Scratch work of Author

NOTES

Bigger Bang:
Proof that Universe (s) of masses ± M are created in pairs.

Scratch work

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Do not read if you are irascible; I apologize for typos.
git clone https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git
"git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git"

Ever wonder why there are 3 space dimensions and 1 time dimension?
Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in$ Nature?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\|=\|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over $\mathbb{R}$) real numbers $\mathbb{R}$,the 2-dim complex numbers,the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the

8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in "A Remarkable Representation of the 3+2 de Sitter Group" by P.A.M.Dirac, J.Math.-Phys.4,901–909 (1963).

Here, Dirac's gamma matrices have been extended into this paper's real tau8 8×8 and tau16 = τ 16×16 matrices, in order to simply incorporate and
couple the WAVE FUNCTION OF the UNIVERSE to Gravity via a term resembling
ψ^† (mass matrix for universe) ψ :
(mass matrix for universe) ~ (τ0 τ1 τ2 τ3)(τ5 τ6 τ7),
[τ5 τ6 τ7 breaks "x5, x6, x7" Lorentz invariance].

Here we use spacetime coordinates that are selected so that we are somewhat consistent with the
xact libraries,
(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):
Cartesian coordinates:
x0 = hidden space (a small circle); x1, x2, x3 are the usual 3-space coords;
x4 = time coord,
and x5, x6, x7= superluminal deflating time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{SO(4, 4; \mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field

equations on a spacetime M8 , whose tangent bundle has
$SO(4, 4; \mathbb{R}) \approx$ Spin(4,4;ℝ)
as iso group, and in which
3 of the 4 space dimensions superluminally INFLATE,
3 of the 4 time dimensions superluminally DEFLATE (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),
and the 4th space dim, x0, curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

In passing we remark that an analogous construction may be defined on an octonion space of one time dimension and seven space dimensions, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group
$SU(3) \otimes SU(2)_L \otimes U(1)_Y$;
The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

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The Einstein - Lovelock vacuum field equations are
$A^b_a = 0$, where

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has bee
completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta^{hh_1 \cdots h_{2k}}_{jj_1 \cdots j_{2k}} R^{j_1 j_2}{}_{h_1 h_2} \cdots R^{j_{2k-1} j_{2k}}{}_{h_{2k-1} h_{2k}} + \lambda \sqrt{g}\, g^{lh}, \quad (4.3$$

where $\alpha_{(k)}$, λ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd:} \end{cases}$$

and $m - 1 = \frac{8}{2} - 1 = 3$.

Citation : Tensors, Differential Forms,
and Variational Principles (Dover Books on Mathematics),
by David Lovelock and Hanno Rund

Let {w1, w2, w3, Λ} be pure numbers;
then by Eq. [4.38] the vacuum equations are
$0 == -\Lambda + H^{-2}\ w1\ \text{Lovelock1} + H^{-4}\ w2\ \text{Lovelock2} + H^{-6}\ w3\ \text{Lovelock3}$
in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein - Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre - Hubble parameter. For now, let's pretend that they are independent.

NOTES :

Under the spacetime coordinate transformation

$x^k \mapsto \overline{x}^k \left(x^j\right)$

according to

$$\Psi 16\,(x) \;\mapsto\; \overline{\Psi 16\,(\overline{x})} \;=\; S\,(\,x\,(\overline{x})\,)\,.\Psi 16\,(\,x\,(\overline{x})\,)\;,\ \text{i.e.,}$$

$$\overline{\Psi 16^{(a)}\,(\overline{x})} \;=\; \left(\, S^{(a)}_{\ (b)}\,\Psi 16^{(b)}\,(\,x\,(\overline{x})\,)\,\right)$$

Therefore,

$$\frac{\partial}{\partial \overline{x}^k}\,\overline{\Psi 16\,(\overline{x})} \;=\; \frac{\partial x^j}{\partial \overline{x}^k}\,\frac{\partial}{\partial x^j}\,S\,(x)\,.\Psi 16\,(x) \;=\; \frac{\partial x^j}{\partial \overline{x}^k}\,\left\{\, s\,(x)\,.\,\frac{\partial}{\partial x^j}\,\Psi 16\,(x) + \left[\,\frac{\partial}{\partial x^j}\,s\,(x)\,\right].\Psi 16\,(x)\,\right\}$$

Identify the Octad connection : $\frac{\partial}{\partial x^j}\,s\,(x) \;=\; -\,s\,.\,\Gamma_j\,(\,x\,) + \frac{\partial \overline{x}^k}{\partial x^j}\,\overline{\Gamma_k}\,(\overline{x})\,.\,s$

hence

$$\frac{\partial}{\partial \overline{x}^k}\,\overline{\Psi 16\,(\overline{x})} - \overline{\Gamma_k}\,.\,\overline{\Psi 16\,(\overline{x})} \;=\; S\,(\,x\,(\overline{x})\,)\,.\,\frac{\partial x^j}{\partial \overline{x}^k}\,\left\{\,\frac{\partial}{\partial x^j}\,\Psi 16\,(\,x\,(\overline{x})\,) - \Gamma_j\,.\Psi 16\,(\,x\,(\overline{x})\,)\,\right\}$$

or

$$\left(\, 1_{16\times 16}\,\frac{\partial}{\partial \overline{x}^k} - \overline{\Gamma_k}\,\right).\,\overline{\Psi 16\,(\overline{x})} \;=$$

$$S(x(\overline{x})) \cdot \left[\frac{\partial x^j}{\partial \overline{x}^k} \left\{ 1_{16\times 16} \frac{\partial}{\partial x^j} - \Gamma_j \right\} \cdot \Psi 16(x(\overline{x})) \right]$$

The connection transforms according to

$$\overline{\Gamma_k}(\overline{x}) = \frac{\partial \overline{x}^j}{\partial x^k} \left\{ S \cdot \Gamma_j(x) \cdot S^{-1} + S_{,j} \cdot S^{-1} \right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2} \left[e_k^{(a)} \nabla_j e^k_{(b)} \right] SAB^{(b)}{}_{(a)}$$

Here, spin connection

coefficients are $e_k^{(a)} \left(\nabla_j e^k_{(b)} \right)$

and

$$\mathbf{S = exp}\left(\frac{1}{2} \omega_{(A)(B)}(x) \; SAB^{(A)(B)} \right),$$

where $\omega_{(B)(A)} = -\omega_{(A)(B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the Ψ16 field [A.K.A., wave function characterizing the Universe(s) of masses ±M];

TODO: prove **Universe(s) of masses ±M are created in pairs!**

K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility**: The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
  - $\nabla_\mu e^a{}_\nu = 0$
  - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

In[ ]:=
```
I lost the citation for this :
```

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J}\, e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistancy" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (neverthless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will neverthless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

SOURCE terms $TU^{\mu\nu}$ for $g_{\alpha\beta}$ (hope that $TU^{\mu\nu} = \Lambda g^{\mu\nu}$) ;

Universes Ψ16 source $g_{\alpha\beta}$;
WARNING: all off–diagonal terms MUST BE ZERO in order for the 'solutions,' TODO: find them, to be valid

```
Lg[] = LagrangianF16massive[3 H M, 134]
  (please see previous notebook)
```

$$\sqrt{\mathrm{Det}[g_{\mu\nu}]} * \mathrm{LagrangianF16massive}[\mathrm{mASs_}, \mathrm{j_}] = \sqrt{\mathrm{Det}[g_{\mu\nu}]} * \left(\mathrm{Transpose}[\Psi 16].\sigma 16.\mathrm{Sum}\left[\mathrm{T16}^{\alpha}[\alpha 1-1].\left\{1_{16\times 16}\frac{\partial}{\partial x^{\alpha 1-1}} - \Gamma_{\alpha 1-1}\right\}.\Psi 16, \{\alpha 1, 1, \mathrm{Length}[X]\}\right] + \frac{\mathrm{mASs}}{2} * \mathrm{Transpose}[\Psi 16].\mathrm{symm16}[[134, 1]].\Psi 16\right)$$

Let j = 134,

$\sigma 16.(\text{mass Matrix}) = \mathrm{T16}^{\alpha}[5].\mathrm{T16}^{\alpha}[6].\mathrm{T16}^{\alpha}[7]$

and

usingLagrangianF16massive =

$$\left(\mathrm{Transpose}[\Psi 16].\sigma 16.\mathrm{Sum}[\mathrm{T}^{\alpha}[\alpha 1-1].\mathrm{D}[\Psi 16, \mathrm{vars}[[\alpha 1]]], \{\alpha 1, 1, \mathrm{Length}[\mathrm{vars}]\}] + \mathrm{mASs} * \mathrm{Transpose}[\Psi 16].\mathrm{symm16}[[j = 134, 1]].\Psi 16\right);$$

where it is assumed that $\Psi 16$ solves the Euler - Lagrange equations, above;

then usingLagrangianF16massive = 0,

as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] (usingLagrangianF16massive ≡ Lg[] |Ψ16=solution-toEL-eqs) :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\ \mathrm{LagrangianF16massive} \right)$$

$$= \left(\mathrm{LagrangianF16massive} * \left[\frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]} \right) \right] + \frac{\partial}{\partial g_{\mu\nu}} \left(\mathrm{LagrangianF16massive} \right) \right)$$

| LagrangianF16massive=usingLagrangianF16massive

$$= \left(0\,? * \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]} \right) + \frac{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \right.$$

$$\frac{\partial}{\partial g_{\mu\nu}} (\text{LagrangianF16massive}) \Big|_{\text{LagrangianF16massive=usingLagrangianF16massive}}$$

$$= \frac{\partial}{\partial g_{\mu\nu}} \Big(\text{Transpose}[\Psi 16].\sigma 16.\text{Sum}\Big[\Big(D[\Psi 16, X[\![\alpha 1]\!]] + \Big(\Big(\frac{1}{2}\Big)\text{connectionMatrix}\Big).\text{Sum}[\omega[\![\alpha 1, a, b]\!] * SAB[\![a, b]\!], \{a, 1, 8\}, \{b, 1, 8\}].\Psi 16\Big), \{\alpha 1, 1, \text{Length}[X]\}\Big] + mASs * \text{Transpose}[\Psi 16].\text{symm16}[\![j, 1]\!].\Psi 16\Big) \Big|_{\text{LagrangianF16massive=usingLagrangianF16massiv}}$$

$$= \text{Transpose}[\Psi 16].\sigma 16.\text{Sum}\Big[\frac{\partial}{\partial g_{\mu\nu}} (T16^{\alpha}[\alpha 1 - 1]) . \Psi 16_{\alpha 1-1}, \{\alpha 1, 1, \text{Length}[X]\}\Big] \Big|_{\Psi 16=\text{solution-toEL-eqs}}$$

$$= \tilde{\Psi}16 .\sigma 16. T16^{A} . \Psi 16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1\,\alpha\beta} e^{B}_{\beta} \eta_{BA} \right) =$$

$$\tilde{\Psi}16 \,.\, \sigma 16 \,.\, T16^{A} \,.\, \Psi 16_{,\alpha} \quad \frac{\partial}{\partial\, g_{\mu\nu}} \left(e^{\alpha}_{(A)} \right)$$

NOTES :

Let g be a square matrix;

we could use : $\frac{\partial}{\partial\, q} \left(g^{-1} \right) = - g^{-1} . \frac{\partial\, g}{\partial\, q} . g^{-1}$,

where q is a parameter

$\left(\text{result from } \frac{\partial}{\partial\, q} \left(g . g^{-1} \right) = 0 \right)$,

In this notebook, g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] , g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial\, g_{\mu\nu}} \left(g^{-1} \right) = - g^{-1} . \frac{\partial\, g}{\partial\, g_{\mu\nu}} . g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In metric matrix g, we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$, and then differentiate, for example.

This should be understood before reading further.

Therefore $\frac{\partial}{\partial g_{\mu\nu}}\left(e^{\alpha}_{(A)}\right) =$

$$\frac{\partial}{\partial g_{\mu\nu}}\left(\text{Inverse}\left[e^{(A)}_{\alpha}\right]\right) =$$

$$-\text{Inverse}\left[e^{(A)}_{\alpha}\right].\frac{\partial e^{(A)}_{\alpha}}{\partial g_{\mu\nu}}.\text{Inverse}\left[e^{(A)}_{\alpha}\right] =$$

$$-e^{\alpha}_{(A)}[\![\alpha 1, A1]\!]\frac{\partial e^{(A)}_{\alpha}[\![A1, \alpha 2]\!]}{\partial g_{\mu\nu}}.e^{\alpha}_{(A)}[\![\alpha 2, A2]\!]$$

Consider $\frac{\partial}{\partial g_{\mu\nu}}\left(g_{\alpha\beta}\right) = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e^{(A)}_{\alpha}\, e^{B}_{\beta}\right) =$

$$\eta_{AB}\, e^{B}_{\beta}\frac{\partial}{\partial g_{\mu\nu}}\left(e^{(A)}_{\alpha}\right) + \eta_{AB}\, e^{B}_{\alpha}\frac{\partial}{\partial g_{\mu\nu}}\left(e^{A}_{\beta}\right);$$

$$e^{\beta}_{C}\frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e^{(A)}_{\alpha}\, e^{B}_{\beta}\right) =$$

$$\eta_{AC}\frac{\partial}{\partial g_{\mu\nu}}\left(e^{(A)}_{\alpha}\right) + \eta_{AB}\, e^{B}_{\alpha}\, e^{\beta}_{C}\frac{\partial}{\partial g_{\mu\nu}}\left(e^{A}_{\beta}\right)$$

In metric matrix g,

we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate.

Some of my calculations employ "http://www.xact.es/download-/xAct_1.2.0.tgz" ; also, see
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Institut d'Astrophysique de Paris (France)
pitrou@iap.fr
http://www2.iap.fr/users/pitrou/
"FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb"
MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify

Universe sources for $g_{\alpha\beta}$;
be sure to append these to the Einstein and/or Einstein–Lovelock field equations.
WARNING: all off–diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous 'hacked solutions' for Ψ16 to be valid

Let T^{α} = {$T16^{\alpha}$[0], $T16^{\alpha}$[4]} and

```
√Det[g_μν] * LagrangianF16massive[mASs_, j_] =
  √Det[g_μν] * ( Transpose[Ψ16].σ16.
      Sum[T^α[α1 - 1].D[ Ψ16, vars[[α1]]], {α1, 1, Length[vars]}] +
    mASs * Transpose[Ψ16].symm16[[134, 1]].Ψ16 )
```

Let j = 134, σ16.(mass Matrix) = $T16^{\alpha}$[5].$T16^{\alpha}$[6].$T16^{\alpha}$[7]
and

```
usingLagrangianF16massive = ( Transpose[Ψ16].σ16.
      Sum[T^α[α1 - 1].D[ Ψ16, vars[[α1]]], {α1, 1, Length[vars]}] +
    mASs * Transpose[Ψ16].symm16[[j = 134, 1]].Ψ16);
```

where it is assumed that Ψ16 solves the Euler - Lagrange equations, above; then usingLagrangianF16massive = 0, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\ \mathrm{LagrangianF16massive} \right)$$

$$= \left(\mathrm{LagrangianF16massive} * \left[\frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]} \right) \right] + \frac{\partial}{\partial g_{\mu\nu}} \left(\mathrm{LagrangianF16massive} \right) \right)$$

LagrangianF16massive=usingLagrangianF16massive

$$= \left(0 * \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]} \right) + \frac{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\mathrm{LagrangianF16massive} \right) \right)$$

LagrangianF16massive=usingLagrangianF16massive

remark: the term

$$\tilde{\Psi}16 \,.\sigma16.\, T16^{A} \,.\, \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(e^{\alpha}_{A}\right) = \frac{\partial}{\partial g_{\mu\nu}} \tilde{\Psi}16 \,.\sigma16.\, T16^{A} \,.\, \Psi16_{,\alpha}\, e^{\alpha}_{A} =$$

$$\frac{\partial}{\partial g_{\mu\nu}} \Bigg(\text{Transpose}[\Psi16].\sigma16.$$

$$\text{Sum}\Big[T^{\alpha}[\overbrace{\alpha1-1].D[\Psi16, \text{vars}[\![\alpha1]\!]]}^{T16^{A}[A1-1].\Psi16_{,\alpha1}\,\eta_{A1\,B}\,e^{B}_{\beta}\,g^{-1\,\alpha1\,\beta}}, \{\alpha1, 1, \text{Length}[\text{vars}]\}\Big]$$

Incomplete theories [show]
Scientists [show]
v • t • e

Frame fields [edit]

We use a set of vierbein or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab} e^a_\mu e^b_\nu = \eta_{\mu\nu}.$$

The vierbein defines a local rest frame, allowing the constant Gamma matrices to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a section of the frame bundle, and so defines a local trivialization of the frame bundle.

Spin connection [edit]

To write down the equation we also need the spin connection, also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e^\mu_a e^a_\nu = \delta^\mu{}_\nu.$$

The connection 1-form is then

$$\omega^\mu{}_{\nu a} := e^\mu_b \nabla_a e^b_\nu$$

where ∇_a is a covariant derivative, or equivalently a choice of connection on the frame bundle, most often taken to be the Levi-Civita connection.

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space $T_p M$ to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

Raising and lowering indices is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract connection on a principal bundle, specifically on the frame bundle, which is defined on any smooth manifold, but which restricts to an *orthonormal* frame bundle on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e^\alpha_\mu \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu{}_{\nu\rho} = e^\alpha_\rho \omega^\mu{}_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2}(\omega_{\nu\rho})_\mu \sigma^{\nu\rho}\psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu R(M^{\nu\rho})v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $\mathrm{SO}(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

Begin

In[ ]:=
```
Print["CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]
```

CopyRight (C) 2022, Patrick L. Nash, under the General Public License.

In[ ]:=
```
Print["Please  cite  this  work,  and this web page, if you use it"]
```

Please cite this work, and this web page, if you use it

In[ ]:=
```
Print["For simplicity, metric  gαβ=  gαβ(x0,x1,x2,x3,x4,x5,x6,x7)= gαβ(x0,x4) = diag{
   g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4) },"]
```

For simplicity, metric $g_{\alpha\beta}$= $g_{\alpha\beta}$(x0,x1,x2,x3,x4,x5,x6,x7)= $g_{\alpha\beta}$(x0,x4) = diag{ g_{00}(x0),g_{11}(x0,x4),g_{11}(x0,x4),g_{11}(x0,x4),-1, g_{77}(x0,x4),g_{77}(x0,x4),g_{77}(x0,x4) },

In[ ]:=
```
Print["where g77(x0,x4) = g11(x0,-x4) "]
```

where g_{77}(x0,x4) = g_{11}(x0,-x4)

In[ ]:=
```
Unprotect[dir];
```

In[ ]:=
```
dir = FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]];
Protect[dir];
If[$Failed == SetDirectory[dir], {CreateDirectory[dir], SetDirectory[dir]}];
```

In[∘]:=
```
names = FileNameSplit[NotebookFileName[]]
```
Out[∘]=
```
{C:, Users, nsh, Documents, 8-dim, 2025-12-13-Partitioned-Determinant,
 Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT.nb}
```
In[∘]:=
```
name = StringReplace[names[[-1]], "nb" → "mx"]
```
Out[∘]=
```
Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT.mx
```
In[∘]:=
```
header = StringReplace[names[[-1]], ".nb" → "-"]
```
Out[∘]=
```
Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT-
```
In[∘]:=
```
DIM8 = 8;
```
In[∘]:=
```
Protect[DIM8, M, K, H]
```
Out[∘]=
```
{DIM8, M, K, H}
```

Coordinates :

$$\{\underbrace{\text{hidden space}}, \underbrace{3-\text{space}}, \underbrace{\text{time}} \underbrace{\text{superluminal deflating time}}\}$$

$$\{x0, \overbrace{x1, x2, x3}, x4, \overbrace{x5, x6, x7}\} :$$

In[∘]:=
```
X = {x0, x1, x2, x3, x4, x5, x6, x7};
Protect[X];
Protect[x0, x1, x2, x3, x4, x5, x6, x7];
```
In[∘]:=
```
sX0 = And @@ Thread[X > 0]
```
Out[∘]=
```
x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 && x7 > 0
```
In[∘]:=
```
ssX = H > 0 && sX0 && 6 H x0 > 0 && 2 la[x4] > 0 && Cot[6 H x0] > 0 && Sin[6 H x0] > 0
```
Out[∘]=
```
H > 0 && x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 &&
 x7 > 0 && 6 H x0[] > 0 && 2 la[x4[]] > 0 && Cot[6 H x0[]] > 0 && Sin[6 H x0[]] > 0
```

For Maple_2025 :

In[∘]:=
```
sreplaceZ = Z[#] → ToExpression["Z" <> ToString[#]] & /@ Range[0, 15]
```
Out[∘]=
```
{Z[0] → Z0, Z[1] → Z1, Z[2] → Z2, Z[3] → Z3, Z[4] → Z4, Z[5] → Z5, Z[6] → Z6, Z[7] → Z7, Z[8] → Z8,
 Z[9] → Z9, Z[10] → Z10, Z[11] → Z11, Z[12] → Z12, Z[13] → Z13, Z[14] → Z14, Z[15] → Z15}
```
In[∘]:=
```
sreplacenZ = nZ[#] → ToExpression["nZ" <> ToString[#]] & /@ Range[0, 15]
```
Out[∘]=
```
{nZ[0] → nZ0, nZ[1] → nZ1, nZ[2] → nZ2, nZ[3] → nZ3, nZ[4] → nZ4,
 nZ[5] → nZ5, nZ[6] → nZ6, nZ[7] → nZ7, nZ[8] → nZ8, nZ[9] → nZ9, nZ[10] → nZ10,
 nZ[11] → nZ11, nZ[12] → nZ12, nZ[13] → nZ13, nZ[14] → nZ14, nZ[15] → nZ15}
```

hacks (a.k.a., lies):

In[ ]:=
```
constraintTrig = Sin[6*H*x0] > 0 && Cos[6*H*x0] > 0 && Csc[6*H*x0] > 0 &&
  Sec[6*H*x0] > 0 && Tan[6*H*x0] > 0 && Cot[6*H*x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 &&
  Sin[z]^(3/2) > 0 && Sin[z]^(1/2) > 0 && Sin[z]^(-3/2) > 0 && Sin[z]^(-1/2) > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
  Cot[z]^(3/2) > 0 && Cot[z]^(1/2) > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

Out[ ]=

$$\mathrm{Sin}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Cos}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Csc}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Sec}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Tan}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Cot}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Sin}[z] > 0 \;\&\&\; \sqrt{\mathrm{Sin}[z]} > 0 \;\&\&\; \mathrm{Sin}[z]^{3/2} > 0 \;\&\&\; \sqrt{\mathrm{Sin}[z]} > 0 \;\&\&\; \frac{1}{\mathrm{Sin}[z]^{3/2}} > 0 \;\&\&\; \frac{1}{\sqrt{\mathrm{Sin}[z]}} > 0 \;\&\&\; \mathrm{Cot}[z] > 0 \;\&\&\; \sqrt{\mathrm{Cot}[z]} > 0 \;\&\&\; \mathrm{Cot}[z]^{3/2} > 0 \;\&\&\; \sqrt{\mathrm{Cot}[z]} > 0 \;\&\&\; \mathrm{Tan}[z] > 0 \;\&\&\; \mathrm{Sec}[z] > 0 \;\&\&\; \sqrt{\mathrm{Csc}[z]} > 0 \;\&\&\; \mathrm{Csc}[z] > 0$$

more hacks (damn lies):

In[ ]:=
```
(*&&K^2-M^2>0&& Sqrt[K^2-M^2] >0&&e^(2 H Sqrt[K^2-M^2] x4>0&&2 H Sqrt[K^2-M^2] x4>0&&
 2 H Sqrt[1-M^2] x4>0&& Sqrt[1-M^2] >0&&e^(2 H Sqrt[1-M^2] x4>0&&2 H Sqrt[1-M^2] x4>0*)
```

In[ ]:=
```
constraintX =
 x0 > 0 && x4 > 0 && H > 0 && 6*H *x0 > 0 && 3*H*x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
  z > 0 && t > 0 && M > 0 && K > 0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0 && e^(2 a4[H x4]) > 0 && e^(a4[H x4]) > 0
```

Out[ ]=

$$x0 > 0 \;\&\&\; x4 > 0 \;\&\&\; H > 0 \;\&\&\; 6\,H\,x0 > 0 \;\&\&\; 3\,H\,x4 > 0 \;\&\&\; a4[x4] > 0 \;\&\&\; A4[t] > 0 \;\&\&\; Q > 0 \;\&\&\; z > 0 \;\&\&\; t > 0 \;\&\&\; M > 0 \;\&\&\; K > 0 \;\&\&\; e^{-2\,a4[H\,x4]} > 0 \;\&\&\; e^{-a4[H\,x4]} > 0 \;\&\&\; e^{2\,a4[H\,x4]} > 0 \;\&\&\; e^{a4[H\,x4]} > 0$$

statistics :

In[ ]:=
```
constraintVars = constraintX && constraintTrig
```

Out[ ]=

$$x0 > 0 \;\&\&\; x4 > 0 \;\&\&\; H > 0 \;\&\&\; 6\,H\,x0 > 0 \;\&\&\; 3\,H\,x4 > 0 \;\&\&\; a4[x4] > 0 \;\&\&\; A4[t] > 0 \;\&\&\; Q > 0 \;\&\&\; z > 0 \;\&\&\; t > 0 \;\&\&\; M > 0 \;\&\&\; K > 0 \;\&\&\; e^{-2\,a4[H\,x4]} > 0 \;\&\&\; e^{-a4[H\,x4]} > 0 \;\&\&\; e^{2\,a4[H\,x4]} > 0 \;\&\&\; e^{a4[H\,x4]} > 0 \;\&\&\; \mathrm{Sin}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Cos}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Csc}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Sec}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Tan}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Cot}[6\,H\,x0] > 0 \;\&\&\; \mathrm{Sin}[z] > 0 \;\&\&\; \sqrt{\mathrm{Sin}[z]} > 0 \;\&\&\; \mathrm{Sin}[z]^{3/2} > 0 \;\&\&\; \sqrt{\mathrm{Sin}[z]} > 0 \;\&\&\; \frac{1}{\mathrm{Sin}[z]^{3/2}} > 0 \;\&\&\; \frac{1}{\sqrt{\mathrm{Sin}[z]}} > 0 \;\&\&\; \mathrm{Cot}[z] > 0 \;\&\&\; \sqrt{\mathrm{Cot}[z]} > 0 \;\&\&\; \mathrm{Cot}[z]^{3/2} > 0 \;\&\&\; \sqrt{\mathrm{Cot}[z]} > 0 \;\&\&\; \mathrm{Tan}[z] > 0 \;\&\&\; \mathrm{Sec}[z] > 0 \;\&\&\; \sqrt{\mathrm{Csc}[z]} > 0 \;\&\&\; \mathrm{Csc}[z] > 0$$

In[ ]:=
```
subsDefects =
{Sqrt[e^(2 a4[H x4])] → e^(a4[H x4]), Sqrt[e^(-2 a4[H x4])] → e^(-a4[H x4]), Sqrt[e^(2 a4[H x4]) Sin[6 H x4]^(1/3)] → e^(a4[H x4]) Sin[6 H x4]^(1/6),
 1/Sqrt[e^(2 a4[H x4]) Sin[6 H x4]^(1/3)] → 1/(e^(a4[H x4]) Sin[6 H x4]^(1/6)),
 1/Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/6)] → 1/(e^(a4[H x4]) Sin[6 H x0]^(1/6)),
 1/Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)] → 1/(e^(-a4[H x4]) Sin[6 H x0]^(1/6)), Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/6)] →
 e^(a4[H x4]) Sin[6 H x0]^(1/6), Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)] → e^(-a4[H x4]) Sin[6 H x0]^(1/6)}
```

Out[ ]=

$$\left\{\sqrt{e^{2\,a4[H\,x4]}} \to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}} \to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}} \to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6},\right.$$
$$\frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}} \to \frac{1}{e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6}},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}} \to \frac{1}{e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}},$$
$$\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}} \to \frac{1}{e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}} \to$$
$$\left.e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \sqrt{e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}} \to e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\right\}$$

Out[ ]=

$$\left\{\sqrt{e^{2\,a4[H\,x4]}} \to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}} \to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}} \to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6},\right.$$
$$\frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}} \to \frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x4]^{1/6}},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}} \to \frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},$$
$$\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}} \to \frac{e^{a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}} \to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},$$
$$\left.\sqrt{e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}} \to e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\right\}$$

future coordinate transformation :

In[ ]:=
```
6 H x0 == z && H x4 == t
szt = Solve[%, {z, t}][[1]]
sx0x4 = Solve[%%, {x0, x4}][[1]]
Protect[sx0x4, szt];
```

Out[ ]=

$$6\,H\,x0 == z\ \&\&\ H\,x4 == t$$

Out[ ]=

$$\{z \to 6\,H\,x0,\ t \to H\,x4\}$$

Out[ ]=

$$\left\{x0 \to \frac{z}{6\,H},\ x4 \to \frac{t}{H}\right\}$$

In[ ]:=
```
sx0x4
```

Out[ ]=

$$\left\{x0 \to \frac{z}{6\,H},\ x4 \to \frac{t}{H}\right\}$$

In[ ]:=
```
(*sa4={a4→((A4[3 H #2])&)}*)
```

In[ ]:=
```
Protect[sx0x4, szt]
```

Out[ ]=

$$\{\}$$

SO (4, 4) Minkowski Lorentz metric

η4488 : 4 + 4 spacetime; 8 × 8 dimensional :

In[ ]:=
```
(η4488 = ArrayFlatten[{{IdentityMatrix[4], 0}, {0, -IdentityMatrix[4]}}]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

SO (4, 4) Spinor Lorentz metric σ;

τ^A are analogous to

the Dirac gamma matrices :

all true : Table[{A, B, FullSimplify[

$$\frac{1}{2}\left(\tau[A].\overline{\tau}[B] + \tau[B].\overline{\tau}[A]\right) == \eta 4488[[A+1, B+1]] * ID8]\},$$

{A, 0, 7}, {B, 0, 7}]

Type - 1 and type -

2 employ σ (the ususal notational abuse),

since $\sigma == \sigma^{-1}$: 4 + 4 spacetime,

8 × 8 dimensional and 16 × 16 dimensional :

[illegible] indices. We define the reduced Brauer-Weyl generators $\{\bar{\tau}^{A'}, \tau^{A'}\}$ of the generators of the two real 8×8 inequivalent irreducible representations of $\overline{SO(4,4)}$ (see eqs. (2.6) and (2.7)) by demanding that the tau matrices satisfy (the tilde denotes transpose)

(2.3) $$\tilde{\bar{\tau}}^{A'} \sigma = \sigma \bar{\tau}^{A'}$$

and

(2.4) $$\tau^{A'} \bar{\tau}^{B'} + \tau^{B'} \bar{\tau}^{A'} = 2IG^{A'B'} = \bar{\tau}^{A'} \tau^{B'} + \bar{\tau}^{B'} \tau^{A'},$$

where I denotes the 8×8 unit matrix. Denoting the matrix elements of $\tau^{A'}$ by ${\tau^{A'a}}_b$, we may write eq. (3) as

(2.5) $$\bar{\tau}^{A'}{}_{ab} = \tau^{A'}{}_{ba},$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}{}^{AB} & 0 \\ 0 & D_{(2)}{}^{AB} \end{pmatrix}$$

$$4D_{(1)}{}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}{}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \to SO(4,4)$ is given by

$$8L^{A'}{}_{B'}(g) = \operatorname{tr} \widetilde{D}^{(2)}(g)\, \tau^{A'} D^{(1)}(g)\, \bar{\tau}_{B'} . \tag{2.8}$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$\widetilde{D}^{(1)} \sigma = \sigma D^{(1)-1} , \tag{2.9}$$

$$\sigma^{-1} \widetilde{D}^{(2)} = D^{(2)-1} \sigma^{-1} , \tag{2.10}$$

$$L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'} , \tag{2.11}$$

$$L^{A'}{}_{B'} \tau^{B'} = \widetilde{D}^{(2)} \tau^{A'} D^{(1)} , \tag{2.12}$$

$$L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \widetilde{D}^{(2)-1} . \tag{2.13}$$

In[ ]:=
```
(σ = ArrayFlatten[{{ConstantArray[0, {4, 4}], IdentityMatrix[4]},
    {IdentityMatrix[4], ConstantArray[0, {4, 4}]}}]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

below :

In[ ]:=
```
σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

below :

In[ ]:=
```
Table[T16^A[A1] = ArrayFlatten[
    {{0, τ̄[A1]}, {τ[A1], 0}}], {A1, 0, 7}];
```

below :

In[ ]:=
```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@
 Range[0, 7]
```

Out[ ]=

{True, True, True, True, True, True, True, True}

below :

In[ ]:=

```
Table[{{A1, B1}, FullSimplify[
    ExpandAll[1/2 (T16^A[A1].T16^A[B1] +
          T16^A[B1].T16^A[A1]) ==
      η4488[[A1 + 1, B1 + 1]] * ID16]]},
  {A1, 0, 7}, {B1, 0, 7}] //
 MatrixForm
```

Out[ ]//MatrixForm=

{0, 0} True	{0, 1} True	{0, 2} True	{0, 3} True	{0, 4 Tru
{1, 0} True	{1, 1} True	{1, 2} True	{1, 3} True	{1, 4 Tru
{2, 0} True	{2, 1} True	{2, 2} True	{2, 3} True	{2, 4 Tru
{3, 0} True	{3, 1} True	{3, 2} True	{3, 3} True	{3, 4 Tru
{4, 0} True	{4, 1} True	{4, 2} True	{4, 3} True	{4, 4 Tru
{5, 0} True	{5, 1} True	{5, 2} True	{5, 3} True	{5, 4 Tru
{6, 0} True	{6, 1} True	{6, 2} True	{6, 3} True	{6, 4 Tru
{7, 0} True	{7, 1} True	{7, 2} True	{7, 3} True	{7, 4 Tru

defs; some Symbols; metric

$$\left\{\left\{a4[t] \to \frac{M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} + c_1 + \frac{1}{2}\,\text{ProductLog}\left[-e^{-\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1}-2\,c_1}\right]\right\}\right\}$$

```
DSolve[
 0 == -M (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7]) +
   (Q1 - e^(-2 a4[t]) Q1) a4'[t], a4[t], t]
```

M c[j] = (j + 1)st Energy Eigenvalue for Transpose[cayZ].Ψ16 , j = 0, ..., 7

ProductLog[z]

gives the principal solution for w in $z = we^w$.

if z and w are any complex numbers, then

$$we^w = z$$

holds if and only if

$$w = W_k(z) \quad \text{for some integer } k.$$

When dealing with real numbers only, the two branches W_0 and W_{-1} suffice: for real numbers x and y the equation

$$ye^y = x$$

can be solved for y only if $x \geq \frac{-1}{e}$; yields $y = W_0(x)$ if $x \geq 0$ and the two values $y = W_0(x)$ and $y = W_{-1}(x)$ if $\frac{-1}{e} \leq x < 0$.

$$-\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} - 2\,c_1 == -1$$

$$\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} + 2\,c_1 == 1, \;,\; t = 0 \;\Rightarrow\; c_1 = \frac{1}{2}$$

$$\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} + 1 == 1$$

$$\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} == 0 \;\Rightarrow\; \text{sum Energy Eigenvalues} = \sum_{j=0}^{7} M\,c[j] = 0 \;\overset{?}{\Rightarrow}\; \text{if } +M \text{ then } -M$$

?todo: NEED DefScalarFunction[#]; & /@ (f16[#]&/@Range[0,15]), and allow xact to compute?

Introduce the wave function, Ψ16, for a Universe:

In[ ]:=
```
Ψ16 = f16[#][x0, x4] & /@ Range[0, 15]
```

Out[ ]=
```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

In[ ]:=
```
(*Ψnew16=F16[#][x0,x4]&/@Range[0,15]*)
```

In[ ]:=
```
Clear[sf16Aa];
sfψ16Aa = f16[#] → ToExpression[
    "((Z[" <> ToString[#] <> "][6*H*#1,H*#2] 1/(Sin[6*H*#1]^(1/2)))&)"] & /@ Range[0, 15]
```

Out[ ]=

$$\left\{f16[0] \to \left(\frac{Z[0][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[1] \to \left(\frac{Z[1][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\right.$$

$$f16[2] \to \left(\frac{Z[2][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[3] \to \left(\frac{Z[3][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$

$$f16[4] \to \left(\frac{Z[4][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[5] \to \left(\frac{Z[5][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$

$$f16[6] \to \left(\frac{Z[6][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[7] \to \left(\frac{Z[7][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$

$$f16[8] \to \left(\frac{Z[8][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[9] \to \left(\frac{Z[9][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$

$$f16[10] \to \left(\frac{Z[10][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[11] \to \left(\frac{Z[11][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$

$$f16[12] \to \left(\frac{Z[12][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[13] \to \left(\frac{Z[13][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$

$$\left. f16[14] \to \left(\frac{Z[14][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[15] \to \left(\frac{Z[15][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right)\right\}$$

In[ ]:=
```
Clear[snewfψ16Aa];
snewfψ16Aa = f16[#] → ToExpression[
    "((nZ[" <> ToString[#] <> "][6*H*#1,H*#2] 1/(Sin[6*H*#1]^(1/2))&)"] & /@ Range[0, 15]
```

Out[ ]=

$$\left\{f16[0] \to \left(\frac{nZ[0][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[1] \to \left(\frac{nZ[1][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\right.$$
$$f16[2] \to \left(\frac{nZ[2][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[3] \to \left(\frac{nZ[3][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$
$$f16[4] \to \left(\frac{nZ[4][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[5] \to \left(\frac{nZ[5][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$
$$f16[6] \to \left(\frac{nZ[6][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[7] \to \left(\frac{nZ[7][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$
$$f16[8] \to \left(\frac{nZ[8][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[9] \to \left(\frac{nZ[9][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$
$$f16[10] \to \left(\frac{nZ[10][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[11] \to \left(\frac{nZ[11][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$
$$f16[12] \to \left(\frac{nZ[12][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[13] \to \left(\frac{nZ[13][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),$$
$$\left.f16[14] \to \left(\frac{nZ[14][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right),\ f16[15] \to \left(\frac{nZ[15][6\,H\,\#1,\ H\,\#2]}{\sqrt{Sin[6\,H\,\#1]}}\ \&\right)\right\}$$

0 (4, 4) related :

In[ ]:= `Needs["Notation`"]`

In[ ]:= Symbolize[$\bar{\tau}$]

In[ ]:= Symbolize[$\overline{\text{new}\tau}$]

In[ ]:= Symbolize[$T16^{A}$]

In[ ]:= Symbolize[$T16^{\alpha}$]

In[ ]:= Symbolize[$newT16^{A}$]

In[ ]:= Symbolize[$newT16^{\alpha}$]

NEED DefScalarFunction[#]; & /@ Flatten[Universe]

In[ ]:= Universe = ToExpression["U" <> ToString[#]] & /@ Range[0, 7]

In[ ]:= Symbolize[$\bar{r}^{A}_{a}$]

In[ ]:= Symbolize[$\bar{r}^{a}_{A}$]

In[∘]:= Symbolize[$gtrye_{\alpha}^{(A)}$]

In[∘]:= Symbolize[$gtrye^{\alpha}_{(A)}$]

SPACETIME METRIC

In[∘]:= (*einsteinMetric=Array[g[#1-1,#2-1]&,{8,8}]*)

SPACETIME METRIC:

In[∘]:=
```
g4488 = Array[g[#1 - 1][#2 - 1][x0, x4] &, {8, 8}];
```

SPACETIME METRIC values:

MatrixMetric44

$$6\,H\,x0 == z \;\&\&\; H\,x4 == t$$

$$\left\{\left\{a4''[t] \to 0,\ a4'[t] \to \frac{2}{3}\,(-1+M)\right\},\ \left\{a4''[t] \to 0,\ a4'[t] \to \frac{2\,(1+M)}{3}\right\}\right\}$$

In[∘]:= (*β3=Exp$\left[2\ H\ x4\ \sqrt{K^2-M^2}\ \right]$;*)

(*β3=Exp[2 * a4[3*H*x4]];*)

$\beta 3$ = Exp[2 * a4[H * x4]](*/.$\left\{a4 \to \left(\left(\left(K1 * \frac{2\ (1+M)}{3} + K2 * \frac{2}{3}\ (-1+M)\right)\#\right)\&\right)\right\}$*)

$\beta 1 = \text{Sin}[6 * H * x0]^{\frac{1}{3}}$;

$\beta 2 = \text{Cot}[6 * H * x0]^{2}$;

Out[∘]=

$e^{2\,a4[H\,x4]}$

In[∘]:=
```
MatrixForm[MatrixMetric44 =
  {{β2, 0, 0, 0, 0, 0, 0, 0}, {0, β1 β3, 0, 0, 0, 0, 0, 0}, {0, 0, β1 β3, 0, 0, 0, 0, 0},
   {0, 0, 0, β1 β3, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -β1/β3, 0, 0}, {0, 0,
    0, 0, 0, 0, -β1/β3, 0}, {0, 0, 0, 0, 0, 0, 0, -β1/β3}} // FullSimplify[#, constraintVars] &]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6\,H\,x0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad -e^{-2\,a4[H\,x4]}\ \text{S}$$

In[ ]:=
```
(*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"][6*H*#1,3*H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@
  Range[0,15]*)
```

In[ ]:=
```
(*Clear[sf16Aa];
sfψ16Aa=
 f16[#]→ToExpression["((Z["<>ToString[#]<>"][6*H*#1,3*H*#2])&)"]&/@Range[0,15]*)
```

In[ ]:=
```
(*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"][6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@
  Range[0,15]*)
```

In[ ]:=
```
(*Inactivate[((q)&)]Evaluate[Activate[Evaluate[q]]]*)
```

In[ ]:=
```
Clear[makeSubstitution];
makeSubstitution[] := Module[{met, mat, t, f, p},
  met = Flatten[g4488][[All, 0]];
  mat = Flatten[MatrixMetric44];
  t = Thread[met → mat];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
     expr = Inactivate[((q) &)];
     s = f[[j]][[1]][[1]] → expr;
     r = Activate[s // Evaluate] // Evaluate;
     r], {j, 1, Length[f]}];
  Return[p];
 ]
```

In[ ]:=
```
makeSubstitution[a_, b_] := Module[{t, f, p},
  t = Thread[a → b];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
     expr = Inactivate[((q) &)];
     s = f[[j]][[1]][[1]] → expr;
     r = Activate[s // Evaluate] // Evaluate;
     r], {j, 1, Length[f]}];
  Return[p];
 ]
```

In[∘]:=
```
ssgm4488 = makeSubstitution[]
```

Out[∘]=

$$\{g[0][0] \to (\mathrm{Cot}[6\,H\,\#1]^2\,\&),\ g[0][1] \to (0\,\&),\ g[0][2] \to (0\,\&),\ g[0][3] \to (0\,\&),$$
$$g[0][4] \to (0\,\&),\ g[0][5] \to (0\,\&),\ g[0][6] \to (0\,\&),\ g[0][7] \to (0\,\&),\ g[1][0] \to (0\,\&),$$
$$g[1][1] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[1][2] \to (0\,\&),\ g[1][3] \to (0\,\&),$$
$$g[1][4] \to (0\,\&),\ g[1][5] \to (0\,\&),\ g[1][6] \to (0\,\&),\ g[1][7] \to (0\,\&),\ g[2][0] \to (0\,\&),$$
$$g[2][1] \to (0\,\&),\ g[2][2] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[2][3] \to (0\,\&),\ g[2][4] \to (0\,\&),$$
$$g[2][5] \to (0\,\&),\ g[2][6] \to (0\,\&),\ g[2][7] \to (0\,\&),\ g[3][0] \to (0\,\&),\ g[3][1] \to (0\,\&),$$
$$g[3][2] \to (0\,\&),\ g[3][3] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[3][4] \to (0\,\&),\ g[3][5] \to (0\,\&),$$
$$g[3][6] \to (0\,\&),\ g[3][7] \to (0\,\&),\ g[4][0] \to (0\,\&),\ g[4][1] \to (0\,\&),\ g[4][2] \to (0\,\&),$$
$$g[4][3] \to (0\,\&),\ g[4][4] \to (-1\,\&),\ g[4][5] \to (0\,\&),\ g[4][6] \to (0\,\&),\ g[4][7] \to (0\,\&),$$
$$g[5][0] \to (0\,\&),\ g[5][1] \to (0\,\&),\ g[5][2] \to (0\,\&),\ g[5][3] \to (0\,\&),\ g[5][4] \to (0\,\&),$$
$$g[5][5] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[5][6] \to (0\,\&),\ g[5][7] \to (0\,\&),$$
$$g[6][0] \to (0\,\&),\ g[6][1] \to (0\,\&),\ g[6][2] \to (0\,\&),\ g[6][3] \to (0\,\&),\ g[6][4] \to (0\,\&),$$
$$g[6][5] \to (0\,\&),\ g[6][6] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[6][7] \to (0\,\&),$$
$$g[7][0] \to (0\,\&),\ g[7][1] \to (0\,\&),\ g[7][2] \to (0\,\&),\ g[7][3] \to (0\,\&),\ g[7][4] \to (0\,\&),$$
$$g[7][5] \to (0\,\&),\ g[7][6] \to (0\,\&),\ g[7][7] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&)\}$$

In[∘]:= $\mathrm{Symbolize}[E_\alpha^{(A)}]$

In[∘]:= $\mathrm{Symbolize}[e_\alpha^{(A)}]$

In[∘]:= $e_\alpha^{(A)} = \mathrm{Array}[E_\alpha^{(A)}[\#1-1, \#2-1]\,\&, \{8, 8\}]$

Out[∘]=

$$\{\{E_\alpha^{(A)}[0,0], E_\alpha^{(A)}[0,1], E_\alpha^{(A)}[0,2], E_\alpha^{(A)}[0,3], E_\alpha^{(A)}[0,4], E_\alpha^{(A)}[0,5], E_\alpha^{(A)}[0,6], E_\alpha^{(A)}[0,7]\},$$
$$\{E_\alpha^{(A)}[1,0], E_\alpha^{(A)}[1,1], E_\alpha^{(A)}[1,2], E_\alpha^{(A)}[1,3], E_\alpha^{(A)}[1,4], E_\alpha^{(A)}[1,5], E_\alpha^{(A)}[1,6], E_\alpha^{(A)}[1,7]\},$$
$$\{E_\alpha^{(A)}[2,0], E_\alpha^{(A)}[2,1], E_\alpha^{(A)}[2,2], E_\alpha^{(A)}[2,3], E_\alpha^{(A)}[2,4], E_\alpha^{(A)}[2,5], E_\alpha^{(A)}[2,6], E_\alpha^{(A)}[2,7]\},$$
$$\{E_\alpha^{(A)}[3,0], E_\alpha^{(A)}[3,1], E_\alpha^{(A)}[3,2], E_\alpha^{(A)}[3,3], E_\alpha^{(A)}[3,4], E_\alpha^{(A)}[3,5], E_\alpha^{(A)}[3,6], E_\alpha^{(A)}[3,7]\},$$
$$\{E_\alpha^{(A)}[4,0], E_\alpha^{(A)}[4,1], E_\alpha^{(A)}[4,2], E_\alpha^{(A)}[4,3], E_\alpha^{(A)}[4,4], E_\alpha^{(A)}[4,5], E_\alpha^{(A)}[4,6], E_\alpha^{(A)}[4,7]\},$$
$$\{E_\alpha^{(A)}[5,0], E_\alpha^{(A)}[5,1], E_\alpha^{(A)}[5,2], E_\alpha^{(A)}[5,3], E_\alpha^{(A)}[5,4], E_\alpha^{(A)}[5,5], E_\alpha^{(A)}[5,6], E_\alpha^{(A)}[5,7]\},$$
$$\{E_\alpha^{(A)}[6,0], E_\alpha^{(A)}[6,1], E_\alpha^{(A)}[6,2], E_\alpha^{(A)}[6,3], E_\alpha^{(A)}[6,4], E_\alpha^{(A)}[6,5], E_\alpha^{(A)}[6,6], E_\alpha^{(A)}[6,7]\},$$
$$\{E_\alpha^{(A)}[7,0], E_\alpha^{(A)}[7,1], E_\alpha^{(A)}[7,2], E_\alpha^{(A)}[7,3], E_\alpha^{(A)}[7,4], E_\alpha^{(A)}[7,5], E_\alpha^{(A)}[7,6], E_\alpha^{(A)}[7,7]\}\}$$

In[∘]:= $(*\mathrm{Symbolize}[se_\alpha^{(A)}]*)$

In[∘]:= $\mathrm{Symbolize}[sge_\alpha^{(A)}]$

In[∘]:= $\mathrm{Symbolize}[sgtrye_\alpha^{(A)}]$

In[∘]:= $\mathrm{Symbolize}[E^\alpha_{(A)}]$

In[∘]:= $\mathrm{Symbolize}[sge^\alpha_{(A)}]$

In[∘]:= $\mathrm{Symbolize}[sgtrye^\alpha_{(A)}]$

In[∘]:= $\mathrm{Symbolize}[e^\alpha_{(A)}]$

In[∘]:=
```
e^α_(A) = Array[E^α_(A)[#1 - 1, #2 - 1] &, {8, 8}]
```

Out[∘]=
```
{{E^α_(A)[0, 0], E^α_(A)[0, 1], E^α_(A)[0, 2], E^α_(A)[0, 3], E^α_(A)[0, 4], E^α_(A)[0, 5], E^α_(A)[0, 6], E^α_(A)[0, 7]},
 {E^α_(A)[1, 0], E^α_(A)[1, 1], E^α_(A)[1, 2], E^α_(A)[1, 3], E^α_(A)[1, 4], E^α_(A)[1, 5], E^α_(A)[1, 6], E^α_(A)[1, 7]},
 {E^α_(A)[2, 0], E^α_(A)[2, 1], E^α_(A)[2, 2], E^α_(A)[2, 3], E^α_(A)[2, 4], E^α_(A)[2, 5], E^α_(A)[2, 6], E^α_(A)[2, 7]},
 {E^α_(A)[3, 0], E^α_(A)[3, 1], E^α_(A)[3, 2], E^α_(A)[3, 3], E^α_(A)[3, 4], E^α_(A)[3, 5], E^α_(A)[3, 6], E^α_(A)[3, 7]},
 {E^α_(A)[4, 0], E^α_(A)[4, 1], E^α_(A)[4, 2], E^α_(A)[4, 3], E^α_(A)[4, 4], E^α_(A)[4, 5], E^α_(A)[4, 6], E^α_(A)[4, 7]},
 {E^α_(A)[5, 0], E^α_(A)[5, 1], E^α_(A)[5, 2], E^α_(A)[5, 3], E^α_(A)[5, 4], E^α_(A)[5, 5], E^α_(A)[5, 6], E^α_(A)[5, 7]},
 {E^α_(A)[6, 0], E^α_(A)[6, 1], E^α_(A)[6, 2], E^α_(A)[6, 3], E^α_(A)[6, 4], E^α_(A)[6, 5], E^α_(A)[6, 6], E^α_(A)[6, 7]},
 {E^α_(A)[7, 0], E^α_(A)[7, 1], E^α_(A)[7, 2], E^α_(A)[7, 3], E^α_(A)[7, 4], E^α_(A)[7, 5], E^α_(A)[7, 6], E^α_(A)[7, 7]}}
```

In[∘]:=
```
(*gtrye^(A)_α = (e^(A)_α /.sgtrye^(A)_α)*)
```

In[∘]:=
```
(*gtrye^α_(A) = (e^α_(A) /.sgtrye^α_(A))*)
```

In[∘]:=
```
(*Symbolize[Γ^κ_α β]*)
```

In[∘]:=
```
(*preSpinConnection=Array[Γ^κ_α β[##]&,{8,8,8}];*)
```

In[∘]:=
```
(*Symbolize[ E^A_a ]*)
```

In[∘]:=
```
(*Symbolize[ E^a_A ]*)
```

In[∘]:=
```
(* F̄^A_a F̄^a_A *)
```

In[∘]:=
```
(*Do[ F̄^A_a =E^A_a[h]=Table[(u[[h]].σ.(τ̄[B])),{B,1,8}],{h,1,Length[u]}];
Do[E^a_A[h]=FullSimplify[Inverse[E^A_a[h]]],{h,1,Length[u]}];
Table[ F̄^a_A =E^a_A[h]== (u[[h]].σ.u[[h]])
    Transpose[Table[FullSimplify[ExpandAll[ η8[[B,B]]τ[B].u[[h]] ]],
      {B,1,8}]],{h,1,Length[u]}]*)
```

In[∘]:=
```
(*Table[FullSimplify[ExpandAll[Transpose[E^A_a[h]].η8.E^A_a[h]]]-
    (u[[h]].σ.u[[h]])σ== Zero,{h,1,Length[u]}]*)
```

In[∘]:=
```
(*iη88=FullSimplify[Inverse[η8]];*)
```

In[∘]:=
```
(*Clear[ω^(a)_μ (b)];*)Clear[ω]; Symbolize[ω^(a)_μ (b)]
```

In[∘]:=
```
ω = Array[ω^(a)_μ (b)[##] &, {8, 8, 8}];
Protect[ω]
```

Out[∘]=
```
{ω}
```

constants

In[ ]:=
```
similarityTransformation00 =
 {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

← ERROR : O8similarityTransformation has 1 st index that transforms as $\frac{\partial}{\partial \Psi}$, not as Ψs

In[ ]:=
```
(* O8similarityTransformation has 1st
 index that transforms as ∂/∂Ψ , not as Ψ *)
```

O8similarityTransformation has 1 st index that transforms as $\frac{\partial}{\partial \Psi}$, not as Ψ :

In[∘]:=
```
O8similarityTransformation = Transpose[similarityTransformation00]
Transpose[O8similarityTransformation].O8similarityTransformation === ID16
O8similarityTransformation.σ16.Transpose[O8similarityTransformation] === σ16
(*O8similarityTransformation.σ16.Transpose[O8similarityTransformation]-σ16//MatrixForm*)
```

Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

Out[∘]=
```
False
```

Out[∘]=
```
False
```

In[∘]:=
```
σ16 === ArrayFlatten[{{-σ, 0}, {0, σ}}]
σ16 // MatrixForm
O8similarityTransformation.σ16.Transpose[O8similarityTransformation] // MatrixForm
```

Out[∘]=
```
False
```

Out[∘]//MatrixForm=
```
σ16
```

Out[∘]//MatrixForm=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}.σ16.
 {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[∘]:=
```
(*ArrayFlatten[{{IdentityMatrix[4],0},{0,-IdentityMatrix[4]}}]*)
```

In[∘]:=
```
ID4 = IdentityMatrix[4];
ID8 = IdentityMatrix[8];
```

In[ ]:=
```
η4488 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[ ]:=
```
η4488[[#, #]] & /@ Range[8]
```

Out[ ]=
```
{1, 1, 1, 1, -1, -1, -1, -1}
```

In[ ]:=
```
Zero4 = ConstantArray[0, {4, 4}]; Zero8 = ConstantArray[0, {8, 8}];
mid = ExpandAll[-1 * ID4];
ϵ3 = Array[Signature[{##}] &, {3, 3, 3}]
ϵ4 = Array[Signature[{##}] &, {4, 4, 4, 4}];
MId = ExpandAll[-1 * ID8];
simp = {Zero4 → 0, ID4 → 1, mid → -1};
Simp = {Zero8 → ZERO, ID8 → ONE, MId → MONE};
```

Out[ ]=
```
{{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}},
 {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}}, {{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}
```

In[ ]:=
```
Zero16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η8, 0}, {0, -η8}}];
Id16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * Id16];
Simp16 = {Zero16 → ZERO16, Id16 → ONE16, MId16 → MONE16};
```

In[ ]:=
```
ZERO16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η4488, 0}, {0, -η4488}}];
ID16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * ID16];
Simp16 = {ZERO16 → ZERO16, ID16 → ONE16, MId16 → MONE16};
```

In[ ]:=
```
ωs = Flatten[
  Table[ToExpression["ω" <> ToString[A1] <> ToString[B1]], {A1, 1, 7}, {B1, A1 + 1, 8}]]
Length[ωs]
```

Out[ ]=
```
{ω12, ω13, ω14, ω15, ω16, ω17, ω18, ω23, ω24, ω25, ω26, ω27, ω28,
 ω34, ω35, ω36, ω37, ω38, ω45, ω46, ω47, ω48, ω56, ω57, ω58, ω67, ω68, ω78}
```

Out[ ]=
```
28
```

In[ ]:=
```
σ.σ - ID8 == Zero8
Tr[σ] == 0
```
Out[ ]=
```
True
```
Out[ ]=
```
True
```

some function definitions :

In[ ]:=
```
blockPartition[mat_] := Module[{(*blocks,block11,block12,block21,block22,*)ret},
  blocks = Partition[mat, {8, 8}];
  (*block11=blocks⟦1,1⟧ ;*)(*Top-left block*)
  (*block12=blocks⟦1,2⟧ ;*)(*Top-right block*)
  (*block21=blocks⟦2,1⟧ ;*)(*Bottom-left block*)
  (*block22=blocks⟦2,2⟧ ;*)(*Bottom-right block*)
  ret =
   ArrayFlatten[{{blocks⟦1, 1⟧, blocks⟦1, 2⟧}, {blocks⟦2, 1⟧, blocks⟦2, 2⟧}}] === mat;
  If[ret, Return[{{blocks⟦1, 1⟧, blocks⟦1, 2⟧}, {blocks⟦2, 1⟧, blocks⟦2, 2⟧}}], ## &[]];
  Return[Null];]
```

In[ ]:=
```
eextract[a_] := Extract[a, {#}] & /@ Range[0, Length[a]]
```

In[ ]:=
```
Clear[imes]
```

In[ ]:=
```
imes[a_] := {a}
```

In[ ]:=
```
imes[a_, b_] := {a, b}
```

In[ ]:=
```
imes[a_, b_, c_] := {a, b, c}
```

In[ ]:=
```
imes[a_, b_, c_, d_] := {a, b, c, d}
```

In[ ]:=
```
imes[a_, b_, c_, d_, f_] := {a, b, c, d, f}
```

In[ ]:=
```
imes[a_, b_, c_, d_, f_, h_] := {a, b, c, d, f, h}
```

In[ ]:=
```
imes[a_, b_, c_, d_, f_, h_, j_] := {a, b, c, d, f, h, j}
```

In[ ]:=
```
times[a__] := Flatten[{Flatten[#] & /@ a}]
```

In[ ]:=
```
(*times[a_,b_,c_,d_]:={a,b}*)
```

In[ ]:=
```
(*imes[a_,b_,c_,d_][0]:={a,d}*)
```

In[ ]:=
```
(*imes[a_,b_][0]^:={a,b}*)
```

In[∘]:=
```
iid[a_, b_] := a
```

In[∘]:=
```
iid[a_, b_, c_] := {a, b}
```

In[∘]:=
```
iid[a_List] := a[[1]]
```

In[∘]:=
```
qid[a__] := a
```

In[∘]:=
```
Clear[der]; der[a__] [c_][b__] := c
```

In[∘]:=
```
Clear[der3]; der3[a__] [c_][b__] := {c, a, b}
```

In[∘]:=
```
(*der[a_, b_] [c_]:= c*)
```

In[∘]:=
```
(*derz[a_, b_] := a*)
```

In[∘]:=
```
(*dert[a_, b_] := b*)
```

In[∘]:=
```
(*derivative[a__] [c_]:= FullForm[c]/.{Derivative→der}*)
```

In[∘]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
(*ToString[#]&/@%*)
% /. {Derivative → der}
```

Out[∘]=

$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{Z[0][z, t], Z[0], Z[0]}
```

In[∘]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
%* (Prime[#] & /@ Range[Length[%]])
Plus @@ %
FullForm[%] (* #]&/@%*)
% /. {Plus → qid, Times → qid, Derivative → der}
List[%]
Partition[%, 2]
eextract[#] & /@ %
(*eextract[#]&/@%*)
(*#__List[[0]]&/@%*)
#[[2]] & /@ %
#[[3]] & /@ %%
```

Out[∘]=
$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$

Out[∘]=
$\{2\, Z[0][z, t], 3\, Z[0]^{(1,0)}[z, t], 5\, Z[0]^{(0,1)}[z, t]\}$

Out[∘]=
$2\, Z[0][z, t] + 5\, Z[0]^{(0,1)}[z, t] + 3\, Z[0]^{(1,0)}[z, t]$

Out[∘]//FullForm=
```
Plus[Times[2, Z[0][z, t]], Times[5, Derivative[0, 1][Z[0]][z, t]],
 Times[3, Derivative[1, 0][Z[0]][z, t]]]
```

Out[∘]=
```
Sequence[2, Z[0][z, t], 5, Z[0], 3, Z[0]]
```

Out[∘]=
```
{2, Z[0][z, t], 5, Z[0], 3, Z[0]}
```

Out[∘]=
```
{{2, Z[0][z, t]}, {5, Z[0]}, {3, Z[0]}}
```

Out[∘]=
```
{{List, 2, Z[0][z, t]}, {List, 5, Z[0]}, {List, 3, Z[0]}}
```

Out[∘]=
```
{2, 5, 3}
```

Out[∘]=
```
{Z[0][z, t], Z[0], Z[0]}
```

Sequence[Subscript[expr, 1], Subscript[expr, 2], …]
represents a sequence of arguments to be spliced automatically into any function .

In[∘]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
(*ToString[#]&/@%*)
% /. {Derivative → der3}
```

Out[∘]=
$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{Z[0][z, t], List[Z[0], 1, 0, z, t], List[Z[0], 0, 1, z, t]}
```

In[ ]:=
```
helpf[x__] := Length[{x}]
```

In[ ]:=
```
{helpf[x, y, z], helpf[]}
```

Out[ ]=
```
{3, helpf[]}
```

In[ ]:=
```
f[##] & @@ X
```

Out[ ]=
```
f[x0, x1, x2, x3, x4, x5, x6, x7]
```

In[ ]:=
```
StringExtract["a--bbb--ccc--dddd", "--" → 3]
```

Out[ ]=
```
ccc
```

In[ ]:=
```
StringExtract["a bbb  cccc aa   d", -1]
```

Out[ ]=
```
d
```

In[ ]:=
```
StringRiffle[{{"a", "b", "c"}, {"d", "e", "f"}}, "\n", "\t"]
```

Out[ ]=
```
a    b    c
d    e    f
```

In[ ]:=
```
{"", "z, t"}
```

Out[ ]=
```
{, z, t}
```

```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
StringSplit[#, "]"] & /@ %
StringExtract[#, "[" → All] & /@ %
StringExtract[#,
   "\!\(\*StyleBox[\"
      \",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"{\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*
      StyleBox[\",\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"
      \",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"z\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*
      StyleBox[\",\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"
      \",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"t\",ShowStringCharacters->True,NumberMarks->True]\)" →
    All] & /@ %
InputForm[TextString[#]] & /@ %
```

In[∘]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
StringSplit[#, "]"] & /@ %
StringExtract[#, "[" → All] & /@ %
(*StringExtract[#," {, z, t"→All]&/@%*)
InputForm[TextString[#]] & /@ %
```

Out[∘]=
```
{Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}
```

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{{Z[0, [z, t}, {Derivative[1, 0, [Z[0, , [z, t}, {Derivative[0, 1, [Z[0, , [z, t}}
```

Out[∘]=
```
{{{Z, 0}, {, z, t}}, {{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}},
 {{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}}
```

Out[∘]=
```
{"{{Z, 0}, {, z, t}}", "{{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}",
 "{{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}"}
```

In[∘]:=
```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
(*InputForm[TextString[#]]&/@%*)
StringExtract[#, "[" → All] & /@ %
StringExtract[#, "]" → All] & /@ %%
StringExtract[#, "[" → 1] & /@ %%%
StringExtract[#, "]" → 1] & /@ %%%%
StringExtract[#, "Z[" → 1] & /@ %%%%%
```

Out[∘]=
```
{Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}
```

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[∘]=
```
{{Z, 0], z, t]}, {Derivative, 1, 0], Z, 0]], z, t]}, {Derivative, 0, 1], Z, 0]], z, t]}}
```

Out[∘]=
```
{{Z[0, [z, t, }, {Derivative[1, 0, [Z[0, , [z, t, }, {Derivative[0, 1, [Z[0, , [z, t, }}
```

Out[∘]=
```
{Z, Derivative, Derivative}
```

Out[∘]=
```
{Z[0, Derivative[1, 0, Derivative[0, 1}
```

Out[∘]=
```
{, Derivative[1, 0][, Derivative[0, 1][}
```

In[∘]:=
```
(*TemplateApply[StringTemplate["Z `[` then `b`"],
 <|"a"→1234,"b"→5678|>]*)
```

In[ ]:=
```
skelx[x_, matQ_ : True] := Module[{t = {666}},
  If[matQ == True, t = Block[{r},
     r = x;
     Table[Block[{q}, q = SameQ[0, r⟦j, i⟧];
       If[True == q, Style[0, Blue], Style[Length[r⟦j, i⟧], Red]]],
      {j, 1, Length[r]}, {i, 1, Length[r⟦-1⟧]}]]];
  If[matQ == False, t = Block[{r},
     r = x;
     Table[Block[{q}, q = SameQ[0, r⟦j⟧];
       If[True == q, Style[0, {RGBColor → {0, 1 / 3, 0}, Bold}], Style[Length[r⟦j⟧], Red]]],
      {j, 1, Length[r]}]]];
  MatrixForm[t, TableAlignments → Left]]
```

In[ ]:=
```
makeSym[size_, fn_] :=
 Module[{rtmp}, rtmp = Table[fn[i, j], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[rtmp, {{2}, {1}}]}]]
```

In[ ]:=
```
makeAntiSym[size_, fn_] :=
 Module[{rtmp}, rtmp = Table[If[i == j, 0, fn[i, j]], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[-rtmp, {{2}, {1}}]}]]
```

In[ ]:=
```
Block[{MX}, MX = makeAntiSym[8, Subscript[ωω, ##] &]]
```

Out[ ]=

$$\{\{0, -\omega\omega_{2,1}, -\omega\omega_{3,1}, -\omega\omega_{4,1}, -\omega\omega_{5,1}, -\omega\omega_{6,1}, -\omega\omega_{7,1}, -\omega\omega_{8,1}\},$$
$$\{\omega\omega_{2,1}, 0, -\omega\omega_{3,2}, -\omega\omega_{4,2}, -\omega\omega_{5,2}, -\omega\omega_{6,2}, -\omega\omega_{7,2}, -\omega\omega_{8,2}\},$$
$$\{\omega\omega_{3,1}, \omega\omega_{3,2}, 0, -\omega\omega_{4,3}, -\omega\omega_{5,3}, -\omega\omega_{6,3}, -\omega\omega_{7,3}, -\omega\omega_{8,3}\},$$
$$\{\omega\omega_{4,1}, \omega\omega_{4,2}, \omega\omega_{4,3}, 0, -\omega\omega_{5,4}, -\omega\omega_{6,4}, -\omega\omega_{7,4}, -\omega\omega_{8,4}\},$$
$$\{\omega\omega_{5,1}, \omega\omega_{5,2}, \omega\omega_{5,3}, \omega\omega_{5,4}, 0, -\omega\omega_{6,5}, -\omega\omega_{7,5}, -\omega\omega_{8,5}\},$$
$$\{\omega\omega_{6,1}, \omega\omega_{6,2}, \omega\omega_{6,3}, \omega\omega_{6,4}, \omega\omega_{6,5}, 0, -\omega\omega_{7,6}, -\omega\omega_{8,6}\},$$
$$\{\omega\omega_{7,1}, \omega\omega_{7,2}, \omega\omega_{7,3}, \omega\omega_{7,4}, \omega\omega_{7,5}, \omega\omega_{7,6}, 0, -\omega\omega_{8,7}\},$$
$$\{\omega\omega_{8,1}, \omega\omega_{8,2}, \omega\omega_{8,3}, \omega\omega_{8,4}, \omega\omega_{8,5}, \omega\omega_{8,6}, \omega\omega_{8,7}, 0\}\}$$

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(\frac{\partial g_{\mu\alpha}}{\partial x^{\beta}} + \frac{\partial g_{\mu\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\mu}}\right),$$

$$\Gamma^{\mu}{}_{\alpha\beta} = g^{\mu\nu}\Gamma_{\nu\alpha\beta},$$

$$R^{\mu}{}_{\nu\alpha\beta} = \frac{\partial \Gamma^{\mu}{}_{\nu\beta}}{\partial x^{\alpha}} - \frac{\partial \Gamma^{\mu}{}_{\nu\alpha}}{\partial x^{\beta}} + \Gamma^{\mu}{}_{\rho\alpha}\Gamma^{\rho}{}_{\nu\beta} - \Gamma^{\mu}{}_{\rho\beta}\Gamma^{\rho}{}_{\nu\alpha}.$$

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}, \qquad R = g^{\mu\nu}R_{\mu\nu},$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

In[ ]:=
```
(*Christoffel, Einstein tensor, ...*)
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, ΓΓ, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    ΓΓ = FullSimplify[Table[(1 / 2) * Sum[(ginv[[i, s]]) *
          (D[g[[s, j]], X[[k]] ] + D[g[[s, k]], X[[j]] ] - D[g[[j, k]], X[[s]] ]), {s, 1, DIM8}],
       {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] , ass];
    RicciΓ = ParallelTable[FullSimplify[D[ΓΓ[[μ, ν, β]], X[[α]] ] - D[ΓΓ[[μ, ν, α]], X[[β]] ] +
         Sum[ΓΓ[[μ, s, α]] × ΓΓ[[s, ν, β]] - ΓΓ[[μ, s, β]] × ΓΓ[[s, ν, α]], {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}] ;
    RieΓ = ParallelTable[ FullSimplify[Sum[ RicciΓ[[α, μ, α, ν]], {α, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}] ;
    RS = FullSimplify[Tr[ ginv.RieΓ] , ass];
    G = ParallelTable[FullSimplify[RieΓ[[α]] - 1/2 g[[α]] RS, ass], {α, 1, Length[RieΓ]}];];
  Print[t];
  Print[Now];
  Return[{ginv, ΓΓ, RicciΓ, RieΓ, RS, G}]]
```

If ‘paste’ Maple MathML to ToExpression[MathMLForm[]] fails, then try to edit this :

In[ ]:=
```
Clear[mapleztStringToMathematica];
mapleztStringToMathematica[input_String, toExpression_ : False, debug_ : False] :=
 Module[{prepare, post, custom, use = input, save = input, linenumber = 0},
  (*use=ToString[InputForm[TextString[input]]];*) (*Print[use];*)
  prepare[in_String] := Module[{s = in}, linenumber++;
    s = in;
    (*If[s==ToString[InputForm[TextString[s]]],
      (*##&[]*)Print[s],Print[ToUpperCase[s]]];*)
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,z)" →
        "D[Z[" <> ToString[j] <> "][z,t],{z,2}]"], {j, 0, 15}];
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,t)" →
        "D[D[Z[" <> ToString[j] <> "][z,t],z],t]"], {j, 0, 15}];
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,z)" →
        "D[D[Z[" <> ToString[j] <> "][z,t],t],z]"], {j, 0, 15}];
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,t)" →
        "D[Z[" <> ToString[j] <> "][z,t],{t,2}]"], {j, 0, 15}];

    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t)" →
        "D[Z[" <> ToString[j] <> "][z,t],t]"], {j, 0, 15}];
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z)" →
        "D[Z[" <> ToString[j] <> "][z,t],z]"], {j, 0, 15}];
```

```
    s = StringReplace[s, "tan(z)" → "Tan[z]"];
    s = StringReplace[s, "cot(z)" → "Cot[z]"];
    s = StringReplace[s, "sin(z)" → "Sin[z]"];
    s = StringReplace[s, "cos(z)" → "Cos[z]"];
    s = StringReplace[s, "sec(z)" → "Sec[z]"];
    s = StringReplace[s, "csc(z)" → "Csc[z]"];

    s = StringReplace[s, "=" → "->"];

    (*Do[s=StringReplace[s,"Z"<>ToString[j]<>"(z,t)"→"Z["<>ToString[j]<>"[z,t]"],
      {j,0,15}];*)
    (*s=StringReplace[s,"diff(a4(t),t)"→"D[a4[t],t]"];*)
    (*s=StringReplace[s,"exp(-2*a4(t))"→"Exp[-2a4[t]]"];*)
    (*s=StringReplace[s,"a4(t))"→"a4[t]"];*)

    Return[s];
   ];
  post[in_String] := Module[{s = in}, linenumber++;
    s = in;
    s = StringReplace[s, "a4(t))" → "a4[t]"];
    Do[s = StringReplace[s, "Z" <> ToString[j] <> "(z,t)" → "Z[" <> ToString[j] <> "][z,t]"],
     {j, 0, 15}];
    Return[s];
   ];
  custom[in_String] := Module[{s = in}, linenumber++;
    s = in;

    s = StringReplace[s, "diff(a4(t),t)" → "D[a4[t],t]"];
    s = StringReplace[s, "exp(-2*a4(t))" → "Exp[-2a4[t]]"];

    Return[s];
   ];
  use = prepare[use];
  use = custom[use];
  Return[post[use]];
  (*If[toExpression,Return[ToExpression[s/.{z→#1,t→#2}],##&[]];*)
 ]
```

Mathematica Lexer and Parser for Maple - like Syntax Supports : Basic arithmetic, function calls, lists {}, equations =, and specific Maple keywords .

In[∘]:=
```
(*---LEXER---*)(*Helper:Check if character is a digit*)
IsDigit[char_] := StringMatchQ[char, RegularExpression["\\d"]];

(*Helper:Check if character is a letter*)
IsAlpha[char_] := StringMatchQ[char, RegularExpression["[a-zA-Z_]"]];

(*Helper:Check if character is whitespace*)
IsSpace[char_] := StringMatchQ[char, RegularExpression["\\s"]];

(*Main Lexer Function*)
GetTokens[str_String] :=
  Module[{chars, len, i, char, tokens = {}, token, numStr, idStr}, chars = Characters[str];
   len = Length[chars];
   i = 1;
   While[i ≤ len, char = chars⟦i⟧;
    Which[(*Skip Whitespace*)IsSpace[char], i++, (*Numbers*)IsDigit[char], numStr = char;
     i++;
     While[i ≤ len && IsDigit[chars⟦i⟧], numStr = numStr <> chars⟦i⟧;
      i++;];
     AppendTo[tokens, {"NUMBER", ToExpression[numStr]}],
     (*Identifiers and Keywords*)IsAlpha[char], idStr = char;
     i++;
     While[i ≤ len && (IsAlpha[chars⟦i⟧] || IsDigit[chars⟦i⟧]), idStr = idStr <> chars⟦i⟧;
      i++;];
     (*Check for specific keywords if needed,
     otherwise IDENTIFIER*)AppendTo[tokens, {"IDENTIFIER", idStr}],
     (*Operators and Punctuation*)char == "+", AppendTo[tokens, {"PLUS", "+"}];
     i++, char == "-", AppendTo[tokens, {"MINUS", "-"}];
     i++, char == "*", AppendTo[tokens, {"STAR", "*"}];
     i++, char == "/", AppendTo[tokens, {"SLASH", "/"}];
     i++, char == "^", AppendTo[tokens, {"CARET", "^"}];
     i++, char == "(", AppendTo[tokens, {"LPAREN", "("}];
     i++, char == ")", AppendTo[tokens, {"RPAREN", ")"}];
     i++, char == "{", AppendTo[tokens, {"LBRACE", "{"}];
     i++, char == "}", AppendTo[tokens, {"RBRACE", "}"}];
     i++, char == "=", AppendTo[tokens, {"EQUALS", "="}];
     i++, char == ",", AppendTo[tokens, {"COMMA", ","}];
     i++, char == "$", AppendTo[tokens, {"DOLLAR", "$"}];
     i++, (*Ellipsis... or Dot.*)char == ".", If[i + 2 ≤ len &&
       chars⟦i + 1⟧ == "." && chars⟦i + 2⟧ == ".", AppendTo[tokens, {"ELLIPSIS", "..."}];
      i += 3, AppendTo[tokens, {"DOT", "."}];
      i++], True, Print["Unknown character: ", char];
     i++];];
   AppendTo[tokens, {"EOF", "EOF"}];
   tokens];
```

In[∘]:=
```
(*---PARSER---*)(*Grammar:Expression→
 Equation Equation→AddExp["=" AddExp] AddExp→MulExp {("+"|"-") MulExp} MulExp→
```

```
    PowerExp {("*"|"/") PowerExp} PowerExp→Primary["^" PowerExp] (Right associative)
       Primary→NUMBER|IDENTIFIER|"(" Expression ")"|IDENTIFIER "(" ArgList ")"|
        "{" ArgList "}" ArgList→Expression {"," Expression}|Empty*)
ParseTokens[tokens_List] := Module[{pos = 1, currentToken, eat, peek, parseExpression,
    parseEquation, parseAddExp, parseMulExp, parsePowerExp, parseUnaryExp,
    parsePrimary, parseArgs, parseList}, currentToken := tokens⟦pos⟧;
   peek[] := tokens⟦pos⟧;
   eat[type_] := If[currentToken⟦1⟧ == type, pos++;
     True, Print["Error: Expected ", type, " but found ", currentToken⟦1⟧];
     False];
   (*Entry point*)parseExpression[] := parseEquation[];
   (*Equation:a=b*)parseEquation[] := Module[{left, right}, left = parseAddExp[];
     If[currentToken⟦1⟧ == "EQUALS", eat["EQUALS"];
      right = parseAddExp[];
      {"Equation", left, right}, left]];
   parseAddExp[] := Module[{node, right, op}, node = parseMulExp[];
     While[MemberQ[{"PLUS", "MINUS"}, currentToken⟦1⟧], op = currentToken⟦2⟧;
      eat[currentToken⟦1⟧];
      right = parseMulExp[];
      node = {"BinaryOp", op, node, right};];
     node];
   parseMulExp[] := Module[{node, right, op}, node = parsePowerExp[];
     While[True, If[MemberQ[{"STAR", "SLASH"}, currentToken⟦1⟧], op = currentToken⟦2⟧;
       eat[currentToken⟦1⟧];
       right = parsePowerExp[];
       node = {"BinaryOp", op, node, right}, (*Implicit Multiplication*)
       If[MemberQ[{"IDENTIFIER", "NUMBER", "LPAREN", "LBRACE", "ELLIPSIS"},
         currentToken⟦1⟧], right = parsePowerExp[];
        node = {"BinaryOp", "*", node, right},
        Break[] (*Not an operator or start of expression*)]]];
     node];
   parsePowerExp[] := Module[{node, right}, node = parseUnaryExp[];
     If[currentToken⟦1⟧ == "CARET", eat["CARET"];
      right = parsePowerExp[];
      (*Right associative recursion*){"BinaryOp", "^", node, right}, node]];
   parseUnaryExp[] := Module[{op, node},
     If[MemberQ[{"PLUS", "MINUS"}, currentToken⟦1⟧], op = currentToken⟦2⟧;
      eat[currentToken⟦1⟧];
      node = parseUnaryExp[];
      {"UnaryOp", op, node}, parsePrimary[]]];
   parsePrimary[] := Module[{token, node, name, args}, token = currentToken;
     Switch[token⟦1⟧, "NUMBER", eat["NUMBER"];
      {"Number", token⟦2⟧}, "IDENTIFIER", eat["IDENTIFIER"];
      name = token⟦2⟧;
      (*Check if it's a function call:IDENTIFIER followed by LPAREN*)
      If[currentToken⟦1⟧ == "LPAREN", eat["LPAREN"];
       args = parseArgs[];
```

```
        eat["RPAREN"];
        {"Call", name, args}, {"Identifier", name}], "LPAREN", eat["LPAREN"];
      node = parseExpression[];
      eat["RPAREN"];
      node, "LBRACE", parseList[], "ELLIPSIS", eat["ELLIPSIS"];
      {"Identifier", "..."}, "EOF", Print["Error: Unexpected EOF in Primary"];
      {"Error", "EOF"}, (*Do NOT eat EOF*)
      _, Print["Error: Unexpected token in Primary: ", token];
      eat[token[[1]]];
      (*Advance to avoid infinite loop*){"Error", token}]];
  (*Parse List:{a,b,c}*)parseList[] := Module[{elements}, eat["LBRACE"];
    elements = parseArgs[];
    eat["RBRACE"];
    {"List", elements}];
  (*Parse Argument List:expr,expr,...*)parseArgs[] := Module[{args = {}, arg},
    If[currentToken[[1]] ≠ "RPAREN" && currentToken[[1]] ≠ "RBRACE", arg = parseExpression[];
     AppendTo[args, arg];
     While[currentToken[[1]] == "COMMA", eat["COMMA"];
      arg = parseExpression[];
      AppendTo[args, arg];];];
    args];
  parseExpression[]];
```

X

```
In[•]:= (*---CONVERTER---*)(*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunction[funcName_String] :=
  Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate", "DESol",
   "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan", "Tan", "sin",
   "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc", "Csc", "csch", "Csch",
   "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh", "coth", "Coth", "sech", "Sech",
   "det", "Det", "transpose", "Transpose", "sphericalbessel", "SphericalBesselJ",
   "lambertw", "ProductLog", "gamma", "Gamma", "airy", "AiryAi", "besselJ",
   "BesselJ", "besselY", "BesselY", "hypergeometricU", "HypergeometricU", "zeta",
   "Zeta", "erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNode[nodeType_, nodeData_, childResults_] :=
  Module[{funcName, cleanFuncName, mathFunc, argsStr},
   Switch[nodeType, "Number", ToString[nodeData], "Identifier",
    (*Handle special identifiers like _Y→Y*)If[StringLength[nodeData] > 0 &&
      StringTake[nodeData, 1] == "_", StringDrop[nodeData, 1], nodeData], "BinaryOp",
    "(" <> childResults[[1]] <> " " <> nodeData <> " " <> childResults[[2]] <> ")",
    "UnaryOp", nodeData <> "(" <> childResults[[1]] <> ")", "Equation",
    childResults[[1]] <> " == " <> childResults[[2]], "List",
    "{" <> StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
    (*Handle special identifiers like _Y→Y for function names too*)
    cleanFuncName = If[StringLength[funcName] > 0 && StringTake[funcName, 1] == "_",
```

```
      StringDrop[funcName, 1], funcName];
    mathFunc = MapleToMathematicaFunction[cleanFuncName];
    (*Special handling for DSolve-requires 3 arguments*)
    If[cleanFuncName == "DESol" || mathFunc == "DSolve",
     (*DSolve[equations,functions,variables]*)(*Ensure we have exactly 3 arguments,
     pad with empty lists if needed*)argsStr = StringRiffle[
       Join[childResults, Table["{}", {i, 1, Max[0, 3 - Length[childResults]]}]], ", "];
     mathFunc <> "[" <> argsStr <> "]",
     (*Default function call*)argsStr = StringRiffle[childResults, ", "];
     mathFunc <> "[" <> argsStr <> "]"], _, "Error"]];

(*Convert AST to Mathematica String-Non-
 recursive iterative version using post-order traversal*)
ToMathematicaString[ast_] := Module[{stack, outputStack, currentItem,
    node, nodeType, children, childCount, childResults, i, result, nodeData},
   (*Use two stacks:one for traversal,one for results*)stack = {{ast, False}};
   (*{node,visited}*)outputStack = {};
   (*Post-order traversal:process children before parents*)
   While[Length[stack] > 0, currentItem = Last[stack];
    node = currentItem[[1]];
    nodeType = node[[1]];
    If[currentItem[[2]], (*Node already visited,process it*)stack = Most[stack];
     (*Determine children and extract data*)
     {children, nodeData} = Switch[nodeType, "Number", {{}, node[[2]]},
       "Identifier", {{}, node[[2]]}, "BinaryOp", {{node[[3]], node[[4]]}, node[[2]]},
       "UnaryOp", {{node[[3]]}, node[[2]]}, "Equation", {{node[[2]], node[[3]]}, ""},
       "List", {node[[2]], ""}, "Call", {node[[3]], node[[2]]}, _, {{}, ""}];
     (*Pop child results from output stack*)childCount = Length[children];
     childResults = {};
     If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
        outputStack = Most[outputStack];, {i, 1, childCount}];];
     (*Process this node*)result = ProcessASTNode[nodeType, nodeData, childResults];
     AppendTo[outputStack, result];, (*Node not visited,
     mark as visited and push children*)stack[[Length[stack]]] = {node, True};
     (*Push children onto stack in reverse order (so they process left-to-right)*)
     children = Switch[nodeType, "Number", {}, "Identifier", {},
       "BinaryOp", {node[[3]], node[[4]]}, "UnaryOp", {node[[3]]}, "Equation",
       {node[[2]], node[[3]]}, "List", node[[2]], "Call", node[[3]], _, {}];
     Do[AppendTo[stack, {children[[i]], False}];, {i, Length[children], 1, -1}];];];
   (*Return the final result*)If[Length[outputStack] > 0, Last[outputStack], "Error"]];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematica[inputStr_String] :=
  Module[{tokens, ast}, tokens = GetTokens[inputStr];
   ast = ParseTokens[tokens];
   ToMathematicaString[ast]];
```

In[ ]:=
```
(*---CONVERTER---*)(*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunctionOLD[funcName_String] :=
```

```
  Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate", "DESol",
   "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan", "Tan", "sin",
   "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc", "Csc", "csch", "Csch",
   "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh", "coth", "Coth", "sech", "Sech",
   "det", "Det", "transpose", "Transpose", "sphericalbessel", "SphericalBesselJ",
   "lambertw", "ProductLog", "gamma", "Gamma", "airy", "AiryAi", "besselJ",
   "BesselJ", "besselY", "BesselY", "hypergeometricU", "HypergeometricU", "zeta",
   "Zeta", "erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNodeOLD[nodeType_, nodeData_, childResults_] :=
  Module[{funcName, cleanFuncName, mathFunc, argsStr},
   Switch[nodeType, "Number", ToString[nodeData], "Identifier",
    (*Handle special identifiers like _Y→Y*)If[StringLength[nodeData] > 0 &&
      StringTake[nodeData, 1] == "_", StringDrop[nodeData, 1], nodeData], "BinaryOp",
    "(" <> childResults⟦1⟧ <> " " <> nodeData <> " " <> childResults⟦2⟧ <> ")",
    "UnaryOp", nodeData <> "(" <> childResults⟦1⟧ <> ")", "Equation",
    childResults⟦1⟧ <> " == " <> childResults⟦2⟧, "List",
    "{" <> StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
    (*Handle special identifiers like _Y→Y for function names too*)
    cleanFuncName = If[StringLength[funcName] > 0 && StringTake[funcName, 1] == "_",
      StringDrop[funcName, 1], funcName];
    mathFunc = MapleToMathematicaFunction[cleanFuncName];
    argsStr = StringRiffle[childResults, ", "];
    mathFunc <> "[" <> argsStr <> "]", _, "Error"]];

(*Convert AST to Mathematica String-Non-
 recursive iterative version using post-order traversal*)
ToMathematicaStringOLD[ast_] := Module[{stack, outputStack, currentItem,
    node, nodeType, children, childCount, childResults, i, result, nodeData},
   (*Use two stacks:one for traversal,one for results*)stack = {{ast, False}};
   (*{node,visited}*)outputStack = {};
   (*Post-order traversal:process children before parents*)
   While[Length[stack] > 0, currentItem = Last[stack];
    node = currentItem⟦1⟧;
    nodeType = node⟦1⟧;
    If[currentItem⟦2⟧, (*Node already visited,process it*)stack = Most[stack];
     (*Determine children and extract data*)
     {children, nodeData} = Switch[nodeType, "Number", {{}, node⟦2⟧},
       "Identifier", {{}, node⟦2⟧}, "BinaryOp", {{node⟦3⟧, node⟦4⟧}, node⟦2⟧},
       "UnaryOp", {{node⟦3⟧}, node⟦2⟧}, "Equation", {{node⟦2⟧, node⟦3⟧}, ""},
       "List", {node⟦2⟧, ""}, "Call", {node⟦3⟧, node⟦2⟧}, _, {{}, ""}];
     (*Pop child results from output stack*)childCount = Length[children];
     childResults = {};
     If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
        outputStack = Most[outputStack];, {i, 1, childCount}];];
     (*Process this node*)result = ProcessASTNode[nodeType, nodeData, childResults];
```

```
      AppendTo[outputStack, result];, (*Node not visited,
      mark as visited and push children*)stack⟦Length[stack]⟧ = {node, True};
      (*Push children onto stack in reverse order (so they process left-to-right)*)
      children = Switch[nodeType, "Number", {}, "Identifier", {},
        "BinaryOp", {node⟦3⟧, node⟦4⟧}, "UnaryOp", {node⟦3⟧}, "Equation",
        {node⟦2⟧, node⟦3⟧}, "List", node⟦2⟧, "Call", node⟦3⟧, _, {}];
      Do[AppendTo[stack, {children⟦i⟧, False}];, {i, Length[children], 1, -1}];];];
    (*Return the final result*)If[Length[outputStack] > 0, Last[outputStack], "Error"]];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematicaOLD[inputStr_String] :=
  Module[{tokens, ast}, tokens = GetTokens[inputStr];
   ast = ParseTokens[tokens];
   ToMathematicaString[ast]];
```

In[ ]:=

```
sta = "{nZ10(z, t) = (c2C7*sin(C2Q1*t) + c2C8*cos(C2Q1*t))*(c2C5*sin(z)^(sqrt(-C2Q1^2
    + M^2)/6) + c2C6*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ11(z, t) =
    (c2C3*sin(C2Q1*t) + c2C4*cos(C2Q1*t))*(c2C1*sin(z)^(sqrt(-C2Q1^2
    + M^2)/6) + c2C2*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ8(z, t) =
    (((-C2Q1*c2C2*c2C3 + M*c2C6*c2C8)*cos(C2Q1*t) + sin(C2Q1*t)*(C2Q1*c2C2*c2C4
    + M*c2C6*c2C7))*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) + ((C2Q1*c2C1*c2C3
    - M*c2C5*c2C8)*cos(C2Q1*t) - sin(C2Q1*t)*(C2Q1*c2C1*c2C4 +
    M*c2C5*c2C7))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) + (-cos(M*t)*c2C10 +
    sin(M*t)*c2C9)*sqrt(-C2Q1^2 + M^2))/sqrt(-C2Q1^2 + M^2), nZ9(z, t) =
    (((C2Q1*c2C6*c2C7 + M*c2C2*c2C4)*cos(C2Q1*t) + sin(C2Q1*t)*(-C2Q1*c2C6*c2C8
    + M*c2C2*c2C3))*sin(z)^(-sqrt(-C2Q1^2 + M^2)/3) + sqrt(-C2Q1^2 +
    M^2)*(cos(M*t)*c2C9 + sin(M*t)*c2C10)*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) +
    (-C2Q1*c2C5*c2C7 - M*c2C1*c2C4)*cos(C2Q1*t) - sin(C2Q1*t)*(-C2Q1*c2C5*c2C8
    + M*c2C1*c2C3))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6)/sqrt(-C2Q1^2 + M^2)}";
```

In[ ]:=
```
ConvertMapleToMathematica[sta]
```

Out[ ]=
```
{nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5 * (Sin[z]
  ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) + (c2C6 * (Sin[z] ^ (-(Sqrt[((-(C2Q1)
  ^ 2) + (M ^ 2))]) / 6))))), nZ11[z, t] == (((c2C3 * Sin[(C2Q1 * t)]) + (c2C4 *
  Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6)))
  + (c2C2 * (Sin[z] ^ (-(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) / 6))))), nZ8[z, t] ==
  (((((((((-(C2Q1) * c2C2) * c2C3) + ((M * c2C6) * c2C8)) * Cos[(C2Q1 * t)]) +
  (Sin[(C2Q1 * t)] * (((C2Q1 * c2C2) * c2C4) + ((M * c2C6) * c2C7)))) * (Sin[z] ^
  (-(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) / 6))) + ((((((C2Q1 * c2C1) * c2C3) - ((M *
  c2C5) * c2C8)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C1) * c2C4)
  + ((M * c2C5) * c2C7)))) * (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6)))) +
  (((-(Cos[(M * t)]) * c2C10) + (Sin[(M * t)] * c2C9)) * Sqrt[((-(C2Q1) ^ 2) + (M ^
  2))])) / Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]), nZ9[z, t] == ((((((((((C2Q1 * c2C6) *
  c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1)
  * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (-(Sqrt[((-(C2Q1) ^ 2) +
  (M ^ 2))]) / 3))) + ((Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] * ((Cos[(M * t)] * c2C9)
  + (Sin[(M * t)] * c2C10))) * (Sin[z] ^ (-(Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) /
  6)))) + ((((-(C2Q1) * c2C5) * c2C7) - ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)])) -
  (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z]
  ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))] / 6))) / Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])}
```

In[ ]:=
```
sti =
  "{nZ10(z,t)=(c2C7*sin(C2Q1*t)+c2C8*cos(C2Q1*t))*(c2C5*sin(z)^(1/6*sqrt(-C2Q1^2+M^2))+
    c2C6*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))),nZ11(z,t)=(c2C3*sin(C2Q1*t)+c2C4*cos(C2Q1*t)
    )*(c2C1*sin(z)^(1/6*sqrt(-C2Q1^2+M^2))+c2C2*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))),nZ8(z
    ,t)=1/6*(6*sqrt(-C2Q1^2+M^2)*c2C6*(c2C7*sin(C2Q1*t)+c2C8*cos(C2Q1*t))*sin(z)^(-1/6
    *sqrt(-C2Q1^2+M^2))-6*sqrt(-C2Q1^2+M^2)*c2C5*(c2C7*sin(C2Q1*t)+c2C8*cos(C2Q1*t))*
    sin(z)^(1/6*sqrt(-C2Q1^2+M^2))+(-6*M*c2C10-Int(-cos(M*t)*((-C2Q1^2+M^2)*Int(-cos(z
    )*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*
    c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)
    +sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6
    *sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*
    c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M
    ^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1
    *c2C3)))),t))*cos(M*t)+(6*M*c2C9-Int(-sin(M*t)*((-C2Q1^2+M^2)*Int(-cos(z)*(((C2Q1*
    c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(
    z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t
    )*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1
    ^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*
    c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M^2))*((C2Q1*
    c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3)))),t)
    )*sin(M*t)-C2Q1*Int(-cos(z)*(((-C2Q1*c2C6*c2C8+M*c2C2*c2C3)*cos(C2Q1*t)-sin(C2Q1*t
    )*(C2Q1*c2C6*c2C7+M*c2C2*c2C4))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((-C2Q1*c2C5*
    c2C8+M*c2C1*c2C3)*cos(C2Q1*t)-sin(C2Q1*t)*(C2Q1*c2C5*c2C7+M*c2C1*c2C4))*sin(z)^(1/
    6*sqrt(-C2Q1^2+M^2)-1)),z))/M,nZ9(z,t)=1/6*((6*M*c2C9-Int(-sin(M*t)*((-C2Q1^2+M^2)
    *Int(-cos(z)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*
    c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)
    *cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M
    ^2)-1)),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1
    *t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*
    sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*
    c2C5*c2C8+M*c2C1*c2C3)))),t))*cos(M*t)+(6*M*c2C10+Int(-cos(M*t)*((-C2Q1^2+M^2)*Int(-cc
    *c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(
    C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1
    )),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q
     2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8
    +M*c2C1*c2C3)))),t))*sin(M*t)+M*Int(-cos(z)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(
    C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-
    1)+((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*
    c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z))/M}";
```

In[ ]:=
```
ConvertMapleToMathematica[sti]
```

Out[ ]=
```
{nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5 * (Sin[z]
  ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))) + (c2C6 * (Sin[z] ^ ((-(1) / 6)
  * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])))), nZ11[z, t] == (((c2C3 * Sin[(C2Q1 * t)])
  + (c2C4 * Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^
  2) + (M ^ 2))]))) + (c2C2 * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
  2))])))), nZ8[z, t] == (((1 / 6) * ((((((((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])
```

```
* c2C6) * ((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)]))) * (Sin[z] ^
((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))) - ((((6 * Sqrt[((-(C2Q1) ^ 2)
+ (M ^ 2))]) * c2C5) * ((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)]))) *
(Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))))) + ((((-(6) * M) * c2C10) -
Integrate[(-(Cos[(M * t)]) * ((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) *
(((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1
* t)] * (((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1)
/ 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))) + ((((((C2Q1 * c2C5) * c2C7) +
((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5)
* c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) +
(M ^ 2))]) - 1))))), z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * (((((((C2Q1
* c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] *
(((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((-(1) / 6) *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) +
(M ^ 2))])) * (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)])
+ (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))))),
t]) * Cos[(M * t)])) + ((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) *
((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * (((((((C2Q1 * c2C6) * c2C7)
+ ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) *
c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]) - 1))) + ((((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) *
Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1)
* c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))),
z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * (((((((C2Q1 * c2C6) * c2C7) +
((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6)
* c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))])))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) *
(((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1
* t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))))))), t]) * Sin[(M
* t)])) - (C2Q1 * Integrate[(-(Cos[z]) * (((((((-(C2Q1) * c2C6) * c2C8) + ((M *
c2C2) * c2C3)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C6) * c2C7)
+ ((M * c2C2) * c2C4)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]) - 1))) + ((((((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)) * Cos[(C2Q1
* t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))), z]))) / M),
nZ9[z, t] == (((1 / 6) * ((((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) *
((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * (((((((C2Q1 * c2C6) * c2C7)
+ ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) *
c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]) - 1))) + ((((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) *
Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1)
* c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))),
z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * (((((((C2Q1 * c2C6) * c2C7) +
((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6)
* c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))])))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) *
(((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1
```

```
* t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))))), t]) * Cos[(M
* t)]) + ((((6 * M) * c2C10) + Integrate[(-(Cos[(M * t)]) * ((((-(C2Q1) ^ 2)
+ (M ^ 2)) * Integrate[(-(Cos[z]) * (((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]) - 1))) + ((((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))
* (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))), z]) - ((6 *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * (((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))])))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))) * (((((C2Q1
* c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)]
* (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))))), t]) * Sin[(M *
t)])) + (M * Integrate[(-(Cos[z]) * (((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]) - 1))) + ((((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))
* (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))))), z])))) / M)}
```

gtry and Γ and ...

In[∘]:=
```
gtry = MatrixMetric44
```

Out[∘]=

$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\}, \{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

In[∘]:=
```
Protect[gtry]
```

Out[∘]=

{gtry}

In[∘]:=
```
Unprotect[ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG]
```

Out[∘]=

{}

In[∘]:=
```
result = {ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG} = rt[gtry];
```

Mon 15 Dec 2025 12:05:25 GMT-8

{5.16043, Null}

Mon 15 Dec 2025 12:05:31 GMT-8

In[∘]:=
```
Protect[ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG]
```

Out[∘]=

{ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG}

verify $g_{\mu\nu|\alpha} = 0$:

In[∘]:=
```
Simplify[Table[D[gtry[[j, k]], X[[i]] ] -
     Sum[ gtry[[k, s]] × Γ[[s, i, j]], {s, 1, DIM8}] - Sum[ gtry[[j, s]] × Γ[[s, i, k]], {s, 1, DIM8}],
    {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Out[∘]=
```
{0}
```

In[∘]:=
```
Table[g[j][k] →
   If[j == k, ToExpression["((g[" <> ToString[j] <> "][" <> ToString[k] <> "][#1,#2])&)"],
    (0 &)], {j, 0, 7}, {k, 0, 7}];
sg = % // Flatten
```

Out[∘]=
```
{g[0][0] → (g[0][0][#1, #2] &), g[0][1] → (0 &), g[0][2] → (0 &),
 g[0][3] → (0 &), g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &),
 g[0][7] → (0 &), g[1][0] → (0 &), g[1][1] → (g[1][1][#1, #2] &), g[1][2] → (0 &),
 g[1][3] → (0 &), g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &),
 g[1][7] → (0 &), g[2][0] → (0 &), g[2][1] → (0 &), g[2][2] → (g[2][2][#1, #2] &),
 g[2][3] → (0 &), g[2][4] → (0 &), g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &),
 g[3][0] → (0 &), g[3][1] → (0 &), g[3][2] → (0 &), g[3][3] → (g[3][3][#1, #2] &),
 g[3][4] → (0 &), g[3][5] → (0 &), g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &),
 g[4][1] → (0 &), g[4][2] → (0 &), g[4][3] → (0 &), g[4][4] → (g[4][4][#1, #2] &),
 g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &), g[5][0] → (0 &), g[5][1] → (0 &),
 g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &), g[5][5] → (g[5][5][#1, #2] &),
 g[5][6] → (0 &), g[5][7] → (0 &), g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &),
 g[6][3] → (0 &), g[6][4] → (0 &), g[6][5] → (0 &), g[6][6] → (g[6][6][#1, #2] &),
 g[6][7] → (0 &), g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &),
 g[7][4] → (0 &), g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (g[7][7][#1, #2] &)}
```

In[∘]:=
```
g[7][7][x0, x4] /. ssgm4488
% /. sx0x4
```

Out[∘]=

$$-e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}$$

Out[∘]=

$$-e^{-2\,a4[t]}\,\mathrm{Sin}[z]^{1/3}$$

In[∘]:=
```
MatrixMetric44
```

Out[∘]=

$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

In[ ]:=
```
ssgm4488[x0, x4] /. sx0x4
```

Out[ ]=

$$\{g[0][0] \to (\mathrm{Cot}[6\,H\,\#1]^2\,\&),\ g[0][1] \to (0\,\&),\ g[0][2] \to (0\,\&),\ g[0][3] \to (0\,\&),$$
$$g[0][4] \to (0\,\&),\ g[0][5] \to (0\,\&),\ g[0][6] \to (0\,\&),\ g[0][7] \to (0\,\&),\ g[1][0] \to (0\,\&),$$
$$g[1][1] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[1][2] \to (0\,\&),\ g[1][3] \to (0\,\&),$$
$$g[1][4] \to (0\,\&),\ g[1][5] \to (0\,\&),\ g[1][6] \to (0\,\&),\ g[1][7] \to (0\,\&),\ g[2][0] \to (0\,\&),$$
$$g[2][1] \to (0\,\&),\ g[2][2] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[2][3] \to (0\,\&),\ g[2][4] \to (0\,\&),$$
$$g[2][5] \to (0\,\&),\ g[2][6] \to (0\,\&),\ g[2][7] \to (0\,\&),\ g[3][0] \to (0\,\&),\ g[3][1] \to (0\,\&),$$
$$g[3][2] \to (0\,\&),\ g[3][3] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[3][4] \to (0\,\&),\ g[3][5] \to (0\,\&),$$
$$g[3][6] \to (0\,\&),\ g[3][7] \to (0\,\&),\ g[4][0] \to (0\,\&),\ g[4][1] \to (0\,\&),\ g[4][2] \to (0\,\&),$$
$$g[4][3] \to (0\,\&),\ g[4][4] \to (-1\,\&),\ g[4][5] \to (0\,\&),\ g[4][6] \to (0\,\&),\ g[4][7] \to (0\,\&),$$
$$g[5][0] \to (0\,\&),\ g[5][1] \to (0\,\&),\ g[5][2] \to (0\,\&),\ g[5][3] \to (0\,\&),\ g[5][4] \to (0\,\&),$$
$$g[5][5] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[5][6] \to (0\,\&),\ g[5][7] \to (0\,\&),$$
$$g[6][0] \to (0\,\&),\ g[6][1] \to (0\,\&),\ g[6][2] \to (0\,\&),\ g[6][3] \to (0\,\&),\ g[6][4] \to (0\,\&),$$
$$g[6][5] \to (0\,\&),\ g[6][6] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[6][7] \to (0\,\&),$$
$$g[7][0] \to (0\,\&),\ g[7][1] \to (0\,\&),\ g[7][2] \to (0\,\&),\ g[7][3] \to (0\,\&),\ g[7][4] \to (0\,\&),$$
$$g[7][5] \to (0\,\&),\ g[7][6] \to (0\,\&),\ g[7][7] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&)\}\left[\frac{z}{6\,H},\ \frac{t}{H}\right]$$

In[ ]:=
```
(*ssgGzt={G[0][0]→((Cot[#1]^2)&),G[0][1]→((0)&),G[0][2]→((0)&),
  G[0][3]→((0)&),G[0][4]→((0)&),G[0][5]→((0)&),G[0][6]→((0)&),G[0][7]→((0)&),
  G[1][0]→((0)&),G[1][1]→((e^(2*A4[#2]) Sin[#1]^(1/3))&),G[1][2]→((0)&),G[1][3]→((0)&),
  G[1][4]→((0)&),G[1][5]→((0)&),G[1][6]→((0)&),G[1][7]→((0)&),G[2][0]→((0)&),
  G[2][1]→((0)&),G[2][2]→((e^(2*A4[#2]) Sin[#1]^(1/3))&),G[2][3]→((0)&),G[2][4]→((0)&),
  G[2][5]→((0)&),G[2][6]→((0)&),G[2][7]→((0)&),G[3][0]→((0)&),G[3][1]→((0)&),
  G[3][2]→((0)&),G[3][3]→((e^(2*A4[#2]) Sin[#1]^(1/3))&),G[3][4]→((0)&),G[3][5]→((0)&),
  G[3][6]→((0)&),G[3][7]→((0)&),G[4][0]→((0)&),G[4][1]→((0)&),G[4][2]→((0)&),
  G[4][3]→((0)&),G[4][4]→((-1)&),G[4][5]→((0)&),G[4][6]→((0)&),G[4][7]→((0)&),
  G[5][0]→((0)&),G[5][1]→((0)&),G[5][2]→((0)&),G[5][3]→((0)&),G[5][4]→((0)&),
  G[5][5]→((-e^(-2*A4[#2]) Sin[#1]^(1/3))&),G[5][6]→((0)&),G[5][7]→((0)&),
  G[6][0]→((0)&),G[6][1]→((0)&),G[6][2]→((0)&),G[6][3]→((0)&),G[6][4]→((0)&),
  G[6][5]→((0)&),G[6][6]→((-e^(-2*A4[#2]) Sin[#1]^(1/3))&),G[6][7]→((0)&),
  G[7][0]→((0)&),G[7][1]→((0)&),G[7][2]→((0)&),G[7][3]→((0)&),G[7][4]→((0)&),
  G[7][5]→((0)&),G[7][6]→((0)&),G[7][7]→((-e^(-2*A4[#2]) Sin[#1]^(1/3))&)}*)
```

In[ ]:=
```
(*G[#][#][z,t]/.ssgGzt&/@Range[0,7]*)
```

In[ ]:=
```
detgg = Det[g4488 /. sg] // FullSimplify[#, constraintVars] &
```

Out[ ]=

$$g[0][0][x0, x4] \times g[1][1][x0, x4] \times g[2][2][x0, x4] \times g[3][3][x0, x4] \times g[4][4][x0, x4] \times g[5][5][x0, x4] \times g[6][6][x0, x4] \times g[7][7][x0, x4]$$

In[ ]:=
```
g4488 /. sg // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} g[0][0][x0, x4] & 0 & 0 & 0 & 0 & 0 \\ 0 & g[1][1][x0, x4] & 0 & 0 & 0 & 0 \\ 0 & 0 & g[2][2][x0, x4] & 0 & 0 & 0 \\ 0 & 0 & 0 & g[3][3][x0, x4] & 0 & 0 \\ 0 & 0 & 0 & 0 & g[4][4][x0, x4] & 0 \\ 0 & 0 & 0 & 0 & 0 & g[5][5][x0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
MatrixMetric44.η4488
(*eAa=*)FullSimplify[√%, constraintVars]
```

Out[ ]=

$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\}, \{0, 0, 0, 0, 0, 0, e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

Out[ ]=

$$\{\{\mathrm{Cot}[6\,H\,x0], 0, 0, 0, 0, 0, 0, 0\}, \{0, \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \{0, 0, \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}, 0, 0, 0, 0, 0\}, \{0, 0, 0, \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}, 0, 0\}, \{0, 0, 0, 0, 0, 0, \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\}\}$$

eAa

In[ ]:=
```
g4488.η4488 /. sg
(eAa = FullSimplify[√%, constraintVars]) // MatrixForm
```

Out[ ]=

$$\{\{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \{0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0\}, \{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0\}, \{0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0\}, \{0, 0, 0, 0, -g[4][4][x0, x4], 0, 0, 0\}, \{0, 0, 0, 0, 0, -g[5][5][x0, x4], 0, 0\}, \{0, 0, 0, 0, 0, 0, -g[6][6][x0, x4], 0\}, \{0, 0, 0, 0, 0, 0, 0, -g[7][7][x0, x4]\}\}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \sqrt{g[0][0][x0, x4]} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{g[1][1][x0, x4]} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{g[2][2][x0, x4]} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{g[3][3][x0, x4]} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{-g[4][4][x0, x4]} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
eAa
ssgm4488
(eAa04 = %% /. % // FullSimplify[#, constraintVars] &) // MatrixForm
```

Out[ ]=

$$\{\{\sqrt{g[0][0][x0,x4]},0,0,0,0,0,0,0\},\{0,\sqrt{g[1][1][x0,x4]},0,0,0,0,0,0\},\{0,0,\sqrt{g[2][2][x0,x4]},0,0,0,0,0\},\{0,0,0,\sqrt{g[3][3][x0,x4]},0,0,0,0\},\{0,0,0,0,\sqrt{-g[4][4][x0,x4]},0,0,0\},\{0,0,0,0,0,\sqrt{-g[5][5][x0,x4]},0,0\},\{0,0,0,0,0,0,\sqrt{-g[6][6][x0,x4]},0\},\{0,0,0,0,0,0,0,\sqrt{-g[7][7][x0,x4]}\}\}$$

Out[ ]=

$$\{g[0][0]\to(\mathrm{Cot}[6\,H\,\#1]^2\,\&),\ g[0][1]\to(0\,\&),\ g[0][2]\to(0\,\&),\ g[0][3]\to(0\,\&),$$
$$g[0][4]\to(0\,\&),\ g[0][5]\to(0\,\&),\ g[0][6]\to(0\,\&),\ g[0][7]\to(0\,\&),\ g[1][0]\to(0\,\&),$$
$$g[1][1]\to(e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[1][2]\to(0\,\&),\ g[1][3]\to(0\,\&),$$
$$g[1][4]\to(0\,\&),\ g[1][5]\to(0\,\&),\ g[1][6]\to(0\,\&),\ g[1][7]\to(0\,\&),\ g[2][0]\to(0\,\&),$$
$$g[2][1]\to(0\,\&),\ g[2][2]\to(e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[2][3]\to(0\,\&),\ g[2][4]\to(0\,\&),$$
$$g[2][5]\to(0\,\&),\ g[2][6]\to(0\,\&),\ g[2][7]\to(0\,\&),\ g[3][0]\to(0\,\&),\ g[3][1]\to(0\,\&),$$
$$g[3][2]\to(0\,\&),\ g[3][3]\to(e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[3][4]\to(0\,\&),\ g[3][5]\to(0\,\&),$$
$$g[3][6]\to(0\,\&),\ g[3][7]\to(0\,\&),\ g[4][0]\to(0\,\&),\ g[4][1]\to(0\,\&),\ g[4][2]\to(0\,\&),$$
$$g[4][3]\to(0\,\&),\ g[4][4]\to(-1\,\&),\ g[4][5]\to(0\,\&),\ g[4][6]\to(0\,\&),\ g[4][7]\to(0\,\&),$$
$$g[5][0]\to(0\,\&),\ g[5][1]\to(0\,\&),\ g[5][2]\to(0\,\&),\ g[5][3]\to(0\,\&),\ g[5][4]\to(0\,\&),$$
$$g[5][5]\to(-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[5][6]\to(0\,\&),\ g[5][7]\to(0\,\&),$$
$$g[6][0]\to(0\,\&),\ g[6][1]\to(0\,\&),\ g[6][2]\to(0\,\&),\ g[6][3]\to(0\,\&),\ g[6][4]\to(0\,\&),$$
$$g[6][5]\to(0\,\&),\ g[6][6]\to(-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[6][7]\to(0\,\&),$$
$$g[7][0]\to(0\,\&),\ g[7][1]\to(0\,\&),\ g[7][2]\to(0\,\&),\ g[7][3]\to(0\,\&),\ g[7][4]\to(0\,\&),$$
$$g[7][5]\to(0\,\&),\ g[7][6]\to(0\,\&),\ g[7][7]\to(-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&)\}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0] & 0 & 0 & 0 & 0 \\ 0 & \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \sqrt{e^{-2\,a4}}$$

OCTAD $e_{\alpha}^{(A)}$ (A) ← Lorentz, α ← spacetime :

- $e_{\alpha}^{(A)} = e_{\alpha}^{(A)} = eAa$

In[∘]:=
```
(sge_α^(A) = Thread[Flatten[e_α^(A)] → Flatten[eAa]]) (*//Column*)
```

Out[∘]=

$$\{E_{\alpha}^{(A)}[0,0] \to \sqrt{g[0][0][x0,x4]},\ E_{\alpha}^{(A)}[0,1] \to 0,\ E_{\alpha}^{(A)}[0,2] \to 0,\ E_{\alpha}^{(A)}[0,3] \to 0,\ E_{\alpha}^{(A)}[0,4] \to 0,$$
$$E_{\alpha}^{(A)}[0,5] \to 0,\ E_{\alpha}^{(A)}[0,6] \to 0,\ E_{\alpha}^{(A)}[0,7] \to 0,\ E_{\alpha}^{(A)}[1,0] \to 0,\ E_{\alpha}^{(A)}[1,1] \to \sqrt{g[1][1][x0,x4]},$$
$$E_{\alpha}^{(A)}[1,2] \to 0,\ E_{\alpha}^{(A)}[1,3] \to 0,\ E_{\alpha}^{(A)}[1,4] \to 0,\ E_{\alpha}^{(A)}[1,5] \to 0,\ E_{\alpha}^{(A)}[1,6] \to 0,\ E_{\alpha}^{(A)}[1,7] \to 0,$$
$$E_{\alpha}^{(A)}[2,0] \to 0,\ E_{\alpha}^{(A)}[2,1] \to 0,\ E_{\alpha}^{(A)}[2,2] \to \sqrt{g[2][2][x0,x4]},\ E_{\alpha}^{(A)}[2,3] \to 0,\ E_{\alpha}^{(A)}[2,4] \to 0,$$
$$E_{\alpha}^{(A)}[2,5] \to 0,\ E_{\alpha}^{(A)}[2,6] \to 0,\ E_{\alpha}^{(A)}[2,7] \to 0,\ E_{\alpha}^{(A)}[3,0] \to 0,\ E_{\alpha}^{(A)}[3,1] \to 0,\ E_{\alpha}^{(A)}[3,2] \to 0,$$
$$E_{\alpha}^{(A)}[3,3] \to \sqrt{g[3][3][x0,x4]},\ E_{\alpha}^{(A)}[3,4] \to 0,\ E_{\alpha}^{(A)}[3,5] \to 0,\ E_{\alpha}^{(A)}[3,6] \to 0,\ E_{\alpha}^{(A)}[3,7] \to 0,$$
$$E_{\alpha}^{(A)}[4,0] \to 0,\ E_{\alpha}^{(A)}[4,1] \to 0,\ E_{\alpha}^{(A)}[4,2] \to 0,\ E_{\alpha}^{(A)}[4,3] \to 0,\ E_{\alpha}^{(A)}[4,4] \to \sqrt{-g[4][4][x0,x4]},$$
$$E_{\alpha}^{(A)}[4,5] \to 0,\ E_{\alpha}^{(A)}[4,6] \to 0,\ E_{\alpha}^{(A)}[4,7] \to 0,\ E_{\alpha}^{(A)}[5,0] \to 0,\ E_{\alpha}^{(A)}[5,1] \to 0,\ E_{\alpha}^{(A)}[5,2] \to 0,$$
$$E_{\alpha}^{(A)}[5,3] \to 0,\ E_{\alpha}^{(A)}[5,4] \to 0,\ E_{\alpha}^{(A)}[5,5] \to \sqrt{-g[5][5][x0,x4]},\ E_{\alpha}^{(A)}[5,6] \to 0,\ E_{\alpha}^{(A)}[5,7] \to 0,$$
$$E_{\alpha}^{(A)}[6,0] \to 0,\ E_{\alpha}^{(A)}[6,1] \to 0,\ E_{\alpha}^{(A)}[6,2] \to 0,\ E_{\alpha}^{(A)}[6,3] \to 0,\ E_{\alpha}^{(A)}[6,4] \to 0,\ E_{\alpha}^{(A)}[6,5] \to 0,$$
$$E_{\alpha}^{(A)}[6,6] \to \sqrt{-g[6][6][x0,x4]},\ E_{\alpha}^{(A)}[6,7] \to 0,\ E_{\alpha}^{(A)}[7,0] \to 0,\ E_{\alpha}^{(A)}[7,1] \to 0,\ E_{\alpha}^{(A)}[7,2] \to 0,$$
$$E_{\alpha}^{(A)}[7,3] \to 0,\ E_{\alpha}^{(A)}[7,4] \to 0,\ E_{\alpha}^{(A)}[7,5] \to 0,\ E_{\alpha}^{(A)}[7,6] \to 0,\ E_{\alpha}^{(A)}[7,7] \to \sqrt{-g[7][7][x0,x4]}\}$$

In[∘]:=
```
(*Symbolize[sge_α^(A)]*)
```

In[∘]:=
```
(*Symbolize[sgtrye_α^(A)]*)
```

In[∘]:=
```
(sgtrye_α^(A) = Thread[Flatten[e_α^(A)] → Flatten[eAa04]]) (*//Column*)
```

Out[∘]=

$$\{E_{\alpha}^{(A)}[0,0] \to \mathrm{Cot}[6\,H\,x0],\ E_{\alpha}^{(A)}[0,1] \to 0,\ E_{\alpha}^{(A)}[0,2] \to 0,\ E_{\alpha}^{(A)}[0,3] \to 0,$$
$$E_{\alpha}^{(A)}[0,4] \to 0,\ E_{\alpha}^{(A)}[0,5] \to 0,\ E_{\alpha}^{(A)}[0,6] \to 0,\ E_{\alpha}^{(A)}[0,7] \to 0,\ E_{\alpha}^{(A)}[1,0] \to 0,$$
$$E_{\alpha}^{(A)}[1,1] \to \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6},\ E_{\alpha}^{(A)}[1,2] \to 0,\ E_{\alpha}^{(A)}[1,3] \to 0,\ E_{\alpha}^{(A)}[1,4] \to 0,$$
$$E_{\alpha}^{(A)}[1,5] \to 0,\ E_{\alpha}^{(A)}[1,6] \to 0,\ E_{\alpha}^{(A)}[1,7] \to 0,\ E_{\alpha}^{(A)}[2,0] \to 0,\ E_{\alpha}^{(A)}[2,1] \to 0,$$
$$E_{\alpha}^{(A)}[2,2] \to \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6},\ E_{\alpha}^{(A)}[2,3] \to 0,\ E_{\alpha}^{(A)}[2,4] \to 0,\ E_{\alpha}^{(A)}[2,5] \to 0,$$
$$E_{\alpha}^{(A)}[2,6] \to 0,\ E_{\alpha}^{(A)}[2,7] \to 0,\ E_{\alpha}^{(A)}[3,0] \to 0,\ E_{\alpha}^{(A)}[3,1] \to 0,\ E_{\alpha}^{(A)}[3,2] \to 0,$$
$$E_{\alpha}^{(A)}[3,3] \to \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6},\ E_{\alpha}^{(A)}[3,4] \to 0,\ E_{\alpha}^{(A)}[3,5] \to 0,\ E_{\alpha}^{(A)}[3,6] \to 0,$$
$$E_{\alpha}^{(A)}[3,7] \to 0,\ E_{\alpha}^{(A)}[4,0] \to 0,\ E_{\alpha}^{(A)}[4,1] \to 0,\ E_{\alpha}^{(A)}[4,2] \to 0,\ E_{\alpha}^{(A)}[4,3] \to 0,$$
$$E_{\alpha}^{(A)}[4,4] \to 1,\ E_{\alpha}^{(A)}[4,5] \to 0,\ E_{\alpha}^{(A)}[4,6] \to 0,\ E_{\alpha}^{(A)}[4,7] \to 0,\ E_{\alpha}^{(A)}[5,0] \to 0,\ E_{\alpha}^{(A)}[5,1] \to 0,$$
$$E_{\alpha}^{(A)}[5,2] \to 0,\ E_{\alpha}^{(A)}[5,3] \to 0,\ E_{\alpha}^{(A)}[5,4] \to 0,\ E_{\alpha}^{(A)}[5,5] \to \sqrt{e^{-2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6},$$
$$E_{\alpha}^{(A)}[5,6] \to 0,\ E_{\alpha}^{(A)}[5,7] \to 0,\ E_{\alpha}^{(A)}[6,0] \to 0,\ E_{\alpha}^{(A)}[6,1] \to 0,\ E_{\alpha}^{(A)}[6,2] \to 0,$$
$$E_{\alpha}^{(A)}[6,3] \to 0,\ E_{\alpha}^{(A)}[6,4] \to 0,\ E_{\alpha}^{(A)}[6,5] \to 0,\ E_{\alpha}^{(A)}[6,6] \to \sqrt{e^{-2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6},$$
$$E_{\alpha}^{(A)}[6,7] \to 0,\ E_{\alpha}^{(A)}[7,0] \to 0,\ E_{\alpha}^{(A)}[7,1] \to 0,\ E_{\alpha}^{(A)}[7,2] \to 0,\ E_{\alpha}^{(A)}[7,3] \to 0,$$
$$E_{\alpha}^{(A)}[7,4] \to 0,\ E_{\alpha}^{(A)}[7,5] \to 0,\ E_{\alpha}^{(A)}[7,6] \to 0,\ E_{\alpha}^{(A)}[7,7] \to \sqrt{e^{-2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}\}$$

In[ ]:=
```
(sgtrye^α_(A) = Thread[Flatten[e^α_(A)] → Flatten[Inverse[e^(A)_α /. sgtrye^(A)_α]]]) (*//Column*)
```

Out[ ]=

$$\left\{E^{\alpha}_{(A)}[0,0]\to \text{Tan}[6\,H\,x0],\ E^{\alpha}_{(A)}[0,1]\to 0,\ E^{\alpha}_{(A)}[0,2]\to 0,\ E^{\alpha}_{(A)}[0,3]\to 0,\right.$$
$$E^{\alpha}_{(A)}[0,4]\to 0,\ E^{\alpha}_{(A)}[0,5]\to 0,\ E^{\alpha}_{(A)}[0,6]\to 0,\ E^{\alpha}_{(A)}[0,7]\to 0,\ E^{\alpha}_{(A)}[1,0]\to 0,$$
$$E^{\alpha}_{(A)}[1,1]\to \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[1,2]\to 0,\ E^{\alpha}_{(A)}[1,3]\to 0,\ E^{\alpha}_{(A)}[1,4]\to 0,$$
$$E^{\alpha}_{(A)}[1,5]\to 0,\ E^{\alpha}_{(A)}[1,6]\to 0,\ E^{\alpha}_{(A)}[1,7]\to 0,\ E^{\alpha}_{(A)}[2,0]\to 0,\ E^{\alpha}_{(A)}[2,1]\to 0,$$
$$E^{\alpha}_{(A)}[2,2]\to \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[2,3]\to 0,\ E^{\alpha}_{(A)}[2,4]\to 0,\ E^{\alpha}_{(A)}[2,5]\to 0,$$
$$E^{\alpha}_{(A)}[2,6]\to 0,\ E^{\alpha}_{(A)}[2,7]\to 0,\ E^{\alpha}_{(A)}[3,0]\to 0,\ E^{\alpha}_{(A)}[3,1]\to 0,\ E^{\alpha}_{(A)}[3,2]\to 0,$$
$$E^{\alpha}_{(A)}[3,3]\to \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[3,4]\to 0,\ E^{\alpha}_{(A)}[3,5]\to 0,\ E^{\alpha}_{(A)}[3,6]\to 0,$$
$$E^{\alpha}_{(A)}[3,7]\to 0,\ E^{\alpha}_{(A)}[4,0]\to 0,\ E^{\alpha}_{(A)}[4,1]\to 0,\ E^{\alpha}_{(A)}[4,2]\to 0,\ E^{\alpha}_{(A)}[4,3]\to 0,$$
$$E^{\alpha}_{(A)}[4,4]\to 1,\ E^{\alpha}_{(A)}[4,5]\to 0,\ E^{\alpha}_{(A)}[4,6]\to 0,\ E^{\alpha}_{(A)}[4,7]\to 0,\ E^{\alpha}_{(A)}[5,0]\to 0,\ E^{\alpha}_{(A)}[5,1]\to 0,$$
$$E^{\alpha}_{(A)}[5,2]\to 0,\ E^{\alpha}_{(A)}[5,3]\to 0,\ E^{\alpha}_{(A)}[5,4]\to 0,\ E^{\alpha}_{(A)}[5,5]\to \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},$$
$$E^{\alpha}_{(A)}[5,6]\to 0,\ E^{\alpha}_{(A)}[5,7]\to 0,\ E^{\alpha}_{(A)}[6,0]\to 0,\ E^{\alpha}_{(A)}[6,1]\to 0,\ E^{\alpha}_{(A)}[6,2]\to 0,$$
$$E^{\alpha}_{(A)}[6,3]\to 0,\ E^{\alpha}_{(A)}[6,4]\to 0,\ E^{\alpha}_{(A)}[6,5]\to 0,\ E^{\alpha}_{(A)}[6,6]\to \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},$$
$$E^{\alpha}_{(A)}[6,7]\to 0,\ E^{\alpha}_{(A)}[7,0]\to 0,\ E^{\alpha}_{(A)}[7,1]\to 0,\ E^{\alpha}_{(A)}[7,2]\to 0,\ E^{\alpha}_{(A)}[7,3]\to 0,$$
$$\left.E^{\alpha}_{(A)}[7,4]\to 0,\ E^{\alpha}_{(A)}[7,5]\to 0,\ E^{\alpha}_{(A)}[7,6]\to 0,\ E^{\alpha}_{(A)}[7,7]\to \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}}\right\}$$

In[ ]:=
```
gtrye^(A)_α = (e^(A)_α /. sgtrye^(A)_α) /. subsDefects
```

Out[ ]=

$$\{\{\text{Cot}[6\,H\,x0],0,0,0,0,0,0,0\},\{0,e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0,0\},$$
$$\{0,0,e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0\},\{0,0,0,e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0,0,0,0\},$$
$$\{0,0,0,0,1,0,0,0\},\{0,0,0,0,0,e^{-a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0,0\},$$
$$\{0,0,0,0,0,0,e^{-a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},0\},\{0,0,0,0,0,0,0,e^{-a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6}\}\}$$

In[ ]:=
```
gtrye^α_(A) = (e^α_(A) /. sgtrye^α_(A)) /. subsDefects
```

Out[ ]=

$$\left\{\{\text{Tan}[6\,H\,x0],0,0,0,0,0,0,0\},\left\{0,\frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0,0\right\},\right.$$
$$\left\{0,0,\frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0\right\},\left\{0,0,0,\frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0\right\},$$
$$\{0,0,0,0,1,0,0,0\},\left\{0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0\right\},$$
$$\left.\left\{0,0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0\right\},\left\{0,0,0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}}\right\}\right\}$$

In[ ]:=
```
e_α^(A)
% /. sgtrye_α^(A) // MatrixForm
```

Out[ ]=

$$\{\{E_\alpha^{(A)}[0,0], E_\alpha^{(A)}[0,1], E_\alpha^{(A)}[0,2], E_\alpha^{(A)}[0,3], E_\alpha^{(A)}[0,4], E_\alpha^{(A)}[0,5], E_\alpha^{(A)}[0,6], E_\alpha^{(A)}[0,7]\},$$
$$\{E_\alpha^{(A)}[1,0], E_\alpha^{(A)}[1,1], E_\alpha^{(A)}[1,2], E_\alpha^{(A)}[1,3], E_\alpha^{(A)}[1,4], E_\alpha^{(A)}[1,5], E_\alpha^{(A)}[1,6], E_\alpha^{(A)}[1,7]\},$$
$$\{E_\alpha^{(A)}[2,0], E_\alpha^{(A)}[2,1], E_\alpha^{(A)}[2,2], E_\alpha^{(A)}[2,3], E_\alpha^{(A)}[2,4], E_\alpha^{(A)}[2,5], E_\alpha^{(A)}[2,6], E_\alpha^{(A)}[2,7]\},$$
$$\{E_\alpha^{(A)}[3,0], E_\alpha^{(A)}[3,1], E_\alpha^{(A)}[3,2], E_\alpha^{(A)}[3,3], E_\alpha^{(A)}[3,4], E_\alpha^{(A)}[3,5], E_\alpha^{(A)}[3,6], E_\alpha^{(A)}[3,7]\},$$
$$\{E_\alpha^{(A)}[4,0], E_\alpha^{(A)}[4,1], E_\alpha^{(A)}[4,2], E_\alpha^{(A)}[4,3], E_\alpha^{(A)}[4,4], E_\alpha^{(A)}[4,5], E_\alpha^{(A)}[4,6], E_\alpha^{(A)}[4,7]\},$$
$$\{E_\alpha^{(A)}[5,0], E_\alpha^{(A)}[5,1], E_\alpha^{(A)}[5,2], E_\alpha^{(A)}[5,3], E_\alpha^{(A)}[5,4], E_\alpha^{(A)}[5,5], E_\alpha^{(A)}[5,6], E_\alpha^{(A)}[5,7]\},$$
$$\{E_\alpha^{(A)}[6,0], E_\alpha^{(A)}[6,1], E_\alpha^{(A)}[6,2], E_\alpha^{(A)}[6,3], E_\alpha^{(A)}[6,4], E_\alpha^{(A)}[6,5], E_\alpha^{(A)}[6,6], E_\alpha^{(A)}[6,7]\},$$
$$\{E_\alpha^{(A)}[7,0], E_\alpha^{(A)}[7,1], E_\alpha^{(A)}[7,2], E_\alpha^{(A)}[7,3], E_\alpha^{(A)}[7,4], E_\alpha^{(A)}[7,5], E_\alpha^{(A)}[7,6], E_\alpha^{(A)}[7,7]\}\}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0] & 0 & 0 & 0 & 0 & \\ 0 & \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & 0 & 0 & \\ 0 & 0 & \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & 0 & \\ 0 & 0 & 0 & \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6} & 0 & \\ 0 & 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 0 & \sqrt{e^{-2\,a4}} \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{pmatrix}$$

In[ ]:=
```
eηe = FullSimplify[Transpose[e_α^(A)].η4488.e_α^(A), constraintVars];
% /. sge_α^(A)
%% /. sgtrye_α^(A)
% == gtry // FullSimplify[#, constraintVars] &
```

Out[ ]=

```
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}
```

Out[ ]=

$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

Out[ ]=

True

In[ ]:=
```
gtry
```

Out[ ]=

$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

In[ ]:=
```
(*FullSimplify[Transpose[e_α^(A)].η4488.e_α^(A),constraintVars]-MatrixMetric44*)
```

In[ ]:=
```
ssgm4488
```

Out[ ]=
```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
 g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
 g[1][1] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
 g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
 g[2][1] → (0 &), g[2][2] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
 g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
 g[3][2] → (0 &), g[3][3] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
 g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
 g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
 g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
 g[5][5] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
 g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
 g[6][5] → (0 &), g[6][6] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
 g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
 g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &)}
```

In[ ]:=
```
MatrixMetric44 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad -e^{-2\,a4[H\,x4]}\,S$$

In[ ]:=
```
e_α^(A)
```

Out[ ]=
```
{{E_α^(A)[0, 0], E_α^(A)[0, 1], E_α^(A)[0, 2], E_α^(A)[0, 3], E_α^(A)[0, 4], E_α^(A)[0, 5], E_α^(A)[0, 6], E_α^(A)[0, 7]},
 {E_α^(A)[1, 0], E_α^(A)[1, 1], E_α^(A)[1, 2], E_α^(A)[1, 3], E_α^(A)[1, 4], E_α^(A)[1, 5], E_α^(A)[1, 6], E_α^(A)[1, 7]},
 {E_α^(A)[2, 0], E_α^(A)[2, 1], E_α^(A)[2, 2], E_α^(A)[2, 3], E_α^(A)[2, 4], E_α^(A)[2, 5], E_α^(A)[2, 6], E_α^(A)[2, 7]},
 {E_α^(A)[3, 0], E_α^(A)[3, 1], E_α^(A)[3, 2], E_α^(A)[3, 3], E_α^(A)[3, 4], E_α^(A)[3, 5], E_α^(A)[3, 6], E_α^(A)[3, 7]},
 {E_α^(A)[4, 0], E_α^(A)[4, 1], E_α^(A)[4, 2], E_α^(A)[4, 3], E_α^(A)[4, 4], E_α^(A)[4, 5], E_α^(A)[4, 6], E_α^(A)[4, 7]},
 {E_α^(A)[5, 0], E_α^(A)[5, 1], E_α^(A)[5, 2], E_α^(A)[5, 3], E_α^(A)[5, 4], E_α^(A)[5, 5], E_α^(A)[5, 6], E_α^(A)[5, 7]},
 {E_α^(A)[6, 0], E_α^(A)[6, 1], E_α^(A)[6, 2], E_α^(A)[6, 3], E_α^(A)[6, 4], E_α^(A)[6, 5], E_α^(A)[6, 6], E_α^(A)[6, 7]},
 {E_α^(A)[7, 0], E_α^(A)[7, 1], E_α^(A)[7, 2], E_α^(A)[7, 3], E_α^(A)[7, 4], E_α^(A)[7, 5], E_α^(A)[7, 6], E_α^(A)[7, 7]}}
```

In[ ]:=
```
Block[{s, r}, s = FullSimplify[e_α^(A) /. sgtrye_α^(A), constraintVars];
 r = FullSimplify[Transpose[s].η4488.s, constraintVars] - MatrixMetric44;
 FullSimplify[r, constraintVars]]
```

Out[ ]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[ ]:=
```
(*(e_α^(A) =eAa)//MatrixForm*)
```

In[ ]:=
```
(*e_α^(A)
 Inverse[%]*)
```

In[ ]:=
```
(*(e^α_(A) =Inverse[e_α^(A) /.sge_Subsuperscript_α_and_LeftParenthesis_A_RightParenthesis])//
 MatrixForm*)
```

In[ ]:= $e^{\alpha}_{(A)}$

Out[ ]=

$$\begin{aligned}
&\{\{E^{\alpha}_{(A)}[0,0], E^{\alpha}_{(A)}[0,1], E^{\alpha}_{(A)}[0,2], E^{\alpha}_{(A)}[0,3], E^{\alpha}_{(A)}[0,4], E^{\alpha}_{(A)}[0,5], E^{\alpha}_{(A)}[0,6], E^{\alpha}_{(A)}[0,7]\},\\
&\{E^{\alpha}_{(A)}[1,0], E^{\alpha}_{(A)}[1,1], E^{\alpha}_{(A)}[1,2], E^{\alpha}_{(A)}[1,3], E^{\alpha}_{(A)}[1,4], E^{\alpha}_{(A)}[1,5], E^{\alpha}_{(A)}[1,6], E^{\alpha}_{(A)}[1,7]\},\\
&\{E^{\alpha}_{(A)}[2,0], E^{\alpha}_{(A)}[2,1], E^{\alpha}_{(A)}[2,2], E^{\alpha}_{(A)}[2,3], E^{\alpha}_{(A)}[2,4], E^{\alpha}_{(A)}[2,5], E^{\alpha}_{(A)}[2,6], E^{\alpha}_{(A)}[2,7]\},\\
&\{E^{\alpha}_{(A)}[3,0], E^{\alpha}_{(A)}[3,1], E^{\alpha}_{(A)}[3,2], E^{\alpha}_{(A)}[3,3], E^{\alpha}_{(A)}[3,4], E^{\alpha}_{(A)}[3,5], E^{\alpha}_{(A)}[3,6], E^{\alpha}_{(A)}[3,7]\},\\
&\{E^{\alpha}_{(A)}[4,0], E^{\alpha}_{(A)}[4,1], E^{\alpha}_{(A)}[4,2], E^{\alpha}_{(A)}[4,3], E^{\alpha}_{(A)}[4,4], E^{\alpha}_{(A)}[4,5], E^{\alpha}_{(A)}[4,6], E^{\alpha}_{(A)}[4,7]\},\\
&\{E^{\alpha}_{(A)}[5,0], E^{\alpha}_{(A)}[5,1], E^{\alpha}_{(A)}[5,2], E^{\alpha}_{(A)}[5,3], E^{\alpha}_{(A)}[5,4], E^{\alpha}_{(A)}[5,5], E^{\alpha}_{(A)}[5,6], E^{\alpha}_{(A)}[5,7]\},\\
&\{E^{\alpha}_{(A)}[6,0], E^{\alpha}_{(A)}[6,1], E^{\alpha}_{(A)}[6,2], E^{\alpha}_{(A)}[6,3], E^{\alpha}_{(A)}[6,4], E^{\alpha}_{(A)}[6,5], E^{\alpha}_{(A)}[6,6], E^{\alpha}_{(A)}[6,7]\},\\
&\{E^{\alpha}_{(A)}[7,0], E^{\alpha}_{(A)}[7,1], E^{\alpha}_{(A)}[7,2], E^{\alpha}_{(A)}[7,3], E^{\alpha}_{(A)}[7,4], E^{\alpha}_{(A)}[7,5], E^{\alpha}_{(A)}[7,6], E^{\alpha}_{(A)}[7,7]\}\}
\end{aligned}$$

In[ ]:=
```
(sge^α_(A) = Thread[Flatten[e^α_(A)] → Flatten[Inverse[e_α^(A) /. sge_α^(A)]]])(*//Column*)
```

Out[ ]=

$$\begin{aligned}
&\{E^{\alpha}_{(A)}[0,0] \to \frac{1}{\sqrt{g[0][0][x0,x4]}}, E^{\alpha}_{(A)}[0,1] \to 0, E^{\alpha}_{(A)}[0,2] \to 0, E^{\alpha}_{(A)}[0,3] \to 0, E^{\alpha}_{(A)}[0,4] \to 0,\\
&E^{\alpha}_{(A)}[0,5] \to 0, E^{\alpha}_{(A)}[0,6] \to 0, E^{\alpha}_{(A)}[0,7] \to 0, E^{\alpha}_{(A)}[1,0] \to 0, E^{\alpha}_{(A)}[1,1] \to \frac{1}{\sqrt{g[1][1][x0,x4]}},\\
&E^{\alpha}_{(A)}[1,2] \to 0, E^{\alpha}_{(A)}[1,3] \to 0, E^{\alpha}_{(A)}[1,4] \to 0, E^{\alpha}_{(A)}[1,5] \to 0, E^{\alpha}_{(A)}[1,6] \to 0, E^{\alpha}_{(A)}[1,7] \to 0,\\
&E^{\alpha}_{(A)}[2,0] \to 0, E^{\alpha}_{(A)}[2,1] \to 0, E^{\alpha}_{(A)}[2,2] \to \frac{1}{\sqrt{g[2][2][x0,x4]}}, E^{\alpha}_{(A)}[2,3] \to 0, E^{\alpha}_{(A)}[2,4] \to 0,\\
&E^{\alpha}_{(A)}[2,5] \to 0, E^{\alpha}_{(A)}[2,6] \to 0, E^{\alpha}_{(A)}[2,7] \to 0, E^{\alpha}_{(A)}[3,0] \to 0, E^{\alpha}_{(A)}[3,1] \to 0, E^{\alpha}_{(A)}[3,2] \to 0,\\
&E^{\alpha}_{(A)}[3,3] \to \frac{1}{\sqrt{g[3][3][x0,x4]}}, E^{\alpha}_{(A)}[3,4] \to 0, E^{\alpha}_{(A)}[3,5] \to 0, E^{\alpha}_{(A)}[3,6] \to 0, E^{\alpha}_{(A)}[3,7] \to 0,\\
&E^{\alpha}_{(A)}[4,0] \to 0, E^{\alpha}_{(A)}[4,1] \to 0, E^{\alpha}_{(A)}[4,2] \to 0, E^{\alpha}_{(A)}[4,3] \to 0, E^{\alpha}_{(A)}[4,4] \to \frac{1}{\sqrt{-g[4][4][x0,x4]}},\\
&E^{\alpha}_{(A)}[4,5] \to 0, E^{\alpha}_{(A)}[4,6] \to 0, E^{\alpha}_{(A)}[4,7] \to 0, E^{\alpha}_{(A)}[5,0] \to 0, E^{\alpha}_{(A)}[5,1] \to 0, E^{\alpha}_{(A)}[5,2] \to 0,\\
&E^{\alpha}_{(A)}[5,3] \to 0, E^{\alpha}_{(A)}[5,4] \to 0, E^{\alpha}_{(A)}[5,5] \to \frac{1}{\sqrt{-g[5][5][x0,x4]}}, E^{\alpha}_{(A)}[5,6] \to 0, E^{\alpha}_{(A)}[5,7] \to 0,\\
&E^{\alpha}_{(A)}[6,0] \to 0, E^{\alpha}_{(A)}[6,1] \to 0, E^{\alpha}_{(A)}[6,2] \to 0, E^{\alpha}_{(A)}[6,3] \to 0, E^{\alpha}_{(A)}[6,4] \to 0, E^{\alpha}_{(A)}[6,5] \to 0,\\
&E^{\alpha}_{(A)}[6,6] \to \frac{1}{\sqrt{-g[6][6][x0,x4]}}, E^{\alpha}_{(A)}[6,7] \to 0, E^{\alpha}_{(A)}[7,0] \to 0, E^{\alpha}_{(A)}[7,1] \to 0, E^{\alpha}_{(A)}[7,2] \to 0,\\
&E^{\alpha}_{(A)}[7,3] \to 0, E^{\alpha}_{(A)}[7,4] \to 0, E^{\alpha}_{(A)}[7,5] \to 0, E^{\alpha}_{(A)}[7,6] \to 0, E^{\alpha}_{(A)}[7,7] \to \frac{1}{\sqrt{-g[7][7][x0,x4]}}\}
\end{aligned}$$

In[ ]:=
```
(*Symbolize[sge^α_(A)]*)
```

In[∘]:=
```
(*Symbolize[sgtrye^α_(A)]*)
```

In[∘]:=
```
(*%/.sge^(A)_α
 %%/.sgtrye^(A)_α*)
```

In[∘]:=
```
(*FullSimplify[Transpose[e^(A)_α].η4488.e^(A)_α,constraintVars]-MatrixMetric44*)
```

For Spin (4, 4); τ tau; T16; OCTAD : Nash; Introduce the wave function, Ψ16, for this Universe::

O(4,4); evalues, evecs of σ

In[∘]:=
```
{evalues, evecs} = Eigensystem[σ]
```

Out[∘]=
```
{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
  {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
  {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}
```

In[∘]:=
```
(u = ExpandAll[1/√2 evecs]) // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

In[∘]:=
```
Table[u[[h]].σ.u[[h]], {h, 1, Length[u]}]
```

Out[∘]=
```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[∘]:=
```
Table[(Transpose[u][[h]]).σ.u[[h]], {h, 1, Length[u]}]
```

Out[∘]=
```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[∘]:=
```
Table[(Transpose[u][[A]]) == -σ.u[[A]], {A, 1, 4}]
Table[(Transpose[u][[A]]) == σ.u[[A]], {A, 5, Length[u]}]
```

Out[∘]=
```
{True, True, True, True}
```

Out[∘]=
```
{True, True, True, True}
```

```
Ax = (1/2 (KroneckerProduct[dX, X] -
```

```
        KroneckerProduct[X, dX])).σ;
Au = (1/2 (KroneckerProduct[dU, U] -
        KroneckerProduct[U, dU])).σ;
Lsquared = ẋ^A ẋ_A = dX.σ.dX +
    X.σ.X (dU.σ.dU/(U.σ.U) - (U.σ.dU/(U.σ.U))^2) + 2/(U.σ.U) Tr[τ̄^A.Ax.τ_A.Au]
Timing[
 Simplify[ExpandAll[dY.η8.dY - Lsquared]]]
{10.733, 0}
```

In[∘]:=
```
FullSimplify[Sqrt[Flatten[MatrixMetric44.η4488]], constraintVars]
```

Out[∘]=

$$\{\mathrm{Cot}[6\,H\,x0],\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$$
$$\sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ 0,\ 0,\ 0,$$
$$0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$$
$$0,\ 0,\ \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\}$$

SO(4), γ; M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+,+,+,+,-,-,-,-}, {x0, x1, x2, x3, x4, x5, x6, x7}

In[∘]:=
```
Qa[h_, p_, q_] := Signature[{h, p, q, 4}];
Qb[h_, p_, q_] := ID4[[p, 4]] * ID4[[q, h]] - ID4[[p, h]] * ID4[[q, 4]];
SelfDualAntiSymmetric[h_, p_, q_] := Qa[h, p, q] - Qb[h, p, q] ;
AntiSelfDualAntiSymmetric[h_, p_, q_] := (Qa[h, p, q] + Qb[h, p, q] );
```

In[∘]:=
```
Qa[1, 2, 3]
```

Out[∘]=
```
1
```

In[∘]:=
```
ϵ4[[1, 2, 3, 4]]
```

Out[∘]=
```
1
```

In[∘]:=
```
Do[s4by4[h] = Table[Table[SelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
 {h, 1, 3} ]
```

In[ ]:=
```
Do[
 t4by4[h] = Table[Table[AntiSelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
 {h, 1, 3} ]
```

In[ ]:=
```
(*Protect[{s4by4[1],s4by4[2],s4by4[3],t4by4[1],t4by4[2],t4by4[3]}]*)
```

In[ ]:=
```
Protect[s4by4, t4by4]
```

Out[ ]=

{s4by4, t4by4}

In[ ]:=
```
{s4by4[#] // MatrixForm, MatrixForm[t4by4[#]]} & /@ Range[3]
```

Out[ ]=

$$\left\{\left\{\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}\right\},\right.$$

$$\left.\left\{\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\right\}, \left\{\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\right\}\right\}$$

In[ ]:=
```
Table[{{J, K}, MatrixForm[s4by4[J].t4by4[K]]}, {J, 1, 3}, {K, 1, 3}]
```

Out[ ]=

$$\left\{\left\{\left\{\{1, 1\}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}, \left\{\{1, 2\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\right\}, \left\{\{1, 3\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}\right\}\right\},\right.$$

$$\left\{\left\{\{2, 1\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\right\}, \left\{\{2, 2\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}, \left\{\{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}\right\}\right\},$$

$$\left.\left\{\left\{\{3, 1\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\right\}, \left\{\{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}\right\}, \left\{\{3, 3\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}\right\}\right\}$$

In[ ]:=
```
Do[st[J, K] = s4by4[J].t4by4[K], {J, 1, 3}, {K, 1, 3}]
```

In[ ]:=
```
Table[{{J, K}, MatrixForm[st[J, K]]}, {J, 1, 3}, {K, 1, 3}]
```

Out[ ]=

$$\left\{\left\{\left\{\{1, 1\}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}, \left\{\{1, 2\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\right\}, \left\{\{1, 3\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}\right\}\right\},\right.$$

$$\left\{\left\{\{2, 1\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\right\}, \left\{\{2, 2\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}, \left\{\{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}\right\}\right\},$$

$$\left.\left\{\left\{\{3, 1\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\right\}, \left\{\{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}\right\}, \left\{\{3, 3\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}\right\}\right\}$$

In[ ]:=
```
Protect[st]
```

Out[ ]=
```
{st}
```

In[ ]:=
```
Table[{{J, K}, MatrixForm[st[J, K] - Transpose[st[J, K]]]}, {J, 1, 3}, {K, 1, 3}]
```

Out[ ]=

$$\left\{\left\{\left\{\{1,1\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{1,2\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{1,3\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\}\right\},\right.$$

$$\left\{\left\{\{2,1\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{2,2\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{2,3\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\}\right\},$$

$$\left.\left\{\left\{\{3,1\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{3,2\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\},\left\{\{3,3\},\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\right\}\right\}\right\}$$

In[ ]:=
```
Permutations[Range[3], 2]
```

Out[ ]=
```
{{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}
```

In[ ]:=
```
KroneckerProduct[Range[3], Range[3]]
```

Out[ ]=
```
{{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}
```

In[ ]:=
```
Table[{J, K}, {J, 1, 3}, {K, 1, 3}]
st[##] & /@ %
st[##] & @@ %%
```

Out[ ]=
```
{{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}}
```

Out[ ]=
```
{st[{{1, 1}, {1, 2}, {1, 3}}], st[{{2, 1}, {2, 2}, {2, 3}}], st[{{3, 1}, {3, 2}, {3, 3}}]}
```

Out[ ]=
```
st[{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}]
```

In[ ]:=
```
Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
Length[%]
```

Out[ ]=
```
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

Out[ ]=
```
16
```

In[ ]:=
```
Unprotect[γ];
γ = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
   Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
```

Out[ ]=
```
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

In[ ]:=
```
Length[γ]
```

Out[ ]=
```
16
```

In[ ]:=
```
Protect[γ]
```
Out[ ]=
```
{γ}
```
In[ ]:=
```
γ[[1]]
```
Out[ ]=
```
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}
```
In[ ]:=
```
γ[[-1]]
```
Out[ ]=
```
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Self Dual Anti-Symmetric

In[ ]:=
$$\text{Table}\left[\left(\frac{1}{2}\sum_{j1=1}^{4}\left(\sum_{j2=1}^{4}\epsilon 4[[p,q,j1,j2]]\times \text{s4by4}[h][[j1,j2]]\right)\right)-\text{s4by4}[h][[p,q]],\ \{h,1,3\},\ \{q,4\},\ \{p,4\}\right]$$

Out[ ]=
```
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}
```

Anti SelfDual Anti-Symmetric

In[ ]:=
$$\text{Table}\left[\left(\frac{1}{2}\sum_{j1=1}^{4}\left(\sum_{j2=1}^{4}\epsilon 4[[p,q,j1,j2]]\times \text{t4by4}[h][[j1,j2]]\right)\right)+\text{t4by4}[h][[p,q]],\ \{h,1,3\},\ \{q,4\},\ \{p,4\}\right]$$

Out[ ]=
```
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}
```

SO(4,4), Spin(4,4) = $\overline{\text{SO}(4,4)}$, τ ; if using xact: M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+,+,+,+,-,-,-,-}, {x0, x1, x2, x3, x4, x5, x6, x7}

In[ ]:=
```
τ̄ === τ̄
```
Out[ ]=
```
False
```
In[ ]:=
```
τ̄
Head[%]
```
Out[ ]=
```
τ̄
```
Out[ ]=
```
Symbol
```

In[ ]:=
```
τ̄
Head[%]
```

Out[ ]=
```
τ̄
```

Out[ ]=
```
ParsedBoxWrapper
```

In[ ]:=
```
τ̄
Head[%]
```

Out[ ]=
```
τ̄
```

Out[ ]=
```
Symbol
```

In[ ]:=
```
Symbol
```

Out[ ]=
```
Symbol
```

In[ ]:=
```
τ̄ === τ̄
```

Out[ ]=
```
True
```

In[ ]:=
```
τ̄ === τ̄
```

Out[ ]=
```
False
```

In[ ]:=
```
η4488 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[ ]:=
```
η4488 == DiagonalMatrix[RotateRight[evalues, 4]]
```

Out[ ]=
```
True
```

In[ ]:=
```
DiagonalMatrix[RotateRight[evalues, 3]] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

these also yield correct reps:

```
ζ[0] = ID8;
Table[ζ[7 - h] = ArrayFlatten[{{0, -t4by4[h]}, {t4by4[h], 0}}],
{h, 1, 3} ];
Table[ζ[h] = ArrayFlatten[{{0, s4by4[h]}, {s4by4[h], 0}}], {h,
1, 3} ];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm
```

```
ζ[0] = ID8;
Table[ζ[h] = ArrayFlatten[{{0, t4by4[h]}, {t4by4[h], 0}}], {h, 1,
3} ];
Table[ζ[7 - h] = ArrayFlatten[{{0, -s4by4[h]}, {s4by4[h], 0}}],
{h, 1, 3} ];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm
```

```
ζ[0] = ID8;
Table[ζ[h] = ArrayFlatten[{{0, t4by4[h]}, {t4by4[h], 0}}], {h, 1,
3} ];
Table[ζ[7 - h] = ArrayFlatten[{{0, s4by4[h]}, {-s4by4[h], 0}}],
{h, 1, 3} ];
(ζ[7] = ζ[1].ζ[2].ζ[3].ζ[4].ζ[5].ζ[6]) // MatrixForm
```

In[ ]:=
```
ξ[0] = ID8;
Table[
  ξ[7 - h] = ArrayFlatten[{{0, t4by4[h]}, {-t4by4[h], 0}}], {h, 1, 3} ];
Table[ξ[h] = ArrayFlatten[{{0, s4by4[h]}, {s4by4[h], 0}}], {h, 1, 3} ];
(ξ[7] = ξ[1].ξ[2].ξ[3].ξ[4].ξ[5].ξ[6]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
σ == ξ[1].ξ[2].ξ[3]
```

Out[ ]=
```
True
```

In[∘]:=
```
Table[{A, B, FullSimplify[
    ExpandAll[1/2 (ξ[A].ξ[B] + ξ[B].ξ[A]) == -η4488[[A + 1, B + 1]] * ID8]]},
  {A, 1, 7}, {B, 1, 7}]
```

Out[∘]=
```
{{{1, 1, True}, {1, 2, True}, {1, 3, True}, {1, 4, True},
  {1, 5, True}, {1, 6, True}, {1, 7, True}}, {{2, 1, True}, {2, 2, True},
  {2, 3, True}, {2, 4, True}, {2, 5, True}, {2, 6, True}, {2, 7, True}},
 {{3, 1, True}, {3, 2, True}, {3, 3, True}, {3, 4, True}, {3, 5, True}, {3, 6, True},
  {3, 7, True}}, {{4, 1, True}, {4, 2, True}, {4, 3, True}, {4, 4, True},
  {4, 5, True}, {4, 6, True}, {4, 7, True}}, {{5, 1, True}, {5, 2, True},
  {5, 3, True}, {5, 4, True}, {5, 5, True}, {5, 6, True}, {5, 7, True}},
 {{6, 1, True}, {6, 2, True}, {6, 3, True}, {6, 4, True}, {6, 5, True},
  {6, 6, True}, {6, 7, True}}, {{7, 1, True}, {7, 2, True},
  {7, 3, True}, {7, 4, True}, {7, 5, True}, {7, 6, True}, {7, 7, True}}}
```

In[∘]:=
```
η4488 // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
Table[{A, -η4488[[A + 1, A + 1]] * ξ[A] == Transpose[ξ[A]]} , {A, 1, 7}]
```

Out[∘]=
```
{{1, True}, {2, True}, {3, True}, {4, True}, {5, True}, {6, True}, {7, True}}
```

In[∘]:=
```
(σ == ξ[1].ξ[2].ξ[3])
```

Out[∘]=
```
True
```

In[∘]:=
```
σ.ξ[1].ξ[2].ξ[3] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
τ[0] = ID8;
Do[
 {τ[A] = FullSimplify[ExpandAll[(ξ[A])]]} ,
 {A, 1, 7}]
(*Do[
 {τ[A] = FullSimplify[ExpandAll[-(σ.ξ[1].
        ξ[2].ξ[3].ξ[A])]]} , {A, 1, 7}]*)
```

In[ ]:=
```
(σ == τ[1].τ[2].τ[3])
```

Out[ ]=
```
True
```

In[ ]:=
```
(*Symbolize[τ̅]*)
```

In[ ]:=
```
τ̅[0] = ID8;
Do[{τ̅[A] = FullSimplify[ExpandAll[
      (σ.Transpose[τ[A]].σ)]]} , {A, 1, 7}]
```

In[ ]:=
```
σ.τ̅[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
```

Out[ ]=
```
{True, True, True, True, True, True, True, True}
```

In[ ]:=
```
Table[{A, B, FullSimplify[
    ExpandAll[1/2 (τ[A].τ̄[B] + τ[B].τ̄[A]) == η4488[[A + 1, B + 1]] * ID8]]},
  {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
\begin{pmatrix}0\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}0\\7\\\text{True}\end{pmatrix}\\
\begin{pmatrix}1\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}1\\7\\\text{True}\end{pmatrix}\\
\begin{pmatrix}2\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}2\\7\\\text{True}\end{pmatrix}\\
\begin{pmatrix}3\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}3\\7\\\text{True}\end{pmatrix}\\
\begin{pmatrix}4\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}4\\7\\\text{True}\end{pmatrix}\\
\begin{pmatrix}5\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}5\\7\\\text{True}\end{pmatrix}\\
\begin{pmatrix}6\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}6\\7\\\text{True}\end{pmatrix}\\
\begin{pmatrix}7\\0\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\1\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\2\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\3\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\4\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\5\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\6\\\text{True}\end{pmatrix} & \begin{pmatrix}7\\7\\\text{True}\end{pmatrix}
\end{pmatrix}$$

In[ ]:=
```
τ[7] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

In[ ]:=
```
(Ω = σ.τ[7]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0
\end{pmatrix}$$

In[ ]:=
```
Ω == τ[4].τ[5].τ[6]
```

Out[ ]=

True

In[ ]:=
```
τ[5].τ[6].τ[7] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

In[ ]:=
```
(σΩ = σ.Ω) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
Inverse[σΩ] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
τ[1].τ[2].τ[3].τ[4].τ[5].τ[6].τ[7] == τ[0] == ID8
```

Out[ ]=

True

In[ ]:=
```
(σ == τ[1].τ[2].τ[3])
```

Out[ ]=

True

In[ ]:=
```
(σ == τ[4].τ[5].τ[6].τ[7])
```

Out[ ]=

True

In[ ]:=
```
τ[1].τ[2].τ[3].τ[#] == -Transpose[τ[1].τ[2].τ[3].τ[#]] & /@ Range[0, 7]
```

Out[ ]=

{False, True, True, True, True, True, True, True}

In[ ]:=
```
σ.τ̄[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
```
Out[ ]=
```
{True, True, True, True, True, True, True, True}
```
In[ ]:=
```
τ[5].τ[6].τ[7].τ̄[#] == Transpose[τ[5].τ[6].τ[7].τ[#]] & /@
 Range[0, 7]
```
Out[ ]=
```
{False, True, True, True, False, True, True, True}
```
In[ ]:=
```
τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```
Out[ ]=
```
{True, True, True, True, True, False, False, False}
```
In[ ]:=
```
τ[4].τ̄[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```
Out[ ]=
```
{True, False, False, False, False, True, True, True}
```
In[ ]:=
```
τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
```
Out[ ]=
```
{True, True, True, True, True, False, False, False}
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$; O (4, 4) : SAB ;

covariantDiffMatrix = $T16^A[5].T16^A[6].T16^A[7]$

$S = \begin{pmatrix} a & b \\ c & d \end{pmatrix};$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} . \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix} . \begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix} = \begin{pmatrix} b.\tau^A & a.\bar{\tau}^A \\ d.\tau^A & c.\bar{\tau}^A \end{pmatrix} . \begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix}$$

$$= \begin{pmatrix} b.\tau^A.\tilde{a} + a.\bar{\tau}^A.\tilde{b} & b.\tau^A.\tilde{c} + a.\bar{\tau}^A.\tilde{d} \\ d.\tau^A.\tilde{a} + c.\bar{\tau}^A.\tilde{b} & d.\tau^A.\tilde{c} + c.\bar{\tau}^A.\tilde{d} \end{pmatrix}$$

In[ ]:=
```
O8similarityTransformation // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

In[ ]:=
```
blocks = Partition[O8similarityTransformation, {8, 8}];
```

In[ ]:=
```
(*To access the individual blocks:*)
block11 = blocks[[1, 1]] (*Top-left block*)
block12 = blocks[[1, 2]] (*Top-right block*)
block21 = blocks[[2, 1]] (*Bottom-left block*)
block22 = blocks[[2, 2]] (*Bottom-right block*)
```

Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

Out[ ]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}}
```

Out[ ]=
```
{{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

Out[ ]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[ ]:=
```
ArrayFlatten[{{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}] ===
 O8similarityTransformation
```

Out[ ]=
```
True
```

In[ ]:=

$$\begin{pmatrix} \text{blocks}[[1,1]] & \text{blocks}[[1,2]] \\ \text{blocks}[[2,1]] & \text{blocks}[[2,2]] \end{pmatrix} . \begin{pmatrix} 0 & \overline{\tau}^{A} \\ \tau^{A} & 0 \end{pmatrix} . \begin{pmatrix} \tilde{\text{blocks}}[[1,1]] & \tilde{\text{blocks}}[[2,1]] \\ \tilde{\text{blocks}}[[1,2]] & \tilde{\text{blocks}}[[2,2]] \end{pmatrix} =$$

$$\begin{pmatrix} \text{blocks}[[1,2]].\tau^{A} & \text{blocks}[[1,1]].\overline{\tau}^{A} \\ \text{blocks}[[2,2]].\tau^{A} & \text{blocks}[[2,1]].\overline{\tau}^{A} \end{pmatrix} .$$

$$\begin{pmatrix} \widetilde{\text{blocks}}[[1,1]] & \widetilde{\text{blocks}}[[2,1]] \\ \widetilde{\text{blocks}}[[1,2]] & \widetilde{\text{blocks}}[[2,2]] \end{pmatrix}$$

$$= \begin{pmatrix} \text{blocks}[[1,2]].\tau^A.\widetilde{\text{blocks}}[[1,1]] + \text{blocks}[[1,1]]. \\ \text{blocks}[[2,2]].\tau^A.\widetilde{\text{blocks}}[[1,1]] + \text{blocks}[[2,1]]. \end{pmatrix}$$

In[∘]:=
```
Table[
 newT16^A[A1] === ArrayFlatten[{{blocks[[1, 2]].τ[A1].Transpose[blocks[[1, 1]]] + blocks[[1, 1]].
      τ̄[A1].Transpose[blocks[[1, 2]]], blocks[[1, 2]].τ[A1].Transpose[blocks[[2, 1]]] +
     blocks[[1, 1]].τ̄[A1].Transpose[blocks[[2, 2]]]},
    {blocks[[2, 2]].τ[A1].Transpose[blocks[[1, 1]]] + blocks[[2, 1]].τ̄[A1].
      Transpose[blocks[[1, 2]]], blocks[[2, 2]].τ[A1].Transpose[blocks[[2, 1]]] +
     blocks[[2, 1]].τ̄[A1].Transpose[blocks[[2, 2]]]}}], {A1, 0, 7}]
```

Out[∘]=
```
{False, False, False, False, False, False, False, False}
```

In[∘]:=
```
MatrixForm[newT16^A[#]] & /@ Range[0, 8]
```

Out[∘]=
```
{newT16^A[0], newT16^A[1], newT16^A[2], newT16^A[3],
 newT16^A[4], newT16^A[5], newT16^A[6], newT16^A[7], newT16^A[8]}
```

In[∘]:=
```
(*Example 16x16 matrix*)(*matrix=Table[RandomInteger[{1,100}],{16},{16}];*)

(*Partition the matrix into 8x8 blocks*)
(*blocks=Partition[matrix,{8,8}];*)

(*The result is a 2x2 list of 8x8 matrices*)
(*blocks*)
```

In[∘]:=
```
(*TakeList[O8similarityTransformation,{8,8}]*)
```

blocks = TakeList[O8similarityTransformation, {8, 8}, {8, 8}]
ArrayFlatten[%] === O8similarityTransformation

Out[∘]=

{{{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0,

0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}}}

Out[ ]=

True

In[ ]:=
```
Length[blocks]
```

Out[ ]=

2

In[ ]:=
```
(*O8similarityTransformation==ArrayFlatten[{{Take[],Take[]},{Take[],Take[]}}]*)
```

Take [*list*, seq_1, seq_2]

gives a nested list in which elements specified by seq_i are taken at level i in *list*.

effectively extracts a submatrix from *list*.

In[ ]:=
```
Take[]
```

Take: Take called with 0 arguments; 1 or more arguments are expected.

Out[ ]=

Take[]

Need these in order to define the Lagrangian for the "universe," later :

In[ ]:=
```
(σ16 = T16^A[0].T16^A[1].T16^A[2].T16^A[3]) // MatrixForm
```

Out[ ]//MatrixForm=

$T16^A[0].T16^A[1].T16^A[2].T16^A[3]$

In[ ]:=
```
Table[T16^A[A1] = ArrayFlatten[{{0, τ̄[A1]}, {τ[A1], 0}}],
  {A1, 0, 7}];
```

In[ ]:=
```
(T16^A[8] = T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
Table[newT16^A[A1] = O8similarityTransformation. T16^A[A1].
    Transpose[O8similarityTransformation], {A1, 0, 8}];
```

In[ ]:=
```
Transpose[Ψ16]. σ16 ===
 Transpose[Transpose[O8similarityTransformation].O8similarityTransformation.Ψ16]. σ16 ===
 Transpose[O8similarityTransformation.Ψ16].O8similarityTransformation. σ16
```

Out[ ]=

True

In[ ]:=
```
Transpose[Ψ16]. σ16 .Transpose[O8similarityTransformation] ===
 Transpose[Transpose[O8similarityTransformation].O8similarityTransformation.Ψ16]. σ16.
  Transpose[O8similarityTransformation] === Transpose[O8similarityTransformation.Ψ16].
  O8similarityTransformation. σ16.Transpose[O8similarityTransformation]
```

Out[ ]=

True

In[ ]:=
```
(newσ16 = O8similarityTransformation. σ16.Transpose[O8similarityTransformation]) //
 MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
σ16 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#] ==
   -Transpose[T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#]] & /@ Range[0, 8]
```

Out[ ]=

{True, True, True, True, True, True, True, True, False}

In[ ]:=
```
T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#] ==
   Transpose[T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#]] & /@ Range[0, 8]
```

Out[ ]=

{True, True, True, True, True, True, True, True, True}

In[ ]:=
```
T16^A[8] == σ16.T16^A[4].T16^A[5].T16^A[6].T16^A[7]
```

Out[ ]=

True

In[∘]:=
```
T16^A[#] == Transpose[T16^A[#]] & /@ Range[0, 8]
```

Out[∘]=
```
{True, True, True, True, False, False, False, False, True}
```

In[∘]:=
```
T16^A[#] == -Transpose[T16^A[#]] & /@ Range[0, 8]
```

Out[∘]=
```
{False, False, False, False, True, True, True, True, False}
```

In[∘]:=
```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
```

Out[∘]=
```
{True, True, True, True, True, True, True, True}
```

In[∘]:=
```
(covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]) // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[∘]:=
```
σ16.covariantDiffMatrix == Transpose[σ16.covariantDiffMatrix]
```

Out[∘]=
```
True
```

In[∘]:=
```
ID16 // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
(σ16(*=T16^A[0].T16^A[1].T16^A[2].T16^A[3]*)) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

Out[ ]=

True

later, after defining $e^{\alpha}_{(A)}$:

In[ ]:=
```
(*Table[T16^α[α1-1]=
   Sum[(e^α_(A)[[α1,A1]]) T16^A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

In[ ]:=
```
(*(T16^α[8]=T16^α[0].T16^α[1].T16^α[2].T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7])//
 MatrixForm*)
```

In[ ]:=
```
(*eLpairs*)
```

In[ ]:=
```
(*Ω16a=Array[0&,{16,16}];
 {1+#[[1]],1+#[[2]]}&/@eLpairs
 (Ω16a[[#[[1]],#[[2]]]]=1)&/@%*)
```

In[ ]:=
```
(*Ω16a//MatrixForm*)
```

In[ ]:=
```
(*(Ω16=σ16.(Ω16a+Transpose[Ω16a]))//MatrixForm*)
```

In[ ]:=
```
T16^A[0].T16^A[4] // MatrixForm
T16^A[0].T16^A[4].σ16
(Ω16 = %) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[ ]=

```
{{0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}}
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
Det[Ω16]
```

Out[ ]=

1

In[ ]:=
```
σ16.Ω16 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@
 Range[0, 7]
```

Out[ ]=
```
{True, True, True, True, True, True, True, True}
```

In[ ]:=
```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
```

Out[ ]=
```
{True, True, True, True, True, True, True, True}
```

In[ ]:=
```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
```

Out[ ]=
```
{True, True, True, True, True, True, True, True}
```

In[ ]:=
```
σ16.newT16^A[#] == -Transpose[σ16.newT16^A[#]] & /@ Range[0, 7]
```

Out[ ]=
```
{False, False, False, True, False, True, True, False}
```

In[ ]:=
```
newσ16.newT16^A[#] == -Transpose[newσ16.newT16^A[#]] & /@ Range[0, 7]
```

Out[ ]=
```
{True, True, True, True, True, True, True, True}
```

In[ ]:=
```
Table[{{A1, B1}, FullSimplify[
    ExpandAll[1/2 (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) ==
      η4488[[A1 + 1, B1 + 1]] * ID16]]},
  {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[ ]//MatrixForm=

{0, 0} True	{0, 1} True	{0, 2} True	{0, 3} True	{0, 4} True	{0, 5} True	{0, 6} True	{0, 7} True
{1, 0} True	{1, 1} True	{1, 2} True	{1, 3} True	{1, 4} True	{1, 5} True	{1, 6} True	{1, 7} True
{2, 0} True	{2, 1} True	{2, 2} True	{2, 3} True	{2, 4} True	{2, 5} True	{2, 6} True	{2, 7} True
{3, 0} True	{3, 1} True	{3, 2} True	{3, 3} True	{3, 4} True	{3, 5} True	{3, 6} True	{3, 7} True
{4, 0} True	{4, 1} True	{4, 2} True	{4, 3} True	{4, 4} True	{4, 5} True	{4, 6} True	{4, 7} True
{5, 0} True	{5, 1} True	{5, 2} True	{5, 3} True	{5, 4} True	{5, 5} True	{5, 6} True	{5, 7} True
{6, 0} True	{6, 1} True	{6, 2} True	{6, 3} True	{6, 4} True	{6, 5} True	{6, 6} True	{6, 7} True
{7, 0} True	{7, 1} True	{7, 2} True	{7, 3} True	{7, 4} True	{7, 5} True	{7, 6} True	{7, 7} True

In[ ]:=
```
Table[{{A1, B1}, FullSimplify[ExpandAll[
      1/2 (newT16^A[A1].newT16^A[B1] + newT16^A[B1].newT16^A[A1]) ==
       η4488[[A1 + 1, B1 + 1]] * ID16]]},
  {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[ ]//MatrixForm=

{0, 0} True	{0, 1} True	{0, 2} True	{0, 3} True	{0, 4} True	{0, 5} True	{0, 6} True	{0, 7} True
{1, 0} True	{1, 1} True	{1, 2} True	{1, 3} True	{1, 4} True	{1, 5} True	{1, 6} True	{1, 7} True
{2, 0} True	{2, 1} True	{2, 2} True	{2, 3} True	{2, 4} True	{2, 5} True	{2, 6} True	{2, 7} True
{3, 0} True	{3, 1} True	{3, 2} True	{3, 3} True	{3, 4} True	{3, 5} True	{3, 6} True	{3, 7} True
{4, 0} True	{4, 1} True	{4, 2} True	{4, 3} True	{4, 4} True	{4, 5} True	{4, 6} True	{4, 7} True
{5, 0} True	{5, 1} True	{5, 2} True	{5, 3} True	{5, 4} True	{5, 5} True	{5, 6} True	{5, 7} True
{6, 0} True	{6, 1} True	{6, 2} True	{6, 3} True	{6, 4} True	{6, 5} True	{6, 6} True	{6, 7} True
{7, 0} True	{7, 1} True	{7, 2} True	{7, 3} True	{7, 4} True	{7, 5} True	{7, 6} True	{7, 7} True

In[ ]:=
```
Ψ16upper = Take[Ψ16, 8]
Ψ16lower = Take[Ψ16, -8]
Ψ16 == Flatten[{Ψ16upper, Ψ16lower}]
```

Out[ ]=
```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4]}
```

Out[ ]=
```
{f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

Out[ ]=
```
True
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
Table[T16^A[A1] = ArrayFlatten[
    {{0, τ̄[A1]}, {τ[A1], 0}}], {A1, 0, 7}]
σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

$$\begin{pmatrix} 0 & \bar{\tau}[A1] \\ \tau[A1] & 0 \end{pmatrix} \begin{pmatrix} \Psi16upper \\ \Psi16lower \end{pmatrix} = \begin{pmatrix} \bar{\tau}[A1].\Psi16lower \\ \tau[A1].\Psi16upper \end{pmatrix}$$

$$\sigma16.\begin{pmatrix} 0 & \bar{\tau}[A1] \\ \tau[A1] & 0 \end{pmatrix} \begin{pmatrix} \Psi16upper \\ \Psi16lower \end{pmatrix} = \begin{pmatrix} -\sigma\,\bar{\tau}[A1].\Psi16lower \\ \sigma\,\tau[A1].\Psi16upper \end{pmatrix}$$

$$\Psi16.\sigma16.\begin{pmatrix} 0 & \bar{\tau}[A1] \\ \tau[A1] & 0 \end{pmatrix}.\Psi16 = \begin{pmatrix} -\Psi16upper.\sigma\,\bar{\tau}[A1].\Psi16lower \\ \Psi16lower.\sigma\,\tau[A1]\,\Psi16upper \end{pmatrix}$$

In[ ]:=
```
(*σ16.T16^A[#]&/@Range[0,7]*)
```

In[ ]:=
```
(*
(T16^A[8]=FullSimplify[
    T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]])//MatrixForm*)
```

In[ ]:=
```
η4488
```

Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
```

In[ ]:=
```
η99 = DiagonalMatrix[{1, 1, 1, 1, -1, -1, -1, -1, 1}]
```

Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[ ]:=
```
(*Table[{A1,B1,
   FullSimplify[ExpandAll[1/2 ( T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]) == η99[[A1,B1]]*ID16]]},
  {A1,0, 8},{B1,0, 8}]//MatrixForm*)
```

In[ ]:=
```
(*Table[1/4 (T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]), {A1, 8},{B1,8}]*)
```

In[ ]:=
```
(P_L = 1/2 (ID16 - T16^A[8])) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

In[ ]:=
```
(P_R = 1/2 (ID16 + T16^A[8])) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

In[ ]:=
```
{ID16 == P_L + P_R, P_L.P_L == P_L, P_L.P_R == P_R.P_L == ZERO16}
```

Out[ ]=
```
{True, True, True}
```

```
SAB = Table[
  1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]), {A1, 0, 7}, {B1, 0, 7}];
```

In[ ]:=
```
SAB = Table[1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]),
   {A1, 0, 7}, {B1, 0, 7}];
```

In[ ]:=
```
ParallelTable[FullSimplify[σ16.SAB[[A1, B1]] === -Transpose[σ16.SAB[[A1, B1]]]],
   {A1, 1, 8}, {B1, 1, 8}] // Flatten // Union
```

Out[ ]=
```
{True}
```

In[ ]:=
```
ParallelTable[{{A1, B1}, FullSimplify[SAB⟦A1, B1⟧ === -Transpose[SAB⟦A1, B1⟧]]},
 {A1, 1, 8}, {B1, 1, 8}]
```

Out[ ]=
```
{{{{1, 1}, True}, {{1, 2}, True}, {{1, 3}, True}, {{1, 4}, True},
  {{1, 5}, False}, {{1, 6}, False}, {{1, 7}, False}, {{1, 8}, False}},
 {{{2, 1}, True}, {{2, 2}, True}, {{2, 3}, True}, {{2, 4}, True},
  {{2, 5}, False}, {{2, 6}, False}, {{2, 7}, False}, {{2, 8}, False}},
 {{{3, 1}, True}, {{3, 2}, True}, {{3, 3}, True}, {{3, 4}, True},
  {{3, 5}, False}, {{3, 6}, False}, {{3, 7}, False}, {{3, 8}, False}},
 {{{4, 1}, True}, {{4, 2}, True}, {{4, 3}, True}, {{4, 4}, True},
  {{4, 5}, False}, {{4, 6}, False}, {{4, 7}, False}, {{4, 8}, False}},
 {{{5, 1}, False}, {{5, 2}, False}, {{5, 3}, False}, {{5, 4}, False},
  {{5, 5}, True}, {{5, 6}, True}, {{5, 7}, True}, {{5, 8}, True}},
 {{{6, 1}, False}, {{6, 2}, False}, {{6, 3}, False}, {{6, 4}, False},
  {{6, 5}, True}, {{6, 6}, True}, {{6, 7}, True}, {{6, 8}, True}},
 {{{7, 1}, False}, {{7, 2}, False}, {{7, 3}, False}, {{7, 4}, False},
  {{7, 5}, True}, {{7, 6}, True}, {{7, 7}, True}, {{7, 8}, True}},
 {{{8, 1}, False}, {{8, 2}, False}, {{8, 3}, False}, {{8, 4}, False},
  {{8, 5}, True}, {{8, 6}, True}, {{8, 7}, True}, {{8, 8}, True}}}
```

In[ ]:=
```
forO44Similarity =
 ParallelTable[If[FullSimplify[SAB⟦A1, B1⟧ === -Transpose[SAB⟦A1, B1⟧]],
    {A1, B1}, ## &[]], {A1, 1, 7}, {B1, A1 + 1, 8}] // Flatten[#, 1] &
Length[%]
```

Out[ ]=
```
{{1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {5, 6}, {5, 7}, {5, 8}, {6, 7}, {6, 8}, {7, 8}}
```

Out[ ]=
```
12
```

In[ ]:=
```
SAB⟦##⟧ & @@ forO44Similarity⟦1⟧
MatrixExp[%]
```

Out[ ]=

$$\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},\right.$$

$$\left\{0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\left\{0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\left\{-\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0\right\},$$

$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0\right\},$$

$$\left.\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0\right\}\right\}$$

Out[ ]=

$$\left\{\left\{\text{Cos}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, \text{Sin}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, \text{Cos}\left[\frac{1}{2}\right], 0, 0, 0, 0, \text{Sin}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, \text{Cos}\left[\frac{1}{2}\right], 0, 0, -\text{Sin}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, 0, \text{Cos}\left[\frac{1}{2}\right], -\text{Sin}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, 0, \text{Sin}\left[\frac{1}{2}\right], \text{Cos}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, \text{Sin}\left[\frac{1}{2}\right], 0, 0, \text{Cos}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, -\text{Sin}\left[\frac{1}{2}\right], 0, 0, 0, 0, \text{Cos}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{-\text{Sin}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, \text{Cos}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, \text{Cos}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, -\text{Sin}\left[\frac{1}{2}\right]\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Cos}\left[\frac{1}{2}\right], 0, 0, 0, 0, -\text{Sin}\left[\frac{1}{2}\right], 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Cos}\left[\frac{1}{2}\right], 0, 0, \text{Sin}\left[\frac{1}{2}\right], 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Cos}\left[\frac{1}{2}\right], \text{Sin}\left[\frac{1}{2}\right], 0, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\text{Sin}\left[\frac{1}{2}\right], \text{Cos}\left[\frac{1}{2}\right], 0, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\text{Sin}\left[\frac{1}{2}\right], 0, 0, \text{Cos}\left[\frac{1}{2}\right], 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \text{Sin}\left[\frac{1}{2}\right], 0, 0, 0, 0, \text{Cos}\left[\frac{1}{2}\right], 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, 0, 0, 0, \text{Sin}\left[\frac{1}{2}\right], 0, 0, 0, 0, 0, 0, \text{Cos}\left[\frac{1}{2}\right]\right\}\right\}$$

In[ ]:=

```
ParallelTable[FullSimplify[SAB⟦A1, B1⟧.SAB⟦A2, B2⟧ - SAB⟦A2, B2⟧.SAB⟦A1, B1⟧ ==
      - (η4488⟦A1, A2⟧ × SAB⟦B1, B2⟧ - η4488⟦A1, B2⟧ × SAB⟦B1, A2⟧ -
         η4488⟦B1, A2⟧ × SAB⟦A1, B2⟧ + η4488⟦B1, B2⟧ × SAB⟦A1, A2⟧)],
   {A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}] // Flatten // Union
```

Out[ ]=

{True}

In[ ]:=

```
ParallelTable[FullSimplify[SAB⟦A1, B1⟧.T16^A[B2 - 1] - T16^A[B2 - 1].SAB⟦A1, B1⟧ ==
      (-η4488⟦B2, A1⟧ T16^A[B1 - 1] + η4488⟦B2, B1⟧ T16^A[A1 - 1])],
   {A1, 1, 8}, {B1, 1, 8}, {B2, 1, 8}] // Flatten // Union
```

Out[ ]=

{True}

In[∘]:=
```
(*ParallelTable[{{A1,B1},FullSimplify[σ16.SAB⟦A1,B1⟧==-Transpose[σ16.SAB⟦A1,B1⟧]]},
 {A1,1, 8},{B1,1,8}]*)
```

In[∘]:=
```
(*ParallelTable[{{A1,B1,A2,B2},
  FullSimplify[SAB⟦A1,B1⟧.SAB⟦A2,B2⟧ -SAB⟦A2,B2⟧.SAB⟦A1,B1⟧==-(η4488⟦A1,A2⟧SAB⟦B1,B2⟧-
       η4488⟦A1,B2⟧SAB⟦B1,A2⟧-η4488⟦B1,A2⟧SAB⟦A1,B2⟧+η4488⟦B1,B2⟧SAB⟦A1,A2⟧)]},
 {A1,1, 7},{B1,A1+1,8},{A2,1, 7},{B2,A2+1,8}]*)
```

In[∘]:=
```
(*ParallelTable[{{A1,B1,B2},FullSimplify[SAB⟦A1,B1⟧.T16^A[B2-1] -T16^A[B2-1].SAB⟦A1,B1⟧==
    (-η4488⟦B2,A1⟧T16^A[B1-1]+η4488⟦B2,B1⟧T16^A[A1-1])]}, {A1,1, 8},{B1,1,8},{B2,1,8}]*)
```

In[∘]:=
```
Do[sAB[A1][B1] = 1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]),
  {A1, 0, 7}, {B1, 0, 7}];
```

In[∘]:=
```
O8similarityTransformation
```

Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[∘]:=
```
O8similarityTransformation.Ψ16 // Column
```

Out[∘]=
```
f16[0][x0, x4]
f16[1][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
f16[2][x0, x4]
f16[3][x0, x4]
f16[6][x0, x4]
f16[7][x0, x4]
f16[10][x0, x4]
f16[11][x0, x4]
f16[14][x0, x4]
f16[15][x0, x4]
```

In[∘]:=
```
scale16 = {1, 1, -1, -1, 1, 1, -1, -1, 1, 1, -1, -1, 1, 1, -1, -1};
Length[scale16]
```

Out[∘]=
```
16
```

In[ ]:=
```
scale16mat = DiagonalMatrix[scale16]
```

Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}
```

In[ ]:=
```
O8similarityTransformation.(scale16 * Ψ16) // Column
```

Out[ ]=
```
f16[0][x0, x4]
f16[1][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
-f16[2][x0, x4]
-f16[3][x0, x4]
-f16[6][x0, x4]
-f16[7][x0, x4]
-f16[10][x0, x4]
-f16[11][x0, x4]
-f16[14][x0, x4]
-f16[15][x0, x4]
```

In[ ]:=
```
Dimensions[ArrayFlatten[{{0, ID8}, {ID8, 0}}]]
```

Out[ ]=
```
{16, 16}
```

In[ ]:=
```
(try1616 =
   Table[Block[{m = O8similarityTransformation.(scale16 * Ψ16), d}, d = D[m[[j]], #] & /@ Ψ16;
     d], {j, 1, 16}]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}$$

In[ ]:=
```
try1616.Transpose[try1616] === ID16
```

Out[ ]=
```
True
```

In[ ]:=
```
(try1616.Ψ16) // Column
```

Out[ ]=
```
f16[0][x0, x4]
f16[1][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
-f16[2][x0, x4]
-f16[3][x0, x4]
-f16[6][x0, x4]
-f16[7][x0, x4]
-f16[10][x0, x4]
-f16[11][x0, x4]
-f16[14][x0, x4]
-f16[15][x0, x4]
```

In[ ]:=
```
(sqrta = ArrayFlatten[{{0, ID8}, {ID8, 0}}]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
{sJordanDecomposition, jJordanDecomposition} = JordanDecomposition[sqrta]
```

Out[ ]=
```
{{{0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {-1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}},
 {{-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}}
```

In[ ]:=
```
sJordanDecomposition.jJordanDecomposition.Inverse[sJordanDecomposition] === sqrta
```

Out[ ]=
```
True
```

In[ ]:=
```
(sqrtb = sJordanDecomposition.√jJordanDecomposition.Inverse[sJordanDecomposition]) //
  MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
\frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} \\
\frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2}
\end{pmatrix}$$

In[ ]:=
```
O44similarityTransformation00 =
 Table[Block[{m = O8similarityTransformation.(scale16 * Ψ16), d}, d = D[m[[j]], #] & /@ Ψ16;
   d], {j, 1, 16}]
```

Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}
```

In[ ]:=
```
(O44similarityTransformation = sqrtb.O44similarityTransformation00.Inverse[sqrtb]) //
 MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
\frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} \\
\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} \\
-\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} \\
\frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2}
\end{pmatrix}$$

In[ ]:=
```
(*O44similarityTransformation=ArrayFlatten[{{0,ID8},{ID8,0}}].
  Table[Block[{m=O8similarityTransformation.(scale16*Ψ16),d},d=D[m[[j]],#]&/@Ψ16;
    d],{j,1,16}]*)
```

In[ ]:=
```
O44similarityTransformation.Transpose[O44similarityTransformation] === Id16
```

Out[ ]=

False

In[ ]:=
```
(O44similarityTransformation = sqrtb.O44similarityTransformation00.Transpose[sqrtb]);
O44similarityTransformation.Transpose[O44similarityTransformation] === Id16
```

Out[ ]=

False

In[ ]:=
```
(O44similarityTransformation = sqrtb.O44similarityTransformation00.sqrtb);
O44similarityTransformation.Transpose[O44similarityTransformation] === Id16
```

Out[ ]=

False

In[ ]:=
```
SAB[[1, 2]] // MatrixForm
O8similarityTransformation. SAB[[1, 2]].
  Transpose[O8similarityTransformation] // MatrixForm
O44similarityTransformation. SAB[[1, 2]].Transpose[O44similarityTransformation] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

In[ ]:=
```
sAB[-1 + 1][-1 + 2] == SAB[[1, 2]]
```

Out[ ]=

True

In[ ]:=
```
σ16.SAB[[1, 2]] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0
\end{pmatrix}$$

In[ ]:=
```
σ16.sAB[1][2] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula:

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+,+,+,-,-,-)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \mathbf{diag}(1,1,1,-1,-1,-1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties.

The final answer is $[\mathbf{J_{ab}}, \mathbf{J_{cd}}] = \mathbf{i}(\boldsymbol{\eta}_{\mathbf{ac}}\mathbf{J_{bd}} - \boldsymbol{\eta}_{\mathbf{ad}}\mathbf{J_{bc}} - \boldsymbol{\eta}_{\mathbf{bc}}\mathbf{J_{ad}} + \boldsymbol{\eta}_{\mathbf{bd}}\mathbf{J_{ac}})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4).

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \ldots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \mathrm{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention)

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$.

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa).

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not).

what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the $so(4,4)$ Lie algebra, where $\mu, \nu \in \{0, 1, \ldots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the standard relations for orthogonal Lie algebras, given by a generalization of the angular momentum commutation relations:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for $so(8)$ in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \mathrm{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).

↪ Why is the metric signature significant for $so(4,4)$ commutation relations?

Why is the metric signature significant for $so(4,4)$ commutation relations?

The metric signature is significant for the $so(p,q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable).

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p,q)$ is given by:
  $[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$
  The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4,4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.

- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
  - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[ ]:=
```
η4488
```
Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
```

orthogonal Lie algebra $so(p,q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

In[ ]:=
```
SAB[[6, 2]].SAB[[6, 7]] - SAB[[6, 7]].SAB[[6, 2]] == η4488[[6, 6]] × SAB[[7, 2]]
```
Out[ ]=
```
True
```
In[ ]:=
```
{{6, 2, 6, 7}, False}
```
Out[ ]=
```
{{6, 2, 6, 7}, False}
```
In[ ]:=
```
SAB[[1, 2]].SAB[[1, 3]] - SAB[[1, 3]].SAB[[1, 2]] == η4488[[1, 1]] × SAB[[2, 3]]
```
Out[ ]=
```
False
```

In[ ]:=
```
sAB[0][0]
```

Out[ ]=

$$\{\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}\}$$

In[ ]:=
```
sAB[0][1]
```

Out[ ]=

$$\left\{\left\{0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0\right\},\left\{0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0\right\},\right.$$
$$\left\{0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0\right\},\left\{0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2}\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0\right\},\left\{0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0\right\},$$
$$\left.\left\{0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0\right\},\left\{0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0\right\}\right\}$$

In[ ]:=
```
Table[Transpose[T16^A[8].sAB[A1][B1]] == T16^A[8].sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
```

Out[ ]=

```
{{True, False, False, False, True, True, True, True},
 {False, True, False, False, True, True, True, True},
 {False, False, True, False, True, True, True, True},
 {False, False, False, True, True, True, True, True},
 {True, True, True, True, True, False, False, False},
 {True, True, True, True, False, True, False, False},
 {True, True, True, True, False, False, True, False},
 {True, True, True, True, False, False, False, True}}
```

In[∘]:=
```
Table[Transpose[T16^A[7].sAB[A1][B1]] == T16^A[7].sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
```

Out[∘]=
```
{{True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {False, False, False, False, True, True, True, False},
 {False, False, False, False, True, True, True, False},
 {False, False, False, False, True, True, True, False},
 {True, True, True, True, False, False, False, True}}
```

In[∘]:=
```
Table[Transpose[σ16.sAB[A1][B1]] == σ16.sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
```

Out[∘]=
```
{{True, False, False, False, False, False, False, False},
 {False, True, False, False, False, False, False, False},
 {False, False, True, False, False, False, False, False},
 {False, False, False, True, False, False, False, False},
 {False, False, False, False, True, False, False, False},
 {False, False, False, False, False, True, False, False},
 {False, False, False, False, False, False, True, False},
 {False, False, False, False, False, False, False, True}}
```

In[∘]:=
```
Table[Transpose[σ16.SAB⟦A1, B1⟧] == σ16.SAB⟦A1, B1⟧, {A1, 1, 8}, {B1, 1, 8}]
```

Out[∘]=
```
{{True, False, False, False, False, False, False, False},
 {False, True, False, False, False, False, False, False},
 {False, False, True, False, False, False, False, False},
 {False, False, False, True, False, False, False, False},
 {False, False, False, False, True, False, False, False},
 {False, False, False, False, False, True, False, False},
 {False, False, False, False, False, False, True, False},
 {False, False, False, False, False, False, False, True}}
```

In[∘]:=
```
SAB⟦1, 2⟧⟦1⟧
```

Out[∘]=

$$\left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0\right\}$$

In[∘]:=
```
SAB⟦1, 2⟧⟦2⟧
```

Out[∘]=

$$\left\{0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

In[∘]:=
```
SAB1 = Table[Take[SAB⟦A1, B1⟧⟦C1⟧, 8], {A1, 8}, {B1, 8}, {C1, 1, 8}];
```

In[∘]:=
```
SAB2 = Table[Take[SAB⟦A1, B1⟧⟦C1⟧, -8], {A1, 8}, {B1, 8}, {C1, 9, 16}];
```

In[ ]:=
```
SAB⟦1, 2⟧ == ArrayFlatten[{{SAB1⟦1, 2⟧, 0}, {0, SAB2⟦1, 2⟧}}]
```

Out[ ]=
```
True
```

In[ ]:=
```
(*Table[
 SAB⟦A1,B1⟧== ArrayFlatten[{{SAB1⟦A1,B1⟧,0},{0,SAB2⟦A1,B1⟧}}],
 {A1,0, 7},{B1,0,7}]*)
```

$$4D_{(1)}{}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}{}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A .$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \to SO(4,4)$ is given by

(2.8) $$8L^{A'}{}_{B'}(g) = \operatorname{tr} \widetilde{D}^{(2)}(g)\, \tau^{A'} D^{(1)}(g)\, \bar{\tau}_{B'} .$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

(2.9) $$\widetilde{D}^{(1)} \sigma = \sigma D^{(1)-1} ,$$

(2.10) $$\sigma^{-1} \widetilde{D}^{(2)} = D^{(2)-1} \sigma^{-1} ,$$

(2.11) $$L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'} ,$$

(2.12) $$L^{A'}{}_{B'} \tau^{B'} = \widetilde{D}^{(2)} \tau^{A'} D^{(1)} ,$$

(2.13) $$L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \widetilde{D}^{(2)-1} .$$

In[ ]:=
```
SAB2⟦1, 2⟧
```

Out[ ]=

$$\left\{\left\{0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}\right\}, \left\{0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0\right\}, \left\{0, 0, 0, 0, 0, \frac{1}{2}, 0, 0\right\}, \left\{0, 0, 0, 0, \frac{1}{2}, 0, 0, 0\right\}, \left\{0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0\right\}, \left\{0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{2}, 0, 0, 0, 0, 0, 0\right\}, \left\{\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0\right\}\right\}$$

In[ ]:=
```
τ[0]
```

Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[ ]:=
```
(*Block[{b,A,F_a^(μ),F^a_(μ),ψ,Ψ},
 b=Array[A,{8}];
 ψ[1]=Array[Ψ[1],{8}];
 ψ[2]=Array[Ψ[2],{8}];
 F_a^(μ)=Table[(1/Sqrt[ψ[1].σ.ψ[1]] ψ[1].σ.(τ̄[μL])),{μL,1,8}];
 F^a_(μ)= Transpose[Table[(1/Sqrt[ψ[1].σ.ψ[1]] η4488[[μL,μL]]τ[μL].ψ[1]),{μL,1,8}]];
 b===FullSimplify[F_a^(μ).ψ[2]/.Thread[ψ[2]→F^a_(μ).b]]]*)
```

In[ ]:=
```
(*Block[{b,A,B,q,F^a_(μ),ψ,φ,Ψ},
 b=Array[A,{8}];
 ψ[1]=Array[Ψ[1],{8}];
 φ[1]=Array[Ψ[2],{8}];
 q=ParallelTable[FullSimplify[
    (1/Sqrt[ψ[1].σ.ψ[1]] ψ[1].σ.1/2 (τ̄[A1].τ[B1]-τ̄[B1].τ[A1]).φ[1])],{A1,1,8},{B1,1,8}];
 B=FullSimplify[(1/Sqrt[ψ[1].σ.ψ[1]] ParallelSum[η4488[[μL,μL]]τ[μL]b[[μL]],{μL,1,8}].ψ[1])];
 FullSimplify[q/.Thread[φ[1]→B]]]*)
```

In[ ]:=
```
(*Block[{b,A,B,q,F^a_(μ),ψ,φ,Ψ},
 b=Array[A,{8}];
 ψ[1]=Array[Ψ[1],{8}];
 φ[1]=Array[Ψ[2],{8}];
 q=ParallelTable[FullSimplify[(ψ[1].σ.(τ̄[A1].τ[B1]).φ[1])],{A1,1,8},{B1,1,8}];
 B=FullSimplify[( ParallelSum[η4488[[μL,μL]]τ[μL]b[[μL]],{μL,1,8}].ψ[1])];
 FullSimplify[1/Sqrt[ψ[1].σ.ψ[1]] q/.Thread[φ[1]→B]]]*)
```

$T16^{\alpha}$

In[ ]:=
```
ssgm4488
```

Out[ ]=

$$\{g[0][0] \to (\mathrm{Cot}[6\,H\,\#1]^2\,\&),\ g[0][1] \to (0\,\&),\ g[0][2] \to (0\,\&),\ g[0][3] \to (0\,\&),$$
$$g[0][4] \to (0\,\&),\ g[0][5] \to (0\,\&),\ g[0][6] \to (0\,\&),\ g[0][7] \to (0\,\&),\ g[1][0] \to (0\,\&),$$
$$g[1][1] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[1][2] \to (0\,\&),\ g[1][3] \to (0\,\&),$$
$$g[1][4] \to (0\,\&),\ g[1][5] \to (0\,\&),\ g[1][6] \to (0\,\&),\ g[1][7] \to (0\,\&),\ g[2][0] \to (0\,\&),$$
$$g[2][1] \to (0\,\&),\ g[2][2] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[2][3] \to (0\,\&),\ g[2][4] \to (0\,\&),$$
$$g[2][5] \to (0\,\&),\ g[2][6] \to (0\,\&),\ g[2][7] \to (0\,\&),\ g[3][0] \to (0\,\&),\ g[3][1] \to (0\,\&),$$
$$g[3][2] \to (0\,\&),\ g[3][3] \to (e^{2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[3][4] \to (0\,\&),\ g[3][5] \to (0\,\&),$$
$$g[3][6] \to (0\,\&),\ g[3][7] \to (0\,\&),\ g[4][0] \to (0\,\&),\ g[4][1] \to (0\,\&),\ g[4][2] \to (0\,\&),$$
$$g[4][3] \to (0\,\&),\ g[4][4] \to (-1\,\&),\ g[4][5] \to (0\,\&),\ g[4][6] \to (0\,\&),\ g[4][7] \to (0\,\&),$$
$$g[5][0] \to (0\,\&),\ g[5][1] \to (0\,\&),\ g[5][2] \to (0\,\&),\ g[5][3] \to (0\,\&),\ g[5][4] \to (0\,\&),$$
$$g[5][5] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[5][6] \to (0\,\&),\ g[5][7] \to (0\,\&),$$
$$g[6][0] \to (0\,\&),\ g[6][1] \to (0\,\&),\ g[6][2] \to (0\,\&),\ g[6][3] \to (0\,\&),\ g[6][4] \to (0\,\&),$$
$$g[6][5] \to (0\,\&),\ g[6][6] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&),\ g[6][7] \to (0\,\&),$$
$$g[7][0] \to (0\,\&),\ g[7][1] \to (0\,\&),\ g[7][2] \to (0\,\&),\ g[7][3] \to (0\,\&),\ g[7][4] \to (0\,\&),$$
$$g[7][5] \to (0\,\&),\ g[7][6] \to (0\,\&),\ g[7][7] \to (-e^{-2\,a4[H\,\#2]}\,\mathrm{Sin}[6\,H\,\#1]^{1/3}\,\&)\}$$

In[ ]:= $e^{\alpha}_{(A)}$ /. $sge^{\alpha}_{(A)}$

Out[ ]=

$$\left\{\left\{\frac{1}{\sqrt{g[0][0][x0,x4]}},0,0,0,0,0,0,0\right\},\left\{0,\frac{1}{\sqrt{g[1][1][x0,x4]}},0,0,0,0,0,0\right\},\right.$$
$$\left\{0,0,\frac{1}{\sqrt{g[2][2][x0,x4]}},0,0,0,0,0\right\},\left\{0,0,0,\frac{1}{\sqrt{g[3][3][x0,x4]}},0,0,0,0\right\},$$
$$\left\{0,0,0,0,\frac{1}{\sqrt{-g[4][4][x0,x4]}},0,0,0\right\},\left\{0,0,0,0,0,\frac{1}{\sqrt{-g[5][5][x0,x4]}},0,0\right\},$$
$$\left.\left\{0,0,0,0,0,0,\frac{1}{\sqrt{-g[6][6][x0,x4]}},0\right\},\left\{0,0,0,0,0,0,0,\frac{1}{\sqrt{-g[7][7][x0,x4]}}\right\}\right\}$$

In[ ]:=
```
Table[T16^α[α1 - 1] =
   Sum[((e^α_(A) /. sge^α_(A))[[α1, A1]]) T16^A[A1 - 1], {A1, 1, 8}],
  {α1, 1, 8}];(*/.ssgm4488*)
```

In[ ]:=
```
(T16^α[8] = T16^α[0].T16^α[1].T16^α[2].T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7]);
```

In[ ]:=
```
Table[newT16^α[A1] = O8similarityTransformation. T16^α[A1].
    Transpose[O8similarityTransformation], {A1, 0, 8} ];
```

In[ ]:=
```
(*T16^α[8]==T16^A[8]*Sec[6 H x0]*)
```

CHECK

Table[T16$^{\alpha}$[α1 - 1] = Sum[(e$^{\alpha}_{(A)}$[[α1, A1]]) T16^{A}[A1 - 1], {A1, 1, 8}], {α1, 1, 8}]; and ● ● ● :

In[ ]:=
```
(*Table[T16^α[α1-1]=
    Sum[(e^α_(A)[[α1,A1]]) T16^A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

In[ ]:=
```
(*(T16^α[8]=T16^α[0].T16^α[1].T16^α[2].T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7])//
 MatrixForm*)
```

In[ ]:=
```
Table[{{A1, B1}, FullSimplify[
    ExpandAll[1/2 (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) ==
      η4488[[A1 + 1, B1 + 1]] * ID16]]},
  {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[ ]//MatrixForm=

{0, 0} True	{0, 1} True	{0, 2} True	{0, 3} True	{0, 4} True	{0, 5} True	{0, 6} True	{0, 7} True
{1, 0} True	{1, 1} True	{1, 2} True	{1, 3} True	{1, 4} True	{1, 5} True	{1, 6} True	{1, 7} True
{2, 0} True	{2, 1} True	{2, 2} True	{2, 3} True	{2, 4} True	{2, 5} True	{2, 6} True	{2, 7} True
{3, 0} True	{3, 1} True	{3, 2} True	{3, 3} True	{3, 4} True	{3, 5} True	{3, 6} True	{3, 7} True
{4, 0} True	{4, 1} True	{4, 2} True	{4, 3} True	{4, 4} True	{4, 5} True	{4, 6} True	{4, 7} True
{5, 0} True	{5, 1} True	{5, 2} True	{5, 3} True	{5, 4} True	{5, 5} True	{5, 6} True	{5, 7} True
{6, 0} True	{6, 1} True	{6, 2} True	{6, 3} True	{6, 4} True	{6, 5} True	{6, 6} True	{6, 7} True
{7, 0} True	{7, 1} True	{7, 2} True	{7, 3} True	{7, 4} True	{7, 5} True	{7, 6} True	{7, 7} True

In[ ]:=
```
Table[{{α1, β1}, FullSimplify[
    1/2 FullSimplify[((T16^α[α1].T16^α[β1] + T16^α[β1].T16^α[α1]) /.
        ssgm4488), constraintVars] ==
     Inverse[MatrixMetric44][[α1 + 1, β1 + 1]] * ID16]},
  {α1, 0, 7}, {β1, 0, 7}] // MatrixForm
```

Out[ ]//MatrixForm=

{0, 0} True	{0, 1} True	{0, 2} True	{0, 3} True	{0, 4} True	{0, 5} True	{0, 6} True	{0, 7} True
{1, 0} True	{1, 1} True	{1, 2} True	{1, 3} True	{1, 4} True	{1, 5} True	{1, 6} True	{1, 7} True
{2, 0} True	{2, 1} True	{2, 2} True	{2, 3} True	{2, 4} True	{2, 5} True	{2, 6} True	{2, 7} True
{3, 0} True	{3, 1} True	{3, 2} True	{3, 3} True	{3, 4} True	{3, 5} True	{3, 6} True	{3, 7} True
{4, 0} True	{4, 1} True	{4, 2} True	{4, 3} True	{4, 4} True	{4, 5} True	{4, 6} True	{4, 7} True
{5, 0} True	{5, 1} True	{5, 2} True	{5, 3} True	{5, 4} True	{5, 5} True	{5, 6} True	{5, 7} True
{6, 0} True	{6, 1} True	{6, 2} True	{6, 3} True	{6, 4} True	{6, 5} True	{6, 6} True	{6, 7} True
{7, 0} True	{7, 1} True	{7, 2} True	{7, 3} True	{7, 4} True	{7, 5} True	{7, 6} True	{7, 7} True

In[ ]:=
```
Table[{{α1, β1}, FullSimplify[
    1/2 FullSimplify[((newT16^α[α1].newT16^α[β1] + newT16^α[β1].
          newT16^α[α1]) /. ssgm4488), constraintVars] ==
     Inverse[MatrixMetric44][[α1 + 1, β1 + 1]] * ID16]},
  {α1, 0, 7}, {β1, 0, 7}] // MatrixForm
```

Out[ ]//MatrixForm=

{0, 0} True	{0, 1} True	{0, 2} True	{0, 3} True	{0, 4} True	{0, 5} True	{0, 6} True	{0, 7} True
{1, 0} True	{1, 1} True	{1, 2} True	{1, 3} True	{1, 4} True	{1, 5} True	{1, 6} True	{1, 7} True
{2, 0} True	{2, 1} True	{2, 2} True	{2, 3} True	{2, 4} True	{2, 5} True	{2, 6} True	{2, 7} True
{3, 0} True	{3, 1} True	{3, 2} True	{3, 3} True	{3, 4} True	{3, 5} True	{3, 6} True	{3, 7} True
{4, 0} True	{4, 1} True	{4, 2} True	{4, 3} True	{4, 4} True	{4, 5} True	{4, 6} True	{4, 7} True
{5, 0} True	{5, 1} True	{5, 2} True	{5, 3} True	{5, 4} True	{5, 5} True	{5, 6} True	{5, 7} True
{6, 0} True	{6, 1} True	{6, 2} True	{6, 3} True	{6, 4} True	{6, 5} True	{6, 6} True	{6, 7} True
{7, 0} True	{7, 1} True	{7, 2} True	{7, 3} True	{7, 4} True	{7, 5} True	{7, 6} True	{7, 7} True

In[ ]:=
```
MatrixMetric44 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0]^2 & 0 & 0 & 0 & 0 & \\ 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 & \\ 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & \\ 0 & 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & \\ 0 & 0 & 0 & 0 & -1 & \\ 0 & 0 & 0 & 0 & 0 & -e^{-2\,a4[H\,x4]}\,S \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{pmatrix}$$

In[ ]:=
```
((e^α_(A) /. sgtrye^α_(A)).MatrixMetric44.(e^α_(A) /. sgtrye^α_(A))) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
Transpose[(e_α^(A) /. sgtrye_α^(A))].η4488.(e_α^(A) /. sgtrye_α^(A)) - MatrixMetric44 //
  FullSimplify[#, constraintVars] &
```

Out[∘]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[∘]:=
```
Transpose[(e^α_(A) /. sgtrye^α_(A))].MatrixMetric44.(e^α_(A) /. sgtrye^α_(A)) - η4488 //
  (*Full*)Simplify[#, constraintVars] &
```

Out[∘]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[∘]:=
```
Inverse[(e^α_(A) /. sgtrye^α_(A))] == (e_α^(A) /. sgtrye_α^(A))
```

Out[∘]=
```
True
```

In[∘]:=
```
Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]
```

Out[∘]=
```
{{0, 0}, {0, 0}}
```

BELOW : LagrangianΨ16 =

$$\kappa \sqrt{\mathrm{Det}[g_{\mu\nu}]}\ \mathrm{Transpose}[\Psi 16].\sigma 16.T16^{A}[A1].\Psi 16_{|\alpha}\, g^{-1\ \alpha\beta}\, \eta_{A1\,B1}\, E^{B1}_{\beta} + \text{mass term}$$

In[∘]:=
```
e_α^(A) /. sgtrye_α^(A)
```

Out[∘]=

$$\{\{\mathrm{Cot}[6\,H\,x0],0,0,0,0,0,0,0\},\{0,\sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0,0\},$$
$$\{0,0,\sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0\},\{0,0,0,\sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0\},$$
$$\{0,0,0,0,1,0,0,0\},\{0,0,0,0,0,\sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0\},$$
$$\{0,0,0,0,0,0,\sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0\},$$
$$\{0,0,0,0,0,0,0,\sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\}\}$$

In[ ]:= $e^{\alpha}_{(A)}$ /. sgtrye$^{\alpha}_{(A)}$

Out[ ]=

$$\left\{\{\text{Tan}[6\,H\,x0],0,0,0,0,0,0,0\},\left\{0,\frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0,0\right\},\right.$$
$$\left\{0,0,\frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0\right\},\left\{0,0,0,\frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0\right\},$$
$$\{0,0,0,0,1,0,0,0\},\left\{0,0,0,0,0,\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},0,0\right\},$$
$$\left\{0,0,0,0,0,0,\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}},0\right\},$$
$$\left.\left\{0,0,0,0,0,0,0,\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\text{Sin}[6\,H\,x0]^{1/6}}\right\}\right\}$$

In[ ]:= $e^{\alpha}_{(A)}$ /. sge$^{\alpha}_{(A)}$

Out[ ]=

$$\left\{\left\{\frac{1}{\sqrt{g[0][0][x0,x4]}},0,0,0,0,0,0,0\right\},\left\{0,\frac{1}{\sqrt{g[1][1][x0,x4]}},0,0,0,0,0,0\right\},\right.$$
$$\left\{0,0,\frac{1}{\sqrt{g[2][2][x0,x4]}},0,0,0,0,0\right\},\left\{0,0,0,\frac{1}{\sqrt{g[3][3][x0,x4]}},0,0,0,0\right\},$$
$$\left\{0,0,0,0,\frac{1}{\sqrt{-g[4][4][x0,x4]}},0,0,0\right\},\left\{0,0,0,0,0,\frac{1}{\sqrt{-g[5][5][x0,x4]}},0,0\right\},$$
$$\left.\left\{0,0,0,0,0,0,\frac{1}{\sqrt{-g[6][6][x0,x4]}},0\right\},\left\{0,0,0,0,0,0,0,\frac{1}{\sqrt{-g[7][7][x0,x4]}}\right\}\right\}$$

In[ ]:=

```
Clear[con];
con[g_, ass_ : {}] := Module[{t, ΓΓ, ginv},
  ginv = FullSimplify[Inverse[g], ass];
  ΓΓ = FullSimplify[Table[(1 / 2) * Sum[(ginv⟦i, s⟧) *
        (D[g⟦s, j⟧, X⟦k⟧ ] + D[g⟦s, k⟧, X⟦j⟧ ] - D[g⟦j, k⟧, X⟦s⟧ ]), {s, 1, DIM8}],
     {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] , ass];
  Return[{ginv, ΓΓ}]]
```

In[ ]:=
```
ssgm4488
```

Out[ ]=
```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
 g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
 g[1][1] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
 g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
 g[2][1] → (0 &), g[2][2] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
 g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
 g[3][2] → (0 &), g[3][3] → (e^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
 g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
 g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
 g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
 g[5][5] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
 g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
 g[6][5] → (0 &), g[6][6] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
 g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
 g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &)}
```

In[ ]:=
```
g4488
g4488 /. sg
```

Out[ ]=
```
{{g[0][0][x0, x4], g[0][1][x0, x4], g[0][2][x0, x4], g[0][3][x0, x4],
   g[0][4][x0, x4], g[0][5][x0, x4], g[0][6][x0, x4], g[0][7][x0, x4]},
 {g[1][0][x0, x4], g[1][1][x0, x4], g[1][2][x0, x4], g[1][3][x0, x4],
   g[1][4][x0, x4], g[1][5][x0, x4], g[1][6][x0, x4], g[1][7][x0, x4]},
 {g[2][0][x0, x4], g[2][1][x0, x4], g[2][2][x0, x4], g[2][3][x0, x4],
   g[2][4][x0, x4], g[2][5][x0, x4], g[2][6][x0, x4], g[2][7][x0, x4]},
 {g[3][0][x0, x4], g[3][1][x0, x4], g[3][2][x0, x4], g[3][3][x0, x4],
   g[3][4][x0, x4], g[3][5][x0, x4], g[3][6][x0, x4], g[3][7][x0, x4]},
 {g[4][0][x0, x4], g[4][1][x0, x4], g[4][2][x0, x4], g[4][3][x0, x4],
   g[4][4][x0, x4], g[4][5][x0, x4], g[4][6][x0, x4], g[4][7][x0, x4]},
 {g[5][0][x0, x4], g[5][1][x0, x4], g[5][2][x0, x4], g[5][3][x0, x4],
   g[5][4][x0, x4], g[5][5][x0, x4], g[5][6][x0, x4], g[5][7][x0, x4]},
 {g[6][0][x0, x4], g[6][1][x0, x4], g[6][2][x0, x4], g[6][3][x0, x4],
   g[6][4][x0, x4], g[6][5][x0, x4], g[6][6][x0, x4], g[6][7][x0, x4]},
 {g[7][0][x0, x4], g[7][1][x0, x4], g[7][2][x0, x4], g[7][3][x0, x4],
   g[7][4][x0, x4], g[7][5][x0, x4], g[7][6][x0, x4], g[7][7][x0, x4]}}
```

Out[ ]=
```
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}
```

In[ ]:=
```
MatrixForm[#] & /@ Block[{eAα, eαA, (*ginv,Γ,gg,*)colVecs, rowVecs, ab, ba, r},
  (*gg=g4488/.sg;*)
  (*{ginv,Γ}=con[gg];*)
  eαA = e^α_(A) /. sge^α_(A);
  eAα = e^(A)_α /. sge^(A)_α;
  rowVecs = Table[eAα[[b, All]], {b, 1, Length[eAα]}];
  colVecs = Table[eαA[[All, b]], {b, 1, Length[eαA]}];
  ab = rowVecs.colVecs // FullSimplify[#, constraintVars] &;
  ba = Transpose[colVecs].Transpose[rowVecs] // FullSimplify[#, constraintVars] &;
  (*r={{ab},{ba}};*)
  r = {ab, ba};
  r]
```

Out[ ]=

$$\left\{\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}\right\}$$

metric compatibility condition for the Octad

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

In[ ]:=
```
Clear[spinCoefficients];
spinCoefficients[eIν_] := Module[{eνI, w, r, s},
  If[Dimensions[eIν] ≠ {8, 8}, Return[{}]];
  If[Det[eIν] == 0, Return[{}]];
  eνI = FullSimplify[Inverse[eIν], constraintVars] /. subsDefects;
  w = Table[-Table[
      FullSimplify[Sum[D[eIν[[I1, ν1]], X[[μ1]]] * eνI[[ν1, Jprime]], {ν1, 1, Length[eνI]}] -
         Sum[  eIν[[I1, ρ]] * Γ[[ρ, μ1, ν1]] * eνI[[ν1, Jprime]], {ν1, 1, Length[eνI]},
          {ρ, 1, Length[X]}], constraintVars] /. subsDefects,
      {I1, 1, Length[eIν]}, {Jprime, 1, Length[eνI]}], {μ1, 1, Length[X]}];
  Assert[Dimensions[w] == {8, 8, 8}];
  Return[w];
 ]
```

In[ ]:=
```
spinCoeffs = spinCoefficients[gtrye_α^(A)]
```

Out[ ]=
```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, -e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0,
   0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, -e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, -e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {e^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4],
   0, 0, 0}, {0, 0, 0, e^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0},
  {e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6)},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4]},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {e^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -e^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}}
```

In[∘]:=
```
Table[
 ωμIJ[μ] = FullSimplify[-((D[gtrye_α^(A), X[[μ]]].gtrye^α_(A)) - (gtrye_α^(A).Γ[[All, μ, All]].gtrye^α_(A))),
    constraintVars] /. subsDefects, {μ, 1, Length[X]}]
```

Out[∘]=

$$\{\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0,0\},\{e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,$$
$$0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,0,-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$$
$$0,0,0\},\{0,0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0\},$$
$$\{e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,-e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0\},\{0,0,0,0,0,0,0,0\},$$
$$\{e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,-e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\}\},\{\{0,0,0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$
$$\{e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,-e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\}\}\}$$

In[∘]:= (*$gtrye^{\alpha}_{(A)}$.$gtrye^{(A)}_{\alpha}$*)

In[∘]:= (*$gtrye^{(A)}_{\alpha}$.$gtrye^{\alpha}_{(A)}$*)

In[∘]:= (*$gtrye^{(A)}_{\alpha}$*)

In[∘]:= (*Γ
 Dimensions[%]*)

In[∘]:= (*Table[Γ[[All,μ,All]],{μ,1,Length[X]}]*)

```
In[•]:= (*Γ⟦All,5,All⟧*)
In[•]:= (*Γ⟦All,1,All⟧*)
In[•]:= (*spinCoeffs⟦1⟧*)
In[•]:= (*spinCoeffs⟦5⟧*)
In[•]:= (*{X⟦1⟧,X⟦5⟧}*)
In[•]:= (*spinCoeffs⟦1⟧⟦1⟧⟦1⟧*)
In[•]:= (*ωμIJ[1]⟦1⟧⟦1⟧*)
In[•]:= (*Block[{μ=1},FullSimplify[-((D[gtrye_α^(A),X⟦μ⟧].gtrye^α_(A))-(gtrye_α^(A).Γ⟦All,μ,All⟧.gtrye^α_(A))),
          constraintVars]/.subsDefects]
        Dimensions[%]*)
In[•]:= (*Block[{μ=1},FullSimplify[
          -((D[gtrye_α^(A),X⟦μ⟧].Transpose[gtrye^α_(A)])-(gtrye_α^(A).Γ⟦All,μ,All⟧.Transpose[gtrye^α_(A)])),
          constraintVars]/.subsDefects]*)
In[•]:= (*Block[{μ=1},
          FullSimplify[-((Sum[    D[gtrye_α^(A)⟦All,ν⟧,X⟦μ⟧]*gtrye^α_(A)⟦ν,All⟧    ,{ν,1,8}]  )-
              (gtrye_α^(A).Γ⟦All,μ,All⟧.gtrye^α_(A))),constraintVars]/.subsDefects]
        Dimensions[%]*)
In[•]:= (*Table[,{A1,1,8},{B1,1,8}]*)
In[•]:= (*Block[{μ=1},
          Table[FullSimplify[-((Sum[    D[gtrye_α^(A)⟦A1,ν⟧,X⟦μ⟧]*gtrye^α_(A)⟦ν,B1⟧    ,{ν,1,8}]  )-
                (Sum[    gtrye_α^(A)⟦A1,ρ⟧*Γ⟦ρ,μ,ν⟧*gtrye^α_(A)⟦ν,B1⟧    ,{ρ,1,8},{ν,1,8}]  )),
             constraintVars]/.subsDefects,{A1,1,8},{B1,1,8}]]
        Dimensions[%]*)
In[•]:= Table[ ( ωμIJ[μ] - spinCoeffs⟦μ⟧ ) /. subsDefects , {μ, 1, Length[X]}] // Flatten // Union
Out[•]=
{0}
```

- **Metric compatibility**: The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
  - $\nabla_\mu e^a{}_\nu = 0$
  - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

○ Metric compatibility : The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame . The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is : ○ V

не "
v=0
• Here, Vu is the covariant derivative with respect to the connection F" bv, and ea v is
the tetrad component .

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistancy" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (neverthless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will neverthless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

In[ ]:=
```
Block[{eAα, eαA, t, ginv, Γ, gg},
 gg = g4488 /. sg;
 {ginv, Γ} = con[gg];
 eαA = e^α_(A) /. sge^α_(A); eAα = e^(A)_α /. sge^(A)_α;
 t = Table[ Block[{v, V, contra, covari},
    V = eAα[[b, All]];
    v = eαA[[All, b]];
    contra = Table[D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before ω terms*)
    covari = Table[D[V[[k]], X[[μ]] ] - Sum[ V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before ω terms*)
    {b - 1, X[[μ]], {{contra}, {covari}}}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
 t]
FullSimplify[# /. ssgm4488, constraintVars] & /@ %
```

Out[ ]=

$$\left\{\left\{\left\{0, x0, \left\{\left\{0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\; g[4][4][x0, x4]}, 0, 0, 0\right\}\right\},\right.\right.\right.$$

$$\left\{\left\{0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0\right\}\right\}\right\},$$

$$\left\{0, x1, \left\{\left\{0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0\right\}\right\},\right.$$

$$\left.\left\{\left\{0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}\right\}\right\},$$

$$\left\{0, x2, \left\{\left\{0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[2][2][x0, x4]}, 0, 0, 0, 0, 0\right\}\right\},\right.$$

$$\left.\left\{\left\{0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0\right\}\right\}\right\},$$

$$\left\{0, x3, \left\{\left\{0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[3][3][x0, x4]}, 0, 0, 0, 0\right\}\right\},\right.$$

$$\left.\left\{\left\{0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0\right\}\right\}\right\},$$

$$\left\{0, x4, \left\{\left\{0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[4][4][x0, x4]}, 0, 0, 0\right\}\right\},\right.$$

$$\left.\left\{\left\{0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0\right\}\right\}\right\},$$

$$\left\{0, x5, \left\{\left\{0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[5][5][x0, x4]}, 0, 0\right\}\right\},\right.$$

$$\left.\left\{\left\{0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0\right\}\right\}\right\},$$

$$\left\{0, x6, \left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[6][6][x0, x4]}, 0\right\}\right\},\right.$$

$$\left.\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0\right\}\right\}\right\},$$

$$\left\{0, x7, \left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[7][7][x0, x4]}\right\}\right\},\right.$$

$$\left.\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}\right\}\right\}\right\},$$

$$\left\{\{1, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \left\{1, x1,\right.\right.$$

$$\left\{\left\{\left\{-\frac{g[1][1]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}\, g[4][4][x0, x4]},\right.\right.\right.$$

$$\left.\left.\left.0, 0, 0\right\}\right\}, \left\{\left\{-\frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0\right\}\right\}\right\},$$

$$\{1, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{1, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

```
{1, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{1, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{1, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{1, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{2, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {2, x2,
```

$$\left\{\left\{\left\{-\frac{g[2][2]^{(1,0)}[x0,x4]}{2\,g[0][0][x0,x4]\,\sqrt{g[2][2][x0,x4]}},0,0,0,-\frac{g[2][2]^{(0,1)}[x0,x4]}{2\,\sqrt{g[2][2][x0,x4]}\,g[4][4][x0,x4]},0,0,0\right\}\right\},\left\{\left\{-\frac{g[2][2]^{(1,0)}[x0,x4]}{2\,\sqrt{g[2][2][x0,x4]}},0,0,0,-\frac{g[2][2]^{(0,1)}[x0,x4]}{2\,\sqrt{g[2][2][x0,x4]}},0,0,0\right\}\right\}\right\},$$

```
{2, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{2, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{3, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {3, x3,
```

$$\left\{\left\{\left\{-\frac{g[3][3]^{(1,0)}[x0,x4]}{2\,g[0][0][x0,x4]\,\sqrt{g[3][3][x0,x4]}},0,0,0,-\frac{g[3][3]^{(0,1)}[x0,x4]}{2\,\sqrt{g[3][3][x0,x4]}\,g[4][4][x0,x4]},0,0,0\right\}\right\},\left\{\left\{-\frac{g[3][3]^{(1,0)}[x0,x4]}{2\,\sqrt{g[3][3][x0,x4]}},0,0,0,-\frac{g[3][3]^{(0,1)}[x0,x4]}{2\,\sqrt{g[3][3][x0,x4]}},0,0,0\right\}\right\}\right\},$$

```
{3, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{3, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
```

$$\left\{\left\{4,x0,\left\{\left\{\frac{g[0][0]^{(0,1)}[x0,x4]}{2\,g[0][0][x0,x4]\,\sqrt{-g[4][4][x0,x4]}},0,0,0,\frac{g[4][4]^{(1,0)}[x0,x4]}{2\,(-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(1,0)}[x0,x4]}{2\,\sqrt{-g[4][4][x0,x4]}\,g[4][4][x0,x4]},0,0,0\right\}\right\},\right.\right.$$

$$\left.\left.\left\{\left\{\frac{g[0][0]^{(0,1)}[x0,x4]\,\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]},0,0,0,-\frac{g[4][4]^{(1,0)}[x0,x4]}{2\,\sqrt{-g[4][4][x0,x4]}}-\frac{g[4][4]^{(1,0)}[x0,x4]\,\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]},0,0,0\right\}\right\}\right\},$$

$$\left\{4,x1,\left\{\left\{\left\{0,\frac{g[1][1]^{(0,1)}[x0,x4]}{2\,g[1][1][x0,x4]\,\sqrt{-g[4][4][x0,x4]}},0,0,0,0,0,0\right\}\right\},\right.\right.$$

$$\left.\left.\left\{\left\{0,\frac{g[1][1]^{(0,1)}[x0,x4]\,\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]},0,0,0,0,0,0\right\}\right\}\right\},$$

$$\left\{4, x2, \left\{\left\{\left\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]}{2\, g[2][2][x0, x4]\, \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0\right\}\right\},\right.\right.$$

$$\left.\left.\left\{\left\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]\, \sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]}, 0, 0, 0, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{4, x3, \left\{\left\{\left\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2\, g[3][3][x0, x4]\, \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0\right\}\right\},\right.\right.$$

$$\left.\left.\left\{\left\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]\, \sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]}, 0, 0, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{4, x4, \left\{\left\{\left\{-\frac{g[4][4]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\, \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0,\right.\right.\right.\right.$$

$$\left.\left.\frac{g[4][4]^{(0,1)}[x0, x4]}{2\, (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\, \sqrt{-g[4][4][x0, x4]}\, g[4][4][x0, x4]}, 0, 0, 0\right\}\right\},$$

$$\left.\left\{\left\{-\frac{g[4][4]^{(1,0)}[x0, x4]\, \sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]}, 0, 0, 0,\right.\right.$$

$$\left.\left.\left.\left.-\frac{g[4][4]^{(0,1)}[x0, x4]}{2\, \sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(0,1)}[x0, x4]\, \sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]}, 0, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{4, x5, \left\{\left\{\left\{0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2\, \sqrt{-g[4][4][x0, x4]}\, g[5][5][x0, x4]}, 0, 0\right\}\right\},\right.\right.$$

$$\left.\left.\left\{\left\{0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]\, \sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]}, 0, 0\right\}\right\}\right\}\right\},$$

$$\left\{4, x6, \left\{\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2\, \sqrt{-g[4][4][x0, x4]}\, g[6][6][x0, x4]}, 0\right\}\right\},\right.\right.$$

$$\left.\left.\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]\, \sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]}, 0\right\}\right\}\right\}\right\},$$

$$\left\{4, x7, \left\{\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4]}{2\, \sqrt{-g[4][4][x0, x4]}\, g[7][7][x0, x4]}\right\}\right\},\right.\right.$$

$$\left.\left.\left.\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4]\, \sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]}\right\}\right\}\right\}\right\}, \left\{\left\{5, x0,\right.\right.$$

$$\left.\left.\left\{\left\{\left\{0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2\, (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2\, \sqrt{-g[5][5][x0, x4]}\, g[5][5][x0, x4]}, 0, 0\right\}\right\},\right.\right.$$

$$\left.\left.\left\{\left\{0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\, \sqrt{-g[5][5][x0, x4]}} - \frac{g[5][5]^{(1,0)}[x0, x4]\, \sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]},\right.\right.\right.$$

$$\left.\left.\left.\left.0, 0\right\}\right\}\right\}\right\}, \{5, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}

$$\left\{\left\{0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}} - \frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]}, 0, 0\right\}\right\}\right\}, \left\{5, x5, \left\{\left\{-\frac{g[5][5]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2\, g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0\right\}\right\},$$

$$\left\{\left\{-\frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]}, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]}, 0, 0, 0\right\}\right\}\right\}\right\},$$

$$\{5, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\Big\}, \Big\{\Big\{6, x0,$$

$$\left\{\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\,(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}\; g[6][6][x0, x4]}, 0\right\}\right\},\right.$$

$$\left\{\left\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]}, 0\right\}\right\}\right\}, \{6, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{6, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{6, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \Big\{6, x4,$$

$$\left\{\left\{\left\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2\,(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}\; g[6][6][x0, x4]}, 0\right\}\right\},\right.$$

$$\left\{\left\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]}, 0\right\}\right\}\right\}, \{6, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \Big\{6, x6,$$

$$\left\{\left\{\left\{-\frac{g[6][6]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2\, g[4][4][x0, x4]\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0\right\}\right\}, \left\{\left\{-\frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]}, 0,\right.\right.\right.$$

$$\left.\left.\left.

```
{7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {7, x4,
 {{{0, 0, 0, 0, 0, 0, 0, g[7][7]^(0,1)[x0, x4]/(2 (-g[7][7][x0, x4])^(3/2)) + g[7][7]^(0,1)[x0, x4]/(2 Sqrt[-g[7][7][x0, x4]] g[7][7][x0, x4])}},
  {{0, 0, 0, 0, 0, 0, 0,
    -g[7][7]^(0,1)[x0, x4]/(2 Sqrt[-g[7][7][x0, x4]]) - (g[7][7]^(0,1)[x0, x4] Sqrt[-g[7][7][x0, x4]])/(2 g[7][7][x0, x4])}}}},
{7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {7, x7,
 {{{-g[7][7]^(1,0)[x0, x4]/(2 g[0][0][x0, x4] Sqrt[-g[7][7][x0, x4]]), 0, 0, 0, -g[7][7]^(0,1)[x0, x4]/(2 g[4][4][x0, x4] Sqrt[-g[7][7][x0, x4]]),
    0, 0, 0}}, {{-(g[7][7]^(1,0)[x0, x4] Sqrt[-g[7][7][x0, x4]])/(2 g[7][7][x0, x4]), 0, 0,
    0, -(g[7][7]^(0,1)[x0, x4] Sqrt[-g[7][7][x0, x4]])/(2 g[7][7][x0, x4]), 0, 0, 0}}}}}}
```

Out[ ]=

```
{{{0, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {0, x1, {{{0, H, 0, 0, 0, 0, 0, 0}}, {{0, e^(2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}}}},
 {0, x2, {{{0, 0, H, 0, 0, 0, 0, 0}}, {{0, 0, e^(2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}}}},
 {0, x3, {{{0, 0, 0, H, 0, 0, 0, 0}}, {{0, 0, 0, e^(2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0, 0, 0}}}},
 {0, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {0, x5, {{{0, 0, 0, 0, 0, H, 0, 0}}, {{0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0}}}},
 {0, x6, {{{0, 0, 0, 0, 0, 0, H, 0}}, {{0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3), 0}}}},
 {0, x7, {{{0, 0, 0, 0, 0, 0, 0, H}}, {{0, 0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3)}}}}}},
{{1, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x1, {{{-Sqrt[e^(2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
     0, 0, 0, Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}},
   {{-(Sqrt[e^(2 a4[H x4])] H Cos[6 H x0])/Sin[6 H x0]^(5/6), 0, 0, 0, -Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}}},
 {1, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{2, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {2, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {2, x2, {{{-Sqrt[e^(2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
     0, 0, 0, Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}},
```

```
    {{-Sqrt[e^(2 a4[H x4])] H Cos[6 H x0]/Sin[6 H x0]^(5/6), 0, 0, 0, -Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}}}},
  {2, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{3, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x3, {{{-Sqrt[e^(2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
      0, 0, 0, Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}},
    {{-Sqrt[e^(2 a4[H x4])] H Cos[6 H x0]/Sin[6 H x0]^(5/6), 0, 0, 0, -Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}}}},
  {3, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{4, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {4, x1, {{{0, H a4'[H x4], 0, 0, 0, 0, 0, 0}},
    {{0, -e^(2 a4[H x4]) H Sin[6 H x0]^(1/3) a4'[H x4], 0, 0, 0, 0, 0, 0}}}},
  {4, x2, {{{0, 0, H a4'[H x4], 0, 0, 0, 0, 0}},
    {{0, 0, -e^(2 a4[H x4]) H Sin[6 H x0]^(1/3) a4'[H x4], 0, 0, 0, 0, 0}}}},
  {4, x3, {{{0, 0, 0, H a4'[H x4], 0, 0, 0, 0}},
    {{0, 0, 0, -e^(2 a4[H x4]) H Sin[6 H x0]^(1/3) a4'[H x4], 0, 0, 0, 0}}}},
  {4, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {4, x5, {{{0, 0, 0, 0, 0, -H a4'[H x4], 0, 0}},
    {{0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3) a4'[H x4], 0, 0}}}},
  {4, x6, {{{0, 0, 0, 0, 0, 0, -H a4'[H x4], 0}},
    {{0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3) a4'[H x4], 0}}}},
  {4, x7, {{{0, 0, 0, 0, 0, 0, 0, -H a4'[H x4]}},
    {{0, 0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3) a4'[H x4]}}}}},
 {{5, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x5, {{{Sqrt[e^(-2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
      0, 0, 0, Sqrt[e^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}},
    {{-Sqrt[e^(-2 a4[H x4])] H Cos[6 H x0]/Sin[6 H x0]^(5/6), 0, 0, 0, Sqrt[e^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}}},
```

```
    {5, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {5, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
   {{6, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {6, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {6, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {6, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {6, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {6, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {6, x6, {{{Sqrt[E^(-2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
        0, 0, 0, Sqrt[E^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}},
      {{-(Sqrt[E^(-2 a4[H x4])] H Cos[6 H x0])/Sin[6 H x0]^(5/6), 0, 0, 0, Sqrt[E^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}}}},
    {6, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
   {{7, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {7, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {7, x7, {{{Sqrt[E^(-2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
        0, 0, 0, Sqrt[E^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}},
      {{-(Sqrt[E^(-2 a4[H x4])] H Cos[6 H x0])/Sin[6 H x0]^(5/6), 0, 0, 0, Sqrt[E^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}}}}}
```

In[∘]:=

```
(*Block[{eAα,eαA,t,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=e^α_(A)/.sge^α_(A);
  eAα=e_α^(A)/.sge_α^(A);
  t=Table[ Block[{v,V,d},v=eαA[[All,b]];
     d=Table[D[v[[k]],X[[μ]] ]+Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}],{k,1,DIM8}];
     {b-1,X[[μ]],d}],{b,1,Length[eαA]},{μ,1,DIM8}];t]
 FullSimplify[#/.ssgm4488,constraintVars]&/@%*)
```

In[∘]:=

```
(*Block[{eAα,eαA,t,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=e^α_(A)/.sge^α_(A);
  eAα=e_α^(A)/.sge_α^(A);
  t=Table[ Block[{v,V,d},v=eAα[[b,All]];
     d=Table[D[v[[k]],X[[μ]] ]-Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}],{k,1,DIM8}];
     {b-1,X[[μ]],d}],{b,1,Length[eαA]},{μ,1,DIM8}];t]
 FullSimplify[#/.ssgm4488,constraintVars]&/@%*)
```

In[∘]:=
```
Symbolize[ω^(a)_μ (b)]
```

In[∘]:=
```
ω = Array[ω^(a)_μ (b)[###] &, {8, 8, 8}];
```

In[∘]:=
```
ω[[1]]
```

Out[∘]=

$$\{\{\omega^{(a)}_{\mu\,(b)}[1,1,1],\ \omega^{(a)}_{\mu\,(b)}[1,1,2],\ \omega^{(a)}_{\mu\,(b)}[1,1,3],\ \omega^{(a)}_{\mu\,(b)}[1,1,4],$$
$$\omega^{(a)}_{\mu\,(b)}[1,1,5],\ \omega^{(a)}_{\mu\,(b)}[1,1,6],\ \omega^{(a)}_{\mu\,(b)}[1,1,7],\ \omega^{(a)}_{\mu\,(b)}[1,1,8]\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[1,2,1],\ \omega^{(a)}_{\mu\,(b)}[1,2,2],\ \omega^{(a)}_{\mu\,(b)}[1,2,3],\ \omega^{(a)}_{\mu\,(b)}[1,2,4],$$
$$\omega^{(a)}_{\mu\,(b)}[1,2,5],\ \omega^{(a)}_{\mu\,(b)}[1,2,6],\ \omega^{(a)}_{\mu\,(b)}[1,2,7],\ \omega^{(a)}_{\mu\,(b)}[1,2,8]\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[1,3,1],\ \omega^{(a)}_{\mu\,(b)}[1,3,2],\ \omega^{(a)}_{\mu\,(b)}[1,3,3],\ \omega^{(a)}_{\mu\,(b)}[1,3,4],$$
$$\omega^{(a)}_{\mu\,(b)}[1,3,5],\ \omega^{(a)}_{\mu\,(b)}[1,3,6],\ \omega^{(a)}_{\mu\,(b)}[1,3,7],\ \omega^{(a)}_{\mu\,(b)}[1,3,8]\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[1,4,1],\ \omega^{(a)}_{\mu\,(b)}[1,4,2],\ \omega^{(a)}_{\mu\,(b)}[1,4,3],\ \omega^{(a)}_{\mu\,(b)}[1,4,4],$$
$$\omega^{(a)}_{\mu\,(b)}[1,4,5],\ \omega^{(a)}_{\mu\,(b)}[1,4,6],\ \omega^{(a)}_{\mu\,(b)}[1,4,7],\ \omega^{(a)}_{\mu\,(b)}[1,4,8]\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[1,5,1],\ \omega^{(a)}_{\mu\,(b)}[1,5,2],\ \omega^{(a)}_{\mu\,(b)}[1,5,3],\ \omega^{(a)}_{\mu\,(b)}[1,5,4],$$
$$\omega^{(a)}_{\mu\,(b)}[1,5,5],\ \omega^{(a)}_{\mu\,(b)}[1,5,6],\ \omega^{(a)}_{\mu\,(b)}[1,5,7],\ \omega^{(a)}_{\mu\,(b)}[1,5,8]\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[1,6,1],\ \omega^{(a)}_{\mu\,(b)}[1,6,2],\ \omega^{(a)}_{\mu\,(b)}[1,6,3],\ \omega^{(a)}_{\mu\,(b)}[1,6,4],$$
$$\omega^{(a)}_{\mu\,(b)}[1,6,5],\ \omega^{(a)}_{\mu\,(b)}[1,6,6],\ \omega^{(a)}_{\mu\,(b)}[1,6,7],\ \omega^{(a)}_{\mu\,(b)}[1,6,8]\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[1,7,1],\ \omega^{(a)}_{\mu\,(b)}[1,7,2],\ \omega^{(a)}_{\mu\,(b)}[1,7,3],\ \omega^{(a)}_{\mu\,(b)}[1,7,4],$$
$$\omega^{(a)}_{\mu\,(b)}[1,7,5],\ \omega^{(a)}_{\mu\,(b)}[1,7,6],\ \omega^{(a)}_{\mu\,(b)}[1,7,7],\ \omega^{(a)}_{\mu\,(b)}[1,7,8]\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[1,8,1],\ \omega^{(a)}_{\mu\,(b)}[1,8,2],\ \omega^{(a)}_{\mu\,(b)}[1,8,3],\ \omega^{(a)}_{\mu\,(b)}[1,8,4],$$
$$\omega^{(a)}_{\mu\,(b)}[1,8,5],\ \omega^{(a)}_{\mu\,(b)}[1,8,6],\ \omega^{(a)}_{\mu\,(b)}[1,8,7],\ \omega^{(a)}_{\mu\,(b)}[1,8,8]\}\}$$

In[∘]:=
```
sω = Block[{eAα, eαA, t, V, ginv, Γ, gg},
  gg = g4488 /. sg;
  {ginv, Γ} = con[gg];
  eαA = e^α_(A) /. sge^α_(A);
  eAα = e^(A)_α /. sge^(A)_α;
  Table[ω[[μ, a, b]] → Block[{v, d}, v = eαA[[All, b]];
     d = Sum[(D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}]) * eAα[[a, k]], {k, 1, DIM8}];
     d], {μ, 1, DIM8}, {a, 1, Length[eαA]}, {b, 1, Length[eαA]}]]
```

Out[∘]=

$$\{\{\{\omega^{(a)}_{\mu\,(b)}[1,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,1,3]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[1,1,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,1,5]\to \frac{g[0][0]^{(0,1)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ \sqrt{-g[4][4][x0,x4]}},$$

$$\omega^{(a)}_{\mu\,(b)}[1,1,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,1,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[1,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,2,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[1,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,2,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[1,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,3,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[1,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,3,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[1,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,4,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[1,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,4,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[1,5,1]\to -\frac{g[0][0]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[4][4][x0,x4]},\ \omega^{(a)}_{\mu\,(b)}[1,5,2]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[1,5,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,5,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,5,5]\to$$

$$\left(\frac{g[4][4]^{(1,0)}[x0,x4]}{2\,(-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}\,g[4][4][x0,x4]}\right)\sqrt{-g[4][4][x0,x4]},$$

$$\omega^{(a)}_{\mu\,(b)}[1,5,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,5,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[1,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,6,3]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[1,6,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,6,6]\to$$

$$\left(\frac{g[5][5]^{(1,0)}[x0,x4]}{2\,(-g[5][5][x0,x4])^{3/2}}+\frac{g[5][5]^{(1,0)}[x0,x4]}{2\sqrt{-g[5][5][x0,x4]}\,g[5][5][x0,x4]}\right)\sqrt{-g[5][5][x0,x4]},$$

$$\omega^{(a)}_{\mu\,(b)}[1,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,6,8]\to 0\Big\},\ \Big\{\omega^{(a)}_{\mu\,(b)}[1,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,7,2]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[1,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,7,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,7,6]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[1,7,7]\to\left(\frac{g[6][6]^{(1,0)}[x0,x4]}{2\,(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(1,0)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}\,g[6][6][x0,x4]}\right)$$

$$\sqrt{-g[6][6][x0,x4]},\ \omega^{(a)}_{\mu\,(b)}[1,7,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[1,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,8,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[1,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[1,8,8]\to$$

$$\left(\frac{g[7][7]^{(1,0)}[x0,x4]}{2\,(-g[7][7][x0,x4])^{3/2}}+\frac{g[7][7]^{(1,0)}[x0,x4]}{2\sqrt{-g[7][7][x0,x4]}\,g[7][7][x0,x4]}\right)\sqrt{-g[7][7][x0,x4]}\Big\}\Big\},$$

$$\Big\{\Big\{\omega^{(a)}_{\mu\,(b)}[2,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,1,2]\to -\frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\,\sqrt{g[1][1][x0,x4]}},$$

$$\omega^{(a)}_{\mu\,(b)}[2,1,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,1,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,1,5]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[2,1,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,1,8]\to 0\Big\},$$

$$\left\{ \omega^{(a)}_{\mu\,(b)}[2,2,1] \to \frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\,\sqrt{g[1][1][x0,x4]}},\right.$$

$$\omega^{(a)}_{\mu\,(b)}[2,2,2] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,2,3] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,2,4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[2,2,5] \to \frac{g[1][1]^{(0,1)}[x0,x4]}{2\sqrt{g[1][1][x0,x4]}\,\sqrt{-g[4][4][x0,x4]}},$$

$$\left.\omega^{(a)}_{\mu\,(b)}[2,2,6] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,2,7] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,2,8] \to 0\right\},$$

$$\left\{\omega^{(a)}_{\mu\,(b)}[2,3,1] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,2] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,3] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,4] \to 0,\right.$$

$$\left.\omega^{(a)}_{\mu\,(b)}[2,3,5] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,6] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,7] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,8] \to 0\right\},$$

$$\left\{\omega^{(a)}_{\mu\,(b)}[2,4,1] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,2] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,3] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,4] \to 0,\right.$$

$$\left.\omega^{(a)}_{\mu\,(b)}[2,4,5] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,6] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,7] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,8] \to 0\right\},$$

$$\left\{\omega^{(a)}_{\mu\,(b)}[2,5,1] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,2] \to -\frac{g[1][1]^{(0,1)}[x0,x4]\,\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[1][1][x0,x4]}\,g[4][4][x0,x4]},\right.$$

$$\omega^{(a)}_{\mu\,(b)}[2,5,3] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,4] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,5] \to 0,$$

$$\left.\omega^{(a)}_{\mu\,(b)}[2,5,6] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,7] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,8] \to 0\right\},$$

$$\left\{\omega^{(a)}_{\mu\,(b)}[2,6,1] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,2] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,3] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,4] \to 0,\right.$$

$$\left.\omega^{(a)}_{\mu\,(b)}[2,6,5] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,6] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,7] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,8] \to 0\right\},$$

$$\left\{\omega^{(a)}_{\mu\,(b)}[2,7,1] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,2] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,3] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,4] \to 0,\right.$$

$$\left.\omega^{(a)}_{\mu\,(b)}[2,7,5] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,6] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,7] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,8] \to 0\right\},$$

$$\left\{\omega^{(a)}_{\mu\,(b)}[2,8,1] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,2] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,3] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,4] \to 0,\right.$$

$$\left.\left.\omega^{(a)}_{\mu\,(b)}[2,8,5] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,6] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,7] \to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,8] \to 0\right\}\right\},$$

$$\left\{\left\{\omega^{(a)}_{\mu\,(b)}[3,1,1] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,1,2] \to 0,\right.\right.$$

$$\omega^{(a)}_{\mu\,(b)}[3,1,3] \to -\frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\,\sqrt{g[2][2][x0,x4]}},\ \omega^{(a)}_{\mu\,(b)}[3,1,4] \to 0,$$

$$\left.\omega^{(a)}_{\mu\,(b)}[3,1,5] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,1,6] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,1,7] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,1,8] \to 0\right\},$$

$$\left\{\omega^{(a)}_{\mu\,(b)}[3,2,1] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,2] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,3] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,4] \to 0,\right.$$

$$\left.\omega^{(a)}_{\mu\,(b)}[3,2,5] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,6] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,7] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,8] \to 0\right\},$$

$$\left\{\omega^{(a)}_{\mu\,(b)}[3,3,1] \to \frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\,\sqrt{g[2][2][x0,x4]}},\right.$$

$$\omega^{(a)}_{\mu\,(b)}[3,3,2] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,3] \to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[3,3,5] \to \frac{g[2][2]^{(0,1)}[x0,x4]}{2\sqrt{g[2][2][x0,x4]}\,\sqrt{-g[4][4][x0,x4]}},$$

$$\omega^{(a)}_{\mu\,(b)}[3,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[3,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[3,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[3,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,2]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[3,5,3]\to -\frac{g[2][2]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[2][2][x0,x4]}\,g[4][4][x0,x4]},\ \omega^{(a)}_{\mu\,(b)}[3,5,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[3,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[3,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[3,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[3,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[3,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[3,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[3,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,8]\to 0\}\},$$

$$\{\{\omega^{(a)}_{\mu\,(b)}[4,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,3]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[4,1,4]\to -\frac{g[3][3]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[3][3][x0,x4]}},\ \omega^{(a)}_{\mu\,(b)}[4,1,5]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[4,1,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[4,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[4,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[4,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[4,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[4,4,1]\to \frac{g[3][3]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[3][3][x0,x4]}},$$

$$\omega^{(a)}_{\mu\,(b)}[4,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,4,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[4,4,5]\to \frac{g[3][3]^{(0,1)}[x0,x4]}{2\sqrt{g[3][3][x0,x4]}\sqrt{-g[4][4][x0,x4]}},\ \omega^{(a)}_{\mu\,(b)}[4,4,6]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[4,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,4,8]\to 0\},\ \{\omega^{(a)}_{\mu\,(b)}[4,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,2]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[4,5,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,4]\to -\frac{g[3][3]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[3][3][x0,x4]}\,g[4][4][x0,x4]},$$

$$\omega^{(a)}_{\mu\,(b)}[4,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,8]\to 0\},$$

$$\left\{\omega^{(a)}_{\mu\,(b)}[4,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,4]\to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[4,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,8]\to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[4,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,4]\to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[4,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,8]\to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[4,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,4]\to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[4,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,8]\to 0\right\}\right\},$$
$$\left\{\left\{\omega^{(a)}_{\mu\,(b)}[5,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,4]\to 0,\right.\right.$$
$$\omega^{(a)}_{\mu\,(b)}[5,1,5]\to -\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ \sqrt{-g[4][4][x0,x4]}},$$
$$\left.\omega^{(a)}_{\mu\,(b)}[5,1,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,8]\to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[5,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,4]\to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[5,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,8]\to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[5,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,4]\to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[5,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,8]\to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[5,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,4]\to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[5,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,8]\to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[5,5,1]\to \frac{g[4][4]^{(1,0)}[x0,x4]\ \sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\ g[4][4][x0,x4]},\ \omega^{(a)}_{\mu\,(b)}[5,5,2]\to 0,\right.$$
$$\omega^{(a)}_{\mu\,(b)}[5,5,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,5]\to$$
$$\left(\frac{g[4][4]^{(0,1)}[x0,x4]}{2\,(-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(0,1)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}\ g[4][4][x0,x4]}\right)\sqrt{-g[4][4][x0,x4]},$$
$$\left.\omega^{(a)}_{\mu\,(b)}[5,5,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,8]\to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[5,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,3]\to 0,\right.$$
$$\omega^{(a)}_{\mu\,(b)}[5,6,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,6]\to$$
$$\left(\frac{g[5][5]^{(0,1)}[x0,x4]}{2\,(-g[5][5][x0,x4])^{3/2}}+\frac{g[5][5]^{(0,1)}[x0,x4]}{2\sqrt{-g[5][5][x0,x4]}\ g[5][5][x0,x4]}\right)\sqrt{-g[5][5][x0,x4]},$$
$$\left.\omega^{(a)}_{\mu\,(b)}[5,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,8]\to 0\right\},\ \left\{\omega^{(a)}_{\mu\,(b)}[5,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,2]\to 0,\right.$$
$$\omega^{(a)}_{\mu\,(b)}[5,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,6]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[5,7,7]\to\left(\frac{g[6][6]^{(0,1)}[x0,x4]}{2\,(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(0,1)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}\ g[6][6][x0,x4]}\right)$$

$$\sqrt{-g[6][6][x0, x4]}, \ \omega^{(a)}_{\mu\,(b)}[5, 7, 8] \to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[5, 8, 1] \to 0, \ \omega^{(a)}_{\mu\,(b)}[5, 8, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[5, 8, 3] \to 0, \ \omega^{(a)}_{\mu\,(b)}[5, 8, 4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[5, 8, 5] \to 0, \ \omega^{(a)}_{\mu\,(b)}[5, 8, 6] \to 0, \ \omega^{(a)}_{\mu\,(b)}[5, 8, 7] \to 0, \ \omega^{(a)}_{\mu\,(b)}[5, 8, 8] \to$$

$$\left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}\;g[7][7][x0, x4]}\right)\sqrt{-g[7][7][x0, x4]}\Big\}\Big\},$$

$$\Big\{\Big\{\omega^{(a)}_{\mu\,(b)}[6, 1, 1] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 1, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 1, 3] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 1, 4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[6, 1, 5] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 1, 6] \to -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\;\sqrt{-g[5][5][x0, x4]}},$$

$$\omega^{(a)}_{\mu\,(b)}[6, 1, 7] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 1, 8] \to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[6, 2, 1] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 2, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 2, 3] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 2, 4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[6, 2, 5] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 2, 6] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 2, 7] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 2, 8] \to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[6, 3, 1] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 3, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 3, 3] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 3, 4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[6, 3, 5] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 3, 6] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 3, 7] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 3, 8] \to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[6, 4, 1] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 4, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 4, 3] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 4, 4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[6, 4, 5] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 4, 6] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 4, 7] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 4, 8] \to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[6, 5, 1] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 5, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 5, 3] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 5, 4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[6, 5, 5] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 5, 6] \to -\frac{g[5][5]^{(0,1)}[x0, x4]\;\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]\;\sqrt{-g[5][5][x0, x4]}},$$

$$\omega^{(a)}_{\mu\,(b)}[6, 5, 7] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 5, 8] \to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[6, 6, 1] \to \frac{g[5][5]^{(1,0)}[x0, x4]\;\sqrt{-g[5][5][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}\;g[5][5][x0, x4]},$$

$$\omega^{(a)}_{\mu\,(b)}[6, 6, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 6, 3] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 6, 4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[6, 6, 5] \to \frac{g[5][5]^{(0,1)}[x0, x4]\;\sqrt{-g[5][5][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}\;g[5][5][x0, x4]},$$

$$\omega^{(a)}_{\mu\,(b)}[6, 6, 6] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 6, 7] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 6, 8] \to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[6, 7, 1] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 7, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 7, 3] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 7, 4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[6, 7, 5] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 7, 6] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 7, 7] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 7, 8] \to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[6, 8, 1] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 8, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 8, 3] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 8, 4] \to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[6, 8, 5] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 8, 6] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 8, 7] \to 0, \ \omega^{(a)}_{\mu\,(b)}[6, 8, 8] \to 0\Big\}\Big\},$$

$$\Big\{\Big\{\omega^{(a)}_{\mu\,(b)}[7, 1, 1] \to 0, \ \omega^{(a)}_{\mu\,(b)}[7, 1, 2] \to 0, \ \omega^{(a)}_{\mu\,(b)}[7, 1, 3] \to 0,$$

$\omega^{(a)}_{\mu\,(b)}[7,1,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,1,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,1,6]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,1,7]\to -\frac{g[6][6]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[6][6][x0,x4]}},\ \omega^{(a)}_{\mu\,(b)}[7,1,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,5,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,5,3]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,5,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,5,6]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,5,7]\to -\frac{g[6][6]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]\sqrt{-g[6][6][x0,x4]}},\ \omega^{(a)}_{\mu\,(b)}[7,5,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,7,1]\to \frac{g[6][6]^{(1,0)}[x0,x4]\sqrt{-g[6][6][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[6][6][x0,x4]},$

$\omega^{(a)}_{\mu\,(b)}[7,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,7,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,7,5]\to \frac{g[6][6]^{(0,1)}[x0,x4]\sqrt{-g[6][6][x0,x4]}}{2\sqrt{-g[4][4][x0,x4]}\,g[6][6][x0,x4]},$

$\omega^{(a)}_{\mu\,(b)}[7,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,7,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,8]\to 0\}\},$

$\{\{\omega^{(a)}_{\mu\,(b)}[8,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[8,1,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,7]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[8,1,8]\to -\frac{g[7][7]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[7][7][x0,x4]}}\},$

$\{\omega^{(a)}_{\mu\,(b)}[8,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[8,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[8,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,4]\to 0,$

$$\omega^{(a)}_{\mu\,(b)}[8,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[8,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[8,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,7]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,5,8]\to -\frac{g[7][7]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]\sqrt{-g[7][7][x0,x4]}}\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[8,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[8,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[8,8,1]\to \frac{g[7][7]^{(1,0)}[x0,x4]\sqrt{-g[7][7][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[7][7][x0,x4]},$$

$$\omega^{(a)}_{\mu\,(b)}[8,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,8,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,8,5]\to \frac{g[7][7]^{(0,1)}[x0,x4]\sqrt{-g[7][7][x0,x4]}}{2\sqrt{-g[4][4][x0,x4]}\,g[7][7][x0,x4]},$$

$$\omega^{(a)}_{\mu\,(b)}[8,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,8,8]\to 0\}\}\}$$

In[∘]:=
```
(*sω=Block[{eAα,eαA,t,V,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=e^α_(A)/.sge^α_(A);
  eAα=e^(A)_α/.sge^(A)_α;
  Table[ ω^(a)_μ(b) [μ,a,b]→Block[{v,d},v=eαA[[All,b]];
     d=Sum[(D[v[[k]],X[[μ]] ]+Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}])*eAα[[a,k]],{k,1,DIM8}];
     d],{μ,1,DIM8},{a,1,Length[eαA]},{b,1,Length[eαA]}]]*)
```

In[∘]:=
```
sωf = Flatten[sω];
```

In[∘]:=
```
(*ω^(a)_μ(b)*)
ω;
ωmat = % /. sωf
Dimensions[%]
```

Out[∘]=

$$\{\{\{0,0,0,0,\frac{g[0][0]^{(0,1)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[4][4][x0,x4]}},0,0,0\},$$

$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},$$

$$\left\{-\frac{g[0][0]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[4][4][x0,x4]},0,0,0,\right.$$
$$\left(\frac{g[4][4]^{(1,0)}[x0,x4]}{2(-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}\,g[4][4][x0,x4]}\right)\sqrt{-g[4][4][x0,x4]},$$
$$\left.0,0,0\right\},\left\{0,0,0,0,0,\right.$$
$$\left(\frac{g[5][5]^{(1,0)}[x0,x4]}{2(-g[5][5][x0,x4])^{3/2}}+\frac{g[5][5]^{(1,0)}[x0,x4]}{2\sqrt{-g[5][5][x0,x4]}\,g[5][5][x0,x4]}\right)\sqrt{-g[5][5][x0,x4]},0,$$
$$\left.0\right\},\left\{0,0,0,0,0,0,\left(\frac{g[6][6]^{(1,0)}[x0,x4]}{2(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(1,0)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}\,g[6][6][x0,x4]}\right)\right.$$
$$\left.\sqrt{-g[6][6][x0,x4]},0\right\},\left\{0,0,0,0,0,0,0,\right.$$
$$\left.\left(\frac{g[7][7]^{(1,0)}[x0,x4]}{2(-g[7][7][x0,x4])^{3/2}}+\frac{g[7][7]^{(1,0)}[x0,x4]}{2\sqrt{-g[7][7][x0,x4]}\,g[7][7][x0,x4]}\right)\sqrt{-g[7][7][x0,x4]}\right\}\right\},$$
$$\left\{\left\{0,-\frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[1][1][x0,x4]}},0,0,0,0,0,0\right\},\right.$$
$$\left\{\frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[1][1][x0,x4]}},0,0,0,\right.$$
$$\left.\frac{g[1][1]^{(0,1)}[x0,x4]}{2\sqrt{g[1][1][x0,x4]}\sqrt{-g[4][4][x0,x4]}},0,0,0\right\},\{0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0\},\left\{0,-\frac{g[1][1]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[1][1][x0,x4]}\,g[4][4][x0,x4]},0,0,0,0,0,0\right\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\Big\},$$
$$\left\{\left\{0,0,-\frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[2][2][x0,x4]}},0,0,0,0,0\right\},\right.$$
$$\{0,0,0,0,0,0,0,0\},\left\{\frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[2][2][x0,x4]}},0,0,0,\right.$$
$$\left.\frac{g[2][2]^{(0,1)}[x0,x4]}{2\sqrt{g[2][2][x0,x4]}\sqrt{-g[4][4][x0,x4]}},0,0,0\right\},\{0,0,0,0,0,0,0,0\},$$
$$\left\{0,0,-\frac{g[2][2]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[2][2][x0,x4]}\,g[4][4][x0,x4]},0,0,0,0,0\right\},$$
$$\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\Big\},$$
$$\left\{\left\{0,0,0,-\frac{g[3][3]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[3][3][x0,x4]}},0,0,0,0\right\},\{0,0,0,0,0,0,0,0\},\right.$$
$$\{0,0,0,0,0,0,0,0\},\left\{\frac{g[3][3]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[3][3][x0,x4]}},\right.$$
$$\left.0,0,0,\frac{g[3][3]^{(0,1)}[x0,x4]}{2\sqrt{g[3][3][x0,x4]}\sqrt{-g[4][4][x0,x4]}},0,0,0\right\},$$

$$\left\{0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]\ \sqrt{-g[4][4][x0, x4]}}{2\ \sqrt{g[3][3][x0, x4]}\ g[4][4][x0, x4]}, 0, 0, 0, 0\right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\Big\},$$

$$\Big\{\Big\{0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\ \sqrt{g[0][0][x0, x4]}\ \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\Big\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\Big\{\frac{g[4][4]^{(1,0)}[x0, x4]\ \sqrt{-g[4][4][x0, x4]}}{2\ \sqrt{g[0][0][x0, x4]}\ g[4][4][x0, x4]}, 0, 0, 0,$$

$$\left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2\ (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\ \sqrt{-g[4][4][x0, x4]}\ g[4][4][x0, x4]}\right) \sqrt{-g[4][4][x0, x4]}, 0,$$

$$0, 0\Big\}, \Big\{0, 0, 0, 0, 0, \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2\ (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2\ \sqrt{-g[5][5][x0, x4]}\ g[5][5][x0, x4]}\right)$$

$$\sqrt{-g[5][5][x0, x4]}, 0, 0\Big\}, \Big\{0, 0, 0, 0, 0, 0,$$

$$\left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2\ (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\ \sqrt{-g[6][6][x0, x4]}\ g[6][6][x0, x4]}\right) \sqrt{-g[6][6][x0, x4]}, 0\Big\},$$

$$\Big\{0, 0, 0, 0, 0, 0, 0, \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2\ (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\ \sqrt{-g[7][7][x0, x4]}\ g[7][7][x0, x4]}\right)$$

$$\sqrt{-g[7][7][x0, x4]}\Big\}\Big\}, \Big\{\Big\{0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\ \sqrt{g[0][0][x0, x4]}\ \sqrt{-g[5][5][x0, x4]}}, 0, 0\Big\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\Big\{0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]\ \sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]\ \sqrt{-g[5][5][x0, x4]}}, 0, 0\Big\},$$

$$\Big\{\frac{g[5][5]^{(1,0)}[x0, x4]\ \sqrt{-g[5][5][x0, x4]}}{2\ \sqrt{g[0][0][x0, x4]}\ g[5][5][x0, x4]}, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]\ \sqrt{-g[5][5][x0, x4]}}{2\ \sqrt{-g[4][4][x0, x4]}\ g[5][5][x0, x4]},$$

$$0, 0, 0\Big\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\Big\},$$

$$\Big\{\Big\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\ \sqrt{g[0][0][x0, x4]}\ \sqrt{-g[6][6][x0, x4]}}, 0\Big\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\Big\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]\ \sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]\ \sqrt{-g[6][6][x0, x4]}}, 0\Big\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \Big\{\frac{g[6][6]^{(1,0)}[x0, x4]\ \sqrt{-g[6][6][x0, x4]}}{2\ \sqrt{g[0][0][x0, x4]}\ g[6][6][x0, x4]}, 0, 0, 0,$$

$$\frac{g[6][6]^{(0,1)}[x0, x4]\ \sqrt{-g[6][6][x0, x4]}}{2\ \sqrt{-g[4][4][x0, x4]}\ g[6][6][x0, x4]}, 0, 0, 0\Big\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\Big\},$$

$$\Big\{\Big\{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4]}{2\ \sqrt{g[0][0][x0, x4]}\ \sqrt{-g[7][7][x0, x4]}}\Big\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$
$$\left\{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]\sqrt{-g[7][7][x0, x4]}}\right\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \left\{\frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}\, g[7][7][x0, x4]},\right.$$
$$\left.\left.\left. 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}\, g[7][7][x0, x4]}, 0, 0, 0\right\}\right\}\right\}$$

Out[ ]=

$$\{8, 8, 8\}$$

In[ ]:=
```
sωgtry = Block[{eAα, eαA, t, V},
  eαA = e^α_(A) /. sgtrye^α_(A);
  eAα = e^(A)_α /. sgtrye^(A)_α;
  Table[(*V=eAα[[a,All]];*) ω^(a)_μ(b) [μ, a, b] → Block[{v, d}, v = eαA[[All, b]];
     d = Sum[(D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}]) * eAα[[a, k]], {k, 1, DIM8}];
     FullSimplify[d, constraintVars] /. {Sqrt[e^(2 a4[H x4])] → e^(a4[H x4]), Sqrt[e^(-2 a4[H x4])] → e^(-a4[H x4])}],
   {μ, 1, DIM8}, {a, 1, Length[eAα]}, {b, 1, Length[eαA]}]]
```

Out[ ]=

$$\left\{\left\{\left\{\omega^{(a)}_{\mu\,(b)}[1, 1, 1] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 1, 2] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 1, 3] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 1, 4] \to 0,\right.\right.\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[1, 1, 5] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 1, 6] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 1, 7] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 1, 8] \to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[1, 2, 1] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 2, 2] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 2, 3] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 2, 4] \to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[1, 2, 5] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 2, 6] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 2, 7] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 2, 8] \to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[1, 3, 1] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 3, 2] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 3, 3] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 3, 4] \to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[1, 3, 5] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 3, 6] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 3, 7] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 3, 8] \to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[1, 4, 1] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 4, 2] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 4, 3] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 4, 4] \to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[1, 4, 5] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 4, 6] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 4, 7] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 4, 8] \to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[1, 5, 1] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 5, 2] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 5, 3] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 5, 4] \to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[1, 5, 5] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 5, 6] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 5, 7] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 5, 8] \to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[1, 6, 1] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 6, 2] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 6, 3] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 6, 4] \to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[1, 6, 5] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 6, 6] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 6, 7] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 6, 8] \to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[1, 7, 1] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 7, 2] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 7, 3] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 7, 4] \to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[1, 7, 5] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 7, 6] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 7, 7] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 7, 8] \to 0\right\},$$
$$\left\{\omega^{(a)}_{\mu\,(b)}[1, 8, 1] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 8, 2] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 8, 3] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 8, 4] \to 0,\right.$$
$$\left.\omega^{(a)}_{\mu\,(b)}[1, 8, 5] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 8, 6] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 8, 7] \to 0, \omega^{(a)}_{\mu\,(b)}[1, 8, 8] \to 0\right\}\right\},$$

$$\{\{\omega^{(a)}_{\mu\,(b)}[2,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,1,2]\to -e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[2,1,3]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[2,1,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,1,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,1,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,1,7]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[2,1,8]\to 0\},\ \{\omega^{(a)}_{\mu\,(b)}[2,2,1]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[2,2,2]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[2,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,2,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,2,5]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$$
$$\omega^{(a)}_{\mu\,(b)}[2,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,2,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[2,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[2,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,3,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[2,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[2,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,4,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[2,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,2]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$$
$$\omega^{(a)}_{\mu\,(b)}[2,5,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,5]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[2,5,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,5,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[2,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[2,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,6,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[2,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[2,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,7,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[2,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[2,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[2,8,8]\to 0\}\},$$
$$\{\{\omega^{(a)}_{\mu\,(b)}[3,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,1,3]\to -e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},$$
$$\omega^{(a)}_{\mu\,(b)}[3,1,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,1,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,1,6]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[3,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,1,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[3,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[3,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,2,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[3,3,1]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[3,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,3]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[3,3,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,5]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$$
$$\omega^{(a)}_{\mu\,(b)}[3,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[3,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[3,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,8]\to 0\},$$

$$\{\omega^{(a)}_{\mu\,(b)}[3,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,3]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$$
$$\omega^{(a)}_{\mu\,(b)}[3,5,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,6]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[3,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[3,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[3,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[3,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[3,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[3,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[3,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,8]\to 0\}\},$$
$$\{\{\omega^{(a)}_{\mu\,(b)}[4,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,3]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,1,4]\to -e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[4,1,5]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,1,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[4,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[4,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[4,4,1]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[4,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,4,3]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,4,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,4,5]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ \omega^{(a)}_{\mu\,(b)}[4,4,6]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,4,8]\to 0\},\ \{\omega^{(a)}_{\mu\,(b)}[4,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,2]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,5,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,4]\to e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$$
$$\omega^{(a)}_{\mu\,(b)}[4,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[4,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[4,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,8]\to 0\},$$
$$\{\omega^{(a)}_{\mu\,(b)}[4,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,4]\to 0,$$
$$\omega^{(a)}_{\mu\,(b)}[4,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,8,8]\to 0\}\},$$
$$\{\{\omega^{(a)}_{\mu\,(b)}[5,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,4]\to 0,$$

$\omega^{(a)}_{\mu\,(b)}[5,1,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,1,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[5,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[5,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,2,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[5,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[5,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,3,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[5,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[5,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,4,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[5,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[5,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,5,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[5,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[5,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,6,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[5,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[5,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,7,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[5,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,8,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[5,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[5,8,8]\to 0\}\},$

$\{\{\omega^{(a)}_{\mu\,(b)}[6,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,1,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,1,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[6,1,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,1,6]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},$

$\omega^{(a)}_{\mu\,(b)}[6,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,1,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[6,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,2,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[6,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,2,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[6,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,3,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[6,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,3,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[6,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,4,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[6,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,4,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[6,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,5,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,5,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,5,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[6,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,5,6]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ \omega^{(a)}_{\mu\,(b)}[6,5,7]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[6,5,8]\to 0\},\ \{\omega^{(a)}_{\mu\,(b)}[6,6,1]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[6,6,2]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[6,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,6,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,6,5]\to -e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$

$\omega^{(a)}_{\mu\,(b)}[6,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,6,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[6,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,7,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[6,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,7,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[6,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,8,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[6,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[6,8,8]\to 0\}\},$

$\{\{\omega^{(a)}_{\mu\,(b)}[7,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,1,3]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,1,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,1,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,1,6]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,1,7]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[7,1,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,2,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,3,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,4,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,5,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,5,3]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,5,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,5,6]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,5,7]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ \omega^{(a)}_{\mu\,(b)}[7,5,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,6,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,7,1]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[7,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,7,3]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,7,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,7,5]\to -e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$

$\omega^{(a)}_{\mu\,(b)}[7,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,7,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[7,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[7,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[7,8,8]\to 0\}\},$

$\{\{\omega^{(a)}_{\mu\,(b)}[8,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,3]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[8,1,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,6]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[8,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,1,8]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\},$

$\{\omega^{(a)}_{\mu\,(b)}[8,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,4]\to 0,$

$$\omega^{(a)}_{\mu\,(b)}[8,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,2,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[8,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,3,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[8,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,4,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[8,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,3]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,5,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,6]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,5,8]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[8,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,6,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[8,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,4]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,7,8]\to 0\Big\},$$

$$\Big\{\omega^{(a)}_{\mu\,(b)}[8,8,1]\to e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[8,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,8,3]\to 0,$$

$$\omega^{(a)}_{\mu\,(b)}[8,8,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,8,5]\to -e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$$

$$\omega^{(a)}_{\mu\,(b)}[8,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[8,8,8]\to 0\Big\}\Big\}\Big\}$$

In[ ]:=
```
sωfgtry = Flatten[sωgtry];
```

In[ ]:=

```
(*ω_μ^(a)_(b)*)
ω;
ωω = % /. sωfgtry // FullSimplify[#, constraintVars] &
Dimensions[%]
```

Out[ ]=

$$\{\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0,0\},\{e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,0,-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\},\{0,0,0,e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0\},\{e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,-e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\}\},$$
$$\{\{0,0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0\},\{0,0,0,0,0,0,0,0\},\{e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,-e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\},\{0,0,0,0,0,0,0,0\}\},\{\{0,0,0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,-e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\}\}\}$$

Out[ ]=

{8, 8, 8}

In[ ]:=
```
g4488 /. sg
% /. ssgm4488
```

Out[ ]=
```
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}
```

Out[ ]=

$$\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \{0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\}, \{0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}$$

check metric compatibility condition for the Octad

In[ ]:=
```
Block[{eAα, eαA, t, ginv, Γ, gg(*,ωω*)},
  (*ωω=ω_μ^(a)_(b) /.sωf;*)
  gg = g4488 /. sg /. ssgm4488;
  {ginv, Γ} = con[gg];
  eαA = e^α_(A) /. sge^α_(A);
  eAα = e_α^(A) /. sge_α^(A);
  t = Table[ Block[{v, V, contra, covari},
      V = eAα[[b, All]];
      v = eαA[[All, b]];
      contra = Table[
        D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]],
          {i, 1, DIM8}], {k, 1, DIM8}];
      (*no ω terms*)
      covari = Table[
        D[V[[k]], X[[μ]] ] - Sum[ V[[i]] × Γ[[i, k, μ]],
            {i, 1, DIM8}] + Sum[ eAα[[J, All]][[k]] ×
```

```
            ωmat⟦μ, b, J⟧, {J, 1, DIM8}] //
          FullSimplify[#, constraintVars &&
            e^(2 a4[H x4]) > 0 && e^(a4[H x4]) > 0 &&
            e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0] &,
        {k, 1, DIM8}]; (*ω terms*)
      (*{b-1,X⟦μ⟧,{{contra},{covari}}}],
        {b,1,Length[eαA]},{μ,1,DIM8}];*)
      {b - 1, X⟦μ⟧, covari}],
    {b, 1, Length[eαA]}, {μ, 1, DIM8}];
  t]
FullSimplify[# /. ssgm4488,
   constraintVars && e^(2 a4[H x4]) > 0 && e^(a4[H x4]) >
     0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0] & /@ %
```

Out[•]=

$$\Big\{\Big\{\Big\{0, x0, \Big\{\Big(g[0][0]^{(1,0)}[x0, x4] + 24\, H\, \mathrm{Csc}[12\, H\, x0]\, g[0][0][x0, x4]\Big) \Big/ \Big(2\sqrt{g[0][0][x0, x4]}\Big), 0, 0, 0, \frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0\Big\}\Big\},$$

$$\Big\{0, x1, \Big\{0, \Big(-g[1][1]^{(1,0)}[x0, x4] + 2\, e^{2\, a4[H\, x4]}\, H\, \mathrm{Sec}[6\, H\, x0]\, \mathrm{Sin}[6\, H\, x0]^{4/3}\, g[0][0][x0, x4]\Big) \Big/$$

```
    (2 Sqrt[g[0][0][x0, x4]]), 0, 0, 0, 0, 0, 0}},
{0, x2, {0, 0, (-g[2][2]^(1,0)[x0, x4] +
      2 e^(2 a4[H x4]) H Sec[6 H x0]
       Sin[6 H x0]^(4/3) g[0][0][x0, x4]) /
    (2 Sqrt[g[0][0][x0, x4]]), 0, 0, 0, 0, 0}},
{0, x3, {0, 0, 0, (-g[3][3]^(1,0)[x0, x4] +
      2 e^(2 a4[H x4]) H Sec[6 H x0]
       Sin[6 H x0]^(4/3) g[0][0][x0, x4]) /
    (2 Sqrt[g[0][0][x0, x4]]), 0, 0, 0, 0}},
{0, x4, {g[0][0]^(0,1)[x0, x4] / (2 Sqrt[g[0][0][x0, x4]]), 0, 0, 0,
   - g[4][4]^(1,0)[x0, x4] / (2 Sqrt[g[0][0][x0, x4]]), 0, 0, 0}},
{0, x5, {0, 0, 0, 0, 0,
   (-g[5][5]^(1,0)[x0, x4] - 2 e^(-2 a4[H x4]) H Sec[6 H
        x0] Sin[6 H x0]^(4/3) g[0][0][x0, x4]) /
    (2 Sqrt[g[0][0][x0, x4]]), 0, 0}},
{0, x6, {0, 0, 0, 0, 0, 0,
   (-g[6][6]^(1,0)[x0, x4] - 2 e^(-2 a4[H x4]) H Sec[6 H
        x0] Sin[6 H x0]^(4/3) g[0][0][x0, x4]) /
    (2 Sqrt[g[0][0][x0, x4]]), 0}},
```

```
{0, x7, {0, 0, 0, 0, 0, 0, 0,
   (-g[7][7]^(1,0)[x0, x4] - 2 e^(-2 a4[H x4]) H Sec[6 H
         x0] Sin[6 H x0]^(4/3) g[0][0][x0, x4]) /
    (2 Sqrt[g[0][0][x0, x4]])}}},
{{1, x0, {0, (g[1][1]^(1,0)[x0, x4] -
      2 H Cot[6 H x0] g[1][1][x0, x4]) /
    (2 Sqrt[g[1][1][x0, x4]]), 0, 0, 0, 0, 0, 0}},
 {1, x1, {(g[1][1]^(1,0)[x0, x4] -
      2 H Cot[6 H x0] g[1][1][x0, x4]) /
    (2 Sqrt[g[1][1][x0, x4]]), 0, 0,
   0, (g[1][1]^(0,1)[x0, x4] -
      6 H a4'[H x4] g[1][1][x0, x4]) /
    (2 Sqrt[g[1][1][x0, x4]]), 0, 0, 0}},
 {1, x2, {0, 0, 0, 0, 0, 0, 0, 0}},
 {1, x3,
  {0, 0, 0, 0, 0, 0, 0, 0}},
 {1, x4, {0, (g[1][1]^(0,1)[x0, x4] -
      6 H a4'[H x4] g[1][1][x0, x4]) /
    (2 Sqrt[g[1][1][x0, x4]]), 0, 0, 0, 0, 0, 0}},
 {1, x5, {0, 0, 0, 0, 0, 0, 0, 0}},
 {1, x6,
```

```
    {0, 0, 0, 0, 0, 0, 0, 0}},
   {1, x7, {0, 0, 0, 0, 0, 0, 0, 0}}},
  {{2, x0, {0, 0, (g[2][2]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[2][2][x0, x4]) /
      (2 Sqrt[g[2][2][x0, x4]]), 0, 0, 0, 0, 0}},
   {2, x1, {0, 0, 0, 0, 0, 0, 0, 0}},
   {2, x2, {(g[2][2]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[2][2][x0, x4]) /
      (2 Sqrt[g[2][2][x0, x4]]), 0, 0,
     0, (g[2][2]^(0,1)[x0, x4] -
        6 H a4'[H x4] g[2][2][x0, x4]) /
      (2 Sqrt[g[2][2][x0, x4]]), 0, 0, 0}},
   {2, x3, {0, 0, 0, 0, 0, 0, 0, 0}},
   {2, x4,
    {0, 0, (g[2][2]^(0,1)[x0, x4] -
        6 H a4'[H x4] g[2][2][x0, x4]) /
      (2 Sqrt[g[2][2][x0, x4]]), 0, 0, 0, 0, 0}},
   {2, x5, {0, 0, 0, 0, 0, 0, 0, 0}},
   {2, x6, {0, 0, 0, 0, 0, 0, 0, 0}},
   {2, x7, {0, 0, 0, 0, 0, 0, 0, 0}}},
  {{3, x0, {0, 0, 0, (g[3][3]^(1,0)[x0, x4] -
```

```
      2 H Cot[6 H x0] g[3][3][x0, x4]) /
    (2 Sqrt[g[3][3][x0, x4]]), 0, 0, 0, 0}},
 {3, x1, {0, 0, 0, 0, 0, 0, 0, 0}},
 {3, x2, {0, 0, 0, 0, 0, 0, 0, 0}},
 {3, x3, {(g[3][3]^(1,0)[x0, x4] -
      2 H Cot[6 H x0] g[3][3][x0, x4]) /
    (2 Sqrt[g[3][3][x0, x4]]), 0, 0,
   0, (g[3][3]^(0,1)[x0, x4] -
      6 H a4'[H x4] g[3][3][x0, x4]) /
    (2 Sqrt[g[3][3][x0, x4]]), 0, 0, 0}},
 {3, x4, {0, 0, 0, (g[3][3]^(0,1)[x0, x4] -
      6 H a4'[H x4] g[3][3][x0, x4]) /
    (2 Sqrt[g[3][3][x0, x4]]), 0, 0, 0, 0}},
 {3, x5, {0, 0, 0, 0, 0, 0, 0, 0}},
 {3, x6, {0, 0, 0, 0, 0, 0, 0, 0}},
 {3, x7, {0, 0, 0, 0, 0, 0, 0, 0}}},
{{4, x0, {g[0][0]^(0,1)[x0, x4] / (2 Sqrt[-g[4][4][x0, x4]]), 0, 0,
   0, -g[4][4]^(1,0)[x0, x4] / (2 Sqrt[-g[4][4][x0, x4]]), 0, 0, 0}},
 {4, x1, {0, (g[1][1]^(0,1)[x0, x4] +
      6 e^(2 a4[H x4]) H Sin[6 H x0]^(1/3)
```

```
        a4'[H x4] g[4][4][x0, x4]) /
    (2 Sqrt[-g[4][4][x0, x4]]), 0,
  0, 0, 0, 0, 0}},
{4, x2, {0, 0, (g[2][2]^(0,1)[x0, x4] +
      6 e^(2 a4[H x4]) H Sin[6 H x0]^(1/3)
        a4'[H x4] g[4][4][x0, x4]) /
    (2 Sqrt[-g[4][4][x0, x4]]), 0, 0, 0, 0, 0}},
{4, x3, {0, 0, 0, (g[3][3]^(0,1)[x0, x4] +
      6 e^(2 a4[H x4]) H Sin[6 H x0]^(1/3)
        a4'[H x4] g[4][4][x0, x4]) /
    (2 Sqrt[-g[4][4][x0, x4]]), 0, 0, 0, 0}},
{4, x4, {-g[4][4]^(1,0)[x0, x4] / (2 Sqrt[-g[4][4][x0, x4]]), 0, 0,
  0, -g[4][4]^(0,1)[x0, x4] / (2 Sqrt[-g[4][4][x0, x4]]), 0, 0, 0}},
{4, x5, {0, 0, 0, 0, 0,
  1/2 (-6 e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3) a4'[H x4] -
      g[5][5]^(0,1)[x0, x4] / g[4][4][x0, x4])
    Sqrt[-g[4][4][x0, x4]], 0, 0}},
```

$$\left\{4, x6, \left\{0, 0, 0, 0, 0, 0, \frac{1}{2}\left(-6\, e^{-2\, a4[H\, x4]}\, H\, \mathrm{Sin}[6\, H\, x0]^{1/3}\, a4'[H\, x4] - \frac{g[6][6]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]}\right)\sqrt{-g[4][4][x0, x4]}, 0\right\}\right\},$$

$$\left\{4, x7, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{2}\left(-6\, e^{-2\, a4[H\, x4]}\, H\, \mathrm{Sin}[6\, H\, x0]^{1/3}\, a4'[H\, x4] - \frac{g[7][7]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]}\right)\sqrt{-g[4][4][x0, x4]}\right\}\right\}, \left\{\left\{5, x0, \left\{0, 0, 0, 0, 0, -\left(\left(g[5][5]^{(1,0)}[x0, x4] - 2\, H\, \mathrm{Cot}[6\, H\, x0]\, g[5][5][x0, x4]\right) / \left(2\sqrt{-g[5][5][x0, x4]}\right)\right), 0, 0\right\}\right\},$$

$$\{5, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{5, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{5, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\left\{5, x4, \left\{0, 0, 0, 0, 0, -\left(\left(g[5][5]^{(0,1)}[x0, x4] + 6\, H\, a4'[H\, x4]\, g[5][5][x0, x4]\right) / \left(2\sqrt{-g[5][5][x0, x4]}\right)\right), 0, 0\right\}\right\},$$

```
{5, x5, {-((g[5][5]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[5][5][x0, x4])/
      (2 Sqrt[-g[5][5][x0, x4]])), 0,
   0, 0, -((g[5][5]^(0,1)[x0, x4] +
        6 H a4'[H x4] g[5][5][x0, x4])/
      (2 Sqrt[-g[5][5][x0, x4]])), 0, 0, 0}},
 {5, x6, {0, 0, 0, 0, 0, 0, 0, 0}},
 {5, x7,
  {0, 0, 0, 0, 0, 0, 0, 0}}},
{{6, x0, {0, 0, 0, 0, 0, 0,
   -((g[6][6]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[6][6][x0, x4])/
      (2 Sqrt[-g[6][6][x0, x4]])), 0}},
 {6, x1, {0, 0, 0, 0, 0, 0, 0, 0}},
 {6, x2,
  {0, 0, 0, 0, 0, 0, 0, 0}},
 {6, x3, {0, 0, 0, 0, 0, 0, 0, 0}},
 {6, x4,
  {0, 0, 0, 0, 0, 0, -((g[6][6]^(0,1)[x0, x4] +
        6 H a4'[H x4] g[6][6][x0, x4])/
      (2 Sqrt[-g[6][6][x0, x4]])), 0}},
 {6, x5, {0, 0, 0, 0, 0, 0, 0, 0}},
```

```
{6, x6,
 {-((g[6][6]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[6][6][x0, x4])/
      (2 Sqrt[-g[6][6][x0, x4]])), 0,
   0, 0, -((g[6][6]^(0,1)[x0, x4] +
        6 H a4'[H x4] g[6][6][x0, x4])/
      (2 Sqrt[-g[6][6][x0, x4]])), 0, 0, 0}},
 {6, x7, {0, 0, 0, 0, 0, 0, 0, 0}}},
{{7, x0, {0, 0, 0, 0, 0, 0, 0,
   -((g[7][7]^(1,0)[x0, x4] -
        2 H Cot[6 H x0] g[7][7][x0, x4])/
      (2 Sqrt[-g[7][7][x0, x4]]))}},
 {7, x1, {0, 0, 0, 0, 0, 0, 0, 0}},
 {7, x2,
  {0, 0, 0, 0, 0, 0, 0, 0}},
 {7, x3, {0, 0, 0, 0, 0, 0, 0, 0}},
 {7, x4, {0, 0, 0, 0, 0, 0,
   0, -((g[7][7]^(0,1)[x0, x4] +
        6 H a4'[H x4] g[7][7][x0, x4])/
      (2 Sqrt[-g[7][7][x0, x4]]))}},
 {7, x5, {0, 0, 0, 0, 0, 0, 0, 0}},
 {7, x6,
```

```
    {0, 0, 0, 0, 0, 0, 0, 0}},
  {7, x7, {-((g[7][7]^(1,0)[x0, x4] -
          2 H Cot[6 H x0] g[7][7][x0, x4])/
        (2 Sqrt[-g[7][7][x0, x4]])), 0,
     0, 0, -((g[7][7]^(0,1)[x0, x4] +
          6 H a4'[H x4] g[7][7][x0, x4])/
        (2 Sqrt[-g[7][7][x0, x4]])), 0, 0, 0}}}}
```

Out[ ]=

```
{{{0, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x1, {0, 0, 0, 0, 0, 0, 0, 0}},
{0, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {0,
x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x6,
{0, 0, 0, 0, 0, 0, 0, 0}}, {0, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x0,
{0, 0, 0, 0, 0, 0, 0, 0}}, {1, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x2, {0,
0, 0, 0, 0, 0, 0, 0}}, {1, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x4, {0, 0, 0,
0, 0, 0, 0, 0}}, {1, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x6, {0, 0, 0, 0, 0,
0, 0, 0}}, {1, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{2, x0, {0, 0, 0, 0, 0, 0,
0, 0}}, {2, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x2, {0, 0, 0, 0, 0, 0, 0,
0}}, {2, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x4, {0, 0, 0, 0, 0, 0, 0, 0}},
{2, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {2,
x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{3, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {3,
x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x3,
{0, 0, 0, 0, 0, 0, 0, 0}}, {3, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x5, {0,
0, 0, 0, 0, 0, 0, 0}}, {3, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x7, {0, 0, 0,
0, 0, 0, 0, 0}}}, {{4, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x1, {0, 0, 0, 0,
0, 0, 0, 0}}, {4, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x3, {0, 0, 0, 0, 0, 0,
0, 0}}, {4, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x5, {0, 0, 0, 0, 0, 0, 0,
0}}, {4, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x7, {0, 0, 0, 0, 0, 0, 0,
0}}}, {{5, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x1, {0, 0, 0, 0, 0, 0, 0,
0}}, {5, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x3, {0, 0, 0, 0, 0, 0, 0, 0}},
```

```
{5, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {5,
x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{6,
x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x2,
{0, 0, 0, 0, 0, 0, 0, 0}}, {6, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x4, {0,
0, 0, 0, 0, 0, 0, 0}}, {6, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x6, {0, 0, 0,
0, 0, 0, 0, 0}}, {6, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{7, x0, {0, 0, 0, 0,
0, 0, 0, 0}}, {7, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x2, {0, 0, 0, 0, 0, 0,
0, 0}}, {7, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x4, {0, 0, 0, 0, 0, 0, 0,
0}}, {7, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x6, {0, 0, 0, 0, 0, 0, 0, 0}},
{7, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}}
```

In[ ]:=

```
Block[{eAα, eαA, t, ginv, Γ, gg(*,ωω*)},
 (*ωω=ω_μ^(a)_(b) /.sωf;*)
 gg = g4488 /. sg;
 {ginv, Γ} = con[gg];
 eαA = e^α_(A) /. sge^α_(A);
 eAα = e^(A)_α /. sge^(A)_α;
 t = Table[ Block[{v, V, contra, covari},
    V = eAα[[b, All]];
    v = eαA[[All, b]];
    contra = Table[
      D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]],
        {i, 1, DIM8}], {k, 1, DIM8}];
    (*no ω terms*)
    covari = Table[D[V[[k]], X[[μ]] ] -
       Sum[ V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}] +
       Sum[ eAα[[J, All]][[k]] × ωmat[[μ, b, J]],
```

```
            {J, 1, DIM8}],
          {k, 1, DIM8}]; (*ω terms*)
        {b - 1, X[[μ]], {{contra}, {covari}}}],
      {b, 1, Length[eαA]}, {μ, 1, DIM8}];
  t]
FullSimplify[# /. ssgm4488,
   constraintVars && e^(2 a4[H x4]) > 0 && e^(a4[H x4]) >
    0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0] & /@ %
```

Out[•]=

$$\{\{\{0, x0, \{\{\{0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[4][4][x0, x4]}, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{0, x1, \{\{\{0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{0, x2, \{\{\{0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[2][2][x0, x4]},$$

```
      0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x3, {{{0, 0, 0,
      g[3][3]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[3][3][x0, x4]), 0,
      0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x4, {{{0, 0, 0, 0,
      g[4][4]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[4][4][x0, x4]),
      0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x5, {{{0, 0, 0, 0, 0,
      g[5][5]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[5][5][x0, x4]),
      0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x6, {{{0, 0, 0, 0, 0, 0,
      g[6][6]^(1,0)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[6][6][x0, x4]),
      0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{0, x7, {{{0, 0, 0, 0, 0, 0, 0,
```

$$\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\, g[7][7][x0, x4]}\Big\}\Big\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\Big\}\Big\}\Big\},$$

$$\Big\{\{1, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \Big\{1, x1,$$

$$\Big\{\Big\{\Big\{-\frac{g[1][1]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\sqrt{g[1][1][x0, x4]}},$$

$$0, 0, 0,$$

$$-\frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}\, g[4][4][x0, x4]},$$

$$0, 0, 0\Big\}\Big\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\Big\}\Big\},$$

$$\{1, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{1, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{1, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{1, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{1, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{1, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

```
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{2, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {2, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {2, x2,
  {{{{-g[2][2]^(1,0)[x0, x4]/(2 g[0][0][x0, x4] Sqrt[g[2][2][x0, x4]]),
      0, 0, 0,
      -g[2][2]^(0,1)[x0, x4]/(2 Sqrt[g[2][2][x0, x4]] g[4][4][x0, x4]),
      0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {2, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {2, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {2, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {2, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {2, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
```

$$\Big\{\{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$
$$\{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$
$$\{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \Big\{3, x3,$$
$$\Big\{\Big\{\Big\{-\frac{g[3][3]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\, \sqrt{g[3][3][x0, x4]}},$$
$$0, 0, 0,$$
$$-\frac{g[3][3]^{(0,1)}[x0, x4]}{2\, \sqrt{g[3][3][x0, x4]}\, g[4][4][x0, x4]},$$
$$0, 0, 0\Big\}\Big\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\Big\}\Big\},$$
$$\{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$
$$\{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$
$$\{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$
$$\{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$
$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\Big\}, \Big\{\Big\{4, x0,$$

$$\left\{\left\{\left\{\frac{g[0][0]^{(0,1)}[x0,x4]}{2\,g[0][0][x0,x4]\sqrt{-g[4][4][x0,x4]}},\right.\right.\right.$$

$$0,\ 0,\ 0,\ \frac{g[4][4]^{(1,0)}[x0,x4]}{2\,(-g[4][4][x0,x4])^{3/2}}+$$

$$\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}\ g[4][4][x0,x4]},$$

$$\left.\left.0,\ 0,\ 0\right\}\right\},$$

$$\left\{\left\{0,\ 0,\ 0,\ 0,\ -\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}}-\right.\right.$$

$$\frac{g[4][4]^{(1,0)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]}$$

$$-\left(\frac{g[4][4]^{(1,0)}[x0,x4]}{2\,(-g[4][4][x0,x4])^{3/2}}+\right.$$

$$g[4][4]^{(1,0)}[x0,x4]\Big/$$

$$\left(2\sqrt{-g[4][4][x0,x4]}\right.$$

$$\left.\left.g[4][4][x0,x4]\right)\right)$$

$$\left.\left.\left.g[4][4][x0,x4],\ 0,\ 0,\ 0\right\}\right\}\right\},\ \{4,\ x1,$$

$$\left\{\left\{\left\{0,\ \frac{g[1][1]^{(0,1)}[x0,x4]}{2\,g[1][1][x0,x4]\sqrt{-g[4][4][x0,x4]}},\right.\right.\right.$$

$$\left.\left.0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}\right\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{4, x2, \{\{\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]}{2\, g[2][2][x0, x4]\, \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{4, x3, \{\{\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2\, g[3][3][x0, x4]\, \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{4, x4,$$

$$\{\{\{-\frac{g[4][4]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\, \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \frac{g[4][4]^{(0,1)}[x0, x4]}{2\, (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\, \sqrt{-g[4][4][x0, x4]}\, g[4][4][x0, x4]}, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x0, x4]}{2\, \sqrt{-g[4][4][x0, x4]}} -$$

$$\frac{g[4][4]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\, g[4][4][x0, x4]}$$

$$-\left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2\,(-g[4][4][x0, x4])^{3/2}} + g[4][4]^{(0,1)}[x0, x4] \Big/ \left(2\sqrt{-g[4][4][x0, x4]}\; g[4][4][x0, x4]\right)\right)$$

$$g[4][4][x0, x4], 0, 0, 0\}\}\}\},$$

$$\{4, x5, \{\{\{0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}\; g[5][5][x0, x4]}, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{4, x6, \{\{\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}\; g[6][6][x0, x4]}, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{4, x7, \{\{\{0, 0, 0, 0, 0, 0, 0,$$

$$\frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}\, g[7][7][x0, x4]}\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{\{5, x0, \{\{\{0, 0, 0, 0, 0,$$

$$\frac{g[5][5]^{(1,0)}[x0, x4]}{2\,(-g[5][5][x0, x4])^{3/2}} +$$

$$\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}\, g[5][5][x0, x4]},$$

$$0, 0\}\}, \{\{0, 0, 0, 0, 0,$$

$$-\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}} -$$

$$\frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]}$$

$$-\left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2\,(-g[5][5][x0, x4])^{3/2}} + g[5][5]^{(1,0)}[x0, x4] \Big/ \left(2\sqrt{-g[5][5][x0, x4]}\; g[5][5][x0, x4]\right)\right)$$

$$g[5][5][x0, x4], 0, 0\}\}\}\},$$

$$\{5, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

```
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{5, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {5, x4,
 {{{0, 0, 0, 0, 0, g[5][5]^(0,1)[x0, x4]/(2 (-g[5][5][x0, x4])^(3/2)) +
      g[5][5]^(0,1)[x0, x4]/(2 Sqrt[-g[5][5][x0, x4]] g[5][5][x0, x4]),
    0, 0}}, {{0, 0, 0, 0, 0,
    -g[5][5]^(0,1)[x0, x4]/(2 Sqrt[-g[5][5][x0, x4]]) -
     g[5][5]^(0,1)[x0, x4] Sqrt[-g[5][5][x0, x4]]/(2 g[5][5][x0, x4])
     - (g[5][5]^(0,1)[x0, x4]/(2 (-g[5][5][x0, x4])^(3/2)) +
        g[5][5]^(0,1)[x0, x4] /
         (2 Sqrt[-g[5][5][x0, x4]]
           g[5][5][x0, x4]))
     g[5][5][x0, x4], 0, 0}}}}, {5, x5,
```

$$\{\{\{-\frac{g[5][5]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\, \sqrt{-g[5][5][x0, x4]}},$$

$$0, 0, 0,$$

$$-\frac{g[5][5]^{(0,1)}[x0, x4]}{2\, g[4][4][x0, x4]\, \sqrt{-g[5][5][x0, x4]}},$$

$$0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{5, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\},$$

$$\{\{6, x0, \{\{\{0, 0, 0, 0, 0, 0,$$

$$\frac{g[6][6]^{(1,0)}[x0, x4]}{2\, (-g[6][6][x0, x4])^{3/2}} +$$

$$\frac{g[6][6]^{(1,0)}[x0, x4]}{2\, \sqrt{-g[6][6][x0, x4]}\; g[6][6][x0, x4]},$$

$$0\}\}, \{\{0, 0, 0, 0, 0, 0,$$

$$-\frac{g[6][6]^{(1,0)}[x0, x4]}{2\, \sqrt{-g[6][6][x0, x4]}} -$$

$$\frac{g[6][6]^{(1,0)}[x0, x4]\, \sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]}$$

$$-\left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2\,(-g[6][6][x0, x4])^{3/2}} + g[6][6]^{(1,0)}[x0, x4] \Big/ \left(2\sqrt{-g[6][6][x0, x4]}\; g[6][6][x0, x4]\right)\right)$$

$$g[6][6][x0, x4], 0\}\}\}\},$$

$$\{6, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{6, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{6, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{6, x4, \{\{\{0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2\,(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}\; g[6][6][x0, x4]}, 0\}\}, \{\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} -$$

```
                g[6][6]^(0,1)[x0, x4] Sqrt[-g[6][6][x0, x4]]
                ---------------------------------------------
                          2 g[6][6][x0, x4]
              - (g[6][6]^(0,1)[x0, x4]
                ----------------------------------------
                2 (-g[6][6][x0, x4])^(3/2) +
                  g[6][6]^(0,1)[x0, x4] /
                    (2 Sqrt[-g[6][6][x0, x4]]
                      g[6][6][x0, x4]))
              g[6][6][x0, x4], 0}}}},
  {6, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {6, x6,
   {{{-                g[6][6]^(1,0)[x0, x4]
      ---------------------------------------------,
      2 g[0][0][x0, x4] Sqrt[-g[6][6][x0, x4]]
      0, 0, 0,
      -                g[6][6]^(0,1)[x0, x4]
      ---------------------------------------------,
      2 g[4][4][x0, x4] Sqrt[-g[6][6][x0, x4]]
      0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {6, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{7, x0, {{{0, 0, 0, 0, 0, 0, 0,
```

```
        g[7][7]^(1,0)[x0, x4]
        ---------------------------------------- +
        2 (-g[7][7][x0, x4])^(3/2)
          g[7][7]^(1,0)[x0, x4]
          ---------------------------------------------------------
          2 Sqrt[-g[7][7][x0, x4]] g[7][7][x0, x4]}}},
  {{{0, 0, 0, 0, 0, 0, 0,
      -g[7][7]^(1,0)[x0, x4]
      ---------------------------------------- -
      2 Sqrt[-g[7][7][x0, x4]]
        g[7][7]^(1,0)[x0, x4] Sqrt[-g[7][7][x0, x4]]
        ---------------------------------------------
                2 g[7][7][x0, x4]
      -(g[7][7]^(1,0)[x0, x4]
        ---------------------------------------- +
        2 (-g[7][7][x0, x4])^(3/2)
          g[7][7]^(1,0)[x0, x4] /
            (2 Sqrt[-g[7][7][x0, x4]] g[7][7][x0,
                x4]) g[7][7][x0, x4]}}}},
 {7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {7, x4, {{{0, 0, 0, 0, 0, 0, 0,
```

$$\frac{g[7][7]^{(0,1)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}\; g[7][7][x0, x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2\,g[7][7][x0, x4]} - \Big(\frac{g[7][7]^{(0,1)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}} + g[7][7]^{(0,1)}[x0, x4] \Big/ \big(2\sqrt{-g[7][7][x0, x4]}\; g[7][7][x0, x4]\big)\Big)\, g[7][7][x0, x4]\Big\}\Big\}\Big\},$$

$$\{7, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{7, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \Big\{7, x7,$$

$$\Big\{\Big\{\Big\{-\frac{g[7][7]^{(1,0)}[x0, x4]}{2\,g[0][0][x0, x4]\sqrt{-g[7][7][x0, x4]}},$$

$$0, 0, 0,$$

$$-\frac{g[7][7]^{(0,1)}[x0, x4]}{2\, g[4][4][x0, x4]\, \sqrt{-g[7][7][x0, x4]}},$$

```
0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}}}
```

Out[ ]=

```
{{{0, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {0, x1, {{{0, H, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {0, x2, {{{0, 0, H, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {0, x3, {{{0, 0, 0, H, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {0, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {0, x5, {{{0, 0, 0, 0, 0, H, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {0, x6, {{{0, 0, 0, 0, 0, 0, H, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {0, x7, {{{0, 0, 0, 0, 0, 0, 0, H}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{1, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}}, {1, x1,
```

```
{{{{-Sqrt[e^(2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
    0, 0, 0, 3 Sqrt[e^(2 a4[H x4])] H
     Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {1, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{2, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {2, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {2, x2, {{{-Sqrt[e^(2 a4[H x4])] H Sec[6 H x0]
```

```
        Sin[6 H x0]^(7/6), 0, 0, 0, 3 Sqrt[e^(2 a4[H x4])]
        H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {2, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {2, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {2, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {2, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{3, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {3, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {3, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {3, x3, {{{{- Sqrt[e^(2 a4[H x4])] H Sec[6 H x0]
        Sin[6 H x0]^(7/6), 0, 0, 0, 3 Sqrt[e^(2 a4[H x4])]
```

```
        H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}},
    {3, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
    {3, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
    {3, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
    {3, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
   {{4, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
    {4, x1, {{{0, 3 H a4′[H x4], 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
    {4, x2, {{{0, 0, 3 H a4′[H x4], 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
    {4, x3, {{{0, 0, 0, 3 H a4′[H x4], 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
    {4, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}}, {4, x5,
     {{{0, 0, 0, 0, 0, -3 H a4′[H x4], 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}}, {4, x6,
```

```
{{{{0, 0, 0, 0, 0, 0, -3 H a4′[H x4], 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {4, x7,
 {{{0, 0, 0, 0, 0, 0, 0, -3 H a4′[H x4]}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
{{5, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {5, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {5, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {5, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {5, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {5, x5,
 {{{{Sqrt[e^(-2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
     0, 0, 0, 3 Sqrt[e^(-2 a4[H x4])] H
      Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {5, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},
  {{0, 0, 0, 0, 0, 0, 0, 0}}}},
 {5, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
```

```
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
  {{6, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
   {6, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
   {6, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
   {6, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
   {6, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
   {6, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}}, {6, x6,
    {{{{Sqrt[e^(-2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
        0, 0, 0, 3 Sqrt[e^(-2 a4[H x4])] H
         Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
   {6, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}},
      {{0, 0, 0, 0, 0, 0, 0, 0}}}}}},
  {{7, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}},
```

```
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}, {7, x7,
   {{{{Sqrt[e^(-2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
      0, 0, 0, 3 Sqrt[e^(-2 a4[H x4])] H
       Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0}},
    {{0, 0, 0, 0, 0, 0, 0, 0}}}}}}
```

In[ ]:=
```
(*ωΨ=
 Table[Sum[ExpandAll[(ω_μ^(a)_(b) [μ,a,b]/.sωf) Simplify[(Transpose[Ψ16].σ16.SAB[[a,b]].Ψ16)]],
   {a,1,8},{b,1,8}],{μ,1,8}]*)
```

In[ ]:=
```
(*ωΨ=Table[Sum[ExpandAll[(ωmat[[μ,a,b]])Simplify[(Transpose[Ψ16].σ16.SAB[[a,b]].Ψ16)]],
   {a,1,8},{b,1,8}],{μ,1,8}]*)
```

In[ ]:=
```
(*jωΨ=ParallelTable[{j,base16[[j,2]],Table[
    {X[[μ]],Simplify[(Transpose[Ψ16].σ16.(ZZZ0 *T16^α[0]+T16^α[4]* ZZZ4 ).base16[[j,1]].(Sum[
          ωmat[[μ,a,b]]SAB[[a,b]],{a,1,8},{b,1,8}])Ψ16)]},{μ,1,8}]},{j,1,Length[base16]}]*)
```

```
In[•]:= (*jωΨ=ParallelTable[{j,base16[[j,2]],Table[Sum[ExpandAll[(ω_μ^(a)_(b) [μ,a,b]/.sωf)Simplify[
          (Transpose[Ψ16].σ16.(ZZZ0 *T16^α[0]+T16^α[4]* ZZZ4 ).base16[[j,1]].SAB[[a,b]].Ψ16)]],
        {a,1,8},{b,1,8}],{μ,1,8}]},{j,1,Length[base16]}]*)
In[•]:= (*jωΨ[[92]]*)
In[•]:= (*jωΨ[[92]]/.ssgm4488//FullSimplify[#,constraintVars]&*)
In[•]:= SAB[[1]][[2]]
Out[•]=
```

$$\left\{\left\{0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0\right\},\left\{0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0\right\},\right.$$
$$\left\{0,0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,0,0,0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0\right\},\left\{0,0,\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{-\frac{1}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2}\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0\right\},\left\{0,0,0,0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0\right\},$$
$$\left\{0,0,0,0,0,0,0,0,0,0,-\frac{1}{2},0,0,0,0,0\right\},$$
$$\left.\left\{0,0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0\right\},\left\{0,0,0,0,0,0,0,0,\frac{1}{2},0,0,0,0,0,0,0\right\}\right\}$$

```
In[•]:= (ω_μ^(a)_(b) [8, 8, 1] /. Flatten[sωgtry])
Out[•]=
```

$$e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}$$

```
In[•]:= SAB[[1, 2]].Ψ16
Out[•]=
```

$$\left\{\frac{1}{2}\,f16[7][x0,x4],\frac{1}{2}\,f16[6][x0,x4],-\frac{1}{2}\,f16[5][x0,x4],-\frac{1}{2}\,f16[4][x0,x4],\right.$$
$$\frac{1}{2}\,f16[3][x0,x4],\frac{1}{2}\,f16[2][x0,x4],-\frac{1}{2}\,f16[1][x0,x4],-\frac{1}{2}\,f16[0][x0,x4],$$
$$-\frac{1}{2}\,f16[15][x0,x4],-\frac{1}{2}\,f16[14][x0,x4],\frac{1}{2}\,f16[13][x0,x4],\frac{1}{2}\,f16[12][x0,x4],$$
$$\left.-\frac{1}{2}\,f16[11][x0,x4],-\frac{1}{2}\,f16[10][x0,x4],\frac{1}{2}\,f16[9][x0,x4],\frac{1}{2}\,f16[8][x0,x4]\right\}$$

In[ ]:=
```
T16^A[0]
```

Out[ ]=
```
{{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
 {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}}
```

In[ ]:=
```
SAB = Table[1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]),
   {A1, 0, 7}, {B1, 0, 7}];
```

In[ ]:=
```
Ψ16
```

Out[ ]=
```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

In[ ]:=
```
Sum[ExpandAll[(ω^(a)_μ(b) [4, a, b] /. sωfgtry) Simplify[(SAB[[a, b]])]], {a, 1, 8}, {b, 1, 8}].
  Ψ16 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[5][x0,x4] \\
e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[4][x0,x4] \\
-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[7][x0,x4] \\
e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[6][x0,x4] \\
-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[1][x0,x4] \\
e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[0][x0,x4] \\
-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[3][x0,x4] \\
e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[2][x0,x4] \\
e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[13][x0,x4] \\
-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[12][x0,x4] \\
e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[15][x0,x4] \\
-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[14][x0,x4] \\
e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[9][x0,x4] \\
-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[8][x0,x4] \\
e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[11][x0,x4] \\
-e^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,f16[10][x0,x4]
\end{pmatrix}$$

In[ ]:=
```
(*Table[Block[{eAα,eαA,t,V},eαA=e^α_(A) /.sgtrye^α_(A);
  eAα=e^(A)_α /.sgtrye^(A)_α;
  V=eAα[[a,All]];
  t=Table[Block[{v,d},v=eαA[[All,b]];
     d=D[v[[k]],X[[j]] ]+Sum[ v[[i]] Γ[[k,i,j]],{i,1,DIM8}];
     d.V],{k,1,DIM8}];
  t],{j,1,DIM8},{a,1,Length[eαA]},{b,1,Length[eαA]}]*)
```

In[ ]:=
```
Clear[rt];
rt[g_, ass_ : {}] :=
 Module[{t, Γ, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    Γ = FullSimplify[
      Table[(1 / 2) * Sum[(ginv[[i, s]]) *
          (D[g[[s, j]], X[[k]] ] +
            D[g[[s, k]], X[[j]] ] - D[g[[j, k]],
             X[[s]] ]), {s, 1, DIM8}],
       {i, 1, DIM8},
       {j, 1, DIM8}, {k, 1, DIM8}] , ass];
    RicciΓ = ParallelTable[
      FullSimplify[D[Γ[[μ, ν, β]], X[[α]] ] -
        D[Γ[[μ, ν, α]], X[[β]] ] + Sum[
         Γ[[μ, s, α]] × Γ[[s, ν, β]] - Γ[[μ, s, β]] ×
          Γ[[s, ν, α]], {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8},
```

```
        {α, 1, DIM8}, {β, 1, DIM8}] ;
      RieΓ = ParallelTable[ FullSimplify[
          Sum[ RicciΓ⟦α, μ, α, ν⟧, {α, 1, DIM8}],
          ass], {μ, 1, DIM8}, {ν, 1, DIM8}] ;
      RS = FullSimplify[Tr[ ginv.RieΓ] , ass];
      G = ParallelTable[
        FullSimplify[RieΓ⟦α⟧ - 1/2 g⟦α⟧ RS, ass],
        {α, 1, Length[RieΓ]}];];
    Print[t];
    Print[Now];
    Return[{ginv, Γ, RicciΓ, RieΓ, RS, G}]]
```

verify $g_{\mu\nu|\alpha} = 0$:

In[∘]:=
```
Simplify[Table[D[gtry⟦j, k⟧, X⟦i⟧ ] -
    Sum[ gtry⟦k, s⟧ × Γ⟦s, i, j⟧, {s, 1, DIM8}] - Sum[ gtry⟦j, s⟧ × Γ⟦s, i, k⟧, {s, 1, DIM8}],
   {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Out[∘]=

{0}

- **CHECK**

In[∘]:=
```
MatrixMetric44 // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad -e^{-2\,a4[H\,x4]}\,S$$

In[∘]:=
```
(e^α_(A).MatrixMetric44.e^α_(A) /. sgtrye^α_(A)) // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
Transpose[e^(A)_α].η4488.e^(A)_α - MatrixMetric44 /. sgtrye^(A)_α // FullSimplify[#, constraintVars] &
```

Out[∘]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[∘]:=
```
Transpose[e^α_(A)].MatrixMetric44.e^α_(A) - η4488 /. sgtrye^α_(A) // FullSimplify[#, constraintVars] &
```

Out[∘]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[∘]:=
```
Inverse[e^α_(A) /. sgtrye^α_(A)] == e^(A)_α /. sgtrye^(A)_α // FullSimplify[#, constraintVars] &
```

Out[∘]=
```
True
```

In[∘]:=
```
Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]
```

Out[∘]=
```
{{0, 0}, {0, 0}}
```

In[∘]:=
```
(*Inverse[Transpose[e^α_(A)].MatrixMetric44.e^α_(A)]==
  e^(A)_α.Inverse[MatrixMetric44].Transpose[e^(A)_α]//FullSimplify[#,constraintVars]&*)
```

In[∘]:=
```
(*e^(A)_α.Inverse[MatrixMetric44].Transpose[e^(A)_α]//FullSimplify[#,constraintVars]&
   %-Inverse[η4488]//Union[Flatten[#]]&*)
```

In[∘]:=
```
(*Transpose[e^(A)_α]==e^(A)_α//FullSimplify[#,constraintVars]&*)
```

In[∘]:=
```
(*(FullSimplify[e^(A)_α.Inverse[MatrixMetric44].Transpose[e^(A)_α] -Inverse[η4488] ,
    constraintVars]//Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)
```

In[∘]:=
```
(*(FullSimplify[Transpose[e^(A)_α].Inverse[MatrixMetric44].e^(A)_α,constraintVars]-
    Inverse[η4488]//Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)
```

In[∘]:=
```
(*(FullSimplify[Transpose[e^(A)_α].ginv.e^(A)_α,constraintVars]-Inverse[η4488]//
   Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)
```

In[∘]:=
```
(E^α_(A) = Table[Sum[e^(A)_α[[A1, α1]] ginv[[α1, β1]] ×
    η4488[[A1, B1]] /. sgtrye^(A)_α,
```

```
{α1, 1, 8}, {A1, 1, 8}], {β1, 1, 8},
{B1, 1, 8}] // FullSimplify[#,
constraintVars] &) // Column
```

Out[ ]=

$$\{\mathrm{Tan}[6\,H\,x0],\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}$$

$$\left\{0,\ \frac{e^{-a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{0,\ 0,\ \frac{e^{-a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

$$\left\{0,\ 0,\ 0,\ \frac{e^{-a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ 0,\ 0,\ 0,\ 0\right\}$$

$$\{0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0\}$$

$$\left\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{e^{a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ 0,\ 0\right\}$$

$$\left\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{e^{a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ 0\right\}$$

$$\left\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{e^{a4[x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}}\right\}$$

$$\mathrm{LagrangianΨ16} = \kappa\sqrt{\mathrm{Det}[g_{\mu\nu}]}\ \mathrm{Transpose}[\Psi16].\sigma16.T16^{A}[A1].\Psi16_{,\alpha}\,g^{-1\ \alpha\,\beta}\,\eta_{A1\,B1}\,E^{B1}_{\beta} + \text{mass term}$$

In[ ]:=
```
ass = constraintVars
```

Out[ ]=

$$x0 > 0\ \&\&\ x4 > 0\ \&\&\ H > 0\ \&\&\ 6\,H\,x0 > 0\ \&\&\ 3\,H\,x4 > 0\ \&\&\ a4[x4] > 0\ \&\&\ A4[t] > 0\ \&\&\ Q > 0\ \&\&\ z > 0\ \&\&\ t > 0\ \&\&\ M > 0\ \&\&\ K > 0\ \&\&\ e^{-2\,a4[H\,x4]} > 0\ \&\&\ e^{-a4[H\,x4]} > 0\ \&\&\ e^{2\,a4[H\,x4]} > 0\ \&\&\ e^{a4[H\,x4]} > 0\ \&\&\ \mathrm{Sin}[6\,H\,x0] > 0\ \&\&\ \mathrm{Cos}[6\,H\,x0] > 0\ \&\&\ \mathrm{Csc}[6\,H\,x0] > 0\ \&\&\ \mathrm{Sec}[6\,H\,x0] > 0\ \&\&\ \mathrm{Tan}[6\,H\,x0] > 0\ \&\&\ \mathrm{Cot}[6\,H\,x0] > 0\ \&\&\ \mathrm{Sin}[z] > 0\ \&\&\ \sqrt{\mathrm{Sin}[z]} > 0\ \&\&\ \mathrm{Sin}[z]^{3/2} > 0\ \&\&\ \sqrt{\mathrm{Sin}[z]} > 0\ \&\&\ \frac{1}{\mathrm{Sin}[z]^{3/2}} > 0\ \&\&\ \frac{1}{\sqrt{\mathrm{Sin}[z]}} > 0\ \&\&\ \mathrm{Cot}[z] > 0\ \&\&\ \sqrt{\mathrm{Cot}[z]} > 0\ \&\&\ \mathrm{Cot}[z]^{3/2} > 0\ \&\&\ \sqrt{\mathrm{Cot}[z]} > 0\ \&\&\ \mathrm{Tan}[z] > 0\ \&\&\ \mathrm{Sec}[z] > 0\ \&\&\ \sqrt{\mathrm{Csc}[z]} > 0\ \&\&\ \mathrm{Csc}[z] > 0$$

In[ ]:=
```
x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
 Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e^(-2 a4[H x4]) > 0 && e^(-a4[H x4]) > 0 && e^(2 a4[H x4]) > 0 &&
 e^(a4[H x4]) > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
 Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 &&
 Sqrt[Sin[z]] > 0 && 1/Sin[z]^(3/2) > 0 && 1/Sqrt[Sin[z]] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
 Cot[z]^(3/2) > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

Out[ ]=

$$x0 > 0 \,\&\&\, x4 > 0 \,\&\&\, H > 0 \,\&\&\, 6\,H\,x0 > 0 \,\&\&\, H\,x4 > 0 \,\&\&\, a4[x4] > 0 \,\&\&\, A4[t] > 0 \,\&\&$$
$$Q > 0 \,\&\&\, z > 0 \,\&\&\, t > 0 \,\&\&\, M > 0 \,\&\&\, K > 0 \,\&\&\, e^{-2\,a4[H\,x4]} > 0 \,\&\&\, e^{-a4[H\,x4]} > 0 \,\&\&\, e^{2\,a4[H\,x4]} > 0 \,\&\&$$
$$e^{a4[H\,x4]} > 0 \,\&\&\, Sin[6\,H\,x0] > 0 \,\&\&\, Cos[6\,H\,x0] > 0 \,\&\&\, Csc[6\,H\,x0] > 0 \,\&\&\, Sec[6\,H\,x0] > 0 \,\&\&$$
$$Tan[6\,H\,x0] > 0 \,\&\&\, Cot[6\,H\,x0] > 0 \,\&\&\, Sin[z] > 0 \,\&\&\, \sqrt{Sin[z]} > 0 \,\&\&\, Sin[z]^{3/2} > 0 \,\&\&$$
$$\sqrt{Sin[z]} > 0 \,\&\&\, \frac{1}{Sin[z]^{3/2}} > 0 \,\&\&\, \frac{1}{\sqrt{Sin[z]}} > 0 \,\&\&\, Cot[z] > 0 \,\&\&\, \sqrt{Cot[z]} > 0 \,\&\&$$
$$Cot[z]^{3/2} > 0 \,\&\&\, \sqrt{Cot[z]} > 0 \,\&\&\, Tan[z] > 0 \,\&\&\, Sec[z] > 0 \,\&\&\, \sqrt{Csc[z]} > 0 \,\&\&\, Csc[z] > 0$$

In[ ]:=
```
ginv == FullSimplify[Inverse[gtry], ass]
```

Out[ ]=

True

In[ ]:=
```
ginv === Transpose[ginv]
```

Out[ ]=

True

verify metric covariant derivative again:

In[ ]:=
```
Simplify[Table[D[gtry[[j, k]], X[[i]] ] -
     Sum[ gtry[[k, s]] × Γ[[s, i, j]], {s, 1, DIM8}] - Sum[ gtry[[j, s]] × Γ[[s, i, k]], {s, 1, DIM8}],
    {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Out[ ]=

{0}

In[ ]:=
```
RS
```

Out[ ]=

$$6\,H^2\left(-7 + a4'[H\,x4]^2\right)$$

In[ ]:=
```
EinsteinG
skelx[%]
```

Out[ ]=

$$\{\{-3 H^2 \mathrm{Cot}[6 H x0]^2 (-5 + a4'[H x4]^2), 0, 0, 0, 0, 0, 0, 0\},$$
$$\{0, -e^{2 a4[H x4]} H^2 \mathrm{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, -e^{2 a4[H x4]} H^2 \mathrm{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0\},$$
$$\{0, 0, 0, -e^{2 a4[H x4]} H^2 \mathrm{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, -3 H^2 (7 + a4'[H x4]^2), 0, 0, 0\},$$
$$\{0, 0, 0, 0, 0, e^{-2 a4[H x4]} H^2 \mathrm{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4]), 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, e^{-2 a4[H x4]} H^2 \mathrm{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4]), 0\},$$
$$\{0, 0, 0, 0, 0, 0, 0, e^{-2 a4[H x4]} H^2 \mathrm{Sin}[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4])\}\}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

$$\mathrm{Union}\Big[\mathrm{Table}\Big[\mathrm{Sum}[\tau[A].\mathrm{bas64}[[k, 1]].\sigma.\overline{\tau}[A] \times \eta 8[[A, A]], \{A, 1, 8\}] - \mathrm{ID8}\,\mathrm{Tr}[\mathrm{bas64}[[k, 1]].\sigma] + 4\,\mathrm{eg}[[k]]\,\frac{\mathrm{bas64}[[k, 1]] - \mathrm{Transpose}[\mathrm{bas64}[[k, 1]]]}{2}.\sigma, \{k, 1, 64\}]\Big]$$

$$\{\{\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\}$$

BASIS of 16 × 16 matrices :

In[ ]:=
```
16 * 16
```

Out[ ]=

256

In[ ]:=
```
2^8
```
Out[ ]=
```
256
```
In[ ]:=
```
Sum[Binomial[8, p], {p, 0, 8}]
```
Out[ ]=
```
256
```
In[ ]:=
```
Id16 == ID16 == Dot @@ (T16^A[#] & /@ Range[0, 8])
```
Out[ ]=
```
True
```

In[ ]:=
```
Clear[t160]; t160 = {{Id16, {Range[0, 8]}}}; Length[t160]
Clear[t16A];
t16A = {};
Do[AppendTo[t16A, {T16^A[j], {j}}], {j, 0, 7}];
Length[t16A]
Clear[t16AB];
t16AB = {};
Do[AppendTo[t16AB, {T16^A[j].T16^A[k], {j, k}}], {j, 0, 6}, {k, j + 1, 7}];
Length[t16AB]
Clear[t16ABC];
t16ABC = {};
Do[AppendTo[t16ABC, {T16^A[j].T16^A[k].T16^A[h], {j, k, h}}],
 {j, 0, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[t16ABC]
Clear[t16ABCD];
t16ABCD = {};
Do[AppendTo[t16ABCD, {T16^A[j].T16^A[k].T16^A[h].T16^A[i], {j, k, h, i}}],
 {j, 0, 4}, {k, j + 1, 5}, {h, k + 1, 6}, {i, h + 1, 7}];
Length[t16ABCD]
Clear[t16ABCDE];
t16ABCDE = {};
Do[AppendTo[t16ABCDE, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l], {j, k, h, i, l}}],
 {j, 0, 3}, {k, j + 1, 4}, {h, k + 1, 5}, {i, h + 1, 6}, {l, i + 1, 7}];
Length[t16ABCDE]
Clear[t16ABCDEF];
t16ABCDEF = {};
Do[AppendTo[t16ABCDEF, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q], {j, k, h, i, l,
    q}}], {j, 0, 2}, {k, j + 1, 3}, {h, k + 1, 4}, {i, h + 1, 5}, {l, i + 1, 6}, {q, l + 1, 7}];
Length[t16ABCDEF]
Clear[t16ABCDEFG];
t16ABCDEFG = {};
Do[AppendTo[t16ABCDEFG,
  {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].T16^A[r], {j, k, h, i, l, q, r}}],
 {j, 0, 1}, {k, j + 1, 2}, {h, k + 1, 3}, {i, h + 1, 4}, {l, i + 1, 5}, {q, l + 1, 6}, {r, q + 1, 7}];
Length[t16ABCDEFG]
Clear[t16ABCDEFGH];
t16ABCDEFGH = {};
Do[AppendTo[t16ABCDEFGH, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].
    T16^A[q].T16^A[r].T16^A[s], {j, k, h, i, l, q, r, s}}], {j, 0, 1}, {k, j + 1, 1},
 {h, k + 1, 2}, {i, h + 1, 3}, {l, i + 1, 4}, {q, l + 1, 5}, {r, q + 1, 6}, {s, r + 1, 7}];
Length[t16ABCDEFGH]
% + %% + %%% + %%%% + %%%%% + %%%%%% + %%%%%%% + %%%%%%%% + %%%%%%%%%
```

Out[ ]=

1

Out[ ]=
8

Out[ ]=
28

Out[ ]=
56

Out[ ]=
70

Out[ ]=
56

Out[ ]=
28

Out[ ]=
8

Out[ ]=
1

Out[ ]=
256

In[ ]:=
```
(*base16=Flatten[
  {t160,t16A,t16AB,t16ABC,t16ABCD,t16ABCDE,t16ABCDEF,t16ABCDEFG,t16ABCDEFGH},1];
Length[base16]*)
```

In[ ]:=
```
base16 = Flatten[
  {t16A, t16AB, t16ABC, t16ABCD, t16ABCDE, t16ABCDEF, t16ABCDEFG, t16ABCDEFGH, t160}, 1];
Length[base16]
```

Out[ ]=
256

In[ ]:=
```
Length[Union[base16⟦All, 1⟧]]
```

Out[ ]=
256

In[ ]:=
```
Tr[#.# / 16] & /@ base16⟦All, 1⟧ // Flatten // Union
```

Out[ ]=
{-1, 1}

In[ ]:=
```
positiveTrMM = Select[base16, Tr[#⟦1⟧.#⟦1⟧] > 0 & → "Index"]
```

Out[ ]=
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[ ]:=
```
negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]
```

Out[ ]=
```
{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58,
 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97,
 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145,
 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182,
 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214,
 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}
```

In[ ]:=
```
{Length[positiveTrMM], Length[negativeTrMM]}
```

Out[ ]=
```
{136, 120}
```

In[ ]:=
```
dups = Block[{s}, s = base16[[All, 1]]; Select[s, MemberQ[s, -#] &]]; Length[dups]
```

Out[ ]=
```
0
```

In[ ]:=
```
dups = Select[base16, MemberQ[base16[[All, 1]], -#[[1]]] &]; Length[dups]
```

Out[ ]=
```
0
```

In[ ]:=
```
BASE16 = use16 = Complement[base16, dups]; Length[use16]
```

Out[ ]=
```
256
```

In[ ]:=
```
(*(T16^A[8]=T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7])//MatrixForm*)
```

In[ ]:=
```
MatrixForm[T16^A[#]] & /@ Range[0, 7]
```

Out[ ]=

$$\left\{\begin{pmatrix} 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\left.\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}\right\}$$

In[ ]:=
```
(*BASE16[[-1]]*)
```

In[ ]:=
```
(*BASE16[[-2]]
 %[[1]]==BASE16[[-1]][[1]]*)
```

In[ ]:=
```
16 * (16 + 1) / 2
16 * (16 - 1) / 2
% + %%
```

Out[ ]=
```
136
```

Out[ ]=
```
120
```

Out[ ]=
```
256
```

In[ ]:=
```
Clear[antisymm16];
antisymm16 = {};
Do[If[base16⟦k, 1⟧ === - Transpose[base16⟦k, 1⟧],
  AppendTo[antisymm16, {base16⟦k, 1⟧, {{k}, base16⟦k, 2⟧}}] ], {k, 1, Length[base16]}];
Length[antisymm16]
antisymm16⟦%⟧
```

Out[ ]=
```
120
```

Out[ ]=
```
{{{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
  {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}}, {{254}, {1, 2, 3, 4, 5, 6, 7}}}
```

In[ ]:=
```
Clear[symm16];
symm16 = {};
Do[If[base16⟦k, 1⟧ === Transpose[base16⟦k, 1⟧],
  AppendTo[symm16, {base16⟦k, 1⟧, {{k}, base16⟦k, 2⟧}}] ], {k, 1, Length[base16]}];
Length[symm16]
symm16⟦%⟧
```

Out[ ]=
```
136
```

Out[ ]=
```
{{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

In[ ]:=
```
dupssymm16 = Select[symm16, MemberQ[symm16⟦All, 1⟧, -#⟦1⟧] &]; Length[dupssymm16]
```

Out[ ]=
```
0
```

In[ ]:=
```
#[[2]] & /@ symm16
```

Out[ ]=
```
{{{1}, {0}}, {{2}, {1}}, {{3}, {2}}, {{4}, {3}}, {{12}, {0, 4}}, {{13}, {0, 5}},
 {{14}, {0, 6}}, {{15}, {0, 7}}, {{18}, {1, 4}}, {{19}, {1, 5}}, {{20}, {1, 6}},
 {{21}, {1, 7}}, {{23}, {2, 4}}, {{24}, {2, 5}}, {{25}, {2, 6}}, {{26}, {2, 7}},
 {{27}, {3, 4}}, {{28}, {3, 5}}, {{29}, {3, 6}}, {{30}, {3, 7}}, {{39}, {0, 1, 4}},
 {{40}, {0, 1, 5}}, {{41}, {0, 1, 6}}, {{42}, {0, 1, 7}}, {{44}, {0, 2, 4}}, {{45}, {0, 2, 5}},
 {{46}, {0, 2, 6}}, {{47}, {0, 2, 7}}, {{48}, {0, 3, 4}}, {{49}, {0, 3, 5}}, {{50}, {0, 3, 6}},
 {{51}, {0, 3, 7}}, {{59}, {1, 2, 4}}, {{60}, {1, 2, 5}}, {{61}, {1, 2, 6}}, {{62}, {1, 2, 7}},
 {{63}, {1, 3, 4}}, {{64}, {1, 3, 5}}, {{65}, {1, 3, 6}}, {{66}, {1, 3, 7}}, {{73}, {2, 3, 4}},
 {{74}, {2, 3, 5}}, {{75}, {2, 3, 6}}, {{76}, {2, 3, 7}}, {{89}, {4, 5, 6}}, {{90}, {4, 5, 7}},
 {{91}, {4, 6, 7}}, {{92}, {5, 6, 7}}, {{93}, {0, 1, 2, 3}}, {{102}, {0, 1, 4, 5}},
 {{103}, {0, 1, 4, 6}}, {{104}, {0, 1, 4, 7}}, {{105}, {0, 1, 5, 6}}, {{106}, {0, 1, 5, 7}},
 {{107}, {0, 1, 6, 7}}, {{112}, {0, 2, 4, 5}}, {{113}, {0, 2, 4, 6}}, {{114}, {0, 2, 4, 7}},
 {{115}, {0, 2, 5, 6}}, {{116}, {0, 2, 5, 7}}, {{117}, {0, 2, 6, 7}}, {{118}, {0, 3, 4, 5}},
 {{119}, {0, 3, 4, 6}}, {{120}, {0, 3, 4, 7}}, {{121}, {0, 3, 5, 6}}, {{122}, {0, 3, 5, 7}},
 {{123}, {0, 3, 6, 7}}, {{132}, {1, 2, 4, 5}}, {{133}, {1, 2, 4, 6}}, {{134}, {1, 2, 4, 7}},
 {{135}, {1, 2, 5, 6}}, {{136}, {1, 2, 5, 7}}, {{137}, {1, 2, 6, 7}}, {{138}, {1, 3, 4, 5}},
 {{139}, {1, 3, 4, 6}}, {{140}, {1, 3, 4, 7}}, {{141}, {1, 3, 5, 6}}, {{142}, {1, 3, 5, 7}},
 {{143}, {1, 3, 6, 7}}, {{148}, {2, 3, 4, 5}}, {{149}, {2, 3, 4, 6}}, {{150}, {2, 3, 4, 7}},
 {{151}, {2, 3, 5, 6}}, {{152}, {2, 3, 5, 7}}, {{153}, {2, 3, 6, 7}}, {{162}, {4, 5, 6, 7}},
 {{167}, {0, 1, 2, 4, 5}}, {{168}, {0, 1, 2, 4, 6}}, {{169}, {0, 1, 2, 4, 7}},
 {{170}, {0, 1, 2, 5, 6}}, {{171}, {0, 1, 2, 5, 7}}, {{172}, {0, 1, 2, 6, 7}},
 {{173}, {0, 1, 3, 4, 5}}, {{174}, {0, 1, 3, 4, 6}}, {{175}, {0, 1, 3, 4, 7}},
 {{176}, {0, 1, 3, 5, 6}}, {{177}, {0, 1, 3, 5, 7}}, {{178}, {0, 1, 3, 6, 7}},
 {{183}, {0, 2, 3, 4, 5}}, {{184}, {0, 2, 3, 4, 6}}, {{185}, {0, 2, 3, 4, 7}},
 {{186}, {0, 2, 3, 5, 6}}, {{187}, {0, 2, 3, 5, 7}}, {{188}, {0, 2, 3, 6, 7}},
 {{197}, {0, 4, 5, 6, 7}}, {{198}, {1, 2, 3, 4, 5}}, {{199}, {1, 2, 3, 4, 6}},
 {{200}, {1, 2, 3, 4, 7}}, {{201}, {1, 2, 3, 5, 6}}, {{202}, {1, 2, 3, 5, 7}},
 {{203}, {1, 2, 3, 6, 7}}, {{212}, {1, 4, 5, 6, 7}}, {{217}, {2, 4, 5, 6, 7}},
 {{218}, {3, 4, 5, 6, 7}}, {{225}, {0, 1, 2, 4, 5, 6}}, {{226}, {0, 1, 2, 4, 5, 7}},
 {{227}, {0, 1, 2, 4, 6, 7}}, {{228}, {0, 1, 2, 5, 6, 7}}, {{229}, {0, 1, 3, 4, 5, 6}},
 {{230}, {0, 1, 3, 4, 5, 7}}, {{231}, {0, 1, 3, 4, 6, 7}}, {{232}, {0, 1, 3, 5, 6, 7}},
 {{234}, {0, 2, 3, 4, 5, 6}}, {{235}, {0, 2, 3, 4, 5, 7}}, {{236}, {0, 2, 3, 4, 6, 7}},
 {{237}, {0, 2, 3, 5, 6, 7}}, {{240}, {1, 2, 3, 4, 5, 6}}, {{241}, {1, 2, 3, 4, 5, 7}},
 {{242}, {1, 2, 3, 4, 6, 7}}, {{243}, {1, 2, 3, 5, 6, 7}}, {{247}, {0, 1, 2, 3, 4, 5, 6}},
 {{248}, {0, 1, 2, 3, 4, 5, 7}}, {{249}, {0, 1, 2, 3, 4, 6, 7}}, {{250}, {0, 1, 2, 3, 5, 6, 7}},
 {{255}, {0, 1, 2, 3, 4, 5, 6, 7}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

In[ ]:=
```
σ16 == T16^A[0].T16^A[1].T16^A[2].T16^A[3]
```

Out[ ]=
```
True
```

In[ ]:=
```
σ16 == base16[[93]][[1]]
```

Out[ ]=
```
True
```

In[∘]:=
```
Select[symm16, #[[2]][[1]][[1]] == 93 & → "Index"]
```
Out[∘]=
```
{49}
```
In[∘]:=
```
σ16 == symm16[[49]][[1]]
```
Out[∘]=
```
True
```
In[∘]:=
```
symm16[[134]]
```
Out[∘]=
```
{{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{250}, {0, 1, 2, 3, 5, 6, 7}}}
```
In[∘]:=
```
Select[symm16, #[[2]][[1]][[1]] == 250 & → "Index"]
```
Out[∘]=
```
{134}
```
In[∘]:=
```
T16__Superscript__A
```
Out[∘]=
$T16^A$

In[∘]:=
```
σ16.T16^A[5].T16^A[6].T16^A[7] == symm16[[134]][[1]]
```
Out[∘]=
```
True
```
In[∘]:=
```
T16^A[8] == base16[[255]][[1]]
```
Out[∘]=
```
True
```
In[∘]:=
```
Select[symm16, #[[2]][[1]][[1]] == 255 & → "Index"]
```
Out[∘]=
```
{135}
```

- **similarly :**

In[∘]:=
```
coupled1 = {4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13} // Union
```
Out[∘]=
```
{0, 1, 4, 5, 8, 9, 12, 13}
```
In[∘]:=
```
coupled2 = Complement[Range[0, 15], coupled1]
```
Out[∘]=
```
{2, 3, 6, 7, 10, 11, 14, 15}
```

In[ ]:=
```
coupled = {coupled1, coupled2}
```

Out[ ]=
```
{{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}}
```

In[ ]:=
```
Block[{m, vars, eqs, eqs1, eqs2, s, Φ16},
 Φ16 = f[#] & /@ Range[16];
 m = Array[qb[#1, #2] &, {16, 16}];
 vars = Flatten[m];
 eqs1 = And @@ Thread[m.Φ16 == Flatten[{Φ16⟦# + 1⟧ & /@ coupled1, Φ16⟦# + 1⟧ & /@ coupled2}]];
 eqs2 = And @@ Thread[m.Transpose[m] == Flatten[ID16]];
 eqs = eqs1 && eqs2;
 s = Solve[eqs, vars] // FullSimplify;
 s
]
```

Out[ ]=
```
{}
```

In[ ]:=
```
linTrans = Block[{m, b, vars, eqs, eqs1, eqs2, s, Φ16, t},
  Φ16 = f[#] & /@ Range[16];
  m = Array[qb[#1, #2] &, {16, 16}];
  b = Flatten[{Φ16⟦# + 1⟧ & /@ coupled1, Φ16⟦# + 1⟧ & /@ coupled2}];
  t = Table[Block[{sv, eq, d}, d = D[ m⟦j⟧.b, Φ16⟦j⟧];
     sv = Flatten[{d → 1, Thread[Complement[m⟦j⟧, {d}] → 0]}];
     (*Print[{m⟦j⟧.b},{Φ16⟦j⟧},{sv}];*)
     (*eq=Thread[m⟦j⟧.b==Φ16⟦j⟧];
     sv=Solve[eq,Flatten[m⟦j⟧]]//FullSimplify;
     sv*)(*{v.b,Φ16⟦j⟧}*)sv], {j, 1, 16}];
  (*
  vars=Flatten[m];
  eqs1=And@@Thread[m.Φ16==Flatten[{Φ16⟦#+1⟧&/@coupled1,Φ16⟦#+1⟧&/@coupled2}]];
  eqs2=And@@Thread[m.Transpose[m]==Flatten[ID16]];
  eqs=eqs1&&eqs2;
  s=Solve[eqs,vars]//FullSimplify;*)
  m /. Flatten[t]
 ]
```

Out[ ]=
```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[ ]:=
```
linTrans // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
linTrans.Transpose[linTrans] // FullSimplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=
```
linTrans.Ψ16 // Column
Union[%[[1]]]
```

Out[ ]=
```
f16[0][x0, x4]
f16[1][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[2][x0, x4]
f16[3][x0, x4]
f16[10][x0, x4]
f16[11][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
f16[6][x0, x4]
f16[7][x0, x4]
f16[14][x0, x4]
f16[15][x0, x4]
```

Out[ ]=
```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

In[ ]:=
```
positiveTrMM = Select[base16, Tr[#[[1]] . #[[1]]] > 0 & ->
"Index"]
```

Out[ ]=
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[ ]:=
```
negativeTrMM = Select[base16, Tr[#[[1]] . #[[1]]] < 0 & ->
"Index"]
```

Out[ ]=
{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81,

82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[ ]:=
```
{Length[positiveTrMM], Length[negativeTrMM]}
```

Out[ ]=

{136, 120}

In[ ]:=
```
positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]
```

Out[ ]=

{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[ ]:=
```
negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]
```

Out[ ]=

{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[ ]:=
```
{Length[positiveTrMM], Length[negativeTrMM]}
```

Out[ ]=

{136, 120}

In[∘]:=
```
base16[[#]] & /@ positiveTrMM;
{#[[2]], Tr[Transpose[#[[1]]].linTrans]} & /@ %
Select[%, #[[2]] ≠ 0 &]
linTransSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];
```

Out[∘]=
```
{{{0}, 0}, {{1}, 0}, {{2}, 0}, {{3}, 0}, {{0, 4}, 0}, {{0, 5}, 0}, {{0, 6}, 0}, {{0, 7}, -4},
 {{1, 4}, 0}, {{1, 5}, 0}, {{1, 6}, 0}, {{1, 7}, 0}, {{2, 4}, 0}, {{2, 5}, 0}, {{2, 6}, 0},
 {{2, 7}, 0}, {{3, 4}, -4}, {{3, 5}, 0}, {{3, 6}, 0}, {{3, 7}, 0}, {{0, 1, 4}, 0},
 {{0, 1, 5}, 0}, {{0, 1, 6}, 0}, {{0, 1, 7}, 0}, {{0, 2, 4}, 0}, {{0, 2, 5}, 0}, {{0, 2, 6}, 0},
 {{0, 2, 7}, 0}, {{0, 3, 4}, 0}, {{0, 3, 5}, 0}, {{0, 3, 6}, 0}, {{0, 3, 7}, 0},
 {{1, 2, 4}, 0}, {{1, 2, 5}, 0}, {{1, 2, 6}, 0}, {{1, 2, 7}, 0}, {{1, 3, 4}, 0},
 {{1, 3, 5}, 0}, {{1, 3, 6}, 0}, {{1, 3, 7}, 0}, {{2, 3, 4}, 0}, {{2, 3, 5}, 0},
 {{2, 3, 6}, 0}, {{2, 3, 7}, 0}, {{4, 5, 6}, -4}, {{4, 5, 7}, 0}, {{4, 6, 7}, 0},
 {{5, 6, 7}, 0}, {{0, 1, 2, 3}, 0}, {{0, 1, 4, 5}, 0}, {{0, 1, 4, 6}, 0}, {{0, 1, 4, 7}, 0},
 {{0, 1, 5, 6}, 0}, {{0, 1, 5, 7}, 0}, {{0, 1, 6, 7}, 0}, {{0, 2, 4, 5}, 0}, {{0, 2, 4, 6}, 0},
 {{0, 2, 4, 7}, 0}, {{0, 2, 5, 6}, 0}, {{0, 2, 5, 7}, 0}, {{0, 2, 6, 7}, 0}, {{0, 3, 4, 5}, 0},
 {{0, 3, 4, 6}, 0}, {{0, 3, 4, 7}, 4}, {{0, 3, 5, 6}, 0}, {{0, 3, 5, 7}, 0}, {{0, 3, 6, 7}, 0},
 {{1, 2, 4, 5}, 0}, {{1, 2, 4, 6}, 0}, {{1, 2, 4, 7}, 0}, {{1, 2, 5, 6}, 0}, {{1, 2, 5, 7}, 0},
 {{1, 2, 6, 7}, -4}, {{1, 3, 4, 5}, 0}, {{1, 3, 4, 6}, 0}, {{1, 3, 4, 7}, 0}, {{1, 3, 5, 6}, 0},
 {{1, 3, 5, 7}, 0}, {{1, 3, 6, 7}, 0}, {{2, 3, 4, 5}, 0}, {{2, 3, 4, 6}, 0}, {{2, 3, 4, 7}, 0},
 {{2, 3, 5, 6}, 0}, {{2, 3, 5, 7}, 0}, {{2, 3, 6, 7}, 0}, {{4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5}, 4},
 {{0, 1, 2, 4, 6}, 0}, {{0, 1, 2, 4, 7}, 0}, {{0, 1, 2, 5, 6}, 0}, {{0, 1, 2, 5, 7}, 0},
 {{0, 1, 2, 6, 7}, 0}, {{0, 1, 3, 4, 5}, 0}, {{0, 1, 3, 4, 6}, 0}, {{0, 1, 3, 4, 7}, 0},
 {{0, 1, 3, 5, 6}, 0}, {{0, 1, 3, 5, 7}, 0}, {{0, 1, 3, 6, 7}, 0}, {{0, 2, 3, 4, 5}, 0},
 {{0, 2, 3, 4, 6}, 0}, {{0, 2, 3, 4, 7}, 0}, {{0, 2, 3, 5, 6}, 0}, {{0, 2, 3, 5, 7}, 0},
 {{0, 2, 3, 6, 7}, 0}, {{0, 4, 5, 6, 7}, 4}, {{1, 2, 3, 4, 5}, 0}, {{1, 2, 3, 4, 6}, 0},
 {{1, 2, 3, 4, 7}, 0}, {{1, 2, 3, 5, 6}, 0}, {{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 6, 7}, 0},
 {{1, 4, 5, 6, 7}, 0}, {{2, 4, 5, 6, 7}, 0}, {{3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6}, 0},
 {{0, 1, 2, 4, 5, 7}, 0}, {{0, 1, 2, 4, 6, 7}, 0}, {{0, 1, 2, 5, 6, 7}, 0},
 {{0, 1, 3, 4, 5, 6}, 0}, {{0, 1, 3, 4, 5, 7}, 0}, {{0, 1, 3, 4, 6, 7}, 0},
 {{0, 1, 3, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6}, 0}, {{0, 2, 3, 4, 5, 7}, 0},
 {{0, 2, 3, 4, 6, 7}, 0}, {{0, 2, 3, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6}, 0},
 {{1, 2, 3, 4, 5, 7}, 0}, {{1, 2, 3, 4, 6, 7}, -4}, {{1, 2, 3, 5, 6, 7}, 0},
 {{0, 1, 2, 3, 4, 5, 6}, 0}, {{0, 1, 2, 3, 4, 5, 7}, 0}, {{0, 1, 2, 3, 4, 6, 7}, 0},
 {{0, 1, 2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6, 7}, 0}, {{{0, 1, 2, 3, 4, 5, 6, 7, 8}}, 4}}
```

Out[∘]=
```
{{{0, 7}, -4}, {{3, 4}, -4}, {{4, 5, 6}, -4}, {{0, 3, 4, 7}, 4},
 {{1, 2, 6, 7}, -4}, {{0, 1, 2, 4, 5}, 4}, {{0, 4, 5, 6, 7}, 4},
 {{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 4, 6, 7}, -4}, {{{0, 1, 2, 3, 4, 5, 6, 7, 8}}, 4}}
```

In[ ]:=
```
linTransSymmetric[[1]]
```

Out[ ]=
```
{{{{-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0,
    0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}, {0, 7}}}
```

In[ ]:=
```
base16[[#]] & /@ negativeTrMM;
{#[[2]], Tr[Transpose[#[[1]]].linTrans]} & /@ %
Select[%, #[[2]] ≠ 0 &]
linTransAntiSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];
```

Out[ ]=
```
{{{4}, 0}, {{5}, 0}, {{6}, 0}, {{7}, 0}, {{0, 1}, 0}, {{0, 2}, 0}, {{0, 3}, 0}, {{1, 2}, 0},
 {{1, 3}, 0}, {{2, 3}, 0}, {{4, 5}, 0}, {{4, 6}, 0}, {{4, 7}, 0}, {{5, 6}, 0}, {{5, 7}, 0},
 {{6, 7}, 0}, {{0, 1, 2}, 0}, {{0, 1, 3}, 0}, {{0, 2, 3}, 0}, {{0, 4, 5}, 0}, {{0, 4, 6}, 0},
 {{0, 4, 7}, 0}, {{0, 5, 6}, 0}, {{0, 5, 7}, 0}, {{0, 6, 7}, 0}, {{1, 2, 3}, 0}, {{1, 4, 5}, 0},
 {{1, 4, 6}, 0}, {{1, 4, 7}, 0}, {{1, 5, 6}, 0}, {{1, 5, 7}, 0}, {{1, 6, 7}, 0},
 {{2, 4, 5}, 0}, {{2, 4, 6}, 0}, {{2, 4, 7}, 0}, {{2, 5, 6}, 0}, {{2, 5, 7}, 0},
 {{2, 6, 7}, 0}, {{3, 4, 5}, 0}, {{3, 4, 6}, 0}, {{3, 4, 7}, 0}, {{3, 5, 6}, -4},
 {{3, 5, 7}, 0}, {{3, 6, 7}, 0}, {{0, 1, 2, 4}, 0}, {{0, 1, 2, 5}, 0}, {{0, 1, 2, 6}, -4},
 {{0, 1, 2, 7}, 0}, {{0, 1, 3, 4}, 0}, {{0, 1, 3, 5}, 0}, {{0, 1, 3, 6}, 0}, {{0, 1, 3, 7}, 0},
 {{0, 2, 3, 4}, 0}, {{0, 2, 3, 5}, 0}, {{0, 2, 3, 6}, 0}, {{0, 2, 3, 7}, 0}, {{0, 4, 5, 6}, 0},
 {{0, 4, 5, 7}, 0}, {{0, 4, 6, 7}, 0}, {{0, 5, 6, 7}, 0}, {{1, 2, 3, 4}, 0}, {{1, 2, 3, 5}, 0},
 {{1, 2, 3, 6}, 0}, {{1, 2, 3, 7}, 0}, {{1, 4, 5, 6}, 0}, {{1, 4, 5, 7}, 0}, {{1, 4, 6, 7}, 0},
 {{1, 5, 6, 7}, 0}, {{2, 4, 5, 6}, 0}, {{2, 4, 5, 7}, 0}, {{2, 4, 6, 7}, 0}, {{2, 5, 6, 7}, 0},
 {{3, 4, 5, 6}, 0}, {{3, 4, 5, 7}, 0}, {{3, 4, 6, 7}, 0}, {{3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4}, 0},
 {{0, 1, 2, 3, 5}, -4}, {{0, 1, 2, 3, 6}, 0}, {{0, 1, 2, 3, 7}, 0}, {{0, 1, 4, 5, 6}, 0},
 {{0, 1, 4, 5, 7}, 0}, {{0, 1, 4, 6, 7}, 0}, {{0, 1, 5, 6, 7}, 0}, {{0, 2, 4, 5, 6}, 0},
 {{0, 2, 4, 5, 7}, 0}, {{0, 2, 4, 6, 7}, 0}, {{0, 2, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6}, 0},
 {{0, 3, 4, 5, 7}, 0}, {{0, 3, 4, 6, 7}, 0}, {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 6}, 0},
 {{1, 2, 4, 5, 7}, 4}, {{1, 2, 4, 6, 7}, 0}, {{1, 2, 5, 6, 7}, 0}, {{1, 3, 4, 5, 6}, 0},
 {{1, 3, 4, 5, 7}, 0}, {{1, 3, 4, 6, 7}, 0}, {{1, 3, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6}, 0},
 {{2, 3, 4, 5, 7}, 0}, {{2, 3, 4, 6, 7}, 0}, {{2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5}, 0},
 {{0, 1, 2, 3, 4, 6}, -4}, {{0, 1, 2, 3, 4, 7}, 0}, {{0, 1, 2, 3, 5, 6}, 0},
 {{0, 1, 2, 3, 5, 7}, 0}, {{0, 1, 2, 3, 6, 7}, 0}, {{0, 1, 4, 5, 6, 7}, 0},
 {{0, 2, 4, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6, 7}, 0}, {{1, 2, 4, 5, 6, 7}, 0},
 {{1, 3, 4, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6, 7}, 0},
 {{0, 1, 3, 4, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6, 7}, 0}}
```

Out[ ]=
```
{{{3, 5, 6}, -4}, {{0, 1, 2, 6}, -4}, {{0, 1, 2, 3, 5}, -4},
 {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 7}, 4}, {{0, 1, 2, 3, 4, 6}, -4}}
```

In[ ]:=
```
{Length[linTransSymmetric], Length[linTransAntiSymmetric]}
```

Out[ ]=
```
{10, 6}
```

BASIS of 8 × 8 matrices :

In[∘]:=
```
tA = {}; Do[AppendTo[tA, {τ[j], {j}}], {j, 1, 7}]
Length[tA]
tAB = {}; Do[AppendTo[tAB, {τ[j].τ[k], {j, k}}], {j, 1, 6}, {k, j + 1, 7}]
Length[tAB]
tABC = {};
Do[AppendTo[tABC, {τ[j].τ[k].τ[h], {j, k, h}}], {j, 1, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[tABC]
```

Out[∘]=
```
7
```

Out[∘]=
```
21
```

Out[∘]=
```
35
```

In[∘]:=
```
tA[[1]]
```

Out[∘]=
```
{{{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}}, {1}}
```

In[∘]:=
```
tA[[#]][[1]] == -Transpose[tA[[#]][[1]]] & /@ Range[Length[tA]]
```

Out[∘]=
```
{True, True, True, False, False, False, False}
```

In[∘]:=
```
bas64 = Flatten[{tA, tAB, tABC, {{Id, {64}}}}, 1]; Length[bas64]
```

Out[∘]=
```
64
```

In[∘]:=
```
determineIF8AntiSymmetric[m_] := Module[{t, tt, sum, res, r},
  t = m.τ[#] & /@ Range[0, 7];
  tt = Transpose[#] & /@ t;
  sum = FullSimplify[t[[#]] + tt[[#]]] & /@ Range[Length[t]];
  res = Union[Flatten[#]] & /@ sum;
  If[Length[#] > 1, Return[False]] & /@ res;
  r = Union[Flatten[res]];
  Return[r == {0}];
 ]
```

In[∘]:=
```
determineIF8AntiSymmetric[Zero8]
```

Out[∘]=
```
True
```

In[∘]:=
```
determineIF8AntiSymmetric[ID8]
```

Out[∘]=
```
False
```

In[ ]:=
```
find8AntiSymmetric = If[determineIF8AntiSymmetric[#[[1]]], #[[2]], ## &] & /@ bas64
```

Out[ ]=
```
{###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &,
 ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &,
 ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &,
 ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &,
 ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &, ###1 &}
```

In[ ]:=
```
anti = {};
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],
  AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}] ], {k, 1, 64}];
Length[anti]
anti[[28]]
```

Out[ ]=
```
28
```

Out[ ]=
```
{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}
```

In[ ]:=
```
symm = {};
Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
  AppendTo[symm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}] ], {k, 1, 64}];
Length[symm]
symm[[%]]
```

Out[ ]=
```
35
```

Out[ ]=
```
{{{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0},
  {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}}, {{59}, {3, 6, 7}}}
```

In[ ]:=
```
η64 = DiagonalMatrix[
  Table[FullSimplify[1/8 Tr[bas64[[A, 1]].bas64[[A, 1]]]], {A, 64}]];
Tr[η64]
```

Out[ ]=
$$7 + \frac{1}{8}\,\mathrm{Tr}[\mathrm{Id}.\mathrm{Id}]$$

In[ ]:=
```
(*(*(*(*(*(*(countTrace=
   Table[{A,B,FullSimplify[1/8 Tr[bas64[[A,1]].bas64[[B,1]]]] },
    {A,1, 64},{B,A, 64}])//MatrixForm*)*)*)*)*)*)
```

In[ ]:=
```
(*■*)
```

In[∘]:=
```
32 × 65
```
Out[∘]=
```
2080
```
In[∘]:=
```
(*Length[Flatten[countTrace,1]]*)
```
In[∘]:=
```
(*{Length[countTrace],Length[countTrace[[1]]],Length[countTrace[[1,1]]]}*)
```
In[∘]:=
```
(*{Length[countTrace],Length[countTrace[[2]]],Length[countTrace[[2,1]]]}*)
```
In[∘]:=
```
(*{Length[countTrace],Length[countTrace[[64]]],Length[countTrace[[64,1]]]}*)
```
In[∘]:=
```
(*{Length[Flatten[countTrace,1][[All,3]]],Count[Flatten[countTrace,1][[All,3]],0],
 Count[Flatten[countTrace,1][[All,3]],1],Count[Flatten[countTrace,1][[All,3]],-1]}*)
```
In[∘]:=
```
32 × 63
```
Out[∘]=
```
2016
```
In[∘]:=
```
anti = {};
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],
  AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}] ], {k, 1, 64}];
Length[anti]
anti[[28]]
```
Out[∘]=
```
28
```
Out[∘]=
```
{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}
```
In[∘]:=
```
anti[[28, 1]]
```
Out[∘]=
```
{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}
```
In[∘]:=
```
anti[[28, 2]]
```
Out[∘]=
```
{{63}, {5, 6, 7}}
```
In[∘]:=
```
symm = {};
Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
  AppendTo[symm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}] ], {k, 1, 64}];
Length[symm]
symm[[28]]
```
Out[∘]=
```
35
```
Out[∘]=
```
{{{-1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, {{52}, {2, 5, 7}}}
```

Spinor Lagrangian

In[ ]:=
```
usegT16 = Table[FullSimplify[(T16^α[α1 - 1]), constraintVars], {α1, 1, Length[X]}];
```

In[ ]:=
```
useT16 = Table[FullSimplify[(T16^α[α1 - 1] /. ssgm4488), constraintVars] /.
    {Sqrt[e^(-2 a4[H x4])] → e^(-a4[H x4]), Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/3)] → e^(a4[H x4]) Sin[6 H x0]^(1/6),
     1/Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/3)] → 1/(e^(a4[H x4]) Sin[6 H x0]^(1/6))}, {α1, 1, Length[X]}] /.
  {1/Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/3)] → 1/(e^(a4[H x4]) Sin[6 H x0]^(1/6)), Sqrt[e^(2 a4[H x4])] → e^(a4[H x4])};
```

In[ ]:=
```
newuseT16 = Table[FullSimplify[(newT16^α[α1 - 1] /. ssgm4488), constraintVars] /.
    {Sqrt[e^(-2 a4[H x4])] → e^(-a4[H x4]), Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/3)] → e^(a4[H x4]) Sin[6 H x0]^(1/6),
     1/Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/3)] → 1/(e^(a4[H x4]) Sin[6 H x0]^(1/6))}, {α1, 1, Length[X]}] /.
  {1/Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/3)] → 1/(e^(a4[H x4]) Sin[6 H x0]^(1/6)), Sqrt[e^(2 a4[H x4])] → e^(a4[H x4])};
```

In[ ]:=
```
Dimensions[useT16]
```

Out[ ]=
```
{8, 16, 16}
```

In[ ]:=
```
Det[MatrixMetric44]
```

Out[ ]=

$\text{Cos}[6\,H\,x0]^2$

In[ ]:=
```
useDSQRT = Cos[6 H x0]
```

Out[ ]=

$\text{Cos}[6\,H\,x0]$

In[ ]:=
```
ωmat /. sg /. ssgm4488 /. subsDefects;
(% /. {Sqrt[e^(2 a4[H x4])] → e^(a4[H x4]), Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) → e^(a4[H x4]) H Sin[6 H x0]^(1/6)}) -
    spinCoeffs // FullSimplify[#, constraintVars] & /. subsDefects /.
    {Sqrt[e^(2 a4[H x4])] → e^(a4[H x4]), Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) → e^(a4[H x4]) H Sin[6 H x0]^(1/6)} //
  FullSimplify[#, constraintVars] & /. subsDefects;
% // Flatten // Union
% // ExpandAll /. subsDefects /.
   {Sqrt[e^(2 a4[H x4])] → e^(a4[H x4]), Sqrt[e^(2 a4[H x4])] H Sin[6 H x0]^(1/6) → e^(a4[H x4]) H Sin[6 H x0]^(1/6)} /. subsDefects
% /. {a4[H x4] → π^4, Sin[6 H x0] → e^12}
```

Out[ ]=

$$\left\{0,\ e^{-a4[H\,x4]}\left(-1+e^{a4[H\,x4]}\sqrt{e^{-2\,a4[H\,x4]}}\right)H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ -\left(\left(-e^{a4[H\,x4]}+\sqrt{e^{2\,a4[H\,x4]}}\right)H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\right),\ \left(-e^{a4[H\,x4]}+\sqrt{e^{2\,a4[H\,x4]}}\right)H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ e^{-a4[H\,x4]}\left(1-e^{a4[H\,x4]}\sqrt{e^{-2\,a4[H\,x4]}}\right)H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ e^{-a4[H\,x4]}\left(-1+e^{a4[H\,x4]}\sqrt{e^{-2\,a4[H\,x4]}}\right)H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ \left(-e^{a4[H\,x4]}+\sqrt{e^{2\,a4[H\,x4]}}\right)H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]\right\}$$

Out[ ]=

$$\left\{0,\ -e^{-a4[H\,x4]}H\,\mathrm{Sin}[6\,H\,x0]^{1/6}+\sqrt{e^{-2\,a4[H\,x4]}}H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ e^{a4[H\,x4]}H\,\mathrm{Sin}[6\,H\,x0]^{1/6}-\sqrt{e^{2\,a4[H\,x4]}}H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ -e^{a4[H\,x4]}H\,\mathrm{Sin}[6\,H\,x0]^{1/6}+\sqrt{e^{2\,a4[H\,x4]}}H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ e^{-a4[H\,x4]}H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]-\sqrt{e^{-2\,a4[H\,x4]}}H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ -e^{-a4[H\,x4]}H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]+\sqrt{e^{-2\,a4[H\,x4]}}H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ -e^{a4[H\,x4]}H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]+\sqrt{e^{2\,a4[H\,x4]}}H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]\right\}$$

Out[ ]=

{0, 0, 0, 0, 0, 0, 0}

In[ ]:=
```
Q2 = 0; Protect[Q1, Q2]
```

Out[ ]=

{Q1, Q2}

In[ ]:=
```
(matTest = (Q1 * ID16 )) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 \end{pmatrix}$$

In[ ]:=
```
(*Protect[Q1,Q2]*)
```

In[ ]:=
```
(*(matTest=(Q1*ID16 +Q2* covariantDiffMatrix))//MatrixForm*)
```

In[ ]:=
```
(newmatTest = O8similarityTransformation.(Q1 * ID16 + Q2 * covariantDiffMatrix).
    Transpose[O8similarityTransformation]) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 \end{pmatrix}$$

In[ ]:=
```
Clear[Lg];
Lg[] :=
  √detgg * (Transpose[Ψ16].σ16.Sum[FullSimplify[(T16^α[α1 - 1] /. sg), constraintVars].
        (D[ Ψ16, X[[α1]]] + (K/2 /. {K → 1}) * Sum[ωmat[[α1, a, b]] * SAB[[a, b]].matTest.Ψ16,
            {a, 1, 8}, {b, 1, 8}]), {α1, 1, Length[X]}] +
      (H * M) * Transpose[Ψ16].σ16.Ψ16) // Simplify[#, constraintVars] &
```

In[ ]:=
```
Clear[newLg];
newLg[] := √detgg *
    (Transpose[Ψ16].newσ16.Sum[FullSimplify[(newT16^α[α1 - 1] /. sg), constraintVars].
        (D[ Ψ16, X[[α1]]] + (K/2 /. {K → 1}) * Sum[ωmat[[α1, a, b]] * O8similarityTransformation.
              SAB[[a, b]].Transpose[O8similarityTransformation].newmatTest.Ψ16,
            {a, 1, 8}, {b, 1, 8}]), {α1, 1, Length[X]}] +
      (H * M) * Transpose[Ψ16].newσ16.Ψ16) // Simplify[#, constraintVars] &
```

In[ ]:=
```
Block[{A1 = 1, B1 = 2}, O8similarityTransformation.
  SAB⟦A1, B1⟧.Transpose[O8similarityTransformation] // MatrixForm]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

In[ ]:=
```
Block[{A1 = 1, B1 = 2, m1, ms, m, j1, s1},
 m1 = ArrayFlatten[{{0, ID8}, {ID8, 0}}]; Print[m1 // MatrixForm];
 {s1, j1} = JordanDecomposition[m1];
 Print[m1 == s1.j1.Inverse[s1]];
 ms = Sqrt[j1];
 m = s1.ms.Inverse[s1];
 Print[m // MatrixForm];
 Print[Sqrt[m1] // MatrixForm];
 m.O8similarityTransformation.SAB⟦A1, B1⟧.
   Transpose[m.O8similarityTransformation] // MatrixForm]
```

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

True

$$\begin{pmatrix}
\frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} \\
\frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}+\frac{i}{2}
\end{pmatrix}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

In[ ]:=
```
ωμIJ[1]
```

Out[ ]=
```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[ ]:=
```
Clear[La];
La[] :=
 useDSQRT * (Transpose[Ψ16].σ16.Sum[useT16[[α1]].(D[ Ψ16, X[[α1]]] + (K/2 /. {K → 1}) * Sum[
              ωμIJ[α1][[A1, B1]] * SAB[[A1, B1]], {A1, 1, 8}, {B1, 1, 8}].matTest.Ψ16), {α1, 1,
          Length[X]}] + (H * M) * Transpose[Ψ16].σ16.Ψ16) // Simplify[#, constraintVars] &
```

NOT O8similarityTransformation . Ψ16 :

In[ ]:=
```
Clear[newLa];
newLa[] :=
 useDSQRT * (Transpose[Ψ16].newσ16.Sum[newuseT16[[α1]].(D[ Ψ16, X[[α1]]] + (K/2 /. {K → 1}) *
            Sum[ωμIJ[α1][[A1, B1]] * (O8similarityTransformation.
                 SAB[[A1, B1]].Transpose[O8similarityTransformation]),
              {A1, 1, 8}, {B1, 1, 8}].newmatTest.Ψ16), {α1, 1, Length[X]}] +
      (H * M) * Transpose[Ψ16].newσ16.Ψ16) // Simplify[#, constraintVars] &
```

```
In[•]:= (*testLa=La[]*)

In[•]:= (*testLg=Lg[];*)

In[•]:= (*D[testLa,a4[H x4]]//Flatten//Union*)
        (*D[testLa,a4′[H x4]]//(*Full*)Simplify[#,constraintVars]&//Flatten//Union*)

In[•]:= (*testLgm=testL/.sg/.ssgm4488//(*Full*)Simplify[#,constraintVars]&/.subsDefects*)

In[•]:= (*D[testLgm,a4[H x4]]
         D[testLgm,a4′[H x4]]//(*Full*)Simplify[#,constraintVars]&*)

In[•]:= Clear[Lj];
        Lj[j_] := useDSQRT * (((
                Transpose[Ψ16].σ16.useT16⟦1⟧.D[Ψ16, X⟦1⟧] +

                  Transpose[Ψ16].σ16.useT16⟦5⟧.D[Ψ16, X⟦5⟧] +
                  (K0/2) * Sum[(*useω⟦1,A1,B1⟧*)ωμIJ[1]⟦A1, B1⟧ × Transpose[Ψ16].σ16.
                      useT16⟦1⟧.SAB⟦A1, B1⟧.(base16⟦j, 1⟧).Ψ16, {A1, 1, 8}, {B1, 1, 8}]) +
              Transpose[Ψ16].σ16.useT16⟦5⟧.D[Ψ16, X⟦5⟧] + (K4/2) * Sum[(*useω⟦5,A1,B1⟧*)
                ωμIJ[5]⟦A1, B1⟧ × Transpose[Ψ16].σ16.useT16⟦5⟧.SAB⟦A1, B1⟧.(base16⟦j, 1⟧).Ψ16,
                {A1, 1, 8}, {B1, 1, 8}]) +
            (H * M) * Transpose[Ψ16].σ16.Ψ16) // Simplify[#, constraintVars] &
```

In[∘]:=
```
Lj[92]
```

Out[∘]=
```
Cos[6 H x0] (-2 H M f16[2][x0, x4] × f16[6][x0, x4] -
    2 H M f16[3][x0, x4] × f16[7][x0, x4] + 2 H M f16[8][x0, x4] × f16[12][x0, x4] +
    2 H M f16[9][x0, x4] × f16[13][x0, x4] + 2 H M f16[10][x0, x4] × f16[14][x0, x4] +
    2 H M f16[11][x0, x4] × f16[15][x0, x4] + 2 f16[9][x0, x4] f16[0]^(0,1)[x0, x4] -
    2 f16[8][x0, x4] f16[1]^(0,1)[x0, x4] - 2 f16[11][x0, x4] f16[2]^(0,1)[x0, x4] +
    2 f16[10][x0, x4] f16[3]^(0,1)[x0, x4] - 2 f16[13][x0, x4] f16[4]^(0,1)[x0, x4] +
    2 f16[12][x0, x4] f16[5]^(0,1)[x0, x4] + 2 f16[15][x0, x4] f16[6]^(0,1)[x0, x4] -
    2 f16[14][x0, x4] f16[7]^(0,1)[x0, x4] - 2 f16[3][x0, x4] f16[10]^(0,1)[x0, x4] +
    2 f16[2][x0, x4] f16[11]^(0,1)[x0, x4] - 2 f16[5][x0, x4] f16[12]^(0,1)[x0, x4] +
    2 f16[4][x0, x4] f16[13]^(0,1)[x0, x4] + 2 f16[7][x0, x4] f16[14]^(0,1)[x0, x4] -
    2 f16[6][x0, x4] f16[15]^(0,1)[x0, x4] + Tan[6 H x0] f16[12][x0, x4] f16[0]^(1,0)[x0, x4] +
    Tan[6 H x0] f16[13][x0, x4] f16[1]^(1,0)[x0, x4] +
    Tan[6 H x0] f16[14][x0, x4] f16[2]^(1,0)[x0, x4] + Tan[6 H x0] f16[15][x0, x4]
     f16[3]^(1,0)[x0, x4] + Tan[6 H x0] f16[8][x0, x4] f16[4]^(1,0)[x0, x4] +
    Tan[6 H x0] f16[9][x0, x4] f16[5]^(1,0)[x0, x4] + Tan[6 H x0] f16[10][x0, x4]
     f16[6]^(1,0)[x0, x4] + Tan[6 H x0] f16[11][x0, x4] f16[7]^(1,0)[x0, x4] -
    Tan[6 H x0] f16[4][x0, x4] f16[8]^(1,0)[x0, x4] - Tan[6 H x0] f16[5][x0, x4]
     f16[9]^(1,0)[x0, x4] - Tan[6 H x0] f16[6][x0, x4] f16[10]^(1,0)[x0, x4] -
    Tan[6 H x0] f16[7][x0, x4] f16[11]^(1,0)[x0, x4] - f16[0][x0, x4]
     (2 H M f16[4][x0, x4] + 2 f16[9]^(0,1)[x0, x4] + Tan[6 H x0] f16[12]^(1,0)[x0, x4]) - f16[1][x0,
      x4] (2 H M f16[5][x0, x4] - 2 f16[8]^(0,1)[x0, x4] + Tan[6 H x0] f16[13]^(1,0)[x0, x4]) -
    Tan[6 H x0] f16[2][x0, x4] f16[14]^(1,0)[x0, x4] -
    Tan[6 H x0] f16[3][x0, x4] f16[15]^(1,0)[x0, x4])
```

In[∘]:=
```
detgg
```

Out[∘]=
```
g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
 g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]
```

In[∘]:=
```
Clear[eL];
eL[Lagrangian_Symbol, detsqrt_] := Module[{L, t},
  L = Lagrangian[];
  t = Table[FullSimplify[1/detsqrt (D[L, f16[k][x0, x4]] - D[D[L, f16[k]^(1,0)[x0, x4]], x0] -
        D[D[L, f16[k]^(0,1)[x0, x4]], x4]), constraintVars], {k, 0, 15}];
  Return[t /. subsDefects];
 ]
```

eLa

In[∘]:=
```
(*Get["2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLa.mx"];*)
```

In[∘]:=
```
eLa = eL[La, useDSQRT];
```

In[∘]:=
```
eLa
```

Out[∘]=
```
{-2 e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[9][x0, x4] a4′[H x4] -
  2 (H M f16[4][x0, x4] + 3 H f16[12][x0, x4] +
     f16[9]^(0,1)[x0, x4] + Tan[6 H x0] f16[12]^(1,0)[x0, x4]),
 2 (-H M f16[5][x0, x4] - 3 H f16[13][x0, x4] + e^(-a4[H x4]) H Q1 Sinh[a4[H x4]]
     f16[8][x0, x4] a4′[H x4] + f16[8]^(0,1)[x0, x4] - Tan[6 H x0] f16[13]^(1,0)[x0, x4]),
 -2 (H M f16[6][x0, x4] + 3 H f16[14][x0, x4] - f16[11]^(0,1)[x0, x4] +
    Tan[6 H x0] f16[14]^(1,0)[x0, x4]),
 -2 (H M f16[7][x0, x4] + 3 H f16[15][x0, x4] + f16[10]^(0,1)[x0, x4] +
    Tan[6 H x0] f16[15]^(1,0)[x0, x4]), -2 H M f16[0][x0, x4] -
  6 H f16[8][x0, x4] - 2 e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[13][x0, x4] a4′[H x4] +
  2 f16[13]^(0,1)[x0, x4] - 2 Tan[6 H x0] f16[8]^(1,0)[x0, x4],
 -2 H M f16[1][x0, x4] - 6 H f16[9][x0, x4] +
  e^(-2 a4[H x4]) (-1 + e^(2 a4[H x4])) H Q1 f16[12][x0, x4] a4′[H x4] -
  2 (f16[12]^(0,1)[x0, x4] + Tan[6 H x0] f16[9]^(1,0)[x0, x4]),
 -2 (H M f16[2][x0, x4] + 3 H f16[10][x0, x4] + f16[15]^(0,1)[x0, x4] +
    Tan[6 H x0] f16[10]^(1,0)[x0, x4]), -2 (H M f16[3][x0, x4] +
    3 H f16[11][x0, x4] - f16[14]^(0,1)[x0, x4] + Tan[6 H x0] f16[11]^(1,0)[x0, x4]),
 2 (3 H f16[4][x0, x4] + H M f16[12][x0, x4] + e^(-a4[H x4]) H Q1 Sinh[a4[H x4]]
     f16[1][x0, x4] a4′[H x4] - f16[1]^(0,1)[x0, x4] + Tan[6 H x0] f16[4]^(1,0)[x0, x4]),
 2 (3 H f16[5][x0, x4] + H M f16[13][x0, x4] - e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[0][x0, x4]
     a4′[H x4] + f16[0]^(0,1)[x0, x4] + Tan[6 H x0] f16[5]^(1,0)[x0, x4]),
 2 (3 H f16[6][x0, x4] + H M f16[14][x0, x4] + f16[3]^(0,1)[x0, x4] +
    Tan[6 H x0] f16[6]^(1,0)[x0, x4]), 2 (3 H f16[7][x0, x4] +
    H M f16[15][x0, x4] - f16[2]^(0,1)[x0, x4] + Tan[6 H x0] f16[7]^(1,0)[x0, x4]),
 2 (3 H f16[0][x0, x4] + H M f16[8][x0, x4] + e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[5][x0, x4] a4′[H x4] +
    f16[5]^(0,1)[x0, x4] + Tan[6 H x0] f16[0]^(1,0)[x0, x4]),
 6 H f16[1][x0, x4] + 2 H M f16[9][x0, x4] - 2 e^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[4][x0, x4] a4′[H x4] -
  2 f16[4]^(0,1)[x0, x4] + 2 Tan[6 H x0] f16[1]^(1,0)[x0, x4],
 2 (3 H f16[2][x0, x4] + H M f16[10][x0, x4] - f16[7]^(0,1)[x0, x4] +
    Tan[6 H x0] f16[2]^(1,0)[x0, x4]), 2 (3 H f16[3][x0, x4] +
    H M f16[11][x0, x4] + f16[6]^(0,1)[x0, x4] + Tan[6 H x0] f16[3]^(1,0)[x0, x4])}
```

In[∘]:=
```
Length[eLa]
```

Out[∘]=
```
16
```

In[∘]:=
```
DumpSave[ToString[header <> "eLa.mx"], eLa];
```

In[∘]:=
```
Print[ToString[header <> "eLa.mx"]]
```

```
Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT-eLa.mx
```

In[∘]:=
```
(*Get["2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix
   3-eLa.mx"]*)
```

In[∘]:=
```
(*newLa[]*)
```

In[ ]:=
```
(*Get["2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-enewLa.mx"]*)
```

In[ ]:=
```
(*enewLa=eL[newLa,useDSQRT]*)
```

In[ ]:=
```
(*DumpSave[ToString[header<>"enewLa.mx"],enewLa];*)
```

In[ ]:=
```
(*Print[ToString[header <> "enewLa.mx"]]*)
```

eLazt

In[ ]:=
```
(*Get["2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLazt.mx"];*)
```

In[ ]:=
```
time1 = Now
AbsoluteTiming[
 eLazt = (2 Sqrt[Sin[z]])/(6 * H) * eLa /. sfψ16Aa /. sx0x4 // FullSimplify[#, constraintVars] &]
Now - time1
```

Out[ ]=

Mon 15 Dec 2025 12:06:23 GMT−8

Out[ ]=

```
{6.03369, {-2/3 e^(-a4[t]) Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] -
    2/3 (M Z[4][z, t] + Z[9]^(0,1)[z, t] + 6 Tan[z] Z[12]^(1,0)[z, t]),
   2/3 (-M Z[5][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + Z[8]^(0,1)[z, t] -
      6 Tan[z] Z[13]^(1,0)[z, t]), -2/3 (M Z[6][z, t] - Z[11]^(0,1)[z, t] + 6 Tan[z] Z[14]^(1,0)[z, t]),
   -2/3 (M Z[7][z, t] + Z[10]^(0,1)[z, t] + 6 Tan[z] Z[15]^(1,0)[z, t]), 2/3 (-M Z[0][z, t] -
      e^(-a4[t]) Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] + Z[13]^(0,1)[z, t] - 6 Tan[z] Z[8]^(1,0)[z, t]),
   -2/3 (M Z[1][z, t] + Q1 Sinh[a4[t]] (-Cosh[a4[t]] + Sinh[a4[t]]) Z[12][z, t] a4'[t] +
      Z[12]^(0,1)[z, t] + 6 Tan[z] Z[9]^(1,0)[z, t]),
   -2/3 (M Z[2][z, t] + Z[15]^(0,1)[z, t] + 6 Tan[z] Z[10]^(1,0)[z, t]),
   -2/3 (M Z[3][z, t] - Z[14]^(0,1)[z, t] + 6 Tan[z] Z[11]^(1,0)[z, t]),
   2/3 (M Z[12][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] -
      Z[1]^(0,1)[z, t] + 6 Tan[z] Z[4]^(1,0)[z, t]), 2/3 (M Z[13][z, t] -
      e^(-a4[t]) Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] + Z[0]^(0,1)[z, t] + 6 Tan[z] Z[5]^(1,0)[z, t]),
   2/3 (M Z[14][z, t] + Z[3]^(0,1)[z, t] + 6 Tan[z] Z[6]^(1,0)[z, t]),
   2/3 (M Z[15][z, t] - Z[2]^(0,1)[z, t] + 6 Tan[z] Z[7]^(1,0)[z, t]),
   2/3 (M Z[8][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] +
      Z[5]^(0,1)[z, t] + 6 Tan[z] Z[0]^(1,0)[z, t]), 2/3 (M Z[9][z, t] -
      e^(-a4[t]) Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] - Z[4]^(0,1)[z, t] + 6 Tan[z] Z[1]^(1,0)[z, t]),
   2/3 (M Z[10][z, t] - Z[7]^(0,1)[z, t] + 6 Tan[z] Z[2]^(1,0)[z, t]),
   2/3 (M Z[11][z, t] + Z[6]^(0,1)[z, t] + 6 Tan[z] Z[3]^(1,0)[z, t])}}
```

Out[ ]=

6.0317244 s

```
{6.06786, {-2/3 e^(-a4[t]) Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] -
    2/3 (M Z[4][z, t] + Z[9]^(0,1)[z, t] + 6 Tan[z] Z[12]^(1,0)[z, t]),
   2/3 (-M Z[5][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] +
      Z[8]^(0,1)[z, t] - 6 Tan[z] Z[13]^(1,0)[z, t]),
```

```
-2/3 (M Z[6][z, t] - Z[11]^(0,1)[z, t] + 6 Tan[z] Z[14]^(1,0)[z, t]),
-2/3 (M Z[7][z, t] + Z[10]^(0,1)[z, t] + 6 Tan[z] Z[15]^(1,0)[z, t]),
2/3 (-M Z[0][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] +
   Z[13]^(0,1)[z, t] - 6 Tan[z] Z[8]^(1,0)[z, t]),
-2/3 (M Z[1][z, t] + Q1 Sinh[a4[t]] (-Cosh[a4[t]] + Sinh[a4[t]]) Z[12][z, t] a4'[t] +
   Z[12]^(0,1)[z, t] + 6 Tan[z] Z[9]^(1,0)[z, t]),
-2/3 (M Z[2][z, t] + Z[15]^(0,1)[z, t] + 6 Tan[z] Z[10]^(1,0)[z, t]),
-2/3 (M Z[3][z, t] - Z[14]^(0,1)[z, t] + 6 Tan[z] Z[11]^(1,0)[z, t]),
2/3 (M Z[12][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] -
   Z[1]^(0,1)[z, t] + 6 Tan[z] Z[4]^(1,0)[z, t]),
2/3 (M Z[13][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] +
   Z[0]^(0,1)[z, t] + 6 Tan[z] Z[5]^(1,0)[z, t]),
2/3 (M Z[14][z, t] + Z[3]^(0,1)[z, t] + 6 Tan[z] Z[6]^(1,0)[z, t]),
2/3 (M Z[15][z, t] - Z[2]^(0,1)[z, t] + 6 Tan[z] Z[7]^(1,0)[z, t]),
2/3 (M Z[8][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] +
   Z[5]^(0,1)[z, t] + 6 Tan[z] Z[0]^(1,0)[z, t]),
2/3 (M Z[9][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] -
   Z[4]^(0,1)[z, t] + 6 Tan[z] Z[1]^(1,0)[z, t]),
2/3 (M Z[10][z, t] - Z[7]^(0,1)[z, t] + 6 Tan[z] Z[2]^(1,0)[z, t]),
2/3 (M Z[11][z, t] + Z[6]^(0,1)[z, t] + 6 Tan[z] Z[3]^(1,0)[z, t])}}
```

Out[ ]=

```
6.0791309 s
```

In[ ]:=

```
DumpSave[ToString[header <> "eLazt.mx"], eLazt];
```

In[ ]:=

```
Print[ToString[header <> "eLazt.mx"]]
```

```
Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT-eLazt.mx
```

In[∘]:=
```
varZ = Z[#] & /@ Range[0, 15]
```

Out[∘]=
```
{Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
 Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}
```

In[∘]:=
```
varZzt = #[z, t] & /@ varZ
```

Out[∘]=
```
{Z[0][z, t], Z[1][z, t], Z[2][z, t], Z[3][z, t], Z[4][z, t],
 Z[5][z, t], Z[6][z, t], Z[7][z, t], Z[8][z, t], Z[9][z, t], Z[10][z, t],
 Z[11][z, t], Z[12][z, t], Z[13][z, t], Z[14][z, t], Z[15][z, t]}
```

In[∘]:=
```
DzvarZ = D[varZzt, z]
```

Out[∘]=
```
{Z[0]^(1,0)[z, t], Z[1]^(1,0)[z, t], Z[2]^(1,0)[z, t], Z[3]^(1,0)[z, t],
 Z[4]^(1,0)[z, t], Z[5]^(1,0)[z, t], Z[6]^(1,0)[z, t], Z[7]^(1,0)[z, t],
 Z[8]^(1,0)[z, t], Z[9]^(1,0)[z, t], Z[10]^(1,0)[z, t], Z[11]^(1,0)[z, t],
 Z[12]^(1,0)[z, t], Z[13]^(1,0)[z, t], Z[14]^(1,0)[z, t], Z[15]^(1,0)[z, t]}
```

In[∘]:=
```
DtvarZ = D[varZzt, t]
```

Out[∘]=
```
{Z[0]^(0,1)[z, t], Z[1]^(0,1)[z, t], Z[2]^(0,1)[z, t], Z[3]^(0,1)[z, t],
 Z[4]^(0,1)[z, t], Z[5]^(0,1)[z, t], Z[6]^(0,1)[z, t], Z[7]^(0,1)[z, t],
 Z[8]^(0,1)[z, t], Z[9]^(0,1)[z, t], Z[10]^(0,1)[z, t], Z[11]^(0,1)[z, t],
 Z[12]^(0,1)[z, t], Z[13]^(0,1)[z, t], Z[14]^(0,1)[z, t], Z[15]^(0,1)[z, t]}
```

In[∘]:=
```
DtvarZsubs = Solve[And @@ Thread[0 == eLazt], DtvarZ][[1]] // FullSimplify[#, constraintVars] &
```

Out[∘]=
```
{Z[0]^(0,1)[z, t] → -M Z[13][z, t] +
    e^(-a4[t]) Q1 Sinh[a4[t]] Z[0][z, t] a4′[t] - 6 Tan[z] Z[5]^(1,0)[z, t], Z[1]^(0,1)[z, t] →
   M Z[12][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[1][z, t] a4′[t] + 6 Tan[z] Z[4]^(1,0)[z, t],
  Z[2]^(0,1)[z, t] → M Z[15][z, t] + 6 Tan[z] Z[7]^(1,0)[z, t],
  Z[3]^(0,1)[z, t] → -M Z[14][z, t] - 6 Tan[z] Z[6]^(1,0)[z, t],
  Z[4]^(0,1)[z, t] → M Z[9][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[4][z, t] a4′[t] + 6 Tan[z] Z[1]^(1,0)[z, t],
  Z[5]^(0,1)[z, t] →
   -M Z[8][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[5][z, t] a4′[t] - 6 Tan[z] Z[0]^(1,0)[z, t],
  Z[6]^(0,1)[z, t] → -M Z[11][z, t] - 6 Tan[z] Z[3]^(1,0)[z, t],
  Z[7]^(0,1)[z, t] → M Z[10][z, t] + 6 Tan[z] Z[2]^(1,0)[z, t], Z[8]^(0,1)[z, t] →
   M Z[5][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[8][z, t] a4′[t] + 6 Tan[z] Z[13]^(1,0)[z, t],
  Z[9]^(0,1)[z, t] → -M Z[4][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[9][z, t] a4′[t] -
    6 Tan[z] Z[12]^(1,0)[z, t], Z[10]^(0,1)[z, t] → -M Z[7][z, t] - 6 Tan[z] Z[15]^(1,0)[z, t],
  Z[11]^(0,1)[z, t] → M Z[6][z, t] + 6 Tan[z] Z[14]^(1,0)[z, t], Z[12]^(0,1)[z, t] →
   -M Z[1][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[12][z, t] a4′[t] - 6 Tan[z] Z[9]^(1,0)[z, t],
  Z[13]^(0,1)[z, t] → M Z[0][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[13][z, t] a4′[t] +
    6 Tan[z] Z[8]^(1,0)[z, t], Z[14]^(0,1)[z, t] → M Z[3][z, t] + 6 Tan[z] Z[11]^(1,0)[z, t],
  Z[15]^(0,1)[z, t] → -M Z[2][z, t] - 6 Tan[z] Z[10]^(1,0)[z, t]}
```

In[ ]:=
```
ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(DtvarZEQS = ToExpression[#] & /@ %) // Column
```

Out[ ]=

$Z[0]^{(0,1)}[z,t] == -M\,Z[13][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[0][z,t]\,a4'[t] - 6\,\mathrm{Tan}[z]\,Z[5]^{(1,0)}[z,t]$

$Z[1]^{(0,1)}[z,t] == M\,Z[12][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[1][z,t]\,a4'[t] + 6\,\mathrm{Tan}[z]\,Z[4]^{(1,0)}[z,t]$

$Z[2]^{(0,1)}[z,t] == M\,Z[15][z,t] + 6\,\mathrm{Tan}[z]\,Z[7]^{(1,0)}[z,t]$

$Z[3]^{(0,1)}[z,t] == -M\,Z[14][z,t] - 6\,\mathrm{Tan}[z]\,Z[6]^{(1,0)}[z,t]$

$Z[4]^{(0,1)}[z,t] == M\,Z[9][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[4][z,t]\,a4'[t] + 6\,\mathrm{Tan}[z]\,Z[1]^{(1,0)}[z,t]$

$Z[5]^{(0,1)}[z,t] == -M\,Z[8][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[5][z,t]\,a4'[t] - 6\,\mathrm{Tan}[z]\,Z[0]^{(1,0)}[z,t]$

$Z[6]^{(0,1)}[z,t] == -M\,Z[11][z,t] - 6\,\mathrm{Tan}[z]\,Z[3]^{(1,0)}[z,t]$

$Z[7]^{(0,1)}[z,t] == M\,Z[10][z,t] + 6\,\mathrm{Tan}[z]\,Z[2]^{(1,0)}[z,t]$

$Z[8]^{(0,1)}[z,t] == M\,Z[5][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[8][z,t]\,a4'[t] + 6\,\mathrm{Tan}[z]\,Z[13]^{(1,0)}[z,t]$

$Z[9]^{(0,1)}[z,t] == -M\,Z[4][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[9][z,t]\,a4'[t] - 6\,\mathrm{Tan}[z]\,Z[12]^{(1,0)}[z,t]$

$Z[10]^{(0,1)}[z,t] == -M\,Z[7][z,t] - 6\,\mathrm{Tan}[z]\,Z[15]^{(1,0)}[z,t]$

$Z[11]^{(0,1)}[z,t] == M\,Z[6][z,t] + 6\,\mathrm{Tan}[z]\,Z[14]^{(1,0)}[z,t]$

$Z[12]^{(0,1)}[z,t] ==$
$-M\,Z[1][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[12][z,t]\,a4'[t] - 6\,\mathrm{Tan}[z]\,Z[9]^{(1,0)}[z,t]$

$Z[13]^{(0,1)}[z,t] == M\,Z[0][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[13][z,t]\,a4'[t] + 6\,\mathrm{Tan}[z]\,Z[8]^{(1,0)}[z,t]$

$Z[14]^{(0,1)}[z,t] == M\,Z[3][z,t] + 6\,\mathrm{Tan}[z]\,Z[11]^{(1,0)}[z,t]$

$Z[15]^{(0,1)}[z,t] == -M\,Z[2][z,t] - 6\,\mathrm{Tan}[z]\,Z[10]^{(1,0)}[z,t]$

In[ ]:=
```
ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(DtvarZrelations = ToExpression[#] & /@ %) // Column
```

Out[ ]=

$M\,Z[13][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[0][z,t]\,a4'[t] + Z[0]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,Z[5]^{(1,0)}[z,t]$

$-M\,Z[12][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[1][z,t]\,a4'[t] + Z[1]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,Z[4]^{(1,0)}[z,t]$

$-M\,Z[15][z,t] + Z[2]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,Z[7]^{(1,0)}[z,t]$

$M\,Z[14][z,t] + Z[3]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,Z[6]^{(1,0)}[z,t]$

$-M\,Z[9][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[4][z,t]\,a4'[t] + Z[4]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,Z[1]^{(1,0)}[z,t]$

$M\,Z[8][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[5][z,t]\,a4'[t] + Z[5]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,Z[0]^{(1,0)}[z,t]$

$M\,Z[11][z,t] + Z[6]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,Z[3]^{(1,0)}[z,t]$

$-M\,Z[10][z,t] + Z[7]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,Z[2]^{(1,0)}[z,t]$

$-M\,Z[5][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[8][z,t]\,a4'[t] + Z[8]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,Z[13]^{(1,0)}[z,t]$

$M\,Z[4][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[9][z,t]\,a4'[t] + Z[9]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,Z[12]^{(1,0)}[z,t]$

$M\,Z[7][z,t] + Z[10]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,Z[15]^{(1,0)}[z,t]$

$-M\,Z[6][z,t] + Z[11]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,Z[14]^{(1,0)}[z,t]$

$M\,Z[1][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[12][z,t]\,a4'[t] + Z[12]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,Z[9]^{(1,0)}[z,t]$

$-M\,Z[0][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,Z[13][z,t]\,a4'[t] + Z[13]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,Z[8]^{(1,0)}[z,t]$

$-M\,Z[3][z,t] + Z[14]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,Z[11]^{(1,0)}[z,t]$

$M\,Z[2][z,t] + Z[15]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,Z[10]^{(1,0)}[z,t]$

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE :

In[ ]:=
```
{{4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13, 0, 1, 4, 13, 13, 8, 0, 5, 12, 1, 12, 9},
 {6, 7, 2, 11, 14, 6, 3, 7, 15, 2, 3, 6, 15, 10, 2, 3, 7, 14, 11}}
eLaztCouplings = Union[Flatten[#]] & /@ %
Union[Flatten[%]] === Range[0, 15]
```

Out[ ]=
```
{{4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13, 0, 1, 4, 13, 13, 8, 0, 5, 12, 1, 12, 9},
 {6, 7, 2, 11, 14, 6, 3, 7, 15, 2, 3, 6, 15, 10, 2, 3, 7, 14, 11}}
```

Out[ ]=
```
{{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}}
```

Out[ ]=
```
True
```

SEPARATE VARIABLES THAT ARE COUPLED TO EACH OTHER :

In[ ]:=
```
Thread[(Z[#] & /@ eLaztCouplings[[1]]) → (yZ[#] & /@ Range[0, 7])]
Thread[(Z[#] & /@ eLaztCouplings[[2]]) → (yZ[#] & /@ Range[8, 15])]
sZtOyZ = Union[Flatten[{%%, %}]]
ToString[FullForm[#]] & /@ sZtOyZ;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(ZyZEQS = ToExpression[#] & /@ %)
syZtoZ = Solve[And @@ %, (yZ[#] & /@ Range[0, 15])][[-1]]
```

Out[ ]=
```
{Z[0] → yZ[0], Z[1] → yZ[1], Z[4] → yZ[2], Z[5] → yZ[3],
 Z[8] → yZ[4], Z[9] → yZ[5], Z[12] → yZ[6], Z[13] → yZ[7]}
```

Out[ ]=
```
{Z[2] → yZ[8], Z[3] → yZ[9], Z[6] → yZ[10], Z[7] → yZ[11],
 Z[10] → yZ[12], Z[11] → yZ[13], Z[14] → yZ[14], Z[15] → yZ[15]}
```

Out[ ]=
```
{Z[0] → yZ[0], Z[1] → yZ[1], Z[2] → yZ[8], Z[3] → yZ[9], Z[4] → yZ[2], Z[5] → yZ[3],
 Z[6] → yZ[10], Z[7] → yZ[11], Z[8] → yZ[4], Z[9] → yZ[5], Z[10] → yZ[12],
 Z[11] → yZ[13], Z[12] → yZ[6], Z[13] → yZ[7], Z[14] → yZ[14], Z[15] → yZ[15]}
```

Out[ ]=
```
{Z[0] == yZ[0], Z[1] == yZ[1], Z[2] == yZ[8], Z[3] == yZ[9], Z[4] == yZ[2], Z[5] == yZ[3],
 Z[6] == yZ[10], Z[7] == yZ[11], Z[8] == yZ[4], Z[9] == yZ[5], Z[10] == yZ[12],
 Z[11] == yZ[13], Z[12] == yZ[6], Z[13] == yZ[7], Z[14] == yZ[14], Z[15] == yZ[15]}
```

Out[ ]=
```
{yZ[0] → Z[0], yZ[1] → Z[1], yZ[2] → Z[4], yZ[3] → Z[5], yZ[4] → Z[8], yZ[5] → Z[9],
 yZ[6] → Z[12], yZ[7] → Z[13], yZ[8] → Z[2], yZ[9] → Z[3], yZ[10] → Z[6],
 yZ[11] → Z[7], yZ[12] → Z[10], yZ[13] → Z[11], yZ[14] → Z[14], yZ[15] → Z[15]}
```

In[ ]:=
```
ToString[FullForm[#]] & /@ sZtOyZ;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(ZyZforCaEQS = -ToExpression[#] & /@ %)
```

Out[ ]=
```
{yZ[0] - Z[0], yZ[1] - Z[1], yZ[8] - Z[2], yZ[9] - Z[3], yZ[2] - Z[4], yZ[3] - Z[5],
 yZ[10] - Z[6], yZ[11] - Z[7], yZ[4] - Z[8], yZ[5] - Z[9], yZ[12] - Z[10],
 yZ[13] - Z[11], yZ[6] - Z[12], yZ[7] - Z[13], yZ[14] - Z[14], yZ[15] - Z[15]}
```

In[ ]:=
```
cayZ = CoefficientArrays[ZyZforCaEQS, (yZ[#] & /@ Range[0, 15])]
```

Out[ ]=

{SparseArray[Specified elements: 16 Dimensions: {16}], SparseArray[Specified elements: 16 Dimensions: {16, 16}]}

In[ ]:=
```
caZ = CoefficientArrays[ZyZforCaEQS, (Z[#] & /@ Range[0, 15])]
```

Out[ ]=

{SparseArray[Specified elements: 16 Dimensions: {16}], SparseArray[Specified elements: 16 Dimensions: {16, 16}]}

In[ ]:=
```
(cayZ2 = cayZ[[2]] // Normal) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

IDENTIFY ORTHOGONAL SIMILARITY TRANSFORMATION FROM SET OF VARIABLES Z[j] TO SET OF VARIABLES yZ[j] :

In[ ]:=
```
cayZ2.(yZ[#] & /@ Range[0, 15]) /. syZtoZ
Transpose[cayZ2].% /. sZtOyZ
```

Out[ ]=
```
{Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
 Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}
```

Out[ ]=
```
{yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
 yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}
```

In[ ]:=
```
cayZ2.Transpose[cayZ2] === ID16
```

Out[ ]=
```
True
```

TRANSFORM Euler Lagrange equations to yZ[j] :

In[ ]:=
```
(DtyZrelations = Transpose[cayZ2].DtvarZrelations /. sZtOyZ // FullSimplify) // Column
```

Out[ ]=

$$M\,yZ[7][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,yZ[0][z,t]\,a4'[t] + yZ[0]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,yZ[3]^{(1,0)}[z,t]$$

$$-M\,yZ[6][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,yZ[1][z,t]\,a4'[t] + yZ[1]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,yZ[2]^{(1,0)}[z,t]$$

$$-M\,yZ[5][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,yZ[2][z,t]\,a4'[t] + yZ[2]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,yZ[1]^{(1,0)}[z,t]$$

$$M\,yZ[4][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,yZ[3][z,t]\,a4'[t] + yZ[3]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,yZ[0]^{(1,0)}[z,t]$$

$$-M\,yZ[3][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,yZ[4][z,t]\,a4'[t] + yZ[4]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,yZ[7]^{(1,0)}[z,t]$$

$$M\,yZ[2][z,t] + e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,yZ[5][z,t]\,a4'[t] + yZ[5]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,yZ[6]^{(1,0)}[z,t]$$

$$M\,yZ[1][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,yZ[6][z,t]\,a4'[t] + yZ[6]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,yZ[5]^{(1,0)}[z,t]$$

$$-M\,yZ[0][z,t] - e^{-a4[t]}\,Q1\,\mathrm{Sinh}[a4[t]]\,yZ[7][z,t]\,a4'[t] + yZ[7]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,yZ[4]^{(1,0)}[z,t]$$

$$-M\,yZ[15][z,t] + yZ[8]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,yZ[11]^{(1,0)}[z,t]$$

$$M\,yZ[14][z,t] + yZ[9]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,yZ[10]^{(1,0)}[z,t]$$

$$M\,yZ[13][z,t] + yZ[10]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,yZ[9]^{(1,0)}[z,t]$$

$$-M\,yZ[12][z,t] + yZ[11]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,yZ[8]^{(1,0)}[z,t]$$

$$M\,yZ[11][z,t] + yZ[12]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,yZ[15]^{(1,0)}[z,t]$$

$$-M\,yZ[10][z,t] + yZ[13]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,yZ[14]^{(1,0)}[z,t]$$

$$-M\,yZ[9][z,t] + yZ[14]^{(0,1)}[z,t] - 6\,\mathrm{Tan}[z]\,yZ[13]^{(1,0)}[z,t]$$

$$M\,yZ[8][z,t] + yZ[15]^{(0,1)}[z,t] + 6\,\mathrm{Tan}[z]\,yZ[12]^{(1,0)}[z,t]$$

CHECK THAT yZ[j] ARE IN PROPER ORDER:

In[ ]:=
```
(DtyZsubs =
   Solve[And @@ Thread[0 == DtyZrelations], (D[(yZ[#][z, t] & /@ Range[0, 15]), t])][[1]] //
    FullSimplify[#, constraintVars] &) // Column
```

Out[ ]=

$yZ[0]^{(0,1)}[z, t] \to$
$-M\, yZ[7][z, t] + e^{-a4[t]}\, Q1\, \mathrm{Sinh}[a4[t]]\, yZ[0][z, t]\, a4'[t] - 6\, \mathrm{Tan}[z]\, yZ[3]^{(1,0)}[z, t]$

$yZ[1]^{(0,1)}[z, t] \to$
$M\, yZ[6][z, t] + e^{-a4[t]}\, Q1\, \mathrm{Sinh}[a4[t]]\, yZ[1][z, t]\, a4'[t] + 6\, \mathrm{Tan}[z]\, yZ[2]^{(1,0)}[z, t]$

$yZ[2]^{(0,1)}[z, t] \to$
$M\, yZ[5][z, t] - e^{-a4[t]}\, Q1\, \mathrm{Sinh}[a4[t]]\, yZ[2][z, t]\, a4'[t] + 6\, \mathrm{Tan}[z]\, yZ[1]^{(1,0)}[z, t]$

$yZ[3]^{(0,1)}[z, t] \to$
$-M\, yZ[4][z, t] - e^{-a4[t]}\, Q1\, \mathrm{Sinh}[a4[t]]\, yZ[3][z, t]\, a4'[t] - 6\, \mathrm{Tan}[z]\, yZ[0]^{(1,0)}[z, t]$

$yZ[4]^{(0,1)}[z, t] \to$
$M\, yZ[3][z, t] - e^{-a4[t]}\, Q1\, \mathrm{Sinh}[a4[t]]\, yZ[4][z, t]\, a4'[t] + 6\, \mathrm{Tan}[z]\, yZ[7]^{(1,0)}[z, t]$

$yZ[5]^{(0,1)}[z, t] \to$
$-M\, yZ[2][z, t] - e^{-a4[t]}\, Q1\, \mathrm{Sinh}[a4[t]]\, yZ[5][z, t]\, a4'[t] - 6\, \mathrm{Tan}[z]\, yZ[6]^{(1,0)}[z, t]$

$yZ[6]^{(0,1)}[z, t] \to$
$-M\, yZ[1][z, t] + e^{-a4[t]}\, Q1\, \mathrm{Sinh}[a4[t]]\, yZ[6][z, t]\, a4'[t] - 6\, \mathrm{Tan}[z]\, yZ[5]^{(1,0)}[z, t]$

$yZ[7]^{(0,1)}[z, t] \to$
$M\, yZ[0][z, t] + e^{-a4[t]}\, Q1\, \mathrm{Sinh}[a4[t]]\, yZ[7][z, t]\, a4'[t] + 6\, \mathrm{Tan}[z]\, yZ[4]^{(1,0)}[z, t]$

$yZ[8]^{(0,1)}[z, t] \to M\, yZ[15][z, t] + 6\, \mathrm{Tan}[z]\, yZ[11]^{(1,0)}[z, t]$

$yZ[9]^{(0,1)}[z, t] \to -M\, yZ[14][z, t] - 6\, \mathrm{Tan}[z]\, yZ[10]^{(1,0)}[z, t]$

$yZ[10]^{(0,1)}[z, t] \to -M\, yZ[13][z, t] - 6\, \mathrm{Tan}[z]\, yZ[9]^{(1,0)}[z, t]$

$yZ[11]^{(0,1)}[z, t] \to M\, yZ[12][z, t] + 6\, \mathrm{Tan}[z]\, yZ[8]^{(1,0)}[z, t]$

$yZ[12]^{(0,1)}[z, t] \to -M\, yZ[11][z, t] - 6\, \mathrm{Tan}[z]\, yZ[15]^{(1,0)}[z, t]$

$yZ[13]^{(0,1)}[z, t] \to M\, yZ[10][z, t] + 6\, \mathrm{Tan}[z]\, yZ[14]^{(1,0)}[z, t]$

$yZ[14]^{(0,1)}[z, t] \to M\, yZ[9][z, t] + 6\, \mathrm{Tan}[z]\, yZ[13]^{(1,0)}[z, t]$

$yZ[15]^{(0,1)}[z, t] \to -M\, yZ[8][z, t] - 6\, \mathrm{Tan}[z]\, yZ[12]^{(1,0)}[z, t]$

In[ ]:=
```
(caZ2 = caZ[[2]] // Normal) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}$$

In[ ]:=
```
sx0x4
```

Out[ ]=

$$\left\{x0 \to \frac{z}{6\,H},\ x4 \to \frac{t}{H}\right\}$$

ENERGY EIGENVALUES OF THE Euler Lagrange equations:

TRY TO FIND ENERGY EIGENFUNCTIONS AND ENERGY EIGENVALUES OF THE Euler Lagrange equations of the form

$$yZ[j] = \frac{1}{\text{Sin}[6*H*x0]^{1/2}} \text{Exp}[E_j[x4] * H] =$$

$$yZ[j] = \frac{1}{\text{Sin}[z]^{1/2}} \text{Exp}[E_j[t]]$$

(recall that

```
sfψ16Aa =
 f16[#] → ToExpression["((Z[" <> ToString[#] <>
      "][6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"] & /@
  Range[0, 15]
```

):

```
In[•]:= syZtOnly = yZ[#] → ToExpression["((Exp[ν[" <> ToString[#] <> "][#2]])&)"] & /@ Range[0, 15]
Out[•]=
{yZ[0] → (Exp[ν[0][#2]] &), yZ[1] → (Exp[ν[1][#2]] &),
 yZ[2] → (Exp[ν[2][#2]] &), yZ[3] → (Exp[ν[3][#2]] &), yZ[4] → (Exp[ν[4][#2]] &),
 yZ[5] → (Exp[ν[5][#2]] &), yZ[6] → (Exp[ν[6][#2]] &), yZ[7] → (Exp[ν[7][#2]] &),
 yZ[8] → (Exp[ν[8][#2]] &), yZ[9] → (Exp[ν[9][#2]] &), yZ[10] → (Exp[ν[10][#2]] &),
 yZ[11] → (Exp[ν[11][#2]] &), yZ[12] → (Exp[ν[12][#2]] &),
 yZ[13] → (Exp[ν[13][#2]] &), yZ[14] → (Exp[ν[14][#2]] &), yZ[15] → (Exp[ν[15][#2]] &)}

In[•]:= syZtc = ν[#][0] → Log[T[#]] & /@ Range[0, 15]
Out[•]=
{ν[0][0] → Log[T[0]], ν[1][0] → Log[T[1]], ν[2][0] → Log[T[2]], ν[3][0] → Log[T[3]],
 ν[4][0] → Log[T[4]], ν[5][0] → Log[T[5]], ν[6][0] → Log[T[6]], ν[7][0] → Log[T[7]],
 ν[8][0] → Log[T[8]], ν[9][0] → Log[T[9]], ν[10][0] → Log[T[10]], ν[11][0] → Log[T[11]],
 ν[12][0] → Log[T[12]], ν[13][0] → Log[T[13]], ν[14][0] → Log[T[14]], ν[15][0] → Log[T[15]]}

In[•]:= DtyZrelations /. syZtOnly
FullSimplify[Normal[Series[#, {t, 0, 0}]]] & /@ %
% /. syZtc
DtyZrelationsA = %
```

Out[ ]=

$$\{e^{\nu[7][t]} M - e^{-a4[t]+\nu[0][t]} Q1 \operatorname{Sinh}[a4[t]]\, a4'[t] + e^{\nu[0][t]} \nu[0]'[t],$$
$$-e^{\nu[6][t]} M - e^{-a4[t]+\nu[1][t]} Q1 \operatorname{Sinh}[a4[t]]\, a4'[t] + e^{\nu[1][t]} \nu[1]'[t],$$
$$-e^{\nu[5][t]} M + e^{-a4[t]+\nu[2][t]} Q1 \operatorname{Sinh}[a4[t]]\, a4'[t] + e^{\nu[2][t]} \nu[2]'[t],$$
$$e^{\nu[4][t]} M + e^{-a4[t]+\nu[3][t]} Q1 \operatorname{Sinh}[a4[t]]\, a4'[t] + e^{\nu[3][t]} \nu[3]'[t],$$
$$-e^{\nu[3][t]} M + e^{-a4[t]+\nu[4][t]} Q1 \operatorname{Sinh}[a4[t]]\, a4'[t] + e^{\nu[4][t]} \nu[4]'[t],$$
$$e^{\nu[2][t]} M + e^{-a4[t]+\nu[5][t]} Q1 \operatorname{Sinh}[a4[t]]\, a4'[t] + e^{\nu[5][t]} \nu[5]'[t],$$
$$e^{\nu[1][t]} M - e^{-a4[t]+\nu[6][t]} Q1 \operatorname{Sinh}[a4[t]]\, a4'[t] + e^{\nu[6][t]} \nu[6]'[t],$$
$$-e^{\nu[0][t]} M - e^{-a4[t]+\nu[7][t]} Q1 \operatorname{Sinh}[a4[t]]\, a4'[t] + e^{\nu[7][t]} \nu[7]'[t],$$
$$-e^{\nu[15][t]} M + e^{\nu[8][t]} \nu[8]'[t],\ e^{\nu[14][t]} M + e^{\nu[9][t]} \nu[9]'[t],\ e^{\nu[13][t]} M + e^{\nu[10][t]} \nu[10]'[t],$$
$$-e^{\nu[12][t]} M + e^{\nu[11][t]} \nu[11]'[t],\ e^{\nu[11][t]} M + e^{\nu[12][t]} \nu[12]'[t],$$
$$-e^{\nu[10][t]} M + e^{\nu[13][t]} \nu[13]'[t],\ -e^{\nu[9][t]} M + e^{\nu[14][t]} \nu[14]'[t],\ e^{\nu[8][t]} M + e^{\nu[15][t]} \nu[15]'[t]\}$$

Out[ ]=

$$\{e^{\nu[7][0]} M + e^{\nu[0][0]} \left(-e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[0]'[0]\right),$$
$$-e^{\nu[6][0]} M + e^{\nu[1][0]} \left(-e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[1]'[0]\right),$$
$$-e^{\nu[5][0]} M + e^{\nu[2][0]} \left(e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[2]'[0]\right),$$
$$e^{\nu[4][0]} M + e^{\nu[3][0]} \left(e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[3]'[0]\right),$$
$$-e^{\nu[3][0]} M + e^{\nu[4][0]} \left(e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[4]'[0]\right),$$
$$e^{\nu[2][0]} M + e^{\nu[5][0]} \left(e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[5]'[0]\right),$$
$$e^{\nu[1][0]} M + e^{\nu[6][0]} \left(-e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[6]'[0]\right),$$
$$-e^{\nu[0][0]} M + e^{\nu[7][0]} \left(-e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[7]'[0]\right),$$
$$-e^{\nu[15][0]} M + e^{\nu[8][0]} \nu[8]'[0],\ e^{\nu[14][0]} M + e^{\nu[9][0]} \nu[9]'[0],\ e^{\nu[13][0]} M + e^{\nu[10][0]} \nu[10]'[0],$$
$$-e^{\nu[12][0]} M + e^{\nu[11][0]} \nu[11]'[0],\ e^{\nu[11][0]} M + e^{\nu[12][0]} \nu[12]'[0],$$
$$-e^{\nu[10][0]} M + e^{\nu[13][0]} \nu[13]'[0],\ -e^{\nu[9][0]} M + e^{\nu[14][0]} \nu[14]'[0],\ e^{\nu[8][0]} M + e^{\nu[15][0]} \nu[15]'[0]\}$$

Out[ ]=

$$\{M\, T[7] + T[0] \left(-e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[0]'[0]\right),$$
$$-M\, T[6] + T[1] \left(-e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[1]'[0]\right),$$
$$-M\, T[5] + T[2] \left(e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[2]'[0]\right),$$
$$M\, T[4] + T[3] \left(e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[3]'[0]\right),$$
$$-M\, T[3] + T[4] \left(e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[4]'[0]\right),$$
$$M\, T[2] + T[5] \left(e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[5]'[0]\right),$$
$$M\, T[1] + T[6] \left(-e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[6]'[0]\right),$$
$$-M\, T[0] + T[7] \left(-e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]]\, a4'[0] + \nu[7]'[0]\right),$$
$$-M\, T[15] + T[8]\, \nu[8]'[0],\ M\, T[14] + T[9]\, \nu[9]'[0],\ M\, T[13] + T[10]\, \nu[10]'[0],$$
$$-M\, T[12] + T[11]\, \nu[11]'[0],\ M\, T[11] + T[12]\, \nu[12]'[0],$$
$$-M\, T[10] + T[13]\, \nu[13]'[0],\ -M\, T[9] + T[14]\, \nu[14]'[0],\ M\, T[8] + T[15]\, \nu[15]'[0]\}$$

Out[•]=

```
{M T[7] + T[0] (-ⅇ^(-a4[0]) Q1 Sinh[a4[0]] a4′[0] + ν[0]′[0]),
 -M T[6] + T[1] (-ⅇ^(-a4[0]) Q1 Sinh[a4[0]] a4′[0] + ν[1]′[0]),
 -M T[5] + T[2] (ⅇ^(-a4[0]) Q1 Sinh[a4[0]] a4′[0] + ν[2]′[0]),
 M T[4] + T[3] (ⅇ^(-a4[0]) Q1 Sinh[a4[0]] a4′[0] + ν[3]′[0]),
 -M T[3] + T[4] (ⅇ^(-a4[0]) Q1 Sinh[a4[0]] a4′[0] + ν[4]′[0]),
 M T[2] + T[5] (ⅇ^(-a4[0]) Q1 Sinh[a4[0]] a4′[0] + ν[5]′[0]),
 M T[1] + T[6] (-ⅇ^(-a4[0]) Q1 Sinh[a4[0]] a4′[0] + ν[6]′[0]),
 -M T[0] + T[7] (-ⅇ^(-a4[0]) Q1 Sinh[a4[0]] a4′[0] + ν[7]′[0]),
 -M T[15] + T[8] ν[8]′[0], M T[14] + T[9] ν[9]′[0], M T[13] + T[10] ν[10]′[0],
 -M T[12] + T[11] ν[11]′[0], M T[11] + T[12] ν[12]′[0],
 -M T[10] + T[13] ν[13]′[0], -M T[9] + T[14] ν[14]′[0], M T[8] + T[15] ν[15]′[0]}
```

In[•]:=

```
(*DtyZrelations/.syZtOnly;
 %/.syZtc
 DtyZrelationsA=%*)
```

In[•]:=

```
Ts = (T[#] & /@ Range[0, 15])
```

Out[•]=

```
{T[0], T[1], T[2], T[3], T[4], T[5], T[6],
 T[7], T[8], T[9], T[10], T[11], T[12], T[13], T[14], T[15]}
```

In[•]:=

```
caDtyZrelationsA = CoefficientArrays[DtyZrelationsA, Ts]
```

Out[•]=

```
{SparseArray[ Specified elements: 0 Dimensions: {16} ], SparseArray[ Specified elements: 32 Dimensions: {16, 16} ]}
```

In[•]:=

```
(caDtyZrelationsA2 = caDtyZrelationsA[[2]] // Normal) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -e^{-a4[0]}\,Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+\nu[0]'[0] & 0 & \\ 0 & -e^{-a4[0]}\,Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+\nu[1]'[0] & \\ 0 & 0 & e^{-a4[0]}\,Q1\,\mathrm{Sinh}[a4 \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & M & \\ -M & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 0 & \end{pmatrix}$$

In[•]:=

```
caDtyZrelationsA2 === ArrayFlatten[
  {{caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]], 0}, {0, caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]]}}]
```

Out[•]=

```
True
```

CHECK THAT THERE ARE NO $\nu[]$ TERMS :

In[ ]:=
```
(caDtyZrelationsA1 = caDtyZrelationsA[[1]] // Normal) // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In[ ]:=
```
time1 = Now
AbsoluteTiming[caDtyZrelationsA21Y0 = Det[caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]] - Y ID8]]
Now
caDtyZrelationsA21Y = (*Full*)Simplify[caDtyZrelationsA21Y0, constraintVars]
Now - time1
```

Out[ ]=

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Out[•]=

$$\{0.0840045,\ e^{-8\,a4[0]}$$
$$\Big(e^{2\,a4[0]}\,M^2\,\Big(e^{2\,a4[0]}\,M^2\,\Big(e^{2\,a4[0]}\,M^2+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[3]'[0]\big)\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[4]'[0]\big)\Big)+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[2]'[0]\big)\Big(e^{2\,a4[0]}\,M^2+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[3]'[0]\big)\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[4]'[0]\big)\Big)\Big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[5]'[0]\Big)\Big)+\big(-e^{a4[0]}\,Y-Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[1]'[0]\big)$$
$$\Big(e^{2\,a4[0]}\,M^2\,\Big(e^{2\,a4[0]}\,M^2+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[3]'[0]\big)\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[4]'[0]\big)\Big)+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[2]'[0]\big)\Big(e^{2\,a4[0]}\,M^2+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[3]'[0]\big)\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[4]'[0]\big)\Big)\Big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[5]'[0]\Big)\Big)\big(-e^{a4[0]}\,Y-Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[6]'[0]\big)\Big)+$$
$$\big(-e^{a4[0]}\,Y-Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[0]'[0]\big)$$
$$\Big(e^{2\,a4[0]}\,M^2\,\Big(e^{2\,a4[0]}\,M^2\,\Big(e^{2\,a4[0]}\,M^2+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[3]'[0]\big)\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[4]'[0]\big)\Big)+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[2]'[0]\big)\Big(e^{2\,a4[0]}\,M^2+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[3]'[0]\big)\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[4]'[0]\big)\Big)\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[5]'[0]\big)\Big)+\big(-e^{a4[0]}\,Y-Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[1]'[0]\big)$$
$$\Big(e^{2\,a4[0]}\,M^2\,\Big(e^{2\,a4[0]}\,M^2+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[3]'[0]\big)\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[4]'[0]\big)\Big)+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[2]'[0]\big)\Big(e^{2\,a4[0]}\,M^2+\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[3]'[0]\big)\big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[4]'[0]\big)\Big)\Big(-e^{a4[0]}\,Y+Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[5]'[0]\Big)\Big)\big(-e^{a4[0]}\,Y-Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[6]'[0]\big)\Big)\big(-e^{a4[0]}\,Y-Q1\,\mathrm{Sinh}[a4[0]]\,a4'[0]+e^{a4[0]}\,\nu[7]'[0]\big)\Big)\}$$

Out[•]=

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Out[•]=

$$\frac{1}{256}\,e^{-16\,a4[0]}\,\Big(Q1^2\,a4'[0]^2+e^{4\,a4[0]}\,\big(4\,M^2+(2\,Y-Q1\,a4'[0]-2\,\nu[3]'[0])\,(2\,Y-Q1\,a4'[0]-2\,\nu[4]'[0])\big)-2\,e^{2\,a4[0]}\,Q1\,a4'[0]\,(-2\,Y+Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0])\Big)$$
$$\Big(Q1^2\,a4'[0]^2+e^{4\,a4[0]}\,\big(4\,M^2+(2\,Y-Q1\,a4'[0]-2\,\nu[2]'[0])\,(2\,Y-Q1\,a4'[0]-2\,\nu[5]'[0])\big)-2\,e^{2\,a4[0]}\,Q1\,a4'[0]\,(-2\,Y+Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0])\Big)$$
$$\Big(Q1^2\,a4'[0]^2+e^{4\,a4[0]}\,\big(4\,M^2+(2\,Y+Q1\,a4'[0]-2\,\nu[1]'[0])\,(2\,Y+Q1\,a4'[0]-2\,\nu[6]'[0])\big)-2\,e^{2\,a4[0]}\,Q1\,a4'[0]\,(2\,Y+Q1\,a4'[0]-\nu[1]'[0]-\nu[6]'[0])\Big)$$
$$\Big(Q1^2\,a4'[0]^2+e^{4\,a4[0]}\,\big(4\,M^2+(2\,Y+Q1\,a4'[0]-2\,\nu[0]'[0])\,(2\,Y+Q1\,a4'[0]-2\,\nu[7]'[0])\big)-2\,e^{2\,a4[0]}\,Q1\,a4'[0]\,(2\,Y+Q1\,a4'[0]-\nu[0]'[0]-\nu[7]'[0])\Big)$$

Out[•]=

1.4006429 s

In[ ]:=
```
sumEvalscaDtyZrelationsA21Y = 1/7! D[caDtyZrelationsA21Y, {Y, 7}] /. {Y → 0} // Simplify
```

Out[ ]=

$$-\nu[0]'[0]-\nu[1]'[0]-\nu[2]'[0]-\nu[3]'[0]-\nu[4]'[0]-\nu[5]'[0]-\nu[6]'[0]-\nu[7]'[0]$$

In[ ]:=
```
time1 = Now
AbsoluteTiming[caDtyZrelationsA22Y0 = Det[caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]] - Y ID8]]
Now
caDtyZrelationsA22Y = (*Full*)Simplify[caDtyZrelationsA22Y0, constraintVars]
Now - time1
```

Out[ ]=

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Out[ ]=

$$\{0.0026515,$$
$$M^2\left(M^2\left(M\left(M^3+M Y^2-M Y\nu[11]'[0]-M Y\nu[12]'[0]+M\nu[11]'[0]\nu[12]'[0]\right)+(-Y+\nu[10]'[0])\left(M^2+Y^2-Y\nu[11]'[0]-Y\nu[12]'[0]+\nu[11]'[0]\nu[12]'[0]\right)(-Y+\nu[13]'[0])\right)+(-Y+\nu[9]'[0])\left(M\left(M^3+M Y^2-M Y\nu[11]'[0]-M Y\nu[12]'[0]+M\nu[11]'[0]\nu[12]'[0]\right)+(-Y+\nu[10]'[0])\left(M^2+Y^2-Y\nu[11]'[0]-Y\nu[12]'[0]+\nu[11]'[0]\nu[12]'[0]\right)(-Y+\nu[13]'[0])\right)(-Y+\nu[14]'[0])\right)+(-Y+\nu[8]'[0])$$
$$\left(M^2\left(M\left(M^3+M Y^2-M Y\nu[11]'[0]-M Y\nu[12]'[0]+M\nu[11]'[0]\nu[12]'[0]\right)+(-Y+\nu[10]'[0])\left(M^2+Y^2-Y\nu[11]'[0]-Y\nu[12]'[0]+\nu[11]'[0]\nu[12]'[0]\right)(-Y+\nu[13]'[0])\right)+(-Y+\nu[9]'[0])\left(M\left(M^3+M Y^2-M Y\nu[11]'[0]-M Y\nu[12]'[0]+M\nu[11]'[0]\nu[12]'[0]\right)+(-Y+\nu[10]'[0])\left(M^2+Y^2-Y\nu[11]'[0]-Y\nu[12]'[0]+\nu[11]'[0]\nu[12]'[0]\right)(-Y+\nu[13]'[0])\right)(-Y+\nu[14]'[0])\right)(-Y+\nu[15]'[0])\}$$

Out[ ]=

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Out[ ]=

$$\left(M^2+(Y-\nu[11]'[0])(Y-\nu[12]'[0])\right)\left(M^2+(Y-\nu[10]'[0])(Y-\nu[13]'[0])\right)\left(M^2+(Y-\nu[9]'[0])(Y-\nu[14]'[0])\right)\left(M^2+(Y-\nu[8]'[0])(Y-\nu[15]'[0])\right)$$

Out[ ]=

0.0473984 s

In[ ]:=
```
sumEvalscaDtyZrelationsA22Y = 1/7! D[caDtyZrelationsA22Y, {Y, 7}] /. {Y → 0} // Simplify
```

Out[ ]=

$$-\nu[8]'[0]-\nu[9]'[0]-\nu[10]'[0]-\nu[11]'[0]-\nu[12]'[0]-\nu[13]'[0]-\nu[14]'[0]-\nu[15]'[0]$$

In[ ]:=
```
sumEvalscaDtyZrelationsA21Y =
  1/7! D[caDtyZrelationsA21Y, {Y, 7}] /. {Y → 0} // Simplify
```

$$-\nu[0]'[t]-\nu[1]'[t]-\nu[2]'[t]-\nu[3]'[t]-\nu[4]'[t]-\nu[5]'[t]-\nu[6]'[t]-\nu[7]'[t]$$

In[ ]:=
```
sumEvalscaDtyZrelationsA22Y =
```

```
1/7! D[caDtyZrelationsA22Y, {Y, 7}] /. {Y → 0} // Simplify
```

```
-ν[8]′[t] - ν[9]′[t] - ν[10]′[t] - ν[11]′[t] -
 ν[12]′[t] - ν[13]′[t] - ν[14]′[t] - ν[15]′[t]
```

{Eigenvals, Eigenvecs} = Eigensystem[m];

Jordan decomposition of a square matrix m .
The result is a list {s, j} where s is a similarity matrix and j is the Jordan canonical form of m.
The *j* matrix is called the Jordan normal form or Jordan canonical form.
j is a special block-diagonal, upper-triangular matrix representation for any square matrix, featuring eigenvalues on the diagonal and ones on the superdiagonal (just above the main diagonal) for non-diagonalizable cases
{s, j} = JordanDecomposition[m]
m== s.j.Inverse[s] **s.j.Inverse[s] - m**

In[ ]:=
```
{sca2, jca2} = JordanDecomposition[caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]]]
```

Out[ ]=

$$\Big\{\Big\{\Big\{0, 0, 0, 0, 0, 0, -\frac{-\nu[8]'[0]+\nu[15]'[0]+\sqrt{-4M^2+\nu[8]'[0]^2-2\nu[8]'[0]\nu[15]'[0]+\nu[15]'[0]^2}}{2M}, -\frac{-\nu[8]'[0]+\nu[15]'[0]-\sqrt{-4M^2+\nu[8]'[0]^2-2\nu[8]'[0]\nu[15]'[0]+\nu[15]'[0]^2}}{2M}\Big\},$$

$$\Big\{0, 0, 0, 0, -\frac{\nu[9]'[0]-\nu[14]'[0]-\sqrt{-4M^2+\nu[9]'[0]^2-2\nu[9]'[0]\nu[14]'[0]+\nu[14]'[0]^2}}{2M}, -\frac{\nu[9]'[0]-\nu[14]'[0]+\sqrt{-4M^2+\nu[9]'[0]^2-2\nu[9]'[0]\nu[14]'[0]+\nu[14]'[0]^2}}{2M}, 0, 0\Big\},$$

$$\Big\{0, 0, -\frac{\nu[10]'[0]-\nu[13]'[0]-\sqrt{-4M^2+\nu[10]'[0]^2-2\nu[10]'[0]\nu[13]'[0]+\nu[13]'[0]^2}}{2M}, -\frac{\nu[10]'[0]-\nu[13]'[0]+\sqrt{-4M^2+\nu[10]'[0]^2-2\nu[10]'[0]\nu[13]'[0]+\nu[13]'[0]^2}}{2M}, 0, 0, 0, 0\Big\},$$

$$\Big\{-\frac{-\nu[11]'[0]+\nu[12]'[0]+\sqrt{-4M^2+\nu[11]'[0]^2-2\nu[11]'[0]\nu[12]'[0]+\nu[12]'[0]^2}}{2M}, -\frac{-\nu[11]'[0]+\nu[12]'[0]-\sqrt{-4M^2+\nu[11]'[0]^2-2\nu[11]'[0]\nu[12]'[0]+\nu[12]'[0]^2}}{2M}, 0, 0, 0, 0, 0, 0\Big\}, \{1, 1, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 1, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 1\}\Big\},$$

$$\Big\{\Big\{\frac{1}{2}\left(\nu[11]'[0]+\nu[12]'[0]-\sqrt{-4M^2+\nu[11]'[0]^2-2\nu[11]'[0]\nu[12]'[0]+\nu[12]'[0]^2}\right), 0, 0, 0, 0, 0, 0, 0\Big\},$$

$$\Big\{0, \frac{1}{2}\left(\nu[11]'[0]+\nu[12]'[0]+\sqrt{-4M^2+\nu[11]'[0]^2-2\nu[11]'[0]\nu[12]'[0]+\nu[12]'[0]^2}\right), 0, 0, 0, 0, 0, 0\Big\},$$

$$\Big\{0, 0, \frac{1}{2}\left(\nu[10]'[0]+\nu[13]'[0]-\sqrt{-4M^2+\nu[10]'[0]^2-2\nu[10]'[0]\nu[13]'[0]+\nu[13]'[0]^2}\right), 0, 0, 0, 0, 0\Big\},$$

$$\Big\{0, 0, 0, \frac{1}{2}\left(\nu[10]'[0]+\nu[13]'[0]+\sqrt{-4M^2+\nu[10]'[0]^2-2\nu[10]'[0]\nu[13]'[0]+\nu[13]'[0]^2}\right), 0, 0, 0, 0\Big\}, \Big\{0, 0, 0, 0, \frac{1}{2}\left(\nu[9]'[0]+\nu[14]'[0]-\sqrt{-4M^2+\nu[9]'[0]^2-2\nu[9]'[0]\nu[14]'[0]+\nu[14]'[0]^2}\right), 0, 0, 0\Big\},$$

$$\Big\{0, 0, 0, 0, 0, \frac{1}{2}\left(\nu[9]'[0]+\nu[14]'[0]+\sqrt{-4M^2+\nu[9]'[0]^2-2\nu[9]'[0]\nu[14]'[0]+\nu[14]'[0]^2}\right), 0, 0\Big\}, \Big\{0, 0, 0, 0, 0, 0, \frac{1}{2}\left(\nu[8]'[0]+\nu[15]'[0]-\sqrt{-4M^2+\nu[8]'[0]^2-2\nu[8]'[0]\nu[15]'[0]+\nu[15]'[0]^2}\right), 0\Big\}, \Big\{0, 0, 0, 0, 0, 0, 0, \frac{1}{2}\left(\nu[8]'[0]+\nu[15]'[0]+\sqrt{-4M^2+\nu[8]'[0]^2-2\nu[8]'[0]\nu[15]'[0]+\nu[15]'[0]^2}\right)\Big\}\Big\}\Big\}$$

In[ ]:=
```
{valscaDtyZrelationsA22, vecscaDtyZrelationsA22} =
 Eigensystem[caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]]]
```

Out[ ]=

$$\Big\{\Big\{\frac{1}{2}\left(\nu[11]'[0]+\nu[12]'[0]-\sqrt{-4\,M^2+\nu[11]'[0]^2-2\,\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[11]'[0]+\nu[12]'[0]+\sqrt{-4\,M^2+\nu[11]'[0]^2-2\,\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[10]'[0]+\nu[13]'[0]-\sqrt{-4\,M^2+\nu[10]'[0]^2-2\,\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[10]'[0]+\nu[13]'[0]+\sqrt{-4\,M^2+\nu[10]'[0]^2-2\,\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[9]'[0]+\nu[14]'[0]-\sqrt{-4\,M^2+\nu[9]'[0]^2-2\,\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[9]'[0]+\nu[14]'[0]+\sqrt{-4\,M^2+\nu[9]'[0]^2-2\,\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[8]'[0]+\nu[15]'[0]-\sqrt{-4\,M^2+\nu[8]'[0]^2-2\,\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}\right),$$
$$\frac{1}{2}\left(\nu[8]'[0]+\nu[15]'[0]+\sqrt{-4\,M^2+\nu[8]'[0]^2-2\,\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}\right)\Big\},$$
$$\Big\{\Big\{0,0,0,-\frac{-\nu[11]'[0]+\nu[12]'[0]+\sqrt{-4\,M^2+\nu[11]'[0]^2-2\,\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}}{2\,M},$$
$$1,0,0,0\Big\},\Big\{0,0,0,$$
$$-\frac{-\nu[11]'[0]+\nu[12]'[0]-\sqrt{-4\,M^2+\nu[11]'[0]^2-2\,\nu[11]'[0]\,\nu[12]'[0]+\nu[12]'[0]^2}}{2\,M},1,0,0,0\Big\},$$
$$\Big\{0,0,-\frac{\nu[10]'[0]-\nu[13]'[0]-\sqrt{-4\,M^2+\nu[10]'[0]^2-2\,\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}}{2\,M},0,0,1,$$
$$0,0\Big\},\Big\{0,0,-\frac{\nu[10]'[0]-\nu[13]'[0]+\sqrt{-4\,M^2+\nu[10]'[0]^2-2\,\nu[10]'[0]\,\nu[13]'[0]+\nu[13]'[0]^2}}{2\,M},$$
$$0,0,1,0,0\Big\},$$
$$\Big\{0,-\frac{\nu[9]'[0]-\nu[14]'[0]-\sqrt{-4\,M^2+\nu[9]'[0]^2-2\,\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}}{2\,M},0,0,0,0,1,$$
$$0\Big\},\Big\{0,-\frac{\nu[9]'[0]-\nu[14]'[0]+\sqrt{-4\,M^2+\nu[9]'[0]^2-2\,\nu[9]'[0]\,\nu[14]'[0]+\nu[14]'[0]^2}}{2\,M},0,0,0,$$
$$0,1,0\Big\},\Big\{-\frac{-\nu[8]'[0]+\nu[15]'[0]+\sqrt{-4\,M^2+\nu[8]'[0]^2-2\,\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}}{2\,M},0,0,$$
$$0,0,0,0,1\Big\},\Big\{-\frac{-\nu[8]'[0]+\nu[15]'[0]-\sqrt{-4\,M^2+\nu[8]'[0]^2-2\,\nu[8]'[0]\,\nu[15]'[0]+\nu[15]'[0]^2}}{2\,M},$$
$$0,0,0,0,0,0,1\Big\}\Big\}\Big\}$$

In[ ]:=
```
{valscaDtyZrelationsA21, vecscaDtyZrelationsA21} =
 Eigensystem[caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]]] //
   FullSimplify[#, e^(6 a4[t]) > 0 && e^(3 a4[t]) > 0 && constraintVars] & //
  FullSimplify[#, e^(6 a4[t]) > 0 && e^(3 a4[t]) > 0 && a4[t] > 0] &
```

Out[ ]=

$$\Big\{\Big\{\frac{1}{2}\,e^{-3\,a4[0]}\,\Big(-e^{a4[0]}\,Q1\,a4'[0]-$$

$$\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0])\Big),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0])\Big),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0])\Big),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0])\Big),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\Big),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\Big),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\Big),$$

$$\frac{1}{2}\,e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\Big)\Big\},$$

$$\Big\{\Big\{0,\,0,\,0,\,\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+e^{3\,a4[0]}\,(-\nu[3]'[0]+\nu[4]'[0])\right)}{2\,M},$$
$$1,\,0,\,0,\,0\Big\},$$

$$\Big\{0,\,0,\,0,\,-\frac{\nu[3]'[0]+e^{-3\,a4[0]}\,\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}-\nu[4]'[0]}{2\,M},\,1,\,0,\,0,\,0\Big\},$$

$$\Big\{0,\,0,\,-\frac{-\nu[2]'[0]+e^{-3\,a4[0]}\,\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+\nu[5]'[0]}{2\,M},\,0,\,0,\,1,\,0,\,0\Big\},$$

$$\Big\{0,\,0,\,\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+e^{3\,a4[0]}\,(\nu[2]'[0]-\nu[5]'[0])\right)}{2\,M},$$
$$0,\,0,\,1,\,0,\,0\Big\},$$

$$\Big\{0,\,-\frac{-\nu[1]'[0]+e^{-3\,a4[0]}\,\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+\nu[6]'[0]}{2\,M},\,0,\,0,\,0,\,0,\,1,\,0\Big\},$$

$$\Big\{0,\,\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+e^{3\,a4[0]}\,(\nu[1]'[0]-\nu[6]'[0])\right)}{2\,M},\,0,\,0,\,0,$$
$$0,\,1,\,0\Big\},\,\Big\{\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+e^{3\,a4[0]}\,(-\nu[0]'[0]+\nu[7]'[0])\right)}{2\,M},$$
$$0,\,0,\,0,\,0,\,0,\,0,\,1\Big\},$$

$$\left\{-\frac{\nu[0]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}-\nu[7]'[0]}{2\,M},0,0,0,0,0,0,1\right\}\Big\}\Big\}$$

In[ ]:=
```
{sca1, jca1} = JordanDecomposition[caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]]] //
  FullSimplify[#, e^(6 a4[t]) > 0 && e^(3 a4[t]) > 0 && constraintVars] & //
 FullSimplify[#, e^(6 a4[t]) > 0 && e^(3 a4[t]) > 0 && a4[t] > 0] &
```

Out[ ]=

$$\Big\{\Big\{\Big\{0,0,0,0,0,0,\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+e^{3\,a4[0]}\left(-\nu[0]'[0]+\nu[7]'[0]\right)\right)}{2\,M},$$
$$-\frac{\nu[0]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}-\nu[7]'[0]}{2\,M}\Big\},$$
$$\Big\{0,0,0,0,-\frac{-\nu[1]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+\nu[6]'[0]}{2\,M},$$
$$\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+e^{3\,a4[0]}\left(\nu[1]'[0]-\nu[6]'[0]\right)\right)}{2\,M},0,0\Big\},$$
$$\Big\{0,0,-\frac{-\nu[2]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+\nu[5]'[0]}{2\,M},$$
$$\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+e^{3\,a4[0]}\left(\nu[2]'[0]-\nu[5]'[0]\right)\right)}{2\,M},0,0,0,0\Big\},$$
$$\Big\{\frac{e^{-3\,a4[0]}\left(\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+e^{3\,a4[0]}\left(-\nu[3]'[0]+\nu[4]'[0]\right)\right)}{2\,M},$$
$$-\frac{\nu[3]'[0]+e^{-3\,a4[0]}\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}-\nu[4]'[0]}{2\,M},0,0,0,0,0,0\Big\},$$
$$\{1,1,0,0,0,0,0,0\},\{0,0,1,1,0,0,0,0\},$$
$$\{0,0,0,0,1,1,0,0\},\{0,0,0,0,0,0,1,1\}\Big\},$$

$$\Big\{\Big\{\frac{1}{2}e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\left(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0]\right)\Big),0,0,0,0,0,0,0\Big\},$$
$$\Big\{0,\frac{1}{2}e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[3]'[0]-\nu[4]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\left(Q1\,a4'[0]+\nu[3]'[0]+\nu[4]'[0]\right)\Big),0,0,0,0,0,0\Big\},$$
$$\Big\{0,0,\frac{1}{2}e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\left(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0]\right)\Big),0,0,0,0,0\Big\},$$
$$\Big\{0,0,0,\frac{1}{2}e^{-3\,a4[0]}\Big(-e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[2]'[0]-\nu[5]'[0])^2\right)}+$$
$$e^{3\,a4[0]}\left(Q1\,a4'[0]+\nu[2]'[0]+\nu[5]'[0]\right)\Big),0,0,0,0\Big\},$$
$$\Big\{0,0,0,0,\frac{1}{2}e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+$$

$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\Big),\ 0,\ 0,\ 0\Big\},$$

$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2}\,e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\,\left(-4\,M^2+(\nu[1]'[0]-\nu[6]'[0])^2\right)}+$$

$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[1]'[0]+\nu[6]'[0])\Big),\ 0,\ 0\Big\},$$

$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2}\,e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]-\sqrt{e^{6\,a4[0]}\,\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+$$

$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\Big),\ 0\Big\},$$

$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2}\,e^{-3\,a4[0]}\Big(e^{a4[0]}\,Q1\,a4'[0]+\sqrt{e^{6\,a4[0]}\,\left(-4\,M^2+(\nu[0]'[0]-\nu[7]'[0])^2\right)}+$$

$$e^{3\,a4[0]}\,(-Q1\,a4'[0]+\nu[0]'[0]+\nu[7]'[0])\Big)\Big\}\Big\}\Big\}$$

In[ ]:=
```
eigenValues = Union[Flatten[{valscaDtyZrelationsA21, valscaDtyZrelationsA22}]]
eigenValues00 = FullSimplify[Normal[Series[#, {t, 0, 0}]]] & /@ eigenValues
```

Out[ ]=

$$\left\{\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[3]'[0] - \nu[4]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[3]'[0] + \nu[4]'[0])\right),\right.$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[3]'[0] - \nu[4]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[3]'[0] + \nu[4]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[2]'[0] - \nu[5]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[2]'[0] + \nu[5]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[2]'[0] - \nu[5]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[2]'[0] + \nu[5]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[1]'[0] - \nu[6]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[1]'[0] + \nu[6]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[1]'[0] - \nu[6]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[1]'[0] + \nu[6]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[0]'[0] - \nu[7]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[0]'[0] + \nu[7]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[0]'[0] - \nu[7]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[0]'[0] + \nu[7]'[0])\right),$$

$$\frac{1}{2}\left(\nu[11]'[0] + \nu[12]'[0] - \sqrt{-4\,M^2 + \nu[11]'[0]^2 - 2\,\nu[11]'[0]\,\nu[12]'[0] + \nu[12]'[0]^2}\right),$$

$$\frac{1}{2}\left(\nu[11]'[0] + \nu[12]'[0] + \sqrt{-4\,M^2 + \nu[11]'[0]^2 - 2\,\nu[11]'[0]\,\nu[12]'[0] + \nu[12]'[0]^2}\right),$$

$$\frac{1}{2}\left(\nu[10]'[0] + \nu[13]'[0] - \sqrt{-4\,M^2 + \nu[10]'[0]^2 - 2\,\nu[10]'[0]\,\nu[13]'[0] + \nu[13]'[0]^2}\right),$$

$$\frac{1}{2}\left(\nu[10]'[0] + \nu[13]'[0] + \sqrt{-4\,M^2 + \nu[10]'[0]^2 - 2\,\nu[10]'[0]\,\nu[13]'[0] + \nu[13]'[0]^2}\right),$$

$$\frac{1}{2}\left(\nu[9]'[0] + \nu[14]'[0] - \sqrt{-4\,M^2 + \nu[9]'[0]^2 - 2\,\nu[9]'[0]\,\nu[14]'[0] + \nu[14]'[0]^2}\right),$$

$$\frac{1}{2}\left(\nu[9]'[0] + \nu[14]'[0] + \sqrt{-4\,M^2 + \nu[9]'[0]^2 - 2\,\nu[9]'[0]\,\nu[14]'[0] + \nu[14]'[0]^2}\right),$$

$$\frac{1}{2}\left(\nu[8]'[0] + \nu[15]'[0] - \sqrt{-4\,M^2 + \nu[8]'[0]^2 - 2\,\nu[8]'[0]\,\nu[15]'[0] + \nu[15]'[0]^2}\right),$$

$$\left.\frac{1}{2}\left(\nu[8]'[0] + \nu[15]'[0] + \sqrt{-4\,M^2 + \nu[8]'[0]^2 - 2\,\nu[8]'[0]\,\nu[15]'[0] + \nu[15]'[0]^2}\right)\right\}$$

Out[ ]=

$$\left\{\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[3]'[0] - \nu[4]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[3]'[0] + \nu[4]'[0])\right),\right.$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[3]'[0] - \nu[4]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[3]'[0] + \nu[4]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[2]'[0] - \nu[5]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[2]'[0] + \nu[5]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(-e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[2]'[0] - \nu[5]'[0])^2\right)} + e^{3\,a4[0]}\,(Q1\,a4'[0] + \nu[2]'[0] + \nu[5]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[1]'[0] - \nu[6]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[1]'[0] + \nu[6]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[1]'[0] - \nu[6]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[1]'[0] + \nu[6]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] - \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[0]'[0] - \nu[7]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[0]'[0] + \nu[7]'[0])\right),$$

$$\frac{1}{2} e^{-3\,a4[0]} \left(e^{a4[0]}\,Q1\,a4'[0] + \sqrt{e^{6\,a4[0]}\left(-4\,M^2 + (\nu[0]'[0] - \nu[7]'[0])^2\right)} + e^{3\,a4[0]}\,(-Q1\,a4'[0] + \nu[0]'[0] + \nu[7]'[0])\right),$$

$$\frac{1}{2}\left(\nu[11]'[0] - \sqrt{-4\,M^2 + (\nu[11]'[0] - \nu[12]'[0])^2} + \nu[12]'[0]\right),$$

$$\frac{1}{2}\left(\nu[11]'[0] + \sqrt{-4\,M^2 + (\nu[11]'[0] - \nu[12]'[0])^2} + \nu[12]'[0]\right),$$

$$\frac{1}{2}\left(\nu[10]'[0] - \sqrt{-4\,M^2 + (\nu[10]'[0] - \nu[13]'[0])^2} + \nu[13]'[0]\right),$$

$$\frac{1}{2}\left(\nu[10]'[0] + \sqrt{-4\,M^2 + (\nu[10]'[0] - \nu[13]'[0])^2} + \nu[13]'[0]\right),$$

$$\frac{1}{2}\left(\nu[9]'[0] - \sqrt{-4\,M^2 + (\nu[9]'[0] - \nu[14]'[0])^2} + \nu[14]'[0]\right),$$

$$\frac{1}{2}\left(\nu[9]'[0] + \sqrt{-4\,M^2 + (\nu[9]'[0] - \nu[14]'[0])^2} + \nu[14]'[0]\right),$$

$$\frac{1}{2}\left(\nu[8]'[0] - \sqrt{-4\,M^2 + (\nu[8]'[0] - \nu[15]'[0])^2} + \nu[15]'[0]\right),$$

$$\left.\frac{1}{2}\left(\nu[8]'[0] + \sqrt{-4\,M^2 + (\nu[8]'[0] - \nu[15]'[0])^2} + \nu[15]'[0]\right)\right\}$$

If superluminal inflation/deflation then pair of universes

with MASSES ± M?

THE FOLLOWING IS CONSISTENT WITH UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES ± M;

Note that we are ONLY looking for superluminal inflation or deflation type solutions,
and NOT solutions that are even/odd functions of t
(like Cos[*v*[*j*]'[0] * t], Sin[*v*[*j*]'[0] * t], Sech[*v*[*j*]'[0] * t], Tanh[*v*[*j*]'[0] * t], ...),
which might also have ± M type eigenvalues,
or even solutions involving the well-known special functions that might also have ± M type eigenvalues.

In[ ]:=
```
sDtv3Dtv4 = Solve[And @@ Thread[0 == eigenValues00[[1 ;; 2]]], {v[3]'[0], v[4]'[0]}] //
  TrigToExp(*//FullSimplify*)
```

Out[ ]=

$$\left\{\left\{\nu[3]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[4]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\},\right.$$
$$\left.\left\{\nu[3]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[4]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\}\right\}$$

In[ ]:=
```
sDtv2Dtv5 = Solve[And @@ Thread[0 == eigenValues00[[3 ;; 4]]], {v[2]'[0], v[5]'[0]}] //
  TrigToExp(*//FullSimplify*)
```

Out[ ]=

$$\left\{\left\{\nu[2]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[5]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\},\right.$$
$$\left.\left\{\nu[2]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[5]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\}\right\}$$

In[ ]:=
```
sDtv1Dtv6 = Solve[And @@ Thread[0 == eigenValues00[[5 ;; 6]]], {v[1]'[0], v[6]'[0]}] //
  TrigToExp(*//FullSimplify*)
```

Out[ ]=

$$\left\{\left\{\nu[1]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[6]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\},\right.$$
$$\left.\left\{\nu[1]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[6]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\}\right\}$$

In[∘]:=
```
sDtv0Dtv7 = Solve[And @@ Thread[0 == eigenValues00[[7 ;; 8]]], {v[0]'[0], v[7]'[0]}] //
  TrigToExp(*//FullSimplify*)
```

Out[∘]=

$$\left\{\left\{v[0]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ v[7]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\},\ \left\{v[0]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ v[7]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\}\right\}$$

In[∘]:=
```
sDtv11Dtv12 =
 Solve[And @@ Thread[0 == eigenValues00[[9 ;; 10]]], {v[11]'[0], v[12]'[0]}] // FullSimplify
```

Out[∘]=

{{v[11]'[0] → M, v[12]'[0] → -M}, {v[11]'[0] → -M, v[12]'[0] → M}}

In[∘]:=
```
sDtv10Dtv13 =
 Solve[And @@ Thread[0 == eigenValues00[[11 ;; 12]]], {v[10]'[0], v[13]'[0]}] // FullSimplify
```

Out[∘]=

{{v[10]'[0] → M, v[13]'[0] → -M}, {v[10]'[0] → -M, v[13]'[0] → M}}

In[∘]:=
```
sDtv9Dtv14 =
 Solve[And @@ Thread[0 == eigenValues00[[13 ;; 14]]], {v[9]'[0], v[14]'[0]}] // FullSimplify
```

Out[∘]=

{{v[9]'[0] → M, v[14]'[0] → -M}, {v[9]'[0] → -M, v[14]'[0] → M}}

In[∘]:=
```
sDtv8Dtv15 =
 Solve[And @@ Thread[0 == eigenValues00[[15 ;; 16]]], {v[8]'[0], v[15]'[0]}] // FullSimplify
```

Out[∘]=

{{v[8]'[0] → M, v[15]'[0] → -M}, {v[8]'[0] → -M, v[15]'[0] → M}}

FYI: wave functions?

In[∘]:=
```
{# + 0, 7 - #} & /@ Range[0, 3]
ToExpression["sDtv" <> ToString[#[[1]]] <> "Dtv" <> ToString[#[[2]]]] & /@ %;
plusM03 = Flatten[#[[1]] & /@ %]
minusM03 = Flatten[#[[2]] & /@ %%]
```

Out[∘]=

$$\{\{0, 7\}, \{1, 6\}, \{2, 5\}, \{3, 4\}\}$$

Out[∘]=

$$\left\{\nu[0]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[7]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\right.$$
$$\nu[1]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[6]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[2]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[5]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\left.\nu[3]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[4]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\}$$

Out[∘]=

$$\left\{\nu[0]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[7]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\right.$$
$$\nu[1]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[6]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[2]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[5]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\left.\nu[3]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[4]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0]\right\}$$

In[∘]:=
```
{# + 8, 15 - #} & /@ Range[0, 3]
ToExpression["sDtv" <> ToString[#[[1]]] <> "Dtv" <> ToString[#[[2]]]] & /@ %
plusM811 = Flatten[#[[1]] & /@ %]
minusM811 = Flatten[#[[2]] & /@ %%]
```

Out[∘]=

$$\{\{8, 15\}, \{9, 14\}, \{10, 13\}, \{11, 12\}\}$$

Out[∘]=

$$\{\{\{\nu[8]'[0] \to M, \nu[15]'[0] \to -M\}, \{\nu[8]'[0] \to -M, \nu[15]'[0] \to M\}\},$$
$$\{\{\nu[9]'[0] \to M, \nu[14]'[0] \to -M\}, \{\nu[9]'[0] \to -M, \nu[14]'[0] \to M\}\},$$
$$\{\{\nu[10]'[0] \to M, \nu[13]'[0] \to -M\}, \{\nu[10]'[0] \to -M, \nu[13]'[0] \to M\}\},$$
$$\{\{\nu[11]'[0] \to M, \nu[12]'[0] \to -M\}, \{\nu[11]'[0] \to -M, \nu[12]'[0] \to M\}\}\}$$

Out[∘]=

$$\{\nu[8]'[0] \to M, \nu[15]'[0] \to -M, \nu[9]'[0] \to M, \nu[14]'[0] \to -M,$$
$$\nu[10]'[0] \to M, \nu[13]'[0] \to -M, \nu[11]'[0] \to M, \nu[12]'[0] \to -M\}$$

Out[∘]=

$$\{\nu[8]'[0] \to -M, \nu[15]'[0] \to M, \nu[9]'[0] \to -M, \nu[14]'[0] \to M,$$
$$\nu[10]'[0] \to -M, \nu[13]'[0] \to M, \nu[11]'[0] \to -M, \nu[12]'[0] \to M\}$$

In[ ]:=
```
plusM03plusM811 = Flatten[{plusM03, plusM811}]
```

Out[ ]=

$$\Big\{\nu[0]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[7]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[1]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[6]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[2]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[5]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[3]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[4]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[8]'[0] \to M,\ \nu[15]'[0] \to -M,$$
$$\nu[9]'[0] \to M,\ \nu[14]'[0] \to -M,\ \nu[10]'[0] \to M,\ \nu[13]'[0] \to -M,\ \nu[11]'[0] \to M,\ \nu[12]'[0] \to -M\Big\}$$

In[ ]:=
```
minusM03minusM811 = Flatten[{minusM03, minusM811}]
```

Out[ ]=

$$\Big\{\nu[0]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[7]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[1]'[0] \to -M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[6]'[0] \to M + \frac{1}{2} Q1\, a4'[0] - \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[2]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[5]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[3]'[0] \to -M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],$$
$$\nu[4]'[0] \to M - \frac{1}{2} Q1\, a4'[0] + \frac{1}{2} e^{-2\, a4[0]} Q1\, a4'[0],\ \nu[8]'[0] \to -M,\ \nu[15]'[0] \to M,$$
$$\nu[9]'[0] \to -M,\ \nu[14]'[0] \to M,\ \nu[10]'[0] \to -M,\ \nu[13]'[0] \to M,\ \nu[11]'[0] \to -M,\ \nu[12]'[0] \to M\Big\}$$

In[ ]:=
```
sfψ16t =
 Table[f16[j] → ToExpression["((Y[" <> ToString[j] <> "][6*H*#1,H*#2]" <> "Exp[" <> ToString[
      "(ν[" <> ToString[j] <> "])'[0]"] <> "*#2]" <> "*1/Sqrt[Sin[6*H*#1]])&)"], {j, 0, 15}]
```

Out[ ]=

$$\left\{f16[0] \to \left(\frac{Y[0][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[0]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\right.$$

$$f16[1] \to \left(\frac{Y[1][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[1]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[2] \to \left(\frac{Y[2][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[2]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[3] \to \left(\frac{Y[3][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[3]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[4] \to \left(\frac{Y[4][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[4]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[5] \to \left(\frac{Y[5][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[5]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[6] \to \left(\frac{Y[6][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[6]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[7] \to \left(\frac{Y[7][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[7]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[8] \to \left(\frac{Y[8][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[8]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[9] \to \left(\frac{Y[9][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[9]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[10] \to \left(\frac{Y[10][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[10]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[11] \to \left(\frac{Y[11][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[11]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[12] \to \left(\frac{Y[12][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[12]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[13] \to \left(\frac{Y[13][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[13]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[14] \to \left(\frac{Y[14][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[14]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$\left.f16[15] \to \left(\frac{Y[15][6\,H\,\#1,\ H\,\#2]\ \mathrm{Exp}[\nu[15]'[0]\ \#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right)\right\}$$

In[ ]:=
```
sfψ16Aa
```

Out[ ]=

$$\left\{f16[0] \to \left(\frac{Z[0][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\ f16[1] \to \left(\frac{Z[1][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\right.$$

$$f16[2] \to \left(\frac{Z[2][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\ f16[3] \to \left(\frac{Z[3][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[4] \to \left(\frac{Z[4][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\ f16[5] \to \left(\frac{Z[5][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[6] \to \left(\frac{Z[6][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\ f16[7] \to \left(\frac{Z[7][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[8] \to \left(\frac{Z[8][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\ f16[9] \to \left(\frac{Z[9][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[10] \to \left(\frac{Z[10][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\ f16[11] \to \left(\frac{Z[11][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$f16[12] \to \left(\frac{Z[12][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\ f16[13] \to \left(\frac{Z[13][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),$$

$$\left.f16[14] \to \left(\frac{Z[14][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right),\ f16[15] \to \left(\frac{Z[15][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}\ \&\right)\right\}$$

In[ ]:=
```
sfψ16t1t = Table[
  f16[j] → (((ToExpression["((Y[" <> ToString[j] <> "][1][6*H*#1,H*#2]" <> "Exp[" <> ToString[
          "(ν[" <> ToString[j] <> "])'[0] "] <> "*#2]" <> "*1/Sqrt[Sin[6*H*#1]]))"] /.
        plusM03plusM811) + (ToExpression["((Y[" <> ToString[j] <> "][2][6*H*#1,H*#2]" <>
          "Exp[" <> ToString["(ν[" <> ToString[j] <> "])'[0] "] <> "*#2]" <>
          "*1/Sqrt[Sin[6*H*#1]]))"] /. minusM03minusM811))), {j, 0, 15}]
sfψ16TRY = Table[Block[{s}, s = %[[j]];
   s[[1]] → ToExpression["((" <> ToString[FullForm[s[[2]]]] <> ")&)"]], {j, 1, Length[%]}]
```

Out[ ]=

$$\left\{f16[0] \to \frac{e^{\#2\left(M+\frac{1}{2}Q1\,a4'[0]-\frac{1}{2}e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[0][1][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}} + \right.$$

$$\frac{e^{\#2\left(-M+\frac{1}{2}Q1\,a4'[0]-\frac{1}{2}e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[0][2][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[1] \to \frac{e^{\#2\left(M+\frac{1}{2}Q1\,a4'[0]-\frac{1}{2}e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[1][1][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}} +$$

$$\frac{e^{\#2\left(-M+\frac{1}{2}Q1\,a4'[0]-\frac{1}{2}e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[1][2][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[2] \to \frac{e^{\#2\left(M-\frac{1}{2}Q1\,a4'[0]+\frac{1}{2}e^{-2\,a4[0]}\,Q1\,a4'[0]\right)}\,Y[2][1][6\,H\,\#1,\ H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}} +$$

$$\frac{e^{\#2\left(-M-\frac{1}{2}Q1\,a4'[0]+\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[2][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[3]\to\frac{e^{\#2\left(M-\frac{1}{2}Q1\,a4'[0]+\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[3][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+$$

$$\frac{e^{\#2\left(-M-\frac{1}{2}Q1\,a4'[0]+\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[3][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[4]\to\frac{e^{\#2\left(-M-\frac{1}{2}Q1\,a4'[0]+\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[4][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+$$

$$\frac{e^{\#2\left(M-\frac{1}{2}Q1\,a4'[0]+\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[4][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[5]\to\frac{e^{\#2\left(-M-\frac{1}{2}Q1\,a4'[0]+\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[5][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+$$

$$\frac{e^{\#2\left(M-\frac{1}{2}Q1\,a4'[0]+\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[5][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[6]\to\frac{e^{\#2\left(-M+\frac{1}{2}Q1\,a4'[0]-\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[6][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+$$

$$\frac{e^{\#2\left(M+\frac{1}{2}Q1\,a4'[0]-\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[6][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[7]\to\frac{e^{\#2\left(-M+\frac{1}{2}Q1\,a4'[0]-\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[7][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+$$

$$\frac{e^{\#2\left(M+\frac{1}{2}Q1\,a4'[0]-\frac{1}{2}e^{-2\,a4[0]}Q1\,a4'[0]\right)}\,Y[7][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[8]\to\frac{e^{M\,\#2}\,Y[8][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+\frac{e^{-M\,\#2}\,Y[8][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[9]\to\frac{e^{M\,\#2}\,Y[9][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+\frac{e^{-M\,\#2}\,Y[9][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[10]\to\frac{e^{M\,\#2}\,Y[10][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+\frac{e^{-M\,\#2}\,Y[10][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[11]\to\frac{e^{M\,\#2}\,Y[11][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+\frac{e^{-M\,\#2}\,Y[11][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

$$f16[12]\to\frac{e^{-M\,\#2}\,Y[12][1][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}}+\frac{e^{M\,\#2}\,Y[12][2][6\,H\,\#1,\,H\,\#2]}{\sqrt{\mathrm{Sin}[6\,H\,\#1]}},$$

```
f16[13] → (e^(-M #2) Y[13][1][6 H #1, H #2])/Sqrt[Sin[6 H #1]] + (e^(M #2) Y[13][2][6 H #1, H #2])/Sqrt[Sin[6 H #1]],
f16[14] → (e^(-M #2) Y[14][1][6 H #1, H #2])/Sqrt[Sin[6 H #1]] + (e^(M #2) Y[14][2][6 H #1, H #2])/Sqrt[Sin[6 H #1]],
f16[15] → (e^(-M #2) Y[15][1][6 H #1, H #2])/Sqrt[Sin[6 H #1]] + (e^(M #2) Y[15][2][6 H #1, H #2])/Sqrt[Sin[6 H #1]]}
```

Out[ ]=

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{f16[0] →
  (e^(#2 (M+Rational[1,2] Q1 a4′[0]+Rational[-1,2] e^(-2 a4[0]) Q1 a4′[0])) Sin[6 H #1]^Rational[-1,2] Y[0][1][6 H #1, H #2] +
    e^(#2 (-M+Rational[1,2] Q1 a4′[0]+Rational[-1,2] e^(-2 a4[0]) Q1 a4′[0]))
     Sin[6 H #1]^Rational[-1,2] Y[0][2][6 H #1, H #2] &), f16[1] →
  (e^(#2 (M+Rational[1,2] Q1 a4′[0]+Rational[-1,2] e^(-2 a4[0]) Q1 a4′[0])) Sin[6 H #1]^Rational[-1,2] Y[1][1][6 H #1, H #2] +
    e^(#2 (-M+Rational[1,2] Q1 a4′[0]+Rational[-1,2] e^(-2 a4[0]) Q1 a4′[0]))
     Sin[6 H #1]^Rational[-1,2] Y[1][2][6 H #1, H #2] &), f16[2] →
  (e^(#2 (M+Rational[-1,2] Q1 a4′[0]+Rational[1,2] e^(-2 a4[0]) Q1 a4′[0])) Sin[6 H #1]^Rational[-1,2] Y[2][1][6 H #1, H #2] +
    e^(#2 (-M+Rational[-1,2] Q1 a4′[0]+Rational[1,2] e^(-2 a4[0]) Q1 a4′[0]))
     Sin[6 H #1]^Rational[-1,2] Y[2][2][6 H #1, H #2] &), f16[3] →
  (e^(#2 (M+Rational[-1,2] Q1 a4′[0]+Rational[1,2] e^(-2 a4[0]) Q1 a4′[0])) Sin[6 H #1]^Rational[-1,2] Y[3][1][6 H #1, H #2] +
    e^(#2 (-M+Rational[-1,2] Q1 a4′[0]+Rational[1,2] e^(-2 a4[0]) Q1 a4′[0]))
     Sin[6 H #1]^Rational[-1,2] Y[3][2][6 H #1, H #2] &),
 f16[4] → (e^(#2 (-M+Rational[-1,2] Q1 a4′[0]+Rational[1,2] e^(-2 a4[0]) Q1 a4′[0])) Sin[6 H #1]^Rational[-1,2]
     Y[4][1][6 H #1, H #2] + e^(#2 (M+Rational[-1,2] Q1 a4′[0]+Rational[1,2] e^(-2 a4[0]) Q1 a4′[0]))
     Sin[6 H #1]^Rational[-1,2] Y[4][2][6 H #1, H #2] &),
 f16[5] → (e^(#2 (-M+Rational[-1,2] Q1 a4′[0]+Rational[1,2] e^(-2 a4[0]) Q1 a4′[0])) Sin[6 H #1]^Rational[-1,2]
     Y[5][1][6 H #1, H #2] + e^(#2 (M+Rational[-1,2] Q1 a4′[0]+Rational[1,2] e^(-2 a4[0]) Q1 a4′[0]))
     Sin[6 H #1]^Rational[-1,2] Y[5][2][6 H #1, H #2] &),
 f16[6] → (e^(#2 (-M+Rational[1,2] Q1 a4′[0]+Rational[-1,2] e^(-2 a4[0]) Q1 a4′[0])) Sin[6 H #1]^Rational[-1,2]
     Y[6][1][6 H #1, H #2] + e^(#2 (M+Rational[1,2] Q1 a4′[0]+Rational[-1,2] e^(-2 a4[0]) Q1 a4′[0]))
     Sin[6 H #1]^Rational[-1,2] Y[6][2][6 H #1, H #2] &),
 f16[7] → (e^(#2 (-M+Rational[1,2] Q1 a4′[0]+Rational[-1,2] e^(-2 a4[0]) Q1 a4′[0])) Sin[6 H #1]^Rational[-1,2]
     Y[7][1][6 H #1, H #2] + e^(#2 (M+Rational[1,2] Q1 a4′[0]+Rational[-1,2] e^(-2 a4[0]) Q1 a4′[0]))
     Sin[6 H #1]^Rational[-1,2] Y[7][2][6 H #1, H #2] &),
 f16[8] → (e^(M #2) Sin[6 H #1]^Rational[-1,2] Y[8][1][6 H #1, H #2] +
    e^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[8][2][6 H #1, H #2] &),
 f16[9] → (e^(M #2) Sin[6 H #1]^Rational[-1,2] Y[9][1][6 H #1, H #2] +
    e^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[9][2][6 H #1, H #2] &),
 f16[10] → (e^(M #2) Sin[6 H #1]^Rational[-1,2] Y[10][1][6 H #1, H #2] +
    e^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[10][2][6 H #1, H #2] &),
 f16[11] → (e^(M #2) Sin[6 H #1]^Rational[-1,2] Y[11][1][6 H #1, H #2] +
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    e^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[11][2][6 H #1, H #2] &),
f16[12] → (e^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[12][1][6 H #1, H #2] +
    e^(M #2) Sin[6 H #1]^Rational[-1,2] Y[12][2][6 H #1, H #2] &),
f16[13] → (e^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[13][1][6 H #1, H #2] +
    e^(M #2) Sin[6 H #1]^Rational[-1,2] Y[13][2][6 H #1, H #2] &),
f16[14] → (e^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[14][1][6 H #1, H #2] +
    e^(M #2) Sin[6 H #1]^Rational[-1,2] Y[14][2][6 H #1, H #2] &),
f16[15] → (e^(-M #2) Sin[6 H #1]^Rational[-1,2] Y[15][1][6 H #1, H #2] +
    e^(M #2) Sin[6 H #1]^Rational[-1,2] Y[15][2][6 H #1, H #2] &)}
```