$$\frac{g[6][6]^{(1,0)}[x0, x4]}{2\, \sqrt{-g[6][6][x0, x4]}\, g[6][6][x0, x4]},$$
$$\left.\left.0\right\}\right\}, \left\{\left\{0, 0, 0, 0, 0, 0,\right.\right.$$
$$-\frac{g[6][6]^{(1,0)}[x0, x4]}{2\, \sqrt{-g[6][6][x0, x4]}} -$$
$$\frac{g[6][6]^{(1,0)}[x0, x4]\, \sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]}$$

$$-\left(\frac{g[6][6]^{(1,0)}[x0,x4]}{2\,(-g[6][6][x0,x4])^{3/2}}+g[6][6]^{(1,0)}[x0,x4]\Big/\left(2\sqrt{-g[6][6][x0,x4]}\;g[6][6][x0,x4]\right)\right)$$

g[6][6][x0, x4], 0}}}},
{6, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{6, x4, {{{0, 0, 0, 0, 0, 0,

$$\frac{g[6][6]^{(0,1)}[x0,x4]}{2\,(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(0,1)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}\;g[6][6][x0,x4]},$$

0}}, {{0, 0, 0, 0, 0, 0,

$$-\frac{g[6][6]^{(0,1)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}}-$$

$$\frac{g[6][6]^{(0,1)}[x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}}{2\ g[6][6][x0,\ x4]}$$

$$-\left(\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ (-g[6][6][x0,\ x4])^{3/2}} + g[6][6]^{(0,1)}[x0,\ x4] \Big/ \left(2\ \sqrt{-g[6][6][x0,\ x4]}\ g[6][6][x0,\ x4]\right)\right)$$

$$g[6][6][x0,\ x4],\ 0\}\}\}\},$$

{6, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\left\{6,\ x6,\ \left\{\left\{\left\{-\frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ g[0][0][x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}},\ 0,\ 0,\ 0,\ -\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ g[4][4][x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}},\ 0,\ 0,\ 0\right\}\right\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\right\}\right\},$$

{6, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}},

{{7, x0, {{{0, 0, 0, 0, 0, 0, 0,

$$\frac{g[7][7]^{(1,0)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}} +$$

$$\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}\; g[7][7][x0, x4]}\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0,$$

$$-\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} -$$

$$\frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2\, g[7][7][x0, x4]}$$

$$-\left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}} + g[7][7]^{(1,0)}[x0, x4] \Big/ \left(2\sqrt{-g[7][7][x0, x4]}\; g[7][7][x0, x4]\right)\right) g[7][7][x0, x4]\}\}\}\},$$

{7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x4, {{{0, 0, 0, 0, 0, 0, 0,

$$\frac{g[7][7]^{(0,1)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}} +$$

$$\frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}\; g[7][7][x0, x4]}\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0,$$

$$-\frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} -$$

$$\frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2\, g[7][7][x0, x4]}$$

$$-\left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}} +\right.$$

$$g[7][7]^{(0,1)}[x0, x4] \Big/$$

$$\left(2\sqrt{-g[7][7][x0, x4]}\; g[7][7][x0,\right.$$

$$\left.\left. x4]\right)\right) g[7][7][x0, x4]\}\}\}\},$$

$$\{7, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{7, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{7, x7,$$

$$\{\{\{-\frac{g[7][7]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\sqrt{-g[7][7][x0, x4]}},$$

$$0, 0, 0,$$

$$-\frac{g[7][7]^{(0,1)}[x0, x4]}{2\, g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}},$$

0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}}}

Out[ ]=

{{{0, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x1, {{{0, H, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x2, {{{0, 0, H, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x3, {{{0, 0, 0, H, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x5, {{{0, 0, 0, 0, 0, H, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x6, {{{0, 0, 0, 0, 0, 0, H, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x7, {{{0, 0, 0, 0, 0, 0, 0, H}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{1, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}, {1, x1,

$$\left\{\left\{\left\{-\sqrt{e^{2\,a4[H\,x4]}}\;H\,Sec[6\,H\,x0]\,Sin[6\,H\,x0]^{7/6},\right.\right.\right.$$
$$0,\,0,\,0,\,3\,\sqrt{e^{2\,a4[H\,x4]}}\;H$$
$$\left.\left.Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4],\,0,\,0,\,0\right\}\right\},$$
$$\left.\left.\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\right\}\right\},$$
$$\{1,\,x2,\,\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$
$$\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\}\},$$
$$\{1,\,x3,\,\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$
$$\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\}\},$$
$$\{1,\,x4,\,\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$
$$\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\}\},$$
$$\{1,\,x5,\,\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$
$$\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\}\},$$
$$\{1,\,x6,\,\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$
$$\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\}\},$$
$$\{1,\,x7,\,\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$
$$\left.\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\}\}\right\},$$
$$\left\{\{2,\,x0,\,\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},\right.$$
$$\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\}\},$$
$$\{2,\,x1,\,\{\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$
$$\{\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\}\},$$
$$\left\{2,\,x2,\,\left\{\left\{\left\{-\sqrt{e^{2\,a4[H\,x4]}}\;H\,Sec[6\,H\,x0]\right.\right.\right.\right.$$

$\text{Sin}[6\,H\,x0]^{7/6}, 0, 0, 0, 3\,\sqrt{e^{2\,a4[H\,x4]}}$
$H\,\text{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4], 0, 0, 0\}\},$
$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
$\{2, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$
$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
$\{2, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$
$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
$\{2, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$
$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
$\{2, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$
$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
$\{2, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$
$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\},$
$\{\{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$
$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
$\{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$
$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
$\{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$
$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
$\{3, x3, \{\{\{-\sqrt{e^{2\,a4[H\,x4]}}\,H\,\text{Sec}[6\,H\,x0]$
$\text{Sin}[6\,H\,x0]^{7/6}, 0, 0, 0, 3\,\sqrt{e^{2\,a4[H\,x4]}}$

```
H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{4, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{4, x1, {{{0, 3 H a4′[H x4], 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{4, x2, {{{0, 0, 3 H a4′[H x4], 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{4, x3, {{{0, 0, 0, 3 H a4′[H x4], 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{4, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}, {4, x5,
{{{0, 0, 0, 0, 0, -3 H a4′[H x4], 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}, {4, x6,
```

$$
\begin{aligned}
&\{\{\{0, 0, 0, 0, 0, 0, -3\,H\,a4'[H\,x4], 0\}\},\\
&\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{4, x7,\\
&\{\{\{0, 0, 0, 0, 0, 0, 0, -3\,H\,a4'[H\,x4]\}\},\\
&\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\},\\
&\Big\{\{5, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},\\
&\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},\\
&\{5, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},\\
&\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},\\
&\{5, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},\\
&\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},\\
&\{5, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},\\
&\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},\\
&\{5, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},\\
&\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \Big\{5, x5,\\
&\Big\{\Big\{\Big\{\sqrt{e^{-2\,a4[H\,x4]}}\;H\,\mathrm{Sec}[6\,H\,x0]\;\mathrm{Sin}[6\,H\,x0]^{7/6},\\
&0, 0, 0, 3\sqrt{e^{-2\,a4[H\,x4]}}\;H\\
&\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4], 0, 0, 0\Big\}\Big\},\\
&\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\Big\}\Big\},\\
&\{5, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},\\
&\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},\\
&\{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},
\end{aligned}
$$

{{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{6, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{6, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{6, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{6, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{6, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{6, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}}, {6, x6,

$\left\{\left\{\left\{\sqrt{e^{-2\,a4[H\,x4]}}\ H\,Sec[6\,H\,x0]\ Sin[6\,H\,x0]^{7/6},\right.\right.\right.$
$0, 0, 0, 3\,\sqrt{e^{-2\,a4[H\,x4]}}\ H$
$\left.\left.Sin[6\,H\,x0]^{1/6}\,a4'[H\,x4], 0, 0, 0\right\}\right\},$
$\left.\left.\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\right\}\right\},$
{6, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 0, 0}}}}}},
{{7, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},

```
{{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {7, x7,
```

$$\left\{\left\{\left\{\sqrt{e^{-2\,a4[H\,x4]}}\;H\,\mathrm{Sec}[6\,H\,x0]\;\mathrm{Sin}[6\,H\,x0]^{7/6},\;0,\;0,\;0,\;3\sqrt{e^{-2\,a4[H\,x4]}}\;H\;\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\;0,\;0,\;0\right\}\right\},\right.$$

```
{{0, 0, 0, 0, 0, 0, 0, 0}}}}}}
```

In[∘]:=
```
(*ωΨ=
 Table[Sum[ExpandAll[(ω_μ^(a)_(b)[μ,a,b]/.sωf)Simplify[(Transpose[Ψ16].σ16.SAB⟦a,b⟧.Ψ16)]],
   {a,1,8},{b,1,8}],{μ,1,8}]*)
```

In[∘]:=
```
(*ωΨ=Table[Sum[ExpandAll[(ωmat⟦μ,a,b⟧)Simplify[(Transpose[Ψ16].σ16.SAB⟦a,b⟧.Ψ16)]],
   {a,1,8},{b,1,8}],{μ,1,8}]*)
```

In[∘]:=
```
(*jωΨ=ParallelTable[{j,base16⟦j,2⟧,Table[
   {X⟦μ⟧,Simplify[(Transpose[Ψ16].σ16.(ZZZ0 *T16^α[0]+T16^α[4]* ZZZ4 ).base16⟦j,1⟧.(Sum[
      ωmat⟦μ,a,b⟧SAB⟦a,b⟧,{a,1,8},{b,1,8}])Ψ16)]},{μ,1,8}]},{j,1,Length[base16]}]*)
```

In[∘]:=
```
(*jωΨ=ParallelTable[{j,base16⟦j,2⟧,Table[Sum[ExpandAll[(ω_μ^(a)_(b)[μ,a,b]/.sωf)Simplify[
        (Transpose[Ψ16].σ16.(ZZZ0 *T16^α[0]+T16^α[4]* ZZZ4 ).base16⟦j,1⟧.SAB⟦a,b⟧.Ψ16)]],
    {a,1,8},{b,1,8}],{μ,1,8}]},{j,1,Length[base16]}]*)
```

In[∘]:=
```
(*jωΨ⟦92⟧*)
```

In[∘]:=
```
(*jωΨ⟦92⟧/.ssgm4488//FullSimplify[#,constraintVars]&*)
```

In[∘]:=
```
SAB⟦1⟧⟦2⟧
```

Out[∘]=

$$\left\{\left\{0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0\right\},\left\{0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0\right\},\right.$$
$$\left\{0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0\right\},\left\{0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2}\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0\right\},\left\{0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0\right\},$$
$$\left.\left\{0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0\right\},\left\{0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0\right\}\right\}$$

In[∘]:=
```
(ω_μ^(a)_(b)[8, 8, 1] /. Flatten[sωgtry])
```

Out[∘]=

$$e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}$$

In[∘]:=
```
SAB⟦1, 2⟧.Ψ16
```

Out[∘]=

$$\left\{\frac{1}{2}\,f16[7][x0,x4],\ \frac{1}{2}\,f16[6][x0,x4],\ -\frac{1}{2}\,f16[5][x0,x4],\ -\frac{1}{2}\,f16[4][x0,x4],\right.$$
$$\frac{1}{2}\,f16[3][x0,x4],\ \frac{1}{2}\,f16[2][x0,x4],\ -\frac{1}{2}\,f16[1][x0,x4],\ -\frac{1}{2}\,f16[0][x0,x4],$$
$$-\frac{1}{2}\,f16[15][x0,x4],\ -\frac{1}{2}\,f16[14][x0,x4],\ \frac{1}{2}\,f16[13][x0,x4],\ \frac{1}{2}\,f16[12][x0,x4],$$
$$\left.-\frac{1}{2}\,f16[11][x0,x4],\ -\frac{1}{2}\,f16[10][x0,x4],\ \frac{1}{2}\,f16[9][x0,x4],\ \frac{1}{2}\,f16[8][x0,x4]\right\}$$

In[ ]:= T16^A[0]

Out[ ]=

```
{{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
 {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}}
```

In[ ]:= SAB = Table[$\frac{1}{4}$ (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]), {A1, 0, 7}, {B1, 0, 7}];

In[ ]:= Ψ16

Out[ ]=

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

In[ ]:= Sum[ExpandAll[($\omega_{\mu\ (b)}^{(a)}$ [4, a, b] /. sωfgtry) Simplify[(SAB⟦a, b⟧)]], {a, 1, 8}, {b, 1, 8}].
Ψ16 // MatrixForm

Out[ ]//MatrixForm=

$$\begin{pmatrix} -e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[5][x0,\ x4] \\ e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[4][x0,\ x4] \\ -e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[7][x0,\ x4] \\ e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[6][x0,\ x4] \\ -e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[1][x0,\ x4] \\ e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[0][x0,\ x4] \\ -e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[3][x0,\ x4] \\ e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[2][x0,\ x4] \\ e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[13][x0,\ x4] \\ -e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[12][x0,\ x4] \\ e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[15][x0,\ x4] \\ -e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[14][x0,\ x4] \\ e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[9][x0,\ x4] \\ -e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[8][x0,\ x4] \\ e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[11][x0,\ x4] \\ -e^{a4[H\,x4]}\ H\ Sin[6\ H\ x0]^{1/6}\ f16[10][x0,\ x4] \end{pmatrix}$$

In[ ]:=
```
(*Table[Block[{eAα,eαA,t,V},eαA=e^α_(A)/.sgtrye^α_(A);
  eAα=e_α^(A)/.sgtrye_α^(A);
  V=eAα[[a,All]];
  t=Table[Block[{v,d},v=eαA[[All,b]];
     d=D[v[[k]],X[[j]] ]+Sum[ v[[i]] Γ[[k,i,j]],{i,1,DIM8}];
     d.V],{k,1,DIM8}];
  t],{j,1,DIM8},{a,1,Length[eαA]},{b,1,Length[eαA]}]*)
```

In[ ]:=
```
Clear[rt];
rt[g_, ass_ : {}] :=
 Module[{t, Γ, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    Γ = FullSimplify[
      Table[(1 / 2) * Sum[(ginv[[i, s]]) *
           (D[g[[s, j]], X[[k]] ] +
              D[g[[s, k]], X[[j]] ] - D[g[[j, k]],
               X[[s]] ]), {s, 1, DIM8}],
        {i, 1, DIM8},
        {j, 1, DIM8}, {k, 1, DIM8}] , ass];
    RicciΓ = ParallelTable[
      FullSimplify[D[Γ[[μ, ν, β]], X[[α]] ] -
         D[Γ[[μ, ν, α]], X[[β]] ] + Sum[
          Γ[[μ, s, α]] × Γ[[s, ν, β]] - Γ[[μ, s, β]] ×
            Γ[[s, ν, α]], {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8},
```

```
      {α, 1, DIM8}, {β, 1, DIM8}] ;
    RieΓ = ParallelTable[ FullSimplify[
       Sum[ RicciΓ⟦α, μ, α, ν⟧, {α, 1, DIM8}],
       ass], {μ, 1, DIM8}, {ν, 1, DIM8}] ;
    RS = FullSimplify[Tr[ ginv.RieΓ] , ass];
    G = ParallelTable[
      FullSimplify[RieΓ⟦α⟧ - 1/2 g⟦α⟧ RS, ass],
      {α, 1, Length[RieΓ]}];];
  Print[t];
  Print[Now];
  Return[{ginv, Γ, RicciΓ, RieΓ, RS, G}]]
```

verify $g_{\mu\nu|\alpha} = 0$:

In[∘]:=
```
Simplify[Table[D[gtry⟦j, k⟧, X⟦i⟧ ] -
    Sum[ gtry⟦k, s⟧ × Γ⟦s, i, j⟧, {s, 1, DIM8}] - Sum[ gtry⟦j, s⟧ × Γ⟦s, i, k⟧, {s, 1, DIM8}],
   {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Out[∘]=
```
{0}
```

- **CHECK**

In[∘]:=
```
MatrixMetric44 // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0]^2 & 0 & 0 & 0 & 0 & \\ 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 & \\ 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & \\ 0 & 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & \\ 0 & 0 & 0 & 0 & -1 & \\ 0 & 0 & 0 & 0 & 0 & -e^{-2\,a4[H\,x4]}\,\mathrm{S} \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{pmatrix}$$

In[∘]:= $\left(e^{\alpha}_{(A)}\text{.MatrixMetric44.}e^{\alpha}_{(A)}\text{ /. sgtrye}^{\alpha}_{(A)}\right)$ // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:= Transpose$\left[e_{\alpha}^{(A)}\right]$.η4488.$e_{\alpha}^{(A)}$ - MatrixMetric44 /. sgtrye$_{\alpha}^{(A)}$ // FullSimplify[#, constraintVars] &

Out[∘]=

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[∘]:= Transpose$\left[e^{\alpha}_{(A)}\right]$.MatrixMetric44.$e^{\alpha}_{(A)}$ - η4488 /. sgtrye$^{\alpha}_{(A)}$ // FullSimplify[#, constraintVars] &

Out[∘]=

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[∘]:= Inverse$\left[e^{\alpha}_{(A)}\text{ /. sgtrye}^{\alpha}_{(A)}\right]$ == $e_{\alpha}^{(A)}$ /. sgtrye$_{\alpha}^{(A)}$ // FullSimplify[#, constraintVars] &

Out[∘]=

True

In[∘]:= Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]

Out[∘]=

```
{{0, 0}, {0, 0}}
```

In[∘]:= (*Inverse$\left[\text{Transpose}\left[e^{\alpha}_{(A)}\right]\text{.MatrixMetric44.}e^{\alpha}_{(A)}\right]$==
$e_{\alpha}^{(A)}$.Inverse[MatrixMetric44].Transpose$\left[e_{\alpha}^{(A)}\right]$//FullSimplify[#,constraintVars]&*)

In[∘]:= (*$e_{\alpha}^{(A)}$.Inverse[MatrixMetric44].Transpose$\left[e_{\alpha}^{(A)}\right]$//FullSimplify[#,constraintVars]&
%-Inverse[η4488]//Union[Flatten[#]]&*)

In[∘]:= (*Transpose$\left[e_{\alpha}^{(A)}\right]$==$e_{\alpha}^{(A)}$//FullSimplify[#,constraintVars]&*)

In[∘]:= (*(FullSimplify$\left[e_{\alpha}^{(A)}\text{.Inverse[MatrixMetric44].Transpose}\left[e_{\alpha}^{(A)}\right]\right.$ -Inverse[η4488] ,
constraintVars]//Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)

In[∘]:= (*(FullSimplify[Transpose$\left[e_{\alpha}^{(A)}\right]$.Inverse[MatrixMetric44].$e_{\alpha}^{(A)}$,constraintVars]-
Inverse[η4488]//Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)

In[∘]:= (*(FullSimplify[Transpose$\left[e_{\alpha}^{(A)}\right]$.ginv.$e_{\alpha}^{(A)}$,constraintVars]-Inverse[η4488]//
Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)

In[∘]:= $\Big(E^{\alpha}_{(A)}$ = Table$\Big[$Sum$\Big[e_{\alpha}^{(A)}$⟦A1, α1⟧ ginv⟦α1, β1⟧ ×
η4488⟦A1, B1⟧ /. sgtrye$_{\alpha}^{(A)}$,

```
{α1, 1, 8}, {A1, 1, 8}], {β1, 1, 8},
{B1, 1, 8}] // FullSimplify[#,
constraintVars] &) // Column
```

Out[ ]=

$$\{\mathrm{Tan}[6\,H\,x0],\,0,\,0,\,0,\,0,\,0,\,0,\,0\}$$
$$\left\{0,\,\frac{e^{-a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\,0,\,0,\,0,\,0,\,0,\,0\right\}$$
$$\left\{0,\,0,\,\frac{e^{-a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\,0,\,0,\,0,\,0,\,0\right\}$$
$$\left\{0,\,0,\,0,\,\frac{e^{-a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\,0,\,0,\,0,\,0\right\}$$
$$\{0,\,0,\,0,\,0,\,1,\,0,\,0,\,0\}$$
$$\left\{0,\,0,\,0,\,0,\,0,\,\frac{e^{a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\,0,\,0\right\}$$
$$\left\{0,\,0,\,0,\,0,\,0,\,0,\,\frac{e^{a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\,0\right\}$$
$$\left\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,\frac{e^{a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}}\right\}$$

$$\mathrm{Lagrangian}\Psi16 = \kappa\sqrt{\mathrm{Det}[g_{\mu\nu}]}\ \mathrm{Transpose}[\Psi16].\sigma16.\mathrm{T16}^{A}[A1].\Psi16_{,\alpha}\, g^{-1\,\alpha\beta}\,\eta_{A1\,B1}\,E^{B1}_{\beta} + \text{mass term}$$

In[ ]:= **ass = constraintVars**

Out[ ]=

$$x0>0\,\&\&\,x4>0\,\&\&\,H>0\,\&\&\,6\,H\,x0>0\,\&\&\,3\,H\,x4>0\,\&\&\,a4[x4]>0\,\&\&\,A4[t]>0\,\&\&\,Q>0\,\&\&\,z>0\,\&\&\,t>0\,\&\&\,M>0\,\&\&\,K>0\,\&\&\,e^{-2\,a4[H\,x4]}>0\,\&\&\,e^{-a4[H\,x4]}>0\,\&\&\,e^{2\,a4[H\,x4]}>0\,\&\&\,e^{a4[H\,x4]}>0\,\&\&\,\mathrm{Sin}[6\,H\,x0]>0\,\&\&\,\mathrm{Cos}[6\,H\,x0]>0\,\&\&\,\mathrm{Csc}[6\,H\,x0]>0\,\&\&\,\mathrm{Sec}[6\,H\,x0]>0\,\&\&\,\mathrm{Tan}[6\,H\,x0]>0\,\&\&\,\mathrm{Cot}[6\,H\,x0]>0\,\&\&\,\mathrm{Sin}[z]>0\,\&\&\,\sqrt{\mathrm{Sin}[z]}>0\,\&\&\,\mathrm{Sin}[z]^{3/2}>0\,\&\&\,\sqrt{\mathrm{Sin}[z]}>0\,\&\&\,\frac{1}{\mathrm{Sin}[z]^{3/2}}>0\,\&\&\,\frac{1}{\sqrt{\mathrm{Sin}[z]}}>0\,\&\&\,\mathrm{Cot}[z]>0\,\&\&\,\sqrt{\mathrm{Cot}[z]}>0\,\&\&\,\mathrm{Cot}[z]^{3/2}>0\,\&\&\,\sqrt{\mathrm{Cot}[z]}>0\,\&\&\,\mathrm{Tan}[z]>0\,\&\&\,\mathrm{Sec}[z]>0\,\&\&\,\sqrt{\mathrm{Csc}[z]}>0\,\&\&\,\mathrm{Csc}[z]>0$$

In[ ]:=
```
x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
 Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && ⅇ^(-2 a4[H x4]) > 0 && ⅇ^(-a4[H x4]) > 0 && ⅇ^(2 a4[H x4]) > 0 &&
 ⅇ^(a4[H x4]) > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
 Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 &&
 Sqrt[Sin[z]] > 0 && 1/Sin[z]^(3/2) > 0 && 1/Sqrt[Sin[z]] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
 Cot[z]^(3/2) > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

Out[ ]=

$x0>0 \;\&\&\; x4>0 \;\&\&\; H>0 \;\&\&\; 6\,H\,x0>0 \;\&\&\; H\,x4>0 \;\&\&\; a4[x4]>0 \;\&\&\; A4[t]>0 \;\&\&\; Q>0 \;\&\&\; z>0 \;\&\&\; t>0 \;\&\&\; M>0 \;\&\&\; K>0 \;\&\&\; e^{-2\,a4[H\,x4]}>0 \;\&\&\; e^{-a4[H\,x4]}>0 \;\&\&\; e^{2\,a4[H\,x4]}>0 \;\&\&\; e^{a4[H\,x4]}>0 \;\&\&\; Sin[6\,H\,x0]>0 \;\&\&\; Cos[6\,H\,x0]>0 \;\&\&\; Csc[6\,H\,x0]>0 \;\&\&\; Sec[6\,H\,x0]>0 \;\&\&\; Tan[6\,H\,x0]>0 \;\&\&\; Cot[6\,H\,x0]>0 \;\&\&\; Sin[z]>0 \;\&\&\; \sqrt{Sin[z]}>0 \;\&\&\; Sin[z]^{3/2}>0 \;\&\&\; \sqrt{Sin[z]}>0 \;\&\&\; \frac{1}{Sin[z]^{3/2}}>0 \;\&\&\; \frac{1}{\sqrt{Sin[z]}}>0 \;\&\&\; Cot[z]>0 \;\&\&\; \sqrt{Cot[z]}>0 \;\&\&\; Cot[z]^{3/2}>0 \;\&\&\; \sqrt{Cot[z]}>0 \;\&\&\; Tan[z]>0 \;\&\&\; Sec[z]>0 \;\&\&\; \sqrt{Csc[z]}>0 \;\&\&\; Csc[z]>0$

In[ ]:=
```
ginv == FullSimplify[Inverse[gtry], ass]
```

Out[ ]=

True

In[ ]:=
```
ginv === Transpose[ginv]
```

Out[ ]=

True

verify metric covariant derivative again:

In[ ]:=
```
Simplify[Table[D[gtry[[j, k]], X[[i]] ] -
    Sum[ gtry[[k, s]] × Γ[[s, i, j]], {s, 1, DIM8}] - Sum[ gtry[[j, s]] × Γ[[s, i, k]], {s, 1, DIM8}],
   {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Out[ ]=

{0}

In[ ]:=
```
RS
```

Out[ ]=

$6\,H^2\,\left(-7+a4'[H\,x4]^2\right)$

In[•]:=
```
EinsteinG
skelx[%]
```

Out[•]=

$$\{\{-3 H^2 \text{Cot}[6 H x0]^2 (-5 + a4'[H x4]^2), 0, 0, 0, 0, 0, 0, 0\},$$
$$\{0, -e^{2 a4[H x4]} H^2 \text{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, -e^{2 a4[H x4]} H^2 \text{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0\},$$
$$\{0, 0, 0, -e^{2 a4[H x4]} H^2 \text{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, -3 H^2 (7 + a4'[H x4]^2), 0, 0, 0\},$$
$$\{0, 0, 0, 0, 0, e^{-2 a4[H x4]} H^2 \text{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4]), 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, e^{-2 a4[H x4]} H^2 \text{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4]), 0\},$$
$$\{0, 0, 0, 0, 0, 0, 0, e^{-2 a4[H x4]} H^2 \text{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4])\}\}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

```
Union[Table[
  Sum[τ[A].bas64[[k, 1]].σ.τ̄[A] × η8[[A, A]], {A,
     1, 8}] - ID8 Tr[bas64[[k, 1]].σ] + 4 eg[[k]]
     (bas64[[k, 1]] - Transpose[bas64[[k, 1]]])/2 .σ,
  {k, 1, 64}]]
```

{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}

BASIS of 16 × 16 matrices :

In[•]:=
```
16 * 16
```

Out[•]=

256

In[•]:= 2^8

Out[•]=

256

In[•]:=
```
Sum[Binomial[8, p], {p, 0, 8}]
```

Out[•]=

256

In[•]:=
```
Id16 == ID16 == Dot @@ (T16^A[#] & /@ Range[0, 8])
```

Out[•]=

True

In[•]:=
```
Clear[t160]; t160 = {{Id16, {Range[0, 8]}}}; Length[t160]
Clear[t16A];
t16A = {};
Do[AppendTo[t16A, {T16^A[j], {j}}], {j, 0, 7}];
Length[t16A]
Clear[t16AB];
t16AB = {};
Do[AppendTo[t16AB, {T16^A[j].T16^A[k], {j, k}}], {j, 0, 6}, {k, j + 1, 7}];
Length[t16AB]
Clear[t16ABC];
t16ABC = {};
Do[AppendTo[t16ABC, {T16^A[j].T16^A[k].T16^A[h], {j, k, h}}],
 {j, 0, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[t16ABC]
Clear[t16ABCD];
t16ABCD = {};
Do[AppendTo[t16ABCD, {T16^A[j].T16^A[k].T16^A[h].T16^A[i], {j, k, h, i}}],
 {j, 0, 4}, {k, j + 1, 5}, {h, k + 1, 6}, {i, h + 1, 7}];
Length[t16ABCD]
Clear[t16ABCDE];
t16ABCDE = {};
Do[AppendTo[t16ABCDE, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l], {j, k, h, i, l}}],
 {j, 0, 3}, {k, j + 1, 4}, {h, k + 1, 5}, {i, h + 1, 6}, {l, i + 1, 7}];
Length[t16ABCDE]
Clear[t16ABCDEF];
t16ABCDEF = {};
Do[AppendTo[t16ABCDEF, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q], {j, k, h, i, l,
     q}}], {j, 0, 2}, {k, j + 1, 3}, {h, k + 1, 4}, {i, h + 1, 5}, {l, i + 1, 6}, {q, l + 1, 7}];
Length[t16ABCDEF]
Clear[t16ABCDEFG];
t16ABCDEFG = {};
Do[AppendTo[t16ABCDEFG,
   {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].T16^A[r], {j, k, h, i, l, q, r}}],
 {j, 0, 1}, {k, j + 1, 2}, {h, k + 1, 3}, {i, h + 1, 4}, {l, i + 1, 5}, {q, l + 1, 6}, {r, q + 1, 7}];
Length[t16ABCDEFG]
Clear[t16ABCDEFGH];
t16ABCDEFGH = {};
Do[AppendTo[t16ABCDEFGH, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].
     T16^A[q].T16^A[r].T16^A[s], {j, k, h, i, l, q, r, s}}], {j, 0, 1}, {k, j + 1, 1},
 {h, k + 1, 2}, {i, h + 1, 3}, {l, i + 1, 4}, {q, l + 1, 5}, {r, q + 1, 6}, {s, r + 1, 7}];
Length[t16ABCDEFGH]
% + %% + %%% + %%%% + %%%%% + %%%%%% + %%%%%%% + %%%%%%%% + %%%%%%%%%
```

Out[•]=

1

Out[ ]=
8

Out[ ]=
28

Out[ ]=
56

Out[ ]=
70

Out[ ]=
56

Out[ ]=
28

Out[ ]=
8

Out[ ]=
1

Out[ ]=
256

In[ ]:=
```
(*base16=Flatten[
  {t160,t16A,t16AB,t16ABC,t16ABCD,t16ABCDE,t16ABCDEF,t16ABCDEFG,t16ABCDEFGH},1];
Length[base16]*)
```

In[ ]:=
```
base16 = Flatten[
  {t16A, t16AB, t16ABC, t16ABCD, t16ABCDE, t16ABCDEF, t16ABCDEFG, t16ABCDEFGH, t160}, 1];
Length[base16]
```

Out[ ]=
256

In[ ]:=
```
Length[Union[base16[[All, 1]]]]
```

Out[ ]=
256

In[ ]:=
```
Tr[#.# / 16] & /@ base16[[All, 1]] // Flatten // Union
```

Out[ ]=
{-1, 1}

In[ ]:=
```
positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]
```

Out[ ]=
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44,
45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91,
92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121,
122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151,
152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185,
186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229,
230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[∘]:=
```
negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]
```
Out[∘]=
```
{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58,
 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97,
 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145,
 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182,
 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214,
 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}
```
In[∘]:=
```
{Length[positiveTrMM], Length[negativeTrMM]}
```
Out[∘]=
```
{136, 120}
```
In[∘]:=
```
dups = Block[{s}, s = base16[[All, 1]]; Select[s, MemberQ[s, -#] &]]; Length[dups]
```
Out[∘]=
```
0
```
In[∘]:=
```
dups = Select[base16, MemberQ[base16[[All, 1]], -#[[1]]] &]; Length[dups]
```
Out[∘]=
```
0
```
In[∘]:=
```
BASE16 = use16 = Complement[base16, dups]; Length[use16]
```
Out[∘]=
```
256
```
In[∘]:=
```
(*(T16^A[8]=T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7])//MatrixForm*)
```
In[∘]:=
```
MatrixForm[T16^A[#]] & /@ Range[0, 7]
```
Out[∘]=

$$\left\{\begin{pmatrix}
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0
\end{pmatrix},
\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix},\right.$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix},$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix},$$

$$\left.\begin{pmatrix}
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0
\end{pmatrix}\right\}$$

In[ ]:= (*BASE16[[-1]]*)

In[ ]:= (*BASE16[[-2]]
 %[[1]]==BASE16[[-1]][[1]]*)

In[ ]:=
```
16 * (16 + 1) / 2
16 * (16 - 1) / 2
% + %%
```

Out[ ]=
```
136
```

Out[ ]=
```
120
```

Out[ ]=
```
256
```

In[ ]:=
```
Clear[antisymm16];
antisymm16 = {};
Do[If[base16⟦k, 1⟧ === - Transpose[base16⟦k, 1⟧],
  AppendTo[antisymm16, {base16⟦k, 1⟧, {{k}, base16⟦k, 2⟧}}] ], {k, 1, Length[base16]}];
Length[antisymm16]
antisymm16⟦%⟧
```

Out[ ]=
```
120
```

Out[ ]=
```
{{{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
  {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}}, {{254}, {1, 2, 3, 4, 5, 6, 7}}}
```

In[ ]:=
```
Clear[symm16];
symm16 = {};
Do[If[base16[[k, 1]] === Transpose[base16[[k, 1]]],
  AppendTo[symm16, {base16[[k, 1]], {{k}, base16[[k, 2]]}}] ], {k, 1, Length[base16]}];
Length[symm16]
symm16[[%]]
```

Out[ ]=
```
136
```

Out[ ]=
```
{{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

In[ ]:=
```
dupssymm16 = Select[symm16, MemberQ[symm16[[All, 1]], -#[[1]]] &]; Length[dupssymm16]
```

Out[ ]=
```
0
```

In[ ]:= #[[2]] & /@ symm16

Out[ ]=

```
{{{1}, {0}}, {{2}, {1}}, {{3}, {2}}, {{4}, {3}}, {{12}, {0, 4}}, {{13}, {0, 5}},
 {{14}, {0, 6}}, {{15}, {0, 7}}, {{18}, {1, 4}}, {{19}, {1, 5}}, {{20}, {1, 6}},
 {{21}, {1, 7}}, {{23}, {2, 4}}, {{24}, {2, 5}}, {{25}, {2, 6}}, {{26}, {2, 7}},
 {{27}, {3, 4}}, {{28}, {3, 5}}, {{29}, {3, 6}}, {{30}, {3, 7}}, {{39}, {0, 1, 4}},
 {{40}, {0, 1, 5}}, {{41}, {0, 1, 6}}, {{42}, {0, 1, 7}}, {{44}, {0, 2, 4}}, {{45}, {0, 2, 5}},
 {{46}, {0, 2, 6}}, {{47}, {0, 2, 7}}, {{48}, {0, 3, 4}}, {{49}, {0, 3, 5}}, {{50}, {0, 3, 6}},
 {{51}, {0, 3, 7}}, {{59}, {1, 2, 4}}, {{60}, {1, 2, 5}}, {{61}, {1, 2, 6}}, {{62}, {1, 2, 7}},
 {{63}, {1, 3, 4}}, {{64}, {1, 3, 5}}, {{65}, {1, 3, 6}}, {{66}, {1, 3, 7}}, {{73}, {2, 3, 4}},
 {{74}, {2, 3, 5}}, {{75}, {2, 3, 6}}, {{76}, {2, 3, 7}}, {{89}, {4, 5, 6}}, {{90}, {4, 5, 7}},
 {{91}, {4, 6, 7}}, {{92}, {5, 6, 7}}, {{93}, {0, 1, 2, 3}}, {{102}, {0, 1, 4, 5}},
 {{103}, {0, 1, 4, 6}}, {{104}, {0, 1, 4, 7}}, {{105}, {0, 1, 5, 6}}, {{106}, {0, 1, 5, 7}},
 {{107}, {0, 1, 6, 7}}, {{112}, {0, 2, 4, 5}}, {{113}, {0, 2, 4, 6}}, {{114}, {0, 2, 4, 7}},
 {{115}, {0, 2, 5, 6}}, {{116}, {0, 2, 5, 7}}, {{117}, {0, 2, 6, 7}}, {{118}, {0, 3, 4, 5}},
 {{119}, {0, 3, 4, 6}}, {{120}, {0, 3, 4, 7}}, {{121}, {0, 3, 5, 6}}, {{122}, {0, 3, 5, 7}},
 {{123}, {0, 3, 6, 7}}, {{132}, {1, 2, 4, 5}}, {{133}, {1, 2, 4, 6}}, {{134}, {1, 2, 4, 7}},
 {{135}, {1, 2, 5, 6}}, {{136}, {1, 2, 5, 7}}, {{137}, {1, 2, 6, 7}}, {{138}, {1, 3, 4, 5}},
 {{139}, {1, 3, 4, 6}}, {{140}, {1, 3, 4, 7}}, {{141}, {1, 3, 5, 6}}, {{142}, {1, 3, 5, 7}},
 {{143}, {1, 3, 6, 7}}, {{148}, {2, 3, 4, 5}}, {{149}, {2, 3, 4, 6}}, {{150}, {2, 3, 4, 7}},
 {{151}, {2, 3, 5, 6}}, {{152}, {2, 3, 5, 7}}, {{153}, {2, 3, 6, 7}}, {{162}, {4, 5, 6, 7}},
 {{167}, {0, 1, 2, 4, 5}}, {{168}, {0, 1, 2, 4, 6}}, {{169}, {0, 1, 2, 4, 7}},
 {{170}, {0, 1, 2, 5, 6}}, {{171}, {0, 1, 2, 5, 7}}, {{172}, {0, 1, 2, 6, 7}},
 {{173}, {0, 1, 3, 4, 5}}, {{174}, {0, 1, 3, 4, 6}}, {{175}, {0, 1, 3, 4, 7}},
 {{176}, {0, 1, 3, 5, 6}}, {{177}, {0, 1, 3, 5, 7}}, {{178}, {0, 1, 3, 6, 7}},
 {{183}, {0, 2, 3, 4, 5}}, {{184}, {0, 2, 3, 4, 6}}, {{185}, {0, 2, 3, 4, 7}},
 {{186}, {0, 2, 3, 5, 6}}, {{187}, {0, 2, 3, 5, 7}}, {{188}, {0, 2, 3, 6, 7}},
 {{197}, {0, 4, 5, 6, 7}}, {{198}, {1, 2, 3, 4, 5}}, {{199}, {1, 2, 3, 4, 6}},
 {{200}, {1, 2, 3, 4, 7}}, {{201}, {1, 2, 3, 5, 6}}, {{202}, {1, 2, 3, 5, 7}},
 {{203}, {1, 2, 3, 6, 7}}, {{212}, {1, 4, 5, 6, 7}}, {{217}, {2, 4, 5, 6, 7}},
 {{218}, {3, 4, 5, 6, 7}}, {{225}, {0, 1, 2, 4, 5, 6}}, {{226}, {0, 1, 2, 4, 5, 7}},
 {{227}, {0, 1, 2, 4, 6, 7}}, {{228}, {0, 1, 2, 5, 6, 7}}, {{229}, {0, 1, 3, 4, 5, 6}},
 {{230}, {0, 1, 3, 4, 5, 7}}, {{231}, {0, 1, 3, 4, 6, 7}}, {{232}, {0, 1, 3, 5, 6, 7}},
 {{234}, {0, 2, 3, 4, 5, 6}}, {{235}, {0, 2, 3, 4, 5, 7}}, {{236}, {0, 2, 3, 4, 6, 7}},
 {{237}, {0, 2, 3, 5, 6, 7}}, {{240}, {1, 2, 3, 4, 5, 6}}, {{241}, {1, 2, 3, 4, 5, 7}},
 {{242}, {1, 2, 3, 4, 6, 7}}, {{243}, {1, 2, 3, 5, 6, 7}}, {{247}, {0, 1, 2, 3, 4, 5, 6}},
 {{248}, {0, 1, 2, 3, 4, 5, 7}}, {{249}, {0, 1, 2, 3, 4, 6, 7}}, {{250}, {0, 1, 2, 3, 5, 6, 7}},
 {{255}, {0, 1, 2, 3, 4, 5, 6, 7}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

In[ ]:= σ16 == $T16^{A}$[0].$T16^{A}$[1].$T16^{A}$[2].$T16^{A}$[3]

Out[ ]=

True

In[ ]:= σ16 == base16[[93]][[1]]

Out[ ]=

True

In[∘]:=
```
Select[symm16, #[[2]][[1]][[1]] == 93 & → "Index"]
```
Out[∘]=
```
{49}
```
In[∘]:=
```
σ16 == symm16[[49]][[1]]
```
Out[∘]=
```
True
```
In[∘]:=
```
symm16[[134]]
```
Out[∘]=
```
{{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{250}, {0, 1, 2, 3, 5, 6, 7}}}
```
In[∘]:=
```
Select[symm16, #[[2]][[1]][[1]] == 250 & → "Index"]
```
Out[∘]=
```
{134}
```
In[∘]:=
```
T16_Superscript_A
```
Out[∘]=
```
T16^A
```
In[∘]:=
```
σ16.T16^A[5].T16^A[6].T16^A[7] == symm16[[134]][[1]]
```
Out[∘]=
```
True
```
In[∘]:=
```
T16^A[8] == base16[[255]][[1]]
```
Out[∘]=
```
True
```
In[∘]:=
```
Select[symm16, #[[2]][[1]][[1]] == 255 & → "Index"]
```
Out[∘]=
```
{135}
```

- **similarly :**

In[∘]:=
```
coupled1 = {4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13} // Union
```
Out[∘]=
```
{0, 1, 4, 5, 8, 9, 12, 13}
```
In[∘]:=
```
coupled2 = Complement[Range[0, 15], coupled1]
```
Out[∘]=
```
{2, 3, 6, 7, 10, 11, 14, 15}
```

In[∘]:=
```
coupled = {coupled1, coupled2}
```
Out[∘]=
```
{{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}}
```
In[∘]:=
```
Block[{m, vars, eqs, eqs1, eqs2, s, Φ16},
 Φ16 = f[#] & /@ Range[16];
 m = Array[qb[#1, #2] &, {16, 16}];
 vars = Flatten[m];
 eqs1 = And @@ Thread[m.Φ16 == Flatten[{Φ16[[# + 1]] & /@ coupled1, Φ16[[# + 1]] & /@ coupled2}]];
 eqs2 = And @@ Thread[m.Transpose[m] == Flatten[ID16]];
 eqs = eqs1 && eqs2;
 s = Solve[eqs, vars] // FullSimplify;
 s
]
```
Out[∘]=
```
{}
```
In[∘]:=
```
linTrans = Block[{m, b, vars, eqs, eqs1, eqs2, s, Φ16, t},
  Φ16 = f[#] & /@ Range[16];
  m = Array[qb[#1, #2] &, {16, 16}];
  b = Flatten[{Φ16[[# + 1]] & /@ coupled1, Φ16[[# + 1]] & /@ coupled2}];
  t = Table[Block[{sv, eq, d}, d = D[ m[[j]].b, Φ16[[j]]];
     sv = Flatten[{d → 1, Thread[Complement[m[[j]], {d}] → 0]}];
     (*Print[{m[[j]].b},{Φ16[[j]]},{sv}];*)
     (*eq=Thread[m[[j]].b==Φ16[[j]]];
     sv=Solve[eq,Flatten[m[[j]]]]//FullSimplify;
     sv*)(*{v.b,Φ16[[j]]}*)sv], {j, 1, 16}];
  (*
  vars=Flatten[m];
  eqs1=And@@Thread[m.Φ16==Flatten[{Φ16[[#+1]]&/@coupled1,Φ16[[#+1]]&/@coupled2}]];
  eqs2=And@@Thread[m.Transpose[m]==Flatten[ID16]];
  eqs=eqs1&&eqs2;
  s=Solve[eqs,vars]//FullSimplify;*)
  m /. Flatten[t]
 ]
```
Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[ ]:= `linTrans // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

In[ ]:= `linTrans.Transpose[linTrans] // FullSimplify // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

In[∘]:=
```
linTrans.Ψ16 // Column
Union[%[[1]]]
```

Out[∘]=
```
f16[0][x0, x4]
f16[1][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[2][x0, x4]
f16[3][x0, x4]
f16[10][x0, x4]
f16[11][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
f16[6][x0, x4]
f16[7][x0, x4]
f16[14][x0, x4]
f16[15][x0, x4]
```

Out[∘]=
```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

In[∘]:=
```
positiveTrMM = Select[base16, Tr[#[[1]] . #[[1]]] > 0 & -> "Index"]
```

Out[∘]=
```
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29,
30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63,
64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105,
106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122,
123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143,
148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172,
173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197,
198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228,
229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247,
248, 249, 250, 255, 256}
```

In[∘]:=
```
negativeTrMM = Select[base16, Tr[#[[1]] . #[[1]]] < 0 & -> "Index"]
```

Out[∘]=
```
{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43,
52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81,
```

82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[ ]:= {Length[positiveTrMM], Length[negativeTrMM]}

Out[ ]=

{136, 120}

```
In[ ]:= positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]
```

Out[ ]=

{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

```
In[ ]:= negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]
```

Out[ ]=

{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

```
In[ ]:= {Length[positiveTrMM], Length[negativeTrMM]}
```

Out[ ]=

{136, 120}

In[∘]:=
```
base16[[#]] & /@ positiveTrMM;
{#[[2]], Tr[Transpose[#[[1]]].linTrans]} & /@ %
Select[%, #[[2]] ≠ 0 &]
linTransSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];
```

Out[∘]=
```
{{{0}, 0}, {{1}, 0}, {{2}, 0}, {{3}, 0}, {{0, 4}, 0}, {{0, 5}, 0}, {{0, 6}, 0}, {{0, 7}, -4},
 {{1, 4}, 0}, {{1, 5}, 0}, {{1, 6}, 0}, {{1, 7}, 0}, {{2, 4}, 0}, {{2, 5}, 0}, {{2, 6}, 0},
 {{2, 7}, 0}, {{3, 4}, -4}, {{3, 5}, 0}, {{3, 6}, 0}, {{3, 7}, 0}, {{0, 1, 4}, 0},
 {{0, 1, 5}, 0}, {{0, 1, 6}, 0}, {{0, 1, 7}, 0}, {{0, 2, 4}, 0}, {{0, 2, 5}, 0}, {{0, 2, 6}, 0},
 {{0, 2, 7}, 0}, {{0, 3, 4}, 0}, {{0, 3, 5}, 0}, {{0, 3, 6}, 0}, {{0, 3, 7}, 0},
 {{1, 2, 4}, 0}, {{1, 2, 5}, 0}, {{1, 2, 6}, 0}, {{1, 2, 7}, 0}, {{1, 3, 4}, 0},
 {{1, 3, 5}, 0}, {{1, 3, 6}, 0}, {{1, 3, 7}, 0}, {{2, 3, 4}, 0}, {{2, 3, 5}, 0},
 {{2, 3, 6}, 0}, {{2, 3, 7}, 0}, {{4, 5, 6}, -4}, {{4, 5, 7}, 0}, {{4, 6, 7}, 0},
 {{5, 6, 7}, 0}, {{0, 1, 2, 3}, 0}, {{0, 1, 4, 5}, 0}, {{0, 1, 4, 6}, 0}, {{0, 1, 4, 7}, 0},
 {{0, 1, 5, 6}, 0}, {{0, 1, 5, 7}, 0}, {{0, 1, 6, 7}, 0}, {{0, 2, 4, 5}, 0}, {{0, 2, 4, 6}, 0},
 {{0, 2, 4, 7}, 0}, {{0, 2, 5, 6}, 0}, {{0, 2, 5, 7}, 0}, {{0, 2, 6, 7}, 0}, {{0, 3, 4, 5}, 0},
 {{0, 3, 4, 6}, 0}, {{0, 3, 4, 7}, 4}, {{0, 3, 5, 6}, 0}, {{0, 3, 5, 7}, 0}, {{0, 3, 6, 7}, 0},
 {{1, 2, 4, 5}, 0}, {{1, 2, 4, 6}, 0}, {{1, 2, 4, 7}, 0}, {{1, 2, 5, 6}, 0}, {{1, 2, 5, 7}, 0},
 {{1, 2, 6, 7}, -4}, {{1, 3, 4, 5}, 0}, {{1, 3, 4, 6}, 0}, {{1, 3, 4, 7}, 0}, {{1, 3, 5, 6}, 0},
 {{1, 3, 5, 7}, 0}, {{1, 3, 6, 7}, 0}, {{2, 3, 4, 5}, 0}, {{2, 3, 4, 6}, 0}, {{2, 3, 4, 7}, 0},
 {{2, 3, 5, 6}, 0}, {{2, 3, 5, 7}, 0}, {{2, 3, 6, 7}, 0}, {{4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5}, 4},
 {{0, 1, 2, 4, 6}, 0}, {{0, 1, 2, 4, 7}, 0}, {{0, 1, 2, 5, 6}, 0}, {{0, 1, 2, 5, 7}, 0},
 {{0, 1, 2, 6, 7}, 0}, {{0, 1, 3, 4, 5}, 0}, {{0, 1, 3, 4, 6}, 0}, {{0, 1, 3, 4, 7}, 0},
 {{0, 1, 3, 5, 6}, 0}, {{0, 1, 3, 5, 7}, 0}, {{0, 1, 3, 6, 7}, 0}, {{0, 2, 3, 4, 5}, 0},
 {{0, 2, 3, 4, 6}, 0}, {{0, 2, 3, 4, 7}, 0}, {{0, 2, 3, 5, 6}, 0}, {{0, 2, 3, 5, 7}, 0},
 {{0, 2, 3, 6, 7}, 0}, {{0, 4, 5, 6, 7}, 4}, {{1, 2, 3, 4, 5}, 0}, {{1, 2, 3, 4, 6}, 0},
 {{1, 2, 3, 4, 7}, 0}, {{1, 2, 3, 5, 6}, 0}, {{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 6, 7}, 0},
 {{1, 4, 5, 6, 7}, 0}, {{2, 4, 5, 6, 7}, 0}, {{3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6}, 0},
 {{0, 1, 2, 4, 5, 7}, 0}, {{0, 1, 2, 4, 6, 7}, 0}, {{0, 1, 2, 5, 6, 7}, 0},
 {{0, 1, 3, 4, 5, 6}, 0}, {{0, 1, 3, 4, 5, 7}, 0}, {{0, 1, 3, 4, 6, 7}, 0},
 {{0, 1, 3, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6}, 0}, {{0, 2, 3, 4, 5, 7}, 0},
 {{0, 2, 3, 4, 6, 7}, 0}, {{0, 2, 3, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6}, 0},
 {{1, 2, 3, 4, 5, 7}, 0}, {{1, 2, 3, 4, 6, 7}, -4}, {{1, 2, 3, 5, 6, 7}, 0},
 {{0, 1, 2, 3, 4, 5, 6}, 0}, {{0, 1, 2, 3, 4, 5, 7}, 0}, {{0, 1, 2, 3, 4, 6, 7}, 0},
 {{0, 1, 2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6, 7}, 0}, {{{0, 1, 2, 3, 4, 5, 6, 7, 8}}, 4}}
```

Out[∘]=
```
{{{0, 7}, -4}, {{3, 4}, -4}, {{4, 5, 6}, -4}, {{0, 3, 4, 7}, 4},
 {{1, 2, 6, 7}, -4}, {{0, 1, 2, 4, 5}, 4}, {{0, 4, 5, 6, 7}, 4},
 {{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 4, 6, 7}, -4}, {{{0, 1, 2, 3, 4, 5, 6, 7, 8}}, 4}}
```

In[ ]:= linTransSymmetric⟦1⟧

Out[ ]=

```
{{{{-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0,
    0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}, {0, 7}}}
```

In[ ]:=
```
base16[[#]] & /@ negativeTrMM;
{#[[2]], Tr[Transpose[#[[1]]].linTrans]} & /@ %
Select[%, #[[2]] ≠ 0 &]
linTransAntiSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];
```

Out[ ]=
```
{{{4}, 0}, {{5}, 0}, {{6}, 0}, {{7}, 0}, {{0, 1}, 0}, {{0, 2}, 0}, {{0, 3}, 0}, {{1, 2}, 0},
 {{1, 3}, 0}, {{2, 3}, 0}, {{4, 5}, 0}, {{4, 6}, 0}, {{4, 7}, 0}, {{5, 6}, 0}, {{5, 7}, 0},
 {{6, 7}, 0}, {{0, 1, 2}, 0}, {{0, 1, 3}, 0}, {{0, 2, 3}, 0}, {{0, 4, 5}, 0}, {{0, 4, 6}, 0},
 {{0, 4, 7}, 0}, {{0, 5, 6}, 0}, {{0, 5, 7}, 0}, {{0, 6, 7}, 0}, {{1, 2, 3}, 0}, {{1, 4, 5}, 0},
 {{1, 4, 6}, 0}, {{1, 4, 7}, 0}, {{1, 5, 6}, 0}, {{1, 5, 7}, 0}, {{1, 6, 7}, 0},
 {{2, 4, 5}, 0}, {{2, 4, 6}, 0}, {{2, 4, 7}, 0}, {{2, 5, 6}, 0}, {{2, 5, 7}, 0},
 {{2, 6, 7}, 0}, {{3, 4, 5}, 0}, {{3, 4, 6}, 0}, {{3, 4, 7}, 0}, {{3, 5, 6}, -4},
 {{3, 5, 7}, 0}, {{3, 6, 7}, 0}, {{0, 1, 2, 4}, 0}, {{0, 1, 2, 5}, 0}, {{0, 1, 2, 6}, -4},
 {{0, 1, 2, 7}, 0}, {{0, 1, 3, 4}, 0}, {{0, 1, 3, 5}, 0}, {{0, 1, 3, 6}, 0}, {{0, 1, 3, 7}, 0},
 {{0, 2, 3, 4}, 0}, {{0, 2, 3, 5}, 0}, {{0, 2, 3, 6}, 0}, {{0, 2, 3, 7}, 0}, {{0, 4, 5, 6}, 0},
 {{0, 4, 5, 7}, 0}, {{0, 4, 6, 7}, 0}, {{0, 5, 6, 7}, 0}, {{1, 2, 3, 4}, 0}, {{1, 2, 3, 5}, 0},
 {{1, 2, 3, 6}, 0}, {{1, 2, 3, 7}, 0}, {{1, 4, 5, 6}, 0}, {{1, 4, 5, 7}, 0}, {{1, 4, 6, 7}, 0},
 {{1, 5, 6, 7}, 0}, {{2, 4, 5, 6}, 0}, {{2, 4, 5, 7}, 0}, {{2, 4, 6, 7}, 0}, {{2, 5, 6, 7}, 0},
 {{3, 4, 5, 6}, 0}, {{3, 4, 5, 7}, 0}, {{3, 4, 6, 7}, 0}, {{3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4}, 0},
 {{0, 1, 2, 3, 5}, -4}, {{0, 1, 2, 3, 6}, 0}, {{0, 1, 2, 3, 7}, 0}, {{0, 1, 4, 5, 6}, 0},
 {{0, 1, 4, 5, 7}, 0}, {{0, 1, 4, 6, 7}, 0}, {{0, 1, 5, 6, 7}, 0}, {{0, 2, 4, 5, 6}, 0},
 {{0, 2, 4, 5, 7}, 0}, {{0, 2, 4, 6, 7}, 0}, {{0, 2, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6}, 0},
 {{0, 3, 4, 5, 7}, 0}, {{0, 3, 4, 6, 7}, 0}, {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 6}, 0},
 {{1, 2, 4, 5, 7}, 4}, {{1, 2, 4, 6, 7}, 0}, {{1, 2, 5, 6, 7}, 0}, {{1, 3, 4, 5, 6}, 0},
 {{1, 3, 4, 5, 7}, 0}, {{1, 3, 4, 6, 7}, 0}, {{1, 3, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6}, 0},
 {{2, 3, 4, 5, 7}, 0}, {{2, 3, 4, 6, 7}, 0}, {{2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5}, 0},
 {{0, 1, 2, 3, 4, 6}, -4}, {{0, 1, 2, 3, 4, 7}, 0}, {{0, 1, 2, 3, 5, 6}, 0},
 {{0, 1, 2, 3, 5, 7}, 0}, {{0, 1, 2, 3, 6, 7}, 0}, {{0, 1, 4, 5, 6, 7}, 0},
 {{0, 2, 4, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6, 7}, 0}, {{1, 2, 4, 5, 6, 7}, 0},
 {{1, 3, 4, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6, 7}, 0},
 {{0, 1, 3, 4, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6, 7}, 0}}
```

Out[ ]=
```
{{{3, 5, 6}, -4}, {{0, 1, 2, 6}, -4}, {{0, 1, 2, 3, 5}, -4},
 {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 7}, 4}, {{0, 1, 2, 3, 4, 6}, -4}}
```

In[ ]:=
```
{Length[linTransSymmetric], Length[linTransAntiSymmetric]}
```

Out[ ]=
```
{10, 6}
```

BASIS of 8 × 8 matrices :

In[∘]:=
```
tA = {}; Do[AppendTo[tA, {τ[j], {j}}], {j, 1, 7}]
Length[tA]
tAB = {}; Do[AppendTo[tAB, {τ[j].τ[k], {j, k}}], {j, 1, 6}, {k, j + 1, 7}]
Length[tAB]
tABC = {};
Do[AppendTo[tABC, {τ[j].τ[k].τ[h], {j, k, h}}], {j, 1, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[tABC]
```

Out[∘]=
```
7
```

Out[∘]=
```
21
```

Out[∘]=
```
35
```

In[∘]:=
```
tA⟦1⟧
```

Out[∘]=
```
{{{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}}, {1}}
```

In[∘]:=
```
tA⟦#⟧⟦1⟧ == -Transpose[tA⟦#⟧⟦1⟧] & /@ Range[Length[tA]]
```

Out[∘]=
```
{True, True, True, False, False, False, False}
```

In[∘]:=
```
bas64 = Flatten[{tA, tAB, tABC, {{Id, {64}}}}, 1]; Length[bas64]
```

Out[∘]=
```
64
```

In[∘]:=
```
determineIF8AntiSymmetric[m_] := Module[{t, tt, sum, res, r},
  t = m.τ[#] & /@ Range[0, 7];
  tt = Transpose[#] & /@ t;
  sum = FullSimplify[t⟦#⟧ + tt⟦#⟧] & /@ Range[Length[t]];
  res = Union[Flatten[#]] & /@ sum;
  If[Length[#] > 1, Return[False]] & /@ res;
  r = Union[Flatten[res]];
  Return[r == {0}];
 ]
```

In[∘]:=
```
determineIF8AntiSymmetric[Zero8]
```

Out[∘]=
```
True
```

In[∘]:=
```
determineIF8AntiSymmetric[ID8]
```

Out[∘]=
```
False
```

In[∘]:=
```
find8AntiSymmetric = If[determineIF8AntiSymmetric[#⟦1⟧], #⟦2⟧, ## &] & /@ bas64
```

Out[∘]=
```
{##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &,
 ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &,
 ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &,
 ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &,
 ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &}
```

In[∘]:=
```
anti = {};
Do[If[bas64⟦k, 1⟧ == -Transpose[bas64⟦k, 1⟧],
  AppendTo[anti, {bas64⟦k, 1⟧, {{k}, bas64⟦k, 2⟧}}] ], {k, 1, 64}];
Length[anti]
anti⟦28⟧
```

Out[∘]=
```
28
```

Out[∘]=
```
{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}
```

In[∘]:=
```
symm = {};
Do[If[bas64⟦k, 1⟧ == Transpose[bas64⟦k, 1⟧],
  AppendTo[symm, {bas64⟦k, 1⟧, {{k}, bas64⟦k, 2⟧}}] ], {k, 1, 64}];
Length[symm]
symm⟦%⟧
```

Out[∘]=
```
35
```

Out[∘]=
```
{{{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0},
  {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}}, {{59}, {3, 6, 7}}}
```

In[∘]:=
```
η64 = DiagonalMatrix[
  Table[FullSimplify[1/8 Tr[bas64⟦A, 1⟧.bas64⟦A, 1⟧]], {A, 64}]];
Tr[η64]
```

Out[∘]=

$$7 + \frac{1}{8}\,\mathrm{Tr}[\mathrm{Id}.\mathrm{Id}]$$

In[∘]:=
```
(*(*(*(*(*(*(countTrace=
  Table[{A,B,FullSimplify[1/8 Tr[bas64⟦A,1⟧.bas64⟦B,1⟧]] },
   {A,1, 64},{B,A, 64}])//MatrixForm*)*)*)*)*)*)
```

In[∘]:=
```
(*■*)
```

In[ ]:= 32 × 65

Out[ ]= 2080

In[ ]:= (*Length[Flatten[countTrace,1]]*)

In[ ]:= (*{Length[countTrace],Length[countTrace⟦1⟧],Length[countTrace⟦1,1⟧]}*)

In[ ]:= (*{Length[countTrace],Length[countTrace⟦2⟧],Length[countTrace⟦2,1⟧]}*)

In[ ]:= (*{Length[countTrace],Length[countTrace⟦64⟧],Length[countTrace⟦64,1⟧]}*)

In[ ]:=
```
(*{Length[Flatten[countTrace,1]⟦All,3⟧],Count[Flatten[countTrace,1]⟦All,3⟧,0],
 Count[Flatten[countTrace,1]⟦All,3⟧,1],Count[Flatten[countTrace,1]⟦All,3⟧,-1]}*)
```

In[ ]:= 32 × 63

Out[ ]= 2016

In[ ]:=
```
anti = {};
Do[If[bas64⟦k, 1⟧ == -Transpose[bas64⟦k, 1⟧],
  AppendTo[anti, {bas64⟦k, 1⟧, {{k}, bas64⟦k, 2⟧}}] ], {k, 1, 64}];
Length[anti]
anti⟦28⟧
```

Out[ ]= 28

Out[ ]=
```
{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}
```

In[ ]:= anti⟦28, 1⟧

Out[ ]=
```
{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}
```

In[ ]:= anti⟦28, 2⟧

Out[ ]= {{63}, {5, 6, 7}}

In[ ]:=
```
symm = {};
Do[If[bas64⟦k, 1⟧ == Transpose[bas64⟦k, 1⟧],
  AppendTo[symm, {bas64⟦k, 1⟧, {{k}, bas64⟦k, 2⟧}}] ], {k, 1, 64}];
Length[symm]
symm⟦28⟧
```

Out[ ]= 35

Out[ ]=
```
{{{-1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, {{52}, {2, 5, 7}}}
```

Spinor Lagrangian

In[∘]:=
```
usegT16 = Table[FullSimplify[(T16^α[α1 - 1]), constraintVars], {α1, 1, Length[X]}];
```

In[∘]:=
$$\text{useT16} = \text{Table}\Big[\text{FullSimplify}[(\text{T16}^{\alpha}[\alpha1 - 1] \;/.\; \text{ssgm4488}), \text{constraintVars}] \;/.\; \Big\{\sqrt{e^{-2\,a4[H\,x4]}} \to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}} \to e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}}} \to \frac{1}{e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6}}\Big\}, \{\alpha1, 1, \text{Length}[X]\}\Big] \;/.\; \Big\{\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}}} \to \frac{1}{e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}} \to e^{a4[H\,x4]}\Big\};$$

In[∘]:=
$$\text{newuseT16} = \text{Table}\Big[\text{FullSimplify}[(\text{newT16}^{\alpha}[\alpha1 - 1] \;/.\; \text{ssgm4488}), \text{constraintVars}] \;/.\; \Big\{\sqrt{e^{-2\,a4[H\,x4]}} \to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}} \to e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}}} \to \frac{1}{e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6}}\Big\}, \{\alpha1, 1, \text{Length}[X]\}\Big] \;/.\; \Big\{\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}}} \to \frac{1}{e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}} \to e^{a4[H\,x4]}\Big\};$$

In[∘]:=
```
Dimensions[useT16]
```

Out[∘]=
```
{8, 16, 16}
```

In[∘]:=
```
Det[MatrixMetric44]
```

Out[∘]=
$$\text{Cos}[6\,H\,x0]^2$$

In[∘]:=
```
useDSQRT = Cos[6 H x0]
```

Out[∘]=
```
Cos[6 H x0]
```

In[◦]:=
```
ωmat /. sg /. ssgm4488 /. subsDefects;
(% /. {Sqrt[e^(2 a4[H x4])] → e^a4[H x4], Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) → e^a4[H x4] H Sin[6 H x0]^(1/6)}) -
    spinCoeffs // FullSimplify[#, constraintVars] & /. subsDefects /.
    {Sqrt[e^(2 a4[H x4])] → e^a4[H x4], Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) → e^a4[H x4] H Sin[6 H x0]^(1/6)} //
  FullSimplify[#, constraintVars] & /. subsDefects;
% // Flatten // Union
% // ExpandAll /. subsDefects /.
   {Sqrt[e^(2 a4[H x4])] → e^a4[H x4], Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) → e^a4[H x4] H Sin[6 H x0]^(1/6)} /. subsDefects
% /. {a4[H x4] → π^4, Sin[6 H x0] → e^12}
```

Out[◦]=
```
{0, e^-a4[H x4] (-1 + e^a4[H x4] Sqrt[e^(-2 a4[H x4])]) H Sin[6 H x0]^(1/6),
 -((-e^a4[H x4] + Sqrt[e^(2 a4[H x4])]) H Sin[6 H x0]^(1/6)), (-e^a4[H x4] + Sqrt[e^(2 a4[H x4])]) H Sin[6 H x0]^(1/6),
 e^-a4[H x4] (1 - e^a4[H x4] Sqrt[e^(-2 a4[H x4])]) H Sin[6 H x0]^(1/6) a4′[H x4],
 e^-a4[H x4] (-1 + e^a4[H x4] Sqrt[e^(-2 a4[H x4])]) H Sin[6 H x0]^(1/6) a4′[H x4],
 (-e^a4[H x4] + Sqrt[e^(2 a4[H x4])]) H Sin[6 H x0]^(1/6) a4′[H x4]}
```

Out[◦]=
```
{0, -e^-a4[H x4] H Sin[6 H x0]^(1/6) + Sqrt[e^(-2 a4[H x4])] H Sin[6 H x0]^(1/6),
 e^a4[H x4] H Sin[6 H x0]^(1/6) - Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6),
 -e^a4[H x4] H Sin[6 H x0]^(1/6) + Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6),
 e^-a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4] - Sqrt[e^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4′[H x4],
 -e^-a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4] + Sqrt[e^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4′[H x4],
 -e^a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4] + Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4′[H x4]}
```

Out[◦]=
```
{0, 0, 0, 0, 0, 0, 0}
```

In[◦]:=
```
Q2 = 0; Protect[Q1, Q2]
```

Out[◦]=
```
{Q1, Q2}
```

In[◦]:=
```
(matTest = (Q1 * ID16)) // MatrixForm
```

Out[◦]//MatrixForm=

$$\begin{pmatrix}
Q1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&Q1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&Q1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&Q1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&Q1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&Q1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&Q1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&Q1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&Q1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&Q1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&Q1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&Q1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&Q1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&Q1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&Q1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&Q1
\end{pmatrix}$$

In[ ]:=
```
(*Protect[Q1,Q2]*)
```

In[ ]:=
```
(*(matTest=(Q1*ID16 +Q2* covariantDiffMatrix))//MatrixForm*)
```

In[ ]:=
```
(newmatTest = O8similarityTransformation.(Q1 * ID16 + Q2 * covariantDiffMatrix).
   Transpose[O8similarityTransformation]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1
\end{pmatrix}$$

In[ ]:=
```
Clear[Lg];
Lg[] :=
  √detgg * (Transpose[Ψ16].σ16.Sum[FullSimplify[(T16^α[α1 - 1] /. sg), constraintVars].
        (D[ Ψ16, X[[α1]]] + (K/2 /. {K → 1}) * Sum[ωmat[[α1, a, b]] * SAB[[a, b]].matTest.Ψ16,
            {a, 1, 8}, {b, 1, 8}]), {α1, 1, Length[X]}] +
      (H * M) * Transpose[Ψ16].σ16.Ψ16) // Simplify[#, constraintVars] &
```

In[ ]:=
```
Clear[newLg];
newLg[] := √detgg *
    (Transpose[Ψ16].newσ16.Sum[FullSimplify[(newT16^α[α1 - 1] /. sg), constraintVars].
        (D[ Ψ16, X[[α1]]] + (K/2 /. {K → 1}) * Sum[ωmat[[α1, a, b]] * O8similarityTransformation.
              SAB[[a, b]].Transpose[O8similarityTransformation].newmatTest.Ψ16,
            {a, 1, 8}, {b, 1, 8}]), {α1, 1, Length[X]}] +
      (H * M) * Transpose[Ψ16].newσ16.Ψ16) // Simplify[#, constraintVars] &
```

In[∘]:=
```
Block[{A1 = 1, B1 = 2}, O8similarityTransformation.
   SAB⟦A1, B1⟧.Transpose[O8similarityTransformation] // MatrixForm]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

In[∘]:=
```
Block[{A1 = 1, B1 = 2, m1, ms, m, j1, s1},
  m1 = ArrayFlatten[{{0, ID8}, {ID8, 0}}]; Print[m1 // MatrixForm];
  {s1, j1} = JordanDecomposition[m1];
  Print[m1 == s1.j1.Inverse[s1]];
  ms = √j1;
  m = s1.ms.Inverse[s1];
  Print[m // MatrixForm];
  Print[√m1 // MatrixForm];
  m.O8similarityTransformation.SAB⟦A1, B1⟧.
    Transpose[m.O8similarityTransformation] // MatrixForm]
```

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

True

$$\begin{pmatrix}
\frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} \\
\frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2}
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Out[∘]//MatrixForm=

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&\frac{i}{2}&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&\frac{i}{2}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&\frac{i}{2}&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\frac{i}{2}&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{i}{2}\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{i}{2}&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{i}{2}&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&-\frac{i}{2}&0&0&0\\
0&0&0&-\frac{i}{2}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-\frac{i}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-\frac{i}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
-\frac{i}{2}&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&\frac{i}{2}&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&\frac{i}{2}&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&\frac{i}{2}&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&\frac{i}{2}&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

In[∘]:=
```
ωμIJ[1]
```

Out[∘]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[∘]:=
```
Clear[La];
La[] :=
 useDSQRT * (Transpose[Ψ16].σ16.Sum[useT16[[α1]].(D[ Ψ16, X[[α1]]] + (K/2 /. {K → 1}) * Sum[
         ωμIJ[α1][[A1, B1]] * SAB[[A1, B1]], {A1, 1, 8}, {B1, 1, 8}].matTest.Ψ16), {α1, 1,
       Length[X]}] + (H * M) * Transpose[Ψ16].σ16.Ψ16) // Simplify[#, constraintVars] &
```

NOT O8similarityTransformation . Ψ16 :

In[∘]:=
```
Clear[newLa];
newLa[] :=
 useDSQRT * (Transpose[Ψ16].newσ16.Sum[newuseT16[[α1]].(D[ Ψ16, X[[α1]]] + (K/2 /. {K → 1}) *
          Sum[ωμIJ[α1][[A1, B1]] * (O8similarityTransformation.
               SAB[[A1, B1]].Transpose[O8similarityTransformation]),
           {A1, 1, 8}, {B1, 1, 8}].newmatTest.Ψ16), {α1, 1, Length[X]}] +
      (H * M) * Transpose[Ψ16].newσ16.Ψ16) // Simplify[#, constraintVars] &
```

```
In[*]:= (*testLa=La[]*)

In[*]:= (*testLg=Lg[];*)

In[*]:= (*D[testLa,a4[H x4]]//Flatten//Union*)
        (*D[testLa,a4′[H x4]]//(*Full*)Simplify[#,constraintVars]&//Flatten//Union*)

In[*]:= (*testLgm=testL/.sg/.ssgm4488//(*Full*)Simplify[#,constraintVars]&/.subsDefects*)

In[*]:= (*D[testLgm,a4[H x4]]
         D[testLgm,a4′[H x4]]//(*Full*)Simplify[#,constraintVars]&*)

In[*]:= Clear[Lj];
        Lj[j_] := useDSQRT * (((
              Transpose[Ψ16].σ16.useT16[[1]].D[ Ψ16, X[[1]]] +
                Transpose[Ψ16].σ16.useT16[[5]].D[ Ψ16, X[[5]]] +
                (K0/2) * Sum[(*useω[[1,A1,B1]]*)ωμIJ[1][[A1, B1]] × Transpose[Ψ16].σ16.
                    useT16[[1]].SAB[[A1, B1]].(base16[[j, 1]]).Ψ16, {A1, 1, 8}, {B1, 1, 8}]) +
             Transpose[Ψ16].σ16.useT16[[5]].D[ Ψ16, X[[5]]] + (K4/2) * Sum[(*useω[[5,A1,B1]]*)
               ωμIJ[5][[A1, B1]] × Transpose[Ψ16].σ16.useT16[[5]].SAB[[A1, B1]].(base16[[j, 1]]).Ψ16,
               {A1, 1, 8}, {B1, 1, 8}]) +
           (H * M) * Transpose[Ψ16].σ16.Ψ16) // Simplify[#, constraintVars] &
```

In[•]:= **Lj[92]**

Out[•]=

```
Cos[6 H x0] (-2 H M f16[2][x0, x4] × f16[6][x0, x4] -
   2 H M f16[3][x0, x4] × f16[7][x0, x4] + 2 H M f16[8][x0, x4] × f16[12][x0, x4] +
   2 H M f16[9][x0, x4] × f16[13][x0, x4] + 2 H M f16[10][x0, x4] × f16[14][x0, x4] +
   2 H M f16[11][x0, x4] × f16[15][x0, x4] + 2 f16[9][x0, x4] f16[0]^(0,1)[x0, x4] -
   2 f16[8][x0, x4] f16[1]^(0,1)[x0, x4] - 2 f16[11][x0, x4] f16[2]^(0,1)[x0, x4] +
   2 f16[10][x0, x4] f16[3]^(0,1)[x0, x4] - 2 f16[13][x0, x4] f16[4]^(0,1)[x0, x4] +
   2 f16[12][x0, x4] f16[5]^(0,1)[x0, x4] + 2 f16[15][x0, x4] f16[6]^(0,1)[x0, x4] -
   2 f16[14][x0, x4] f16[7]^(0,1)[x0, x4] - 2 f16[3][x0, x4] f16[10]^(0,1)[x0, x4] +
   2 f16[2][x0, x4] f16[11]^(0,1)[x0, x4] - 2 f16[5][x0, x4] f16[12]^(0,1)[x0, x4] +
   2 f16[4][x0, x4] f16[13]^(0,1)[x0, x4] + 2 f16[7][x0, x4] f16[14]^(0,1)[x0, x4] -
   2 f16[6][x0, x4] f16[15]^(0,1)[x0, x4] + Tan[6 H x0] f16[12][x0, x4] f16[0]^(1,0)[x0, x4] +
   Tan[6 H x0] f16[13][x0, x4] f16[1]^(1,0)[x0, x4] +
   Tan[6 H x0] f16[14][x0, x4] f16[2]^(1,0)[x0, x4] + Tan[6 H x0] f16[15][x0, x4]
    f16[3]^(1,0)[x0, x4] + Tan[6 H x0] f16[8][x0, x4] f16[4]^(1,0)[x0, x4] +
   Tan[6 H x0] f16[9][x0, x4] f16[5]^(1,0)[x0, x4] + Tan[6 H x0] f16[10][x0, x4]
    f16[6]^(1,0)[x0, x4] + Tan[6 H x0] f16[11][x0, x4] f16[7]^(1,0)[x0, x4] -
   Tan[6 H x0] f16[4][x0, x4] f16[8]^(1,0)[x0, x4] - Tan[6 H x0] f16[5][x0, x4]
    f16[9]^(1,0)[x0, x4] - Tan[6 H x0] f16[6][x0, x4] f16[10]^(1,0)[x0, x4] -
   Tan[6 H x0] f16[7][x0, x4] f16[11]^(1,0)[x0, x4] - f16[0][x0, x4]
    (2 H M f16[4][x0, x4] + 2 f16[9]^(0,1)[x0, x4] + Tan[6 H x0] f16[12]^(1,0)[x0, x4]) - f16[1][x0,
     x4] (2 H M f16[5][x0, x4] - 2 f16[8]^(0,1)[x0, x4] + Tan[6 H x0] f16[13]^(1,0)[x0, x4]) -
   Tan[6 H x0] f16[2][x0, x4] f16[14]^(1,0)[x0, x4] -
   Tan[6 H x0] f16[3][x0, x4] f16[15]^(1,0)[x0, x4])
```

In[•]:= **detgg**

Out[•]=

```
g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
 g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]
```

In[•]:=

```
Clear[eL];
eL[Lagrangian_Symbol, detsqrt_] := Module[{L, t},
  L = Lagrangian[];
  t = Table[FullSimplify[1/detsqrt (D[L, f16[k][x0, x4]] - D[D[L, f16[k]^(1,0)[x0, x4]], x0] -
        D[D[L, f16[k]^(0,1)[x0, x4]], x4]), constraintVars], {k, 0, 15}];
  Return[t /. subsDefects];
 ]
```

eLa

In[•]:= **(*Get["2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLa.mx"];*)**

In[•]:= **eLa = eL[La, useDSQRT];**

```
In[∘]:= eLa

Out[∘]=
{-2 e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[9][x0, x4] a4′[H x4] -
   2 (H M f16[4][x0, x4] + 3 H f16[12][x0, x4] +
      f16[9]^(0,1)[x0, x4] + Tan[6 H x0] f16[12]^(1,0)[x0, x4]),
  2 (-H M f16[5][x0, x4] - 3 H f16[13][x0, x4] + e^(-a4[H x4]) H Q1 Sinh[a4[H x4]]
      f16[8][x0, x4] a4′[H x4] + f16[8]^(0,1)[x0, x4] - Tan[6 H x0] f16[13]^(1,0)[x0, x4]),
  -2 (H M f16[6][x0, x4] + 3 H f16[14][x0, x4] - f16[11]^(0,1)[x0, x4] +
     Tan[6 H x0] f16[14]^(1,0)[x0, x4]),
  -2 (H M f16[7][x0, x4] + 3 H f16[15][x0, x4] + f16[10]^(0,1)[x0, x4] +
     Tan[6 H x0] f16[15]^(1,0)[x0, x4]), -2 H M f16[0][x0, x4] -
   6 H f16[8][x0, x4] - 2 e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[13][x0, x4] a4′[H x4] +
   2 f16[13]^(0,1)[x0, x4] - 2 Tan[6 H x0] f16[8]^(1,0)[x0, x4],
  -2 H M f16[1][x0, x4] - 6 H f16[9][x0, x4] +
   e^(-2 a4[H x4]) (-1 + e^(2 a4[H x4])) H Q1 f16[12][x0, x4] a4′[H x4] -
   2 (f16[12]^(0,1)[x0, x4] + Tan[6 H x0] f16[9]^(1,0)[x0, x4]),
  -2 (H M f16[2][x0, x4] + 3 H f16[10][x0, x4] + f16[15]^(0,1)[x0, x4] +
     Tan[6 H x0] f16[10]^(1,0)[x0, x4]), -2 (H M f16[3][x0, x4] +
     3 H f16[11][x0, x4] - f16[14]^(0,1)[x0, x4] + Tan[6 H x0] f16[11]^(1,0)[x0, x4]),
  2 (3 H f16[4][x0, x4] + H M f16[12][x0, x4] + e^(-a4[H x4]) H Q1 Sinh[a4[H x4]]
      f16[1][x0, x4] a4′[H x4] - f16[1]^(0,1)[x0, x4] + Tan[6 H x0] f16[4]^(1,0)[x0, x4]),
  2 (3 H f16[5][x0, x4] + H M f16[13][x0, x4] - e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[0][x0, x4]
      a4′[H x4] + f16[0]^(0,1)[x0, x4] + Tan[6 H x0] f16[5]^(1,0)[x0, x4]),
  2 (3 H f16[6][x0, x4] + H M f16[14][x0, x4] + f16[3]^(0,1)[x0, x4] +
     Tan[6 H x0] f16[6]^(1,0)[x0, x4]), 2 (3 H f16[7][x0, x4] +
     H M f16[15][x0, x4] - f16[2]^(0,1)[x0, x4] + Tan[6 H x0] f16[7]^(1,0)[x0, x4]),
  2 (3 H f16[0][x0, x4] + H M f16[8][x0, x4] + e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[5][x0, x4] a4′[H x4] +
     f16[5]^(0,1)[x0, x4] + Tan[6 H x0] f16[0]^(1,0)[x0, x4]),
  6 H f16[1][x0, x4] + 2 H M f16[9][x0, x4] - 2 e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[4][x0, x4] a4′[H x4] -
   2 f16[4]^(0,1)[x0, x4] + 2 Tan[6 H x0] f16[1]^(1,0)[x0, x4],
  2 (3 H f16[2][x0, x4] + H M f16[10][x0, x4] - f16[7]^(0,1)[x0, x4] +
     Tan[6 H x0] f16[2]^(1,0)[x0, x4]), 2 (3 H f16[3][x0, x4] +
     H M f16[11][x0, x4] + f16[6]^(0,1)[x0, x4] + Tan[6 H x0] f16[3]^(1,0)[x0, x4])}

In[∘]:= Length[eLa]

Out[∘]=
16

In[∘]:= DumpSave[ToString[header <> "eLa.mx"], eLa];

In[∘]:= Print[ToString[header <> "eLa.mx"]]

Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT-eLa.mx

In[∘]:= (*Get["2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix
    3-eLa.mx"]*)

In[∘]:= (*newLa[]*)
```

In[ ]:=
```
(*Get["2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-enewLa.mx"]*)
```

In[ ]:=
```
(*enewLa=eL[newLa,useDSQRT]*)
```

In[ ]:=
```
(*DumpSave[ToString[header<>"enewLa.mx"],enewLa];*)
```

In[ ]:=
```
(*Print[ToString[header <> "enewLa.mx"]]*)
```

eLazt

In[ ]:=
```
(*Get["2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLazt.mx"];*)
```

In[ ]:=
```
time1 = Now
AbsoluteTiming[
 eLazt = (2 Sqrt[Sin[z]])/(6*H) * eLa /. sfψ16Aa /. sx0x4 // FullSimplify[#, constraintVars] &]
Now - time1
```

$$eLazt = \frac{2\sqrt{Sin[z]}}{6*H} * eLa$$

Out[ ]=

Mon 15 Dec 2025 12:06:23 GMT-8

Out[ ]=

$$\{6.03369, \{-\frac{2}{3} e^{-a4[t]} Q1 \, Sinh[a4[t]] \, Z[9][z,t] \, a4'[t] - \frac{2}{3} (M \, Z[4][z,t] + Z[9]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[12]^{(1,0)}[z,t]),$$
$$\frac{2}{3} (-M \, Z[5][z,t] + e^{-a4[t]} Q1 \, Sinh[a4[t]] \, Z[8][z,t] \, a4'[t] + Z[8]^{(0,1)}[z,t] - 6 \, Tan[z] \, Z[13]^{(1,0)}[z,t]), -\frac{2}{3} (M \, Z[6][z,t] - Z[11]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[14]^{(1,0)}[z,t]),$$
$$-\frac{2}{3} (M \, Z[7][z,t] + Z[10]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[15]^{(1,0)}[z,t]), \frac{2}{3} (-M \, Z[0][z,t] - e^{-a4[t]} Q1 \, Sinh[a4[t]] \, Z[13][z,t] \, a4'[t] + Z[13]^{(0,1)}[z,t] - 6 \, Tan[z] \, Z[8]^{(1,0)}[z,t]),$$
$$-\frac{2}{3} (M \, Z[1][z,t] + Q1 \, Sinh[a4[t]] \, (-Cosh[a4[t]] + Sinh[a4[t]]) \, Z[12][z,t] \, a4'[t] + Z[12]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[9]^{(1,0)}[z,t]),$$
$$-\frac{2}{3} (M \, Z[2][z,t] + Z[15]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[10]^{(1,0)}[z,t]),$$
$$-\frac{2}{3} (M \, Z[3][z,t] - Z[14]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[11]^{(1,0)}[z,t]),$$
$$\frac{2}{3} (M \, Z[12][z,t] + e^{-a4[t]} Q1 \, Sinh[a4[t]] \, Z[1][z,t] \, a4'[t] - Z[1]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[4]^{(1,0)}[z,t]), \frac{2}{3} (M \, Z[13][z,t] - e^{-a4[t]} Q1 \, Sinh[a4[t]] \, Z[0][z,t] \, a4'[t] + Z[0]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[5]^{(1,0)}[z,t]),$$
$$\frac{2}{3} (M \, Z[14][z,t] + Z[3]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[6]^{(1,0)}[z,t]),$$
$$\frac{2}{3} (M \, Z[15][z,t] - Z[2]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[7]^{(1,0)}[z,t]),$$
$$\frac{2}{3} (M \, Z[8][z,t] + e^{-a4[t]} Q1 \, Sinh[a4[t]] \, Z[5][z,t] \, a4'[t] + Z[5]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[0]^{(1,0)}[z,t]), \frac{2}{3} (M \, Z[9][z,t] - e^{-a4[t]} Q1 \, Sinh[a4[t]] \, Z[4][z,t] \, a4'[t] - Z[4]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[1]^{(1,0)}[z,t]),$$
$$\frac{2}{3} (M \, Z[10][z,t] - Z[7]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[2]^{(1,0)}[z,t]),$$
$$\frac{2}{3} (M \, Z[11][z,t] + Z[6]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[3]^{(1,0)}[z,t])\}\}$$

Out[ ]=

6.0317244 s

$$\{6.06786, \{-\frac{2}{3} e^{-a4[t]} Q1 \, Sinh[a4[t]] \, Z[9][z,t] \, a4'[t] - \frac{2}{3} (M \, Z[4][z,t] + Z[9]^{(0,1)}[z,t] + 6 \, Tan[z] \, Z[12]^{(1,0)}[z,t]),$$
$$\frac{2}{3} (-M \, Z[5][z,t] + e^{-a4[t]} Q1 \, Sinh[a4[t]] \, Z[8][z,t] \, a4'[t] + Z[8]^{(0,1)}[z,t] - 6 \, Tan[z] \, Z[13]^{(1,0)}[z,t]),$$

$-\frac{2}{3}\left(M\, Z[6][z,t] - Z[11]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[14]^{(1,0)}[z,t]\right),$

$-\frac{2}{3}\left(M\, Z[7][z,t] + Z[10]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[15]^{(1,0)}[z,t]\right),$

$\frac{2}{3}\left(-M\, Z[0][z,t] - e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, Z[13][z,t]\, a4'[t] + Z[13]^{(0,1)}[z,t] - 6\, Tan[z]\, Z[8]^{(1,0)}[z,t]\right),$

$-\frac{2}{3}\left(M\, Z[1][z,t] + Q1\, Sinh[a4[t]]\, (-Cosh[a4[t]] + Sinh[a4[t]])\, Z[12][z,t]\, a4'[t] + Z[12]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[9]^{(1,0)}[z,t]\right),$

$-\frac{2}{3}\left(M\, Z[2][z,t] + Z[15]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[10]^{(1,0)}[z,t]\right),$

$-\frac{2}{3}\left(M\, Z[3][z,t] - Z[14]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[11]^{(1,0)}[z,t]\right),$

$\frac{2}{3}\left(M\, Z[12][z,t] + e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, Z[1][z,t]\, a4'[t] - Z[1]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[4]^{(1,0)}[z,t]\right),$

$\frac{2}{3}\left(M\, Z[13][z,t] - e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, Z[0][z,t]\, a4'[t] + Z[0]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[5]^{(1,0)}[z,t]\right),$

$\frac{2}{3}\left(M\, Z[14][z,t] + Z[3]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[6]^{(1,0)}[z,t]\right),$

$\frac{2}{3}\left(M\, Z[15][z,t] - Z[2]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[7]^{(1,0)}[z,t]\right),$

$\frac{2}{3}\left(M\, Z[8][z,t] + e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, Z[5][z,t]\, a4'[t] + Z[5]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[0]^{(1,0)}[z,t]\right),$

$\frac{2}{3}\left(M\, Z[9][z,t] - e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, Z[4][z,t]\, a4'[t] - Z[4]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[1]^{(1,0)}[z,t]\right),$

$\frac{2}{3}\left(M\, Z[10][z,t] - Z[7]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[2]^{(1,0)}[z,t]\right),$

$\frac{2}{3}\left(M\, Z[11][z,t] + Z[6]^{(0,1)}[z,t] + 6\, Tan[z]\, Z[3]^{(1,0)}[z,t]\right)\}\}$

Out[∘]=

6.0791309 s

In[∘]:=
```
DumpSave[ToString[header <> "eLazt.mx"], eLazt];
```

In[∘]:=
```
Print[ToString[header <> "eLazt.mx"]]
```

Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT-eLazt.mx

In[•]:= **varZ = Z[#] & /@ Range[0, 15]**

Out[•]=
```
{Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
 Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}
```

In[•]:= **varZzt = #[z, t] & /@ varZ**

Out[•]=
```
{Z[0][z, t], Z[1][z, t], Z[2][z, t], Z[3][z, t], Z[4][z, t],
 Z[5][z, t], Z[6][z, t], Z[7][z, t], Z[8][z, t], Z[9][z, t], Z[10][z, t],
 Z[11][z, t], Z[12][z, t], Z[13][z, t], Z[14][z, t], Z[15][z, t]}
```

In[•]:= **DzvarZ = D[varZzt, z]**

Out[•]=

$\{Z[0]^{(1,0)}[z,t], Z[1]^{(1,0)}[z,t], Z[2]^{(1,0)}[z,t], Z[3]^{(1,0)}[z,t], Z[4]^{(1,0)}[z,t], Z[5]^{(1,0)}[z,t], Z[6]^{(1,0)}[z,t], Z[7]^{(1,0)}[z,t], Z[8]^{(1,0)}[z,t], Z[9]^{(1,0)}[z,t], Z[10]^{(1,0)}[z,t], Z[11]^{(1,0)}[z,t], Z[12]^{(1,0)}[z,t], Z[13]^{(1,0)}[z,t], Z[14]^{(1,0)}[z,t], Z[15]^{(1,0)}[z,t]\}$

In[•]:= **DtvarZ = D[varZzt, t]**

Out[•]=

$\{Z[0]^{(0,1)}[z,t], Z[1]^{(0,1)}[z,t], Z[2]^{(0,1)}[z,t], Z[3]^{(0,1)}[z,t], Z[4]^{(0,1)}[z,t], Z[5]^{(0,1)}[z,t], Z[6]^{(0,1)}[z,t], Z[7]^{(0,1)}[z,t], Z[8]^{(0,1)}[z,t], Z[9]^{(0,1)}[z,t], Z[10]^{(0,1)}[z,t], Z[11]^{(0,1)}[z,t], Z[12]^{(0,1)}[z,t], Z[13]^{(0,1)}[z,t], Z[14]^{(0,1)}[z,t], Z[15]^{(0,1)}[z,t]\}$

In[•]:= **DtvarZsubs = Solve[And @@ Thread[0 == eLazt], DtvarZ][[1]] // FullSimplify[#, constraintVars] &**

Out[•]=

$$\begin{aligned}\{&Z[0]^{(0,1)}[z,t] \to -M\,Z[13][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[0][z,t]\,a4'[t] - 6\,\mathrm{Tan}[z]\,Z[5]^{(1,0)}[z,t],\\ &Z[1]^{(0,1)}[z,t] \to M\,Z[12][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[1][z,t]\,a4'[t] + 6\,\mathrm{Tan}[z]\,Z[4]^{(1,0)}[z,t],\\ &Z[2]^{(0,1)}[z,t] \to M\,Z[15][z,t] + 6\,\mathrm{Tan}[z]\,Z[7]^{(1,0)}[z,t],\\ &Z[3]^{(0,1)}[z,t] \to -M\,Z[14][z,t] - 6\,\mathrm{Tan}[z]\,Z[6]^{(1,0)}[z,t],\\ &Z[4]^{(0,1)}[z,t] \to M\,Z[9][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[4][z,t]\,a4'[t] + 6\,\mathrm{Tan}[z]\,Z[1]^{(1,0)}[z,t],\\ &Z[5]^{(0,1)}[z,t] \to -M\,Z[8][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[5][z,t]\,a4'[t] - 6\,\mathrm{Tan}[z]\,Z[0]^{(1,0)}[z,t],\\ &Z[6]^{(0,1)}[z,t] \to -M\,Z[11][z,t] - 6\,\mathrm{Tan}[z]\,Z[3]^{(1,0)}[z,t],\\ &Z[7]^{(0,1)}[z,t] \to M\,Z[10][z,t] + 6\,\mathrm{Tan}[z]\,Z[2]^{(1,0)}[z,t],\\ &Z[8]^{(0,1)}[z,t] \to M\,Z[5][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[8][z,t]\,a4'[t] + 6\,\mathrm{Tan}[z]\,Z[13]^{(1,0)}[z,t],\\ &Z[9]^{(0,1)}[z,t] \to -M\,Z[4][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[9][z,t]\,a4'[t] - 6\,\mathrm{Tan}[z]\,Z[12]^{(1,0)}[z,t],\\ &Z[10]^{(0,1)}[z,t] \to -M\,Z[7][z,t] - 6\,\mathrm{Tan}[z]\,Z[15]^{(1,0)}[z,t],\\ &Z[11]^{(0,1)}[z,t] \to M\,Z[6][z,t] + 6\,\mathrm{Tan}[z]\,Z[14]^{(1,0)}[z,t],\\ &Z[12]^{(0,1)}[z,t] \to -M\,Z[1][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[12][z,t]\,a4'[t] - 6\,\mathrm{Tan}[z]\,Z[9]^{(1,0)}[z,t],\\ &Z[13]^{(0,1)}[z,t] \to M\,Z[0][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[13][z,t]\,a4'[t] + 6\,\mathrm{Tan}[z]\,Z[8]^{(1,0)}[z,t],\\ &Z[14]^{(0,1)}[z,t] \to M\,Z[3][z,t] + 6\,\mathrm{Tan}[z]\,Z[11]^{(1,0)}[z,t],\\ &Z[15]^{(0,1)}[z,t] \to -M\,Z[2][z,t] - 6\,\mathrm{Tan}[z]\,Z[10]^{(1,0)}[z,t]\}\end{aligned}$$

In[ ]:=
```
ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(DtvarZEQS = ToExpression[#] & /@ %) // Column
```

Out[ ]=

$Z[0]^{(0,1)}[z, t] == -M\, Z[13][z, t] + e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[0][z, t]\, a4'[t] - 6\, \text{Tan}[z]\, Z[5]^{(1,0)}[z, t]$

$Z[1]^{(0,1)}[z, t] == M\, Z[12][z, t] + e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[1][z, t]\, a4'[t] + 6\, \text{Tan}[z]\, Z[4]^{(1,0)}[z, t]$

$Z[2]^{(0,1)}[z, t] == M\, Z[15][z, t] + 6\, \text{Tan}[z]\, Z[7]^{(1,0)}[z, t]$

$Z[3]^{(0,1)}[z, t] == -M\, Z[14][z, t] - 6\, \text{Tan}[z]\, Z[6]^{(1,0)}[z, t]$

$Z[4]^{(0,1)}[z, t] == M\, Z[9][z, t] - e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[4][z, t]\, a4'[t] + 6\, \text{Tan}[z]\, Z[1]^{(1,0)}[z, t]$

$Z[5]^{(0,1)}[z, t] == -M\, Z[8][z, t] - e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[5][z, t]\, a4'[t] - 6\, \text{Tan}[z]\, Z[0]^{(1,0)}[z, t]$

$Z[6]^{(0,1)}[z, t] == -M\, Z[11][z, t] - 6\, \text{Tan}[z]\, Z[3]^{(1,0)}[z, t]$

$Z[7]^{(0,1)}[z, t] == M\, Z[10][z, t] + 6\, \text{Tan}[z]\, Z[2]^{(1,0)}[z, t]$

$Z[8]^{(0,1)}[z, t] == M\, Z[5][z, t] - e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[8][z, t]\, a4'[t] + 6\, \text{Tan}[z]\, Z[13]^{(1,0)}[z, t]$

$Z[9]^{(0,1)}[z, t] == -M\, Z[4][z, t] - e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[9][z, t]\, a4'[t] - 6\, \text{Tan}[z]\, Z[12]^{(1,0)}[z, t]$

$Z[10]^{(0,1)}[z, t] == -M\, Z[7][z, t] - 6\, \text{Tan}[z]\, Z[15]^{(1,0)}[z, t]$

$Z[11]^{(0,1)}[z, t] == M\, Z[6][z, t] + 6\, \text{Tan}[z]\, Z[14]^{(1,0)}[z, t]$

$Z[12]^{(0,1)}[z, t] ==$
$-M\, Z[1][z, t] + e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[12][z, t]\, a4'[t] - 6\, \text{Tan}[z]\, Z[9]^{(1,0)}[z, t]$

$Z[13]^{(0,1)}[z, t] == M\, Z[0][z, t] + e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[13][z, t]\, a4'[t] + 6\, \text{Tan}[z]\, Z[8]^{(1,0)}[z, t]$

$Z[14]^{(0,1)}[z, t] == M\, Z[3][z, t] + 6\, \text{Tan}[z]\, Z[11]^{(1,0)}[z, t]$

$Z[15]^{(0,1)}[z, t] == -M\, Z[2][z, t] - 6\, \text{Tan}[z]\, Z[10]^{(1,0)}[z, t]$

In[ ]:=
```
ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(DtvarZrelations = ToExpression[#] & /@ %) // Column
```

Out[ ]=

$M\, Z[13][z, t] - e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[0][z, t]\, a4'[t] + Z[0]^{(0,1)}[z, t] + 6\, \text{Tan}[z]\, Z[5]^{(1,0)}[z, t]$

$-M\, Z[12][z, t] - e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[1][z, t]\, a4'[t] + Z[1]^{(0,1)}[z, t] - 6\, \text{Tan}[z]\, Z[4]^{(1,0)}[z, t]$

$-M\, Z[15][z, t] + Z[2]^{(0,1)}[z, t] - 6\, \text{Tan}[z]\, Z[7]^{(1,0)}[z, t]$

$M\, Z[14][z, t] + Z[3]^{(0,1)}[z, t] + 6\, \text{Tan}[z]\, Z[6]^{(1,0)}[z, t]$

$-M\, Z[9][z, t] + e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[4][z, t]\, a4'[t] + Z[4]^{(0,1)}[z, t] - 6\, \text{Tan}[z]\, Z[1]^{(1,0)}[z, t]$

$M\, Z[8][z, t] + e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[5][z, t]\, a4'[t] + Z[5]^{(0,1)}[z, t] + 6\, \text{Tan}[z]\, Z[0]^{(1,0)}[z, t]$

$M\, Z[11][z, t] + Z[6]^{(0,1)}[z, t] + 6\, \text{Tan}[z]\, Z[3]^{(1,0)}[z, t]$

$-M\, Z[10][z, t] + Z[7]^{(0,1)}[z, t] - 6\, \text{Tan}[z]\, Z[2]^{(1,0)}[z, t]$

$-M\, Z[5][z, t] + e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[8][z, t]\, a4'[t] + Z[8]^{(0,1)}[z, t] - 6\, \text{Tan}[z]\, Z[13]^{(1,0)}[z, t]$

$M\, Z[4][z, t] + e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[9][z, t]\, a4'[t] + Z[9]^{(0,1)}[z, t] + 6\, \text{Tan}[z]\, Z[12]^{(1,0)}[z, t]$

$M\, Z[7][z, t] + Z[10]^{(0,1)}[z, t] + 6\, \text{Tan}[z]\, Z[15]^{(1,0)}[z, t]$

$-M\, Z[6][z, t] + Z[11]^{(0,1)}[z, t] - 6\, \text{Tan}[z]\, Z[14]^{(1,0)}[z, t]$

$M\, Z[1][z, t] - e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[12][z, t]\, a4'[t] + Z[12]^{(0,1)}[z, t] + 6\, \text{Tan}[z]\, Z[9]^{(1,0)}[z, t]$

$-M\, Z[0][z, t] - e^{-a4[t]}\, Q1\, \text{Sinh}[a4[t]]\, Z[13][z, t]\, a4'[t] + Z[13]^{(0,1)}[z, t] - 6\, \text{Tan}[z]\, Z[8]^{(1,0)}[z, t]$

$-M\, Z[3][z, t] + Z[14]^{(0,1)}[z, t] - 6\, \text{Tan}[z]\, Z[11]^{(1,0)}[z, t]$

$M\, Z[2][z, t] + Z[15]^{(0,1)}[z, t] + 6\, \text{Tan}[z]\, Z[10]^{(1,0)}[z, t]$

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE :

In[∘]:=
```
{{4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13, 0, 1, 4, 13, 13, 8, 0, 5, 12, 1, 12, 9},
 {6, 7, 2, 11, 14, 6, 3, 7, 15, 2, 3, 6, 15, 10, 2, 3, 7, 14, 11}}
eLaztCouplings = Union[Flatten[#]] & /@ %
Union[Flatten[%]] === Range[0, 15]
```

Out[∘]=
```
{{4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13, 0, 1, 4, 13, 13, 8, 0, 5, 12, 1, 12, 9},
 {6, 7, 2, 11, 14, 6, 3, 7, 15, 2, 3, 6, 15, 10, 2, 3, 7, 14, 11}}
```

Out[∘]=
```
{{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}}
```

Out[∘]=
```
True
```

SEPARATE VARIABLES THAT ARE COUPLED TO EACH OTHER :

In[∘]:=
```
Thread[(Z[#] & /@ eLaztCouplings[[1]]) → (yZ[#] & /@ Range[0, 7])]
Thread[(Z[#] & /@ eLaztCouplings[[2]]) → (yZ[#] & /@ Range[8, 15])]
sZtOyZ = Union[Flatten[{%%, %}]]
ToString[FullForm[#]] & /@ sZtOyZ;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(ZyZEQS = ToExpression[#] & /@ %)
syZtoZ = Solve[And @@ %, (yZ[#] & /@ Range[0, 15])][[-1]]
```

Out[∘]=
```
{Z[0] → yZ[0], Z[1] → yZ[1], Z[4] → yZ[2], Z[5] → yZ[3],
 Z[8] → yZ[4], Z[9] → yZ[5], Z[12] → yZ[6], Z[13] → yZ[7]}
```

Out[∘]=
```
{Z[2] → yZ[8], Z[3] → yZ[9], Z[6] → yZ[10], Z[7] → yZ[11],
 Z[10] → yZ[12], Z[11] → yZ[13], Z[14] → yZ[14], Z[15] → yZ[15]}
```

Out[∘]=
```
{Z[0] → yZ[0], Z[1] → yZ[1], Z[2] → yZ[8], Z[3] → yZ[9], Z[4] → yZ[2], Z[5] → yZ[3],
 Z[6] → yZ[10], Z[7] → yZ[11], Z[8] → yZ[4], Z[9] → yZ[5], Z[10] → yZ[12],
 Z[11] → yZ[13], Z[12] → yZ[6], Z[13] → yZ[7], Z[14] → yZ[14], Z[15] → yZ[15]}
```

Out[∘]=
```
{Z[0] == yZ[0], Z[1] == yZ[1], Z[2] == yZ[8], Z[3] == yZ[9], Z[4] == yZ[2], Z[5] == yZ[3],
 Z[6] == yZ[10], Z[7] == yZ[11], Z[8] == yZ[4], Z[9] == yZ[5], Z[10] == yZ[12],
 Z[11] == yZ[13], Z[12] == yZ[6], Z[13] == yZ[7], Z[14] == yZ[14], Z[15] == yZ[15]}
```

Out[∘]=
```
{yZ[0] → Z[0], yZ[1] → Z[1], yZ[2] → Z[4], yZ[3] → Z[5], yZ[4] → Z[8], yZ[5] → Z[9],
 yZ[6] → Z[12], yZ[7] → Z[13], yZ[8] → Z[2], yZ[9] → Z[3], yZ[10] → Z[6],
 yZ[11] → Z[7], yZ[12] → Z[10], yZ[13] → Z[11], yZ[14] → Z[14], yZ[15] → Z[15]}
```

In[∘]:=
```
ToString[FullForm[#]] & /@ sZtOyZ;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(ZyZforCaEQS = -ToExpression[#] & /@ %)
```

Out[∘]=
```
{yZ[0] - Z[0], yZ[1] - Z[1], yZ[8] - Z[2], yZ[9] - Z[3], yZ[2] - Z[4], yZ[3] - Z[5],
 yZ[10] - Z[6], yZ[11] - Z[7], yZ[4] - Z[8], yZ[5] - Z[9], yZ[12] - Z[10],
 yZ[13] - Z[11], yZ[6] - Z[12], yZ[7] - Z[13], yZ[14] - Z[14], yZ[15] - Z[15]}
```

```
In[•]:= cayZ = CoefficientArrays[ZyZforCaEQS, (yZ[#] & /@ Range[0, 15])]

Out[•]= {SparseArray[ Specified elements: 16  Dimensions: {16} ], SparseArray[ Specified elements: 16  Dimensions: {16, 16} ]}

In[•]:= caZ = CoefficientArrays[ZyZforCaEQS, (Z[#] & /@ Range[0, 15])]

Out[•]= {SparseArray[ Specified elements: 16  Dimensions: {16} ], SparseArray[ Specified elements: 16  Dimensions: {16, 16} ]}

In[•]:= (cayZ2 = cayZ[[2]] // Normal) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

IDENTIFY ORTHOGONAL SIMILARITY TRANSFORMATION FROM SET OF VARIABLES Z[j] TO SET OF VARIABLES yZ[j] :

```
In[•]:= cayZ2.(yZ[#] & /@ Range[0, 15]) /. syZtoZ
        Transpose[cayZ2].% /. sZtOyZ

Out[•]= {Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
         Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}

Out[•]= {yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
         yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}

In[•]:= cayZ2.Transpose[cayZ2] === ID16

Out[•]= True
```

TRANSFORM Euler Lagrange equations to yZ[j] :

In[ ]:= (DtyZrelations = Transpose[cayZ2].DtvarZrelations /. sZtOyZ // FullSimplify) // Column

Out[ ]=

$$M\, yZ[7][z,t] - e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, yZ[0][z,t]\, a4'[t] + yZ[0]^{(0,1)}[z,t] + 6\, Tan[z]\, yZ[3]^{(1,0)}[z,t]$$
$$-M\, yZ[6][z,t] - e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, yZ[1][z,t]\, a4'[t] + yZ[1]^{(0,1)}[z,t] - 6\, Tan[z]\, yZ[2]^{(1,0)}[z,t]$$
$$-M\, yZ[5][z,t] + e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, yZ[2][z,t]\, a4'[t] + yZ[2]^{(0,1)}[z,t] - 6\, Tan[z]\, yZ[1]^{(1,0)}[z,t]$$
$$M\, yZ[4][z,t] + e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, yZ[3][z,t]\, a4'[t] + yZ[3]^{(0,1)}[z,t] + 6\, Tan[z]\, yZ[0]^{(1,0)}[z,t]$$
$$-M\, yZ[3][z,t] + e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, yZ[4][z,t]\, a4'[t] + yZ[4]^{(0,1)}[z,t] - 6\, Tan[z]\, yZ[7]^{(1,0)}[z,t]$$
$$M\, yZ[2][z,t] + e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, yZ[5][z,t]\, a4'[t] + yZ[5]^{(0,1)}[z,t] + 6\, Tan[z]\, yZ[6]^{(1,0)}[z,t]$$
$$M\, yZ[1][z,t] - e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, yZ[6][z,t]\, a4'[t] + yZ[6]^{(0,1)}[z,t] + 6\, Tan[z]\, yZ[5]^{(1,0)}[z,t]$$
$$-M\, yZ[0][z,t] - e^{-a4[t]}\, Q1\, Sinh[a4[t]]\, yZ[7][z,t]\, a4'[t] + yZ[7]^{(0,1)}[z,t] - 6\, Tan[z]\, yZ[4]^{(1,0)}[z,t]$$
$$-M\, yZ[15][z,t] + yZ[8]^{(0,1)}[z,t] - 6\, Tan[z]\, yZ[11]^{(1,0)}[z,t]$$
$$M\, yZ[14][z,t] + yZ[9]^{(0,1)}[z,t] + 6\, Tan[z]\, yZ[10]^{(1,0)}[z,t]$$
$$M\, yZ[13][z,t] + yZ[10]^{(0,1)}[z,t] + 6\, Tan[z]\, yZ[9]^{(1,0)}[z,t]$$
$$-M\, yZ[12][z,t] + yZ[11]^{(0,1)}[z,t] - 6\, Tan[z]\, yZ[8]^{(1,0)}[z,t]$$
$$M\, yZ[11][z,t] + yZ[12]^{(0,1)}[z,t] + 6\, Tan[z]\, yZ[15]^{(1,0)}[z,t]$$
$$-M\, yZ[10][z,t] + yZ[13]^{(0,1)}[z,t] - 6\, Tan[z]\, yZ[14]^{(1,0)}[z,t]$$
$$-M\, yZ[9][z,t] + yZ[14]^{(0,1)}[z,t] - 6\, Tan[z]\, yZ[13]^{(1,0)}[z,t]$$
$$M\, yZ[8][z,t] + yZ[15]^{(0,1)}[z,t] + 6\, Tan[z]\, yZ[12]^{(1,0)}[z,t]$$

CHECK THAT yZ[j] ARE IN PROPER ORDER:

In[∘]:=
```
(DtyZsubs =
   Solve[And @@ Thread[0 == DtyZrelations], (D[(yZ[#][z, t] & /@ Range[0, 15]), t])][[1]] //
    FullSimplify[#, constraintVars] &) // Column
```

Out[∘]=
```
yZ[0]^(0,1)[z, t] →
 -M yZ[7][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] yZ[0][z, t] a4′[t] - 6 Tan[z] yZ[3]^(1,0)[z, t]
yZ[1]^(0,1)[z, t] →
 M yZ[6][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] yZ[1][z, t] a4′[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]
yZ[2]^(0,1)[z, t] →
 M yZ[5][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] yZ[2][z, t] a4′[t] + 6 Tan[z] yZ[1]^(1,0)[z, t]
yZ[3]^(0,1)[z, t] →
 -M yZ[4][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] yZ[3][z, t] a4′[t] - 6 Tan[z] yZ[0]^(1,0)[z, t]
yZ[4]^(0,1)[z, t] →
 M yZ[3][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] yZ[4][z, t] a4′[t] + 6 Tan[z] yZ[7]^(1,0)[z, t]
yZ[5]^(0,1)[z, t] →
 -M yZ[2][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] yZ[5][z, t] a4′[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]
yZ[6]^(0,1)[z, t] →
 -M yZ[1][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] yZ[6][z, t] a4′[t] - 6 Tan[z] yZ[5]^(1,0)[z, t]
yZ[7]^(0,1)[z, t] →
 M yZ[0][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] yZ[7][z, t] a4′[t] + 6 Tan[z] yZ[4]^(1,0)[z, t]
yZ[8]^(0,1)[z, t] → M yZ[15][z, t] + 6 Tan[z] yZ[11]^(1,0)[z, t]
yZ[9]^(0,1)[z, t] → -M yZ[14][z, t] - 6 Tan[z] yZ[10]^(1,0)[z, t]
yZ[10]^(0,1)[z, t] → -M yZ[13][z, t] - 6 Tan[z] yZ[9]^(1,0)[z, t]
yZ[11]^(0,1)[z, t] → M yZ[12][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t]
yZ[12]^(0,1)[z, t] → -M yZ[11][z, t] - 6 Tan[z] yZ[15]^(1,0)[z, t]
yZ[13]^(0,1)[z, t] → M yZ[10][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]
yZ[14]^(0,1)[z, t] → M yZ[9][z, t] + 6 Tan[z] yZ[13]^(1,0)[z, t]
yZ[15]^(0,1)[z, t] → -M yZ[8][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t]
```

In[∘]:=
```
(caZ2 = caZ[[2]] // Normal) // MatrixForm
```

Out[∘]//MatrixForm=

$$
\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}
$$

In[∘]:=
```
sx0x4
```

Out[∘]=

$$\left\{x0 \to \frac{z}{6\,H},\; x4 \to \frac{t}{H}\right\}$$

ENERGY EIGENVALUES OF THE Euler Lagrange equations:

TRY TO FIND ENERGY EIGENFUNCTIONS AND ENERGY EIGENVALUES OF THE Euler Lagrange equations of the form

$$yZ[j] = \frac{1}{\text{Sin}[6*H*x0]^{1/2}} \text{Exp}[E_j\,[x4] * H\,] =$$

$$yZ[j] = \frac{1}{\text{Sin}[z]^{1/2}} \text{Exp}[E_j\,[\,t]]$$

(recall that

```
sfψ16Aa =
  f16[#] → ToExpression["((Z[" <> ToString[#] <>
        "][6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"] & /@
   Range[0, 15]
```

):

```
In[•]:= syZtOnly = yZ[#] → ToExpression["((Exp[ν[" <> ToString[#] <> "][#2]])&)"] & /@ Range[0, 15]
Out[•]=
{yZ[0] → (Exp[ν[0][#2]] &), yZ[1] → (Exp[ν[1][#2]] &),
 yZ[2] → (Exp[ν[2][#2]] &), yZ[3] → (Exp[ν[3][#2]] &), yZ[4] → (Exp[ν[4][#2]] &),
 yZ[5] → (Exp[ν[5][#2]] &), yZ[6] → (Exp[ν[6][#2]] &), yZ[7] → (Exp[ν[7][#2]] &),
 yZ[8] → (Exp[ν[8][#2]] &), yZ[9] → (Exp[ν[9][#2]] &), yZ[10] → (Exp[ν[10][#2]] &),
 yZ[11] → (Exp[ν[11][#2]] &), yZ[12] → (Exp[ν[12][#2]] &),
 yZ[13] → (Exp[ν[13][#2]] &), yZ[14] → (Exp[ν[14][#2]] &), yZ[15] → (Exp[ν[15][#2]] &)}
In[•]:= syZtc = ν[#][0] → Log[T[#]] & /@ Range[0, 15]
Out[•]=
{ν[0][0] → Log[T[0]], ν[1][0] → Log[T[1]], ν[2][0] → Log[T[2]], ν[3][0] → Log[T[3]],
 ν[4][0] → Log[T[4]], ν[5][0] → Log[T[5]], ν[6][0] → Log[T[6]], ν[7][0] → Log[T[7]],
 ν[8][0] → Log[T[8]], ν[9][0] → Log[T[9]], ν[10][0] → Log[T[10]], ν[11][0] → Log[T[11]],
 ν[12][0] → Log[T[12]], ν[13][0] → Log[T[13]], ν[14][0] → Log[T[14]], ν[15][0] → Log[T[15]]}
In[•]:= DtyZrelations /. syZtOnly
FullSimplify[Normal[Series[#, {t, 0, 0}]]] & /@ %
% /. syZtc
DtyZrelationsA = %
```

Out[ ]=

$$\{e^{\nu[7][t]} M - e^{-a4[t]+\nu[0][t]} Q1 \sinh[a4[t]] a4'[t] + e^{\nu[0][t]} \nu[0]'[t],$$
$$-e^{\nu[6][t]} M - e^{-a4[t]+\nu[1][t]} Q1 \sinh[a4[t]] a4'[t] + e^{\nu[1][t]} \nu[1]'[t],$$
$$-e^{\nu[5][t]} M + e^{-a4[t]+\nu[2][t]} Q1 \sinh[a4[t]] a4'[t] + e^{\nu[2][t]} \nu[2]'[t],$$
$$e^{\nu[4][t]} M + e^{-a4[t]+\nu[3][t]} Q1 \sinh[a4[t]] a4'[t] + e^{\nu[3][t]} \nu[3]'[t],$$
$$-e^{\nu[3][t]} M + e^{-a4[t]+\nu[4][t]} Q1 \sinh[a4[t]] a4'[t] + e^{\nu[4][t]} \nu[4]'[t],$$
$$e^{\nu[2][t]} M + e^{-a4[t]+\nu[5][t]} Q1 \sinh[a4[t]] a4'[t] + e^{\nu[5][t]} \nu[5]'[t],$$
$$e^{\nu[1][t]} M - e^{-a4[t]+\nu[6][t]} Q1 \sinh[a4[t]] a4'[t] + e^{\nu[6][t]} \nu[6]'[t],$$
$$-e^{\nu[0][t]} M - e^{-a4[t]+\nu[7][t]} Q1 \sinh[a4[t]] a4'[t] + e^{\nu[7][t]} \nu[7]'[t],$$
$$-e^{\nu[15][t]} M + e^{\nu[8][t]} \nu[8]'[t], e^{\nu[14][t]} M + e^{\nu[9][t]} \nu[9]'[t], e^{\nu[13][t]} M + e^{\nu[10][t]} \nu[10]'[t],$$
$$-e^{\nu[12][t]} M + e^{\nu[11][t]} \nu[11]'[t], e^{\nu[11][t]} M + e^{\nu[12][t]} \nu[12]'[t],$$
$$-e^{\nu[10][t]} M + e^{\nu[13][t]} \nu[13]'[t], -e^{\nu[9][t]} M + e^{\nu[14][t]} \nu[14]'[t], e^{\nu[8][t]} M + e^{\nu[15][t]} \nu[15]'[t]\}$$

Out[ ]=

$$\{e^{\nu[7][0]} M + e^{\nu[0][0]} \left(-e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[0]'[0]\right),$$
$$-e^{\nu[6][0]} M + e^{\nu[1][0]} \left(-e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[1]'[0]\right),$$
$$-e^{\nu[5][0]} M + e^{\nu[2][0]} \left(e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[2]'[0]\right),$$
$$e^{\nu[4][0]} M + e^{\nu[3][0]} \left(e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[3]'[0]\right),$$
$$-e^{\nu[3][0]} M + e^{\nu[4][0]} \left(e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[4]'[0]\right),$$
$$e^{\nu[2][0]} M + e^{\nu[5][0]} \left(e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[5]'[0]\right),$$
$$e^{\nu[1][0]} M + e^{\nu[6][0]} \left(-e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[6]'[0]\right),$$
$$-e^{\nu[0][0]} M + e^{\nu[7][0]} \left(-e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[7]'[0]\right),$$
$$-e^{\nu[15][0]} M + e^{\nu[8][0]} \nu[8]'[0], e^{\nu[14][0]} M + e^{\nu[9][0]} \nu[9]'[0], e^{\nu[13][0]} M + e^{\nu[10][0]} \nu[10]'[0],$$
$$-e^{\nu[12][0]} M + e^{\nu[11][0]} \nu[11]'[0], e^{\nu[11][0]} M + e^{\nu[12][0]} \nu[12]'[0],$$
$$-e^{\nu[10][0]} M + e^{\nu[13][0]} \nu[13]'[0], -e^{\nu[9][0]} M + e^{\nu[14][0]} \nu[14]'[0], e^{\nu[8][0]} M + e^{\nu[15][0]} \nu[15]'[0]\}$$

Out[ ]=

$$\{M T[7] + T[0] \left(-e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[0]'[0]\right),$$
$$-M T[6] + T[1] \left(-e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[1]'[0]\right),$$
$$-M T[5] + T[2] \left(e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[2]'[0]\right),$$
$$M T[4] + T[3] \left(e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[3]'[0]\right),$$
$$-M T[3] + T[4] \left(e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[4]'[0]\right),$$
$$M T[2] + T[5] \left(e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[5]'[0]\right),$$
$$M T[1] + T[6] \left(-e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[6]'[0]\right),$$
$$-M T[0] + T[7] \left(-e^{-a4[0]} Q1 \sinh[a4[0]] a4'[0] + \nu[7]'[0]\right),$$
$$-M T[15] + T[8] \nu[8]'[0], M T[14] + T[9] \nu[9]'[0], M T[13] + T[10] \nu[10]'[0],$$
$$-M T[12] + T[11] \nu[11]'[0], M T[11] + T[12] \nu[12]'[0],$$
$$-M T[10] + T[13] \nu[13]'[0], -M T[9] + T[14] \nu[14]'[0], M T[8] + T[15] \nu[15]'[0]\}$$

Out[•]=

```
{M T[7] + T[0] (-e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[0]′[0]),
 -M T[6] + T[1] (-e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[1]′[0]),
 -M T[5] + T[2] (e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[2]′[0]),
 M T[4] + T[3] (e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[3]′[0]),
 -M T[3] + T[4] (e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[4]′[0]),
 M T[2] + T[5] (e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[5]′[0]),
 M T[1] + T[6] (-e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[6]′[0]),
 -M T[0] + T[7] (-e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[7]′[0]),
 -M T[15] + T[8] ν[8]′[0], M T[14] + T[9] ν[9]′[0], M T[13] + T[10] ν[10]′[0],
 -M T[12] + T[11] ν[11]′[0], M T[11] + T[12] ν[12]′[0],
 -M T[10] + T[13] ν[13]′[0], -M T[9] + T[14] ν[14]′[0], M T[8] + T[15] ν[15]′[0]}
```

In[•]:=

```
(*DtyZrelations/.syZtOnly;
 %/.syZtc
 DtyZrelationsA=%*)
```

In[•]:=

```
Ts = (T[#] & /@ Range[0, 15])
```

Out[•]=

```
{T[0], T[1], T[2], T[3], T[4], T[5], T[6],
 T[7], T[8], T[9], T[10], T[11], T[12], T[13], T[14], T[15]}
```

In[•]:=

```
caDtyZrelationsA = CoefficientArrays[DtyZrelationsA, Ts]
```

Out[•]=

```
{SparseArray[ Specified elements: 0
              Dimensions: {16} ], SparseArray[ Specified elements: 32
                                               Dimensions: {16, 16} ]}
```

In[•]:=

```
(caDtyZrelationsA2 = caDtyZrelationsA[[2]] // Normal) // MatrixForm
```

Out[•]//MatrixForm=

```
-e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[0]′[0]    0
0                                              -e^-a4[0] Q1 Sinh[a4[0]] a4′[0] + ν[1]′[0]
0                                              0                                              e^-a4[0] Q1 Sinh[a4
0                                              0
0                                              0
0                                              0
0                                              M
-M                                             0
0                                              0
0                                              0
0                                              0
0                                              0
0                                              0
0                                              0
0                                              0
0                                              0
```

In[•]:=

```
caDtyZrelationsA2 === ArrayFlatten[
   {{caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]], 0}, {0, caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]]}}]
```

Out[•]=

```
True
```

CHECK THAT THERE ARE NO ν[] TERMS :

In[ ]:=
```
(caDtyZrelationsA1 = caDtyZrelationsA⟦1⟧ // Normal) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In[ ]:=
```
time1 = Now
AbsoluteTiming[caDtyZrelationsA21Y0 = Det[caDtyZrelationsA2⟦1 ;; 8, 1 ;; 8⟧ - Y ID8]]
Now
caDtyZrelationsA21Y = (*Full*)Simplify[caDtyZrelationsA21Y0, constraintVars]
Now - time1
```

Out[ ]=

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Out[•]=

$$\{0.0840045,\ e^{-8\,a4[0]}$$
$$(e^{2\,a4[0]} M^2\ (e^{2\,a4[0]} M^2\ (e^{2\,a4[0]} M^2\ (e^{2\,a4[0]} M^2 + (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[3]'[0])$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[4]'[0])) +$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[2]'[0])\ (e^{2\,a4[0]} M^2 +$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[3]'[0])\ (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]$$
$$a4'[0] + e^{a4[0]}\, \nu[4]'[0]))\ (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[5]'[0])) +$$
$$(-e^{a4[0]}\, Y - Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[1]'[0])$$
$$(e^{2\,a4[0]} M^2\ (e^{2\,a4[0]} M^2 + (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[3]'[0])$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[4]'[0])) +$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[2]'[0])\ (e^{2\,a4[0]} M^2 +$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[3]'[0])\ (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]$$
$$a4'[0] + e^{a4[0]}\, \nu[4]'[0]))\ (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[5]'[0]))$$
$$(-e^{a4[0]}\, Y - Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[6]'[0])) +$$
$$(-e^{a4[0]}\, Y - Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[0]'[0])$$
$$(e^{2\,a4[0]} M^2\ (e^{2\,a4[0]} M^2\ (e^{2\,a4[0]} M^2 + (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[3]'[0])$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[4]'[0])) +$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[2]'[0])$$
$$(e^{2\,a4[0]} M^2 + (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[3]'[0])$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[4]'[0]))$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[5]'[0])) + (-e^{a4[0]}\, Y -$$
$$Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[1]'[0])$$
$$(e^{2\,a4[0]} M^2\ (e^{2\,a4[0]} M^2 + (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[3]'[0])$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[4]'[0])) +$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[2]'[0])\ (e^{2\,a4[0]} M^2 +$$
$$(-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[3]'[0])\ (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]$$
$$a4'[0] + e^{a4[0]}\, \nu[4]'[0]))\ (-e^{a4[0]}\, Y + Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[5]'[0]))$$
$$(-e^{a4[0]}\, Y - Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[6]'[0]))\ (-e^{a4[0]}\, Y -$$
$$Q1\,\text{Sinh}[a4[0]]\, a4'[0] + e^{a4[0]}\, \nu[7]'[0])))\}$$

Out[•]=

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Out[•]=

$$\frac{1}{256}\, e^{-16\,a4[0]}\ (Q1^2\, a4'[0]^2 + e^{4\,a4[0]}\ (4\,M^2 + (2\,Y - Q1\,a4'[0] - 2\,\nu[3]'[0])\ (2\,Y - Q1\,a4'[0] - 2\,\nu[4]'[0])) -$$
$$2\,e^{2\,a4[0]}\, Q1\, a4'[0]\ (-2\,Y + Q1\,a4'[0] + \nu[3]'[0] + \nu[4]'[0]))$$
$$(Q1^2\, a4'[0]^2 + e^{4\,a4[0]}\ (4\,M^2 + (2\,Y - Q1\,a4'[0] - 2\,\nu[2]'[0])\ (2\,Y - Q1\,a4'[0] - 2\,\nu[5]'[0])) -$$
$$2\,e^{2\,a4[0]}\, Q1\, a4'[0]\ (-2\,Y + Q1\,a4'[0] + \nu[2]'[0] + \nu[5]'[0]))$$
$$(Q1^2\, a4'[0]^2 + e^{4\,a4[0]}\ (4\,M^2 + (2\,Y + Q1\,a4'[0] - 2\,\nu[1]'[0])\ (2\,Y + Q1\,a4'[0] - 2\,\nu[6]'[0])) -$$
$$2\,e^{2\,a4[0]}\, Q1\, a4'[0]\ (2\,Y + Q1\,a4'[0] - \nu[1]'[0] - \nu[6]'[0]))$$
$$(Q1^2\, a4'[0]^2 + e^{4\,a4[0]}\ (4\,M^2 + (2\,Y + Q1\,a4'[0] - 2\,\nu[0]'[0])\ (2\,Y + Q1\,a4'[0] - 2\,\nu[7]'[0])) -$$
$$2\,e^{2\,a4[0]}\, Q1\, a4'[0]\ (2\,Y + Q1\,a4'[0] - \nu[0]'[0] - \nu[7]'[0]))$$

Out[•]=

1.4006429 s

In[∘]:=
```
sumEvalscaDtyZrelationsA21Y = 1/7! D[caDtyZrelationsA21Y, {Y, 7}] /. {Y → 0} // Simplify
```

Out[∘]=

$$-\nu[0]'[0]-\nu[1]'[0]-\nu[2]'[0]-\nu[3]'[0]-\nu[4]'[0]-\nu[5]'[0]-\nu[6]'[0]-\nu[7]'[0]$$

In[∘]:=
```
time1 = Now
AbsoluteTiming[caDtyZrelationsA22Y0 = Det[caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]] - Y ID8]]
Now
caDtyZrelationsA22Y = (*Full*)Simplify[caDtyZrelationsA22Y0, constraintVars]
Now - time1
```

Out[∘]=

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Out[∘]=

$$\{0.0026515,\ M^2\Big(M^2\big(M\,(M^3+M\,Y^2-M\,Y\,\nu[11]'[0]-M\,Y\,\nu[12]'[0]+M\,\nu[11]'[0]\,\nu[12]'[0])+(-Y+\nu[10]'[0])\,(M^2+Y^2-Y\,\nu[11]'[0]-Y\,\nu[12]'[0]+\nu[11]'[0]\,\nu[12]'[0])\,(-Y+\nu[13]'[0])\big)+(-Y+\nu[9]'[0])\,\big(M\,(M^3+M\,Y^2-M\,Y\,\nu[11]'[0]-M\,Y\,\nu[12]'[0]+M\,\nu[11]'[0]\,\nu[12]'[0])+(-Y+\nu[10]'[0])\,(M^2+Y^2-Y\,\nu[11]'[0]-Y\,\nu[12]'[0]+\nu[11]'[0]\,\nu[12]'[0])\,(-Y+\nu[13]'[0])\big)\,(-Y+\nu[14]'[0])\Big)+(-Y+\nu[8]'[0])\Big(M^2\big(M\,(M^3+M\,Y^2-M\,Y\,\nu[11]'[0]-M\,Y\,\nu[12]'[0]+M\,\nu[11]'[0]\,\nu[12]'[0])+(-Y+\nu[10]'[0])\,(M^2+Y^2-Y\,\nu[11]'[0]-Y\,\nu[12]'[0]+\nu[11]'[0]\,\nu[12]'[0])\,(-Y+\nu[13]'[0])\big)+(-Y+\nu[9]'[0])\,\big(M\,(M^3+M\,Y^2-M\,Y\,\nu[11]'[0]-M\,Y\,\nu[12]'[0]+M\,\nu[11]'[0]\,\nu[12]'[0])+(-Y+\nu[10]'[0])\,(M^2+Y^2-Y\,\nu[11]'[0]-Y\,\nu[12]'[0]+\nu[11]'[0]\,\nu[12]'[0])\,(-Y+\nu[13]'[0])\big)\,(-Y+\nu[14]'[0])\Big)\,(-Y+\nu[15]'[0])\}$$

Out[∘]=

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Out[∘]=

$$(M^2+(Y-\nu[11]'[0])\,(Y-\nu[12]'[0]))\,(M^2+(Y-\nu[10]'[0])\,(Y-\nu[13]'[0]))\,(M^2+(Y-\nu[9]'[0])\,(Y-\nu[14]'[0]))\,(M^2+(Y-\nu[8]'[0])\,(Y-\nu[15]'[0]))$$

Out[∘]=

0.0473984 s

In[∘]:=
```
sumEvalscaDtyZrelationsA22Y = 1/7! D[caDtyZrelationsA22Y, {Y, 7}] /. {Y → 0} // Simplify
```

Out[∘]=

$$-\nu[8]'[0]-\nu[9]'[0]-\nu[10]'[0]-\nu[11]'[0]-\nu[12]'[0]-\nu[13]'[0]-\nu[14]'[0]-\nu[15]'[0]$$

In[∘]:=
```
sumEvalscaDtyZrelationsA21Y =
 1/7! D[caDtyZrelationsA21Y, {Y, 7}] /. {Y → 0} // Simplify
```

$$-\nu[0]'[t]-\nu[1]'[t]-\nu[2]'[t]-\nu[3]'[t]-\nu[4]'[t]-\nu[5]'[t]-\nu[6]'[t]-\nu[7]'[t]$$

In[∘]:=
```
sumEvalscaDtyZrelationsA22Y =
```

```
1/7! D[caDtyZrelationsA22Y, {Y, 7}] /. {Y → 0} // Simplify
```

```
-ν[8]′[t] - ν[9]′[t] - ν[10]′[t] - ν[11]′[t] -
 ν[12]′[t] - ν[13]′[t] - ν[14]′[t] - ν[15]′[t]
```

{Eigenvals, Eigenvecs} = Eigensystem[m];

Jordan decomposition of a square matrix m .
The result is a list {s, j} where s is a similarity matrix and j is the Jordan canonical form of m.
The *j* matrix is called the Jordan normal form or Jordan canonical form.
j is a special block-diagonal, upper-triangular matrix representation for any square matrix, featuring eigenvalues on the diagonal and ones on the superdiagonal (just above the main diagonal) for non-diagonalizable cases
{s, j} = JordanDecomposition[m]
m== s.j.Inverse[s]   **s.j.Inverse[s] - m**

```
In[∘]:= {sca2, jca2} = JordanDecomposition[caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]]]
```

Out[ ]=

$$\Big\{\Big\{\Big\{0,\,0,\,0,\,0,\,0,\,0,$$
$$-\frac{-\nu[8]'[0]+\nu[15]'[0]+\sqrt{-4\,M^2+\nu[8]'[0]^2-2\,\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}}{2\,M},$$
$$-\frac{-\nu[8]'[0]+\nu[15]'[0]-\sqrt{-4\,M^2+\nu[8]'[0]^2-2\,\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}}{2\,M}\Big\},$$
$$\Big\{0,\,0,\,0,\,0,\,-\frac{\nu[9]'[0]-\nu[14]'[0]-\sqrt{-4\,M^2+\nu[9]'[0]^2-2\,\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}}{2\,M},$$
$$-\frac{\nu[9]'[0]-\nu[14]'[0]+\sqrt{-4\,M^2+\nu[9]'[0]^2-2\,\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}}{2\,M},\,0,\,0\Big\},$$
$$\Big\{0,\,0,\,-\frac{\nu[10]'[0]-\nu[13]'[0]-\sqrt{-4\,M^2+\nu[10]'[0]^2-2\,\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}}{2\,M},$$
$$-\frac{\nu[10]'[0]-\nu[13]'[0]+\sqrt{-4\,M^2+\nu[10]'[0]^2-2\,\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}}{2\,M},\,0,\,0,\,0,\,0\Big\},$$
$$\Big\{-\frac{-\nu[11]'[0]+\nu[12]'[0]+\sqrt{-4\,M^2+\nu[11]'[0]^2-2\,\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}}{2\,M},$$
$$-\frac{-\nu[11]'[0]+\nu[12]'[0]-\sqrt{-4\,M^2+\nu[11]'[0]^2-2\,\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}}{2\,M},$$
$$0,\,0,\,0,\,0,\,0,\,0\Big\},\,\{1,\,1,\,0,\,0,\,0,\,0,\,0,\,0\},$$
$$\{0,\,0,\,1,\,1,\,0,\,0,\,0,\,0\},\,\{0,\,0,\,0,\,0,\,1,\,1,\,0,\,0\},\,\{0,\,0,\,0,\,0,\,0,\,0,\,1,\,1\}\Big\},$$
$$\Big\{\Big\{\frac{1}{2}\left(\nu[11]'[0]+\nu[12]'[0]-\sqrt{-4\,M^2+\nu[11]'[0]^2-2\,\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}\right),$$
$$0,\,0,\,0,\,0,\,0,\,0,\,0\Big\},$$
$$\Big\{0,\,\frac{1}{2}\left(\nu[11]'[0]+\nu[12]'[0]+\sqrt{-4\,M^2+\nu[11]'[0]^2-2\,\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}\right),$$
$$0,\,0,\,0,\,0,\,0,\,0\Big\},$$
$$\Big\{0,\,0,\,\frac{1}{2}\left(\nu[10]'[0]+\nu[13]'[0]-\sqrt{-4\,M^2+\nu[10]'[0]^2-2\,\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}\right),$$
$$0,\,0,\,0,\,0,\,0\Big\},$$
$$\Big\{0,\,0,\,0,\,\frac{1}{2}\left(\nu[10]'[0]+\nu[13]'[0]+\sqrt{-4\,M^2+\nu[10]'[0]^2-2\,\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}\right),$$
$$0,\,0,\,0,\,0\Big\},\,\Big\{0,\,0,\,0,\,0,$$
$$\frac{1}{2}\left(\nu[9]'[0]+\nu[14]'[0]-\sqrt{-4\,M^2+\nu[9]'[0]^2-2\,\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}\right),\,0,\,0,\,0\Big\},$$
$$\Big\{0,\,0,\,0,\,0,\,0,\,\frac{1}{2}\left(\nu[9]'[0]+\nu[14]'[0]+\sqrt{-4\,M^2+\nu[9]'[0]^2-2\,\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}\right),$$
$$0,\,0\Big\},\,\Big\{0,\,0,\,0,\,0,\,0,\,0,$$
$$\frac{1}{2}\left(\nu[8]'[0]+\nu[15]'[0]-\sqrt{-4\,M^2+\nu[8]'[0]^2-2\,\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}\right),\,0\Big\},\,\Big\{0,\,0,\,0,$$
$$0,\,0,\,0,\,0,\,\frac{1}{2}\left(\nu[8]'[0]+\nu[15]'[0]+\sqrt{-4\,M^2+\nu[8]'[0]^2-2\,\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}\right)\Big\}\Big\}\Big\}$$

In[∘]:= {valscaDtyZrelationsA22, vecscaDtyZrelationsA22} =
Eigensystem[caDtyZrelationsA2⟦9 ;; 16, 9 ;; 16⟧]

Out[∘]=

$$\left\{\left\{\frac{1}{2}\left(\nu[11]'[0]+\nu[12]'[0]-\sqrt{-4M^2+\nu[11]'[0]^2-2\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}\right),\right.\right.$$
$$\frac{1}{2}\left(\nu[11]'[0]+\nu[12]'[0]+\sqrt{-4M^2+\nu[11]'[0]^2-2\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[10]'[0]+\nu[13]'[0]-\sqrt{-4M^2+\nu[10]'[0]^2-2\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[10]'[0]+\nu[13]'[0]+\sqrt{-4M^2+\nu[10]'[0]^2-2\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[9]'[0]+\nu[14]'[0]-\sqrt{-4M^2+\nu[9]'[0]^2-2\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[9]'[0]+\nu[14]'[0]+\sqrt{-4M^2+\nu[9]'[0]^2-2\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[8]'[0]+\nu[15]'[0]-\sqrt{-4M^2+\nu[8]'[0]^2-2\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}\right),$$
$$\left.\frac{1}{2}\left(\nu[8]'[0]+\nu[15]'[0]+\sqrt{-4M^2+\nu[8]'[0]^2-2\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}\right)\right\},$$
$$\left\{\left\{0,0,0,-\frac{-\nu[11]'[0]+\nu[12]'[0]+\sqrt{-4M^2+\nu[11]'[0]^2-2\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}}{2M},\right.\right.$$
$$\left.1,0,0,0\right\},\left\{0,0,0,\right.$$
$$\left.-\frac{-\nu[11]'[0]+\nu[12]'[0]-\sqrt{-4M^2+\nu[11]'[0]^2-2\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}}{2M},1,0,0,0\right\},$$
$$\left\{0,0,-\frac{\nu[10]'[0]-\nu[13]'[0]-\sqrt{-4M^2+\nu[10]'[0]^2-2\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}}{2M},0,0,1,\right.$$
$$\left.0,0\right\},\left\{0,0,-\frac{\nu[10]'[0]-\nu[13]'[0]+\sqrt{-4M^2+\nu[10]'[0]^2-2\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}}{2M},\right.$$
$$\left.0,0,1,0,0\right\},$$
$$\left\{0,-\frac{\nu[9]'[0]-\nu[14]'[0]-\sqrt{-4M^2+\nu[9]'[0]^2-2\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}}{2M},0,0,0,0,1,\right.$$
$$\left.0\right\},\left\{0,-\frac{\nu[9]'[0]-\nu[14]'[0]+\sqrt{-4M^2+\nu[9]'[0]^2-2\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}}{2M},0,0,0,\right.$$
$$\left.0,1,0\right\},\left\{-\frac{-\nu[8]'[0]+\nu[15]'[0]+\sqrt{-4M^2+\nu[8]'[0]^2-2\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}}{2M},0,0,\right.$$
$$\left.0,0,0,0,1\right\},\left\{-\frac{-\nu[8]'[0]+\nu[15]'[0]-\sqrt{-4M^2+\nu[8]'[0]^2-2\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}}{2M},\right.$$
$$\left.\left.\left.0,0,0,0,0,0,1\right\}\right\}\right\}$$

In[∘]:= {valscaDtyZrelationsA21, vecscaDtyZrelationsA21} =
Eigensystem[caDtyZrelationsA2⟦1 ;; 8, 1 ;; 8⟧] //
FullSimplify[#, $e^{6\,a4[t]}$ > 0 && $e^{3\,a4[t]}$ > 0 && constraintVars] & //
FullSimplify[#, $e^{6\,a4[t]}$ > 0 && $e^{3\,a4[t]}$ > 0 && a4[t] > 0] &

Out[∘]=

$$\left\{\left\{\frac{1}{2}e^{-3\,a4[0]}\left(-e^{a4[0]}\,Q1\,a4'[0]-\right.\right.\right.$$

$$\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0])\Big),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\left(-e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0])\right),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\left(-e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0])\right),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\left(-e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0])\right),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\left(e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\right),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\left(e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\right),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\left(e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\right),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\left(e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\right)\Big\},$$

$$\Big\{\Big\{0,0,0,\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+e^{3\,a4[0]}\,(-\nu[3]'[0]+\nu[4]'[0])\right)}{2\,M},1,0,0,0\Big\},$$

$$\Big\{0,0,0,-\frac{\nu[3]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}-\nu[4]'[0]}{2\,M},1,0,0,0\Big\},$$

$$\Big\{0,0,-\frac{-\nu[2]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+\nu[5]'[0]}{2\,M},0,0,1,0,0\Big\},$$

$$\Big\{0,0,\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+e^{3\,a4[0]}\,(\nu[2]'[0]-\nu[5]'[0])\right)}{2\,M},0,0,1,0,0\Big\},$$

$$\Big\{0,-\frac{-\nu[1]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+\nu[6]'[0]}{2\,M},0,0,0,0,1,0\Big\},$$

$$\Big\{0,\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+e^{3\,a4[0]}\,(\nu[1]'[0]-\nu[6]'[0])\right)}{2\,M},0,0,0,0,1,0\Big\},\ \Big\{\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+e^{3\,a4[0]}\,(-\nu[0]'[0]+\nu[7]'[0])\right)}{2\,M},$$

$$0,0,0,0,0,0,1\Big\},$$

$$\left\{-\frac{\nu[0]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}-\nu[7]'[0]}{2\,M},0,0,0,0,0,0,1\right\}\}\}$$

In[∘]:= **{sca1, jca1} = JordanDecomposition[caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]]] // FullSimplify[#, $e^{6\,a4[t]}$ > 0 && $e^{3\,a4[t]}$ > 0 && constraintVars] & // FullSimplify[#, $e^{6\,a4[t]}$ > 0 && $e^{3\,a4[t]}$ > 0 && a4[t] > 0] &**

Out[∘]=

$$\Big\{\Big\{\Big\{0,0,0,0,0,0,\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+e^{3\,a4[0]}(-\nu[0]'[0]+\nu[7]'[0])\right)}{2\,M},$$
$$-\frac{\nu[0]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}-\nu[7]'[0]}{2\,M}\Big\},$$
$$\Big\{0,0,0,0,-\frac{-\nu[1]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+\nu[6]'[0]}{2\,M},$$
$$\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+e^{3\,a4[0]}(\nu[1]'[0]-\nu[6]'[0])\right)}{2\,M},0,0\Big\},$$
$$\Big\{0,0,-\frac{-\nu[2]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+\nu[5]'[0]}{2\,M},$$
$$\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+e^{3\,a4[0]}(\nu[2]'[0]-\nu[5]'[0])\right)}{2\,M},0,0,0,0\Big\},$$
$$\Big\{\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+e^{3\,a4[0]}(-\nu[3]'[0]+\nu[4]'[0])\right)}{2\,M},$$
$$-\frac{\nu[3]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}-\nu[4]'[0]}{2\,M},0,0,0,0,0,0\Big\},$$
$$\{1,1,0,0,0,0,0,0\},\{0,0,1,1,0,0,0,0\},$$
$$\{0,0,0,0,1,1,0,0\},\{0,0,0,0,0,0,1,1\}\Big\},$$
$$\Big\{\Big\{\frac{1}{2}e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+$$
$$e^{3\,a4[0]}(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0])\Big),0,0,0,0,0,0,0\Big\},$$
$$\Big\{0,\frac{1}{2}e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+$$
$$e^{3\,a4[0]}(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0])\Big),0,0,0,0,0,0\Big\},$$
$$\Big\{0,0,\frac{1}{2}e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+$$
$$e^{3\,a4[0]}(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0])\Big),0,0,0,0,0\Big\},$$
$$\Big\{0,0,0,\frac{1}{2}e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+$$
$$e^{3\,a4[0]}(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0])\Big),0,0,0,0\Big\},$$
$$\Big\{0,0,0,0,\frac{1}{2}e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+$$

$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\Big),0,0,0\Big\},$$

$$\Big\{0,0,0,0,0,\frac{1}{2}e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\Big),0,0\Big\},$$

$$\Big\{0,0,0,0,0,0,\frac{1}{2}e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\Big),0\Big\},$$

$$\Big\{0,0,0,0,0,0,0,\frac{1}{2}e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\Big)\Big\}\Big\}\Big\}$$

In[ ]:=
```
eigenValues = Union[Flatten[{valscaDtyZrelationsA21, valscaDtyZrelationsA22}]]
eigenValues00 = FullSimplify[Normal[Series[#, {t, 0, 0}]]] & /@ eigenValues
```

Out[ ]=

$$\Big\{\frac{1}{2} e^{-3\,a4[0]} \Big(-e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[3]'[0] - \nu[4]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[3]'[0] + \nu[4]'[0])\Big),$$

$$\frac{1}{2} e^{-3\,a4[0]} \Big(-e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[3]'[0] - \nu[4]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[3]'[0] + \nu[4]'[0])\Big),$$

$$\frac{1}{2} e^{-3\,a4[0]} \Big(-e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[2]'[0] - \nu[5]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[2]'[0] + \nu[5]'[0])\Big),$$

$$\frac{1}{2} e^{-3\,a4[0]} \Big(-e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[2]'[0] - \nu[5]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[2]'[0] + \nu[5]'[0])\Big),$$

$$\frac{1}{2} e^{-3\,a4[0]} \Big(e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[1]'[0] - \nu[6]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[1]'[0] + \nu[6]'[0])\Big),$$

$$\frac{1}{2} e^{-3\,a4[0]} \Big(e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[1]'[0] - \nu[6]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[1]'[0] + \nu[6]'[0])\Big),$$

$$\frac{1}{2} e^{-3\,a4[0]} \Big(e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[0]'[0] - \nu[7]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[0]'[0] + \nu[7]'[0])\Big),$$

$$\frac{1}{2} e^{-3\,a4[0]} \Big(e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[0]'[0] - \nu[7]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[0]'[0] + \nu[7]'[0])\Big),$$

$$\frac{1}{2}\Big(\nu[11]'[0] + \nu[12]'[0] - \sqrt{-4\,M^2 + \nu[11]'[0]^2 - 2\,\nu[11]'[0]\,\nu[12]'[0] + \nu[12]'[0]^2}\Big),$$

$$\frac{1}{2}\Big(\nu[11]'[0] + \nu[12]'[0] + \sqrt{-4\,M^2 + \nu[11]'[0]^2 - 2\,\nu[11]'[0]\,\nu[12]'[0] + \nu[12]'[0]^2}\Big),$$

$$\frac{1}{2}\Big(\nu[10]'[0] + \nu[13]'[0] - \sqrt{-4\,M^2 + \nu[10]'[0]^2 - 2\,\nu[10]'[0]\,\nu[13]'[0] + \nu[13]'[0]^2}\Big),$$

$$\frac{1}{2}\Big(\nu[10]'[0] + \nu[13]'[0] + \sqrt{-4\,M^2 + \nu[10]'[0]^2 - 2\,\nu[10]'[0]\,\nu[13]'[0] + \nu[13]'[0]^2}\Big),$$

$$\frac{1}{2}\Big(\nu[9]'[0] + \nu[14]'[0] - \sqrt{-4\,M^2 + \nu[9]'[0]^2 - 2\,\nu[9]'[0]\,\nu[14]'[0] + \nu[14]'[0]^2}\Big),$$

$$\frac{1}{2}\Big(\nu[9]'[0] + \nu[14]'[0] + \sqrt{-4\,M^2 + \nu[9]'[0]^2 - 2\,\nu[9]'[0]\,\nu[14]'[0] + \nu[14]'[0]^2}\Big),$$

$$\frac{1}{2}\Big(\nu[8]'[0] + \nu[15]'[0] - \sqrt{-4\,M^2 + \nu[8]'[0]^2 - 2\,\nu[8]'[0]\,\nu[15]'[0] + \nu[15]'[0]^2}\Big),$$

$$\frac{1}{2}\Big(\nu[8]'[0] + \nu[15]'[0] + \sqrt{-4\,M^2 + \nu[8]'[0]^2 - 2\,\nu[8]'[0]\,\nu[15]'[0] + \nu[15]'[0]^2}\Big)\Big\}$$

Out[ ]=

$$\left\{\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0])\right),\right.$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\right),$$

$$\frac{1}{2}\left(\nu[11]'[0] - \sqrt{-4\,M^2+(\nu[11]'[0]-\nu[12]'[0])^2} + \nu[12]'[0]\right),$$

$$\frac{1}{2}\left(\nu[11]'[0] + \sqrt{-4\,M^2+(\nu[11]'[0]-\nu[12]'[0])^2} + \nu[12]'[0]\right),$$

$$\frac{1}{2}\left(\nu[10]'[0] - \sqrt{-4\,M^2+(\nu[10]'[0]-\nu[13]'[0])^2} + \nu[13]'[0]\right),$$

$$\frac{1}{2}\left(\nu[10]'[0] + \sqrt{-4\,M^2+(\nu[10]'[0]-\nu[13]'[0])^2} + \nu[13]'[0]\right),$$

$$\frac{1}{2}\left(\nu[9]'[0] - \sqrt{-4\,M^2+(\nu[9]'[0]-\nu[14]'[0])^2} + \nu[14]'[0]\right),$$

$$\frac{1}{2}\left(\nu[9]'[0] + \sqrt{-4\,M^2+(\nu[9]'[0]-\nu[14]'[0])^2} + \nu[14]'[0]\right),$$

$$\frac{1}{2}\left(\nu[8]'[0] - \sqrt{-4\,M^2+(\nu[8]'[0]-\nu[15]'[0])^2} + \nu[15]'[0]\right),$$

$$\left.\frac{1}{2}\left(\nu[8]'[0] + \sqrt{-4\,M^2+(\nu[8]'[0]-\nu[15]'[0])^2} + \nu[15]'[0]\right)\right\}$$

If superluminal inflation/deflation then pair of universes with MASSES ± M?

THE FOLLOWING IS CONSISTENT WITH UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES ± M;

Note that we are ONLY looking for superluminal inflation or deflation type solutions,
and NOT solutions that are even/odd functions of t
(like Cos[*v[j]*'[0] * t], Sin[*v[j]*'[0] * t], Sech[*v[j]*'[0] * t], Tanh[*v[j]*'[0] * t], ...),
which might also have ± M type eigenvalues,
or even solutions involving the well-known special functions that might also have ± M type eigenvalues.

In[∘]:=
```
sDtv3Dtv4 = Solve[And @@ Thread[0 == eigenValues00[[1 ;; 2]]], {v[3]′[0], v[4]′[0]}] //
  TrigToExp(*//FullSimplify*)
```

Out[∘]=

$$\left\{\left\{v[3]'[0]\to M-\frac{1}{2}\,Q1\,a4'[0]+\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ v[4]'[0]\to -M-\frac{1}{2}\,Q1\,a4'[0]+\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\},\left\{v[3]'[0]\to -M-\frac{1}{2}\,Q1\,a4'[0]+\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ v[4]'[0]\to M-\frac{1}{2}\,Q1\,a4'[0]+\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\}\right\}$$

In[∘]:=
```
sDtv2Dtv5 = Solve[And @@ Thread[0 == eigenValues00[[3 ;; 4]]], {v[2]′[0], v[5]′[0]}] //
  TrigToExp(*//FullSimplify*)
```

Out[∘]=

$$\left\{\left\{v[2]'[0]\to M-\frac{1}{2}\,Q1\,a4'[0]+\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ v[5]'[0]\to -M-\frac{1}{2}\,Q1\,a4'[0]+\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\},\left\{v[2]'[0]\to -M-\frac{1}{2}\,Q1\,a4'[0]+\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ v[5]'[0]\to M-\frac{1}{2}\,Q1\,a4'[0]+\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\}\right\}$$

In[∘]:=
```
sDtv1Dtv6 = Solve[And @@ Thread[0 == eigenValues00[[5 ;; 6]]], {v[1]′[0], v[6]′[0]}] //
  TrigToExp(*//FullSimplify*)
```

Out[∘]=

$$\left\{\left\{v[1]'[0]\to M+\frac{1}{2}\,Q1\,a4'[0]-\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ v[6]'[0]\to -M+\frac{1}{2}\,Q1\,a4'[0]-\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\},\left\{v[1]'[0]\to -M+\frac{1}{2}\,Q1\,a4'[0]-\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ v[6]'[0]\to M+\frac{1}{2}\,Q1\,a4'[0]-\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\}\right\}$$

In[ ]:=
```
sDtv0Dtv7 = Solve[And @@ Thread[0 == eigenValues00[[7 ;; 8]]], {ν[0]′[0], ν[7]′[0]}] //
  TrigToExp(*//FullSimplify*)
```

Out[ ]=

$$\left\{\left\{\nu[0]'[0] \to M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[7]'[0] \to -M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\},\right.$$
$$\left.\left\{\nu[0]'[0] \to -M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[7]'[0] \to M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\}\right\}$$

In[ ]:=
```
sDtv11Dtv12 =
 Solve[And @@ Thread[0 == eigenValues00[[9 ;; 10]]], {ν[11]′[0], ν[12]′[0]}] // FullSimplify
```

Out[ ]=

$$\{\{\nu[11]'[0] \to M,\ \nu[12]'[0] \to -M\},\ \{\nu[11]'[0] \to -M,\ \nu[12]'[0] \to M\}\}$$

In[ ]:=
```
sDtv10Dtv13 =
 Solve[And @@ Thread[0 == eigenValues00[[11 ;; 12]]], {ν[10]′[0], ν[13]′[0]}] // FullSimplify
```

Out[ ]=

$$\{\{\nu[10]'[0] \to M,\ \nu[13]'[0] \to -M\},\ \{\nu[10]'[0] \to -M,\ \nu[13]'[0] \to M\}\}$$

In[ ]:=
```
sDtv9Dtv14 =
 Solve[And @@ Thread[0 == eigenValues00[[13 ;; 14]]], {ν[9]′[0], ν[14]′[0]}] // FullSimplify
```

Out[ ]=

$$\{\{\nu[9]'[0] \to M,\ \nu[14]'[0] \to -M\},\ \{\nu[9]'[0] \to -M,\ \nu[14]'[0] \to M\}\}$$

In[ ]:=
```
sDtv8Dtv15 =
 Solve[And @@ Thread[0 == eigenValues00[[15 ;; 16]]], {ν[8]′[0], ν[15]′[0]}] // FullSimplify
```

Out[ ]=

$$\{\{\nu[8]'[0] \to M,\ \nu[15]'[0] \to -M\},\ \{\nu[8]'[0] \to -M,\ \nu[15]'[0] \to M\}\}$$

FYI: wave functions?

In[•]:=
```
{# + 0, 7 - #} & /@ Range[0, 3]
ToExpression["sDtν" <> ToString[#[[1]]] <> "Dtν" <> ToString[#[[2]]]] & /@ %;
plusM03 = Flatten[#[[1]] & /@ %]
minusM03 = Flatten[#[[2]] & /@ %%]
```

Out[•]=

{{0, 7}, {1, 6}, {2, 5}, {3, 4}}

Out[•]=

$$\left\{\nu[0]'[0] \to M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[7]'[0] \to -M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\right.$$
$$\nu[1]'[0] \to M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[6]'[0] \to -M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],$$
$$\nu[2]'[0] \to M - \frac{1}{2}\,Q1\,a4'[0] + \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[5]'[0] \to -M - \frac{1}{2}\,Q1\,a4'[0] + \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],$$
$$\left.\nu[3]'[0] \to M - \frac{1}{2}\,Q1\,a4'[0] + \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[4]'[0] \to -M - \frac{1}{2}\,Q1\,a4'[0] + \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\}$$

Out[•]=

$$\left\{\nu[0]'[0] \to -M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[7]'[0] \to M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\right.$$
$$\nu[1]'[0] \to -M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[6]'[0] \to M + \frac{1}{2}\,Q1\,a4'[0] - \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],$$
$$\nu[2]'[0] \to -M - \frac{1}{2}\,Q1\,a4'[0] + \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[5]'[0] \to M - \frac{1}{2}\,Q1\,a4'[0] + \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],$$
$$\left.\nu[3]'[0] \to -M - \frac{1}{2}\,Q1\,a4'[0] + \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0],\ \nu[4]'[0] \to M - \frac{1}{2}\,Q1\,a4'[0] + \frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right\}$$

In[•]:=
```
{# + 8, 15 - #} & /@ Range[0, 3]
ToExpression["sDtν" <> ToString[#[[1]]] <> "Dtν" <> ToString[#[[2]]]] & /@ %
plusM811 = Flatten[#[[1]] & /@ %]
minusM811 = Flatten[#[[2]] & /@ %%]
```

Out[•]=

{{8, 15}, {9, 14}, {10, 13}, {11, 12}}

Out[•]=

$$\{\{\{\nu[8]'[0] \to M,\ \nu[15]'[0] \to -M\},\ \{\nu[8]'[0] \to -M,\ \nu[15]'[0] \to M\}\},$$
$$\{\{\nu[9]'[0] \to M,\ \nu[14]'[0] \to -M\},\ \{\nu[9]'[0] \to -M,\ \nu[14]'[0] \to M\}\},$$
$$\{\{\nu[10]'[0] \to M,\ \nu[13]'[0] \to -M\},\ \{\nu[10]'[0] \to -M,\ \nu[13]'[0] \to M\}\},$$
$$\{\{\nu[11]'[0] \to M,\ \nu[12]'[0] \to -M\},\ \{\nu[11]'[0] \to -M,\ \nu[12]'[0] \to M\}\}\}$$

Out[•]=

$$\{\nu[8]'[0] \to M,\ \nu[15]'[0] \to -M,\ \nu[9]'[0] \to M,\ \nu[14]'[0] \to -M,$$
$$\nu[10]'[0] \to M,\ \nu[13]'[0] \to -M,\ \nu[11]'[0] \to M,\ \nu[12]'[0] \to -M\}$$

Out[•]=

$$\{\nu[8]'[0] \to -M,\ \nu[15]'[0] \to M,\ \nu[9]'[0] \to -M,\ \nu[14]'[0] \to M,$$
$$\nu[10]'[0] \to -M,\ \nu[13]'[0] \to M,\ \nu[11]'[0] \to -M,\ \nu[12]'[0] \to M\}$$

In[∘]:= **plusM03plusM811 = Flatten[{plusM03, plusM811}]**

Out[∘]=

$$\{\nu[0]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[7]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[1]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[6]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[2]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[5]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[3]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[4]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[8]'[0] \to M,\ \nu[15]'[0] \to -M,$$
$$\nu[9]'[0] \to M,\ \nu[14]'[0] \to -M,\ \nu[10]'[0] \to M,\ \nu[13]'[0] \to -M,\ \nu[11]'[0] \to M,\ \nu[12]'[0] \to -M\}$$

In[∘]:= **minusM03minusM811 = Flatten[{minusM03, minusM811}]**

Out[∘]=

$$\{\nu[0]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[7]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[1]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[6]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[2]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[5]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[3]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[4]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[8]'[0] \to -M,\ \nu[15]'[0] \to M,$$
$$\nu[9]'[0] \to -M,\ \nu[14]'[0] \to M,\ \nu[10]'[0] \to -M,\ \nu[13]'[0] \to M,\ \nu[11]'[0] \to -M,\ \nu[12]'[0] \to M\}$$

In[ ]:=
```
sfψ16t =
  Table[f16[j] → ToExpression["((Y[" <> ToString[j] <> "][6*H*#1,H*#2]" <> "Exp[" <> ToString[
      "(ν[" <> ToString[j] <> "])'[0]"] <> "*#2]" <> "*1/Sqrt[Sin[6*H*#1]])&)"], {j, 0, 15}]
```

Out[ ]=

$$\left\{f16[0] \to \left(\frac{Y[0][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[0]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),\right.$$
$$f16[1] \to \left(\frac{Y[1][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[1]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[2] \to \left(\frac{Y[2][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[2]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[3] \to \left(\frac{Y[3][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[3]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[4] \to \left(\frac{Y[4][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[4]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[5] \to \left(\frac{Y[5][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[5]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[6] \to \left(\frac{Y[6][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[6]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[7] \to \left(\frac{Y[7][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[7]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[8] \to \left(\frac{Y[8][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[8]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[9] \to \left(\frac{Y[9][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[9]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[10] \to \left(\frac{Y[10][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[10]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[11] \to \left(\frac{Y[11][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[11]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[12] \to \left(\frac{Y[12][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[12]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[13] \to \left(\frac{Y[13][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[13]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$f16[14] \to \left(\frac{Y[14][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[14]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right),$$
$$\left.f16[15] \to \left(\frac{Y[15][6\,H\,\#1,\,H\,\#2]\,\mathrm{Exp}[\nu[15]'[0]\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\,\&\right)\right\}$$

In[•]:= sfψ16Aa

Out[•]=

$$\left\{f16[0]\to\left(\frac{Z[0][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\,f16[1]\to\left(\frac{Z[1][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\right.$$
$$f16[2]\to\left(\frac{Z[2][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\,f16[3]\to\left(\frac{Z[3][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),$$
$$f16[4]\to\left(\frac{Z[4][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\,f16[5]\to\left(\frac{Z[5][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),$$
$$f16[6]\to\left(\frac{Z[6][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\,f16[7]\to\left(\frac{Z[7][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),$$
$$f16[8]\to\left(\frac{Z[8][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\,f16[9]\to\left(\frac{Z[9][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),$$
$$f16[10]\to\left(\frac{Z[10][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\,f16[11]\to\left(\frac{Z[11][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),$$
$$f16[12]\to\left(\frac{Z[12][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\,f16[13]\to\left(\frac{Z[13][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),$$
$$\left.f16[14]\to\left(\frac{Z[14][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right),\,f16[15]\to\left(\frac{Z[15][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\,\&\right)\right\}$$

In[•]:=
```
sfψ16t1t = Table[
   f16[j] → (((ToExpression["((Y[" <> ToString[j] <> "][1][6*H*#1,H*#2]" <> "Exp[" <> ToString[
            "(ν[" <> ToString[j] <> "])'[0] "] <> "*#2]" <> "*1/Sqrt[Sin[6*H*#1]]))"] /.
         plusM03plusM811) + (ToExpression["((Y[" <> ToString[j] <> "][2][6*H*#1,H*#2]" <>
            "Exp[" <> ToString["(ν[" <> ToString[j] <> "])'[0] "] <> "*#2]" <>
            "*1/Sqrt[Sin[6*H*#1]]))"] /. minusM03minusM811))), {j, 0, 15}]
sfψ16TRY = Table[Block[{s}, s = %[[j]];
    s[[1]] → ToExpression["((" <> ToString[FullForm[s[[2]]]] <> ")&)"]], {j, 1, Length[%]}]
```

Out[•]=

$$\left\{f16[0]\to\frac{e^{\#2\left(M+\frac{1}{2}\,Q1\,a4'[0]-\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[0][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}+\right.$$
$$\frac{e^{\#2\left(-M+\frac{1}{2}\,Q1\,a4'[0]-\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[0][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}},$$
$$f16[1]\to\frac{e^{\#2\left(M+\frac{1}{2}\,Q1\,a4'[0]-\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[1][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}+$$
$$\frac{e^{\#2\left(-M+\frac{1}{2}\,Q1\,a4'[0]-\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[1][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}},$$
$$f16[2]\to\frac{e^{\#2\left(M-\frac{1}{2}\,Q1\,a4'[0]+\frac{1}{2}\,e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[2][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}+$$

$$\frac{e^{\#2 \left(-M-\frac{1}{2} Q1\, a4'[0]+\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[2][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[3] \to \frac{e^{\#2 \left(M-\frac{1}{2} Q1\, a4'[0]+\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[3][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} +$$

$$\frac{e^{\#2 \left(-M-\frac{1}{2} Q1\, a4'[0]+\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[3][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[4] \to \frac{e^{\#2 \left(-M-\frac{1}{2} Q1\, a4'[0]+\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[4][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} +$$

$$\frac{e^{\#2 \left(M-\frac{1}{2} Q1\, a4'[0]+\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[4][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[5] \to \frac{e^{\#2 \left(-M-\frac{1}{2} Q1\, a4'[0]+\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[5][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} +$$

$$\frac{e^{\#2 \left(M-\frac{1}{2} Q1\, a4'[0]+\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[5][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[6] \to \frac{e^{\#2 \left(-M+\frac{1}{2} Q1\, a4'[0]-\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[6][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} +$$

$$\frac{e^{\#2 \left(M+\frac{1}{2} Q1\, a4'[0]-\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[6][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[7] \to \frac{e^{\#2 \left(-M+\frac{1}{2} Q1\, a4'[0]-\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[7][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} +$$

$$\frac{e^{\#2 \left(M+\frac{1}{2} Q1\, a4'[0]-\frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right)} Y[7][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[8] \to \frac{e^{M \#2} Y[8][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} + \frac{e^{-M \#2} Y[8][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[9] \to \frac{e^{M \#2} Y[9][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} + \frac{e^{-M \#2} Y[9][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[10] \to \frac{e^{M \#2} Y[10][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} + \frac{e^{-M \#2} Y[10][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[11] \to \frac{e^{M \#2} Y[11][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} + \frac{e^{-M \#2} Y[11][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$f16[12] \to \frac{e^{-M \#2} Y[12][1][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}} + \frac{e^{M \#2} Y[12][2][6 H \#1, H \#2]}{\sqrt{Sin[6 H \#1]}},$$

$$\text{f16}[13]\to\frac{e^{-M\,\#2}\,Y[13][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\text{Sin}[6\,H\,\#1]}}+\frac{e^{M\,\#2}\,Y[13][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\text{Sin}[6\,H\,\#1]}},$$

$$\text{f16}[14]\to\frac{e^{-M\,\#2}\,Y[14][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\text{Sin}[6\,H\,\#1]}}+\frac{e^{M\,\#2}\,Y[14][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\text{Sin}[6\,H\,\#1]}},$$

$$\text{f16}[15]\to\frac{e^{-M\,\#2}\,Y[15][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\text{Sin}[6\,H\,\#1]}}+\frac{e^{M\,\#2}\,Y[15][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\text{Sin}[6\,H\,\#1]}}\Big\}$$

Out[∘]=

$$\Big\{\text{f16}[0]\to\Big(e^{\#2\,(M+\text{Rational}[1,2]\,Q1\,a4'[0]+\text{Rational}[-1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[0][1][6\,H\,\#1,\,H\,\#2]+e^{\#2\,(-M+\text{Rational}[1,2]\,Q1\,a4'[0]+\text{Rational}[-1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[0][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[1]\to\Big(e^{\#2\,(M+\text{Rational}[1,2]\,Q1\,a4'[0]+\text{Rational}[-1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[1][1][6\,H\,\#1,\,H\,\#2]+e^{\#2\,(-M+\text{Rational}[1,2]\,Q1\,a4'[0]+\text{Rational}[-1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[1][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[2]\to\Big(e^{\#2\,(M+\text{Rational}[-1,2]\,Q1\,a4'[0]+\text{Rational}[1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[2][1][6\,H\,\#1,\,H\,\#2]+e^{\#2\,(-M+\text{Rational}[-1,2]\,Q1\,a4'[0]+\text{Rational}[1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[2][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[3]\to\Big(e^{\#2\,(M+\text{Rational}[-1,2]\,Q1\,a4'[0]+\text{Rational}[1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[3][1][6\,H\,\#1,\,H\,\#2]+e^{\#2\,(-M+\text{Rational}[-1,2]\,Q1\,a4'[0]+\text{Rational}[1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[3][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[4]\to\Big(e^{\#2\,(-M+\text{Rational}[-1,2]\,Q1\,a4'[0]+\text{Rational}[1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[4][1][6\,H\,\#1,\,H\,\#2]+e^{\#2\,(M+\text{Rational}[-1,2]\,Q1\,a4'[0]+\text{Rational}[1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[4][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[5]\to\Big(e^{\#2\,(-M+\text{Rational}[-1,2]\,Q1\,a4'[0]+\text{Rational}[1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[5][1][6\,H\,\#1,\,H\,\#2]+e^{\#2\,(M+\text{Rational}[-1,2]\,Q1\,a4'[0]+\text{Rational}[1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[5][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[6]\to\Big(e^{\#2\,(-M+\text{Rational}[1,2]\,Q1\,a4'[0]+\text{Rational}[-1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[6][1][6\,H\,\#1,\,H\,\#2]+e^{\#2\,(M+\text{Rational}[1,2]\,Q1\,a4'[0]+\text{Rational}[-1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[6][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[7]\to\Big(e^{\#2\,(-M+\text{Rational}[1,2]\,Q1\,a4'[0]+\text{Rational}[-1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[7][1][6\,H\,\#1,\,H\,\#2]+e^{\#2\,(M+\text{Rational}[1,2]\,Q1\,a4'[0]+\text{Rational}[-1,2]\,e^{-2\,a4[0]}\,Q1\,a4'[0])}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[7][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[8]\to\Big(e^{M\,\#2}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[8][1][6\,H\,\#1,\,H\,\#2]+e^{-M\,\#2}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[8][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[9]\to\Big(e^{M\,\#2}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[9][1][6\,H\,\#1,\,H\,\#2]+e^{-M\,\#2}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[9][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[10]\to\Big(e^{M\,\#2}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[10][1][6\,H\,\#1,\,H\,\#2]+e^{-M\,\#2}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[10][2][6\,H\,\#1,\,H\,\#2]\;\&\Big),$$

$$\text{f16}[11]\to\Big(e^{M\,\#2}\,\text{Sin}[6\,H\,\#1]^{\text{Rational}[-1,2]}\,Y[11][1][6\,H\,\#1,\,H\,\#2]+$$

```
    ⅇ^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[11][2][6 H #1, H #2] &),
f16[12] → (ⅇ^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[12][1][6 H #1, H #2] +
    ⅇ^(M #2) Sin[6 H #1]^Rational[-1,2] Y[12][2][6 H #1, H #2] &),
f16[13] → (ⅇ^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[13][1][6 H #1, H #2] +
    ⅇ^(M #2) Sin[6 H #1]^Rational[-1,2] Y[13][2][6 H #1, H #2] &),
f16[14] → (ⅇ^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[14][1][6 H #1, H #2] +
    ⅇ^(M #2) Sin[6 H #1]^Rational[-1,2] Y[14][2][6 H #1, H #2] &),
f16[15] → (ⅇ^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[15][1][6 H #1, H #2] +
    ⅇ^(M #2) Sin[6 H #1]^Rational[-1,2] Y[15][2][6 H #1, H #2] &)}
```