ALT

NOTES

HYPOTHESIS : If , employing the Einstein eqs (or Einstein – Lovelock eqs) ,
superluminal inflation / deflation exists,

then at time x4 = 0 (**before** the particles of the standard model exist)
a pair of universes with MASSES ± M is created
(i.e., universes are created in pairs)
Dark matter / dark energy may be
related to wave function overlap of these two universes

THE FOLLOWING SCRATCH IS CONSISTENT WITH
UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES ± M.

Note that we are ONLY looking for
superluminal inflation or deflation type solutions,
and NOT solutions that are even / odd functions of t
(like Cos[ν[j]'[0] * t], Sin[ν[j]'[0] * t],
Sech[ν[j]'[0] * t], Tanh[ν[j]'[0] * t], ...),
which also have ± M type eigenvalues,
or even solutions involving the
well – known special functions that might also have ± M type parameters.

Bigger Bang:
Question: Are Universe (s) of masses ± M created in pairs at time x4 = 0
(before the particles of the standard model exist) ?

Scratch work

Patrick L. Nash, Ph.D.

Professor, UTSA Physics and Astronomy, Retired (UTSA)
Patrick299Nash at gmail ...
Do not read if you are irascible; I apologize for typos and duplications and ....
git clone https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git
“git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git”

Ever wonder why there are 3 space dimensions and 1 time dimension?

Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in$ Nature?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\|=\|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over $\mathbb{R}$) real numbers $\mathbb{R}$,the 2-dim complex numbers,the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in "A Remarkable Representation of the 3+2 de Sitter Group" by P.A.M.Dirac, J.Math.Phys.4,901–909 (1963).

Comment: The term "reduced Brauer–Weyl generators", below, refers to a specific set of irreducible generators for certain complex Clifford algebras, particularly in the context of mathematical physics and the study of Dirac spinors. These generators were introduced by **E.A. Lord** (E. A. Lord. "The Dirac spinor in six dimensions". Mathematical Proceedings of the Cambridge Philosophical Society, vol. 64, pp. 765–778, 1968) and are described as "reduced" because they form an irreducible representation of the Clifford algebra C_{2n} from $C_{2(n-1)}$.

Here, we employ Lord's idea of reduced Brauer–Weyl generators to extend Dirac's 4×4 gamma matrices into this notebook's real tau8 8×8 and tau16 16×16 matrices,
which are employed to write down the 'Dirac Equation for the Universe',
[remember, this is before the particles of the standard model exist]
formulated in terms of a $\overline{O(4,4)}$ spinor Ψ16
(Ψ16 AKA 'WAVE FUNCTION OF the UNIVERSE,' which could possibly be named something more pompous).
Then this equation is used to
[WARNING: syncope, presyncope AHEAD]
couple the WAVE FUNCTION OF the UNIVERSE to Gravity.

THIS WORK IS PARTIALLY BASED ON :

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. Dirac
Cambridge University, Cambridge, England and
*Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

P. A. M. Dirac
Cambridge University, Cambridge, England and
*Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

Proc. Camb. Phil. Soc. (1968), **64**, 765
PCPS 64–96
Printed in Great Britain

765

The Dirac spinor in six dimensions

By E. A. LORD
Department of Mathematics, King's College, University of London

(*Received* 17 *November* 1966)

JOURNAL OF MATHEMATICAL PHYSICS **51**, 042501 (2010)

Second gravity

Patrick L. Nash[a)]
Department of Physics and Astronomy, The University of Texas at San Antonio, San Antonio, Texas 78249-0697, USA

(Received 29 July 2009; accepted 8 February 2010; published online 5 April 2010)

IL NUOVO CIMENTO VOL. 105 B, N. 1 Gennaio 1990

On the Structure of the Split Octonion Algebra.

P. L. Nash
Division of Earth and Physical Sciences
University of Texas at San Antonio, TX 78285-0663

(ricevuto il 24 Luglio 1989)

Identities satisfied by the generators of the Dirac algebra

Patrick L. Nash
M. S. 474, NASA Langley Research Center, Hampton, Virginia 23665

(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin-$\frac{1}{2}$ wave equation

Patrick L. Nash
Division of Earth and Physical Sciences, University of Texas at San Antonio, San Antonio, Texas 78285

(Received 24 January 1984; accepted for publication 16 November 1984)

A new spin-$\frac{1}{2}$ wave equation

Patrick L. Nash
Division of Earth and Physical Sciences, University of Texas at San Antonio, San Antonio, Texas 78285

(Received 24 January 1984; accepted for publication 16 November 1984)

Here we use spacetime coordinates that are selected so that we are somewhat consistent with the xact libraries,
(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):
Cartesian coordinates:
x0 = hidden space (a small circle); x1, x2, x3 are the usual 3-space coords;
x4 = time coord,
and x5, x6, x7= superluminal deflating time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{\text{SO}(4, 4; \mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M8 , whose tangent bundle has
$\overline{\text{SO}(4, 4; \mathbb{R})} \approx \text{Spin}(4,4;\mathbb{R})$
as iso group, and in which
3 of the 4 space dimensions superluminally INFLATE,
3 of the 4 time dimensions superluminally DEFLATE (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),
and the 4th space dim, x0, curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

In passing we remark that an analogous construction may be defined on an octonion space of one time dimension and seven space dimensions, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group
$\text{SU}(3) \otimes \text{SU}(2)_L \otimes U(1)_Y$;
The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary *n* has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum^{m-1} \alpha_{(k)} g^{jl} \delta^{hh_1 \cdots h_{2k}}_{jj_1 \cdots j_{2k}} R^{j_1 j_2}{}_{h_1 h_2} \cdots R^{j_{2k-1} j_{2k}}{}_{h_{2k-1} h_{2k}} + \lambda \sqrt{g}\, g^{lh}, \quad (4.38)$$

The Einstein – Lovelock vacuum field equations are
$A_a^b = 0$, where

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta^{hh_1 \cdots h_{2k}}_{jj_1 \cdots j_{2k}} R^{j_1 j_2}{}_{h_1 h_2} \cdots R^{j_{2k-1} j_{2k}}{}_{h_{2k-1} h_{2k}} + \lambda \sqrt{g}\, g^{lh}, \quad (4.38)$$

where $\alpha_{(k)}$, λ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

and $m - 1 = \frac{8}{2} - 1 = 3$.
Citation : Tensors, Differential Forms,
and Variational Principles (Dover Books on Mathematics),
by David Lovelock and Hanno Rund

Let {w1, w2, w3, Λ} be pure numbers;
then by Eq. [4.38] the vacuum equations are
$0 == -\Lambda + H^{-2}\ w1\ \text{Lovelock1} + H^{-4}\ w2\ \text{Lovelock2} + H^{-6}\ w3\ \text{Lovelock3}$
in a hopefully obvious notation.
Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein – Lovelock tensors have different dimensions.
For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre – Hubble parameter. For now, let's pretend that they are independent.

NOTES :

Under the spacetime coordinate transformation

$x^k \mapsto \overline{x}^k (x^j)$
according to
$\Psi16\ (x) \mapsto \overline{\Psi16}\,(\overline{x}) = S\ (x\ (\overline{x})\ ) . \Psi16\ (x\ (\overline{x})\ )$,
i.e., $\overline{\Psi16}^{(a)}\ (\overline{x}) = \left(S^{(a)}_{\ (b)}\ \Psi16^{(b)}\ (x\ (\overline{x})\ ) \right)$

Therefore,

$$\frac{\partial}{\partial \overline{x}^k}\overline{\Psi16(\overline{x})} = \frac{\partial x^j}{\partial \overline{x}^k}\frac{\partial}{\partial x^j}\,S(x).\Psi16(x) =$$

$$\frac{\partial x^j}{\partial \overline{x}^k}\left\{ S(x)\,.\frac{\partial}{\partial x^j}\Psi16(x) + \left[\frac{\partial}{\partial x^j}S(x)\right].\Psi16(x)\right\}$$

Identify the Octad connection : $\frac{\partial}{\partial x^j}S(x) = -S\,.\,\Gamma_j(x) + \frac{\partial \overline{x}^k}{\partial x^j}\overline{\Gamma_k}(\overline{x})\,.\,S$

hence

$$\frac{\partial}{\partial \overline{x}^k}\overline{\Psi16(\overline{x})} - \overline{\Gamma_k}\,.\,\overline{\Psi16(\overline{x})} =$$

$$S(x(\overline{x}))\,.\,\frac{\partial x^j}{\partial \overline{x}^k}\left\{\frac{\partial}{\partial x^j}\Psi16(x(\overline{x})) - \Gamma_j.\Psi16(x(\overline{x}))\right\}$$

or

$$\left(1_{16\times16}\frac{\partial}{\partial \overline{x}^k} - \overline{\Gamma_k}\right).\,\overline{\Psi16(\overline{x})} =$$

$$S(x(\overline{x}))\,.\left[\frac{\partial x^j}{\partial \overline{x}^k}\left\{1_{16\times16}\frac{\partial}{\partial x^j} - \Gamma_j\right\}.\Psi16(x(\overline{x}))\right]$$

The connection transforms according to

$$\overline{\Gamma_k}(\overline{x}) = \frac{\partial x^j}{\partial \overline{x}^k}\left\{ S\,.\,\Gamma_j(x)\,.\,S^{-1} + S_{,j}\,.\,S^{-1}\right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2}\left[e_k^{(a)}\,\nabla_j\,e^k_{(b)}\right]\,SAB^{(b)}{}_{(a)}$$

Here, spin connection coefficients are $e_k^{(a)}\left(\nabla_j e^k_{(b)}\right)$

and

$$S = \exp\left(\frac{1}{2}\,\omega_{(A)(B)}(x)\;SAB^{(A)(B)}\right),$$

where $\omega_{(B)(A)} = -\omega_{(A)(B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the Ψ16 field [A.K.A., wave function characterizing the Universe(s) of masses ±M];

TODO: prove **Universe(s) of masses ±M are created in pairs!**

K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility**: The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
  - $\nabla_\mu e^a{}_\nu = 0$
  - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

apologies, I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu}e^I_{\nu]} = \partial_{[\mu}e^I_{\nu]} + \omega^I_{[\mu J}e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu}e^I_\rho + \omega^I_{\mu J}\, e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistancy" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (neverthless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will neverthless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called ‘Tetrad Postulate’ and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

? SOURCE terms $TU^{\mu\nu}$ for $g_{\alpha\beta}$ come from "Universes' Wave Function Ψ16 Lagrangian"

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]}\ \text{LagrangianΨ16} \right)$$

(hope that $TU^{\mu\nu} = \Lambda\, g^{\mu\nu}$, and ×

H = some function of M, where
Universe (s) of masses ± M created in pairs at time x4 = 0,
before the particles of the standard model exist) ;

WARNING:

Universes Ψ16 source $g_{\alpha\beta}$;
The Euler–Lagrange equations for Ψ16 must have 'solutions' such that
all off–diagonal terms of $TU^{\mu\nu}$ ARE ZERO .

The Ψ16 Lagrangian (please see below) =

$$\sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg}[] =$$

$$\sqrt{\text{Det}[g_{\mu\nu}]} * \left(\text{Transpose}[\Psi16].\sigma16.\text{Sum}\left[T16^{\alpha}[\alpha1-1].\left\{1_{16\times16}\frac{\partial}{\partial x^{\alpha1-1}} - \Gamma_{\alpha1-1}\right\}.\Psi16, \{\alpha1, 1, \text{Length}[X]\}\right] + \frac{\text{mASs}}{2} * \text{Transpose}[\Psi16].\sigma16.\Psi16\right)$$

Next,
assume that Ψ16soln solves the Euler – Lagrange equations.

SOURCE terms
$TU^{\mu\nu}$ [evaluate terms after performing differentiation] (using LagrangianΨ16 ≡
$\left(\sqrt{\text{Det}[g_{\alpha\beta}]}\ \text{Lg}[]\right)\big|_{\Psi16=\text{solution-toEL-eqs}}$) :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]}\ \text{Lg}[]\right)$$

$$= \left(\text{Lg}[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]}\right)\right] + \frac{\partial}{\partial g_{\mu\nu}}(\text{Lg}[])\right)\Bigg|_{\Psi16=\Psi16\text{soln}}$$

$$= \left(0 ? * \frac{1}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\mathrm{Det}[g_{\alpha\beta}]} \right) + \frac{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}}{\sqrt{\mathrm{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\mathrm{Lg}[\,] \right) \right) \Bigg|_{\Psi16=\Psi16\mathrm{soln}}$$

$$= \frac{\partial}{\partial g_{\mu\nu}} \Big(\mathrm{Transpose}[\Psi16].\sigma16.\mathrm{Sum}\Big[\Big(\mathrm{D}[\Psi16, X[\![\alpha1]\!]] + \Big(\Big(\frac{1}{2}\Big) \mathrm{connectionMatrix}\Big).\mathrm{Sum}[$$

$$\omega[\![\alpha1, a, b]\!] * \mathrm{SAB}[\![a, b]\!], \{a, 1, 8\}, \{b, 1, 8\}].\Psi16\Big), \{\alpha1, 1, \mathrm{Length}[X]\}\Big] +$$

$$\mathrm{mASs} * \mathrm{Transpose}[\Psi16].\mathrm{symm16}[\![j, 1]\!].\Psi16\Big) \Bigg|_{\Psi16=\Psi16\mathrm{soln}}$$

$$= \mathrm{Transpose}[\Psi16].\sigma16.\mathrm{Sum}\Big[\frac{\partial}{\partial g_{\mu\nu}} \left(\mathrm{T16}^{\alpha}[\alpha1 - 1] \right) . \Psi16_{\alpha1-1},$$

$$\{\alpha1, 1, \mathrm{Length}[X]\}\Big] \Big|_{\Psi16=\mathrm{solution-toEL-eqs}}$$

$$= \tilde{\Psi}16 .\sigma16. \mathrm{T16}^{A} . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1\,\alpha\beta}\, e^{B}_{\beta}\, \eta_{BA} \right) = \tilde{\Psi}16 .\sigma16. \mathrm{T16}^{A} . \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} \left(e^{\alpha}_{(A)} \right)$$

NOTES :

Let g be a square matrix; we could use : $\frac{\partial}{\partial q} \left(g^{-1} \right) = - g^{-1} . \frac{\partial g}{\partial q} . g^{-1}$,

where q is a parameter (result from $\frac{\partial}{\partial q} \left(g.g^{-1} \right) = 0$),

In this notebook,

g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] ,

g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1} \right) = - g^{-1} . \frac{\partial g}{\partial g_{\mu\nu}} . g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq$

$g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In metric matrix g,

we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$, and then differentiate, for example.

This should be understood before reading further.

Therefore $\frac{\partial}{\partial g_{\mu\nu}} \left(e^{\alpha}_{(A)} \right) = \frac{\partial}{\partial g_{\mu\nu}} \left(\mathrm{Inverse}\left[e^{(A)}_{\alpha}\right] \right) =$

$$- \mathrm{Inverse}\left[e^{(A)}_{\alpha}\right] . \frac{\partial e^{(A)}_{\alpha}}{\partial g_{\mu\nu}} . \mathrm{Inverse}\left[e^{(A)}_{\alpha}\right] = - e^{\alpha}_{(A)} [\![\alpha1, A1]\!] \frac{\partial e^{(A)}_{\alpha} [\![A1, \alpha2]\!]}{\partial g_{\mu\nu}} . e^{\alpha}_{(A)} [\![\alpha2, A2]\!]$$

Consider $\frac{\partial}{\partial g_{\mu\nu}}(g_{\alpha\beta}) = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e_\alpha^{(A)}\ e_\beta^B\right) = \eta_{AB}\ e_\beta^B \frac{\partial}{\partial g_{\mu\nu}}\left(e_\alpha^{(A)}\right) + \eta_{AB}\, e_\alpha^B \frac{\partial}{\partial g_{\mu\nu}}\left(e_\beta^A\right)$;

$e_C^\beta \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} = \frac{\partial}{\partial g_{\mu\nu}}\left(\eta_{AB}\, e_\alpha^{(A)}\ e_\beta^B\right) = \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}}\left(e_\alpha^{(A)}\right) + \eta_{AB}\, e_\alpha^B\, e_C^\beta \frac{\partial}{\partial g_{\mu\nu}}\left(e_\beta^A\right)$

In metric matrix g, we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate.

Some of my calculations employ “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see
Cyril Pitrou
Institut d’Astrophysique de Paris (France)
pitrou@iap.fr
http://www2.iap.fr/users/pitrou/
“FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb”
MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify

I copied and pasted too often; however, I am going to leave this here.

Universe sources for $g_{\alpha\beta}$; be sure to append these to the Einstein and/or Einstein–Lovelock field equations.

WARNING: all off–diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous ‘hacked solutions’ for Ψ16 to be valid

Let $T^{\alpha} = \{T16^{\alpha}[0], T16^{\alpha}[4]\}$ and

$\sqrt{Det[g_{\mu\nu}]} * Lg[][mASs\_, j\_] =$

$\sqrt{Det[g_{\mu\nu}]} * (Transpose[\Psi16].\sigma16.Sum[T^{\alpha}[\alpha1-1].D[\Psi16, vars[[\alpha1]]], \{\alpha1, 1, Length[vars]\}] + mASs * Transpose[\Psi16].symm16[[134, 1]].\Psi16)$

Let j = 134, $\sigma16.(mass Matrix) = T16^{\alpha}[5].T16^{\alpha}[6].T16^{\alpha}[7]$

and

usingLagrangianF16massive = (Transpose[Ψ16].σ16.
Sum[T^{α}[α1 - 1].D[Ψ16, vars[[α1]]], {α1, 1, Length[vars]}] +
mASs * Transpose[Ψ16].symm16[[j = 134, 1]].Ψ16);

where it is assumed that Ψ16 solves the Euler - Lagrange equations, above; then usingLagrangianF16massive = 0, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{Det[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{Det[g_{\alpha\beta}]}\, Lg[] \right)$$

$$= \left(Lg[] * \left[\frac{1}{\sqrt{Det[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{Det[g_{\alpha\beta}]} \right) \right] + \frac{\partial}{\partial g_{\mu\nu}} (Lg[]) \right) \Bigg|_{Lg[]=usingLagrangianF16massive}$$

$$= \left(0 * \frac{1}{\sqrt{Det[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{Det[g_{\alpha\beta}]} \right) + \right.$$

remark: the term

$$\tilde{\Psi 16}\,.\sigma 16.\,T16^{A}\,.\,\Psi 16_{,\alpha}\frac{\partial}{\partial g_{\mu\nu}}(e^{\alpha}_{A}) = \frac{\partial}{\partial g_{\mu\nu}}\tilde{\Psi 16}\,.\sigma 16.\,T16^{A}\,.\,\Psi 16_{,\alpha}\,e^{\alpha}_{A} =$$

$$\frac{\partial}{\partial g_{\mu\nu}}\left(\text{Transpose}[\Psi 16].\sigma 16.\right.$$

$$\left.\text{Sum}\left[T^{\alpha}[\overbrace{\alpha 1-1].D[\Psi 16,\text{vars}[\![\alpha 1]\!]}^{T16^{A}[A1-1].\Psi 16_{,\alpha 1}\,\eta_{A1\,B}\,e^{B}_{\beta}\,g^{-1\,\alpha 1\,\beta}}],\{\alpha 1,1,\text{Length}[\text{vars}]\}\right]\right)$$

Frame fields [edit]

We use a set of vierbein or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab}e^a_\mu e^b_\nu = \eta_{\mu\nu}.$$

The vierbein defines a local rest frame, allowing the constant Gamma matrices to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a section of the frame bundle, and so defines a local trivialization of the frame bundle.

Incomplete theories	[show]
Scientists	[show]
	v • t • e

Spin connection [edit]

To write down the equation we also need the spin connection, also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e^\mu_a e^a_\nu = \delta^\mu{}_\nu.$$

The connection 1-form is then

$$\omega^\mu{}_{\nu a} := e^\mu_b \nabla_a e^b_\nu$$

where ∇_a is a covariant derivative, or equivalently a choice of connection on the frame bundle, most often taken to be the Levi-Civita connection.

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space T_pM to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

Raising and lowering indices is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract connection on a principal bundle, specifically on the frame bundle, which is defined on any smooth manifold, but which restricts to an *orthonormal* frame bundle on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

Incomplete theories	[show]
Scientists	[show]

V • T • E

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$$\partial_\mu \psi = e^\alpha_\mu \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu{}_{\nu\rho} = e^\alpha_\rho \omega^\mu{}_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2}(\omega_{\nu\rho})_\mu \sigma^{\nu\rho}\psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu R(M^{\nu\rho})v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $\mathrm{SO}(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu{}_{\nu\rho} = e_\rho^\alpha \omega^\mu{}_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2}(\omega_{\nu\rho})_\mu \sigma^{\nu\rho} \psi,$$

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$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu R(M^{\nu\rho}) v = \partial_\mu v + \frac{1}{2}(\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $\mathrm{SO}(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

OTHER WORK THAT I USED :

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Grou

P. A. M. DIRAC

Cambridge University, Cambridge, England and *Institute for Advanced Study, Princeton, New Jersey**

(Received 20 February 1963)

Proc. Camb. Phil. Soc. (1968), **64**, 765 765

PCPS 64–96

Printed in Great Britain

The Dirac spinor in six dimensions

BY E. A. LORD

Department of Mathematics, King's College, University of London

(*Received* 17 *November* 1966)

Identities satisfied by the generators of the Dirac algebra

Patrick L. Nash
M. S. 474, NASA Langley Research Center, Hampton, Virginia 23665

(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin-$\frac{1}{2}$ wave equation

Patrick L. Nash
Division of Earth and Physical Sciences, University of Texas at San Antonio, San Antonio, Texas 78285

(Received 24 January 1984; accepted for publication 16 November 1984)

In[ ]:=
```
(* ================================================================================ *)
(*                          Dirac-Lord-Nash_4+4.wl                                *)
(* ======================  a few references:   ================================ *)
(*JOURNAL OF NATHENATICAL PHYSICS, VOLUME 4, NUMBER 7, JULY 1963*)
(*"A Remarkable Representation of the 3 + 2 de Sitter Group"*)
(*P. A. M. DIrac*)
(* ================================================================================ *)
(*Proc. Camb. Phil. Soc. (1968), 64, 765*)
(*"The Dirac spinor in six dimensions"*)
(*E. A. LORD*)
(*Department of Mathematics, King's College, University of London*)
(* ================================================================================ *)
(*J. Math. Phys. 25 (2), February 1984*)
(*"Identities satisfied by the generators of the Dirac algebra"*)
(*Patrick L. Nash*)
(* ================================================================================ *)
(*IL NUOVO CIMENTO, VoL. 105 B, N. 1, Gennaio 1990*)
(*"On the Structure of the Split Octonion Algebra"*)
(*P. L. NASH*)
(*University of Texas at San Antonio, TX 78285-0663*)
(* ================================================================================ *)
(*JOURNAL OF MATHEMATICAL PHYSICS 51, 042501 (2010)*)
(*"Second gravity"*)
(*Patrick L. Nash*)
(* ================================================================================ *)
(*                                                                              *)
```

Load the Package

In[ ]:=
```
SetDirectory[NotebookDirectory[]];
```

In[ ]:=
```
Get["Dirac-Lord-Nash_4+4.wl"]
```

```
Dirac-Lord-Nash_4+4 loaded successfully!
  BUT, WARNING:  DO NOT USE IF YOU WANT A CORRECT RESULT!
```

```
In[ ]:= Clear[rawSets];
rawSets[l_, f_, o_] := Module[{t, h},
   t = Table[Cases[
  l⟦j⟧,
h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], f] :> n + o,
  Infinity,
  Heads → True
], {j, 1, Length[l]}];
   Return[t];
  ]
```

```
In[ ]:= (*rawSetsel16=rawSets[el16,"f16",0]*)
```

```
In[ ]:= (*couplings=showCoupledEquations[rawSetsel16]
 %//Flatten//Sort
 %===Range[0,15]*)
```

```
In[ ]:= Clear[dualRawSets];
dualRawSets[l_, f_, o_, h_, oh_] := Module[{r, r1, r2},
   r1 = rawSets[l, f, o];
   r2 = rawSets[l, h, oh];
   r = Union[Flatten[{r1, r2}, 1]] // Drop[#, 1] &;
   Return[r];
  ]
```

```
In[ ]:= (* Helper to perform one pass of merging *)
MergeSetsStep[currentSets_List] :=
   Module[{n = Length[currentSets], i, j, merged = currentSets},
  (* We look for the first pair (i, j) that intersects *)
  Catch[
   For[i = 1, i ≤ Length[merged], i++,
    For[j = i + 1, j ≤ Length[merged], j++,
     If[Intersection[merged⟦i⟧, merged⟦j⟧] =!= {},
      (* Found intersection: Merge and Throw to restart/finish this step *)
      merged = Delete[merged, {{i}, {j}}];
      AppendTo[merged, Sort[Union[currentSets⟦i⟧, currentSets⟦j⟧]]];
      Throw[Sort[merged]]; (* Sort for canonical form *)
     ];
    ];
   ];
   (* If no intersection found, return original sorted *)
   Sort[merged]
  ]
];
```

```
In[ ]:= showCoupledEquations[items_List] :=
  FixedPoint[MergeSetsStep, items];
```

SEVERAL SIMPLE TESTS, before we proceed :

In[•]:=
```
splitOctonionMultTable
```

Out[•]=

times	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

In[•]:=

Under the involutive automorphism $\gamma^{AB} \to -\tilde{\gamma}^{AB}$ of so(3,3), the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra so(4), which is the Lie algebra of $\overline{\mathrm{SO}(4)}$, the maximal compact subgroup of $\overline{\mathrm{SL}(4,\mathbf{R})}$. The subalgebra so(4)$\cong$su(2) $\dot{+}$ su(2) may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of so(4) whereby $\tau\in$so(4) is mapped into its dual, $^*\tau$. The even subspace under * of so(4) corresponds to self-dual tensors, and, say, the first su(2) in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second su(2) in the direct sum. A basis for so(4) may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by $(h=1,2,3)$,

$$2s^h = (\gamma^{23},\gamma^{31},\gamma^{21}), \tag{4}$$

and

$$2t^h = (\gamma^{45},\gamma^{64},\gamma^{65}). \tag{5}$$

Out[•]=

Under the involutive automorphism $\gamma^{AB} \to -\tilde{\gamma}^{AB}$ of so(3,3), the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra so(4), which is the Lie algebra of $\overline{\mathrm{SO}(4)}$, the maximal compact subgroup of $\overline{\mathrm{SL}(4,\mathbf{R})}$. The subalgebra $\mathrm{so}(4) \cong \mathrm{su}(2) + \mathrm{su}(2)$ may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of so(4) whereby $\tau \in \mathrm{so}(4)$ is mapped into its dual, ${}^*\tau$. The even subspace under * of so(4) corresponds to self-dual tensors, and, say, the first su(2) in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second su(2) in the direct sum. A basis for so(4) may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by $(h = 1,2,3)$,

$$2s^h = (\gamma^{23}, \gamma^{31}, \gamma^{21}), \tag{4}$$

and

$$2t^h = (\gamma^{45}, \gamma^{64}, \gamma^{65}). \tag{5}$$

In[•]:=
```
MatrixForm[#] & /@ allS4by4
```

Out[•]=

$$\left\{\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\right\}$$

In[•]:=
```
MatrixForm[#] & /@ allT4by4
```

Out[•]=

$$\left\{\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\right\}$$

In[ ]:= **MatrixForm[#] & /@ allQ**

Out[ ]=

$$\left\{\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}, \begin{pmatrix}0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\end{pmatrix}, \begin{pmatrix}0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&0&1\\0&0&0&0&1&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\end{pmatrix}, \begin{pmatrix}0&0&0&0&0&1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&-1&0\\0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0\end{pmatrix}, \begin{pmatrix}0&0&0&0&0&1&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&1&0\\0&-1&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0\end{pmatrix}, \begin{pmatrix}0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&0&-1\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\end{pmatrix}, \begin{pmatrix}0&0&0&0&0&0&0&-1\\0&0&0&0&0&0&1&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&1&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&1&0&0&0&0&0&0\\-1&0&0&0&0&0&0&0\end{pmatrix}, \begin{pmatrix}-1&0&0&0&0&0&0&0\\0&-1&0&0&0&0&0&0\\0&0&-1&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix}\right\}$$

In[ ]:=
```
MatrixForm[#] & /@ allQBar
```

Out[ ]=

$$\left\{\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \begin{pmatrix} 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0 \end{pmatrix},\right.$$

$$\begin{pmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0&0&0&-1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&0&0&1&0\\ 0&-1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&1&0&0&0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0&0&0&-1&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&-1&0\\ 0&1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&1&0&0&0&0&0 \end{pmatrix},$$

$$\left.\begin{pmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&-1 \end{pmatrix}\right\}$$

Verify Unit Spinor and Projections

In[ ]:=
```
verifyAntiCommutation[]
```

Out[ ]=

True

In[ ]:=
```
(* Check unit is eigenspinor of σ8 *)
Print["Unit spinor: ", unit]
Print["σ8.unit = ", Simplify[σ8.unit]]
Print["Eigenvalue check (difference should be 0): ", Simplify[σ8.unit - unit]]
```

Unit spinor: $\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}$

σ8.unit = $\left\{\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0\right\}$

Eigenvalue check (difference should be 0): {0, 0, 0, 0, 0, 0, 0, 0}

```
In[ ]:= (* Check realProjection8 is a projector *)
Print["realProjection8^2 - realProjection8 (should be 0): ",
 Simplify[realProjection8.realProjection8 - realProjection8]]
Print["Tr[realProjection8] = ", Tr[realProjection8]]

realProjection8^2 - realProjection8 (should be 0):
 {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}

Tr[realProjection8] = 1
```

Verify Fundamental Identity

```
In[ ]:= (* Test fundamental identity on several matrices *)
Print["FundamentalIdentity test on ID8: ", fundamentalIdentityTest1]
Print["FundamentalIdentity test on σ8: ", fundamentalIdentityTest2]
Print["FundamentalIdentity test on tau[1].tau[2]: ", fundamentalIdentityTest3]

FundamentalIdentity test on ID8: True

FundamentalIdentity test on σ8: True

FundamentalIdentity test on tau[1].tau[2]: False

In[ ]:= Q[1] . Q[2] === -Transpose[Q[1] . Q[2]]

Out[ ]=
True
```

Define Test Spinors and Coordinates

```
In[ ]:= (* Define symbolic 8-component spinors *)
(*psi1 = f8[#][x0, x4] & /@ Range[0, 7];
psi2 = h8[#][x0, x4] & /@ Range[0, 7];
Print["psi1 = ", psi1]
 Print["psi2 = ", psi2]*)

In[ ]:= (* Define symbolic 16-component spinor *)
Ψ16 = f16[#][x0, x4] & /@ Range[0, 15];
Print["Ψ16 = ", Ψ16]

Ψ16 = {f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4], f16[4][x0, x4],
  f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4], f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4],
  f16[11][x0, x4], f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}

In[ ]:= (* Define 8 coordinates X^A *)
X = {x0, x1, x2, x3, x4, x5, x6, x7};
Print["Coordinates X = ", X]

Coordinates X = {x0, x1, x2, x3, x4, x5, x6, x7}
```

In[ ]:=
```
Protect[Q1, M, Ψ16, f16, X, x0, x1, x2, x3, x4, x5, x6, x7]
```
Out[ ]=
```
{Q1, M, Ψ16, f16, X, x0, x1, x2, x3, x4, x5, x6, x7}
```

Simple example
Lagrangian L16
todo (trivial, but tiresome): Construction does not enploy Octad,...
L16 ≠ La , FROM “Pair_Creation_of_Universes_WaveFunctionOfUniverse-4+4-Einstein-Lovelock-Nash.nb”!

```
FROM "Pair_Creation_of_Universes_WaveFunctionOfUniverse-4+4-
   Einstein-Lovelock-Nash.nb"

Clear[La];
La[] :=
 useDSQRT * (Transpose[Ψ16].σ16.Sum[useT16[[α1]].(D[ Ψ16,
              X[[α1]]] + (Q1/2) * Sum[ωμIJ[α1][[A1, B1]] * SAB[[A1, B1]],
                {A1, 1, 8}, {B1, 1, 8}].Ψ16), {α1, 1, Length[X]}] +
       (H * M) * Transpose[Ψ16].σ16.Ψ16) //
   Simplify[#, constraintVars] &
```

The L16 Lagrangian for 16-component Spin(4,4) spinors:

$$L_{16} = \Psi^T.\sigma_{16}.\sum_{A=0}^{7} T_{16}[A].\partial_{X^A}\Psi + \frac{M}{2}\Psi^T.\sigma_{16}.\Psi$$

In[ ]:=
```
(* Define mass parameter M *)
```

In[•]:=
```
(* Mass term: (M/2) Ψ16^T . σ16 . Ψ16 *)
L16mass = M / 2 * Transpose[Ψ16].σ16.Ψ16;
Print["L16 M term: ", Simplify[L16mass]]
```

```
L16 M term: -M (f16[0][x0, x4] × f16[4][x0, x4] + f16[1][x0, x4] × f16[5][x0, x4] +
    f16[2][x0, x4] × f16[6][x0, x4] + f16[3][x0, x4] × f16[7][x0, x4] -
    f16[8][x0, x4] × f16[12][x0, x4] - f16[9][x0, x4] × f16[13][x0, x4] -
    f16[10][x0, x4] × f16[14][x0, x4] - f16[11][x0, x4] × f16[15][x0, x4])
```

In[•]:=
```
(* Kinetic term: Ψ16^T . σ16 . Sum[T16A[A].D[Ψ16,X⟦A+1⟧],{A,0,7}] *)
L16kinetic = Transpose[Ψ16].σ16.Sum[T16A[A].D[Ψ16, X⟦A + 1⟧], {A, 0, 7}];
Print["L16 kinetic term: ", Simplify[L16kinetic]]
```

```
L16 kinetic term: f16[12][x0, x4] (f16[5]^(0,1)[x0, x4] + f16[0]^(1,0)[x0, x4]) +
  f16[13][x0, x4] (-f16[4]^(0,1)[x0, x4] + f16[1]^(1,0)[x0, x4]) +
  f16[14][x0, x4] (-f16[7]^(0,1)[x0, x4] + f16[2]^(1,0)[x0, x4]) +
  f16[15][x0, x4] (f16[6]^(0,1)[x0, x4] + f16[3]^(1,0)[x0, x4]) +
  f16[8][x0, x4] (-f16[1]^(0,1)[x0, x4] + f16[4]^(1,0)[x0, x4]) +
  f16[9][x0, x4] (f16[0]^(0,1)[x0, x4] + f16[5]^(1,0)[x0, x4]) +
  f16[10][x0, x4] (f16[3]^(0,1)[x0, x4] + f16[6]^(1,0)[x0, x4]) +
  f16[11][x0, x4] (-f16[2]^(0,1)[x0, x4] + f16[7]^(1,0)[x0, x4]) +
  f16[4][x0, x4] (f16[13]^(0,1)[x0, x4] - f16[8]^(1,0)[x0, x4]) -
  f16[5][x0, x4] (f16[12]^(0,1)[x0, x4] + f16[9]^(1,0)[x0, x4]) -
  f16[6][x0, x4] (f16[15]^(0,1)[x0, x4] + f16[10]^(1,0)[x0, x4]) +
  f16[7][x0, x4] (f16[14]^(0,1)[x0, x4] - f16[11]^(1,0)[x0, x4]) -
  f16[0][x0, x4] (f16[9]^(0,1)[x0, x4] + f16[12]^(1,0)[x0, x4]) +
  f16[1][x0, x4] (f16[8]^(0,1)[x0, x4] - f16[13]^(1,0)[x0, x4]) +
  f16[2][x0, x4] (f16[11]^(0,1)[x0, x4] - f16[14]^(1,0)[x0, x4]) -
  f16[3][x0, x4] (f16[10]^(0,1)[x0, x4] + f16[15]^(1,0)[x0, x4])
```

```
(* Full L16 Lagrangian,  NOT  La  from
 "Pair_Creation_of_Universes_WaveFunctionOfUniverse-4+4-Einstein-Lovelock-Nash.nb"  *)
L16 = L16kinetic + L16mass;
Print["\n=== Full L16 Lagrangian ==="]
Print[Simplify[L16]]
```

```
=== Full L16 Lagrangian ===

-M (f16[0][x0, x4] × f16[4][x0, x4] + f16[1][x0, x4] × f16[5][x0, x4] +
    f16[2][x0, x4] × f16[6][x0, x4] + f16[3][x0, x4] × f16[7][x0, x4] -
    f16[8][x0, x4] × f16[12][x0, x4] - f16[9][x0, x4] × f16[13][x0, x4] -
    f16[10][x0, x4] × f16[14][x0, x4] - f16[11][x0, x4] × f16[15][x0, x4]) +
 f16[12][x0, x4] (f16[5]^(0,1)[x0, x4] + f16[0]^(1,0)[x0, x4]) +
 f16[13][x0, x4] (-f16[4]^(0,1)[x0, x4] + f16[1]^(1,0)[x0, x4]) +
 f16[14][x0, x4] (-f16[7]^(0,1)[x0, x4] + f16[2]^(1,0)[x0, x4]) +
 f16[15][x0, x4] (f16[6]^(0,1)[x0, x4] + f16[3]^(1,0)[x0, x4]) +
 f16[8][x0, x4] (-f16[1]^(0,1)[x0, x4] + f16[4]^(1,0)[x0, x4]) +
 f16[9][x0, x4] (f16[0]^(0,1)[x0, x4] + f16[5]^(1,0)[x0, x4]) +
 f16[10][x0, x4] (f16[3]^(0,1)[x0, x4] + f16[6]^(1,0)[x0, x4]) +
 f16[11][x0, x4] (-f16[2]^(0,1)[x0, x4] + f16[7]^(1,0)[x0, x4]) +
 f16[4][x0, x4] (f16[13]^(0,1)[x0, x4] - f16[8]^(1,0)[x0, x4]) -
 f16[5][x0, x4] (f16[12]^(0,1)[x0, x4] + f16[9]^(1,0)[x0, x4]) -
 f16[6][x0, x4] (f16[15]^(0,1)[x0, x4] + f16[10]^(1,0)[x0, x4]) +
 f16[7][x0, x4] (f16[14]^(0,1)[x0, x4] - f16[11]^(1,0)[x0, x4]) -
 f16[0][x0, x4] (f16[9]^(0,1)[x0, x4] + f16[12]^(1,0)[x0, x4]) +
 f16[1][x0, x4] (f16[8]^(0,1)[x0, x4] - f16[13]^(1,0)[x0, x4]) +
 f16[2][x0, x4] (f16[11]^(0,1)[x0, x4] - f16[14]^(1,0)[x0, x4]) -
 f16[3][x0, x4] (f16[10]^(0,1)[x0, x4] + f16[15]^(1,0)[x0, x4])
```

```
Clear[eL16];
```

Euler–Lagrange expressions:

```
eL16[] := Module[{L, t, useDSQRT = 1},
  L = L16;
  t = (*Parallel*)Table[(*Full*)
    Simplify[1/useDSQRT (D[L, f16[k][x0, x4]] - D[D[L, f16[k]^(1,0)[x0, x4]], x0] -
        D[D[L, f16[k]^(0,1)[x0, x4]], x4])], {k, 0, 15}];
  Return[t];
 ]
```

In[ ]:=
```
(el16 = eL16[]) // Column
```

Out[ ]=
```
-M f16[4][x0, x4] - 2 (f16[9]^(0,1)[x0, x4] + f16[12]^(1,0)[x0, x4])
-M f16[5][x0, x4] + 2 f16[8]^(0,1)[x0, x4] - 2 f16[13]^(1,0)[x0, x4]
-M f16[6][x0, x4] + 2 f16[11]^(0,1)[x0, x4] - 2 f16[14]^(1,0)[x0, x4]
-M f16[7][x0, x4] - 2 (f16[10]^(0,1)[x0, x4] + f16[15]^(1,0)[x0, x4])
-M f16[0][x0, x4] + 2 f16[13]^(0,1)[x0, x4] - 2 f16[8]^(1,0)[x0, x4]
-M f16[1][x0, x4] - 2 (f16[12]^(0,1)[x0, x4] + f16[9]^(1,0)[x0, x4])
-M f16[2][x0, x4] - 2 (f16[15]^(0,1)[x0, x4] + f16[10]^(1,0)[x0, x4])
-M f16[3][x0, x4] + 2 f16[14]^(0,1)[x0, x4] - 2 f16[11]^(1,0)[x0, x4]
M f16[12][x0, x4] - 2 f16[1]^(0,1)[x0, x4] + 2 f16[4]^(1,0)[x0, x4]
M f16[13][x0, x4] + 2 (f16[0]^(0,1)[x0, x4] + f16[5]^(1,0)[x0, x4])
M f16[14][x0, x4] + 2 (f16[3]^(0,1)[x0, x4] + f16[6]^(1,0)[x0, x4])
M f16[15][x0, x4] - 2 f16[2]^(0,1)[x0, x4] + 2 f16[7]^(1,0)[x0, x4]
M f16[8][x0, x4] + 2 (f16[5]^(0,1)[x0, x4] + f16[0]^(1,0)[x0, x4])
M f16[9][x0, x4] - 2 f16[4]^(0,1)[x0, x4] + 2 f16[1]^(1,0)[x0, x4]
M f16[10][x0, x4] - 2 f16[7]^(0,1)[x0, x4] + 2 f16[2]^(1,0)[x0, x4]
M f16[11][x0, x4] + 2 (f16[6]^(0,1)[x0, x4] + f16[3]^(1,0)[x0, x4])
```

In[ ]:=
```
rawSets = Cases[
  #,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
  Infinity,
  Heads → True
] & /@ el16
```

Out[ ]=
```
{{4, 9, 12}, {5, 8, 13}, {6, 11, 14}, {7, 10, 15},
 {0, 13, 8}, {1, 12, 9}, {2, 15, 10}, {3, 14, 11}, {12, 1, 4}, {13, 0, 5},
 {14, 3, 6}, {15, 2, 7}, {8, 5, 0}, {9, 4, 1}, {10, 7, 2}, {11, 6, 3}}
```

In[ ]:=
```
Dimensions[rawSets]
```

Out[ ]=
```
{16, 3}
```

In[ ]:=
```
couplings = showCoupledEquations[rawSetsel16]
% // Flatten // Sort
% === Range[0, 15]
```

In[240]:=

```
(*processRawSets[]:=Module[{s1,s2,J,L=Range[Length[rawSets]],r={}},
  While[Length[L]>1,
   s1=Union[rawSets[[L[[1]]]]];
   L=Drop[L,1];
   J=L[[1]];
   While[J≤16,
    If[MemberQ[L,J],
     s2=Union[rawSets[[J]]];
     If[Intersection[s1,s2]≠{},s1=Union[Flatten[{s1,s2}]];
      L=Complement[L,{J}];,##&[]];,
     ##&[]];
    J++;];

   AppendTo[r,s1];
  ];
  Return[r];
 ]*)
```

In[241]:=

```
(*couplings=Sort[processRawSets[],#1[[1]]<#2[[1]]&]*)
```

In[ ]:=

```
Z16 = Z[#][x0, x4] & /@ Range[0, 15]
```

Out[ ]=

```
{Z[0][x0, x4], Z[1][x0, x4], Z[2][x0, x4], Z[3][x0, x4], Z[4][x0, x4], Z[5][x0, x4],
 Z[6][x0, x4], Z[7][x0, x4], Z[8][x0, x4], Z[9][x0, x4], Z[10][x0, x4],
 Z[11][x0, x4], Z[12][x0, x4], Z[13][x0, x4], Z[14][x0, x4], Z[15][x0, x4]}
```

In[ ]:=

```
Z16s = Partition[Z16, Length[couplings]]
```

Out[ ]=

```
{{Z[0][x0, x4], Z[1][x0, x4], Z[2][x0, x4], Z[3][x0, x4]},
 {Z[4][x0, x4], Z[5][x0, x4], Z[6][x0, x4], Z[7][x0, x4]},
 {Z[8][x0, x4], Z[9][x0, x4], Z[10][x0, x4], Z[11][x0, x4]},
 {Z[12][x0, x4], Z[13][x0, x4], Z[14][x0, x4], Z[15][x0, x4]}}
```

In[ ]:=

```
sf16toZ = Thread[(f16[#] & /@ Flatten[couplings]) →
   Table[ToExpression["((Z[" <> ToString[j] <> "][#1,#2])&)"], {j, 0, 15}]]
```

Out[ ]=

```
{f16[0] → (Z[0][#1, #2] &), f16[5] → (Z[1][#1, #2] &),
 f16[8] → (Z[2][#1, #2] &), f16[13] → (Z[3][#1, #2] &), f16[1] → (Z[4][#1, #2] &),
 f16[4] → (Z[5][#1, #2] &), f16[9] → (Z[6][#1, #2] &), f16[12] → (Z[7][#1, #2] &),
 f16[2] → (Z[8][#1, #2] &), f16[7] → (Z[9][#1, #2] &), f16[10] → (Z[10][#1, #2] &),
 f16[15] → (Z[11][#1, #2] &), f16[3] → (Z[12][#1, #2] &), f16[6] → (Z[13][#1, #2] &),
 f16[11] → (Z[14][#1, #2] &), f16[14] → (Z[15][#1, #2] &)}
```

In[ ]:=

```
elZ = el16 /. sf16toZ
(sDx4Z = Solve[And @@ Thread[0 == %], Z[#]^(0,1)[x0, x4] & /@ Range[0, 15]][[-1]]) // Column
eqsDx4Z = Partition[sDx4Z /. {Rule → Equal}, Length[couplings]]
```

Out[∘]=

$$\{-M\,Z[5][x0,x4]-2\left(Z[6]^{(0,1)}[x0,x4]+Z[7]^{(1,0)}[x0,x4]\right),$$
$$-M\,Z[1][x0,x4]+2\,Z[2]^{(0,1)}[x0,x4]-2\,Z[3]^{(1,0)}[x0,x4],$$
$$-M\,Z[13][x0,x4]+2\,Z[14]^{(0,1)}[x0,x4]-2\,Z[15]^{(1,0)}[x0,x4],$$
$$-M\,Z[9][x0,x4]-2\left(Z[10]^{(0,1)}[x0,x4]+Z[11]^{(1,0)}[x0,x4]\right),$$
$$-M\,Z[0][x0,x4]+2\,Z[3]^{(0,1)}[x0,x4]-2\,Z[2]^{(1,0)}[x0,x4],$$
$$-M\,Z[4][x0,x4]-2\left(Z[7]^{(0,1)}[x0,x4]+Z[6]^{(1,0)}[x0,x4]\right),$$
$$-M\,Z[8][x0,x4]-2\left(Z[11]^{(0,1)}[x0,x4]+Z[10]^{(1,0)}[x0,x4]\right),$$
$$-M\,Z[12][x0,x4]+2\,Z[15]^{(0,1)}[x0,x4]-2\,Z[14]^{(1,0)}[x0,x4],$$
$$M\,Z[7][x0,x4]-2\,Z[4]^{(0,1)}[x0,x4]+2\,Z[5]^{(1,0)}[x0,x4],$$
$$M\,Z[3][x0,x4]+2\left(Z[0]^{(0,1)}[x0,x4]+Z[1]^{(1,0)}[x0,x4]\right),$$
$$M\,Z[15][x0,x4]+2\left(Z[12]^{(0,1)}[x0,x4]+Z[13]^{(1,0)}[x0,x4]\right),$$
$$M\,Z[11][x0,x4]-2\,Z[8]^{(0,1)}[x0,x4]+2\,Z[9]^{(1,0)}[x0,x4],$$
$$M\,Z[2][x0,x4]+2\left(Z[1]^{(0,1)}[x0,x4]+Z[0]^{(1,0)}[x0,x4]\right),$$
$$M\,Z[6][x0,x4]-2\,Z[5]^{(0,1)}[x0,x4]+2\,Z[4]^{(1,0)}[x0,x4],$$
$$M\,Z[10][x0,x4]-2\,Z[9]^{(0,1)}[x0,x4]+2\,Z[8]^{(1,0)}[x0,x4],$$
$$M\,Z[14][x0,x4]+2\left(Z[13]^{(0,1)}[x0,x4]+Z[12]^{(1,0)}[x0,x4]\right)\}$$

Out[∘]=

$$Z[0]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(-M\,Z[3][x0,x4]-2\,Z[1]^{(1,0)}[x0,x4]\right)$$
$$Z[1]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(-M\,Z[2][x0,x4]-2\,Z[0]^{(1,0)}[x0,x4]\right)$$
$$Z[2]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(M\,Z[1][x0,x4]+2\,Z[3]^{(1,0)}[x0,x4]\right)$$
$$Z[3]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(M\,Z[0][x0,x4]+2\,Z[2]^{(1,0)}[x0,x4]\right)$$
$$Z[4]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(M\,Z[7][x0,x4]+2\,Z[5]^{(1,0)}[x0,x4]\right)$$
$$Z[5]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(M\,Z[6][x0,x4]+2\,Z[4]^{(1,0)}[x0,x4]\right)$$
$$Z[6]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(-M\,Z[5][x0,x4]-2\,Z[7]^{(1,0)}[x0,x4]\right)$$
$$Z[7]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(-M\,Z[4][x0,x4]-2\,Z[6]^{(1,0)}[x0,x4]\right)$$
$$Z[8]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(M\,Z[11][x0,x4]+2\,Z[9]^{(1,0)}[x0,x4]\right)$$
$$Z[9]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(M\,Z[10][x0,x4]+2\,Z[8]^{(1,0)}[x0,x4]\right)$$
$$Z[10]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(-M\,Z[9][x0,x4]-2\,Z[11]^{(1,0)}[x0,x4]\right)$$
$$Z[11]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(-M\,Z[8][x0,x4]-2\,Z[10]^{(1,0)}[x0,x4]\right)$$
$$Z[12]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(-M\,Z[15][x0,x4]-2\,Z[13]^{(1,0)}[x0,x4]\right)$$
$$Z[13]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(-M\,Z[14][x0,x4]-2\,Z[12]^{(1,0)}[x0,x4]\right)$$
$$Z[14]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(M\,Z[13][x0,x4]+2\,Z[15]^{(1,0)}[x0,x4]\right)$$
$$Z[15]^{(0,1)}[x0,x4]\to\frac{1}{2}\left(M\,Z[12][x0,x4]+2\,Z[14]^{(1,0)}[x0,x4]\right)$$

Out[ ]=

$$\left\{\left\{Z[0]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[3][x0, x4] - 2\,Z[1]^{(1,0)}[x0, x4]\right),\right.\right.$$
$$Z[1]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[2][x0, x4] - 2\,Z[0]^{(1,0)}[x0, x4]\right),$$
$$Z[2]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[1][x0, x4] + 2\,Z[3]^{(1,0)}[x0, x4]\right),$$
$$\left.Z[3]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[0][x0, x4] + 2\,Z[2]^{(1,0)}[x0, x4]\right)\right\},$$
$$\left\{Z[4]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[7][x0, x4] + 2\,Z[5]^{(1,0)}[x0, x4]\right),\right.$$
$$Z[5]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[6][x0, x4] + 2\,Z[4]^{(1,0)}[x0, x4]\right),$$
$$Z[6]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[5][x0, x4] - 2\,Z[7]^{(1,0)}[x0, x4]\right),$$
$$\left.Z[7]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[4][x0, x4] - 2\,Z[6]^{(1,0)}[x0, x4]\right)\right\},$$
$$\left\{Z[8]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[11][x0, x4] + 2\,Z[9]^{(1,0)}[x0, x4]\right),\right.$$
$$Z[9]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[10][x0, x4] + 2\,Z[8]^{(1,0)}[x0, x4]\right),$$
$$Z[10]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[9][x0, x4] - 2\,Z[11]^{(1,0)}[x0, x4]\right),$$
$$\left.Z[11]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[8][x0, x4] - 2\,Z[10]^{(1,0)}[x0, x4]\right)\right\},$$
$$\left\{Z[12]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[15][x0, x4] - 2\,Z[13]^{(1,0)}[x0, x4]\right),\right.$$
$$Z[13]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[14][x0, x4] - 2\,Z[12]^{(1,0)}[x0, x4]\right),$$
$$Z[14]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[13][x0, x4] + 2\,Z[15]^{(1,0)}[x0, x4]\right),$$
$$\left.\left.Z[15]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[12][x0, x4] + 2\,Z[14]^{(1,0)}[x0, x4]\right)\right\}\right\}$$

DSolve fails; try to solve using Maple?

In[ ]:= **DSolve[eqsDx4Z[[#]], Z16s[[#]], {x0, x4}] & /@ Range[Length[eqsDx4Z]]**

Out[ ]=

$$\left\{\text{DSolve}\left[\left\{Z[0]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[3][x0, x4] - 2\,Z[1]^{(1,0)}[x0, x4]\right),\right.\right.\right.$$
$$Z[1]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[2][x0, x4] - 2\,Z[0]^{(1,0)}[x0, x4]\right),$$
$$Z[2]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[1][x0, x4] + 2\,Z[3]^{(1,0)}[x0, x4]\right),$$
$$\left.Z[3]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[0][x0, x4] + 2\,Z[2]^{(1,0)}[x0, x4]\right)\right\},$$
$$\left.\{Z[0][x0, x4], Z[1][x0, x4], Z[2][x0, x4], Z[3][x0, x4]\}, \{x0, x4\}\right],$$
$$\text{DSolve}\left[\left\{Z[4]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[7][x0, x4] + 2\,Z[5]^{(1,0)}[x0, x4]\right),\right.\right.$$
$$Z[5]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[6][x0, x4] + 2\,Z[4]^{(1,0)}[x0, x4]\right),$$
$$Z[6]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[5][x0, x4] - 2\,Z[7]^{(1,0)}[x0, x4]\right),$$
$$\left.Z[7]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[4][x0, x4] - 2\,Z[6]^{(1,0)}[x0, x4]\right)\right\},$$
$$\left.\{Z[4][x0, x4], Z[5][x0, x4], Z[6][x0, x4], Z[7][x0, x4]\}, \{x0, x4\}\right],$$
$$\text{DSolve}\left[\left\{Z[8]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[11][x0, x4] + 2\,Z[9]^{(1,0)}[x0, x4]\right),\right.\right.$$
$$Z[9]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[10][x0, x4] + 2\,Z[8]^{(1,0)}[x0, x4]\right),$$
$$Z[10]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[9][x0, x4] - 2\,Z[11]^{(1,0)}[x0, x4]\right),$$
$$\left.Z[11]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[8][x0, x4] - 2\,Z[10]^{(1,0)}[x0, x4]\right)\right\},$$
$$\left.\{Z[8][x0, x4], Z[9][x0, x4], Z[10][x0, x4], Z[11][x0, x4]\}, \{x0, x4\}\right],$$
$$\text{DSolve}\left[\left\{Z[12]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[15][x0, x4] - 2\,Z[13]^{(1,0)}[x0, x4]\right),\right.\right.$$
$$Z[13]^{(0,1)}[x0, x4] == \frac{1}{2}\left(-M\,Z[14][x0, x4] - 2\,Z[12]^{(1,0)}[x0, x4]\right),$$
$$Z[14]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[13][x0, x4] + 2\,Z[15]^{(1,0)}[x0, x4]\right),$$
$$\left.Z[15]^{(0,1)}[x0, x4] == \frac{1}{2}\left(M\,Z[12][x0, x4] + 2\,Z[14]^{(1,0)}[x0, x4]\right)\right\},$$
$$\left.\left.\{Z[12][x0, x4], Z[13][x0, x4], Z[14][x0, x4], Z[15][x0, x4]\}, \{x0, x4\}\right]\right\}$$

In[•]:=
```
(sDx4Dx4Z =
   Solve[And @@ Thread[0 == D[elZ, x4]], Z[#]^(0,2) [x0, x4] & /@ Range[0, 15]][[-1]] /. sDx4Z //
     FullSimplify // ExpandAll) // Column
eqsDx4Dx4Z = sDx4Dx4Z /. {Rule → Equal}
zeroeqsDx4Dx4Z = sDx4Dx4Z /. {Rule → Subtract}
```

Out[•]=

$$Z[0]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[0][x0, x4] - \frac{1}{2} M Z[2]^{(1,0)}[x0, x4] - Z[1]^{(1,1)}[x0, x4]$$
$$Z[1]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[1][x0, x4] - \frac{1}{2} M Z[3]^{(1,0)}[x0, x4] - Z[0]^{(1,1)}[x0, x4]$$
$$Z[2]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[2][x0, x4] - \frac{1}{2} M Z[0]^{(1,0)}[x0, x4] + Z[3]^{(1,1)}[x0, x4]$$
$$Z[3]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[3][x0, x4] - \frac{1}{2} M Z[1]^{(1,0)}[x0, x4] + Z[2]^{(1,1)}[x0, x4]$$
$$Z[4]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[4][x0, x4] - \frac{1}{2} M Z[6]^{(1,0)}[x0, x4] + Z[5]^{(1,1)}[x0, x4]$$
$$Z[5]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[5][x0, x4] - \frac{1}{2} M Z[7]^{(1,0)}[x0, x4] + Z[4]^{(1,1)}[x0, x4]$$
$$Z[6]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[6][x0, x4] - \frac{1}{2} M Z[4]^{(1,0)}[x0, x4] - Z[7]^{(1,1)}[x0, x4]$$
$$Z[7]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[7][x0, x4] - \frac{1}{2} M Z[5]^{(1,0)}[x0, x4] - Z[6]^{(1,1)}[x0, x4]$$
$$Z[8]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[8][x0, x4] - \frac{1}{2} M Z[10]^{(1,0)}[x0, x4] + Z[9]^{(1,1)}[x0, x4]$$
$$Z[9]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[9][x0, x4] - \frac{1}{2} M Z[11]^{(1,0)}[x0, x4] + Z[8]^{(1,1)}[x0, x4]$$
$$Z[10]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[10][x0, x4] - \frac{1}{2} M Z[8]^{(1,0)}[x0, x4] - Z[11]^{(1,1)}[x0, x4]$$
$$Z[11]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[11][x0, x4] - \frac{1}{2} M Z[9]^{(1,0)}[x0, x4] - Z[10]^{(1,1)}[x0, x4]$$
$$Z[12]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[12][x0, x4] - \frac{1}{2} M Z[14]^{(1,0)}[x0, x4] - Z[13]^{(1,1)}[x0, x4]$$
$$Z[13]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[13][x0, x4] - \frac{1}{2} M Z[15]^{(1,0)}[x0, x4] - Z[12]^{(1,1)}[x0, x4]$$
$$Z[14]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[14][x0, x4] - \frac{1}{2} M Z[12]^{(1,0)}[x0, x4] + Z[15]^{(1,1)}[x0, x4]$$
$$Z[15]^{(0,2)}[x0, x4] \to -\frac{1}{4} M^2 Z[15][x0, x4] - \frac{1}{2} M Z[13]^{(1,0)}[x0, x4] + Z[14]^{(1,1)}[x0, x4]$$

Out[ ]=

$$\left\{Z[0]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[0][x0, x4] - \frac{1}{2} M Z[2]^{(1,0)}[x0, x4] - Z[1]^{(1,1)}[x0, x4],\right.$$
$$Z[1]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[1][x0, x4] - \frac{1}{2} M Z[3]^{(1,0)}[x0, x4] - Z[0]^{(1,1)}[x0, x4],$$
$$Z[2]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[2][x0, x4] - \frac{1}{2} M Z[0]^{(1,0)}[x0, x4] + Z[3]^{(1,1)}[x0, x4],$$
$$Z[3]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[3][x0, x4] - \frac{1}{2} M Z[1]^{(1,0)}[x0, x4] + Z[2]^{(1,1)}[x0, x4],$$
$$Z[4]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[4][x0, x4] - \frac{1}{2} M Z[6]^{(1,0)}[x0, x4] + Z[5]^{(1,1)}[x0, x4],$$
$$Z[5]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[5][x0, x4] - \frac{1}{2} M Z[7]^{(1,0)}[x0, x4] + Z[4]^{(1,1)}[x0, x4],$$
$$Z[6]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[6][x0, x4] - \frac{1}{2} M Z[4]^{(1,0)}[x0, x4] - Z[7]^{(1,1)}[x0, x4],$$
$$Z[7]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[7][x0, x4] - \frac{1}{2} M Z[5]^{(1,0)}[x0, x4] - Z[6]^{(1,1)}[x0, x4],$$
$$Z[8]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[8][x0, x4] - \frac{1}{2} M Z[10]^{(1,0)}[x0, x4] + Z[9]^{(1,1)}[x0, x4],$$
$$Z[9]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[9][x0, x4] - \frac{1}{2} M Z[11]^{(1,0)}[x0, x4] + Z[8]^{(1,1)}[x0, x4],$$
$$Z[10]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[10][x0, x4] - \frac{1}{2} M Z[8]^{(1,0)}[x0, x4] - Z[11]^{(1,1)}[x0, x4],$$
$$Z[11]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[11][x0, x4] - \frac{1}{2} M Z[9]^{(1,0)}[x0, x4] - Z[10]^{(1,1)}[x0, x4],$$
$$Z[12]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[12][x0, x4] - \frac{1}{2} M Z[14]^{(1,0)}[x0, x4] - Z[13]^{(1,1)}[x0, x4],$$
$$Z[13]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[13][x0, x4] - \frac{1}{2} M Z[15]^{(1,0)}[x0, x4] - Z[12]^{(1,1)}[x0, x4],$$
$$Z[14]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[14][x0, x4] - \frac{1}{2} M Z[12]^{(1,0)}[x0, x4] + Z[15]^{(1,1)}[x0, x4],$$
$$\left.Z[15]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[15][x0, x4] - \frac{1}{2} M Z[13]^{(1,0)}[x0, x4] + Z[14]^{(1,1)}[x0, x4]\right\}$$

Out[∘]=

$$\left\{\frac{1}{4} M^2 Z[0][x0, x4] + Z[0]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[2]^{(1,0)}[x0, x4] + Z[1]^{(1,1)}[x0, x4],\right.$$
$$\frac{1}{4} M^2 Z[1][x0, x4] + Z[1]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[3]^{(1,0)}[x0, x4] + Z[0]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[2][x0, x4] + Z[2]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[0]^{(1,0)}[x0, x4] - Z[3]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[3][x0, x4] + Z[3]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[1]^{(1,0)}[x0, x4] - Z[2]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[4][x0, x4] + Z[4]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[6]^{(1,0)}[x0, x4] - Z[5]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[5][x0, x4] + Z[5]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[7]^{(1,0)}[x0, x4] - Z[4]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[6][x0, x4] + Z[6]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[4]^{(1,0)}[x0, x4] + Z[7]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[7][x0, x4] + Z[7]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[5]^{(1,0)}[x0, x4] + Z[6]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[8][x0, x4] + Z[8]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[10]^{(1,0)}[x0, x4] - Z[9]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[9][x0, x4] + Z[9]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[11]^{(1,0)}[x0, x4] - Z[8]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[10][x0, x4] + Z[10]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[8]^{(1,0)}[x0, x4] + Z[11]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[11][x0, x4] + Z[11]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[9]^{(1,0)}[x0, x4] + Z[10]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[12][x0, x4] + Z[12]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[14]^{(1,0)}[x0, x4] + Z[13]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[13][x0, x4] + Z[13]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[15]^{(1,0)}[x0, x4] + Z[12]^{(1,1)}[x0, x4],$$
$$\frac{1}{4} M^2 Z[14][x0, x4] + Z[14]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[12]^{(1,0)}[x0, x4] - Z[15]^{(1,1)}[x0, x4],$$
$$\left.\frac{1}{4} M^2 Z[15][x0, x4] + Z[15]^{(0,2)}[x0, x4] + \frac{1}{2} M Z[13]^{(1,0)}[x0, x4] - Z[14]^{(1,1)}[x0, x4]\right\}$$

In[∘]:= **DSolve$\left[Z[15]^{(0,2)}[x0, x4] == -\frac{1}{4} M^2 Z[15][x0, x4], Z[15][x0, x4], \{x0, x4\}\right]$**

Out[∘]=

$$\left\{\left\{Z[15][x0, x4] \to \mathrm{Cos}\left[\frac{M\,x4}{2}\right] c_1[x0] + \mathrm{Sin}\left[\frac{M\,x4}{2}\right] c_2[x0]\right\}\right\}$$

In[∘]:=
```
(*
sZtoY=Thread[(Z[#]&/@Flatten[couplings])→
    Table[ToExpression["((Y["<>ToString[j]<>",1][#1]*Cos[M #2/2]+Y["<>
        ToString[j]<>",2][#1]*Sin[M #2/2])&)"],{j,0,15}]]*)
```

In[ ]:=

```
sZtoY = Thread[(Z[#] & /@ Flatten[couplings]) →
   Table[ToExpression["((Exp[1/2*Y[" <> ToString[j] <> ",1]Log[Sin[#1]]+ⅈ (M #2)/2]+Exp[1/2*Y[" <>
      ToString[j] <> ",2]Log[Sin[#1]]-ⅈ (M #2)/2])&)"], {j, 0, 15}]]
```

Out[ ]=

$$\left\{Z[0] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[0,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[0,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),\right.$$

$$Z[5] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[1,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[1,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[8] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[2,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[2,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[13] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[3,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[3,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[1] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[4,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[4,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[4] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[5,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[5,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[9] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[6,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[6,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[12] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[7,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[7,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[2] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[8,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[8,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[7] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[9,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[9,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[10] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[10,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[10,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[15] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[11,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[11,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[3] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[12,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[12,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[6] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[13,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[13,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$Z[11] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[14,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[14,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right),$$

$$\left.Z[14] \to \left(\mathrm{Exp}\left[\frac{1}{2} Y[15,1] \,\mathrm{Log}[\mathrm{Sin}[\#1]] + \frac{1}{2} i (M \#2)\right] + \mathrm{Exp}\left[\frac{1}{2} Y[15,2] \,\mathrm{Log}[\mathrm{Sin}[\#1]] - \frac{1}{2} i (M \#2)\right] \&\right)\right\}$$

In[•]:=
```
Ys = Union[Flatten[{ Y[#, 1], Y[#, 2]} & /@ Range[0, 15]]]
```

Out[•]=
```
{Y[0, 1], Y[0, 2], Y[1, 1], Y[1, 2], Y[2, 1], Y[2, 2], Y[3, 1], Y[3, 2],
 Y[4, 1], Y[4, 2], Y[5, 1], Y[5, 2], Y[6, 1], Y[6, 2], Y[7, 1], Y[7, 2],
 Y[8, 1], Y[8, 2], Y[9, 1], Y[9, 2], Y[10, 1], Y[10, 2], Y[11, 1], Y[11, 2],
 Y[12, 1], Y[12, 2], Y[13, 1], Y[13, 2], Y[14, 1], Y[14, 2], Y[15, 1], Y[15, 2]}
```

In[•]:=

$$\left(\text{zeroeqsDx4Dx4Y} = \frac{4}{e^{-\frac{1}{2} i M x4} M \,\text{Cot}[x0]} \text{zeroeqsDx4Dx4Z} \;/.\; \text{sZtoY} \;//\; \text{FullSimplify}\right) \;//\; \text{Column}$$

Out[•]=

$$-i\, \text{Sin}[x0]^{\frac{1}{2} Y[4,2]}\, Y[4,2] + e^{i M x4} \left(i\, \text{Sin}[x0]^{\frac{1}{2} Y[4,1]}\, Y[4,1] + \text{Sin}[x0]^{\frac{1}{2} Y[8,1]}\, Y[8,1]\right) + \text{Sin}[x0]^{\frac{1}{2} Y[8,2]}\, Y[8,2]$$

$$-i\, \text{Sin}[x0]^{\frac{1}{2} Y[0,2]}\, Y[0,2] + e^{i M x4} \left(i\, \text{Sin}[x0]^{\frac{1}{2} Y[0,1]}\, Y[0,1] + \text{Sin}[x0]^{\frac{1}{2} Y[12,1]}\, Y[12,1]\right) + \text{Sin}[x0]^{\frac{1}{2} Y[12,2]}\, Y[12,2]$$

$$\text{Sin}[x0]^{\frac{1}{2} Y[0,2]}\, Y[0,2] + e^{i M x4} \left(\text{Sin}[x0]^{\frac{1}{2} Y[0,1]}\, Y[0,1] - i\, \text{Sin}[x0]^{\frac{1}{2} Y[12,1]}\, Y[12,1]\right) + i\, \text{Sin}[x0]^{\frac{1}{2} Y[12,2]}\, Y[12,2]$$

$$\text{Sin}[x0]^{\frac{1}{2} Y[4,2]}\, Y[4,2] + e^{i M x4} \left(\text{Sin}[x0]^{\frac{1}{2} Y[4,1]}\, Y[4,1] - i\, \text{Sin}[x0]^{\frac{1}{2} Y[8,1]}\, Y[8,1]\right) + i\, \text{Sin}[x0]^{\frac{1}{2} Y[8,2]}\, Y[8,2]$$

$$i\, \text{Sin}[x0]^{\frac{1}{2} Y[1,2]}\, Y[1,2] + e^{i M x4} \left(-i\, \text{Sin}[x0]^{\frac{1}{2} Y[1,1]}\, Y[1,1] + \text{Sin}[x0]^{\frac{1}{2} Y[13,1]}\, Y[13,1]\right) + \text{Sin}[x0]^{\frac{1}{2} Y[13,2]}\, Y[13,2]$$

$$i\, \text{Sin}[x0]^{\frac{1}{2} Y[5,2]}\, Y[5,2] + e^{i M x4} \left(-i\, \text{Sin}[x0]^{\frac{1}{2} Y[5,1]}\, Y[5,1] + \text{Sin}[x0]^{\frac{1}{2} Y[9,1]}\, Y[9,1]\right) + \text{Sin}[x0]^{\frac{1}{2} Y[9,2]}\, Y[9,2]$$

$$\text{Sin}[x0]^{\frac{1}{2} Y[5,2]}\, Y[5,2] + e^{i M x4} \left(\text{Sin}[x0]^{\frac{1}{2} Y[5,1]}\, Y[5,1] + i\, \text{Sin}[x0]^{\frac{1}{2} Y[9,1]}\, Y[9,1]\right) - i\, \text{Sin}[x0]^{\frac{1}{2} Y[9,2]}\, Y[9,2]$$

$$\text{Sin}[x0]^{\frac{1}{2} Y[1,2]}\, Y[1,2] + e^{i M x4} \left(\text{Sin}[x0]^{\frac{1}{2} Y[1,1]}\, Y[1,1] + i\, \text{Sin}[x0]^{\frac{1}{2} Y[13,1]}\, Y[13,1]\right) - i\, \text{Sin}[x0]^{\frac{1}{2} Y[13,2]}\, Y[13,2]$$

$$i\, \text{Sin}[x0]^{\frac{1}{2} Y[6,2]}\, Y[6,2] + e^{i M x4} \left(-i\, \text{Sin}[x0]^{\frac{1}{2} Y[6,1]}\, Y[6,1] + \text{Sin}[x0]^{\frac{1}{2} Y[10,1]}\, Y[10,1]\right) + \text{Sin}[x0]^{\frac{1}{2} Y[10,2]}\, Y[10,2]$$

$$i\, \text{Sin}[x0]^{\frac{1}{2} Y[2,2]}\, Y[2,2] + e^{i M x4} \left(-i\, \text{Sin}[x0]^{\frac{1}{2} Y[2,1]}\, Y[2,1] + \text{Sin}[x0]^{\frac{1}{2} Y[14,1]}\, Y[14,1]\right) + \text{Sin}[x0]^{\frac{1}{2} Y[14,2]}\, Y[14,2]$$

$$\text{Sin}[x0]^{\frac{1}{2} Y[2,2]}\, Y[2,2] + e^{i M x4} \left(\text{Sin}[x0]^{\frac{1}{2} Y[2,1]}\, Y[2,1] + i\, \text{Sin}[x0]^{\frac{1}{2} Y[14,1]}\, Y[14,1]\right) - i\, \text{Sin}[x0]^{\frac{1}{2} Y[14,2]}\, Y[14,2]$$

$$\text{Sin}[x0]^{\frac{1}{2} Y[6,2]}\, Y[6,2] + e^{i M x4} \left(\text{Sin}[x0]^{\frac{1}{2} Y[6,1]}\, Y[6,1] + i\, \text{Sin}[x0]^{\frac{1}{2} Y[10,1]}\, Y[10,1]\right) - i\, \text{Sin}[x0]^{\frac{1}{2} Y[10,2]}\, Y[10,2]$$

$$-\mathrm{i}\,\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[3,2]}\,Y[3,2]+$$
$$e^{\mathrm{i}\,M\,x4}\left(\mathrm{i}\,\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[3,1]}\,Y[3,1]+\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[15,1]}\,Y[15,1]\right)+\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[15,2]}\,Y[15,2]$$
$$-\mathrm{i}\,\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[7,2]}\,Y[7,2]+$$
$$e^{\mathrm{i}\,M\,x4}\left(\mathrm{i}\,\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[7,1]}\,Y[7,1]+\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[11,1]}\,Y[11,1]\right)+\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[11,2]}\,Y[11,2]$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[7,2]}\,Y[7,2]+$$
$$e^{\mathrm{i}\,M\,x4}\left(\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[7,1]}\,Y[7,1]-\mathrm{i}\,\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[11,1]}\,Y[11,1]\right)+\mathrm{i}\,\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[11,2]}\,Y[11,2]$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[3,2]}\,Y[3,2]+$$
$$e^{\mathrm{i}\,M\,x4}\left(\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[3,1]}\,Y[3,1]-\mathrm{i}\,\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[15,1]}\,Y[15,1]\right)+\mathrm{i}\,\mathrm{Sin}[x0]^{\frac{1}{2}\,Y[15,2]}\,Y[15,2]$$

In[ ]:= $$\mathbf{Solve}\left[y[3,1] == \mathrm{Sin}[x0]^{\frac{1}{2}\,Y[3,1]}\,Y[3,1],\ Y[3,1]\right]$$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[ ]=
$$\left\{\left\{Y[3,1]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[3,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]}\right\}\right\}$$

In[ ]:= $$\mathbf{sY = Union}\left[\mathrm{Flatten}\left[\left\{Y[\#,1]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[\#,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},\ Y[\#,2]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[\#,2]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]}\right\}\ \&\ /@\ \mathrm{Range}[0,15]\right]\right]$$

Out[ ]=
$$\left\{Y[0,1]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[0,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},\right.$$
$$Y[0,2]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[0,2]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$
$$Y[1,1]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[1,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$
$$Y[1,2]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[1,2]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$
$$Y[2,1]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[2,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$
$$Y[2,2]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[2,2]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$
$$Y[3,1]\to\frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\,y[3,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[3, 2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[3, 2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[4, 1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[4, 1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[4, 2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[4, 2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[5, 1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[5, 1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[5, 2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[5, 2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[6, 1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[6, 1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[6, 2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[6, 2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[7, 1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[7, 1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[7, 2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[7, 2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[8, 1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[8, 1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[8, 2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[8, 2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[9, 1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[9, 1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[9, 2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[9, 2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[10, 1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[10, 1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[10, 2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[10, 2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[11, 1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[11, 1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[11, 2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[11, 2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[12,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[12,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$
$$Y[12,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[12,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$
$$Y[13,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[13,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$
$$Y[13,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[13,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$
$$Y[14,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[14,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$
$$Y[14,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[14,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$
$$Y[15,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[15,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$
$$Y[15,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[15,2]\right]}{\text{Log}[\text{Sin}[x0]]}\}$$

In[ ]:=
$$\textbf{sy = Union}\Big[\textbf{Flatten}\Big[\left\{\text{Sin}[x0]^{\frac{1}{2}Y[\#,1]}\,Y[\#,1] \to y[\#,1],\ \text{Sin}[x0]^{\frac{1}{2}Y[\#,2]}\,Y[\#,2] \to y[\#,2]\right\}\ \&\ /@\ \text{Range}[0,15]\Big]\Big]$$

Out[ ]=

$$\{\mathrm{Sin}[x0]^{\frac{1}{2}Y[0,1]}\,Y[0,1]\to y[0,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[0,2]}\,Y[0,2]\to y[0,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[1,1]}\,Y[1,1]\to y[1,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[1,2]}\,Y[1,2]\to y[1,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[2,1]}\,Y[2,1]\to y[2,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[2,2]}\,Y[2,2]\to y[2,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[3,1]}\,Y[3,1]\to y[3,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[3,2]}\,Y[3,2]\to y[3,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[4,1]}\,Y[4,1]\to y[4,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[4,2]}\,Y[4,2]\to y[4,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[5,1]}\,Y[5,1]\to y[5,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[5,2]}\,Y[5,2]\to y[5,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[6,1]}\,Y[6,1]\to y[6,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[6,2]}\,Y[6,2]\to y[6,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[7,1]}\,Y[7,1]\to y[7,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[7,2]}\,Y[7,2]\to y[7,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[8,1]}\,Y[8,1]\to y[8,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[8,2]}\,Y[8,2]\to y[8,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[9,1]}\,Y[9,1]\to y[9,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[9,2]}\,Y[9,2]\to y[9,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[10,1]}\,Y[10,1]\to y[10,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[10,2]}\,Y[10,2]\to y[10,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[11,1]}\,Y[11,1]\to y[11,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[11,2]}\,Y[11,2]\to y[11,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[12,1]}\,Y[12,1]\to y[12,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[12,2]}\,Y[12,2]\to y[12,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[13,1]}\,Y[13,1]\to y[13,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[13,2]}\,Y[13,2]\to y[13,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[14,1]}\,Y[14,1]\to y[14,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[14,2]}\,Y[14,2]\to y[14,2],$$
$$\mathrm{Sin}[x0]^{\frac{1}{2}Y[15,1]}\,Y[15,1]\to y[15,1],\ \mathrm{Sin}[x0]^{\frac{1}{2}Y[15,2]}\,Y[15,2]\to y[15,2]\}$$

In[ ]:= **zeroeqsDx4Dx4y = zeroeqsDx4Dx4Y /. sy**

Out[ ]=

$$\{-i\,y[4,2]+e^{i\,M\,x4}\,(i\,y[4,1]+y[8,1])+y[8,2],$$
$$-i\,y[0,2]+e^{i\,M\,x4}\,(i\,y[0,1]+y[12,1])+y[12,2],$$
$$y[0,2]+e^{i\,M\,x4}\,(y[0,1]-i\,y[12,1])+i\,y[12,2],$$
$$y[4,2]+e^{i\,M\,x4}\,(y[4,1]-i\,y[8,1])+i\,y[8,2],$$
$$i\,y[1,2]+e^{i\,M\,x4}\,(-i\,y[1,1]+y[13,1])+y[13,2],$$
$$i\,y[5,2]+e^{i\,M\,x4}\,(-i\,y[5,1]+y[9,1])+y[9,2],$$
$$y[5,2]+e^{i\,M\,x4}\,(y[5,1]+i\,y[9,1])-i\,y[9,2],$$
$$y[1,2]+e^{i\,M\,x4}\,(y[1,1]+i\,y[13,1])-i\,y[13,2],$$
$$i\,y[6,2]+e^{i\,M\,x4}\,(-i\,y[6,1]+y[10,1])+y[10,2],$$
$$i\,y[2,2]+e^{i\,M\,x4}\,(-i\,y[2,1]+y[14,1])+y[14,2],$$
$$y[2,2]+e^{i\,M\,x4}\,(y[2,1]+i\,y[14,1])-i\,y[14,2],$$
$$y[6,2]+e^{i\,M\,x4}\,(y[6,1]+i\,y[10,1])-i\,y[10,2],$$
$$-i\,y[3,2]+e^{i\,M\,x4}\,(i\,y[3,1]+y[15,1])+y[15,2],$$
$$-i\,y[7,2]+e^{i\,M\,x4}\,(i\,y[7,1]+y[11,1])+y[11,2],$$
$$y[7,2]+e^{i\,M\,x4}\,(y[7,1]-i\,y[11,1])+i\,y[11,2],$$
$$y[3,2]+e^{i\,M\,x4}\,(y[3,1]-i\,y[15,1])+i\,y[15,2]\}$$

In[•]:=
```
ys = Union[Flatten[{ y[#, 1], y[#, 2]} & /@ Range[0, 15]]]
```
Out[•]=
```
{y[0, 1], y[0, 2], y[1, 1], y[1, 2], y[2, 1], y[2, 2], y[3, 1], y[3, 2],
 y[4, 1], y[4, 2], y[5, 1], y[5, 2], y[6, 1], y[6, 2], y[7, 1], y[7, 2],
 y[8, 1], y[8, 2], y[9, 1], y[9, 2], y[10, 1], y[10, 2], y[11, 1], y[11, 2],
 y[12, 1], y[12, 2], y[13, 1], y[13, 2], y[14, 1], y[14, 2], y[15, 1], y[15, 2]}
```
In[•]:=
```
sys = Solve[And @@ Thread[0 == zeroeqsDx4Dx4y], ys][[-1]]
```
••• Solve: Equations may not give solutions for all "solve" variables.

Out[•]=
```
{y[8, 1] → -ⅈ y[4, 1], y[8, 2] → ⅈ y[4, 2], y[9, 1] → ⅈ y[5, 1], y[9, 2] → -ⅈ y[5, 2],
 y[10, 1] → ⅈ y[6, 1], y[10, 2] → -ⅈ y[6, 2], y[11, 1] → -ⅈ y[7, 1], y[11, 2] → ⅈ y[7, 2],
 y[12, 1] → -ⅈ y[0, 1], y[12, 2] → ⅈ y[0, 2], y[13, 1] → ⅈ y[1, 1], y[13, 2] → -ⅈ y[1, 2],
 y[14, 1] → ⅈ y[2, 1], y[14, 2] → -ⅈ y[2, 2], y[15, 1] → -ⅈ y[3, 1], y[15, 2] → ⅈ y[3, 2]}
```
In[•]:=
```
ssY = sY /. sys
```
Out[•]=

$$\Big\{Y[0,1] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[0,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[0,2] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[0,2]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[1,1] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[1,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[1,2] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[1,2]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[2,1] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[2,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[2,2] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[2,2]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[3,1] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[3,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[3,2] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[3,2]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[4,1] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[4,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[4,2] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[4,2]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[5,1] \to \frac{2\,\mathrm{ProductLog}\left[\frac{1}{2}\,\mathrm{Log}[\mathrm{Sin}[x0]]\;y[5,1]\right]}{\mathrm{Log}[\mathrm{Sin}[x0]]},$$

$$Y[5,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[5,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[6,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[6,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[6,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[6,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[7,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[7,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[7,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[7,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[8,1] \to \frac{2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[4,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[8,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[4,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[9,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[5,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[9,2] \to \frac{2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[5,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[10,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[6,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[10,2] \to \frac{2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[6,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[11,1] \to \frac{2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[7,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[11,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[7,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[12,1] \to \frac{2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[0,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[12,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[0,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[13,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[1,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[13,2] \to \frac{2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[1,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[14,1] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\mathbb{i}\,\text{Log}[\text{Sin}[x0]]\,y[2,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[14,2] \to \frac{2\,\text{ProductLog}\left[-\frac{1}{2}\,\mathbb{i}\,\text{Log}[\text{Sin}[x0]]\,y[2,2]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[15,1] \to \frac{2\,\text{ProductLog}\left[-\frac{1}{2}\,\mathbb{i}\,\text{Log}[\text{Sin}[x0]]\,y[3,1]\right]}{\text{Log}[\text{Sin}[x0]]},$$

$$Y[15,2] \to \frac{2\,\text{ProductLog}\left[\frac{1}{2}\,\mathbb{i}\,\text{Log}[\text{Sin}[x0]]\,y[3,2]\right]}{\text{Log}[\text{Sin}[x0]]}\Big\}$$

In[ ]:=
```
zeroeqsDx4Dx4Y /. ssY // FullSimplify
```

Out[ ]=

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

In[ ]:=
```
sZtoY /. ssY // FullSimplify
zeroeqsDx4Dx4Z /. % // FullSimplify
```

Out[ ]=

$$\Big\{Z[0] \to \left(\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[0,1]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\mathbb{i}\,(M\,\#2)\right] + \text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[0,2]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\mathbb{i}\,(M\,\#2)\right]\,\&\right),$$

$$Z[5] \to \left(\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[1,1]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\mathbb{i}\,(M\,\#2)\right] + \text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[1,2]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\mathbb{i}\,(M\,\#2)\right]\,\&\right),$$

$$Z[8] \to \left(\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[2,1]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\mathbb{i}\,(M\,\#2)\right] + \text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[2,2]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\mathbb{i}\,(M\,\#2)\right]\,\&\right),$$

$$Z[13] \to \left(\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[3,1]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\mathbb{i}\,(M\,\#2)\right] + \text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[3,2]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\mathbb{i}\,(M\,\#2)\right]\,\&\right),$$

$$Z[1] \to \left(\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[4,1]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\mathbb{i}\,(M\,\#2)\right] +\right.$$

$$\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[4,2]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big]\,\&\Big),$$

$$Z[4] \to \Big(\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[5,1]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big] +$$

$$\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[5,2]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big]\,\&\Big),$$

$$Z[9] \to \Big(\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[6,1]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big] +$$

$$\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[6,2]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big]\,\&\Big),$$

$$Z[12] \to \Big(\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[7,1]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big] +$$

$$\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{Log}[\text{Sin}[x0]]\,y[7,2]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big]\,\&\Big),$$

$$Z[2] \to \Big(\text{Exp}\Big[\frac{(2\,\text{ProductLog}[-\frac{1}{2}\,\text{i}\,\text{Log}[\text{Sin}[x0]]\,y[4,1]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big] +$$

$$\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{i}\,\text{Log}[\text{Sin}[x0]]\,y[4,2]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big]\,\&\Big),$$

$$Z[7] \to \Big(\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{i}\,\text{Log}[\text{Sin}[x0]]\,y[5,1]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big] +$$

$$\text{Exp}\Big[\frac{(2\,\text{ProductLog}[-\frac{1}{2}\,\text{i}\,\text{Log}[\text{Sin}[x0]]\,y[5,2]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big]\,\&\Big),$$

$$Z[10] \to \Big(\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{i}\,\text{Log}[\text{Sin}[x0]]\,y[6,1]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big] +$$

$$\text{Exp}\Big[\frac{(2\,\text{ProductLog}[-\frac{1}{2}\,\text{i}\,\text{Log}[\text{Sin}[x0]]\,y[6,2]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big]\,\&\Big),$$

$$Z[15] \to \Big(\text{Exp}\Big[\frac{(2\,\text{ProductLog}[-\frac{1}{2}\,\text{i}\,\text{Log}[\text{Sin}[x0]]\,y[7,1]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} + \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big] +$$

$$\text{Exp}\Big[\frac{(2\,\text{ProductLog}[\frac{1}{2}\,\text{i}\,\text{Log}[\text{Sin}[x0]]\,y[7,2]])\,\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]} - \frac{1}{2}\,\text{i}\,(\text{M}\,\#2)\Big]\,\&\Big),$$

$$Z[3] \to \left(\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[0,1]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]}+\frac{1}{2}\,i\,(M\,\#2)\right]+\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[0,2]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]}-\frac{1}{2}\,i\,(M\,\#2)\right]\,\&\right),$$

$$Z[6] \to \left(\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[1,1]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]}+\frac{1}{2}\,i\,(M\,\#2)\right]+\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[1,2]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]}-\frac{1}{2}\,i\,(M\,\#2)\right]\,\&\right),$$

$$Z[11] \to \left(\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[2,1]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]}+\frac{1}{2}\,i\,(M\,\#2)\right]+\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[2,2]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]}-\frac{1}{2}\,i\,(M\,\#2)\right]\,\&\right),$$

$$Z[14] \to \left(\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[-\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[3,1]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]}+\frac{1}{2}\,i\,(M\,\#2)\right]+\text{Exp}\left[\frac{\left(2\,\text{ProductLog}\left[\frac{1}{2}\,i\,\text{Log}[\text{Sin}[x0]]\,y[3,2]\right]\right)\text{Log}[\text{Sin}[\#1]]}{2\,\text{Log}[\text{Sin}[x0]]}-\frac{1}{2}\,i\,(M\,\#2)\right]\,\&\right)\}$$

Out[ ]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

zeroeqsDx4Dx4Z

In[ ]:=

```
(*sZtoY/.{Rule→Equal}
 Solve[And@@%,Ys]*)
```