

Scratch work of Author

NOTES

HYPOTHESIS : If ,
employing the Einstein eqs (or Einstein - Lovelock eqs) ,
superluminal inflation / deflation exists,
then at time $x_4 = 0$ (before the particles of the standard model exist)
a pair of universes with MASSES $\pm M$ is created
(i.e., universes are created in pairs)
Dark matter / dark energy may be
related to wave function overlap of these two universes

THE FOLLOWING SCRATCH IS CONSISTENT WITH
UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES $\pm M$.

Note that we are ONLY looking for
superluminal inflation or deflation type solutions ,
and NOT solutions that are even / odd functions of t
(like $\cos[\nu[j]^\theta * t]$, $\sin[\nu[j]^\theta * t]$,
 $\text{Sech}[\nu[j]^\theta * t]$, $\text{Tanh}[\nu[j]^\theta * t]$, ...),
which also have $\pm M$ type eigenvalues ,
or even solutions involving the
well -
known special functions that might also have $\pm M$ type parameters .

Bigger Bang:

Question: Are Universe (s) of masses $\pm M$ created in pairs at time $x_4 = 0$ (before the particles of the standard model exist) ?

Scratch work

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Do not read if you are irascible; I apologize for typos and duplications and

git clone <https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git>

“git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git”

Ever wonder why there are 3 space dimensions and 1 time dimension?

Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in \text{Nature}$?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\| = \|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over \mathbb{R}) real numbers \mathbb{R} , the 2-dim complex numbers, the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in "A Remarkable Representation of the 3+2 de Sitter Group" by P.A.M.Dirac, J.Math.Phys.4,901–909 (1963).

Comment: The term "reduced Brauer–Weyl generators", below, refers to a specific set of irreducible generators for certain complex Clifford algebras, particularly in the context of mathematical physics and the study of Dirac spinors. These generators were introduced by **E.A. Lord** (E. A. Lord. "The Dirac spinor in six dimensions". Mathematical Proceedings of the Cambridge Philosophical Society, vol. 64, pp. 765–778, 1968) and are described as "reduced" because they form an irreducible representation of the Clifford algebra C_{2n} from $C_{2(n-1)}$.

Here, we employ Lord's idea of reduced Brauer–Weyl generators to extend Dirac's 4×4 gamma matrices into this notebook's real tau8 8×8 and tau16 16×16 matrices,
which are employed to write down the 'Dirac Equation for the Universe',
[remember, this is before the particles of the standard model exist]

formulated in terms of a $O(4, 4)$ spinor Ψ_{16}
(Ψ_{16} AKA '**WAVE FUNCTION OF the UNIVERSE**', which could possibly be named something more pompous).

Then this equation is used to
[WARNING: syncope, presyncope AHEAD]
couple the **WAVE FUNCTION OF the UNIVERSE** to Gravity.

THIS WORK IS PARTIALLY BASED ON :

JOURNAL OF MATHEMATICAL PHYSICS

VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. DIRAC

*Cambridge University, Cambridge, England and
Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

Proc. Camb. Phil. Soc. (1968), 64, 765

765

PCPS 64–96

Printed in Great Britain

The Dirac spinor in six dimensions

BY E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

JOURNAL OF MATHEMATICAL PHYSICS **51**, 042501 (2010)

Second gravity

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IL NUOVO CIMENTO

VOL. 105 B, N. 1

Gennaio 1990

On the Structure of the Split Octonion Algebra.

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(ricevuto il 24 Luglio 1989)

Identities satisfied by the generators of the Dirac algebra

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(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin- $\frac{1}{2}$ wave equation

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(Received 24 January 1984; accepted for publication 16 November 1984)

Here we use [spacetime coordinates](#) that are selected so that we are somewhat consistent with the [exact libraries](#),

(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):

[Cartesian coordinates](#):

$x0$ = hidden space (a small circle); $x1, x2, x3$ are the usual 3-space coords;

$x4$ = time coord,

and $x5, x6, x7$ = superluminal [deflating](#) time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{\text{SO}(4,4;\mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M8 , whose tangent bundle has

$$\text{SO}(4, 4; \mathbb{R}) \approx \text{Spin}(4,4;\mathbb{R})$$

as iso group, and in which

3 of the 4 space dimensions superluminally INFLATE,

3 of the 4 time dimensions superluminally DEFLATE (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),

and the 4th space dim, x_0 , curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

In passing we remark that an analogous construction may be defined on an octonion space of one time dimension and seven space dimensions, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group

$$\text{SU}(3) \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y;$$

The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

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The Einstein - Lovelock vacuum field equations are

$$A_a^b = 0, \text{ where}$$

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta_{jj_1 \dots j_{2k}}^{hh_1 \dots h_{2k}} R^{j_1 j_2}_{ h_1 h_2} \dots R^{j_{2k-1} j_{2k}}_{\phantom{j_{2k-1} j_{2k}} h_{2k-1} h_{2k}} + \lambda \sqrt{g} g^{lh}, \quad (4.3)$$

where $\alpha_{(k)}, \lambda$ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

and $m-1 = \frac{n}{2} - 1 = 3$.

Citation: Tensors, Differential Forms,
and Variational Principles (Dover Books on Mathematics),
by David Lovelock and Hanno Rund

Let {w1, w2, w3, Δ} be pure numbers;

then by Eq. [4.38] the vacuum equations are

$$0 = -\Delta + H^{-2} w1 \text{ Lovelock1} + H^{-4} w2 \text{ Lovelock2} + H^{-6} w3 \text{ Lovelock3}$$

in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein - Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre - Hubble parameter. For now, let's pretend that they are independent.

NOTES :

Under the spacetime coordinate transformation

$$x^k \mapsto \bar{x}^k (x^j)$$

according to

$$\Psi16(x) \mapsto \bar{\Psi}16(\bar{x}) = S(x(\bar{x})) \cdot \Psi16(x(\bar{x})),$$

$$\text{i.e., } \overline{\Psi 16^{(a)}(\bar{x})} = \left(S_{(b)}^{(a)} \Psi 16^{(b)}(x) \right)$$

Therefore,

$$\frac{\partial}{\partial \bar{x}^k} \overline{\Psi 16(\bar{x})} = \frac{\partial x^j}{\partial \bar{x}^k} \frac{\partial}{\partial x^j} S(x) \cdot \Psi 16(x) =$$

$$\frac{\partial x^j}{\partial \bar{x}^k} \left\{ S(x) \cdot \frac{\partial}{\partial x^j} \Psi 16(x) + \left[\frac{\partial}{\partial x^j} S(x) \right] \cdot \Psi 16(x) \right\}$$

$$\text{Identify the Octad connection : } \frac{\partial}{\partial x^j} S(x) =$$

$$- S \cdot \Gamma_j(x) + \frac{\partial \bar{x}^k}{\partial x^j} \bar{\Gamma}_k(\bar{x}) \cdot S$$

hence

$$\frac{\partial}{\partial \bar{x}^k} \overline{\Psi 16(\bar{x})} = \bar{\Gamma}_k \cdot \overline{\Psi 16(\bar{x})} =$$

$$S(x(\bar{x})) \cdot \frac{\partial x^j}{\partial \bar{x}^k} \left\{ \frac{\partial}{\partial x^j} \Psi 16(x(\bar{x})) - \Gamma_j \cdot \Psi 16(x(\bar{x})) \right\}$$

or

$$\left(1_{16 \times 16} \frac{\partial}{\partial \bar{x}^k} - \bar{\Gamma}_k \right) \cdot \overline{\Psi 16(\bar{x})} =$$

$$S(x(\bar{x})) \cdot \left[\frac{\partial x^j}{\partial \bar{x}^k} \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^j} - \Gamma_j \right\} \cdot \Psi 16(x(\bar{x})) \right]$$

The connection transforms according to

$$\bar{\Gamma}_k(\bar{x}) = \frac{\partial \bar{x}^j}{\partial x^k} \left\{ S \cdot \Gamma_j(x) \cdot S^{-1} + S_{,j} \cdot S^{-1} \right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2} [e_k^{(a)} \nabla_j e_{(b)}^k] \text{ SAB}^{(b)(a)}$$

Here, spin connection coefficients are $e_k^{(a)} (\nabla_j e_{(b)}^k)$

and

$$\mathbf{S} = \exp \left(\frac{1}{2} \omega_{(A)(B)}(x) \text{ SAB}^{(A)(B)} \right),$$

where $\omega_{(B)(A)} = -\omega_{(A)(B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the $\Psi 16$ field [A.K.A., wave function characterizing the Universe(s) of masses $\pm M$];

TODO: prove Universe(s) of masses $\pm M$ are created in pairs!

K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:

- $\nabla_\mu e^a{}_\nu = 0$
- Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

apologies, I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

? SOURCE terms
 TU^{μν} for $g_{\alpha\beta}$ come from
 "Universes' Wave Function
 Ψ16 Lagrangian"

$$\frac{1}{\kappa} \text{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lagrangian}\Psi16)$$

(hope that $\text{TU}^{\mu\nu} = \Lambda g^{\mu\nu}$, and $\text{H} = \text{some function of M, where }$

$\text{Universe (s) of masses } \pm M \text{ created in pairs at time } x4 = 0,$

$\text{before the particles of the standard model exist}) ;$

WARNING:

Universes $\Psi16$ source $g_{\alpha\beta}$;

The Euler–Lagrange equations for $\Psi16$ must have ‘solutions’ such that

all off-diagonal terms of $\text{TU}^{\mu\nu}$ ARE ZERO .

The $\Psi16$ Lagrangian (please see below) =

$$\begin{aligned} \sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg[]} = \\ \sqrt{\text{Det}[g_{\mu\nu}]} * \left(\text{Transpose}[\Psi16].\sigma16.\text{Sum}\left[T16^\alpha[\alpha1 - 1].\left\{1_{16 \times 16} \frac{\partial}{\partial x^{\alpha1-1}} - \Gamma_{\alpha1-1}\right\}.\Psi16, \right. \right. \\ \left. \left. \{\alpha1, 1, \text{Length}[X]\}\right] + \frac{\text{mAsS}}{2} * \text{Transpose}[\Psi16].\sigma16.\Psi16 \right) \end{aligned}$$

Next,

assume that $\Psi16\text{soln}$ solves the Euler – Lagrange equations.

SOURCE terms

$\text{TU}^{\mu\nu}$ [evaluate terms after performing differentiation] (using Lagrangian $\Psi16 \equiv$

$\left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{ Lg[]} \right) |_{\Psi16=\text{solution-toEL-eqs}}$:

$$\frac{1}{\kappa} \text{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]} \text{ Lg[]})$$

$$= \left(\text{Lg}[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]}) \right] + \frac{\partial}{\partial g_{\mu\nu}} (\text{Lg}[]) \right) \quad \boxed{\Psi16=\Psi16soln}$$

$$= \left(\Theta ? * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]}) + \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\text{Lg}[]) \right) \quad \boxed{\Psi16=\Psi16soln}$$

$$= \frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\Psi16].\sigma16. \text{Sum} \left[\left(D[\Psi16, X[\alpha1]] + \left(\frac{1}{2} \right) \text{connectionMatrix} \right). \text{Sum}[\omega[\alpha1, a, b] * \text{SAB}[a, b], \{a, 1, 8\}, \{b, 1, 8\}] . \Psi16 \right], \{\alpha1, 1, \text{Length}[X]\} \right] +$$

$$\text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[j, 1].\Psi16 \quad \boxed{\Psi16=\Psi16soln}$$

$$= \text{Transpose}[\Psi16].\sigma16.\text{Sum} \left[\frac{\partial}{\partial g_{\mu\nu}} (\text{T16}^\alpha[\alpha1 - 1]) . \Psi16_{\alpha1-1}, \{\alpha1, 1, \text{Length}[X]\} \right] \quad \boxed{\Psi16=\text{solution-toEL-eqs}}$$

$$= \sim \Psi16.\sigma16.\text{T16}^A.\Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (g^{-1}^{\alpha\beta} e_\beta^B \eta_{BA}) = \sim \Psi16.\sigma16.\text{T16}^A.\Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha)$$

NOTES :

Let g be a square matrix; we could use : $\frac{\partial}{\partial q} (g^{-1}) = -g^{-1} \cdot \frac{\partial g}{\partial q} \cdot g^{-1}$,

where q is a parameter (result from $\frac{\partial}{\partial q} (g \cdot g^{-1}) = 0$),

In this notebook,

g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] ,
 g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}} (g^{-1}) = -g^{-1} \cdot \frac{\partial g}{\partial g_{\mu\nu}} \cdot g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq g_{\mu\nu}$
since the $g_{\mu\nu}$ are independent. In metric matrix g ,
we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate, for example.

This should be understood before reading further.

$$\text{Therefore } \frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha) = \frac{\partial}{\partial g_{\mu\nu}} (\text{Inverse}[e_\alpha^{(A)}]) = -\text{Inverse}[e_\alpha^{(A)}] \cdot \frac{\partial e_\alpha^{(A)}}{\partial g_{\mu\nu}} \cdot \text{Inverse}[e_\alpha^{(A)}] = -e_{(A)}^\alpha [[\alpha 1, A1]] \frac{\partial e_\alpha^{(A)} [[A1, \alpha 2]]}{\partial g_{\mu\nu}} \cdot e_{(A)}^\alpha [[\alpha 2, A2]]$$

$$\text{Consider } \frac{\partial}{\partial g_{\mu\nu}} (g_{\alpha\beta}) = \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^{(B)}) = \eta_{AB} e_\beta^{(B)} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^{(A)} \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^{(B)});$$

$$e_\beta^{(B)} \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} = \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^{(B)}) = \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^{(A)} e_\beta^{(B)} \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^{(B)})$$

In metric matrix g , we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate.

Some of my calculations employ “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see Cyril Pitrou

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<http://www2.iap.fr/users/pitrou/>

“FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb”

[MyArrayComponents\[expr_\] := expr //ToBasis\[BS\] //ComponentArray //ToValues //ToValues //Simplify](#)

I copied and pasted too often; however, I am going to leave this here.

Universe sources for $g_{\alpha\beta}$;

be sure to append these to the Einstein and/or Einstein–Love–lock field equations.

WARNING: all off–diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous ‘hacked solutions’ for $\Psi16$ to be valid

Let $T^\alpha = \{T16^\alpha[0], T16^\alpha[4]\}$ and

```
 $\sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg[]}[\text{mASs}_, j_] =$ 
 $\sqrt{\text{Det}[g_{\mu\nu}]} * (\text{Transpose}[\Psi16].\sigma16.\text{Sum}[$ 
 $T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\}] +$ 
 $\text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[134, 1].\Psi16)$ 
```

Let $j = 134$, $\sigma16.\text{(mass Matrix)} = T16^\alpha[5].T16^\alpha[6].T16^\alpha[7]$

and

```
usingLagrangianF16massive = (Transpose[\Psi16].\sigma16.
Sum[T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\}] +
mASs * Transpose[\Psi16].\text{symm16}[j = 134, 1].\Psi16);
```

where it is assumed that $\Psi16$ solves the Euler – Lagrange equations, above; then `usingLagrangianF16massive = 0`, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lg}[] \right)$$

$$= \left(\text{Lg}[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \right. \\ \left. \frac{\partial}{\partial g_{\mu\nu}} (\text{Lg}[]) \right) \quad \mid \text{Lg}[] = \text{usingLagrangianF16massive}$$

$$= \left(0 * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \right. \\ \left. \sqrt{\text{Det}[g_{\alpha\beta}]} \frac{\partial}{\partial g_{\mu\nu}} (\text{Lg}[]) \right) \quad \mid \dots \dots \dots \dots \dots \dots \dots \dots$$

remark: the term

$$\overset{\sim}{\Psi}16 \cdot \sigma16 \cdot T16^A \cdot \Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (e_A^\alpha) = \frac{\partial}{\partial g_{\mu\nu}} \overset{\sim}{\Psi}16 \cdot \sigma16 \cdot T16^A \cdot \Psi16_{,\alpha} e_A^\alpha =$$

$$\frac{\partial}{\partial g_{\mu\nu}} \left(\begin{array}{l} \text{Transpose}[\Psi16] \cdot \sigma16 \cdot \\ T16^A[A1-1] \cdot \underbrace{\Psi16_{,\alpha1} \eta_{A1B} e_B^\beta}_{g^{-1}\alpha1\beta} \\ \text{Sum} \left[T^\alpha[\alpha1 - 1] \cdot D[\Psi16, vars[\alpha1]], \{\alpha1, 1, \text{Length}[vars]\} \right] \end{array} \right)$$

Frame fields [edit]

We use a set of [vierbein](#) or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab} e_\mu^a e_\nu^b = \eta_{\mu\nu}.$$

The vierbein defines a local rest [frame](#), allowing the constant [Gamma matrices](#) to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a [section](#) of the [frame bundle](#), and so defines a local trivialization of the frame bundle.

Incomplete theories**Scientists**

[snow]

[show]

v • t • e

Spin connection [edit]

To write down the equation we also need the [spin connection](#), also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e_a^\mu e_\nu^a = \delta_\nu^\mu.$$

The connection 1-form is then

$$\omega^\mu{}_{\nu a} := e_b^\mu \nabla_a e_\nu^b$$

where ∇_a is a [covariant derivative](#), or equivalently a choice of [connection](#) on the frame bundle, most often taken to be the [Levi-Civita connection](#).

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space $T_p M$ to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

[Raising and lowering indices](#) is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract [connection on a principal bundle](#), specifically on the [frame bundle](#), which is defined on any smooth manifold, but which restricts to an [orthonormal frame bundle](#) on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu_{\nu\rho} = e_\rho^\alpha \omega^\mu_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2} (\omega_{\nu\rho})_\mu \sigma^{\nu\rho} \psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu R(M^{\nu\rho}) v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $\text{SO}(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

OTHER WORK THAT I USED :

JOURNAL OF MATHEMATICAL PHYSICS

VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. DIRAC

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Institute for Advanced Study, Princeton, New Jersey**
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Proc. Camb. Phil. Soc. (1968), 64, 765

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The Dirac spinor in six dimensions

By E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

Identities satisfied by the generators of the Dirac algebra

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A new spin- $\frac{1}{2}$ wave equation

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Begin

```
In[4]:= Print[
  "CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]
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In[5]:= Print["Please cite this work, and this web page, if you use it"]
Please cite this work, and this web page, if you use it

In[6]:= Print["For simplicity, metric gαβ = gαβ(x0,x1,x2,x3,x4,x5,x6,x7) =
gαβ(x0,x4) = diag{ g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1,
g77(x0,x4),g77(x0,x4),g77(x0,x4) },"]
For simplicity, metric gαβ = gαβ(x0,x1,x2,x3,x4,x5,x6,x7) = gαβ(x0,x4) = diag{
g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4) },
g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4) },

In[7]:= Print["where g77(x0,x4) = g11(x0,-x4) "]
where g77(x0,x4) = g11(x0,-x4)
ConvertMapleToMathematicaV2

In[1182]:= FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]]
mapleToMathematicaV2PATH =
FileNameJoin[{%, "ConvertMapleToMathematicaV2.wl"}]

Out[1182]=
/Users/nsh/Documents/8-dim/2026-02-09-Eternal-DEFLATION-Inflation-M4-MAX

Out[1183]=
/Users/nsh/Documents/8-dim/2026-02-09-Eternal-DEFLATION-Inflation-M4-MAX/
ConvertMapleToMathematicaV2.wl

In[1184]:= Get[mapleToMathematicaV2PATH]      (*Loads new maple to mathematica parser*)
::: ConvertMapleToMathematicaV2: Symbol ConvertMapleToMathematicaV2 appears in multiple contexts
{ConvertMapleToMathematicaV2`, Global`}; definitions in context ConvertMapleToMathematicaV2` may
shadow or be shadowed by other definitions. ??
ConvertMapleToMathematicaV2 loaded successfully!
BUT, WARNING: DO NOT USE IF YOU WANT A CORRECT RESULT!
```

Load the Maple output strings.

```
In[11]:= (*SetDirectory[NotebookDirectory[]];*)

In[12]:= (*maplestringEQ1=Get["string-seq1.txt"]
    maplestringEQ2=Get["thinkpad_stringEQ2.txt"]*)

In[13]:= NotebookFileName[]

Out[13]= /Users/nsh/Documents/8-dim/2026-02-09-Eternal-DEFLATION-Inflation-M4-MAX/2026-02-13-Pair-
    Crtn-Univ-same_E-L-eqs-alt-approach-mmM4p.nb

In[14]:= Unprotect[dir];

In[15]:= dir = FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]];
    Protect[dir];
    If[$Failed == SetDirectory[dir], {.CreateDirectory[dir], SetDirectory[dir]}];

In[18]:= (*FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]]*)

In[19]:= names = FileNameSplit[NotebookFileName[]]

Out[19]= {, Users, nsh, Documents, 8-dim, 2026-02-09-Eternal-DEFLATION-Inflation-M4-MAX,
    2026-02-13-Pair-Crtn-Univ-same_E-L-eqs-alt-approach-mmM4p.nb}

In[20]:= name = StringReplace[names[[-1]], "nb" → "mx"]

Out[20]= 2026-02-13-Pair-Crtn-Univ-same_E-L-eqs-alt-approach-mmM4p.mx

In[21]:= header = StringReplace[names[[-1]], ".nb" → "-"]

Out[21]= 2026-02-13-Pair-Crtn-Univ-same_E-L-eqs-alt-approach-mmM4p-

In[22]:= SetOptions[Simplify, TimeConstraint → 1]

Out[22]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {},
    TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[23]:= SetOptions[FullSimplify, TimeConstraint → 3]

Out[23]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {},
    TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}

In[24]:= Options[Simplify]

Out[24]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {},
    TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[25]:= Options[FullSimplify]

Out[25]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {},
    TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}

In[26]:= {Off[General::spell], Off[General::spell1]};
```

```
In[27]:= DIM8 = 8;
In[28]:= Protect[DIM8, M, K, H]
Out[28]= {DIM8, M, K, H}
```

Coordinates :

$\underbrace{\text{hidden space}}, \underbrace{\text{3-space}}, \underbrace{\text{time}} \quad \underbrace{\text{superluminal deflating time}}$
 $\{x_0, \overbrace{x_1, x_2, x_3}^{\text{3-space}}, x_4, \overbrace{x_5, x_6, x_7}^{\text{superluminal deflating time}}\}:$

```
In[29]:= X = {x0, x1, x2, x3, x4, x5, x6, x7};
Protect[X];
Protect[x0, x1, x2, x3, x4, x5, x6, x7];
```

```
In[32]:= sX0 = And @@ Thread[X > 0]
```

```
Out[32]= x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 && x7 > 0
```

```
In[33]:= ssX = H > 0 && sX0 && 6 H x0 > 0 && 2 la[x4] > 0 && Cot[6 H x0] > 0 && Sin[6 H x0] > 0
```

```
Out[33]= H > 0 && x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 &&
x7 > 0 && 6 H x0[] > 0 && 2 la[x4[]] > 0 && Cot[6 H x0[]] > 0 && Sin[6 H x0[]] > 0
```

For Maple_2025 :

```
In[34]:= sreplaceZ = Z[#] → ToExpression["Z" <> ToString[#]] & /@ Range[0, 15]
```

```
Out[34]= {Z[0] → Z0, Z[1] → Z1, Z[2] → Z2, Z[3] → Z3, Z[4] → Z4, Z[5] → Z5, Z[6] → Z6, Z[7] → Z7, Z[8] → Z8,
Z[9] → Z9, Z[10] → Z10, Z[11] → Z11, Z[12] → Z12, Z[13] → Z13, Z[14] → Z14, Z[15] → Z15}
```

```
In[35]:= sreplacenZ = nZ[#] → ToExpression["nZ" <> ToString[#]] & /@ Range[0, 15]
```

```
Out[35]= {nZ[0] → nZ0, nZ[1] → nZ1, nZ[2] → nZ2, nZ[3] → nZ3, nZ[4] → nZ4,
nZ[5] → nZ5, nZ[6] → nZ6, nZ[7] → nZ7, nZ[8] → nZ8, nZ[9] → nZ9, nZ[10] → nZ10,
nZ[11] → nZ11, nZ[12] → nZ12, nZ[13] → nZ13, nZ[14] → nZ14, nZ[15] → nZ15}
```

hacks (a.k.a., lies):

```
In[36]:= constraintTrig = Sin[6 * H * x0] > 0 && Cos[6 * H * x0] > 0 && Csc[6 * H * x0] > 0 &&
Sec[6 * H * x0] > 0 && Tan[6 * H * x0] > 0 && Cot[6 * H * x0] > 0 && Sin[z] > 0 &&
Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 && Sin[z]^(1/2) > 0 && Sin[z]^(−3/2) > 0 &&
Sin[z]^(−1/2) > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 && Cot[z]^(3/2) > 0 &&
Cot[z]^(1/2) > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

```
Out[36]= Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^(3/2) > 0 &&
Sqrt[Sin[z]] > 0 && 1/(Sin[z]^(3/2)) > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]^(3/2) > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

more hacks (damn lies):

```
In[37]:= (*&&K^2-M^2>0&& √(K^2-M^2)>0&&e^(H ∙ √(K^2-M^2) ∙ x^4)>0&&2 H ∙ √(K^2-M^2) ∙ x^4>0&&
2 H ∙ √(1-M^2) ∙ x^4>0&& √(1-M^2)>0&&e^(H ∙ √(1-M^2) ∙ x^4)>0&&2 H ∙ √(1-M^2) ∙ x^4>0*)
```

```
In[38]:= constraintX = x0 > 0 && x4 > 0 && H > 0 && 6 * H * x0 > 0 &&
3 * H * x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 && t > 0 && M > 0 &&
K > 0 && e^{-2 a4[H x4]} > 0 && e^{-a4[H x4]} > 0 && e^{2 a4[H x4]} > 0 && e^{a4[H x4]} > 0
```

```
Out[38]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e^{-2 a4[H x4]} > 0 && e^{-a4[H x4]} > 0 && e^{2 a4[H x4]} > 0 && e^{a4[H x4]} > 0
```

statistics :

```
In[39]:= constraintVars = constraintX && constraintTrig
```

```
Out[39]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e^{-2 a4[H x4]} > 0 && e^{-a4[H x4]} > 0 && e^{2 a4[H x4]} > 0 &&
e^{a4[H x4]} > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && √Sin[z] > 0 && Sin[z]^{3/2} > 0 &&
√Sin[z] > 0 && 1/Sin[z]^{3/2} > 0 && 1/√Sin[z] > 0 && Cot[z] > 0 && √Cot[z] > 0 &&
Cot[z]^{3/2} > 0 && √Cot[z] > 0 && Tan[z] > 0 && Sec[z] > 0 && √Sec[z] > 0 && Csc[z] > 0
```

```
In[40]:= subsDefects = { √e^{2 a4[H x4]} → e^{a4[H x4]}, √e^{-2 a4[H x4]} → e^{-a4[H x4]},
```

$$\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}}} \rightarrow \frac{1}{e^{a4[H x4]} \sin[6 H x4]^{1/6}},$$

$$\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}}} \rightarrow \frac{1}{e^{a4[H x4]} \sin[6 H x4]^{1/6}},$$

$$\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{1}{e^{a4[H x4]} \sin[6 H x0]^{1/6}},$$

$$\frac{1}{\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{1}{e^{-a4[H x4]} \sin[6 H x0]^{1/6}},$$

$$\sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}} \rightarrow e^{a4[H x4]} \sin[6 H x0]^{1/6},$$

$$\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/6}} \rightarrow e^{-a4[H x4]} \sin[6 H x0]^{1/6}\}$$

```
Out[40]= { √e^{2 a4[H x4]} → e^{a4[H x4]}, √e^{-2 a4[H x4]} → e^{-a4[H x4]}, √e^{2 a4[H x4]} Sin[6 H x4]^{1/3} → e^{a4[H x4]} Sin[6 H x4]^{1/6},
1/√e^{2 a4[H x4]} Sin[6 H x4]^{1/3} → e^{-a4[H x4]} / Sin[6 H x4]^{1/6}, 1/√e^{2 a4[H x4]} Sin[6 H x0]^{1/6} → e^{-a4[H x4]} / Sin[6 H x0]^{1/6},
1/√e^{-2 a4[H x4]} Sin[6 H x0]^{1/6} → e^{a4[H x4]} / Sin[6 H x0]^{1/6}, √e^{2 a4[H x4]} Sin[6 H x0]^{1/6} → e^{a4[H x4]} Sin[6 H x0]^{1/6},
√e^{-2 a4[H x4]} Sin[6 H x0]^{1/6} → e^{-a4[H x4]} Sin[6 H x0]^{1/6}\}
```

future coordinate transformation :

```
In[41]:= 6 H x0 == z && H x4 == t
szt = Solve[% , {z, t}] [[1]]
sx0x4 = Solve[%%, {x0, x4}] [[1]]
Protect[sx0x4, szt];
```

```
Out[41]= 6 H x0 == z && H x4 == t
```

```
Out[42]= {z → 6 H x0, t → H x4}
```

```
Out[43]= {x0 →  $\frac{z}{6H}$ , x4 →  $\frac{t}{H}$ }
```

```
In[45]:= sx0x4
```

```
Out[45]= {x0 →  $\frac{z}{6H}$ , x4 →  $\frac{t}{H}$ }
```

```
In[46]:= (*sa4={a4→((A4[3 H #2])&)}*)
```

```
In[47]:= Protect[sx0x4, szt]
```

```
Out[47]= {}
```

SO (4, 4) Minkowski Lorentz metric η_{4488} : 4 + 4 spacetime; 8 × 8 dimensional :

```
In[48]:= ( $\eta_{4488}$  = ArrayFlatten[
{{IdentityMatrix[4], 0}, {0, -IdentityMatrix[4]}}]) // MatrixForm
```

```
Out[48]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```

SO (4, 4) Spinor Lorentz metric σ ;

τ^A are analogous to the Dirac gamma matrices :

all true : Table[

```
{A, B, FullSimplify[ $\frac{1}{2} (\tau[A].\bar{\tau}[B] + \tau[B].\bar{\tau}[A]) = \eta_{4488}[A+1, B+1]*ID8$ ],  
{A, 0, 7}, {B, 0, 7}]
```

Type - 1 and type - 2 employ σ (the usual notational abuse),

since $\sigma = \sigma^{-1}$: 4 + 4 spacetime, 8 × 8 dimensional and 16 × 16 dimensional :

indices. We define the reduced Brauer-Weyl generators $\{\bar{\tau}^{A'}, \tau^{A'}\}$ of the generators of the two real 8×8 inequivalent irreducible representations of $\overline{SO(4, 4)}$ (see eqs. (2.6) and (2.7)) by demanding that the tau matrices satisfy (the tilde denotes transpose)

$$(2.3) \quad \tilde{\tau}^{A'} \sigma = \sigma \bar{\tau}^{A'}$$

and

$$(2.4) \quad \tau^{A'} \bar{\tau}^{B'} + \tau^{B'} \bar{\tau}^{A'} = 2IG^{A'B'} = \bar{\tau}^{A'} \tau^{B'} + \bar{\tau}^{B'} \tau^{A'},$$

where I denotes the 8×8 unit matrix. Denoting the matrix elements of $\tau^{A'}$ by $\tau^{A'a}_b$, we may write eq. (3) as

$$(2.5) \quad \bar{\tau}^{A'}_{ab} = \tau^{A'}_{ba},$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix}$$

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4, 4)}$ and $L \in SO(4, 4)$. The canonical 2-1 homomorphism $\overline{SO(4, 4)} \rightarrow SO(4, 4)$ is given by

$$(2.8) \quad 8L^{A'}_{\ B'}(g) = \text{tr} \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4, 4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}_{\ C'} G_{A'B'} L^{B'}_{\ D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}_{\ B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}_{\ B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

```
In[49]:= (σ = ArrayFlatten[{{ConstantArray[0, {4, 4}], IdentityMatrix[4]}, {IdentityMatrix[4], ConstantArray[0, {4, 4}]}}]) // MatrixForm
```

```
Out[49]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

below :

```
In[50]:= σ16 == ArrayFlatten[{{{-σ, 0}, {0, σ}}}]
```

below :

```
In[51]:= Table[T16^A[A1] = ArrayFlatten[{{(0, τ[A1]), (τ[A1], 0)}}, {A1, 0, 7}];
```

below :

```
In[52]:= σ16.T16^A[#[ ] = -Transpose[σ16.T16^A[#[ ]]] & /@ Range[0, 7]
```

```
Out[52]= {True, True, True, True, True, True, True, True}
```

below :

```
In[53]:= Table[{A1, B1}, FullSimplify[ExpandAll[1/2 (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1])] ==
```

```


$$\eta 4488 \left[ A1 + 1, B1 + 1 \right] * ID16 \right] \right], \{A1, 0, 7\}, \{B1, 0, 7\} \right] // MatrixForm$$

```

Out[*]//MatrixForm=

$$\begin{pmatrix} \{0, 0\} & \{0, 1\} & \{0, 2\} & \{0, 3\} & \{0, 4\} & \{0, 5\} & \{0, 6\} & \{0, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{1, 0\} & \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} & \{1, 5\} & \{1, 6\} & \{1, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{2, 0\} & \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} & \{2, 5\} & \{2, 6\} & \{2, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{3, 0\} & \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} & \{3, 5\} & \{3, 6\} & \{3, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{4, 0\} & \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} & \{4, 5\} & \{4, 6\} & \{4, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{5, 0\} & \{5, 1\} & \{5, 2\} & \{5, 3\} & \{5, 4\} & \{5, 5\} & \{5, 6\} & \{5, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{6, 0\} & \{6, 1\} & \{6, 2\} & \{6, 3\} & \{6, 4\} & \{6, 5\} & \{6, 6\} & \{6, 7\} \\ \text{True} & \text{True} \end{pmatrix} \\ \begin{pmatrix} \{7, 0\} & \{7, 1\} & \{7, 2\} & \{7, 3\} & \{7, 4\} & \{7, 5\} & \{7, 6\} & \{7, 7\} \\ \text{True} & \text{True} \end{pmatrix}$$

defs; some Symbols; metric

$$\left\{ \left\{ a4[t] \rightarrow \frac{M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} + c_1 + \frac{1}{2} \text{ProductLog} \left[-e^{-\frac{2 M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} - 2 c_1} \right] \right\} \right\}$$

$$\text{DSolve}[0 == -M (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7]) + (Q1 - e^{-2 a4[t]} Q1) a4'[t], a4[t], t]$$

$$Mc[j] = (j + 1)^{\text{st}} \text{ Energy Eigenvalue for Transpose[cayZ].}\Psi16, j = 0, \dots, 7$$

ProductLog[z]

gives the principal solution for w in z = we^w.

if z and w are any complex numbers, then

$$we^w = z$$

holds if and only if

$$w = W_k(z) \text{ for some integer } k.$$

When dealing with real numbers only, the two branches W_0 and W_{-1} suffice: for real numbers x and y the equation

$$ye^y = x$$

can be solved for y only if $x \geq \frac{-1}{e}$; yields $y = W_0(x)$ if $x \geq 0$ and the two values $y = W_0(x)$ and $y = W_{-1}(x)$ if $\frac{-1}{e} \leq x < 0$.

?todo: NEED DefScalarFunction[#]; & /@ (f16[#]&/@Range[0,15]), and allow xact to compute?

Introduce the wave function, Ψ_{16} , for a Universe:

```
In[50]:=  $\Psi_{16} = f16[\#][x0, x4] \& /@ Range[0, 15]$ 
Out[50]= {f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4], f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4], f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4], f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
In[51]:= processRawSets[rawSets_] := Module[{s1, s2, J, ret, L = Range[Length[rawSets]], r = {}},
  While[Length[L] > 1,
    s1 = Union[rawSets[[L[[1]]]]];
    L = Drop[L, 1];
    J = L[[1]];
    While[J <= 16,
      If[MemberQ[L, J],
        s2 = Union[rawSets[[J]]];
        If[Intersection[s1, s2] != {}, s1 = Union[Flatten[{s1, s2}]];
         L = Complement[L, {J}]; ##&[]];
        ##&[]];
      J++];
      AppendTo[r, s1];
    ];
    ret = Sort[r, #1[[1]] < #2[[1]] &];
    Return[ret];
  ]
]
```

```
In[52]:= rawSets¶16 = Cases[
  #,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
  Infinity,
  Heads → True
] & /@ ¶16

Out[52]= {{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}, {15}}

In[53]:= processRawSets[rawSets¶16]

Out[53]= {{0}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}}

In[54]:= (*¶new16=F16[#] [x0,x4]&/@Range[0,15]*)

In[55]:= Clear[sf16Aa];
sf¶16Aa = f16[#] → ToExpression[
  "((Z[" <> ToString[#] <> "] [6*H#1,H#2])&)"] & /@ Range[0, 15]

Out[55]= {f16[0] → (Z[0][6 H#1, H#2] &), f16[1] → (Z[1][6 H#1, H#2] &),
f16[2] → (Z[2][6 H#1, H#2] &), f16[3] → (Z[3][6 H#1, H#2] &),
f16[4] → (Z[4][6 H#1, H#2] &), f16[5] → (Z[5][6 H#1, H#2] &), f16[6] → (Z[6][6 H#1, H#2] &),
f16[7] → (Z[7][6 H#1, H#2] &), f16[8] → (Z[8][6 H#1, H#2] &), f16[9] → (Z[9][6 H#1, H#2] &),
f16[10] → (Z[10][6 H#1, H#2] &), f16[11] → (Z[11][6 H#1, H#2] &),
f16[12] → (Z[12][6 H#1, H#2] &), f16[13] → (Z[13][6 H#1, H#2] &),
f16[14] → (Z[14][6 H#1, H#2] &), f16[15] → (Z[15][6 H#1, H#2] &)}

In[56]:= (*Clear[sf16Aa];
sf¶16Aa=f16[#]→ToExpression[
  "((Z["<>ToString[#]<>"] [6*H#1,H#2] 1/Sin[6*H#1]^(1/2))&)"]&/@Range[0,15]*)
```

```
In[57]:= Clear[snewfψ16Aa];
snewfψ16Aa = f16[#] → ToExpression["((nZ[" <> ToString[#] <>
"] [6*H#1,H#2] 1
Sin[6 * H #1]1/2 )&)" ] & /@ Range[0, 15]
```

```
Out[57]= {f16[0] → (nZ[0] [6 H #1, H #2] &), f16[1] → (nZ[1] [6 H #1, H #2] &),
f16[2] → (nZ[2] [6 H #1, H #2] &), f16[3] → (nZ[3] [6 H #1, H #2] &),
f16[4] → (nZ[4] [6 H #1, H #2] &), f16[5] → (nZ[5] [6 H #1, H #2] &),
f16[6] → (nZ[6] [6 H #1, H #2] &), f16[7] → (nZ[7] [6 H #1, H #2] &),
f16[8] → (nZ[8] [6 H #1, H #2] &), f16[9] → (nZ[9] [6 H #1, H #2] &),
f16[10] → (nZ[10] [6 H #1, H #2] &), f16[11] → (nZ[11] [6 H #1, H #2] &),
f16[12] → (nZ[12] [6 H #1, H #2] &), f16[13] → (nZ[13] [6 H #1, H #2] &),
f16[14] → (nZ[14] [6 H #1, H #2] &), f16[15] → (nZ[15] [6 H #1, H #2] &)}
```

0 (4, 4) related :

```
In[58]:= Needs["Notation`"]
```

```
In[59]:= Symbolize[σga1A]
```

```
In[60]:= Symbolize[σga2A]
```

```
In[61]:= Symbolize[τ]
```

```
In[62]:= Symbolize[T16A]
```

```
In[63]:= Symbolize[T16α]
```

```
In[64]:= Symbolize[ua]
```

```
In[65]:= Symbolize[Jy]
```

```
In[66]:= Symbolize[FaA]
```

```
In[67]:= Symbolize[FAa]
```

```
In[68]:= Symbolize[ σ̄₂₂ ]
```

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1,1,1, - 1) \quad (5)$$

```
In[69]:= Symbolize[ gαβ ]
```

note below : g_{AB} is really $g_{(A)(B)}$:: Minkowski - Lorentz metric

```
In[70]:= Symbolize[ gAB ]
```

note below : G_{AB} is really $G_{(A)(B)}$:: Minkowski - Lorentz

```
In[71]:= (*Symbolize[ GAB ]*)
```

```
In[72]:= Symbolize[ S44αβ ]
```

$$\Sigma^{\alpha\beta} = \xi S^{\alpha\beta} u,$$

```
In[73]:= Symbolize[ Σαβ ]
```

```
In[74]:= (*Symbolize[ Σαβ ]*)
```

$$\frac{1}{2}\Sigma^{jk}\Sigma_{jk} = \frac{1}{4}\partial_a u^a u^b \partial_b - \frac{1}{4}\partial_a u^b u^b \partial_a,$$

```
In[75]:= Symbolize[ Σ² ]
```

NEED DefScalarFunction[#]; & /@ Flatten[Universe]

```
In[76]:= Universe = ToExpression["U" <> ToString[#]] & /@ Range[0, 7]
```

```
In[76]:= Symbolize[ F_a^A ]
```

```
In[77]:= Symbolize[ F_A^a ]
```

```
In[78]:= Symbolize[gtrye_α^(A)]
```

```
In[79]:= Symbolize[gtrye_(A)^α]
```

SPACETIME METRIC

```
In[80]:= (*einsteinsteinMetric=Array[g[#,1-1, #2-1]&, {8,8}]*)
```

SPACETIME METRIC:

```
In[81]:= g4488 = Array[g[#,1-1][#,2-1][x0, x4] &, {8, 8}];
```

SPACETIME METRIC values:

MatrixMetric44

$$6 H x0 = z \&& H x4 = t$$

$$\left\{ \left\{ a4''[t] \rightarrow 0, a4'[t] \rightarrow \frac{2}{3} (-1 + M) \right\}, \left\{ a4''[t] \rightarrow 0, a4'[t] \rightarrow \frac{2 (1 + M)}{3} \right\} \right\}$$

```
In[82]:= (*β3=Exp[2 H x4 - √(K²-M²)] ;*)
(*β3=Exp[2 * a4[3 H x4]] ;*)
β3 = Exp[2 * a4[H * x4]] (* /. {a4 → ((K1 * 2^(1+M)/3 + K2 * 2/3 (-1+M)) #) &} *)
β1 = Sin[6 * H * x0]^(1/3);
β2 = Cot[6 * H * x0]^2;
```

Out[82]= $e^{2 a4[H x4]}$

```
In[85]:= MatrixForm[
MatrixMetric44 = {{β2, 0, 0, 0, 0, 0, 0, 0, 0}, {0, β1 β3, 0, 0, 0, 0, 0, 0, 0}, {0, 0, β1 β3, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, β1 β3, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -β1/β3, 0, 0}, {0, 0, 0, 0, 0, 0, -β1/β3, 0}, {0, 0, 0, 0, 0, 0, 0, -β1/β3}} // FullSimplify[#, constraintVars] &]
```

$$\begin{pmatrix} \cot[6 H x0]^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{2 a4[H x4]} \sin[6 H x0]^{1/3} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{2 a4[H x4]} \sin[6 H x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{2 a4[H x4]} \sin[6 H x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -e^{-2 a4[H x4]} \sin[6 H x0]^{1/3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[86]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression[
"((Z["<>ToString[#]<>"] [6*H#1,3*H#2] 1/Sin[6*H#1]^(1/2))&)"]&/@Range[0,15]*)
```

```
In[87]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"] [6*H#1,3*H#2])&)"]&/@Range[0,15]*)
```

```
In[88]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression[
"((Z["<>ToString[#]<>"] [6*H#1,H#2] 1/Sin[6*H#1]^(1/2))&)"]&/@Range[0,15]*)
```

In[89]:= (*Inactivate[((q)&)]Evaluate[Activate[Evaluate[q]]]*)

```
In[90]:= Clear[makeSubstitution];
makeSubstitution[] := Module[{met, mat, t, f, p},
  met = Flatten[g4488][[All, 0]];
  mat = Flatten[MatrixMetric44];
  t = Thread[met → mat];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]

In[92]:= makeSubstitution[a_, b_] := Module[{t, f, p},
  t = Thread[a → b];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]


```

```
In[93]:= ssgm4488 = makeSubstitution[]
```

```
Out[93]= {g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
g[1][1] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
g[2][1] → (0 &), g[2][2] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
g[3][2] → (0 &), g[3][3] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
g[5][5] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
g[6][5] → (0 &), g[6][6] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &)}
```

```
In[94]:= Symbolize[E(A)α]
```

```

In[95]:= Symbolize[eα(A)]
In[96]:= eα(A) = Array[Eα(A) [#1 - 1, #2 - 1] &, {8, 8}]
Out[96]=
{ {Eα(A) [0, 0], Eα(A) [0, 1], Eα(A) [0, 2], Eα(A) [0, 3], Eα(A) [0, 4], Eα(A) [0, 5], Eα(A) [0, 6], Eα(A) [0, 7]}, {Eα(A) [1, 0], Eα(A) [1, 1], Eα(A) [1, 2], Eα(A) [1, 3], Eα(A) [1, 4], Eα(A) [1, 5], Eα(A) [1, 6], Eα(A) [1, 7]}, {Eα(A) [2, 0], Eα(A) [2, 1], Eα(A) [2, 2], Eα(A) [2, 3], Eα(A) [2, 4], Eα(A) [2, 5], Eα(A) [2, 6], Eα(A) [2, 7]}, {Eα(A) [3, 0], Eα(A) [3, 1], Eα(A) [3, 2], Eα(A) [3, 3], Eα(A) [3, 4], Eα(A) [3, 5], Eα(A) [3, 6], Eα(A) [3, 7]}, {Eα(A) [4, 0], Eα(A) [4, 1], Eα(A) [4, 2], Eα(A) [4, 3], Eα(A) [4, 4], Eα(A) [4, 5], Eα(A) [4, 6], Eα(A) [4, 7]}, {Eα(A) [5, 0], Eα(A) [5, 1], Eα(A) [5, 2], Eα(A) [5, 3], Eα(A) [5, 4], Eα(A) [5, 5], Eα(A) [5, 6], Eα(A) [5, 7]}, {Eα(A) [6, 0], Eα(A) [6, 1], Eα(A) [6, 2], Eα(A) [6, 3], Eα(A) [6, 4], Eα(A) [6, 5], Eα(A) [6, 6], Eα(A) [6, 7]}, {Eα(A) [7, 0], Eα(A) [7, 1], Eα(A) [7, 2], Eα(A) [7, 3], Eα(A) [7, 4], Eα(A) [7, 5], Eα(A) [7, 6], Eα(A) [7, 7]}}

In[97]:= (*Symbolize[seα(A)]*)
In[98]:= Symbolize[sgeα(A)]
In[99]:= Symbolize[sgtryeα(A)]
In[100]:= Symbolize[E(A)α]
In[101]:= Symbolize[sge(A)α]
In[102]:= Symbolize[sgtrye(A)α]
In[103]:= Symbolize[e(A)α]
In[104]:= e(A)α = Array[E(A)α [#1 - 1, #2 - 1] &, {8, 8}]
Out[104]=
{ {E(A)α [0, 0], E(A)α [0, 1], E(A)α [0, 2], E(A)α [0, 3], E(A)α [0, 4], E(A)α [0, 5], E(A)α [0, 6], E(A)α [0, 7]}, {E(A)α [1, 0], E(A)α [1, 1], E(A)α [1, 2], E(A)α [1, 3], E(A)α [1, 4], E(A)α [1, 5], E(A)α [1, 6], E(A)α [1, 7]}, {E(A)α [2, 0], E(A)α [2, 1], E(A)α [2, 2], E(A)α [2, 3], E(A)α [2, 4], E(A)α [2, 5], E(A)α [2, 6], E(A)α [2, 7]}, {E(A)α [3, 0], E(A)α [3, 1], E(A)α [3, 2], E(A)α [3, 3], E(A)α [3, 4], E(A)α [3, 5], E(A)α [3, 6], E(A)α [3, 7]}, {E(A)α [4, 0], E(A)α [4, 1], E(A)α [4, 2], E(A)α [4, 3], E(A)α [4, 4], E(A)α [4, 5], E(A)α [4, 6], E(A)α [4, 7]}, {E(A)α [5, 0], E(A)α [5, 1], E(A)α [5, 2], E(A)α [5, 3], E(A)α [5, 4], E(A)α [5, 5], E(A)α [5, 6], E(A)α [5, 7]}, {E(A)α [6, 0], E(A)α [6, 1], E(A)α [6, 2], E(A)α [6, 3], E(A)α [6, 4], E(A)α [6, 5], E(A)α [6, 6], E(A)α [6, 7]}, {E(A)α [7, 0], E(A)α [7, 1], E(A)α [7, 2], E(A)α [7, 3], E(A)α [7, 4], E(A)α [7, 5], E(A)α [7, 6], E(A)α [7, 7]}}

In[105]:= (*gtryeα(A) = (eα(A) /. sgtryeα(A))*)
In[106]:= (*gtrye(A)α = (e(A)α /. sgtrye(A)α)*)
In[107]:= (*Symbolize[Iα βκ]*)
In[108]:= (*preSpinConnection=Array[Iα βκ [##]&, {8, 8, 8}];*)

```

```

In[109]:= (*Symbolize[ EAa ]*)

In[110]:= (*Symbolize[ EaA ]*)

In[111]:= (* FAa FaA *)

In[112]:= (*Do[ FAa=EAa[h]=Table[ (u[[h]].σ.(τ[B])), {B,1,8}], {h,1,Length[u]}];
Do[ EaA[h]=FullSimplify[Inverse[EAa[h]]], {h,1,Length[u]}];
Table[ FaA=EaA[h]== (u[[h]].σ.u[[h]])
Transpose[Table[FullSimplify[ExpandAll[ η8[[B,B]τ[B].u[[h]] ]],
{B,1,8}]], {h,1,Length[u]}]*)

In[113]:= (*Table[
FullSimplify[ExpandAll[Transpose[EAa[h]].η8.EAa[h]]]- (u[[h]].σ.u[[h]]) σ== Zero,
{h,1,Length[u]}]*)

In[114]:= (*inη88=FullSimplify[Inverse[η8]];*)

In[115]:= (*Clear[ω(a)(b)];*) Clear[ω]; Symbolize[ω(a)(b)]

In[116]:= ω = Array[ω(a)(b) & , {8, 8, 8}];
Protect[ω]

Out[117]= {ω}

```

constants

```

← ERROR : 08similarityTransformation has 1 st index that transforms as  $\frac{\partial}{\partial \Psi}$  ,
not as  $\Psi$ 

In[118]:= (* 08similarityTransformation has 1st index that transforms as  $\frac{\partial}{\partial \Psi}$  ,
not as  $\Psi$  *)

08similarityTransformation has 1 st index that transforms as  $\frac{\partial}{\partial \Psi}$  , not as  $\Psi$  :

In[119]:= (*ArrayFlatten[{{IdentityMatrix[4],0},{0,-IdentityMatrix[4]}}]*)

```

```

In[120]:= ID4 = IdentityMatrix[4];
ID8 = IdentityMatrix[8];

In[122]:= η4488 // MatrixForm
Out[122]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


In[123]:= η4488[[#, #]] & /@ Range[8]
Out[123]= {1, 1, 1, 1, -1, -1, -1, -1}

In[124]:= 

Zero4 = ConstantArray[0, {4, 4}]; Zero8 = ConstantArray[0, {8, 8}];
mid = ExpandAll[-1 * ID4];
ε3 = Array[Signature[{##}] &, {3, 3, 3}]
ε4 = Array[Signature[{##}] &, {4, 4, 4, 4}];
MId = ExpandAll[-1 * ID8];
simp = {Zero4 → 0, ID4 → 1, mid → -1};
Simp = {Zero8 → ZERO, ID8 → ONE, MId → MONE};

Out[126]= {{{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}}, {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}}}, {{{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}}
```

In[131]:= Zero16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η8, 0}, {0, -η8}}];
Id16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * Id16];
Simp16 = {Zero16 → ZERO16, Id16 → ONE16, MId16 → MONE16};

In[135]:= ZERO16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η4488, 0}, {0, -η4488}}];
ID16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * ID16];
Simp16 = {ZERO16 → ZERO16, ID16 → ONE16, MId16 → MONE16};

```
In[139]:= ws = Flatten[Table[ToExpression["w" <> ToString[A1] <> ToString[B1]], {A1, 1, 7}, {B1, A1 + 1, 8}]];
Length[ws]

Out[139]= {w12, w13, w14, w15, w16, w17, w18, w23, w24, w25, w26, w27, w28, w34, w35, w36, w37, w38, w45, w46, w47, w48, w56, w57, w58, w67, w68, w78}

Out[140]= 28

In[141]=

σ.σ - ID8 == Zero8
Tr[σ] == 0

Out[141]= True

Out[142]= True
```

some function definitions :

```
In[143]:= Clear[rawSets];
rawSets[l_, f_, o_: 0] := Module[{t, h},
  t = Table[Cases[
    l[[j]],
    h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], f] & n + o,
    Infinity,
    Heads → True
  ], {j, 1, Length[l]}];
  Return[t];
]

In[145]:= (*rawSetsel16=rawSets[el16,"f16",0]*)

In[146]:= (*couplings=showCoupledEquations[rawSetsel16]
%//Flatten//Sort
%==Range[0,15]*)
```

```
In[147]:= Clear[dualRawSets];
dualRawSets[l_, f_, o_, h_, oh_] := Module[{r, r1, r2},
  r1 = rawSets[l, f, o];
  r2 = rawSets[l, h, oh];
  r = Union[Flatten[{r1, r2}, 1]] // Drop[#, 1] &;
  Return[r];
]
```

'FixedPoint', below, uses 'MergeSetsStep', below.
 According to Wolfram docs, one uses 'Throw' to exit from 'FixedPoint' before it is finished.

FixedPoint really uses Catch and Throw, so what could go wrong

▼ Details and Options

- FixedPoint always returns the last result it gets.
- You can use Throw to exit from FixedPoint before it is finished.
- FixedPoint[f, expr] applies SameQ to successive pairs of results to determine whether a fixed point has been reached.
- FixedPoint[f, expr, ..., SameTest -> s] applies s to successive pairs of results.

In[149]:=

```
(* Helper to perform one pass of merging *)
MergeSetsStep[currentSets_List] :=
  Module[{n = Length[currentSets], i, j, merged = currentSets},
(* We look for the first pair (i, j) that intersects *)
  Catch[
    For[i = 1, i <= Length[merged], i++,
      For[j = i + 1, j <= Length[merged], j++,
        If[Intersection[merged[[i]], merged[[j]]] != {}.,
          (* Found intersection: Merge and Throw to restart/finish this step *)
          merged = Delete[merged, {{i}, {j}}];
          AppendTo[merged, Sort[Union[currentSets[[i]], currentSets[[j]]]]];
          Throw[Sort[merged]]; (* Sort for canonical form *)
        ];
      ];
    ];
    (* If no intersection found, return original sorted *)
    Sort[merged]
  ];
];
```

In[150]:=

```
showCoupledEquations[items_List] :=
  FixedPoint[MergeSetsStep, items];
```

```
In[151]:= blockPartition[mat_] :=
  Module[{(*blocks,block11,block12,block21,block22,*)ret},
    blocks = Partition[mat, {8, 8}];
    (*block11=blocks[[1,1]] ;*)(*Top-left block*)
    (*block12=blocks[[1,2]] ;*)(*Top-right block*)
    (*block21=blocks[[2,1]] ;*)(*Bottom-left block*)
    (*block22=blocks[[2,2]] ;*)(*Bottom-right block*)
    ret = ArrayFlatten[
      {{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}] === mat;
  If[ret, Return[
    {{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}], ## &[]];
  Return[Null];]

In[152]:= eextract[a_] := Extract[a, {#}] & /@ Range[0, Length[a]]

In[153]:= Clear[imes]

In[154]:= imes[a_] := {a}

In[155]:= imes[a_, b_] := {a, b}

In[156]:= imes[a_, b_, c_] := {a, b, c}

In[157]:= imes[a_, b_, c_, d_] := {a, b, c, d}

In[158]:= imes[a_, b_, c_, d_, f_] := {a, b, c, d, f}

In[159]:= imes[a_, b_, c_, d_, f_, h_] := {a, b, c, d, f, h}

In[160]:= imes[a_, b_, c_, d_, f_, h_, j_] := {a, b, c, d, f, h, j}

In[161]:= times[a__] := Flatten[{Flatten[#] & /@ a}]

In[162]:= (*times[a_,b_,c_,d_]:= {a,b}*)

In[163]:= (*imes[a_,b_,c_,d_][0]:= {a,d}*)
```

In[164]:=

```
(*imes[a_,b_][θ]:= {a,b}*)
```

In[165]:=

```
iid[a_, b_] := a
```

In[166]:=

```
iid[a_, b_, c_] := {a, b}
```

In[167]:=

```
iid[a_List] := a[[1]]
```

In[168]:=

```
qid[a__] := a
```

In[169]:=

```
Clear[der]; der[a__][c_][b__] := c
```

In[170]:=

```
Clear[der3]; der3[a__][c_][b__] := {c, a, b}
```

In[171]:=

```
(*der[a_, b_][c_]:= c*)
```

In[172]:=

```
(*derz[a_, b_]:= a*)
```

In[173]:=

```
(*dert[a_, b_]:= b*)
```

In[174]:=

```
(*derivative[a__][c_]:= FullForm[c]/.{Derivative→der}*)
```

In[175]:=

```
{Z[θ][z, t], D[Z[θ][z, t], z], D[Z[θ][z, t], t]}
```

```
FullForm[#]& /@ %
```

```
(*ToString[#]&/@%*)
```

```
% /. {Derivative→der}
```

Out[175]=

```
{Z[θ][z, t], Z[θ]^(1,0)[z, t], Z[θ]^(0,1)[z, t]}
```

Out[176]=

```
{Z[θ][z, t], Derivative[1, 0][Z[θ]][z, t], Derivative[0, 1][Z[θ]][z, t]}
```

Out[177]=

```
{Z[θ][z, t], Z[θ], Z[θ]}
```

In[178]:=

```

{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
%* (Prime[#] & /@ Range[Length[%]])
Plus @@ %
FullForm[%] (* #]&/@%*)
% /. {Plus -> qid, Times -> qid, Derivative -> der}
List[%]
Partition[%, 2]
eextract[#] & /@ %
(*eextract[#]&/@%*)
(*#__List[[0]]&/@%*)
#[[2]] & /@ %
#[[3]] & /@ %

```

Out[178]=

 $\{Z[0][z, t], Z[0]^{(1,0)}[z, t], Z[0]^{(0,1)}[z, t]\}$

Out[179]=

 $\{2 Z[0][z, t], 3 Z[0]^{(1,0)}[z, t], 5 Z[0]^{(0,1)}[z, t]\}$

Out[180]=

 $2 Z[0][z, t] + 5 Z[0]^{(0,1)}[z, t] + 3 Z[0]^{(1,0)}[z, t]$

Out[181]/.FullForm=

```

Plus[Times[2, Z[0][z, t]], Times[5, Derivative[0, 1][Z[0]][z, t]],
Times[3, Derivative[1, 0][Z[0]][z, t]]]

```

Out[182]=

 $\text{Sequence}[2, Z[0][z, t], 5, Z[0], 3, Z[0]]$

Out[183]=

 $\{2, Z[0][z, t], 5, Z[0], 3, Z[0]\}$

Out[184]=

 $\{\{2, Z[0][z, t]\}, \{5, Z[0]\}, \{3, Z[0]\}\}$

Out[185]=

 $\{\{\text{List}, 2, Z[0][z, t]\}, \{\text{List}, 5, Z[0]\}, \{\text{List}, 3, Z[0]\}\}$

Out[186]=

 $\{2, 5, 3\}$

Out[187]=

 $\{Z[0][z, t], Z[0], Z[0]\}$

Sequence[Subscript[expr, 1], Subscript[expr, 2], ...]

represents a sequence of arguments to be spliced automatically into any function .


```
In[198]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}

FullForm[#] & /@ %

ToString[#] & /@ %

StringSplit[#, "]\"] & /@ %

StringExtract[#, "[" → All] & /@ %

(*StringExtract[#, " {, z, t"} → All]&/@%*)

InputForm[TextString[#]] & /@ %

Out[198]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[199]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[200]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[201]= {{Z[0], [z, t], {Derivative[1, 0, [Z[0], , [z, t], {Derivative[0, 1, [Z[0], , [z, t]}}

Out[202]= {{Z[0], {, z, t}}, {{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}, {{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}}

Out[203]= {"{{Z, 0}, {, z, t}}", "{{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}",
"{{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}"}
```

```

In[204]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
(*InputForm[TextString[#]]&/@%*)
StringExtract[#, "[" → All] & /@ %
StringExtract[#, "]" → All] & /@ %
StringExtract[#, "[" → 1] & /@ %%%
StringExtract[#, "]" → 1] & /@ %%%%
StringExtract[#, "Z[" → 1] & /@ %%%%%

Out[204]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[205]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[206]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[207]= {{Z, 0}, z, t}}, {Derivative, 1, 0], Z, 0]], z, t}}, {Derivative, 0, 1], Z, 0]], z, t}}}

Out[208]= {{Z[0], [z, t, ], {Derivative[1, 0], [Z[0], , [z, t, ], {Derivative[0, 1], [Z[0], , [z, t, ]}

Out[209]= {Z, Derivative, Derivative}

Out[210]= {Z[0, Derivative[1, 0, Derivative[0, 1]

Out[211]= , Derivative[1, 0][, Derivative[0, 1][}

In[212]:= (*TemplateApply[StringTemplate["Z `['` then `b`"],
<|"a"→1234,"b"→5678|>]*)
```

```

In[213]:= skelx[x_, matQ_ : True] := Module[{t = {666}},
  If[matQ == True, t = Block[{r},
    r = x;
    Table[Block[{q}, q = SameQ[0, r[[j, i]]];
      If[True == q, Style[0, Blue], Style[Length[r[[j, i]], Red]]],
      {j, 1, Length[r]}, {i, 1, Length[r[[ -1]]}]]];
  If[matQ == False, t = Block[{r},
    r = x;
    Table[Block[{q}, q = SameQ[0, r[[j]]];
      If[True == q, Style[0, {RGBColor → {0, 1/3, 0}, Bold}],
      Style[Length[r[[j]]], Red]]], {j, 1, Length[r]}]];
  MatrixForm[t, TableAlignments → Left]]
```

```
In[214]:= makeSym[size_, fn_] :=
Module[{rtmp}, rtmp = Table[fn[i, j], {i, 1, size}, {j, 1, i}];
MapThread[Join, {rtmp, Rest /@ Flatten[rtmp, {{2}, {1}}]}]];

In[215]:= makeAntiSym[size_, fn_] :=
Module[{rtmp}, rtmp = Table[If[i == j, 0, fn[i, j]], {i, 1, size}, {j, 1, i}];
MapThread[Join, {rtmp, Rest /@ Flatten[-rtmp, {{2}, {1}}]}]];

In[216]:= Block[{MX}, MX = makeAntiSym[8, Subscript[\[omega], ##] &]]

Out[216]=
{ {0, -\omega_{2,1}, -\omega_{3,1}, -\omega_{4,1}, -\omega_{5,1}, -\omega_{6,1}, -\omega_{7,1}, -\omega_{8,1}}, { \omega_{2,1}, 0, -\omega_{3,2}, -\omega_{4,2}, -\omega_{5,2}, -\omega_{6,2}, -\omega_{7,2}, -\omega_{8,2}}, { \omega_{3,1}, \omega_{3,2}, 0, -\omega_{4,3}, -\omega_{5,3}, -\omega_{6,3}, -\omega_{7,3}, -\omega_{8,3}}, { \omega_{4,1}, \omega_{4,2}, \omega_{4,3}, 0, -\omega_{5,4}, -\omega_{6,4}, -\omega_{7,4}, -\omega_{8,4}}, { \omega_{5,1}, \omega_{5,2}, \omega_{5,3}, \omega_{5,4}, 0, -\omega_{6,5}, -\omega_{7,5}, -\omega_{8,5}}, { \omega_{6,1}, \omega_{6,2}, \omega_{6,3}, \omega_{6,4}, \omega_{6,5}, 0, -\omega_{7,6}, -\omega_{8,6}}, { \omega_{7,1}, \omega_{7,2}, \omega_{7,3}, \omega_{7,4}, \omega_{7,5}, \omega_{7,6}, 0, -\omega_{8,7}}, { \omega_{8,1}, \omega_{8,2}, \omega_{8,3}, \omega_{8,4}, \omega_{8,5}, \omega_{8,6}, \omega_{8,7}, 0} }
```

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2} \left(\frac{\partial g_{\mu\alpha}}{\partial x^\beta} + \frac{\partial g_{\mu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\mu} \right),$$

$$\Gamma^\mu{}_{\alpha\beta} = g^{\mu\nu} \Gamma_{\nu\alpha\beta},$$

$$R^\mu{}_{\nu\alpha\beta} = \frac{\partial \Gamma^\mu{}_{\nu\beta}}{\partial x^\alpha} - \frac{\partial \Gamma^\mu{}_{\nu\alpha}}{\partial x^\beta} + \Gamma^\mu{}_{\rho\alpha} \Gamma^\rho{}_{\nu\beta} - \Gamma^\mu{}_{\rho\beta} \Gamma^\rho{}_{\nu\alpha}.$$

$$R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}, \quad R = g^{\mu\nu} R_{\mu\nu},$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

In[217]:=

```
(*Christoffel, Einstein tensor, ...*)
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, rΓ, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    rΓ = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
      (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8} *
      {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
    RicciΓ =
      ParallelTable[FullSimplify[D[rΓ[[μ, ν, β]], X[[α]]] - D[rΓ[[μ, ν, α]], X[[β]]] +
        Sum[rΓ[[μ, s, α]] * rΓ[[s, ν, β]] - rΓ[[μ, s, β]] * rΓ[[s, ν, α]],
        {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}];
    RieΓ = ParallelTable[FullSimplify[Sum[RicciΓ[[α, μ, α, ν]],
      {α, 1, DIM8}], ass], {μ, 1, DIM8}, {ν, 1, DIM8}];
    RS = FullSimplify[Tr[ginv.RieΓ], ass];
    G = ParallelTable[
      FullSimplify[RieΓ[[α]] - 1/2 g[[α]] RS, ass], {α, 1, Length[RieΓ]}];
    ];
  Print[t];
  Print[Now];
  Return[{ginv, rΓ, RicciΓ, RieΓ, RS, G}]
]
```

If ‘paste’ Maple MathML to ToExpression[MathMLForm[]] fails, then try to edit this :

In[219]:=

```
Clear[mapleztStringToMathematica];
mapleztStringToMathematica[
  input_String, toExpression_: False, debug_: False] :=
Module[{prepare, post, custom, use = input, save = input, linenumber = 0},
(*use=ToString[InputForm[TextString[input]]];*)(*Print[use];*)
  prepare[in_String] := Module[{s = in}, linenumber++;
  s = in;
  (*If[s==ToString[InputForm[TextString[s]]]],*
   (*###&[]*)Print[s],Print[ToUpperCase[s]]];*)
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,z)" >
    "D[Z[" <> ToString[j] <> "][z,t],{z,2}]]"], {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,t)" >
    "D[D[Z[" <> ToString[j] <> "][z,t],z],t]"], {j, 0, 15}];
```

```

Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,z)" >
    "D[D[Z[" <> ToString[j] <> "] [z,t],t],z]", {j, 0, 15}];
Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,t)" >
    "D[Z[" <> ToString[j] <> "] [z,t],{t,2}]"], {j, 0, 15}];

Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t)" >
    "D[Z[" <> ToString[j] <> "] [z,t],t]", {j, 0, 15}];
Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z)" >
    "D[Z[" <> ToString[j] <> "] [z,t],z]", {j, 0, 15}];

s = StringReplace[s, "tan(z)" >> "Tan[z]"];
s = StringReplace[s, "cot(z)" >> "Cot[z]"];
s = StringReplace[s, "sin(z)" >> "Sin[z]"];
s = StringReplace[s, "cos(z)" >> "Cos[z]"];
s = StringReplace[s, "sec(z)" >> "Sec[z]"];
s = StringReplace[s, "csc(z)" >> "Csc[z]"];

s = StringReplace[s, "=" >> "->"];

(*Do[s=StringReplace[s,
    "Z"<>ToString[j]<>"(z,t)">>Z["<>ToString[j]<>"] [z,t]",{j,0,15}];*)
(*s=StringReplace[s,"diff(a4(t),t)">>"D[a4[t],t]"];*)
(*s=StringReplace[s,"exp(-2*a4(t))">>"Exp[-2a4[t]]"];*)
(*s=StringReplace[s,"a4(t)">>"a4[t]"];*)

Return[s];
];
post[in_String] := Module[{s = in}, linenumber++;
  s = in;
  s = StringReplace[s, "a4(t)" >> "a4[t]"];
  Do[s = StringReplace[s, "Z" <> ToString[j] <> "(z,t)" >
      "Z[" <> ToString[j] <> "] [z,t]", {j, 0, 15}];
  Return[s];
];
custom[in_String] := Module[{s = in}, linenumber++;
  s = in;

  s = StringReplace[s, "diff(a4(t),t)" >> "D[a4[t],t]"];
  s = StringReplace[s, "exp(-2*a4(t))" >> "Exp[-2a4[t]]"];

```

```

    Return[s];
];
use = prepare[use];
use = custom[use];
Return[post[use]];
(*If[toExpression,Return[ToExpression[s/.{z→#1,t→#2}],##&[]];*)
]

```

gtry and Γ and ...

```

In[221]:= gtry = MatrixMetric44
Out[221]=
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

In[222]:= Protect[gtry]
Out[222]=
{gtry}

In[223]:= Unprotect[ginv,  $\Gamma$ , Ricci $\Gamma$ , Rie $\Gamma$ , RS, EinsteinG]
Out[223]=
{}

In[224]:= result = {ginv,  $\Gamma$ , Ricci $\Gamma$ , Rie $\Gamma$ , RS, EinsteinG} = rt[gtry];
Fri 13 Feb 2026 02:36:43 GMT-8
{3.59788, Null}
Fri 13 Feb 2026 02:36:46 GMT-8

In[225]:= Protect[ginv,  $\Gamma$ , Ricci $\Gamma$ , Rie $\Gamma$ , RS, EinsteinG]
Out[225]=
{ginv,  $\Gamma$ , Ricci $\Gamma$ , Rie $\Gamma$ , RS, EinsteinG}

verify g $\mu\nu$  | $\alpha$  = 0 :
In[226]:= Simplify[
  Table[D[gtry[[j, k]], X[[i]]] - Sum[gtry[[k, s]]  $\times$   $\Gamma$ [[s, i, j]], {s, 1, DIM8}] -
    Sum[gtry[[j, s]]  $\times$   $\Gamma$ [[s, i, k]], {s, 1, DIM8}],
  {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union
Simplify[
  Table[D[gtry[[j, k]], X[[i]]] - Sum[gtry[[k, s]]  $\times$   $\Gamma$ [[s, i, j]], {s, 1, DIM8}] -
    Sum[gtry[[j, s]]  $\times$   $\Gamma$ [[s, i, k]], {s, 1, DIM8}],
  {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union

Out[226]=
{0}

```

```
In[227]:= 
Table[g[j][k] → If[j == k,
  ToExpression["((g[" <> ToString[j] <> "] [" <> ToString[k] <> "] [#1, #2]) &)"],
  (0 &)], {j, 0, 7}, {k, 0, 7}];
sg = % // Flatten

Out[228]=
{g[0][0] → (g[0][0][#1, #2] &), g[0][1] → (0 &), g[0][2] → (0 &),
g[0][3] → (0 &), g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &),
g[0][7] → (0 &), g[1][0] → (0 &), g[1][1] → (g[1][1][#1, #2] &), g[1][2] → (0 &),
g[1][3] → (0 &), g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &),
g[1][7] → (0 &), g[2][0] → (0 &), g[2][1] → (0 &), g[2][2] → (g[2][2][#1, #2] &),
g[2][3] → (0 &), g[2][4] → (0 &), g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &),
g[3][0] → (0 &), g[3][1] → (0 &), g[3][2] → (0 &), g[3][3] → (g[3][3][#1, #2] &),
g[3][4] → (0 &), g[3][5] → (0 &), g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &),
g[4][1] → (0 &), g[4][2] → (0 &), g[4][3] → (0 &), g[4][4] → (g[4][4][#1, #2] &),
g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &), g[5][0] → (0 &), g[5][1] → (0 &),
g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &), g[5][5] → (g[5][5][#1, #2] &),
g[5][6] → (0 &), g[5][7] → (0 &), g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &),
g[6][3] → (0 &), g[6][4] → (0 &), g[6][5] → (0 &), g[6][6] → (g[6][6][#1, #2] &),
g[6][7] → (0 &), g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &),
g[7][4] → (0 &), g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (g[7][7][#1, #2] &)}

In[229]:= 
g[7][7][x0, x4] /. ssgm4488
% /. sx0x4

Out[229]=
- e-2 a4[H x4] Sin[6 H x0]1/3

Out[230]=
- e-2 a4[t] Sin[z]1/3

In[231]:= 
MatrixMetric44

Out[231]=
{{Cot[6 H x0]2, 0, 0, 0, 0, 0, 0, 0}, {0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0, 0}, 
{0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0}, {0, 0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0}, 
{0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0, 0}, 
{0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0}, {0, 0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3}}
```

In[232]:=

ssgm4488[x0, x4] /. sx0x4

Out[232]=

$$\begin{aligned} & \{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (e^{2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ & g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ & g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ & g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ & g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-e^{-2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2A4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2A4[H\#2]} \sin[6H\#1]^{1/3} \&) \} \left[\frac{z}{6H}, \frac{t}{H} \right] \end{aligned}$$

In[233]:=

$$\begin{aligned} (*ssgGzt = \{ G[0][0] \rightarrow (\text{Cot}[\#1]^2 \&), G[0][1] \rightarrow ((0) \&), G[0][2] \rightarrow ((0) \&), \\ G[0][3] \rightarrow ((0) \&), G[0][4] \rightarrow ((0) \&), G[0][5] \rightarrow ((0) \&), G[0][6] \rightarrow ((0) \&), \\ G[0][7] \rightarrow ((0) \&), G[1][0] \rightarrow ((0) \&), G[1][1] \rightarrow ((e^{2A4[\#2]} \sin[\#1]^{1/3}) \&), \\ G[1][2] \rightarrow ((0) \&), G[1][3] \rightarrow ((0) \&), G[1][4] \rightarrow ((0) \&), G[1][5] \rightarrow ((0) \&), \\ G[1][6] \rightarrow ((0) \&), G[1][7] \rightarrow ((0) \&), G[2][0] \rightarrow ((0) \&), G[2][1] \rightarrow ((0) \&), \\ G[2][2] \rightarrow ((e^{2A4[\#2]} \sin[\#1]^{1/3}) \&), G[2][3] \rightarrow ((0) \&), G[2][4] \rightarrow ((0) \&), \\ G[2][5] \rightarrow ((0) \&), G[2][6] \rightarrow ((0) \&), G[2][7] \rightarrow ((0) \&), G[3][0] \rightarrow ((0) \&), \\ G[3][1] \rightarrow ((0) \&), G[3][2] \rightarrow ((0) \&), G[3][3] \rightarrow ((e^{2A4[\#2]} \sin[\#1]^{1/3}) \&), \\ G[3][4] \rightarrow ((0) \&), G[3][5] \rightarrow ((0) \&), G[3][6] \rightarrow ((0) \&), G[3][7] \rightarrow ((0) \&), \\ G[4][0] \rightarrow ((0) \&), G[4][1] \rightarrow ((0) \&), G[4][2] \rightarrow ((0) \&), G[4][3] \rightarrow ((0) \&), \\ G[4][4] \rightarrow ((-1) \&), G[4][5] \rightarrow ((0) \&), G[4][6] \rightarrow ((0) \&), G[4][7] \rightarrow ((0) \&), \\ G[5][0] \rightarrow ((0) \&), G[5][1] \rightarrow ((0) \&), G[5][2] \rightarrow ((0) \&), G[5][3] \rightarrow ((0) \&), \\ G[5][4] \rightarrow ((0) \&), G[5][5] \rightarrow ((-e^{-2A4[\#2]} \sin[\#1]^{1/3}) \&), \\ G[5][6] \rightarrow ((0) \&), G[5][7] \rightarrow ((0) \&), G[6][0] \rightarrow ((0) \&), G[6][1] \rightarrow ((0) \&), \\ G[6][2] \rightarrow ((0) \&), G[6][3] \rightarrow ((0) \&), G[6][4] \rightarrow ((0) \&), G[6][5] \rightarrow ((0) \&), \\ G[6][6] \rightarrow ((-e^{-2A4[\#2]} \sin[\#1]^{1/3}) \&), G[6][7] \rightarrow ((0) \&), G[7][0] \rightarrow ((0) \&), \\ G[7][1] \rightarrow ((0) \&), G[7][2] \rightarrow ((0) \&), G[7][3] \rightarrow ((0) \&), G[7][4] \rightarrow ((0) \&), \\ G[7][5] \rightarrow ((0) \&), G[7][6] \rightarrow ((0) \&), G[7][7] \rightarrow ((-e^{-2A4[\#2]} \sin[\#1]^{1/3}) \&) \} *) \end{aligned}$$

In[234]:=

(*G[#][#][z,t] /. ssgGzt & /@ Range[0, 7]*)

```
In[235]:= detgg = Det[g4488 /. sg] // FullSimplify[#, constraintVars] &
Out[235]= 
$$g[0][0][x_0, x_4] \times g[1][1][x_0, x_4] \times g[2][2][x_0, x_4] \times g[3][3][x_0, x_4] \times g[4][4][x_0, x_4] \times g[5][5][x_0, x_4] \times g[6][6][x_0, x_4] \times g[7][7][x_0, x_4]$$

In[236]:= g4488 /. sg // MatrixForm
Out[236]//MatrixForm=

$$\begin{pmatrix} g[0][0][x_0, x_4] & 0 & 0 & 0 & 0 & 0 \\ 0 & g[1][1][x_0, x_4] & 0 & 0 & 0 & 0 \\ 0 & 0 & g[2][2][x_0, x_4] & 0 & 0 & 0 \\ 0 & 0 & 0 & g[3][3][x_0, x_4] & 0 & 0 \\ 0 & 0 & 0 & 0 & g[4][4][x_0, x_4] & 0 \\ 0 & 0 & 0 & 0 & 0 & g[5][5][x_0, x_4] \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[237]:= MatrixMetric44.η4488
(*eAa=*) FullSimplify[ Sqrt[%], constraintVars ]
Out[237]= 
$$\left\{ \left\{ \text{Cot}[6Hx_0]^2, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, e^{2a4[Hx_4]} \sin[6Hx_0]^{1/3}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, e^{2a4[Hx_4]} \sin[6Hx_0]^{1/3}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, e^{2a4[Hx_4]} \sin[6Hx_0]^{1/3}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, e^{-2a4[Hx_4]} \sin[6Hx_0]^{1/3}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, e^{-2a4[Hx_4]} \sin[6Hx_0]^{1/3}, 0, 0, 0 \right\} \right\}$$

Out[238]= 
$$\left\{ \left\{ \text{Cot}[6Hx_0], 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \sqrt{e^{2a4[Hx_4]}} \sin[6Hx_0]^{1/6}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, \sqrt{e^{2a4[Hx_4]}} \sin[6Hx_0]^{1/6}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, \sqrt{e^{2a4[Hx_4]}} \sin[6Hx_0]^{1/6}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \sqrt{e^{-2a4[Hx_4]}} \sin[6Hx_0]^{1/6}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, \sqrt{e^{-2a4[Hx_4]}} \sin[6Hx_0]^{1/6}, 0, 0, 0 \right\} \right\}$$

eAa
```

In[239]:=

```
g4488.η4488 /. sg
( eAa = FullSimplify[ √%, constraintVars] ) // MatrixForm
```

Out[239]=

```
{ {g[0][0][x0, x4], 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, -g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, -g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, -g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, -g[7][7][x0, x4]} }
```

Out[240]/MatrixForm=

$$\begin{pmatrix} \sqrt{g[0][0][x0, x4]} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{g[1][1][x0, x4]} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{g[2][2][x0, x4]} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{g[3][3][x0, x4]} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{-g[4][4][x0, x4]} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[241]:=

eAa

ssgm4488

(eAa04 = %% /. % // FullSimplify[#, constraintVars] &) // MatrixForm

Out[241]=

$$\left\{ \left\{ \sqrt{g[0][0][x0, x4]}, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \sqrt{g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, \sqrt{g[2][2][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \sqrt{g[3][3][x0, x4]}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, \sqrt{-g[4][4][x0, x4]}, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \sqrt{-g[5][5][x0, x4]}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, \sqrt{-g[6][6][x0, x4]}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \sqrt{-g[7][7][x0, x4]} \right\} \right\}$$

Out[242]=

$$\begin{aligned} g[0][0] &\rightarrow (\text{Cot}[6 H \#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] &\rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] &\rightarrow (\text{e}^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ g[1][4] &\rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ g[2][1] &\rightarrow (0 \&), g[2][2] \rightarrow (\text{e}^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ g[2][5] &\rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ g[3][2] &\rightarrow (0 \&), g[3][3] \rightarrow (\text{e}^{2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] &\rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] &\rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] &\rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] &\rightarrow (-\text{e}^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] &\rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] &\rightarrow (0 \&), g[6][6] \rightarrow (-\text{e}^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] &\rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] &\rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-\text{e}^{-2 a4[H \#2]} \sin[6 H \#1]^{1/3} \&) \end{aligned}$$

Out[243]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6 H x0] & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\text{e}^{2 a4[H x4]}} \sin[6 H x0]^{1/6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\text{e}^{2 a4[H x4]}} \sin[6 H x0]^{1/6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\text{e}^{2 a4[H x4]}} \sin[6 H x0]^{1/6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\text{e}^{-2 a4}} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

OCTAD $e_{\alpha \leftarrow \text{spacetime}}^{(A) \leftarrow \text{Lorentz}}$:

■ $e_{\alpha}^{(A)} = e_{\alpha}^{(A)} = eAa$

In[244]:=

$$(sge_{\alpha}^{(A)} = \text{Thread}[\text{Flatten}[e_{\alpha}^{(A)}] \rightarrow \text{Flatten}[eAa]])) (*//Column*)$$

Out[244]=

$$\begin{aligned} & \left\{ E_{\alpha}^{(A)}[0, 0] \rightarrow \sqrt{g[0][0][x0, x4]}, E_{\alpha}^{(A)}[0, 1] \rightarrow 0, E_{\alpha}^{(A)}[0, 2] \rightarrow 0, E_{\alpha}^{(A)}[0, 3] \rightarrow 0, E_{\alpha}^{(A)}[0, 4] \rightarrow 0, \right. \\ & E_{\alpha}^{(A)}[0, 5] \rightarrow 0, E_{\alpha}^{(A)}[0, 6] \rightarrow 0, E_{\alpha}^{(A)}[0, 7] \rightarrow 0, E_{\alpha}^{(A)}[1, 0] \rightarrow 0, E_{\alpha}^{(A)}[1, 1] \rightarrow \sqrt{g[1][1][x0, x4]}, \\ & E_{\alpha}^{(A)}[1, 2] \rightarrow 0, E_{\alpha}^{(A)}[1, 3] \rightarrow 0, E_{\alpha}^{(A)}[1, 4] \rightarrow 0, E_{\alpha}^{(A)}[1, 5] \rightarrow 0, E_{\alpha}^{(A)}[1, 6] \rightarrow 0, E_{\alpha}^{(A)}[1, 7] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 0] \rightarrow 0, E_{\alpha}^{(A)}[2, 1] \rightarrow 0, E_{\alpha}^{(A)}[2, 2] \rightarrow \sqrt{g[2][2][x0, x4]}, E_{\alpha}^{(A)}[2, 3] \rightarrow 0, E_{\alpha}^{(A)}[2, 4] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 5] \rightarrow 0, E_{\alpha}^{(A)}[2, 6] \rightarrow 0, E_{\alpha}^{(A)}[2, 7] \rightarrow 0, E_{\alpha}^{(A)}[3, 0] \rightarrow 0, E_{\alpha}^{(A)}[3, 1] \rightarrow 0, E_{\alpha}^{(A)}[3, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[3, 3] \rightarrow \sqrt{g[3][3][x0, x4]}, E_{\alpha}^{(A)}[3, 4] \rightarrow 0, E_{\alpha}^{(A)}[3, 5] \rightarrow 0, E_{\alpha}^{(A)}[3, 6] \rightarrow 0, E_{\alpha}^{(A)}[3, 7] \rightarrow 0, \\ & E_{\alpha}^{(A)}[4, 0] \rightarrow 0, E_{\alpha}^{(A)}[4, 1] \rightarrow 0, E_{\alpha}^{(A)}[4, 2] \rightarrow 0, E_{\alpha}^{(A)}[4, 3] \rightarrow 0, E_{\alpha}^{(A)}[4, 4] \rightarrow \sqrt{-g[4][4][x0, x4]}, \\ & E_{\alpha}^{(A)}[4, 5] \rightarrow 0, E_{\alpha}^{(A)}[4, 6] \rightarrow 0, E_{\alpha}^{(A)}[4, 7] \rightarrow 0, E_{\alpha}^{(A)}[5, 0] \rightarrow 0, E_{\alpha}^{(A)}[5, 1] \rightarrow 0, E_{\alpha}^{(A)}[5, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[5, 3] \rightarrow 0, E_{\alpha}^{(A)}[5, 4] \rightarrow 0, E_{\alpha}^{(A)}[5, 5] \rightarrow \sqrt{-g[5][5][x0, x4]}, E_{\alpha}^{(A)}[5, 6] \rightarrow 0, E_{\alpha}^{(A)}[5, 7] \rightarrow 0, \\ & E_{\alpha}^{(A)}[6, 0] \rightarrow 0, E_{\alpha}^{(A)}[6, 1] \rightarrow 0, E_{\alpha}^{(A)}[6, 2] \rightarrow 0, E_{\alpha}^{(A)}[6, 3] \rightarrow 0, E_{\alpha}^{(A)}[6, 4] \rightarrow 0, E_{\alpha}^{(A)}[6, 5] \rightarrow 0, \\ & E_{\alpha}^{(A)}[6, 6] \rightarrow \sqrt{-g[6][6][x0, x4]}, E_{\alpha}^{(A)}[6, 7] \rightarrow 0, E_{\alpha}^{(A)}[7, 0] \rightarrow 0, E_{\alpha}^{(A)}[7, 1] \rightarrow 0, E_{\alpha}^{(A)}[7, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[7, 3] \rightarrow 0, E_{\alpha}^{(A)}[7, 4] \rightarrow 0, E_{\alpha}^{(A)}[7, 5] \rightarrow 0, E_{\alpha}^{(A)}[7, 6] \rightarrow 0, E_{\alpha}^{(A)}[7, 7] \rightarrow \sqrt{-g[7][7][x0, x4]} \} \end{aligned}$$

In[245]:=

$$(*\text{Symbolize}[sge_{\alpha}^{(A)}]*)$$

In[246]:=

$$(*\text{Symbolize}[sgtrye_{\alpha}^{(A)}]*)$$

In[247]:=

$$(sgtrye_{\alpha}^{(A)} = \text{Thread}[\text{Flatten}[e_{\alpha}^{(A)}] \rightarrow \text{Flatten}[eAa04]]) (*//Column*)$$

Out[247]=

$$\begin{aligned} & \left\{ E_{\alpha}^{(A)}[0, 0] \rightarrow \text{Cot}[6Hx0], E_{\alpha}^{(A)}[0, 1] \rightarrow 0, E_{\alpha}^{(A)}[0, 2] \rightarrow 0, E_{\alpha}^{(A)}[0, 3] \rightarrow 0, \right. \\ & E_{\alpha}^{(A)}[0, 4] \rightarrow 0, E_{\alpha}^{(A)}[0, 5] \rightarrow 0, E_{\alpha}^{(A)}[0, 6] \rightarrow 0, E_{\alpha}^{(A)}[0, 7] \rightarrow 0, E_{\alpha}^{(A)}[1, 0] \rightarrow 0, \\ & E_{\alpha}^{(A)}[1, 1] \rightarrow \sqrt{e^{2a4[Hx4]}}, E_{\alpha}^{(A)}[1, 2] \rightarrow 0, E_{\alpha}^{(A)}[1, 3] \rightarrow 0, E_{\alpha}^{(A)}[1, 4] \rightarrow 0, \\ & E_{\alpha}^{(A)}[1, 5] \rightarrow 0, E_{\alpha}^{(A)}[1, 6] \rightarrow 0, E_{\alpha}^{(A)}[1, 7] \rightarrow 0, E_{\alpha}^{(A)}[2, 0] \rightarrow 0, E_{\alpha}^{(A)}[2, 1] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 2] \rightarrow \sqrt{e^{2a4[Hx4]}}, E_{\alpha}^{(A)}[2, 3] \rightarrow 0, E_{\alpha}^{(A)}[2, 4] \rightarrow 0, E_{\alpha}^{(A)}[2, 5] \rightarrow 0, \\ & E_{\alpha}^{(A)}[2, 6] \rightarrow 0, E_{\alpha}^{(A)}[2, 7] \rightarrow 0, E_{\alpha}^{(A)}[3, 0] \rightarrow 0, E_{\alpha}^{(A)}[3, 1] \rightarrow 0, E_{\alpha}^{(A)}[3, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[3, 3] \rightarrow \sqrt{e^{2a4[Hx4]}}, E_{\alpha}^{(A)}[3, 4] \rightarrow 0, E_{\alpha}^{(A)}[3, 5] \rightarrow 0, E_{\alpha}^{(A)}[3, 6] \rightarrow 0, \\ & E_{\alpha}^{(A)}[3, 7] \rightarrow 0, E_{\alpha}^{(A)}[4, 0] \rightarrow 0, E_{\alpha}^{(A)}[4, 1] \rightarrow 0, E_{\alpha}^{(A)}[4, 2] \rightarrow 0, E_{\alpha}^{(A)}[4, 3] \rightarrow 0, \\ & E_{\alpha}^{(A)}[4, 4] \rightarrow 1, E_{\alpha}^{(A)}[4, 5] \rightarrow 0, E_{\alpha}^{(A)}[4, 6] \rightarrow 0, E_{\alpha}^{(A)}[4, 7] \rightarrow 0, E_{\alpha}^{(A)}[5, 0] \rightarrow 0, E_{\alpha}^{(A)}[5, 1] \rightarrow 0, \\ & E_{\alpha}^{(A)}[5, 2] \rightarrow 0, E_{\alpha}^{(A)}[5, 3] \rightarrow 0, E_{\alpha}^{(A)}[5, 4] \rightarrow 0, E_{\alpha}^{(A)}[5, 5] \rightarrow \sqrt{e^{-2a4[Hx4]}}, E_{\alpha}^{(A)}[6Hx0]^{1/6}, \\ & E_{\alpha}^{(A)}[5, 6] \rightarrow 0, E_{\alpha}^{(A)}[5, 7] \rightarrow 0, E_{\alpha}^{(A)}[6, 0] \rightarrow 0, E_{\alpha}^{(A)}[6, 1] \rightarrow 0, E_{\alpha}^{(A)}[6, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)}[6, 3] \rightarrow 0, E_{\alpha}^{(A)}[6, 4] \rightarrow 0, E_{\alpha}^{(A)}[6, 5] \rightarrow 0, E_{\alpha}^{(A)}[6, 6] \rightarrow \sqrt{e^{-2a4[Hx4]}}, E_{\alpha}^{(A)}[6Hx0]^{1/6}, \\ & E_{\alpha}^{(A)}[6, 7] \rightarrow 0, E_{\alpha}^{(A)}[7, 0] \rightarrow 0, E_{\alpha}^{(A)}[7, 1] \rightarrow 0, E_{\alpha}^{(A)}[7, 2] \rightarrow 0, E_{\alpha}^{(A)}[7, 3] \rightarrow 0, \\ & E_{\alpha}^{(A)}[7, 4] \rightarrow 0, E_{\alpha}^{(A)}[7, 5] \rightarrow 0, E_{\alpha}^{(A)}[7, 6] \rightarrow 0, E_{\alpha}^{(A)}[7, 7] \rightarrow \sqrt{e^{-2a4[Hx4]}}, E_{\alpha}^{(A)}[6Hx0]^{1/6} \} \end{aligned}$$

In[248]:=

$$\begin{aligned} \text{sgtrye}_{(A)}^{\alpha} = \text{Thread}[\text{Flatten}[e_{(A)}^{\alpha}] \rightarrow \text{Flatten}[\text{Inverse}[e_{\alpha}^{(A)} / . \text{sgtrye}_{\alpha}^{(A)}]]] \\ (*//Column*) \end{aligned}$$

Out[248]=

$$\begin{aligned} & \left\{ E_{(A)}^{\alpha}[0, 0] \rightarrow \tan[6 H x 0], E_{(A)}^{\alpha}[0, 1] \rightarrow 0, E_{(A)}^{\alpha}[0, 2] \rightarrow 0, E_{(A)}^{\alpha}[0, 3] \rightarrow 0, \right. \\ & E_{(A)}^{\alpha}[0, 4] \rightarrow 0, E_{(A)}^{\alpha}[0, 5] \rightarrow 0, E_{(A)}^{\alpha}[0, 6] \rightarrow 0, E_{(A)}^{\alpha}[0, 7] \rightarrow 0, E_{(A)}^{\alpha}[1, 0] \rightarrow 0, \\ & E_{(A)}^{\alpha}[1, 1] \rightarrow \frac{1}{\sqrt{e^{2 a 4 [H x 4]}} \sin[6 H x 0]^{1/6}}, E_{(A)}^{\alpha}[1, 2] \rightarrow 0, E_{(A)}^{\alpha}[1, 3] \rightarrow 0, E_{(A)}^{\alpha}[1, 4] \rightarrow 0, \\ & E_{(A)}^{\alpha}[1, 5] \rightarrow 0, E_{(A)}^{\alpha}[1, 6] \rightarrow 0, E_{(A)}^{\alpha}[1, 7] \rightarrow 0, E_{(A)}^{\alpha}[2, 0] \rightarrow 0, E_{(A)}^{\alpha}[2, 1] \rightarrow 0, \\ & E_{(A)}^{\alpha}[2, 2] \rightarrow \frac{1}{\sqrt{e^{2 a 4 [H x 4]}} \sin[6 H x 0]^{1/6}}, E_{(A)}^{\alpha}[2, 3] \rightarrow 0, E_{(A)}^{\alpha}[2, 4] \rightarrow 0, E_{(A)}^{\alpha}[2, 5] \rightarrow 0, \\ & E_{(A)}^{\alpha}[2, 6] \rightarrow 0, E_{(A)}^{\alpha}[2, 7] \rightarrow 0, E_{(A)}^{\alpha}[3, 0] \rightarrow 0, E_{(A)}^{\alpha}[3, 1] \rightarrow 0, E_{(A)}^{\alpha}[3, 2] \rightarrow 0, \\ & E_{(A)}^{\alpha}[3, 3] \rightarrow \frac{1}{\sqrt{e^{2 a 4 [H x 4]}} \sin[6 H x 0]^{1/6}}, E_{(A)}^{\alpha}[3, 4] \rightarrow 0, E_{(A)}^{\alpha}[3, 5] \rightarrow 0, E_{(A)}^{\alpha}[3, 6] \rightarrow 0, \\ & E_{(A)}^{\alpha}[3, 7] \rightarrow 0, E_{(A)}^{\alpha}[4, 0] \rightarrow 0, E_{(A)}^{\alpha}[4, 1] \rightarrow 0, E_{(A)}^{\alpha}[4, 2] \rightarrow 0, E_{(A)}^{\alpha}[4, 3] \rightarrow 0, \\ & E_{(A)}^{\alpha}[4, 4] \rightarrow 1, E_{(A)}^{\alpha}[4, 5] \rightarrow 0, E_{(A)}^{\alpha}[4, 6] \rightarrow 0, E_{(A)}^{\alpha}[4, 7] \rightarrow 0, E_{(A)}^{\alpha}[5, 0] \rightarrow 0, E_{(A)}^{\alpha}[5, 1] \rightarrow 0, \\ & E_{(A)}^{\alpha}[5, 2] \rightarrow 0, E_{(A)}^{\alpha}[5, 3] \rightarrow 0, E_{(A)}^{\alpha}[5, 4] \rightarrow 0, E_{(A)}^{\alpha}[5, 5] \rightarrow \frac{1}{\sqrt{e^{-2 a 4 [H x 4]}} \sin[6 H x 0]^{1/6}}, \\ & E_{(A)}^{\alpha}[5, 6] \rightarrow 0, E_{(A)}^{\alpha}[5, 7] \rightarrow 0, E_{(A)}^{\alpha}[6, 0] \rightarrow 0, E_{(A)}^{\alpha}[6, 1] \rightarrow 0, E_{(A)}^{\alpha}[6, 2] \rightarrow 0, \\ & E_{(A)}^{\alpha}[6, 3] \rightarrow 0, E_{(A)}^{\alpha}[6, 4] \rightarrow 0, E_{(A)}^{\alpha}[6, 5] \rightarrow 0, E_{(A)}^{\alpha}[6, 6] \rightarrow \frac{1}{\sqrt{e^{-2 a 4 [H x 4]}} \sin[6 H x 0]^{1/6}}, \\ & E_{(A)}^{\alpha}[6, 7] \rightarrow 0, E_{(A)}^{\alpha}[7, 0] \rightarrow 0, E_{(A)}^{\alpha}[7, 1] \rightarrow 0, E_{(A)}^{\alpha}[7, 2] \rightarrow 0, E_{(A)}^{\alpha}[7, 3] \rightarrow 0, \\ & E_{(A)}^{\alpha}[7, 4] \rightarrow 0, E_{(A)}^{\alpha}[7, 5] \rightarrow 0, E_{(A)}^{\alpha}[7, 6] \rightarrow 0, E_{(A)}^{\alpha}[7, 7] \rightarrow \frac{1}{\sqrt{e^{-2 a 4 [H x 4]}} \sin[6 H x 0]^{1/6}} \} \end{aligned}$$

In[249]:=

$$\text{gtrye}_{\alpha}^{(A)} = (e_{\alpha}^{(A)} / . \text{sgtrye}_{\alpha}^{(A)}) / . \text{subsDefects}$$

Out[249]=

$$\begin{aligned} & \left\{ \{\cot[6 H x 0], 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \right. \\ & \{0, 0, e^{a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0, 0\}, \\ & \left. \{0, 0, 0, 0, 0, 0, e^{-a 4 [H x 4]} \sin[6 H x 0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, 0, e^{-a 4 [H x 4]} \sin[6 H x 0]^{1/6}\} \right\} \end{aligned}$$

In[250]:=

$$\text{gtrye}_{(A)}^{\alpha} = (e_{(A)}^{\alpha} / . \text{sgtrye}_{(A)}^{\alpha}) / . \text{subsDefects}$$

Out[250]=

$$\begin{aligned} & \left\{ \{\tan[6 H x 0], 0, 0, 0, 0, 0, 0, 0\}, \left\{0, \frac{e^{-a 4 [H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0\right\}, \right. \\ & \left\{0, 0, \frac{e^{-a 4 [H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{e^{-a 4 [H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0, 0, 0\right\}, \\ & \{0, 0, 0, 0, 1, 0, 0, 0\}, \left\{0, 0, 0, 0, 0, \frac{e^{a 4 [H x 4]}}{\sin[6 H x 0]^{1/6}}, 0, 0\right\}, \\ & \left. \left\{0, 0, 0, 0, 0, 0, \frac{e^{a 4 [H x 4]}}{\sin[6 H x 0]^{1/6}}, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{e^{a 4 [H x 4]}}{\sin[6 H x 0]^{1/6}}\right\} \right\} \end{aligned}$$

In[251]:=

$$\mathbf{e}_\alpha^{(A)}$$

$$\% /. \text{sgtrye}_\alpha^{(A)} // \text{MatrixForm}$$

Out[251]=

$$\left\{ \begin{array}{l} \{E_\alpha^{(A)}[0, 0], E_\alpha^{(A)}[0, 1], E_\alpha^{(A)}[0, 2], E_\alpha^{(A)}[0, 3], E_\alpha^{(A)}[0, 4], E_\alpha^{(A)}[0, 5], E_\alpha^{(A)}[0, 6], E_\alpha^{(A)}[0, 7]\}, \\ \{E_\alpha^{(A)}[1, 0], E_\alpha^{(A)}[1, 1], E_\alpha^{(A)}[1, 2], E_\alpha^{(A)}[1, 3], E_\alpha^{(A)}[1, 4], E_\alpha^{(A)}[1, 5], E_\alpha^{(A)}[1, 6], E_\alpha^{(A)}[1, 7]\}, \\ \{E_\alpha^{(A)}[2, 0], E_\alpha^{(A)}[2, 1], E_\alpha^{(A)}[2, 2], E_\alpha^{(A)}[2, 3], E_\alpha^{(A)}[2, 4], E_\alpha^{(A)}[2, 5], E_\alpha^{(A)}[2, 6], E_\alpha^{(A)}[2, 7]\}, \\ \{E_\alpha^{(A)}[3, 0], E_\alpha^{(A)}[3, 1], E_\alpha^{(A)}[3, 2], E_\alpha^{(A)}[3, 3], E_\alpha^{(A)}[3, 4], E_\alpha^{(A)}[3, 5], E_\alpha^{(A)}[3, 6], E_\alpha^{(A)}[3, 7]\}, \\ \{E_\alpha^{(A)}[4, 0], E_\alpha^{(A)}[4, 1], E_\alpha^{(A)}[4, 2], E_\alpha^{(A)}[4, 3], E_\alpha^{(A)}[4, 4], E_\alpha^{(A)}[4, 5], E_\alpha^{(A)}[4, 6], E_\alpha^{(A)}[4, 7]\}, \\ \{E_\alpha^{(A)}[5, 0], E_\alpha^{(A)}[5, 1], E_\alpha^{(A)}[5, 2], E_\alpha^{(A)}[5, 3], E_\alpha^{(A)}[5, 4], E_\alpha^{(A)}[5, 5], E_\alpha^{(A)}[5, 6], E_\alpha^{(A)}[5, 7]\}, \\ \{E_\alpha^{(A)}[6, 0], E_\alpha^{(A)}[6, 1], E_\alpha^{(A)}[6, 2], E_\alpha^{(A)}[6, 3], E_\alpha^{(A)}[6, 4], E_\alpha^{(A)}[6, 5], E_\alpha^{(A)}[6, 6], E_\alpha^{(A)}[6, 7]\}, \\ \{E_\alpha^{(A)}[7, 0], E_\alpha^{(A)}[7, 1], E_\alpha^{(A)}[7, 2], E_\alpha^{(A)}[7, 3], E_\alpha^{(A)}[7, 4], E_\alpha^{(A)}[7, 5], E_\alpha^{(A)}[7, 6], E_\alpha^{(A)}[7, 7]\} \end{array} \right.$$

Out[252]//MatrixForm=

$$\begin{array}{cccccc} \text{Cot}[6Hx0] & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

In[253]:=

```
eηe = FullSimplify[Transpose[e_\alpha^{(A)}].η4488.e_\alpha^{(A)}, constraintVars];
% /. sge_\alpha^{(A)}
%% /. sgtrye_\alpha^{(A)}
% == gtry // FullSimplify[#, constraintVars] &
```

Out[254]=

$$\{\{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \{0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0\}, \{0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0\}, \{0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0\}, \{0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]\}\}$$

Out[255]=

$$\{\{\text{Cot}[6Hx0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}\}\}$$

Out[256]=

True

In[257]:=

gtry

Out[257]=

$$\{\{\text{Cot}[6Hx0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}\}\}$$

In[258]:=

$$(*\text{FullSimplify}[\text{Transpose}[e_{\alpha}^{(A)}] . \eta 4488 . e_{\alpha}^{(A)}, \text{constraintVars}] - \text{MatrixMetric44}*)$$

In[259]:=

ssgm4488

Out[259]=

$$\begin{aligned} & \{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (\text{e}^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ & g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ & g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (\text{e}^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ & g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ & g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (\text{e}^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-\text{e}^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-\text{e}^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-\text{e}^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&)\} \end{aligned}$$

In[260]:=

MatrixMetric44 // MatrixForm

Out[260]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & \text{e}^{2a4[Hx4]} \text{Sin}[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & \text{e}^{2a4[Hx4]} \text{Sin}[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & \text{e}^{2a4[Hx4]} \text{Sin}[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[261]:=

 $e_{\alpha}^{(A)}$

Out[261]=

$$\begin{aligned} & \{ \{E_{\alpha}^{(A)}[0, 0], E_{\alpha}^{(A)}[0, 1], E_{\alpha}^{(A)}[0, 2], E_{\alpha}^{(A)}[0, 3], E_{\alpha}^{(A)}[0, 4], E_{\alpha}^{(A)}[0, 5], E_{\alpha}^{(A)}[0, 6], E_{\alpha}^{(A)}[0, 7]\}, \\ & \{E_{\alpha}^{(A)}[1, 0], E_{\alpha}^{(A)}[1, 1], E_{\alpha}^{(A)}[1, 2], E_{\alpha}^{(A)}[1, 3], E_{\alpha}^{(A)}[1, 4], E_{\alpha}^{(A)}[1, 5], E_{\alpha}^{(A)}[1, 6], E_{\alpha}^{(A)}[1, 7]\}, \\ & \{E_{\alpha}^{(A)}[2, 0], E_{\alpha}^{(A)}[2, 1], E_{\alpha}^{(A)}[2, 2], E_{\alpha}^{(A)}[2, 3], E_{\alpha}^{(A)}[2, 4], E_{\alpha}^{(A)}[2, 5], E_{\alpha}^{(A)}[2, 6], E_{\alpha}^{(A)}[2, 7]\}, \\ & \{E_{\alpha}^{(A)}[3, 0], E_{\alpha}^{(A)}[3, 1], E_{\alpha}^{(A)}[3, 2], E_{\alpha}^{(A)}[3, 3], E_{\alpha}^{(A)}[3, 4], E_{\alpha}^{(A)}[3, 5], E_{\alpha}^{(A)}[3, 6], E_{\alpha}^{(A)}[3, 7]\}, \\ & \{E_{\alpha}^{(A)}[4, 0], E_{\alpha}^{(A)}[4, 1], E_{\alpha}^{(A)}[4, 2], E_{\alpha}^{(A)}[4, 3], E_{\alpha}^{(A)}[4, 4], E_{\alpha}^{(A)}[4, 5], E_{\alpha}^{(A)}[4, 6], E_{\alpha}^{(A)}[4, 7]\}, \\ & \{E_{\alpha}^{(A)}[5, 0], E_{\alpha}^{(A)}[5, 1], E_{\alpha}^{(A)}[5, 2], E_{\alpha}^{(A)}[5, 3], E_{\alpha}^{(A)}[5, 4], E_{\alpha}^{(A)}[5, 5], E_{\alpha}^{(A)}[5, 6], E_{\alpha}^{(A)}[5, 7]\}, \\ & \{E_{\alpha}^{(A)}[6, 0], E_{\alpha}^{(A)}[6, 1], E_{\alpha}^{(A)}[6, 2], E_{\alpha}^{(A)}[6, 3], E_{\alpha}^{(A)}[6, 4], E_{\alpha}^{(A)}[6, 5], E_{\alpha}^{(A)}[6, 6], E_{\alpha}^{(A)}[6, 7]\}, \\ & \{E_{\alpha}^{(A)}[7, 0], E_{\alpha}^{(A)}[7, 1], E_{\alpha}^{(A)}[7, 2], E_{\alpha}^{(A)}[7, 3], E_{\alpha}^{(A)}[7, 4], E_{\alpha}^{(A)}[7, 5], E_{\alpha}^{(A)}[7, 6], E_{\alpha}^{(A)}[7, 7]\} \} \end{aligned}$$

In[262]:=

```
Block[{s, r}, s = FullSimplify[eα(A) /. sgtryeα(A), constraintVars];
r = FullSimplify[Transpose[s].η4488.s, constraintVars] - MatrixMetric44;
FullSimplify[r, constraintVars]]
```

Out[262]=

```
{ {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0} }
```

In[263]:=

```
(* (eα(A) = eAa) // MatrixForm *)
```

In[264]:=

```
(* eα(A)
Inverse[%]*)
```

In[265]:=

```
(* (e(A)α = Inverse[eα(A) /.
sge/Subscript/α/and/LeftParenthesis/A/RightParenthesis]) //
MatrixForm *)
```

In[266]:=

$e_{(A)}^{\alpha}$

Out[266]=

```
{ {E(A)α [0, 0], E(A)α [0, 1], E(A)α [0, 2], E(A)α [0, 3], E(A)α [0, 4], E(A)α [0, 5], E(A)α [0, 6], E(A)α [0, 7]}, {E(A)α [1, 0], E(A)α [1, 1], E(A)α [1, 2], E(A)α [1, 3], E(A)α [1, 4], E(A)α [1, 5], E(A)α [1, 6], E(A)α [1, 7]}, {E(A)α [2, 0], E(A)α [2, 1], E(A)α [2, 2], E(A)α [2, 3], E(A)α [2, 4], E(A)α [2, 5], E(A)α [2, 6], E(A)α [2, 7]}, {E(A)α [3, 0], E(A)α [3, 1], E(A)α [3, 2], E(A)α [3, 3], E(A)α [3, 4], E(A)α [3, 5], E(A)α [3, 6], E(A)α [3, 7]}, {E(A)α [4, 0], E(A)α [4, 1], E(A)α [4, 2], E(A)α [4, 3], E(A)α [4, 4], E(A)α [4, 5], E(A)α [4, 6], E(A)α [4, 7]}, {E(A)α [5, 0], E(A)α [5, 1], E(A)α [5, 2], E(A)α [5, 3], E(A)α [5, 4], E(A)α [5, 5], E(A)α [5, 6], E(A)α [5, 7]}, {E(A)α [6, 0], E(A)α [6, 1], E(A)α [6, 2], E(A)α [6, 3], E(A)α [6, 4], E(A)α [6, 5], E(A)α [6, 6], E(A)α [6, 7]}, {E(A)α [7, 0], E(A)α [7, 1], E(A)α [7, 2], E(A)α [7, 3], E(A)α [7, 4], E(A)α [7, 5], E(A)α [7, 6], E(A)α [7, 7]} }
```

In[267]:=

$$(\text{sge}_{(A)}^{\alpha} = \text{Thread}[\text{Flatten}[e_{(A)}^{\alpha}] \rightarrow \text{Flatten}[\text{Inverse}[e_{\alpha}^{(A)} / . \text{sge}_{\alpha}^{(A)}]]]) (*//Column*)$$

Out[267]=

$$\begin{aligned} & \left\{ E_{(A)}^{\alpha} [0, 0] \rightarrow \frac{1}{\sqrt{g[0][0][x0, x4]}}, E_{(A)}^{\alpha} [0, 1] \rightarrow 0, E_{(A)}^{\alpha} [0, 2] \rightarrow 0, E_{(A)}^{\alpha} [0, 3] \rightarrow 0, E_{(A)}^{\alpha} [0, 4] \rightarrow 0, \right. \\ & E_{(A)}^{\alpha} [0, 5] \rightarrow 0, E_{(A)}^{\alpha} [0, 6] \rightarrow 0, E_{(A)}^{\alpha} [0, 7] \rightarrow 0, E_{(A)}^{\alpha} [1, 0] \rightarrow 0, E_{(A)}^{\alpha} [1, 1] \rightarrow \frac{1}{\sqrt{g[1][1][x0, x4]}}, \\ & E_{(A)}^{\alpha} [1, 2] \rightarrow 0, E_{(A)}^{\alpha} [1, 3] \rightarrow 0, E_{(A)}^{\alpha} [1, 4] \rightarrow 0, E_{(A)}^{\alpha} [1, 5] \rightarrow 0, E_{(A)}^{\alpha} [1, 6] \rightarrow 0, E_{(A)}^{\alpha} [1, 7] \rightarrow 0, \\ & E_{(A)}^{\alpha} [2, 0] \rightarrow 0, E_{(A)}^{\alpha} [2, 1] \rightarrow 0, E_{(A)}^{\alpha} [2, 2] \rightarrow -\frac{1}{\sqrt{g[2][2][x0, x4]}}, E_{(A)}^{\alpha} [2, 3] \rightarrow 0, E_{(A)}^{\alpha} [2, 4] \rightarrow 0, \\ & E_{(A)}^{\alpha} [2, 5] \rightarrow 0, E_{(A)}^{\alpha} [2, 6] \rightarrow 0, E_{(A)}^{\alpha} [2, 7] \rightarrow 0, E_{(A)}^{\alpha} [3, 0] \rightarrow 0, E_{(A)}^{\alpha} [3, 1] \rightarrow 0, E_{(A)}^{\alpha} [3, 2] \rightarrow 0, \\ & E_{(A)}^{\alpha} [3, 3] \rightarrow \frac{1}{\sqrt{g[3][3][x0, x4]}}, E_{(A)}^{\alpha} [3, 4] \rightarrow 0, E_{(A)}^{\alpha} [3, 5] \rightarrow 0, E_{(A)}^{\alpha} [3, 6] \rightarrow 0, E_{(A)}^{\alpha} [3, 7] \rightarrow 0, \\ & E_{(A)}^{\alpha} [4, 0] \rightarrow 0, E_{(A)}^{\alpha} [4, 1] \rightarrow 0, E_{(A)}^{\alpha} [4, 2] \rightarrow 0, E_{(A)}^{\alpha} [4, 3] \rightarrow 0, E_{(A)}^{\alpha} [4, 4] \rightarrow \frac{1}{\sqrt{-g[4][4][x0, x4]}}, \\ & E_{(A)}^{\alpha} [4, 5] \rightarrow 0, E_{(A)}^{\alpha} [4, 6] \rightarrow 0, E_{(A)}^{\alpha} [4, 7] \rightarrow 0, E_{(A)}^{\alpha} [5, 0] \rightarrow 0, E_{(A)}^{\alpha} [5, 1] \rightarrow 0, E_{(A)}^{\alpha} [5, 2] \rightarrow 0, \\ & E_{(A)}^{\alpha} [5, 3] \rightarrow 0, E_{(A)}^{\alpha} [5, 4] \rightarrow 0, E_{(A)}^{\alpha} [5, 5] \rightarrow -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, E_{(A)}^{\alpha} [5, 6] \rightarrow 0, E_{(A)}^{\alpha} [5, 7] \rightarrow 0, \\ & E_{(A)}^{\alpha} [6, 0] \rightarrow 0, E_{(A)}^{\alpha} [6, 1] \rightarrow 0, E_{(A)}^{\alpha} [6, 2] \rightarrow 0, E_{(A)}^{\alpha} [6, 3] \rightarrow 0, E_{(A)}^{\alpha} [6, 4] \rightarrow 0, E_{(A)}^{\alpha} [6, 5] \rightarrow 0, \\ & E_{(A)}^{\alpha} [6, 6] \rightarrow \frac{1}{\sqrt{-g[6][6][x0, x4]}}, E_{(A)}^{\alpha} [6, 7] \rightarrow 0, E_{(A)}^{\alpha} [7, 0] \rightarrow 0, E_{(A)}^{\alpha} [7, 1] \rightarrow 0, E_{(A)}^{\alpha} [7, 2] \rightarrow 0, \\ & \left. E_{(A)}^{\alpha} [7, 3] \rightarrow 0, E_{(A)}^{\alpha} [7, 4] \rightarrow 0, E_{(A)}^{\alpha} [7, 5] \rightarrow 0, E_{(A)}^{\alpha} [7, 6] \rightarrow 0, E_{(A)}^{\alpha} [7, 7] \rightarrow \frac{1}{\sqrt{-g[7][7][x0, x4]}} \right\} \end{aligned}$$

In[268]:=

$$(*\text{Symbolize}[\text{sge}_{(A)}^{\alpha}]*)$$

In[269]:=

$$(*\text{Symbolize}[\text{sgtrye}_{(A)}^{\alpha}]*)$$

In[270]:=

$$(*%/. \text{sge}_{\alpha}^{(A)} \newline %%/. \text{sgtrye}_{\alpha}^{(A)} *)$$

In[271]:=

$$(*\text{FullSimplify}[\text{Transpose}[e_{\alpha}^{(A)}] . \eta 4488.e_{\alpha}^{(A)}, \text{constraintVars}] - \text{MatrixMetric44}*)$$

For Spin (4, 4); τ tau; T16; OCTAD : Nash; Introduce the wave function, Ψ_{16} , for this Universe::

O(4,4); evals, evecs of σ

In[272]:=

```
{evals, evecs} = Eigensystem[\sigma]
```

Out[272]=

```
{ {-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0}, {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1}, {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}} }
```

In[273]:=

$$\left(u = \text{ExpandAll} \left[\frac{1}{\sqrt{2}} \text{evecs} \right] \right) // \text{MatrixForm}$$

Out[273]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

In[274]:=

```
Table[u[[h]].\sigma.u[[h]], {h, 1, Length[u]}]
```

Out[274]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[275]:=

```
Table[(Transpose[u][[h]]).\sigma.u[[h]], {h, 1, Length[u]}]
```

Out[275]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[276]:=

```
Table[(Transpose[u][[A]]) == -\sigma.u[[A]], {A, 1, 4}]
```

```
Table[(Transpose[u][[A]]) == \sigma.u[[A]], {A, 5, Length[u]}]
```

Out[276]=

```
{True, True, True, True}
```

Out[277]=

```
{True, True, True, True}
```

$$Ax = \left(\frac{\text{KroneckerProduct}[dX, X] - \text{KroneckerProduct}[X, dX]}{2} \right) . \sigma;$$

$$Au = \left(\frac{\text{KroneckerProduct}[dU, U] - \text{KroneckerProduct}[U, dU]}{2} \right) . \sigma;$$

$$Lsquared = \dot{x}^A \dot{x}_A = dX . \sigma . dX +$$

$$X \cdot \sigma \cdot X \left(\frac{dU \cdot \sigma \cdot dU}{U \cdot \sigma \cdot U} - \left(\frac{U \cdot \sigma \cdot dU}{U \cdot \sigma \cdot U} \right)^2 \right) + \frac{2}{U \cdot \sigma \cdot U} \text{Tr} [\tau^A \cdot Ax \cdot \tau_A \cdot Au]$$

```
Timing[Simplify[ExpandAll[dY.η8.dY - Lsquared]]]
{10.733, 0}
```

In[278]:=

```
FullSimplify[Sqrt[Flatten[MatrixMetric44.η4488]], constraintVars]
```

Out[278]=

$$\begin{aligned} & \left\{ \text{Cot}[6 H x \theta], 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{2 a4[H x 4]}} \sin[6 H x \theta]^{1/6}, 0, 0, 0, 0, 0, 0, 0, 0, \right. \\ & \sqrt{e^{2 a4[H x 4]}} \sin[6 H x \theta]^{1/6}, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{2 a4[H x 4]}} \sin[6 H x \theta]^{1/6}, 0, 0, 0, \\ & 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x 4]}} \sin[6 H x \theta]^{1/6}, 0, 0, 0, 0, 0, 0, \\ & 0, 0, \sqrt{e^{-2 a4[H x 4]}} \sin[6 H x \theta]^{1/6}, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x 4]}} \sin[6 H x \theta]^{1/6} \} \end{aligned}$$

SO(4), γ ; M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+, +, +, +, -, -, -}, {x0, x1, x2, x3, x4, x5, x6, x7}

In[279]:=

Under the involutive automorphism $\gamma^{AB} \rightarrow -\tilde{\gamma}^{AB}$ of $\text{so}(3,3)$, the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra $\text{so}(4)$, which is the Lie algebra of $\overline{\text{SO}(4)}$, the maximal compact subgroup of $\overline{\text{SL}(4, \mathbb{R})}$. The subalgebra $\text{so}(4) \cong \text{su}(2) + \text{su}(2)$ may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of $\text{so}(4)$ whereby $\tau \in \text{so}(4)$ is mapped into its dual, ${}^*\tau$. The even subspace under * of $\text{so}(4)$ corresponds to self-dual tensors, and, say, the first $\text{su}(2)$ in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second $\text{su}(2)$ in the direct sum. A basis for $\text{so}(4)$ may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by ($h = 1, 2, 3$),

$$2s^h = (\gamma^{23}, \gamma^{31}, \gamma^{21}), \quad (4)$$

and

$$2t^h = (\gamma^{45}, \gamma^{64}, \gamma^{65}). \quad (5)$$

Out[279]=

Under the involutive automorphism $\gamma^{AB} \rightarrow -\tilde{\gamma}^{AB}$ of $so(3,3)$, the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra $so(4)$, which is the Lie algebra of $\overline{SO(4)}$, the maximal compact subgroup of $\overline{SL(4,\mathbb{R})}$. The subalgebra $so(4) \cong su(2) + su(2)$ may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of $so(4)$ whereby $\tau \in so(4)$ is mapped into its dual, ${}^*\tau$. The even subspace under * of $so(4)$ corresponds to self-dual tensors, and, say, the first $su(2)$ in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second $su(2)$ in the direct sum. A basis for $so(4)$ may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by ($h = 1, 2, 3$),

$$2s^h = (\gamma^{23}, \gamma^{31}, \gamma^{21}), \quad (4)$$

and

$$2t^h = (\gamma^{45}, \gamma^{64}, \gamma^{65}). \quad (5)$$

In[280]:=

```
Qa[h_, p_, q_] := Signature[{h, p, q, 4}];
Qb[h_, p_, q_] := ID4[p, 4] * ID4[q, h] - ID4[p, h] * ID4[q, 4];
SelfDualAntiSymmetric[h_, p_, q_] := Qa[h, p, q] - Qb[h, p, q];
AntiSelfDualAntiSymmetric[h_, p_, q_] := (Qa[h, p, q] + Qb[h, p, q]);
```

In[284]:=

```
Qa[1, 2, 3]
```

Out[284]=

```
1
```

In[285]:=

```
e4[[1, 2, 3, 4]]
```

Out[285]=

```
1
```

In[286]:=

```
Do[s4by4[h] = Table[Table[SelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
{h, 1, 3}]
```

```
In[287]:= 
Do[
  t4by4[h] = Table[Table[AntiSelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
  {h, 1, 3}]

In[288]:= (*Protect[{s4by4[1], s4by4[2], s4by4[3], t4by4[1], t4by4[2], t4by4[3]}]*)

In[289]:= Protect[s4by4, t4by4]

Out[289]= {s4by4, t4by4}

In[290]:= {s4by4[#] // MatrixForm, MatrixForm[t4by4[#]]} & /@ Range[3]

Out[290]=
{{\left(\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right), \left(\begin{array}{cccc} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)\}, 
{\left(\begin{array}{cccc} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right), \left(\begin{array}{cccc} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)\}, 
{\left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{array}\right), \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)\}}}

In[291]:= 
Table[{{J, K}, MatrixForm[s4by4[J].t4by4[K]]}, {J, 1, 3}, {K, 1, 3}]

Out[291]=
{{{{1, 1}, \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}, {{1, 2}, \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)}, {{1, 3}, \left(\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right)}}, 
{{{2, 1}, \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{array}\right)}, {{2, 2}, \left(\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}, {{2, 3}, \left(\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)}}, 
{{{3, 1}, \left(\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)}, {{3, 2}, \left(\begin{array}{cccc} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right)}, {{3, 3}, \left(\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)}}}]

In[292]:= 
Do[st[J, K] = s4by4[J].t4by4[K], {J, 1, 3}, {K, 1, 3}]
```

In[293]:=

```
Table[{{J, K}, MatrixForm[st[J, K]]}, {J, 1, 3}, {K, 1, 3}]
```

Out[293]=

$$\left\{ \left\{ \left\{ \{1, 1\}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}, \{1, 2\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right\}, \{1, 3\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \right\} \right\},$$

$$\left\{ \left\{ \{2, 1\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \right\}, \{2, 2\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}, \{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right\},$$

$$\left\{ \left\{ \{3, 1\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\}, \{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \right\}, \{3, 3\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\} \right\}$$

In[294]:=

```
Protect[st]
```

Out[294]=

```
{st}
```

In[295]:=

```
Table[{{J, K}, MatrixForm[st[J, K] - Transpose[st[J, K]]]}, {J, 1, 3}, {K, 1, 3}]
```

Out[295]=

$$\left\{ \left\{ \left\{ \{1, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \{1, 2\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \{1, 3\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\} \right\},$$

$$\left\{ \left\{ \{2, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \{2, 2\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\},$$

$$\left\{ \left\{ \{3, 1\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\}, \{3, 3\}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right\} \right\}$$

In[296]:=

```
Permutations[Range[3], 2]
```

Out[296]=

```
{{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}
```

In[297]:=

```
KroneckerProduct[Range[3], Range[3]]
```

Out[297]=

```
{ {1, 2, 3}, {2, 4, 6}, {3, 6, 9} }
```

```
In[298]:= Table[{J, K}, {J, 1, 3}, {K, 1, 3}]
st[##] & /@ %
st[##] & @@ %

Out[298]= {{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}}

Out[299]= {st[{{1, 1}, {1, 2}, {1, 3}}], st[{{2, 1}, {2, 2}, {2, 3}}], st[{{3, 1}, {3, 2}, {3, 3}}]}

Out[300]= st[{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}]

In[301]:= Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}], 1]
Length[%]

Out[301]= {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
{{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
{{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
{{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}
```

Out[302]=

16

```
In[303]:= Unprotect[ $\gamma$ ];
 $\gamma = \text{Flatten}[\{\text{s4by4}[\#] & /@ \text{Range}[3], \text{t4by4}[\#] & /@ \text{Range}[3],$ 
 $\text{Flatten}[\text{Table}[\text{st}[J, K], \{J, 1, 3\}, \{K, 1, 3\}], 1], \{\text{ID4}\}], 1]$ 
```

```
Out[304]=  $\{\{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-1, 0, 0, 0\}\},$ 
 $\{\{0, 0, -1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\},$ 
 $\{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\},$ 
 $\{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{1, 0, 0, 0\}\},$ 
 $\{\{0, 0, -1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\},$ 
 $\{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 1, 0\}\},$ 
 $\{\{1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\},$ 
 $\{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\},$ 
 $\{\{0, 0, 1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\},$ 
 $\{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, -1, 0\}\},$ 
 $\{\{-1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\},$ 
 $\{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{1, 0, 0, 0\}\},$ 
 $\{\{0, 0, 1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\},$ 
 $\{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\},$ 
 $\{\{-1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\},$ 
 $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$ 
```

```
In[305]:= Length[ $\gamma$ ]
Out[305]= 16
```

```
In[306]:= Protect[ $\gamma$ ]
Out[306]=  $\{\gamma\}$ 
```

```
In[307]:=  $\gamma[[1]]$ 
Out[307]=  $\{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-1, 0, 0, 0\}\}$ 
```

```
In[308]:=  $\gamma[[-1]]$ 
Out[308]=  $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}$ 
Self Dual Anti-Symmetric
```

```
In[309]:= Table $\left[\left(\frac{1}{2} \sum_{j1=1}^4 \left(\sum_{j2=1}^4 \epsilon4[p, q, j1, j2] \times \text{s4by4}[h][j1, j2]\right)\right) - \text{s4by4}[h][p, q],$ 
 $\{h, 1, 3\}, \{q, 4\}, \{p, 4\}\right]$ 
```

```
Out[309]=  $\{\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$ 
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\},$ 
 $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}\}$ 
```

Anti SelfDual Anti-Symmetric

```
In[310]:= Table[ $\left(\frac{1}{2} \sum_{j_1=1}^4 \left(\sum_{j_2=1}^4 \epsilon 4 [[p, q, j_1, j_2] \times t4by4[h][j_1, j_2]]\right)\right) + t4by4[h][p, q],$   

 $\{h, 1, 3\}, \{q, 4\}, \{p, 4\}\]$ 
Out[310]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},  

{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}},  

{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}
```

SO(4,4), Spin(4,4) = $\overline{SO(4,4)}$, τ ; if using xact: M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+, +, +, +, -, -, -}, {x0, x1, x2, x3, x4, x5, x6, x7}

```
In[311]:=  $\overline{\tau} == \tau$ 
Out[311]= False
In[312]:=  $\overline{\tau}$ 
Head[%]
Out[312]=  $\overline{\tau}$ 
Out[313]= Symbol
In[314]:=  $\overline{\tau}$ 
Head[%]
Out[314]=  $\overline{\tau}$ 
Out[315]= ParsedBoxWrapper
In[316]:=  $\overline{\tau}$ 
Head[%]
Out[316]=  $\overline{\tau}$ 
Out[317]= Symbol
In[318]:=  $\overline{\tau}$ 
Symbol
Out[318]= Symbol
```

In[319]:=

$$\overline{\tau} === \overline{\tau}$$

Out[319]=

True

In[320]:=

$$\overline{\tau} == \overline{\tau}$$

Out[320]=

False

In[321]:=

 $\eta4488 // \text{MatrixForm}$

Out[321]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[322]:=

$$\eta4488 = \text{DiagonalMatrix}[\text{RotateRight}[\text{values}, 4]]$$

Out[322]=

True

In[323]:=

 $\text{DiagonalMatrix}[\text{RotateRight}[\text{values}, 3]] // \text{MatrixForm}$

Out[323]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[324]:=

```
 $\tau[0] = \text{ID8};$ 
 $\text{Table}[$ 
 $\tau[7-h] = \text{ArrayFlatten}[\{\{0, t4by4[h]\}, \{-t4by4[h], 0\}\}], \{h, 1, 3\}];$ 
 $\text{Table}[\tau[h] = \text{ArrayFlatten}[\{\{0, s4by4[h]\}, \{s4by4[h], 0\}\}], \{h, 1, 3\}];$ 
 $(\tau[7] = \tau[1].\tau[2].\tau[3].\tau[4].\tau[5].\tau[6]) // \text{MatrixForm}$ 
```

Out[327]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[328]:=  $\sigma = \tau[1] \cdot \tau[2] \cdot \tau[3]$ 
Out[328]= True

In[329]:= Table[{A, B, FullSimplify[
  ExpandAll[\frac{1}{2} (\tau[A] \cdot \tau[B] + \tau[B] \cdot \tau[A]) == -\eta4488 [A+1, B+1] * ID8]]}, {A, 1, 7}, {B, 1, 7}]
Out[329]= {{{1, 1, True}, {1, 2, True}, {1, 3, True}, {1, 4, True}, {1, 5, True}, {1, 6, True}, {1, 7, True}}, {{2, 1, True}, {2, 2, True}, {2, 3, True}, {2, 4, True}, {2, 5, True}, {2, 6, True}, {2, 7, True}}, {{3, 1, True}, {3, 2, True}, {3, 3, True}, {3, 4, True}, {3, 5, True}, {3, 6, True}, {3, 7, True}}, {{4, 1, True}, {4, 2, True}, {4, 3, True}, {4, 4, True}, {4, 5, True}, {4, 6, True}, {4, 7, True}}, {{5, 1, True}, {5, 2, True}, {5, 3, True}, {5, 4, True}, {5, 5, True}, {5, 6, True}, {5, 7, True}}, {{6, 1, True}, {6, 2, True}, {6, 3, True}, {6, 4, True}, {6, 5, True}, {6, 6, True}, {6, 7, True}}, {{7, 1, True}, {7, 2, True}, {7, 3, True}, {7, 4, True}, {7, 5, True}, {7, 6, True}, {7, 7, True}}}

In[330]:=  $\eta4488 // \text{MatrixForm}$ 
Out[330]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


In[331]:= Table[{A, -\eta4488 [A+1, A+1] * \tau[A] == Transpose[\tau[A]]}, {A, 1, 7}]
Out[331]= {{1, True}, {2, True}, {3, True}, {4, True}, {5, True}, {6, True}, {7, True}}
In[332]:=  $(\sigma = \tau[1] \cdot \tau[2] \cdot \tau[3])$ 
Out[332]= True
```

In[333]:= $\sigma \cdot \tau[1] \cdot \tau[2] \cdot \tau[3] // \text{MatrixForm}$

Out[333]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[334]:=

```
(*τ[0]=ID8;
Do[{τ[A] = FullSimplify[ExpandAll[(τ[A])]]}, {A, 1, 7}]
(*Do[{τ[A] = FullSimplify[ExpandAll[-(σ.τ[1].τ[2].τ[3].τ[A])]]}, {A, 1, 7}]*)
```

In[335]:=

$(\sigma == \tau[1] \cdot \tau[2] \cdot \tau[3])$

Out[335]=

True

In[336]:=

(*Symbolize[τ]*)

In[337]:=

```
τ[0] = ID8;
Do[{τ[A] = FullSimplify[ExpandAll[(σ.Transpose[τ[A]].σ)]]}, {A, 1, 7}]
```

In[339]:=

$\sigma \cdot \bar{\tau}[\#] == \text{Transpose}[\sigma \cdot \tau[\#]] \& /@ \text{Range}[0, 7]$

Out[339]=

{True, True, True, True, True, True, True}

In[340]:=

```
Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\tau[A] \cdot \bar{\tau}[B] + \tau[B] \cdot \bar{\tau}[A]) = \eta 4488 [A+1, B+1] * ID8]$ ]}],
 {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

Out[340]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \\ 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \\ 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \end{pmatrix}$$

In[341]:=

```
 $\tau[7]$  // MatrixForm
```

Out[341]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[342]:=

```
( $\Omega = \sigma \cdot \tau[7]$ ) // MatrixForm
```

Out[342]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[343]:=
 $\Omega == \tau[4] \cdot \tau[5] \cdot \tau[6]$

Out[343]=
 True

In[344]:=
 $\tau[5] \cdot \tau[6] \cdot \tau[7] // \text{MatrixForm}$

Out[344]/MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

In[345]:=
 $(\sigma \Omega = \sigma \cdot \Omega) // \text{MatrixForm}$

Out[345]/MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[346]:=
 $\text{Inverse}[\sigma \Omega] // \text{MatrixForm}$

Out[346]/MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[347]:=
 $\tau[1] \cdot \tau[2] \cdot \tau[3] \cdot \tau[4] \cdot \tau[5] \cdot \tau[6] \cdot \tau[7] == \tau[0] == \text{ID8}$

Out[347]=
 True

In[348]:=

$(\sigma == \tau[1] \cdot \tau[2] \cdot \tau[3])$

Out[348]=
 True

In[349]:=

$(\sigma == \tau[4] \cdot \tau[5] \cdot \tau[6] \cdot \tau[7])$

Out[349]=
 True

```
In[350]:=  $\tau[1].\tau[2].\tau[3].\tau[\#] == -\text{Transpose}[\tau[1].\tau[2].\tau[3].\tau[\#]] \& /@ \text{Range}[0, 7]$ 

Out[350]= {False, True, True, True, True, True, True}

In[351]:=  $\sigma.\bar{\tau}[\#] == \text{Transpose}[\sigma.\tau[\#]] \& /@ \text{Range}[0, 7]$ 

Out[351]= {True, True, True, True, True, True, True}

In[352]:=  $\tau[5].\tau[6].\tau[7].\bar{\tau}[\#] == \text{Transpose}[\tau[5].\tau[6].\tau[7].\tau[\#]] \& /@ \text{Range}[0, 7]$ 

Out[352]= {False, True, True, True, False, True, True}

In[353]:=  $\tau[4].\tau[\#] == \text{Transpose}[\tau[4].\tau[\#]] \& /@ \text{Range}[0, 7]$ 

Out[353]= {True, True, True, True, False, False, False}

In[354]:=  $\tau[4].\bar{\tau}[\#] == \text{Transpose}[\tau[4].\tau[\#]] \& /@ \text{Range}[0, 7]$ 

Out[354]= {True, False, False, False, True, True, True}

In[355]:=  $\tau[4].\tau[\#] == \text{Transpose}[\tau[4].\tau[\#]] \& /@ \text{Range}[0, 7]$ 

Out[355]= {True, True, True, True, False, False, False}
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}. \quad ; \quad \mathbf{0}(4, 4) : \mathbf{SAB} ;$$

covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]

$$\mathbf{S} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix} ;$$

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix} \cdot \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix} \cdot \begin{pmatrix} \tilde{\mathbf{a}} & \tilde{\mathbf{c}} \\ \tilde{\mathbf{b}} & \tilde{\mathbf{d}} \end{pmatrix} = \begin{pmatrix} \mathbf{b} \cdot \tau^A & \mathbf{a} \cdot \bar{\tau}^A \\ \mathbf{d} \cdot \tau^A & \mathbf{c} \cdot \bar{\tau}^A \end{pmatrix} \cdot \begin{pmatrix} \tilde{\mathbf{a}} & \tilde{\mathbf{c}} \\ \tilde{\mathbf{b}} & \tilde{\mathbf{d}} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{b} \cdot \tau^A \cdot \tilde{\mathbf{a}} + \mathbf{a} \cdot \bar{\tau}^A \cdot \tilde{\mathbf{b}} & \mathbf{b} \cdot \tau^A \cdot \tilde{\mathbf{c}} + \mathbf{a} \cdot \bar{\tau}^A \cdot \tilde{\mathbf{d}} \\ \mathbf{d} \cdot \tau^A \cdot \tilde{\mathbf{a}} + \mathbf{c} \cdot \bar{\tau}^A \cdot \tilde{\mathbf{b}} & \mathbf{d} \cdot \tau^A \cdot \tilde{\mathbf{c}} + \mathbf{c} \cdot \bar{\tau}^A \cdot \tilde{\mathbf{d}} \end{pmatrix}$$

Need these in order to define the Lagrangian for the "universe," later :

In[356]:=

```
(σ16 = T16A[0].T16A[1].T16A[2].T16A[3]) // MatrixForm
```

Out[356]//MatrixForm=

T16^A[0].T16^A[1].T16^A[2].T16^A[3]

In[357]:=

```
Table[T16A[A1] = ArrayFlatten[{{0, τ[A1]}, {τ[A1], 0}}], {A1, 0, 7}];
```

In[358]:=

```
(T16A[8] = T16A[0].T16A[1].T16A[2].T16A[3].T16A[4].T16A[5].T16A[6].T16A[7]) // MatrixForm
```

Out[358]//MatrixForm=

In[359]:=

$\sigma16 // \text{MatrixForm}$

Out[359]//MatrixForm=

0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0
-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

In[360]:=

```
T16A[0].T16A[1].T16A[2].T16A[3].T16A[#] ==
-Transpose[T16A[0].T16A[1].T16A[2].T16A[3].T16A[#]] & /@ Range[0, 8]
```

Out[360]=

{True, True, True, True, True, True, True, True, False}

In[361]:=

```
T16A[4].T16A[5].T16A[6].T16A[7].T16A[#] ==
Transpose[T16A[4].T16A[5].T16A[6].T16A[7].T16A[#]] & /@ Range[0, 8]
```

Out[361]=

{True, True, True, True, True, True, True, True, True}

In[362]:=

$T16^A[8] = \sigma_{16}.T16^A[4].T16^A[5].T16^A[6].T16^A[7]$

Out[362]=

True

In[363]:=

`T16A[#] == Transpose[T16A[#]] & /@ Range[0, 8]`

Out[363]=

{True, True, True, True, False, False, False, False, True}

In[364]:=

```
T16A[#] == -Transpose[T16A[#]] & /@ Range[0, 8]
```

Out[364]=

```
{False, False, False, False, True, True, True, True, False}
```

In[365]:=

```
 $\sigma16.T16^A[\#] == -Transpose[\sigma16.T16^A[\#]] \& /@ Range[0, 7]$ 
```

Out[365]=

{True, True, True, True, True, True, True, True}

In[366]:=

```
(covariantDiffMatrix = T16A[5].T16A[6].T16A[7]) // MatrixForm
```

Out[366]//MatrixForm=

In[367]:=

```
σ16.covariantDiffMatrix = Transpose[σ16.covariantDiffMatrix]
```

Out[367]=

```
True
```

In[368]:=

```
ID16 // MatrixForm
```

Out[368]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[369]:=

```
(σ16(*=T16A[0].T16A[1].T16A[2].T16A[3]*)) // MatrixForm
```

Out[369]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}$$

In[370]:=

```
σ16 = ArrayFlatten[{{{-σ, 0}, {0, σ}}}]
```

Out[370]=

```
True
```

later, after defining $e_{(A)}^\alpha$:

In[371]:=

```
(*Table[T16α[α1-1]= Sum[(eα(A)[[α1,A1]] ) T16A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

In[372]:=

```
(* (T16α[8]=T16α[0].T16α[1].T16α[2].
   T16α[3].T16α[4].T16α[5].T16α[6].T16α[7])//MatrixForm*)
```

In[373]:=

```
(*eLpairs*)
```

In[374]:=

```
(*Ω16a=Array[0&,{16,16}];
 {1+#[[1]],1+#[[2]]} &/@eLpairs
 (Ω16a[[#\[[1]],#[[2]]]]=1) &/@%*)
```

In[375]:=

```
(*Ω16a//MatrixForm*)
```

In[376]:=

```
(* (Ω16=σ16.(Ω16a+Transpose[Ω16a]))//MatrixForm*)
```

In[377]:=

```
T16A[0].T16A[4] // MatrixForm
```

T16^A[0].T16^A[4].σ16

```
(Ω16 = %) // MatrixForm
```

Out[377]//MatrixForm=

0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	-1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	-1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-1	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	-1
0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-1	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0

Out[378]=

Out[379]//MatrixForm=

In[380]:=

Det [Ω16]

Out[380]=

1

In[381]:=

$\sigma16.\Omega16 // \text{MatrixForm}$

Out[381]//MatrixForm=

In[382]:=

```
 $\sigma16.T16^A[\#] == -Transpose[\sigma16.T16^A[\#]] \& /@ Range[0, 7]$ 
```

Out[382]=

{True, True, True, True, True, True, True, True}

In[383]:=

```
 $\sigma16.T16^A[\#] == -Transpose[\sigma16.T16^A[\#]] \& /@ Range[0, 7]$ 
```

Out[383]=

{True, True, True, True, True, True, True, True}

In[384]:=

```
σ16.T16A[#] == -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

Out[384]=

{True, True, True, True, True, True, True, True}

In[385]:=

```
σ16.newT16A[#] == -Transpose[σ16.newT16A[#]] & /@ Range[0, 7]
```

Out[385]=

In[386]:=

```
newσ16.newT16A[#] == -Transpose[newσ16.newT16A[#]] & /@ Range[0, 7]
```

Out[386]=

$$\left\{ \begin{aligned} \text{new}\sigma16.\text{newT16}^A[0] &= -(\text{new}\sigma16.\text{newT16}^A[0])^\top, & \text{new}\sigma16.\text{newT16}^A[1] &= -(\text{new}\sigma16.\text{newT16}^A[1])^\top, \\ \text{new}\sigma16.\text{newT16}^A[2] &= -(\text{new}\sigma16.\text{newT16}^A[2])^\top, & \text{new}\sigma16.\text{newT16}^A[3] &= -(\text{new}\sigma16.\text{newT16}^A[3])^\top, \\ \text{new}\sigma16.\text{newT16}^A[4] &= -(\text{new}\sigma16.\text{newT16}^A[4])^\top, & \text{new}\sigma16.\text{newT16}^A[5] &= -(\text{new}\sigma16.\text{newT16}^A[5])^\top, \\ \text{new}\sigma16.\text{newT16}^A[6] &= -(\text{new}\sigma16.\text{newT16}^A[6])^\top, & \text{new}\sigma16.\text{newT16}^A[7] &= -(\text{new}\sigma16.\text{newT16}^A[7])^\top \end{aligned} \right\}$$

In[387]:=

```
Table[{{A1, B1},  
FullSimplify[ExpandAll[ $\frac{1}{2} (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) =$   
 $\eta 4488 [A1 + 1, B1 + 1] * ID16$ ]], {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[387]/MatrixForm=

In[388]:=

```
Table[{{A1, B1}, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\text{newT16}^A[\text{A1}] \cdot \text{newT16}^A[\text{B1}] + \text{newT16}^A[\text{B1}] \cdot \text{newT16}^A[\text{A1}]) =$ 
     $\eta 4488 [\text{A1} + 1, \text{B1} + 1] * \text{ID16}] }], {A1, 0, 7}, {B1, 0, 7}] // MatrixForm$ 
```

Out[388]/MatrixForm=

```

newT16A[0]^2 = {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0,
(1/2) (newT16A[0].newT16A[1] + newT16A[1].newT16A[0]) = {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(1/2) (newT16A[0].newT16A[2] + newT16A[2].newT16A[0]) = {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(1/2) (newT16A[0].newT16A[3] + newT16A[3].newT16A[0]) = {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(1/2) (newT16A[0].newT16A[4] + newT16A[4].newT16A[0]) = {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(1/2) (newT16A[0].newT16A[5] + newT16A[5].newT16A[0]) = {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(1/2) (newT16A[0].newT16A[6] + newT16A[6].newT16A[0]) = {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(1/2) (newT16A[0].newT16A[7] + newT16A[7].newT16A[0]) = {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

```

```
In[389]:=  $\Psi_{16upper} = \text{Take}[\Psi_{16}, 8]$ 
 $\Psi_{16lower} = \text{Take}[\Psi_{16}, -8]$ 
 $\Psi_{16} == \text{Flatten}[\{\Psi_{16upper}, \Psi_{16lower}\}]$ 

Out[389]= {f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4]}

Out[390]= {f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}

Out[391]= True
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

<pre>Table[T16^A[A1] = ArrayFlatten[{{0, \tau[A1]}, {\tau[A1], 0}}], {A1, 0, 7}] \sigma_{16} = ArrayFlatten[{{{-\sigma, 0}, {0, \sigma}}}] \begin{pmatrix} 0 & \tau[A1] \\ \tau[A1] & 0 \end{pmatrix} \begin{pmatrix} \Psi_{16upper} \\ \Psi_{16lower} \end{pmatrix} = \begin{pmatrix} \tau[A1].\Psi_{16lower} \\ \tau[A1].\Psi_{16upper} \end{pmatrix} \sigma_{16}. \begin{pmatrix} 0 & \tau[A1] \\ \tau[A1] & 0 \end{pmatrix} \begin{pmatrix} \Psi_{16upper} \\ \Psi_{16lower} \end{pmatrix} = \begin{pmatrix} -\sigma \tau[A1].\Psi_{16lower} \\ \sigma \tau[A1].\Psi_{16upper} \end{pmatrix} \Psi_{16}.\sigma_{16}. \begin{pmatrix} 0 & \tau[A1] \\ \tau[A1] & 0 \end{pmatrix} .\Psi_{16} = \begin{pmatrix} -\Psi_{16upper}.\sigma \tau[A1].\Psi_{16lower} \\ \Psi_{16lower}.\sigma \tau[A1].\Psi_{16upper} \end{pmatrix}</pre>

```
In[392]:= (*\sigma_{16}.T16^A[#]&/@Range[0,7]*)

In[393]:=  $(*$ 
 $(T16^A[8]=\text{FullSimplify}[T16^A[0].T16^A[1].T16^A[2].$ 
 $T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]])//\text{MatrixForm}*)$ 
```

```
In[394]:=  $\eta_{4488}$ 

Out[394]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
```

```
In[395]:=  $\eta_{99} = \text{DiagonalMatrix}[\{1, 1, 1, 1, -1, -1, -1, -1, 1\}]$ 

Out[395]= {{1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```



```
SAB = Table[ $\frac{1}{4} (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1])$ , {A1, 0, 7}, {B1, 0, 7}];
```

In[401]:=

```
SAB =
Table[ $\frac{1}{4} (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1])$ , {A1, 0, 7}, {B1, 0, 7}];
```

In[402]:=

```
ParallelTable[
FullSimplify[σ16.SAB[A1, B1] == -Transpose[σ16.SAB[A1, B1]]],
{A1, 1, 8}, {B1, 1, 8}] // Flatten // Union
```

Out[402]=

{True}

In[403]:=

```
ParallelTable[
FullSimplify[SAB[A1, B1].SAB[A2, B2] - SAB[A2, B2].SAB[A1, B1] ==
- (η4488[A1, A2] × SAB[B1, B2] - η4488[A1, B2] × SAB[B1, A2] -
η4488[B1, A2] × SAB[A1, B2] + η4488[B1, B2] × SAB[A1, A2])],
{A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}] // Flatten // Union
```

Out[403]=

{True}

In[404]:=

```
ParallelTable[
FullSimplify[SAB[A1, B1].T16^A[B2 - 1] - T16^A[B2 - 1].SAB[A1, B1] ==
(-η4488[B2, A1] T16^A[B1 - 1] + η4488[B2, B1] T16^A[A1 - 1])],
{A1, 1, 8}, {B1, 1, 8}, {B2, 1, 8}] // Flatten // Union
```

Out[404]=

{True}

In[405]:=

```
(*ParallelTable[
{{A1, B1}, FullSimplify[σ16.SAB[A1, B1] == -Transpose[σ16.SAB[A1, B1]]]}, {A1, 1, 8}, {B1, 1, 8}]*)
```

In[406]:=

```
(*ParallelTable[
{{A1, B1, A2, B2}, FullSimplify[SAB[A1, B1].SAB[A2, B2] - SAB[A2, B2].SAB[A1, B1] ==
- (η4488[A1, A2] SAB[B1, B2] - η4488[A1, B2] SAB[B1, A2] -
η4488[B1, A2] SAB[A1, B2] + η4488[B1, B2] SAB[A1, A2])]}, {A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}]*)
```

In[407]:=

```
(*ParallelTable[
{ {A1,B1,B2}, FullSimplify[SAB[A1,B1].T16^A[B2-1] - T16^A[B2-1].SAB[A1,B1]==
(-η4488[B2,A1]T16^A[B1-1]+η4488[B2,B1]T16^A[A1-1]) ] },
{A1,1, 8},{B1,1,8},{B2,1,8} ]*)
```

In[408]:=

```
Do[sAB[A1][B1] =  $\frac{1}{4} (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1])$ ,  
 {A1, 0, 7}, {B1, 0, 7}];
```

In[409]:=

```
Dimensions[ArrayFlatten[{{0, ID8}, {ID8, 0}}]]
```

Out[409]=

{ 16, 16 }

In[410]:=

sAB [-1 + 1] [-1 + 2] == **SAB** [1, 2]

Out[410]=

True

In[411]:=

```
 $\sigma16.SAB[1, 2] // MatrixForm$ 
```

Out[411]/

10

0	0	- $\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0
0	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	- $\frac{1}{2}$	0	0	0	0	0	0	0	0
0	0	0	0	0	0	- $\frac{1}{2}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	- $\frac{1}{2}$	0	0	0	0
0	0	0	0	0	0	0	0	0	0	- $\frac{1}{2}$	0	0	0	0	0
0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	0
0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	- $\frac{1}{2}$	0
0	0	0	0	0	0	0	0	0	0	0	0	- $\frac{1}{2}$	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	0	0

In[412]:=

σ16.sAB[1] [2] // MatrixForm

Out[412]//MatrixForm=

$$\left(\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3, 3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula: \otimes

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. \otimes

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4). ⊗

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) ⊗

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. ⊗

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa). ⊗

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). ⊗

what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the *so(4,4)* Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: ⊗

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for *so(8)* in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



↳ Why is the metric signature significant for \$so(4,4)\$ commutation relations?

Why is the metric signature significant for \$so(4,4)\$ commutation relations?

The metric signature is significant for the $so(p, q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). ⓘ

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$
The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4, 4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[413]:=

 $\eta4488$

Out[413]=

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1} }
```

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

In[414]:=

$$\text{SAB}[6, 2] \cdot \text{SAB}[6, 7] - \text{SAB}[6, 7] \cdot \text{SAB}[6, 2] = \eta4488[6, 6] \times \text{SAB}[7, 2]$$

Out[414]=

True

In[415]:=

$$\{ \{6, 2, 6, 7\}, \text{False} \}$$

Out[415]=

$$\{ \{6, 2, 6, 7\}, \text{False} \}$$

In[416]:=

$$\text{SAB}[1, 2] \cdot \text{SAB}[1, 3] - \text{SAB}[1, 3] \cdot \text{SAB}[1, 2] = \eta4488[1, 1] \times \text{SAB}[2, 3]$$

Out[416]=

False

In[417]:=

sAB [θ] [θ]

Out[417]=

In[418]:=

sAB[0][1]

Out[418]=

$$\begin{aligned}
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0 \right\}
\end{aligned}$$

```
In[419]:= Table[Transpose[T16A[8].sAB[A1][B1]] == T16A[8].sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[419]= {{True, False, False, False, True, True, True}, {False, True, False, False, True, True, True}, {False, False, True, False, True, True, True}, {False, False, False, True, True, True, True}, {True, True, True, True, False, False, False}, {True, True, True, True, False, False, False}, {True, True, True, True, False, False, True}, {True, True, True, True, False, False, True}}
In[420]:= Table[Transpose[T16A[7].sAB[A1][B1]] == T16A[7].sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[420]= {{True, True, True, True, False, False, True}, {True, True, True, True, False, False, True}, {True, True, True, True, False, False, True}, {True, True, True, True, False, False, True}, {False, False, False, False, True, True, False}, {False, False, False, False, True, True, False}, {False, False, False, False, True, True, False}, {True, True, True, True, False, False, True}}
In[421]:= Table[Transpose[\sigma16.sAB[A1][B1]] == \sigma16.sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[421]= {{True, False, False, False, False, False, False}, {False, True, False, False, False, False, False}, {False, False, True, False, False, False, False}, {False, False, False, True, False, False, False}, {False, False, False, False, True, False, False}, {False, False, False, False, True, False, False}, {False, False, False, False, False, True, False}, {False, False, False, False, False, False, True}}
In[422]:= Table[Transpose[\sigma16.SAB[A1, B1]] == \sigma16.SAB[A1, B1], {A1, 1, 8}, {B1, 1, 8}]
Out[422]= {{True, False, False, False, False, False, False, False}, {False, True, False, False, False, False, False, False}, {False, False, True, False, False, False, False, False}, {False, False, False, True, False, False, False, False}, {False, False, False, False, True, False, False, False}, {False, False, False, False, False, True, False, False}, {False, False, False, False, False, False, True, False}, {False, False, False, False, False, False, False, True}}
```

```

In[423]:= SAB[[1, 2][1]]

Out[423]=  $\left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}$ 

In[424]:= SAB[[1, 2][2]]

Out[424]=  $\left\{0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}$ 


$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$


In[425]:= SAB1 = Table[Take[SAB[[A1, B1]][C1], 8], {A1, 8}, {B1, 8}, {C1, 1, 8}];

In[426]:= SAB2 = Table[Take[SAB[[A1, B1]][C1], -8], {A1, 8}, {B1, 8}, {C1, 9, 16}];

In[427]:= SAB[[1, 2]] = ArrayFlatten[{{SAB1[[1, 2]], 0}, {0, SAB2[[1, 2]]}}]

Out[427]= True

In[428]:= (*Table[SAB[[A1,B1]]= ArrayFlatten[{{SAB1[[A1,B1]],0},{0,SAB2[[A1,B1]]}}],
{A1,0, 7},{B1,0,7}]*)

```

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \rightarrow SO(4,4)$ is given by

$$(2.8) \quad 8L^{A'}_{\ B'}(g) = \text{tr} \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}_{\ C'} G_{A'B'} L^{B'}_{\ D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}_{\ B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}_{\ B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

In[429]:=

SAB2[[1, 2]]

Out[429]=

$$\left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2} \right\}, \right. \\ \left\{ 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0 \right\}, \left\{ 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0 \right\}, \\ \left. \left\{ 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}$$

In[430]:=

τ[θ]

Out[430]=

$$\{ \{ 1, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 1, 0, 0, 0, 0, 0, 0 \}, \\ \{ 0, 0, 1, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 1, 0, 0, 0 \}, \\ \{ 0, 0, 0, 0, 0, 1, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 1, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 1 \} \}$$

In[431]:=

```
(*Block[{b,A,F_a^(μ),F_a^a,ψ,Ψ},
b=Array[A,{8}];
ψ[1]=Array[Ψ[1],{8}];
ψ[2]=Array[Ψ[2],{8}];
F_a^(μ)=Table[(1/(Sqrt[ψ[1].σ.ψ[1]])ψ[1].σ.(τ[μL])),{μL,1,8}];
F_a^a=Transpose[Table[(1/(Sqrt[ψ[1].σ.ψ[1]])η4488[[μL,μL]τ[μL].ψ[1]]),{μL,1,8}]];
b==FullSimplify[F_a^(μ).ψ[2]/.Thread[ψ[2]→F_a^a.b]]]*)
```

In[432]:=

```
(*Block[{b,A,B,q,Fa(μ),ψ,φ,Π},
b=Array[A,{8}];
ψ[1]=Array[Π[1],{8}];
φ[1]=Array[Π[2],{8}];
q=ParallelTable[
  FullSimplify[(1/(ψ[1].σ.ψ[1])ψ[1].σ.1/2(τ[A1].τ[B1]-τ[B1].τ[A1]).φ[1])],
  {A1,1,8},{B1,1,8}];
B=FullSimplify[
  (1/(ψ[1].σ.ψ[1]) ParallelSum[η4488[[μL,μL]]τ[μL]b[[μL]],{μL,1,8}].ψ[1])];
FullSimplify[q/.Thread[φ[1]→B]]*)
```

In[433]:=

```
(*Block[{b,A,B,q,Fa(μ),ψ,φ,Π},
b=Array[A,{8}];
ψ[1]=Array[Π[1],{8}];
φ[1]=Array[Π[2],{8}];
q=ParallelTable[
  FullSimplify[(ψ[1].σ.(τ[A1].τ[B1]).φ[1])],{A1,1,8},{B1,1,8}];
B=FullSimplify[( ParallelSum[η4488[[μL,μL]]τ[μL]b[[μL]],{μL,1,8}].ψ[1])];
FullSimplify[(1/(ψ[1].σ.ψ[1])q/.Thread[φ[1]→B])]*)
```

T16^α

In[434]:=

ssgm4488

Out[434]=

```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
g[1][1] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
g[2][1] → (0 &), g[2][2] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
g[3][2] → (0 &), g[3][3] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
g[5][5] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
g[6][5] → (0 &), g[6][6] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &) }
```

In[435]:=

e_(A)^α / . sge_(A)^α

Out[435]=

```
{ { 1 / √g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0 }, { 0, 1 / √g[1][1][x0, x4], 0, 0, 0, 0, 0, 0 },
{ 0, 0, 1 / √g[2][2][x0, x4], 0, 0, 0, 0, 0, 0 }, { 0, 0, 0, 1 / √g[3][3][x0, x4], 0, 0, 0, 0 },
{ 0, 0, 0, 0, 1 / √-g[4][4][x0, x4], 0, 0, 0, 0 }, { 0, 0, 0, 0, 0, 1 / √-g[5][5][x0, x4], 0, 0 },
{ 0, 0, 0, 0, 0, 0, 1 / √-g[6][6][x0, x4], 0 }, { 0, 0, 0, 0, 0, 0, 0, 1 / √-g[7][7][x0, x4] } }
```

In[436]:=

```
Table[T16α[α1 - 1] = Sum[((e(A)α / . sge(A)α) [[α1, A1]]) T16A[A1 - 1], {A1, 1, 8}], {α1, 1, 8}]; (* /. ssgm4488 *)
```

In[437]:=

```
(T16α[8] = T16α[0].T16α[1].T16α[2].T16α[3].T16α[4].T16α[5].T16α[6].T16α[7]);
```

In[438]:=

```
(*T16α[8]==T16A[8]*Sec[6 H x0]*)
```

CHECK

Table[$T16^\alpha[\alpha_1 - 1] = \text{Sum}[(e_{(A)}^\alpha [[\alpha_1, A1]]) T16^A[A1 - 1], \{A1, 1, 8\}], \{\alpha_1, 1, 8\}]$;
and ● ● ● :

In[439]:=

```
(*Table[T16α[α1-1]= Sum[(eα(A)[[α1,A1]] ) T16A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

In[440]:=

```
(* (T16α[8]=T16α[0].T16α[1].T16α[2].
T16α[3].T16α[4].T16α[5].T16α[6].T16α[7])//MatrixForm*)
```

In[441]:=

```
Table[{ {A1, B1},
FullSimplify[ExpandAll[ $\frac{1}{2} (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) =$ 
 $\eta 4488 [[A1 + 1, B1 + 1]] * ID16$ ]}], {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

Out[441]/MatrixForm=

$\begin{pmatrix} \{0, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{0, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{1, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{2, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{3, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{4, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{5, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{5, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{6, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{6, 7\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{7, 0\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 5\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 6\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{7, 7\} \\ \text{True} \end{pmatrix}$

In[442]:=

```
Table[{α1, β1}, FullSimplify[
  1/2 FullSimplify[((T16^α[α1].T16^α[β1] + T16^α[β1].T16^α[α1]) /. ssgm4488),
  constraintVars == Inverse[MatrixMetric44][α1 + 1, β1 + 1] * ID16]},{α1, 0, 7}, {β1, 0, 7}] // MatrixForm
```

Out[442]/MatrixForm=

$$\begin{pmatrix} \{0, 0\} & \{0, 1\} & \{0, 2\} & \{0, 3\} & \{0, 4\} & \{0, 5\} & \{0, 6\} & \{0, 7\} \\ \text{True} & \text{True} \\ \{1, 0\} & \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} & \{1, 5\} & \{1, 6\} & \{1, 7\} \\ \text{True} & \text{True} \\ \{2, 0\} & \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} & \{2, 5\} & \{2, 6\} & \{2, 7\} \\ \text{True} & \text{True} \\ \{3, 0\} & \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} & \{3, 5\} & \{3, 6\} & \{3, 7\} \\ \text{True} & \text{True} \\ \{4, 0\} & \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} & \{4, 5\} & \{4, 6\} & \{4, 7\} \\ \text{True} & \text{True} \\ \{5, 0\} & \{5, 1\} & \{5, 2\} & \{5, 3\} & \{5, 4\} & \{5, 5\} & \{5, 6\} & \{5, 7\} \\ \text{True} & \text{True} \\ \{6, 0\} & \{6, 1\} & \{6, 2\} & \{6, 3\} & \{6, 4\} & \{6, 5\} & \{6, 6\} & \{6, 7\} \\ \text{True} & \text{True} \\ \{7, 0\} & \{7, 1\} & \{7, 2\} & \{7, 3\} & \{7, 4\} & \{7, 5\} & \{7, 6\} & \{7, 7\} \\ \text{True} & \text{True} \end{pmatrix}$$

In[443]:=

MatrixMetric44 // MatrixForm

Out[443]/MatrixForm=

$$\begin{pmatrix} \cot[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[444]:=

$$((e_{(A)}^\alpha / . \text{sgtrye}_{(A)}^\alpha) . \text{MatrixMetric44} . (e_{(A)}^\alpha / . \text{sgtrye}_{(A)}^\alpha)) // \text{MatrixForm}$$

Out[444]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[450]:=

$$\mathbf{e}_{(A)}^{\alpha} / . \text{sgtrye}_{(A)}^{\alpha}$$

Out[450]=

$$\begin{aligned} & \left\{ \{ \text{Tan}[6 H x \theta], 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ 0, \frac{1}{\sqrt{e^{2 a 4 [H x 4]}} \sin[6 H x \theta]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{e^{-2 a 4 [H x 4]}} \sin[6 H x \theta]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{e^{2 a 4 [H x 4]}} \sin[6 H x \theta]^{1/6}}, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2 a 4 [H x 4]}} \sin[6 H x \theta]^{1/6}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2 a 4 [H x 4]}} \sin[6 H x \theta]^{1/6}}, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2 a 4 [H x 4]}} \sin[6 H x \theta]^{1/6}} \right\} \right\} \end{aligned}$$

In[451]:=

$$\mathbf{e}_{(A)}^{\alpha} / . \text{sge}_{(A)}^{\alpha}$$

Out[451]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{g[0][0][x \theta, x 4]}}, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{g[1][1][x \theta, x 4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{g[2][2][x \theta, x 4]}}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{g[3][3][x \theta, x 4]}}, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x \theta, x 4]}}, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x \theta, x 4]}}, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x \theta, x 4]}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x \theta, x 4]}} \right\} \right\} \end{aligned}$$

In[452]:=

```

Clear[con];
con[g_, ass_ : {}] := Module[{t, rΓ, ginv},
  ginv = FullSimplify[Inverse[g], ass];
  rΓ = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
    (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}] *
    {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
  Return[{ginv, rΓ}]];

```

In[454]:=

ssgm4488

Out[454]=

$$\{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&)\}$$

In[455]:=

g4488**g4488 /. sg**

Out[455]=

$$\{\{g[0][0][x0, x4], g[0][1][x0, x4], g[0][2][x0, x4], g[0][3][x0, x4], \\ g[0][4][x0, x4], g[0][5][x0, x4], g[0][6][x0, x4], g[0][7][x0, x4]\}, \\ \{g[1][0][x0, x4], g[1][1][x0, x4], g[1][2][x0, x4], g[1][3][x0, x4], \\ g[1][4][x0, x4], g[1][5][x0, x4], g[1][6][x0, x4], g[1][7][x0, x4]\}, \\ \{g[2][0][x0, x4], g[2][1][x0, x4], g[2][2][x0, x4], g[2][3][x0, x4], \\ g[2][4][x0, x4], g[2][5][x0, x4], g[2][6][x0, x4], g[2][7][x0, x4]\}, \\ \{g[3][0][x0, x4], g[3][1][x0, x4], g[3][2][x0, x4], g[3][3][x0, x4], \\ g[3][4][x0, x4], g[3][5][x0, x4], g[3][6][x0, x4], g[3][7][x0, x4]\}, \\ \{g[4][0][x0, x4], g[4][1][x0, x4], g[4][2][x0, x4], g[4][3][x0, x4], \\ g[4][4][x0, x4], g[4][5][x0, x4], g[4][6][x0, x4], g[4][7][x0, x4]\}, \\ \{g[5][0][x0, x4], g[5][1][x0, x4], g[5][2][x0, x4], g[5][3][x0, x4], \\ g[5][4][x0, x4], g[5][5][x0, x4], g[5][6][x0, x4], g[5][7][x0, x4]\}, \\ \{g[6][0][x0, x4], g[6][1][x0, x4], g[6][2][x0, x4], g[6][3][x0, x4], \\ g[6][4][x0, x4], g[6][5][x0, x4], g[6][6][x0, x4], g[6][7][x0, x4]\}, \\ \{g[7][0][x0, x4], g[7][1][x0, x4], g[7][2][x0, x4], g[7][3][x0, x4], \\ g[7][4][x0, x4], g[7][5][x0, x4], g[7][6][x0, x4], g[7][7][x0, x4]\}\}$$

Out[456]=

$$\{\{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \{0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0\}, \{0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0\}, \{0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0\}, \{0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]\}\}$$

In[457]:=

```

MatrixForm[#] & /@ Block[{eA $\alpha$ , e $\alpha$ A, (*g $\text{inv}$ , $\Gamma$ ,gg,*) colVecs, rowVecs, ab, ba, r},
  (*gg=g4488/.sg;*)
  (*{g $\text{inv}$ , $\Gamma$ }=con[gg];*)
  e $\alpha$ A = e $_{(A)}^{\alpha}$  /. sge $_{(A)}^{\alpha}$ ;
  eA $\alpha$  = e $_{\alpha}^{(A)}$  /. sge $_{\alpha}^{(A)}$ ;
  rowVecs = Table[eA $\alpha$ [[b, All]], {b, 1, Length[eA $\alpha$ ] }];
  colVecs = Table[e $\alpha$ A[[All, b]], {b, 1, Length[e $\alpha$ A]}];
  ab = rowVecs.colVecs // FullSimplify[#, constraintVars] &;
  ba = Transpose[colVecs].Transpose[rowVecs] // FullSimplify[#, constraintVars] &;
  (*r={{ab},{ba}};*)
  r = {ab, ba};
  r]

```

Out[457]=

$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

metric compatibility condition for the Octad

$$\nabla_\mu e_\nu^I = \partial_\mu e_\nu^I - \Gamma_{\mu\nu}^\rho e_\rho^I + \omega_{\mu J}^I e_\nu^J = 0$$

In[458]:=

```

Clear[spinCoefficients];
spinCoefficients[eIv_] := Module[{evI, w, r, s},
  If[Dimensions[eIv] != {8, 8}, Return[{[]}]];
  If[Det[eIv] == 0, Return[{[]}]];
  evI = FullSimplify[Inverse[eIv], constraintVars] /. subsDefects;
  w = Table[
    -Table[FullSimplify[Sum[D[eIv][I1, v1], X][μ1]] * evI[v1, Jprime], {v1, 1,
      Length[evI]}] - Sum[ eIv[I1, ρ] * Γ[ρ, μ1, v1] * evI[v1, Jprime],
      {v1, 1, Length[evI]}], {ρ, 1, Length[X]}],
    constraintVars] /. subsDefects, {I1, 1, Length[eIv]},
    {Jprime, 1, Length[evI]}, {μ1, 1, Length[X]}];
  Assert[Dimensions[w] == {8, 8, 8}];
  Return[w];
]

```


In[461]:=

```
Table[ $\omega_{\mu IJ}[\mu] = \text{FullSimplify}\left[-\left(\left(D[gtrye_{\alpha}^{(A)}, X[\mu]]\right).gtrye_{(A)}^{\alpha}\right) - \left(gtrye_{\alpha}^{(A)}.I[All, \mu, All].gtrye_{(A)}^{\alpha}\right)\right], constraintVars] /. subsDefects, \{\mu, 1, Length[X]\}]$ 
```

Out[461]=

In[462]:=

(*gtrye $_{(A)}^{\alpha}$ • gtrye $_{\alpha}^{(A)}$ *)

In[463]:=

(*gtrye _{α} ^(A) . gtrye_(A) ^{α} *)

In[464]:=

(*gtrye _{α} ^(A)*)

```

In[465]:= (*Γ
Dimensions[%]*)

In[466]:= (*Table[Γ[All,μ,All],{μ,1,Length[X]}]*)

In[467]:= (*Γ[All,5,All]*)

In[468]:= (*Γ[All,1,All]*)

In[469]:= (*spinCoeffs[1]*)

In[470]:= (*spinCoeffs[5]*)

In[471]:= (*{X[1],X[5]}*)

In[472]:= (*spinCoeffs[1][1][1]*)

In[473]:= (*ωμIJ[1][1][1]*)

In[474]:= (*Block[{μ=1},
FullSimplify[-((D[gtrye_α^(A),X[μ]].gtrye_α^(A))-(gtrye_α^(A).Γ[All,μ,All].gtrye_α^(A))),constraintVars]/.subsDefects]
Dimensions[%]*)

In[475]:= (*Block[{μ=1},
FullSimplify[-((D[gtrye_α^(A),X[μ]].Transpose[gtrye_α^(A)])-(gtrye_α^(A).Γ[All,μ,All].Transpose[gtrye_α^(A)])),constraintVars]/.subsDefects]*)

In[476]:= (*Block[{μ=1},FullSimplify[
-((Sum[D[gtrye_α^(A)][All,ν],X[μ]]*gtrye_α^(A)[ν,All]),{ν,1,8}]-(
gtrye_α^(A).Γ[All,μ,All].gtrye_α^(A))),constraintVars]/.subsDefects]
Dimensions[%]*)

In[477]:= (*Table[,{A1,1,8},{B1,1,8}]*)

In[478]:= (*Block[{μ=1},Table[FullSimplify[
-((Sum[D[gtrye_α^(A)][A1,ν],X[μ]]*gtrye_α^(A)[ν,B1]),{ν,1,8}]-(
Sum[gtrye_α^(A)[A1,ρ]*Γ[ρ,μ,ν]*gtrye_α^(A)[ν,B1]),{ρ,1,8},
{ν,1,8})),constraintVars]/.subsDefects,{A1,1,8},{B1,1,8}]]
Dimensions[%]*)

```

In[479]:=

```
Table[ ( wμIJ[μ] - spinCoeffs[[μ]] ) /. subsDefects , {μ, 1, Length[X]} ] //  
Flatten // Union
```

Out[479]=

{}

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:

- $\nabla_\mu e^a{}_\nu = 0$
- Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

- Metric compatibility : The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame . The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is : ◦ V he "
v=0
- Here, V_μ is the covariant derivative with respect to the connection $F^\mu{}_\nu$, and $e^a{}_\nu$ is the tetrad component .

No it is not the torsionless condition which is:

$$T_{\mu\nu}^I = D_\mu^\omega e_\nu^I = \partial_{[\mu} e_{\nu]}^I + \omega_{[\mu}^I e_{\nu]}^J = 0$$

This postulate says:

$$\nabla_\mu e_\nu^I = \partial_\mu e_\nu^I - \Gamma_{\mu\nu}^\rho e_\rho^I + \omega_{\mu J}^I e_\nu^J = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D_\mu^\omega \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

In[480]:=

```

Block[{eAα, eαA, t, ginv, Γ, gg},
  gg = g4488 /. sg;
  {ginv, Γ} = con[gg];
  eαA = eα(A) /. sgeα(A); eAα = eAα /. sgeAα;
  t = Table[ Block[{v, V, contra, covari},
    V = eAα[[b, All]];
    v = eαA[[All, b]];
    contra =
      Table[D[v[[k]], X[[μ]]] + Sum[v[[i]] × Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
      (*before ω terms*)
    covari =
      Table[D[V[[k]], X[[μ]]] - Sum[V[[i]] × Γ[[i, k, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
      (*before ω terms*)
    {b - 1, X[[μ]], {{contra}, {covari}}}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
  t]
FullSimplify[#, # /. ssgm4488, constraintVars] & /@ %

```

Out[480]=

$$\begin{aligned}
& \left\{ \left\{ 0, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \right. \right. \\
& \left. \left. \left\{ 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 0, x_1, \left\{ \left\{ 0, \frac{g[1][1]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[1][1][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \right. \\
& \left. \left. \left\{ 0, \frac{g[1][1]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 0, x_2, \left\{ \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[2][2][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \right. \\
& \left. \left. \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 0, x_3, \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[3][3][x_0, x_4]}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \right. \\
& \left. \left. \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 0, x_4, \left\{ \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0, 0 \right\} \right\}, \right. \right. \\
& \left. \left. \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0, 0, 0, 0 \right\} \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 0, x_5, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 0, x_6, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}}, 0 \right\} \right\} \right\}, \\
& \left\{ 0, x_7, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]}} \right\} \right\} \right\}, \\
& \left\{ 1, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{1, x_1, \right. \\
& \quad \left. \left\{ \left\{ -\frac{g[1][1]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[1][1][x_0, x_4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[1][1][x_0, x_4]} g[4][4][x_0, x_4]}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0 \right\} \right\}, \left\{ \left\{ -\frac{g[1][1]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[1][1][x_0, x_4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[1][1][x_0, x_4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{1, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \left\{ 2, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \quad \left. \left\{ 2, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{2, x_2, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ -\frac{g[2][2]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]} g[4][4][x_0, x_4]}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0 \right\} \right\}, \left\{ \left\{ -\frac{g[2][2]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{2, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \left\{ 3, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \quad \left. \left\{ 3, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \right. \\
& \quad \left. \left. \left\{ 3, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{3, x_3, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ \left\{ -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, \right. \right. \right. \\
& \left. \left. \left. 0, 0, 0 \right\} \right\}, \left\{ \left\{ -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x0, \left\{ \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \left. \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, \right. \right. \\
& \left. \left. \left. -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{4, x1, \left\{ \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x0, x4]}{2 g[1][1][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, \frac{g[1][1]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{4, x2, \left\{ \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x0, x4]}{2 g[2][2][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{4, x3, \left\{ \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2 g[3][3][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{4, x4, \left\{ \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \left. \left. \left. \frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0, \right. \right. \\
& \left. \left. -\frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}} - \frac{g[4][4]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 4, x_5, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \right. \\
& \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_6, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\}, \right. \right. \\
& \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_7, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\}, \right. \right. \\
& \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]} \right\} \right\} \right\}, \left\{ 5, x_0, \right. \\
& \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}} - \frac{g[5][5]^{(1,0)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]}, \right. \right. \\
& \left. \left. \left. 0, 0 \right\} \right\} \right\}, \left\{ 5, x_1, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 5, x_2, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 5, x_3, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \left\{ 5, x_4, \right. \\
& \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}} - \frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]}, \right. \right. \\
& \left. \left. 0, 0 \right\} \right\} \right\}, \left\{ 5, x_5, \left\{ \left\{ -\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}, \right. \right. \right. \\
& \left. \left. \left. 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \left\{ -\frac{g[5][5]^{(1,0)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]}, 0, 0, 0, \right. \\
& \left. -\frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ 5, x_6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 5, x_7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \left\{ 6, x_0, \right. \\
& \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 (-g[6][6][x_0, x_4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]}} - \frac{g[6][6]^{(1,0)}[x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}{2 g[6][6][x_0, x_4]}, \right. \right. \\
& \left. \left. 0, 0 \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \{ \{ \{ \{ \{ \{ \{ 0 \} \} \} \}, \{ 6, x1, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 6, x2, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 6, x3, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \{ 6, x4, \\
& \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0 \} \}, \\
& \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, \\
& \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 6, x5, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \{ 6, x6, \\
& \{ \{ \{ -\frac{g[6][6]^{(1,0)}[x0, x4]}{2g[6][6][x0, x4]\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2g[4][4][x0, x4]\sqrt{-g[6][6][x0, x4]}}, \\
& 0, 0, 0 \}, \{ \{ -\frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, 0, \\
& 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, 0, 0, 0 \} \} \}, \\
& \{ 6, x7, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \{ 7, x0, \\
& \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \frac{g[7][7]^{(1,0)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \} \}, \\
& \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& -\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]} \} \} \}, \\
& \{ 7, x1, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 7, x2, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 7, x3, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \{ 7, x4, \\
& \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \} \}, \\
& \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& -\frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]} \} \} \}, \\
& \{ 7, x5, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 7, x6, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \{ 7, x7, \\
& \{ \{ \{ -\frac{g[7][7]^{(1,0)}[x0, x4]}{2g[6][6][x0, x4]\sqrt{-g[7][7][x0, x4]}}, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]}{2g[4][4][x0, x4]\sqrt{-g[7][7][x0, x4]}}, \\
& 0, 0, 0 \}, \{ \{ -\frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]}, 0, 0, \\
& 0, -\frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]}, 0, 0, 0 \} \} \} \}
\end{aligned}$$

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$$\{ \{ \{ 0, x0, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \},$$

$$\begin{aligned}
& \{0, x1, \{\{\{0, H, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, e^{2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x2, \{\{\{0, 0, H, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, e^{2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x3, \{\{\{0, 0, 0, H, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, e^{2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{0, x5, \{\{\{0, 0, 0, 0, 0, H, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0, 0, 0\}\}\}\}, \\
& \{0, x6, \{\{\{0, 0, 0, 0, 0, 0, H, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3}, 0, 0, 0\}\}\}\}, \\
& \{0, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, H, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3}\}\}\}\}, \\
& \{1, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x1, \{\{\{-\sqrt{e^{2 a4[H x4]}} H \sec[6 H x0] \sin[6 H x0]^{7/6}, \\
& \quad 0, 0, 0, \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\}\}\}, \\
& \left\{ \left\{ -\frac{\sqrt{e^{2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, 0, 0, 0, -\sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \{1, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{1, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x2, \{\{\{-\sqrt{e^{2 a4[H x4]}} H \sec[6 H x0] \sin[6 H x0]^{7/6}, \\
& \quad 0, 0, 0, \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\}\}\}, \\
& \left\{ \left\{ -\frac{\sqrt{e^{2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, 0, 0, 0, -\sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \{2, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x3, \{\{\{-\sqrt{e^{2 a4[H x4]}} H \sec[6 H x0] \sin[6 H x0]^{7/6}, \\
& \quad 0, 0, 0, \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\}\}\}, \\
& \left\{ \left\{ -\frac{\sqrt{e^{2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, 0, 0, 0, -\sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},
\end{aligned}$$


```

{7, x5, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {7, x6, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {7, x7, {{{{e^-2 a^4 [H x4]}^H} Sec[6 H x0] Sin[6 H x0]}^{7/6}, 0, 0, 0, Sqrt[e^-2 a^4 [H x4]] H Sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0}}, {{-Sqrt[e^-2 a^4 [H x4]] H Cos[6 H x0] Sin[6 H x0]^{5/6}, 0, 0, 0, Sqrt[e^-2 a^4 [H x4]] H Sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0}}}}}
In[482]:= (*Block[{eAα,eαA,t,ginv,Γ,gg},
gg=g4488/.sg;
{ginv,Γ}=con[gg];
eαA=e^(A)/(sge^(A));
eAα=e^(A)/(sge_α^(A));
t=Table[ Block[{v,V,d},v=eαA[[All,b]];
d=Table[D[v[[k]],X[[μ]] ]+Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}],{k,1,DIM8}];
{b-1,X[[μ]],d}],{b,1,Length[eαA]}, {μ,1,DIM8}];t]
FullSimplify[#/ssgm4488,constraintVars]&/@%*)
In[483]:= (*Block[{eAα,eαA,t,ginv,Γ,gg},
gg=g4488/.sg;
{ginv,Γ}=con[gg];
eαA=e^(A)/(sge^(A));
eAα=e^(A)/(sge_α^(A));
t=Table[ Block[{v,V,d},v=eAα[[b,All]];
d=Table[D[v[[k]],X[[μ]] ]-Sum[ v[[i]] Γ[[k,i,μ]],{i,1,DIM8}],{k,1,DIM8}];
{b-1,X[[μ]],d}],{b,1,Length[eαA]}, {μ,1,DIM8}];t]
FullSimplify[#/ssgm4488,constraintVars]&/@%*)
In[]:= Symbolize[w^(a) μ_(b)]
In[]:= w=Array[w^(a) μ_(b)[##]&,{8,8,8}];
```

In[484]:=

 ω_{II}

Out[484]=

$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 1, 1], \omega_{\mu(b)}^{(a)} [1, 1, 2], \omega_{\mu(b)}^{(a)} [1, 1, 3], \omega_{\mu(b)}^{(a)} [1, 1, 4], \\ \omega_{\mu(b)}^{(a)} [1, 1, 5], \omega_{\mu(b)}^{(a)} [1, 1, 6], \omega_{\mu(b)}^{(a)} [1, 1, 7], \omega_{\mu(b)}^{(a)} [1, 1, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 2, 1], \omega_{\mu(b)}^{(a)} [1, 2, 2], \omega_{\mu(b)}^{(a)} [1, 2, 3], \omega_{\mu(b)}^{(a)} [1, 2, 4], \\ \omega_{\mu(b)}^{(a)} [1, 2, 5], \omega_{\mu(b)}^{(a)} [1, 2, 6], \omega_{\mu(b)}^{(a)} [1, 2, 7], \omega_{\mu(b)}^{(a)} [1, 2, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 3, 1], \omega_{\mu(b)}^{(a)} [1, 3, 2], \omega_{\mu(b)}^{(a)} [1, 3, 3], \omega_{\mu(b)}^{(a)} [1, 3, 4], \\ \omega_{\mu(b)}^{(a)} [1, 3, 5], \omega_{\mu(b)}^{(a)} [1, 3, 6], \omega_{\mu(b)}^{(a)} [1, 3, 7], \omega_{\mu(b)}^{(a)} [1, 3, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 4, 1], \omega_{\mu(b)}^{(a)} [1, 4, 2], \omega_{\mu(b)}^{(a)} [1, 4, 3], \omega_{\mu(b)}^{(a)} [1, 4, 4], \\ \omega_{\mu(b)}^{(a)} [1, 4, 5], \omega_{\mu(b)}^{(a)} [1, 4, 6], \omega_{\mu(b)}^{(a)} [1, 4, 7], \omega_{\mu(b)}^{(a)} [1, 4, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 5, 1], \omega_{\mu(b)}^{(a)} [1, 5, 2], \omega_{\mu(b)}^{(a)} [1, 5, 3], \omega_{\mu(b)}^{(a)} [1, 5, 4], \\ \omega_{\mu(b)}^{(a)} [1, 5, 5], \omega_{\mu(b)}^{(a)} [1, 5, 6], \omega_{\mu(b)}^{(a)} [1, 5, 7], \omega_{\mu(b)}^{(a)} [1, 5, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 6, 1], \omega_{\mu(b)}^{(a)} [1, 6, 2], \omega_{\mu(b)}^{(a)} [1, 6, 3], \omega_{\mu(b)}^{(a)} [1, 6, 4], \\ \omega_{\mu(b)}^{(a)} [1, 6, 5], \omega_{\mu(b)}^{(a)} [1, 6, 6], \omega_{\mu(b)}^{(a)} [1, 6, 7], \omega_{\mu(b)}^{(a)} [1, 6, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 7, 1], \omega_{\mu(b)}^{(a)} [1, 7, 2], \omega_{\mu(b)}^{(a)} [1, 7, 3], \omega_{\mu(b)}^{(a)} [1, 7, 4], \\ \omega_{\mu(b)}^{(a)} [1, 7, 5], \omega_{\mu(b)}^{(a)} [1, 7, 6], \omega_{\mu(b)}^{(a)} [1, 7, 7], \omega_{\mu(b)}^{(a)} [1, 7, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 8, 1], \omega_{\mu(b)}^{(a)} [1, 8, 2], \omega_{\mu(b)}^{(a)} [1, 8, 3], \omega_{\mu(b)}^{(a)} [1, 8, 4], \\ \omega_{\mu(b)}^{(a)} [1, 8, 5], \omega_{\mu(b)}^{(a)} [1, 8, 6], \omega_{\mu(b)}^{(a)} [1, 8, 7], \omega_{\mu(b)}^{(a)} [1, 8, 8] \end{array} \right\} \end{array} \right\}$$

In[485]:=

```

sw = Block[{eAα, eαA, t, V, ginv, Γ, gg},
gg = g4488 /. sg;
{ginv, Γ} = con[gg];
eαA = eα(A) /. sge(A);
eAα = eA(A) /. sge(A);
Table[ω[μ, a, b] → Block[{v, d}, v = eαA[[All, b]];
d = Sum[(D[v[k], X[μ]] + Sum[v[i] × Γ[k, i, μ], {i, 1, DIM8}]) * eAα[a, k],
{k, 1, DIM8}];
d], {μ, 1, DIM8}, {a, 1, Length[eαA]}, {b, 1, Length[eAα]}]

```

Out[485]=

$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 1, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 5] \rightarrow \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \\ \omega_{\mu(b)}^{(a)} [1, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 8] \rightarrow 0 \end{array} \right\}, \end{array} \right\}$$

$$\begin{aligned}
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 2, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 3, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 4, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 5, 1] \rightarrow -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [1, 5, 2] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 5, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 5, 5] \rightarrow \\ \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [1, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 5, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 6, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 6, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 6, 6] \rightarrow \\ \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [1, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 6, 8] \rightarrow 0, \quad \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 7, 2] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 7, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 7, 6] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 7, 7] \rightarrow \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\ \left. \sqrt{-g[6][6][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [1, 7, 8] \rightarrow 0 \right\}, \\ \omega_{\mu(b)}^{(a)} [1, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 8, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [1, 8, 8] \rightarrow \\ \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [2, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 1, 2] \rightarrow -\frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, \\ \omega_{\mu(b)}^{(a)} [2, 1, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 1, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 1, 5] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [2, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 1, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [2, 2, 1] \rightarrow \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}} \end{array} \right\}
\end{aligned}$$

$$\begin{aligned}
& \omega_{\mu}^{(a)} [2, 2, 2] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 2, 3] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 2, 4] \rightarrow 0, \\
& \omega_{\mu}^{(a)} [2, 2, 5] \rightarrow \frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [2, 2, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 2, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 2, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [2, 3, 1] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 3, 2] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 3, 3] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 3, 4] \rightarrow 0, \\ \omega_{\mu}^{(a)} [2, 3, 5] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 3, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 3, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 3, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [2, 4, 1] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 4, 2] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 4, 3] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 4, 4] \rightarrow 0, \\ \omega_{\mu}^{(a)} [2, 4, 5] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 4, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 4, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 4, 8] \rightarrow 0 \end{array} \right\}, \\
& \omega_{\mu}^{(a)} [2, 5, 1] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 5, 2] \rightarrow -\frac{g[1][1]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [2, 5, 3] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 5, 4] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 5, 5] \rightarrow 0, \\ \omega_{\mu}^{(a)} [2, 5, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 5, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 5, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [2, 6, 1] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 6, 2] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 6, 3] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 6, 4] \rightarrow 0, \\ \omega_{\mu}^{(a)} [2, 6, 5] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 6, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 6, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 6, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [2, 7, 1] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 7, 2] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 7, 3] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 7, 4] \rightarrow 0, \\ \omega_{\mu}^{(a)} [2, 7, 5] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 7, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 7, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 7, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [2, 8, 1] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 8, 2] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 8, 3] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 8, 4] \rightarrow 0, \\ \omega_{\mu}^{(a)} [2, 8, 5] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 8, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 8, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [2, 8, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [3, 1, 1] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 1, 2] \rightarrow 0, \\ \omega_{\mu}^{(a)} [3, 1, 3] \rightarrow -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, \quad \omega_{\mu}^{(a)} [3, 1, 4] \rightarrow 0, \\ \omega_{\mu}^{(a)} [3, 1, 5] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 1, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 1, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 1, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [3, 2, 1] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 2, 2] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 2, 3] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 2, 4] \rightarrow 0, \\ \omega_{\mu}^{(a)} [3, 2, 5] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 2, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 2, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 2, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu}^{(a)} [3, 3, 1] \rightarrow -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, \\ \omega_{\mu}^{(a)} [3, 3, 2] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 3, 3] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 3, 4] \rightarrow 0, \\ \omega_{\mu}^{(a)} [3, 3, 5] \rightarrow -\frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \quad \omega_{\mu}^{(a)} [3, 3, 6] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 3, 7] \rightarrow 0, \quad \omega_{\mu}^{(a)} [3, 3, 8] \rightarrow 0 \end{array} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 5, 2] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 5, 3] \rightarrow -\frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [3, 5, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 5, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 1, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 1, 4] \rightarrow -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [4, 1, 5] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 1, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 4, 1] \rightarrow \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \\ \omega_{\mu(b)}^{(a)} [4, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 4, 5] \rightarrow \frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [4, 4, 6] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 8] \rightarrow 0 \end{array} \right\}, \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 5, 2] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 5, 4] \rightarrow -\frac{g[3][3]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [4, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 5, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 6, 4] \rightarrow 0, \end{array} \right\}
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [4, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [4, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [4, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [5, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 1, 5] \rightarrow -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 5, 1] \rightarrow \frac{g[4][4]^{(1,0)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[4][4][x0, x4]}, \omega_{\mu(b)}^{(a)} [5, 5, 2] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 5, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 5, 5] \rightarrow \right. \right. \\
& \quad \left. \left. \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 5, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 3] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 6, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 6] \rightarrow \right. \right. \\
& \quad \left. \left. \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 8] \rightarrow 0 \right\}, \left\{ \omega_{\mu(b)}^{(a)} [5, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 7, 2] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 7, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 7, 6] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [5, 7, 7] \rightarrow \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}g[6][6][x0, x4]} \right) \right. \right. \\
& \quad \left. \left. \sqrt{-g[6][6][x0, x4]}, \omega_{\mu(b)}^{(a)} [5, 7, 8] \rightarrow 0 \right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu}^{(a)} [5, 8, 1] \rightarrow 0, \omega_{\mu}^{(a)} [5, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)} [5, 8, 3] \rightarrow 0, \omega_{\mu}^{(a)} [5, 8, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)} [5, 8, 5] \rightarrow 0, \omega_{\mu}^{(a)} [5, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)} [5, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)} [5, 8, 8] \rightarrow \right. \\
& \left. \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ \left\{ \omega_{\mu}^{(a)} [6, 1, 1] \rightarrow 0, \omega_{\mu}^{(a)} [6, 1, 2] \rightarrow 0, \omega_{\mu}^{(a)} [6, 1, 3] \rightarrow 0, \omega_{\mu}^{(a)} [6, 1, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu}^{(a)} [6, 1, 5] \rightarrow 0, \omega_{\mu}^{(a)} [6, 1, 6] \rightarrow -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[5][5][x0, x4]}}, \right. \right. \\
& \left. \left. \omega_{\mu}^{(a)} [6, 1, 7] \rightarrow 0, \omega_{\mu}^{(a)} [6, 1, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \omega_{\mu}^{(a)} [6, 2, 1] \rightarrow 0, \omega_{\mu}^{(a)} [6, 2, 2] \rightarrow 0, \omega_{\mu}^{(a)} [6, 2, 3] \rightarrow 0, \omega_{\mu}^{(a)} [6, 2, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 2, 5] \rightarrow 0, \omega_{\mu}^{(a)} [6, 2, 6] \rightarrow 0, \omega_{\mu}^{(a)} [6, 2, 7] \rightarrow 0, \omega_{\mu}^{(a)} [6, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [6, 3, 1] \rightarrow 0, \omega_{\mu}^{(a)} [6, 3, 2] \rightarrow 0, \omega_{\mu}^{(a)} [6, 3, 3] \rightarrow 0, \omega_{\mu}^{(a)} [6, 3, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 3, 5] \rightarrow 0, \omega_{\mu}^{(a)} [6, 3, 6] \rightarrow 0, \omega_{\mu}^{(a)} [6, 3, 7] \rightarrow 0, \omega_{\mu}^{(a)} [6, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [6, 4, 1] \rightarrow 0, \omega_{\mu}^{(a)} [6, 4, 2] \rightarrow 0, \omega_{\mu}^{(a)} [6, 4, 3] \rightarrow 0, \omega_{\mu}^{(a)} [6, 4, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 4, 5] \rightarrow 0, \omega_{\mu}^{(a)} [6, 4, 6] \rightarrow 0, \omega_{\mu}^{(a)} [6, 4, 7] \rightarrow 0, \omega_{\mu}^{(a)} [6, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [6, 5, 1] \rightarrow 0, \omega_{\mu}^{(a)} [6, 5, 2] \rightarrow 0, \omega_{\mu}^{(a)} [6, 5, 3] \rightarrow 0, \omega_{\mu}^{(a)} [6, 5, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 5, 5] \rightarrow 0, \omega_{\mu}^{(a)} [6, 5, 6] \rightarrow -\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 5, 7] \rightarrow 0, \omega_{\mu}^{(a)} [6, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [6, 6, 1] \rightarrow \frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[5][5][x0, x4]}, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)} [6, 6, 3] \rightarrow 0, \omega_{\mu}^{(a)} [6, 6, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 6, 5] \rightarrow \frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}g[5][5][x0, x4]}, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 6, 6] \rightarrow 0, \omega_{\mu}^{(a)} [6, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)} [6, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [6, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)} [6, 7, 2] \rightarrow 0, \omega_{\mu}^{(a)} [6, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)} [6, 7, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)} [6, 7, 6] \rightarrow 0, \omega_{\mu}^{(a)} [6, 7, 7] \rightarrow 0, \omega_{\mu}^{(a)} [6, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [6, 8, 1] \rightarrow 0, \omega_{\mu}^{(a)} [6, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)} [6, 8, 3] \rightarrow 0, \omega_{\mu}^{(a)} [6, 8, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu}^{(a)} [6, 8, 5] \rightarrow 0, \omega_{\mu}^{(a)} [6, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)} [6, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)} [6, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu}^{(a)} [7, 1, 1] \rightarrow 0, \omega_{\mu}^{(a)} [7, 1, 2] \rightarrow 0, \omega_{\mu}^{(a)} [7, 1, 3] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu}^{(a)} [7, 1, 4] \rightarrow 0, \omega_{\mu}^{(a)} [7, 1, 5] \rightarrow 0, \omega_{\mu}^{(a)} [7, 1, 6] \rightarrow 0, \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [7, 1, 7] \rightarrow -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[6][6][x0, x4]}}, \omega_{\mu(b)}^{(a)} [7, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 5, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 5, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 5, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 5, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 5, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 5, 7] \rightarrow -\frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[6][6][x0, x4]}}, \omega_{\mu(b)}^{(a)} [7, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 7, 1] \rightarrow \frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[6][6][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 7, 5] \rightarrow \frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}g[6][6][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 7, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 7, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [7, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [8, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [8, 1, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 1, 7] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [8, 1, 8] \rightarrow -\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[7][7][x0, x4]}} \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [8, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [8, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [8, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [8, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [8, 3, 8] \rightarrow 0 \right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu}^{(a)} [8, 4, 1] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 3] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 4, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 5, 1] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 3] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 5, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 7] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 5, 8] \rightarrow -\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}} \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 6, 1] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 3] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 6, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 8, 1] \rightarrow \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 8, 3] \rightarrow 0, \omega_{\mu}^{(a)} [8, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 8, 5] \rightarrow \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 8, 8] \rightarrow 0 \right\} \}
\end{aligned}$$

In[486]:=

```
(*sw=Block[{eAα,eαA,t,V,ginv,Γ,gg},
  gg=g4488/.sg;
  {ginv,Γ}=con[gg];
  eAα=e(A) /.sge(A);
  eαA=e(A) /.sge(A);
  Table[ ω(a)μ(b) [μ,a,b]→Block[{v,d},v=eAα[[a,k]],{k,1,DIM8}];
    d=Sum[
      (D[v[[k]],X[[μ]]]+Sum[ v[[i]] Γ[[k,i,μ]],[i,1,DIM8]])*eAα[[a,k]],{k,1,DIM8}];
      d],{μ,1,DIM8},{a,1,Length[eAα]},{b,1,Length[eαA]}]*)
```

In[487]:=

swf = Flatten[sw];

In[488]:=

```
(*ω(a)μ(b)*)
ω;
wmat=%/.swf
Dimensions[%]
```

Out[489]=

$$\left\{ \left\{ 0, 0, 0, 0, \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \right.$$

$$\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, \right.$$

$$\left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]},$$

$$0, 0, 0\}, \{0, 0, 0, 0, 0, 0,$$

$$\left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, 0,$$

$$0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \sqrt{-g[6][6][x0, x4]}, 0,$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]}, \},$$

$$\left\{ 0, -\frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\left\{ \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, \right.$$

$$\left. \frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, -\frac{g[1][1]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0, 0\right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ 0, 0, -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, \right.$$

$$\left. \frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ 0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0\right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ 0, 0, 0, -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \right.$$

$$\begin{aligned}
& 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \}, \\
& \left\{ 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0 \right\}, \\
& \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \\
& \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, \right. \\
& \left. \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, 0, \right. \\
& \left. 0, 0 \right\}, \{ 0, 0, 0, 0, 0, 0, \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[5][5][x0, x4]}, 0, 0 \right\}, \{ 0, 0, 0, 0, 0, 0, 0, \\
& \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[6][6][x0, x4]}, 0 \right\}, \\
& \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \\
& \left\{ \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \\
& \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \\
& \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \\
& \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[6][6][x0, x4]}, 0, 0, 0, \right. \\
& \left. \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[6][6][x0, x4]}, 0, 0, 0 \right\}, \\
& \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}
\end{aligned}$$

$$\left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[7][7][x0, x4]}} \right\}, \right.$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left. \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}} \right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \right.$$

$$\{0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]}, 0, 0, 0 \right\} \}$$

$$0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}, 0, 0, 0 \}$$

Out[490]=

{8, 8, 8}

In[491]:=

```
sugtry = Block[{eA $\alpha$ , e $\alpha$ A, t, V},
```

 $e\alpha A = e_{(A)}^\alpha / . \text{sgtry} e_{(A)}^\alpha;$
 $eA\alpha = e_\alpha^{(A)} / . \text{sgtry} e_\alpha^{(A)};$

```
Table[(*V=eA $\alpha$ [[a,All]];*)  $\omega_{\mu(b)}^{(a)}$  [μ, a, b]  $\rightarrow$  Block[{v, d}, v = e $\alpha$ A[[All, b]];
```

 $d = \text{Sum}[(D[v[k], X[\mu]] + \text{Sum}[v[i] \times r[k, i, \mu], \{i, 1, \text{DIM8}\}] * eA\alpha[[a, k]],$
 $\{k, 1, \text{DIM8}\});$
 $\text{FullSimplify}[d, \text{constraintVars}] /. \{ \sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}, \sqrt{e^{-2 a4[H x4]}} \rightarrow$
 $e^{-a4[H x4]} \}]$, {μ, 1, DIM8}, {a, 1, Length[eA α]}, {b, 1, Length[e α A]}]

Out[491]=

$$\left\{ \left\{ \omega_{\mu(b)}^{(a)} [1, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 4] \rightarrow 0, \right. \right.$$
 $\omega_{\mu(b)}^{(a)} [1, 1, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 8] \rightarrow 0 \},$
 $\left. \left\{ \omega_{\mu(b)}^{(a)} [1, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 4] \rightarrow 0, \right. \right.$
 $\omega_{\mu(b)}^{(a)} [1, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 8] \rightarrow 0 \},$
 $\left. \left\{ \omega_{\mu(b)}^{(a)} [1, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 4] \rightarrow 0, \right. \right.$
 $\omega_{\mu(b)}^{(a)} [1, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 8] \rightarrow 0 \},$
 $\left. \left\{ \omega_{\mu(b)}^{(a)} [1, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 4] \rightarrow 0, \right. \right.$
 $\omega_{\mu(b)}^{(a)} [1, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 8] \rightarrow 0 \},$
 $\left. \left\{ \omega_{\mu(b)}^{(a)} [1, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 4] \rightarrow 0, \right. \right.$
 $\omega_{\mu(b)}^{(a)} [1, 5, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 8] \rightarrow 0 \},$
 $\left. \left\{ \omega_{\mu(b)}^{(a)} [1, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 4] \rightarrow 0, \right. \right.$
 $\omega_{\mu(b)}^{(a)} [1, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 8] \rightarrow 0 \},$

$$\left\{ \begin{array}{l}
\left\{ \begin{array}{l}
\omega_{\mu(b)}^{(a)} [8, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 3] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 1, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 6] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 8] \rightarrow e^{-a4[H \times 4]} H \sin[6H \times 0]^{1/6}
\end{array} \right\}, \\
\left\{ \begin{array}{l}
\omega_{\mu(b)}^{(a)} [8, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 8] \rightarrow 0
\end{array} \right\}, \\
\left\{ \begin{array}{l}
\omega_{\mu(b)}^{(a)} [8, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 8] \rightarrow 0
\end{array} \right\}, \\
\left\{ \begin{array}{l}
\omega_{\mu(b)}^{(a)} [8, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 8] \rightarrow 0
\end{array} \right\}, \\
\left\{ \begin{array}{l}
\omega_{\mu(b)}^{(a)} [8, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 3] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 5, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 6] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 8] \rightarrow e^{-a4[H \times 4]} H \sin[6H \times 0]^{1/6} a4'[H \times 4]
\end{array} \right\}, \\
\left\{ \begin{array}{l}
\omega_{\mu(b)}^{(a)} [8, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 8] \rightarrow 0
\end{array} \right\}, \\
\left\{ \begin{array}{l}
\omega_{\mu(b)}^{(a)} [8, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 8] \rightarrow 0
\end{array} \right\}, \\
\left\{ \begin{array}{l}
\omega_{\mu(b)}^{(a)} [8, 8, 1] \rightarrow e^{-a4[H \times 4]} H \sin[6H \times 0]^{1/6}, \quad \omega_{\mu(b)}^{(a)} [8, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 3] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 8, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 5] \rightarrow -e^{-a4[H \times 4]} H \sin[6H \times 0]^{1/6} a4'[H \times 4], \\
\omega_{\mu(b)}^{(a)} [8, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 8] \rightarrow 0
\end{array} \right\} \}
\end{array} \right\}$$

In[492]:=

```
swnfgtry = Flatten[swnfgtry];
```

In[493]:=

```

(* $\omega_{\mu}^{(a)}$ *)  

(* $\omega_{\mu}^{(b)}$ *)  

 $\omega$ ;  

 $\omega\omega = \%$  /. swfgtry // FullSimplify[#, constraintVars] &  

Dimensions[%]

```

Out[494]=

Out[495]=

$$\{8, 8, 8\}$$

```
In[496]:= g4488 /. sg
% /. ssgm4488

Out[496]= {{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}

Out[497]= {{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

check metric compatibility condition for the Octad

In[498]:= Block[{eAα, eαA, t, ginv, Γ, gg(*, ωω*)},
(*ωω=ωμ^(a)(b)/.swf;*)
gg = g4488 /. sg /. ssgm4488;
{ginv, Γ} = con[gg];
eαA = eα^(A) /. sge^(A); eAα = eA^(A) /. sge^(A);
t = Table[Block[{v, V, contra, covari},
V = eAα[b, All];
v = eαA[All, b];
contra =
Table[D[v[k], X[μ]] + Sum[v[i] × Γ[k, i, μ], {i, 1, DIM8}], {k, 1, DIM8}];
(*no ω terms*)
covari = Table[D[V[k], X[μ]] - Sum[V[i] × Γ[i, k, μ], {i, 1, DIM8}] +
Sum[eAα[J, All][k] × ωmat[μ, b, J], {J, 1, DIM8}] //
FullSimplify[#, constraintVars && e^2 a4[H x4] > 0 && e^a4[H x4] > 0 &&
e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0] &, {k, 1, DIM8}]; (*ω terms*)
(*{b-1,X[μ]},{contra},{covari}}],{b,1,Length[eαA]}, {μ,1,DIM8}];*
{b-1, X[μ], covari}, {b, 1, Length[eαA]}, {μ, 1, DIM8}];

t]
FullSimplify[#/ . ssgm4488,
constraintVars && e^2 a4[H x4] > 0 && e^a4[H x4] > 0 && e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0] & /@ %

Out[498]= {{0, x0, {g[0][0]^(1,0)[x0, x4] + 24 H Csc[12 H x0] g[0][0][x0, x4],
2 √g[0][0][x0, x4]}, {0, 0, 0, {g[0][0]^(0,1)[x0, x4],
2 √g[0][0][x0, x4]}, {0, 0, 0}}, {0, x1, {0, (-g[1][1]^(1,0)[x0, x4] + 2 e^2 a4[H x4] H Sec[6 H x0] Sin[6 H x0]^(4/3)}}}}
```

$$\begin{aligned}
& g[0][0][x0, x4]) / (2 \sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0, 0, 0, 0 \}, \\
& \{0, x2, \{0, 0, (-g[2][2]^{(1,0)}[x0, x4] + 2 e^{2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} \\
& g[0][0][x0, x4]) / (2 \sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0, 0\}, \\
& \{0, x3, \{0, 0, 0, 0, (-g[3][3]^{(1,0)}[x0, x4] + 2 e^{2 a4[H x4]} H \operatorname{Sec}[6 H x0] \\
& \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4]) / (2 \sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0, 0\}, \\
& \{0, x4, \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\}, \\
& \{0, x5, \{0, 0, 0, 0, 0, 0, (-g[5][5]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H x0] \\
& \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4]) / (2 \sqrt{g[0][0][x0, x4]}), 0, 0\}, \\
& \{0, x6, \{0, 0, 0, 0, 0, 0, 0, (-g[6][6]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H x0] \\
& \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4]) / (2 \sqrt{g[0][0][x0, x4]}), 0\}, \\
& \{0, x7, \{0, 0, 0, 0, 0, 0, 0, 0, (-g[7][7]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \\
& \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4]) / (2 \sqrt{g[0][0][x0, x4]} \} \}, \\
& \left\{ 1, x0, \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, \right. \right. \\
& \left. \left. 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 1, x1, \left\{ \frac{g[1][1]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, \right. \right. \\
& \left. \left. 0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0 \right\} \right\}, \\
& \{1, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{1, x4, \\
& \left\{ 0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \{1, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{1, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{1, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{ 2, x0, \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, \right. \right. \\
& \left. \left. 0, 0, 0, 0, 0 \right\} \right\}, \{2, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{ 2, x2, \left\{ \frac{g[2][2]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, \right. \right. \\
& \left. \left. 0, \frac{g[2][2]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0 \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \{2, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_4, \\
& \left\{0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[2][2][x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0, 0, 0\right\}\}, \\
& \{2, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_0, \\
& \left\{0, 0, 0, \frac{g[3][3]^{(1,0)}[x_0, x_4] - 2 H \text{Cot}[6 H x_0] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, 0\right\}\}, \\
& \{3, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x_3, \left\{\frac{g[3][3]^{(1,0)}[x_0, x_4] - 2 H \text{Cot}[6 H x_0] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}\right., 0, 0, 0, \\
& \left.\frac{g[3][3]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0\right\}\}, \{3, x_4, \\
& \left\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4] - 6 H a4'[H x_4] g[3][3][x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, 0\right\}\}, \\
& \{3, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{4, x_0, \left\{\frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0\right\}\}, \\
& \{4, x_1, \\
& \left\{0, \frac{(g[1][1]^{(0,1)}[x_0, x_4] + 6 e^{2 a4[H x_4]} H \text{Sin}[6 H x_0]^{1/3} a4'[H x_4] g[4][4][x_0, x_4]) /}{(2 \sqrt{-g[4][4][x_0, x_4]})}, 0, 0, 0, 0, 0, 0\right\}, \{4, x_2, \{0, 0, \\
& (g[2][2]^{(0,1)}[x_0, x_4] + 6 e^{2 a4[H x_4]} H \text{Sin}[6 H x_0]^{1/3} a4'[H x_4] g[4][4][x_0, x_4]) / \\
& (2 \sqrt{-g[4][4][x_0, x_4]}), 0, 0, 0, 0, 0, 0\}, \{4, x_3, \{0, 0, 0, \\
& (g[3][3]^{(0,1)}[x_0, x_4] + 6 e^{2 a4[H x_4]} H \text{Sin}[6 H x_0]^{1/3} a4'[H x_4] g[4][4][x_0, x_4]) / \\
& (2 \sqrt{-g[4][4][x_0, x_4]}), 0, 0, 0, 0, 0, 0\}, \\
& \{4, x_4, \left\{-\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0\right\}\}, \\
& \{4, x_5, \\
& \left\{0, 0, 0, 0, 0, 0, \frac{1}{2} \left(-6 e^{-2 a4[H x_4]} H \text{Sin}[6 H x_0]^{1/3} a4'[H x_4] - \frac{g[5][5]^{(0,1)}[x_0, x_4]}{g[4][4][x_0, x_4]}\right) \right. \\
& \left. \sqrt{-g[4][4][x_0, x_4]}, 0, 0\right\}, \{4, x_6, \{0, 0, 0, 0, 0, 0, 0, \\
& \frac{1}{2} \left(-6 e^{-2 a4[H x_4]} H \text{Sin}[6 H x_0]^{1/3} a4'[H x_4] - \frac{g[6][6]^{(0,1)}[x_0, x_4]}{g[4][4][x_0, x_4]}\right) \right. \\
& \left. \sqrt{-g[4][4][x_0, x_4]}, 0\right\}, \{4, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left(-6 e^{-2 a4[Hx4]} H \sin[6Hx0]^{1/3} a4'[Hx4] - \frac{g[7][7]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \\
& \sqrt{-g[4][4][x0, x4]} \} \}, \{ \{ 5, x0, \right. \\
& \left. \{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4] - 2 H \cot[6Hx0] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0 \} \}, \right. \\
& \{ 5, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 5, x2, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 5, x3, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 5, x4, \\
& \left. \{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4] + 6 H a4'[Hx4] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0 \} \}, \right. \\
& \{ 5, x5, \left\{ -\frac{g[5][5]^{(1,0)}[x0, x4] - 2 H \cot[6Hx0] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0, \right. \\
& \left. 0, -\frac{g[5][5]^{(0,1)}[x0, x4] + 6 H a4'[Hx4] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0 \} \}, \right. \\
& \{ 5, x6, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 5, x7, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 6, x0, \right. \\
& \left. \{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4] - 2 H \cot[6Hx0] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0 \} \}, \right. \\
& \{ 6, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 6, x2, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 6, x3, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 6, x4, \\
& \left. \{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4] + 6 H a4'[Hx4] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0 \} \}, \right. \\
& \{ 6, x5, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 6, x6, \left\{ -\frac{g[6][6]^{(1,0)}[x0, x4] - 2 H \cot[6Hx0] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0, 0, \right. \\
& \left. 0, -\frac{g[6][6]^{(0,1)}[x0, x4] + 6 H a4'[Hx4] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0 \} \}, \right. \\
& \{ 6, x7, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ \{ 7, x0, \right. \\
& \left. \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4] - 2 H \cot[6Hx0] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}} \} \}, \right. \\
& \{ 7, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 7, x2, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 7, x3, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 7, x4, \\
& \left. \{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4] + 6 H a4'[Hx4] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}} \} \}, \right. \\
& \{ 7, x5, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \{ 7, x6, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \},
\end{aligned}$$

$$\left\{ 7, x_7, \left\{ -\frac{g[7][7]^{(1,0)}[x_0, x_4] - 2 H \operatorname{Cot}[6 H x_0] g[7][7][x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]}}, 0, 0, \right. \right.$$

$$\left. \left. 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4] + 6 H a4'[H x_4] g[7][7][x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]}}, 0, 0, 0 \right\} \right\}$$

Out[8]=

$$\{\{0, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{2, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{2, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{2, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{2, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{2, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{2, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{2, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{3, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{3, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{3, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{3, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{3, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{3, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{3, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{4, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{4, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{4, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{4, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{4, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{4, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{4, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{5, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{5, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{5, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{5, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{5, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{5, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{5, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{6, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{6, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{6, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{6, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{6, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{6, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{6, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x_0, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{7, x_1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{7, x_2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{7, x_3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{7, x_4, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{7, x_5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{7, x_6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{7, x_7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}$$

In[9]:=

```
Block[{eA $\alpha$ , e $\alpha$ A, t, ginv,  $\Gamma$ , gg(*,  $\omega$  $\omega$ *), 
(* $\omega$  $\omega$ = $\omega$  $_{\mu}^{(a)}$  $_{(b)}$  /. s $\omega$ f;*)
gg = g4488 /. sg;
{ginv,  $\Gamma$ } = con[gg];
e $\alpha$ A = e $_{(A)}^{\alpha}$  /. sge $_{(A)}^{\alpha}$ ; eA $\alpha$  = e $_{\alpha}^{(A)}$  /. sge $_{\alpha}^{(A)}$ ;
t = Table[Block[{v, V, contra, covari},
V = eA $\alpha$ [b, All];
v = e $\alpha$ A[All, b];
contra =
Table[D[v[k], X[ $\mu$ ]] + Sum[v[i] $\times$  $\Gamma$ [k, i,  $\mu$ ], {i, 1, DIM8}], {k, 1, DIM8}];
(*no  $\omega$  terms*)
covari = Table[D[V[k], X[ $\mu$ ]] - Sum[V[i] $\times$  $\Gamma$ [i, k,  $\mu$ ], {i, 1, DIM8}] +
Sum[eA $\alpha$ [j, All][k] $\times$  $\omega$ mat[ $\mu$ , b, j], {j, 1, DIM8}],
{k, 1, DIM8}]; (* $\omega$  terms*)
{b - 1, X[ $\mu$ ], {{contra}, {covari}}}], {b, 1, Length[e $\alpha$ A]}, { $\mu$ , 1, DIM8}];
t]
FullSimplify[#, ss $\omega$ g4488,
constraintVars && e $^{2 a4[H x4]} > 0$  && e $^{a4[H x4]} > 0$  && e $^{-2 a4[H x4]} > 0$  && e $^{-a4[H x4]} > 0$ ] & /@ %

```

Out[9]=

$$\left\{ \left\{ 0, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \right. \right.$$

$$\begin{aligned}
& \{1, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x_2, \left\{ \left\{ -\frac{g[2][2]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[2][2][x_0, x_4]}}, 0, \right. \right. \\
& \left. \left. 0, 0, -\frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{2, x_3, \\
& \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x_3, \left\{ \left\{ -\frac{g[3][3]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[3][3][x_0, x_4]}}, 0, \right. \right. \\
& \left. \left. 0, 0, -\frac{g[3][3]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{3, x_4, \\
& \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{3, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x_0, \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, \right. \right. \\
& \left. \left. \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} - \right. \\
& \left. \frac{g[4][4]^{(1,0)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]} - \left(\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \right. \right. \\
& \left. \left. \frac{g[4][4]^{(1,0)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]} \right) g[4][4][x_0, x_4], 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 4, x_1, \left\{ \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x_0, x_4]}{2 g[1][1][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 4, x_2, \left\{ \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 g[2][2][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 4, x_3, \left\{ \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4]}{2 g[3][3][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 4, x_4, \left\{ \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, \right. \right. \right. \\
& \left. \left. \left. \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}} - \right. \\
& \left. \frac{g[4][4]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]} - \left(\frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \right. \right. \\
& \left. \left. \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]} \right) g[4][4][x_0, x_4], 0, 0, 0 \} \} \} \}, \\
& \{ 4, x_5, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\} \right\}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 4, x_6, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\} \right\}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 4, x_7, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\} \right\}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 5, x_0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \right. \right. \right. \\
& \left. \left. \left. \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}
\end{aligned}$$

$$\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, \{ \{0, 0, 0, 0, 0, 0, 0, 0\} \},$$

$$- \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}} - \frac{g[5][5]^{(1,0)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]} -$$

$$\left(\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]} \right) g[5][5][x_0, x_4], \{ \{0, 0, 0, 0, 0, 0, 0, 0\} \} \},$$

$$\{5, x_1, \{ \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \} \},$$

$$\{5, x_2, \{ \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \} \},$$

$$\{5, x_3, \{ \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \} \},$$

$$\{5, x_4, \{ \{ \{0, 0, 0, 0, 0, 0,$$

$$\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, \{ \{0, 0, 0, 0, 0, 0, 0, 0\} \},$$

$$\left\{ \{0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}} - \right.$$

$$\frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]} - \left(\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \right.$$

$$\left. \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]} \right) g[5][5][x_0, x_4], \{ \{0, 0, 0, 0, 0, 0\} \} \},$$

$$\{5, x_5, \{ \{ \{ -\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}, \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \},$$

$$-\frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}, \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \},$$

$$\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \}, \{5, x_6,$$

$$\{ \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \} \},$$

$$\{5, x_7, \{ \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}, \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \} \},$$

$$\{6, x_0, \{ \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \},$$

$$\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 (-g[6][6][x_0, x_4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]} g[6][6][x_0, x_4]}, \{ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \},$$

$$\left\{ \{0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[6][6][x_0, x_4]}} - \right.$$

$$\frac{g[6][6]^{(1,0)}[x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}{2 g[6][6][x_0, x_4]} - \left(\frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 (-g[6][6][x_0, x_4])^{3/2}} + \right.$$

$$\left. \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) g[6][6][x0, x4], 0 \} \} \},$$

$$\{6, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{6, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{6, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{6, x4, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. ,$$

$$\frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0 \} \},$$

$$\left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}} - \right. \right. \right. \right. \right. ,$$

$$\frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]} - \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \right.$$

$$\left. \left. \left. \left. \left. \left. \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) g[6][6][x0, x4], 0 \} \} \right\} \right\} ,$$

$$\{6, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{6, x6, \left\{ \left\{ \left\{ -\frac{g[6][6]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0, \right. \right. \right. \right. \right. ,$$

$$0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0 \} \},$$

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\} \} \}, \{6, x7,$$

$$\{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{7, x0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. , \frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} +$$

$$\frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \} \},$$

$$\left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}} - \right. \right. \right. \right. \right. ,$$

$$\frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 g[7][7][x0, x4]} - \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \right.$$

$$\left. \left. \left. \left. \left. \left. \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) g[7][7][x0, x4] \right\} \right\} \right\} ,$$

$$\{7, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{7, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\{7, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$$

$$\begin{aligned}
& \left\{ 7, x_4, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 (-g[7][7][x_0, x_4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right\}, \\
& \quad \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]}} - \right. \right. \\
& \quad \left. \frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2 g[7][7][x_0, x_4]} - \left(\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 (-g[7][7][x_0, x_4])^{3/2}} + \right. \right. \\
& \quad \left. \left. \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right) g[7][7][x_0, x_4] \right\} \right\}, \\
& \{7, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \left\{ 7, x_7, \left\{ \left\{ \left\{ -\frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}, 0, \right. \right. \right. \right. \right. \\
& \quad 0, 0, -\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}, 0, 0, 0 \right\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\} \right\} \right\}
\end{aligned}$$

Out[•]=

$$\begin{aligned}
& \{ \{ \{ 0, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 0, x_1, \{\{\{0, H, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 0, x_2, \{\{\{0, 0, H, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 0, x_3, \{\{\{0, 0, 0, H, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 0, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 0, x_5, \{\{\{0, 0, 0, 0, 0, H, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 0, x_6, \{\{\{0, 0, 0, 0, 0, 0, H, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 0, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, H\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 1, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 1, x_1, \left\{ \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{2 a4[H x4]}} \right. \right. \right. \right. \right. \\
& \quad H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{ 1, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 1, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 1, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 1, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 1, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 1, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{ 2, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},
\end{aligned}$$


```

{6, x2, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{6, x3, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{6, x4, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{6, x5, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{6, x6, {{\sqrt{e^{-2 a4[H x4]}} H Sec[6 H x0] Sin[6 H x0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{-2 a4[H x4]}} H
Sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, ,
{6, x7, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{7, x0, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{7, x1, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{7, x2, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{7, x3, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{7, x4, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{7, x5, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{7, x6, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, ,
{7, x7, {{\sqrt{e^{-2 a4[H x4]}} H Sec[6 H x0] Sin[6 H x0]^{7/6}, 0, 0, 0, 3 \sqrt{e^{-2 a4[H x4]}} H
Sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}

```

In[498]:=

```

(*w\Psi=Table[Sum[ExpandAll[
  \left( \omega_{\mu}^{(a)}_{(b)} [\mu,a,b]/.swf \right) Simplify[(Transpose[\Psi16].\sigma16.SAB[a,b].\Psi16)], 
  {a,1,8},{b,1,8}],{\mu,1,8}]*)

```

In[499]:=

```

(*w\Psi=Table[
  Sum[ExpandAll[(\omega mat[[\mu,a,b]]) Simplify[(Transpose[\Psi16].\sigma16.SAB[a,b].\Psi16)]], 
  {a,1,8},{b,1,8}],{\mu,1,8}]*)

```

In[500]:=

```

(*jw\Psi=ParallelTable[{j,base16[[j,2]],
  Table[{X[[\mu]],Simplify[(Transpose[\Psi16].\sigma16.(ZZZ0*T16^\alpha[0]+T16^\alpha[4]* ZZZ4 ) .
  base16[[j,1]].(Sum[\omega mat[[\mu,a,b]]SAB[a,b],{a,1,8},{b,1,8}])\Psi16)]},
  {\mu,1,8}]},{j,1,Length[base16}]*)

```

In[501]:=

```

(*jw\Psi=ParallelTable[{j,base16[[j,2]],
  Table[Sum[ExpandAll[\left( \omega_{\mu}^{(a)}_{(b)} [\mu,a,b]/.swf \right) Simplify[(Transpose[\Psi16].\sigma16.
  (ZZZ0*T16^\alpha[0]+T16^\alpha[4]* ZZZ4 ).base16[[j,1]].SAB[a,b].\Psi16)]],
  {a,1,8},{b,1,8}],{\mu,1,8}},{j,1,Length[base16}]}*)

```

In[502]:=

```
(*jw\Psi[[92]]*)
```


In[507]:=

T16^A [θ]

Out[507]=

In[•]:=

SAB =

$$\text{Table}\left[\frac{1}{4} \left(\text{T16}^{\text{A}}[\text{A1}].\text{T16}^{\text{A}}[\text{B1}]-\text{T16}^{\text{A}}[\text{B1}].\text{T16}^{\text{A}}[\text{A1}]\right), \{\text{A1}, 0, 7\}, \{\text{B1}, 0, 7\}\right];$$

In[508]:=

Ψ16

Out[508]=

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

In[509]:=

```
Sum[ExpandAll[(wμ(a)(b) [4, a, b] /. swfgtry) Simplify[(SAB[[a, b]])]], {a, 1, 8}, {b, 1, 8}].Π16 // MatrixForm
```

Out[509]//MatrixForm=

```

- ea4[H x4] H Sin[6 H x0]1/6 f16[5][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[4][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[7][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[6][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[1][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[0][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[3][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[2][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[13][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[12][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[15][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[14][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[9][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[8][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[11][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[10][x0, x4]

```

```
In[510]:= (*Table[Block[{eA $\alpha$ , e $\alpha$ A, t, V}, e $\alpha$ A=e $\alpha$ (A) /. sgtrye $\alpha$ (A);
eA $\alpha$ =e $\alpha$ (A) /. sgtrye $\alpha$ (A);
V=eA $\alpha$ [[a,All]];
t=Table[Block[{v,d},v=e $\alpha$ A[[All,b]];
d=D[v[[k]],X[[j]]]+Sum[v[[i]] $\Gamma$ [[k,i,j]],{i,1,DIM8}];
d.V],{k,1,DIM8}];
t],[{j,1,DIM8},{a,1,Length[e $\alpha$ A]},{b,1,Length[e $\alpha$ A]}]*)

In[511]:= Clear[rt];
rt[g_, ass_ : {}] := Module[{t,  $\Gamma$ , Ricci $\Gamma$ , Rie $\Gamma$ , RS, G, ginv},
Print[Now];
t = AbsoluteTiming[
ginv = FullSimplify[Inverse[g], ass];
 $\Gamma$  = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]) *
(D[g[[s, j], X[[k]]] + D[g[[s, k], X[[j]]] - D[g[[j, k], X[[s]]]), {s, 1, DIM8
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
Ricci $\Gamma$  = ParallelTable[
FullSimplify[D[ $\Gamma$ [[ $\mu$ , v,  $\beta$ ], X[[ $\alpha$ ]] - D[ $\Gamma$ [[ $\mu$ , v,  $\alpha$ ], X[[ $\beta$ ]] + Sum[
 $\Gamma$ [[ $\mu$ , s,  $\alpha$ ] $\times$  $\Gamma$ [[s, v,  $\beta$ ] -  $\Gamma$ [[ $\mu$ , s,  $\beta$ ] $\times$  $\Gamma$ [[s, v,  $\alpha$ ], {s, 1, DIM8}], ass],
{ $\mu$ , 1, DIM8}, {v, 1, DIM8}, { $\alpha$ , 1, DIM8}, { $\beta$ , 1, DIM8}]];
Rie $\Gamma$  =
ParallelTable[ FullSimplify[Sum[Ricci $\Gamma$ [[ $\alpha$ ,  $\mu$ ,  $\alpha$ , v], { $\alpha$ , 1, DIM8}], ass],
{ $\mu$ , 1, DIM8}, {v, 1, DIM8}];
RS = FullSimplify[Tr[ginv.Rie $\Gamma$ ], ass];
G = ParallelTable[
FullSimplify[Rie $\Gamma$ [[ $\alpha$ ]] -  $\frac{1}{2}$  g[[ $\alpha$ ]] RS, ass], { $\alpha$ , 1, Length[Rie $\Gamma$ ]}];
Print[t];
Print[Now];
Return[{ginv,  $\Gamma$ , Ricci $\Gamma$ , Rie $\Gamma$ , RS, G}]]
```

verify $g_{\mu\nu|\alpha} = 0$:

```
In[511]:= Simplify[
Table[D[gtry[[j, k], X[[i]]] - Sum[gtry[[k, s]] $\times$  $\Gamma$ [[s, i, j]], {s, 1, DIM8}] -
Sum[gtry[[j, s]] $\times$  $\Gamma$ [[s, i, k]], {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union

Out[511]= {0}
```

■ CHECK

In[512]:=

MatrixMetric44 // MatrixForm

Out[512]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6Hx\theta]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a^4[Hx^4]} \sin[6Hx\theta]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a^4[Hx^4]} \sin[6Hx\theta]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a^4[Hx^4]} \sin[6Hx\theta]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[513]:=

(e^α_(A).MatrixMetric44.e^α_(A) /. sgtrye^α_(A)) // MatrixForm

Out[513]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[514]:=

```
Transpose[eα(A)].η4488.eα(A) - MatrixMetric44 /. sgtryeα(A) //  
FullSimplify[#, constraintVars] &
```

Out[514]=

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\}$$

In[515]:=

```
Transpose[eα(A)].MatrixMetric44.eα(A) - η4488 /. sgtryeα(A) //  
FullSimplify[#, constraintVars] &
```

Out[515]=

$$\{\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}\}$$

In[516]:=

```
Inverse[eα(A) /. sgtryeα(A)] = eα(A) /. sgtryeα(A) // FullSimplify[#, constraintVars] &
```

Out[516]=

True

In[517]:=

```
Block[{a, b, c, d, m}, m = {{a, b}, {c, d}};  
Inverse[Transpose[m]] - Transpose[Inverse[m]]]
```

Out[517]=

$$\{\{0, 0\}, \{0, 0\}\}$$

```
In[518]:= (*Inverse[Transpose[e(A)].MatrixMetric44.e(A)] ==
e(A).Inverse[MatrixMetric44].Transpose[e(A)] //*
FullSimplify[#,constraintVars]&*)

In[519]:= (*e(A).Inverse[MatrixMetric44].Transpose[e(A)] //*
FullSimplify[#,constraintVars]&
%-Inverse[η4488]//Union[Flatten[#]]&*)

In[520]:= (*Transpose[e(A)]==e(A)//FullSimplify[#,constraintVars]&*)

In[521]:= (* (FullSimplify[e(A).Inverse[MatrixMetric44].Transpose[e(A)] -
Inverse[η4488],constraintVars] //
Union[Flatten[#]]&) //FullSimplify[#,constraintVars]&*)

In[522]:= (* (FullSimplify[Transpose[e(A)].Inverse[MatrixMetric44].e(A),
constraintVars]-Inverse[η4488]//
Union[Flatten[#]]&) //FullSimplify[#,constraintVars]&*)

In[523]:= (* (FullSimplify[Transpose[e(A)].ginv.e(A),constraintVars]-Inverse[η4488]//
Union[Flatten[#]]&) //FullSimplify[#,constraintVars]&*)

In[*]:= (E(A)α=Table[Sum[e(A)α1A1,α1] ginv[α1,β1]×η4488[A1,B1] /.sgtrye(A)α,
{α1,1,8},{A1,1,8}],{β1,1,8},{B1,1,8}]//*
FullSimplify[#,constraintVars]&) //Column
```

Out[*]=

```
{Tan[6 H x0], 0, 0, 0, 0, 0, 0, 0}
{0, e-a4[x4]/Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0}
{0, 0, e-a4[x4]/Sin[6 H x0]1/6, 0, 0, 0, 0, 0}
{0, 0, 0, e-a4[x4]/Sin[6 H x0]1/6, 0, 0, 0, 0}
{0, 0, 0, 0, 1, 0, 0, 0}
{0, 0, 0, 0, 0, ea4[x4]/Sin[6 H x0]1/6, 0, 0}
{0, 0, 0, 0, 0, 0, ea4[x4]/Sin[6 H x0]1/6, 0}
{0, 0, 0, 0, 0, 0, 0, ea4[x4]/Sin[6 H x0]1/6}
```

Lagrangian_{¶16} =

$\kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi16].\sigma16.T16^A[A1].\Psi16_{,\alpha} g^{-1}{}^{\alpha\beta} \eta_{A1 B1} E_\beta^{B1} + \text{mass term}$

In[524]:=

ass = constraintVars

Out[524]=

$$\begin{aligned}
 & x_0 > 0 \&& x_4 > 0 \&& H > 0 \&& 6Hx_0 > 0 \&& 3Hx_4 > 0 \&& a4[x4] > 0 \&& A4[t] > 0 \&& \\
 & Q > 0 \&& z > 0 \&& t > 0 \&& M > 0 \&& K > 0 \&& e^{-2a4[Hx4]} > 0 \&& e^{-a4[Hx4]} > 0 \&& e^{2a4[Hx4]} > 0 \&& \\
 & e^{a4[Hx4]} > 0 \&& \sin[6Hx_0] > 0 \&& \cos[6Hx_0] > 0 \&& \csc[6Hx_0] > 0 \&& \sec[6Hx_0] > 0 \&& \\
 & \tan[6Hx_0] > 0 \&& \cot[6Hx_0] > 0 \&& \sin[z] > 0 \&& \sqrt{\sin[z]} > 0 \&& \sin[z]^{3/2} > 0 \&& \\
 & \sqrt{\sin[z]} > 0 \&& \frac{1}{\sin[z]^{3/2}} > 0 \&& \frac{1}{\sqrt{\sin[z]}} > 0 \&& \cot[z] > 0 \&& \sqrt{\cot[z]} > 0 \&& \\
 & \cot[z]^{3/2} > 0 \&& \sqrt{\cot[z]} > 0 \&& \tan[z] > 0 \&& \sec[z] > 0 \&& \sqrt{\csc[z]} > 0 \&& \csc[z] > 0
 \end{aligned}$$

In[525]:=

$$\begin{aligned}
 & x_0 > 0 \&& x_4 > 0 \&& H > 0 \&& 6Hx_0 > 0 \&& Hx_4 > 0 \&& a4[x4] > 0 \&& A4[t] > 0 \&& Q > 0 \&& \\
 & z > 0 \&& t > 0 \&& M > 0 \&& K > 0 \&& e^{-2a4[Hx4]} > 0 \&& e^{-a4[Hx4]} > 0 \&& e^{2a4[Hx4]} > 0 \&& \\
 & e^{a4[Hx4]} > 0 \&& \sin[6Hx_0] > 0 \&& \cos[6Hx_0] > 0 \&& \csc[6Hx_0] > 0 \&& \sec[6Hx_0] > 0 \&& \\
 & \tan[6Hx_0] > 0 \&& \cot[6Hx_0] > 0 \&& \sin[z] > 0 \&& \sqrt{\sin[z]} > 0 \&& \sin[z]^{3/2} > 0 \&& \\
 & \sqrt{\sin[z]} > 0 \&& \frac{1}{\sin[z]^{3/2}} > 0 \&& \frac{1}{\sqrt{\sin[z]}} > 0 \&& \cot[z] > 0 \&& \sqrt{\cot[z]} > 0 \&& \\
 & \cot[z]^{3/2} > 0 \&& \sqrt{\cot[z]} > 0 \&& \tan[z] > 0 \&& \sec[z] > 0 \&& \sqrt{\csc[z]} > 0 \&& \csc[z] > 0
 \end{aligned}$$

Out[525]=

$$\begin{aligned}
 & x_0 > 0 \&& x_4 > 0 \&& H > 0 \&& 6Hx_0 > 0 \&& Hx_4 > 0 \&& a4[x4] > 0 \&& A4[t] > 0 \&& \\
 & Q > 0 \&& z > 0 \&& t > 0 \&& M > 0 \&& K > 0 \&& e^{-2a4[Hx4]} > 0 \&& e^{-a4[Hx4]} > 0 \&& e^{2a4[Hx4]} > 0 \&& \\
 & e^{a4[Hx4]} > 0 \&& \sin[6Hx_0] > 0 \&& \cos[6Hx_0] > 0 \&& \csc[6Hx_0] > 0 \&& \sec[6Hx_0] > 0 \&& \\
 & \tan[6Hx_0] > 0 \&& \cot[6Hx_0] > 0 \&& \sin[z] > 0 \&& \sqrt{\sin[z]} > 0 \&& \sin[z]^{3/2} > 0 \&& \\
 & \sqrt{\sin[z]} > 0 \&& \frac{1}{\sin[z]^{3/2}} > 0 \&& \frac{1}{\sqrt{\sin[z]}} > 0 \&& \cot[z] > 0 \&& \sqrt{\cot[z]} > 0 \&& \\
 & \cot[z]^{3/2} > 0 \&& \sqrt{\cot[z]} > 0 \&& \tan[z] > 0 \&& \sec[z] > 0 \&& \sqrt{\csc[z]} > 0 \&& \csc[z] > 0
 \end{aligned}$$

In[526]:=

ginv = FullSimplify[Inverse[gtry], ass]

Out[526]=

True

In[527]:=

ginv === Transpose[ginv]

Out[527]=

True

verify metric covariant derivative again:

In[528]:=

```

Simplify[
  Table[D[gtry[[j, k]], X[[i]]] - Sum[gtry[[k, s]] \Gamma[[s, i, j]], {s, 1, DIM8}] -
    Sum[gtry[[j, s]] \Gamma[[s, i, k]], {s, 1, DIM8}],
  {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union

```

Out[528]=

{0}

In[529]:=

RS

Out[529]=

$$6 H^2 (-7 + a4' [H x4]^2)$$

In[530]:=

EinsteinG**skelx[%]**

Out[530]=

$$\begin{aligned} & \left\{ -3 H^2 \operatorname{Cot}[6 H x0]^2 (-5 + a4' [H x4]^2), 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, -e^{2 a4 [H x4]} H^2 \sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 - a4'' [H x4]), 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, -e^{2 a4 [H x4]} H^2 \sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 - a4'' [H x4]), 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, -e^{2 a4 [H x4]} H^2 \sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 - a4'' [H x4]), 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, -3 H^2 (7 + a4' [H x4]^2), 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, e^{-2 a4 [H x4]} H^2 \sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 + a4'' [H x4]), 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, e^{-2 a4 [H x4]} H^2 \sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 + a4'' [H x4]), 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, e^{-2 a4 [H x4]} H^2 \sin[6 H x0]^{1/3} (-15 + 3 a4' [H x4]^2 + a4'' [H x4]) \right\} \end{aligned}$$

Out[531]/MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{pmatrix}$$

$$\begin{aligned} & \text{Union}\left[\text{Table}\left[\right.\right. \\ & \left. \left. \text{Sum}\left[\tau[A].\text{bas64}[k, 1].\sigma.\bar{\tau}[A] \times \eta8[A, A], \{A, 1, 8\}\right] - \text{ID8}\text{Tr}[\text{bas64}[k, 1].\sigma] + \right. \right. \\ & \left. \left. 4 \text{ eg}[k] \frac{\text{bas64}[k, 1] - \text{Transpose}[\text{bas64}[k, 1]]}{2} .\sigma, \{k, 1, 64\}\right]\right] \\ & \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \end{aligned}$$

BASIS of 16×16 matrices :

In[532]:=

16 * 16

Out[532]=

256

In[533]:=

 2^8

Out[533]=

256

```

In[534]:= Sum[Binomial[8, p], {p, 0, 8}]
Out[534]= 256

In[535]:= Id16 == ID16 == Dot @@ (T16A[#] & /@ Range[0, 8])
Out[535]= True

In[536]:= Clear[t160]; t160 = {{Id16, {Range[0, 8]}}}; Length[t160]
Clear[t16A];
t16A = {};
Do[AppendTo[t16A, {T16A[j], {j}}], {j, 0, 7}];
Length[t16A]
Clear[t16AB];
t16AB = {};
Do[AppendTo[t16AB, {T16A[j].T16A[k], {j, k}}], {j, 0, 6}, {k, j + 1, 7}];
Length[t16AB]
Clear[t16ABC];
t16ABC = {};
Do[AppendTo[t16ABC, {T16A[j].T16A[k].T16A[h], {j, k, h}}], {j, 0, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[t16ABC]
Clear[t16ABCD];
t16ABCD = {};
Do[AppendTo[t16ABCD, {T16A[j].T16A[k].T16A[h].T16A[i], {j, k, h, i}}], {j, 0, 4}, {k, j + 1, 5}, {h, k + 1, 6}, {i, h + 1, 7}];
Length[t16ABCD]
Clear[t16ABCDE];
t16ABCDE = {};
Do[AppendTo[t16ABCDE, {T16A[j].T16A[k].T16A[h].T16A[i].T16A[l], {j, k, h, i, l}}], {j, 0, 3}, {k, j + 1, 4}, {h, k + 1, 5}, {i, h + 1, 6}, {l, i + 1, 7}];
Length[t16ABCDE]
Clear[t16ABCDEF];
t16ABCDEF = {};
Do[AppendTo[t16ABCDEF,
{T16A[j].T16A[k].T16A[h].T16A[i].T16A[l].T16A[q], {j, k, h, i, l, q}}], {j, 0, 2}, {k, j + 1, 3}, {h, k + 1, 4}, {i, h + 1, 5}, {l, i + 1, 6}, {q, l + 1, 7}];
Length[t16ABCDEF]
Clear[t16ABCDEFG];

```

```
t16ABCDEFG = {};
Do[AppendTo[t16ABCDEFG, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].
    T16^A[q].T16^A[r], {j, k, h, i, l, q, r}}], {j, 0, 1}, {k, j + 1, 2},
    {h, k + 1, 3}, {i, h + 1, 4}, {l, i + 1, 5}, {q, l + 1, 6}, {r, q + 1, 7}];
Length[t16ABCDEFG]
Clear[t16ABCDEFGH];
t16ABCDEFGH = {};
Do[AppendTo[t16ABCDEFGH, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].
    T16^A[r].T16^A[s], {j, k, h, i, l, q, r, s}}], {j, 0, 1}, {k, j + 1, 1}, {h,
    k + 1, 2}, {i, h + 1, 3}, {l, i + 1, 4}, {q, l + 1, 5}, {r, q + 1, 6}, {s, r + 1, 7}];
Length[t16ABCDEFGH]
% + %% + %% + %%% + %%%% + %%%%% + %%%%%% + %%%%%%% + %%%%%%%%
```

Out[536]=

1

Out[537]=

8

Out[538]=

28

Out[539]=

56

Out[540]=

70

Out[541]=

56

Out[542]=

28

Out[543]=

8

Out[544]=

1

Out[545]=

256

In[546]:=

```
(*base16=Flatten[{t160,t16A,t16AB,t16ABC,
    t16ABCD,t16ABCDE,t16ABCDEF,t16ABCDEFG,t16ABCDEFGH},1];
Length[base16]*)
```

In[547]:=

```
base16 = Flatten[{t16A, t16AB, t16ABC, t16ABCD,
    t16ABCDE, t16ABCDEF, t16ABCDEFG, t16ABCDEFGH, t160}, 1];
Length[base16]
```

Out[547]=

256

In[558]:=

(*BASE16[-1]*)

In[566]:=

```
Clear[symmm16];
symmm16 = {};
Do[If[base16[[k, 1]] === Transpose[base16[[k, 1]]], AppendTo[symmm16,
    {base16[[k, 1]], {{k}, base16[[k, 2]]}}], {k, 1, Length[base16]}];
Length[symmm16]
symmm16[[%]]
```

Out[567]=

136

Out[568]=

```
{{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

In[569]:=

```
dupssymmm16 = Select[symmm16, MemberQ[symmm16[[All, 1]], -#[[1]]] &];
Length[dupssymmm16]
```

Out[569]=

0

```
In[570]:= #[[2]] & /@ symm16
Out[570]= {{}, {{1}, {0}}, {{2}, {1}}, {{3}, {2}}, {{4}, {3}}, {{12}, {0, 4}}, {{13}, {0, 5}}, {{14}, {0, 6}}, {{15}, {0, 7}}, {{18}, {1, 4}}, {{19}, {1, 5}}, {{20}, {1, 6}}, {{21}, {1, 7}}, {{23}, {2, 4}}, {{24}, {2, 5}}, {{25}, {2, 6}}, {{26}, {2, 7}}, {{27}, {3, 4}}, {{28}, {3, 5}}, {{29}, {3, 6}}, {{30}, {3, 7}}, {{39}, {0, 1, 4}}, {{40}, {0, 1, 5}}, {{41}, {0, 1, 6}}, {{42}, {0, 1, 7}}, {{44}, {0, 2, 4}}, {{45}, {0, 2, 5}}, {{46}, {0, 2, 6}}, {{47}, {0, 2, 7}}, {{48}, {0, 3, 4}}, {{49}, {0, 3, 5}}, {{50}, {0, 3, 6}}, {{51}, {0, 3, 7}}, {{59}, {1, 2, 4}}, {{60}, {1, 2, 5}}, {{61}, {1, 2, 6}}, {{62}, {1, 2, 7}}, {{63}, {1, 3, 4}}, {{64}, {1, 3, 5}}, {{65}, {1, 3, 6}}, {{66}, {1, 3, 7}}, {{73}, {2, 3, 4}}, {{74}, {2, 3, 5}}, {{75}, {2, 3, 6}}, {{76}, {2, 3, 7}}, {{89}, {4, 5, 6}}, {{90}, {4, 5, 7}}, {{91}, {4, 6, 7}}, {{92}, {5, 6, 7}}, {{93}, {0, 1, 2, 3}}, {{102}, {0, 1, 4, 5}}, {{103}, {0, 1, 4, 6}}, {{104}, {0, 1, 4, 7}}, {{105}, {0, 1, 5, 6}}, {{106}, {0, 1, 5, 7}}, {{107}, {0, 1, 6, 7}}, {{112}, {0, 2, 4, 5}}, {{113}, {0, 2, 4, 6}}, {{114}, {0, 2, 4, 7}}, {{115}, {0, 2, 5, 6}}, {{116}, {0, 2, 5, 7}}, {{117}, {0, 2, 6, 7}}, {{118}, {0, 3, 4, 5}}, {{119}, {0, 3, 4, 6}}, {{120}, {0, 3, 4, 7}}, {{121}, {0, 3, 5, 6}}, {{122}, {0, 3, 5, 7}}, {{123}, {0, 3, 6, 7}}, {{132}, {1, 2, 4, 5}}, {{133}, {1, 2, 4, 6}}, {{134}, {1, 2, 4, 7}}, {{135}, {1, 2, 5, 6}}, {{136}, {1, 2, 5, 7}}, {{137}, {1, 2, 6, 7}}, {{138}, {1, 3, 4, 5}}, {{139}, {1, 3, 4, 6}}, {{140}, {1, 3, 4, 7}}, {{141}, {1, 3, 5, 6}}, {{142}, {1, 3, 5, 7}}, {{143}, {1, 3, 6, 7}}, {{148}, {2, 3, 4, 5}}, {{149}, {2, 3, 4, 6}}, {{150}, {2, 3, 4, 7}}, {{151}, {2, 3, 5, 6}}, {{152}, {2, 3, 5, 7}}, {{153}, {2, 3, 6, 7}}, {{162}, {4, 5, 6, 7}}, {{167}, {0, 1, 2, 4, 5}}, {{168}, {0, 1, 2, 4, 6}}, {{169}, {0, 1, 2, 4, 7}}, {{170}, {0, 1, 2, 5, 6}}, {{171}, {0, 1, 2, 5, 7}}, {{172}, {0, 1, 2, 6, 7}}, {{173}, {0, 1, 3, 4, 5}}, {{174}, {0, 1, 3, 4, 6}}, {{175}, {0, 1, 3, 4, 7}}, {{176}, {0, 1, 3, 5, 6}}, {{177}, {0, 1, 3, 5, 7}}, {{178}, {0, 1, 3, 6, 7}}, {{183}, {0, 2, 3, 4, 5}}, {{184}, {0, 2, 3, 4, 6}}, {{185}, {0, 2, 3, 4, 7}}, {{186}, {0, 2, 3, 5, 6}}, {{187}, {0, 2, 3, 5, 7}}, {{188}, {0, 2, 3, 6, 7}}, {{197}, {0, 4, 5, 6, 7}}, {{198}, {1, 2, 3, 4, 5}}, {{199}, {1, 2, 3, 4, 6}}, {{200}, {1, 2, 3, 4, 7}}, {{201}, {1, 2, 3, 5, 6}}, {{202}, {1, 2, 3, 5, 7}}, {{203}, {1, 2, 3, 6, 7}}, {{212}, {1, 4, 5, 6, 7}}, {{217}, {2, 4, 5, 6, 7}}, {{218}, {3, 4, 5, 6, 7}}, {{225}, {0, 1, 2, 4, 5, 6}}, {{226}, {0, 1, 2, 4, 5, 7}}, {{227}, {0, 1, 2, 4, 6, 7}}, {{228}, {0, 1, 2, 5, 6, 7}}, {{229}, {0, 1, 3, 4, 5, 6}}, {{230}, {0, 1, 3, 4, 5, 7}}, {{231}, {0, 1, 3, 4, 6, 7}}, {{232}, {0, 1, 3, 5, 6, 7}}, {{234}, {0, 2, 3, 4, 5, 6}}, {{235}, {0, 2, 3, 4, 5, 7}}, {{236}, {0, 2, 3, 4, 6, 7}}, {{237}, {0, 2, 3, 5, 6, 7}}, {{240}, {1, 2, 3, 4, 5, 6}}, {{241}, {1, 2, 3, 4, 5, 7}}, {{242}, {1, 2, 3, 4, 6, 7}}, {{243}, {1, 2, 3, 5, 6, 7}}, {{247}, {0, 1, 2, 3, 4, 5, 6}}, {{248}, {0, 1, 2, 3, 4, 5, 7}}, {{249}, {0, 1, 2, 3, 4, 6, 7}}, {{250}, {0, 1, 2, 3, 5, 6, 7}}, {{255}, {0, 1, 2, 3, 4, 5, 6, 7}}, {{256}, {0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

```
In[571]:= σ16 = T16^A[0].T16^A[1].T16^A[2].T16^A[3]
```

```
Out[571]= True
```

```
In[572]:= σ16 == base16[[93]][1]
```

```
Out[572]= True
```


■ similarly :

```
In[581]:= 
  coupled1 = {4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13} // Union
Out[581]=
{0, 1, 4, 5, 8, 9, 12, 13}

In[582]:= 
  coupled2 = Complement[Range[0, 15], coupled1]
Out[582]=
{2, 3, 6, 7, 10, 11, 14, 15}

In[583]:= 
  coupled = {coupled1, coupled2}
Out[583]=
{{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15} }

In[584]:= 
  Block[{m, vars, eqs, eqs1, eqs2, s, #16},
    #16 = f[#] & /@ Range[16];
    m = Array[qb[#1, #2] &, {16, 16}];
    vars = Flatten[m];
    eqs1 = And @@ Thread[m.#16 == Flatten[{#16[[# + 1]] & /@ coupled1, #16[[# + 1]] & /@ coupled2}]];
    eqs2 = And @@ Thread[m.Transpose[m] == Flatten[ID16]];
    eqs = eqs1 && eqs2;
    s = Solve[eqs, vars] // FullSimplify;
    s
  ]
Out[584]=
{ }
```


In[586]:=

`linTrans // MatrixForm`

Out[586]//MatrixForm=

In[587]:=

```
linTrans.Transpose[linTrans] // FullSimplify // MatrixForm
```

Out[587]//MatrixForm=

```
In[588]:= linTrans.¶16 // Column
Union[%[[1]]]

Out[588]= f16[0][x0, x4]
f16[1][x0, x4]
f16[8][x0, x4]
f16[9][x0, x4]
f16[2][x0, x4]
f16[3][x0, x4]
f16[10][x0, x4]
f16[11][x0, x4]
f16[4][x0, x4]
f16[5][x0, x4]
f16[12][x0, x4]
f16[13][x0, x4]
f16[6][x0, x4]
f16[7][x0, x4]
f16[14][x0, x4]
f16[15][x0, x4]

Out[589]= {f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}

In[=]:= positiveTrMM = Select[base16, Tr#[[1]] . #[[1]]] > 0 & -> "Index"]

Out[=]= {1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50,
51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114,
115, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149,
150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188,
197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240,
241, 242, 243, 247, 248, 249, 250, 255, 256}

In[=]:= negativeTrMM = Select[base16, Tr#[[1]] . #[[1]]] < 0 & -> "Index"]

Out[=]= {5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72,
77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126,
127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180,
181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219,
220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[=]:= {Length[positiveTrMM], Length[negativeTrMM]}

Out[=]= {136, 120}
```

```
In[590]:= positiveTrMM = Select[base16, Tr[#[1].#[1]] > 0 & → "Index"]
Out[590]= {1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[591]:= negativeTrMM = Select[base16, Tr[#[1].#[1]] < 0 & → "Index"]
Out[591]= {5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[592]:= {Length[positiveTrMM], Length[negativeTrMM]}
Out[592]= {136, 120}
```



```
In[598]:= base16[[#]] & /@ negativeTrMM;
{#[[2]], Tr[Transpose[#[[1]].linTrans]} & /@ %
Select[% , #[[2]] ≠ 0 &]
linTransAntiSymmetric =
Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];

Out[599]= {{{4}, 0}, {{5}, 0}, {{6}, 0}, {{7}, 0}, {{0, 1}, 0}, {{0, 2}, 0}, {{0, 3}, 0}, {{1, 2}, 0},
{{1, 3}, 0}, {{2, 3}, 0}, {{4, 5}, 0}, {{4, 6}, 0}, {{4, 7}, 0}, {{5, 6}, 0}, {{5, 7}, 0},
{{6, 7}, 0}, {{0, 1, 2}, 0}, {{0, 1, 3}, 0}, {{0, 2, 3}, 0}, {{0, 4, 5}, 0}, {{0, 4, 6}, 0},
{{0, 4, 7}, 0}, {{0, 5, 6}, 0}, {{0, 5, 7}, 0}, {{0, 6, 7}, 0}, {{1, 2, 3}, 0}, {{1, 4, 5}, 0},
{{1, 4, 6}, 0}, {{1, 4, 7}, 0}, {{1, 5, 6}, 0}, {{1, 5, 7}, 0}, {{1, 6, 7}, 0},
{{2, 4, 5}, 0}, {{2, 4, 6}, 0}, {{2, 4, 7}, 0}, {{2, 5, 6}, 0}, {{2, 5, 7}, 0},
{{2, 6, 7}, 0}, {{3, 4, 5}, 0}, {{3, 4, 6}, 0}, {{3, 4, 7}, 0}, {{3, 5, 6}, -4},
{{3, 5, 7}, 0}, {{3, 6, 7}, 0}, {{0, 1, 2, 4}, 0}, {{0, 1, 2, 5}, 0}, {{0, 1, 2, 6}, -4},
{{0, 1, 2, 7}, 0}, {{0, 1, 3, 4}, 0}, {{0, 1, 3, 5}, 0}, {{0, 1, 3, 6}, 0}, {{0, 1, 3, 7}, 0},
{{0, 2, 3, 4}, 0}, {{0, 2, 3, 5}, 0}, {{0, 2, 3, 6}, 0}, {{0, 2, 3, 7}, 0}, {{0, 4, 5, 6}, 0},
{{0, 4, 5, 7}, 0}, {{0, 4, 6, 7}, 0}, {{0, 5, 6, 7}, 0}, {{1, 2, 3, 4}, 0}, {{1, 2, 3, 5}, 0},
{{1, 2, 3, 6}, 0}, {{1, 2, 3, 7}, 0}, {{1, 4, 5, 6}, 0}, {{1, 4, 5, 7}, 0}, {{1, 4, 6, 7}, 0},
{{1, 5, 6, 7}, 0}, {{2, 4, 5, 6}, 0}, {{2, 4, 5, 7}, 0}, {{2, 4, 6, 7}, 0}, {{2, 5, 6, 7}, 0},
{{3, 4, 5, 6}, 0}, {{3, 4, 5, 7}, 0}, {{3, 4, 6, 7}, 0}, {{3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4}, 0},
{{0, 1, 2, 3, 5}, -4}, {{0, 1, 2, 3, 6}, 0}, {{0, 1, 2, 3, 7}, 0}, {{0, 1, 4, 5, 6}, 0},
{{0, 1, 4, 5, 7}, 0}, {{0, 1, 4, 6, 7}, 0}, {{0, 1, 5, 6, 7}, 0}, {{0, 2, 4, 5, 6}, 0},
{{0, 2, 4, 5, 7}, 0}, {{0, 2, 4, 6, 7}, 0}, {{0, 2, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6}, 0},
{{0, 3, 4, 5, 7}, 0}, {{0, 3, 4, 6, 7}, 0}, {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 6}, 0},
{{1, 2, 4, 5, 7}, 4}, {{1, 2, 4, 6, 7}, 0}, {{1, 2, 5, 6, 7}, 0}, {{1, 3, 4, 5, 6}, 0},
{{1, 3, 4, 5, 7}, 0}, {{1, 3, 4, 6, 7}, 0}, {{1, 3, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6}, 0},
{{2, 3, 4, 5, 7}, 0}, {{2, 3, 4, 6, 7}, 0}, {{2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5}, 0},
{{0, 1, 2, 3, 4, 6}, -4}, {{0, 1, 2, 3, 4, 7}, 0}, {{0, 1, 2, 3, 5, 6}, 0},
{{0, 1, 2, 3, 5, 7}, 0}, {{0, 1, 2, 3, 6, 7}, 0}, {{0, 1, 4, 5, 6, 7}, 0},
{{0, 2, 4, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6, 7}, 0}, {{1, 2, 4, 5, 6, 7}, 0},
{{1, 3, 4, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6, 7}, 0},
{{0, 1, 3, 4, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6, 7}, 0}}
Out[600]= {{3, 5, 6}, -4}, {{0, 1, 2, 6}, -4}, {{0, 1, 2, 3, 5}, -4},
{{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 7}, 4}, {{0, 1, 2, 3, 4, 6}, -4}}
In[602]:= {Length[linTransSymmetric], Length[linTransAntiSymmetric]}
Out[602]= {10, 6}
```

BASIS of 8×8 matrices :

```
In[603]:= tA = {} ; Do[AppendTo[tA, {τ[j], {j}}], {j, 1, 7}]
Length[tA]
tAB = {};
Do[AppendTo[tAB, {τ[j].τ[k], {j, k}}], {j, 1, 6}, {k, j+1, 7}]
Length[tAB]
tABC = {};
Do[AppendTo[tABC, {τ[j].τ[k].τ[h], {j, k, h}}], {j, 1, 5}, {k, j+1, 6}, {h, k+1, 7}];
Length[tABC]

Out[604]= 7

Out[606]= 21

Out[608]= 35

In[609]:= tA[[1]]
Out[609]= {{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {1} }

In[610]:= tA[[#]] [[1]] == -Transpose[tA[[#]] [[1]]] & /@ Range[Length[tA]]
Out[610]= {True, True, True, False, False, False}

In[611]:= bas64 = Flatten[{tA, tAB, tABC, {{Id, {64}}}}, 1]; Length[bas64]

Out[611]= 64

In[612]:= determineIF8AntiSymmetric[m_] := Module[{t, tt, sum, res, r},
  t = m.τ[#] & /@ Range[0, 7];
  tt = Transpose[#] & /@ t;
  sum = FullSimplify[t[[#]] + tt[[#]]] & /@ Range[Length[t]];
  res = Union[Flatten[#]] & /@ sum;
  If[Length[#] > 1, Return[False]] & /@ res;
  r = Union[Flatten[res]];
  Return[r == {0}];
]
```


In[622]:=

```

η64 = DiagonalMatrix[
  Table[FullSimplify[ $\frac{1}{8} \text{Tr}[\text{bas64}[A, 1].\text{bas64}[A, 1]]$ ], {A, 64}];
Tr[η64]

```

Out[622]=

$$7 + \frac{1}{8} \text{Tr}[\text{Id}.\text{Id}]$$

In[623]:=

```

(* (* (* (* (* (* (countTrace=
Table[{A,B,FullSimplify[ $\frac{1}{8} \text{Tr}[\text{bas64}[A,1].\text{bas64}[B,1]]$ ] }, {A,1, 64},{B,A, 64}]) //MatrixForm*) *) *) *) *)

```

In[624]:=

$$(*■*)$$

In[625]:=

$$32 \times 65$$

Out[625]=

$$2080$$

In[626]:=

$$(*\text{Length}[\text{Flatten}[\text{countTrace}, 1]]*)$$

In[627]:=

$$(*\{\text{Length}[\text{countTrace}], \text{Length}[\text{countTrace}\llbracket 1 \rrbracket], \text{Length}[\text{countTrace}\llbracket 1,1 \rrbracket]\}*)$$

In[628]:=

$$(*\{\text{Length}[\text{countTrace}], \text{Length}[\text{countTrace}\llbracket 2 \rrbracket], \text{Length}[\text{countTrace}\llbracket 2,1 \rrbracket]\}*)$$

In[629]:=

$$(*\{\text{Length}[\text{countTrace}], \text{Length}[\text{countTrace}\llbracket 64 \rrbracket], \text{Length}[\text{countTrace}\llbracket 64,1 \rrbracket]\}*)$$

In[630]:=

$$(*\{\text{Length}[\text{Flatten}[\text{countTrace}, 1]\llbracket \text{All}, 3 \rrbracket], \text{Count}[\text{Flatten}[\text{countTrace}, 1]\llbracket \text{All}, 3 \rrbracket, 0], \text{Count}[\text{Flatten}[\text{countTrace}, 1]\llbracket \text{All}, 3 \rrbracket, 1], \text{Count}[\text{Flatten}[\text{countTrace}, 1]\llbracket \text{All}, 3 \rrbracket, -1]\}*)$$

In[631]:=

$$32 \times 63$$

Out[631]=

$$2016$$

In[632]:=

```

anti = {};
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],
    AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[anti]
anti[[28]]

```

Out[633]=

28

Out[634]=

```

{{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}}

```

In[635]:=

```
anti[[28, 1]]
```

Out[635]=

```

{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0},
{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}}

```

In[636]:=

```
anti[[28, 2]]
```

Out[636]=

```
{{{63}, {5, 6, 7}}}
```

In[637]:=

```

symm = {};
Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
    AppendTo[symm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[symm]
symm[[28]]

```

Out[638]=

35

Out[639]=

```

{{{-1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
{0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, {{52}, {2, 5, 7}}}}

```

For the split orthogonal group $\text{Spin}(4,4)$ and its associated Lie algebra, there exist three fundamentally equivalent 8-dimensional representations:

1. The vector representation V
2. The type-1 spinor representation $S+$ (or $S1$), positive chirality
3. The type-2 spinor representation $S-$ (or $S2$), negative chirality

These representations are related by outer automorphisms of $\text{Spin}(4,4)$. All three representations have the same dimension: $\dim(V) = \dim(S1) = \dim(S2) = 8$

This is unique to dimension 8 and forms the basis of triality: Cartan, E. (1925). La théorie des spineurs. Hermann, Paris.

metric-preserving transformations; find automorphism $\sigma g \longleftrightarrow \sigma g_A^a$

$$\begin{aligned}\sigma.\sigma g &== \text{Transpose}[\sigma g^{-1}].g \\ \sigma &== \text{Transpose}[\sigma g^{-1}].g.\sigma g^{-1} \\ (\sigma g)^a{}_A &\equiv \sigma g^a{}_A \\ (\sigma g^{-1})^A{}_a &\equiv \sigma g^A{}_a \quad ; \quad (\text{Transpose}[\sigma g^{-1}])^A{}_a \equiv \sigma g_a{}^A \\ \sigma_{ab} &== \sigma g_a{}^A g_{AB} \sigma g_b{}^B\end{aligned}$$

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$N = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[640]:=

$$\text{Clear}[\sigma g_1{}^a]; \sigma g_1{}^a = \frac{1}{\sqrt{2}} (\eta 4488 + \sigma)$$

Out[640]=

$$\begin{aligned}&\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ &\left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}\end{aligned}$$

In[641]:=

$$\sigma g_{2A}^a = \frac{1}{\sqrt{2}} \{ \{1, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 1, 0\}, \\ \{0, 1, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 1\}, \\ \{1, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, -1, 0\}, \\ \{0, 1, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, -1\}\}$$

Out[641]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ \left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

In[642]:=

$$(s1s2 = \sigma g_{1A}^a.\text{Transpose}[\sigma g_{2A}^a]) // \text{MatrixForm}$$

Out[642]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[643]:=

$$(* (s1s2 = \text{Inverse}[\sigma g_{1A}^a.\text{Transpose}[\sigma g_{2A}^a]]) // \text{MatrixForm} *)$$

In[644]:=

$$s1s2.s1s2 === \text{ID8}$$

Out[644]=

True

```
In[645]:= (Thread[¶16[[1 ;; 8]] → s1s2.¶16[[9 ;; 16]]]) // Column
%[[1]] [[All, 1, 0]]
%[[1]] [[All, 2]] /. {x0 → #1, x4 → #2}
ToExpression["((" <> ToString[#] <> ")") &] & /@ %
forceDimensionReductionA = Thread[%%% → %]

Out[645]= f16[0][x0, x4] → f16[8][x0, x4]
f16[1][x0, x4] → f16[10][x0, x4]
f16[2][x0, x4] → f16[9][x0, x4]
f16[3][x0, x4] → f16[11][x0, x4]
f16[4][x0, x4] → f16[12][x0, x4]
f16[5][x0, x4] → f16[14][x0, x4]
f16[6][x0, x4] → f16[13][x0, x4]
f16[7][x0, x4] → f16[15][x0, x4]

Out[646]= {f16[0], f16[1], f16[2], f16[3], f16[4], f16[5], f16[6], f16[7]}

Out[647]= {f16[8][#1, #2], f16[10][#1, #2], f16[9][#1, #2], f16[11][#1, #2],
f16[12][#1, #2], f16[14][#1, #2], f16[13][#1, #2], f16[15][#1, #2]}

Out[648]= {f16[8][#1, #2] &, f16[10][#1, #2] &, f16[9][#1, #2] &, f16[11][#1, #2] &,
f16[12][#1, #2] &, f16[14][#1, #2] &, f16[13][#1, #2] &, f16[15][#1, #2] &}

Out[649]= {f16[0] → (f16[8][#1, #2] &), f16[1] → (f16[10][#1, #2] &),
f16[2] → (f16[9][#1, #2] &), f16[3] → (f16[11][#1, #2] &), f16[4] → (f16[12][#1, #2] &),
f16[5] → (f16[14][#1, #2] &), f16[6] → (f16[13][#1, #2] &), f16[7] → (f16[15][#1, #2] &)}
```

```
In[650]:= (Thread[¶16[[9 ;; 16]] → s1s2.¶16[[1 ;; 8]]]) // Column
%[[1]] [[All, 1, 0]]
%[[1]] [[All, 2]] /. {x0 → #1, x4 → #2}
ToExpression["((" <> ToString[#] <> ")") &] & /@ %
forceDimensionReduction = Thread[% % % → %]

Out[650]= f16[8][x0, x4] → f16[0][x0, x4]
f16[9][x0, x4] → f16[2][x0, x4]
f16[10][x0, x4] → f16[1][x0, x4]
f16[11][x0, x4] → f16[3][x0, x4]
f16[12][x0, x4] → f16[4][x0, x4]
f16[13][x0, x4] → f16[6][x0, x4]
f16[14][x0, x4] → f16[5][x0, x4]
f16[15][x0, x4] → f16[7][x0, x4]

Out[651]= {f16[8], f16[9], f16[10], f16[11], f16[12], f16[13], f16[14], f16[15]}

Out[652]= {f16[0][#1, #2], f16[2][#1, #2], f16[1][#1, #2], f16[3][#1, #2],
f16[4][#1, #2], f16[6][#1, #2], f16[5][#1, #2], f16[7][#1, #2]}

Out[653]= {f16[0][#1, #2] &, f16[2][#1, #2] &, f16[1][#1, #2] &, f16[3][#1, #2] &,
f16[4][#1, #2] &, f16[6][#1, #2] &, f16[5][#1, #2] &, f16[7][#1, #2] &}

Out[654]= {f16[8] → (f16[0][#1, #2] &), f16[9] → (f16[2][#1, #2] &),
f16[10] → (f16[1][#1, #2] &), f16[11] → (f16[3][#1, #2] &), f16[12] → (f16[4][#1, #2] &),
f16[13] → (f16[6][#1, #2] &), f16[14] → (f16[5][#1, #2] &), f16[15] → (f16[7][#1, #2] &)}

In[655]:= (*σga2A == -# + σga1A & /@ bas64[[All, 1]]*)

In[656]:= σga2A.Transpose[σga2A] === ID8
```

$\sigma g_{2A}^a \cdot \text{Transpose}[\sigma g_{2A}^a] == \text{ID8}$

```
Out[656]= True

In[657]:= σga1A.Transpose[σga1A] === ID8
```

$\sigma g_{1A}^a \cdot \text{Transpose}[\sigma g_{1A}^a] == \text{ID8}$

```
Out[657]= True
```

metric - preserving transformation:

```
In[658]:= σga1A.σ.Transpose[σga1A] === η4488
```

$\sigma g_{1A}^a \cdot \sigma \cdot \text{Transpose}[\sigma g_{1A}^a] == \eta 4488$

```
Out[658]= True
```

metric - preserving transformation:

In[659]:=

$$\sigma g_{2A}^a \cdot \sigma . \text{Transpose}[\sigma g_{2A}^a] == \eta 4488$$

Out[659]=

True

In[660]:=

$$Mc = \frac{1}{\sqrt{2}} (\eta 4488 + \sigma)$$

Out[660]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

In[661]:=

$$Nc = \frac{1}{\sqrt{2}} \{ \{ 1, 0, 0, 0, 1, 0, 0, 0 \}, \{ 0, 0, 1, 0, 0, 0, 1, 0 \}, \{ 0, 1, 0, 0, 0, 1, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, 1 \}, \{ 1, 0, 0, 0, -1, 0, 0, 0 \}, \{ 0, 0, 1, 0, 0, 0, -1, 0 \}, \{ 0, 1, 0, 0, 0, -1, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, -1 \} \}$$

Out[661]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

In[662]:=

$$(* (s1s2=Mc.Transpose[Nc]) // MatrixForm *)$$

In[663]:=

$$Nc . \text{Transpose}[Nc] // \text{MatrixForm}$$

Out[663]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```

In[664]:= Mc.Transpose[Mc] // MatrixForm
Out[664]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


In[665]:= Mc.σ.Transpose[Mc] === η4488
Out[665]= True

In[666]:= Nc.σ.Transpose[Nc] === η4488
Out[666]= True

In[667]:= (Thread[Π16[[9 ;; 16] → s1s2.Π16[[1 ;; 8]]]) // Column
Out[667]=
f16[8][x0, x4] → f16[0][x0, x4]
f16[9][x0, x4] → f16[2][x0, x4]
f16[10][x0, x4] → f16[1][x0, x4]
f16[11][x0, x4] → f16[3][x0, x4]
f16[12][x0, x4] → f16[4][x0, x4]
f16[13][x0, x4] → f16[6][x0, x4]
f16[14][x0, x4] → f16[5][x0, x4]
f16[15][x0, x4] → f16[7][x0, x4]

In[668]:= f8[#][x0, x4] & /@ Range[0, 7] // Column
          (s1s2.%[[1]]) // Column
Out[668]=
f8[0][x0, x4]
f8[1][x0, x4]
f8[2][x0, x4]
f8[3][x0, x4]
f8[4][x0, x4]
f8[5][x0, x4]
f8[6][x0, x4]
f8[7][x0, x4]

Out[669]=
f8[0][x0, x4]
f8[2][x0, x4]
f8[1][x0, x4]
f8[3][x0, x4]
f8[4][x0, x4]
f8[6][x0, x4]
f8[5][x0, x4]
f8[7][x0, x4]

```

Killing Vector Fields

In[670]:=

$$\begin{aligned}
 & -DG\left(\left[\left["vector", M8, []\right], \left[\left[\left[6\right], \right.\right.\right.\right. \\
 & \left. \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}}(x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right.\right.\right. \\
 & \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right.\right.\right. \\
 & \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})\right], \left[\left[7\right], \right.\right.\right. \\
 & \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}(x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right.\right.\right. \\
 & \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right.\right.\right. \\
 & \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})\right]\right], _DG\left(\left[\left["vector", \right.\right.\right. \\
 & \left. \left. \left. M8, []\right], \left[\left[\left[3\right], \right.\right.\right. \\
 & \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}(x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right.\right.\right. \\
 & \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right.\right.\right. \\
 & \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})\right], \left[\left[4\right], \right.\right.\right. \\
 & \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}}(x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right.\right.\right. \\
 & \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right.\right.\right. \\
 & \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})\right]\right], _DG\left(\left[\left["vector", \right.\right.\right. \\
 & \left. \left. \left. M8, []\right], \left[\left[\left[3\right], \frac{1}{\sin(6Hx0)^{1/3}}((2 \cos(Hx0) - 1)^{1/3} \right.\right.\right. \\
 & \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right.\right.\right. \\
 & \left. \left. \left. /^3 (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3})\right]\right], \right.\right.\right. \\
 & \left. \left. \left. _DG\left(\left[\left["vector", M8, []\right], \left[\left[\left[4\right], \right.\right.\right. \right.\right.\right. \\
 & \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) \right.\right.\right. \\
 & \left. \left. \left. + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (\cos(Hx0) - 1)^{1/6} \right.\right.\right. \\
 & \left. \left. \left. (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3})\right]\right], \right.\right.\right. \\
 & \left. \left. \left. _DG\left(\left[\left["vector", M8, []\right], \left[\left[\left[6\right], \right.\right.\right. \right.\right.\right. \\
 & \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}}(x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right.\right.\right. \\
 & \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right.\right.\right. \\
 & \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})\right], \left[\left[8\right], \right.\right.\right. \\
 & \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}(x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} \right.\right.\right. \\
 & \left. \left. \left. (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right.\right.\right. \\
 & \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})\right]\right], _DG\left(\left[\left["vector", \right.\right.\right. \\
 & \left. \left. \left. M8, []\right], \left[\left[\left[6\right], -\frac{1}{\sin(6Hx0)^{1/3}}((2 \cos(Hx0) \right.\right.\right. \\
 & \left. \left. \left. - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right.\right.\right. \\
 & \left. \left. \left. /^3 (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3})\right]\right]\right]
 \end{aligned}$$

```

), _DG( [{"vector", M8, []}, {[7],
- 1/(sin(6Hx0)^1/3)(x7 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0))^(1/3)], [8],
1/(sin(6Hx0)^1/3)(x6 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0))^(1/3)]}], _DG( [{"vector",
M8, []}, {[7], - 1/(sin(6Hx0)^1/3)((2 cos(Hx0)
- 1)^1/3 (cos(Hx0) + 1)^1/6 (-1 + 2 cos(2 Hx0))^(1/3
(cos(Hx0) - 1)^1/6 (2 cos(Hx0) + 1)^1/3 cos(Hx0)^(1/3)]}]
), _DG( [{"vector", M8, []}, {[8],
- 1/(sin(6Hx0)^1/3)((2 cos(Hx0) - 1)^1/3 (cos(Hx0)
+ 1)^1/6 (-1 + 2 cos(2 Hx0))^(1/3 (cos(Hx0) - 1)^1
/6 (2 cos(Hx0) + 1)^1/3 cos(Hx0)^(1/3)]]),
_DG( [{"vector", M8, []}, {[2],
1/(sin(6Hx0)^1/3)(x2 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0))^(1/3)], [3],
- 1/(sin(6Hx0)^1/3)(x1 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0))^(1/3)]}], _DG( [{"vector",
M8, []}, {[2],
1/(sin(6Hx0)^1/3)(x3 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0))^(1/3)]}],
), _DG( [{"vector", M8, []}, {[2], 1/(sin(6Hx0)^1/3)((2 cos(Hx0) - 1)^1
/3 (cos(Hx0) + 1)^1/6 (-1 + 2 cos(2 Hx0))^(1
/3 (cos(Hx0) - 1)^1/6 (2 cos(Hx0) + 1)^1/3 cos(Hx0)^(1/3)]]
})

```

Out[670]=

```

_DG( [{"vector", M8, []}, {[6],

```

$$-\frac{1}{\sin(6Hx\theta)^{1/3}}(x_6 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [7],$$

$$\frac{1}{\sin(6Hx\theta)^{1/3}}(x_5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]], _DG\left([["vector", M8, []], [[3],$$

$$\frac{1}{\sin(6Hx\theta)^{1/3}}(x_3 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [4],$$

$$-\frac{1}{\sin(6Hx\theta)^{1/3}}(x_2 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]], _DG\left([["vector", M8, []], [[3], \frac{1}{\sin(6Hx\theta)^{1/3}}((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} (\cos(Hx\theta) - 1)^{1/6} (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})], [4],$$

$$\frac{1}{\sin(6Hx\theta)^{1/3}}((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} (\cos(Hx\theta) - 1)^{1/6} (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})]], _DG\left([["vector", M8, []], [[6],$$

$$-\frac{1}{\sin(6Hx\theta)^{1/3}}(x_7 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [8],$$

$$\frac{1}{\sin(6Hx\theta)^{1/3}}(x_5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]], _DG\left([["vector", M8, []], [[6], -\frac{1}{\sin(6Hx\theta)^{1/3}}((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} (\cos(Hx\theta) - 1)^{1/6} (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})], [7],$$

$$-\frac{1}{\sin(6Hx\theta)^{1/3}}(x_7 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (2 \cos(Hx\theta) + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3} (\cos(Hx\theta) - 1)^{1/6} (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})]]$$

$$\begin{aligned}
& \frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}), \left[[8], \right. \\
& \left. \frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \\
& \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right]], _DG \left(\left[["vector", \right. \right. \\
& M8, []], \left[[7], - \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) \right. \\
& \left. - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} \right. \\
& \left. /^3 (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) \right]] \\
&), _DG \left(\left[["vector", M8, []], \left[[8], \right. \right. \\
& \left. \left. - \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^6 (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) \right]] \right), \\
& _DG \left(\left[["vector", M8, []], \left[[2], \right. \right. \\
& \left. \left. - \frac{1}{\sin(6Hx0)^{1/3}} (x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right]], \left[[3], \right. \right. \\
& \left. \left. - \frac{1}{\sin(6Hx0)^{1/3}} (x1 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right]] \right), _DG \left(\left[["vector", \right. \right. \\
& M8, []], \left[[2], \right. \right. \\
& \left. \left. - \frac{1}{\sin(6Hx0)^{1/3}} (x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right]], \left[[4], \right. \right. \\
& \left. \left. - \frac{1}{\sin(6Hx0)^{1/3}} (x1 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \\
& \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \\
& \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right]] \right), _DG \left(\left[["vector", \right. \right. \\
& M8, []], \left[[2], \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^1 \right. \\
& \left. /^3 (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} \right. \\
& \left. /^3 (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3}) \right]] \\
&]
\end{aligned}$$

In[671]:=

(*ToExpression[MathMLForm[{"http://www.w3.org/TR/MathML","math"}]()]*)

In[672]:=

(*ToExpression[MathMLForm[\square]]*)

In[673]:=

```
"[_DG([["vector"], M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
```

```

1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"], M8,
[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"], M8,
[]], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],
M8, []], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],
M8, []], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"], M8,
[]], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],
M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"], M8,
[]], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],
M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],
M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]])

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+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],  

[[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)  

+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +  

1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],  

[[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)  

+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])"

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In[674]:=

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killingVectors =
"[_DG([[\"vector\"], M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -  

1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)  

+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],  

[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)  

+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +  

1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],  

[[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +  

1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"],  

M8, []], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)  

+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)  

+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],  

[[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]], _DG([[\"vector\"], M8,  

[]], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
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+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\",
M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\",
M8, []], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\",
M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\",
M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\",
M8, []], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\",
M8, []], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])"

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Out[674]=

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[_DG([[{"vector", M8, []}], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]], _DG([[{"vector", M8, []}], [[[3],
x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [
[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]], _DG([[{"vector", M8, []}], [[[3],
(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], _DG([[{"vector", M8, []}], [[[4],
(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*x0)^(1/3)]], _DG([[{"vector", M8, []}], [[[6],
-x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]], _DG([[{"vector", M8, []}], [[[6],
-(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*x0))^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*x0)^(1/3)]], _DG([[{"vector", M8, []}], [[[7],
-x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]], _DG([[ {"vector", M8, []}], [[[7],
-(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*x0))^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*x0)^(1/3)]], _DG([[ {"vector", M8, []}], [[[8],
-(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]], _DG([[ {"vector", M8, []}], [[[2],
x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]], _DG([[ {"vector", M8, []}], [[[2],
x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]], _DG([[ {"vector", M8, []}], [[[2],
(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*x0))^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*x0)^(1/3)]]

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```
In[676]:= ConvertMapleToMathematicaV2[
  " -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
  1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
  2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3), , x5*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
  1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)"]]

Out[676]= ConvertMapleToMathematicaV2[
  -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3),
  , x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3) ]

In[677]:= ConvertMapleToMathematica[
  "x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
  1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
  2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3), -x2*cos(H*x0)^(1/3)*(cos(H*x0) -
  1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
  1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)"]

Out[677]= ConvertMapleToMathematica[
  x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3),
  -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
  1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3) ]

In[678]:= ConvertMapleToMathematica[
  "(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
  2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
  1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)"]

Out[678]= ConvertMapleToMathematica[
  (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) +
  1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3) ]
```

In[679]:=

M8 > $KV := \text{KillingVectors}(g);$

$$KV := \left[-\frac{x6 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x5}}{\sin(6 Hx0)^{1/6}}, \right.$$

$$+ \frac{x5 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x6}}{\sin(6 Hx0)^{1/6}},$$

$$\frac{x3 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x2}}{\sin(6 Hx0)^{1/6}},$$

$$-\frac{x2 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x3}}{\sin(6 Hx0)^{1/6}},$$

$$\frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x2}}{\sin(6 Hx0)^{1/6}},$$

$$\frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x3}}{\sin(6 Hx0)^{1/6}},$$

$$-\frac{x7 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x5}}{\sin(6 Hx0)^{1/6}},$$

$$+\frac{x5 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x7}}{\sin(6 Hx0)^{1/6}},$$

$$-\frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x5}}{\sin(6 Hx0)^{1/6}},$$

$$-\frac{x7 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x6}}{\sin(6 Hx0)^{1/6}},$$

$$+\frac{x6 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x7}}{\sin(6 Hx0)^{1/6}},$$

$$-\frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x6}}{\sin(6 Hx0)^{1/6}},$$

$$-\frac{(2 \cos(Hx0) - 1)^{1/6} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} (2 \cos(Hx0) + 1)^{1/6} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/6} D_{x7}}{\sin(6 Hx0)^{1/6}},$$

$$\left. \frac{x2 \cos(Hx0)^{1/6} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/6} D_{x1}}{\sin(6 Hx0)^{1/6}} \right]$$

Out[679]=

M8 > $KV := \text{KillingVectors}(g);$

$$KV := \left[-\frac{x6 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} D_{x5}}{\sin(6 Hx0)^{1/\beta}} \right.$$

$$+ \frac{x5 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} D_{x6}}{\sin(6 Hx0)^{1/\beta}},$$

$$\frac{x3 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} D_{x2}}{\sin(6 Hx0)^{1/\beta}}$$

$$- \frac{x2 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} D_{x3}}{\sin(6 Hx0)^{1/\beta}},$$

$$\frac{(2 \cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} \cos(Hx0)^{1/\beta} D_{x2}}{\sin(6 Hx0)^{1/\beta}},$$

$$\frac{(2 \cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} \cos(Hx0)^{1/\beta} D_{x3}}{\sin(6 Hx0)^{1/\beta}},$$

$$- \frac{x7 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} D_{x5}}{\sin(6 Hx0)^{1/\beta}}$$

$$+ \frac{x5 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} D_{x7}}{\sin(6 Hx0)^{1/\beta}},$$

$$- \frac{(2 \cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} \cos(Hx0)^{1/\beta} D_{x5}}{\sin(6 Hx0)^{1/\beta}},$$

$$- \frac{x7 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} D_{x6}}{\sin(6 Hx0)^{1/\beta}}$$

$$+ \frac{x6 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} D_{x7}}{\sin(6 Hx0)^{1/\beta}},$$

$$- \frac{(2 \cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} \cos(Hx0)^{1/\beta} D_{x6}}{\sin(6 Hx0)^{1/\beta}},$$

$$- \frac{(2 \cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} \cos(Hx0)^{1/\beta} D_{x7}}{\sin(6 Hx0)^{1/\beta}},$$

$$\frac{x2 \cos(Hx0)^{1/\beta} (\cos(Hx0) - 1)^{1/\beta} (\cos(Hx0) + 1)^{1/\beta} (2 \cos(Hx0) - 1)^{1/\beta} (2 \cos(Hx0) + 1)^{1/\beta} (-1 + 2 \cos(2 Hx0))^{1/\beta} D_{x1}}{\sin(6 Hx0)^{1/\beta}}$$

In[680]:=

$$\begin{aligned}
& - \frac{x^4 \cos(Hx\theta)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} (2 \cos(Hx\theta) - 1)^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} D_{x^2}}{\sin(6Hx\theta)^{1/\beta}}, \\
& x^3 \cos(Hx\theta)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} (2 \cos(Hx\theta) - 1)^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} D_{x^3}, \\
& - \frac{x^4 \cos(Hx\theta)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} (2 \cos(Hx\theta) - 1)^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} D_{x^3}}{\sin(6Hx\theta)^{1/\beta}}, \\
& \left. \frac{(2 \cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} \cos(Hx\theta)^{1/\beta} D_{x^3}}{\sin(6Hx\theta)^{1/\beta}} \right]
\end{aligned}$$

Out[680]=

$$\begin{aligned}
& - \frac{x^3 \cos(Hx\theta)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} (2 \cos(Hx\theta) - 1)^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} D_{x^2}}{\sin(6Hx\theta)^{1/\beta}}, \\
& - \frac{x^3 \cos(Hx\theta)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} (2 \cos(Hx\theta) - 1)^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} D_{xI}}{\sin(6Hx\theta)^{1/\beta}}, \\
& - \frac{x^3 \cos(Hx\theta)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} (2 \cos(Hx\theta) - 1)^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} D_{x3}}{\sin(6Hx\theta)^{1/\beta}}, \\
& \left[\frac{(2 \cos(Hx\theta) - 1)^{1/\beta} (\cos(Hx\theta) - 1)^{1/\beta} (-1 + 2 \cos(2Hx\theta))^{1/\beta} (2 \cos(Hx\theta) + 1)^{1/\beta} (\cos(Hx\theta) + 1)^{1/\beta} \cos(Hx\theta)^{1/\beta} D_{xI}}{\sin(6Hx\theta)^{1/\beta}} \right]
\end{aligned}$$

In[681]:=

8 >

Out[681]=

8 >

4×4 Dirac γ matrices :

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta], \quad (22)$$

$$S^{\alpha\beta} = -\frac{1}{2}\gamma^{\alpha\beta}. \quad (23)$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \quad (24)$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^\alpha \epsilon = -\epsilon \gamma^\alpha, \quad (25)$$

$$\tilde{S}^{\alpha\beta} \epsilon = -\epsilon S^{\alpha\beta}, \quad (26)$$

$$[S^{\alpha\beta}, \gamma_\mu] = \delta_\mu^\alpha \gamma^\beta - \delta_\mu^\beta \gamma^\alpha, \quad (27)$$

$$[S^{\alpha\beta}, S^{\mu\nu}] = g^{\alpha\mu} S^{\beta\nu} - g^{\alpha\nu} S^{\beta\mu} - g^{\beta\mu} S^{\alpha\nu} + g^{\beta\nu} S^{\alpha\mu}, \quad (28)$$

and

$$\gamma^5 S^{\alpha\beta} = \frac{1}{2} g^{\alpha\mu} g^{\beta\nu} \epsilon_{\mu\nu\lambda\sigma} S^{\lambda\sigma}. \quad (29)$$

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

```
In[682]:= (*Symbolize[ gαβ]*)
```

```
In[683]:= η4488
Dimensions[%]
gαβ = η4488[[2;;5, 2;;5]]
```

```
Out[683]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}}
```

```
Out[684]= {8, 8}
```

```
Out[685]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}
```

```
In[686]:= Protect[gαβ]
```

```
Out[686]= {gSubscriptαβ}
```

```
In[687]:= g3 = DiagonalMatrix[{1, 1, -1}]
```

```
Out[687]= {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}
```

```
In[688]:= Do[Do[Gr[h, k] = t4by4[h].s4by4[k]*(-g3[[h, h]]*g3[[k, k]]), {k, 1, 3}], {h, 1, 3}]
```

In[689]:= **Gr[1, 1] // MatrixForm**

Out[689]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In[690]:=

$$\gamma = \{\text{Gr}[1, 1], \text{Gr}[1, 2], \text{Gr}[1, 3], \text{t4by4}[2]\};$$

Set: Symbol γ is Protected.

In[691]:=

$$\text{Table}[\text{FullSimplify}[\gamma[h].\gamma[h] == g_{\alpha\beta}[h, h] * \text{ID4}], \{h, 1, 4\}]$$

Out[691]=

$$\{\text{False}, \text{False}, \text{False}, \text{True}\}$$

$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\gamma_0 g^{\alpha\beta}, \quad (4)$$

where

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

and γ_0 denotes the 4×4 unit matrix. The $\Sigma^{\alpha\beta}$ of Eq. (1)

In[692]:=

$$\text{Table}\left[\left\{\{h, k\}, \text{FullSimplify}\left[\frac{\gamma[h].\gamma[k] + \gamma[k].\gamma[h]}{2}\right] == g_{\alpha\beta}[h, k] \text{ ID4}\right\}, \{h, 1, 4\}, \{k, 1, 4\}\right] // \text{MatrixForm}$$

Out[692]//MatrixForm=

$$\begin{pmatrix} (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) \\ (\text{False}) & (\text{True}) & (\text{True}) & (\text{False}) \\ (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) \\ (\text{True}) & (\text{False}) & (\text{True}) & (\text{False}) \\ (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) \\ (\text{True}) & (\text{True}) & (\text{False}) & (\text{False}) \\ (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) \\ (\text{False}) & (\text{False}) & (\text{False}) & (\text{True}) \end{pmatrix}$$

In[693]:=

$$\text{Table}\left[\left\{\{h, k\}, \text{FullSimplify}\left[\frac{\gamma[h].\gamma[k] + \gamma[k].\gamma[h]}{2}\right] == \eta 4488 [1 + h, 1 + k] \text{ ID4}\right\}, \{h, 1, 4\}, \{k, 1, 4\}\right] // \text{MatrixForm}$$

Out[693]//MatrixForm=

$$\begin{pmatrix} (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) \\ (\text{False}) & (\text{True}) & (\text{True}) & (\text{False}) \\ (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) \\ (\text{True}) & (\text{False}) & (\text{True}) & (\text{False}) \\ (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) \\ (\text{True}) & (\text{True}) & (\text{False}) & (\text{False}) \\ (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) \\ (\text{False}) & (\text{False}) & (\text{False}) & (\text{True}) \end{pmatrix}$$

$$\gamma^5 = -\gamma^1 \gamma^2 \gamma^3 \gamma^4$$

In[694]:=

$$-\gamma[1] \cdot \gamma[2] \cdot \gamma[3] \cdot \gamma[4]$$

Out[694]=

$$\{\{0, 0, 0, 1\}, \{0, 0, -1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\}$$

In[695]:=

$$\text{AppendTo}[\gamma, (-\gamma[1] \cdot \gamma[2] \cdot \gamma[3] \cdot \gamma[4])]$$

Set: Symbol γ is Protected. [i](#)

Out[695]=

$$\begin{aligned} & \{\{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-1, 0, 0, 0\}\}, \\ & \quad \{\{0, 0, -1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\}, \\ & \quad \{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, -1, 0\}\}, \\ & \quad \{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{1, 0, 0, 0\}\}, \\ & \quad \{\{0, 0, -1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\}, \\ & \quad \{\{0, 1, 0, 0\}, \{-1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, 1, 0\}\}, \\ & \quad \{\{1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\}, \\ & \quad \{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 1, 0\}\}, \\ & \quad \{\{0, 0, 1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, -1, 0, 0\}\}, \\ & \quad \{\{0, 1, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, -1\}, \{0, 0, -1, 0\}\}, \\ & \quad \{\{-1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, 1\}\}, \\ & \quad \{\{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{1, 0, 0, 0\}\}, \\ & \quad \{\{0, 0, 1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\}, \\ & \quad \{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\}, \\ & \quad \{\{-1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}, \\ & \quad \{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\}\}, \\ & \quad \{\{0, 0, 0, 1\}, \{0, 0, -1, 0\}, \{0, 1, 0, 0\}, \{-1, 0, 0, 0\}\}\} \end{aligned}$$

In[696]:=

$$\text{Protect}[\gamma]$$

Out[696]=

$$\{\}$$

In[697]:=

$$\gamma[5]$$

Out[697]=

$$\{\{0, 0, -1, 0\}, \{0, 0, 0, -1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}\}$$

In[698]:=

$$\gamma[5] == -t4by4[3]$$

Out[698]=

$$\text{False}$$

In[699]:=

```
Table[
  {{h, k}, FullSimplify[\gamma[h].\gamma[k] + \gamma[k].\gamma[h]] === 2 * \eta4488[[1 + h, 1 + k] ID4],
  {h, 1, 5}, {k, 1, 5}] // MatrixForm
```

Out[699]//MatrixForm=

$$\begin{pmatrix} (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) & (\{1, 5\}) \\ (\text{False}) & (\text{True}) & (\text{True}) & (\text{False}) & (\text{False}) \\ (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) & (\{2, 5\}) \\ (\text{True}) & (\text{False}) & (\text{True}) & (\text{False}) & (\text{False}) \\ (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) & (\{3, 5\}) \\ (\text{True}) & (\text{True}) & (\text{False}) & (\text{False}) & (\text{False}) \\ (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) & (\{4, 5\}) \\ (\text{False}) & (\text{False}) & (\text{False}) & (\text{True}) & (\text{True}) \\ (\{5, 1\}) & (\{5, 2\}) & (\{5, 3\}) & (\{5, 4\}) & (\{5, 5\}) \\ (\text{False}) & (\text{False}) & (\text{False}) & (\text{True}) & (\text{True}) \end{pmatrix}$$

$$\gamma^\alpha = \gamma^{\alpha 6}, \quad (19)$$

$$\gamma^5 = \gamma^{56}, \quad (20)$$

$$\gamma^\alpha \gamma^5 = \gamma^{\alpha 5}, \quad (21)$$

$$\epsilon = \gamma^{64}.$$

In[700]:=

```
-\gamma[[4]] // MatrixForm
```

Out[700]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

In[701]:=

```
s\epsilon = {\epsilon \rightarrow t4by4[2]}
```

Out[701]=

```
{\epsilon \rightarrow {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}
```

In[702]:=

```
Transpose[\epsilon /. s\epsilon] === (-\epsilon /. s\epsilon)
```

Out[702]=

```
True
```

In[703]:=

```
ParallelTable[{{A1},
  FullSimplify[(\epsilon /. s\epsilon).\gamma[A1] === Transpose[(\epsilon /. s\epsilon).\gamma[A1]]]}, {A1, 1, 5}]
```

Out[703]=

```
{{{1}, True}, {{2}, True}, {{3}, True}, {{4}, False}, {{5}, True}}
```

```

In[704]:=  $\gamma[4] == \text{t4by4}[2]$ 
Out[704]= False

In[705]:=  $\text{t4by4}[1]$ 
Out[705]=  $\{\{0, 0, 0, -1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{1, 0, 0, 0\}\}$ 

 $S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta].$  (3)

In[706]:=  $S44\alpha\beta = -\frac{1}{4} \text{Table}[\text{FullSimplify}[\gamma[h].\gamma[k] - \gamma[k].\gamma[h]], \{h, 1, 4\}, \{k, 1, 4\}];$ 
In[707]:= Protect[S44\alpha\beta]
Out[707]= {S44\alpha\beta}

In[708]:=  $\eta44 = \text{DiagonalMatrix}[\{1, 1, 1, -1\}]$ 
Out[708]=  $\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, -1\}\}$ 
In[709]:= Protect[\eta44]
Out[709]= {\eta44}

```

S44\alpha\beta commutation relations; misc :

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta], \quad (22)$$

$$S^{\alpha\beta} = -\frac{1}{2}\gamma^{\alpha\beta}. \quad (23)$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \quad (24)$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^\alpha \epsilon = -\epsilon \gamma^\alpha, \quad (25)$$

$$\tilde{S}^{\alpha\beta} \epsilon = -\epsilon S^{\alpha\beta}, \quad (26)$$

$$[S^{\alpha\beta}, \gamma_\mu] = \delta_\mu^\alpha \gamma^\beta - \delta_\mu^\beta \gamma^\alpha, \quad (27)$$

$$[S^{\alpha\beta}, S^{\mu\nu}] = g^{\alpha\mu} S^{\beta\nu} - g^{\alpha\nu} S^{\beta\mu} - g^{\beta\mu} S^{\alpha\nu} + g^{\beta\nu} S^{\alpha\mu}, \quad (28)$$

and

$$\gamma^\delta S^{\alpha\beta} = \frac{1}{2} g^{\alpha\mu} g^{\beta\nu} \epsilon_{\mu\nu\lambda\sigma} S^{\lambda\sigma}. \quad (29)$$

```
S44 $\alpha\beta = -\frac{1}{4} \text{Table}[\text{FullSimplify}[\gamma[h].\gamma[k] - \gamma[k].\gamma[h]], \{h, 1, 4\}, \{k, 1, 4\}];$ 
```

In[710]:=

```
(*SAB=
Table[ $\frac{1}{4}$  (T16A[A1].T16A[B1]-T16A[B1].T16A[A1]), {A1,0, 7},{B1,0, 7}];*)
```

In[711]:=

```
(*ParallelTable[
FullSimplify[SAB[A1,B1]==-Transpose[SAB[A1,B1]]],{A1,1, 8},{B1,1,8}]/.Flatten/.Union*)
```

In[712]:=

```
(*ParallelTable[
{{A1,B1},FullSimplify[SAB[A1,B1]==-Transpose[SAB[A1,B1]]]}, {A1,1, 8},{B1,1,8}]*)
```

In[713]:=

```
ParallelTable[
FullSimplify[S44 $\alpha\beta$ [A1, B1].S44 $\alpha\beta$ [A2, B2] - S44 $\alpha\beta$ [A2, B2].S44 $\alpha\beta$ [A1, B1] ==
(η44[A1, A2] × S44 $\alpha\beta$ [B1, B2] - η44[A1, B2] × S44 $\alpha\beta$ [B1, A2] -
η44[B1, A2] × S44 $\alpha\beta$ [A1, B2] + η44[B1, B2] × S44 $\alpha\beta$ [A1, A2])], {A1, 1, 3},
{B1, A1 + 1, 4}, {A2, 1, 3}, {B2, A2 + 1, 4}] // Flatten // Union
```

Out[713]=

```
{False, True}
```

In[714]:=

```

ParallelTable[{{A1, B1, A2, B2},

  FullSimplify[S44 $\alpha\beta$ [[A1, B1].S44 $\alpha\beta$ [[A2, B2] - S44 $\alpha\beta$ [[A2, B2].S44 $\alpha\beta$ [[A1, B1] ==

  ( $\eta_{44}$ [[A1, A2]]  $\times$  S44 $\alpha\beta$ [[B1, B2]] -  $\eta_{44}$ [[A1, B2]]  $\times$  S44 $\alpha\beta$ [[B1, A2]] -  

    $\eta_{44}$ [[B1, A2]]  $\times$  S44 $\alpha\beta$ [[A1, B2]] +  $\eta_{44}$ [[B1, B2]]  $\times$  S44 $\alpha\beta$ [[A1, A2]])]},

{A1, 1, 4}, {B1, 1, 4}, {A2, 1, 4}, {B2, 1, 4}] // MatrixForm

```

Out[714]//MatrixForm=

In[715]:=

```
ParallelTable[FullSimplify[S44 $\alpha\beta$ [[A1, B1]. $\gamma$ [[B2]] -  $\gamma$ [[B2]].S44 $\alpha\beta$ [[A1, B1]] ==  
( $\eta$ 44[[B2, A1]]  $\times$   $\gamma$ [[B1]] -  $\eta$ 44[[B2, B1]]  $\times$   $\gamma$ [[A1]])],  
{A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // Flatten // Union
```

Out[715]=

{False, True}

In[716]:=

```
ParallelTable[
 {{A1, B1, B2}, FullSimplify[S44αβ[A1, B1].γ[B2] - γ[B2].S44αβ[A1, B1] ==
 (η44[B2, A1] × γ[B1] - η44[B2, B1] × γ[A1])]}, {A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // MatrixForm
```

Out[716]//MatrixForm=

$$\begin{pmatrix} \{1, 1, 1\} & \text{True} & \{1, 2, 1\} & \text{False} & \{1, 3, 1\} & \text{False} & \{1, 4, 1\} & \text{False} \\ \{1, 1, 2\} & \text{True} & \{1, 2, 2\} & \text{False} & \{1, 3, 2\} & \text{True} & \{1, 4, 2\} & \text{True} \\ \{1, 1, 3\} & \text{True} & \{1, 2, 3\} & \text{True} & \{1, 3, 3\} & \text{False} & \{1, 4, 3\} & \text{True} \\ \{1, 1, 4\} & \text{True} & \{1, 2, 4\} & \text{True} & \{1, 3, 4\} & \text{True} & \{1, 4, 4\} & \text{False} \\ \{2, 1, 1\} & \text{False} & \{2, 2, 1\} & \text{True} & \{2, 3, 1\} & \text{True} & \{2, 4, 1\} & \text{True} \\ \{2, 1, 2\} & \text{False} & \{2, 2, 2\} & \text{True} & \{2, 3, 2\} & \text{False} & \{2, 4, 2\} & \text{False} \\ \{2, 1, 3\} & \text{True} & \{2, 2, 3\} & \text{True} & \{2, 3, 3\} & \text{False} & \{2, 4, 3\} & \text{True} \\ \{2, 1, 4\} & \text{True} & \{2, 2, 4\} & \text{True} & \{2, 3, 4\} & \text{True} & \{2, 4, 4\} & \text{False} \\ \{3, 1, 1\} & \text{False} & \{3, 2, 1\} & \text{True} & \{3, 3, 1\} & \text{True} & \{3, 4, 1\} & \text{True} \\ \{3, 1, 2\} & \text{True} & \{3, 2, 2\} & \text{False} & \{3, 3, 2\} & \text{True} & \{3, 4, 2\} & \text{True} \\ \{3, 1, 3\} & \text{False} & \{3, 2, 3\} & \text{False} & \{3, 3, 3\} & \text{True} & \{3, 4, 3\} & \text{False} \\ \{3, 1, 4\} & \text{True} & \{3, 2, 4\} & \text{True} & \{3, 3, 4\} & \text{True} & \{3, 4, 4\} & \text{False} \\ \{4, 1, 1\} & \text{False} & \{4, 2, 1\} & \text{True} & \{4, 3, 1\} & \text{True} & \{4, 4, 1\} & \text{True} \\ \{4, 1, 2\} & \text{True} & \{4, 2, 2\} & \text{False} & \{4, 3, 2\} & \text{True} & \{4, 4, 2\} & \text{True} \\ \{4, 1, 3\} & \text{True} & \{4, 2, 3\} & \text{True} & \{4, 3, 3\} & \text{False} & \{4, 4, 3\} & \text{True} \\ \{4, 1, 4\} & \text{False} & \{4, 2, 4\} & \text{False} & \{4, 3, 4\} & \text{False} & \{4, 4, 4\} & \text{True} \end{pmatrix}$$

In[717]:=

```
ParallelTable[{{A1, B1}, FullSimplify[(ε /. se).S44αβ[A1, B1] ===
 Transpose[(ε /. se).S44αβ[A1, B1]]]}, {A1, 1, 3}, {B1, A1 + 1, 4}]
```

Out[717]=

$$\{\{\{1, 2\}, \text{True}\}, \{\{1, 3\}, \text{True}\}, \{\{1, 4\}, \text{True}\}\}, \{\{\{2, 3\}, \text{True}\}, \{\{2, 4\}, \text{True}\}\}, \{\{\{3, 4\}, \text{True}\}\}$$

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula: \mathcal{O}

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. \mathcal{O}

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4). ∅

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) ∅

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. ∅

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa). ∅

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). ∅

what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the *so(4,4)* Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: ∅

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for *so(8)* in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



↳ Why is the metric signature significant for \$so(4,4)\$ commutation relations?

Why is the metric signature significant for \$so(4,4)\$ commutation relations?

The metric signature is significant for the $so(p, q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). ⓘ

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$
The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4, 4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[718]:=

 η_{4488}

Out[718]=

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1} }
```

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

$$t^A = \begin{pmatrix} 0 & \vec{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

In[719]:=

```
(*Table[SAB[[A1,B1]]== ArrayFlatten[{{SAB1[[A1,B1]],0},{0,SAB2[[A1,B1]]}}],{A1,0,7},{B1,0,7}]*)
```

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4, 4)}$ and $L \in SO(4, 4)$. The canonical 2-1 homomorphism $\overline{SO(4, 4)} \rightarrow SO(4, 4)$ is given by

$$(2.8) \quad 8L^{A'}{}_{B'}(g) = \text{tr } \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4, 4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}{}_{B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

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A Remarkable Representation of the $3 + 2$ de Sitter Group

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Identities satisfied by the generators of the Dirac algebra

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```
In[720]:= Unprotect[ $\gamma$ ABs];
 $\gamma$ ABs = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]

Out[721]= {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
{{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
{{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
{{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

In[722]:= Length[ $\gamma$ ABs]
Out[722]= 16

In[723]:= Protect[ $\gamma$ ABs]
Out[723]= { $\gamma$ ABs}

In[724]:=  $\gamma$ ABs[[1]]
Out[724]= {0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}

In[725]:=  $\gamma$ ABs[[-1]]
Out[725]= {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}

Recall Self Dual Anti-Symmetric

In[726]:= Table

```

Recall Anti SelfDual Anti-Symmetric

```
In[727]:= Table[(1/2) Sum[Sum[epsilon4[p, q, j1, j2] * t4by4[h][j1, j2], {j2, 1, 4}], {j1, 1, 4}] + t4by4[h][p, q], {h, 1, 3}, {q, 4}, {p, 4}]

Out[727]= {{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

BASIS of 4×4 matrices :

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The Dirac spinor in six dimensions

BY E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

3. Decomposition of the basic spinor representations. We have already stated that the basic spinor representations of $SO(n, c)$, for n even, are reducible. We shall consider here the reduction of the basic 8-component spinor representation of $SO(6, c)$ into constituent 4-component representations. The procedure is easily generalized to apply to any even-dimensional space. An irreducible representation of the Clifford algebra C_6 can be constructed from a given irreducible representation of C_5 . The $\gamma_\mu (\mu = 1, \dots, 5)$ of an irreducible representation of C_5 satisfy (2.5), so that the quantities $e_\alpha (\alpha = 1, \dots, 6)$ defined by

$$\left. \begin{aligned} e_\mu &= (-i) \begin{pmatrix} \gamma_\mu B \\ -B^{-1}\gamma_\mu \end{pmatrix} \quad (\mu = 1, \dots, 5), \\ e_6 &= \begin{pmatrix} B \\ B^{-1} \end{pmatrix}, \end{aligned} \right\} \quad (3.1)$$

satisfy

$$\frac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \dots, 6).$$

For the moment the matrix B is completely arbitrary. The e_α therefore give an irreducible representation of C_6 . They can be written more concisely in terms of the sets of matrices a_α and \bar{a}_α , defined as follows:

$$\left. \begin{aligned} a_\mu &= \gamma_\mu B, & \bar{a}_\mu &= -B^{-1}\gamma_\mu \quad (\mu = 1, \dots, 5), \\ a_6 &= iB, & \bar{a}_6 &= iB^{-1}. \end{aligned} \right\} \quad (3.2)$$

The generator e_α of C_6 can then be written

$$e_\alpha = (-i) \begin{pmatrix} a_\alpha \\ \bar{a}_\alpha \end{pmatrix} \quad (\alpha = 1, \dots, 6). \quad (3.3)$$

With $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1e_2e_3e_4e_5e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

and $e_{\alpha\beta} = \frac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2} \begin{pmatrix} a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha & \\ \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha & \end{pmatrix}.$

This latter expression shows that the infinitesimal generators of the 8-component spinor representation of $SO(6, c)$ is the direct sum of two 4-component representations. The basic spinor can be written

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix},$$

where ϕ and χ are four component spinors which transform according to representations with infinitesimal generators

$$\left. \begin{aligned} G_{\alpha\beta}^{(1)} &= -\frac{1}{4}(a_\alpha \bar{a} - a_\beta \bar{a}_\alpha) \\ \text{and} \quad G_{\alpha\beta}^{(2)} &= -\frac{1}{4}(\bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha), \end{aligned} \right\} \quad (3.4)$$

$$\begin{matrix} \text{sy} & \text{sy} & \text{asy} & \text{sy} & \text{sy} \\ \underline{1} & \underline{4} & \underline{6} & \underline{4} & \underline{1} \end{matrix}$$

```

In[728]:= Plus @@ {1, 4, 6, 4, 1}
Out[728]= 16
          {sy, sy, asy, sy, sy}
          1   4   6   4   1

In[729]:= Plus @@ {1, 4, 6, 4, 1}
Out[729]= 16

In[730]:= MatrixForm[If[# == 2, i, 1] * PauliMatrix[#]] & /@ Range[3]
Out[730]= { ( 0  1 ), ( 0  1 ), ( 1  0 ) }
          1  0   -1  0   0  -1

In[731]:= σ22 = Flatten[
  {{IdentityMatrix[2]}, If[# == 2, i, 1] * PauliMatrix[#] & /@ Range[3]}, 1]
Out[731]= {{{1, 0}, {0, 1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}}}

In[732]:= σ22̄ = Flatten[
  {{-IdentityMatrix[2]}, If[# == 2, i, 1] * PauliMatrix[#] & /@ Range[3]}, 1]
Out[732]= {{{-1, 0}, {0, -1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}}}

In[733]:= -IdentityMatrix[2] === Dot @@ (σ22[[#]] & /@ Range[Length[σ22]])
Out[733]= True

In[734]:= Table[{{A, B}, FullSimplify[1/2 Tr[1/2 (σ22[[A]].σ22̄[[B]] + σ22[[B]].σ22̄[[A]])]}], {{A, 1, 4}, {B, 1, 4}}] // MatrixForm
Out[734]//MatrixForm= {{ ( {1, 1} ) ( {1, 2} ) ( {1, 3} ) ( {1, 4} ) , ( {2, 1} ) ( {2, 2} ) ( {2, 3} ) ( {2, 4} ) , ( {3, 1} ) ( {3, 2} ) ( {3, 3} ) ( {3, 4} ) , ( {4, 1} ) ( {4, 2} ) ( {4, 3} ) ( {4, 4} ) }, { ( {1, 1} ) ( {1, 2} ) ( {1, 3} ) ( {1, 4} ) , ( {2, 1} ) ( {2, 2} ) ( {2, 3} ) ( {2, 4} ) , ( {3, 1} ) ( {3, 2} ) ( {3, 3} ) ( {3, 4} ) , ( {4, 1} ) ( {4, 2} ) ( {4, 3} ) ( {4, 4} ) } }

In[735]:= η2244 = DiagonalMatrix[{-1, 1, -1, 1}]
Out[735]= {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}

```

In[736]:=

```
Table[{ {A, B}, FullSimplify[ExpandAll[1/2 ( σ22[A].σ22[B] + σ22[B].σ22[A]) == η2244[A, B]*IdentityMatrix[2]]]}, {A, 1, 4}, {B, 1, 4}] // MatrixForm
```

Out[736]//MatrixForm=

$$\left(\begin{array}{c} \left(\begin{array}{cc} \{1, 1\} & \{1, 2\} \\ \text{True} & \text{True} \end{array} \right) \left(\begin{array}{cc} \{1, 3\} & \{1, 4\} \\ \text{True} & \text{True} \end{array} \right) \\ \left(\begin{array}{cc} \{2, 1\} & \{2, 2\} \\ \text{True} & \text{True} \end{array} \right) \left(\begin{array}{cc} \{2, 3\} & \{2, 4\} \\ \text{True} & \text{True} \end{array} \right) \\ \left(\begin{array}{cc} \{3, 1\} & \{3, 2\} \\ \text{True} & \text{True} \end{array} \right) \left(\begin{array}{cc} \{3, 3\} & \{3, 4\} \\ \text{True} & \text{True} \end{array} \right) \\ \left(\begin{array}{cc} \{4, 1\} & \{4, 2\} \\ \text{True} & \text{True} \end{array} \right) \left(\begin{array}{cc} \{4, 3\} & \{4, 4\} \\ \text{True} & \text{True} \end{array} \right) \end{array} \right)$$

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The Dirac spinor in six dimensions

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(Received 17 November 1966)

$$\left. \begin{aligned} e_\mu &= (-i) \begin{pmatrix} \gamma_\mu B \\ -B^{-1}\gamma_\mu \end{pmatrix} \quad (\mu = 1, \dots, 5), \\ e_6 &= \begin{pmatrix} B \\ B^{-1} \end{pmatrix}, \end{aligned} \right\}$$

$$\frac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \dots, 6).$$

$$\left. \begin{aligned} \alpha_\mu &= \gamma_\mu B, \quad \bar{a}_\mu = -B^{-1}\gamma_\mu \quad (\mu = 1, \dots, 5), \\ a_6 &= iB, \quad \bar{a}_6 = iB^{-1}. \end{aligned} \right\}$$

generator e_α of C_6 can then be written

$$e_\alpha = (-i) \begin{pmatrix} a_\alpha \\ \bar{a}_\alpha \end{pmatrix} \quad (\alpha = 1, \dots, 6).$$

Since $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1 e_2 e_3 e_4 e_5 e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$e_{\alpha\beta} = \frac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2} \begin{pmatrix} a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha & \\ \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha & \end{pmatrix}.$$

```
In[737]:= 
Do[yyy[j] = ArrayFlatten[{{{0, σ22[[j]]}, {-σ22[[j]], 0}}}], {j, 4}]; 
yyy[5] = ArrayFlatten[{{0, IdentityMatrix[2]}, {IdentityMatrix[2], 0}}]

Out[737]= 
{{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}}

In[738]:= 
yyy[5] = {{0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0}]

Out[738]= 
{{0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0}}

In[739]:= 
(yyy[6] = Dot @@ (yyy[#[ ] & /@ Range[5])) // MatrixForm

Out[739]//MatrixForm= 

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$


In[740]:= 
MatrixForm[yyy[#[ ] ] & /@ Range[6]]

Out[740]= 
{ $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$ }

In[741]:= 
Do[xxx[j] = ArrayFlatten[{{0, σ22[[j]]}, {σ22[[j]], 0}}], {j, 4}]

In[742]:= 
MatrixForm[xxx[#[ ] ] & /@ Range[4]]

Out[742]= 
{ $\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$ }

In[743]:= 
xxx[5] = Dot @@ (xxx[#[ ] & /@ Range[4]])

Out[743]= 
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1} }

In[744]:= 
IdentityMatrix[4]

Out[744]= 
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

In[745]:=

```

txA0 = {{IdentityMatrix[4], {0}}};
Length[txA0]
txA = {} ; Do[AppendTo[txA, {xxx[j], {j}}], {j, 1, 4}]
Length[txA]
txAB = {};
Do[AppendTo[txAB, {xxx[j].xxx[k], {j, k}}], {j, 1, 3}, {k, j + 1, 4}]
Length[txAB]
txABC = {};
Do[AppendTo[txABC, {xxx[j].xxx[k].xxx[h], {j, k, h}}], {j, 1, 2}, {k, j + 1, 3}, {h, k + 1, 4}];
Length[txABC]
txABCD = {{xxx[1].xxx[2].xxx[3].xxx[4], {1, 2, 3, 4}}};
Length[txABCD]

```

Out[746]=

1

Out[748]=

4

Out[750]=

6

Out[752]=

4

Out[754]=

1

In[755]:=

```
base4by4 = Flatten[{txA0, txA, txAB, txBC, txABCD}, 1]; Length[base4by4]
```

Out[755]=

16

In[756]:=

```

Table[{{A, B}, Block[{r}, r = FullSimplify[ $\frac{1}{4}$  Tr[
 $\frac{1}{2}$  (base4by4[[A, 1]].base4by4[[B, 1]] + base4by4[[B, 1]].base4by4[[A, 1]])];
If[r == 0, Style[r, Red], Style[r, Darker[Green]]]], {A, 1, Length[base4by4]}, {B, 1, Length[base4by4]}] // MatrixForm

```

Out[756]//MatrixForm=

In[757]:=

base4by4[[16][1]]

Out[757]=

$$\{\{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, -1\}\}$$

In[758]:=

```
anti4by4 = {};
Do[If[base4by4[[k, 1]] == -Transpose[base4by4[[k, 1]]], AppendTo[anti4by4,
  {base4by4[[k, 1]], {{k}, base4by4[[k, 2]]}}], {k, 1, Length[base4by4]}];
Length[anti4by4]
anti4by4[[%]]
```

Out[759]=

6

Out[760]=

{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}, {{14}, {1, 3, 4}}}}

In[761]:=

```
symm4by4 = {};
Do[If[base4by4[[k, 1]] == Transpose[base4by4[[k, 1]]], AppendTo[symm4by4,
  {base4by4[[k, 1]], {{k}, base4by4[[k, 2]]}}], {k, 1, Length[base4by4]}];
Length[symm4by4]
symm4by4[[%]]
```

Out[762]=

10

Out[763]=

{{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}, {{16}, {1, 2, 3, 4}}}

In[764]:=

```
 $\eta_{16} =$ 
DiagonalMatrix[Table[FullSimplify[ $\frac{1}{4} \text{Tr}[\text{symm4by4}[A, 1] \cdot \text{symm4by4}[A, 1]]$ ],
{A, Length[symm4by4]}]];
\text{Tr}[\eta_{16}]
```

Out[764]=

10

O(4,4); values, evecs of σ

In[765]:=

{values, evecs} = Eigensystem[σ]

Out[765]=

{{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
{0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
{0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}}

In[766]:=

$$\left(\text{u} = \text{ExpandAll} \left[\frac{1}{\sqrt{2}} \text{evecs} \right] \right) // \text{MatrixForm}$$

Out[766]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

In[767]:=

```
Table[u[[h]].σ.u[[h]], {h, 1, Length[u]}]
```

Out[767]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[768]:=

```
Table[(Transpose[u][[h]]).σ.u[[h]], {h, 1, Length[u]}]
```

Out[768]=

```
{-1, -1, -1, -1, 1, 1, 1, 1}
```

In[769]:=

```
Table[(Transpose[u][[A]]) == -σ.u[[A]], {A, 1, 4}]
```

```
Table[(Transpose[u][[A]]) == σ.u[[A]], {A, 5, Length[u]}]
```

Out[769]=

```
{True, True, True, True}
```

Out[770]=

```
{True, True, True, True}
```

In[771]:=

```
hUSE = 8;
```

In[772]:=

```
Protect[hUSE]
```

Out[772]=

```
{hUSE}
```

In[773]:=

```
unit = u[[hUSE]]
```

Out[773]=

```
{1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
```

In[774]:=

```
Protect[unit]
```

Out[774]=

```
{unit}
```

```
In[775]:= (Transpose[unit].σ.unit)
Out[775]= 1
In[776]:= (Transpose[unit].σ.(τ[θ]))
Out[776]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
In[777]:= myid[MX_]:= Sum[τ[A].MX.τ[A] η4488[A, A], {A, 1, 8}]
Myid[MX_]:= Sum[τ[A].MX.τ[A] × η448[A, A], {A, 1, 8}]
Union[Table[
  Sum[τ[A].bas64[k, 1].σ.τ[A] × η8[A, A], {A, 1, 8}] - IdTr[bas64[k, 1].σ] +
  bas64[k, 1] - Transpose[bas64[k, 1]]] .σ, {k, 1, 64}]]
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}}
τ[θ]
Out[779]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
(Transpose[unit].σ.(τ[θ]))
Out[780]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
FaA = Transpose[η4488[#, 1] * (τ[#].unit) & /@ Range[0, 7]]
Out[781]= {{1/√2, 0, 0, 0, 0, 0, 0, 1/√2}, {0, 0, 0, -1/√2, 1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, -1/√2, 0, 0}, {0, -1/√2, 0, 0, 0, 0, 1/√2, 0}, {1/√2, 0, 0, 0, 0, 0, 0, -1/√2}, {0, 0, -1/√2, -1/√2, 0, 0, 0, 1/√2}, {0, 0, 1/√2, 0, 0, 1/√2, 0, 0}, {0, -1/√2, 0, 0, 0, 0, 1/√2, 0}}
```

In[782]:=

$$F_a^A = \text{Transpose}[\text{unit}].\sigma.(\bar{\tau}[\#]) \& /@ \text{Range}[0, 7]$$

Out[782]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\}$$

In[783]:=

$$F_A^a.F_a^A == \text{ID8}$$

Out[783]=

True

pretend that the X are Minkowski Coordinates :

In[784]:=

$$\text{Block}[\{U, u = U[\#] \& /@ \text{Range}[8], fx = F_A^a.X, eq, sx, su, ret\}, \\ eq = \text{And} @@ \text{Thread}[u == fx]; \\ sx = \text{Solve}[eq, X][[-1]]; \\ su = \text{Solve}[eq, u][[-1]]; \\ ret = \{sx, su\}; \\ ret]$$

Out[784]=

$$\left\{ \begin{aligned} x0 &\rightarrow \frac{U[1]}{\sqrt{2}} + \frac{U[5]}{\sqrt{2}}, x1 &\rightarrow -\frac{U[4]}{\sqrt{2}} - \frac{U[8]}{\sqrt{2}}, x2 &\rightarrow \frac{U[3]}{\sqrt{2}} + \frac{U[7]}{\sqrt{2}}, \\ x3 &\rightarrow -\frac{U[2]}{\sqrt{2}} - \frac{U[6]}{\sqrt{2}}, x4 &\rightarrow \frac{1}{2} (\sqrt{2} U[2] - \sqrt{2} U[6]), x5 &\rightarrow \frac{1}{2} (-\sqrt{2} U[3] + \sqrt{2} U[7]), \\ x6 &\rightarrow \frac{1}{2} (-\sqrt{2} U[4] + \sqrt{2} U[8]), x7 &\rightarrow \frac{1}{2} (\sqrt{2} U[1] - \sqrt{2} U[5]) \end{aligned} \right\}, \\ \left\{ \begin{aligned} U[1] &\rightarrow \frac{1}{2} (\sqrt{2} x0 + \sqrt{2} x7), U[2] &\rightarrow \frac{1}{2} (-\sqrt{2} x3 + \sqrt{2} x4), U[3] &\rightarrow \frac{1}{2} (\sqrt{2} x2 - \sqrt{2} x5), \\ U[4] &\rightarrow \frac{1}{2} (-\sqrt{2} x1 - \sqrt{2} x6), U[5] &\rightarrow \frac{1}{2} (\sqrt{2} x0 - \sqrt{2} x7), U[6] &\rightarrow \frac{1}{2} (-\sqrt{2} x3 - \sqrt{2} x4), \\ U[7] &\rightarrow \frac{1}{2} (\sqrt{2} x2 + \sqrt{2} x5), U[8] &\rightarrow \frac{1}{2} (-\sqrt{2} x1 + \sqrt{2} x6) \end{aligned} \right\}$$

In[785]:=

$$(sgtrye_{\alpha}^{(A)} = \text{Thread}[\text{Flatten}[e_{\alpha}^{(A)}] \rightarrow \text{Flatten}[eAa04]]) (* // Column *)$$

In[785]:=

subsDefects

Out[785]=

$$\left\{ \begin{aligned} \sqrt{e^{2a4[Hx4]}} &\rightarrow e^{a4[Hx4]}, \quad \sqrt{e^{-2a4[Hx4]}} \rightarrow e^{-a4[Hx4]}, \quad \sqrt{e^{2a4[Hx4]} \sin[6Hx4]^{1/3}} \rightarrow e^{a4[Hx4]} \sin[6Hx4]^{1/6}, \\ \frac{1}{\sqrt{e^{2a4[Hx4]} \sin[6Hx4]^{1/3}}} &\rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx4]^{1/6}}, \quad \frac{1}{\sqrt{e^{2a4[Hx4]} \sin[6Hx0]^{1/6}}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, \\ \frac{1}{\sqrt{e^{-2a4[Hx4]} \sin[6Hx0]^{1/6}}} &\rightarrow \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, \quad \sqrt{e^{2a4[Hx4]} \sin[6Hx0]^{1/6}} \rightarrow e^{a4[Hx4]} \sin[6Hx0]^{1/6}, \\ \sqrt{e^{-2a4[Hx4]} \sin[6Hx0]^{1/6}} &\rightarrow e^{-a4[Hx4]} \sin[6Hx0]^{1/6} \end{aligned} \right\}$$

In[786]:=

Thread[X → 0]

Out[786]=

$$\{x0 \rightarrow 0, x1 \rightarrow 0, x2 \rightarrow 0, x3 \rightarrow 0, x4 \rightarrow 0, x5 \rightarrow 0, x6 \rightarrow 0, x7 \rightarrow 0\}$$

Stop pretending :

In[787]:=

```
FAa.eα(A).X /. sgtryeα(A) // FullSimplify[#, constraintVars] &;
frameOctadX = # /. { $\sqrt{e^{2a4[Hx4]}} \rightarrow e^{a4[Hx4]}$ ,  $\sqrt{e^{-2a4[Hx4]}} \rightarrow e^{-a4[Hx4]}$ } & /@ %
% /. {x1 → 0, x2 → 0, x3 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Out[788]=

$$\left\{ \begin{aligned} \frac{x0 \cot[6Hx0] + e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \quad &\frac{x4 - e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\ \frac{(e^{a4[Hx4]} x2 - e^{-a4[Hx4]} x5) \sin[6Hx0]^{1/6}}{\sqrt{2}}, \quad &-\frac{(e^{a4[Hx4]} x1 + e^{-a4[Hx4]} x6) \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\ \frac{x0 \cot[6Hx0] - e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \quad &-\frac{x4 + e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\ \frac{(e^{a4[Hx4]} x2 + e^{-a4[Hx4]} x5) \sin[6Hx0]^{1/6}}{\sqrt{2}}, \quad &\frac{(-e^{a4[Hx4]} x1 + e^{-a4[Hx4]} x6) \sin[6Hx0]^{1/6}}{\sqrt{2}} \end{aligned} \right\}$$

Out[789]=

$$\left\{ \frac{x0 \cot[6Hx0]}{\sqrt{2}}, \frac{x4}{\sqrt{2}}, 0, 0, \frac{x0 \cot[6Hx0]}{\sqrt{2}}, -\frac{x4}{\sqrt{2}}, 0, 0 \right\}$$

In[790]:=

F_A^a.eAα.X - frameOctadX // FullSimplify[#, constraintVars] &

Out[790]=

$$\left\{ \begin{aligned} \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}.eA\alpha. \end{aligned} \right.$$

$$\begin{aligned}
& \text{eA}\alpha.\{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} + \frac{x_4 + e^{a4[H x 4]} x_3 \sin[6 H x 0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \right. \right. \\
& \left. \left. -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \\
& \text{eA}\alpha.\{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} - \frac{(e^{a4[H x 4]} x_2 + e^{-a4[H x 4]} x_5) \sin[6 H x 0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \right. \right. \\
& \left. \left. -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \\
& \text{eA}\alpha.\{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\} + \frac{e^{-a4[H x 4]} (e^{2 a4[H x 4]} x_1 - x_6) \sin[6 H x 0]^{1/6}}{\sqrt{2}}
\end{aligned}$$

In[791]:=

{\{F_A^a\}, {eA}\alpha, {X}} // Column

Out[791]=

$$\begin{aligned}
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\
& \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \\
& \{eA}\alpha \\
& \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\}
\end{aligned}$$

In[792]:=

```
{x2u, u2x} = Block[{U, u = U[#] & /@ Range[8],
  fx = frameOctadX, eq, sx, su, ret, altEQ, sCot, skot},
  altEQ = And @@ {Cot[6 H x0] == kot, Sin[6 H x0] == sin, a4[H x4] == A44};
  sCot = Solve[altEQ, {Cot[6 H x0], Sin[6 H x0], a4[H x4]}][[1]];
  skot = Solve[altEQ, {kot, sin, A44}][[1]];
  eq = And @@ Thread[u == (fx /. sCot)];
  sx = Solve[eq, X][[-1]];
  su = Solve[eq, u][[-1]];
  ret = {sx /. skot // FullSimplify, su /. skot // ExpandAll};
  ret]
```

Out[792]=

$$\begin{aligned} \left\{ \begin{aligned} x0 &\rightarrow \frac{\tan[6 H x0] (U[1] + U[5])}{\sqrt{2}}, \quad x1 \rightarrow -\frac{e^{-a4[H x4]} (U[4] + U[8])}{\sqrt{2} \sin[6 H x0]^{1/6}}, \\ x2 &\rightarrow -\frac{e^{-a4[H x4]} (U[3] + U[7])}{\sqrt{2} \sin[6 H x0]^{1/6}}, \quad x3 \rightarrow -\frac{e^{-a4[H x4]} (U[2] + U[6])}{\sqrt{2} \sin[6 H x0]^{1/6}}, \quad x4 \rightarrow \frac{U[2] - U[6]}{\sqrt{2}}, \\ x5 &\rightarrow \frac{e^{a4[H x4]} (-U[3] + U[7])}{\sqrt{2} \sin[6 H x0]^{1/6}}, \quad x6 \rightarrow \frac{e^{a4[H x4]} (-U[4] + U[8])}{\sqrt{2} \sin[6 H x0]^{1/6}}, \quad x7 \rightarrow \frac{e^{a4[H x4]} (U[1] - U[5])}{\sqrt{2} \sin[6 H x0]^{1/6}} \}, \\ \{U[1] &\rightarrow \frac{x0 \cot[6 H x0]}{\sqrt{2}} + \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \quad U[2] \rightarrow \frac{x4}{\sqrt{2}} - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[3] &\rightarrow \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[4] &\rightarrow -\frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[5] &\rightarrow \frac{x0 \cot[6 H x0]}{\sqrt{2}} - \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \quad U[6] \rightarrow -\frac{x4}{\sqrt{2}} - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[7] &\rightarrow \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[8] &\rightarrow -\frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}} \} \end{aligned} \right.$$

In[793]:=

u2x

Out[793]=

$$\begin{aligned} U[1] &\rightarrow \frac{x_0 \operatorname{Cot}[6 H x_0]}{\sqrt{2}} + \frac{e^{-a4[H x4]} x_7 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \quad U[2] \rightarrow \frac{x_4}{\sqrt{2}} - \frac{e^{a4[H x4]} x_3 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[3] &\rightarrow \frac{e^{a4[H x4]} x_2 \sin[6 H x_0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x_5 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[4] &\rightarrow -\frac{e^{a4[H x4]} x_1 \sin[6 H x_0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x_6 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[5] &\rightarrow \frac{x_0 \operatorname{Cot}[6 H x_0]}{\sqrt{2}} - \frac{e^{-a4[H x4]} x_7 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \quad U[6] \rightarrow -\frac{x_4}{\sqrt{2}} - \frac{e^{a4[H x4]} x_3 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[7] &\rightarrow \frac{e^{a4[H x4]} x_2 \sin[6 H x_0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x_5 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ U[8] &\rightarrow -\frac{e^{a4[H x4]} x_1 \sin[6 H x_0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x_6 \sin[6 H x_0]^{1/6}}{\sqrt{2}} \} \end{aligned}$$

In[794]:=

```

{sumu2x, diffu2x} = Block[{U, u, a1, a2, a3 = {}, aa4 = {}},
  (*Clear[U,u,a1,a2,a3,a4];*)
  u = U[#] & /@ Range[8]; Print[u];
  a1 = u /. u2x; Print[a1];
  a2 = Transpose[{{Take[a1, 4]}, {Take[a1, -4]} }]; Print[a2];
  a3 = ExpandAll[FullSimplify[\frac{#1 + #2}{2}]] & /@ a2;
  aa4 = ExpandAll[FullSimplify[\frac{#1 - #2}{2}]] & /@ a2;
  {a3, aa4}]

```

$$\begin{aligned}
& \{U[1], U[2], U[3], U[4], U[5], U[6], U[7], U[8]\} \\
& \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \\
& \quad \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad - \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad \left. - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\} \\
& \left\{ \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \right. \\
& \quad \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad \left. \left. \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}} - \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, - \frac{x4 - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}}{\sqrt{2}}, \right. \right. \right. \\
& \quad \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \quad - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}} \} \right\} \} \\
\end{aligned}$$

Out[794]=

$$\begin{aligned}
& \left\{ \left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}}, - \frac{e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right. \right. \\
& \quad \frac{e^{a4[Hx4]} x2 \sin[6Hx0]^{1/6}}{\sqrt{2}}, - \frac{e^{a4[Hx4]} x1 \sin[6Hx0]^{1/6}}{\sqrt{2}} \}, \\
& \quad \left. \left. \left\{ \frac{e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{x4}{\sqrt{2}}, - \frac{e^{-a4[Hx4]} x5 \sin[6Hx0]^{1/6}}{\sqrt{2}}, - \frac{e^{-a4[Hx4]} x6 \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\} \right\} \right\}
\end{aligned}$$

split octonions; evals, evecs of σ

```

# Type - 1 spinor basis (constructed from Clifford algebra considerations)
# These create a maximal isotropic subspace with respect to the split form
self . type1_spinor _basis _matrix = sqrt2_inv*np . array ([[1, 1, 0, 0, 0, 0, 0, 0], # s1_ 0[1, -1, 0, 0, 0, 0, 0, 0],
s1_ 1[0, 0, 1, 1, 0, 0, 0, 0], # s1_ 2[0, 0, 1, -1, 0, 0, 0, 0], # s1_ 3[0, 0, 0, 1, 1, 0, 0], # s1_ 4[0, 0, 0, 0, 1, -1, 0, 0],
# s1_ 5[0, 0, 0, 0, 0, 1, 1], # s1_ 6[0, 0, 0, 0, 0, 1, -1], # s1_ 7])

```

```
# Type - 2 spinor basis (the complementary spinor representation)
```

```
# Related to Type - 1 by another triality automorphism
```

```
self . type2_spinor _basis _matrix = sqrt2_inv*np . array ([[1, 0, 1, 0, 0, 0, 0, 0], # s2_ 0[1, 0, -1, 0, 0, 0, 0, 0],
s2_ 1[0, 1, 0, 1, 0, 0, 0, 0], # s2_ 2[0, 1, 0, -1, 0, 0, 0, 0], # s2_ 3[0, 0, 0, 0, 1, 0, 1, 0], # s2_ 4[0, 0, 0, 0, 1, 0, -1, 0],
# s2_ 5[0, 0, 0, 0, 0, 1, 0, 1], # s2_ 6[0, 0, 0, 0, 0, 1, 0, -1], # s2_ 7])
```

In[795]:=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

Out[795]=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

In[796]:=

4 Transformation Matrices

We provide explicit 8×8 matrices that transform coordinates between the three bases.

4.1 Vector to Type-1 Spinor

The transformation matrix from vector basis to type-1 spinor basis is:

$$M_{V \rightarrow S_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (11)$$

4.2 Vector to Type-2 Spinor

The transformation matrix from vector basis to type-2 spinor basis is:

$$M_{V \rightarrow S_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (12)$$

Out[796]=

4 Transformation Matrices

We provide explicit 8×8 matrices that transform coordinates between the three bases.

4.1 Vector to Type-1 Spinor

The transformation matrix from vector basis to type-1 spinor basis is:

$$M_{V \rightarrow S_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (11)$$

4.2 Vector to Type-2 Spinor

The transformation matrix from vector basis to type-2 spinor basis is:

$$M_{V \rightarrow S_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (12)$$

In[797]:=

```
(type1SpinorBasisMatrix =
 {{1, 1, 0, 0, 0, 0, 0, 0}, {1, -1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 1, 0, 0, 0, 0}, {0, 0, 1, -1, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 1, 0, 0}, {0, 0, 0, 0, 1, -1, 0, 0},
 {0, 0, 0, 0, 0, 0, 1, 1}, {0, 0, 0, 0, 0, 0, 1, -1}}) // MatrixForm
```

Out[797]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

In[798]:=

```
(type2SpinorBasisMatrix =
 {{1, 0, 1, 0, 0, 0, 0, 0}, {1, 0, -1, 0, 0, 0, 0, 0},
 {0, 1, 0, 1, 0, 0, 0, 0}, {0, 1, 0, -1, 0, 0, 0, 0},
 {0, 0, 0, 0, 1, 0, 1, 0}, {0, 0, 0, 0, 1, 0, -1, 0},
 {0, 0, 0, 0, 0, 1, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0, -1}}) // MatrixForm
```

Out[798]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix}$$

In[799]:=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

Out[799]=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

```
In[800]:= MS1S2 = type1SpinorBasisMatrix.Transpose[type2SpinorBasisMatrix] // MatrixForm
Out[800]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$


In[801]:= gAB = η4488
Out[801]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[802]:= **{evalsAB, evecsAB} = Eigensystem[gAB]**

```
Out[802]= {{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}}}
```

In[803]:= **{evals, evecs} = Eigensystem[σ]**

```
Out[803]= {{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0}, {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1}, {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}}}
```

In[804]:= **u = ExpandAll[1/Sqrt[2] evecs]** // MatrixForm

```
Out[804]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$

```

In[805]:= **Table[u[[h]].σ.u[[h]], {h, 1, Length[u]}]**

```
Out[805]= {-1, -1, -1, -1, 1, 1, 1, 1}
```

```
In[806]:= Table[(Transpose[u][h]).σ.u[[h]], {h, 1, Length[u]}]
Out[806]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[807]:= Table[(Transpose[u][A]) === -σ.u[[A]], {A, 1, 4}]
Table[(Transpose[u][A]) === σ.u[[A]], {A, 5, Length[u]}]
Out[807]= {True, True, True, True}

Out[808]= {True, True, True, True}

In[809]:= hUSE = 8;
*** Set: Symbol hUSE is Protected. i

In[810]:= Protect[hUSE]
Out[810]= {}

In[811]:= unit = u[[hUSE]]
*** Set: Symbol unit is Protected. i
Out[811]=  $\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$ 

In[812]:= Protect[unit]
Out[812]= {}

In[813]:= (Transpose[unit].σ.unit)
Out[813]= 1

In[814]:= (Transpose[unit].σ.(τ[θ]))
Out[814]=  $\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$ 
```

In[815]:=

```

Do[E^A_a[h] = Table[(u[[h]].\sigma.(\tau[B])), {B, 0, 7}], {h, 1, Length[u]}];
\mathfrak{F}_a^A = \frac{1}{\sqrt{\mu}\sigma u} u^c \sigma_{c0} \tau^A_b u^b_a
Do[E^A_A[h] = FullSimplify[Inverse[E^A_a[h]]], {h, 1, Length[u]}];
\mathfrak{F}_a^0 = \frac{1}{\sqrt{\mu}\sigma u} \tau_A^a_b u^b
Table[E^a_A[h] === (u[[h]].\sigma.u[[h]])
Transpose[Table[FullSimplify[ExpandAll[\eta4488[B, B]\times\tau[B-1].u[[h]]]],
{B, 1, 8}]], {h, 1, Length[u]}]

```

Out[815]=

$$\mathfrak{F}_a^A = \frac{1}{\sqrt{\tilde{u}\sigma u}} u^c \sigma_{cb} \bar{\tau}^{Ab}{}_a$$

Out[816]=

$$\tilde{\mathfrak{F}}_A^a = \frac{1}{\sqrt{\tilde{u}\sigma u}} \tau_A{}^a{}_b u^b$$

Out[817]=

{True, True, True, True, True, True, True, True}

In[818]:=

```
(*Do[E^A_a[h]=Table[(u[[h]].\sigma.(T[B])),{B,0,7}],{h,1,Length[u]}];
Do[E^a_A[h]=FullSimplify[Inverse[E^A_a[h]]],{h,1,Length[u]}];
Table[E^a_A[h]==(u[[h]].\sigma.u[[h]])
    Transpose[Table[FullSimplify[ExpandAll[\eta4488[B,B]\tau[B-1].u[[h]]]],
    {B,1,8})]],{h,1,Length[u]}]*)
```

In[819]:=

```

myid[MX_] := Sum[ $\bar{\tau}[A].MX.\tau[A] \eta4488[A, A]$ , {A, 1, 8}]
Myid[MX_] := Sum[ $\tau[A].MX.\bar{\tau}[A] \times \eta4488[A, A]$ , {A, 1, 8}]

```

```

Union[Table[
  Sum[τ[A].bas64[[k, 1]].σ.τ[A] × η8[[A, A]], {A, 1, 8}] - Id Tr[bas64[[k, 1]].σ] +
  4 eg[[k]] 
$$\frac{\text{bas64}[[k, 1]] - \text{Transpose}[\text{bas64}[[k, 1]]]}{2} \cdot \sigma, \{k, 1, 64\}\Big]$$

```

In[82]:=

τ[θ]

Out[821]=

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},  
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},  
{0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1} }
```

In[822]:=

(Transpose[unit].σ.(τ[θ]))

Out[822]=

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

In[823]:=

 $F_A^a = \text{Transpose}[\eta 4488[\#, 1, \# + 1] * (\tau[\#].unit) \& /@ \text{Range}[0, 7]]$

Out[823]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ & \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \end{aligned}$$

In[824]:=

 $F_a^A = \text{Transpose}[unit].\sigma.(\tau[\#]) \& /@ \text{Range}[0, 7]$

Out[824]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\} \end{aligned}$$

In[825]:=

 $F_A^a.F_a^A == ID8$

Out[825]=

True

 $gtrye_{\alpha}^{(A)} = (e_{\alpha}^{(A)} /. sgtrye_{\alpha}^{(A)}) /. \text{subsDefects}$ $gtrye_{(A)}^{\alpha} = (e_{(A)}^{\alpha} /. sgtrye_{(A)}^{\alpha}) /. \text{subsDefects}$ $(sgtrye_{(A)}^{\alpha} = \text{Thread}[\text{Flatten}[e_{(A)}^{\alpha}] \rightarrow \text{Flatten}[\text{Inverse}[e_{\alpha}^{(A)} /. sgtrye_{\alpha}^{(A)}]]])$
 $(*//Column*)$ $gtrye_{(A)}^{\alpha} = (e_{(A)}^{\alpha} /. sgtrye_{(A)}^{\alpha}) /. \text{subsDefects}$

Out[825]=

$$\begin{aligned} & \left\{ \{ \text{Tan}[6 H x 0], 0, 0, 0, 0, 0, 0, 0, 0 \}, \left\{ 0, \frac{e^{-a4[H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{e^{-a4[H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{e^{-a4[H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{e^{a4[H x 4]}}{\text{Sin}[6 H x 0]^{1/6}}, 0, 0 \right\}, \right. \end{aligned}$$

$$\left\{0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}\right\}$$

pretend that the X are Minkowski Coordinates :

In[826]:=

```
Block[{U, u = U[#] & /@ Range[8], fx = Fa.X, eq, sx, su, ret},
  eq = And @@ Thread[u == fx];
  sx = Solve[eq, X][[-1]];
  su = Solve[eq, u][[-1]];
  ret = {sx, su};
  ret]
```

Out[826]=

$$\begin{aligned} &\left\{x0 \rightarrow \frac{U[1]}{\sqrt{2}} + \frac{U[5]}{\sqrt{2}}, x1 \rightarrow -\frac{U[4]}{\sqrt{2}} - \frac{U[8]}{\sqrt{2}}, x2 \rightarrow \frac{U[3]}{\sqrt{2}} + \frac{U[7]}{\sqrt{2}}, \right. \\ &x3 \rightarrow -\frac{U[2]}{\sqrt{2}} - \frac{U[6]}{\sqrt{2}}, x4 \rightarrow \frac{1}{2} (\sqrt{2} U[2] - \sqrt{2} U[6]), x5 \rightarrow \frac{1}{2} (-\sqrt{2} U[3] + \sqrt{2} U[7]), \\ &x6 \rightarrow \frac{1}{2} (-\sqrt{2} U[4] + \sqrt{2} U[8]), x7 \rightarrow \frac{1}{2} (\sqrt{2} U[1] - \sqrt{2} U[5])\}, \\ &\left\{U[1] \rightarrow \frac{1}{2} (\sqrt{2} x0 + \sqrt{2} x7), U[2] \rightarrow \frac{1}{2} (-\sqrt{2} x3 + \sqrt{2} x4), U[3] \rightarrow \frac{1}{2} (\sqrt{2} x2 - \sqrt{2} x5), \right. \\ &U[4] \rightarrow \frac{1}{2} (-\sqrt{2} x1 - \sqrt{2} x6), U[5] \rightarrow \frac{1}{2} (\sqrt{2} x0 - \sqrt{2} x7), U[6] \rightarrow \frac{1}{2} (-\sqrt{2} x3 - \sqrt{2} x4), \\ &\left. U[7] \rightarrow \frac{1}{2} (\sqrt{2} x2 + \sqrt{2} x5), U[8] \rightarrow \frac{1}{2} (-\sqrt{2} x1 + \sqrt{2} x6)\right\} \end{aligned}$$

In[827]:=

```
sgtrye(A) = Thread[Flatten[e(A)] → Flatten[eAa04]] (* //Column*)
```

Out[827]=

$$\begin{aligned} &\left\{\sqrt{e^{2a4[Hx4]}} \rightarrow e^{a4[Hx4]}, \sqrt{e^{-2a4[Hx4]}} \rightarrow e^{-a4[Hx4]}, \sqrt{e^{2a4[Hx4]}} \sin[6Hx4]^{1/3} \rightarrow e^{a4[Hx4]} \sin[6Hx4]^{1/6}, \right. \\ &\frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx4]^{1/3}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx4]^{1/6}}, \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, \\ &\frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}} \rightarrow \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{a4[Hx4]} \sin[6Hx0]^{1/6}, \\ &\left.\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{-a4[Hx4]} \sin[6Hx0]^{1/6}\right\} \end{aligned}$$

In[828]:=

```
Thread[X → 0]
```

Out[828]=

$$\{x0 \rightarrow 0, x1 \rightarrow 0, x2 \rightarrow 0, x3 \rightarrow 0, x4 \rightarrow 0, x5 \rightarrow 0, x6 \rightarrow 0, x7 \rightarrow 0\}$$

Stop pretending :

```
(sgtrye(A) = Thread[Flatten[e(A)] → Flatten[Inverse[e(A) /. sgtrye(A)]]]) (* //Column*)
```


In[831]:=

```
FAa.eα(A).X /. sgtryeα(A) // FullSimplify[#, constraintVars] &;
frameOctadX = # /. {Sqrt[e2 a4[H x4]] → ea4[H x4], Sqrt[e-2 a4[H x4]] → e-a4[H x4]} & /@ %
% /. {x1 → 0, x2 → 0, x3 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Out[832]=

$$\left\{ \frac{x_0 \operatorname{Cot}[6 H x_0] + e^{-a4[H x4]} x_7 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \frac{x_4 - e^{a4[H x4]} x_3 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \right. \\ \frac{e^{-a4[H x4]} (e^{2 a4[H x4]} x_2 - x_5) \sin[6 H x_0]^{1/6}}{\sqrt{2}}, -\frac{e^{-a4[H x4]} (e^{2 a4[H x4]} x_1 + x_6) \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ \frac{x_0 \operatorname{Cot}[6 H x_0] - e^{-a4[H x4]} x_7 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, -\frac{x_4 + e^{a4[H x4]} x_3 \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \\ \left. \frac{e^{-a4[H x4]} (e^{2 a4[H x4]} x_2 + x_5) \sin[6 H x_0]^{1/6}}{\sqrt{2}}, \frac{e^{-a4[H x4]} (-e^{2 a4[H x4]} x_1 + x_6) \sin[6 H x_0]^{1/6}}{\sqrt{2}} \right\}$$

Out[833]=

$$\left\{ \frac{x_0 \operatorname{Cot}[6 H x_0]}{\sqrt{2}}, \frac{x_4}{\sqrt{2}}, 0, 0, \frac{x_0 \operatorname{Cot}[6 H x_0]}{\sqrt{2}}, -\frac{x_4}{\sqrt{2}}, 0, 0 \right\}$$

In[834]:=

```
FAa.eAα.X - frameOctadX // FullSimplify[#, constraintVars] &
```

Out[834]=

$$\{0, 0, 0, 0, 0, 0, 0, 0\}$$

In[835]:=

eA_α

Out[835]=

$$\begin{aligned} &\{\{\operatorname{Cot}[6 H x_0], 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{a4[H x4]} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, e^{a4[H x4]} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a4[H x4]} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-a4[H x4]} \sin[6 H x_0]^{1/6}, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, e^{-a4[H x4]} \sin[6 H x_0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, e^{-a4[H x4]} \sin[6 H x_0]^{1/6}\} \end{aligned}$$

In[836]:=

```
\{\{FAa\}, \{eAα\}, \{X\}\} // Column
```

Out[836]=

$$\begin{aligned} &\left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\ &\left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \right. \\ &\left. \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \right\} \\ &\{\{\operatorname{Cot}[6 H x_0], 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{a4[H x4]} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, e^{a4[H x4]} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a4[H x4]} \sin[6 H x_0]^{1/6}, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-a4[H x4]} \sin[6 H x_0]^{1/6}, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, e^{-a4[H x4]} \sin[6 H x_0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, e^{-a4[H x4]} \sin[6 H x_0]^{1/6}\} \} \\ &\{\{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\}\} \end{aligned}$$

split octonion multiplication constants :

$$\begin{aligned} m_{ab}^c &= \mathbb{F}_a^A \tau_A{}_b{}^c \\ m_{AB}^C &= \mathbb{F}_c^C \tau_A{}_b{}^c \mathbb{F}_B^b \end{aligned}$$

In[837]:=

$$(xw)w = xw^2$$

Out[837]=

$$(xw)w = xw^2$$

In[838]:=

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

Out[838]=

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In[839]:=

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex structure J_u . If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on \mathbf{O} . However, because S^6 is connected, we see that the orientation of \mathbf{O} induced by the natural orientation of \mathbf{O}_u as a complex vector space does not depend on u . We refer to this orientation as the natural orientation of \mathbf{O} .

Out[839]=

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In[840]:=

Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

so J_u is an isometry for each $u \in S$. Moreover, it follows that \mathbf{O}_u is endowed with a natural Hermitian structure with respect to the inner product $\langle \cdot, \cdot \rangle$. We denote the group of complex linear transformations of \mathbf{O}_u by $GL(\mathbf{O}_u)$ and the special unitary transformations of \mathbf{O}_u with its Hermitian metric by $SU(\mathbf{O}_u)$.

Out[840]=

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J_u

In[841]:=

```
Table[FullSimplify[
Sum[\psi[A1] \times J[B1] gAB[[A1, A1]] E^A_a[hUSE][[C]][c] \tau[A1 - 1][[c]][b] E^a_A[hUSE][[b]][B1],
{c, 1, 8}, {b, 1, 8}, {A1, 1, 8}, {B1, 1, 8}}], {C, 1, 8}]
```

Out[841]=

$$\begin{aligned}
& \{J[1] \times \psi[1] - J[2] \times \psi[2] - J[3] \times \psi[3] - J[4] \times \psi[4] + \\
& J[5] \times \psi[5] + J[6] \times \psi[6] + J[7] \times \psi[7] + J[8] \times \psi[8], J[2] \times \psi[1] + J[1] \times \psi[2] - \\
& J[4] \times \psi[3] + J[3] \times \psi[4] + J[6] \times \psi[5] - J[5] \times \psi[6] - J[8] \times \psi[7] + J[7] \times \psi[8], \\
& J[3] \times \psi[1] + J[4] \times \psi[2] + J[1] \times \psi[3] - J[2] \times \psi[4] + J[7] \times \psi[5] + J[8] \times \psi[6] - \\
& J[5] \times \psi[7] - J[6] \times \psi[8], J[4] \times \psi[1] - J[3] \times \psi[2] + J[2] \times \psi[3] + J[1] \times \psi[4] + \\
& J[8] \times \psi[5] - J[7] \times \psi[6] + J[6] \times \psi[7] - J[5] \times \psi[8], J[5] \times \psi[1] + J[6] \times \psi[2] + \\
& J[7] \times \psi[3] + J[8] \times \psi[4] + J[1] \times \psi[5] - J[2] \times \psi[6] - J[3] \times \psi[7] - J[4] \times \psi[8], \\
& J[6] \times \psi[1] - J[5] \times \psi[2] + J[8] \times \psi[3] - J[7] \times \psi[4] + J[2] \times \psi[5] + J[1] \times \psi[6] + \\
& J[4] \times \psi[7] - J[3] \times \psi[8], J[7] \times \psi[1] - J[8] \times \psi[2] - J[5] \times \psi[3] + J[6] \times \psi[4] + \\
& J[3] \times \psi[5] - J[4] \times \psi[6] + J[1] \times \psi[7] + J[2] \times \psi[8], J[8] \times \psi[1] + J[7] \times \psi[2] - \\
& J[6] \times \psi[3] - J[5] \times \psi[4] + J[4] \times \psi[5] + J[3] \times \psi[6] - J[2] \times \psi[7] + J[1] \times \psi[8] \}
\end{aligned}$$

In[842]:=

```
Table[  
 FullSimplify[Sum[ gAB[[B1, B1]] E^A_a[hUSE][[C]][[c]] τ[B1 - 1][[c]][[b]] E^a_A[hUSE][[b]][[B1]],  
 {c, 1, 8}, {b, 1, 8}]], {C, 1, 8}, {A1, 1, 8}, {B1, 1, 8}]
```

Out[842]=

In[843]:=

Table 1

```
FullSimplify[ExpandAll[ gAB[[B, B]] E^A_a[hUSE].\tau[B-1].E^a_A[hUSE] ]], {B, 1, 8}]
```

Out[843]=

```

{{{1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0},

{0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0},

{0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}},

{{0, -1, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0},

{0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0},

{0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 0, 1, 0}},

{{0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0},

{0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1},

{0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}},

{{0, 0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0},

{1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1},

{0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}},

{{0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1},

{0, 0, 0, 0, 0, 0, 1}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},

{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}},

{{0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},

{0, 0, 0, 0, 0, 0, -1}, {0, -1, 0, 0, 0, 0, 0, 0},

{1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0},

{{0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1},

{0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},

{0, 0, -1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}}}}

```

In[844]:=

Table 1

```
Transpose[FullSimplify[ExpandAll[gAB[B, B] EAa[hUSE].\u03c4[B - 1].EaA[hUSE]]], {B, 1, 8}] // MatrixForm
```

Out[844]//MatrixForm=

$$\left(\begin{array}{c|c|c|c|c|c|c|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

$$\left(\begin{array}{c|c|c|c|c|c|c|c} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{array} \right)$$

In[845]:=

EA = Array [eA, 8]

Out[845]=

```
{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}
```

```

In[846]:=  $e_{\alpha}^{(A)}$ 
Out[846]=  $\left\{ \left\{ \text{Cot}[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right.$ 
 $\left. \left\{ 0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0 \right\}, \right.$ 
 $\left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0 \right\}, \right.$ 
 $\left. \left\{ 0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6} \right\} \right\}$ 
In[847]:= h = hUSE;
B = 1; FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]
B = 2; FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]
B = 3; FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]
B = 4; FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]
B = 5; FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]
B = 6; FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]
B = 7; FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]
B = 8;
FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]
Out[848]= {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}
Out[849]= {eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}
Out[850]= {eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}
Out[851]= {eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]}
Out[852]= {eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]}
Out[853]= {eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]}
Out[854]= {eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]}
Out[855]= {eA[8], eA[7], -eA[6], -eA[5], -eA[4], -eA[3], eA[2], eA[1]}
In[856]:= B = 1;
Grid[
{EA, FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]}, Frame -> All]
Out[856]= 

|       |       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|
| eA[1] | eA[2] | eA[3] | eA[4] | eA[5] | eA[6] | eA[7] | eA[8] |
| eA[1] | eA[2] | eA[3] | eA[4] | eA[5] | eA[6] | eA[7] | eA[8] |


```

In[857]:=

```
gridx = Grid[Partition[Flatten[{{x}, EA}], Table[
  ({x}, FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]) /.
    {x → ToExpression["eA[" <> ToString[B] <> "]"]}),
  {B, 1, 8}]], 9], Frame → All]
```

Out[857]=

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

In[858]:=

$\epsilon_B =$ $\epsilon_A =$	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8
ϵ_1	$-\epsilon_8$	ϵ_3	$-\epsilon_2$	$-\epsilon_5$	ϵ_4	$-\epsilon_7$	ϵ_6	ϵ_1
ϵ_2	$-\epsilon_3$	$-\epsilon_8$	ϵ_1	$-\epsilon_6$	ϵ_7	ϵ_4	$-\epsilon_5$	ϵ_2
ϵ_3	ϵ_2	$-\epsilon_1$	$-\epsilon_8$	$-\epsilon_7$	$-\epsilon_6$	ϵ_5	ϵ_4	ϵ_3
ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4
ϵ_5	$-\epsilon_4$	$-\epsilon_7$	ϵ_6	$-\epsilon_1$	ϵ_8	ϵ_3	$-\epsilon_2$	ϵ_5
ϵ_6	ϵ_7	$-\epsilon_4$	$-\epsilon_5$	$-\epsilon_2$	$-\epsilon_3$	ϵ_8	ϵ_1	ϵ_6
ϵ_7	$-\epsilon_6$	ϵ_5	$-\epsilon_4$	$-\epsilon_3$	ϵ_2	$-\epsilon_1$	ϵ_8	ϵ_7
ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8

Out[858]=

$\epsilon_B =$ $\epsilon_A =$	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8
ϵ_1	$-\epsilon_8$	ϵ_3	$-\epsilon_2$	$-\epsilon_5$	ϵ_4	$-\epsilon_7$	ϵ_6	ϵ_1
ϵ_2	$-\epsilon_3$	$-\epsilon_8$	ϵ_1	$-\epsilon_6$	ϵ_7	ϵ_4	$-\epsilon_5$	ϵ_2
ϵ_3	ϵ_2	$-\epsilon_1$	$-\epsilon_8$	$-\epsilon_7$	$-\epsilon_6$	ϵ_5	ϵ_4	ϵ_3
ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4
ϵ_5	$-\epsilon_4$	$-\epsilon_7$	ϵ_6	$-\epsilon_1$	ϵ_8	ϵ_3	$-\epsilon_2$	ϵ_5
ϵ_6	ϵ_7	$-\epsilon_4$	$-\epsilon_5$	$-\epsilon_2$	$-\epsilon_3$	ϵ_8	ϵ_1	ϵ_6
ϵ_7	$-\epsilon_6$	ϵ_5	$-\epsilon_4$	$-\epsilon_3$	ϵ_2	$-\epsilon_1$	ϵ_8	ϵ_7
ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8

1	i	j	k	ℓ	li	ℓj	ℓk
i	-1	k	$-j$	$-li$	ℓ	$-\ell k$	ℓj
j	$-k$	-1	i	$-\ell j$	ℓk	ℓ	$-li$
k	j	$-i$	-1	$-\ell k$	$-\ell j$	li	ℓ
ℓ	li	ℓj	ℓk	1	i	j	k
li	$-\ell$	$-\ell k$	ℓj	$-i$	1	k	$-j$
ℓj	ℓk	$-\ell$	$-li$	$-j$	$-k$	1	i
ℓk	$-\ell j$	li	$-\ell$	$-k$	j	$-i$	1

In[859]:=

```
(τABC = Table[ExpandAll[gAB[[B, B]] E^A_a[hUSE].τ[B - 1].E^a_A[hUSE]], {B, 1, 8}]);  
(τABC[[#]] // MatrixForm) & /@ Range[8]
```

Out[859]=

$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\left\{ \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\left\{ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \}$$

```

In[860]:= EA
Out[860]= {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[861]:= EA.τABC[1]
Out[861]= {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[862]:= EA.τABC[2]
Out[862]= {eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}

In[863]:= EA.τABC[3]
Out[863]= {eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}

In[864]:= EA.τABC[4]
Out[864]= {eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]}

In[865]:= EA.τABC[5]
Out[865]= {eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]}

In[866]:= EA.τABC[6]
Out[866]= {eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]}

In[867]:= EA.τABC[7]
Out[867]= {eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]}

In[868]:= TeXForm[gridx]
Out[868]//TeXForm=
\begin{array}{cccccccc}
x & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \text{eA}(4) & \text{eA}(5) & \text{eA}(6) & \text{eA}(7) \\
& \text{eA}(1) & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \text{eA}(4) & \text{eA}(5) & \text{eA}(6) \\
& \text{eA}(2) & \text{eA}(2) & \text{eA}(1) & \text{eA}(4) & \text{eA}(3) & \text{eA}(5) & \text{eA}(6) \\
& \text{eA}(3) & \text{eA}(3) & \text{eA}(4) & \text{eA}(1) & \text{eA}(2) & \text{eA}(7) & \text{eA}(8) \\
& \text{eA}(4) & \text{eA}(4) & \text{eA}(3) & \text{eA}(2) & \text{eA}(1) & \text{eA}(8) & \text{eA}(7) \\
& \text{eA}(5) & \text{eA}(5) & \text{eA}(6) & \text{eA}(7) & \text{eA}(8) & \text{eA}(1) & \text{eA}(6) \\
& \text{eA}(6) & \text{eA}(6) & \text{eA}(5) & \text{eA}(4) & \text{eA}(3) & \text{eA}(2) & \text{eA}(7) \\
& \text{eA}(7) & \text{eA}(7) & \text{eA}(8) & \text{eA}(7) & \text{eA}(6) & \text{eA}(5) & \text{eA}(4) \\
& \text{eA}(8) & \text{eA}(8) & \text{eA}(7) & \text{eA}(6) & \text{eA}(5) & \text{eA}(4) & \text{eA}(3)
\end{array}

(*StringReplace[ToString[TeXForm[gridx]], "\text{eA}" → "\epsilon"]*)

```

In[870]:=

```
Grid[Partition[
  Flatten[{{{{x}, EA}, Table[({{x}, Table[Sum[FullSimplify[ExpandAll[gAB[[B, B]
    EA[[C1]] E^A_a[hUSE][[C1, c1]] \[Tau][B - 1][[c1, d1]] E^a_A[hUSE][[d1, B1]]]], {C1, 1, 8}, {c1, 1, 8}, {d1, 1, 8}], {B1, 1, 8}}]} /. 
    {x \[Rule] ToExpression["eA[" \<> ToString[B] \<> "]"]}], {B, 1, 8}]}], 9], Frame \[Rule] All]
```

Out[870]=

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

In[871]:=

gridx

Out[871]=

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

Complex Structure

J_u

In[872]:=

J[#] \[Rule] 0 & /@ Range[0, 7]

Out[872]=

{J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 0, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}

In[873]:=

sJu = {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 1, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}

Out[873]=

{J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 1, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}

```
for  $\frac{1}{i}$  :
```

In[874]:=

```

Block[\{\psi, J, t, r, sJu\},
sJu =
{J[0] \rightarrow 0, J[1] \rightarrow 0, J[2] \rightarrow 0, J[3] \rightarrow 0, J[4] \rightarrow 1, J[5] \rightarrow 0, J[6] \rightarrow 0, J[7] \rightarrow 0};
t = Table[FullSimplify[Sum[\psi[A1 - 1] \times J[B1 - 1] gAB[[A1, A1]] E^A_a[hUSE][[C]][c] \tau[A1 - 1][c][b] E^a_A[hUSE][[b]][B1]],
{c, 1, 8}, {b, 1, 8}, {A1, 1, 8}, {B1, 1, 8}]], {C, 1, 8}];
r = t /. sJu
]

```

Out[874]=

$$\{\psi[4], -\psi[5], -\psi[6], -\psi[7], \psi[0], -\psi[1], -\psi[2], -\psi[3]\}$$

In[875]:=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \widetilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elemen

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

042501-7 Second gravity

J. Math. Phys. 51, 042501 (2010)

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix} \quad (10)$$

of the 16-component spinor representation of $\overline{\text{SO}(4,4;\mathbb{R})}$. We see that, as is, in fact, well known from the general theory, the 16-component spinor representation of $\overline{\text{SO}(4,4;\mathbb{R})}$ is the direct sum of two (inequivalent) real 8×8 irreducible spinor representations $D_{(1)}=D_{(1)}(g)$ and $D_{(2)}=D_{(2)}(g)$ of $\overline{\text{SO}(4,4;\mathbb{R})} \ni g$ that are generated by $D_{(1)}^{AB}$ and $D_{(2)}^{AB}$, respectively. This we record as

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A \quad (11)$$

and

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A. \quad (12)$$

Out[875]=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \widetilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elements

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

042501-7 Second gravity

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$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A \quad (11)$$

and

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A. \quad (12)$$

In[876]:=

042501-8 Patrick L. Nash

J. Math. Phys. 51, 042501 (2010)

$$\widetilde{D_{(1)}}^{AB}\sigma = -\sigma D_{(1)}^{AB} \Rightarrow \tilde{D}_{(1)}\sigma = \sigma D_{(1)}^{-1}, \quad (18)$$

$$\widetilde{D_{(2)}}^{AB}\sigma = -\sigma D_{(2)}^{AB} \Rightarrow \tilde{D}_{(2)}\sigma = \sigma D_{(2)}^{-1}, \quad (19)$$

$$L^A{}_C G_{AB} L^B{}_D = G_{CD} = \{\tilde{L}GL\}_{CD}, \quad (20)$$

$$L^A{}_B \bar{\tau}^B = D_{(1)}^{-1} \bar{\tau}^A D_{(2)}, \quad (21)$$

$$L^A{}_B \tau^B = D_{(2)}^{-1} \tau^A D_{(1)}, \quad (22)$$

The canonical 2-1 homomorphism $\overline{\mathrm{SO}(4,4;\mathbb{R})} \rightarrow \mathrm{SO}(4,4;\mathbb{R}): g \mapsto L(g)$ is given by

$$8L^A{}_B = \mathrm{tr}(D_{(1)}^{-1} \bar{\tau}^A D_{(2)} \tau^B) G_{CB}, \quad (23)$$

where tr denotes the trace. Note that $D_{(1)}(g(\omega)) = D_{(2)}(g(\omega))$ when $\omega_{A8} = 0$, i.e., when one restricts $\overline{\mathrm{SO}(4,4;\mathbb{R})}$ to

$$\overline{\mathrm{SO}(3,4;\mathbb{R})} = \{g \in \overline{\mathrm{SO}(4,4;\mathbb{R})} \mid ,$$

$$g = \begin{pmatrix} \exp\left(\frac{1}{4}\omega_{AB} D_{(1)}^{AB}\right) & 0 \\ 0 & \exp\left(\frac{1}{4}\omega_{AB} D_{(2)}^{AB}\right) \end{pmatrix} \quad \text{and} \quad \omega_{A8} = 0 \} \quad (24)$$

[one of the real forms of spin (7, C)].

Out[876]=

042501-8 Patrick L. Nash

J. Math. Phys. 51, 042501 (2010)

$$\widetilde{D}_{(1)}^{AB}\sigma = -\sigma D_{(1)}^{AB} \Rightarrow \tilde{D}_{(1)}\sigma = \sigma D_{(1)}^{-1}, \quad (18)$$

$$\widetilde{D}_{(2)}^{AB}\sigma = -\sigma D_{(2)}^{AB} \Rightarrow \tilde{D}_{(2)}\sigma = \sigma D_{(2)}^{-1}, \quad (19)$$

$$L^A{}_C G_{AB} L^B{}_D = G_{CD} = \{\tilde{L}GL\}_{CD}, \quad (20)$$

$$L^A{}_B \bar{\tau}^B = D_{(1)}^{-1} \bar{\tau}^A D_{(2)}, \quad (21)$$

$$L^A{}_B \tau^B = D_{(2)}^{-1} \tau^A D_{(1)}, \quad (22)$$

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$$8L^A{}_B = \mathrm{tr}(D_{(1)}^{-1} \bar{\tau}^A D_{(2)} \tau^B) G_{CB}, \quad (23)$$

where tr denotes the trace. Note that $D_{(1)}(g(\omega)) = D_{(2)}(g(\omega))$ when $\omega_{A8} = 0$, i.e., when one restricts $\overline{\mathrm{SO}(4,4;\mathbb{R})}$ to

$$\overline{\mathrm{SO}(3,4;\mathbb{R})} = \{g \in \overline{\mathrm{SO}(4,4;\mathbb{R})} \mid$$

$$g = \begin{pmatrix} \exp\left(\frac{1}{4}\omega_{AB} D_{(1)}^{AB}\right) & 0 \\ 0 & \exp\left(\frac{1}{4}\omega_{AB} D_{(2)}^{AB}\right) \end{pmatrix} \quad \text{and} \quad \omega_{A8} = 0 \} \quad (24)$$

[one of the real forms of spin (7, C)].

In[877]:=

```
Block[{\psi, J, t, r, sJu},
sJu = {J[0] → 0, J[1] → 0, J[2] → 0,
       J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0};
r = {\psi[4], -ψ[5], -ψ[6], -ψ[7], ψ[0], -ψ[1], -ψ[2], -ψ[3]};
ψ = τ[A].τ[B].¶16[[1;;8]];
]
```

Out[877]=

```
τ[A].{f16[7][x0, x4], f16[6][x0, x4], -f16[5][x0, x4], -f16[4][x0, x4],
      f16[3][x0, x4], f16[2][x0, x4], -f16[1][x0, x4], -f16[0][x0, x4]}
τ[A1].τ[B1] → D_{(1)}^{-1}.τ[A1].τ[B1].D_{(1)};
τ[A1].τ[B1] → D_{(2)}^{-1}.τ[A1].τ[B1].D_{(2)};
σ.τ[A1].τ[B1] = Transpose[τ[A1]].σ.τ[B1] =
    Transpose[τ[A1]].Transpose[σ.τ[B1]] = Transpose[σ.τ[B1].τ[A1]];
σ.τ[A1].τ[B1] = Transpose[τ[B1].τ[A1]].Transpose[σ];
σ.τ[A1].τ[B1].ψ1 = Transpose[τ[B1].τ[A1]].Transpose[σ].ψ1;
Transpose[ψ2].σ ↔ Transpose[σ].ψ1 ? ;
σ.τ[A1].τ[B1].ψ1 ↔ Transpose[τ[B1].τ[A1]].Transpose[ψ2].σ;
σ.τ[A1].τ[B1].ψ1 ↔ Transpose[ψ2.τ[B1].τ[A1]].σ;
```

```

B1 = A1
 $\sigma.\psi_1 \leftrightarrow \text{Transpose}[\psi_2].\sigma^{-1}$  ?;
 $\psi_1 \leftrightarrow \text{Transpose}[\psi_2].\sigma^{-1}$  ?;
 $\text{Transpose}[\psi_2] \leftrightarrow \sigma.\psi_1$  ?;

In[878]:= {ψ1[4], -ψ1[5], -ψ1[6], -ψ1[7], ψ1[0], -ψ1[1], -ψ1[2], -ψ1[3]} /. {ψ1[#] →};

In[878]:= (*Block[{ψ1,ψ2,σψ,sσψ,r,sJψ},*
  σψ=σ.(ψ2[#]&/@Range[0,7]);
  sσψ=(ψ1[#]→σψ[[#+1]]&/@Range[0,7]);
  r={ψ1[4],-ψ1[5],-ψ1[6],-ψ1[7],ψ1[0],-ψ1[1],-ψ1[2],-ψ1[3]};
  (*ψ=τ[A].τ[B].Π16[[1;;8]]*)
  r/.sσψ
]*)

In[879]:= Block[{ψ1,ψ2,σψ,sσψ,r},
  σψ=σ.(ψ2[#]&/@Range[0,7]);
  sσψ=(ψ1[#]→σψ[[#+1]]&/@Range[0,7]);
  r={ψ1[4],-ψ1[5],-ψ1[6],-ψ1[7],ψ1[0],-ψ1[1],-ψ1[2],-ψ1[3]};
  (*ψ=τ[A].τ[B].Π16[[1;;8]]*)
  r/.sσψ
]

Out[879]= {ψ2[0], -ψ2[1], -ψ2[2], -ψ2[3], ψ2[4], -ψ2[5], -ψ2[6], -ψ2[7]}

for  $\frac{1}{i}$  :

In[880]:= Π16cccc = Block[{ψ1,ψ2,σψ,sσψ,sψ1,sψ2,r},
  σψ=σ.(Π16[[9;;16]]);
  sσψ=(ψ1[#]→σψ[[#+1]]&/@Range[0,7]);
  sψ1=Thread[(ψ1[#]&/@Range[0,7])→Π16[[1;;8]]];
  sψ2=Thread[(ψ1[#]&/@Range[0,7])→Π16[[9;;16]]];
  r={ψ1[4],-ψ1[5],-ψ1[6],-ψ1[7],ψ1[0],-ψ1[1],-ψ1[2],-ψ1[3]};
  (*ψ=τ[A].τ[B].*)
  Flatten[{r/.sψ1,r/.sψ2}]
]

Out[880]= {f16[4][x0,x4], -f16[5][x0,x4], -f16[6][x0,x4], -f16[7][x0,x4],
  f16[0][x0,x4], -f16[1][x0,x4], -f16[2][x0,x4], -f16[3][x0,x4],
  f16[12][x0,x4], -f16[13][x0,x4], -f16[14][x0,x4], -f16[15][x0,x4],
  f16[8][x0,x4], -f16[9][x0,x4], -f16[10][x0,x4], -f16[11][x0,x4]}

```


In[886]:=

caΨccΨ2.caΨccΨ2 // MatrixForm

Out[886]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

In[887]:=

$$\begin{aligned} & \mathbf{caΨccΨ2.Ψ16cccc - Ψ16 + Ψ16cccc} \\ & \mathbf{caΨccΨ2.Ψ16 - Ψ16cccc + Ψ16} \end{aligned}$$

Out[887]=

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

Out[888]=

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

for $\frac{1}{i}$:

In[889]:=

jjcc = caΨccΨ2 + ID16;

In[890]:=

jjcc.Ψ16 === Ψ16cccc

Out[890]=

True

In[891]:=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

Out[891]=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

In[892]:=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

Out[892]=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

$$F_A^a = \text{Transpose}[\eta 4488[\# + 1, \# + 1] * (\tau[\#].\text{unit}) \& /@ \text{Range}[0, 7]]$$

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right.$$

$$\left\{ \theta, \theta, \frac{1}{\sqrt{2}}, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta \right\}, \left\{ \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta \right\}, \\ \left\{ \frac{1}{\sqrt{2}}, \theta, \theta, \theta, \theta, \theta, -\frac{1}{\sqrt{2}} \right\}, \left\{ \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta \right\}, \\ \left\{ \theta, \theta, \frac{1}{\sqrt{2}}, \theta, \theta, \frac{1}{\sqrt{2}}, \theta, \theta \right\}, \left\{ \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, \theta, \frac{1}{\sqrt{2}}, \theta \right\} \}$$

```
FaA = Transpose[unit].σ.(τ[#]) & /@ Range[0, 7]
```

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, \theta, \theta, \theta, \frac{1}{\sqrt{2}}, \theta, \theta, \theta \right\}, \left\{ \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, -\frac{1}{\sqrt{2}} \right\}, \right. \\ \left\{ \theta, \theta, \frac{1}{\sqrt{2}}, \theta, \theta, \theta, \frac{1}{\sqrt{2}}, \theta \right\}, \left\{ \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta \right\}, \\ \left\{ \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta \right\}, \left\{ \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, \frac{1}{\sqrt{2}}, \theta \right\}, \\ \left. \left\{ \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta \right\} \right\}$$

In[893]:=

F_a^A

Out[893]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, \theta, \theta, \theta, \frac{1}{\sqrt{2}}, \theta, \theta, \theta \right\}, \left\{ \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, -\frac{1}{\sqrt{2}} \right\}, \right. \\ \left\{ \theta, \theta, \frac{1}{\sqrt{2}}, \theta, \theta, \theta, \frac{1}{\sqrt{2}}, \theta \right\}, \left\{ \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta \right\}, \left\{ \theta, \frac{1}{\sqrt{2}}, \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta \right\}, \\ \left. \left\{ \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, \frac{1}{\sqrt{2}}, \theta \right\}, \left\{ \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, \theta, \theta, \theta, -\frac{1}{\sqrt{2}}, \theta, \theta, \theta \right\} \right\}$$

In[894]:=

unit

Out[894]=

$$\left\{ \frac{1}{\sqrt{2}}, \theta, \theta, \theta, \frac{1}{\sqrt{2}}, \theta, \theta, \theta \right\}$$

In[895]:=

(F_a^A.unit)

Out[895]=

$$\{1, \theta, \theta, \theta, \theta, \theta, \theta, \theta\}$$

In[896]:=

(F_a^A[8].unit)

Out[896]=

$$0$$

In[897]:=

unit.F_A

Out[897]=

$$\{1, \theta, \theta, \theta, \theta, \theta, \theta, \theta\}$$

In[905]:=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

Out[905]=

$$\bar{x} = 2\langle x, 1 \rangle - x.$$

In[906]:=

$$\Psi16cc = 2 \text{realProjection16.}\Psi16 - \Psi16 // \text{FullSimplify}$$

Out[906]=

$$\begin{aligned} & \{f16[4][x_0, x_4], -f16[1][x_0, x_4], -f16[2][x_0, x_4], -f16[3][x_0, x_4], \\ & f16[0][x_0, x_4], -f16[5][x_0, x_4], -f16[6][x_0, x_4], -f16[7][x_0, x_4], \\ & f16[12][x_0, x_4], -f16[9][x_0, x_4], -f16[10][x_0, x_4], -f16[11][x_0, x_4], \\ & f16[8][x_0, x_4], -f16[13][x_0, x_4], -f16[14][x_0, x_4], -f16[15][x_0, x_4]\} \end{aligned}$$

In[907]:=

$$\begin{aligned} \Psi16ccA = & 2 (\text{Flatten}[\{\text{unit.}\Psi16[[1;;8]] * (\text{unit.}\text{F}_A^a), \text{unit.}\Psi16[[9;;16]] * (\text{unit.}\text{F}_A^a)\}\}) - \\ & \Psi16 // \text{FullSimplify} \end{aligned}$$

Out[907]=

$$\begin{aligned} & \{(-1 + \sqrt{2}) f16[0][x_0, x_4] + \sqrt{2} f16[4][x_0, x_4], -f16[1][x_0, x_4], -f16[2][x_0, x_4], \\ & -f16[3][x_0, x_4], -f16[4][x_0, x_4], -f16[5][x_0, x_4], -f16[6][x_0, x_4], -f16[7][x_0, x_4], \\ & (-1 + \sqrt{2}) f16[8][x_0, x_4] + \sqrt{2} f16[12][x_0, x_4], -f16[9][x_0, x_4], -f16[10][x_0, x_4], \\ & -f16[11][x_0, x_4], -f16[12][x_0, x_4], -f16[13][x_0, x_4], -f16[14][x_0, x_4], -f16[15][x_0, x_4]\} \end{aligned}$$

In[908]:=

$$\Psi16cc - \Psi16ccA // \text{FullSimplify}$$

Out[908]=

$$\begin{aligned} & \{-((-1 + \sqrt{2}) (f16[0][x_0, x_4] + f16[4][x_0, x_4])), 0, 0, 0, \\ & f16[0][x_0, x_4] + f16[4][x_0, x_4], 0, 0, 0, -((-1 + \sqrt{2}) (f16[8][x_0, x_4] + f16[12][x_0, x_4])), \\ & 0, 0, 0, f16[8][x_0, x_4] + f16[12][x_0, x_4], 0, 0, 0\} \end{aligned}$$

In[909]:=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \tilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elemen

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

Out[909]=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \tilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elemen

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

```
Table[{A, B, FullSimplify[
  ExpandAll[1/2 (τ[B].τ[A] + τ[A].τ[B]) == η4488[[A+1, B+1]*ID8]]}], {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

```
Table[{A, B, FullSimplify[
  ExpandAll[1/2 (τ[A].τ[B] + τ[B].τ[A]) == η4488[[A+1, B+1]*ID8]]}], {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

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17 (1982) 185–232

In[]:=

SUBMANIFOLDS AND SPECIAL STRUCTURES ON THE OCTONIANS

ROBERT L. BRYANT

In[]:=

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

Classically, this observation was used to define an almost complex structure on S^6 as follows: If $u \in S^6$, then R_u preserves the 2-plane spanned by 1 and u and therefore preserves its orthogonal 6-plane, which may be identified with $T_u S^6 \subseteq \text{Im } \mathbf{O}$ after translation to the origin. Thus R_u induces a complex

structure on $T_u S^6$ for each $u \in S^6$. This almost complex structure is not integrable (even locally) to a complex structure (see below). In [Ca], Calabi noticed that for *any* oriented $M^6 \subseteq \text{Im } \mathbf{O}$, $R_{N(x)}$ induces a complex structure on $T_x M^6$ (where $N(x)$ is the oriented unit normal). Thus every oriented $M^6 \subseteq \text{Im } \mathbf{O}$ inherits an almost complex structure. Moreover, M^6 inherits a metric from $\text{Im } \mathbf{O}$, so we actually have a $U(3)$ -structure on M^6 . (Calabi calls these structures “almost Hermitian.” He also proves that such M^6 possess a canonical $SU(3)$ -substructure but we will not need this.) Calabi shows that the second fundamental form Π of M decomposes with respect to the $U(3)$ -structure into a piece $\Pi^{1,1}$ of type (1,1) and a piece $\Pi^{0,2}$ of type (0,2). He then shows that the almost complex structure of M is integrable if and only if $\Pi^{1,1} = 0$ and that the canonical 2-form of the $U(3)$ -structure, say Ω , is closed if and only if $\Pi^{0,2} = 0$ and $\text{tr}_1 \Pi^{1,1} = 0$. From this it follows that the $U(3)$ -structure on M^6 is Kähler if and only if $\Pi \equiv 0$, so that M^6 is a hyperplane (or a union of pieces of hyperplanes). Calabi then constructs nontrivial examples of $M^6 \subseteq \text{Im } \mathbf{O}$ for which the almost complex structure is integrable. His examples are of the form $S \times \mathbf{R}^4 \subseteq \text{Im } \mathbf{O}$, where $S \subseteq \mathbf{R}^3$ is a minimal surface, $\mathbf{R}^3 \subseteq \text{Im } \mathbf{O}$ is an *associative* 3-plane, and $\mathbf{R}^4 = (\mathbf{R}^3)^\perp$. Calabi leaves open the problem of determining whether or not there are nontrivial $M^6 \subseteq \text{Im } \mathbf{O}$ for which the canonical 2-form is closed.

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

In[]:=

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex structure J_u . If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on \mathbf{O} . However, because S^6 is connected, we see that the orientation of \mathbf{O} induced by the natural orientation of \mathbf{O}_u as a complex vector space does not depend on u . We refer to this orientation as the natural orientation of \mathbf{O} .

Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

so J_u is an isometry for each $u \in S$. Moreover, it follows that \mathbf{O}_u is endowed with a natural Hermitian structure with respect to the inner product $\langle \cdot, \cdot \rangle$. We denote the group of complex linear transformations of \mathbf{O}_u by $GL(\mathbf{O}_u)$ and the special unitary transformations of \mathbf{O}_u with its Hermitian metric by $SU(\mathbf{O}_u)$.

We let $\text{Spin}(7) \subseteq SO(8)$ denote the subgroup generated by the set $\{J_u \mid u \in S^6\} \subseteq SO(8)$. It is known (see [12]) that $\text{Spin}(7)$ is a connected, simply connected, compact Lie group of real dimension 21. Its center is $\{\pm I_8\} \cong \mathbb{Z}/2$ and $\text{Spin}(7)/\{\pm I_8\}$ is isomorphic to $SO(7)$, a simple group. We want to make explicit the structure equations of $\text{Spin}(7)$ as a subgroup of $SO(8)$ in such a way that its relationship with the complex structures J_u is made clear.

In[1]:= Let $u \in S^6$ be an imaginary unit which is orthogonal to $\epsilon \in \mathbf{O}$. For each $\lambda \in \mathbb{R}$, $(\cos \lambda \epsilon + \sin \lambda u)$ is an imaginary unit. Hence $J_\epsilon \circ J_{(\cos \lambda \epsilon + \sin \lambda u)} = -\cos \lambda I + \sin \lambda J_\epsilon \circ J_u$ is an element of $\text{Spin}(7)$. We easily compute that $J_\epsilon \circ J_u + J_u \circ J_\epsilon = 0$ by using (1.10). Thus $(J_\epsilon \circ J_u)^2 = J_\epsilon \circ J_u \circ J_\epsilon \circ J_u = -J_\epsilon^2 \circ J_u^2 = -I$. It follows that

$$(1.16) \quad \exp(\lambda J_\epsilon \circ J_u) = \cos \lambda I + \sin \lambda J_\epsilon \circ J_u.$$

Thus, if $\text{spin}(7) \subseteq so(8)$ is the Lie algebra of $\text{Spin}(7)$, we see that $J_\epsilon \circ J_u \in \text{spin}(7)$ for all $u \in S^6$ with $\langle u, \epsilon \rangle = 0$. Since $\text{spin}(7)$ is a vector space, we see that $L \subseteq \text{spin}(7)$ where

$$(1.17) \quad L = \{J_\epsilon \circ J_w \mid w \in \text{Im } \mathbf{O}, \langle \epsilon, w \rangle = 0\}.$$

Note that $\dim_{\mathbb{R}} L = 6$.

Transpose[a].σ16.b == Transpose[J.a].σ16.J.b = a.Transpose[J].σ16.J.b
want ψcc.σ16.?Ψ16 = Ψ16.J.σ16.?Ψ16 ≠ 0

need 1 irrep for {i, u}

need 1 irrep for {i, u} _perp

1 == x0 or x4; u == x4 or x0

x0 has periodic fns; use x4

In[910]:=

t16A[All, 2]

Out[910]=

{ {0}, {1}, {2}, {3}, {4}, {5}, {6}, {7} }

In[911]:=

T16^A[4].T16^A[4] == -Id16

Out[911]=

True

In[912]:=

(*JT4=T16^A[4] *) (* complex structure on S^6 *)

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex

J.J == -ID16

matrix multiplication is associative

(split) octonion multiplication is NOT associative

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

$$\text{Transpose}[J].\sigma16.J == \sigma16$$

In[913]:=

```
Clear[possibleComplexStructureIndex2];
(*Clear[possibleComplexStructureMatrix2];*)
possibleComplexStructureIndex2[j_] := Module[{m, r, jsj, jj},
  J = base16[[j]][1];
  jj = (FullSimplify[J.J] === -ID16);
  jsj = FullSimplify[Transpose[J].σ16.J] === σ16;
  Return[{jj, jsj}]]
```

In[915]:=

```
possibleComplexStructureIndex2[#] & /@ {12, 97, 100, 109, 127, 225, 230, 236}
Out[915]=
{{False, False}, {True, False}, {True, False}, {True, False},
 {True, False}, {False, False}, {False, False}, {False, False}}
```

In[916]:=

```
Clear[possibleComplexStructureIndex];
Clear[possibleComplexStructureMatrix];
possibleComplexStructureIndex = (*Parallel*)Table[
  Block[{J, r, m},
    Clear[J, r, m];
    m = r = {};
    J = base16[[j]][1];
    If[(J.J === -ID16) &&
      (FullSimplify[Transpose[J].σ16.J] === σ16), r = base16[[j]][2]];
    m = σ16.J;, ##&[]];
    (*If[!(m===-Transpose[m]),r=base16[[j]][2],r={}]*)
    r]
  , {j, 1, Length[base16]}] // Union // Drop[#, 1] &
```

Out[918]=

```
{ {4}, {5}, {6}, {7}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {4, 5}, {4, 6},
 {4, 7}, {5, 6}, {5, 7}, {6, 7}, {0, 1, 2, 3, 4}, {0, 1, 2, 3, 5}, {0, 1, 2, 3, 6},
 {0, 1, 2, 3, 7}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7}, {0, 1, 5, 6, 7},
 {0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 2, 5, 6, 7}, {0, 3, 4, 5, 6},
 {0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {0, 3, 5, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7},
 {1, 2, 4, 6, 7}, {1, 2, 5, 6, 7}, {1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7},
 {1, 3, 5, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7}, {2, 3, 4, 6, 7}, {2, 3, 5, 6, 7},
 {0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 6}, {0, 1, 2, 3, 4, 7}, {0, 1, 2, 3, 5, 6},
 {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}, {0, 1, 4, 5, 6, 7}, {0, 2, 4, 5, 6, 7},
 {0, 3, 4, 5, 6, 7}, {1, 2, 4, 5, 6, 7}, {1, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 7}}
```

In[919]:=

```
Length[possibleComplexStructureIndex]
```

Out[919]=

56

```
In[920]:= sixersJ = If[Length[#] ≥ 6, #, ## &[]] & /@ possibleComplexStructureIndex
Out[920]= {{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 6}, {0, 1, 2, 3, 4, 7}, {0, 1, 2, 3, 5, 6},
{0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}, {0, 1, 4, 5, 6, 7}, {0, 2, 4, 5, 6, 7},
{0, 3, 4, 5, 6, 7}, {1, 2, 4, 5, 6, 7}, {1, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 7}}
```

```
In[921]:= Clear[possibleT4ComplexStructureIndex];
Clear[possibleT4ComplexStructureMatrix];
possibleComplexT4StructureIndex = (*Parallel*)Table[
  Block[{J, a1, a2, b1, b2, c1, c2,
    c3, c3ori, c3s, c2ed, found, r, m, debug = False},
    Clear[J, a1, a2, b1, b2, c1, c2, c3, c3ori, c3s, c2ed, found, r, m];
    c3s = found = r = {};
    If[FullSimplify[T16A[4].base16[[j]][1]] === base16[[j]][1].T16A[4]],
      J = T16A[4].base16[[j]][1] // FullSimplify;
      a1 = J.J // FullSimplify;
      If[(a1 === -ID16),
        a2 = Transpose[J].σ16.J // FullSimplify;
        If[(a2 === σ16),
          m = σ16.J;
          c1 = Append[base16[[j]][2], 4] // Sort; (* for J = T16A[4]*#*[1]*)]
          c3ori = c3 = Select[base16, #[[2]] == c1 & → "Index"];
          (*Print["c1= ",c1];*)
          If[c3 === {},
            c3s = StringReplace[ToString[c1], "4, 4" → "256"] //
              ToExpression // Sort // Drop[#, -1] &;
            c3 = Select[base16, #[[2]] == c3s & → "Index"],
            ## &[]];
          If[debug, Print["Append[base16[[j]][[2]],4]//Sort=c1= ",
            c1, "Select[base16,#[[2]]==c1&= ", c3ori,
            ";StringReplace[c1,'4, 4'->'256']= ",
            c3s, ";c3= ", c3], ## &[]];
          If[! (c3 === {}),
            c2 = c3[[1]];
            (*Print["c2= ",c2];*)
            If[c2 > 0 && c2 ≤ Length[base16], found = base16[[c2]], ## &[]];
            If[! (found === {}) && (J === found[[1]] || J === -found[[1]])
              (*&&!(m===-Transpose[m])*),
              r = found[[2]];
              (*Print[base16[[j]][2],";Tr[J.J]/16=",Tr[a1]/16,
```

```

";c1=",c1,";c2=Select[base16,#[[2]]==c1&=  ",c2,
";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
Tr[found[[1]].found[[1]]/16];*)

",
## &[]];
, ## &[], ## &[], ## &[], ## &[], ## &[]];

r]
, {j, 1, Length[base16]}] // Union // Drop[#, 1] &

```

Out[923]=

```

{{4}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {5, 6}, {5, 7},
{6, 7}, {0, 1, 2, 3, 4}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7},
{0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 3, 4, 5, 6},
{0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7}, {1, 2, 4, 6, 7},
{1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7},
{2, 3, 4, 6, 7}, {0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}

```

In[924]:=

```
JT4 = T16^A[4] ;(* complex structure on S^6 *)
```

```
Jcomplement  $\longleftrightarrow (\hat{q}[7]*\text{Cos}[Q7] + \hat{q}[6]*\text{Sin}[Q7]\text{Cos}[Q8] + \hat{q}[5]*\text{Sin}[Q7]\text{Sin}[Q8])$ 
```

In[925]:=

```
basisProjections16 =
Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@
Range[16];
```

In[926]:=

```
basisProjections8 =
Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@*
Range[8];
```

In[927]:=

```
JcomplementProjection8 =
(basisProjections8[[7]] + basisProjections8[[6]] + basisProjections8[[5]]);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8
```

Out[928]=

```
True
```

In[929]:=

```
JcomplementProjection8 = (basisProjections8[[7]] +
basisProjections8[[6]] + basisProjections8[[5]] + basisProjections8[[4]]);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8
```

Out[930]=

```
True
```

In[931]:=

```
(*JcomplementProjection8=
(basisProjections8[[7]]*\text{Cos}[Q7]+basisProjections8[[6]]*\text{Sin}[Q7]\text{Cos}[Q8]+
basisProjections8[[5]]*\text{Sin}[Q7]\text{Sin}[Q8]);*)
```

In[932]:=

```
JcomplementProjection8 // FullSimplify // MatrixForm  
JcomplementProjection8 // MatrixForm  
%% - % :  
%
```

Out[932]//MatrixForm=

Out[933]//MatrixForm=

Out[934]=

In[935]:=

```
(parallel04 =
  Block[{x = Table[KroneckerDelta[1, j], {j, 16}]}, KroneckerProduct[x, x]] +
  Block[{x = Table[KroneckerDelta[5, j], {j, 16}]},
    KroneckerProduct[x, x]]) // MatrixForm
```

Out[935]//MatrixForm=

In[936]:=

```
(projection04 = ID16 -  
  Block[{x = Table[KroneckerDelta[1, j], {j, 16}]}, KroneckerProduct[x, x]] -  
  Block[{x = Table[KroneckerDelta[5, j], {j, 16}]},  
    KroneckerProduct[x, x]]) // MatrixForm
```

Out[936]//MatrixForm=

In[937]:=

`projection04.parallel04 === Zero16`

Out[937]=

True

$\{\{0, 1, 2, 3, 5, 6\}, \{0, 1, 2, 3, 5, 7\}, \{0, 1, 2, 3, 6, 7\}\}$

```
In[938]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 5, 6};  
% === -T16^A[4].T16^A[7].T16^A[8]  
Out[939]= True  
In[940]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 5, 7};  
% === T16^A[4].T16^A[6].T16^A[8]  
Out[941]= True  
In[942]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 6, 7};  
% === -T16^A[4].T16^A[5].T16^A[8]  
Out[943]= True
```


In[960]:=

$$\mathbf{T4cc} = \mathbf{T16^A}[4] - (\mathbf{realProjection16.T16^A}[4]) // \mathbf{FullSimplify}$$

Out[960]=

$$\left\{ \begin{array}{l} \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0 \right\}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0 \}, \\ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0 \right\}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0 \}, \\ \left\{ 0, \frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ \{ 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \{ 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \left\{ 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ \{ 1, 0 \}, \\ \{ 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \end{array} \right\}$$

In[961]:=

$$\mathbf{T4cc} \cdot \mathbf{T4cc}$$

Out[961]=

$$\left\{ \begin{array}{l} \left\{ -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ \left\{ 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ \{ 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \{ 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \left\{ \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ \left\{ 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ \{ 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0 \right\}, \\ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\}, \\ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{1}{2}, 0, 0, 0, -\frac{1}{2}, 0, 0, 0 \right\}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0 \} \end{array} \right\}$$

In[966]:=

```
Jcomplementcc = Jcomplement - (realProjection16.Jcomplement) // FullSimplify
```

Out[966]=

$$\begin{aligned} & \left\{ \left\{ 0, \frac{\cos(Q7)}{2}, -\frac{1}{2} \cos(Q8) \sin(Q7), -\frac{1}{2} \sin(Q7) \sin(Q8), 0, \right. \right. \\ & \quad \left. \left. -\frac{\cos(Q7)}{2}, \frac{1}{2} \cos(Q8) \sin(Q7), \frac{1}{2} \sin(Q7) \sin(Q8), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \sin(Q7) \sin(Q8), -\cos(Q8) \sin(Q7), \cos(Q7), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ \cos(Q8) \sin(Q7), -\sin(Q7) \sin(Q8), 0, 0, 0, 0, 0, \cos(Q7), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ \sin(Q7) \sin(Q8), \cos(Q8) \sin(Q7), 0, 0, 0, 0, -\cos(Q7), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, -\frac{\cos(Q7)}{2}, \frac{1}{2} \cos(Q8) \sin(Q7), \frac{1}{2} \sin(Q7) \sin(Q8), 0, \frac{\cos(Q7)}{2}, \right. \\ & \quad \left. -\frac{1}{2} \cos(Q8) \sin(Q7), -\frac{1}{2} \sin(Q7) \sin(Q8), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ \cos(Q7), 0, 0, 0, 0, 0, \sin(Q7) \sin(Q8), -\cos(Q8) \sin(Q7), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, \cos(Q7), \cos(Q8) \sin(Q7), -\sin(Q7) \sin(Q8), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, -\cos(Q7), 0, \sin(Q7) \sin(Q8), \cos(Q8) \sin(Q7), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{\cos(Q7)}{2}, \frac{1}{2} \cos(Q8) \sin(Q7), \right. \\ & \quad \left. \frac{1}{2} \sin(Q7) \sin(Q8), 0, \frac{\cos(Q7)}{2}, -\frac{1}{2} \cos(Q8) \sin(Q7), -\frac{1}{2} \sin(Q7) \sin(Q8) \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -\sin(Q7) \sin(Q8), \cos(Q8) \sin(Q7), -\cos(Q7), 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -\cos(Q8) \sin(Q7), \sin(Q7) \sin(Q8), 0, 0, 0, 0, 0, -\cos(Q7) \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -\sin(Q7) \sin(Q8), -\cos(Q8) \sin(Q7), 0, 0, 0, 0, \cos(Q7), 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\cos(Q7)}{2}, -\frac{1}{2} \cos(Q8) \sin(Q7), \right. \\ & \quad \left. -\frac{1}{2} \sin(Q7) \sin(Q8), 0, -\frac{\cos(Q7)}{2}, \frac{1}{2} \cos(Q8) \sin(Q7), \frac{1}{2} \sin(Q7) \sin(Q8) \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -\cos(Q7), 0, 0, 0, 0, 0, -\sin(Q7) \sin(Q8), \cos(Q8) \sin(Q7) \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, -\cos(Q7), -\cos(Q8) \sin(Q7), \sin(Q7) \sin(Q8), 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \cos(Q7), 0, -\sin(Q7) \sin(Q8), -\cos(Q8) \sin(Q7), 0, 0 \right\} \end{aligned}$$

In[967]:=

```
FullSimplify[Jcomplementcc.Jcomplementcc]
```

Out[967]=

$$\begin{aligned} & \left\{ \left\{ -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, \frac{1}{4} (-3 + \cos(2 Q7)), \frac{1}{2} \cos(Q7) \cos(Q8) \sin(Q7), \frac{1}{2} \cos(Q7) \sin(Q7) \sin(Q8), 0, \frac{\cos(Q7)^2}{2}, \right. \\ & \quad \left. \frac{1}{2} \cos(Q7) \cos(Q8) \sin(Q7), \frac{1}{2} \cos(Q7) \sin(Q7) \sin(Q8), 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, \frac{1}{2} \cos(Q7) \cos(Q8) \sin(Q7), -\cos(Q7)^2 + \frac{1}{4} (-3 + \cos(2 Q8)) \sin(Q7)^2, \right. \\ & \quad \left. \frac{1}{2} \cos(Q8) \sin(Q7)^2 \sin(Q8), 0, \frac{1}{2} \cos(Q7) \cos(Q8) \sin(Q7), \right. \\ & \quad \left. \frac{1}{2} \cos(Q8)^2 \sin(Q7)^2, \frac{1}{2} \cos(Q8) \sin(Q7)^2 \sin(Q8), 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \\
& -\cos[Q7]^2 + \frac{1}{4} (-3 + \cos[2 Q8]) \sin[Q7]^2, \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8] \}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \\
& \frac{1}{2} \sin[Q7]^2 \sin[Q8]^2, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \\
& \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], -\cos[Q7]^2 - \frac{1}{4} (3 + \cos[2 Q8]) \sin[Q7]^2 \}
\end{aligned}$$

In[968]:=

T16^A[4].Jcomplement // MatrixForm

Out[968]/MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\cos[Q7] & 0 & 0 & 0 & 0 \\
0 & \cos[Q7] & 0 & 0 & 0 \\
0 & 0 & \cos[Q7] & 0 & -\sin[Q7] \sin[Q8] - \cos[Q7] \cos[Q8] \sin[Q7] \sin[Q8] \\
0 & 0 & 0 & \cos[Q7] & \cos[Q8] \sin[Q7] - \sin[Q7] \cos[Q8] \sin[Q7] \\
0 & 0 & -\sin[Q7] \sin[Q8] & \cos[Q8] \sin[Q7] & -\cos[Q7] \\
0 & 0 & -\cos[Q8] \sin[Q7] & -\sin[Q7] \sin[Q8] & 0 \\
\sin[Q7] \sin[Q8] & \cos[Q8] \sin[Q7] & 0 & 0 & 0 \\
-\cos[Q8] \sin[Q7] & \sin[Q7] \sin[Q8] & 0 & 0 & 0
\end{pmatrix}$$

In[969]:=

T16^A[4].Jcomplement === Jcomplement.T16^A[4]**FullSimplify[Jcomplement.Jcomplement] === -ID16****T16^A[4].T16^A[4] === -ID16**

Out[969]=

True

Out[970]=

True

Out[971]=

True

In[972]:=

unit3 = {Cos[Q7], Sin[Q7] Cos[Q8], Sin[Q7] Sin[Q8]}

Out[972]=

{Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}

In[973]:=

unit3.unit3 // FullSimplify

Out[973]=

1

```
In[974]:= unit3
sixers
Complement[{0, 1, 2, 3, 5, 6, 7}, #] & /@ %
eqsRQQ7Q8 = Thread[(Subscript[q, #][1]] & /@ %) == RQ * unit3]
sRQQ7Q8 = Solve[And @@ %, {RQ, Q7, Q8}] // FullSimplify
seqsRQQ7Q8 = %% /. {Equal -> Rule}

Out[974]= {Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}

Out[975]= {{0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7} }

Out[976]= {{7}, {6}, {5} }

Out[977]= {q7 == RQ Cos[Q7], q6 == RQ Cos[Q8] Sin[Q7], q5 == RQ Sin[Q7] Sin[Q8]}

Out[978]= 
$$\left\{ \begin{array}{l} RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \operatorname{Log} \left[ \frac{-i\sqrt{q_5^2 + q_6^2 - q_7^2}}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, \\ Q8 \rightarrow \operatorname{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \end{array} \right\}, \left\{ \begin{array}{l} RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, \\ Q7 \rightarrow 2\pi c_1 - i \operatorname{Log} \left[ \frac{i(\sqrt{q_5^2 + q_6^2} + i q_7)}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, Q8 \rightarrow 2\pi c_2 - i \operatorname{Log} \left[ \frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}} \right] \text{ if } c_2 \in \mathbb{Z} \end{array} \right\}, \\ \left\{ \begin{array}{l} RQ \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \operatorname{Log} \left[ \frac{-i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, \\ Q8 \rightarrow 2\pi c_2 - i \operatorname{Log} \left[ \frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}} \right] \text{ if } c_2 \in \mathbb{Z} \end{array} \right\}, \left\{ \begin{array}{l} RQ \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, \\ Q7 \rightarrow 2\pi c_1 - i \operatorname{Log} \left[ \frac{i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}} \right] \text{ if } c_1 \in \mathbb{Z}, Q8 \rightarrow \operatorname{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \end{array} \right\} \right\}$$

Out[979]= {q7 -> RQ Cos[Q7], q6 -> RQ Cos[Q8] Sin[Q7], q5 -> RQ Sin[Q7] Sin[Q8]}
```

```

In[980]:= FullSimplify[ ( √(q62 + q52) /. seqsRQQ7Q8 ), Q7 > 0 && Sin[Q7] > 0 && RQ > 0 ]
% / q7 /. seqsRQQ7Q8
Solve[ √(q62 + q52) == FullSimplify[%, Q7 > 0 && Sin[Q7] > 0 && RQ > 0], Q7 ]
sQ7 = {Q7 → ArcTan[q7, √(q52 + q62)]}

Out[980]= RQ Sin[Q7]
Out[981]= Tan[Q7]
Out[982]= { {Q7 → ArcTan[ √(q52 + q62) / q7 ] + π c1 if c1 ∈ ℤ] } }

Out[983]= {Q7 → ArcTan[q7, √(q52 + q62)]}

In[984]:= (*sQ7=*) Solve[ √(q62 + q52) == FullSimplify[ ( √(q62 + q52) / q7 ) /. seqsRQQ7Q8 ], Q7 > 0 && Sin[Q7] > 0 && RQ > 0 ], Q7 ] // FullSimplify
Out[984]= { {Q7 → ArcCot[ q7 / √(q52 + q62) ] + π c1 if c1 ∈ ℤ] } }

```

```
In[985]:= eqsRQQ7Q8[[2;;3]]
Transpose[{{%[All, 1], %[All, 2]}]]

$$\%[2][1] / \%[1][1] == \%[2][2] / \%[1][2] // \text{FullSimplify}$$

sQ8 = Solve[%, Q8][1] // FullSimplify
sQ8 = {Q8 \rightarrow \text{ArcTan}[q_6, q_5]}
sRQQ7Q8 /. % // FullSimplify

Out[985]= {q_6 == RQ \text{Cos}[Q8] \text{Sin}[Q7], q_5 == RQ \text{Sin}[Q7] \text{Sin}[Q8]}

Out[986]= {{q_6, RQ \text{Cos}[Q8] \text{Sin}[Q7]}, {q_5, RQ \text{Sin}[Q7] \text{Sin}[Q8]}}
```

Out[987]=
$$\frac{q_5}{q_6} == \text{Tan}[Q8]$$

Out[988]=
$$\left\{ Q8 \rightarrow \text{ArcTan}\left[\frac{q_5}{q_6}\right] + \pi c_1 \text{ if } c_1 \in \mathbb{Z} \right\}$$

Out[989]= {Q8 \rightarrow \text{ArcTan}[q_6, q_5]}

Out[990]=
$$\left\{ \begin{array}{l} RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \text{Log}\left[\frac{-i\sqrt{q_5^2 + q_6^2} - q_7}{\sqrt{q_5^2 + q_6^2 + q_7}}\right] \text{ if } c_1 \in \mathbb{Z}, \\ \text{ArcTan}[q_6, q_5] \rightarrow \text{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \end{array} \right\},$$

$$\left\{ \begin{array}{l} RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \text{Log}\left[\frac{i(\sqrt{q_5^2 + q_6^2} + i q_7)}{\sqrt{q_5^2 + q_6^2 + q_7}}\right] \text{ if } c_1 \in \mathbb{Z}, \\ \text{ArcTan}[q_6, q_5] \rightarrow \text{ArcTan}[q_6, q_5] + 2\pi c_2 - i \text{Log}\left[\frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z} \end{array} \right\},$$

$$\left\{ \begin{array}{l} RQ \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \text{Log}\left[\frac{-i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7}}\right] \text{ if } c_1 \in \mathbb{Z}, \\ \text{ArcTan}[q_6, q_5] \rightarrow \text{ArcTan}[q_6, q_5] + 2\pi c_2 - i \text{Log}\left[\frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z} \end{array} \right\},$$

$$\left\{ \begin{array}{l} Q7 \rightarrow 2\pi c_1 - i \text{Log}\left[\frac{i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7}}\right] \text{ if } c_1 \in \mathbb{Z}, \\ \text{ArcTan}[q_6, q_5] \rightarrow \text{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \end{array} \right\}$$

In[991]:= **sQ7Q8 = Flatten[{sQ7, sQ8}]**

Out[991]= {Q7 \rightarrow \text{ArcTan}[q_7, \sqrt{q_5^2 + q_6^2}], Q8 \rightarrow \text{ArcTan}[q_6, q_5]}

```

In[992]:= Block[{a, b, c},
  {a, b, c} * unit3]
Out[992]= {a Cos[Q7], b Cos[Q8] Sin[Q7], c Sin[Q7] Sin[Q8]}

In[993]:= base16[[#][1]].base16[[#][1]] === -ID16 & /@ {222, 223, 224}
Out[993]= {True, True, True}

In[994]:= (*sixersT4=%*{Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}
{Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}.
{Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}/.FullSimplify*)
Out[995]:= Block[{J = T16^A[4], m = σ16.J},
  {J.J === -ID16, σ16 === Transpose[J].σ16.J, !(m === -Transpose[m])}]
Out[995]= {True, True, False}

In[996]:= (*Clear[possibleComplexStructureIndex];
Clear[possibleComplexStructureMatrix];
possibleComplexStructureIndex=Select[base16,Block[{J,a0,a1,a2,b1,b2,r},
  Clear[J,a0,a1,a2,b1,b2,r];
  a0=#[[1]];
  J=T16^A[4]*a0//FullSimplify;
  a1=J.J//FullSimplify;
  b1=(a1===-ID16);
  a2=Transpose[J].σ16.J//FullSimplify;
  b2=(a2====σ16);
  r=b1&&b2;
  If[r,Block[{c1,c2,c3},
    c1=Append[#[2],4]//Sort; (* for J = T16^A[4]*#[1]*)
    c2=Select[base16,#[2]==c1&→"Index"][[1]];
    Print[#[2],";",Tr[a1]/16,";c1=",c1,";c2=",
    c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
    Tr[base16[[c2]][1].base16[[c2]][1]]/16],##&[]];
    r
  ]&→"Index"]
possibleComplexStructureMatrix=base16[[2]]&/@%*)

In[997]:= (*Tr[(T16^A[3].T16^A[4]).(T16^A[3].T16^A[4])]/16*)

```

```

In[998]:= (*base16[[5]][[2]]*)
In[999]:= (*Select[base16,#[[2]]=={1,2,3,4,5,6}&→"Index"]*)
In[1000]:= (*base16[[240]][[2]]
Tr[base16[[240]][[1]].base16[[240]][[1]]/16*)
In[1001]:= (*(*Parallel*)Do[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
Clear[J,a0,a1,a2,b1,b2,r];
a0=base16[[j]][[1]];
J=T16^4*a0//FullSimplify;
a1=J.J//FullSimplify;
If[!(a1===-ID16),Continue[],##&[]];
(*b1=(a1===-ID16);*)
a2=Transpose[J].σ16.J//FullSimplify;
If[!(a2===σ16),Continue[],##&[]];
(*b2=(a2===σ16);*)
(*r=b1&&b2;*)

c1=Append[base16[[j]][[2]],4]//Sort; (* for J = T16^4*#[[1]]*)
c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
Print[base16[[j]][[2]],";",Tr[a1]/16,";c1=",c1,
";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]/16=",
Tr[base16[[c2]][[1]].base16[[c2]][[1]]/16];
{J,base16[[c2]][[1]]}];
,{j,1,Length[base16}}]*)

In[1002]:= (*ParallelDo[
Block[{r,t,a1},
t=Tr[base16[[j]][[1]].base16[[k]][[1]]/16];
r=t^2==KroneckerDelta[j,k];
a1=If[r,##&[],Print[{{j,k},t,{base16[[j]][[2]],base16[[k]][[2]]}}]];
],{j,1,Length[base16]},{k,1,Length[base16}}]*)

```

In[1003]:=

```
(* (*Parallel*)Table[Catch[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
  Clear[J,a0,a1,a2,b1,b2,r];
  a0=base16[[j]][1]];
  J=T16^A[4]*a0//FullSimplify;
  a1=J.J//FullSimplify;
  If[!(a1===-ID16),Throw[Null],##&[]];
  (*b1=(a1===-ID16);*)
  a2=Transpose[J].σ16.J//FullSimplify;
  If[!(a2===σ16),Throw[Null],##&[]];
  (*b2=(a2===σ16);*)
  (*r=b1&&b2;*)

  c1=Append[base16[[j]][2],4]//Sort; (* for J = T16^A[4]*#[[1]]*)
  c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
  Print[base16[[j]][2],";",Tr[a1]/16,";c1=",c1,
  ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
  Tr[base16[[c2]][1].base16[[c2]][1]]/16];
  {{J},{base16[[c2]][1]},{J==base16[[c2]][1]||J===-base16[[c2]][1]}}]];
  ,{j,1,Length[base16]/128}]*)
```

In[1004]:=

```
(*(*Parallel*)Table[
  Block[{J,a1,a2,b1,b2,c1,c2,c3,r},
    Clear[J,a1,a2,b1,b2,r];
    r={};
    J=T16^A[4].base16[[j]][1]//FullSimplify;
    a1=J.J//FullSimplify;
    If[(a1===-ID16),
      a2=Transpose[J].σ16.J//FullSimplify;
      If[(a2==σ16),
        c1=Append[base16[[j]][2],4]//Sort; (* for J = T16^A[4]*#[1]*)
        c3=Select[base16,##[2]==c1&→"Index"];
        If[!(c3=={}),
          c2=c3[[1]];
          Print[base16[[j]][2],";Tr[J.J]/16=",Tr[a1]/16,";c1=",c1,
            ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
            Tr[base16[[c2]][1].base16[[c2]][1]]/16];
          r={{}J},{base16[[c2]][1]},{J==base16[[c2]][1]||J===-base16[[c2]][1]}];
          ,##&[],##&[],##&[]];
        r]
      ,{j,1,Length[base16}]]//Union*)
```

In[1005]:=

```
(*possibleComplexStructureIndex=
Select[base16,Block[{J=T16^A[4]*#[1]],a1,a2,b1,b2,r},
  a1=J.J;
  a2=Transpose[J].σ16.J;
  b1=a1===-ID16;
  b2=a2==σ16;
  r=b1&&b2;
  (*If[r,Print[a1]];*)
  r
  ]&→"Index"]
possibleComplexStructureMatrix=base16[[#][2]]&/@%*)
```

In[1006]:=

```
base16[[201]][2]
```

Out[1006]=

```
{1, 2, 3, 5, 6}
```

In[1007]:=

$$\mathbf{T16}^A[4].\mathbf{base16}[[201][1]] // \text{MatrixForm}$$

Out[1007]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[1008]:=

$$\mathbf{T16}^A[1].\mathbf{T16}^A[2].\mathbf{T16}^A[3].\mathbf{T16}^A[4].\mathbf{T16}^A[5].\mathbf{T16}^A[6] // \text{MatrixForm}$$

Out[1008]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

In[1009]:=

```
(*possibleComplexStructureIndex=
Select[base16,Block[{J=T16^A[4]*#[[1]],a1=J.J,a2=Transpose[J].σ16.J,
b1=a1===-ID16,b2=a2==σ16},b1&&b2]&→"Index"]
possibleComplexStructureMatrix=base16[[#][2]]&/@%*)
```

In[1010]:=

```
possibleComplexStructureIndex =
  Select[base16, Block[{J = T16^A[4] * #\[LeftDoubleBracket]1\[RightDoubleBracket], m = σ16.J}, J.J === -ID16 &&
    σ16 === Transpose[J].σ16.J(*&&! (m === -Transpose[m]) *)] & → "Index"]
possibleComplexStructureMatrix = base16[[#]\[LeftDoubleBracket]2\[RightDoubleBracket]] & /@ %
```

Out[1010]=

{4, 178, 187, 201}

Out[1011]=

{ {3}, {0, 1, 3, 6, 7}, {0, 2, 3, 5, 7}, {1, 2, 3, 5, 6} }

In[1012]:=

Select[base16, #\[LeftDoubleBracket]2\[RightDoubleBracket] == {1, 2, 3, 4, 5, 6} & → "Index"]

Out[1012]=

{240}

In[1013]:=

base16[[240]]\[LeftDoubleBracket]2\[RightDoubleBracket]

Out[1013]=

{1, 2, 3, 4, 5, 6}

In[1014]:=

base16[[240]]\[LeftDoubleBracket]1\[RightDoubleBracket] // MatrixForm

Out[1014]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1015]:=

```
(*possibleComplexStructureIndex=
  Select[base16,Block[{J=#\[LeftDoubleBracket]1\[RightDoubleBracket],m=σ16.#\[LeftDoubleBracket]1\[RightDoubleBracket]},J.J===-ID16&&
  σ16==Transposed[J].σ16.J&&! (m===-Transposed[m]) ]&→"Index"]
possibleComplexStructureMatrix=base16[[#]\[LeftDoubleBracket]2\[RightDoubleBracket]&/@%*)
```

```
findComplexStucture[] := Module[{list, filter(*,innerProduct*)},
  (*innerProduct[a_,b_]:=FullSimplify[Transposed[a].σ16.b];*)
  (*filter[J_][a_,b_]:=J.J===-ID16&&innerProduct[a,b]==innerProduct[J.a,J.b];*)
  filter[J_] := J . J === -ID16 && σ16 === Transpose[J] . σ16 . J;
  Return[list];
]
```

```
In[1016]:= Ω16 === T16A[1].T16A[2].T16A[3].T16A[4]
Out[1016]= True
```

Spinor Lagrangian

```
In[1017]:= usegT16 =
  Table[FullSimplify[(T16α[α1 - 1]), constraintVars], {α1, 1, Length[X]}];
In[1018]:= useT16 = Table[FullSimplify[(T16α[α1 - 1] /. ssgm4488), constraintVars] /.
  {Sqrt[e-2 a4[H x4]] → e-a4[H x4], Sqrt[e2 a4[H x4] Sin[6 H x0]1/3] → ea4[H x4] Sin[6 H x0]1/6,
   1/Sqrt[e2 a4[H x4] Sin[6 H x0]1/3] → 1/ea4[H x4] Sin[6 H x0]1/6}, {α1, 1, Length[X]}] /.
  {1/Sqrt[e-2 a4[H x4] Sin[6 H x0]1/3] → 1/ea4[H x4] Sin[6 H x0]1/6, Sqrt[e2 a4[H x4]] → ea4[H x4]};
In[1019]:= Dimensions[useT16]
Out[1019]= {8, 16, 16}
In[1020]:= Det[MatrixMetric44]
Out[1020]= Cos[6 H x0]2
In[1021]:= useDSQRT = Cos[6 H x0]
Out[1021]= Cos[6 H x0]
```

```

In[1022]:= 
wmat /. sg /. ssgm4488 /. subsDefects;

$$\left( \frac{\sqrt{e^{2 a4[H x4]}}}{\sqrt{e^{2 a4[H x4]}}} \rightarrow e^{a4[H x4]}, \right.$$


$$\left. \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} \rightarrow e^{a4[H x4]} H \sin[6 H x0]^{1/6} \right) - spinCoeffs //$$

```

```

FullSimplify[#, constraintVars] & /. subsDefects /.

$$\left\{ \sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}, \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} \rightarrow e^{a4[H x4]} H \sin[6 H x0]^{1/6} \right\} //$$

```

```

FullSimplify[#, constraintVars] & /. subsDefects;
% // Flatten // Union
% // ExpandAll /. subsDefects /.

$$\left\{ \sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}, \right.$$


$$\left. \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} \rightarrow e^{a4[H x4]} H \sin[6 H x0]^{1/6} \right\} /. subsDefects
% /. {a4[H x4] \rightarrow \pi^4, Sin[6 H x0] \rightarrow e^12}

Out[1024]=

$$\left\{ 0, e^{-a4[H x4]} \left( -1 + e^{a4[H x4]} \sqrt{e^{-2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6}, \right.$$


$$- \left( \left( -e^{a4[H x4]} + \sqrt{e^{2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6}, \left( -e^{a4[H x4]} + \sqrt{e^{2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6}, \right.$$


$$e^{-a4[H x4]} \left( 1 - e^{a4[H x4]} \sqrt{e^{-2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6} a4'[H x4],$$


$$e^{-a4[H x4]} \left( -1 + e^{a4[H x4]} \sqrt{e^{-2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6} a4'[H x4],$$


$$\left. \left( -e^{a4[H x4]} + \sqrt{e^{2 a4[H x4]}} \right) H \sin[6 H x0]^{1/6} a4'[H x4] \right\}$$


Out[1025]=

$$\left\{ 0, -e^{-a4[H x4]} H \sin[6 H x0]^{1/6} + \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6}, \right.$$


$$e^{a4[H x4]} H \sin[6 H x0]^{1/6} - \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6},$$


$$-e^{a4[H x4]} H \sin[6 H x0]^{1/6} + \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6},$$


$$e^{-a4[H x4]} H \sin[6 H x0]^{1/6} a4'[H x4] - \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4],$$


$$-e^{-a4[H x4]} H \sin[6 H x0]^{1/6} a4'[H x4] + \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4],$$


$$\left. -e^{a4[H x4]} H \sin[6 H x0]^{1/6} a4'[H x4] + \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4] \right\}$$


Out[1026]=
{0, 0, 0, 0, 0, 0, 0}

In[1027]:= 
Q2 = 0; Protect[Q1, Q2]

Out[1027]=
{Q1, Q2}$$

```

In[1028]:=

```

Clear[Lg];
Lg[] :=  $\sqrt{\det gg} \cdot \left( \text{Transpose}[\Psi_{16}] \cdot \sigma_{16} \cdot \right.$ 
 $\text{Sum}\left[ \text{FullSimplify}[(T_{16}^\alpha[\alpha_1 - 1] /. sg), \text{constraintVars}] \cdot \left( D[\Psi_{16}, X[\alpha_1]] + \right. \right.$ 
 $\left( \frac{Q_1}{2} \right) \cdot \text{Sum}[\omega_{\text{mat}}[\alpha_1, a, b] \cdot SAB[a, b] \cdot \Psi_{16}, \{a, 1, 8\}, \{b, 1, 8\}],$ 
 $\left. \left. \{ \alpha_1, 1, \text{Length}[X] \} \right] + (H \cdot M) \cdot \text{Transpose}[\Psi_{16}] \cdot \sigma_{16} \cdot \Psi_{16} \right) //$ 
 $\text{Simplify}[\#, \text{constraintVars}] \&$ 

```

In[1030]:=

$\omega\mu\mathbf{IJ}[1]$

Out[1030]=

```

{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}

```

In[1031]:=

```

Clear[La];
La[] :=
useDSQRT * ( Transpose[\!`16].\!\sigma16.Sum[useT16[\!\alpha1].(D[\!`16, X[\!\alpha1]] + (\!\frac{Q1}{2}) *
Sum[\!\omega\muIJ[\!\alpha1][A1, B1]*SAB[A1, B1], {A1, 1, 8}, {B1, 1, 8}].\!`16),
{\!\alpha1, 1, Length[X]}] + (H*M)*Transpose[\!`16].\!\sigma16.\!`16) //
Simplify[#, constraintVars] &

```

In[1033]:=

(*testLa=La[]*)

In[1034]:=

```
(*testLg=Lg[];*)
```

In[1035]:=

```
(*D[testLa,a4[H x4]]//Flatten//Union*)
(*D[testLa,a4'[H x4]]//(*Full*)Simplify[#,constraintVars]&//Flatten//Union*)
```

In[1036]:=

(*testLgm=

```
testL/.sg/.ssgm4488//(*Full*)Simplify[#,constraintVars]&/.subsDefects*)
```

In[1037]:=

(*D[testLgm,a4[H x4]]
D[testLgm,a4'[H x4]]

In[1038]:=

```

Clear[Lj];
Lj[j_] := useDSQRT * ((((
Transpose[\!`16].\!`16.useT16[1].D[\!`16, X[1]] +
Transpose[\!`16].\!`16.useT16[5].D[\!`16, X[5]] + \!\left(\frac{K0}{2}\right)*
Sum[(\!`usew[1,A1,B1]*)\!`muIJ[1][A1, B1]\!`*Transpose[\!`16].\!`16.useT16[1].
SAB[A1, B1].(base16[j, 1]).\!`16, {A1, 1, 8}, {B1, 1, 8}]) +
Transpose[\!`16].\!`16.useT16[5].D[\!`16, X[5]] + \!\left(\frac{K4}{2}\right)*
Sum[(\!`usew[5,A1,B1]*)\!`muIJ[5][A1, B1]\!`*Transpose[\!`16].\!`16.useT16[5].
SAB[A1, B1].(base16[j, 1]).\!`16, {A1, 1, 8}, {B1, 1, 8}]) +
(H*M)*Transpose[\!`16].\!`16.\!`16) // Simplify[#, constraintVars] &

```

In[1040]:=

(*Lj[92]*)

In[1041]:=

detgg

Out[1041]=

$$g[0][0][x_0, x_4] \times g[1][1][x_0, x_4] \times g[2][2][x_0, x_4] \times g[3][3][x_0, x_4] \times \\ g[4][4][x_0, x_4] \times g[5][5][x_0, x_4] \times g[6][6][x_0, x_4] \times g[7][7][x_0, x_4]$$

In[1042]:=

Clear[eL];

```

eL[Lagrangian_Symbol, detsqrt_] := Module[{L, t},
  L = Lagrangian[];
  t = Table[FullSimplify[
    \!\left(\frac{1}{detsqrt}\right)\!\left(D[L, f16[k][x_0, x_4]] - D[D[L, f16[k]^{(1,0)}[x_0, x_4]], x_0] - \!\right.
    D[D[L, f16[k]^{(0,1)}[x_0, x_4]], x_4]), constraintVars], {k, 0, 15}];
  Return[t /. subsDefects];
]

```

eLa

In[1044]:=

```

(*Get[
"2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLa.
mx"] ;*)

```

```

In[1045]:= eLa = eL[La, useDSQRT];

In[1046]:= eLa

Out[1046]=
{-2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[9][x0, x4] a4'[H x4] -
 2 (H M f16[4][x0, x4] + 3 H f16[12][x0, x4] +
   f16[9](0,1)[x0, x4] + Tan[6 H x0] f16[12](1,0)[x0, x4]),
 2 (-H M f16[5][x0, x4] - 3 H f16[13][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[8][x0, x4] a4'[H x4] + f16[8](0,1)[x0, x4] - Tan[6 H x0] f16[13](1,0)[x0, x4]),
 -2 (H M f16[6][x0, x4] + 3 H f16[14][x0, x4] - f16[11](0,1)[x0, x4] +
   Tan[6 H x0] f16[14](1,0)[x0, x4]),
 -2 (H M f16[7][x0, x4] + 3 H f16[15][x0, x4] + f16[10](0,1)[x0, x4] +
   Tan[6 H x0] f16[15](1,0)[x0, x4]), -2 H M f16[0][x0, x4] -
 6 H f16[8][x0, x4] - 2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[13][x0, x4] a4'[H x4] +
 2 f16[13](0,1)[x0, x4] - 2 Tan[6 H x0] f16[8](1,0)[x0, x4],
 -2 H M f16[1][x0, x4] - 6 H f16[9][x0, x4] +
 e-2 a4[H x4] (-1 + e2 a4[H x4]) H Q1 f16[12][x0, x4] a4'[H x4] -
 2 (f16[12](0,1)[x0, x4] + Tan[6 H x0] f16[9](1,0)[x0, x4]),
 -2 (H M f16[2][x0, x4] + 3 H f16[10][x0, x4] + f16[15](0,1)[x0, x4] +
 Tan[6 H x0] f16[10](1,0)[x0, x4]), -2 (H M f16[3][x0, x4] +
 3 H f16[11][x0, x4] - f16[14](0,1)[x0, x4] + Tan[6 H x0] f16[11](1,0)[x0, x4]),
 2 (3 H f16[4][x0, x4] + H M f16[12][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[1][x0, x4] a4'[H x4] - f16[1](0,1)[x0, x4] + Tan[6 H x0] f16[4](1,0)[x0, x4]),
 2 (3 H f16[5][x0, x4] + H M f16[13][x0, x4] - e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[0][x0, x4] a4'[H x4] + f16[0](0,1)[x0, x4] + Tan[6 H x0] f16[5](1,0)[x0, x4]),
 2 (3 H f16[6][x0, x4] + H M f16[14][x0, x4] + f16[3](0,1)[x0, x4] +
 Tan[6 H x0] f16[6](1,0)[x0, x4]), 2 (3 H f16[7][x0, x4] +
 H M f16[15][x0, x4] - f16[2](0,1)[x0, x4] + Tan[6 H x0] f16[7](1,0)[x0, x4]),
 2 (3 H f16[0][x0, x4] + H M f16[8][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[5][x0, x4] a4'[H x4] + f16[5](0,1)[x0, x4] + Tan[6 H x0] f16[0](1,0)[x0, x4]),
 6 H f16[1][x0, x4] + 2 H M f16[9][x0, x4] - 2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[4][x0, x4] a4'[H x4] -
 2 f16[4](0,1)[x0, x4] + 2 Tan[6 H x0] f16[1](1,0)[x0, x4],
 2 (3 H f16[2][x0, x4] + H M f16[10][x0, x4] - f16[7](0,1)[x0, x4] +
 Tan[6 H x0] f16[2](1,0)[x0, x4]), 2 (3 H f16[3][x0, x4] +
 H M f16[11][x0, x4] + f16[6](0,1)[x0, x4] + Tan[6 H x0] f16[3](1,0)[x0, x4])}

In[1047]:= Length[eLa]
Out[1047]= 16

In[1048]:= DumpSave[ToString[header <> "eLa.mx"], eLa];

In[1049]:= Print[ToString[header <> "eLa.mx"]]

```

```
In[1050]:= (*Cases[
  eLa,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
  Infinity,
  Heads → True
]*)

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE :

In[1051]:= eLaRawSets = Cases[
  #,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
  Infinity,
  Heads → True
] & /@ eLa

Out[1051]= {{9, 4, 12, 9, 12}, {5, 13, 8, 8, 13}, {6, 14, 11, 14}, {7, 15, 10, 15}, {0, 8, 13, 13, 8},
{1, 9, 12, 12, 9}, {2, 10, 15, 10}, {3, 11, 14, 11}, {4, 12, 1, 1, 4}, {5, 13, 0, 0, 5},
{6, 14, 3, 6}, {7, 15, 2, 7}, {0, 8, 5, 5, 0}, {1, 9, 4, 4, 1}, {2, 10, 7, 2}, {3, 11, 6, 3}]

In[1052]:= Dimensions[eLaRawSets]

Out[1052]= {16}

In[1053]:= rawSetsSel16 = rawSets[eLa, "f16", 0]
% === eLaRawSets

Out[1053]= {{9, 4, 12, 9, 12}, {5, 13, 8, 8, 13}, {6, 14, 11, 14}, {7, 15, 10, 15}, {0, 8, 13, 13, 8},
{1, 9, 12, 12, 9}, {2, 10, 15, 10}, {3, 11, 14, 11}, {4, 12, 1, 1, 4}, {5, 13, 0, 0, 5},
{6, 14, 3, 6}, {7, 15, 2, 7}, {0, 8, 5, 5, 0}, {1, 9, 4, 4, 1}, {2, 10, 7, 2}, {3, 11, 6, 3}]

Out[1054]= True

In[1055]:= eLaCouplings = showCoupledEquations[eLaRawSets]
% // Flatten // Sort
% === Range[0, 15]

Out[1055]= {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}]

Out[1056]= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

Out[1057]= True
```

```
In[1058]:= eLaztCouplings = eLaCouplings
Out[1058]= {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
In[1059]:= (*processRawSets[]:=Module[{s1,s2,J,L=Range[Length[eLaRawSets]],r={}}, While[Length[L]>1, s1=Union[eLaRawSets[[L[[1]]]]]; L=Drop[L,1]; J=L[[1]]; While[J≤16, If[MemberQ[L,J], s2=Union[eLaRawSets[[J]]]; If[Intersection[s1,s2]≠{},s1=Union[Flatten[{s1,s2}]]; L=Complement[L,{J}];,##&[]];,##&[]]; J++;]; AppendTo[r,s1]; ]; Return[r]; ]*)

```

eLazt

```
In[1089]:= (*Get[
"2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLazt
.mx"] ;*)
```

In[1090]:=

```

time1 = Now
AbsoluteTiming[
  eLazt =  $\frac{1}{2 \cdot H} \cdot eLa / . sfy16Aa / . sx0x4 // FullSimplify[\#, constraintVars] &$ 
Now - time1

```

Out[1090]=

Fri 13 Feb 2026 02:42:24 GMT-8

Out[1091]=

$$\begin{aligned}
& \{2.12171, \{-M Z[4][z, t] - 3 Z[12][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] - \\
& \quad Z[9]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t], -M Z[5][z, t] - 3 Z[13][z, t] + \\
& \quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + Z[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t], \\
& \quad -M Z[6][z, t] - 3 Z[14][z, t] + Z[11]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t], \\
& \quad -M Z[7][z, t] - 3 Z[15][z, t] - Z[10]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t], \\
& \quad -M Z[0][z, t] - 3 Z[8][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + \\
& \quad Z[13]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t], -M Z[1][z, t] - 3 Z[9][z, t] + \\
& \quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] - Z[12]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t], \\
& \quad -M Z[2][z, t] - 3 Z[10][z, t] - Z[15]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t], \\
& \quad -M Z[3][z, t] - 3 Z[11][z, t] + Z[14]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t], \\
& \quad 3 Z[4][z, t] + M Z[12][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] - \\
& \quad Z[1]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t], 3 Z[5][z, t] + M Z[13][z, t] - \\
& \quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] + Z[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t], \\
& \quad 3 Z[6][z, t] + M Z[14][z, t] + Z[3]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t], \\
& \quad 3 Z[7][z, t] + M Z[15][z, t] - Z[2]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t], \\
& \quad 3 Z[0][z, t] + M Z[8][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] + \\
& \quad Z[5]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t], 3 Z[1][z, t] + M Z[9][z, t] - \\
& \quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] - Z[4]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t], \\
& \quad 3 Z[2][z, t] + M Z[10][z, t] - Z[7]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t], \\
& \quad 3 Z[3][z, t] + M Z[11][z, t] + Z[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t]\}\}
\end{aligned}$$

Out[1092]=

2.124982 s

In[1093]:=

DumpSave[ToString[header <> "eLazt.mx"], eLazt];

In[1094]:=

Print[ToString[header <> "eLazt.mx"]]

2026-02-13-Pair-Crtn-Univ-same_E-L-eqs-alt-approach-mmM4p-eLazt.mx

```
In[1095]:= eLast // Column
Out[1095]=
-M Z[4][z, t] - 3 Z[12][z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] - Z[9](0,1)[z, t] - 6 Tan[z] Z[12](1,0)[z, t]
-M Z[5][z, t] - 3 Z[13][z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + Z[8](0,1)[z, t] - 6 Tan[z] Z[13](1,0)[z, t]
-M Z[6][z, t] - 3 Z[14][z, t] + Z[11](0,1)[z, t] - 6 Tan[z] Z[14](1,0)[z, t]
-M Z[7][z, t] - 3 Z[15][z, t] - Z[10](0,1)[z, t] - 6 Tan[z] Z[15](1,0)[z, t]
-M Z[0][z, t] - 3 Z[8][z, t] - e-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] +
Z[13](0,1)[z, t] - 6 Tan[z] Z[8](1,0)[z, t]
-M Z[1][z, t] - 3 Z[9][z, t] + e-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] -
Z[12](0,1)[z, t] - 6 Tan[z] Z[9](1,0)[z, t]
-M Z[2][z, t] - 3 Z[10][z, t] - Z[15](0,1)[z, t] - 6 Tan[z] Z[10](1,0)[z, t]
-M Z[3][z, t] - 3 Z[11][z, t] + Z[14](0,1)[z, t] - 6 Tan[z] Z[11](1,0)[z, t]
3 Z[4][z, t] + M Z[12][z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] - Z[1](0,1)[z, t] + 6 Tan[z] Z[4](1,0)[z, t]
3 Z[5][z, t] + M Z[13][z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] + Z[0](0,1)[z, t] + 6 Tan[z] Z[5](1,0)[z, t]
3 Z[6][z, t] + M Z[14][z, t] + Z[3](0,1)[z, t] + 6 Tan[z] Z[6](1,0)[z, t]
3 Z[7][z, t] + M Z[15][z, t] - Z[2](0,1)[z, t] + 6 Tan[z] Z[7](1,0)[z, t]
3 Z[0][z, t] + M Z[8][z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] + Z[5](0,1)[z, t] + 6 Tan[z] Z[0](1,0)[z, t]
3 Z[1][z, t] + M Z[9][z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] - Z[4](0,1)[z, t] + 6 Tan[z] Z[1](1,0)[z, t]
3 Z[2][z, t] + M Z[10][z, t] - Z[7](0,1)[z, t] + 6 Tan[z] Z[2](1,0)[z, t]
3 Z[3][z, t] + M Z[11][z, t] + Z[6](0,1)[z, t] + 6 Tan[z] Z[3](1,0)[z, t]

In[1096]:= varZ = Z[#] & /@ Range[0, 15]
Out[1096]=
{Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}

In[1097]:= varZzt = #[z, t] & /@ varZ
Out[1097]=
{Z[0][z, t], Z[1][z, t], Z[2][z, t], Z[3][z, t], Z[4][z, t],
Z[5][z, t], Z[6][z, t], Z[7][z, t], Z[8][z, t], Z[9][z, t], Z[10][z, t],
Z[11][z, t], Z[12][z, t], Z[13][z, t], Z[14][z, t], Z[15][z, t]}

In[1098]:= DzvarZ = D[varZzt, z]
Out[1098]=
{Z[0](1,0)[z, t], Z[1](1,0)[z, t], Z[2](1,0)[z, t], Z[3](1,0)[z, t],
Z[4](1,0)[z, t], Z[5](1,0)[z, t], Z[6](1,0)[z, t], Z[7](1,0)[z, t],
Z[8](1,0)[z, t], Z[9](1,0)[z, t], Z[10](1,0)[z, t], Z[11](1,0)[z, t],
Z[12](1,0)[z, t], Z[13](1,0)[z, t], Z[14](1,0)[z, t], Z[15](1,0)[z, t]}
```

```

In[1099]:= DtvarZ = D[varZzt, t]
Out[1099]= {Z[0]^(0,1)[z, t], Z[1]^(0,1)[z, t], Z[2]^(0,1)[z, t], Z[3]^(0,1)[z, t],
Z[4]^(0,1)[z, t], Z[5]^(0,1)[z, t], Z[6]^(0,1)[z, t], Z[7]^(0,1)[z, t],
Z[8]^(0,1)[z, t], Z[9]^(0,1)[z, t], Z[10]^(0,1)[z, t], Z[11]^(0,1)[z, t],
Z[12]^(0,1)[z, t], Z[13]^(0,1)[z, t], Z[14]^(0,1)[z, t], Z[15]^(0,1)[z, t]}

In[1100]:= (DtvarZsubs = Solve[And @@ Thread[θ == eLazt], DtvarZ][1] // FullSimplify[#, constraintVars] &) // Column
DtvarZEQS = %[[1]] /. {Rule → Equal}
DtvarZrelations = DtvarZsubs /. {Rule → Subtract}

Out[1100]= Z[0]^(0,1)[z, t] →
-3 Z[5][z, t] - M Z[13][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] - 6 Tan[z] Z[5]^(1,0)[z, t]
Z[1]^(0,1)[z, t] →
3 Z[4][z, t] + M Z[12][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] + 6 Tan[z] Z[4]^(1,0)[z, t]
Z[2]^(0,1)[z, t] → 3 Z[7][z, t] + M Z[15][z, t] + 6 Tan[z] Z[7]^(1,0)[z, t]
Z[3]^(0,1)[z, t] → -3 Z[6][z, t] - M Z[14][z, t] - 6 Tan[z] Z[6]^(1,0)[z, t]
Z[4]^(0,1)[z, t] →
3 Z[1][z, t] + M Z[9][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] + 6 Tan[z] Z[1]^(1,0)[z, t]
Z[5]^(0,1)[z, t] →
-3 Z[0][z, t] - M Z[8][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] - 6 Tan[z] Z[0]^(1,0)[z, t]
Z[6]^(0,1)[z, t] → -3 Z[3][z, t] - M Z[11][z, t] - 6 Tan[z] Z[3]^(1,0)[z, t]
Z[7]^(0,1)[z, t] → 3 Z[2][z, t] + M Z[10][z, t] + 6 Tan[z] Z[2]^(1,0)[z, t]
Z[8]^(0,1)[z, t] →
M Z[5][z, t] + 3 Z[13][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + 6 Tan[z] Z[13]^(1,0)[z, t]
Z[9]^(0,1)[z, t] →
-M Z[4][z, t] - 3 Z[12][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] - 6 Tan[z] Z[12]^(1,0)[z, t]
Z[10]^(0,1)[z, t] → -M Z[7][z, t] - 3 (Z[15][z, t] + 2 Tan[z] Z[15]^(1,0)[z, t])
Z[11]^(0,1)[z, t] → M Z[6][z, t] + 3 Z[14][z, t] + 6 Tan[z] Z[14]^(1,0)[z, t]
Z[12]^(0,1)[z, t] →
-M Z[1][z, t] - 3 Z[9][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] - 6 Tan[z] Z[9]^(1,0)[z, t]
Z[13]^(0,1)[z, t] →
M Z[0][z, t] + 3 Z[8][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] + 6 Tan[z] Z[8]^(1,0)[z, t]
Z[14]^(0,1)[z, t] → M Z[3][z, t] + 3 Z[11][z, t] + 6 Tan[z] Z[11]^(1,0)[z, t]
Z[15]^(0,1)[z, t] → -M Z[2][z, t] - 3 (Z[10][z, t] + 2 Tan[z] Z[10]^(1,0)[z, t])

```

Out[1101]=

$$\begin{aligned} \{Z[0]^{(0,1)}[z, t] &= -3 Z[5][z, t] - M Z[13][z, t] + \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t], Z[1]^{(0,1)}[z, t] = \\ &\quad 3 Z[4][z, t] + M Z[12][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t], \\ Z[2]^{(0,1)}[z, t] &= 3 Z[7][z, t] + M Z[15][z, t] + 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t], \\ Z[3]^{(0,1)}[z, t] &= -3 Z[6][z, t] - M Z[14][z, t] - 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t], \\ Z[4]^{(0,1)}[z, t] &= 3 Z[1][z, t] + M Z[9][z, t] - \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t], Z[5]^{(0,1)}[z, t] = \\ &\quad -3 Z[0][z, t] - M Z[8][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t], \\ Z[6]^{(0,1)}[z, t] &= -3 Z[3][z, t] - M Z[11][z, t] - 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t], \\ Z[7]^{(0,1)}[z, t] &= 3 Z[2][z, t] + M Z[10][z, t] + 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t], Z[8]^{(0,1)}[z, t] = \\ &\quad M Z[5][z, t] + 3 Z[13][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t], \\ Z[9]^{(0,1)}[z, t] &= -M Z[4][z, t] - 3 Z[12][z, t] - \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t], \\ Z[10]^{(0,1)}[z, t] &= -M Z[7][z, t] - 3 (Z[15][z, t] + 2 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t]), \\ Z[11]^{(0,1)}[z, t] &= M Z[6][z, t] + 3 Z[14][z, t] + 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t], \\ Z[12]^{(0,1)}[z, t] &= -M Z[1][z, t] - 3 Z[9][z, t] + \\ &\quad e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t], Z[13]^{(0,1)}[z, t] = \\ &\quad M Z[0][z, t] + 3 Z[8][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t], \\ Z[14]^{(0,1)}[z, t] &= M Z[3][z, t] + 3 Z[11][z, t] + 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t], \\ Z[15]^{(0,1)}[z, t] &= -M Z[2][z, t] - 3 (Z[10][z, t] + 2 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t]) \} \end{aligned}$$

Out[1102]=

$$\begin{aligned} \{3 Z[5][z, t] + M Z[13][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] + \\ Z[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t], -3 Z[4][z, t] - M Z[12][z, t] - \\ e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] + Z[1]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t], \\ -3 Z[7][z, t] - M Z[15][z, t] + Z[2]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t], \\ 3 Z[6][z, t] + M Z[14][z, t] + Z[3]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t], \\ -3 Z[1][z, t] - M Z[9][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] + \\ Z[4]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t], 3 Z[0][z, t] + M Z[8][z, t] + \\ e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] + Z[5]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t], \\ 3 Z[3][z, t] + M Z[11][z, t] + Z[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t], \\ -3 Z[2][z, t] - M Z[10][z, t] + Z[7]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t], \\ -M Z[5][z, t] - 3 Z[13][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + \\ Z[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t], M Z[4][z, t] + 3 Z[12][z, t] + \\ e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] + Z[9]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t], \\ M Z[7][z, t] + Z[10]^{(0,1)}[z, t] + 3 (Z[15][z, t] + 2 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t]), \\ -M Z[6][z, t] - 3 Z[14][z, t] + Z[11]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t], \\ M Z[1][z, t] + 3 Z[9][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] + \\ Z[12]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t], -M Z[0][z, t] - 3 Z[8][z, t] - \\ e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + Z[13]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t], \\ -M Z[3][z, t] - 3 Z[11][z, t] + Z[14]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t], \\ M Z[2][z, t] + Z[15]^{(0,1)}[z, t] + 3 (Z[10][z, t] + 2 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t]) \} \end{aligned}$$

In[1103]:=

```
(*ToString[FullForm[#]]&/@DtvarZsubs;
StringReplace[#, "Rule" → "Subtract"]&/@%;
(DtvarZrelations=ToExpression[#]&/@%) //Column*)
```

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE :

```
In[1104]:= eLaZtCouplings = eLaCouplings
Out[1104]= {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
```

SEPARATE VARIABLES THAT ARE COUPLED TO EACH OTHER (reorder) :

```
In[1105]:= yZdef = yZ[#] & /@ Range[0, 15]
Out[1105]= {yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}
In[1106]:= Zdef = Z[#] & /@ Range[0, 15]
Out[1106]= {Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}
```

```
In[1107]:= sZt0yZ =
  Z[#\[If[1]]] \[Rule] yZ[#\[If[2]]] & /@ Transpose[{Flatten[eLaZtCouplings], Range[0, 15]}]
syZtoZ = Solve[And @@ (sZt0yZ /. {Rule \[Rule] Equal}), (yZ[#] & /@ Range[0, 15])][[-1]]
zeroZyZforCaExpression = -sZt0yZ /. {Rule \[Rule] Subtract}
zeroZyZEQS = sZt0yZ /. {Rule \[Rule] Equal}
```

```
Out[1107]= {Z[0] \[Rule] yZ[0], Z[5] \[Rule] yZ[1], Z[8] \[Rule] yZ[2], Z[13] \[Rule] yZ[3], Z[1] \[Rule] yZ[4],
Z[4] \[Rule] yZ[5], Z[9] \[Rule] yZ[6], Z[12] \[Rule] yZ[7], Z[2] \[Rule] yZ[8], Z[7] \[Rule] yZ[9], Z[10] \[Rule] yZ[10],
Z[15] \[Rule] yZ[11], Z[3] \[Rule] yZ[12], Z[6] \[Rule] yZ[13], Z[11] \[Rule] yZ[14], Z[14] \[Rule] yZ[15]}
```

```
Out[1108]= {yZ[0] \[Rule] Z[0], yZ[1] \[Rule] Z[5], yZ[2] \[Rule] Z[8], yZ[3] \[Rule] Z[13], yZ[4] \[Rule] Z[1],
yZ[5] \[Rule] Z[4], yZ[6] \[Rule] Z[9], yZ[7] \[Rule] Z[12], yZ[8] \[Rule] Z[2], yZ[9] \[Rule] Z[7], yZ[10] \[Rule] Z[10],
yZ[11] \[Rule] Z[15], yZ[12] \[Rule] Z[3], yZ[13] \[Rule] Z[6], yZ[14] \[Rule] Z[11], yZ[15] \[Rule] Z[14]}
```

```
Out[1109]= {yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1],
yZ[5] - Z[4], yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}
```

```
Out[1110]= {Z[0] == yZ[0], Z[5] == yZ[1], Z[8] == yZ[2], Z[13] == yZ[3], Z[1] == yZ[4], Z[4] == yZ[5],
Z[9] == yZ[6], Z[12] == yZ[7], Z[2] == yZ[8], Z[7] == yZ[9], Z[10] == yZ[10],
Z[15] == yZ[11], Z[3] == yZ[12], Z[6] == yZ[13], Z[11] == yZ[14], Z[14] == yZ[15]}
```

```
In[1111]:= cayZ = CoefficientArrays[zeroZyZforCaExpression, yZdef]
```

```
Out[1111]= {SparseArray[ Specified elements: 16 Dimensions: {16}], SparseArray[ Specified elements: 16 Dimensions: {16, 16}]}
```

In[1112]:=

```
caZ = CoefficientArrays[zeroZyZforCaExpression, Zdef]
```

Out[1112]=

```
{SparseArray[ +  Specified elements: 16
Dimensions: {16} ], SparseArray[ +  Specified elements: 16
Dimensions: {16, 16} ]}
```

In[1113]:=

```
zeroZyZforCaExpression === cayZ[[1]] + cayZ[[2]].yzdef // Expand
```

Out[1113]=

True

In[1114]:=

```
zeroZyZforCaExpression === caZ[[1]] + caZ[[2]].Zdef // Expand
```

Out[1114]=

True

[p[1115]:-

(cavZ2 = cavZ[2] // Normal) === ID16

Out[1115]=

True

[n[1116]:=

```
(cayZ1 = cayZ[1] // Normal)
```

Out[1116]=

[1117]

```
(ca72 = ca7[[2]] // Normal) // MatrixForm
```

Out[1117]//MatrixForm=

In[1125]:=

```
Transpose[almightyS].almightyS === ID16
```

Out[1125]=

```
True
```

In[1126]:=

```
Transpose[almightyS].σ16.almightyS === σ16
```

Out[1126]=

```
False
```

In[1127]:=

```
σ16 // MatrixForm
```

Out[1127]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[1128]:=

```
Transpose[almightyS].ArrayFlatten[{{0, σ}, {σ, 0}}].almightyS ===
ArrayFlatten[{{0, σ}, {σ, 0}}]
```

Out[1128]=

```
False
```

In[*]:=

```
Clear[La];
La[] := 
( Transpose[Ψ16].σ16.Sum[T16^A[α1 - 1].(D[Ψ16, X[α1]]), {α1, 1, Length[X]}] +
(H * M) * Transpose[Ψ16].σ16.Ψ16) // Simplify[#, constraintVars] &
```

In[1129]:=

```
σ16.T16^A[#] === -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]
```

Out[1129]=

```
{True, True, True, True, True, True, True, True}
```

In[1130]:=

```
(*La[];
 - Sum[(D[Transpose[\!16], X[\!1]]).o16.T16^[\!1-1], {\!1, 1, Length[X]}].\!16+
 (H*M)*Transpose[\!16].o16.\!16//FullSimplify[#, constraintVars]&;
 ExpandAll[%-%]//FullSimplify[#, constraintVars]&*)
```

In[1131]:=

```
(* (Transpose[\!16].o16.Sum[T16^[\!1-1].(D[\!16, X[\!1]]), {\!1, 1, Length[X]}]+
 (H*M)*Transpose[\!16].o16.\!16)/.sf\!16Aa/
 sx0x4//FullSimplify[#, constraintVars]&;
 - Sum[(D[Transpose[\!16], X[\!1]]).o16.T16^[\!1-1], {\!1, 1, Length[X]}].\!16+
 (H*M)*Transpose[\!16].o16.\!16/.sf\!16Aa/
 sx0x4//FullSimplify[#, constraintVars]&;
 ExpandAll[%-%]//FullSimplify[#, constraintVars]&*)
```

In[1132]:=

```
Thread[Zdef \rightarrow -Transpose[caZ2].caZ[\!1]]
```

Out[1132]=

```
{Z[0] \rightarrow yZ[0], Z[1] \rightarrow yZ[4], Z[2] \rightarrow yZ[8], Z[3] \rightarrow yZ[12], Z[4] \rightarrow yZ[5],
 Z[5] \rightarrow yZ[1], Z[6] \rightarrow yZ[13], Z[7] \rightarrow yZ[9], Z[8] \rightarrow yZ[2], Z[9] \rightarrow yZ[6], Z[10] \rightarrow yZ[10],
 Z[11] \rightarrow yZ[14], Z[12] \rightarrow yZ[7], Z[13] \rightarrow yZ[3], Z[14] \rightarrow yZ[15], Z[15] \rightarrow yZ[11]}
```

In[1133]:=

```
zeroZyZforCaExpression === caZ[\!1] + caZ[\!2].Zdef // Expand
```

Out[1133]=

```
True
```

In[1134]:=

```
(caZ1 = caZ[\!1] // Normal) === yZdef
```

Out[1134]=

```
True
```

In[1135]:=

```
caZ[\!1] + caZ[\!2].Zdef // Expand
% /. sZt0yZ
```

Out[1135]=

```
{yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1],
 yZ[5] - Z[4], yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
 yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}
```

Out[1136]=

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
In[1137]:= almightyS.Zdef
% /. sZt0yZ
yZdef === (% // Sort)

Out[1137]= {Z[0], Z[4], Z[8], Z[12], Z[5], Z[1], Z[13],
Z[9], Z[2], Z[6], Z[10], Z[14], Z[7], Z[3], Z[15], Z[11]}

Out[1138]= {yZ[0], yZ[5], yZ[2], yZ[7], yZ[1], yZ[4], yZ[3], yZ[6],
yZ[8], yZ[13], yZ[10], yZ[15], yZ[9], yZ[12], yZ[11], yZ[14]}

Out[1139]= True

In[1140]:= cayZ2 === ID16

Out[1140]= True

In[1141]:= caZ2 // MatrixForm

Out[1141]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```

ORTHOGONAL O(16) SIMILARITY TRANSFORMATION :

```
In[1142]:= caZ2.Transpose[caZ2] === ID16
```

```
Out[1142]= True
```

BUT NOT ORTHOGONAL O(8,8) SIMILARITY TRANSFORMATION :

```
In[1143]:= caZ2.σ16.Transpose[caZ2] === σ16
```

```
Out[1143]= False
```

Also, Not a Direct Sum :

```
In[1144]:= caZ2 // MatrixForm
Out[1144]//MatrixForm=

$$\left( \begin{array}{cccccccccccccccc} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{array} \right)$$


In[1145]:= caZ2 === ArrayFlatten[{{caZ2[[1 ;; 8, 1 ;; 8]], caZ2[[1 ;; 8, 9 ;; 16]]}, {caZ2[[9 ;; 16, 1 ;; 8]], caZ2[[9 ;; 16, 9 ;; 16]]}}]
Out[1145]= True

In[1146]:= caZ2 === ArrayFlatten[{{caZ2[[1 ;; 8, 1 ;; 8]], 0}, {0, caZ2[[9 ;; 16, 9 ;; 16]]}}]
Out[1146]= False
```

TRANSFORM Euler Lagrange equations to yZ[j] :

```
In[1147]:= DtvarZrelations // Column
Out[1147]=
3 Z[5] [z, t] + M Z[13] [z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[0] [z, t] a4'[t] + Z[0] (0,1) [z, t] + 6 Tan[z] Z[5] (1,0) [z, t]
-3 Z[4] [z, t] - M Z[12] [z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[1] [z, t] a4'[t] + Z[1] (0,1) [z, t] - 6 Tan[z] Z[4] (1,0) [z, t]
-3 Z[7] [z, t] - M Z[15] [z, t] + Z[2] (0,1) [z, t] - 6 Tan[z] Z[7] (1,0) [z, t]
3 Z[6] [z, t] + M Z[14] [z, t] + Z[3] (0,1) [z, t] + 6 Tan[z] Z[6] (1,0) [z, t]
-3 Z[1] [z, t] - M Z[9] [z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[4] [z, t] a4'[t] + Z[4] (0,1) [z, t] - 6 Tan[z] Z[1] (1,0) [z, t]
3 Z[0] [z, t] + M Z[8] [z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[5] [z, t] a4'[t] + Z[5] (0,1) [z, t] + 6 Tan[z] Z[0] (1,0) [z, t]
3 Z[3] [z, t] + M Z[11] [z, t] + Z[6] (0,1) [z, t] + 6 Tan[z] Z[3] (1,0) [z, t]
-3 Z[2] [z, t] - M Z[10] [z, t] + Z[7] (0,1) [z, t] - 6 Tan[z] Z[2] (1,0) [z, t]
-M Z[5] [z, t] - 3 Z[13] [z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[8] [z, t] a4'[t] + Z[8] (0,1) [z, t] - 6 Tan[z] Z[13] (1,0) [z, t]
M Z[4] [z, t] + 3 Z[12] [z, t] + e-a4[t] Q1 Sinh[a4[t]] Z[9] [z, t] a4'[t] +
Z[9] (0,1) [z, t] + 6 Tan[z] Z[12] (1,0) [z, t]
M Z[7] [z, t] + Z[10] (0,1) [z, t] + 3 (Z[15] [z, t] + 2 Tan[z] Z[15] (1,0) [z, t])
-M Z[6] [z, t] - 3 Z[14] [z, t] + Z[11] (0,1) [z, t] - 6 Tan[z] Z[14] (1,0) [z, t]
M Z[1] [z, t] + 3 Z[9] [z, t] - e-a4[t] Q1 Sinh[a4[t]] Z[12] [z, t] a4'[t] +
Z[12] (0,1) [z, t] + 6 Tan[z] Z[9] (1,0) [z, t]
-M Z[0] [z, t] - 3 Z[8] [z, t] - e-a4[t] Q1 Sinh[a4[t]] Z[13] [z, t] a4'[t] +
Z[13] (0,1) [z, t] - 6 Tan[z] Z[8] (1,0) [z, t]
-M Z[3] [z, t] - 3 Z[11] [z, t] + Z[14] (0,1) [z, t] - 6 Tan[z] Z[11] (1,0) [z, t]
M Z[2] [z, t] + Z[15] (0,1) [z, t] + 3 (Z[10] [z, t] + 2 Tan[z] Z[10] (1,0) [z, t])
```

In[1148]:=

```
zeroZyZforCaExpression === caZ[[1]] + caZ[[2]].Zdef // Expand
```

Out[1148]=

True

In[1149]:=

```
caZ2.DtvarZrelations // Column
(DtyZrelations = %[[1]] /. sZt0yZ // FullSimplify) // Column
```

```

Out[1149]=
-3 Z[5] [z, t] - M Z[13] [z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[0] [z, t] a4'[t] - Z[0] (0,1) [z, t] - 6 Tan[z] Z[5] (1,0) [z, t]
-3 Z[0] [z, t] - M Z[8] [z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[5] [z, t] a4'[t] - Z[5] (0,1) [z, t] - 6 Tan[z] Z[0] (1,0) [z, t]
M Z[5] [z, t] + 3 Z[13] [z, t] - e-a4[t] Q1 Sinh[a4[t]] Z[8] [z, t] a4'[t] -
Z[8] (0,1) [z, t] + 6 Tan[z] Z[13] (1,0) [z, t]
M Z[0] [z, t] + 3 Z[8] [z, t] + e-a4[t] Q1 Sinh[a4[t]] Z[13] [z, t] a4'[t] -
Z[13] (0,1) [z, t] + 6 Tan[z] Z[8] (1,0) [z, t]
3 Z[4] [z, t] + M Z[12] [z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[1] [z, t] a4'[t] - Z[1] (0,1) [z, t] + 6 Tan[z] Z[4] (1,0) [z, t]
3 Z[1] [z, t] + M Z[9] [z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[4] [z, t] a4'[t] - Z[4] (0,1) [z, t] + 6 Tan[z] Z[1] (1,0) [z, t]
-M Z[4] [z, t] - 3 Z[12] [z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[9] [z, t] a4'[t] - Z[9] (0,1) [z, t] - 6 Tan[z] Z[12] (1,0) [z, t]
-M Z[1] [z, t] - 3 Z[9] [z, t] + e-a4[t] Q1 Sinh[a4[t]] Z[12] [z, t] a4'[t] -
Z[12] (0,1) [z, t] - 6 Tan[z] Z[9] (1,0) [z, t]
3 Z[7] [z, t] + M Z[15] [z, t] - Z[2] (0,1) [z, t] + 6 Tan[z] Z[7] (1,0) [z, t]
3 Z[2] [z, t] + M Z[10] [z, t] - Z[7] (0,1) [z, t] + 6 Tan[z] Z[2] (1,0) [z, t]
-M Z[7] [z, t] - Z[10] (0,1) [z, t] - 3 (Z[15] [z, t] + 2 Tan[z] Z[15] (1,0) [z, t])
-M Z[2] [z, t] - Z[15] (0,1) [z, t] - 3 (Z[10] [z, t] + 2 Tan[z] Z[10] (1,0) [z, t])
-3 Z[6] [z, t] - M Z[14] [z, t] - Z[3] (0,1) [z, t] - 6 Tan[z] Z[6] (1,0) [z, t]
-3 Z[3] [z, t] - M Z[11] [z, t] - Z[6] (0,1) [z, t] - 6 Tan[z] Z[3] (1,0) [z, t]
M Z[6] [z, t] + 3 Z[14] [z, t] - Z[11] (0,1) [z, t] + 6 Tan[z] Z[14] (1,0) [z, t]
M Z[3] [z, t] + 3 Z[11] [z, t] - Z[14] (0,1) [z, t] + 6 Tan[z] Z[11] (1,0) [z, t]

```

```

Out[1150]=
-3 yZ[1][z, t] - M yZ[3][z, t] +
e-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - yZ[0](0,1)[z, t] - 6 Tan[z] yZ[1](1,0)[z, t]
-3 yZ[0][z, t] - M yZ[2][z, t] -
e-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - yZ[1](0,1)[z, t] - 6 Tan[z] yZ[0](1,0)[z, t]
M yZ[1][z, t] + 3 yZ[3][z, t] - e-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] -
yZ[2](0,1)[z, t] + 6 Tan[z] yZ[3](1,0)[z, t]
M yZ[0][z, t] + 3 yZ[2][z, t] + e-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] -
yZ[3](0,1)[z, t] + 6 Tan[z] yZ[2](1,0)[z, t]
3 yZ[5][z, t] + M yZ[7][z, t] + e-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] -
yZ[4](0,1)[z, t] + 6 Tan[z] yZ[5](1,0)[z, t]
3 yZ[4][z, t] + M yZ[6][z, t] - e-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] -
yZ[5](0,1)[z, t] + 6 Tan[z] yZ[4](1,0)[z, t]
-M yZ[5][z, t] - 3 yZ[7][z, t] -
e-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - yZ[6](0,1)[z, t] - 6 Tan[z] yZ[7](1,0)[z, t]
-M yZ[4][z, t] - 3 yZ[6][z, t] +
e-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - yZ[7](0,1)[z, t] - 6 Tan[z] yZ[6](1,0)[z, t]
3 yZ[9][z, t] + M yZ[11][z, t] - yZ[8](0,1)[z, t] + 6 Tan[z] yZ[9](1,0)[z, t]
3 yZ[8][z, t] + M yZ[10][z, t] - yZ[9](0,1)[z, t] + 6 Tan[z] yZ[8](1,0)[z, t]
-M yZ[9][z, t] - 3 yZ[11][z, t] - yZ[10](0,1)[z, t] - 6 Tan[z] yZ[11](1,0)[z, t]
-M yZ[8][z, t] - 3 yZ[10][z, t] - yZ[11](0,1)[z, t] - 6 Tan[z] yZ[10](1,0)[z, t]
-3 yZ[13][z, t] - M yZ[15][z, t] - yZ[12](0,1)[z, t] - 6 Tan[z] yZ[13](1,0)[z, t]
-3 yZ[12][z, t] - M yZ[14][z, t] - yZ[13](0,1)[z, t] - 6 Tan[z] yZ[12](1,0)[z, t]
M yZ[13][z, t] + 3 yZ[15][z, t] - yZ[14](0,1)[z, t] + 6 Tan[z] yZ[15](1,0)[z, t]
M yZ[12][z, t] + 3 yZ[14][z, t] - yZ[15](0,1)[z, t] + 6 Tan[z] yZ[14](1,0)[z, t]

In[1151]:= (* (DtyZrelations=Transpose[caZ2].DtvarZrelations/.sZt0yZ//FullSimplify) // Column*)

```

CHECK THAT yZ[j] ARE IN PROPER ORDER (4 things , 4 at a time):

```
In[1152]:= (DtYzsubs = Solve[And @@ Thread[θ == DtYzrelations],  
  (D[(yZ[#][z, t] & /@ Range[0, 15]), t])[[1]] //  
  FullSimplify[#, constraintVars] &) // Column  
  
Out[1152]=  
yZ[0]^(0,1)[z, t] → -3 yZ[1][z, t] - M yZ[3][z, t] +  
  e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - 6 Tan[z] yZ[1]^(1,0)[z, t]  
yZ[1]^(0,1)[z, t] → -3 yZ[0][z, t] - M yZ[2][z, t] -  
  e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t]  
yZ[2]^(0,1)[z, t] →  
  M yZ[1][z, t] + 3 yZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] + 6 Tan[z] yZ[3]^(1,0)[z, t]  
yZ[3]^(0,1)[z, t] →  
  M yZ[0][z, t] + 3 yZ[2][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]  
yZ[4]^(0,1)[z, t] →  
  3 yZ[5][z, t] + M yZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] + 6 Tan[z] yZ[5]^(1,0)[z, t]  
yZ[5]^(0,1)[z, t] →  
  3 yZ[4][z, t] + M yZ[6][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t]  
yZ[6]^(0,1)[z, t] → -M yZ[5][z, t] - 3 yZ[7][z, t] -  
  e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - 6 Tan[z] yZ[7]^(1,0)[z, t]  
yZ[7]^(0,1)[z, t] → -M yZ[4][z, t] - 3 yZ[6][z, t] +  
  e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]  
yZ[8]^(0,1)[z, t] → 3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t]  
yZ[9]^(0,1)[z, t] → 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t]  
yZ[10]^(0,1)[z, t] → -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t])  
yZ[11]^(0,1)[z, t] → -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])  
yZ[12]^(0,1)[z, t] → -3 yZ[13][z, t] - M yZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t]  
yZ[13]^(0,1)[z, t] → -3 yZ[12][z, t] - M yZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t]  
yZ[14]^(0,1)[z, t] → M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t]  
yZ[15]^(0,1)[z, t] → M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]  
  
In[1153]:= DtvaryZEQS = DtYzsubs /. {Rule → Equal};  
zeroDtYzeqs = DtYzsubs /. {Rule → Subtract}  
coupledDtYzeqs = Partition[DtvaryZEQS, 4]
```

Out[1154]=

$$\begin{aligned} & \{3 yZ[1][z, t] + M yZ[3][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[0][z, t] a4'[t] + \\ & yZ[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[1]^{(1,0)}[z, t], 3 yZ[0][z, t] + M yZ[2][z, t] + \\ & e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[1][z, t] a4'[t] + yZ[1]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[0]^{(1,0)}[z, t], \\ & -M yZ[1][z, t] - 3 yZ[3][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[2][z, t] a4'[t] + \\ & yZ[2]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[3]^{(1,0)}[z, t], -M yZ[0][z, t] - 3 yZ[2][z, t] - \\ & e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[3][z, t] a4'[t] + yZ[3]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[2]^{(1,0)}[z, t], \\ & -3 yZ[5][z, t] - M yZ[7][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[4][z, t] a4'[t] + \\ & yZ[4]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[5]^{(1,0)}[z, t], -3 yZ[4][z, t] - M yZ[6][z, t] + \\ & e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[5][z, t] a4'[t] + yZ[5]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[4]^{(1,0)}[z, t], \\ & M yZ[5][z, t] + 3 yZ[7][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[6][z, t] a4'[t] + \\ & yZ[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[7]^{(1,0)}[z, t], M yZ[4][z, t] + 3 yZ[6][z, t] - \\ & e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[7][z, t] a4'[t] + yZ[7]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[6]^{(1,0)}[z, t], \\ & -3 yZ[9][z, t] - M yZ[11][z, t] + yZ[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[9]^{(1,0)}[z, t], \\ & -3 yZ[8][z, t] - M yZ[10][z, t] + yZ[9]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[8]^{(1,0)}[z, t], \\ & M yZ[9][z, t] + yZ[10]^{(0,1)}[z, t] + 3 (yZ[11][z, t] + 2 \operatorname{Tan}[z] yZ[11]^{(1,0)}[z, t]), \\ & M yZ[8][z, t] + yZ[11]^{(0,1)}[z, t] + 3 (yZ[10][z, t] + 2 \operatorname{Tan}[z] yZ[10]^{(1,0)}[z, t]), \\ & 3 yZ[13][z, t] + M yZ[15][z, t] + yZ[12]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[13]^{(1,0)}[z, t], \\ & 3 yZ[12][z, t] + M yZ[14][z, t] + yZ[13]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[12]^{(1,0)}[z, t], \\ & -M yZ[13][z, t] - 3 yZ[15][z, t] + yZ[14]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[15]^{(1,0)}[z, t], \\ & -M yZ[12][z, t] - 3 yZ[14][z, t] + yZ[15]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[14]^{(1,0)}[z, t]\} \end{aligned}$$

Out[1155]=

$$\begin{aligned} & \{yZ[0]^{(0,1)}[z, t] == -3 yZ[1][z, t] - M yZ[3][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[0][z, t] a4'[t] - \\ & 6 \operatorname{Tan}[z] yZ[1]^{(1,0)}[z, t], yZ[1]^{(0,1)}[z, t] == -3 yZ[0][z, t] - M yZ[2][z, t] - \\ & e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[1][z, t] a4'[t] - 6 \operatorname{Tan}[z] yZ[0]^{(1,0)}[z, t], \\ & yZ[2]^{(0,1)}[z, t] == M yZ[1][z, t] + 3 yZ[3][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[2][z, t] a4'[t] + \\ & 6 \operatorname{Tan}[z] yZ[3]^{(1,0)}[z, t], yZ[3]^{(0,1)}[z, t] == M yZ[0][z, t] + 3 yZ[2][z, t] + \\ & e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[3][z, t] a4'[t] + 6 \operatorname{Tan}[z] yZ[2]^{(1,0)}[z, t]\}, \\ & \{yZ[4]^{(0,1)}[z, t] == 3 yZ[5][z, t] + M yZ[7][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[4][z, t] a4'[t] + \\ & 6 \operatorname{Tan}[z] yZ[5]^{(1,0)}[z, t], yZ[5]^{(0,1)}[z, t] == 3 yZ[4][z, t] + M yZ[6][z, t] - \\ & e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[5][z, t] a4'[t] + 6 \operatorname{Tan}[z] yZ[4]^{(1,0)}[z, t], \\ & yZ[6]^{(0,1)}[z, t] == -M yZ[5][z, t] - 3 yZ[7][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[6][z, t] a4'[t] - \\ & 6 \operatorname{Tan}[z] yZ[7]^{(1,0)}[z, t], yZ[7]^{(0,1)}[z, t] == -M yZ[4][z, t] - 3 yZ[6][z, t] + \\ & e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[7][z, t] a4'[t] - 6 \operatorname{Tan}[z] yZ[6]^{(1,0)}[z, t]\}, \\ & \{yZ[8]^{(0,1)}[z, t] == 3 yZ[9][z, t] + M yZ[11][z, t] + 6 \operatorname{Tan}[z] yZ[9]^{(1,0)}[z, t], \\ & yZ[9]^{(0,1)}[z, t] == 3 yZ[8][z, t] + M yZ[10][z, t] + 6 \operatorname{Tan}[z] yZ[8]^{(1,0)}[z, t], \\ & yZ[10]^{(0,1)}[z, t] == -M yZ[9][z, t] - 3 (yZ[11][z, t] + 2 \operatorname{Tan}[z] yZ[11]^{(1,0)}[z, t]), \\ & yZ[11]^{(0,1)}[z, t] == -M yZ[8][z, t] - 3 (yZ[10][z, t] + 2 \operatorname{Tan}[z] yZ[10]^{(1,0)}[z, t])\}, \\ & \{yZ[12]^{(0,1)}[z, t] == -3 yZ[13][z, t] - M yZ[15][z, t] - 6 \operatorname{Tan}[z] yZ[13]^{(1,0)}[z, t], \\ & yZ[13]^{(0,1)}[z, t] == -3 yZ[12][z, t] - M yZ[14][z, t] - 6 \operatorname{Tan}[z] yZ[12]^{(1,0)}[z, t], \\ & yZ[14]^{(0,1)}[z, t] == M yZ[13][z, t] + 3 yZ[15][z, t] + 6 \operatorname{Tan}[z] yZ[15]^{(1,0)}[z, t], \\ & yZ[15]^{(0,1)}[z, t] == M yZ[12][z, t] + 3 yZ[14][z, t] + 6 \operatorname{Tan}[z] yZ[14]^{(1,0)}[z, t]\}\} \end{aligned}$$

```

In[1165]:= 
eqF = F[t] == e-a4[t] * Q1 * Sinh[a4[t]] * a4'[t]
sQ1 = Solve[eqF, Q1][[1]]
coupledYZFeqs = coupledYZeqs /. sQ1 // FullSimplify
Out[1165]=
F[t] == e-a4[t] Q1 Sinh[a4[t]] a4'[t]
Out[1166]=
{Q1 →  $\frac{e^{a4[t]} \operatorname{Csch}[a4[t]] F[t]}{a4'[t]}$ }
Out[1167]=
{ {F[t] × yZ[0][z, t] == 3 yZ[1][z, t] + M yZ[3][z, t] + yZ[0](0,1)[z, t] + 6 Tan[z] yZ[1](1,0)[z, t], 
  3 yZ[0][z, t] + F[t] × yZ[1][z, t] + M yZ[2][z, t] + yZ[1](0,1)[z, t] + 6 Tan[z] yZ[0](1,0)[z, t] == 
  0, M yZ[1][z, t] + 3 yZ[3][z, t] + 6 Tan[z] yZ[3](1,0)[z, t] == 
  F[t] × yZ[2][z, t] + yZ[2](0,1)[z, t], 
  M yZ[0][z, t] + 3 yZ[2][z, t] + F[t] × yZ[3][z, t] + 6 Tan[z] yZ[2](1,0)[z, t] == yZ[3](0,1)[z, t]}, 
 {F[t] × yZ[4][z, t] + 3 yZ[5][z, t] + M yZ[7][z, t] + 6 Tan[z] yZ[5](1,0)[z, t] == yZ[4](0,1)[z, t], 
  3 yZ[4][z, t] + M yZ[6][z, t] + 6 Tan[z] yZ[4](1,0)[z, t] == F[t] × yZ[5][z, t] + yZ[5](0,1)[z, t], 
  M yZ[5][z, t] + F[t] × yZ[6][z, t] + 3 yZ[7][z, t] + 
  yZ[6](0,1)[z, t] + 6 Tan[z] yZ[7](1,0)[z, t] == 0, 
  M yZ[4][z, t] + 3 yZ[6][z, t] + yZ[7](0,1)[z, t] + 6 Tan[z] yZ[6](1,0)[z, t] == F[t] × yZ[7][z, t]}, 
 {3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9](1,0)[z, t] == yZ[8](0,1)[z, t], 
  3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8](1,0)[z, t] == yZ[9](0,1)[z, t], 
  M yZ[9][z, t] + 3 yZ[11][z, t] + yZ[10](0,1)[z, t] + 6 Tan[z] yZ[11](1,0)[z, t] == 0, 
  M yZ[8][z, t] + 3 yZ[10][z, t] + yZ[11](0,1)[z, t] + 6 Tan[z] yZ[10](1,0)[z, t] == 0}, 
 {3 yZ[13][z, t] + M yZ[15][z, t] + yZ[12](0,1)[z, t] + 6 Tan[z] yZ[13](1,0)[z, t] == 0, 
  3 yZ[12][z, t] + M yZ[14][z, t] + yZ[13](0,1)[z, t] + 6 Tan[z] yZ[12](1,0)[z, t] == 0, 
  M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15](1,0)[z, t] == yZ[14](0,1)[z, t], 
  M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14](1,0)[z, t] == yZ[15](0,1)[z, t]}}
In[1156]:= 
Clear[yZ, yZvar];
yZvar = (yZ[#][z, t] & /@ Range[0, 15])
Out[1157]=
{yZ[0][z, t], yZ[1][z, t], yZ[2][z, t], yZ[3][z, t], yZ[4][z, t],
 yZ[5][z, t], yZ[6][z, t], yZ[7][z, t], yZ[8][z, t], yZ[9][z, t], yZ[10][z, t],
 yZ[11][z, t], yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t]}
In[1168]:= 
DSolve[coupledYZFeqs[[1]], yZvar[[1 ;; 4]], {z, t}]
Out[1168]=
DSolve[
 {F[t] × yZ[0][z, t] == 3 yZ[1][z, t] + M yZ[3][z, t] + yZ[0](0,1)[z, t] + 6 Tan[z] yZ[1](1,0)[z, t], 
  3 yZ[0][z, t] + F[t] × yZ[1][z, t] + M yZ[2][z, t] + yZ[1](0,1)[z, t] + 6 Tan[z] yZ[0](1,0)[z, t] == 
  0, M yZ[1][z, t] + 3 yZ[3][z, t] + 6 Tan[z] yZ[3](1,0)[z, t] == 
  F[t] × yZ[2][z, t] + yZ[2](0,1)[z, t], 
  M yZ[0][z, t] + 3 yZ[2][z, t] + F[t] × yZ[3][z, t] + 6 Tan[z] yZ[2](1,0)[z, t] == yZ[3](0,1)[z, t]}, 
 {yZ[0][z, t], yZ[1][z, t], yZ[2][z, t], yZ[3][z, t]}, {z, t}]

```

```

In[1169]:= DSolve[coupledYZeqs[[2]], yZvar[[5 ;; 8]], {z, t}]
Out[1169]= DSolve[
{yZ[4]^(0,1)[z, t] == 3 yZ[5][z, t] + M yZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
6 Tan[z] yZ[5]^(1,0)[z, t], yZ[5]^(0,1)[z, t] == 3 yZ[4][z, t] + M yZ[6][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t],
yZ[6]^(0,1)[z, t] == -M yZ[5][z, t] - 3 yZ[7][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] -
6 Tan[z] yZ[7]^(1,0)[z, t], yZ[7]^(0,1)[z, t] == -M yZ[4][z, t] - 3 yZ[6][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]}, {yZ[4][z, t], yZ[5][z, t], yZ[6][z, t], yZ[7][z, t]}, {z, t}]
In[1170]:= DSolve[coupledYZFeqs[[3]], yZvar[[9 ;; 12]], {z, t}]
Out[1170]= DSolve[{3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t] == yZ[8]^(0,1)[z, t],
3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t] == yZ[9]^(0,1)[z, t],
M yZ[9][z, t] + 3 yZ[11][z, t] + yZ[10]^(0,1)[z, t] + 6 Tan[z] yZ[11]^(1,0)[z, t] == 0,
M yZ[8][z, t] + 3 yZ[10][z, t] + yZ[11]^(0,1)[z, t] + 6 Tan[z] yZ[10]^(1,0)[z, t] == 0},
{yZ[8][z, t], yZ[9][z, t], yZ[10][z, t], yZ[11][z, t]}, {z, t}]
In[1171]:= DSolve[coupledYZFeqs[[4]], yZvar[[13 ;; 16]], {z, t}]
Out[1171]= DSolve[{3 yZ[13][z, t] + M yZ[15][z, t] + yZ[12]^(0,1)[z, t] + 6 Tan[z] yZ[13]^(1,0)[z, t] == 0,
3 yZ[12][z, t] + M yZ[14][z, t] + yZ[13]^(0,1)[z, t] + 6 Tan[z] yZ[12]^(1,0)[z, t] == 0,
M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t] == yZ[14]^(0,1)[z, t],
M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t] == yZ[15]^(0,1)[z, t}],
{yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t]}, {z, t}]

```

TRY MAPLE : MUST use ` List[Equal[Derivative[0,1][yZ8][z,t],Pl...`

```
In[1172]:= syZzt = yZ[#] → ToExpression["((yZ" <> ToString[#] <> "[#1,#2])&)"] & /@ Range[0, 15]
maplexyZeqs = FullForm[coupledelyZFeqs /. % // FullSimplify]

Out[1172]= {yZ[0] → (yZ0[#1, #2] &), yZ[1] → (yZ1[#1, #2] &),
yZ[2] → (yZ2[#1, #2] &), yZ[3] → (yZ3[#1, #2] &), yZ[4] → (yZ4[#1, #2] &),
yZ[5] → (yZ5[#1, #2] &), yZ[6] → (yZ6[#1, #2] &), yZ[7] → (yZ7[#1, #2] &),
yZ[8] → (yZ8[#1, #2] &), yZ[9] → (yZ9[#1, #2] &), yZ[10] → (yZ10[#1, #2] &),
yZ[11] → (yZ11[#1, #2] &), yZ[12] → (yZ12[#1, #2] &),
yZ[13] → (yZ13[#1, #2] &), yZ[14] → (yZ14[#1, #2] &), yZ[15] → (yZ15[#1, #2] &)}

Out[1173]//FullForm=
List[List[Equal[Times[F[t], yZ0[z, t]], Plus[Times[3, yZ1[z, t]], Times[M, yZ3[z, t]]],
Derivative[0, 1][yZ0][z, t], Times[6, Tan[z], Derivative[1, 0][yZ1][z, t]]], Equal[Plus[Times[3, yZ0[z, t]], Times[F[t], yZ1[z, t]], Times[M, yZ2[z, t]]],
Derivative[0, 1][yZ1][z, t], Times[6, Tan[z], Derivative[1, 0][yZ0][z, t]]], 0], Equal[Plus[Times[M, yZ1[z, t]], Times[3, yZ3[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ3][z, t]]], Plus[Times[F[t], yZ2[z, t]], Derivative[0, 1][yZ2][z, t]], Equal[Plus[Times[M, yZ0[z, t]], Times[3, yZ2[z, t]], Times[F[t], yZ3[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ2][z, t]], Derivative[0, 1][yZ3][z, t]], List[Equal[Plus[Times[F[t], yZ4[z, t]], Times[3, yZ5[z, t]], Times[M, yZ7[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ5][z, t]], Derivative[0, 1][yZ4][z, t]], Equal[Plus[Times[3, yZ4[z, t]], Times[M, yZ6[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ4][z, t]]], Plus[Times[F[t], yZ5[z, t]], Derivative[0, 1][yZ5][z, t]], Equal[Plus[Times[M, yZ5[z, t]], Times[F[t], yZ6[z, t]], Times[3, yZ7[z, t]]],
Derivative[0, 1][yZ6][z, t], Times[6, Tan[z], Derivative[1, 0][yZ7][z, t]]], 0], Equal[Plus[Times[M, yZ4[z, t]], Times[3, yZ6[z, t]], Derivative[0, 1][yZ7][z, t]],
Times[6, Tan[z], Derivative[1, 0][yZ6][z, t]], Times[F[t], yZ7[z, t]]], List[Equal[Plus[Times[M, yZ11[z, t]], Times[3, yZ9[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ9][z, t]], Derivative[0, 1][yZ8][z, t]], Equal[Plus[Times[M, yZ10[z, t]], Times[3, yZ8[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ8][z, t]], Derivative[0, 1][yZ9][z, t]], Equal[Plus[Times[3, yZ11[z, t]], Times[M, yZ9[z, t]], Derivative[0, 1][yZ10][z, t]],
Times[6, Tan[z], Derivative[1, 0][yZ10][z, t]], 0], Equal[Plus[Times[3, yZ10[z, t]], Times[M, yZ8[z, t]], Derivative[0, 1][yZ11][z, t]],
Times[6, Tan[z], Derivative[1, 0][yZ11][z, t]], 0], Equal[Plus[Times[3, yZ11[z, t]], Times[6, Tan[z], Derivative[1, 0][yZ10][z, t]]], Times[F[t], yZ12[z, t]], Derivative[0, 1][yZ11][z, t]], List[Equal[Plus[Times[3, yZ13[z, t]], Times[M, yZ15[z, t]]], Derivative[0, 1][yZ12][z, t]],
Times[6, Tan[z], Derivative[1, 0][yZ13][z, t]], 0], Equal[Plus[Times[3, yZ12[z, t]], Times[M, yZ14[z, t]], Derivative[0, 1][yZ13][z, t]],
Times[6, Tan[z], Derivative[1, 0][yZ12][z, t]], 0], Equal[Plus[Times[M, yZ13[z, t]], Times[3, yZ15[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ15][z, t]], Derivative[0, 1][yZ14][z, t]], Equal[Plus[Times[M, yZ12[z, t]], Times[3, yZ14[z, t]]], Times[6, Tan[z], Derivative[1, 0][yZ14][z, t]], Derivative[0, 1][yZ15][z, t]]]]]
```

```
In[1174]:= coupledyzFeqs
Out[1174]= { {F[t] × yZ[0][z, t] = 3 yZ[1][z, t] + M yZ[3][z, t] + yZ[0]^(0,1)[z, t] + 6 Tan[z] yZ[1]^(1,0)[z, t], 3 yZ[0][z, t] + F[t] × yZ[1][z, t] + M yZ[2][z, t] + yZ[1]^(0,1)[z, t] + 6 Tan[z] yZ[0]^(1,0)[z, t] == 0, M yZ[1][z, t] + 3 yZ[3][z, t] + 6 Tan[z] yZ[3]^(1,0)[z, t] == F[t] × yZ[2][z, t] + yZ[2]^(0,1)[z, t], M yZ[0][z, t] + 3 yZ[2][z, t] + F[t] × yZ[3][z, t] + 6 Tan[z] yZ[2]^(1,0)[z, t] == yZ[3]^(0,1)[z, t]}, {F[t] × yZ[4][z, t] + 3 yZ[5][z, t] + M yZ[7][z, t] + 6 Tan[z] yZ[5]^(1,0)[z, t] == yZ[4]^(0,1)[z, t], 3 yZ[4][z, t] + M yZ[6][z, t] + 6 Tan[z] yZ[4]^(1,0)[z, t] == F[t] × yZ[5][z, t] + yZ[5]^(0,1)[z, t], M yZ[5][z, t] + F[t] × yZ[6][z, t] + 3 yZ[7][z, t] + yZ[6]^(0,1)[z, t] + 6 Tan[z] yZ[7]^(1,0)[z, t] == 0, M yZ[4][z, t] + 3 yZ[6][z, t] + yZ[7]^(0,1)[z, t] + 6 Tan[z] yZ[6]^(1,0)[z, t] == F[t] × yZ[7][z, t]}, {3 yZ[9][z, t] + M yZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t] == yZ[8]^(0,1)[z, t], 3 yZ[8][z, t] + M yZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t] == yZ[9]^(0,1)[z, t], M yZ[9][z, t] + 3 yZ[11][z, t] + yZ[10]^(0,1)[z, t] + 6 Tan[z] yZ[11]^(1,0)[z, t] == 0, M yZ[8][z, t] + 3 yZ[10][z, t] + yZ[11]^(0,1)[z, t] + 6 Tan[z] yZ[10]^(1,0)[z, t] == 0}, {3 yZ[13][z, t] + M yZ[15][z, t] + yZ[12]^(0,1)[z, t] + 6 Tan[z] yZ[13]^(1,0)[z, t] == 0, 3 yZ[12][z, t] + M yZ[14][z, t] + yZ[13]^(0,1)[z, t] + 6 Tan[z] yZ[12]^(1,0)[z, t] == 0, M yZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t] == yZ[14]^(0,1)[z, t], M yZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t] == yZ[15]^(0,1)[z, t]} }
```

```
In[1175]:= mapleyZvars =
FullForm[yZ[#][z, t] & /@ Range[4 * (# - 1), 4 * (# - 1) + 3] /. syZzt] & /@ Range[4]
Out[1175]= {List[yZ0[z, t], yZ1[z, t], yZ2[z, t], yZ3[z, t]], List[yZ4[z, t], yZ5[z, t], yZ6[z, t], yZ7[z, t]], List[yZ8[z, t], yZ9[z, t], yZ10[z, t], yZ11[z, t]], List[yZ12[z, t], yZ13[z, t], yZ14[z, t], yZ15[z, t]]}
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

```
“maple-output.py”:
import textwrap
filename = "maple-output.txt"
maple_long_string = "paste maple string here"
# Wraps the text and joins with newlines
with open(filename, "w") as f:
    f . write ("\n" . join (textwrap . wrap (maple_long_string, width = 80)))
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

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TRY MAPLE : MUST use ` List[Equal[Derivative[0,1][yZ8][z,t],Pl...`

```
In[*]:= syZzt = yZ[#] → ToExpression["((yZ" <> ToString[#] <> "[#1,#2])&)"] & /@ Range[0, 15]
maplexyZeqs = FullForm[coupled yZFeqs /. % // FullSimplify]

Out[*]= {yZ[0] → (yZ0[#1, #2] &), yZ[1] → (yZ1[#1, #2] &),
yZ[2] → (yZ2[#1, #2] &), yZ[3] → (yZ3[#1, #2] &), yZ[4] → (yZ4[#1, #2] &),
yZ[5] → (yZ5[#1, #2] &), yZ[6] → (yZ6[#1, #2] &), yZ[7] → (yZ7[#1, #2] &),
yZ[8] → (yZ8[#1, #2] &), yZ[9] → (yZ9[#1, #2] &), yZ[10] → (yZ10[#1, #2] &),
yZ[11] → (yZ11[#1, #2] &), yZ[12] → (yZ12[#1, #2] &),
yZ[13] → (yZ13[#1, #2] &), yZ[14] → (yZ14[#1, #2] &), yZ[15] → (yZ15[#1, #2] &)}
```

```

Out[=]//FullForm=
List[List[Equal[Plus[Times[3, yZ1[z, t]], Times[M, yZ3[z, t]]],
Derivative[0, 1][yZ0][z, t], Times[6, Tan[z], Derivative[1, 0][yZ1][z, t]]],
Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ0[z, t], Derivative[1][a4][t]]],
Equal[Plus[Times[3, yZ0[z, t]], Times[M, yZ2[z, t]]],
Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ1[z, t], Derivative[1][a4][t]],
Derivative[0, 1][yZ1][z, t], Times[6, Tan[z], Derivative[1, 0][yZ0][z, t]]], 0],
Equal[Plus[Times[M, yZ1[z, t]], Times[3, yZ3[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ3][z, t]]], Plus[Times[Power[E, Times[-1, a4[t]]],
Q1, Sinh[a4[t]], yZ2[z, t], Derivative[1][a4][t]], Derivative[0, 1][yZ2][z, t]]],
Equal[Plus[Times[M, yZ0[z, t]], Times[3, yZ2[z, t]]],
Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ3[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][yZ2][z, t]]], Derivative[0, 1][yZ3][z, t]],
List[Equal[Plus[Times[3, yZ5[z, t]], Times[M, yZ7[z, t]]],
Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ4[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][yZ5][z, t]]], Derivative[0, 1][yZ4][z, t]],
Equal[Plus[Times[3, yZ4[z, t]], Times[M, yZ6[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ4][z, t]]], Plus[Times[Power[E, Times[-1, a4[t]]],
Q1, Sinh[a4[t]], yZ5[z, t], Derivative[1][a4][t]], Derivative[0, 1][yZ5][z, t]]],
Equal[Plus[Times[M, yZ5[z, t]], Times[3, yZ7[z, t]]],
Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ6[z, t], Derivative[1][a4][t]],
Derivative[0, 1][yZ6][z, t], Times[6, Tan[z], Derivative[1, 0][yZ7][z, t]]], 0],
Equal[Plus[Times[M, yZ4[z, t]], Times[3, yZ6[z, t]]], Derivative[0, 1][yZ7][z, t],
Times[6, Tan[z], Derivative[1, 0][yZ6][z, t]]],
Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ7[z, t], Derivative[1][a4][t]]],
List[Equal[Plus[Times[M, yZ11[z, t]], Times[3, yZ9[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ9][z, t]]], Derivative[0, 1][yZ8][z, t]],
Equal[Plus[Times[M, yZ10[z, t]], Times[3, yZ8[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ8][z, t]]], Derivative[0, 1][yZ9][z, t]],
Equal[Plus[Times[3, yZ11[z, t]], Times[M, yZ10[z, t]]], Derivative[0, 1][yZ10][z, t],
Times[6, Tan[z], Derivative[1, 0][yZ11][z, t]]], 0],
Equal[Plus[Times[3, yZ10[z, t]], Times[M, yZ8[z, t]]], Derivative[0, 1][yZ11][z, t],
Times[6, Tan[z], Derivative[1, 0][yZ10][z, t]]], 0],
List[Equal[Plus[Times[3, yZ13[z, t]], Times[M, yZ15[z, t]]], Derivative[0, 1][yZ12][z, t],
Times[6, Tan[z], Derivative[1, 0][yZ13][z, t]]], 0],
Equal[Plus[Times[3, yZ12[z, t]], Times[M, yZ14[z, t]]], Derivative[0, 1][yZ13][z, t],
Times[6, Tan[z], Derivative[1, 0][yZ12][z, t]]], 0],
Equal[Plus[Times[M, yZ13[z, t]], Times[3, yZ15[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ15][z, t]]], Derivative[0, 1][yZ14][z, t]],
Equal[Plus[Times[M, yZ12[z, t]], Times[3, yZ14[z, t]]],
Times[6, Tan[z], Derivative[1, 0][yZ14][z, t]]], Derivative[0, 1][yZ15][z, t]]]

```

```
In[1]:= mapleyZvars =
FullForm[yZ[#][z, t] & /@ Range[4 * (# - 1), 4 * (# - 1) + 3] /. syZzt] & /@ Range[4]
Out[1]= {List[yZ0[z, t], yZ1[z, t], yZ2[z, t], yZ3[z, t]],  

         List[yZ4[z, t], yZ5[z, t], yZ6[z, t], yZ7[z, t]],  

         List[yZ8[z, t], yZ9[z, t], yZ10[z, t], yZ11[z, t]],  

         List[yZ12[z, t], yZ13[z, t], yZ14[z, t], yZ15[z, t]]}
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

```
“maple-output.py”:
import textwrap
filename = "maple-output.txt"
maple_long_string = "paste maple string here"
# Wraps the text and joins with newlines
with open (filename, "w") as f :
    f . write ("\"n" . join (textwrap . wrap (maple_long_string, width = 80)))
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

"Solution" by Maple

```
In[1176]:= (*\$16;
%/.sf\$16Aa/.sx0x4;
%/.sZt0yZ;
Thread[%==%%];
Solve[And@@%,yZ[#][z,t]&/@Range[0,15]][[1]]
%/.szt/.{Rule→Equal};
Solve[And@@%,f16[#][x0,x4]&/@Range[0,15]][[1]];
%/.{Rule→Equal}*)
```

Load the Maple output strings. Get is a lexer and parer and ... , so use ReadString

```
In[1177]:= SetDirectory[NotebookDirectory[]];
```

```
In[1178]:= maplestringEQ3 =
"{{yZ10(z, t) = (c37*sin(sqrt(M^2 - 36*C3 - 9)*t) + c38*cos(sqrt(M^2 - 36*C3 - 9)*t))*(c36*sin(z)^(-sqrt(1 + 4*C3)/2) + c35*sin(z)^(sqrt(1 + 4*C3)/2))/sqrt(sin(z)), yZ11(z, t) = (c33*sin(sqrt(M^2 - 36*C3 - 9)*t) + c34*cos(sqrt(M^2 - 36*C3 - 9)*t))*(sin(z)^(-sqrt(1 + 4*C3)/2)*c32 + sin(z)^(sqrt(1 + 4*C3)/2)*c31)/sqrt(sin(z)), yZ8(z, t) = -3*((sin(z)^(-sqrt(1 + 4*C3)/2)*c32 + sin(z)^(sqrt(1 + 4*C3)/2)*c31)*cos(sqrt(M^2 - 36*C3 - 9)) + sin(sqrt(M^2 - 36*C3 - 9)*t)*c34)*sqrt(M^2 - 36*C3 - 9)/3 + sqrt(1 + 4*C3)*(c35*sin(z)^(sqrt(1 + 4*C3)/2) - c36*sin(z)^(-sqrt(1 + 4*C3)/2))*c37)/(sqrt(sin(z))*M), yZ9(z, t) = -((cos(sqrt(M^2 - 36*C3 - 9)*t)*c37 - sin(sqrt(M^2 - 36*C3 - 9)*t)*c38)*(c36*sin(z)^(-sqrt(1 + 4*C3)/2) + c35*sin(z)^(sqrt(1 + 4*C3)/2))*sqrt(M^2 - 36*C3 - 9) + 3*(c33*sin(sqrt(M^2 - 36*C3 - 9)*t) + c34*cos(sqrt(M^2 - 36*C3 - 9)*t))*sqrt(1 + 4*C3)*(sin(z)^(sqrt(1 + 4*C3)/2)*c31 - sin(z)^(-sqrt(1 + 4*C3)/2)*c32))/(sqrt(sin(z))*M)}";
```

```
In[1179]:= maplestringEQ4 =
"{{yZ12(z, t) = -3*(-((cos(sqrt(M^2 - 36*C4 - 9)*t)*c43 - sin(sqrt(M^2 - 36*C4 - 9)*t)*c44)*(c42*sin(z)^(-sqrt(1 + 4*C4)/2) + c41*sin(z)^(sqrt(1 + 4*C4)/2))/sqrt(M^2 - 36*C4 - 9))/3 + (c45*sin(z)^(sqrt(1 + 4*C4)/2) - c46*sin(z)^(-sqrt(1 + 4*C4)/2))*sqrt(1 + 4*C4)*(c47*sin(sqrt(M^2 - 36*C4 - 9)*t) + c48*cos(sqrt(M^2 - 36*C4 - 9)*t)))/(sqrt(sin(z))*M), yZ13(z, t) = ((cos(sqrt(M^2 - 36*C4 - 9)*t)*c47 - sin(sqrt(M^2 - 36*C4 - 9)*t)*c48)*(c46*sin(z)^(-sqrt(1 + 4*C4)/2) + c45*sin(z)^(sqrt(1 + 4*C4)/2))/sqrt(M^2 - 36*C4 - 9) - 3*(c41*sin(z)^(sqrt(1 + 4*C4)/2) - c42*sin(z)^(-sqrt(1 + 4*C4)/2))*(c43*sin(sqrt(M^2 - 36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4 - 9)*t))*sqrt(1 + 4*C4))/(sqrt(sin(z))*M), yZ14(z, t) = (c47*sin(sqrt(M^2 - 36*C4 - 9)*t) + c48*cos(sqrt(M^2 - 36*C4 - 9)*t))*(c46*sin(z)^(-sqrt(1 + 4*C4)/2) + c45*sin(z)^(sqrt(1 + 4*C4)/2))/sqrt(sin(z)), yZ15(z, t) = (c43*sin(sqrt(M^2 - 36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4 - 9)*t))*(c42*sin(z)^(-sqrt(1 + 4*C4)/2) + c41*sin(z)^(sqrt(1 + 4*C4)/2))/sqrt(sin(z))}}";
```

```
In[1185]:= solvedEQ3 = ConvertMapleToMathematicaV2[maplestringEQ3]
Out[1185]=
{YZ10[z, t] ==  $\frac{1}{\sqrt{\sin[z]}} \left( c38 \cos[\sqrt{-9 - 36 C3 + M^2} t] + c37 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right)$ 
 $\left( c36 \sin[z]^{-\frac{1}{2}} \sqrt{1+4 C3} + c35 \sin[z]^{\frac{1}{2}} \sqrt{1+4 C3} \right),$ 
YZ11[z, t] ==  $\frac{1}{\sqrt{\sin[z]}} \left( c34 \cos[\sqrt{-9 - 36 C3 + M^2} t] + c33 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right)$ 
 $\left( c32 \sin[z]^{-\frac{1}{2}} \sqrt{1+4 C3} + c31 \sin[z]^{\frac{1}{2}} \sqrt{1+4 C3} \right),$ 
YZ8[z, t] ==
 $- \frac{1}{M \sqrt{\sin[z]}} 3 \left( \frac{1}{3} \sqrt{-9 - 36 C3 + M^2} \left( c33 \cos[\sqrt{-9 - 36 C3 + M^2} t] - c34 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right) + \right.$ 
 $\left( c32 \sin[z]^{-\frac{1}{2}} \sqrt{1+4 C3} + c31 \sin[z]^{\frac{1}{2}} \sqrt{1+4 C3} \right) + \sqrt{1+4 C3} \left( c38 \cos[\sqrt{-9 - 36 C3 + M^2} t] + \right.$ 
 $\left. c37 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right) \left( -c36 \sin[z]^{-\frac{1}{2}} \sqrt{1+4 C3} + c35 \sin[z]^{\frac{1}{2}} \sqrt{1+4 C3} \right),$ 
YZ9[z, t] ==
 $\frac{1}{M \sqrt{\sin[z]}} \left( -3 \sqrt{1+4 C3} \left( c34 \cos[\sqrt{-9 - 36 C3 + M^2} t] + c33 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right) \right.$ 
 $\left( -c32 \sin[z]^{-\frac{1}{2}} \sqrt{1+4 C3} + c31 \sin[z]^{\frac{1}{2}} \sqrt{1+4 C3} \right) -$ 
 $\sqrt{-9 - 36 C3 + M^2} \left( c37 \cos[\sqrt{-9 - 36 C3 + M^2} t] - c38 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right)$ 
 $\left. \left( c36 \sin[z]^{-\frac{1}{2}} \sqrt{1+4 C3} + c35 \sin[z]^{\frac{1}{2}} \sqrt{1+4 C3} \right) \right) \}$ 
```

```
In[1186]:= solvedEQ4 = ConvertMapleToMathematicaV2[maplestringEQ4]

Out[1186]=
{YZ12[z, t] ==
 - $\frac{1}{M \sqrt{\sin[z]}} 3 \left( -\frac{1}{3} \sqrt{-9 - 36 C4 + M^2} \left( c43 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] - c44 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \right.$ 
  $\left( c42 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c41 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) + \sqrt{1 + 4 C4} \left( c48 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c47 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \left( -c46 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c45 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right),$ 
 YZ13[z, t] ==  $\frac{1}{M \sqrt{\sin[z]}} \left( -3 \sqrt{1 + 4 C4} \left( c44 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c43 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \right.$ 
  $\left( -c42 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c41 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) +$ 
  $\sqrt{-9 - 36 C4 + M^2} \left( c47 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] - c48 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right)$ 
  $\left( c46 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c45 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right),$ 
 YZ14[z, t] ==  $\frac{1}{\sqrt{\sin[z]}} \left( c48 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c47 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right)$ 
  $\left( c46 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c45 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right),$ 
 YZ15[z, t] ==  $\frac{1}{\sqrt{\sin[z]}} \left( c44 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c43 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right)$ 
  $\left( c42 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c41 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) \}$ 
```

```
In[1192]:= maplestringEQ1 = ReadString["maple-textwrap-stringEQ1_mmM4pro_2026-02-13.txt"];
maplestringEQ2 = ReadString["maple-textwrap-stringEQ2_mmM4pro_2026-02-13.txt"];
```

In[1193]:=

```
time1 = Now
If[FindFile["solvedEQ1.mx"] === $Failed,
 AbsoluteTiming[solvedEQ1 = ConvertMapleToMathematicaV2[maplestringEQ1]];
 DumpSave[ToString[header <> "solvedEQ1.mx"], solvedEQ1];
 Print[ToString[header <> "solvedEQ1.mx"]];
 , Get["solvedEQ1.mx"]]
Now - time1
```

Out[1193]=

Fri 13 Feb 2026 04:11:15 GMT-8

2026-02-13-Pair-Crtn-Univ-same_E-L-eqs-alt-approach-mmM4p-solvedEQ1.mx

Out[1195]=

12.51934473 min

In[1196]:=

solvedEQ1

Out[1196]=

```
{YZ0[z, t] ==  $\frac{1}{2 F M t} \left( -F YZ3[z, t] + (-9 + M^2 - F t^2) YZ3[z, t] + YZ3^{(0,2)}[z, t] + \right.$ 
```

$$\begin{aligned}
& \left(-72 \tan[z] - 36 \tan[z]^3 \right) YZ3^{(1,0)}[z, t] - 36 \tan[z]^2 YZ3^{(2,0)}[z, t] \Big), \\
YZ1[z, t] = & \frac{1}{F M t^2 \sqrt{\sin[z]}} \sec[z]^5 \left(-3 e^{-\frac{F t^2}{2}} t^2 \cos[M t] \right. \\
& \int e^{\frac{F t^2}{2}} t^3 \cos[M t] \left(\frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{1}{2} (2 F^2 - F^2 t^2 + F t^2 (-9 + M^2 + F t^2)) \right. \right. \\
& \cos[z]^3 YZ3^{(0,2)}[z, t] + F t (F + F t^2) \cos[z]^3 YZ3^{(0,3)}[z, t] - \\
& \frac{1}{2} F t^2 \cos[z]^3 YZ3^{(0,4)}[z, t] + F (F t (-9 - F + M^2 - F t^2) \cos[z]^3 YZ3^{(0,1)}[z, t] - \\
& F ((-9 - F + M^2) \cos[z]^3 YZ3[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) \\
& YZ3^{(1,0)}[z, t]) - 36 F^2 t (1 + \cos[z]^2) \sin[z] YZ3^{(1,1)}[z, t] + \\
& 18 F t^2 (1 + \cos[z]^2) \sin[z] YZ3^{(1,2)}[z, t] - 36 F^2 (-1 + \cos[z]) \\
& \cos[z] (1 + \cos[z]) YZ3^{(2,0)}[z, t] + F (-36 F t \cos[z] + 36 F t \cos[z]^3) \\
& YZ3^{(2,1)}[z, t] + (18 F t^2 \cos[z] - 18 F t^2 \cos[z]^3) YZ3^{(2,2)}[z, t] \Big) dz + \\
& F t^2 \left(-\frac{1}{36} (F + M^2 + F t^2) \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 (\cos[z]^3 \right. \\
& ((-9 - F + M^2 + F t^2) YZ3[z, t] - 2 F t YZ3^{(0,1)}[z, t]) + \\
& \cos[z]^3 YZ3^{(0,2)}[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ3^{(1,0)}[z, t] + \\
& (-36 \cos[z] + 36 \cos[z]^3) YZ3^{(2,0)}[z, t] \Big) dz + \frac{1}{18} \cos[z]^5 \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-F t (-9 - F + M^2 + F t^2) \cos[z]^3 YZ3^{(0,1)}[z, t] + 2 \left(\frac{F}{2} + F t^2 \right) \right. \\
& \cos[z]^3 YZ3^{(0,2)}[z, t] - F t \cos[z]^3 YZ3^{(0,3)}[z, t] + F ((-9 - F + M^2 - F t^2) \\
& \cos[z]^3 YZ3[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ3^{(1,0)}[z, t]) + \\
& 36 F t (1 + \cos[z]^2) \sin[z] YZ3^{(1,1)}[z, t] - F (36 \cos[z] - 36 \cos[z]^3) \\
& YZ3^{(2,0)}[z, t] - (-36 F t \cos[z] + 36 F t \cos[z]^3) YZ3^{(2,1)}[z, t] \Big) dz - \\
& 198 \cos[z]^2 \sin[z]^{7/2} YZ3^{(1,0)}[z, t] - 108 \sin[z]^{11/2} YZ3^{(1,0)}[z, t] + \\
& \cos[z]^4 \sin[z]^{3/2} ((-117 - F + M^2 - 3 F t^2) YZ3^{(1,0)}[z, t] - 2 F t YZ3^{(1,1)}[z, t]) + \\
& \cos[z]^4 \sin[z]^{3/2} YZ3^{(1,2)}[z, t] + \frac{1}{2} \cos[z] (\cos[z]^4 \sqrt{\sin[z]} \\
& ((-9 - F + M^2 - 3 F t^2) YZ3[z, t] - 2 F t YZ3^{(0,1)}[z, t] + YZ3^{(0,2)}[z, t]) - \\
& 324 \cos[z]^2 \sin[z]^{5/2} YZ3^{(2,0)}[z, t] - 216 \sin[z]^{9/2} YZ3^{(2,0)}[z, t]) - 36 \\
& \cos[z]^2 \sin[z]^{7/2} YZ3^{(3,0)}[z, t] \Big) \Big) dt - 3 e^{-\frac{F t^2}{2}} t^2 \\
& \left(\int e^{\frac{F t^2}{2}} t^3 \sin[M t] \left(\frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{1}{2} (2 F^2 - F^2 t^2 + F t^2 (-9 + M^2 + F t^2)) \right. \right. \right. \\
& \cos[z]^3 YZ3^{(0,2)}[z, t] + F t (F + F t^2) \cos[z]^3 YZ3^{(0,3)}[z, t] - \frac{1}{2} F t^2 \\
& \cos[z]^3 YZ3^{(0,4)}[z, t] + F (F t (-9 - F + M^2 - F t^2) \cos[z]^3 YZ3^{(0,1)}[z, t] - \\
& F ((-9 - F + M^2) \cos[z]^3 YZ3[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) \\
& YZ3^{(1,0)}[z, t]) - 36 F^2 t (1 + \cos[z]^2) \sin[z] YZ3^{(1,1)}[z, t] +
\end{aligned}$$

$$\begin{aligned}
& 18 F t^2 (1 + \cos[z]^2) \sin[z] YZ3^{(1,2)}[z, t] - 36 F^2 (-1 + \cos[z]) \\
& \cos[z] (1 + \cos[z]) YZ3^{(2,0)}[z, t] + F (-36 F t \cos[z] + 36 F t \cos[z]^3) \\
& YZ3^{(2,1)}[z, t] + (18 F t^2 \cos[z] - 18 F t^2 \cos[z]^3) YZ3^{(2,2)}[z, t] \Big) dz + \\
& F t^2 \left(-\frac{1}{36} (F + M^2 + F t^2) \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 (\cos[z]^3 \right. \\
& ((-9 - F + M^2 + F t^2) YZ3[z, t] - 2 F t YZ3^{(0,1)}[z, t]) + \cos[z]^3 \\
& YZ3^{(0,2)}[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ3^{(1,0)}[z, t] + \\
& (-36 \cos[z] + 36 \cos[z]^3) YZ3^{(2,0)}[z, t] \Big) dz + \frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \\
& \sec[z]^2 \left(-F t (-9 - F + M^2 + F t^2) \cos[z]^3 YZ3^{(0,1)}[z, t] + 2 \left(\frac{F}{2} + F t^2 \right) \cos[z]^3 \right. \\
& YZ3^{(0,2)}[z, t] - F t \cos[z]^3 YZ3^{(0,3)}[z, t] + F ((-9 - F + M^2 - F t^2) \cos[z]^3 \\
& YZ3[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ3^{(1,0)}[z, t]) + \\
& 36 F t (1 + \cos[z]^2) \sin[z] YZ3^{(1,1)}[z, t] - F (36 \cos[z] - 36 \cos[z]^3) \\
& YZ3^{(2,0)}[z, t] - (-36 F t \cos[z] + 36 F t \cos[z]^3) YZ3^{(2,1)}[z, t] \Big) dz - \\
& 198 \cos[z]^2 \sin[z]^{7/2} YZ3^{(1,0)}[z, t] - 108 \sin[z]^{11/2} YZ3^{(1,0)}[z, t] + \\
& \cos[z]^4 \sin[z]^{3/2} ((-117 - F + M^2 - 3 F t^2) YZ3^{(1,0)}[z, t] - 2 F t YZ3^{(1,1)}[z, t]) + \\
& \cos[z]^4 \sin[z]^{3/2} YZ3^{(1,2)}[z, t] + \frac{1}{2} \cos[z] (\cos[z]^4 \sqrt{\sin[z]} \\
& ((-9 - F + M^2 - 3 F t^2) YZ3[z, t] - 2 F t YZ3^{(0,1)}[z, t] + YZ3^{(0,2)}[z, t]) - \\
& 324 \cos[z]^2 \sin[z]^{5/2} YZ3^{(2,0)}[z, t] - 216 \sin[z]^{9/2} YZ3^{(2,0)}[z, t]) - \\
& 36 \cos[z]^2 \sin[z]^{7/2} YZ3^{(3,0)}[z, t] \Big) dt \Big) \sin[M t] + \\
& \cos[z]^4 \left(\frac{1}{8} \cos[z] \int \frac{1}{(1 + \cos[2 z]) \sqrt{\sin[z]}} \left(-F t (-9 - F + M^2 + F t^2) \cos[z] \right. \right. \\
& YZ3^{(0,1)}[z, t] + \cos[3 z] \left(\frac{1}{3} F (-9 - F + M^2 - F t^2) YZ3[z, t] - \right. \\
& \left. \left. \frac{1}{3} F t (-9 - F + M^2 + F t^2) YZ3^{(0,1)}[z, t] \right) + (F + 2 F t^2) \left(\cos[z] + \frac{1}{3} \cos[3 z] \right) \right. \\
& YZ3^{(0,2)}[z, t] + F t \left(-\cos[z] - \frac{1}{3} \cos[3 z] \right) YZ3^{(0,3)}[z, t] + \\
& F ((-9 - F + M^2 - F t^2) \cos[z] YZ3[z, t] + (-60 \sin[z] - 12 \sin[3 z]) YZ3^{(1,0)}[z, t]) + \\
& 12 F t (5 \sin[z] + \sin[3 z]) YZ3^{(1,1)}[z, t] - 12 F (\cos[z] - \cos[3 z]) YZ3^{(2,0)}[z, t] + \\
& 12 F t (\cos[z] - \cos[3 z]) YZ3^{(2,1)}[z, t] \Big) dz + \\
& F t^2 \left(-\frac{1}{8} \cos[z] \int \frac{1}{(1 + \cos[2 z]) \sqrt{\sin[z]}} \left((-9 - F + M^2 + F t^2) \cos[z] \right. \right. \\
& YZ3[z, t] - 2 F t \cos[z] YZ3^{(0,1)}[z, t] + \\
& \cos[3 z] \left(\frac{1}{3} (-9 - F + M^2 + F t^2) YZ3[z, t] - \frac{2}{3} F t YZ3^{(0,1)}[z, t] \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(\cos[z] + \frac{1}{3} \cos[3z] \right) YZ3^{(0,2)}[z, t] + (-60 \sin[z] - 12 \sin[3z]) \\
& YZ3^{(1,0)}[z, t] + (-12 \cos[z] + 12 \cos[3z]) YZ3^{(2,0)}[z, t] \Big) dz + \\
& e^{-\frac{Ft^2}{2}} M \cos[z] (c12 \cos[Mt] - c11 \sin[Mt]) - 3 \cos[z] \sqrt{\sin[z]} YZ3[z, t] - \\
& 6 \sin[z]^{3/2} YZ3^{(1,0)}[z, t] \Bigg) \Bigg), \\
YZ2[z, t] = & \frac{1}{FMt \sqrt{\sin[z]}} \sec[z]^5 \left(3 \right. \\
& e^{-\frac{Ft^2}{2}} \\
& t \\
& \cos[\\
& M \\
& t] \\
& \int e^{\frac{Ft^2}{2}} t^3 \sin[Mt] \left(\frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{1}{2} (2F^2 - F^2 t^2 + F t^2 (-9 + M^2 + F t^2)) \right. \right. \\
& \cos[z]^3 YZ3^{(0,2)}[z, t] + F t (F + F t^2) \cos[z]^3 YZ3^{(0,3)}[z, t] - \\
& \frac{1}{2} F t^2 \cos[z]^3 YZ3^{(0,4)}[z, t] + F (F t (-9 - F + M^2 - F t^2) \cos[z]^3 YZ3^{(0,1)}[z, t] - \\
& F ((-9 - F + M^2) \cos[z]^3 YZ3[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) \\
& YZ3^{(1,0)}[z, t]) - 36 F^2 t (1 + \cos[z]^2) \sin[z] YZ3^{(1,1)}[z, t] + \\
& 18 F t^2 (1 + \cos[z]^2) \sin[z] YZ3^{(1,2)}[z, t] - 36 F^2 (-1 + \cos[z]) \\
& \cos[z] (1 + \cos[z]) YZ3^{(2,0)}[z, t] + F (-36 F t \cos[z] + 36 F t \cos[z]^3) \\
& YZ3^{(2,1)}[z, t] + (18 F t^2 \cos[z] - 18 F t^2 \cos[z]^3) YZ3^{(2,2)}[z, t] \Big) dz + \\
& F t^2 \left(-\frac{1}{36} (F + M^2 + F t^2) \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 (\cos[z]^3 \right. \\
& ((-9 - F + M^2 + F t^2) YZ3[z, t] - 2 F t YZ3^{(0,1)}[z, t]) + \\
& \cos[z]^3 YZ3^{(0,2)}[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ3^{(1,0)}[z, t] + \\
& (-36 \cos[z] + 36 \cos[z]^3) YZ3^{(2,0)}[z, t] \Big) dz + \frac{1}{18} \cos[z]^5 \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-F t (-9 - F + M^2 + F t^2) \cos[z]^3 YZ3^{(0,1)}[z, t] + 2 \left(\frac{F}{2} + F t^2 \right) \right. \\
& \cos[z]^3 YZ3^{(0,2)}[z, t] - F t \cos[z]^3 YZ3^{(0,3)}[z, t] + F ((-9 - F + M^2 - F t^2) \\
& \cos[z]^3 YZ3[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ3^{(1,0)}[z, t]) + \\
& 36 F t (1 + \cos[z]^2) \sin[z] YZ3^{(1,1)}[z, t] - F (36 \cos[z] - 36 \cos[z]^3) \\
& YZ3^{(2,0)}[z, t] - (-36 F t \cos[z] + 36 F t \cos[z]^3) YZ3^{(2,1)}[z, t] \Big) dz - \\
& 198 \cos[z]^2 \sin[z]^{7/2} YZ3^{(1,0)}[z, t] - 108 \sin[z]^{11/2} YZ3^{(1,0)}[z, t] + \\
& \cos[z]^4 \sin[z]^{3/2} ((-117 - F + M^2 - 3 F t^2) YZ3^{(1,0)}[z, t] - 2 F t YZ3^{(1,1)}[z, t]) + \\
& \cos[z]^4 \sin[z]^{3/2} YZ3^{(1,2)}[z, t] + \frac{1}{2} \cos[z]
\end{aligned}$$

$$\begin{aligned}
& \left(\cos[z]^4 \sqrt{\sin[z]} \left((-9 - F + M^2 - 3 F t^2) YZ3[z, t] - 2 F t YZ3^{(0,1)}[z, t] + \right. \right. \\
& \quad \left. \left. YZ3^{(0,2)}[z, t] \right) - 324 \cos[z]^2 \sin[z]^{5/2} YZ3^{(2,0)}[z, t] - \right. \\
& \quad \left. 216 \sin[z]^{9/2} YZ3^{(2,0)}[z, t] \right) - 36 \cos[z]^2 \sin[z]^{7/2} YZ3^{(3,0)}[z, t] - \\
& 3 e^{-\frac{F t^2}{2}} t \left(\int e^{\frac{F t^2}{2}} t^3 \cos[M t] \left(\frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{1}{2} (2 F^2 - F^2 t^2 + \right. \right. \right. \\
& \quad \left. \left. F t^2 (-9 + M^2 + F t^2)) \cos[z]^3 YZ3^{(0,2)}[z, t] + \right. \right. \\
& \quad \left. \left. F t (F + F t^2) \cos[z]^3 YZ3^{(0,3)}[z, t] - \frac{1}{2} F t^2 \cos[z]^3 YZ3^{(0,4)}[z, t] + \right. \right. \\
& \quad \left. \left. F (F t (-9 - F + M^2 - F t^2) \cos[z]^3 YZ3^{(0,1)}[z, t] - F ((-9 - F + M^2) \cos[z]^3 \right. \right. \\
& \quad \left. \left. YZ3[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ3^{(1,0)}[z, t]) \right) - \right. \\
& \quad 36 F^2 t (1 + \cos[z]^2) \sin[z] YZ3^{(1,1)}[z, t] + 18 F t^2 (1 + \cos[z]^2) \\
& \quad \sin[z] YZ3^{(1,2)}[z, t] - 36 F^2 (-1 + \cos[z]) \cos[z] (1 + \cos[z]) \\
& \quad YZ3^{(2,0)}[z, t] + F (-36 F t \cos[z] + 36 F t \cos[z]^3) YZ3^{(2,1)}[z, t] + \\
& \quad \left. (18 F t^2 \cos[z] - 18 F t^2 \cos[z]^3) YZ3^{(2,2)}[z, t] \right) dz + \\
& F t^2 \left(-\frac{1}{36} (F + M^2 + F t^2) \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 (\cos[z]^3 \right. \\
& \quad \left. ((-9 - F + M^2 + F t^2) YZ3[z, t] - 2 F t YZ3^{(0,1)}[z, t]) + \cos[z]^3 \right. \\
& \quad \left. YZ3^{(0,2)}[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ3^{(1,0)}[z, t] + \right. \\
& \quad \left. (-36 \cos[z] + 36 \cos[z]^3) YZ3^{(2,0)}[z, t] \right) dz + \frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \\
& \quad \sec[z]^2 \left(-F t (-9 - F + M^2 + F t^2) \cos[z]^3 YZ3^{(0,1)}[z, t] + 2 \left(\frac{F}{2} + F t^2 \right) \cos[z]^3 \right. \\
& \quad \left. YZ3^{(0,2)}[z, t] - F t \cos[z]^3 YZ3^{(0,3)}[z, t] + F ((-9 - F + M^2 - F t^2) \cos[z]^3 \right. \\
& \quad \left. YZ3[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ3^{(1,0)}[z, t]) + \right. \\
& \quad 36 F t (1 + \cos[z]^2) \sin[z] YZ3^{(1,1)}[z, t] - F (36 \cos[z] - 36 \cos[z]^3) \\
& \quad YZ3^{(2,0)}[z, t] - (-36 F t \cos[z] + 36 F t \cos[z]^3) YZ3^{(2,1)}[z, t] \right) dz - \\
& 198 \cos[z]^2 \sin[z]^{7/2} YZ3^{(1,0)}[z, t] - 108 \sin[z]^{11/2} YZ3^{(1,0)}[z, t] + \\
& \cos[z]^4 \sin[z]^{3/2} ((-117 - F + M^2 - 3 F t^2) YZ3^{(1,0)}[z, t] - 2 F t YZ3^{(1,1)}[z, t]) + \\
& \cos[z]^4 \sin[z]^{3/2} YZ3^{(1,2)}[z, t] + \frac{1}{2} \cos[z] (\cos[z]^4 \sqrt{\sin[z]} \\
& \quad ((-9 - F + M^2 - 3 F t^2) YZ3[z, t] - 2 F t YZ3^{(0,1)}[z, t] + YZ3^{(0,2)}[z, t]) - \\
& \quad 324 \cos[z]^2 \sin[z]^{5/2} YZ3^{(2,0)}[z, t] - 216 \sin[z]^{9/2} YZ3^{(2,0)}[z, t]) - \\
& \quad 36 \cos[z]^2 \sin[z]^{7/2} YZ3^{(3,0)}[z, t] \right) dt \right) \sin[M t] + \\
& M \cos[z]^5 \left(-\frac{1}{8} \int \frac{1}{(1 + \cos[2 z]) \sqrt{\sin[z]}} \left((-9 - F + M^2 + F t^2) \cos[z] YZ3[z, t] - \right. \right. \\
& \quad \left. \left. 2 F t \cos[z] YZ3^{(0,1)}[z, t] + \right. \right. \\
& \quad \left. \left. \cos[3 z] \left(\frac{1}{3} (-9 - F + M^2 + F t^2) YZ3[z, t] - \frac{2}{3} F t YZ3^{(0,1)}[z, t] \right) + \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\cos[z] + \frac{1}{3} \cos[3z] \right) YZ3^{(0,2)}[z, t] + (-60 \sin[z] - 12 \sin[3z]) YZ3^{(1,0)}[z, t] + \\
& (-12 \cos[z] + 12 \cos[3z]) YZ3^{(2,0)}[z, t] \Bigg) dz + \\
& e^{-\frac{Ft^2}{2}} F t (c11 \cos[Mt] + c12 \sin[Mt]) \Bigg) \Bigg\}, YZ3^{(0,4)}[z, t] = \frac{1}{F} \\
& t^2 \left(- \left((-2F^3 - 2F^2 t^2 (-9 + M^2 - Ft^2)) + F^2 (-18 + 2M^2 + Ft^2) \right. \right. + \\
& F t^2 ((-3 + M)^2 + Ft^2) ((3 + M)^2 + Ft^2) \Big) \\
& YZ3[z, t] \Big) + 2F^2 t (-9 - F + M^2 + \\
& F \\
& t^2) YZ3^{(0,1)}[z, t] + (-2 \\
& F^2 + 2 \\
& F^2 \\
& t^2 - 2 \\
& F \\
& t^2 \\
& (-9 + M^2 - Ft^2)) \\
& YZ3^{(0,2)}[z, t] + 2F^2 t YZ3^{(0,3)}[\\
& z, \\
& t] + 72 \\
& \tan[z] \\
& (F^2 (2 + \tan[z]^2) - \\
& F^2 \\
& t^2 \\
& (2 + \tan[z]^2) + F \\
& t^2 \\
& (-90 + 2M^2 + (-405 + M^2) \tan[z]^2 - 594 \tan[z]^4 - \\
& 270 \tan[z]^6 + Ft^2 (-2 - \tan[z]^2))) \\
& YZ3^{(1,0)}[z, t] - 72 F^2 t \tan[z] (2 + \tan[z]^2) \\
& YZ3^{(1,1)}[\\
& z, \\
& t] + 72 \\
& F \\
& t^2 \\
& \tan[z] \\
& (2 + \tan[z]^2) YZ3^{(1,2)}[z, t] + \\
& 72 \\
& \tan[z]^2 \\
& (F^2 - F^2 t^2 + Ft^2 (-261 + M^2 - Ft^2 - 504 \tan[z]^2 - 270 \tan[z]^4)) \\
& YZ3^{(2,0)}[\\
& z,
\end{aligned}$$

```

t] - 72
F^2
t
Tan[
z]^2
YZ3^(2,1) [z, t] + 72 F t^2 Tan[z]^2 YZ3^(2,2) [
z,
t] - 7776
F
t^2
Tan[
z]^3
 $\left(\frac{4}{3} + \tan[z]^2\right) YZ3^{(3,0)} [z, t] - 1296$ 
F
t^2
Tan[
z]^4 YZ3^(4,0) [z, t] } }

```

In[1188]:=

```

time1 = Now
If[FindFile["solvedEQ2.mx"] === $Failed,
  AbsoluteTiming[solvedEQ2 = ConvertMapleToMathematicaV2[maplestringEQ2]];
  DumpSave[ToString[header <> "solvedEQ2.mx"], solvedEQ2];
  Print[ToString[header <> "solvedEQ2.mx"]];
  , Get["solvedEQ2.mx"]]
Now - time1

```

Out[1188]=

Fri 13 Feb 2026 03:58:51 GMT-8

2026-02-13-Pair-Crtn-Univ-same_E-L-eqs-alt-approach-mmM4p-solvedEQ2.mx

Out[1190]=

12.39105020 min

In[1191]:=

solvedEQ2

Out[1191]=

$$\begin{aligned}
YZ4[z, t] &= \frac{1}{2 F M t} (F YZ7[z, t] + (9 - M^2 + F t^2) YZ7[z, t] - \\
&\quad YZ7^{(0,2)} [z, t] + (72 \tan[z] + 36 \tan[z]^3) YZ7^{(1,0)} [z, t] + 36 \tan[z]^2 YZ7^{(2,0)} [z, t]), \\
YZ5[z, t] &= \frac{1}{F M t^2 \sqrt{\sin[z]}} \sec[z]^5 \left(-3 e^{-\frac{F t^2}{2}} t^2 \cos[M t] \right. \\
&\quad \left. \int e^{\frac{F t^2}{2}} t^3 \cos[M t] \left(\frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{1}{2} (2 F^2 - F^2 t^2 + F t^2 (-9 + M^2 + F t^2)) \right. \right. \right. \\
&\quad \left. \left. \left. \cos[z]^3 YZ7^{(0,2)} [z, t] + F t (F + F t^2) \cos[z]^3 YZ7^{(0,3)} [z, t] - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} F t^2 \cos[z]^3 YZ7^{(0,4)}[z, t] + F (F t (-9 - F + M^2 - F t^2) \cos[z]^3 YZ7^{(0,1)}[z, t] - \\
& F ((-9 - F + M^2) \cos[z]^3 YZ7[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) \\
& YZ7^{(1,0)}[z, t])) - 36 F^2 t (1 + \cos[z]^2) \sin[z] YZ7^{(1,1)}[z, t] + \\
& 18 F t^2 (1 + \cos[z]^2) \sin[z] YZ7^{(1,2)}[z, t] - 36 F^2 (-1 + \cos[z]) \\
& \cos[z] (1 + \cos[z]) YZ7^{(2,0)}[z, t] + F (-36 F t \cos[z] + 36 F t \cos[z]^3) \\
& YZ7^{(2,1)}[z, t] + (18 F t^2 \cos[z] - 18 F t^2 \cos[z]^3) YZ7^{(2,2)}[z, t] \Big) dz + \\
& F t^2 \left(-\frac{1}{36} (F + M^2 + F t^2) \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 (\cos[z]^3 \right. \\
& ((-9 - F + M^2 + F t^2) YZ7[z, t] - 2 F t YZ7^{(0,1)}[z, t]) + \\
& \cos[z]^3 YZ7^{(0,2)}[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ7^{(1,0)}[z, t] + \\
& (-36 \cos[z] + 36 \cos[z]^3) YZ7^{(2,0)}[z, t] \Big) dz - \frac{1}{18} \cos[z]^5 \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(F t (-9 - F + M^2 + F t^2) \cos[z]^3 YZ7^{(0,1)}[z, t] - 2 \left(\frac{F}{2} + F t^2 \right) \right. \\
& \cos[z]^3 YZ7^{(0,2)}[z, t] + F t \cos[z]^3 YZ7^{(0,3)}[z, t] - F ((-9 - F + M^2 - F t^2) \\
& \cos[z]^3 YZ7[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ7^{(1,0)}[z, t]) - \\
& 36 F t (1 + \cos[z]^2) \sin[z] YZ7^{(1,1)}[z, t] + F (36 \cos[z] - 36 \cos[z]^3) \\
& YZ7^{(2,0)}[z, t] + (-36 F t \cos[z] + 36 F t \cos[z]^3) YZ7^{(2,1)}[z, t] \Big) dz - \\
& 198 \cos[z]^2 \sin[z]^{7/2} YZ7^{(1,0)}[z, t] - 108 \sin[z]^{11/2} YZ7^{(1,0)}[z, t] + \\
& \cos[z]^4 \sin[z]^{3/2} ((-117 - F + M^2 - 3 F t^2) YZ7^{(1,0)}[z, t] - 2 F t YZ7^{(1,1)}[z, t]) + \\
& \cos[z]^4 \sin[z]^{3/2} YZ7^{(1,2)}[z, t] + \frac{1}{2} \cos[z] (\cos[z]^4 \sqrt{\sin[z]} \\
& ((-9 - F + M^2 - 3 F t^2) YZ7[z, t] - 2 F t YZ7^{(0,1)}[z, t] + YZ7^{(0,2)}[z, t]) - \\
& 324 \cos[z]^2 \sin[z]^{5/2} YZ7^{(2,0)}[z, t] - 216 \sin[z]^{9/2} YZ7^{(2,0)}[z, t] - 36 \\
& \cos[z]^2 \sin[z]^{7/2} YZ7^{(3,0)}[z, t] \Big) dt - 3 e^{-\frac{F t^2}{2}} t^2 \\
& \left(\int e^{\frac{F t^2}{2}} t^3 \sin[M t] \left(\frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{1}{2} (2 F^2 - F^2 t^2 + F t^2 (-9 + M^2 + F t^2)) \right. \right. \right. \\
& \cos[z]^3 YZ7^{(0,2)}[z, t] + F t (F + F t^2) \cos[z]^3 YZ7^{(0,3)}[z, t] - \frac{1}{2} F t^2 \\
& \cos[z]^3 YZ7^{(0,4)}[z, t] + F (F t (-9 - F + M^2 - F t^2) \cos[z]^3 YZ7^{(0,1)}[z, t] - \\
& F ((-9 - F + M^2) \cos[z]^3 YZ7[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) \\
& YZ7^{(1,0)}[z, t])) - 36 F^2 t (1 + \cos[z]^2) \sin[z] YZ7^{(1,1)}[z, t] + \\
& 18 F t^2 (1 + \cos[z]^2) \sin[z] YZ7^{(1,2)}[z, t] - 36 F^2 (-1 + \cos[z]) \\
& \cos[z] (1 + \cos[z]) YZ7^{(2,0)}[z, t] + F (-36 F t \cos[z] + 36 F t \cos[z]^3) \\
& YZ7^{(2,1)}[z, t] + (18 F t^2 \cos[z] - 18 F t^2 \cos[z]^3) YZ7^{(2,2)}[z, t] \Big) dz + \\
& F t^2 \left(-\frac{1}{36} (F + M^2 + F t^2) \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 (\cos[z]^3 \right. \\
& ((-9 - F + M^2 + F t^2) YZ7[z, t] - 2 F t YZ7^{(0,1)}[z, t]) + \cos[z]^3
\end{aligned}$$

$$\begin{aligned}
& YZ7^{(0,2)}[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ7^{(1,0)}[z, t] + \\
& (-36 \cos[z] + 36 \cos[z]^3) YZ7^{(2,0)}[z, t] \) dz - \frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \\
& \sec[z]^2 \left(F t (-9 - F + M^2 + F t^2) \cos[z]^3 YZ7^{(0,1)}[z, t] - 2 \left(\frac{F}{2} + F t^2 \right) \cos[z]^3 \right. \\
& YZ7^{(0,2)}[z, t] + F t \cos[z]^3 YZ7^{(0,3)}[z, t] - F ((-9 - F + M^2 - F t^2) \cos[z]^3 \\
& YZ7[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ7^{(1,0)}[z, t]) - \\
& 36 F t (1 + \cos[z]^2) \sin[z] YZ7^{(1,1)}[z, t] + F (36 \cos[z] - 36 \cos[z]^3) \\
& YZ7^{(2,0)}[z, t] + (-36 F t \cos[z] + 36 F t \cos[z]^3) YZ7^{(2,1)}[z, t] \) dz - \\
& 198 \cos[z]^2 \sin[z]^{7/2} YZ7^{(1,0)}[z, t] - 108 \sin[z]^{11/2} YZ7^{(1,0)}[z, t] + \\
& \cos[z]^4 \sin[z]^{3/2} ((-117 - F + M^2 - 3 F t^2) YZ7^{(1,0)}[z, t] - 2 F t YZ7^{(1,1)}[z, t]) + \\
& \cos[z]^4 \sin[z]^{3/2} YZ7^{(1,2)}[z, t] + \frac{1}{2} \cos[z] (\cos[z]^4 \sqrt{\sin[z]} \\
& ((-9 - F + M^2 - 3 F t^2) YZ7[z, t] - 2 F t YZ7^{(0,1)}[z, t] + YZ7^{(0,2)}[z, t]) - \\
& 324 \cos[z]^2 \sin[z]^{5/2} YZ7^{(2,0)}[z, t] - 216 \sin[z]^{9/2} YZ7^{(2,0)}[z, t]) - \\
& 36 \cos[z]^2 \sin[z]^{7/2} YZ7^{(3,0)}[z, t] \) \) dt \) \sin[M t] - \\
& \cos[z]^4 \left(-\frac{1}{8} \cos[z] \int \frac{1}{(1 + \cos[2 z]) \sqrt{\sin[z]}} \left(-F t (-9 - F + M^2 + F t^2) \cos[z] \right. \right. \\
& YZ7^{(0,1)}[z, t] + \cos[3 z] \left(\frac{1}{3} F (-9 - F + M^2 - F t^2) YZ7[z, t] - \right. \\
& \left. \left. \frac{1}{3} F t (-9 - F + M^2 + F t^2) YZ7^{(0,1)}[z, t] \right) + (F + 2 F t^2) \left(\cos[z] + \frac{1}{3} \cos[3 z] \right) \right. \\
& YZ7^{(0,2)}[z, t] + F t \left(-\cos[z] - \frac{1}{3} \cos[3 z] \right) YZ7^{(0,3)}[z, t] + \\
& F ((-9 - F + M^2 - F t^2) \cos[z] YZ7[z, t] + (-60 \sin[z] - 12 \sin[3 z]) YZ7^{(1,0)}[z, t]) + \\
& 12 F t (5 \sin[z] + \sin[3 z]) YZ7^{(1,1)}[z, t] - 12 F (\cos[z] - \cos[3 z]) YZ7^{(2,0)}[z, t] + \\
& 12 F t (\cos[z] - \cos[3 z]) YZ7^{(2,1)}[z, t] \) dz + \\
& F t^2 \left(\frac{1}{8} \cos[z] \int \frac{1}{(1 + \cos[2 z]) \sqrt{\sin[z]}} \left((-9 - F + M^2 + F t^2) \cos[z] \right. \right. \\
& YZ7[z, t] - 2 F t \cos[z] YZ7^{(0,1)}[z, t] + \\
& \cos[3 z] \left(\frac{1}{3} (-9 - F + M^2 + F t^2) YZ7[z, t] - \frac{2}{3} F t YZ7^{(0,1)}[z, t] \right) + \\
& \left. \left. \left(\cos[z] + \frac{1}{3} \cos[3 z] \right) YZ7^{(0,2)}[z, t] + (-60 \sin[z] - 12 \sin[3 z]) \right. \right. \\
& YZ7^{(1,0)}[z, t] + (-12 \cos[z] + 12 \cos[3 z]) YZ7^{(2,0)}[z, t] \) dz + \\
& e^{-\frac{F t^2}{2}} M \cos[z] (c22 \cos[M t] - c21 \sin[M t]) + 3 \cos[z] \sqrt{\sin[z]} YZ7[z, t] + \\
& 6 \sin[z]^{3/2} YZ7^{(1,0)}[z, t] \) \) \),
\end{aligned}$$

$$\begin{aligned}
YZ6[z, t] = & \frac{1}{F M t \sqrt{\sin[z]}} \sec[z]^5 \left(-3 e^{-\frac{F t^2}{2}} \right. \\
& t \cos[M t] \\
& \left. \int e^{\frac{F t^2}{2}} t^3 \sin[M t] \left(\frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{1}{2} (2 F^2 - F^2 t^2 + F t^2 (-9 + M^2 + F t^2)) \right. \right. \right. \\
& \cos[z]^3 YZ7^{(0,2)}[z, t] + F t (F + F t^2) \cos[z]^3 YZ7^{(0,3)}[z, t] - \\
& \frac{1}{2} F t^2 \cos[z]^3 YZ7^{(0,4)}[z, t] + F (F t (-9 - F + M^2 - F t^2) \cos[z]^3 YZ7^{(0,1)}[z, t] - \\
& F ((-9 - F + M^2) \cos[z]^3 YZ7[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) \\
& \quad YZ7^{(1,0)}[z, t])) - 36 F^2 t (1 + \cos[z]^2) \sin[z] YZ7^{(1,1)}[z, t] + \\
& 18 F t^2 (1 + \cos[z]^2) \sin[z] YZ7^{(1,2)}[z, t] - 36 F^2 (-1 + \cos[z]) \\
& \cos[z] (1 + \cos[z]) YZ7^{(2,0)}[z, t] + F (-36 F t \cos[z] + 36 F t \cos[z]^3) \\
& \quad YZ7^{(2,1)}[z, t] + (18 F t^2 \cos[z] - 18 F t^2 \cos[z]^3) YZ7^{(2,2)}[z, t] \Big) dz + \\
& F t^2 \left(-\frac{1}{36} (F + M^2 + F t^2) \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 (\cos[z]^3 \right. \\
& \quad ((-9 - F + M^2 + F t^2) YZ7[z, t] - 2 F t YZ7^{(0,1)}[z, t]) + \\
& \cos[z]^3 YZ7^{(0,2)}[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ7^{(1,0)}[z, t] + \\
& \quad (-36 \cos[z] + 36 \cos[z]^3) YZ7^{(2,0)}[z, t] \Big) dz - \frac{1}{18} \cos[z]^5 \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(F t (-9 - F + M^2 + F t^2) \cos[z]^3 YZ7^{(0,1)}[z, t] - 2 \left(\frac{F}{2} + F t^2 \right) \right. \\
& \quad \cos[z]^3 YZ7^{(0,2)}[z, t] + F t \cos[z]^3 YZ7^{(0,3)}[z, t] - F ((-9 - F + M^2 - F t^2) \\
& \quad \cos[z]^3 YZ7[z, t] + (-36 \sin[z] - 36 \cos[z]^2 \sin[z]) YZ7^{(1,0)}[z, t]) - \\
& 36 F t (1 + \cos[z]^2) \sin[z] YZ7^{(1,1)}[z, t] + F (36 \cos[z] - 36 \cos[z]^3) \\
& \quad YZ7^{(2,0)}[z, t] + (-36 F t \cos[z] + 36 F t \cos[z]^3) YZ7^{(2,1)}[z, t] \Big) dz - \\
& 198 \cos[z]^2 \sin[z]^{7/2} YZ7^{(1,0)}[z, t] - 108 \sin[z]^{11/2} YZ7^{(1,0)}[z, t] + \\
& \cos[z]^4 \sin[z]^{3/2} ((-117 - F + M^2 - 3 F t^2) YZ7^{(1,0)}[z, t] - 2 F t YZ7^{(1,1)}[z, t]) + \\
& \cos[z]^4 \sin[z]^{3/2} YZ7^{(1,2)}[z, t] + \frac{1}{2} \cos[z] \\
& (\cos[z]^4 \sqrt{\sin[z]} ((-9 - F + M^2 - 3 F t^2) YZ7[z, t] - 2 F t YZ7^{(0,1)}[z, t] + \\
& \quad YZ7^{(0,2)}[z, t]) - 324 \cos[z]^2 \sin[z]^{5/2} YZ7^{(2,0)}[z, t] - \\
& \quad 216 \sin[z]^{9/2} YZ7^{(2,0)}[z, t]) - 36 \cos[z]^2 \sin[z]^{7/2} YZ7^{(3,0)}[z, t] \Big) dt + \\
& 3 e^{-\frac{F t^2}{2}} t \left(\int e^{\frac{F t^2}{2}} t^3 \cos[M t] \left(\frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(-\frac{1}{2} (2 F^2 - F^2 t^2 + \right. \right. \right. \\
& \quad F t^2 (-9 + M^2 + F t^2)) \cos[z]^3 YZ7^{(0,2)}[z, t] +
\end{aligned}$$

$$\begin{aligned}
& F t \left(F + F t^2 \right) \cos[z]^3 YZ7^{(0,3)}[z, t] - \frac{1}{2} F t^2 \cos[z]^3 YZ7^{(0,4)}[z, t] + \\
& F \left(F t \left(-9 - F + M^2 - F t^2 \right) \cos[z]^3 YZ7^{(0,1)}[z, t] - F \left(\left(-9 - F + M^2 \right) \cos[z]^3 \right. \right. \\
& \left. \left. YZ7[z, t] + \left(-36 \sin[z] - 36 \cos[z]^2 \sin[z] \right) YZ7^{(1,0)}[z, t] \right) \right) - \\
& 36 F^2 t \left(1 + \cos[z]^2 \right) \sin[z] YZ7^{(1,1)}[z, t] + 18 F t^2 \left(1 + \cos[z]^2 \right) \\
& \sin[z] YZ7^{(1,2)}[z, t] - 36 F^2 \left(-1 + \cos[z] \right) \cos[z] \left(1 + \cos[z] \right) \\
& YZ7^{(2,0)}[z, t] + F \left(-36 F t \cos[z] + 36 F t \cos[z]^3 \right) YZ7^{(2,1)}[z, t] + \\
& \left. \left(18 F t^2 \cos[z] - 18 F t^2 \cos[z]^3 \right) YZ7^{(2,2)}[z, t] \right) dz + \\
& F t^2 \left(-\frac{1}{36} \left(F + M^2 + F t^2 \right) \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left(\cos[z]^3 \right. \right. \\
& \left. \left. \left(-9 - F + M^2 + F t^2 \right) YZ7[z, t] - 2 F t YZ7^{(0,1)}[z, t] \right) + \cos[z]^3 \right. \\
& \left. YZ7^{(0,2)}[z, t] + \left(-36 \sin[z] - 36 \cos[z]^2 \sin[z] \right) YZ7^{(1,0)}[z, t] + \right. \\
& \left. \left(-36 \cos[z] + 36 \cos[z]^3 \right) YZ7^{(2,0)}[z, t] \right) dz - \frac{1}{18} \cos[z]^5 \int \frac{1}{\sqrt{\sin[z]}} \\
& \sec[z]^2 \left(F t \left(-9 - F + M^2 + F t^2 \right) \cos[z]^3 YZ7^{(0,1)}[z, t] - 2 \left(\frac{F}{2} + F t^2 \right) \cos[z]^3 \right. \\
& \left. YZ7^{(0,2)}[z, t] + F t \cos[z]^3 YZ7^{(0,3)}[z, t] - F \left(\left(-9 - F + M^2 - F t^2 \right) \cos[z]^3 \right. \right. \\
& \left. \left. YZ7[z, t] + \left(-36 \sin[z] - 36 \cos[z]^2 \sin[z] \right) YZ7^{(1,0)}[z, t] \right) - \right. \\
& \left. 36 F t \left(1 + \cos[z]^2 \right) \sin[z] YZ7^{(1,1)}[z, t] + F \left(36 \cos[z] - 36 \cos[z]^3 \right) \right. \\
& \left. YZ7^{(2,0)}[z, t] + \left(-36 F t \cos[z] + 36 F t \cos[z]^3 \right) YZ7^{(2,1)}[z, t] \right) dz - \\
& 198 \cos[z]^2 \sin[z]^{7/2} YZ7^{(1,0)}[z, t] - 108 \sin[z]^{11/2} YZ7^{(1,0)}[z, t] + \\
& \cos[z]^4 \sin[z]^{3/2} \left(\left(-117 - F + M^2 - 3 F t^2 \right) YZ7^{(1,0)}[z, t] - 2 F t YZ7^{(1,1)}[z, t] \right) + \\
& \cos[z]^4 \sin[z]^{3/2} YZ7^{(1,2)}[z, t] + \frac{1}{2} \cos[z] \left(\cos[z]^4 \sqrt{\sin[z]} \right. \\
& \left. \left(\left(-9 - F + M^2 - 3 F t^2 \right) YZ7[z, t] - 2 F t YZ7^{(0,1)}[z, t] + YZ7^{(0,2)}[z, t] \right) - \right. \\
& \left. 324 \cos[z]^2 \sin[z]^{5/2} YZ7^{(2,0)}[z, t] - 216 \sin[z]^{9/2} YZ7^{(2,0)}[z, t] \right) - \\
& \left. \left. 36 \cos[z]^2 \sin[z]^{7/2} YZ7^{(3,0)}[z, t] \right) \right) dt \right) \sin[M t] + \\
& M \cos[z]^5 \left(\frac{1}{8} \int \frac{1}{(1 + \cos[2 z]) \sqrt{\sin[z]}} \left(\left(-9 - F + M^2 + F t^2 \right) \cos[z] YZ7[z, t] - \right. \right. \\
& \left. \left. 2 F t \cos[z] YZ7^{(0,1)}[z, t] + \right. \right. \\
& \cos[3 z] \left(\frac{1}{3} \left(-9 - F + M^2 + F t^2 \right) YZ7[z, t] - \frac{2}{3} F t YZ7^{(0,1)}[z, t] \right) + \\
& \left. \left. \left(\cos[z] + \frac{1}{3} \cos[3 z] \right) YZ7^{(0,2)}[z, t] + \left(-60 \sin[z] - 12 \sin[3 z] \right) YZ7^{(1,0)}[z, t] + \right. \right. \\
& \left. \left. \left(-12 \cos[z] + 12 \cos[3 z] \right) YZ7^{(2,0)}[z, t] \right) dz + \right. \\
& \left. \left. e^{-\frac{F t^2}{2}} F t \left(c21 \cos[M t] + c22 \sin[M t] \right) \right) \right\}, YZ7^{(0,4)}[z, t] = \frac{1}{F}
\end{aligned}$$

$$t^2 \left(- \left(\left(-2 F^3 - 2 F^2 t^2 \left(-9 + M^2 - F t^2 \right) + F^2 \left(-18 + 2 M^2 + F t^2 \right) + \right. \right. \right. \right.$$

$$\begin{aligned}
& F t^2 ((-3 + M)^2 + F t^2) ((3 + M)^2 + F t^2) \\
& YZ7[z, t] + 2 F^2 t (-9 - F + M^2 + \\
& F t^2) YZ7^{(0,1)}[z, t] + (-2 F^2 + 2 F^2 \\
& t^2 - 2 F t^2 \\
& (-9 + M^2 - F t^2)) \\
& YZ7^{(0,2)}[z, t] + 2 F^2 t YZ7^{(0,3)}[\\
& z, t] + 72 \\
& \text{Tan}[\\
& z] \\
& (F^2 (2 + \text{Tan}[z]^2) - \\
& F^2 t^2 \\
& (2 + \text{Tan}[z]^2) + F t^2 \\
& (-90 + 2 M^2 + (-405 + M^2) \text{Tan}[z]^2 - 594 \text{Tan}[z]^4 - \\
& 270 \text{Tan}[z]^6 + F t^2 (-2 - \text{Tan}[z]^2))) \\
& YZ7^{(1,0)}[z, t] - 72 F^2 t \text{Tan}[z] (2 + \text{Tan}[z]^2) \\
& YZ7^{(1,1)}[\\
& z, t] + 72 \\
& F t^2 \\
& \text{Tan}[\\
& z] \\
& (2 + \text{Tan}[z]^2) YZ7^{(1,2)}[z, t] + \\
& 72 \\
& \text{Tan}[\\
& z]^2 \\
& (F^2 - F^2 t^2 + F t^2 (-261 + M^2 - F t^2 - 504 \text{Tan}[z]^2 - 270 \text{Tan}[z]^4)) \\
& YZ7^{(2,0)}[\\
& z, t] - 72 \\
& F^2 t \\
& \text{Tan}[\\
& z]^2 \\
& YZ7^{(2,1)}[z, t] + 72 F t^2 \text{Tan}[z]^2 YZ7^{(2,2)}[\\
& z, t] - 7776 \\
& F
\end{aligned}$$

$$\begin{aligned} & t^2 \\ & \operatorname{Tan}[z]^3 \\ & \left(\frac{4}{3} + \operatorname{Tan}[z]^2\right) Y Z \mathcal{T}^{(3,0)}[z, t] - 1296 \\ & F \\ & t^2 \\ & \operatorname{Tan}[z]^4 Y Z \mathcal{T}^{(4,0)}[z, t] \Bigg) \Bigg\} \end{aligned}$$