Scratch work of Author

NOTES

HYPOTHESIS : If ,
employing the Einstein eqs (or Einstein - Lovelock eqs) ,
superluminal inflation / deflation exists,
then at time x4 =
0 (**before** the particles of the standard model exist)
a pair of universes with MASSES ± M is created
(i.e., universes are created in pairs)
Dark matter / dark energy may be
related to wave function overlap of these two universes

THE FOLLOWING SCRATCH IS CONSISTENT WITH
UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES ± M.

Note that we are ONLY looking for
superluminal inflation or deflation type solutions,
and NOT solutions that are even / odd functions of t
(like Cos[ν[j]'[0] * t], Sin[ν[j]'[0] * t],
Sech[ν[j]'[0] * t], Tanh[ν[j]'[0] * t], ...),
which also have ± M type eigenvalues,
or even solutions involving the
well-known special functions that might also have ± M type parameters.

Bigger Bang:
Question: **Are Universe (s) of masses ± M created in pairs at time x4 = 0 (before the particles of the standard model exist) ?**

Scratch work

Patrick L. Nash, Ph.D.

Professor, UTSA Physics and Astronomy, Retired (UTSA)
Patrick299Nash at gmail ...
Do not read if you are irascible; I apologize for typos and duplications and ....
git clone https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git
“git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git”

Ever wonder why there are 3 space dimensions and 1 time dimension?
Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in$ Nature?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\|=\|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over $\mathbb{R}$) real numbers $\mathbb{R}$,the 2-dim complex numbers,the 4-dim quaternions, and the 8-dim octonions.

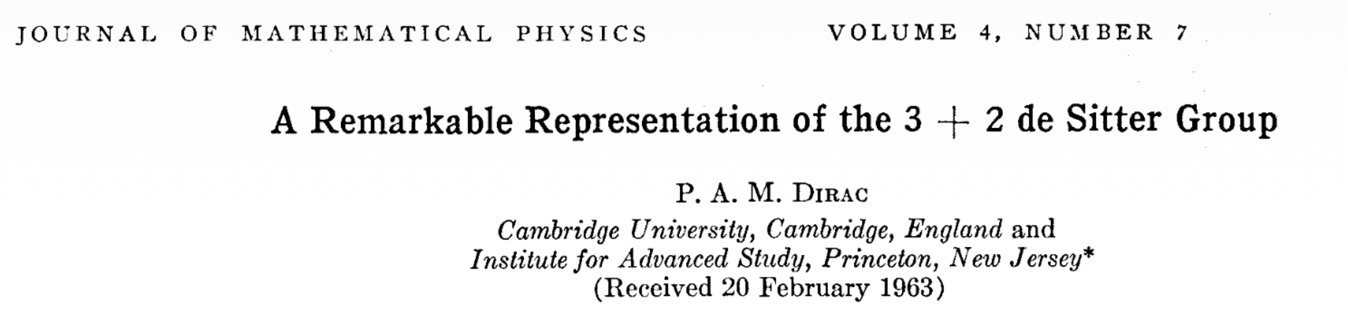
Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in "A Remarkable Representation of the 3+2 de Sitter Group" by P.A.M.Dirac, J.Math.Phys.4,901–909 (1963).

Comment: The term "reduced Brauer–Weyl generators", below, refers to a specific set of irreducible generators for certain complex Clifford algebras, particularly in the context of mathematical physics and the study of Dirac spinors. These generators were introduced by **E.A. Lord** (E. A. Lord. "The Dirac spinor in six dimensions". Mathematical Proceedings of the Cambridge Philosophical Society, vol. 64, pp. 765–778, 1968) and are described as "reduced" because they form an irreducible representation of the Clifford algebra C_{2n} from $C_{2(n-1)}$.

Here, we employ Lord's idea of reduced Brauer–Weyl generators to extend Dirac's 4×4 gamma matrices into this notebook's real tau8 8×8 and tau16 16×16 matrices,
which are employed to write down the 'Dirac Equation for the Universe',
[remember, this is before the particles of the standard model exist]
formulated in terms of a $O(4, 4)$ spinor $\Psi16$
($\Psi16$ AKA 'WAVE FUNCTION OF the UNIVERSE,' which could possibly be named something more pompous).
Then this equation is used to
[WARNING: syncope, presyncope AHEAD]
couple the WAVE FUNCTION OF the UNIVERSE to Gravity.

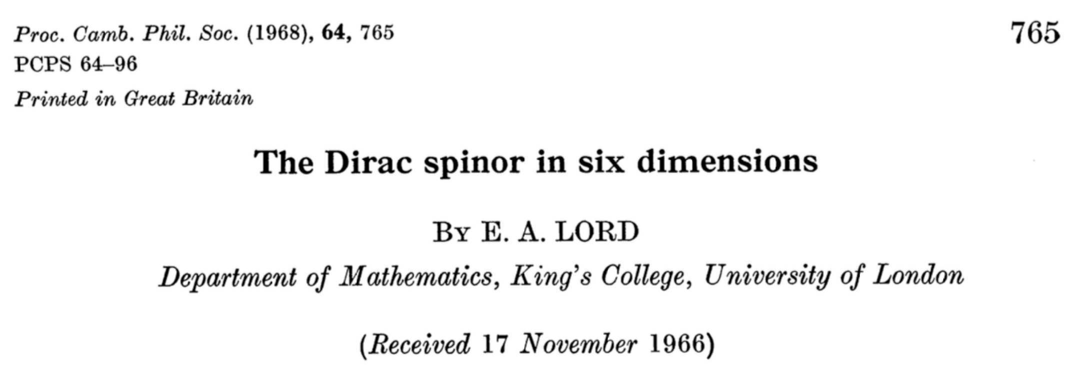
THIS WORK IS PARTIALLY BASED ON :



JOURNAL OF MATHEMATICAL PHYSICS VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. Dirac
Cambridge University, Cambridge, England and
*Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)



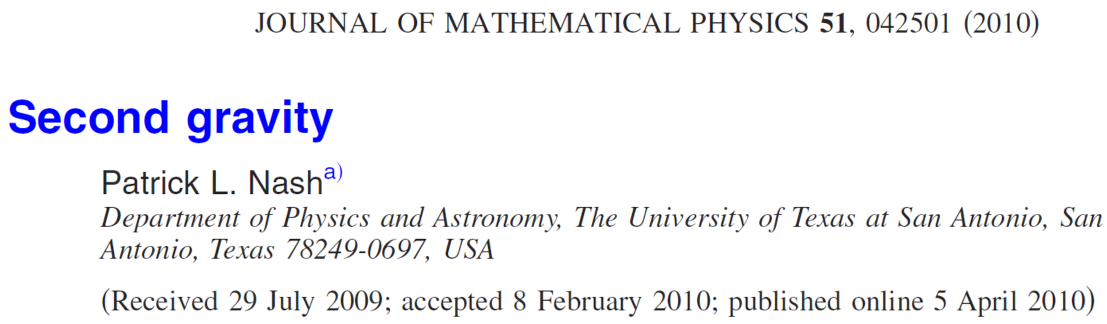
Proc. Camb. Phil. Soc. (1968), **64**, 765
PCPS 64–96
Printed in Great Britain

765

The Dirac spinor in six dimensions

By E. A. LORD
Department of Mathematics, King's College, University of London

(*Received* 17 *November* 1966)



JOURNAL OF MATHEMATICAL PHYSICS **51**, 042501 (2010)

Second gravity

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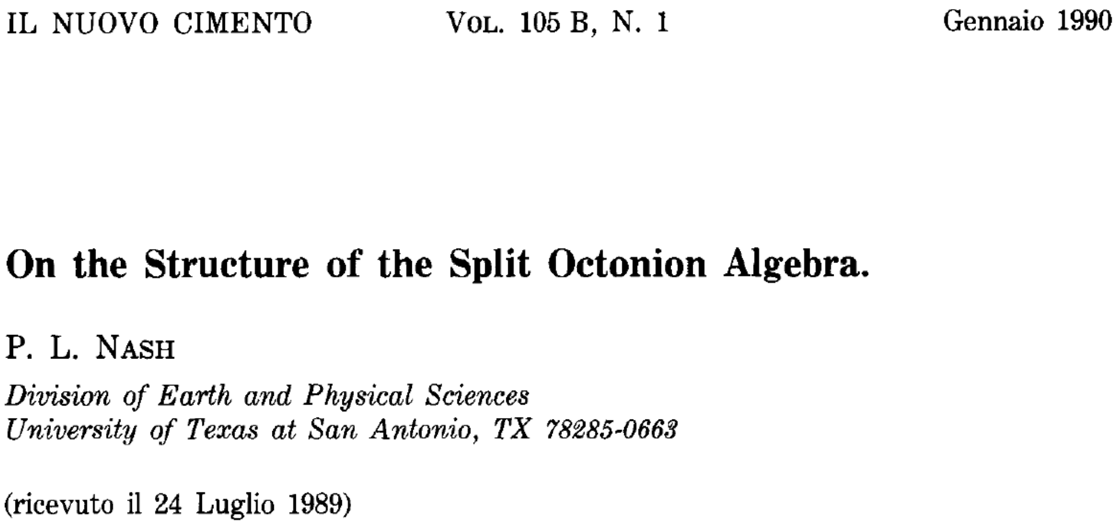
(Received 29 July 2009; accepted 8 February 2010; published online 5 April 2010)

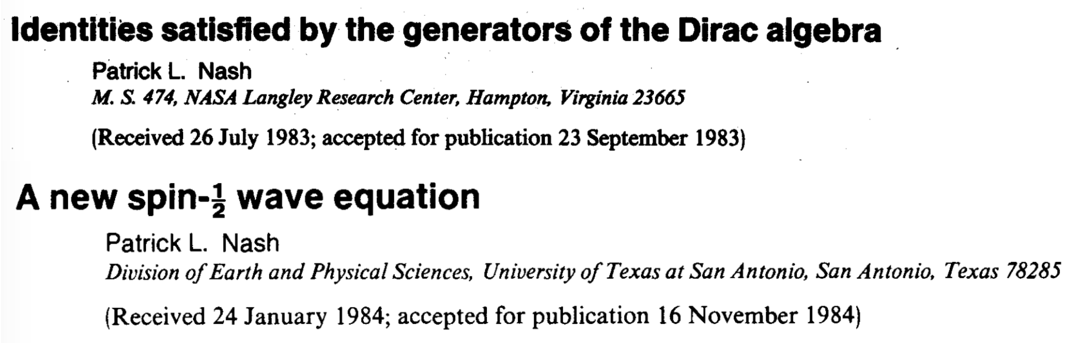


IL NUOVO CIMENTO VOL. 105 B, N. 1 Gennaio 1990

On the Structure of the Split Octonion Algebra.

P. L. NASH
Division of Earth and Physical Sciences
University of Texas at San Antonio, TX 78285-0663

(ricevuto il 24 Luglio 1989)



Identities satisfied by the generators of the Dirac algebra

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(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin-$\frac{1}{2}$ wave equation

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(Received 24 January 1984; accepted for publication 16 November 1984)

Here we use spacetime coordinates that are selected so that we are somewhat consistent with the xact libraries,
(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):
Cartesian coordinates:
x0 = hidden space (a small circle); x1, x2, x3 are the usual 3-space coords;
x4 = time coord,
and x5, x6, x7= superluminal deflating time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{\mathrm{SO}(4, 4; \mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

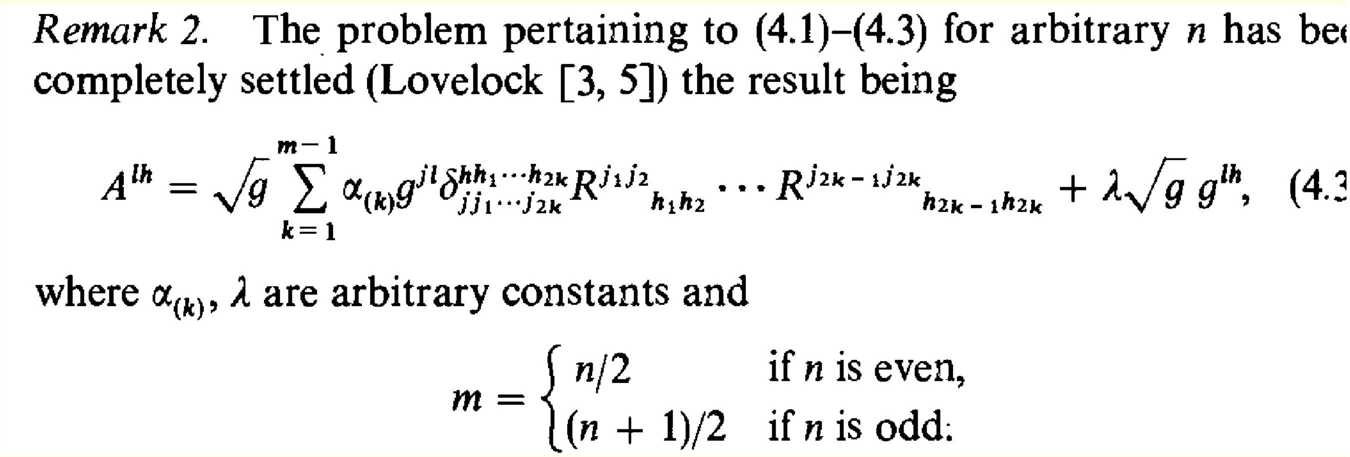
Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M8 , whose tangent bundle has
$\overline{\mathrm{SO}(4,4;\mathbb{R})} \approx \mathrm{Spin}(4,4;\mathbb{R})$
as iso group, and in which
3 of the 4 space dimensions superluminally INFLATE,
3 of the 4 time dimensions superluminally DEFLATE (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),
and the 4th space dim, x0, curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

In passing we remark that an analogous construction may be defined on an octonion space of one time dimension and seven space dimensions, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group
$\mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes U(1)_Y$;
The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

Please cite this work and this web page if you use it.

The Einstein - Lovelock vacuum field equations are
A_a^b = 0, where



Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has bee
completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta^{hh_1 \cdots h_{2k}}_{jj_1 \cdots j_{2k}} R^{j_1 j_2}{}_{h_1 h_2} \cdots R^{j_{2k-1} j_{2k}}{}_{h_{2k-1} h_{2k}} + \lambda \sqrt{g}\, g^{lh}, \quad (4.3$$

where $\alpha_{(k)}$, λ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even}, \\ (n+1)/2 & \text{if } n \text{ is odd}. \end{cases}$$

and $m - 1 = \frac{8}{2} - 1 = 3$.

Citation : Tensors, Differential Forms,
and Variational Principles (Dover Books on Mathematics),
by David Lovelock and Hanno Rund

Let {w1, w2, w3, Λ} be pure numbers;
then by Eq. [4.38] the vacuum equations are
$0 == -\Lambda + H^{-2}$ w1 Lovelock1 $+ H^{-4}$ w2 Lovelock2 $+ H^{-6}$ w3 Lovelock3
in a hopefully obvious notation.
Here H is a fundamental inverse length that
exists in virtue of the fact that the Einstein -
Lovelock tensors have different dimensions.
For simplicity, we do not attempt to associate
H with the familiar inverse Planck length,
or the Lemaitre - Hubble parameter. For now,
let's pretend that they are independent.

NOTES :

Under the spacetime coordinate transformation

$x^k \mapsto \overline{x}^k\left(x^j\right)$

according to

$\Psi 16\,(x) \mapsto \overline{\Psi 16\,(\overline{x})} = S\,(x\,(\overline{x}))\,.\Psi 16\,(x\,(\overline{x}))$, i.e., $\overline{\Psi16^{(a)}(\overline{x})} = \left(S^{(a)}_{(b)}\ \Psi16^{(b)}(x(\overline{x}))\right)$

Therefore,

$$\frac{\partial}{\partial \overline{x}^k}\overline{\Psi16(\overline{x})} = \frac{\partial x^j}{\partial \overline{x}^k}\ \frac{\partial}{\partial x^j}\ S(x).\Psi16(x) =$$

$$\frac{\partial x^j}{\partial \overline{x}^k}\left\{ S(x) \bullet \frac{\partial}{\partial x^j}\Psi16(x) + \left[\frac{\partial}{\partial x^j} S(x)\right]\bullet\Psi16(x)\right\}$$

Identify the Octad connection : $\frac{\partial}{\partial x^j} S(x) =$

$$- S \bullet \Gamma_j(x) + \frac{\partial \overline{x}^k}{\partial x^j}\overline{\Gamma_k}(\overline{x}) . S$$

hence

$$\frac{\partial}{\partial \overline{x}^k}\overline{\Psi16(\overline{x})} - \overline{\Gamma_k} \bullet \overline{\Psi16(\overline{x})} =$$

$$S(x(\overline{x})) . \frac{\partial x^j}{\partial \overline{x}^k}\left\{\frac{\partial}{\partial x^j}\Psi16\left(x(\overline{x})\right) - \Gamma_j .\Psi16\left(x(\overline{x})\right)\right\}$$

or

$$\left(1_{16\times16}\frac{\partial}{\partial \overline{x}^k} - \overline{\Gamma_k}\right)\bullet \overline{\Psi16(\overline{x})} =$$

$$S(x(\overline{x})) . \left[\frac{\partial x^j}{\partial \overline{x}^k}\left\{1_{16\times16}\frac{\partial}{\partial x^j} - \Gamma_j\right\}.\Psi16\left(x(\overline{x})\right)\right]$$

The connection transforms according to

$$\overline{\Gamma_k}(\overline{x}) = \frac{\partial \overline{x}^j}{\partial x^k}\left\{ S \bullet \Gamma_j(x) . S^{-1} + S_{,j} . S^{-1}\right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2}\left[e_k^{(a)}\ \nabla_j\ e^k_{(b)}\right]\ SAB^{(b)}{}_{(a)}$$

Here, spin connection coefficients are $e_k^{(a)}\ (\nabla_j\ e^k_{(b)}\Big|$

and

$$\mathbf{S = exp}\left(\frac{1}{2}\omega_{(A)\ (B)}\ (x)\ SAB^{(A)\ (B)}\right)\ ,$$

where $\omega_{(B)\ (A)} = -\omega_{(A)\ (B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the Ψ16 field [A.K.A., wave function characterizing the Universe(s) of masses ±M];

TODO: prove **Universe(s) of masses ±M are created in pairs!**
K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility**: The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
    - $\nabla_\mu e^a{}_\nu = 0$
    - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

apologies, I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^{\omega}_{[\mu}e^I_{\nu]} = \partial_{[\mu}e^I_{\nu]} + \omega^I_{[\mu J}e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu}e^I_\rho + \omega^I_{\mu J}\, e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistancy" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^{\omega}_\mu \eta_{IJ} = 0$ (neverthless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will neverthless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called ‘Tetrad Postulate’ and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

? SOURCE terms $TU^{\mu\nu}$ for $g_{\alpha\beta}$ come from "Universes' Wave Function Ψ16 Lagrangian"

$$\frac{1}{\kappa}\,\mathrm{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial\, g_{\mu\nu}}\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\ \mathrm{Lagrangian}\Psi 16\right)$$

(hope that $\mathrm{TU}^{\mu\nu} = \Lambda\, g^{\mu\nu}$, and

H = some function of M, where

Universe (s) of masses ± M created in pairs at time x4 = 0,

before the particles of the standard model exist) ;

WARNING:

Universes Ψ16 source $g_{\alpha\beta}$;

The Euler–Lagrange equations for Ψ16 must have ‘solutions’ such that

all off–diagonal terms of $\mathrm{TU}^{\mu\nu}$ ARE ZERO .

The Ψ16 Lagrangian (please see below) =

$$\sqrt{\mathrm{Det}[g_{\mu\nu}]}\ *\ \mathrm{Lg}[\,] =$$

$$\sqrt{\mathrm{Det}[g_{\mu\nu}]}\ *\left(\mathrm{Transpose}[\Psi 16].\sigma 16.\mathrm{Sum}\left[\mathrm{T16}^{\alpha}[\alpha 1-1].\left\{1_{16\times 16}\,\frac{\partial}{\partial x^{\alpha 1-1}} - \Gamma_{\alpha 1-1}\right\}.\Psi 16,\ \{\alpha 1, 1, \mathrm{Length}[X]\}\right] + \frac{\mathrm{mASs}}{2} * \mathrm{Transpose}[\Psi 16].\sigma 16.\Psi 16\right)$$

Next,

assume that Ψ16soln solves the Euler – Lagrange equations.

SOURCE terms

$\mathrm{TU}^{\mu\nu}$ [evaluate terms after performing differentiation] (using LagrangianΨ16 ≡

$$\left(\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\ \mathrm{Lg}[\,]\right)\Big|_{\Psi 16=\text{solution-toEL-eqs}}\right):$$

$$\frac{1}{\kappa}\,\mathrm{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial\, g_{\mu\nu}}\left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]}\ \mathrm{Lg}[\,]\right)$$

$$= \left(\mathrm{Lg}[\,] * \left[\frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]} \right) \right] + \frac{\partial}{\partial g_{\mu\nu}} (\mathrm{Lg}[\,]) \right) \Bigg|_{\Psi16=\Psi16soln}$$

$$= \left(0\,?\, * \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]} \right) + \frac{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\mathrm{Lg}[\,]) \right) \Bigg|_{\Psi16=\Psi16soln}$$

$$= \frac{\partial}{\partial g_{\mu\nu}} \Bigg(\mathrm{Transpose}[\Psi16].\sigma16. \mathrm{Sum}\Big[\Big(\mathrm{D}[\Psi16, \mathrm{X}[\![\alpha1]\!]] + \Big(\Big(\frac{1}{2}\Big) \mathrm{connectionMatrix} \Big). \mathrm{Sum}[\omega[\![\alpha1, a, b]\!] * \mathrm{SAB}[\![a, b]\!], \{a, 1, 8\}, \{b, 1, 8\}].\Psi16 \Big), \{\alpha1, 1, \mathrm{Length}[\mathrm{X}]\} \Big] + \mathrm{mASs} * \mathrm{Transpose}[\Psi16].\mathrm{symm16}[\![j, 1]\!].\Psi16 \Bigg) \Bigg|_{\Psi16=\Psi16soln}$$

$$= \mathrm{Transpose}[\Psi16].\sigma16.\mathrm{Sum}\Big[\frac{\partial}{\partial g_{\mu\nu}} (\mathrm{T16}^{\alpha}[\alpha1 - 1]) \;.\; \Psi16_{\alpha1-1}\,, \{\alpha1, 1, \mathrm{Length}[\mathrm{X}]\} \Big] \Big|_{\Psi16=solution-toEL-eqs}$$

$$= \tilde{\Psi}16\,.\sigma16.\,\mathrm{T16}^{A}\,.\,\Psi16_{,\alpha}\, \frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1\,\alpha\beta}\, e^{B}_{\beta}\, \eta_{BA} \right) = \tilde{\Psi}16\,.\sigma16.\,\mathrm{T16}^{A}\,.\,\Psi16_{,\alpha}\, \frac{\partial}{\partial g_{\mu\nu}} \left(e^{\alpha}_{(A)} \right)$$

NOTES :

Let g be a square matrix; we could use : $\frac{\partial}{\partial q} (g^{-1}) = - g^{-1} . \frac{\partial g}{\partial q} . g^{-1}$,

where q is a parameter (result from $\frac{\partial}{\partial q} (g . g^{-1}) = 0$),

In this notebook,
g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] ,
g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}} (g^{-1}) = - g^{-1} . \frac{\partial g}{\partial g_{\mu\nu}} . g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In metric matrix g, we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$, and then differentiate, for example.

This should be understood before reading further.

Therefore $\frac{\partial}{\partial g_{\mu\nu}}\left(e^{\alpha}_{(A)}\right) = \frac{\partial}{\partial g_{\mu\nu}}\left(\text{Inverse}\left[e^{(A)}_{\alpha}\right]\right) =$

$$- \text{Inverse}\left[e^{(A)}_{\alpha}\right].\frac{\partial e^{(A)}_{\alpha}}{\partial g_{\mu\nu}}.\text{Inverse}\left[e^{(A)}_{\alpha}\right] = - e^{\alpha}_{(A)}[\![\alpha 1, A1]\!] \frac{\partial e^{(A)}_{\alpha}[\![A1, \alpha 2]\!]}{\partial g_{\mu\nu}}.e^{\alpha}_{(A)}[\![\alpha 2, A2]\!]$$

Consider $\frac{\partial}{\partial g_{\mu\nu}}(g_{\alpha\beta}) = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e^{(A)}_{\alpha}\; e^{B}_{\beta}\right) = \eta_{AB}\; e^{B}_{\beta} \frac{\partial}{\partial g_{\mu\nu}}\left(e^{(A)}_{\alpha}\right) + \eta_{AB}\, e^{B}_{\alpha} \frac{\partial}{\partial g_{\mu\nu}}\left(e^{A}_{\beta}\right);$

$$e^{\beta}_{C} \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e^{(A)}_{\alpha}\; e^{B}_{\beta}\right) = \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}}\left(e^{(A)}_{\alpha}\right) + \eta_{AB}\, e^{B}_{\alpha}\, e^{\beta}_{C} \frac{\partial}{\partial g_{\mu\nu}}\left(e^{A}_{\beta}\right)$$

In metric matrix g, we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate.

Some of my calculations employ “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see
Cyril Pitrou
Institut d’Astrophysique de Paris (France)
pitrou@iap.fr
http://www2.iap.fr/users/pitrou/
“FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb”
MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify

---

I copied and pasted too often; however, I am going to leave this here.

Universe sources for $g_{\alpha\beta}$;
be sure to append these to the Einstein and/or Einstein–Lovelock field equations.

WARNING: all off–diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous ‘hacked solutions’ for Ψ16 to be valid

Let T^{α} = {$T16^{\alpha}$[0], $T16^{\alpha}$[4]} and

$\sqrt{\text{Det}[g_{\mu\nu}]}$ * Lg[][mASs_, j_] =
$\sqrt{\text{Det}[g_{\mu\nu}]}$ * (Transpose[Ψ16].σ16.Sum[
T^{α}[α1 – 1].D[Ψ16, vars⟦α1⟧], {α1, 1, Length[vars]}] +
mASs * Transpose[Ψ16].symm16⟦134, 1⟧.Ψ16)

Let j = 134, σ16.(mass Matrix) = $T16^{\alpha}$[5].$T16^{\alpha}$[6].$T16^{\alpha}$[7]
and
usingLagrangianF16massive = (Transpose[Ψ16].σ16.
Sum[T^{α}[α1 – 1].D[Ψ16, vars⟦α1⟧], {α1, 1, Length[vars]}] +
mASs * Transpose[Ψ16].symm16⟦j = 134, 1⟧.Ψ16);
where it is assumed that Ψ16 solves the Euler – Lagrange equations, above; then usingLagrangianF16massive = 0, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa}\,TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial\, g_{\mu\nu}}\left(\sqrt{\text{Det}[g_{\alpha\beta}]}\ \text{Lg}[]\right)$$

$$= \left(\text{Lg}[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial\, g_{\mu\nu}}\left(\sqrt{\text{Det}[g_{\alpha\beta}]}\right)\right] + \frac{\partial}{\partial\, g_{\mu\nu}}\left(\text{Lg}[]\right)\right)\Bigg|_{\text{Lg}[]=\text{usingLagrangianF16massive}}$$

$$= \left(0 * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}}\,\frac{\partial}{\partial\, g_{\mu\nu}}\left(\sqrt{\text{Det}[g_{\alpha\beta}]}\right) + \right.$$

$$\sqrt{\text{Det}[g_{\alpha\beta}]} \quad \partial \quad (\text{Lg}[])\Big|$$

remark: the term

$$\tilde{\Psi}16 .\sigma16. T16^{A} . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (e^{\alpha}_{A}) = \frac{\partial}{\partial g_{\mu\nu}} \tilde{\Psi}16 .\sigma16. T16^{A} . \Psi16_{,\alpha} e^{\alpha}_{A} =$$

$$\frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\Psi16].\sigma16. \right.$$

$$\text{Sum}\left[T^{\alpha}\overbrace{[\alpha1-1].\text{D}[\Psi16, \text{vars}[[\alpha1]]]}^{T16^{A}[A1-1].\Psi16_{,\alpha1}\,\eta_{A1\,B}\,e^{B}_{\beta}\,g^{-1\,\alpha1\,\beta}}, \{\alpha1, 1, \text{Length}[\text{vars}]\}\right]$$

Frame fields [edit]

We use a set of vierbein or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab} e^a_\mu e^b_\nu = \eta_{\mu\nu}.$$

The vierbein defines a local rest frame, allowing the constant Gamma matrices to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a section of the frame bundle, and so defines a local trivialization of the frame bundle.

Spin connection [edit]

To write down the equation we also need the spin connection, also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e^\mu_a e^a_\nu = \delta^\mu{}_\nu.$$

The connection 1-form is then

$$\omega^\mu{}_{\nu a} := e^\mu_b \nabla_a e^b_\nu$$

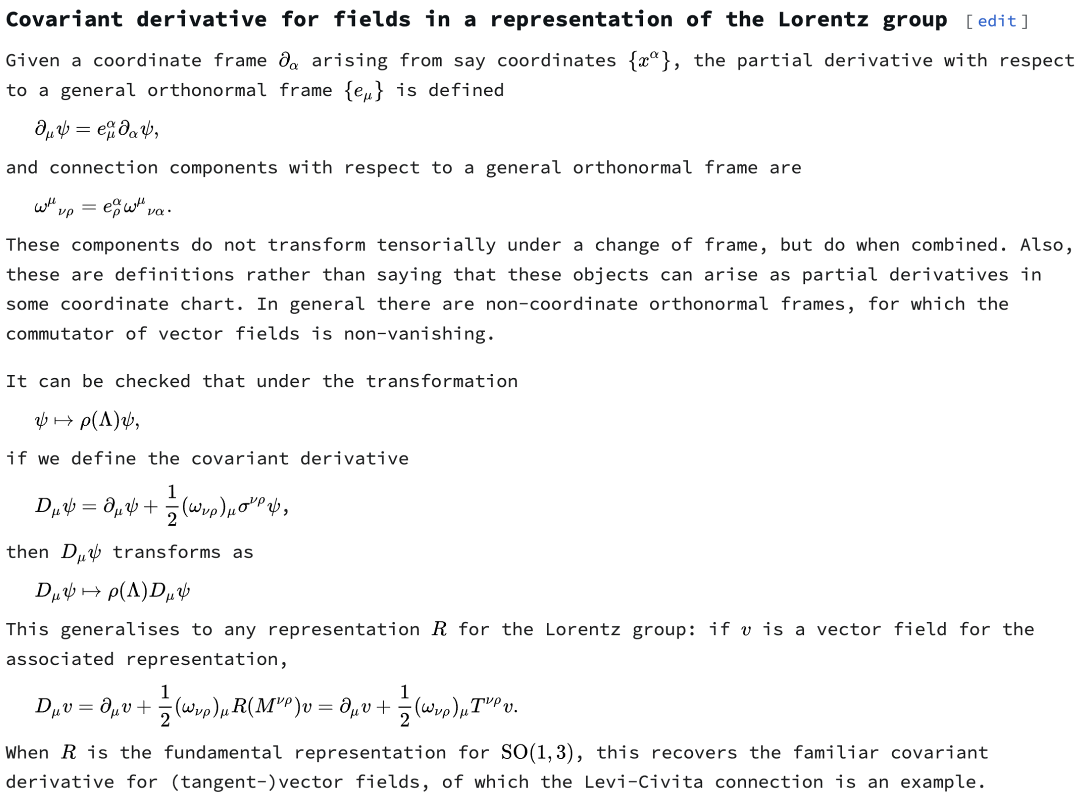
where ∇_a is a covariant derivative, or equivalently a choice of connection on the frame bundle, most often taken to be the Levi-Civita connection.

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space T_pM to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

Raising and lowering indices is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract connection on a principal bundle, specifically on the frame bundle, which is defined on any smooth manifold, but which restricts to an *orthonormal* frame bundle on pseudo-Riemannian manifolds.



Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu{}_{\nu\rho} = e_\rho^\alpha \omega^\mu{}_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2}(\omega_{\nu\rho})_\mu \sigma^{\nu\rho}\psi,$$

then $D_\mu \psi$ transforms as

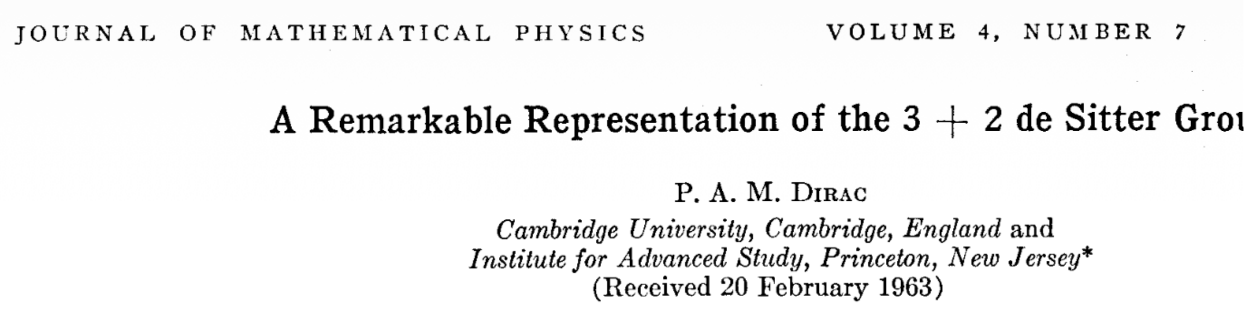
$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu R(M^{\nu\rho})v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $\mathrm{SO}(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

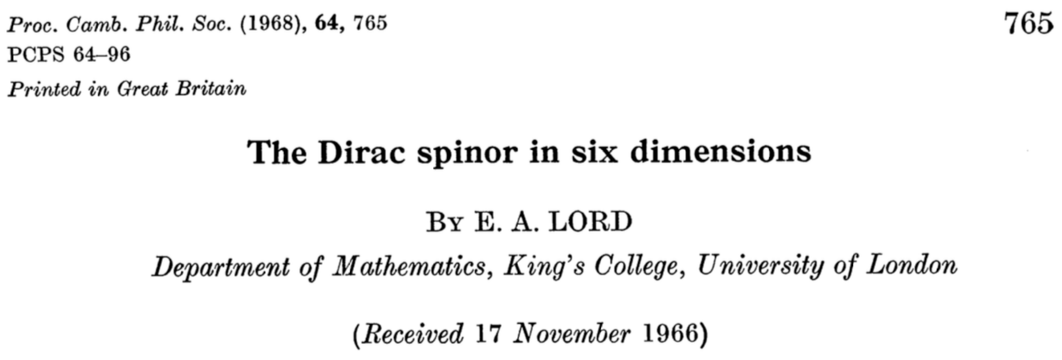
OTHER WORK THAT I USED :



JOURNAL OF MATHEMATICAL PHYSICS VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Gro

P. A. M. Dirac
Cambridge University, Cambridge, England and
*Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)



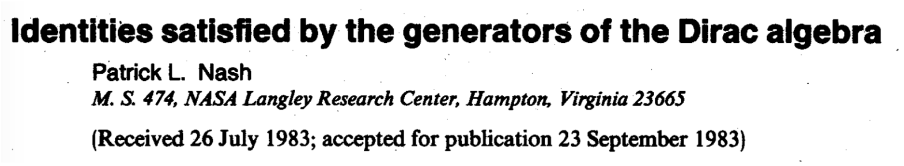
Proc. Camb. Phil. Soc. (1968), **64**, 765
PCPS 64–96
Printed in Great Britain

765

The Dirac spinor in six dimensions

By E. A. LORD
Department of Mathematics, King's College, University of London

(*Received* 17 *November* 1966)

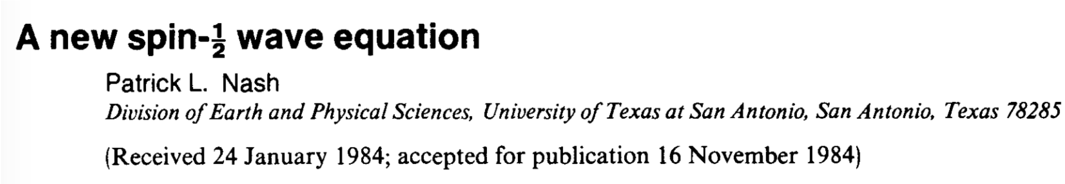


Identities satisfied by the generators of the Dirac algebra

Patrick L. Nash

M. S. 474, NASA Langley Research Center, Hampton, Virginia 23665

(Received 26 July 1983; accepted for publication 23 September 1983)



A new spin-$\frac{1}{2}$ wave equation

Patrick L. Nash

Division of Earth and Physical Sciences, University of Texas at San Antonio, San Antonio, Texas 78285

(Received 24 January 1984; accepted for publication 16 November 1984)

Begin

```
In[1]:= Print[
         "CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]

        CopyRight (C) 2022, Patrick L. Nash, under the General Public License.

In[2]:= Print["Please  cite  this  work,  and this web page, if you use it"]

        Please  cite  this  work,  and this web page, if you use it

In[3]:= Print["For simplicity, metric  gαβ=  gαβ(x0,x1,x2,x3,x4,x5,x6,x7)=
           gαβ(x0,x4) = diag{ g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1,
           g77(x0,x4),g77(x0,x4),g77(x0,x4) },"]

        For simplicity, metric  gαβ=  gαβ(x0,x1,x2,x3,x4,x5,x6,x7)= gαβ(x0,x4) = diag{
          g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4) },

In[4]:= Print["where g77(x0,x4) = g11(x0,-x4) "]

        where g77(x0,x4) = g11(x0,-x4)
```

ConvertMapleToMathematicaV2

```
In[5]:= FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]]
        mapleToMathematicaV2PATH =
         FileNameJoin[{%, "ConvertMapleToMathematicaV2.wl"}]

Out[5]= /Users/nsh/Public/Eternal-DEFLATION-Inflation

Out[6]= /Users/nsh/Public/Eternal-DEFLATION-Inflation/ConvertMapleToMathematicaV2.wl

In[7]:= Get[mapleToMathematicaV2PATH]    (*Loads new maple to mathematica parser*)

        ConvertMapleToMathematicaV2 loaded successfully!
          BUT, WARNING:  DO NOT USE IF YOU WANT A CORRECT RESULT!
```

Load the Maple output strings.

```
In[8]:= (*SetDirectory[NotebookDirectory[]];*)

In[9]:= (*maplestringEQ1=Get["string-seq1.txt"]
         maplestringEQ2=Get["thinkpad_stringEQ2.txt"]*)
```

```
In[10]:= NotebookFileName[]
Out[10]=
         /Users/nsh/Public/Eternal-DEFLATION-Inflation/Pair-Crtn-Univ-same_E-L-eqs-alt-approach.nb

In[11]:= Unprotect[dir];

In[12]:= dir = FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]];
         Protect[dir];
         If[$Failed ⩵ SetDirectory[dir], {CreateDirectory[dir], SetDirectory[dir]}];

In[15]:= (*FileNameJoin[Drop[FileNameSplit[NotebookFileName[]],-1]]*)

In[16]:= names = FileNameSplit[NotebookFileName[]]
Out[16]=
         {, Users, nsh, Public, Eternal-DEFLATION-Inflation,
          Pair-Crtn-Univ-same_E-L-eqs-alt-approach.nb}

In[17]:= name = StringReplace[names⟦-1⟧, "nb" → "mx"]
Out[17]=
         Pair-Crtn-Univ-same_E-L-eqs-alt-approach.mx

In[18]:= header = StringReplace[names⟦-1⟧, ".nb" → "-"]
Out[18]=
         Pair-Crtn-Univ-same_E-L-eqs-alt-approach-

In[19]:= SetOptions[Simplify, TimeConstraint → 1]
Out[19]=
         {Assumptions ⧴ $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {},
          TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[20]:= SetOptions[FullSimplify, TimeConstraint → 3]
Out[20]=
         {Assumptions ⧴ $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {},
          TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}

In[21]:= Options[Simplify]
Out[21]=
         {Assumptions ⧴ $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {},
          TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[22]:= Options[FullSimplify]
Out[22]=
         {Assumptions ⧴ $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {},
          TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}

In[23]:= {Off[General::spell], Off[General::spell1]};

In[24]:= DIM8 = 8;

In[25]:= Protect[DIM8, M, K, H]
Out[25]=
         {DIM8, M, K, H}
```

Coordinates :

$$\left\{\underbrace{\text{hidden space}}_{},\ \underbrace{3-\text{space}}_{},\quad \underbrace{\text{time}}_{}\ \ \underbrace{\text{superluminal deflating time}}_{}\right\}$$

$$\{x0,\ \overbrace{x1,\ x2,\ x3},\ x4,\ \ \overbrace{x5,\ x6,\ x7}\}\ :$$

```
In[26]:= X = {x0, x1, x2, x3, x4, x5, x6, x7};
         Protect[X];
         Protect[x0, x1, x2, x3, x4, x5, x6, x7];
```

```
In[29]:= sX0 = And @@ Thread[X > 0]
Out[29]=
x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 && x7 > 0
```

```
In[30]:= ssX = H > 0 && sX0 && 6 H x0 > 0 && 2 la[x4] > 0 && Cot[6 H x0] > 0 && Sin[6 H x0] > 0
Out[30]=
H > 0 && x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 &&
 x7 > 0 && 6 H x0[] > 0 && 2 la[x4[]] > 0 && Cot[6 H x0[]] > 0 && Sin[6 H x0[]] > 0
```

For Maple_2025 :

```
In[31]:= sreplaceZ = Z[#] → ToExpression["Z" <> ToString[#]] & /@ Range[0, 15]
Out[31]=
{Z[0] → Z0, Z[1] → Z1, Z[2] → Z2, Z[3] → Z3, Z[4] → Z4, Z[5] → Z5, Z[6] → Z6, Z[7] → Z7, Z[8] → Z8,
 Z[9] → Z9, Z[10] → Z10, Z[11] → Z11, Z[12] → Z12, Z[13] → Z13, Z[14] → Z14, Z[15] → Z15}
```

```
In[32]:= sreplacenZ = nZ[#] → ToExpression["nZ" <> ToString[#]] & /@ Range[0, 15]
Out[32]=
{nZ[0] → nZ0, nZ[1] → nZ1, nZ[2] → nZ2, nZ[3] → nZ3, nZ[4] → nZ4,
 nZ[5] → nZ5, nZ[6] → nZ6, nZ[7] → nZ7, nZ[8] → nZ8, nZ[9] → nZ9, nZ[10] → nZ10,
 nZ[11] → nZ11, nZ[12] → nZ12, nZ[13] → nZ13, nZ[14] → nZ14, nZ[15] → nZ15}
```

hacks (a.k.a., lies):

In[33]:=

$$\begin{aligned}&\text{constraintTrig} = \text{Sin}[6*H*x0] > 0\ \&\&\ \text{Cos}[6*H*x0] > 0\ \&\&\ \text{Csc}[6*H*x0] > 0\ \&\&\\ &\quad \text{Sec}[6*H*x0] > 0\ \&\&\ \text{Tan}[6*H*x0] > 0\ \&\&\ \text{Cot}[6*H*x0] > 0\ \&\&\ \text{Sin}[z] > 0\ \&\&\\ &\quad \sqrt{\text{Sin}[z]} > 0\ \&\&\ \text{Sin}[z]^{3/2} > 0\ \&\&\ \text{Sin}[z]^{1/2} > 0\ \&\&\ \text{Sin}[z]^{-3/2} > 0\ \&\&\\ &\quad \text{Sin}[z]^{-1/2} > 0\ \&\&\ \text{Cot}[z] > 0\ \&\&\ \sqrt{\text{Cot}[z]} > 0\ \&\&\ \text{Cot}[z]^{3/2} > 0\ \&\&\\ &\quad \text{Cot}[z]^{1/2} > 0\ \&\&\ \text{Tan}[z] > 0\ \&\&\ \text{Sec}[z] > 0\ \&\&\ \sqrt{\text{Csc}[z]} > 0\ \&\&\ \text{Csc}[z] > 0\end{aligned}$$

Out[33]=

$$\begin{aligned}&\text{Sin}[6\,H\,x0] > 0\ \&\&\ \text{Cos}[6\,H\,x0] > 0\ \&\&\ \text{Csc}[6\,H\,x0] > 0\ \&\&\ \text{Sec}[6\,H\,x0] > 0\ \&\&\\ &\quad \text{Tan}[6\,H\,x0] > 0\ \&\&\ \text{Cot}[6\,H\,x0] > 0\ \&\&\ \text{Sin}[z] > 0\ \&\&\ \sqrt{\text{Sin}[z]} > 0\ \&\&\ \text{Sin}[z]^{3/2} > 0\ \&\&\\ &\quad \sqrt{\text{Sin}[z]} > 0\ \&\&\ \frac{1}{\text{Sin}[z]^{3/2}} > 0\ \&\&\ \frac{1}{\sqrt{\text{Sin}[z]}} > 0\ \&\&\ \text{Cot}[z] > 0\ \&\&\ \sqrt{\text{Cot}[z]} > 0\ \&\&\\ &\quad \text{Cot}[z]^{3/2} > 0\ \&\&\ \sqrt{\text{Cot}[z]} > 0\ \&\&\ \text{Tan}[z] > 0\ \&\&\ \text{Sec}[z] > 0\ \&\&\ \sqrt{\text{Csc}[z]} > 0\ \&\&\ \text{Csc}[z] > 0\end{aligned}$$

more hacks (damn lies):

In[34]:=

$$\begin{aligned}&(*\&\&K^2-M^2>0\&\&\sqrt{K^2-M^2}>0\&\&e^{2\ H\ \sqrt{K^2-M^2}\ x4}>0\&\&2\ H\ \sqrt{K^2-M^2}\ \ x4>0\&\&\\ &\ 2\ H\ \sqrt{1-M^2}\ \ x4>0\&\&\sqrt{1-M^2}>0\&\&e^{2\ H\ \sqrt{1-M^2}\ x4}>0\&\&2\ H\ \sqrt{1-M^2}\ \ x4>0*)\end{aligned}$$

In[35]:=

```
constraintX = x0 > 0 && x4 > 0 && H > 0 && 6 * H * x0 > 0 &&
  3 * H * x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 && t > 0 && M > 0 &&
  K > 0 && ⅇ^(-2 a4[H x4]) > 0 && ⅇ^(-a4[H x4]) > 0 && ⅇ^(2 a4[H x4]) > 0 && ⅇ^(a4[H x4]) > 0
```

Out[35]=

$$x0>0\ \&\&\ x4>0\ \&\&\ H>0\ \&\&\ 6\,H\,x0>0\ \&\&\ 3\,H\,x4>0\ \&\&\ a4[x4]>0\ \&\&\ A4[t]>0\ \&\&\ Q>0\ \&\&\ z>0\ \&\&\ t>0\ \&\&\ M>0\ \&\&\ K>0\ \&\&\ e^{-2\,a4[H\,x4]}>0\ \&\&\ e^{-a4[H\,x4]}>0\ \&\&\ e^{2\,a4[H\,x4]}>0\ \&\&\ e^{a4[H\,x4]}>0$$

statistics :

In[36]:=

```
constraintVars = constraintX && constraintTrig
```

Out[36]=

$$x0>0\ \&\&\ x4>0\ \&\&\ H>0\ \&\&\ 6\,H\,x0>0\ \&\&\ 3\,H\,x4>0\ \&\&\ a4[x4]>0\ \&\&\ A4[t]>0\ \&\&\ Q>0\ \&\&\ z>0\ \&\&\ t>0\ \&\&\ M>0\ \&\&\ K>0\ \&\&\ e^{-2\,a4[H\,x4]}>0\ \&\&\ e^{-a4[H\,x4]}>0\ \&\&\ e^{2\,a4[H\,x4]}>0\ \&\&\ e^{a4[H\,x4]}>0\ \&\&\ \mathrm{Sin}[6\,H\,x0]>0\ \&\&\ \mathrm{Cos}[6\,H\,x0]>0\ \&\&\ \mathrm{Csc}[6\,H\,x0]>0\ \&\&\ \mathrm{Sec}[6\,H\,x0]>0\ \&\&\ \mathrm{Tan}[6\,H\,x0]>0\ \&\&\ \mathrm{Cot}[6\,H\,x0]>0\ \&\&\ \mathrm{Sin}[z]>0\ \&\&\ \sqrt{\mathrm{Sin}[z]}>0\ \&\&\ \mathrm{Sin}[z]^{3/2}>0\ \&\&\ \sqrt{\mathrm{Sin}[z]}>0\ \&\&\ \frac{1}{\mathrm{Sin}[z]^{3/2}}>0\ \&\&\ \frac{1}{\sqrt{\mathrm{Sin}[z]}}>0\ \&\&\ \mathrm{Cot}[z]>0\ \&\&\ \sqrt{\mathrm{Cot}[z]}>0\ \&\&\ \mathrm{Cot}[z]^{3/2}>0\ \&\&\ \sqrt{\mathrm{Cot}[z]}>0\ \&\&\ \mathrm{Tan}[z]>0\ \&\&\ \mathrm{Sec}[z]>0\ \&\&\ \sqrt{\mathrm{Csc}[z]}>0\ \&\&\ \mathrm{Csc}[z]>0$$

In[37]:=

$$\mathrm{subsDefects}=\Big\{\sqrt{e^{2\,a4[H\,x4]}}\to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}}\to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}}\to\frac{1}{e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6}},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{1}{e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{1}{e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\Big\}$$

Out[37]=

$$\Big\{\sqrt{e^{2\,a4[H\,x4]}}\to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}}\to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}}\to\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x4]^{1/6}},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{e^{a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\Big\}$$

future coordinate transformation :

In[38]:=
```
6 H x0 == z && H x4 == t
szt = Solve[%, {z, t}][[1]]
sx0x4 = Solve[%%, {x0, x4}][[1]]
Protect[sx0x4, szt];
```

Out[38]=
```
6 H x0 == z && H x4 == t
```

Out[39]=
```
{z → 6 H x0, t → H x4}
```

Out[40]=

$$\left\{x0\to\frac{z}{6\,H},\ x4\to\frac{t}{H}\right\}$$

In[42]:=
```
sx0x4
```

Out[42]=

$$\left\{x0\to\frac{z}{6\,H},\ x4\to\frac{t}{H}\right\}$$

In[43]:=
```
(*sa4={a4→((A4[3 H #2])&)}*)
```

In[44]:=
```
Protect[sx0x4, szt]
```

Out[44]=
```
{}
```

$\overline{\text{SO }(4,\ 4)}$ Minkowski Lorentz metric η4488 : 4 + 4 spacetime; 8 × 8 dimensional :

In[45]:=
```
(η4488 = ArrayFlatten[
    {{IdentityMatrix[4], 0}, {0, -IdentityMatrix[4]}}]) // MatrixForm
```

Out[45]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$\overline{\text{SO }(4,\ 4)}$ Spinor Lorentz metric σ;

τ^A are analogous to the Dirac gamma matrices :

all true : Table[

$$\left\{A,\ B,\ \text{FullSimplify}\left[\frac{1}{2}\left(\tau[A].\overline{\tau}[B]+\tau[B].\overline{\tau}[A]\right) == \eta4488[\![A+1,\ B+1]\!]*\text{ID8}\right]\right\},$$

{A, 0, 7}, {B, 0, 7}]

Type - 1 and type - 2 employ σ (the ususal notational abuse),

since σ == σ^{-1} : 4 + 4 spacetime, 8 × 8 dimensional and 16 × 16 dimensional :

indices. We define the reduced Brauer-Weyl generators $\{\bar{\tau}^{A'}, \tau^{A'}\}$ of the generators of the two real 8×8 inequivalent irreducible representations of $\overline{SO(4,4)}$ (see eqs. (2.6) and (2.7)) by demanding that the tau matrices satisfy (the tilde denotes transpose)

(2.3) $$\tilde{\tau}^{A'}\sigma = \sigma\bar{\tau}^{A'}$$

and

(2.4) $$\tau^{A'}\bar{\tau}^{B'} + \tau^{B'}\bar{\tau}^{A'} = 2IG^{A'B'} = \bar{\tau}^{A'}\tau^{B'} + \bar{\tau}^{B'}\tau^{A'},$$

where I denotes the 8×8 unit matrix. Denoting the matrix elements of $\tau^{A'}$ by ${\tau^{A'a}}_b$, we may write eq. (3) as

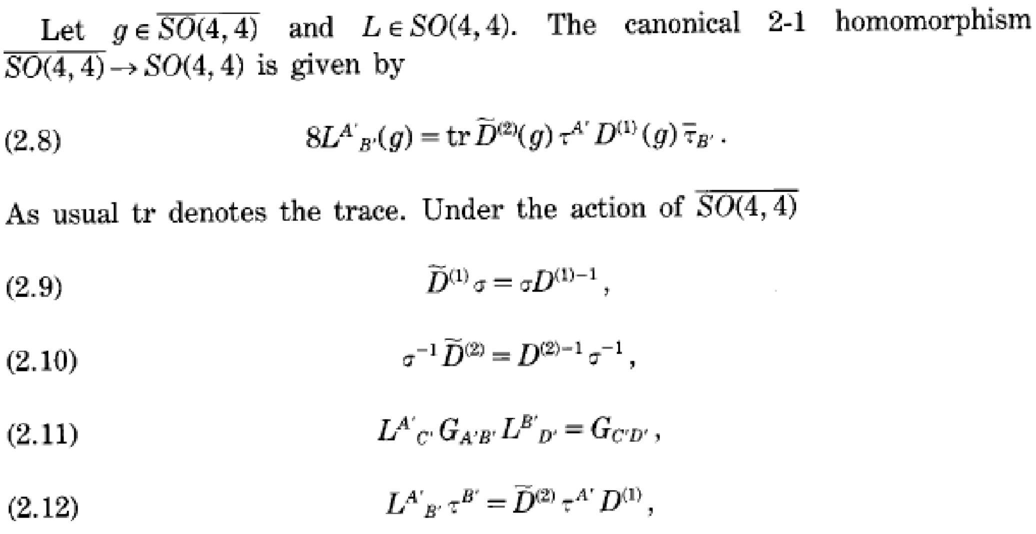
(2.5) $$\bar{\tau}^{A'}{}_{ab} = \tau^{A'}{}_{ba},$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4\begin{pmatrix} {D_{(1)}}^{AB} & 0 \\ 0 & {D_{(2)}}^{AB} \end{pmatrix}$$

$$4{D_{(1)}}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4{D_{(2)}}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$



Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \to SO(4,4)$ is given by

(2.8) $$8L^{A'}{}_{B'}(g) = \mathrm{tr}\,\widetilde{D}^{(2)}(g)\,\tau^{A'}\,D^{(1)}(g)\,\overline{\tau}_{B'}\,.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

(2.9) $$\widetilde{D}^{(1)}\sigma = \sigma D^{(1)-1}\,,$$

(2.10) $$\sigma^{-1}\widetilde{D}^{(2)} = D^{(2)-1}\sigma^{-1}\,,$$

(2.11) $$L^{A'}{}_{C'}\,G_{A'B'}\,L^{B'}{}_{D'} = G_{C'D'}\,,$$

(2.12) $$L^{A'}{}_{B'}\,\tau^{B'} = \widetilde{D}^{(2)}\,\tau^{A'}\,D^{(1)}\,,$$

(2.13) $$L^{A'}{}_{B'}\,\overline{\tau}^{B'} = D^{(1)-1}\,\overline{\tau}^{A'}\,\widetilde{D}^{(2)-1}\,.$$

```
In[46]:= (σ = ArrayFlatten[{{ConstantArray[0, {4, 4}], IdentityMatrix[4]},
            {IdentityMatrix[4], ConstantArray[0, {4, 4}]}}]) // MatrixForm
```

```
Out[46]//MatrixForm=
 0 0 0 0 1 0 0 0
 0 0 0 0 0 1 0 0
 0 0 0 0 0 0 1 0
 0 0 0 0 0 0 0 1
 1 0 0 0 0 0 0 0
 0 1 0 0 0 0 0 0
 0 0 1 0 0 0 0 0
 0 0 0 1 0 0 0 0
```

below :

```
In[∘]:= σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

below :

```
In[∘]:= Table[T16^A[A1] = ArrayFlatten[{{0, τ̄[A1]}, {τ[A1], 0}}], {A1, 0, 7}];
```

below :

```
In[∘]:= σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
```

```
Out[∘]= {True, True, True, True, True, True, True, True}
```

below :

```
In[∘]:= Table[{{A1, B1},
          FullSimplify[ExpandAll[1/2 (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) ==
```

```
η4488⟦A1 + 1, B1 + 1⟧ * ID16]]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

```
Out[∘]//MatrixForm=
 {0, 0}  {0, 1}  {0, 2}  {0, 3}  {0, 4}  {0, 5}  {0, 6}  {0, 7}
  True    True    True    True    True    True    True    True
 {1, 0}  {1, 1}  {1, 2}  {1, 3}  {1, 4}  {1, 5}  {1, 6}  {1, 7}
  True    True    True    True    True    True    True    True
 {2, 0}  {2, 1}  {2, 2}  {2, 3}  {2, 4}  {2, 5}  {2, 6}  {2, 7}
  True    True    True    True    True    True    True    True
 {3, 0}  {3, 1}  {3, 2}  {3, 3}  {3, 4}  {3, 5}  {3, 6}  {3, 7}
  True    True    True    True    True    True    True    True
 {4, 0}  {4, 1}  {4, 2}  {4, 3}  {4, 4}  {4, 5}  {4, 6}  {4, 7}
  True    True    True    True    True    True    True    True
 {5, 0}  {5, 1}  {5, 2}  {5, 3}  {5, 4}  {5, 5}  {5, 6}  {5, 7}
  True    True    True    True    True    True    True    True
 {6, 0}  {6, 1}  {6, 2}  {6, 3}  {6, 4}  {6, 5}  {6, 6}  {6, 7}
  True    True    True    True    True    True    True    True
 {7, 0}  {7, 1}  {7, 2}  {7, 3}  {7, 4}  {7, 5}  {7, 6}  {7, 7}
  True    True    True    True    True    True    True    True
```

---

defs; some Symbols; metric

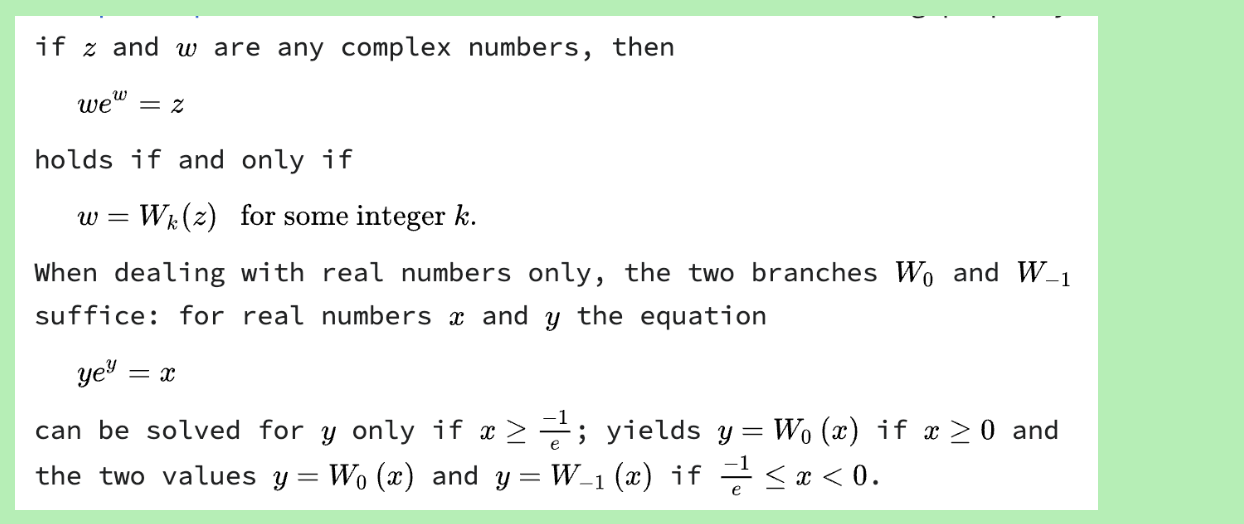
$$\left\{\left\{a4[t] \to \frac{M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1} + \mathbb{c}_1 + \frac{1}{2}\,\mathrm{ProductLog}\left[-e^{-\frac{2\,M\,t\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1}-2\,\mathbb{c}_1}\right]\right\}\right\}$$

$$\mathrm{DSolve}\left[0 == -M\,(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7]) + \left(Q1 - e^{-2\,a4[t]}\,Q1\right)\,a4'[t],\ a4[t],\ t\right]$$

$M\,c[j]\ =\ (j+1)^{st}$ Energy Eigenvalue for Transpose[cayZ].Ψ16 , j = 0, ..., 7

ProductLog[z]

gives the principal solution for w in $z = w e^{w}$.



if z and w are any complex numbers, then

$$we^w = z$$

holds if and only if

$$w = W_k(z) \quad \text{for some integer } k.$$

When dealing with real numbers only, the two branches W_0 and W_{-1} suffice: for real numbers x and y the equation

$$ye^y = x$$

can be solved for y only if $x \geq \frac{-1}{e}$; yields $y = W_0\left(x\right)$ if $x \geq 0$ and the two values $y = W_0\left(x\right)$ and $y = W_{-1}\left(x\right)$ if $\frac{-1}{e} \leq x < 0$.

?todo: NEED DefScalarFunction[#]; & /@ (f16[#]&/@Range[0,15]), and allow xact to compute?

Introduce the wave function, Ψ16, for a Universe:

```
In[47]:= Ψ16 = f16[#][x0, x4] & /@ Range[0, 15]

Out[47]=
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

```
In[48]:= processRawSets[rawSets_] := Module[{s1, s2, J, ret, L = Range[Length[rawSets]], r = {}},
           While[Length[L] > 1,
            s1 = Union[rawSets⟦L⟦1⟧⟧];
            L = Drop[L, 1];
            J = L⟦1⟧;
            While[J ≤ 16,
             If[MemberQ[L, J],
              s2 = Union[rawSets⟦J⟧];
              If[Intersection[s1, s2] ≠ {}, s1 = Union[Flatten[{s1, s2}]];
               L = Complement[L, {J}];, ## &[]];,
              ## &[]];
             J++;];

            AppendTo[r, s1];
           ];
           ret = Sort[r, #1⟦1⟧ < #2⟦1⟧ &];
           Return[ret];
          ]
```

```
In[49]:= rawSetsΨ16 = Cases[
           #,
           h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :→ n,
           Infinity,
           Heads → True
         ] & /@ Ψ16
Out[49]=
         {{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}, {15}}
In[50]:= processRawSets[rawSetsΨ16]
Out[50]=
         {{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}}
In[51]:= (*Ψnew16=F16[#][x0,x4]&/@Range[0,15]*)
In[52]:= Clear[sf16Aa];
         sfψ16Aa = f16[#] → ToExpression[
               "((Z[" <> ToString[#] <> "][6*H*#1,H*#2])&)"] & /@ Range[0, 15]
Out[52]=
         {f16[0] → (Z[0][6 H #1, H #2] &), f16[1] → (Z[1][6 H #1, H #2] &),
          f16[2] → (Z[2][6 H #1, H #2] &), f16[3] → (Z[3][6 H #1, H #2] &),
          f16[4] → (Z[4][6 H #1, H #2] &), f16[5] → (Z[5][6 H #1, H #2] &), f16[6] → (Z[6][6 H #1, H #2] &),
          f16[7] → (Z[7][6 H #1, H #2] &), f16[8] → (Z[8][6 H #1, H #2] &), f16[9] → (Z[9][6 H #1, H #2] &),
          f16[10] → (Z[10][6 H #1, H #2] &), f16[11] → (Z[11][6 H #1, H #2] &),
          f16[12] → (Z[12][6 H #1, H #2] &), f16[13] → (Z[13][6 H #1, H #2] &),
          f16[14] → (Z[14][6 H #1, H #2] &), f16[15] → (Z[15][6 H #1, H #2] &)}
In[53]:= (*Clear[sf16Aa];
         sfψ16Aa=f16[#]→ToExpression[
               "((Z["<>ToString[#]<>"][6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@Range[0,15]*)
```

In[54]:=

```
Clear[snewfψ16Aa];
snewfψ16Aa = f16[#] → ToExpression["((nZ[" <> ToString[#] <>
                                           1
        "][6*H*#1,H*#2] ─────────────────)&)"] & /@ Range[0, 15]
                        Sin[6 * H * #1]^1/2
```

Out[54]=

$$\left\{\mathrm{f16}[0]\to\left(\frac{\mathrm{nZ}[0][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),\ \mathrm{f16}[1]\to\left(\frac{\mathrm{nZ}[1][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),\right.$$

$$\mathrm{f16}[2]\to\left(\frac{\mathrm{nZ}[2][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),\ \mathrm{f16}[3]\to\left(\frac{\mathrm{nZ}[3][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),$$

$$\mathrm{f16}[4]\to\left(\frac{\mathrm{nZ}[4][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),\ \mathrm{f16}[5]\to\left(\frac{\mathrm{nZ}[5][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),$$

$$\mathrm{f16}[6]\to\left(\frac{\mathrm{nZ}[6][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),\ \mathrm{f16}[7]\to\left(\frac{\mathrm{nZ}[7][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),$$

$$\mathrm{f16}[8]\to\left(\frac{\mathrm{nZ}[8][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),\ \mathrm{f16}[9]\to\left(\frac{\mathrm{nZ}[9][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),$$

$$\mathrm{f16}[10]\to\left(\frac{\mathrm{nZ}[10][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),\ \mathrm{f16}[11]\to\left(\frac{\mathrm{nZ}[11][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),$$

$$\mathrm{f16}[12]\to\left(\frac{\mathrm{nZ}[12][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),\ \mathrm{f16}[13]\to\left(\frac{\mathrm{nZ}[13][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),$$

$$\left.\mathrm{f16}[14]\to\left(\frac{\mathrm{nZ}[14][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right),\ \mathrm{f16}[15]\to\left(\frac{\mathrm{nZ}[15][6\,\mathrm{H}\,\#1,\ \mathrm{H}\,\#2]}{\sqrt{\mathrm{Sin}[6\,\mathrm{H}\,\#1]}}\ \&\right)\right\}$$

$\overline{\mathrm{O\ (4,\ 4)}}$ related :

In[55]:=

```
Needs["Notation`"]
```

In[56]:= Symbolize[$\boxed{\sigma g^{a}_{1\,A}}$]

In[57]:= Symbolize[$\boxed{\sigma g^{a}_{2\,A}}$]

In[58]:= Symbolize[$\boxed{\overline{\tau}}$]

In[59]:= Symbolize[$\boxed{\mathrm{T16}^{A}}$]

In[60]:= Symbolize[$\boxed{\mathrm{T16}^{\alpha}}$]

In[61]:= Symbolize[$\boxed{u^{a}}$]

In[62]:= Symbolize[$\boxed{J_{y}}$]

In[63]:= Symbolize[$\boxed{F^{A}_{a}}$]

In[64]:= Symbolize[$\boxed{F^{a}_{A}}$]

In[65]:= Symbolize[$\overline{\sigma22}$]

$$g^{\alpha\beta} = g_{\alpha\beta} = \mathrm{diag}(1,1,1,-1) \qquad (5)$$

In[66]:= Symbolize[$g_{\alpha\beta}$]

note below : g_{AB} is really $g_{(A)\,(B)}$ :: Minkowski – Lorentz metric

In[67]:= Symbolize[g_{AB}]

note below : G_{AB} is really $G_{(A)\,(B)}$ :: Minkowski – Lorentz

In[68]:= (*Symbolize[G_{AB}]*)

In[69]:= Symbolize[S44αβ]

$$\Sigma^{\alpha\beta} = \xi S^{\alpha\beta} u,$$

In[70]:= Symbolize[$\Sigma^{\alpha\beta}$]

In[71]:= (*Symbolize[$\Sigma^{\alpha\beta}$]*)

$$\tfrac{1}{2}\Sigma^{jk}\Sigma_{jk} = \tfrac{1}{4}\,\partial_a\, u^a u^b\, \partial_b - \tfrac{1}{4}\partial_a\, u^b u^b\, \partial_a,$$

In[72]:= Symbolize[Σ^2]

NEED DefScalarFunction[#]; & /@ Flatten[Universe]

In[∘]:= Universe = ToExpression["U" <> ToString[#]] & /@ Range[0, 7]

In[73]:= Symbolize[$Ƒ^{A}_{a}$]

In[74]:= Symbolize[$Ƒ^{a}_{A}$]

In[75]:= Symbolize[$\mathrm{gtrye}^{(A)}_{\alpha}$]

In[76]:= Symbolize[$\mathrm{gtrye}^{\alpha}_{(A)}$]

SPACETIME METRIC

In[77]:= (*einsteinMetric=Array[g[#1-1,#2-1]&,{8,8}]*)

SPACETIME METRIC:

```
In[78]:= g4488 = Array[g[#1 - 1][#2 - 1][x0, x4] &, {8, 8}];
```

SPACETIME METRIC values:

MatrixMetric44

6 H x0 == z && H x4 == t

$$\left\{\left\{a4''[t]\to 0,\ a4'[t]\to \frac{2}{3}\,(-1+M)\right\},\ \left\{a4''[t]\to 0,\ a4'[t]\to \frac{2\,(1+M)}{3}\right\}\right\}$$

In[79]:=
```
(*β3=Exp[2 H x4 Sqrt[K^2-M^2] ];*)
(*β3=Exp[2 * a4[3*H*x4] ];*)
β3 = Exp[2 * a4[H * x4] ] (*/.{a4→(((K1*(2 (1+M))/3+K2*2/3 (-1+M))#)&)}*)
β1 = Sin[6 * H * x0]^(1/3);
β2 = Cot[6 * H * x0]^2;
```

Out[79]=
$$e^{2\,a4[H\,x4]}$$

In[82]:=
```
MatrixForm[
 MatrixMetric44 = {{β2, 0, 0, 0, 0, 0, 0, 0}, {0, β1 β3, 0, 0, 0, 0, 0, 0},
    {0, 0, β1 β3, 0, 0, 0, 0, 0}, {0, 0, 0, β1 β3, 0, 0, 0, 0},
    {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -β1/β3, 0, 0},
    {0, 0, 0, 0, 0, 0, -β1/β3, 0}, {0, 0, 0, 0, 0, 0, 0, -β1/β3}} //
   FullSimplify[#, constraintVars] &]
```

Out[82]//MatrixForm=
```
Cot[6 H x0]^2   0                               0                               0                               0
0               e^(2 a4[H x4]) Sin[6 H x0]^(1/3) 0                               0                               0
0               0                               e^(2 a4[H x4]) Sin[6 H x0]^(1/3) 0                               0
0               0                               0                               e^(2 a4[H x4]) Sin[6 H x0]^(1/3) 0
0               0                               0                               0                               -1
0               0                               0                               0                               0     -e^(-2 a4[H x4]) S
0               0                               0                               0                               0
0               0                               0                               0                               0
```

In[83]:=
```
(*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression[
    "((Z["<>ToString[#]<>"][6*H*#1,3*H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@Range[0,15]*)
```

In[84]:=
```
(*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"][6*H*#1,3*H*#2])&)"]&/@
  Range[0,15]*)
```

In[85]:=
```
(*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression[
    "((Z["<>ToString[#]<>"][6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@Range[0,15]*)
```

In[86]:=
```
(*Inactivate[((q)&)]Evaluate[Activate[Evaluate[q]]]*)
```

```
In[87]:= Clear[makeSubstitution];
         makeSubstitution[] := Module[{met, mat, t, f, p},
           met = Flatten[g4488]〚All, 0〛;
           mat = Flatten[MatrixMetric44];
           t = Thread[met → mat];
           f = Table[FullForm[t〚j〛 /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
           p = Table[Block[{s, r, q, expr}, q = f〚j〛〚1〛〚2〛;
              expr = Inactivate[((q) &)];
              s = f〚j〛〚1〛〚1〛 → expr;
              r = Activate[s // Evaluate] // Evaluate;
              r], {j, 1, Length[f]}];
           Return[p];
          ]

In[89]:= makeSubstitution[a_, b_] := Module[{t, f, p},
           t = Thread[a → b];
           f = Table[FullForm[t〚j〛 /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
           p = Table[Block[{s, r, q, expr}, q = f〚j〛〚1〛〚2〛;
              expr = Inactivate[((q) &)];
              s = f〚j〛〚1〛〚1〛 → expr;
              r = Activate[s // Evaluate] // Evaluate;
              r], {j, 1, Length[f]}];
           Return[p];
          ]

In[90]:= ssgm4488 = makeSubstitution[]
```

```
Out[90]=
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
 g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
 g[1][1] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
 g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
 g[2][1] → (0 &), g[2][2] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
 g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
 g[3][2] → (0 &), g[3][3] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
 g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
 g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
 g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
 g[5][5] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
 g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
 g[6][5] → (0 &), g[6][6] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
 g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
 g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &)}
```

In[91]:= Symbolize[$E_{\alpha}^{(A)}$]

In[92]:= Symbolize[$e_{\alpha}^{(A)}$]

In[93]:= $e_{\alpha}^{(A)}$ = Array[$E_{\alpha}^{(A)}$[#1 - 1, #2 - 1] &, {8, 8}]

Out[93]=

$\{\{E_{\alpha}^{(A)}[0, 0], E_{\alpha}^{(A)}[0, 1], E_{\alpha}^{(A)}[0, 2], E_{\alpha}^{(A)}[0, 3], E_{\alpha}^{(A)}[0, 4], E_{\alpha}^{(A)}[0, 5], E_{\alpha}^{(A)}[0, 6], E_{\alpha}^{(A)}[0, 7]\},$
$\{E_{\alpha}^{(A)}[1, 0], E_{\alpha}^{(A)}[1, 1], E_{\alpha}^{(A)}[1, 2], E_{\alpha}^{(A)}[1, 3], E_{\alpha}^{(A)}[1, 4], E_{\alpha}^{(A)}[1, 5], E_{\alpha}^{(A)}[1, 6], E_{\alpha}^{(A)}[1, 7]\},$
$\{E_{\alpha}^{(A)}[2, 0], E_{\alpha}^{(A)}[2, 1], E_{\alpha}^{(A)}[2, 2], E_{\alpha}^{(A)}[2, 3], E_{\alpha}^{(A)}[2, 4], E_{\alpha}^{(A)}[2, 5], E_{\alpha}^{(A)}[2, 6], E_{\alpha}^{(A)}[2, 7]\},$
$\{E_{\alpha}^{(A)}[3, 0], E_{\alpha}^{(A)}[3, 1], E_{\alpha}^{(A)}[3, 2], E_{\alpha}^{(A)}[3, 3], E_{\alpha}^{(A)}[3, 4], E_{\alpha}^{(A)}[3, 5], E_{\alpha}^{(A)}[3, 6], E_{\alpha}^{(A)}[3, 7]\},$
$\{E_{\alpha}^{(A)}[4, 0], E_{\alpha}^{(A)}[4, 1], E_{\alpha}^{(A)}[4, 2], E_{\alpha}^{(A)}[4, 3], E_{\alpha}^{(A)}[4, 4], E_{\alpha}^{(A)}[4, 5], E_{\alpha}^{(A)}[4, 6], E_{\alpha}^{(A)}[4, 7]\},$
$\{E_{\alpha}^{(A)}[5, 0], E_{\alpha}^{(A)}[5, 1], E_{\alpha}^{(A)}[5, 2], E_{\alpha}^{(A)}[5, 3], E_{\alpha}^{(A)}[5, 4], E_{\alpha}^{(A)}[5, 5], E_{\alpha}^{(A)}[5, 6], E_{\alpha}^{(A)}[5, 7]\},$
$\{E_{\alpha}^{(A)}[6, 0], E_{\alpha}^{(A)}[6, 1], E_{\alpha}^{(A)}[6, 2], E_{\alpha}^{(A)}[6, 3], E_{\alpha}^{(A)}[6, 4], E_{\alpha}^{(A)}[6, 5], E_{\alpha}^{(A)}[6, 6], E_{\alpha}^{(A)}[6, 7]\},$
$\{E_{\alpha}^{(A)}[7, 0], E_{\alpha}^{(A)}[7, 1], E_{\alpha}^{(A)}[7, 2], E_{\alpha}^{(A)}[7, 3], E_{\alpha}^{(A)}[7, 4], E_{\alpha}^{(A)}[7, 5], E_{\alpha}^{(A)}[7, 6], E_{\alpha}^{(A)}[7, 7]\}\}$

In[94]:= (*Symbolize[$se_{\alpha}^{(A)}$]*)

In[95]:= Symbolize[$sge_{\alpha}^{(A)}$]

In[96]:= Symbolize[$sgtrye_{\alpha}^{(A)}$]

In[97]:= Symbolize[$E_{(A)}^{\alpha}$]

In[98]:= Symbolize[$sge_{(A)}^{\alpha}$]

In[99]:= Symbolize[$sgtrye_{(A)}^{\alpha}$]

In[100]:=

Symbolize[$e_{(A)}^{\alpha}$]

In[101]:=

$e_{(A)}^{\alpha}$ = Array[$E_{(A)}^{\alpha}$[#1 - 1, #2 - 1] &, {8, 8}]

Out[101]=

$\{\{E_{(A)}^{\alpha}[0, 0], E_{(A)}^{\alpha}[0, 1], E_{(A)}^{\alpha}[0, 2], E_{(A)}^{\alpha}[0, 3], E_{(A)}^{\alpha}[0, 4], E_{(A)}^{\alpha}[0, 5], E_{(A)}^{\alpha}[0, 6], E_{(A)}^{\alpha}[0, 7]\},$
$\{E_{(A)}^{\alpha}[1, 0], E_{(A)}^{\alpha}[1, 1], E_{(A)}^{\alpha}[1, 2], E_{(A)}^{\alpha}[1, 3], E_{(A)}^{\alpha}[1, 4], E_{(A)}^{\alpha}[1, 5], E_{(A)}^{\alpha}[1, 6], E_{(A)}^{\alpha}[1, 7]\},$
$\{E_{(A)}^{\alpha}[2, 0], E_{(A)}^{\alpha}[2, 1], E_{(A)}^{\alpha}[2, 2], E_{(A)}^{\alpha}[2, 3], E_{(A)}^{\alpha}[2, 4], E_{(A)}^{\alpha}[2, 5], E_{(A)}^{\alpha}[2, 6], E_{(A)}^{\alpha}[2, 7]\},$
$\{E_{(A)}^{\alpha}[3, 0], E_{(A)}^{\alpha}[3, 1], E_{(A)}^{\alpha}[3, 2], E_{(A)}^{\alpha}[3, 3], E_{(A)}^{\alpha}[3, 4], E_{(A)}^{\alpha}[3, 5], E_{(A)}^{\alpha}[3, 6], E_{(A)}^{\alpha}[3, 7]\},$
$\{E_{(A)}^{\alpha}[4, 0], E_{(A)}^{\alpha}[4, 1], E_{(A)}^{\alpha}[4, 2], E_{(A)}^{\alpha}[4, 3], E_{(A)}^{\alpha}[4, 4], E_{(A)}^{\alpha}[4, 5], E_{(A)}^{\alpha}[4, 6], E_{(A)}^{\alpha}[4, 7]\},$
$\{E_{(A)}^{\alpha}[5, 0], E_{(A)}^{\alpha}[5, 1], E_{(A)}^{\alpha}[5, 2], E_{(A)}^{\alpha}[5, 3], E_{(A)}^{\alpha}[5, 4], E_{(A)}^{\alpha}[5, 5], E_{(A)}^{\alpha}[5, 6], E_{(A)}^{\alpha}[5, 7]\},$
$\{E_{(A)}^{\alpha}[6, 0], E_{(A)}^{\alpha}[6, 1], E_{(A)}^{\alpha}[6, 2], E_{(A)}^{\alpha}[6, 3], E_{(A)}^{\alpha}[6, 4], E_{(A)}^{\alpha}[6, 5], E_{(A)}^{\alpha}[6, 6], E_{(A)}^{\alpha}[6, 7]\},$
$\{E_{(A)}^{\alpha}[7, 0], E_{(A)}^{\alpha}[7, 1], E_{(A)}^{\alpha}[7, 2], E_{(A)}^{\alpha}[7, 3], E_{(A)}^{\alpha}[7, 4], E_{(A)}^{\alpha}[7, 5], E_{(A)}^{\alpha}[7, 6], E_{(A)}^{\alpha}[7, 7]\}\}$

In[102]:=

(*$gtrye_{\alpha}^{(A)}$=($e_{\alpha}^{(A)}$/.$sgtrye_{\alpha}^{(A)}$)*)

In[103]:=

(*$gtrye_{(A)}^{\alpha}$=($e_{(A)}^{\alpha}$/.$sgtrye_{(A)}^{\alpha}$)*)

In[104]:=

(*Symbolize[$\Gamma^{\kappa}_{\alpha\ \beta}$]*)

In[105]:=

(*preSpinConnection=Array[$\Gamma^{\kappa}_{\alpha\ \beta}$[##]&,{8,8,8}];*)

In[106]:=

(*Symbolize[$E^{A}{}_{a}$]*)

In[107]:=

```
(*Symbolize[ E^a_A ]*)
```

In[108]:=

```
(* Ƒ^A_a  Ƒ^a_A *)
```

In[109]:=

```
(*Do[ Ƒ^A_a =E^A_a[h]=Table[(u⟦h⟧.σ.(τ̄[B])),{B,1,8}],{h,1,Length[u]}];
Do[E^a_A[h]=FullSimplify[Inverse[E^A_a[h]]],{h,1,Length[u]}];
Table[ Ƒ^a_A =E^a_A[h]== (u⟦h⟧.σ.u⟦h⟧)
     Transpose[Table[FullSimplify[ExpandAll[ η8⟦B,B⟧τ[B].u⟦h⟧ ]],
       {B,1,8}]],{h,1,Length[u]}]*)
```

In[110]:=

```
(*Table[
 FullSimplify[ExpandAll[Transpose[E^A_a[h]].η8.E^A_a[h]]]- (u⟦h⟧.σ.u⟦h⟧)σ== Zero,
 {h,1,Length[u]}]*)
```

In[111]:=

```
(*iη88=FullSimplify[Inverse[η8]];*)
```

In[112]:=

```
(*Clear[ω_μ^(a)_(b)];*)Clear[ω]; Symbolize[ω_μ^(a)_(b)]
```

In[113]:=

```
ω = Array[ω_μ^(a)_(b)[##] &, {8, 8, 8}];
Protect[ω]
```

Out[114]=

```
{ω}
```

constants

← **ERROR : O8similarityTransformation has 1 st index that transforms as $\frac{\partial}{\partial \Psi}$,**

not as Ψs

In[115]:=

```
(* O8similarityTransformation has 1st index that transforms as ∂/∂Ψ ,
not  as Ψ   *)
```

O8similarityTransformation has 1 st index that transforms as $\frac{\partial}{\partial \Psi}$, not as Ψ :

In[116]:=

```
(*ArrayFlatten[{{IdentityMatrix[4],0},{0,-IdentityMatrix[4]}}]*)
```

In[117]:=

```
ID4 = IdentityMatrix[4];
ID8 = IdentityMatrix[8];
```

```
In[119]:=
        η4488 // MatrixForm
Out[119]//MatrixForm=
        ⎛1 0 0 0  0   0   0   0 ⎞
        ⎜0 1 0 0  0   0   0   0 ⎟
        ⎜0 0 1 0  0   0   0   0 ⎟
        ⎜0 0 0 1  0   0   0   0 ⎟
        ⎜0 0 0 0 -1   0   0   0 ⎟
        ⎜0 0 0 0  0  -1   0   0 ⎟
        ⎜0 0 0 0  0   0  -1   0 ⎟
        ⎝0 0 0 0  0   0   0  -1 ⎠
In[120]:=
        η4488⟦#, #⟧ & /@ Range[8]
Out[120]=
        {1, 1, 1, 1, -1, -1, -1, -1}
In[121]:=


        Zero4 = ConstantArray[0, {4, 4}]; Zero8 = ConstantArray[0, {8, 8}];
        mid = ExpandAll[-1 * ID4];
        ϵ3 = Array[Signature[{##}] &, {3, 3, 3}]
        ϵ4 = Array[Signature[{##}] &, {4, 4, 4, 4}];
        MId = ExpandAll[-1 * ID8];
        simp = {Zero4 → 0, ID4 → 1, mid → -1};
        Simp = {Zero8 → ZERO, ID8 → ONE, MId → MONE};
Out[123]=
        {{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}},
         {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}}, {{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}
In[128]:=
        Zero16 = ConstantArray[0, {16, 16}];
        G16 = ArrayFlatten[{{η8, 0}, {0, -η8}}];
        Id16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * Id16];
        Simp16 = {Zero16 → ZERO16, Id16 → ONE16, MId16 → MONE16};
In[132]:=
        ZERO16 = ConstantArray[0, {16, 16}];
        G16 = ArrayFlatten[{{η4488, 0}, {0, -η4488}}];
        ID16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * ID16];
        Simp16 = {ZERO16 → ZERO16, ID16 → ONE16, MId16 → MONE16};
In[136]:=
        ωs = Flatten[Table[ToExpression["ω" <> ToString[A1] <> ToString[B1]],
            {A1, 1, 7}, {B1, A1 + 1, 8}]]
        Length[ωs]
Out[136]=
        {ω12, ω13, ω14, ω15, ω16, ω17, ω18, ω23, ω24, ω25, ω26, ω27, ω28,
         ω34, ω35, ω36, ω37, ω38, ω45, ω46, ω47, ω48, ω56, ω57, ω58, ω67, ω68, ω78}
Out[137]=
        28
```

In[138]:=

```
σ.σ - ID8 == Zero8
Tr[σ]  == 0
```

Out[138]=

```
True
```

Out[139]=

```
True
```

---

some function definitions :

In[140]:=

```
Clear[rawSets];
rawSets[l_, f_, o_] := Module[{t, h},
   t = Table[Cases[
  l⟦j⟧,
h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], f]  :→ n + o,
  Infinity,
  Heads → True
], {j, 1, Length[l]}];
   Return[t];
  ]
```

In[142]:=

```
(*rawSetsel16=rawSets[el16,"f16",0]*)
```

In[143]:=

```
(*couplings=showCoupledEquations[rawSetsel16]
 %//Flatten//Sort
 %===Range[0,15]*)
```

In[144]:=

```
Clear[dualRawSets];
dualRawSets[l_, f_, o_, h_, oh_] := Module[{r, r1, r2},
   r1 = rawSets[l, f, o];
   r2 = rawSets[l, h, oh];
   r = Union[Flatten[{r1, r2}, 1]] // Drop[#, 1] &;
   Return[r];
  ]
```

FixedPoint
uses MergeSetsStep, **below**

You can use Throw to exit from FixedPoint before it is finished.
FixedPoint really uses Catch and Throw, so what could go wrong?

Wolfram Documentation: Details and Options

- ○ FixedPoint always returns the last result it gets.
- ▪ You can use Throw to exit from Fixed Point before it is finished.
- ▪ FixedPoint[f, expr] applies SameQ to successive pairs of results to determine whether a fixed point has been reached.
- ▪ FixedPointLf, expr,..., SameTest -> s] applies s to successive pairs of results.

In[146]:=

```
(* Helper to perform one pass of merging *)
MergeSetsStep[currentSets_List] :=
  Module[{n = Length[currentSets], i, j, merged = currentSets},
 (* We look for the first pair (i, j) that intersects *)
 Catch[
  For[i = 1, i ≤ Length[merged], i++,
   For[j = i + 1, j ≤ Length[merged], j++,
    If[Intersection[merged⟦i⟧, merged⟦j⟧] =!= {},
     (* Found intersection: Merge and Throw to restart/finish this step *)
     merged = Delete[merged, {{i}, {j}}];
     AppendTo[merged, Sort[Union[currentSets⟦i⟧, currentSets⟦j⟧]]];
     Throw[Sort[merged]]; (* Sort for canonical form *)
    ];
   ];
  ];
  (* If no intersection found, return original sorted *)
  Sort[merged]
 ]
];
```

In[147]:=

```
showCoupledEquations[items_List] :=
 FixedPoint[MergeSetsStep, items];
```

In[148]:=

```
blockPartition[mat_] :=
 Module[{(*blocks,block11,block12,block21,block22,*)ret},
  blocks = Partition[mat, {8, 8}];
  (*block11=blocks⟦1,1⟧ ;*)(*Top-left block*)
  (*block12=blocks⟦1,2⟧ ;*)(*Top-right block*)
  (*block21=blocks⟦2,1⟧ ;*)(*Bottom-left block*)
  (*block22=blocks⟦2,2⟧ ;*)(*Bottom-right block*)
  ret = ArrayFlatten[
     {{blocks⟦1, 1⟧, blocks⟦1, 2⟧}, {blocks⟦2, 1⟧, blocks⟦2, 2⟧}}] === mat;
  If[ret, Return[
    {{blocks⟦1, 1⟧, blocks⟦1, 2⟧}, {blocks⟦2, 1⟧, blocks⟦2, 2⟧}}], ## &[]];
  Return[Null];]
```

In[149]:=

```
eextract[a_] := Extract[a, {#}] & /@ Range[0, Length[a]]
```

In[150]:=

```
Clear[imes]
```

In[151]:=

```
imes[a_] := {a}
```

In[152]:=

```
imes[a_, b_] := {a, b}
```

In[153]:=

```
imes[a_, b_, c_] := {a, b, c}
```

In[154]:=

```
imes[a_, b_, c_, d_] := {a, b, c, d}
```

In[155]:=

```
imes[a_, b_, c_, d_, f_] := {a, b, c, d, f}
```

In[156]:=

```
imes[a_, b_, c_, d_, f_, h_] := {a, b, c, d, f, h}
```

In[157]:=

```
imes[a_, b_, c_, d_, f_, h_, j_] := {a, b, c, d, f, h, j}
```

In[158]:=

```
times[a__] := Flatten[{Flatten[#] & /@ a}]
```

In[159]:=

```
(*times[a_,b_,c_,d_]:={a,b}*)
```

In[160]:=

```
(*imes[a_,b_,c_,d_][0]:={a,d}*)
```

In[161]:=

```
(*imes[a_,b_][0]^:={a,b}*)
```

In[162]:=

```
iid[a_, b_] := a
```

In[163]:=

```
iid[a_, b_, c_] := {a, b}
```

In[164]:=

```
iid[a_List] := a〚1〛
```

In[165]:=

```
qid[a__] := a
```

In[166]:=

```
Clear[der]; der[a__] [c_][b__] := c
```

In[167]:=

```
Clear[der3]; der3[a__] [c_][b__] := {c, a, b}
```

In[168]:=

```
(*der[a_, b_] [c_]:= c*)
```

In[169]:=

```
(*derz[a_, b_] := a*)
```

In[170]:=

```
(*dert[a_, b_] := b*)
```

In[171]:=

```
(*derivative[a__] [c_]:= FullForm[c]/.{Derivative→der}*)
```

In[172]:=

```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
(*ToString[#]&/@%*)
% /. {Derivative → der}
```

Out[172]=

```
{Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}
```

Out[173]=

```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[174]=

```
{Z[0][z, t], Z[0], Z[0]}
```

In[175]:=

```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
% * (Prime[#] & /@ Range[Length[%]])
Plus @@ %
FullForm[%]  (* #]&/@%*)
% /. {Plus → qid, Times → qid, Derivative → der}
List[%]
Partition[%, 2]
eextract[#] & /@ %
(*eextract[#]&/@%*)
(*#__List⟦0⟧&/@%*)
#⟦2⟧ & /@ %
#⟦3⟧ & /@ %%
```

Out[175]=

```
{Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}
```

Out[176]=

```
{2 Z[0][z, t], 3 Z[0]^(1,0)[z, t], 5 Z[0]^(0,1)[z, t]}
```

Out[177]=

```
2 Z[0][z, t] + 5 Z[0]^(0,1)[z, t] + 3 Z[0]^(1,0)[z, t]
```

Out[178]//FullForm=

```
Plus[Times[2, Z[0][z, t]], Times[5, Derivative[0, 1][Z[0]][z, t]],
 Times[3, Derivative[1, 0][Z[0]][z, t]]]
```

Out[179]=

```
Sequence[2, Z[0][z, t], 5, Z[0], 3, Z[0]]
```

Out[180]=

```
{2, Z[0][z, t], 5, Z[0], 3, Z[0]}
```

Out[181]=

```
{{2, Z[0][z, t]}, {5, Z[0]}, {3, Z[0]}}
```

Out[182]=

```
{{List, 2, Z[0][z, t]}, {List, 5, Z[0]}, {List, 3, Z[0]}}
```

Out[183]=

```
{2, 5, 3}
```

Out[184]=

```
{Z[0][z, t], Z[0], Z[0]}
```

Sequence[Subscript[expr, 1], Subscript[expr, 2], …]
represents a sequence of arguments to be spliced automatically into any function .

In[185]:=

```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
(*ToString[#]&/@%*)
% /. {Derivative → der3}
```

Out[185]=

$$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$$

Out[186]=

```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[187]=

```
{Z[0][z, t], List[Z[0], 1, 0, z, t], List[Z[0], 0, 1, z, t]}
```

In[188]:=

```
helpf[x__] := Length[{x}]
```

In[189]:=

```
{helpf[x, y, z], helpf[]}
```

Out[189]=

```
{3, helpf[]}
```

In[190]:=

```
f[##] & @@ X
```

Out[190]=

```
f[x0, x1, x2, x3, x4, x5, x6, x7]
```

In[191]:=

```
StringExtract["a--bbb--ccc--dddd", "--" → 3]
```

Out[191]=

```
ccc
```

In[192]:=

```
StringExtract["a bbb  cccc aa   d", -1]
```

Out[192]=

```
d
```

In[193]:=

```
StringRiffle[{{"a", "b", "c"}, {"d", "e", "f"}}, "\n", "\t"]
```

Out[193]=

```
a	b	c
d	e	f
```

In[194]:=

```
{"", "z, t"}
```

Out[194]=

```
{, z, t}
```

```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
StringSplit[#, "]"] & /@ %
StringExtract[#, "[" → All] & /@ %
StringExtract[#,
   "\!\(\*StyleBox[\"
      \",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"{\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*
      StyleBox[\",\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"
      \",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"z\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*
      StyleBox[\",\",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"
      \",ShowStringCharacters->True,NumberMarks->True]\)\!\(\*StyleBox[\"t\",ShowStringCharacters->True,NumberMarks->True]\)" →
    All] & /@ %
InputForm[TextString[#]] & /@ %
```

In[195]:=

```
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
StringSplit[#, "]"] & /@ %
StringExtract[#, "[" → All] & /@ %
(*StringExtract[#," {, z, t"→All]&/@%*)
InputForm[TextString[#]] & /@ %
```

Out[195]=

$$\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$$

Out[196]=

```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[197]=

```
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
```

Out[198]=

```
{{Z[0, [z, t}, {Derivative[1, 0, [Z[0, , [z, t}, {Derivative[0, 1, [Z[0, , [z, t}}
```

Out[199]=

```
{{{Z, 0}, {, z, t}}, {{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}},
 {{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}}
```

Out[200]=

```
{"{{Z, 0}, {, z, t}}", "{{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}",
 "{{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}"}
```

```
In[201]:=
{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
(*InputForm[TextString[#]]&/@%*)
StringExtract[#, "[" → All] & /@ %
StringExtract[#, "]" → All] & /@ %%
StringExtract[#, "[" → 1] & /@ %%%
StringExtract[#, "]" → 1] & /@ %%%%
StringExtract[#, "Z[" → 1] & /@ %%%%%
Out[201]=
{Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}
Out[202]=
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
Out[203]=
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
Out[204]=
{{Z, 0], z, t]}, {Derivative, 1, 0], Z, 0]], z, t]}, {Derivative, 0, 1], Z, 0]], z, t]}}
Out[205]=
{{Z[0, [z, t, }, {Derivative[1, 0, [Z[0, , [z, t, }, {Derivative[0, 1, [Z[0, , [z, t, }}
Out[206]=
{Z, Derivative, Derivative}
Out[207]=
{Z[0, Derivative[1, 0, Derivative[0, 1}
Out[208]=
{, Derivative[1, 0][, Derivative[0, 1][}
In[209]:=
(*TemplateApply[StringTemplate["Z `[` then `b`"],
 <|"a"→1234,"b"→5678|>]*)
In[210]:=
skelx[x_, matQ_ : True] := Module[{t = {666}},
  If[matQ == True, t = Block[{r},
     r = x;
     Table[Block[{q}, q = SameQ[0, r⟦j, i⟧];
       If[True == q, Style[0, Blue], Style[Length[r⟦j, i⟧], Red]]],
      {j, 1, Length[r]}, {i, 1, Length[r⟦-1⟧]}]]];
  If[matQ == False, t = Block[{r},
     r = x;
     Table[Block[{q}, q = SameQ[0, r⟦j⟧];
       If[True == q, Style[0, {RGBColor → {0, 1 / 3, 0}, Bold}],
        Style[Length[r⟦j⟧], Red]]], {j, 1, Length[r]}]]];
  MatrixForm[t, TableAlignments → Left]]
```

```
In[211]:=
makeSym[size_, fn_] :=
 Module[{rtmp}, rtmp = Table[fn[i, j], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[rtmp, {{2}, {1}}]}]]
```

```
In[212]:=
makeAntiSym[size_, fn_] :=
 Module[{rtmp}, rtmp = Table[If[i == j, 0, fn[i, j]], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[-rtmp, {{2}, {1}}]}]]
```

```
In[213]:=
Block[{MX}, MX = makeAntiSym[8, Subscript[ωω, ##] &]]
```

```
Out[213]=
{{0, -ωω_{2,1}, -ωω_{3,1}, -ωω_{4,1}, -ωω_{5,1}, -ωω_{6,1}, -ωω_{7,1}, -ωω_{8,1}},
 {ωω_{2,1}, 0, -ωω_{3,2}, -ωω_{4,2}, -ωω_{5,2}, -ωω_{6,2}, -ωω_{7,2}, -ωω_{8,2}},
 {ωω_{3,1}, ωω_{3,2}, 0, -ωω_{4,3}, -ωω_{5,3}, -ωω_{6,3}, -ωω_{7,3}, -ωω_{8,3}},
 {ωω_{4,1}, ωω_{4,2}, ωω_{4,3}, 0, -ωω_{5,4}, -ωω_{6,4}, -ωω_{7,4}, -ωω_{8,4}},
 {ωω_{5,1}, ωω_{5,2}, ωω_{5,3}, ωω_{5,4}, 0, -ωω_{6,5}, -ωω_{7,5}, -ωω_{8,5}},
 {ωω_{6,1}, ωω_{6,2}, ωω_{6,3}, ωω_{6,4}, ωω_{6,5}, 0, -ωω_{7,6}, -ωω_{8,6}},
 {ωω_{7,1}, ωω_{7,2}, ωω_{7,3}, ωω_{7,4}, ωω_{7,5}, ωω_{7,6}, 0, -ωω_{8,7}},
 {ωω_{8,1}, ωω_{8,2}, ωω_{8,3}, ωω_{8,4}, ωω_{8,5}, ωω_{8,6}, ωω_{8,7}, 0}}
```

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2}\left(\frac{\partial g_{\mu\alpha}}{\partial x^{\beta}} + \frac{\partial g_{\mu\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\mu}}\right),$$

$$\Gamma^{\mu}{}_{\alpha\beta} = g^{\mu\nu}\Gamma_{\nu\alpha\beta},$$

$$R^{\mu}{}_{\nu\alpha\beta} = \frac{\partial \Gamma^{\mu}{}_{\nu\beta}}{\partial x^{\alpha}} - \frac{\partial \Gamma^{\mu}{}_{\nu\alpha}}{\partial x^{\beta}} + \Gamma^{\mu}{}_{\rho\alpha}\Gamma^{\rho}{}_{\nu\beta} - \Gamma^{\mu}{}_{\rho\beta}\Gamma^{\rho}{}_{\nu\alpha}.$$

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}, \qquad R = g^{\mu\nu}R_{\mu\nu},$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

In[214]:=

```
(*Christoffel, Einstein tensor, ...*)
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, ΓΓ, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    ΓΓ = FullSimplify[Table[(1 / 2) * Sum[(ginv⟦i, s⟧) *
          (D[g⟦s, j⟧, X⟦k⟧ ] + D[g⟦s, k⟧, X⟦j⟧ ] - D[g⟦j, k⟧, X⟦s⟧ ]), {s, 1, DIM8
       {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] , ass];
    RicciΓ =
     ParallelTable[FullSimplify[D[ΓΓ⟦μ, ν, β⟧, X⟦α⟧ ] - D[ΓΓ⟦μ, ν, α⟧, X⟦β⟧ ] +
        Sum[ΓΓ⟦μ, s, α⟧ × ΓΓ⟦s, ν, β⟧ - ΓΓ⟦μ, s, β⟧ × ΓΓ⟦s, ν, α⟧,
         {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}] ;
    RieΓ = ParallelTable[ FullSimplify[Sum[ RicciΓ⟦α, μ, α, ν⟧,
        {α, 1, DIM8}], ass], {μ, 1, DIM8}, {ν, 1, DIM8}] ;
    RS = FullSimplify[Tr[ ginv.RieΓ] , ass];
    G = ParallelTable[
      FullSimplify[RieΓ⟦α⟧ - 1/2 g⟦α⟧ RS, ass], {α, 1, Length[RieΓ]}];];
  Print[t];
  Print[Now];
  Return[{ginv, ΓΓ, RicciΓ, RieΓ, RS, G}]]
```

If ‘paste’ Maple MathML to ToExpression[MathMLForm[]] fails, then try to edit this :

In[216]:=

```
Clear[mapleztStringToMathematica];
mapleztStringToMathematica[
  input_String, toExpression_ : False, debug_ : False] :=
 Module[{prepare, post, custom, use = input, save = input, linenumber = 0},
  (*use=ToString[InputForm[TextString[input]]];*)(*Print[use];*)
  prepare[in_String] := Module[{s = in}, linenumber ++;
    s = in;
    (*If[s==ToString[InputForm[TextString[s]]],
      (*##&[]*)Print[s],Print[ToUpperCase[s]]];*)
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,z)" →
        "D[Z[" <> ToString[j] <> "][z,t],{z,2}]"], {j, 0, 15}];
    Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,t)" →
        "D[D[Z[" <> ToString[j] <> "][z,t],z],t]"], {j, 0, 15}];
```

```
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,z)" →
      "D[D[Z[" <> ToString[j] <> "][z,t],t],z]"], {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,t)" →
      "D[Z[" <> ToString[j] <> "][z,t],{t,2}]"], {j, 0, 15}];

  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t)" →
      "D[Z[" <> ToString[j] <> "][z,t],t]"], {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z)" →
      "D[Z[" <> ToString[j] <> "][z,t],z]"], {j, 0, 15}];

  s = StringReplace[s, "tan(z)" → "Tan[z]"];
  s = StringReplace[s, "cot(z)" → "Cot[z]"];
  s = StringReplace[s, "sin(z)" → "Sin[z]"];
  s = StringReplace[s, "cos(z)" → "Cos[z]"];
  s = StringReplace[s, "sec(z)" → "Sec[z]"];
  s = StringReplace[s, "csc(z)" → "Csc[z]"];

  s = StringReplace[s, "=" → "->"];

  (*Do[s=StringReplace[s,
      "Z"<>ToString[j]<>"(z,t)"→"Z["<>ToString[j]<>"[z,t]"],{j,0,15}];*)
  (*s=StringReplace[s,"diff(a4(t),t)"→"D[a4[t],t]"];*)
  (*s=StringReplace[s,"exp(-2*a4(t))"→"Exp[-2a4[t]]"];*)
  (*s=StringReplace[s,"a4(t))"→"a4[t]"];*)

  Return[s];
 ];
post[in_String] := Module[{s = in}, linenumber++;
  s = in;
  s = StringReplace[s, "a4(t))" → "a4[t]"];
  Do[s = StringReplace[s, "Z" <> ToString[j] <> "(z,t)" →
      "Z[" <> ToString[j] <> "][z,t]"], {j, 0, 15}];
  Return[s];
 ];
custom[in_String] := Module[{s = in}, linenumber++;
  s = in;

  s = StringReplace[s, "diff(a4(t),t)" → "D[a4[t],t]"];
  s = StringReplace[s, "exp(-2*a4(t))" → "Exp[-2a4[t]]"];
```

```
      Return[s];
    ];
  use = prepare[use];
  use = custom[use];
  Return[post[use]];
  (*If[toExpression,Return[ToExpression[s/.{z→#1,t→#2}],##&[]];*)
 ]
```

---

gtry and Γ and ...

In[218]:=

```
gtry = MatrixMetric44
```

Out[218]=

```
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0},
 {0, 0, ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0},
 {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0},
 {0, 0, 0, 0, 0, 0, -ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3)}}
```

In[219]:=

```
Protect[gtry]
```

Out[219]=

```
{gtry}
```

In[220]:=

```
Unprotect[ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG]
```

Out[220]=

```
{}
```

In[221]:=

```
result = {ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG} = rt[gtry];
```

Wed 11 Feb 2026 05:17:54 GMT−8

```
{3.3646, Null}
```

Wed 11 Feb 2026 05:17:58 GMT−8

In[222]:=

```
Protect[ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG]
```

Out[222]=

```
{ginv, Γ, RicciΓ, RieΓ, RS, EinsteinG}
```

verify $g_{\mu\nu\,|\alpha} = 0$:

In[223]:=

```
Simplify[
   Table[D[gtry⟦j, k⟧, X⟦i⟧ ] - Sum[ gtry⟦k, s⟧ × Γ⟦s, i, j⟧, {s, 1, DIM8}] -
     Sum[ gtry⟦j, s⟧ × Γ⟦s, i, k⟧, {s, 1, DIM8}],
    {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Out[223]=

```
{0}
```

In[224]:=

```
Table[g[j][k] → If[j == k,
    ToExpression["((g[" <> ToString[j] <> "][" <> ToString[k] <> "][#1,#2])&)"],
    (0 &)], {j, 0, 7}, {k, 0, 7}];
sg = % // Flatten
```

Out[225]=

```
{g[0][0] → (g[0][0][#1, #2] &), g[0][1] → (0 &), g[0][2] → (0 &),
 g[0][3] → (0 &), g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &),
 g[0][7] → (0 &), g[1][0] → (0 &), g[1][1] → (g[1][1][#1, #2] &), g[1][2] → (0 &),
 g[1][3] → (0 &), g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &),
 g[1][7] → (0 &), g[2][0] → (0 &), g[2][1] → (0 &), g[2][2] → (g[2][2][#1, #2] &),
 g[2][3] → (0 &), g[2][4] → (0 &), g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &),
 g[3][0] → (0 &), g[3][1] → (0 &), g[3][2] → (0 &), g[3][3] → (g[3][3][#1, #2] &),
 g[3][4] → (0 &), g[3][5] → (0 &), g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &),
 g[4][1] → (0 &), g[4][2] → (0 &), g[4][3] → (0 &), g[4][4] → (g[4][4][#1, #2] &),
 g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &), g[5][0] → (0 &), g[5][1] → (0 &),
 g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &), g[5][5] → (g[5][5][#1, #2] &),
 g[5][6] → (0 &), g[5][7] → (0 &), g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &),
 g[6][3] → (0 &), g[6][4] → (0 &), g[6][5] → (0 &), g[6][6] → (g[6][6][#1, #2] &),
 g[6][7] → (0 &), g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &),
 g[7][4] → (0 &), g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (g[7][7][#1, #2] &)}
```

In[226]:=

```
g[7][7][x0, x4] /. ssgm4488
% /. sx0x4
```

Out[226]=

```
-ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3)
```

Out[227]=

```
-ⅇ^(-2 a4[t]) Sin[z]^(1/3)
```

In[228]:=

```
MatrixMetric44
```

Out[228]=

```
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0},
 {0, 0, ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0},
 {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0},
 {0, 0, 0, 0, 0, 0, -ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3)}}
```

In[229]:=

```
ssgm4488[x0, x4] /. sx0x4
```

Out[229]=

```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
  g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
  g[1][1] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
  g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
  g[2][1] → (0 &), g[2][2] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
  g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
  g[3][2] → (0 &), g[3][3] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
  g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
  g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
  g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
  g[5][5] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
  g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
  g[6][5] → (0 &), g[6][6] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
  g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
  g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &)}[z/(6 H), t/H]
```

In[230]:=

```
(*ssgGzt={G[0][0]→((Cot[#1]^2)&),G[0][1]→((0)&),G[0][2]→((0)&),
  G[0][3]→((0)&),G[0][4]→((0)&),G[0][5]→((0)&),G[0][6]→((0)&),
  G[0][7]→((0)&),G[1][0]→((0)&),G[1][1]→((ⅇ^(2*A4[#2]) Sin[#1]^(1/3))&),
  G[1][2]→((0)&),G[1][3]→((0)&),G[1][4]→((0)&),G[1][5]→((0)&),
  G[1][6]→((0)&),G[1][7]→((0)&),G[2][0]→((0)&),G[2][1]→((0)&),
  G[2][2]→((ⅇ^(2*A4[#2]) Sin[#1]^(1/3))&),G[2][3]→((0)&),G[2][4]→((0)&),
  G[2][5]→((0)&),G[2][6]→((0)&),G[2][7]→((0)&),G[3][0]→((0)&),
  G[3][1]→((0)&),G[3][2]→((0)&),G[3][3]→((ⅇ^(2*A4[#2]) Sin[#1]^(1/3))&),
  G[3][4]→((0)&),G[3][5]→((0)&),G[3][6]→((0)&),G[3][7]→((0)&),
  G[4][0]→((0)&),G[4][1]→((0)&),G[4][2]→((0)&),G[4][3]→((0)&),
  G[4][4]→((-1)&),G[4][5]→((0)&),G[4][6]→((0)&),G[4][7]→((0)&),
  G[5][0]→((0)&),G[5][1]→((0)&),G[5][2]→((0)&),G[5][3]→((0)&),
  G[5][4]→((0)&),G[5][5]→((-ⅇ^(-2*A4[#2]) Sin[#1]^(1/3))&),
  G[5][6]→((0)&),G[5][7]→((0)&),G[6][0]→((0)&),G[6][1]→((0)&),
  G[6][2]→((0)&),G[6][3]→((0)&),G[6][4]→((0)&),G[6][5]→((0)&),
  G[6][6]→((-ⅇ^(-2*A4[#2]) Sin[#1]^(1/3))&),G[6][7]→((0)&),G[7][0]→((0)&),
  G[7][1]→((0)&),G[7][2]→((0)&),G[7][3]→((0)&),G[7][4]→((0)&),
  G[7][5]→((0)&),G[7][6]→((0)&),G[7][7]→((-ⅇ^(-2*A4[#2]) Sin[#1]^(1/3))&)}*)
```

In[231]:=

```
(*G[#][#][z,t]/.ssgGzt&/@Range[0,7]*)
```

In[232]:=

```
detgg = Det[g4488 /. sg] // FullSimplify[#, constraintVars] &
```

Out[232]=

```
g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
 g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]
```

In[233]:=

```
g4488 /. sg // MatrixForm
```

Out[233]//MatrixForm=

```
( g[0][0][x0, x4]         0                  0                  0                  0                  0
         0         g[1][1][x0, x4]           0                  0                  0                  0
         0                0          g[2][2][x0, x4]            0                  0                  0
         0                0                  0           g[3][3][x0, x4]           0                  0
         0                0                  0                  0           g[4][4][x0, x4]           0
         0                0                  0                  0                  0           g[5][5][x0
         0                0                  0                  0                  0                  0
         0                0                  0                  0                  0                  0
```

In[234]:=

```
MatrixMetric44.η4488
(*eAa=*)FullSimplify[√%, constraintVars]
```

Out[234]=

```
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0},
 {0, 0, e^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0},
 {0, 0, 0, 0, 0, 0, e^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, e^(-2 a4[H x4]) Sin[6 H x0]^(1/3)}}
```

Out[235]=

```
{{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0}, {0, √(e^(2 a4[H x4])) Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0},
 {0, 0, √(e^(2 a4[H x4])) Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, √(e^(2 a4[H x4])) Sin[6 H x0]^(1/6), 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, √(e^(-2 a4[H x4])) Sin[6 H x0]^(1/6), 0, 0},
 {0, 0, 0, 0, 0, 0, √(e^(-2 a4[H x4])) Sin[6 H x0]^(1/6), 0},
 {0, 0, 0, 0, 0, 0, 0, √(e^(-2 a4[H x4])) Sin[6 H x0]^(1/6)}}
eAa
```

In[236]:=

```
g4488.η4488 /. sg
(eAa = FullSimplify[√%, constraintVars]) // MatrixForm
```

Out[236]=

```
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, -g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, -g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, -g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, -g[7][7][x0, x4]}}
```

Out[237]//MatrixForm=

```
( √g[0][0][x0, x4]          0                   0                   0                   0
          0          √g[1][1][x0, x4]          0                   0                   0
          0                   0          √g[2][2][x0, x4]          0                   0
          0                   0                   0          √g[3][3][x0, x4]          0
          0                   0                   0                   0          √-g[4][4][x0, x4]
          0                   0                   0                   0                   0
          0                   0                   0                   0                   0
          0                   0                   0                   0                   0
```

```
In[238]:=
eAa
ssgm4488
(eAa04 = %% /. % // FullSimplify[#, constraintVars] &) // MatrixForm
```

```
Out[238]=
{{√g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, √g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, √g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, √g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, √-g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, √-g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, √-g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, √-g[7][7][x0, x4]}}
```

```
Out[239]=
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
 g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
 g[1][1] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
 g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
 g[2][1] → (0 &), g[2][2] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
 g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
 g[3][2] → (0 &), g[3][3] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
 g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
 g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
 g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
 g[5][5] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
 g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
 g[6][5] → (0 &), g[6][6] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
 g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
 g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &)}
```

```
Out[240]//MatrixForm=
( Cot[6 H x0]   0                                  0                                  0                                  0
  0             √ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/6)  0                                  0                                  0
  0             0                                  √ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/6)  0                                  0
  0             0                                  0                                  √ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/6)  0
  0             0                                  0                                  0                                  1
  0             0                                  0                                  0                                  0  √ⅇ^(-2 a4
  0             0                                  0                                  0                                  0
  0             0                                  0                                  0                                  0
```

OCTAD $e_{\alpha\leftarrow\text{spacetime}}^{(A)\longleftarrow\text{Lorentz}}$:

- **$e_\alpha{}^{(A)} = e_\alpha^{(A)}$ = eAa**

In[241]:=

```
(sge_α^(A) = Thread[Flatten[e_α^(A)] → Flatten[eAa]]) (*//Column*)
```

Out[241]=

```
{E_α^(A)[0, 0] → √(g[0][0][x0, x4]), E_α^(A)[0, 1] → 0, E_α^(A)[0, 2] → 0, E_α^(A)[0, 3] → 0, E_α^(A)[0, 4] → 0,
 E_α^(A)[0, 5] → 0, E_α^(A)[0, 6] → 0, E_α^(A)[0, 7] → 0, E_α^(A)[1, 0] → 0, E_α^(A)[1, 1] → √(g[1][1][x0, x4]),
 E_α^(A)[1, 2] → 0, E_α^(A)[1, 3] → 0, E_α^(A)[1, 4] → 0, E_α^(A)[1, 5] → 0, E_α^(A)[1, 6] → 0, E_α^(A)[1, 7] → 0,
 E_α^(A)[2, 0] → 0, E_α^(A)[2, 1] → 0, E_α^(A)[2, 2] → √(g[2][2][x0, x4]), E_α^(A)[2, 3] → 0, E_α^(A)[2, 4] → 0,
 E_α^(A)[2, 5] → 0, E_α^(A)[2, 6] → 0, E_α^(A)[2, 7] → 0, E_α^(A)[3, 0] → 0, E_α^(A)[3, 1] → 0, E_α^(A)[3, 2] → 0,
 E_α^(A)[3, 3] → √(g[3][3][x0, x4]), E_α^(A)[3, 4] → 0, E_α^(A)[3, 5] → 0, E_α^(A)[3, 6] → 0, E_α^(A)[3, 7] → 0,
 E_α^(A)[4, 0] → 0, E_α^(A)[4, 1] → 0, E_α^(A)[4, 2] → 0, E_α^(A)[4, 3] → 0, E_α^(A)[4, 4] → √(-g[4][4][x0, x4]),
 E_α^(A)[4, 5] → 0, E_α^(A)[4, 6] → 0, E_α^(A)[4, 7] → 0, E_α^(A)[5, 0] → 0, E_α^(A)[5, 1] → 0, E_α^(A)[5, 2] → 0,
 E_α^(A)[5, 3] → 0, E_α^(A)[5, 4] → 0, E_α^(A)[5, 5] → √(-g[5][5][x0, x4]), E_α^(A)[5, 6] → 0, E_α^(A)[5, 7] → 0,
 E_α^(A)[6, 0] → 0, E_α^(A)[6, 1] → 0, E_α^(A)[6, 2] → 0, E_α^(A)[6, 3] → 0, E_α^(A)[6, 4] → 0, E_α^(A)[6, 5] → 0,
 E_α^(A)[6, 6] → √(-g[6][6][x0, x4]), E_α^(A)[6, 7] → 0, E_α^(A)[7, 0] → 0, E_α^(A)[7, 1] → 0, E_α^(A)[7, 2] → 0,
 E_α^(A)[7, 3] → 0, E_α^(A)[7, 4] → 0, E_α^(A)[7, 5] → 0, E_α^(A)[7, 6] → 0, E_α^(A)[7, 7] → √(-g[7][7][x0, x4])}
```

In[242]:=

```
(*Symbolize[sge_α^(A)]*)
```

In[243]:=

```
(*Symbolize[sgtrye_α^(A)]*)
```

In[244]:=

```
(sgtrye_α^(A) = Thread[Flatten[e_α^(A)] → Flatten[eAa04]]) (*//Column*)
```

Out[244]=

```
{E_α^(A)[0, 0] → Cot[6 H x0], E_α^(A)[0, 1] → 0, E_α^(A)[0, 2] → 0, E_α^(A)[0, 3] → 0,
 E_α^(A)[0, 4] → 0, E_α^(A)[0, 5] → 0, E_α^(A)[0, 6] → 0, E_α^(A)[0, 7] → 0, E_α^(A)[1, 0] → 0,
 E_α^(A)[1, 1] → √(ⅇ^(2 a4[H x4])) Sin[6 H x0]^(1/6), E_α^(A)[1, 2] → 0, E_α^(A)[1, 3] → 0, E_α^(A)[1, 4] → 0,
 E_α^(A)[1, 5] → 0, E_α^(A)[1, 6] → 0, E_α^(A)[1, 7] → 0, E_α^(A)[2, 0] → 0, E_α^(A)[2, 1] → 0,
 E_α^(A)[2, 2] → √(ⅇ^(2 a4[H x4])) Sin[6 H x0]^(1/6), E_α^(A)[2, 3] → 0, E_α^(A)[2, 4] → 0, E_α^(A)[2, 5] → 0,
 E_α^(A)[2, 6] → 0, E_α^(A)[2, 7] → 0, E_α^(A)[3, 0] → 0, E_α^(A)[3, 1] → 0, E_α^(A)[3, 2] → 0,
 E_α^(A)[3, 3] → √(ⅇ^(2 a4[H x4])) Sin[6 H x0]^(1/6), E_α^(A)[3, 4] → 0, E_α^(A)[3, 5] → 0, E_α^(A)[3, 6] → 0,
 E_α^(A)[3, 7] → 0, E_α^(A)[4, 0] → 0, E_α^(A)[4, 1] → 0, E_α^(A)[4, 2] → 0, E_α^(A)[4, 3] → 0,
 E_α^(A)[4, 4] → 1, E_α^(A)[4, 5] → 0, E_α^(A)[4, 6] → 0, E_α^(A)[4, 7] → 0, E_α^(A)[5, 0] → 0, E_α^(A)[5, 1] → 0,
 E_α^(A)[5, 2] → 0, E_α^(A)[5, 3] → 0, E_α^(A)[5, 4] → 0, E_α^(A)[5, 5] → √(ⅇ^(-2 a4[H x4])) Sin[6 H x0]^(1/6),
 E_α^(A)[5, 6] → 0, E_α^(A)[5, 7] → 0, E_α^(A)[6, 0] → 0, E_α^(A)[6, 1] → 0, E_α^(A)[6, 2] → 0,
 E_α^(A)[6, 3] → 0, E_α^(A)[6, 4] → 0, E_α^(A)[6, 5] → 0, E_α^(A)[6, 6] → √(ⅇ^(-2 a4[H x4])) Sin[6 H x0]^(1/6),
 E_α^(A)[6, 7] → 0, E_α^(A)[7, 0] → 0, E_α^(A)[7, 1] → 0, E_α^(A)[7, 2] → 0, E_α^(A)[7, 3] → 0,
 E_α^(A)[7, 4] → 0, E_α^(A)[7, 5] → 0, E_α^(A)[7, 6] → 0, E_α^(A)[7, 7] → √(ⅇ^(-2 a4[H x4])) Sin[6 H x0]^(1/6)}
```

In[245]:=

$$\left(\texttt{sgtrye}^{\alpha}_{\;(A)} = \texttt{Thread}\left[\texttt{Flatten}\left[\texttt{e}^{\alpha}_{\;(A)}\right] \rightarrow \texttt{Flatten}\left[\texttt{Inverse}\left[\texttt{e}_{\alpha}^{\;(A)}\;\texttt{/.}\;\texttt{sgtrye}_{\alpha}^{\;(A)}\right]\right]\right]\right)$$

`(*//Column*)`

Out[245]=

$$\Big\{E^{\alpha}_{(A)}[0,0]\rightarrow \text{Tan}[6\,H\,x0],\ E^{\alpha}_{(A)}[0,1]\rightarrow 0,\ E^{\alpha}_{(A)}[0,2]\rightarrow 0,\ E^{\alpha}_{(A)}[0,3]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[0,4]\rightarrow 0,\ E^{\alpha}_{(A)}[0,5]\rightarrow 0,\ E^{\alpha}_{(A)}[0,6]\rightarrow 0,\ E^{\alpha}_{(A)}[0,7]\rightarrow 0,\ E^{\alpha}_{(A)}[1,0]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[1,1]\rightarrow \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[1,2]\rightarrow 0,\ E^{\alpha}_{(A)}[1,3]\rightarrow 0,\ E^{\alpha}_{(A)}[1,4]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[1,5]\rightarrow 0,\ E^{\alpha}_{(A)}[1,6]\rightarrow 0,\ E^{\alpha}_{(A)}[1,7]\rightarrow 0,\ E^{\alpha}_{(A)}[2,0]\rightarrow 0,\ E^{\alpha}_{(A)}[2,1]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[2,2]\rightarrow \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[2,3]\rightarrow 0,\ E^{\alpha}_{(A)}[2,4]\rightarrow 0,\ E^{\alpha}_{(A)}[2,5]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[2,6]\rightarrow 0,\ E^{\alpha}_{(A)}[2,7]\rightarrow 0,\ E^{\alpha}_{(A)}[3,0]\rightarrow 0,\ E^{\alpha}_{(A)}[3,1]\rightarrow 0,\ E^{\alpha}_{(A)}[3,2]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[3,3]\rightarrow \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},\ E^{\alpha}_{(A)}[3,4]\rightarrow 0,\ E^{\alpha}_{(A)}[3,5]\rightarrow 0,\ E^{\alpha}_{(A)}[3,6]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[3,7]\rightarrow 0,\ E^{\alpha}_{(A)}[4,0]\rightarrow 0,\ E^{\alpha}_{(A)}[4,1]\rightarrow 0,\ E^{\alpha}_{(A)}[4,2]\rightarrow 0,\ E^{\alpha}_{(A)}[4,3]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[4,4]\rightarrow 1,\ E^{\alpha}_{(A)}[4,5]\rightarrow 0,\ E^{\alpha}_{(A)}[4,6]\rightarrow 0,\ E^{\alpha}_{(A)}[4,7]\rightarrow 0,\ E^{\alpha}_{(A)}[5,0]\rightarrow 0,\ E^{\alpha}_{(A)}[5,1]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[5,2]\rightarrow 0,\ E^{\alpha}_{(A)}[5,3]\rightarrow 0,\ E^{\alpha}_{(A)}[5,4]\rightarrow 0,\ E^{\alpha}_{(A)}[5,5]\rightarrow \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},$$

$$E^{\alpha}_{(A)}[5,6]\rightarrow 0,\ E^{\alpha}_{(A)}[5,7]\rightarrow 0,\ E^{\alpha}_{(A)}[6,0]\rightarrow 0,\ E^{\alpha}_{(A)}[6,1]\rightarrow 0,\ E^{\alpha}_{(A)}[6,2]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[6,3]\rightarrow 0,\ E^{\alpha}_{(A)}[6,4]\rightarrow 0,\ E^{\alpha}_{(A)}[6,5]\rightarrow 0,\ E^{\alpha}_{(A)}[6,6]\rightarrow \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}},$$

$$E^{\alpha}_{(A)}[6,7]\rightarrow 0,\ E^{\alpha}_{(A)}[7,0]\rightarrow 0,\ E^{\alpha}_{(A)}[7,1]\rightarrow 0,\ E^{\alpha}_{(A)}[7,2]\rightarrow 0,\ E^{\alpha}_{(A)}[7,3]\rightarrow 0,$$

$$E^{\alpha}_{(A)}[7,4]\rightarrow 0,\ E^{\alpha}_{(A)}[7,5]\rightarrow 0,\ E^{\alpha}_{(A)}[7,6]\rightarrow 0,\ E^{\alpha}_{(A)}[7,7]\rightarrow \frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}}\Big\}$$

In[246]:=

$$\texttt{gtrye}_{\alpha}^{\;(A)} = \left(\texttt{e}_{\alpha}^{\;(A)}\;\texttt{/.}\;\texttt{sgtrye}_{\alpha}^{\;(A)}\right)\;\texttt{/.}\;\texttt{subsDefects}$$

Out[246]=

$$\big\{\{\text{Cot}[6\,H\,x0],0,0,0,0,0,0,0\},\ \{0,\ e^{a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0,0\},$$

$$\{0,0,\ e^{a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0\},\ \{0,0,0,\ e^{a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},0,0,0,0\},$$

$$\{0,0,0,0,1,0,0,0\},\ \{0,0,0,0,0,\ e^{-a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},0,0\},$$

$$\{0,0,0,0,0,0,\ e^{-a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},0\},\ \{0,0,0,0,0,0,0,\ e^{-a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6}\}\big\}$$

In[247]:=

$$\texttt{gtrye}^{\alpha}_{\;(A)} = \left(\texttt{e}^{\alpha}_{\;(A)}\;\texttt{/.}\;\texttt{sgtrye}^{\alpha}_{\;(A)}\right)\;\texttt{/.}\;\texttt{subsDefects}$$

Out[247]=

$$\Big\{\{\text{Tan}[6\,H\,x0],0,0,0,0,0,0,0\},\ \Big\{0,\ \frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0,0\Big\},$$

$$\Big\{0,0,\ \frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0\Big\},\ \Big\{0,0,0,\ \frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0\Big\},$$

$$\{0,0,0,0,1,0,0,0\},\ \Big\{0,0,0,0,0,\ \frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0\Big\},$$

$$\Big\{0,0,0,0,0,0,\ \frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0\Big\},\ \Big\{0,0,0,0,0,0,0,\ \frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}}\Big\}\Big\}$$

In[248]:=

```
e_α^(A)
% /. sgtrye_α^(A) // MatrixForm
```

Out[248]=

```
{{E_α^(A)[0, 0], E_α^(A)[0, 1], E_α^(A)[0, 2], E_α^(A)[0, 3], E_α^(A)[0, 4], E_α^(A)[0, 5], E_α^(A)[0, 6], E_α^(A)[0, 7]},
 {E_α^(A)[1, 0], E_α^(A)[1, 1], E_α^(A)[1, 2], E_α^(A)[1, 3], E_α^(A)[1, 4], E_α^(A)[1, 5], E_α^(A)[1, 6], E_α^(A)[1, 7]},
 {E_α^(A)[2, 0], E_α^(A)[2, 1], E_α^(A)[2, 2], E_α^(A)[2, 3], E_α^(A)[2, 4], E_α^(A)[2, 5], E_α^(A)[2, 6], E_α^(A)[2, 7]},
 {E_α^(A)[3, 0], E_α^(A)[3, 1], E_α^(A)[3, 2], E_α^(A)[3, 3], E_α^(A)[3, 4], E_α^(A)[3, 5], E_α^(A)[3, 6], E_α^(A)[3, 7]},
 {E_α^(A)[4, 0], E_α^(A)[4, 1], E_α^(A)[4, 2], E_α^(A)[4, 3], E_α^(A)[4, 4], E_α^(A)[4, 5], E_α^(A)[4, 6], E_α^(A)[4, 7]},
 {E_α^(A)[5, 0], E_α^(A)[5, 1], E_α^(A)[5, 2], E_α^(A)[5, 3], E_α^(A)[5, 4], E_α^(A)[5, 5], E_α^(A)[5, 6], E_α^(A)[5, 7]},
 {E_α^(A)[6, 0], E_α^(A)[6, 1], E_α^(A)[6, 2], E_α^(A)[6, 3], E_α^(A)[6, 4], E_α^(A)[6, 5], E_α^(A)[6, 6], E_α^(A)[6, 7]},
 {E_α^(A)[7, 0], E_α^(A)[7, 1], E_α^(A)[7, 2], E_α^(A)[7, 3], E_α^(A)[7, 4], E_α^(A)[7, 5], E_α^(A)[7, 6], E_α^(A)[7, 7]}}
```

Out[249]//MatrixForm=

```
( Cot[6 H x0]              0                               0                               0                               0
       0      Sqrt[e^(2 a4[H x4])] Sin[6 H x0]^(1/6)       0                               0                               0
       0                   0              Sqrt[e^(2 a4[H x4])] Sin[6 H x0]^(1/6)           0                               0
       0                   0                               0              Sqrt[e^(2 a4[H x4])] Sin[6 H x0]^(1/6)           0
       0                   0                               0                               0                               1
       0                   0                               0                               0                               0   Sqrt[e^(-2 a4
       0                   0                               0                               0                               0
       0                   0                               0                               0                               0
```

In[250]:=

```
eηe = FullSimplify[Transpose[e_α^(A)].η4488.e_α^(A), constraintVars];
% /. sge_α^(A)
%% /. sgtrye_α^(A)
% == gtry // FullSimplify[#, constraintVars] &
```

Out[251]=

```
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}
```

Out[252]=

```
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0},
 {0, 0, e^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0},
 {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0},
 {0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) Sin[6 H x0]^(1/3)}}
```

Out[253]=

```
True
```

In[254]:=

```
gtry
```

Out[254]=

```
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0},
 {0, 0, e^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0},
 {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0},
 {0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) Sin[6 H x0]^(1/3)}}
```

In[255]:=

```
(*FullSimplify[Transpose[e_α^(A)].η4488.e_α^(A),constraintVars]-MatrixMetric44*)
```

In[256]:=

```
ssgm4488
```

Out[256]=

```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
 g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
 g[1][1] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
 g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
 g[2][1] → (0 &), g[2][2] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
 g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
 g[3][2] → (0 &), g[3][3] → (ⅇ^(2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
 g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
 g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
 g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
 g[5][5] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
 g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
 g[6][5] → (0 &), g[6][6] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
 g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
 g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-ⅇ^(-2 a4[H #2]) Sin[6 H #1]^(1/3) &)}
```

In[257]:=

```
MatrixMetric44 // MatrixForm
```

Out[257]//MatrixForm=

```
( Cot[6 H x0]^2            0                            0                            0                        0
      0        ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3)         0                            0                        0
      0                    0            ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3)             0                        0
      0                    0                            0           ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3)          0
      0                    0                            0                            0                       -1
      0                    0                            0                            0                        0   -ⅇ^(-2 a4[H x4]) S
      0                    0                            0                            0                        0
      0                    0                            0                            0                        0
```

In[258]:=

```
e_α^(A)
```

Out[258]=

```
{{E_α^(A)[0, 0], E_α^(A)[0, 1], E_α^(A)[0, 2], E_α^(A)[0, 3], E_α^(A)[0, 4], E_α^(A)[0, 5], E_α^(A)[0, 6], E_α^(A)[0, 7]},
 {E_α^(A)[1, 0], E_α^(A)[1, 1], E_α^(A)[1, 2], E_α^(A)[1, 3], E_α^(A)[1, 4], E_α^(A)[1, 5], E_α^(A)[1, 6], E_α^(A)[1, 7]},
 {E_α^(A)[2, 0], E_α^(A)[2, 1], E_α^(A)[2, 2], E_α^(A)[2, 3], E_α^(A)[2, 4], E_α^(A)[2, 5], E_α^(A)[2, 6], E_α^(A)[2, 7]},
 {E_α^(A)[3, 0], E_α^(A)[3, 1], E_α^(A)[3, 2], E_α^(A)[3, 3], E_α^(A)[3, 4], E_α^(A)[3, 5], E_α^(A)[3, 6], E_α^(A)[3, 7]},
 {E_α^(A)[4, 0], E_α^(A)[4, 1], E_α^(A)[4, 2], E_α^(A)[4, 3], E_α^(A)[4, 4], E_α^(A)[4, 5], E_α^(A)[4, 6], E_α^(A)[4, 7]},
 {E_α^(A)[5, 0], E_α^(A)[5, 1], E_α^(A)[5, 2], E_α^(A)[5, 3], E_α^(A)[5, 4], E_α^(A)[5, 5], E_α^(A)[5, 6], E_α^(A)[5, 7]},
 {E_α^(A)[6, 0], E_α^(A)[6, 1], E_α^(A)[6, 2], E_α^(A)[6, 3], E_α^(A)[6, 4], E_α^(A)[6, 5], E_α^(A)[6, 6], E_α^(A)[6, 7]},
 {E_α^(A)[7, 0], E_α^(A)[7, 1], E_α^(A)[7, 2], E_α^(A)[7, 3], E_α^(A)[7, 4], E_α^(A)[7, 5], E_α^(A)[7, 6], E_α^(A)[7, 7]}}
```

In[259]:=

```
Block[{s, r}, s = FullSimplify[e_α^(A) /. sgtrye_α^(A), constraintVars];
 r = FullSimplify[Transpose[s].η4488.s, constraintVars] - MatrixMetric44;
 FullSimplify[r, constraintVars]]
```

Out[259]=

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[260]:=

```
(*(e_α^(A) =eAa)//MatrixForm*)
```

In[261]:=

```
(*e_α^(A)
 Inverse[%]*)
```

In[262]:=

```
(*(e^α_(A) =Inverse[e_α^(A) /.
     sge␣Subsuperscript␣α␣and␣LeftParenthesis␣A␣RightParenthesis])//
 MatrixForm*)
```

In[263]:=

```
e^α_(A)
```

Out[263]=

```
{{E^α_(A)[0, 0], E^α_(A)[0, 1], E^α_(A)[0, 2], E^α_(A)[0, 3], E^α_(A)[0, 4], E^α_(A)[0, 5], E^α_(A)[0, 6], E^α_(A)[0, 7]},
 {E^α_(A)[1, 0], E^α_(A)[1, 1], E^α_(A)[1, 2], E^α_(A)[1, 3], E^α_(A)[1, 4], E^α_(A)[1, 5], E^α_(A)[1, 6], E^α_(A)[1, 7]},
 {E^α_(A)[2, 0], E^α_(A)[2, 1], E^α_(A)[2, 2], E^α_(A)[2, 3], E^α_(A)[2, 4], E^α_(A)[2, 5], E^α_(A)[2, 6], E^α_(A)[2, 7]},
 {E^α_(A)[3, 0], E^α_(A)[3, 1], E^α_(A)[3, 2], E^α_(A)[3, 3], E^α_(A)[3, 4], E^α_(A)[3, 5], E^α_(A)[3, 6], E^α_(A)[3, 7]},
 {E^α_(A)[4, 0], E^α_(A)[4, 1], E^α_(A)[4, 2], E^α_(A)[4, 3], E^α_(A)[4, 4], E^α_(A)[4, 5], E^α_(A)[4, 6], E^α_(A)[4, 7]},
 {E^α_(A)[5, 0], E^α_(A)[5, 1], E^α_(A)[5, 2], E^α_(A)[5, 3], E^α_(A)[5, 4], E^α_(A)[5, 5], E^α_(A)[5, 6], E^α_(A)[5, 7]},
 {E^α_(A)[6, 0], E^α_(A)[6, 1], E^α_(A)[6, 2], E^α_(A)[6, 3], E^α_(A)[6, 4], E^α_(A)[6, 5], E^α_(A)[6, 6], E^α_(A)[6, 7]},
 {E^α_(A)[7, 0], E^α_(A)[7, 1], E^α_(A)[7, 2], E^α_(A)[7, 3], E^α_(A)[7, 4], E^α_(A)[7, 5], E^α_(A)[7, 6], E^α_(A)[7, 7]}}
```

In[264]:=

$$\left(\mathtt{sge}^{\alpha}_{(A)}=\mathtt{Thread}\left[\mathtt{Flatten}\left[e^{\alpha}_{(A)}\right]\to\mathtt{Flatten}\left[\mathtt{Inverse}\left[e_{\alpha}^{(A)}\ /.\ \mathtt{sge}_{\alpha}^{(A)}\right]\right]\right]\right)\ (*//\mathtt{Column}*)$$

Out[264]=

$$\left\{E^{\alpha}_{(A)}[0,0]\to\frac{1}{\sqrt{g[0][0][x0,x4]}},\ E^{\alpha}_{(A)}[0,1]\to 0,\ E^{\alpha}_{(A)}[0,2]\to 0,\ E^{\alpha}_{(A)}[0,3]\to 0,\ E^{\alpha}_{(A)}[0,4]\to 0,\right.$$

$$E^{\alpha}_{(A)}[0,5]\to 0,\ E^{\alpha}_{(A)}[0,6]\to 0,\ E^{\alpha}_{(A)}[0,7]\to 0,\ E^{\alpha}_{(A)}[1,0]\to 0,\ E^{\alpha}_{(A)}[1,1]\to\frac{1}{\sqrt{g[1][1][x0,x4]}},$$

$$E^{\alpha}_{(A)}[1,2]\to 0,\ E^{\alpha}_{(A)}[1,3]\to 0,\ E^{\alpha}_{(A)}[1,4]\to 0,\ E^{\alpha}_{(A)}[1,5]\to 0,\ E^{\alpha}_{(A)}[1,6]\to 0,\ E^{\alpha}_{(A)}[1,7]\to 0,$$

$$E^{\alpha}_{(A)}[2,0]\to 0,\ E^{\alpha}_{(A)}[2,1]\to 0,\ E^{\alpha}_{(A)}[2,2]\to\frac{1}{\sqrt{g[2][2][x0,x4]}},\ E^{\alpha}_{(A)}[2,3]\to 0,\ E^{\alpha}_{(A)}[2,4]\to 0,$$

$$E^{\alpha}_{(A)}[2,5]\to 0,\ E^{\alpha}_{(A)}[2,6]\to 0,\ E^{\alpha}_{(A)}[2,7]\to 0,\ E^{\alpha}_{(A)}[3,0]\to 0,\ E^{\alpha}_{(A)}[3,1]\to 0,\ E^{\alpha}_{(A)}[3,2]\to 0,$$

$$E^{\alpha}_{(A)}[3,3]\to\frac{1}{\sqrt{g[3][3][x0,x4]}},\ E^{\alpha}_{(A)}[3,4]\to 0,\ E^{\alpha}_{(A)}[3,5]\to 0,\ E^{\alpha}_{(A)}[3,6]\to 0,\ E^{\alpha}_{(A)}[3,7]\to 0,$$

$$E^{\alpha}_{(A)}[4,0]\to 0,\ E^{\alpha}_{(A)}[4,1]\to 0,\ E^{\alpha}_{(A)}[4,2]\to 0,\ E^{\alpha}_{(A)}[4,3]\to 0,\ E^{\alpha}_{(A)}[4,4]\to\frac{1}{\sqrt{-g[4][4][x0,x4]}},$$

$$E^{\alpha}_{(A)}[4,5]\to 0,\ E^{\alpha}_{(A)}[4,6]\to 0,\ E^{\alpha}_{(A)}[4,7]\to 0,\ E^{\alpha}_{(A)}[5,0]\to 0,\ E^{\alpha}_{(A)}[5,1]\to 0,\ E^{\alpha}_{(A)}[5,2]\to 0,$$

$$E^{\alpha}_{(A)}[5,3]\to 0,\ E^{\alpha}_{(A)}[5,4]\to 0,\ E^{\alpha}_{(A)}[5,5]\to\frac{1}{\sqrt{-g[5][5][x0,x4]}},\ E^{\alpha}_{(A)}[5,6]\to 0,\ E^{\alpha}_{(A)}[5,7]\to 0,$$

$$E^{\alpha}_{(A)}[6,0]\to 0,\ E^{\alpha}_{(A)}[6,1]\to 0,\ E^{\alpha}_{(A)}[6,2]\to 0,\ E^{\alpha}_{(A)}[6,3]\to 0,\ E^{\alpha}_{(A)}[6,4]\to 0,\ E^{\alpha}_{(A)}[6,5]\to 0,$$

$$E^{\alpha}_{(A)}[6,6]\to\frac{1}{\sqrt{-g[6][6][x0,x4]}},\ E^{\alpha}_{(A)}[6,7]\to 0,\ E^{\alpha}_{(A)}[7,0]\to 0,\ E^{\alpha}_{(A)}[7,1]\to 0,\ E^{\alpha}_{(A)}[7,2]\to 0,$$

$$\left.E^{\alpha}_{(A)}[7,3]\to 0,\ E^{\alpha}_{(A)}[7,4]\to 0,\ E^{\alpha}_{(A)}[7,5]\to 0,\ E^{\alpha}_{(A)}[7,6]\to 0,\ E^{\alpha}_{(A)}[7,7]\to\frac{1}{\sqrt{-g[7][7][x0,x4]}}\right\}$$

In[265]:=

$(*\mathtt{Symbolize}[\mathtt{sge}^{\alpha}_{(A)}]*)$

In[266]:=

$(*\mathtt{Symbolize}[\mathtt{sgtrye}^{\alpha}_{(A)}]*)$

In[267]:=

$(*\%/.\mathtt{sge}_{\alpha}^{(A)}$

$\%\%/.\mathtt{sgtrye}_{\alpha}^{(A)}*)$

In[268]:=

$(*\mathtt{FullSimplify}[\mathtt{Transpose}[e_{\alpha}^{(A)}].\eta 4488.e_{\alpha}^{(A)},\mathtt{constraintVars}]-\mathtt{MatrixMetric44}*)$

For Spin (4, 4); τ tau; T16; OCTAD : Nash; Introduce the wave function, Ψ16, for this Universe::

O(4,4); evalues, evecs of σ

In[269]:=

```
{evalues, evecs} = Eigensystem[σ]
```

Out[269]=

```
{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
  {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
  {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}
```

In[270]:=

$$\left(\text{u} = \text{ExpandAll}\left[\frac{1}{\sqrt{2}}\ \text{evecs}\right]\right)\ //\ \text{MatrixForm}$$

Out[270]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

In[271]:=

```
Table[u⟦h⟧.σ.u⟦h⟧, {h, 1, Length[u]}]
```

Out[271]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[272]:=

```
Table[(Transpose[u]⟦h⟧).σ.u⟦h⟧, {h, 1, Length[u]}]
```

Out[272]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[273]:=

```
Table[(Transpose[u]⟦A⟧) == -σ.u⟦A⟧, {A, 1, 4}]
Table[(Transpose[u]⟦A⟧) == σ.u⟦A⟧, {A, 5, Length[u]}]
```

Out[273]=

```
{True, True, True, True}
```

Out[274]=

```
{True, True, True, True}
```

$$\text{Ax} = \left(\frac{\text{KroneckerProduct}[\text{dX},\ \text{X}]\ -\ \text{KroneckerProduct}[\text{X},\ \text{dX}]}{2}\right).\sigma;$$

$$\text{Au} = \left(\frac{\text{KroneckerProduct}[\text{dU},\ \text{U}]\ -\ \text{KroneckerProduct}[\text{U},\ \text{dU}]}{2}\right).\sigma;$$

$$\text{Lsquared} = \dot{x}^{A}\ \dot{x}_{A} = \text{dX}.\sigma\ .\text{dX} +$$

$$X.\sigma.X\left(\frac{dU.\sigma.dU}{U.\sigma.U}-\left(\frac{U.\sigma.dU}{U.\sigma.U}\right)^{2}\right)+\frac{2}{U.\sigma.U}\,\mathrm{Tr}\left[\tau^{A}.Ax.\tau_{A}.Au\right]$$

```
Timing[Simplify[ExpandAll[dY.η8.dY - Lsquared]]]
{10.733, 0}
```

In[275]:=

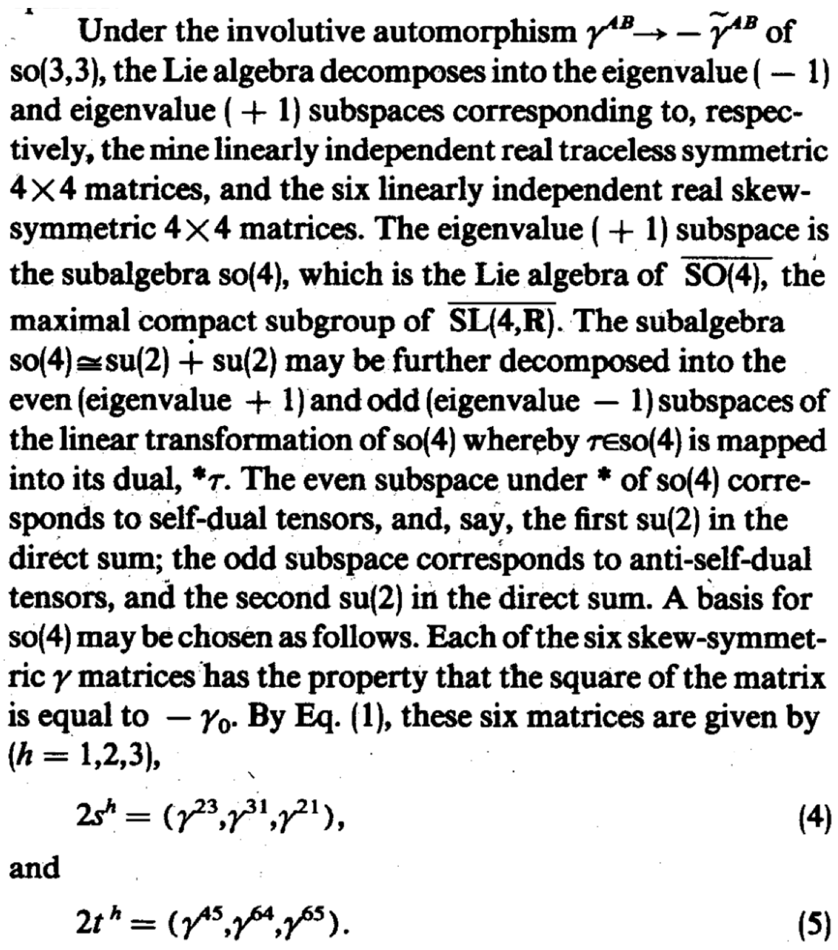
```
FullSimplify[Sqrt[Flatten[MatrixMetric44.η4488]], constraintVars]
```

Out[275]=

```
{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0, 0, √(ⅇ^(2 a4[H x4])) Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0, 0, 0,
 √(ⅇ^(2 a4[H x4])) Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0, 0, 0, √(ⅇ^(2 a4[H x4])) Sin[6 H x0]^(1/6), 0, 0, 0,
 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, √(ⅇ^(-2 a4[H x4])) Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0,
 0, 0, √(ⅇ^(-2 a4[H x4])) Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0, 0, 0, √(ⅇ^(-2 a4[H x4])) Sin[6 H x0]^(1/6)}
```

SO(4), γ; M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+,+,+,+,-,-,-,-}, {x0, x1, x2, x3, x4, x5, x6, x7}

In[276]:=



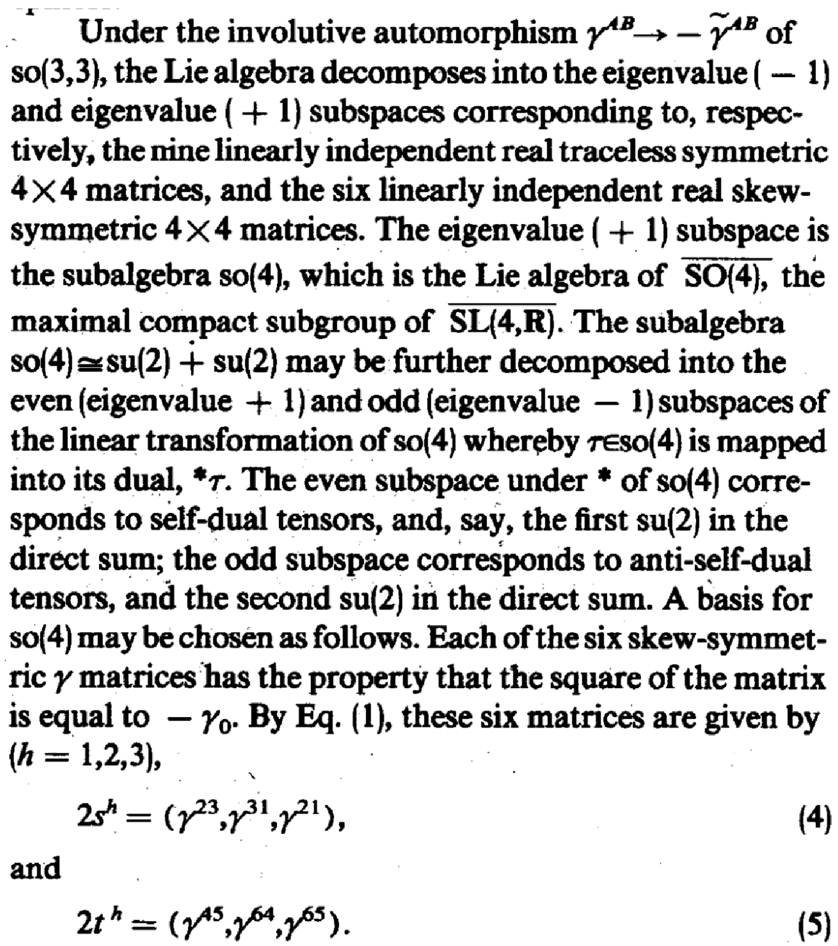
Under the involutive automorphism $\gamma^{AB}\to-\tilde{\gamma}^{AB}$ of so(3,3), the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra so(4), which is the Lie algebra of $\overline{\mathrm{SO}(4)}$, the maximal compact subgroup of $\overline{\mathrm{SL}(4,\mathbf{R})}$. The subalgebra so(4)≅su(2) + su(2) may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of so(4) whereby $\tau\in$so(4) is mapped into its dual, $^*\tau$. The even subspace under * of so(4) corresponds to self-dual tensors, and, say, the first su(2) in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second su(2) in the direct sum. A basis for so(4) may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by $(h=1,2,3)$,

$$2s^h=(\gamma^{23},\gamma^{31},\gamma^{21}),\qquad(4)$$

and

$$2t^h=(\gamma^{45},\gamma^{64},\gamma^{65}).\qquad(5)$$

Out[276]=



Under the involutive automorphism $\gamma^{AB} \to -\tilde{\gamma}^{AB}$ of so(3,3), the Lie algebra decomposes into the eigenvalue (− 1) and eigenvalue (+ 1) subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue (+ 1) subspace is the subalgebra so(4), which is the Lie algebra of $\overline{\mathrm{SO}(4)}$, the maximal compact subgroup of $\overline{\mathrm{SL}(4,\mathbf{R})}$. The subalgebra so(4)≅su(2) $\dotplus$ su(2) may be further decomposed into the even (eigenvalue + 1) and odd (eigenvalue − 1) subspaces of the linear transformation of so(4) whereby $\tau \in$ so(4) is mapped into its dual, $^*\tau$. The even subspace under * of so(4) corresponds to self-dual tensors, and, say, the first su(2) in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second su(2) in the direct sum. A basis for so(4) may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by ($h = 1,2,3$),

$$2s^h = (\gamma^{23},\gamma^{31},\gamma^{21}), \tag{4}$$

and

$$2t^h = (\gamma^{45},\gamma^{64},\gamma^{65}). \tag{5}$$

In[277]:=

```
Qa[h_, p_, q_] := Signature[{h, p, q, 4}];
Qb[h_, p_, q_] := ID4⟦p, 4⟧ * ID4⟦q, h⟧ - ID4⟦p, h⟧ * ID4⟦q, 4⟧;
SelfDualAntiSymmetric[h_, p_, q_] := Qa[h, p, q] - Qb[h, p, q] ;
AntiSelfDualAntiSymmetric[h_, p_, q_] := (Qa[h, p, q] + Qb[h, p, q] );
```

In[281]:=

```
Qa[1, 2, 3]
```

Out[281]=

```
1
```

In[282]:=

```
ϵ4⟦1, 2, 3, 4⟧
```

Out[282]=

```
1
```

In[283]:=

```
Do[s4by4[h] = Table[Table[SelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
 {h, 1, 3} ]
```

```
In[284]:=
Do[
 t4by4[h] = Table[Table[AntiSelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
 {h, 1, 3} ]
In[285]:=
(*Protect[{s4by4[1],s4by4[2],s4by4[3],t4by4[1],t4by4[2],t4by4[3]}]*)
In[286]:=
Protect[s4by4, t4by4]
Out[286]=
{s4by4, t4by4}
In[287]:=
{s4by4[#] // MatrixForm, MatrixForm[t4by4[#]]} & /@ Range[3]
Out[287]=
   ( 0  0  0  1)   (0  0  0 -1)
{{ ( 0  0  1  0) , (0  0  1  0) },
   ( 0 -1  0  0)   (0 -1  0  0)
   (-1  0  0  0)   (1  0  0  0)

  (0  0 -1  0)   (0  0 -1  0)     ( 0  1  0  0)   ( 0  1  0  0)
 {(0  0  0  1) , (0  0  0 -1) }, {(-1  0  0  0) , (-1  0  0  0) }}
  (1  0  0  0)   (1  0  0  0)     ( 0  0  0  1)   ( 0  0  0 -1)
  (0 -1  0  0)   (0  1  0  0)     ( 0  0 -1  0)   ( 0  0  1  0)
In[288]:=
Table[{{J, K}, MatrixForm[s4by4[J].t4by4[K]]}, {J, 1, 3}, {K, 1, 3}]
Out[288]=
             (1  0  0  0)            (0 1 0 0)            (0  0  1  0)
{{{{1, 1},   (0 -1  0  0) }, {{1, 2}, (1 0 0 0) }, {{1, 3}, (0  0  0 -1) }},
             (0  0 -1  0)            (0 0 0 1)            (1  0  0  0)
             (0  0  0  1)            (0 0 1 0)            (0 -1  0  0)

             (0 1  0  0)            (-1 0  0 0)            (0 0 0 1)
 {{{2, 1},   (1 0  0  0) }, {{2, 2}, ( 0 1  0 0) }, {{2, 3}, (0 0 1 0) }},
             (0 0  0 -1)            ( 0 0 -1 0)            (0 1 0 0)
             (0 0 -1  0)            ( 0 0  0 1)            (1 0 0 0)

             (0 0 1 0)            ( 0 0 0 -1)            (-1  0 0 0)
 {{{3, 1},   (0 0 0 1) }, {{3, 2}, ( 0 0 1  0) }, {{3, 3}, ( 0 -1 0 0) }}}
             (1 0 0 0)            ( 0 1 0  0)            ( 0  0 1 0)
             (0 1 0 0)            (-1 0 0  0)            ( 0  0 0 1)
In[289]:=
Do[st[J, K] = s4by4[J].t4by4[K], {J, 1, 3}, {K, 1, 3}]
```

In[290]:=

```
Table[{{J, K}, MatrixForm[st[J, K]]}, {J, 1, 3}, {K, 1, 3}]
```

Out[290]=

```
            ⎛1  0   0  0⎞             ⎛0 1 0 0⎞             ⎛0  0  1  0⎞
{{{{1, 1},  ⎜0 -1   0  0⎟}, {{1, 2},  ⎜1 0 0 0⎟}, {{1, 3},  ⎜0  0  0 -1⎟}},
            ⎜0  0  -1  0⎟             ⎜0 0 0 1⎟             ⎜1  0  0  0⎟
            ⎝0  0   0  1⎠             ⎝0 0 1 0⎠             ⎝0 -1  0  0⎠

            ⎛0 1  0  0⎞             ⎛-1 0  0 0⎞             ⎛0 0 0 1⎞
 {{{2, 1},  ⎜1 0  0  0⎟}, {{2, 2},  ⎜ 0 1  0 0⎟}, {{2, 3},  ⎜0 0 1 0⎟}},
            ⎜0 0  0 -1⎟             ⎜ 0 0 -1 0⎟             ⎜0 1 0 0⎟
            ⎝0 0 -1  0⎠             ⎝ 0 0  0 1⎠             ⎝1 0 0 0⎠

            ⎛0 0 1 0⎞             ⎛ 0  0 0 -1⎞             ⎛-1  0 0 0⎞
 {{{3, 1},  ⎜0 0 0 1⎟}, {{3, 2},  ⎜ 0  0 1  0⎟}, {{3, 3},  ⎜ 0 -1 0 0⎟}}}
            ⎜1 0 0 0⎟             ⎜ 0  1 0  0⎟             ⎜ 0  0 1 0⎟
            ⎝0 1 0 0⎠             ⎝-1  0 0  0⎠             ⎝ 0  0 0 1⎠
```

In[291]:=

```
Protect[st]
```

Out[291]=

```
{st}
```

In[292]:=

```
Table[{{J, K}, MatrixForm[st[J, K] - Transpose[st[J, K]]]}, {J, 1, 3}, {K, 1, 3}]
```

Out[292]=

```
            ⎛0 0 0 0⎞             ⎛0 0 0 0⎞             ⎛0 0 0 0⎞
{{{{1, 1},  ⎜0 0 0 0⎟}, {{1, 2},  ⎜0 0 0 0⎟}, {{1, 3},  ⎜0 0 0 0⎟}},
            ⎜0 0 0 0⎟             ⎜0 0 0 0⎟             ⎜0 0 0 0⎟
            ⎝0 0 0 0⎠             ⎝0 0 0 0⎠             ⎝0 0 0 0⎠

            ⎛0 0 0 0⎞             ⎛0 0 0 0⎞             ⎛0 0 0 0⎞
 {{{2, 1},  ⎜0 0 0 0⎟}, {{2, 2},  ⎜0 0 0 0⎟}, {{2, 3},  ⎜0 0 0 0⎟}},
            ⎜0 0 0 0⎟             ⎜0 0 0 0⎟             ⎜0 0 0 0⎟
            ⎝0 0 0 0⎠             ⎝0 0 0 0⎠             ⎝0 0 0 0⎠

            ⎛0 0 0 0⎞             ⎛0 0 0 0⎞             ⎛0 0 0 0⎞
 {{{3, 1},  ⎜0 0 0 0⎟}, {{3, 2},  ⎜0 0 0 0⎟}, {{3, 3},  ⎜0 0 0 0⎟}}}
            ⎜0 0 0 0⎟             ⎜0 0 0 0⎟             ⎜0 0 0 0⎟
            ⎝0 0 0 0⎠             ⎝0 0 0 0⎠             ⎝0 0 0 0⎠
```

In[293]:=

```
Permutations[Range[3], 2]
```

Out[293]=

```
{{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}
```

In[294]:=

```
KroneckerProduct[Range[3], Range[3]]
```

Out[294]=

```
{{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}
```

In[295]:=

```
Table[{J, K}, {J, 1, 3}, {K, 1, 3}]
st[##] & /@ %
st[##] & @@ %%
```

Out[295]=

```
{{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}}
```

Out[296]=

```
{st[{{1, 1}, {1, 2}, {1, 3}}], st[{{2, 1}, {2, 2}, {2, 3}}], st[{{3, 1}, {3, 2}, {3, 3}}]}
```

Out[297]=

```
st[{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}]
```

In[298]:=

```
Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
Length[%]
```

Out[298]=

```
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

Out[299]=

```
16
```

In[300]:=

```
Unprotect[ɣ];
ɣ = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
   Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
```

Out[301]=

```
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

In[302]:=

```
Length[ɣ]
```

Out[302]=

```
16
```

In[303]:=

```
Protect[ɣ]
```

Out[303]=

```
{ɣ}
```

In[304]:=

```
ɣ⟦1⟧
```

Out[304]=

```
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}
```

In[305]:=

```
ɣ⟦-1⟧
```

Out[305]=

```
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Self Dual Anti-Symmetric

In[306]:=

$$\text{Table}\left[\left(\frac{1}{2}\sum_{j1=1}^{4}\left(\sum_{j2=1}^{4}\epsilon 4[\![p,\ q,\ j1,\ j2]\!]\times \text{s4by4}[h][\![j1,\ j2]\!]\right)\right)-\text{s4by4}[h][\![p,\ q]\!],\ \{h,\ 1,\ 3\},\ \{q,\ 4\},\ \{p,\ 4\}\right]$$

Out[306]=

```
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}
```

Anti SelfDual Anti-Symmetric

In[307]:=

$$\text{Table}\left[\left(\frac{1}{2}\sum_{j1=1}^{4}\left(\sum_{j2=1}^{4}\epsilon4[[p, q, j1, j2]]\times t4by4[h][[j1, j2]]\right)\right)+t4by4[h][[p, q]],\ \{h, 1, 3\}, \{q, 4\}, \{p, 4\}\right]$$

Out[307]=

```
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}
```

SO(4,4), Spin(4,4) = $\overline{SO(4,4)}$, τ ; if using xact: M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+,+,+,+,-,-,-,-}, {x0, x1, x2, x3, x4, x5, x6, x7}

In[308]:=

```
τ̅ === τ̅
```

Out[308]=

```
False
```

In[309]:=

```
τ̅
Head[%]
```

Out[309]=

```
τ̅
```

Out[310]=

```
Symbol
```

In[311]:=

```
τ̅
Head[%]
```

Out[311]=

```
τ̅
```

Out[312]=

```
ParsedBoxWrapper
```

In[313]:=

```
τ̅
Head[%]
```

Out[313]=

```
τ̅
```

Out[314]=

```
Symbol
```

In[315]:=

```
Symbol
```

Out[315]=

```
Symbol
```

```
In[316]:=
τ̄ === τ̄
Out[316]=
True
In[317]:=
τ̄ === τ̄
Out[317]=
False
In[318]:=
η4488 // MatrixForm
Out[318]//MatrixForm=
1 0 0 0  0  0  0  0
0 1 0 0  0  0  0  0
0 0 1 0  0  0  0  0
0 0 0 1  0  0  0  0
0 0 0 0 -1  0  0  0
0 0 0 0  0 -1  0  0
0 0 0 0  0  0 -1  0
0 0 0 0  0  0  0 -1
In[319]:=
η4488 == DiagonalMatrix[RotateRight[evalues, 4]]
Out[319]=
True
In[320]:=
DiagonalMatrix[RotateRight[evalues, 3]] // MatrixForm
Out[320]//MatrixForm=
1 0 0  0  0  0  0 0
0 1 0  0  0  0  0 0
0 0 1  0  0  0  0 0
0 0 0 -1  0  0  0 0
0 0 0  0 -1  0  0 0
0 0 0  0  0 -1  0 0
0 0 0  0  0  0 -1 0
0 0 0  0  0  0  0 1
In[321]:=
τ[0] = ID8;
Table[
  τ[7 - h] = ArrayFlatten[{{0, t4by4[h]}, {-t4by4[h], 0}}], {h, 1, 3}];
Table[τ[h] = ArrayFlatten[{{0, s4by4[h]}, {s4by4[h], 0}}], {h, 1, 3}];
(τ[7] = τ[1].τ[2].τ[3].τ[4].τ[5].τ[6]) // MatrixForm
Out[324]//MatrixForm=
-1  0  0  0 0 0 0 0
 0 -1  0  0 0 0 0 0
 0  0 -1  0 0 0 0 0
 0  0  0 -1 0 0 0 0
 0  0  0  0 1 0 0 0
 0  0  0  0 0 1 0 0
 0  0  0  0 0 0 1 0
 0  0  0  0 0 0 0 1
```

In[325]:=

```
σ == τ[1].τ[2].τ[3]
```

Out[325]=

```
True
```

In[326]:=

```
Table[{A, B, FullSimplify[
    ExpandAll[1/2 (τ[A].τ[B] + τ[B].τ[A]) == -η4488[[A + 1, B + 1]] * ID8]]},
  {A, 1, 7}, {B, 1, 7}]
```

Out[326]=

```
{{{1, 1, True}, {1, 2, True}, {1, 3, True}, {1, 4, True},
   {1, 5, True}, {1, 6, True}, {1, 7, True}}, {{2, 1, True}, {2, 2, True},
   {2, 3, True}, {2, 4, True}, {2, 5, True}, {2, 6, True}, {2, 7, True}},
  {{3, 1, True}, {3, 2, True}, {3, 3, True}, {3, 4, True}, {3, 5, True}, {3, 6, True},
   {3, 7, True}}, {{4, 1, True}, {4, 2, True}, {4, 3, True}, {4, 4, True},
   {4, 5, True}, {4, 6, True}, {4, 7, True}}, {{5, 1, True}, {5, 2, True},
   {5, 3, True}, {5, 4, True}, {5, 5, True}, {5, 6, True}, {5, 7, True}},
  {{6, 1, True}, {6, 2, True}, {6, 3, True}, {6, 4, True}, {6, 5, True},
   {6, 6, True}, {6, 7, True}}, {{7, 1, True}, {7, 2, True},
   {7, 3, True}, {7, 4, True}, {7, 5, True}, {7, 6, True}, {7, 7, True}}}
```

In[327]:=

```
η4488 // MatrixForm
```

Out[327]//MatrixForm=

```
( 1  0  0  0   0   0   0   0 )
( 0  1  0  0   0   0   0   0 )
( 0  0  1  0   0   0   0   0 )
( 0  0  0  1   0   0   0   0 )
( 0  0  0  0  -1   0   0   0 )
( 0  0  0  0   0  -1   0   0 )
( 0  0  0  0   0   0  -1   0 )
( 0  0  0  0   0   0   0  -1 )
```

In[328]:=

```
Table[{A, -η4488[[A + 1, A + 1]] * τ[A]  == Transpose[τ[A]]} , {A, 1, 7}]
```

Out[328]=

```
{{1, True}, {2, True}, {3, True}, {4, True}, {5, True}, {6, True}, {7, True}}
```

In[329]:=

```
(σ == τ[1].τ[2].τ[3])
```

Out[329]=

```
True
```

In[330]:=

```
σ.τ[1].τ[2].τ[3] // MatrixForm
```

Out[330]//MatrixForm=

```
1 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0
0 0 1 0 0 0 0 0
0 0 0 1 0 0 0 0
0 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 1
```

In[331]:=

```
(*τ[0]=ID8;
Do[{τ[A] = FullSimplify[ExpandAll[(τ[A])]]} , {A, 1, 7}]*)
(*Do[{τ[A] = FullSimplify[ExpandAll[-(σ.τ[1].τ[2].τ[3].τ[A])]]} ,
 {A, 1, 7}]*)
```

In[332]:=

```
(σ == τ[1].τ[2].τ[3])
```

Out[332]=

```
True
```

In[333]:=

```
(*Symbolize[τ̄]*)
```

In[334]:=

```
τ̄[0] = ID8;
Do[{τ̄[A] = FullSimplify[ExpandAll[(σ.Transpose[τ[A]].σ)]]} , {A, 1, 7}]
```

In[336]:=

```
σ.τ̄[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
```

Out[336]=

```
{True, True, True, True, True, True, True, True}
```

In[337]:=

```
Table[{A, B, FullSimplify[
              1
    ExpandAll[ ─ (τ[A].τ̄[B] + τ[B].τ̄[A]) == η4488⟦A + 1, B + 1⟧ * ID8]]},
              2
   {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

Out[337]//MatrixForm=

```
   0      0      0      0      0      0      0      0
   0      1      2      3      4      5      6      7
  True   True   True   True   True   True   True   True
   1      1      1      1      1      1      1      1
   0      1      2      3      4      5      6      7
  True   True   True   True   True   True   True   True
   2      2      2      2      2      2      2      2
   0      1      2      3      4      5      6      7
  True   True   True   True   True   True   True   True
   3      3      3      3      3      3      3      3
   0      1      2      3      4      5      6      7
  True   True   True   True   True   True   True   True
   4      4      4      4      4      4      4      4
   0      1      2      3      4      5      6      7
  True   True   True   True   True   True   True   True
   5      5      5      5      5      5      5      5
   0      1      2      3      4      5      6      7
  True   True   True   True   True   True   True   True
   6      6      6      6      6      6      6      6
   0      1      2      3      4      5      6      7
  True   True   True   True   True   True   True   True
   7      7      7      7      7      7      7      7
   0      1      2      3      4      5      6      7
  True   True   True   True   True   True   True   True
```

In[338]:=

```
τ[7] // MatrixForm
```

Out[338]//MatrixForm=

```
-1   0   0   0  0  0  0  0
 0  -1   0   0  0  0  0  0
 0   0  -1   0  0  0  0  0
 0   0   0  -1  0  0  0  0
 0   0   0   0  1  0  0  0
 0   0   0   0  0  1  0  0
 0   0   0   0  0  0  1  0
 0   0   0   0  0  0  0  1
```

In[339]:=

```
(Ω = σ.τ[7]) // MatrixForm
```

Out[339]//MatrixForm=

```
 0   0   0   0  1  0  0  0
 0   0   0   0  0  1  0  0
 0   0   0   0  0  0  1  0
 0   0   0   0  0  0  0  1
-1   0   0   0  0  0  0  0
 0  -1   0   0  0  0  0  0
 0   0  -1   0  0  0  0  0
 0   0   0  -1  0  0  0  0
```

```
In[340]:=
        Ω == τ[4].τ[5].τ[6]
Out[340]=
        True
In[341]:=
        τ[5].τ[6].τ[7] // MatrixForm
Out[341]//MatrixForm=
        ( 0  1 0  0  0  0   0  0 )
        (-1  0 0  0  0  0   0  0 )
        ( 0  0 0 -1  0  0   0  0 )
        ( 0  0 1  0  0  0   0  0 )
        ( 0  0 0  0  0 -1   0  0 )
        ( 0  0 0  0  1  0   0  0 )
        ( 0  0 0  0  0  0   0  1 )
        ( 0  0 0  0  0  0  -1  0 )
In[342]:=
        (σΩ = σ.Ω) // MatrixForm
Out[342]//MatrixForm=
        (-1  0   0   0  0 0 0 0 )
        ( 0 -1   0   0  0 0 0 0 )
        ( 0  0  -1   0  0 0 0 0 )
        ( 0  0   0  -1  0 0 0 0 )
        ( 0  0   0   0  1 0 0 0 )
        ( 0  0   0   0  0 1 0 0 )
        ( 0  0   0   0  0 0 1 0 )
        ( 0  0   0   0  0 0 0 1 )
In[343]:=
        Inverse[σΩ] // MatrixForm
Out[343]//MatrixForm=
        (-1  0   0   0  0 0 0 0 )
        ( 0 -1   0   0  0 0 0 0 )
        ( 0  0  -1   0  0 0 0 0 )
        ( 0  0   0  -1  0 0 0 0 )
        ( 0  0   0   0  1 0 0 0 )
        ( 0  0   0   0  0 1 0 0 )
        ( 0  0   0   0  0 0 1 0 )
        ( 0  0   0   0  0 0 0 1 )
In[344]:=
        τ[1].τ[2].τ[3].τ[4].τ[5].τ[6].τ[7] == τ[0] == ID8
Out[344]=
        True
In[345]:=
        (σ == τ[1].τ[2].τ[3])
Out[345]=
        True
In[346]:=
        (σ == τ[4].τ[5].τ[6].τ[7])
Out[346]=
        True
```

```
In[347]:=
τ[1].τ[2].τ[3].τ[#] == -Transpose[τ[1].τ[2].τ[3].τ[#]] & /@ Range[0, 7]
Out[347]=
{False, True, True, True, True, True, True, True}
In[348]:=
σ.τ̄[#]  == Transpose[σ.τ[#]] & /@ Range[0, 7]
Out[348]=
{True, True, True, True, True, True, True, True}
In[349]:=
τ[5].τ[6].τ[7].τ̄[#]  == Transpose[τ[5].τ[6].τ[7].τ[#]] & /@ Range[0, 7]
Out[349]=
{False, True, True, True, False, True, True, True}
In[350]:=
τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
Out[350]=
{True, True, True, True, True, False, False, False}
In[351]:=
τ[4].τ̄[#]  == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
Out[351]=
{True, False, False, False, False, True, True, True}
In[352]:=
τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
Out[352]=
{True, True, True, True, True, False, False, False}
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$ **; O (4, 4) : SAB ;**

$$\textbf{covariantDiffMatrix} = \mathrm{T16}^A[5].\mathrm{T16}^A[6].\mathrm{T16}^A[7]$$

$$S = \begin{pmatrix} a & b \\ c & d \end{pmatrix};$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}.\begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.\begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix} = \begin{pmatrix} b.\tau^A & a.\bar{\tau}^A \\ d.\tau^A & c.\bar{\tau}^A \end{pmatrix}.\begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix}$$

$$= \begin{pmatrix} b.\tau^A.\tilde{a} + a.\bar{\tau}^A.\tilde{b} & b.\tau^A.\tilde{c} + a.\bar{\tau}^A.\tilde{d} \\ d.\tau^A.\tilde{a} + c.\bar{\tau}^A.\tilde{b} & d.\tau^A.\tilde{c} + c.\bar{\tau}^A.\tilde{d} \end{pmatrix}$$

Need these in order to define the Lagrangian for the "universe," later :

In[353]:=

```
(σ16 = T16^A[0].T16^A[1].T16^A[2].T16^A[3]) // MatrixForm
```

Out[353]//MatrixForm=

```
T16^A[0].T16^A[1].T16^A[2].T16^A[3]
```

In[354]:=

```
Table[T16^A[A1] = ArrayFlatten[{{0, τ̅[A1]}, {τ[A1], 0}}], {A1, 0, 7}];
```

In[355]:=

```
(T16^A[8] = T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]) //
 MatrixForm
```

Out[355]//MatrixForm=

```
⎛-1   0   0   0   0   0   0   0  0 0 0 0 0 0 0 0⎞
⎜ 0  -1   0   0   0   0   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0  -1   0   0   0   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0  -1   0   0   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0  -1   0   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0  -1   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0  -1   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0  -1  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  1 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 1 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 1 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 0 1 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 0 0 1 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 0 0 0 1 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 0 0 0 0 1 0⎟
⎝ 0   0   0   0   0   0   0   0  0 0 0 0 0 0 0 1⎠
```

In[356]:=

```
σ16 // MatrixForm
```

Out[356]//MatrixForm=

```
⎛ 0   0   0   0  -1   0   0   0  0 0 0 0 0 0 0 0⎞
⎜ 0   0   0   0   0  -1   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0  -1   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0  -1  0 0 0 0 0 0 0 0⎟
⎜-1   0   0   0   0   0   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0  -1   0   0   0   0   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0  -1   0   0   0   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0  -1   0   0   0   0  0 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 0 0 1 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 0 0 0 1 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 0 0 0 0 1 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 0 0 0 0 0 1⎟
⎜ 0   0   0   0   0   0   0   0  1 0 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 1 0 0 0 0 0 0⎟
⎜ 0   0   0   0   0   0   0   0  0 0 1 0 0 0 0 0⎟
⎝ 0   0   0   0   0   0   0   0  0 0 0 1 0 0 0 0⎠
```

```
In[357]:=
T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#] ==
   -Transpose[T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#]] & /@ Range[0, 8]
Out[357]=
{True, True, True, True, True, True, True, True, False}
In[358]:=
T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#] ==
   Transpose[T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#]] & /@ Range[0, 8]
Out[358]=
{True, True, True, True, True, True, True, True, True}
In[359]:=
T16^A[8] == σ16.T16^A[4].T16^A[5].T16^A[6].T16^A[7]
Out[359]=
True
In[360]:=
T16^A[#] == Transpose[T16^A[#]] & /@ Range[0, 8]
Out[360]=
{True, True, True, True, False, False, False, False, True}
In[361]:=
T16^A[#] == -Transpose[T16^A[#]] & /@ Range[0, 8]
Out[361]=
{False, False, False, False, True, True, True, True, False}
In[362]:=
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
Out[362]=
{True, True, True, True, True, True, True, True}
In[363]:=
(covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]) // MatrixForm
Out[363]//MatrixForm=
( 0   0   0  0  0  0  0  0   0  1  0   0  0   0   0  0 )
( 0   0   0  0  0  0  0  0  -1  0  0   0  0   0   0  0 )
( 0   0   0  0  0  0  0  0   0  0  0  -1  0   0   0  0 )
( 0   0   0  0  0  0  0  0   0  0  1   0  0   0   0  0 )
( 0   0   0  0  0  0  0  0   0  0  0   0  0  -1   0  0 )
( 0   0   0  0  0  0  0  0   0  0  0   0  1   0   0  0 )
( 0   0   0  0  0  0  0  0   0  0  0   0  0   0   0  1 )
( 0   0   0  0  0  0  0  0   0  0  0   0  0   0  -1  0 )
( 0  -1   0  0  0  0  0  0   0  0  0   0  0   0   0  0 )
( 1   0   0  0  0  0  0  0   0  0  0   0  0   0   0  0 )
( 0   0   0  1  0  0  0  0   0  0  0   0  0   0   0  0 )
( 0   0  -1  0  0  0  0  0   0  0  0   0  0   0   0  0 )
( 0   0   0  0  0  1  0  0   0  0  0   0  0   0   0  0 )
( 0   0   0  0 -1  0  0  0   0  0  0   0  0   0   0  0 )
( 0   0   0  0  0  0  0 -1   0  0  0   0  0   0   0  0 )
( 0   0   0  0  0  0  1  0   0  0  0   0  0   0   0  0 )
```

In[364]:=

```
σ16.covariantDiffMatrix == Transpose[σ16.covariantDiffMatrix]
```

Out[364]=

```
True
```

In[365]:=

```
ID16 // MatrixForm
```

Out[365]//MatrixForm=

```
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
```

In[366]:=

```
(σ16(*=T16^A[0].T16^A[1].T16^A[2].T16^A[3]*)) // MatrixForm
```

Out[366]//MatrixForm=

```
 0   0   0   0  -1   0   0   0  0 0 0 0 0 0 0 0
 0   0   0   0   0  -1   0   0  0 0 0 0 0 0 0 0
 0   0   0   0   0   0  -1   0  0 0 0 0 0 0 0 0
 0   0   0   0   0   0   0  -1  0 0 0 0 0 0 0 0
-1   0   0   0   0   0   0   0  0 0 0 0 0 0 0 0
 0  -1   0   0   0   0   0   0  0 0 0 0 0 0 0 0
 0   0  -1   0   0   0   0   0  0 0 0 0 0 0 0 0
 0   0   0  -1   0   0   0   0  0 0 0 0 0 0 0 0
 0   0   0   0   0   0   0   0  0 0 0 0 1 0 0 0
 0   0   0   0   0   0   0   0  0 0 0 0 0 1 0 0
 0   0   0   0   0   0   0   0  0 0 0 0 0 0 1 0
 0   0   0   0   0   0   0   0  0 0 0 0 0 0 0 1
 0   0   0   0   0   0   0   0  1 0 0 0 0 0 0 0
 0   0   0   0   0   0   0   0  0 1 0 0 0 0 0 0
 0   0   0   0   0   0   0   0  0 0 1 0 0 0 0 0
 0   0   0   0   0   0   0   0  0 0 0 1 0 0 0 0
```

In[367]:=

```
σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
```

Out[367]=

```
True
```

later, after defining $e^{\alpha}_{(A)}$:

In[368]:=

```
(*Table[T16^α[α1-1]=  Sum[(e^α_(A)[[α1,A1]] ) T16^A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

In[369]:=

```
(*(T16α[8]=T16α[0].T16α[1].T16α[2].
    T16α[3].T16α[4].T16α[5].T16α[6].T16α[7])//MatrixForm*)
```

In[370]:=

```
(*eLpairs*)
```

In[371]:=

```
(*Ω16a=Array[0&,{16,16}];
 {1+#⟦1⟧,1+#⟦2⟧}&/@eLpairs
 (Ω16a⟦#⟦1⟧,#⟦2⟧⟧=1)&/@%*)
```

In[372]:=

```
(*Ω16a//MatrixForm*)
```

In[373]:=

```
(*(Ω16=σ16.(Ω16a+Transpose[Ω16a]))//MatrixForm*)
```

In[374]:=

```
T16^A[0].T16^A[4] // MatrixForm
T16^A[0].T16^A[4].σ16
(Ω16 = %) // MatrixForm
```

Out[374]//MatrixForm=

```
⎛0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0⎞
 0  0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0 -1  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0
 0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0
 0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0 -1  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
 0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1  0
 0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0
 0  0  0  0  0  0  0  0 -1  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0  0
⎝0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0⎠
```

Out[375]=

```
{{0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}}
```

Out[376]//MatrixForm=

```
⎛0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0  0⎞
 1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0
 0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0
 0  0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0 -1  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0
 0  0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0
 0  0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1
⎝0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0⎠
```

In[377]:=

```
Det[Ω16]
```

Out[377]=

```
1
```

In[378]:=

```
σ16.Ω16 // MatrixForm
```

Out[378]//MatrixForm=

```
⎛ 0  0  0  0  0 -1  0  0  0  0  0  0  0  0  0  0 ⎞
  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0
  0  0  0  0  0  0 -1  0  0  0  0  0  0  0  0  0
  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 -1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0
  0  0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0
  0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0
⎝ 0  0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0 ⎠
```

In[379]:=

```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
```

Out[379]=

```
{True, True, True, True, True, True, True, True}
```

In[380]:=

```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
```

Out[380]=

```
{True, True, True, True, True, True, True, True}
```

In[381]:=

```
σ16.T16^A[#] == -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
```

Out[381]=

```
{True, True, True, True, True, True, True, True}
```

In[382]:=

```
σ16.newT16^A[#] == -Transpose[σ16.newT16^A[#]] & /@ Range[0, 7]
```

Out[382]=

```
{{{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0,
     0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
    {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
     0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[0] ==
```

```
-({{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},                    ,
    {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0,
     0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
    {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
     1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[0])^T

{{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
   {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[1] ==

-({{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},                    ,
    {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0,
     0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
    {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
     0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
     1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[1])^T

{{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
   {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[2] ==
```

```
 - ({{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},                                    ,
     {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0,
      0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
     {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
     {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
     {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
     {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[2])^T

{{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
   1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[3] ==

 - ({{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},                                    ,
     {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0,
      0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
     {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
     {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
     {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
     {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[3])^T

{{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
   1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[4] ==
```

```
 -({{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},                                   ,
     {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0,
      0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
     {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
     {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
     {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
     {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[4])^T

{{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
   1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[5] ==

 -({{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},                                   ,
     {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0,
      0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
     {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
     {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
     {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
     {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
      1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[5])^T

{{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
   0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
   1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[6] ==
```

```
 -({{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},                                    ,
      {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0,
       0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
      {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[6])^T

{{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
   {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
    1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[7] ==

 -({{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},                                    }
      {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0,
       0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
      {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
      {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}}.newT16^A[7])^T
```

In[383]:=

```
newσ16.newT16^A[#] == -Transpose[newσ16.newT16^A[#]] & /@ Range[0, 7]
```

Out[383]=

```
{newσ16.newT16^A[0] == -(newσ16.newT16^A[0])^T, newσ16.newT16^A[1] == -(newσ16.newT16^A[1])^T,
 newσ16.newT16^A[2] == -(newσ16.newT16^A[2])^T, newσ16.newT16^A[3] == -(newσ16.newT16^A[3])^T,
 newσ16.newT16^A[4] == -(newσ16.newT16^A[4])^T, newσ16.newT16^A[5] == -(newσ16.newT16^A[5])^T,
 newσ16.newT16^A[6] == -(newσ16.newT16^A[6])^T, newσ16.newT16^A[7] == -(newσ16.newT16^A[7])^T}
```

In[384]:=

```
Table[{{A1, B1},
   FullSimplify[ExpandAll[1/2 (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) ==
      η4488[[A1 + 1, B1 + 1]] * ID16]]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[384]//MatrixForm=

```
( {0, 0}  {0, 1}  {0, 2}  {0, 3}  {0, 4}  {0, 5}  {0, 6}  {0, 7} )
(  True    True    True    True    True    True    True    True  )
( {1, 0}  {1, 1}  {1, 2}  {1, 3}  {1, 4}  {1, 5}  {1, 6}  {1, 7} )
(  True    True    True    True    True    True    True    True  )
( {2, 0}  {2, 1}  {2, 2}  {2, 3}  {2, 4}  {2, 5}  {2, 6}  {2, 7} )
(  True    True    True    True    True    True    True    True  )
( {3, 0}  {3, 1}  {3, 2}  {3, 3}  {3, 4}  {3, 5}  {3, 6}  {3, 7} )
(  True    True    True    True    True    True    True    True  )
( {4, 0}  {4, 1}  {4, 2}  {4, 3}  {4, 4}  {4, 5}  {4, 6}  {4, 7} )
(  True    True    True    True    True    True    True    True  )
( {5, 0}  {5, 1}  {5, 2}  {5, 3}  {5, 4}  {5, 5}  {5, 6}  {5, 7} )
(  True    True    True    True    True    True    True    True  )
( {6, 0}  {6, 1}  {6, 2}  {6, 3}  {6, 4}  {6, 5}  {6, 6}  {6, 7} )
(  True    True    True    True    True    True    True    True  )
( {7, 0}  {7, 1}  {7, 2}  {7, 3}  {7, 4}  {7, 5}  {7, 6}  {7, 7} )
(  True    True    True    True    True    True    True    True  )
```

In[385]:=

```
Table[{{A1, B1}, FullSimplify[
    ExpandAll[1/2 (newT16^A[A1].newT16^A[B1] + newT16^A[B1].newT16^A[A1]) ==
      η4488[[A1 + 1, B1 + 1]] * ID16]]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[385]//MatrixForm=

```
(                        (newT16^A[0]^2 == {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, (
( (1/2 (newT16^A[0].newT16^A[1] + newT16^A[1].newT16^A[0]) == {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
( (1/2 (newT16^A[0].newT16^A[2] + newT16^A[2].newT16^A[0]) == {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
( (1/2 (newT16^A[0].newT16^A[3] + newT16^A[3].newT16^A[0]) == {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
( (1/2 (newT16^A[0].newT16^A[4] + newT16^A[4].newT16^A[0]) == {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
( (1/2 (newT16^A[0].newT16^A[5] + newT16^A[5].newT16^A[0]) == {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
( (1/2 (newT16^A[0].newT16^A[6] + newT16^A[6].newT16^A[0]) == {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
( (1/2 (newT16^A[0].newT16^A[7] + newT16^A[7].newT16^A[0]) == {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
```

In[386]:=

```
Ψ16upper = Take[Ψ16, 8]
Ψ16lower = Take[Ψ16, -8]
Ψ16 == Flatten[{Ψ16upper, Ψ16lower}]
```

Out[386]=

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4]}
```

Out[387]=

```
{f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

Out[388]=

```
True
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
Table[T16^A[A1] = ArrayFlatten[{{0, τ̄[A1]}, {τ[A1], 0}}], {A1, 0, 7}]
σ16 == ArrayFlatten[{{-σ, 0}, {0, σ}}]
( 0      τ̄[A1] ) ( Ψ16upper )   ( τ̄[A1]. Ψ16lower )
( τ[A1]  0     ) ( Ψ16lower ) = ( τ[A1].Ψ16upper  )
σ16.( 0      τ̄[A1] ) ( Ψ16upper )   ( -σ τ̄[A1]. Ψ16lower )
    ( τ[A1]  0     ) ( Ψ16lower ) = (  σ τ[A1].Ψ16upper  )
Ψ16.σ16.( 0      τ̄[A1] ).Ψ16 = ( -Ψ16upper.σ τ̄[A1] .Ψ16lower )
        ( τ[A1]  0     )       (  Ψ16lower.σ τ[A1] Ψ16upper  )
```

In[389]:=

```
(*σ16.T16^A[#]&/@Range[0,7]*)
```

In[390]:=

```
(*
(T16^A[8]=FullSimplify[T16^A[0].T16^A[1].T16^A[2].
    T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]])//MatrixForm*)
```

In[391]:=

```
η4488
```

Out[391]=

```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
```

In[392]:=

```
η99 = DiagonalMatrix[{1, 1, 1, 1, -1, -1, -1, -1, 1}]
```

Out[392]=

```
{{1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[393]:=

```
(*Table[{A1,B1,FullSimplify[
    ExpandAll[1/2 ( T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1])== η99[[A1,B1]]*ID16]]},
  {A1,0, 8},{B1,0, 8}]//MatrixForm*)
```

In[394]:=

```
(*Table[1/4 (T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]), {A1, 8},{B1,8}]*)
```

In[395]:=

```
(P_L = 1/2 (ID16 - T16^A[8])) // MatrixForm
```

Out[395]//MatrixForm=

```
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

In[396]:=

```
(P_R = 1/2 (ID16 + T16^A[8])) // MatrixForm
```

Out[396]//MatrixForm=

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
```

In[397]:=

```
{ID16 == P_L + P_R, P_L.P_L == P_L, P_L.P_R == P_R.P_L == ZERO16}
```

Out[397]=

```
{True, True, True}
```

```
SAB = Table[1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]), {A1, 0, 7}, {B1, 0, 7}];
```

In[398]:=

```
SAB =
  Table[1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]), {A1, 0, 7}, {B1, 0, 7}];
```

In[399]:=

```
ParallelTable[
   FullSimplify[σ16.SAB⟦A1, B1⟧ === -Transpose[σ16.SAB⟦A1, B1⟧]],
   {A1, 1, 8}, {B1, 1, 8}] // Flatten // Union
```

Out[399]=

```
{True}
```

In[400]:=

```
ParallelTable[
   FullSimplify[SAB⟦A1, B1⟧.SAB⟦A2, B2⟧ - SAB⟦A2, B2⟧.SAB⟦A1, B1⟧ ==
     -(η4488⟦A1, A2⟧ × SAB⟦B1, B2⟧ - η4488⟦A1, B2⟧ × SAB⟦B1, A2⟧ -
        η4488⟦B1, A2⟧ × SAB⟦A1, B2⟧ + η4488⟦B1, B2⟧ × SAB⟦A1, A2⟧)],
   {A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}] // Flatten // Union
```

Out[400]=

```
{True}
```

In[401]:=

```
ParallelTable[
   FullSimplify[SAB⟦A1, B1⟧.T16^A[B2 - 1] - T16^A[B2 - 1].SAB⟦A1, B1⟧ ==
     (-η4488⟦B2, A1⟧ T16^A[B1 - 1] + η4488⟦B2, B1⟧ T16^A[A1 - 1])],
   {A1, 1, 8}, {B1, 1, 8}, {B2, 1, 8}] // Flatten // Union
```

Out[401]=

```
{True}
```

In[402]:=

```
(*ParallelTable[
 {{A1,B1},FullSimplify[σ16.SAB⟦A1,B1⟧==-Transpose[σ16.SAB⟦A1,B1⟧]]},
 {A1,1, 8},{B1,1,8}]*)
```

In[403]:=

```
(*ParallelTable[
 {{A1,B1,A2,B2},FullSimplify[SAB⟦A1,B1⟧.SAB⟦A2,B2⟧ -SAB⟦A2,B2⟧.SAB⟦A1,B1⟧==
    -(η4488⟦A1,A2⟧SAB⟦B1,B2⟧-η4488⟦A1,B2⟧SAB⟦B1,A2⟧-
       η4488⟦B1,A2⟧SAB⟦A1,B2⟧+η4488⟦B1,B2⟧SAB⟦A1,A2⟧)]},
 {A1,1, 7},{B1,A1+1,8},{A2,1, 7},{B2,A2+1,8}]*)
```

In[404]:=

```
(*ParallelTable[
 {{A1,B1,B2},FullSimplify[SAB⟦A1,B1⟧.T16^A[B2-1] -T16^A[B2-1].SAB⟦A1,B1⟧==
    (-η4488⟦B2,A1⟧T16^A[B1-1]+η4488⟦B2,B1⟧T16^A[A1-1])]},
 {A1,1, 8},{B1,1,8},{B2,1,8}]*)
```

In[405]:=

```
Do[sAB[A1][B1] = 1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]),
  {A1, 0, 7}, {B1, 0, 7}];
```

In[406]:=

```
Dimensions[ArrayFlatten[{{0, ID8}, {ID8, 0}}]]
```

Out[406]=

```
{16, 16}
```

In[407]:=

```
sAB[-1 + 1][-1 + 2] == SAB⟦1, 2⟧
```

Out[407]=

```
True
```

In[408]:=

```
σ16.SAB⟦1, 2⟧ // MatrixForm
```

Out[408]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0
\end{pmatrix}$$

In[409]:=

```
σ16.sAB[1][2] // MatrixForm
```

Out[409]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

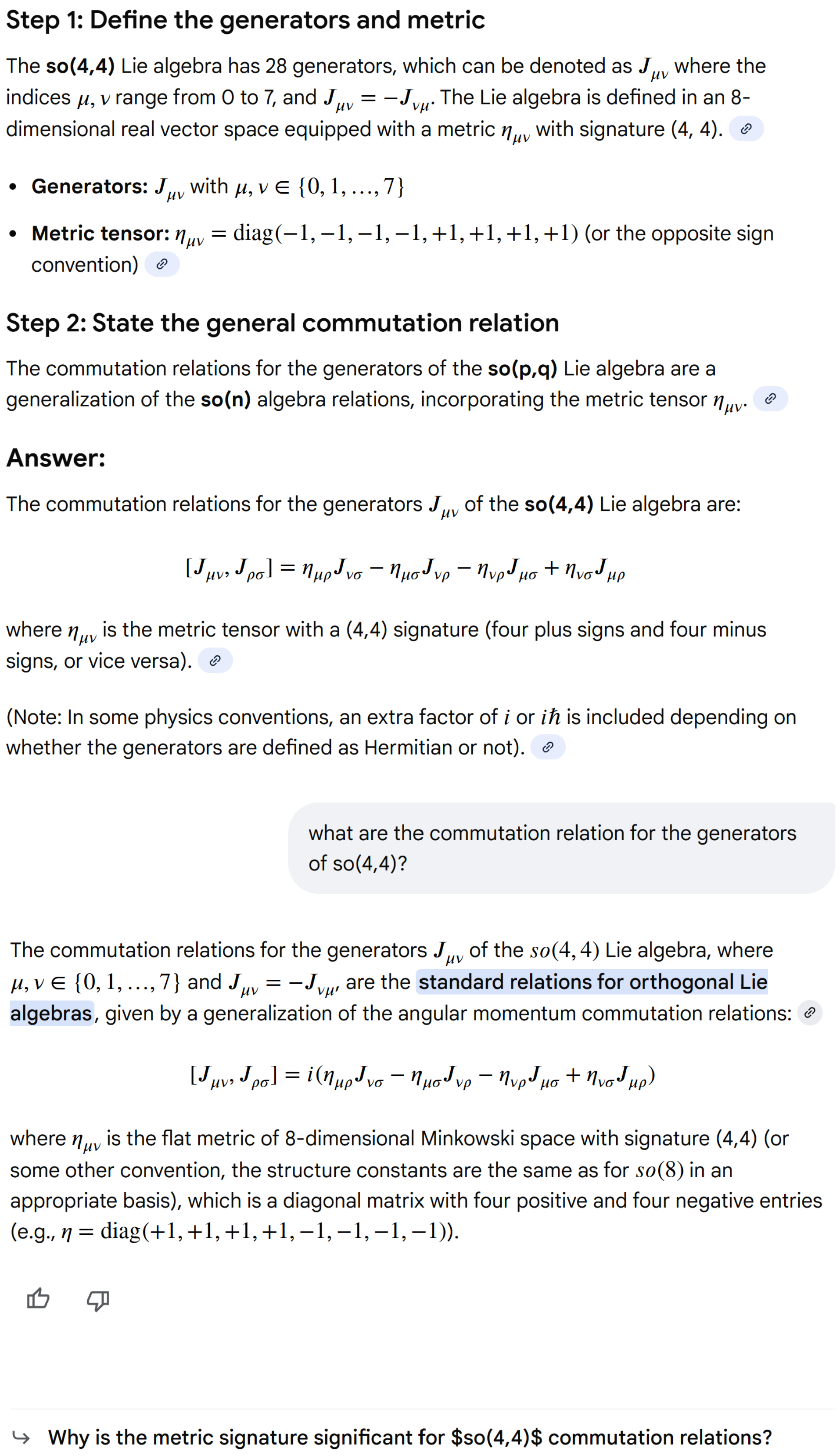
The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula:

$$[J_{ab}, J_{cd}] = i(\eta_{ac} J_{bd} - \eta_{ad} J_{bc} - \eta_{bc} J_{ad} + \eta_{bd} J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \mathbf{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties.

The final answer is $[\mathbf{J_{ab}}, \mathbf{J_{cd}}] = \mathbf{i}(\boldsymbol{\eta}_{\mathbf{ac}} \mathbf{J_{bd}} - \boldsymbol{\eta}_{\mathbf{ad}} \mathbf{J_{bc}} - \boldsymbol{\eta}_{\mathbf{bc}} \mathbf{J_{ad}} + \boldsymbol{\eta}_{\mathbf{bd}} \mathbf{J_{ac}})$.



Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4).

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \ldots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \mathrm{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention)

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$.

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa).

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not).

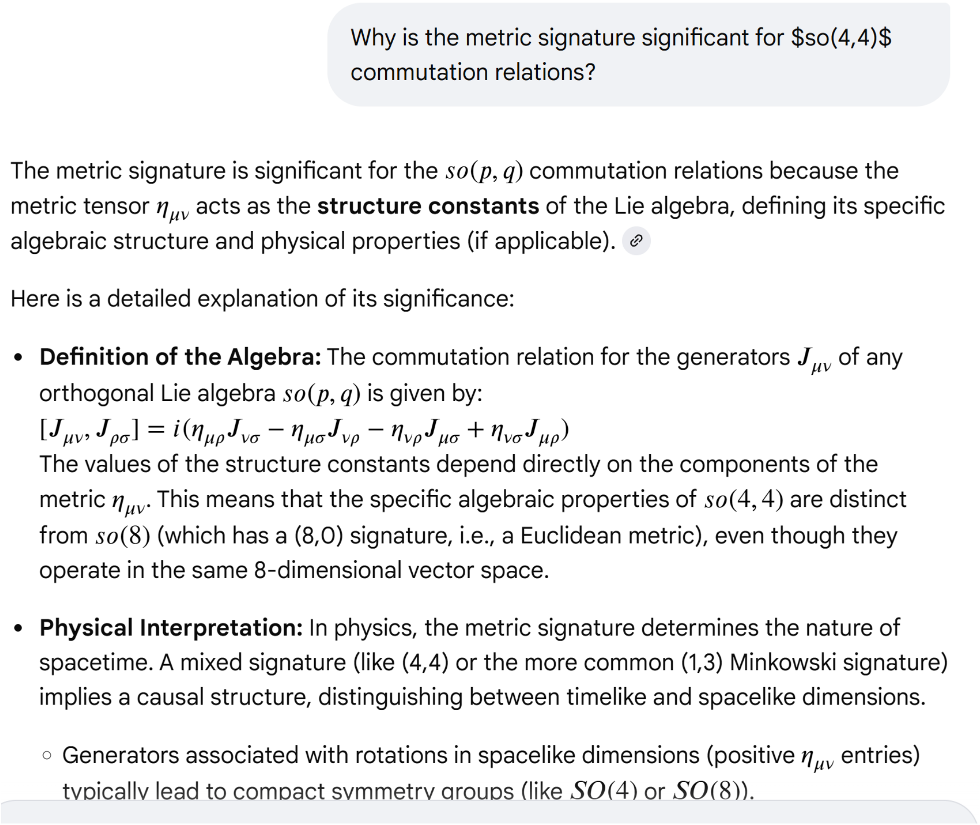
what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the $so(4,4)$ Lie algebra, where $\mu, \nu \in \{0, 1, \ldots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the standard relations for orthogonal Lie algebras, given by a generalization of the angular momentum commutation relations:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for $so(8)$ in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \mathrm{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).

↪ Why is the metric signature significant for $so(4,4)$ commutation relations?



Why is the metric signature significant for $so(4,4)$ commutation relations?

The metric signature is significant for the $so(p,q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable).

Here is a detailed explanation of its significance:

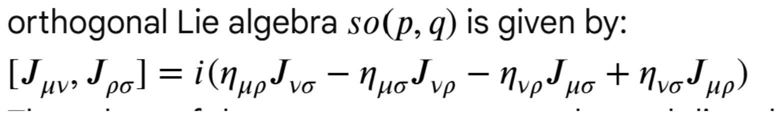
- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p,q)$ is given by:
  $[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$
  The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4,4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
  - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[410]:=

```
η4488
```

Out[410]=

```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
```



orthogonal Lie algebra $so(p,q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

In[411]:=

```
SAB⟦6, 2⟧.SAB⟦6, 7⟧ - SAB⟦6, 7⟧.SAB⟦6, 2⟧ == η4488⟦6, 6⟧ × SAB⟦7, 2⟧
```

Out[411]=

```
True
```

In[412]:=

```
{{6, 2, 6, 7}, False}
```

Out[412]=

```
{{6, 2, 6, 7}, False}
```

In[413]:=

```
SAB⟦1, 2⟧.SAB⟦1, 3⟧ - SAB⟦1, 3⟧.SAB⟦1, 2⟧ == η4488⟦1, 1⟧ × SAB⟦2, 3⟧
```

Out[413]=

```
False
```

In[414]:=

```
sAB[0][0]
```

Out[414]=

```
{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

In[415]:=

```
sAB[0][1]
```

Out[415]=

```
{{0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {-1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0}}
```

In[416]:=

```
Table[Transpose[T16^A[8].sAB[A1][B1]] == T16^A[8].sAB[A1][B1],
 {A1, 0, 7}, {B1, 0, 7}]
```

Out[416]=

```
{{True, False, False, False, True, True, True, True},
 {False, True, False, False, True, True, True, True},
 {False, False, True, False, True, True, True, True},
 {False, False, False, True, True, True, True, True},
 {True, True, True, True, True, False, False, False},
 {True, True, True, True, False, True, False, False},
 {True, True, True, True, False, False, True, False},
 {True, True, True, True, False, False, False, True}}
```

In[417]:=

```
Table[Transpose[T16^A[7].sAB[A1][B1]] == T16^A[7].sAB[A1][B1],
 {A1, 0, 7}, {B1, 0, 7}]
```

Out[417]=

```
{{True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {True, True, True, True, False, False, False, True},
 {False, False, False, False, True, True, True, False},
 {False, False, False, False, True, True, True, False},
 {False, False, False, False, True, True, True, False},
 {True, True, True, True, False, False, False, True}}
```

In[418]:=

```
Table[Transpose[σ16.sAB[A1][B1]] == σ16.sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
```

Out[418]=

```
{{True, False, False, False, False, False, False, False},
 {False, True, False, False, False, False, False, False},
 {False, False, True, False, False, False, False, False},
 {False, False, False, True, False, False, False, False},
 {False, False, False, False, True, False, False, False},
 {False, False, False, False, False, True, False, False},
 {False, False, False, False, False, False, True, False},
 {False, False, False, False, False, False, False, True}}
```

In[419]:=

```
Table[Transpose[σ16.SAB⟦A1, B1⟧] == σ16.SAB⟦A1, B1⟧, {A1, 1, 8}, {B1, 1, 8}]
```

Out[419]=

```
{{True, False, False, False, False, False, False, False},
 {False, True, False, False, False, False, False, False},
 {False, False, True, False, False, False, False, False},
 {False, False, False, True, False, False, False, False},
 {False, False, False, False, True, False, False, False},
 {False, False, False, False, False, True, False, False},
 {False, False, False, False, False, False, True, False},
 {False, False, False, False, False, False, False, True}}
```

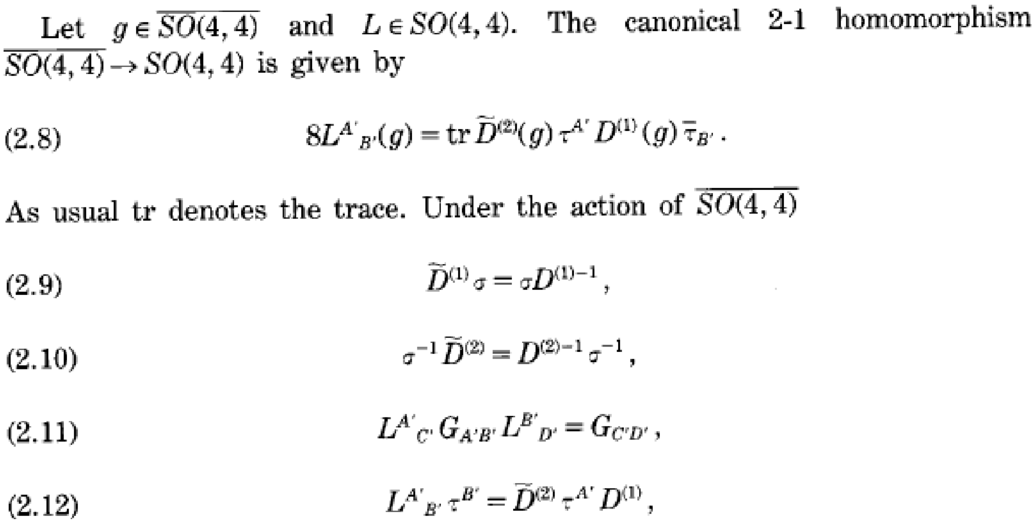
```
In[420]:=
SAB⟦1, 2⟧⟦1⟧
Out[420]=
                      1
{0, 0, 0, 0, 0, 0, 0, -, 0, 0, 0, 0, 0, 0, 0, 0}
                      2
In[421]:=
SAB⟦1, 2⟧⟦2⟧
Out[421]=
                   1
{0, 0, 0, 0, 0, 0, -, 0, 0, 0, 0, 0, 0, 0, 0, 0}
                   2
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
In[422]:=
SAB1 = Table[Take[SAB⟦A1, B1⟧⟦C1⟧, 8],  {A1,  8}, {B1, 8}, {C1, 1, 8}];
In[423]:=
SAB2 = Table[Take[SAB⟦A1, B1⟧⟦C1⟧, -8],  {A1,  8}, {B1, 8}, {C1, 9, 16}];
In[424]:=
SAB⟦1, 2⟧ == ArrayFlatten[{{SAB1⟦1, 2⟧, 0}, {0, SAB2⟦1, 2⟧}}]
Out[424]=
True
In[425]:=
(*Table[SAB⟦A1,B1⟧== ArrayFlatten[{{SAB1⟦A1,B1⟧,0},{0,SAB2⟦A1,B1⟧}}],
 {A1,0, 7},{B1,0,7}]*)
```

$$4{D_{(1)}}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4{D_{(2)}}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$



Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \to SO(4,4)$ is given by

(2.8) $$8L^{A'}{}_{B'}(g) = \mathrm{tr}\,\widetilde{D}^{(2)}(g)\,\tau^{A'}D^{(1)}(g)\,\overline{\tau}_{B'}\,.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

(2.9) $$\widetilde{D}^{(1)}\sigma = \sigma D^{(1)-1}\,,$$

(2.10) $$\sigma^{-1}\widetilde{D}^{(2)} = D^{(2)-1}\sigma^{-1}\,,$$

(2.11) $$L^{A'}{}_{C'}G_{A'B'}L^{B'}{}_{D'} = G_{C'D'}\,,$$

(2.12) $$L^{A'}{}_{B'}\tau^{B'} = \widetilde{D}^{(2)}\tau^{A'}D^{(1)}\,,$$

(2.13) $$L^{A'}{}_{B'}\overline{\tau}^{B'} = D^{(1)-1}\overline{\tau}^{A'}\widetilde{D}^{(2)-1}\,.$$

In[426]:=

```
SAB2[[1, 2]]
```

Out[426]=

```
{{0, 0, 0, 0, 0, 0, 0, -1/2}, {0, 0, 0, 0, 0, 0, -1/2, 0},
 {0, 0, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, -1/2, 0, 0, 0, 0},
 {0, 0, -1/2, 0, 0, 0, 0, 0}, {0, 1/2, 0, 0, 0, 0, 0, 0}, {1/2, 0, 0, 0, 0, 0, 0, 0}}
```

In[427]:=

```
τ[0]
```

Out[427]=

```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[428]:=

```
(*Block[{b,A,F_a^(μ),F^a_(μ),ψ,Ψ},
 b=Array[A,{8}];
 ψ[1]=Array[Ψ[1],{8}];
 ψ[2]=Array[Ψ[2],{8}];
 F_a^(μ)=Table[(1/Sqrt[ψ[1].σ.ψ[1]] ψ[1].σ.(τ̅[μL])),{μL,1,8}];
 F^a_(μ)= Transpose[Table[(1/Sqrt[ψ[1].σ.ψ[1]] η4488[[μL,μL]]τ[μL].ψ[1]),{μL,1,8}]];
 b===FullSimplify[F_a^(μ).ψ[2]/.Thread[ψ[2]→F^a_(μ).b]]*)
```

In[429]:=

```
(*Block[{b,A,B,q,F^a_(μ),ψ,φ,Ψ},
 b=Array[A,{8}];
 ψ[1]=Array[Ψ[1],{8}];
 φ[1]=Array[Ψ[2],{8}];
 q=ParallelTable[
   FullSimplify[(1/√(ψ[1].σ.ψ[1]) ψ[1].σ.1/2 (τ̄[A1].τ[B1]-τ̄[B1].τ[A1]).φ[1])],
   {A1,1,8},{B1,1,8}];
 B=FullSimplify[
   (1/√(ψ[1].σ.ψ[1]) ParallelSum[η4488⟦μL,μL⟧τ[μL]b⟦μL⟧,{μL,1,8}].ψ[1])];
 FullSimplify[q/.Thread[φ[1]→B]]]*)
```

In[430]:=

```
(*Block[{b,A,B,q,F^a_(μ),ψ,φ,Ψ},
 b=Array[A,{8}];
 ψ[1]=Array[Ψ[1],{8}];
 φ[1]=Array[Ψ[2],{8}];
 q=ParallelTable[
   FullSimplify[(ψ[1].σ.(τ̄[A1].τ[B1]).φ[1])],{A1,1,8},{B1,1,8}];
 B=FullSimplify[( ParallelSum[η4488⟦μL,μL⟧τ[μL]b⟦μL⟧,{μL,1,8}].ψ[1])];
 FullSimplify[1/(ψ[1].σ.ψ[1]) q/.Thread[φ[1]→B]]]*)
```

T16$^{\alpha}$

In[431]:=

```
ssgm4488
```

Out[431]=

```
{g[0][0] → (Cot[6 H ♯1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
 g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
 g[1][1] → (ⅇ^(2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
 g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
 g[2][1] → (0 &), g[2][2] → (ⅇ^(2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
 g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
 g[3][2] → (0 &), g[3][3] → (ⅇ^(2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
 g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
 g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
 g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
 g[5][5] → (-ⅇ^(-2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
 g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
 g[6][5] → (0 &), g[6][6] → (-ⅇ^(-2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[6][7] → (0 &),
 g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
 g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-ⅇ^(-2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &)}
```

In[432]:=

```
e^α_(A) /. sge^α_(A)
```

Out[432]=

```
{{1/√(g[0][0][x0, x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 1/√(g[1][1][x0, x4]), 0, 0, 0, 0, 0, 0},
 {0, 0, 1/√(g[2][2][x0, x4]), 0, 0, 0, 0, 0}, {0, 0, 0, 1/√(g[3][3][x0, x4]), 0, 0, 0, 0},
 {0, 0, 0, 0, 1/√(-g[4][4][x0, x4]), 0, 0, 0}, {0, 0, 0, 0, 0, 1/√(-g[5][5][x0, x4]), 0, 0},
 {0, 0, 0, 0, 0, 0, 1/√(-g[6][6][x0, x4]), 0}, {0, 0, 0, 0, 0, 0, 0, 1/√(-g[7][7][x0, x4])}}
```

In[433]:=

```
Table[T16^α[α1 - 1] =  Sum[((e^α_(A) /. sge^α_(A)) [[α1, A1]] ) T16^A[A1 - 1], {A1, 1, 8}],
   {α1, 1, 8}];(*/.ssgm4488*)
```

In[434]:=

```
(T16^α[8] = T16^α[0].T16^α[1].T16^α[2].T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7]);
```

In[435]:=

```
(*T16^α[8]==T16^A[8]*Sec[6 H x0]*)
```

CHECK

```
Table[T16^α[α1 - 1] = Sum[(e^α_(A)[[α1, A1]]) T16^A[A1 - 1], {A1, 1, 8}], {α1, 1, 8}];
```

and ● ● ● :

In[436]:=

```
(*Table[T16^α[α1-1]=  Sum[(e^α_(A)[[α1,A1]] ) T16^A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

In[437]:=

```
(*(T16^α[8]=T16^α[0].T16^α[1].T16^α[2].
    T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7])//MatrixForm*)
```

In[438]:=

```
Table[{{A1, B1},
   FullSimplify[ExpandAll[1/2 (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) ==
      η4488[[A1 + 1, B1 + 1]] * ID16]]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[438]//MatrixForm=

```
({0, 0}) ({0, 1}) ({0, 2}) ({0, 3}) ({0, 4}) ({0, 5}) ({0, 6}) ({0, 7})
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
({1, 0}) ({1, 1}) ({1, 2}) ({1, 3}) ({1, 4}) ({1, 5}) ({1, 6}) ({1, 7})
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
({2, 0}) ({2, 1}) ({2, 2}) ({2, 3}) ({2, 4}) ({2, 5}) ({2, 6}) ({2, 7})
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
({3, 0}) ({3, 1}) ({3, 2}) ({3, 3}) ({3, 4}) ({3, 5}) ({3, 6}) ({3, 7})
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
({4, 0}) ({4, 1}) ({4, 2}) ({4, 3}) ({4, 4}) ({4, 5}) ({4, 6}) ({4, 7})
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
({5, 0}) ({5, 1}) ({5, 2}) ({5, 3}) ({5, 4}) ({5, 5}) ({5, 6}) ({5, 7})
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
({6, 0}) ({6, 1}) ({6, 2}) ({6, 3}) ({6, 4}) ({6, 5}) ({6, 6}) ({6, 7})
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
({7, 0}) ({7, 1}) ({7, 2}) ({7, 3}) ({7, 4}) ({7, 5}) ({7, 6}) ({7, 7})
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
```

In[439]:=

```
Table[{{α1, β1}, FullSimplify[
    1
    ─ FullSimplify[((T16^α[α1].T16^α[β1] + T16^α[β1].T16^α[α1]) /. ssgm4488),
    2
      constraintVars] == Inverse[MatrixMetric44][[α1 + 1, β1 + 1]] * ID16]},
  {α1, 0, 7}, {β1, 0, 7}] // MatrixForm
```

Out[439]//MatrixForm=

```
( {0, 0}  {0, 1}  {0, 2}  {0, 3}  {0, 4}  {0, 5}  {0, 6}  {0, 7} )
(  True    True    True    True    True    True    True    True  )
( {1, 0}  {1, 1}  {1, 2}  {1, 3}  {1, 4}  {1, 5}  {1, 6}  {1, 7} )
(  True    True    True    True    True    True    True    True  )
( {2, 0}  {2, 1}  {2, 2}  {2, 3}  {2, 4}  {2, 5}  {2, 6}  {2, 7} )
(  True    True    True    True    True    True    True    True  )
( {3, 0}  {3, 1}  {3, 2}  {3, 3}  {3, 4}  {3, 5}  {3, 6}  {3, 7} )
(  True    True    True    True    True    True    True    True  )
( {4, 0}  {4, 1}  {4, 2}  {4, 3}  {4, 4}  {4, 5}  {4, 6}  {4, 7} )
(  True    True    True    True    True    True    True    True  )
( {5, 0}  {5, 1}  {5, 2}  {5, 3}  {5, 4}  {5, 5}  {5, 6}  {5, 7} )
(  True    True    True    True    True    True    True    True  )
( {6, 0}  {6, 1}  {6, 2}  {6, 3}  {6, 4}  {6, 5}  {6, 6}  {6, 7} )
(  True    True    True    True    True    True    True    True  )
( {7, 0}  {7, 1}  {7, 2}  {7, 3}  {7, 4}  {7, 5}  {7, 6}  {7, 7} )
(  True    True    True    True    True    True    True    True  )
```

In[440]:=

```
MatrixMetric44 // MatrixForm
```

Out[440]//MatrixForm=

```
( Cot[6 H x0]^2   0                              0                              0                              0
  0               e^(2 a4[H x4]) Sin[6 H x0]^(1/3)  0                           0                              0
  0               0                              e^(2 a4[H x4]) Sin[6 H x0]^(1/3)  0                           0
  0               0                              0                              e^(2 a4[H x4]) Sin[6 H x0]^(1/3)  0
  0               0                              0                              0                              -1
  0               0                              0                              0                              0    - e^(-2 a4[H x4]) S
  0               0                              0                              0                              0
  0               0                              0                              0                              0
```

In[441]:=

```
((e^α_(A) /. sgtrye^α_(A)).MatrixMetric44.(e^α_(A) /. sgtrye^α_(A))) // MatrixForm
```

Out[441]//MatrixForm=

```
( 1  0  0  0   0   0   0   0 )
( 0  1  0  0   0   0   0   0 )
( 0  0  1  0   0   0   0   0 )
( 0  0  0  1   0   0   0   0 )
( 0  0  0  0  -1   0   0   0 )
( 0  0  0  0   0  -1   0   0 )
( 0  0  0  0   0   0  -1   0 )
( 0  0  0  0   0   0   0  -1 )
```

In[442]:=

```
Transpose[(e_α^(A) /. sgtrye_α^(A))].η4488.(e_α^(A) /. sgtrye_α^(A)) - MatrixMetric44 //
  FullSimplify[#, constraintVars] &
```

Out[442]=

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[443]:=

```
Transpose[(e^α_(A) /. sgtrye^α_(A))].MatrixMetric44.(e^α_(A) /. sgtrye^α_(A)) - η4488 //
  (*Full*)Simplify[#, constraintVars] &
```

Out[443]=

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[444]:=

```
Inverse[(e^α_(A) /. sgtrye^α_(A))] == (e_α^(A) /. sgtrye_α^(A))
```

Out[444]=

```
True
```

In[445]:=

```
Block[{a, b, c, d, m}, m = {{a, b}, {c, d}};
 Inverse[Transpose[m]] - Transpose[Inverse[m]]]
```

Out[445]=

```
{{0, 0}, {0, 0}}
```

BELOW : LagrangianΨ16 =

$\kappa \sqrt{\text{Det}[g_{\mu\nu}]}$ Transpose[Ψ16].σ16.T16^{A}[A1]. Ψ16$_{|\alpha}$ g$^{-1\ \alpha\beta}$ $\eta_{A1\,B1}$ E^{B1}_{β} + mass term

In[446]:=

```
e_α^(A) /. sgtrye_α^(A)
```

Out[446]=

$$\left\{\{\text{Cot}[6\,H\,x0], 0, 0, 0, 0, 0, 0, 0\}, \left\{0, \sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}, 0, 0, 0, 0, 0, 0\right\},\right.$$
$$\left\{0, 0, \sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \sqrt{e^{2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}, 0, 0, 0, 0\right\},$$
$$\{0, 0, 0, 0, 1, 0, 0, 0\}, \left\{0, 0, 0, 0, 0, \sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}, 0, 0\right\},$$
$$\left\{0, 0, 0, 0, 0, 0, \sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}, 0\right\},$$
$$\left.\left\{0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2\,a4[H\,x4]}}\ \text{Sin}[6\,H\,x0]^{1/6}\right\}\right\}$$

In[447]:=

$$e^{\alpha}_{(A)}\ \text{/.}\ \text{sgtrye}^{\alpha}_{(A)}$$

Out[447]=

$$\Big\{\{\text{Tan}[6\,\text{H}\,\text{x0}],0,0,0,0,0,0,0\},\Big\{0,\frac{1}{\sqrt{\mathbb{e}^{2\,\text{a4}[\text{H}\,\text{x4}]}}\ \text{Sin}[6\,\text{H}\,\text{x0}]^{1/6}},0,0,0,0,0,0\Big\},$$
$$\Big\{0,0,\frac{1}{\sqrt{\mathbb{e}^{2\,\text{a4}[\text{H}\,\text{x4}]}}\ \text{Sin}[6\,\text{H}\,\text{x0}]^{1/6}},0,0,0,0,0\Big\},\Big\{0,0,0,\frac{1}{\sqrt{\mathbb{e}^{2\,\text{a4}[\text{H}\,\text{x4}]}}\ \text{Sin}[6\,\text{H}\,\text{x0}]^{1/6}},0,0,0,0\Big\},$$
$$\{0,0,0,0,1,0,0,0\},\Big\{0,0,0,0,0,\frac{1}{\sqrt{\mathbb{e}^{-2\,\text{a4}[\text{H}\,\text{x4}]}}\ \text{Sin}[6\,\text{H}\,\text{x0}]^{1/6}},0,0\Big\},$$
$$\Big\{0,0,0,0,0,0,\frac{1}{\sqrt{\mathbb{e}^{-2\,\text{a4}[\text{H}\,\text{x4}]}}\ \text{Sin}[6\,\text{H}\,\text{x0}]^{1/6}},0\Big\},$$
$$\Big\{0,0,0,0,0,0,0,\frac{1}{\sqrt{\mathbb{e}^{-2\,\text{a4}[\text{H}\,\text{x4}]}}\ \text{Sin}[6\,\text{H}\,\text{x0}]^{1/6}}\Big\}\Big\}$$

In[448]:=

$$e^{\alpha}_{(A)}\ \text{/.}\ \text{sge}^{\alpha}_{(A)}$$

Out[448]=

$$\Big\{\Big\{\frac{1}{\sqrt{\text{g}[0][0][\text{x0},\text{x4}]}},0,0,0,0,0,0,0\Big\},\Big\{0,\frac{1}{\sqrt{\text{g}[1][1][\text{x0},\text{x4}]}},0,0,0,0,0,0\Big\},$$
$$\Big\{0,0,\frac{1}{\sqrt{\text{g}[2][2][\text{x0},\text{x4}]}},0,0,0,0,0\Big\},\Big\{0,0,0,\frac{1}{\sqrt{\text{g}[3][3][\text{x0},\text{x4}]}},0,0,0,0\Big\},$$
$$\Big\{0,0,0,0,\frac{1}{\sqrt{-\text{g}[4][4][\text{x0},\text{x4}]}},0,0,0\Big\},\Big\{0,0,0,0,0,\frac{1}{\sqrt{-\text{g}[5][5][\text{x0},\text{x4}]}},0,0\Big\},$$
$$\Big\{0,0,0,0,0,0,\frac{1}{\sqrt{-\text{g}[6][6][\text{x0},\text{x4}]}},0\Big\},\Big\{0,0,0,0,0,0,0,\frac{1}{\sqrt{-\text{g}[7][7][\text{x0},\text{x4}]}}\Big\}\Big\}$$

In[449]:=

```
Clear[con];
con[g_, ass_ : {}] := Module[{t, ΓΓ, ginv},
  ginv = FullSimplify[Inverse[g], ass];
  ΓΓ = FullSimplify[Table[(1 / 2) * Sum[(ginv⟦i, s⟧) *
          (D[g⟦s, j⟧, X⟦k⟧ ] + D[g⟦s, k⟧, X⟦j⟧ ] - D[g⟦j, k⟧, X⟦s⟧ ]), {s, 1, DIM8}]
      {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] , ass];
  Return[{ginv, ΓΓ}]]
```

```
In[451]:=
        ssgm4488
Out[451]=
        {g[0][0] → (Cot[6 H ♯1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
         g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
         g[1][1] → (ⅇ^(2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
         g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
         g[2][1] → (0 &), g[2][2] → (ⅇ^(2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
         g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
         g[3][2] → (0 &), g[3][3] → (ⅇ^(2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
         g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
         g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
         g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
         g[5][5] → (-ⅇ^(-2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
         g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
         g[6][5] → (0 &), g[6][6] → (-ⅇ^(-2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &), g[6][7] → (0 &),
         g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
         g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-ⅇ^(-2 a4[H ♯2]) Sin[6 H ♯1]^(1/3) &)}
In[452]:=
        g4488
        g4488 /. sg
Out[452]=
        {{g[0][0][x0, x4], g[0][1][x0, x4], g[0][2][x0, x4], g[0][3][x0, x4],
          g[0][4][x0, x4], g[0][5][x0, x4], g[0][6][x0, x4], g[0][7][x0, x4]},
         {g[1][0][x0, x4], g[1][1][x0, x4], g[1][2][x0, x4], g[1][3][x0, x4],
          g[1][4][x0, x4], g[1][5][x0, x4], g[1][6][x0, x4], g[1][7][x0, x4]},
         {g[2][0][x0, x4], g[2][1][x0, x4], g[2][2][x0, x4], g[2][3][x0, x4],
          g[2][4][x0, x4], g[2][5][x0, x4], g[2][6][x0, x4], g[2][7][x0, x4]},
         {g[3][0][x0, x4], g[3][1][x0, x4], g[3][2][x0, x4], g[3][3][x0, x4],
          g[3][4][x0, x4], g[3][5][x0, x4], g[3][6][x0, x4], g[3][7][x0, x4]},
         {g[4][0][x0, x4], g[4][1][x0, x4], g[4][2][x0, x4], g[4][3][x0, x4],
          g[4][4][x0, x4], g[4][5][x0, x4], g[4][6][x0, x4], g[4][7][x0, x4]},
         {g[5][0][x0, x4], g[5][1][x0, x4], g[5][2][x0, x4], g[5][3][x0, x4],
          g[5][4][x0, x4], g[5][5][x0, x4], g[5][6][x0, x4], g[5][7][x0, x4]},
         {g[6][0][x0, x4], g[6][1][x0, x4], g[6][2][x0, x4], g[6][3][x0, x4],
          g[6][4][x0, x4], g[6][5][x0, x4], g[6][6][x0, x4], g[6][7][x0, x4]},
         {g[7][0][x0, x4], g[7][1][x0, x4], g[7][2][x0, x4], g[7][3][x0, x4],
          g[7][4][x0, x4], g[7][5][x0, x4], g[7][6][x0, x4], g[7][7][x0, x4]}}
Out[453]=
        {{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
         {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
         {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
         {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}
```

```
In[454]:=
      MatrixForm[#] & /@ Block[{eAα, eαA, (*ginv,Γ,gg,*)colVecs, rowVecs, ab, ba, r},
        (*gg=g4488/.sg;*)
        (*{ginv,Γ}=con[gg];*)
        eαA = e^α_(A) /. sge^α_(A);
        eAα = e^(A)_α /. sge^(A)_α;
        rowVecs = Table[eAα[[b, All]], {b, 1, Length[eAα]}];
        colVecs = Table[eαA[[All, b]], {b, 1, Length[eαA]}];
        ab = rowVecs.colVecs // FullSimplify[#, constraintVars] &;
        ba = Transpose[colVecs].Transpose[rowVecs] //
          FullSimplify[#, constraintVars] &;
        (*r={{ab},{ba}};*)
        r = {ab, ba};
        r]
```

```
Out[454]=
   ( 1 0 0 0 0 0 0 0 )   ( 1 0 0 0 0 0 0 0 )
   ( 0 1 0 0 0 0 0 0 )   ( 0 1 0 0 0 0 0 0 )
   ( 0 0 1 0 0 0 0 0 )   ( 0 0 1 0 0 0 0 0 )
  {( 0 0 0 1 0 0 0 0 ) , ( 0 0 0 1 0 0 0 0 )}
   ( 0 0 0 0 1 0 0 0 )   ( 0 0 0 0 1 0 0 0 )
   ( 0 0 0 0 0 1 0 0 )   ( 0 0 0 0 0 1 0 0 )
   ( 0 0 0 0 0 0 1 0 )   ( 0 0 0 0 0 0 1 0 )
   ( 0 0 0 0 0 0 0 1 )   ( 0 0 0 0 0 0 0 1 )
```

---

metric compatibility condition for the Octad

$$\nabla_\mu e_\nu^I = \partial_\mu e_\nu^I - \Gamma_{\mu\nu}^\rho e_\rho^I + \omega_{\mu J}^I e_\nu^J = 0$$

In[455]:=

```
Clear[spinCoefficients];
spinCoefficients[eIν_] := Module[{eνI, w, r, s},
  If[Dimensions[eIν] ≠ {8, 8}, Return[{}]];
  If[Det[eIν] == 0, Return[{}]];
  eνI = FullSimplify[Inverse[eIν], constraintVars] /. subsDefects;
  w = Table[
    -Table[FullSimplify[Sum[D[eIν⟦I1, ν1⟧, X⟦μ1⟧] ∗ eνI⟦ν1, Jprime⟧, {ν1, 1,
           Length[eνI]}] - Sum[ eIν⟦I1, ρ⟧ ∗ Γ⟦ρ, μ1, ν1⟧ ∗ eνI⟦ν1, Jprime⟧,
          {ν1, 1, Length[eνI]}, {ρ, 1, Length[X]}],
        constraintVars] /. subsDefects, {I1, 1, Length[eIν]},
      {Jprime, 1, Length[eνI]}], {μ1, 1, Length[X]}];
  Assert[Dimensions[w] == {8, 8, 8}];
  Return[w];
 ]
```

In[457]:=

```
spinCoeffs = spinCoefficients[gtrye_α^(A)]
```

Out[457]=

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, -ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0,
   0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, -ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, -ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4],
   0, 0, 0}, {0, 0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0},
  {ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6)},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4]},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}}
```

In[458]:=

```
Table[ωμIJ[μ] = FullSimplify[
    -((D[gtrye_α^(A), X⟦μ⟧].gtrye^α_(A)) - (gtrye_α^(A).Γ⟦All, μ, All⟧.gtrye^α_(A))),
    constraintVars] /. subsDefects, {μ, 1, Length[X]}]
```

Out[458]=

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {ⅇ^a4[H x4] H Sin[6 H x0]^(1/6), 0,
   0, 0, ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^a4[H x4] H Sin[6 H x0]^(1/6), 0, 0, 0, ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {ⅇ^a4[H x4] H Sin[6 H x0]^(1/6), 0, 0, 0, ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4],
   0, 0, 0}, {0, 0, 0, ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0},
  {ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6)},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4]},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^-a4[H x4] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}}
```

In[459]:=

```
(*gtrye^α_(A).gtrye_α^(A)*)
```

In[460]:=

```
(*gtrye_α^(A).gtrye^α_(A)*)
```

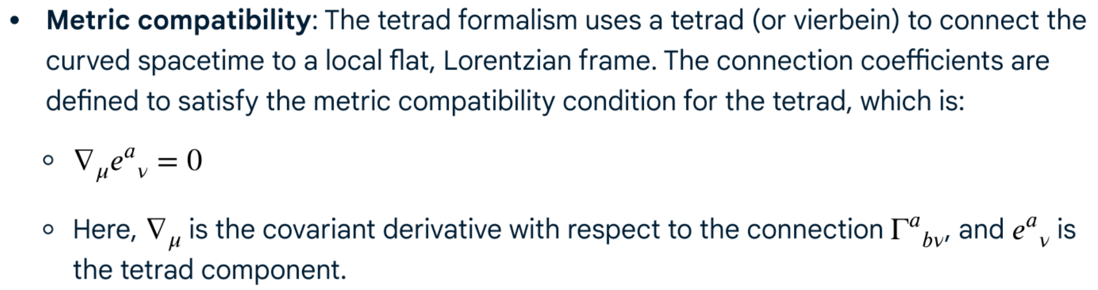
In[461]:=

```
(*gtrye_α^(A)*)
```

```
In[462]:=
(*Γ
 Dimensions[%]*)
In[463]:=
(*Table[Γ⟦All,μ,All⟧,{μ,1,Length[X]}]*)
In[464]:=
(*Γ⟦All,5,All⟧*)
In[465]:=
(*Γ⟦All,1,All⟧*)
In[466]:=
(*spinCoeffs⟦1⟧*)
In[467]:=
(*spinCoeffs⟦5⟧*)
In[468]:=
(*{X⟦1⟧,X⟦5⟧}*)
In[469]:=
(*spinCoeffs⟦1⟧⟦1⟧⟦1⟧*)
In[470]:=
(*ωμIJ[1]⟦1⟧⟦1⟧*)
In[471]:=
(*Block[{μ=1},
  FullSimplify[-((D[gtrye_α^(A),X⟦μ⟧].gtrye^α_(A))-(gtrye_α^(A).Γ⟦All,μ,All⟧.gtrye^α_(A))),
    constraintVars]/.subsDefects]
 Dimensions[%]*)
In[472]:=
(*Block[{μ=1},
 FullSimplify[-((D[gtrye_α^(A),X⟦μ⟧].Transpose[gtrye^α_(A)])-(gtrye_α^(A).Γ⟦All,μ,All⟧.
         Transpose[gtrye^α_(A)])),constraintVars]/.subsDefects]*)
In[473]:=
(*Block[{μ=1},FullSimplify[
     -((Sum[     D[gtrye_α^(A)⟦All,ν⟧,X⟦μ⟧]*gtrye^α_(A)⟦ν,All⟧     ,{ν,1,8}]   )-
        (gtrye_α^(A).Γ⟦All,μ,All⟧.gtrye^α_(A))),constraintVars]/.subsDefects]
 Dimensions[%]*)
In[474]:=
(*Table[,{A1,1,8},{B1,1,8}]*)
In[475]:=
(*Block[{μ=1},Table[FullSimplify[
      -((Sum[     D[gtrye_α^(A)⟦A1,ν⟧,X⟦μ⟧]*gtrye^α_(A)⟦ν,B1⟧     ,{ν,1,8}]   )-
          (Sum[     gtrye_α^(A)⟦A1,ρ⟧*Γ⟦ρ,μ,ν⟧*gtrye^α_(A)⟦ν,B1⟧    ,{ρ,1,8},
            {ν,1,8}]   )),constraintVars]/.subsDefects,{A1,1,8},{B1,1,8}]]
 Dimensions[%]*)
```

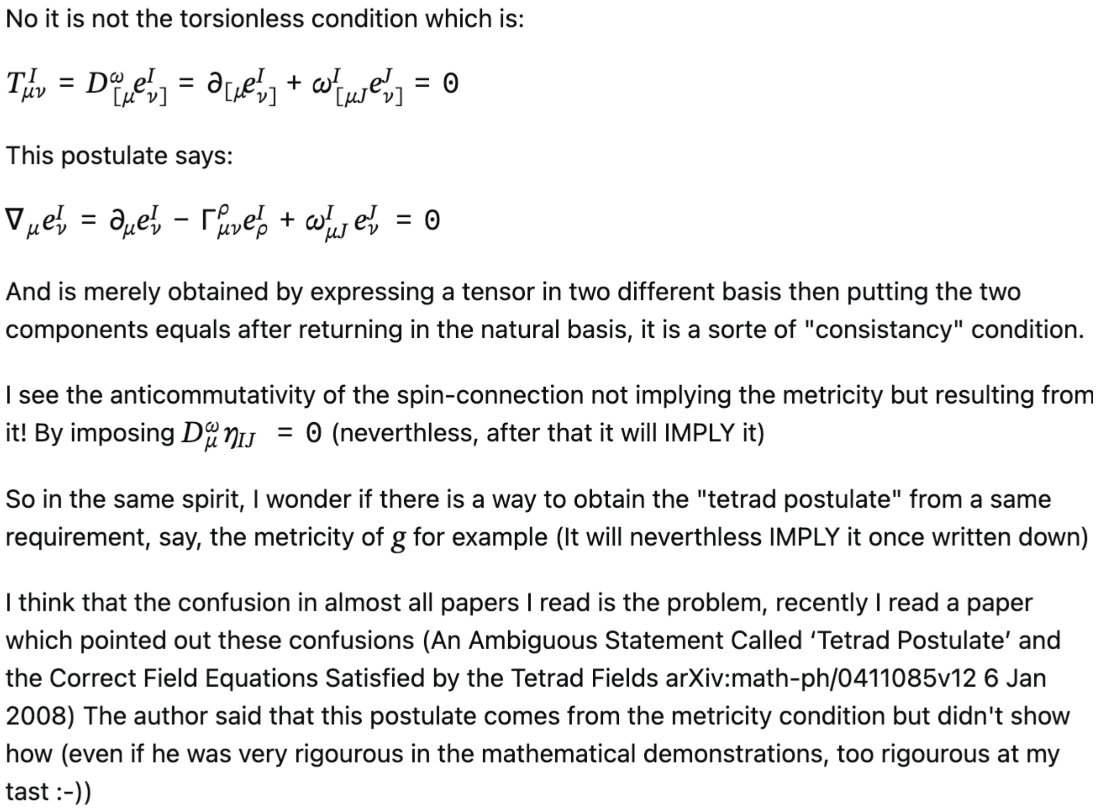
```
In[476]:=
Table[ ( ωμIJ[μ] - spinCoeffs⟦μ⟧ ) /. subsDefects , {μ, 1, Length[X]}] //
  Flatten // Union
```

```
Out[476]=
{0}
```



- **Metric compatibility**: The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
  - $\nabla_\mu e^a{}_\nu = 0$
  - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

○ Metric compatibility : The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame . The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is : ○ V
нe "
v=0
• Here, Vu is the covariant derivative with respect to the connection F" bv, and ea v is
the tetrad component .



No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu}e^I_{\nu]} = \partial_{[\mu}e^I_{\nu]} + \omega^I_{[\mu J}e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu}e^I_\rho + \omega^I_{\mu J}\, e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistancy" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (neverthless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will neverthless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called ‘Tetrad Postulate’ and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

In[477]:=

```
Block[{eAα, eαA, t, ginv, Γ, gg},
 gg = g4488 /. sg;
 {ginv, Γ} = con[gg];
 eαA = e^α_(A) /. sge^α_(A); eAα = e_α^(A) /. sge_α^(A);
 t = Table[ Block[{v, V, contra, covari},
    V = eAα[[b, All]];
    v = eαA[[All, b]];
    contra =
     Table[D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before ω terms*)
    covari =
     Table[D[V[[k]], X[[μ]] ] - Sum[ V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*before ω terms*)
    {b - 1, X[[μ]], {{contra}, {covari}}}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
 t]
FullSimplify[# /. ssgm4488, constraintVars] & /@ %
```

Out[477]=

```
                                g[0][0]^(0,1)[x0, x4]
{{{0, x0, {{{0, 0, 0, 0, - ─────────────────────────────────────── , 0, 0, 0}},
                           2 √g[0][0][x0, x4]  g[4][4][x0, x4]

                  g[0][0]^(0,1)[x0, x4]
    {{0, 0, 0, 0, - ───────────────────── , 0, 0, 0}}}},
                    2 √g[0][0][x0, x4]

                          g[1][1]^(1,0)[x0, x4]
  {0, x1, {{{0, ─────────────────────────────────────── , 0, 0, 0, 0, 0, 0}},
                2 √g[0][0][x0, x4]  g[1][1][x0, x4]

            g[1][1]^(1,0)[x0, x4]
    {{0, ───────────────────── , 0, 0, 0, 0, 0, 0}}}},
         2 √g[0][0][x0, x4]

                             g[2][2]^(1,0)[x0, x4]
  {0, x2, {{{0, 0, ─────────────────────────────────────── , 0, 0, 0, 0, 0}},
                   2 √g[0][0][x0, x4]  g[2][2][x0, x4]

               g[2][2]^(1,0)[x0, x4]
    {{0, 0, ───────────────────── , 0, 0, 0, 0, 0}}}},
            2 √g[0][0][x0, x4]

                                g[3][3]^(1,0)[x0, x4]
  {0, x3, {{{0, 0, 0, ─────────────────────────────────────── , 0, 0, 0, 0}},
                      2 √g[0][0][x0, x4]  g[3][3][x0, x4]

                  g[3][3]^(1,0)[x0, x4]
    {{0, 0, 0, ───────────────────── , 0, 0, 0, 0}}}},
               2 √g[0][0][x0, x4]

                                   g[4][4]^(1,0)[x0, x4]
  {0, x4, {{{0, 0, 0, 0, ─────────────────────────────────────── , 0, 0, 0}},
                         2 √g[0][0][x0, x4]  g[4][4][x0, x4]

                     g[4][4]^(1,0)[x0, x4]
    {{0, 0, 0, 0, ───────────────────── , 0, 0, 0}}}},
                  2 √g[0][0][x0, x4]
```

$$\Big\{0,\ x5,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}},\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{0,\ x6,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}\ g[6][6][x0,\ x4]},\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}},\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{0,\ x7,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}\ g[7][7][x0,\ x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}}\Big\}\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{\{1,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\ \Big\{1,\ x1,$$

$$\Big\{\Big\{\Big\{-\frac{g[1][1]^{(1,0)}[x0,\ x4]}{2\,g[0][0][x0,\ x4]\ \sqrt{g[1][1][x0,\ x4]}},\ 0,\ 0,\ 0,\ -\frac{g[1][1]^{(0,1)}[x0,\ x4]}{2\ \sqrt{g[1][1][x0,\ x4]}\ g[4][4][x0,\ x4]},$$

$$0,\ 0,\ 0\Big\}\Big\},\ \Big\{\Big\{-\frac{g[1][1]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[1][1][x0,\ x4]}},\ 0,\ 0,\ 0,\ -\frac{g[1][1]^{(0,1)}[x0,\ x4]}{2\ \sqrt{g[1][1][x0,\ x4]}},\ 0,\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\{1,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x3,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x4,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\Big\},$$

$$\Big\{\{2,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{2,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\ \Big\{2,\ x2,$$

$$\Big\{\Big\{\Big\{-\frac{g[2][2]^{(1,0)}[x0,\ x4]}{2\,g[0][0][x0,\ x4]\ \sqrt{g[2][2][x0,\ x4]}},\ 0,\ 0,\ 0,\ -\frac{g[2][2]^{(0,1)}[x0,\ x4]}{2\ \sqrt{g[2][2][x0,\ x4]}\ g[4][4][x0,\ x4]},$$

$$0,\ 0,\ 0\Big\}\Big\},\ \Big\{\Big\{-\frac{g[2][2]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[2][2][x0,\ x4]}},\ 0,\ 0,\ 0,\ -\frac{g[2][2]^{(0,1)}[x0,\ x4]}{2\ \sqrt{g[2][2][x0,\ x4]}},\ 0,\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\{2,\ x3,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{2,\ x4,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{2,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{2,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{2,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\Big\},$$

$$\Big\{\{3,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{3,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{3,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\ \Big\{3,\ x3,$$

$$\{\{\{-\frac{g[3][3]^{(1,0)}[x0, x4]}{2\,g[0][0][x0, x4]\,\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]}{2\,\sqrt{g[3][3][x0, x4]}\,g[4][4][x0, x4]},$$

$$0, 0, 0\}\}, \{\{-\frac{g[3][3]^{(1,0)}[x0, x4]}{2\,\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]}{2\,\sqrt{g[3][3][x0, x4]}}, 0, 0, 0\}\}\}\},$$

$$\{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\},$$

$$\{\{4, x0, \{\{\{\frac{g[0][0]^{(0,1)}[x0, x4]}{2\,g[0][0][x0, x4]\,\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0,$$

$$\frac{g[4][4]^{(1,0)}[x0, x4]}{2\,(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2\,\sqrt{-g[4][4][x0, x4]}\,g[4][4][x0, x4]}, 0, 0, 0\}\},$$

$$\{\{\frac{g[0][0]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0,$$

$$-\frac{g[4][4]^{(1,0)}[x0, x4]}{2\,\sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(1,0)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0\}\}\}\},$$

$$\{4, x1, \{\{\{0, \frac{g[1][1]^{(0,1)}[x0, x4]}{2\,g[1][1][x0, x4]\,\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, \frac{g[1][1]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{4, x2, \{\{\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]}{2\,g[2][2][x0, x4]\,\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{4, x3, \{\{\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2\,g[3][3][x0, x4]\,\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0, 0\}\}\}\},$$

$$\{4, x4, \{\{\{-\frac{g[4][4]^{(1,0)}[x0, x4]}{2\,g[0][0][x0, x4]\,\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0,$$

$$\frac{g[4][4]^{(0,1)}[x0, x4]}{2\,(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\,\sqrt{-g[4][4][x0, x4]}\,g[4][4][x0, x4]}, 0, 0, 0\}\},$$

$$\{\{-\frac{g[4][4]^{(1,0)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0,$$

$$-\frac{g[4][4]^{(0,1)}[x0, x4]}{2\,\sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]}, 0, 0, 0\}\}\}\},$$

$$\Big\{4,\ x5,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]},\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{4,\ x6,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[6][6][x0,\ x4]},\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]},\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{4,\ x7,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[7][7][x0,\ x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[7][7]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]}\Big\}\Big\}\Big\}\Big\}\Big\},\ \Big\{\Big\{5,\ x0,$$

$$\Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ (-g[5][5][x0,\ x4])^{3/2}}+\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}}-\frac{g[5][5]^{(1,0)}[x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}}{2\,g[5][5][x0,\ x4]},$$

$$0,\ 0\Big\}\Big\}\Big\}\Big\},\ \{5,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{5,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{5,\ x3,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\ \Big\{5,\ x4,$$

$$\Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ (-g[5][5][x0,\ x4])^{3/2}}+\frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}}-\frac{g[5][5]^{(0,1)}[x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}}{2\,g[5][5][x0,\ x4]},$$

$$0,\ 0\Big\}\Big\}\Big\}\Big\},\ \Big\{5,\ x5,\ \Big\{\Big\{\Big\{-\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\,g[0][0][x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}},$$

$$0,\ 0,\ 0,\ -\frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\,g[4][4][x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}},\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{-\frac{g[5][5]^{(1,0)}[x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}}{2\,g[5][5][x0,\ x4]},\ 0,\ 0,\ 0,$$

$$-\frac{g[5][5]^{(0,1)}[x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}}{2\,g[5][5][x0,\ x4]},\ 0,\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\{5,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{5,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\Big\},\ \Big\{\Big\{6,\ x0,$$

$$\Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ (-g[6][6][x0,\ x4])^{3/2}}+\frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[6][6][x0,\ x4]}\ g[6][6][x0,\ x4]},\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[6][6][x0,\ x4]}}-\frac{g[6][6]^{(1,0)}[x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}}{2\,g[6][6][x0,\ x4]},$$

$$0\}\}\}\},\ \{6,\ x1,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\{6,\ x2,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\{6,\ x3,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\ \Big\{6,\ x4,$$

$$\Big\{\Big\{\Big\{0,0,0,0,0,0,\ \frac{g[6][6]^{(0,1)}[x0,x4]}{2\,(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(0,1)}[x0,x4]}{2\,\sqrt{-g[6][6][x0,x4]}\;g[6][6][x0,x4]},\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,0,0,0,0,0,\ -\frac{g[6][6]^{(0,1)}[x0,x4]}{2\,\sqrt{-g[6][6][x0,x4]}}-\frac{g[6][6]^{(0,1)}[x0,x4]\,\sqrt{-g[6][6][x0,x4]}}{2\,g[6][6][x0,x4]},$$

$$0\Big\}\Big\}\Big\}\Big\},\ \{6,\ x5,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\ \Big\{6,\ x6,$$

$$\Big\{\Big\{\Big\{-\frac{g[6][6]^{(1,0)}[x0,x4]}{2\,g[0][0][x0,x4]\,\sqrt{-g[6][6][x0,x4]}},\ 0,0,0,\ -\frac{g[6][6]^{(0,1)}[x0,x4]}{2\,g[4][4][x0,x4]\,\sqrt{-g[6][6][x0,x4]}},$$

$$0,0,0\Big\}\Big\},\ \Big\{\Big\{-\frac{g[6][6]^{(1,0)}[x0,x4]\,\sqrt{-g[6][6][x0,x4]}}{2\,g[6][6][x0,x4]},\ 0,$$

$$0,0,\ -\frac{g[6][6]^{(0,1)}[x0,x4]\,\sqrt{-g[6][6][x0,x4]}}{2\,g[6][6][x0,x4]},\ 0,0,0\Big\}\Big\}\Big\}\Big\},$$

$$\{6,\ x7,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\}\Big\},\ \Big\{\Big\{7,\ x0,$$

$$\Big\{\Big\{\Big\{0,0,0,0,0,0,0,\ \frac{g[7][7]^{(1,0)}[x0,x4]}{2\,(-g[7][7][x0,x4])^{3/2}}+\frac{g[7][7]^{(1,0)}[x0,x4]}{2\,\sqrt{-g[7][7][x0,x4]}\;g[7][7][x0,x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0,0,0,0,0,0,0,$$

$$-\frac{g[7][7]^{(1,0)}[x0,x4]}{2\,\sqrt{-g[7][7][x0,x4]}}-\frac{g[7][7]^{(1,0)}[x0,x4]\,\sqrt{-g[7][7][x0,x4]}}{2\,g[7][7][x0,x4]}\Big\}\Big\}\Big\}\Big\},$$

$$\{7,\ x1,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\{7,\ x2,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\{7,\ x3,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\ \Big\{7,\ x4,$$

$$\Big\{\Big\{\Big\{0,0,0,0,0,0,0,\ \frac{g[7][7]^{(0,1)}[x0,x4]}{2\,(-g[7][7][x0,x4])^{3/2}}+\frac{g[7][7]^{(0,1)}[x0,x4]}{2\,\sqrt{-g[7][7][x0,x4]}\;g[7][7][x0,x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0,0,0,0,0,0,0,$$

$$-\frac{g[7][7]^{(0,1)}[x0,x4]}{2\,\sqrt{-g[7][7][x0,x4]}}-\frac{g[7][7]^{(0,1)}[x0,x4]\,\sqrt{-g[7][7][x0,x4]}}{2\,g[7][7][x0,x4]}\Big\}\Big\}\Big\}\Big\},$$

$$\{7,\ x5,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\{7,\ x6,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\ \Big\{7,\ x7,$$

$$\Big\{\Big\{\Big\{-\frac{g[7][7]^{(1,0)}[x0,x4]}{2\,g[0][0][x0,x4]\,\sqrt{-g[7][7][x0,x4]}},\ 0,0,0,\ -\frac{g[7][7]^{(0,1)}[x0,x4]}{2\,g[4][4][x0,x4]\,\sqrt{-g[7][7][x0,x4]}},$$

$$0,0,0\Big\}\Big\},\ \Big\{\Big\{-\frac{g[7][7]^{(1,0)}[x0,x4]\,\sqrt{-g[7][7][x0,x4]}}{2\,g[7][7][x0,x4]},\ 0,0,$$

$$0,\ -\frac{g[7][7]^{(0,1)}[x0,x4]\,\sqrt{-g[7][7][x0,x4]}}{2\,g[7][7][x0,x4]},\ 0,0,0\Big\}\Big\}\Big\}\Big\}\Big\}\Big\}$$

Out[478]=

$$\Big\{\Big\{\{0,\ x0,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\begin{array}{l}
\{0,\ x1,\ \{\{\{0,\ H,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ \mathbb{e}^{2\,a4[H\,x4]}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/3},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{0,\ x2,\ \{\{\{0,\ 0,\ H,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ \mathbb{e}^{2\,a4[H\,x4]}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/3},\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{0,\ x3,\ \{\{\{0,\ 0,\ 0,\ H,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ \mathbb{e}^{2\,a4[H\,x4]}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/3},\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{0,\ x4,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{0,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ H,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ -\mathbb{e}^{-2\,a4[H\,x4]}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/3},\ 0,\ 0\}\}\}\},\\
\{0,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ H,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\mathbb{e}^{-2\,a4[H\,x4]}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/3},\ 0\}\}\}\},\\
\{0,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ H\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\mathbb{e}^{-2\,a4[H\,x4]}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\}\}\}\},\\
\{\{1,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{1,\ x1,\ \{\{\{-\sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Sec}[6\,H\,x0]\ \mathrm{Sin}[6\,H\,x0]^{7/6},\\
\quad 0,\ 0,\ 0,\ \sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\}\},\\
\quad \{\{-\frac{\sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Cos}[6\,H\,x0]}{\mathrm{Sin}[6\,H\,x0]^{5/6}},\ 0,\ 0,\ 0,\ -\sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\}\}\}\},\\
\{1,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{1,\ x3,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{1,\ x4,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{1,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{1,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{1,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\},\\
\{\{2,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{2,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{2,\ x2,\ \{\{\{-\sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Sec}[6\,H\,x0]\ \mathrm{Sin}[6\,H\,x0]^{7/6},\\
\quad 0,\ 0,\ 0,\ \sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\}\},\\
\quad \{\{-\frac{\sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Cos}[6\,H\,x0]}{\mathrm{Sin}[6\,H\,x0]^{5/6}},\ 0,\ 0,\ 0,\ -\sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\}\}\}\},\\
\{2,\ x3,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{2,\ x4,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{2,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{2,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{2,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\},\\
\{\{3,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{3,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{3,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{3,\ x3,\ \{\{\{-\sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Sec}[6\,H\,x0]\ \mathrm{Sin}[6\,H\,x0]^{7/6},\\
\quad 0,\ 0,\ 0,\ \sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\}\},\\
\quad \{\{-\frac{\sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Cos}[6\,H\,x0]}{\mathrm{Sin}[6\,H\,x0]^{5/6}},\ 0,\ 0,\ 0,\ -\sqrt{\mathbb{e}^{2\,a4[H\,x4]}}\ H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\}\}\}\},\\
\{3,\ x4,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{3,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{3,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},\\
\{3,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\},
\end{array}$$

$$
\begin{array}{l}
\{\{4,\ x0,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{4,\ x1,\ \{\{\{0,\ H\,a4'[H\,x4],0,0,0,0,0,0\}\},\\
\quad \{\{0,\ -e^{2\,a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,a4'[H\,x4],0,0,0,0,0,0\}\}\}\},\\
\ \{4,\ x2,\ \{\{\{0,0,\ H\,a4'[H\,x4],0,0,0,0,0\}\},\\
\quad \{\{0,0,\ -e^{2\,a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,a4'[H\,x4],0,0,0,0,0\}\}\}\},\\
\ \{4,\ x3,\ \{\{\{0,0,0,\ H\,a4'[H\,x4],0,0,0,0\}\},\\
\quad \{\{0,0,0,\ -e^{2\,a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,a4'[H\,x4],0,0,0,0\}\}\}\},\\
\ \{4,\ x4,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{4,\ x5,\ \{\{\{0,0,0,0,0,\ -H\,a4'[H\,x4],0,0\}\},\\
\quad \{\{0,0,0,0,0,\ -e^{-2\,a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,a4'[H\,x4],0,0\}\}\}\},\\
\ \{4,\ x6,\ \{\{\{0,0,0,0,0,0,\ -H\,a4'[H\,x4],0\}\},\\
\quad \{\{0,0,0,0,0,0,\ -e^{-2\,a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,a4'[H\,x4],0\}\}\}\},\\
\ \{4,\ x7,\ \{\{\{0,0,0,0,0,0,0,\ -H\,a4'[H\,x4]\}\},\\
\quad \{\{0,0,0,0,0,0,0,\ -e^{-2\,a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,a4'[H\,x4]\}\}\}\}\},
\end{array}
$$

$$
\begin{array}{l}
\{\{5,\ x0,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{5,\ x1,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{5,\ x2,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{5,\ x3,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{5,\ x4,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{5,\ x5,\ \{\{\{\sqrt{e^{-2\,a4[H\,x4]}}\,H\,\mathrm{Sec}[6\,H\,x0]\,\mathrm{Sin}[6\,H\,x0]^{7/6},\\
\qquad 0,0,0,\ \sqrt{e^{-2\,a4[H\,x4]}}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\}\},\\
\quad \{\{-\dfrac{\sqrt{e^{-2\,a4[H\,x4]}}\,H\,\mathrm{Cos}[6\,H\,x0]}{\mathrm{Sin}[6\,H\,x0]^{5/6}},0,0,0,\ \sqrt{e^{-2\,a4[H\,x4]}}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\}\}\}\},\\
\ \{5,\ x6,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{5,\ x7,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\}\},
\end{array}
$$

$$
\begin{array}{l}
\{\{6,\ x0,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{6,\ x1,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{6,\ x2,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{6,\ x3,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{6,\ x4,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{6,\ x5,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{6,\ x6,\ \{\{\{\sqrt{e^{-2\,a4[H\,x4]}}\,H\,\mathrm{Sec}[6\,H\,x0]\,\mathrm{Sin}[6\,H\,x0]^{7/6},\\
\qquad 0,0,0,\ \sqrt{e^{-2\,a4[H\,x4]}}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\}\},\\
\quad \{\{-\dfrac{\sqrt{e^{-2\,a4[H\,x4]}}\,H\,\mathrm{Cos}[6\,H\,x0]}{\mathrm{Sin}[6\,H\,x0]^{5/6}},0,0,0,\ \sqrt{e^{-2\,a4[H\,x4]}}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],0,0,0\}\}\}\},\\
\ \{6,\ x7,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\}\},
\end{array}
$$

$$
\begin{array}{l}
\{\{7,\ x0,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{7,\ x1,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{7,\ x2,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{7,\ x3,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},\\
\ \{7,\ x4,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},
\end{array}
$$

```
{7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
{7, x7, {{{Sqrt[e^(-2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6),
    0, 0, 0, Sqrt[e^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}},
  {{-(Sqrt[e^(-2 a4[H x4])] H Cos[6 H x0])/(Sin[6 H x0]^(5/6)), 0, 0, 0, Sqrt[e^(-2 a4[H x4])] H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}}}}
```

In[479]:=

```
(*Block[{eAα,eαA,t,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=e^α_(A)/.sge^α_(A);
  eAα=e^(A)_α/.sge^(A)_α;
  t=Table[ Block[{v,V,d},v=eαA[[All,b]];
     d=Table[D[v[[k]],X[[μ]] ]+Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}],{k,1,DIM8}];
     {b-1,X[[μ]],d}],{b,1,Length[eαA]},{μ,1,DIM8}];t]
 FullSimplify[#/.ssgm4488,constraintVars]&/@%*)
```

In[480]:=

```
(*Block[{eAα,eαA,t,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=e^α_(A)/.sge^α_(A);
  eAα=e^(A)_α/.sge^(A)_α;
  t=Table[ Block[{v,V,d},v=eAα[[b,All]];
     d=Table[D[v[[k]],X[[μ]] ]-Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}],{k,1,DIM8}];
     {b-1,X[[μ]],d}],{b,1,Length[eαA]},{μ,1,DIM8}];t]
 FullSimplify[#/.ssgm4488,constraintVars]&/@%*)
```

In[∘]:=

```
Symbolize[ω_μ^(a)_(b)]
```

In[∘]:=

```
ω = Array[ω_μ^(a)_(b)[##] &, {8, 8, 8}];
```

In[481]:=

```
ω⟦1⟧
```

Out[481]=

```
{{ω_μ^(a)_(b)[1, 1, 1], ω_μ^(a)_(b)[1, 1, 2], ω_μ^(a)_(b)[1, 1, 3], ω_μ^(a)_(b)[1, 1, 4],
  ω_μ^(a)_(b)[1, 1, 5], ω_μ^(a)_(b)[1, 1, 6], ω_μ^(a)_(b)[1, 1, 7], ω_μ^(a)_(b)[1, 1, 8]},
 {ω_μ^(a)_(b)[1, 2, 1], ω_μ^(a)_(b)[1, 2, 2], ω_μ^(a)_(b)[1, 2, 3], ω_μ^(a)_(b)[1, 2, 4],
  ω_μ^(a)_(b)[1, 2, 5], ω_μ^(a)_(b)[1, 2, 6], ω_μ^(a)_(b)[1, 2, 7], ω_μ^(a)_(b)[1, 2, 8]},
 {ω_μ^(a)_(b)[1, 3, 1], ω_μ^(a)_(b)[1, 3, 2], ω_μ^(a)_(b)[1, 3, 3], ω_μ^(a)_(b)[1, 3, 4],
  ω_μ^(a)_(b)[1, 3, 5], ω_μ^(a)_(b)[1, 3, 6], ω_μ^(a)_(b)[1, 3, 7], ω_μ^(a)_(b)[1, 3, 8]},
 {ω_μ^(a)_(b)[1, 4, 1], ω_μ^(a)_(b)[1, 4, 2], ω_μ^(a)_(b)[1, 4, 3], ω_μ^(a)_(b)[1, 4, 4],
  ω_μ^(a)_(b)[1, 4, 5], ω_μ^(a)_(b)[1, 4, 6], ω_μ^(a)_(b)[1, 4, 7], ω_μ^(a)_(b)[1, 4, 8]},
 {ω_μ^(a)_(b)[1, 5, 1], ω_μ^(a)_(b)[1, 5, 2], ω_μ^(a)_(b)[1, 5, 3], ω_μ^(a)_(b)[1, 5, 4],
  ω_μ^(a)_(b)[1, 5, 5], ω_μ^(a)_(b)[1, 5, 6], ω_μ^(a)_(b)[1, 5, 7], ω_μ^(a)_(b)[1, 5, 8]},
 {ω_μ^(a)_(b)[1, 6, 1], ω_μ^(a)_(b)[1, 6, 2], ω_μ^(a)_(b)[1, 6, 3], ω_μ^(a)_(b)[1, 6, 4],
  ω_μ^(a)_(b)[1, 6, 5], ω_μ^(a)_(b)[1, 6, 6], ω_μ^(a)_(b)[1, 6, 7], ω_μ^(a)_(b)[1, 6, 8]},
 {ω_μ^(a)_(b)[1, 7, 1], ω_μ^(a)_(b)[1, 7, 2], ω_μ^(a)_(b)[1, 7, 3], ω_μ^(a)_(b)[1, 7, 4],
  ω_μ^(a)_(b)[1, 7, 5], ω_μ^(a)_(b)[1, 7, 6], ω_μ^(a)_(b)[1, 7, 7], ω_μ^(a)_(b)[1, 7, 8]},
 {ω_μ^(a)_(b)[1, 8, 1], ω_μ^(a)_(b)[1, 8, 2], ω_μ^(a)_(b)[1, 8, 3], ω_μ^(a)_(b)[1, 8, 4],
  ω_μ^(a)_(b)[1, 8, 5], ω_μ^(a)_(b)[1, 8, 6], ω_μ^(a)_(b)[1, 8, 7], ω_μ^(a)_(b)[1, 8, 8]}}
```

In[482]:=

```
sω = Block[{eAα, eαA, t, V, ginv, Γ, gg},
   gg = g4488 /. sg;
   {ginv, Γ} = con[gg];
   eαA = e^α_(A) /. sge^α_(A);
   eAα = e_α^(A) /. sge_α^(A);
   Table[ω⟦μ, a, b⟧ → Block[{v, d}, v = eαA⟦All, b⟧;
      d = Sum[(D[v⟦k⟧, X⟦μ⟧ ] + Sum[ v⟦i⟧ × Γ⟦k, i, μ⟧, {i, 1, DIM8}]) * eAα⟦a, k⟧,
         {k, 1, DIM8}];
      d], {μ, 1, DIM8}, {a, 1, Length[eαA]}, {b, 1, Length[eαA]}]]
```

Out[482]=

```
{{{ω_μ^(a)_(b)[1, 1, 1] → 0, ω_μ^(a)_(b)[1, 1, 2] → 0, ω_μ^(a)_(b)[1, 1, 3] → 0,
   ω_μ^(a)_(b)[1, 1, 4] → 0, ω_μ^(a)_(b)[1, 1, 5] →

                    g[0][0]^(0,1)[x0, x4]
     ─────────────────────────────────────────────────── ,
     2 Sqrt[g[0][0][x0, x4]] Sqrt[-g[4][4][x0, x4]]

   ω_μ^(a)_(b)[1, 1, 6] → 0, ω_μ^(a)_(b)[1, 1, 7] → 0, ω_μ^(a)_(b)[1, 1, 8] → 0},
```

$$\Big\{\omega_{\mu\ (b)}^{(a)}[1,2,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,2,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[1,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,2,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[1,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,3,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[1,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,3,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[1,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,4,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[1,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,4,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[1,5,1]\to -\frac{g[0][0]^{(0,1)}[x0,x4]\ \sqrt{-g[4][4][x0,x4]}}{2\ \sqrt{g[0][0][x0,x4]}\ g[4][4][x0,x4]},\ \omega_{\mu\ (b)}^{(a)}[1,5,2]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[1,5,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,5,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,5,5]\to$$

$$\left(\frac{g[4][4]^{(1,0)}[x0,x4]}{2\ (-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(1,0)}[x0,x4]}{2\ \sqrt{-g[4][4][x0,x4]}\ g[4][4][x0,x4]}\right)\sqrt{-g[4][4][x0,x4]},$$

$$\omega_{\mu\ (b)}^{(a)}[1,5,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,5,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,5,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[1,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,6,3]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[1,6,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,6,6]\to$$

$$\left(\frac{g[5][5]^{(1,0)}[x0,x4]}{2\ (-g[5][5][x0,x4])^{3/2}}+\frac{g[5][5]^{(1,0)}[x0,x4]}{2\ \sqrt{-g[5][5][x0,x4]}\ g[5][5][x0,x4]}\right)\sqrt{-g[5][5][x0,x4]},$$

$$\omega_{\mu\ (b)}^{(a)}[1,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,6,8]\to 0\Big\},\ \Big\{\omega_{\mu\ (b)}^{(a)}[1,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,7,2]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[1,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,7,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,7,6]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[1,7,7]\to\left(\frac{g[6][6]^{(1,0)}[x0,x4]}{2\ (-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(1,0)}[x0,x4]}{2\ \sqrt{-g[6][6][x0,x4]}\ g[6][6][x0,x4]}\right)$$

$$\sqrt{-g[6][6][x0,x4]},\ \omega_{\mu\ (b)}^{(a)}[1,7,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[1,8,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,8,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[1,8,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[1,8,8]\to$$

$$\left(\frac{g[7][7]^{(1,0)}[x0,x4]}{2\ (-g[7][7][x0,x4])^{3/2}}+\frac{g[7][7]^{(1,0)}[x0,x4]}{2\ \sqrt{-g[7][7][x0,x4]}\ g[7][7][x0,x4]}\right)\sqrt{-g[7][7][x0,x4]}\Big\}\Big\},$$

$$\Big\{\Big\{\omega_{\mu\ (b)}^{(a)}[2,1,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,1,2]\to -\frac{g[1][1]^{(1,0)}[x0,x4]}{2\ \sqrt{g[0][0][x0,x4]}\ \sqrt{g[1][1][x0,x4]}},$$

$$\omega_{\mu\ (b)}^{(a)}[2,1,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,1,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,1,5]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[2,1,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,1,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[2,1,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[2,2,1]\to\frac{g[1][1]^{(1,0)}[x0,x4]}{2\ \sqrt{g[0][0][x0,x4]}\ \sqrt{g[1][1][x0,x4]}},$$

$\omega^{(a)}_{\mu\ (b)}[2,2,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,2,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,2,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,2,5]\to \frac{g[1][1]^{(0,1)}[x0,x4]}{2\sqrt{g[1][1][x0,x4]}\ \sqrt{-g[4][4][x0,x4]}},$

$\omega^{(a)}_{\mu\ (b)}[2,2,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,2,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,2,8]\to 0\Big\},$

$\Big\{\omega^{(a)}_{\mu\ (b)}[2,3,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,3,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,3,8]\to 0\Big\},$

$\Big\{\omega^{(a)}_{\mu\ (b)}[2,4,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,4,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,4,8]\to 0\Big\},$

$\Big\{\omega^{(a)}_{\mu\ (b)}[2,5,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,2]\to -\frac{g[1][1]^{(0,1)}[x0,x4]\ \sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[1][1][x0,x4]}\ g[4][4][x0,x4]},$

$\omega^{(a)}_{\mu\ (b)}[2,5,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,4]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,5]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,5,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,5,8]\to 0\Big\},$

$\Big\{\omega^{(a)}_{\mu\ (b)}[2,6,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,6,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,6,8]\to 0\Big\},$

$\Big\{\omega^{(a)}_{\mu\ (b)}[2,7,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,7,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,7,8]\to 0\Big\},$

$\Big\{\omega^{(a)}_{\mu\ (b)}[2,8,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[2,8,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[2,8,8]\to 0\Big\}\Big\},$

$\Big\{\Big\{\omega^{(a)}_{\mu\ (b)}[3,1,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,1,2]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[3,1,3]\to -\frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ \sqrt{g[2][2][x0,x4]}},\ \omega^{(a)}_{\mu\ (b)}[3,1,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[3,1,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,1,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,1,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,1,8]\to 0\Big\},$

$\Big\{\omega^{(a)}_{\mu\ (b)}[3,2,1]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[3,2,5]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,2,8]\to 0\Big\},$

$\Big\{\omega^{(a)}_{\mu\ (b)}[3,3,1]\to \frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ \sqrt{g[2][2][x0,x4]}},$

$\omega^{(a)}_{\mu\ (b)}[3,3,2]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,3,3]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,3,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}[3,3,5]\to \frac{g[2][2]^{(0,1)}[x0,x4]}{2\sqrt{g[2][2][x0,x4]}\ \sqrt{-g[4][4][x0,x4]}},$

$\omega^{(a)}_{\mu\ (b)}[3,3,6]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,3,7]\to 0,\ \omega^{(a)}_{\mu\ (b)}[3,3,8]\to 0\Big\},$

$\{\omega_{\mu\ (b)}^{(a)}[3,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,4,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,4,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[3,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,5,2]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,5,3]\to -\frac{g[2][2]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ \sqrt{g[2][2][x0,\ x4]}\ g[4][4][x0,\ x4]},\ \omega_{\mu\ (b)}^{(a)}[3,5,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,5,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,5,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,5,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[3,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,6,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[3,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,7,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[3,8,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[3,8,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[3,8,8]\to 0\}\},$
$\{\{\omega_{\mu\ (b)}^{(a)}[4,1,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,1,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,1,3]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,1,4]\to -\frac{g[3][3]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}\ \sqrt{g[3][3][x0,\ x4]}},\ \omega_{\mu\ (b)}^{(a)}[4,1,5]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,1,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,1,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,1,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,2,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,2,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,3,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,4,1]\to \frac{g[3][3]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}\ \sqrt{g[3][3][x0,\ x4]}},$
$\omega_{\mu\ (b)}^{(a)}[4,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,4,4]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,4,5]\to \frac{g[3][3]^{(0,1)}[x0,\ x4]}{2\ \sqrt{g[3][3][x0,\ x4]}\ \sqrt{-g[4][4][x0,\ x4]}},\ \omega_{\mu\ (b)}^{(a)}[4,4,6]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,4,8]\to 0\},\ \{\omega_{\mu\ (b)}^{(a)}[4,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,2]\to 0,$
$\omega_{\mu\ (b)}^{(a)}[4,5,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,4]\to -\frac{g[3][3]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ \sqrt{g[3][3][x0,\ x4]}\ g[4][4][x0,\ x4]},$
$\omega_{\mu\ (b)}^{(a)}[4,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,5,8]\to 0\},$
$\{\omega_{\mu\ (b)}^{(a)}[4,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[4,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,6,8]\to 0\Big\},$

$\Big\{\omega_{\mu\ (b)}^{(a)}[4,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[4,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,7,8]\to 0\Big\},$

$\Big\{\omega_{\mu\ (b)}^{(a)}[4,8,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[4,8,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[4,8,8]\to 0\Big\}\Big\},$

$\Big\{\Big\{\omega_{\mu\ (b)}^{(a)}[5,1,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,4]\to 0,$

$$\omega_{\mu\ (b)}^{(a)}[5,1,5]\to -\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ \sqrt{-g[4][4][x0,x4]}},$$

$\omega_{\mu\ (b)}^{(a)}[5,1,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,1,8]\to 0\Big\},$

$\Big\{\omega_{\mu\ (b)}^{(a)}[5,2,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,2,8]\to 0\Big\},$

$\Big\{\omega_{\mu\ (b)}^{(a)}[5,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,3,8]\to 0\Big\},$

$\Big\{\omega_{\mu\ (b)}^{(a)}[5,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,4]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,4,8]\to 0\Big\},$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[5,5,1]\to \frac{g[4][4]^{(1,0)}[x0,x4]\ \sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\ g[4][4][x0,x4]},\ \omega_{\mu\ (b)}^{(a)}[5,5,2]\to 0,$$

$\omega_{\mu\ (b)}^{(a)}[5,5,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,5]\to$

$$\left(\frac{g[4][4]^{(0,1)}[x0,x4]}{2\,(-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(0,1)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}\ g[4][4][x0,x4]}\right)\sqrt{-g[4][4][x0,x4]},$$

$\omega_{\mu\ (b)}^{(a)}[5,5,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,5,8]\to 0\Big\},$

$\Big\{\omega_{\mu\ (b)}^{(a)}[5,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,3]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,6,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,6]\to$

$$\left(\frac{g[5][5]^{(0,1)}[x0,x4]}{2\,(-g[5][5][x0,x4])^{3/2}}+\frac{g[5][5]^{(0,1)}[x0,x4]}{2\sqrt{-g[5][5][x0,x4]}\ g[5][5][x0,x4]}\right)\sqrt{-g[5][5][x0,x4]},$$

$\omega_{\mu\ (b)}^{(a)}[5,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,6,8]\to 0\Big\},\ \Big\{\omega_{\mu\ (b)}^{(a)}[5,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,2]\to 0,$

$\omega_{\mu\ (b)}^{(a)}[5,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[5,7,6]\to 0,$

$$\omega_{\mu\ (b)}^{(a)}[5,7,7]\to\left(\frac{g[6][6]^{(0,1)}[x0,x4]}{2\,(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(0,1)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}\ g[6][6][x0,x4]}\right)$$

$\sqrt{-g[6][6][x0,x4]},\ \omega_{\mu\ (b)}^{(a)}[5,7,8]\to 0\Big\},$

$$\Big\{\omega_{\mu\,(b)}^{(a)}[5,8,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,8,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,8,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,8,4]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[5,8,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,8,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,8,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[5,8,8]\to$$

$$\left(\frac{g[7][7]^{(0,1)}[x0,x4]}{2\,(-g[7][7][x0,x4])^{3/2}}+\frac{g[7][7]^{(0,1)}[x0,x4]}{2\,\sqrt{-g[7][7][x0,x4]}\;g[7][7][x0,x4]}\right)\sqrt{-g[7][7][x0,x4]}\Big\}\Big\},$$

$$\Big\{\Big\{\omega_{\mu\,(b)}^{(a)}[6,1,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,1,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,1,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,1,4]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[6,1,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,1,6]\to -\frac{g[5][5]^{(1,0)}[x0,x4]}{2\,\sqrt{g[0][0][x0,x4]}\,\sqrt{-g[5][5][x0,x4]}},$$

$$\omega_{\mu\,(b)}^{(a)}[6,1,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,1,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\,(b)}^{(a)}[6,2,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,2,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,2,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,2,4]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[6,2,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,2,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,2,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,2,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\,(b)}^{(a)}[6,3,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,3,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,3,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,3,4]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[6,3,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,3,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,3,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,3,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\,(b)}^{(a)}[6,4,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,4,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,4,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,4,4]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[6,4,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,4,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,4,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,4,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\,(b)}^{(a)}[6,5,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,5,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,5,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,5,4]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[6,5,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,5,6]\to -\frac{g[5][5]^{(0,1)}[x0,x4]\,\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]\,\sqrt{-g[5][5][x0,x4]}},$$

$$\omega_{\mu\,(b)}^{(a)}[6,5,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,5,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\,(b)}^{(a)}[6,6,1]\to \frac{g[5][5]^{(1,0)}[x0,x4]\,\sqrt{-g[5][5][x0,x4]}}{2\,\sqrt{g[0][0][x0,x4]}\;g[5][5][x0,x4]},$$

$$\omega_{\mu\,(b)}^{(a)}[6,6,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,6,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,6,4]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[6,6,5]\to \frac{g[5][5]^{(0,1)}[x0,x4]\,\sqrt{-g[5][5][x0,x4]}}{2\,\sqrt{-g[4][4][x0,x4]}\;g[5][5][x0,x4]},$$

$$\omega_{\mu\,(b)}^{(a)}[6,6,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,6,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,6,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\,(b)}^{(a)}[6,7,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,7,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,7,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,7,4]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[6,7,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,7,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,7,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,7,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\,(b)}^{(a)}[6,8,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,8,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,8,3]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,8,4]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[6,8,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,8,6]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,8,7]\to 0,\ \omega_{\mu\,(b)}^{(a)}[6,8,8]\to 0\Big\}\Big\},$$

$$\Big\{\Big\{\omega_{\mu\,(b)}^{(a)}[7,1,1]\to 0,\ \omega_{\mu\,(b)}^{(a)}[7,1,2]\to 0,\ \omega_{\mu\,(b)}^{(a)}[7,1,3]\to 0,$$

$$\omega_{\mu\,(b)}^{(a)}[7,1,4]\to 0,\ \omega_{\mu\,(b)}^{(a)}[7,1,5]\to 0,\ \omega_{\mu\,(b)}^{(a)}[7,1,6]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[7,1,7]\to -\frac{g[6][6]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\,\sqrt{-g[6][6][x0,x4]}},\ \omega_{\mu\ (b)}^{(a)}[7,1,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[7,2,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,2,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[7,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,2,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[7,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,3,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[7,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,3,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[7,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,4,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[7,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,4,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[7,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,5,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,5,3]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[7,5,4]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,5,6]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[7,5,7]\to -\frac{g[6][6]^{(0,1)}[x0,x4]\,\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]\,\sqrt{-g[6][6][x0,x4]}},\ \omega_{\mu\ (b)}^{(a)}[7,5,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[7,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,6,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[7,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,6,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[7,7,1]\to \frac{g[6][6]^{(1,0)}[x0,x4]\,\sqrt{-g[6][6][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[6][6][x0,x4]},$$

$$\omega_{\mu\ (b)}^{(a)}[7,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,7,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[7,7,5]\to \frac{g[6][6]^{(0,1)}[x0,x4]\,\sqrt{-g[6][6][x0,x4]}}{2\sqrt{-g[4][4][x0,x4]}\,g[6][6][x0,x4]},$$

$$\omega_{\mu\ (b)}^{(a)}[7,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,7,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[7,8,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,8,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[7,8,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[7,8,8]\to 0\Big\}\Big\},$$

$$\Big\{\Big\{\omega_{\mu\ (b)}^{(a)}[8,1,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,1,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,1,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,1,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,1,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,1,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,1,7]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,1,8]\to -\frac{g[7][7]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\,\sqrt{-g[7][7][x0,x4]}}\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[8,2,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,2,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,2,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,2,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,2,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,2,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,2,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,2,8]\to 0\Big\},$$

$$\Big\{\omega_{\mu\ (b)}^{(a)}[8,3,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,3,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,3,8]\to 0\Big\},$$

$$\{\omega_{\mu\ (b)}^{(a)}[8,4,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,4,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,4,8]\to 0\},$$

$$\{\omega_{\mu\ (b)}^{(a)}[8,5,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,5,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,5,7]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,5,8]\to -\frac{g[7][7]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\,g[4][4][x0,x4]\sqrt{-g[7][7][x0,x4]}}\},$$

$$\{\omega_{\mu\ (b)}^{(a)}[8,6,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,6,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,6,8]\to 0\},$$

$$\{\omega_{\mu\ (b)}^{(a)}[8,7,1]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,7,5]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,7,8]\to 0\},$$

$$\{\omega_{\mu\ (b)}^{(a)}[8,8,1]\to \frac{g[7][7]^{(1,0)}[x0,x4]\sqrt{-g[7][7][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}\,g[7][7][x0,x4]},$$

$$\omega_{\mu\ (b)}^{(a)}[8,8,2]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,3]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,4]\to 0,$$

$$\omega_{\mu\ (b)}^{(a)}[8,8,5]\to \frac{g[7][7]^{(0,1)}[x0,x4]\sqrt{-g[7][7][x0,x4]}}{2\sqrt{-g[4][4][x0,x4]}\,g[7][7][x0,x4]},$$

$$\omega_{\mu\ (b)}^{(a)}[8,8,6]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,7]\to 0,\ \omega_{\mu\ (b)}^{(a)}[8,8,8]\to 0\}\}\}$$

In[483]:=

```
(*sω=Block[{eAα,eαA,t,V,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eαA=e^α_(A)/.sge^α_(A);
  eAα=e^(A)_α/.sge^(A)_α;
  Table[ω^(a)_μ (b)[μ,a,b]→Block[{v,d},v=eαA⟦All,b⟧;
     d=Sum[
       (D[v⟦k⟧,X⟦μ⟧ ]+Sum[ v⟦i⟧ Γ⟦k,i,μ⟧,{i,1,DIM8}])*eAα⟦a,k⟧,{k,1,DIM8}];
     d],{μ,1,DIM8},{a,1,Length[eαA]},{b,1,Length[eαA]}]]*)
```

In[484]:=

```
sωf = Flatten[sω];
```

In[485]:=

```
(*ω^(a)_μ (b)*)
ω;
ωmat = % /. sωf
Dimensions[%]
```

Out[486]=

$$\{\{\{0,0,0,0,\frac{g[0][0]^{(0,1)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[4][4][x0,x4]}},0,0,0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\Big\{-\frac{g[0][0]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,\sqrt{g[0][0][x0, x4]}\,g[4][4][x0, x4]}, 0, 0, 0,$$

$$\left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2\,(-g[4][4][x0, x4])^{3/2}}+\frac{g[4][4]^{(1,0)}[x0, x4]}{2\,\sqrt{-g[4][4][x0, x4]}\,g[4][4][x0, x4]}\right)\sqrt{-g[4][4][x0, x4]},$$

$$0, 0, 0\Big\}, \Big\{0, 0, 0, 0, 0,$$

$$\left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2\,(-g[5][5][x0, x4])^{3/2}}+\frac{g[5][5]^{(1,0)}[x0, x4]}{2\,\sqrt{-g[5][5][x0, x4]}\,g[5][5][x0, x4]}\right)\sqrt{-g[5][5][x0, x4]}, 0,$$

$$0\Big\}, \Big\{0, 0, 0, 0, 0, 0, \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2\,(-g[6][6][x0, x4])^{3/2}}+\frac{g[6][6]^{(1,0)}[x0, x4]}{2\,\sqrt{-g[6][6][x0, x4]}\,g[6][6][x0, x4]}\right)$$

$$\sqrt{-g[6][6][x0, x4]}, 0\Big\}, \Big\{0, 0, 0, 0, 0, 0, 0,$$

$$\left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2\,(-g[7][7][x0, x4])^{3/2}}+\frac{g[7][7]^{(1,0)}[x0, x4]}{2\,\sqrt{-g[7][7][x0, x4]}\,g[7][7][x0, x4]}\right)\sqrt{-g[7][7][x0, x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0, -\frac{g[1][1]^{(1,0)}[x0, x4]}{2\,\sqrt{g[0][0][x0, x4]}\,\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0\Big\},$$

$$\Big\{\frac{g[1][1]^{(1,0)}[x0, x4]}{2\,\sqrt{g[0][0][x0, x4]}\,\sqrt{g[1][1][x0, x4]}}, 0, 0, 0,$$

$$\frac{g[1][1]^{(0,1)}[x0, x4]}{2\,\sqrt{g[1][1][x0, x4]}\,\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\Big\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \Big\{0, -\frac{g[1][1]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,\sqrt{g[1][1][x0, x4]}\,g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0\Big\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\Big\},$$

$$\Big\{\Big\{0, 0, -\frac{g[2][2]^{(1,0)}[x0, x4]}{2\,\sqrt{g[0][0][x0, x4]}\,\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0\Big\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \Big\{\frac{g[2][2]^{(1,0)}[x0, x4]}{2\,\sqrt{g[0][0][x0, x4]}\,\sqrt{g[2][2][x0, x4]}}, 0, 0, 0,$$

$$\frac{g[2][2]^{(0,1)}[x0, x4]}{2\,\sqrt{g[2][2][x0, x4]}\,\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\Big\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\Big\{0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4]\,\sqrt{-g[4][4][x0, x4]}}{2\,\sqrt{g[2][2][x0, x4]}\,g[4][4][x0, x4]}, 0, 0, 0, 0, 0\Big\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\Big\},$$

$$\Big\{\Big\{0, 0, 0, -\frac{g[3][3]^{(1,0)}[x0, x4]}{2\,\sqrt{g[0][0][x0, x4]}\,\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\Big\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \Big\{\frac{g[3][3]^{(1,0)}[x0, x4]}{2\,\sqrt{g[0][0][x0, x4]}\,\sqrt{g[3][3][x0, x4]}},$$

$$0,\ 0,\ 0,\ \frac{g[3][3]^{(0,1)}[x0,\ x4]}{2\ \sqrt{g[3][3][x0,\ x4]}\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0\Big\},$$

$$\Big\{0,\ 0,\ 0,\ -\frac{g[3][3]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ \sqrt{g[3][3][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0,\ 0\Big\},$$

$$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ -\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0\Big\},$$

$$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},$$

$$\Big\{\frac{g[4][4]^{(1,0)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ \sqrt{g[0][0][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0,$$

$$\left(\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\ (-g[4][4][x0,\ x4])^{3/2}}+\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[4][4][x0,\ x4]}\right)\ \sqrt{-g[4][4][x0,\ x4]},\ 0,$$

$$0,\ 0\Big\},\ \Big\{0,\ 0,\ 0,\ 0,\ 0,\ \left(\frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ (-g[5][5][x0,\ x4])^{3/2}}+\frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[5][5][x0,\ x4]}\ g[5][5][x0,\ x4]}\right)$$

$$\sqrt{-g[5][5][x0,\ x4]},\ 0,\ 0\Big\},\ \Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,$$

$$\left(\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ (-g[6][6][x0,\ x4])^{3/2}}+\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[6][6][x0,\ x4]}\ g[6][6][x0,\ x4]}\right)\ \sqrt{-g[6][6][x0,\ x4]},\ 0\Big\},$$

$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \left(\frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ (-g[7][7][x0,\ x4])^{3/2}}+\frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[7][7][x0,\ x4]}\ g[7][7][x0,\ x4]}\right)$$

$$\sqrt{-g[7][7][x0,\ x4]}\Big\}\Big\},\ \Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}\ \sqrt{-g[5][5][x0,\ x4]}},\ 0,\ 0\Big\},$$

$$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},$$

$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[5][5]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ g[4][4][x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}},\ 0,\ 0\Big\},$$

$$\Big\{\frac{g[5][5]^{(1,0)}[x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}}{2\ \sqrt{g[0][0][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(0,1)}[x0,\ x4]\ \sqrt{-g[5][5][x0,\ x4]}}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[5][5][x0,\ x4]},$$

$$0,\ 0,\ 0\Big\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ \sqrt{g[0][0][x0,\ x4]}\ \sqrt{-g[6][6][x0,\ x4]}},\ 0\Big\},$$

$$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},$$

$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[6][6]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ g[4][4][x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}},\ 0\Big\},$$

$$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \Big\{\frac{g[6][6]^{(1,0)}[x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}}{2\ \sqrt{g[0][0][x0,\ x4]}\ g[6][6][x0,\ x4]},\ 0,\ 0,\ 0,$$

$$\frac{g[6][6]^{(0,1)}[x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[6][6][x0,\ x4]},\ 0,\ 0,\ 0\Big\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\Big\},$$

$$\begin{aligned}
&\Big\{\Big\{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\ \sqrt{-g[7][7][x0, x4]}}\Big\},\\
&\ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},\\
&\ \Big\{0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]\ \sqrt{-g[4][4][x0, x4]}}{2\,g[4][4][x0, x4]\ \sqrt{-g[7][7][x0, x4]}}\Big\}, \{0, 0, 0, 0, 0, 0, 0, 0\},\\
&\ \{0, 0, 0, 0, 0, 0, 0, 0\}, \Big\{\frac{g[7][7]^{(1,0)}[x0, x4]\ \sqrt{-g[7][7][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}\ g[7][7][x0, x4]},\\
&\quad 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4]\ \sqrt{-g[7][7][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}\ g[7][7][x0, x4]}, 0, 0, 0\Big\}\Big\}\Big\}
\end{aligned}$$

Out[487]=

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{8, 8, 8}
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In[488]:=

$$\begin{aligned}
&\text{s}\omega\text{gtry} = \text{Block}\Big[\{\text{eA}\alpha, \text{e}\alpha\text{A}, \text{t}, \text{V}\},\\
&\quad \text{e}\alpha\text{A} = \text{e}^{\alpha}_{(\text{A})}\ /.\ \text{sgtrye}^{\alpha}_{(\text{A})};\\
&\quad \text{eA}\alpha = \text{e}^{(\text{A})}_{\alpha}\ /.\ \text{sgtrye}^{(\text{A})}_{\alpha};\\
&\quad \text{Table}\Big[(*\text{V=eA}\alpha[\![\text{a,All}]\!];*)\ \omega^{(\text{a})}_{\mu\ (\text{b})}[\mu, \text{a}, \text{b}] \to \text{Block}\Big[\{\text{v}, \text{d}\}, \text{v} = \text{e}\alpha\text{A}[\![\text{All}, \text{b}]\!];\\
&\qquad \text{d} = \text{Sum}[(\text{D}[\text{v}[\![\text{k}]\!], \text{X}[\![\mu]\!]] + \text{Sum}[\text{v}[\![\text{i}]\!] \times \Gamma[\![\text{k}, \text{i}, \mu]\!], \{\text{i}, 1, \text{DIM8}\}]) * \text{eA}\alpha[\![\text{a}, \text{k}]\!],\\
&\qquad\quad \{\text{k}, 1, \text{DIM8}\}];\\
&\qquad \text{FullSimplify}[\text{d}, \text{constraintVars}]\ /.\ \Big\{\sqrt{e^{2\,\text{a4}[\text{H x4}]}} \to e^{\text{a4}[\text{H x4}]}, \sqrt{e^{-2\,\text{a4}[\text{H x4}]}} \to\\
&\qquad\quad e^{-\text{a4}[\text{H x4}]}\Big\}\Big], \{\mu, 1, \text{DIM8}\}, \{\text{a}, 1, \text{Length}[\text{eA}\alpha]\}, \{\text{b}, 1, \text{Length}[\text{e}\alpha\text{A}]\}\Big]\Big]
\end{aligned}$$

Out[488]=

$$\begin{aligned}
&\Big\{\Big\{\Big\{\omega^{(a)}_{\mu\ (b)}[1, 1, 1] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 1, 2] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 1, 3] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 1, 4] \to 0,\\
&\quad \omega^{(a)}_{\mu\ (b)}[1, 1, 5] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 1, 6] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 1, 7] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 1, 8] \to 0\Big\},\\
&\ \Big\{\omega^{(a)}_{\mu\ (b)}[1, 2, 1] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 2, 2] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 2, 3] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 2, 4] \to 0,\\
&\quad \omega^{(a)}_{\mu\ (b)}[1, 2, 5] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 2, 6] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 2, 7] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 2, 8] \to 0\Big\},\\
&\ \Big\{\omega^{(a)}_{\mu\ (b)}[1, 3, 1] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 3, 2] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 3, 3] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 3, 4] \to 0,\\
&\quad \omega^{(a)}_{\mu\ (b)}[1, 3, 5] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 3, 6] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 3, 7] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 3, 8] \to 0\Big\},\\
&\ \Big\{\omega^{(a)}_{\mu\ (b)}[1, 4, 1] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 4, 2] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 4, 3] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 4, 4] \to 0,\\
&\quad \omega^{(a)}_{\mu\ (b)}[1, 4, 5] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 4, 6] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 4, 7] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 4, 8] \to 0\Big\},\\
&\ \Big\{\omega^{(a)}_{\mu\ (b)}[1, 5, 1] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 5, 2] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 5, 3] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 5, 4] \to 0,\\
&\quad \omega^{(a)}_{\mu\ (b)}[1, 5, 5] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 5, 6] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 5, 7] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 5, 8] \to 0\Big\},\\
&\ \Big\{\omega^{(a)}_{\mu\ (b)}[1, 6, 1] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 6, 2] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 6, 3] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 6, 4] \to 0,\\
&\quad \omega^{(a)}_{\mu\ (b)}[1, 6, 5] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 6, 6] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 6, 7] \to 0, \omega^{(a)}_{\mu\ (b)}[1, 6, 8] \to 0\Big\},
\end{aligned}$$

{ $\omega^{(a)}_{\mu\,(b)}$ [1, 7, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 7, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 7, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 7, 4] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [1, 7, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 7, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 7, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 7, 8] → 0},
{ $\omega^{(a)}_{\mu\,(b)}$ [1, 8, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 8, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 8, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 8, 4] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [1, 8, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 8, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 8, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [1, 8, 8] → 0}},
{{ $\omega^{(a)}_{\mu\,(b)}$ [2, 1, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 1, 2] → $-e^{a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$, $\omega^{(a)}_{\mu\,(b)}$ [2, 1, 3] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [2, 1, 4] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 1, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 1, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 1, 7] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [2, 1, 8] → 0}, { $\omega^{(a)}_{\mu\,(b)}$ [2, 2, 1] → $e^{a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$, $\omega^{(a)}_{\mu\,(b)}$ [2, 2, 2] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [2, 2, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 2, 4] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 2, 5] → $e^{a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$ a4′[H x4],
$\omega^{(a)}_{\mu\,(b)}$ [2, 2, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 2, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 2, 8] → 0},
{ $\omega^{(a)}_{\mu\,(b)}$ [2, 3, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 3, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 3, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 3, 4] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [2, 3, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 3, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 3, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 3, 8] → 0},
{ $\omega^{(a)}_{\mu\,(b)}$ [2, 4, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 4, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 4, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 4, 4] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [2, 4, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 4, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 4, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 4, 8] → 0},
{ $\omega^{(a)}_{\mu\,(b)}$ [2, 5, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 5, 2] → $e^{a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$ a4′[H x4],
$\omega^{(a)}_{\mu\,(b)}$ [2, 5, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 5, 4] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 5, 5] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [2, 5, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 5, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 5, 8] → 0},
{ $\omega^{(a)}_{\mu\,(b)}$ [2, 6, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 6, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 6, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 6, 4] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [2, 6, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 6, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 6, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 6, 8] → 0},
{ $\omega^{(a)}_{\mu\,(b)}$ [2, 7, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 7, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 7, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 7, 4] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [2, 7, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 7, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 7, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 7, 8] → 0},
{ $\omega^{(a)}_{\mu\,(b)}$ [2, 8, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 8, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 8, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 8, 4] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [2, 8, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 8, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 8, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [2, 8, 8] → 0}},
{{ $\omega^{(a)}_{\mu\,(b)}$ [3, 1, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 1, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 1, 3] → $-e^{a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$,
$\omega^{(a)}_{\mu\,(b)}$ [3, 1, 4] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 1, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 1, 6] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [3, 1, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 1, 8] → 0},
{ $\omega^{(a)}_{\mu\,(b)}$ [3, 2, 1] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 2, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 2, 3] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 2, 4] → 0,
$\omega^{(a)}_{\mu\,(b)}$ [3, 2, 5] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 2, 6] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 2, 7] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 2, 8] → 0},
{ $\omega^{(a)}_{\mu\,(b)}$ [3, 3, 1] → $e^{a4[H\,x4]}$ H Sin[6 H x0]$^{1/6}$, $\omega^{(a)}_{\mu\,(b)}$ [3, 3, 2] → 0, $\omega^{(a)}_{\mu\,(b)}$ [3, 3, 3] → 0,

$\omega^{(a)}_{\mu\,(b)}[3,3,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,5]\to \mathbb{e}^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$

$\omega^{(a)}_{\mu\,(b)}[3,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,3,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[3,4,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[3,4,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,4,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[3,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,3]\to \mathbb{e}^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$

$\omega^{(a)}_{\mu\,(b)}[3,5,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,6]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[3,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,5,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[3,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[3,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,6,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[3,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[3,7,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,7,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[3,8,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[3,8,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[3,8,8]\to 0\}\},$

$\{\{\omega^{(a)}_{\mu\,(b)}[4,1,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,3]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[4,1,4]\to -\mathbb{e}^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[4,1,5]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[4,1,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,1,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[4,2,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[4,2,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,2,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[4,3,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[4,3,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,3,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[4,4,1]\to \mathbb{e}^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6},\ \omega^{(a)}_{\mu\,(b)}[4,4,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,4,3]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[4,4,4]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,4,5]\to \mathbb{e}^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],\ \omega^{(a)}_{\mu\,(b)}[4,4,6]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[4,4,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,4,8]\to 0\},\ \{\omega^{(a)}_{\mu\,(b)}[4,5,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,2]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[4,5,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,4]\to \mathbb{e}^{a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4],$

$\omega^{(a)}_{\mu\,(b)}[4,5,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,5,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[4,6,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,4]\to 0,$

$\omega^{(a)}_{\mu\,(b)}[4,6,5]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,6]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,7]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,6,8]\to 0\},$

$\{\omega^{(a)}_{\mu\,(b)}[4,7,1]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,2]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,3]\to 0,\ \omega^{(a)}_{\mu\,(b)}[4,7,4]\to 0,$

$\omega^{(a)}_{\mu\ (b)}$[4, 7, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[4, 7, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[4, 7, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[4, 7, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[4, 8, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[4, 8, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[4, 8, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[4, 8, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[4, 8, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[4, 8, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[4, 8, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[4, 8, 8] → 0}},
{{$\omega^{(a)}_{\mu\ (b)}$[5, 1, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 1, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 1, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 1, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[5, 1, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 1, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 1, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 1, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[5, 2, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 2, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 2, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 2, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[5, 2, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 2, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 2, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 2, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[5, 3, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 3, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 3, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 3, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[5, 3, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 3, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 3, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 3, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[5, 4, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 4, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 4, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 4, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[5, 4, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 4, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 4, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 4, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[5, 5, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 5, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 5, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 5, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[5, 5, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 5, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 5, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 5, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[5, 6, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 6, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 6, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 6, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[5, 6, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 6, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 6, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 6, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[5, 7, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 7, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 7, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 7, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[5, 7, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 7, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 7, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 7, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[5, 8, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 8, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 8, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 8, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[5, 8, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 8, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 8, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[5, 8, 8] → 0}},
{{$\omega^{(a)}_{\mu\ (b)}$[6, 1, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 1, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 1, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 1, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[6, 1, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 1, 6] → $e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}$,
$\omega^{(a)}_{\mu\ (b)}$[6, 1, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 1, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[6, 2, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 2, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 2, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 2, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[6, 2, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 2, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 2, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 2, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[6, 3, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 3, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 3, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 3, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[6, 3, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 3, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 3, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 3, 8] → 0},
{$\omega^{(a)}_{\mu\ (b)}$[6, 4, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 4, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 4, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 4, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[6, 4, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 4, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 4, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 4, 8] → 0},

{ $\omega^{(a)}_{\mu\ (b)}$[6, 5, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 5, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 5, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 5, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[6, 5, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 5, 6] → $e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]$, $\omega^{(a)}_{\mu\ (b)}$[6, 5, 7] → 0,
$\omega^{(a)}_{\mu\ (b)}$[6, 5, 8] → 0}, { $\omega^{(a)}_{\mu\ (b)}$[6, 6, 1] → $e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}$, $\omega^{(a)}_{\mu\ (b)}$[6, 6, 2] → 0,
$\omega^{(a)}_{\mu\ (b)}$[6, 6, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 6, 4] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 6, 5] → $-e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]$,
$\omega^{(a)}_{\mu\ (b)}$[6, 6, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 6, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 6, 8] → 0},
{ $\omega^{(a)}_{\mu\ (b)}$[6, 7, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 7, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 7, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 7, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[6, 7, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 7, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 7, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 7, 8] → 0},
{ $\omega^{(a)}_{\mu\ (b)}$[6, 8, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 8, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 8, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 8, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[6, 8, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 8, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 8, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[6, 8, 8] → 0}},
{{ $\omega^{(a)}_{\mu\ (b)}$[7, 1, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 1, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 1, 3] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 1, 4] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 1, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 1, 6] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 1, 7] → $e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}$, $\omega^{(a)}_{\mu\ (b)}$[7, 1, 8] → 0},
{ $\omega^{(a)}_{\mu\ (b)}$[7, 2, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 2, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 2, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 2, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 2, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 2, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 2, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 2, 8] → 0},
{ $\omega^{(a)}_{\mu\ (b)}$[7, 3, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 3, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 3, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 3, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 3, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 3, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 3, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 3, 8] → 0},
{ $\omega^{(a)}_{\mu\ (b)}$[7, 4, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 4, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 4, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 4, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 4, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 4, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 4, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 4, 8] → 0},
{ $\omega^{(a)}_{\mu\ (b)}$[7, 5, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 5, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 5, 3] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 5, 4] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 5, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 5, 6] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 5, 7] → $e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]$, $\omega^{(a)}_{\mu\ (b)}$[7, 5, 8] → 0},
{ $\omega^{(a)}_{\mu\ (b)}$[7, 6, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 6, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 6, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 6, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 6, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 6, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 6, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 6, 8] → 0},
{ $\omega^{(a)}_{\mu\ (b)}$[7, 7, 1] → $e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}$, $\omega^{(a)}_{\mu\ (b)}$[7, 7, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 7, 3] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 7, 4] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 7, 5] → $-e^{-a4[H\,x4]}\,H\,\mathrm{Sin}[6\,H\,x0]^{1/6}\,a4'[H\,x4]$,
$\omega^{(a)}_{\mu\ (b)}$[7, 7, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 7, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 7, 8] → 0},
{ $\omega^{(a)}_{\mu\ (b)}$[7, 8, 1] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 8, 2] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 8, 3] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 8, 4] → 0,
$\omega^{(a)}_{\mu\ (b)}$[7, 8, 5] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 8, 6] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 8, 7] → 0, $\omega^{(a)}_{\mu\ (b)}$[7, 8, 8] → 0}},

```
{{ω^(a)_μ(b)[8, 1, 1] → 0, ω^(a)_μ(b)[8, 1, 2] → 0, ω^(a)_μ(b)[8, 1, 3] → 0,
  ω^(a)_μ(b)[8, 1, 4] → 0, ω^(a)_μ(b)[8, 1, 5] → 0, ω^(a)_μ(b)[8, 1, 6] → 0,
  ω^(a)_μ(b)[8, 1, 7] → 0, ω^(a)_μ(b)[8, 1, 8] → ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6)},
 {ω^(a)_μ(b)[8, 2, 1] → 0, ω^(a)_μ(b)[8, 2, 2] → 0, ω^(a)_μ(b)[8, 2, 3] → 0, ω^(a)_μ(b)[8, 2, 4] → 0,
  ω^(a)_μ(b)[8, 2, 5] → 0, ω^(a)_μ(b)[8, 2, 6] → 0, ω^(a)_μ(b)[8, 2, 7] → 0, ω^(a)_μ(b)[8, 2, 8] → 0},
 {ω^(a)_μ(b)[8, 3, 1] → 0, ω^(a)_μ(b)[8, 3, 2] → 0, ω^(a)_μ(b)[8, 3, 3] → 0, ω^(a)_μ(b)[8, 3, 4] → 0,
  ω^(a)_μ(b)[8, 3, 5] → 0, ω^(a)_μ(b)[8, 3, 6] → 0, ω^(a)_μ(b)[8, 3, 7] → 0, ω^(a)_μ(b)[8, 3, 8] → 0},
 {ω^(a)_μ(b)[8, 4, 1] → 0, ω^(a)_μ(b)[8, 4, 2] → 0, ω^(a)_μ(b)[8, 4, 3] → 0, ω^(a)_μ(b)[8, 4, 4] → 0,
  ω^(a)_μ(b)[8, 4, 5] → 0, ω^(a)_μ(b)[8, 4, 6] → 0, ω^(a)_μ(b)[8, 4, 7] → 0, ω^(a)_μ(b)[8, 4, 8] → 0},
 {ω^(a)_μ(b)[8, 5, 1] → 0, ω^(a)_μ(b)[8, 5, 2] → 0, ω^(a)_μ(b)[8, 5, 3] → 0,
  ω^(a)_μ(b)[8, 5, 4] → 0, ω^(a)_μ(b)[8, 5, 5] → 0, ω^(a)_μ(b)[8, 5, 6] → 0,
  ω^(a)_μ(b)[8, 5, 7] → 0, ω^(a)_μ(b)[8, 5, 8] → ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4]},
 {ω^(a)_μ(b)[8, 6, 1] → 0, ω^(a)_μ(b)[8, 6, 2] → 0, ω^(a)_μ(b)[8, 6, 3] → 0, ω^(a)_μ(b)[8, 6, 4] → 0,
  ω^(a)_μ(b)[8, 6, 5] → 0, ω^(a)_μ(b)[8, 6, 6] → 0, ω^(a)_μ(b)[8, 6, 7] → 0, ω^(a)_μ(b)[8, 6, 8] → 0},
 {ω^(a)_μ(b)[8, 7, 1] → 0, ω^(a)_μ(b)[8, 7, 2] → 0, ω^(a)_μ(b)[8, 7, 3] → 0, ω^(a)_μ(b)[8, 7, 4] → 0,
  ω^(a)_μ(b)[8, 7, 5] → 0, ω^(a)_μ(b)[8, 7, 6] → 0, ω^(a)_μ(b)[8, 7, 7] → 0, ω^(a)_μ(b)[8, 7, 8] → 0},
 {ω^(a)_μ(b)[8, 8, 1] → ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), ω^(a)_μ(b)[8, 8, 2] → 0, ω^(a)_μ(b)[8, 8, 3] → 0,
  ω^(a)_μ(b)[8, 8, 4] → 0, ω^(a)_μ(b)[8, 8, 5] → -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4],
  ω^(a)_μ(b)[8, 8, 6] → 0, ω^(a)_μ(b)[8, 8, 7] → 0, ω^(a)_μ(b)[8, 8, 8] → 0}}}
```

In[489]:=

```
sωfgtry = Flatten[sωgtry];
```

In[490]:=

```
(*ω_μ^(a)_(b)*)
ω;
ωω = % /. sωfgtry // FullSimplify[#, constraintVars] &
Dimensions[%]
```

Out[491]=

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, -ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0,
   0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, -ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, -ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4],
   0, 0, 0}, {0, 0, 0, ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0},
  {ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6)},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4]},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6), 0, 0, 0, -ⅇ^(-a4[H x4]) H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}}
```

Out[492]=

```
{8, 8, 8}
```

In[493]:=

```
g4488 /. sg
% /. ssgm4488
```

Out[493]=

```
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},
 {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},
 {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0},
 {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}
```

Out[494]=

```
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0},
 {0, 0, ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, ⅇ^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0},
 {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0},
 {0, 0, 0, 0, 0, 0, -ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -ⅇ^(-2 a4[H x4]) Sin[6 H x0]^(1/3)}}
```

check metric compatibility condition for the Octad

In[∘]:=

```
Block[{eAα, eαA, t, ginv, Γ, gg(*,ωω*)},
 (*ωω=ω_μ^(a)_(b) /.sωf;*)
 gg = g4488 /. sg /. ssgm4488;
 {ginv, Γ} = con[gg];
 eαA = e^α_(A) /. sge^α_(A); eAα = e^(A)_α /. sge^(A)_α;
 t = Table[ Block[{v, V, contra, covari},
    V = eAα[[b, All]];
    v = eαA[[All, b]];
    contra =
     Table[D[v[[k]], X[[μ]] ] + Sum[ v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
    (*no ω terms*)
    covari = Table[D[V[[k]], X[[μ]] ] - Sum[ V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}] +
        Sum[ eAα[[J, All]][[k]] × ωmat[[μ, b, J]], {J, 1, DIM8}] //
       FullSimplify[#, constraintVars && ⅇ^(2 a4[H x4]) > 0 && ⅇ^(a4[H x4]) > 0 &&
          ⅇ^(-2 a4[H x4]) > 0 && ⅇ^(-a4[H x4]) > 0] &, {k, 1, DIM8}]; (*ω terms*)
    (*{b-1,X[[μ]],{{contra},{covari}}}],{b,1,Length[eαA]},{μ,1,DIM8}];*)
    {b - 1, X[[μ]], covari}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
 t]
FullSimplify[# /. ssgm4488,
   constraintVars && ⅇ^(2 a4[H x4]) > 0 && ⅇ^(a4[H x4]) > 0 && ⅇ^(-2 a4[H x4]) > 0 && ⅇ^(-a4[H x4]) > 0] & /@ %
```

Out[∘]=

```
{{{0, x0, {(g[0][0]^(1,0)[x0, x4] + 24 H Csc[12 H x0] g[0][0][x0, x4])/(2 Sqrt[g[0][0][x0, x4]]),

    0, 0, 0, g[0][0]^(0,1)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]]), 0, 0, 0}},

  {0, x1, {0, (-g[1][1]^(1,0)[x0, x4] + 2 ⅇ^(2 a4[H x4]) H Sec[6 H x0] Sin[6 H x0]^(4/3)
```

$$\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]\big) \big/ \left(2\ \sqrt{\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]}\right),\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\big\}\big\},$$

$$\big\{0,\ \mathrm{x2},\ \big\{0,\ 0,\ \big(-\mathrm{g}[2][2]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}] + 2\ e^{2\ \mathrm{a4}[\mathrm{H}\ \mathrm{x4}]}\ \mathrm{H}\ \mathrm{Sec}[6\ \mathrm{H}\ \mathrm{x0}]\ \mathrm{Sin}[6\ \mathrm{H}\ \mathrm{x0}]^{4/3}$$
$$\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]\big) \big/ \left(2\ \sqrt{\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]}\right),\ 0,\ 0,\ 0,\ 0,\ 0\big\}\big\},$$

$$\big\{0,\ \mathrm{x3},\ \big\{0,\ 0,\ 0,\ \big(-\mathrm{g}[3][3]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}] + 2\ e^{2\ \mathrm{a4}[\mathrm{H}\ \mathrm{x4}]}\ \mathrm{H}\ \mathrm{Sec}[6\ \mathrm{H}\ \mathrm{x0}]$$
$$\mathrm{Sin}[6\ \mathrm{H}\ \mathrm{x0}]^{4/3}\ \mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]\big) \big/ \left(2\ \sqrt{\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]}\right),\ 0,\ 0,\ 0,\ 0\big\}\big\},$$

$$\left\{0,\ \mathrm{x4},\ \left\{\frac{\mathrm{g}[0][0]^{(0,1)}[\mathrm{x0},\ \mathrm{x4}]}{2\ \sqrt{\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]}},\ 0,\ 0,\ 0,\ -\frac{\mathrm{g}[4][4]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}]}{2\ \sqrt{\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]}},\ 0,\ 0,\ 0\right\}\right\},$$

$$\big\{0,\ \mathrm{x5},\ \big\{0,\ 0,\ 0,\ 0,\ 0,\ \big(-\mathrm{g}[5][5]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}] - 2\ e^{-2\ \mathrm{a4}[\mathrm{H}\ \mathrm{x4}]}\ \mathrm{H}\ \mathrm{Sec}[6\ \mathrm{H}\ \mathrm{x0}]$$
$$\mathrm{Sin}[6\ \mathrm{H}\ \mathrm{x0}]^{4/3}\ \mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]\big) \big/ \left(2\ \sqrt{\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]}\right),\ 0,\ 0\big\}\big\},$$

$$\big\{0,\ \mathrm{x6},\ \big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \big(-\mathrm{g}[6][6]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}] - 2\ e^{-2\ \mathrm{a4}[\mathrm{H}\ \mathrm{x4}]}\ \mathrm{H}\ \mathrm{Sec}[6\ \mathrm{H}\ \mathrm{x0}]$$
$$\mathrm{Sin}[6\ \mathrm{H}\ \mathrm{x0}]^{4/3}\ \mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]\big) \big/ \left(2\ \sqrt{\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]}\right),\ 0\big\}\big\},$$

$$\big\{0,\ \mathrm{x7},\ \big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \big(-\mathrm{g}[7][7]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}] - 2\ e^{-2\ \mathrm{a4}[\mathrm{H}\ \mathrm{x4}]}\ \mathrm{H}$$
$$\mathrm{Sec}[6\ \mathrm{H}\ \mathrm{x0}]\ \mathrm{Sin}[6\ \mathrm{H}\ \mathrm{x0}]^{4/3}\ \mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]\big) \big/ \left(2\ \sqrt{\mathrm{g}[0][0][\mathrm{x0},\ \mathrm{x4}]}\right)\big\}\big\}\big\},$$

$$\left\{\left\{1,\ \mathrm{x0},\ \left\{0,\ \frac{\mathrm{g}[1][1]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}] - 2\ \mathrm{H}\ \mathrm{Cot}[6\ \mathrm{H}\ \mathrm{x0}]\ \mathrm{g}[1][1][\mathrm{x0},\ \mathrm{x4}]}{2\ \sqrt{\mathrm{g}[1][1][\mathrm{x0},\ \mathrm{x4}]}},\right.\right.\right.$$
$$\left.\left.0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}\right\},$$

$$\left\{1,\ \mathrm{x1},\ \left\{\frac{\mathrm{g}[1][1]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}] - 2\ \mathrm{H}\ \mathrm{Cot}[6\ \mathrm{H}\ \mathrm{x0}]\ \mathrm{g}[1][1][\mathrm{x0},\ \mathrm{x4}]}{2\ \sqrt{\mathrm{g}[1][1][\mathrm{x0},\ \mathrm{x4}]}},\ 0,\ 0,\right.\right.$$
$$\left.\left.0,\ \frac{\mathrm{g}[1][1]^{(0,1)}[\mathrm{x0},\ \mathrm{x4}] - 6\ \mathrm{H}\ \mathrm{a4}'[\mathrm{H}\ \mathrm{x4}]\ \mathrm{g}[1][1][\mathrm{x0},\ \mathrm{x4}]}{2\ \sqrt{\mathrm{g}[1][1][\mathrm{x0},\ \mathrm{x4}]}},\ 0,\ 0,\ 0\right\}\right\},$$

$$\{1,\ \mathrm{x2},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{1,\ \mathrm{x3},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},$$

$$\Big\{1,\ \mathrm{x4},$$
$$\left.\left\{0,\ \frac{\mathrm{g}[1][1]^{(0,1)}[\mathrm{x0},\ \mathrm{x4}] - 6\ \mathrm{H}\ \mathrm{a4}'[\mathrm{H}\ \mathrm{x4}]\ \mathrm{g}[1][1][\mathrm{x0},\ \mathrm{x4}]}{2\ \sqrt{\mathrm{g}[1][1][\mathrm{x0},\ \mathrm{x4}]}},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}\right\},$$

$$\{1,\ \mathrm{x5},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},$$
$$\{1,\ \mathrm{x6},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},$$
$$\{1,\ \mathrm{x7},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\},$$

$$\left\{\left\{2,\ \mathrm{x0},\ \left\{0,\ 0,\ \frac{\mathrm{g}[2][2]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}] - 2\ \mathrm{H}\ \mathrm{Cot}[6\ \mathrm{H}\ \mathrm{x0}]\ \mathrm{g}[2][2][\mathrm{x0},\ \mathrm{x4}]}{2\ \sqrt{\mathrm{g}[2][2][\mathrm{x0},\ \mathrm{x4}]}},\right.\right.\right.$$
$$\left.\left.0,\ 0,\ 0,\ 0,\ 0\right\}\right\},\ \{2,\ \mathrm{x1},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},$$

$$\left\{2,\ \mathrm{x2},\ \left\{\frac{\mathrm{g}[2][2]^{(1,0)}[\mathrm{x0},\ \mathrm{x4}] - 2\ \mathrm{H}\ \mathrm{Cot}[6\ \mathrm{H}\ \mathrm{x0}]\ \mathrm{g}[2][2][\mathrm{x0},\ \mathrm{x4}]}{2\ \sqrt{\mathrm{g}[2][2][\mathrm{x0},\ \mathrm{x4}]}},\ 0,\ 0,\right.\right.$$
$$\left.\left.0,\ \frac{\mathrm{g}[2][2]^{(0,1)}[\mathrm{x0},\ \mathrm{x4}] - 6\ \mathrm{H}\ \mathrm{a4}'[\mathrm{H}\ \mathrm{x4}]\ \mathrm{g}[2][2][\mathrm{x0},\ \mathrm{x4}]}{2\ \sqrt{\mathrm{g}[2][2][\mathrm{x0},\ \mathrm{x4}]}},\ 0,\ 0,\ 0\right\}\right\},$$

$$\{2, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x4,$$

$$\{0, 0, \frac{g[2][2]^{(0,1)}[x0, x4] - 6\, H\, a4'[H\, x4]\, g[2][2][x0, x4]}{2\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0\}\},$$

$$\{2, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{2, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{\{3, x0,$$

$$\{0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4] - 2\, H\, Cot[6\, H\, x0]\, g[3][3][x0, x4]}{2\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\}\},$$

$$\{3, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{3, x3, \{\frac{g[3][3]^{(1,0)}[x0, x4] - 2\, H\, Cot[6\, H\, x0]\, g[3][3][x0, x4]}{2\sqrt{g[3][3][x0, x4]}}, 0, 0, 0,$$

$$\frac{g[3][3]^{(0,1)}[x0, x4] - 6\, H\, a4'[H\, x4]\, g[3][3][x0, x4]}{2\sqrt{g[3][3][x0, x4]}}, 0, 0, 0\}\}, \{3, x4,$$

$$\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4] - 6\, H\, a4'[H\, x4]\, g[3][3][x0, x4]}{2\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\}\},$$

$$\{3, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{3, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\},$$

$$\{\{4, x0, \{\frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\}\},$$

$$\{4, x1,$$

$$\{0, \left(g[1][1]^{(0,1)}[x0, x4] + 6\, e^{2\, a4[H\, x4]}\, H\, Sin[6\, H\, x0]^{1/3}\, a4'[H\, x4]\, g[4][4][x0, x4]\right) /$$

$$\left(2\sqrt{-g[4][4][x0, x4]}\right), 0, 0, 0, 0, 0, 0\}\}, \{4, x2, \{0, 0,$$

$$\left(g[2][2]^{(0,1)}[x0, x4] + 6\, e^{2\, a4[H\, x4]}\, H\, Sin[6\, H\, x0]^{1/3}\, a4'[H\, x4]\, g[4][4][x0, x4]\right) /$$

$$\left(2\sqrt{-g[4][4][x0, x4]}\right), 0, 0, 0, 0, 0\}\}, \{4, x3, \{0, 0, 0,$$

$$\left(g[3][3]^{(0,1)}[x0, x4] + 6\, e^{2\, a4[H\, x4]}\, H\, Sin[6\, H\, x0]^{1/3}\, a4'[H\, x4]\, g[4][4][x0, x4]\right) /$$

$$\left(2\sqrt{-g[4][4][x0, x4]}\right), 0, 0, 0, 0\}\},$$

$$\{4, x4, \{-\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\}\},$$

$$\{4, x5,$$

$$\{0, 0, 0, 0, 0, \frac{1}{2}\left(-6\, e^{-2\, a4[H\, x4]}\, H\, Sin[6\, H\, x0]^{1/3}\, a4'[H\, x4] - \frac{g[5][5]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]}\right)$$

$$\sqrt{-g[4][4][x0, x4]}, 0, 0\}\}, \{4, x6, \{0, 0, 0, 0, 0, 0,$$

$$\frac{1}{2}\left(-6\, e^{-2\, a4[H\, x4]}\, H\, Sin[6\, H\, x0]^{1/3}\, a4'[H\, x4] - \frac{g[6][6]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]}\right)$$

$$\sqrt{-g[4][4][x0, x4]}, 0\}\}, \{4, x7, \{0, 0, 0, 0, 0, 0, 0,$$

$$\frac{1}{2}\left(-6\,e^{-2\,a4[H\,x4]}\,H\,Sin[6\,H\,x0]^{1/3}\,a4'[H\,x4]-\frac{g[7][7]^{(0,1)}[x0,\,x4]}{g[4][4][x0,\,x4]}\right)$$

$$\sqrt{-g[4][4][x0,\,x4]}\}\}\},\ \{\{5,\,x0,$$

$$\{0,\,0,\,0,\,0,\,0,\,-\frac{g[5][5]^{(1,0)}[x0,\,x4]-2\,H\,Cot[6\,H\,x0]\,g[5][5][x0,\,x4]}{2\,\sqrt{-g[5][5][x0,\,x4]}},\,0,\,0\}\},$$

$$\{5,\,x1,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},\ \{5,\,x2,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$

$$\{5,\,x3,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},\ \{5,\,x4,$$

$$\{0,\,0,\,0,\,0,\,0,\,-\frac{g[5][5]^{(0,1)}[x0,\,x4]+6\,H\,a4'[H\,x4]\,g[5][5][x0,\,x4]}{2\,\sqrt{-g[5][5][x0,\,x4]}},\,0,\,0\}\},$$

$$\{5,\,x5,\,\{-\frac{g[5][5]^{(1,0)}[x0,\,x4]-2\,H\,Cot[6\,H\,x0]\,g[5][5][x0,\,x4]}{2\,\sqrt{-g[5][5][x0,\,x4]}},\,0,\,0,$$

$$0,\,-\frac{g[5][5]^{(0,1)}[x0,\,x4]+6\,H\,a4'[H\,x4]\,g[5][5][x0,\,x4]}{2\,\sqrt{-g[5][5][x0,\,x4]}},\,0,\,0,\,0\}\},$$

$$\{5,\,x6,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},\ \{5,\,x7,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\},\ \{\{6,\,x0,$$

$$\{0,\,0,\,0,\,0,\,0,\,0,\,-\frac{g[6][6]^{(1,0)}[x0,\,x4]-2\,H\,Cot[6\,H\,x0]\,g[6][6][x0,\,x4]}{2\,\sqrt{-g[6][6][x0,\,x4]}},\,0\}\},$$

$$\{6,\,x1,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},\ \{6,\,x2,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$

$$\{6,\,x3,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},\ \{6,\,x4,$$

$$\{0,\,0,\,0,\,0,\,0,\,0,\,-\frac{g[6][6]^{(0,1)}[x0,\,x4]+6\,H\,a4'[H\,x4]\,g[6][6][x0,\,x4]}{2\,\sqrt{-g[6][6][x0,\,x4]}},\,0\}\},$$

$$\{6,\,x5,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$

$$\{6,\,x6,\,\{-\frac{g[6][6]^{(1,0)}[x0,\,x4]-2\,H\,Cot[6\,H\,x0]\,g[6][6][x0,\,x4]}{2\,\sqrt{-g[6][6][x0,\,x4]}},\,0,\,0,$$

$$0,\,-\frac{g[6][6]^{(0,1)}[x0,\,x4]+6\,H\,a4'[H\,x4]\,g[6][6][x0,\,x4]}{2\,\sqrt{-g[6][6][x0,\,x4]}},\,0,\,0,\,0\}\},$$

$$\{6,\,x7,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\}\},\ \{\{7,\,x0,$$

$$\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,-\frac{g[7][7]^{(1,0)}[x0,\,x4]-2\,H\,Cot[6\,H\,x0]\,g[7][7][x0,\,x4]}{2\,\sqrt{-g[7][7][x0,\,x4]}}\}\},$$

$$\{7,\,x1,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},\ \{7,\,x2,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$

$$\{7,\,x3,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},\ \{7,\,x4,$$

$$\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,-\frac{g[7][7]^{(0,1)}[x0,\,x4]+6\,H\,a4'[H\,x4]\,g[7][7][x0,\,x4]}{2\,\sqrt{-g[7][7][x0,\,x4]}}\}\},$$

$$\{7,\,x5,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},\ \{7,\,x6,\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,0\}\},$$

```
{7, x7, {-(g[7][7]^(1,0)[x0, x4] - 2 H Cot[6 H x0] g[7][7][x0, x4])/(2 Sqrt[-g[7][7][x0, x4]]), 0, 0,
   0, -(g[7][7]^(0,1)[x0, x4] + 6 H a4′[H x4] g[7][7][x0, x4])/(2 Sqrt[-g[7][7][x0, x4]]), 0, 0, 0}}}}
```

Out[∘]=

```
{{{0, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x3, {0, 0, 0, 0,
0, 0, 0, 0}}, {0, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, x7,
{0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x2, {0, 0, 0, 0, 0, 0, 0,
0}}, {1, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {1, x6, {0, 0, 0,
0, 0, 0, 0, 0}}, {1, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{2, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {2,
x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x5, {0, 0, 0, 0, 0, 0, 0,
0}}, {2, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {2, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{3, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x1, {0, 0,
0, 0, 0, 0, 0, 0}}, {3, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {3,
x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {3, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{4, x0, {0, 0, 0, 0, 0, 0,
0, 0}}, {4, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x4, {0, 0,
0, 0, 0, 0, 0, 0}}, {4, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {4, x7, {0, 0, 0, 0, 0, 0, 0, 0}}},
{{5, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x2, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x3, {0, 0, 0, 0, 0,
0, 0, 0}}, {5, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x7, {0,
0, 0, 0, 0, 0, 0, 0}}}, {{6, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x2, {0, 0, 0, 0, 0, 0, 0, 0}},
{6, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x6, {0, 0, 0, 0, 0,
0, 0, 0}}, {6, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{7, x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x2,
{0, 0, 0, 0, 0, 0, 0, 0}}, {7, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x5, {0, 0, 0, 0, 0, 0, 0,
0}}, {7, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}}
```

In[∘]:=

```
Block[{eAα, eαA, t, ginv, Γ, gg(*,ωω*)},
 (*ωω=ω_μ^(a)_(b) /.sωf;*)
 gg = g4488 /. sg;
 {ginv, Γ} = con[gg];
 eαA = e^α_(A) /. sge^α_(A); eAα = e^(A)_α /. sge^(A)_α;
 t = Table[ Block[{v, V, contra, covari},
    V = eAα⟦b, All⟧;
    v = eαA⟦All, b⟧;
    contra =
     Table[D[v⟦k⟧, X⟦μ⟧ ] + Sum[ v⟦i⟧ × Γ⟦k, i, μ⟧, {i, 1, DIM8}], {k, 1, DIM8}];
    (*no ω terms*)
    covari = Table[D[V⟦k⟧, X⟦μ⟧ ] - Sum[ V⟦i⟧ × Γ⟦i, k, μ⟧, {i, 1, DIM8}] +
       Sum[ eAα⟦J, All⟧⟦k⟧ × ωmat⟦μ, b, J⟧, {J, 1, DIM8}],
      {k, 1, DIM8}]; (*ω terms*)
    {b - 1, X⟦μ⟧, {{contra}, {covari}}}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
 t]
FullSimplify[# /. ssgm4488,
   constraintVars && ⅇ^(2 a4[H x4]) > 0 && ⅇ^(a4[H x4]) > 0 && ⅇ^(-2 a4[H x4]) > 0 && ⅇ^(-a4[H x4]) > 0] & /@ %
```

Out[∘]=

```
{{{0, x0, {{{0, 0, 0, 0, -g[0][0]^(0,1)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[4][4][x0, x4]), 0, 0, 0}},
```

$$\{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\Big\{0,\ x1,\ \Big\{\Big\{\Big\{0,\ \frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ g[1][1][x0,x4]},\ 0,0,0,0,0,0\Big\}\Big\},$$

$$\{\{0,0,0,0,0,0,0,0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x2,\ \Big\{\Big\{\Big\{0,\ 0,\ \frac{g[2][2]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ g[2][2][x0,x4]},\ 0,0,0,0,0\Big\}\Big\},$$

$$\{\{0,0,0,0,0,0,0,0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x3,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ \frac{g[3][3]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ g[3][3][x0,x4]},\ 0,0,0,0\Big\}\Big\},$$

$$\{\{0,0,0,0,0,0,0,0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x4,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ \frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ g[4][4][x0,x4]},\ 0,0,0\Big\}\Big\},$$

$$\{\{0,0,0,0,0,0,0,0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x5,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ g[5][5][x0,x4]},\ 0,0\Big\}\Big\},$$

$$\{\{0,0,0,0,0,0,0,0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x6,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ g[6][6][x0,x4]},\ 0\Big\}\Big\},$$

$$\{\{0,0,0,0,0,0,0,0\}\}\Big\}\Big\},$$

$$\Big\{0,\ x7,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[7][7]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\ g[7][7][x0,x4]}\Big\}\Big\},$$

$$\{\{0,0,0,0,0,0,0,0\}\}\Big\}\Big\}\Big\},$$

$$\Big\{\{1,\ x0,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\Big\{1,\ x1,\ \Big\{\Big\{\Big\{-\frac{g[1][1]^{(1,0)}[x0,x4]}{2\,g[0][0][x0,x4]\ \sqrt{g[1][1][x0,x4]}},\ 0,$$

$$0,\ 0,\ -\frac{g[1][1]^{(0,1)}[x0,x4]}{2\sqrt{g[1][1][x0,x4]}\ g[4][4][x0,x4]},\ 0,0,0\Big\}\Big\},$$

$$\{\{0,0,0,0,0,0,0,0\}\}\Big\}\Big\},\ \{1,\ x2,$$

$$\{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\{1,\ x3,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\{1,\ x4,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\{1,\ x5,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

$$\{1,\ x6,\ \{\{\{0,0,0,0,0,0,0,0\}\},\ \{\{0,0,0,0,0,0,0,0\}\}\}\},$$

{1, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},

{{2, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{2, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\Big\{2,\ x2,\ \Big\{\Big\{\Big\{-\frac{g[2][2]^{(1,0)}[x0,\ x4]}{2\,g[0][0][x0,\ x4]\ \sqrt{g[2][2][x0,\ x4]}},\ 0,$$

$$0,\ 0,\ -\frac{g[2][2]^{(0,1)}[x0,\ x4]}{2\ \sqrt{g[2][2][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0\Big\}\Big\},$$

{{0, 0, 0, 0, 0, 0, 0, 0}}}}, {2, x3,

{{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{2, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{2, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{2, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{2, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},

{{3, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{3, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{3, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

$$\Big\{3,\ x3,\ \Big\{\Big\{\Big\{-\frac{g[3][3]^{(1,0)}[x0,\ x4]}{2\,g[0][0][x0,\ x4]\ \sqrt{g[3][3][x0,\ x4]}},\ 0,$$

$$0,\ 0,\ -\frac{g[3][3]^{(0,1)}[x0,\ x4]}{2\ \sqrt{g[3][3][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0\Big\}\Big\},$$

{{0, 0, 0, 0, 0, 0, 0, 0}}}}, {3, x4,

{{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{3, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{3, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},

{3, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},

$$\Big\{\Big\{4,\ x0,\ \Big\{\Big\{\Big\{\frac{g[0][0]^{(0,1)}[x0,\ x4]}{2\,g[0][0][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0,$$

$$\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ (-g[4][4][x0,\ x4])^{3/2}}+\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ -\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}}-$$

$$\frac{g[4][4]^{(1,0)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\,g[4][4][x0,\ x4]}-\Bigg(\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ (-g[4][4][x0,\ x4])^{3/2}}+$$

$$\left.\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[4][4][x0,\ x4]}\right)\ g[4][4][x0,\ x4],\ 0,\ 0,\ 0\}\}\}\},$$

$$\Big\{4,\ x1,\ \Big\{\Big\{\Big\{0,\ \frac{g[1][1]^{(0,1)}[x0,\ x4]}{2\ g[1][1][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{4,\ x2,\ \Big\{\Big\{\Big\{0,\ 0,\ \frac{g[2][2]^{(0,1)}[x0,\ x4]}{2\ g[2][2][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{4,\ x3,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ \frac{g[3][3]^{(0,1)}[x0,\ x4]}{2\ g[3][3][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{4,\ x4,\ \Big\{\Big\{\Big\{-\frac{g[4][4]^{(1,0)}[x0,\ x4]}{2\ g[0][0][x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}},\ 0,\ 0,\ 0,$$

$$\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\ (-g[4][4][x0,\ x4])^{3/2}}+\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[4][4][x0,\ x4]},\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ -\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}}-$$

$$\frac{g[4][4]^{(0,1)}[x0,\ x4]\ \sqrt{-g[4][4][x0,\ x4]}}{2\ g[4][4][x0,\ x4]}-\left(\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\ (-g[4][4][x0,\ x4])^{3/2}}+\right.$$

$$\left.\frac{g[4][4]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[4][4][x0,\ x4]}\right)\ g[4][4][x0,\ x4],\ 0,\ 0,\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\Big\{4,\ x5,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[5][5][x0,\ x4]},\ 0,\ 0\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{4,\ x6,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[6][6][x0,\ x4]},\ 0\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\Big\{4,\ x7,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[4][4][x0,\ x4]}\ g[7][7][x0,\ x4]}\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\}\Big\},$$

$$\Big\{\Big\{5,\ x0,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[5][5]^{(1,0)}[x0,\ x4]}{2\ (-g[5][5][x0,\ x4])^{3/2}}+$$

$$\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}\; g[5][5][x0, x4]}, 0, 0\}\}, \{\{0, 0, 0, 0, 0,$$

$$-\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}} - \frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]} -$$

$$\left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2\,(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}\; g[5][5][x0, x4]}\right)$$

$$g[5][5][x0, x4], 0, 0\}\}\}\},$$

$$\{5, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{5, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{5, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{5, x4, \{\{\{0, 0, 0, 0, 0,$$

$$\frac{g[5][5]^{(0,1)}[x0, x4]}{2\,(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}\; g[5][5][x0, x4]}, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}} -$$

$$\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\, g[5][5][x0, x4]} - \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2\,(-g[5][5][x0, x4])^{3/2}} + \right.$$

$$\left.\frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}\; g[5][5][x0, x4]}\right) g[5][5][x0, x4], 0, 0\}\}\}\},$$

$$\{5, x5, \{\{\{-\frac{g[5][5]^{(1,0)}[x0, x4]}{2\, g[0][0][x0, x4]\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0,$$

$$-\frac{g[5][5]^{(0,1)}[x0, x4]}{2\, g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{5, x6,$$

$$\{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\},$$

$$\{\{6, x0, \{\{\{0, 0, 0, 0, 0, 0,$$

$$\frac{g[6][6]^{(1,0)}[x0, x4]}{2\,(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}\; g[6][6][x0, x4]}, 0\}\},$$

$$\{\{0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} -$$

$$\frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\, g[6][6][x0, x4]} - \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2\,(-g[6][6][x0, x4])^{3/2}} + \right.$$

$$\frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[6][6][x0,\ x4]}\ g[6][6][x0,\ x4]}\Bigg)\ g[6][6][x0,\ x4],\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\{6,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{6,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{6,\ x3,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\Big\{6,\ x4,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,$$

$$\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ (-g[6][6][x0,\ x4])^{3/2}}+\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[6][6][x0,\ x4]}\ g[6][6][x0,\ x4]},\ 0\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[6][6][x0,\ x4]}}-$$

$$\frac{g[6][6]^{(0,1)}[x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}}{2\ g[6][6][x0,\ x4]}-\Bigg(\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ (-g[6][6][x0,\ x4])^{3/2}}+$$

$$\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[6][6][x0,\ x4]}\ g[6][6][x0,\ x4]}\Bigg)\ g[6][6][x0,\ x4],\ 0\Big\}\Big\}\Big\}\Big\},$$

$$\{6,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\Big\{6,\ x6,\ \Big\{\Big\{\Big\{-\frac{g[6][6]^{(1,0)}[x0,\ x4]}{2\ g[0][0][x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}},\ 0,$$

$$0,\ 0,\ -\frac{g[6][6]^{(0,1)}[x0,\ x4]}{2\ g[4][4][x0,\ x4]\ \sqrt{-g[6][6][x0,\ x4]}},\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},\ \{6,\ x7,$$

$$\{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\Big\},$$

$$\Big\{\Big\{7,\ x0,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ (-g[7][7][x0,\ x4])^{3/2}}+$$

$$\frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[7][7][x0,\ x4]}\ g[7][7][x0,\ x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[7][7][x0,\ x4]}}-$$

$$\frac{g[7][7]^{(1,0)}[x0,\ x4]\ \sqrt{-g[7][7][x0,\ x4]}}{2\ g[7][7][x0,\ x4]}-\Bigg(\frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ (-g[7][7][x0,\ x4])^{3/2}}+$$

$$\frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ \sqrt{-g[7][7][x0,\ x4]}\ g[7][7][x0,\ x4]}\Bigg)\ g[7][7][x0,\ x4]\Big\}\Big\}\Big\}\Big\},$$

$$\{7,\ x1,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{7,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{7,\ x3,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\Big\{7,\ x4,\ \Big\{\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$$

$$\frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ (-g[7][7][x0,\ x4])^{3/2}}+\frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[7][7][x0,\ x4]}\ g[7][7][x0,\ x4]}\Big\}\Big\},$$

$$\Big\{\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[7][7][x0,\ x4]}}-$$

$$\frac{g[7][7]^{(0,1)}[x0,\ x4]\ \sqrt{-g[7][7][x0,\ x4]}}{2\ g[7][7][x0,\ x4]}-\left(\frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ (-g[7][7][x0,\ x4])^{3/2}}+\right.$$

$$\left.\frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ \sqrt{-g[7][7][x0,\ x4]}\ g[7][7][x0,\ x4]}\right)g[7][7][x0,\ x4]\Big\}\Big\}\Big\}\Big\},$$

$$\{7,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{7,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\Big\{7,\ x7,\ \Big\{\Big\{\Big\{-\frac{g[7][7]^{(1,0)}[x0,\ x4]}{2\ g[0][0][x0,\ x4]\ \sqrt{-g[7][7][x0,\ x4]}},\ 0,$$

$$0,\ 0,\ -\frac{g[7][7]^{(0,1)}[x0,\ x4]}{2\ g[4][4][x0,\ x4]\ \sqrt{-g[7][7][x0,\ x4]}},\ 0,\ 0,\ 0\Big\}\Big\},$$

$$\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\}\Big\}\Big\}$$

Out[∘]=

$$\Big\{\{\{0,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{0,\ x1,\ \{\{\{0,\ H,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{0,\ x2,\ \{\{\{0,\ 0,\ H,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{0,\ x3,\ \{\{\{0,\ 0,\ 0,\ H,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{0,\ x4,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{0,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ H,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{0,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ H,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{0,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ H\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\},$$

$$\Big\{\{1,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\Big\{1,\ x1,\ \Big\{\Big\{\Big\{-\sqrt{e^{2\,a4[H\,x4]}}\ H\,Sec[6\,H\,x0]\ Sin[6\,H\,x0]^{7/6},\ 0,\ 0,\ 0,\ 3\ \sqrt{e^{2\,a4[H\,x4]}}$$

$$H\,Sin[6\,H\,x0]^{1/6}\ a4'[H\,x4],\ 0,\ 0,\ 0\Big\}\Big\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\Big\}\Big\},$$

$$\{1,\ x2,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x3,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x4,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x5,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x6,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

$$\{1,\ x7,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\}\Big\},$$

$$\Big\{\{2,\ x0,\ \{\{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\},\ \{\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}\}\}\},$$

```
  {2, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x2, {{{-Sqrt[ⅇ^(2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6), 0, 0, 0, 3 Sqrt[ⅇ^(2 a4[H x4])]
       H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {2, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{3, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x3, {{{-Sqrt[ⅇ^(2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6), 0, 0, 0, 3 Sqrt[ⅇ^(2 a4[H x4])]
       H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {3, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{4, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {4, x1, {{{0, 3 H a4′[H x4], 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {4, x2, {{{0, 0, 3 H a4′[H x4], 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {4, x3, {{{0, 0, 0, 3 H a4′[H x4], 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {4, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {4, x5, {{{0, 0, 0, 0, 0, -3 H a4′[H x4], 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {4, x6, {{{0, 0, 0, 0, 0, 0, -3 H a4′[H x4], 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {4, x7, {{{0, 0, 0, 0, 0, 0, 0, -3 H a4′[H x4]}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{5, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x5, {{{Sqrt[ⅇ^(-2 a4[H x4])] H Sec[6 H x0] Sin[6 H x0]^(7/6), 0, 0, 0, 3 Sqrt[ⅇ^(-2 a4[H x4])]
       H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {5, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{6, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {6, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
```

```
  {6, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {6, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {6, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {6, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {6, x6, {{{√(ⅇ^(-2 a4[H x4])) H Sec[6 H x0] Sin[6 H x0]^(7/6), 0, 0, 0, 3 √(ⅇ^(-2 a4[H x4]))
      H Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {6, x7, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}},
 {{7, x0, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}},
  {7, x7, {{{√(ⅇ^(-2 a4[H x4])) H Sec[6 H x0] Sin[6 H x0]^(7/6), 0, 0, 0, 3 √(ⅇ^(-2 a4[H x4])) H
      Sin[6 H x0]^(1/6) a4′[H x4], 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}}}}}}
```

```
In[495]:=
(*ωΨ=Table[Sum[ExpandAll[
    (ω_μ^(a)_(b)[μ,a,b]/.sωf)Simplify[(Transpose[Ψ16].σ16.SAB⟦a,b⟧.Ψ16)]],
   {a,1,8},{b,1,8}],{μ,1,8}]*)
```

```
In[496]:=
(*ωΨ=Table[
  Sum[ExpandAll[(ωmat⟦μ,a,b⟧)Simplify[(Transpose[Ψ16].σ16.SAB⟦a,b⟧.Ψ16)]],
   {a,1,8},{b,1,8}],{μ,1,8}]*)
```

```
In[497]:=
(*jωΨ=ParallelTable[{j,base16⟦j,2⟧,
   Table[{X⟦μ⟧,Simplify[(Transpose[Ψ16].σ16.(ZZZ0 *T16^α[0]+T16^α[4]* ZZZ4 ).
         base16⟦j,1⟧.(Sum[ωmat⟦μ,a,b⟧SAB⟦a,b⟧,{a,1,8},{b,1,8}])Ψ16)]},
    {μ,1,8}]},{j,1,Length[base16]}]*)
```

```
In[498]:=
(*jωΨ=ParallelTable[{j,base16⟦j,2⟧,
   Table[Sum[ExpandAll[(ω_μ^(a)_(b)[μ,a,b]/.sωf)Simplify[(Transpose[Ψ16].σ16.
          (ZZZ0 *T16^α[0]+T16^α[4]* ZZZ4 ).base16⟦j,1⟧.SAB⟦a,b⟧.Ψ16)]],
     {a,1,8},{b,1,8}],{μ,1,8}]},{j,1,Length[base16]}]*)
```

```
In[499]:=
(*jωΨ⟦92⟧*)
```

In[500]:=

```
(*jωΨ⟦92⟧/.ssgm4488//FullSimplify[#,constraintVars]&*)
```

In[501]:=

```
SAB⟦1⟧⟦2⟧
```

Out[501]=

```
{{0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {-1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0}}
```

In[502]:=

$$\left(\omega^{(a)}_{\mu\ (b)}\,[8,\ 8,\ 1]\ \text{/. Flatten[s}\omega\text{gtry]}\right)$$

Out[502]=

$$e^{-\text{a4}[\text{H x4}]}\ \text{H Sin}[6\ \text{H x0}]^{1/6}$$

In[503]:=

```
SAB⟦1, 2⟧.Ψ16
```

Out[503]=

```
{1/2 f16[7][x0, x4], 1/2 f16[6][x0, x4], -1/2 f16[5][x0, x4], -1/2 f16[4][x0, x4],
 1/2 f16[3][x0, x4], 1/2 f16[2][x0, x4], -1/2 f16[1][x0, x4], -1/2 f16[0][x0, x4],
 -1/2 f16[15][x0, x4], -1/2 f16[14][x0, x4], 1/2 f16[13][x0, x4], 1/2 f16[12][x0, x4],
 -1/2 f16[11][x0, x4], -1/2 f16[10][x0, x4], 1/2 f16[9][x0, x4], 1/2 f16[8][x0, x4]}
```

```
In[504]:=
T16^A[0]
Out[504]=
{{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
 {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[∘]:=
SAB =
  Table[1/4 (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]), {A1, 0, 7}, {B1, 0, 7}];
```

```
In[505]:=
Ψ16
Out[505]=
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

```
In[506]:=
Sum[ExpandAll[(ω_μ^(a)_(b)[4, a, b] /. sωfgtry) Simplify[(SAB⟦a, b⟧)]],
   {a, 1, 8}, {b, 1, 8}].Ψ16 // MatrixForm
Out[506]//MatrixForm=
 -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[5][x0, x4]
  ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[4][x0, x4]
 -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[7][x0, x4]
  ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[6][x0, x4]
 -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[1][x0, x4]
  ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[0][x0, x4]
 -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[3][x0, x4]
  ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[2][x0, x4]
  ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[13][x0, x4]
 -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[12][x0, x4]
  ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[15][x0, x4]
 -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[14][x0, x4]
  ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[9][x0, x4]
 -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[8][x0, x4]
  ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[11][x0, x4]
 -ⅇ^a4[H x4] H Sin[6 H x0]^(1/6) f16[10][x0, x4]
```

In[507]:=

```
(*Table[Block[{eAα,eαA,t,V},eαA=e^α_(A)/.sgtrye^α_(A);
  eAα=e_α^(A)/.sgtrye_α^(A);
  V=eAα[[a,All]];
  t=Table[Block[{v,d},v=eαA[[All,b]];
     d=D[v[[k]],X[[j]] ]+Sum[ v[[i]] Γ[[k,i,j]],{i,1,DIM8}];
     d.V],{k,1,DIM8}];
  t],{j,1,DIM8},{a,1,Length[eαA]},{b,1,Length[eαA]}]*)
```

In[∘]:=

```
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, Γ, RicciΓ, RieΓ, RS, G, ginv},
   Print[Now];
   t = AbsoluteTiming[
     ginv = FullSimplify[Inverse[g], ass];
     Γ = FullSimplify[Table[(1 / 2) * Sum[(ginv[[i, s]]) *
            (D[g[[s, j]], X[[k]] ] + D[g[[s, k]], X[[j]] ] - D[g[[j, k]], X[[s]] ]), {s, 1, DIM8
         {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] , ass];
     RicciΓ = ParallelTable[
       FullSimplify[D[Γ[[μ, ν, β]], X[[α]] ] - D[Γ[[μ, ν, α]], X[[β]] ] + Sum[
          Γ[[μ, s, α]] × Γ[[s, ν, β]] - Γ[[μ, s, β]] × Γ[[s, ν, α]], {s, 1, DIM8}], ass],
       {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}] ;
     RieΓ =
      ParallelTable[ FullSimplify[Sum[ RicciΓ[[α, μ, α, ν]], {α, 1, DIM8}], ass],
       {μ, 1, DIM8}, {ν, 1, DIM8}] ;
     RS = FullSimplify[Tr[ ginv.RieΓ] , ass];
     G = ParallelTable[
       FullSimplify[RieΓ[[α]] - 1/2 g[[α]] RS, ass], {α, 1, Length[RieΓ]}];];
   Print[t];
   Print[Now];
   Return[{ginv, Γ, RicciΓ, RieΓ, RS, G}]]
```

verify $g_{\mu\nu|\alpha} = 0$:

In[508]:=

```
Simplify[
   Table[D[gtry[[j, k]], X[[i]] ] - Sum[ gtry[[k, s]] × Γ[[s, i, j]], {s, 1, DIM8}] -
     Sum[ gtry[[j, s]] × Γ[[s, i, k]], {s, 1, DIM8}],
    {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Out[508]=

```
{0}
```

CHECK

In[509]:=

```
MatrixMetric44 // MatrixForm
```

Out[509]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0]^{2} & 0 & 0 & 0 & 0 \\ 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -e^{-2\,a4[H\,x4]}\,\mathrm{S} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[510]:=

$$\left(\mathrm{e}^{\alpha}_{(A)}.\mathrm{MatrixMetric44}.\mathrm{e}^{\alpha}_{(A)}\ \text{/. }\ \mathrm{sgtrye}^{\alpha}_{(A)}\right)\ \text{// MatrixForm}$$

Out[510]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[511]:=

$$\mathrm{Transpose}\left[\mathrm{e}_{\alpha}^{(A)}\right].\eta4488.\mathrm{e}_{\alpha}^{(A)} - \mathrm{MatrixMetric44}\ \text{/. }\ \mathrm{sgtrye}_{\alpha}^{(A)}\ \text{//}\ \mathrm{FullSimplify}[\#, \mathrm{constraintVars}]\ \&$$

Out[511]=

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[512]:=

$$\mathrm{Transpose}\left[\mathrm{e}^{\alpha}_{(A)}\right].\mathrm{MatrixMetric44}.\mathrm{e}^{\alpha}_{(A)} - \eta4488\ \text{/. }\ \mathrm{sgtrye}^{\alpha}_{(A)}\ \text{//}\ \mathrm{FullSimplify}[\#, \mathrm{constraintVars}]\ \&$$

Out[512]=

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

In[513]:=

$$\mathrm{Inverse}\left[\mathrm{e}^{\alpha}_{(A)}\ \text{/. }\ \mathrm{sgtrye}^{\alpha}_{(A)}\right] == \mathrm{e}_{\alpha}^{(A)}\ \text{/. }\ \mathrm{sgtrye}_{\alpha}^{(A)}\ \text{//}\ \mathrm{FullSimplify}[\#, \mathrm{constraintVars}]\ \&$$

Out[513]=

```
True
```

In[514]:=

```
Block[{a, b, c, d, m}, m = {{a, b}, {c, d}};
 Inverse[Transpose[m]] - Transpose[Inverse[m]]]
```

Out[514]=

```
{{0, 0}, {0, 0}}
```

In[515]:=

```
(*Inverse[Transpose[e^α_(A)].MatrixMetric44.e^α_(A)]==
  e_α^(A).Inverse[MatrixMetric44].Transpose[e_α^(A)]//
 FullSimplify[#,constraintVars]&*)
```

In[516]:=

```
(*e_α^(A).Inverse[MatrixMetric44].Transpose[e_α^(A)]//
     FullSimplify[#,constraintVars]&
   %-Inverse[η4488]//Union[Flatten[#]]&*)
```

In[517]:=

```
(*Transpose[e_α^(A)]==e_α^(A)//FullSimplify[#,constraintVars]&*)
```

In[518]:=

```
(*(FullSimplify[e_α^(A).Inverse[MatrixMetric44].Transpose[e_α^(A)] -
     Inverse[η4488] ,constraintVars]//
   Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)
```

In[519]:=

```
(*(FullSimplify[Transpose[e_α^(A)].Inverse[MatrixMetric44].e_α^(A),
     constraintVars]-Inverse[η4488]//
   Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)
```

In[520]:=

```
(*(FullSimplify[Transpose[e_α^(A)].ginv.e_α^(A),constraintVars]-Inverse[η4488]//
   Union[Flatten[#]]&)//FullSimplify[#,constraintVars]&*)
```

In[∘]:=

```
(E^α_(A) = Table[Sum[e_α^(A)⟦A1, α1⟧ ginv⟦α1, β1⟧ × η4488⟦A1, B1⟧ /. sgtrye_α^(A),
       {α1, 1, 8}, {A1, 1, 8}], {β1, 1, 8}, {B1, 1, 8}] //
    FullSimplify[#, constraintVars] &) // Column
```

Out[∘]=

```
{Tan[6 H x0], 0, 0, 0, 0, 0, 0, 0}
{0, ⅇ^(-a4[x4])/Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}
{0, 0, ⅇ^(-a4[x4])/Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}
{0, 0, 0, ⅇ^(-a4[x4])/Sin[6 H x0]^(1/6), 0, 0, 0, 0}
{0, 0, 0, 0, 1, 0, 0, 0}
{0, 0, 0, 0, 0, ⅇ^(a4[x4])/Sin[6 H x0]^(1/6), 0, 0}
{0, 0, 0, 0, 0, 0, ⅇ^(a4[x4])/Sin[6 H x0]^(1/6), 0}
{0, 0, 0, 0, 0, 0, 0, ⅇ^(a4[x4])/Sin[6 H x0]^(1/6)}
```

```
LagrangianΨ16 =
 κ Sqrt[Det[g_μν]] Transpose[Ψ16].σ16.T16^A[A1]. Ψ16_,α g^-1 αβ η_A1 B1 E^B1_β + mass term
```

In[521]:=

```
ass = constraintVars
```

Out[521]=

$$\begin{aligned}&x0>0\ \&\&\ x4>0\ \&\&\ H>0\ \&\&\ 6\,H\,x0>0\ \&\&\ 3\,H\,x4>0\ \&\&\ a4[x4]>0\ \&\&\ A4[t]>0\ \&\&\\ &Q>0\ \&\&\ z>0\ \&\&\ t>0\ \&\&\ M>0\ \&\&\ K>0\ \&\&\ e^{-2\,a4[H\,x4]}>0\ \&\&\ e^{-a4[H\,x4]}>0\ \&\&\ e^{2\,a4[H\,x4]}>0\ \&\&\\ &e^{a4[H\,x4]}>0\ \&\&\ \mathrm{Sin}[6\,H\,x0]>0\ \&\&\ \mathrm{Cos}[6\,H\,x0]>0\ \&\&\ \mathrm{Csc}[6\,H\,x0]>0\ \&\&\ \mathrm{Sec}[6\,H\,x0]>0\ \&\&\\ &\mathrm{Tan}[6\,H\,x0]>0\ \&\&\ \mathrm{Cot}[6\,H\,x0]>0\ \&\&\ \mathrm{Sin}[z]>0\ \&\&\ \sqrt{\mathrm{Sin}[z]}>0\ \&\&\ \mathrm{Sin}[z]^{3/2}>0\ \&\&\\ &\sqrt{\mathrm{Sin}[z]}>0\ \&\&\ \frac{1}{\mathrm{Sin}[z]^{3/2}}>0\ \&\&\ \frac{1}{\sqrt{\mathrm{Sin}[z]}}>0\ \&\&\ \mathrm{Cot}[z]>0\ \&\&\ \sqrt{\mathrm{Cot}[z]}>0\ \&\&\\ &\mathrm{Cot}[z]^{3/2}>0\ \&\&\ \sqrt{\mathrm{Cot}[z]}>0\ \&\&\ \mathrm{Tan}[z]>0\ \&\&\ \mathrm{Sec}[z]>0\ \&\&\ \sqrt{\mathrm{Csc}[z]}>0\ \&\&\ \mathrm{Csc}[z]>0\end{aligned}$$

In[522]:=

$$\begin{aligned}&\mathbf{x0>0\ \&\&\ x4>0\ \&\&\ H>0\ \&\&\ 6\,H\,x0>0\ \&\&\ H\,x4>0\ \&\&\ a4[x4]>0\ \&\&\ A4[t]>0\ \&\&\ Q>0\ \&\&}\\ &\mathbf{z>0\ \&\&\ t>0\ \&\&\ M>0\ \&\&\ K>0\ \&\&\ e^{-2\,a4[H\,x4]}>0\ \&\&\ e^{-a4[H\,x4]}>0\ \&\&\ e^{2\,a4[H\,x4]}>0\ \&\&}\\ &\mathbf{e^{a4[H\,x4]}>0\ \&\&\ Sin[6\,H\,x0]>0\ \&\&\ Cos[6\,H\,x0]>0\ \&\&\ Csc[6\,H\,x0]>0\ \&\&\ Sec[6\,H\,x0]>0\ \&\&}\\ &\mathbf{Tan[6\,H\,x0]>0\ \&\&\ Cot[6\,H\,x0]>0\ \&\&\ Sin[z]>0\ \&\&\ \sqrt{Sin[z]}>0\ \&\&\ Sin[z]^{3/2}>0\ \&\&}\\ &\mathbf{\sqrt{Sin[z]}>0\ \&\&\ \frac{1}{Sin[z]^{3/2}}>0\ \&\&\ \frac{1}{\sqrt{Sin[z]}}>0\ \&\&\ Cot[z]>0\ \&\&\ \sqrt{Cot[z]}>0\ \&\&}\\ &\mathbf{Cot[z]^{3/2}>0\ \&\&\ \sqrt{Cot[z]}>0\ \&\&\ Tan[z]>0\ \&\&\ Sec[z]>0\ \&\&\ \sqrt{Csc[z]}>0\ \&\&\ Csc[z]>0}\end{aligned}$$

Out[522]=

$$\begin{aligned}&x0>0\ \&\&\ x4>0\ \&\&\ H>0\ \&\&\ 6\,H\,x0>0\ \&\&\ H\,x4>0\ \&\&\ a4[x4]>0\ \&\&\ A4[t]>0\ \&\&\\ &Q>0\ \&\&\ z>0\ \&\&\ t>0\ \&\&\ M>0\ \&\&\ K>0\ \&\&\ e^{-2\,a4[H\,x4]}>0\ \&\&\ e^{-a4[H\,x4]}>0\ \&\&\ e^{2\,a4[H\,x4]}>0\ \&\&\\ &e^{a4[H\,x4]}>0\ \&\&\ \mathrm{Sin}[6\,H\,x0]>0\ \&\&\ \mathrm{Cos}[6\,H\,x0]>0\ \&\&\ \mathrm{Csc}[6\,H\,x0]>0\ \&\&\ \mathrm{Sec}[6\,H\,x0]>0\ \&\&\\ &\mathrm{Tan}[6\,H\,x0]>0\ \&\&\ \mathrm{Cot}[6\,H\,x0]>0\ \&\&\ \mathrm{Sin}[z]>0\ \&\&\ \sqrt{\mathrm{Sin}[z]}>0\ \&\&\ \mathrm{Sin}[z]^{3/2}>0\ \&\&\\ &\sqrt{\mathrm{Sin}[z]}>0\ \&\&\ \frac{1}{\mathrm{Sin}[z]^{3/2}}>0\ \&\&\ \frac{1}{\sqrt{\mathrm{Sin}[z]}}>0\ \&\&\ \mathrm{Cot}[z]>0\ \&\&\ \sqrt{\mathrm{Cot}[z]}>0\ \&\&\\ &\mathrm{Cot}[z]^{3/2}>0\ \&\&\ \sqrt{\mathrm{Cot}[z]}>0\ \&\&\ \mathrm{Tan}[z]>0\ \&\&\ \mathrm{Sec}[z]>0\ \&\&\ \sqrt{\mathrm{Csc}[z]}>0\ \&\&\ \mathrm{Csc}[z]>0\end{aligned}$$

In[523]:=

```
ginv == FullSimplify[Inverse[gtry], ass]
```

Out[523]=

```
True
```

In[524]:=

```
ginv === Transpose[ginv]
```

Out[524]=

```
True
```

verify metric covariant derivative again:

In[525]:=

```
Simplify[
   Table[D[gtry⟦j, k⟧, X⟦i⟧ ] - Sum[ gtry⟦k, s⟧ × Γ⟦s, i, j⟧, {s, 1, DIM8}] -
      Sum[ gtry⟦j, s⟧ × Γ⟦s, i, k⟧, {s, 1, DIM8}],
     {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
```

Out[525]=

```
{0}
```

```
In[526]:=
RS
Out[526]=
6 H^2 (-7 + a4′[H x4]^2)
In[527]:=
EinsteinG
skelx[%]
Out[527]=
{{-3 H^2 Cot[6 H x0]^2 (-5 + a4′[H x4]^2), 0, 0, 0, 0, 0, 0, 0},
 {0, -ⅇ^(2 a4[H x4]) H^2 Sin[6 H x0]^(1/3) (-15 + 3 a4′[H x4]^2 - a4″[H x4]), 0, 0, 0, 0, 0, 0},
 {0, 0, -ⅇ^(2 a4[H x4]) H^2 Sin[6 H x0]^(1/3) (-15 + 3 a4′[H x4]^2 - a4″[H x4]), 0, 0, 0, 0, 0},
 {0, 0, 0, -ⅇ^(2 a4[H x4]) H^2 Sin[6 H x0]^(1/3) (-15 + 3 a4′[H x4]^2 - a4″[H x4]), 0, 0, 0, 0},
 {0, 0, 0, 0, -3 H^2 (7 + a4′[H x4]^2), 0, 0, 0},
 {0, 0, 0, 0, 0, ⅇ^(-2 a4[H x4]) H^2 Sin[6 H x0]^(1/3) (-15 + 3 a4′[H x4]^2 + a4″[H x4]), 0, 0},
 {0, 0, 0, 0, 0, 0, ⅇ^(-2 a4[H x4]) H^2 Sin[6 H x0]^(1/3) (-15 + 3 a4′[H x4]^2 + a4″[H x4]), 0},
 {0, 0, 0, 0, 0, 0, 0, ⅇ^(-2 a4[H x4]) H^2 Sin[6 H x0]^(1/3) (-15 + 3 a4′[H x4]^2 + a4″[H x4])}}
Out[528]//MatrixForm=
( 4 0 0 0 0 0 0 0 )
( 0 5 0 0 0 0 0 0 )
( 0 0 5 0 0 0 0 0 )
( 0 0 0 5 0 0 0 0 )
( 0 0 0 0 3 0 0 0 )
( 0 0 0 0 0 4 0 0 )
( 0 0 0 0 0 0 4 0 )
( 0 0 0 0 0 0 0 4 )
```

```
Union[Table[
  Sum[τ[A].bas64⟦k, 1⟧.σ.τ̄[A] × η8⟦A, A⟧, {A, 1, 8}] - ID8 Tr[bas64⟦k, 1⟧.σ] +
   4 eg⟦k⟧ (bas64⟦k, 1⟧ - Transpose[bas64⟦k, 1⟧])/2 .σ, {k, 1, 64}]]
```

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}
```

BASIS of 16 × 16 matrices :

```
In[529]:=
16 * 16
Out[529]=
256
In[530]:=
2^8
Out[530]=
256
```

In[531]:=

```
Sum[Binomial[8, p], {p, 0, 8}]
```

Out[531]=

```
256
```

In[532]:=

```
Id16 == ID16 == Dot @@ (T16^A[#] & /@ Range[0, 8])
```

Out[532]=

```
True
```

In[533]:=

```
Clear[t160]; t160 = {{Id16, {Range[0, 8]}}}; Length[t160]
Clear[t16A];
t16A = {};
Do[AppendTo[t16A, {T16^A[j], {j}}], {j, 0, 7}];
Length[t16A]
Clear[t16AB];
t16AB = {};
Do[AppendTo[t16AB, {T16^A[j].T16^A[k], {j, k}}], {j, 0, 6}, {k, j + 1, 7}];
Length[t16AB]
Clear[t16ABC];
t16ABC = {};
Do[AppendTo[t16ABC, {T16^A[j].T16^A[k].T16^A[h], {j, k, h}}],
 {j, 0, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[t16ABC]
Clear[t16ABCD];
t16ABCD = {};
Do[AppendTo[t16ABCD, {T16^A[j].T16^A[k].T16^A[h].T16^A[i], {j, k, h, i}}],
 {j, 0, 4}, {k, j + 1, 5}, {h, k + 1, 6}, {i, h + 1, 7}];
Length[t16ABCD]
Clear[t16ABCDE];
t16ABCDE = {};
Do[AppendTo[t16ABCDE, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l], {j, k, h, i,
    l}}], {j, 0, 3}, {k, j + 1, 4}, {h, k + 1, 5}, {i, h + 1, 6}, {l, i + 1, 7}];
Length[t16ABCDE]
Clear[t16ABCDEF];
t16ABCDEF = {};
Do[AppendTo[t16ABCDEF,
  {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q], {j, k, h, i, l, q}}],
 {j, 0, 2}, {k, j + 1, 3}, {h, k + 1, 4}, {i, h + 1, 5}, {l, i + 1, 6}, {q, l + 1, 7}];
Length[t16ABCDEF]
Clear[t16ABCDEFG];
```

```
t16ABCDEFG = {};
Do[AppendTo[t16ABCDEFG, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].
    T16^A[q].T16^A[r], {j, k, h, i, l, q, r}}], {j, 0, 1}, {k, j + 1, 2},
 {h, k + 1, 3}, {i, h + 1, 4}, {l, i + 1, 5}, {q, l + 1, 6}, {r, q + 1, 7}];
Length[t16ABCDEFG]
Clear[t16ABCDEFGH];
t16ABCDEFGH = {};
Do[AppendTo[t16ABCDEFGH, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].
    T16^A[r].T16^A[s], {j, k, h, i, l, q, r, s}}], {j, 0, 1}, {k, j + 1, 1}, {h,
  k + 1, 2}, {i, h + 1, 3}, {l, i + 1, 4}, {q, l + 1, 5}, {r, q + 1, 6}, {s, r + 1, 7}];
Length[t16ABCDEFGH]
% + %% + %%% + %%%% + %%%%% + %%%%%% + %%%%%%% + %%%%%%%% + %%%%%%%%%
```

```
Out[533]=
1
Out[534]=
8
Out[535]=
28
Out[536]=
56
Out[537]=
70
Out[538]=
56
Out[539]=
28
Out[540]=
8
Out[541]=
1
Out[542]=
256
```

```
In[543]:=
(*base16=Flatten[{t160,t16A,t16AB,t16ABC,
   t16ABCD,t16ABCDE,t16ABCDEF,t16ABCDEFG,t16ABCDEFGH},1];
Length[base16]*)
```

```
In[544]:=
base16 = Flatten[{t16A, t16AB, t16ABC, t16ABCD,
   t16ABCDE, t16ABCDEF, t16ABCDEFG, t16ABCDEFGH, t160}, 1];
Length[base16]
```

```
Out[544]=
256
```

```
In[545]:=
Length[Union[base16[[All, 1]]]]
Out[545]=
256
In[546]:=
Tr[#.# / 16] & /@ base16[[All, 1]] // Flatten // Union
Out[546]=
{-1, 1}
In[547]:=
positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]
Out[547]=
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44,
 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91,
 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121,
 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151,
 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185,
 186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229,
 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}
In[548]:=
negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]
Out[548]=
{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58,
 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97,
 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145,
 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182,
 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214,
 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}
In[549]:=
{Length[positiveTrMM], Length[negativeTrMM]}
Out[549]=
{136, 120}
In[550]:=
dups = Block[{s}, s = base16[[All, 1]]; Select[s, MemberQ[s, -#] &]]; Length[dups]
Out[550]=
0
In[551]:=
dups = Select[base16, MemberQ[base16[[All, 1]], -#[[1]]] &]; Length[dups]
Out[551]=
0
In[552]:=
BASE16 = use16 = Complement[base16, dups]; Length[use16]
Out[552]=
256
In[553]:=
(*(T16^A[8]=T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7])//
 MatrixForm*)
In[554]:=
MatrixForm[T16^A[#]] & /@ Range[0, 7]
Out[554]=
(0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0     (0  0  0 0  0  0  0 0 0 0  0  0  0 0  0  -1
 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0      0  0  0 0  0  0  0 0 0 0  0  0  0 0 -1  0
 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0      0  0  0 0  0  0  0 0 0 0  0  0  0 1  0  0
```

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}\}$$

In[555]:=

```
(*BASE16[[-1]]*)
```

In[556]:=

```
(*BASE16〚-2〛
 %〚1〛==BASE16〚-1〛〚1〛*)
```

In[557]:=

```
16 * (16 + 1) / 2
16 * (16 - 1) / 2
% + %%
```

Out[557]=

```
136
```

Out[558]=

```
120
```

Out[559]=

```
256
```

In[560]:=

```
Clear[antisymm16];
antisymm16 = {};
Do[If[base16〚k, 1〛 === - Transpose[base16〚k, 1〛], AppendTo[antisymm16,
    {base16〚k, 1〛, {{k}, base16〚k, 2〛}}] ], {k, 1, Length[base16]}];
Length[antisymm16]
antisymm16〚%〛
```

Out[561]=

```
120
```

Out[562]=

```
{{{0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
  {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}}, {{254}, {1, 2, 3, 4, 5, 6, 7}}}
```

In[563]:=

```
Clear[symm16];
symm16 = {};
Do[If[base16⟦k, 1⟧ === Transpose[base16⟦k, 1⟧], AppendTo[symm16,
   {base16⟦k, 1⟧, {{k}, base16⟦k, 2⟧}}] ], {k, 1, Length[base16]}];
Length[symm16]
symm16⟦%⟧
```

Out[564]=

```
136
```

Out[565]=

```
{{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

In[566]:=

```
dupssymm16 = Select[symm16, MemberQ[symm16⟦All, 1⟧, -#⟦1⟧] &];
Length[dupssymm16]
```

Out[566]=

```
0
```

In[567]:=

```
#⟦2⟧ & /@ symm16
```

Out[567]=

```
{{{1}, {0}}, {{2}, {1}}, {{3}, {2}}, {{4}, {3}}, {{12}, {0, 4}}, {{13}, {0, 5}},
 {{14}, {0, 6}}, {{15}, {0, 7}}, {{18}, {1, 4}}, {{19}, {1, 5}}, {{20}, {1, 6}},
 {{21}, {1, 7}}, {{23}, {2, 4}}, {{24}, {2, 5}}, {{25}, {2, 6}}, {{26}, {2, 7}},
 {{27}, {3, 4}}, {{28}, {3, 5}}, {{29}, {3, 6}}, {{30}, {3, 7}}, {{39}, {0, 1, 4}},
 {{40}, {0, 1, 5}}, {{41}, {0, 1, 6}}, {{42}, {0, 1, 7}}, {{44}, {0, 2, 4}}, {{45}, {0, 2, 5}},
 {{46}, {0, 2, 6}}, {{47}, {0, 2, 7}}, {{48}, {0, 3, 4}}, {{49}, {0, 3, 5}}, {{50}, {0, 3, 6}},
 {{51}, {0, 3, 7}}, {{59}, {1, 2, 4}}, {{60}, {1, 2, 5}}, {{61}, {1, 2, 6}}, {{62}, {1, 2, 7}},
 {{63}, {1, 3, 4}}, {{64}, {1, 3, 5}}, {{65}, {1, 3, 6}}, {{66}, {1, 3, 7}}, {{73}, {2, 3, 4}},
 {{74}, {2, 3, 5}}, {{75}, {2, 3, 6}}, {{76}, {2, 3, 7}}, {{89}, {4, 5, 6}}, {{90}, {4, 5, 7}},
 {{91}, {4, 6, 7}}, {{92}, {5, 6, 7}}, {{93}, {0, 1, 2, 3}}, {{102}, {0, 1, 4, 5}},
 {{103}, {0, 1, 4, 6}}, {{104}, {0, 1, 4, 7}}, {{105}, {0, 1, 5, 6}}, {{106}, {0, 1, 5, 7}},
 {{107}, {0, 1, 6, 7}}, {{112}, {0, 2, 4, 5}}, {{113}, {0, 2, 4, 6}}, {{114}, {0, 2, 4, 7}},
 {{115}, {0, 2, 5, 6}}, {{116}, {0, 2, 5, 7}}, {{117}, {0, 2, 6, 7}}, {{118}, {0, 3, 4, 5}},
 {{119}, {0, 3, 4, 6}}, {{120}, {0, 3, 4, 7}}, {{121}, {0, 3, 5, 6}}, {{122}, {0, 3, 5, 7}},
 {{123}, {0, 3, 6, 7}}, {{132}, {1, 2, 4, 5}}, {{133}, {1, 2, 4, 6}}, {{134}, {1, 2, 4, 7}},
 {{135}, {1, 2, 5, 6}}, {{136}, {1, 2, 5, 7}}, {{137}, {1, 2, 6, 7}}, {{138}, {1, 3, 4, 5}},
 {{139}, {1, 3, 4, 6}}, {{140}, {1, 3, 4, 7}}, {{141}, {1, 3, 5, 6}}, {{142}, {1, 3, 5, 7}},
 {{143}, {1, 3, 6, 7}}, {{148}, {2, 3, 4, 5}}, {{149}, {2, 3, 4, 6}}, {{150}, {2, 3, 4, 7}},
 {{151}, {2, 3, 5, 6}}, {{152}, {2, 3, 5, 7}}, {{153}, {2, 3, 6, 7}}, {{162}, {4, 5, 6, 7}},
 {{167}, {0, 1, 2, 4, 5}}, {{168}, {0, 1, 2, 4, 6}}, {{169}, {0, 1, 2, 4, 7}},
 {{170}, {0, 1, 2, 5, 6}}, {{171}, {0, 1, 2, 5, 7}}, {{172}, {0, 1, 2, 6, 7}},
 {{173}, {0, 1, 3, 4, 5}}, {{174}, {0, 1, 3, 4, 6}}, {{175}, {0, 1, 3, 4, 7}},
 {{176}, {0, 1, 3, 5, 6}}, {{177}, {0, 1, 3, 5, 7}}, {{178}, {0, 1, 3, 6, 7}},
 {{183}, {0, 2, 3, 4, 5}}, {{184}, {0, 2, 3, 4, 6}}, {{185}, {0, 2, 3, 4, 7}},
 {{186}, {0, 2, 3, 5, 6}}, {{187}, {0, 2, 3, 5, 7}}, {{188}, {0, 2, 3, 6, 7}},
 {{197}, {0, 4, 5, 6, 7}}, {{198}, {1, 2, 3, 4, 5}}, {{199}, {1, 2, 3, 4, 6}},
 {{200}, {1, 2, 3, 4, 7}}, {{201}, {1, 2, 3, 5, 6}}, {{202}, {1, 2, 3, 5, 7}},
 {{203}, {1, 2, 3, 6, 7}}, {{212}, {1, 4, 5, 6, 7}}, {{217}, {2, 4, 5, 6, 7}},
 {{218}, {3, 4, 5, 6, 7}}, {{225}, {0, 1, 2, 4, 5, 6}}, {{226}, {0, 1, 2, 4, 5, 7}},
 {{227}, {0, 1, 2, 4, 6, 7}}, {{228}, {0, 1, 2, 5, 6, 7}}, {{229}, {0, 1, 3, 4, 5, 6}},
 {{230}, {0, 1, 3, 4, 5, 7}}, {{231}, {0, 1, 3, 4, 6, 7}}, {{232}, {0, 1, 3, 5, 6, 7}},
 {{234}, {0, 2, 3, 4, 5, 6}}, {{235}, {0, 2, 3, 4, 5, 7}}, {{236}, {0, 2, 3, 4, 6, 7}},
 {{237}, {0, 2, 3, 5, 6, 7}}, {{240}, {1, 2, 3, 4, 5, 6}}, {{241}, {1, 2, 3, 4, 5, 7}},
 {{242}, {1, 2, 3, 4, 6, 7}}, {{243}, {1, 2, 3, 5, 6, 7}}, {{247}, {0, 1, 2, 3, 4, 5, 6}},
 {{248}, {0, 1, 2, 3, 4, 5, 7}}, {{249}, {0, 1, 2, 3, 4, 6, 7}}, {{250}, {0, 1, 2, 3, 5, 6, 7}},
 {{255}, {0, 1, 2, 3, 4, 5, 6, 7}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

In[568]:=

```
σ16 ⩵ T16^A[0].T16^A[1].T16^A[2].T16^A[3]
```

Out[568]=

```
True
```

In[569]:=

```
σ16 ⩵ base16⟦93⟧⟦1⟧
```

Out[569]=

```
True
```

```
In[570]:=
Select[symm16, #⟦2⟧⟦1⟧⟦1⟧ == 93 & → "Index"]
Out[570]=
{49}
In[571]:=
σ16 == symm16⟦49⟧⟦1⟧
Out[571]=
True
In[572]:=
symm16⟦134⟧
Out[572]=
{{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{250}, {0, 1, 2, 3, 5, 6, 7}}}
In[573]:=
Select[symm16, #⟦2⟧⟦1⟧⟦1⟧ == 250 & → "Index"]
Out[573]=
{134}
In[574]:=
T16⎵Superscript⎵A
Out[574]=
T16^A
In[575]:=
σ16.T16^A[5].T16^A[6].T16^A[7] == symm16⟦134⟧⟦1⟧
Out[575]=
True
In[576]:=
T16^A[8] == base16⟦255⟧⟦1⟧
Out[576]=
True
In[577]:=
Select[symm16, #⟦2⟧⟦1⟧⟦1⟧ == 255 & → "Index"]
Out[577]=
{135}
```

- **similarly :**

```
In[578]:=
coupled1 = {4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13} // Union
Out[578]=
{0, 1, 4, 5, 8, 9, 12, 13}
In[579]:=
coupled2 = Complement[Range[0, 15], coupled1]
Out[579]=
{2, 3, 6, 7, 10, 11, 14, 15}
In[580]:=
coupled = {coupled1, coupled2}
Out[580]=
{{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}}
In[581]:=
Block[{m, vars, eqs, eqs1, eqs2, s, Φ16},
 Φ16 = f[#] & /@ Range[16];
 m = Array[qb[#1, #2] &, {16, 16}];
 vars = Flatten[m];
 eqs1 = And @@
   Thread[m.Φ16 ⩵ Flatten[{Φ16⟦# + 1⟧ & /@ coupled1, Φ16⟦# + 1⟧ & /@ coupled2}]];
 eqs2 = And @@ Thread[m.Transpose[m] ⩵ Flatten[ID16]];
 eqs = eqs1 && eqs2;
 s = Solve[eqs, vars] // FullSimplify;
 s
]
Out[581]=
{}
```

In[582]:=

```
linTrans = Block[{m, b, vars, eqs, eqs1, eqs2, s, Φ16, t},
  Φ16 = f[#] & /@ Range[16];
  m = Array[qb[#1, #2] &, {16, 16}];
  b = Flatten[{Φ16⟦# + 1⟧ & /@ coupled1, Φ16⟦# + 1⟧ & /@ coupled2}];
  t = Table[Block[{sv, eq, d}, d = D[ m⟦j⟧.b, Φ16⟦j⟧];
     sv = Flatten[{d → 1, Thread[Complement[m⟦j⟧, {d}] → 0]}];
     (*Print[{m⟦j⟧.b},{Φ16⟦j⟧},{sv}];*)
     (*eq=Thread[m⟦j⟧.b==Φ16⟦j⟧];
     sv=Solve[eq,Flatten[m⟦j⟧]]//FullSimplify;
     sv*)(*{v.b,Φ16⟦j⟧}*)sv], {j, 1, 16}];
  (*
  vars=Flatten[m];
  eqs1=
   And@@Thread[m.Φ16==Flatten[{Φ16⟦#+1⟧&/@coupled1,Φ16⟦#+1⟧&/@coupled2}]];
  eqs2=And@@Thread[m.Transpose[m]==Flatten[ID16]];
  eqs=eqs1&&eqs2;
  s=Solve[eqs,vars]//FullSimplify;*)
  m /. Flatten[t]
 ]
```

Out[582]=

```
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

In[583]:=

```
linTrans // MatrixForm
```

Out[583]//MatrixForm=

```
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
```

In[584]:=

```
linTrans.Transpose[linTrans] // FullSimplify // MatrixForm
```

Out[584]//MatrixForm=

```
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
```

```
In[585]:=
linTrans.Ψ16 // Column
Union[%⟦1⟧]
Out[585]=
f16[0][x0, x4]
f16[1][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[2][x0, x4]
f16[3][x0, x4]
f16[10][x0, x4]
f16[11][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
f16[6][x0, x4]
f16[7][x0, x4]
f16[14][x0, x4]
f16[15][x0, x4]
Out[586]=
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
In[∘]:= positiveTrMM = Select[base16, Tr[#[[1]] . #[[1]]] > 0 & -> "Index"]
Out[∘]=
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50,
51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114,
115, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149,
150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188,
197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240,
241, 242, 243, 247, 248, 249, 250, 255, 256}
In[∘]:= negativeTrMM = Select[base16, Tr[#[[1]] . #[[1]]] < 0 & -> "Index"]
Out[∘]=
{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72,
77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126,
127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180,
181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219,
220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}
In[∘]:= {Length[positiveTrMM], Length[negativeTrMM]}
Out[∘]=
{136, 120}
```

```
In[587]:=
        positiveTrMM = Select[base16, Tr[#⟦1⟧.#⟦1⟧] > 0 & → "Index"]
Out[587]=
        {1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44,
         45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91,
         92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121,
         122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151,
         152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185,
         186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229,
         230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}
In[588]:=
        negativeTrMM = Select[base16, Tr[#⟦1⟧.#⟦1⟧] < 0 & → "Index"]
Out[588]=
        {5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58,
         67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97,
         98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145,
         146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182,
         189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214,
         215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}
In[589]:=
        {Length[positiveTrMM], Length[negativeTrMM]}
Out[589]=
        {136, 120}
```

```
In[590]:=
base16⟦#⟧ & /@ positiveTrMM;
{#⟦2⟧, Tr[Transpose[#⟦1⟧].linTrans]} & /@ %
Select[%, #⟦2⟧ ≠ 0 &]
linTransSymmetric = Table[Select[base16, #⟦2⟧ == %⟦j⟧⟦1⟧ &], {j, 1, Length[%]}];
```

```
Out[591]=
{{{0}, 0}, {{1}, 0}, {{2}, 0}, {{3}, 0}, {{0, 4}, 0}, {{0, 5}, 0}, {{0, 6}, 0}, {{0, 7}, -4},
 {{1, 4}, 0}, {{1, 5}, 0}, {{1, 6}, 0}, {{1, 7}, 0}, {{2, 4}, 0}, {{2, 5}, 0}, {{2, 6}, 0},
 {{2, 7}, 0}, {{3, 4}, -4}, {{3, 5}, 0}, {{3, 6}, 0}, {{3, 7}, 0}, {{0, 1, 4}, 0},
 {{0, 1, 5}, 0}, {{0, 1, 6}, 0}, {{0, 1, 7}, 0}, {{0, 2, 4}, 0}, {{0, 2, 5}, 0}, {{0, 2, 6}, 0},
 {{0, 2, 7}, 0}, {{0, 3, 4}, 0}, {{0, 3, 5}, 0}, {{0, 3, 6}, 0}, {{0, 3, 7}, 0},
 {{1, 2, 4}, 0}, {{1, 2, 5}, 0}, {{1, 2, 6}, 0}, {{1, 2, 7}, 0}, {{1, 3, 4}, 0},
 {{1, 3, 5}, 0}, {{1, 3, 6}, 0}, {{1, 3, 7}, 0}, {{2, 3, 4}, 0}, {{2, 3, 5}, 0},
 {{2, 3, 6}, 0}, {{2, 3, 7}, 0}, {{4, 5, 6}, -4}, {{4, 5, 7}, 0}, {{4, 6, 7}, 0},
 {{5, 6, 7}, 0}, {{0, 1, 2, 3}, 0}, {{0, 1, 4, 5}, 0}, {{0, 1, 4, 6}, 0}, {{0, 1, 4, 7}, 0},
 {{0, 1, 5, 6}, 0}, {{0, 1, 5, 7}, 0}, {{0, 1, 6, 7}, 0}, {{0, 2, 4, 5}, 0}, {{0, 2, 4, 6}, 0},
 {{0, 2, 4, 7}, 0}, {{0, 2, 5, 6}, 0}, {{0, 2, 5, 7}, 0}, {{0, 2, 6, 7}, 0}, {{0, 3, 4, 5}, 0},
 {{0, 3, 4, 6}, 0}, {{0, 3, 4, 7}, 4}, {{0, 3, 5, 6}, 0}, {{0, 3, 5, 7}, 0}, {{0, 3, 6, 7}, 0},
 {{1, 2, 4, 5}, 0}, {{1, 2, 4, 6}, 0}, {{1, 2, 4, 7}, 0}, {{1, 2, 5, 6}, 0}, {{1, 2, 5, 7}, 0},
 {{1, 2, 6, 7}, -4}, {{1, 3, 4, 5}, 0}, {{1, 3, 4, 6}, 0}, {{1, 3, 4, 7}, 0}, {{1, 3, 5, 6}, 0},
 {{1, 3, 5, 7}, 0}, {{1, 3, 6, 7}, 0}, {{2, 3, 4, 5}, 0}, {{2, 3, 4, 6}, 0}, {{2, 3, 4, 7}, 0},
 {{2, 3, 5, 6}, 0}, {{2, 3, 5, 7}, 0}, {{2, 3, 6, 7}, 0}, {{4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5}, 4},
 {{0, 1, 2, 4, 6}, 0}, {{0, 1, 2, 4, 7}, 0}, {{0, 1, 2, 5, 6}, 0}, {{0, 1, 2, 5, 7}, 0},
 {{0, 1, 2, 6, 7}, 0}, {{0, 1, 3, 4, 5}, 0}, {{0, 1, 3, 4, 6}, 0}, {{0, 1, 3, 4, 7}, 0},
 {{0, 1, 3, 5, 6}, 0}, {{0, 1, 3, 5, 7}, 0}, {{0, 1, 3, 6, 7}, 0}, {{0, 2, 3, 4, 5}, 0},
 {{0, 2, 3, 4, 6}, 0}, {{0, 2, 3, 4, 7}, 0}, {{0, 2, 3, 5, 6}, 0}, {{0, 2, 3, 5, 7}, 0},
 {{0, 2, 3, 6, 7}, 0}, {{0, 4, 5, 6, 7}, 4}, {{1, 2, 3, 4, 5}, 0}, {{1, 2, 3, 4, 6}, 0},
 {{1, 2, 3, 4, 7}, 0}, {{1, 2, 3, 5, 6}, 0}, {{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 6, 7}, 0},
 {{1, 4, 5, 6, 7}, 0}, {{2, 4, 5, 6, 7}, 0}, {{3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6}, 0},
 {{0, 1, 2, 4, 5, 7}, 0}, {{0, 1, 2, 4, 6, 7}, 0}, {{0, 1, 2, 5, 6, 7}, 0},
 {{0, 1, 3, 4, 5, 6}, 0}, {{0, 1, 3, 4, 5, 7}, 0}, {{0, 1, 3, 4, 6, 7}, 0},
 {{0, 1, 3, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6}, 0}, {{0, 2, 3, 4, 5, 7}, 0},
 {{0, 2, 3, 4, 6, 7}, 0}, {{0, 2, 3, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6}, 0},
 {{1, 2, 3, 4, 5, 7}, 0}, {{1, 2, 3, 4, 6, 7}, -4}, {{1, 2, 3, 5, 6, 7}, 0},
 {{0, 1, 2, 3, 4, 5, 6}, 0}, {{0, 1, 2, 3, 4, 5, 7}, 0}, {{0, 1, 2, 3, 4, 6, 7}, 0},
 {{0, 1, 2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6, 7}, 0}, {{{0, 1, 2, 3, 4, 5, 6, 7, 8}}, 4}}
```

```
Out[592]=
{{{0, 7}, -4}, {{3, 4}, -4}, {{4, 5, 6}, -4}, {{0, 3, 4, 7}, 4},
 {{1, 2, 6, 7}, -4}, {{0, 1, 2, 4, 5}, 4}, {{0, 4, 5, 6, 7}, 4},
 {{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 4, 6, 7}, -4}, {{{0, 1, 2, 3, 4, 5, 6, 7, 8}}, 4}}
```

In[594]:=

```
linTransSymmetric⟦1⟧
```

Out[594]=

```
{{{{-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
    {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0,
     0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0,
     0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
     1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0,
     0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
    {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}, {0, 7}}}
```

In[595]:=

```
base16⟦#⟧ & /@ negativeTrMM;
{#⟦2⟧, Tr[Transpose[#⟦1⟧].linTrans]} & /@ %
Select[%, #⟦2⟧ ≠ 0 &]
linTransAntiSymmetric =
  Table[Select[base16, #⟦2⟧ ⩵ %⟦j⟧⟦1⟧ &], {j, 1, Length[%]}];
```

Out[596]=

```
{{{4}, 0}, {{5}, 0}, {{6}, 0}, {{7}, 0}, {{0, 1}, 0}, {{0, 2}, 0}, {{0, 3}, 0}, {{1, 2}, 0},
 {{1, 3}, 0}, {{2, 3}, 0}, {{4, 5}, 0}, {{4, 6}, 0}, {{4, 7}, 0}, {{5, 6}, 0}, {{5, 7}, 0},
 {{6, 7}, 0}, {{0, 1, 2}, 0}, {{0, 1, 3}, 0}, {{0, 2, 3}, 0}, {{0, 4, 5}, 0}, {{0, 4, 6}, 0},
 {{0, 4, 7}, 0}, {{0, 5, 6}, 0}, {{0, 5, 7}, 0}, {{0, 6, 7}, 0}, {{1, 2, 3}, 0}, {{1, 4, 5}, 0},
 {{1, 4, 6}, 0}, {{1, 4, 7}, 0}, {{1, 5, 6}, 0}, {{1, 5, 7}, 0}, {{1, 6, 7}, 0},
 {{2, 4, 5}, 0}, {{2, 4, 6}, 0}, {{2, 4, 7}, 0}, {{2, 5, 6}, 0}, {{2, 5, 7}, 0},
 {{2, 6, 7}, 0}, {{3, 4, 5}, 0}, {{3, 4, 6}, 0}, {{3, 4, 7}, 0}, {{3, 5, 6}, -4},
 {{3, 5, 7}, 0}, {{3, 6, 7}, 0}, {{0, 1, 2, 4}, 0}, {{0, 1, 2, 5}, 0}, {{0, 1, 2, 6}, -4},
 {{0, 1, 2, 7}, 0}, {{0, 1, 3, 4}, 0}, {{0, 1, 3, 5}, 0}, {{0, 1, 3, 6}, 0}, {{0, 1, 3, 7}, 0},
 {{0, 2, 3, 4}, 0}, {{0, 2, 3, 5}, 0}, {{0, 2, 3, 6}, 0}, {{0, 2, 3, 7}, 0}, {{0, 4, 5, 6}, 0},
 {{0, 4, 5, 7}, 0}, {{0, 4, 6, 7}, 0}, {{0, 5, 6, 7}, 0}, {{1, 2, 3, 4}, 0}, {{1, 2, 3, 5}, 0},
 {{1, 2, 3, 6}, 0}, {{1, 2, 3, 7}, 0}, {{1, 4, 5, 6}, 0}, {{1, 4, 5, 7}, 0}, {{1, 4, 6, 7}, 0},
 {{1, 5, 6, 7}, 0}, {{2, 4, 5, 6}, 0}, {{2, 4, 5, 7}, 0}, {{2, 4, 6, 7}, 0}, {{2, 5, 6, 7}, 0},
 {{3, 4, 5, 6}, 0}, {{3, 4, 5, 7}, 0}, {{3, 4, 6, 7}, 0}, {{3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4}, 0},
 {{0, 1, 2, 3, 5}, -4}, {{0, 1, 2, 3, 6}, 0}, {{0, 1, 2, 3, 7}, 0}, {{0, 1, 4, 5, 6}, 0},
 {{0, 1, 4, 5, 7}, 0}, {{0, 1, 4, 6, 7}, 0}, {{0, 1, 5, 6, 7}, 0}, {{0, 2, 4, 5, 6}, 0},
 {{0, 2, 4, 5, 7}, 0}, {{0, 2, 4, 6, 7}, 0}, {{0, 2, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6}, 0},
 {{0, 3, 4, 5, 7}, 0}, {{0, 3, 4, 6, 7}, 0}, {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 6}, 0},
 {{1, 2, 4, 5, 7}, 4}, {{1, 2, 4, 6, 7}, 0}, {{1, 2, 5, 6, 7}, 0}, {{1, 3, 4, 5, 6}, 0},
 {{1, 3, 4, 5, 7}, 0}, {{1, 3, 4, 6, 7}, 0}, {{1, 3, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6}, 0},
 {{2, 3, 4, 5, 7}, 0}, {{2, 3, 4, 6, 7}, 0}, {{2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5}, 0},
 {{0, 1, 2, 3, 4, 6}, -4}, {{0, 1, 2, 3, 4, 7}, 0}, {{0, 1, 2, 3, 5, 6}, 0},
 {{0, 1, 2, 3, 5, 7}, 0}, {{0, 1, 2, 3, 6, 7}, 0}, {{0, 1, 4, 5, 6, 7}, 0},
 {{0, 2, 4, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6, 7}, 0}, {{1, 2, 4, 5, 6, 7}, 0},
 {{1, 3, 4, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6, 7}, 0},
 {{0, 1, 3, 4, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6, 7}, 0}}
```

Out[597]=

```
{{{3, 5, 6}, -4}, {{0, 1, 2, 6}, -4}, {{0, 1, 2, 3, 5}, -4},
 {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 7}, 4}, {{0, 1, 2, 3, 4, 6}, -4}}
```

In[599]:=

```
{Length[linTransSymmetric], Length[linTransAntiSymmetric]}
```

Out[599]=

```
{10, 6}
```

BASIS of 8 × 8 matrices :

In[600]:=

```
tA = {}; Do[AppendTo[tA, {τ[j], {j}}], {j, 1, 7}]
Length[tA]
tAB = {};
Do[AppendTo[tAB, {τ[j].τ[k], {j, k}}], {j, 1, 6}, {k, j + 1, 7}]
Length[tAB]
tABC = {};
Do[AppendTo[tABC, {τ[j].τ[k].τ[h], {j, k, h}}],
  {j, 1, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[tABC]
```

Out[601]=

```
7
```

Out[603]=

```
21
```

Out[605]=

```
35
```

In[606]:=

```
tA⟦1⟧
```

Out[606]=

```
{{{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}}, {1}}
```

In[607]:=

```
tA⟦#⟧⟦1⟧ == -Transpose[tA⟦#⟧⟦1⟧] & /@ Range[Length[tA]]
```

Out[607]=

```
{True, True, True, False, False, False, False}
```

In[608]:=

```
bas64 = Flatten[{tA, tAB, tABC, {{Id, {64}}}}, 1]; Length[bas64]
```

Out[608]=

```
64
```

In[609]:=

```
determineIF8AntiSymmetric[m_] := Module[{t, tt, sum, res, r},
  t = m.τ[#] & /@ Range[0, 7];
  tt = Transpose[#] & /@ t;
  sum = FullSimplify[t⟦#⟧ + tt⟦#⟧] & /@ Range[Length[t]];
  res = Union[Flatten[#]] & /@ sum;
  If[Length[#] > 1, Return[False]] & /@ res;
  r = Union[Flatten[res]];
  Return[r == {0}];
 ]
```

In[610]:=

```
determineIF8AntiSymmetric[Zero8]
```

Out[610]=

```
True
```

In[611]:=

```
determineIF8AntiSymmetric[ID8]
```

Out[611]=

```
False
```

In[ ]:=

```
(*find8AntiSymmetric=
 If[determineIF8AntiSymmetric[#⟦1⟧],#⟦2⟧,##&]&/@bas64*)
```

Out[612]=

```
{##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &,
 ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &,
 ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &,
 ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &,
 ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &, ##1 &}
```

In[613]:=

```
anti = {};
Do[If[bas64⟦k, 1⟧ == -Transpose[bas64⟦k, 1⟧],
   AppendTo[anti, {bas64⟦k, 1⟧, {{k}, bas64⟦k, 2⟧}}] ], {k, 1, 64}];
Length[anti]
anti⟦28⟧
```

Out[614]=

```
28
```

Out[615]=

```
{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}
```

In[616]:=

```
symm = {};
Do[If[bas64⟦k, 1⟧ == Transpose[bas64⟦k, 1⟧],
   AppendTo[symm, {bas64⟦k, 1⟧, {{k}, bas64⟦k, 2⟧}}] ], {k, 1, 64}];
Length[symm]
symm⟦%⟧
```

Out[617]=

```
35
```

Out[618]=

```
{{{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0},
  {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}}, {{59}, {3, 6, 7}}}
```

In[619]:=

```
η64 = DiagonalMatrix[
  Table[FullSimplify[1/8 Tr[bas64⟦A, 1⟧.bas64⟦A, 1⟧]], {A, 64}]];
Tr[η64]
```

Out[619]=

$$7+\frac{1}{8}\,\mathrm{Tr}[\mathrm{Id}.\mathrm{Id}]$$

In[620]:=

```
(*(*(*(*(*(*(countTrace=
  Table[{A,B,FullSimplify[1/8 Tr[bas64⟦A,1⟧.bas64⟦B,1⟧]] },
   {A,1, 64},{B,A, 64}])//MatrixForm*)*)*)*)*)*)
```

In[621]:=

```
(*■*)
```

In[622]:=

```
32 × 65
```

Out[622]=

```
2080
```

In[623]:=

```
(*Length[Flatten[countTrace,1]]*)
```

In[624]:=

```
(*{Length[countTrace],Length[countTrace⟦1⟧],Length[countTrace⟦1,1⟧]}*)
```

In[625]:=

```
(*{Length[countTrace],Length[countTrace⟦2⟧],Length[countTrace⟦2,1⟧]}*)
```

In[626]:=

```
(*{Length[countTrace],Length[countTrace⟦64⟧],Length[countTrace⟦64,1⟧]}*)
```

In[627]:=

```
(*{Length[Flatten[countTrace,1]⟦All,3⟧],
 Count[Flatten[countTrace,1]⟦All,3⟧,0],
 Count[Flatten[countTrace,1]⟦All,3⟧,1],
 Count[Flatten[countTrace,1]⟦All,3⟧,-1]}*)
```

In[628]:=

```
32 × 63
```

Out[628]=

```
2016
```

In[629]:=

```
anti = {};
Do[If[bas64⟦k, 1⟧ == -Transpose[bas64⟦k, 1⟧],
  AppendTo[anti, {bas64⟦k, 1⟧, {{k}, bas64⟦k, 2⟧}}] ], {k, 1, 64}];
Length[anti]
anti⟦28⟧
```

Out[630]=

```
28
```

Out[631]=

```
{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}
```

In[632]:=

```
anti⟦28, 1⟧
```

Out[632]=

```
{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
 {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}
```

In[633]:=

```
anti⟦28, 2⟧
```

Out[633]=

```
{{63}, {5, 6, 7}}
```

In[634]:=

```
symm = {};
Do[If[bas64⟦k, 1⟧ == Transpose[bas64⟦k, 1⟧],
  AppendTo[symm, {bas64⟦k, 1⟧, {{k}, bas64⟦k, 2⟧}}] ], {k, 1, 64}];
Length[symm]
symm⟦28⟧
```

Out[635]=

```
35
```

Out[636]=

```
{{{-1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0},
  {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
  {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, {{52}, {2, 5, 7}}}
```

For the split orthogonal group Spin(4,4) and its associated Lie algebra, there exist three fundamentally equivalent 8-dimensional representations:

1. The vector representation V
2. The type-1 spinor representation S+ (or S1), positive chirality
3. The type-2 spinor representation S− (or S2), negative chirality

These representations are related by outer automorphisms of Spin(4,4). All three representations have the same dimension: dim(V) = dim(S1) = dim(S2) = 8
This is unique to dimension 8 and forms the basis of triality: Cartan, E. (1925). La th´eorie des spineurs. Hermann, Paris.

metric-preserving transformations; find automorphism $\sigma g \longleftrightarrow \sigma g^{a}{}_{A}$

$\sigma.\sigma\mathrm{g} == \mathrm{Transpose}[\sigma\mathrm{g}^{-1}].\mathrm{g}$

$\sigma == \mathrm{Transpose}[\sigma\mathrm{g}^{-1}].\mathrm{g}.\sigma\mathrm{g}^{-1}$

$(\sigma\mathrm{g})^{a}{}_{A} \equiv \sigma\mathrm{g}^{a}{}_{A}$

$\left(\sigma\mathrm{g}^{-1}\right)^{A}{}_{a} \equiv \sigma\mathrm{g}^{A}{}_{a} \quad ; \quad \left(\mathrm{Transpose}\left[\sigma\mathrm{g}^{-1}\right]\right)^{A}{}_{a} \equiv \sigma\mathrm{g}\,{}_{a}{}^{A}$

$\sigma_{\mathrm{ab}} == \ \sigma\mathrm{g}\,{}_{a}{}^{A}\ g_{\mathrm{AB}}\ \sigma\mathrm{g}^{B}{}_{b}$

$$M = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$N = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[637]:=

$$\mathbf{Clear}\left[\sigma \mathrm{g}^{\mathrm{a}}_{1\,\mathrm{A}}\right];\ \sigma \mathrm{g}^{\mathrm{a}}_{1\,\mathrm{A}} = \frac{1}{\sqrt{2}}\ (\eta 4488 + \sigma)$$

Out[637]=

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0\right\},\right.$$
$$\left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\},$$
$$\left.\left\{0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\}\right\}$$

In[638]:=

$$\sigma g^{a}_{2\,A} = \frac{1}{\sqrt{2}}\, \{\{1,0,0,0,1,0,0,0\},\{0,0,1,0,0,0,1,0\},\{0,1,0,0,0,1,0,0\},\{0,0,0,1,0,0,0,1\},\{1,0,0,0,-1,0,0,0\},\{0,0,1,0,0,0,-1,0\},\{0,1,0,0,0,-1,0,0\},\{0,0,0,1,0,0,0,-1\}\}$$

Out[638]=

$$\left\{\left\{\frac{1}{\sqrt{2}},0,0,0,\frac{1}{\sqrt{2}},0,0,0\right\},\left\{0,0,\frac{1}{\sqrt{2}},0,0,0,\frac{1}{\sqrt{2}},0\right\},\left\{0,\frac{1}{\sqrt{2}},0,0,0,\frac{1}{\sqrt{2}},0,0\right\},\left\{0,0,0,\frac{1}{\sqrt{2}},0,0,0,\frac{1}{\sqrt{2}}\right\},\left\{\frac{1}{\sqrt{2}},0,0,0,-\frac{1}{\sqrt{2}},0,0,0\right\},\left\{0,0,\frac{1}{\sqrt{2}},0,0,0,-\frac{1}{\sqrt{2}},0\right\},\left\{0,\frac{1}{\sqrt{2}},0,0,0,-\frac{1}{\sqrt{2}},0,0\right\},\left\{0,0,0,\frac{1}{\sqrt{2}},0,0,0,-\frac{1}{\sqrt{2}}\right\}\right\}$$

In[639]:=

$$\left(\mathtt{s1s2} = \sigma g^{a}_{1\,A}.\mathtt{Transpose}\left[\sigma g^{a}_{2\,A}\right]\right) \mathbin{//} \mathtt{MatrixForm}$$

Out[639]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[640]:=

$$\mathtt{(*(s1s2=Inverse[}\sigma g^{a}_{1A}\mathtt{.Transpose[}\sigma g^{a}_{2A}\mathtt{]])//MatrixForm*)}$$

In[641]:=

```
s1s2.s1s2 === ID8
```

Out[641]=

```
True
```

In[642]:=

```
(Thread[Ψ16⟦1 ;; 8⟧ → s1s2.Ψ16⟦9 ;; 16⟧]) // Column
%⟦1⟧⟦All, 1, 0⟧
%%⟦1⟧⟦All, 2⟧ /. {x0 → #1, x4 → #2}
ToExpression["((" <> ToString[#] <> ")&)"] & /@ %
forceDimensionReductionA = Thread[%%% → %]
```

Out[642]=

```
f16[0][x0, x4] → f16[8][x0, x4]
f16[1][x0, x4] → f16[10][x0, x4]
f16[2][x0, x4] → f16[9][x0, x4]
f16[3][x0, x4] → f16[11][x0, x4]
f16[4][x0, x4] → f16[12][x0, x4]
f16[5][x0, x4] → f16[14][x0, x4]
f16[6][x0, x4] → f16[13][x0, x4]
f16[7][x0, x4] → f16[15][x0, x4]
```

Out[643]=

```
{f16[0], f16[1], f16[2], f16[3], f16[4], f16[5], f16[6], f16[7]}
```

Out[644]=

```
{f16[8][#1, #2], f16[10][#1, #2], f16[9][#1, #2], f16[11][#1, #2],
 f16[12][#1, #2], f16[14][#1, #2], f16[13][#1, #2], f16[15][#1, #2]}
```

Out[645]=

```
{f16[8][#1, #2] &, f16[10][#1, #2] &, f16[9][#1, #2] &, f16[11][#1, #2] &,
 f16[12][#1, #2] &, f16[14][#1, #2] &, f16[13][#1, #2] &, f16[15][#1, #2] &}
```

Out[646]=

```
{f16[0] → (f16[8][#1, #2] &), f16[1] → (f16[10][#1, #2] &),
 f16[2] → (f16[9][#1, #2] &), f16[3] → (f16[11][#1, #2] &), f16[4] → (f16[12][#1, #2] &),
 f16[5] → (f16[14][#1, #2] &), f16[6] → (f16[13][#1, #2] &), f16[7] → (f16[15][#1, #2] &)}
```

In[647]:=

```
(Thread[Ψ16⟦9 ;; 16⟧ → s1s2.Ψ16⟦1 ;; 8⟧]) // Column
%⟦1⟧⟦All, 1, 0⟧
%%⟦1⟧⟦All, 2⟧ /. {x0 → #1, x4 → #2}
ToExpression["((" <> ToString[#] <> ")&)"] & /@ %
forceDimensionReduction = Thread[%%% → %]
```

Out[647]=

```
f16[8][x0, x4] → f16[0][x0, x4]
f16[9][x0, x4] → f16[2][x0, x4]
f16[10][x0, x4] → f16[1][x0, x4]
f16[11][x0, x4] → f16[3][x0, x4]
f16[12][x0, x4] → f16[4][x0, x4]
f16[13][x0, x4] → f16[6][x0, x4]
f16[14][x0, x4] → f16[5][x0, x4]
f16[15][x0, x4] → f16[7][x0, x4]
```

Out[648]=

```
{f16[8], f16[9], f16[10], f16[11], f16[12], f16[13], f16[14], f16[15]}
```

Out[649]=

```
{f16[0][#1, #2], f16[2][#1, #2], f16[1][#1, #2], f16[3][#1, #2],
 f16[4][#1, #2], f16[6][#1, #2], f16[5][#1, #2], f16[7][#1, #2]}
```

Out[650]=

```
{f16[0][#1, #2] &, f16[2][#1, #2] &, f16[1][#1, #2] &, f16[3][#1, #2] &,
 f16[4][#1, #2] &, f16[6][#1, #2] &, f16[5][#1, #2] &, f16[7][#1, #2] &}
```

Out[651]=

```
{f16[8] → (f16[0][#1, #2] &), f16[9] → (f16[2][#1, #2] &),
 f16[10] → (f16[1][#1, #2] &), f16[11] → (f16[3][#1, #2] &), f16[12] → (f16[4][#1, #2] &),
 f16[13] → (f16[6][#1, #2] &), f16[14] → (f16[5][#1, #2] &), f16[15] → (f16[7][#1, #2] &)}
```

In[652]:=

$(*\sigma g^{a}_{2A}===-\#+\sigma g^{a}_{1A}\&/@bas64⟦All,1⟧*)$

In[653]:=

$\sigma g^{a}_{2\,A}.\mathrm{Transpose}\left[\sigma g^{a}_{2\,A}\right]$ === ID8

Out[653]=

```
True
```

In[654]:=

$\sigma g^{a}_{1\,A}.\mathrm{Transpose}\left[\sigma g^{a}_{1\,A}\right]$ === ID8

Out[654]=

```
True
```

metric - preserving transformation:

In[655]:=

$\sigma g^{a}_{1\,A}.\sigma.\mathrm{Transpose}\left[\sigma g^{a}_{1\,A}\right]$ === η4488

Out[655]=

```
True
```

metric - preserving transformation:

In[656]:=

$$\sigma g^{a}_{2\,A}.\sigma.\mathrm{Transpose}\left[\sigma g^{a}_{2\,A}\right] === \eta 4488$$

Out[656]=

```
True
```

In[657]:=

$$\mathrm{Mc} = \frac{1}{\sqrt{2}}\ (\eta 4488 + \sigma)$$

Out[657]=

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0\right\},\right.$$
$$\left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\},$$
$$\left.\left\{0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\}\right\}$$

In[658]:=

$$\mathrm{Nc} = \frac{1}{\sqrt{2}}\ \{\{1, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 0\},$$
$$\{0, 1, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 1\},$$
$$\{1, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, -1, 0\},$$
$$\{0, 1, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, -1\}\}$$

Out[658]=

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\},\right.$$
$$\left\{0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\},$$
$$\left.\left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\}\right\}$$

In[659]:=

```
(*(s1s2=Mc.Transpose[Nc])//MatrixForm*)
```

In[660]:=

```
Nc.Transpose[Nc] // MatrixForm
```

Out[660]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[661]:=
Mc.Transpose[Mc] // MatrixForm
Out[661]//MatrixForm=
(1 0 0 0 0 0 0 0)
(0 1 0 0 0 0 0 0)
(0 0 1 0 0 0 0 0)
(0 0 0 1 0 0 0 0)
(0 0 0 0 1 0 0 0)
(0 0 0 0 0 1 0 0)
(0 0 0 0 0 0 1 0)
(0 0 0 0 0 0 0 1)
In[662]:=
Mc.σ.Transpose[Mc] === η4488
Out[662]=
True
In[663]:=
Nc.σ.Transpose[Nc] === η4488
Out[663]=
True
In[664]:=
(Thread[Ψ16⟦9 ;; 16⟧ → s1s2.Ψ16⟦1 ;; 8⟧]) // Column
Out[664]=
f16[8][x0, x4] → f16[0][x0, x4]
f16[9][x0, x4] → f16[2][x0, x4]
f16[10][x0, x4] → f16[1][x0, x4]
f16[11][x0, x4] → f16[3][x0, x4]
f16[12][x0, x4] → f16[4][x0, x4]
f16[13][x0, x4] → f16[6][x0, x4]
f16[14][x0, x4] → f16[5][x0, x4]
f16[15][x0, x4] → f16[7][x0, x4]
In[665]:=
f8[#][x0, x4] & /@ Range[0, 7] // Column
(s1s2.%⟦1⟧) // Column
Out[665]=
f8[0][x0, x4]
f8[1][x0, x4]
f8[2][x0, x4]
f8[3][x0, x4]
f8[4][x0, x4]
f8[5][x0, x4]
f8[6][x0, x4]
f8[7][x0, x4]
Out[666]=
f8[0][x0, x4]
f8[2][x0, x4]
f8[1][x0, x4]
f8[3][x0, x4]
f8[4][x0, x4]
f8[6][x0, x4]
f8[5][x0, x4]
f8[7][x0, x4]
```

Killing Vector Fields

In[667]:=

$[\_DG([["vector", M8, [\,]], [[[6],$

$-\frac{1}{\sin(6\,Hx0)^{1/3}}(x6\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}(\cos(Hx0)+1)^{1/6}(2\cos(Hx0)-1)^{1/3}(2\cos(Hx0)$

$+1)^{1/3}(-1+2\cos(2\,Hx0))^{1/3})], [[7],$

$\frac{1}{\sin(6\,Hx0)^{1/3}}(x5\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}(\cos(Hx0)+1)^{1/6}(2\cos(Hx0)-1)^{1/3}(2\cos(Hx0)$

$+1)^{1/3}(-1+2\cos(2\,Hx0))^{1/3})]]]), \_DG([["vector",$

$M8, [\,]], [[[3],$

$\frac{1}{\sin(6\,Hx0)^{1/3}}(x3\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}(\cos(Hx0)+1)^{1/6}(2\cos(Hx0)-1)^{1/3}(2\cos(Hx0)$

$+1)^{1/3}(-1+2\cos(2\,Hx0))^{1/3})], [[4],$

$-\frac{1}{\sin(6\,Hx0)^{1/3}}(x2\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}(\cos(Hx0)+1)^{1/6}(2\cos(Hx0)-1)^{1/3}(2\cos(Hx0)$

$+1)^{1/3}(-1+2\cos(2\,Hx0))^{1/3})]]]), \_DG([["vector",$

$M8, [\,]], [[[3], \frac{1}{\sin(6\,Hx0)^{1/3}}((2\cos(Hx0)-1)^{1}$

$^{/3}(\cos(Hx0)+1)^{1/6}(-1+2\cos(2\,Hx0))^{1}$

$^{/3}\,(\cos(Hx0)-1)^{1/6}\,(2\cos(Hx0)+1)^{1/3}\cos(Hx0)^{1/3})]]$

$]), \_DG([["vector", M8, [\,]], [[[4],$

$\frac{1}{\sin(6\,Hx0)^{1/3}}((2\cos(Hx0)-1)^{1/3}(\cos(Hx0)$

$+1)^{1/6}\,(-1+2\cos(2\,Hx0))^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}(2\cos(Hx0)+1)^{1/3}\cos(Hx0)^{1/3})]]]),$

$\_DG([["vector", M8, [\,]], [[[6],$

$-\frac{1}{\sin(6\,Hx0)^{1/3}}(x7\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}(\cos(Hx0)+1)^{1/6}(2\cos(Hx0)-1)^{1/3}(2\cos(Hx0)$

$+1)^{1/3}(-1+2\cos(2\,Hx0))^{1/3})], [[8],$

$\frac{1}{\sin(6\,Hx0)^{1/3}}(x5\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}(\cos(Hx0)+1)^{1/6}(2\cos(Hx0)-1)^{1/3}(2\cos(Hx0)$

$+1)^{1/3}(-1+2\cos(2\,Hx0))^{1/3})]]]), \_DG([["vector",$

$M8, [\,]], [[[6], -\frac{1}{\sin(6\,Hx0)^{1/3}}((2\cos(Hx0)$

$-1)^{1/3}(\cos(Hx0)+1)^{1/6}(-1+2\cos(2\,Hx0))^{1}$

$^{/3}(\cos(Hx0)-1)^{1/6}(2\cos(Hx0)+1)^{1/3}\cos(Hx0)^{1/3})]]]$

$), \_DG\Big(\Big[["vector", M8, [\,]], \Big[\Big[[7],$

$-\frac{1}{\sin(6\,H x0)^{1/3}}\big(x7\cos(H x0)^{1/3}\,(\cos(H x0)-1)^{1}$

${}^{/6}\,(\cos(H x0)+1)^{1/6}\,(2\cos(H x0)-1)^{1/3}\,(2\cos(H x0)$

$+1)^{1/3}\,(-1+2\cos(2\,H x0))^{1/3}\big)\Big], \Big[[8],$

$\frac{1}{\sin(6\,H x0)^{1/3}}\big(x6\cos(H x0)^{1/3}\,(\cos(H x0)-1)^{1}$

${}^{/6}\,(\cos(H x0)+1)^{1/6}\,(2\cos(H x0)-1)^{1/3}\,(2\cos(H x0)$

$+1)^{1/3}\,(-1+2\cos(2\,H x0))^{1/3}\big)\Big]\Big]\Big]\Big), \_DG\Big(\Big[["vector",$

$M8, [\,]], \Big[\Big[[7], -\frac{1}{\sin(6\,H x0)^{1/3}}\big((2\cos(H x0)$

$-1)^{1/3}\,(\cos(H x0)+1)^{1/6}\,(-1+2\cos(2\,H x0))^{1}$

${}^{/3}\,(\cos(H x0)-1)^{1/6}\,(2\cos(H x0)+1)^{1/3}\cos(H x0)^{1/3}\big)\Big]\Big]\Big]$

$), \_DG\Big(\Big[["vector", M8, [\,]], \Big[\Big[[8],$

$-\frac{1}{\sin(6\,H x0)^{1/3}}\big((2\cos(H x0)-1)^{1/3}\,(\cos(H x0)$

$+1)^{1/6}\,(-1+2\cos(2\,H x0))^{1/3}\,(\cos(H x0)-1)^{1}$

${}^{/6}\,(2\cos(H x0)+1)^{1/3}\cos(H x0)^{1/3}\big)\Big]\Big]\Big]\Big),$

$\_DG\Big(\Big[["vector", M8, [\,]], \Big[\Big[[2],$

$\frac{1}{\sin(6\,H x0)^{1/3}}\big(x2\cos(H x0)^{1/3}\,(\cos(H x0)-1)^{1}$

${}^{/6}\,(\cos(H x0)+1)^{1/6}\,(2\cos(H x0)-1)^{1/3}\,(2\cos(H x0)$

$+1)^{1/3}\,(-1+2\cos(2\,H x0))^{1/3}\big)\Big], \Big[[3],$

$-\frac{1}{\sin(6\,H x0)^{1/3}}\big(x1\cos(H x0)^{1/3}\,(\cos(H x0)-1)^{1}$

${}^{/6}\,(\cos(H x0)+1)^{1/6}\,(2\cos(H x0)-1)^{1/3}\,(2\cos(H x0)$

$+1)^{1/3}\,(-1+2\cos(2\,H x0))^{1/3}\big)\Big]\Big]\Big]\Big), \_DG\Big(\Big[["vector",$

$M8, [\,]], \Big[\Big[[2],$

$\frac{1}{\sin(6\,H x0)^{1/3}}\big(x3\cos(H x0)^{1/3}\,(\cos(H x0)-1)^{1}$

${}^{/6}\,(\cos(H x0)+1)^{1/6}\,(2\cos(H x0)-1)^{1/3}\,(2\cos(H x0)$

$+1)^{1/3}\,(-1+2\cos(2\,H x0))^{1/3}\big)\Big], \Big[[4],$

$-\frac{1}{\sin(6\,H x0)^{1/3}}\big(x1\cos(H x0)^{1/3}\,(\cos(H x0)-1)^{1}$

${}^{/6}\,(\cos(H x0)+1)^{1/6}\,(2\cos(H x0)-1)^{1/3}\,(2\cos(H x0)$

$+1)^{1/3}\,(-1+2\cos(2\,H x0))^{1/3}\big)\Big]\Big]\Big]\Big), \_DG\Big(\Big[["vector",$

$M8, [\,]], \Big[\Big[[2], \frac{1}{\sin(6\,H x0)^{1/3}}\big((2\cos(H x0)-1)^{1}$

${}^{/3}\,(\cos(H x0)+1)^{1/6}\,(-1+2\cos(2\,H x0))^{1}$

${}^{/3}\,(\cos(H x0)-1)^{1/6}\,(2\cos(H x0)+1)^{1/3}\cos(H x0)^{1/3}\big)\Big]\Big]$

$\Big]\Big)\Big]$

Out[667]=

$\Big[\_DG\Big(\Big[["vector", M8, [\,]], \Big[\Big[[6],$

$-\frac{1}{\sin(6\,Hx0)^{1/3}}\big(x6\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$
$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$
$+1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)\big],\Big[[7],$
$\frac{1}{\sin(6\,Hx0)^{1/3}}\big(x5\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$
$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$
$+1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)]]]\big),\ \_DG\Big(\Big[[\text{"vector"},$
$M8,[\ ]],\Big[\Big[[3],$
$\frac{1}{\sin(6\,Hx0)^{1/3}}\big(x3\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$
$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$
$+1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)\big],\Big[[4],$
$-\frac{1}{\sin(6\,Hx0)^{1/3}}\big(x2\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$
$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$
$+1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)]]]\big),\ \_DG\Big(\Big[[\text{"vector"},$
$M8,[\ ]],\Big[\Big[[3],\ \frac{1}{\sin(6\,Hx0)^{1/3}}\big((2\cos(Hx0)-1)^{1}$
$^{/3}\,(\cos(Hx0)+1)^{1/6}\,(-1+2\cos(2\,Hx0))^{1}$
$^{/3}\,(\cos(Hx0)-1)^{1/6}\,(2\cos(Hx0)+1)^{1/3}\cos(Hx0)^{1/3}\big)]]$
$]\big),\ \_DG\Big(\Big[[\text{"vector"},M8,[\ ]],\Big[\Big[[4],$
$\frac{1}{\sin(6\,Hx0)^{1/3}}\big((2\cos(Hx0)-1)^{1/3}\,(\cos(Hx0)$
$+1)^{1/6}\,(-1+2\cos(2\,Hx0))^{1/3}\,(\cos(Hx0)-1)^{1}$
$^{/6}\,(2\cos(Hx0)+1)^{1/3}\cos(Hx0)^{1/3}\big)]]]\big),$
$\_DG\Big(\Big[[\text{"vector"},M8,[\ ]],\Big[\Big[[6],$
$-\frac{1}{\sin(6\,Hx0)^{1/3}}\big(x7\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$
$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$
$+1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)\big],\Big[[8],$
$\frac{1}{\sin(6\,Hx0)^{1/3}}\big(x5\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$
$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$
$+1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)]]]\big),\ \_DG\Big(\Big[[\text{"vector"},$
$M8,[\ ]],\Big[\Big[[6],-\frac{1}{\sin(6\,Hx0)^{1/3}}\big((2\cos(Hx0)$
$-1)^{1/3}\,(\cos(Hx0)+1)^{1/6}\,(-1+2\cos(2\,Hx0))^{1}$
$^{/3}\,(\cos(Hx0)-1)^{1/6}\,(2\cos(Hx0)+1)^{1/3}\cos(Hx0)^{1/3}\big)]]]$
$\big),\ \_DG\Big(\Big[[\text{"vector"},M8,[\ ]],\Big[\Big[[7],$
$-\frac{1}{\sin(6\,Hx0)^{1/3}}\big(x7\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})\,],\,\Big[[8],$

$\frac{1}{\sin(6Hx0)^{1/3}}(x6\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})\,]]]),\,\_DG\Big(\Big[["vector",$

$M8,[\,]],\Big[\Big[[7],-\frac{1}{\sin(6Hx0)^{1/3}}((2\cos(Hx0)$

$-1)^{1/3}\,(\cos(Hx0)+1)^{1/6}\,(-1+2\cos(2Hx0))^{1}$

$^{/3}\,(\cos(Hx0)-1)^{1/6}\,(2\cos(Hx0)+1)^{1/3}\cos(Hx0)^{1/3})\,]]]$

$),\,\_DG\Big(\Big[["vector",M8,[\,]],\Big[\Big[[8],$

$-\frac{1}{\sin(6Hx0)^{1/3}}((2\cos(Hx0)-1)^{1/3}\,(\cos(Hx0)$

$+1)^{1/6}\,(-1+2\cos(2Hx0))^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(2\cos(Hx0)+1)^{1/3}\cos(Hx0)^{1/3})\,]]]),$

$\_DG\Big(\Big[["vector",M8,[\,]],\Big[\Big[[2],$

$\frac{1}{\sin(6Hx0)^{1/3}}(x2\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})\,],\,\Big[[3],$

$-\frac{1}{\sin(6Hx0)^{1/3}}(x1\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})\,]]]),\,\_DG\Big(\Big[["vector",$

$M8,[\,]],\Big[\Big[[2],$

$\frac{1}{\sin(6Hx0)^{1/3}}(x3\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})\,],\,\Big[[4],$

$-\frac{1}{\sin(6Hx0)^{1/3}}(x1\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})\,]]]),\,\_DG\Big(\Big[["vector",$

$M8,[\,]],\Big[\Big[[2],\frac{1}{\sin(6Hx0)^{1/3}}((2\cos(Hx0)-1)^{1}$

$^{/3}\,(\cos(Hx0)+1)^{1/6}\,(-1+2\cos(2Hx0))^{1}$

$^{/3}\,(\cos(Hx0)-1)^{1/6}\,(2\cos(Hx0)+1)^{1/3}\cos(Hx0)^{1/3})\,]]$

$])\,]$

In[668]:=

```
(*ToExpression[MathMLForm[{"http://www.w3.org/TR/MathML","math"}()]]*)
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In[669]:=

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(*ToExpression[MathMLForm[□]]*)
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In[670]:=

```
"[_DG([[\"vector\", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
```

```
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\",
M8, []], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
_DG([[\"vector\", M8, []], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
_DG([[\"vector\", M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\",
M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
_DG([[\"vector\", M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
```

```
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)
+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]])]"
```

Out[670]=

```
[_DG([["vector", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
  + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
  2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[3],
  x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
  [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) +
  1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
  1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[4],
  (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0)
  - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[6],
  -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0)
  - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[7],
  -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0)
  - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) +
  1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
  1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[2],
  x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
  [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
  + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
  2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[2],
  (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0)
  - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]])]
```

In[671]:=

```
killingVectors =
 "[_DG([[\"vector\", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
   1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
```

```
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\",
M8, []], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
_DG([[\"vector\", M8, []], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
_DG([[\"vector\", M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\",
M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
_DG([[\"vector\", M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
```

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[[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([[\"vector\", M8,
[]], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]])]"
```

Out[671]=

```
[_DG([["vector", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
  + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
  2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[3],
  x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
  [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
  1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[4],
  (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0)
  + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[6],
  -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0)
  + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[7],
  -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0)
  + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
  1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[2],
  x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
  [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]),
  _DG([["vector", M8, []], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
  + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
  2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1
  + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]), _DG([["vector", M8, []], [[[2],
  (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0)
  + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]])]
```

In[672]:=

$$\Big[\_DG\Big(\Big[\big["vector", M8, [\ ]\big], \Big[\Big[[6],$$

$$-\frac{1}{\quad}\left(x6\cos(Hx0)^{1/3}\ (\cos(Hx0)-1)^{1}\right.$$

$\sin(6\,Hx0)^{1/3}$

${}^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+\,1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3})\,],\Big[[7],$

$\frac{1}{\sin(6\,Hx0)^{1/3}}(x5\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

${}^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+\,1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3})\,]]]),\,\_DG\Big(\Big[["vector",$

$M8,[\,]],\Big[\Big[[3],$

$\frac{1}{\sin(6\,Hx0)^{1/3}}(x3\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

${}^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+\,1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3})\,],\Big[[4],$

$-\frac{1}{\sin(6\,Hx0)^{1/3}}(x2\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

${}^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+\,1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3})\,]]]),\,\_DG\Big(\Big[["vector",$

$M8,[\,]],\Big[\Big[[3],\,\frac{1}{\sin(6\,Hx0)^{1/3}}((2\cos(Hx0)-1)^{1}$

${}^{/3}\,(\cos(Hx0)-1)^{1/6}\,(-1+2\cos(2\,Hx0))^{1}$

${}^{/3}\,(2\cos(Hx0)+1)^{1/3}\,(\cos(Hx0)+1)^{1/6}\cos(Hx0)^{1/3})\,]]$

$]),\,\_DG\Big(\Big[["vector",M8,[\,]],\Big[\Big[[4],$

$\frac{1}{\sin(6\,Hx0)^{1/3}}((2\cos(Hx0)-1)^{1/3}\,(\cos(Hx0)$

$-\,1)^{1/6}\,(-1+2\cos(2\,Hx0))^{1/3}\,(2\cos(Hx0)+1)^{1}$

${}^{/3}\,(\cos(Hx0)+1)^{1/6}\cos(Hx0)^{1/3})\,]]]),\,\_DG\Big(\Big[["vector",$

$(\cos(Hx0)+1) \quad \cos(Hx0) \quad)]]]), \_DG\Big(\Big[[\text{"vector"},$

$M8, [\;]], \Big[\Big[[6],$

$-\frac{1}{\sin(6Hx0)^{1/3}}(x7\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})], \Big[[8],$

$\frac{1}{\sin(6Hx0)^{1/3}}(x5\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})]]]), \_DG\Big(\Big[[\text{"vector"},$

$M8, [\;]], \Big[\Big[[6], -\frac{1}{\sin(6Hx0)^{1/3}}((2\cos(Hx0)$

$-1)^{1/3}\,(\cos(Hx0)-1)^{1/6}\,(-1+2\cos(2Hx0))^{1}$

$^{/3}\,(2\cos(Hx0)+1)^{1/3}\,(\cos(Hx0)+1)^{1/6}\cos(Hx0)^{1/3})]]]$

$), \_DG\Big(\Big[[\text{"vector"}, M8, [\;]], \Big[\Big[[7],$

$-\frac{1}{\sin(6Hx0)^{1/3}}(x7\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})], \Big[[8],$

$\frac{1}{\sin(6Hx0)^{1/3}}(x6\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+1)^{1/3}\,(-1+2\cos(2Hx0))^{1/3})]]]), \_DG\Big(\Big[[\text{"vector"},$

$M8, [\;]], \Big[\Big[[7], -\frac{1}{\sin(6Hx0)^{1/3}}((2\cos(Hx0)$

$-1)^{1/3}\,(\cos(Hx0)-1)^{1/6}\,(-1+2\cos(2Hx0))^{1}$

$-\ 1)\ (\cos(Hx0) - 1)\ (-1 + 2\cos(2\,Hx0))$

${}^{/3}\,(2\cos(Hx0) + 1)^{1/3}\,(\cos(Hx0) + 1)^{1/6}\cos(Hx0)^{1/3})\,]]]$

$),\ \_DG\Big(\Big[\,[\text{"vector"}, M8, [\ ]], \Big[\Big[\,[8],$

$-\dfrac{1}{\sin(6\,Hx0)^{1/3}}\,(\,(2\cos(Hx0) - 1)^{1/3}\,(\cos(Hx0)$

$-\ 1)^{1/6}\,(-1 + 2\cos(2\,Hx0))^{1/3}\,(2\cos(Hx0) + 1)^{1}$

${}^{/3}\,(\cos(Hx0) + 1)^{1/6}\cos(Hx0)^{1/3})\,]]]),\ \_DG\Big(\Big[\,[\text{"vector"},$

$M8, [\ ]], \Big[\Big[\,[2],$

$\dfrac{1}{\sin(6\,Hx0)^{1/3}}\,(x2\cos(Hx0)^{1/3}\,(\cos(Hx0) - 1)^{1}$

${}^{/6}\,(\cos(Hx0) + 1)^{1/6}\,(2\cos(Hx0) - 1)^{1/3}\,(2\cos(Hx0)$

$+\ 1)^{1/3}\,(-1 + 2\cos(2\,Hx0))^{1/3})\,], \Big[\,[3],$

$-\dfrac{1}{\sin(6\,Hx0)^{1/3}}\,(x1\cos(Hx0)^{1/3}\,(\cos(Hx0) - 1)^{1}$

${}^{/6}\,(\cos(Hx0) + 1)^{1/6}\,(2\cos(Hx0) - 1)^{1/3}\,(2\cos(Hx0)$

$+\ 1)^{1/3}\,(-1 + 2\cos(2\,Hx0))^{1/3})\,]]]),\ \_DG\Big(\Big[\,[\text{"vector"},$

$M8, [\ ]], \Big[\Big[\,[2],$

$\dfrac{1}{\sin(6\,Hx0)^{1/3}}\,(x3\cos(Hx0)^{1/3}\,(\cos(Hx0) - 1)^{1}$

${}^{/6}\,(\cos(Hx0) + 1)^{1/6}\,(2\cos(Hx0) - 1)^{1/3}\,(2\cos(Hx0)$

$+\ 1)^{1/3}\,(-1 + 2\cos(2\,Hx0))^{1/3})\,], \Big[\,[4],$

$-\dfrac{1}{\sin(6\,Hx0)^{1/3}}\,(x1\cos(Hx0)^{1/3}\,(\cos(Hx0) - 1)^{1}$

${}^{/6}\,(\cos(Hx0) + 1)^{1/6}\,(2\cos(Hx0) - 1)^{1/3}\,(2\cos(Hx0)$

$+\,1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3})\,]]]),\ \_DG\Big(\Big[\big["\text{vector}",$

$M8,[\,]\big],\Big[\Big[[2],\ \dfrac{1}{\sin(6\,Hx0)^{1/3}}\big(\,(2\cos(Hx0)-1)^{1}$

$^{/3}\,(\cos(Hx0)-1)^{1/6}\,(-1+2\cos(2\,Hx0))^{1}$

$^{/3}\,(2\cos(Hx0)+1)^{1/3}\,(\cos(Hx0)+1)^{1/6}\cos(Hx0)^{1/3}\big)\,\Big]\Big]$

$\Big]\Big)\,]$

Out[672]=

$\Big[\_DG\Big(\Big[\big["\text{vector}",M8,[\,]\big],\Big[\Big[[6],$

$-\dfrac{1}{\sin(6\,Hx0)^{1/3}}\big(x6\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+\,1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)\,\Big],\Big[[7],$

$\dfrac{1}{\sin(6\,Hx0)^{1/3}}\big(x5\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+\,1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)\,\Big]\Big]\Big]\Big),\ \_DG\Big(\Big[\big["\text{vector}",$

$M8,[\,]\big],\Big[\Big[[3],$

$\dfrac{1}{\sin(6\,Hx0)^{1/3}}\big(x3\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+\,1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)\,\Big],\Big[[4],$

$-\dfrac{1}{\sin(6\,Hx0)^{1/3}}\big(x2\cos(Hx0)^{1/3}\,(\cos(Hx0)-1)^{1}$

$^{/6}\,(\cos(Hx0)+1)^{1/6}\,(2\cos(Hx0)-1)^{1/3}\,(2\cos(Hx0)$

$+\,1)^{1/3}\,(-1+2\cos(2\,Hx0))^{1/3}\big)\,\Big]\Big]\Big]\Big),\ \_DG\Big(\Big[\big["\text{vector}",$

$M8,[\,]\big],\Big[\Big[[3],\ \dfrac{1}{\sin(6\,Hx0)^{1/3}}\big(\,(2\cos(Hx0)-1)^{1}$

$^{/3}\,(\cos(Hx0)-1)^{1/6}\,(-1+2\cos(2\,Hx0))^{1}$

$$(\cos(H x0) - 1) \quad (-1 + 2\cos(2 H x0))$$

$$^{/3}\ (2\cos(H x0) + 1)^{1/3}\ (\cos(H x0) + 1)^{1/6} \cos(H x0)^{1/3})\,]]$$

$$]),\ \_DG\Big(\Big[\,[\text{"vector"}, M8, [\ ]], \Big[\Big[\,[4],$$

$$\frac{1}{\sin(6 H x0)^{1/3}}((2\cos(H x0) - 1)^{1/3}\ (\cos(H x0)$$

$$-\ 1)^{1/6}\ (-1 + 2\cos(2 H x0))^{1/3}\ (2\cos(H x0) + 1)^{1}$$

$$^{/3}\ (\cos(H x0) + 1)^{1/6} \cos(H x0)^{1/3})\,]]]),\ \_DG\Big(\Big[\,[\text{"vector"},$$

$$M8, [\ ]], \Big[\Big[\,[6],$$

$$-\frac{1}{\sin(6 H x0)^{1/3}}(x7 \cos(H x0)^{1/3}\ (\cos(H x0) - 1)^{1}$$

$$^{/6}\ (\cos(H x0) + 1)^{1/6}\ (2\cos(H x0) - 1)^{1/3}\ (2\cos(H x0)$$

$$+\ 1)^{1/3}\ (-1 + 2\cos(2 H x0))^{1/3})\,],\ \Big[\,[8],$$

$$\frac{1}{\sin(6 H x0)^{1/3}}(x5 \cos(H x0)^{1/3}\ (\cos(H x0) - 1)^{1}$$

$$^{/6}\ (\cos(H x0) + 1)^{1/6}\ (2\cos(H x0) - 1)^{1/3}\ (2\cos(H x0)$$

$$+\ 1)^{1/3}\ (-1 + 2\cos(2 H x0))^{1/3})\,]]]),\ \_DG\Big(\Big[\,[\text{"vector"},$$

$$M8, [\ ]], \Big[\Big[\,[6], -\frac{1}{\sin(6 H x0)^{1/3}}((2\cos(H x0)$$

$$-\ 1)^{1/3}\ (\cos(H x0) - 1)^{1/6}\ (-1 + 2\cos(2 H x0))^{1}$$

$$^{/3}\ (2\cos(H x0) + 1)^{1/3}\ (\cos(H x0) + 1)^{1/6} \cos(H x0)^{1/3})\,]]]$$

$$),\ \_DG\Big(\Big[\,[\text{"vector"}, M8, [\ ]], \Big[\Big[\,[7],$$

$$-\frac{1}{\sin(6 H x0)^{1/3}}(x7 \cos(H x0)^{1/3}\ (\cos(H x0) - 1)^{1}$$

$$^{/6}\ (\cos(H x0) + 1)^{1/6}\ (2\cos(H x0) - 1)^{1/3}\ (2\cos(H x0)$$

$+\ 1)^{1/3}\ (-1 + 2\cos(2\,Hx0))^{1/3})\ ],\ \Big[\ [8],$

$\frac{1}{\sin(6\,Hx0)^{1/3}}(x6\cos(Hx0)^{1/3}\ (\cos(Hx0) - 1)^{1}$

${}^{/6}\ (\cos(Hx0) + 1)^{1/6}\ (2\cos(Hx0) - 1)^{1/3}\ (2\cos(Hx0)$

$+\ 1)^{1/3}\ (-1 + 2\cos(2\,Hx0))^{1/3})\ ]]]),\ \_DG\Big(\Big[\ [\text{"vector"},$

$M8,\ [\ ]],\ \Big[\Big[\ [7],\ -\frac{1}{\sin(6\,Hx0)^{1/3}}((2\cos(Hx0)$

$-\ 1)^{1/3}\ (\cos(Hx0) - 1)^{1/6}\ (-1 + 2\cos(2\,Hx0))^{1}$

${}^{/3}\ (2\cos(Hx0) + 1)^{1/3}\ (\cos(Hx0) + 1)^{1/6}\cos(Hx0)^{1/3})\ ]]]$

$),\ \_DG\Big(\Big[\ [\text{"vector"},\ M8,\ [\ ]],\ \Big[\Big[\ [8],$

$-\frac{1}{\sin(6\,Hx0)^{1/3}}((2\cos(Hx0) - 1)^{1/3}\ (\cos(Hx0)$

$-\ 1)^{1/6}\ (-1 + 2\cos(2\,Hx0))^{1/3}\ (2\cos(Hx0) + 1)^{1}$

${}^{/3}\ (\cos(Hx0) + 1)^{1/6}\cos(Hx0)^{1/3})\ ]]]),\ \_DG\Big(\Big[\ [\text{"vector"},$

$M8,\ [\ ]],\ \Big[\Big[\ [2],$

$\frac{1}{\sin(6\,Hx0)^{1/3}}(x2\cos(Hx0)^{1/3}\ (\cos(Hx0) - 1)^{1}$

${}^{/6}\ (\cos(Hx0) + 1)^{1/6}\ (2\cos(Hx0) - 1)^{1/3}\ (2\cos(Hx0)$

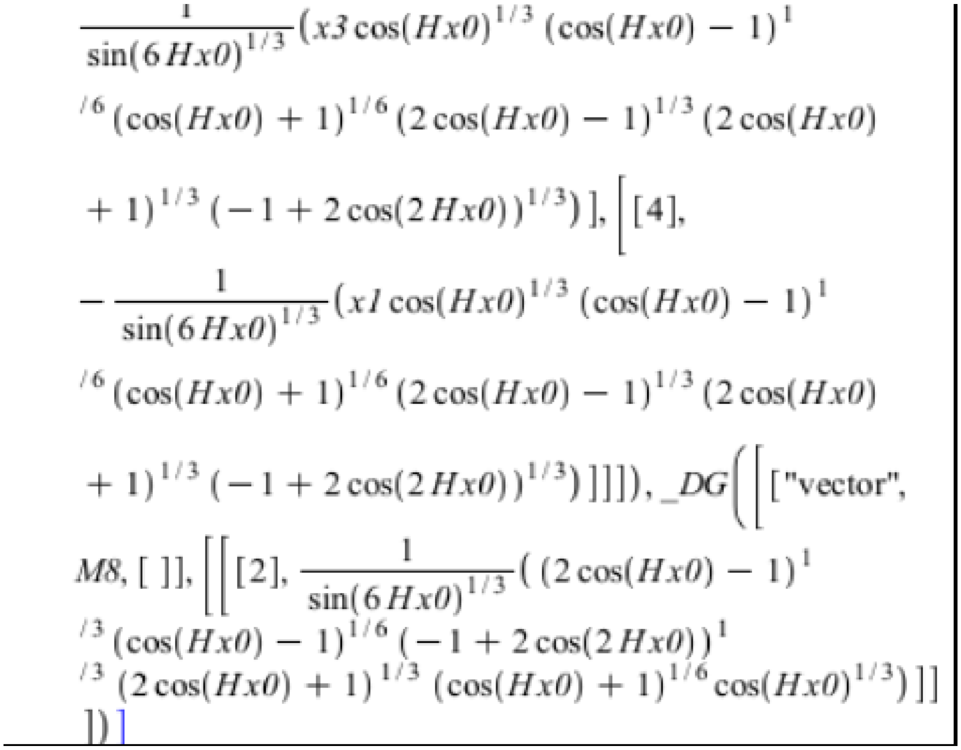
$+\ 1)^{1/3}\ (-1 + 2\cos(2\,Hx0))^{1/3})\ ],\ \Big[\ [3],$

$-\frac{1}{\sin(6\,Hx0)^{1/3}}(x1\cos(Hx0)^{1/3}\ (\cos(Hx0) - 1)^{1}$

${}^{/6}\ (\cos(Hx0) + 1)^{1/6}\ (2\cos(Hx0) - 1)^{1/3}\ (2\cos(Hx0)$

$+\ 1)^{1/3}\ (-1 + 2\cos(2\,Hx0))^{1/3})\ ]]]),\ \_DG\Big(\Big[\ [\text{"vector"},$

$M8,\ [\ ]],\ \Big[\Big[\ [2],$



$\frac{1}{\sin(6\,Hx0)^{1/3}}(x3\cos(Hx0)^{1/3}\ (\cos(Hx0)-1)^{1}$

$^{/6}\ (\cos(Hx0)+1)^{1/6}\ (2\cos(Hx0)-1)^{1/3}\ (2\cos(Hx0)$

$+\ 1)^{1/3}\ (-1+2\cos(2\,Hx0))^{1/3})\ ],\ \big[\,[4],$

$-\frac{1}{\sin(6\,Hx0)^{1/3}}(x1\cos(Hx0)^{1/3}\ (\cos(Hx0)-1)^{1}$

$^{/6}\ (\cos(Hx0)+1)^{1/6}\ (2\cos(Hx0)-1)^{1/3}\ (2\cos(Hx0)$

$+\ 1)^{1/3}\ (-1+2\cos(2\,Hx0))^{1/3})\ ]]]),\ \_DG\Big(\big[\,["vector",$

$M8,\ [\ ]],\ \big[\big[\,[2],\ \frac{1}{\sin(6\,Hx0)^{1/3}}(\ (2\cos(Hx0)-1)^{1}$

$^{/3}\ (\cos(Hx0)-1)^{1/6}\ (-1+2\cos(2\,Hx0))^{1}$

$^{/3}\ (2\cos(Hx0)+1)^{1/3}\ (\cos(Hx0)+1)^{1/6}\cos(Hx0)^{1/3})\ ]]$

$])\,]$

In[673]:=

```
ConvertMapleToMathematicaV2[
 " -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
   1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
   2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3), , x5*cos(H*x0)^(1/3)*(cos(H*x0)
   - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
   + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)"]
```

Out[673]=

$$-\frac{1}{\mathrm{Sin}[6\,H\,x0]^{1/3}}x6\,(-1+\mathrm{Cos}[H\,x0])^{1/6}\,\mathrm{Cos}[H\,x0]^{1/3}\,(1+\mathrm{Cos}[H\,x0])^{1/6}\,(-1+2\,\mathrm{Cos}[H\,x0])^{1/3}\,(1+2\,\mathrm{Cos}[H\,x0])^{1/3}\,(-1+2\,\mathrm{Cos}[2\,H\,x0])^{1/3}$$

In[674]:=

```
ConvertMapleToMathematica[
 "x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
   1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
   2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3),  -x2*cos(H*x0)^(1/3)*(cos(H*x0)
   - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
   + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)"]
```

Out[674]=

```
ConvertMapleToMathematica[
 x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
   1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3),
   -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
   1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]
```

In[675]:=

```
ConvertMapleToMathematica[
 "(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
   2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
   + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)"]
```

Out[675]=

```
ConvertMapleToMathematica[
 (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0)
   + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]
```

In[676]:=

M8 > $KV := KillingVectors(g);$

$$KV := \Big[-\frac{x6 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x5}{\sin(6 H x0)^{1/3}}$$

$$+ \frac{x5 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x6}{\sin(6 H x0)^{1/3}},$$

$$\frac{x3 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x2}{\sin(6 H x0)^{1/3}}$$

$$- \frac{x2 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x3}{\sin(6 H x0)^{1/3}},$$

$$\frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x2}{\sin(6 H x0)^{1/3}},$$

$$\frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x3}{\sin(6 H x0)^{1/3}},$$

$$- \frac{x7 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x5}{\sin(6 H x0)^{1/3}}$$

$$+ \frac{x5 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x7}{\sin(6 H x0)^{1/3}},$$

$$- \frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x5}{\sin(6 H x0)^{1/3}},$$

$$- \frac{x7 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x6}{\sin(6 H x0)^{1/3}}$$

$$+ \frac{x6 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x7}{\sin(6 H x0)^{1/3}},$$

$$- \frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x6}{\sin(6 H x0)^{1/3}},$$

$$- \frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x7}{\sin(6 H x0)^{1/3}},$$

$$\frac{x2 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x1}{\sin(6 H x0)^{1/3}}$$

Out[676]=

M8 > $KV := KillingVectors(g);$

$$KV := \left[-\frac{x6 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x5}{\sin(6 H x0)^{1/3}} \right.$$

$$+ \frac{x5 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x6}{\sin(6 H x0)^{1/3}},$$

$$\frac{x3 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x2}{\sin(6 H x0)^{1/3}}$$

$$- \frac{x2 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x3}{\sin(6 H x0)^{1/3}},$$

$$\frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x2}{\sin(6 H x0)^{1/3}},$$

$$\frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x3}{\sin(6 H x0)^{1/3}},$$

$$- \frac{x7 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x5}{\sin(6 H x0)^{1/3}}$$

$$+ \frac{x5 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x7}{\sin(6 H x0)^{1/3}},$$

$$- \frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x5}{\sin(6 H x0)^{1/3}},$$

$$- \frac{x7 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x6}{\sin(6 H x0)^{1/3}}$$

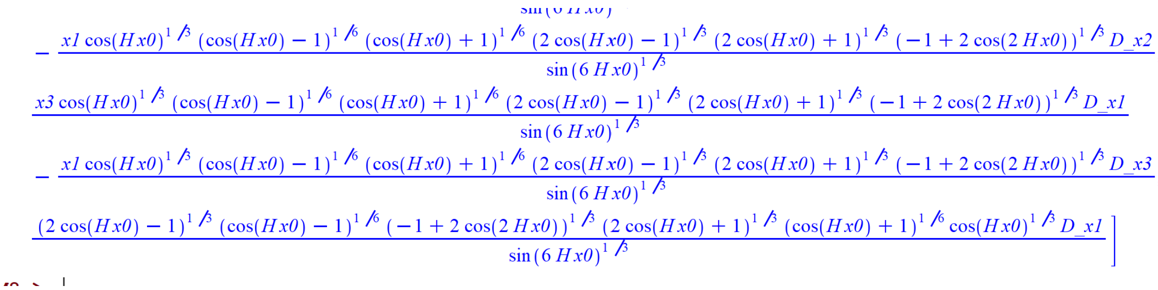
$$+ \frac{x6 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x7}{\sin(6 H x0)^{1/3}},$$

$$- \frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x6}{\sin(6 H x0)^{1/3}},$$

$$- \frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x7}{\sin(6 H x0)^{1/3}},$$

$$\frac{x2 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x1}{\sin(6 H x0)^{1/3}}$$

In[677]:=



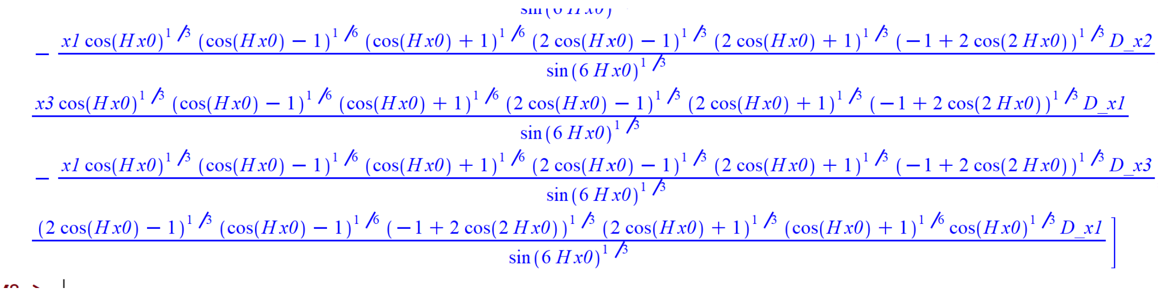
$$- \frac{x1 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x2}{\sin(6 H x0)^{1/3}},$$

$$\frac{x3 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x1}{\sin(6 H x0)^{1/3}}$$

$$- \frac{x1 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x3}{\sin(6 H x0)^{1/3}},$$

$$\left. \frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x1}{\sin(6 H x0)^{1/3}} \right]$$

Out[677]=



$$- \frac{x1 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x2}{\sin(6 H x0)^{1/3}},$$

$$\frac{x3 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x1}{\sin(6 H x0)^{1/3}}$$

$$- \frac{x1 \cos(H x0)^{1/3} (\cos(H x0)-1)^{1/6} (\cos(H x0)+1)^{1/6} (2\cos(H x0)-1)^{1/3} (2\cos(H x0)+1)^{1/3} (-1+2\cos(2 H x0))^{1/3} D\_x3}{\sin(6 H x0)^{1/3}},$$

$$\left. \frac{(2\cos(H x0)-1)^{1/3} (\cos(H x0)-1)^{1/6} (-1+2\cos(2 H x0))^{1/3} (2\cos(H x0)+1)^{1/3} (\cos(H x0)+1)^{1/6} \cos(H x0)^{1/3} D\_x1}{\sin(6 H x0)^{1/3}} \right]$$

In[678]:=

$$\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x2}{\sin(6\,H\,x0)^{1/3}},$$

$$\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x3}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{x7\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x5}{\sin(6\,H\,x0)^{1/3}}$$

$$+\frac{x5\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x7}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x5}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{x7\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x6}{\sin(6\,H\,x0)^{1/3}}$$

$$+\frac{x6\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x7}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x6}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x7}{\sin(6\,H\,x0)^{1/3}},$$

$$\frac{x2\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x1}{\sin(6\,H\,x0)^{1/3}}$$

$$-\frac{x1\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x2}{\sin(6\,H\,x0)^{1/3}},$$

$$\frac{x3\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x1}{\sin(6\,H\,x0)^{1/3}}$$

$$-\frac{x1\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x3}{\sin(6\,H\,x0)^{1/3}},$$

$$\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x1}{\sin(6\,H\,x0)^{1/3}}\Bigg]$$

8 > |

Out[678]=

$$\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x2}{\sin(6\,H\,x0)^{1/3}},$$

$$\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x3}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{x7\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x5}{\sin(6\,H\,x0)^{1/3}}$$

$$+\frac{x5\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x7}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x5}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{x7\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x6}{\sin(6\,H\,x0)^{1/3}}$$

$$+\frac{x6\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x7}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x6}{\sin(6\,H\,x0)^{1/3}},$$

$$-\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x7}{\sin(6\,H\,x0)^{1/3}},$$

$$\frac{x2\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x1}{\sin(6\,H\,x0)^{1/3}}$$

$$-\frac{x1\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x2}{\sin(6\,H\,x0)^{1/3}},$$

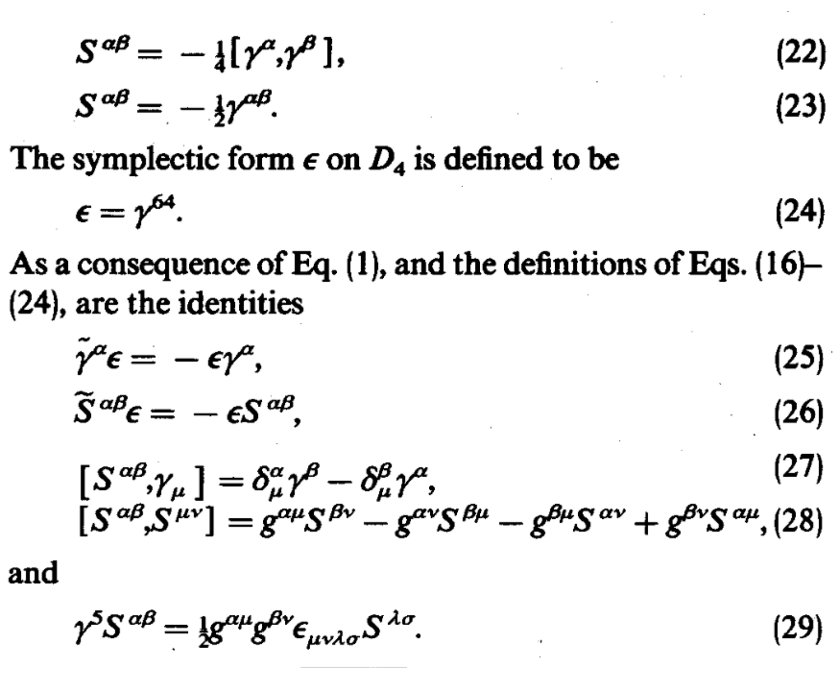
$$\frac{x3\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x1}{\sin(6\,H\,x0)^{1/3}}$$

$$-\frac{x1\cos(H\,x0)^{1/3}(\cos(H\,x0)-1)^{1/6}(\cos(H\,x0)+1)^{1/6}(2\cos(H\,x0)-1)^{1/3}(2\cos(H\,x0)+1)^{1/3}(-1+2\cos(2\,H\,x0))^{1/3}D\_x3}{\sin(6\,H\,x0)^{1/3}},$$

$$\frac{(2\cos(H\,x0)-1)^{1/3}(\cos(H\,x0)-1)^{1/6}(-1+2\cos(2\,H\,x0))^{1/3}(2\cos(H\,x0)+1)^{1/3}(\cos(H\,x0)+1)^{1/6}\cos(H\,x0)^{1/3}D\_x1}{\sin(6\,H\,x0)^{1/3}}\Bigg]$$

8 > |

4×4 Dirac γ matrices :



$$S^{\alpha\beta} = -\tfrac{1}{4}[\gamma^{\alpha},\gamma^{\beta}], \tag{22}$$

$$S^{\alpha\beta} = -\tfrac{1}{2}\gamma^{\alpha\beta}. \tag{23}$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \tag{24}$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^{\alpha}\epsilon = -\epsilon\gamma^{\alpha}, \tag{25}$$

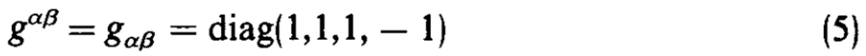
$$\tilde{S}^{\alpha\beta}\epsilon = -\epsilon S^{\alpha\beta}, \tag{26}$$

$$[S^{\alpha\beta},\gamma_{\mu}] = \delta^{\alpha}_{\mu}\gamma^{\beta} - \delta^{\beta}_{\mu}\gamma^{\alpha}, \tag{27}$$

$$[S^{\alpha\beta},S^{\mu\nu}] = g^{\alpha\mu}S^{\beta\nu} - g^{\alpha\nu}S^{\beta\mu} - g^{\beta\mu}S^{\alpha\nu} + g^{\beta\nu}S^{\alpha\mu}, \tag{28}$$

and

$$\gamma^{5}S^{\alpha\beta} = \tfrac{1}{2}g^{\alpha\mu}g^{\beta\nu}\epsilon_{\mu\nu\lambda\sigma}S^{\lambda\sigma}. \tag{29}$$



$$g^{\alpha\beta} = g_{\alpha\beta} = \mathrm{diag}(1,1,1,-1) \tag{5}$$

In[679]:=

```
(*Symbolize[ gαβ ]*)
```

In[680]:=

```
η4488
Dimensions[%]
gαβ = η4488⟦2 ;; 5, 2 ;; 5⟧
```

Out[680]=

```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
```

Out[681]=

```
{8, 8}
```

Out[682]=

```
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}
```

In[683]:=

```
Protect[gαβ]
```

Out[683]=

```
{g⎵Subscript⎵αβ}
```

In[684]:=

```
g3 = DiagonalMatrix[{1, 1, -1}]
```

Out[684]=

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}
```

In[685]:=

```
Do[Do[Gr[h, k] = t4by4[h].s4by4[k] * (-g3⟦h, h⟧ * g3⟦k, k⟧), {k, 1, 3}], {h, 1, 3}]
```

In[686]:=

```
Gr[1, 1] // MatrixForm
```

Out[686]//MatrixForm=

```
-1  0  0   0
 0  1  0   0
 0  0  1   0
 0  0  0  -1
```

In[687]:=

```
γ = {Gr[1, 1], Gr[1, 2], Gr[1, 3], t4by4[2]};
```

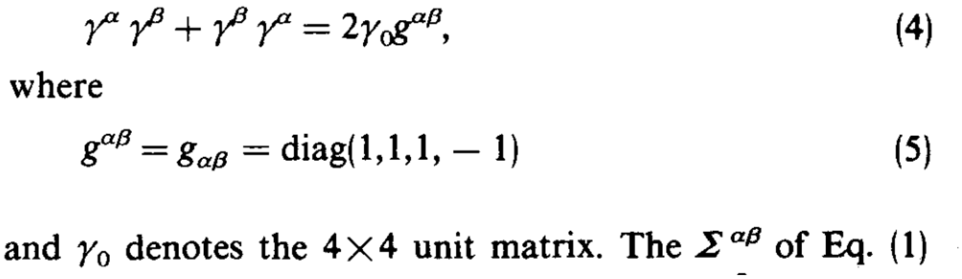
Set: Symbol γ is Protected.

In[688]:=

```
Table[FullSimplify[γ⟦h⟧.γ⟦h⟧ === gαβ⟦h, h⟧ * ID4], {h, 1, 4}]
```

Out[688]=

```
{False, False, False, True}
```



$$\gamma^{\alpha}\gamma^{\beta}+\gamma^{\beta}\gamma^{\alpha}=2\gamma_0 g^{\alpha\beta}, \qquad (4)$$

where

$$g^{\alpha\beta}=g_{\alpha\beta}=\mathrm{diag}(1,1,1,-1) \qquad (5)$$

and γ_0 denotes the 4×4 unit matrix. The $\Sigma^{\alpha\beta}$ of Eq. (1)

In[689]:=

```
Table[{{h, k}, FullSimplify[(γ⟦h⟧.γ⟦k⟧ + γ⟦k⟧.γ⟦h⟧)/2] === gαβ⟦h, k⟧ ID4},
  {h, 1, 4}, {k, 1, 4}] // MatrixForm
```

Out[689]//MatrixForm=

```
( {1, 1} ) ( {1, 2} ) ( {1, 3} ) ( {1, 4} )
(  False ) (  True  ) (  True  ) (  False )
( {2, 1} ) ( {2, 2} ) ( {2, 3} ) ( {2, 4} )
(  True  ) (  False ) (  True  ) (  False )
( {3, 1} ) ( {3, 2} ) ( {3, 3} ) ( {3, 4} )
(  True  ) (  True  ) (  False ) (  False )
( {4, 1} ) ( {4, 2} ) ( {4, 3} ) ( {4, 4} )
(  False ) (  False ) (  False ) (  True  )
```

In[690]:=

```
Table[{{h, k}, FullSimplify[(γ⟦h⟧.γ⟦k⟧ + γ⟦k⟧.γ⟦h⟧)/2] === η4488⟦1 + h, 1 + k⟧ ID4},
  {h, 1, 4}, {k, 1, 4}] // MatrixForm
```

Out[690]//MatrixForm=

```
( {1, 1} ) ( {1, 2} ) ( {1, 3} ) ( {1, 4} )
(  False ) (  True  ) (  True  ) (  False )
( {2, 1} ) ( {2, 2} ) ( {2, 3} ) ( {2, 4} )
(  True  ) (  False ) (  True  ) (  False )
( {3, 1} ) ( {3, 2} ) ( {3, 3} ) ( {3, 4} )
(  True  ) (  True  ) (  False ) (  False )
( {4, 1} ) ( {4, 2} ) ( {4, 3} ) ( {4, 4} )
(  False ) (  False ) (  False ) (  True  )
```

$$\gamma^5 = -\gamma^1\gamma^2\gamma^3\gamma^4$$

In[691]:=

```
-γ⟦1⟧.γ⟦2⟧.γ⟦3⟧.γ⟦4⟧
```

Out[691]=

```
{{0, 0, 0, 1}, {0, 0, -1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}
```

In[692]:=

```
AppendTo[γ, (-γ⟦1⟧.γ⟦2⟧.γ⟦3⟧.γ⟦4⟧)]
```

```
Set: Symbol γ is Protected.
```

Out[692]=

```
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, -1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}}
```

In[693]:=

```
Protect[γ]
```

Out[693]=

```
{}
```

In[694]:=

```
γ⟦5⟧
```

Out[694]=

```
{{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}
```

In[695]:=

```
γ⟦5⟧ === -t4by4[3]
```

Out[695]=

```
False
```

In[696]:=

```
Table[
  {{h, k}, FullSimplify[γ[[h]].γ[[k]] + γ[[k]].γ[[h]]] === 2 * η4488[[1 + h, 1 + k]] ID4},
  {h, 1, 5}, {k, 1, 5}] // MatrixForm
```

Out[696]//MatrixForm=

```
( ({1, 1})  ({1, 2})  ({1, 3})  ({1, 4})  ({1, 5}) )
( ( False)  (  True)  (  True)  ( False)  ( False) )
( ({2, 1})  ({2, 2})  ({2, 3})  ({2, 4})  ({2, 5}) )
( (  True)  ( False)  (  True)  ( False)  ( False) )
( ({3, 1})  ({3, 2})  ({3, 3})  ({3, 4})  ({3, 5}) )
( (  True)  (  True)  ( False)  ( False)  ( False) )
( ({4, 1})  ({4, 2})  ({4, 3})  ({4, 4})  ({4, 5}) )
( ( False)  ( False)  ( False)  (  True)  (  True) )
( ({5, 1})  ({5, 2})  ({5, 3})  ({5, 4})  ({5, 5}) )
( ( False)  ( False)  ( False)  (  True)  (  True) )
```

$$\gamma^a = \gamma^{a6}, \qquad (19)$$

$$\gamma^5 = \gamma^{56}, \qquad (20)$$

$$\gamma^a\gamma^5 = \gamma^{a5}, \qquad (21)$$

$$\epsilon = \gamma^{64}.$$

In[697]:=

```
-γ[[4]] // MatrixForm
```

Out[697]//MatrixForm=

```
(  0  0  0  1 )
(  0  0 -1  0 )
(  0  1  0  0 )
( -1  0  0  0 )
```

In[698]:=

```
sϵ = {ϵ → t4by4[2]}
```

Out[698]=

```
{ϵ → {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}
```

In[699]:=

```
Transpose[ϵ /. sϵ] === (-ϵ /. sϵ)
```

Out[699]=

```
True
```

In[700]:=

```
ParallelTable[{{A1},
  FullSimplify[(ϵ /. sϵ).γ[[A1]] === Transpose[(ϵ /. sϵ).γ[[A1]]]]}, {A1, 1, 5}]
```

Out[700]=

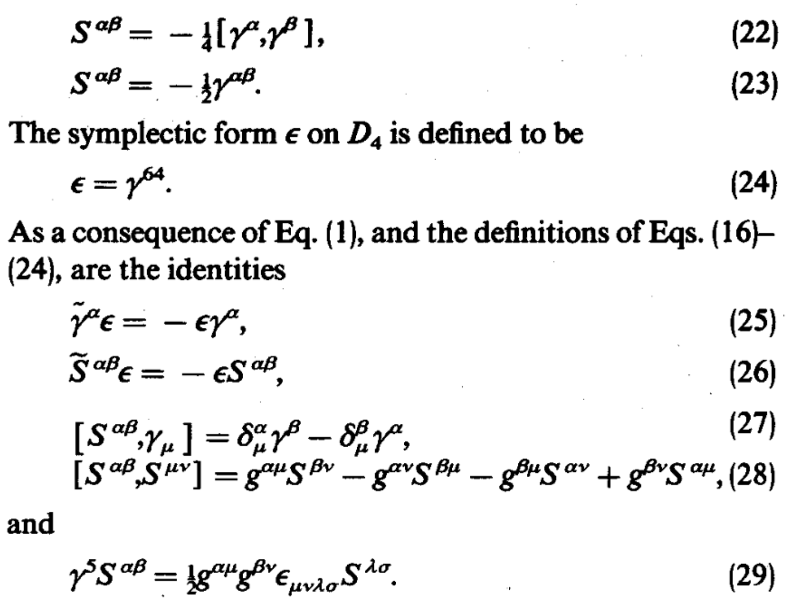
```
{{{1}, True}, {{2}, True}, {{3}, True}, {{4}, False}, {{5}, True}}
```

```
In[701]:=
γ⟦4⟧ === t4by4[2]
Out[701]=
False
In[702]:=
t4by4[1]
Out[702]=
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}}
```

$$S^{\alpha\beta} = -\tfrac{1}{4}[\gamma^\alpha, \gamma^\beta]. \qquad (3)$$

```
In[703]:=
          1
S44αβ = - — Table[FullSimplify[γ⟦h⟧.γ⟦k⟧ - γ⟦k⟧.γ⟦h⟧], {h, 1, 4}, {k, 1, 4}];
          4
In[704]:=
Protect[S44αβ]
Out[704]=
{S44αβ}
In[705]:=
η44 = DiagonalMatrix[{1, 1, 1, -1}]
Out[705]=
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}
In[706]:=
Protect[η44]
Out[706]=
{η44}
```

S44αβ commutation relations; misc :



$$S^{\alpha\beta} = -\tfrac{1}{4}[\gamma^\alpha,\gamma^\beta], \qquad (22)$$

$$S^{\alpha\beta} = -\tfrac{1}{2}\gamma^{\alpha\beta}. \qquad (23)$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \qquad (24)$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^\alpha \epsilon = -\epsilon\gamma^\alpha, \qquad (25)$$

$$\tilde{S}^{\alpha\beta}\epsilon = -\epsilon S^{\alpha\beta}, \qquad (26)$$

$$[S^{\alpha\beta},\gamma_\mu] = \delta^\alpha_\mu\gamma^\beta - \delta^\beta_\mu\gamma^\alpha, \qquad (27)$$

$$[S^{\alpha\beta},S^{\mu\nu}] = g^{\alpha\mu}S^{\beta\nu} - g^{\alpha\nu}S^{\beta\mu} - g^{\beta\mu}S^{\alpha\nu} + g^{\beta\nu}S^{\alpha\mu}, \qquad (28)$$

and

$$\gamma^5 S^{\alpha\beta} = \tfrac{1}{2}g^{\alpha\mu}g^{\beta\nu}\epsilon_{\mu\nu\lambda\sigma}S^{\lambda\sigma}. \qquad (29)$$

```
          1
S44αβ = - — Table[FullSimplify[γ⟦h⟧.γ⟦k⟧ - γ⟦k⟧.γ⟦h⟧], {h, 1, 4}, {k, 1, 4}];
          4
```

In[707]:=

```
(*SAB=
  Table[1/4 (T16^A[A1].T16^A[B1]-T16^A[B1].T16^A[A1]), {A1,0, 7},{B1,0, 7}];*)
```

In[708]:=

```
(*ParallelTable[
   FullSimplify[σ16.SAB⟦A1,B1⟧===-Transpose[σ16.SAB⟦A1,B1⟧]],
   {A1,1, 8},{B1,1,8}]//Flatten//Union*)
```

In[709]:=

```
(*ParallelTable[
 {{A1,B1},FullSimplify[SAB⟦A1,B1⟧===-Transpose[SAB⟦A1,B1⟧]]},
 {A1,1, 8},{B1,1,8}]*)
```

In[710]:=

```
ParallelTable[
   FullSimplify[S44αβ⟦A1, B1⟧.S44αβ⟦A2, B2⟧ - S44αβ⟦A2, B2⟧.S44αβ⟦A1, B1⟧ ==
     (η44⟦A1, A2⟧ × S44αβ⟦B1, B2⟧ - η44⟦A1, B2⟧ × S44αβ⟦B1, A2⟧ -
       η44⟦B1, A2⟧ × S44αβ⟦A1, B2⟧ + η44⟦B1, B2⟧ × S44αβ⟦A1, A2⟧)], {A1, 1, 3},
   {B1, A1 + 1, 4}, {A2, 1, 3}, {B2, A2 + 1, 4}] // Flatten // Union
```

Out[710]=

```
{False, True}
```

In[711]:=

```
ParallelTable[{{A1, B1, A2, B2},
   FullSimplify[S44αβ⟦A1, B1⟧.S44αβ⟦A2, B2⟧ - S44αβ⟦A2, B2⟧.S44αβ⟦A1, B1⟧ ==
     (η44⟦A1, A2⟧ × S44αβ⟦B1, B2⟧ - η44⟦A1, B2⟧ × S44αβ⟦B1, A2⟧ -
       η44⟦B1, A2⟧ × S44αβ⟦A1, B2⟧ + η44⟦B1, B2⟧ × S44αβ⟦A1, A2⟧)]},
  {A1, 1, 4}, {B1, 1, 4}, {A2, 1, 4}, {B2, 1, 4}] // MatrixForm
```

Out[711]//MatrixForm=

```
{1, 1, 1, 1}  {1, 1, 1, 2}  {1, 1, 1, 3}  {1, 1, 1, 4}   {1, 2, 1, 1}  {1, 2, 1,
    True          True          True          True           True          True
{1, 1, 2, 1}  {1, 1, 2, 2}  {1, 1, 2, 3}  {1, 1, 2, 4}   {1, 2, 2, 1}  {1, 2, 2,
    True          True          True          True           True          True
{1, 1, 3, 1}  {1, 1, 3, 2}  {1, 1, 3, 3}  {1, 1, 3, 4}   {1, 2, 3, 1}  {1, 2, 3,
    True          True          True          True           False         False
{1, 1, 4, 1}  {1, 1, 4, 2}  {1, 1, 4, 3}  {1, 1, 4, 4}   {1, 2, 4, 1}  {1, 2, 4,
    True          True          True          True           True          True
{2, 1, 1, 1}  {2, 1, 1, 2}  {2, 1, 1, 3}  {2, 1, 1, 4}   {2, 2, 1, 1}  {2, 2, 1,
    True          True          False         True           True          True
{2, 1, 2, 1}  {2, 1, 2, 2}  {2, 1, 2, 3}  {2, 1, 2, 4}   {2, 2, 2, 1}  {2, 2, 2,
    True          True          False         True           True          True
{2, 1, 3, 1}  {2, 1, 3, 2}  {2, 1, 3, 3}  {2, 1, 3, 4}   {2, 2, 3, 1}  {2, 2, 3,
    False         False         True          True           True          True
{2, 1, 4, 1}  {2, 1, 4, 2}  {2, 1, 4, 3}  {2, 1, 4, 4}   {2, 2, 4, 1}  {2, 2, 4,
    True          True          True          True           True          True
{3, 1, 1, 1}  {3, 1, 1, 2}  {3, 1, 1, 3}  {3, 1, 1, 4}   {3, 2, 1, 1}  {3, 2, 1,
    True          False         True          True           True          False
{3, 1, 2, 1}  {3, 1, 2, 2}  {3, 1, 2, 3}  {3, 1, 2, 4}   {3, 2, 2, 1}  {3, 2, 2,
    False         True          False         True           False         True
{3, 1, 3, 1}  {3, 1, 3, 2}  {3, 1, 3, 3}  {3, 1, 3, 4}   {3, 2, 3, 1}  {3, 2, 3,
    True          False         True          True           False         True
{3, 1, 4, 1}  {3, 1, 4, 2}  {3, 1, 4, 3}  {3, 1, 4, 4}   {3, 2, 4, 1}  {3, 2, 4,
    True          True          True          True           True          True
{4, 1, 1, 1}  {4, 1, 1, 2}  {4, 1, 1, 3}  {4, 1, 1, 4}   {4, 2, 1, 1}  {4, 2, 1,
    True          True          True          True           True          True
{4, 1, 2, 1}  {4, 1, 2, 2}  {4, 1, 2, 3}  {4, 1, 2, 4}   {4, 2, 2, 1}  {4, 2, 2,
    True          True          True          False          True          True
{4, 1, 3, 1}  {4, 1, 3, 2}  {4, 1, 3, 3}  {4, 1, 3, 4}   {4, 2, 3, 1}  {4, 2, 3,
    True          True          True          False          True          True
{4, 1, 4, 1}  {4, 1, 4, 2}  {4, 1, 4, 3}  {4, 1, 4, 4}   {4, 2, 4, 1}  {4, 2, 4,
    True          False         False         True           False         True
```

In[712]:=

```
ParallelTable[FullSimplify[S44αβ⟦A1, B1⟧.ɣ⟦B2⟧ - ɣ⟦B2⟧.S44αβ⟦A1, B1⟧ ==
    (η44⟦B2, A1⟧ × ɣ⟦B1⟧ - η44⟦B2, B1⟧ × ɣ⟦A1⟧)],
  {A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // Flatten // Union
```

Out[712]=

```
{False, True}
```

In[713]:=

```
ParallelTable[
  {{A1, B1, B2}, FullSimplify[S44αβ⟦A1, B1⟧.ɣ⟦B2⟧ - ɣ⟦B2⟧.S44αβ⟦A1, B1⟧ ==
     (η44⟦B2, A1⟧ × ɣ⟦B1⟧ - η44⟦B2, B1⟧ × ɣ⟦A1⟧)]},
  {A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // MatrixForm
```

Out[713]//MatrixForm=

```
( ({1, 1, 1}  True )  ({1, 2, 1}  False)  ({1, 3, 1}  False)  ({1, 4, 1}  False) )
( ({1, 1, 2}  True )  ({1, 2, 2}  False)  ({1, 3, 2}  True )  ({1, 4, 2}  True ) )
( ({1, 1, 3}  True )  ({1, 2, 3}  True )  ({1, 3, 3}  False)  ({1, 4, 3}  True ) )
( ({1, 1, 4}  True )  ({1, 2, 4}  True )  ({1, 3, 4}  True )  ({1, 4, 4}  False) )
( ({2, 1, 1}  False)  ({2, 2, 1}  True )  ({2, 3, 1}  True )  ({2, 4, 1}  True ) )
( ({2, 1, 2}  False)  ({2, 2, 2}  True )  ({2, 3, 2}  False)  ({2, 4, 2}  False) )
( ({2, 1, 3}  True )  ({2, 2, 3}  True )  ({2, 3, 3}  False)  ({2, 4, 3}  True ) )
( ({2, 1, 4}  True )  ({2, 2, 4}  True )  ({2, 3, 4}  True )  ({2, 4, 4}  False) )
( ({3, 1, 1}  False)  ({3, 2, 1}  True )  ({3, 3, 1}  True )  ({3, 4, 1}  True ) )
( ({3, 1, 2}  True )  ({3, 2, 2}  False)  ({3, 3, 2}  True )  ({3, 4, 2}  True ) )
( ({3, 1, 3}  False)  ({3, 2, 3}  False)  ({3, 3, 3}  True )  ({3, 4, 3}  False) )
( ({3, 1, 4}  True )  ({3, 2, 4}  True )  ({3, 3, 4}  True )  ({3, 4, 4}  False) )
( ({4, 1, 1}  False)  ({4, 2, 1}  True )  ({4, 3, 1}  True )  ({4, 4, 1}  True ) )
( ({4, 1, 2}  True )  ({4, 2, 2}  False)  ({4, 3, 2}  True )  ({4, 4, 2}  True ) )
( ({4, 1, 3}  True )  ({4, 2, 3}  True )  ({4, 3, 3}  False)  ({4, 4, 3}  True ) )
( ({4, 1, 4}  False)  ({4, 2, 4}  False)  ({4, 3, 4}  False)  ({4, 4, 4}  True ) )
```

In[714]:=

```
ParallelTable[{{A1, B1}, FullSimplify[(ϵ /. sϵ).S44αβ⟦A1, B1⟧ ===
    Transpose[(ϵ /. sϵ).S44αβ⟦A1, B1⟧]]}, {A1, 1, 3}, {B1, A1 + 1, 4}]
```

Out[714]=

```
{{{{1, 2}, True}, {{1, 3}, True}, {{1, 4}, True}},
 {{{2, 3}, True}, {{2, 4}, True}}, {{{3, 4}, True}}}
```

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula:

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+,+,+,-,-,-)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \mathbf{diag}(1,1,1,-1,-1,-1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties.

The final answer is $[\mathbf{J_{ab}}, \mathbf{J_{cd}}] = \mathbf{i}(\boldsymbol{\eta}_{\mathbf{ac}}\mathbf{J_{bd}} - \boldsymbol{\eta}_{\mathbf{ad}}\mathbf{J_{bc}} - \boldsymbol{\eta}_{\mathbf{bc}}\mathbf{J_{ad}} + \boldsymbol{\eta}_{\mathbf{bd}}\mathbf{J_{ac}})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4).

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \ldots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \mathrm{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention)

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$.

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa).

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not).

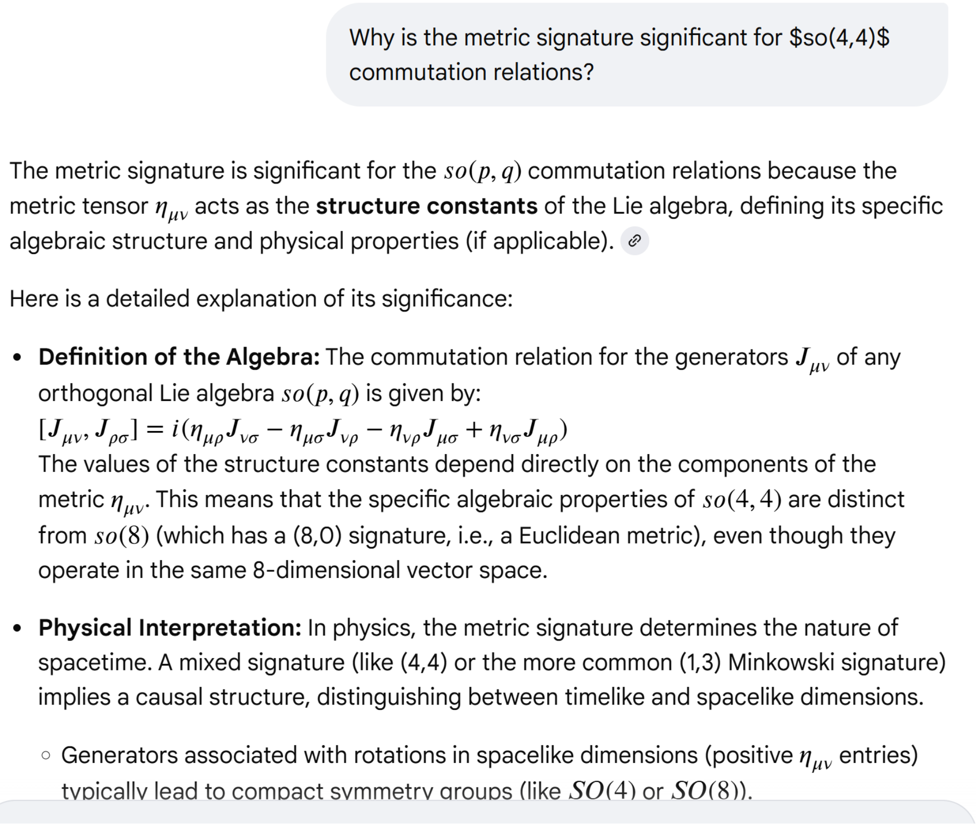
what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the $so(4,4)$ Lie algebra, where $\mu, \nu \in \{0, 1, \ldots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the standard relations for orthogonal Lie algebras, given by a generalization of the angular momentum commutation relations:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for $so(8)$ in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \mathrm{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).

↳ Why is the metric signature significant for $so(4,4)$ commutation relations?



Why is the metric signature significant for $so(4,4)$ commutation relations?

The metric signature is significant for the $so(p,q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable).

Here is a detailed explanation of its significance:

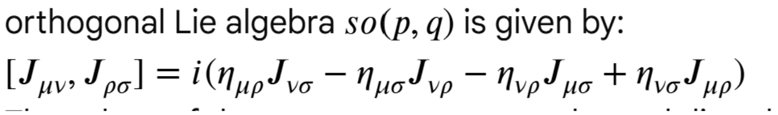
- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p,q)$ is given by:
  $[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$
  The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4,4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
  - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[715]:=

```
η4488
```

Out[715]=

```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
```



orthogonal Lie algebra $so(p,q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

In[716]:=

```
(*Table[SAB⟦A1,B1⟧⩵ ArrayFlatten[{{SAB1⟦A1,B1⟧,0},{0,SAB2⟦A1,B1⟧}}],
 {A1,0, 7},{B1,0,7}]*)
```

$$4D_{(1)}{}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}{}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A .$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \to SO(4,4)$ is given by

$$8L^{A'}{}_{B'}(g) = \mathrm{tr}\,\tilde{D}^{(2)}(g)\,\tau^{A'}\,D^{(1)}(g)\,\bar{\tau}_{B'} . \tag{2.8}$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$\tilde{D}^{(1)}\sigma = \sigma D^{(1)-1} , \tag{2.9}$$

$$\sigma^{-1}\tilde{D}^{(2)} = D^{(2)-1}\sigma^{-1} , \tag{2.10}$$

$$L^{A'}{}_{C'}\,G_{A'B'}\,L^{B'}{}_{D'} = G_{C'D'} , \tag{2.11}$$

$$L^{A'}{}_{B'}\,\tau^{B'} = \tilde{D}^{(2)}\,\tau^{A'}\,D^{(1)} , \tag{2.12}$$

$$L^{A'}{}_{B'}\,\bar{\tau}^{B'} = D^{(1)-1}\,\bar{\tau}^{A'}\,\tilde{D}^{(2)-1} . \tag{2.13}$$

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A Remarkable Representation of the 3 + 2 de Sitter Group

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(Received 20 February 1963)

Identities satisfied by the generators of the Dirac algebra

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(Received 26 July 1983; accepted for publication 23 September 1983)

204 J. Math. Phys. 25 (2), February 1984 0022-2488/84/020204-06$02.50 © 1984 American Institute of Physic

In[717]:=

```
Unprotect[γABs];
γABs = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
   Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
```

Out[718]=

```
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

In[719]:=

```
Length[γABs]
```

Out[719]=

```
16
```

In[720]:=

```
Protect[γABs]
```

Out[720]=

```
{γABs}
```

In[721]:=

```
γABs[[1]]
```

Out[721]=

```
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}
```

In[722]:=

```
γABs[[-1]]
```

Out[722]=

```
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Recall Self Dual Anti-Symmetric

In[723]:=

$$\text{Table}\left[\left(\frac{1}{2}\sum_{j1=1}^{4}\left(\sum_{j2=1}^{4}\epsilon4[\![p, q, j1, j2]\!]\times \text{s4by4}[h][\![j1, j2]\!]\right)\right)-\text{s4by4}[h][\![p, q]\!],\ \{h, 1, 3\}, \{q, 4\}, \{p, 4\}\right]$$

Out[723]=

```
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}
```

Recall Anti SelfDual Anti-Symmetric

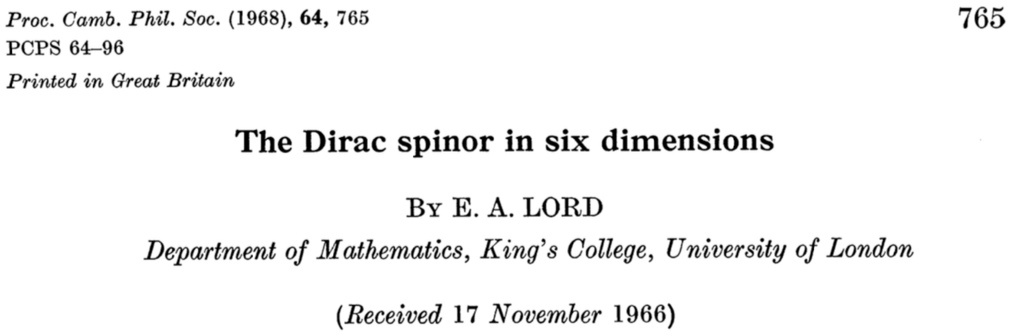
In[724]:=

```
Table[(1/2 Sum[(Sum[ϵ4[[p, q, j1, j2]] × t4by4[h][[j1, j2]], {j2, 1, 4}]), {j1, 1, 4}]) + t4by4[h][[p, q]],
  {h, 1, 3}, {q, 4}, {p, 4}]
```

Out[724]=

```
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},
 {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}
```

BASIS of 4 × 4 matrices :



Proc. Camb. Phil. Soc. (1968), **64**, 765 765
PCPS 64–96
Printed in Great Britain

The Dirac spinor in six dimensions

By E. A. LORD

Department of Mathematics, King's College, University of London

(*Received* 17 *November* 1966)

3. *Decomposition of the basic spinor representations.* We have already stated that the basic spinor representations of $SO(n,c)$, for n even, are reducible. We shall consider here the reduction of the basic 8-component spinor representation of $SO(6,c)$ into constituent 4-component representations. The procedure is easily generalized to apply to any even-dimensional space. An irreducible representation of the Clifford algebra C_6 can be constructed from a given irreducible representation of C_5. The $\gamma_\mu (\mu = 1, \dots 5)$ of an irreducible representation of C_5 satisfy (2·5), so that the quantities e_α $(\alpha = 1, \dots 6)$ defined by

$$\left.\begin{aligned} e_\mu &= (-i)\begin{pmatrix} & \gamma_\mu B \\ -B^{-1}\gamma_\mu & \end{pmatrix} \quad (\mu = 1, \dots 5), \\ e_6 &= \begin{pmatrix} & B \\ B^{-1} & \end{pmatrix}, \end{aligned}\right\} \tag{3·1}$$

satisfy
$$\tfrac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \dots 6).$$

For the moment the matrix B is completely arbitrary. The e_α therefore give an irreducible representation of C_6. They can be written more concisely in terms of the sets of matrices a_α and $\bar{a}_\alpha$, defined as follows:

$$\left.\begin{aligned} \alpha_\mu &= \gamma_\mu B, \quad \bar{a}_\mu = -B^{-1}\gamma_\mu \quad (\mu = 1, \dots 5), \\ a_6 &= iB, \qquad \bar{a}_6 = iB^{-1}. \end{aligned}\right\} \tag{3·2}$$

The generator e_α of C_6 can then be written

$$e_\alpha = (-i)\begin{pmatrix} & a_\alpha \\ \bar{a}_\alpha & \end{pmatrix} \quad (\alpha = 1, \dots 6). \tag{3·3}$$

With $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1e_2e_3e_4e_5e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

and
$$e_{\alpha\beta} = \tfrac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2}\begin{pmatrix} a_\alpha\bar{a}_\beta - a_\beta\bar{a}_\alpha & \\ & \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha \end{pmatrix}.$$

This latter expression shows that the infinitesimal generators of the 8-component spinor representation of $SO(6,c)$ is the direct sum of two 4-component representations. The basic spinor can be written

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix},$$

where ϕ and χ are four component spinors which transform according to representations with infinitesimal generators

$$\left.\begin{aligned} G^{(1)}_{\alpha\beta} &= -\tfrac{1}{4}(a_\alpha\bar{a} - a_\beta\bar{a}_\alpha) \\ G^{(2)}_{\alpha\beta} &= -\tfrac{1}{4}(\bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha), \end{aligned}\right\} \tag{3·4}$$

and

$$\{\underbrace{\text{sy}}_{1}, \underbrace{\text{sy}}_{4}, \underbrace{\text{asy}}_{6}, \underbrace{\text{sy}}_{4}, \underbrace{\text{sy}}_{1}\}$$

In[725]:=

```
Plus @@ {1, 4, 6, 4, 1}
```

Out[725]=

```
16
```

$$\{\underbrace{\text{sy}}_{1}, \underbrace{\text{sy}}_{4}, \underbrace{\text{asy}}_{6}, \underbrace{\text{sy}}_{4}, \underbrace{\text{sy}}_{1}\}$$

In[726]:=

```
Plus @@ {1, 4, 6, 4, 1}
```

Out[726]=

```
16
```

In[727]:=

```
MatrixForm[If[# == 2, ⅈ, 1] * PauliMatrix[#]] & /@ Range[3]
```

Out[727]=

$$\left\{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\}$$

In[728]:=

```
σ22 = Flatten[
  {{IdentityMatrix[2]}, If[# == 2, ⅈ, 1] * PauliMatrix[#] & /@ Range[3]}, 1]
```

Out[728]=

```
{{{1, 0}, {0, 1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}}}
```

In[729]:=

$$\overline{\sigma22} = \texttt{Flatten[}$$

```
  {{-IdentityMatrix[2]}, If[# == 2, ⅈ, 1] * PauliMatrix[#] & /@ Range[3]}, 1]
```

Out[729]=

```
{{{-1, 0}, {0, -1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}}}
```

In[730]:=

```
-IdentityMatrix[2] === Dot @@ (σ22[[#]] & /@ Range[Length[σ22]])
```

Out[730]=

```
True
```

In[731]:=

$$\texttt{Table}\left[\left\{\{\texttt{A, B}\}, \texttt{FullSimplify}\left[\frac{1}{2}\,\texttt{Tr}\left[\frac{1}{2}\left(\sigma22[\![\texttt{A}]\!].\overline{\sigma22}[\![\texttt{B}]\!] + \sigma22[\![\texttt{B}]\!].\overline{\sigma22}[\![\texttt{A}]\!]\right)\right]\right]\right\}, \{\texttt{A, 1, 4}\}, \{\texttt{B, 1, 4}\}\right] \texttt{ // MatrixForm}$$

Out[731]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} \{1,1\} \\ -1 \end{pmatrix} & \begin{pmatrix} \{1,2\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{1,3\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{1,4\} \\ 0 \end{pmatrix} \\ \begin{pmatrix} \{2,1\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{2,2\} \\ 1 \end{pmatrix} & \begin{pmatrix} \{2,3\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{2,4\} \\ 0 \end{pmatrix} \\ \begin{pmatrix} \{3,1\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{3,2\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{3,3\} \\ -1 \end{pmatrix} & \begin{pmatrix} \{3,4\} \\ 0 \end{pmatrix} \\ \begin{pmatrix} \{4,1\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{4,2\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{4,3\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{4,4\} \\ 1 \end{pmatrix} \end{pmatrix}$$

In[732]:=

```
η2244 = DiagonalMatrix[{-1, 1, -1, 1}]
```

Out[732]=

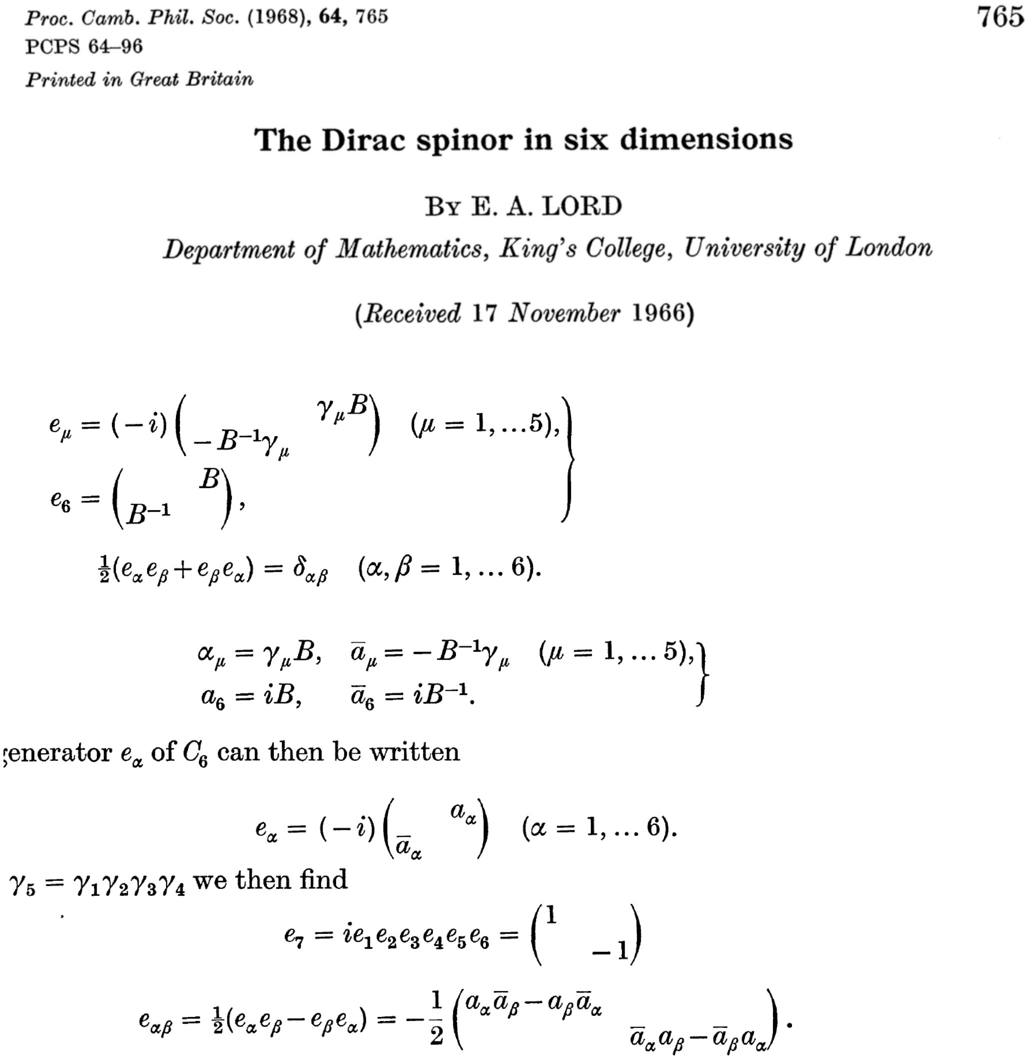
```
{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}
```

In[733]:=

```
Table[{{A, B}, FullSimplify[ExpandAll[1/2 (σ22⟦A⟧.σ̅2̅2̅⟦B⟧ + σ22⟦B⟧.σ̅2̅2̅⟦A⟧) ==
      η2244⟦A, B⟧ * IdentityMatrix[2]]]}, {A, 1, 4}, {B, 1, 4}] // MatrixForm
```

Out[733]//MatrixForm=

```
( {1, 1}   {1, 2}   {1, 3}   {1, 4} )
(  True     True     True     True  )
( {2, 1}   {2, 2}   {2, 3}   {2, 4} )
(  True     True     True     True  )
( {3, 1}   {3, 2}   {3, 3}   {3, 4} )
(  True     True     True     True  )
( {4, 1}   {4, 2}   {4, 3}   {4, 4} )
(  True     True     True     True  )
```



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765

The Dirac spinor in six dimensions

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$$\left.\begin{aligned} e_\mu &= (-i)\begin{pmatrix} & \gamma_\mu B\\ -B^{-1}\gamma_\mu & \end{pmatrix} \quad (\mu = 1,\ldots 5),\\ e_6 &= \begin{pmatrix} & B\\ B^{-1} & \end{pmatrix}, \end{aligned}\right\}$$

$$\tfrac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \ldots 6).$$

$$\left.\begin{aligned} \alpha_\mu &= \gamma_\mu B, & \bar{a}_\mu &= -B^{-1}\gamma_\mu \quad (\mu = 1, \ldots 5),\\ a_6 &= iB, & \bar{a}_6 &= iB^{-1}. \end{aligned}\right\}$$

enerator e_α of C_6 can then be written

$$e_\alpha = (-i)\begin{pmatrix} & a_\alpha\\ \bar{a}_\alpha & \end{pmatrix} \quad (\alpha = 1, \ldots 6).$$

$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1e_2e_3e_4e_5e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$e_{\alpha\beta} = \tfrac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2}\begin{pmatrix} a_\alpha\bar{a}_\beta - a_\beta\bar{a}_\alpha & \\ & \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha \end{pmatrix}.$$

```
In[734]:=
Do[yyy[j] = ArrayFlatten[{{0, σ22⟦j⟧}, {-σ22⟦j⟧, 0}}], {j, 4}];
yyy[5] = ArrayFlatten[{{0, IdentityMatrix[2]}, {IdentityMatrix[2], 0}}]
Out[734]=
{{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}}
In[735]:=
yyy[5] = {{0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0}}
Out[735]=
{{0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0}}
In[736]:=
(yyy[6] = Dot @@ (yyy[#] & /@ Range[5])) // MatrixForm
Out[736]//MatrixForm=
( 0  0  1  0 )
( 0  0  0 -1 )
( 1  0  0  0 )
( 0 -1  0  0 )
In[737]:=
MatrixForm[yyy[#]] & /@ Range[6]
Out[737]=
 ( 0  0  1  0 )  ( 0  0  0  1 )  ( 0  0  0  1 )  ( 0  0  1  0 )  ( 0  0 -1  0 )  ( 0  0  1  0 )
{( 0  0  0  1 ), ( 0  0  1  0 ), ( 0  0 -1  0 ), ( 0  0  0 -1 ), ( 0  0  0  1 ), ( 0  0  0 -1 )}
 (-1  0  0  0 )  ( 0 -1  0  0 )  ( 0 -1  0  0 )  (-1  0  0  0 )  (-1  0  0  0 )  ( 1  0  0  0 )
 ( 0 -1  0  0 )  (-1  0  0  0 )  ( 1  0  0  0 )  ( 0  1  0  0 )  ( 0  1  0  0 )  ( 0 -1  0  0 )
In[738]:=
Do[xxx[j] = ArrayFlatten[{{0, σ̅2̅2̅⟦j⟧}, {σ22⟦j⟧, 0}}], {j, 4}]
In[739]:=
MatrixForm[xxx[#]] & /@ Range[4]
Out[739]=
 ( 0  0 -1  0 )  ( 0  0  0  1 )  ( 0  0  0  1 )  ( 0  0  1  0 )
{( 0  0  0 -1 ), ( 0  0  1  0 ), ( 0  0 -1  0 ), ( 0  0  0 -1 )}
 ( 1  0  0  0 )  ( 0  1  0  0 )  ( 0  1  0  0 )  ( 1  0  0  0 )
 ( 0  1  0  0 )  ( 1  0  0  0 )  (-1  0  0  0 )  ( 0 -1  0  0 )
In[740]:=
xxx[5] = Dot @@ (xxx[#] & /@ Range[4])
Out[740]=
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}
In[741]:=
IdentityMatrix[4]
Out[741]=
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

In[742]:=

```
txA0 = {{IdentityMatrix[4], {0}}};
Length[txA0]
txA = {}; Do[AppendTo[txA, {xxx[j], {j}}], {j, 1, 4}]
Length[txA]
txAB = {};
Do[AppendTo[txAB, {xxx[j].xxx[k], {j, k}}], {j, 1, 3}, {k, j + 1, 4}]
Length[txAB]
txABC = {};
Do[AppendTo[txABC, {xxx[j].xxx[k].xxx[h], {j, k, h}}],
  {j, 1, 2}, {k, j + 1, 3}, {h, k + 1, 4}];
Length[txABC]
txABCD = {{xxx[1].xxx[2].xxx[3].xxx[4], {1, 2, 3, 4}}};
Length[txABCD]
```

Out[743]=

```
1
```

Out[745]=

```
4
```

Out[747]=

```
6
```

Out[749]=

```
4
```

Out[751]=

```
1
```

In[752]:=

```
base4by4 = Flatten[{txA0, txA, txAB, txABC, txABCD}, 1]; Length[base4by4]
```

Out[752]=

```
16
```

In[753]:=

```
Table[{{A, B}, Block[{r}, r = FullSimplify[1/4 Tr[
          1/2 (base4by4⟦A, 1⟧.base4by4⟦B, 1⟧ + base4by4⟦B, 1⟧.base4by4⟦A, 1⟧)]];
     If[r == 0, Style[r, Red], Style[r, Darker[Green]]]]},
   {A, 1, Length[base4by4]}, {B, 1, Length[base4by4]}] // MatrixForm
```

Out[753]//MatrixForm=

```
 {1, 1}   {1, 2}   {1, 3}   {1, 4}   {1, 5}   {1, 6}   {1, 7}   {1, 8}   {1
   1        0        0        0        0        0        0        0
 {2, 1}   {2, 2}   {2, 3}   {2, 4}   {2, 5}   {2, 6}   {2, 7}   {2, 8}   {2
   0       -1        0        0        0        0        0        0
 {3, 1}   {3, 2}   {3, 3}   {3, 4}   {3, 5}   {3, 6}   {3, 7}   {3, 8}   {3
   0        0        1        0        0        0        0        0
 {4, 1}   {4, 2}   {4, 3}   {4, 4}   {4, 5}   {4, 6}   {4, 7}   {4, 8}   {4
   0        0        0       -1        0        0        0        0
 {5, 1}   {5, 2}   {5, 3}   {5, 4}   {5, 5}   {5, 6}   {5, 7}   {5, 8}   {5
   0        0        0        0        1        0        0        0
 {6, 1}   {6, 2}   {6, 3}   {6, 4}   {6, 5}   {6, 6}   {6, 7}   {6, 8}   {6
   0        0        0        0        0        1        0        0
 {7, 1}   {7, 2}   {7, 3}   {7, 4}   {7, 5}   {7, 6}   {7, 7}   {7, 8}   {7
   0        0        0        0        0        0       -1        0
 {8, 1}   {8, 2}   {8, 3}   {8, 4}   {8, 5}   {8, 6}   {8, 7}   {8, 8}   {8
   0        0        0        0        0        0        0        1
 {9, 1}   {9, 2}   {9, 3}   {9, 4}   {9, 5}   {9, 6}   {9, 7}   {9, 8}   {9
   0        0        0        0        0        0        0        0
{10, 1}  {10, 2}  {10, 3}  {10, 4}  {10, 5}  {10, 6}  {10, 7}  {10, 8}  {10
   0        0        0        0        0        0        0        0
{11, 1}  {11, 2}  {11, 3}  {11, 4}  {11, 5}  {11, 6}  {11, 7}  {11, 8}  {11
   0        0        0        0        0        0        0        0
{12, 1}  {12, 2}  {12, 3}  {12, 4}  {12, 5}  {12, 6}  {12, 7}  {12, 8}  {12
   0        0        0        0        0        0        0        0
{13, 1}  {13, 2}  {13, 3}  {13, 4}  {13, 5}  {13, 6}  {13, 7}  {13, 8}  {13
   0        0        0        0        0        0        0        0
{14, 1}  {14, 2}  {14, 3}  {14, 4}  {14, 5}  {14, 6}  {14, 7}  {14, 8}  {14
   0        0        0        0        0        0        0        0
{15, 1}  {15, 2}  {15, 3}  {15, 4}  {15, 5}  {15, 6}  {15, 7}  {15, 8}  {15
   0        0        0        0        0        0        0        0
{16, 1}  {16, 2}  {16, 3}  {16, 4}  {16, 5}  {16, 6}  {16, 7}  {16, 8}  {16
   0        0        0        0        0        0        0        0
```

In[754]:=

```
base4by4⟦16⟧⟦1⟧
```

Out[754]=

```
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}
```

In[755]:=

```
anti4by4 = {};
Do[If[base4by4⟦k, 1⟧ == -Transpose[base4by4⟦k, 1⟧], AppendTo[anti4by4,
   {base4by4⟦k, 1⟧, {{k}, base4by4⟦k, 2⟧}}] ], {k, 1, Length[base4by4]}];
Length[anti4by4]
anti4by4⟦%⟧
```

Out[756]=

```
6
```

Out[757]=

```
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}, {{14}, {1, 3, 4}}}
```

In[758]:=

```
symm4by4 = {};
Do[If[base4by4⟦k, 1⟧ == Transpose[base4by4⟦k, 1⟧], AppendTo[symm4by4,
   {base4by4⟦k, 1⟧, {{k}, base4by4⟦k, 2⟧}}] ], {k, 1, Length[base4by4]}];
Length[symm4by4]
symm4by4⟦%⟧
```

Out[759]=

```
10
```

Out[760]=

```
{{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}, {{16}, {1, 2, 3, 4}}}
```

In[761]:=

```
η16 =
  DiagonalMatrix[Table[FullSimplify[1/4 Tr[symm4by4⟦A, 1⟧.symm4by4⟦A, 1⟧]],
    {A, Length[symm4by4]}]];
Tr[η16]
```

Out[761]=

```
10
```

O(4,4); evalues, evecs of σ

In[762]:=

```
{evalues, evecs} = Eigensystem[σ]
```

Out[762]=

```
{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
  {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
  {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}
```

In[763]:=

$$\left(𝕦 = \text{ExpandAll}\left[\frac{1}{\sqrt{2}}\ \text{evecs}\right]\right)\ //\ \text{MatrixForm}$$

Out[763]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

In[764]:=

```
Table[𝕦⟦h⟧.σ.𝕦⟦h⟧, {h, 1, Length[𝕦]}]
```

Out[764]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[765]:=

```
Table[(Transpose[𝕦]⟦h⟧).σ.𝕦⟦h⟧, {h, 1, Length[𝕦]}]
```

Out[765]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[766]:=

```
Table[(Transpose[𝕦]⟦A⟧) == -σ.𝕦⟦A⟧, {A, 1, 4}]
Table[(Transpose[𝕦]⟦A⟧) == σ.𝕦⟦A⟧, {A, 5, Length[𝕦]}]
```

Out[766]=

```
{True, True, True, True}
```

Out[767]=

```
{True, True, True, True}
```

In[768]:=

```
hUSE = 8;
```

In[769]:=

```
Protect[hUSE]
```

Out[769]=

```
{hUSE}
```

In[770]:=

```
unit = 𝕦⟦hUSE⟧
```

Out[770]=

$$\left\{\frac{1}{\sqrt{2}},\ 0,\ 0,\ 0,\ \frac{1}{\sqrt{2}},\ 0,\ 0,\ 0\right\}$$

In[771]:=

```
Protect[unit]
```

Out[771]=

```
{unit}
```

In[772]:=

```
(Transpose[unit].σ.unit)
```

Out[772]=

```
1
```

In[773]:=

```
(Transpose[unit].σ.(τ̄[0]))
```

Out[773]=

$$\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}$$

In[774]:=

```
myid[MX_] := Sum[τ̄[A].MX.τ[A] η4488⟦A, A⟧, {A, 1, 8}]
Myid[MX_] := Sum[τ[A].MX.τ̄[A] × η448⟦A,  A⟧, {A, 1, 8}]
```

```
Union[Table[
  Sum[τ[A].bas64⟦k, 1⟧.σ.τ̄[A] × η8⟦A,  A⟧, {A, 1, 8}] - Id Tr[bas64⟦k, 1⟧.σ ] +
           bas64⟦k, 1⟧ - Transpose[bas64⟦k, 1⟧]
   4 eg⟦k⟧ ──────────────────────────────────── .σ, {k, 1, 64}]]
                            2
```

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}
```

In[776]:=

```
τ̄[0]
```

Out[776]=

```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[777]:=

```
(Transpose[unit].σ.(τ̄[0]))
```

Out[777]=

$$\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}$$

In[778]:=

```
F^a_A = Transpose[η4488⟦# + 1, # + 1⟧ * (τ[#].unit) & /@ Range[0, 7]]
```

Out[778]=

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\},\right.$$
$$\left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\},$$
$$\left.\left\{0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}\right\}$$

In[779]:=

```
F_a^A = Transpose[unit].σ.(τ̄[#]) & /@ Range[0, 7]
```

Out[779]=

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\},\right.$$
$$\left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\},$$
$$\left.\left\{0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\}\right\}$$

In[780]:=

```
F_A^a.F_a^A === ID8
```

Out[780]=

```
True
```

pretend that the X are Minkowski Coordinates :

In[781]:=

```
Block[{U, u = U[#] & /@ Range[8], fx = F_A^a.X, eq, sx, su, ret},
 eq = And @@ Thread[u == fx];
 sx = Solve[eq, X][[-1]];
 su = Solve[eq, u][[-1]];
 ret = {sx, su};
 ret]
```

Out[781]=

$$\left\{\left\{x0 \to \frac{U[1]}{\sqrt{2}} + \frac{U[5]}{\sqrt{2}}, x1 \to -\frac{U[4]}{\sqrt{2}} - \frac{U[8]}{\sqrt{2}}, x2 \to \frac{U[3]}{\sqrt{2}} + \frac{U[7]}{\sqrt{2}},\right.\right.$$
$$x3 \to -\frac{U[2]}{\sqrt{2}} - \frac{U[6]}{\sqrt{2}}, x4 \to \frac{1}{2}\left(\sqrt{2}\,U[2] - \sqrt{2}\,U[6]\right), x5 \to \frac{1}{2}\left(-\sqrt{2}\,U[3] + \sqrt{2}\,U[7]\right),$$
$$\left. x6 \to \frac{1}{2}\left(-\sqrt{2}\,U[4] + \sqrt{2}\,U[8]\right), x7 \to \frac{1}{2}\left(\sqrt{2}\,U[1] - \sqrt{2}\,U[5]\right)\right\},$$
$$\left\{U[1] \to \frac{1}{2}\left(\sqrt{2}\,x0 + \sqrt{2}\,x7\right), U[2] \to \frac{1}{2}\left(-\sqrt{2}\,x3 + \sqrt{2}\,x4\right), U[3] \to \frac{1}{2}\left(\sqrt{2}\,x2 - \sqrt{2}\,x5\right),\right.$$
$$U[4] \to \frac{1}{2}\left(-\sqrt{2}\,x1 - \sqrt{2}\,x6\right), U[5] \to \frac{1}{2}\left(\sqrt{2}\,x0 - \sqrt{2}\,x7\right), U[6] \to \frac{1}{2}\left(-\sqrt{2}\,x3 - \sqrt{2}\,x4\right),$$
$$\left.\left. U[7] \to \frac{1}{2}\left(\sqrt{2}\,x2 + \sqrt{2}\,x5\right), U[8] \to \frac{1}{2}\left(-\sqrt{2}\,x1 + \sqrt{2}\,x6\right)\right\}\right\}$$

In[∘]:=

```
(sgtrye_α^(A) = Thread[Flatten[e_α^(A)] → Flatten[eAa04]]) (*//Column*)
```

In[782]:=

```
subsDefects
```

Out[782]=

$$\left\{\sqrt{e^{2\,a4[H\,x4]}}\to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}}\to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\ \mathrm{Sin}[6\,H\,x4]^{1/3}}\to e^{a4[H\,x4]}\ \mathrm{Sin}[6\,H\,x4]^{1/6},\right.$$
$$\frac{1}{\sqrt{e^{2\,a4[H\,x4]}\ \mathrm{Sin}[6\,H\,x4]^{1/3}}}\to\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x4]^{1/6}},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},$$
$$\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{e^{a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{a4[H\,x4]}\ \mathrm{Sin}[6\,H\,x0]^{1/6},$$
$$\left.\sqrt{e^{-2\,a4[H\,x4]}}\ \mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{-a4[H\,x4]}\ \mathrm{Sin}[6\,H\,x0]^{1/6}\right\}$$

In[783]:=

```
Thread[X → 0]
```

Out[783]=

```
{x0 → 0, x1 → 0, x2 → 0, x3 → 0, x4 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Stop pretending :

In[784]:=

$$F^{a}_{A}.e^{(A)}_{\alpha}.X\ /.\ \mathrm{sgtrye}^{(A)}_{\alpha}\ //\ \mathrm{FullSimplify}[\#,\ \mathrm{constraintVars}]\ \&;$$
$$\mathrm{frameOctadX}=\#\ /.\ \left\{\sqrt{e^{2\,a4[H\,x4]}}\to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}}\to e^{-a4[H\,x4]}\right\}\ \&\ /@\ \%$$
$$\%\ /.\ \{x1\to 0,\ x2\to 0,\ x3\to 0,\ x5\to 0,\ x6\to 0,\ x7\to 0\}$$

Out[785]=

$$\left\{\frac{x0\ \mathrm{Cot}[6\,H\,x0]+e^{-a4[H\,x4]}\ x7\ \mathrm{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{x4-e^{a4[H\,x4]}\ x3\ \mathrm{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\right.$$
$$\frac{\left(e^{a4[H\,x4]}\ x2-e^{-a4[H\,x4]}\ x5\right)\ \mathrm{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{\left(e^{a4[H\,x4]}\ x1+e^{-a4[H\,x4]}\ x6\right)\ \mathrm{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$
$$\frac{x0\ \mathrm{Cot}[6\,H\,x0]-e^{-a4[H\,x4]}\ x7\ \mathrm{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{x4+e^{a4[H\,x4]}\ x3\ \mathrm{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$
$$\left.\frac{\left(e^{a4[H\,x4]}\ x2+e^{-a4[H\,x4]}\ x5\right)\ \mathrm{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{\left(-e^{a4[H\,x4]}\ x1+e^{-a4[H\,x4]}\ x6\right)\ \mathrm{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}\right\}$$

Out[786]=

$$\left\{\frac{x0\ \mathrm{Cot}[6\,H\,x0]}{\sqrt{2}},\ \frac{x4}{\sqrt{2}},\ 0,\ 0,\ \frac{x0\ \mathrm{Cot}[6\,H\,x0]}{\sqrt{2}},\ -\frac{x4}{\sqrt{2}},\ 0,\ 0\right\}$$

In[787]:=

$$F^{a}_{A}.eA\alpha.X-\mathrm{frameOctadX}\ //\ \mathrm{FullSimplify}[\#,\ \mathrm{constraintVars}]\ \&$$

Out[787]=

$$\left\{\left\{\left\{\frac{1}{\sqrt{2}},0,0,0,0,0,0,\frac{1}{\sqrt{2}}\right\},\left\{0,0,0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,0,0\right\},\right.\right.$$
$$\left\{0,0,\frac{1}{\sqrt{2}},0,0,-\frac{1}{\sqrt{2}},0,0\right\},\left\{0,-\frac{1}{\sqrt{2}},0,0,0,0,-\frac{1}{\sqrt{2}},0\right\},$$
$$\left\{\frac{1}{\sqrt{2}},0,0,0,0,0,0,-\frac{1}{\sqrt{2}}\right\},\left\{0,0,0,-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0,0,0\right\},$$
$$\left.\left\{0,0,\frac{1}{\sqrt{2}},0,0,\frac{1}{\sqrt{2}},0,0\right\},\left\{0,-\frac{1}{\sqrt{2}},0,0,0,0,\frac{1}{\sqrt{2}},0\right\}\right\}.eA\alpha.$$

$$\{x0, x1, x2, x3, x4, x5, x6, x7\} - \frac{x0\,\text{Cot}[6\,H\,x0] + e^{-a4[H\,x4]}\,x7\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}\right\}.$$

$$eA\alpha.\{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{-x4 + e^{a4[H\,x4]}\,x3\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}\right\}.$$

$$eA\alpha.\{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{e^{-a4[H\,x4]}\left(-e^{2\,a4[H\,x4]}\,x2 + x5\right)\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}\right\}.$$

$$eA\alpha.\{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{\left(e^{a4[H\,x4]}\,x1 + e^{-a4[H\,x4]}\,x6\right)\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}\right\}.$$

$$eA\alpha.\{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{-x0\,\text{Cot}[6\,H\,x0] + e^{-a4[H\,x4]}\,x7\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0\right\}, \left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0\right\}, \left\{0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0\right\}, \left\{0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}\right\}.$$

$$\mathrm{eA}\alpha.\{\mathrm{x0},\ \mathrm{x1},\ \mathrm{x2},\ \mathrm{x3},\ \mathrm{x4},\ \mathrm{x5},\ \mathrm{x6},\ \mathrm{x7}\}+\frac{\mathrm{x4}+e^{\mathrm{a4}[\mathrm{H\,x4}]}\ \mathrm{x3}\ \mathrm{Sin}[6\ \mathrm{H\,x0}]^{1/6}}{\sqrt{2}},$$

$$\left\{\left\{\frac{1}{\sqrt{2}},0,0,0,0,0,0,\frac{1}{\sqrt{2}}\right\},\left\{0,0,0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,0,0\right\},\left\{0,0,\frac{1}{\sqrt{2}},0,0,-\frac{1}{\sqrt{2}},0,0\right\},\left\{0,-\frac{1}{\sqrt{2}},0,0,0,0,-\frac{1}{\sqrt{2}},0\right\},\left\{\frac{1}{\sqrt{2}},0,0,0,0,0,0,-\frac{1}{\sqrt{2}}\right\},\left\{0,0,0,-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0,0,0\right\},\left\{0,0,\frac{1}{\sqrt{2}},0,0,\frac{1}{\sqrt{2}},0,0\right\},\left\{0,-\frac{1}{\sqrt{2}},0,0,0,0,\frac{1}{\sqrt{2}},0\right\}\right\}.$$

$$\mathrm{eA}\alpha.\{\mathrm{x0},\ \mathrm{x1},\ \mathrm{x2},\ \mathrm{x3},\ \mathrm{x4},\ \mathrm{x5},\ \mathrm{x6},\ \mathrm{x7}\}-\frac{\left(e^{\mathrm{a4}[\mathrm{H\,x4}]}\ \mathrm{x2}+e^{-\mathrm{a4}[\mathrm{H\,x4}]}\ \mathrm{x5}\right)\ \mathrm{Sin}[6\ \mathrm{H\,x0}]^{1/6}}{\sqrt{2}},$$

$$\left\{\left\{\frac{1}{\sqrt{2}},0,0,0,0,0,0,\frac{1}{\sqrt{2}}\right\},\left\{0,0,0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,0,0\right\},\left\{0,0,\frac{1}{\sqrt{2}},0,0,-\frac{1}{\sqrt{2}},0,0\right\},\left\{0,-\frac{1}{\sqrt{2}},0,0,0,0,-\frac{1}{\sqrt{2}},0\right\},\left\{\frac{1}{\sqrt{2}},0,0,0,0,0,0,-\frac{1}{\sqrt{2}}\right\},\left\{0,0,0,-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0,0,0\right\},\left\{0,0,\frac{1}{\sqrt{2}},0,0,\frac{1}{\sqrt{2}},0,0\right\},\left\{0,-\frac{1}{\sqrt{2}},0,0,0,0,\frac{1}{\sqrt{2}},0\right\}\right\}.$$

$$\mathrm{eA}\alpha.\{\mathrm{x0},\ \mathrm{x1},\ \mathrm{x2},\ \mathrm{x3},\ \mathrm{x4},\ \mathrm{x5},\ \mathrm{x6},\ \mathrm{x7}\}+\frac{e^{-\mathrm{a4}[\mathrm{H\,x4}]}\ \left(e^{2\ \mathrm{a4}[\mathrm{H\,x4}]}\ \mathrm{x1}-\mathrm{x6}\right)\ \mathrm{Sin}[6\ \mathrm{H\,x0}]^{1/6}}{\sqrt{2}}\Bigg\}$$

In[788]:=

$$\left\{\left\{\mathrm{F}^{\mathrm{a}}_{\mathrm{A}}\right\},\ \{\mathrm{eA}\alpha\},\ \{\mathrm{X}\}\right\}\ //\ \mathrm{Column}$$

Out[788]=

$$\left\{\left\{\left\{\frac{1}{\sqrt{2}},0,0,0,0,0,0,\frac{1}{\sqrt{2}}\right\},\left\{0,0,0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,0,0\right\},\left\{0,0,\frac{1}{\sqrt{2}},0,0,-\frac{1}{\sqrt{2}},0,0\right\},\left\{0,-\frac{1}{\sqrt{2}},0,0,0,0,-\frac{1}{\sqrt{2}},0\right\},\left\{\frac{1}{\sqrt{2}},0,0,0,0,0,0,-\frac{1}{\sqrt{2}}\right\},\left\{0,0,0,-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0,0,0\right\},\left\{0,0,\frac{1}{\sqrt{2}},0,0,\frac{1}{\sqrt{2}},0,0\right\},\left\{0,-\frac{1}{\sqrt{2}},0,0,0,0,\frac{1}{\sqrt{2}},0\right\}\right\}\right\}$$

$$\{\mathrm{eA}\alpha\}$$

$$\{\{\mathrm{x0},\ \mathrm{x1},\ \mathrm{x2},\ \mathrm{x3},\ \mathrm{x4},\ \mathrm{x5},\ \mathrm{x6},\ \mathrm{x7}\}\}$$

In[789]:=

```
{x2u, u2x} = Block[{U, u = U[#] & /@ Range[8],
    fx = frameOctadX, eq, sx, su, ret, altEQ, sCot, skot},
   altEQ = And @@ {Cot[6 H x0] == kot, Sin[6 H x0] == sin, a4[H x4] == A44};
   sCot = Solve[altEQ, {Cot[6 H x0], Sin[6 H x0], a4[H x4]}]〚1〛;
   skot = Solve[altEQ, {kot, sin, A44}]〚1〛;
   eq = And @@ Thread[u == (fx /. sCot)];
   sx = Solve[eq, X]〚-1〛;
   su = Solve[eq, u]〚-1〛;
   ret = {sx /. skot // FullSimplify, su /. skot // ExpandAll};
   ret]
```

Out[789]=

$$
\Big\{\Big\{\mathrm{x0}\to\frac{\mathrm{Tan}[6\,\mathrm{H}\,\mathrm{x0}]\;(\mathrm{U}[1]+\mathrm{U}[5])}{\sqrt{2}},\;\mathrm{x1}\to-\frac{e^{-\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;(\mathrm{U}[4]+\mathrm{U}[8])}{\sqrt{2}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}},
$$

$$
\mathrm{x2}\to\frac{e^{-\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;(\mathrm{U}[3]+\mathrm{U}[7])}{\sqrt{2}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}},\;\mathrm{x3}\to-\frac{e^{-\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;(\mathrm{U}[2]+\mathrm{U}[6])}{\sqrt{2}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}},\;\mathrm{x4}\to\frac{\mathrm{U}[2]-\mathrm{U}[6]}{\sqrt{2}},
$$

$$
\mathrm{x5}\to\frac{e^{\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;(-\mathrm{U}[3]+\mathrm{U}[7])}{\sqrt{2}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}},\;\mathrm{x6}\to\frac{e^{\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;(-\mathrm{U}[4]+\mathrm{U}[8])}{\sqrt{2}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}},\;\mathrm{x7}\to\frac{e^{\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;(\mathrm{U}[1]-\mathrm{U}[5])}{\sqrt{2}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}\Big\},
$$

$$
\Big\{\mathrm{U}[1]\to\frac{\mathrm{x0}\;\mathrm{Cot}[6\,\mathrm{H}\,\mathrm{x0}]}{\sqrt{2}}+\frac{e^{-\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x7}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}},\;\mathrm{U}[2]\to\frac{\mathrm{x4}}{\sqrt{2}}-\frac{e^{\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x3}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}},
$$

$$
\mathrm{U}[3]\to\frac{e^{\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x2}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}}-\frac{e^{-\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x5}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}},
$$

$$
\mathrm{U}[4]\to-\frac{e^{\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x1}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}}-\frac{e^{-\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x6}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}},
$$

$$
\mathrm{U}[5]\to\frac{\mathrm{x0}\;\mathrm{Cot}[6\,\mathrm{H}\,\mathrm{x0}]}{\sqrt{2}}-\frac{e^{-\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x7}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}},\;\mathrm{U}[6]\to-\frac{\mathrm{x4}}{\sqrt{2}}-\frac{e^{\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x3}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}},
$$

$$
\mathrm{U}[7]\to\frac{e^{\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x2}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}}+\frac{e^{-\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x5}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}},
$$

$$
\mathrm{U}[8]\to-\frac{e^{\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x1}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}}+\frac{e^{-\mathrm{a4}[\mathrm{H}\,\mathrm{x4}]}\;\mathrm{x6}\;\mathrm{Sin}[6\,\mathrm{H}\,\mathrm{x0}]^{1/6}}{\sqrt{2}}\Big\}\Big\}
$$

In[790]:=

```
u2x
```

Out[790]=

$$\left\{U[1]\to \frac{x0\,\text{Cot}[6\,H\,x0]}{\sqrt{2}}+\frac{e^{-a4[H\,x4]}\,x7\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\; U[2]\to \frac{x4}{\sqrt{2}}-\frac{e^{a4[H\,x4]}\,x3\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\right.$$

$$U[3]\to \frac{e^{a4[H\,x4]}\,x2\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}-\frac{e^{-a4[H\,x4]}\,x5\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$

$$U[4]\to -\frac{e^{a4[H\,x4]}\,x1\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}-\frac{e^{-a4[H\,x4]}\,x6\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$

$$U[5]\to \frac{x0\,\text{Cot}[6\,H\,x0]}{\sqrt{2}}-\frac{e^{-a4[H\,x4]}\,x7\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\; U[6]\to -\frac{x4}{\sqrt{2}}-\frac{e^{a4[H\,x4]}\,x3\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$

$$U[7]\to \frac{e^{a4[H\,x4]}\,x2\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}+\frac{e^{-a4[H\,x4]}\,x5\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$

$$\left.U[8]\to -\frac{e^{a4[H\,x4]}\,x1\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}+\frac{e^{-a4[H\,x4]}\,x6\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}\right\}$$

In[791]:=

```
{sumu2x, diffu2x} = Block[{U, u, a1, a2, a3 = {}, aa4 = {}},
  (*Clear[U,u,a1,a2,a3,a4];*)
  u = U[#] & /@ Range[8]; Print[u];
  a1 = u /. u2x; Print[a1];
  a2 = Transpose[{{Take[a1, 4]}, {Take[a1, -4]}}]; Print[a2];
  a3 = ExpandAll[FullSimplify[(#⟦1⟧ + #⟦2⟧)/2]] & /@ a2;
  aa4 = ExpandAll[FullSimplify[(#⟦1⟧ - #⟦2⟧)/2]] & /@ a2;
  {a3, aa4}]
```

```
{U[1], U[2], U[3], U[4], U[5], U[6], U[7], U[8]}
```

$$\Big\{\frac{x0\,\text{Cot}[6\,H\,x0]}{\sqrt{2}}+\frac{e^{-a4[H\,x4]}\,x7\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{x4}{\sqrt{2}}-\frac{e^{a4[H\,x4]}\,x3\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{e^{a4[H\,x4]}\,x2\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}-\frac{e^{-a4[H\,x4]}\,x5\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{e^{a4[H\,x4]}\,x1\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}-\frac{e^{-a4[H\,x4]}\,x6\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{x0\,\text{Cot}[6\,H\,x0]}{\sqrt{2}}-\frac{e^{-a4[H\,x4]}\,x7\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{x4}{\sqrt{2}}-\frac{e^{a4[H\,x4]}\,x3\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{e^{a4[H\,x4]}\,x2\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}+\frac{e^{-a4[H\,x4]}\,x5\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{e^{a4[H\,x4]}\,x1\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}+\frac{e^{-a4[H\,x4]}\,x6\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}\Big\}$$

$$\Big\{\Big\{\Big\{\frac{x0\,\text{Cot}[6\,H\,x0]}{\sqrt{2}}+\frac{e^{-a4[H\,x4]}\,x7\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{x4}{\sqrt{2}}-\frac{e^{a4[H\,x4]}\,x3\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{e^{a4[H\,x4]}\,x2\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}-\frac{e^{-a4[H\,x4]}\,x5\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{e^{a4[H\,x4]}\,x1\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}-\frac{e^{-a4[H\,x4]}\,x6\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}\Big\},\ \Big\{\frac{x0\,\text{Cot}[6\,H\,x0]}{\sqrt{2}}-\frac{e^{-a4[H\,x4]}\,x7\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{x4}{\sqrt{2}}-\frac{e^{a4[H\,x4]}\,x3\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{e^{a4[H\,x4]}\,x2\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}+\frac{e^{-a4[H\,x4]}\,x5\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{e^{a4[H\,x4]}\,x1\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}+\frac{e^{-a4[H\,x4]}\,x6\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}\Big\}\Big\}\Big\}$$

Out[791]=

$$\Big\{\Big\{\Big\{\frac{x0\,\text{Cot}[6\,H\,x0]}{\sqrt{2}},\ -\frac{e^{a4[H\,x4]}\,x3\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{e^{a4[H\,x4]}\,x2\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{e^{a4[H\,x4]}\,x1\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}\Big\}\Big\},\ \Big\{\Big\{\frac{e^{-a4[H\,x4]}\,x7\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{x4}{\sqrt{2}},\ -\frac{e^{-a4[H\,x4]}\,x5\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{e^{-a4[H\,x4]}\,x6\,\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}\Big\}\Big\}\Big\}$$

---

split octonions; evalues, evecs of σ

```
# Type - 1 spinor basis (constructed from Clifford algebra considerations)
# These create a maximal isotropic subspace with respect to the split form
self . type1_spinor _basis _matrix = sqrt2_inv*np . array ([[1, 1, 0, 0, 0, 0, 0, 0], # s1_ 0[1, -1, 0, 0, 0, 0, 0, 0], # s1_ 1[0, 0, 1, 1, 0, 0, 0, 0], # s1_ 2[0, 0, 1, -1, 0, 0, 0, 0], # s1_ 3[0, 0, 0, 0, 1, 1, 0, 0], # s1_ 4[0, 0, 0, 0, 1, -1, 0, 0], # s1_ 5[0, 0, 0, 0, 0, 0, 1, 1], # s1_ 6[0, 0, 0, 0, 0, 0, 1, -1], # s1_ 7])

# Type - 2 spinor basis (the complementary spinor representation)
# Related to Type - 1 by another triality automorphism
```

self . type2_spinor _basis _matrix = sqrt2_inv*np . array ([[1, 0, 1, 0, 0, 0, 0, 0], # s2_ 0[1, 0, -1, 0, 0, 0, 0, 0], # s2_ 1[0, 1, 0, 1, 0, 0, 0, 0], # s2_ 2[0, 1, 0, -1, 0, 0, 0, 0], # s2_ 3[0, 0, 0, 0, 1, 0, 1, 0], # s2_ 4[0, 0, 0, 0, 1, 0, -1, 0], # s2_ 5[0, 0, 0, 0, 0, 1, 0, 1], # s2_ 6[0, 0, 0, 0, 0, 1, 0, -1], # s2_ 7])

In[792]:=

$$M_{S_1\to S_2} = \frac{1}{2}\begin{matrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{matrix}$$

In[793]:=

4 Transformation Matrices

We provide explicit 8 × 8 matrices that transform coordinates between the three bases.

4.1 Vector to Type-1 Spinor

The transformation matrix from vector basis to type-1 spinor basis is:

$$M_{V\to S_1} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \tag{11}$$

4.2 Vector to Type-2 Spinor

The transformation matrix from vector basis to type-2 spinor basis is:

$$M_{V\to S_2} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \tag{12}$$

In[794]:=

```
(type1SpinorBasisMatrix =
   {{1, 1, 0, 0, 0, 0, 0, 0}, {1, -1, 0, 0, 0, 0, 0, 0},
    {0, 0, 1, 1, 0, 0, 0, 0}, {0, 0, 1, -1, 0, 0, 0, 0},
    {0, 0, 0, 0, 1, 1, 0, 0}, {0, 0, 0, 0, 1, -1, 0, 0},
    {0, 0, 0, 0, 0, 0, 1, 1}, {0, 0, 0, 0, 0, 0, 1, -1}}) // MatrixForm
```

Out[794]//MatrixForm=

```
1  1  0  0  0  0  0  0
1 -1  0  0  0  0  0  0
0  0  1  1  0  0  0  0
0  0  1 -1  0  0  0  0
0  0  0  0  1  1  0  0
0  0  0  0  1 -1  0  0
0  0  0  0  0  0  1  1
0  0  0  0  0  0  1 -1
```

In[795]:=

```
(type2SpinorBasisMatrix =
   {{1, 0, 1, 0, 0, 0, 0, 0}, {1, 0, -1, 0, 0, 0, 0, 0},
    {0, 1, 0, 1, 0, 0, 0, 0}, {0, 1, 0, -1, 0, 0, 0, 0},
    {0, 0, 0, 0, 1, 0, 1, 0}, {0, 0, 0, 0, 1, 0, -1, 0},
    {0, 0, 0, 0, 0,  1, 0, 1}, {0, 0, 0, 0, 0,  1, 0, -1}}) // MatrixForm
```

Out[795]//MatrixForm=

```
1 0  1   0 0 0  0  0
1 0 -1   0 0 0  0  0
0 1  0   1 0 0  0  0
0 1  0  -1 0 0  0  0
0 0  0   0 1 0  1  0
0 0  0   0 1 0 -1  0
0 0  0   0 0 1  0  1
0 0  0   0 0 1  0 -1
```

In[796]:=

$$M_{S_1\rightarrow S_2}=\frac{1}{2}\begin{matrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{matrix}$$

$$M_{S_1\to S_2}=\frac{1}{2}\begin{bmatrix}1&1&1&1&0&0&0&0\\1&1&-1&-1&0&0&0&0\\1&-1&1&-1&0&0&0&0\\1&-1&-1&1&0&0&0&0\\0&0&0&0&1&1&1&1\\0&0&0&0&1&1&-1&-1\\0&0&0&0&1&-1&1&-1\\0&0&0&0&1&-1&-1&1\end{bmatrix}$$

```
In[797]:=
MS1S2 = type1SpinorBasisMatrix.Transpose[type2SpinorBasisMatrix] // MatrixForm
Out[797]//MatrixForm=
 1  1   1   1  0  0   0   0
 1  1  -1  -1  0  0   0   0
 1 -1   1  -1  0  0   0   0
 1 -1  -1   1  0  0   0   0
 0  0   0   0  1  1   1   1
 0  0   0   0  1  1  -1  -1
 0  0   0   0  1 -1   1  -1
 0  0   0   0  1 -1  -1   1
In[798]:=
gAB = η4488
Out[798]=
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
In[799]:=
{evaluesAB, evecsAB} = Eigensystem[gAB]
Out[799]=
{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}}}
In[800]:=
{evalues, evecs} = Eigensystem[σ]
Out[800]=
{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
  {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
  {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}
```

In[801]:=

$$\left(\mathfrak{u} = \texttt{ExpandAll}\left[\frac{1}{\sqrt{2}}\ \texttt{evecs}\right]\right)\ \texttt{// MatrixForm}$$

Out[801]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

In[802]:=

```
Table[𝔲⟦h⟧.σ.𝔲⟦h⟧, {h, 1, Length[𝔲]}]
```

Out[802]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[803]:=

```
Table[(Transpose[𝔲]⟦h⟧).σ.𝔲⟦h⟧, {h, 1, Length[𝔲]}]
```

Out[803]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[804]:=

```
Table[(Transpose[𝔲]⟦A⟧) === -σ.𝔲⟦A⟧, {A, 1, 4}]
Table[(Transpose[𝔲]⟦A⟧) === σ.𝔲⟦A⟧, {A, 5, Length[𝔲]}]
```

Out[804]=

```
{True, True, True, True}
```

Out[805]=

```
{True, True, True, True}
```

In[806]:=

```
hUSE = 8;
```

Set: Symbol hUSE is Protected.

In[807]:=

```
Protect[hUSE]
```

Out[807]=

```
{}
```

In[808]:=

```
unit = 𝔲⟦hUSE⟧
```

Set: Symbol unit is Protected.

Out[808]=

$$\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}$$

```
In[809]:=
    Protect[unit]
Out[809]=
    {}
In[810]:=
    (Transpose[unit].σ.unit)
Out[810]=
    1
In[811]:=
    (Transpose[unit].σ.(τ̄[0]))
Out[811]=
```

$$\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}$$

```
In[812]:=
    Do[E^A_a[h] = Table[(𝕦⟦h⟧.σ.(τ̄[B])), {B, 0, 7}], {h, 1, Length[𝕦]}];
```

$$\mathfrak{F}^A_a = \frac{1}{\sqrt{\tilde{u}\sigma u}} u^c \sigma_{cb} \bar{\tau}^{Ab}{}_a$$

```
    Do[E^a_A[h] = FullSimplify[Inverse[E^A_a[h]]], {h, 1, Length[𝕦]}];
```

$$\mathfrak{F}^a_A = \frac{1}{\sqrt{\tilde{u}\sigma u}} \tau_A{}^a{}_b u^b$$

```
    Table[E^a_A[h] === (𝕦⟦h⟧.σ.𝕦⟦h⟧)
       Transpose[Table[FullSimplify[ExpandAll[ η4488⟦B, B⟧ × τ[B - 1].𝕦⟦h⟧ ]],
         {B, 1, 8}]], {h, 1, Length[𝕦]}]
Out[812]=
```

$$\mathfrak{F}^A_a = \frac{1}{\sqrt{\tilde{u}\sigma u}} u^c \sigma_{cb} \bar{\tau}^{Ab}{}_a$$

```
Out[813]=
```

$$\mathfrak{F}^a_A = \frac{1}{\sqrt{\tilde{u}\sigma u}} \tau_A{}^a{}_b u^b$$

```
Out[814]=
    {True, True, True, True, True, True, True, True}
In[815]:=
    (*Do[E^A_a[h]=Table[(u⟦h⟧.σ.(τ̄[B])),{B,0,7}],{h,1,Length[u]}];
    Do[E^a_A[h]=FullSimplify[Inverse[E^A_a[h]]],{h,1,Length[u]}];
     Table[E^a_A[h]=== (u⟦h⟧.σ.u⟦h⟧)
       Transpose[Table[FullSimplify[ExpandAll[ η4488⟦B,B⟧τ[B-1].u⟦h⟧ ]],
         {B,1,8}]],{h,1,Length[u]}]*)
In[816]:=
    myid[MX_] := Sum[τ̄[A].MX.τ[A] η4488⟦A, A⟧, {A, 1, 8}]
    Myid[MX_] := Sum[τ[A].MX.τ̄[A] × η448⟦A,  A⟧, {A, 1, 8}]

    Union[Table[
      Sum[τ[A].bas64⟦k, 1⟧.σ.τ̄[A] × η8⟦A,  A⟧, {A, 1, 8}] - Id Tr[bas64⟦k, 1⟧.σ ] +
```

```
        bas64⟦k, 1⟧ - Transpose[bas64⟦k, 1⟧]
4 eg⟦k⟧ ──────────────────────────────────── .σ, {k, 1, 64}]]
                         2
```

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}
```

In[818]:=

```
τ̅[0]
```

Out[818]=

```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[819]:=

```
(Transpose[unit].σ.(τ̅[0]))
```

Out[819]=

```
{1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
```

In[820]:=

```
F^a_A = Transpose[η4488⟦# + 1, # + 1⟧ * (τ[#].unit) & /@ Range[0, 7]]
```

Out[820]=

```
{{1/√2, 0, 0, 0, 0, 0, 0, 1/√2}, {0, 0, 0, -1/√2, 1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, -1/√2, 0, 0},
 {0, -1/√2, 0, 0, 0, 0, -1/√2, 0}, {1/√2, 0, 0, 0, 0, 0, 0, -1/√2},
 {0, 0, 0, -1/√2, -1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, 1/√2, 0, 0}, {0, -1/√2, 0, 0, 0, 0, 1/√2, 0}}
```

In[821]:=

```
F^A_a = Transpose[unit].σ.(τ̅[#]) & /@ Range[0, 7]
```

Out[821]=

```
{{1/√2, 0, 0, 0, 1/√2, 0, 0, 0}, {0, 0, 0, -1/√2, 0, 0, 0, -1/√2},
 {0, 0, 1/√2, 0, 0, 0, 1/√2, 0}, {0, -1/√2, 0, 0, 0, -1/√2, 0, 0}, {0, 1/√2, 0, 0, 0, -1/√2, 0, 0},
 {0, 0, -1/√2, 0, 0, 0, 1/√2, 0}, {0, 0, 0, -1/√2, 0, 0, 0, 1/√2}, {1/√2, 0, 0, 0, -1/√2, 0, 0, 0}}
```

In[822]:=

```
F^a_A.F^A_a === ID8
```

Out[822]=

```
True
```

```
gtrye^(A)_α = (e^(A)_α /. sgtrye^(A)_α) /. subsDefects
gtrye^α_(A) = (e^α_(A) /. sgtrye^α_(A)) /. subsDefects
```

```
(sgtrye^α_(A) = Thread[Flatten[e^α_(A)] → Flatten[Inverse[e^(A)_α /. sgtrye^(A)_α]]])
(*//Column*)
```

$$\text{gtrye}^{\alpha}_{(A)} = \left(e^{\alpha}_{(A)}\ \text{/. sgtrye}^{\alpha}_{(A)}\right)\ \text{/. subsDefects}$$

Out[∘]=

$$\left\{\{\text{Tan}[6\,H\,x0],0,0,0,0,0,0,0\},\left\{0,\frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0,0\right\},\left\{0,0,\frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0\right\},\left\{0,0,0,\frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0,0,0\right\},\{0,0,0,0,1,0,0,0\},\left\{0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0,0\right\},\left\{0,0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}},0\right\},\left\{0,0,0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}}\right\}\right\}$$

pretend that the X are Minkowski Coordinates :

In[823]:=

```
Block[{U, u = U[#] & /@ Range[8], fx = F^a_A.X, eq, sx, su, ret},
 eq = And @@ Thread[u == fx];
 sx = Solve[eq, X][[-1]];
 su = Solve[eq, u][[-1]];
 ret = {sx, su};
 ret]
```

Out[823]=

$$\left\{\left\{x0\to\frac{U[1]}{\sqrt{2}}+\frac{U[5]}{\sqrt{2}},\ x1\to-\frac{U[4]}{\sqrt{2}}-\frac{U[8]}{\sqrt{2}},\ x2\to\frac{U[3]}{\sqrt{2}}+\frac{U[7]}{\sqrt{2}},\ x3\to-\frac{U[2]}{\sqrt{2}}-\frac{U[6]}{\sqrt{2}},\ x4\to\frac{1}{2}\left(\sqrt{2}\,U[2]-\sqrt{2}\,U[6]\right),\ x5\to\frac{1}{2}\left(-\sqrt{2}\,U[3]+\sqrt{2}\,U[7]\right),\ x6\to\frac{1}{2}\left(-\sqrt{2}\,U[4]+\sqrt{2}\,U[8]\right),\ x7\to\frac{1}{2}\left(\sqrt{2}\,U[1]-\sqrt{2}\,U[5]\right)\right\},\left\{U[1]\to\frac{1}{2}\left(\sqrt{2}\,x0+\sqrt{2}\,x7\right),\ U[2]\to\frac{1}{2}\left(-\sqrt{2}\,x3+\sqrt{2}\,x4\right),\ U[3]\to\frac{1}{2}\left(\sqrt{2}\,x2-\sqrt{2}\,x5\right),\ U[4]\to\frac{1}{2}\left(-\sqrt{2}\,x1-\sqrt{2}\,x6\right),\ U[5]\to\frac{1}{2}\left(\sqrt{2}\,x0-\sqrt{2}\,x7\right),\ U[6]\to\frac{1}{2}\left(-\sqrt{2}\,x3-\sqrt{2}\,x4\right),\ U[7]\to\frac{1}{2}\left(\sqrt{2}\,x2+\sqrt{2}\,x5\right),\ U[8]\to\frac{1}{2}\left(-\sqrt{2}\,x1+\sqrt{2}\,x6\right)\right\}\right\}$$

In[∘]:=

$$\left(\text{sgtrye}^{(A)}_{\alpha} = \text{Thread}\left[\text{Flatten}\left[e^{(A)}_{\alpha}\right]\to\text{Flatten}[\text{eAa04}]\right]\right)\ \text{(*//Column*)}$$

In[824]:=

subsDefects

Out[824]=

$$\left\{\sqrt{e^{2\,a4[H\,x4]}}\to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}}\to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/6},\right.$$
$$\frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x4]^{1/3}}}\to\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x4]^{1/6}},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},$$
$$\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}}\to\frac{e^{a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},$$
$$\left.\sqrt{e^{-2\,a4[H\,x4]}}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\to e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\right\}$$

In[825]:=

Thread[X → 0]

Out[825]=

```
{x0 → 0, x1 → 0, x2 → 0, x3 → 0, x4 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Stop pretending :

$$\left(\mathrm{sgtrye}^{\alpha}_{(A)}=\mathrm{Thread}\left[\mathrm{Flatten}\left[e^{\alpha}_{(A)}\right]\to\mathrm{Flatten}\left[\mathrm{Inverse}\left[e^{(A)}_{\alpha}\ /.\ \mathrm{sgtrye}^{(A)}_{\alpha}\right]\right]\right]\right)$$

`(*//Column*)`

$$\mathrm{gtrye}^{\alpha}_{(A)}=\left(e^{\alpha}_{(A)}\ /.\ \mathrm{sgtrye}^{\alpha}_{(A)}\right)\ /.\ \mathrm{subsDefects}$$

$$\left\{\{\mathrm{Tan}[6\,H\,x0],0,0,0,0,0,0,0\},\left\{0,\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0,0\right\},\right.$$
$$\left\{0,0,\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},0,0,0,0,0\right\},\left\{0,0,0,\frac{e^{-a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},0,0,0,0\right\},$$
$$\{0,0,0,0,1,0,0,0\},\left\{0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},0,0\right\},$$
$$\left.\left\{0,0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}},0\right\},\left\{0,0,0,0,0,0,0,\frac{e^{a4[H\,x4]}}{\mathrm{Sin}[6\,H\,x0]^{1/6}}\right\}\right\}$$

In[826]:=

$$\boldsymbol{e^{(A)}_{\alpha}=\mathrm{eA}\alpha=\mathrm{gtrye}^{(A)}_{\alpha}}$$

Out[826]=

$$\left\{\{\mathrm{Cot}[6\,H\,x0],0,0,0,0,0,0,0\},\left\{0,e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0,0\right\},\right.$$
$$\left\{0,0,e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0,0\right\},\left\{0,0,0,e^{a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0,0,0\right\},$$
$$\{0,0,0,0,1,0,0,0\},\left\{0,0,0,0,0,e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0,0\right\},$$
$$\left.\left\{0,0,0,0,0,0,e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6},0\right\},\left\{0,0,0,0,0,0,0,e^{-a4[H\,x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/6}\right\}\right\}$$

In[∘]:= $\mathrm{gtrye}^{(A)}_{\alpha}=\left(e^{(A)}_{\alpha}\ /.\ \mathrm{sgtrye}^{(A)}_{\alpha}\right)\ /.\ \mathrm{subsDefects}$

In[∘]:= $\mathrm{gtrye}^{\alpha}_{(A)}=\left(e^{\alpha}_{(A)}\ /.\ \mathrm{sgtrye}^{\alpha}_{(A)}\right)\ /.\ \mathrm{subsDefects}$

In[827]:=

$$\{\text{gtrye}_{\alpha}^{(A)},\ \text{gtrye}^{\alpha}_{(A)}\}\ \text{// MatrixForm}$$

Out[827]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} \text{Cot}[6\,H\,x0] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ e^{a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ e^{a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ e^{a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} \text{Tan}[6\,H\,x0] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ \frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{e^{-a4[H\,x4]}}{\text{Sin}[6\,H\,x0]^{1/6}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix}$$

In[828]:=

$$F^{a}_{A}.e_{\alpha}^{(A)}.X\ /.\ \text{sgtrye}_{\alpha}^{(A)}\ \text{// FullSimplify[\#, constraintVars] \&;}$$
$$\text{frameOctadX} = \#\ /.\ \left\{\sqrt{e^{2\,a4[H\,x4]}} \to e^{a4[H\,x4]},\ \sqrt{e^{-2\,a4[H\,x4]}} \to e^{-a4[H\,x4]}\right\}\ \&\ /@\ \%$$
$$\%\ /.\ \{x1 \to 0,\ x2 \to 0,\ x3 \to 0,\ x5 \to 0,\ x6 \to 0,\ x7 \to 0\}$$

Out[829]=

$$\Big\{\frac{x0\ \text{Cot}[6\,H\,x0] + e^{-a4[H\,x4]}\ x7\ \text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{x4 - e^{a4[H\,x4]}\ x3\ \text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$
$$\frac{e^{-a4[H\,x4]}\left(e^{2\,a4[H\,x4]}\ x2 - x5\right)\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{e^{-a4[H\,x4]}\left(e^{2\,a4[H\,x4]}\ x1 + x6\right)\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$
$$\frac{x0\ \text{Cot}[6\,H\,x0] - e^{-a4[H\,x4]}\ x7\ \text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ -\frac{x4 + e^{a4[H\,x4]}\ x3\ \text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},$$
$$\frac{e^{-a4[H\,x4]}\left(e^{2\,a4[H\,x4]}\ x2 + x5\right)\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}},\ \frac{e^{-a4[H\,x4]}\left(-e^{2\,a4[H\,x4]}\ x1 + x6\right)\text{Sin}[6\,H\,x0]^{1/6}}{\sqrt{2}}\Big\}$$

Out[830]=

$$\left\{\frac{x0\ \text{Cot}[6\,H\,x0]}{\sqrt{2}},\ \frac{x4}{\sqrt{2}},\ 0,\ 0,\ \frac{x0\ \text{Cot}[6\,H\,x0]}{\sqrt{2}},\ -\frac{x4}{\sqrt{2}},\ 0,\ 0\right\}$$

In[831]:=

$$F^{a}_{A}.\text{eA}\alpha.X - \text{frameOctadX}\ \text{// FullSimplify[\#, constraintVars] \&}$$

Out[831]=

```
{0, 0, 0, 0, 0, 0, 0, 0}
```

In[832]:=

$$\text{eA}\alpha$$

Out[832]=

$$\{\{\text{Cot}[6\,H\,x0],\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ e^{a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\},$$
$$\{0,\ 0,\ e^{a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},\ 0,\ 0,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ e^{a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},\ 0,\ 0,\ 0,\ 0\},$$
$$\{0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ e^{-a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},\ 0,\ 0\},$$
$$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ e^{-a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6},\ 0\},\ \{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ e^{-a4[H\,x4]}\ \text{Sin}[6\,H\,x0]^{1/6}\}\}$$

In[833]:=

```
{{F^a_A}, {eAα}, {X}} // Column
```

Out[833]=

```
{{{1/√2, 0, 0, 0, 0, 0, 0, 1/√2}, {0, 0, 0, -1/√2, 1/√2, 0, 0, 0},
  {0, 0, 1/√2, 0, 0, -1/√2, 0, 0}, {0, -1/√2, 0, 0, 0, 0, -1/√2, 0}, {1/√2, 0, 0, 0, 0, 0, 0, -1/√2},
  {0, 0, 0, -1/√2, -1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, 1/√2, 0, 0}, {0, -1/√2, 0, 0, 0, 0, 1/√2, 0}}}
{{{Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0}, {0, e^(a4[H x4]) Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0},
  {0, 0, e^(a4[H x4]) Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, e^(a4[H x4]) Sin[6 H x0]^(1/6), 0, 0, 0, 0},
  {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e^(-a4[H x4]) Sin[6 H x0]^(1/6), 0, 0},
  {0, 0, 0, 0, 0, 0, e^(-a4[H x4]) Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, e^(-a4[H x4]) Sin[6 H x0]^(1/6)}}}
{{x0, x1, x2, x3, x4, x5, x6, x7}}
```

---

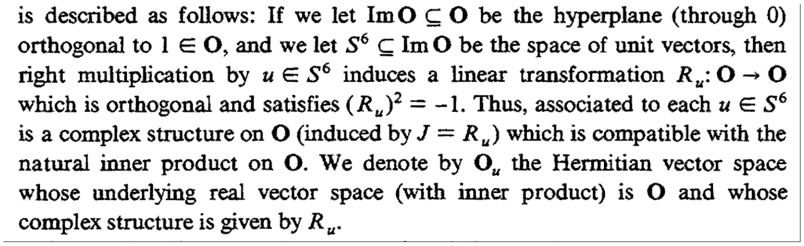
split octonion multiplication constants :

$$m_{ab}^{c} = \mathbb{F}_{a}^{A}\, \tau_{A}{}^{c}{}_{b}$$
$$m_{AB}^{C} = \mathbb{F}_{c}^{C}\, \tau_{A}{}^{c}{}_{b}\, \mathbb{F}_{B}^{b}$$

In[834]:=

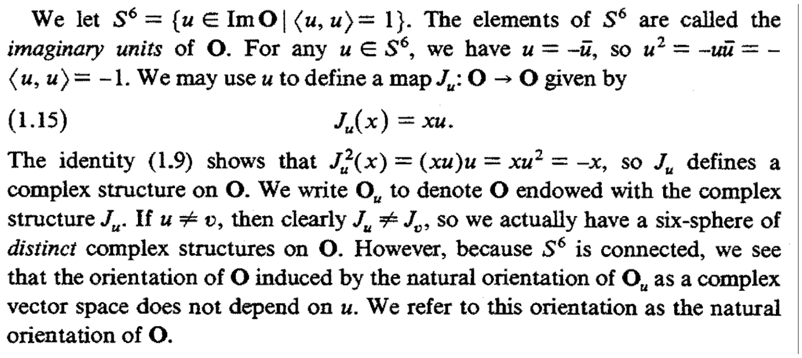
$$(xw)w = xw^2$$

In[835]:=



is described as follows: If we let $\mathrm{Im}\,\mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \mathrm{Im}\,\mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \to \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on $\mathbf{O}$ (induced by $J = R_u$) which is compatible with the natural inner product on $\mathbf{O}$. We denote by $\mathbf{O}_u$ the Hermitian vector space whose underlying real vector space (with inner product) is $\mathbf{O}$ and whose complex structure is given by R_u.

In[836]:=

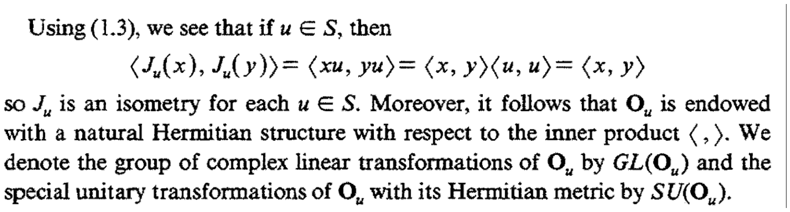


We let $S^6 = \{u \in \mathrm{Im}\,\mathbf{O} \mid \langle u, u\rangle = 1\}$. The elements of S^6 are called the *imaginary units* of $\mathbf{O}$. For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u\rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \to \mathbf{O}$ given by

$$(1.15) \qquad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on $\mathbf{O}$. We write $\mathbf{O}_u$ to denote $\mathbf{O}$ endowed with the complex structure J_u. If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on $\mathbf{O}$. However, because S^6 is connected, we see that the orientation of $\mathbf{O}$ induced by the natural orientation of $\mathbf{O}_u$ as a complex vector space does not depend on u. We refer to this orientation as the natural orientation of $\mathbf{O}$.

In[837]:=



Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y)\rangle = \langle xu, yu\rangle = \langle x, y\rangle\langle u, u\rangle = \langle x, y\rangle$$

so J_u is an isometry for each $u \in S$. Moreover, it follows that $\mathbf{O}_u$ is endowed with a natural Hermitian structure with respect to the inner product $\langle\,,\rangle$. We denote the group of complex linear transformations of $\mathbf{O}_u$ by $GL(\mathbf{O}_u)$ and the special unitary transformations of $\mathbf{O}_u$ with its Hermitian metric by $SU(\mathbf{O}_u)$.

J_u

In[838]:=

```
Table[FullSimplify[
  Sum[ψ[A1] × J[B1] g_AB⟦A1, A1⟧ E^A_a[hUSE]⟦C⟧⟦c⟧ τ[A1 - 1]⟦c⟧⟦b⟧ E^a_A[hUSE]⟦b⟧⟦B1⟧,
   {c, 1, 8}, {b, 1, 8}, {A1, 1, 8}, {B1, 1, 8}]], {C, 1, 8}]
```

Out[838]=

```
{J[1] × ψ[1] - J[2] × ψ[2] - J[3] × ψ[3] - J[4] × ψ[4] +
  J[5] × ψ[5] + J[6] × ψ[6] + J[7] × ψ[7] + J[8] × ψ[8], J[2] × ψ[1] + J[1] × ψ[2] -
  J[4] × ψ[3] + J[3] × ψ[4] + J[6] × ψ[5] - J[5] × ψ[6] - J[8] × ψ[7] + J[7] × ψ[8],
 J[3] × ψ[1] + J[4] × ψ[2] + J[1] × ψ[3] - J[2] × ψ[4] + J[7] × ψ[5] + J[8] × ψ[6] -
  J[5] × ψ[7] - J[6] × ψ[8], J[4] × ψ[1] - J[3] × ψ[2] + J[2] × ψ[3] + J[1] × ψ[4] +
  J[8] × ψ[5] - J[7] × ψ[6] + J[6] × ψ[7] - J[5] × ψ[8], J[5] × ψ[1] + J[6] × ψ[2] +
  J[7] × ψ[3] + J[8] × ψ[4] + J[1] × ψ[5] - J[2] × ψ[6] - J[3] × ψ[7] - J[4] × ψ[8],
 J[6] × ψ[1] - J[5] × ψ[2] + J[8] × ψ[3] - J[7] × ψ[4] + J[2] × ψ[5] + J[1] × ψ[6] +
  J[4] × ψ[7] - J[3] × ψ[8], J[7] × ψ[1] - J[8] × ψ[2] - J[5] × ψ[3] + J[6] × ψ[4] +
  J[3] × ψ[5] - J[4] × ψ[6] + J[1] × ψ[7] + J[2] × ψ[8], J[8] × ψ[1] + J[7] × ψ[2] -
  J[6] × ψ[3] - J[5] × ψ[4] + J[4] × ψ[5] + J[3] × ψ[6] - J[2] × ψ[7] + J[1] × ψ[8]}
```

```
In[839]:=
        Table[
         FullSimplify[Sum[ g_AB⟦B1, B1⟧ E^A_a[hUSE]⟦C⟧⟦c⟧ τ[B1 - 1]⟦c⟧⟦b⟧ E^a_A[hUSE]⟦b⟧⟦B1⟧,
            {c, 1, 8}, {b, 1, 8}]], {C, 1, 8}, {A1, 1, 8}, {B1, 1, 8}]
Out[839]=
        {{{1, -1, -1, -1, 1, 1, 1, 1}, {1, -1, -1, -1, 1, 1, 1, 1},
          {1, -1, -1, -1, 1, 1, 1, 1}, {1, -1, -1, -1, 1, 1, 1, 1}, {1, -1, -1, -1, 1, 1, 1, 1},
          {1, -1, -1, -1, 1, 1, 1, 1}, {1, -1, -1, -1, 1, 1, 1, 1}, {1, -1, -1, -1, 1, 1, 1, 1}},
         {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
         {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
         {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
         {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
         {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
         {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
         {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}
```

In[840]:=

```
Table[
 FullSimplify[ExpandAll[ gAB⟦B, B⟧ EAa[hUSE].τ[B – 1].EaA[hUSE]]], {B, 1, 8}]
```

Out[840]=

```
{{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}},
 {{0, -1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 1, 0}},
 {{0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0},
  {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}},
 {{0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0},
  {1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0},
  {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}},
 {{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0},
  {0, 0, 0, 0, 0, 0, 0, 1}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
  {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1},
  {0, 0, 0, 0, 0, 0, -1, 0}, {0, -1, 0, 0, 0, 0, 0, 0},
  {1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, -1, 0, 0, 0},
  {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
  {1, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0}},
 {{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
  {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
  {0, 0, -1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}}}
```

In[841]:=

```
Table[
  Transpose[FullSimplify[ExpandAll[ gAB⟦B, B⟧ EAa[hUSE].τ[B – 1].EaA[hUSE]]]],
  {B, 1, 8}] // MatrixForm
```

Out[841]//MatrixForm=

```
( 1    0    0    0    0    0    0    0
  0    1    0    0    0    0    0    0
  0    0    1    0    0    0    0    0
  0    0    0    1    0    0    0    0
  0    0    0    0    1    0    0    0
  0    0    0    0    0    1    0    0
  0    0    0    0    0    0    1    0
  0    0    0    0    0    0    0    1 )
( 0   -1    0    0    0    0    0    0
  1    0    0    0    0    0    0    0
  0    0    0    1    0    0    0    0
  0    0   -1    0    0    0    0    0
  0    0    0    0    0    1    0    0
  0    0    0    0   -1    0    0    0
  0    0    0    0    0    0    0   -1
  0    0    0    0    0    0    1    0 )
( 0    0   -1    0    0    0    0    0
```

$$\left(\begin{array}{cccccccc}
\begin{pmatrix}0\\1\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}-1\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\-1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\-1\end{pmatrix} & \begin{pmatrix}0\\0\\0\\1\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\end{pmatrix} \\
\begin{pmatrix}0\\0\\0\\1\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\-1\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}-1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\-1\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\-1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{pmatrix} \\
\begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} & \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\1\\0\\0\\0\\0\end{pmatrix} \\
\begin{pmatrix}0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\-1\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\-1\\0\end{pmatrix} & \begin{pmatrix}0\\-1\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\-1\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} \\
\begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\-1\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\-1\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\-1\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\1\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\-1\\0\\0\\0\\0\\0\\0\end{pmatrix} \\
\begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\0\\-1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\0\\-1\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\-1\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\-1\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} & \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}
\end{array}\right)$$

In[842]:=

```
EA = Array[eA, 8]
```

Out[842]=

```
{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}
```

In[843]:=

$$e_{\alpha}^{(A)}$$

Out[843]=

$$\{\{\text{Cot}[6\,H\,x0],\,0,\,0,\,0,\,0,\,0,\,0,\,0\},\,\{0,\,e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},\,0,\,0,\,0,\,0,\,0,\,0\},$$
$$\{0,\,0,\,e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},\,0,\,0,\,0,\,0,\,0\},\,\{0,\,0,\,0,\,e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},\,0,\,0,\,0,\,0\},$$
$$\{0,\,0,\,0,\,0,\,1,\,0,\,0,\,0\},\,\{0,\,0,\,0,\,0,\,0,\,e^{-a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},\,0,\,0\},$$
$$\{0,\,0,\,0,\,0,\,0,\,0,\,e^{-a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},\,0\},\,\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,e^{-a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6}\}\}$$

In[844]:=

h = hUSE;

B = 1; FullSimplify[ExpandAll[g_{AB}⟦B, B⟧ EA.$E^{A}{}_{a}$[h].τ[B − 1].$E^{a}{}_{A}$[h]]]

B = 2; FullSimplify[ExpandAll[g_{AB}⟦B, B⟧ EA.$E^{A}{}_{a}$[h].τ[B − 1].$E^{a}{}_{A}$[h]]]

B = 3; FullSimplify[ExpandAll[g_{AB}⟦B, B⟧ EA.$E^{A}{}_{a}$[h].τ[B − 1].$E^{a}{}_{A}$[h]]]

B = 4; FullSimplify[ExpandAll[g_{AB}⟦B, B⟧ EA.$E^{A}{}_{a}$[h].τ[B − 1].$E^{a}{}_{A}$[h]]]

B = 5; FullSimplify[ExpandAll[g_{AB}⟦B, B⟧ EA.$E^{A}{}_{a}$[h].τ[B − 1].$E^{a}{}_{A}$[h]]]

B = 6; FullSimplify[ExpandAll[g_{AB}⟦B, B⟧ EA.$E^{A}{}_{a}$[h].τ[B − 1].$E^{a}{}_{A}$[h]]]

B = 7; FullSimplify[ExpandAll[g_{AB}⟦B, B⟧ EA.$E^{A}{}_{a}$[h].τ[B − 1].$E^{a}{}_{A}$[h]]]

B = 8;

FullSimplify[ExpandAll[g_{AB}⟦B, B⟧ EA.$E^{A}{}_{a}$[h].τ[B − 1].$E^{a}{}_{A}$[h]]]

Out[845]=

```
{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}
```

Out[846]=

```
{eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}
```

Out[847]=

```
{eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}
```

Out[848]=

```
{eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]}
```

Out[849]=

```
{eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]}
```

Out[850]=

```
{eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]}
```

Out[851]=

```
{eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]}
```

Out[852]=

```
{eA[8], eA[7], -eA[6], -eA[5], -eA[4], -eA[3], eA[2], eA[1]}
```

In[853]:=

B = 1;

Grid[

{EA, FullSimplify[ExpandAll[g_{AB}⟦B, B⟧ EA.$E^{A}{}_{a}$[h].τ[B − 1].$E^{a}{}_{A}$[h]]]}, Frame → All]

Out[853]=

eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]

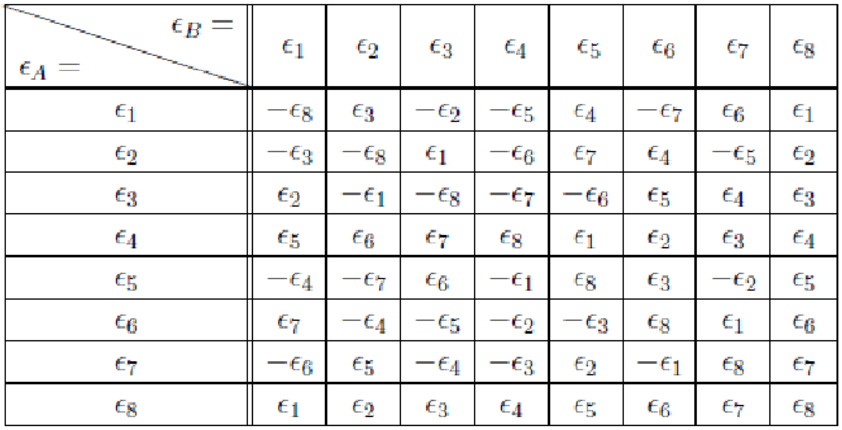
In[854]:=

```
gridx = Grid[Partition[Flatten[{{{x}, EA}, Table[
      ({{x}, FullSimplify[ExpandAll[ gAB⟦B, B⟧ EA.E^A_a[h].τ[B - 1].E^a_A[h]]]} /.
        {x → ToExpression["eA[" <> ToString[B] <> "]"]}),
      {B, 1, 8}]]}], 9], Frame → All]
```

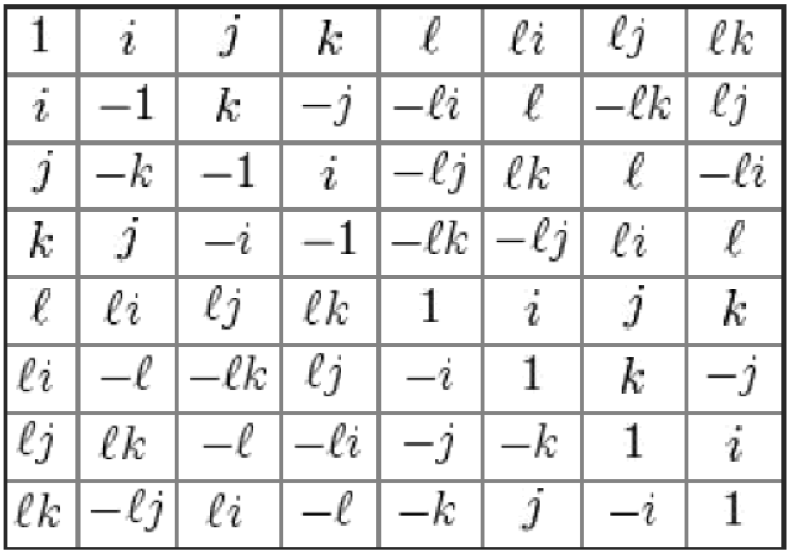
Out[854]=

```
  x    eA[1]  eA[2]  eA[3]  eA[4]  eA[5]  eA[6]  eA[7]  eA[8]
eA[1]  eA[1]  eA[2]  eA[3]  eA[4]  eA[5]  eA[6]  eA[7]  eA[8]
eA[2]  eA[2] -eA[1] -eA[4]  eA[3] -eA[6]  eA[5]  eA[8] -eA[7]
eA[3]  eA[3]  eA[4] -eA[1] -eA[2] -eA[7] -eA[8]  eA[5]  eA[6]
eA[4]  eA[4] -eA[3]  eA[2] -eA[1] -eA[8]  eA[7] -eA[6]  eA[5]
eA[5]  eA[5]  eA[6]  eA[7]  eA[8]  eA[1]  eA[2]  eA[3]  eA[4]
eA[6]  eA[6] -eA[5]  eA[8] -eA[7] -eA[2]  eA[1] -eA[4]  eA[3]
eA[7]  eA[7] -eA[8] -eA[5]  eA[6] -eA[3]  eA[4]  eA[1] -eA[2]
eA[8]  eA[8]  eA[7] -eA[6] -eA[5] -eA[4] -eA[3]  eA[2]  eA[1]
```

In[855]:=



$\epsilon_B =$ $\epsilon_A =$	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8
ϵ_1	$-\epsilon_8$	ϵ_3	$-\epsilon_2$	$-\epsilon_5$	ϵ_4	$-\epsilon_7$	ϵ_6	ϵ_1
ϵ_2	$-\epsilon_3$	$-\epsilon_8$	ϵ_1	$-\epsilon_6$	ϵ_7	ϵ_4	$-\epsilon_5$	ϵ_2
ϵ_3	ϵ_2	$-\epsilon_1$	$-\epsilon_8$	$-\epsilon_7$	$-\epsilon_6$	ϵ_5	ϵ_4	ϵ_3
ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4
ϵ_5	$-\epsilon_4$	$-\epsilon_7$	ϵ_6	$-\epsilon_1$	ϵ_8	ϵ_3	$-\epsilon_2$	ϵ_5
ϵ_6	ϵ_7	$-\epsilon_4$	$-\epsilon_5$	$-\epsilon_2$	$-\epsilon_3$	ϵ_8	ϵ_1	ϵ_6
ϵ_7	$-\epsilon_6$	ϵ_5	$-\epsilon_4$	$-\epsilon_3$	ϵ_2	$-\epsilon_1$	ϵ_8	ϵ_7
ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8



1	i	j	k	ℓ	ℓi	ℓj	ℓk
i	-1	k	$-j$	$-\ell i$	ℓ	$-\ell k$	ℓj
j	$-k$	-1	i	$-\ell j$	ℓk	ℓ	$-\ell i$
k	j	$-i$	-1	$-\ell k$	$-\ell j$	ℓi	ℓ
ℓ	ℓi	ℓj	ℓk	1	i	j	k
ℓi	$-\ell$	$-\ell k$	ℓj	$-i$	1	k	$-j$
ℓj	ℓk	$-\ell$	$-\ell i$	$-j$	$-k$	1	i
ℓk	$-\ell j$	ℓi	$-\ell$	$-k$	j	$-i$	1

In[856]:=

```
(τABC = Table[ExpandAll[ gAB⟦B, B⟧ EAa[hUSE].τ[B - 1].EaA[hUSE]], {B, 1, 8}]);
(τABC⟦#⟧ // MatrixForm) & /@ Range[8]
```

Out[856]=

```
 ⎛1 0 0 0 0 0 0 0⎞   ⎛0 -1  0  0  0  0  0  0⎞
 ⎜0 1 0 0 0 0 0 0⎟   ⎜1  0  0  0  0  0  0  0⎟
 ⎜0 0 1 0 0 0 0 0⎟   ⎜0  0  0  1  0  0  0  0⎟
{⎜0 0 0 1 0 0 0 0⎟ , ⎜0  0 -1  0  0  0  0  0⎟ ,
 ⎜0 0 0 0 1 0 0 0⎟   ⎜0  0  0  0  0  1  0  0⎟
 ⎜0 0 0 0 0 1 0 0⎟   ⎜0  0  0  0 -1  0  0  0⎟
 ⎜0 0 0 0 0 0 1 0⎟   ⎜0  0  0  0  0  0  0 -1⎟
 ⎝0 0 0 0 0 0 0 1⎠   ⎝0  0  0  0  0  0  1  0⎠

 ⎛0  0 -1  0  0  0  0  0⎞   ⎛0  0  0 -1  0  0  0  0⎞   ⎛0 0 0 0 1 0 0 0⎞
 ⎜0  0  0 -1  0  0  0  0⎟   ⎜0  0  1  0  0  0  0  0⎟   ⎜0 0 0 0 0 1 0 0⎟
 ⎜1  0  0  0  0  0  0  0⎟   ⎜0 -1  0  0  0  0  0  0⎟   ⎜0 0 0 0 0 0 1 0⎟
 ⎜0  1  0  0  0  0  0  0⎟ , ⎜1  0  0  0  0  0  0  0⎟ , ⎜0 0 0 0 0 0 0 1⎟ ,
 ⎜0  0  0  0  0  0  1  0⎟   ⎜0  0  0  0  0  0  0  1⎟   ⎜1 0 0 0 0 0 0 0⎟
 ⎜0  0  0  0  0  0  0  1⎟   ⎜0  0  0  0  0  0 -1  0⎟   ⎜0 1 0 0 0 0 0 0⎟
 ⎜0  0  0  0 -1  0  0  0⎟   ⎜0  0  0  0  0  1  0  0⎟   ⎜0 0 1 0 0 0 0 0⎟
 ⎝0  0  0  0  0 -1  0  0⎠   ⎝0  0  0  0 -1  0  0  0⎠   ⎝0 0 0 1 0 0 0 0⎠

 ⎛0  0  0  0  0  1  0  0⎞   ⎛0  0  0  0  0  0  1  0⎞   ⎛0  0  0  0  0  0  0  1⎞
 ⎜0  0  0  0 -1  0  0  0⎟   ⎜0  0  0  0  0  0  0 -1⎟   ⎜0  0  0  0  0  0  1  0⎟
 ⎜0  0  0  0  0  0  0  1⎟   ⎜0  0  0  0 -1  0  0  0⎟   ⎜0  0  0  0  0 -1  0  0⎟
 ⎜0  0  0  0  0  0 -1  0⎟ , ⎜0  0  0  0  0  1  0  0⎟ , ⎜0  0  0  0 -1  0  0  0⎟}
 ⎜0 -1  0  0  0  0  0  0⎟   ⎜0  0 -1  0  0  0  0  0⎟   ⎜0  0  0 -1  0  0  0  0⎟
 ⎜1  0  0  0  0  0  0  0⎟   ⎜0  0  0  1  0  0  0  0⎟   ⎜0  0 -1  0  0  0  0  0⎟
 ⎜0  0  0 -1  0  0  0  0⎟   ⎜1  0  0  0  0  0  0  0⎟   ⎜0  1  0  0  0  0  0  0⎟
 ⎝0  0  1  0  0  0  0  0⎠   ⎝0 -1  0  0  0  0  0  0⎠   ⎝1  0  0  0  0  0  0  0⎠
```

In[857]:=

```
EA
```

Out[857]=

```
{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}
```

In[858]:=

```
EA.τABC⟦1⟧
```

Out[858]=

```
{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}
```

In[859]:=

```
EA.τABC⟦2⟧
```

Out[859]=

```
{eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}
```

In[860]:=

```
EA.τABC⟦3⟧
```

Out[860]=

```
{eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}
```

In[861]:=

```
EA.τABC⟦4⟧
```

Out[861]=

```
{eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]}
```

```
In[862]:=
        EA.τABC⟦5⟧
Out[862]=
        {eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]}
In[863]:=
        EA.τABC⟦6⟧
Out[863]=
        {eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]}
In[864]:=
        EA.τABC⟦7⟧
Out[864]=
        {eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]}
In[865]:=
        TeXForm[gridx]
Out[865]//TeXForm=
        \begin{array}{ccccccccc}
         x & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \text{eA}(4) & \text{eA}(5) & \text{eA}(6) & \te
         \text{eA}(1) & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \text{eA}(4) & \text{eA}(5) & \text{
         \text{eA}(2) & \text{eA}(2) & -\text{eA}(1) & -\text{eA}(4) & \text{eA}(3) & -\text{eA}(6) & \tex
         \text{eA}(3) & \text{eA}(3) & \text{eA}(4) & -\text{eA}(1) & -\text{eA}(2) & -\text{eA}(7) & -\te
         \text{eA}(4) & \text{eA}(4) & -\text{eA}(3) & \text{eA}(2) & -\text{eA}(1) & -\text{eA}(8) & \tex
         \text{eA}(5) & \text{eA}(5) & \text{eA}(6) & \text{eA}(7) & \text{eA}(8) & \text{eA}(1) & \text{
         \text{eA}(6) & \text{eA}(6) & -\text{eA}(5) & \text{eA}(8) & -\text{eA}(7) & -\text{eA}(2) & \tex
         \text{eA}(7) & \text{eA}(7) & -\text{eA}(8) & -\text{eA}(5) & \text{eA}(6) & -\text{eA}(3) & \tex
         \text{eA}(8) & \text{eA}(8) & \text{eA}(7) & -\text{eA}(6) & -\text{eA}(5) & -\text{eA}(4) & -\te
        \end{array}
In[866]:=
        (*StringReplace[ToString[TeXForm[gridx]],"\text{eA}"→ "\epsilon_"]*)
In[867]:=
        Grid[Partition[
          Flatten[{{{x}, EA}, Table[({{x}, Table[Sum[FullSimplify[ExpandAll[ g_AB⟦B, B⟧
                     EA⟦C1⟧ E^A_a[hUSE]⟦C1, c1⟧ τ[B - 1]⟦c1, d1⟧ E^a_A[hUSE]⟦d1, B1⟧]],
                  {C1, 1, 8}, {c1, 1, 8}, {d1, 1, 8}], {B1, 1, 8}]} /.
                {x → ToExpression["eA[" <> ToString[B] <> "]"]}),
              {B, 1, 8}]}], 9], Frame → All]
Out[867]=
```

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

In[868]:=

```
gridx
```

Out[868]=

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

Complex Structure

J_u

In[869]:=

```
J[#] → 0 & /@ Range[0, 7]
```

Out[869]=

```
{J[0] → 0, J[1] → 0, J[2] → 0, J[3] → 0, J[4] → 0, J[5] → 0, J[6] → 0, J[7] → 0}
```

In[870]:=

```
sJu = {J[0] → 0, J[1] → 0, J[2] → 0, J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0}
```

Out[870]=

```
{J[0] → 0, J[1] → 0, J[2] → 0, J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0}
```

for $\frac{1}{i}$:

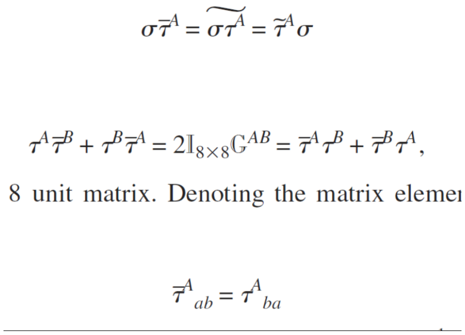
In[871]:=

```
Block[{ψ, J, t, r, sJu},
 sJu =
  {J[0] → 0, J[1] → 0, J[2] → 0, J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0};
 t = Table[FullSimplify[Sum[ψ[A1 - 1] × J[B1 - 1] g_AB[[A1, A1]]
       E^A_a[hUSE][[C]][[c]] τ[A1 - 1][[c]][[b]] E^a_A[hUSE][[b]][[B1]],
      {c, 1, 8}, {b, 1, 8}, {A1, 1, 8}, {B1, 1, 8}]], {C, 1, 8}];
 r = t /. sJu
]
```

Out[871]=

```
{ψ[4], -ψ[5], -ψ[6], -ψ[7], ψ[0], -ψ[1], -ψ[2], -ψ[3]}
```

In[872]:=

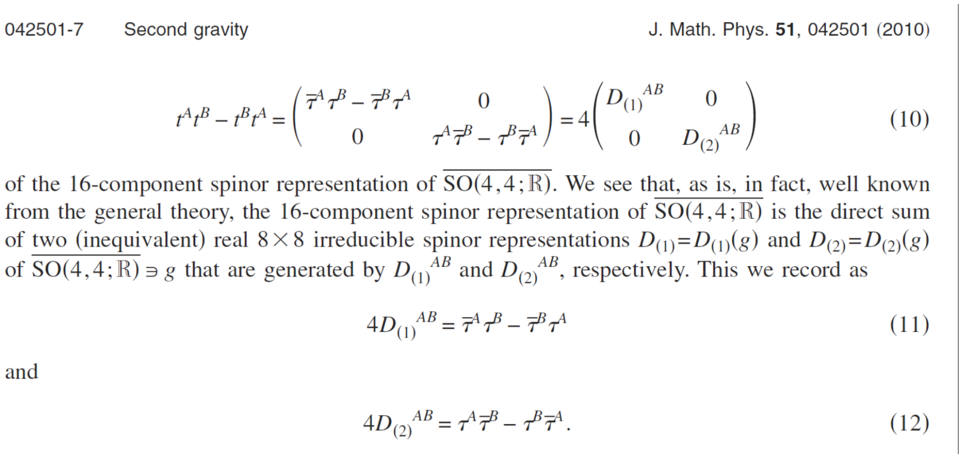


$$\sigma\overline{\tau}^A = \widetilde{\sigma\tau^A} = \widetilde{\tau}^A\sigma$$

$$\tau^A\overline{\tau}^B + \tau^B\overline{\tau}^A = 2\mathbb{I}_{8\times 8}\mathbb{G}^{AB} = \overline{\tau}^A\tau^B + \overline{\tau}^B\tau^A,$$

8 unit matrix. Denoting the matrix eleme

$$\overline{\tau}^A{}_{ab} = \tau^A{}_{ba}$$



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$$t^At^B - t^Bt^A = \begin{pmatrix} \overline{\tau}^A\tau^B - \overline{\tau}^B\tau^A & 0 \\ 0 & \tau^A\overline{\tau}^B - \tau^B\overline{\tau}^A \end{pmatrix} = 4\begin{pmatrix} D_{(1)}{}^{AB} & 0 \\ 0 & D_{(2)}{}^{AB} \end{pmatrix} \tag{10}$$

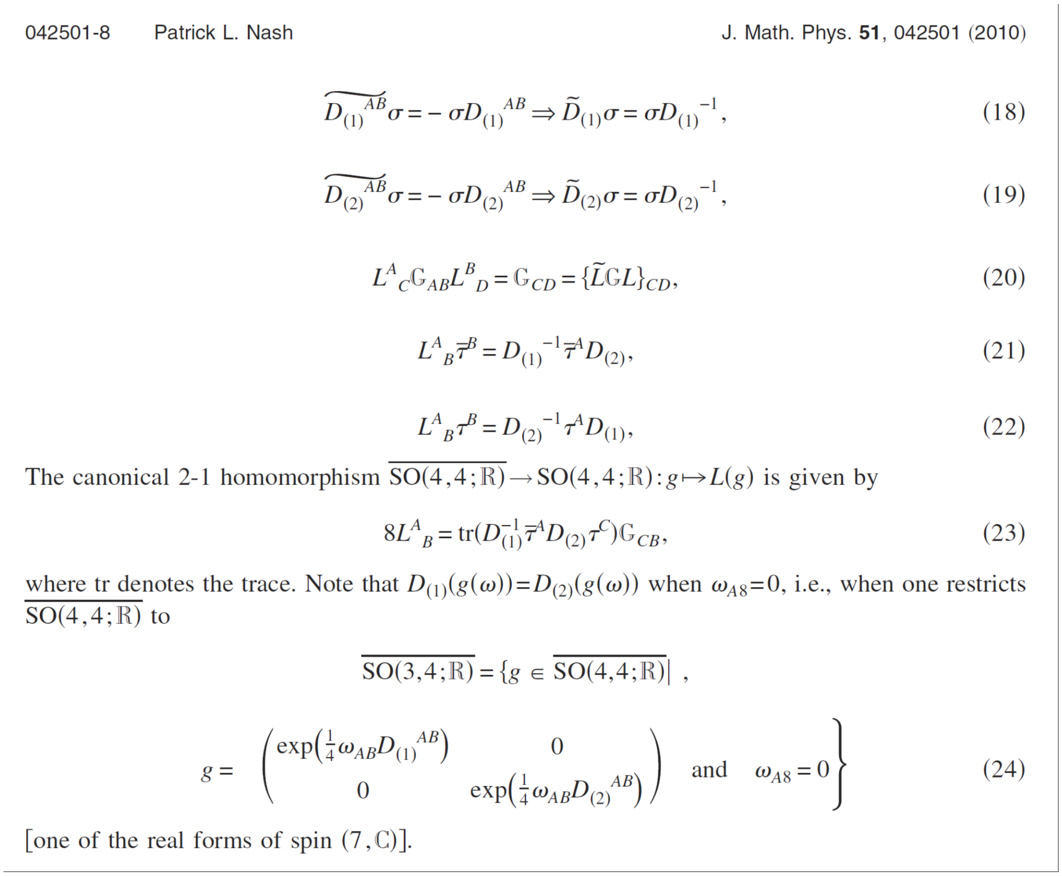
of the 16-component spinor representation of $\overline{\mathrm{SO}(4,4;\mathbb{R})}$. We see that, as is, in fact, well known from the general theory, the 16-component spinor representation of $\overline{\mathrm{SO}(4,4;\mathbb{R})}$ is the direct sum of two (inequivalent) real 8×8 irreducible spinor representations $D_{(1)}=D_{(1)}(g)$ and $D_{(2)}=D_{(2)}(g)$ of $\overline{\mathrm{SO}(4,4;\mathbb{R})} \ni g$ that are generated by $D_{(1)}{}^{AB}$ and $D_{(2)}{}^{AB}$, respectively. This we record as

$$4D_{(1)}{}^{AB} = \overline{\tau}^A\tau^B - \overline{\tau}^B\tau^A \tag{11}$$

and

$$4D_{(2)}{}^{AB} = \tau^A\overline{\tau}^B - \tau^B\overline{\tau}^A. \tag{12}$$

In[873]:=



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$$\widetilde{D_{(1)}{}^{AB}}\sigma = -\sigma D_{(1)}{}^{AB} \Rightarrow \tilde{D}_{(1)}\sigma = \sigma D_{(1)}{}^{-1}, \tag{18}$$

$$\widetilde{D_{(2)}{}^{AB}}\sigma = -\sigma D_{(2)}{}^{AB} \Rightarrow \tilde{D}_{(2)}\sigma = \sigma D_{(2)}{}^{-1}, \tag{19}$$

$$L^{A}{}_{C}\mathbb{G}_{AB}L^{B}{}_{D} = \mathbb{G}_{CD} = \{\tilde{L}\mathbb{G}L\}_{CD}, \tag{20}$$

$$L^{A}{}_{B}\bar{\tau}^{B} = D_{(1)}{}^{-1}\bar{\tau}^{A}D_{(2)}, \tag{21}$$

$$L^{A}{}_{B}\tau^{B} = D_{(2)}{}^{-1}\tau^{A}D_{(1)}, \tag{22}$$

The canonical 2-1 homomorphism $\overline{\mathrm{SO}(4,4;\mathbb{R})} \rightarrow \mathrm{SO}(4,4;\mathbb{R}): g \mapsto L(g)$ is given by

$$8L^{A}{}_{B} = \mathrm{tr}(D_{(1)}^{-1}\bar{\tau}^{A}D_{(2)}\tau^{C})\mathbb{G}_{CB}, \tag{23}$$

where tr denotes the trace. Note that $D_{(1)}(g(\omega)) = D_{(2)}(g(\omega))$ when $\omega_{A8}=0$, i.e., when one restricts $\overline{\mathrm{SO}(4,4;\mathbb{R})}$ to

$$\overline{\mathrm{SO}(3,4;\mathbb{R})} = \{g \in \overline{\mathrm{SO}(4,4;\mathbb{R})}| \ ,$$

$$g = \begin{pmatrix} \exp\left(\frac{1}{4}\omega_{AB}D_{(1)}{}^{AB}\right) & 0 \\ 0 & \exp\left(\frac{1}{4}\omega_{AB}D_{(2)}{}^{AB}\right) \end{pmatrix} \ \text{and} \quad \omega_{A8} = 0 \Big\} \tag{24}$$

[one of the real forms of spin $(7,\mathbb{C})$].

In[874]:=

```
Block[{ψ, J, t, r, sJu},
 sJu = {J[0] → 0, J[1] → 0, J[2] → 0,
   J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0};
 r = {ψ[4], -ψ[5], -ψ[6], -ψ[7], ψ[0], -ψ[1], -ψ[2], -ψ[3]};
 ψ = τ̅[A].τ[B].Ψ16⟦1 ;; 8⟧
]
```

Out[874]=

```
τ̅[A].{f16[7][x0, x4], f16[6][x0, x4], -f16[5][x0, x4], -f16[4][x0, x4],
  f16[3][x0, x4], f16[2][x0, x4], -f16[1][x0, x4], -f16[0][x0, x4]}
```

```
τ̅[A1].τ[B1] → D(1)^-1.τ̅[A1].τ[B1].D(1);
τ[A1].τ̅[B1] → D(2)^-1.τ[A1].τ̅[B1].D(2);
σ.τ̅[A1].τ[B1] = Transpose[τ[A1]].σ.τ[B1] =
   Transpose[τ[A1]].Transpose[σ.τ̅[B1]] = Transpose[σ.τ̅[B1].τ[A1]];
σ.τ̅[A1].τ[B1] = Transpose[τ̅[B1].τ[A1]].Transpose[σ];
σ.τ̅[A1].τ[B1].ψ1 = Transpose[τ̅[B1].τ[A1]].Transpose[σ].ψ1;
Transpose[ψ2].σ ⟷ Transpose[σ].ψ1 ? ;
σ.τ̅[A1].τ[B1].ψ1 ⟷ Transpose[τ̅[B1].τ[A1]].Transpose[ψ2].σ;
σ.τ̅[A1].τ[B1].ψ1 ⟷ Transpose[ψ2.τ̅[B1].τ[A1]].σ;
```

```
B1 = A1
σ.ψ₁ ⟷ Transpose[ψ₂].σ^-1 ?;
ψ₁ ⟷ Transpose[ψ₂].σ^-1 ?;
 Transpose[ψ₂] ⟷  σ.ψ₁ ?;
```

In[875]:=

```
{ψ₁[4], -ψ₁[5], -ψ₁[6], -ψ₁[7], ψ₁[0], -ψ₁[1], -ψ₁[2], -ψ₁[3]} /. {ψ₁[#] →};
```

In[875]:=

```
(*Block[{ψ₁,ψ₂,σψ,sσψ,r,sJu},
 σψ=σ.(ψ₂[#]&/@Range[0,7]);
 sσψ=(ψ₁[#]→σψ⟦#+1⟧&/@Range[0,7]);
 r={ψ₁[4],-ψ₁[5],-ψ₁[6],-ψ₁[7],ψ₁[0],-ψ₁[1],-ψ₁[2],-ψ₁[3]};
 (*ψ=τ̄[A].τ[B].Ψ16⟦1;;8⟧*)
 r/.sσψ
]*)
```

In[876]:=

```
Block[{ψ1, ψ2, σψ, sσψ, r},
 σψ = σ.(ψ2[#] & /@ Range[0, 7]);
 sσψ = (ψ1[#] → σψ⟦# + 1⟧ & /@ Range[0, 7]);
 r = {ψ1[4], -ψ1[5], -ψ1[6], -ψ1[7], ψ1[0], -ψ1[1], -ψ1[2], -ψ1[3]};
 (*ψ=τ̄[A].τ[B].Ψ16⟦1;;8⟧*)
 r /. sσψ
]
```

Out[876]=

```
{ψ2[0], -ψ2[1], -ψ2[2], -ψ2[3], ψ2[4], -ψ2[5], -ψ2[6], -ψ2[7]}
```

for $\frac{1}{i}$:

In[877]:=

```
Ψ16cccc = Block[{ψ1, ψ2, σψ, sσψ, sψ1, sψ2, r},
   σψ = σ.(Ψ16⟦9 ;; 16⟧);
   sσψ = (ψ1[#] → σψ⟦# + 1⟧ & /@ Range[0, 7]);
   sψ1 = Thread[(ψ1[#] & /@ Range[0, 7]) → Ψ16⟦1 ;; 8⟧];
   sψ2 = Thread[(ψ1[#] & /@ Range[0, 7]) → Ψ16⟦9 ;; 16⟧];
   r = {ψ1[4], -ψ1[5], -ψ1[6], -ψ1[7], ψ1[0], -ψ1[1], -ψ1[2], -ψ1[3]};
   (*ψ=τ̄[A].τ[B].*)
   Flatten[{r /. sψ1, r /. sψ2}]
  ]
```

Out[877]=

```
{f16[4][x0, x4], -f16[5][x0, x4], -f16[6][x0, x4], -f16[7][x0, x4],
 f16[0][x0, x4], -f16[1][x0, x4], -f16[2][x0, x4], -f16[3][x0, x4],
 f16[12][x0, x4], -f16[13][x0, x4], -f16[14][x0, x4], -f16[15][x0, x4],
 f16[8][x0, x4], -f16[9][x0, x4], -f16[10][x0, x4], -f16[11][x0, x4]}
```

```
In[878]:=
        caΨccΨ = CoefficientArrays[Ψ16cccc − Ψ16, Ψ16cccc]
Out[878]=
        {SparseArray[ Specified elements: 12  ], SparseArray[ Specified elements: 8       ]}
                      Dimensions: {16}                      Dimensions: {16, 16}
In[879]:=
        (caΨcc2 = Normal[caΨccΨ⟦2⟧]) // MatrixForm
Out[879]//MatrixForm=
        ⎛ 1  0 0 0 -1 0 0 0  0 0 0 0  0 0 0 0⎞
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜-1  0 0 0  1 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  1 0 0 0 -1 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0 -1 0 0 0  1 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎜ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎟
        ⎝ 0  0 0 0  0 0 0 0  0 0 0 0  0 0 0 0⎠
In[880]:=
        caΨccΨ = CoefficientArrays[Ψ16cccc − Ψ16, Ψ16]
Out[880]=
        {SparseArray[ Specified elements: 0  ], SparseArray[ Specified elements: 32      ]}
                      Dimensions: {16}                     Dimensions: {16, 16}
In[881]:=
        (caΨccΨ2 = Normal[caΨccΨ⟦2⟧]) // MatrixForm
Out[881]//MatrixForm=
        ⎛-1  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0⎞
        ⎜ 0 -1  0  0  0 -1  0  0  0  0  0  0  0  0  0  0⎟
        ⎜ 0  0 -1  0  0  0 -1  0  0  0  0  0  0  0  0  0⎟
        ⎜ 0  0  0 -1  0  0  0 -1  0  0  0  0  0  0  0  0⎟
        ⎜ 1  0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0⎟
        ⎜ 0 -1  0  0  0 -1  0  0  0  0  0  0  0  0  0  0⎟
        ⎜ 0  0 -1  0  0  0 -1  0  0  0  0  0  0  0  0  0⎟
        ⎜ 0  0  0 -1  0  0  0 -1  0  0  0  0  0  0  0  0⎟
        ⎜ 0  0  0  0  0  0  0  0 -1  0  0  0  1  0  0  0⎟
        ⎜ 0  0  0  0  0  0  0  0  0 -1  0  0  0 -1  0  0⎟
        ⎜ 0  0  0  0  0  0  0  0  0  0 -1  0  0  0 -1  0⎟
        ⎜ 0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0 -1⎟
        ⎜ 0  0  0  0  0  0  0  0  1  0  0  0 -1  0  0  0⎟
        ⎜ 0  0  0  0  0  0  0  0  0 -1  0  0  0 -1  0  0⎟
        ⎜ 0  0  0  0  0  0  0  0  0  0 -1  0  0  0 -1  0⎟
        ⎝ 0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0 -1⎠
In[882]:=
        caΨccΨ2 === Transpose[caΨccΨ2]
Out[882]=
        True
```

In[883]:=

```
caΨccΨ2.caΨccΨ2 // MatrixForm
```

Out[883]//MatrixForm=

```
 2  0 0 0  -2 0 0 0   0  0 0 0   0  0 0 0
 0  2 0 0   0 2 0 0   0  0 0 0   0  0 0 0
 0  0 2 0   0 0 2 0   0  0 0 0   0  0 0 0
 0  0 0 2   0 0 0 2   0  0 0 0   0  0 0 0
-2  0 0 0   2 0 0 0   0  0 0 0   0  0 0 0
 0  2 0 0   0 2 0 0   0  0 0 0   0  0 0 0
 0  0 2 0   0 0 2 0   0  0 0 0   0  0 0 0
 0  0 0 2   0 0 0 2   0  0 0 0   0  0 0 0
 0  0 0 0   0 0 0 0   2  0 0 0  -2  0 0 0
 0  0 0 0   0 0 0 0   0  2 0 0   0  2 0 0
 0  0 0 0   0 0 0 0   0  0 2 0   0  0 2 0
 0  0 0 0   0 0 0 0   0  0 0 2   0  0 0 2
 0  0 0 0   0 0 0 0  -2  0 0 0   2  0 0 0
 0  0 0 0   0 0 0 0   0  2 0 0   0  2 0 0
 0  0 0 0   0 0 0 0   0  0 2 0   0  0 2 0
 0  0 0 0   0 0 0 0   0  0 0 2   0  0 0 2
```

In[884]:=

```
caΨccΨ2.Ψ16cccc - Ψ16 + Ψ16cccc
caΨccΨ2.Ψ16 - Ψ16cccc + Ψ16
```

Out[884]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

Out[885]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

for $\frac{1}{\mathbb{i}}$:

In[886]:=

```
jjcc = caΨccΨ2 + ID16;
```

In[887]:=

```
jjcc.Ψ16 === Ψ16cccc
```

Out[887]=

```
True
```

In[888]:=

$\bar{x} = 2\langle x, 1\rangle - x.$

Out[888]=

$\bar{x} = 2\langle x, 1\rangle - x.$

In[889]:=

$\bar{x} = 2\langle x, 1\rangle - x.$

Out[889]=

$\bar{x} = 2\langle x, 1\rangle - x.$

```
F^a_A = Transpose[η4488[[# + 1, # + 1]] * (τ[#].unit) & /@ Range[0, 7]]
```

$\left\{\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0\right\},\right.$

$$\left\{0,\,0,\,\frac{1}{\sqrt{2}},\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0\right\},\,\left\{0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0\right\},$$
$$\left\{\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}}\right\},\,\left\{0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0\right\},$$
$$\left\{0,\,0,\,\frac{1}{\sqrt{2}},\,0,\,0,\,\frac{1}{\sqrt{2}},\,0,\,0\right\},\,\left\{0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,0,\,\frac{1}{\sqrt{2}},\,0\right\}\right\}$$

$$F^{A}_{a} = \text{Transpose[unit]}.\sigma.(\overline{\tau}[\#])\ \&\ /@\ \text{Range}[0,\,7]$$

$$\left\{\left\{\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,\frac{1}{\sqrt{2}},\,0,\,0,\,0\right\},\,\left\{0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}}\right\},$$
$$\left\{0,\,0,\,\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,\frac{1}{\sqrt{2}},\,0\right\},\,\left\{0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0\right\},$$
$$\left\{0,\,\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0\right\},\,\left\{0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,\frac{1}{\sqrt{2}},\,0\right\},$$
$$\left\{0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,\frac{1}{\sqrt{2}}\right\},\,\left\{\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0\right\}\right\}$$

In[890]:=

$$F^{A}_{a}$$

Out[890]=

$$\left\{\left\{\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,\frac{1}{\sqrt{2}},\,0,\,0,\,0\right\},\,\left\{0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}}\right\},$$
$$\left\{0,\,0,\,\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,\frac{1}{\sqrt{2}},\,0\right\},\,\left\{0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0\right\},\,\left\{0,\,\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0\right\},$$
$$\left\{0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,\frac{1}{\sqrt{2}},\,0\right\},\,\left\{0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,\frac{1}{\sqrt{2}}\right\},\,\left\{\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,-\frac{1}{\sqrt{2}},\,0,\,0,\,0\right\}\right\}$$

In[891]:=

```
unit
```

Out[891]=

$$\left\{\frac{1}{\sqrt{2}},\,0,\,0,\,0,\,\frac{1}{\sqrt{2}},\,0,\,0,\,0\right\}$$

In[892]:=

$$\left(F^{A}_{a}.\text{unit}\right)$$

Out[892]=

```
{1, 0, 0, 0, 0, 0, 0, 0}
```

In[893]:=

$$\left(F^{A}_{a}[\![8]\!].\text{unit}\right)$$

Out[893]=

```
0
```

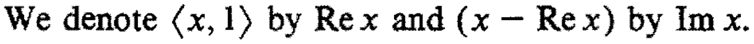
In[894]:=

$$\text{unit}.F^{a}_{A}$$

Out[894]=

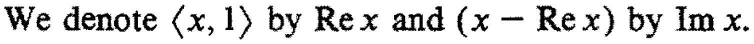
```
{1, 0, 0, 0, 0, 0, 0, 0}
```

```
In[895]:=
unit.unit // FullSimplify
Out[895]=
1
In[896]:=
unit.σ.unit // FullSimplify
Out[896]=
1
In[897]:=
realProjection8 = KroneckerProduct[unit, unit]
Out[897]=
{{1/2, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {1/2, 0, 0, 0, 1/2, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
In[898]:=
realProjection8.realProjection8 === realProjection8
Out[898]=
True
In[899]:=
realProjection16 = ArrayFlatten[{{realProjection8, 0}, {0, realProjection8}}]
Out[899]=
{{1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
In[900]:=
realProjection16.realProjection16 === realProjection16
Out[900]=
True
In[901]:=
```



We denote $\langle x, 1\rangle$ by $\operatorname{Re} x$ and $(x - \operatorname{Re} x)$ by $\operatorname{Im} x$.

```
Out[901]=
```



We denote $\langle x, 1\rangle$ by $\operatorname{Re} x$ and $(x - \operatorname{Re} x)$ by $\operatorname{Im} x$.

In[902]:=

$$\bar{x} = 2\langle x, 1\rangle - x.$$

Out[902]=

$$\bar{x} = 2\langle x, 1\rangle - x.$$

In[903]:=

```
Ψ16cc = 2 realProjection16.Ψ16 - Ψ16 // FullSimplify
```

Out[903]=

```
{f16[4][x0, x4], -f16[1][x0, x4], -f16[2][x0, x4], -f16[3][x0, x4],
 f16[0][x0, x4], -f16[5][x0, x4], -f16[6][x0, x4], -f16[7][x0, x4],
 f16[12][x0, x4], -f16[9][x0, x4], -f16[10][x0, x4], -f16[11][x0, x4],
 f16[8][x0, x4], -f16[13][x0, x4], -f16[14][x0, x4], -f16[15][x0, x4]}
```

In[904]:=

```
Ψ16ccA =
 2 (Flatten[{unit.Ψ16[[1 ;; 8]] * (unit.F^a_A), unit.Ψ16[[9 ;; 16]] * (unit.F^a_A)}]) -
   Ψ16 // FullSimplify
```

Out[904]=

```
{(-1 + √2) f16[0][x0, x4] + √2 f16[4][x0, x4], -f16[1][x0, x4], -f16[2][x0, x4],
 -f16[3][x0, x4], -f16[4][x0, x4], -f16[5][x0, x4], -f16[6][x0, x4], -f16[7][x0, x4],
 (-1 + √2) f16[8][x0, x4] + √2 f16[12][x0, x4], -f16[9][x0, x4], -f16[10][x0, x4],
 -f16[11][x0, x4], -f16[12][x0, x4], -f16[13][x0, x4], -f16[14][x0, x4], -f16[15][x0, x4]}
```

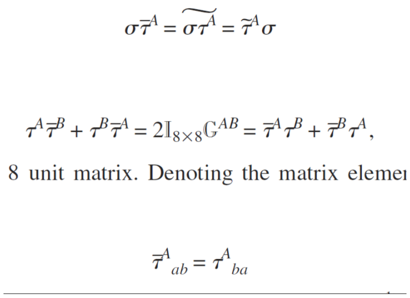
In[905]:=

```
Ψ16cc - Ψ16ccA // FullSimplify
```

Out[905]=

```
{-((-1 + √2) (f16[0][x0, x4] + f16[4][x0, x4])), 0, 0, 0,
 f16[0][x0, x4] + f16[4][x0, x4], 0, 0, 0, -((-1 + √2) (f16[8][x0, x4] + f16[12][x0, x4])),
 0, 0, 0, f16[8][x0, x4] + f16[12][x0, x4], 0, 0, 0}
```

In[906]:=



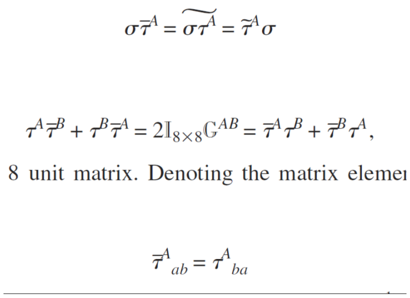
$$\sigma\bar{\tau}^A = \widetilde{\sigma\tau^A} = \tilde{\tau}^A\sigma$$

$$\tau^A\bar{\tau}^B + \tau^B\bar{\tau}^A = 2\mathbb{I}_{8\times 8}\mathbb{G}^{AB} = \bar{\tau}^A\tau^B + \bar{\tau}^B\tau^A,$$

8 unit matrix. Denoting the matrix eleme

$$\bar{\tau}^A{}_{ab} = \tau^A{}_{ba}$$

Out[906]=



$$\sigma\bar{\tau}^A = \widetilde{\sigma\tau^A} = \tilde{\tau}^A\sigma$$

$$\tau^A\bar{\tau}^B + \tau^B\bar{\tau}^A = 2\mathbb{I}_{8\times 8}\mathbb{G}^{AB} = \bar{\tau}^A\tau^B + \bar{\tau}^B\tau^A,$$

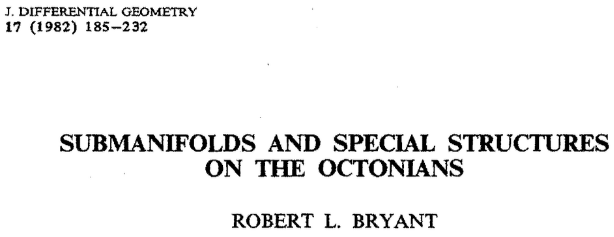
8 unit matrix. Denoting the matrix eleme

$$\bar{\tau}^A{}_{ab} = \tau^A{}_{ba}$$

```
Table[{A, B, FullSimplify[
    ExpandAll[1/2 (τ̄[B].τ[A] + τ̄[A].τ[B]) == η4488⟦A + 1, B + 1⟧ * ID8]]},
  {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

```
Table[{A, B, FullSimplify[
    ExpandAll[1/2 (τ[A].τ̄[B] + τ[B].τ̄[A]) == η4488⟦A + 1, B + 1⟧ * ID8]]},
  {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

In[ ]:=

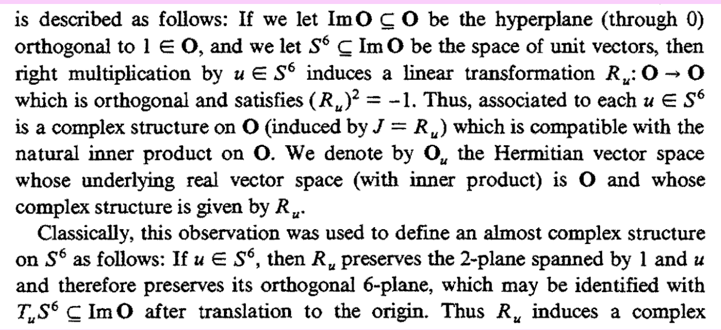


J. DIFFERENTIAL GEOMETRY
17 (1982) 185–232

SUBMANIFOLDS AND SPECIAL STRUCTURES
ON THE OCTONIANS

ROBERT L. BRYANT

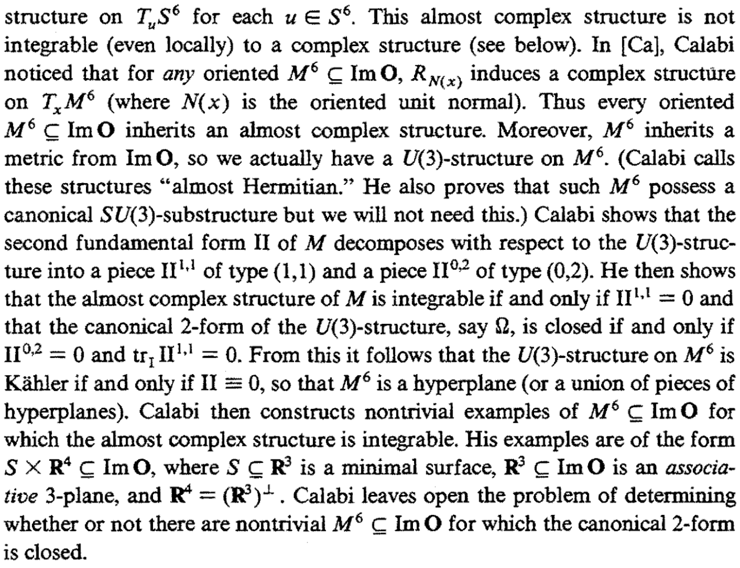
In[∘]:=



is described as follows: If we let $\mathrm{Im}\,\mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \mathrm{Im}\,\mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \to \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on $\mathbf{O}$ (induced by $J = R_u$) which is compatible with the natural inner product on $\mathbf{O}$. We denote by $\mathbf{O}_u$ the Hermitian vector space whose underlying real vector space (with inner product) is $\mathbf{O}$ and whose complex structure is given by R_u.

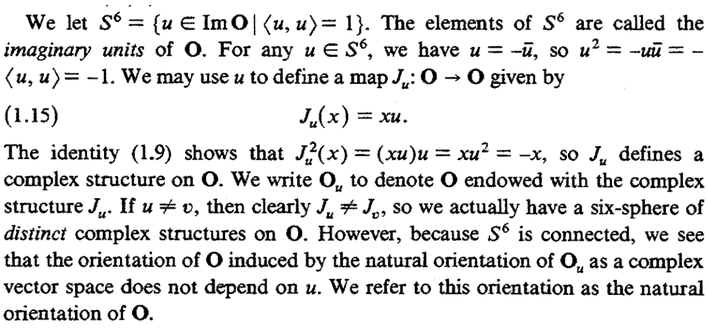
Classically, this observation was used to define an almost complex structure on S^6 as follows: If $u \in S^6$, then R_u preserves the 2-plane spanned by 1 and u and therefore preserves its orthogonal 6-plane, which may be identified with $T_u S^6 \subseteq \mathrm{Im}\,\mathbf{O}$ after translation to the origin. Thus R_u induces a complex

In[∘]:=



structure on $T_u S^6$ for each $u \in S^6$. This almost complex structure is not integrable (even locally) to a complex structure (see below). In [Ca], Calabi noticed that for *any* oriented $M^6 \subseteq \mathrm{Im}\,\mathbf{O}$, $R_{N(x)}$ induces a complex structure on $T_x M^6$ (where $N(x)$ is the oriented unit normal). Thus every oriented $M^6 \subseteq \mathrm{Im}\,\mathbf{O}$ inherits an almost complex structure. Moreover, M^6 inherits a metric from $\mathrm{Im}\,\mathbf{O}$, so we actually have a $U(3)$-structure on M^6. (Calabi calls these structures "almost Hermitian." He also proves that such M^6 possess a canonical $SU(3)$-substructure but we will not need this.) Calabi shows that the second fundamental form II of M decomposes with respect to the $U(3)$-structure into a piece $\mathrm{II}^{1,1}$ of type (1,1) and a piece $\mathrm{II}^{0,2}$ of type (0,2). He then shows that the almost complex structure of M is integrable if and only if $\mathrm{II}^{1,1} = 0$ and that the canonical 2-form of the $U(3)$-structure, say Ω, is closed if and only if $\mathrm{II}^{0,2} = 0$ and $\mathrm{tr}_I\, \mathrm{II}^{1,1} = 0$. From this it follows that the $U(3)$-structure on M^6 is Kähler if and only if $\mathrm{II} \equiv 0$, so that M^6 is a hyperplane (or a union of pieces of hyperplanes). Calabi then constructs nontrivial examples of $M^6 \subseteq \mathrm{Im}\,\mathbf{O}$ for which the almost complex structure is integrable. His examples are of the form $S \times \mathbf{R}^4 \subseteq \mathrm{Im}\,\mathbf{O}$, where $S \subseteq \mathbf{R}^3$ is a minimal surface, $\mathbf{R}^3 \subseteq \mathrm{Im}\,\mathbf{O}$ is an *associative* 3-plane, and $\mathbf{R}^4 = (\mathbf{R}^3)^{\perp}$. Calabi leaves open the problem of determining whether or not there are nontrivial $M^6 \subseteq \mathrm{Im}\,\mathbf{O}$ for which the canonical 2-form is closed.

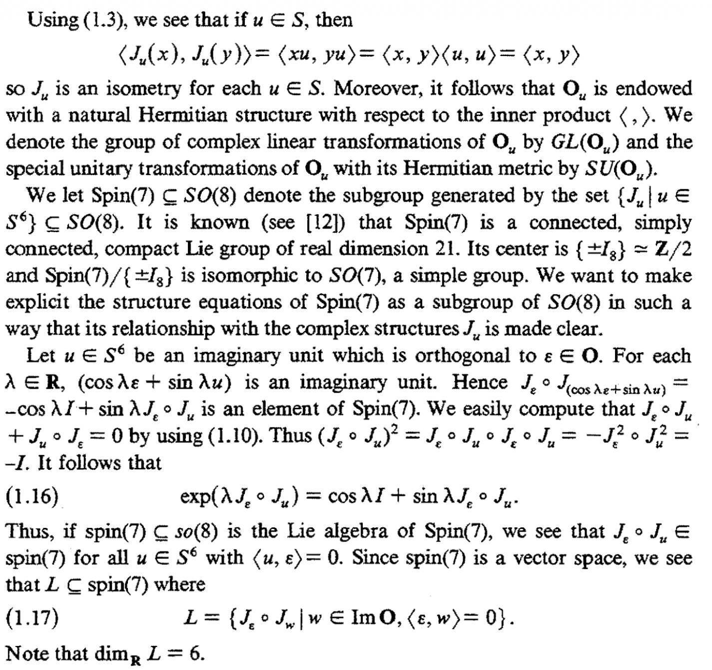
In[∘]:=



We let $S^6 = \{u \in \mathrm{Im}\,\mathbf{O} \mid \langle u, u\rangle = 1\}$. The elements of S^6 are called the *imaginary units* of $\mathbf{O}$. For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u\rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \to \mathbf{O}$ given by

$$J_u(x) = xu. \tag{1.15}$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on $\mathbf{O}$. We write $\mathbf{O}_u$ to denote $\mathbf{O}$ endowed with the complex structure J_u. If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on $\mathbf{O}$. However, because S^6 is connected, we see that the orientation of $\mathbf{O}$ induced by the natural orientation of $\mathbf{O}_u$ as a complex vector space does not depend on u. We refer to this orientation as the natural orientation of $\mathbf{O}$.



Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y)\rangle = \langle xu, yu\rangle = \langle x, y\rangle\langle u, u\rangle = \langle x, y\rangle$$

so J_u is an isometry for each $u \in S$. Moreover, it follows that $\mathbf{O}_u$ is endowed with a natural Hermitian structure with respect to the inner product $\langle\, ,\rangle$. We denote the group of complex linear transformations of $\mathbf{O}_u$ by $GL(\mathbf{O}_u)$ and the special unitary transformations of $\mathbf{O}_u$ with its Hermitian metric by $SU(\mathbf{O}_u)$.

We let $\mathrm{Spin}(7) \subseteq SO(8)$ denote the subgroup generated by the set $\{J_u \mid u \in S^6\} \subseteq SO(8)$. It is known (see [12]) that Spin(7) is a connected, simply connected, compact Lie group of real dimension 21. Its center is $\{\pm I_8\} \simeq \mathbf{Z}/2$ and $\mathrm{Spin}(7)/\{\pm I_8\}$ is isomorphic to $SO(7)$, a simple group. We want to make explicit the structure equations of Spin(7) as a subgroup of $SO(8)$ in such a way that its relationship with the complex structures J_u is made clear.

In[∘]:=

Let $u \in S^6$ be an imaginary unit which is orthogonal to $\varepsilon \in \mathbf{O}$. For each $\lambda \in \mathbf{R}$, $(\cos\lambda\varepsilon + \sin\lambda u)$ is an imaginary unit. Hence $J_\varepsilon \circ J_{(\cos\lambda\varepsilon+\sin\lambda u)} = -\cos\lambda I + \sin\lambda J_\varepsilon \circ J_u$ is an element of Spin(7). We easily compute that $J_\varepsilon \circ J_u + J_u \circ J_\varepsilon = 0$ by using (1.10). Thus $(J_\varepsilon \circ J_u)^2 = J_\varepsilon \circ J_u \circ J_\varepsilon \circ J_u = -J_\varepsilon^2 \circ J_u^2 = -I$. It follows that

$$\exp(\lambda J_\varepsilon \circ J_u) = \cos\lambda I + \sin\lambda J_\varepsilon \circ J_u. \tag{1.16}$$

Thus, if $\mathrm{spin}(7) \subseteq so(8)$ is the Lie algebra of Spin(7), we see that $J_\varepsilon \circ J_u \in \mathrm{spin}(7)$ for all $u \in S^6$ with $\langle u, \varepsilon\rangle = 0$. Since spin(7) is a vector space, we see that $L \subseteq \mathrm{spin}(7)$ where

$$L = \{J_\varepsilon \circ J_w \mid w \in \mathrm{Im}\,\mathbf{O}, \langle \varepsilon, w\rangle = 0\}. \tag{1.17}$$

Note that $\dim_{\mathbf{R}} L = 6$.

Transpose[a] . σ16 . b == Transpose[J . a] . σ16 . J . b = a . Transpose[J] . σ16 . J . b

want ψcc.σ16.?.Ψ16 = Ψ16.J.σ16.?.Ψ16 ≠ 0

need 1 irrep for {i, u}

need 1 irrep for {i, u} _perp

1 == x0 or x4; u == x4 or x0

x0 has periodic fns; use x4

In[907]:=

```
t16A⟦All, 2⟧
```

Out[907]=

```
{{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}}
```

In[908]:=

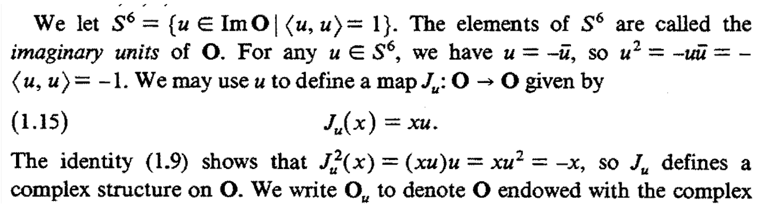
```
T16^A[4].T16^A[4] === -Id16
```

Out[908]=

```
True
```

In[909]:=

```
(*JT4=T16^A[4] *)(* complex structure on S^6 *)
```



We let $S^6 = \{u \in \mathrm{Im}\,\mathbf{O} \mid \langle u, u\rangle = 1\}$. The elements of S^6 are called the *imaginary units* of $\mathbf{O}$. For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u\rangle = -1$. We may use u to define a map $J_u\colon \mathbf{O} \to \mathbf{O}$ given by

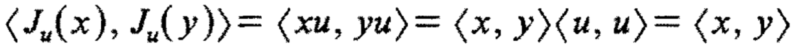
$$J_u(x) = xu. \tag{1.15}$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on $\mathbf{O}$. We write $\mathbf{O}_u$ to denote $\mathbf{O}$ endowed with the complex

J.J === -ID16

matrix multplication is associative

(split) octonion multplication is NOT associative



$$\langle J_u(x), J_u(y)\rangle = \langle xu, yu\rangle = \langle x, y\rangle\langle u, u\rangle = \langle x, y\rangle$$

Transpose[J].σ16.J === σ16

In[910]:=

```
Clear[possibleComplexStructureIndex2];
(*Clear[possibleComplexStructureMatrix2];*)
possibleComplexStructureIndex2[j_] := Module[{m, r, jsj, jj},
  J = base16⟦j⟧⟦1⟧;
  jj = (FullSimplify[J.J] === -ID16);
  jsj = FullSimplify[Transpose[J].σ16.J] === σ16;
  Return[{jj, jsj}];]
```

In[912]:=

```
possibleComplexStructureIndex2[#] & /@ {12, 97, 100, 109, 127, 225, 230, 236}
```

Out[912]=

```
{{False, False}, {True, False}, {True, False}, {True, False},
 {True, False}, {False, False}, {False, False}, {False, False}}
```

In[913]:=

```
Clear[possibleComplexStructureIndex];
Clear[possibleComplexStructureMatrix];
possibleComplexStructureIndex = (*Parallel*)Table[
    Block[{J, r, m},
     Clear[J, r, m];
     m = r = {};
     J = base16⟦j⟧⟦1⟧;
     If[(J.J === -ID16) &&
       (FullSimplify[Transpose[J].σ16.J] === σ16), r = base16⟦j⟧⟦2⟧;
      m = σ16.J;, ## &[]];
     (*If[!(m===-Transpose[m]),r=base16⟦j⟧⟦2⟧,r={}];*)
     r]
    , {j, 1, Length[base16]}] // Union // Drop[#, 1] &
```

Out[915]=

```
{{4}, {5}, {6}, {7}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {4, 5}, {4, 6},
 {4, 7}, {5, 6}, {5, 7}, {6, 7}, {0, 1, 2, 3, 4}, {0, 1, 2, 3, 5}, {0, 1, 2, 3, 6},
 {0, 1, 2, 3, 7}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7}, {0, 1, 5, 6, 7},
 {0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 2, 5, 6, 7}, {0, 3, 4, 5, 6},
 {0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {0, 3, 5, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7},
 {1, 2, 4, 6, 7}, {1, 2, 5, 6, 7}, {1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7},
 {1, 3, 5, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7}, {2, 3, 4, 6, 7}, {2, 3, 5, 6, 7},
 {0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 6}, {0, 1, 2, 3, 4, 7}, {0, 1, 2, 3, 5, 6},
 {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}, {0, 1, 4, 5, 6, 7}, {0, 2, 4, 5, 6, 7},
 {0, 3, 4, 5, 6, 7}, {1, 2, 4, 5, 6, 7}, {1, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 7}}
```

In[916]:=

```
Length[possibleComplexStructureIndex]
```

Out[916]=

```
56
```

In[917]:=

```
sixersJ = If[Length[#] ≥ 6, #, ## &[]] & /@ possibleComplexStructureIndex
```

Out[917]=

```
{{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 6}, {0, 1, 2, 3, 4, 7}, {0, 1, 2, 3, 5, 6},
 {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}, {0, 1, 4, 5, 6, 7}, {0, 2, 4, 5, 6, 7},
 {0, 3, 4, 5, 6, 7}, {1, 2, 4, 5, 6, 7}, {1, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 7}}
```

In[918]:=

```
Clear[possibleT4ComplexStructureIndex];
Clear[possibleT4ComplexStructureMatrix];
possibleComplexT4StructureIndex = (*Parallel*)Table[
    Block[{J, a1, a2, b1, b2, c1, c2,
      c3, c3ori, c3s, c2ed, found, r, m, debug = False},
     Clear[J, a1, a2, b1, b2, c1, c2, c3, c3ori, c3s, c2ed, found, r, m];
     c3s = found = r = {};
     If[FullSimplify[T16^A[4].base16⟦j⟧⟦1⟧ === base16⟦j⟧⟦1⟧.T16^A[4]],
      J = T16^A[4].base16⟦j⟧⟦1⟧ // FullSimplify;
      a1 = J.J // FullSimplify;
      If[(a1 === -ID16),
       a2 = Transpose[J].σ16.J // FullSimplify;
       If[(a2 === σ16),
        m = σ16.J;
        c1 = Append[base16⟦j⟧⟦2⟧, 4] // Sort; (* for J = T16^A[4]*#⟦1⟧*)
        c3ori = c3 = Select[base16, #⟦2⟧ ⩵ c1 & → "Index"];
        (*Print["c1= ",c1];*)
        If[c3 === {},
         c3s = StringReplace[ToString[c1], "4, 4" → "256"] //
             ToExpression // Sort // Drop[#, -1] &;
         c3 = Select[base16, #⟦2⟧ ⩵ c3s & → "Index"],
         ## &[]];
        If[debug, Print["Append[base16[[j]][[2]],4]//Sort=c1= ",
          c1, "Select[base16,#[[2]]==c1&]= ", c3ori,
          ";StringReplace[c1,'4, 4'->'256']= ",
          c3s, ";c3= ", c3], ## &[]];
        If[! (c3 === {}),
         c2 = c3⟦1⟧;
         (*Print["c2= ",c2];*)
         If[c2 > 0 && c2 ≤ Length[base16], found = base16⟦c2⟧, ## &[]];
         If[! (found === {}) && (J === found⟦1⟧  || J === -found⟦1⟧)
           (*&&!(m===-Transpose[m])*),
          r = found⟦2⟧;
          (*Print[base16⟦j⟧⟦2⟧,";Tr[J.J]/16=",Tr[a1]/16,
```

```
              ";c1=",c1,";c2=Select[base16,#[[2]]==c1&=  ",c2,
              ";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
              Tr[found⟦1⟧.found⟦1⟧]/16;*)
            ,
            ## &[];
          , ## &[], ## &[]], ## &[]];, ## &[]];
    r]
   , {j, 1, Length[base16]}] // Union // Drop[#, 1] &
```

Out[920]=

```
{{4}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {5, 6}, {5, 7},
 {6, 7}, {0, 1, 2, 3, 4}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7},
 {0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 3, 4, 5, 6},
 {0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7}, {1, 2, 4, 6, 7},
 {1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7},
 {2, 3, 4, 6, 7}, {0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}
```

In[921]:=

```
JT4 = T16^A[4] ;(* complex structure on S^6 *)
```

Jcomplement ⟷($\hat{q}$[[7]]*Cos[Q7] + $\hat{q}$[[6]]*Sin[Q7] Cos[Q8] + $\hat{q}$[[5]]*Sin[Q7] Sin[Q8])

In[922]:=

```
basisProjections16 =
  Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@
   Range[16];
```

In[923]:=

```
basisProjections8 =
  Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@
   Range[8];
```

In[924]:=

```
JcomplementProjection8 =
  (basisProjections8⟦7⟧ + basisProjections8⟦6⟧ + basisProjections8⟦5⟧);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8
```

Out[925]=

```
True
```

In[926]:=

```
JcomplementProjection8 = (basisProjections8⟦7⟧ +
    basisProjections8⟦6⟧ + basisProjections8⟦5⟧ + basisProjections8⟦4⟧);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8
```

Out[927]=

```
True
```

In[928]:=

```
(*JcomplementProjection8=
  (basisProjections8⟦7⟧*Cos[Q7]+basisProjections8⟦6⟧*Sin[Q7]Cos[Q8]+
    basisProjections8⟦5⟧*Sin[Q7]Sin[Q8]);*)
```

```
In[929]:=
        JcomplementProjection8.JcomplementProjection8 // FullSimplify // MatrixForm
        JcomplementProjection8 // MatrixForm
        %% - %⋮
Out[929]//MatrixForm=
        ⎛0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎞
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎝0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎠
Out[930]//MatrixForm=
        ⎛0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎞
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎜0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎟
        ⎝0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0⎠
Out[931]=
        {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 1 - ⋮, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 0, 1 - ⋮, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 0, 0, 1 - ⋮, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 0, 0, 0, 1 - ⋮, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
         {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

In[932]:=

```
(parallel04 =
   Block[{x = Table[KroneckerDelta[1, j], {j, 16}]}, KroneckerProduct[x, x]] +
    Block[{x = Table[KroneckerDelta[5, j], {j, 16}]},
     KroneckerProduct[x, x]]) // MatrixForm
```

Out[932]//MatrixForm=

```
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

In[933]:=

```
(projection04 = ID16 -
    Block[{x = Table[KroneckerDelta[1, j], {j, 16}]}, KroneckerProduct[x, x]] -
    Block[{x = Table[KroneckerDelta[5, j], {j, 16}]},
     KroneckerProduct[x, x]]) // MatrixForm
```

Out[933]//MatrixForm=

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
```

In[934]:=

```
projection04.parallel04 === Zero16
```

Out[934]=

```
True
```

$\{\{0, 1, 2, 3, 5, 6\}, \{0, 1, 2, 3, 5, 7\}, \{0, 1, 2, 3, 6, 7\}\}$

```
In[935]:=
        T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] & @@ {0, 1, 2, 3, 5, 6};
        % === -T16^A[4].T16^A[7].T16^A[8]
Out[936]=
        True
In[937]:=
        T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] & @@ {0, 1, 2, 3, 5, 7};
        % === T16^A[4].T16^A[6].T16^A[8]
Out[938]=
        True
In[939]:=
        T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] & @@ {0, 1, 2, 3, 6, 7};
        % === -T16^A[4].T16^A[5].T16^A[8]
Out[940]=
        True
```

```
In[941]:=
sixers = If[Length[#] ≥ 6, #, ## &[]] & /@ possibleComplexT4StructureIndex
sixersIndices = Table[Select[base16, #⟦2⟧ === sixers⟦j⟧ & → "Index"],
   {j, 1, Length[sixers]}] // Flatten
base16⟦#⟧⟦1⟧ & /@ sixersIndices;
Jcomplement = (%⟦1⟧ * Cos[Q7] + %⟦2⟧ * Sin[Q7] Cos[Q8] + %⟦3⟧ * Sin[Q7] Sin[Q8])
Dimensions[Jcomplement]
FullSimplify[Jcomplement.Jcomplement] === -ID16
Out[941]=
{{0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}
Out[942]=
{222, 223, 224}
Out[944]=
{{0, 0, -Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, -Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0},
 {Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7], 0, 0, 0, 0, -Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -Cos[Q7], 0, 0, 0, 0, -Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0},
 {Cos[Q7], 0, 0, 0, 0, 0, Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, Cos[Q7], Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -Cos[Q7], 0, Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8], 0, Cos[Q7], 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7], -Cos[Q7], 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, -Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, -Cos[Q7]},
 {0, 0, 0, 0, 0, 0, 0, 0, -Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], 0, 0, 0, 0, Cos[Q7], 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[Q7], 0, 0, 0, 0, Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]},
 {0, 0, 0, 0, 0, 0, 0, 0, -Cos[Q7], 0, 0, 0, 0, 0, -Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7]},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -Cos[Q7], -Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8], 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[Q7], 0, -Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], 0, 0}}
Out[945]=
{16, 16}
Out[946]=
True
In[947]:=
realProjection8 = KroneckerProduct[unit, unit]
Out[947]=
{{1/2, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {1/2, 0, 0, 0, 1/2, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
In[948]:=
realProjection8.realProjection8 === realProjection8
Out[948]=
True
In[949]:=
(*KroneckerProduct[unit,Transpose[unit]]*)
```

In[950]:=

```
realProjection16 = ArrayFlatten[{{realProjection8, 0}, {0, realProjection8}}]
```

Out[950]=

```
{{1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

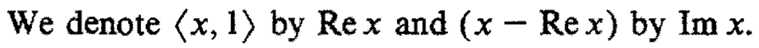
In[951]:=

```
realProjection16.realProjection16 === realProjection16
```

Out[951]=

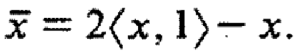
```
True
```

In[952]:=



We denote $\langle x, 1\rangle$ by $\operatorname{Re} x$ and $(x - \operatorname{Re} x)$ by $\operatorname{Im} x$.

In[953]:=



$$\bar{x} = 2\langle x, 1\rangle - x.$$

In[954]:=

```
unit.unit
```

Out[954]=

```
1
```

In[956]:=

```
T16^A[4].T16^A[4] === -ID16
```

Out[956]=

```
True
```

In[957]:=

```
T4cc = T16^A[4] - (realProjection16.T16^A[4]) // FullSimplify
```

Out[957]=

```
{{0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, -1/2, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 1/2, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
 {0, 1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1/2, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

In[958]:=

```
T4cc.T4cc
```

Out[958]=

```
{{-1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1/2, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {1/2, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -1/2, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, 1/2, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, -1/2, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, -1/2, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2, 0, 0, 0, -1/2, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}
```

In[959]:=

```
T16^A[7].T16^A[7] === -ID16
```

Out[959]=

```
True
```

In[960]:=

```
T7cc = T16^A[7] - (realProjection16.T16^A[7]) // FullSimplify
```

Out[960]=

```
                            1           1
{{0, 0, 0, 0, 0, 0, 0, 0, -, 0, 0, 0, -, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
                            2           2
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
                             1            1
 {0, 0, 0, 0, 0, 0, 0, 0, - -, 0, 0, 0, - -, 0, 0, 0},
                             2            2
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1},
    1            1
 {- -, 0, 0, 0, - -, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
    2            2
 {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  1           1
 {-, 0, 0, 0, -, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  2           2
 {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}}
```

In[961]:=

```
T7cc.T7cc
```

Out[961]=

```
{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}
```

In[962]:=

```
FullSimplify[Jcomplement.Jcomplement] === -ID16
```

Out[962]=

```
True
```

In[963]:=

```
Jcomplementcc = Jcomplement - (realProjection16.Jcomplement) // FullSimplify
```

Out[963]=

```
{{0, Cos[Q7]/2, -1/2 Cos[Q8] Sin[Q7], -1/2 Sin[Q7] Sin[Q8], 0,
  -Cos[Q7]/2, 1/2 Cos[Q8] Sin[Q7], 1/2 Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0},
 {Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7], 0, 0, 0, 0, -Cos[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, -Cos[Q7]/2, 1/2 Cos[Q8] Sin[Q7], 1/2 Sin[Q7] Sin[Q8], 0, Cos[Q7]/2,
  -1/2 Cos[Q8] Sin[Q7], -1/2 Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0},
 {Cos[Q7], 0, 0, 0, 0, 0, Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, Cos[Q7], Cos[Q8] Sin[Q7], -Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, -Cos[Q7], 0, Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, -Cos[Q7]/2, 1/2 Cos[Q8] Sin[Q7],
  1/2 Sin[Q7] Sin[Q8], 0, Cos[Q7]/2, -1/2 Cos[Q8] Sin[Q7], -1/2 Sin[Q7] Sin[Q8]},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7], -Cos[Q7], 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, -Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, -Cos[Q7]},
 {0, 0, 0, 0, 0, 0, 0, 0, -Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], 0, 0, 0, 0, Cos[Q7], 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[Q7]/2, -1/2 Cos[Q8] Sin[Q7],
  -1/2 Sin[Q7] Sin[Q8], 0, -Cos[Q7]/2, 1/2 Cos[Q8] Sin[Q7], 1/2 Sin[Q7] Sin[Q8]},
 {0, 0, 0, 0, 0, 0, 0, 0, -Cos[Q7], 0, 0, 0, 0, 0, -Sin[Q7] Sin[Q8], Cos[Q8] Sin[Q7]},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -Cos[Q7], -Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8], 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[Q7], 0, -Sin[Q7] Sin[Q8], -Cos[Q8] Sin[Q7], 0, 0}}
```

In[964]:=

```
FullSimplify[Jcomplementcc.Jcomplementcc]
```

Out[964]=

```
{{-1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1/4 (-3 + Cos[2 Q7]), 1/2 Cos[Q7] Cos[Q8] Sin[Q7], 1/2 Cos[Q7] Sin[Q7] Sin[Q8], 0, Cos[Q7]^2/2,
  1/2 Cos[Q7] Cos[Q8] Sin[Q7], 1/2 Cos[Q7] Sin[Q7] Sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1/2 Cos[Q7] Cos[Q8] Sin[Q7], -Cos[Q7]^2 + 1/4 (-3 + Cos[2 Q8]) Sin[Q7]^2,
  1/2 Cos[Q8] Sin[Q7]^2 Sin[Q8], 0, 1/2 Cos[Q7] Cos[Q8] Sin[Q7],
  1/2 Cos[Q8]^2 Sin[Q7]^2, 1/2 Cos[Q8] Sin[Q7]^2 Sin[Q8], 0, 0, 0, 0, 0, 0, 0, 0},
```

$$\Big\{0,\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},\ \frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]},$$
$$-\text{Cos[Q7]}^2-\frac{1}{4}\,(3+\text{Cos[2 Q8]})\,\text{Sin[Q7]}^2,\ 0,\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},$$
$$\frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]},\ \frac{1}{2}\,\text{Sin[Q7]}^2\,\text{Sin[Q8]}^2,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\Big\},$$
$$\Big\{\frac{1}{2},\ 0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\Big\},$$
$$\Big\{0,\ \frac{\text{Cos[Q7]}^2}{2},\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},\ 0,\ \frac{1}{4}\,(-3+\text{Cos[2 Q7]}),$$
$$\frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\Big\},$$
$$\Big\{0,\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ \frac{1}{2}\,\text{Cos[Q8]}^2\,\text{Sin[Q7]}^2,\ \frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]},$$
$$0,\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ -\text{Cos[Q7]}^2+\frac{1}{4}\,(-3+\text{Cos[2 Q8]})\,\text{Sin[Q7]}^2,$$
$$\frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\Big\},\ \Big\{0,\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},$$
$$\frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]},\ \frac{1}{2}\,\text{Sin[Q7]}^2\,\text{Sin[Q8]}^2,\ 0,\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},$$
$$\frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]},\ -\text{Cos[Q7]}^2-\frac{1}{4}\,(3+\text{Cos[2 Q8]})\,\text{Sin[Q7]}^2,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\Big\},$$
$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0\Big\},\ \Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$$
$$\frac{1}{4}\,(-3+\text{Cos[2 Q7]}),\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},$$
$$0,\ \frac{\text{Cos[Q7]}^2}{2},\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]}\Big\},$$
$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ -\text{Cos[Q7]}^2+\frac{1}{4}\,(-3+\text{Cos[2 Q8]})\,\text{Sin[Q7]}^2,$$
$$\frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]},\ 0,\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ \frac{1}{2}\,\text{Cos[Q8]}^2\,\text{Sin[Q7]}^2,$$
$$\frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]}\Big\},\ \Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},$$
$$\frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]},\ -\text{Cos[Q7]}^2-\frac{1}{4}\,(3+\text{Cos[2 Q8]})\,\text{Sin[Q7]}^2,\ 0,$$
$$\frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},\ \frac{1}{2}\,\text{Cos[Q8] Sin[Q7]}^2\,\text{Sin[Q8]},\ \frac{1}{2}\,\text{Sin[Q7]}^2\,\text{Sin[Q8]}^2\Big\},$$
$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2},\ 0,\ 0,\ 0,\ -\frac{1}{2},\ 0,\ 0,\ 0\Big\},$$
$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{\text{Cos[Q7]}^2}{2},\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]},$$
$$0,\ \frac{1}{4}\,(-3+\text{Cos[2 Q7]}),\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ \frac{1}{2}\,\text{Cos[Q7] Sin[Q7] Sin[Q8]}\Big\},$$
$$\Big\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{2}\,\text{Cos[Q7] Cos[Q8] Sin[Q7]},\ \frac{1}{2}\,\text{Cos[Q8]}^2\,\text{Sin[Q7]}^2,$$

```
 1                              1
 - Cos[Q8] Sin[Q7]^2 Sin[Q8], 0, - Cos[Q7] Cos[Q8] Sin[Q7],
 2                              2
              1                                 1
 -Cos[Q7]^2 + - (-3 + Cos[2 Q8]) Sin[Q7]^2,   - Cos[Q8] Sin[Q7]^2 Sin[Q8]},
              4                                 2
                                1                                 1
{0, 0, 0, 0, 0, 0, 0, 0, 0,     - Cos[Q7] Sin[Q7] Sin[Q8],        - Cos[Q8] Sin[Q7]^2 Sin[Q8],
                                2                                 2
 1                               1
 - Sin[Q7]^2 Sin[Q8]^2, 0,       - Cos[Q7] Sin[Q7] Sin[Q8],
 2                               2
 1                                                 1
 - Cos[Q8] Sin[Q7]^2 Sin[Q8], -Cos[Q7]^2 -         - (3 + Cos[2 Q8]) Sin[Q7]^2}}
 2                                                 4
```

In[965]:=

```
T16^A[4].Jcomplement // MatrixForm
```

Out[965]//MatrixForm=

```
(        0                  0                  0                  0                  0
         0                  0                  0                  0                  0
         0                  0                  0                  0                  0
         0                  0                  0                  0                  0
         0                  0                  0                  0                  0
         0                  0                  0                  0                  0
         0                  0                  0                  0                  0
         0                  0                  0                  0                  0
      Cos[Q7]               0                  0                  0                  0
         0               Cos[Q7]               0                  0                  0
         0                  0               Cos[Q7]               0          -Sin[Q7] Sin[Q8]  -Cos[Q
         0                  0                  0               Cos[Q7]        Cos[Q8] Sin[Q7]  -Sin[Q
         0                  0          -Sin[Q7] Sin[Q8]   Cos[Q8] Sin[Q7]        -Cos[Q7]
         0                  0          -Cos[Q8] Sin[Q7]  -Sin[Q7] Sin[Q8]           0                -Co
  Sin[Q7] Sin[Q8]   Cos[Q8] Sin[Q7]            0                  0                  0
 -Cos[Q8] Sin[Q7]   Sin[Q7] Sin[Q8]            0                  0                  0
```

In[966]:=

```
T16^A[4].Jcomplement === Jcomplement.T16^A[4]
FullSimplify[Jcomplement.Jcomplement] === -ID16
T16^A[4].T16^A[4] === -ID16
```

Out[966]=

```
True
```

Out[967]=

```
True
```

Out[968]=

```
True
```

In[969]:=

```
unit3 = {Cos[Q7], Sin[Q7] Cos[Q8], Sin[Q7] Sin[Q8]}
```

Out[969]=

```
{Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}
```

In[970]:=

```
unit3.unit3 // FullSimplify
```

Out[970]=

```
1
```

In[971]:=

```
unit3
sixers
Complement[{0, 1, 2, 3, 5, 6, 7}, #] & /@ %
eqsRQQ7Q8 = Thread[(Subscript[q, #⟦1⟧] & /@ %) == RQ * unit3]
sRQQ7Q8 = Solve[And @@ %, {RQ, Q7, Q8}] // FullSimplify
seqsRQQ7Q8 = %% /. {Equal → Rule}
```

Out[971]=

```
{Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}
```

Out[972]=

```
{{0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}
```

Out[973]=

```
{{7}, {6}, {5}}
```

Out[974]=

$$\{q_7 == \text{RQ}\,\text{Cos}[\text{Q7}],\ q_6 == \text{RQ}\,\text{Cos}[\text{Q8}]\,\text{Sin}[\text{Q7}],\ q_5 == \text{RQ}\,\text{Sin}[\text{Q7}]\,\text{Sin}[\text{Q8}]\}$$

Out[975]=

$$\Bigl\{\Bigl\{\text{RQ} \to -\sqrt{q_5^2+q_6^2+q_7^2},\ \text{Q7} \to \boxed{2\pi\,c_1 - i\,\text{Log}\Bigl[\frac{-i\,\sqrt{q_5^2+q_6^2}-q_7}{\sqrt{q_5^2+q_6^2+q_7^2}}\Bigr]\ \text{if}\ c_1 \in \mathbb{Z}},$$

$$\text{Q8} \to \boxed{\text{ArcTan}[q_6,\ q_5] + 2\pi\,c_2\ \text{if}\ c_2 \in \mathbb{Z}}\Bigr\},\ \Bigl\{\text{RQ} \to -\sqrt{q_5^2+q_6^2+q_7^2},$$

$$\text{Q7} \to \boxed{2\pi\,c_1 - i\,\text{Log}\Bigl[\frac{i\,\bigl(\sqrt{q_5^2+q_6^2}+i\,q_7\bigr)}{\sqrt{q_5^2+q_6^2+q_7^2}}\Bigr]\ \text{if}\ c_1 \in \mathbb{Z}},\ \text{Q8} \to \boxed{2\pi\,c_2 - i\,\text{Log}\Bigl[\frac{-i\,q_5-q_6}{\sqrt{q_5^2+q_6^2}}\Bigr]\ \text{if}\ c_2 \in \mathbb{Z}}\Bigr\},$$

$$\Bigl\{\text{RQ} \to \sqrt{q_5^2+q_6^2+q_7^2},\ \text{Q7} \to \boxed{2\pi\,c_1 - i\,\text{Log}\Bigl[\frac{-i\,\sqrt{q_5^2+q_6^2}+q_7}{\sqrt{q_5^2+q_6^2+q_7^2}}\Bigr]\ \text{if}\ c_1 \in \mathbb{Z}},$$

$$\text{Q8} \to \boxed{2\pi\,c_2 - i\,\text{Log}\Bigl[\frac{-i\,q_5-q_6}{\sqrt{q_5^2+q_6^2}}\Bigr]\ \text{if}\ c_2 \in \mathbb{Z}}\Bigr\},\ \Bigl\{\text{RQ} \to \sqrt{q_5^2+q_6^2+q_7^2},$$

$$\text{Q7} \to \boxed{2\pi\,c_1 - i\,\text{Log}\Bigl[\frac{i\,\sqrt{q_5^2+q_6^2}+q_7}{\sqrt{q_5^2+q_6^2+q_7^2}}\Bigr]\ \text{if}\ c_1 \in \mathbb{Z}},\ \text{Q8} \to \boxed{\text{ArcTan}[q_6,\ q_5] + 2\pi\,c_2\ \text{if}\ c_2 \in \mathbb{Z}}\Bigr\}\Bigr\}$$

Out[976]=

$$\{q_7 \to \text{RQ}\,\text{Cos}[\text{Q7}],\ q_6 \to \text{RQ}\,\text{Cos}[\text{Q8}]\,\text{Sin}[\text{Q7}],\ q_5 \to \text{RQ}\,\text{Sin}[\text{Q7}]\,\text{Sin}[\text{Q8}]\}$$

In[977]:=

$$\texttt{FullSimplify}\left[\left(\sqrt{q_6{}^2+q_5{}^2}\ \texttt{/. seqsRQQ7Q8}\right),\ \texttt{Q7} > 0\ \&\&\ \texttt{Sin[Q7]} > 0\ \&\&\ \texttt{RQ} > 0\right]$$

$$\texttt{\%} / q_7\ \texttt{/. seqsRQQ7Q8}$$

$$\texttt{Solve}\left[\frac{\sqrt{q_6{}^2+q_5{}^2}}{q_7} == \texttt{FullSimplify[\%, Q7 > 0 \&\& Sin[Q7] > 0 \&\& RQ > 0], Q7}\right]$$

$$\texttt{sQ7} = \left\{\texttt{Q7} \rightarrow \texttt{ArcTan}\left[q_7,\ \sqrt{q_5^2+q_6^2}\right]\right\}$$

Out[977]=

RQ Sin[Q7]

Out[978]=

Tan[Q7]

Out[979]=

$$\left\{\left\{\texttt{Q7} \rightarrow \boxed{\texttt{ArcTan}\left[\frac{\sqrt{q_5^2+q_6^2}}{q_7}\right] + \pi\, c_1\ \text{if}\ c_1 \in \mathbb{Z}}\right\}\right\}$$

Out[980]=

$$\left\{\texttt{Q7} \rightarrow \texttt{ArcTan}\left[q_7,\ \sqrt{q_5^2+q_6^2}\right]\right\}$$

In[981]:=

$$\texttt{(*sQ7=*)Solve}\left[\frac{\sqrt{q_6{}^2+q_5{}^2}}{q_7} == \texttt{FullSimplify}\left[\left(\frac{\sqrt{q_6{}^2+q_5{}^2}}{q_7}\ \texttt{/. seqsRQQ7Q8}\right),\right.\right.$$

$$\left.\left.\texttt{Q7} > 0\ \&\&\ \texttt{Sin[Q7]} > 0\ \&\&\ \texttt{RQ} > 0\right],\ \texttt{Q7}\right]\ \texttt{// FullSimplify}$$

Out[981]=

$$\left\{\left\{\texttt{Q7} \rightarrow \boxed{\texttt{ArcCot}\left[\frac{q_7}{\sqrt{q_5^2+q_6^2}}\right] + \pi\, c_1\ \text{if}\ c_1 \in \mathbb{Z}}\right\}\right\}$$

In[982]:=

```
eqsRQQ7Q8⟦2 ;; 3⟧
Transpose[{%⟦All, 1⟧, %⟦All, 2⟧}]
%⟦2⟧⟦1⟧ / %⟦1⟧⟦1⟧ == %⟦2⟧⟦2⟧ / %⟦1⟧⟦2⟧ // FullSimplify
sQ8 = Solve[%, Q8]⟦1⟧ // FullSimplify
sQ8 = {Q8 → ArcTan[q6, q5]}
sRQQ7Q8 /. % // FullSimplify
```

Out[982]=

$$\{q_6 == \text{RQ}\,\text{Cos}[\text{Q8}]\,\text{Sin}[\text{Q7}],\ q_5 == \text{RQ}\,\text{Sin}[\text{Q7}]\,\text{Sin}[\text{Q8}]\}$$

Out[983]=

$$\{\{q_6,\ \text{RQ}\,\text{Cos}[\text{Q8}]\,\text{Sin}[\text{Q7}]\},\ \{q_5,\ \text{RQ}\,\text{Sin}[\text{Q7}]\,\text{Sin}[\text{Q8}]\}\}$$

Out[984]=

$$\frac{q_5}{q_6} == \text{Tan}[\text{Q8}]$$

Out[985]=

$$\left\{\text{Q8} \to \boxed{\text{ArcTan}\left[\frac{q_5}{q_6}\right] + \pi\, c_1 \text{ if } c_1 \in \mathbb{Z}}\right\}$$

Out[986]=

$$\{\text{Q8} \to \text{ArcTan}[q_6,\ q_5]\}$$

Out[987]=

$$\Bigg\{\Bigg\{\text{RQ} \to -\sqrt{q_5^2 + q_6^2 + q_7^2},\ \text{Q7} \to \boxed{2\pi c_1 - i\,\text{Log}\left[\frac{-i\sqrt{q_5^2 + q_6^2} - q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}},$$

$$\text{ArcTan}[q_6,\ q_5] \to \boxed{\text{ArcTan}[q_6,\ q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z}}\Bigg\},$$

$$\Bigg\{\text{RQ} \to -\sqrt{q_5^2 + q_6^2 + q_7^2},\ \text{Q7} \to \boxed{2\pi c_1 - i\,\text{Log}\left[\frac{i\left(\sqrt{q_5^2 + q_6^2} + i\,q_7\right)}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}},$$

$$\text{ArcTan}[q_6,\ q_5] \to \boxed{2\pi c_2 - i\,\text{Log}\left[\frac{-i\,q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z}}\Bigg\},$$

$$\Bigg\{\text{RQ} \to \sqrt{q_5^2 + q_6^2 + q_7^2},\ \text{Q7} \to \boxed{2\pi c_1 - i\,\text{Log}\left[\frac{-i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}},$$

$$\text{ArcTan}[q_6,\ q_5] \to \boxed{2\pi c_2 - i\,\text{Log}\left[\frac{-i\,q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z}}\Bigg\},\ \Bigg\{\text{RQ} \to \sqrt{q_5^2 + q_6^2 + q_7^2},$$

$$\text{Q7} \to \boxed{2\pi c_1 - i\,\text{Log}\left[\frac{i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}},\ \text{ArcTan}[q_6,\ q_5] \to \boxed{\text{ArcTan}[q_6,\ q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z}}\Bigg\}\Bigg\}$$

In[988]:=

```
sQ7Q8 = Flatten[{sQ7, sQ8}]
```

Out[988]=

$$\left\{\text{Q7} \to \text{ArcTan}\left[q_7,\ \sqrt{q_5^2 + q_6^2}\right],\ \text{Q8} \to \text{ArcTan}[q_6,\ q_5]\right\}$$

```
In[989]:=
Block[{a, b, c},
 {a, b, c} * unit3]
Out[989]=
{a Cos[Q7], b Cos[Q8] Sin[Q7], c Sin[Q7] Sin[Q8]}
In[990]:=
base16⟦#⟧⟦1⟧.base16⟦#⟧⟦1⟧ === -ID16 & /@ {222, 223, 224}
Out[990]=
{True, True, True}
In[991]:=
(*sixersT4=%*{Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}
 {Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}.
   {Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}//FullSimplify*)
In[992]:=
Block[{J = T16^A[4], m = σ16.J},
 {J.J === -ID16, σ16 === Transpose[J].σ16.J, ! (m === -Transpose[m])}]
Out[992]=
{True, True, False}
In[993]:=
(*Clear[possibleComplexStructureIndex];
  Clear[possibleComplexStructureMatrix];
  possibleComplexStructureIndex=Select[base16,Block[{J,a0,a1,a2,b1,b2,r},
      Clear[J,a0,a1,a2,b1,b2,r];
      a0=#⟦1⟧;
      J=T16^A[4]*a0//FullSimplify;
      a1=J.J//FullSimplify;
      b1=(a1===-ID16);
      a2=Transpose[J].σ16.J//FullSimplify;
      b2=(a2===σ16);
      r=b1&&b2;
      If[r,Block[{c1,c2,c3},
         c1=Append[#⟦2⟧,4]//Sort; (* for J = T16^A[4]*#⟦1⟧*)
         c2=Select[base16,#⟦2⟧==c1&→"Index"]⟦1⟧;
         Print[#⟦2⟧,";",Tr[a1]/16,";c1=",c1,";c2=",
          c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
          Tr[base16⟦c2⟧⟦1⟧.base16⟦c2⟧⟦1⟧]/16]],##&[]];
      r
     ]&→"Index"]
  possibleComplexStructureMatrix=base16⟦#⟧⟦2⟧&/@%*)
In[994]:=
(*Tr[(T16^A[3].T16^A[4]).(T16^A[3].T16^A[4])]/16*)
```

In[995]:=

```
(*base16⟦5⟧⟦2⟧*)
```

In[996]:=

```
(*Select[base16,#⟦2⟧⩵{1,2,3,4,5,6}&→"Index"]*)
```

In[997]:=

```
(*base16⟦240⟧⟦2⟧
 Tr[base16⟦240⟧⟦1⟧.base16⟦240⟧⟦1⟧]/16*)
```

In[998]:=

```
(*(*Parallel*)Do[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
   Clear[J,a0,a1,a2,b1,b2,r];
   a0=base16⟦j⟧⟦1⟧;
   J=T16^A[4]*a0//FullSimplify;
   a1=J.J//FullSimplify;
   If[!(a1===-ID16),Continue[],##&[]];
   (*b1=(a1===-ID16);*)
   a2=Transpose[J].σ16.J//FullSimplify;
   If[!(a2===σ16),Continue[],##&[]];
   (*b2=(a2===σ16);*)
   (*r=b1&&b2;*)

   c1=Append[base16⟦j⟧⟦2⟧,4]//Sort; (* for J = T16^A[4]*#⟦1⟧*)
   c2=Select[base16,#⟦2⟧⩵c1&→"Index"]⟦1⟧;
   Print[base16⟦j⟧⟦2⟧,";",Tr[a1]/16,";c1=",c1,
    ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
    Tr[base16⟦c2⟧⟦1⟧.base16⟦c2⟧⟦1⟧]/16];
   {J,base16⟦c2⟧⟦1⟧}];
 ,{j,1,Length[base16]}]*)
```

In[999]:=

```
(*ParallelDo[
 Block[{r,t,a1},
  t=Tr[base16⟦j⟧⟦1⟧.base16⟦k⟧⟦1⟧/16];
  r=t^2===KroneckerDelta[j,k];
  a1=If[r,##&[],Print[{{j,k},t,{base16⟦j⟧⟦2⟧,base16⟦k⟧⟦2⟧}}]];
 ],{j,1,Length[base16]},{k,1,Length[base16]}]*)
```

In[1000]:=

```
(*(*Parallel*)Table[Catch[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
   Clear[J,a0,a1,a2,b1,b2,r];
   a0=base16⟦j⟧⟦1⟧;
   J=T16^A[4]*a0//FullSimplify;
   a1=J.J//FullSimplify;
   If[!(a1===-ID16),Throw[Null],##&[]];
   (*b1=(a1===-ID16);*)
   a2=Transpose[J].σ16.J//FullSimplify;
   If[!(a2===σ16),Throw[Null],##&[]];
   (*b2=(a2===σ16);*)
   (*r=b1&&b2;*)

   c1=Append[base16⟦j⟧⟦2⟧,4]//Sort; (* for J = T16^A[4]*#⟦1⟧*)
   c2=Select[base16,#⟦2⟧⩵c1&→"Index"]⟦1⟧;
   Print[base16⟦j⟧⟦2⟧,";",Tr[a1]/16,";c1=",c1,
    ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
    Tr[base16⟦c2⟧⟦1⟧.base16⟦c2⟧⟦1⟧]/16];
   {{J},{base16⟦c2⟧⟦1⟧},{J===base16⟦c2⟧⟦1⟧  || J===-base16⟦c2⟧⟦1⟧}}]]
 ,{j,1,Length[base16]/128}]*)
```

In[1001]:=

```
(*(*Parallel*)Table[
  Block[{J,a1,a2,b1,b2,c1,c2,c3,r},
   Clear[J,a1,a2,b1,b2,r];
   r={};
   J=T16^A[4].base16⟦j⟧⟦1⟧//FullSimplify;
   a1=J.J//FullSimplify;
   If[(a1===-ID16),
    a2=Transpose[J].σ16.J//FullSimplify;
    If[(a2===σ16),
     c1=Append[base16⟦j⟧⟦2⟧,4]//Sort; (* for J = T16^A[4]*#⟦1⟧*)
     c3=Select[base16,#⟦2⟧==c1&→"Index"];
     If[!(c3==={}),
      c2=c3⟦1⟧;
      Print[base16⟦j⟧⟦2⟧,";Tr[J.J]/16=",Tr[a1]/16,";c1=",c1,
       ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
       Tr[base16⟦c2⟧⟦1⟧.base16⟦c2⟧⟦1⟧]/16];
      r={{J},{base16⟦c2⟧⟦1⟧},{J===base16⟦c2⟧⟦1⟧  || J===-base16⟦c2⟧⟦1⟧}};
      ,##&[]],##&[]],##&[]];
   r]
  ,{j,1,Length[base16]}]//Union*)
```

In[1002]:=

```
(*possibleComplexStructureIndex=
 Select[base16,Block[{J=T16^A[4]*#⟦1⟧,a1,a2,b1,b2,r},
      a1=J.J;
      a2=Transpose[J].σ16.J;
      b1=a1===-ID16;
      b2=a2===σ16;
      r=b1&&b2;
      (*If[r,Print[a1]];*)
      r
     ]&→"Index"]
  possibleComplexStructureMatrix=base16⟦#⟧⟦2⟧&/@%*)
```

In[1003]:=

```
base16⟦201⟧⟦2⟧
```

Out[1003]=

```
{1, 2, 3, 5, 6}
```

In[1004]:=

```
T16^A[4].base16⟦201⟧⟦1⟧ // MatrixForm
```

Out[1004]//MatrixForm=

```
-1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0 -1  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1
```

In[1005]:=

```
T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6] // MatrixForm
```

Out[1005]//MatrixForm=

```
 1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0 -1  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0 -1  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0 -1  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0 -1  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1
```

In[1006]:=

```
(*possibleComplexStructureIndex=
  Select[base16,Block[{J=T16^A[4]*#⟦1⟧,a1=J.J,a2=Transpose[J].σ16.J,
       b1=a1===-ID16,b2=a2⩵σ16},b1&&b2]&→"Index"]
 possibleComplexStructureMatrix=base16⟦#⟧⟦2⟧&/@%*)
```

In[1007]:=

```
possibleComplexStructureIndex =
 Select[base16, Block[{J = T16^A[4] * #[[1]], m = σ16.J}, J.J === -ID16 &&
      σ16 === Transpose[J].σ16.J(*&&!(m===-Transpose[m])  *)] & → "Index"]
possibleComplexStructureMatrix = base16[[#]] [[2]] & /@ %
```

Out[1007]=

```
{4, 178, 187, 201}
```

Out[1008]=

```
{{3}, {0, 1, 3, 6, 7}, {0, 2, 3, 5, 7}, {1, 2, 3, 5, 6}}
```

In[1009]:=

```
Select[base16, #[[2]] == {1, 2, 3, 4, 5, 6} & → "Index"]
```

Out[1009]=

```
{240}
```

In[1010]:=

```
base16[[240]] [[2]]
```

Out[1010]=

```
{1, 2, 3, 4, 5, 6}
```

In[1011]:=

```
base16[[240]] [[1]] // MatrixForm
```

Out[1011]//MatrixForm=

```
( 1  0  0  0  0   0   0   0   0  0  0  0  0   0   0   0  )
( 0  1  0  0  0   0   0   0   0  0  0  0  0   0   0   0  )
( 0  0  1  0  0   0   0   0   0  0  0  0  0   0   0   0  )
( 0  0  0  1  0   0   0   0   0  0  0  0  0   0   0   0  )
( 0  0  0  0  -1  0   0   0   0  0  0  0  0   0   0   0  )
( 0  0  0  0  0   -1  0   0   0  0  0  0  0   0   0   0  )
( 0  0  0  0  0   0   -1  0   0  0  0  0  0   0   0   0  )
( 0  0  0  0  0   0   0   -1  0  0  0  0  0   0   0   0  )
( 0  0  0  0  0   0   0   0   1  0  0  0  0   0   0   0  )
( 0  0  0  0  0   0   0   0   0  1  0  0  0   0   0   0  )
( 0  0  0  0  0   0   0   0   0  0  1  0  0   0   0   0  )
( 0  0  0  0  0   0   0   0   0  0  0  1  0   0   0   0  )
( 0  0  0  0  0   0   0   0   0  0  0  0  -1  0   0   0  )
( 0  0  0  0  0   0   0   0   0  0  0  0  0   -1  0   0  )
( 0  0  0  0  0   0   0   0   0  0  0  0  0   0   -1  0  )
( 0  0  0  0  0   0   0   0   0  0  0  0  0   0   0   -1 )
```

In[1012]:=

```
(*possibleComplexStructureIndex=
  Select[base16,Block[{J=#[[1]],m=σ16.#[[1]]},J.J===-ID16&&
       σ16===Transpose[J].σ16.J&&!(m===-Transpose[m]) ]&→"Index"]
 possibleComplexStructureMatrix=base16[[#]][[2]]&/@%*)
```

```
findComplexStucture[] := Module[{list, filter(*,innerProduct*)},
  (*innerProduct[a_,b_]:=FullSimplify[Transpose[a].σ16.b];*)
  (*filter[J_][a_,b_]:=J.J===-ID16&&innerProduct[a,b]===innerProduct[J.a,J.b];*)
  filter[J_] := J . J === -ID16 && σ16 === Transpose[J] . σ16 . J;
  Return[list];
  ]
```

In[1013]:=

$$\Omega 16 === \mathrm{T16}^{A}[1].\mathrm{T16}^{A}[2].\mathrm{T16}^{A}[3].\mathrm{T16}^{A}[4]$$

Out[1013]=

```
True
```

Spinor Lagrangian

In[1014]:=

$$\text{usegT16} = \text{Table}[\text{FullSimplify}[(\mathrm{T16}^{\alpha}[\alpha 1 - 1]), \text{constraintVars}], \{\alpha 1, 1, \text{Length}[X]\}];$$

In[1015]:=

$$\text{useT16} = \text{Table}\Big[\text{FullSimplify}[(\mathrm{T16}^{\alpha}[\alpha 1 - 1] \text{ /. ssgm4488}), \text{constraintVars}] \text{ /. } \Big\{\sqrt{e^{-2\,a4[H\,x4]}} \to e^{-a4[H\,x4]},\ \sqrt{e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}} \to e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6},\ \frac{1}{\sqrt{e^{2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}}} \to \frac{1}{e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6}}\Big\}, \{\alpha 1, 1, \text{Length}[X]\}\Big] \text{ /. } \Big\{\frac{1}{\sqrt{e^{-2\,a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/3}}} \to \frac{1}{e^{a4[H\,x4]}\,\text{Sin}[6\,H\,x0]^{1/6}},\ \sqrt{e^{2\,a4[H\,x4]}} \to e^{a4[H\,x4]}\Big\};$$

In[1016]:=

```
Dimensions[useT16]
```

Out[1016]=

```
{8, 16, 16}
```

In[1017]:=

```
Det[MatrixMetric44]
```

Out[1017]=

$$\text{Cos}[6\,H\,x0]^{2}$$

In[1018]:=

```
useDSQRT = Cos[6 H x0]
```

Out[1018]=

```
Cos[6 H x0]
```

In[1019]:=

```
ωmat /. sg /. ssgm4488 /. subsDefects;
(% /. {√(ⅇ^(2 a4[H x4])) → ⅇ^(a4[H x4]),
      √(ⅇ^(2 a4[H x4])) H Sin[6 H x0]^(1/6) → ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6)}) - spinCoeffs //
   FullSimplify[#, constraintVars] & /. subsDefects /.
    {√(ⅇ^(2 a4[H x4])) → ⅇ^(a4[H x4]), √(ⅇ^(2 a4[H x4])) H Sin[6 H x0]^(1/6) → ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6)} //
  FullSimplify[#, constraintVars] & /. subsDefects;
% // Flatten // Union
% // ExpandAll /. subsDefects /. {√(ⅇ^(2 a4[H x4])) → ⅇ^(a4[H x4]),
    √(ⅇ^(2 a4[H x4])) H Sin[6 H x0]^(1/6) → ⅇ^(a4[H x4]) H Sin[6 H x0]^(1/6)} /. subsDefects
% /. {a4[H x4] → π^4, Sin[6 H x0] → ⅇ^12}
```

Out[1021]=

$$\Big\{0,\ e^{-\text{a4}[\text{H x4}]}\left(-1+e^{\text{a4}[\text{H x4}]}\sqrt{e^{-2\,\text{a4}[\text{H x4}]}}\right)\text{H Sin}[6\,\text{H x0}]^{1/6},$$
$$-\left(\left(-e^{\text{a4}[\text{H x4}]}+\sqrt{e^{2\,\text{a4}[\text{H x4}]}}\right)\text{H Sin}[6\,\text{H x0}]^{1/6}\right),\ \left(-e^{\text{a4}[\text{H x4}]}+\sqrt{e^{2\,\text{a4}[\text{H x4}]}}\right)\text{H Sin}[6\,\text{H x0}]^{1/6},$$
$$e^{-\text{a4}[\text{H x4}]}\left(1-e^{\text{a4}[\text{H x4}]}\sqrt{e^{-2\,\text{a4}[\text{H x4}]}}\right)\text{H Sin}[6\,\text{H x0}]^{1/6}\,\text{a4}'[\text{H x4}],$$
$$e^{-\text{a4}[\text{H x4}]}\left(-1+e^{\text{a4}[\text{H x4}]}\sqrt{e^{-2\,\text{a4}[\text{H x4}]}}\right)\text{H Sin}[6\,\text{H x0}]^{1/6}\,\text{a4}'[\text{H x4}],$$
$$\left(-e^{\text{a4}[\text{H x4}]}+\sqrt{e^{2\,\text{a4}[\text{H x4}]}}\right)\text{H Sin}[6\,\text{H x0}]^{1/6}\,\text{a4}'[\text{H x4}]\Big\}$$

Out[1022]=

$$\Big\{0,\ -e^{-\text{a4}[\text{H x4}]}\,\text{H Sin}[6\,\text{H x0}]^{1/6}+\sqrt{e^{-2\,\text{a4}[\text{H x4}]}}\,\text{H Sin}[6\,\text{H x0}]^{1/6},$$
$$e^{\text{a4}[\text{H x4}]}\,\text{H Sin}[6\,\text{H x0}]^{1/6}-\sqrt{e^{2\,\text{a4}[\text{H x4}]}}\,\text{H Sin}[6\,\text{H x0}]^{1/6},$$
$$-e^{\text{a4}[\text{H x4}]}\,\text{H Sin}[6\,\text{H x0}]^{1/6}+\sqrt{e^{2\,\text{a4}[\text{H x4}]}}\,\text{H Sin}[6\,\text{H x0}]^{1/6},$$
$$e^{-\text{a4}[\text{H x4}]}\,\text{H Sin}[6\,\text{H x0}]^{1/6}\,\text{a4}'[\text{H x4}]-\sqrt{e^{-2\,\text{a4}[\text{H x4}]}}\,\text{H Sin}[6\,\text{H x0}]^{1/6}\,\text{a4}'[\text{H x4}],$$
$$-e^{-\text{a4}[\text{H x4}]}\,\text{H Sin}[6\,\text{H x0}]^{1/6}\,\text{a4}'[\text{H x4}]+\sqrt{e^{-2\,\text{a4}[\text{H x4}]}}\,\text{H Sin}[6\,\text{H x0}]^{1/6}\,\text{a4}'[\text{H x4}],$$
$$-e^{\text{a4}[\text{H x4}]}\,\text{H Sin}[6\,\text{H x0}]^{1/6}\,\text{a4}'[\text{H x4}]+\sqrt{e^{2\,\text{a4}[\text{H x4}]}}\,\text{H Sin}[6\,\text{H x0}]^{1/6}\,\text{a4}'[\text{H x4}]\Big\}$$

Out[1023]=

```
{0, 0, 0, 0, 0, 0, 0}
```

In[1024]:=

```
Q2 = 0; Protect[Q1, Q2]
```

Out[1024]=

```
{Q1, Q2}
```

```
In[1025]:=
Clear[Lg];
Lg[] := √detgg * (Transpose[Ψ16].σ16.
      Sum[FullSimplify[(T16^α[α1 - 1] /. sg), constraintVars].(D[ Ψ16, X⟦α1⟧] +
           (Q1/2) * Sum[ωmat⟦α1, a, b⟧ * SAB⟦a, b⟧.Ψ16, {a, 1, 8}, {b, 1, 8}]),
        {α1, 1, Length[X]}] + (H * M) * Transpose[Ψ16].σ16.Ψ16) //
   Simplify[#, constraintVars] &
```

```
In[1027]:=
ωμIJ[1]
Out[1027]=
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[1028]:=
Clear[La];
La[] :=
 useDSQRT * (Transpose[Ψ16].σ16.Sum[useT16⟦α1⟧.(D[ Ψ16, X⟦α1⟧] + (Q1/2) *
            Sum[ωμIJ[α1]⟦A1, B1⟧ * SAB⟦A1, B1⟧, {A1, 1, 8}, {B1, 1, 8}].Ψ16),
        {α1, 1, Length[X]}] + (H * M) * Transpose[Ψ16].σ16.Ψ16) //
   Simplify[#, constraintVars] &
```

```
In[1030]:=
(*testLa=La[]*)
In[1031]:=
(*testLg=Lg[];*)
In[1032]:=
(*D[testLa,a4[H x4]]//Flatten//Union*)
(*D[testLa,a4′[H x4]]//(*Full*)Simplify[#,constraintVars]&//Flatten//Union*)
In[1033]:=
(*testLgm=
 testL/.sg/.ssgm4488//(*Full*)Simplify[#,constraintVars]&/.subsDefects*)
In[1034]:=
(*D[testLgm,a4[H x4]]
 D[testLgm,a4′[H x4]]//(*Full*)Simplify[#,constraintVars]&*)
```

In[1035]:=

```
Clear[Lj];
Lj[j_] := useDSQRT * (((
        Transpose[Ψ16].σ16.useT16⟦1⟧.D[ Ψ16, X⟦1⟧] +

         Transpose[Ψ16].σ16.useT16⟦5⟧.D[ Ψ16, X⟦5⟧] + (K0/2) *
          Sum[(*useω⟦1,A1,B1⟧*)ωμIJ[1]⟦A1, B1⟧ × Transpose[Ψ16].σ16.useT16⟦
             1⟧.SAB⟦A1, B1⟧.(base16⟦j, 1⟧).Ψ16, {A1, 1, 8}, {B1, 1, 8}]) +
       Transpose[Ψ16].σ16.useT16⟦5⟧.D[ Ψ16, X⟦5⟧] + (K4/2) * Sum[
         (*useω⟦5,A1,B1⟧*) ωμIJ[5]⟦A1, B1⟧ × Transpose[Ψ16].σ16.useT16⟦5⟧.
          SAB⟦A1, B1⟧.(base16⟦j, 1⟧).Ψ16, {A1, 1, 8}, {B1, 1, 8}]) +
      (H * M) * Transpose[Ψ16].σ16.Ψ16) // Simplify[#, constraintVars] &
```

In[1037]:=

```
(*Lj[92]*)
```

In[1038]:=

```
detgg
```

Out[1038]=

```
g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
 g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]
```

In[1039]:=

```
Clear[eL];
eL[Lagrangian_Symbol, detsqrt_] := Module[{L, t},
   L = Lagrangian[];
   t = Table[FullSimplify[
       1/detsqrt (D[L, f16[k][x0, x4]] - D[D[L, f16[k]^(1,0)[x0, x4]], x0] -
          D[D[L, f16[k]^(0,1)[x0, x4]], x4]), constraintVars], {k, 0, 15}];
   Return[t /. subsDefects];
  ]
```

---

eLa

In[1041]:=

```
(*Get[
  "2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLa.
   mx"];*)
```

```
In[1042]:=
    eLa = eL[La, useDSQRT];
In[1043]:=
    eLa
Out[1043]=
    {-2 ⅇ^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[9][x0, x4] a4′[H x4] -
      2 (H M f16[4][x0, x4] + 3 H f16[12][x0, x4] +
          f16[9]^(0,1)[x0, x4] + Tan[6 H x0] f16[12]^(1,0)[x0, x4]),
     2 (-H M f16[5][x0, x4] - 3 H f16[13][x0, x4] + ⅇ^(-a4[H x4]) H Q1 Sinh[a4[H x4]]
          f16[8][x0, x4] a4′[H x4] + f16[8]^(0,1)[x0, x4] - Tan[6 H x0] f16[13]^(1,0)[x0, x4]),
     -2 (H M f16[6][x0, x4] + 3 H f16[14][x0, x4] - f16[11]^(0,1)[x0, x4] +
        Tan[6 H x0] f16[14]^(1,0)[x0, x4]),
     -2 (H M f16[7][x0, x4] + 3 H f16[15][x0, x4] + f16[10]^(0,1)[x0, x4] +
        Tan[6 H x0] f16[15]^(1,0)[x0, x4]), -2 H M f16[0][x0, x4] -
      6 H f16[8][x0, x4] - 2 ⅇ^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[13][x0, x4] a4′[H x4] +
      2 f16[13]^(0,1)[x0, x4] - 2 Tan[6 H x0] f16[8]^(1,0)[x0, x4],
     -2 H M f16[1][x0, x4] - 6 H f16[9][x0, x4] +
      ⅇ^(-2 a4[H x4]) (-1 + ⅇ^(2 a4[H x4])) H Q1 f16[12][x0, x4] a4′[H x4] -
      2 (f16[12]^(0,1)[x0, x4] + Tan[6 H x0] f16[9]^(1,0)[x0, x4]),
     -2 (H M f16[2][x0, x4] + 3 H f16[10][x0, x4] + f16[15]^(0,1)[x0, x4] +
        Tan[6 H x0] f16[10]^(1,0)[x0, x4]), -2 (H M f16[3][x0, x4] +
        3 H f16[11][x0, x4] - f16[14]^(0,1)[x0, x4] + Tan[6 H x0] f16[11]^(1,0)[x0, x4]),
     2 (3 H f16[4][x0, x4] + H M f16[12][x0, x4] + ⅇ^(-a4[H x4]) H Q1 Sinh[a4[H x4]]
          f16[1][x0, x4] a4′[H x4] - f16[1]^(0,1)[x0, x4] + Tan[6 H x0] f16[4]^(1,0)[x0, x4]),
     2 (3 H f16[5][x0, x4] + H M f16[13][x0, x4] - ⅇ^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[0][x0, x4]
          a4′[H x4] + f16[0]^(0,1)[x0, x4] + Tan[6 H x0] f16[5]^(1,0)[x0, x4]),
     2 (3 H f16[6][x0, x4] + H M f16[14][x0, x4] + f16[3]^(0,1)[x0, x4] +
        Tan[6 H x0] f16[6]^(1,0)[x0, x4]), 2 (3 H f16[7][x0, x4] +
        H M f16[15][x0, x4] - f16[2]^(0,1)[x0, x4] + Tan[6 H x0] f16[7]^(1,0)[x0, x4]),
     2 (3 H f16[0][x0, x4] + H M f16[8][x0, x4] + ⅇ^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[5][x0, x4] a4′[H x4] +
        f16[5]^(0,1)[x0, x4] + Tan[6 H x0] f16[0]^(1,0)[x0, x4]),
     6 H f16[1][x0, x4] + 2 H M f16[9][x0, x4] - 2 ⅇ^(-a4[H x4]) H Q1 Sinh[a4[H x4]] f16[4][x0, x4] a4′[H x4] -
      2 f16[4]^(0,1)[x0, x4] + 2 Tan[6 H x0] f16[1]^(1,0)[x0, x4],
     2 (3 H f16[2][x0, x4] + H M f16[10][x0, x4] - f16[7]^(0,1)[x0, x4] +
        Tan[6 H x0] f16[2]^(1,0)[x0, x4]), 2 (3 H f16[3][x0, x4] +
        H M f16[11][x0, x4] + f16[6]^(0,1)[x0, x4] + Tan[6 H x0] f16[3]^(1,0)[x0, x4])}
In[1044]:=
    Length[eLa]
Out[1044]=
    16
In[1045]:=
    DumpSave[ToString[header <> "eLa.mx"], eLa];
In[1046]:=
    Print[ToString[header <> "eLa.mx"]]
    Pair-Crtn-Univ-same_E-L-eqs-alt-approach-eLa.mx
```

In[1047]:=

```
(*Cases[
    eLa,
    h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] ⧴ n,
    Infinity,
    Heads → True
]*)
```

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE :

In[1048]:=

```
eLaRawSets = Cases[
  #,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] ⧴ n,
  Infinity,
  Heads → True
] & /@ eLa
```

Out[1048]=

```
{{9, 4, 12, 9, 12}, {5, 13, 8, 8, 13}, {6, 14, 11, 14}, {7, 15, 10, 15}, {0, 8, 13, 13, 8},
 {1, 9, 12, 12, 9}, {2, 10, 15, 10}, {3, 11, 14, 11}, {4, 12, 1, 1, 4}, {5, 13, 0, 0, 5},
 {6, 14, 3, 6}, {7, 15, 2, 7}, {0, 8, 5, 5, 0}, {1, 9, 4, 4, 1}, {2, 10, 7, 2}, {3, 11, 6, 3}}
```

In[1049]:=

```
Dimensions[eLaRawSets]
```

Out[1049]=

```
{16}
```

In[1050]:=

```
rawSetsel16 = rawSets[eLa, "f16", 0]
% === eLaRawSets
```

Out[1050]=

```
{{9, 4, 12, 9, 12}, {5, 13, 8, 8, 13}, {6, 14, 11, 14}, {7, 15, 10, 15}, {0, 8, 13, 13, 8},
 {1, 9, 12, 12, 9}, {2, 10, 15, 10}, {3, 11, 14, 11}, {4, 12, 1, 1, 4}, {5, 13, 0, 0, 5},
 {6, 14, 3, 6}, {7, 15, 2, 7}, {0, 8, 5, 5, 0}, {1, 9, 4, 4, 1}, {2, 10, 7, 2}, {3, 11, 6, 3}}
```

Out[1051]=

```
True
```

In[1052]:=

```
eLaCouplings = showCoupledEquations[eLaRawSets]
% // Flatten // Sort
% === Range[0, 15]
```

Out[1052]=

```
{{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
```

Out[1053]=

```
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
```

Out[1054]=

```
True
```

In[1055]:=

```
eLaztCouplings = eLaCouplings
```

Out[1055]=

```
{{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
```

In[1056]:=

```
(*processRawSets[]:=Module[{s1,s2,J,L=Range[Length[eLaRawSets]],r={}},
  While[Length[L]>1,
   s1=Union[eLaRawSets⟦L⟦1⟧⟧];
   L=Drop[L,1];
   J=L⟦1⟧;
   While[J≤16,
    If[MemberQ[L,J],
     s2=Union[eLaRawSets⟦J⟧];
     If[Intersection[s1,s2]≠{},s1=Union[Flatten[{s1,s2}]];
      L=Complement[L,{J}];,##&[]];,
     ##&[]];
    J++;];

   AppendTo[r,s1];
  ];
  Return[r];
 ]*)
```

eLazt

In[1057]:=

```
(*Get[
  "2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLazt
    .mx"];*)
```

In[1058]:=

```
time1 = Now
AbsoluteTiming[
  eLazt = 1/(2 * H) * eLa /. sfψ16Aa /. sx0x4 // FullSimplify[#, constraintVars] &]
Now - time1
```

Out[1058]=

```
Wed 11 Feb 2026 05:18:49 GMT-8
```

Out[1059]=

```
{5.60865, {-M Z[4][z, t] - 3 Z[12][z, t] - ⅇ^-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4′[t] -
    Z[9]^(0,1)[z, t] - 6 Tan[z] Z[12]^(1,0)[z, t], -M Z[5][z, t] - 3 Z[13][z, t] +
    ⅇ^-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4′[t] + Z[8]^(0,1)[z, t] - 6 Tan[z] Z[13]^(1,0)[z, t],
  -M Z[6][z, t] - 3 Z[14][z, t] + Z[11]^(0,1)[z, t] - 6 Tan[z] Z[14]^(1,0)[z, t],
  -M Z[7][z, t] - 3 Z[15][z, t] - Z[10]^(0,1)[z, t] - 6 Tan[z] Z[15]^(1,0)[z, t],
  -M Z[0][z, t] - 3 Z[8][z, t] - ⅇ^-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4′[t] +
    Z[13]^(0,1)[z, t] - 6 Tan[z] Z[8]^(1,0)[z, t], -M Z[1][z, t] - 3 Z[9][z, t] +
    ⅇ^-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4′[t] - Z[12]^(0,1)[z, t] - 6 Tan[z] Z[9]^(1,0)[z, t],
  -M Z[2][z, t] - 3 Z[10][z, t] - Z[15]^(0,1)[z, t] - 6 Tan[z] Z[10]^(1,0)[z, t],
  -M Z[3][z, t] - 3 Z[11][z, t] + Z[14]^(0,1)[z, t] - 6 Tan[z] Z[11]^(1,0)[z, t],
  3 Z[4][z, t] + M Z[12][z, t] + ⅇ^-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4′[t] -
    Z[1]^(0,1)[z, t] + 6 Tan[z] Z[4]^(1,0)[z, t], 3 Z[5][z, t] + M Z[13][z, t] -
    ⅇ^-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4′[t] + Z[0]^(0,1)[z, t] + 6 Tan[z] Z[5]^(1,0)[z, t],
  3 Z[6][z, t] + M Z[14][z, t] + Z[3]^(0,1)[z, t] + 6 Tan[z] Z[6]^(1,0)[z, t],
  3 Z[7][z, t] + M Z[15][z, t] - Z[2]^(0,1)[z, t] + 6 Tan[z] Z[7]^(1,0)[z, t],
  3 Z[0][z, t] + M Z[8][z, t] + ⅇ^-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4′[t] +
    Z[5]^(0,1)[z, t] + 6 Tan[z] Z[0]^(1,0)[z, t], 3 Z[1][z, t] + M Z[9][z, t] -
    ⅇ^-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4′[t] - Z[4]^(0,1)[z, t] + 6 Tan[z] Z[1]^(1,0)[z, t],
  3 Z[2][z, t] + M Z[10][z, t] - Z[7]^(0,1)[z, t] + 6 Tan[z] Z[2]^(1,0)[z, t],
  3 Z[3][z, t] + M Z[11][z, t] + Z[6]^(0,1)[z, t] + 6 Tan[z] Z[3]^(1,0)[z, t]}}
```

Out[1060]=

```
5.611752 s
```

In[1061]:=

```
DumpSave[ToString[header <> "eLazt.mx"], eLazt];
```

In[1062]:=

```
Print[ToString[header <> "eLazt.mx"]]
```

```
Pair-Crtn-Univ-same_E-L-eqs-alt-approach-eLazt.mx
```

In[1063]:=

```
varZ = Z[#] & /@ Range[0, 15]
```

Out[1063]=

```
{Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
 Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}
```

```
In[1064]:=
        varZzt = #[z, t] & /@ varZ
Out[1064]=
        {Z[0][z, t], Z[1][z, t], Z[2][z, t], Z[3][z, t], Z[4][z, t],
         Z[5][z, t], Z[6][z, t], Z[7][z, t], Z[8][z, t], Z[9][z, t], Z[10][z, t],
         Z[11][z, t], Z[12][z, t], Z[13][z, t], Z[14][z, t], Z[15][z, t]}
In[1065]:=
        DzvarZ = D[varZzt, z]
Out[1065]=
        {Z[0]^(1,0)[z, t], Z[1]^(1,0)[z, t], Z[2]^(1,0)[z, t], Z[3]^(1,0)[z, t],
         Z[4]^(1,0)[z, t], Z[5]^(1,0)[z, t], Z[6]^(1,0)[z, t], Z[7]^(1,0)[z, t],
         Z[8]^(1,0)[z, t], Z[9]^(1,0)[z, t], Z[10]^(1,0)[z, t], Z[11]^(1,0)[z, t],
         Z[12]^(1,0)[z, t], Z[13]^(1,0)[z, t], Z[14]^(1,0)[z, t], Z[15]^(1,0)[z, t]}
In[1066]:=
        DtvarZ = D[varZzt, t]
Out[1066]=
        {Z[0]^(0,1)[z, t], Z[1]^(0,1)[z, t], Z[2]^(0,1)[z, t], Z[3]^(0,1)[z, t],
         Z[4]^(0,1)[z, t], Z[5]^(0,1)[z, t], Z[6]^(0,1)[z, t], Z[7]^(0,1)[z, t],
         Z[8]^(0,1)[z, t], Z[9]^(0,1)[z, t], Z[10]^(0,1)[z, t], Z[11]^(0,1)[z, t],
         Z[12]^(0,1)[z, t], Z[13]^(0,1)[z, t], Z[14]^(0,1)[z, t], Z[15]^(0,1)[z, t]}
```

In[1067]:=

```
(DtvarZsubs = Solve[And @@ Thread[0 ⩵ eLazt], DtvarZ]⟦1⟧ //
    FullSimplify[#, constraintVars] &) // Column
```

Out[1067]=

```
Z[0]^(0,1)[z, t] →
 -3 Z[5][z, t] - M Z[13][z, t] + 𝕖^(-a4[t]) Q1 Sinh[a4[t]] Z[0][z, t] a4′[t] - 6 Tan[z] Z[5]^(1,0)[z, t]
Z[1]^(0,1)[z, t] →
 3 Z[4][z, t] + M Z[12][z, t] + 𝕖^(-a4[t]) Q1 Sinh[a4[t]] Z[1][z, t] a4′[t] + 6 Tan[z] Z[4]^(1,0)[z, t]
Z[2]^(0,1)[z, t] → 3 Z[7][z, t] + M Z[15][z, t] + 6 Tan[z] Z[7]^(1,0)[z, t]
Z[3]^(0,1)[z, t] → -3 Z[6][z, t] - M Z[14][z, t] - 6 Tan[z] Z[6]^(1,0)[z, t]
Z[4]^(0,1)[z, t] →
 3 Z[1][z, t] + M Z[9][z, t] - 𝕖^(-a4[t]) Q1 Sinh[a4[t]] Z[4][z, t] a4′[t] + 6 Tan[z] Z[1]^(1,0)[z, t]
Z[5]^(0,1)[z, t] →
 -3 Z[0][z, t] - M Z[8][z, t] - 𝕖^(-a4[t]) Q1 Sinh[a4[t]] Z[5][z, t] a4′[t] - 6 Tan[z] Z[0]^(1,0)[z, t]
Z[6]^(0,1)[z, t] → -3 Z[3][z, t] - M Z[11][z, t] - 6 Tan[z] Z[3]^(1,0)[z, t]
Z[7]^(0,1)[z, t] → 3 Z[2][z, t] + M Z[10][z, t] + 6 Tan[z] Z[2]^(1,0)[z, t]
Z[8]^(0,1)[z, t] →
 M Z[5][z, t] + 3 Z[13][z, t] - 𝕖^(-a4[t]) Q1 Sinh[a4[t]] Z[8][z, t] a4′[t] + 6 Tan[z] Z[13]^(1,0)[z, t]
Z[9]^(0,1)[z, t] →
 -M Z[4][z, t] - 3 Z[12][z, t] - 𝕖^(-a4[t]) Q1 Sinh[a4[t]] Z[9][z, t] a4′[t] - 6 Tan[z] Z[12]^(1,0)[z, t]
Z[10]^(0,1)[z, t] → -M Z[7][z, t] - 3 (Z[15][z, t] + 2 Tan[z] Z[15]^(1,0)[z, t])
Z[11]^(0,1)[z, t] → M Z[6][z, t] + 3 Z[14][z, t] + 6 Tan[z] Z[14]^(1,0)[z, t]
Z[12]^(0,1)[z, t] →
 -M Z[1][z, t] - 3 Z[9][z, t] + 𝕖^(-a4[t]) Q1 Sinh[a4[t]] Z[12][z, t] a4′[t] - 6 Tan[z] Z[9]^(1,0)[z, t]
Z[13]^(0,1)[z, t] →
 M Z[0][z, t] + 3 Z[8][z, t] + 𝕖^(-a4[t]) Q1 Sinh[a4[t]] Z[13][z, t] a4′[t] + 6 Tan[z] Z[8]^(1,0)[z, t]
Z[14]^(0,1)[z, t] → M Z[3][z, t] + 3 Z[11][z, t] + 6 Tan[z] Z[11]^(1,0)[z, t]
Z[15]^(0,1)[z, t] → -M Z[2][z, t] - 3 (Z[10][z, t] + 2 Tan[z] Z[10]^(1,0)[z, t])
```

In[1068]:=

```
ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(DtvarZEQS = ToExpression[#] & /@ %) // Column
```

Out[1070]=

```
Z[0]^(0,1)[z, t] ==
 -3 Z[5][z, t] - M Z[13][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[0][z, t] a4′[t] - 6 Tan[z] Z[5]^(1,0)[z, t]
Z[1]^(0,1)[z, t] ==
 3 Z[4][z, t] + M Z[12][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[1][z, t] a4′[t] + 6 Tan[z] Z[4]^(1,0)[z, t]
Z[2]^(0,1)[z, t] == 3 Z[7][z, t] + M Z[15][z, t] + 6 Tan[z] Z[7]^(1,0)[z, t]
Z[3]^(0,1)[z, t] == -3 Z[6][z, t] - M Z[14][z, t] - 6 Tan[z] Z[6]^(1,0)[z, t]
Z[4]^(0,1)[z, t] ==
 3 Z[1][z, t] + M Z[9][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[4][z, t] a4′[t] + 6 Tan[z] Z[1]^(1,0)[z, t]
Z[5]^(0,1)[z, t] ==
 -3 Z[0][z, t] - M Z[8][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[5][z, t] a4′[t] - 6 Tan[z] Z[0]^(1,0)[z, t]
Z[6]^(0,1)[z, t] == -3 Z[3][z, t] - M Z[11][z, t] - 6 Tan[z] Z[3]^(1,0)[z, t]
Z[7]^(0,1)[z, t] == 3 Z[2][z, t] + M Z[10][z, t] + 6 Tan[z] Z[2]^(1,0)[z, t]
Z[8]^(0,1)[z, t] ==
 M Z[5][z, t] + 3 Z[13][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[8][z, t] a4′[t] + 6 Tan[z] Z[13]^(1,0)[z, t]
Z[9]^(0,1)[z, t] ==
 -M Z[4][z, t] - 3 Z[12][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[9][z, t] a4′[t] - 6 Tan[z] Z[12]^(1,0)[z, t]
Z[10]^(0,1)[z, t] == -M Z[7][z, t] - 3 (Z[15][z, t] + 2 Tan[z] Z[15]^(1,0)[z, t])
Z[11]^(0,1)[z, t] == M Z[6][z, t] + 3 Z[14][z, t] + 6 Tan[z] Z[14]^(1,0)[z, t]
Z[12]^(0,1)[z, t] ==
 -M Z[1][z, t] - 3 Z[9][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[12][z, t] a4′[t] - 6 Tan[z] Z[9]^(1,0)[z, t]
Z[13]^(0,1)[z, t] ==
 M Z[0][z, t] + 3 Z[8][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[13][z, t] a4′[t] + 6 Tan[z] Z[8]^(1,0)[z, t]
Z[14]^(0,1)[z, t] == M Z[3][z, t] + 3 Z[11][z, t] + 6 Tan[z] Z[11]^(1,0)[z, t]
Z[15]^(0,1)[z, t] == -M Z[2][z, t] - 3 (Z[10][z, t] + 2 Tan[z] Z[10]^(1,0)[z, t])
```

In[1071]:=

```
ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(DtvarZrelations = ToExpression[#] & /@ %) // Column
```

Out[1073]=

```
3 Z[5][z, t] + M Z[13][z, t] -
  e^(-a4[t]) Q1 Sinh[a4[t]] Z[0][z, t] a4′[t] + Z[0]^(0,1)[z, t] + 6 Tan[z] Z[5]^(1,0)[z, t]
-3 Z[4][z, t] - M Z[12][z, t] -
  e^(-a4[t]) Q1 Sinh[a4[t]] Z[1][z, t] a4′[t] + Z[1]^(0,1)[z, t] - 6 Tan[z] Z[4]^(1,0)[z, t]
-3 Z[7][z, t] - M Z[15][z, t] + Z[2]^(0,1)[z, t] - 6 Tan[z] Z[7]^(1,0)[z, t]
3 Z[6][z, t] + M Z[14][z, t] + Z[3]^(0,1)[z, t] + 6 Tan[z] Z[6]^(1,0)[z, t]
-3 Z[1][z, t] - M Z[9][z, t] +
  e^(-a4[t]) Q1 Sinh[a4[t]] Z[4][z, t] a4′[t] + Z[4]^(0,1)[z, t] - 6 Tan[z] Z[1]^(1,0)[z, t]
3 Z[0][z, t] + M Z[8][z, t] +
  e^(-a4[t]) Q1 Sinh[a4[t]] Z[5][z, t] a4′[t] + Z[5]^(0,1)[z, t] + 6 Tan[z] Z[0]^(1,0)[z, t]
3 Z[3][z, t] + M Z[11][z, t] + Z[6]^(0,1)[z, t] + 6 Tan[z] Z[3]^(1,0)[z, t]
-3 Z[2][z, t] - M Z[10][z, t] + Z[7]^(0,1)[z, t] - 6 Tan[z] Z[2]^(1,0)[z, t]
-M Z[5][z, t] - 3 Z[13][z, t] +
  e^(-a4[t]) Q1 Sinh[a4[t]] Z[8][z, t] a4′[t] + Z[8]^(0,1)[z, t] - 6 Tan[z] Z[13]^(1,0)[z, t]
M Z[4][z, t] + 3 Z[12][z, t] + e^(-a4[t]) Q1 Sinh[a4[t]] Z[9][z, t] a4′[t] +
  Z[9]^(0,1)[z, t] + 6 Tan[z] Z[12]^(1,0)[z, t]
M Z[7][z, t] + Z[10]^(0,1)[z, t] + 3 (Z[15][z, t] + 2 Tan[z] Z[15]^(1,0)[z, t])
-M Z[6][z, t] - 3 Z[14][z, t] + Z[11]^(0,1)[z, t] - 6 Tan[z] Z[14]^(1,0)[z, t]
M Z[1][z, t] + 3 Z[9][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[12][z, t] a4′[t] +
  Z[12]^(0,1)[z, t] + 6 Tan[z] Z[9]^(1,0)[z, t]
-M Z[0][z, t] - 3 Z[8][z, t] - e^(-a4[t]) Q1 Sinh[a4[t]] Z[13][z, t] a4′[t] +
  Z[13]^(0,1)[z, t] - 6 Tan[z] Z[8]^(1,0)[z, t]
-M Z[3][z, t] - 3 Z[11][z, t] + Z[14]^(0,1)[z, t] - 6 Tan[z] Z[11]^(1,0)[z, t]
M Z[2][z, t] + Z[15]^(0,1)[z, t] + 3 (Z[10][z, t] + 2 Tan[z] Z[10]^(1,0)[z, t])
```

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE :

In[1074]:=

```
eLaztCouplings = eLaCouplings
```

Out[1074]=

```
{{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
```

In[1075]:=

```
(*{{5,13,0,8,5,5,0,5,13,8},{4,12,1,1,9,4},{7,15,2,2,10,7},{6,14,3,3,11,6}}
 eLaztCouplings=Union[#]&/@%
 (*eLaztCouplings=Union[Flatten[#]]&/@%*)
 Union[Flatten[%]]===Range[0,15]*)
```

SEPARATE VARIABLES THAT ARE COUPLED TO EACH OTHER (reorder) :

```
In[1076]:=
Thread[(Z[#] & /@ Flatten[eLaztCouplings]) → (yZ[#] & /@ Range[0, 15])]
% // Sort
sZtOyZ
% - %%
Out[1076]=
{Z[0] → yZ[0], Z[5] → yZ[1], Z[8] → yZ[2], Z[13] → yZ[3], Z[1] → yZ[4],
 Z[4] → yZ[5], Z[9] → yZ[6], Z[12] → yZ[7], Z[2] → yZ[8], Z[7] → yZ[9], Z[10] → yZ[10],
 Z[15] → yZ[11], Z[3] → yZ[12], Z[6] → yZ[13], Z[11] → yZ[14], Z[14] → yZ[15]}
Out[1077]=
{Z[0] → yZ[0], Z[1] → yZ[4], Z[2] → yZ[8], Z[3] → yZ[12], Z[4] → yZ[5],
 Z[5] → yZ[1], Z[6] → yZ[13], Z[7] → yZ[9], Z[8] → yZ[2], Z[9] → yZ[6], Z[10] → yZ[10],
 Z[11] → yZ[14], Z[12] → yZ[7], Z[13] → yZ[3], Z[14] → yZ[15], Z[15] → yZ[11]}
Out[1078]=
sZtOyZ
Out[1079]=
{sZtOyZ - (Z[0] → yZ[0]), sZtOyZ - (Z[1] → yZ[4]),
 sZtOyZ - (Z[2] → yZ[8]), sZtOyZ - (Z[3] → yZ[12]), sZtOyZ - (Z[4] → yZ[5]),
 sZtOyZ - (Z[5] → yZ[1]), sZtOyZ - (Z[6] → yZ[13]), sZtOyZ - (Z[7] → yZ[9]),
 sZtOyZ - (Z[8] → yZ[2]), sZtOyZ - (Z[9] → yZ[6]), sZtOyZ - (Z[10] → yZ[10]),
 sZtOyZ - (Z[11] → yZ[14]), sZtOyZ - (Z[12] → yZ[7]), sZtOyZ - (Z[13] → yZ[3]),
 sZtOyZ - (Z[14] → yZ[15]), sZtOyZ - (Z[15] → yZ[11])}
```

In[1080]:=

```
(*Thread[(Z[#]&/@eLaztCouplings⟦1⟧)→(yZ[#]&/@Range[0,3])]
 Thread[(Z[#]&/@eLaztCouplings⟦2⟧)→(yZ[#]&/@Range[4,7])]
 Thread[(Z[#]&/@eLaztCouplings⟦3⟧)→(yZ[#]&/@Range[8,11])]
 Thread[(Z[#]&/@eLaztCouplings⟦4⟧)→(yZ[#]&/@Range[12,15])]*)
sZtOyZ = Thread[(Z[#] & /@ Flatten[eLaztCouplings]) → (yZ[#] & /@ Range[0, 15])]
(*Union[Flatten[{%,%%,%%%,%%%%}]]*)
ZyZEQS = sZtOyZ /. {Rule → Equal}
ZyZforCaEQS = -sZtOyZ /. {Rule → Subtract}
(*ToString[FullForm[#]]&/@sZtOyZ;
StringReplace[#,"Rule"→ "Equal"]&/@%;*)
(*(ZyZEQS=ToExpression[#]&/@%)*)
syZtoZ = Solve[And @@ ZyZEQS, (yZ[#] & /@ Range[0, 15])]⟦-1⟧
```

Out[1080]=

```
{Z[0] → yZ[0], Z[5] → yZ[1], Z[8] → yZ[2], Z[13] → yZ[3], Z[1] → yZ[4],
 Z[4] → yZ[5], Z[9] → yZ[6], Z[12] → yZ[7], Z[2] → yZ[8], Z[7] → yZ[9], Z[10] → yZ[10],
 Z[15] → yZ[11], Z[3] → yZ[12], Z[6] → yZ[13], Z[11] → yZ[14], Z[14] → yZ[15]}
```

Out[1081]=

```
{Z[0] == yZ[0], Z[5] == yZ[1], Z[8] == yZ[2], Z[13] == yZ[3], Z[1] == yZ[4], Z[4] == yZ[5],
 Z[9] == yZ[6], Z[12] == yZ[7], Z[2] == yZ[8], Z[7] == yZ[9], Z[10] == yZ[10],
 Z[15] == yZ[11], Z[3] == yZ[12], Z[6] == yZ[13], Z[11] == yZ[14], Z[14] == yZ[15]}
```

Out[1082]=

```
{yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1],
 yZ[5] - Z[4], yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
 yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}
```

Out[1083]=

```
{yZ[0] → Z[0], yZ[1] → Z[5], yZ[2] → Z[8], yZ[3] → Z[13], yZ[4] → Z[1],
 yZ[5] → Z[4], yZ[6] → Z[9], yZ[7] → Z[12], yZ[8] → Z[2], yZ[9] → Z[7], yZ[10] → Z[10],
 yZ[11] → Z[15], yZ[12] → Z[3], yZ[13] → Z[6], yZ[14] → Z[11], yZ[15] → Z[14]}
```

In[1084]:=

```
(*Thread[(Z[#]&/@Flatten[eLaztCouplings])→(yZ[#]&/@Range[0,15])]
 %//Sort
 sZtOyZ
 %-%%*)
```

In[1085]:=

```
ToString[FullForm[#]] & /@ sZtOyZ;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(ZyZforCaEQS = -ToExpression[#] & /@ %)
```

Out[1087]=

```
{yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1],
 yZ[5] - Z[4], yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
 yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}
```

In[1088]:=

```
cayZ = CoefficientArrays[ZyZforCaEQS, (yZ[#] & /@ Range[0, 15])]
```

Out[1088]=

```
{SparseArray[ ⊞ ■ Specified elements: 16
                  Dimensions: {16} ], SparseArray[ ⊞ ▨ Specified elements: 16
                                                        Dimensions: {16, 16} ]}
```

In[1089]:=

```
caZ = CoefficientArrays[ZyZforCaEQS, (Z[#] & /@ Range[0, 15])]
```

Out[1089]=

```
{SparseArray[ ⊞ ■ Specified elements: 16
                  Dimensions: {16} ], SparseArray[ ⊞ ▨ Specified elements: 16
                                                        Dimensions: {16, 16} ]}
```

In[1090]:=

```
(cayZ2 = cayZ⟦2⟧ // Normal) // MatrixForm
```

Out[1090]//MatrixForm=

```
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
```

IDENTIFY ORTHOGONAL SIMILARITY TRANSFORMATION FROM SET OF VARIABLES Z[j] TO SET OF VARIABLES yZ[j] :

In[1091]:=

```
cayZ2.(yZ[#] & /@ Range[0, 15])
(*   cayZ2.yZ   === Z :   *)
% /. syZtoZ
(*   inverse: yZ  === Transpose[cayZ2].Z   *)
Transpose[cayZ2].% /. sZtOyZ
```

Out[1091]=

```
{yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
 yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}
```

Out[1092]=

```
{Z[0], Z[5], Z[8], Z[13], Z[1], Z[4], Z[9],
 Z[12], Z[2], Z[7], Z[10], Z[15], Z[3], Z[6], Z[11], Z[14]}
```

Out[1093]=

```
{yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
 yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}
```

ORTHOGONAL O(16) SIMILARITY TRANSFORMATION :

```
In[1094]:=
cayZ2.Transpose[cayZ2] === ID16
Out[1094]=
True
```

BUT NOT ORTHOGONAL O(8,8) SIMILARITY TRANSFORMATION :

```
In[1095]:=
cayZ2.σ16.Transpose[cayZ2] === σ16
Out[1095]=
True
```

Also, Not a Direct Sum :

```
In[1096]:=
cayZ2 === ArrayFlatten[{{cayZ2⟦1 ;; 8, 1 ;; 8⟧, cayZ2⟦1 ;; 8, 9 ;; 16⟧},
    {cayZ2⟦9 ;; 16, 1 ;; 8⟧, cayZ2⟦9 ;; 16, 9 ;; 16⟧}}]
Out[1096]=
True
In[1097]:=
cayZ2 === ArrayFlatten[{{cayZ2⟦1 ;; 8, 1 ;; 8⟧, 0}, {0, cayZ2⟦9 ;; 16, 9 ;; 16⟧}}]
Out[1097]=
True
```

TRANSFORM Euler Lagrange equations to yZ[j] :

```
In[1098]:=
(DtyZrelations = Transpose[cayZ2].DtvarZrelations /. sZtOyZ // FullSimplify) //
 Column
```

```
Out[1098]=
3 yZ[1][z, t] + M yZ[3][z, t] - ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4′[t] +
 yZ[0]^(0,1)[z, t] + 6 Tan[z] yZ[1]^(1,0)[z, t]
-3 yZ[5][z, t] - M yZ[7][z, t] -
 ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4′[t] + yZ[4]^(0,1)[z, t] - 6 Tan[z] yZ[5]^(1,0)[z, t]
-3 yZ[9][z, t] - M yZ[11][z, t] + yZ[8]^(0,1)[z, t] - 6 Tan[z] yZ[9]^(1,0)[z, t]
3 yZ[13][z, t] + M yZ[15][z, t] + yZ[12]^(0,1)[z, t] + 6 Tan[z] yZ[13]^(1,0)[z, t]
-3 yZ[4][z, t] - M yZ[6][z, t] +
 ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4′[t] + yZ[5]^(0,1)[z, t] - 6 Tan[z] yZ[4]^(1,0)[z, t]
3 yZ[0][z, t] + M yZ[2][z, t] + ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4′[t] +
 yZ[1]^(0,1)[z, t] + 6 Tan[z] yZ[0]^(1,0)[z, t]
3 yZ[12][z, t] + M yZ[14][z, t] + yZ[13]^(0,1)[z, t] + 6 Tan[z] yZ[12]^(1,0)[z, t]
-3 yZ[8][z, t] - M yZ[10][z, t] + yZ[9]^(0,1)[z, t] - 6 Tan[z] yZ[8]^(1,0)[z, t]
-M yZ[1][z, t] - 3 yZ[3][z, t] +
 ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4′[t] + yZ[2]^(0,1)[z, t] - 6 Tan[z] yZ[3]^(1,0)[z, t]
M yZ[5][z, t] + 3 yZ[7][z, t] + ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4′[t] +
 yZ[6]^(0,1)[z, t] + 6 Tan[z] yZ[7]^(1,0)[z, t]
M yZ[9][z, t] + 3 yZ[11][z, t] + yZ[10]^(0,1)[z, t] + 6 Tan[z] yZ[11]^(1,0)[z, t]
-M yZ[13][z, t] - 3 yZ[15][z, t] + yZ[14]^(0,1)[z, t] - 6 Tan[z] yZ[15]^(1,0)[z, t]
M yZ[4][z, t] + 3 yZ[6][z, t] - ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4′[t] +
 yZ[7]^(0,1)[z, t] + 6 Tan[z] yZ[6]^(1,0)[z, t]
-M yZ[0][z, t] - 3 yZ[2][z, t] -
 ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4′[t] + yZ[3]^(0,1)[z, t] - 6 Tan[z] yZ[2]^(1,0)[z, t]
-M yZ[12][z, t] - 3 yZ[14][z, t] + yZ[15]^(0,1)[z, t] - 6 Tan[z] yZ[14]^(1,0)[z, t]
M yZ[8][z, t] + 3 yZ[10][z, t] + yZ[11]^(0,1)[z, t] + 6 Tan[z] yZ[10]^(1,0)[z, t]
```

CHECK THAT yZ[j] ARE IN PROPER ORDER (4 things , 4 at a time):

In[1099]:=

```
(DtyZsubs = Solve[And @@ Thread[0 == DtyZrelations],
      (D[(yZ[#][z, t] & /@ Range[0, 15]), t])][[1]] //
    FullSimplify[#, constraintVars] &) // Column
```

Out[1099]=

```
yZ[0]^(0,1)[z, t] → -3 yZ[1][z, t] - M yZ[3][z, t] +
   ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4′[t] - 6 Tan[z] yZ[1]^(1,0)[z, t]
yZ[1]^(0,1)[z, t] → -3 yZ[0][z, t] - M yZ[2][z, t] -
   ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4′[t] - 6 Tan[z] yZ[0]^(1,0)[z, t]
yZ[2]^(0,1)[z, t] →
 M yZ[1][z, t] + 3 yZ[3][z, t] - ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4′[t] + 6 Tan[z] yZ[3]^(1,0)[z, t]
yZ[3]^(0,1)[z, t] →
 M yZ[0][z, t] + 3 yZ[2][z, t] + ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4′[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]
yZ[4]^(0,1)[z, t] →
 3 yZ[5][z, t] + M yZ[7][z, t] + ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4′[t] + 6 Tan[z] yZ[5]^(1,0)[z, t]
yZ[5]^(0,1)[z, t] →
 3 yZ[4][z, t] + M yZ[6][z, t] - ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4′[t] + 6 Tan[z] yZ[4]^(1,0)[z, t]
yZ[6]^(0,1)[z, t] → -M yZ[5][z, t] - 3 yZ[7][z, t] -
   ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4′[t] - 6 Tan[z] yZ[7]^(1,0)[z, t]
yZ[7]^(0,1)[z, t] → -M yZ[4][z, t] - 3 yZ[6][z, t] +
   ⅇ^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4′[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]
yZ[8]^(0,1)[z, t] → 3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t]
yZ[9]^(0,1)[z, t] → 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t]
yZ[10]^(0,1)[z, t] → -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t])
yZ[11]^(0,1)[z, t] → -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])
yZ[12]^(0,1)[z, t] → -3 yZ[13][z, t] - M yZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t]
yZ[13]^(0,1)[z, t] → -3 yZ[12][z, t] - M yZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t]
yZ[14]^(0,1)[z, t] → M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t]
yZ[15]^(0,1)[z, t] → M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]
```

In[1100]:=

```
(caZ2 = caZ[[2]] // Normal) // MatrixForm
```

Out[1100]//MatrixForm=

```
-1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0 -1  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0 -1  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0 -1  0  0
 0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0
 0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0 -1  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1
 0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0 -1  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1  0
```

In[1101]:=

```
sx0x4
```

Out[1101]=

```
        z          t
{x0 → ───, x4 → ─}
       6 H         H
```

In[1102]:=

```
(*DtvarZEQS*)
```

In[1103]:=

```
(*DtyZsubs*)
```

In[1104]:=

```
(*(DtvarZsubs=Solve[And@@Thread[0==eLazt],DtvarZ][[1]]//
    FullSimplify[#,constraintVars]&)//Column*)
```

In[1105]:=

```
(*(DtyZsubs=
   Solve[And@@Thread[0==DtyZrelations],(D[(yZ[#][z,t]&/@Range[0,15]),t])][[
     1]]//FullSimplify[#,constraintVars]&)//Column*)
```

In[1106]:=

```
(*ToString[FullForm[#]]&/@DtyZsubs;
StringReplace[#,"Rule"→ "Equal"]&/@%;
(DtvaryZEQS=ToExpression[#]&/@%)//Column*)
DtvaryZEQS = DtyZsubs /. {Rule → Equal};
zeroDtyZeqs = DtyZsubs /. {Rule → Subtract}
coupledyZeqs = Partition[DtvaryZEQS, 4]
```

Out[1107]=

```
{3 yZ[1][z, t] + M yZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4′[t] +
   yZ[0]^(0,1)[z, t] + 6 Tan[z] yZ[1]^(1,0)[z, t], 3 yZ[0][z, t] + M yZ[2][z, t] +
   e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4′[t] + yZ[1]^(0,1)[z, t] + 6 Tan[z] yZ[0]^(1,0)[z, t],
 -M yZ[1][z, t] - 3 yZ[3][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4′[t] +
   yZ[2]^(0,1)[z, t] - 6 Tan[z] yZ[3]^(1,0)[z, t], -M yZ[0][z, t] - 3 yZ[2][z, t] -
   e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4′[t] + yZ[3]^(0,1)[z, t] - 6 Tan[z] yZ[2]^(1,0)[z, t],
 -3 yZ[5][z, t] - M yZ[7][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4′[t] +
   yZ[4]^(0,1)[z, t] - 6 Tan[z] yZ[5]^(1,0)[z, t], -3 yZ[4][z, t] - M yZ[6][z, t] +
   e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4′[t] + yZ[5]^(0,1)[z, t] - 6 Tan[z] yZ[4]^(1,0)[z, t],
 M yZ[5][z, t] + 3 yZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4′[t] +
   yZ[6]^(0,1)[z, t] + 6 Tan[z] yZ[7]^(1,0)[z, t], M yZ[4][z, t] + 3 yZ[6][z, t] -
   e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4′[t] + yZ[7]^(0,1)[z, t] + 6 Tan[z] yZ[6]^(1,0)[z, t],
 -3 yZ[9][z, t] - M yZ[11][z, t] + yZ[8]^(0,1)[z, t] - 6 Tan[z] yZ[9]^(1,0)[z, t],
 -3 yZ[8][z, t] - M yZ[10][z, t] + yZ[9]^(0,1)[z, t] - 6 Tan[z] yZ[8]^(1,0)[z, t],
 M yZ[9][z, t] + yZ[10]^(0,1)[z, t] + 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),
 M yZ[8][z, t] + yZ[11]^(0,1)[z, t] + 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t]),
 3 yZ[13][z, t] + M yZ[15][z, t] + yZ[12]^(0,1)[z, t] + 6 Tan[z] yZ[13]^(1,0)[z, t],
 3 yZ[12][z, t] + M yZ[14][z, t] + yZ[13]^(0,1)[z, t] + 6 Tan[z] yZ[12]^(1,0)[z, t],
 -M yZ[13][z, t] - 3 yZ[15][z, t] + yZ[14]^(0,1)[z, t] - 6 Tan[z] yZ[15]^(1,0)[z, t],
 -M yZ[12][z, t] - 3 yZ[14][z, t] + yZ[15]^(0,1)[z, t] - 6 Tan[z] yZ[14]^(1,0)[z, t]}
```

Out[1108]=

```
{{yZ[0]^(0,1)[z, t] == -3 yZ[1][z, t] - M yZ[3][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4′[t] -
    6 Tan[z] yZ[1]^(1,0)[z, t], yZ[1]^(0,1)[z, t] == -3 yZ[0][z, t] - M yZ[2][z, t] -
    e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4′[t] - 6 Tan[z] yZ[0]^(1,0)[z, t],
  yZ[2]^(0,1)[z, t] == M yZ[1][z, t] + 3 yZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4′[t] +
    6 Tan[z] yZ[3]^(1,0)[z, t], yZ[3]^(0,1)[z, t] == M yZ[0][z, t] + 3 yZ[2][z, t] +
    e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4′[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]},
 {yZ[4]^(0,1)[z, t] == 3 yZ[5][z, t] + M yZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4′[t] +
    6 Tan[z] yZ[5]^(1,0)[z, t], yZ[5]^(0,1)[z, t] == 3 yZ[4][z, t] + M yZ[6][z, t] -
    e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4′[t] + 6 Tan[z] yZ[4]^(1,0)[z, t],
  yZ[6]^(0,1)[z, t] == -M yZ[5][z, t] - 3 yZ[7][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4′[t] -
    6 Tan[z] yZ[7]^(1,0)[z, t], yZ[7]^(0,1)[z, t] == -M yZ[4][z, t] - 3 yZ[6][z, t] +
    e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4′[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]},
 {yZ[8]^(0,1)[z, t] == 3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t],
  yZ[9]^(0,1)[z, t] == 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t],
  yZ[10]^(0,1)[z, t] == -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),
  yZ[11]^(0,1)[z, t] == -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])},
 {yZ[12]^(0,1)[z, t] == -3 yZ[13][z, t] - M yZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t],
  yZ[13]^(0,1)[z, t] == -3 yZ[12][z, t] - M yZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t],
  yZ[14]^(0,1)[z, t] == M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t],
  yZ[15]^(0,1)[z, t] == M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]}}
```

In[1109]:=

```
(*ToString[FullForm[#]]&/@Take[coupledyZeqs,2]
 StringReplace[#,"Equal"→ "Subtract"]&/@%;
 (ToExpression[#]&/@%)
 #/.{Equal→Subtract}&/@Take[coupledyZeqs,2]
%-%%*)
```

In[1110]:=

```
Clear[yZ, yZvar];
yZvar = (yZ[#][z, t] & /@ Range[0, 15])
```

Out[1111]=

```
{yZ[0][z, t], yZ[1][z, t], yZ[2][z, t], yZ[3][z, t], yZ[4][z, t],
 yZ[5][z, t], yZ[6][z, t], yZ[7][z, t], yZ[8][z, t], yZ[9][z, t], yZ[10][z, t],
 yZ[11][z, t], yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t]}
```

In[1112]:=

```
DSolve[coupledyZeqs⟦1⟧, yZvar⟦1 ;; 4⟧, {z, t}]
```

Out[1112]=

```
DSolve[
 {yZ[0]^(0,1)[z, t] == -3 yZ[1][z, t] - M yZ[3][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4′[t] -
    6 Tan[z] yZ[1]^(1,0)[z, t], yZ[1]^(0,1)[z, t] == -3 yZ[0][z, t] - M yZ[2][z, t] -
    e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4′[t] - 6 Tan[z] yZ[0]^(1,0)[z, t],
  yZ[2]^(0,1)[z, t] == M yZ[1][z, t] + 3 yZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4′[t] +
    6 Tan[z] yZ[3]^(1,0)[z, t], yZ[3]^(0,1)[z, t] == M yZ[0][z, t] + 3 yZ[2][z, t] +
    e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4′[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]},
 {yZ[0][z, t], yZ[1][z, t], yZ[2][z, t], yZ[3][z, t]}, {z, t}]
```

In[1113]:=

```
DSolve[coupledyZeqs⟦2⟧, yZvar⟦5 ;; 8⟧, {z, t}]
```

Out[1113]=

```
DSolve[
 {yZ[4]^(0,1)[z, t] ⩵ 3 yZ[5][z, t] + M yZ[7][z, t] + 𝕖^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4′[t] +
     6 Tan[z] yZ[5]^(1,0)[z, t], yZ[5]^(0,1)[z, t] ⩵ 3 yZ[4][z, t] + M yZ[6][z, t] -
     𝕖^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4′[t] + 6 Tan[z] yZ[4]^(1,0)[z, t],
   yZ[6]^(0,1)[z, t] ⩵ -M yZ[5][z, t] - 3 yZ[7][z, t] - 𝕖^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4′[t] -
     6 Tan[z] yZ[7]^(1,0)[z, t], yZ[7]^(0,1)[z, t] ⩵ -M yZ[4][z, t] - 3 yZ[6][z, t] +
     𝕖^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4′[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]},
 {yZ[4][z, t], yZ[5][z, t], yZ[6][z, t], yZ[7][z, t]}, {z, t}]
```

In[1114]:=

```
DSolve[coupledyZeqs⟦3⟧, yZvar⟦9 ;; 12⟧, {z, t}]
```

Out[1114]=

```
DSolve[{yZ[8]^(0,1)[z, t] ⩵ 3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t],
   yZ[9]^(0,1)[z, t] ⩵ 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t],
   yZ[10]^(0,1)[z, t] ⩵ -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),
   yZ[11]^(0,1)[z, t] ⩵ -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])},
 {yZ[8][z, t], yZ[9][z, t], yZ[10][z, t], yZ[11][z, t]}, {z, t}]
```

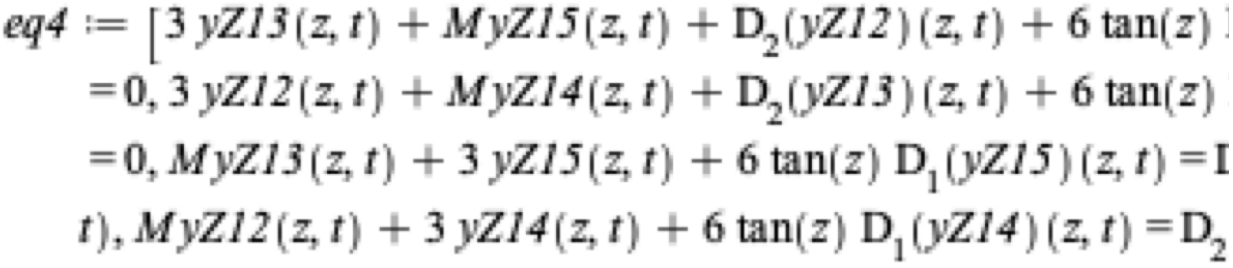
In[1115]:=

```
DSolve[coupledyZeqs⟦4⟧, yZvar⟦13 ;; 16⟧, {z, t}]
```

Out[1115]=

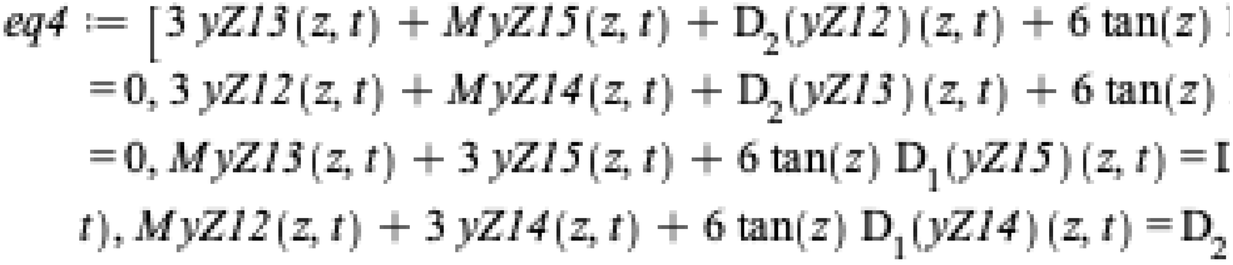
```
DSolve[{yZ[12]^(0,1)[z, t] ⩵ -3 yZ[13][z, t] - M yZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t],
   yZ[13]^(0,1)[z, t] ⩵ -3 yZ[12][z, t] - M yZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t],
   yZ[14]^(0,1)[z, t] ⩵ M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t],
   yZ[15]^(0,1)[z, t] ⩵ M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]},
 {yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t]}, {z, t}]
```

In[1116]:=



```
eq4 := [3 yZ13(z, t) + MyZ15(z, t) + D2(yZ12)(z, t) + 6 tan(z)
     =0, 3 yZ12(z, t) + MyZ14(z, t) + D2(yZ13)(z, t) + 6 tan(z)
     =0, MyZ13(z, t) + 3 yZ15(z, t) + 6 tan(z) D1(yZ15)(z, t) =
     t), MyZ12(z, t) + 3 yZ14(z, t) + 6 tan(z) D1(yZ14)(z, t) = D2
```

Out[1116]=



```
eq4 := [3 yZ13(z, t) + MyZ15(z, t) + D2(yZ12)(z, t) + 6 tan(z)
     =0, 3 yZ12(z, t) + MyZ14(z, t) + D2(yZ13)(z, t) + 6 tan(z)
     =0, MyZ13(z, t) + 3 yZ15(z, t) + 6 tan(z) D1(yZ15)(z, t) =
     t), MyZ12(z, t) + 3 yZ14(z, t) + 6 tan(z) D1(yZ14)(z, t) = D2
```

TRY MAPLE : MUST use `  List[Equal[Derivative[0,1][yZ8][z,t],Pl...  `

```
In[1117]:=
syZzt = yZ[#] → ToExpression["((yZ" <> ToString[#] <> "[#1,#2])&)"] & /@ Range[0, 15]
maplexyZeqs = FullForm[coupledyZeqs /. % // FullSimplify]
```

```
Out[1117]=
{yZ[0] → (yZ0[#1, #2] &), yZ[1] → (yZ1[#1, #2] &),
 yZ[2] → (yZ2[#1, #2] &), yZ[3] → (yZ3[#1, #2] &), yZ[4] → (yZ4[#1, #2] &),
 yZ[5] → (yZ5[#1, #2] &), yZ[6] → (yZ6[#1, #2] &), yZ[7] → (yZ7[#1, #2] &),
 yZ[8] → (yZ8[#1, #2] &), yZ[9] → (yZ9[#1, #2] &), yZ[10] → (yZ10[#1, #2] &),
 yZ[11] → (yZ11[#1, #2] &), yZ[12] → (yZ12[#1, #2] &),
 yZ[13] → (yZ13[#1, #2] &), yZ[14] → (yZ14[#1, #2] &), yZ[15] → (yZ15[#1, #2] &)}
```

Out[1118]//FullForm=

```
List[List[Equal[Plus[Times[3, yZ1[z, t]], Times[M, yZ3[z, t]],
    Derivative[0, 1][yZ0][z, t], Times[6, Tan[z], Derivative[1, 0][yZ1][z, t]]],
   Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ0[z, t], Derivative[1][a4][t]]],
  Equal[Plus[Times[3, yZ0[z, t]], Times[M, yZ2[z, t]],
    Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ1[z, t], Derivative[1][a4][t]],
    Derivative[0, 1][yZ1][z, t], Times[6, Tan[z], Derivative[1, 0][yZ0][z, t]]], 0],
  Equal[Plus[Times[M, yZ1[z, t]], Times[3, yZ3[z, t]],
    Times[6, Tan[z], Derivative[1, 0][yZ3][z, t]]], Plus[Times[Power[E, Times[-1, a4[t]]],
     Q1, Sinh[a4[t]], yZ2[z, t], Derivative[1][a4][t]], Derivative[0, 1][yZ2][z, t]]],
  Equal[Plus[Times[M, yZ0[z, t]], Times[3, yZ2[z, t]],
    Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ3[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][yZ2][z, t]]], Derivative[0, 1][yZ3][z, t]]],
 List[Equal[Plus[Times[3, yZ5[z, t]], Times[M, yZ7[z, t]],
    Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ4[z, t], Derivative[1][a4][t]],
    Times[6, Tan[z], Derivative[1, 0][yZ5][z, t]]], Derivative[0, 1][yZ4][z, t]],
  Equal[Plus[Times[3, yZ4[z, t]], Times[M, yZ6[z, t]],
    Times[6, Tan[z], Derivative[1, 0][yZ4][z, t]]], Plus[Times[Power[E, Times[-1, a4[t]]],
     Q1, Sinh[a4[t]], yZ5[z, t], Derivative[1][a4][t]], Derivative[0, 1][yZ5][z, t]]],
  Equal[Plus[Times[M, yZ5[z, t]], Times[3, yZ7[z, t]],
    Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ6[z, t], Derivative[1][a4][t]],
    Derivative[0, 1][yZ6][z, t], Times[6, Tan[z], Derivative[1, 0][yZ7][z, t]]], 0],
  Equal[Plus[Times[M, yZ4[z, t]], Times[3, yZ6[z, t]], Derivative[0, 1][yZ7][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ6][z, t]]],
   Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ7[z, t], Derivative[1][a4][t]]]],
 List[Equal[Plus[Times[M, yZ11[z, t]], Times[3, yZ9[z, t]],
    Times[6, Tan[z], Derivative[1, 0][yZ9][z, t]]], Derivative[0, 1][yZ8][z, t]],
  Equal[Plus[Times[M, yZ10[z, t]], Times[3, yZ8[z, t]],
    Times[6, Tan[z], Derivative[1, 0][yZ8][z, t]]], Derivative[0, 1][yZ9][z, t]],
  Equal[Plus[Times[3, yZ11[z, t]], Times[M, yZ9[z, t]], Derivative[0, 1][yZ10][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ11][z, t]]], 0],
  Equal[Plus[Times[3, yZ10[z, t]], Times[M, yZ8[z, t]], Derivative[0, 1][yZ11][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ10][z, t]]], 0]],
 List[Equal[Plus[Times[3, yZ13[z, t]], Times[M, yZ15[z, t]], Derivative[0, 1][yZ12][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ13][z, t]]], 0],
  Equal[Plus[Times[3, yZ12[z, t]], Times[M, yZ14[z, t]], Derivative[0, 1][yZ13][z, t],
    Times[6, Tan[z], Derivative[1, 0][yZ12][z, t]]], 0],
  Equal[Plus[Times[M, yZ13[z, t]], Times[3, yZ15[z, t]],
    Times[6, Tan[z], Derivative[1, 0][yZ15][z, t]]], Derivative[0, 1][yZ14][z, t]],
  Equal[Plus[Times[M, yZ12[z, t]], Times[3, yZ14[z, t]],
    Times[6, Tan[z], Derivative[1, 0][yZ14][z, t]]], Derivative[0, 1][yZ15][z, t]]]]
```

In[1119]:=

```
mapleyZvars =
 FullForm[yZ[#][z, t] & /@ Range[4 * (# - 1), 4 * (# - 1) + 3] /. syZzt] & /@ Range[4]
```

Out[1119]=

```
{List[yZ0[z, t], yZ1[z, t], yZ2[z, t], yZ3[z, t]],
 List[yZ4[z, t], yZ5[z, t], yZ6[z, t], yZ7[z, t]],
 List[yZ8[z, t], yZ9[z, t], yZ10[z, t], yZ11[z, t]],
 List[yZ12[z, t], yZ13[z, t], yZ14[z, t], yZ15[z, t]]}
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.
A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

```
“maple-output.py”:
import textwrap
filename = "maple-output.txt"
maple_long _string = "paste maple string here"
# Wraps the text and joins with newlines
with open (filename, "w") as f :
    f . write ("\n" . join (textwrap . wrap (maple_long _string, width = 80)))
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.
A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

"Solution" by Maple

In[1120]:=

```
Ψ16;
% /. sfψ16Aa /. sx0x4;
% /. sZtOyZ;
Thread[% == %%%];
Solve[And @@ %, yZ[#][z, t] & /@ Range[0, 15]][[1]]
% /. szt /. {Rule → Equal};
Solve[And @@ %, f16[#][x0, x4] & /@ Range[0, 15]][[1]];
% /. {Rule → Equal}
```

Out[1124]=

```
{yZ[0][z, t] → f16[0][x0, x4], yZ[1][z, t] → f16[5][x0, x4],
 yZ[2][z, t] → f16[8][x0, x4], yZ[3][z, t] → f16[13][x0, x4], yZ[4][z, t] → f16[1][x0, x4],
 yZ[5][z, t] → f16[4][x0, x4], yZ[6][z, t] → f16[9][x0, x4], yZ[7][z, t] → f16[12][x0, x4],
 yZ[8][z, t] → f16[2][x0, x4], yZ[9][z, t] → f16[7][x0, x4], yZ[10][z, t] → f16[10][x0, x4],
 yZ[11][z, t] → f16[15][x0, x4], yZ[12][z, t] → f16[3][x0, x4], yZ[13][z, t] → f16[6][x0, x4],
 yZ[14][z, t] → f16[11][x0, x4], yZ[15][z, t] → f16[14][x0, x4]}
```

Out[1127]=

```
{f16[0][x0, x4] == yZ[0][6 H x0, H x4], f16[1][x0, x4] == yZ[4][6 H x0, H x4],
 f16[2][x0, x4] == yZ[8][6 H x0, H x4], f16[3][x0, x4] == yZ[12][6 H x0, H x4],
 f16[4][x0, x4] == yZ[5][6 H x0, H x4], f16[5][x0, x4] == yZ[1][6 H x0, H x4],
 f16[6][x0, x4] == yZ[13][6 H x0, H x4], f16[7][x0, x4] == yZ[9][6 H x0, H x4],
 f16[8][x0, x4] == yZ[2][6 H x0, H x4], f16[9][x0, x4] == yZ[6][6 H x0, H x4],
 f16[10][x0, x4] == yZ[10][6 H x0, H x4], f16[11][x0, x4] == yZ[14][6 H x0, H x4],
 f16[12][x0, x4] == yZ[7][6 H x0, H x4], f16[13][x0, x4] == yZ[3][6 H x0, H x4],
 f16[14][x0, x4] == yZ[15][6 H x0, H x4], f16[15][x0, x4] == yZ[11][6 H x0, H x4]}
```

Load the Maple output strings. Get is a lexer and parer and ... , so use ReadString

In[1128]:=

```
SetDirectory[NotebookDirectory[]];
```

In[1129]:=

$$seq3 := \left\{ yZ10(z,t) = \frac{1}{\sqrt{\sin(z)}} \left(\left(c37 \sin\left(\sqrt{M^2 - 36\,C3 - 9}\;\, t \right) \right. \right. \right.$$

$$\left. + c38 \cos\left(\sqrt{M^2 - 36\,C3 - 9}\;\, t \right) \right) \left(c36 \sin(z)^{-\frac{\sqrt{1+4\,C3}}{2}} \right.$$

$$\left. \left. + c35 \sin(z)^{\frac{\sqrt{1+4\,C3}}{2}} \right) \right), yZ11(z,t)$$

$$= \frac{1}{\sqrt{\sin(z)}} \left(\left(c33 \sin\left(\sqrt{M^2 - 36\,C3 - 9}\;\, t \right) \right. \right.$$

$$+\, c34 \cos\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)\Big) \Big(c32 \sin(z)^{-\frac{\sqrt{1+4\, C3}}{2}}$$

$$+\, c31 \sin(z)^{\frac{\sqrt{1+4\, C3}}{2}}\Big)\Big),\; yZ8(z, t) =$$

$$-\frac{1}{\sqrt{\sin(z)}\; M}\Bigg(3\,\Bigg(\frac{1}{3}\Big(\Big(\cos\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)\, c33$$

$$-\, \sin\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)\, c34\Big) \Big(c32 \sin(z)^{-\frac{\sqrt{1+4\, C3}}{2}}$$

$$+\, c31 \sin(z)^{\frac{\sqrt{1+4\, C3}}{2}}\Big) \sqrt{M^2 - 36\, C3 - 9}\,\Big)$$

$$+\, \Big(c37 \sin\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)$$

$$+\, c38 \cos\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)\Big) \sqrt{1 + 4\, C3}\, \Big(c35 \sin(z)^{\frac{\sqrt{1+4\, C3}}{2}}$$

$$-\, c36 \sin(z)^{-\frac{\sqrt{1+4\, C3}}{2}}\Big)\Bigg)\Bigg),\; yZ9(z, t) =$$

$$-\frac{1}{\sqrt{\sin(z)}\; M}\Big(\Big(\Big(c36 \sin(z)^{-\frac{\sqrt{1+4\, C3}}{2}}$$

$$+\, c35 \sin(z)^{\frac{\sqrt{1+4\, C3}}{2}}\Big) \Big(\cos\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)\, c37$$

$$-\, \sin\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)\, c38\Big) \sqrt{M^2 - 36\, C3 - 9}$$

$$+\, 3 \sqrt{1 + 4\, C3}\, \Big(c31 \sin(z)^{\frac{\sqrt{1+4\, C3}}{2}}$$

$$-\, c32 \sin(z)^{-\frac{\sqrt{1+4\, C3}}{2}}\Big) \Big(c33 \sin\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)$$

$$+\, c34 \cos\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)\Big)\Big)\Big)\Big\}$$

Out[1129]=

$$seq3 := \Big\{ yZ10(z, t) = \frac{1}{\sqrt{\sin(z)}}\Big(\Big(c37 \sin\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)$$

$$+\, c38 \cos\Big(\sqrt{M^2 - 36\, C3 - 9}\;\, t\Big)\Big) \Big(c36 \sin(z)^{-\frac{\sqrt{1+4\, C3}}{2}}$$

$$+ c35 \sin(z)^{\frac{\sqrt{1+4\,C3}}{2}}\Big)\Big), yZ11(z,t)$$

$$= \frac{1}{\sqrt{\sin(z)}}\Big(\Big(c33 \sin\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)$$

$$+ c34 \cos\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)\Big)\Big(c32 \sin(z)^{-\frac{\sqrt{1+4\,C3}}{2}}$$

$$+ c31 \sin(z)^{\frac{\sqrt{1+4\,C3}}{2}}\Big)\Big), yZ8(z,t) =$$

$$-\frac{1}{\sqrt{\sin(z)}\;M}\Big(3\Big(\frac{1}{3}\Big(\Big(\cos\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)\,c33$$

$$- \sin\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)\,c34\Big)\Big(c32 \sin(z)^{-\frac{\sqrt{1+4\,C3}}{2}}$$

$$+ c31 \sin(z)^{\frac{\sqrt{1+4\,C3}}{2}}\Big)\sqrt{M^2 - 36\,C3 - 9}\Big)$$

$$+ \Big(c37 \sin\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)$$

$$+ c38 \cos\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)\Big)\sqrt{1 + 4\,C3}\;\Big(c35 \sin(z)^{\frac{\sqrt{1+4\,C3}}{2}}$$

$$- c36 \sin(z)^{-\frac{\sqrt{1+4\,C3}}{2}}\Big)\Big)\Big)\Big), yZ9(z,t) =$$

$$-\frac{1}{\sqrt{\sin(z)}\;M}\Big(\Big(\Big(c36 \sin(z)^{-\frac{\sqrt{1+4\,C3}}{2}}$$

$$+ c35 \sin(z)^{\frac{\sqrt{1+4\,C3}}{2}}\Big)\Big(\cos\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)\,c37$$

$$- \sin\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)\,c38\Big)\sqrt{M^2 - 36\,C3 - 9}$$

$$+ 3\sqrt{1 + 4\,C3}\;\Big(c31 \sin(z)^{\frac{\sqrt{1+4\,C3}}{2}}$$

$$- c32 \sin(z)^{-\frac{\sqrt{1+4\,C3}}{2}}\Big)\Big(c33 \sin\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)$$

$$+ c34 \cos\Big(\sqrt{M^2 - 36\,C3 - 9}\;t\Big)\Big)\Big)\Big)\Big\}$$

In[1130]:=

$$
\begin{aligned}
&seq4 := \Bigg\{ yZ12(z,t) = -\frac{1}{\sqrt{\sin(z)}\,M} \Bigg(3 \Bigg(\frac{1}{3} \Bigg(\Big(\sin\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big)\,c44 \\
&\quad - \cos\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big)\,c43 \Big) \Big(c42\,\sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}} \\
&\quad + c41\,\sin(z)^{\frac{\sqrt{1+4\,C4}}{2}} \Big) \sqrt{M^2 - 36\,C4 - 9} \Bigg) \\
&\quad + \sqrt{1 + 4\,C4}\; \Big(c47\,\sin\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big) \\
&\quad + c48\,\cos\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big) \Big) \Big(c45\,\sin(z)^{\frac{\sqrt{1+4\,C4}}{2}} \\
&\quad - c46\,\sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}} \Big) \Bigg) \Bigg), yZ13(z,t) = \\
&\quad -\frac{1}{\sqrt{\sin(z)}\,M} \Bigg(\Bigg(c46\,\sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}} \\
&\quad + c45\,\sin(z)^{\frac{\sqrt{1+4\,C4}}{2}} \Bigg) \Big(\sin\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big)\,c48 \\
&\quad - \cos\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big)\,c47 \Big) \sqrt{M^2 - 36\,C4 - 9} \\
&\quad + 3\,\sqrt{1 + 4\,C4}\; \Big(c41\,\sin(z)^{\frac{\sqrt{1+4\,C4}}{2}} \\
&\quad - c42\,\sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}} \Big) \Big(c43\,\sin\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big) \\
&\quad + c44\,\cos\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big) \Big) \Bigg), yZ14(z,t) \\
&\quad = \frac{1}{\sqrt{\sin(z)}} \Bigg(\Bigg(c46\,\sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}} \\
&\quad + c45\,\sin(z)^{\frac{\sqrt{1+4\,C4}}{2}} \Bigg) \Big(c47\,\sin\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big) \\
&\quad + c48\,\cos\Big(\sqrt{M^2 - 36\,C4 - 9}\;t \Big) \Big) \Bigg), yZ15(z,t)
\end{aligned}
$$

$$= \frac{1}{\sqrt{\sin(z)}} \Bigg(\Bigg(c42 \sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}}$$

$$+\, c41 \sin(z)^{\frac{\sqrt{1+4\,C4}}{2}}\Bigg) \Big(c43 \sin\Big(\sqrt{M^2-36\,C4-9}\;t\Big)$$

$$+\, c44 \cos\Big(\sqrt{M^2-36\,C4-9}\;t\Big)\Big)\Bigg)\Bigg\}$$

Out[1130]=

$$seq4 := \Bigg\{yZ12(z,t) = -\frac{1}{\sqrt{\sin(z)}\;M}\Bigg(3\,\Bigg(\frac{1}{3}\Bigg(\Big(\sin\Big(\sqrt{M^2-36\,C4-9}\;t\Big)\,c44$$

$$-\, \cos\Big(\sqrt{M^2-36\,C4-9}\;t\Big)\,c43\Big)\,\Bigg(c42 \sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}}$$

$$+\, c41 \sin(z)^{\frac{\sqrt{1+4\,C4}}{2}}\Bigg)\,\sqrt{M^2-36\,C4-9}\,\Bigg)$$

$$+\, \sqrt{1+4\,C4}\,\Big(c47 \sin\Big(\sqrt{M^2-36\,C4-9}\;t\Big)$$

$$+\, c48 \cos\Big(\sqrt{M^2-36\,C4-9}\;t\Big)\Big)\,\Bigg(c45 \sin(z)^{\frac{\sqrt{1+4\,C4}}{2}}$$

$$-\, c46 \sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}}\Bigg)\Bigg)\Bigg)\Bigg),\; yZ13(z,t) =$$

$$-\frac{1}{\sqrt{\sin(z)}\;M}\Bigg(\Bigg(c46 \sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}}$$

$$+\, c45 \sin(z)^{\frac{\sqrt{1+4\,C4}}{2}}\Bigg)\,\Big(\sin\Big(\sqrt{M^2-36\,C4-9}\;t\Big)\,c48$$

$$-\, \cos\Big(\sqrt{M^2-36\,C4-9}\;t\Big)\,c47\Big)\,\sqrt{M^2-36\,C4-9}$$

$$+\, 3\,\sqrt{1+4\,C4}\,\Bigg(c41 \sin(z)^{\frac{\sqrt{1+4\,C4}}{2}}$$

$$-\, c42 \sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}}\Bigg)\,\Big(c43 \sin\Big(\sqrt{M^2-36\,C4-9}\;t\Big)$$

$$+\, c44 \cos\Big(\sqrt{M^2-36\,C4-9}\;t\Big)\Big)\Bigg),\; yZ14(z,t)$$

$$= \frac{1}{\sqrt{\sin(z)}} \left(\left(c46 \sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}} + c45 \sin(z)^{\frac{\sqrt{1+4\,C4}}{2}} \right) \left(c47 \sin\left(\sqrt{M^2 - 36\,C4 - 9}\;t\right) + c48 \cos\left(\sqrt{M^2 - 36\,C4 - 9}\;t\right) \right) \right), yZ15(z,t)$$

$$= \frac{1}{\sqrt{\sin(z)}} \left(\left(c42 \sin(z)^{-\frac{\sqrt{1+4\,C4}}{2}} + c41 \sin(z)^{\frac{\sqrt{1+4\,C4}}{2}} \right) \left(c43 \sin\left(\sqrt{M^2 - 36\,C4 - 9}\;t\right) + c44 \cos\left(\sqrt{M^2 - 36\,C4 - 9}\;t\right) \right) \right) \}$$

In[1131]:=

```
maplestringEQ3 =
  "{{yZ10(z,t)=((c37 sin(sqrt(M^2-36 C3-9) t)+c38 cos(sqrt(M^2-36 C3-9) t))
    (c36 (sin(z))^(-(sqrt(1+4 C3))/2)+c35 (sin(z))^((sqrt(1+4
    C3))/2)))/(sqrt(sin(z))),yZ11(z,t)=((c33 sin(sqrt(M^2-36 C3-9) t)+c34
    cos(sqrt(M^2-36 C3-9) t)) (c32 (sin(z))^(-(sqrt(1+4 C3))/2)+c31
    (sin(z))^((sqrt(1+4 C3))/2)))/(sqrt(sin(z))),yZ8(z,t)=-(3 (((cos(sqrt(M^2-36
    C3-9) t) c33-sin(sqrt(M^2-36 C3-9) t) c34) (c32 (sin(z))^(-(sqrt(1+4 C3))/2)+c31
    (sin(z))^((sqrt(1+4 C3))/2)) sqrt(M^2-36 C3-9))/3+(c37 sin(sqrt(M^2-36 C3-9)
    t)+c38 cos(sqrt(M^2-36 C3-9) t)) sqrt(1+4 C3) (c35 (sin(z))^((sqrt(1+4
    C3))/2)-c36 (sin(z))^(-(sqrt(1+4 C3))/2))))/(sqrt(sin(z)) M),yZ9(z,t)=-((c36
    (sin(z))^(-(sqrt(1+4 C3))/2)+c35 (sin(z))^((sqrt(1+4 C3))/2)) (cos(sqrt(M^2-36
    C3-9) t) c37-sin(sqrt(M^2-36 C3-9) t) c38) sqrt(M^2-36 C3-9)+3 sqrt(1+4 C3)
    (c31 (sin(z))^((sqrt(1+4 C3))/2)-c32 (sin(z))^(-(sqrt(1+4 C3))/2)) (c33
    sin(sqrt(M^2-36 C3-9) t)+c34 cos(sqrt(M^2-36 C3-9) t)))/(sqrt(sin(z)) M)}}";
```

In[1132]:=

```
maplestringEQ4 =
  "{{yZ12(z, t) = -3*(((sin(sqrt(M^2 - 36*C4 - 9)*t)*c44 - cos(sqrt(M^2 - 36*C4
    - 9)*t)*c43)*(c42*sin(z)^(-sqrt(1 + 4*C4)/2) + c41*sin(z)^(sqrt(1 +
    4*C4)/2))*sqrt(M^2 - 36*C4 - 9))/3 + sqrt(1 + 4*C4)*(c47*sin(sqrt(M^2 -
    36*C4 - 9)*t) + c48*cos(sqrt(M^2 - 36*C4 - 9)*t))*(c45*sin(z)^(sqrt(1 +
    4*C4)/2) - c46*sin(z)^(-sqrt(1 + 4*C4)/2)))/(sqrt(sin(z))*M), yZ13(z, t) =
    -((c46*sin(z)^(-sqrt(1 + 4*C4)/2) + c45*sin(z)^(sqrt(1 + 4*C4)/2))*(sin(sqrt(M^2
    - 36*C4 - 9)*t)*c48 - cos(sqrt(M^2 - 36*C4 - 9)*t)*c47)*sqrt(M^2 - 36*C4 -
    9) + 3*sqrt(1 + 4*C4)*(c41*sin(z)^(sqrt(1 + 4*C4)/2) - c42*sin(z)^(-sqrt(1
    + 4*C4)/2))*(c43*sin(sqrt(M^2 - 36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4
    - 9)*t)))/(sqrt(sin(z))*M), yZ14(z, t) = (c46*sin(z)^(-sqrt(1 + 4*C4)/2)
    + c45*sin(z)^(sqrt(1 + 4*C4)/2))*(c47*sin(sqrt(M^2 - 36*C4 - 9)*t) +
    c48*cos(sqrt(M^2 - 36*C4 - 9)*t))/sqrt(sin(z)), yZ15(z, t) = (c42*sin(z)^(-sqrt(1
    + 4*C4)/2) + c41*sin(z)^(sqrt(1 + 4*C4)/2))*(c43*sin(sqrt(M^2 -
    36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4 - 9)*t))/sqrt(sin(z))}}";
```

In[1133]:=

```
solvedEQ3 = ConvertMapleToMathematicaV2[ReleaseHold[maplestringEQ3]]
```

Out[1133]=

$$\Big\{\Big\{\mathrm{YZ10}[z,t]==\frac{1}{\sqrt{\mathrm{Sin}[z]}}\left(\mathrm{C35}[\mathrm{Sin}[z]]^{\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}+\mathrm{C36}[\mathrm{Sin}[z]]^{-\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}\right)\left(\mathrm{c38}\,\mathrm{Cos}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]+\mathrm{c37}\,\mathrm{Sin}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]\right),$$

$$\mathrm{YZ11}[z,t]==\frac{1}{\sqrt{\mathrm{Sin}[z]}}\left(\mathrm{C31}[\mathrm{Sin}[z]]^{\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}+\mathrm{C32}[\mathrm{Sin}[z]]^{-\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}\right)\left(\mathrm{c34}\,\mathrm{Cos}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]+\mathrm{c33}\,\mathrm{Sin}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]\right),$$

$$\mathrm{YZ8}[z,t]==-\frac{1}{M\sqrt{\mathrm{Sin}[z]}}3\left(\frac{1}{3}\sqrt{-9-36\,\mathrm{C3}+M^2}\left(\mathrm{C31}[\mathrm{Sin}[z]]^{\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}+\mathrm{C32}[\mathrm{Sin}[z]]^{-\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}\right)\left(\mathrm{c33}\,\mathrm{Cos}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]-\mathrm{c34}\,\mathrm{Sin}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]\right)+\sqrt{1+4\,\mathrm{C3}}\left(\mathrm{C35}[\mathrm{Sin}[z]]^{\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}-\mathrm{C36}[\mathrm{Sin}[z]]^{-\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}\right)\left(\mathrm{c38}\,\mathrm{Cos}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]+\mathrm{c37}\,\mathrm{Sin}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]\right)\right),$$

$$\mathrm{YZ9}[z,t]==\frac{1}{M\sqrt{\mathrm{Sin}[z]}}\left(-3\sqrt{1+4\,\mathrm{C3}}\left(\mathrm{C31}[\mathrm{Sin}[z]]^{\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}-\mathrm{C32}[\mathrm{Sin}[z]]^{-\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}\right)\left(\mathrm{c34}\,\mathrm{Cos}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]+\mathrm{c33}\,\mathrm{Sin}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]\right)-\sqrt{-9-36\,\mathrm{C3}+M^2}\left(\mathrm{C35}[\mathrm{Sin}[z]]^{\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}+\mathrm{C36}[\mathrm{Sin}[z]]^{-\frac{1}{2}\sqrt{1+4\,\mathrm{C3}}}\right)\left(\mathrm{c37}\,\mathrm{Cos}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]-\mathrm{c38}\,\mathrm{Sin}\Big[\sqrt{-9-36\,\mathrm{C3}+M^2}\;t\Big]\right)\right)\Big\}\Big\}$$

In[1134]:=

```
solvedEQ4 = ConvertMapleToMathematicaV2[ReleaseHold[maplestringEQ4]]
```

Out[1134]=

$$\Big\{\Big\{\text{YZ12}[z,t]==-\frac{1}{M\sqrt{\text{Sin}[z]}}3\left(\frac{1}{3}\sqrt{-9-36\,\text{C4}+M^2}\left(-\text{c43}\,\text{Cos}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]+\text{c44}\,\text{Sin}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]\right)\left(\text{c42}\,\text{Sin}[z]^{-\frac{1}{2}\sqrt{1+4\,\text{C4}}}+\text{c41}\,\text{Sin}[z]^{\frac{1}{2}\sqrt{1+4\,\text{C4}}}\right)+\sqrt{1+4\,\text{C4}}\left(\text{c48}\,\text{Cos}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]+\text{c47}\,\text{Sin}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]\right)\left(-\text{c46}\,\text{Sin}[z]^{-\frac{1}{2}\sqrt{1+4\,\text{C4}}}+\text{c45}\,\text{Sin}[z]^{\frac{1}{2}\sqrt{1+4\,\text{C4}}}\right)\right),$$

$$\text{YZ13}[z,t]==\frac{1}{M\sqrt{\text{Sin}[z]}}\left(-3\sqrt{1+4\,\text{C4}}\left(\text{c44}\,\text{Cos}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]+\text{c43}\,\text{Sin}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]\right)\left(-\text{c42}\,\text{Sin}[z]^{-\frac{1}{2}\sqrt{1+4\,\text{C4}}}+\text{c41}\,\text{Sin}[z]^{\frac{1}{2}\sqrt{1+4\,\text{C4}}}\right)-\sqrt{-9-36\,\text{C4}+M^2}\left(-\text{c47}\,\text{Cos}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]+\text{c48}\,\text{Sin}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]\right)\left(\text{c46}\,\text{Sin}[z]^{-\frac{1}{2}\sqrt{1+4\,\text{C4}}}+\text{c45}\,\text{Sin}[z]^{\frac{1}{2}\sqrt{1+4\,\text{C4}}}\right)\right),$$

$$\text{YZ14}[z,t]==\frac{1}{\sqrt{\text{Sin}[z]}}\left(\text{c48}\,\text{Cos}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]+\text{c47}\,\text{Sin}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]\right)\left(\text{c46}\,\text{Sin}[z]^{-\frac{1}{2}\sqrt{1+4\,\text{C4}}}+\text{c45}\,\text{Sin}[z]^{\frac{1}{2}\sqrt{1+4\,\text{C4}}}\right),$$

$$\text{YZ15}[z,t]==\frac{1}{\sqrt{\text{Sin}[z]}}\left(\text{c44}\,\text{Cos}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]+\text{c43}\,\text{Sin}\left[\sqrt{-9-36\,\text{C4}+M^2}\;t\right]\right)\left(\text{c42}\,\text{Sin}[z]^{-\frac{1}{2}\sqrt{1+4\,\text{C4}}}+\text{c41}\,\text{Sin}[z]^{\frac{1}{2}\sqrt{1+4\,\text{C4}}}\right)\Big\}\Big\}$$

In[1135]:=

```
maplestringEQ2 = ReadString["maple-textwrap-string-EQ2-thinkpad-2026-02-11.txt"];
```

In[1136]:=

```
time1 = Now
If[FindFile["solvedEQ2.mx"] === $Failed,
 AbsoluteTiming[solvedEQ2 = ConvertMapleToMathematicaV2[maplestringEQ2]];
 DumpSave[ToString[header <> "solvedEQ2.mx"], solvedEQ2];
 Print[ToString[header <> "solvedEQ2.mx"]];
 , Get[ "solvedEQ2.mx"]]
Now - time1
```

Out[1136]=

```
Wed 11 Feb 2026 07:00:19 GMT-8
```

```
Pair-Crtn-Univ-same_E-L-eqs-alt-approach-solvedEQ2.mx
```

Out[1138]=

```
57.18618078 min
```

In[1139]:=

```
solvedEQ2
```

Out[1139]=

$$\Big\{\Big\{\text{YZ4}[z,t]==-\frac{1}{2\,M\,\text{Q1}\,\text{A4}'[t]}\text{Csch}[\text{A4}[t]]\left(\frac{1}{2}e^{-\text{A4}[t]}(-2+\text{Q1})\,\text{Q1}\,\text{YZ7}[z,t]\,\text{A4}'[t]^2-\right.$$

$$\frac{1}{4}\,\mathrm{e}^{-3\,\mathrm{A4}[t]}\,\mathrm{Q1}^2\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]^2-\mathrm{Q1}\,\mathrm{Sinh}[\mathrm{A4}[t]]\,\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]+\mathrm{e}^{\mathrm{A4}[t]}\,\mathrm{YZ7}^{(0,2)}[z,t]+$$

$$\mathrm{e}^{\mathrm{A4}[t]}\left(\mathrm{YZ7}[z,t]\left(-9+M^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+\left(-72\,\mathrm{Tan}[z]-36\,\mathrm{Tan}[z]^3\right)\mathrm{YZ7}^{(1,0)}[z,t]\right)-$$

$$36\,\mathrm{e}^{\mathrm{A4}[t]}\,\mathrm{Tan}[z]^2\,\mathrm{YZ7}^{(2,0)}[z,t]\Big),$$

$$\mathrm{YZ5}[z,t]==\frac{1}{\left(-1+\mathrm{e}^{2\,\mathrm{A4}[t]}\right)^2 M\,\mathrm{Q1}\,\sqrt{\mathrm{Sin}[z]}\,\mathrm{A4}'[t]^2}$$

$$\left(\frac{1}{3}\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\mathrm{Cos}[z]\left(-\frac{1}{4}\left(1+\mathrm{e}^{4\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]^2-\frac{1}{2}\left(-\frac{1}{2}+\mathrm{e}^{2\,\mathrm{A4}[t]}-\frac{1}{2}\,\mathrm{e}^{4\,\mathrm{A4}[t]}\right)\mathrm{Q1}\right.\right.$$

$$\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]\,\mathrm{A4}^{(3)}[t]-\frac{1}{2}\,\mathrm{A4}'[t]\left(\mathrm{e}^{4\,\mathrm{A4}[t]}\left(-9+M^2+\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)\mathrm{YZ7}^{(0,1)}[z,t]+\right.$$

$$\mathrm{Q1}\,\mathrm{A4}'[t]^2\left(-\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+\mathrm{Q1}\,\mathrm{Cosh}[2\,\mathrm{A4}[t]]\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+\right.$$

$$\left.\left.\left(1+\frac{3\,\mathrm{Q1}}{4}\right)\mathrm{YZ7}^{(0,1)}[z,t]-\frac{1}{4}\,\mathrm{e}^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ7}^{(0,1)}[z,t]\right)\right)+$$

$$\left(\frac{1}{2}\left(1+\mathrm{e}^{4\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{A4}'[t]^2+\mathrm{e}^{2\,\mathrm{A4}[t]}\left((1-\mathrm{Q1})\,\mathrm{A4}'[t]^2-\frac{\mathrm{A4}''[t]}{2}\right)+\frac{1}{2}\,\mathrm{e}^{4\,\mathrm{A4}[t]}\,\mathrm{A4}''[t]\right)$$

$$\mathrm{YZ7}^{(0,2)}[z,t]+\frac{1}{2}\left(\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]}\right)\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,3)}[z,t]+$$

$$\mathrm{A4}''[t]\left(\frac{1}{8}\,\mathrm{Q1}\,\mathrm{A4}'[t]\left((-4-3\,\mathrm{Q1})\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+\mathrm{e}^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+\right.\right.$$

$$\left.2\,\mathrm{YZ7}^{(0,1)}[z,t]\right)+\mathrm{e}^{4\,\mathrm{A4}[t]}\left(\frac{1}{2}\,\mathrm{YZ7}[z,t]\left(-9+M^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+\right.$$

$$\left.\left.\frac{1}{4}\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]+18\left(-2\,\mathrm{Tan}[z]-\mathrm{Tan}[z]^3\right)\mathrm{YZ7}^{(1,0)}[z,t]\right)\right)+\mathrm{e}^{2\,\mathrm{A4}[t]}$$

$$\left(\frac{1}{2}\,\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]^2+\mathrm{A4}''[t]\left(-\frac{1}{2}\,\mathrm{YZ7}[z,t]\left(-9+M^2+\left(-\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)-\right.\right.$$

$$\left.\frac{1}{2}\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]+18\left(2\,\mathrm{Tan}[z]+\mathrm{Tan}[z]^3\right)\mathrm{YZ7}^{(1,0)}[z,t]\right)+$$

$$\mathrm{A4}'[t]\left(\frac{1}{2}\left(-9+M^2+\left(\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)\mathrm{YZ7}^{(0,1)}[z,t]+\mathrm{A4}'[t]\right.$$

$$\left.\left.\left(\mathrm{YZ7}[z,t]\left(-9+M^2-\mathrm{Q1}\,\mathrm{A4}'[t]^2\right)+36\left(-2\,\mathrm{Tan}[z]-\mathrm{Tan}[z]^3\right)\mathrm{YZ7}^{(1,0)}[z,t]\right)\right)\right)-$$

$$18\left(\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]}\right)\mathrm{Tan}[z]\left(2+\mathrm{Tan}[z]^2\right)\mathrm{A4}'[t]\,\mathrm{YZ7}^{(1,1)}[z,t]-$$

$$36\,\mathrm{Tan}[z]^2\left(\mathrm{e}^{2\,\mathrm{A4}[t]}\left(\mathrm{A4}'[t]^2-\frac{\mathrm{A4}''[t]}{2}\right)+\frac{1}{2}\,\mathrm{e}^{4\,\mathrm{A4}[t]}\,\mathrm{A4}''[t]\right)\mathrm{YZ7}^{(2,0)}[z,t]-$$

$$\left.18\left(\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]}\right)\mathrm{Tan}[z]^2\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(2,1)}[z,t]\right)dz-$$

$$\frac{1}{12}\,\mathrm{Q1}\left(-1-\frac{1}{2}\,\mathrm{e}^{-3\,\mathrm{A4}[t]}\,\mathrm{Csch}[\mathrm{A4}[t]]-\frac{1}{2}\,\mathrm{e}^{\mathrm{A4}[t]}\,\mathrm{Csch}[\mathrm{A4}[t]]+\right.$$

$$\left.\left.\mathrm{e}^{2\,\mathrm{A4}[t]}\left(1+\mathrm{e}^{-3\,\mathrm{A4}[t]}\,\mathrm{Csch}[\mathrm{A4}[t]]\right)\right)\right)$$

$$\Big(\int \frac{1}{\sqrt{\mathrm{Sin}[z]}} \mathrm{Cos}[z] \; \Big(\frac{1}{2} \mathrm{Q1} \; \big(\mathrm{YZ7}[z, t] \; \mathrm{A4}''[t] + \mathrm{A4}'[t] \; \big((-2 - \mathrm{Q1}) \; \mathrm{YZ7}[z, t] \; \mathrm{A4}'[t] +$$

$$\mathrm{Q1} \; \mathrm{Cosh}[2 \; \mathrm{A4}[t]] \; \mathrm{YZ7}[z, t] \; \mathrm{A4}'[t] + 2 \; \mathrm{YZ7}^{(0,1)}[z, t]\big)\big) + \mathrm{e}^{2 \, \mathrm{A4}[t]} \; \mathrm{YZ7}^{(0,2)}[z, t] +$$

$$\mathrm{e}^{2 \, \mathrm{A4}[t]} \; \Big(\big(-9 + \mathrm{M}^2\big) \; \mathrm{YZ7}[z, t] - \frac{1}{2} \mathrm{Q1} \; \mathrm{YZ7}[z, t] \; \mathrm{A4}''[t] - \mathrm{Q1} \; \mathrm{A4}'[t] \; \mathrm{YZ7}^{(0,1)}[z, t] +$$

$$\big(-72 \; \mathrm{Tan}[z] - 36 \; \mathrm{Tan}[z]^3\big) \; \mathrm{YZ7}^{(1,0)}[z, t]\Big) - 36 \; \mathrm{e}^{2 \, \mathrm{A4}[t]} \; \mathrm{Tan}[z]^2 \; \mathrm{YZ7}^{(2,0)}[z, t]\Big) \, dz\Big)$$

$$\mathrm{A4}'[t]^2 - \frac{3}{8} \; \mathrm{e}^{-\frac{1}{4} \mathrm{Q1} \left(\mathrm{e}^{-2 \, \mathrm{A4}[t]} + 2 \, \mathrm{A4}[t]\right)} \; \big(-1 + \mathrm{e}^{2 \, \mathrm{A4}[t]}\big)^2 \; \mathrm{Cos}[\mathrm{M} \; t]$$

$$\Big(\int \frac{1}{\mathrm{A4}'[t]^3} \mathrm{e}^{\frac{1}{4} \mathrm{e}^{-2 \, \mathrm{A4}[t]} \mathrm{Q1} + \frac{1}{2} (-14 + \mathrm{Q1}) \, \mathrm{A4}[t]} \; \mathrm{Cos}[\mathrm{M} \; t] \; \mathrm{Csch}[\mathrm{A4}[t]]^3$$

$$\Big(-\frac{2}{9} \; \mathrm{e}^{4 \, \mathrm{A4}[t]} \int \frac{1}{\sqrt{\mathrm{Sin}[z]}} \mathrm{Sec}[z]^2 \; \Big(-\frac{3}{4} \; \Big(-\frac{1}{3} + \mathrm{e}^{2 \, \mathrm{A4}[t]} - \mathrm{e}^{4 \, \mathrm{A4}[t]} + \frac{1}{3} \; \mathrm{e}^{6 \, \mathrm{A4}[t]}\Big) \; \mathrm{Q1} \; \mathrm{Cos}[z]^3$$

$$\mathrm{YZ7}[z, t] \; \mathrm{A4}''[t]^3 - \frac{3}{8} \; \Big(-\frac{1}{3} + \mathrm{e}^{2 \, \mathrm{A4}[t]} - \mathrm{e}^{4 \, \mathrm{A4}[t]} + \frac{1}{3} \; \mathrm{e}^{6 \, \mathrm{A4}[t]}\Big) \; \mathrm{Q1} \; \mathrm{Cos}[z]^3 \; \mathrm{YZ7}[z, t]$$

$$\mathrm{A4}'[t]^2 \; \mathrm{A4}^{(4)}[t] + \frac{1}{4} \; \mathrm{Cos}[z]^3 \; \Big(\mathrm{A4}'[t]^2 \; \Big(\frac{1}{4} \; \mathrm{Q1} \; \big(-4 - 4 \; \mathrm{Q1} + \mathrm{e}^{-2 \, \mathrm{A4}[t]} \; \mathrm{Q1}\big) \; \mathrm{A4}'[t]^2 +$$

$$\mathrm{e}^{6 \, \mathrm{A4}[t]} \; \Big(-9 + \mathrm{M}^2 + \frac{1}{4} \; \mathrm{Q1}^2 \; \mathrm{A4}'[t]^2\Big) + \mathrm{e}^{4 \, \mathrm{A4}[t]} \; \big(18 - 2 \; \mathrm{M}^2 + \big(4 - \mathrm{Q1} - \mathrm{Q1}^2\big)$$

$$\mathrm{A4}'[t]^2\big) + \mathrm{e}^{2 \, \mathrm{A4}[t]} \; \Big(-9 + \mathrm{M}^2 + \Big(4 + 2 \; \mathrm{Q1} + \frac{3 \; \mathrm{Q1}^2}{2}\Big) \; \mathrm{A4}'[t]^2\Big)\Big) -$$

$$\frac{3}{2} \; \Big(\mathrm{e}^{4 \, \mathrm{A4}[t]} \; \Big(-\frac{4}{3} - \mathrm{Q1}\Big) + \frac{1}{3} \; \big(-1 + \mathrm{e}^{6 \, \mathrm{A4}[t]}\big) \; \mathrm{Q1} + \mathrm{e}^{2 \, \mathrm{A4}[t]} \; \Big(\frac{4}{3} + \mathrm{Q1}\Big)\Big)$$

$$\mathrm{A4}'[t]^2 \; \mathrm{A4}''[t] + 2 \; \big(\mathrm{e}^{2 \, \mathrm{A4}[t]} - 2 \; \mathrm{e}^{4 \, \mathrm{A4}[t]} + \mathrm{e}^{6 \, \mathrm{A4}[t]}\big) \; \mathrm{A4}''[t]^2\Big) \; \mathrm{YZ7}^{(0,2)}[z, t] -$$

$$\frac{3}{4} \; \mathrm{Cos}[z]^3 \; \mathrm{A4}'[t] \; \Big(\Big(\mathrm{e}^{4 \, \mathrm{A4}[t]} \; \Big(\frac{4}{3} - \mathrm{Q1}\Big) + \mathrm{e}^{2 \, \mathrm{A4}[t]} \; \Big(-\frac{4}{3} + \mathrm{Q1}\Big) + \frac{1}{3} \; \big(-1 + \mathrm{e}^{6 \, \mathrm{A4}[t]}\big) \; \mathrm{Q1}\Big)$$

$$\mathrm{A4}'[t]^2 + \frac{2}{3} \; \big(\mathrm{e}^{2 \, \mathrm{A4}[t]} - 2 \; \mathrm{e}^{4 \, \mathrm{A4}[t]} + \mathrm{e}^{6 \, \mathrm{A4}[t]}\big) \; \mathrm{A4}''[t]\Big) \; \mathrm{YZ7}^{(0,3)}[z, t] +$$

$$\frac{1}{4} \; \big(\mathrm{e}^{2 \, \mathrm{A4}[t]} - 2 \; \mathrm{e}^{4 \, \mathrm{A4}[t]} + \mathrm{e}^{6 \, \mathrm{A4}[t]}\big) \; \mathrm{Cos}[z]^3 \; \mathrm{A4}'[t]^2 \; \mathrm{YZ7}^{(0,4)}[z, t] + \mathrm{A4}''[t]^2$$

$$\Big(-\frac{1}{4} \; \mathrm{Q1} \; \mathrm{Cos}[z]^3 \; \mathrm{A4}'[t] \; \mathrm{YZ7}^{(0,1)}[z, t] + \mathrm{e}^{6 \, \mathrm{A4}[t]} \; \Big(\frac{1}{2} \; (-3 + \mathrm{M}) \; (3 + \mathrm{M}) \; \mathrm{Cos}[z]^3$$

$$\mathrm{YZ7}[z, t] + \frac{1}{4} \; \mathrm{Q1} \; \mathrm{Cos}[z]^3 \; \mathrm{A4}'[t] \; \mathrm{YZ7}^{(0,1)}[z, t] - 18 \; \big(1 + \mathrm{Cos}[z]^2\big)$$

$$\mathrm{Sin}[z] \; \mathrm{YZ7}^{(1,0)}[z, t]\Big) + \mathrm{e}^{2 \, \mathrm{A4}[t]} \; \Big(\frac{1}{2} \; (-3 + \mathrm{M}) \; (3 + \mathrm{M}) \; \mathrm{Cos}[z]^3 \; \mathrm{YZ7}[z, t] +$$

$$\frac{3}{4} \; \mathrm{Q1} \; \mathrm{Cos}[z]^3 \; \mathrm{A4}'[t] \; \mathrm{YZ7}^{(0,1)}[z, t] - 18 \; \big(1 + \mathrm{Cos}[z]^2\big) \; \mathrm{Sin}[z] \; \mathrm{YZ7}^{(1,0)}[$$

$$z, t]\Big) + \mathrm{e}^{4 \, \mathrm{A4}[t]} \; \Big(\big(9 - \mathrm{M}^2\big) \; \mathrm{Cos}[z]^3 \; \mathrm{YZ7}[z, t] - \frac{3}{4} \; \mathrm{Q1} \; \mathrm{Cos}[z]^3 \; \mathrm{A4}'[t]$$

$$\mathrm{YZ7}^{(0,1)}[z, t] + 36 \; \big(1 + \mathrm{Cos}[z]^2\big) \; \mathrm{Sin}[z] \; \mathrm{YZ7}^{(1,0)}[z, t]\Big)\Big) + \mathrm{A4}'[t]^3$$

$$\Big(\frac{1}{4}\left(-3+\mathrm{e}^{-2\,A4[t]}\right) Q1^2\,\mathrm{Cos}[z]^3\,A4'[t]^2\left(YZ7[z,t]\,A4'[t]-YZ7^{(0,1)}[z,t]\right)+$$

$$\mathrm{e}^{4\,A4[t]}\Big(-\mathrm{Cos}[z]^3\left(-9+M^2+\left(-Q1-\frac{Q1^2}{4}\right)A4'[t]^2\right)YZ7^{(0,1)}[z,t]+$$

$$A4'[t]\Big(\mathrm{Cos}[z]^3\,YZ7[z,t]\left(-9+M^2+\left(-Q1-\frac{Q1^2}{4}\right)A4'[t]^2\right)-$$

$$36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\Big)\Big)+$$

$$\mathrm{e}^{2\,A4[t]}\Big(\mathrm{Cos}[z]^3\left(-9+M^2+\left(-Q1-\frac{3\,Q1^2}{4}\right)A4'[t]^2\right)YZ7^{(0,1)}[z,t]+$$

$$A4'[t]\Big(\mathrm{Cos}[z]^3\,YZ7[z,t]\left(-9+M^2+\left(-Q1+\frac{3\,Q1^2}{4}\right)A4'[t]^2\right)-$$

$$36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\Big)\Big)\Big)-\frac{1}{2}\,A4'[t]$$

$$A4''[t]\Big(\mathrm{Cos}[z]^3\Big(\mathrm{e}^{6\,A4[t]}\left(-9+M^2-\frac{1}{4}\,Q1^2\,A4'[t]^2\right)YZ7^{(0,1)}[z,t]+$$

$$\frac{3}{4}\,Q1\,A4'[t]^2\Big(-3\,Q1\,YZ7[z,t]\,A4'[t]+\mathrm{e}^{-2\,A4[t]}\,Q1\Big(YZ7[z,t]\,A4'[t]-$$

$$\frac{1}{3}\,YZ7^{(0,1)}[z,t]\Big)+\frac{4}{3}\,(1+Q1)\,YZ7^{(0,1)}[z,t]\Big)\Big)+$$

$$\mathrm{e}^{4\,A4[t]}\Big(-2\,\mathrm{Cos}[z]^3\left(-9+M^2-\frac{1}{2}\left(Q1+Q1^2\right)A4'[t]^2\right)YZ7^{(0,1)}[z,t]-$$

$$A4'[t]\Big(\mathrm{Cos}[z]^3\,YZ7[z,t]\left(-9+M^2+3\left(Q1+\frac{Q1^2}{4}\right)A4'[t]^2\right)-$$

$$36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\Big)\Big)+$$

$$\mathrm{e}^{2\,A4[t]}\Big(\mathrm{Cos}[z]^3\left(-9+M^2+\left(-2\,Q1-\frac{3\,Q1^2}{2}\right)A4'[t]^2\right)YZ7^{(0,1)}[z,t]+$$

$$A4'[t]\Big(\mathrm{Cos}[z]^3\,YZ7[z,t]\left(-9+M^2+3\left(Q1+\frac{3\,Q1^2}{4}\right)A4'[t]^2\right)-$$

$$36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\Big)\Big)\Big)-36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]$$

$$A4'[t]\Big(\left(\mathrm{e}^{2\,A4[t]}-\mathrm{e}^{4\,A4[t]}\right)A4'[t]^2+\left(-\frac{1}{2}\,\mathrm{e}^{2\,A4[t]}+\mathrm{e}^{4\,A4[t]}-\frac{1}{2}\,\mathrm{e}^{6\,A4[t]}\right)$$

$$A4''[t]\Big)\,YZ7^{(1,1)}[z,t]-9\left(\mathrm{e}^{2\,A4[t]}-2\,\mathrm{e}^{4\,A4[t]}+\mathrm{e}^{6\,A4[t]}\right)$$

$$\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,A4'[t]^2\,YZ7^{(1,2)}[z,t]-36\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2$$

$$\Big(\left(\mathrm{e}^{2\,A4[t]}+\mathrm{e}^{4\,A4[t]}\right)A4'[t]^4-\frac{1}{2}\left(\mathrm{e}^{2\,A4[t]}-\mathrm{e}^{4\,A4[t]}\right)A4'[t]^2\,A4''[t]+$$

$$\left(\frac{1}{2}\,\mathrm{e}^{2\,A4[t]}-\mathrm{e}^{4\,A4[t]}+\frac{1}{2}\,\mathrm{e}^{6\,A4[t]}\right)A4''[t]^2\Big)\,YZ7^{(2,0)}[z,t]-\frac{1}{4}\,A4'[t]\,A4^{(3)}[t]$$

$$\Big(-\frac{9}{2}\left(-\frac{1}{3}+e^{2\,A4[t]}-e^{4\,A4[t]}+\frac{1}{3}\,e^{6\,A4[t]}\right)Q1\,\mathrm{Cos}[z]^3\,YZ7[z,t]\,A4''[t]-\frac{1}{4}\,Q1$$

$$\mathrm{Cos}[z]^3\,A4'[t]\,\big(e^{-2\,A4[t]}\,Q1\,YZ7[z,t]\,A4'[t]-4\,(1+Q1)\,YZ7[z,t]\,A4'[t]+$$

$$4\,YZ7^{(0,1)}[z,t]\big)+\left(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\right)\mathrm{Cos}[z]^3\,YZ7^{(0,2)}[z,t]+$$

$$e^{6\,A4[t]}\,\Big(\mathrm{Cos}[z]^3\,YZ7[z,t]\left(-9+M^2-\frac{1}{4}\,Q1^2\,A4'[t]^2\right)+Q1\,\mathrm{Cos}[z]^3$$

$$A4'[t]\,YZ7^{(0,1)}[z,t]-36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\Big)+$$

$$e^{2\,A4[t]}\,\Big(\mathrm{Cos}[z]^3\,YZ7[z,t]\left(-9+M^2+\left(-2\,Q1-\frac{3\,Q1^2}{2}\right)A4'[t]^2\right)+3\,Q1\,\mathrm{Cos}[z]^3$$

$$A4'[t]\,YZ7^{(0,1)}[z,t]-36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\Big)+$$

$$e^{4\,A4[t]}\,\Big(-2\,\mathrm{Cos}[z]^3\,YZ7[z,t]\left(-9+M^2-\frac{1}{2}\left(Q1+Q1^2\right)A4'[t]^2\right)-3\,Q1\,\mathrm{Cos}[z]^3$$

$$A4'[t]\,YZ7^{(0,1)}[z,t]+72\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\Big)-$$

$$36\left(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\right)\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\,YZ7^{(2,0)}[z,t]\Big)-$$

$$36\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\,A4'[t]\,\Big(\left(e^{2\,A4[t]}-e^{4\,A4[t]}\right)A4'[t]^2+$$

$$\left(-\frac{1}{2}\,e^{2\,A4[t]}+e^{4\,A4[t]}-\frac{1}{2}\,e^{6\,A4[t]}\right)A4''[t]\Big)\,YZ7^{(2,1)}[z,t]-$$

$$9\left(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\right)\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\,A4'[t]^2\,YZ7^{(2,2)}[z,t]\Big)\,dz+$$

$$A4'[t]^2\,\Big(\frac{1}{9}\left(e^{4\,A4[t]}-2\,e^{6\,A4[t]}+e^{8\,A4[t]}\right)Q1\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\,\mathrm{Sec}[z]^2$$

$$\Big(-\frac{1}{2}\,Q1\,\mathrm{Cos}[z]^3\,(-1+\mathrm{Cosh}[2\,A4[t]])\,YZ7[z,t]\,A4''[t]^2+$$

$$\frac{1}{2}\,Q1\,\mathrm{Cos}[z]^3\,(-1+\mathrm{Cosh}[2\,A4[t]])\,YZ7[z,t]\,A4'[t]\,A4^{(3)}[t]+$$

$$\mathrm{Cos}[z]^3\left((1-Q1+Q1\,\mathrm{Cosh}[2\,A4[t]])\,A4'[t]^2+\frac{1}{2}\left(-1+e^{2\,A4[t]}\right)A4''[t]\right)$$

$$YZ7^{(0,2)}[z,t]-\frac{1}{2}\left(-1+e^{2\,A4[t]}\right)\mathrm{Cos}[z]^3\,A4'[t]\,YZ7^{(0,3)}[z,t]+A4'[t]$$

$$\Big(-\frac{1}{4}\,e^{-4\,A4[t]}\,Q1^2\,\mathrm{Cos}[z]^3\,A4'[t]^2\left(YZ7[z,t]\,A4'[t]-\frac{1}{2}\,YZ7^{(0,1)}[z,t]\right)-$$

$$\frac{1}{2}\,e^{2\,A4[t]}\,\mathrm{Cos}[z]^3\left(-9+M^2+\frac{1}{4}\,Q1^2\,A4'[t]^2\right)YZ7^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,\mathrm{Cos}[z]^3\left(-9+M^2+\left(Q1+\frac{3\,Q1^2}{4}\right)A4'[t]^2\right)YZ7^{(0,1)}[z,t]+\frac{1}{2}\,e^{-2\,A4[t]}\,Q1$$

$$\mathrm{Cos}[z]^3\,A4'[t]^2\left(Q1\,YZ7[z,t]\,A4'[t]+\left(-1-\frac{3\,Q1}{4}\right)YZ7^{(0,1)}[z,t]\right)+$$

$$\mathrm{A4}'[\mathrm{t}]\ \Big(\mathrm{Cos}[\mathrm{z}]^3\ \mathrm{YZ7}[\mathrm{z},\mathrm{t}]\ \Big(-9+\mathrm{M}^2+\Big(-\mathrm{Q1}-\frac{\mathrm{Q1}^2}{4}\Big)\ \mathrm{A4}'[\mathrm{t}]^2\Big)-$$

$$36\ \big(1+\mathrm{Cos}[\mathrm{z}]^2\big)\ \mathrm{Sin}[\mathrm{z}]\ \mathrm{YZ7}^{(1,0)}[\mathrm{z},\mathrm{t}]\Big)\Big)+$$

$$\mathrm{A4}''[\mathrm{t}]\ \Big(\frac{1}{8}\ \mathbb{e}^{-4\,\mathrm{A4}[\mathrm{t}]}\ \mathrm{Q1}^2\ \mathrm{Cos}[\mathrm{z}]^3\ \mathrm{YZ7}[\mathrm{z},\mathrm{t}]\ \mathrm{A4}'[\mathrm{t}]^2-\frac{3}{8}\ \mathbb{e}^{-2\,\mathrm{A4}[\mathrm{t}]}$$

$$\mathrm{Q1}\ \Big(\frac{4}{3}+\mathrm{Q1}\Big)\ \mathrm{Cos}[\mathrm{z}]^3\ \mathrm{YZ7}[\mathrm{z},\mathrm{t}]\ \mathrm{A4}'[\mathrm{t}]^2-\frac{1}{2}\ \mathrm{Cos}[\mathrm{z}]^3\ \mathrm{YZ7}[\mathrm{z},\mathrm{t}]$$

$$\Big(-9+\mathrm{M}^2+\Big(-\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{4}\Big)\ \mathrm{A4}'[\mathrm{t}]^2\Big)-\frac{1}{2}\ \mathrm{Q1}\,\mathrm{Cos}[\mathrm{z}]^3\ \mathrm{A4}'[\mathrm{t}]\ \mathrm{YZ7}^{(0,1)}[\mathrm{z},\mathrm{t}]+$$

$$\frac{1}{2}\ \mathrm{Q1}\,\mathrm{Cos}[\mathrm{z}]^3\ \mathrm{Cosh}[2\,\mathrm{A4}[\mathrm{t}]]\ \mathrm{A4}'[\mathrm{t}]\ \mathrm{YZ7}^{(0,1)}[\mathrm{z},\mathrm{t}]+18\ \big(1+\mathrm{Cos}[\mathrm{z}]^2\big)$$

$$\mathrm{Sin}[\mathrm{z}]\ \mathrm{YZ7}^{(1,0)}[\mathrm{z},\mathrm{t}]+\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}\ \Big(\frac{1}{2}\ \mathrm{Cos}[\mathrm{z}]^3\ \mathrm{YZ7}[\mathrm{z},\mathrm{t}]\ \Big(-9+\mathrm{M}^2-$$

$$\frac{1}{4}\ \mathrm{Q1}^2\ \mathrm{A4}'[\mathrm{t}]^2\Big)-18\ \big(1+\mathrm{Cos}[\mathrm{z}]^2\big)\ \mathrm{Sin}[\mathrm{z}]\ \mathrm{YZ7}^{(1,0)}[\mathrm{z},\mathrm{t}]\Big)\Big)+$$

$$18\ \big(-1+\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}\big)\ \big(1+\mathrm{Cos}[\mathrm{z}]^2\big)\ \mathrm{Sin}[\mathrm{z}]\ \mathrm{A4}'[\mathrm{t}]\ \mathrm{YZ7}^{(1,1)}[\mathrm{z},\mathrm{t}]-$$

$$36\,\mathrm{Cos}[\mathrm{z}]\ \mathrm{Sin}[\mathrm{z}]^2\ \Big(\mathrm{A4}'[\mathrm{t}]^2+\frac{1}{2}\ \big(-1+\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}\big)\ \mathrm{A4}''[\mathrm{t}]\Big)\ \mathrm{YZ7}^{(2,0)}[\mathrm{z},\mathrm{t}]+$$

$$18\ \big(-1+\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}\big)\ \mathrm{Cos}[\mathrm{z}]\ \mathrm{Sin}[\mathrm{z}]^2\ \mathrm{A4}'[\mathrm{t}]\ \mathrm{YZ7}^{(2,1)}[\mathrm{z},\mathrm{t}]\Big)\,\mathrm{d}\mathrm{z}+\frac{1}{3}$$

$$\Big(\Big(\int\frac{1}{\sqrt{\mathrm{Sin}[\mathrm{z}]}}\mathrm{Cos}[\mathrm{z}]\ \Big(\frac{1}{2}\ \mathrm{Q1}\ \big(\mathrm{YZ7}[\mathrm{z},\mathrm{t}]\ \mathrm{A4}''[\mathrm{t}]+\mathrm{A4}'[\mathrm{t}]\ \big((-2-\mathrm{Q1})\ \mathrm{YZ7}[\mathrm{z},\mathrm{t}]$$

$$\mathrm{A4}'[\mathrm{t}]+\mathrm{Q1}\,\mathrm{Cosh}[2\,\mathrm{A4}[\mathrm{t}]]\ \mathrm{YZ7}[\mathrm{z},\mathrm{t}]\ \mathrm{A4}'[\mathrm{t}]+2\,\mathrm{YZ7}^{(0,1)}[\mathrm{z},\mathrm{t}]\big)\big)+$$

$$\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}\ \mathrm{YZ7}^{(0,2)}[\mathrm{z},\mathrm{t}]+\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}\ \Big(\big(-9+\mathrm{M}^2\big)\ \mathrm{YZ7}[\mathrm{z},\mathrm{t}]-\frac{1}{2}\ \mathrm{Q1}\,\mathrm{YZ7}[$$

$$\mathrm{z},\mathrm{t}]\ \mathrm{A4}''[\mathrm{t}]-\mathrm{Q1}\,\mathrm{A4}'[\mathrm{t}]\ \mathrm{YZ7}^{(0,1)}[\mathrm{z},\mathrm{t}]+\big(-72\,\mathrm{Tan}[\mathrm{z}]-36\,\mathrm{Tan}[\mathrm{z}]^3\big)$$

$$\mathrm{YZ7}^{(1,0)}[\mathrm{z},\mathrm{t}]\Big)-36\,\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}\ \mathrm{Tan}[\mathrm{z}]^2\ \mathrm{YZ7}^{(2,0)}[\mathrm{z},\mathrm{t}]\Big)\,\mathrm{d}\mathrm{z}\Big)$$

$$\Big(\frac{1}{6}\ \Big(\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}\ (-1+\mathrm{Q1})-\frac{\mathrm{Q1}}{4}\Big)\ \mathrm{Q1}\,\mathrm{A4}'[\mathrm{t}]^2+\frac{1}{6}\ \mathbb{e}^{8\,\mathrm{A4}[\mathrm{t}]}\ \Big(-\mathrm{M}^2-\frac{1}{4}\ \mathrm{Q1}^2\ \mathrm{A4}'[\mathrm{t}]^2\Big)+$$

$$\mathbb{e}^{4\,\mathrm{A4}[\mathrm{t}]}\ \Big(-\frac{\mathrm{M}^2}{6}+\Big(\frac{\mathrm{Q1}}{3}-\frac{\mathrm{Q1}^2}{4}\Big)\ \mathrm{A4}'[\mathrm{t}]^2\Big)+\frac{1}{3}\ \mathbb{e}^{6\,\mathrm{A4}[\mathrm{t}]}\ \Big(\mathrm{M}^2+\frac{1}{2}\ \big(-\mathrm{Q1}+\mathrm{Q1}^2\big)\ \mathrm{A4}'[\mathrm{t}]^2\Big)+$$

$$\frac{1}{12}\ \big(\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}-3\,\mathbb{e}^{4\,\mathrm{A4}[\mathrm{t}]}+3\,\mathbb{e}^{6\,\mathrm{A4}[\mathrm{t}]}-\mathbb{e}^{8\,\mathrm{A4}[\mathrm{t}]}\big)\ \mathrm{Q1}\,\mathrm{A4}''[\mathrm{t}]\Big)+\sqrt{\mathrm{Sin}[\mathrm{z}]}$$

$$\Big(\frac{1}{2}\ \big(\mathbb{e}^{4\,\mathrm{A4}[\mathrm{t}]}-3\,\mathbb{e}^{6\,\mathrm{A4}[\mathrm{t}]}+3\,\mathbb{e}^{8\,\mathrm{A4}[\mathrm{t}]}-\mathbb{e}^{10\,\mathrm{A4}[\mathrm{t}]}\big)\ \mathrm{Q1}\,\mathrm{YZ7}[\mathrm{z},\mathrm{t}]\ \mathrm{A4}''[\mathrm{t}]-$$

$$\frac{3}{4}\ \mathrm{Q1}\,\mathrm{A4}'[\mathrm{t}]\ \Big(\mathbb{e}^{2\,\mathrm{A4}[\mathrm{t}]}\ \mathrm{Q1}\,\mathrm{YZ7}[\mathrm{z},\mathrm{t}]\ \mathrm{A4}'[\mathrm{t}]+$$

$$4\,\mathbb{e}^{4\,\mathrm{A4}[\mathrm{t}]}\ \Big(\Big(\frac{1}{3}-\mathrm{Q1}\Big)\ \mathrm{YZ7}[\mathrm{z},\mathrm{t}]\ \mathrm{A4}'[\mathrm{t}]-\frac{1}{3}\ \mathrm{YZ7}^{(0,1)}[\mathrm{z},\mathrm{t}]\Big)\Big)+\mathbb{e}^{6\,\mathrm{A4}[\mathrm{t}]}$$

$$\left(\mathrm{YZ7}[z,t]\left(-9+M^2+\left(2\,\mathrm{Q1}-\frac{9\,\mathrm{Q1}^2}{2}\right)\mathrm{A4}'[t]^2\right)-3\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]\right)+$$

$$e^{10\,\mathrm{A4}[t]}\left(\mathrm{YZ7}[z,t]\left(-9+M^2-\frac{3}{4}\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)-\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]\right)+e^{8\,\mathrm{A4}[t]}$$

$$\left(-2\,\mathrm{YZ7}[z,t]\left(-9+M^2+\frac{1}{2}\left(\mathrm{Q1}-3\,\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+3\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]\right)+$$

$$\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{YZ7}^{(0,2)}[z,t]\Big)-396$$

$$\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^3\,\mathrm{Sin}[z]^{7/2}\,\mathrm{YZ7}^{(1,0)}[z,t]-216$$

$$\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^5\,\mathrm{Sin}[z]^{11/2}\,\mathrm{YZ7}^{(1,0)}[z,t]+2$$

$$\mathrm{Sec}[z]\,\mathrm{Sin}[z]^{3/2}\,\Big(\Big(\Big(-\frac{3}{4}\mathrm{Q1}\left(4\,e^{4\,\mathrm{A4}[t]}\left(\frac{1}{3}-\mathrm{Q1}\right)+e^{2\,\mathrm{A4}[t]}\,\mathrm{Q1}\right)\mathrm{A4}'[t]^2+$$

$$e^{10\,\mathrm{A4}[t]}\left(-117+M^2-\frac{3}{4}\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+e^{6\,\mathrm{A4}[t]}\Big(-117+M^2+$$

$$\left(2\,\mathrm{Q1}-\frac{9\,\mathrm{Q1}^2}{2}\right)\mathrm{A4}'[t]^2\Big)+e^{8\,\mathrm{A4}[t]}\left(234-2\,M^2+\left(-\mathrm{Q1}+3\,\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+$$

$$\frac{1}{2}\left(e^{4\,\mathrm{A4}[t]}-3\,e^{6\,\mathrm{A4}[t]}+3\,e^{8\,\mathrm{A4}[t]}-e^{10\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{A4}''[t]\Big)\,\mathrm{YZ7}^{(1,0)}[z,t]+$$

$$\left(e^{4\,\mathrm{A4}[t]}-3\,e^{6\,\mathrm{A4}[t]}+3\,e^{8\,\mathrm{A4}[t]}-e^{10\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(1,1)}[z,t]\Big)+2$$

$$\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]\,\mathrm{Sin}[z]^{3/2}\,\mathrm{YZ7}^{(1,2)}[z,t]-324$$

$$\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^2\,\mathrm{Sin}[z]^{5/2}\,\mathrm{YZ7}^{(2,0)}[z,t]-216$$

$$\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^4\,\mathrm{Sin}[z]^{9/2}\,\mathrm{YZ7}^{(2,0)}[z,t]-72$$

$$\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^3\,\mathrm{Sin}[z]^{7/2}\,\mathrm{YZ7}^{(3,0)}[z,t]\Big)\Big)\,dt\Big)$$

$$\mathrm{A4}'[t]^2-\frac{3}{8}\,e^{-\frac{1}{4}\mathrm{Q1}\left(e^{-2\,\mathrm{A4}[t]}+2\,\mathrm{A4}[t]\right)}\left(-1+e^{2\,\mathrm{A4}[t]}\right)^2\Big(\int\frac{1}{\mathrm{A4}'[t]^3}$$

$$e^{\frac{1}{4}e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}+\frac{1}{2}(-14+\mathrm{Q1})\,\mathrm{A4}[t]}$$

$$\mathrm{Csch}[\mathrm{A4}[t]]^3$$

$$\mathrm{Sin}[M\,t]$$

$$\Big(-\frac{2}{9}\,e^{4\,\mathrm{A4}[t]}\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\,\mathrm{Sec}[z]^2$$

$$\Big(-\frac{3}{4}\left(-\frac{1}{3}+e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}+\frac{1}{3}e^{6\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]^3-$$

$$\frac{3}{8}\left(-\frac{1}{3}+e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}+\frac{1}{3}e^{6\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]^2\,\mathrm{A4}^{(4)}[t]+$$

$$\frac{1}{4}\mathrm{Cos}[z]^3\,\Big(\mathrm{A4}'[t]^2\,\Big(\frac{1}{4}\mathrm{Q1}\left(-4-4\,\mathrm{Q1}+e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\right)\mathrm{A4}'[t]^2+e^{6\,\mathrm{A4}[t]}$$

$$\left(-9+M^2+\frac{1}{4}\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+e^{4\,\mathrm{A4}[t]}\left(18-2\,M^2+\left(4-\mathrm{Q1}-\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+$$

$$e^{2\,\mathrm{A4}[t]}\left(-9+M^2+\left(4+2\,\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{2}\right)\mathrm{A4}'[t]^2\right)\Big)-$$

$$\frac{3}{2}\ \Big(e^{4\,\mathrm{A4}[t]}\ \Big(-\frac{4}{3}-\mathrm{Q1}\Big)+\frac{1}{3}\ \big(-1+e^{6\,\mathrm{A4}[t]}\big)\ \mathrm{Q1}+e^{2\,\mathrm{A4}[t]}\ \Big(\frac{4}{3}+\mathrm{Q1}\Big)\Big)$$

$$\mathrm{A4}'[t]^2\,\mathrm{A4}''[t]+2\ \big(e^{2\,\mathrm{A4}[t]}-2\,e^{4\,\mathrm{A4}[t]}+e^{6\,\mathrm{A4}[t]}\big)\ \mathrm{A4}''[t]^2\Big)\ \mathrm{YZ7}^{(0,2)}[z,t]-$$

$$\frac{3}{4}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\ \Big(\Big(e^{4\,\mathrm{A4}[t]}\ \Big(\frac{4}{3}-\mathrm{Q1}\Big)+e^{2\,\mathrm{A4}[t]}\ \Big(-\frac{4}{3}+\mathrm{Q1}\Big)+\frac{1}{3}\ \big(-1+e^{6\,\mathrm{A4}[t]}\big)\ \mathrm{Q1}\Big)$$

$$\mathrm{A4}'[t]^2+\frac{2}{3}\ \big(e^{2\,\mathrm{A4}[t]}-2\,e^{4\,\mathrm{A4}[t]}+e^{6\,\mathrm{A4}[t]}\big)\ \mathrm{A4}''[t]\Big)\ \mathrm{YZ7}^{(0,3)}[z,t]+$$

$$\frac{1}{4}\ \big(e^{2\,\mathrm{A4}[t]}-2\,e^{4\,\mathrm{A4}[t]}+e^{6\,\mathrm{A4}[t]}\big)\ \mathrm{Cos}[z]^3\,\mathrm{A4}'[t]^2\,\mathrm{YZ7}^{(0,4)}[z,t]+$$

$$\mathrm{A4}''[t]^2\ \Big(-\frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]+$$

$$e^{6\,\mathrm{A4}[t]}\ \Big(\frac{1}{2}\ (-3+\mathrm{M})\ (3+\mathrm{M})\ \mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]+\frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]$$

$$\mathrm{YZ7}^{(0,1)}[z,t]-18\ \big(1+\mathrm{Cos}[z]^2\big)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t]\Big)+$$

$$e^{2\,\mathrm{A4}[t]}\ \Big(\frac{1}{2}\ (-3+\mathrm{M})\ (3+\mathrm{M})\ \mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]+\frac{3}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3$$

$$\mathrm{A4}'[t]\ \mathrm{YZ7}^{(0,1)}[z,t]-18\ \big(1+\mathrm{Cos}[z]^2\big)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t]\Big)+$$

$$e^{4\,\mathrm{A4}[t]}\ \Big(\big(9-\mathrm{M}^2\big)\ \mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]-\frac{3}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]$$

$$\mathrm{YZ7}^{(0,1)}[z,t]+36\ \big(1+\mathrm{Cos}[z]^2\big)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t]\Big)\Big)+\mathrm{A4}'[t]^3$$

$$\Big(\frac{1}{4}\ \big(-3+e^{-2\,\mathrm{A4}[t]}\big)\ \mathrm{Q1}^2\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]^2\ \big(\mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]-\mathrm{YZ7}^{(0,1)}[z,t]\big)+$$

$$e^{4\,\mathrm{A4}[t]}\ \Big(-\mathrm{Cos}[z]^3\ \Big(-9+\mathrm{M}^2+\Big(-\mathrm{Q1}-\frac{\mathrm{Q1}^2}{4}\Big)\ \mathrm{A4}'[t]^2\Big)\ \mathrm{YZ7}^{(0,1)}[z,t]+$$

$$\mathrm{A4}'[t]\ \Big(\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\ \Big(-9+\mathrm{M}^2+\Big(-\mathrm{Q1}-\frac{\mathrm{Q1}^2}{4}\Big)\ \mathrm{A4}'[t]^2\Big)-$$

$$36\ \big(1+\mathrm{Cos}[z]^2\big)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t]\Big)\Big)+$$

$$e^{2\,\mathrm{A4}[t]}\ \Big(\mathrm{Cos}[z]^3\ \Big(-9+\mathrm{M}^2+\Big(-\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{4}\Big)\ \mathrm{A4}'[t]^2\Big)\ \mathrm{YZ7}^{(0,1)}[z,t]+$$

$$\mathrm{A4}'[t]\ \Big(\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\ \Big(-9+\mathrm{M}^2+\Big(-\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{4}\Big)\ \mathrm{A4}'[t]^2\Big)-$$

$$36\ \big(1+\mathrm{Cos}[z]^2\big)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t]\Big)\Big)\Big)-\frac{1}{2}\,\mathrm{A4}'[t]$$

$$\mathrm{A4}''[t]\ \Big(\mathrm{Cos}[z]^3\ \Big(e^{6\,\mathrm{A4}[t]}\ \Big(-9+\mathrm{M}^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\Big)\ \mathrm{YZ7}^{(0,1)}[z,t]+$$

$$\frac{3}{4}\,\mathrm{Q1}\,\mathrm{A4}'[t]^2\ \Big(-3\,\mathrm{Q1}\,\mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]+e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\ \Big(\mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]-$$

$$\frac{1}{3}\,\mathrm{YZ7}^{(0,1)}[z,t]\Big)+\frac{4}{3}\,(1+\mathrm{Q1})\,\mathrm{YZ7}^{(0,1)}[z,t]\Big)\Big)+$$

$$e^{4\,\mathrm{A4}[t]}\,\Big(-2\,\mathrm{Cos}[z]^{3}\,\Big(-9+M^{2}-\frac{1}{2}\,\big(\mathrm{Q1}+\mathrm{Q1}^{2}\big)\,\mathrm{A4}'[t]^{2}\Big)\,\mathrm{YZ7}^{(0,1)}[z,t]-$$

$$\mathrm{A4}'[t]\,\Big(\mathrm{Cos}[z]^{3}\,\mathrm{YZ7}[z,t]\,\Big(-9+M^{2}+3\,\Big(\mathrm{Q1}+\frac{\mathrm{Q1}^{2}}{4}\Big)\,\mathrm{A4}'[t]^{2}\Big)-$$

$$36\,\big(1+\mathrm{Cos}[z]^{2}\big)\,\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\Big)\Big)+$$

$$e^{2\,\mathrm{A4}[t]}\,\Big(\mathrm{Cos}[z]^{3}\,\Big(-9+M^{2}+\Big(-2\,\mathrm{Q1}-\frac{3\,\mathrm{Q1}^{2}}{2}\Big)\,\mathrm{A4}'[t]^{2}\Big)\,\mathrm{YZ7}^{(0,1)}[z,t]+$$

$$\mathrm{A4}'[t]\,\Big(\mathrm{Cos}[z]^{3}\,\mathrm{YZ7}[z,t]\,\Big(-9+M^{2}+3\,\Big(\mathrm{Q1}+\frac{3\,\mathrm{Q1}^{2}}{4}\Big)\,\mathrm{A4}'[t]^{2}\Big)-$$

$$36\,\big(1+\mathrm{Cos}[z]^{2}\big)\,\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\Big)\Big)\Big)-$$

$$36\,\big(1+\mathrm{Cos}[z]^{2}\big)\,\mathrm{Sin}[z]\,\mathrm{A4}'[t]\,\Big(\big(e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}\big)\,\mathrm{A4}'[t]^{2}+$$

$$\Big(-\frac{1}{2}\,e^{2\,\mathrm{A4}[t]}+e^{4\,\mathrm{A4}[t]}-\frac{1}{2}\,e^{6\,\mathrm{A4}[t]}\Big)\,\mathrm{A4}''[t]\Big)\,\mathrm{YZ7}^{(1,1)}[z,t]-$$

$$9\,\big(e^{2\,\mathrm{A4}[t]}-2\,e^{4\,\mathrm{A4}[t]}+e^{6\,\mathrm{A4}[t]}\big)\,\big(1+\mathrm{Cos}[z]^{2}\big)\,\mathrm{Sin}[z]\,\mathrm{A4}'[t]^{2}\,\mathrm{YZ7}^{(1,2)}[z,t]-$$

$$36\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^{2}\,\Big(\big(e^{2\,\mathrm{A4}[t]}+e^{4\,\mathrm{A4}[t]}\big)\,\mathrm{A4}'[t]^{4}-\frac{1}{2}\,\big(e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}\big)\,\mathrm{A4}'[t]^{2}$$

$$\mathrm{A4}''[t]+\Big(\frac{1}{2}\,e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}+\frac{1}{2}\,e^{6\,\mathrm{A4}[t]}\Big)\,\mathrm{A4}''[t]^{2}\Big)\,\mathrm{YZ7}^{(2,0)}[z,t]-$$

$$\frac{1}{4}\,\mathrm{A4}'[t]\,\mathrm{A4}^{(3)}[t]\,\Big(-\frac{9}{2}\,\Big(-\frac{1}{3}+e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}+\frac{1}{3}\,e^{6\,\mathrm{A4}[t]}\Big)\,\mathrm{Q1}\,\mathrm{Cos}[z]^{3}$$

$$\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]-\frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^{3}\,\mathrm{A4}'[t]\,\Big(e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]-$$

$$4\,(1+\mathrm{Q1})\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+4\,\mathrm{YZ7}^{(0,1)}[z,t]\Big)+$$

$$\big(e^{2\,\mathrm{A4}[t]}-2\,e^{4\,\mathrm{A4}[t]}+e^{6\,\mathrm{A4}[t]}\big)\,\mathrm{Cos}[z]^{3}\,\mathrm{YZ7}^{(0,2)}[z,t]+e^{6\,\mathrm{A4}[t]}$$

$$\Big(\mathrm{Cos}[z]^{3}\,\mathrm{YZ7}[z,t]\,\Big(-9+M^{2}-\frac{1}{4}\,\mathrm{Q1}^{2}\,\mathrm{A4}'[t]^{2}\Big)+\mathrm{Q1}\,\mathrm{Cos}[z]^{3}\,\mathrm{A4}'[t]$$

$$\mathrm{YZ7}^{(0,1)}[z,t]-36\,\big(1+\mathrm{Cos}[z]^{2}\big)\,\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\Big)+e^{2\,\mathrm{A4}[t]}$$

$$\Big(\mathrm{Cos}[z]^{3}\,\mathrm{YZ7}[z,t]\,\Big(-9+M^{2}+\Big(-2\,\mathrm{Q1}-\frac{3\,\mathrm{Q1}^{2}}{2}\Big)\,\mathrm{A4}'[t]^{2}\Big)+3\,\mathrm{Q1}\,\mathrm{Cos}[z]^{3}$$

$$\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]-36\,\big(1+\mathrm{Cos}[z]^{2}\big)\,\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\Big)+$$

$$e^{4\,\mathrm{A4}[t]}\,\Big(-2\,\mathrm{Cos}[z]^{3}\,\mathrm{YZ7}[z,t]\,\Big(-9+M^{2}-\frac{1}{2}\,\big(\mathrm{Q1}+\mathrm{Q1}^{2}\big)\,\mathrm{A4}'[t]^{2}\Big)-3\,\mathrm{Q1}\,\mathrm{Cos}[z]^{3}$$

$$\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]+72\,\big(1+\mathrm{Cos}[z]^{2}\big)\,\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\Big)-$$

$$36\,\big(e^{2\,\mathrm{A4}[t]}-2\,e^{4\,\mathrm{A4}[t]}+e^{6\,\mathrm{A4}[t]}\big)\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^{2}\,\mathrm{YZ7}^{(2,0)}[z,t]\Big)-$$

$$36\,\text{Cos}[z]\,\text{Sin}[z]^2\,\text{A4}'[t]\,\Big(\big(\mathrm{e}^{2\,\text{A4}[t]}-\mathrm{e}^{4\,\text{A4}[t]}\big)\,\text{A4}'[t]^2+$$

$$\Big(-\frac{1}{2}\,\mathrm{e}^{2\,\text{A4}[t]}+\mathrm{e}^{4\,\text{A4}[t]}-\frac{1}{2}\,\mathrm{e}^{6\,\text{A4}[t]}\Big)\,\text{A4}''[t]\Big)\,\text{YZ7}^{(2,1)}[z,t]-$$

$$9\,\big(\mathrm{e}^{2\,\text{A4}[t]}-2\,\mathrm{e}^{4\,\text{A4}[t]}+\mathrm{e}^{6\,\text{A4}[t]}\big)\,\text{Cos}[z]\,\text{Sin}[z]^2\,\text{A4}'[t]^2\,\text{YZ7}^{(2,2)}[z,t]\Big)\,dz+$$

$$\text{A4}'[t]^2\,\Big(\frac{1}{9}\,\big(\mathrm{e}^{4\,\text{A4}[t]}-2\,\mathrm{e}^{6\,\text{A4}[t]}+\mathrm{e}^{8\,\text{A4}[t]}\big)\,\text{Q1}\int\frac{1}{\sqrt{\text{Sin}[z]}}\,\text{Sec}[z]^2$$

$$\Big(-\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,(-1+\text{Cosh}[2\,\text{A4}[t]])\,\text{YZ7}[z,t]\,\text{A4}''[t]^2+$$

$$\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,(-1+\text{Cosh}[2\,\text{A4}[t]])\,\text{YZ7}[z,t]\,\text{A4}'[t]\,\text{A4}^{(3)}[t]+$$

$$\text{Cos}[z]^3\,\Big((1-\text{Q1}+\text{Q1}\,\text{Cosh}[2\,\text{A4}[t]])\,\text{A4}'[t]^2+\frac{1}{2}\,\big(-1+\mathrm{e}^{2\,\text{A4}[t]}\big)\,\text{A4}''[t]\Big)$$

$$\text{YZ7}^{(0,2)}[z,t]-\frac{1}{2}\,\big(-1+\mathrm{e}^{2\,\text{A4}[t]}\big)\,\text{Cos}[z]^3\,\text{A4}'[t]\,\text{YZ7}^{(0,3)}[z,t]+\text{A4}'[t]$$

$$\Big(-\frac{1}{4}\,\mathrm{e}^{-4\,\text{A4}[t]}\,\text{Q1}^2\,\text{Cos}[z]^3\,\text{A4}'[t]^2\,\Big(\text{YZ7}[z,t]\,\text{A4}'[t]-\frac{1}{2}\,\text{YZ7}^{(0,1)}[z,t]\Big)-$$

$$\frac{1}{2}\,\mathrm{e}^{2\,\text{A4}[t]}\,\text{Cos}[z]^3\,\Big(-9+\text{M}^2+\frac{1}{4}\,\text{Q1}^2\,\text{A4}'[t]^2\Big)\,\text{YZ7}^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,\text{Cos}[z]^3\,\Big(-9+\text{M}^2+\Big(\text{Q1}+\frac{3\,\text{Q1}^2}{4}\Big)\,\text{A4}'[t]^2\Big)\,\text{YZ7}^{(0,1)}[z,t]+\frac{1}{2}\,\mathrm{e}^{-2\,\text{A4}[t]}\,\text{Q1}$$

$$\text{Cos}[z]^3\,\text{A4}'[t]^2\,\Big(\text{Q1}\,\text{YZ7}[z,t]\,\text{A4}'[t]+\Big(-1-\frac{3\,\text{Q1}}{4}\Big)\,\text{YZ7}^{(0,1)}[z,t]\Big)+$$

$$\text{A4}'[t]\,\Big(\text{Cos}[z]^3\,\text{YZ7}[z,t]\,\Big(-9+\text{M}^2+\Big(-\text{Q1}-\frac{\text{Q1}^2}{4}\Big)\,\text{A4}'[t]^2\Big)-$$

$$36\,\big(1+\text{Cos}[z]^2\big)\,\text{Sin}[z]\,\text{YZ7}^{(1,0)}[z,t]\Big)\Big)+$$

$$\text{A4}''[t]\,\Big(\frac{1}{8}\,\mathrm{e}^{-4\,\text{A4}[t]}\,\text{Q1}^2\,\text{Cos}[z]^3\,\text{YZ7}[z,t]\,\text{A4}'[t]^2-\frac{3}{8}\,\mathrm{e}^{-2\,\text{A4}[t]}$$

$$\text{Q1}\,\Big(\frac{4}{3}+\text{Q1}\Big)\,\text{Cos}[z]^3\,\text{YZ7}[z,t]\,\text{A4}'[t]^2-\frac{1}{2}\,\text{Cos}[z]^3\,\text{YZ7}[z,t]$$

$$\Big(-9+\text{M}^2+\Big(-\text{Q1}-\frac{3\,\text{Q1}^2}{4}\Big)\,\text{A4}'[t]^2\Big)-\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,\text{A4}'[t]\,\text{YZ7}^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,\text{Cosh}[2\,\text{A4}[t]]\,\text{A4}'[t]\,\text{YZ7}^{(0,1)}[z,t]+18\,\big(1+\text{Cos}[z]^2\big)$$

$$\text{Sin}[z]\,\text{YZ7}^{(1,0)}[z,t]+\mathrm{e}^{2\,\text{A4}[t]}\,\Big(\frac{1}{2}\,\text{Cos}[z]^3\,\text{YZ7}[z,t]\,\Big(-9+\text{M}^2-$$

$$\frac{1}{4}\,\text{Q1}^2\,\text{A4}'[t]^2\Big)-18\,\big(1+\text{Cos}[z]^2\big)\,\text{Sin}[z]\,\text{YZ7}^{(1,0)}[z,t]\Big)\Big)+$$

$$18\,\big(-1+\mathrm{e}^{2\,\text{A4}[t]}\big)\,\big(1+\text{Cos}[z]^2\big)\,\text{Sin}[z]\,\text{A4}'[t]\,\text{YZ7}^{(1,1)}[z,t]-$$

$$36\,\text{Cos}[z]\,\text{Sin}[z]^2\,\Big(\text{A4}'[t]^2+\frac{1}{2}\,\big(-1+\mathrm{e}^{2\,\text{A4}[t]}\big)\,\text{A4}''[t]\Big)\,\text{YZ7}^{(2,0)}[z,t]+$$

$$18\,(-1+e^{2\,A4[t]})\,\text{Cos}[z]\,\text{Sin}[z]^{2}\,A4'[t]\,YZ7^{(2,1)}[z,t])\,\mathrm{d}z+\frac{1}{3}$$

$$((\int\frac{1}{\sqrt{\text{Sin}[z]}}\text{Cos}[z]\,(\frac{1}{2}\,Q1\,(YZ7[z,t]\,A4''[t]+A4'[t]\,((-2-Q1)\,YZ7[z,t]$$

$$A4'[t]+Q1\,\text{Cosh}[2\,A4[t]]\,YZ7[z,t]\,A4'[t]+2\,YZ7^{(0,1)}[z,t])))+$$

$$e^{2\,A4[t]}\,YZ7^{(0,2)}[z,t]+e^{2\,A4[t]}\,(((-9+M^{2})\,YZ7[z,t]-\frac{1}{2}\,Q1\,YZ7[$$

$$z,t]\,A4''[t]-Q1\,A4'[t]\,YZ7^{(0,1)}[z,t]+(-72\,\text{Tan}[z]-36\,\text{Tan}[z]^{3})$$

$$YZ7^{(1,0)}[z,t])-36\,e^{2\,A4[t]}\,\text{Tan}[z]^{2}\,YZ7^{(2,0)}[z,t])\,\mathrm{d}z)$$

$$(\frac{1}{6}\,(e^{2\,A4[t]}\,(-1+Q1)-\frac{Q1}{4})\,Q1\,A4'[t]^{2}+\frac{1}{6}\,e^{8\,A4[t]}\,(-M^{2}-\frac{1}{4}\,Q1^{2}\,A4'[t]^{2})+$$

$$e^{4\,A4[t]}\,(-\frac{M^{2}}{6}+(\frac{Q1}{3}-\frac{Q1^{2}}{4})\,A4'[t]^{2})+\frac{1}{3}\,e^{6\,A4[t]}\,(M^{2}+\frac{1}{2}\,(-Q1+Q1^{2})\,A4'[t]^{2})+$$

$$\frac{1}{12}\,(e^{2\,A4[t]}-3\,e^{4\,A4[t]}+3\,e^{6\,A4[t]}-e^{8\,A4[t]})\,Q1\,A4''[t])+\sqrt{\text{Sin}[z]}$$

$$(\frac{1}{2}\,(e^{4\,A4[t]}-3\,e^{6\,A4[t]}+3\,e^{8\,A4[t]}-e^{10\,A4[t]})\,Q1\,YZ7[z,t]\,A4''[t]-$$

$$\frac{3}{4}\,Q1\,A4'[t]\,(e^{2\,A4[t]}\,Q1\,YZ7[z,t]\,A4'[t]+$$

$$4\,e^{4\,A4[t]}\,((\frac{1}{3}-Q1)\,YZ7[z,t]\,A4'[t]-\frac{1}{3}\,YZ7^{(0,1)}[z,t]))+e^{6\,A4[t]}$$

$$(YZ7[z,t]\,(-9+M^{2}+(2\,Q1-\frac{9\,Q1^{2}}{2})\,A4'[t]^{2})-3\,Q1\,A4'[t]\,YZ7^{(0,1)}[z,t])+$$

$$e^{10\,A4[t]}\,(YZ7[z,t]\,(-9+M^{2}-\frac{3}{4}\,Q1^{2}\,A4'[t]^{2})-Q1\,A4'[t]\,YZ7^{(0,1)}[z,t])+e^{8\,A4[t]}$$

$$(-2\,YZ7[z,t]\,(-9+M^{2}+\frac{1}{2}\,(Q1-3\,Q1^{2})\,A4'[t]^{2})+3\,Q1\,A4'[t]\,YZ7^{(0,1)}[z,t])+$$

$$(e^{6\,A4[t]}-2\,e^{8\,A4[t]}+e^{10\,A4[t]})\,YZ7^{(0,2)}[z,t])-396$$

$$(e^{6\,A4[t]}-2\,e^{8\,A4[t]}+e^{10\,A4[t]})\,\text{Sec}[z]^{3}\,\text{Sin}[z]^{7/2}\,YZ7^{(1,0)}[z,t]-216$$

$$(e^{6\,A4[t]}-2\,e^{8\,A4[t]}+e^{10\,A4[t]})\,\text{Sec}[z]^{5}\,\text{Sin}[z]^{11/2}\,YZ7^{(1,0)}[z,t]+2$$

$$\text{Sec}[z]\,\text{Sin}[z]^{3/2}\,(((-\frac{3}{4}\,Q1\,(4\,e^{4\,A4[t]}\,(\frac{1}{3}-Q1)+e^{2\,A4[t]}\,Q1)\,A4'[t]^{2}+$$

$$e^{10\,A4[t]}\,(-117+M^{2}-\frac{3}{4}\,Q1^{2}\,A4'[t]^{2})+e^{6\,A4[t]}\,(-117+M^{2}+$$

$$(2\,Q1-\frac{9\,Q1^{2}}{2})\,A4'[t]^{2})+e^{8\,A4[t]}\,(234-2\,M^{2}+(-Q1+3\,Q1^{2})\,A4'[t]^{2})+$$

$$\frac{1}{2}\,(e^{4\,A4[t]}-3\,e^{6\,A4[t]}+3\,e^{8\,A4[t]}-e^{10\,A4[t]})\,Q1\,A4''[t])\,YZ7^{(1,0)}[z,t]+$$

$$(e^{4\,A4[t]}-3\,e^{6\,A4[t]}+3\,e^{8\,A4[t]}-e^{10\,A4[t]})\,Q1\,A4'[t]\,YZ7^{(1,1)}[z,t])+2$$

$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]\,\text{Sin}[z]^{3/2}\,\text{YZ7}^{(1,2)}[z,t]-324$

$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]^2\,\text{Sin}[z]^{5/2}\,\text{YZ7}^{(2,0)}[z,t]-216$

$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]^4\,\text{Sin}[z]^{9/2}\,\text{YZ7}^{(2,0)}[z,t]-72$

$\left.\left.\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]^3\,\text{Sin}[z]^{7/2}\,\text{YZ7}^{(3,0)}[z,t]\right)\right)$

$\left.dt\right)\text{Sin}[M\,t]\,\text{A4}'[t]^2-e^{-\frac{1}{4}\text{Q1}\left(e^{-2\,\text{A4}[t]}+2\,\text{A4}[t]\right)}\,M\,\text{Q1}\,\left(\text{c22}\right.$

$\text{Cos}[$

M

$t]-$

$\text{c21}\,\text{Sin}[M\,t]+e^{4\,\text{A4}[t]}\,(\text{c22}\,\text{Cos}[M\,t]-\text{c21}\,\text{Sin}[M\,t])+$

2

$e^{2\,\text{A4}[t]}$

$(-\text{c22}\,\text{Cos}[M\,t]+$

$\left.\text{c21}\,\text{Sin}[M\,t])\right)\text{A4}'[$

$t]^2-6\,e^{2\,\text{A4}[t]}\,\text{Q1}\,\sqrt{\text{Sin}[z]}\,\text{A4}'[$

$t]^2\,\left(-\text{YZ7}[\right.$

$z,$

$t]-2\,\text{Tan}[$

$z]\,\text{YZ7}^{(1,0)}[$

$z,$

$\left.t]\right)-$

$3\,\text{Q1}\,\sqrt{\text{Sin}[z]}\,\text{A4}'[t]^2\,\left(\text{YZ7}[z,t]+\right.$

2

$\text{Tan}[$

$z]\,\text{YZ7}^{(1,0)}[$

$z,$

$\left.t]\right)-$

$3\,e^{4\,\text{A4}[t]}\,\text{Q1}\,\sqrt{\text{Sin}[z]}\,\text{A4}'[t]^2\,\left(\text{YZ7}[\right.$

$z,$

$t]+2$

$\text{Tan}[$

$z]\,\text{YZ7}^{(1,0)}[$

$z,$

$\left.\left.t]\right)\right),$

$\text{YZ6}[z,t]==\dfrac{1}{\left(-1+e^{2\,\text{A4}[t]}\right)M\,\text{Q1}\,\sqrt{\text{Sin}[z]}\,\text{A4}'[t]}$

$\left(-\dfrac{3}{8}\right.$

$e^{-\frac{1}{4}\text{Q1}\left(e^{-2\,\text{A4}[t]}+2\,\text{A4}[t]\right)}$

$\left(-1+\right.$

$\left.e^{2\,\text{A4}[t]}\right)\text{Cos}[$

M

t]

$$\left(\int \frac{1}{\mathrm{A4}'[t]^3} \mathbb{e}^{\frac{1}{4}\,\mathbb{e}^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}+\frac{1}{2}\,(-14+\mathrm{Q1})\,\mathrm{A4}[t]}\right.$$

$$\mathrm{Csch}[\mathrm{A4}[t]]^3$$

$$\mathrm{Sin}[M\,t]$$

$$\left(-\frac{2}{9}\,\mathbb{e}^{4\,\mathrm{A4}[t]}\int \frac{1}{\sqrt{\mathrm{Sin}[z]}}\,\mathrm{Sec}[z]^2\right.$$

$$\left(-\frac{3}{4}\left(-\frac{1}{3}+\mathbb{e}^{2\,\mathrm{A4}[t]}-\mathbb{e}^{4\,\mathrm{A4}[t]}+\frac{1}{3}\,\mathbb{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]^3-\right.$$

$$\frac{3}{8}\left(-\frac{1}{3}+\mathbb{e}^{2\,\mathrm{A4}[t]}-\mathbb{e}^{4\,\mathrm{A4}[t]}+\frac{1}{3}\,\mathbb{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]^2\,\mathrm{A4}^{(4)}[t]+$$

$$\frac{1}{4}\,\mathrm{Cos}[z]^3\left(\mathrm{A4}'[t]^2\left(\frac{1}{4}\,\mathrm{Q1}\left(-4-4\,\mathrm{Q1}+\mathbb{e}^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\right)\mathrm{A4}'[t]^2+\mathbb{e}^{6\,\mathrm{A4}[t]}\right.\right.$$

$$\left(-9+M^2+\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+\mathbb{e}^{4\,\mathrm{A4}[t]}\left(18-2\,M^2+\left(4-\mathrm{Q1}-\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+$$

$$\left.\mathbb{e}^{2\,\mathrm{A4}[t]}\left(-9+M^2+\left(4+2\,\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{2}\right)\mathrm{A4}'[t]^2\right)\right)-$$

$$\frac{3}{2}\left(\mathbb{e}^{4\,\mathrm{A4}[t]}\left(-\frac{4}{3}-\mathrm{Q1}\right)+\frac{1}{3}\left(-1+\mathbb{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{Q1}+\mathbb{e}^{2\,\mathrm{A4}[t]}\left(\frac{4}{3}+\mathrm{Q1}\right)\right)$$

$$\left.\mathrm{A4}'[t]^2\,\mathrm{A4}''[t]+2\left(\mathbb{e}^{2\,\mathrm{A4}[t]}-2\,\mathbb{e}^{4\,\mathrm{A4}[t]}+\mathbb{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{A4}''[t]^2\right)\mathrm{YZ7}^{(0,2)}[z,t]-$$

$$\frac{3}{4}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\left(\left(\mathbb{e}^{4\,\mathrm{A4}[t]}\left(\frac{4}{3}-\mathrm{Q1}\right)+\mathbb{e}^{2\,\mathrm{A4}[t]}\left(-\frac{4}{3}+\mathrm{Q1}\right)+\frac{1}{3}\left(-1+\mathbb{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{Q1}\right)\right.$$

$$\left.\mathrm{A4}'[t]^2+\frac{2}{3}\left(\mathbb{e}^{2\,\mathrm{A4}[t]}-2\,\mathbb{e}^{4\,\mathrm{A4}[t]}+\mathbb{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{A4}''[t]\right)\mathrm{YZ7}^{(0,3)}[z,t]+$$

$$\frac{1}{4}\left(\mathbb{e}^{2\,\mathrm{A4}[t]}-2\,\mathbb{e}^{4\,\mathrm{A4}[t]}+\mathbb{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]^2\,\mathrm{YZ7}^{(0,4)}[z,t]+$$

$$\mathrm{A4}''[t]^2\left(-\frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]+\right.$$

$$\mathbb{e}^{6\,\mathrm{A4}[t]}\left(\frac{1}{2}\,(-3+M)\,(3+M)\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]+\frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\right.$$

$$\left.\mathrm{YZ7}^{(0,1)}[z,t]-18\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\right)+$$

$$\mathbb{e}^{2\,\mathrm{A4}[t]}\left(\frac{1}{2}\,(-3+M)\,(3+M)\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]+\frac{3}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\right.$$

$$\left.\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]-18\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\right)+$$

$$\mathbb{e}^{4\,\mathrm{A4}[t]}\left(\left(9-M^2\right)\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]-\frac{3}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\right.$$

$$\left.\left.\mathrm{YZ7}^{(0,1)}[z,t]+36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\right)\right)+\mathrm{A4}'[t]^3$$

$$\left(\frac{1}{4}\left(-3+\mathbb{e}^{-2\,\mathrm{A4}[t]}\right)\mathrm{Q1}^2\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]^2\left(\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]-\mathrm{YZ7}^{(0,1)}[z,t]\right)+\right.$$

$$\mathrm{e}^{4\,A4[t]}\,\left(-\mathrm{Cos}[z]^3\,\left(-9+M^2+\left(-Q1-\frac{Q1^2}{4}\right)A4'[t]^2\right)YZ7^{(0,1)}[z,t]+\right.$$

$$A4'[t]\,\left(\mathrm{Cos}[z]^3\,YZ7[z,t]\,\left(-9+M^2+\left(-Q1-\frac{Q1^2}{4}\right)A4'[t]^2\right)-\right.$$

$$\left.\left.36\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\right)\right)+$$

$$\mathrm{e}^{2\,A4[t]}\,\left(\mathrm{Cos}[z]^3\,\left(-9+M^2+\left(-Q1-\frac{3\,Q1^2}{4}\right)A4'[t]^2\right)YZ7^{(0,1)}[z,t]+\right.$$

$$A4'[t]\,\left(\mathrm{Cos}[z]^3\,YZ7[z,t]\,\left(-9+M^2+\left(-Q1+\frac{3\,Q1^2}{4}\right)A4'[t]^2\right)-\right.$$

$$\left.\left.\left.36\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\right)\right)\right)-\frac{1}{2}\,A4'[t]$$

$$A4''[t]\,\left(\mathrm{Cos}[z]^3\,\left(\mathrm{e}^{6\,A4[t]}\,\left(-9+M^2-\frac{1}{4}\,Q1^2\,A4'[t]^2\right)YZ7^{(0,1)}[z,t]+\right.\right.$$

$$\frac{3}{4}\,Q1\,A4'[t]^2\,\left(-3\,Q1\,YZ7[z,t]\,A4'[t]+\mathrm{e}^{-2\,A4[t]}\,Q1\,\left(YZ7[z,t]\,A4'[t]-\right.\right.$$

$$\left.\left.\left.\frac{1}{3}\,YZ7^{(0,1)}[z,t]\right)+\frac{4}{3}\,(1+Q1)\,YZ7^{(0,1)}[z,t]\right)\right)+$$

$$\mathrm{e}^{4\,A4[t]}\,\left(-2\,\mathrm{Cos}[z]^3\,\left(-9+M^2-\frac{1}{2}\,\left(Q1+Q1^2\right)A4'[t]^2\right)YZ7^{(0,1)}[z,t]-\right.$$

$$A4'[t]\,\left(\mathrm{Cos}[z]^3\,YZ7[z,t]\,\left(-9+M^2+3\,\left(Q1+\frac{Q1^2}{4}\right)A4'[t]^2\right)-\right.$$

$$\left.\left.36\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\right)\right)+$$

$$\mathrm{e}^{2\,A4[t]}\,\left(\mathrm{Cos}[z]^3\,\left(-9+M^2+\left(-2\,Q1-\frac{3\,Q1^2}{2}\right)A4'[t]^2\right)YZ7^{(0,1)}[z,t]+\right.$$

$$A4'[t]\,\left(\mathrm{Cos}[z]^3\,YZ7[z,t]\,\left(-9+M^2+3\,\left(Q1+\frac{3\,Q1^2}{4}\right)A4'[t]^2\right)-\right.$$

$$\left.\left.\left.36\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ7^{(1,0)}[z,t]\right)\right)\right)-$$

$$36\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,A4'[t]\,\left(\left(\mathrm{e}^{2\,A4[t]}-\mathrm{e}^{4\,A4[t]}\right)A4'[t]^2+\right.$$

$$\left.\left(-\frac{1}{2}\,\mathrm{e}^{2\,A4[t]}+\mathrm{e}^{4\,A4[t]}-\frac{1}{2}\,\mathrm{e}^{6\,A4[t]}\right)A4''[t]\right)YZ7^{(1,1)}[z,t]-$$

$$9\,\left(\mathrm{e}^{2\,A4[t]}-2\,\mathrm{e}^{4\,A4[t]}+\mathrm{e}^{6\,A4[t]}\right)\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,A4'[t]^2\,YZ7^{(1,2)}[z,t]-$$

$$36\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\,\left(\left(\mathrm{e}^{2\,A4[t]}+\mathrm{e}^{4\,A4[t]}\right)A4'[t]^4-\frac{1}{2}\,\left(\mathrm{e}^{2\,A4[t]}-\mathrm{e}^{4\,A4[t]}\right)A4'[t]^2\right.$$

$$\left.A4''[t]+\left(\frac{1}{2}\,\mathrm{e}^{2\,A4[t]}-\mathrm{e}^{4\,A4[t]}+\frac{1}{2}\,\mathrm{e}^{6\,A4[t]}\right)A4''[t]^2\right)YZ7^{(2,0)}[z,t]-$$

$$\frac{1}{4}\,A4'[t]\,A4^{(3)}[t]\,\left(-\frac{9}{2}\,\left(-\frac{1}{3}+\mathrm{e}^{2\,A4[t]}-\mathrm{e}^{4\,A4[t]}+\frac{1}{3}\,\mathrm{e}^{6\,A4[t]}\right)Q1\,\mathrm{Cos}[z]^3\right.$$

$$\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]-\frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\,\Big(e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]-$$

$$4\,(1+\mathrm{Q1})\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+4\,\mathrm{YZ7}^{(0,1)}[z,t]\Big)+$$

$$\Big(e^{2\,\mathrm{A4}[t]}-2\,e^{4\,\mathrm{A4}[t]}+e^{6\,\mathrm{A4}[t]}\Big)\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}^{(0,2)}[z,t]+e^{6\,\mathrm{A4}[t]}$$

$$\Bigg(\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\,\Big(-9+M^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\Big)+\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]$$

$$\mathrm{YZ7}^{(0,1)}[z,t]-36\,\Big(1+\mathrm{Cos}[z]^2\Big)\,\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\Bigg)+e^{2\,\mathrm{A4}[t]}$$

$$\Bigg(\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\,\Bigg(-9+M^2+\Big(-2\,\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{2}\Big)\,\mathrm{A4}'[t]^2\Bigg)+3\,\mathrm{Q1}\,\mathrm{Cos}[z]^3$$

$$\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]-36\,\Big(1+\mathrm{Cos}[z]^2\Big)\,\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\Bigg)+$$

$$e^{4\,\mathrm{A4}[t]}\,\Bigg(-2\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\,\Big(-9+M^2-\frac{1}{2}\,\Big(\mathrm{Q1}+\mathrm{Q1}^2\Big)\,\mathrm{A4}'[t]^2\Big)-3\,\mathrm{Q1}\,\mathrm{Cos}[z]^3$$

$$\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]+72\,\Big(1+\mathrm{Cos}[z]^2\Big)\,\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\Bigg)-$$

$$36\,\Big(e^{2\,\mathrm{A4}[t]}-2\,e^{4\,\mathrm{A4}[t]}+e^{6\,\mathrm{A4}[t]}\Big)\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\,\mathrm{YZ7}^{(2,0)}[z,t]\Bigg)-$$

$$36\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\,\mathrm{A4}'[t]\,\Bigg(\Big(e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}\Big)\,\mathrm{A4}'[t]^2+$$

$$\Big(-\frac{1}{2}\,e^{2\,\mathrm{A4}[t]}+e^{4\,\mathrm{A4}[t]}-\frac{1}{2}\,e^{6\,\mathrm{A4}[t]}\Big)\,\mathrm{A4}''[t]\Bigg)\,\mathrm{YZ7}^{(2,1)}[z,t]-$$

$$9\,\Big(e^{2\,\mathrm{A4}[t]}-2\,e^{4\,\mathrm{A4}[t]}+e^{6\,\mathrm{A4}[t]}\Big)\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\,\mathrm{A4}'[t]^2\,\mathrm{YZ7}^{(2,2)}[z,t]\Bigg)\,dz+$$

$$\mathrm{A4}'[t]^2\,\Bigg(\frac{1}{9}\,\Big(e^{4\,\mathrm{A4}[t]}-2\,e^{6\,\mathrm{A4}[t]}+e^{8\,\mathrm{A4}[t]}\Big)\,\mathrm{Q1}\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\,\mathrm{Sec}[z]^2$$

$$\Bigg(-\frac{1}{2}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,(-1+\mathrm{Cosh}[2\,\mathrm{A4}[t]])\,\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]^2+$$

$$\frac{1}{2}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,(-1+\mathrm{Cosh}[2\,\mathrm{A4}[t]])\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]\,\mathrm{A4}^{(3)}[t]+$$

$$\mathrm{Cos}[z]^3\,\Big((1-\mathrm{Q1}+\mathrm{Q1}\,\mathrm{Cosh}[2\,\mathrm{A4}[t]])\,\mathrm{A4}'[t]^2+\frac{1}{2}\,\Big(-1+e^{2\,\mathrm{A4}[t]}\Big)\,\mathrm{A4}''[t]\Big)$$

$$\mathrm{YZ7}^{(0,2)}[z,t]-\frac{1}{2}\,\Big(-1+e^{2\,\mathrm{A4}[t]}\Big)\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,3)}[z,t]+\mathrm{A4}'[t]$$

$$\Bigg(-\frac{1}{4}\,e^{-4\,\mathrm{A4}[t]}\,\mathrm{Q1}^2\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]^2\,\Big(\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]-\frac{1}{2}\,\mathrm{YZ7}^{(0,1)}[z,t]\Big)-$$

$$\frac{1}{2}\,e^{2\,\mathrm{A4}[t]}\,\mathrm{Cos}[z]^3\,\Big(-9+M^2+\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\Big)\,\mathrm{YZ7}^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,\mathrm{Cos}[z]^3\,\Bigg(-9+M^2+\Big(\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{4}\Big)\,\mathrm{A4}'[t]^2\Bigg)\,\mathrm{YZ7}^{(0,1)}[z,t]+\frac{1}{2}\,e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}$$

$$\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]^2\,\Bigg(\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+\Big(-1-\frac{3\,\mathrm{Q1}}{4}\Big)\,\mathrm{YZ7}^{(0,1)}[z,t]\Bigg)+$$

$$\mathrm{A4}'[t]\ \left(\mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]\ \left(-9+M^2+\left(-\mathrm{Q1}-\frac{\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)-\right.$$

$$\left.\left.36\ \left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t]\right)\right)+$$

$$\mathrm{A4}''[t]\ \left(\frac{1}{8}\ e^{-4\,\mathrm{A4}[t]}\ \mathrm{Q1}^2\ \mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]^2-\frac{3}{8}\ e^{-2\,\mathrm{A4}[t]}\right.$$

$$\mathrm{Q1}\ \left(\frac{4}{3}+\mathrm{Q1}\right)\mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]^2-\frac{1}{2}\ \mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]$$

$$\left(-9+M^2+\left(-\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)-\frac{1}{2}\ \mathrm{Q1}\ \mathrm{Cos}[z]^3\ \mathrm{A4}'[t]\ \mathrm{YZ7}^{(0,1)}[z,t]+$$

$$\frac{1}{2}\ \mathrm{Q1}\ \mathrm{Cos}[z]^3\ \mathrm{Cosh}[2\,\mathrm{A4}[t]]\ \mathrm{A4}'[t]\ \mathrm{YZ7}^{(0,1)}[z,t]+18\ \left(1+\mathrm{Cos}[z]^2\right)$$

$$\mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t]+e^{2\,\mathrm{A4}[t]}\ \left(\frac{1}{2}\ \mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]\ \left(-9+M^2-\right.\right.$$

$$\left.\left.\left.\frac{1}{4}\ \mathrm{Q1}^2\ \mathrm{A4}'[t]^2\right)-18\ \left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t]\right)\right)+$$

$$18\ \left(-1+e^{2\,\mathrm{A4}[t]}\right)\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\ \mathrm{A4}'[t]\ \mathrm{YZ7}^{(1,1)}[z,t]-$$

$$36\ \mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ \left(\mathrm{A4}'[t]^2+\frac{1}{2}\ \left(-1+e^{2\,\mathrm{A4}[t]}\right)\mathrm{A4}''[t]\right)\mathrm{YZ7}^{(2,0)}[z,t]+$$

$$\left.18\ \left(-1+e^{2\,\mathrm{A4}[t]}\right)\mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ \mathrm{A4}'[t]\ \mathrm{YZ7}^{(2,1)}[z,t]\right)\mathrm{d}z+\frac{1}{3}$$

$$\left(\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\mathrm{Cos}[z]\ \left(\frac{1}{2}\ \mathrm{Q1}\ \left(\mathrm{YZ7}[z,t]\ \mathrm{A4}''[t]+\mathrm{A4}'[t]\ \left((-2-\mathrm{Q1})\ \mathrm{YZ7}[z,t]\right.\right.\right.\right.$$

$$\left.\left.\mathrm{A4}'[t]+\mathrm{Q1}\ \mathrm{Cosh}[2\,\mathrm{A4}[t]]\ \mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]+2\ \mathrm{YZ7}^{(0,1)}[z,t]\right)\right)+$$

$$e^{2\,\mathrm{A4}[t]}\ \mathrm{YZ7}^{(0,2)}[z,t]+e^{2\,\mathrm{A4}[t]}\ \left(\left(-9+M^2\right)\mathrm{YZ7}[z,t]-\frac{1}{2}\ \mathrm{Q1}\ \mathrm{YZ7}[\right.$$

$$z,t]\ \mathrm{A4}''[t]-\mathrm{Q1}\ \mathrm{A4}'[t]\ \mathrm{YZ7}^{(0,1)}[z,t]+\left(-72\ \mathrm{Tan}[z]-36\ \mathrm{Tan}[z]^3\right)$$

$$\left.\left.\left.\mathrm{YZ7}^{(1,0)}[z,t]\right)-36\ e^{2\,\mathrm{A4}[t]}\ \mathrm{Tan}[z]^2\ \mathrm{YZ7}^{(2,0)}[z,t]\right)\mathrm{d}z\right)$$

$$\left(\frac{1}{6}\ \left(e^{2\,\mathrm{A4}[t]}\ (-1+\mathrm{Q1})-\frac{\mathrm{Q1}}{4}\right)\mathrm{Q1}\ \mathrm{A4}'[t]^2+\frac{1}{6}\ e^{8\,\mathrm{A4}[t]}\ \left(-M^2-\frac{1}{4}\ \mathrm{Q1}^2\ \mathrm{A4}'[t]^2\right)+\right.$$

$$e^{4\,\mathrm{A4}[t]}\ \left(-\frac{M^2}{6}+\left(\frac{\mathrm{Q1}}{3}-\frac{\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)+\frac{1}{3}\ e^{6\,\mathrm{A4}[t]}\ \left(M^2+\frac{1}{2}\ \left(-\mathrm{Q1}+\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+$$

$$\left.\frac{1}{12}\ \left(e^{2\,\mathrm{A4}[t]}-3\ e^{4\,\mathrm{A4}[t]}+3\ e^{6\,\mathrm{A4}[t]}-e^{8\,\mathrm{A4}[t]}\right)\mathrm{Q1}\ \mathrm{A4}''[t]\right)+\sqrt{\mathrm{Sin}[z]}$$

$$\left(\frac{1}{2}\ \left(e^{4\,\mathrm{A4}[t]}-3\ e^{6\,\mathrm{A4}[t]}+3\ e^{8\,\mathrm{A4}[t]}-e^{10\,\mathrm{A4}[t]}\right)\mathrm{Q1}\ \mathrm{YZ7}[z,t]\ \mathrm{A4}''[t]-\right.$$

$$\frac{3}{4}\ \mathrm{Q1}\ \mathrm{A4}'[t]\ \left(e^{2\,\mathrm{A4}[t]}\ \mathrm{Q1}\ \mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]+\right.$$

$$\left.4\ e^{4\,\mathrm{A4}[t]}\ \left(\left(\frac{1}{3}-\mathrm{Q1}\right)\mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]-\frac{1}{3}\ \mathrm{YZ7}^{(0,1)}[z,t]\right)\right)+e^{6\,\mathrm{A4}[t]}$$

$$\left(\text{YZ7}[z,t]\left(-9+M^2+\left(2\,\text{Q1}-\frac{9\,\text{Q1}^2}{2}\right)\text{A4}'[t]^2\right)-3\,\text{Q1}\,\text{A4}'[t]\,\text{YZ7}^{(0,1)}[z,t]\right)+$$

$$e^{10\,\text{A4}[t]}\left(\text{YZ7}[z,t]\left(-9+M^2-\frac{3}{4}\text{Q1}^2\,\text{A4}'[t]^2\right)-\text{Q1}\,\text{A4}'[t]\,\text{YZ7}^{(0,1)}[z,t]\right)+e^{8\,\text{A4}[t]}$$

$$\left(-2\,\text{YZ7}[z,t]\left(-9+M^2+\frac{1}{2}\left(\text{Q1}-3\,\text{Q1}^2\right)\text{A4}'[t]^2\right)+3\,\text{Q1}\,\text{A4}'[t]\,\text{YZ7}^{(0,1)}[z,t]\right)+$$

$$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{YZ7}^{(0,2)}[z,t]\Big)-396$$

$$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]^3\,\text{Sin}[z]^{7/2}\,\text{YZ7}^{(1,0)}[z,t]-216$$

$$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]^5\,\text{Sin}[z]^{11/2}\,\text{YZ7}^{(1,0)}[z,t]+2$$

$$\text{Sec}[z]\,\text{Sin}[z]^{3/2}\left(\left(\left(-\frac{3}{4}\text{Q1}\left(4\,e^{4\,\text{A4}[t]}\left(\frac{1}{3}-\text{Q1}\right)+e^{2\,\text{A4}[t]}\,\text{Q1}\right)\text{A4}'[t]^2+\right.\right.\right.$$

$$e^{10\,\text{A4}[t]}\left(-117+M^2-\frac{3}{4}\text{Q1}^2\,\text{A4}'[t]^2\right)+e^{6\,\text{A4}[t]}\left(-117+M^2+\right.$$

$$\left.\left(2\,\text{Q1}-\frac{9\,\text{Q1}^2}{2}\right)\text{A4}'[t]^2\right)+e^{8\,\text{A4}[t]}\left(234-2\,M^2+\left(-\text{Q1}+3\,\text{Q1}^2\right)\text{A4}'[t]^2\right)+$$

$$\left.\frac{1}{2}\left(e^{4\,\text{A4}[t]}-3\,e^{6\,\text{A4}[t]}+3\,e^{8\,\text{A4}[t]}-e^{10\,\text{A4}[t]}\right)\text{Q1}\,\text{A4}''[t]\right)\text{YZ7}^{(1,0)}[z,t]+$$

$$\left.\left(e^{4\,\text{A4}[t]}-3\,e^{6\,\text{A4}[t]}+3\,e^{8\,\text{A4}[t]}-e^{10\,\text{A4}[t]}\right)\text{Q1}\,\text{A4}'[t]\,\text{YZ7}^{(1,1)}[z,t]\right)+2$$

$$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]\,\text{Sin}[z]^{3/2}\,\text{YZ7}^{(1,2)}[z,t]-324$$

$$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]^2\,\text{Sin}[z]^{5/2}\,\text{YZ7}^{(2,0)}[z,t]-216$$

$$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]^4\,\text{Sin}[z]^{9/2}\,\text{YZ7}^{(2,0)}[z,t]-72$$

$$\left.\left.\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]^3\,\text{Sin}[z]^{7/2}\,\text{YZ7}^{(3,0)}[z,t]\right)\right)dt\Big)$$

$$\text{A4}'[t]+\frac{3}{8}\,e^{-\frac{1}{4}\text{Q1}\left(e^{-2\,\text{A4}[t]}+2\,\text{A4}[t]\right)}\left(-1+e^{2\,\text{A4}[t]}\right)\left(\int\frac{1}{\text{A4}'[t]^3}\right.$$

$$e^{\frac{1}{4}e^{-2\,\text{A4}[t]}\,\text{Q1}+\frac{1}{2}(-14+\text{Q1})\,\text{A4}[t]}$$

$$\text{Cos}[$$

$$M\,t]\,\text{Csch}[\text{A4}[t]]^3$$

$$\left(-\frac{2}{9}\,e^{4\,\text{A4}[t]}\int\frac{1}{\sqrt{\text{Sin}[z]}}\text{Sec}[z]^2\left(-\frac{3}{4}\left(-\frac{1}{3}+e^{2\,\text{A4}[t]}-e^{4\,\text{A4}[t]}+\frac{1}{3}e^{6\,\text{A4}[t]}\right)\text{Q1}\,\text{Cos}[z]^3\right.\right.$$

$$\text{YZ7}[z,t]\,\text{A4}''[t]^3-\frac{3}{8}\left(-\frac{1}{3}+e^{2\,\text{A4}[t]}-e^{4\,\text{A4}[t]}+\frac{1}{3}e^{6\,\text{A4}[t]}\right)\text{Q1}\,\text{Cos}[z]^3\,\text{YZ7}[z,t]$$

$$\text{A4}'[t]^2\,\text{A4}^{(4)}[t]+\frac{1}{4}\text{Cos}[z]^3\left(\text{A4}'[t]^2\left(\frac{1}{4}\text{Q1}\left(-4-4\,\text{Q1}+e^{-2\,\text{A4}[t]}\,\text{Q1}\right)\text{A4}'[t]^2+\right.\right.$$

$$e^{6\,\text{A4}[t]}\left(-9+M^2+\frac{1}{4}\text{Q1}^2\,\text{A4}'[t]^2\right)+e^{4\,\text{A4}[t]}\left(18-2\,M^2+\left(4-\text{Q1}-\text{Q1}^2\right)\right.$$

$$\left.\left.\text{A4}'[t]^2\right)+e^{2\,\text{A4}[t]}\left(-9+M^2+\left(4+2\,\text{Q1}+\frac{3\,\text{Q1}^2}{2}\right)\text{A4}'[t]^2\right)\right)-$$

$$\frac{3}{2}\left(e^{4\,\text{A4}[t]}\left(-\frac{4}{3}-\text{Q1}\right)+\frac{1}{3}\left(-1+e^{6\,\text{A4}[t]}\right)\text{Q1}+e^{2\,\text{A4}[t]}\left(\frac{4}{3}+\text{Q1}\right)\right)$$

$$\mathrm{A4}'[t]^2\,\mathrm{A4}''[t] + 2\left(\mathrm{e}^{2\,\mathrm{A4}[t]} - 2\,\mathrm{e}^{4\,\mathrm{A4}[t]} + \mathrm{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{A4}''[t]^2\bigg)\,\mathrm{YZ7}^{(0,2)}[z,t] -$$

$$\frac{3}{4}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\,\Bigg(\bigg(\mathrm{e}^{4\,\mathrm{A4}[t]}\left(\frac{4}{3} - \mathrm{Q1}\right) + \mathrm{e}^{2\,\mathrm{A4}[t]}\left(-\frac{4}{3} + \mathrm{Q1}\right) + \frac{1}{3}\left(-1 + \mathrm{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{Q1}\bigg)$$

$$\mathrm{A4}'[t]^2 + \frac{2}{3}\left(\mathrm{e}^{2\,\mathrm{A4}[t]} - 2\,\mathrm{e}^{4\,\mathrm{A4}[t]} + \mathrm{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{A4}''[t]\Bigg)\,\mathrm{YZ7}^{(0,3)}[z,t] +$$

$$\frac{1}{4}\left(\mathrm{e}^{2\,\mathrm{A4}[t]} - 2\,\mathrm{e}^{4\,\mathrm{A4}[t]} + \mathrm{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]^2\,\mathrm{YZ7}^{(0,4)}[z,t] +$$

$$\mathrm{A4}''[t]^2\,\Bigg(-\frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t] +$$

$$\mathrm{e}^{6\,\mathrm{A4}[t]}\,\bigg(\frac{1}{2}\,(-3 + \mathrm{M})\,(3 + \mathrm{M})\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t] + \frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]$$

$$\mathrm{YZ7}^{(0,1)}[z,t] - 18\left(1 + \mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\bigg) +$$

$$\mathrm{e}^{2\,\mathrm{A4}[t]}\,\bigg(\frac{1}{2}\,(-3 + \mathrm{M})\,(3 + \mathrm{M})\,\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t] + \frac{3}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3$$

$$\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t] - 18\left(1 + \mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\bigg) +$$

$$\mathrm{e}^{4\,\mathrm{A4}[t]}\,\bigg(\left(9 - \mathrm{M}^2\right)\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t] - \frac{3}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]$$

$$\mathrm{YZ7}^{(0,1)}[z,t] + 36\left(1 + \mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\bigg)\Bigg) + \mathrm{A4}'[t]^3$$

$$\Bigg(\frac{1}{4}\left(-3 + \mathrm{e}^{-2\,\mathrm{A4}[t]}\right)\mathrm{Q1}^2\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]^2\left(\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t] - \mathrm{YZ7}^{(0,1)}[z,t]\right) +$$

$$\mathrm{e}^{4\,\mathrm{A4}[t]}\,\Bigg(-\mathrm{Cos}[z]^3\left(-9 + \mathrm{M}^2 + \left(-\mathrm{Q1} - \frac{\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)\mathrm{YZ7}^{(0,1)}[z,t] +$$

$$\mathrm{A4}'[t]\,\bigg(\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\left(-9 + \mathrm{M}^2 + \left(-\mathrm{Q1} - \frac{\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right) -$$

$$36\left(1 + \mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\bigg)\Bigg) +$$

$$\mathrm{e}^{2\,\mathrm{A4}[t]}\,\Bigg(\mathrm{Cos}[z]^3\left(-9 + \mathrm{M}^2 + \left(-\mathrm{Q1} - \frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)\mathrm{YZ7}^{(0,1)}[z,t] +$$

$$\mathrm{A4}'[t]\,\bigg(\mathrm{Cos}[z]^3\,\mathrm{YZ7}[z,t]\left(-9 + \mathrm{M}^2 + \left(-\mathrm{Q1} + \frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right) -$$

$$36\left(1 + \mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ7}^{(1,0)}[z,t]\bigg)\Bigg)\Bigg) - \frac{1}{2}\,\mathrm{A4}'[t]$$

$$\mathrm{A4}''[t]\,\Bigg(\mathrm{Cos}[z]^3\,\bigg(\mathrm{e}^{6\,\mathrm{A4}[t]}\left(-9 + \mathrm{M}^2 - \frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)\mathrm{YZ7}^{(0,1)}[z,t] +$$

$$\frac{3}{4}\,\mathrm{Q1}\,\mathrm{A4}'[t]^2\,\bigg(-3\,\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t] + \mathrm{e}^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\bigg(\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t] -$$

$$\frac{1}{3}\,\mathrm{YZ7}^{(0,1)}[z,t]\bigg) + \frac{4}{3}\,(1 + \mathrm{Q1})\,\mathrm{YZ7}^{(0,1)}[z,t]\bigg)\bigg) +$$

$$\mathrm{e}^{4\,\mathrm{A4}[t]}\ (-2\,\mathrm{Cos}[z]^3\ (-9+M^2-\frac{1}{2}\ (\mathrm{Q1}+\mathrm{Q1}^2)\ \mathrm{A4}'[t]^2)\ \mathrm{YZ7}^{(0,1)}[z,t]\ -$$

$$\mathrm{A4}'[t]\ (\mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]\ (-9+M^2+3\ (\mathrm{Q1}+\frac{\mathrm{Q1}^2}{4})\ \mathrm{A4}'[t]^2)\ -$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t]))\ +$$

$$\mathrm{e}^{2\,\mathrm{A4}[t]}\ (\mathrm{Cos}[z]^3\ (-9+M^2+(-2\,\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{2})\ \mathrm{A4}'[t]^2)\ \mathrm{YZ7}^{(0,1)}[z,t]\ +$$

$$\mathrm{A4}'[t]\ (\mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]\ (-9+M^2+3\ (\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{4})\ \mathrm{A4}'[t]^2)\ -$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t])))\ -$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{A4}'[t]\ ((\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]})\ \mathrm{A4}'[t]^2\ +$$

$$(-\frac{1}{2}\ \mathrm{e}^{2\,\mathrm{A4}[t]}+\mathrm{e}^{4\,\mathrm{A4}[t]}-\frac{1}{2}\ \mathrm{e}^{6\,\mathrm{A4}[t]})\ \mathrm{A4}''[t])\ \mathrm{YZ7}^{(1,1)}[z,t]\ -$$

$$9\ (\mathrm{e}^{2\,\mathrm{A4}[t]}-2\ \mathrm{e}^{4\,\mathrm{A4}[t]}+\mathrm{e}^{6\,\mathrm{A4}[t]})\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{A4}'[t]^2\ \mathrm{YZ7}^{(1,2)}[z,t]\ -$$

$$36\,\mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ ((\mathrm{e}^{2\,\mathrm{A4}[t]}+\mathrm{e}^{4\,\mathrm{A4}[t]})\ \mathrm{A4}'[t]^4-\frac{1}{2}\ (\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]})\ \mathrm{A4}'[t]^2$$

$$\mathrm{A4}''[t]+(\frac{1}{2}\ \mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]}+\frac{1}{2}\ \mathrm{e}^{6\,\mathrm{A4}[t]})\ \mathrm{A4}''[t]^2)\ \mathrm{YZ7}^{(2,0)}[z,t]\ -$$

$$\frac{1}{4}\ \mathrm{A4}'[t]\ \mathrm{A4}^{(3)}[t]\ (-\frac{9}{2}\ (-\frac{1}{3}+\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]}+\frac{1}{3}\ \mathrm{e}^{6\,\mathrm{A4}[t]})\ \mathrm{Q1}\,\mathrm{Cos}[z]^3$$

$$\mathrm{YZ7}[z,t]\ \mathrm{A4}''[t]-\frac{1}{4}\ \mathrm{Q1}\,\mathrm{Cos}[z]^3\ \mathrm{A4}'[t]\ (\mathrm{e}^{-2\,\mathrm{A4}[t]}\ \mathrm{Q1}\ \mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]\ -$$

$$4\ (1+\mathrm{Q1})\ \mathrm{YZ7}[z,t]\ \mathrm{A4}'[t]+4\ \mathrm{YZ7}^{(0,1)}[z,t])\ +$$

$$(\mathrm{e}^{2\,\mathrm{A4}[t]}-2\ \mathrm{e}^{4\,\mathrm{A4}[t]}+\mathrm{e}^{6\,\mathrm{A4}[t]})\ \mathrm{Cos}[z]^3\ \mathrm{YZ7}^{(0,2)}[z,t]+\mathrm{e}^{6\,\mathrm{A4}[t]}$$

$$(\mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]\ (-9+M^2-\frac{1}{4}\ \mathrm{Q1}^2\ \mathrm{A4}'[t]^2)+\mathrm{Q1}\,\mathrm{Cos}[z]^3\ \mathrm{A4}'[t]$$

$$\mathrm{YZ7}^{(0,1)}[z,t]-36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t])+\mathrm{e}^{2\,\mathrm{A4}[t]}$$

$$(\mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]\ (-9+M^2+(-2\,\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{2})\ \mathrm{A4}'[t]^2)+3\,\mathrm{Q1}\,\mathrm{Cos}[z]^3$$

$$\mathrm{A4}'[t]\ \mathrm{YZ7}^{(0,1)}[z,t]-36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t])\ +$$

$$\mathrm{e}^{4\,\mathrm{A4}[t]}\ (-2\,\mathrm{Cos}[z]^3\ \mathrm{YZ7}[z,t]\ (-9+M^2-\frac{1}{2}\ (\mathrm{Q1}+\mathrm{Q1}^2)\ \mathrm{A4}'[t]^2)-3\,\mathrm{Q1}\,\mathrm{Cos}[z]^3$$

$$\mathrm{A4}'[t]\ \mathrm{YZ7}^{(0,1)}[z,t]+72\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{YZ7}^{(1,0)}[z,t])\ -$$

$$36\ (\mathrm{e}^{2\,\mathrm{A4}[t]}-2\ \mathrm{e}^{4\,\mathrm{A4}[t]}+\mathrm{e}^{6\,\mathrm{A4}[t]})\ \mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ \mathrm{YZ7}^{(2,0)}[z,t])\ -$$

$$36\,\mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ \mathrm{A4}'[t]\ ((\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]})\ \mathrm{A4}'[t]^2\ +$$

$$\Big(-\frac{1}{2}\, e^{2\,A4[t]} + e^{4\,A4[t]} - \frac{1}{2}\, e^{6\,A4[t]}\Big)\, A4''[t]\Big)\, YZ7^{(2,1)}[z,t] -$$

$$9\,\Big(e^{2\,A4[t]} - 2\,e^{4\,A4[t]} + e^{6\,A4[t]}\Big)\, Cos[z]\, Sin[z]^2\, A4'[t]^2\, YZ7^{(2,2)}[z,t]\Big)\, dz +$$

$$A4'[t]^2\,\Big(\frac{1}{9}\,\Big(e^{4\,A4[t]} - 2\,e^{6\,A4[t]} + e^{8\,A4[t]}\Big)\, Q1 \int \frac{1}{\sqrt{Sin[z]}}\, Sec[z]^2$$

$$\Big(-\frac{1}{2}\, Q1\, Cos[z]^3\, (-1 + Cosh[2\,A4[t]])\, YZ7[z,t]\, A4''[t]^2 +$$

$$\frac{1}{2}\, Q1\, Cos[z]^3\, (-1 + Cosh[2\,A4[t]])\, YZ7[z,t]\, A4'[t]\, A4^{(3)}[t] +$$

$$Cos[z]^3\,\Big((1 - Q1 + Q1\, Cosh[2\,A4[t]])\, A4'[t]^2 + \frac{1}{2}\,\Big(-1 + e^{2\,A4[t]}\Big)\, A4''[t]\Big)$$

$$YZ7^{(0,2)}[z,t] - \frac{1}{2}\,\Big(-1 + e^{2\,A4[t]}\Big)\, Cos[z]^3\, A4'[t]\, YZ7^{(0,3)}[z,t] + A4'[t]$$

$$\Big(-\frac{1}{4}\, e^{-4\,A4[t]}\, Q1^2\, Cos[z]^3\, A4'[t]^2\,\Big(YZ7[z,t]\, A4'[t] - \frac{1}{2}\, YZ7^{(0,1)}[z,t]\Big) -$$

$$\frac{1}{2}\, e^{2\,A4[t]}\, Cos[z]^3\,\Big(-9 + M^2 + \frac{1}{4}\, Q1^2\, A4'[t]^2\Big)\, YZ7^{(0,1)}[z,t] +$$

$$\frac{1}{2}\, Cos[z]^3\,\Big(-9 + M^2 + \Big(Q1 + \frac{3\,Q1^2}{4}\Big)\, A4'[t]^2\Big)\, YZ7^{(0,1)}[z,t] + \frac{1}{2}\, e^{-2\,A4[t]}\, Q1$$

$$Cos[z]^3\, A4'[t]^2\,\Big(Q1\, YZ7[z,t]\, A4'[t] + \Big(-1 - \frac{3\,Q1}{4}\Big)\, YZ7^{(0,1)}[z,t]\Big) +$$

$$A4'[t]\,\Big(Cos[z]^3\, YZ7[z,t]\,\Big(-9 + M^2 + \Big(-Q1 - \frac{Q1^2}{4}\Big)\, A4'[t]^2\Big) -$$

$$36\,\Big(1 + Cos[z]^2\Big)\, Sin[z]\, YZ7^{(1,0)}[z,t]\Big)\Big) +$$

$$A4''[t]\,\Big(\frac{1}{8}\, e^{-4\,A4[t]}\, Q1^2\, Cos[z]^3\, YZ7[z,t]\, A4'[t]^2 - \frac{3}{8}\, e^{-2\,A4[t]}$$

$$Q1\,\Big(\frac{4}{3} + Q1\Big)\, Cos[z]^3\, YZ7[z,t]\, A4'[t]^2 - \frac{1}{2}\, Cos[z]^3\, YZ7[z,t]$$

$$\Big(-9 + M^2 + \Big(-Q1 - \frac{3\,Q1^2}{4}\Big)\, A4'[t]^2\Big) - \frac{1}{2}\, Q1\, Cos[z]^3\, A4'[t]\, YZ7^{(0,1)}[z,t] +$$

$$\frac{1}{2}\, Q1\, Cos[z]^3\, Cosh[2\,A4[t]]\, A4'[t]\, YZ7^{(0,1)}[z,t] + 18\,\Big(1 + Cos[z]^2\Big)$$

$$Sin[z]\, YZ7^{(1,0)}[z,t] + e^{2\,A4[t]}\,\Big(\frac{1}{2}\, Cos[z]^3\, YZ7[z,t]\,\Big(-9 + M^2 -$$

$$\frac{1}{4}\, Q1^2\, A4'[t]^2\Big) - 18\,\Big(1 + Cos[z]^2\Big)\, Sin[z]\, YZ7^{(1,0)}[z,t]\Big)\Big) +$$

$$18\,\Big(-1 + e^{2\,A4[t]}\Big)\,\Big(1 + Cos[z]^2\Big)\, Sin[z]\, A4'[t]\, YZ7^{(1,1)}[z,t] -$$

$$36\, Cos[z]\, Sin[z]^2\,\Big(A4'[t]^2 + \frac{1}{2}\,\Big(-1 + e^{2\,A4[t]}\Big)\, A4''[t]\Big)\, YZ7^{(2,0)}[z,t] +$$

$$18\,\Big(-1 + e^{2\,A4[t]}\Big)\, Cos[z]\, Sin[z]^2\, A4'[t]\, YZ7^{(2,1)}[z,t]\Big)\, dz + \frac{1}{3}$$

$$\left(\int \frac{1}{\sqrt{\mathrm{Sin}[z]}}\mathrm{Cos}[z]\ \left(\frac{1}{2}\,\mathrm{Q1}\,\left(\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]+\mathrm{A4}'[t]\,\left((-2-\mathrm{Q1})\,\mathrm{YZ7}[z,t]\right.\right.\right.\right.$$

$$\left.\left.\mathrm{A4}'[t]+\mathrm{Q1}\,\mathrm{Cosh}[2\,\mathrm{A4}[t]]\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+2\,\mathrm{YZ7}^{(0,1)}[z,t]\right)\right)+$$

$$\mathrm{e}^{2\,\mathrm{A4}[t]}\,\mathrm{YZ7}^{(0,2)}[z,t]+\mathrm{e}^{2\,\mathrm{A4}[t]}\,\left(\left(-9+\mathrm{M}^2\right)\mathrm{YZ7}[z,t]-\frac{1}{2}\,\mathrm{Q1}\,\mathrm{YZ7}[\right.$$

$$z,t]\,\mathrm{A4}''[t]-\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]+\left(-72\,\mathrm{Tan}[z]-36\,\mathrm{Tan}[z]^3\right)$$

$$\left.\left.\left.\mathrm{YZ7}^{(1,0)}[z,t]\right)-36\,\mathrm{e}^{2\,\mathrm{A4}[t]}\,\mathrm{Tan}[z]^2\,\mathrm{YZ7}^{(2,0)}[z,t]\right)\,dz\right)$$

$$\left(\frac{1}{6}\left(\mathrm{e}^{2\,\mathrm{A4}[t]}\,(-1+\mathrm{Q1})-\frac{\mathrm{Q1}}{4}\right)\mathrm{Q1}\,\mathrm{A4}'[t]^2+\frac{1}{6}\,\mathrm{e}^{8\,\mathrm{A4}[t]}\,\left(-\mathrm{M}^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+\right.$$

$$\mathrm{e}^{4\,\mathrm{A4}[t]}\,\left(-\frac{\mathrm{M}^2}{6}+\left(\frac{\mathrm{Q1}}{3}-\frac{\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)+\frac{1}{3}\,\mathrm{e}^{6\,\mathrm{A4}[t]}\,\left(\mathrm{M}^2+\frac{1}{2}\left(-\mathrm{Q1}+\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+$$

$$\left.\frac{1}{12}\left(\mathrm{e}^{2\,\mathrm{A4}[t]}-3\,\mathrm{e}^{4\,\mathrm{A4}[t]}+3\,\mathrm{e}^{6\,\mathrm{A4}[t]}-\mathrm{e}^{8\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{A4}''[t]\right)+\sqrt{\mathrm{Sin}[z]}$$

$$\left(\frac{1}{2}\left(\mathrm{e}^{4\,\mathrm{A4}[t]}-3\,\mathrm{e}^{6\,\mathrm{A4}[t]}+3\,\mathrm{e}^{8\,\mathrm{A4}[t]}-\mathrm{e}^{10\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]-\right.$$

$$\frac{3}{4}\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\left(\mathrm{e}^{2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+\right.$$

$$\left.4\,\mathrm{e}^{4\,\mathrm{A4}[t]}\,\left(\left(\frac{1}{3}-\mathrm{Q1}\right)\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]-\frac{1}{3}\,\mathrm{YZ7}^{(0,1)}[z,t]\right)\right)+\mathrm{e}^{6\,\mathrm{A4}[t]}$$

$$\left(\mathrm{YZ7}[z,t]\,\left(-9+\mathrm{M}^2+\left(2\,\mathrm{Q1}-\frac{9\,\mathrm{Q1}^2}{2}\right)\mathrm{A4}'[t]^2\right)-3\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]\right)+$$

$$\mathrm{e}^{10\,\mathrm{A4}[t]}\,\left(\mathrm{YZ7}[z,t]\,\left(-9+\mathrm{M}^2-\frac{3}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)-\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]\right)+\mathrm{e}^{8\,\mathrm{A4}[t]}$$

$$\left(-2\,\mathrm{YZ7}[z,t]\,\left(-9+\mathrm{M}^2+\frac{1}{2}\left(\mathrm{Q1}-3\,\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+3\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]\right)+$$

$$\left.\left(\mathrm{e}^{6\,\mathrm{A4}[t]}-2\,\mathrm{e}^{8\,\mathrm{A4}[t]}+\mathrm{e}^{10\,\mathrm{A4}[t]}\right)\mathrm{YZ7}^{(0,2)}[z,t]\right)-396$$

$$\left(\mathrm{e}^{6\,\mathrm{A4}[t]}-2\,\mathrm{e}^{8\,\mathrm{A4}[t]}+\mathrm{e}^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^3\,\mathrm{Sin}[z]^{7/2}\,\mathrm{YZ7}^{(1,0)}[z,t]-216$$

$$\left(\mathrm{e}^{6\,\mathrm{A4}[t]}-2\,\mathrm{e}^{8\,\mathrm{A4}[t]}+\mathrm{e}^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^5\,\mathrm{Sin}[z]^{11/2}\,\mathrm{YZ7}^{(1,0)}[z,t]+2$$

$$\mathrm{Sec}[z]\,\mathrm{Sin}[z]^{3/2}\,\left(\left(\left(-\frac{3}{4}\,\mathrm{Q1}\,\left(4\,\mathrm{e}^{4\,\mathrm{A4}[t]}\,\left(\frac{1}{3}-\mathrm{Q1}\right)+\mathrm{e}^{2\,\mathrm{A4}[t]}\,\mathrm{Q1}\right)\mathrm{A4}'[t]^2+\right.\right.\right.$$

$$\mathrm{e}^{10\,\mathrm{A4}[t]}\,\left(-117+\mathrm{M}^2-\frac{3}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+\mathrm{e}^{6\,\mathrm{A4}[t]}\,\left(-117+\mathrm{M}^2+\right.$$

$$\left.\left(2\,\mathrm{Q1}-\frac{9\,\mathrm{Q1}^2}{2}\right)\mathrm{A4}'[t]^2\right)+\mathrm{e}^{8\,\mathrm{A4}[t]}\,\left(234-2\,\mathrm{M}^2+\left(-\mathrm{Q1}+3\,\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+$$

$$\left.\frac{1}{2}\left(\mathrm{e}^{4\,\mathrm{A4}[t]}-3\,\mathrm{e}^{6\,\mathrm{A4}[t]}+3\,\mathrm{e}^{8\,\mathrm{A4}[t]}-\mathrm{e}^{10\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{A4}''[t]\right)\mathrm{YZ7}^{(1,0)}[z,t]+$$

$$\left.\left(\mathrm{e}^{4\,\mathrm{A4}[t]}-3\,\mathrm{e}^{6\,\mathrm{A4}[t]}+3\,\mathrm{e}^{8\,\mathrm{A4}[t]}-\mathrm{e}^{10\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(1,1)}[z,t]\right)+2$$

$$\left(\mathrm{e}^{6\,\mathrm{A4}[t]}-2\,\mathrm{e}^{8\,\mathrm{A4}[t]}+\mathrm{e}^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]\,\mathrm{Sin}[z]^{3/2}\,\mathrm{YZ7}^{(1,2)}[z,t]-324$$

$$\left(\mathrm{e}^{6\,\mathrm{A4}[t]}-2\,\mathrm{e}^{8\,\mathrm{A4}[t]}+\mathrm{e}^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^2\,\mathrm{Sin}[z]^{5/2}$$

$$\mathrm{YZ7}^{(2,0)}[z,t]-216\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^{4}$$

$$\mathrm{Sin}[z]^{9/2}\,\mathrm{YZ7}^{(2,0)}[z,t]-72\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)$$

$$\left.\left.\left.\mathrm{Sec}[z]^{3}\,\mathrm{Sin}[z]^{7/2}\,\mathrm{YZ7}^{(3,0)}[z,t]\right)\right)dt\right)\mathrm{Sin}[M\,t]\,\mathrm{A4}'[t]+$$

$$M\left(\frac{1}{6}\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\mathrm{Cos}[z]\left(\frac{1}{2}\,\mathrm{Q1}\left(\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]+\mathrm{A4}'[t]\left((-2-\mathrm{Q1})\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+\right.\right.\right.\right.$$

$$\left.\left.\mathrm{Q1}\,\mathrm{Cosh}[2\,\mathrm{A4}[t]]\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]+2\,\mathrm{YZ7}^{(0,1)}[z,t]\right)\right)+$$

$$e^{2\,\mathrm{A4}[t]}\,\mathrm{YZ7}^{(0,2)}[z,t]+e^{2\,\mathrm{A4}[t]}\left(\left(-9+M^{2}\right)\mathrm{YZ7}[z,t]-\frac{1}{2}\,\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]-\right.$$

$$\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ7}^{(0,1)}[z,t]+\left(-72\,\mathrm{Tan}[z]-36\,\mathrm{Tan}[z]^{3}\right)$$

$$\left.\left.\mathrm{YZ7}^{(1,0)}[z,t]\right)-36\,e^{2\,\mathrm{A4}[t]}\,\mathrm{Tan}[z]^{2}\,\mathrm{YZ7}^{(2,0)}[z,t]\right)dz+$$

$$e^{-\frac{1}{4}\mathrm{Q1}\left(e^{-2\,\mathrm{A4}[t]}+2\,\mathrm{A4}[t]\right)}\left(-1+e^{2\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,(\mathrm{c21}\,\mathrm{Cos}[M\,t]+\mathrm{c22}\,\mathrm{Sin}[M\,t])$$

$$\left.\left.\mathrm{A4}'[t]\right)\right)\Big\},$$

$$\mathrm{YZ7}^{(0,4)}[z,t]==\frac{1}{4\,\mathrm{A4}'[t]^{2}}\left(-4\left(-1+e^{-2\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}''[t]^{3}-2\left(-1+e^{-2\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{YZ7}[z,t]\,\mathrm{A4}'[t]^{2}\,\mathrm{A4}^{(4)}[t]+4\left(-2\,\mathrm{A4}'[t]^{2}\left(-9+M^{2}+\frac{1}{2}\,e^{-2\,\mathrm{A4}[t]}\,(-2+\mathrm{Q1})\,\mathrm{Q1}\,\mathrm{A4}'[t]^{2}-\frac{1}{4}\,e^{-4\,\mathrm{A4}[t]}\,\mathrm{Q1}^{2}\,\mathrm{A4}'[t]^{2}+\right.\right.\right.$$

$\left(1-\frac{\text{Q1}^2}{4}-\text{Coth}[\text{A4}[\text{t}]]+\text{Csch}[\text{A4}[\text{t}]]^2\right)$

$\text{A4}'[\text{t}]^2\Big)-$

$\left(-1-\text{Q1}+e^{-2\,\text{A4}[\text{t}]}\,\text{Q1}+\text{Coth}[\text{A4}[\text{t}]]\right)\text{A4}'[\text{t}]^2$

$\text{A4}''[$

$\text{t}]-2$

$\text{A4}''[\text{t}]^2\Big)\ \text{YZ7}^{(0,2)}[$

$\text{z},\text{t}]+8\,\text{A4}'[\text{t}]$

$\big((-1+$

$\text{Coth}[$

$\text{A4}[\text{t}]])$

$\text{A4}'[\text{t}]^2+\text{A4}''[\text{t}]\big)\ \text{YZ7}^{(0,3)}[\text{z},\text{t}]+\text{A4}''[$

$\text{t}]^2\ \Big(-e^{-4\,\text{A4}[\text{t}]}$

Q1^2

$\text{YZ7}[$

$\text{z},$

$\text{t}]$

$\text{A4}'[\text{t}]^2-$

8

$\text{YZ7}[$

$\text{z},$

$\text{t}]\ \Big(-9+$

M^2+

$\frac{1}{8}$

Q1^2

$\text{A4}'[\text{t}]^2\Big)-$

$4\,\text{Q1}\,\text{A4}'[\text{t}]\ \text{YZ7}^{(0,1)}[\text{z},\text{t}]+2\,e^{-2\,\text{A4}[\text{t}]}$

Q1

$\text{A4}'[$

$\text{t}]$

$\big(\text{Q1}\,\text{YZ7}[\text{z},\text{t}]\,\text{A4}'[\text{t}]+$

2

$\text{YZ7}^{(0,1)}[\text{z},\text{t}]\big)+$

$288\ \left(2\,\text{Tan}[\text{z}]+\text{Tan}[\text{z}]^3\right)\ \text{YZ7}^{(1,0)}[\text{z},\text{t}]\Big)-4\,\text{A4}'[$

$\text{t}]$

$\text{A4}''[$

$\text{t}]$

$\Big(-\frac{1}{4}$

$e^{-6\,A4[t]}$

$Q1^{3}$

$YZ7[$

$z,$

$t]$

$A4'[t]^{3}\,+$

3

$e^{-A4[t]}$

$Q1$

$\text{Csch}[$

$A4[$

$t]]\;YZ7[$

$z,\,t]\;A4'[t]^{3}-YZ7[z,$

$t]\;A4'[$

$t]$

$\Big((-3+M)\;(3+M)\;(1+Q1-\text{Coth}[A4[t]])\;-$

$\frac{1}{4}$

$Q1^{3}$

$A4'[t]^{2}\Big)\,+$

$\frac{3}{4}\,e^{-4\,A4[t]}\;Q1^{2}\;A4'[t]^{2}\;\Big(\Big(\frac{2}{3}+Q1\Big)\;YZ7[z,\,t]\;A4'[t]\,-$

$\frac{2}{3}$

$YZ7^{(0,1)}[z,\,t]\Big)\,+$

$\Big(18-2\,M^{2}-\frac{1}{2}\,Q1^{2}\,A4'[t]^{2}\Big)\;YZ7^{(0,1)}[z,\,t]\,+$

36

$(1\,+$

$Q1\,-$

$\text{Coth}[$

$A4[t]])$

$\text{Tan}[z]\;\big(2+\text{Tan}[z]^{2}\big)\;A4'[t]\;YZ7^{(1,0)}[$

$z,$

$t]\,+e^{-2\,A4[t]}$

$Q1\,A4'[$

$t]$

$\Big(YZ7[z,\,t]\;\Big(-9+M^{2}+\frac{1}{2}\;\Big(-Q1-\frac{3\,Q1^{2}}{2}\Big)\;A4'[t]^{2}\Big)\,+$

$(-2+Q1)$

$A4'[t]$

$YZ7^{(0,1)}[z,\,t]+36$

$\big(-2\,\text{Tan}[z]-\text{Tan}[z]^{3}\big)$

```
              YZ7(1,0)[z, t]))) -

        4 A4′[t]2 (- 1/4 ⅇ-6 A4[t] (-2 + Q1) Q13
            YZ7[
              z,
              t]
            A4′[t]4 +
           1
           ──
           16
            ⅇ-8 A4[t]
            Q14
            YZ7[
              z,
              t]
            A4′[t]4 +
           YZ7[
              z,
              t]
             ((-3 + M)2 (3 + M)2 +
                (-18 + 2 M2 + 9 Q12/2 + M2 Q12/2 + 18 Coth[A4[t]] - 2 M2 Coth[A4[t]] -
                    18 Csch[A4[t]]2 + 2 M2 Csch[A4[t]]2) A4′[t]2 +
                (Q14/16 - Q1 Csch[A4[t]]2) A4′[t]4) - 2 (-3 + M) (3 +
               M)
             (-1 + Coth[A4[t]]) A4′[t] YZ7(0,1)[z, t] +
           2
            ⅇ-A4[t]
            Q1
            Csch[
              A4[
                t]]
            A4′[t]3 YZ7(0,1)[z, t] - 72 Tan[
              z] (-90 +
                2 M2 +
                (-405 + M2)
                  Tan[z]2 - 594
                  Tan[z]4 - 270
                  Tan[z]6 -
                1
                ─ (-4 + Q12 + 4 Coth[A4[t]] - 4 Csch[A4[t]]2)
                4
```

$$(2+\mathrm{Tan}[z]^{2})$$

$$\mathrm{A4}'[t]^{2})\ \mathrm{YZ7}^{(1,0)}[z,t]+e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{A4}'[$$

$$t]^{2}$$

$$(-\mathrm{YZ7}[z,t]\ (-18+2\,M^{2}+9\,\mathrm{Q1}+M^{2}\,\mathrm{Q1}+(-\mathrm{Q1}-\frac{\mathrm{Q1}^{2}}{2}+\frac{\mathrm{Q1}^{3}}{4}+\mathrm{Csch}[\mathrm{A4}[t]]^{2})\ \mathrm{A4}'[t]^{2})-$$

$$\mathrm{Q1}\,\mathrm{A4}'[t]$$

$$\mathrm{YZ7}^{(0,1)}[z,t]-36$$

$$(-2+\mathrm{Q1})\ \mathrm{Tan}[z]$$

$$(2+\mathrm{Tan}[z]^{2})$$

$$\mathrm{YZ7}^{(1,0)}[z,t])+$$

$$\frac{1}{2}\,e^{-4\,\mathrm{A4}[t]}\,\mathrm{Q1}^{2}\,\mathrm{A4}'[t]^{2}\ (\mathrm{YZ7}[z,t]\ (9+M^{2}+(-2\,\mathrm{Q1}+\frac{3\,\mathrm{Q1}^{2}}{4})\ \mathrm{A4}'[t]^{2})+$$

$$2\,\mathrm{A4}'[t]\ \mathrm{YZ7}^{(0,1)}[z,t]+$$

$$36\ (2\,\mathrm{Tan}[z]+\mathrm{Tan}[z]^{3})$$

$$\mathrm{YZ7}^{(1,0)}[z,t]))-$$

$$288\,\mathrm{Tan}[z]\ (2+\mathrm{Tan}[z]^{2})\ \mathrm{A4}'[t]\ ((-1+\mathrm{Coth}[\mathrm{A4}[t]])$$

$$\mathrm{A4}'[t]^{2}+$$

$$\mathrm{A4}''[$$

$$t])$$

$$\mathrm{YZ7}^{(1,1)}[z,t]+288$$

$$\mathrm{Tan}[$$

$$z]$$

$$(2+$$

$$\mathrm{Tan}[$$

$$z]^{2})\ \mathrm{A4}'[$$

$$t]^{2}\ \mathrm{YZ7}^{(1,2)}[z,t]+288\,\mathrm{Tan}[$$

$$z]^{2}\ (\mathrm{A4}'[t]^{2}$$

$$(-261+$$

$$M^{2}-$$

$$504\,\mathrm{Tan}[z]^{2}-$$

$$270\,\mathrm{Tan}[z]^{4}+$$

$$\frac{1}{2}\,e^{-2\,\mathrm{A4}[t]}\ (-2+\mathrm{Q1})$$

$$\mathrm{Q1}\,\mathrm{A4}'[t]^{2}-\frac{1}{4}$$

$$e^{-4\,\mathrm{A4}[t]}\,\mathrm{Q1}^{2}\,\mathrm{A4}'[t]^{2}+$$

$$(1-\frac{\mathrm{Q1}^{2}}{4}-\mathrm{Coth}[\mathrm{A4}[t]]+\mathrm{Csch}[\mathrm{A4}[t]]^{2})\ \mathrm{A4}'[t]^{2})+$$

```
      1
      — (-1 - Q1 + ⅇ^(-2 A4[t]) Q1 + Coth[A4[t]])
      2
       A4′[t]^2
       A4′′[
         t] +
     A4′′[t]^2 ) YZ7^(2,0)[z, t] + 4 A4′[
  t]
 A4^(3)[
  t]
 ((-9 + M^2) YZ7[
         z,
                1
         t] + — Q1^2 YZ7[
                4
         z,
         t]
                     1
       A4′[t]^2 + —
                     4
       ⅇ^(-4 A4[t])
       Q1^2
       YZ7[
         z,
         t]
                     3
       A4′[t]^2 + —
                     2
       (-1 +
          ⅇ^(-2 A4[t])) Q1 YZ7[
         z, t] A4′′[t] + Q1 A4′[
         t] YZ7^(0,1)[
         z,
         t] +
                              Q1^2
     ⅇ^(-2 A4[t]) ((Q1 - ————) YZ7[z, t] A4′[t]^2 -
                               2
          Q1 A4′[t] YZ7^(0,1)[z, t]) +
     YZ7^(0,2)[z, t] + 36 (-2 Tan[z] - Tan[z]^3)
       YZ7^(1,0)[
         z,
         t] - 36
       Tan[z]^2 YZ7^(2,0)[
         z,
         t]) -
288 Tan[z]^2 A4′[t] ((-1 + Coth[A4[t]])
```

```
   A4′[t]^2 +
  A4″[
   t])
YZ7^(2,1)[z, t] + 288 Tan[
  z]^2 A4′[
  t]^2
YZ7^(2,2)[z, t] - 31104
Tan[
  z]^3 (4/3 +
  Tan[z]^2)
A4′[t]^2 YZ7^(3,0)[z, t] - 5184
Tan[
  z]^4 A4′[
  t]^2 YZ7^(4,0)[z, t])}
```

In[1140]:=

```
maplestringEQ1 = ReadString["maple-textwrap-stringEQ1_L9i_2026-02-05.txt"];
```

In[1141]:=

```
time1 = Now
If[FindFile["solvedEQ1.mx"] === $Failed,
 AbsoluteTiming[solvedEQ1 = ConvertMapleToMathematicaV2[maplestringEQ1]];
 DumpSave[ToString[header <> "solvedEQ1.mx"], solvedEQ1];
 Print[ToString[header <> "solvedEQ1.mx"]];
 , Get[ "solvedEQ1.mx"]]
Now - time1
```

Out[1141]=

```
Wed 11 Feb 2026 07:57:30 GMT-8
```

```
Pair-Crtn-Univ-same_E-L-eqs-alt-approach-solvedEQ1.mx
```

Out[1143]=

```
57.54158187 min
```

In[1144]:=

```
solvedEQ1
```

Out[1144]=

$$\left\{\left\{\text{YZ0}[z,t]==\frac{1}{2\,M\,\text{Q1}\,\text{A4}'[t]}\text{Csch}[\text{A4}[t]]\left(\frac{1}{2}e^{-\text{A4}[t]}(-2+\text{Q1})\,\text{Q1}\,\text{YZ3}[z,t]\,\text{A4}'[t]^2-\frac{1}{4}e^{-3\,\text{A4}[t]}\,\text{Q1}^2\,\text{YZ3}[z,t]\,\text{A4}'[t]^2-\text{Q1}\,\text{Sinh}[\text{A4}[t]]\,\text{YZ3}[z,t]\,\text{A4}''[t]+e^{\text{A4}[t]}\,\text{YZ3}^{(0,2)}[z,t]+e^{\text{A4}[t]}\left(\text{YZ3}[z,t]\left(-9+M^2-\frac{1}{4}\text{Q1}^2\,\text{A4}'[t]^2\right)+\left(-72\,\text{Tan}[z]-36\,\text{Tan}[z]^3\right)\text{YZ3}^{(1,0)}[z,t]\right)-36\,e^{\text{A4}[t]}\,\text{Tan}[z]^2\,\text{YZ3}^{(2,0)}[z,t]\right),\ \text{YZ1}[z,t]==\frac{1}{\left(-1+e^{2\,\text{A4}[t]}\right)^2 M\,\text{Q1}\,\sqrt{\text{Sin}[z]}\,\text{A4}'[t]^2}\right.\right.$$

$$
\begin{aligned}
&\Bigg(\frac{1}{3}\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\mathrm{Cos}[z]\,\Bigg(-\frac{1}{4}\left(1+e^{4\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}''[t]^2-\frac{1}{2}\left(-\frac{1}{2}+e^{2\,\mathrm{A4}[t]}-\frac{1}{2}e^{4\,\mathrm{A4}[t]}\right)\mathrm{Q1}\\
&\quad\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]\,\mathrm{A4}^{(3)}[t]-\frac{1}{2}\mathrm{A4}'[t]\,\Bigg(e^{4\,\mathrm{A4}[t]}\left(-9+M^2+\frac{1}{4}\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)\mathrm{YZ3}^{(0,1)}[z,t]+\\
&\quad\mathrm{Q1}\,\mathrm{A4}'[t]^2\,\Bigg(-\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+\mathrm{Q1}\,\mathrm{Cosh}[2\,\mathrm{A4}[t]]\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+\\
&\quad\left(1+\frac{3\,\mathrm{Q1}}{4}\right)\mathrm{YZ3}^{(0,1)}[z,t]-\frac{1}{4}e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ3}^{(0,1)}[z,t]\Bigg)\Bigg)+\\
&\quad\left(\frac{1}{2}\left(1+e^{4\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{A4}'[t]^2+e^{2\,\mathrm{A4}[t]}\left((1-\mathrm{Q1})\,\mathrm{A4}'[t]^2-\frac{\mathrm{A4}''[t]}{2}\right)+\frac{1}{2}e^{4\,\mathrm{A4}[t]}\,\mathrm{A4}''[t]\right)\\
&\quad\mathrm{YZ3}^{(0,2)}[z,t]+\frac{1}{2}\left(e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}\right)\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,3)}[z,t]+\\
&\quad\mathrm{A4}''[t]\,\Bigg(\frac{1}{8}\mathrm{Q1}\,\mathrm{A4}'[t]\,\Big((-4-3\,\mathrm{Q1})\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+\\
&\quad2\,\mathrm{YZ3}^{(0,1)}[z,t]\Big)+e^{4\,\mathrm{A4}[t]}\,\Bigg(\frac{1}{2}\mathrm{YZ3}[z,t]\left(-9+M^2-\frac{1}{4}\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+\\
&\quad\frac{1}{4}\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]+18\left(-2\,\mathrm{Tan}[z]-\mathrm{Tan}[z]^3\right)\mathrm{YZ3}^{(1,0)}[z,t]\Bigg)\Bigg)+e^{2\,\mathrm{A4}[t]}\\
&\quad\Bigg(\frac{1}{2}\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}''[t]^2+\mathrm{A4}''[t]\,\Bigg(-\frac{1}{2}\mathrm{YZ3}[z,t]\left(-9+M^2+\left(-\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)-\\
&\quad\frac{1}{2}\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]+18\left(2\,\mathrm{Tan}[z]+\mathrm{Tan}[z]^3\right)\mathrm{YZ3}^{(1,0)}[z,t]\Bigg)+\\
&\quad\mathrm{A4}'[t]\,\Bigg(\frac{1}{2}\left(-9+M^2+\left(\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)\mathrm{YZ3}^{(0,1)}[z,t]+\mathrm{A4}'[t]\\
&\quad\left(\mathrm{YZ3}[z,t]\left(-9+M^2-\mathrm{Q1}\,\mathrm{A4}'[t]^2\right)+36\left(-2\,\mathrm{Tan}[z]-\mathrm{Tan}[z]^3\right)\mathrm{YZ3}^{(1,0)}[z,t]\right)\Bigg)\Bigg)-\\
&\quad18\left(e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}\right)\mathrm{Tan}[z]\left(2+\mathrm{Tan}[z]^2\right)\mathrm{A4}'[t]\,\mathrm{YZ3}^{(1,1)}[z,t]-\\
&\quad36\,\mathrm{Tan}[z]^2\left(e^{2\,\mathrm{A4}[t]}\left(\mathrm{A4}'[t]^2-\frac{\mathrm{A4}''[t]}{2}\right)+\frac{1}{2}e^{4\,\mathrm{A4}[t]}\,\mathrm{A4}''[t]\right)\mathrm{YZ3}^{(2,0)}[z,t]-\\
&\quad18\left(e^{2\,\mathrm{A4}[t]}-e^{4\,\mathrm{A4}[t]}\right)\mathrm{Tan}[z]^2\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(2,1)}[z,t]\Bigg)\,dz+\\
&\frac{1}{12}\mathrm{Q1}\left(1+\frac{1}{2}e^{-3\,\mathrm{A4}[t]}\,\mathrm{Csch}[\mathrm{A4}[t]]+\frac{1}{2}e^{\mathrm{A4}[t]}\,\mathrm{Csch}[\mathrm{A4}[t]]-e^{2\,\mathrm{A4}[t]}\left(1+e^{-3\,\mathrm{A4}[t]}\,\mathrm{Csch}[\mathrm{A4}[t]]\right)\right)\\
&\Bigg(\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\mathrm{Cos}[z]\,\Bigg(\frac{1}{2}\mathrm{Q1}\,\Big(\mathrm{YZ3}[z,t]\,\mathrm{A4}''[t]+\mathrm{A4}'[t]\,\Big((-2-\mathrm{Q1})\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+\\
&\quad\mathrm{Q1}\,\mathrm{Cosh}[2\,\mathrm{A4}[t]]\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+2\,\mathrm{YZ3}^{(0,1)}[z,t]\Big)\Big)+e^{2\,\mathrm{A4}[t]}\,\mathrm{YZ3}^{(0,2)}[z,t]+\\
&\quad e^{2\,\mathrm{A4}[t]}\,\Bigg(\left(-9+M^2\right)\mathrm{YZ3}[z,t]-\frac{1}{2}\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}''[t]-\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]+\\
&\quad\left(-72\,\mathrm{Tan}[z]-36\,\mathrm{Tan}[z]^3\right)\mathrm{YZ3}^{(1,0)}[z,t]\Bigg)-36\,e^{2\,\mathrm{A4}[t]}\,\mathrm{Tan}[z]^2\,\mathrm{YZ3}^{(2,0)}[z,t]\Bigg)\,dz\Bigg)\\
&\mathrm{A4}'[t]^2-\frac{3}{8}e^{-\frac{1}{4}\mathrm{Q1}\left(e^{-2\,\mathrm{A4}[t]}+2\,\mathrm{A4}[t]\right)}\left(-1+e^{2\,\mathrm{A4}[t]}\right)^2\mathrm{Cos}[M\,t]
\end{aligned}
$$

$$\Bigg(\int \frac{1}{A4'[t]^3}\, \mathrm{e}^{\frac{1}{4}\, \mathrm{e}^{-2\, A4[t]}\, Q1+\frac{1}{2}\, (-14+Q1)\, A4[t]}\, \mathrm{Cos}[M\, t]\, \mathrm{Csch}[A4[t]]^3$$

$$\Bigg(-\frac{2}{9}\, \mathrm{e}^{4\, A4[t]} \int \frac{1}{\sqrt{\mathrm{Sin}[z]}}\, \mathrm{Sec}[z]^2\, \Bigg(-\frac{3}{4}\, \Bigg(-\frac{1}{3}+\mathrm{e}^{2\, A4[t]}-\mathrm{e}^{4\, A4[t]}+\frac{1}{3}\, \mathrm{e}^{6\, A4[t]}\Bigg)\, Q1\, \mathrm{Cos}[z]^3$$

$$YZ3[z,\, t]\, A4''[t]^3-\frac{3}{8}\, \Bigg(-\frac{1}{3}+\mathrm{e}^{2\, A4[t]}-\mathrm{e}^{4\, A4[t]}+\frac{1}{3}\, \mathrm{e}^{6\, A4[t]}\Bigg)\, Q1\, \mathrm{Cos}[z]^3\, YZ3[z,\, t]$$

$$A4'[t]^2\, A4^{(4)}[t]+\frac{1}{4}\, \mathrm{Cos}[z]^3\, \Bigg(A4'[t]^2\, \Bigg(\frac{1}{4}\, Q1\, \Big(-4-4\, Q1+\mathrm{e}^{-2\, A4[t]}\, Q1\Big)\, A4'[t]^2+$$

$$\mathrm{e}^{6\, A4[t]}\, \Bigg(-9+M^2+\frac{1}{4}\, Q1^2\, A4'[t]^2\Bigg)+\mathrm{e}^{4\, A4[t]}\, \Big(18-2\, M^2+\Big(4-Q1-Q1^2\Big)$$

$$A4'[t]^2\Big)+\mathrm{e}^{2\, A4[t]}\, \Bigg(-9+M^2+\Bigg(4+2\, Q1+\frac{3\, Q1^2}{2}\Bigg)\, A4'[t]^2\Bigg)\Bigg)-$$

$$\frac{3}{2}\, \Bigg(\mathrm{e}^{4\, A4[t]}\, \Bigg(-\frac{4}{3}-Q1\Bigg)+\frac{1}{3}\, \Big(-1+\mathrm{e}^{6\, A4[t]}\Big)\, Q1+\mathrm{e}^{2\, A4[t]}\, \Bigg(\frac{4}{3}+Q1\Bigg)\Bigg)$$

$$A4'[t]^2\, A4''[t]+2\, \Big(\mathrm{e}^{2\, A4[t]}-2\, \mathrm{e}^{4\, A4[t]}+\mathrm{e}^{6\, A4[t]}\Big)\, A4''[t]^2\Bigg)\, YZ3^{(0,2)}[z,\, t]-$$

$$\frac{3}{4}\, \mathrm{Cos}[z]^3\, A4'[t]\, \Bigg(\Bigg(\mathrm{e}^{4\, A4[t]}\, \Bigg(\frac{4}{3}-Q1\Bigg)+\mathrm{e}^{2\, A4[t]}\, \Bigg(-\frac{4}{3}+Q1\Bigg)+\frac{1}{3}\, \Big(-1+\mathrm{e}^{6\, A4[t]}\Big)\, Q1\Bigg)$$

$$A4'[t]^2+\frac{2}{3}\, \Big(\mathrm{e}^{2\, A4[t]}-2\, \mathrm{e}^{4\, A4[t]}+\mathrm{e}^{6\, A4[t]}\Big)\, A4''[t]\Bigg)\, YZ3^{(0,3)}[z,\, t]+$$

$$\frac{1}{4}\, \Big(\mathrm{e}^{2\, A4[t]}-2\, \mathrm{e}^{4\, A4[t]}+\mathrm{e}^{6\, A4[t]}\Big)\, \mathrm{Cos}[z]^3\, A4'[t]^2\, YZ3^{(0,4)}[z,\, t]+A4''[t]^2$$

$$\Bigg(-\frac{1}{4}\, Q1\, \mathrm{Cos}[z]^3\, A4'[t]\, YZ3^{(0,1)}[z,\, t]+\mathrm{e}^{6\, A4[t]}\, \Bigg(\frac{1}{2}\, (-3+M)\, (3+M)\, \mathrm{Cos}[z]^3$$

$$YZ3[z,\, t]+\frac{1}{4}\, Q1\, \mathrm{Cos}[z]^3\, A4'[t]\, YZ3^{(0,1)}[z,\, t]-18\, \Big(1+\mathrm{Cos}[z]^2\Big)$$

$$\mathrm{Sin}[z]\, YZ3^{(1,0)}[z,\, t]\Bigg)+\mathrm{e}^{2\, A4[t]}\, \Bigg(\frac{1}{2}\, (-3+M)\, (3+M)\, \mathrm{Cos}[z]^3\, YZ3[z,\, t]+$$

$$\frac{3}{4}\, Q1\, \mathrm{Cos}[z]^3\, A4'[t]\, YZ3^{(0,1)}[z,\, t]-18\, \Big(1+\mathrm{Cos}[z]^2\Big)\, \mathrm{Sin}[z]$$

$$YZ3^{(1,0)}[z,\, t]\Bigg)+\mathrm{e}^{4\, A4[t]}\, \Bigg(\Big(9-M^2\Big)\, \mathrm{Cos}[z]^3\, YZ3[z,\, t]-\frac{3}{4}\, Q1\, \mathrm{Cos}[z]^3$$

$$A4'[t]\, YZ3^{(0,1)}[z,\, t]+36\, \Big(1+\mathrm{Cos}[z]^2\Big)\, \mathrm{Sin}[z]\, YZ3^{(1,0)}[z,\, t]\Bigg)\Bigg)+$$

$$A4'[t]^3\, \Bigg(\frac{1}{4}\, \Big(-3+\mathrm{e}^{-2\, A4[t]}\Big)\, Q1^2\, \mathrm{Cos}[z]^3\, A4'[t]^2\, \Big(YZ3[z,\, t]\, A4'[t]-$$

$$YZ3^{(0,1)}[z,\, t]\Big)+\mathrm{e}^{4\, A4[t]}\, \Bigg(\mathrm{Cos}[z]^3\, \Bigg(9-M^2-\Bigg(-Q1-\frac{Q1^2}{4}\Bigg)\, A4'[t]^2\Bigg)\, YZ3^{(0,1)}[$$

$$z,\, t]+A4'[t]\, \Bigg(\mathrm{Cos}[z]^3\, YZ3[z,\, t]\, \Bigg(-9+M^2+\Bigg(-Q1-\frac{Q1^2}{4}\Bigg)\, A4'[t]^2\Bigg)-$$

$$36\, \Big(1+\mathrm{Cos}[z]^2\Big)\, \mathrm{Sin}[z]\, YZ3^{(1,0)}[z,\, t]\Bigg)\Bigg)+$$

$$\mathrm{e}^{2\,\mathrm{A4}[t]}\left(\mathrm{Cos}[z]^3\left(-9+M^2+\left(-\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)\mathrm{YZ3}^{(0,1)}[z,t]+\right.$$

$$\mathrm{A4}'[t]\left(\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]\left(-9+M^2+\left(-\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)-\right.$$

$$\left.\left.\left.36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)\right)\right)-\frac{1}{2}\,\mathrm{A4}'[t]$$

$$\mathrm{A4}''[t]\left(\mathrm{Cos}[z]^3\left(\mathrm{e}^{6\,\mathrm{A4}[t]}\left(-9+M^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)\mathrm{YZ3}^{(0,1)}[z,t]+\right.\right.$$

$$\frac{3}{4}\,\mathrm{Q1}\,\mathrm{A4}'[t]^2\left(-3\,\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+\mathrm{e}^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\left(\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]-\right.\right.$$

$$\left.\left.\left.\frac{1}{3}\,\mathrm{YZ3}^{(0,1)}[z,t]\right)+\frac{4}{3}\,(1+\mathrm{Q1})\,\mathrm{YZ3}^{(0,1)}[z,t]\right)\right)+$$

$$\mathrm{e}^{4\,\mathrm{A4}[t]}\left(-2\,\mathrm{Cos}[z]^3\left(-9+M^2-\frac{1}{2}\left(\mathrm{Q1}+\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)\mathrm{YZ3}^{(0,1)}[z,t]-\right.$$

$$\mathrm{A4}'[t]\left(\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]\left(-9+M^2+3\left(\mathrm{Q1}+\frac{\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)-\right.$$

$$\left.\left.36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)\right)+$$

$$\mathrm{e}^{2\,\mathrm{A4}[t]}\left(\mathrm{Cos}[z]^3\left(-9+M^2+\left(-2\,\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{2}\right)\mathrm{A4}'[t]^2\right)\mathrm{YZ3}^{(0,1)}[z,t]+\right.$$

$$\mathrm{A4}'[t]\left(\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]\left(-9+M^2+3\left(\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)-\right.$$

$$\left.\left.\left.36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)\right)\right)-36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]$$

$$\mathrm{A4}'[t]\left(\left(\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]}\right)\mathrm{A4}'[t]^2+\left(-\frac{1}{2}\,\mathrm{e}^{2\,\mathrm{A4}[t]}+\mathrm{e}^{4\,\mathrm{A4}[t]}-\frac{1}{2}\,\mathrm{e}^{6\,\mathrm{A4}[t]}\right)\right.$$

$$\left.\mathrm{A4}''[t]\right)\mathrm{YZ3}^{(1,1)}[z,t]-9\left(\mathrm{e}^{2\,\mathrm{A4}[t]}-2\,\mathrm{e}^{4\,\mathrm{A4}[t]}+\mathrm{e}^{6\,\mathrm{A4}[t]}\right)$$

$$\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{A4}'[t]^2\,\mathrm{YZ3}^{(1,2)}[z,t]-36\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2$$

$$\left(\left(\mathrm{e}^{2\,\mathrm{A4}[t]}+\mathrm{e}^{4\,\mathrm{A4}[t]}\right)\mathrm{A4}'[t]^4-\frac{1}{2}\left(\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]}\right)\mathrm{A4}'[t]^2\,\mathrm{A4}''[t]+\right.$$

$$\left.\left(\frac{1}{2}\,\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]}+\frac{1}{2}\,\mathrm{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{A4}''[t]^2\right)\mathrm{YZ3}^{(2,0)}[z,t]-\frac{1}{4}\,\mathrm{A4}'[t]\,\mathrm{A4}^{(3)}[t]$$

$$\left(-\frac{9}{2}\left(-\frac{1}{3}+\mathrm{e}^{2\,\mathrm{A4}[t]}-\mathrm{e}^{4\,\mathrm{A4}[t]}+\frac{1}{3}\,\mathrm{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]\,\mathrm{A4}''[t]-\frac{1}{4}\,\mathrm{Q1}\right.$$

$$\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\left(\mathrm{e}^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]-4\,(1+\mathrm{Q1})\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+\right.$$

$$\left.4\,\mathrm{YZ3}^{(0,1)}[z,t]\right)+\left(\mathrm{e}^{2\,\mathrm{A4}[t]}-2\,\mathrm{e}^{4\,\mathrm{A4}[t]}+\mathrm{e}^{6\,\mathrm{A4}[t]}\right)\mathrm{Cos}[z]^3\,\mathrm{YZ3}^{(0,2)}[z,t]+$$

$$\mathrm{e}^{6\,\mathrm{A4}[t]}\left(\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]\left(-9+M^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+\mathrm{Q1}\,\mathrm{Cos}[z]^3\right.$$

$$\left.\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]-36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)+$$

$$e^{2\,A4[t]}\;\Big(Cos[z]^3\;YZ3[z,t]\;\Big(-9+M^2+\Big(-2\,Q1-\frac{3\,Q1^2}{2}\Big)\;A4'[t]^2\Big)+3\,Q1\,Cos[z]^3$$

$$A4'[t]\;YZ3^{(0,1)}[z,t]-36\;\big(1+Cos[z]^2\big)\;Sin[z]\;YZ3^{(1,0)}[z,t]\Big)+$$

$$e^{4\,A4[t]}\;\Big(-2\,Cos[z]^3\;YZ3[z,t]\;\Big(-9+M^2-\frac{1}{2}\;\big(Q1+Q1^2\big)\;A4'[t]^2\Big)-3\,Q1\,Cos[z]^3$$

$$A4'[t]\;YZ3^{(0,1)}[z,t]+72\;\big(1+Cos[z]^2\big)\;Sin[z]\;YZ3^{(1,0)}[z,t]\Big)-$$

$$36\;\big(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\big)\;Cos[z]\;Sin[z]^2\;YZ3^{(2,0)}[z,t]\Big)-$$

$$36\,Cos[z]\;Sin[z]^2\;A4'[t]\;\Big(\big(e^{2\,A4[t]}-e^{4\,A4[t]}\big)\;A4'[t]^2+$$

$$\Big(-\frac{1}{2}\;e^{2\,A4[t]}+e^{4\,A4[t]}-\frac{1}{2}\;e^{6\,A4[t]}\Big)\;A4''[t]\Big)\;YZ3^{(2,1)}[z,t]-$$

$$9\;\big(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\big)\;Cos[z]\;Sin[z]^2\;A4'[t]^2\;YZ3^{(2,2)}[z,t]\Big)\;dz+$$

$$A4'[t]^2\;\Big(\frac{1}{9}\;\big(e^{4\,A4[t]}-2\,e^{6\,A4[t]}+e^{8\,A4[t]}\big)\;Q1\int\frac{1}{\sqrt{Sin[z]}}\,Sec[z]^2$$

$$\Big(-\frac{1}{2}\;Q1\,Cos[z]^3\;(-1+Cosh[2\,A4[t]])\;YZ3[z,t]\;A4''[t]^2+$$

$$\frac{1}{2}\;Q1\,Cos[z]^3\;(-1+Cosh[2\,A4[t]])\;YZ3[z,t]\;A4'[t]\;A4^{(3)}[t]+$$

$$Cos[z]^3\;\Big((1-Q1+Q1\,Cosh[2\,A4[t]])\;A4'[t]^2+\frac{1}{2}\;\big(-1+e^{2\,A4[t]}\big)\;A4''[t]\Big)$$

$$YZ3^{(0,2)}[z,t]-\frac{1}{2}\;\big(-1+e^{2\,A4[t]}\big)\;Cos[z]^3\;A4'[t]\;YZ3^{(0,3)}[z,t]+A4'[t]$$

$$\Big(-\frac{1}{4}\;e^{-4\,A4[t]}\;Q1^2\,Cos[z]^3\;A4'[t]^2\;\Big(YZ3[z,t]\;A4'[t]-\frac{1}{2}\;YZ3^{(0,1)}[z,t]\Big)-$$

$$\frac{1}{2}\;e^{2\,A4[t]}\;Cos[z]^3\;\Big(-9+M^2+\frac{1}{4}\;Q1^2\;A4'[t]^2\Big)\;YZ3^{(0,1)}[z,t]+$$

$$\frac{1}{2}\;Cos[z]^3\;\Big(-9+M^2+\Big(Q1+\frac{3\,Q1^2}{4}\Big)\;A4'[t]^2\Big)\;YZ3^{(0,1)}[z,t]+\frac{1}{2}\;e^{-2\,A4[t]}\;Q1$$

$$Cos[z]^3\;A4'[t]^2\;\Big(Q1\,YZ3[z,t]\;A4'[t]+\Big(-1-\frac{3\,Q1}{4}\Big)\;YZ3^{(0,1)}[z,t]\Big)+$$

$$A4'[t]\;\Big(Cos[z]^3\;YZ3[z,t]\;\Big(-9+M^2+\Big(-Q1-\frac{Q1^2}{4}\Big)\;A4'[t]^2\Big)-$$

$$36\;\big(1+Cos[z]^2\big)\;Sin[z]\;YZ3^{(1,0)}[z,t]\Big)\Big)+$$

$$A4''[t]\;\Big(\frac{1}{8}\;e^{-4\,A4[t]}\;Q1^2\,Cos[z]^3\;YZ3[z,t]\;A4'[t]^2-\frac{3}{8}\;e^{-2\,A4[t]}$$

$$Q1\;\Big(\frac{4}{3}+Q1\Big)\;Cos[z]^3\;YZ3[z,t]\;A4'[t]^2-\frac{1}{2}\;Cos[z]^3\;YZ3[z,t]$$

$$\left(-9+M^2+\left(-\text{Q1}-\frac{3\,\text{Q1}^2}{4}\right)\text{A4}'[t]^2\right)-\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,\text{Cosh}[2\,\text{A4}[t]]\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]+18\left(1+\text{Cos}[z]^2\right)$$

$$\text{Sin}[z]\,\text{YZ3}^{(1,0)}[z,t]+e^{2\,\text{A4}[t]}\ (\frac{1}{2}\,\text{Cos}[z]^3\,\text{YZ3}[z,t]\ (-9+M^2-$$

$$\frac{1}{4}\,\text{Q1}^2\,\text{A4}'[t]^2)-18\left(1+\text{Cos}[z]^2\right)\text{Sin}[z]\,\text{YZ3}^{(1,0)}[z,t]))+$$

$$18\left(-1+e^{2\,\text{A4}[t]}\right)\left(1+\text{Cos}[z]^2\right)\text{Sin}[z]\,\text{A4}'[t]\,\text{YZ3}^{(1,1)}[z,t]-$$

$$36\,\text{Cos}[z]\,\text{Sin}[z]^2\left(\text{A4}'[t]^2+\frac{1}{2}\left(-1+e^{2\,\text{A4}[t]}\right)\text{A4}''[t]\right)\text{YZ3}^{(2,0)}[z,t]+$$

$$18\left(-1+e^{2\,\text{A4}[t]}\right)\text{Cos}[z]\,\text{Sin}[z]^2\,\text{A4}'[t]\,\text{YZ3}^{(2,1)}[z,t])\,dz+\frac{1}{3}$$

$$(\int\frac{1}{\sqrt{\text{Sin}[z]}}\text{Cos}[z]\ (\frac{1}{2}\,\text{Q1}\ (\text{YZ3}[z,t]\,\text{A4}''[t]+\text{A4}'[t]\ ((-2-\text{Q1})\,\text{YZ3}[z,t]$$

$$\text{A4}'[t]+\text{Q1}\,\text{Cosh}[2\,\text{A4}[t]]\,\text{YZ3}[z,t]\,\text{A4}'[t]+2\,\text{YZ3}^{(0,1)}[z,t])))+$$

$$e^{2\,\text{A4}[t]}\,\text{YZ3}^{(0,2)}[z,t]+e^{2\,\text{A4}[t]}\ (\left(-9+M^2\right)\text{YZ3}[z,t]-\frac{1}{2}\,\text{Q1}\,\text{YZ3}[$$

$$z,t]\,\text{A4}''[t]-\text{Q1}\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]+\left(-72\,\text{Tan}[z]-36\,\text{Tan}[z]^3\right)$$

$$\text{YZ3}^{(1,0)}[z,t])-36\,e^{2\,\text{A4}[t]}\,\text{Tan}[z]^2\,\text{YZ3}^{(2,0)}[z,t])\,dz)$$

$$(\frac{1}{6}\left(e^{2\,\text{A4}[t]}\left(-1+\text{Q1}\right)-\frac{\text{Q1}}{4}\right)\text{Q1}\,\text{A4}'[t]^2+\frac{1}{6}\,e^{8\,\text{A4}[t]}\left(-M^2-\frac{1}{4}\,\text{Q1}^2\,\text{A4}'[t]^2\right)+$$

$$e^{4\,\text{A4}[t]}\left(-\frac{M^2}{6}+\left(\frac{\text{Q1}}{3}-\frac{\text{Q1}^2}{4}\right)\text{A4}'[t]^2\right)+\frac{1}{3}\,e^{6\,\text{A4}[t]}\left(M^2+\frac{1}{2}\left(-\text{Q1}+\text{Q1}^2\right)\text{A4}'[t]^2\right)+$$

$$\frac{1}{12}\left(e^{2\,\text{A4}[t]}-3\,e^{4\,\text{A4}[t]}+3\,e^{6\,\text{A4}[t]}-e^{8\,\text{A4}[t]}\right)\text{Q1}\,\text{A4}''[t])+\sqrt{\text{Sin}[z]}$$

$$(\frac{1}{2}\left(e^{4\,\text{A4}[t]}-3\,e^{6\,\text{A4}[t]}+3\,e^{8\,\text{A4}[t]}-e^{10\,\text{A4}[t]}\right)\text{Q1}\,\text{YZ3}[z,t]\,\text{A4}''[t]-$$

$$\frac{3}{4}\,\text{Q1}\,\text{A4}'[t]\ (e^{2\,\text{A4}[t]}\,\text{Q1}\,\text{YZ3}[z,t]\,\text{A4}'[t]+$$

$$4\,e^{4\,\text{A4}[t]}\left(\left(\frac{1}{3}-\text{Q1}\right)\text{YZ3}[z,t]\,\text{A4}'[t]-\frac{1}{3}\,\text{YZ3}^{(0,1)}[z,t]\right))+e^{6\,\text{A4}[t]}$$

$$\left(\text{YZ3}[z,t]\left(-9+M^2+\left(2\,\text{Q1}-\frac{9\,\text{Q1}^2}{2}\right)\text{A4}'[t]^2\right)-3\,\text{Q1}\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]\right)+$$

$$e^{10\,\text{A4}[t]}\left(\text{YZ3}[z,t]\left(-9+M^2-\frac{3}{4}\,\text{Q1}^2\,\text{A4}'[t]^2\right)-\text{Q1}\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]\right)+e^{8\,\text{A4}[t]}$$

$$\left(-2\,\text{YZ3}[z,t]\left(-9+M^2+\frac{1}{2}\left(\text{Q1}-3\,\text{Q1}^2\right)\text{A4}'[t]^2\right)+3\,\text{Q1}\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]\right)+$$

$$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{YZ3}^{(0,2)}[z,t])-396$$

$$\left(e^{6\,\text{A4}[t]}-2\,e^{8\,\text{A4}[t]}+e^{10\,\text{A4}[t]}\right)\text{Sec}[z]^3\,\text{Sin}[z]^{7/2}\,\text{YZ3}^{(1,0)}[z,t]-216$$

$$\left(e^{6\,A4[t]}-2\,e^{8\,A4[t]}+e^{10\,A4[t]}\right)\,\text{Sec}[z]^{5}\,\text{Sin}[z]^{11/2}\,\text{YZ3}^{(1,0)}[z,t]+2$$

$$\text{Sec}[z]\,\text{Sin}[z]^{3/2}\,\Bigg(\Bigg(\left(-\frac{3}{4}\,\text{Q1}\,\left(4\,e^{4\,A4[t]}\,\left(\frac{1}{3}-\text{Q1}\right)+e^{2\,A4[t]}\,\text{Q1}\right)\,A4'[t]^{2}+\right.$$

$$e^{10\,A4[t]}\,\left(-117+M^{2}-\frac{3}{4}\,\text{Q1}^{2}\,A4'[t]^{2}\right)+e^{6\,A4[t]}\,\Bigg(-117+M^{2}+$$

$$\left(2\,\text{Q1}-\frac{9\,\text{Q1}^{2}}{2}\right)\,A4'[t]^{2}\Bigg)+e^{8\,A4[t]}\,\left(234-2\,M^{2}+\left(-\text{Q1}+3\,\text{Q1}^{2}\right)\,A4'[t]^{2}\right)+$$

$$\left.\frac{1}{2}\,\left(e^{4\,A4[t]}-3\,e^{6\,A4[t]}+3\,e^{8\,A4[t]}-e^{10\,A4[t]}\right)\,\text{Q1}\,A4''[t]\right)\,\text{YZ3}^{(1,0)}[z,t]+$$

$$\left(e^{4\,A4[t]}-3\,e^{6\,A4[t]}+3\,e^{8\,A4[t]}-e^{10\,A4[t]}\right)\,\text{Q1}\,A4'[t]\,\text{YZ3}^{(1,1)}[z,t]\Bigg)+2$$

$$\left(e^{6\,A4[t]}-2\,e^{8\,A4[t]}+e^{10\,A4[t]}\right)\,\text{Sec}[z]\,\text{Sin}[z]^{3/2}\,\text{YZ3}^{(1,2)}[z,t]-324$$

$$\left(e^{6\,A4[t]}-2\,e^{8\,A4[t]}+e^{10\,A4[t]}\right)\,\text{Sec}[z]^{2}\,\text{Sin}[z]^{5/2}\,\text{YZ3}^{(2,0)}[z,t]-216$$

$$\left(e^{6\,A4[t]}-2\,e^{8\,A4[t]}+e^{10\,A4[t]}\right)\,\text{Sec}[z]^{4}\,\text{Sin}[z]^{9/2}\,\text{YZ3}^{(2,0)}[z,t]-72$$

$$\left(e^{6\,A4[t]}-2\,e^{8\,A4[t]}+e^{10\,A4[t]}\right)\,\text{Sec}[z]^{3}\,\text{Sin}[z]^{7/2}\,\text{YZ3}^{(3,0)}[z,t]\Bigg)\Bigg)\,dt\Bigg)$$

$$A4'[t]^{2}-\frac{3}{8}\,e^{-\frac{1}{4}\,\text{Q1}\,\left(e^{-2\,A4[t]}+2\,A4[t]\right)}\,\left(-1+e^{2\,A4[t]}\right)^{2}\,\Bigg(\int\frac{1}{A4'[t]^{3}}$$

$$e^{\frac{1}{4}\,e^{-2\,A4[t]}\,\text{Q1}+\frac{1}{2}\,(-14+\text{Q1})\,A4[t]}$$

$$\text{Csch}[A4[t]]^{3}$$

$$\text{Sin}[M\,t]$$

$$\Bigg(-\frac{2}{9}\,e^{4\,A4[t]}\,\int\frac{1}{\sqrt{\text{Sin}[z]}}\,\text{Sec}[z]^{2}$$

$$\Bigg(-\frac{3}{4}\,\left(-\frac{1}{3}+e^{2\,A4[t]}-e^{4\,A4[t]}+\frac{1}{3}\,e^{6\,A4[t]}\right)\,\text{Q1}\,\text{Cos}[z]^{3}\,\text{YZ3}[z,t]\,A4''[t]^{3}-$$

$$\frac{3}{8}\,\left(-\frac{1}{3}+e^{2\,A4[t]}-e^{4\,A4[t]}+\frac{1}{3}\,e^{6\,A4[t]}\right)\,\text{Q1}\,\text{Cos}[z]^{3}\,\text{YZ3}[z,t]\,A4'[t]^{2}\,A4^{(4)}[t]+$$

$$\frac{1}{4}\,\text{Cos}[z]^{3}\,\Bigg(A4'[t]^{2}\,\Bigg(\frac{1}{4}\,\text{Q1}\,\left(-4-4\,\text{Q1}+e^{-2\,A4[t]}\,\text{Q1}\right)\,A4'[t]^{2}+e^{6\,A4[t]}$$

$$\left(-9+M^{2}+\frac{1}{4}\,\text{Q1}^{2}\,A4'[t]^{2}\right)+e^{4\,A4[t]}\,\left(18-2\,M^{2}+\left(4-\text{Q1}-\text{Q1}^{2}\right)\,A4'[t]^{2}\right)+$$

$$e^{2\,A4[t]}\,\left(-9+M^{2}+\left(4+2\,\text{Q1}+\frac{3\,\text{Q1}^{2}}{2}\right)\,A4'[t]^{2}\right)\Bigg)-$$

$$\frac{3}{2}\,\left(e^{4\,A4[t]}\,\left(-\frac{4}{3}-\text{Q1}\right)+\frac{1}{3}\,\left(-1+e^{6\,A4[t]}\right)\,\text{Q1}+e^{2\,A4[t]}\,\left(\frac{4}{3}+\text{Q1}\right)\right)$$

$$A4'[t]^{2}\,A4''[t]+2\,\left(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\right)\,A4''[t]^{2}\Bigg)\,\text{YZ3}^{(0,2)}[z,t]-$$

$$\frac{3}{4}\,\text{Cos}[z]^{3}\,A4'[t]\,\Bigg(\Bigg(\left(e^{4\,A4[t]}\,\left(\frac{4}{3}-\text{Q1}\right)+e^{2\,A4[t]}\,\left(-\frac{4}{3}+\text{Q1}\right)+\frac{1}{3}\,\left(-1+e^{6\,A4[t]}\right)\,\text{Q1}\right)$$

$$A4'[t]^{2}+\frac{2}{3}\,\left(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\right)\,A4''[t]\Bigg)\,\text{YZ3}^{(0,3)}[z,t]+$$

$$\begin{aligned}
&\frac{1}{4}\left(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\right)\mathrm{Cos}[z]^3\,A4'[t]^2\,YZ3^{(0,4)}[z,t]+\\
&A4''[t]^2\,\Big(-\frac{1}{4}\,Q1\,\mathrm{Cos}[z]^3\,A4'[t]\,YZ3^{(0,1)}[z,t]+\\
&\quad e^{6\,A4[t]}\,\Big(\frac{1}{2}\,(-3+M)\,(3+M)\,\mathrm{Cos}[z]^3\,YZ3[z,t]+\frac{1}{4}\,Q1\,\mathrm{Cos}[z]^3\,A4'[t]\\
&\qquad YZ3^{(0,1)}[z,t]-18\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Big)+\\
&\quad e^{2\,A4[t]}\,\Big(\frac{1}{2}\,(-3+M)\,(3+M)\,\mathrm{Cos}[z]^3\,YZ3[z,t]+\frac{3}{4}\,Q1\,\mathrm{Cos}[z]^3\\
&\qquad A4'[t]\,YZ3^{(0,1)}[z,t]-18\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Big)+\\
&\quad e^{4\,A4[t]}\,\Big(\left(9-M^2\right)\mathrm{Cos}[z]^3\,YZ3[z,t]-\frac{3}{4}\,Q1\,\mathrm{Cos}[z]^3\,A4'[t]\\
&\qquad YZ3^{(0,1)}[z,t]+36\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Big)\Big)+A4'[t]^3\\
&\Big(\frac{1}{4}\,\left(-3+e^{-2\,A4[t]}\right)Q1^2\,\mathrm{Cos}[z]^3\,A4'[t]^2\,\left(YZ3[z,t]\,A4'[t]-YZ3^{(0,1)}[z,t]\right)+\\
&\quad e^{4\,A4[t]}\,\Big(\mathrm{Cos}[z]^3\,\Big(9-M^2-\Big(-Q1-\frac{Q1^2}{4}\Big)A4'[t]^2\Big)YZ3^{(0,1)}[z,t]+\\
&\qquad A4'[t]\,\Big(\mathrm{Cos}[z]^3\,YZ3[z,t]\,\Big(-9+M^2+\Big(-Q1-\frac{Q1^2}{4}\Big)A4'[t]^2\Big)-\\
&\qquad\quad 36\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Big)\Big)+\\
&\quad e^{2\,A4[t]}\,\Big(\mathrm{Cos}[z]^3\,\Big(-9+M^2+\Big(-Q1-\frac{3\,Q1^2}{4}\Big)A4'[t]^2\Big)YZ3^{(0,1)}[z,t]+\\
&\qquad A4'[t]\,\Big(\mathrm{Cos}[z]^3\,YZ3[z,t]\,\Big(-9+M^2+\Big(-Q1+\frac{3\,Q1^2}{4}\Big)A4'[t]^2\Big)-\\
&\qquad\quad 36\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Big)\Big)\Big)-\frac{1}{2}\,A4'[t]\\
&A4''[t]\,\Big(\mathrm{Cos}[z]^3\,\Big(e^{6\,A4[t]}\,\Big(-9+M^2-\frac{1}{4}\,Q1^2\,A4'[t]^2\Big)YZ3^{(0,1)}[z,t]+\\
&\qquad \frac{3}{4}\,Q1\,A4'[t]^2\,\Big(-3\,Q1\,YZ3[z,t]\,A4'[t]+e^{-2\,A4[t]}\,Q1\,\Big(YZ3[z,t]\,A4'[t]-\\
&\qquad\quad \frac{1}{3}\,YZ3^{(0,1)}[z,t]\Big)+\frac{4}{3}\,(1+Q1)\,YZ3^{(0,1)}[z,t]\Big)\Big)+\\
&\quad e^{4\,A4[t]}\,\Big(-2\,\mathrm{Cos}[z]^3\,\Big(-9+M^2-\frac{1}{2}\,\left(Q1+Q1^2\right)A4'[t]^2\Big)YZ3^{(0,1)}[z,t]-\\
&\qquad A4'[t]\,\Big(\mathrm{Cos}[z]^3\,YZ3[z,t]\,\Big(-9+M^2+3\,\Big(Q1+\frac{Q1^2}{4}\Big)A4'[t]^2\Big)-\\
&\qquad\quad 36\,\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Big)\Big)+
\end{aligned}$$

$$e^{2\,A4[t]}\ (\mathrm{Cos}[z]^3\ (-9+M^2+(-2\,Q1-\frac{3\,Q1^2}{2})\ A4'[t]^2)\ YZ3^{(0,1)}[z,t]+$$

$$A4'[t]\ (\mathrm{Cos}[z]^3\ YZ3[z,t]\ (-9+M^2+3\ (Q1+\frac{3\,Q1^2}{4})\ A4'[t]^2)-$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ YZ3^{(1,0)}[z,t])))-$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ A4'[t]\ ((e^{2\,A4[t]}-e^{4\,A4[t]})\ A4'[t]^2+$$

$$(-\frac{1}{2}\ e^{2\,A4[t]}+e^{4\,A4[t]}-\frac{1}{2}\ e^{6\,A4[t]})\ A4''[t])\ YZ3^{(1,1)}[z,t]-$$

$$9\ (e^{2\,A4[t]}-2\ e^{4\,A4[t]}+e^{6\,A4[t]})\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ A4'[t]^2\ YZ3^{(1,2)}[z,t]-$$

$$36\,\mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ ((e^{2\,A4[t]}+e^{4\,A4[t]})\ A4'[t]^4-\frac{1}{2}\ (e^{2\,A4[t]}-e^{4\,A4[t]})\ A4'[t]^2$$

$$A4''[t]+(\frac{1}{2}\ e^{2\,A4[t]}-e^{4\,A4[t]}+\frac{1}{2}\ e^{6\,A4[t]})\ A4''[t]^2)\ YZ3^{(2,0)}[z,t]-$$

$$\frac{1}{4}\ A4'[t]\ A4^{(3)}[t]\ (-\frac{9}{2}\ (-\frac{1}{3}+e^{2\,A4[t]}-e^{4\,A4[t]}+\frac{1}{3}\ e^{6\,A4[t]})\ Q1\,\mathrm{Cos}[z]^3$$

$$YZ3[z,t]\ A4''[t]-\frac{1}{4}\ Q1\,\mathrm{Cos}[z]^3\ A4'[t]\ (e^{-2\,A4[t]}\ Q1\ YZ3[z,t]\ A4'[t]-$$

$$4\ (1+Q1)\ YZ3[z,t]\ A4'[t]+4\ YZ3^{(0,1)}[z,t])+$$

$$(e^{2\,A4[t]}-2\ e^{4\,A4[t]}+e^{6\,A4[t]})\ \mathrm{Cos}[z]^3\ YZ3^{(0,2)}[z,t]+e^{6\,A4[t]}$$

$$(\mathrm{Cos}[z]^3\ YZ3[z,t]\ (-9+M^2-\frac{1}{4}\ Q1^2\ A4'[t]^2)+Q1\,\mathrm{Cos}[z]^3\ A4'[t]$$

$$YZ3^{(0,1)}[z,t]-36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ YZ3^{(1,0)}[z,t])+e^{2\,A4[t]}$$

$$(\mathrm{Cos}[z]^3\ YZ3[z,t]\ (-9+M^2+(-2\,Q1-\frac{3\,Q1^2}{2})\ A4'[t]^2)+3\,Q1\,\mathrm{Cos}[z]^3$$

$$A4'[t]\ YZ3^{(0,1)}[z,t]-36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ YZ3^{(1,0)}[z,t])+$$

$$e^{4\,A4[t]}\ (-2\,\mathrm{Cos}[z]^3\ YZ3[z,t]\ (-9+M^2-\frac{1}{2}\ (Q1+Q1^2)\ A4'[t]^2)-3\,Q1\,\mathrm{Cos}[z]^3$$

$$A4'[t]\ YZ3^{(0,1)}[z,t]+72\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ YZ3^{(1,0)}[z,t])-$$

$$36\ (e^{2\,A4[t]}-2\ e^{4\,A4[t]}+e^{6\,A4[t]})\ \mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ YZ3^{(2,0)}[z,t])-$$

$$36\,\mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ A4'[t]\ ((e^{2\,A4[t]}-e^{4\,A4[t]})\ A4'[t]^2+$$

$$(-\frac{1}{2}\ e^{2\,A4[t]}+e^{4\,A4[t]}-\frac{1}{2}\ e^{6\,A4[t]})\ A4''[t])\ YZ3^{(2,1)}[z,t]-$$

$$9\ (e^{2\,A4[t]}-2\ e^{4\,A4[t]}+e^{6\,A4[t]})\ \mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ A4'[t]^2\ YZ3^{(2,2)}[z,t])\ dz+$$

$$A4'[t]^2\ (\frac{1}{9}\ (e^{4\,A4[t]}-2\ e^{6\,A4[t]}+e^{8\,A4[t]})\ Q1\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\mathrm{Sec}[z]^2$$

$$\Big(-\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,(-1+\text{Cosh}[2\,\text{A4}[t]])\,\text{YZ3}[z,t]\,\text{A4}''[t]^2+$$

$$\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,(-1+\text{Cosh}[2\,\text{A4}[t]])\,\text{YZ3}[z,t]\,\text{A4}'[t]\,\text{A4}^{(3)}[t]+$$

$$\text{Cos}[z]^3\,\Big((1-\text{Q1}+\text{Q1}\,\text{Cosh}[2\,\text{A4}[t]])\,\text{A4}'[t]^2+\frac{1}{2}\,\big(-1+e^{2\,\text{A4}[t]}\big)\,\text{A4}''[t]\Big)$$

$$\text{YZ3}^{(0,2)}[z,t]-\frac{1}{2}\,\big(-1+e^{2\,\text{A4}[t]}\big)\,\text{Cos}[z]^3\,\text{A4}'[t]\,\text{YZ3}^{(0,3)}[z,t]+\text{A4}'[t]$$

$$\Big(-\frac{1}{4}\,e^{-4\,\text{A4}[t]}\,\text{Q1}^2\,\text{Cos}[z]^3\,\text{A4}'[t]^2\,\Big(\text{YZ3}[z,t]\,\text{A4}'[t]-\frac{1}{2}\,\text{YZ3}^{(0,1)}[z,t]\Big)-$$

$$\frac{1}{2}\,e^{2\,\text{A4}[t]}\,\text{Cos}[z]^3\,\Big(-9+M^2+\frac{1}{4}\,\text{Q1}^2\,\text{A4}'[t]^2\Big)\,\text{YZ3}^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,\text{Cos}[z]^3\,\Big(-9+M^2+\Big(\text{Q1}+\frac{3\,\text{Q1}^2}{4}\Big)\,\text{A4}'[t]^2\Big)\,\text{YZ3}^{(0,1)}[z,t]+\frac{1}{2}\,e^{-2\,\text{A4}[t]}\,\text{Q1}$$

$$\text{Cos}[z]^3\,\text{A4}'[t]^2\,\Big(\text{Q1}\,\text{YZ3}[z,t]\,\text{A4}'[t]+\Big(-1-\frac{3\,\text{Q1}}{4}\Big)\,\text{YZ3}^{(0,1)}[z,t]\Big)+$$

$$\text{A4}'[t]\,\Big(\text{Cos}[z]^3\,\text{YZ3}[z,t]\,\Big(-9+M^2+\Big(-\text{Q1}-\frac{\text{Q1}^2}{4}\Big)\,\text{A4}'[t]^2\Big)-$$

$$36\,\big(1+\text{Cos}[z]^2\big)\,\text{Sin}[z]\,\text{YZ3}^{(1,0)}[z,t]\Big)\Big)+$$

$$\text{A4}''[t]\,\Big(\frac{1}{8}\,e^{-4\,\text{A4}[t]}\,\text{Q1}^2\,\text{Cos}[z]^3\,\text{YZ3}[z,t]\,\text{A4}'[t]^2-\frac{3}{8}\,e^{-2\,\text{A4}[t]}$$

$$\text{Q1}\,\Big(\frac{4}{3}+\text{Q1}\Big)\,\text{Cos}[z]^3\,\text{YZ3}[z,t]\,\text{A4}'[t]^2-\frac{1}{2}\,\text{Cos}[z]^3\,\text{YZ3}[z,t]$$

$$\Big(-9+M^2+\Big(-\text{Q1}-\frac{3\,\text{Q1}^2}{4}\Big)\,\text{A4}'[t]^2\Big)-\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,\text{Q1}\,\text{Cos}[z]^3\,\text{Cosh}[2\,\text{A4}[t]]\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]+18\,\big(1+\text{Cos}[z]^2\big)$$

$$\text{Sin}[z]\,\text{YZ3}^{(1,0)}[z,t]+e^{2\,\text{A4}[t]}\,\Big(\frac{1}{2}\,\text{Cos}[z]^3\,\text{YZ3}[z,t]\,\Big(-9+M^2-$$

$$\frac{1}{4}\,\text{Q1}^2\,\text{A4}'[t]^2\Big)-18\,\big(1+\text{Cos}[z]^2\big)\,\text{Sin}[z]\,\text{YZ3}^{(1,0)}[z,t]\Big)\Big)+$$

$$18\,\big(-1+e^{2\,\text{A4}[t]}\big)\,\big(1+\text{Cos}[z]^2\big)\,\text{Sin}[z]\,\text{A4}'[t]\,\text{YZ3}^{(1,1)}[z,t]-$$

$$36\,\text{Cos}[z]\,\text{Sin}[z]^2\,\Big(\text{A4}'[t]^2+\frac{1}{2}\,\big(-1+e^{2\,\text{A4}[t]}\big)\,\text{A4}''[t]\Big)\,\text{YZ3}^{(2,0)}[z,t]+$$

$$18\,\big(-1+e^{2\,\text{A4}[t]}\big)\,\text{Cos}[z]\,\text{Sin}[z]^2\,\text{A4}'[t]\,\text{YZ3}^{(2,1)}[z,t]\Big)\,dz+\frac{1}{3}$$

$$\Big(\Big(\int\frac{1}{\sqrt{\text{Sin}[z]}}\,\text{Cos}[z]\,\Big(\frac{1}{2}\,\text{Q1}\,\big(\text{YZ3}[z,t]\,\text{A4}''[t]+\text{A4}'[t]\,\big((-2-\text{Q1})\,\text{YZ3}[z,t]$$

$$\text{A4}'[t]+\text{Q1}\,\text{Cosh}[2\,\text{A4}[t]]\,\text{YZ3}[z,t]\,\text{A4}'[t]+2\,\text{YZ3}^{(0,1)}[z,t]\big)\big)+$$

$$e^{2\,\text{A4}[t]}\,\text{YZ3}^{(0,2)}[z,t]+e^{2\,\text{A4}[t]}\,\Big(\big(-9+M^2\big)\,\text{YZ3}[z,t]-\frac{1}{2}\,\text{Q1}\,\text{YZ3}[$$

$$z,t]\,\text{A4}''[t]-\text{Q1}\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]+\big(-72\,\text{Tan}[z]-36\,\text{Tan}[z]^3\big)$$

$$\text{YZ3}^{(1,0)}[z,t]) - 36\, e^{2\,\text{A4}[t]}\,\text{Tan}[z]^2\,\text{YZ3}^{(2,0)}[z,t])\, dz)$$

$$(\frac{1}{6}\,(e^{2\,\text{A4}[t]}\,(-1+\text{Q1}) - \frac{\text{Q1}}{4})\,\text{Q1}\,\text{A4}'[t]^2 + \frac{1}{6}\,e^{8\,\text{A4}[t]}\,(-M^2 - \frac{1}{4}\,\text{Q1}^2\,\text{A4}'[t]^2) +$$

$$e^{4\,\text{A4}[t]}\,(-\frac{M^2}{6} + (\frac{\text{Q1}}{3} - \frac{\text{Q1}^2}{4})\,\text{A4}'[t]^2) + \frac{1}{3}\,e^{6\,\text{A4}[t]}\,(M^2 + \frac{1}{2}\,(-\text{Q1}+\text{Q1}^2)\,\text{A4}'[t]^2) +$$

$$\frac{1}{12}\,(e^{2\,\text{A4}[t]} - 3\,e^{4\,\text{A4}[t]} + 3\,e^{6\,\text{A4}[t]} - e^{8\,\text{A4}[t]})\,\text{Q1}\,\text{A4}''[t]) + \sqrt{\text{Sin}[z]}$$

$$(\frac{1}{2}\,(e^{4\,\text{A4}[t]} - 3\,e^{6\,\text{A4}[t]} + 3\,e^{8\,\text{A4}[t]} - e^{10\,\text{A4}[t]})\,\text{Q1}\,\text{YZ3}[z,t]\,\text{A4}''[t] -$$

$$\frac{3}{4}\,\text{Q1}\,\text{A4}'[t]\,(e^{2\,\text{A4}[t]}\,\text{Q1}\,\text{YZ3}[z,t]\,\text{A4}'[t] +$$

$$4\,e^{4\,\text{A4}[t]}\,((\frac{1}{3} - \text{Q1})\,\text{YZ3}[z,t]\,\text{A4}'[t] - \frac{1}{3}\,\text{YZ3}^{(0,1)}[z,t])) + e^{6\,\text{A4}[t]}$$

$$(\text{YZ3}[z,t]\,(-9 + M^2 + (2\,\text{Q1} - \frac{9\,\text{Q1}^2}{2})\,\text{A4}'[t]^2) - 3\,\text{Q1}\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]) +$$

$$e^{10\,\text{A4}[t]}\,(\text{YZ3}[z,t]\,(-9 + M^2 - \frac{3}{4}\,\text{Q1}^2\,\text{A4}'[t]^2) - \text{Q1}\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]) + e^{8\,\text{A4}[t]}$$

$$(-2\,\text{YZ3}[z,t]\,(-9 + M^2 + \frac{1}{2}\,(\text{Q1} - 3\,\text{Q1}^2)\,\text{A4}'[t]^2) + 3\,\text{Q1}\,\text{A4}'[t]\,\text{YZ3}^{(0,1)}[z,t]) +$$

$$(e^{6\,\text{A4}[t]} - 2\,e^{8\,\text{A4}[t]} + e^{10\,\text{A4}[t]})\,\text{YZ3}^{(0,2)}[z,t]) - 396$$

$$(e^{6\,\text{A4}[t]} - 2\,e^{8\,\text{A4}[t]} + e^{10\,\text{A4}[t]})\,\text{Sec}[z]^3\,\text{Sin}[z]^{7/2}\,\text{YZ3}^{(1,0)}[z,t] - 216$$

$$(e^{6\,\text{A4}[t]} - 2\,e^{8\,\text{A4}[t]} + e^{10\,\text{A4}[t]})\,\text{Sec}[z]^5\,\text{Sin}[z]^{11/2}\,\text{YZ3}^{(1,0)}[z,t] + 2$$

$$\text{Sec}[z]\,\text{Sin}[z]^{3/2}\,((-\frac{3}{4}\,\text{Q1}\,(4\,e^{4\,\text{A4}[t]}\,(\frac{1}{3} - \text{Q1}) + e^{2\,\text{A4}[t]}\,\text{Q1})\,\text{A4}'[t]^2 +$$

$$e^{10\,\text{A4}[t]}\,(-117 + M^2 - \frac{3}{4}\,\text{Q1}^2\,\text{A4}'[t]^2) + e^{6\,\text{A4}[t]}\,(-117 + M^2 +$$

$$(2\,\text{Q1} - \frac{9\,\text{Q1}^2}{2})\,\text{A4}'[t]^2) + e^{8\,\text{A4}[t]}\,(234 - 2\,M^2 + (-\text{Q1} + 3\,\text{Q1}^2)\,\text{A4}'[t]^2) +$$

$$\frac{1}{2}\,(e^{4\,\text{A4}[t]} - 3\,e^{6\,\text{A4}[t]} + 3\,e^{8\,\text{A4}[t]} - e^{10\,\text{A4}[t]})\,\text{Q1}\,\text{A4}''[t])\,\text{YZ3}^{(1,0)}[z,t] +$$

$$(e^{4\,\text{A4}[t]} - 3\,e^{6\,\text{A4}[t]} + 3\,e^{8\,\text{A4}[t]} - e^{10\,\text{A4}[t]})\,\text{Q1}\,\text{A4}'[t]\,\text{YZ3}^{(1,1)}[z,t]) + 2$$

$$(e^{6\,\text{A4}[t]} - 2\,e^{8\,\text{A4}[t]} + e^{10\,\text{A4}[t]})\,\text{Sec}[z]\,\text{Sin}[z]^{3/2}\,\text{YZ3}^{(1,2)}[z,t] - 324$$

$$(e^{6\,\text{A4}[t]} - 2\,e^{8\,\text{A4}[t]} + e^{10\,\text{A4}[t]})\,\text{Sec}[z]^2\,\text{Sin}[z]^{5/2}\,\text{YZ3}^{(2,0)}[z,t] - 216$$

$$(e^{6\,\text{A4}[t]} - 2\,e^{8\,\text{A4}[t]} + e^{10\,\text{A4}[t]})\,\text{Sec}[z]^4\,\text{Sin}[z]^{9/2}\,\text{YZ3}^{(2,0)}[z,t] - 72$$

$$(e^{6\,\text{A4}[t]} - 2\,e^{8\,\text{A4}[t]} + e^{10\,\text{A4}[t]})\,\text{Sec}[z]^3\,\text{Sin}[z]^{7/2}\,\text{YZ3}^{(3,0)}[z,t]))$$

$$dt)\,\text{Sin}[M\,t]\,\text{A4}'[t]^2 + e^{-\frac{1}{4}\,\text{Q1}\,(e^{-2\,\text{A4}[t]} + 2\,\text{A4}[t])}\,M\,\text{Q1}\,(\text{c12}$$

$$\text{Cos}[$$

```
          M
            t] -
        c11 Sin[M t] + 𝕖^(4 A4[t]) (c12 Cos[M t] - c11 Sin[M t]) +
        2
          𝕖^(2 A4[t])
          (-c12 Cos[M t] +
            c11 Sin[M t]))
    A4′[t]^2 + 3 Q1 √(Sin[z]) A4′[t]^2 (-YZ3[
          z,
          t] - 2 Tan[
          z] YZ3^(1,0)[
          z,
          t]) +
   3 𝕖^(4 A4[t]) Q1 √(Sin[z]) A4′[t]^2 (-YZ3[
          z,
          t] - 2 Tan[
          z] YZ3^(1,0)[
          z,
          t]) +
   6 𝕖^(2 A4[t]) Q1 √(Sin[z]) A4′[t]^2 (YZ3[
        z,
        t] + 2
        Tan[
          z] YZ3^(1,0)[
          z,
          t])),

YZ2[z, t] == 1 / ((-1 + 𝕖^(2 A4[t])) M Q1 √(Sin[z]) A4′[t])
  (3/8
     𝕖^(-1/4 Q1 (𝕖^(-2 A4[t]) + 2 A4[t]))
     (-1 +
        𝕖^(2 A4[t])) Cos[
      M
        t]
     (∫ 1/A4′[t]^3 𝕖^(1/4 𝕖^(-2 A4[t]) Q1 + 1/2 (-14+Q1) A4[t])
          Csch[A4[t]]^3
          Sin[M t]
          (-2/9 𝕖^(4 A4[t]) ∫ 1/√(Sin[z]) Sec[z]^2
```

$$\left(-\frac{3}{4}\left(-\frac{1}{3}+e^{2\,A4[t]}-e^{4\,A4[t]}+\frac{1}{3}e^{6\,A4[t]}\right)Q1\,\mathrm{Cos}[z]^3\,YZ3[z,t]\,A4''[t]^3-\right.$$

$$\frac{3}{8}\left(-\frac{1}{3}+e^{2\,A4[t]}-e^{4\,A4[t]}+\frac{1}{3}e^{6\,A4[t]}\right)Q1\,\mathrm{Cos}[z]^3\,YZ3[z,t]\,A4'[t]^2\,A4^{(4)}[t]+$$

$$\frac{1}{4}\mathrm{Cos}[z]^3\,\Bigg(A4'[t]^2\,\Bigg(\frac{1}{4}Q1\left(-4-4\,Q1+e^{-2\,A4[t]}Q1\right)A4'[t]^2+e^{6\,A4[t]}$$

$$\left(-9+M^2+\frac{1}{4}Q1^2\,A4'[t]^2\right)+e^{4\,A4[t]}\left(18-2\,M^2+\left(4-Q1-Q1^2\right)A4'[t]^2\right)+$$

$$e^{2\,A4[t]}\left(-9+M^2+\left(4+2\,Q1+\frac{3\,Q1^2}{2}\right)A4'[t]^2\right)\Bigg)-$$

$$\frac{3}{2}\left(e^{4\,A4[t]}\left(-\frac{4}{3}-Q1\right)+\frac{1}{3}\left(-1+e^{6\,A4[t]}\right)Q1+e^{2\,A4[t]}\left(\frac{4}{3}+Q1\right)\right)$$

$$A4'[t]^2\,A4''[t]+2\left(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\right)A4''[t]^2\Bigg)\,YZ3^{(0,2)}[z,t]-$$

$$\frac{3}{4}\mathrm{Cos}[z]^3\,A4'[t]\,\Bigg(\Bigg(e^{4\,A4[t]}\left(\frac{4}{3}-Q1\right)+e^{2\,A4[t]}\left(-\frac{4}{3}+Q1\right)+\frac{1}{3}\left(-1+e^{6\,A4[t]}\right)Q1\Bigg)$$

$$A4'[t]^2+\frac{2}{3}\left(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\right)A4''[t]\Bigg)\,YZ3^{(0,3)}[z,t]+$$

$$\frac{1}{4}\left(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\right)\mathrm{Cos}[z]^3\,A4'[t]^2\,YZ3^{(0,4)}[z,t]+$$

$$A4''[t]^2\,\Bigg(-\frac{1}{4}Q1\,\mathrm{Cos}[z]^3\,A4'[t]\,YZ3^{(0,1)}[z,t]+$$

$$e^{6\,A4[t]}\Bigg(\frac{1}{2}(-3+M)\,(3+M)\,\mathrm{Cos}[z]^3\,YZ3[z,t]+\frac{1}{4}Q1\,\mathrm{Cos}[z]^3\,A4'[t]$$

$$YZ3^{(0,1)}[z,t]-18\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Bigg)+$$

$$e^{2\,A4[t]}\Bigg(\frac{1}{2}(-3+M)\,(3+M)\,\mathrm{Cos}[z]^3\,YZ3[z,t]+\frac{3}{4}Q1\,\mathrm{Cos}[z]^3$$

$$A4'[t]\,YZ3^{(0,1)}[z,t]-18\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Bigg)+$$

$$e^{4\,A4[t]}\Bigg(\left(9-M^2\right)\mathrm{Cos}[z]^3\,YZ3[z,t]-\frac{3}{4}Q1\,\mathrm{Cos}[z]^3\,A4'[t]$$

$$YZ3^{(0,1)}[z,t]+36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Bigg)\Bigg)+A4'[t]^3$$

$$\Bigg(\frac{1}{4}\left(-3+e^{-2\,A4[t]}\right)Q1^2\,\mathrm{Cos}[z]^3\,A4'[t]^2\left(YZ3[z,t]\,A4'[t]-YZ3^{(0,1)}[z,t]\right)+$$

$$e^{4\,A4[t]}\Bigg(\mathrm{Cos}[z]^3\left(9-M^2-\left(-Q1-\frac{Q1^2}{4}\right)A4'[t]^2\right)YZ3^{(0,1)}[z,t]+$$

$$A4'[t]\,\Bigg(\mathrm{Cos}[z]^3\,YZ3[z,t]\left(-9+M^2+\left(-Q1-\frac{Q1^2}{4}\right)A4'[t]^2\right)-$$

$$36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Bigg)\Bigg)+$$

$$\mathbb{e}^{2\,A4[t]}\ (\mathrm{Cos}[z]^3\ (-9+M^2+(-Q1-\frac{3\,Q1^2}{4})\ A4'[t]^2)\ YZ3^{(0,1)}[z,t]+$$

$$A4'[t]\ (\mathrm{Cos}[z]^3\ YZ3[z,t]\ (-9+M^2+(-Q1+\frac{3\,Q1^2}{4})\ A4'[t]^2)-$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ YZ3^{(1,0)}[z,t])))-\frac{1}{2}\ A4'[t]$$

$$A4''[t]\ (\mathrm{Cos}[z]^3\ (\mathbb{e}^{6\,A4[t]}\ (-9+M^2-\frac{1}{4}\ Q1^2\ A4'[t]^2)\ YZ3^{(0,1)}[z,t]+$$

$$\frac{3}{4}\ Q1\ A4'[t]^2\ (-3\ Q1\ YZ3[z,t]\ A4'[t]+\mathbb{e}^{-2\,A4[t]}\ Q1\ (YZ3[z,t]\ A4'[t]-$$

$$\frac{1}{3}\ YZ3^{(0,1)}[z,t])+\frac{4}{3}\ (1+Q1)\ YZ3^{(0,1)}[z,t]))+$$

$$\mathbb{e}^{4\,A4[t]}\ (-2\ \mathrm{Cos}[z]^3\ (-9+M^2-\frac{1}{2}\ (Q1+Q1^2)\ A4'[t]^2)\ YZ3^{(0,1)}[z,t]-$$

$$A4'[t]\ (\mathrm{Cos}[z]^3\ YZ3[z,t]\ (-9+M^2+3\ (Q1+\frac{Q1^2}{4})\ A4'[t]^2)-$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ YZ3^{(1,0)}[z,t]))+$$

$$\mathbb{e}^{2\,A4[t]}\ (\mathrm{Cos}[z]^3\ (-9+M^2+(-2\ Q1-\frac{3\,Q1^2}{2})\ A4'[t]^2)\ YZ3^{(0,1)}[z,t]+$$

$$A4'[t]\ (\mathrm{Cos}[z]^3\ YZ3[z,t]\ (-9+M^2+3\ (Q1+\frac{3\,Q1^2}{4})\ A4'[t]^2)-$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ YZ3^{(1,0)}[z,t])))-$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ A4'[t]\ ((\mathbb{e}^{2\,A4[t]}-\mathbb{e}^{4\,A4[t]})\ A4'[t]^2+$$

$$(-\frac{1}{2}\ \mathbb{e}^{2\,A4[t]}+\mathbb{e}^{4\,A4[t]}-\frac{1}{2}\ \mathbb{e}^{6\,A4[t]})\ A4''[t])\ YZ3^{(1,1)}[z,t]-$$

$$9\ (\mathbb{e}^{2\,A4[t]}-2\ \mathbb{e}^{4\,A4[t]}+\mathbb{e}^{6\,A4[t]})\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ A4'[t]^2\ YZ3^{(1,2)}[z,t]-$$

$$36\ \mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ ((\mathbb{e}^{2\,A4[t]}+\mathbb{e}^{4\,A4[t]})\ A4'[t]^4-\frac{1}{2}\ (\mathbb{e}^{2\,A4[t]}-\mathbb{e}^{4\,A4[t]})\ A4'[t]^2$$

$$A4''[t]+(\frac{1}{2}\ \mathbb{e}^{2\,A4[t]}-\mathbb{e}^{4\,A4[t]}+\frac{1}{2}\ \mathbb{e}^{6\,A4[t]})\ A4''[t]^2)\ YZ3^{(2,0)}[z,t]-$$

$$\frac{1}{4}\ A4'[t]\ A4^{(3)}[t]\ (-\frac{9}{2}\ (-\frac{1}{3}+\mathbb{e}^{2\,A4[t]}-\mathbb{e}^{4\,A4[t]}+\frac{1}{3}\ \mathbb{e}^{6\,A4[t]})\ Q1\ \mathrm{Cos}[z]^3$$

$$YZ3[z,t]\ A4''[t]-\frac{1}{4}\ Q1\ \mathrm{Cos}[z]^3\ A4'[t]\ (\mathbb{e}^{-2\,A4[t]}\ Q1\ YZ3[z,t]\ A4'[t]-$$

$$4\ (1+Q1)\ YZ3[z,t]\ A4'[t]+4\ YZ3^{(0,1)}[z,t])+$$

$$(\mathbb{e}^{2\,A4[t]}-2\ \mathbb{e}^{4\,A4[t]}+\mathbb{e}^{6\,A4[t]})\ \mathrm{Cos}[z]^3\ YZ3^{(0,2)}[z,t]+\mathbb{e}^{6\,A4[t]}$$

$$(\mathrm{Cos}[z]^3\ YZ3[z,t]\ (-9+M^2-\frac{1}{4}\ Q1^2\ A4'[t]^2)+Q1\ \mathrm{Cos}[z]^3\ A4'[t]$$

$$YZ3^{(0,1)}[z,t]-36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ YZ3^{(1,0)}[z,t])+\mathbb{e}^{2\,A4[t]}$$

$$\Bigg(Cos[z]^3\, YZ3[z,t]\, \Bigg(-9+M^2+\Bigg(-2\,Q1-\frac{3\,Q1^2}{2}\Bigg)\,A4'[t]^2\Bigg)+3\,Q1\,Cos[z]^3$$

$$A4'[t]\,YZ3^{(0,1)}[z,t]-36\,\big(1+Cos[z]^2\big)\,Sin[z]\,YZ3^{(1,0)}[z,t]\Bigg)+$$

$$e^{4\,A4[t]}\,\Bigg(-2\,Cos[z]^3\,YZ3[z,t]\,\Bigg(-9+M^2-\frac{1}{2}\,\big(Q1+Q1^2\big)\,A4'[t]^2\Bigg)-3\,Q1\,Cos[z]^3$$

$$A4'[t]\,YZ3^{(0,1)}[z,t]+72\,\big(1+Cos[z]^2\big)\,Sin[z]\,YZ3^{(1,0)}[z,t]\Bigg)-$$

$$36\,\big(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\big)\,Cos[z]\,Sin[z]^2\,YZ3^{(2,0)}[z,t]\Bigg)-$$

$$36\,Cos[z]\,Sin[z]^2\,A4'[t]\,\Bigg(\Big(\big(e^{2\,A4[t]}-e^{4\,A4[t]}\big)\,A4'[t]^2+$$

$$\Bigg(-\frac{1}{2}\,e^{2\,A4[t]}+e^{4\,A4[t]}-\frac{1}{2}\,e^{6\,A4[t]}\Bigg)\,A4''[t]\Big)\,YZ3^{(2,1)}[z,t]-$$

$$9\,\big(e^{2\,A4[t]}-2\,e^{4\,A4[t]}+e^{6\,A4[t]}\big)\,Cos[z]\,Sin[z]^2\,A4'[t]^2\,YZ3^{(2,2)}[z,t]\Bigg)\,dz+$$

$$A4'[t]^2\,\Bigg(\frac{1}{9}\,\big(e^{4\,A4[t]}-2\,e^{6\,A4[t]}+e^{8\,A4[t]}\big)\,Q1\int\frac{1}{\sqrt{Sin[z]}}Sec[z]^2$$

$$\Bigg(-\frac{1}{2}\,Q1\,Cos[z]^3\,(-1+Cosh[2\,A4[t]])\,YZ3[z,t]\,A4''[t]^2+$$

$$\frac{1}{2}\,Q1\,Cos[z]^3\,(-1+Cosh[2\,A4[t]])\,YZ3[z,t]\,A4'[t]\,A4^{(3)}[t]+$$

$$Cos[z]^3\,\Bigg((1-Q1+Q1\,Cosh[2\,A4[t]])\,A4'[t]^2+\frac{1}{2}\,\big(-1+e^{2\,A4[t]}\big)\,A4''[t]\Bigg)$$

$$YZ3^{(0,2)}[z,t]-\frac{1}{2}\,\big(-1+e^{2\,A4[t]}\big)\,Cos[z]^3\,A4'[t]\,YZ3^{(0,3)}[z,t]+A4'[t]$$

$$\Bigg(-\frac{1}{4}\,e^{-4\,A4[t]}\,Q1^2\,Cos[z]^3\,A4'[t]^2\,\Bigg(YZ3[z,t]\,A4'[t]-\frac{1}{2}\,YZ3^{(0,1)}[z,t]\Bigg)-$$

$$\frac{1}{2}\,e^{2\,A4[t]}\,Cos[z]^3\,\Bigg(-9+M^2+\frac{1}{4}\,Q1^2\,A4'[t]^2\Bigg)\,YZ3^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,Cos[z]^3\,\Bigg(-9+M^2+\Bigg(Q1+\frac{3\,Q1^2}{4}\Bigg)\,A4'[t]^2\Bigg)\,YZ3^{(0,1)}[z,t]+\frac{1}{2}\,e^{-2\,A4[t]}\,Q1$$

$$Cos[z]^3\,A4'[t]^2\,\Bigg(Q1\,YZ3[z,t]\,A4'[t]+\Bigg(-1-\frac{3\,Q1}{4}\Bigg)\,YZ3^{(0,1)}[z,t]\Bigg)+$$

$$A4'[t]\,\Bigg(Cos[z]^3\,YZ3[z,t]\,\Bigg(-9+M^2+\Bigg(-Q1-\frac{Q1^2}{4}\Bigg)\,A4'[t]^2\Bigg)-$$

$$36\,\big(1+Cos[z]^2\big)\,Sin[z]\,YZ3^{(1,0)}[z,t]\Bigg)\Bigg)+$$

$$A4''[t]\,\Bigg(\frac{1}{8}\,e^{-4\,A4[t]}\,Q1^2\,Cos[z]^3\,YZ3[z,t]\,A4'[t]^2-\frac{3}{8}\,e^{-2\,A4[t]}$$

$$Q1\,\Bigg(\frac{4}{3}+Q1\Bigg)\,Cos[z]^3\,YZ3[z,t]\,A4'[t]^2-\frac{1}{2}\,Cos[z]^3\,YZ3[z,t]$$

$$\left(-9+M^2+\left(-\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)-\frac{1}{2}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{Cosh}[2\,\mathrm{A4}[t]]\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]+18\left(1+\mathrm{Cos}[z]^2\right)$$

$$\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]+e^{2\,\mathrm{A4}[t]}\ (\frac{1}{2}\,\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]\ (-9+M^2-$$

$$\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2)-18\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]))+$$

$$18\left(-1+e^{2\,\mathrm{A4}[t]}\right)\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(1,1)}[z,t]-$$

$$36\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\left(\mathrm{A4}'[t]^2+\frac{1}{2}\left(-1+e^{2\,\mathrm{A4}[t]}\right)\mathrm{A4}''[t]\right)\mathrm{YZ3}^{(2,0)}[z,t]+$$

$$18\left(-1+e^{2\,\mathrm{A4}[t]}\right)\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(2,1)}[z,t])\,dz+\frac{1}{3}$$

$$(\left(\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\mathrm{Cos}[z]\ (\frac{1}{2}\,\mathrm{Q1}\ (\mathrm{YZ3}[z,t]\,\mathrm{A4}''[t]+\mathrm{A4}'[t]\ ((-2-\mathrm{Q1})\,\mathrm{YZ3}[z,t]\right.$$

$$\mathrm{A4}'[t]+\mathrm{Q1}\,\mathrm{Cosh}[2\,\mathrm{A4}[t]]\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+2\,\mathrm{YZ3}^{(0,1)}[z,t]))+$$

$$e^{2\,\mathrm{A4}[t]}\,\mathrm{YZ3}^{(0,2)}[z,t]+e^{2\,\mathrm{A4}[t]}\ (\left(-9+M^2\right)\mathrm{YZ3}[z,t]-\frac{1}{2}\,\mathrm{Q1}\,\mathrm{YZ3}[$$

$$z,t]\,\mathrm{A4}''[t]-\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]+\left(-72\,\mathrm{Tan}[z]-36\,\mathrm{Tan}[z]^3\right)$$

$$\mathrm{YZ3}^{(1,0)}[z,t])-36\,e^{2\,\mathrm{A4}[t]}\,\mathrm{Tan}[z]^2\,\mathrm{YZ3}^{(2,0)}[z,t])\,dz)$$

$$(\frac{1}{6}\left(e^{2\,\mathrm{A4}[t]}\left(-1+\mathrm{Q1}\right)-\frac{\mathrm{Q1}}{4}\right)\mathrm{Q1}\,\mathrm{A4}'[t]^2+\frac{1}{6}\,e^{8\,\mathrm{A4}[t]}\left(-M^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)+$$

$$e^{4\,\mathrm{A4}[t]}\left(-\frac{M^2}{6}+\left(\frac{\mathrm{Q1}}{3}-\frac{\mathrm{Q1}^2}{4}\right)\mathrm{A4}'[t]^2\right)+\frac{1}{3}\,e^{6\,\mathrm{A4}[t]}\left(M^2+\frac{1}{2}\left(-\mathrm{Q1}+\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+$$

$$\frac{1}{12}\left(e^{2\,\mathrm{A4}[t]}-3\,e^{4\,\mathrm{A4}[t]}+3\,e^{6\,\mathrm{A4}[t]}-e^{8\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{A4}''[t])+\sqrt{\mathrm{Sin}[z]}$$

$$(\frac{1}{2}\left(e^{4\,\mathrm{A4}[t]}-3\,e^{6\,\mathrm{A4}[t]}+3\,e^{8\,\mathrm{A4}[t]}-e^{10\,\mathrm{A4}[t]}\right)\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}''[t]-$$

$$\frac{3}{4}\,\mathrm{Q1}\,\mathrm{A4}'[t]\ (e^{2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+$$

$$4\,e^{4\,\mathrm{A4}[t]}\left(\left(\frac{1}{3}-\mathrm{Q1}\right)\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]-\frac{1}{3}\,\mathrm{YZ3}^{(0,1)}[z,t]\right))+e^{6\,\mathrm{A4}[t]}$$

$$\left(\mathrm{YZ3}[z,t]\left(-9+M^2+\left(2\,\mathrm{Q1}-\frac{9\,\mathrm{Q1}^2}{2}\right)\mathrm{A4}'[t]^2\right)-3\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]\right)+$$

$$e^{10\,\mathrm{A4}[t]}\left(\mathrm{YZ3}[z,t]\left(-9+M^2-\frac{3}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)-\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]\right)+e^{8\,\mathrm{A4}[t]}$$

$$\left(-2\,\mathrm{YZ3}[z,t]\left(-9+M^2+\frac{1}{2}\left(\mathrm{Q1}-3\,\mathrm{Q1}^2\right)\mathrm{A4}'[t]^2\right)+3\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]\right)+$$

$$\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{YZ3}^{(0,2)}[z,t])-396$$

$$\left(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\right)\mathrm{Sec}[z]^3\,\mathrm{Sin}[z]^{7/2}\,\mathrm{YZ3}^{(1,0)}[z,t]-216$$

$$\left(e^{6\,A4[t]} - 2\,e^{8\,A4[t]} + e^{10\,A4[t]}\right)\,\mathrm{Sec}[z]^5\,\mathrm{Sin}[z]^{11/2}\,YZ3^{(1,0)}[z,t] + 2$$

$$\mathrm{Sec}[z]\,\mathrm{Sin}[z]^{3/2}\,\Bigg(\Bigg(\left(-\frac{3}{4}\,Q1\,\left(4\,e^{4\,A4[t]}\,\left(\frac{1}{3} - Q1\right) + e^{2\,A4[t]}\,Q1\right)\,A4'[t]^2 +\right.$$

$$e^{10\,A4[t]}\,\left(-117 + M^2 - \frac{3}{4}\,Q1^2\,A4'[t]^2\right) + e^{6\,A4[t]}\,\Bigg(-117 + M^2 +$$

$$\left.\left(2\,Q1 - \frac{9\,Q1^2}{2}\right)\,A4'[t]^2\Bigg) + e^{8\,A4[t]}\,\left(234 - 2\,M^2 + \left(-Q1 + 3\,Q1^2\right)\,A4'[t]^2\right) +\right.$$

$$\left.\frac{1}{2}\,\left(e^{4\,A4[t]} - 3\,e^{6\,A4[t]} + 3\,e^{8\,A4[t]} - e^{10\,A4[t]}\right)\,Q1\,A4''[t]\right)\,YZ3^{(1,0)}[z,t] +$$

$$\left.\left(e^{4\,A4[t]} - 3\,e^{6\,A4[t]} + 3\,e^{8\,A4[t]} - e^{10\,A4[t]}\right)\,Q1\,A4'[t]\,YZ3^{(1,1)}[z,t]\right) + 2$$

$$\left(e^{6\,A4[t]} - 2\,e^{8\,A4[t]} + e^{10\,A4[t]}\right)\,\mathrm{Sec}[z]\,\mathrm{Sin}[z]^{3/2}\,YZ3^{(1,2)}[z,t] - 324$$

$$\left(e^{6\,A4[t]} - 2\,e^{8\,A4[t]} + e^{10\,A4[t]}\right)\,\mathrm{Sec}[z]^2\,\mathrm{Sin}[z]^{5/2}\,YZ3^{(2,0)}[z,t] - 216$$

$$\left(e^{6\,A4[t]} - 2\,e^{8\,A4[t]} + e^{10\,A4[t]}\right)\,\mathrm{Sec}[z]^4\,\mathrm{Sin}[z]^{9/2}\,YZ3^{(2,0)}[z,t] - 72$$

$$\left.\left.\left.\left(e^{6\,A4[t]} - 2\,e^{8\,A4[t]} + e^{10\,A4[t]}\right)\,\mathrm{Sec}[z]^3\,\mathrm{Sin}[z]^{7/2}\,YZ3^{(3,0)}[z,t]\right)\right)\,dt\right)$$

$$A4'[t] - \frac{3}{8}\,e^{-\frac{1}{4}\,Q1\,\left(e^{-2\,A4[t]}+2\,A4[t]\right)}\,\left(-1 + e^{2\,A4[t]}\right)\,\Bigg(\int\frac{1}{A4'[t]^3}$$

$$e^{\frac{1}{4}\,e^{-2\,A4[t]}\,Q1+\frac{1}{2}\,(-14+Q1)\,A4[t]}$$

$$\mathrm{Cos}[$$

$$M\,t]\,\mathrm{Csch}[A4[t]]^3$$

$$\Bigg(-\frac{2}{9}\,e^{4\,A4[t]}\,\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\,\mathrm{Sec}[z]^2\,\Bigg(-\frac{3}{4}\,\left(-\frac{1}{3} + e^{2\,A4[t]} - e^{4\,A4[t]} + \frac{1}{3}\,e^{6\,A4[t]}\right)\,Q1\,\mathrm{Cos}[z]^3$$

$$YZ3[z,t]\,A4''[t]^3 - \frac{3}{8}\,\left(-\frac{1}{3} + e^{2\,A4[t]} - e^{4\,A4[t]} + \frac{1}{3}\,e^{6\,A4[t]}\right)\,Q1\,\mathrm{Cos}[z]^3\,YZ3[z,t]$$

$$A4'[t]^2\,A4^{(4)}[t] + \frac{1}{4}\,\mathrm{Cos}[z]^3\,\Bigg(A4'[t]^2\,\Bigg(\frac{1}{4}\,Q1\,\left(-4 - 4\,Q1 + e^{-2\,A4[t]}\,Q1\right)\,A4'[t]^2 +$$

$$e^{6\,A4[t]}\,\left(-9 + M^2 + \frac{1}{4}\,Q1^2\,A4'[t]^2\right) + e^{4\,A4[t]}\,\Big(18 - 2\,M^2 + \left(4 - Q1 - Q1^2\right)$$

$$\left.\left.A4'[t]^2\Big) + e^{2\,A4[t]}\,\left(-9 + M^2 + \left(4 + 2\,Q1 + \frac{3\,Q1^2}{2}\right)\,A4'[t]^2\right)\right)\right. -$$

$$\frac{3}{2}\,\left(e^{4\,A4[t]}\,\left(-\frac{4}{3} - Q1\right) + \frac{1}{3}\,\left(-1 + e^{6\,A4[t]}\right)\,Q1 + e^{2\,A4[t]}\,\left(\frac{4}{3} + Q1\right)\right)$$

$$\left.A4'[t]^2\,A4''[t] + 2\,\left(e^{2\,A4[t]} - 2\,e^{4\,A4[t]} + e^{6\,A4[t]}\right)\,A4''[t]^2\right)\,YZ3^{(0,2)}[z,t] -$$

$$\frac{3}{4}\,\mathrm{Cos}[z]^3\,A4'[t]\,\Bigg(\Bigg(e^{4\,A4[t]}\,\left(\frac{4}{3} - Q1\right) + e^{2\,A4[t]}\,\left(-\frac{4}{3} + Q1\right) + \frac{1}{3}\,\left(-1 + e^{6\,A4[t]}\right)\,Q1\Bigg)$$

$$\left.A4'[t]^2 + \frac{2}{3}\,\left(e^{2\,A4[t]} - 2\,e^{4\,A4[t]} + e^{6\,A4[t]}\right)\,A4''[t]\right)\,YZ3^{(0,3)}[z,t] +$$

$$\frac{1}{4}\,\left(e^{2\,A4[t]} - 2\,e^{4\,A4[t]} + e^{6\,A4[t]}\right)\,\mathrm{Cos}[z]^3\,A4'[t]^2\,YZ3^{(0,4)}[z,t] +$$

$$\mathrm{A4}''[t]^2\ \left(-\frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]+\right.$$

$$e^{6\,\mathrm{A4}[t]}\ \left(\frac{1}{2}\,(-3+M)\,(3+M)\,\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]+\frac{1}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\right.$$

$$\left.\mathrm{YZ3}^{(0,1)}[z,t]-18\,\left(1+\mathrm{Cos}[z]^2\right)\,\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)+$$

$$e^{2\,\mathrm{A4}[t]}\ \left(\frac{1}{2}\,(-3+M)\,(3+M)\,\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]+\frac{3}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\right.$$

$$\left.\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]-18\,\left(1+\mathrm{Cos}[z]^2\right)\,\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)+$$

$$e^{4\,\mathrm{A4}[t]}\ \left(\left(9-M^2\right)\,\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]-\frac{3}{4}\,\mathrm{Q1}\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]\right.$$

$$\left.\left.\mathrm{YZ3}^{(0,1)}[z,t]+36\,\left(1+\mathrm{Cos}[z]^2\right)\,\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)\right)+\mathrm{A4}'[t]^3$$

$$\left(\frac{1}{4}\,\left(-3+e^{-2\,\mathrm{A4}[t]}\right)\,\mathrm{Q1}^2\,\mathrm{Cos}[z]^3\,\mathrm{A4}'[t]^2\,\left(\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]-\mathrm{YZ3}^{(0,1)}[z,t]\right)+\right.$$

$$e^{4\,\mathrm{A4}[t]}\ \left(\mathrm{Cos}[z]^3\,\left(9-M^2-\left(-\mathrm{Q1}-\frac{\mathrm{Q1}^2}{4}\right)\,\mathrm{A4}'[t]^2\right)\,\mathrm{YZ3}^{(0,1)}[z,t]+\right.$$

$$\mathrm{A4}'[t]\ \left(\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]\ \left(-9+M^2+\left(-\mathrm{Q1}-\frac{\mathrm{Q1}^2}{4}\right)\,\mathrm{A4}'[t]^2\right)-\right.$$

$$\left.\left.36\,\left(1+\mathrm{Cos}[z]^2\right)\,\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)\right)+$$

$$e^{2\,\mathrm{A4}[t]}\ \left(\mathrm{Cos}[z]^3\,\left(-9+M^2+\left(-\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{4}\right)\,\mathrm{A4}'[t]^2\right)\,\mathrm{YZ3}^{(0,1)}[z,t]+\right.$$

$$\mathrm{A4}'[t]\ \left(\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]\ \left(-9+M^2+\left(-\mathrm{Q1}+\frac{3\,\mathrm{Q1}^2}{4}\right)\,\mathrm{A4}'[t]^2\right)-\right.$$

$$\left.\left.\left.36\,\left(1+\mathrm{Cos}[z]^2\right)\,\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)\right)\right)-\frac{1}{2}\,\mathrm{A4}'[t]$$

$$\mathrm{A4}''[t]\ \left(\mathrm{Cos}[z]^3\ \left(e^{6\,\mathrm{A4}[t]}\ \left(-9+M^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\right)\,\mathrm{YZ3}^{(0,1)}[z,t]+\right.\right.$$

$$\frac{3}{4}\,\mathrm{Q1}\,\mathrm{A4}'[t]^2\ \left(-3\,\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\ \left(\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]-\right.\right.$$

$$\left.\left.\left.\frac{1}{3}\,\mathrm{YZ3}^{(0,1)}[z,t]\right)+\frac{4}{3}\,(1+\mathrm{Q1})\,\mathrm{YZ3}^{(0,1)}[z,t]\right)\right)+$$

$$e^{4\,\mathrm{A4}[t]}\ \left(-2\,\mathrm{Cos}[z]^3\ \left(-9+M^2-\frac{1}{2}\,\left(\mathrm{Q1}+\mathrm{Q1}^2\right)\,\mathrm{A4}'[t]^2\right)\,\mathrm{YZ3}^{(0,1)}[z,t]-\right.$$

$$\mathrm{A4}'[t]\ \left(\mathrm{Cos}[z]^3\,\mathrm{YZ3}[z,t]\ \left(-9+M^2+3\,\left(\mathrm{Q1}+\frac{\mathrm{Q1}^2}{4}\right)\,\mathrm{A4}'[t]^2\right)-\right.$$

$$\left.\left.36\,\left(1+\mathrm{Cos}[z]^2\right)\,\mathrm{Sin}[z]\,\mathrm{YZ3}^{(1,0)}[z,t]\right)\right)+$$

$$e^{2\,\mathrm{A4}[t]}\ \left(\mathrm{Cos}[z]^3\ \left(-9+M^2+\left(-2\,\mathrm{Q1}-\frac{3\,\mathrm{Q1}^2}{2}\right)\,\mathrm{A4}'[t]^2\right)\,\mathrm{YZ3}^{(0,1)}[z,t]+\right.$$

$$A4'[t]\ (\mathrm{Cos}[z]^3\ \mathrm{YZ3}[z,t]\ (-9+M^2+3\ (Q1+\frac{3\ Q1^2}{4})\ A4'[t]^2)-$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{YZ3}^{(1,0)}[z,t])))-$$

$$36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ A4'[t]\ ((e^{2\,A4[t]}-e^{4\,A4[t]})\ A4'[t]^2+$$

$$(-\frac{1}{2}\ e^{2\,A4[t]}+e^{4\,A4[t]}-\frac{1}{2}\ e^{6\,A4[t]})\ A4''[t])\ \mathrm{YZ3}^{(1,1)}[z,t]-$$

$$9\ (e^{2\,A4[t]}-2\ e^{4\,A4[t]}+e^{6\,A4[t]})\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ A4'[t]^2\ \mathrm{YZ3}^{(1,2)}[z,t]-$$

$$36\ \mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ ((e^{2\,A4[t]}+e^{4\,A4[t]})\ A4'[t]^4-\frac{1}{2}\ (e^{2\,A4[t]}-e^{4\,A4[t]})\ A4'[t]^2$$

$$A4''[t]+(\frac{1}{2}\ e^{2\,A4[t]}-e^{4\,A4[t]}+\frac{1}{2}\ e^{6\,A4[t]})\ A4''[t]^2)\ \mathrm{YZ3}^{(2,0)}[z,t]-$$

$$\frac{1}{4}\ A4'[t]\ A4^{(3)}[t]\ (-\frac{9}{2}\ (-\frac{1}{3}+e^{2\,A4[t]}-e^{4\,A4[t]}+\frac{1}{3}\ e^{6\,A4[t]})\ Q1\ \mathrm{Cos}[z]^3$$

$$\mathrm{YZ3}[z,t]\ A4''[t]-\frac{1}{4}\ Q1\ \mathrm{Cos}[z]^3\ A4'[t]\ (e^{-2\,A4[t]}\ Q1\ \mathrm{YZ3}[z,t]\ A4'[t]-$$

$$4\ (1+Q1)\ \mathrm{YZ3}[z,t]\ A4'[t]+4\ \mathrm{YZ3}^{(0,1)}[z,t])+$$

$$(e^{2\,A4[t]}-2\ e^{4\,A4[t]}+e^{6\,A4[t]})\ \mathrm{Cos}[z]^3\ \mathrm{YZ3}^{(0,2)}[z,t]+e^{6\,A4[t]}$$

$$(\mathrm{Cos}[z]^3\ \mathrm{YZ3}[z,t]\ (-9+M^2-\frac{1}{4}\ Q1^2\ A4'[t]^2)+Q1\ \mathrm{Cos}[z]^3\ A4'[t]$$

$$\mathrm{YZ3}^{(0,1)}[z,t]-36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{YZ3}^{(1,0)}[z,t])+e^{2\,A4[t]}$$

$$(\mathrm{Cos}[z]^3\ \mathrm{YZ3}[z,t]\ (-9+M^2+(-2\ Q1-\frac{3\ Q1^2}{2})\ A4'[t]^2)+3\ Q1\ \mathrm{Cos}[z]^3$$

$$A4'[t]\ \mathrm{YZ3}^{(0,1)}[z,t]-36\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{YZ3}^{(1,0)}[z,t])+$$

$$e^{4\,A4[t]}\ (-2\ \mathrm{Cos}[z]^3\ \mathrm{YZ3}[z,t]\ (-9+M^2-\frac{1}{2}\ (Q1+Q1^2)\ A4'[t]^2)-3\ Q1\ \mathrm{Cos}[z]^3$$

$$A4'[t]\ \mathrm{YZ3}^{(0,1)}[z,t]+72\ (1+\mathrm{Cos}[z]^2)\ \mathrm{Sin}[z]\ \mathrm{YZ3}^{(1,0)}[z,t])-$$

$$36\ (e^{2\,A4[t]}-2\ e^{4\,A4[t]}+e^{6\,A4[t]})\ \mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ \mathrm{YZ3}^{(2,0)}[z,t])-$$

$$36\ \mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ A4'[t]\ ((e^{2\,A4[t]}-e^{4\,A4[t]})\ A4'[t]^2+$$

$$(-\frac{1}{2}\ e^{2\,A4[t]}+e^{4\,A4[t]}-\frac{1}{2}\ e^{6\,A4[t]})\ A4''[t])\ \mathrm{YZ3}^{(2,1)}[z,t]-$$

$$9\ (e^{2\,A4[t]}-2\ e^{4\,A4[t]}+e^{6\,A4[t]})\ \mathrm{Cos}[z]\ \mathrm{Sin}[z]^2\ A4'[t]^2\ \mathrm{YZ3}^{(2,2)}[z,t])\ dz+$$

$$A4'[t]^2\ (\frac{1}{9}\ (e^{4\,A4[t]}-2\ e^{6\,A4[t]}+e^{8\,A4[t]})\ Q1\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\mathrm{Sec}[z]^2$$

$$(-\frac{1}{2}\ Q1\ \mathrm{Cos}[z]^3\ (-1+\mathrm{Cosh}[2\ A4[t]])\ \mathrm{YZ3}[z,t]\ A4''[t]^2+$$

$$\frac{1}{2}\,Q1\,\mathrm{Cos}[z]^3\,(-1+\mathrm{Cosh}[2\,A4[t]])\,YZ3[z,t]\,A4'[t]\,A4^{(3)}[t]+$$

$$\mathrm{Cos}[z]^3\left((1-Q1+Q1\,\mathrm{Cosh}[2\,A4[t]])\,A4'[t]^2+\frac{1}{2}\left(-1+e^{2\,A4[t]}\right)A4''[t]\right)$$

$$YZ3^{(0,2)}[z,t]-\frac{1}{2}\left(-1+e^{2\,A4[t]}\right)\mathrm{Cos}[z]^3\,A4'[t]\,YZ3^{(0,3)}[z,t]+A4'[t]$$

$$\Bigg(-\frac{1}{4}\,e^{-4\,A4[t]}\,Q1^2\,\mathrm{Cos}[z]^3\,A4'[t]^2\left(YZ3[z,t]\,A4'[t]-\frac{1}{2}\,YZ3^{(0,1)}[z,t]\right)-$$

$$\frac{1}{2}\,e^{2\,A4[t]}\,\mathrm{Cos}[z]^3\left(-9+M^2+\frac{1}{4}\,Q1^2\,A4'[t]^2\right)YZ3^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,\mathrm{Cos}[z]^3\left(-9+M^2+\left(Q1+\frac{3\,Q1^2}{4}\right)A4'[t]^2\right)YZ3^{(0,1)}[z,t]+\frac{1}{2}\,e^{-2\,A4[t]}\,Q1$$

$$\mathrm{Cos}[z]^3\,A4'[t]^2\left(Q1\,YZ3[z,t]\,A4'[t]+\left(-1-\frac{3\,Q1}{4}\right)YZ3^{(0,1)}[z,t]\right)+$$

$$A4'[t]\,\Bigg(\mathrm{Cos}[z]^3\,YZ3[z,t]\left(-9+M^2+\left(-Q1-\frac{Q1^2}{4}\right)A4'[t]^2\right)-$$

$$36\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Bigg)\Bigg)+$$

$$A4''[t]\,\Bigg(\frac{1}{8}\,e^{-4\,A4[t]}\,Q1^2\,\mathrm{Cos}[z]^3\,YZ3[z,t]\,A4'[t]^2-\frac{3}{8}\,e^{-2\,A4[t]}$$

$$Q1\left(\frac{4}{3}+Q1\right)\mathrm{Cos}[z]^3\,YZ3[z,t]\,A4'[t]^2-\frac{1}{2}\,\mathrm{Cos}[z]^3\,YZ3[z,t]$$

$$\left(-9+M^2+\left(-Q1-\frac{3\,Q1^2}{4}\right)A4'[t]^2\right)-\frac{1}{2}\,Q1\,\mathrm{Cos}[z]^3\,A4'[t]\,YZ3^{(0,1)}[z,t]+$$

$$\frac{1}{2}\,Q1\,\mathrm{Cos}[z]^3\,\mathrm{Cosh}[2\,A4[t]]\,A4'[t]\,YZ3^{(0,1)}[z,t]+18\left(1+\mathrm{Cos}[z]^2\right)$$

$$\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]+e^{2\,A4[t]}\,\Bigg(\frac{1}{2}\,\mathrm{Cos}[z]^3\,YZ3[z,t]\,\Bigg(-9+M^2-$$

$$\frac{1}{4}\,Q1^2\,A4'[t]^2\Bigg)-18\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,YZ3^{(1,0)}[z,t]\Bigg)\Bigg)+$$

$$18\left(-1+e^{2\,A4[t]}\right)\left(1+\mathrm{Cos}[z]^2\right)\mathrm{Sin}[z]\,A4'[t]\,YZ3^{(1,1)}[z,t]-$$

$$36\,\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\left(A4'[t]^2+\frac{1}{2}\left(-1+e^{2\,A4[t]}\right)A4''[t]\right)YZ3^{(2,0)}[z,t]+$$

$$18\left(-1+e^{2\,A4[t]}\right)\mathrm{Cos}[z]\,\mathrm{Sin}[z]^2\,A4'[t]\,YZ3^{(2,1)}[z,t]\Bigg)\,dz+\frac{1}{3}$$

$$\Bigg(\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\,\mathrm{Cos}[z]\,\Bigg(\frac{1}{2}\,Q1\,\Big(YZ3[z,t]\,A4''[t]+A4'[t]\,\big((-2-Q1)\,YZ3[z,t]$$

$$A4'[t]+Q1\,\mathrm{Cosh}[2\,A4[t]]\,YZ3[z,t]\,A4'[t]+2\,YZ3^{(0,1)}[z,t]\big)\Big)+$$

$$e^{2\,A4[t]}\,YZ3^{(0,2)}[z,t]+e^{2\,A4[t]}\,\Bigg(\left(-9+M^2\right)YZ3[z,t]-\frac{1}{2}\,Q1\,YZ3[$$

$$z,t]\,A4''[t]-Q1\,A4'[t]\,YZ3^{(0,1)}[z,t]+\left(-72\,\mathrm{Tan}[z]-36\,\mathrm{Tan}[z]^3\right)$$

$$YZ3^{(1,0)}[z,t]\Bigg)-36\,e^{2\,A4[t]}\,\mathrm{Tan}[z]^2\,YZ3^{(2,0)}[z,t]\Bigg)\,dz\Bigg)$$

$$\Big(\frac{1}{6}\Big(e^{2\,\mathrm{A4}[t]}\,(-1+\mathrm{Q1})-\frac{\mathrm{Q1}}{4}\Big)\,\mathrm{Q1}\,\mathrm{A4}'[t]^2+\frac{1}{6}\,e^{8\,\mathrm{A4}[t]}\,\Big(-M^2-\frac{1}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\Big)+$$

$$e^{4\,\mathrm{A4}[t]}\,\Big(-\frac{M^2}{6}+\Big(\frac{\mathrm{Q1}}{3}-\frac{\mathrm{Q1}^2}{4}\Big)\,\mathrm{A4}'[t]^2\Big)+\frac{1}{3}\,e^{6\,\mathrm{A4}[t]}\,\Big(M^2+\frac{1}{2}\,\big(-\mathrm{Q1}+\mathrm{Q1}^2\big)\,\mathrm{A4}'[t]^2\Big)+$$

$$\frac{1}{12}\,\big(e^{2\,\mathrm{A4}[t]}-3\,e^{4\,\mathrm{A4}[t]}+3\,e^{6\,\mathrm{A4}[t]}-e^{8\,\mathrm{A4}[t]}\big)\,\mathrm{Q1}\,\mathrm{A4}''[t]\Big)+\sqrt{\mathrm{Sin}[z]}$$

$$\Big(\frac{1}{2}\,\big(e^{4\,\mathrm{A4}[t]}-3\,e^{6\,\mathrm{A4}[t]}+3\,e^{8\,\mathrm{A4}[t]}-e^{10\,\mathrm{A4}[t]}\big)\,\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}''[t]-$$

$$\frac{3}{4}\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\Big(e^{2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+$$

$$4\,e^{4\,\mathrm{A4}[t]}\,\Big(\Big(\frac{1}{3}-\mathrm{Q1}\Big)\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]-\frac{1}{3}\,\mathrm{YZ3}^{(0,1)}[z,t]\Big)\Big)+e^{6\,\mathrm{A4}[t]}$$

$$\Big(\mathrm{YZ3}[z,t]\,\Big(-9+M^2+\Big(2\,\mathrm{Q1}-\frac{9\,\mathrm{Q1}^2}{2}\Big)\,\mathrm{A4}'[t]^2\Big)-3\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]\Big)+$$

$$e^{10\,\mathrm{A4}[t]}\,\Big(\mathrm{YZ3}[z,t]\,\Big(-9+M^2-\frac{3}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\Big)-\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]\Big)+e^{8\,\mathrm{A4}[t]}$$

$$\Big(-2\,\mathrm{YZ3}[z,t]\,\Big(-9+M^2+\frac{1}{2}\,\big(\mathrm{Q1}-3\,\mathrm{Q1}^2\big)\,\mathrm{A4}'[t]^2\Big)+3\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z,t]\Big)+$$

$$\big(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\big)\,\mathrm{YZ3}^{(0,2)}[z,t]\Big)-396$$

$$\big(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\big)\,\mathrm{Sec}[z]^3\,\mathrm{Sin}[z]^{7/2}\,\mathrm{YZ3}^{(1,0)}[z,t]-216$$

$$\big(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\big)\,\mathrm{Sec}[z]^5\,\mathrm{Sin}[z]^{11/2}\,\mathrm{YZ3}^{(1,0)}[z,t]+2$$

$$\mathrm{Sec}[z]\,\mathrm{Sin}[z]^{3/2}\,\Big(\Big(-\frac{3}{4}\,\mathrm{Q1}\,\Big(4\,e^{4\,\mathrm{A4}[t]}\,\Big(\frac{1}{3}-\mathrm{Q1}\Big)+e^{2\,\mathrm{A4}[t]}\,\mathrm{Q1}\Big)\,\mathrm{A4}'[t]^2+$$

$$e^{10\,\mathrm{A4}[t]}\,\Big(-117+M^2-\frac{3}{4}\,\mathrm{Q1}^2\,\mathrm{A4}'[t]^2\Big)+e^{6\,\mathrm{A4}[t]}\,\Big(-117+M^2+$$

$$\Big(2\,\mathrm{Q1}-\frac{9\,\mathrm{Q1}^2}{2}\Big)\,\mathrm{A4}'[t]^2\Big)+e^{8\,\mathrm{A4}[t]}\,\big(234-2\,M^2+\big(-\mathrm{Q1}+3\,\mathrm{Q1}^2\big)\,\mathrm{A4}'[t]^2\big)+$$

$$\frac{1}{2}\,\big(e^{4\,\mathrm{A4}[t]}-3\,e^{6\,\mathrm{A4}[t]}+3\,e^{8\,\mathrm{A4}[t]}-e^{10\,\mathrm{A4}[t]}\big)\,\mathrm{Q1}\,\mathrm{A4}''[t]\Big)\,\mathrm{YZ3}^{(1,0)}[z,t]+$$

$$\big(e^{4\,\mathrm{A4}[t]}-3\,e^{6\,\mathrm{A4}[t]}+3\,e^{8\,\mathrm{A4}[t]}-e^{10\,\mathrm{A4}[t]}\big)\,\mathrm{Q1}\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(1,1)}[z,t]\Big)+2$$

$$\big(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\big)\,\mathrm{Sec}[z]\,\mathrm{Sin}[z]^{3/2}\,\mathrm{YZ3}^{(1,2)}[z,t]-324$$

$$\big(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\big)\,\mathrm{Sec}[z]^2\,\mathrm{Sin}[z]^{5/2}$$

$$\mathrm{YZ3}^{(2,0)}[z,t]-216\,\big(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\big)\,\mathrm{Sec}[z]^4$$

$$\mathrm{Sin}[z]^{9/2}\,\mathrm{YZ3}^{(2,0)}[z,t]-72\,\big(e^{6\,\mathrm{A4}[t]}-2\,e^{8\,\mathrm{A4}[t]}+e^{10\,\mathrm{A4}[t]}\big)$$

$$\mathrm{Sec}[z]^3\,\mathrm{Sin}[z]^{7/2}\,\mathrm{YZ3}^{(3,0)}[z,t]\Big)\Big)\,dt\Big)\,\mathrm{Sin}[M\,t]\,\mathrm{A4}'[t]+$$

$$M\,\Big(-\frac{1}{6}\int\frac{1}{\sqrt{\mathrm{Sin}[z]}}\,\mathrm{Cos}[z]\,\Big(\frac{1}{2}\,\mathrm{Q1}\,\big(\mathrm{YZ3}[z,t]\,\mathrm{A4}''[t]+\mathrm{A4}'[t]\,\big((-2-\mathrm{Q1})\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+$$

$$\mathrm{Q1}\,\mathrm{Cosh}[2\,\mathrm{A4}[t]]\,\mathrm{YZ3}[z,t]\,\mathrm{A4}'[t]+2\,\mathrm{YZ3}^{(0,1)}[z,t]\big)\big)+$$

$$e^{2\,A4[t]}\,YZ3^{(0,2)}[z,t]+e^{2\,A4[t]}\left(\left(-9+M^2\right)YZ3[z,t]-\frac{1}{2}\,Q1\,YZ3[z,t]\,A4''[t]-\right.$$

$$Q1\,A4'[t]\,YZ3^{(0,1)}[z,t]+\left(-72\,Tan[z]-36\,Tan[z]^3\right)$$

$$\left.YZ3^{(1,0)}[z,t]\right)-36\,e^{2\,A4[t]}\,Tan[z]^2\,YZ3^{(2,0)}[z,t]\Big)\,dz+$$

$$e^{-\frac{1}{4}\,Q1\,\left(e^{-2\,A4[t]}+2\,A4[t]\right)}\left(-1+e^{2\,A4[t]}\right)Q1\,(c11\,Cos[M\,t]+c12\,Sin[M\,t])$$

$$A4'[t]\Big)\Big)\Big\},$$

$$YZ3^{(0,4)}[z,t]==\frac{1}{4\,A4'[t]^2}\Big(-4\left(-1+e^{-2\,A4[t]}\right)$$

$$Q1\,YZ3[z,t]\,A4''[t]^3-2\left(-1+e^{-2\,A4[t]}\right)$$

$$Q1\,YZ3[z,t]\,A4'[t]^2\,A4^{(4)}[t]+4\Big(-2$$

$$A4'[t]^2\Big(-9+M^2+\frac{1}{2}\,e^{-2\,A4[t]}\,(-2+Q1)\,Q1\,A4'[t]^2-\frac{1}{4}\,e^{-4\,A4[t]}\,Q1^2\,A4'[t]^2+$$

$$\left(1-\frac{Q1^2}{4}-Coth[A4[t]]+Csch[A4[t]]^2\right)$$

$$A4'[t]^2\Big)-$$

$$\left(-1-Q1+e^{-2\,A4[t]}\,Q1+Coth[A4[t]]\right)A4'[t]^2$$

$$A4''[t]-2$$

```
     A4″[t]²) YZ3^(0,2)[

  z, t] + 8 A4′[t]
 ((-1 +
       Coth[
         A4[t]])
     A4′[t]² + A4″[t]) YZ3^(0,3)[z, t] + A4″[
    t]² (-𝕖^(-4 A4[t])
     Q1²
     YZ3[
       z,
       t]
     A4′[t]² -
    8
     YZ3[
       z,
       t] (-9 +
         M² +
         1
         ─
         8
           Q1²
           A4′[t]²) -
    4 Q1 A4′[t] YZ3^(0,1)[z, t] + 2 𝕖^(-2 A4[t])
     Q1
     A4′[
       t]
     (Q1 YZ3[z, t] A4′[t] +
        2
          YZ3^(0,1)[z, t]) +
    288 (2 Tan[z] + Tan[z]³) YZ3^(1,0)[z, t]) - 4 A4′[
  t]
 A4″[
  t]
 (- 1
    ─
    4
     𝕖^(-6 A4[t])
     Q1³
     YZ3[
       z,
       t]
     A4′[t]³ +
    3
```

```
      ⅇ^(-A4[t])
      Q1
      Csch[
        A4[
          t]] YZ3[
        z, t] A4′[t]^3 - YZ3[z,
        t] A4′[
        t]
       ((-3 + M) (3 + M) (1 + Q1 - Coth[A4[t]]) -
          1
          —
          4
           Q1^3
           A4′[t]^2) +
     3
     — ⅇ^(-4 A4[t]) Q1^2 A4′[t]^2 ((2
     4                              — + Q1) YZ3[z, t] A4′[t] -
                                    3
          2
          —
          3
           YZ3^(0,1)[z, t]) +
                          1
     (18 - 2 M^2 - — Q1^2 A4′[t]^2) YZ3^(0,1)[z, t] +
                          2
     36
      (1 +
         Q1 -
         Coth[
           A4[t]])
      Tan[z] (2 + Tan[z]^2) A4′[t] YZ3^(1,0)[
        z,
        t] + ⅇ^(-2 A4[t])
      Q1 A4′[
        t]
                                 1           3 Q1^2
       (YZ3[z, t] (-9 + M^2 + — (-Q1 - ——————) A4′[t]^2) +
                                 2             2
          (-2 + Q1)
           A4′[t]
           YZ3^(0,1)[z, t] + 36
           (-2 Tan[z] - Tan[z]^3)
           YZ3^(1,0)[z, t])) -
                       1
  4 A4′[t]^2 (- — ⅇ^(-6 A4[t]) (-2 + Q1) Q1^3
                       4
      YZ3[
        z,
        t]
```

$\mathrm{A4}'[t]^4 +$

$\frac{1}{16}$

$e^{-8\,\mathrm{A4}[t]}$

$\mathrm{Q1}^4$

$\mathrm{YZ3}[$

$z,$

$t]$

$\mathrm{A4}'[t]^4 +$

$\mathrm{YZ3}[$

$z,$

$t]$

$\Big((-3+M)^2\,(3+M)^2 +$

$\Big(-18 + 2\,M^2 + \frac{9\,\mathrm{Q1}^2}{2} + \frac{M^2\,\mathrm{Q1}^2}{2} + 18\,\mathrm{Coth}[\mathrm{A4}[t]] - 2\,M^2\,\mathrm{Coth}[\mathrm{A4}[t]] -$

$18\,\mathrm{Csch}[\mathrm{A4}[t]]^2 + 2\,M^2\,\mathrm{Csch}[\mathrm{A4}[t]]^2\Big)\,\mathrm{A4}'[t]^2 +$

$\Big(\frac{\mathrm{Q1}^4}{16} - \mathrm{Q1}\,\mathrm{Csch}[\mathrm{A4}[t]]^2\Big)\,\mathrm{A4}'[t]^4\Big) - 2\,(-3+M)\,(3 +$

$M)$

$(-1 + \mathrm{Coth}[\mathrm{A4}[t]])\,\mathrm{A4}'[t]\,\mathrm{YZ3}^{(0,1)}[z, t] +$

2

$e^{-\mathrm{A4}[t]}$

$\mathrm{Q1}$

$\mathrm{Csch}[$

$\mathrm{A4}[$

$t]]$

$\mathrm{A4}'[t]^3\,\mathrm{YZ3}^{(0,1)}[z, t] - 72\,\mathrm{Tan}[$

$z]\,\Big(-90 +$

$2\,M^2 +$

$(-405 + M^2)$

$\mathrm{Tan}[z]^2 - 594$

$\mathrm{Tan}[z]^4 - 270$

$\mathrm{Tan}[z]^6 -$

$\frac{1}{4}\,\big(-4 + \mathrm{Q1}^2 + 4\,\mathrm{Coth}[\mathrm{A4}[t]] - 4\,\mathrm{Csch}[\mathrm{A4}[t]]^2\big)$

$\big(2 + \mathrm{Tan}[z]^2\big)$

$\mathrm{A4}'[t]^2\Big)\,\mathrm{YZ3}^{(1,0)}[z, t] - e^{-2\,\mathrm{A4}[t]}\,\mathrm{Q1}\,\mathrm{A4}'[$

$t]^2$

$\Big(\mathrm{YZ3}[z, t]\,\Big(-18 + 2\,M^2 + 9\,\mathrm{Q1} + M^2\,\mathrm{Q1} + \Big(-\mathrm{Q1} - \frac{\mathrm{Q1}^2}{2} + \frac{\mathrm{Q1}^3}{4} + \mathrm{Csch}[\mathrm{A4}[t]]^2\Big)\,\mathrm{A4}'[t]^2\Big) +$

$\mathrm{Q1}\,\mathrm{A4}'[t]$

```
            YZ3^(0,1)[z, t] + 36
            (-2 + Q1) Tan[z]
            (2 + Tan[z]^2)
            YZ3^(1,0)[z, t]) +
        1
        - e^(-4 A4[t]) Q1^2 A4'[t]^2 (YZ3[z, t] (9 + M^2 + (-2 Q1 + 3 Q1^2/4) A4'[t]^2) +
        2
           2 A4'[t] YZ3^(0,1)[z, t] +
           36 (2 Tan[z] + Tan[z]^3)
            YZ3^(1,0)[z, t])) -
288 Tan[z] (2 + Tan[z]^2) A4'[t] ((-1 + Coth[A4[t]])
      A4'[t]^2 +
    A4''[
      t])
  YZ3^(1,1)[z, t] + 288
  Tan[
   z]
  (2 +
    Tan[
      z]^2) A4'[
    t]^2 YZ3^(1,2)[z, t] + 288 Tan[
    z]^2 (A4'[t]^2
     (-261 +
       M^2 -
       504 Tan[z]^2 -
       270 Tan[z]^4 +
       1
       - e^(-2 A4[t]) (-2 + Q1)
       2
                    1
        Q1 A4'[t]^2 - -
                    4
        e^(-4 A4[t]) Q1^2 A4'[t]^2 +
       (1 - Q1^2/4 - Coth[A4[t]] + Csch[A4[t]]^2) A4'[t]^2) +
    1
    - (-1 - Q1 + e^(-2 A4[t]) Q1 + Coth[A4[t]])
    2
     A4'[t]^2
     A4''[
       t] +
    A4''[t]^2) YZ3^(2,0)[z, t] + 4 A4'[
   t]
```

```
 A4^(3)[
  t]
 ((-9 + M^2) YZ3[
      z,
      t] + 1/4 Q1^2 YZ3[
      z,
      t]
     A4′[t]^2 + 1/4
     ⅇ^(-4 A4[t])
     Q1^2
     YZ3[
      z,
      t]
     A4′[t]^2 + 3/2
     (-1 +
        ⅇ^(-2 A4[t])) Q1 YZ3[
      z, t] A4′′[t] + Q1 A4′[
      t] YZ3^(0,1)[
      z,
      t] +
    ⅇ^(-2 A4[t]) ((Q1 - Q1^2/2) YZ3[z, t] A4′[t]^2 -
        Q1 A4′[t] YZ3^(0,1)[z, t]) +
    YZ3^(0,2)[z, t] + 36 (-2 Tan[z] - Tan[z]^3)
     YZ3^(1,0)[
      z,
      t] - 36
     Tan[z]^2 YZ3^(2,0)[
      z,
      t]) -
288 Tan[z]^2 A4′[t] ((-1 + Coth[A4[t]])
     A4′[t]^2 +
    A4′′[
     t])
 YZ3^(2,1)[z, t] + 288 Tan[
   z]^2 A4′[
   t]^2
 YZ3^(2,2)[z, t] - 31 104
 Tan[
```

```
    z]^3 (4/3 +
    Tan[z]^2)
A4′[t]^2 YZ3^(3,0)[z, t] - 5184
Tan[
    z]^4 A4′[
    t]^2 YZ3^(4,0)[z, t])}
```