

NOTES

HYPOTHESIS : If ,
employing the Einstein eqs (or Einstein - Lovelock eqs) ,
superluminal inflation / deflation exists,
then at time $x_4 = 0$ (before the particles of the standard model exist)
a pair of universes with $MASSES \pm M$ is created
(i.e., universes are created in pairs)
Dark matter / dark energy may be
related to wave function overlap of these two universes

THE FOLLOWING SCRATCH IS CONSISTENT WITH
UNIVERSES THAT ARE CREATED IN PAIRS, WITH $MASSES \pm M$.

Note that we are ONLY looking for
superluminal inflation or deflation type solutions ,
and NOT solutions that are even / odd functions of t
(like $\cos[\nu[j]^\theta * t]$, $\sin[\nu[j]^\theta * t]$,
 $\text{Sech}[\nu[j]^\theta * t]$, $\tanh[\nu[j]^\theta * t]$, ...),
which also have $\pm M$ type eigenvalues ,
or even solutions involving the
well -
known special functions that might also have $\pm M$ type parameters.

Bigger Bang:

Question: Are Universe (s) of masses $\pm M$ created in pairs at time $x_4 = 0$
(before the particles of the standard model exist) ?

Scratch work

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Do not read if you are irascible; I apologize for typos and duplications and

git clone <https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git>

“git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git”

Ever wonder why there are 3 space dimensions and 1 time dimension?

Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in \text{Nature}$?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\| = \|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over \mathbb{R}) real numbers \mathbb{R} , the 2-dim complex numbers, the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in "A Remarkable Representation of the 3+2 de Sitter Group" by P.A.M.Dirac, J.Math.Phys.4,901–909 (1963).

Comment: The term "reduced Brauer–Weyl generators", below, refers to a specific set of irreducible generators for certain complex Clifford algebras, particularly in the context of mathematical physics and the study of Dirac spinors. These generators were introduced by **E.A. Lord** (E. A. Lord. "The Dirac spinor in six dimensions". Mathematical Proceedings of the Cambridge Philosophical Society, vol. 64, pp. 765–778, 1968) and are described as "reduced" because they form an irreducible representation of the Clifford algebra C_{2n} from $C_{2(n-1)}$.

Here, we employ Lord's idea of reduced Brauer–Weyl generators to extend Dirac's 4×4 gamma matrices into this notebook's real tau8 8×8 and tau16 16×16 matrices,

which are employed to write down the 'Dirac Equation for the Universe',

[remember, this is before the particles of the standard model exist]

formulated in terms of a $O(4, 4)$ spinor Ψ_6

(Ψ_6 AKA '**WAVE FUNCTION OF the UNIVERSE**', which could possibly be named something more pompous).

Then this equation is used to

[WARNING: syncope, presyncope AHEAD]

couple the **WAVE FUNCTION OF the UNIVERSE** to Gravity.

THIS WORK IS PARTIALLY BASED ON :

JOURNAL OF MATHEMATICAL PHYSICS

VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. DIRAC

*Cambridge University, Cambridge, England and
Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

Proc. Camb. Phil. Soc. (1968), **64**, 765
 PCPS 64–96
Printed in Great Britain

765

The Dirac spinor in six dimensions

By E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

JOURNAL OF MATHEMATICAL PHYSICS **51**, 042501 (2010)

Second gravity

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IL NUOVO CIMENTO

VOL. 105 B, N. 1

Gennaio 1990

On the Structure of the Split Octonion Algebra.

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(ricevuto il 24 Luglio 1989)

Identities satisfied by the generators of the Dirac algebra

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(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin- $\frac{1}{2}$ wave equation

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(Received 24 January 1984; accepted for publication 16 November 1984)

Here we use [spacetime coordinates](#) that are selected so that we are somewhat consistent with the [exact libraries](#),

(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):

[Cartesian coordinates:](#)

x_0 = hidden space (a small circle); x_1, x_2, x_3 are the usual 3-space coords;

x_4 = time coord,

and x_5, x_6, x_7 = superluminal [deflating](#) time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{SO(4, 4; \mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M8 , whose tangent bundle has

$\overline{SO(4, 4; \mathbb{R})} \approx \text{Spin}(4,4;\mathbb{R})$

as iso group, and in which

3 of the 4 space dimensions [superluminally INFLATE](#),

3 of the 4 time dimensions [superluminally DEFLATE](#) (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),

and the 4th space dim, x_0 , curls up into a ring ([hence a particle whose wave function penetrates this ring acquires a mass contribution](#)).

In passing we remark that an analogous construction may be defined on an octonion space of [one time dimension and seven space dimensions](#), if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

[Unsolved problem 1](#) (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group

$SU(3) \otimes SU(2)_L \otimes U(1)$;

The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

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The Einstein - Lovelock vacuum field equations are

$$A_a^b = 0, \text{ where}$$

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta_{jj_1 \dots j_{2k}}^{hh_1 \dots h_{2k}} R^{j_1 j_2}_{\quad h_1 h_2} \dots R^{j_{2k-1} j_{2k}}_{\quad h_{2k-1} h_{2k}} + \lambda \sqrt{g} g^{lh}, \quad (4.3)$$

where $\alpha_{(k)}, \lambda$ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

$$\text{and } m-1 = \frac{n}{2} - 1 = 3.$$

Citation: Tensors, Differential Forms,
and Variational Principles (Dover Books on Mathematics),
by David Lovelock and Hanno Rund

Let $\{w1, w2, w3, \Delta\}$ be pure numbers;

then by Eq. [4.38] the vacuum equations are

$$0 = -\Delta + H^{-2} w1 \text{ Lovelock1} + H^{-4} w2 \text{ Lovelock2} + H^{-6} w3 \text{ Lovelock3}$$

in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein - Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre - Hubble parameter. For now, let's pretend that they are independent.

NOTES:

Under the spacetime coordinate transformation

$$x^k \mapsto \bar{x}^k (x^j)$$

according to

$$\Psi16(x) \mapsto \bar{\Psi}16(\bar{x}) = S(x(\bar{x})) \cdot \Psi16(x(\bar{x})),$$

$$\text{i.e., } \overline{\Psi 16^{(a)}(\bar{x})} = \left(S_{(b)}^{(a)} \Psi 16^{(b)}(x) \right)$$

Therefore,

$$\frac{\partial}{\partial \bar{x}^k} \overline{\Psi 16(\bar{x})} = \frac{\partial x^j}{\partial \bar{x}^k} \frac{\partial}{\partial x^j} S(x) \cdot \Psi 16(x) = \\ \frac{\partial x^j}{\partial \bar{x}^k} \left\{ S(x) \cdot \frac{\partial}{\partial x^j} \Psi 16(x) + \left[\frac{\partial}{\partial x^j} S(x) \right] \cdot \Psi 16(x) \right\}$$

$$\text{Identify the Octad connection : } \frac{\partial}{\partial x^j} S(x) =$$

$$- S \cdot \Gamma_j(x) + \frac{\partial \bar{x}^k}{\partial x^j} \bar{\Gamma}_k(\bar{x}) \cdot S$$

hence

$$\frac{\partial}{\partial \bar{x}^k} \overline{\Psi 16(\bar{x})} = \bar{\Gamma}_k \cdot \overline{\Psi 16(\bar{x})} = \\ S(x(\bar{x})) \cdot \frac{\partial x^j}{\partial \bar{x}^k} \left\{ \frac{\partial}{\partial x^j} \Psi 16(x(\bar{x})) - \Gamma_j \cdot \Psi 16(x(\bar{x})) \right\}$$

or

$$\left(1_{16 \times 16} \frac{\partial}{\partial \bar{x}^k} - \bar{\Gamma}_k \right) \cdot \overline{\Psi 16(\bar{x})} = \\ S(x(\bar{x})) \cdot \left[\frac{\partial x^j}{\partial \bar{x}^k} \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^j} - \Gamma_j \right\} \cdot \Psi 16(x(\bar{x})) \right]$$

The connection transforms according to

$$\bar{\Gamma}_k(\bar{x}) = \frac{\partial \bar{x}^j}{\partial x^k} \left\{ S \cdot \Gamma_j(x) \cdot S^{-1} + S_{,j} \cdot S^{-1} \right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2} [e_k^{(a)} \nabla_j e_{(b)}^k] \text{SAB}^{(b)(a)}$$

Here, spin connection coefficients are $e_k^{(a)} (\nabla_j e_{(b)}^k)$

and

$$S = \exp \left(\frac{1}{2} \omega_{(A)(B)}(x) \text{SAB}^{(A)(B)} \right),$$

where $\omega_{(B)(A)} = -\omega_{(A)(B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the $\Psi 16$ field [A.K.A., wave function characterizing the Universe(s) of masses $\pm M$];

TODO: prove Universe(s) of masses $\pm M$ are created in pairs!

K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:

- $\nabla_\mu e^a{}_\nu = 0$
- Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

apologies, I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

? SOURCE terms
 $T^{\mu\nu}$ for $g_{\alpha\beta}$ come from
 "Universes' Wave Function
 Ψ^{16} Lagrangian"

$$\frac{1}{\kappa} \text{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lagrangian}\Psi16)$$

(hope that $\text{TU}^{\mu\nu} = \Lambda g^{\mu\nu}$, and \times

$H = \text{some function of } M, \text{ where } \times$

Universe (s) of masses $\pm M$ created in pairs at time $x4 = 0$,

before the particles of the standard model exist) ;

WARNING:

Universes $\Psi16$ source $g_{\alpha\beta}$;

The Euler–Lagrange equations for $\Psi16$ must have ‘solutions’ such that

all off-diagonal terms of $\text{TU}^{\mu\nu}$ ARE ZERO .

The $\Psi16$ Lagrangian (please see below) =

$$\begin{aligned} \sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg[]} = & \\ \sqrt{\text{Det}[g_{\mu\nu}]} * \left(\text{Transpose}[\Psi16].\circ16.\text{Sum} \left[T16^\alpha [\alpha1 - 1] . \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^{\alpha1-1}} - \Gamma_{\alpha1-1} \right\} . \Psi16, \right. \right. \\ \left. \left. \{\alpha1, 1, \text{Length}[X]\} \right] + \frac{\text{mASs}}{2} * \text{Transpose}[\Psi16].\circ16.\Psi16 \right) \end{aligned}$$

Next,

assume that $\Psi16\text{soln}$ solves the Euler – Lagrange equations.

SOURCE terms

$\text{TU}^{\mu\nu}$ [evaluate terms after performing differentiation] (using $\text{Lagrangian}\Psi16 \equiv$
 $\left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lg[]} \right) |_{\Psi16=\text{solution-toEL-eqs}}$) :

$$\frac{1}{\kappa} \text{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lg[]})$$

$$\begin{aligned}
&= \left(\mathcal{L}g[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(-\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \frac{\partial}{\partial g_{\mu\nu}} (\mathcal{L}g[]) \right) \quad \mid \quad \text{¶16=¶16soln} \\
&= \left(0 ? * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(-\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\mathcal{L}g[]) \right) \quad \mid \quad \text{¶16=¶16soln} \\
&= \frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\text{¶16}].\sigma16. \right. \\
&\quad \left. \text{Sum} \left[\left(D[\text{¶16}, X[\alpha1]] + \left(\frac{1}{2} \right) \text{connectionMatrix} \right). \text{Sum}[\omega[\alpha1, a, b] * \text{SAB}[a, b], \right. \right. \\
&\quad \left. \left. \{a, 1, 8\}, \{b, 1, 8\}] . \text{¶16} \right), \{\alpha1, 1, \text{Length}[X]\} \right] + \\
&\quad \text{mASs} * \text{Transpose}[\text{¶16}].\text{symm16}[j, 1].\text{¶16} \right) \quad \mid \quad \text{¶16=¶16soln} \\
&= \text{Transpose}[\text{¶16}].\sigma16.\text{Sum} \left[\frac{\partial}{\partial g_{\mu\nu}} (\text{T16}^\alpha[\alpha1 - 1]) . \text{¶16}_{\alpha1-1}, \right. \\
&\quad \left. \{\alpha1, 1, \text{Length}[X]\} \right] \quad \mid \quad \text{¶16=solution-toEL-eqs} \\
&= \underset{\sim}{\text{¶16}} . \sigma16. \text{T16}^A . \text{¶16}_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1\alpha\beta} e_\beta^B \eta_{BA} \right) = \underset{\sim}{\text{¶16}} . \sigma16. \text{T16}^A . \text{¶16}_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha)
\end{aligned}$$

NOTES :

Let g be a square matrix; we could use: $\frac{\partial}{\partial q} (g^{-1}) = -g^{-1} \cdot \frac{\partial g}{\partial q} \cdot g^{-1}$,

where q is a parameter (result from $\frac{\partial}{\partial q} (g \cdot g^{-1}) = 0$),

In this notebook,

g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] ,
 g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}} (g^{-1}) = -g^{-1} \cdot \frac{\partial g}{\partial g_{\mu\nu}} \cdot g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In metric matrix g ,

we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate, for example.

This should be understood before reading further.

$$\text{Therefore } \frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha) = \frac{\partial}{\partial g_{\mu\nu}} (\text{Inverse}[e_\alpha^{(A)}]) = -\text{Inverse}[e_\alpha^{(A)}] \cdot \frac{\partial e_\alpha^{(A)}}{\partial g_{\mu\nu}} \cdot \text{Inverse}[e_\alpha^{(A)}] = -e_{(A)}^\alpha [[\alpha 1, A1]] \frac{\partial e_\alpha^{(A)} [[A1, \alpha 2]]}{\partial g_{\mu\nu}} \cdot e_{(A)}^\alpha [[\alpha 2, A2]]$$

$$\text{Consider } \frac{\partial}{\partial g_{\mu\nu}} (g_{\alpha\beta}) = \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^{(B)}) = \eta_{AB} e_\beta^{(B)} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^{(A)} \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^{(B)}) ; \\ e_C^\beta \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} = \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^{(B)}) = \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^{(A)} e_C^\beta \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^{(B)})$$

In metric matrix g , we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate.

Some of my calculations employ “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see
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“FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb”

`MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify`

I copied and pasted too often; however, I am going to leave this here.

Universe sources for $g_{\alpha\beta}$;
be sure to append these to the Einstein and/or Einstein–Lovelock field equations.

WARNING: all off-diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous ‘hacked solutions’ for $\Psi16$ to be valid

Let $T^\alpha = \{T16^\alpha[0], T16^\alpha[4]\}$ and

```
 $\sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg}[][\text{mASs}_-, j_-] =$ 
 $\sqrt{\text{Det}[g_{\mu\nu}]} * (\text{Transpose}[\Psi16].\sigma16.\text{Sum}[$ 
 $T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\}] +$ 
 $\text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[134, 1].\Psi16)$ 
```

Let $j = 134$, $\sigma16.(\text{mass Matrix}) = T16^\alpha[5].T16^\alpha[6].T16^\alpha[7]$

and

```
usingLagrangianF16massive = (Transpose[\Psi16].\sigma16.
 $\text{Sum}[T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\}] +$ 
 $\text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[j = 134, 1].\Psi16);$ 
```

where it is assumed that $\Psi16$ solves the Euler – Lagrange equations, above; then $\text{usingLagrangianF16massive} = 0$, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lg}[] \right)$$

$$= \left(\text{Lg}[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \right.$$

remark: the term

$$\underset{\partial g_{\mu\nu}}{\overset{\sim}{\partial}} \Psi^{16} \cdot \sigma^{16} \cdot T^{16^A} \cdot \Psi^{16}_{,\alpha} \underset{\partial g_{\mu\nu}}{\overset{\sim}{\partial}} (e_A^\alpha) = \underset{\partial g_{\mu\nu}}{\overset{\sim}{\partial}} \Psi^{16} \cdot \sigma^{16} \cdot T^{16^A} \cdot \Psi^{16}_{,\alpha} e_A^\alpha =$$

$$\frac{\partial}{\partial g_{\mu\nu}} \left(\begin{array}{l} \text{Transpose}[\Psi^{16}] \cdot \sigma^{16} \cdot \\ \\ T^{16^A}[A1-1] \cdot \Psi^{16}_{,\alpha1} \eta_{A1B} e_B^\beta g^{-1} \alpha1 \beta \\ \text{Sum} \left[T^\alpha [\underbrace{\alpha1 - 1}_{} \cdot D[\Psi^{16}, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\} \right] \end{array} \right)$$

Frame fields [edit]

We use a set of [vierbein](#) or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab} e_\mu^a e_\nu^b = \eta_{\mu\nu}.$$

The vierbein defines a local rest [frame](#), allowing the constant [Gamma matrices](#) to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a [section](#) of the [frame bundle](#), and so defines a local trivialization of the frame bundle.

Incomplete theories	[show]
Scientists	v • t • e

Spin connection [edit]

To write down the equation we also need the [spin connection](#), also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e_a^\mu e_\nu^a = \delta_\nu^\mu.$$

The connection 1-form is then

$$\omega^\mu{}_{\nu a} := e_b^\mu \nabla_a e_\nu^b$$

where ∇_a is a [covariant derivative](#), or equivalently a choice of [connection](#) on the frame bundle, most often taken to be the [Levi-Civita connection](#).

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space $T_p M$ to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

[Raising and lowering indices](#) is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract [connection on a principal bundle](#), specifically on the [frame bundle](#), which is defined on any smooth manifold, but which restricts to an [orthonormal frame bundle](#) on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu_{\nu\rho} = e_\rho^\alpha \omega^\mu_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2} (\omega_{\nu\rho})_\mu \sigma^{\nu\rho} \psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu R(M^{\nu\rho}) v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $SO(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

OTHER WORK THAT I USED:

JOURNAL OF MATHEMATICAL PHYSICS

VOLUME 4, NUMBER 7

A Remarkable Representation of the $3 + 2$ de Sitter Group

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Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

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765

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The Dirac spinor in six dimensions

By E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

Identities satisfied by the generators of the Dirac algebra

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A new spin- $\frac{1}{2}$ wave equation

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Begin

```
In[1]:= Print[
  "CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]
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In[2]:= Print["Please cite this work, and this web page, if you use it"]
Please cite this work, and this web page, if you use it

In[3]:= Print["For simplicity, metric gαβ= gαβ(x0,x1,x2,x3,x4,x5,x6,x7)=
           gαβ(x0,x4) = diag{ g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1,
           g77(x0,x4),g77(x0,x4),g77(x0,x4) },"]
For simplicity, metric gαβ= gαβ(x0,x1,x2,x3,x4,x5,x6,x7)= gαβ(x0,x4) = diag{
           g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4) } ,
In[4]:= Print["where g77(x0,x4) = g11(x0,-x4) "]
where g77(x0,x4) = g11(x0,-x4)
ConvertMapleToMathematicaV2

In[5]:= FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]]
mapleToMathematicaV2PATH =
  FileNameJoin[{%, "ConvertMapleToMathematicaV2.m"}]

Out[5]= /Users/nsh/Documents/8-dim/2026-01-04-Complex

Out[6]= /Users/nsh/Documents/8-dim/2026-01-04-Complex/ConvertMapleToMathematicaV2.m

In[7]:= Get[mapleToMathematicaV2PATH] (*Loads new maple to mathematica parser*)
ConvertMapleToMathematicaV2 loaded successfully!
BUT, WARNING: DO NOT USE IF YOU WANT A CORRECT RESULT!
Load the Maple output strings.

In[8]:= (*SetDirectory[NotebookDirectory[]];*)

In[9]:= (*maplestringEQ1=Get["string-seq1.txt"]
maplestringEQ2=Get["thinkpad_stringEQ2.txt"]*)
```

```
In[5]:= NotebookFileName[]
Out[5]= /Users/nsh/Documents/8-dim/2026-01-04-Complex/Pair-Crtn-Univ-same_E-L-eqs-alt-
approach.nb

In[6]:= Unprotect[dir];
Protect[dir];
If[$Failed == SetDirectory[dir], {.CreateDirectory[dir], SetDirectory[dir]}];

In[10]:= (*FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]]*)

In[11]:= names = FileNameSplit[NotebookFileName[]]
Out[11]= {, Users, nsh, Documents, 8-dim, 2026-01-04-Complex,
Pair-Crtn-Univ-same_E-L-eqs-alt-approach.nb}

In[12]:= name = StringReplace[names[[-1]], "nb" → "mx"]
Out[12]= Pair-Crtn-Univ-same_E-L-eqs-alt-approach.mx

In[13]:= header = StringReplace[names[[-1]], ".nb" → "-"]
Out[13]= Pair-Crtn-Univ-same_E-L-eqs-alt-approach-

In[1]:= SetOptions[Simplify, TimeConstraint → 1]
Out[1]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[2]:= SetOptions[FullSimplify, TimeConstraint → 3]
Out[2]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}

In[3]:= Options[Simplify]
Out[3]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[4]:= Options[FullSimplify]
Out[4]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}

In[5]:= {Off[General::spell], Off[General::spell1]};
```

```
In[1]:= DIM8 = 8;
In[2]:= Protect[DIM8, M, K, H]
Out[2]= {DIM8, M, K, H}
```

Coordinates :

$\left\{ \underbrace{\text{hidden space}}, \underbrace{\text{3-space}}, \underbrace{\text{time}}_{\text{superluminal deflating time}} \right\}$
 $\{x_0, \overrightarrow{x_1}, \overrightarrow{x_2}, \overrightarrow{x_3}, x_4, \overrightarrow{x_5}, \overrightarrow{x_6}, \overrightarrow{x_7}\} :$

```
In[3]:= X = {x0, x1, x2, x3, x4, x5, x6, x7};
Protect[X];
Protect[x0, x1, x2, x3, x4, x5, x6, x7];
```

```
In[4]:= sX0 = And @@ Thread[X > 0]
Out[4]= x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 && x7 > 0
```

```
In[5]:= ssX = H > 0 && sX0 && 6 H x0 > 0 && 2 la[x4] > 0 && Cot[6 H x0] > 0 && Sin[6 H x0] > 0
Out[5]= H > 0 && x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 &&
x7 > 0 && 6 H x0[] > 0 && 2 la[x4[]] > 0 && Cot[6 H x0[]] > 0 && Sin[6 H x0[]] > 0
```

For Maple_2025:

```
In[6]:= sreplaceZ = Z[#] → ToExpression["Z" <> ToString[#]] & /@ Range[0, 15]
Out[6]= {Z[0] → Z0, Z[1] → Z1, Z[2] → Z2, Z[3] → Z3, Z[4] → Z4,
Z[5] → Z5, Z[6] → Z6, Z[7] → Z7, Z[8] → Z8, Z[9] → Z9, Z[10] → Z10,
Z[11] → Z11, Z[12] → Z12, Z[13] → Z13, Z[14] → Z14, Z[15] → Z15}
```

```
In[7]:= sreplacenZ = nZ[#] → ToExpression["nZ" <> ToString[#]] & /@ Range[0, 15]
Out[7]= {nZ[0] → nZ0, nZ[1] → nZ1, nZ[2] → nZ2, nZ[3] → nZ3, nZ[4] → nZ4,
nZ[5] → nZ5, nZ[6] → nZ6, nZ[7] → nZ7, nZ[8] → nZ8, nZ[9] → nZ9, nZ[10] → nZ10,
nZ[11] → nZ11, nZ[12] → nZ12, nZ[13] → nZ13, nZ[14] → nZ14, nZ[15] → nZ15}
```

hacks (a.k.a., lies):

```
In[1]:= constraintTrig = Sin[6 * H * x0] > 0 && Cos[6 * H * x0] > 0 && Csc[6 * H * x0] > 0 &&
Sec[6 * H * x0] > 0 && Tan[6 * H * x0] > 0 && Cot[6 * H * x0] > 0 && Sin[z] > 0 &&
Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 && Sin[z]^(1/2) > 0 && Sin[z]^(−3/2) > 0 &&
Sin[z]^(−1/2) > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 && Cot[z]^(3/2) > 0 &&
Cot[z]^(1/2) > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

```
Out[1]= Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]^(3/2) > 0 && 1/Sin[z] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]^(3/2) > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

more hacks (damn lies):

```
In[2]:= (*&&K^2-M^2>0&&Sqrt[K^2-M^2]>0&&e^(H Sqrt[K^2-M^2])^(x4)>0&&2 H Sqrt[K^2-M^2]^(x4)>0&&
2 H Sqrt[1-M^2]^(x4)>0&&Sqrt[1-M^2]>0&&e^(H Sqrt[1-M^2])^(x4)>0&&2 H Sqrt[1-M^2]^(x4)>0*)
```

```
In[3]:= constraintX = x0 > 0 && x4 > 0 && H > 0 && 6 * H * x0 > 0 &&
3 * H * x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 && t > 0 && M > 0 &&
K > 0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0 && e^(2 a4[H x4]) > 0 && e^(a4[H x4]) > 0
```

```
Out[3]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 &&
t > 0 && M > 0 && K > 0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0 && e^(2 a4[H x4]) > 0 && e^(a4[H x4]) > 0
```

statistics:

```
In[4]:= constraintVars = constraintX && constraintTrig
```

```
Out[4]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0 && e^(2 a4[H x4]) > 0 &&
e^(a4[H x4]) > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]^(3/2) > 0 && 1/Sin[z] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]^(3/2) > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

```
In[1]:= subsDefects = { Sqrt[e^(2 a4[H x4])] → e^(a4[H x4]), Sqrt[e^(-2 a4[H x4])] → e^(-a4[H x4]),
Sqrt[e^(2 a4[H x4]) Sin[6 H x4]^(1/3)] → e^(a4[H x4]) Sin[6 H x4]^(1/6),
1/Sqrt[e^(2 a4[H x4]) Sin[6 H x4]^(1/3)] → 1/(e^(a4[H x4]) Sin[6 H x4]^(1/6)),
1/Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/6)] → 1/(e^(a4[H x4]) Sin[6 H x0]^(1/6)),
1/Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)] → 1/(e^(-a4[H x4]) Sin[6 H x0]^(1/6)),
Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/6)] → e^(a4[H x4]) Sin[6 H x0]^(1/6),
Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)] → e^(-a4[H x4]) Sin[6 H x0]^(1/6) }
```

```
Out[1]= { Sqrt[e^(2 a4[H x4])] → e^(a4[H x4]), Sqrt[e^(-2 a4[H x4])] → e^(-a4[H x4]),
Sqrt[e^(2 a4[H x4]) Sin[6 H x4]^(1/3)] → e^(a4[H x4]) Sin[6 H x4]^(1/6),
1/Sqrt[e^(2 a4[H x4]) Sin[6 H x4]^(1/3)] → 1/(Sin[6 H x4]^(1/6)), 1/Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)] → 1/(Sin[6 H x0]^(1/6)),
1/Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)] → 1/(Sin[6 H x0]^(1/6)),
Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/6)] → e^(a4[H x4]) Sin[6 H x0]^(1/6),
Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)] → e^(-a4[H x4]) Sin[6 H x0]^(1/6) }
```

future coordinate transformation :

```
In[2]:= 6 H x0 == z && H x4 == t
szt = Solve[%, {z, t}] [[1]]
sx0x4 = Solve[%%, {x0, x4}] [[1]]
Protect[sx0x4, szt];
```

```
Out[2]= 6 H x0 == z && H x4 == t
```

```
Out[3]= {z → 6 H x0, t → H x4}
```

```
Out[4]= {x0 → z/(6 H), x4 → t/H}
```

```
In[5]:= sx0x4
```

```
Out[5]= {x0 → z/(6 H), x4 → t/H}
```

```
In[6]:= (*sa4={a4→((A4[3 H #2])&)}*)
```

```
In[]:= Protect[sx0x4, szt]
Out[]= {}

SO(4, 4) Minkowski Lorentz metric  $\eta_{4488}$  : 4+4 spacetime; 8×8 dimensional :

In[]:= ( $\eta_{4488}$  = ArrayFlatten[
  {{IdentityMatrix[4], 0}, {0, -IdentityMatrix[4]}}]) // MatrixForm
Out[//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


SO(4, 4) Spinor Lorentz metric  $\sigma$ ;
 $\tau^A$  are analogous to the Dirac gamma matrices :

all true : Table[
```

$$\left\{ A, B, \text{FullSimplify}\left[\frac{1}{2} (\tau[A].\bar{\tau}[B] + \tau[B].\bar{\tau}[A]) == \eta_{4488}[[A+1, B+1]*ID8]\right], \{A, 0, 7\}, \{B, 0, 7\} \right]$$

Type - 1 and type - 2 employ σ (the usual notational abuse),
since $\sigma = \sigma^{-1}$: 4+4 spacetime, 8×8 dimensional and 16×16 dimensional :

indices. We define the reduced Brauer-Weyl generators $\{\tilde{\tau}^{A'}, \tau^{A'}\}$ of the generators of the two real 8×8 inequivalent irreducible representations of $\overline{SO}(4, 4)$ (see eqs. (2.6) and (2.7)) by demanding that the tau matrices satisfy (the tilde denotes transpose)

$$(2.3) \quad \tilde{\tau}^{A'} \sigma = \sigma \bar{\tau}^{A'}$$

and

$$(2.4) \quad \tau^{A'} \bar{\tau}^{B'} + \tau^{B'} \bar{\tau}^{A'} = 2IG^{A'B'} = \bar{\tau}^{A'} \tau^{B'} + \bar{\tau}^{B'} \tau^{A'},$$

where I denotes the 8×8 unit matrix. Denoting the matrix elements of $\tau^{A'}$ by $\tau^{A'a}_b$, we may write eq. (3) as

$$(2.5) \quad \bar{\tau}^{A'}_{ab} = \tau^{A'}_{ba},$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix}$$

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4, 4)}$ and $L \in SO(4, 4)$. The canonical 2-1 homomorphism $\overline{SO(4, 4)} \rightarrow SO(4, 4)$ is given by

$$(2.8) \quad 8L^{A'}{}_{B'}(g) = \text{tr } \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4, 4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}{}_{B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

```
In[=]:= σ = ArrayFlatten[{{ConstantArray[0, {4, 4}], IdentityMatrix[4]}, {IdentityMatrix[4], ConstantArray[0, {4, 4}]}}] // MatrixForm
```

Out[=]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

below:

```
In[=]:= σ16 == ArrayFlatten[{{{-σ, 0}, {0, σ}}]}
```

below:

```
In[=]:= Table[T16A[A1] = ArrayFlatten[{{0, τ[A1]}, {τ[A1], 0}}], {A1, 0, 7}];
```

below:

```
In[=]:= σ16.T16A[#] = -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

```
Out[=]= {True, True, True, True, True, True, True}
```

below:

```
In[=]:= Table[{A1, B1},  
FullSimplify[ExpandAll[ $\frac{1}{2} (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) =$   
 $\eta 4488 [A1+1, B1+1] * ID16$ ]], {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

```
Out[=]//MatrixForm=
```

({0, 0})	({0, 1})	({0, 2})	({0, 3})	({0, 4})	({0, 5})	({0, 6})	({0, 7})
(True)							
({1, 0})	({1, 1})	({1, 2})	({1, 3})	({1, 4})	({1, 5})	({1, 6})	({1, 7})
(True)							
({2, 0})	({2, 1})	({2, 2})	({2, 3})	({2, 4})	({2, 5})	({2, 6})	({2, 7})
(True)							
({3, 0})	({3, 1})	({3, 2})	({3, 3})	({3, 4})	({3, 5})	({3, 6})	({3, 7})
(True)							
({4, 0})	({4, 1})	({4, 2})	({4, 3})	({4, 4})	({4, 5})	({4, 6})	({4, 7})
(True)							
({5, 0})	({5, 1})	({5, 2})	({5, 3})	({5, 4})	({5, 5})	({5, 6})	({5, 7})
(True)							
({6, 0})	({6, 1})	({6, 2})	({6, 3})	({6, 4})	({6, 5})	({6, 6})	({6, 7})
(True)							
({7, 0})	({7, 1})	({7, 2})	({7, 3})	({7, 4})	({7, 5})	({7, 6})	({7, 7})
(True)							

defs; some Symbols; metric

$$\left\{ \left\{ a4[t] \rightarrow \frac{M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} + c_1 + \frac{1}{2} \text{ProductLog}\left[-e^{-\frac{2 M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} - 2 c_1}\right]\right\} \right\}$$

$$\text{DSolve}[\theta == -M (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7]) + (Q1 - e^{-2 a4[t]} Q1) a4'[t], a4[t], t]$$

$$M c[j] = (j + 1)^{\text{st}} \text{ Energy Eigenvalue for Transpose[cayZ].}\Psi16, j = 0, \dots, 7$$

`ProductLog[z]`

gives the principal solution for w in $z = we^w$.

if z and w are any complex numbers, then

$$we^w = z$$

holds if and only if

$$w = W_k(z) \text{ for some integer } k.$$

When dealing with real numbers only, the two branches W_0 and W_{-1} suffice: for real numbers x and y the equation

$$ye^y = x$$

can be solved for y only if $x \geq \frac{-1}{e}$; yields $y = W_0(x)$ if $x \geq 0$ and the two values $y = W_0(x)$ and $y = W_{-1}(x)$ if $\frac{-1}{e} \leq x < 0$.

?todo: NEED `DefScalarFunction[#]; & /@ (f16[#]&/@Range[0,15])`, and allow `xact` to compute?

Introduce the wave function, $\Psi16$, for a Universe:

`In[1]:= Ψ16 = f16[#][x0, x4] & /@ Range[0, 15]`

`Out[1]=`

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

```

In[1]:= processRawSets[rawSets_] :=
Module[{s1, s2, J, ret, L = Range[Length[rawSets]], r = {}},
  While[Length[L] > 1,
    s1 = Union[rawSets[[L[[1]]]]];
    L = Drop[L, 1];
    J = L[[1]];
    While[J ≤ 16,
      If[MemberQ[L, J],
        s2 = Union[rawSets[[J]]];
        If[Intersection[s1, s2] ≠ {}, s1 = Union[Flatten[{s1, s2}]]];
        L = Complement[L, {J}];, ##&[]];
      ##&[]];
    J++];
    AppendTo[r, s1];
  ];
  ret = Sort[r, #1[[1]] < #2[[1]] &];
  Return[ret];
]

In[2]:= rawSets¶16 = Cases[
  #,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] ↦ n,
  Infinity,
  Heads → True
] & /@ ¶16
Out[2]= {{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}, {15} }

In[3]:= processRawSets[rawSets¶16]
Out[3]= {{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14} }

In[4]:= (*¶new16=F16[#] [x0,x4]&/@Range[0,15]*)

```

```
In[1]:= Clear[sf16Aa];
sf16Aa = f16[#] & /@ Range[0, 15]
```

```
Out[1]= {f16[0] >> (Z[0][6 H#1, H#2] &), f16[1] >> (Z[1][6 H#1, H#2] &),
f16[2] >> (Z[2][6 H#1, H#2] &), f16[3] >> (Z[3][6 H#1, H#2] &),
f16[4] >> (Z[4][6 H#1, H#2] &), f16[5] >> (Z[5][6 H#1, H#2] &),
f16[6] >> (Z[6][6 H#1, H#2] &), f16[7] >> (Z[7][6 H#1, H#2] &),
f16[8] >> (Z[8][6 H#1, H#2] &), f16[9] >> (Z[9][6 H#1, H#2] &),
f16[10] >> (Z[10][6 H#1, H#2] &), f16[11] >> (Z[11][6 H#1, H#2] &),
f16[12] >> (Z[12][6 H#1, H#2] &), f16[13] >> (Z[13][6 H#1, H#2] &),
f16[14] >> (Z[14][6 H#1, H#2] &), f16[15] >> (Z[15][6 H#1, H#2] &)}
```

```
In[2]:= (*Clear[sf16Aa];
sf16Aa=f16[#]&ToExpression[
"((Z["<>ToString[#]<>"] [6*H#1,H#2] 1/Sin[6*H#1]^(1/2))&)">]& /@Range[0,15]*)
```

```
In[1]:= Clear[snewf16Aa];
snewf16Aa = f16[#] → ToExpression["((nZ[" <> ToString[#] <>
""] [6*H#1, H#2] 1
Sin[6 * H * #1]1/2 )&)"]
& /@ Range[0, 15]
```

```
Out[1]= {f16[0] → (nZ[0] [6 H#1, H#2] &),
f16[1] → (nZ[1] [6 H#1, H#2] &),
f16[2] → (nZ[2] [6 H#1, H#2] &),
f16[3] → (nZ[3] [6 H#1, H#2] &),
f16[4] → (nZ[4] [6 H#1, H#2] &),
f16[5] → (nZ[5] [6 H#1, H#2] &),
f16[6] → (nZ[6] [6 H#1, H#2] &),
f16[7] → (nZ[7] [6 H#1, H#2] &),
f16[8] → (nZ[8] [6 H#1, H#2] &),
f16[9] → (nZ[9] [6 H#1, H#2] &),
f16[10] → (nZ[10] [6 H#1, H#2] &),
f16[11] → (nZ[11] [6 H#1, H#2] &),
f16[12] → (nZ[12] [6 H#1, H#2] &),
f16[13] → (nZ[13] [6 H#1, H#2] &),
f16[14] → (nZ[14] [6 H#1, H#2] &),
f16[15] → (nZ[15] [6 H#1, H#2] &)}
```

0 (4, 4) related:

```
In[2]:= Needs["Notation`"]
```

```
In[3]:= Symbolize[σga1 A]
```

```
In[4]:= Symbolize[σga2 A]
```

```
In[5]:= Symbolize[τ]
```

```
In[6]:= Symbolize[T16A]
```

```
In[7]:= Symbolize[T16α]
```

```
In[8]:= Symbolize[ua]
```

```
In[9]:= Symbolize[Jy]
```

In[1]:= Symbolize[F^A_a]

In[2]:= Symbolize[F^a_A]

In[3]:= Symbolize[σ²²]

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

In[4]:= Symbolize[g_{αβ}]

note below: g_{AB} is really $g_{(A)(B)}$:: Minkowski – Lorentz metric

In[5]:= Symbolize[g_{AB}]

note below: G_{AB} is really $G_{(A)(B)}$:: Minkowski – Lorentz

In[6]:= (*Symbolize[G_{AB}]*)

In[7]:= Symbolize[S_{44αβ}]

$$\Sigma^{\alpha\beta} = \xi S^{\alpha\beta} u,$$

In[8]:= Symbolize[Σ^{αβ}]

In[9]:= (*Symbolize[Σ^{αβ}]*)

$$\frac{1}{2}\Sigma^{jk}\Sigma_{jk} = \frac{1}{4}\partial_a u^a u^b \partial_b - \frac{1}{4}\partial_a u^b u^b \partial_a,$$

In[10]:= Symbolize[Σ²]

NEED DefScalarFunction[#]; & /@ Flatten[Universe]

In[11]:= Universe = ToExpression["U" <> ToString[#]] & /@ Range[0, 7]

In[12]:= Symbolize[F^A_a]

In[13]:= Symbolize[F^a_A]

In[14]:= Symbolize[gtrye^(A)_α]

In[15]:= Symbolize[gtrye^α_(A)]

SPACETIME METRIC

In[16]:= (*einsteinsteinMetric=Array[g[#1-1, #2-1]&, {8, 8}]*)

SPACETIME METRIC:

```
In[8]:= g4488 = Array[g[#1 - 1] [#2 - 1] [x0, x4] &, {8, 8}];
```

SPACETIME METRIC values:

MatrixMetric44

```
6 H x0 == z && H x4 == t
{{a4''[t] == 0, a4'[t] == 2/3 (-1 + M)}, {a4''[t] == 0, a4'[t] == 2 (1 + M)/3}}
In[9]:= (*β3=Exp[2 H x4 - √(K²-M²)];*)
(*β3=Exp[2 * a4[3 H x4]];*)
β3 = Exp[2 * a4[H * x4]] (*/.{a4→((K1 2 (1+M)/3+K2 2/3 (-1+M))#)&})*)
β1 = Sin[6 * H * x0]^(1/3);
β2 = Cot[6 * H * x0]^2;
Out[9]= e^2 a4[H x4]
```

```
In[10]:= MatrixForm[
MatrixMetric44 = {{β2, 0, 0, 0, 0, 0, 0, 0}, {0, β1 β3, 0, 0, 0, 0, 0, 0},
{0, 0, β1 β3, 0, 0, 0, 0, 0}, {0, 0, 0, β1 β3, 0, 0, 0, 0},
{0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -β1/β3, 0, 0},
{0, 0, 0, 0, 0, 0, -β1/β3, 0}, {0, 0, 0, 0, 0, 0, 0, -β1/β3}} //.
FullSimplify[#, constraintVars] &]
```

```
Out[10]//MatrixForm=
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[11]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression[
"((Z["<>ToString[#]<>"] [6 H #1, 3 H #2] 1/Sin[6 H #1]^(1/2))&)"]&/@Range[0,15]*)

```

```

In[1]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"] [6*H*#1,3*H*#2])&)"]&/@Range[0,15]*)

In[2]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression[
"((Z["<>ToString[#]<>"] [6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@Range[0,15]*)

In[3]:= (*Inactivate[((q)&)]Evaluate[Activate[Evaluate[q]]]*)

In[4]:= Clear[makeSubstitution];
makeSubstitution[] := Module[{met, mat, t, f, p},
  met = Flatten[g4488][[All, 0]];
  mat = Flatten[MatrixMetric44];
  t = Thread[met → mat];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]

In[5]:= makeSubstitution[a_, b_] := Module[{t, f, p},
  t = Thread[a → b];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]

```

In[•]:= **ssgm4488 = makeSubstitution[]**

Out[•]=

$$\{g[0][0] \rightarrow (\text{Cot}[6 H \#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] \rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&), \\ g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&), \\ g[2][2] \rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&), \\ g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&), \\ g[3][3] \rightarrow (e^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&)\}$$

In[•]:= **Symbolize[E_α^(A)]**

In[•]:= **Symbolize[e_α^(A)]**

In[•]:= **e_α^(A) = Array[E_α^(A) [#, # - 1] &, {8, 8}]**

Out[•]=

$$\{\{E_{\alpha}^{(A)}[0, 0], E_{\alpha}^{(A)}[0, 1], E_{\alpha}^{(A)}[0, 2], E_{\alpha}^{(A)}[0, 3], \\ E_{\alpha}^{(A)}[0, 4], E_{\alpha}^{(A)}[0, 5], E_{\alpha}^{(A)}[0, 6], E_{\alpha}^{(A)}[0, 7]\}, \{E_{\alpha}^{(A)}[1, 0], E_{\alpha}^{(A)}[1, 1], \\ E_{\alpha}^{(A)}[1, 2], E_{\alpha}^{(A)}[1, 3], E_{\alpha}^{(A)}[1, 4], E_{\alpha}^{(A)}[1, 5], E_{\alpha}^{(A)}[1, 6], E_{\alpha}^{(A)}[1, 7]\}, \\ \{E_{\alpha}^{(A)}[2, 0], E_{\alpha}^{(A)}[2, 1], E_{\alpha}^{(A)}[2, 2], E_{\alpha}^{(A)}[2, 3], E_{\alpha}^{(A)}[2, 4], E_{\alpha}^{(A)}[2, 5], \\ E_{\alpha}^{(A)}[2, 6], E_{\alpha}^{(A)}[2, 7]\}, \{E_{\alpha}^{(A)}[3, 0], E_{\alpha}^{(A)}[3, 1], E_{\alpha}^{(A)}[3, 2], E_{\alpha}^{(A)}[3, 3], \\ E_{\alpha}^{(A)}[3, 4], E_{\alpha}^{(A)}[3, 5], E_{\alpha}^{(A)}[3, 6], E_{\alpha}^{(A)}[3, 7]\}, \{E_{\alpha}^{(A)}[4, 0], E_{\alpha}^{(A)}[4, 1], \\ E_{\alpha}^{(A)}[4, 2], E_{\alpha}^{(A)}[4, 3], E_{\alpha}^{(A)}[4, 4], E_{\alpha}^{(A)}[4, 5], E_{\alpha}^{(A)}[4, 6], E_{\alpha}^{(A)}[4, 7]\}, \\ \{E_{\alpha}^{(A)}[5, 0], E_{\alpha}^{(A)}[5, 1], E_{\alpha}^{(A)}[5, 2], E_{\alpha}^{(A)}[5, 3], E_{\alpha}^{(A)}[5, 4], E_{\alpha}^{(A)}[5, 5], \\ E_{\alpha}^{(A)}[5, 6], E_{\alpha}^{(A)}[5, 7]\}, \{E_{\alpha}^{(A)}[6, 0], E_{\alpha}^{(A)}[6, 1], E_{\alpha}^{(A)}[6, 2], E_{\alpha}^{(A)}[6, 3], \\ E_{\alpha}^{(A)}[6, 4], E_{\alpha}^{(A)}[6, 5], E_{\alpha}^{(A)}[6, 6], E_{\alpha}^{(A)}[6, 7]\}, \{E_{\alpha}^{(A)}[7, 0], E_{\alpha}^{(A)}[7, 1], \\ E_{\alpha}^{(A)}[7, 2], E_{\alpha}^{(A)}[7, 3], E_{\alpha}^{(A)}[7, 4], E_{\alpha}^{(A)}[7, 5], E_{\alpha}^{(A)}[7, 6], E_{\alpha}^{(A)}[7, 7]\}\}$$

In[•]:= **(*Symbolize[se_α^(A)]*)**

In[•]:= **Symbolize[sge_α^(A)]**

In[•]:= **Symbolize[sgtrye_α^(A)]**

In[•]:= **Symbolize[E_(A)^α]**

```

In[1]:= Symbolize[sge $\alpha$ (A)]
In[2]:= Symbolize[sgtrye $\alpha$ (A)]
In[3]:= Symbolize[e $\alpha$ (A)]
In[4]:= e $\alpha$ (A) = Array[E $\alpha$ (A) [#[1 - 1, #2 - 1] &, {8, 8}]]

Out[4]= {{E $\alpha$ (A) [0, 0], E $\alpha$ (A) [0, 1], E $\alpha$ (A) [0, 2], E $\alpha$ (A) [0, 3], E $\alpha$ (A) [0, 4], E $\alpha$ (A) [0, 5], E $\alpha$ (A) [0, 6], E $\alpha$ (A) [0, 7]}, {E $\alpha$ (A) [1, 0], E $\alpha$ (A) [1, 1], E $\alpha$ (A) [1, 2], E $\alpha$ (A) [1, 3], E $\alpha$ (A) [1, 4], E $\alpha$ (A) [1, 5], E $\alpha$ (A) [1, 6], E $\alpha$ (A) [1, 7]}, {E $\alpha$ (A) [2, 0], E $\alpha$ (A) [2, 1], E $\alpha$ (A) [2, 2], E $\alpha$ (A) [2, 3], E $\alpha$ (A) [2, 4], E $\alpha$ (A) [2, 5], E $\alpha$ (A) [2, 6], E $\alpha$ (A) [2, 7]}, {E $\alpha$ (A) [3, 0], E $\alpha$ (A) [3, 1], E $\alpha$ (A) [3, 2], E $\alpha$ (A) [3, 3], E $\alpha$ (A) [3, 4], E $\alpha$ (A) [3, 5], E $\alpha$ (A) [3, 6], E $\alpha$ (A) [3, 7]}, {E $\alpha$ (A) [4, 0], E $\alpha$ (A) [4, 1], E $\alpha$ (A) [4, 2], E $\alpha$ (A) [4, 3], E $\alpha$ (A) [4, 4], E $\alpha$ (A) [4, 5], E $\alpha$ (A) [4, 6], E $\alpha$ (A) [4, 7]}, {E $\alpha$ (A) [5, 0], E $\alpha$ (A) [5, 1], E $\alpha$ (A) [5, 2], E $\alpha$ (A) [5, 3], E $\alpha$ (A) [5, 4], E $\alpha$ (A) [5, 5], E $\alpha$ (A) [5, 6], E $\alpha$ (A) [5, 7]}, {E $\alpha$ (A) [6, 0], E $\alpha$ (A) [6, 1], E $\alpha$ (A) [6, 2], E $\alpha$ (A) [6, 3], E $\alpha$ (A) [6, 4], E $\alpha$ (A) [6, 5], E $\alpha$ (A) [6, 6], E $\alpha$ (A) [6, 7]}, {E $\alpha$ (A) [7, 0], E $\alpha$ (A) [7, 1], E $\alpha$ (A) [7, 2], E $\alpha$ (A) [7, 3], E $\alpha$ (A) [7, 4], E $\alpha$ (A) [7, 5], E $\alpha$ (A) [7, 6], E $\alpha$ (A) [7, 7]}}
In[5]:= (*gtrye $\alpha$ (A) = (e $\alpha$ (A) /. sgtrye $\alpha$ (A)) *)
In[6]:= (*gtrye $\alpha$ (A) = (e $\alpha$ (A) /. sgtrye $\alpha$ (A)) *)
In[7]:= (*Symbolize[R $\alpha$  $\beta$ ] *)
In[8]:= (*preSpinConnection=Array[R $\alpha$  $\beta$  [##]&,{8,8,8}] ;*)
In[9]:= (*Symbolize[EAa] *)
In[10]:= (*Symbolize[EaA] *)
In[11]:= (* FAa FaA *)
In[12]:= (*Do[FAa=EAa[h]=Table[(u[[h]].σ.(τ[B])),{B,1,8}],{h,1,Length[u]}];
Do[EaA[h]=FullSimplify[Inverse[EAa[h]]],{h,1,Length[u]}];
Table[FaA=EaA[h]=(u[[h]].σ.u[[h]]);
Transpose[Table[FullSimplify[ExpandAll[η8[[B,B]]τ[B].u[[h]]]],
{B,1,8}]],{h,1,Length[u]}]*)
In[13]:= (*Table[
FullSimplify[ExpandAll[Transpose[EAa[h]].η8.EAa[h]]]- (u[[h]].σ.u[[h]])σ== Zero,
{h,1,Length[u]}]*)
```

```
In[=]:= (* In88=FullSimplify[Inverse[n8]]; *)
In[=]:= (* Clear[wμ(a)(b)]; *) Clear[w]; Symbolize[wμ(a)(b)]
In[=]:= w = Array[wμ(a)(b) &##8000; {8, 8, 8}];
Protect[w]
Out[=]= {w}
```

constants

```
← ERROR : 08similarityTransformation has 1 st index that transforms as  $\frac{\partial}{\partial \Psi}$  ,
not as  $\Psi$ 

In[=]:= (* 08similarityTransformation has 1st index that transforms as  $\frac{\partial}{\partial \Psi}$  ,
not as  $\Psi$  *)
08similarityTransformation has 1 st index that transforms as  $\frac{\partial}{\partial \Psi}$  , not as  $\Psi$  :

In[=]:= (* ArrayFlatten[{IdentityMatrix[4], 0}, {0, -IdentityMatrix[4]}] *)
In[=]:= ID4 = IdentityMatrix[4];
ID8 = IdentityMatrix[8];

In[=]:= n4488 // MatrixForm
Out[=]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


In[=]:= n4488[[#, #]] & /@ Range[8]
Out[=]= {1, 1, 1, 1, -1, -1, -1, -1}
```

```

In[]:= Zero4 = ConstantArray[0, {4, 4}]; Zero8 = ConstantArray[0, {8, 8}];
mid = ExpandAll[-1 * ID4];
ε3 = Array[Signature[{##}] &, {3, 3, 3}];
ε4 = Array[Signature[{##}] &, {4, 4, 4, 4}];
MId = ExpandAll[-1 * ID8];
simp = {Zero4 → 0, ID4 → 1, mid → -1};
Simp = {Zero8 → ZERO, ID8 → ONE, MId → MONE};

Out[]= {{0, 0, 0}, {0, 0, 1}, {0, -1, 0}},
{{0, 0, -1}, {0, 0, 0}, {1, 0, 0}}, {{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}

In[]:= Zero16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η8, 0}, {0, -η8}}];
Id16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * Id16];
Simp16 = {Zero16 → ZERO16, Id16 → ONE16, MId16 → MONE16};

In[]:= ZERO16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η4488, 0}, {0, -η4488}}];
ID16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * ID16];
Simp16 = {ZERO16 → ZERO16, ID16 → ONE16, MId16 → MONE16};

In[]:= ws = Flatten[Table[ToExpression["ω" <> ToString[A1] <> ToString[B1]],
{A1, 1, 7}, {B1, A1 + 1, 8}]];
Length[ws]

Out[]= {ω12, ω13, ω14, ω15, ω16, ω17, ω18, ω23, ω24, ω25, ω26, ω27, ω28, ω34,
ω35, ω36, ω37, ω38, ω45, ω46, ω47, ω48, ω56, ω57, ω58, ω67, ω68, ω78}

Out[]= 28

In[]:= σ.σ - ID8 == Zero8
Tr[σ] == 0

Out[]= True

Out[]= True

```

some function definitions:

```

In[]:= Clear[rawSets];
rawSets[l_, f_, o_] := Module[{t, h},
  t = Table[Cases[
    l[[j]],
    h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], f] & n + o,
    Infinity,
    Heads → True
  ], {j, 1, Length[l]}];
  Return[t];
]

In[]:= (*rawSetsel16=rawSets[el16,"f16",0]*)

In[]:= (*couplings=showCoupledEquations[rawSetsel16]
//Flatten//Sort
====Range[0,15]*)
```

```

In[]:= Clear[dualRawSets];
dualRawSets[l_, f_, o_, h_, oh_] := Module[{r, r1, r2},
  r1 = rawSets[l, f, o];
  r2 = rawSets[l, h, oh];
  r = Union[Flatten[{r1, r2}, 1]] // Drop[#, 1] &;
  Return[r];
]

In[]:= (* Helper to perform one pass of merging *)
MergeSetsStep[currentSets_List] :=
  Module[{n = Length[currentSets], i, j, merged = currentSets},
  (* We look for the first pair (i, j) that intersects *)
  Catch[
    For[i = 1, i ≤ Length[merged], i++,
      For[j = i + 1, j ≤ Length[merged], j++,
        If[Intersection[merged[[i]], merged[[j]]] != {}, 
          (* Found intersection: Merge and Throw to restart/finish this step *)
          merged = Delete[merged, {{i}, {j}}];
          AppendTo[merged, Sort[Union[currentSets[[i]], currentSets[[j]]]]];
          Throw[Sort[merged]]; (* Sort for canonical form *)
        ];
      ];
    ];
    (* If no intersection found, return original sorted *)
    Sort[merged]
  ];
];

```

```

In[1]:= showCoupledEquations[items_List] :=
  FixedPoint[MergeSetsStep, items];

In[2]:= blockPartition[mat_] :=
  Module[{(*blocks,block11,block12,block21,block22,*)ret},
    blocks = Partition[mat, {8, 8}];
    (*block11=blocks[[1,1]] ;*)(*Top-left block*)
    (*block12=blocks[[1,2]] ;*)(*Top-right block*)
    (*block21=blocks[[2,1]] ;*)(*Bottom-left block*)
    (*block22=blocks[[2,2]] ;*)(*Bottom-right block*)
    ret = ArrayFlatten[
      {{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}] === mat;
    If[ret, Return[
      {{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}], ## &[]];
    Return[Null]];

In[3]:= eextract[a_] := Extract[a, {#}] & /@ Range[0, Length[a]]

In[4]:= Clear[iimes]

In[5]:= iimes[a_] := {a}

In[6]:= iimes[a_, b_] := {a, b}

In[7]:= iimes[a_, b_, c_] := {a, b, c}

In[8]:= iimes[a_, b_, c_, d_] := {a, b, c, d}

In[9]:= iimes[a_, b_, c_, d_, f_] := {a, b, c, d, f}

In[10]:= iimes[a_, b_, c_, d_, f_, h_] := {a, b, c, d, f, h}

In[11]:= iimes[a_, b_, c_, d_, f_, h_, j_] := {a, b, c, d, f, h, j}

In[12]:= times[a__] := Flatten[{Flatten[#] & /@ a}]

In[13]:= (*times[a_,b_,c_,d_]:= {a,b}*)

In[14]:= (*iimes[a_,b_,c_,d_][0]:= {a,d}*)

In[15]:= (*iimes[a_,b_][0]^:= {a,b}*)

```

```

In[1]:= iid[a_, b_] := a

In[2]:= iid[a_, b_, c_] := {a, b}

In[3]:= iid[a_List] := a[[1]]

In[4]:= qid[a__] := a

In[5]:= Clear[der]; der[a__] [c_] [b__] := c

In[6]:= Clear[der3]; der3[a__] [c_] [b__] := {c, a, b}

In[7]:= (*der[a_, b_] [c_]:= c*)

In[8]:= (*derz[a_, b_] := a*)

In[9]:= (*dert[a_, b_] := b*)

In[10]:= (*derivative[a__] [c_]:= FullForm[c]/.{Derivative->der}*)

In[11]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
(*ToString[#]&/@%*)
% /. {Derivative -> der}

Out[10]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[11]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[12]= {Z[0][z, t], Z[0], Z[0]}

```

```
In[ ]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}

%* (Prime[#] & /@ Range[Length[%]])

Plus @@ %

FullForm[%] (* #]&/@%*)

% /. {Plus -> qid, Times -> qid, Derivative -> der}

List[%]

Partition[%, 2]

eextract[#] & /@ %

(*eextract[#]&/@%*)

(*#__List[[0]]&/@%*)

#[[2]] & /@ %

#[[3]] & /@ %

Out[ ]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[ ]= {2 Z[0][z, t], 3 Z[0]^(1,0)[z, t], 5 Z[0]^(0,1)[z, t]}

Out[ ]= 2 Z[0][z, t] + 5 Z[0]^(0,1)[z, t] + 3 Z[0]^(1,0)[z, t]

Out[ ]//FullForm=
Plus[Times[2, Z[0][z, t]], Times[5, Derivative[0, 1][Z[0]][z, t]],
Times[3, Derivative[1, 0][Z[0]][z, t]]]

Out[ ]=
Sequence[2, Z[0][z, t], 5, Z[0], 3, Z[0]]

Out[ ]=
{2, Z[0][z, t], 5, Z[0], 3, Z[0]}

Out[ ]=
{{2, Z[0][z, t]}, {5, Z[0]}, {3, Z[0]}}

Out[ ]=
{{List, 2, Z[0][z, t]}, {List, 5, Z[0]}, {List, 3, Z[0]}}

Out[ ]=
{2, 5, 3}

Out[ ]=
{Z[0][z, t], Z[0], Z[0]}

Sequence[Subscript[expr, 1], Subscript[expr, 2], ...]

represents a sequence of arguments to be spliced automatically into any function.
```

```

In[1]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[##] & /@ %
(*ToString[##]&/@%*)
% /. {Derivative -> der3}

Out[1]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[2]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[3]= {Z[0][z, t], List[Z[0], 1, 0, z, t], List[Z[0], 0, 1, z, t]}

In[4]:= helpf[x__] := Length[{x}]

In[5]:= {helpf[x, y, z], helpf[]}

Out[5]= {3, helpf[]}

In[6]:= f[##] & @@ X

Out[6]= f[x0, x1, x2, x3, x4, x5, x6, x7]

In[7]:= StringExtract["a--bbb--ccc--dddd", "--" -> 3]

Out[7]= ccc

In[8]:= StringExtract["a bbb cccc aa d", -1]

Out[8]= d

In[9]:= StringRiffle[{{"a", "b", "c"}, {"d", "e", "f"}}, "\n", "\t"]

Out[9]= a      b      c
          d      e      f

In[10]:= {"", "z, t"}

Out[10]= {, z, t}

{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[##] & /@ %
ToString[##] & /@ %
StringSplit[#, "]"] & /@ %
StringExtract[#, "[" -> All] & /@ %
StringExtract[#, "
"\!`\\(*StyleBox[\"
  ",ShowStringCharacters->True,NumberMarks->True]\)`\!\`\\(*StyleBox[\\"\",ShowStringCharacters->True,NumberMarks->True]\`\!\`\\(*
  StyleBox[\\"\",\\!`ShowStringCharacters->True,NumberMarks->True]\`\!\`\\(*StyleBox[\\"z\\\",ShowStringCharacters->True,NumberMarks->True]\`\!\`\\(*
  StyleBox[\\"\",\\!`ShowStringCharacters->True,NumberMarks->True]\`\!\`\\(*StyleBox[\\"t\\\",ShowStringCharacters->True,NumberMarks->True]\`\!\`\\(*
  StyleBox[\\"\",\\!`ShowStringCharacters->True,NumberMarks->True]\`\!\`\\(*StyleBox[\\"t\\\",ShowStringCharacters->True,NumberMarks->True]\`\!\`\\(*
  All] & /@ %

InputForm[TextString[##]] & /@ %

```

```

In[=]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}

FullForm[#] & /@ %

ToString[#] & /@ %

StringSplit[#, "]"] & /@ %

StringExtract[#, "[" → All] & /@ %

(*StringExtract[#, {" ", z, t"} → All]&/@%*)

InputForm[TextString[#]] & /@ %

Out[=]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[=]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]= {{Z[0], [z, t], {Derivative[1, 0, [Z[0], [z, t], {Derivative[0, 1, [Z[0], [z, t]}}

Out[=]= {{{Z, 0}, {, z, t}}, {{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}, {{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}}

Out[=]= {"{{Z, 0}, {, z, t}}", "{{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}",
"{{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}"}

```

```

In[=]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
(*InputForm[TextString[#]]&/@%*)
StringExtract[#, "[" → All] & /@ %
StringExtract[#, "]" → All] & /@ %
StringExtract[#, "[" → 1] & /@ %%%
StringExtract[#, "]" → 1] & /@ %%%%
StringExtract[#, "Z[" → 1] & /@ %%%%%
Out[=]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[=]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]= {{Z, 0}, z, t}, {Derivative, 1, 0}, Z, 0]], z, t}, ,
{Derivative, 0, 1}, Z, 0]], z, t}]

Out[=]= {{Z[0, [z, t, ], {Derivative[1, 0, [Z[0, , [z, t, ],
{Derivative[0, 1, [Z[0, , [z, t, ]]

Out[=]= {Z, Derivative, Derivative}

Out[=]= {Z[0, Derivative[1, 0, Derivative[0, 1]

Out[=]= {, Derivative[1, 0][, Derivative[0, 1][}

In[=]:= (*TemplateApply[StringTemplate["Z `["` then `b`"],
<|"a"→1234,"b"→5678|>]*)

In[=]:= skelx[x_, matQ_ : True] := Module[{t = {666}},
If[matQ == True, t = Block[{r},
r = x;
Table[Block[{q}, q = SameQ[0, r[[j, i]]];
If[True == q, Style[0, Blue], Style[Length[r[[j, i]], Red]],
{j, 1, Length[r]}, {i, 1, Length[r[[ -1]]}]];
If[matQ == False, t = Block[{r},
r = x;
Table[Block[{q}, q = SameQ[0, r[[j]]];
If[True == q, Style[0, {RGBColor → {0, 1/3, 0}, Bold}],
Style[Length[r[[j]]], Red]], {j, 1, Length[r]}];
MatrixForm[t, TableAlignments → Left]

```

```
In[1]:= makeSym[size_, fn_] :=
Module[{rtmp}, rtmp = Table[fn[i, j], {i, 1, size}, {j, 1, i}];
MapThread[Join, {rtmp, Rest /@ Flatten[rtmp, {{2}, {1}}]}]]

In[2]:= makeAntiSym[size_, fn_] :=
Module[{rtmp}, rtmp = Table[If[i == j, 0, fn[i, j]], {i, 1, size}, {j, 1, i}];
MapThread[Join, {rtmp, Rest /@ Flatten[-rtmp, {{2}, {1}}]}]]

In[3]:= Block[{MX}, MX = makeAntiSym[8, Subscript[\[omega], ##] &]]
Out[3]=
{ {0, -\omega_{2,1}, -\omega_{3,1}, -\omega_{4,1}, -\omega_{5,1}, -\omega_{6,1}, -\omega_{7,1}, -\omega_{8,1}},
{ \omega_{2,1}, 0, -\omega_{3,2}, -\omega_{4,2}, -\omega_{5,2}, -\omega_{6,2}, -\omega_{7,2}, -\omega_{8,2}},
{ \omega_{3,1}, \omega_{3,2}, 0, -\omega_{4,3}, -\omega_{5,3}, -\omega_{6,3}, -\omega_{7,3}, -\omega_{8,3}},
{ \omega_{4,1}, \omega_{4,2}, \omega_{4,3}, 0, -\omega_{5,4}, -\omega_{6,4}, -\omega_{7,4}, -\omega_{8,4}},
{ \omega_{5,1}, \omega_{5,2}, \omega_{5,3}, \omega_{5,4}, 0, -\omega_{6,5}, -\omega_{7,5}, -\omega_{8,5}},
{ \omega_{6,1}, \omega_{6,2}, \omega_{6,3}, \omega_{6,4}, \omega_{6,5}, 0, -\omega_{7,6}, -\omega_{8,6}},
{ \omega_{7,1}, \omega_{7,2}, \omega_{7,3}, \omega_{7,4}, \omega_{7,5}, \omega_{7,6}, 0, -\omega_{8,7}},
{ \omega_{8,1}, \omega_{8,2}, \omega_{8,3}, \omega_{8,4}, \omega_{8,5}, \omega_{8,6}, \omega_{8,7}, 0} }
```

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2} \left(\frac{\partial g_{\mu\alpha}}{\partial x^\beta} + \frac{\partial g_{\mu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\mu} \right),$$

$$\Gamma^\mu{}_{\alpha\beta} = g^{\mu\nu} \Gamma_{\nu\alpha\beta},$$

$$R^\mu{}_{\nu\alpha\beta} = \frac{\partial \Gamma^\mu{}_{\nu\beta}}{\partial x^\alpha} - \frac{\partial \Gamma^\mu{}_{\nu\alpha}}{\partial x^\beta} + \Gamma^\mu{}_{\rho\alpha} \Gamma^\rho{}_{\nu\beta} - \Gamma^\mu{}_{\rho\beta} \Gamma^\rho{}_{\nu\alpha}.$$

$$R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}, \quad R = g^{\mu\nu} R_{\mu\nu},$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

```
In[1]:= (*Christoffel, Einstein tensor, ...*)
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, rr, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    ΓΓ = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
      D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8} *
      {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
    RicciΓ =
      ParallelTable[FullSimplify[D[ΓΓ[[μ, ν, β]], X[[α]]] - D[ΓΓ[[μ, ν, α]], X[[β]]] +
        Sum[ΓΓ[[μ, s, α]] * ΓΓ[[s, ν, β]] - ΓΓ[[μ, s, β]] * ΓΓ[[s, ν, α]],
        {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}];
    RieΓ = ParallelTable[ FullSimplify[Sum[ RicciΓ[[α, μ, α, ν]], {α, 1, DIM8}],
      ass], {μ, 1, DIM8}, {ν, 1, DIM8}];
    RS = FullSimplify[Tr[ ginv.RieΓ], ass];
    G = ParallelTable[
      FullSimplify[RieΓ[[α]] - 1/2 g[[α]] RS, ass], {α, 1, Length[RieΓ]}];
    ];
  Print[t];
  Print[Now];
  Return[{ginv, ΓΓ, RicciΓ, RieΓ, RS, G}]
]
```

If ‘paste’ Maple MathML to ToExpression[MathMLForm[]] fails, then try to edit this :

```
In[2]:= Clear[mapleztStringToMathematica];
mapleztStringToMathematica[
  input_String, toExpression_: False, debug_: False] :=
Module[{prepare, post, custom, use = input, save = input, linenumber = 0},
(*use=ToString[InputForm[TextString[input]]];*)(*Print[use];*)
  prepare[in_String] := Module[{s = in}, linenumber++;
  s = in;
  (*If[s==ToString[InputForm[TextString[s]]]],(*###&[]*)(*Print[s],Print[ToUpperCase[s]]*)(*)*)
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,z)" >
    "D[Z[" <> ToString[j] <> "][z,t],{z,2}]]"], {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,t)" >
    "D[D[Z[" <> ToString[j] <> "][z,t],z],t]"], {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,z)" >
```

```

"D[D[Z[" <> ToString[j] <> "][z,t],t],z]", {j, 0, 15}];

Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,t)" >
  "D[Z[" <> ToString[j] <> "][z,t],{t,2}]]", {j, 0, 15}];

Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t)" >
  "D[Z[" <> ToString[j] <> "][z,t],t]", {j, 0, 15}];

Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z)" >
  "D[Z[" <> ToString[j] <> "][z,t],z]", {j, 0, 15}];

s = StringReplace[s, "tan(z)" >> "Tan[z]"];
s = StringReplace[s, "cot(z)" >> "Cot[z]"];
s = StringReplace[s, "sin(z)" >> "Sin[z]"];
s = StringReplace[s, "cos(z)" >> "Cos[z]"];
s = StringReplace[s, "sec(z)" >> "Sec[z]"];
s = StringReplace[s, "csc(z)" >> "Csc[z]"];

s = StringReplace[s, "=" >> "->"];

(*Do[s=StringReplace[s,
  "Z"<>ToString[j]<>"(z,t)">>"Z["<>ToString[j]<>"[z,t]",{j,0,15}];*)
(*s=StringReplace[s,"diff(a4(t),t)">>"D[a4[t],t]"];*)
(*s=StringReplace[s,"exp(-2*a4(t))">>"Exp[-2a4[t]]"];*)
(*s=StringReplace[s,"a4(t)">>"a4[t]"];*)

Return[s];
];

post[in_String] := Module[{s = in}, linenumber++;
  s = in;
  s = StringReplace[s, "a4(t)" >> "a4[t]"];
  Do[s = StringReplace[s, "Z" <> ToString[j] <> "(z,t)" >
    "Z[" <> ToString[j] <> "][z,t]", {j, 0, 15}];
  Return[s];
];
custom[in_String] := Module[{s = in}, linenumber++;
  s = in;

  s = StringReplace[s, "diff(a4(t),t)" >> "D[a4[t],t]"];
  s = StringReplace[s, "exp(-2*a4(t))" >> "Exp[-2a4[t]]"];

Return[s];
]

```

```

];
use = prepare[use];
use = custom[use];
Return[post[use]];
(*If[toExpression,Return[ToExpression[s/.{z→#1,t→#2}],##&[]];*)
]

```

Mathematica Lexer and Parser for Maple - like Syntax Supports : Basic arithmetic, function calls, lists {}, equations =, and specific Maple keywords .

```

In[1]:= (*---LEXER---*) (*Helper:Check if character is a digit*)
IsDigit[char_] := StringMatchQ[char, RegularExpression["\\d"]];

(*Helper:Check if character is a letter*)
IsAlpha[char_] := StringMatchQ[char, RegularExpression"[a-zA-Z_"]];

(*Helper:Check if character is whitespace*)
IsSpace[char_] := StringMatchQ[char, RegularExpression["\\s"]];

(*Main Lexer Function*)
GetTokens[str_String] :=
Module[{chars, len, i, char, tokens = {}, token, numStr, idStr},
  chars = Characters[str];
  len = Length[chars];
  i = 1;
  While[i ≤ len, char = chars[[i]];
    Which[(*Skip Whitespace*) IsSpace[char],
      i++, (*Numbers*) IsDigit[char], numStr = char;
      i++;
      While[i ≤ len && IsDigit[chars[[i]]], numStr = numStr <> chars[[i]];
      i++];
    AppendTo[tokens, {"NUMBER", ToExpression[numStr]}],
    (*Identifiers and Keywords*) IsAlpha[char], idStr = char;
    i++;
    While[i ≤ len && (IsAlpha[chars[[i]]] || IsDigit[chars[[i]]]),
      idStr = idStr <> chars[[i]];
      i++];
    (*Check for specific keywords if needed,
    otherwise IDENTIFIER*) AppendTo[tokens, {"IDENTIFIER", idStr}],
    (*Operators and Punctuation*) char == "+", AppendTo[tokens, {"PLUS", "+"}];
    i++, char == "-", AppendTo[tokens, {"MINUS", "-"}];
  ]
]

```

```

i++, char == "*", AppendTo[tokens, {"STAR", "*"}];
i++, char == "/", AppendTo[tokens, {"SLASH", "/" }];
i++, char == "^", AppendTo[tokens, {"CARET", "^"}];
i++, char == "(", AppendTo[tokens, {"LPAREN", "(" }];
i++, char == ")", AppendTo[tokens, {"RPAREN", ")" }];
i++, char == "{", AppendTo[tokens, {"LBRACE", "{" }];
i++, char == "}", AppendTo[tokens, {"RBRACE", "}" }];
i++, char == "=", AppendTo[tokens, {"EQUALS", "=" }];
i++, char == ",", AppendTo[tokens, {"COMMA", "," }];
i++, char == "$", AppendTo[tokens, {"DOLLAR", "$"}];
i++, (*Ellipsis... or Dot.*) char == ".", If[i + 2 ≤ len && chars[[i + 1]] ==
".." && chars[[i + 2]] == ".", AppendTo[tokens, {"ELLIPSIS", "..."}]];
i += 3, AppendTo[tokens, {"DOT", "."}];
i++], True, Print["Unknown character: ", char];
i++]];
AppendTo[tokens, {"EOF", "EOF"}];
tokens];

In[1]:= (*---PARSER---*)
(*Grammar: Expression→Equation Equation→AddExp["=" AddExp] AddExp→
MulExp {("+"|"-") MulExp} MulExp→PowerExp {("*"|"/") PowerExp} PowerExp→
Primary["^" PowerExp] (Right associative) Primary→
NUMBER|IDENTIFIER|"(" Expression ")"|IDENTIFIER "(" ArgList ")"
"{" ArgList "}" ArgList→Expression {," Expression}|Empty*)
ParseTokens[tokens_List] := Module[{pos = 1, currentToken, eat,
peek, parseExpression, parseEquation, parseAddExp,
parseMulExp, parsePowerExp, parseUnaryExp, parsePrimary,
parseArgs, parseList}, currentToken := tokens[[pos]];
peek[] := tokens[[pos]];
eat[type_] := If[currentToken[[1]] == type, pos++;
True, Print["Error: Expected ", type, " but found ", currentToken[[1]]];
False];
(*Entry point*)parseExpression[] := parseEquation[];
(*Equation:a=b*)
parseEquation[] := Module[{left, right}, left = parseAddExp[];
If[currentToken[[1]] == "EQUALS", eat["EQUALS"];
right = parseAddExp[];
{"Equation", left, right}, left]];
parseAddExp[] := Module[{node, right, op}, node = parseMulExp[];
While[MemberQ[{"PLUS", "MINUS"}, currentToken[[1]]], op = currentToken[[2]];
eat[currentToken[[1]]];

```

```

right = parseMulExp[];
node = {"BinaryOp", op, node, right};};
node];
parseMulExp[] := Module[{node, right, op}, node = parsePowerExp[];
While[True,
If[MemberQ[{"STAR", "SLASH"}, currentToken[[1]]], op = currentToken[[2]];
eat[currentToken[[1]]];
right = parsePowerExp[];
node = {"BinaryOp", op, node, right}, (*Implicit Multiplication*)
If[MemberQ[{"IDENTIFIER", "NUMBER", "LPAREN", "LBRACE", "ELLIPSIS"}, currentToken[[1]]], right = parsePowerExp[];
node = {"BinaryOp", "*", node, right},
Break[] (*Not an operator or start of expression*)]];
node];
parsePowerExp[] := Module[{node, right}, node = parseUnaryExp[];
If[currentToken[[1]] == "CARET", eat["CARET"]];
right = parsePowerExp[];
(*Right associative recursion*) {"BinaryOp", "^", node, right}, node]];
parseUnaryExp[] := Module[{op, node},
If[MemberQ[{"PLUS", "MINUS"}, currentToken[[1]]], op = currentToken[[2]];
eat[currentToken[[1]]];
node = parseUnaryExp[];
{"UnaryOp", op, node}, parsePrimary[]]];
parsePrimary[] := Module[{token, node, name, args}, token = currentToken;
Switch[token[[1]], "NUMBER", eat["NUMBER"]];
{"Number", token[[2]]}, "IDENTIFIER", eat["IDENTIFIER"]];
name = token[[2]];
(*Check if it's a function call:IDENTIFIER followed by LPAREN*)
If[currentToken[[1]] == "LPAREN", eat["LPAREN"]];
args = parseArgs[];
eat["RPAREN"];
{"Call", name, args}, {"Identifier", name}], "LPAREN", eat["LPAREN"]];
node = parseExpression[];
eat["RPAREN"];
node, "LBRACE", parseList[], "ELLIPSIS", eat["ELLIPSIS"]];
{"Identifier", "..."},
"EOF", Print["Error: Unexpected EOF in Primary"];
{"Error", "EOF"}, (*Do NOT eat EOF*)_,
Print["Error: Unexpected token in Primary: ", token];
eat[token[[1]]];

```

```

(*Advance to avoid infinite loop*){"Error", token}]];
(*Parse List:{a,b,c}* )parseList[] := Module[{elements}, eat["LBRACE"]];
elements = parseArgs[];
eat["RBRACE"];
 {"List", elements}];
(*Parse Argument List:expr,expr,...*)
parseArgs[] := Module[{args = {}, arg}, If[currentToken[[1]] != "RPAREN" &&
currentToken[[1]] != "RBRACE", arg = parseExpression[];
AppendTo[args, arg];
While[currentToken[[1]] == "COMMA", eat["COMMA"]];
arg = parseExpression[];
AppendTo[args, arg];];
args];
parseExpression[]];

X

In[]:= (*---CONVERTER---*)
(*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunction[funcName_String] :=
Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate",
"DESol", "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan",
"Tan", "sin", "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc",
"Csc", "csch", "Csch", "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh",
"coth", "Coth", "sech", "Sech", "det", "Det", "transpose", "Transpose",
"sphericalbessel", "SphericalBesselJ", "lambertw", "ProductLog",
"gamma", "Gamma", "airy", "AiryAi", "besselJ", "BesselJ", "besselY",
"BesselY", "hypergeometricU", "HypergeometricU", "zeta", "Zeta",
"erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNode[nodeType_, nodeData_, childResults_] :=
Module[{funcName, cleanFuncName, mathFunc, argsStr},
Switch[nodeType, "Number", ToString[nodeData],
"Identifier", (*Handle special identifiers like _Y→Y*)
If[StringLength[nodeData] > 0 && StringTake[nodeData, 1] == "_",
StringDrop[nodeData, 1], nodeData], "BinaryOp",
"(" <> childResults[[1]] <> " " <> nodeData <> " " <> childResults[[2]] <> ")",
"UnaryOp", nodeData <> "(" <> childResults[[1]] <> ")", "Equation",
childResults[[1]] <> " == " <> childResults[[2]], "List", "{" <>
StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
(*Handle special identifiers like _Y→Y for function names too*)]

```

```

cleanFuncName = If[StringLength[funcName] > 0 &&
    StringTake[funcName, 1] == "_", StringDrop[funcName, 1], funcName];
mathFunc = MapleToMathematicaFunction[cleanFuncName];
(*Special handling for DSolve-requires 3 arguments*)
If[cleanFuncName == "DESol" || mathFunc == "DSolve",
    (*DSolve[equations,functions,variables]*)
    (*Ensure we have exactly 3 arguments,pad with empty lists if needed*)
    argsStr = StringRiffle[Join[childResults,
        Table["{}", {i, 1, Max[0, 3 - Length[childResults]]}]], ", "];
    mathFunc <> "[" <> argsStr <> "]",
    (*Default function call*)argsStr = StringRiffle[childResults, ", "];
    mathFunc <> "[" <> argsStr <> "]"], _, "Error"]];

(*Convert AST to Mathematica String-Non-
recursive iterative version using post-order traversal*)
ToMathematicaString[ast_] :=
Module[{stack, outputStack, currentItem, node,.nodeType, children,
    childCount, childResults, i, result, nodeData}, (*Use two
    stacks:one for traversal,one for results*)stack = {{ast, False}};
    (*{node,visited}*)
    outputStack = {};
    (*Post-order traversal:process children before parents*)
    While[Length[stack] > 0, currentItem = Last[stack];
        node = currentItem[[1]];
        nodeType = node[[1]];
        If[currentItem[[2]],
            (*Node already visited,process it*)stack = Most[stack];
            (*Determine children and extract data*)
            {children, nodeData} = Switch[nodeType, "Number", {}, node[[2]],
                "Identifier", {}, node[[2]], "BinaryOp", {{node[[3]], node[[4]]}, node[[2]]},
                "UnaryOp", {{node[[3]]}, node[[2]]}, "Equation", {{node[[2]], node[[3]]}}, ""),
                "List", {node[[2]], ""}, "Call", {node[[3]], node[[2]]}, _, {}, ""}];
            (*Pop child results from output stack*)
            childCount = Length[children];
            childResults = {};
            If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
                outputStack = Most[outputStack];, {i, 1, childCount}]];
            (*Process this node*)
            result = ProcessASTNode[nodeType, nodeData, childResults];
            AppendTo[outputStack, result];, (*Node not visited,
            mark as visited and push children*)stack[[Length[stack]]] = {node, True};
            (*Push children onto stack

```

```

    in reverse order (so they process left-to-right)*)
children = Switch[nodeType, "Number", {}, "Identifier", {},
  "BinaryOp", {node[[3]], node[[4]]}, "UnaryOp", {node[[3]]}, "Equation",
  {node[[2]], node[[3]]}, "List", node[[2]], "Call", node[[3]], _, {}];
Do[AppendTo[stack, {children[[i]], False}]]|,
{i, Length[children], 1, -1}];];
(*Return the final result*)
If[Length[outputStack] > 0, Last[outputStack], "Error"]];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematica[inputStr_String] :=
Module[{tokens, ast}, tokens = GetTokens[inputStr];
ast = ParseTokens[tokens];
ToMathematicaString[ast]];

In[1]:= (*---CONVERTER---*)
(*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunctionOLD[funcName_String] :=
Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate",
"DESol", "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan",
"Tan", "sin", "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc",
"Csc", "csch", "Csch", "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh",
"coth", "Coth", "sech", "Sech", "det", "Det", "transpose", "Transpose",
"sphericalbessel", "SphericalBesselJ", "lambertw", "ProductLog",
"gamma", "Gamma", "airy", "AiryAi", "besselJ", "BesselJ", "besselY",
"BesselY", "hypergeometricU", "HypergeometricU", "zeta", "Zeta",
"erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNodeOLD[nodeType_, nodeData_, childResults_] :=
Module[{funcName, cleanFuncName, mathFunc, argsStr},
Switch[nodeType, "Number", ToString[nodeData],
"Identifier", (*Handle special identifiers like _Y→Y*)
If[StringLength[nodeData] > 0 && StringTake[nodeData, 1] == "_",
StringDrop[nodeData, 1], nodeData], "BinaryOp",
"(" <> childResults[[1]] <> " " <> nodeData <> " " <> childResults[[2]] <> ")",
"UnaryOp", nodeData <> "(" <> childResults[[1]] <> ")",
"Equation", childResults[[1]] <> " == " <> childResults[[2]], "List", "{" <>
StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
(*Handle special identifiers like _Y→Y for function names too*)
cleanFuncName = If[StringLength[funcName] > 0 &&

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StringTake[funcName, 1] = "_", StringDrop[funcName, 1], funcName];
mathFunc = MapleToMathematicaFunction[cleanFuncName];
argsStr = StringRiffle[childResults, ", "];
mathFunc <> "[" <> argsStr <> "]", _, "Error"]];

(*Convert AST to Mathematica String-Non-
recursive iterative version using post-order traversal*)
ToMathematicaStringOLD[ast_] :=
Module[{stack, outputStack, currentItem, node,.nodeType, children,
childCount, childResults, i, result, nodeData}, (*Use two
stacks:one for traversal,one for results*)stack = {{ast, False}};
(*{node,visited}*)
outputStack = {};
(*Post-order traversal:process children before parents*)
While[Length[stack] > 0, currentItem = Last[stack];
node = currentItem[[1]];
nodeType = node[[1]];
If[currentItem[[2]],
(*Node already visited,process it*)stack = Most[stack];
(*Determine children and extract data*)
{children, nodeData} = Switch[nodeType, "Number", {}, node[[2]],
"Identifier", {}, node[[2]], "BinaryOp", {{node[[3]], node[[4]]}, node[[2]]},
"UnaryOp", {{node[[3]]}, node[[2]]}, "Equation", {{node[[2]], node[[3]]}}, ""),
"List", {node[[2]], ""}, "Call", {node[[3]], node[[2]]}, _, {}, ""}];
(*Pop child results from output stack*)childCount = Length[children];
childResults = {};
If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
outputStack = Most[outputStack];, {i, 1, childCount}]];
(*Process this node*)
result = ProcessASTNode[nodeType, nodeData, childResults];
AppendTo[outputStack, result];, (*Node not visited,
mark as visited and push children*)stack[[Length[stack]]] = {node, True};
(*Push children onto stack
in reverse order (so they process left-to-right)*)
children = Switch[nodeType, "Number", {}, "Identifier", {},
"BinaryOp", {node[[3]], node[[4]]}, "UnaryOp", {node[[3]]}, "Equation",
{node[[2]], node[[3]]}, "List", node[[2]], "Call", node[[3]], _, {}];
Do[AppendTo[stack, {children[[i]], False}];,
{i, Length[children], 1, -1}];];
(*Return the final result*)
If[Length[outputStack] > 0, Last[outputStack], "Error"]];

```

```
(*---MAIN WRAPPER---*)
ConvertMapleToMathematicaOLD[inputStr_String] :=
Module[{tokens, ast}, tokens = GetTokens[inputStr];
ast = ParseTokens[tokens];
ToMathematicaString[ast]];

In[]:= sta = "nZ10(z, t) = (c2C7*sin(C2Q1*t) +
c2C8*cos(C2Q1*t))*(c2C5*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) +
c2C6*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ11(z, t) = (c2C3*sin(C2Q1*t) +
c2C4*cos(C2Q1*t))*(c2C1*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) +
c2C2*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ8(z, t) = (((-C2Q1*c2C2*c2C3 +
M*c2C6*c2C8)*cos(C2Q1*t) + sin(C2Q1*t)*(C2Q1*c2C2*c2C4 +
M*c2C6*c2C7))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) + ((C2Q1*c2C1*c2C3 -
M*c2C5*c2C8)*cos(C2Q1*t) - sin(C2Q1*t)*(C2Q1*c2C1*c2C4 +
M*c2C5*c2C7))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) + (-cos(M*t)*c2C10 +
sin(M*t)*c2C9)*sqrt(-C2Q1^2 + M^2))/sqrt(-C2Q1^2 + M^2),
nZ9(z, t) = (((C2Q1*c2C6*c2C7 + M*c2C2*c2C4)*cos(C2Q1*t) +
sin(C2Q1*t)*(-C2Q1*c2C6*c2C8 + M*c2C2*c2C3))*sin(z)^(-sqrt(-C2Q1^2 +
M^2)/3) + sqrt(-C2Q1^2 + M^2)*(cos(M*t)*c2C9 +
sin(M*t)*c2C10)*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) + (-C2Q1*c2C5*c2C7 -
M*c2C1*c2C4)*cos(C2Q1*t) - sin(C2Q1*t)*(-C2Q1*c2C5*c2C8 +
M*c2C1*c2C3))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6)/sqrt(-C2Q1^2 + M^2)}";
```

```

In[]:= ConvertMapleToMathematica[sta]
Out[]=

{nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5
* (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) + (c2C6 * (Sin[z]
^ (-Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6)))), nZ11[z, t] == (((c2C3
* Sin[(C2Q1 * t)]) + (c2C4 * Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z] ^
(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) + (c2C2 * (Sin[z] ^ (-Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))] / 6)))), nZ8[z, t] == ((((((-C2Q1) * c2C2) * c2C3)
+ ((M * c2C6) * c2C8)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((C2Q1 *
c2C2) * c2C4) + ((M * c2C6) * c2C7))) * (Sin[z] ^ (-Sqrt[((-(C2Q1) ^ 2)
+ (M ^ 2))] / 6))) + (((((C2Q1 * c2C1) * c2C3) - ((M * c2C5) * c2C8))
* Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C1) * c2C4) + ((M *
c2C5) * c2C7))) * (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) +
(((-(Cos[(M * t)]) * c2C10) + (Sin[(M * t)] * c2C9)) * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]) / Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]), nZ9[z, t] ==
(((((((-C2Q1) * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)])
+ (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) *
(Sin[z] ^ (-Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 3))) + ((Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))] * ((Cos[(M * t)] * c2C9) + (Sin[(M * t)] * c2C10))) *
(Sin[z] ^ (-Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) + (((-(C2Q1) *
c2C5) * c2C7) - ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 *
t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))) * (Sin[z] ^
(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) / Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])

In[]:= sti =
```
{`nZ10(z,t)=(c2C7*`sin(C2Q1*t)+c2C8*`cos(C2Q1*t))*(`c2C5*`sin(z)`^(1/6*`sqrt(-C2Q1^2+M^2))+c2C6*`sin(z)`^(-1/6*`sqrt(-C2Q1^2+M^2))),`nZ11(z,t)=(c2C3*`sin(C2Q1*t)+c2C4*`cos(C2Q1*t))*(`c2C1*`sin(z)`^(1/6*`sqrt(-C2Q1^2+M^2))+c2C2*`sin(z)`^(-1/6*`sqrt(-C2Q1^2+M^2))),`nZ8(z,t)=1/6*(6*`sqrt(-C2Q1^2+M^2)*c2C6*(c2C7*`sin(C2Q1*t)+c2C8*`cos(C2Q1*t))*`sin(z)`^(-1/6*`sqrt(-C2Q1^2+M^2))-6*`sqrt(-C2Q1^2+M^2)*c2C5*(c2C7*`sin(C2Q1*t)+c2C8*`cos(C2Q1*t))*`sin(z)`^(1/6*`sqrt(-C2Q1^2+M^2))+(-6*M*c2C10-`Int(-`cos(M*t)*((-C2Q1^2+M^2)*`Int(-`cos(M*t)*(`c2C6*c2C8+M*c2C2*c2C3))*`sin(z)`^(-1/6*`sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*`cos(C2Q1*t)+`sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*`sin(z)`^(1/6*`sqrt(-C2Q1^2+M^2)-1)),z)-6*`sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*`cos(C2Q1*t)+`sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*`sin(z)`^(-1/6*`sqrt(-C2Q1^2+M^2))-`sin(z)`^(1/6*`sqrt(-C2Q1^2+M^2))*(C2Q1*c2C5*c2C7+M*c2C1*c2C4)*`cos(C2Q1*t)+`sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*`cos(M*t)+(6*M*c2C9-`Int(-`sin(M*t)*((-C2Q1^2+M^2)*`Int(-`cos(z)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*`cos(C2Q1*t)+`sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*`sin(z)`^(-1/6*`sqrt(-C2Q1^2+M^2))-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*`cos(C2Q1*t)+`sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*`cos(M*t)+(6*M*c2C9-`Int(-`sin(M*t)*((-C2Q1*c2C6*c2C7+M*c2C2*c2C4)*`cos(C2Q1*t)+`sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*`sin(z)`^(-1/6*`sqrt(-C2Q1^2+M^2))-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*`cos(C2Q1*t)+`sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3)))
```

```

```

c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1^2
+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*
c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*
sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*
(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*sin(M*t)-C2Q1*Int(-cos(z)*((-C2Q1*c2C6*c2C8+M*c2C2*c2C3)*cos(C2Q1*t)-sin(C2Q1*t)*(C2Q1*c2C6*c2C7+M*c2C2*c2C4))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((-C2Q1*c2C5*c2C8+M*c2C1*c2C3)*cos(C2Q1*t)-sin(C2Q1*t)*(C2Q1*c2C5*c2C7+M*c2C1*c2C4))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1))/M,nZ9[z,t]=1/6*((6*M*c2C9-Int(-sin(M*t)*((-C2Q1*sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*cos(M*t)+(6*M*c2C10+Int(-cos(M*t)*((-C2Q1^2+M^2)*Int(-cos(z)*((-C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*cos(M*t)+6*M*c2C10+Int(-cos(M*t)*((-C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*sin(M*t)+M*Int(-cos(z)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1))+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)))/M}";
```

In[•]:= ConvertMapleToMathematica[sti]

Out[•]=

```

{nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5 *
(Sin[z]^(1/6) * Sqrt[((-(C2Q1)^2 + (M^2)))]))) + (c2C6 * (Sin[z]^
((-1)/6) * Sqrt[((-(C2Q1)^2 + (M^2)))]))), nZ11[z, t] == (((c2C3 *
Sin[(C2Q1 * t)]) + (c2C4 * Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z]^(1/
6) * Sqrt[((-(C2Q1)^2 + (M^2)))]))) + (c2C2 * (Sin[z]^((-1)/6) *
Sqrt[((-(C2Q1)^2 + (M^2)))]))), nZ8[z, t] == (((1/6) * (((((6 *
Sqrt[((-(C2Q1)^2 + (M^2)))] * c2C6) * ((c2C7 * Sin[(C2Q1 * t)])
+ (c2C8 * Cos[(C2Q1 * t)]))) * (Sin[z]^((-1)/6) * Sqrt[((-(C2Q1)^
2 + (M^2)))])) - (((6 * Sqrt[((-(C2Q1)^2 + (M^2)))] * c2C5) *
((c2C7 * Sin[(C2Q1 * t)] + (c2C8 * Cos[(C2Q1 * t)]))) * (Sin[z]^((1/
6) * Sqrt[((-(C2Q1)^2 + (M^2)))])))) + ((((-6) * M) * c2C10) -
```

```

Integrate[(-(Cos[(M * t)]) * ((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z])
* (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)])]
+ (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) *
(Sin[z] ^ (((-1) / 6) * Sqrt[(((-(C2Q1) ^ 2) + (M ^ 2))] - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) +
(Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[(((-(C2Q1) ^ 2) + (M ^ 2))] - 1))))), z]) - ((6
* Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * (((((C2Q1 * c2C6) * c2C7) + ((M *
c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) *
c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-1) / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))]))) - ((Sin[z] ^ ((1 / 6) * Sqrt[(((-(C2Q1) ^ 2) + (M ^ 2)))]))
* (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) +
(Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))))), t]) * Cos[(M * t)]) + (((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) *
(((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6)
* c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] *
(((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((((-1)
/ 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)))] - 1))) + (((((C2Q1 * c2C5) *
c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] *
(((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z] ^ (((1 / 6)
* Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)))] - 1)))), z]) - ((6 * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) *
Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ ((((-1) / 6) * Sqrt[((-(C2Q1) ^ 2) +
(M ^ 2)))] - 1)))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) *
((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) +
(Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))), t]) * Sin[(M * t)]) - (C2Q1 * Integrate[(-(Cos[z]) * (((((-(C2Q1) *
c2C6) * c2C8) + ((M * c2C2) * c2C3)) * Cos[(C2Q1 * t)])) - (Sin[(C2Q1 * t)] *
(((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)))) * (Sin[z] ^ ((((-1)
/ 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)))] - 1))) + (((((-(C2Q1) * c2C5)
* c2C8) + ((M * c2C1) * c2C3)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] *
(((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)))) * (Sin[z] ^ (((1 / 6) *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)))] - 1)))), z])) / M), nZ9[z, t] == (((1
/ 6) * (((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) * (((-(C2Q1) ^
2) + (M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M *
c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6)
* c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((((-1) / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2)))] - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) *
c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8)
+ ((M * c2C1) * c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2)
+ (M ^ 2)))] - 1)))), z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) *
((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)])) +
(Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) *

```

$$\begin{aligned}
& (\sin[z] \wedge ((-(1) / 6) * \sqrt{((-C2Q1) \wedge 2) + (M \wedge 2))})) - ((\sin[z] \wedge \\
& ((1 / 6) * \sqrt{((-C2Q1) \wedge 2) + (M \wedge 2))})) * (((((C2Q1 * c2C5) * c2C7) + \\
& ((M * c2C1) * c2C4)) * \cos[(C2Q1 * t)]) + (\sin[(C2Q1 * t)] * (((-(C2Q1) * \\
& c2C5) * c2C8) + ((M * c2C1) * c2C3))))))) * (\sin[(C2Q1 * t)] * \cos[(M * t)]) + (((6 \\
& * M) * c2C10) + \text{Integrate}[-(\cos[(M * t)]) * (((-(C2Q1) \wedge 2) + (M \wedge 2)) \\
& * \text{Integrate}[(-\cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * \\
& c2C4)) * \cos[(C2Q1 * t)]) + (\sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) \\
& + ((M * c2C2) * c2C3)))) * (\sin[z] \wedge (((-(1) / 6) * \sqrt{((-C2Q1) \wedge 2) + \\
& (M \wedge 2))}) - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * \\
& \cos[(C2Q1 * t)]) + (\sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * \\
& c2C1) * c2C3)))) * (\sin[z] \wedge (((1 / 6) * \sqrt{((-C2Q1) \wedge 2) + (M \wedge 2))}) - \\
& 1))), z]) - ((6 * \sqrt{((-C2Q1) \wedge 2) + (M \wedge 2))}) * (((((C2Q1 * c2C6) \\
& * c2C7) + ((M * c2C2) * c2C4)) * \cos[(C2Q1 * t)]) + (\sin[(C2Q1 * t)] * \\
& (((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (\sin[z] \wedge (((-(1) / 6) \\
& * \sqrt{((-C2Q1) \wedge 2) + (M \wedge 2))}) - ((\sin[z] \wedge ((1 / 6) * \sqrt{((-C2Q1) \\
& \wedge 2) + (M \wedge 2))}) * (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * \\
& \cos[(C2Q1 * t)]) + (\sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * \\
& c2C1) * c2C3)))) * (\sin[z] \wedge (((-(1) / 6) * \sqrt{((-C2Q1) \wedge 2) + (M \wedge 2))}) - \\
& 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * \cos[(C2Q1 * t)]) + \\
& (\sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * \\
& (\sin[z] \wedge (((1 / 6) * \sqrt{((-C2Q1) \wedge 2) + (M \wedge 2))}) - 1)))), z])) / M)
\end{aligned}$$

gtry and Γ and ...

```
In[1]:= gtry = MatrixMetric44
Out[1]=
{{Cot[6 H x 0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, E^(2 a4[H x 4]) Sin[6 H x 0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, E^(2 a4[H x 4]) Sin[6 H x 0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, E^(2 a4[H x 4]) Sin[6 H x 0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -E^(-2 a4[H x 4]) Sin[6 H x 0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, -E^(-2 a4[H x 4]) Sin[6 H x 0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, -E^(-2 a4[H x 4]) Sin[6 H x 0]^(1/3)}}}
```

```
In[2]:= Protect[gtry]
Out[2]=
{gtry}
```

```
In[3]:= Unprotect[ginv, \[Gamma], Ricci\[Gamma], Rie\[Gamma], RS, EinsteinG]
Out[3]=
{}
```

```
In[4]:= result = {ginv, \[Gamma], Ricci\[Gamma], Rie\[Gamma], RS, EinsteinG} = rt[gtry];
```

Tue 3 Feb 2026 15:34:36 GMT-8

{3.63541, Null}

Tue 3 Feb 2026 15:34:40 GMT-8

In[•]:= **Protect[ginv, Γ , Ricci Γ , Rie Γ , RS, EinsteinG]**
Out[•]= {ginv, Γ , Ricci Γ , Rie Γ , RS, EinsteinG}

verify $g_{\mu\nu|\alpha} = 0$:

In[•]:= **Simplify[**
Table[D[gtry[[j, k]], X[i]] - Sum[gtry[[k, s]] \times Γ [[s, i, j]], {s, 1, DIM8}] -
Sum[gtry[[j, s]] \times Γ [[s, i, k]], {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union

Out[•]= {0}

In[•]:= **Table[g[j][k] \rightarrow If[j == k,**
ToExpression["((g[" <> ToString[j] <> "] [" <> ToString[k] <> "] [#1, #2]) &)"],
(0 &)], {j, 0, 7}, {k, 0, 7}];
sg = % // Flatten

Out[•]= {g[0][0] \rightarrow (g[0][0][#1, #2] &), g[0][1] \rightarrow (0 &), g[0][2] \rightarrow (0 &), g[0][3] \rightarrow (0 &),
g[0][4] \rightarrow (0 &), g[0][5] \rightarrow (0 &), g[0][6] \rightarrow (0 &), g[0][7] \rightarrow (0 &),
g[1][0] \rightarrow (0 &), g[1][1] \rightarrow (g[1][1][#1, #2] &), g[1][2] \rightarrow (0 &), g[1][3] \rightarrow (0 &),
g[1][4] \rightarrow (0 &), g[1][5] \rightarrow (0 &), g[1][6] \rightarrow (0 &), g[1][7] \rightarrow (0 &),
g[2][0] \rightarrow (0 &), g[2][1] \rightarrow (0 &), g[2][2] \rightarrow (g[2][2][#1, #2] &), g[2][3] \rightarrow (0 &),
g[2][4] \rightarrow (0 &), g[2][5] \rightarrow (0 &), g[2][6] \rightarrow (0 &), g[2][7] \rightarrow (0 &),
g[3][0] \rightarrow (0 &), g[3][1] \rightarrow (0 &), g[3][2] \rightarrow (0 &), g[3][3] \rightarrow (g[3][3][#1, #2] &),
g[3][4] \rightarrow (0 &), g[3][5] \rightarrow (0 &), g[3][6] \rightarrow (0 &), g[3][7] \rightarrow (0 &), g[4][0] \rightarrow (0 &),
g[4][1] \rightarrow (0 &), g[4][2] \rightarrow (0 &), g[4][3] \rightarrow (0 &), g[4][4] \rightarrow (g[4][4][#1, #2] &),
g[4][5] \rightarrow (0 &), g[4][6] \rightarrow (0 &), g[4][7] \rightarrow (0 &), g[5][0] \rightarrow (0 &), g[5][1] \rightarrow (0 &),
g[5][2] \rightarrow (0 &), g[5][3] \rightarrow (0 &), g[5][4] \rightarrow (0 &), g[5][5] \rightarrow (g[5][5][#1, #2] &),
g[5][6] \rightarrow (0 &), g[5][7] \rightarrow (0 &), g[6][0] \rightarrow (0 &), g[6][1] \rightarrow (0 &), g[6][2] \rightarrow (0 &),
g[6][3] \rightarrow (0 &), g[6][4] \rightarrow (0 &), g[6][5] \rightarrow (0 &), g[6][6] \rightarrow (g[6][6][#1, #2] &),
g[6][7] \rightarrow (0 &), g[7][0] \rightarrow (0 &), g[7][1] \rightarrow (0 &), g[7][2] \rightarrow (0 &), g[7][3] \rightarrow (0 &),
g[7][4] \rightarrow (0 &), g[7][5] \rightarrow (0 &), g[7][6] \rightarrow (0 &), g[7][7] \rightarrow (g[7][7][#1, #2] &)}

In[•]:= **g[7][7][x0, x4] /. ssgm4488**
% /. sx0x4

Out[•]= $-\epsilon^{-2} a4[H x4] \sin[6 H x0]^{1/3}$

Out[•]= $-\epsilon^{-2} a4[t] \sin[z]^{1/3}$

In[1]:= **MatrixMetric44**

Out[1]=

$$\left\{ \left\{ \text{Cot}[6Hx0]^2, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3} \right\} \right\}$$

In[2]:= **ssgm4488[x0, x4] /. sx0x4**

Out[2]=

$$\begin{aligned} & \left\{ g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \right. \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&), \\ & g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&), \\ & g[2][2] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&), \\ & g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&), \\ & g[3][3] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & \left. g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&) \right\} \left[\frac{z}{6H}, \frac{t}{H} \right] \end{aligned}$$

```
In[8]:= (*ssgGzt={G[0][0]→((Cot[#1]^2)&),G[0][1]→((0)&),G[0][2]→((0)&),
G[0][3]→((0)&),G[0][4]→((0)&),G[0][5]→((0)&),G[0][6]→((0)&),
G[0][7]→((0)&),G[1][0]→((0)&),G[1][1]→((e2*A4[#2] Sin[#1]1/3)&),
G[1][2]→((0)&),G[1][3]→((0)&),G[1][4]→((0)&),G[1][5]→((0)&),
G[1][6]→((0)&),G[1][7]→((0)&),G[2][0]→((0)&),G[2][1]→((0)&),
G[2][2]→((e2*A4[#2] Sin[#1]1/3)&),G[2][3]→((0)&),G[2][4]→((0)&),
G[2][5]→((0)&),G[2][6]→((0)&),G[2][7]→((0)&),G[3][0]→((0)&),
G[3][1]→((0)&),G[3][2]→((0)&),G[3][3]→((e2*A4[#2] Sin[#1]1/3)&),
G[3][4]→((0)&),G[3][5]→((0)&),G[3][6]→((0)&),G[3][7]→((0)&),
G[4][0]→((0)&),G[4][1]→((0)&),G[4][2]→((0)&),G[4][3]→((0)&),
G[4][4]→((-1)&),G[4][5]→((0)&),G[4][6]→((0)&),G[4][7]→((0)&),
G[5][0]→((0)&),G[5][1]→((0)&),G[5][2]→((0)&),G[5][3]→((0)&),
G[5][4]→((0)&),G[5][5]→((-e-2*A4[#2] Sin[#1]1/3)&),
G[5][6]→((0)&),G[5][7]→((0)&),G[6][0]→((0)&),G[6][1]→((0)&),
G[6][2]→((0)&),G[6][3]→((0)&),G[6][4]→((0)&),G[6][5]→((0)&),
G[6][6]→((-e-2*A4[#2] Sin[#1]1/3)&),G[6][7]→((0)&),G[7][0]→((0)&),
G[7][1]→((0)&),G[7][2]→((0)&),G[7][3]→((0)&),G[7][4]→((0)&),
G[7][5]→((0)&),G[7][6]→((0)&),G[7][7]→((-e-2*A4[#2] Sin[#1]1/3)&})*)
```

```
In[9]:= (*G[#[#][z,t]/.ssgGzt&/@Range[0,7]*)
```

```
In[10]:= detgg = Det[g4488 /. sg] // FullSimplify[#, constraintVars] &
```

Out[10]=

$$g[0][0][x_0, x_4] \times g[1][1][x_0, x_4] \times g[2][2][x_0, x_4] \times g[3][3][x_0, x_4] \times \\ g[4][4][x_0, x_4] \times g[5][5][x_0, x_4] \times g[6][6][x_0, x_4] \times g[7][7][x_0, x_4]$$

```
In[11]:= g4488 /. sg // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix} g[0][0][x_0, x_4] & 0 & 0 & 0 & 0 \\ 0 & g[1][1][x_0, x_4] & 0 & 0 & 0 \\ 0 & 0 & g[2][2][x_0, x_4] & 0 & 0 \\ 0 & 0 & 0 & g[3][3][x_0, x_4] & 0 \\ 0 & 0 & 0 & 0 & g[4][4][x_0, x_4] \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} g[5]$$

In[1]:= **MatrixMetric44.**η4488

(*eAa=*) **FullSimplify**[$\sqrt{\%}$, constraintVars]

Out[1]=

$$\left\{ \left\{ \text{Cot}[6Hx0]^2, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, e^{-2a4[Hx4]} \sin[6Hx0]^{1/3} \right\} \right\}$$

Out[2]=

$$\left\{ \left\{ \text{Cot}[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, e^{-2a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, e^{-2a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, e^{-2a4[Hx4]} \sin[6Hx0]^{1/6} \right\} \right\}$$

eAa

In[3]:= **g4488.**η4488 /. sg

(eAa = FullSimplify[$\sqrt{\%}$, constraintVars]**) // MatrixForm**

Out[3]=

$$\left\{ \left\{ g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, -g[4][4][x0, x4], 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, -g[5][5][x0, x4], 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, -g[6][6][x0, x4], 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, -g[7][7][x0, x4], 0, 0, 0 \right\} \right\}$$

Out[4]//MatrixForm=

$$\begin{pmatrix} \sqrt{g[0][0][x0, x4]} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{g[1][1][x0, x4]} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{g[2][2][x0, x4]} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{g[3][3][x0, x4]} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{-g[4][4][x0, x4]} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

In[=]:= eAa
ssgm4488
(eAa04 = %% /. % // FullSimplify[#, constraintVars] &) // MatrixForm

Out[=]=
{ { √g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, √g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, √g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, √g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, √-g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, √-g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, √-g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, √-g[7][7][x0, x4]} }

Out[=]=
{ g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
  g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
  g[1][1] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &), g[1][4] → (0 &),
  g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &), g[2][1] → (0 &),
  g[2][2] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &), g[2][5] → (0 &),
  g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &), g[3][2] → (0 &),
  g[3][3] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
  g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
  g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
  g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
  g[5][5] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
  g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
  g[6][5] → (0 &), g[6][6] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
  g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
  g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &) }

Out[=]//MatrixForm=

$$\begin{array}{cccccc} \text{Cot}[6 H x0] & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$


```

OCTAD $e_{\alpha \leftarrow \text{spacetime}}^{(A) \leftarrow \text{Lorentz}}$:

$$e_{\alpha}^{(A)} = e_{\alpha}^{(A)} = eAa$$

In[•]:= $(\text{sge}_{\alpha}^{(A)} = \text{Thread}[\text{Flatten}[e_{\alpha}^{(A)}] \rightarrow \text{Flatten}[eAa]])) (*//Column*)$

Out[•]=

$$\begin{aligned} & \left\{ E_{\alpha}^{(A)}[0, 0] \rightarrow \sqrt{g[0][0][x0, x4]}, E_{\alpha}^{(A)}[0, 1] \rightarrow 0, E_{\alpha}^{(A)}[0, 2] \rightarrow 0, \right. \\ & E_{\alpha}^{(A)}[0, 3] \rightarrow 0, E_{\alpha}^{(A)}[0, 4] \rightarrow 0, E_{\alpha}^{(A)}[0, 5] \rightarrow 0, E_{\alpha}^{(A)}[0, 6] \rightarrow 0, E_{\alpha}^{(A)}[0, 7] \rightarrow 0, \\ & E_{\alpha}^{(A)}[1, 0] \rightarrow 0, E_{\alpha}^{(A)}[1, 1] \rightarrow \sqrt{g[1][1][x0, x4]}, E_{\alpha}^{(A)}[1, 2] \rightarrow 0, E_{\alpha}^{(A)}[1, 3] \rightarrow 0, \\ & E_{\alpha}^{(A)}[1, 4] \rightarrow 0, E_{\alpha}^{(A)}[1, 5] \rightarrow 0, E_{\alpha}^{(A)}[1, 6] \rightarrow 0, E_{\alpha}^{(A)}[1, 7] \rightarrow 0, E_{\alpha}^{(A)}[2, 0] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 1] \rightarrow 0, E_{\alpha}^{(A)}[2, 2] \rightarrow \sqrt{g[2][2][x0, x4]}, E_{\alpha}^{(A)}[2, 3] \rightarrow 0, E_{\alpha}^{(A)}[2, 4] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 5] \rightarrow 0, E_{\alpha}^{(A)}[2, 6] \rightarrow 0, E_{\alpha}^{(A)}[2, 7] \rightarrow 0, E_{\alpha}^{(A)}[3, 0] \rightarrow 0, E_{\alpha}^{(A)}[3, 1] \rightarrow 0, \\ & E_{\alpha}^{(A)}[3, 2] \rightarrow 0, E_{\alpha}^{(A)}[3, 3] \rightarrow \sqrt{g[3][3][x0, x4]}, E_{\alpha}^{(A)}[3, 4] \rightarrow 0, E_{\alpha}^{(A)}[3, 5] \rightarrow 0, \\ & E_{\alpha}^{(A)}[3, 6] \rightarrow 0, E_{\alpha}^{(A)}[3, 7] \rightarrow 0, E_{\alpha}^{(A)}[4, 0] \rightarrow 0, E_{\alpha}^{(A)}[4, 1] \rightarrow 0, E_{\alpha}^{(A)}[4, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[4, 3] \rightarrow 0, E_{\alpha}^{(A)}[4, 4] \rightarrow \sqrt{-g[4][4][x0, x4]}, E_{\alpha}^{(A)}[4, 5] \rightarrow 0, \\ & E_{\alpha}^{(A)}[4, 6] \rightarrow 0, E_{\alpha}^{(A)}[4, 7] \rightarrow 0, E_{\alpha}^{(A)}[5, 0] \rightarrow 0, E_{\alpha}^{(A)}[5, 1] \rightarrow 0, E_{\alpha}^{(A)}[5, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[5, 3] \rightarrow 0, E_{\alpha}^{(A)}[5, 4] \rightarrow 0, E_{\alpha}^{(A)}[5, 5] \rightarrow \sqrt{-g[5][5][x0, x4]}, \\ & E_{\alpha}^{(A)}[5, 6] \rightarrow 0, E_{\alpha}^{(A)}[5, 7] \rightarrow 0, E_{\alpha}^{(A)}[6, 0] \rightarrow 0, E_{\alpha}^{(A)}[6, 1] \rightarrow 0, E_{\alpha}^{(A)}[6, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[6, 3] \rightarrow 0, E_{\alpha}^{(A)}[6, 4] \rightarrow 0, E_{\alpha}^{(A)}[6, 5] \rightarrow 0, E_{\alpha}^{(A)}[6, 6] \rightarrow \sqrt{-g[6][6][x0, x4]}, \\ & E_{\alpha}^{(A)}[6, 7] \rightarrow 0, E_{\alpha}^{(A)}[7, 0] \rightarrow 0, E_{\alpha}^{(A)}[7, 1] \rightarrow 0, E_{\alpha}^{(A)}[7, 2] \rightarrow 0, E_{\alpha}^{(A)}[7, 3] \rightarrow 0, \\ & \left. E_{\alpha}^{(A)}[7, 4] \rightarrow 0, E_{\alpha}^{(A)}[7, 5] \rightarrow 0, E_{\alpha}^{(A)}[7, 6] \rightarrow 0, E_{\alpha}^{(A)}[7, 7] \rightarrow \sqrt{-g[7][7][x0, x4]} \right\} \end{aligned}$$

In[•]:= $(*\text{Symbolize}[\text{sge}_{\alpha}^{(A)}]*)$

In[•]:= $(*\text{Symbolize}[\text{sgtrye}_{\alpha}^{(A)}]*)$

In[•]:= $(\text{sgtrye}_{\alpha}^{(A)} = \text{Thread}[\text{Flatten}[e_{\alpha}^{(A)}] \rightarrow \text{Flatten}[eAa04]])) (*//Column*)$

Out[•]=

$$\begin{aligned} & \left\{ E_{\alpha}^{(A)}[0, 0] \rightarrow \text{Cot}[6Hx0], E_{\alpha}^{(A)}[0, 1] \rightarrow 0, E_{\alpha}^{(A)}[0, 2] \rightarrow 0, E_{\alpha}^{(A)}[0, 3] \rightarrow 0, \right. \\ & E_{\alpha}^{(A)}[0, 4] \rightarrow 0, E_{\alpha}^{(A)}[0, 5] \rightarrow 0, E_{\alpha}^{(A)}[0, 6] \rightarrow 0, E_{\alpha}^{(A)}[0, 7] \rightarrow 0, E_{\alpha}^{(A)}[1, 0] \rightarrow 0, \\ & E_{\alpha}^{(A)}[1, 1] \rightarrow \sqrt{e^{2 a4[H x4]}} \text{Sin}[6Hx0]^{1/6}, E_{\alpha}^{(A)}[1, 2] \rightarrow 0, E_{\alpha}^{(A)}[1, 3] \rightarrow 0, E_{\alpha}^{(A)}[1, 4] \rightarrow 0, \\ & E_{\alpha}^{(A)}[1, 5] \rightarrow 0, E_{\alpha}^{(A)}[1, 6] \rightarrow 0, E_{\alpha}^{(A)}[1, 7] \rightarrow 0, E_{\alpha}^{(A)}[2, 0] \rightarrow 0, E_{\alpha}^{(A)}[2, 1] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 2] \rightarrow \sqrt{e^{2 a4[H x4]}} \text{Sin}[6Hx0]^{1/6}, E_{\alpha}^{(A)}[2, 3] \rightarrow 0, E_{\alpha}^{(A)}[2, 4] \rightarrow 0, E_{\alpha}^{(A)}[2, 5] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 6] \rightarrow 0, E_{\alpha}^{(A)}[2, 7] \rightarrow 0, E_{\alpha}^{(A)}[3, 0] \rightarrow 0, E_{\alpha}^{(A)}[3, 1] \rightarrow 0, E_{\alpha}^{(A)}[3, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[3, 3] \rightarrow \sqrt{e^{2 a4[H x4]}} \text{Sin}[6Hx0]^{1/6}, E_{\alpha}^{(A)}[3, 4] \rightarrow 0, E_{\alpha}^{(A)}[3, 5] \rightarrow 0, E_{\alpha}^{(A)}[3, 6] \rightarrow 0, \\ & E_{\alpha}^{(A)}[3, 7] \rightarrow 0, E_{\alpha}^{(A)}[4, 0] \rightarrow 0, E_{\alpha}^{(A)}[4, 1] \rightarrow 0, E_{\alpha}^{(A)}[4, 2] \rightarrow 0, E_{\alpha}^{(A)}[4, 3] \rightarrow 0, E_{\alpha}^{(A)}[4, 4] \rightarrow 1, \\ & E_{\alpha}^{(A)}[4, 5] \rightarrow 0, E_{\alpha}^{(A)}[4, 6] \rightarrow 0, E_{\alpha}^{(A)}[4, 7] \rightarrow 0, E_{\alpha}^{(A)}[5, 0] \rightarrow 0, E_{\alpha}^{(A)}[5, 1] \rightarrow 0, \\ & E_{\alpha}^{(A)}[5, 2] \rightarrow 0, E_{\alpha}^{(A)}[5, 3] \rightarrow 0, E_{\alpha}^{(A)}[5, 4] \rightarrow 0, E_{\alpha}^{(A)}[5, 5] \rightarrow \sqrt{e^{-2 a4[H x4]}} \text{Sin}[6Hx0]^{1/6}, \\ & E_{\alpha}^{(A)}[5, 6] \rightarrow 0, E_{\alpha}^{(A)}[5, 7] \rightarrow 0, E_{\alpha}^{(A)}[6, 0] \rightarrow 0, E_{\alpha}^{(A)}[6, 1] \rightarrow 0, E_{\alpha}^{(A)}[6, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[6, 3] \rightarrow 0, E_{\alpha}^{(A)}[6, 4] \rightarrow 0, E_{\alpha}^{(A)}[6, 5] \rightarrow 0, E_{\alpha}^{(A)}[6, 6] \rightarrow \sqrt{e^{-2 a4[H x4]}} \text{Sin}[6Hx0]^{1/6}, \\ & E_{\alpha}^{(A)}[6, 7] \rightarrow 0, E_{\alpha}^{(A)}[7, 0] \rightarrow 0, E_{\alpha}^{(A)}[7, 1] \rightarrow 0, E_{\alpha}^{(A)}[7, 2] \rightarrow 0, E_{\alpha}^{(A)}[7, 3] \rightarrow 0, \\ & \left. E_{\alpha}^{(A)}[7, 4] \rightarrow 0, E_{\alpha}^{(A)}[7, 5] \rightarrow 0, E_{\alpha}^{(A)}[7, 6] \rightarrow 0, E_{\alpha}^{(A)}[7, 7] \rightarrow \sqrt{e^{-2 a4[H x4]}} \text{Sin}[6Hx0]^{1/6} \right\} \end{aligned}$$

```
In[ ]:= (sgtrye $_{(A)}^{\alpha}$  = Thread[Flatten[e $_{(A)}^{\alpha}$ ]  $\rightarrow$  Flatten[Inverse[e $_{\alpha}^{(A)}$  /. sgtrye $_{\alpha}^{(A)}$ ]]])  
(*//Column*)
```

```
Out[ ]= {E $_{(A)}^{\alpha}$  [0, 0]  $\rightarrow$  Tan[6 H x0], E $_{(A)}^{\alpha}$  [0, 1]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [0, 2]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [0, 3]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [0, 4]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [0, 5]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [0, 6]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [0, 7]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [1, 0]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [1, 1]  $\rightarrow$   $\frac{1}{\sqrt{e^{2 a4[H \times 4]} \sin[6 H x0]^{1/6}}}$ , E $_{(A)}^{\alpha}$  [1, 2]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [1, 3]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [1, 4]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [1, 5]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [1, 6]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [1, 7]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [2, 0]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [2, 1]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [2, 2]  $\rightarrow$   $\frac{1}{\sqrt{e^{2 a4[H \times 4]} \sin[6 H x0]^{1/6}}}$ , E $_{(A)}^{\alpha}$  [2, 3]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [2, 4]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [2, 5]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [2, 6]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [2, 7]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [3, 0]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [3, 1]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [3, 2]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [3, 3]  $\rightarrow$   $\frac{1}{\sqrt{e^{2 a4[H \times 4]} \sin[6 H x0]^{1/6}}}$ , E $_{(A)}^{\alpha}$  [3, 4]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [3, 5]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [3, 6]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [3, 7]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [4, 0]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [4, 1]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [4, 2]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [4, 3]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [4, 4]  $\rightarrow$  1,  
E $_{(A)}^{\alpha}$  [4, 5]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [4, 6]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [4, 7]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [5, 0]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [5, 1]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [5, 2]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [5, 3]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [5, 4]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [5, 5]  $\rightarrow$   $\frac{1}{\sqrt{e^{-2 a4[H \times 4]} \sin[6 H x0]^{1/6}}}$ ,  
E $_{(A)}^{\alpha}$  [5, 6]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [5, 7]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [6, 0]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [6, 1]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [6, 2]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [6, 3]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [6, 4]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [6, 5]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [6, 6]  $\rightarrow$   $\frac{1}{\sqrt{e^{-2 a4[H \times 4]} \sin[6 H x0]^{1/6}}}$ ,  
E $_{(A)}^{\alpha}$  [6, 7]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [7, 0]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [7, 1]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [7, 2]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [7, 3]  $\rightarrow$  0,  
E $_{(A)}^{\alpha}$  [7, 4]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [7, 5]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [7, 6]  $\rightarrow$  0, E $_{(A)}^{\alpha}$  [7, 7]  $\rightarrow$   $\frac{1}{\sqrt{e^{-2 a4[H \times 4]} \sin[6 H x0]^{1/6}}}$ }
```

```
In[ ]:= gtrye $_{\alpha}^{(A)}$  = (e $_{\alpha}^{(A)}$  /. sgtrye $_{\alpha}^{(A)}$ ) /. subsDefects
```

```
Out[ ]= {{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0, 0}, {0, e $^{a4[H \times 4]}$  Sin[6 H x0] $^{1/6}$ , 0, 0, 0, 0, 0, 0},  
{0, 0, e $^{a4[H \times 4]}$  Sin[6 H x0] $^{1/6}$ , 0, 0, 0, 0, 0}, {0, 0, 0, e $^{a4[H \times 4]}$  Sin[6 H x0] $^{1/6}$ , 0, 0, 0, 0},  
{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e $^{-a4[H \times 4]}$  Sin[6 H x0] $^{1/6}$ , 0, 0},  
{0, 0, 0, 0, 0, 0, e $^{-a4[H \times 4]}$  Sin[6 H x0] $^{1/6}$ , 0},  
{0, 0, 0, 0, 0, 0, 0, e $^{-a4[H \times 4]}$  Sin[6 H x0] $^{1/6}$ }}
```

```

In[]:= gtrye(A) = (e(A) /. sgtrye(A)) /. subsDefects
Out[]=
{ {Tan[6 H x 0], 0, 0, 0, 0, 0, 0, 0}, {0, e-a4[H x 4]/Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, e-a4[H x 4]/Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, 0, e-a4[H x 4]/Sin[6 H x 0]1/6, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e-a4[H x 4]/Sin[6 H x 0]1/6, 0, 0}, {0, 0, 0, 0, 0, 0, ea4[H x 4]/Sin[6 H x 0]1/6, 0}, {0, 0, 0, 0, 0, 0, 0, ea4[H x 4]/Sin[6 H x 0]1/6} }

In[]:= e(A)
% /. sgtrye(A) // MatrixForm
Out[=
{{E(A)[0, 0], E(A)[0, 1], E(A)[0, 2], E(A)[0, 3], E(A)[0, 4], E(A)[0, 5], E(A)[0, 6], E(A)[0, 7]}, {E(A)[1, 0], E(A)[1, 1], E(A)[1, 2], E(A)[1, 3], E(A)[1, 4], E(A)[1, 5], E(A)[1, 6], E(A)[1, 7]}, {E(A)[2, 0], E(A)[2, 1], E(A)[2, 2], E(A)[2, 3], E(A)[2, 4], E(A)[2, 5], E(A)[2, 6], E(A)[2, 7]}, {E(A)[3, 0], E(A)[3, 1], E(A)[3, 2], E(A)[3, 3], E(A)[3, 4], E(A)[3, 5], E(A)[3, 6], E(A)[3, 7]}, {E(A)[4, 0], E(A)[4, 1], E(A)[4, 2], E(A)[4, 3], E(A)[4, 4], E(A)[4, 5], E(A)[4, 6], E(A)[4, 7]}, {E(A)[5, 0], E(A)[5, 1], E(A)[5, 2], E(A)[5, 3], E(A)[5, 4], E(A)[5, 5], E(A)[5, 6], E(A)[5, 7]}, {E(A)[6, 0], E(A)[6, 1], E(A)[6, 2], E(A)[6, 3], E(A)[6, 4], E(A)[6, 5], E(A)[6, 6], E(A)[6, 7]}, {E(A)[7, 0], E(A)[7, 1], E(A)[7, 2], E(A)[7, 3], E(A)[7, 4], E(A)[7, 5], E(A)[7, 6], E(A)[7, 7]}}

Out[=]//MatrixForm=
Cot[6 H x 0] 0 0 0 0 0
0 √e2 a4[H x 4] Sin[6 H x 0]1/6 0 0 0
0 0 √e2 a4[H x 4] Sin[6 H x 0]1/6 0 0 0
0 0 0 √e2 a4[H x 4] Sin[6 H x 0]1/6 0 0
0 0 0 0 0 1
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0

```

```
In[]:= eηe = FullSimplify[Transpose[e_α^(A)] . η4488.e_α^(A), constraintVars];
% /. sge_α^(A)
%% /. sgtrye_α^(A)
% == gtry // FullSimplify[#, constraintVars] &

Out[]= {{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}

Out[=]
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

Out[=]
True

In[=]
gtry

Out[=]
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

In[=]
(*FullSimplify[Transpose[e_α^(A)] . η4488.e_α^(A),constraintVars]-MatrixMetric44*)
```

In[1]:= **ssgm4488**

Out[1]=

$$\{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&), \\ g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&), \\ g[2][2] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&), \\ g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&), \\ g[3][3] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&)\}$$

In[2]:= **MatrixMetric44 // MatrixForm**

Out[2]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[3]:= **e^(A)_α**

Out[3]=

$$\{\{E_{\alpha}^{(A)}[0, 0], E_{\alpha}^{(A)}[0, 1], E_{\alpha}^{(A)}[0, 2], E_{\alpha}^{(A)}[0, 3], \\ E_{\alpha}^{(A)}[0, 4], E_{\alpha}^{(A)}[0, 5], E_{\alpha}^{(A)}[0, 6], E_{\alpha}^{(A)}[0, 7]\}, \{E_{\alpha}^{(A)}[1, 0], E_{\alpha}^{(A)}[1, 1], \\ E_{\alpha}^{(A)}[1, 2], E_{\alpha}^{(A)}[1, 3], E_{\alpha}^{(A)}[1, 4], E_{\alpha}^{(A)}[1, 5], E_{\alpha}^{(A)}[1, 6], E_{\alpha}^{(A)}[1, 7]\}, \\ \{E_{\alpha}^{(A)}[2, 0], E_{\alpha}^{(A)}[2, 1], E_{\alpha}^{(A)}[2, 2], E_{\alpha}^{(A)}[2, 3], E_{\alpha}^{(A)}[2, 4], E_{\alpha}^{(A)}[2, 5], \\ E_{\alpha}^{(A)}[2, 6], E_{\alpha}^{(A)}[2, 7]\}, \{E_{\alpha}^{(A)}[3, 0], E_{\alpha}^{(A)}[3, 1], E_{\alpha}^{(A)}[3, 2], E_{\alpha}^{(A)}[3, 3], \\ E_{\alpha}^{(A)}[3, 4], E_{\alpha}^{(A)}[3, 5], E_{\alpha}^{(A)}[3, 6], E_{\alpha}^{(A)}[3, 7]\}, \{E_{\alpha}^{(A)}[4, 0], E_{\alpha}^{(A)}[4, 1], \\ E_{\alpha}^{(A)}[4, 2], E_{\alpha}^{(A)}[4, 3], E_{\alpha}^{(A)}[4, 4], E_{\alpha}^{(A)}[4, 5], E_{\alpha}^{(A)}[4, 6], E_{\alpha}^{(A)}[4, 7]\}, \\ \{E_{\alpha}^{(A)}[5, 0], E_{\alpha}^{(A)}[5, 1], E_{\alpha}^{(A)}[5, 2], E_{\alpha}^{(A)}[5, 3], E_{\alpha}^{(A)}[5, 4], E_{\alpha}^{(A)}[5, 5], \\ E_{\alpha}^{(A)}[5, 6], E_{\alpha}^{(A)}[5, 7]\}, \{E_{\alpha}^{(A)}[6, 0], E_{\alpha}^{(A)}[6, 1], E_{\alpha}^{(A)}[6, 2], E_{\alpha}^{(A)}[6, 3], \\ E_{\alpha}^{(A)}[6, 4], E_{\alpha}^{(A)}[6, 5], E_{\alpha}^{(A)}[6, 6], E_{\alpha}^{(A)}[6, 7]\}, \{E_{\alpha}^{(A)}[7, 0], E_{\alpha}^{(A)}[7, 1], \\ E_{\alpha}^{(A)}[7, 2], E_{\alpha}^{(A)}[7, 3], E_{\alpha}^{(A)}[7, 4], E_{\alpha}^{(A)}[7, 5], E_{\alpha}^{(A)}[7, 6], E_{\alpha}^{(A)}[7, 7]\}\}$$

```
In[]:= Block[{s, r}, s = FullSimplify[eα(A) /. sgtryeα(A), constraintVars];
r = FullSimplify[Transpose[s].η4488.s, constraintVars] - MatrixMetric44;
FullSimplify[r, constraintVars]]
```

```
Out[]:= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[]:= (* (eα(A)=eAa) //MatrixForm*)
```

```
In[]:= (*eα(A)
Inverse[%]*)
```

```
In[]:= (* (e(A)α=Inverse[
eα(A) /. sge/Subsuperscript α and LeftParenthesis A RightParenthesis])//MatrixForm*)
```

```
In[]:= e(A)α
```

```
Out[]:= {{E(A)α[0, 0], E(A)α[0, 1], E(A)α[0, 2], E(A)α[0, 3],
E(A)α[0, 4], E(A)α[0, 5], E(A)α[0, 6], E(A)α[0, 7]}, {E(A)α[1, 0], E(A)α[1, 1],
E(A)α[1, 2], E(A)α[1, 3], E(A)α[1, 4], E(A)α[1, 5], E(A)α[1, 6], E(A)α[1, 7]},
{E(A)α[2, 0], E(A)α[2, 1], E(A)α[2, 2], E(A)α[2, 3], E(A)α[2, 4], E(A)α[2, 5],
E(A)α[2, 6], E(A)α[2, 7]}, {E(A)α[3, 0], E(A)α[3, 1], E(A)α[3, 2], E(A)α[3, 3],
E(A)α[3, 4], E(A)α[3, 5], E(A)α[3, 6], E(A)α[3, 7]}, {E(A)α[4, 0], E(A)α[4, 1],
E(A)α[4, 2], E(A)α[4, 3], E(A)α[4, 4], E(A)α[4, 5], E(A)α[4, 6], E(A)α[4, 7]},
{E(A)α[5, 0], E(A)α[5, 1], E(A)α[5, 2], E(A)α[5, 3], E(A)α[5, 4], E(A)α[5, 5],
E(A)α[5, 6], E(A)α[5, 7]}, {E(A)α[6, 0], E(A)α[6, 1], E(A)α[6, 2], E(A)α[6, 3],
E(A)α[6, 4], E(A)α[6, 5], E(A)α[6, 6], E(A)α[6, 7]}, {E(A)α[7, 0], E(A)α[7, 1],
E(A)α[7, 2], E(A)α[7, 3], E(A)α[7, 4], E(A)α[7, 5], E(A)α[7, 6], E(A)α[7, 7]}}
```

```
In[]:= (sgeα(A) = Thread[Flatten[eα(A)] → Flatten[Inverse[e(A)α /. sge(A)α]]]) (*//Column*)
```

Out[]:=

$$\begin{aligned} & \left\{ E_{(A)}^{\alpha}[0, 0] \rightarrow \frac{1}{\sqrt{g[0][0][x0, x4]}}, E_{(A)}^{\alpha}[0, 1] \rightarrow 0, E_{(A)}^{\alpha}[0, 2] \rightarrow 0, \right. \\ & E_{(A)}^{\alpha}[0, 3] \rightarrow 0, E_{(A)}^{\alpha}[0, 4] \rightarrow 0, E_{(A)}^{\alpha}[0, 5] \rightarrow 0, E_{(A)}^{\alpha}[0, 6] \rightarrow 0, E_{(A)}^{\alpha}[0, 7] \rightarrow 0, \\ & E_{(A)}^{\alpha}[1, 0] \rightarrow 0, E_{(A)}^{\alpha}[1, 1] \rightarrow \frac{1}{\sqrt{g[1][1][x0, x4]}}, E_{(A)}^{\alpha}[1, 2] \rightarrow 0, E_{(A)}^{\alpha}[1, 3] \rightarrow 0, \\ & E_{(A)}^{\alpha}[1, 4] \rightarrow 0, E_{(A)}^{\alpha}[1, 5] \rightarrow 0, E_{(A)}^{\alpha}[1, 6] \rightarrow 0, E_{(A)}^{\alpha}[1, 7] \rightarrow 0, E_{(A)}^{\alpha}[2, 0] \rightarrow 0, \\ & E_{(A)}^{\alpha}[2, 1] \rightarrow 0, E_{(A)}^{\alpha}[2, 2] \rightarrow \frac{1}{\sqrt{g[2][2][x0, x4]}}, E_{(A)}^{\alpha}[2, 3] \rightarrow 0, E_{(A)}^{\alpha}[2, 4] \rightarrow 0, \\ & E_{(A)}^{\alpha}[2, 5] \rightarrow 0, E_{(A)}^{\alpha}[2, 6] \rightarrow 0, E_{(A)}^{\alpha}[2, 7] \rightarrow 0, E_{(A)}^{\alpha}[3, 0] \rightarrow 0, E_{(A)}^{\alpha}[3, 1] \rightarrow 0, \\ & E_{(A)}^{\alpha}[3, 2] \rightarrow 0, E_{(A)}^{\alpha}[3, 3] \rightarrow \frac{1}{\sqrt{g[3][3][x0, x4]}}, E_{(A)}^{\alpha}[3, 4] \rightarrow 0, E_{(A)}^{\alpha}[3, 5] \rightarrow 0, \\ & E_{(A)}^{\alpha}[3, 6] \rightarrow 0, E_{(A)}^{\alpha}[3, 7] \rightarrow 0, E_{(A)}^{\alpha}[4, 0] \rightarrow 0, E_{(A)}^{\alpha}[4, 1] \rightarrow 0, E_{(A)}^{\alpha}[4, 2] \rightarrow 0, \\ & E_{(A)}^{\alpha}[4, 3] \rightarrow 0, E_{(A)}^{\alpha}[4, 4] \rightarrow \frac{1}{\sqrt{-g[4][4][x0, x4]}}, E_{(A)}^{\alpha}[4, 5] \rightarrow 0, \\ & E_{(A)}^{\alpha}[4, 6] \rightarrow 0, E_{(A)}^{\alpha}[4, 7] \rightarrow 0, E_{(A)}^{\alpha}[5, 0] \rightarrow 0, E_{(A)}^{\alpha}[5, 1] \rightarrow 0, E_{(A)}^{\alpha}[5, 2] \rightarrow 0, \\ & E_{(A)}^{\alpha}[5, 3] \rightarrow 0, E_{(A)}^{\alpha}[5, 4] \rightarrow 0, E_{(A)}^{\alpha}[5, 5] \rightarrow \frac{1}{\sqrt{-g[5][5][x0, x4]}}, \\ & E_{(A)}^{\alpha}[5, 6] \rightarrow 0, E_{(A)}^{\alpha}[5, 7] \rightarrow 0, E_{(A)}^{\alpha}[6, 0] \rightarrow 0, E_{(A)}^{\alpha}[6, 1] \rightarrow 0, E_{(A)}^{\alpha}[6, 2] \rightarrow 0, \\ & E_{(A)}^{\alpha}[6, 3] \rightarrow 0, E_{(A)}^{\alpha}[6, 4] \rightarrow 0, E_{(A)}^{\alpha}[6, 5] \rightarrow 0, E_{(A)}^{\alpha}[6, 6] \rightarrow \frac{1}{\sqrt{-g[6][6][x0, x4]}}, \\ & E_{(A)}^{\alpha}[6, 7] \rightarrow 0, E_{(A)}^{\alpha}[7, 0] \rightarrow 0, E_{(A)}^{\alpha}[7, 1] \rightarrow 0, E_{(A)}^{\alpha}[7, 2] \rightarrow 0, E_{(A)}^{\alpha}[7, 3] \rightarrow 0, \\ & \left. E_{(A)}^{\alpha}[7, 4] \rightarrow 0, E_{(A)}^{\alpha}[7, 5] \rightarrow 0, E_{(A)}^{\alpha}[7, 6] \rightarrow 0, E_{(A)}^{\alpha}[7, 7] \rightarrow \frac{1}{\sqrt{-g[7][7][x0, x4]}} \right\} \end{aligned}$$

```
In[]:= (*Symbolize[sgeα(A)]*)
```

```
In[]:= (*Symbolize[sgtryeα(A)]*)
```

```
In[]:= (*%/.sge(A)α
%%/.sgtrye(A)α*)
```

```
In[]:= (*FullSimplify[Transpose[e(A)α].η4488.e(A)α, constraintVars]-MatrixMetric44*)
```

For Spin (4, 4); τ tau; T16; OCTAD : Nash; Introduce the wave function, Ψ_{16} , for this Universe:

O(4,4); evals, evecs of σ

```
In[1]:= {evals, evecs} = Eigensystem[\sigma]
Out[1]= {{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0}, {0, -1, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1}, {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0}}}
```

```
In[2]:= u = ExpandAll[(1/Sqrt[2]) evecs] // MatrixForm
```

$$\text{Out[2]//MatrixForm=}$$

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

```
In[3]:= Table[u[[h]].\sigma.u[[h]], {h, 1, Length[u]}]
```

```
Out[3]= {-1, -1, -1, -1, 1, 1, 1, 1}
```

```
In[4]:= Table[(Transpose[u][[h]]).\sigma.u[[h]], {h, 1, Length[u]}]
```

```
Out[4]= {-1, -1, -1, -1, 1, 1, 1, 1}
```

```
In[5]:= Table[(Transpose[u][[A]]) == -\sigma.u[[A]], {A, 1, 4}]
```

```
Table[(Transpose[u][[A]]) == \sigma.u[[A]], {A, 5, Length[u]}]
```

```
Out[5]= {True, True, True, True}
```

```
Out[6]= {True, True, True, True}
```

$$\text{Ax} = \left(\frac{\text{KroneckerProduct}[dX, X] - \text{KroneckerProduct}[X, dX]}{2} \right) . \sigma;$$

$$\text{Au} = \left(\frac{\text{KroneckerProduct}[dU, U] - \text{KroneckerProduct}[U, dU]}{2} \right) . \sigma;$$

$$\text{Lsquared} =$$

$$\dot{x}^A \dot{x}_A = dX.\sigma.dX + X.\sigma.X \left(\frac{dU.\sigma.dU}{U.\sigma.U} - \left(\frac{U.\sigma.dU}{U.\sigma.U} \right)^2 \right) + \frac{2}{U.\sigma.U} \text{Tr}[\bar{\tau}^A.Ax.\tau_A.Au]$$

```
Timing[Simplify[ExpandAll[dY.η8.dY - Lsquared]]]
{10.733, 0}
```

$$SO(4), \quad \gamma; M8, \{0, 1, 2, 3, 4, 5, 6, 7\}, \{+, +, +, -, -, -, -\}, \{x0, x1, x2, x3, x4, x5, x6, x7\}$$

Under the involutive automorphism $\gamma^{AB} \rightarrow -\tilde{\gamma}^{AB}$ of $so(3,3)$, the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra $so(4)$, which is the Lie algebra of $\overline{SO(4)}$, the maximal compact subgroup of $\overline{SL(4, \mathbb{R})}$. The subalgebra $so(4) \cong su(2) + su(2)$ may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of $so(4)$ whereby $\tau \in so(4)$ is mapped into its dual, ${}^*\tau$. The even subspace under $*$ of $so(4)$ corresponds to self-dual tensors, and, say, the first $su(2)$ in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second $su(2)$ in the direct sum. A basis for $so(4)$ may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by ($h = 1, 2, 3$).

$$2s^h = (\gamma^{23}, \gamma^{31}, \gamma^{21}), \quad (4)$$

and

$$2t^h = (\gamma^{45}, \gamma^{64}, \gamma^{65}). \quad (5)$$

Out[]=

Under the involutive automorphism $\gamma^{AB} \rightarrow -\tilde{\gamma}^{AB}$ of $so(3,3)$, the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra $so(4)$, which is the Lie algebra of $\overline{SO(4)}$, the maximal compact subgroup of $\overline{SL(4,\mathbb{R})}$. The subalgebra $so(4) \cong su(2) + su(2)$ may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of $so(4)$ whereby $\tau \in so(4)$ is mapped into its dual, ${}^*\tau$. The even subspace under * of $so(4)$ corresponds to self-dual tensors, and, say, the first $su(2)$ in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second $su(2)$ in the direct sum. A basis for $so(4)$ may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by ($h = 1, 2, 3$),

$$2s^h = (\gamma^{23}, \gamma^{31}, \gamma^{21}), \quad (4)$$

and

$$2t^h = (\gamma^{45}, \gamma^{64}, \gamma^{65}). \quad (5)$$

```
In[ ]:= Qa[h_, p_, q_] := Signature[{h, p, q, 4}];
Qb[h_, p_, q_] := ID4[[p, 4]] * ID4[[q, h]] - ID4[[p, h]] * ID4[[q, 4]];
SelfDualAntiSymmetric[h_, p_, q_] := Qa[h, p, q] - Qb[h, p, q];
AntiSelfDualAntiSymmetric[h_, p_, q_] := (Qa[h, p, q] + Qb[h, p, q]);
```

```
In[ ]:= Qa[1, 2, 3]
```

Out[]=

1

```
In[ ]:= e4[[1, 2, 3, 4]]
```

Out[]=

1

```
In[ ]:= Do[s4by4[h] = Table[Table[SelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
{h, 1, 3}]
```

```
In[ ]:= Do[
```

```
t4by4[h] = Table[Table[AntiSelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
{h, 1, 3}]
```

```

In[]:= (*Protect[{s4by4[1],s4by4[2],s4by4[3],t4by4[1],t4by4[2],t4by4[3]}]*)

In[]:= Protect[s4by4, t4by4]
Out[]= {s4by4, t4by4}

In[]:= {s4by4[#] // MatrixForm, MatrixForm[t4by4[#]]} & /@ Range[3]
Out[=]

{{\left(\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right)}, \left(\begin{array}{cccc} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)}}, {\left(\begin{array}{cccc} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right)}, \left(\begin{array}{cccc} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)}, {\left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{array}\right)}, \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)}}

In[]:= Table[{{J, K}, MatrixForm[s4by4[J].t4by4[K]]}, {J, 1, 3}, {K, 1, 3}]
Out[=]

{{{\{1, 1\}, \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}}, {{1, 2\}, \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)}}, {{1, 3\}, \left(\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right)}}, {{\{2, 1\}, \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{array}\right)}}, {{\{2, 2\}, \left(\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}}, {{\{2, 3\}, \left(\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)}}, {{\{3, 1\}, \left(\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)}}, {{\{3, 2\}, \left(\begin{array}{cccc} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right)}}, {{\{3, 3\}, \left(\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}}}},

In[]:= Do[st[J, K] = s4by4[J].t4by4[K], {J, 1, 3}, {K, 1, 3}]

In[]:= Table[{{J, K}, MatrixForm[st[J, K]]}, {J, 1, 3}, {K, 1, 3}]
Out[=]

{{{\{1, 1\}, \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}}, {{1, 2\}, \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)}}, {{1, 3\}, \left(\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right)}}, {{\{2, 1\}, \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{array}\right)}}, {{\{2, 2\}, \left(\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}}, {{\{2, 3\}, \left(\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)}}, {{\{3, 1\}, \left(\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)}}, {{\{3, 2\}, \left(\begin{array}{cccc} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right)}}, {{\{3, 3\}, \left(\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}}}

```

```

In[]:= Protect[st]
Out[]= {st}

In[]:= Table[{{J, K}, MatrixForm[st[J, K] - Transpose[st[J, K]]]}, {J, 1, 3}, {K, 1, 3}]
Out[=]

$$\left\{ \left\{ \{1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{1, 2\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{1, 3\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \right.$$


$$\left. \left\{ \{2, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{2, 2\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \right.$$


$$\left. \left. \left\{ \{3, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \left\{ \{3, 3\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\} \right\} \right\}$$


In[]:= Permutations[Range[3], 2]
Out[=]
{{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}KroneckerProduct[Range[3], Range[3]]
Out[=]
{{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}Table[{J, K}, {J, 1, 3}, {K, 1, 3}]
st[##] & /@%
st[##] & @@ %%
Out[=]
{{{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}}, {{{3, 1}, {3, 2}, {3, 3}}}}
Out[=]
{st[{{1, 1}, {1, 2}, {1, 3}}], st[{{2, 1}, {2, 2}, {2, 3}}], st[{{3, 1}, {3, 2}, {3, 3}}]}
Out[=]
st[{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}]

```

```

In[]:= Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
Length[%]

Out[]=
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

Out[=]
16

In[]:= Unprotect[y];
y = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]

Out[=]
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

```

```
In[1]:= Length[\gamma]
Out[1]= 16

In[2]:= Protect[\gamma]
Out[2]= {\gamma}

In[3]:= \gamma[[1]]
Out[3]= {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}

In[4]:= \gamma[[-1]]
Out[4]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Self Dual Anti-Symmetric

```
In[5]:= Table[ $\left(\frac{1}{2} \sum_{j_1=1}^4 \left( \sum_{j_2=1}^4 \epsilon 4[p, q, j_1, j_2] \times s4by4[h][j_1, j_2] \right) - s4by4[h][p, q], \{h, 1, 3\}, \{q, 4\}, \{p, 4\} \right)$ , Out[5]= {{ { {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} }, { {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} }, { {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} } }}
```

Anti SelfDual Anti-Symmetric

```
In[6]:= Table[ $\left(\frac{1}{2} \sum_{j_1=1}^4 \left( \sum_{j_2=1}^4 \epsilon 4[p, q, j_1, j_2] \times t4by4[h][j_1, j_2] \right) + t4by4[h][p, q], \{h, 1, 3\}, \{q, 4\}, \{p, 4\} \right)$ , Out[6]= {{ { {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} }, { {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} }, { {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} } }}
```

$SO(4,4), Spin(4,4) = \overline{SO(4,4)}, \tau$; if using xact: M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+, +, +, -, -, -}, {x0, x1, x2, x3, x4, x5, x6, x7}

```
In[1]:= \tau === \tau
Out[1]= False
```

In[1]:= $\overline{\tau}$

Head[%]

Out[1]=

$\overline{\tau}$

Out[1]=

Symbol

In[2]:= $\overline{\tau}$

Head[%]

Out[2]=

$\overline{\tau}$

Out[2]=

ParsedBoxWrapper

In[3]:= $\overline{\tau}$

Head[%]

Out[3]=

$\overline{\tau}$

Out[3]=

Symbol

In[4]:= **Symbol**

Out[4]=

Symbol

In[5]:= $\overline{\tau} == \overline{\tau}$

Out[5]=

True

In[6]:= $\overline{\tau} == \overline{\tau}$

Out[6]=

False

In[7]:= **η4488 // MatrixForm**

Out[7]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[8]:= η4488 == DiagonalMatrix[RotateRight[values, 4]]
```

Out[8]=

True

```
In[9]:= DiagonalMatrix[RotateRight[values, 3]] // MatrixForm
```

Out[9]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[10]:= τ[0] = ID8;
```

Table[

τ[7-h] = ArrayFlatten[{{0, t4by4[h]}, {-t4by4[h], 0}}], {h, 1, 3}];

Table[τ[h] = ArrayFlatten[{{0, s4by4[h]}, {s4by4[h], 0}}]], {h, 1, 3}];

(τ[7] = τ[1].τ[2].τ[3].τ[4].τ[5].τ[6]) // MatrixForm

Out[10]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[11]:= σ == τ[1].τ[2].τ[3]
```

Out[11]=

True

```
In[1]:= Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\tau[A].\tau[B] + \tau[B].\tau[A]) == -\eta 4488 [A+1, B+1] * ID8]$ ]}],
  {A, 1, 7}, {B, 1, 7}]

Out[1]= {{{1, 1, True}, {1, 2, True}, {1, 3, True},
  {1, 4, True}, {1, 5, True}, {1, 6, True}, {1, 7, True}},
  {{2, 1, True}, {2, 2, True}, {2, 3, True}, {2, 4, True}, {2, 5, True},
  {2, 6, True}, {2, 7, True}}, {{3, 1, True}, {3, 2, True},
  {3, 3, True}, {3, 4, True}, {3, 5, True}, {3, 6, True}, {3, 7, True}},
  {{4, 1, True}, {4, 2, True}, {4, 3, True}, {4, 4, True}, {4, 5, True},
  {4, 6, True}, {4, 7, True}}, {{5, 1, True}, {5, 2, True},
  {5, 3, True}, {5, 4, True}, {5, 5, True}, {5, 6, True}, {5, 7, True}},
  {{6, 1, True}, {6, 2, True}, {6, 3, True}, {6, 4, True}, {6, 5, True},
  {6, 6, True}, {6, 7, True}}, {{7, 1, True}, {7, 2, True},
  {7, 3, True}, {7, 4, True}, {7, 5, True}, {7, 6, True}, {7, 7, True}}}

In[2]:=  $\eta 4488 // MatrixForm$ 

Out[2]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

```
In[3]:= Table[{A, - $\eta 4488 [A+1, A+1] * \tau[A] == \text{Transpose}[\tau[A]]$ }, {A, 1, 7}]
```

```
Out[3]= {{1, True}, {2, True}, {3, True}, {4, True}, {5, True}, {6, True}, {7, True}}
```

```
In[4]:= ( $\sigma == \tau[1].\tau[2].\tau[3]$ )
```

```
Out[4]= True
```

```
In[5]:=  $\sigma.\tau[1].\tau[2].\tau[3] // MatrixForm$ 
```

```
Out[5]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```

```
In[8]:= (*τ[0]=ID8;
Do[{τ[A] = FullSimplify[ExpandAll[(τ[A])]]} , {A, 1, 7}]*)
(*Do[{τ[A] = FullSimplify[ExpandAll[-(σ.τ[1].τ[2].τ[3].τ[A])]]} ,
{A, 1, 7}]*)
```

```
In[9]:= (σ == τ[1].τ[2].τ[3])
```

Out[9]=

True

```
In[10]:= (*Symbolize[τ]*)
```

```
In[11]:= τ[0] = ID8;
Do[{τ[A] = FullSimplify[ExpandAll[(σ.Transpose[τ[A]].σ)]]} , {A, 1, 7}]
```

```
In[12]:= σ.τ[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
```

Out[12]=

{True, True, True, True, True, True, True, True}

```
In[8]:= Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\tau[A] \cdot \tau[B] + \tau[B] \cdot \tau[A]) = \eta 4488 [A+1, B+1] * ID8]$ ]}},
  {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

```
In[•]:= τ[7] // MatrixForm
```

```
Out[•]//MatrixForm=
```

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[8]:= $(\Omega = \sigma \cdot \tau[7]) // \text{MatrixForm}$

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[9]:= $\Omega == \tau[4] \cdot \tau[5] \cdot \tau[6]$

Out[9]=

True

In[10]:= $\tau[5] \cdot \tau[6] \cdot \tau[7] // \text{MatrixForm}$

Out[10]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

In[11]:= $(\sigma \Omega = \sigma \cdot \Omega) // \text{MatrixForm}$

Out[11]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[12]:= $\text{Inverse}[\sigma \Omega] // \text{MatrixForm}$

Out[12]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[13]:= $\tau[1] \cdot \tau[2] \cdot \tau[3] \cdot \tau[4] \cdot \tau[5] \cdot \tau[6] \cdot \tau[7] == \tau[0] == \text{ID8}$

Out[13]=

True

```
In[1]:= (σ == τ[1].τ[2].τ[3])
```

Out[1]=

True

```
In[2]:= (σ == τ[4].τ[5].τ[6].τ[7])
```

Out[2]=

True

```
In[3]:= τ[1].τ[2].τ[3].τ[#] == -Transpose[τ[1].τ[2].τ[3].τ[#]] & /@ Range[0, 7]
```

Out[3]=

{False, True, True, True, True, True, True}

```
In[4]:= σ.τ[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
```

Out[4]=

{True, True, True, True, True, True, True}

```
In[5]:= τ[5].τ[6].τ[7].τ[#] == Transpose[τ[5].τ[6].τ[7].τ[#]] & /@ Range[0, 7]
```

Out[5]=

{False, True, True, True, False, True, True, True}

```
In[6]:= τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```

Out[6]=

{True, True, True, True, True, False, False, False}

```
In[7]:= τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```

Out[7]=

{True, False, False, False, False, True, True, True}

```
In[8]:= τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```

Out[8]=

{True, True, True, True, True, False, False, False}

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}. \quad ; \text{ O (4, 4) : SAB ;}$$

covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]

$$S = \begin{pmatrix} a & b \\ c & d \end{pmatrix};$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix} \cdot \begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix} = \begin{pmatrix} b \cdot \tau^A & a \cdot \bar{\tau}^A \\ d \cdot \tau^A & c \cdot \bar{\tau}^A \end{pmatrix} \cdot \begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix}$$

$$= \begin{pmatrix} b \cdot \tau^A \cdot \tilde{a} + a \cdot \bar{\tau}^A \cdot \tilde{b} & b \cdot \tau^A \cdot \tilde{c} + a \cdot \bar{\tau}^A \cdot \tilde{d} \\ d \cdot \tau^A \cdot \tilde{a} + c \cdot \bar{\tau}^A \cdot \tilde{b} & d \cdot \tau^A \cdot \tilde{c} + c \cdot \bar{\tau}^A \cdot \tilde{d} \end{pmatrix}$$

Need these in order to define the Lagrangian for the "universe," later:

```
In[8]:= 
$$(\sigma16 = T16^A[0].T16^A[1].T16^A[2].T16^A[3]) // \text{MatrixForm}$$

```

```
Out[•]//MatrixForm=
```

```
In[•]:= Table[T16^A[A1] = ArrayFlatten[{{0, τ[A1]}, {τ[A1], 0}}], {A1, 0, 7}];
```

```
In[6]:= (T16A[8] = T16A[0].T16A[1].T16A[2].T16A[3].T16A[4].T16A[5].T16A[6].T16A[7]) // MatrixForm
```

In[1]:= $\sigma_{16} // \text{MatrixForm}$

Out[1]//MatrixForm=

$$\left(\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

In[2]:= $T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[\#] ==$
 $- \text{Transpose}[T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[\#]] \& /@ \text{Range}[0, 8]$

Out[2]= {True, True, True, True, True, True, True, False}

In[3]:= $T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[\#] ==$
 $\text{Transpose}[T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[\#]] \& /@ \text{Range}[0, 8]$

Out[3]= {True, True, True, True, True, True, True, True}

In[4]:= $T16^A[8] == \sigma_{16}.T16^A[4].T16^A[5].T16^A[6].T16^A[7]$

Out[4]= True

In[5]:= $T16^A[\#] == \text{Transpose}[T16^A[\#]] \& /@ \text{Range}[0, 8]$

Out[5]= {True, True, True, True, False, False, False, True}

In[6]:= $T16^A[\#] == -\text{Transpose}[T16^A[\#]] \& /@ \text{Range}[0, 8]$

Out[6]= {False, False, False, False, True, True, True, False}

In[7]:= $\sigma_{16}.T16^A[\#] == -\text{Transpose}[\sigma_{16}.T16^A[\#]] \& /@ \text{Range}[0, 7]$

Out[7]= {True, True, True, True, True, True, True, True}

```
In[5]:= (covariantDiffMatrix = T16A[5].T16A[6].T16A[7]) // MatrixForm
```

```
In[=]:= σ16.covariantDiffMatrix = Transpose[σ16.covariantDiffMatrix]
```

Out[•] =

True

In[•]:= ID16 // MatrixForm

Out[•]//MatrixForm=

(1 0

```
In[8]:= (σ16(*=T16A[0].T16A[1].T16A[2].T16A[3]*)) // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[9]:= σ16 == ArrayFlatten[{{{-σ, 0}, {0, σ}}}]
```

Out[9]=

True

later, after defining $e_{(A)}^\alpha$:

```
In[10]:= (*Table[T16α[α1-1]= Sum[(eα(A)[[α1,A1]] ) T16A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

```
In[11]:= (*(T16α[8]=T16α[0].T16α[1].T16α[2].T16α[3].T16α[4].T16α[5].T16α[6].T16α[7])//MatrixForm*)
```

```
In[12]:= (*eLpairs*)
```

```
In[13]:= (*Ω16a=Array[0&, {16,16}];  
{1+#[[1]],1+#[[2]]}&/@eLpairs  
(Ω16a[[#1],#[[2]]]=1)&/@%*)
```

```
In[14]:= (*Ω16a//MatrixForm*)
```

```
In[15]:= (*(Ω16=σ16.(Ω16a+Transpose[Ω16a]))//MatrixForm*)
```

```
In[16]:= T16A[0].T16A[4] // MatrixForm  
T16A[0].T16A[4].σ16  
(Ω16 = %) // MatrixForm
```

```

Out[=]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


Out[=]=
{{0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}

```

In[•]:= Det[Ω16]

Out[•] =

1

```
In[•]:= σ16.Ω16 // MatrixForm
```

Out[•]//MatrixForm=

0	0	0	0	0	-1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	-1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0
-1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	-1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1
0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	-1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	-1	0	0	0	0

```
In[6]:= σ16.T16A[#] == -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

Out[•]=

{True, True, True, True, True, True, True, True}

```
In[1]:= σ16.T16A[#] == -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

Out[•] =

{True, True, True, True, True, True, True, True}

```
In[6]:= σ16.T16A[#] == -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

```
Out[=]= {True, True, True, True, True, True, True, True}
```

```
In[•]:= σ16.newT16A[#] = -Transpose[σ16.newT16A[#]] & /@ Range[0, 7]
```



```
In[1]:= newσ16.newT16A[#] == -Transpose[newσ16.newT16A[#]] & /@ Range[0, 7]
```

Out[1]=

$$\left\{ \begin{aligned} \text{new}\sigma_{16}.\text{newT16}^A[0] &== -(\text{new}\sigma_{16}.\text{newT16}^A[0])^T, \\ \text{new}\sigma_{16}.\text{newT16}^A[1] &== -(\text{new}\sigma_{16}.\text{newT16}^A[1])^T, \\ \text{new}\sigma_{16}.\text{newT16}^A[2] &== -(\text{new}\sigma_{16}.\text{newT16}^A[2])^T, \\ \text{new}\sigma_{16}.\text{newT16}^A[3] &== -(\text{new}\sigma_{16}.\text{newT16}^A[3])^T, \\ \text{new}\sigma_{16}.\text{newT16}^A[4] &== -(\text{new}\sigma_{16}.\text{newT16}^A[4])^T, \\ \text{new}\sigma_{16}.\text{newT16}^A[5] &== -(\text{new}\sigma_{16}.\text{newT16}^A[5])^T, \\ \text{new}\sigma_{16}.\text{newT16}^A[6] &== -(\text{new}\sigma_{16}.\text{newT16}^A[6])^T, \\ \text{new}\sigma_{16}.\text{newT16}^A[7] &== -(\text{new}\sigma_{16}.\text{newT16}^A[7])^T \end{aligned} \right\}$$

```
In[2]:= Table[{ {A1, B1}, FullSimplify[ExpandAll[1/2 (T16A[A1].T16A[B1] + T16A[B1].T16A[A1]) == η4488[[A1 + 1, B1 + 1] * ID16]]]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} (\{0, 0\}) & (\{0, 1\}) & (\{0, 2\}) & (\{0, 3\}) & (\{0, 4\}) & (\{0, 5\}) & (\{0, 6\}) & (\{0, 7\}) \\ \text{True} & \text{True} \\ (\{1, 0\}) & (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) & (\{1, 5\}) & (\{1, 6\}) & (\{1, 7\}) \\ \text{True} & \text{True} \\ (\{2, 0\}) & (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) & (\{2, 5\}) & (\{2, 6\}) & (\{2, 7\}) \\ \text{True} & \text{True} \\ (\{3, 0\}) & (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) & (\{3, 5\}) & (\{3, 6\}) & (\{3, 7\}) \\ \text{True} & \text{True} \\ (\{4, 0\}) & (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) & (\{4, 5\}) & (\{4, 6\}) & (\{4, 7\}) \\ \text{True} & \text{True} \\ (\{5, 0\}) & (\{5, 1\}) & (\{5, 2\}) & (\{5, 3\}) & (\{5, 4\}) & (\{5, 5\}) & (\{5, 6\}) & (\{5, 7\}) \\ \text{True} & \text{True} \\ (\{6, 0\}) & (\{6, 1\}) & (\{6, 2\}) & (\{6, 3\}) & (\{6, 4\}) & (\{6, 5\}) & (\{6, 6\}) & (\{6, 7\}) \\ \text{True} & \text{True} \\ (\{7, 0\}) & (\{7, 1\}) & (\{7, 2\}) & (\{7, 3\}) & (\{7, 4\}) & (\{7, 5\}) & (\{7, 6\}) & (\{7, 7\}) \\ \text{True} & \text{True} \end{pmatrix}$$

```
In[3]:= Table[{ {A1, B1}, FullSimplify[ ExpandAll[1/2 (newT16A[A1].newT16A[B1] + newT16A[B1].newT16A[A1]) == η4488[[A1 + 1, B1 + 1] * ID16]]]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[3]//MatrixForm=

(... 1 ...)

Full expression not available (original memory size: 0.5 MB)



```
In[8]:=  $\Psi16upper = \text{Take}[\Psi16, 8]$ 
 $\Psi16lower = \text{Take}[\Psi16, -8]$ 
 $\Psi16 == \text{Flatten}[\{\Psi16upper, \Psi16lower\}]$ 

Out[8]=
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4]}

Out[9]=
{f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}

Out[10]=
True
```

$$t^A = \begin{pmatrix} 0 & \vec{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
Table[T16^A[A1] = ArrayFlatten[{{0, \tau[A1]}, {\tau[A1], 0}}], {A1, 0, 7}]
\sigma16 = ArrayFlatten[{{-\sigma, 0}, {0, \sigma}}]
\left( \begin{array}{cc} 0 & \tau[A1] \\ \tau[A1] & 0 \end{array} \right) \left( \begin{array}{c} \Psi16upper \\ \Psi16lower \end{array} \right) = \left( \begin{array}{c} \tau[A1].\Psi16lower \\ \tau[A1].\Psi16upper \end{array} \right)
\sigma16.\left( \begin{array}{cc} 0 & \tau[A1] \\ \tau[A1] & 0 \end{array} \right) \left( \begin{array}{c} \Psi16upper \\ \Psi16lower \end{array} \right) = \left( \begin{array}{c} -\sigma \tau[A1].\Psi16lower \\ \sigma \tau[A1].\Psi16upper \end{array} \right)
\Psi16.\sigma16.\left( \begin{array}{cc} 0 & \tau[A1] \\ \tau[A1] & 0 \end{array} \right).\Psi16 = \left( \begin{array}{c} -\Psi16upper.\sigma \tau[A1].\Psi16lower \\ \Psi16lower.\sigma \tau[A1].\Psi16upper \end{array} \right)
```

```
In[11]:= (*\sigma16.T16^A[#]&/@Range[0,7]*)
```

```
In[12]:= (*
(T16^A[8]=FullSimplify[T16^A[0].T16^A[1].T16^A[2].
T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]])//MatrixForm*)
```

```
In[13]:=  $\eta4488$ 
Out[13]=
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, -1}}
In[14]:=  $\eta99 = \text{DiagonalMatrix}[\{1, 1, 1, 1, -1, -1, -1, 1\}]$ 
Out[14]=
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
 {0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```



```
SAB = Table[ $\frac{1}{4} (\text{T16}^A[\text{A1}].\text{T16}^A[\text{B1}] - \text{T16}^A[\text{B1}].\text{T16}^A[\text{A1}])$ , {A1, 0, 7}, {B1, 0, 7}];
```

```
In[1]:= SAB = Table[ $\frac{1}{4} (\text{T16}^A[\text{A1}].\text{T16}^A[\text{B1}] - \text{T16}^A[\text{B1}].\text{T16}^A[\text{A1}])$ , {A1, 0, 7}, {B1, 0, 7}];
```

```
In[2]:= ParallelTable[
  FullSimplify[σ16.SAB[A1, B1] === -Transpose[σ16.SAB[A1, B1]]],
  {A1, 1, 8}, {B1, 1, 8}] // Flatten // Union
```

```
Out[2]= {True}
```

```
In[3]:= ParallelTable[
  FullSimplify[SAB[A1, B1].SAB[A2, B2] - SAB[A2, B2].SAB[A1, B1] ==
    - (η4488[A1, A2] × SAB[B1, B2] - η4488[A1, B2] × SAB[B1, A2] -
      η4488[B1, A2] × SAB[A1, B2] + η4488[B1, B2] × SAB[A1, A2])], {A1, 1, 7},
  {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}] // Flatten // Union
```

```
Out[3]= {True}
```

```
In[4]:= ParallelTable[
  FullSimplify[SAB[A1, B1].T16^A[B2 - 1] - T16^A[B2 - 1].SAB[A1, B1] ==
    (-η4488[B2, A1] T16^A[B1 - 1] + η4488[B2, B1] T16^A[A1 - 1])], {A1, 1, 8}, {B1, 1, 8}, {B2, 1, 8}] // Flatten // Union
```

```
Out[4]= {True}
```

```
In[5]:= (*ParallelTable[
  {{A1, B1}}, FullSimplify[σ16.SAB[A1, B1] == -Transpose[σ16.SAB[A1, B1]]]], {A1, 1, 8}, {B1, 1, 8}]*)
```

```
In[6]:= (*ParallelTable[
  {{A1, B1, A2, B2}}, FullSimplify[SAB[A1, B1].SAB[A2, B2] - SAB[A2, B2].SAB[A1, B1] ==
    - (η4488[A1, A2] SAB[B1, B2] - η4488[A1, B2] SAB[B1, A2] -
      η4488[B1, A2] SAB[A1, B2] + η4488[B1, B2] SAB[A1, A2])], {A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}]*)
```

```
In[8]:= (*ParallelTable[
  { {A1,B1,B2}, FullSimplify[SAB[A1,B1].T16^A[B2-1] - T16^A[B2-1].SAB[A1,B1]==
    (-η4488[B2,A1]T16^A[B1-1]+η4488[B2,B1]T16^A[A1-1]) ] },
  {A1,1, 8},{B1,1,8},{B2,1,8}]*)
```

```
In[9]:= Do[sAB[A1][B1] = 1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]),
  {A1, 0, 7}, {B1, 0, 7}];
```

```
In[10]:= Dimensions[ArrayFlatten[{{0, ID8}, {ID8, 0}}]]
Out[10]= {16, 16}
```

```
In[11]:= sAB[-1+1][-1+2] == SAB[1, 2]
```

Out[11]=

True

```
In[12]:= σ16.SAB[1, 2] // MatrixForm
```

Out[12]//MatrixForm=

$$\left(\begin{array}{cccccccccccccccc} 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{array} \right)$$

In[=]:= $\sigma16.sAB[1][2] // \text{MatrixForm}$

Out[=]//MatrixForm=

$$\left(\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula: [🔗](#)

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. [🔗](#)

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4). 🔗

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) 🔗

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. 🔗

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa). 🔗

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). 🔗

what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the *so*(4, 4) Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: 🔗

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for *so*(8) in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g. $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



↳ Why is the metric signature significant for \$so(4,4)\$ commutation relations?

Why is the metric signature significant for \$so(4,4)\$ commutation relations?

The metric signature is significant for the $so(p, q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). ☀

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$
The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4, 4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[1]:= **η4488**

Out[1]=

$$\{\{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 0, 0, -1, 0\}, \{0, 0, 0, 0, 0, 0, 0, -1\}\}$$

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

In[2]:= SAB[6, 2].SAB[6, 7] - SAB[6, 7].SAB[6, 2] = η4488[6, 6] × SAB[7, 2]

Out[2]=

True

In[3]:= {6, 2, 6, 7}, False

Out[3]=

{6, 2, 6, 7}, False

In[4]:= SAB[1, 2].SAB[1, 3] - SAB[1, 3].SAB[1, 2] = η4488[1, 1] × SAB[2, 3]

Out[4]=

False

In[•]:= sAB[θ][1]

Out[•] =

```

In[]:= Table[Transpose[T16^A[8].sAB[A1][B1]] == T16^A[8].sAB[A1][B1],
{A1, 0, 7}, {B1, 0, 7}]

Out[]= {{True, False, False, False, True, True, True, True},
{False, True, False, False, True, True, True, True},
{False, False, True, False, True, True, True, True},
{False, False, False, True, True, True, True, True},
{True, True, True, True, False, False, False, False},
{True, True, True, True, False, True, False, False},
{True, True, True, True, False, False, True, False},
{True, True, True, True, False, False, False, True}]

In[]:= Table[Transpose[T16^A[7].sAB[A1][B1]] == T16^A[7].sAB[A1][B1],
{A1, 0, 7}, {B1, 0, 7}]

Out[]= {{True, True, True, True, False, False, True, True},
{True, True, True, True, False, False, False, True},
{True, True, True, True, False, False, False, True},
{True, True, True, True, False, False, False, True},
{False, False, False, False, True, True, False, False},
{False, False, False, False, True, True, True, False},
{False, False, False, False, True, True, True, False},
{True, True, True, True, False, False, True, True}]

In[]:= Table[Transpose[\sigma16.sAB[A1][B1]] == \sigma16.sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]

Out[=] {{True, False, False, False, False, False, False, False},
{False, True, False, False, False, False, False, False},
{False, False, True, False, False, False, False, False},
{False, False, False, True, False, False, False, False},
{False, False, False, False, True, False, False, False},
{False, False, False, False, False, True, False, False},
{False, False, False, False, False, False, True, False},
{False, False, False, False, False, False, False, True}}

```

```
In[]:= Table[Transpose[\sigma16.SAB[A1, B1]] == \sigma16.SAB[A1, B1], {A1, 1, 8}, {B1, 1, 8}]
Out[]= {{True, False, False, False, False, False, False, False}, {False, True, False, False, False, False, False, False}, {False, False, True, False, False, False, False, False}, {False, False, False, True, False, False, False, False}, {False, False, False, False, True, False, False, False}, {False, False, False, False, False, True, False, False}, {False, False, False, False, False, False, True, False}, {False, False, False, False, False, False, False, True}}
```



```
In[]:= SAB[[1, 2]][1]
Out[]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```



```
In[]:= SAB[[1, 2]][2]
Out[]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```


$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$


```
In[]:= SAB1 = Table[Take[SAB[[A1, B1]][C1]], 8], {A1, 8}, {B1, 8}, {C1, 1, 8}];
In[]:= SAB2 = Table[Take[SAB[[A1, B1]][C1]], -8], {A1, 8}, {B1, 8}, {C1, 9, 16}];
In[]:= SAB[[1, 2]] == ArrayFlatten[{{SAB1[[1, 2]], 0}, {0, SAB2[[1, 2]]}}]
Out[]= True
```

(*Table[SAB[[A1,B1]]== ArrayFlatten[{{SAB1[[A1,B1]],0},{0,SAB2[[A1,B1]]}}], {A1,0, 7},{B1,0,7}]*)

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \rightarrow SO(4,4)$ is given by

$$(2.8) \quad 8L^{A'}_{\ B'}(g) = \text{tr} \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}_{\ C'} G_{A'B'} L^{B'}_{\ D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}_{\ B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}_{\ B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

In[1]:= SAB2[[1, 2]]

Out[1]=

$$\left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0 \right\}, \right. \\ \left. \left\{ 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0 \right\}, \left\{ 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}$$

In[2]:= $\tau[\theta]$

Out[2]=

$$\{ \{ 1, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 1, 0, 0, 0, 0, 0, 0 \}, \\ \{ 0, 0, 1, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 1, 0, 0, 0 \}, \\ \{ 0, 0, 0, 0, 0, 1, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 1, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 1 \} \}$$

```
In[3]:= (*Block[{b,A,F_a^(μ),F_(μ),ψ,Π},

b=Array[A,{8}];

ψ[1]=Array[Π[1],{8}];

ψ[2]=Array[Π[2],{8}];

F_a^(μ)=Table[(1/(Sqrt[ψ[1].σ.ψ[1]])ψ[1].σ.(τ[μL])),{μL,1,8}];

F_(μ)=Transpose[Table[(1/(Sqrt[ψ[1].σ.ψ[1]])η4488[[μL,μL]]τ[μL].ψ[1]),{μL,1,8}]];

b==FullSimplify[F_a^(μ).ψ[2]/.Thread[ψ[2]→F_(μ).b]]]*)
```

```
In[8]:= (*Block[{b,A,B,q,F^a_{(\mu)},\psi,\varphi,\Psi}, 
b=Array[A,{8}]; 
\psi[1]=Array[\Psi[1],{8}]; 
\varphi[1]=Array[\Psi[2],{8}]; 
q=ParallelTable[
  FullSimplify[(1/\sqrt{\psi[1].\sigma.\psi[1]})\psi[1].\sigma.\frac{1}{2}(\tau[A1].\tau[B1]-\tau[B1].\tau[A1]).\varphi[1]],
  {A1,1,8},{B1,1,8}]];
B=FullSimplify[
  (1/\sqrt{\psi[1].\sigma.\psi[1]}) ParallelSum[\eta4488[[\mu L,\mu L]]\tau[\mu L]b[[\mu L]],{\mu L,1,8}].\psi[1]]];
FullSimplify[q/.Thread[\varphi[1]\rightarrow B]]*)
```

```
In[9]:= (*Block[{b,A,B,q,F^a_{(\mu)},\psi,\varphi,\Psi}, 
b=Array[A,{8}]; 
\psi[1]=Array[\Psi[1],{8}]; 
\varphi[1]=Array[\Psi[2],{8}]; 
q=ParallelTable[
  FullSimplify[(\psi[1].\sigma.(\tau[A1].\tau[B1]).\varphi[1])],{A1,1,8},{B1,1,8}];
B=FullSimplify[( ParallelSum[\eta4488[[\mu L,\mu L]]\tau[\mu L]b[[\mu L]],{\mu L,1,8}].\psi[1])];
FullSimplify[(1/\psi[1].\sigma.\psi[1])q/.Thread[\varphi[1]\rightarrow B]]]*)
```

T16^α

In[•]:= **ssgm4488**

Out[•]=

```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
g[1][1] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &), g[1][4] → (0 &),
g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &), g[2][1] → (0 &),
g[2][2] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &), g[2][5] → (0 &),
g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &), g[3][2] → (0 &),
g[3][3] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
g[5][5] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
g[6][5] → (0 &), g[6][6] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &)}
```

In[•]:= **e^α_(A) / . sge^α_(A)**

Out[•]=

```
{ { 1 / √g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0 }, { 0, 1 / √g[1][1][x0, x4], 0, 0, 0, 0, 0, 0 },
{ 0, 0, 1 / √g[2][2][x0, x4], 0, 0, 0, 0, 0, 0 }, { 0, 0, 0, 1 / √g[3][3][x0, x4], 0, 0, 0, 0 },
{ 0, 0, 0, 0, 1 / √-g[4][4][x0, x4], 0, 0, 0, 0 }, { 0, 0, 0, 0, 0, 1 / √-g[5][5][x0, x4], 0, 0 },
{ 0, 0, 0, 0, 0, 0, 1 / √-g[6][6][x0, x4], 0 }, { 0, 0, 0, 0, 0, 0, 0, 1 / √-g[7][7][x0, x4] } }
```

In[•]:= **Table[T16^α[α1 - 1] = Sum[((e^α_(A) / . sge^α_(A)) [[α1, A1]]) T16^A[A1 - 1], {A1, 1, 8}], {α1, 1, 8}]; (*./ssgm4488*)**

In[•]:= **(T16^α[8] = T16^α[0].T16^α[1].T16^α[2].T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7]);**

In[•]:= **(*T16^α[8]==T16^A[8]*Sec[6 H x0]*)**

CHECK

Table[T16 ^{α} [α 1 - 1] = Sum[(e ^{α} _(A)[[α 1, A1]]) T16^A[A1 - 1], {A1, 1, 8}], { α 1, 1, 8}];
and • • • :

```
In[8]:= (*Table[T16^{\alpha}[\alpha1-1]= Sum[(e_{(A)}^{\alpha}[[\alpha1,A1]] ) T16^A[A1-1],{A1,1,8}],{\alpha1,1,8}];*)
```

```
In[8]:= (*(T16α[8]=T16α[0].T16α[1].T16α[2].T16α[3].T16α[4].T16α[5].T16α[6].T16α[7])//  
MatrixForm*)
```

```
In[6]:= Table[{A1, B1},  
  FullSimplify[ExpandAll[ $\frac{1}{2} (\text{T16}^A[\text{A1}].\text{T16}^A[\text{B1}] + \text{T16}^A[\text{B1}].\text{T16}^A[\text{A1}]) =$   
     $\eta 4488[\text{A1} + 1, \text{B1} + 1] * \text{ID16}]]]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm$ 
```

```
Out[•]//MatrixForm=
{{{{0, 0}}, {{0, 1}}, {{0, 2}}, {{0, 3}}, {{0, 4}}, {{0, 5}}, {{0, 6}}, {{0, 7}}}, {{{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}}, {{{{1, 0}}}, {{1, 1}}, {{1, 2}}, {{1, 3}}, {{1, 4}}, {{1, 5}}, {{1, 6}}, {{1, 7}}}, {{{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}}, {{{{2, 0}}}, {{2, 1}}, {{2, 2}}, {{2, 3}}, {{2, 4}}, {{2, 5}}, {{2, 6}}, {{2, 7}}}, {{{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}}, {{{{3, 0}}}, {{3, 1}}, {{3, 2}}, {{3, 3}}, {{3, 4}}, {{3, 5}}, {{3, 6}}, {{3, 7}}}, {{{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}}, {{{{4, 0}}}, {{4, 1}}, {{4, 2}}, {{4, 3}}, {{4, 4}}, {{4, 5}}, {{4, 6}}, {{4, 7}}}, {{{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}}, {{{{5, 0}}}, {{5, 1}}, {{5, 2}}, {{5, 3}}, {{5, 4}}, {{5, 5}}, {{5, 6}}, {{5, 7}}}, {{{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}}, {{{{6, 0}}}, {{6, 1}}, {{6, 2}}, {{6, 3}}, {{6, 4}}, {{6, 5}}, {{6, 6}}, {{6, 7}}}, {{{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}}, {{{{7, 0}}}, {{7, 1}}, {{7, 2}}, {{7, 3}}, {{7, 4}}, {{7, 5}}, {{7, 6}}, {{7, 7}}}, {{{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}, {{True}}}}
```

```
In[•]:= Table[{α1, β1}, FullSimplify[
  1/2 FullSimplify[((T16^α[α1].T16^α[β1] + T16^α[β1].T16^α[α1]) /. ssgm4488),
  constraintVars] == Inverse[MatrixMetric44][α1+1, β1+1]*ID16],{α1, 0, 7}, {β1, 0, 7}] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} (\{0, 0\}) & (\{0, 1\}) & (\{0, 2\}) & (\{0, 3\}) & (\{0, 4\}) & (\{0, 5\}) & (\{0, 6\}) & (\{0, 7\}) \\ \text{True} & \text{True} \\ (\{1, 0\}) & (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) & (\{1, 5\}) & (\{1, 6\}) & (\{1, 7\}) \\ \text{True} & \text{True} \\ (\{2, 0\}) & (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) & (\{2, 5\}) & (\{2, 6\}) & (\{2, 7\}) \\ \text{True} & \text{True} \\ (\{3, 0\}) & (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) & (\{3, 5\}) & (\{3, 6\}) & (\{3, 7\}) \\ \text{True} & \text{True} \\ (\{4, 0\}) & (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) & (\{4, 5\}) & (\{4, 6\}) & (\{4, 7\}) \\ \text{True} & \text{True} \\ (\{5, 0\}) & (\{5, 1\}) & (\{5, 2\}) & (\{5, 3\}) & (\{5, 4\}) & (\{5, 5\}) & (\{5, 6\}) & (\{5, 7\}) \\ \text{True} & \text{True} \\ (\{6, 0\}) & (\{6, 1\}) & (\{6, 2\}) & (\{6, 3\}) & (\{6, 4\}) & (\{6, 5\}) & (\{6, 6\}) & (\{6, 7\}) \\ \text{True} & \text{True} \\ (\{7, 0\}) & (\{7, 1\}) & (\{7, 2\}) & (\{7, 3\}) & (\{7, 4\}) & (\{7, 5\}) & (\{7, 6\}) & (\{7, 7\}) \\ \text{True} & \text{True} \end{pmatrix}$$

In[•]:= MatrixMetric44 // MatrixForm

Out[•]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6 H x 0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2 a4[H x 4]} \sin[6 H x 0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2 a4[H x 4]} \sin[6 H x 0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2 a4[H x 4]} \sin[6 H x 0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:= ((e^α_(A) /. sgtrye^α_(A)).MatrixMetric44.(e^α_(A) /. sgtrye^α_(A))) // MatrixForm

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[1]:= $\mathbf{e}_{(A)}^\alpha / . \mathbf{sgtrye}_{(A)}^\alpha$

Out[1]=

$$\begin{aligned} & \left\{ \{\tan[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0 \right\}, \{0, 0, 0, 0, 1, 0, 0, 0, 0\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}}, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}} \right\} \right\} \end{aligned}$$

In[2]:= $\mathbf{e}_{(A)}^\alpha / . \mathbf{sge}_{(A)}^\alpha$

Out[2]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}} \right\} \right\} \end{aligned}$$

```
Clear[con];
con[g_, ass_ : {}] := Module[{t, II, ginv},
  ginv = FullSimplify[Inverse[g], ass];
  II = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
    (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}],
    {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
  Return[{ginv, II}]]
```

In[•]:= **ssgm4488**

Out[•]=

$$\begin{aligned} & \{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), g[1][4] \rightarrow (0 \&), \\ & g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), g[2][1] \rightarrow (0 \&), \\ & g[2][2] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), g[2][5] \rightarrow (0 \&), \\ & g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), g[3][2] \rightarrow (0 \&), \\ & g[3][3] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&)\} \end{aligned}$$

In[•]:= **g4488**

g4488 /. sg

Out[•]=

$$\begin{aligned} & \{ \{g[0][0][x0, x4], g[0][1][x0, x4], g[0][2][x0, x4], g[0][3][x0, x4], \\ & g[0][4][x0, x4], g[0][5][x0, x4], g[0][6][x0, x4], g[0][7][x0, x4]\}, \\ & \{g[1][0][x0, x4], g[1][1][x0, x4], g[1][2][x0, x4], g[1][3][x0, x4], \\ & g[1][4][x0, x4], g[1][5][x0, x4], g[1][6][x0, x4], g[1][7][x0, x4]\}, \\ & \{g[2][0][x0, x4], g[2][1][x0, x4], g[2][2][x0, x4], g[2][3][x0, x4], \\ & g[2][4][x0, x4], g[2][5][x0, x4], g[2][6][x0, x4], g[2][7][x0, x4]\}, \\ & \{g[3][0][x0, x4], g[3][1][x0, x4], g[3][2][x0, x4], g[3][3][x0, x4], \\ & g[3][4][x0, x4], g[3][5][x0, x4], g[3][6][x0, x4], g[3][7][x0, x4]\}, \\ & \{g[4][0][x0, x4], g[4][1][x0, x4], g[4][2][x0, x4], g[4][3][x0, x4], \\ & g[4][4][x0, x4], g[4][5][x0, x4], g[4][6][x0, x4], g[4][7][x0, x4]\}, \\ & \{g[5][0][x0, x4], g[5][1][x0, x4], g[5][2][x0, x4], g[5][3][x0, x4], \\ & g[5][4][x0, x4], g[5][5][x0, x4], g[5][6][x0, x4], g[5][7][x0, x4]\}, \\ & \{g[6][0][x0, x4], g[6][1][x0, x4], g[6][2][x0, x4], g[6][3][x0, x4], \\ & g[6][4][x0, x4], g[6][5][x0, x4], g[6][6][x0, x4], g[6][7][x0, x4]\}, \\ & \{g[7][0][x0, x4], g[7][1][x0, x4], g[7][2][x0, x4], g[7][3][x0, x4], \\ & g[7][4][x0, x4], g[7][5][x0, x4], g[7][6][x0, x4], g[7][7][x0, x4]\}\} \end{aligned}$$

Out[•]=

$$\begin{aligned} & \{ \{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \{0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0\}, \{0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0\}, \{0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0\}, \{0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]\}\} \end{aligned}$$

```
In[6]:= MatrixForm[#] & /@ Block[{eAα, eαA, (*ginv,Γ,gg,*) colVecs, rowVecs, ab, ba, r},
(*gg=g4488/.sg;*)
(*{ginv,Γ}=con[gg];*)
eαA = eα(A) /. sgeα(A);
eAα = e(A)α /. sge(A)α;
rowVecs = Table[eAα[[b, All]], {b, 1, Length[eAα]}];
colVecs = Table[eαA[[All, b]], {b, 1, Length[eαA]}];
ab = rowVecs.colVecs // FullSimplify[#, constraintVars] &;
ba = Transpose[colVecs].Transpose[rowVecs] // FullSimplify[#, constraintVars] &;
(*r={{ab},{ba}};*)
r = {ab, ba};
r]
```

Out[6]=

$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

metric compatibility condition for the Octad

$$\nabla_\mu e_\nu^I = \partial_\mu e_\nu^I - \Gamma_{\mu\nu}^\rho e_\rho^I + \omega_{\mu J}^I e_\nu^J = 0$$

```

In[8]:= Clear[spinCoefficients];
spinCoefficients[eIv_] := Module[{evI, w, r, s},
  If[Dimensions[eIv] != {8, 8}, Return[]];
  If[Det[eIv] == 0, Return[]];
  evI = FullSimplify[Inverse[eIv], constraintVars] /. subsDefects;
  w = Table[
    -Table[FullSimplify[Sum[D[eIv][I1, v1], X][μ1]] * evI[v1, Jprime], {v1, 1,
      Length[evI]}] - Sum[ eIv[I1, ρ] * Γ[ρ, μ1, v1] * evI[v1, Jprime],
    {v1, 1, Length[evI]}, {ρ, 1, Length[X]}],
    constraintVars] /. subsDefects, {I1, 1, Length[eIv]},
    {Jprime, 1, Length[evI]}], {μ1, 1, Length[X]}];
  Assert[Dimensions[w] == {8, 8, 8}];
  Return[w];
]

```


In[•]:= (*gtrye $_{(A)}^{\alpha}$. gtrye $_{\alpha}^{(A)}$ *)

```

In[1]:= (*gtrye(A).gtrye(A)*)

In[2]:= (*gtrye(A)*)

In[3]:= (*Γ
Dimensions[%]*)

In[4]:= (*Table[Γ[All,μ,All],{μ,1,Length[X]}]*)

In[5]:= (*Γ[All,5,All]*)

In[6]:= (*Γ[All,1,All]*)

In[7]:= (*spinCoeffs[1]*)

In[8]:= (*spinCoeffs[5]*)

In[9]:= (*{X[1],X[5]}*)

In[10]:= (*spinCoeffs[1][1][1]*)

In[11]:= (*ωμIJ[1][1][1]*)

In[12]:= (*Block[{μ=1},
FullSimplify[-((D[gtrye(A),X[μ]].gtrye(A))-(gtrye(A).Γ[All,μ,All].gtrye(A))),
constraintVars]/.subsDefects]
Dimensions[%]*)

In[13]:= (*Block[{μ=1},
FullSimplify[-((D[gtrye(A),X[μ]].Transpose[gtrye(A)])-(gtrye(A).Γ[All,μ,All].
Transpose[gtrye(A)])),constraintVars]/.subsDefects]*)

In[14]:= (*Block[{μ=1},FullSimplify[
-((Sum[D[gtrye(A)[All,v],X[μ]]*gtrye(A)[v,All],{v,1,8}]-)
(gtrye(A).Γ[All,μ,All].gtrye(A))),constraintVars]/.subsDefects]
Dimensions[%]*)

In[15]:= (*Table[,{A1,1,8},{B1,1,8}]*)

In[16]:= (*Block[{μ=1},Table[FullSimplify[
-((Sum[D[gtrye(A)[A1,v],X[μ]]*gtrye(A)[v,B1],{v,1,8}]-)
(Sum[gtrye(A)[A1,ρ]*Γ[ρ,μ,v]*gtrye(A)[v,B1],{ρ,1,8},
{v,1,8}]),constraintVars]/.subsDefects,{A1,1,8},{B1,1,8}]]
Dimensions[%]*)

```

```
In[8]:= Table[ ( wμIJ[μ] - spinCoeffs[[μ]] ) /. subsDefects , {μ, 1, Length[X]}] // Flatten // Union
```

Out[8]=

{}

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:

- $\nabla_\mu e^a{}_\nu = 0$
- Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

◦ Metric compatibility : The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame . The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is :

He "

v=0

• Here, V_μ is the covariant derivative with respect to the connection $F^\nu{}_\mu$, and $e_\mu{}^\nu$ is the tetrad component .

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

```
In[=]:= Block[{eAα, eαA, t, ginv, Γ, gg},
  gg = g4488 /. sg;
  {ginv, Γ} = con[gg];
  eαA = eα(A) /. sgeα(A); eAα = e(A)α /. sge(A)α;
  t = Table[ Block[{v, V, contra, covari},
    V = eAα[[b, All]];
    v = eαA[[All, b]];
    contra =
      Table[D[v[[k]], X[[μ]]] + Sum[v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before ω terms*)
    covari =
      Table[D[V[[k]], X[[μ]]] - Sum[V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before ω terms*)
    {b - 1, X[[μ]], {{contra}, {covari}}}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
  t]
FullSimplify[#, # /. ssgm4488, constraintVars] & /@ %
```

```
Out[=]=
{ {{0, x0, {{0, 0, 0, 0, -g[0][0]^(0,1)[x0, x4]/(2 √g[0][0][x0, x4]) g[4][4][x0, x4], 0, 0, 0}}}, {{0, 0, 0, 0, -g[0][0]^(0,1)[x0, x4]/(2 √g[0][0][x0, x4]), 0, 0, 0}}}, {{0, x1, {{0, g[1][1]^(1,0)[x0, x4]/(2 √g[0][0][x0, x4]) g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}}}, {{0, g[1][1]^(1,0)[x0, x4]/(2 √g[0][0][x0, x4]), 0, 0, 0, 0, 0, 0}}}}, {{0, x2, {{0, 0, g[2][2]^(1,0)[x0, x4]/(2 √g[0][0][x0, x4]) g[2][2][x0, x4], 0, 0, 0, 0, 0, 0}}}, {{0, 0, g[2][2]^(1,0)[x0, x4]/(2 √g[0][0][x0, x4]), 0, 0, 0, 0, 0, 0}}}}, {{0, x3, {{0, 0, 0, g[3][3]^(1,0)[x0, x4]/(2 √g[0][0][x0, x4]) g[3][3][x0, x4], 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, g[3][3]^(1,0)[x0, x4]/(2 √g[0][0][x0, x4]), 0, 0, 0, 0, 0, 0}}}}, {{0, x4, {{0, 0, 0, 0, g[4][4]^(1,0)[x0, x4]/(2 √g[0][0][x0, x4]) g[4][4][x0, x4], 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, g[4][4]^(1,0)[x0, x4]/(2 √g[0][0][x0, x4]), 0, 0, 0, 0, 0, 0}}}}
```

$$\begin{aligned}
& \left\{ 0, x_5, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 0, x_6, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0 \right\} \right\} \right\}, \\
& \left\{ 0, x_7, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0 \right\} \right\} \right\}, \\
& \left\{ 1, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \quad \left\{ 1, x_1, \left\{ \left\{ -\frac{g[1][1]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[1][1][x_0, x_4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[1][1][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \quad \left\{ \left\{ -\frac{g[1][1]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[1][1][x_0, x_4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[1][1][x_0, x_4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 1, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 1, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 2, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 2, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \quad \left\{ 2, x_2, \left\{ \left\{ -\frac{g[2][2]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[2][2][x_0, x_4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \quad \left\{ \left\{ -\frac{g[2][2]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 2, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 2, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 2, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \{2, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\}, \\
& \left\{ \begin{array}{l} \{3, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_3, \left\{ \begin{array}{l} \left\{ \left\{ -\frac{g[3][3]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[3][3][x_0, x_4]}}, \right. \right. \\ \left. \left. 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\ \left\{ \left\{ -\frac{g[3][3]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0 \right\} \right\} \right\}, \\ \{3, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{4, x_0, \left\{ \begin{array}{l} \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, \right. \right. \\ \left. \left. \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\ \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, \right. \right. \\ \left. \left. -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}} - \frac{g[4][4]^{(1,0)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\} \right\}, \\ \{4, x_1, \left\{ \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x_0, x_4]}{2 g[1][1][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\ \left. \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ \{4, x_2, \left\{ \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 g[2][2][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\ \left. \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\ \{4, x_3, \left\{ \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4]}{2 g[3][3][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\ \left. \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0 \right\} \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 4, x_4, \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. + \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. - \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}} - \frac{g[4][4]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_5, \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \\
& \quad \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\}, \right. \\
& \quad \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\}, \right. \\
& \quad \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]} \right\} \right\} \right\}, \\
& \left\{ 5, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, \right. \\
& \quad \left. \left. - \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}} - \frac{g[5][5]^{(1,0)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]}, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 5, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 5, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 5, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left\{ 5, x_4, \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, \right. \\
& \quad \left. \left. - \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}} - \frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]}, 0, 0 \right\} \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 5, x_5, \left\{ \left\{ -\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ -\frac{g[5][5]^{(1,0)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. -\frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]}, 0, 0, 0 \right\} \right\} \}, \\
& \{5, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \left\{ 6, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 (-g[6][6][x_0, x_4])^{3/2}} + \right. \right. \right. \\
& \quad \left. \left. \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\}, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \\
& \quad \left. \left. -\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]} - \frac{g[6][6]^{(1,0)}[x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}{2 g[6][6][x_0, x_4]}, 0 \right\} \right\} \}, \\
& \{6, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_4, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 (-g[6][6][x_0, x_4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\}, \left\{ \left\{ 0, 0, 0, \right. \right. \\
& \quad \left. \left. 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]} - \frac{g[6][6]^{(0,1)}[x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}{2 g[6][6][x_0, x_4]}, 0 \right\} \right\} \}, \\
& \{6, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x_6, \left\{ \left\{ -\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}, 0, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ -\frac{g[6][6]^{(1,0)}[x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}{2 g[6][6][x_0, x_4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. -\frac{g[6][6]^{(0,1)}[x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}{2 g[6][6][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \{6, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \frac{g[7][7]^{(1,0)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \} \}, \{ \{ 0, 0, 0, 0, 0, \\
& 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]} \} \}, \\
& \{ 7, x1, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 7, x2, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 7, x3, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 7, x4, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \} \}, \{ \{ 0, 0, 0, 0, 0, \\
& 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]} \} \}, \\
& \{ 7, x5, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 7, x6, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 7, x7, \{ \{ \{ -\frac{g[7][7]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4]\sqrt{-g[7][7][x0, x4]}}, 0, \\
& 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]}{2g[4][4][x0, x4]\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0 \} \}, \\
& \{ \{ -\frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]}, 0, 0, 0, \\
& -\frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]}, 0, 0, 0 \} \} \} \} \}
\end{aligned}$$

Out[•]=

$$\begin{aligned}
& \{ \{ 0, x0, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 0, x1, \{ \{ \{ 0, H, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, e^{2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 0, x2, \{ \{ \{ 0, 0, H, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, e^{2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 0, x3, \{ \{ \{ 0, 0, 0, H, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, e^{2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 0, x4, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 0, x5, \{ \{ \{ 0, 0, 0, 0, 0, H, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0, 0 \} \} \}, \{ 0, x6, \\
& \{ \{ \{ 0, 0, 0, 0, 0, 0, H, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} H \sin[6Hx0]^{1/3}, 0 \} \} \}, \{ 0, \\
& x7, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, H \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} H \sin[6Hx0]^{1/3} \} \} \} \}, \\
& \{ 1, x0, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 1, x1, \{ \{ \{ -\sqrt{e^{2a4[Hx4]}} H \sec[6Hx0] \sin[6Hx0]^{7/6}, 0, 0, 0, \\
& \sqrt{e^{2a4[Hx4]}} H \sin[6Hx0]^{1/6} a4'[Hx4], 0, 0, 0 \} \}, \{ \{ -\frac{\sqrt{e^{2a4[Hx4]}} H \cos[6Hx0]}{\sin[6Hx0]^{5/6}},
\end{aligned}$$

In[1]:= $w[[1]]$

Out[1]=

$$\left\{ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 1, 1], \omega_{\mu(b)}^{(a)} [1, 1, 2], \omega_{\mu(b)}^{(a)} [1, 1, 3], \omega_{\mu(b)}^{(a)} [1, 1, 4], \\ \omega_{\mu(b)}^{(a)} [1, 1, 5], \omega_{\mu(b)}^{(a)} [1, 1, 6], \omega_{\mu(b)}^{(a)} [1, 1, 7], \omega_{\mu(b)}^{(a)} [1, 1, 8] \end{array} \right\}, \right. \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 2, 1], \omega_{\mu(b)}^{(a)} [1, 2, 2], \omega_{\mu(b)}^{(a)} [1, 2, 3], \omega_{\mu(b)}^{(a)} [1, 2, 4], \\ \omega_{\mu(b)}^{(a)} [1, 2, 5], \omega_{\mu(b)}^{(a)} [1, 2, 6], \omega_{\mu(b)}^{(a)} [1, 2, 7], \omega_{\mu(b)}^{(a)} [1, 2, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 3, 1], \omega_{\mu(b)}^{(a)} [1, 3, 2], \omega_{\mu(b)}^{(a)} [1, 3, 3], \omega_{\mu(b)}^{(a)} [1, 3, 4], \\ \omega_{\mu(b)}^{(a)} [1, 3, 5], \omega_{\mu(b)}^{(a)} [1, 3, 6], \omega_{\mu(b)}^{(a)} [1, 3, 7], \omega_{\mu(b)}^{(a)} [1, 3, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 4, 1], \omega_{\mu(b)}^{(a)} [1, 4, 2], \omega_{\mu(b)}^{(a)} [1, 4, 3], \omega_{\mu(b)}^{(a)} [1, 4, 4], \\ \omega_{\mu(b)}^{(a)} [1, 4, 5], \omega_{\mu(b)}^{(a)} [1, 4, 6], \omega_{\mu(b)}^{(a)} [1, 4, 7], \omega_{\mu(b)}^{(a)} [1, 4, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 5, 1], \omega_{\mu(b)}^{(a)} [1, 5, 2], \omega_{\mu(b)}^{(a)} [1, 5, 3], \omega_{\mu(b)}^{(a)} [1, 5, 4], \\ \omega_{\mu(b)}^{(a)} [1, 5, 5], \omega_{\mu(b)}^{(a)} [1, 5, 6], \omega_{\mu(b)}^{(a)} [1, 5, 7], \omega_{\mu(b)}^{(a)} [1, 5, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 6, 1], \omega_{\mu(b)}^{(a)} [1, 6, 2], \omega_{\mu(b)}^{(a)} [1, 6, 3], \omega_{\mu(b)}^{(a)} [1, 6, 4], \\ \omega_{\mu(b)}^{(a)} [1, 6, 5], \omega_{\mu(b)}^{(a)} [1, 6, 6], \omega_{\mu(b)}^{(a)} [1, 6, 7], \omega_{\mu(b)}^{(a)} [1, 6, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 7, 1], \omega_{\mu(b)}^{(a)} [1, 7, 2], \omega_{\mu(b)}^{(a)} [1, 7, 3], \omega_{\mu(b)}^{(a)} [1, 7, 4], \\ \omega_{\mu(b)}^{(a)} [1, 7, 5], \omega_{\mu(b)}^{(a)} [1, 7, 6], \omega_{\mu(b)}^{(a)} [1, 7, 7], \omega_{\mu(b)}^{(a)} [1, 7, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 8, 1], \omega_{\mu(b)}^{(a)} [1, 8, 2], \omega_{\mu(b)}^{(a)} [1, 8, 3], \omega_{\mu(b)}^{(a)} [1, 8, 4], \\ \omega_{\mu(b)}^{(a)} [1, 8, 5], \omega_{\mu(b)}^{(a)} [1, 8, 6], \omega_{\mu(b)}^{(a)} [1, 8, 7], \omega_{\mu(b)}^{(a)} [1, 8, 8] \end{array} \right\} \right\}$$

In[2]:= $sw = \text{Block}[\{\text{eA}\alpha, \text{e}\alpha A, t, V, \text{ginv}, \Gamma, gg\},$

$$\begin{aligned} gg &= \text{g4488} /. sg; \\ \{\text{ginv}, \Gamma\} &= \text{con}[gg]; \\ \text{e}\alpha A &= e_{(A)}^\alpha /. sge_{(A)}^\alpha; \\ \text{eA}\alpha &= e_\alpha^{(A)} /. sge_\alpha^{(A)}; \\ \text{Table}[w[[\mu, a, b]] \rightarrow \text{Block}[\{v, d\}, v = \text{e}\alpha A[\text{All}, b]], \\ d &= \text{Sum}[(D[v[k], X[\mu]] + \text{Sum}[v[i] \times \Gamma[k, i, \mu], \{i, 1, \text{DIM8}\}]) * \text{eA}\alpha[a, k], \\ &\quad \{k, 1, \text{DIM8}\}]; \\ d], \{\mu, 1, \text{DIM8}\}, \{a, 1, \text{Length}[\text{e}\alpha A]\}, \{b, 1, \text{Length}[\text{eA}\alpha]\}] \end{aligned}$$

Out[2]=

$$\left\{ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 1, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 5] \rightarrow \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}} \end{array} \right\} \right\}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [1, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 5, 1] \rightarrow -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 5, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 5, 5] \rightarrow \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[4][4][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 6, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 5] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 6, 6] \rightarrow \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[5][5][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 7, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 7, 7] \rightarrow \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[6][6][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 7] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 8, 8] \rightarrow \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[7][7][x0, x4]} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left[\omega_{\mu(b)}^{(a)} [2, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 2] \rightarrow -\frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[1][1][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 5] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \left[\omega_{\mu(b)}^{(a)} [2, 2, 1] \rightarrow \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[1][1][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 2, 5] \rightarrow \frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 2, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \left[\omega_{\mu(b)}^{(a)} [2, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 3, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \left[\omega_{\mu(b)}^{(a)} [2, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 4, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \left[\omega_{\mu(b)}^{(a)} [2, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 2] \rightarrow -\frac{g[1][1]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[1][1][x0, x4]}\sqrt{g[4][4][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 5] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 5, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \left[\omega_{\mu(b)}^{(a)} [2, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 6, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 6, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \left[\omega_{\mu(b)}^{(a)} [2, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 7, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 7, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 7, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 7, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \left[\omega_{\mu(b)}^{(a)} [2, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 8, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 8, 8] \rightarrow 0 \right\} \right\}, \\
& \left\{ \left[\omega_{\mu(b)}^{(a)} [3, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 1, 2] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 1, 3] \rightarrow -\frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[2][2][x0, x4]}}, \omega_{\mu(b)}^{(a)} [3, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 1, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 1, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \left[\omega_{\mu(b)}^{(a)} [3, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 2, 4] \rightarrow 0, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [3, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [3, 3, 1] \rightarrow \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [3, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [3, 3, 5] \rightarrow \frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [3, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [3, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [3, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [3, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 5, 2] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [3, 5, 3] \rightarrow -\frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, \omega_{\mu(b)}^{(a)} [3, 5, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [3, 5, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [3, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [3, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [3, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [3, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 7, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 7, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [3, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [3, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [3, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [4, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 1, 3] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 1, 4] \rightarrow -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \omega_{\mu(b)}^{(a)} [4, 1, 5] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [4, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 1, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left. \left\{ \omega_{\mu(b)}^{(a)} [4, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 4] \rightarrow 0, \right. \right. \right. \\
& \quad \left. \left. \left. \omega_{\mu(b)}^{(a)} [4, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left. \left\{ \omega_{\mu(b)}^{(a)} [4, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 4] \rightarrow 0, \right. \right. \right. \\
& \quad \left. \left. \left. \omega_{\mu(b)}^{(a)} [4, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 3, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left. \left\{ \omega_{\mu(b)}^{(a)} [4, 4, 1] \rightarrow \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \right. \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \omega_{\mu(b)}^{(a)} [4, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 4] \rightarrow 0, \\
& \omega_{\mu(b)}^{(a)} [4, 4, 5] \rightarrow \frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 5, 3] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 5, 4] \rightarrow -\frac{g[3][3]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [4, 5, 5] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 6, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 7, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 8, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [5, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 1, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 1, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 1, 5] \rightarrow -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 1, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 2, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 3, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 4, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 5, 1] \rightarrow \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 5, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 5, 5] \rightarrow \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \right\} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-g[4][4][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [5, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 5, 8] \rightarrow 0, \\
& \left\{ \omega_{\mu(b)}^{(a)} [5, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 6, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [5, 6, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 6, 5] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [5, 6, 6] \rightarrow \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[5][5][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [5, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [5, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 7, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [5, 7, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 7, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [5, 7, 7] \rightarrow \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[6][6][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [5, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [5, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [5, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [5, 8, 7] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [5, 8, 8] \rightarrow \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [6, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [6, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 6] \rightarrow -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[5][5][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [6, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 1, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [6, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [6, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [6, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [6, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 3, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [6, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [6, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 4, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [6, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [6, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 6] \rightarrow -\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [6, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 6, 1] \rightarrow \frac{g[5][5]^{(1,0)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[5][5][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 6, 5] \rightarrow \frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[5][5][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [6, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [7, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 3] \rightarrow 0, \right. \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 1, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 1, 7] \rightarrow -\frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[6][6][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [7, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 5, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 5, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 5, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 5, 7] \rightarrow -\frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[6][6][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [7, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 7, 1] \rightarrow \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[6][6][x0, x4]}, \right.
\end{aligned}$$

$$\begin{aligned}
& \omega_{\mu(b)}^{(a)} [7, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 4] \rightarrow 0, \\
& \omega_{\mu(b)}^{(a)} [7, 7, 5] \rightarrow \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[6][6][x0, x4]}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [7, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [7, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [7, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 7] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 1, 8] \rightarrow -\frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[7][7][x0, x4]}} \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 7] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 5, 8] \rightarrow -\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}} \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 8, 1] \rightarrow \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [8, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 8, 5] \rightarrow \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}, \end{array} \right\},
\end{aligned}$$

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 $\omega_{\mu}^{(a)}(b) [8, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)}(b) [8, 8, 8] \rightarrow 0 \} \} \}$ 

In[1]:= (*sw=Block[{eAα,eαA,t,V,ginv,Γ,gg},  

gg=g4488/.sg;  

{ginv,Γ}=con[gg];  

eαA=e_{(A)}^α/.sge_{(A)}^α;  

eAα=e_{α}^{(A)}/.sge_{α}^{(A)};  

Table[ $\omega_{\mu}^{(a)}(b)$  [μ,a,b]→Block[{v,d},v=eαA[[All,b]];  

d=Sum[  

(D[v[[k]],X[[μ]]]+Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}])*eAα[[a,k]],{k,1,DIM8}];  

d],{μ,1,DIM8},{a,1,Length[eαA]},{b,1,Length[eαA]}]]*)

In[2]:= swf=Flatten[sw];

(* $\omega_{\mu}^{(a)}(b)$ *)  

ω;  

wmat=%/.swf  

Dimensions[%]

Out[2]=

$$\left\{ \left\{ 0, 0, 0, 0, \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \right.$$


$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\left\{ -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, \right.$$


$$\left. \left( \frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \right.$$


$$\sqrt{-g[4][4][x0, x4]}, 0, 0, 0 \},$$


$$\{0, 0, 0, 0, 0, \left( \frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \right.$$


$$\sqrt{-g[5][5][x0, x4]}, 0, 0 \},$$


$$\{0, 0, 0, 0, 0, 0, \left( \frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right.$$


$$\sqrt{-g[6][6][x0, x4]}, 0 \}, \{0, 0, 0, 0, 0, 0, 0,$$


$$\left. \left( \frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \right.$$


$$\sqrt{-g[7][7][x0, x4]} \} \},$$


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$$\begin{aligned}
& \left\{ \left\{ 0, -\frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\
& \left\{ \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, \right. \\
& \left. \left. \frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \right. \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \right. \\
& \left. \left\{ 0, -\frac{g[1][1]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \right\}, \\
& \left\{ \left\{ 0, 0, -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0 \right\}, \right. \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, \right. \right. \\
& \left. \left. \frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \right. \\
& \left. \left\{ 0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0, 0 \right\}, \right. \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \right\}, \\
& \left\{ \left\{ 0, 0, 0, -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0 \right\}, \right. \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \right. \\
& \left. \left\{ \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, \right. \right. \\
& \left. \left. \frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0 \right\}, \right. \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \right\}, \\
& \left\{ \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \right. \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \right. \\
& \left. \left\{ \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, \right. \right. \\
& \left. \left. \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \right. \\
& \left. \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \right\},
\end{aligned}$$


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$$\left\{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}}\right\},$$


$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$


$$\left\{\frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]}, 0, 0, 0,$$


$$\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}, 0, 0, 0\}\}\}$$

Out[8]= {8, 8, 8}

In[9]:= swgtry = Block[{eAα, eαA, t, v},
  eαA = eα(A) /. sgtryeα(A);
  eAα = e(A)α /. sgtrye(A)α;
  Table[(*v=eAα[[a,All]];*) ω(a)μ(b) [μ, a, b] → Block[{v, d}, v = eαA[[All, b]];
    d = Sum[(D[v[[k]], X[[μ]]] + Sum[v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}]) * eAα[[a, k]],
      {k, 1, DIM8}];
    FullSimplify[d, constraintVars] /. {Sqrt[e2 a4[H x4]] → ea4[H x4], Sqrt[e-2 a4[H x4]] →
      e-a4[H x4]}], {μ, 1, DIM8}, {a, 1, Length[eAα]}, {b, 1, Length[eαA]}]]
Out[9]= {{ω(a)μ(b) [1, 1, 1] → 0, ω(a)μ(b) [1, 1, 2] → 0, ω(a)μ(b) [1, 1, 3] → 0, ω(a)μ(b) [1, 1, 4] → 0,
  ω(a)μ(b) [1, 1, 5] → 0, ω(a)μ(b) [1, 1, 6] → 0, ω(a)μ(b) [1, 1, 7] → 0, ω(a)μ(b) [1, 1, 8] → 0},
  {ω(a)μ(b) [1, 2, 1] → 0, ω(a)μ(b) [1, 2, 2] → 0, ω(a)μ(b) [1, 2, 3] → 0, ω(a)μ(b) [1, 2, 4] → 0,
  ω(a)μ(b) [1, 2, 5] → 0, ω(a)μ(b) [1, 2, 6] → 0, ω(a)μ(b) [1, 2, 7] → 0, ω(a)μ(b) [1, 2, 8] → 0},
  {ω(a)μ(b) [1, 3, 1] → 0, ω(a)μ(b) [1, 3, 2] → 0, ω(a)μ(b) [1, 3, 3] → 0, ω(a)μ(b) [1, 3, 4] → 0,
  ω(a)μ(b) [1, 3, 5] → 0, ω(a)μ(b) [1, 3, 6] → 0, ω(a)μ(b) [1, 3, 7] → 0, ω(a)μ(b) [1, 3, 8] → 0},
  {ω(a)μ(b) [1, 4, 1] → 0, ω(a)μ(b) [1, 4, 2] → 0, ω(a)μ(b) [1, 4, 3] → 0, ω(a)μ(b) [1, 4, 4] → 0,
  ω(a)μ(b) [1, 4, 5] → 0, ω(a)μ(b) [1, 4, 6] → 0, ω(a)μ(b) [1, 4, 7] → 0, ω(a)μ(b) [1, 4, 8] → 0},
  {ω(a)μ(b) [1, 5, 1] → 0, ω(a)μ(b) [1, 5, 2] → 0, ω(a)μ(b) [1, 5, 3] → 0, ω(a)μ(b) [1, 5, 4] → 0,
  ω(a)μ(b) [1, 5, 5] → 0, ω(a)μ(b) [1, 5, 6] → 0, ω(a)μ(b) [1, 5, 7] → 0, ω(a)μ(b) [1, 5, 8] → 0},
  {ω(a)μ(b) [1, 6, 1] → 0, ω(a)μ(b) [1, 6, 2] → 0, ω(a)μ(b) [1, 6, 3] → 0, ω(a)μ(b) [1, 6, 4] → 0,
  ω(a)μ(b) [1, 6, 5] → 0, ω(a)μ(b) [1, 6, 6] → 0, ω(a)μ(b) [1, 6, 7] → 0, ω(a)μ(b) [1, 6, 8] → 0},
  {ω(a)μ(b) [1, 7, 1] → 0, ω(a)μ(b) [1, 7, 2] → 0, ω(a)μ(b) [1, 7, 3] → 0, ω(a)μ(b) [1, 7, 4] → 0,
  ω(a)μ(b) [1, 7, 5] → 0, ω(a)μ(b) [1, 7, 6] → 0, ω(a)μ(b) [1, 7, 7] → 0, ω(a)μ(b) [1, 7, 8] → 0}

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$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [7, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 7, 1] \rightarrow e^{-a4[H \times 4]} H \sin[6Hx0]^{1/6}, \quad \omega_{\mu(b)}^{(a)} [7, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 7, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 5] \rightarrow -e^{-a4[H \times 4]} H \sin[6Hx0]^{1/6} a4'[H \times 4], \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [8, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 3] \rightarrow 0, \right. \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 1, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 8] \rightarrow e^{-a4[H \times 4]} H \sin[6Hx0]^{1/6} \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 5, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 8] \rightarrow e^{-a4[H \times 4]} H \sin[6Hx0]^{1/6} a4'[H \times 4] \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [8, 8, 1] \rightarrow e^{-a4[H \times 4]} H \sin[6Hx0]^{1/6}, \quad \omega_{\mu(b)}^{(a)} [8, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 8, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 5] \rightarrow -e^{-a4[H \times 4]} H \sin[6Hx0]^{1/6} a4'[H \times 4], \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [8, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 8] \rightarrow 0 \right\} \}
\end{aligned}$$

In[8]:= **swfgtry = Flatten[swgtry];**


```

Out[=]= {8, 8, 8}

In[=]:= g4488 /. sg  

% /. ssgm4488

Out[=]= {{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0},  

{0, 0, g[2][2][x0, x4], 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},  

{0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},  

{0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}

Out[=]= {{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0},  

{0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0},  

{0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},  

{0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0},  

{0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0},  

{0, 0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

check metric compatibility condition for the Octad

In[=]:= Block[{eAα, eαA, t, ginv, Γ, gg(*, ωω*)},  

(*ωω = ω^(a)_μ(b) /. swf; *)  

gg = g4488 /. sg /. ssgm4488;  

{ginv, Γ} = con[gg];  

eAα = eA^(α) /._ sge^(α); eAα = e^(A) /._ sge^(A);  

t = Table[Block[{v, V, contra, covari},  

V = eAα[[b, All]];  

v = eA[[All, b]];  

contra =  

Table[D[v[[k]], X[[μ]]] + Sum[v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];  

(*no ω terms*)  

covari = Table[D[V[[k]], X[[μ]]] - Sum[V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}] +  

Sum[eAα[[J, All]][[k]] × ωmat[[μ, b, J]], {J, 1, DIM8}] //  

FullSimplify[#, constraintVars && e^2 a4[H x4] > 0 && e^a4[H x4] > 0 &&  

e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0] &, {k, 1, DIM8}]; (*ω terms*)  

(*{b-1, X[[μ]], {{contra}, {covari}}}, {b, 1, Length[eA]}, {μ, 1, DIM8}];*)  

{b - 1, X[[μ]], covari}, {b, 1, Length[eA]}, {μ, 1, DIM8}];  

t]  

FullSimplify[#/._ ssgm4488,  

constraintVars && e^2 a4[H x4] > 0 && e^a4[H x4] > 0 && e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0] & /@ %

Out[=]= {{0, x0, {g[0][0]^(1,0)[x0, x4] + 24 H Csc[12 H x0] g[0][0][x0, x4],  

2 √g[0][0][x0, x4]}}

```

$$\begin{aligned}
& \left. 0, 0, 0, \frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \Big\}, \\
& \{0, x1, \{0, \{-g[1][1]^{(1,0)}[x0, x4] + 2e^{2a4[Hx4]} H \operatorname{Sec}[6Hx0] \operatorname{Sin}[6Hx0]^{4/3} \\
& \quad g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0, 0\}\}, \\
& \{0, x2, \{0, 0, \{-g[2][2]^{(1,0)}[x0, x4] + 2e^{2a4[Hx4]} H \operatorname{Sec}[6Hx0] \operatorname{Sin}[6Hx0]^{4/3} \\
& \quad g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0, 0\}\}, \\
& \{0, x3, \{0, 0, 0, \{-g[3][3]^{(1,0)}[x0, x4] + 2e^{2a4[Hx4]} H \operatorname{Sec}[6Hx0] \\
& \quad \operatorname{Sin}[6Hx0]^{4/3} g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0\}\}, \\
& \{0, x4, \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \Big\}, \\
& \{0, x5, \{0, 0, 0, 0, 0, \{-g[5][5]^{(1,0)}[x0, x4] - 2e^{-2a4[Hx4]} H \operatorname{Sec}[6Hx0] \\
& \quad \operatorname{Sin}[6Hx0]^{4/3} g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0, 0\}\}, \\
& \{0, x6, \{0, 0, 0, 0, 0, 0, \{-g[6][6]^{(1,0)}[x0, x4] - 2e^{-2a4[Hx4]} H \operatorname{Sec}[6Hx0] \\
& \quad \operatorname{Sin}[6Hx0]^{4/3} g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]}), 0\}\}, \\
& \{0, x7, \{0, 0, 0, 0, 0, 0, 0, \{-g[7][7]^{(1,0)}[x0, x4] - 2e^{-2a4[Hx4]} H \\
& \quad \operatorname{Sec}[6Hx0] \operatorname{Sin}[6Hx0]^{4/3} g[0][0][x0, x4]\} / (2\sqrt{g[0][0][x0, x4]})\}\}, \\
& \left\{ \left\{ 1, x0, \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4] - 2H \operatorname{Cot}[6Hx0] g[1][1][x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 1, x1, \left\{ \frac{g[1][1]^{(1,0)}[x0, x4] - 2H \operatorname{Cot}[6Hx0] g[1][1][x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, \right. \right. \\
& \quad \left. \left. 0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6Ha4'[Hx4] g[1][1][x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0 \right\} \right\}, \\
& \{1, x2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{1, x4, \\
& \quad \left. \left. \left. 0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6Ha4'[Hx4] g[1][1][x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \{1, x5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{1, x6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{1, x7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \left\{ \left\{ 2, x0, \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4] - 2H \operatorname{Cot}[6Hx0] g[2][2][x0, x4]}{2\sqrt{g[2][2][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, 0, 0 \right\} \right\}, \{2, x1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 2, x_2, \left\{ \frac{g[2][2]^{(1,0)}[x_0, x_4] - 2 H \operatorname{Cot}[6 H x_0] g[2][2][x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, \right. \right. \\
& \quad \left. \left. 0, \frac{g[2][2]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[2][2][x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \{2, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_4, \\
& \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[2][2][x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0, 0, 0 \right\} \}, \\
& \{2, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \left\{ \{3, x_0, \right. \\
& \quad \left. \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x_0, x_4] - 2 H \operatorname{Cot}[6 H x_0] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, 0 \right\} \right\}, \\
& \{3, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{ 3, x_3, \left\{ \frac{g[3][3]^{(1,0)}[x_0, x_4] - 2 H \operatorname{Cot}[6 H x_0] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}} \right., 0, 0, 0, \right. \\
& \quad \left. \left. \frac{g[3][3]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \{3, x_4, \\
& \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, 0 \right\} \}, \\
& \{3, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{ \{4, x_0, \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \{4, x_1, \\
& \left\{ 0, (g[1][1]^{(0,1)}[x_0, x_4] + 6 e^{2 a4[H x_4]} H \operatorname{Sin}[6 H x_0]^{1/3} a4'[H x_4] g[4][4][x_0, x_4]) / \right. \\
& \quad \left. (2 \sqrt{-g[4][4][x_0, x_4]}), 0, 0, 0, 0, 0, 0 \right\}, \{4, x_2, \{0, 0, \\
& \quad (g[2][2]^{(0,1)}[x_0, x_4] + 6 e^{2 a4[H x_4]} H \operatorname{Sin}[6 H x_0]^{1/3} a4'[H x_4] g[4][4][x_0, x_4]) / \right. \\
& \quad \left. (2 \sqrt{-g[4][4][x_0, x_4]}), 0, 0, 0, 0, 0 \right\}, \{4, x_3, \{0, 0, 0, \\
& \quad (g[3][3]^{(0,1)}[x_0, x_4] + 6 e^{2 a4[H x_4]} H \operatorname{Sin}[6 H x_0]^{1/3} a4'[H x_4] g[4][4][x_0, x_4]) / \right. \\
& \quad \left. (2 \sqrt{-g[4][4][x_0, x_4]}), 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 4, x_4, \left\{ -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \{4, x_5, \\
& \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{2} \left(-6 e^{-2 a4[H x_4]} H \operatorname{Sin}[6 H x_0]^{1/3} a4'[H x_4] - \frac{g[5][5]^{(0,1)}[x_0, x_4]}{g[4][4][x_0, x_4]} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-g[4][4][x0, x4]}, 0, 0 \} \}, \{ 4, x6, \{ 0, 0, 0, 0, 0, 0, 0, \\
& \frac{1}{2} \left(-6 e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4] - \frac{g[6][6]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]}, 0 \} \}, \{ 4, x7, \{ 0, 0, 0, 0, 0, 0, 0, 0, \\
& \frac{1}{2} \left(-6 e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4] - \frac{g[7][7]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]} \right\} \}, \{ \{ 5, x0, \\
& \{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0 \} \}, \\
& \{ 5, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 5, x2, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 5, x3, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 5, x4, \\
& \{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0 \} \}, \\
& \{ 5, x5, \{ -\frac{g[5][5]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0, \\
& 0, -\frac{g[5][5]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0 \} \}, \\
& \{ 5, x6, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 5, x7, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \{ \{ 6, x0, \\
& \{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0 \} \}, \\
& \{ 6, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 6, x2, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 6, x3, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 6, x4, \\
& \{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0 \} \}, \\
& \{ 6, x5, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 6, x6, \{ -\frac{g[6][6]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0, 0, \\
& 0, -\frac{g[6][6]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0 \} \}, \\
& \{ 6, x7, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \{ \{ 7, x0, \\
& \{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4] - 2 H \cot[6 H x0] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} \} \}, \\
& \{ 7, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 7, x2, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \},
\end{aligned}$$

$$\{7, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, -\frac{g[7][7]^{(0,1)}[x_0, x_4] + 6 H a4' [H x4] g[7][7][x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]}}\},$$

$$\{7, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{7, x_7, \left\{ \begin{array}{l} \left\{ -\frac{g[7][7]^{(1,0)}[x_0, x_4] - 2 H \text{Cot}[6 H x0] g[7][7][x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]}} \right., \\ \left. 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4] + 6 H a4' [H x4] g[7][7][x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]}} \right\}, 0, 0, 0 \end{array} \right\}$$

Out[8]=

$$\{\{0, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{0, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{1, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}$$

In[9]:=

```
Block[{eAα, eαA, t, ginv, Γ, gg(*, ωω*)},
(*ωω=ω(a)μ(b) /. sωf;*)
gg = g4488 /. sg;
{ginv, Γ} = con[gg];
eαA = eα(A) /. sgeα(A); eAα = e(A)α /. sge(A)α;
t = Table[Block[{v, V, contra, covari},
V = eAα[b, All];
v = eαA[All, b];
contra =
Table[D[v[k], X[μ]] + Sum[v[i] × T[k, i, μ], {i, 1, DIM8}], {k, 1, DIM8}];
(*no ω terms*)
covari = Table[D[V[k], X[μ]] - Sum[V[i] × T[i, k, μ], {i, 1, DIM8}] +
Sum[eAα[J, All][k] × ωmat[μ, b, J], {J, 1, DIM8}],
{k, 1, DIM8}]; (*ω terms*)
{b - 1, X[μ]}, {{contra}, {covari}}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
```

$t]$

```

FullSimplify[#, ssym4488,
  constraintVars &&  $e^{2 a4[x4]} > 0 \&\& e^{a4[x4]} > 0 \&\& e^{-2 a4[x4]} > 0 \&\& e^{-a4[x4]} > 0$ ] & /@ %
Out[#]=
{{{{0, x0, {{0, 0, 0, 0, -g[0][0]^(0,1)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[4][4][x0, x4]), 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0}}}, {0, x1, {{0, g[1][1]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[1][1][x0, x4]), 0, 0, 0, 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x2, {{0, 0, g[2][2]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[2][2][x0, x4]), 0, 0, 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x3, {{0, 0, 0, g[3][3]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[3][3][x0, x4]), 0, 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x4, {{0, 0, 0, 0, g[4][4]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[4][4][x0, x4]), 0, 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x5, {{0, 0, 0, 0, 0, g[5][5]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[5][5][x0, x4]), 0, 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x6, {{0, 0, 0, 0, 0, 0, g[6][6]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[6][6][x0, x4]), 0}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {0, x7, {{0, 0, 0, 0, 0, 0, 0, g[7][7]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[7][7][x0, x4])}}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x0, {{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x1, {{-g[1][1]^(1,0)[x0, x4]/(2 g[0][0][x0, x4] Sqrt[g[1][1][x0, x4]]), 0, 0, 0, 0, 0, 0, 0}}, {0, 0, -g[1][1]^(0,1)[x0, x4]/(2 Sqrt[g[1][1][x0, x4]] g[4][4][x0, x4]), 0, 0, 0, 0}}}}}

```

```

{ { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { 1, x2,
  { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 1, x3, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 1, x4, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 1, x5, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 1, x6, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 1, x7, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } } ,
{ { 2, x0, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x1, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x2, { { { -  $\frac{g[2][2]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[2][2][x0, x4]}}$  , 0,
  0, 0, -  $\frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}$  , 0, 0, 0 } } } } ,
{ { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } , { 2, x3,
  { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x4, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x5, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x6, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 2, x7, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } } ,
{ { 3, x0, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x1, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x2, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x3, { { { -  $\frac{g[3][3]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[3][3][x0, x4]}}$  , 0,
  0, 0, -  $\frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}$  , 0, 0, 0 } } } } ,
{ { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } , { 3, x4,
  { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x5, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x6, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } ,
{ 3, x7, { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } , { { { 0, 0, 0, 0, 0, 0, 0, 0 } } } } } } ,
{ { 4, x0, { { {  $\frac{g[0][0]^{(0,1)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[4][4][x0, x4]}}$  , 0, 0, 0,
   $\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]}$  , 0, 0, 0 } } } } } ,

```

$$\begin{aligned}
& \left\{ \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}} - \right. \right. \\
& \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]} - \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \right. \right. \\
& \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]} \right) g[4][4][x0, x4], 0, 0, 0 \right\} \} \}, \\
& \left\{ 4, x1, \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x0, x4]}{2g[1][1][x0, x4]\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x2, \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]}{2g[2][2][x0, x4]\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x3, \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2g[3][3][x0, x4]\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x4, \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4]\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \left. \left. \left. \frac{g[4][4]^{(0,1)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}} - \right. \right. \\
& \left. \left. \frac{g[4][4]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]} - \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \right. \right. \\
& \left. \left. \frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]} \right) g[4][4][x0, x4], 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x5, \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[5][5][x0, x4]}, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 4, x6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[6][6][x0, x4]}, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 4, x_7, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[4][4][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\} \right\}, \\
& \left\{ 5, x_0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2(-g[5][5][x_0, x_4])^{3/2}} + \right. \right. \right. \right. \\
& \quad \left. \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \quad -\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]}} - \frac{g[5][5]^{(1,0)}[x_0, x_4]\sqrt{-g[5][5][x_0, x_4]}}{2g[5][5][x_0, x_4]} - \\
& \quad \left(\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2(-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]} \right) \\
& \quad \left. \left. \left. \left. g[5][5][x_0, x_4], 0, 0 \right\} \right\} \right\}, \\
& \{5, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x_4, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2(-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \\
& \quad \left\{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]}} - \right. \\
& \quad \left. \frac{g[5][5]^{(0,1)}[x_0, x_4]\sqrt{-g[5][5][x_0, x_4]}}{2g[5][5][x_0, x_4]} - \left(\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2(-g[5][5][x_0, x_4])^{3/2}} + \right. \right. \\
& \quad \left. \left. \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]} \right) g[5][5][x_0, x_4], 0, 0 \right\} \right\}, \\
& \{5, x_5, \left\{ \left\{ \left\{ -\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2g[0][0][x_0, x_4]\sqrt{-g[5][5][x_0, x_4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2g[4][4][x_0, x_4]\sqrt{-g[5][5][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_6, \\
& \quad \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \quad \{5, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \quad \left\{ 6, x_0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2(-g[6][6][x_0, x_4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[6][6][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \} \}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[6][6][x_0, x_4]}} - \right. \right. \\
& \left. \left. \frac{g[6][6]^{(1,0)}[x_0, x_4]\sqrt{-g[6][6][x_0, x_4]}}{2g[6][6][x_0, x_4]} - \left(\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2(-g[6][6][x_0, x_4])^{3/2}} + \right. \right. \right. \\
& \left. \left. \left. \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[6][6][x_0, x_4]} g[6][6][x_0, x_4]} \right) g[6][6][x_0, x_4], 0 \} \} \right\}, \\
& \{6, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x_4, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \\
& \left. \left. \left. \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2(-g[6][6][x_0, x_4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[6][6][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \} \right\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[6][6][x_0, x_4]}} - \right. \right. \\
& \left. \left. \frac{g[6][6]^{(0,1)}[x_0, x_4]\sqrt{-g[6][6][x_0, x_4]}}{2g[6][6][x_0, x_4]} - \left(\frac{g[6][6]^{(0,1)}[x_0, x_4]}{2(-g[6][6][x_0, x_4])^{3/2}} + \right. \right. \right. \\
& \left. \left. \left. \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[6][6][x_0, x_4]} g[6][6][x_0, x_4]} \right) g[6][6][x_0, x_4], 0 \} \} \right\}, \\
& \{6, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{6, x_6, \left\{ \left\{ \left\{ -\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2g[0][0][x_0, x_4]\sqrt{-g[6][6][x_0, x_4]}}, 0, \right. \right. \right. \\
& \left. \left. \left. 0, 0, -\frac{g[6][6]^{(0,1)}[x_0, x_4]}{2g[4][4][x_0, x_4]\sqrt{-g[6][6][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_7, \\
& \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \\
& \left. \left. \left. \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2(-g[7][7][x_0, x_4])^{3/2}} + \right. \right. \right. \\
& \left. \left. \left. \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]}} - \right. \right. \\
& \left. \left. \frac{g[7][7]^{(1,0)}[x_0, x_4]\sqrt{-g[7][7][x_0, x_4]}}{2g[7][7][x_0, x_4]} \right) g[7][7][x_0, x_4], 0 \} \right\}
\end{aligned}$$

$$\begin{aligned}
& \frac{g[7][7]^{(1,0)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2 g[7][7][x_0, x_4]} - \left(\frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 (-g[7][7][x_0, x_4])^{3/2}} + \right. \\
& \left. \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right) g[7][7][x_0, x_4] \} \}, \\
& \{7, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 (-g[7][7][x_0, x_4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]}} - \right. \\
& \left. \frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2 g[7][7][x_0, x_4]} - \right. \\
& \left. \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right) g[7][7][x_0, x_4] \} \}, \\
& \{7, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_7, \{\{\left\{ -\frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}, 0, \right. \right. \\
& \left. \left. 0, 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}, 0, 0, 0 \right\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\} \} \} \} \}
\end{aligned}$$

Out[=]

$$\begin{aligned}
& \{\{\{0, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_1, \{\{\{0, H, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_2, \{\{\{0, 0, H, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_3, \{\{\{0, 0, 0, H, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_5, \{\{\{0, 0, 0, 0, 0, H, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_6, \{\{\{0, 0, 0, 0, 0, 0, H, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, H\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x_1, \{\{\left\{ -\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{2 a4[H x4]}} \right. \right. \\
& \left. \left. H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\} \} \} \},
\end{aligned}$$


```

{5, x5, {{{{\sqrt{e^{-2 a4[H x4]}} H Sec[6 H x0] Sin[6 H x0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{-2 a4[H x4]}} H Sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{5, x6, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{5, x7, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{6, x0, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{6, x1, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{6, x2, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{6, x3, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{6, x4, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{6, x5, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{6, x6, {{{\sqrt{e^{-2 a4[H x4]}} H Sec[6 H x0] Sin[6 H x0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{-2 a4[H x4]}} H Sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{6, x7, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{7, x0, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{7, x1, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{7, x2, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{7, x3, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{7, x4, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{7, x5, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{7, x6, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{7, x7, {{{\sqrt{e^{-2 a4[H x4]}} H Sec[6 H x0] Sin[6 H x0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{-2 a4[H x4]}} H Sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}}}
In[=]:= (*w\Psi=Table[Sum[ExpandAll[
  \left( \omega_{\mu}^{(a)}_{(b)} [\mu,a,b]/.swf \right) Simplify[(Transpose[\Psi16].\sigma16.SAB[a,b].\Psi16)], {a,1,8},{b,1,8}],{\mu,1,8}]*]
In[=]:= (*w\Psi=Table[
  Sum[ExpandAll[(\omega mat[\mu,a,b]) Simplify[(Transpose[\Psi16].\sigma16.SAB[a,b].\Psi16)]], {a,1,8},{b,1,8}],{\mu,1,8}]*)
In[=]:= (*jw\Psi=ParallelTable[{j,base16[[j,2]],
  Table[{X[\mu]},Simplify[(Transpose[\Psi16].\sigma16.(ZZZ0*T16^\alpha[0]+T16^\alpha[4]*ZZZ4 ) .
    base16[[j,1]].(Sum[\omega mat[\mu,a,b]SAB[a,b],{a,1,8},{b,1,8}])\Psi16)]}, {\mu,1,8}]},{j,1,Length[base16}]]*)

```



```

In[1]:=  $\left( \omega_{\mu}^{(a)} {}_{(b)} [8, 8, 1] /. \text{Flatten}[swgtry] \right)$ 
Out[1]=  $e^{-a4[H x 4]} H \sin[6 H x 0]^{1/6}$ 

In[2]:= SAB[[1, 2]].¶16
Out[2]=

$$\begin{aligned} & \left\{ \frac{1}{2} f16[7][x0, x4], \frac{1}{2} f16[6][x0, x4], -\frac{1}{2} f16[5][x0, x4], -\frac{1}{2} f16[4][x0, x4], \right. \\ & \frac{1}{2} f16[3][x0, x4], \frac{1}{2} f16[2][x0, x4], -\frac{1}{2} f16[1][x0, x4], -\frac{1}{2} f16[0][x0, x4], \\ & -\frac{1}{2} f16[15][x0, x4], -\frac{1}{2} f16[14][x0, x4], \frac{1}{2} f16[13][x0, x4], \frac{1}{2} f16[12][x0, x4], \\ & \left. -\frac{1}{2} f16[11][x0, x4], -\frac{1}{2} f16[10][x0, x4], \frac{1}{2} f16[9][x0, x4], \frac{1}{2} f16[8][x0, x4] \right\} \end{aligned}$$


```

```
In[8]:= SAB = Table[ $\frac{1}{4} (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]), \{A1, 0, 7\}, \{B1, 0, 7\}]$ ;
```

```
In[1]:= #16  
Out[1]= {f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],  
f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],  
f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],  
f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

```
In[1]:= Sum[ExpandAll[( $\omega_{\mu}^{(a)}_{(b)}$  [4, a, b] /. swfgtry) Simplify[(SAB[a, b])]], {a, 1, 8}, {b, 1, 8}] . \!f16 // MatrixForm
```

```
Out[1]//MatrixForm=
```

$$\left(\begin{array}{c} -e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[5][x0, x4] \\ e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[4][x0, x4] \\ -e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[7][x0, x4] \\ e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[6][x0, x4] \\ -e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[1][x0, x4] \\ e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[0][x0, x4] \\ -e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[3][x0, x4] \\ e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[2][x0, x4] \\ e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[13][x0, x4] \\ -e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[12][x0, x4] \\ e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[15][x0, x4] \\ -e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[14][x0, x4] \\ e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[9][x0, x4] \\ -e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[8][x0, x4] \\ e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[11][x0, x4] \\ -e^{a4[H x4]} H \sin[6 H x0]^{1/6} f16[10][x0, x4] \end{array} \right)$$

```
In[2]:= (*Table[Block[{eA\alpha,e\alpha A,t,V},e\alpha A=e^{\alpha}_{(A)}/.sgtry e^{\alpha}_{(A)}; eA\alpha=e^{\alpha}_{\alpha}/.sgtry e^{\alpha}_{\alpha}; V=eA\alpha[[a,All]]; t=Table[Block[{v,d},v=e\alpha A[[All,b]]; d=D[v[[k]],X[[j]]]+Sum[v[[i]]\Gamma[[k,i,j]],{i,1,DIM8}]; d.V],{k,1,DIM8}]; t],[{j,1,DIM8},{a,1,Length[e\alpha A]},{b,1,Length[e\alpha A]}]*)
```

```
In[3]:= Clear[rt];
rt[g_, ass_ : {}] := Module[{t, \Gamma, Ricci\Gamma, Rie\Gamma, RS, G, ginv},
Print[Now];
t = AbsoluteTiming[
ginv = FullSimplify[Inverse[g], ass];
\Gamma = FullSimplify[Table[(1/2)*Sum[(ginv[[i, s])* (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}]* {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
Ricci\Gamma = ParallelTable[
FullSimplify[D[\Gamma[[\mu, \nu, \beta]], X[[\alpha]]] - D[\Gamma[[\mu, \nu, \alpha]], X[[\beta]]] + Sum[\Gamma[[\mu, s, \alpha]] \times \Gamma[[s, \nu, \beta]] - \Gamma[[\mu, s, \beta]] \times \Gamma[[s, \nu, \alpha]], {s, 1, DIM8}], ass],
{\mu, 1, DIM8}, {\nu, 1, DIM8}, {\alpha, 1, DIM8}, {\beta, 1, DIM8}];
Rie\Gamma =
ParallelTable[FullSimplify[Sum[Ricci\Gamma[[\alpha, \mu, \alpha, \nu]], {\alpha, 1, DIM8}], ass],
```

```

{μ, 1, DIM8}, {ν, 1, DIM8}] ;
RS = FullSimplify[Tr[ginv.RieΓ], ass];
G = ParallelTable[
  FullSimplify[RieΓ[α] - 1/2 g[α] RS, ass], {α, 1, Length[RieΓ]}];
Print[t];
Print[Now];
Return[{ginv, Γ, RicciΓ, RieΓ, RS, G}]

```

verify $g_{\mu\nu|\alpha} = 0$:

```

In[]:= Simplify[
  Table[D[gtry[j, k], X[i]] - Sum[gtry[k, s] × Γ[s, i, j], {s, 1, DIM8}] -
    Sum[gtry[j, s] × Γ[s, i, k], {s, 1, DIM8}],
  {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union

```

Out[]=

{0}

CHECK

In[]:= MatrixMetric44 // MatrixForm

```

Out[//MatrixForm=]

$$\begin{pmatrix} \text{Cot}[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a^4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a^4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a^4[Hx4]} \sin[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


```

In[]:= (e^α_(A) . MatrixMetric44.e^α_(A) /. sgtrye^α_(A)) // MatrixForm

Out[//MatrixForm=]

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


```
In[1]:=  $E_{(A)}^{\alpha} = \text{Table}\left[\text{Sum}\left[e_{\alpha}^{(A)}[A1, \alpha1] \text{ginv}[\alpha1, \beta1] \times \eta 4488[A1, B1] \text{sgtrye}_{\alpha}^{(A)}, \{\alpha1, 1, 8\}, \{A1, 1, 8\}\right], \{\beta1, 1, 8\}, \{B1, 1, 8\}\right] // \text{FullSimplify}[\#, \text{constraintVars}] \& \right] // \text{Column}$ 
```

```
Out[1]=  $\{\tan[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0\}$   

 $\left\{0, \frac{e^{-a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0\right\}$   

 $\left\{0, 0, \frac{e^{-a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0\right\}$   

 $\left\{0, 0, 0, \frac{e^{-a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0\right\}$   

 $\{0, 0, 0, 0, 1, 0, 0, 0, 0\}$   

 $\left\{0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0\right\}$   

 $\left\{0, 0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}, 0\right\}$   

 $\left\{0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}\right\}$ 
```

Lagrangian Ψ 16 =

$$\kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi16].\sigma16.T16^A[A1].\Psi16_{,\alpha} g^{-1}{}^{\alpha\beta} \eta_{A1 B1} E_{\beta}^{B1} + \text{mass term}$$

```
In[2]:= ass = constraintVars
```

```
Out[2]=  $x0 > 0 \&& x4 > 0 \&& H > 0 \&& 6Hx0 > 0 \&& 3Hx4 > 0 \&& a4[x4] > 0 \&& A4[t] > 0 \&& Q > 0 \&&$   

 $z > 0 \&& t > 0 \&& M > 0 \&& K > 0 \&& e^{-2a4[Hx4]} > 0 \&& e^{-a4[Hx4]} > 0 \&& e^{2a4[Hx4]} > 0 \&&$   

 $e^{a4[Hx4]} > 0 \&& \sin[6Hx0] > 0 \&& \cos[6Hx0] > 0 \&& \csc[6Hx0] > 0 \&& \sec[6Hx0] > 0 \&&$   

 $\tan[6Hx0] > 0 \&& \cot[6Hx0] > 0 \&& \sin[z] > 0 \&& \sqrt{\sin[z]} > 0 \&& \sin[z]^{3/2} > 0 \&&$   

 $\sqrt{\sin[z]} > 0 \&& \frac{1}{\sin[z]^{3/2}} > 0 \&& \frac{1}{\sqrt{\sin[z]}} > 0 \&& \cot[z] > 0 \&& \sqrt{\cot[z]} > 0 \&&$   

 $\cot[z]^{3/2} > 0 \&& \sqrt{\cot[z]} > 0 \&& \tan[z] > 0 \&& \sec[z] > 0 \&& \sqrt{\csc[z]} > 0 \&& \csc[z] > 0$ 
```

```
In[]:= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && √Sin[z] > 0 && Sin[z]3/2 > 0 &&
√Sin[z] > 0 && 1/Sin[z]3/2 > 0 && 1/√Sin[z] > 0 && Cot[z] > 0 && √Cot[z] > 0 &&
Cot[z]3/2 > 0 && √Cot[z] > 0 && Tan[z] > 0 && Sec[z] > 0 && √Csc[z] > 0 && Csc[z] > 0
Out[]=
x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && √Sin[z] > 0 && Sin[z]3/2 > 0 &&
√Sin[z] > 0 && 1/Sin[z]3/2 > 0 && 1/√Sin[z] > 0 && Cot[z] > 0 && √Cot[z] > 0 &&
Cot[z]3/2 > 0 && √Cot[z] > 0 && Tan[z] > 0 && Sec[z] > 0 && √Csc[z] > 0 && Csc[z] > 0
In[]:= ginv == FullSimplify[Inverse[gtry], ass]
Out[]=
True
In[]:= ginv === Transpose[ginv]
Out[]=
True
verify metric covariant derivative again:
```

```
In[]:= Simplify[
  Table[D[gtry[[j, k]], X[[i]]] - Sum[ gtry[[k, s]] × Γ[[s, i, j]], {s, 1, DIM8}] -
    Sum[ gtry[[j, s]] × Γ[[s, i, k]], {s, 1, DIM8}],
  {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

```
Out[]=
{0}
In[]:= RS
Out[]=
6 H2 (-7 + a4'[H x4]2)
```

```
In[1]:= EinsteinG
skelx[%]

Out[1]=
{ {-3 H^2 Cot[6 H x0]^2 (-5 + a4'[H x4]^2), 0, 0, 0, 0, 0, 0, 0, 0}, {0, -e^2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 0, -e^2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -e^2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -3 H^2 (7 + a4'[H x4]^2), 0, 0, 0, 0}, {0, 0, 0, 0, 0, e^-2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4]), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, e^-2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4]), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, e^-2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4])} }
```

```
Out[1]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{pmatrix}$$

```

```
Union[Table[
Sum[\tau[A].bas64[[k, 1]].\sigma.\tau[A]\times\eta8[[A, A]], {A, 1, 8}] - ID8 Tr[bas64[[k, 1]].\sigma] +
bas64[[k, 1]] - Transpose[bas64[[k, 1]]]
4 eg[[k]] \frac{\sigma, {k, 1, 64}]}{2}]]
```

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}
```

BASIS of 16×16 matrices :

```
In[2]:= 16 * 16
```

```
Out[2]=
256
```

```
In[3]:= 2^8
```

```
Out[3]=
256
```

```
In[4]:= Sum[Binomial[8, p], {p, 0, 8}]
```

```
Out[4]=
256
```

```
In[]:= Id16 == ID16 == Dot @@ (T16^A[#] & /@ Range[0, 8])
```

```
Out[]:=
```

```
True
```

```
In[]:= Clear[t160]; t160 = {{Id16, {Range[0, 8]}}}; Length[t160]
Clear[t16A];
t16A = {};
Do[AppendTo[t16A, {T16^A[j], {j}}], {j, 0, 7}];
Length[t16A]
Clear[t16AB];
t16AB = {};
Do[AppendTo[t16AB, {T16^A[j].T16^A[k], {j, k}}], {j, 0, 6}, {k, j + 1, 7}];
Length[t16AB]
Clear[t16ABC];
t16ABC = {};
Do[AppendTo[t16ABC, {T16^A[j].T16^A[k].T16^A[h], {j, k, h}}], {j, 0, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[t16ABC]
Clear[t16ABCD];
t16ABCD = {};
Do[AppendTo[t16ABCD, {T16^A[j].T16^A[k].T16^A[h].T16^A[i], {j, k, h, i}}], {j, 0, 4}, {k, j + 1, 5}, {h, k + 1, 6}, {i, h + 1, 7}];
Length[t16ABCD]
Clear[t16ABCDE];
t16ABCDE = {};
Do[AppendTo[t16ABCDE, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l], {j, k, h, i, l}}], {j, 0, 3}, {k, j + 1, 4}, {h, k + 1, 5}, {i, h + 1, 6}, {l, i + 1, 7}];
Length[t16ABCDE]
Clear[t16ABCDEF];
t16ABCDEF = {};
Do[AppendTo[t16ABCDEF,
{T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q], {j, k, h, i, l, q}}], {j, 0, 2}, {k, j + 1, 3}, {h, k + 1, 4}, {i, h + 1, 5}, {l, i + 1, 6}, {q, l + 1, 7}];
Length[t16ABCDEF]
Clear[t16ABCDEFG];
t16ABCDEFG = {};
Do[AppendTo[t16ABCDEFG, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].T16^A[r], {j, k, h, i, l, q, r}}], {j, 0, 1}, {k, j + 1, 2}, {h, k + 1, 3}, {i, h + 1, 4}, {l, i + 1, 5}, {q, l + 1, 6}, {r, q + 1, 7}];
```


$$\left(\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \}$$

In[6]:= (*BASE16[-1]*)

In[7]:= (*BASE16[-2]

%[1]==BASE16[-1][1]*)

In[8]:= 16 * (16 + 1) / 2

16 * (16 - 1) / 2

% + %%

Out[8]=

136

Out[9]=

120

Out[10]=

256

```
In[1]:= Clear[antisymm16];
antisymm16 = {};
Do[If[base16[[k, 1]] === -Transpose[base16[[k, 1]]], AppendTo[antisymm16,
    {base16[[k, 1]], {{k}, base16[[k, 2]]}}]], {k, 1, Length[base16]}];
Length[antisymm16]
antisymm16[[%]]
```

Out[1]= 120

```
Out[2]= {{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}, {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{254}, {1, 2, 3, 4, 5, 6, 7}}}
```

```

In[1]:= Clear[symmm16];
symmm16 = {};
Do[If[base16[[k, 1]] === Transpose[base16[[k, 1]]], AppendTo[symmm16,
    {base16[[k, 1]], {{k}, base16[[k, 2]]}}], {k, 1, Length[base16]}];
Length[symmm16]
symmm16[[%]]

Out[1]=
136

Out[2]=
{{{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}}

In[3]:= dupssymmm16 = Select[symmm16, MemberQ[symmm16[[All, 1]], -#[[1]]] &];
Length[dupssymmm16]

Out[3]=
0

```

```
In[]:= ##[2] & /@ symm16
Out[]=
{{{{1}, {0}}, {{2}, {1}}, {{3}, {2}}, {{4}, {3}}, {{12}, {0, 4}}, {{13}, {0, 5}}, {{14}, {0, 6}}, {{15}, {0, 7}}, {{18}, {1, 4}}, {{19}, {1, 5}}, {{20}, {1, 6}}, {{21}, {1, 7}}, {{23}, {2, 4}}, {{24}, {2, 5}}, {{25}, {2, 6}}, {{26}, {2, 7}}, {{27}, {3, 4}}, {{28}, {3, 5}}, {{29}, {3, 6}}, {{30}, {3, 7}}, {{39}, {0, 1, 4}}, {{40}, {0, 1, 5}}, {{41}, {0, 1, 6}}, {{42}, {0, 1, 7}}, {{44}, {0, 2, 4}}, {{45}, {0, 2, 5}}, {{46}, {0, 2, 6}}, {{47}, {0, 2, 7}}, {{48}, {0, 3, 4}}, {{49}, {0, 3, 5}}, {{50}, {0, 3, 6}}, {{51}, {0, 3, 7}}, {{59}, {1, 2, 4}}, {{60}, {1, 2, 5}}, {{61}, {1, 2, 6}}, {{62}, {1, 2, 7}}, {{63}, {1, 3, 4}}, {{64}, {1, 3, 5}}, {{65}, {1, 3, 6}}, {{66}, {1, 3, 7}}, {{73}, {2, 3, 4}}, {{74}, {2, 3, 5}}, {{75}, {2, 3, 6}}, {{76}, {2, 3, 7}}, {{89}, {4, 5, 6}}, {{90}, {4, 5, 7}}, {{91}, {4, 6, 7}}, {{92}, {5, 6, 7}}, {{93}, {0, 1, 2, 3}}, {{102}, {0, 1, 4, 5}}, {{103}, {0, 1, 4, 6}}, {{104}, {0, 1, 4, 7}}, {{105}, {0, 1, 5, 6}}, {{106}, {0, 1, 5, 7}}, {{107}, {0, 1, 6, 7}}, {{112}, {0, 2, 4, 5}}, {{113}, {0, 2, 4, 6}}, {{114}, {0, 2, 4, 7}}, {{115}, {0, 2, 5, 6}}, {{116}, {0, 2, 5, 7}}, {{117}, {0, 2, 6, 7}}, {{118}, {0, 3, 4, 5}}, {{119}, {0, 3, 4, 6}}, {{120}, {0, 3, 4, 7}}, {{121}, {0, 3, 5, 6}}, {{122}, {0, 3, 5, 7}}, {{123}, {0, 3, 6, 7}}, {{132}, {1, 2, 4, 5}}, {{133}, {1, 2, 4, 6}}, {{134}, {1, 2, 4, 7}}, {{135}, {1, 2, 5, 6}}, {{136}, {1, 2, 5, 7}}, {{137}, {1, 2, 6, 7}}, {{138}, {1, 3, 4, 5}}, {{139}, {1, 3, 4, 6}}, {{140}, {1, 3, 4, 7}}, {{141}, {1, 3, 5, 6}}, {{142}, {1, 3, 5, 7}}, {{143}, {1, 3, 6, 7}}, {{148}, {2, 3, 4, 5}}, {{149}, {2, 3, 4, 6}}, {{150}, {2, 3, 4, 7}}, {{151}, {2, 3, 5, 6}}, {{152}, {2, 3, 5, 7}}, {{153}, {2, 3, 6, 7}}, {{162}, {4, 5, 6, 7}}, {{167}, {0, 1, 2, 4, 5}}, {{168}, {0, 1, 2, 4, 6}}, {{169}, {0, 1, 2, 4, 7}}, {{170}, {0, 1, 2, 5, 6}}, {{171}, {0, 1, 2, 5, 7}}, {{172}, {0, 1, 2, 6, 7}}, {{173}, {0, 1, 3, 4, 5}}, {{174}, {0, 1, 3, 4, 6}}, {{175}, {0, 1, 3, 4, 7}}, {{176}, {0, 1, 3, 5, 6}}, {{177}, {0, 1, 3, 5, 7}}, {{178}, {0, 1, 3, 6, 7}}, {{183}, {0, 2, 3, 4, 5}}, {{184}, {0, 2, 3, 4, 6}}, {{185}, {0, 2, 3, 4, 7}}, {{186}, {0, 2, 3, 5, 6}}, {{187}, {0, 2, 3, 5, 7}}, {{188}, {0, 2, 3, 6, 7}}, {{197}, {0, 4, 5, 6, 7}}, {{198}, {1, 2, 3, 4, 5}}, {{199}, {1, 2, 3, 4, 6}}, {{200}, {1, 2, 3, 4, 7}}, {{201}, {1, 2, 3, 5, 6}}, {{202}, {1, 2, 3, 5, 7}}, {{203}, {1, 2, 3, 6, 7}}, {{212}, {1, 4, 5, 6, 7}}, {{217}, {2, 4, 5, 6, 7}}, {{218}, {3, 4, 5, 6, 7}}, {{225}, {0, 1, 2, 4, 5, 6}}, {{226}, {0, 1, 2, 4, 5, 7}}, {{227}, {0, 1, 2, 4, 6, 7}}, {{228}, {0, 1, 2, 5, 6, 7}}, {{229}, {0, 1, 3, 4, 5, 6}}, {{230}, {0, 1, 3, 4, 5, 7}}, {{231}, {0, 1, 3, 4, 6, 7}}, {{232}, {0, 1, 3, 5, 6, 7}}, {{234}, {0, 2, 3, 4, 5, 6}}, {{235}, {0, 2, 3, 4, 5, 7}}, {{236}, {0, 2, 3, 4, 6, 7}}, {{237}, {0, 2, 3, 5, 6, 7}}, {{240}, {1, 2, 3, 4, 5, 6}}, {{241}, {1, 2, 3, 4, 5, 7}}, {{242}, {1, 2, 3, 4, 6, 7}}, {{243}, {1, 2, 3, 5, 6, 7}}, {{247}, {0, 1, 2, 3, 4, 5, 6}}, {{248}, {0, 1, 2, 3, 4, 5, 7}}, {{249}, {0, 1, 2, 3, 4, 6, 7}}, {{250}, {0, 1, 2, 3, 5, 6, 7}}, {{255}, {0, 1, 2, 3, 4, 5, 6, 7}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7}}}}
```



```
In[8]:= T16^A[8] == base16[[255][1]]
Out[8]= True

In[9]:= Select[symmm16, #[[2]][1]] == 255 & -> "Index"]
Out[9]= {135}
```

similarly:

```
In[10]:= coupled1 = {4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13} // Union
Out[10]= {0, 1, 4, 5, 8, 9, 12, 13}

In[11]:= coupled2 = Complement[Range[0, 15], coupled1]
Out[11]= {2, 3, 6, 7, 10, 11, 14, 15}

In[12]:= coupled = {coupled1, coupled2}
Out[12]= {{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}]

In[13]:= Block[{m, vars, eqs, eqs1, eqs2, s, #16},
  #16 = f[#] & /@ Range[16];
  m = Array[qb[#1, #2] &, {16, 16}];
  vars = Flatten[m];
  eqs1 = And @@ Thread[m.#16 == Flatten[{#16[[# + 1]] & /@ coupled1, #16[[# + 1]] & /@ coupled2}]];
  eqs2 = And @@ Thread[m.Transpose[m] == Flatten[ID16]];
  eqs = eqs1 && eqs2;
  s = Solve[eqs, vars] // FullSimplify;
  s
]
Out[13]= {}
```

```
In[]:= linTrans = Block[{m, b, vars, eqs, eqs1, eqs2, s, #16, t},
  #16 = f[#] & /@ Range[16];
  m = Array[qb[#1, #2] &, {16, 16}];
  b = Flatten[{#16[[# + 1]] & /@ coupled1, #16[[# + 1]] & /@ coupled2}];
  t = Table[Block[{sv, eq, d}, d = D[m[[j]].b, #16[[j]]];
    sv = Flatten[{d -> 1, Thread[Complement[m[[j]], {d}] -> 0]}];
    (*Print[{m[[j]].b}, {#16[[j]]}, {sv}];*)
    (*eq=Thread[m[[j]].b==#16[[j]]];
    sv=Solve[eq,Flatten[m[[j]]]]//FullSimplify;
    sv*) (*{v.b,#16[[j]]}*) sv], {j, 1, 16}];
  (*
  vars=Flatten[m];
  eqs1=
  And@@Thread[m.#16==Flatten[{#16[[#+1]]&/@coupled1,#16[[#+1]]&/@coupled2}]];
  eqs2=And@@Thread[m.Transpose[m]==Flatten[ID16]];
  eqs=eqs1&&eqs2;
  s=Solve[eqs,vars]//FullSimplify;*)
  m /. Flatten[t]
]

Out[]=
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}}
```

```
In[•]:= linTrans // MatrixForm
```

Out[•]//MatrixForm=

```
In[•]:= linTrans.Transpose[linTrans] // FullSimplify // MatrixForm
```

Out[•]//MatrixForm=

```

In[]:= linTrans.¶16 // Column
Union[%[[1]]]

Out[]=
f16[0] [x0, x4]
f16[1] [x0, x4]
f16[8] [x0, x4]
f16[9] [x0, x4]
f16[2] [x0, x4]
f16[3] [x0, x4]
f16[10] [x0, x4]
f16[11] [x0, x4]
f16[4] [x0, x4]
f16[5] [x0, x4]
f16[12] [x0, x4]
f16[13] [x0, x4]
f16[6] [x0, x4]
f16[7] [x0, x4]
f16[14] [x0, x4]
f16[15] [x0, x4]

Out[=]
{f16[0] [x0, x4], f16[1] [x0, x4], f16[2] [x0, x4], f16[3] [x0, x4],
 f16[4] [x0, x4], f16[5] [x0, x4], f16[6] [x0, x4], f16[7] [x0, x4],
 f16[8] [x0, x4], f16[9] [x0, x4], f16[10] [x0, x4], f16[11] [x0, x4],
 f16[12] [x0, x4], f16[13] [x0, x4], f16[14] [x0, x4], f16[15] [x0, x4]}

In[]:= positiveTrMM = Select[base16, Tr[#[[1]] . #[[1]]] > 0 & -> "Index"]

Out[=
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59,
60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116,
117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151,
152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198,
199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242,
243, 247, 248, 249, 250, 255, 256}

In[]:= negativeTrMM = Select[base16, Tr[#[[1]] . #[[1]]] < 0 & -> "Index"]

Out[=
{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77,
78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128,
129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182,
189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221,
222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[]:= {Length[positiveTrMM], Length[negativeTrMM]}

Out[=
{136, 120}

```

```
In[]:= positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]
Out[]= {1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40,
41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74,
75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115,
116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139,
140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171,
172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198,
199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231,
232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[]:= negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]
Out[]= {5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54,
55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86,
87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126,
127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160,
161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194,
195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219,
220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[]:= {Length[positiveTrMM], Length[negativeTrMM]}
Out[=] {136, 120}
```

```

In[]:= base16[[#]] & /@ positiveTrMM;
{#[[2]], Tr[Transpose[#[[1]].linTrans]]} & /@%
Select[% , #[[2]] ≠ 0 &]
linTransSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];
Out[]=
{{{{0}, 0}, {{1}, 0}, {{2}, 0}, {{3}, 0}, {{0, 4}, 0}, {{0, 5}, 0}, {{0, 6}, 0},
{{0, 7}, -4}, {{1, 4}, 0}, {{1, 5}, 0}, {{1, 6}, 0}, {{1, 7}, 0}, {{2, 4}, 0},
{{2, 5}, 0}, {{2, 6}, 0}, {{2, 7}, 0}, {{3, 4}, -4}, {{3, 5}, 0}, {{3, 6}, 0},
{{3, 7}, 0}, {{0, 1, 4}, 0}, {{0, 1, 5}, 0}, {{0, 1, 6}, 0}, {{0, 1, 7}, 0},
{{0, 2, 4}, 0}, {{0, 2, 5}, 0}, {{0, 2, 6}, 0}, {{0, 2, 7}, 0}, {{0, 3, 4}, 0},
{{0, 3, 5}, 0}, {{0, 3, 6}, 0}, {{0, 3, 7}, 0}, {{1, 2, 4}, 0}, {{1, 2, 5}, 0},
{{1, 2, 6}, 0}, {{1, 2, 7}, 0}, {{1, 3, 4}, 0}, {{1, 3, 5}, 0}, {{1, 3, 6}, 0},
{{1, 3, 7}, 0}, {{2, 3, 4}, 0}, {{2, 3, 5}, 0}, {{2, 3, 6}, 0}, {{2, 3, 7}, 0},
{{4, 5, 6}, -4}, {{4, 5, 7}, 0}, {{4, 6, 7}, 0}, {{5, 6, 7}, 0}, {{0, 1, 2, 3}, 0},
{{0, 1, 4, 5}, 0}, {{0, 1, 4, 6}, 0}, {{0, 1, 4, 7}, 0}, {{0, 1, 5, 6}, 0},
{{0, 1, 5, 7}, 0}, {{0, 1, 6, 7}, 0}, {{0, 2, 4, 5}, 0}, {{0, 2, 4, 6}, 0},
{{0, 2, 4, 7}, 0}, {{0, 2, 5, 6}, 0}, {{0, 2, 5, 7}, 0}, {{0, 2, 6, 7}, 0},
{{0, 3, 4, 5}, 0}, {{0, 3, 4, 6}, 0}, {{0, 3, 4, 7}, 4}, {{0, 3, 5, 6}, 0},
{{0, 3, 5, 7}, 0}, {{0, 3, 6, 7}, 0}, {{1, 2, 4, 5}, 0}, {{1, 2, 4, 6}, 0},
{{1, 2, 4, 7}, 0}, {{1, 2, 5, 6}, 0}, {{1, 2, 5, 7}, 0}, {{1, 2, 6, 7}, -4},
{{1, 3, 4, 5}, 0}, {{1, 3, 4, 6}, 0}, {{1, 3, 4, 7}, 0}, {{1, 3, 5, 6}, 0},
{{1, 3, 5, 7}, 0}, {{1, 3, 6, 7}, 0}, {{2, 3, 4, 5}, 0}, {{2, 3, 4, 6}, 0},
{{2, 3, 4, 7}, 0}, {{2, 3, 5, 6}, 0}, {{2, 3, 5, 7}, 0}, {{2, 3, 6, 7}, 0},
{{4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5}, 4}, {{0, 1, 2, 4, 6}, 0}, {{0, 1, 2, 4, 7}, 0},
{{0, 1, 2, 5, 6}, 0}, {{0, 1, 2, 5, 7}, 0}, {{0, 1, 2, 6, 7}, 0}, {{0, 1, 3, 4, 5}, 0},
{{0, 1, 3, 4, 6}, 0}, {{0, 1, 3, 4, 7}, 0}, {{0, 1, 3, 5, 6}, 0}, {{0, 1, 3, 5, 7}, 0},
{{0, 1, 3, 6, 7}, 0}, {{0, 2, 3, 4, 5}, 0}, {{0, 2, 3, 4, 6}, 0}, {{0, 2, 3, 4, 7}, 0},
{{0, 2, 3, 5, 6}, 0}, {{0, 2, 3, 5, 7}, 0}, {{0, 2, 3, 6, 7}, 0}, {{0, 4, 5, 6, 7}, 4},
{{1, 2, 3, 4, 5}, 0}, {{1, 2, 3, 4, 6}, 0}, {{1, 2, 3, 4, 7}, 0}, {{1, 2, 3, 5, 6}, 0},
{{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 6, 7}, 0}, {{1, 4, 5, 6, 7}, 0}, {{2, 4, 5, 6, 7}, 0},
{{3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6}, 0}, {{0, 1, 2, 4, 5, 7}, 0},
{{0, 1, 2, 4, 6, 7}, 0}, {{0, 1, 2, 5, 6, 7}, 0}, {{0, 1, 3, 4, 5, 6}, 0},
{{0, 1, 3, 4, 5, 7}, 0}, {{0, 2, 3, 4, 5, 6}, 0}, {{0, 2, 3, 4, 5, 7}, 0},
{{0, 2, 3, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6}, 0}, {{1, 2, 3, 4, 5, 7}, 0},
{{1, 2, 3, 4, 6, 7}, -4}, {{1, 2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6}, 0},
{{0, 1, 2, 3, 4, 5, 7}, 0}, {{0, 1, 2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6, 7}, 0}

```



```

In[]:= base16[[#]] & /@ negativeTrMM;
{#[[2]], Tr[Transpose[#[[1]].linTrans]]} & /@%
Select[% , #[[2]] ≠ 0 &]
linTransAntiSymmetric =
Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];

Out[]=
{{{4}, 0}, {{5}, 0}, {{6}, 0}, {{7}, 0}, {{0, 1}, 0}, {{0, 2}, 0}, {{0, 3}, 0},
{{1, 2}, 0}, {{1, 3}, 0}, {{2, 3}, 0}, {{4, 5}, 0}, {{4, 6}, 0}, {{4, 7}, 0},
{{5, 6}, 0}, {{5, 7}, 0}, {{6, 7}, 0}, {{0, 1, 2}, 0}, {{0, 1, 3}, 0},
{{0, 2, 3}, 0}, {{0, 4, 5}, 0}, {{0, 4, 6}, 0}, {{0, 4, 7}, 0}, {{0, 5, 6}, 0},
{{0, 5, 7}, 0}, {{0, 6, 7}, 0}, {{1, 2, 3}, 0}, {{1, 4, 5}, 0}, {{1, 4, 6}, 0},
{{1, 4, 7}, 0}, {{1, 5, 6}, 0}, {{1, 5, 7}, 0}, {{1, 6, 7}, 0}, {{2, 4, 5}, 0},
{{2, 4, 6}, 0}, {{2, 4, 7}, 0}, {{2, 5, 6}, 0}, {{2, 5, 7}, 0}, {{2, 6, 7}, 0},
{{3, 4, 5}, 0}, {{3, 4, 6}, 0}, {{3, 4, 7}, 0}, {{3, 5, 6}, -4}, {{3, 5, 7}, 0},
{{3, 6, 7}, 0}, {{0, 1, 2, 4}, 0}, {{0, 1, 2, 5}, 0}, {{0, 1, 2, 6}, -4},
{{0, 1, 2, 7}, 0}, {{0, 1, 3, 4}, 0}, {{0, 1, 3, 5}, 0}, {{0, 1, 3, 6}, 0},
{{0, 1, 3, 7}, 0}, {{0, 2, 3, 4}, 0}, {{0, 2, 3, 5}, 0}, {{0, 2, 3, 6}, 0},
{{0, 2, 3, 7}, 0}, {{0, 4, 5, 6}, 0}, {{0, 4, 5, 7}, 0}, {{0, 4, 6, 7}, 0},
{{0, 5, 6, 7}, 0}, {{1, 2, 3, 4}, 0}, {{1, 2, 3, 5}, 0}, {{1, 2, 3, 6}, 0},
{{1, 2, 3, 7}, 0}, {{1, 4, 5, 6}, 0}, {{1, 4, 5, 7}, 0}, {{1, 4, 6, 7}, 0},
{{1, 5, 6, 7}, 0}, {{2, 4, 5, 6}, 0}, {{2, 4, 5, 7}, 0}, {{2, 4, 6, 7}, 0},
{{2, 5, 6, 7}, 0}, {{3, 4, 5, 6}, 0}, {{3, 4, 5, 7}, 0}, {{3, 4, 6, 7}, 0},
{{3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4}, 0}, {{0, 1, 2, 3, 5}, -4}, {{0, 1, 2, 3, 6}, 0},
{{0, 1, 2, 3, 7}, 0}, {{0, 1, 4, 5, 6}, 0}, {{0, 1, 4, 5, 7}, 0},
{{0, 1, 4, 6, 7}, 0}, {{0, 1, 5, 6, 7}, 0}, {{0, 2, 4, 5, 6}, 0}, {{0, 2, 4, 5, 7}, 0},
{{0, 2, 4, 6, 7}, 0}, {{0, 2, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6}, 0}, {{0, 3, 4, 5, 7}, 0},
{{0, 3, 4, 6, 7}, 0}, {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 6}, 0}, {{1, 2, 4, 5, 7}, 4},
{{1, 2, 4, 6, 7}, 0}, {{1, 2, 5, 6, 7}, 0}, {{1, 3, 4, 5, 6}, 0}, {{1, 3, 4, 5, 7}, 0},
{{1, 3, 4, 6, 7}, 0}, {{1, 3, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6}, 0}, {{2, 3, 4, 5, 7}, 0},
{{2, 3, 4, 6, 7}, 0}, {{2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5}, 0},
{{0, 1, 2, 3, 4, 6}, -4}, {{0, 1, 2, 3, 4, 7}, 0}, {{0, 1, 2, 3, 5, 6}, 0},
{{0, 1, 2, 3, 5, 7}, 0}, {{0, 1, 2, 3, 6, 7}, 0}, {{0, 1, 4, 5, 6, 7}, 0},
{{0, 2, 4, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6, 7}, 0}, {{1, 2, 4, 5, 6, 7}, 0},
{{1, 3, 4, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6, 7}, 0},
{{0, 1, 2, 3, 4, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6, 7}, 0}}

```

BASIS of 8×8 matrices :

```
In[1]:= tA = {} ; Do[AppendTo[tA, {τ[j], {j}}], {j, 1, 7}]
Length[tA]
tAB = {};
Do[AppendTo[tAB, {τ[j].τ[k], {j, k}}], {j, 1, 6}, {k, j + 1, 7}]
Length[tAB]
tABC = {};
Do[AppendTo[tABC, {τ[j].τ[k].τ[h], {j, k, h}}], {j, 1, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[tABC]

Out[1]=
7

Out[2]=
21

Out[3]=
35

In[4]:= tA[[1]]
Out[4]=
{{{{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
{0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}}, {1}}}

In[5]:= tA[[#]] [[1]] == -Transpose[tA[[#]] [[1]]] & /@ Range[Length[tA]]
Out[5]=
{True, True, True, False, False, False}

In[6]:= bas64 = Flatten[{tA, tAB, tABC, {{Id, {64}}}}, 1]; Length[bas64]

Out[6]=
64

In[7]:= determineIF8AntiSymmetric[m_] := Module[{t, tt, sum, res, r},
t = m.τ[#] & /@ Range[0, 7];
tt = Transpose[#] & /@ t;
sum = FullSimplify[t[[#]] + tt[[#]]] & /@ Range[Length[t]];
res = Union[Flatten[#]] & /@ sum;
If[Length[#] > 1, Return[False]] & /@ res;
r = Union[Flatten[res]];
Return[r == {0}];
]
```

```
In[6]:= determineIF8AntiSymmetric[Zero8]
```

Out[•]=

```
determineTF8AntiSymmetric[TD8]
```

8 [1]

False

```
In[6]:= find8AntiSymmetric =  
If[determineIF8AntiSymmetric[#[[1]], #[[2]], ## &] & /@ bas64
```

Out[•]=

```
In[8]:= anti = {};
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],
    AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[anti]
anti[[28]]
```

Out[1]=

28

Out[•] =

```

{{{0, 1, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}}, {5, 6, 7}}}}

```

$|D[\bullet]| :=$

```

Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
    AppendTo[symmm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}] ], {k, 1, 64}];
Length[symmm]
symmm[[%]]

```

Out[•]=

35

Out[•]=

```

{{{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1},
{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}}, {{59}, {3, 6, 7}}}}

```

```
In[6]:= η64 = DiagonalMatrix[
  Table[FullSimplify[ $\frac{1}{8} \text{Tr}[\text{bas64}[A, 1].\text{bas64}[A, 1]]$ ], {A, 64}]];
Tr[η64]
```

$$Out[\cdot] = \frac{1}{7 + \frac{8}{\text{Tr}[\text{Id}.\text{Id}]}}$$

```
In[=]:= (*(*(*(*(*(*(*countTrace=
Table[{A,B,FullSimplify[\frac{1}{8}Tr[bas64[A,1].bas64[B,1]]]},{{A,1, 64},{B,A, 64}})]//MatrixForm*)*)*)*)*)*)
```

In[•]:= (*■*)

In[•]:= 32 × 65

Out[•] =

2080

```
In[•]:= (*Length[Flatten[countTrace,1]]*)
```

```
In[6]:= (* {Length[countTrace], Length[countTrace[[1]]], Length[countTrace[[1, 1]]]} *)
```

```
In[1]:= (* {Length[countTrace], Length[countTrace[[2]]], Length[countTrace[[2,1]]]} *)
```

```
In[=]:= (* {Length[countTrace], Length[countTrace[[64]], Length[countTrace[[64,1]]]} *)
```

```

Count[Flatten[countTrace,1][All,3],0],
Count[Flatten[countTrace,1][All,3],1],
Count[Flatten[countTrace,1][All,3],-1}])

```

In[•]:= 32 × 63

Out[•] =

2016

```
In[1]:= anti = {};
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],
 AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[anti]
anti[[28]]

Out[1]=
28

Out[2]=
{{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}}
```

```
In[3]:= anti[[28, 1]]

Out[3]=
{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}}

In[4]:= anti[[28, 2]]

Out[4]=
{{63}, {5, 6, 7}}
```

```
In[5]:= symm = {};
Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
 AppendTo[symm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[symm]
symm[[28]]
```

```
Out[5]=
35

Out[6]=
{{{ -1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, {{52}, {2, 5, 7}}}}
```

For the split orthogonal group $\text{Spin}(4,4)$ and its associated Lie algebra, there exist three fundamentally equivalent 8-dimensional representations:

1. The vector representation V
2. The type-1 spinor representation $S+$ (or $S1$), positive chirality
3. The type-2 spinor representation $S-$ (or $S2$), negative chirality

These representations are related by outer automorphisms of $\text{Spin}(4,4)$. All three representations have the same dimension: $\dim(V) = \dim(S1) = \dim(S2) = 8$

This is unique to dimension 8 and forms the basis of triality: Cartan, E. (1925). La th'eorie des spineurs. Hermann, Paris.

metric-preserving transformations; find automorphism $\sigma g \leftrightarrow \sigma g_A^a$

$$\sigma \cdot \sigma g = \text{Transpose}[\sigma g^{-1}] \cdot g$$

$$\sigma = \text{Transpose}[\sigma g^{-1}] \cdot g \cdot \sigma g^{-1}$$

$$(\sigma g)_A^a \equiv \sigma g_A^a$$

$$(\sigma g^{-1})_a^A \equiv \sigma g_A^a ; (\text{Transpose}[\sigma g^{-1}])_a^A \equiv \sigma g_a^A$$

$$\sigma_{ab} = \sigma g_a^A g_{AB} \sigma g_b^B$$

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$N = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\text{In}[1]:= \text{Clear}[\sigma g_1^a]; \sigma g_1^a = \frac{1}{\sqrt{2}} (\eta 4488 + \sigma)$$

Out[1]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \right. \\ \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

```

In[1]:= σg2Aa = 1/√2 {{1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 1, 0},
{0, 1, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
{1, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 1, 0, 0, 0, -1, 0},
{0, 1, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 1, 0, 0, 0, -1}};

Out[1]= {{1/√2, 0, 0, 0, 1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, 0, 1/√2, 0},
{0, 1/√2, 0, 0, 0, 1/√2, 0, 0}, {0, 0, 0, 1/√2, 0, 0, 0, 1},
{1/√2, 0, 0, 0, -1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, 0, -1/√2, 0},
{0, 1/√2, 0, 0, 0, -1/√2, 0, 0}, {0, 0, 0, 1/√2, 0, 0, 0, -1}};

In[2]:= (s1s2 = σg1Aa.Transpose[σg2Aa]) // MatrixForm

Out[2]//MatrixForm=
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}};

In[3]:= (* (s1s2=Inverse[σg1Aa.Transpose[σg2Aa]])//MatrixForm*)

In[4]:= s1s2.s1s2 === ID8

Out[4]= True

```

```
In[]:= (Thread[¶16[[1 ;; 8]] → s1s2.¶16[[9 ;; 16]]]) // Column
%[[1]] [[All, 1, 0]]
%%[[1]] [[All, 2]] /. {x0 → #1, x4 → #2}
ToExpression["((" <> ToString[#] <> ")") &] & /@ %
forceDimensionReductionA = Thread[%%% → %]

Out[]=
f16[0][x0, x4] → f16[8][x0, x4]
f16[1][x0, x4] → f16[10][x0, x4]
f16[2][x0, x4] → f16[9][x0, x4]
f16[3][x0, x4] → f16[11][x0, x4]
f16[4][x0, x4] → f16[12][x0, x4]
f16[5][x0, x4] → f16[14][x0, x4]
f16[6][x0, x4] → f16[13][x0, x4]
f16[7][x0, x4] → f16[15][x0, x4]

Out[]=
{f16[0], f16[1], f16[2], f16[3], f16[4], f16[5], f16[6], f16[7]}

Out[=]
{f16[8][#1, #2], f16[10][#1, #2], f16[9][#1, #2], f16[11][#1, #2],
 f16[12][#1, #2], f16[14][#1, #2], f16[13][#1, #2], f16[15][#1, #2]}

Out[=]
{f16[8][#1, #2] &, f16[10][#1, #2] &, f16[9][#1, #2] &, f16[11][#1, #2] &,
 f16[12][#1, #2] &, f16[14][#1, #2] &, f16[13][#1, #2] &, f16[15][#1, #2] &}

Out[=]
{f16[0] → (f16[8][#1, #2] &), f16[1] → (f16[10][#1, #2] &),
 f16[2] → (f16[9][#1, #2] &), f16[3] → (f16[11][#1, #2] &),
 f16[4] → (f16[12][#1, #2] &), f16[5] → (f16[14][#1, #2] &),
 f16[6] → (f16[13][#1, #2] &), f16[7] → (f16[15][#1, #2] &)}
```

```
In[]:= (Thread[¶16[[9 ;; 16] → s1s2.¶16[[1 ;; 8]]]) // Column
%[[1]] [[All, 1, 0]]
%[[1]] [[All, 2]] /. {x0 → #1, x4 → #2}
ToExpression["((" <> ToString[#] <> ")") &] & /@ %
forceDimensionReduction = Thread[%%% → %]

Out[]=
f16[8][x0, x4] → f16[0][x0, x4]
f16[9][x0, x4] → f16[2][x0, x4]
f16[10][x0, x4] → f16[1][x0, x4]
f16[11][x0, x4] → f16[3][x0, x4]
f16[12][x0, x4] → f16[4][x0, x4]
f16[13][x0, x4] → f16[6][x0, x4]
f16[14][x0, x4] → f16[5][x0, x4]
f16[15][x0, x4] → f16[7][x0, x4]

Out[=]
{f16[8], f16[9], f16[10], f16[11], f16[12], f16[13], f16[14], f16[15]}

Out[=]
{f16[0][#1, #2], f16[2][#1, #2], f16[1][#1, #2], f16[3][#1, #2],
 f16[4][#1, #2], f16[6][#1, #2], f16[5][#1, #2], f16[7][#1, #2]}

Out[=]
{f16[0][#1, #2] &, f16[2][#1, #2] &, f16[1][#1, #2] &, f16[3][#1, #2] &,
 f16[4][#1, #2] &, f16[6][#1, #2] &, f16[5][#1, #2] &, f16[7][#1, #2] &}

Out[=]
{f16[8] → (f16[0][#1, #2] &), f16[9] → (f16[2][#1, #2] &),
 f16[10] → (f16[1][#1, #2] &), f16[11] → (f16[3][#1, #2] &),
 f16[12] → (f16[4][#1, #2] &), f16[13] → (f16[6][#1, #2] &),
 f16[14] → (f16[5][#1, #2] &), f16[15] → (f16[7][#1, #2] &)}
```

In[]:= (* $\sigma g_{2A}^a == -\# + \sigma g_{1A}^a \& /@ \text{base64}[[\text{All}, 1]]$ *)

In[]:= $\sigma g_{2A}^a \cdot \text{Transpose}[\sigma g_{2A}^a] == \text{ID8}$

Out[=]
True

In[]:= $\sigma g_{1A}^a \cdot \text{Transpose}[\sigma g_{1A}^a] == \text{ID8}$

Out[=]
True

metric - preserving transformation:

In[]:= $\sigma g_{1A}^a \cdot \sigma \cdot \text{Transpose}[\sigma g_{1A}^a] == \eta 4488$

Out[=]
True

metric - preserving transformation:

In[1]:= $\sigma g_{2A}^a \cdot \sigma \cdot \text{Transpose}[\sigma g_{2A}^a] == \eta 4488$

Out[1]=

True

In[2]:= $Mc = \frac{1}{\sqrt{2}} (\eta 4488 + \sigma)$

Out[2]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \\ & \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\} \end{aligned}$$

In[3]:= $Nc = \frac{1}{\sqrt{2}} \{ \{1, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 0\}, \{0, 1, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 1\}, \{1, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, -1, 0\}, \{0, 1, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, -1\} \}$

Out[3]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ & \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \\ & \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \\ & \left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\} \end{aligned}$$

In[4]:= (* (s1s2=Mc.Transpose[Nc]) // MatrixForm *)

In[5]:= Nc.Transpose[Nc] // MatrixForm

Out[5]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```

In[]:= Mc.Transpose[Mc] // MatrixForm
Out[//MatrixForm]=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


In[]:= Mc.σ.Transpose[Mc] === η4488
Out[=]
True

In[]:= Nc.σ.Transpose[Nc] === η4488
Out[=]
True

In[]:= (Thread[Π16[[9 ;; 16]] → s1s2.Π16[[1 ;; 8]]) // Column
Out[=]
f16[8][x0, x4] → f16[0][x0, x4]
f16[9][x0, x4] → f16[2][x0, x4]
f16[10][x0, x4] → f16[1][x0, x4]
f16[11][x0, x4] → f16[3][x0, x4]
f16[12][x0, x4] → f16[4][x0, x4]
f16[13][x0, x4] → f16[6][x0, x4]
f16[14][x0, x4] → f16[5][x0, x4]
f16[15][x0, x4] → f16[7][x0, x4]

In[]:= f8[#][x0, x4] & /@ Range[0, 7] // Column
(s1s2.%[[1]]) // Column
Out[=]
f8[0][x0, x4]
f8[1][x0, x4]
f8[2][x0, x4]
f8[3][x0, x4]
f8[4][x0, x4]
f8[5][x0, x4]
f8[6][x0, x4]
f8[7][x0, x4]

Out[=]
f8[0][x0, x4]
f8[2][x0, x4]
f8[1][x0, x4]
f8[3][x0, x4]
f8[4][x0, x4]
f8[6][x0, x4]
f8[5][x0, x4]
f8[7][x0, x4]

```

Killing Vector Fields

$$\begin{aligned}
& -DG\left(\left[["vector", M8, []], \left[\left[[6], \right.\right.\right. \right. \\
& \left. \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}}(x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \right. \\
& \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3} \right] \right], \left[[7], \right. \right. \\
& \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}(x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \\
& \left. \left. \left. + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3} \right] \right]]], _DG\left(\left[["vector", \right. \right. \\
& M8, []], \left[[3], \right. \right. \\
& \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}(x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \\
& \left. \left. + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3} \right] \right], \left[[4], \right. \right. \\
& \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}}(x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \\
& \left. \left. + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3} \right] \right]], _DG\left(\left[["vector", \right. \right. \\
& M8, []], \left[[3], \frac{1}{\sin(6Hx0)^{1/3}}((2 \cos(Hx0) - 1)^{1/3} \right. \\
& \left. \left. (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \right. \\
& \left. \left. /^3 (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) \right] \right]), _DG\left(\left[["vector", M8, []], \left[[4], \right. \right. \\
& \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \\
& \left. \left. (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) \right] \right]], _DG\left(\left[["vector", M8, []], \left[[6], \right. \right. \\
& \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}}(x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3} \right] \right], \left[[8], \right. \right. \\
& \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}(x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right. \right. \\
& \left. \left. + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3} \right] \right]], _DG\left(\left[["vector", \right. \right. \\
& M8, []], \left[[6], -\frac{1}{\sin(6Hx0)^{1/3}}((2 \cos(Hx0) \right. \right. \\
& \left. \left. - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \right. \\
& \left. \left. /^3 (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) \right] \right])
\end{aligned}$$

Out[•]=

```
  _DG( [ "vector", M8, [ ]], [ [ 6],
```

$$\begin{aligned}
& - \frac{1}{\sin(6Hx\theta)^{1/3}} (x6 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [[7], \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]], _DG\left(\left[["vector",\right. \right. \\
& M8, []], \left[[3], \right. \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x3 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [[4], \\
& - \frac{1}{\sin(6Hx\theta)^{1/3}} (x2 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]], _DG\left(\left[["vector",\right. \right. \\
& M8, []], \left[[3], \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^1 \\
& /3 (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} \\
& /3 (\cos(Hx\theta) - 1)^{1/6} (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})] \\
& \left. \right. \left. \right)], _DG\left(\left[["vector", M8, []], \left[[4], \right. \right. \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) \\
& + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /6 (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})]]], _DG\left(\left[["vector",\right. \right. \\
& M8, []], \left[[6], \right. \\
& - \frac{1}{\sin(6Hx\theta)^{1/3}} (x7 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [[8], \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]], _DG\left(\left[["vector",\right. \right. \\
& M8, []], \left[[6], - \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) \\
& - 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} \\
& /3 (\cos(Hx\theta) - 1)^{1/6} (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})] \\
& \left. \right. \left. \right)], _DG\left(\left[["vector", M8, []], \left[[7], \right. \right. \\
& - \frac{1}{\sin(6Hx\theta)^{1/3}} (x7 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^1 \\
& /6 (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} (\cos(Hx\theta) - 1)^{1/3} \cos(Hx\theta)^{1/3})
\end{aligned}$$

$$\begin{aligned}
& \cdot \cdot \cdot (\cos(Hx0) + 1) \cdot \cdot \cdot (2 \cos(Hx0) - 1) \cdot \cdot \cdot (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}), [[8], \\
& \frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})]]], _DG\left([["vector",\right. \\
& M8, []], \left[[7], -\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) \\
& - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} \\
& (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3})]\right] \\
&), _DG\left([["vector", M8, []], [[8],\right. \\
& -\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) \\
& + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3})]\right], \\
& _DG\left([["vector", M8, []], [[2],\right. \\
& \frac{1}{\sin(6Hx0)^{1/3}} (x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})], [[3], \\
& -\frac{1}{\sin(6Hx0)^{1/3}} (x1 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})]\right], _DG\left([["vector",\right. \\
& M8, []], \left[[2],\right. \\
& \frac{1}{\sin(6Hx0)^{1/3}} (x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})]\right], \\
& _DG\left([["vector", M8, []], [[4],\right. \\
& -\frac{1}{\sin(6Hx0)^{1/3}} (x1 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \\
& (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})]\right], _DG\left([["vector",\right. \\
& M8, []], \left[[2], \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} \\
& (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} \\
& (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3})]\right]
\end{aligned}$$

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ln[= (*ToExpression[MathMLForm[{"http://www.w3.org/TR/MathML","math"}]()])
ln[= (*ToExpression[MathMLForm[\square]])
ln[= "[_DG([[\"vector\", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)

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+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],  

[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)  

+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +  

1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],  

[[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +  

1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"],  

M8, []], [[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)  

+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"],  

M8, []], [[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],  

[[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)  

+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"],  

M8, []], [[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/3)*(2*cos(H*x0) - 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],  

[[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +  

1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"],  

M8, []], [[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +  

1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"],  

M8, []], [[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],

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[[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"], M8,
[]], [[[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],

[[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,
[]], [[[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])]

Out[=]
[_DG([[["vector"], M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],

[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[["vector"], M8,
[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0)
- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[["vector"], M8,
[]], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[["vector"], M8,
[]], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[["vector"], M8,
[]], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[["vector"], M8,
[]], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x5*cos(H*x0)^(1/3)*(cos(H*x0)
- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[["vector"], M8,
[]], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[["vector"], M8,
[]], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x6*cos(H*x0)^(1/3)*(cos(H*x0)
- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]])

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_mDG([["vector", M8, []], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) -
1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])

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In[6]:= killingVectors =
"<[_DG([["vector\"", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector\"", M8,
[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector\"", M8,
[]], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector\"", M8,
[]], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector\"", M8,
[]], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]],

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[[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,
[]], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], _DG([[\"vector\"], M8, []], [[[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,
[]], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], _DG([[\"vector\"], M8, []], [[[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,
[]], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], _DG([[\"vector\"], M8, []], [[[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,
[]], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]]"
```

Out[•]=

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[_DG([[\"vector\"], M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], _DG([[\"vector\"], M8, []], [[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,
[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
```

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+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], ,
_DG([[["vector", M8, []], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], , _DG([[["vector",
M8, []], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], , _DG([[["vector",
M8, []], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], ,
_DG([[["vector", M8, []], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], , _DG([[["vector",
M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], ,
_DG([[["vector", M8, []], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], , _DG([[["vector",
M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], , _DG([[["vector",
M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], ,
_DG([[["vector", M8, []], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], ,
_DG([[["vector", M8, []], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])

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$\text{DG}[[["vector", M8, []], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]]$

$$\begin{aligned}
& \left[-\frac{1}{\sin(6Hx\theta)^{1/3}} (x6 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \right. \\
& \quad \left. + 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \right. \\
& \quad \left. + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3}) \right], [[7], \\
& \quad \frac{1}{\sin(6Hx\theta)^{1/3}} (x5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \\
& \quad (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& \quad + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]], _DG\left(\left[\begin{array}{l} "vector", \right. \right. \\
& M8, [], \left[\begin{array}{l} [3], \right. \\
& \quad \frac{1}{\sin(6Hx\theta)^{1/3}} (x3 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \\
& \quad (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& \quad + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3}) \right], [[4], \\
& \quad \left. \frac{1}{\sin(6Hx\theta)^{1/3}} (x2 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \right. \\
& \quad (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& \quad + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]], _DG\left(\left[\begin{array}{l} "vector", \right. \right. \\
& M8, [], \left[\begin{array}{l} [3], \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^{1/3} \right. \\
& \quad (\cos(Hx\theta) - 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} \\
& \quad (2 \cos(Hx\theta) + 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3}) \right] \\
& \quad \left. \right], _DG\left(\left[\begin{array}{l} "vector", M8, [], \right. \right. \\
& \quad \left[\begin{array}{l} [4], \right. \\
& \quad \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) \\
& \quad - 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} (2 \cos(Hx\theta) - 1)^{1/3} \right. \\
& \quad \left. \right]
\end{aligned}$$

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- 1) - (- 1 + z cos(z PI x u)) - (z cos(Pi x u) + 1)

/3 (cos(Hx0) + 1)1/6 cos(Hx0)1/3 ]]]), _DG( [ [ "vector",
M8, [ ]], [ [ [ 6],
- 1 / sin(6 Hx0)1/3 (x7 cos(Hx0)1/3 (cos(Hx0) - 1)1
/6 (cos(Hx0) + 1)1/6 (2 cos(Hx0) - 1)1/3 (2 cos(Hx0)
+ 1)1/3 (- 1 + 2 cos(2 Hx0))1/3 ], [ 8 ],
1 / sin(6 Hx0)1/3 (x5 cos(Hx0)1/3 (cos(Hx0) - 1)1
/6 (cos(Hx0) + 1)1/6 (2 cos(Hx0) - 1)1/3 (2 cos(Hx0)
+ 1)1/3 (- 1 + 2 cos(2 Hx0))1/3 ]]]), _DG( [ [ "vector",
M8, [ ]], [ [ [ 6], - 1 / sin(6 Hx0)1/3 ((2 cos(Hx0)
- 1)1/3 (cos(Hx0) - 1)1/6 (- 1 + 2 cos(2 Hx0))1
/3 (2 cos(Hx0) + 1)1/3 (cos(Hx0) + 1)1/6 cos(Hx0)1/3 ]]]
),
_DG( [ [ "vector", M8, [ ]], [ [ [ 7],
- 1 / sin(6 Hx0)1/3 (x7 cos(Hx0)1/3 (cos(Hx0) - 1)1
/6 (cos(Hx0) + 1)1/6 (2 cos(Hx0) - 1)1/3 (2 cos(Hx0)
+ 1)1/3 (- 1 + 2 cos(2 Hx0))1/3 ], [ 8 ],
1 / sin(6 Hx0)1/3 (x6 cos(Hx0)1/3 (cos(Hx0) - 1)1
/6 (cos(Hx0) + 1)1/6 (2 cos(Hx0) - 1)1/3 (2 cos(Hx0)
+ 1)1/3 (- 1 + 2 cos(2 Hx0))1/3 ]]]), _DG( [ [ "vector",

```

$M8, []], \left[\left[[7], -\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \right. \\ \left. \left. /^3 (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \\), _DG \left(\left[["vector", M8, []], \left[\left[[8], \right. \right. \right. \\ \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \right. \right. \\ \left. \left. \left. /^3 (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \right), _DG \left(\left[["vector", \right. \\ M8, []], \left[\left[[2], \right. \right. \right. \\ \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}} (x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \\ \left. \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^1 \right) \right] \right], \left[[3], \right. \\ \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}} (x1 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \\ \left. \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^1 \right) \right] \right], _DG \left(\left[["vector", \right. \\ M8, []], \left[\left[[2], \right. \right. \right. \\ \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}} (x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \\ \left. \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^1 \right) \right] \right], \left[[4], \right. \\ \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}} (x1 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \\ \left. \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^1 \right) \right] \right]$

$$\begin{aligned}
 & /6 (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
 & + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]]), _DG\left(\left[["vector", \right. \right. \\
 & M8, []], \left[[2], \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^{1/3} \right. \\
 & (\cos(Hx\theta) - 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} \\
 & \left. \left. (2 \cos(Hx\theta) + 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3}) \right] \right] \\
 & \left. \left. \right] \right]
 \end{aligned}$$

Out[=]=

$$\begin{aligned}
 & \left[_DG\left(\left[["vector", M8, []], \left[[6], \right. \right. \right. \right. \\
 & \left. \left. \left. \left. - \frac{1}{\sin(6Hx\theta)^{1/3}} (x6 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \right. \right. \right. \\
 & (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
 & + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3}) \right], \left[[7], \right. \right. \\
 & \left. \left. \frac{1}{\sin(6Hx\theta)^{1/3}} (x5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \right. \right. \\
 & (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
 & + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3}) \right]]], _DG\left(\left[["vector", \right. \right. \\
 & M8, []], \left[[3], \right. \right. \\
 & \left. \left. \frac{1}{\sin(6Hx\theta)^{1/3}} (x3 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \right. \right. \\
 & (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
 & + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3}) \right], \left[[4], \right. \right. \\
 & \left. \left. - \frac{1}{\sin(6Hx\theta)^{1/3}} (x2 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \right. \right. \\
 & (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
 & + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3}) \right]], _DG\left(\left[["vector", \right. \right. \\
 & \dots \dots \left. \left. \right] \right]
 \end{aligned}$$

$$\begin{aligned}
& M8, [], \left[\left[[3], \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \right. \\
& \left. \left. /^3 (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \\
& \left. \right]), _DG \left(\left[["vector", M8, []], \left[\left[[4], \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \right. \right. \right. \\
& \left. \left. \left. \left. /^3 (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \right]), _DG \left(\left[["vector", \right. \right. \\
& M8, [], \left[\left[[6], \right. \right. \right. \right. \\
& \left. \left. \left. \left. - \frac{1}{\sin(6Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \\
& \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^1 \right) \right] \right], \left[[8], \right. \right. \\
& \left. \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}} (x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \\
& \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^1 \right) \right] \right]), _DG \left(\left[["vector", \right. \right. \\
& M8, [], \left[\left[[6], - \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \right. \\
& \left. \left. /^3 (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \\
& \left. \right]), _DG \left(\left[["vector", M8, []], \left[\left[[7], \right. \right. \right. \right. \\
& \left. \left. \left. \left. - \frac{1}{\sin(6Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \right. \right. \right. \\
& \left. \left. \left. \left. /^3 (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \right]
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x6\cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \\
& + 1)^{1/3} (-1 + 2\cos(2Hx\theta))^{1/3})], [[8], \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x6\cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \\
& + 1)^{1/3} (-1 + 2\cos(2Hx\theta))^{1/3})]]], _DG\left(\left[["vector", \right. \right. \\
& M8, []], \left[[7], -\frac{1}{\sin(6Hx\theta)^{1/3}} ((2\cos(Hx\theta) \\
& - 1)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (-1 + 2\cos(2Hx\theta))^{1/3} \\
& /3 (2\cos(Hx\theta) + 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3})] \right] \right. \\
&), _DG\left(\left[["vector", M8, []], \left[[8], \right. \right. \\
& -\frac{1}{\sin(6Hx\theta)^{1/3}} ((2\cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) \\
& - 1)^{1/6} (-1 + 2\cos(2Hx\theta))^{1/3} (2\cos(Hx\theta) + 1)^{1/3} \\
& /3 (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3})] \right] \right. \\
& M8, []], \left[[2], \right. \right. \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x2\cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \\
& + 1)^{1/3} (-1 + 2\cos(2Hx\theta))^{1/3})], [[3], \\
& -\frac{1}{\sin(6Hx\theta)^{1/3}} (x1\cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} \\
& + 1)^{1/3} (-1 + 2\cos(2Hx\theta))^{1/3})]]], _DG\left(\left[["vector", \right. \right. \\
& \left. \left. \right]
\end{aligned}$$

```

M8, [ ], [[2],
  
$$\frac{1}{\sin(6Hx\theta)^{1/3}} (x^3 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [[4],
  
$$-\frac{1}{\sin(6Hx\theta)^{1/3}} (x^I \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]], _DG[[["vector",
  M8, [ ], [[2], 
$$\frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} \cos(Hx\theta)^{1/3})]]]
]]]$$$$$$

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In[=]:= ConvertMapleToMathematicaV2[
" -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3), , x5*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)"]
Out[=]=

$$-\frac{1}{\sin[6 H x \theta]^{1/3}} x^6 (-1 + \cos[H x \theta])^{1/6} \cos[H x \theta]^{1/3} (1 + \cos[H x \theta])^{1/6}$$


$$(-1 + 2 \cos[H x \theta])^{1/3} (1 + 2 \cos[H x \theta])^{1/3} (-1 + 2 \cos[2 H x \theta])^{1/3}$$


```

```
In[]:= ConvertMapleToMathematica[
  "x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
  1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
  2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3), -x2*cos(H*x0)^(1/3)*(cos(H*x0)
  - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
  + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)"]

Out[]= (((((x3 * (Cos[(H * x0]) ^ (1 / 3))) * ((Cos[(H * x0]) - 1) ^ (1 /
6))) * ((Cos[(H * x0]) + 1) ^ (1 / 6))) * (((2 * Cos[(H * x0)]) - 1)
^ (1 / 3))) * (((2 * Cos[(H * x0)]) + 1) ^ (1 / 3))) * ((-(1) + (2 *
Cos[((2 * H) * x0)])) ^ (1 / 3))) / (Sin[((6 * H) * x0)] ^ (1 / 3))

In[]:= ConvertMapleToMathematica[
  "(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
  2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
  + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)"]

Out[]= ((((((2 * Cos[(H * x0)] - 1) ^ (1 / 3)) * ((Cos[(H * x0)] -
1) ^ (1 / 6))) * ((-(1) + (2 * Cos[((2 * H) * x0)])) ^ (1 / 3))) *
(((2 * Cos[(H * x0)] + 1) ^ (1 / 3))) * ((Cos[(H * x0)] + 1) ^ (1 /
6))) * (Cos[(H * x0)] ^ (1 / 3))) / (Sin[((6 * H) * x0)] ^ (1 / 3))
```

M8 > $KV := \text{KillingVectors}(g);$

$$KV := \left[\begin{array}{l} -\frac{x6 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x5}}{\sin(6 Hx0)^1 \wedge} \\ + \frac{x5 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x6}}{\sin(6 Hx0)^1 \wedge}, \\ \frac{x3 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x2}}{\sin(6 Hx0)^1 \wedge} \\ - \frac{x2 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x3}}{\sin(6 Hx0)^1 \wedge}, \\ \frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x2}}{\sin(6 Hx0)^1 \wedge}, \\ \frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x3}}{\sin(6 Hx0)^1 \wedge}, \\ \frac{x7 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x5}}{\sin(6 Hx0)^1 \wedge} \\ + \frac{x5 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x7}}{\sin(6 Hx0)^1 \wedge}, \\ - \frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x5}}{\sin(6 Hx0)^1 \wedge}, \\ - \frac{x7 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x6}}{\sin(6 Hx0)^1 \wedge} \\ + \frac{x6 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x7}}{\sin(6 Hx0)^1 \wedge}, \\ - \frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x6}}{\sin(6 Hx0)^1 \wedge}, \\ - \frac{(2 \cos(Hx0) - 1)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge \cos(Hx0)^1 \wedge D_{x7}}{\sin(6 Hx0)^1 \wedge}, \\ x2 \cos(Hx0)^1 \wedge (\cos(Hx0) - 1)^1 \wedge (\cos(Hx0) + 1)^1 \wedge (2 \cos(Hx0) - 1)^1 \wedge (2 \cos(Hx0) + 1)^1 \wedge (-1 + 2 \cos(2 Hx0))^1 \wedge D_{x1}}{\sin(6 Hx0)^1 \wedge} \end{array} \right]$$

Out[•] =

$$\frac{x^3 \cos(Hx\theta)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} (2 \cos(Hx\theta) - 1)^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} D_{x^2}}{\sin(6Hx\theta)^{1/\beta}} - \frac{x^3 \cos(Hx\theta)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} (2 \cos(Hx\theta) - 1)^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} D_{xI}}{\sin(6Hx\theta)^{1/\beta}} - \frac{x^3 \cos(Hx\theta)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} (2 \cos(Hx\theta) - 1)^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} D_{x3}}{\sin(6Hx\theta)^{1/\beta}} + \frac{(2 \cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} \cos(Hx\theta)^{1/\beta} D_{xI}}{\sin(6Hx\theta)^{1/\beta}}$$

In[•]:=

Out[•]=

$$\begin{aligned}
& - \frac{x^3 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2Hx0))^{1/\beta} D_{x2}}{\sin(6Hx0)^{1/\beta}}, \\
& \frac{x^3 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2Hx0))^{1/\beta} D_{x1}}{\sin(6Hx0)^{1/\beta}}, \\
& - \frac{x^3 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2Hx0))^{1/\beta} D_{x3}}{\sin(6Hx0)^{1/\beta}}, \\
& \frac{(2 \cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (-1 + 2 \cos(2Hx0))^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} \cos(Hx0)^{1/\beta} D_{x1}}{\sin(6Hx0)^{1/\beta}}
\end{aligned}$$

8 > |

8 > |

4×4 Dirac γ matrices:

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta], \quad (22)$$

$$S^{\alpha\beta} = -\frac{1}{2}\gamma^{\alpha\beta}. \quad (23)$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \quad (24)$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^\alpha \epsilon = -\epsilon \gamma^\alpha, \quad (25)$$

$$\tilde{S}^{\alpha\beta} \epsilon = -\epsilon S^{\alpha\beta}, \quad (26)$$

$$[S^{\alpha\beta}, \gamma_\mu] = \delta_\mu^\alpha \gamma^\beta - \delta_\mu^\beta \gamma^\alpha, \quad (27)$$

$$[S^{\alpha\beta}, S^{\mu\nu}] = g^{\alpha\mu} S^{\beta\nu} - g^{\alpha\nu} S^{\beta\mu} - g^{\beta\mu} S^{\alpha\nu} + g^{\beta\nu} S^{\alpha\mu}, \quad (28)$$

and

$$\gamma^5 S^{\alpha\beta} = \frac{1}{2} g^{\alpha\mu} g^{\beta\nu} \epsilon_{\mu\nu\lambda\sigma} S^{\lambda\sigma}. \quad (29)$$

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

```
In[1]:= (*Symbolize[ g<sub>αβ</sub> ]*)

In[2]:= η4488
Dimensions[%]
g<sub>αβ</sub> = η4488[[2;;5, 2;;5]]

Out[2]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
          {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
          {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}

Out[3]= {8, 8}

Out[4]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}

In[5]:= Protect[g<sub>αβ</sub>]
Out[5]= {g<sub>Subscript</sub>αβ}

In[6]:= g3 = DiagonalMatrix[{1, 1, -1}]
Out[6]= {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}

In[7]:= Do[Do[Gr[h, k] = t4by4[h].s4by4[k]*(-g3[[h, h]]*g3[[k, k]]), {k, 1, 3}], {h, 1, 3}]
```

```
In[=]:= Gr[1, 1] // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[=]:= γ = {Gr[1, 1], Gr[1, 2], Gr[1, 3], t4by4[2]};
```

Set: Symbol γ is Protected. [i](#)

```
In[=]:= Table[FullSimplify[γ[[h]].γ[[h]] === gαβ[[h, h]] ID4], {h, 1, 4}]
```

```
Out[=]= {False, False, False, True}
```

$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\gamma_0 g^{\alpha\beta}, \quad (4)$$

where

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

and γ_0 denotes the 4×4 unit matrix. The $\Sigma^{\alpha\beta}$ of Eq. (1)

```
In[=]:= Table[{{h, k}, FullSimplify[(γ[[h]].γ[[k]] + γ[[k]].γ[[h]])/2] === gαβ[[h, k]] ID4}, {h, 1, 4}, {k, 1, 4}] // MatrixForm
```

$$\begin{pmatrix} (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) \\ \text{False} & \text{True} & \text{True} & \text{False} \\ (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) \\ \text{True} & \text{False} & \text{True} & \text{False} \\ (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) \\ \text{True} & \text{True} & \text{False} & \text{False} \\ (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) \\ \text{False} & \text{False} & \text{False} & \text{True} \end{pmatrix}$$

```
In[1]:= Table[{h, k}, FullSimplify[(γ[h].γ[k] + γ[k].γ[h])/2] === η4488[[1 + h, 1 + k]] ID4},
{h, 1, 4}, {k, 1, 4}] // MatrixForm
```

Out[1]//MatrixForm=

$$\begin{pmatrix} \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} \\ \text{False} & \text{True} & \text{True} & \text{False} \\ \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} \\ \text{True} & \text{False} & \text{True} & \text{False} \\ \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} \\ \text{True} & \text{True} & \text{False} & \text{False} \\ \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} \\ \text{False} & \text{False} & \text{False} & \text{True} \end{pmatrix}$$

$$\gamma^5 = -\gamma^1 \gamma^2 \gamma^3 \gamma^4$$

```
In[2]:= -γ[1].γ[2].γ[3].γ[4]
```

Out[2]=

$$\{\{0, 0, 0, 1\}, \{0, 0, -1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\}$$

```
In[3]:= AppendTo[γ, (-γ[1].γ[2].γ[3].γ[4])]
```

Set: Symbol γ is Protected. [i](#)

Out[3]=

$$\{\{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-1, 0, 0, 0\}\}, \{\{0, 0, -1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\}, \{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\}, \{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{1, 0, 0, 0\}\}, \{\{0, 0, -1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\}, \{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 1, 0\}\}, \{\{1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\}, \{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\}, \{\{0, 0, 1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\}, \{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, -1, 0\}\}, \{\{-1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\}, \{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{1, 0, 0, 0\}\}, \{\{0, 0, 1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\}, \{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\}, \{\{-1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}, \{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}, \{\{0, 0, 0, 1\}, \{0, 0, -1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\}\}$$

```
In[4]:= Protect[γ]
```

Out[4]=

$$\{\}$$

```
In[5]:= γ[[5]]
Out[5]= {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}
```

```
In[6]:= γ[[5]] === -t4by4[3]
Out[6]= False
```

```
In[7]:= Table[
  {{h, k}, FullSimplify[γ[[h]].γ[[k]] + γ[[k]].γ[[h]]] === 2 * η4488[[1 + h, 1 + k]] ID4},
  {h, 1, 5}, {k, 1, 5}] // MatrixForm
```

```
Out[7]//MatrixForm=
{{{{1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}}, {False, True, True, False, False}},
 {{{2, 1}, {2, 2}, {2, 3}, {2, 4}, {2, 5}}, {True, False, True, False, False}},
 {{{3, 1}, {3, 2}, {3, 3}, {3, 4}, {3, 5}}, {True, True, False, False, False}},
 {{{4, 1}, {4, 2}, {4, 3}, {4, 4}, {4, 5}}, {False, False, False, True, True}},
 {{{5, 1}, {5, 2}, {5, 3}, {5, 4}, {5, 5}}, {False, False, False, True, True}}}}
```

$$\gamma^\alpha = \gamma^{\alpha 6}, \quad (19)$$

$$\gamma^5 = \gamma^{56}, \quad (20)$$

$$\gamma^\alpha \gamma^5 = \gamma^{\alpha 5}, \quad (21)$$

$$\epsilon = \gamma^{64}.$$

```
In[8]:= -γ[[4]] // MatrixForm
```

```
Out[8]//MatrixForm=
{{0, 0, 0, 1},
 {0, 0, -1, 0},
 {0, 1, 0, 0},
 {-1, 0, 0, 0}}
```

```
In[9]:= sε = {ε → t4by4[2]}
Out[9]= {ε → {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}
```

```
In[10]:= Transpose[ε /. sε] === (-ε /. sε)
Out[10]= True
```

```
In[1]:= ParallelTable[{{A1},
  FullSimplify[(e /. se).γ[A1] === Transpose[(e /. se).γ[A1]]]}, {A1, 1, 5}]
```

```
Out[1]= {{1}, True}, {{2}, True}, {{3}, True}, {{4}, False}, {{5}, True}}
```

```
In[2]:= γ[4] === t4by4[2]
```

```
Out[2]= False
```

```
In[3]:= t4by4[1]
```

```
Out[3]= {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}}
```

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta]. \quad (3)$$

```
In[4]:= S44αβ = -1/4 Table[FullSimplify[γ[h].γ[k] - γ[k].γ[h]], {h, 1, 4}, {k, 1, 4}];
```

```
In[5]:= Protect[S44αβ]
```

```
Out[5]= {S44αβ}
```

```
In[6]:= η44 = DiagonalMatrix[{1, 1, 1, -1}]
```

```
Out[6]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}
```

```
In[7]:= Protect[η44]
```

```
Out[7]= {η44}
```

S44αβ commutation relations; misc :

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta], \quad (22)$$

$$S^{\alpha\beta} = -\frac{1}{2}\gamma^{\alpha\beta}. \quad (23)$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \quad (24)$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^\alpha \epsilon = -\epsilon \gamma^\alpha, \quad (25)$$

$$\tilde{S}^{\alpha\beta} \epsilon = -\epsilon S^{\alpha\beta}, \quad (26)$$

$$[S^{\alpha\beta}, \gamma_\mu] = \delta_\mu^\alpha \gamma^\beta - \delta_\mu^\beta \gamma^\alpha, \quad (27)$$

$$[S^{\alpha\beta}, S^{\mu\nu}] = g^{\alpha\mu} S^{\beta\nu} - g^{\alpha\nu} S^{\beta\mu} - g^{\beta\mu} S^{\alpha\nu} + g^{\beta\nu} S^{\alpha\mu}, \quad (28)$$

and

$$\gamma^5 S^{\alpha\beta} = \frac{1}{2} g^{\alpha\mu} g^{\beta\nu} \epsilon_{\mu\nu\lambda\sigma} S^{\lambda\sigma}. \quad (29)$$

$$S44\alpha\beta = -\frac{1}{4} \text{Table}[\text{FullSimplify}[\gamma[h].\gamma[k] - \gamma[k].\gamma[h]], \{h, 1, 4\}, \{k, 1, 4\}];$$

```
In[1]:= (*SAB=
Table[1/4 (T16^A[A1].T16^A[B1]-T16^A[B1].T16^A[A1]), {A1,0, 7},{B1,0, 7}];*)
```

```
In[2]:= (*ParallelTable[
FullSimplify[\sigma16.SAB[A1,B1]==-Transpose[\sigma16.SAB[A1,B1]]],
{A1,1, 8},{B1,1,8}]/.Flatten//Union*)
```

```
In[3]:= (*ParallelTable[
{{A1,B1},FullSimplify[SAB[A1,B1]==-Transpose[SAB[A1,B1]]]}, {A1,1, 8},{B1,1,8}]*)
```

```
In[4]:= ParallelTable[
FullSimplify[S44\alpha\beta[A1, B1].S44\alpha\beta[A2, B2] - S44\alpha\beta[A2, B2].S44\alpha\beta[A1, B1] ==
(\eta44[A1, A2]\times S44\alpha\beta[B1, B2] - \eta44[A1, B2]\times S44\alpha\beta[B1, A2] -
\eta44[B1, A2]\times S44\alpha\beta[A1, B2] + \eta44[B1, B2]\times S44\alpha\beta[A1, A2])], {A1, 1, 3},
{B1, A1 + 1, 4}, {A2, 1, 3}, {B2, A2 + 1, 4}] // Flatten // Union
```

```
Out[4]= {False, True}
```

```
In[=]:= ParallelTable[{{A1, B1, A2, B2}, 
  FullSimplify[S44αβ[A1, B1].S44αβ[A2, B2] - S44αβ[A2, B2].S44αβ[A1, B1] == 
    (η44[A1, A2] × S44αβ[B1, B2] - η44[A1, B2] × S44αβ[B1, A2] - 
     η44[B1, A2] × S44αβ[A1, B2] + η44[B1, B2] × S44αβ[A1, A2])]}, 
  {A1, 1, 4}, {B1, 1, 4}, {A2, 1, 4}, {B2, 1, 4}] // MatrixForm
```

```
Out[=]//MatrixForm=
```

$$\begin{pmatrix} \begin{pmatrix} \{1, 1, 1, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 1, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 1, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 1, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{1, 1, 2, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 2, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 2, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 2, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{1, 1, 3, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 3, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 3, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 3, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{1, 1, 4, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 4, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 4, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{1, 1, 4, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{2, 1, 1, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1, 1, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1, 1, 3\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{2, 1, 1, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{2, 1, 2, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1, 2, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1, 2, 3\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{2, 1, 2, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{2, 1, 3, 1\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{2, 1, 3, 2\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{2, 1, 3, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1, 3, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{2, 1, 4, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1, 4, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1, 4, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{2, 1, 4, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{3, 1, 1, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1, 1, 2\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{3, 1, 1, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1, 1, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{3, 1, 2, 1\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{3, 1, 2, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1, 2, 3\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{3, 1, 2, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{3, 1, 3, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1, 3, 2\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{3, 1, 3, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1, 3, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{3, 1, 4, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1, 4, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1, 4, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{3, 1, 4, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{4, 1, 1, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 1, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 1, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 1, 4\} \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} \{4, 1, 2, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 2, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 2, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 2, 4\} \\ \text{False} \end{pmatrix} \\ \begin{pmatrix} \{4, 1, 3, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 3, 2\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 3, 3\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 3, 4\} \\ \text{False} \end{pmatrix} \\ \begin{pmatrix} \{4, 1, 4, 1\} \\ \text{True} \end{pmatrix} & \begin{pmatrix} \{4, 1, 4, 2\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{4, 1, 4, 3\} \\ \text{False} \end{pmatrix} & \begin{pmatrix} \{4, 1, 4, 4\} \\ \text{True} \end{pmatrix} \end{pmatrix}$$

```
In[=]:= ParallelTable[FullSimplify[S44αβ[A1, B1].γ[B2] - γ[B2].S44αβ[A1, B1] == 
  (η44[B2, A1] × γ[B1] - η44[B2, B1] × γ[A1])], 
  {A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // Flatten // Union
```

```
Out[=]= {False, True}
```

```
In[=]:= ParallelTable[
  {{A1, B1, B2}, FullSimplify[S44αβ[A1, B1].γ[B2] - γ[B2].S44αβ[A1, B1] ==
    (η44[B2, A1] × γ[B1] - η44[B2, B1] × γ[A1])]}, {A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // MatrixForm
```

Out[=]//MatrixForm=

$$\begin{pmatrix} \{1, 1, 1\} & \text{True} & \{1, 2, 1\} & \text{False} & \{1, 3, 1\} & \text{False} & \{1, 4, 1\} & \text{False} \\ \{1, 1, 2\} & \text{True} & \{1, 2, 2\} & \text{False} & \{1, 3, 2\} & \text{True} & \{1, 4, 2\} & \text{True} \\ \{1, 1, 3\} & \text{True} & \{1, 2, 3\} & \text{True} & \{1, 3, 3\} & \text{False} & \{1, 4, 3\} & \text{True} \\ \{1, 1, 4\} & \text{True} & \{1, 2, 4\} & \text{True} & \{1, 3, 4\} & \text{True} & \{1, 4, 4\} & \text{False} \\ \{2, 1, 1\} & \text{False} & \{2, 2, 1\} & \text{True} & \{2, 3, 1\} & \text{True} & \{2, 4, 1\} & \text{True} \\ \{2, 1, 2\} & \text{False} & \{2, 2, 2\} & \text{True} & \{2, 3, 2\} & \text{False} & \{2, 4, 2\} & \text{False} \\ \{2, 1, 3\} & \text{True} & \{2, 2, 3\} & \text{True} & \{2, 3, 3\} & \text{False} & \{2, 4, 3\} & \text{True} \\ \{2, 1, 4\} & \text{True} & \{2, 2, 4\} & \text{True} & \{2, 3, 4\} & \text{True} & \{2, 4, 4\} & \text{False} \\ \{3, 1, 1\} & \text{False} & \{3, 2, 1\} & \text{True} & \{3, 3, 1\} & \text{True} & \{3, 4, 1\} & \text{True} \\ \{3, 1, 2\} & \text{True} & \{3, 2, 2\} & \text{False} & \{3, 3, 2\} & \text{True} & \{3, 4, 2\} & \text{True} \\ \{3, 1, 3\} & \text{False} & \{3, 2, 3\} & \text{False} & \{3, 3, 3\} & \text{True} & \{3, 4, 3\} & \text{False} \\ \{3, 1, 4\} & \text{True} & \{3, 2, 4\} & \text{True} & \{3, 3, 4\} & \text{True} & \{3, 4, 4\} & \text{False} \\ \{4, 1, 1\} & \text{False} & \{4, 2, 1\} & \text{True} & \{4, 3, 1\} & \text{True} & \{4, 4, 1\} & \text{True} \\ \{4, 1, 2\} & \text{True} & \{4, 2, 2\} & \text{False} & \{4, 3, 2\} & \text{True} & \{4, 4, 2\} & \text{True} \\ \{4, 1, 3\} & \text{True} & \{4, 2, 3\} & \text{True} & \{4, 3, 3\} & \text{False} & \{4, 4, 3\} & \text{True} \\ \{4, 1, 4\} & \text{False} & \{4, 2, 4\} & \text{False} & \{4, 3, 4\} & \text{False} & \{4, 4, 4\} & \text{True} \end{pmatrix}$$

```
In[=]:= ParallelTable[{{A1, B1}, FullSimplify[(ε /. se).S44αβ[A1, B1] ===
  Transpose[(ε /. se).S44αβ[A1, B1]]]}, {A1, 1, 3}, {B1, A1 + 1, 4}]
```

Out[=]=

$$\{\{\{1, 2\}, \text{True}\}, \{\{1, 3\}, \text{True}\}, \{\{1, 4\}, \text{True}\}\}, \{\{\{2, 3\}, \text{True}\}, \{\{2, 4\}, \text{True}\}\}, \{\{\{3, 4\}, \text{True}\}\}$$

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula: \mathcal{O}

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. \mathcal{O}

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4). 🔗

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) 🔗

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. 🔗

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa). 🔗

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). 🔗

what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the *so*(4, 4) Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: 🔗

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for *so*(8) in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



↳ Why is the metric signature significant for \$so(4,4)\$ commutation relations?

Why is the metric signature significant for \$so(4,4)\$ commutation relations?

The metric signature is significant for the $so(p, q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). 

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p, q)$ is given by:
 $[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$
The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4, 4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[1]:= **η4488**

Out[1]=
 $\{\{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\},$
 $\{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\},$
 $\{0, 0, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 0, 0, -1, 0\}, \{0, 0, 0, 0, 0, 0, 0, -1\}\}$

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

$$t^A = \begin{pmatrix} 0 & \vec{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
In[2]:= (*Table[SAB[[A1,B1]] = ArrayFlatten[{{SAB1[[A1,B1]],0},{0,SAB2[[A1,B1]]}}], {A1,0, 7},{B1,0,7}]*)
```

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4, 4)}$ and $L \in SO(4, 4)$. The canonical 2-1 homomorphism $\overline{SO(4, 4)} \rightarrow SO(4, 4)$ is given by

$$(2.8) \quad 8L^{A'}{}_{B'}(g) = \text{tr } \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4, 4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}{}_{B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

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A Remarkable Representation of the 3 + 2 de Sitter Group

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(Received 20 February 1963)

Identities satisfied by the generators of the Dirac algebra

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```
In[]:= Unprotect[ $\gamma$ ABs];
 $\gamma$ ABs = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]

Out[]=
{{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}, {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}}, {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}}, {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}}, {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}, {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}}, {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}, {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}, {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}}, {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}}, {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}, {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}}, {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}}, {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}, {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}, {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}}
```

In[]:= **Length**[γ ABs]

Out[]=

16

In[]:= **Protect**[γ ABs]

Out[]=

{ γ ABs}

In[]:= γ ABs[[1]]

Out[]=

{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}}

In[]:= γ ABs[[-1]]

Out[]=

{{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

Recall Self Dual Anti-Symmetric

```
In[]:= Table[ $\left(\frac{1}{2} \sum_{j1=1}^4 \left(\sum_{j2=1}^4 \epsilon4[p, q, j1, j2] \times s4by4[h][j1, j2]\right)\right) - s4by4[h][p, q],$ 
{h, 1, 3}, {q, 4}, {p, 4}]
```

Out[]=

```
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

Recall Anti SelfDual Anti-Symmetric

```
In[ ]:= Table[(1/2) Sum[(Sum[epsilon4[p, q, j1, j2] * t4by4[h][j1, j2], {j2, 1, 4}], {j1, 1, 4}) + t4by4[h][p, q], {h, 1, 3}, {q, 4}, {p, 4}]

Out[ ]= {{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

BASIS of 4×4 matrices :

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765

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The Dirac spinor in six dimensions

BY E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

3. Decomposition of the basic spinor representations. We have already stated that the basic spinor representations of $SO(n, c)$, for n even, are reducible. We shall consider here the reduction of the basic 8-component spinor representation of $SO(6, c)$ into constituent 4-component representations. The procedure is easily generalized to apply to any even-dimensional space. An irreducible representation of the Clifford algebra C_6 can be constructed from a given irreducible representation of C_5 . The $\gamma_\mu (\mu = 1, \dots, 5)$ of an irreducible representation of C_5 satisfy (2.5), so that the quantities $e_\alpha (\alpha = 1, \dots, 6)$ defined by

$$\left. \begin{aligned} e_\mu &= (-i) \begin{pmatrix} & \gamma_\mu B \\ -B^{-1}\gamma_\mu & \end{pmatrix} \quad (\mu = 1, \dots, 5), \\ e_6 &= \begin{pmatrix} & B \\ B^{-1} & \end{pmatrix}, \end{aligned} \right\} \quad (3.1)$$

satisfy

$$\frac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \dots, 6).$$

For the moment the matrix B is completely arbitrary. The e_α therefore give an irreducible representation of C_6 . They can be written more concisely in terms of the sets of matrices a_α and \bar{a}_α , defined as follows:

$$\left. \begin{aligned} a_\mu &= \gamma_\mu B, \quad \bar{a}_\mu = -B^{-1}\gamma_\mu \quad (\mu = 1, \dots, 5), \\ a_6 &= iB, \quad \bar{a}_6 = iB^{-1}. \end{aligned} \right\} \quad (3.2)$$

The generator e_α of C_6 can then be written

$$e_\alpha = (-i) \begin{pmatrix} & a_\alpha \\ \bar{a}_\alpha & \end{pmatrix} \quad (\alpha = 1, \dots, 6). \quad (3.3)$$

With $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1e_2e_3e_4e_5e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

and $e_{\alpha\beta} = \frac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2} \begin{pmatrix} a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha & \\ \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha & \end{pmatrix}.$

This latter expression shows that the infinitesimal generators of the 8-component spinor representation of $SO(6, c)$ is the direct sum of two 4-component representations. The basic spinor can be written

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix},$$

where ϕ and χ are four component spinors which transform according to representations with infinitesimal generators

$$\left. \begin{aligned} G_{\alpha\beta}^{(1)} &= -\frac{1}{4}(a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha) \\ G_{\alpha\beta}^{(2)} &= -\frac{1}{4}(\bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha), \end{aligned} \right\} \quad (3.4)$$

and $\underbrace{\text{sy}}_1, \underbrace{\text{sy}}_4, \underbrace{\text{asy}}_6, \underbrace{\text{sy}}_4, \underbrace{\text{sy}}_1$

```

In[]:= Plus @@ {1, 4, 6, 4, 1}
Out[]= 16

{sy, sy, asy, sy, sy}
  1   4   6   4   1

In[]:= Plus @@ {1, 4, 6, 4, 1}
Out[]= 16

In[]:= MatrixForm[If[# == 2, i, 1] * PauliMatrix[#]] & /@ Range[3]
Out[=]
{ ( 0  1 ), ( 0  1 ), ( 1  0 ) }
  1  0      -1  0      0  -1

In[]:= σ22 =
Flatten[{{IdentityMatrix[2]}, If[# == 2, i, 1] * PauliMatrix[#] & /@ Range[3]}, 1]
Out[=]
{{{1, 0}, {0, 1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}}}

In[]:= σ22 = Flatten[
{{-IdentityMatrix[2]}, If[# == 2, i, 1] * PauliMatrix[#] & /@ Range[3]}, 1]
Out[=]
{{{ -1, 0}, {0, -1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}}}

In[]:= -IdentityMatrix[2] === Dot @@ (σ22[[#]] & /@ Range[Length[σ22]])
Out[=]
True

In[]:= Table[{{A, B}, FullSimplify[1/2 Tr[1/2 (σ22[[A]].σ22[[B]] + σ22[[B]].σ22[[A]])]}], 
{A, 1, 4}, {B, 1, 4}] // MatrixForm

```

$$\begin{pmatrix} \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```

In[]:= η2244 = DiagonalMatrix[{-1, 1, -1, 1}]
Out[=]
{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}

```

```
In[=]:= Table[{{A, B}, FullSimplify[ExpandAll[
  1/2 (σ22[A].σ22[B] + σ22[B].σ22[A]) == η2244[A, B]*IdentityMatrix[2]]]}, {{A, 1, 4}, {B, 1, 4}}] // MatrixForm
```

```
Out[=]//MatrixForm=
{{{{1, 1}, True}, {{1, 2}, True}, {{1, 3}, True}, {{1, 4}, True}}, {{{2, 1}, True}, {{2, 2}, True}, {{2, 3}, True}, {{2, 4}, True}}, {{{3, 1}, True}, {{3, 2}, True}, {{3, 3}, True}, {{3, 4}, True}}, {{{4, 1}, True}, {{4, 2}, True}, {{4, 3}, True}, {{4, 4}, True}}}}
```

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The Dirac spinor in six dimensions

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$$\left. \begin{aligned} e_\mu &= (-i) \begin{pmatrix} \gamma_\mu B \\ -B^{-1}\gamma_\mu \end{pmatrix} \quad (\mu = 1, \dots, 5), \\ e_6 &= \begin{pmatrix} B \\ B^{-1} \end{pmatrix}, \end{aligned} \right\}$$

$$\frac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \dots, 6).$$

$$\left. \begin{aligned} \alpha_\mu &= \gamma_\mu B, & \bar{a}_\mu &= -B^{-1}\gamma_\mu \quad (\mu = 1, \dots, 5), \\ a_6 &= iB, & \bar{a}_6 &= iB^{-1}. \end{aligned} \right\}$$

Generator e_α of C_6 can then be written

$$e_\alpha = (-i) \begin{pmatrix} a_\alpha \\ \bar{a}_\alpha \end{pmatrix} \quad (\alpha = 1, \dots, 6).$$

Since $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1e_2e_3e_4e_5e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$e_{\alpha\beta} = \frac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2} \begin{pmatrix} a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha & \\ \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha & \end{pmatrix}.$$

```

In[]:= Do[yyy[j] = ArrayFlatten[{{0, σ22[[j]]}, {-σ22[[j]], 0}}], {j, 4}];
yyy[5] = ArrayFlatten[{{0, IdentityMatrix[2]}, {IdentityMatrix[2], 0}}]
Out[=]= {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}};

In[]:= yyy[5] = {{0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0}}
Out[=]= {{0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0}};

In[]:= (yyy[6] = Dot @@ (yyy[#] & /@ Range[5])) // MatrixForm
Out[//MatrixForm]=

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$


In[]:= MatrixForm[yyy[#]] & /@ Range[6]
Out[=]=

$$\left\{ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \right.$$


$$\left. \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \right\}$$


In[]:= Do[xxx[j] = ArrayFlatten[{{0, σ22[[j]]}, {σ22[[j]], 0}}], {j, 4}]
Out[=]= MatrixForm[xxx[#]] & /@ Range[4]
Out[=]=

$$\left\{ \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \right\}$$


In[]:= xxx[5] = Dot @@ (xxx[#] & /@ Range[4])
Out[=]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};

In[]:= IdentityMatrix[4]
Out[=]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}

```

```
In[ ]:= txA0 = {{IdentityMatrix[4], {0}}};  
Length[txA0]  
txA = {}; Do[AppendTo[txA, {xxx[j], {j}}], {j, 1, 4}]  
Length[txA]  
txAB = {};  
Do[AppendTo[txAB, {xxx[j].xxx[k], {j, k}}], {j, 1, 3}, {k, j + 1, 4}]  
Length[txAB]  
txABC = {};  
Do[AppendTo[txABC, {xxx[j].xxx[k].xxx[h], {j, k, h}}],  
{j, 1, 2}, {k, j + 1, 3}, {h, k + 1, 4}];  
Length[txABC]  
txABCD = {{xxx[1].xxx[2].xxx[3].xxx[4], {1, 2, 3, 4}}};  
Length[txABCD]
```

Out[]=

1

Out[]=

4

Out[]=

6

Out[]=

4

Out[]=

1

```
In[ ]:= base4by4 = Flatten[{txA0, txA, txAB, txBC, txABCD}, 1]; Length[base4by4]
```

Out[]=

16

```
In[6]:= Table[{{A, B}, Block[{r}, r = FullSimplify[ $\frac{1}{4}$  Tr[
 $\frac{1}{2}$  (base4by4[[A, 1]].base4by4[[B, 1]] + base4by4[[B, 1]].base4by4[[A, 1]])]];
If[r == 0, Style[r, Red], Style[r, Darker[Green]]]], {A, 1, Length[base4by4]}, {B, 1, Length[base4by4]}] // MatrixForm
```

```
In[•]:= base4by4[[16]][1]
Out[•]= { {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1} }
```

```
In[1]:= anti4by4 = {};
Do[If[base4by4[[k, 1]] == -Transpose[base4by4[[k, 1]]], AppendTo[anti4by4,
{base4by4[[k, 1]], {{k}, base4by4[[k, 2]]}}], {k, 1, Length[base4by4]}];
Length[anti4by4]
anti4by4[[%]]
```

```
Out[1]=
6
Out[2]=
{{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}, {{14}, {1, 3, 4}}}}
```

```
In[3]:= symm4by4 = {};
Do[If[base4by4[[k, 1]] == Transpose[base4by4[[k, 1]]], AppendTo[symm4by4,
{base4by4[[k, 1]], {{k}, base4by4[[k, 2]]}}], {k, 1, Length[base4by4]}];
Length[symm4by4]
symm4by4[[%]]
```

```
Out[3]=
10
Out[4]=
{{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}, {{16}, {1, 2, 3, 4}}}
```

```
In[5]:= η16 =
DiagonalMatrix[Table[FullSimplify[ $\frac{1}{4} \text{Tr}[\text{symm4by4}[[A, 1]] . \text{symm4by4}[[A, 1]]]$ ],
{A, Length[symm4by4]}]];
Tr[η16]
```

```
Out[5]=
10
```

O(4,4); evaluations, evecs of σ

```
In[6]:= {evals, evecs} = Eigensystem[σ]
Out[6]=
{{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
{0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
{0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}}
```

```
In[1]:= 
$$\mathbf{u} = \text{ExpandAll}\left[\frac{1}{\sqrt{2}} \mathbf{evecs}\right] // \text{MatrixForm}$$

```

Out[1]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

```
In[2]:= Table[u[[h]].σ.u[[h]], {h, 1, Length[u]}]
```

Out[2]=

$$\{-1, -1, -1, -1, 1, 1, 1, 1\}$$

```
In[3]:= Table[(Transpose[u][[h]]).σ.u[[h]], {h, 1, Length[u]}]
```

Out[3]=

$$\{-1, -1, -1, -1, 1, 1, 1, 1\}$$

```
In[4]:= Table[(Transpose[u][[A]]) == -σ.u[[A]], {A, 1, 4}]
```

```
Table[(Transpose[u][[A]]) == σ.u[[A]], {A, 5, Length[u]}]
```

Out[4]=

$$\{\text{True}, \text{True}, \text{True}, \text{True}\}$$

Out[5]=

$$\{\text{True}, \text{True}, \text{True}, \text{True}\}$$

```
In[6]:= hUSE = 8;
```

```
In[7]:= Protect[hUSE]
```

Out[7]=

$$\{\text{hUSE}\}$$

```
In[8]:= unit = u[[hUSE]]
```

Out[8]=

$$\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}$$

```
In[9]:= Protect[unit]
```

Out[9]=

$$\{\text{unit}\}$$

In[8]:= (Transpose[unit].σ.unit)

Out[8]=

1

In[9]:= (Transpose[unit].σ.(τ[0]))

Out[9]=

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

In[10]:= myid[MX_] := Sum[τ[A].MX.τ[A] η4488[A, A], {A, 1, 8}]

Myid[MX_] := Sum[τ[A].MX.τ[A] × η448[A, A], {A, 1, 8}]

Union[Table[

$$\begin{aligned} & \text{Sum}[\tau[A].\text{bas64}[k, 1].\sigma.\tau[A] \times \eta8[A, A], \{A, 1, 8\}] - \text{IdTr}[\text{bas64}[k, 1].\sigma] + \\ & 4 \text{ eg}[k] \frac{\text{bas64}[k, 1] - \text{Transpose}[\text{bas64}[k, 1]]}{2} .\sigma, \{k, 1, 64\}] \end{aligned}$$

$\{\{\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\}$

In[11]:= τ[0]

Out[11]=

$$\begin{aligned} & \{\{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 0\}, \{0, 0, 0, 0, 0, 0, 0, 1\}\} \end{aligned}$$

In[12]:= (Transpose[unit].σ.(τ[0]))

Out[12]=

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

In[13]:= FA = Transpose[η4488[#, 1, #, 1] * (τ[#].unit) & /@ Range[0, 7]]

Out[13]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \\ & \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \end{aligned}$$

In[1]:= $F_a^A = \text{Transpose}[\text{unit}.\sigma.(\bar{\tau}[\#]) \& /@ \text{Range}[0, 7]]$

Out[1]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\}$$

In[2]:= $F_A^a.F_a^A == \text{ID8}$

Out[2]=

True

pretend that the X are Minkowski Coordinates :

In[3]:= $\text{Block}[\{U, u = U[\#] \& /@ \text{Range}[8], fx = F_A^a.X, eq, sx, su, ret\},$
 $\text{eq} = \text{And} @@ \text{Thread}[u == fx];$
 $sx = \text{Solve}[eq, X][[-1]]; \quad su = \text{Solve}[eq, u][[-1]]; \quad ret = \{sx, su\}; \quad ret]$

Out[3]=

$$\left\{ \begin{aligned} x0 &\rightarrow \frac{U[1]}{\sqrt{2}} + \frac{U[5]}{\sqrt{2}}, \quad x1 \rightarrow -\frac{U[4]}{\sqrt{2}} - \frac{U[8]}{\sqrt{2}}, \quad x2 \rightarrow \frac{U[3]}{\sqrt{2}} + \frac{U[7]}{\sqrt{2}}, \\ x3 &\rightarrow -\frac{U[2]}{\sqrt{2}} - \frac{U[6]}{\sqrt{2}}, \quad x4 \rightarrow \frac{1}{2} (\sqrt{2} U[2] - \sqrt{2} U[6]), \quad x5 \rightarrow \frac{1}{2} (-\sqrt{2} U[3] + \sqrt{2} U[7]), \\ x6 &\rightarrow \frac{1}{2} (-\sqrt{2} U[4] + \sqrt{2} U[8]), \quad x7 \rightarrow \frac{1}{2} (\sqrt{2} U[1] - \sqrt{2} U[5]), \\ U[1] &\rightarrow \frac{1}{2} (\sqrt{2} x0 + \sqrt{2} x7), \quad U[2] \rightarrow \frac{1}{2} (-\sqrt{2} x3 + \sqrt{2} x4), \quad U[3] \rightarrow \frac{1}{2} (\sqrt{2} x2 - \sqrt{2} x5), \\ U[4] &\rightarrow \frac{1}{2} (-\sqrt{2} x1 - \sqrt{2} x6), \quad U[5] \rightarrow \frac{1}{2} (\sqrt{2} x0 - \sqrt{2} x7), \quad U[6] \rightarrow \frac{1}{2} (-\sqrt{2} x3 - \sqrt{2} x4), \\ U[7] &\rightarrow \frac{1}{2} (\sqrt{2} x2 + \sqrt{2} x5), \quad U[8] \rightarrow \frac{1}{2} (-\sqrt{2} x1 + \sqrt{2} x6) \end{aligned} \right\}$$

In[4]:= $(\text{sgtrye}_\alpha^{(A)} = \text{Thread}[\text{Flatten}[e_\alpha^{(A)}] \rightarrow \text{Flatten}[eAa04]]) (* // \text{Column} *)$

In[1]:= **subsDefects**

Out[1]=

$$\left\{ \begin{array}{l} \sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}, \sqrt{e^{-2 a4[H x4]}} \rightarrow e^{-a4[H x4]}, \\ \sqrt{e^{2 a4[H x4]}} \sin[6 H x4]^{1/3} \rightarrow e^{a4[H x4]} \sin[6 H x4]^{1/6}, \\ \frac{1}{\sqrt{e^{2 a4[H x4]}} \sin[6 H x4]^{1/3}} \rightarrow \frac{e^{-a4[H x4]}}{\sin[6 H x4]^{1/6}}, \frac{1}{\sqrt{e^{2 a4[H x4]}} \sin[6 H x0]^{1/6}} \rightarrow \frac{e^{-a4[H x4]}}{\sin[6 H x0]^{1/6}}, \\ \frac{1}{\sqrt{e^{-2 a4[H x4]}} \sin[6 H x0]^{1/6}} \rightarrow \frac{e^{a4[H x4]}}{\sin[6 H x0]^{1/6}}, \\ \sqrt{e^{2 a4[H x4]}} \sin[6 H x0]^{1/6} \rightarrow e^{a4[H x4]} \sin[6 H x0]^{1/6}, \\ \sqrt{e^{-2 a4[H x4]}} \sin[6 H x0]^{1/6} \rightarrow e^{-a4[H x4]} \sin[6 H x0]^{1/6} \end{array} \right\}$$

In[2]:= **Thread[X → 0]**

Out[2]=

$$\{x0 \rightarrow 0, x1 \rightarrow 0, x2 \rightarrow 0, x3 \rightarrow 0, x4 \rightarrow 0, x5 \rightarrow 0, x6 \rightarrow 0, x7 \rightarrow 0\}$$

Stop pretending:

```
In[3]:= Fa.eα(A).X /. sgtryeα(A) // FullSimplify[#, constraintVars] &;
frameOctadX = # /. {sqrt{e2 a4[H x4]} → ea4[H x4], sqrt{e-2 a4[H x4]} → e-a4[H x4]} & /@ %
% /. {x1 → 0, x2 → 0, x3 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Out[3]=

$$\left\{ \begin{array}{l} \frac{x0 \cot[6 H x0] + e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \frac{x4 - e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ \frac{(e^{a4[H x4]} x2 - e^{-a4[H x4]} x5) \sin[6 H x0]^{1/6}}{\sqrt{2}}, -\frac{(e^{a4[H x4]} x1 + e^{-a4[H x4]} x6) \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ \frac{x0 \cot[6 H x0] - e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, -\frac{x4 + e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ \frac{(e^{a4[H x4]} x2 + e^{-a4[H x4]} x5) \sin[6 H x0]^{1/6}}{\sqrt{2}}, -\frac{(-e^{a4[H x4]} x1 + e^{-a4[H x4]} x6) \sin[6 H x0]^{1/6}}{\sqrt{2}} \end{array} \right\}$$

Out[4]=

$$\left\{ \frac{x0 \cot[6 H x0]}{\sqrt{2}}, \frac{x4}{\sqrt{2}}, 0, 0, \frac{x0 \cot[6 H x0]}{\sqrt{2}}, -\frac{x4}{\sqrt{2}}, 0, 0 \right\}$$

In[5]:= **F^a.eA_α.X - frameOctadX // FullSimplify[#, constraintVars] &**

Out[5]=

$$\left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \right.$$

$$\left. \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \right. \right.$$

$$\begin{aligned}
& \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \}. eA\alpha. \\
& \{x0, x1, x2, x3, x4, x5, x6, x7\} - \frac{x0 \operatorname{Cot}[6 H x0] + e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. eA\alpha. \\
& eA\alpha. \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{-x4 + e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. eA\alpha. \\
& \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{e^{-a4[H x4]} (-e^{2 a4[H x4]} x2 + x5) \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. eA\alpha. \\
& \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{(e^{a4[H x4]} x1 + e^{-a4[H x4]} x6) \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \\
& \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \}. eA\alpha. \\
& \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{-x0 \operatorname{Cot}[6 H x0] + e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. eA\alpha. \\
& eA\alpha. \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{x4 + e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. eA\alpha. \\
& \{x0, x1, x2, x3, x4, x5, x6, x7\} - \frac{(e^{a4[H x4]} x2 + e^{-a4[H x4]} x5) \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\
& \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. eA\alpha. \\
& \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{e^{-a4[H x4]} (e^{2 a4[H x4]} x1 - x6) \sin[6 H x0]^{1/6}}{\sqrt{2}}
\end{aligned}$$

```
In[8]:= {{F_A^a}, {eA\alpha}, {X}} // Column
Out[8]= {{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}}}, {eA\alpha}, {{x0, x1, x2, x3, x4, x5, x6, x7}}}
```

```
In[]:= {x2u, u2x} = Block[{U, u = U[#] & /@ Range[8],  

  fx = frameOctadX, eq, sx, su, ret, altEQ, sCot, skot},  

  altEQ = And @@ {Cot[6 H x0] == kot, Sin[6 H x0] == sin, a4[H x4] == A44};  

  sCot = Solve[altEQ, {Cot[6 H x0], Sin[6 H x0], a4[H x4]}][[1]];  

  skot = Solve[altEQ, {kot, sin, A44}][[1]];  

  eq = And @@ Thread[u == (fx /. sCot)];  

  sx = Solve[eq, X][[-1]];  

  su = Solve[eq, u][[-1]];  

  ret = {sx /. skot // FullSimplify, su /. skot // ExpandAll};  

  ret]
```

Out[]:=

$$\left\{ \begin{array}{l} x_0 \rightarrow \frac{\tan[6Hx_0](U[1] + U[5])}{\sqrt{2}}, \quad x_1 \rightarrow -\frac{e^{-a4[Hx4]}(U[4] + U[8])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, \\ x_2 \rightarrow \frac{e^{-a4[Hx4]}(U[3] + U[7])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, \quad x_3 \rightarrow -\frac{e^{-a4[Hx4]}(U[2] + U[6])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, \quad x_4 \rightarrow \frac{U[2] - U[6]}{\sqrt{2}}, \\ x_5 \rightarrow \frac{e^{a4[Hx4]}(-U[3] + U[7])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, \quad x_6 \rightarrow \frac{e^{a4[Hx4]}(-U[4] + U[8])}{\sqrt{2}\sin[6Hx_0]^{1/6}}, \quad x_7 \rightarrow \frac{e^{a4[Hx4]}(U[1] - U[5])}{\sqrt{2}\sin[6Hx_0]^{1/6}} \end{array} \right\},$$

$$\left\{ \begin{array}{l} U[1] \rightarrow \frac{x_0 \cot[6Hx_0]}{\sqrt{2}} + \frac{e^{-a4[Hx4]}x_7 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \quad U[2] \rightarrow \frac{x_4}{\sqrt{2}} - \frac{e^{a4[Hx4]}x_3 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[3] \rightarrow \frac{e^{a4[Hx4]}x_2 \sin[6Hx_0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]}x_5 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[4] \rightarrow -\frac{e^{a4[Hx4]}x_1 \sin[6Hx_0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]}x_6 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[5] \rightarrow \frac{x_0 \cot[6Hx_0]}{\sqrt{2}} - \frac{e^{-a4[Hx4]}x_7 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[6] \rightarrow -\frac{x_4}{\sqrt{2}} - \frac{e^{a4[Hx4]}x_3 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[7] \rightarrow \frac{e^{a4[Hx4]}x_2 \sin[6Hx_0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]}x_5 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\ U[8] \rightarrow -\frac{e^{a4[Hx4]}x_1 \sin[6Hx_0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]}x_6 \sin[6Hx_0]^{1/6}}{\sqrt{2}} \end{array} \right\}$$

In[=]:= u2x

Out[=]=

$$\begin{aligned} U[1] &\rightarrow \frac{x0 \operatorname{Cot}[6 H x0]}{\sqrt{2}} + \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \quad U[2] \rightarrow \frac{x4}{\sqrt{2}} - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[3] &\rightarrow \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[4] &\rightarrow -\frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[5] &\rightarrow \frac{x0 \operatorname{Cot}[6 H x0]}{\sqrt{2}} - \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \quad U[6] \rightarrow -\frac{x4}{\sqrt{2}} - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[7] &\rightarrow \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[8] &\rightarrow -\frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}} \} \end{aligned}$$

```
{sumu2x, diffu2x} = Block[{U, u, a1, a2, a3 = {}, aa4 = {}}, 
  (*Clear[U,u,a1,a2,a3,a4];*)
  u = U[#] & /@ Range[8]; Print[u];
  a1 = u /. u2x; Print[a1];
  a2 = Transpose[{{Take[a1, 4]}, {Take[a1, -4]}}]; Print[a2];
  a3 = ExpandAll[FullSimplify[(#\[LeftDoubleBracket]1\[RightDoubleBracket] + #\[LeftDoubleBracket]2\[RightDoubleBracket])/2]] & /@ a2;
  aa4 = ExpandAll[FullSimplify[(#\[LeftDoubleBracket]1\[RightDoubleBracket] - #\[LeftDoubleBracket]2\[RightDoubleBracket])/2]] & /@ a2;
  {a3, aa4}]
```

$$\begin{aligned}
& \{U[1], U[2], U[3], U[4], U[5], U[6], U[7], U[8]\} \\
& \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \\
& \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& - \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \left. - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\} \\
& \left\{ \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \right. \\
& \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \left. \left. \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, - \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \right. \right. \right. \\
& \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\} \right\} \\
& Out[=]
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}}, - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \right. \\
& - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} \}, \left\{ \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \\
& \left. \left. \frac{x4 - \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, - \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\} \right\}
\end{aligned}$$

split octonions; evals, evecs of σ

```

# Type - 1 spinor basis (constructed from Clifford algebra considerations)
# These create a maximal isotropic subspace with respect to the split form
self.type1_spinor_matrix = sqrt2_inv*np.array([[1, 1, 0, 0, 0, 0, 0, 0], # s1_0[1, -1, 0, 0, 0, 0, 0, 0], # s1_
1[0, 0, 1, 1, 0, 0, 0, 0], # s1_2[0, 0, 1, -1, 0, 0, 0, 0], # s1_3[0, 0, 0, 1, 1, 0, 0], # s1_4[0, 0, 0, 1, -1, 0, 0], # s1_
5[0, 0, 0, 0, 0, 1, 1], # s1_6[0, 0, 0, 0, 0, 1, -1], # s1_7])

```

Type - 2 spinor basis (the complementary spinor representation)

```
# Related to Type - 1 by another triality automorphism
self.type2_spinor_matrix = sqrt2_inv*np.array([[1, 0, 1, 0, 0, 0, 0, 0], # s2_0[1, 0, -1, 0, 0, 0, 0, 0], # s2_
1[0, 1, 0, 1, 0, 0, 0, 0], # s2_2[0, 1, 0, -1, 0, 0, 0, 0], # s2_3[0, 0, 0, 1, 0, 1, 0], # s2_4[0, 0, 0, 1, 0, -1, 0], # s2_
5[0, 0, 0, 0, 1, 0, 1], # s2_6[0, 0, 0, 0, 1, 0, -1], # s2_7])
```

$$In[\circ]:= M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

$$Out[\circ]= M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

4 Transformation Matrices

We provide explicit 8×8 matrices that transform coordinates between the three bases.

4.1 Vector to Type-1 Spinor

The transformation matrix from vector basis to type-1 spinor basis is:

$$M_{V \rightarrow S_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (11)$$

In[1]:=

4.2 Vector to Type-2 Spinor

The transformation matrix from vector basis to type-2 spinor basis is:

$$M_{V \rightarrow S_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (12)$$

Out[1]:=

4 Transformation Matrices

We provide explicit 8×8 matrices that transform coordinates between the three bases.

4.1 Vector to Type-1 Spinor

The transformation matrix from vector basis to type-1 spinor basis is:

$$M_{V \rightarrow S_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (11)$$

4.2 Vector to Type-2 Spinor

The transformation matrix from vector basis to type-2 spinor basis is:

$$M_{V \rightarrow S_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (12)$$

```
In[=]:= (type1SpinorBasisMatrix =
  {{1, 1, 0, 0, 0, 0, 0, 0}, {1, -1, 0, 0, 0, 0, 0, 0},
   {0, 0, 1, 1, 0, 0, 0, 0}, {0, 0, 1, -1, 0, 0, 0, 0},
   {0, 0, 0, 0, 1, 1, 0, 0}, {0, 0, 0, 0, 1, -1, 0, 0},
   {0, 0, 0, 0, 0, 0, 1, 1}, {0, 0, 0, 0, 0, 0, 1, -1}}) // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$


In[=]:= (type2SpinorBasisMatrix =
  {{1, 0, 1, 0, 0, 0, 0, 0}, {1, 0, -1, 0, 0, 0, 0, 0},
   {0, 1, 0, 1, 0, 0, 0, 0}, {0, 1, 0, -1, 0, 0, 0, 0},
   {0, 0, 0, 0, 1, 0, 1, 0}, {0, 0, 0, 0, 1, 0, -1, 0},
   {0, 0, 0, 0, 0, 1, 0, 1}, {0, 0, 0, 0, 0, 1, 0, -1}}) // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix}$$

```

$$\text{In}[\#]:= M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

Out[\#]=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\text{In}[\#]:= M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

```
In[]:= MS1S2 = type1SpinorBasisMatrix.Transpose[type2SpinorBasisMatrix] // MatrixForm
Out[//MatrixForm=
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$


```
In[]:= gAB = η4488
Out[=
```

$$\{\{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 0, 0, -1, 0\}, \{0, 0, 0, 0, 0, 0, 0, -1\}\}$$


```
In[]:= {evalsAB, evecsAB} = Eigensystem[gAB]
Out[=
```

$$\{\{-1, -1, -1, -1, 1, 1, 1, 1\}, \{\{0, 0, 0, 0, 0, 0, 0, 1\}, \{0, 0, 0, 0, 0, 0, 1, 0\},$$

$$\{0, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\},$$

$$\{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\}, \{1, 0, 0, 0, 0, 0, 0, 0\}\}\}$$


```
In[]:= {evals, evecs} = Eigensystem[σ]
Out[=
```

$$\{\{-1, -1, -1, -1, 1, 1, 1, 1\}, \{\{0, 0, 0, -1, 0, 0, 0, 1\}, \{0, 0, -1, 0, 0, 0, 1, 0\},$$

$$\{0, -1, 0, 0, 0, 1, 0, 0\}, \{-1, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 1\},$$

$$\{0, 0, 1, 0, 0, 0, 1, 0\}, \{0, 1, 0, 0, 0, 1, 0, 0\}, \{1, 0, 0, 0, 1, 0, 0, 0\}\}\}$$


```
In[]:= u = ExpandAll[1/Sqrt[2] evecs] // MatrixForm
Out[//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$


```
In[]:= Table[u[[h]].σ.u[[h]], {h, 1, Length[u]}]
Out[=
```

$$\{-1, -1, -1, -1, 1, 1, 1, 1\}$$

```
In[1]:= Table[(Transpose[u][h]).σ.u[[h]], {h, 1, Length[u]}]
Out[1]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[2]:= Table[(Transpose[u][A]) === -σ.u[[A]], {A, 1, 4}]
Table[(Transpose[u][A]) === σ.u[[A]], {A, 5, Length[u]}]

Out[2]= {True, True, True, True}

Out[3]= {True, True, True, True}
```

In[4]:= **hUSE = 8;**

Set: Symbol hUSE is Protected. [i](#)

```
In[5]:= Protect[hUSE]
```

```
Out[5]= {}
```

In[6]:= **unit = u[[hUSE]]**

Set: Symbol unit is Protected. [i](#)

```
Out[6]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
```

In[7]:= Protect[unit]

```
Out[7]= {}
```

In[8]:= (Transpose[unit].σ.unit)

```
Out[8]= 1
```

In[9]:= (Transpose[unit].σ.(τ[0]))

```
Out[9]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
```

```
In[1]:= Do[E^A_a[h] = Table[(u[[h]].σ.(τ[B])), {B, 0, 7}], {h, 1, Length[u]}];

$$\mathfrak{F}_a^A = \frac{1}{\sqrt{\tilde{u}\sigma u}} u^c \sigma_{cb} \bar{\tau}^{Ab}_a$$

Do[E^a_A[h] = FullSimplify[Inverse[E^A_a[h]]], {h, 1, Length[u]}];

$$\mathfrak{F}_A^a = \frac{1}{\sqrt{\tilde{u}\sigma u}} \tau_A^a{}_b u^b$$

Table[E^a_A[h] === (u[[h]].σ.u[[h]])
Transpose[Table[FullSimplify[ExpandAll[η4488[[B, B]] × τ[B - 1].u[[h]]]],
{B, 1, 8}]], {h, 1, Length[u]}]
```

Out[1]=

$$\mathfrak{F}_a^A = \frac{1}{\sqrt{\tilde{u}\sigma u}} u^c \sigma_{cb} \bar{\tau}^{Ab}_a$$

Out[2]=

$$\mathfrak{F}_A^a = \frac{1}{\sqrt{\tilde{u}\sigma u}} \tau_A^a{}_b u^b$$

Out[3]=

{True, True, True, True, True, True, True, True}

```
In[2]:= (*Do[E^A_a[h]=Table[(u[[h]].σ.(τ[B])),{B,0,7}],{h,1,Length[u]}];
Do[E^a_A[h]=FullSimplify[Inverse[E^A_a[h]]],{h,1,Length[u]}];
Table[E^a_A[h]===(u[[h]].σ.u[[h]])*
Transpose[Table[FullSimplify[ExpandAll[η4488[[B,B]]τ[B-1].u[[h]]]],
{B,1,8}]],{h,1,Length[u]}]*)
```

```
In[3]:= myid[MX_] := Sum[τ[A].MX.τ[A] η4488[[A, A]], {A, 1, 8}]
Myid[MX_] := Sum[τ[A].MX.τ[A] × η448[[A, A]], {A, 1, 8}]
```

```
Union[Table[
Sum[τ[A].bas64[[k, 1]].σ.τ[A] × η8[[A, A]], {A, 1, 8}] - Id Tr[bas64[[k, 1]].σ] +
4 eg[[k]] 
$$\frac{\text{bas64}[[k, 1]] - \text{Transpose}[\text{bas64}[[k, 1]]]}{2} .\sigma, \{k, 1, 64\}]$$
]
```

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[4]:= τ[θ]

Out[4]=

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1} }
```

In[1]:= **(Transpose[unit].σ. (τ[θ]))**

Out[1]=

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

In[2]:= **F_A^a = Transpose[η4488[[#, 1, # + 1] * (τ[#].unit) & /@ Range[0, 7]]]**

Out[2]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \\ & \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \end{aligned}$$

In[3]:= **F_a^A = Transpose[unit].σ. (τ[#]) & /@ Range[0, 7]**

Out[3]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \\ & \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \\ & \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\} \end{aligned}$$

In[4]:= **F_A^a.F_a^A == ID8**

Out[4]=

True

gtrye_α^(A) = (e_α^(A) /. sgtrye_α^(A)) /. subsDefects

gtrye_(A)^α = (e_(A)^α /. sgtrye_(A)^α) /. subsDefects

(sgtrye_(A)^α = Thread[Flatten[e_(A)^α] → Flatten[Inverse[e_α^(A) /. sgtrye_α^(A)]]])
(*//Column*)

gtrye_(A)^α = (e_(A)^α /. sgtrye_(A)^α) /. subsDefects

Out[5]=

$$\left\{ \{ \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ 0, \frac{e^{-a4[H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right.$$

$$\left\{0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0\right\}, \\ \left\{0, 0, 0, 0, 0, 1, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0\right\}, \\ \left\{0, 0, 0, 0, 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}\right\}$$

pretend that the X are Minkowski Coordinates:

```
In[]:= Block[{U, u = U[#] & /@ Range[8], fx = Fa.X, eq, sx, su, ret},
  eq = And @@ Thread[u == fx];
  sx = Solve[eq, X][[-1]];
  su = Solve[eq, u][[-1]];
  ret = {sx, su};
  ret]
```

Out[=]=

$$\left\{x0 \rightarrow \frac{U[1]}{\sqrt{2}} + \frac{U[5]}{\sqrt{2}}, x1 \rightarrow -\frac{U[4]}{\sqrt{2}} - \frac{U[8]}{\sqrt{2}}, x2 \rightarrow \frac{U[3]}{\sqrt{2}} + \frac{U[7]}{\sqrt{2}}, \right. \\ x3 \rightarrow -\frac{U[2]}{\sqrt{2}} - \frac{U[6]}{\sqrt{2}}, x4 \rightarrow \frac{1}{2} (\sqrt{2} U[2] - \sqrt{2} U[6]), x5 \rightarrow \frac{1}{2} (-\sqrt{2} U[3] + \sqrt{2} U[7]), \\ x6 \rightarrow \frac{1}{2} (-\sqrt{2} U[4] + \sqrt{2} U[8]), x7 \rightarrow \frac{1}{2} (\sqrt{2} U[1] - \sqrt{2} U[5])\}, \\ \left\{U[1] \rightarrow \frac{1}{2} (\sqrt{2} x0 + \sqrt{2} x7), U[2] \rightarrow \frac{1}{2} (-\sqrt{2} x3 + \sqrt{2} x4), U[3] \rightarrow \frac{1}{2} (\sqrt{2} x2 - \sqrt{2} x5), \right. \\ U[4] \rightarrow \frac{1}{2} (-\sqrt{2} x1 - \sqrt{2} x6), U[5] \rightarrow \frac{1}{2} (\sqrt{2} x0 - \sqrt{2} x7), U[6] \rightarrow \frac{1}{2} (-\sqrt{2} x3 - \sqrt{2} x4), \\ U[7] \rightarrow \frac{1}{2} (\sqrt{2} x2 + \sqrt{2} x5), U[8] \rightarrow \frac{1}{2} (-\sqrt{2} x1 + \sqrt{2} x6)\}$$

```
In[]:= (sgtrye(A) = Thread[Flatten[e(A)] → Flatten[eAa04]]) (* //Column*)
```

In[]:= **subsDefects**

Out[=]=

$$\left\{\sqrt{e^{2a4[Hx4]}} \rightarrow e^{a4[Hx4]}, \sqrt{e^{-2a4[Hx4]}} \rightarrow e^{-a4[Hx4]}, \right. \\ \sqrt{e^{2a4[Hx4]}} \sin[6Hx4]^{1/3} \rightarrow e^{a4[Hx4]} \sin[6Hx4]^{1/6}, \\ \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx4]^{1/3}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx4]^{1/6}}, \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, \\ \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}} \rightarrow \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, \\ \left.\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{a4[Hx4]} \sin[6Hx0]^{1/6}, \sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{-a4[Hx4]} \sin[6Hx0]^{1/6}\right\}$$

```
In[1]:= Thread[X → 0]
Out[1]= {x0 → 0, x1 → 0, x2 → 0, x3 → 0, x4 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Stop pretending:

$$(sgtrye_{(A)}^{\alpha} = Thread[Flatten[e_{(A)}^{\alpha}] \rightarrow Flatten[Inverse[e_{\alpha}^{(A)} /. sgtrye_{\alpha}^{(A)}]]]) \\ (*//Column*)$$

$$\begin{aligned} gtrye_{(A)}^{\alpha} &= (e_{(A)}^{\alpha} /. sgtrye_{(A)}^{\alpha}) /. \text{subsDefects} \\ &\left\{ \{ \text{Tan}[6 H x0], 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ 0, \frac{e^{-a4[H x4]}}{\text{Sin}[6 H x0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, \frac{e^{-a4[H x4]}}{\text{Sin}[6 H x0]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{e^{-a4[H x4]}}{\text{Sin}[6 H x0]^{1/6}}, 0, 0, 0, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{e^{a4[H x4]}}{\text{Sin}[6 H x0]^{1/6}}, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 0, 0, \frac{e^{a4[H x4]}}{\text{Sin}[6 H x0]^{1/6}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[H x4]}}{\text{Sin}[6 H x0]^{1/6}} \right\} \right\} \end{aligned}$$

$$In[2]:= e_{\alpha}^{(A)} = e A \alpha = gtrye_{\alpha}^{(A)}$$

$$\begin{aligned} Out[2]= &\left\{ \{ \text{Cot}[6 H x0], 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ 0, e^{a4[H x4]} \text{Sin}[6 H x0]^{1/6}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, e^{a4[H x4]} \text{Sin}[6 H x0]^{1/6}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, e^{a4[H x4]} \text{Sin}[6 H x0]^{1/6}, 0, 0, 0, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, e^{-a4[H x4]} \text{Sin}[6 H x0]^{1/6}, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 0, 0, e^{-a4[H x4]} \text{Sin}[6 H x0]^{1/6}, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, 0, 0, 0, 0, 0, e^{-a4[H x4]} \text{Sin}[6 H x0]^{1/6} \right\} \right\} \end{aligned}$$

$$In[3]:= gtrye_{\alpha}^{(A)} = (e_{\alpha}^{(A)} /. sgtrye_{\alpha}^{(A)}) /. \text{subsDefects}$$

$$In[4]:= gtrye_{(A)}^{\alpha} = (e_{(A)}^{\alpha} /. sgtrye_{(A)}^{\alpha}) /. \text{subsDefects}$$

$$\begin{aligned}
In[=] &:= \{\mathbf{gtrye}_{\alpha}^{(A)}, \mathbf{gtrye}_{(A)}^{\alpha}\} // \text{MatrixForm} \\
Out[=] &= \text{MatrixForm} = \\
&\left(\begin{array}{c} \text{Cot}[6 H x \theta] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right) \left(\begin{array}{c} 0 \\ e^{a4[H x 4]} \sin[6 H x \theta]^{1/6} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right) \left(\begin{array}{c} 0 \\ 0 \\ e^{a4[H x 4]} \sin[6 H x \theta]^{1/6} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right) \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right) \\
&\left(\begin{array}{c} \tan[6 H x \theta] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right) \left(\begin{array}{c} 0 \\ \frac{e^{-a4[H x 4]}}{\sin[6 H x \theta]^{1/6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right) \left(\begin{array}{c} 0 \\ 0 \\ \frac{e^{-a4[H x 4]}}{\sin[6 H x \theta]^{1/6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right) \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ \frac{e^{-a4[H x 4]}}{\sin[6 H x \theta]^{1/6}} \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right)
\end{aligned}$$

```

In[=]:= F_A^a.e_{\alpha}^{(A)}.X /. sgtrye_{\alpha}^{(A)} // FullSimplify[#, constraintVars] &;
frameOctadX = # /. {Sqrt[e^2 a4[H x 4]] -> e^{a4[H x 4]}, Sqrt[e^-2 a4[H x 4]] -> e^-a4[H x 4]} & /@ %
% /. {x1 -> 0, x2 -> 0, x3 -> 0, x5 -> 0, x6 -> 0, x7 -> 0}

```

$$\begin{aligned}
Out[=] &= \left\{ \frac{x_0 \cot[6 H x \theta] + e^{-a4[H x 4]} x_7 \sin[6 H x \theta]^{1/6}}{\sqrt{2}}, \frac{x_4 - e^{a4[H x 4]} x_3 \sin[6 H x \theta]^{1/6}}{\sqrt{2}}, \right. \\
&\quad \frac{e^{-a4[H x 4]} (e^{2 a4[H x 4]} x_2 - x_5) \sin[6 H x \theta]^{1/6}}{\sqrt{2}}, -\frac{e^{-a4[H x 4]} (e^{2 a4[H x 4]} x_1 + x_6) \sin[6 H x \theta]^{1/6}}{\sqrt{2}}, \\
&\quad \frac{x_0 \cot[6 H x \theta] - e^{-a4[H x 4]} x_7 \sin[6 H x \theta]^{1/6}}{\sqrt{2}}, -\frac{x_4 + e^{a4[H x 4]} x_3 \sin[6 H x \theta]^{1/6}}{\sqrt{2}}, \\
&\quad \left. \frac{e^{-a4[H x 4]} (e^{2 a4[H x 4]} x_2 + x_5) \sin[6 H x \theta]^{1/6}}{\sqrt{2}}, \frac{e^{-a4[H x 4]} (-e^{2 a4[H x 4]} x_1 + x_6) \sin[6 H x \theta]^{1/6}}{\sqrt{2}} \right\}
\end{aligned}$$

$$Out[=] = \left\{ \frac{x_0 \cot[6 H x \theta]}{\sqrt{2}}, \frac{x_4}{\sqrt{2}}, 0, 0, \frac{x_0 \cot[6 H x \theta]}{\sqrt{2}}, -\frac{x_4}{\sqrt{2}}, 0, 0 \right\}$$

```

In[=]:= F_A^a.eA\alpha.X - frameOctadX // FullSimplify[#, constraintVars] &
Out[=] = {0, 0, 0, 0, 0, 0, 0, 0}

```

In[1]:= $\mathbf{eA}\alpha$

Out[1]=

$$\begin{aligned} & \left\{ \{\text{Cot}[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0\}, \right. \\ & \left. \{0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \right. \\ & \left. \{0, 0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0\}, \right. \\ & \left. \{0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0\}, \right. \\ & \left. \{0, 0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}\} \right\} \end{aligned}$$

In[2]:= $\{\{F_A^a\}, \{\mathbf{eA}\alpha\}, \{X\}\} // \text{Column}$

Out[2]=

$$\begin{aligned} & \left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\ & \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \right. \\ & \left. \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\ & \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \right\} \\ & \left\{ \{\text{Cot}[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0\}, \right. \\ & \left. \{0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \right. \\ & \left. \{0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0\}, \right. \\ & \left. \{0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0\}, \right. \\ & \left. \{0, 0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}\} \right\} \\ & \{x0, x1, x2, x3, x4, x5, x6, x7\} \end{aligned}$$

split octonion multiplication constants:

$$m_{ab}^c = \mathbb{F}_a^A \tau_A{}_b{}^c$$

$$m_{AB}^C = \mathbb{F}_c^C \tau_A{}_b \mathbb{F}_B^b$$

In[3]:= $(\mathbf{xw})w = \mathbf{xw}^2$

Out[3]=

$$(\mathbf{xw})w = \mathbf{xw}^2$$

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

In[1]:=

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex structure J_u . If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on \mathbf{O} . However, because S^6 is connected, we see that the orientation of \mathbf{O} induced by the natural orientation of \mathbf{O}_u as a complex vector space does not depend on u . We refer to this orientation as the natural orientation of \mathbf{O} .

Out[1]:=

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Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

In[]:= so J_u is an isometry for each $u \in S$. Moreover, it follows that \mathbf{O}_u is endowed with a natural Hermitian structure with respect to the inner product $\langle \cdot, \cdot \rangle$. We denote the group of complex linear transformations of \mathbf{O}_u by $GL(\mathbf{O}_u)$ and the special unitary transformations of \mathbf{O}_u with its Hermitian metric by $SU(\mathbf{O}_u)$.

Out[]=

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J_u

In[]:=

Table[FullSimplify[

$$\text{Sum}[\psi[A1] \times J[B1] g_{AB}[A1, A1] E^A_a[\text{hUSE}] \llbracket C \rrbracket \llbracket c \rrbracket \tau[A1 - 1] \llbracket c \rrbracket \llbracket b \rrbracket E^a_A[\text{hUSE}] \llbracket b \rrbracket \llbracket B1 \rrbracket, \\ \{c, 1, 8\}, \{b, 1, 8\}, \{A1, 1, 8\}, \{B1, 1, 8\}], \{C, 1, 8\}]$$

Out[]=

$$\{J[1] \times \psi[1] - J[2] \times \psi[2] - J[3] \times \psi[3] - J[4] \times \psi[4] + J[5] \times \psi[5] + \\ J[6] \times \psi[6] + J[7] \times \psi[7] + J[8] \times \psi[8], J[2] \times \psi[1] + J[1] \times \psi[2] - \\ J[4] \times \psi[3] + J[3] \times \psi[4] + J[6] \times \psi[5] - J[5] \times \psi[6] - J[8] \times \psi[7] + J[7] \times \psi[8], \\ J[3] \times \psi[1] + J[4] \times \psi[2] + J[1] \times \psi[3] - J[2] \times \psi[4] + J[7] \times \psi[5] + J[8] \times \psi[6] - \\ J[5] \times \psi[7] - J[6] \times \psi[8], J[4] \times \psi[1] - J[3] \times \psi[2] + J[2] \times \psi[3] + \\ J[1] \times \psi[4] + J[8] \times \psi[5] - J[7] \times \psi[6] + J[6] \times \psi[7] - J[5] \times \psi[8], \\ J[5] \times \psi[1] + J[6] \times \psi[2] + J[7] \times \psi[3] + J[8] \times \psi[4] + J[1] \times \psi[5] - J[2] \times \psi[6] - \\ J[3] \times \psi[7] - J[4] \times \psi[8], J[6] \times \psi[1] - J[5] \times \psi[2] + J[8] \times \psi[3] - \\ J[7] \times \psi[4] + J[2] \times \psi[5] + J[1] \times \psi[6] + J[4] \times \psi[7] - J[3] \times \psi[8], \\ J[7] \times \psi[1] - J[8] \times \psi[2] - J[5] \times \psi[3] + J[6] \times \psi[4] + J[3] \times \psi[5] - J[4] \times \psi[6] + \\ J[1] \times \psi[7] + J[2] \times \psi[8], J[8] \times \psi[1] + J[7] \times \psi[2] - J[6] \times \psi[3] - \\ J[5] \times \psi[4] + J[4] \times \psi[5] + J[3] \times \psi[6] - J[2] \times \psi[7] + J[1] \times \psi[8]\}$$


```
In[8]:= Table[
  Transpose[FullSimplify[ExpandAll[gAB[[B, B]] E^A_a[hUSE].\tau[B - 1].E^a_A[hUSE]]]],
  {B, 1, 8}] // MatrixForm
```

```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$


$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$


```

$\backslash \cup /$	$\backslash \perp /$	$\backslash \cup /$					
(0)	(0)	(-1)	(0)	(0)	(0)	(0)	(0)
0	0	0	-1	0	0	0	0
1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1
0	0	0	0	-1	0	0	0
0	0	0	0	0	-1	0	0
(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
(0)	(0)	(0)	(-1)	(0)	(0)	(0)	(0)
0	0	1	0	0	0	0	0
0	-1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1
0	0	0	0	0	0	-1	0
0	0	0	0	0	1	0	0
0	0	0	0	-1	0	0	0
(0)	(0)	(0)	(0)	(1)	(0)	(0)	(0)
0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	0
(0)	(0)	(0)	(0)	(0)	(1)	(0)	(0)
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1
0	0	0	0	0	0	-1	0
0	-1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	0
0	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0
(0)	(0)	(0)	(0)	(0)	(0)	(1)	(0)
0	0	0	0	0	0	0	-1
0	0	0	0	-1	0	0	0
0	0	0	0	0	1	0	0
0	0	-1	0	0	0	0	0
0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0
(0)	(0)	(0)	(0)	(0)	(0)	(0)	(1)
0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0
0	0	0	0	-1	0	0	0
0	0	0	-1	0	0	0	0
0	0	-1	0	0	0	0	0
0	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0

In[8]:= EA = Array[eA, 8]

```

Out[ ]=
{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[ ]:= e(A)α
Out[ ]=
{{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0}, {0, ea4[H x4] Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, ea4[H x4] Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, 0, ea4[H x4] Sin[6 H x0]1/6, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e-a4[H x4] Sin[6 H x0]1/6, 0, 0}, {0, 0, 0, 0, 0, 0, e-a4[H x4] Sin[6 H x0]1/6, 0}, {0, 0, 0, 0, 0, 0, 0, e-a4[H x4] Sin[6 H x0]1/6}}

In[ ]:= h = hUSE;
B = 1; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 2; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 3; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 4; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 5; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 6; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 7; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 8;
FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]

Out[ ]=
{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

Out[ ]=
{eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}

Out[ ]=
{eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}

Out[ ]=
{eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]}

Out[ ]=
{eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]}

Out[ ]=
{eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]}

Out[ ]=
{eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]}

Out[ ]=
{eA[8], eA[7], -eA[6], -eA[5], -eA[4], -eA[3], eA[2], eA[1]}

```

```
In[]:= B = 1;
Grid[
{EA, FullSimplify[ExpandAll[ gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h] ]], Frame -> All}]
```

Out[]=

eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]

```
In[]:= gridx = Grid[Partition[Flatten[{{{x}, EA}, Table[
({{x}, FullSimplify[ExpandAll[ gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h] ]]} /.
{x \rightarrow ToExpression["eA[" <> ToString[B] <> "]"]}),
{B, 1, 8}]}], 9], Frame -> All]
```

Out[]=

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

$\epsilon_A =$	$\epsilon_B =$							
	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8
ϵ_1	$-\epsilon_8$	ϵ_3	$-\epsilon_2$	$-\epsilon_5$	ϵ_4	$-\epsilon_7$	ϵ_6	ϵ_1
ϵ_2	$-\epsilon_3$	$-\epsilon_8$	ϵ_1	$-\epsilon_6$	ϵ_7	ϵ_4	$-\epsilon_5$	ϵ_2
ϵ_3	ϵ_2	$-\epsilon_1$	$-\epsilon_8$	$-\epsilon_7$	$-\epsilon_6$	ϵ_5	ϵ_4	ϵ_3
ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4
ϵ_5	$-\epsilon_4$	$-\epsilon_7$	ϵ_6	$-\epsilon_1$	ϵ_8	ϵ_3	$-\epsilon_2$	ϵ_5
ϵ_6	ϵ_7	$-\epsilon_4$	$-\epsilon_5$	$-\epsilon_2$	$-\epsilon_3$	ϵ_8	ϵ_1	ϵ_6
ϵ_7	$-\epsilon_6$	ϵ_5	$-\epsilon_4$	$-\epsilon_3$	ϵ_2	$-\epsilon_1$	ϵ_8	ϵ_7
ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8

Out[]=

$\epsilon_A =$	$\epsilon_B =$							
	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8
ϵ_1	$-\epsilon_8$	ϵ_3	$-\epsilon_2$	$-\epsilon_5$	ϵ_4	$-\epsilon_7$	ϵ_6	ϵ_1
ϵ_2	$-\epsilon_3$	$-\epsilon_8$	ϵ_1	$-\epsilon_6$	ϵ_7	ϵ_4	$-\epsilon_5$	ϵ_2
ϵ_3	ϵ_2	$-\epsilon_1$	$-\epsilon_8$	$-\epsilon_7$	$-\epsilon_6$	ϵ_5	ϵ_4	ϵ_3
ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4
ϵ_5	$-\epsilon_4$	$-\epsilon_7$	ϵ_6	$-\epsilon_1$	ϵ_8	ϵ_3	$-\epsilon_2$	ϵ_5
ϵ_6	ϵ_7	$-\epsilon_4$	$-\epsilon_5$	$-\epsilon_2$	$-\epsilon_3$	ϵ_8	ϵ_1	ϵ_6
ϵ_7	$-\epsilon_6$	ϵ_5	$-\epsilon_4$	$-\epsilon_3$	ϵ_2	$-\epsilon_1$	ϵ_8	ϵ_7
ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8

1	i	j	k	ℓ	li	ℓj	ℓk
i	-1	k	$-j$	$-li$	ℓ	$-\ell k$	ℓj
j	$-k$	-1	i	$-\ell j$	ℓk	ℓ	$-li$
k	j	$-i$	-1	$-\ell k$	$-\ell j$	li	ℓ
ℓ	li	ℓj	ℓk	1	i	j	k
li	$-\ell$	$-\ell k$	ℓj	$-i$	1	k	$-j$
ℓj	ℓk	$-\ell$	$-li$	$-j$	$-k$	1	i
ℓk	$-\ell j$	li	$-\ell$	$-k$	j	$-i$	1

```
In[8]:= (τABC = Table[ExpandAll[gAB[[B, B]] E^A_a[hUSE].τ[B - 1].E^a_A[hUSE]], {B, 1, 8}]);  
(τABC[[#]] // MatrixForm) & /@ Range[8]
```

Out[8]=

$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\}$$

In[9]:= EA

Out[9]= {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[10]:= EA.τABC[[1]]

Out[10]= {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[11]:= EA.τABC[[2]]

Out[11]= {eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}

In[12]:= EA.τABC[[3]]

Out[12]= {eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}

In[1]:= EA. τ ABC[[4]]

Out[1]=

$$\{eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]\}$$

In[2]:= EA. τ ABC[[5]]

Out[2]=

$$\{eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]\}$$

In[3]:= EA. τ ABC[[6]]

Out[3]=

$$\{eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]\}$$

In[4]:= EA. τ ABC[[7]]

Out[4]=

$$\{eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]\}$$

In[5]:= TeXForm[gridx]

Out[5]//TeXForm=

```
\begin{array}{cccccccccc}
x & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \text{eA}(4) & \\
& \text{eA}(5) & \text{eA}(6) & \text{eA}(7) & \text{eA}(8) \\
\text{eA}(1) & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \\
& \text{eA}(4) & \text{eA}(5) & \text{eA}(6) & \text{eA}(7) & \\
& \text{eA}(8) \\
\text{eA}(2) & \text{eA}(2) & -\text{eA}(1) & -\text{eA}(4) & \\
& \text{eA}(3) & -\text{eA}(6) & \text{eA}(5) & \text{eA}(8) & \\
& -\text{eA}(7) \\
\text{eA}(3) & \text{eA}(3) & \text{eA}(4) & -\text{eA}(1) & \\
& -\text{eA}(2) & -\text{eA}(7) & -\text{eA}(8) & \text{eA}(5) & \\
& \text{eA}(6) \\
\text{eA}(4) & \text{eA}(4) & -\text{eA}(3) & \text{eA}(2) & \\
& -\text{eA}(1) & -\text{eA}(8) & \text{eA}(7) & -\text{eA}(6) & \\
& \text{eA}(5) \\
\text{eA}(5) & \text{eA}(5) & \text{eA}(6) & \text{eA}(7) & \\
& \text{eA}(8) & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \\
& \text{eA}(4) \\
\text{eA}(6) & \text{eA}(6) & -\text{eA}(5) & \text{eA}(8) & \\
& -\text{eA}(7) & -\text{eA}(2) & \text{eA}(1) & -\text{eA}(4) & \\
& \text{eA}(3) \\
\text{eA}(7) & \text{eA}(7) & -\text{eA}(8) & -\text{eA}(5) & \\
& \text{eA}(6) & -\text{eA}(3) & \text{eA}(4) & \text{eA}(1) & \\
& -\text{eA}(2) \\
\text{eA}(8) & \text{eA}(8) & \text{eA}(7) & -\text{eA}(6) & \\
& -\text{eA}(5) & -\text{eA}(4) & -\text{eA}(3) & \text{eA}(2) & \\
& \text{eA}(1)
\end{array}
```

In[6]:= (*StringReplace[ToString[TeXForm[gridx]], "\text{eA}" \rightarrow "\epsilon"]*)

```
In[=]:= Grid[Partition[
  Flatten[{{{x}, EA}, Table[{({{x}, Table[Sum[FullSimplify[ExpandAll[gAB[[B, B]] EA[[C1]] E^A_a[hUSE][[C1, c1]] \[Tau][B - 1][[c1, d1]] E^a_A[hUSE][[d1, B1]]]], {C1, 1, 8}, {c1, 1, 8}, {d1, 1, 8}], {B1, 1, 8}}] /. {x \[Rule] ToExpression["eA[" \<> ToString[B] \<> "]"]}, {B, 1, 8}]}}], 9], Frame \[Rule] All]
Out[=]=
```

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

```
In[=]:= gridx
Out[=]=
```

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

Complex Structure

J_u

```
In[=]:= J[#] \[Rule] 0 & /@ Range[0, 7]
Out[=]= {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 0, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}

In[=]:= sJu = {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 1, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}
Out[=]= {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 1, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}
```

for $\frac{1}{i}$:

```
In[8]:= Block[{ψ, J, t, r, sJu},
  sJu =
  {J[0] → 0, J[1] → 0, J[2] → 0, J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0};
  t = Table[FullSimplify[Sum[ψ[A1 - 1] × J[B1 - 1] gAB[[A1, A1]] E^A_a[hUSE][[C]][c] τ[A1 - 1][c][b] E^a_A[hUSE][[b]][B1]],
    {c, 1, 8}, {b, 1, 8}, {A1, 1, 8}, {B1, 1, 8}]], {C, 1, 8}];
  r = t /. sJu
]
```

Out[8]=
 $\{\psi[4], -\psi[5], -\psi[6], -\psi[7], \psi[0], -\psi[1], -\psi[2], -\psi[3]\}$

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \tilde{\tau}^A \sigma$$

In[9]:= $\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2I_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A$,

8 unit matrix. Denoting the matrix elemen

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

042501-7 Second gravity

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$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix} \quad (10)$$

of the 16-component spinor representation of $\overline{\text{SO}(4,4;\mathbb{R})}$. We see that, as is, in fact, well known from the general theory, the 16-component spinor representation of $\text{SO}(4,4;\mathbb{R})$ is the direct sum of two (inequivalent) real 8×8 irreducible spinor representations $D_{(1)}=D_{(1)}(g)$ and $D_{(2)}=D_{(2)}(g)$ of $\text{SO}(4,4;\mathbb{R}) \ni g$ that are generated by $D_{(1)}^{AB}$ and $D_{(2)}^{AB}$, respectively. This we record as

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A \quad (11)$$

and

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A. \quad (12)$$

Out[]=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \widetilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elements

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

042501-7 Second gravity

J. Math. Phys. 51, 042501 (2010)

$$\tau^A t^B - t^B \tau^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix} \quad (10)$$

of the 16-component spinor representation of $\overline{\text{SO}(4,4;\mathbb{R})}$. We see that, as is, in fact, well known from the general theory, the 16-component spinor representation of $\text{SO}(4,4;\mathbb{R})$ is the direct sum of two (inequivalent) real 8×8 irreducible spinor representations $D_{(1)} = D_{(1)}(g)$ and $D_{(2)} = D_{(2)}(g)$ of $\text{SO}(4,4;\mathbb{R}) \ni g$ that are generated by $D_{(1)}^{AB}$ and $D_{(2)}^{AB}$, respectively. This we record as

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A \quad (11)$$

and

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A. \quad (12)$$

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$$\widetilde{D}_{(1)}^{AB}\sigma = -\sigma D_{(1)}^{AB} \Rightarrow \tilde{D}_{(1)}\sigma = \sigma D_{(1)}^{-1}, \quad (18)$$

$$\widetilde{D}_{(2)}^{AB}\sigma = -\sigma D_{(2)}^{AB} \Rightarrow \tilde{D}_{(2)}\sigma = \sigma D_{(2)}^{-1}, \quad (19)$$

$$L^A{}_C G_{AB} L^B{}_D = G_{CD} = \{\tilde{L}GL\}_{CD}, \quad (20)$$

$$L^A{}_B \bar{\tau}^B = D_{(1)}^{-1} \bar{\tau}^A D_{(2)}, \quad (21)$$

$$L^A{}_B \tau^B = D_{(2)}^{-1} \tau^A D_{(1)}, \quad (22)$$

The canonical 2-1 homomorphism $\overline{\mathrm{SO}(4,4;\mathbb{R})} \rightarrow \mathrm{SO}(4,4;\mathbb{R}): g \mapsto L(g)$ is given by

$$8L^A{}_B = \mathrm{tr}(D_{(1)}^{-1} \bar{\tau}^A D_{(2)} \tau^B) G_{CB}, \quad (23)$$

where tr denotes the trace. Note that $D_{(1)}(g(\omega)) = D_{(2)}(g(\omega))$ when $\omega_{A8} = 0$, i.e., when one restricts $\overline{\mathrm{SO}(4,4;\mathbb{R})}$ to

$$\overline{\mathrm{SO}(3,4;\mathbb{R})} = \{g \in \overline{\mathrm{SO}(4,4;\mathbb{R})} \mid ,$$

$$g = \begin{pmatrix} \exp\left(\frac{1}{4}\omega_{AB} D_{(1)}^{AB}\right) & 0 \\ 0 & \exp\left(\frac{1}{4}\omega_{AB} D_{(2)}^{AB}\right) \end{pmatrix} \quad \text{and} \quad \omega_{A8} = 0 \} \quad (24)$$

[one of the real forms of spin (7, C)].

Out[]=

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$$\widetilde{D}_{(1)}^{AB}\sigma = -\sigma D_{(1)}^{AB} \Rightarrow \tilde{D}_{(1)}\sigma = \sigma D_{(1)}^{-1}, \quad (18)$$

$$\widetilde{D}_{(2)}^{AB}\sigma = -\sigma D_{(2)}^{AB} \Rightarrow \tilde{D}_{(2)}\sigma = \sigma D_{(2)}^{-1}, \quad (19)$$

$$L^A{}_C G_{AB} L^B{}_D = G_{CD} = \{\tilde{L}GL\}_{CD}, \quad (20)$$

$$L^A{}_B \bar{\tau}^B = D_{(1)}^{-1} \bar{\tau}^A D_{(2)}, \quad (21)$$

$$L^A{}_B \tau^B = D_{(2)}^{-1} \tau^A D_{(1)}, \quad (22)$$

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where tr denotes the trace. Note that $D_{(1)}(g(\omega)) = D_{(2)}(g(\omega))$ when $\omega_{A8} = 0$, i.e., when one restricts $\overline{\mathrm{SO}(4,4;\mathbb{R})}$ to

$$\overline{\mathrm{SO}(3,4;\mathbb{R})} = \{g \in \overline{\mathrm{SO}(4,4;\mathbb{R})} \mid$$

$$g = \begin{pmatrix} \exp\left(\frac{1}{4}\omega_{AB} D_{(1)}^{AB}\right) & 0 \\ 0 & \exp\left(\frac{1}{4}\omega_{AB} D_{(2)}^{AB}\right) \end{pmatrix} \quad \text{and} \quad \omega_{A8} = 0 \} \quad (24)$$

[one of the real forms of spin (7, C)].

```
In[ ]:=
Block[{\psi, J, t, r, sJu},
sJu = {J[0] \rightarrow 0, J[1] \rightarrow 0, J[2] \rightarrow 0,
       J[3] \rightarrow 0, J[4] \rightarrow 1, J[5] \rightarrow 0, J[6] \rightarrow 0, J[7] \rightarrow 0};
r = {\psi[4], -\psi[5], -\psi[6], -\psi[7], \psi[0], -\psi[1], -\psi[2], -\psi[3]};
\psi = \tau[A].\tau[B].\#16[[1;;8]];
]
```

Out[]=

```
\tau[A].{f16[7][x0, x4], f16[6][x0, x4], -f16[5][x0, x4], -f16[4][x0, x4],
      f16[3][x0, x4], f16[2][x0, x4], -f16[1][x0, x4], -f16[0][x0, x4]}

\tau[A1].\tau[B1] \rightarrow D_{(1)}^{-1}.\tau[A1].\tau[B1].D_{(1)};
\tau[A1].\tau[B1] \rightarrow D_{(2)}^{-1}.\tau[A1].\tau[B1].D_{(2)};
\sigma.\tau[A1].\tau[B1] = Transpose[\tau[A1]].\sigma.\tau[B1] =
      Transpose[\tau[A1]].Transpose[\sigma.\tau[B1]] = Transpose[\sigma.\tau[B1].\tau[A1]];
\sigma.\tau[A1].\tau[B1] = Transpose[\tau[B1].\tau[A1]].Transpose[\sigma];
\sigma.\tau[A1].\tau[B1].\psi_1 = Transpose[\tau[B1].\tau[A1]].Transpose[\sigma].\psi_1;
Transpose[\psi_2].\sigma \leftrightarrow Transpose[\sigma].\psi_1 ? ;
\sigma.\tau[A1].\tau[B1].\psi_1 \leftrightarrow Transpose[\tau[B1].\tau[A1]].Transpose[\psi_2].\sigma;
\sigma.\tau[A1].\tau[B1].\psi_1 \leftrightarrow Transpose[\psi_2.\tau[B1].\tau[A1]].\sigma;
```

```

B1 = A1
 $\sigma.\psi_1 \leftrightarrow \text{Transpose}[\psi_2].\sigma^{-1}$  ?;
 $\psi_1 \leftrightarrow \text{Transpose}[\psi_2].\sigma^{-1}$  ?;
 $\text{Transpose}[\psi_2] \leftrightarrow \sigma.\psi_1$  ?;

In[1]:= {ψ1[4], -ψ1[5], -ψ1[6], -ψ1[7], ψ1[0], -ψ1[1], -ψ1[2], -ψ1[3]} /. {ψ1[#] →};

In[2]:= (*Block[{ψ1,ψ2,σψ,sσψ,r,sJ,u},
σψ=σ.(ψ2[#]&/@Range[0,7]);
sσψ=(ψ1[#]→σψ[[#+1]]&/@Range[0,7]);
r={ψ1[4],-ψ1[5],-ψ1[6],-ψ1[7],ψ1[0],-ψ1[1],-ψ1[2],-ψ1[3]};
(*ψ=τ[A].τ[B].Π16[[1;;8]]*)
r/.sσψ
]*)

In[3]:= Block[{ψ1,ψ2,σψ,sσψ,r},
σψ=σ.(ψ2[#]&/@Range[0,7]);
sσψ=(ψ1[#]→σψ[[#+1]]&/@Range[0,7]);
r={ψ1[4],-ψ1[5],-ψ1[6],-ψ1[7],ψ1[0],-ψ1[1],-ψ1[2],-ψ1[3]};
(*ψ=τ[A].τ[B].Π16[[1;;8]]*)
r/.sσψ
]

Out[3]= {ψ2[0], -ψ2[1], -ψ2[2], -ψ2[3], ψ2[4], -ψ2[5], -ψ2[6], -ψ2[7]}

```

for $\frac{1}{i}$:

```

Π16cccc = Block[{ψ1,ψ2,σψ,sσψ,sψ1,sψ2,r},
σψ=σ.(Π16[[9;;16]]);
sσψ=(ψ1[#]→σψ[[#+1]]&/@Range[0,7]);
sψ1=Thread[(ψ1[#]&/@Range[0,7])→Π16[[1;;8]]];
sψ2=Thread[(ψ1[#]&/@Range[0,7])→Π16[[9;;16]]];
r={ψ1[4],-ψ1[5],-ψ1[6],-ψ1[7],ψ1[0],-ψ1[1],-ψ1[2],-ψ1[3]};
(*ψ=τ[A].τ[B].*)
Flatten[{r/.sψ1,r/.sψ2}]
]

Out[4]= {f16[4][x0,x4], -f16[5][x0,x4], -f16[6][x0,x4], -f16[7][x0,x4],
f16[0][x0,x4], -f16[1][x0,x4], -f16[2][x0,x4], -f16[3][x0,x4],
f16[12][x0,x4], -f16[13][x0,x4], -f16[14][x0,x4], -f16[15][x0,x4],
f16[8][x0,x4], -f16[9][x0,x4], -f16[10][x0,x4], -f16[11][x0,x4]}

```

```
In[•]:= caccc = CoefficientArrays[¶16cccc - ¶16, ¶16cccc]
Out[•]= {SparseArray[ Specified elements: 12 Dimensions: {16}], SparseArray[ Specified elements: 8 Dimensions: {16, 16}]}
```

```
In[•]:= (caΨccΨ2 = Normal[caΨccΨ[[2]]]) // MatrixForm
```

Out[•]//MatrixForm=

```
In[•]:= caΨccΨ = CoefficientArrays[Ψ16cccc - Ψ16, Ψ16]
```

Out[•]=

`{SparseArray[+ Specified elements: 0 Dimensions: {16}], SparseArray[+ Specified elements: 32 Dimensions: {16, 16}]}`

```
In[•]:= (ca甫cc甫2 = Normal[ca甫cc甫[2]]) // MatrixForm
```

Out[•]//MatrixForm=

In[1]:= $\text{ca}\Psi\text{cc}\Psi 2 == \text{Transpose}[\text{ca}\Psi\text{cc}\Psi 2]$

Out[1]=

True

In[2]:= $\text{ca}\Psi\text{cc}\Psi 2.\text{ca}\Psi\text{cc}\Psi 2 // \text{MatrixForm}$

Out[2]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In[3]:= $\text{ca}\Psi\text{cc}\Psi 2.\Psi 16cccc - \Psi 16 + \Psi 16cccc$

$\text{ca}\Psi\text{cc}\Psi 2.\Psi 16 - \Psi 16cccc + \Psi 16$

Out[3]=

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

Out[4]=

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

for $\frac{1}{i}$:

In[5]:= $\text{jjcc} = \text{ca}\Psi\text{cc}\Psi 2 + \text{ID16};$

In[6]:= $\text{jjcc}.\Psi 16 == \Psi 16cccc$

Out[6]=

True

In[7]:= $\bar{x} = 2\langle x, 1 \rangle - x.$

Out[7]=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

In[8]:= $\bar{x} = 2\langle x, 1 \rangle - x.$

Out[8]=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

$F_A^a = \text{Transpose}[\eta 4488[\#, 1, \# + 1] * (\tau[\#].\text{unit}) \& /@ \text{Range}[0, 7]]$

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}$$

```
FaA = Transpose[unit].σ.(π[#]) & /@ Range[0, 7]
```

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\}$$

In[8]:= F_a^A

Out[8]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\}$$

In[9]:= unit

Out[9]=

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

In[10]:= (F_a^A.unit)

Out[10]=

$$\{1, 0, 0, 0, 0, 0, 0, 0\}$$

In[11]:= (F_a^A[8].unit)

Out[11]=

$$0$$


```
In[1]:= realProjection16.realProjection16 === realProjection16
Out[1]= True
```

In[2]:= We denote $\langle x, 1 \rangle$ by $\text{Re } x$ and $(x - \text{Re } x)$ by $\text{Im } x$.

```
Out[2]= We denote  $\langle x, 1 \rangle$  by  $\text{Re } x$  and  $(x - \text{Re } x)$  by  $\text{Im } x$ .
```

```
In[3]:=  $\bar{x} = 2\langle x, 1 \rangle - x.$ 
```

```
In[4]:=  $\Psi16cc = 2 \text{realProjection16.}\Psi16 - \Psi16 // \text{FullSimplify}$ 
Out[4]= {f16[4][x0, x4], -f16[1][x0, x4], -f16[2][x0, x4], -f16[3][x0, x4],
f16[0][x0, x4], -f16[5][x0, x4], -f16[6][x0, x4], -f16[7][x0, x4],
f16[12][x0, x4], -f16[9][x0, x4], -f16[10][x0, x4], -f16[11][x0, x4],
f16[8][x0, x4], -f16[13][x0, x4], -f16[14][x0, x4], -f16[15][x0, x4]}
```

```
In[5]:=  $\Psi16ccA =$ 
2 (Flatten[{unit.\Psi16[[1;;8]] * (unit.F_A^a), unit.\Psi16[[9;;16]] * (unit.F_A^a)}]) -
\Psi16 // \text{FullSimplify}
```

```
Out[5]= {(-1 +  $\sqrt{2}$ ) f16[0][x0, x4] +  $\sqrt{2}$  f16[4][x0, x4], -f16[1][x0, x4], -f16[2][x0, x4],
-f16[3][x0, x4], -f16[4][x0, x4], -f16[5][x0, x4], -f16[6][x0, x4],
-f16[7][x0, x4], (-1 +  $\sqrt{2}$ ) f16[8][x0, x4] +  $\sqrt{2}$  f16[12][x0, x4],
-f16[9][x0, x4], -f16[10][x0, x4], -f16[11][x0, x4], -f16[12][x0, x4],
-f16[13][x0, x4], -f16[14][x0, x4], -f16[15][x0, x4]}
```

```
In[6]:=  $\Psi16cc - \Psi16ccA // \text{FullSimplify}$ 
Out[6]= {-((-1 +  $\sqrt{2}$ ) (f16[0][x0, x4] + f16[4][x0, x4])), 0, 0, 0, f16[0][x0, x4] + f16[4][x0, x4], 0, 0, 0,
-((-1 +  $\sqrt{2}$ ) (f16[8][x0, x4] + f16[12][x0, x4])), 0, 0, 0, f16[8][x0, x4] + f16[12][x0, x4], 0, 0, 0}
```

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \widetilde{\tau}^A \sigma$$

$$In[\#]:= \tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elemen

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

Out[\#]=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \widetilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elemen

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

```
Table[{A, B, FullSimplify[
  ExpandAll[1/2 (τ[B].τ[A] + τ[A].τ[B]) == η4488[[A+1, B+1]]*ID8]],
 {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

```
Table[{A, B, FullSimplify[
  ExpandAll[1/2 (τ[A].τ[B] + τ[B].τ[A]) == η4488[[A+1, B+1]]*ID8]],
 {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

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In[\#]:=

SUBMANIFOLDS AND SPECIAL STRUCTURES ON THE OCTONIANS

ROBERT L. BRYANT

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

Classically, this observation was used to define an almost complex structure on S^6 as follows: If $u \in S^6$, then R_u preserves the 2-plane spanned by 1 and u and therefore preserves its orthogonal 6-plane, which may be identified with $T_u S^6 \subseteq \text{Im } \mathbf{O}$ after translation to the origin. Thus R_u induces a complex

structure on $T_u S^6$ for each $u \in S^6$. This almost complex structure is not integrable (even locally) to a complex structure (see below). In [Ca], Calabi noticed that for *any* oriented $M^6 \subseteq \text{Im } \mathbf{O}$, $R_{N(x)}$ induces a complex structure on $T_x M^6$ (where $N(x)$ is the oriented unit normal). Thus every oriented $M^6 \subseteq \text{Im } \mathbf{O}$ inherits an almost complex structure. Moreover, M^6 inherits a metric from $\text{Im } \mathbf{O}$, so we actually have a $U(3)$ -structure on M^6 . (Calabi calls these structures “almost Hermitian.” He also proves that such M^6 possess a canonical $SU(3)$ -substructure but we will not need this.) Calabi shows that the second fundamental form Π of M decomposes with respect to the $U(3)$ -structure into a piece $\Pi^{1,1}$ of type (1,1) and a piece $\Pi^{0,2}$ of type (0,2). He then shows that the almost complex structure of M is integrable if and only if $\Pi^{1,1} = 0$ and that the canonical 2-form of the $U(3)$ -structure, say Ω , is closed if and only if $\Pi^{0,2} = 0$ and $\text{tr}_1 \Pi^{1,1} = 0$. From this it follows that the $U(3)$ -structure on M^6 is Kähler if and only if $\Pi \equiv 0$, so that M^6 is a hyperplane (or a union of pieces of hyperplanes). Calabi then constructs nontrivial examples of $M^6 \subseteq \text{Im } \mathbf{O}$ for which the almost complex structure is integrable. His examples are of the form $S \times \mathbf{R}^4 \subseteq \text{Im } \mathbf{O}$, where $S \subseteq \mathbf{R}^3$ is a minimal surface, $\mathbf{R}^3 \subseteq \text{Im } \mathbf{O}$ is an *associative* 3-plane, and $\mathbf{R}^4 = (\mathbf{R}^3)^\perp$. Calabi leaves open the problem of determining whether or not there are nontrivial $M^6 \subseteq \text{Im } \mathbf{O}$ for which the canonical 2-form is closed.

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex structure J_u . If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on \mathbf{O} . However, because S^6 is connected, we see that the orientation of \mathbf{O} induced by the natural orientation of \mathbf{O}_u as a complex vector space does not depend on u . We refer to this orientation as the natural orientation of \mathbf{O} .

Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

so J_u is an isometry for each $u \in S$. Moreover, it follows that \mathbf{O}_u is endowed with a natural Hermitian structure with respect to the inner product $\langle \cdot, \cdot \rangle$. We denote the group of complex linear transformations of \mathbf{O}_u by $GL(\mathbf{O}_u)$ and the special unitary transformations of \mathbf{O}_u with its Hermitian metric by $SU(\mathbf{O}_u)$.

We let $\text{Spin}(7) \subseteq SO(8)$ denote the subgroup generated by the set $\{J_u \mid u \in S^6\} \subseteq SO(8)$. It is known (see [12]) that $\text{Spin}(7)$ is a connected, simply connected, compact Lie group of real dimension 21. Its center is $\{\pm I_8\} \simeq \mathbb{Z}/2$ and $\text{Spin}(7)/\{\pm I_8\}$ is isomorphic to $SO(7)$, a simple group. We want to make explicit the structure equations of $\text{Spin}(7)$ as a subgroup of $SO(8)$ in such a way that its relationship with the complex structures J_u is made clear.

In[]:=

Let $u \in S^6$ be an imaginary unit which is orthogonal to $\epsilon \in \mathbf{O}$. For each $\lambda \in \mathbb{R}$, $(\cos \lambda \epsilon + \sin \lambda u)$ is an imaginary unit. Hence $J_\epsilon \circ J_{(\cos \lambda \epsilon + \sin \lambda u)} = -\cos \lambda I + \sin \lambda J_\epsilon \circ J_u$ is an element of $\text{Spin}(7)$. We easily compute that $J_\epsilon \circ J_u + J_u \circ J_\epsilon = 0$ by using (1.10). Thus $(J_\epsilon \circ J_u)^2 = J_\epsilon \circ J_u \circ J_\epsilon \circ J_u = -J_\epsilon^2 \circ J_u^2 = -I$. It follows that

$$(1.16) \quad \exp(\lambda J_\epsilon \circ J_u) = \cos \lambda I + \sin \lambda J_\epsilon \circ J_u.$$

Thus, if $\text{spin}(7) \subseteq so(8)$ is the Lie algebra of $\text{Spin}(7)$, we see that $J_\epsilon \circ J_u \in \text{spin}(7)$ for all $u \in S^6$ with $\langle u, \epsilon \rangle = 0$. Since $\text{spin}(7)$ is a vector space, we see that $L \subseteq \text{spin}(7)$ where

$$(1.17) \quad L = \{J_\epsilon \circ J_w \mid w \in \text{Im } \mathbf{O}, \langle \epsilon, w \rangle = 0\}.$$

Note that $\dim_{\mathbb{R}} L = 6$.

`Transpose[a].σ16.b == Transpose[J.a].σ16.J.b = a.Transpose[J].σ16.J.b`

want $\psicc.\sigma16.\Psi16 = \Psi16.J.\sigma16.\Psi16 \neq 0$

need 1 irrep for {i, u}

need 1 irrep for {i, u} _perp

1 == x0 or x4; u == x4 or x0

x0 has periodic fns; use x4

In[]:= t16A[[All, 2]]

Out[]=

{ {0}, {1}, {2}, {3}, {4}, {5}, {6}, {7} }

In[]:= T16^A[4].T16^A[4] === -Id16

Out[]=

True

*In[]:= (*JT4=T16^A[4] *) (* complex structure on S^6 *)*

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex

$$J.J == -ID16$$

matrix multiplication is associative

(split) octonion multiplication is NOT associative

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

Transpose[J].σ16.J] === σ16

```
In[1]:= Clear[possibleComplexStructureIndex2];
(*Clear[possibleComplexStructureMatrix2];*)
possibleComplexStructureIndex2[j_] := Module[{m, r, jsj, jj},
  J = base16[[j]][1];
  jj = (FullSimplify[J.J] === -ID16);
  jsj = FullSimplify[Transpose[J].σ16.J] === σ16;
  Return[{jj, jsj}];]
```

```
In[2]:= possibleComplexStructureIndex2[#] & /@ {12, 97, 100, 109, 127, 225, 230, 236}
Out[2]= {{False, False}, {True, False}, {True, False}, {True, False},
{True, False}, {False, False}, {False, False}}
```

```
In[3]:= Clear[possibleComplexStructureIndex];
Clear[possibleComplexStructureMatrix];
possibleComplexStructureIndex = (*Parallel*)Table[
  Block[{J, r, m},
    Clear[J, r, m];
    m = r = {};
    J = base16[[j]][1];
    If[(J.J === -ID16) &&
      (FullSimplify[Transpose[J].σ16.J] === σ16), r = base16[[j]][2]];
    m = σ16.J;, ##&[]];
    (*If[!(m===-Transpose[m]),r=base16[[j]][2],r={}]*)
    r]
  , {j, 1, Length[base16]}] // Union // Drop[#, 1] &
```

```
Out[3]= {{4}, {5}, {6}, {7}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {4, 5}, {4, 6},
{4, 7}, {5, 6}, {5, 7}, {6, 7}, {0, 1, 2, 3, 4}, {0, 1, 2, 3, 5}, {0, 1, 2, 3, 6},
{0, 1, 2, 3, 7}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7}, {0, 1, 5, 6, 7},
{0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 2, 5, 6, 7}, {0, 3, 4, 5, 6},
{0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {0, 3, 5, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7},
{1, 2, 4, 6, 7}, {1, 2, 5, 6, 7}, {1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7},
{1, 3, 5, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7}, {2, 3, 4, 6, 7}, {2, 3, 5, 6, 7},
{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 6}, {0, 1, 2, 3, 4, 7}, {0, 1, 2, 3, 5, 6},
{0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}, {0, 1, 4, 5, 6, 7}, {0, 2, 4, 5, 6, 7},
{0, 3, 4, 5, 6, 7}, {1, 2, 4, 5, 6, 7}, {1, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 7}}
```



```

r = found[[2]];
(*Print[base16[[j]][[2]],";Tr[J.J]/16=",Tr[a1]/16,
";c1=",c1,";c2=Select[base16,#[[2]]==c1&= ",c2,
";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
Tr[found[[1]].found[[1]]/16];*)

,
## &[]];
, ## &[], ## &[], ## &[], ## &[], ## &[]];
r]
, {j, 1, Length[base16]}] // Union // Drop[#, 1] &

Out[=]=
{{4}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {5, 6}, {5, 7},
{6, 7}, {0, 1, 2, 3, 4}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7},
{0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 3, 4, 5, 6},
{0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7}, {1, 2, 4, 6, 7},
{1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7},
{2, 3, 4, 6, 7}, {0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}


In[=]:= JT4 = T16^A[4] ;(* complex structure on S^6 *)
Jcomplement <-> ( q[[7]]*Cos[Q7] + q[[6]]*Sin[Q7] Cos[Q8] + q[[5]]*Sin[Q7] Sin[Q8] )

In[=]:= basisProjections16 =
Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@
Range[16];

In[=]:= basisProjections8 =
Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@
Range[8];

In[=]:= JcomplementProjection8 =
(basisProjections8[[7]] + basisProjections8[[6]] + basisProjections8[[5]]);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8

Out[=]=
True

In[=]:= JcomplementProjection8 = (basisProjections8[[7]] +
basisProjections8[[6]] + basisProjections8[[5]] + basisProjections8[[4]]);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8

Out[=]=
True

In[=]:= (*JcomplementProjection8=
(basisProjections8[[7]]*Cos[Q7]+basisProjections8[[6]]*Sin[Q7]Cos[Q8]+
basisProjections8[[5]]*Sin[Q7]Sin[Q8]);*)

```



```
In[=]:= (projection04 = ID16 - Block[{x = Table[KroneckerDelta[1, j], {j, 16}]}, 
    KroneckerProduct[x, x]] - Block[{x = Table[KroneckerDelta[5, j], {j, 16}]}, 
    KroneckerProduct[x, x]]) // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


In[=]:= projection04.parallel04 === Zero16

Out[=]=
True

{{0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}
```

(Note: The matrix shown is a 16x16 identity matrix, indicating that the projection is zero.)

```
In[=]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 5, 6};
% === -T16^A[4].T16^A[7].T16^A[8]

Out[=]=
True
```

```
In[=]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 5, 7};
% === T16^A[4].T16^A[6].T16^A[8]

Out[=]=
True
```

```
In[=]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 6, 7};
% === -T16^A[4].T16^A[5].T16^A[8]

Out[=]=
True
```



```
In[6]:= realProjection16.realProjection16 === realProjection16
Out[6]= True
```

In[•]:= We denote $\langle x, 1 \rangle$ by $\text{Re } x$ and $(x - \text{Re } x)$ by $\text{Im } x$.

```
In[1]:=  $\bar{x} = 2\langle x, 1 \rangle - x.$ 
Out[1]=  $\bar{x} = 2\langle x, 1 \rangle - x.$ 
```

```
In[1]:= unit.unit  
Out[1]= 1
```

```
In[•]:= unit. $\sigma$ .unit  
Out[•]= 1
```

```

In[]:= T16^A[4].T16^A[4] === -ID16
Out[]=
True

In[]:= T4cc = T16^A[4] - (realProjection16.T16^A[4]) // FullSimplify
Out[=
{ {0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, -1/2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 1/2, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1/2, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0} }

```

In[1]:= **T4cc.T4cc**

Out[1]=

$$\left\{ \left\{ -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right.$$

$$\left\{ 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\{0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\{0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0 \right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0 \right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0\},$$

$$\left. \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1\} \right\}$$

In[2]:= **T16^A[7].T16^A[7] === -ID16**

Out[2]=

True

$$\begin{aligned}
& \left. \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \frac{1}{2} \sin[Q7]^2 \sin[Q8]^2 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& 0, \frac{\cos[Q7]^2}{2}, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], 0, \\
& \frac{1}{4} (-3 + \cos[2 Q7]), \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8] \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q8]^2 \sin[Q7]^2, \right. \\
& \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \\
& \left. -\cos[Q7]^2 + \frac{1}{4} (-3 + \cos[2 Q8]) \sin[Q7]^2, \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8] \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \right. \\
& \frac{1}{2} \sin[Q7]^2 \sin[Q8]^2, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \\
& \left. \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], -\cos[Q7]^2 - \frac{1}{4} (3 + \cos[2 Q8]) \sin[Q7]^2 \right\}
\end{aligned}$$

In[4]:= **T16^A[4].Jcomplement // MatrixForm**

Out[4]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\cos[Q7] & 0 & 0 & 0 & 0 \\
0 & \cos[Q7] & 0 & 0 & 0 \\
0 & 0 & \cos[Q7] & 0 & -\sin[Q7] \sin[Q8] \\
0 & 0 & 0 & \cos[Q7] & \cos[Q8] \sin[Q7] \\
0 & 0 & -\sin[Q7] \sin[Q8] & \cos[Q8] \sin[Q7] & -\cos[Q7] \\
0 & 0 & -\cos[Q8] \sin[Q7] & -\sin[Q7] \sin[Q8] & 0 \\
\sin[Q7] \sin[Q8] & \cos[Q8] \sin[Q7] & 0 & 0 & 0 \\
-\cos[Q8] \sin[Q7] & \sin[Q7] \sin[Q8] & 0 & 0 & 0
\end{pmatrix}$$

```
In[]:= T16^A[4].Jcomplement === Jcomplement.T16^A[4]
FullSimplify[Jcomplement.Jcomplement] === -ID16
T16^A[4].T16^A[4] === -ID16

Out[=]=
True

Out[=]=
True

Out[=]=
True

In[]:= unit3 = {Cos[Q7], Sin[Q7] Cos[Q8], Sin[Q7] Sin[Q8]}

Out[=]=
{Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}

In[]:= unit3.unit3 // FullSimplify

Out[=]=
1
```

```

In[]:= unit3
sixers
Complement[{0, 1, 2, 3, 5, 6, 7}, #] & /@ %
eqsRQQ7Q8 = Thread[Subscript[q, #[1]] & /@ %] == RQ * unit3
sRQQ7Q8 = Solve[And @@ %, {RQ, Q7, Q8}] // FullSimplify
seqsRQQ7Q8 = %% /. {Equal -> Rule}

Out[]=
{Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}

Out[]=
{{0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7} }

Out[]=
{{7}, {6}, {5} }

Out[]=
{q7 == RQ Cos[Q7], q6 == RQ Cos[Q8] Sin[Q7], q5 == RQ Sin[Q7] Sin[Q8]}

Out[]=

$$\left\{ \begin{array}{l} \text{RQ} \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, \quad Q7 \rightarrow 2\pi c_1 - i \operatorname{Log} \left[ \frac{-i \sqrt{q_5^2 + q_6^2} - q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, \\ \end{array} \right.$$


Q8 -> 
$$\operatorname{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z}$$
, 
$$\left\{ \begin{array}{l} \text{RQ} \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, \\ \end{array} \right.$$


Q7 -> 
$$2\pi c_1 - i \operatorname{Log} \left[ \frac{i (\sqrt{q_5^2 + q_6^2} + i q_7)}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}$$
, 
$$Q8 \rightarrow 2\pi c_2 - i \operatorname{Log} \left[ \frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}} \right] \text{ if } c_2 \in \mathbb{Z}$$



$$\left\{ \begin{array}{l} \text{RQ} \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, \quad Q7 \rightarrow 2\pi c_1 - i \operatorname{Log} \left[ \frac{-i \sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, \\ \end{array} \right.$$


Q8 -> 
$$2\pi c_2 - i \operatorname{Log} \left[ \frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}} \right] \text{ if } c_2 \in \mathbb{Z}$$
, 
$$\left\{ \begin{array}{l} \text{RQ} \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, \\ \end{array} \right.$$


Q7 -> 
$$2\pi c_1 - i \operatorname{Log} \left[ \frac{i \sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}$$
, 
$$Q8 \rightarrow \operatorname{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \} \}$$


Out[=]
{q7 -> RQ Cos[Q7], q6 -> RQ Cos[Q8] Sin[Q7], q5 -> RQ Sin[Q7] Sin[Q8] }

```

```

In[1]:= FullSimplify[(Sqrt[q6^2 + q5^2] /. seqsRQQ7Q8), Q7 > 0 && Sin[Q7] > 0 && RQ > 0]
% / q7 /. seqsRQQ7Q8
Solve[(Sqrt[q6^2 + q5^2]/q7 == FullSimplify[%], Q7 > 0 && Sin[Q7] > 0 && RQ > 0], Q7]
sQ7 = {Q7 -> ArcTan[q7, Sqrt[q5^2 + q6^2]]}

Out[1]=
RQ Sin[Q7]

Out[2]=
Tan[Q7]

Out[3]=
{Q7 -> ArcTan[Sqrt[q5^2 + q6^2]/q7] + Pi c1 if c1 ∈ ℤ}

Out[4]=
{Q7 -> ArcTan[q7, Sqrt[q5^2 + q6^2]]}

(*sQ7=*) Solve[(Sqrt[q6^2 + q5^2]/q7 == FullSimplify[(Sqrt[q6^2 + q5^2]/q7 /. seqsRQQ7Q8),
Q7 > 0 && Sin[Q7] > 0 && RQ > 0], Q7] // FullSimplify
Out[5]=
{Q7 -> ArcCot[q7/Sqrt[q5^2 + q6^2]] + Pi c1 if c1 ∈ ℤ}

```

```
In[]:= eqsRQQ7Q8[[2 ;; 3]]
Transpose[{{All, 1}, {All, 2}}]
%[[2]][[1]] / %[[1]][[1]] == %[[2]][[2]] / %[[1]][[2]] // FullSimplify
sQ8 = Solve[%, Q8][[1]] // FullSimplify
sQ8 = {Q8 → ArcTan[q6, q5]}
sRQQ7Q8 /. % // FullSimplify

Out[]= {q6 == RQ Cos[Q8] Sin[Q7], q5 == RQ Sin[Q7] Sin[Q8]}

Out[=]
{ {q6, RQ Cos[Q8] Sin[Q7]}, {q5, RQ Sin[Q7] Sin[Q8]} }

Out[=]

$$\frac{q_5}{q_6} == \tan[Q8]$$


Out[=]

$$\left\{ Q8 \rightarrow \text{ArcTan}\left[\frac{q_5}{q_6}\right] + \pi c_1 \text{ if } c_1 \in \mathbb{Z} \right\}$$


Out[=]
{Q8 → ArcTan[q6, q5]}

Out[=]

$$\left\{ RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \log\left[\frac{-i\sqrt{q_5^2 + q_6^2} - q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z} \right\},$$

ArcTan[q6, q5] → 
$$\text{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z}$$
 ,  


$$\left\{ RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \log\left[\frac{i\left(\sqrt{q_5^2 + q_6^2} + i q_7\right)}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z} \right\},$$

ArcTan[q6, q5] → 
$$2\pi c_2 - i \log\left[\frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z}$$
 ,  


$$\left\{ RQ \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \log\left[\frac{-i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z} \right\},$$

ArcTan[q6, q5] → 
$$2\pi c_2 - i \log\left[\frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z}$$
 ,  


$$\left\{ RQ \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \log\left[\frac{i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z} \right\},$$

ArcTan[q6, q5] → 
$$\text{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \}$$

```

```

In[]:= sQ7Q8 = Flatten[{sQ7, sQ8}]
Out[]= {Q7 → ArcTan[q7, √(q52 + q62)], Q8 → ArcTan[q6, q5]}

In[]:= Block[{a, b, c},
  {a, b, c} * unit3]
Out[]= {a Cos[Q7], b Cos[Q8] Sin[Q7], c Sin[Q7] Sin[Q8]}

In[]:= base16[[#][1]].base16[[#][1]] === -ID16 & /@ {222, 223, 224}
Out[]= {True, True, True}

In[]:= (*sixersT4=%*{Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}
          {Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}.
          {Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]})//FullSimplify*)
          Block[{J = T16A[4], m = σ16.J},
            {J.J === -ID16, σ16 === Transpose[J].σ16.J, !(m === -Transpose[m])}]
Out[]= {True, True, False}

In[]:= (*Clear[possibleComplexStructureIndex];
  Clear[possibleComplexStructureMatrix];
  possibleComplexStructureIndex=Select[base16,Block[{J,a0,a1,a2,b1,b2,r},
    Clear[J,a0,a1,a2,b1,b2,r];
    a0=#[[1]];
    J=T16A[4]*a0//FullSimplify;
    a1=J.J//FullSimplify;
    b1=(a1===-ID16);
    a2=Transpose[J].σ16.J//FullSimplify;
    b2=(a2===σ16);
    r=b1&&b2;
    If[r,Block[{c1,c2,c3},
      c1=Append[#[[2]],4]//Sort; (* for J = T16A[4]*#[[1]]*)
      c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
      Print[#[[2]],";",Tr[a1]/16,";c1=",c1,";c2=",
        c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
        Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16],##&[]];
      r
    ]&→"Index"]
  possibleComplexStructureMatrix=base16[[2]]&/@%*)

```

```

In[1]:= (*Tr[(T16^A[3].T16^A[4]).(T16^A[3].T16^A[4])]/16*)

In[2]:= (*base16[[5]][[2]]*)

In[3]:= (*Select[base16,#[[2]]=={1,2,3,4,5,6}&→"Index"]*)

In[4]:= (*base16[[240]][[2]]
Tr[base16[[240]][[1]].base16[[240]][[1]]]/16*)

In[5]:= (*(*Parallel*)Do[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
Clear[J,a0,a1,a2,b1,b2,r];
a0=base16[[j]][[1]];
J=T16^A[4]*a0//FullSimplify;
a1=J.J//FullSimplify;
If[!(a1===-ID16),Continue[],##&[]];
(*b1=(a1===-ID16);*)
a2=Transpose[J].σ16.J//FullSimplify;
If[!(a2===σ16),Continue[],##&[]];
(*b2=(a2===σ16);*)
(*r=b1&&b2;*)

c1=Append[base16[[j]][[2]],4]//Sort; (* for J = T16^A[4]*#[[1]]*)
c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
Print[base16[[j]][[2]],";",Tr[a1]/16,";c1=",c1,
";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16];
{J,base16[[c2]][[1]]}];
,{j,1,Length[base16}}]*)

(*ParallelDo[
Block[{r,t,a1},
t=Tr[base16[[j]][[1]].base16[[k]][[1]]/16];
r=t^2==KroneckerDelta[j,k];
a1=If[r,##&[],Print[{{j,k},t,{base16[[j]][[2]],base16[[k]][[2]]}}]];
],{j,1,Length[base16]}, {k,1,Length[base16}}]*)

```

```
In[ ]:= (*(*Parallel*)Table[Catch[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
  Clear[J,a0,a1,a2,b1,b2,r];
  a0=base16[[j]][1];
  J=T16^A[4]*a0//FullSimplify;
  a1=J.J//FullSimplify;
  If[!(a1===-ID16),Throw[Null],##&[]];
  (*b1=(a1===-ID16);*)
  a2=Transpose[J].σ16.J//FullSimplify;
  If[!(a2==σ16),Throw[Null],##&[]];
  (*b2=(a2==σ16);*)
  (*r=b1&&b2;*)

  c1=Append[base16[[j]][2],4]//Sort; (* for J = T16^A[4]*#[[1]]*)
  c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
  Print[base16[[j]][2],";",Tr[a1]/16,";c1=",c1,
  ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
  Tr[base16[[c2]][1].base16[[c2]][1]]/16];
  {{J},{base16[[c2]][1]},{J==base16[[c2]][1]||J===-base16[[c2]][1]}}]]
, {j,1,Length[base16]/128}]*)
```

```
In[1]:= (*(*Parallel*)Table[
  Block[{J,a1,a2,b1,b2,c1,c2,c3,r},
  Clear[J,a1,a2,b1,b2,r];
  r={};
  J=T16^A[4].base16[[j]][1]//FullSimplify;
  a1=J.J//FullSimplify;
  If[(a1===-ID16),
    a2=Transpose[J].σ16.J//FullSimplify;
  If[(a2==σ16),
    c1=Append[base16[[j]][2],4]//Sort; (* for J = T16^A[4]*#[1]*)
    c3=Select[base16,#[2]==c1&&"Index"];
    If[!(c3=={}),
      c2=c3[[1]];
      Print[base16[[j]][2],"";Tr[J.J]/16=",Tr[a1]/16,";c1=",c1,
        ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
        Tr[base16[[c2]][1].base16[[c2]][1]]/16];
      r={J}, {base16[[c2]][1]}, {J==base16[[c2]][1] || J===-base16[[c2]][1]}];
      ,##&[],##&[],##&[]];
    ];
  ];
  ,{j,1,Length[base16}]]//Union*)


```

```
In[2]:= (*possibleComplexStructureIndex=
  Select[base16,Block[{J=T16^A[4]*#[1],a1,a2,b1,b2,r},
  a1=J.J;
  a2=Transpose[J].σ16.J;
  b1=a1===-ID16;
  b2=a2==σ16;
  r=b1&&b2;
  (*If[r,Print[a1]];*)
  r
  ]&&"Index"]
  possibleComplexStructureMatrix=base16[[#][2]]&/@%*)


```

```
In[3]:= base16[[201]][2]
Out[3]= {1, 2, 3, 5, 6}
```

```
In[8]:= (*possibleComplexStructureIndex=
    Select[base16,Block[{J=T16^A[4]*#[1]],a1=J.J,a2=Transpose[J].σ16.J,
        b1=a1===-ID16,b2=a2==σ16},b1&&b2]&→"Index"]
possibleComplexStructureMatrix=base16[[#]][[2]]&/@%*)
```

```
In[]:= possibleComplexStructureIndex =
  Select[base16, Block[{J = T16^A[4] * #[[1]], m = σ16.J}, J.J === -ID16 &&
    σ16 === Transpose[J].σ16.J (*&&! (m === -Transpose[m]) *)] & → "Index"]
possibleComplexStructureMatrix = base16[[#][[2]] & /@ %
```

```
Out[]= {4, 178, 187, 201}
```

```
Out[]= {{3}, {0, 1, 3, 6, 7}, {0, 2, 3, 5, 7}, {1, 2, 3, 5, 6}}
```

```
In[]:= Select[base16, #[[2]] == {1, 2, 3, 4, 5, 6} & → "Index"]
```

```
Out[]= {240}
```

```
In[]:= base16[[240]][[2]]
```

```
Out[]= {1, 2, 3, 4, 5, 6}
```

```
In[]:= base16[[240]][[1]] // MatrixForm
```

```
Out[//MatrixForm=
```

$$\left(\begin{array}{cccccccccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

```
In[]:= (*possibleComplexStructureIndex=
  Select[base16,Block[{J=#[[1]],m=σ16.#[[1]]},J.J===-ID16&&
    σ16==Transposed[J].σ16.J&&! (m===-Transposed[m]) ]&→"Index"]
possibleComplexStructureMatrix=base16[[#][[2]]&/@%*)
```

```
findComplexStucture[] := Module[{list, filter(*, innerProduct*)},
  (*innerProduct[a_, b_] := FullSimplify[Transpose[a].σ16.b];*)
  (*filter[J_][a_, b_] := J.J == -ID16 && innerProduct[a, b] == innerProduct[J.a, J.b];*)
  filter[J_] := J.J == -ID16 && σ16 == Transpose[J].σ16.J;
  Return[list];
]
```

```
In[1]:= Ω16 === T16^A[1].T16^A[2].T16^A[3].T16^A[4]
Out[1]= True
```

Spinor Lagrangian

```
In[2]:= usegT16 =
Table[FullSimplify[(T16^α[α1 - 1]), constraintVars], {α1, 1, Length[X]}];
In[3]:= useT16 = Table[FullSimplify[(T16^α[α1 - 1] /. ssgm4488), constraintVars] /.
{Sqrt[e^-2 a4[H x4]] → e^-a4[H x4], Sqrt[e^2 a4[H x4] Sin[6 H x0]^1/3] → e^a4[H x4] Sin[6 H x0]^1/6,
1/Sqrt[e^2 a4[H x4] Sin[6 H x0]^1/3] → 1/(e^a4[H x4] Sin[6 H x0]^1/6)}, {α1, 1, Length[X]}] /.
{1/Sqrt[e^-2 a4[H x4] Sin[6 H x0]^1/3] → 1/(e^a4[H x4] Sin[6 H x0]^1/6), Sqrt[e^2 a4[H x4]] → e^a4[H x4]}];
```

```
In[4]:= Dimensions[useT16]
```

```
Out[4]= {8, 16, 16}
```

```
In[5]:= Det[MatrixMetric44]
```

```
Out[5]= Cos[6 H x0]^2
```

```
In[6]:= useDSQRT = Cos[6 H x0]
```

```
Out[6]= Cos[6 H x0]
```

```

In[]:= wmat /. sg /. ssgm4488 /. subsDefects;
          (% /. {Sqrt[e^2 a4[H x4]] -> e^a4[H x4], 
          Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) -> e^a4[H x4] H Sin[6 H x0]^(1/6)}) - spinCoeffs //.
FullSimplify[#, constraintVars] & /. subsDefects //.
{Sqrt[e^2 a4[H x4]] -> e^a4[H x4], Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) -> e^a4[H x4] H Sin[6 H x0]^(1/6)} //.
FullSimplify[#, constraintVars] & /. subsDefects;
% // Flatten // Union
% // ExpandAll /. subsDefects /. {Sqrt[e^2 a4[H x4]] -> e^a4[H x4], 
          Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) -> e^a4[H x4] H Sin[6 H x0]^(1/6)} /. subsDefects
% /. {a4[H x4] -> π^4, Sin[6 H x0] -> e^12}

Out[]= {0, e^-a4[H x4] (-1 + e^a4[H x4] Sqrt[e^-2 a4[H x4]]) H Sin[6 H x0]^(1/6),
-((-e^a4[H x4] + Sqrt[e^2 a4[H x4]]) H Sin[6 H x0]^(1/6)), (-e^a4[H x4] + Sqrt[e^2 a4[H x4]]) H Sin[6 H x0]^(1/6),
e^-a4[H x4] (1 - e^a4[H x4] Sqrt[e^-2 a4[H x4]]) H Sin[6 H x0]^(1/6) a4'[H x4],
e^-a4[H x4] (-1 + e^a4[H x4] Sqrt[e^-2 a4[H x4]]) H Sin[6 H x0]^(1/6) a4'[H x4],
(-e^a4[H x4] + Sqrt[e^2 a4[H x4]]) H Sin[6 H x0]^(1/6) a4'[H x4]}

Out[]= {0, -e^-a4[H x4] H Sin[6 H x0]^(1/6) + Sqrt[e^-2 a4[H x4]] H Sin[6 H x0]^(1/6),
e^a4[H x4] H Sin[6 H x0]^(1/6) - Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6),
-e^a4[H x4] H Sin[6 H x0]^(1/6) + Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6),
e^-a4[H x4] H Sin[6 H x0]^(1/6) a4'[H x4] - Sqrt[e^-2 a4[H x4]] H Sin[6 H x0]^(1/6) a4'[H x4],
-e^-a4[H x4] H Sin[6 H x0]^(1/6) a4'[H x4] + Sqrt[e^-2 a4[H x4]] H Sin[6 H x0]^(1/6) a4'[H x4],
-e^a4[H x4] H Sin[6 H x0]^(1/6) a4'[H x4] + Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) a4'[H x4]}

Out[=]
{0, 0, 0, 0, 0, 0, 0}

```

In[]:= Q2 = 0; Protect[Q1, Q2]

Out[=]
{Q1, Q2}

```
In[=]:= Clear[Lg];
Lg[] :=  $\sqrt{\det gg} * \left( \text{Transpose}[\Psi_{16}] . \sigma_{16} . \right.$ 
 $\text{Sum}\left[ \text{FullSimplify}\left[ (T16^\alpha[\alpha_1 - 1] /. sg), \text{constraintVars} \right] . \left( D[\Psi_{16}, X[\alpha_1]] + \right. \right.$ 
 $\left. \left( \frac{Q1}{2} \right) * \text{Sum}[\omega_{\mu IJ}[\alpha_1, a, b] * SAB[a, b].\Psi_{16}, \{a, 1, 8\}, \{b, 1, 8\}] \right),$ 
 $\left. \left. \{\alpha_1, 1, \text{Length}[X]\} \right] + (H * M) * \text{Transpose}[\Psi_{16}] . \sigma_{16} . \Psi_{16} \right) //$ 
Simplify[#, constraintVars] &
```

```
In[=]:=  $\omega_{\mu IJ}[1]$ 
Out[=]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[=]:= Clear[La];
La[] :=
useDSQRT *  $\left( \text{Transpose}[\Psi_{16}] . \sigma_{16} . \text{Sum}\left[ \text{useT16}[\alpha_1] . \left( D[\Psi_{16}, X[\alpha_1]] + \left( \frac{Q1}{2} \right) * \right. \right. \right.$ 
 $\left. \left. \left. \text{Sum}[\omega_{\mu IJ}[\alpha_1][A1, B1] * SAB[A1, B1], \{A1, 1, 8\}, \{B1, 1, 8\}] . \Psi_{16} \right), \right. \right.$ 
 $\left. \left. \left. \{\alpha_1, 1, \text{Length}[X]\} \right] + (H * M) * \text{Transpose}[\Psi_{16}] . \sigma_{16} . \Psi_{16} \right) //$ 
Simplify[#, constraintVars] &
```

```
In[=]:= (*testLa=La[]*)
In[=]:= (*testLg=Lg[]*)
In[=]:= (*D[testLa,a4[H x4]]//Flatten//Union*)
(*D[testLa,a4'[H x4]]//(*Full*)Simplify[#,constraintVars]&//Flatten//Union*)
In[=]:= (*testLgm=
testL/.sg/.ssgm4488//(*Full*)Simplify[#,constraintVars]&/.subsDefects*)
In[=]:= (*D[testLgm,a4[H x4]]
D[testLgm,a4'[H x4]]//(*Full*)Simplify[#,constraintVars]&*)
```

```
In[=]:= Clear[Lj];
Lj[j_] := useDSQRT * ((((
Transpose[\!`16].\!`16.useT16[1].D[\!`16, X[1]] +
Transpose[\!`16].\!`16.useT16[5].D[\!`16, X[5]] + (K0/2) *
Sum[(\!`usew[1,A1,B1]*)\!`muIJ[1][A1, B1]\!`times Transpose[\!`16].\!`16.useT16[1].SAB[A1, B1].(base16[j, 1]).\!`16, {A1, 1, 8}, {B1, 1, 8}]) +
Transpose[\!`16].\!`16.useT16[5].D[\!`16, X[5]] + (K4/2)*Sum[
(\!`usew[5,A1,B1]*)\!`muIJ[5][A1, B1]\!`times Transpose[\!`16].\!`16.useT16[5].SAB[A1, B1].(base16[j, 1]).\!`16, {A1, 1, 8}, {B1, 1, 8}]) +
(H*M)*Transpose[\!`16].\!`16.\!`16) // Simplify[#, constraintVars] &
```

```
In[=]:= (*Lj[92]*)
In[=]:= detgg
Out[=]=
g[0][0][x0, x4]\!`times g[1][1][x0, x4]\!`times g[2][2][x0, x4]\!`times g[3][3][x0, x4]\!`times
g[4][4][x0, x4]\!`times g[5][5][x0, x4]\!`times g[6][6][x0, x4]\!`times g[7][7][x0, x4]

In[=]:= Clear[eL];
eL[Lagrangian_Symbol, detsqrt_]:= Module[{L, t},
L = Lagrangian[];
t = Table[FullSimplify[
1/detsqrt (D[L, f16[k][x0, x4]] - D[D[L, f16[k]^(1,0)[x0, x4]], x0] -
D[D[L, f16[k]^(0,1)[x0, x4]], x4]), constraintVars], {k, 0, 15}];
Return[t /. subsDefects];
]

```

eLa

```
In[=]:= (*Get[
"2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLa.
mx"]*)

In[=]:= eLa = eL[La, useDSQRT];
```

```
In[]:= eLa
Out[]=
{-2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[9][x0, x4] a4'[H x4] -
2 (H M f16[4][x0, x4] + 3 H f16[12][x0, x4] +
f16[9](0,1)[x0, x4] + Tan[6 H x0] f16[12](1,0)[x0, x4]),
2 (-H M f16[5][x0, x4] - 3 H f16[13][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[8][x0, x4] a4'[H x4] + f16[8](0,1)[x0, x4] - Tan[6 H x0] f16[13](1,0)[x0, x4]),
-2 (H M f16[6][x0, x4] + 3 H f16[14][x0, x4] - f16[11](0,1)[x0, x4] +
Tan[6 H x0] f16[14](1,0)[x0, x4]),
-2 (H M f16[7][x0, x4] + 3 H f16[15][x0, x4] + f16[10](0,1)[x0, x4] +
Tan[6 H x0] f16[15](1,0)[x0, x4]), -2 H M f16[0][x0, x4] -
6 H f16[8][x0, x4] - 2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[13][x0, x4] a4'[H x4] +
2 f16[13](0,1)[x0, x4] - 2 Tan[6 H x0] f16[8](1,0)[x0, x4],
-2 H M f16[1][x0, x4] - 6 H f16[9][x0, x4] +
e-2 a4[H x4] (-1 + e2 a4[H x4]) H Q1 f16[12][x0, x4] a4'[H x4] -
2 (f16[12](0,1)[x0, x4] + Tan[6 H x0] f16[9](1,0)[x0, x4]),
-2 (H M f16[2][x0, x4] + 3 H f16[10][x0, x4] +
f16[15](0,1)[x0, x4] + Tan[6 H x0] f16[10](1,0)[x0, x4]),
-2 (H M f16[3][x0, x4] + 3 H f16[11][x0, x4] - f16[14](0,1)[x0, x4] +
Tan[6 H x0] f16[11](1,0)[x0, x4]),
2 (3 H f16[4][x0, x4] + H M f16[12][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[1][x0, x4] a4'[H x4] - f16[1](0,1)[x0, x4] + Tan[6 H x0] f16[4](1,0)[x0, x4]),
2 (3 H f16[5][x0, x4] + H M f16[13][x0, x4] - e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[0][x0, x4] a4'[H x4] + f16[0](0,1)[x0, x4] + Tan[6 H x0] f16[5](1,0)[x0, x4]),
2 (3 H f16[6][x0, x4] + H M f16[14][x0, x4] + f16[3](0,1)[x0, x4] +
Tan[6 H x0] f16[6](1,0)[x0, x4]), 2 (3 H f16[7][x0, x4] +
H M f16[15][x0, x4] - f16[2](0,1)[x0, x4] + Tan[6 H x0] f16[7](1,0)[x0, x4]),
2 (3 H f16[0][x0, x4] + H M f16[8][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[5][x0, x4] a4'[H x4] + f16[5](0,1)[x0, x4] + Tan[6 H x0] f16[0](1,0)[x0, x4]),
6 H f16[1][x0, x4] + 2 H M f16[9][x0, x4] - 2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[4][x0, x4] a4'[H x4] - 2 f16[4](0,1)[x0, x4] + 2 Tan[6 H x0] f16[1](1,0)[x0, x4],
2 (3 H f16[2][x0, x4] + H M f16[10][x0, x4] - f16[7](0,1)[x0, x4] +
Tan[6 H x0] f16[2](1,0)[x0, x4]), 2 (3 H f16[3][x0, x4] +
H M f16[11][x0, x4] + f16[6](0,1)[x0, x4] + Tan[6 H x0] f16[3](1,0)[x0, x4])}
```

```
In[]:= Length[eLa]
```

```
Out[]=
```

16

```
In[]:= DumpSave[ToString[header <> "eLa.mx"], eLa];
```

```
In[1]:= Print[ToString[header <> "eLa.mx"]]
Pair-Crtn-Univ-same_E-L-eqs-alt-approach-eLa.mx

In[2]:= (*Cases[
  eLa,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
  Infinity,
  Heads → True
]*)

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE:

In[3]:= eLaRawSets = Cases[
  #,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
  Infinity,
  Heads → True
] & /@ eLa
Out[3]= {{9, 4, 12, 9, 12}, {5, 13, 8, 8, 13}, {6, 14, 11, 14},
{7, 15, 10, 15}, {0, 8, 13, 13, 8}, {1, 9, 12, 12, 9}, {2, 10, 15, 10},
{3, 11, 14, 11}, {4, 12, 1, 1, 4}, {5, 13, 0, 0, 5}, {6, 14, 3, 6},
{7, 15, 2, 7}, {0, 8, 5, 5, 0}, {1, 9, 4, 4, 1}, {2, 10, 7, 2}, {3, 11, 6, 3}]

In[4]:= Dimensions[eLaRawSets]
Out[4]= {16}

In[5]:= rawSetsSel16 = rawSets[eLa, "f16", 0]
% === eLaRawSets
Out[5]= {{9, 4, 12, 9, 12}, {5, 13, 8, 8, 13}, {6, 14, 11, 14},
{7, 15, 10, 15}, {0, 8, 13, 13, 8}, {1, 9, 12, 12, 9}, {2, 10, 15, 10},
{3, 11, 14, 11}, {4, 12, 1, 1, 4}, {5, 13, 0, 0, 5}, {6, 14, 3, 6},
{7, 15, 2, 7}, {0, 8, 5, 5, 0}, {1, 9, 4, 4, 1}, {2, 10, 7, 2}, {3, 11, 6, 3}]

Out[6]=
True
```

```
In[1]:= eLaCouplings = showCoupledEquations[eLaRawSets]
% // Flatten // Sort
% === Range[0, 15]

Out[1]= {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
```

```
Out[2]= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
```

```
Out[3]=
```

```
True
```

```
In[4]:= eLaztCouplings = eLaCouplings

Out[4]= {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}


In[5]:= (*processRawSets[]:=Module[{s1,s2,J,L=Range[Length[eLaRawSets]],r={}},
While[Length[L]>1,
s1=Union[eLaRawSets[[L[[1]]]];
L=Drop[L,1];
J=L[[1]];
While[J<=16,
If[MemberQ[L,J],
s2=Union[eLaRawSets[[J]]];
If[Intersection[s1,s2]!={},s1=Union[Flatten[{s1,s2}]];
L=Complement[L,{J}];,##&[]];
##&[]];
J++]];
AppendTo[r,s1];
];
Return[r];
]*)
```

eLazt

```
In[6]:= (*Get[
"2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLazt
.mx"]*)
```

```
In[1]:= time1 = Now
AbsoluteTiming[
  eLazt =  $\frac{1}{2 \cdot H} \cdot eLa /. sfy16Aa /. sx0x4 // FullSimplify[\#, constraintVars] &$ 
Now - time1

Out[1]= Tue 3 Feb 2026 15:35:31 GMT-8

Out[2]= {5.68581, { $-M Z[4][z, t] - 3 Z[12][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] - Z[9]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t], -M Z[5][z, t] - 3 Z[13][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + Z[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t], -M Z[6][z, t] - 3 Z[14][z, t] + Z[11]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t], -M Z[7][z, t] - 3 Z[15][z, t] - Z[10]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t], -M Z[0][z, t] - 3 Z[8][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + Z[13]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t], -M Z[1][z, t] - 3 Z[9][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] - Z[12]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t], -M Z[2][z, t] - 3 Z[10][z, t] - Z[15]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t], -M Z[3][z, t] - 3 Z[11][z, t] + Z[14]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t], 3 Z[4][z, t] + M Z[12][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] - Z[1]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t], 3 Z[5][z, t] + M Z[13][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] + Z[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t], 3 Z[6][z, t] + M Z[14][z, t] + Z[3]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t], 3 Z[7][z, t] + M Z[15][z, t] - Z[2]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t], 3 Z[0][z, t] + M Z[8][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] + Z[5]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t], 3 Z[1][z, t] + M Z[9][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] - Z[4]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t], 3 Z[2][z, t] + M Z[10][z, t] - Z[7]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t], 3 Z[3][z, t] + M Z[11][z, t] + Z[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t]\}}}

Out[3]= 5.688943 s

In[4]:= DumpSave[ToString[header <> "eLazt.mx"], eLazt];
In[5]:= Print[ToString[header <> "eLazt.mx"]]
Pair-Crtn-Univ-same_E-L-eqs-alt-approach-eLazt.mx

In[6]:= varZ = Z[#] & /@ Range[0, 15]
Out[6]= {Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6], Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}$ 
```

```
In[]:= varZzt = #[z, t] & /@ varZ
Out[]:= {Z[0][z, t], Z[1][z, t], Z[2][z, t], Z[3][z, t], Z[4][z, t],
Z[5][z, t], Z[6][z, t], Z[7][z, t], Z[8][z, t], Z[9][z, t], Z[10][z, t],
Z[11][z, t], Z[12][z, t], Z[13][z, t], Z[14][z, t], Z[15][z, t]}

In[]:= DzvarZ = D[varZzt, z]
Out[]:= {Z[0]^(1,0)[z, t], Z[1]^(1,0)[z, t], Z[2]^(1,0)[z, t], Z[3]^(1,0)[z, t],
Z[4]^(1,0)[z, t], Z[5]^(1,0)[z, t], Z[6]^(1,0)[z, t], Z[7]^(1,0)[z, t],
Z[8]^(1,0)[z, t], Z[9]^(1,0)[z, t], Z[10]^(1,0)[z, t], Z[11]^(1,0)[z, t],
Z[12]^(1,0)[z, t], Z[13]^(1,0)[z, t], Z[14]^(1,0)[z, t], Z[15]^(1,0)[z, t]}

In[]:= DtvarZ = D[varZzt, t]
Out[]:= {Z[0]^(0,1)[z, t], Z[1]^(0,1)[z, t], Z[2]^(0,1)[z, t], Z[3]^(0,1)[z, t],
Z[4]^(0,1)[z, t], Z[5]^(0,1)[z, t], Z[6]^(0,1)[z, t], Z[7]^(0,1)[z, t],
Z[8]^(0,1)[z, t], Z[9]^(0,1)[z, t], Z[10]^(0,1)[z, t], Z[11]^(0,1)[z, t],
Z[12]^(0,1)[z, t], Z[13]^(0,1)[z, t], Z[14]^(0,1)[z, t], Z[15]^(0,1)[z, t]}
```

```
In[]:= (DtvarZsubs = Solve[And @@ Thread[θ == eLazt], DtvarZ] [[1]] //  
FullSimplify[#, constraintVars] &) // Column  
  
Out[]=  
Z[0]^(0,1)[z, t] → -3 Z[5][z, t] - M Z[13][z, t] +  
e^-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] - 6 Tan[z] Z[5]^(1,0)[z, t]  
Z[1]^(0,1)[z, t] → 3 Z[4][z, t] + M Z[12][z, t] +  
e^-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] + 6 Tan[z] Z[4]^(1,0)[z, t]  
Z[2]^(0,1)[z, t] → 3 Z[7][z, t] + M Z[15][z, t] + 6 Tan[z] Z[7]^(1,0)[z, t]  
Z[3]^(0,1)[z, t] → -3 Z[6][z, t] - M Z[14][z, t] - 6 Tan[z] Z[6]^(1,0)[z, t]  
Z[4]^(0,1)[z, t] → 3 Z[1][z, t] + M Z[9][z, t] -  
e^-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] + 6 Tan[z] Z[1]^(1,0)[z, t]  
Z[5]^(0,1)[z, t] → -3 Z[0][z, t] - M Z[8][z, t] -  
e^-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] - 6 Tan[z] Z[0]^(1,0)[z, t]  
Z[6]^(0,1)[z, t] → -3 Z[3][z, t] - M Z[11][z, t] - 6 Tan[z] Z[3]^(1,0)[z, t]  
Z[7]^(0,1)[z, t] → 3 Z[2][z, t] + M Z[10][z, t] + 6 Tan[z] Z[2]^(1,0)[z, t]  
Z[8]^(0,1)[z, t] → M Z[5][z, t] + 3 Z[13][z, t] -  
e^-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + 6 Tan[z] Z[13]^(1,0)[z, t]  
Z[9]^(0,1)[z, t] → -M Z[4][z, t] - 3 Z[12][z, t] -  
e^-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] - 6 Tan[z] Z[12]^(1,0)[z, t]  
Z[10]^(0,1)[z, t] → -M Z[7][z, t] - 3 (Z[15][z, t] + 2 Tan[z] Z[15]^(1,0)[z, t])  
Z[11]^(0,1)[z, t] → M Z[6][z, t] + 3 Z[14][z, t] + 6 Tan[z] Z[14]^(1,0)[z, t]  
Z[12]^(0,1)[z, t] → -M Z[1][z, t] - 3 Z[9][z, t] +  
e^-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] - 6 Tan[z] Z[9]^(1,0)[z, t]  
Z[13]^(0,1)[z, t] → M Z[0][z, t] + 3 Z[8][z, t] +  
e^-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] + 6 Tan[z] Z[8]^(1,0)[z, t]  
Z[14]^(0,1)[z, t] → M Z[3][z, t] + 3 Z[11][z, t] + 6 Tan[z] Z[11]^(1,0)[z, t]  
Z[15]^(0,1)[z, t] → -M Z[2][z, t] - 3 (Z[10][z, t] + 2 Tan[z] Z[10]^(1,0)[z, t])
```

```
In[ ]:= ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(DtvarZEQS = ToExpression[#] & /@ %) // Column
```

```
Out[ ]=

$$\begin{aligned} Z[0]^{(0,1)}[z, t] &= -3 Z[5][z, t] - M Z[13][z, t] + \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t] \\ Z[1]^{(0,1)}[z, t] &= 3 Z[4][z, t] + M Z[12][z, t] + \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t] \\ Z[2]^{(0,1)}[z, t] &= 3 Z[7][z, t] + M Z[15][z, t] + 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t] \\ Z[3]^{(0,1)}[z, t] &= -3 Z[6][z, t] - M Z[14][z, t] - 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t] \\ Z[4]^{(0,1)}[z, t] &= 3 Z[1][z, t] + M Z[9][z, t] - \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t] \\ Z[5]^{(0,1)}[z, t] &= -3 Z[0][z, t] - M Z[8][z, t] - \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t] \\ Z[6]^{(0,1)}[z, t] &= -3 Z[3][z, t] - M Z[11][z, t] - 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t] \\ Z[7]^{(0,1)}[z, t] &= 3 Z[2][z, t] + M Z[10][z, t] + 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t] \\ Z[8]^{(0,1)}[z, t] &= M Z[5][z, t] + 3 Z[13][z, t] - \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t] \\ Z[9]^{(0,1)}[z, t] &= -M Z[4][z, t] - 3 Z[12][z, t] - \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t] \\ Z[10]^{(0,1)}[z, t] &= -M Z[7][z, t] - 3 (Z[15][z, t] + 2 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t]) \\ Z[11]^{(0,1)}[z, t] &= M Z[6][z, t] + 3 Z[14][z, t] + 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t] \\ Z[12]^{(0,1)}[z, t] &= -M Z[1][z, t] - 3 Z[9][z, t] + \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t] \\ Z[13]^{(0,1)}[z, t] &= M Z[0][z, t] + 3 Z[8][z, t] + \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t] \\ Z[14]^{(0,1)}[z, t] &= M Z[3][z, t] + 3 Z[11][z, t] + 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t] \\ Z[15]^{(0,1)}[z, t] &= -M Z[2][z, t] - 3 (Z[10][z, t] + 2 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t]) \end{aligned}$$

```

```
In[1]:= ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(DtvarZrelations = ToExpression[#] & /@ %) // Column
```

```
Out[1]=
3 Z[5][z, t] + M Z[13][z, t] - e^{-a4[t]} Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] +
Z[0]^{(0,1)}[z, t] + 6 Tan[z] Z[5]^{(1,0)}[z, t]
-3 Z[4][z, t] - M Z[12][z, t] -
e^{-a4[t]} Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] + Z[1]^{(0,1)}[z, t] - 6 Tan[z] Z[4]^{(1,0)}[z, t]
-3 Z[7][z, t] - M Z[15][z, t] + Z[2]^{(0,1)}[z, t] - 6 Tan[z] Z[7]^{(1,0)}[z, t]
3 Z[6][z, t] + M Z[14][z, t] + Z[3]^{(0,1)}[z, t] + 6 Tan[z] Z[6]^{(1,0)}[z, t]
-3 Z[1][z, t] - M Z[9][z, t] + e^{-a4[t]} Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] +
Z[4]^{(0,1)}[z, t] - 6 Tan[z] Z[1]^{(1,0)}[z, t]
3 Z[0][z, t] + M Z[8][z, t] + e^{-a4[t]} Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] +
Z[5]^{(0,1)}[z, t] + 6 Tan[z] Z[0]^{(1,0)}[z, t]
3 Z[3][z, t] + M Z[11][z, t] + Z[6]^{(0,1)}[z, t] + 6 Tan[z] Z[3]^{(1,0)}[z, t]
-3 Z[2][z, t] - M Z[10][z, t] + Z[7]^{(0,1)}[z, t] - 6 Tan[z] Z[2]^{(1,0)}[z, t]
-M Z[5][z, t] - 3 Z[13][z, t] + e^{-a4[t]} Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] +
Z[8]^{(0,1)}[z, t] - 6 Tan[z] Z[13]^{(1,0)}[z, t]
M Z[4][z, t] + 3 Z[12][z, t] + e^{-a4[t]} Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] +
Z[9]^{(0,1)}[z, t] + 6 Tan[z] Z[12]^{(1,0)}[z, t]
M Z[7][z, t] + Z[10]^{(0,1)}[z, t] + 3 (Z[15][z, t] + 2 Tan[z] Z[15]^{(1,0)}[z, t])
-M Z[6][z, t] - 3 Z[14][z, t] + Z[11]^{(0,1)}[z, t] - 6 Tan[z] Z[14]^{(1,0)}[z, t]
M Z[1][z, t] + 3 Z[9][z, t] - e^{-a4[t]} Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] +
Z[12]^{(0,1)}[z, t] + 6 Tan[z] Z[9]^{(1,0)}[z, t]
-M Z[0][z, t] - 3 Z[8][z, t] - e^{-a4[t]} Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] +
Z[13]^{(0,1)}[z, t] - 6 Tan[z] Z[8]^{(1,0)}[z, t]
-M Z[3][z, t] - 3 Z[11][z, t] + Z[14]^{(0,1)}[z, t] - 6 Tan[z] Z[11]^{(1,0)}[z, t]
M Z[2][z, t] + Z[15]^{(0,1)}[z, t] + 3 (Z[10][z, t] + 2 Tan[z] Z[10]^{(1,0)}[z, t])
```

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE:

```
In[2]:= eLaztCouplings = eLaCouplings
Out[2]= {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
```

```
In[3]:= (*{{5,13,0,8,5,5,0,5,13,8},{4,12,1,1,9,4},{7,15,2,2,10,7},{6,14,3,3,11,6}}
eLaztCouplings=Union[#]&/@%
(*eLaztCouplings=Union[Flatten[#]]&/@%*)
Union[Flatten[%]]==Range[0,15]*)
```

SEPARATE VARIABLES THAT ARE COUPLED TO EACH OTHER (reorder):

```
In[]:= Thread[(Z[#] & /@ Flatten[eLastCouplings]) → (yZ[#] & /@ Range[0, 15])]
% // Sort
sZt0yZ
% - %%
```

Out[]=

```
{Z[0] → yZ[0], Z[5] → yZ[1], Z[8] → yZ[2], Z[13] → yZ[3], Z[1] → yZ[4], Z[4] → yZ[5],
Z[9] → yZ[6], Z[12] → yZ[7], Z[2] → yZ[8], Z[7] → yZ[9], Z[10] → yZ[10],
Z[15] → yZ[11], Z[3] → yZ[12], Z[6] → yZ[13], Z[11] → yZ[14], Z[14] → yZ[15]}
```

Out[]=

```
{Z[0] → yZ[0], Z[1] → yZ[4], Z[2] → yZ[8], Z[3] → yZ[12], Z[4] → yZ[5], Z[5] → yZ[1],
Z[6] → yZ[13], Z[7] → yZ[9], Z[8] → yZ[2], Z[9] → yZ[6], Z[10] → yZ[10],
Z[11] → yZ[14], Z[12] → yZ[7], Z[13] → yZ[3], Z[14] → yZ[15], Z[15] → yZ[11]}
```

Out[]=

```
sZt0yZ
```

Out[]=

```
{sZt0yZ - (Z[0] → yZ[0]), sZt0yZ - (Z[1] → yZ[4]),
sZt0yZ - (Z[2] → yZ[8]), sZt0yZ - (Z[3] → yZ[12]), sZt0yZ - (Z[4] → yZ[5]),
sZt0yZ - (Z[5] → yZ[1]), sZt0yZ - (Z[6] → yZ[13]), sZt0yZ - (Z[7] → yZ[9]),
sZt0yZ - (Z[8] → yZ[2]), sZt0yZ - (Z[9] → yZ[6]), sZt0yZ - (Z[10] → yZ[10]),
sZt0yZ - (Z[11] → yZ[14]), sZt0yZ - (Z[12] → yZ[7]), sZt0yZ - (Z[13] → yZ[3]),
sZt0yZ - (Z[14] → yZ[15]), sZt0yZ - (Z[15] → yZ[11])}
```

```

In[=]:= (*Thread[(Z[#]&/@eLaztCouplings[[1]])>(yZ[#]&/@Range[0,3])]
Thread[(Z[#]&/@eLaztCouplings[[2]])>(yZ[#]&/@Range[4,7])]
Thread[(Z[#]&/@eLaztCouplings[[3]])>(yZ[#]&/@Range[8,11])]
Thread[(Z[#]&/@eLaztCouplings[[4]])>(yZ[#]&/@Range[12,15])]*)
sZt0yZ = Thread[(Z[#] & /@ Flatten[eLaztCouplings]) > (yZ[#] & /@ Range[0, 15])]
(*Union[Flatten[{%,%%,%%%,%%%%}]]*)
ZyZEQS = sZt0yZ /. {Rule → Equal}
ZyZforCaEQS = -sZt0yZ /. {Rule → Subtract}
(*ToString[FullForm[#]]&/@sZt0yZ;
StringReplace[#, "Rule"→ "Equal"]&/@%;*)
(*(ZyZEQS=ToExpression[#]&/@%)*)
syZtoZ = Solve[And @@ ZyZEQS, (yZ[#] & /@ Range[0, 15])] [[1]]
Out[=]= {Z[0] → yZ[0], Z[5] → yZ[1], Z[8] → yZ[2], Z[13] → yZ[3], Z[1] → yZ[4], Z[4] → yZ[5],
Z[9] → yZ[6], Z[12] → yZ[7], Z[2] → yZ[8], Z[7] → yZ[9], Z[10] → yZ[10],
Z[15] → yZ[11], Z[3] → yZ[12], Z[6] → yZ[13], Z[11] → yZ[14], Z[14] → yZ[15]}

Out[=]= {Z[0] == yZ[0], Z[5] == yZ[1], Z[8] == yZ[2], Z[13] == yZ[3], Z[1] == yZ[4], Z[4] == yZ[5],
Z[9] == yZ[6], Z[12] == yZ[7], Z[2] == yZ[8], Z[7] == yZ[9], Z[10] == yZ[10],
Z[15] == yZ[11], Z[3] == yZ[12], Z[6] == yZ[13], Z[11] == yZ[14], Z[14] == yZ[15]}

Out[=]= {yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1], yZ[5] - Z[4],
yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}

Out[=]= {yZ[0] → Z[0], yZ[1] → Z[5], yZ[2] → Z[8], yZ[3] → Z[13], yZ[4] → Z[1], yZ[5] → Z[4],
yZ[6] → Z[9], yZ[7] → Z[12], yZ[8] → Z[2], yZ[9] → Z[7], yZ[10] → Z[10],
yZ[11] → Z[15], yZ[12] → Z[3], yZ[13] → Z[6], yZ[14] → Z[11], yZ[15] → Z[14]}

(*Thread[(Z[#]&/@Flatten[eLaztCouplings])>(yZ[#]&/@Range[0,15])]
%//Sort
sZt0yZ
%-%%*)

In[=]:= ToString[FullForm[#]] & /@ sZt0yZ;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(ZyZforCaEQS = -ToExpression[#] & /@ %)

Out[=]= {yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1], yZ[5] - Z[4],
yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}

```

```
In[•]:= cayZ = CoefficientArrays[ZyZforCaEQS, (yZ[#] & /@ Range[0, 15])]
```

Out[•]=

{SparseArray[ Specified elements: 16 Dimensions: {16}], SparseArray[ Specified elements: 16 Dimensions: {16, 16}]}

```
In[•]:= caZ = CoefficientArrays[ZyZforCaEQS, (Z[#] & /@ Range[0, 15])]
```

Out[•]=

{SparseArray[ Specified elements: 16 Dimensions: {16}], SparseArray[ Specified elements: 16 Dimensions: {16, 16}]}

```
In[•]:= (cayZ2 = cayZ[[2]] // Normal) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

IDENTIFY ORTHOGONAL SIMILARITY TRANSFORMATION FROM SET OF VARIABLES Z[j] TO SET OF VARIABLES yZ[j] :

```
In[•]:= cayZ2.(yZ[#] & /@ Range[0, 15])
(*   cayZ2.yZ === Z : *)
% /. syZtoZ
(*   inverse: yZ === Transpose[cayZ2].Z *)
Transpose[cayZ2].% /. sZt0yZ
```

Out[•]=

{yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}

Out[•]=

{Z[0], Z[5], Z[8], Z[13], Z[1], Z[4], Z[9],
Z[12], Z[2], Z[7], Z[10], Z[15], Z[3], Z[6], Z[11], Z[14]}

Out[•]=

{yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}

ORTHOGONAL O(16) SIMILARITY TRANSFORMATION :

```
In[ ]:= cayZ2.Transpose[cayZ2] === ID16
```

Out[]=

True

BUT NOT ORTHOGONAL O(8,8) SIMILARITY TRANSFORMATION :

```
In[ ]:= cayZ2.σ16.Transpose[cayZ2] === σ16
```

Out[]=

True

Also, Not a Direct Sum :

```
In[ ]:= cayZ2 === ArrayFlatten[{{cayZ2[[1;;8, 1;;8]], cayZ2[[1;;8, 9;;16]]}, {cayZ2[[9;;16, 1;;8]], cayZ2[[9;;16, 9;;16]]}}]
```

Out[]=

True

```
In[ ]:= cayZ2 === ArrayFlatten[{{cayZ2[[1;;8, 1;;8]], 0}, {0, cayZ2[[9;;16, 9;;16]]}}]
```

Out[]=

True

TRANSFORM Euler Lagrange equations to yZ[j] :

```
In[]:= (DtyZrelations = Transpose[cayZ2].DtvarZrelations /. sZt0yZ // FullSimplify) // Column
Out[]=
3 yZ[1][z, t] + M yZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] +
yZ[0]^(0,1)[z, t] + 6 Tan[z] yZ[1]^(1,0)[z, t]
-3 yZ[5][z, t] - M yZ[7][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
yZ[4]^(0,1)[z, t] - 6 Tan[z] yZ[5]^(1,0)[z, t]
-3 yZ[9][z, t] - M yZ[11][z, t] + yZ[8]^(0,1)[z, t] - 6 Tan[z] yZ[9]^(1,0)[z, t]
3 yZ[13][z, t] + M yZ[15][z, t] + yZ[12]^(0,1)[z, t] + 6 Tan[z] yZ[13]^(1,0)[z, t]
-3 yZ[4][z, t] - M yZ[6][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] +
yZ[5]^(0,1)[z, t] - 6 Tan[z] yZ[4]^(1,0)[z, t]
3 yZ[0][z, t] + M yZ[2][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] +
yZ[1]^(0,1)[z, t] + 6 Tan[z] yZ[0]^(1,0)[z, t]
3 yZ[12][z, t] + M yZ[14][z, t] + yZ[13]^(0,1)[z, t] + 6 Tan[z] yZ[12]^(1,0)[z, t]
-3 yZ[8][z, t] - M yZ[10][z, t] + yZ[9]^(0,1)[z, t] - 6 Tan[z] yZ[8]^(1,0)[z, t]
-M yZ[1][z, t] - 3 yZ[3][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] +
yZ[2]^(0,1)[z, t] - 6 Tan[z] yZ[3]^(1,0)[z, t]
M yZ[5][z, t] + 3 yZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] +
yZ[6]^(0,1)[z, t] + 6 Tan[z] yZ[7]^(1,0)[z, t]
M yZ[9][z, t] + 3 yZ[11][z, t] + yZ[10]^(0,1)[z, t] + 6 Tan[z] yZ[11]^(1,0)[z, t]
-M yZ[13][z, t] - 3 yZ[15][z, t] + yZ[14]^(0,1)[z, t] - 6 Tan[z] yZ[15]^(1,0)[z, t]
M yZ[4][z, t] + 3 yZ[6][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] +
yZ[7]^(0,1)[z, t] + 6 Tan[z] yZ[6]^(1,0)[z, t]
-M yZ[0][z, t] - 3 yZ[2][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] +
yZ[3]^(0,1)[z, t] - 6 Tan[z] yZ[2]^(1,0)[z, t]
-M yZ[12][z, t] - 3 yZ[14][z, t] + yZ[15]^(0,1)[z, t] - 6 Tan[z] yZ[14]^(1,0)[z, t]
M yZ[8][z, t] + 3 yZ[10][z, t] + yZ[11]^(0,1)[z, t] + 6 Tan[z] yZ[10]^(1,0)[z, t]
```

CHECK THAT $yZ[j]$ ARE IN PROPER ORDER (4 things, 4 at a time):

```

In[]:= (DtyZsubs = Solve[And @@ Thread[θ == DtyZrelations],
  (D[(yZ[#][z, t] & /@ Range[0, 15]), t])[[1]] //*
  FullSimplify[#, constraintVars] &) // Column

Out[=]
yZ[0]^(0,1) [z, t] → -3 yZ[1] [z, t] - M yZ[3] [z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[0] [z, t] a4'[t] - 6 Tan[z] yZ[1]^(1,0) [z, t]
yZ[1]^(0,1) [z, t] → -3 yZ[0] [z, t] - M yZ[2] [z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[1] [z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0) [z, t]
yZ[2]^(0,1) [z, t] → M yZ[1] [z, t] + 3 yZ[3] [z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[2] [z, t] a4'[t] + 6 Tan[z] yZ[3]^(1,0) [z, t]
yZ[3]^(0,1) [z, t] → M yZ[0] [z, t] + 3 yZ[2] [z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[3] [z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0) [z, t]
yZ[4]^(0,1) [z, t] → 3 yZ[5] [z, t] + M yZ[7] [z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[4] [z, t] a4'[t] + 6 Tan[z] yZ[5]^(1,0) [z, t]
yZ[5]^(0,1) [z, t] → 3 yZ[4] [z, t] + M yZ[6] [z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[5] [z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0) [z, t]
yZ[6]^(0,1) [z, t] → -M yZ[5] [z, t] - 3 yZ[7] [z, t] -
  e^-a4[t] Q1 Sinh[a4[t]] yZ[6] [z, t] a4'[t] - 6 Tan[z] yZ[7]^(1,0) [z, t]
yZ[7]^(0,1) [z, t] → -M yZ[4] [z, t] - 3 yZ[6] [z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[7] [z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0) [z, t]
yZ[8]^(0,1) [z, t] → 3 yZ[9] [z, t] + M yZ[11] [z, t] + 6 Tan[z] yZ[9]^(1,0) [z, t]
yZ[9]^(0,1) [z, t] → 3 yZ[8] [z, t] + M yZ[10] [z, t] + 6 Tan[z] yZ[8]^(1,0) [z, t]
yZ[10]^(0,1) [z, t] → -M yZ[9] [z, t] - 3 (yZ[11] [z, t] + 2 Tan[z] yZ[11]^(1,0) [z, t])
yZ[11]^(0,1) [z, t] → -M yZ[8] [z, t] - 3 (yZ[10] [z, t] + 2 Tan[z] yZ[10]^(1,0) [z, t])
yZ[12]^(0,1) [z, t] → -3 yZ[13] [z, t] - M yZ[15] [z, t] - 6 Tan[z] yZ[13]^(1,0) [z, t]
yZ[13]^(0,1) [z, t] → -3 yZ[12] [z, t] - M yZ[14] [z, t] - 6 Tan[z] yZ[12]^(1,0) [z, t]
yZ[14]^(0,1) [z, t] → M yZ[13] [z, t] + 3 yZ[15] [z, t] + 6 Tan[z] yZ[15]^(1,0) [z, t]
yZ[15]^(0,1) [z, t] → M yZ[12] [z, t] + 3 yZ[14] [z, t] + 6 Tan[z] yZ[14]^(1,0) [z, t]

```

```
In[]:= (caZ2 = caZ[[2]] // Normal) // MatrixForm
Out[//MatrixForm=
```

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


```
In[]:= sx0x4
Out[=
```

$$\left\{ x0 \rightarrow \frac{z}{6H}, x4 \rightarrow \frac{t}{H} \right\}$$


```
In[]:= (*DtvarZEQS*)
In[]:= (*DtyZsubs*)
In[]:= (*DtvarZsubs=Solve[And@@Thread[0==eLast],DtvarZ][[1]]//FullSimplify[#,constraintVars]&)//Column*)
In[]:= (*(DtyZsubs=
Solve[And@@Thread[0==DtyZrelations],(D[(yZ[#][z,t]&/@Range[0,15]),t])][[1]]//FullSimplify[#,constraintVars]&)//Column*)
```

```
In[]:= (*ToString[FullForm[#]]&/@DtyZsubs;
StringReplace[#, "Rule" → "Equal"]&/@%;
(DtvaryZEQS=ToExpression[#]&/%) //Column*)
DtvaryZEQS = DtyZsubs /. {Rule → Equal};
zeroDtyZeqs = DtyZsubs /. {Rule → Subtract};
coupledyZeqs = Partition[DtvaryZEQS, 4]
```

```

Out[ ]=
{3 yZ[1][z, t] + M yZ[3][z, t] - e^{-a4[t]} Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] +
yZ[0]^{(0,1)}[z, t] + 6 Tan[z] yZ[1]^{(1,0)}[z, t], 3 yZ[0][z, t] + M yZ[2][z, t] +
e^{-a4[t]} Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] + yZ[1]^{(0,1)}[z, t] + 6 Tan[z] yZ[0]^{(1,0)}[z, t],
-M yZ[1][z, t] - 3 yZ[3][z, t] + e^{-a4[t]} Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] +
yZ[2]^{(0,1)}[z, t] - 6 Tan[z] yZ[3]^{(1,0)}[z, t], -M yZ[0][z, t] - 3 yZ[2][z, t] -
e^{-a4[t]} Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + yZ[3]^{(0,1)}[z, t] - 6 Tan[z] yZ[2]^{(1,0)}[z, t],
-3 yZ[5][z, t] - M yZ[7][z, t] - e^{-a4[t]} Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
yZ[4]^{(0,1)}[z, t] - 6 Tan[z] yZ[5]^{(1,0)}[z, t], -3 yZ[4][z, t] - M yZ[6][z, t] +
e^{-a4[t]} Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + yZ[5]^{(0,1)}[z, t] - 6 Tan[z] yZ[4]^{(1,0)}[z, t],
M yZ[5][z, t] + 3 yZ[7][z, t] + e^{-a4[t]} Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] +
yZ[6]^{(0,1)}[z, t] + 6 Tan[z] yZ[7]^{(1,0)}[z, t], M yZ[4][z, t] + 3 yZ[6][z, t] -
e^{-a4[t]} Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] + yZ[7]^{(0,1)}[z, t] + 6 Tan[z] yZ[6]^{(1,0)}[z, t],
-3 yZ[9][z, t] - M yZ[11][z, t] + yZ[8]^{(0,1)}[z, t] - 6 Tan[z] yZ[9]^{(1,0)}[z, t],
-3 yZ[8][z, t] - M yZ[10][z, t] + yZ[9]^{(0,1)}[z, t] - 6 Tan[z] yZ[8]^{(1,0)}[z, t],
M yZ[9][z, t] + yZ[10]^{(0,1)}[z, t] + 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^{(1,0)}[z, t]),
M yZ[8][z, t] + yZ[11]^{(0,1)}[z, t] + 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^{(1,0)}[z, t]),
3 yZ[13][z, t] + M yZ[15][z, t] + yZ[12]^{(0,1)}[z, t] + 6 Tan[z] yZ[13]^{(1,0)}[z, t],
3 yZ[12][z, t] + M yZ[14][z, t] + yZ[13]^{(0,1)}[z, t] + 6 Tan[z] yZ[12]^{(1,0)}[z, t],
-M yZ[13][z, t] - 3 yZ[15][z, t] + yZ[14]^{(0,1)}[z, t] - 6 Tan[z] yZ[15]^{(1,0)}[z, t],
-M yZ[12][z, t] - 3 yZ[14][z, t] + yZ[15]^{(0,1)}[z, t] - 6 Tan[z] yZ[14]^{(1,0)}[z, t]}

```

```

Out[ ]= { {yZ[0]^(0,1)[z, t] == -3 yZ[1][z, t] - M yZ[3][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - 6 Tan[z] yZ[1]^(1,0)[z, t],  

yZ[1]^(0,1)[z, t] == -3 yZ[0][z, t] - M yZ[2][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t],  

yZ[2]^(0,1)[z, t] == M yZ[1][z, t] + 3 yZ[3][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] + 6 Tan[z] yZ[3]^(1,0)[z, t],  

yZ[3]^(0,1)[z, t] == M yZ[0][z, t] + 3 yZ[2][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]},  

{yZ[4]^(0,1)[z, t] == 3 yZ[5][z, t] + M yZ[7][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] + 6 Tan[z] yZ[5]^(1,0)[z, t],  

yZ[5]^(0,1)[z, t] == 3 yZ[4][z, t] + M yZ[6][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t],  

yZ[6]^(0,1)[z, t] == -M yZ[5][z, t] - 3 yZ[7][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - 6 Tan[z] yZ[7]^(1,0)[z, t],  

yZ[7]^(0,1)[z, t] == -M yZ[4][z, t] - 3 yZ[6][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]},  

{yZ[8]^(0,1)[z, t] == 3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t],  

yZ[9]^(0,1)[z, t] == 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t],  

yZ[10]^(0,1)[z, t] == -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),  

yZ[11]^(0,1)[z, t] == -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])},  

{yZ[12]^(0,1)[z, t] == -3 yZ[13][z, t] - M yZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t],  

yZ[13]^(0,1)[z, t] == -3 yZ[12][z, t] - M yZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t],  

yZ[14]^(0,1)[z, t] == M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t],  

yZ[15]^(0,1)[z, t] == M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]}}

In[ ]:= (*ToString[FullForm[#]]&/@Take[coupledYzeqs,2]
StringReplace[#, "Equal"→ "Subtract"]&/@%;
(ToExpression[#]&/@%)
#/.{Equal→Subtract}&/@Take[coupledYzeqs,2]
%-%%*)

In[ ]:= Clear[yZ, yZvar];
yZvar = (yZ[#][z, t] & /@ Range[0, 15])

Out[ ]=
{yZ[0][z, t], yZ[1][z, t], yZ[2][z, t], yZ[3][z, t], yZ[4][z, t],
yZ[5][z, t], yZ[6][z, t], yZ[7][z, t], yZ[8][z, t], yZ[9][z, t], yZ[10][z, t],
yZ[11][z, t], yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t]}

```

```

In[1]:= DSolve[coupledYZeqs[[1]], yZvar[[1 ;; 4]], {z, t}]
Out[1]= DSolve[{yZ[0]^(0,1)[z, t] == -3 yZ[1][z, t] - M yZ[3][z, t] +
  e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - 6 Tan[z] yZ[1]^(1,0)[z, t],  

  yZ[1]^(0,1)[z, t] == -3 yZ[0][z, t] - M yZ[2][z, t] -  

  e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t],  

  yZ[2]^(0,1)[z, t] == M yZ[1][z, t] + 3 yZ[3][z, t] -  

  e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] + 6 Tan[z] yZ[3]^(1,0)[z, t],  

  yZ[3]^(0,1)[z, t] == M yZ[0][z, t] + 3 yZ[2][z, t] +  

  e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]},  

{yZ[0][z, t], yZ[1][z, t], yZ[2][z, t], yZ[3][z, t]}, {z, t}]

In[2]:= DSolve[coupledYZeqs[[2]], yZvar[[5 ;; 8]], {z, t}]
Out[2]= DSolve[{yZ[4]^(0,1)[z, t] == 3 yZ[5][z, t] + M yZ[7][z, t] +  

  e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] + 6 Tan[z] yZ[5]^(1,0)[z, t],  

  yZ[5]^(0,1)[z, t] == 3 yZ[4][z, t] + M yZ[6][z, t] -  

  e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t],  

  yZ[6]^(0,1)[z, t] == -M yZ[5][z, t] - 3 yZ[7][z, t] -  

  e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - 6 Tan[z] yZ[7]^(1,0)[z, t],  

  yZ[7]^(0,1)[z, t] == -M yZ[4][z, t] - 3 yZ[6][z, t] +  

  e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]},  

{yZ[4][z, t], yZ[5][z, t], yZ[6][z, t], yZ[7][z, t]}, {z, t}]

In[3]:= DSolve[coupledYZeqs[[3]], yZvar[[9 ;; 12]], {z, t}]
Out[3]= DSolve[{yZ[8]^(0,1)[z, t] == 3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t],  

  yZ[9]^(0,1)[z, t] == 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t],  

  yZ[10]^(0,1)[z, t] == -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),  

  yZ[11]^(0,1)[z, t] == -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])},  

{yZ[8][z, t], yZ[9][z, t], yZ[10][z, t], yZ[11][z, t]}, {z, t}]

In[4]:= DSolve[coupledYZeqs[[4]], yZvar[[13 ;; 16]], {z, t}]
Out[4]= DSolve[{yZ[12]^(0,1)[z, t] == -3 yZ[13][z, t] - M yZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t],  

  yZ[13]^(0,1)[z, t] == -3 yZ[12][z, t] - M yZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t],  

  yZ[14]^(0,1)[z, t] == M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t],  

  yZ[15]^(0,1)[z, t] == M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]},  

{yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t]}, {z, t}]

```

$$\begin{aligned}
eq4 := & [3 yZ13(z, t) + MyZ15(z, t) + D_2(yZ12)(z, t) + 6 \tan(z) I \\
& = 0, 3 yZ12(z, t) + MyZ14(z, t) + D_2(yZ13)(z, t) + 6 \tan(z) I \\
& = 0, MyZ13(z, t) + 3 yZ15(z, t) + 6 \tan(z) D_1(yZ15)(z, t) = 0 \\
& t), MyZ12(z, t) + 3 yZ14(z, t) + 6 \tan(z) D_1(yZ14)(z, t) = D_2(t)
\end{aligned}$$

TRY MAPLE: MUST use ` List[Equal[Derivative[0,1][yZ8][z,t],Pl...`

```

In[8]:= syZzt = yZ[#] → ToExpression["((yZ" <> ToString[#] <> "[#1,#2])&)"] & /@ Range[0, 15]
maplexyZeqs = FullForm[coupledyyZeqs /. % // FullSimplify]

Out[8]=
{yZ[0] → (yZ0[#1, #2] &), yZ[1] → (yZ1[#1, #2] &),
 yZ[2] → (yZ2[#1, #2] &), yZ[3] → (yZ3[#1, #2] &), yZ[4] → (yZ4[#1, #2] &),
 yZ[5] → (yZ5[#1, #2] &), yZ[6] → (yZ6[#1, #2] &), yZ[7] → (yZ7[#1, #2] &),
 yZ[8] → (yZ8[#1, #2] &), yZ[9] → (yZ9[#1, #2] &), yZ[10] → (yZ10[#1, #2] &),
 yZ[11] → (yZ11[#1, #2] &), yZ[12] → (yZ12[#1, #2] &),
 yZ[13] → (yZ13[#1, #2] &), yZ[14] → (yZ14[#1, #2] &), yZ[15] → (yZ15[#1, #2] &)}

```

```

Out[=]//FullForm=
List[List[
  Equal[Plus[Times[3, yZ1[z, t]], Times[M, yZ3[z, t]], Derivative[0, 1][yZ0][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ1][z, t]]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ0[z, t], Derivative[1][a4][t]]], 
  Equal[Plus[Times[3, yZ0[z, t]], Times[M, yZ2[z, t]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ1[z, t], Derivative[1][a4][t]],
    Derivative[0, 1][yZ1][z, t], Times[6, Tan[z], Derivative[1, 0][yZ0][z, t]]], 0],
  Equal[Plus[Times[M, yZ1[z, t]], Times[3, yZ3[z, t]], 
    Times[6, Tan[z], Derivative[1, 0][yZ3][z, t]]], 
  Plus[Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ2[z, t],
    Derivative[1][a4][t]], Derivative[0, 1][yZ2][z, t]]], 
  Equal[Plus[Times[M, yZ0[z, t]], Times[3, yZ2[z, t]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ3[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][yZ2][z, t]]], Derivative[0, 1][yZ3][z, t]]], 
  List[Equal[Plus[Times[3, yZ5[z, t]], Times[M, yZ7[z, t]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ4[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][yZ5][z, t]]], Derivative[0, 1][yZ4][z, t]],
  Equal[Plus[Times[3, yZ4[z, t]], Times[M, yZ6[z, t]], 
    Times[6, Tan[z], Derivative[1, 0][yZ4][z, t]]], 
  Plus[Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ5[z, t],
    Derivative[1][a4][t]], Derivative[0, 1][yZ5][z, t]]], 
  Equal[Plus[Times[M, yZ5[z, t]], Times[3, yZ7[z, t]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ6[z, t], Derivative[1][a4][t]],
    Derivative[0, 1][yZ6][z, t], Times[6, Tan[z], Derivative[1, 0][yZ7][z, t]]], 0],
  Equal[Plus[Times[M, yZ4[z, t]], Times[3, yZ6[z, t]], Derivative[0, 1][yZ7][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ6][z, t]]], Times[
    Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ7[z, t], Derivative[1][a4][t]]], 
  List[Equal[Plus[Times[M, yZ11[z, t]], Times[3, yZ9[z, t]], 
    Times[6, Tan[z], Derivative[1, 0][yZ9][z, t]]], Derivative[0, 1][yZ8][z, t]],
  Equal[Plus[Times[M, yZ10[z, t]], Times[3, yZ8[z, t]], 
    Times[6, Tan[z], Derivative[1, 0][yZ8][z, t]]], Derivative[0, 1][yZ9][z, t]],
  Equal[Plus[Times[3, yZ11[z, t]], Times[M, yZ9[z, t]], Derivative[0, 1][yZ10][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ11][z, t]]], 0],
  Equal[Plus[Times[3, yZ10[z, t]], Times[M, yZ8[z, t]], Derivative[0, 1][yZ11][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ10][z, t]]], 0]], 
  List[Equal[Plus[Times[3, yZ13[z, t]], Times[M, yZ15[z, t]], 
    Derivative[0, 1][yZ12][z, t], Times[6, Tan[z], Derivative[1, 0][yZ13][z, t]]], 0],
  Equal[Plus[Times[3, yZ12[z, t]], Times[M, yZ14[z, t]], 
    Derivative[0, 1][yZ13][z, t], Times[6, Tan[z], Derivative[1, 0][yZ12][z, t]]], 0],
  Equal[Plus[Times[M, yZ13[z, t]], Times[3, yZ15[z, t]], 
    Times[6, Tan[z], Derivative[1, 0][yZ15][z, t]]], Derivative[0, 1][yZ14][z, t]],
  Equal[Plus[Times[M, yZ12[z, t]], Times[3, yZ14[z, t]], 
    Times[6, Tan[z], Derivative[1, 0][yZ14][z, t]]], Derivative[0, 1][yZ15][z, t]]]
]

```

```
In[]:= mapleyZvars =
FullForm[yZ[#][z, t] & /@ Range[4 * (# - 1), 4 * (# - 1) + 3] /. syZzt] & /@ Range[4]
Out[]:= {List[yZ0[z, t], yZ1[z, t], yZ2[z, t], yZ3[z, t]], 
List[yZ4[z, t], yZ5[z, t], yZ6[z, t], yZ7[z, t]], 
List[yZ8[z, t], yZ9[z, t], yZ10[z, t], yZ11[z, t]], 
List[yZ12[z, t], yZ13[z, t], yZ14[z, t], yZ15[z, t]]}
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

```
“maple-output.py”:
import textwrap
filename = "maple-output.txt"
maple_long_string = "paste maple string here"
# Wraps the text and joins with newlines
with open (filename, "w") as f:
    f . write ("\n" . join (textwrap . wrap (maple_long_string, width = 80)))
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

"Solution" by Maple

```
In[]:= Φ16;
% /. sfψ16Aa /. sx0x4;
% /. sZt0yZ;
Thread[% == %%%];
Solve[And @@ %, yZ[#][z, t] & /@ Range[0, 15]] [[1]]
% /. szt /. {Rule → Equal};
Solve[And @@ %, f16[#][x0, x4] & /@ Range[0, 15]] [[1]];
% /. {Rule → Equal}

Out[=]=
{yZ[0][z, t] → f16[0][x0, x4], yZ[1][z, t] → f16[5][x0, x4],
 yZ[2][z, t] → f16[8][x0, x4], yZ[3][z, t] → f16[13][x0, x4],
 yZ[4][z, t] → f16[1][x0, x4], yZ[5][z, t] → f16[4][x0, x4],
 yZ[6][z, t] → f16[9][x0, x4], yZ[7][z, t] → f16[12][x0, x4],
 yZ[8][z, t] → f16[2][x0, x4], yZ[9][z, t] → f16[7][x0, x4],
 yZ[10][z, t] → f16[10][x0, x4], yZ[11][z, t] → f16[15][x0, x4],
 yZ[12][z, t] → f16[3][x0, x4], yZ[13][z, t] → f16[6][x0, x4],
 yZ[14][z, t] → f16[11][x0, x4], yZ[15][z, t] → f16[14][x0, x4]}

Out[=]=
{f16[0][x0, x4] == yZ[0][6 H x0, H x4], f16[1][x0, x4] == yZ[4][6 H x0, H x4],
 f16[2][x0, x4] == yZ[8][6 H x0, H x4], f16[3][x0, x4] == yZ[12][6 H x0, H x4],
 f16[4][x0, x4] == yZ[5][6 H x0, H x4], f16[5][x0, x4] == yZ[1][6 H x0, H x4],
 f16[6][x0, x4] == yZ[13][6 H x0, H x4], f16[7][x0, x4] == yZ[9][6 H x0, H x4],
 f16[8][x0, x4] == yZ[2][6 H x0, H x4], f16[9][x0, x4] == yZ[6][6 H x0, H x4],
 f16[10][x0, x4] == yZ[10][6 H x0, H x4], f16[11][x0, x4] == yZ[14][6 H x0, H x4],
 f16[12][x0, x4] == yZ[7][6 H x0, H x4], f16[13][x0, x4] == yZ[3][6 H x0, H x4],
 f16[14][x0, x4] == yZ[15][6 H x0, H x4], f16[15][x0, x4] == yZ[11][6 H x0, H x4]}

Load the Maple output strings. Get is a lexer and parer and ... , so use ReadString
```

```
In[]:= SetDirectory[NotebookDirectory[]];
```

$$\begin{aligned} seq3 := \left\{ yZ10(z, t) = \frac{1}{\sqrt{\sin(z)}} \left((c37 \sin(\sqrt{M^2 - 36 C3 - 9} t) \right. \right. \\ \left. \left. + c38 \cos(\sqrt{M^2 - 36 C3 - 9} t) \right) \left(c36 \sin(z)^{-\frac{\sqrt{1+4C3}}{2}} \right. \right. \\ \left. \left. + c35 \sin(z)^{\frac{\sqrt{1+4C3}}{2}} \right) \right), yZ11(z, t) \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\sqrt{\sin(z)}} \left(\left(c33 \sin(\sqrt{M^2 - 36 C3 - 9} t) \right. \right. \\
&\quad \left. \left. + c34 \cos(\sqrt{M^2 - 36 C3 - 9} t) \right) \right) \left(c32 \sin(z) - \frac{\sqrt{1+4C3}}{2} \right. \\
&\quad \left. \left. + c31 \sin(z) \frac{\sqrt{1+4C3}}{2} \right) \right), yZ8(z, t) = \\
&\quad - \frac{1}{\sqrt{\sin(z)} M} \left(3 \left(\frac{1}{3} \left((\cos(\sqrt{M^2 - 36 C3 - 9} t) c33 \right. \right. \right. \right. \\
&\quad \left. \left. \left. \left. - \sin(\sqrt{M^2 - 36 C3 - 9} t) c34 \right) \right) \left(c32 \sin(z) - \frac{\sqrt{1+4C3}}{2} \right. \right. \\
&\quad \left. \left. \left. + c31 \sin(z) \frac{\sqrt{1+4C3}}{2} \right) \right) \sqrt{M^2 - 36 C3 - 9} \right) \\
&\quad + (c37 \sin(\sqrt{M^2 - 36 C3 - 9} t) \\
&\quad + c38 \cos(\sqrt{M^2 - 36 C3 - 9} t)) \sqrt{1+4C3} \left(c35 \sin(z) \frac{\sqrt{1+4C3}}{2} \right. \\
&\quad \left. \left. - c36 \sin(z) - \frac{\sqrt{1+4C3}}{2} \right) \right), yZ9(z, t) = \\
&\quad - \frac{1}{\sqrt{\sin(z)} M} \left(\left(c36 \sin(z) - \frac{\sqrt{1+4C3}}{2} \right. \right. \\
&\quad \left. \left. + c35 \sin(z) \frac{\sqrt{1+4C3}}{2} \right) (\cos(\sqrt{M^2 - 36 C3 - 9} t) c37 \right. \\
&\quad \left. - \sin(\sqrt{M^2 - 36 C3 - 9} t) c38) \right) \sqrt{M^2 - 36 C3 - 9} \\
&\quad + 3 \sqrt{1+4C3} \left(c31 \sin(z) \frac{\sqrt{1+4C3}}{2} \right. \\
&\quad \left. - c32 \sin(z) - \frac{\sqrt{1+4C3}}{2} \right) \left(c33 \sin(\sqrt{M^2 - 36 C3 - 9} t) \right. \\
&\quad \left. + c34 \cos(\sqrt{M^2 - 36 C3 - 9} t) \right) \Big) \\
q4 := & \left\{ yZ12(z, t) = - \frac{1}{\sqrt{\sin(z)} M} \left(3 \left(\frac{1}{3} \left((\sin(\sqrt{M^2 - 36 C4 - 9} t) c44 \right. \right. \right. \right. \right. \right. \\
&\quad \left. \left. \left. \left. \left. \left. - \cos(\sqrt{M^2 - 36 C4 - 9} t) c45 \right) \right) \right) \sqrt{M^2 - 36 C4 - 9} \right) \\
&\quad + (c47 \sin(\sqrt{M^2 - 36 C4 - 9} t) \\
&\quad + c48 \cos(\sqrt{M^2 - 36 C4 - 9} t)) \sqrt{1+4C4} \left(c49 \sin(z) \frac{\sqrt{1+4C4}}{2} \right. \\
&\quad \left. \left. - c46 \sin(z) - \frac{\sqrt{1+4C4}}{2} \right) \right), yZ10(z, t) = \\
&\quad - \frac{1}{\sqrt{\sin(z)} M} \left(\left(c46 \sin(z) - \frac{\sqrt{1+4C4}}{2} \right. \right. \\
&\quad \left. \left. + c49 \sin(z) \frac{\sqrt{1+4C4}}{2} \right) (\cos(\sqrt{M^2 - 36 C4 - 9} t) c47 \right. \\
&\quad \left. - \sin(\sqrt{M^2 - 36 C4 - 9} t) c48) \right) \sqrt{M^2 - 36 C4 - 9} \\
&\quad + 3 \sqrt{1+4C4} \left(c41 \sin(z) \frac{\sqrt{1+4C4}}{2} \right. \\
&\quad \left. - c42 \sin(z) - \frac{\sqrt{1+4C4}}{2} \right) \left(c43 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right. \\
&\quad \left. + c45 \cos(\sqrt{M^2 - 36 C4 - 9} t) \right) \Big)
\end{aligned}$$

$$\begin{aligned}
& - \cos(\sqrt{M^2 - 36 C4 - 9} t) c43 \Big| c42 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}} \\
& + c41 \sin(z)^{\frac{\sqrt{1+4C4}}{2}} \Big) \sqrt{M^2 - 36 C4 - 9} \Big) \\
& + \sqrt{1+4C4} \left(c47 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right. \\
& + c48 \cos(\sqrt{M^2 - 36 C4 - 9} t) \left. \left(c45 \sin(z)^{\frac{\sqrt{1+4C4}}{2}} \right. \right. \\
& \left. \left. - c46 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}} \right) \right), yZ13(z, t) = \\
& - \frac{1}{\sqrt{\sin(z)} M} \left(\left(c46 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}} \right. \right. \\
& \left. \left. + c45 \sin(z)^{\frac{\sqrt{1+4C4}}{2}} \right) \left(\sin(\sqrt{M^2 - 36 C4 - 9} t) c48 \right. \right. \\
& \left. \left. - \cos(\sqrt{M^2 - 36 C4 - 9} t) c47 \right) \sqrt{M^2 - 36 C4 - 9} \right. \\
& \left. + 3 \sqrt{1+4C4} \left(c41 \sin(z)^{\frac{\sqrt{1+4C4}}{2}} \right. \right. \\
& \left. \left. - c42 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}} \right) \left(c43 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right. \right. \\
& \left. \left. + c44 \cos(\sqrt{M^2 - 36 C4 - 9} t) \right) \right), yZ14(z, t) \\
& = \frac{1}{\sqrt{\sin(z)}} \left(\left(c46 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}} \right. \right. \\
& \left. \left. + c45 \sin(z)^{\frac{\sqrt{1+4C4}}{2}} \right) \left(c47 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right. \right. \\
& \left. \left. + c48 \cos(\sqrt{M^2 - 36 C4 - 9} t) \right) \right), yZ15(z, t) \\
& = \frac{1}{\sqrt{\sin(z)}} \left(\left(c42 \sin(z)^{-\frac{\sqrt{1+4C4}}{2}} \right. \right. \\
& \left. \left. - \sqrt{1+4C4} \right) \right)
\end{aligned}$$

$$\left. \left. + c41 \sin(z)^{\frac{M^2 - 1 - 36 C4 - 9}{2}} \right\} \left(c43 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right. \right. \\ \left. \left. + c44 \cos(\sqrt{M^2 - 36 C4 - 9} t) \right) \right\}$$

```

In[8]:= maplestringEQ3 =
"{{yZ10(z,t)=((c37 sin(sqrt(M^2-36 C3-9) t)+c38 cos(sqrt(M^2-36 C3-9)
t)) (c36 (sin(z))^(-(sqrt(1+4 C3))/2)+c35 (sin(z))^((sqrt(1+4
C3))/2))/sqrt(sin(z))),yZ11(z,t)=((c33 sin(sqrt(M^2-36 C3-9)
t)+c34 cos(sqrt(M^2-36 C3-9) t)) (c32 (sin(z))^(-(sqrt(1+4
C3))/2)+c31 (sin(z))^((sqrt(1+4 C3))/2))/sqrt(sin(z))),yZ8(z,t)=-(3
(((cos(sqrt(M^2-36 C3-9) t) c33-sin(sqrt(M^2-36 C3-9) t) c34) (c32
(sin(z))^(-(sqrt(1+4 C3))/2)+c31 (sin(z))^((sqrt(1+4 C3))/2)) sqrt(M^2-36
C3-9))/3+(c37 sin(sqrt(M^2-36 C3-9) t)+c38 cos(sqrt(M^2-36 C3-9) t))
sqrt(1+4 C3) (c35 (sin(z))^((sqrt(1+4 C3))/2)-c36 (sin(z))^(-(sqrt(1+4
C3))/2)))/sqrt(sin(z)) M),yZ9(z,t)=-(c36 (sin(z))^(-(sqrt(1+4
C3))/2)+c35 (sin(z))^((sqrt(1+4 C3))/2)) (cos(sqrt(M^2-36 C3-9) t)
c37-sin(sqrt(M^2-36 C3-9) t) c38) sqrt(M^2-36 C3-9)+3 sqrt(1+4 C3) (c31
(sin(z))^((sqrt(1+4 C3))/2)-c32 (sin(z))^(-(sqrt(1+4 C3))/2)) (c33
sin(sqrt(M^2-36 C3-9) t)+c34 cos(sqrt(M^2-36 C3-9) t)))/sqrt(sin(z)) M)}};

In[9]:= maplestringEQ4 =
"{{yZ12(z, t) = -3*((sin(sqrt(M^2 - 36*C4 - 9)*t)*c44 - cos(sqrt(M^2 - 36*C4
- 9)*t)*c43)*(c42*sin(z)^(-sqrt(1 + 4*C4)/2) + c41*sin(z)^(sqrt(1 +
4*C4)/2))*sqrt(M^2 - 36*C4 - 9))/3 + sqrt(1 + 4*C4)*(c47*sin(sqrt(M^2 -
36*C4 - 9)*t) + c48*cos(sqrt(M^2 - 36*C4 - 9)*t))*(c45*sin(z)^(sqrt(1
+ 4*C4)/2) - c46*sin(z)^(-sqrt(1 + 4*C4)/2))/sqrt(sin(z))*M),
yZ13(z, t) = -(c46*sin(z)^(-sqrt(1 + 4*C4)/2) + c45*sin(z)^(sqrt(1 +
4*C4)/2))*(sin(sqrt(M^2 - 36*C4 - 9)*t)*c48 - cos(sqrt(M^2 - 36*C4 -
9)*t)*c47)*sqrt(M^2 - 36*C4 - 9) + 3*sqrt(1 + 4*C4)*(c41*sin(z)^(sqrt(1
+ 4*C4)/2) - c42*sin(z)^(-sqrt(1 + 4*C4)/2))*(c43*sin(sqrt(M^2 -
36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4 - 9)*t))/sqrt(sin(z))*M),
yZ14(z, t) = (c46*sin(z)^(-sqrt(1 + 4*C4)/2) + c45*sin(z)^(sqrt(1
+ 4*C4)/2))*(c47*sin(sqrt(M^2 - 36*C4 - 9)*t) + c48*cos(sqrt(M^2 -
36*C4 - 9)*t))/sqrt(sin(z)), yZ15(z, t) = (c42*sin(z)^(-sqrt(1
+ 4*C4)/2) + c41*sin(z)^(sqrt(1 + 4*C4)/2))*(c43*sin(sqrt(M^2 -
36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4 - 9)*t))/sqrt(sin(z)))}}";

```

```
In[]:= solvedEQ3 = ConvertMapleToMathematicaV2[ReleaseHold[maplestringEQ3]]
```

Out[]=

$$\left\{ \begin{aligned} YZ10[z, t] &= \frac{1}{\sqrt{\sin[z]}} \left(C35[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} + C36[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right) \\ &\quad \left(c38 \cos\left[\sqrt{-9 - 36C3 + M^2} t\right] + c37 \sin\left[\sqrt{-9 - 36C3 + M^2} t\right] \right), \\ YZ11[z, t] &= \frac{1}{\sqrt{\sin[z]}} \left(C31[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} + C32[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right) \\ &\quad \left(c34 \cos\left[\sqrt{-9 - 36C3 + M^2} t\right] + c33 \sin\left[\sqrt{-9 - 36C3 + M^2} t\right] \right), YZ8[z, t] = \\ &- \frac{1}{M \sqrt{\sin[z]}} 3 \left(\frac{1}{3} \sqrt{-9 - 36C3 + M^2} \left(C31[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} + C32[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right) \right. \\ &\quad \left(c33 \cos\left[\sqrt{-9 - 36C3 + M^2} t\right] - c34 \sin\left[\sqrt{-9 - 36C3 + M^2} t\right] \right) + \\ &\quad \sqrt{1+4C3} \left(C35[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} - C36[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right) \\ &\quad \left(c38 \cos\left[\sqrt{-9 - 36C3 + M^2} t\right] + c37 \sin\left[\sqrt{-9 - 36C3 + M^2} t\right] \right) \Big), \\ YZ9[z, t] &= \frac{1}{M \sqrt{\sin[z]}} \left(-3 \sqrt{1+4C3} \left(C31[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} - C32[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right) \right. \\ &\quad \left(c34 \cos\left[\sqrt{-9 - 36C3 + M^2} t\right] + c33 \sin\left[\sqrt{-9 - 36C3 + M^2} t\right] \right) - \\ &\quad \sqrt{-9 - 36C3 + M^2} \left(C35[\sin[z]]^{\frac{1}{2} \sqrt{1+4C3}} + C36[\sin[z]]^{-\frac{1}{2} \sqrt{1+4C3}} \right) \\ &\quad \left. \left(c37 \cos\left[\sqrt{-9 - 36C3 + M^2} t\right] - c38 \sin\left[\sqrt{-9 - 36C3 + M^2} t\right] \right) \right) \Big) \} \}$$

```
In[1]:= solvedEQ4 = ConvertMapleToMathematicaV2[ReleaseHold[maplestringEQ4]]
Out[1]=

$$\left\{ \begin{aligned} YZ12[z, t] &= -\frac{1}{M \sqrt{\sin[z]}} \\ &\quad 3 \left( \frac{1}{3} \sqrt{-9 - 36 C4 + M^2} \left( -c43 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c44 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \right. \\ &\quad \left( c42 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c41 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) + \\ &\quad \sqrt{1 + 4 C4} \left( c48 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c47 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \\ &\quad \left. \left( -c46 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c45 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) \right), YZ13[z, t] = \\ &\quad \frac{1}{M \sqrt{\sin[z]}} \left( -3 \sqrt{1 + 4 C4} \left( c44 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c43 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \right. \\ &\quad \left( -c42 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c41 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) - \\ &\quad \sqrt{-9 - 36 C4 + M^2} \left( -c47 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c48 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \\ &\quad \left. \left( c46 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c45 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) \right), \\ YZ14[z, t] &= \frac{1}{\sqrt{\sin[z]}} \left( c48 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c47 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \\ &\quad \left( c46 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c45 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right), \\ YZ15[z, t] &= \frac{1}{\sqrt{\sin[z]}} \left( c44 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c43 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \\ &\quad \left( c42 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c41 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) \} \} \right\}$$

```

```
In[15]:= maplestringEQ1 = ReadString["maple-textwrap-stringEQ1_L9i_2026-02-05.txt"];
```

```
In[16]:= maplestringEQ2 = ReadString["thinkpad_stringEQ2.txt"];
```

```
In[16]:= time1 = Now
If[FindFile["solvedEQ1.mx"] === $Failed,
 AbsoluteTiming[solvedEQ1 = ConvertMapleToMathematicaV2[maplestringEQ1]];
 DumpSave[ToString[header <> "solvedEQ1.mx"], solvedEQ1];
 Print[ToString[header <> "solvedEQ1.mx"]];
 , Get["solvedEQ1.mx"]]
Now - time1
```

```
Out[16]=
```

Thu 5 Feb 2026 18:23:59 GMT-8

Pair-Crtn-Univ-same_E-L-eqs-alt-approach-solvedEQ1.mx

```
Out[18]=
```

57.90493793 min

In[19]:= solvedEQ1

Out[19]=

$$\begin{aligned}
YZ0[z, t] &= \frac{1}{2 M Q1 A4'[t]} \\
\text{Csch}[A4[t]] \left(\frac{1}{2} e^{-A4[t]} (-2 + Q1) Q1 YZ3[z, t] A4'[t]^2 - \frac{1}{4} e^{-3 A4[t]} Q1^2 YZ3[z, t] A4'[t]^2 - \right. \\
&\quad Q1 \operatorname{Sinh}[A4[t]] YZ3[z, t] A4''[t] + e^{A4[t]} YZ3^{(0,2)}[z, t] + \\
&\quad e^{A4[t]} \left(YZ3[z, t] \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) + (-72 \operatorname{Tan}[z] - 36 \operatorname{Tan}[z]^3) YZ3^{(1,0)}[z, t] \right) - \\
&\quad \left. 36 e^{A4[t]} \operatorname{Tan}[z]^2 YZ3^{(2,0)}[z, t] \right), \\
YZ1[z, t] &= \left(\frac{1}{3} \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left(-\frac{1}{4} (1 + e^{4 A4[t]}) Q1 YZ3[z, t] A4''[t]^2 - \right. \right. \\
&\quad \frac{1}{2} \left(-\frac{1}{2} + e^{2 A4[t]} - \frac{1}{2} e^{4 A4[t]} \right) Q1 YZ3[z, t] A4'[t] A4^{(3)}[t] - \\
&\quad \frac{1}{2} A4'[t] \left(e^{4 A4[t]} \left(-9 + M^2 + \frac{1}{4} Q1^2 A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + Q1 A4'[t]^2 \right. \\
&\quad \left. \left. \left(-Q1 YZ3[z, t] A4'[t] + Q1 \cosh[2 A4[t]] YZ3[z, t] A4'[t] + \right. \right. \right. \\
&\quad \left. \left. \left. \left(1 + \frac{3 Q1}{4} \right) YZ3^{(0,1)}[z, t] - \frac{1}{4} e^{-2 A4[t]} Q1 YZ3^{(0,1)}[z, t] \right) + \right. \\
&\quad \left(\frac{1}{2} (1 + e^{4 A4[t]}) Q1 A4'[t]^2 + e^{2 A4[t]} \left((1 - Q1) A4'[t]^2 - \frac{A4''[t]}{2} \right) + \frac{1}{2} \right. \\
&\quad \left. e^{4 A4[t]} A4''[t] \right) YZ3^{(0,2)}[z, t] + \frac{1}{2} (e^{2 A4[t]} - e^{4 A4[t]}) A4'[t] YZ3^{(0,3)}[z, t] + \\
&\quad A4''[t] \left(\frac{1}{8} Q1 A4'[t] ((-4 - 3 Q1) YZ3[z, t] A4'[t] + e^{-2 A4[t]} Q1 YZ3[z, t] A4'[t] \right. \\
&\quad \left. \left. + 2 YZ3^{(0,1)}[z, t]) + e^{4 A4[t]} \left(\frac{1}{2} YZ3[z, t] \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) + \right. \right. \\
&\quad \left. \left. \frac{1}{4} Q1 A4'[t] YZ3^{(0,1)}[z, t] + 18 (-2 \operatorname{Tan}[z] - \operatorname{Tan}[z]^3) YZ3^{(1,0)}[z, t] \right) + \right. \\
&\quad \left. e^{2 A4[t]} \left(\frac{1}{2} Q1 YZ3[z, t] A4''[t]^2 + A4''[t] \left(-\frac{1}{2} YZ3[z, t] \right. \right. \right. \\
&\quad \left. \left. \left. \left(-9 + M^2 + \left(-Q1 - \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) - \frac{1}{2} Q1 A4'[t] YZ3^{(0,1)}[z, t] + \right. \right. \\
&\quad \left. \left. \left. 18 (2 \operatorname{Tan}[z] + \operatorname{Tan}[z]^3) YZ3^{(1,0)}[z, t] \right) + A4'[t] \right. \\
&\quad \left(\frac{1}{2} \left(-9 + M^2 + \left(Q1 + \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + A4'[t] (YZ3[z, t] \right. \\
&\quad \left. \left. \left(-9 + M^2 - Q1 A4'[t]^2 \right) + 36 (-2 \operatorname{Tan}[z] - \operatorname{Tan}[z]^3) YZ3^{(1,0)}[z, t] \right) \right) - \\
&\quad \left. 18 (e^{2 A4[t]} - e^{4 A4[t]}) \operatorname{Tan}[z] (2 + \operatorname{Tan}[z]^2) A4'[t] YZ3^{(1,1)}[z, t] - \right)
\end{aligned}$$

$$\begin{aligned}
& 36 \tan[z]^2 \left(e^{2A4[t]} \left(A4'[t]^2 - \frac{A4''[t]}{2} \right) + \frac{1}{2} e^{4A4[t]} A4''[t] \right) YZ3^{(2,0)}[z, t] - \\
& 18 \left(e^{2A4[t]} - e^{4A4[t]} \right) \tan[z]^2 A4'[t] YZ3^{(2,1)}[z, t] \Big) dz + \\
& \frac{1}{12} Q1 \left(1 + \frac{1}{2} e^{-3A4[t]} \operatorname{Csch}[A4[t]] + \frac{1}{2} e^{A4[t]} \operatorname{Csch}[A4[t]] - \right. \\
& \quad \left. e^{2A4[t]} (1 + e^{-3A4[t]} \operatorname{Csch}[A4[t]]) \right) \\
& \left(\int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left(\frac{1}{2} Q1 (YZ3[z, t] A4''[t] + A4'[t] ((-2 - Q1) YZ3[z, t] A4'[t] + \right. \right. \\
& \quad \left. \left. Q1 \cosh[2A4[t]] YZ3[z, t] A4'[t] + 2YZ3^{(0,1)}[z, t]) \right) + \right. \\
& \quad \left. e^{2A4[t]} YZ3^{(0,2)}[z, t] + e^{2A4[t]} \left((-9 + M^2) YZ3[z, t] - \frac{1}{2} Q1 YZ3[z, t] A4''[t] - \right. \right. \\
& \quad \left. \left. Q1 A4'[t] YZ3^{(0,1)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3) YZ3^{(1,0)}[z, t] \right) - \right. \\
& \quad \left. 36 e^{2A4[t]} \tan[z]^2 YZ3^{(2,0)}[z, t] \right) dz \Big) A4'[t]^2 - \\
& \frac{3}{8} e^{-\frac{1}{4} Q1 (e^{-2A4[t]} + 2A4[t])} (-1 + e^{2A4[t]})^2 \cos[M t] \\
& \left(\int \frac{1}{A4'[t]^3} \frac{1}{e^{\frac{1}{4} e^{-2A4[t]} Q1 + \frac{1}{2} (-14+Q1) A4[t]}} \cos[M t] \operatorname{Csch}[A4[t]]^3 \right. \\
& \quad \left(-\frac{2}{9} e^{4A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{3}{4} \left(-\frac{1}{3} + e^{2A4[t]} - e^{4A4[t]} + \frac{1}{3} e^{6A4[t]} \right) Q1 \right. \right. \\
& \quad \left. \left. \cos[z]^3 YZ3[z, t] A4''[t]^3 - \frac{3}{8} \left(-\frac{1}{3} + e^{2A4[t]} - e^{4A4[t]} + \frac{1}{3} e^{6A4[t]} \right) \right. \right. \\
& \quad \left. \left. Q1 \cos[z]^3 YZ3[z, t] A4'[t]^2 A4^{(4)}[t] + \frac{1}{4} \cos[z]^3 \right. \right. \\
& \quad \left. \left(A4'[t]^2 \left(\frac{1}{4} Q1 (-4 - 4 Q1 + e^{-2A4[t]} Q1) A4'[t]^2 + e^{6A4[t]} \left(-9 + M^2 + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{1}{4} Q1^2 A4'[t]^2 \right) + e^{4A4[t]} (18 - 2 M^2 + (4 - Q1 - Q1^2) A4'[t]^2) + \right. \right. \\
& \quad \left. \left. e^{2A4[t]} \left(-9 + M^2 + \left(4 + 2 Q1 + \frac{3 Q1^2}{2} \right) A4'[t]^2 \right) \right) - \frac{3}{2} \left(e^{4A4[t]} \left(-\frac{4}{3} - \right. \right. \\
& \quad \left. \left. Q1 \right) + \frac{1}{3} (-1 + e^{6A4[t]}) Q1 + e^{2A4[t]} \left(\frac{4}{3} + Q1 \right) \right) A4'[t]^2 A4''[t] + \\
& \quad \left. 2 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) A4''[t]^2 \right) YZ3^{(0,2)}[z, t] - \frac{3}{4} \cos[z]^3 \\
& A4'[t] \left(\left(e^{4A4[t]} \left(\frac{4}{3} - Q1 \right) + e^{2A4[t]} \left(-\frac{4}{3} + Q1 \right) + \frac{1}{3} (-1 + e^{6A4[t]}) Q1 \right) \right. \\
& \quad \left. A4'[t]^2 + \frac{2}{3} (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) A4''[t] \right) YZ3^{(0,3)}[z, t] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} \left(e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \cos[z]^3 A4'[t]^2 YZ3^{(0,4)}[z, t] + \\
& A4''[t]^2 \left(-\frac{1}{4} Q1 \cos[z]^3 A4'[t] YZ3^{(0,1)}[z, t] + \right. \\
& e^{6A4[t]} \left(\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 YZ3[z, t] + \frac{1}{4} Q1 \cos[z]^3 A4'[t] \right. \\
& \left. \left. YZ3^{(0,1)}[z, t] - 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \right. \\
& e^{2A4[t]} \left(\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 YZ3[z, t] + \frac{3}{4} Q1 \cos[z]^3 A4'[t] \right. \\
& \left. t] YZ3^{(0,1)}[z, t] - 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \\
& e^{4A4[t]} \left((9 - M^2) \cos[z]^3 YZ3[z, t] - \frac{3}{4} Q1 \cos[z]^3 A4'[t] \right. \\
& \left. YZ3^{(0,1)}[z, t] + 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \\
& A4'[t]^3 \left(\frac{1}{4} (-3 + e^{-2A4[t]}) Q1^2 \cos[z]^3 A4'[t]^2 (YZ3[z, t] \right. \\
& \left. A4'[t] - YZ3^{(0,1)}[z, t]) + e^{4A4[t]} \right. \\
& \left. \left(\cos[z]^3 \left(9 - M^2 - \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \right. \\
& \left. \left. A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + e^{2A4[t]} \\
& \left(\cos[z]^3 \left(-9 + M^2 + \left(-Q1 - \frac{3Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \\
& \left. A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 + \frac{3Q1^2}{4} \right) A4'[t]^2 \right) - 36 \right. \right. \\
& \left. \left. (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) \left. \right) - \frac{1}{2} A4'[t] A4''[t] \\
& \left(\cos[z]^3 \left(e^{6A4[t]} \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \frac{3}{4} \right. \right. \\
& \left. \left. Q1 A4'[t]^2 \left(-3 Q1 YZ3[z, t] A4'[t] + e^{-2A4[t]} Q1 \left(YZ3[z, t] A4'[t] \right. \right. \right. \\
& \left. \left. \left. - \frac{1}{3} YZ3^{(0,1)}[z, t] \right) + \frac{4}{3} (1 + Q1) YZ3^{(0,1)}[z, t] \right) \right) + \\
& e^{4A4[t]} \left(-2 \cos[z]^3 \left(-9 + M^2 - \frac{1}{2} (Q1 + Q1^2) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] - \right. \\
& \left. A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + 3 \left(Q1 + \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{1}{2} e^{2 A4[t]} + e^{4 A4[t]} - \frac{1}{2} e^{6 A4[t]} \right) A4''[t] \Big) YZ3^{(2,1)}[z, t] - \\
& 9 \left(e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]} \right) \cos[z] \sin[z]^2 A4'[t]^2 YZ3^{(2,2)}[z, t] \Big) \\
& d z + A4'[t]^2 \left(\frac{1}{9} \left(e^{4 A4[t]} - 2 e^{6 A4[t]} + e^{8 A4[t]} \right) Q1 \int \frac{1}{\sqrt{\sin[z]}} \right. \\
& \sec[z]^2 \left(-\frac{1}{2} Q1 \cos[z]^3 (-1 + \cosh[2 A4[t]]) YZ3[z, t] A4''[t]^2 + \right. \\
& \left. \frac{1}{2} Q1 \cos[z]^3 (-1 + \cosh[2 A4[t]]) YZ3[z, t] A4'[t] A4^{(3)}[t] + \right. \\
& \left. \cos[z]^3 \left((1 - Q1 + Q1 \cosh[2 A4[t]]) A4'[t]^2 + \frac{1}{2} (-1 + e^{2 A4[t]}) \right. \right. \\
& \left. \left. A4''[t] \right) YZ3^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2 A4[t]}) \cos[z]^3 A4'[t] \right. \\
& \left. YZ3^{(0,3)}[z, t] + A4'[t] \left(-\frac{1}{4} e^{-4 A4[t]} Q1^2 \cos[z]^3 A4'[t]^2 \right. \right. \\
& \left. \left. \left(YZ3[z, t] A4'[t] - \frac{1}{2} YZ3^{(0,1)}[z, t] \right) - \frac{1}{2} e^{2 A4[t]} \cos[z]^3 \right. \right. \\
& \left. \left. \left(-9 + M^2 + \frac{1}{4} Q1^2 A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \frac{1}{2} \cos[z]^3 \left(-9 + M^2 + \right. \right. \right. \\
& \left. \left. \left. \left(Q1 + \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \frac{1}{2} e^{-2 A4[t]} Q1 \cos[z]^3 \right. \right. \\
& \left. \left. A4'[t]^2 \left(Q1 YZ3[z, t] A4'[t] + \left(-1 - \frac{3 Q1}{4} \right) YZ3^{(0,1)}[z, t] \right) + \right. \right. \\
& \left. \left. A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \right. \right. \\
& \left. \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + \right. \\
& A4''[t] \left(\frac{1}{8} e^{-4 A4[t]} Q1^2 \cos[z]^3 YZ3[z, t] A4'[t]^2 - \right. \\
& \left. \frac{3}{8} e^{-2 A4[t]} Q1 \left(\frac{4}{3} + Q1 \right) \cos[z]^3 YZ3[z, t] A4'[t]^2 - \frac{1}{2} \cos[z]^3 \right. \\
& \left. YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) - \frac{1}{2} Q1 \cos[z]^3 \right. \\
& \left. A4'[t] YZ3^{(0,1)}[z, t] + \frac{1}{2} Q1 \cos[z]^3 \cosh[2 A4[t]] A4'[t] \right. \\
& \left. YZ3^{(0,1)}[z, t] + 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] + \right. \\
& \left. e^{2 A4[t]} \left(\frac{1}{2} \cos[z]^3 YZ3[z, t] \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) - \right. \right. \\
& \left. \left. 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + 18 (-1 + e^{2 A4[t]})
\end{aligned}$$

$$\begin{aligned}
& \left(1 + \cos[z]^2 \right) \sin[z] A4'[t] YZ3^{(1,1)}[z, t] - 36 \cos[z] \\
& \sin[z]^2 \left(A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) YZ3^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] YZ3^{(2,1)}[z, t] \Bigg) dz + \\
& \frac{1}{3} \left(\int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left(\frac{1}{2} Q1 (YZ3[z, t] A4''[t] + A4'[t] \right. \right. \\
& \left. \left. ((-2 - Q1) YZ3[z, t] A4'[t] + Q1 \cosh[2A4[t]] YZ3[z, t] \right. \right. \\
& \left. \left. A4'[t] + 2 YZ3^{(0,1)}[z, t]) \right) + e^{2A4[t]} YZ3^{(0,2)}[z, t] + \\
& e^{2A4[t]} \left((-9 + M^2) YZ3[z, t] - \frac{1}{2} Q1 YZ3[z, t] A4''[t] - \right. \\
& \left. Q1 A4'[t] YZ3^{(0,1)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3) \right. \\
& \left. \left. YZ3^{(1,0)}[z, t] \right) - 36 e^{2A4[t]} \tan[z]^2 YZ3^{(2,0)}[z, t] \right) dz \Bigg) \\
& \left(\frac{1}{6} \left(e^{2A4[t]} (-1 + Q1) - \frac{Q1}{4} \right) Q1 A4'[t]^2 + \frac{1}{6} e^{8A4[t]} \left(-M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) + \right. \\
& e^{4A4[t]} \left(-\frac{M^2}{6} + \left(\frac{Q1}{3} - \frac{Q1^2}{4} \right) A4'[t]^2 \right) + \frac{1}{3} e^{6A4[t]} \left(M^2 + \frac{1}{2} (-Q1 + Q1^2) \right. \\
& \left. A4'[t]^2 \right) + \frac{1}{12} \left(e^{2A4[t]} - 3 e^{4A4[t]} + 3 e^{6A4[t]} - e^{8A4[t]} \right) Q1 A4''[t] \Bigg) + \\
& \sqrt{\sin[z]} \left(\frac{1}{2} \left(e^{4A4[t]} - 3 e^{6A4[t]} + 3 e^{8A4[t]} - e^{10A4[t]} \right) Q1 YZ3[z, t] \right. \\
& \left. A4''[t] - \frac{3}{4} Q1 A4'[t] \left(e^{2A4[t]} Q1 YZ3[z, t] A4'[t] + \right. \right. \\
& \left. \left. 4 e^{4A4[t]} \left(\left(\frac{1}{3} - Q1 \right) YZ3[z, t] A4'[t] - \frac{1}{3} YZ3^{(0,1)}[z, t] \right) \right) + \right. \\
& e^{6A4[t]} \left(YZ3[z, t] \left(-9 + M^2 + \left(2 Q1 - \frac{9 Q1^2}{2} \right) A4'[t]^2 \right) - \right. \\
& \left. \left. 3 Q1 A4'[t] YZ3^{(0,1)}[z, t] \right) + e^{10A4[t]} \right. \\
& \left. \left(YZ3[z, t] \left(-9 + M^2 - \frac{3}{4} Q1^2 A4'[t]^2 \right) - Q1 A4'[t] YZ3^{(0,1)}[z, t] \right) + \right. \\
& e^{8A4[t]} \left(-2 YZ3[z, t] \left(-9 + M^2 + \frac{1}{2} (Q1 - 3 Q1^2) A4'[t]^2 \right) + 3 Q1 A4'[t] \right. \\
& \left. YZ3^{(0,1)}[z, t] \right) + \left(e^{6A4[t]} - 2 e^{8A4[t]} + e^{10A4[t]} \right) YZ3^{(0,2)}[z, t] \Bigg) - \\
& 396 \left(e^{6A4[t]} - 2 e^{8A4[t]} + e^{10A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} YZ3^{(1,0)}[z, t] - \\
& 216 \left(e^{6A4[t]} - 2 e^{8A4[t]} + e^{10A4[t]} \right) \sec[z]^5 \\
& \sin[z]^{11/2} YZ3^{(1,0)}[z, t] + 2 \sec[z] \sin[z]^{3/2}
\end{aligned}$$

$$\begin{aligned}
& \left(\left(-\frac{3}{4} Q1 \left(4 e^{4 A4[t]} \left(\frac{1}{3} - Q1 \right) + e^{2 A4[t]} Q1 \right) A4'[t]^2 + e^{10 A4[t]} \left(-117 + M^2 - \right. \right. \right. \\
& \left. \left. \left. \frac{3}{4} Q1^2 A4'[t]^2 \right) + e^{6 A4[t]} \left(-117 + M^2 + \left(2 Q1 - \frac{9 Q1^2}{2} \right) A4'[t]^2 \right) + \right. \\
& \left. e^{8 A4[t]} \left(234 - 2 M^2 + (-Q1 + 3 Q1^2) A4'[t]^2 \right) + \frac{1}{2} \right. \\
& \left. \left(e^{4 A4[t]} - 3 e^{6 A4[t]} + 3 e^{8 A4[t]} - e^{10 A4[t]} \right) Q1 A4''[t] \right) YZ3^{(1,0)}[z, t] + \\
& \left. \left(e^{4 A4[t]} - 3 e^{6 A4[t]} + 3 e^{8 A4[t]} - e^{10 A4[t]} \right) Q1 A4'[t] YZ3^{(1,1)}[z, t] \right) + \\
& 2 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z] \sin[z]^{3/2} YZ3^{(1,2)}[z, t] - \\
& 324 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z]^2 \sin[z]^{5/2} YZ3^{(2,0)}[z, t] - \\
& 216 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z]^4 \sin[z]^{9/2} YZ3^{(2,0)}[z, t] - \\
& 72 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z]^3 \sin[z]^{7/2} YZ3^{(3,0)}[z, t] \right) dt \\
& A4'[t]^2 - \frac{3}{8} e^{-\frac{1}{4} Q1 (e^{-2 A4[t]} + 2 A4[t])} \left(-1 + e^{2 A4[t]} \right)^2 \\
& \left(\int \frac{1}{A4'[t]^3} \right. \\
& e^{\frac{1}{4} e^{-2 A4[t]} Q1 + \frac{1}{2} (-14 + Q1) A4[t]} \\
& \operatorname{Csch}[A4[t]]^3 \\
& \sin[M t] \\
& \left(-\frac{2}{9} e^{4 A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \operatorname{Sec}[z]^2 \left(-\frac{3}{4} \left(-\frac{1}{3} + e^{2 A4[t]} - e^{4 A4[t]} + \frac{1}{3} e^{6 A4[t]} \right) Q1 \right. \right. \\
& \left. \left. \cos[z]^3 YZ3[z, t] A4''[t]^3 - \frac{3}{8} \left(-\frac{1}{3} + e^{2 A4[t]} - e^{4 A4[t]} + \frac{1}{3} e^{6 A4[t]} \right) \right. \right. \\
& Q1 \cos[z]^3 YZ3[z, t] A4'[t]^2 A4^{(4)}[t] + \frac{1}{4} \cos[z]^3 \\
& \left(A4'[t]^2 \left(\frac{1}{4} Q1 (-4 - 4 Q1 + e^{-2 A4[t]} Q1) A4'[t]^2 + e^{6 A4[t]} \left(-9 + M^2 + \right. \right. \right. \\
& \left. \left. \left. \frac{1}{4} Q1^2 A4'[t]^2 \right) + e^{4 A4[t]} \left(18 - 2 M^2 + (4 - Q1 - Q1^2) A4'[t]^2 \right) + \right. \\
& \left. e^{2 A4[t]} \left(-9 + M^2 + \left(4 + 2 Q1 + \frac{3 Q1^2}{2} \right) A4'[t]^2 \right) \right) - \\
& \frac{3}{2} \left(e^{4 A4[t]} \left(-\frac{4}{3} - Q1 \right) + \frac{1}{3} (-1 + e^{6 A4[t]}) Q1 + e^{2 A4[t]} \left(\frac{4}{3} + Q1 \right) \right) \\
& A4'[t]^2 A4''[t] + 2 \left(e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]} \right) A4''[t]^2 \Big) \\
& YZ3^{(0,2)}[z, t] - \frac{3}{4} \cos[z]^3 A4'[t] \left(\left(e^{4 A4[t]} \left(\frac{4}{3} - Q1 \right) + \right. \right. \\
& \left. \left. \left(e^{6 A4[t]} \left(-\frac{4}{3} - Q1 \right) + \frac{1}{3} (-1 + e^{8 A4[t]}) Q1 + e^{10 A4[t]} \right) A4''[t] \right) \right. \\
& \left. \left(e^{8 A4[t]} - 2 e^{10 A4[t]} + e^{12 A4[t]} \right) A4''[t]^2 \right) \\
& \left(e^{4 A4[t]} - 3 e^{6 A4[t]} + 3 e^{8 A4[t]} - e^{10 A4[t]} \right) Q1 A4''[t] \Big)
\end{aligned}$$

$$\begin{aligned}
& \left(e^{2A4[t]} \left(-\frac{4}{3} + Q1 \right) + \frac{1}{3} (-1 + e^{6A4[t]}) Q1 \right) A4'[t]^2 + \\
& \frac{2}{3} \left((e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]}) A4''[t] \right) YZ3^{(0,3)}[z, t] + \\
& \frac{1}{4} \left((e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]}) \cos[z]^3 A4'[t]^2 YZ3^{(0,4)}[z, t] + \right. \\
& A4''[t]^2 \left(-\frac{1}{4} Q1 \cos[z]^3 A4'[t] YZ3^{(0,1)}[z, t] + \right. \\
& e^{6A4[t]} \left(\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 YZ3[z, t] + \frac{1}{4} Q1 \cos[z]^3 A4'[t] \right. \\
& \left. \left. YZ3^{(0,1)}[z, t] - 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \right. \\
& e^{2A4[t]} \left(\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 YZ3[z, t] + \frac{3}{4} Q1 \cos[z]^3 A4'[t] \right. \\
& \left. YZ3^{(0,1)}[z, t] - 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \\
& e^{4A4[t]} \left((9 - M^2) \cos[z]^3 YZ3[z, t] - \frac{3}{4} Q1 \cos[z]^3 A4'[t] \right. \\
& \left. YZ3^{(0,1)}[z, t] + 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \\
& A4'[t]^3 \left(\frac{1}{4} (-3 + e^{-2A4[t]}) Q1^2 \cos[z]^3 A4'[t]^2 (YZ3[z, t] \right. \\
& \left. A4'[t] - YZ3^{(0,1)}[z, t]) + e^{4A4[t]} \right. \\
& \left(\cos[z]^3 \left(9 - M^2 - \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \\
& A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + e^{2A4[t]} \\
& \left(\cos[z]^3 \left(-9 + M^2 + \left(-Q1 - \frac{3Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \\
& A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 + \frac{3Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) - \\
& \frac{1}{2} A4'[t] A4''[t] \left(\cos[z]^3 \left(e^{6A4[t]} \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) \right. \right. \\
& \left. YZ3^{(0,1)}[z, t] + \frac{3}{4} Q1 A4'[t]^2 \left(-3 Q1 YZ3[z, t] A4'[t] + \right. \right. \\
& \left. \left. e^{-2A4[t]} Q1 \left(YZ3[z, t] A4'[t] - \frac{1}{3} YZ3^{(0,1)}[z, t] \right) + \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{4}{3} (1 + Q1) YZ3^{(0,1)}[z, t] \right) \Bigg) + e^{4A4[t]} \\
& \left(-2 \cos[z]^3 \left(-9 + M^2 - \frac{1}{2} (Q1 + Q1^2) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] - \right. \\
& A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + 3 \left(Q1 + \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + e^{2A4[t]} \\
& \left(\cos[z]^3 \left(-9 + M^2 + \left(-2 Q1 - \frac{3 Q1^2}{2} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \\
& A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + 3 \left(Q1 + \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) - \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left((e^{2A4[t]} - e^{4A4[t]}) A4'[t]^2 + \right. \\
& \left. \left(-\frac{1}{2} e^{2A4[t]} + e^{4A4[t]} - \frac{1}{2} e^{6A4[t]} \right) A4''[t] \right) YZ3^{(1,1)}[z, t] - \\
& 9 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) (1 + \cos[z]^2) \sin[z] \\
& A4'[t]^2 YZ3^{(1,2)}[z, t] - 36 \cos[z] \sin[z]^2 \\
& \left((e^{2A4[t]} + e^{4A4[t]}) A4'[t]^4 - \frac{1}{2} (e^{2A4[t]} - e^{4A4[t]}) A4'[t]^2 A4''[t] + \right. \\
& \left. \left(\frac{1}{2} e^{2A4[t]} - e^{4A4[t]} + \frac{1}{2} e^{6A4[t]} \right) A4''[t]^2 \right) YZ3^{(2,0)}[z, t] - \\
& \frac{1}{4} A4'[t] A4^{(3)}[t] \left(-\frac{9}{2} \left(-\frac{1}{3} + e^{2A4[t]} - e^{4A4[t]} + \frac{1}{3} e^{6A4[t]} \right) Q1 \cos[z]^3 \right. \\
& \left. YZ3[z, t] A4''[t] - \frac{1}{4} Q1 \cos[z]^3 A4'[t] (e^{-2A4[t]} Q1 YZ3[z, t] \right. \\
& A4'[t] - 4 (1 + Q1) YZ3[z, t] A4'[t] + 4 YZ3^{(0,1)}[z, t]) + \\
& (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \cos[z]^3 YZ3^{(0,2)}[z, t] + e^{6A4[t]} \\
& \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) + Q1 \cos[z]^3 A4'[t] \right. \\
& \left. YZ3^{(0,1)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \\
& e^{2A4[t]} \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-2 Q1 - \frac{3 Q1^2}{2} \right) A4'[t]^2 \right) + \right. \\
& 3 Q1 \cos[z]^3 A4'[t] YZ3^{(0,1)}[z, t] - 36 (1 + \cos[z]^2) \\
& \sin[z] YZ3^{(1,0)}[z, t] \Bigg) + e^{4A4[t]} \left(-2 \cos[z]^3 YZ3[z, t] \right. \\
& \left. \left(-9 + M^2 - \frac{1}{2} (Q1 + Q1^2) A4'[t]^2 \right) - 3 Q1 \cos[z]^3 A4'[t] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(YZ3^{(0,1)}[z, t] + 72 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) - \right. \\
& \left. 36 (\mathrm{e}^{2A4[t]} - 2 \mathrm{e}^{4A4[t]} + \mathrm{e}^{6A4[t]}) \cos[z] \sin[z]^2 YZ3^{(2,0)}[z, t] \right) - \\
& 36 \cos[z] \sin[z]^2 A4'[t] \left((\mathrm{e}^{2A4[t]} - \mathrm{e}^{4A4[t]}) A4'[t]^2 + \right. \\
& \left(-\frac{1}{2} \mathrm{e}^{2A4[t]} + \mathrm{e}^{4A4[t]} - \frac{1}{2} \mathrm{e}^{6A4[t]} \right) A4''[t] \Big) \\
& YZ3^{(2,1)}[z, t] - 9 (\mathrm{e}^{2A4[t]} - 2 \mathrm{e}^{4A4[t]} + \mathrm{e}^{6A4[t]}) \cos[z] \\
& \sin[z]^2 A4'[t]^2 YZ3^{(2,2)}[z, t] \Big) dz + \\
& A4'[t]^2 \left(\frac{1}{9} (\mathrm{e}^{4A4[t]} - 2 \mathrm{e}^{6A4[t]} + \mathrm{e}^{8A4[t]}) Q1 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \right. \\
& \left(-\frac{1}{2} Q1 \cos[z]^3 (-1 + \cosh[2A4[t]]) YZ3[z, t] A4''[t]^2 + \frac{1}{2} Q1 \right. \\
& \left. \cos[z]^3 (-1 + \cosh[2A4[t]]) YZ3[z, t] A4'[t] A4^{(3)}[t] + \cos[z]^3 \right. \\
& \left. \left((1 - Q1 + Q1 \cosh[2A4[t]]) A4'[t]^2 + \frac{1}{2} (-1 + \mathrm{e}^{2A4[t]}) A4''[t] \right) \right. \\
& YZ3^{(0,2)}[z, t] - \frac{1}{2} (-1 + \mathrm{e}^{2A4[t]}) \cos[z]^3 A4'[t] \\
& YZ3^{(0,3)}[z, t] + A4'[t] \left(-\frac{1}{4} \mathrm{e}^{-4A4[t]} Q1^2 \cos[z]^3 A4'[t]^2 \right. \\
& \left(YZ3[z, t] A4'[t] - \frac{1}{2} YZ3^{(0,1)}[z, t] \right) - \frac{1}{2} \mathrm{e}^{2A4[t]} \cos[z]^3 \\
& \left(-9 + M^2 + \frac{1}{4} Q1^2 A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \frac{1}{2} \cos[z]^3 \left(-9 + M^2 + \right. \\
& \left. \left(Q1 + \frac{3Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \frac{1}{2} \mathrm{e}^{-2A4[t]} Q1 \cos[z]^3 \\
& A4'[t]^2 \left(Q1 YZ3[z, t] A4'[t] + \left(-1 - \frac{3Q1}{4} \right) YZ3^{(0,1)}[z, t] \right) + \\
& A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \\
& A4''[t] \left(\frac{1}{8} \mathrm{e}^{-4A4[t]} Q1^2 \cos[z]^3 YZ3[z, t] A4'[t]^2 - \right. \\
& \left. \frac{3}{8} \mathrm{e}^{-2A4[t]} Q1 \left(\frac{4}{3} + Q1 \right) \cos[z]^3 YZ3[z, t] A4'[t]^2 - \frac{1}{2} \cos[z]^3 \right. \\
& YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{3Q1^2}{4} \right) A4'[t]^2 \right) - \frac{1}{2} Q1 \cos[z]^3
\end{aligned}$$

$$\begin{aligned}
& A4'[t] YZ3^{(0,1)}[z, t] + \frac{1}{2} Q1 \cos[z]^3 \cosh[2 A4[t]] A4'[t] \\
& YZ3^{(0,1)}[z, t] + 18 (\sin[z]^2) \sin[z] YZ3^{(1,0)}[z, t] + \\
& e^{2 A4[t]} \left(\frac{1}{2} \cos[z]^3 YZ3[z, t] \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) - \right. \\
& \left. 18 (\sin[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + 18 (-1 + e^{2 A4[t]}) \\
& (\sin[z]^2) \sin[z] A4'[t] YZ3^{(1,1)}[z, t] - 36 \cos[z] \\
& \sin[z]^2 \left(A4'[t]^2 + \frac{1}{2} (-1 + e^{2 A4[t]}) A4''[t] \right) YZ3^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2 A4[t]}) \cos[z] \sin[z]^2 A4'[t] YZ3^{(2,1)}[z, t] \Big) dz + \\
& \frac{1}{3} \left(\int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left(\frac{1}{2} Q1 (YZ3[z, t] A4''[t] + A4'[t] \right. \right. \\
& ((-2 - Q1) YZ3[z, t] A4'[t] + Q1 \cosh[2 A4[t]] YZ3[z, t] \\
& A4'[t] + 2 YZ3^{(0,1)}[z, t]) \Big) + e^{2 A4[t]} YZ3^{(0,2)}[z, t] + \\
& e^{2 A4[t]} \left((-9 + M^2) YZ3[z, t] - \frac{1}{2} Q1 YZ3[z, t] A4''[t] - \right. \\
& \left. Q1 A4'[t] YZ3^{(0,1)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3) \right. \\
& \left. YZ3^{(1,0)}[z, t] \right) - 36 e^{2 A4[t]} \tan[z]^2 YZ3^{(2,0)}[z, t] \Big) dz \Bigg) \\
& \left(\frac{1}{6} \left(e^{2 A4[t]} (-1 + Q1) - \frac{Q1}{4} \right) Q1 A4'[t]^2 + \frac{1}{6} e^{8 A4[t]} \left(-M^2 - \right. \right. \\
& \left. \frac{1}{4} Q1^2 A4'[t]^2 \right) + e^{4 A4[t]} \left(-\frac{M^2}{6} + \left(\frac{Q1}{3} - \frac{Q1^2}{4} \right) A4'[t]^2 \right) + \\
& \frac{1}{3} e^{6 A4[t]} \left(M^2 + \frac{1}{2} (-Q1 + Q1^2) A4'[t]^2 \right) + \frac{1}{12} \\
& \left. \left(e^{2 A4[t]} - 3 e^{4 A4[t]} + 3 e^{6 A4[t]} - e^{8 A4[t]} \right) Q1 A4''[t] \right) + \\
& \sqrt{\sin[z]} \left(\frac{1}{2} \left(e^{4 A4[t]} - 3 e^{6 A4[t]} + 3 e^{8 A4[t]} - e^{10 A4[t]} \right) Q1 YZ3[z, t] \right. \\
& A4''[t] - \frac{3}{4} Q1 A4'[t] \left(e^{2 A4[t]} Q1 YZ3[z, t] A4'[t] + \right. \\
& \left. 4 e^{4 A4[t]} \left(\left(\frac{1}{3} - Q1 \right) YZ3[z, t] A4'[t] - \frac{1}{3} YZ3^{(0,1)}[z, t] \right) \right) + \\
& e^{6 A4[t]} \left(YZ3[z, t] \left(-9 + M^2 + \left(2 Q1 - \frac{9 Q1^2}{2} \right) A4'[t]^2 \right) - \right. \\
& \left. 3 Q1 A4'[t] YZ3^{(0,1)}[z, t] \right) + e^{10 A4[t]} \\
& \left(YZ3[z, t] \left(-9 + M^2 - \frac{3}{4} Q1^2 A4'[t]^2 \right) - Q1 A4'[t] YZ3^{(0,1)}[z, t] \right) +
\end{aligned}$$

$$\begin{aligned}
& e^{8A4[t]} \left(-2YZ3[z, t] \left(-9 + M^2 + \frac{1}{2} (Q1 - 3Q1^2) A4'[t]^2 \right) + 3Q1 A4'[t] \right. \\
& \quad \left. YZ3^{(0,1)}[z, t] \right) + \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) YZ3^{(0,2)}[z, t] \Big) - \\
& 396 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \operatorname{Sec}[z]^3 \sin[z]^{7/2} YZ3^{(1,0)}[z, t] - \\
& 216 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \operatorname{Sec}[z]^5 \\
& \sin[z]^{11/2} YZ3^{(1,0)}[z, t] + 2 \operatorname{Sec}[z] \sin[z]^{3/2} \\
& \left(\left(-\frac{3}{4} Q1 \left(4e^{4A4[t]} \left(\frac{1}{3} - Q1 \right) + e^{2A4[t]} Q1 \right) A4'[t]^2 + e^{10A4[t]} \left(-117 + M^2 - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3}{4} Q1^2 A4'[t]^2 \right) + e^{6A4[t]} \left(-117 + M^2 + \left(2Q1 - \frac{9Q1^2}{2} \right) A4'[t]^2 \right) + \right. \\
& \quad \left. e^{8A4[t]} \left(234 - 2M^2 + (-Q1 + 3Q1^2) A4'[t]^2 \right) + \frac{1}{2} \right. \\
& \quad \left. \left(e^{4A4[t]} - 3e^{6A4[t]} + 3e^{8A4[t]} - e^{10A4[t]} \right) Q1 A4''[t] \right) YZ3^{(1,0)}[z, t] + \\
& \quad \left. \left(e^{4A4[t]} - 3e^{6A4[t]} + 3e^{8A4[t]} - e^{10A4[t]} \right) Q1 A4'[t] YZ3^{(1,1)}[z, t] \right) + \\
& 2 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \operatorname{Sec}[z] \sin[z]^{3/2} YZ3^{(1,2)}[z, t] - \\
& 324 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \operatorname{Sec}[z]^2 \sin[z]^{5/2} YZ3^{(2,0)}[z, t] - \\
& 216 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \operatorname{Sec}[z]^4 \sin[z]^{9/2} YZ3^{(2,0)}[z, t] - \\
& 72 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \operatorname{Sec}[z]^3 \sin[z]^{7/2} YZ3^{(3,0)}[z, t] \Big) \Big) dt \\
& \sin[Mt] A4'[t]^2 + e^{-\frac{1}{4} Q1 (e^{-2A4[t]} + 2A4[t])} M Q1 (c12 \\
& \cos[\\
& M \\
& t] - \\
& c11 \sin[Mt] + e^{4A4[t]} (c12 \cos[Mt] - c11 \sin[Mt]) + \\
& 2 \\
& e^{2A4[t]} \\
& (-c12 \cos[Mt] + \\
& c11 \sin[Mt])) \\
& A4'[t]^2 + 3Q1 \sqrt{\sin[z]} A4'[t]^2 (-YZ3[z, t] - 2 \tan[\\
& z] YZ3^{(1,0)}[\\
& z, \\
& t]) + \\
& 3e^{4A4[t]} Q1 \sqrt{\sin[z]} A4'[t]^2 (-YZ3[z, t] - 2 \\
& \tan[\\
& z] YZ3^{(1,0)}[\\
& z, \\
& t]) + \\
& 6e^{2A4[t]} Q1 \sqrt{\sin[z]} A4'[t]^2 (YZ3[
\end{aligned}$$

$$\begin{aligned}
& z, \\
& t] + 2 \\
& \text{Tan} [\\
& z] \text{YZ3}^{(1,0)} [\\
& z, \\
& t]) \Big) / \\
& \left((-1 + e^{2 A4[t]})^2 M Q1 \sqrt{\sin[z]} A4'[t]^2 \right), \text{YZ2}[\\
& z, \\
& t] == \\
& \left(\frac{3}{8} \right. \\
& e^{-\frac{1}{4} Q1 (e^{-2 A4[t]} + 2 A4[t])} \\
& (-1 + \\
& e^{2 A4[t]}) \cos[M \\
& t] \\
& \left(\int \frac{1}{A4'[t]^3} e^{\frac{1}{4} e^{-2 A4[t]} Q1 + \frac{1}{2} (-14 + Q1) A4[t]} \right. \\
& \left. \cosh[A4[t]]^3 \right. \\
& \left. \sin[M t] \right. \\
& \left(-\frac{2}{9} e^{4 A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{3}{4} \left(-\frac{1}{3} + e^{2 A4[t]} - e^{4 A4[t]} + \frac{1}{3} e^{6 A4[t]} \right) Q1 \right. \right. \\
& \left. \left. \cos[z]^3 \text{YZ3}[z, t] A4''[t]^3 - \frac{3}{8} \left(-\frac{1}{3} + e^{2 A4[t]} - e^{4 A4[t]} + \frac{1}{3} e^{6 A4[t]} \right) \right. \right. \\
& \left. \left. Q1 \cos[z]^3 \text{YZ3}[z, t] A4'[t]^2 A4^{(4)}[t] + \frac{1}{4} \cos[z]^3 \right. \right. \\
& \left. \left(A4'[t]^2 \left(\frac{1}{4} Q1 (-4 - 4 Q1 + e^{-2 A4[t]} Q1) A4'[t]^2 + e^{6 A4[t]} (-9 + M^2 + \right. \right. \right. \\
& \left. \left. \left. \frac{1}{4} Q1^2 A4'[t]^2 \right) + e^{4 A4[t]} (18 - 2 M^2 + (4 - Q1 - Q1^2) A4'[t]^2) + \right. \right. \\
& \left. \left. \left. e^{2 A4[t]} \left(-9 + M^2 + \left(4 + 2 Q1 + \frac{3 Q1^2}{2} \right) A4'[t]^2 \right) \right) - \right. \right. \\
& \left. \left. \frac{3}{2} \left(e^{4 A4[t]} \left(-\frac{4}{3} - Q1 \right) + \frac{1}{3} (-1 + e^{6 A4[t]}) Q1 + e^{2 A4[t]} \left(\frac{4}{3} + Q1 \right) \right) \right. \\
& \left. \left. A4'[t]^2 A4''[t] + 2 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) A4''[t]^2 \right) \right. \\
& \text{YZ3}^{(0,2)}[z, t] - \frac{3}{4} \cos[z]^3 A4'[t] \left(\left(e^{4 A4[t]} \left(\frac{4}{3} - Q1 \right) + \right. \right. \\
& \left. \left. e^{2 A4[t]} \left(-\frac{4}{3} + Q1 \right) + \frac{1}{3} (-1 + e^{6 A4[t]}) Q1 \right) A4'[t]^2 + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2}{3} \left(e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) A4''[t] \Big) YZ3^{(0,3)}[z, t] + \\
& \frac{1}{4} \left(e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \cos[z]^3 A4'[t]^2 YZ3^{(0,4)}[z, t] + \\
& A4''[t]^2 \left(-\frac{1}{4} Q1 \cos[z]^3 A4'[t] YZ3^{(0,1)}[z, t] + \right. \\
& e^{6A4[t]} \left(\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 YZ3[z, t] + \frac{1}{4} Q1 \cos[z]^3 A4'[t] \right. \\
& \left. \left. YZ3^{(0,1)}[z, t] - 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \right. \\
& e^{2A4[t]} \left(\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 YZ3[z, t] + \frac{3}{4} Q1 \cos[z]^3 A4'[t] \right. \\
& \left. \left. t] YZ3^{(0,1)}[z, t] - 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \right. \\
& e^{4A4[t]} \left((9 - M^2) \cos[z]^3 YZ3[z, t] - \frac{3}{4} Q1 \cos[z]^3 A4'[t] \right. \\
& \left. YZ3^{(0,1)}[z, t] + 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \\
& A4'[t]^3 \left(\frac{1}{4} (-3 + e^{-2A4[t]}) Q1^2 \cos[z]^3 A4'[t]^2 (YZ3[z, t] \right. \\
& \left. A4'[t] - YZ3^{(0,1)}[z, t]) + e^{4A4[t]} \right. \\
& \left(\cos[z]^3 \left(9 - M^2 - \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \\
& A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + e^{2A4[t]} \\
& \left(\cos[z]^3 \left(-9 + M^2 + \left(-Q1 - \frac{3Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \\
& A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 + \frac{3Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) - \\
& \frac{1}{2} A4'[t] A4''[t] \left(\cos[z]^3 \left(e^{6A4[t]} \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) \right. \right. \\
& \left. YZ3^{(0,1)}[z, t] + \frac{3}{4} Q1 A4'[t]^2 \left(-3 Q1 YZ3[z, t] A4'[t] + \right. \right. \\
& \left. \left. e^{-2A4[t]} Q1 \left(YZ3[z, t] A4'[t] - \frac{1}{3} YZ3^{(0,1)}[z, t] \right) + \right. \right. \\
& \left. \left. \frac{4}{3} (1 + Q1) YZ3^{(0,1)}[z, t] \right) \right) + e^{4A4[t]}
\end{aligned}$$

$$\begin{aligned}
& \left(-2 \cos[z]^3 \left(-9 + M^2 - \frac{1}{2} (Q1 + Q1^2) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] - \right. \\
& \quad A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + 3 \left(Q1 + \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \quad \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + e^{2A4[t]} \\
& \quad \left(\cos[z]^3 \left(-9 + M^2 + \left(-2 Q1 - \frac{3 Q1^2}{2} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \\
& \quad A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + 3 \left(Q1 + \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \quad \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) - \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left((e^{2A4[t]} - e^{4A4[t]}) A4'[t]^2 + \right. \\
& \quad \left. \left(-\frac{1}{2} e^{2A4[t]} + e^{4A4[t]} - \frac{1}{2} e^{6A4[t]} \right) A4''[t] \right) YZ3^{(1,1)}[z, t] - \\
& 9 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) (1 + \cos[z]^2) \sin[z] \\
& A4'[t]^2 YZ3^{(1,2)}[z, t] - 36 \cos[z] \sin[z]^2 \\
& \left((e^{2A4[t]} + e^{4A4[t]}) A4'[t]^4 - \frac{1}{2} (e^{2A4[t]} - e^{4A4[t]}) A4'[t]^2 A4''[t] + \right. \\
& \quad \left. \left(\frac{1}{2} e^{2A4[t]} - e^{4A4[t]} + \frac{1}{2} e^{6A4[t]} \right) A4''[t]^2 \right) YZ3^{(2,0)}[z, t] - \\
& \frac{1}{4} A4'[t] A4^{(3)}[t] \left(-\frac{9}{2} \left(-\frac{1}{3} + e^{2A4[t]} - e^{4A4[t]} + \frac{1}{3} e^{6A4[t]} \right) Q1 \cos[z]^3 \right. \\
& \quad YZ3[z, t] A4''[t] - \frac{1}{4} Q1 \cos[z]^3 A4'[t] (e^{-2A4[t]} Q1 YZ3[z, t] \\
& \quad A4'[t] - 4 (1 + Q1) YZ3[z, t] A4'[t] + 4 YZ3^{(0,1)}[z, t]) + \\
& \quad (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \cos[z]^3 YZ3^{(0,2)}[z, t] + e^{6A4[t]} \\
& \quad \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) + Q1 \cos[z]^3 A4'[t] \right. \\
& \quad YZ3^{(0,1)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \left. \right) + \\
& e^{2A4[t]} \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-2 Q1 - \frac{3 Q1^2}{2} \right) A4'[t]^2 \right) + \right. \\
& \quad 3 Q1 \cos[z]^3 A4'[t] YZ3^{(0,1)}[z, t] - 36 (1 + \cos[z]^2) \\
& \quad \sin[z] YZ3^{(1,0)}[z, t] \left. \right) + e^{4A4[t]} \left(-2 \cos[z]^3 YZ3[z, t] \right. \\
& \quad \left. \left(-9 + M^2 - \frac{1}{2} (Q1 + Q1^2) A4'[t]^2 \right) - 3 Q1 \cos[z]^3 A4'[t] \right. \\
& \quad YZ3^{(0,1)}[z, t] + 72 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \left. \right) -
\end{aligned}$$

$$\begin{aligned}
& \left. \left(36 \left(e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \cos[z] \sin[z]^2 YZ3^{(2,0)}[z, t] \right) - \right. \\
& 36 \cos[z] \sin[z]^2 A4'[t] \left(\left(e^{2A4[t]} - e^{4A4[t]} \right) A4'[t]^2 + \right. \\
& \left. \left(-\frac{1}{2} e^{2A4[t]} + e^{4A4[t]} - \frac{1}{2} e^{6A4[t]} \right) A4''[t] \right) \\
& YZ3^{(2,1)}[z, t] - 9 \left(e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \cos[z] \\
& \sin[z]^2 A4'[t]^2 YZ3^{(2,2)}[z, t] \Bigg) dz + \\
& A4'[t]^2 \left(\frac{1}{9} \left(e^{4A4[t]} - 2e^{6A4[t]} + e^{8A4[t]} \right) Q1 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \right. \\
& \left. \left(-\frac{1}{2} Q1 \cos[z]^3 (-1 + \cosh[2A4[t]]) YZ3[z, t] A4''[t]^2 + \frac{1}{2} Q1 \right. \right. \\
& \cos[z]^3 (-1 + \cosh[2A4[t]]) YZ3[z, t] A4'[t] A4^{(3)}[t] + \cos[z]^3 \\
& \left. \left((1 - Q1 + Q1 \cosh[2A4[t]]) A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \\
& YZ3^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] \\
& YZ3^{(0,3)}[z, t] + A4'[t] \left(-\frac{1}{4} e^{-4A4[t]} Q1^2 \cos[z]^3 A4'[t]^2 \right. \\
& \left. \left(YZ3[z, t] A4'[t] - \frac{1}{2} YZ3^{(0,1)}[z, t] \right) - \frac{1}{2} e^{2A4[t]} \cos[z]^3 \right. \\
& \left. \left(-9 + M^2 + \frac{1}{4} Q1^2 A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \frac{1}{2} \cos[z]^3 \left(-9 + M^2 + \right. \right. \\
& \left. \left. \left(Q1 + \frac{3Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \frac{1}{2} e^{-2A4[t]} Q1 \cos[z]^3 \right. \\
& A4'[t]^2 \left(Q1 YZ3[z, t] A4'[t] + \left(-1 - \frac{3Q1}{4} \right) YZ3^{(0,1)}[z, t] \right) + \\
& A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + \\
& A4''[t] \left(\frac{1}{8} e^{-4A4[t]} Q1^2 \cos[z]^3 YZ3[z, t] A4'[t]^2 - \right. \\
& \left. \frac{3}{8} e^{-2A4[t]} Q1 \left(\frac{4}{3} + Q1 \right) \cos[z]^3 YZ3[z, t] A4'[t]^2 - \frac{1}{2} \cos[z]^3 \right. \\
& YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{3Q1^2}{4} \right) A4'[t]^2 \right) - \frac{1}{2} Q1 \cos[z]^3 \\
& A4'[t] YZ3^{(0,1)}[z, t] + \frac{1}{2} Q1 \cos[z]^3 \cosh[2A4[t]] A4'[t] \\
& YZ3^{(0,1)}[z, t] + 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] +
\end{aligned}$$

$$\begin{aligned}
& e^{2 A4[t]} \left(\frac{1}{2} \cos[z]^3 YZ3[z, t] \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) - \right. \\
& \quad \left. 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + 18 (-1 + e^{2 A4[t]}) \\
& \quad (1 + \cos[z]^2) \sin[z] A4'[t] YZ3^{(1,1)}[z, t] - 36 \cos[z] \\
& \quad \sin[z]^2 \left(A4'[t]^2 + \frac{1}{2} (-1 + e^{2 A4[t]}) A4''[t] \right) YZ3^{(2,0)}[z, t] + \\
& \quad \left. 18 (-1 + e^{2 A4[t]}) \cos[z] \sin[z]^2 A4'[t] YZ3^{(2,1)}[z, t] \right) dz + \\
& \frac{1}{3} \left(\int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left(\frac{1}{2} Q1 (YZ3[z, t] A4''[t] + A4'[t] \right. \right. \\
& \quad \left((-2 - Q1) YZ3[z, t] A4'[t] + Q1 \cosh[2 A4[t]] YZ3[z, t] \right. \\
& \quad \left. \left. A4'[t] + 2 YZ3^{(0,1)}[z, t] \right) \right) + e^{2 A4[t]} YZ3^{(0,2)}[z, t] + \\
& \quad e^{2 A4[t]} \left((-9 + M^2) YZ3[z, t] - \frac{1}{2} Q1 YZ3[z, t] A4''[t] - \right. \\
& \quad \left. Q1 A4'[t] YZ3^{(0,1)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3) \right. \\
& \quad \left. YZ3^{(1,0)}[z, t] \right) - 36 e^{2 A4[t]} \tan[z]^2 YZ3^{(2,0)}[z, t] \right) dz \Bigg) \\
& \left(\frac{1}{6} \left(e^{2 A4[t]} (-1 + Q1) - \frac{Q1}{4} \right) Q1 A4'[t]^2 + \frac{1}{6} e^{8 A4[t]} \left(-M^2 - \right. \right. \\
& \quad \left. \frac{1}{4} Q1^2 A4'[t]^2 \right) + e^{4 A4[t]} \left(-\frac{M^2}{6} + \left(\frac{Q1}{3} - \frac{Q1^2}{4} \right) A4'[t]^2 \right) + \\
& \quad \frac{1}{3} e^{6 A4[t]} \left(M^2 + \frac{1}{2} (-Q1 + Q1^2) A4'[t]^2 \right) + \frac{1}{12} \\
& \quad \left. \left(e^{2 A4[t]} - 3 e^{4 A4[t]} + 3 e^{6 A4[t]} - e^{8 A4[t]} \right) Q1 A4''[t] \right) + \\
& \quad \sqrt{\sin[z]} \left(\frac{1}{2} \left(e^{4 A4[t]} - 3 e^{6 A4[t]} + 3 e^{8 A4[t]} - e^{10 A4[t]} \right) Q1 YZ3[z, t] \right. \\
& \quad A4''[t] - \frac{3}{4} Q1 A4'[t] \left(e^{2 A4[t]} Q1 YZ3[z, t] A4'[t] + \right. \\
& \quad \left. 4 e^{4 A4[t]} \left(\left(\frac{1}{3} - Q1 \right) YZ3[z, t] A4'[t] - \frac{1}{3} YZ3^{(0,1)}[z, t] \right) \right) + \\
& \quad e^{6 A4[t]} \left(YZ3[z, t] \left(-9 + M^2 + \left(2 Q1 - \frac{9 Q1^2}{2} \right) A4'[t]^2 \right) - \right. \\
& \quad \left. 3 Q1 A4'[t] YZ3^{(0,1)}[z, t] \right) + e^{10 A4[t]} \\
& \quad \left(YZ3[z, t] \left(-9 + M^2 - \frac{3}{4} Q1^2 A4'[t]^2 \right) - Q1 A4'[t] YZ3^{(0,1)}[z, t] \right) + \\
& \quad e^{8 A4[t]} \left(-2 YZ3[z, t] \left(-9 + M^2 + \frac{1}{2} (Q1 - 3 Q1^2) A4'[t]^2 \right) + 3 Q1 A4'[t] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(YZ3^{(0,1)}[z, t] + \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) YZ3^{(0,2)}[z, t] \right) - \right. \\
& 396 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} YZ3^{(1,0)}[z, t] - \\
& 216 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \sec[z]^5 \\
& \sin[z]^{11/2} YZ3^{(1,0)}[z, t] + 2 \sec[z] \sin[z]^{3/2} \\
& \left(\left(-\frac{3}{4} Q1 \left(4e^{4A4[t]} \left(\frac{1}{3} - Q1 \right) + e^{2A4[t]} Q1 \right) A4'[t]^2 + e^{10A4[t]} \left(-117 + M^2 - \right. \right. \right. \\
& \left. \left. \left. \frac{3}{4} Q1^2 A4'[t]^2 \right) + e^{6A4[t]} \left(-117 + M^2 + \left(2Q1 - \frac{9Q1^2}{2} \right) A4'[t]^2 \right) + \right. \\
& e^{8A4[t]} \left(234 - 2M^2 + (-Q1 + 3Q1^2) A4'[t]^2 \right) + \frac{1}{2} \\
& \left. \left(e^{4A4[t]} - 3e^{6A4[t]} + 3e^{8A4[t]} - e^{10A4[t]} \right) Q1 A4''[t] \right) YZ3^{(1,0)}[z, t] + \\
& \left. \left(e^{4A4[t]} - 3e^{6A4[t]} + 3e^{8A4[t]} - e^{10A4[t]} \right) Q1 A4'[t] YZ3^{(1,1)}[z, t] \right) + \\
& 2 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \sec[z] \sin[z]^{3/2} YZ3^{(1,2)}[z, t] - \\
& 324 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \sec[z]^2 \sin[z]^{5/2} YZ3^{(2,0)}[z, t] - \\
& 216 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \sec[z]^4 \sin[z]^{9/2} YZ3^{(2,0)}[z, t] - \\
& 72 \left(e^{6A4[t]} - 2e^{8A4[t]} + e^{10A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} YZ3^{(3,0)}[z, t] \Bigg) dt \\
& A4'[t] - \frac{3}{8} e^{-\frac{1}{4} Q1 (e^{-2A4[t]} + 2A4[t])} (-1 + e^{2A4[t]}) \left(\int \frac{1}{A4'[t]^3} \right. \\
& \left. \left. \frac{1}{e^{\frac{1}{4} e^{-2A4[t]} Q1 + \frac{1}{2} (-14+Q1) A4[t]}} \right. \right. \\
& \cos[M t] \\
& \cosh[A4[t]]^3 \\
& \left(-\frac{2}{9} e^{4A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{3}{4} \left(-\frac{1}{3} + e^{2A4[t]} - e^{4A4[t]} + \frac{1}{3} e^{6A4[t]} \right) Q1 \right. \right. \\
& \cos[z]^3 YZ3[z, t] A4''[t]^3 - \frac{3}{8} \left(-\frac{1}{3} + e^{2A4[t]} - e^{4A4[t]} + \frac{1}{3} e^{6A4[t]} \right) \\
& Q1 \cos[z]^3 YZ3[z, t] A4'[t]^2 A4^{(4)}[t] + \frac{1}{4} \cos[z]^3 \\
& \left(A4'[t]^2 \left(\frac{1}{4} Q1 (-4 - 4Q1 + e^{-2A4[t]} Q1) A4'[t]^2 + e^{6A4[t]} \left(-9 + M^2 + \right. \right. \right. \\
& \left. \left. \left. \frac{1}{4} Q1^2 A4'[t]^2 \right) + e^{4A4[t]} (18 - 2M^2 + (4 - Q1 - Q1^2) A4'[t]^2) + \right. \\
& e^{2A4[t]} \left(-9 + M^2 + \left(4 + 2Q1 + \frac{3Q1^2}{2} \right) A4'[t]^2 \right) \Bigg) - \\
& \frac{3}{2} \left(e^{4A4[t]} \left(-\frac{4}{3} - Q1 \right) + \frac{1}{3} (-1 + e^{6A4[t]}) Q1 + e^{2A4[t]} \left(\frac{4}{3} + Q1 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& A4'[t]^2 A4''[t] + 2 \left(e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) A4''[t]^2 \\
& YZ3^{(0,2)}[z, t] - \frac{3}{4} \cos[z]^3 A4'[t] \left(\left(e^{4A4[t]} \left(\frac{4}{3} - Q1 \right) + \right. \right. \\
& \left. \left. e^{2A4[t]} \left(-\frac{4}{3} + Q1 \right) + \frac{1}{3} (-1 + e^{6A4[t]}) Q1 \right) A4'[t]^2 + \right. \\
& \left. \frac{2}{3} \left(e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) A4''[t] \right) YZ3^{(0,3)}[z, t] + \\
& \frac{1}{4} \left(e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \cos[z]^3 A4'[t]^2 YZ3^{(0,4)}[z, t] + \\
& A4''[t]^2 \left(-\frac{1}{4} Q1 \cos[z]^3 A4'[t] YZ3^{(0,1)}[z, t] + \right. \\
& \left. e^{6A4[t]} \left(\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 YZ3[z, t] + \frac{1}{4} Q1 \cos[z]^3 A4'[t] \right. \right. \\
& \left. \left. YZ3^{(0,1)}[z, t] - 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \right. \\
& \left. e^{2A4[t]} \left(\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 YZ3[z, t] + \frac{3}{4} Q1 \cos[z]^3 A4'[t] \right. \right. \\
& \left. \left. YZ3^{(0,1)}[z, t] - 18 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \right. \\
& \left. e^{4A4[t]} \left((9 - M^2) \cos[z]^3 YZ3[z, t] - \frac{3}{4} Q1 \cos[z]^3 A4'[t] \right. \right. \\
& \left. \left. YZ3^{(0,1)}[z, t] + 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + \right. \\
& A4'[t]^3 \left(\frac{1}{4} \left(-3 + e^{-2A4[t]} \right) Q1^2 \cos[z]^3 A4'[t]^2 (YZ3[z, t] \right. \\
& \left. A4'[t] - YZ3^{(0,1)}[z, t]) + e^{4A4[t]} \right. \\
& \left. \left(\cos[z]^3 \left(9 - M^2 - \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \right. \\
& \left. \left. A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + e^{2A4[t]} \\
& \left. \left(\cos[z]^3 \left(-9 + M^2 + \left(-Q1 - \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \right. \\
& \left. \left. A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 + \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) - \right. \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) \right) - \\
& \frac{1}{2} A4'[t] A4''[t] \left(\cos[z]^3 \left(e^{6A4[t]} \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& YZ3^{(0,1)}[z, t] + \frac{3}{4} Q1 A4'[t]^2 \left(-3 Q1 YZ3[z, t] A4'[t] + \right. \\
& \quad e^{-2 A4[t]} Q1 \left(YZ3[z, t] A4'[t] - \frac{1}{3} YZ3^{(0,1)}[z, t] \right) + \\
& \quad \left. \frac{4}{3} (1 + Q1) YZ3^{(0,1)}[z, t] \right) + e^{4 A4[t]} \\
& \left(-2 \cos[z]^3 \left(-9 + M^2 - \frac{1}{2} (Q1 + Q1^2) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] - \right. \\
& \quad A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + 3 \left(Q1 + \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \quad \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + e^{2 A4[t]} \\
& \left(\cos[z]^3 \left(-9 + M^2 + \left(-2 Q1 - \frac{3 Q1^2}{2} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \right. \\
& \quad A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + 3 \left(Q1 + \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) - \right. \\
& \quad \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) - \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left((e^{2 A4[t]} - e^{4 A4[t]}) A4'[t]^2 + \right. \\
& \quad \left. \left(-\frac{1}{2} e^{2 A4[t]} + e^{4 A4[t]} - \frac{1}{2} e^{6 A4[t]} \right) A4''[t] \right) YZ3^{(1,1)}[z, t] - \\
& 9 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) (1 + \cos[z]^2) \sin[z] \\
& A4'[t]^2 YZ3^{(1,2)}[z, t] - 36 \cos[z] \sin[z]^2 \\
& \left((e^{2 A4[t]} + e^{4 A4[t]}) A4'[t]^4 - \frac{1}{2} (e^{2 A4[t]} - e^{4 A4[t]}) A4'[t]^2 A4''[t] + \right. \\
& \quad \left. \left(\frac{1}{2} e^{2 A4[t]} - e^{4 A4[t]} + \frac{1}{2} e^{6 A4[t]} \right) A4''[t]^2 \right) YZ3^{(2,0)}[z, t] - \\
& \frac{1}{4} A4'[t] A4^{(3)}[t] \left(-\frac{9}{2} \left(-\frac{1}{3} + e^{2 A4[t]} - e^{4 A4[t]} + \frac{1}{3} e^{6 A4[t]} \right) Q1 \cos[z]^3 \right. \\
& \quad YZ3[z, t] A4''[t] - \frac{1}{4} Q1 \cos[z]^3 A4'[t] (e^{-2 A4[t]} Q1 YZ3[z, t] \\
& \quad A4'[t] - 4 (1 + Q1) YZ3[z, t] A4'[t] + 4 YZ3^{(0,1)}[z, t]) + \\
& \quad (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \cos[z]^3 YZ3^{(0,2)}[z, t] + e^{6 A4[t]} \\
& \quad \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) + Q1 \cos[z]^3 A4'[t] \right. \\
& \quad YZ3^{(0,1)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \left. \right) + \\
& e^{2 A4[t]} \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-2 Q1 - \frac{3 Q1^2}{2} \right) A4'[t]^2 \right) + \right. \\
& \quad \left. 3 Q1 \cos[z]^3 A4'[t] YZ3^{(0,1)}[z, t] - 36 (1 + \cos[z]^2) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \sin[z] YZ3^{(1,0)}[z, t] \right) + e^{4A4[t]} \left(-2 \cos[z]^3 YZ3[z, t] \right. \\
& \left. \left(-9 + M^2 - \frac{1}{2} (Q1 + Q1^2) A4'[t]^2 \right) - 3 Q1 \cos[z]^3 A4'[t] \right. \\
& \left. YZ3^{(0,1)}[z, t] + 72 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) - \\
& \left. 36 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \cos[z] \sin[z]^2 YZ3^{(2,0)}[z, t] \right) - \\
& 36 \cos[z] \sin[z]^2 A4'[t] \left((e^{2A4[t]} - e^{4A4[t]}) A4'[t]^2 + \right. \\
& \left. \left(-\frac{1}{2} e^{2A4[t]} + e^{4A4[t]} - \frac{1}{2} e^{6A4[t]} \right) A4''[t] \right) \\
& YZ3^{(2,1)}[z, t] - 9 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \cos[z] \\
& \sin[z]^2 A4'[t]^2 YZ3^{(2,2)}[z, t] \Big) dz + \\
& A4'[t]^2 \left(\frac{1}{9} (e^{4A4[t]} - 2 e^{6A4[t]} + e^{8A4[t]}) Q1 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \right. \\
& \left. \left(-\frac{1}{2} Q1 \cos[z]^3 (-1 + \cosh[2A4[t]]) YZ3[z, t] A4''[t]^2 + \frac{1}{2} Q1 \right. \right. \\
& \left. \left. \cos[z]^3 (-1 + \cosh[2A4[t]]) YZ3[z, t] A4'[t] A4^{(3)}[t] + \cos[z]^3 \right. \right. \\
& \left. \left. \left((1 - Q1 + Q1 \cosh[2A4[t]]) A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \right. \\
& \left. \left. YZ3^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] \right. \right. \\
& \left. \left. YZ3^{(0,3)}[z, t] + A4'[t] \left(-\frac{1}{4} e^{-4A4[t]} Q1^2 \cos[z]^3 A4'[t]^2 \right. \right. \right. \\
& \left. \left. \left(YZ3[z, t] A4'[t] - \frac{1}{2} YZ3^{(0,1)}[z, t] \right) - \frac{1}{2} e^{2A4[t]} \cos[z]^3 \right. \right. \\
& \left. \left. \left(-9 + M^2 + \frac{1}{4} Q1^2 A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \frac{1}{2} \cos[z]^3 \left(-9 + M^2 + \right. \right. \right. \\
& \left. \left. \left. \left(Q1 + \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) YZ3^{(0,1)}[z, t] + \frac{1}{2} e^{-2A4[t]} Q1 \cos[z]^3 \right. \right. \\
& \left. \left. A4'[t]^2 \left(Q1 YZ3[z, t] A4'[t] + \left(-1 - \frac{3 Q1}{4} \right) YZ3^{(0,1)}[z, t] \right) + \right. \right. \\
& \left. \left. A4'[t] \left(\cos[z]^3 YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{Q1^2}{4} \right) A4'[t]^2 \right) - \right. \right. \right. \\
& \left. \left. \left. 36 (1 + \cos[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) \right) + \right. \\
& \left. A4''[t] \left(\frac{1}{8} e^{-4A4[t]} Q1^2 \cos[z]^3 YZ3[z, t] A4'[t]^2 - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{3}{8} e^{-2 A4[t]} Q1 \left(\frac{4}{3} + Q1 \right) \cos[z]^3 YZ3[z, t] A4'[t]^2 - \frac{1}{2} \cos[z]^3 \\
& YZ3[z, t] \left(-9 + M^2 + \left(-Q1 - \frac{3 Q1^2}{4} \right) A4'[t]^2 \right) - \frac{1}{2} Q1 \cos[z]^3 \\
& A4'[t] YZ3^{(0,1)}[z, t] + \frac{1}{2} Q1 \cos[z]^3 \cosh[2 A4[t]] A4'[t] \\
& YZ3^{(0,1)}[z, t] + 18 (\sin[z]^2) \sin[z] YZ3^{(1,0)}[z, t] + \\
& e^{2 A4[t]} \left(\frac{1}{2} \cos[z]^3 YZ3[z, t] \left(-9 + M^2 - \frac{1}{4} Q1^2 A4'[t]^2 \right) - \right. \\
& \left. 18 (\sin[z]^2) \sin[z] YZ3^{(1,0)}[z, t] \right) + 18 (-1 + e^{2 A4[t]}) \\
& (\sin[z]^2) \sin[z] A4'[t] YZ3^{(1,1)}[z, t] - 36 \cos[z] \\
& \sin[z]^2 \left(A4'[t]^2 + \frac{1}{2} (-1 + e^{2 A4[t]}) A4''[t] \right) YZ3^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2 A4[t]}) \cos[z] \sin[z]^2 A4'[t] YZ3^{(2,1)}[z, t] \Big) dz + \\
& \frac{1}{3} \left(\int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left(\frac{1}{2} Q1 (YZ3[z, t] A4''[t] + A4'[t] \right. \right. \\
& \left. \left((-2 - Q1) YZ3[z, t] A4'[t] + Q1 \cosh[2 A4[t]] YZ3[z, t] \right. \right. \\
& \left. \left. A4'[t] + 2 YZ3^{(0,1)}[z, t] \right) \right) + e^{2 A4[t]} YZ3^{(0,2)}[z, t] + \\
& e^{2 A4[t]} \left((-9 + M^2) YZ3[z, t] - \frac{1}{2} Q1 YZ3[z, t] A4''[t] - \right. \\
& \left. Q1 A4'[t] YZ3^{(0,1)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3) \right. \\
& \left. YZ3^{(1,0)}[z, t] \right) - 36 e^{2 A4[t]} \tan[z]^2 YZ3^{(2,0)}[z, t] \Big) dz \Bigg) \\
& \left(\frac{1}{6} \left(e^{2 A4[t]} (-1 + Q1) - \frac{Q1}{4} \right) Q1 A4'[t]^2 + \frac{1}{6} e^{8 A4[t]} \left(-M^2 - \right. \right. \\
& \left. \left. \frac{1}{4} Q1^2 A4'[t]^2 \right) + e^{4 A4[t]} \left(-\frac{M^2}{6} + \left(\frac{Q1}{3} - \frac{Q1^2}{4} \right) A4'[t]^2 \right) + \right. \\
& \left. \frac{1}{3} e^{6 A4[t]} \left(M^2 + \frac{1}{2} (-Q1 + Q1^2) A4'[t]^2 \right) + \frac{1}{12} \right. \\
& \left. \left(e^{2 A4[t]} - 3 e^{4 A4[t]} + 3 e^{6 A4[t]} - e^{8 A4[t]} \right) Q1 A4''[t] \right) + \\
& \sqrt{\sin[z]} \left(\frac{1}{2} \left(e^{4 A4[t]} - 3 e^{6 A4[t]} + 3 e^{8 A4[t]} - e^{10 A4[t]} \right) Q1 YZ3[z, t] \right. \\
& A4''[t] - \frac{3}{4} Q1 A4'[t] \left(e^{2 A4[t]} Q1 YZ3[z, t] A4'[t] + \right. \\
& \left. \left. 4 e^{4 A4[t]} \left(\left(\frac{1}{3} - Q1 \right) YZ3[z, t] A4'[t] - \frac{1}{3} YZ3^{(0,1)}[z, t] \right) \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& e^{6 A4[t]} \left(YZ3[z, t] \left(-9 + M^2 + \left(2 Q1 - \frac{9 Q1^2}{2} \right) A4'[t]^2 \right) - \right. \\
& \quad \left. 3 Q1 A4'[t] YZ3^{(0,1)}[z, t] \right) + e^{10 A4[t]} \\
& \quad \left(YZ3[z, t] \left(-9 + M^2 - \frac{3}{4} Q1^2 A4'[t]^2 \right) - Q1 A4'[t] YZ3^{(0,1)}[z, t] \right) + \\
& \quad e^{8 A4[t]} \left(-2 YZ3[z, t] \left(-9 + M^2 + \frac{1}{2} (Q1 - 3 Q1^2) A4'[t]^2 \right) + 3 Q1 A4'[t] \right. \\
& \quad \left. YZ3^{(0,1)}[z, t] \right) + \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) YZ3^{(0,2)}[z, t] - \\
& 396 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z]^3 \sin[z]^{7/2} YZ3^{(1,0)}[z, t] - \\
& 216 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z]^5 \\
& \sin[z]^{11/2} YZ3^{(1,0)}[z, t] + 2 \operatorname{Sec}[z] \sin[z]^{3/2} \\
& \left(\left(-\frac{3}{4} Q1 \left(4 e^{4 A4[t]} \left(\frac{1}{3} - Q1 \right) + e^{2 A4[t]} Q1 \right) A4'[t]^2 + e^{10 A4[t]} \left(-117 + M^2 - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3}{4} Q1^2 A4'[t]^2 \right) + e^{6 A4[t]} \left(-117 + M^2 + \left(2 Q1 - \frac{9 Q1^2}{2} \right) A4'[t]^2 \right) + \right. \\
& \quad \left. e^{8 A4[t]} \left(234 - 2 M^2 + (-Q1 + 3 Q1^2) A4'[t]^2 \right) + \frac{1}{2} \right. \\
& \quad \left. \left(e^{4 A4[t]} - 3 e^{6 A4[t]} + 3 e^{8 A4[t]} - e^{10 A4[t]} \right) Q1 A4''[t] \right) YZ3^{(1,0)}[z, t] + \\
& \quad \left. \left(e^{4 A4[t]} - 3 e^{6 A4[t]} + 3 e^{8 A4[t]} - e^{10 A4[t]} \right) Q1 A4'[t] YZ3^{(1,1)}[z, t] \right) + \\
& 2 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z] \sin[z]^{3/2} YZ3^{(1,2)}[z, t] - \\
& 324 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z]^2 \sin[z]^{5/2} YZ3^{(2,0)}[z, t] - \\
& 216 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z]^4 \sin[z]^{9/2} YZ3^{(2,0)}[z, t] - \\
& 72 \left(e^{6 A4[t]} - 2 e^{8 A4[t]} + e^{10 A4[t]} \right) \operatorname{Sec}[z]^3 \\
& \sin[z]^{7/2} YZ3^{(3,0)}[z, t] \Bigg) \Bigg) \sin[M t] A4'[t] + \\
& M \left(-\frac{1}{6} \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left(\frac{1}{2} Q1 (YZ3[z, t] A4''[t] + A4'[t] ((-2 - Q1) YZ3[z, t] \right. \right. \\
& \quad \left. \left. A4'[t] + Q1 \cosh[2 A4[t]] YZ3[z, t] A4'[t] + 2 YZ3^{(0,1)}[z, t]) + \right. \right. \\
& \quad \left. e^{2 A4[t]} YZ3^{(0,2)}[z, t] + e^{2 A4[t]} \left((-9 + M^2) YZ3[z, t] - \frac{1}{2} Q1 YZ3[z, t] \right. \right. \\
& \quad \left. \left. A4''[t] - Q1 A4'[t] YZ3^{(0,1)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3) \right. \right. \\
& \quad \left. \left. YZ3^{(1,0)}[z, t] \right) - 36 e^{2 A4[t]} \tan[z]^2 YZ3^{(2,0)}[z, t] \right) dz + \\
& e^{-\frac{1}{4} Q1 (e^{-2 A4[t]} + 2 A4[t])} (-1 + e^{2 A4[t]}) Q1 (c11 \cos[M t] + c12 \sin[M t]) \\
& A4' [
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\text{t}]}{\left(\left(-1 + e^{2 A4[t]} \right) M Q1 \sqrt{\sin[z]} A4'[t] \right)} \right) \right\} \\
& YZ3^{(0,4)}[z, t] = \frac{1}{4 A4'[t]^2} \\
& \left(-4 \right. \\
& \left. \left(-1 + e^{-2 A4[t]} \right) \right. \\
& Q1 \\
& YZ3[z, t] \\
& A4''[t]^3 - 2 \left(-1 + e^{-2 A4[t]} \right) \\
& Q1 \\
& YZ3[z, t] \\
& A4'[t]^2 A4^{(4)}[t] \\
& t] + 4 \left(-2 \right. \\
& \left. A4'[t]^2 \left(-9 + M^2 + \frac{1}{2} e^{-2 A4[t]} \right. \right. \\
& \left. \left. (-2 + Q1) \right) \right. \\
& Q1 \\
& A4'[t]^2 - \frac{1}{4} e^{-4 A4[t]} \\
& Q1^2 \\
& A4'[t]^2 + \\
& \left(1 - \frac{Q1^2}{4} - \operatorname{Coth}[A4[t]] + \operatorname{Csch}[A4[t]]^2 \right) \\
& A4'[t]^2 \left. \right) - \\
& \left(-1 - Q1 + e^{-2 A4[t]} Q1 + \operatorname{Coth}[A4[t]] \right) A4'[t]^2 A4''[t] -
\end{aligned}$$

$$\begin{aligned}
& \frac{2}{A4''[t]^2} \\
& YZ3^{(0,2)}[z, t] + 8 A4'[t] \left((-1 + \operatorname{Coth}[A4[t]]) \right. \\
& \quad \left. A4'[t]^2 + A4''[t] \right) YZ3^{(0,3)}[z, t] + A4''[t]^2 \\
& \left(-e^{-4 A4[t]} \right. \\
& \quad Q1^2 \\
& \quad YZ3[z, t] \\
& \quad A4'[t]^2 - 8 YZ3[z, t] \\
& \quad \left. \left(-9 + \frac{1}{8} M^2 + Q1^2 \right. \right. \\
& \quad \left. \left. A4'[t]^2 \right) - 4 \right. \\
& Q1 A4'[t] YZ3^{(0,1)}[z, t] + 2 e^{-2 A4[t]} \\
& Q1 \\
& A4'[t] \\
& (Q1 YZ3[z, t] \\
& \quad A4'[t] + 2 YZ3^{(0,1)}[z, t]) + \\
& 288 (2 \operatorname{Tan}[z] + \operatorname{Tan}[z]^3) YZ3^{(1,0)}[z, t] \\
& \left. \left. - 4 A4'[t] A4''[t] \left(-\frac{1}{4} e^{-6 A4[t]} \right. \right. \right. \\
& \quad Q1^3 \\
& \quad YZ3[z, t] \\
& \quad A4'[t]^3 + 3 e^{-A4[t]}
\end{aligned}$$

Q1
 $\text{Csch}[\text{A4}[t]] \text{YZ3}[z, t] \text{A4}'[t]^3 - \text{YZ3}[z, t]$
 $\text{A4}'[t] \left((-3 + M) (3 + M) (1 + Q1 - \text{Coth}[\text{A4}[t]]) - \frac{1}{4} \frac{Q1^3}{\text{A4}'[t]^2} \right) +$
 $\frac{3}{4} e^{-4 \text{A4}[t]} Q1^2 \text{A4}'[t]^2 \left(\left(\frac{2}{3} + Q1 \right) \text{YZ3}[z, t] \text{A4}'[t] - \frac{2}{3} \text{YZ3}^{(0,1)}[z, t] \right) +$
 $\left(18 - 2 M^2 - \frac{1}{2} Q1^2 \text{A4}'[t]^2 \right) \text{YZ3}^{(0,1)}[z, t] +$
 36
 $(1 + Q1 - \text{Coth}[\text{A4}[t]])$
 $\text{Tan}[z] (2 + \text{Tan}[z]^2) \text{A4}'[t] \text{YZ3}^{(1,0)}[z, t] + e^{-2 \text{A4}[t]}$
 $Q1 \text{A4}'[t] \left(\text{YZ3}[z, t] \left(-9 + M^2 + \frac{1}{2} \left(-Q1 - \frac{3 Q1^2}{2} \right) \text{A4}'[t]^2 \right) + (-2 + Q1) \text{A4}'[t] \text{YZ3}^{(0,1)}[z, t] + 36 (-2 \text{Tan}[z] - \text{Tan}[z]^3) \text{YZ3}^{(1,0)}[z, t] \right) -$

$$\begin{aligned}
& 4 A4'[t]^2 \left(-\frac{1}{4} e^{-6 A4[t]} (-2 + Q1) \right. \\
& Q1^3 \\
& YZ3[z, t] \\
& A4'[t]^4 + \\
& \frac{1}{16} e^{-8 A4[t]} \\
& Q1^4 \\
& YZ3[z, t] \\
& A4'[t]^4 + \\
& YZ3[z, t] \\
& \left. \left((-3 + M)^2 (3 + M)^2 + \right. \right. \\
& \left. \left. \left(-18 + 2 M^2 + \frac{9 Q1^2}{2} + \frac{M^2 Q1^2}{2} + 18 \operatorname{Coth}[A4[t]] - \right. \right. \right. \\
& \left. \left. \left. 2 M^2 \operatorname{Coth}[A4[t]] - 18 \operatorname{Csch}[A4[t]]^2 + \right. \right. \right. \\
& \left. \left. \left. 2 M^2 \operatorname{Csch}[A4[t]]^2 \right) A4'[t]^2 + \right. \\
& \left. \left. \left(\frac{Q1^4}{16} - Q1 \operatorname{Csch}[A4[t]]^2 \right) A4'[t]^4 \right) - 2 (-3 + M) (3 + M) \\
& (-1 + \operatorname{Coth}[A4[t]]) A4'[t] YZ3^{(0,1)}[z, t] + \\
& 2 e^{-A4[t]} \\
& Q1 \\
& \operatorname{Csch}[A4[t]] \\
& A4'[t]^3 YZ3^{(0,1)}[z, t] - 72 \\
& \operatorname{Tan}[z] \\
& \left(-90 + \right. \\
& \left. 2 M^2 + (-405 + M^2) \operatorname{Tan}[z]^2 - 594 \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Tan}[z]^4 - 270 \\
& \operatorname{Tan}[z]^6 - \\
& \frac{1}{4} (-4 + Q1^2 + 4 \operatorname{Coth}[A4[t]] - 4 \operatorname{Csch}[A4[t]]^2) \\
& (2 + \operatorname{Tan}[z]^2) \\
& A4'[t]^2 \\
& YZ3^{(1,0)}[z, t] - e^{-2A4[t]} Q1 A4'[t]^2 \\
& \left(YZ3[z, t] \right. \\
& \left. \left(-18 + 2M^2 + 9Q1 + M^2Q1 + \right. \right. \\
& \left. \left(-Q1 - \frac{Q1^2}{2} + \frac{Q1^3}{4} + \operatorname{Csch}[A4[t]]^2 \right) A4'[t]^2 \right) + \\
& Q1 A4'[t] YZ3^{(0,1)}[z, t] + 36(-2 + Q1) \operatorname{Tan}[z] \\
& (2 + \operatorname{Tan}[z]^2) \\
& YZ3^{(1,0)}[z, t] \Big) + \\
& \frac{1}{2} e^{-4A4[t]} Q1^2 A4'[t]^2 \left(YZ3[z, t] \left(9 + M^2 + \left(-2Q1 + \frac{3Q1^2}{4} \right) A4'[t]^2 \right) + \right. \\
& \left. \left. 2 A4'[t] \right. \right. \\
& \left. YZ3^{(0,1)}[z, t] + 36 \right. \\
& \left. (2 \operatorname{Tan}[z] + \operatorname{Tan}[z]^3) \right. \\
& \left. YZ3^{(1,0)}[z, t] \right) \Big) - \\
& 288 \operatorname{Tan}[z] (2 + \operatorname{Tan}[z]^2) A4'[t] \\
& ((-1 + \\
& \operatorname{Coth}[\\
& A4[t]]) A4'[\\
& t]^2 + \\
& A4''[t]) YZ3^{(1,1)}[z, t] + 288 \operatorname{Tan}[z] (2 + \\
& \operatorname{Tan}[\\
& z]^2) \\
& A4'[t]^2 YZ3^{(1,2)}[z, t] + 288 \operatorname{Tan}[\\
& z]^2 \\
& \left(A4'[t]^2 \left(-261 + M^2 - 504 \operatorname{Tan}[z]^2 - \right. \right. \\
& \left. \left. 270 \right. \right. \\
& \left. \left. \operatorname{Tan}[z]^4 + \frac{1}{2} \right. \right)
\end{aligned}$$

$$\begin{aligned}
& e^{-2 A4[t]} \\
& (-2 + Q1) \\
& Q1 \\
& A4'[t]^2 - \frac{1}{4} \\
& e^{-4 A4[t]} \\
& Q1^2 \\
& A4'[t]^2 + \\
& \left(1 - \frac{Q1^2}{4} - \operatorname{Coth}[A4[t]] + \operatorname{Csch}[A4[t]]^2 \right) \\
& A4'[t]^2 \Bigg) + \\
& \frac{1}{2} \left(-1 - Q1 + e^{-2 A4[t]} Q1 + \operatorname{Coth}[A4[t]] \right) \\
& A4' [\\
& t]^2 A4''[t] + \\
& A4''[t]^2 \right) YZ3^{(2,0)}[z, t] + 4 A4'[t] A4^{(3)}[t] \\
& \left((-9 + M^2) YZ3[\right. \\
& z, \\
& t] + \frac{1}{4} \\
& Q1^2 YZ3[\\
& z, \\
& t] \\
& A4'[t]^2 + \\
& \frac{1}{4} \\
& e^{-4 A4[t]} \\
& Q1^2 \\
& YZ3[\\
& z, \\
& t] \\
& A4'[t]^2 + \\
& \frac{3}{2} \\
& (-1 + \\
& e^{-2 A4[t]}) \\
& Q1 YZ3[z, t] A4''[t] + Q1 A4' [\\
& t] \\
& YZ3^{(0,1)}[\\
& z,
\end{aligned}$$

$$\begin{aligned}
& t] + \\
& e^{-2 A4[t]} \left(\left(Q1 - \frac{Q1^2}{2} \right) YZ3[z, t] A4'[t]^2 - \right. \\
& Q1 \\
& A4'[t] \\
& \left. YZ3^{(0,1)}[z, t] \right) + \\
& YZ3^{(0,2)}[z, t] + 36 (-2 \operatorname{Tan}[z] - \operatorname{Tan}[z]^3) \\
& YZ3^{(1,0)}[\\
& z, \\
& t] - 36 \\
& \operatorname{Tan}[z]^2 \\
& YZ3^{(2,0)}[\\
& z, \\
& t] \Big) - \\
& 288 \operatorname{Tan}[z]^2 A4'[t] ((-1 + \operatorname{Coth}[A4[t]]) \\
& A4'[\\
& t]^2 + \\
& A4''[t]) YZ3^{(2,1)}[z, t] + 288 \operatorname{Tan}[\\
& z]^2 A4'[\\
& t]^2 \\
& YZ3^{(2,2)}[z, t] - 31104 \\
& \operatorname{Tan}[\\
& z]^3 \left(\frac{4}{3} + \right. \\
& \left. \operatorname{Tan}[z]^2 \right) A4'[t]^2 \\
& YZ3^{(3,0)}[\\
& z, \\
& t] - \\
& 5184 \\
& \operatorname{Tan}[\\
& z]^4 A4'[\\
& t]^2 YZ3^{(4,0)}[z, t] \Big)
\end{aligned}$$

```
time1 = Now
If[FindFile["solvedEQ2.mx"] === $Failed,
 AbsoluteTiming[solvedEQ2 = ConvertMapleToMathematicaV2[maplestringEQ2]];
 DumpSave[ToString[header <> "solvedEQ2.mx"], solvedEQ2];
 Print[ToString[header <> "solvedEQ2.mx"]];
 , Get[ "solvedEQ2.mx"]]
Now - time1
```