Scratch work of Author

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Do not read if you are irascible.
git clone https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git
"git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git"

Ever wonder why there are 3 space dimensions and 1 time dimension?
Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for X, Y ∊ Nature?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\|=\|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over $\mathbb{R}$) real numbers $\mathbb{R}$,the 2-dim complex numbers,the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in "A Remarkable Representation of the 3+2 de Sitter Group" by P.A.M.Dirac, J.Math.Phys.4,901–909 (1963).

Here, Dirac's gamma matrices have been extended into this paper's real tau8 8×8 and tau16 = τ 16×16 matrices, in order to simply incorporate and couple the WAVE FUNCTION OF the UNIVERSE to Gravity via a term resembling

ψ^† (mass matrix for universe) ψ :
mass matrix = (τ0 τ1 τ2 τ3)(τ5 τ6 τ7),
[τ5 τ6 τ7 breaks "x5, x6, x7" Lorentz invariance].

Here we use spacetime coordinates that are selected so that we are somewhat consistent with the
xact libraries,
(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):
Cartesian coordinates:
x0 = hidden space (a small circle); x1, x2, x3 are the usual 3-space coords;
x4 = time coord,
and x5, x6, x7= superluminal deflating time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{\mathrm{SO}}(4, 4; \mathbb{R})$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M8 , whose tangent bundle has
$\overline{\mathrm{SO}}(4, 4; \mathbb{R}) \approx \mathrm{Spin}(4,4;\mathbb{R})$
as iso group, and in which
3 of the 4 space dimensions superluminally INFLATE,
3 of the 4 time dimensions superluminally DEFLATE (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),
and the 4th space dim, x0, curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

In passing we remark that an analogous construction may be defined on an octonion space of one time dimension and seven space dimensions, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group
$SU(3) \otimes SU(2)_L \otimes U(1)_Y$;
The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

The Einstein - Lovelock vacuum field equations are
$A_a^b = 0$, where

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has l
completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta^{hh_1 \cdots h_{2k}}_{jj_1 \cdots j_{2k}} R^{j_1 j_2}{}_{h_1 h_2} \cdots R^{j_{2k-1} j_{2k}}{}_{h_{2k-1} h_{2k}} + \lambda \sqrt{g}\, g^{lh}, \quad ($$

where $\alpha_{(k)}$, λ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

and $m - 1 = \frac{8}{2} - 1 = 3$.

Citation : Tensors, Differential Forms,
and Variational Principles (Dover Books on Mathematics),
by David Lovelock and Hanno Rund

Let {w1, w2, w3, Λ} be pure numbers;
then by Eq. [4.38] the vacuum equations are
$0 == -\Lambda + H^{-2}\ w1\ \mathrm{Lovelock1} + H^{-4}\ w2\ \mathrm{Lovelock2} + H^{-6}\ w3\ \mathrm{Lovelock3}$
in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein - Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre - Hubble parameter. For now, let's pretend that they are independent.

- **My calculation employs “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see**
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  pitrou@iap.fr
  http://www2.iap.fr/users/pitrou/
  “FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb”

MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify

Begin

```
In[∘]:= Print["CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]

        CopyRight (C) 2022, Patrick L. Nash, under the General Public License.

In[∘]:= Unprotect[dir];

In[∘]:= dir = FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]];
        Protect[dir];
        If[$Failed == SetDirectory[dir], {CreateDirectory[dir], SetDirectory[dir]}];

In[∘]:= names = FileNameSplit[NotebookFileName[]];

In[∘]:= name = StringReplace[names[[-1]], "nb" → "mx"];

In[∘]:= header = StringReplace[names[[-1]], ".nb" → "-"]

Out[∘]=
        2025-08-26-Einstein-Lovelock-4+4-Nash-
```

indices

```
In[∘]:= DIM8 = 8;

In[∘]:= Protect[DIM8];

In[∘]:= indices = {a, b, c, d, f, i, j, k, l, m, n, p, q, r, s, y}
        Length[%]

Out[∘]=
        {a, b, c, d, f, i, j, k, l, m, n, p, q, r, s, y}

Out[∘]=
        16
```

Initialization

```
In[∘]:= Print["CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]

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In[∘]:= << xAct`xCoba`
```

```
------------------------------------------------------------
Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}
```


```
Connecting to external mac executable...
Connection established.
------------------------------------------------------------
Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}
```


```
------------------------------------------------------------
Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}
```


In[ ]:=
```
<< xAct`xTras`
```

```
------------------------------------------------------------
Package xAct`xPert`  version 1.0.6, {2018, 2, 28}
CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia
  and Guillermo A. Mena Marugan, under the General Public License.
** Variable $PrePrint assigned value ScreenDollarIndices
** Variable $CovDFormat changed from Prefix to Postfix
** Option AllowUpperDerivatives of ContractMetric changed from False to True
** Option MetricOn of MakeRule changed from None to All
** Option ContractMetrics of MakeRule changed from False to True
------------------------------------------------------------
Package xAct`Invar`  version 2.0.5, {2013, 7, 1}
CopyRight (C) 2006-2020, J. M. Martin-Garcia,
  D. Yllanes and R. Portugal, under the General Public License.
** DefConstantSymbol: Defining constant symbol sigma.
** DefConstantSymbol: Defining constant symbol dim.
** Option CurvatureRelations of DefCovD changed from True to False
** Variable $CommuteCovDsOnScalars changed from True to False
------------------------------------------------------------
Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}
CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.
------------------------------------------------------------
Package xAct`xTras`  version 1.4.2, {2014, 10, 30}
CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.
** Variable $CovDFormat changed from Postfix to Prefix
** Option CurvatureRelations of DefCovD changed from False to True
------------------------------------------------------------
These packages come with ABSOLUTELY NO WARRANTY; for details type
  Disclaimer[]. This is free software, and you are welcome to redistribute
  it under certain conditions. See the General Public License for details.
------------------------------------------------------------
```

```
In[∘]:= $CVVerbose = False;
```

```
In[∘]:= << xAct`ShowTime1`
```

```
In[∘]:= $ShowTimeThreshold = 0.1;
```

We define a function to collect and canonicalize equal-order terms:

```
In[∘]:= org[expr_] := Collect[ContractMetric[expr], $PerturbationParameter, ToCanonical]
```

We change the options of ToCanonical to prevent indices inside the derivations from moving up and down.

```
In[•]:= SetOptions[ToCanonical, UseMetricOnVBundle → None, Verbose → False]
Out[•]=
{Verbose → False, UseMetricOnVBundle → None,
 Method → {ChangeCovD, ExpandChristoffel → False}, MathLink :→ $xpermQ, TimeVerbose → False}
```

We define a differentiable manifold of symbolic dimension dimM8 and introduce some differential forms on it.

```
In[•]:= DefConstantSymbol[dimM8, PrintAs → "DIM8"];
** DefConstantSymbol: Defining constant symbol dimM8.
In[•]:= dimM8 = 8;
In[•]:= DefManifold[M8, dimM8, indices]
** DefManifold: Defining manifold M8.
** DefVBundle: Defining vbundle TangentM8.
```

We change the options of ToCanonical to prevent indices inside the derivations from moving up and down.

```
In[•]:= SetOptions[ToCanonical, UseMetricOnVBundle → None]
Out[•]=
{Verbose → False, UseMetricOnVBundle → None,
 Method → {ChangeCovD, ExpandChristoffel → False}, MathLink :→ $xpermQ, TimeVerbose → False}
```

DEFCHART

```
In[•]:= X = {x0[], x1[], x2[], x3[], x4[], x5[], x6[], x7[]}
Out[•]=
{x0[], x1[], x2[], x3[], x4[], x5[], x6[], x7[]}
In[•]:= sX0 = And @@ Thread[X > 0]
Out[•]=
x0[] > 0 && x1[] > 0 && x2[] > 0 && x3[] > 0 && x4[] > 0 && x5[] > 0 && x6[] > 0 && x7[] > 0
In[•]:= ssX = H > 0 && sX0 && 6 H x0 > 0 && 2 la[x4] > 0 && Cot[6 H x0] > 0 && Sin[6 H x0] > 0
Out[•]=
H > 0 && x0[] > 0 && x1[] > 0 && x2[] > 0 && x3[] > 0 && x4[] > 0 && x5[] > 0 && x6[] > 0 &&
 x7[] > 0 && 6 H x0[] > 0 && 2 la[x4[]] > 0 && Cot[6 H x0[]] > 0 && Sin[6 H x0[]] > 0
In[•]:= DefChart[chartM8, M8, {0, 1, 2, 3, 4, 5, 6, 7}, X, ExtendedCoordinateDerivatives → True,
  FormatBasis → {"Partials", "Differentials"}, ChartColor → RGBColor[0, 0, 1]]
```

```
** DefChart: Defining chart chartM8.
** DefTensor: Defining coordinate scalar x0[].
** DefTensor: Defining coordinate scalar x1[].
** DefTensor: Defining coordinate scalar x2[].
** DefTensor: Defining coordinate scalar x3[].
** DefTensor: Defining coordinate scalar x4[].
** DefTensor: Defining coordinate scalar x5[].
** DefTensor: Defining coordinate scalar x6[].
** DefTensor: Defining coordinate scalar x7[].
** DefMapping: Defining mapping chartM8.
** DefMapping: Defining inverse mapping ichartM8.
** DefTensor: Defining mapping differential tensor dichartM8[-a, ichartM8a].
** DefTensor: Defining mapping differential tensor dchartM8[-a, chartM8a].
** DefBasis: Defining basis chartM8. Coordinated basis.
** DefCovD: Defining parallel derivative PDchartM8[-a].
** DefTensor: Defining vanishing torsion tensor TorsionPDchartM8[a, -b, -c].
** DefTensor: Defining symmetric Christoffel tensor ChristoffelPDchartM8[a, -b, -c].
** DefTensor: Defining vanishing Riemann tensor RiemannPDchartM8[-a, -b, -c, d].
** DefTensor: Defining vanishing Ricci tensor RicciPDchartM8[-a, -b].
** DefTensor: Defining antisymmetric +1 density etaUpchartM8[a, b, c, d, f, i, j, k].
** DefTensor: Defining antisymmetric -1 density
 etaDownchartM8[-a, -b, -c, -d, -f, -i, -j, -k].
```

MyArrayComponents

Let us wrap all these steps into one single function

```
In[∘]:= Clear[MyArrayComponents]
```

```
In[∘]:= MyArrayComponents[expr_] :=
         expr // ToBasis[chartM8] // ComponentArray // ToValues // ToValues // Simplify
```

```
In[∘]:= My4ArrayComponents[expr_] :=
         expr // ToBasis[chartM8] // ComponentArray // ToValues // ToValues // ToValues //
           ToValues // Simplify
```

DefMetric

```
In[∘]:= DefMetric[{4, 4, 0}, g44[-a, -b], CD, {";", "D"}, WeightedWithBasis → AIndex]
```

```
** DefTensor: Defining symmetric metric tensor g44[-a, -b].
** DefTensor: Defining antisymmetric tensor epsilong44[-a, -b, -c, -d, -f, -i, -j, -k].
** DefCovD: Defining covariant derivative CD[-a].
** DefTensor: Defining vanishing torsion tensor TorsionCD[a, -b, -c].
** DefTensor: Defining symmetric Christoffel tensor ChristoffelCD[a, -b, -c].
** DefTensor: Defining Riemann tensor RiemannCD[-a, -b, -c, -d].
** DefTensor: Defining symmetric Ricci tensor RicciCD[-a, -b].
** DefCovD:  Contractions of Riemann automatically replaced by Ricci.
** DefTensor: Defining Ricci scalar RicciScalarCD[].
** DefCovD:  Contractions of Ricci automatically replaced by RicciScalar.
** DefTensor: Defining symmetric Einstein tensor EinsteinCD[-a, -b].
** DefTensor: Defining Weyl tensor WeylCD[-a, -b, -c, -d].
** DefTensor: Defining symmetric TFRicci tensor TFRicciCD[-a, -b].
** DefTensor: Defining Kretschmann scalar KretschmannCD[].
** DefCovD:  Computing RiemannToWeylRules for dim 8
** DefCovD:  Computing RicciToTFRicci for dim 8
** DefCovD:  Computing RicciToEinsteinRules for dim 8
** DefTensor: Defining symmetrized Riemann tensor SymRiemannCD[-a, -b, -c, -d].
** DefTensor: Defining symmetric Schouten tensor SchoutenCD[-a, -b].
** DefTensor: Defining
 symmetric cosmological Schouten tensor SchoutenCCCD[LI[_], -a, -b].
** DefTensor: Defining
 symmetric cosmological Einstein tensor EinsteinCCCD[LI[_], -a, -b].
** DefTensor: Defining weight +2 density Detg44[]. Determinant.
** DefParameter: Defining parameter PerturbationParameterg44.
** DefTensor: Defining tensor Perturbationg44[LI[order], -a, -b].
0.153873
```

In[∘]:= **?SignatureOfMetric**

Out[∘]=

Symbol

SignatureOfMetric[metric] gives the signature of the metric, in the form of a list of three elements: {p1s, m1s, zeros} giving the numbers of +1's, −1's and zeros, respectively, always in this order.

In[◦]:=
```
(*Abs[signDetM]^=1
DefMetric[signDetM,g44[-a,-b],CD,{";","D"},WeightedWithBasis→AIndex]
  (*DefMetric[{1,7,0},g44[-a,-b],CD,{";","D"},WeightedWithBasis→AIndex]*)
signDetM=(-1)^(dimM8/2)
SignatureOfMetric[g44]^={dimM8/2,dimM8/2,0}*)
```

In[◦]:=
```
SignatureOfMetric[g44]
```

Out[◦]=

{4, 4, 0}

Check

In[◦]:=
```
g44[{-0, -chartM8}, {-0, -chartM8}]
```

Out[◦]=

$g44_{00}$

In[◦]:=
```
MatrixForm[TableMetric = g44[-a, -b] // ToBasis[chartM8] // ComponentArray]
% // ToValues // ToValues // Simplify // MatrixForm[#, TableAlignments → Left] &
```

** DefTensor: Defining weight +2 density Detg44chartM8[]. Determinant.

Out[◦]//MatrixForm=

$$\begin{pmatrix} g44_{00} & g44_{01} & g44_{02} & g44_{03} & g44_{04} & g44_{05} & g44_{06} & g44_{07} \\ g44_{10} & g44_{11} & g44_{12} & g44_{13} & g44_{14} & g44_{15} & g44_{16} & g44_{17} \\ g44_{20} & g44_{21} & g44_{22} & g44_{23} & g44_{24} & g44_{25} & g44_{26} & g44_{27} \\ g44_{30} & g44_{31} & g44_{32} & g44_{33} & g44_{34} & g44_{35} & g44_{36} & g44_{37} \\ g44_{40} & g44_{41} & g44_{42} & g44_{43} & g44_{44} & g44_{45} & g44_{46} & g44_{47} \\ g44_{50} & g44_{51} & g44_{52} & g44_{53} & g44_{54} & g44_{55} & g44_{56} & g44_{57} \\ g44_{60} & g44_{61} & g44_{62} & g44_{63} & g44_{64} & g44_{65} & g44_{66} & g44_{67} \\ g44_{70} & g44_{71} & g44_{72} & g44_{73} & g44_{74} & g44_{75} & g44_{76} & g44_{77} \end{pmatrix}$$

Out[◦]//MatrixForm=

$$\begin{pmatrix} g44_{00} & g44_{01} & g44_{02} & g44_{03} & g44_{04} & g44_{05} & g44_{06} & g44_{07} \\ g44_{10} & g44_{11} & g44_{12} & g44_{13} & g44_{14} & g44_{15} & g44_{16} & g44_{17} \\ g44_{20} & g44_{21} & g44_{22} & g44_{23} & g44_{24} & g44_{25} & g44_{26} & g44_{27} \\ g44_{30} & g44_{31} & g44_{32} & g44_{33} & g44_{34} & g44_{35} & g44_{36} & g44_{37} \\ g44_{40} & g44_{41} & g44_{42} & g44_{43} & g44_{44} & g44_{45} & g44_{46} & g44_{47} \\ g44_{50} & g44_{51} & g44_{52} & g44_{53} & g44_{54} & g44_{55} & g44_{56} & g44_{57} \\ g44_{60} & g44_{61} & g44_{62} & g44_{63} & g44_{64} & g44_{65} & g44_{66} & g44_{67} \\ g44_{70} & g44_{71} & g44_{72} & g44_{73} & g44_{74} & g44_{75} & g44_{76} & g44_{77} \end{pmatrix}$$

In[◦]:=
```
g44[-a, -f] × g44[f, b] // Simplify
FullForm[%]
```

Out[◦]=

$\delta_a{}^b$

Out[◦]//FullForm=

delta[Times[-1, a], b]

In[∘]:=
```
MatrixForm[TableDelta = delta[Times[-1, a], b] // ToBasis[chartM8] // ComponentArray]
% // ToValues // ToValues // Simplify
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[∘]:=
```
MyArrayComponents@g44[-a, b]
```

Out[∘]=
```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

$$\delta^{j_1\cdots j_r}_{h_1\cdots h_r} = \begin{vmatrix} \delta^{j_1}_{h_1} & \delta^{j_1}_{h_2} & \cdots & \delta^{j_1}_{h_r} \\ \delta^{j_2}_{h_1} & \delta^{j_2}_{h_2} & \cdots & \delta^{j_2}_{h_r} \\ \cdots & \cdots & \cdots & \cdots \\ \delta^{j_r}_{h_1} & \delta^{j_r}_{h_2} & \cdots & \delta^{j_r}_{h_r} \end{vmatrix}.$$

The generalized Kronecker delta or multi-index Kronecker delta of order $2p$ is a type (p, p) tensor that is a completely antisymmetric in its p upper indices, and also in its p lower indices.[8] In terms of the indices, the generalized Kronecker delta is defined as (Frankel 2011):

$$\delta^{\mu_1\ldots\mu_p}_{\nu_1\ldots\nu_p} = \begin{cases} +1 & \text{if } (\nu_1\ldots\nu_p) \text{ is an even permutation of } (\mu_1\ldots\mu_p) \\ -1 & \text{if } (\nu_1\ldots\nu_p) \text{ is an odd permutation of } (\mu_1\ldots\mu_p) \\ 0 & \text{otherwise} \end{cases}$$

When $p = 1$, the definition reduces to the standard Kronecker delta that corresponds to the n x n identity matrix $I_{ij} = \delta^i_j$ where i and j take the values $1, 2, \ldots, n$.

In[∘]:=
```
Clear[kδ];
kδ[lower_, upper_] /; Length[lower] == Length[upper] := Det[Outer[delta, lower, upper]]
```

In[∘]:=
```
kδ[{-a}, {b}]
% // Simplification
MyArrayComponents@%% // MatrixForm
```

Out[∘]=

$\delta_a{}^b$

Out[∘]=

$\delta_a{}^b$

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[∘]:=
```
kδ[{-a, -f}, {b, c}]
% // Simplification
(MyArrayComponents@%%)⟦2, 1⟧
```

Out[∘]=

$-\delta_a{}^c\ \delta_f{}^b + \delta_a{}^b\ \delta_f{}^c$

Out[∘]=

$-\delta_a{}^c\ \delta^b{}_f + \delta_a{}^b\ \delta^c{}_f$

Out[∘]=

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

Einstein - Lovelock

In[∘]:=
```
1/4 RiemannCD[-c, -d, f, i] × kδ[{-a, -f, -i}, {b, c, d}]
Lovelock1 = -%(* √Detg[] *) // Simplification
```

Out[∘]=

$$\frac{1}{4}\left(-\delta_a{}^d\ \delta_f{}^c\ \delta_i{}^b + \delta_a{}^c\ \delta_f{}^d\ \delta_i{}^b + \delta_a{}^d\ \delta_f{}^b\ \delta_i{}^c - \delta_a{}^b\ \delta_f{}^d\ \delta_i{}^c - \delta_a{}^c\ \delta_f{}^b\ \delta_i{}^d + \delta_a{}^b\ \delta_f{}^c\ \delta_i{}^d\right) \mathrm{R[D]}_{cd}{}^{fi}$$

Out[∘]=

$$\mathrm{R[D]}_a{}^b - \frac{1}{2}\ \delta_a{}^b\ \mathrm{R[D]}$$

In[∘]:=

```
1
— RiemannCD[-c, -d, f, i] × RiemannCD[-j, -k, l, m] ×
8
 kδ[{-a, -f, -i, -l, -m}, {b, c, d, j, k}]
Lovelock2 = -%(* √Detg[] *) // Simplification
```

Out[∘]=

$$\begin{aligned}
&\frac{1}{8}\Big(\big(\delta_a{}^k\,\delta_f{}^j\,\delta_i{}^d\,\delta_l{}^c-\delta_a{}^j\,\delta_f{}^k\,\delta_i{}^d\,\delta_l{}^c-\delta_a{}^k\,\delta_f{}^d\,\delta_i{}^j\,\delta_l{}^c+\delta_a{}^d\,\delta_f{}^k\,\delta_i{}^j\,\delta_l{}^c+\delta_a{}^j\,\delta_f{}^d\,\delta_i{}^k\,\delta_l{}^c-\\
&\quad\delta_a{}^d\,\delta_f{}^j\,\delta_i{}^k\,\delta_l{}^c-\delta_a{}^k\,\delta_f{}^j\,\delta_i{}^c\,\delta_l{}^d+\delta_a{}^j\,\delta_f{}^k\,\delta_i{}^c\,\delta_l{}^d+\delta_a{}^k\,\delta_f{}^c\,\delta_i{}^j\,\delta_l{}^d-\delta_a{}^c\,\delta_f{}^k\,\delta_i{}^j\,\delta_l{}^d-\\
&\quad\delta_a{}^j\,\delta_f{}^c\,\delta_i{}^k\,\delta_l{}^d+\delta_a{}^c\,\delta_f{}^j\,\delta_i{}^k\,\delta_l{}^d+\delta_a{}^k\,\delta_f{}^d\,\delta_i{}^c\,\delta_l{}^j-\delta_a{}^d\,\delta_f{}^k\,\delta_i{}^c\,\delta_l{}^j-\delta_a{}^k\,\delta_f{}^c\,\delta_i{}^d\,\delta_l{}^j+\\
&\quad\delta_a{}^c\,\delta_f{}^k\,\delta_i{}^d\,\delta_l{}^j+\delta_a{}^d\,\delta_f{}^c\,\delta_i{}^k\,\delta_l{}^j-\delta_a{}^c\,\delta_f{}^d\,\delta_i{}^k\,\delta_l{}^j-\delta_a{}^j\,\delta_f{}^d\,\delta_i{}^c\,\delta_l{}^k+\delta_a{}^d\,\delta_f{}^j\,\delta_i{}^c\,\delta_l{}^k+\\
&\quad\delta_a{}^j\,\delta_f{}^c\,\delta_i{}^d\,\delta_l{}^k-\delta_a{}^c\,\delta_f{}^j\,\delta_i{}^d\,\delta_l{}^k-\delta_a{}^d\,\delta_f{}^c\,\delta_i{}^j\,\delta_l{}^k+\delta_a{}^c\,\delta_f{}^d\,\delta_i{}^j\,\delta_l{}^k\big)\,\delta_m{}^b-\\
&\big(\delta_a{}^k\,\delta_f{}^j\,\delta_i{}^d\,\delta_l{}^b-\delta_a{}^j\,\delta_f{}^k\,\delta_i{}^d\,\delta_l{}^b-\delta_a{}^k\,\delta_f{}^d\,\delta_i{}^j\,\delta_l{}^b+\delta_a{}^d\,\delta_f{}^k\,\delta_i{}^j\,\delta_l{}^b+\delta_a{}^j\,\delta_f{}^d\,\delta_i{}^k\,\delta_l{}^b-\\
&\quad\delta_a{}^d\,\delta_f{}^j\,\delta_i{}^k\,\delta_l{}^b-\delta_a{}^k\,\delta_f{}^j\,\delta_i{}^b\,\delta_l{}^d+\delta_a{}^j\,\delta_f{}^k\,\delta_i{}^b\,\delta_l{}^d+\delta_a{}^k\,\delta_f{}^b\,\delta_i{}^j\,\delta_l{}^d-\delta_a{}^b\,\delta_f{}^k\,\delta_i{}^j\,\delta_l{}^d-\\
&\quad\delta_a{}^j\,\delta_f{}^b\,\delta_i{}^k\,\delta_l{}^d+\delta_a{}^b\,\delta_f{}^j\,\delta_i{}^k\,\delta_l{}^d+\delta_a{}^k\,\delta_f{}^d\,\delta_i{}^b\,\delta_l{}^j-\delta_a{}^d\,\delta_f{}^k\,\delta_i{}^b\,\delta_l{}^j-\delta_a{}^k\,\delta_f{}^b\,\delta_i{}^d\,\delta_l{}^j+\\
&\quad\delta_a{}^b\,\delta_f{}^k\,\delta_i{}^d\,\delta_l{}^j+\delta_a{}^d\,\delta_f{}^b\,\delta_i{}^k\,\delta_l{}^j-\delta_a{}^b\,\delta_f{}^d\,\delta_i{}^k\,\delta_l{}^j-\delta_a{}^j\,\delta_f{}^d\,\delta_i{}^b\,\delta_l{}^k+\delta_a{}^d\,\delta_f{}^j\,\delta_i{}^b\,\delta_l{}^k+\\
&\quad\delta_a{}^j\,\delta_f{}^b\,\delta_i{}^d\,\delta_l{}^k-\delta_a{}^b\,\delta_f{}^j\,\delta_i{}^d\,\delta_l{}^k-\delta_a{}^d\,\delta_f{}^b\,\delta_i{}^j\,\delta_l{}^k+\delta_a{}^b\,\delta_f{}^d\,\delta_i{}^j\,\delta_l{}^k\big)\,\delta_m{}^c+\\
&\big(\delta_a{}^k\,\delta_f{}^j\,\delta_i{}^c\,\delta_l{}^b-\delta_a{}^j\,\delta_f{}^k\,\delta_i{}^c\,\delta_l{}^b-\delta_a{}^k\,\delta_f{}^c\,\delta_i{}^j\,\delta_l{}^b+\delta_a{}^c\,\delta_f{}^k\,\delta_i{}^j\,\delta_l{}^b+\delta_a{}^j\,\delta_f{}^c\,\delta_i{}^k\,\delta_l{}^b-\\
&\quad\delta_a{}^c\,\delta_f{}^j\,\delta_i{}^k\,\delta_l{}^b-\delta_a{}^k\,\delta_f{}^j\,\delta_i{}^b\,\delta_l{}^c+\delta_a{}^j\,\delta_f{}^k\,\delta_i{}^b\,\delta_l{}^c+\delta_a{}^k\,\delta_f{}^b\,\delta_i{}^j\,\delta_l{}^c-\delta_a{}^b\,\delta_f{}^k\,\delta_i{}^j\,\delta_l{}^c-\\
&\quad\delta_a{}^j\,\delta_f{}^b\,\delta_i{}^k\,\delta_l{}^c+\delta_a{}^b\,\delta_f{}^j\,\delta_i{}^k\,\delta_l{}^c+\delta_a{}^k\,\delta_f{}^c\,\delta_i{}^b\,\delta_l{}^j-\delta_a{}^c\,\delta_f{}^k\,\delta_i{}^b\,\delta_l{}^j-\delta_a{}^k\,\delta_f{}^b\,\delta_i{}^c\,\delta_l{}^j+\\
&\quad\delta_a{}^b\,\delta_f{}^k\,\delta_i{}^c\,\delta_l{}^j+\delta_a{}^c\,\delta_f{}^b\,\delta_i{}^k\,\delta_l{}^j-\delta_a{}^b\,\delta_f{}^c\,\delta_i{}^k\,\delta_l{}^j-\delta_a{}^j\,\delta_f{}^c\,\delta_i{}^b\,\delta_l{}^k+\delta_a{}^c\,\delta_f{}^j\,\delta_i{}^b\,\delta_l{}^k+\\
&\quad\delta_a{}^j\,\delta_f{}^b\,\delta_i{}^c\,\delta_l{}^k-\delta_a{}^b\,\delta_f{}^j\,\delta_i{}^c\,\delta_l{}^k-\delta_a{}^c\,\delta_f{}^b\,\delta_i{}^j\,\delta_l{}^k+\delta_a{}^b\,\delta_f{}^c\,\delta_i{}^j\,\delta_l{}^k\big)\,\delta_m{}^d-\\
&\big(\delta_a{}^k\,\delta_f{}^d\,\delta_i{}^c\,\delta_l{}^b-\delta_a{}^d\,\delta_f{}^k\,\delta_i{}^c\,\delta_l{}^b-\delta_a{}^k\,\delta_f{}^c\,\delta_i{}^d\,\delta_l{}^b+\delta_a{}^c\,\delta_f{}^k\,\delta_i{}^d\,\delta_l{}^b+\delta_a{}^d\,\delta_f{}^c\,\delta_i{}^k\,\delta_l{}^b-\\
&\quad\delta_a{}^c\,\delta_f{}^d\,\delta_i{}^k\,\delta_l{}^b-\delta_a{}^k\,\delta_f{}^d\,\delta_i{}^b\,\delta_l{}^c+\delta_a{}^d\,\delta_f{}^k\,\delta_i{}^b\,\delta_l{}^c+\delta_a{}^k\,\delta_f{}^b\,\delta_i{}^d\,\delta_l{}^c-\delta_a{}^b\,\delta_f{}^k\,\delta_i{}^d\,\delta_l{}^c-\\
&\quad\delta_a{}^d\,\delta_f{}^b\,\delta_i{}^k\,\delta_l{}^c+\delta_a{}^b\,\delta_f{}^d\,\delta_i{}^k\,\delta_l{}^c+\delta_a{}^k\,\delta_f{}^c\,\delta_i{}^b\,\delta_l{}^d-\delta_a{}^c\,\delta_f{}^k\,\delta_i{}^b\,\delta_l{}^d-\delta_a{}^k\,\delta_f{}^b\,\delta_i{}^c\,\delta_l{}^d+\\
&\quad\delta_a{}^b\,\delta_f{}^k\,\delta_i{}^c\,\delta_l{}^d+\delta_a{}^c\,\delta_f{}^b\,\delta_i{}^k\,\delta_l{}^d-\delta_a{}^b\,\delta_f{}^c\,\delta_i{}^k\,\delta_l{}^d-\delta_a{}^d\,\delta_f{}^c\,\delta_i{}^b\,\delta_l{}^k+\delta_a{}^c\,\delta_f{}^d\,\delta_i{}^b\,\delta_l{}^k+\\
&\quad\delta_a{}^d\,\delta_f{}^b\,\delta_i{}^c\,\delta_l{}^k-\delta_a{}^b\,\delta_f{}^d\,\delta_i{}^c\,\delta_l{}^k-\delta_a{}^c\,\delta_f{}^b\,\delta_i{}^d\,\delta_l{}^k+\delta_a{}^b\,\delta_f{}^c\,\delta_i{}^d\,\delta_l{}^k\big)\,\delta_m{}^j+\\
&\big(\delta_a{}^j\,\delta_f{}^d\,\delta_i{}^c\,\delta_l{}^b-\delta_a{}^d\,\delta_f{}^j\,\delta_i{}^c\,\delta_l{}^b-\delta_a{}^j\,\delta_f{}^c\,\delta_i{}^d\,\delta_l{}^b+\delta_a{}^c\,\delta_f{}^j\,\delta_i{}^d\,\delta_l{}^b+\delta_a{}^d\,\delta_f{}^c\,\delta_i{}^j\,\delta_l{}^b-\\
&\quad\delta_a{}^c\,\delta_f{}^d\,\delta_i{}^j\,\delta_l{}^b-\delta_a{}^j\,\delta_f{}^d\,\delta_i{}^b\,\delta_l{}^c+\delta_a{}^d\,\delta_f{}^j\,\delta_i{}^b\,\delta_l{}^c+\delta_a{}^j\,\delta_f{}^b\,\delta_i{}^d\,\delta_l{}^c-\\
&\quad\delta_a{}^b\,\delta_f{}^j\,\delta_i{}^d\,\delta_l{}^c-\delta_a{}^d\,\delta_f{}^b\,\delta_i{}^j\,\delta_l{}^c+\delta_a{}^b\,\delta_f{}^d\,\delta_i{}^j\,\delta_l{}^c+\delta_a{}^j\,\delta_f{}^c\,\delta_i{}^b\,\delta_l{}^d-\\
&\quad\delta_a{}^c\,\delta_f{}^j\,\delta_i{}^b\,\delta_l{}^d-\delta_a{}^j\,\delta_f{}^b\,\delta_i{}^c\,\delta_l{}^d+\delta_a{}^b\,\delta_f{}^j\,\delta_i{}^c\,\delta_l{}^d+\delta_a{}^c\,\delta_f{}^b\,\delta_i{}^j\,\delta_l{}^d-\\
&\quad\delta_a{}^b\,\delta_f{}^c\,\delta_i{}^j\,\delta_l{}^d-\delta_a{}^d\,\delta_f{}^c\,\delta_i{}^b\,\delta_l{}^j+\delta_a{}^c\,\delta_f{}^d\,\delta_i{}^b\,\delta_l{}^j+\delta_a{}^d\,\delta_f{}^b\,\delta_i{}^c\,\delta_l{}^j-\\
&\quad\delta_a{}^b\,\delta_f{}^d\,\delta_i{}^c\,\delta_l{}^j-\delta_a{}^c\,\delta_f{}^b\,\delta_i{}^d\,\delta_l{}^j+\delta_a{}^b\,\delta_f{}^c\,\delta_i{}^d\,\delta_l{}^j\big)\,\delta_m{}^k\Big)\;R[D]_{cd}{}^{fi}\;R[D]_{jk}{}^{lm}
\end{aligned}$$

0.200409

Out[∘]=

$$\begin{aligned}
&-4\;R[D]_a{}^j\;R[D]^b{}_j+2\left(R[D]_a{}^b\;R[D]-2\;R[D]^j{}_k\;R[D]_{aj}{}^{bk}+R[D]_{aj}{}^{kl}\;R[D]^{bj}{}_{kl}\right)+\\
&\frac{1}{2}\;\delta_a{}^b\;\left(4\;R[D]_j{}^k\;R[D]^j{}_k-R[D]^2-R[D]_{jk}{}^{lm}\;R[D]^{jk}{}_{lm}\right)
\end{aligned}$$

In[◦]:=

```
1/16 RiemannCD[-c, -d, f, i] × RiemannCD[-j, -k, l, m] ×
 RiemannCD[-n, -p, q, r] × kδ[{-a, -f, -i, -l, -m, -q, -r}, {b, c, d, j, k, n, p}]
Lovelock3 = -%(* √Detg[] *) // Simplification
```

0.220275

Out[◦]=

$$\frac{1}{16}\left(\cdots 9 \cdots + (\cdots 1 \cdots)\ \cdots 1 \cdots\right) R[D]_{cd}{}^{fi}\ R[D]_{jk}{}^{lm}\ R[D]_{np}{}^{qr}$$

Full expression not available (original memory size: 3.1 MB)

16.9354

Out[◦]=

$$\begin{aligned}
&-8\,\delta_a{}^b\, R[D]_j{}^l\, R[D]^j{}_k\, R[D]^k{}_l + 6\,\delta_a{}^b\, R[D]_j{}^k\, R[D]^j{}_k\, R[D] - \frac{1}{2}\,\delta_a{}^b\, R[D]^3 - \\
&12\, R[D]^j{}_k\, R[D]\, R[D]_{aj}{}^{bk} + 24\, R[D]^b{}_j\, R[D]^k{}_l\, R[D]_{ak}{}^{jl} + 24\, R[D]_j{}^l\, R[D]^j{}_k\, R[D]_{al}{}^{bk} + \\
&6\, R[D]\, R[D]_{aj}{}^{kl}\, R[D]^{bj}{}_{kl} - 12\, R[D]^j{}_k\, R[D]_{aj}{}^{lm}\, R[D]^{bk}{}_{lm} - 24\, R[D]^j{}_k\, R[D]_{al}{}^{km}\, R[D]^{bl}{}_{jm} + \\
&12\, R[D]_a{}^j\left(2\, R[D]^b{}_k\, R[D]_j{}^k - R[D]^b{}_j\, R[D] + 2\, R[D]^k{}_l\, R[D]^{bl}{}_{jk} - R[D]^{bk}{}_{lm}\, R[D]_{jk}{}^{lm}\right) - \\
&12\,\delta_a{}^b\, R[D]^j{}_k\, R[D]^l{}_m\, R[D]_{jl}{}^{km} + 24\, R[D]^j{}_k\, R[D]_{al}{}^{bm}\, R[D]_{jm}{}^{kl} - 12\, R[D]^b{}_j\, R[D]_{ak}{}^{lm}\, R[D]^{jk}{}_{lm} - \\
&\frac{3}{2}\,\delta_a{}^b\, R[D]\, R[D]_{jk}{}^{lm}\, R[D]^{jk}{}_{lm} + 3\, R[D]_a{}^b\left(-4\, R[D]_j{}^k\, R[D]^j{}_k + R[D]^2 + R[D]_{jk}{}^{lm}\, R[D]^{jk}{}_{lm}\right) - \\
&24\, R[D]_{aj}{}^{kl}\, R[D]^{bm}{}_{kn}\, R[D]^{jn}{}_{lm} + 6\, R[D]_{aj}{}^{kl}\, R[D]^{bj}{}_{mn}\, R[D]_{kl}{}^{mn} - \\
&12\, R[D]_{aj}{}^{bk}\, R[D]^{jl}{}_{mn}\, R[D]_{kl}{}^{mn} + 4\,\delta_a{}^b\, R[D]_{jn}{}^{lp}\, R[D]^{jk}{}_{lm}\, R[D]_{kp}{}^{mn} + \\
&12\,\delta_a{}^b\, R[D]^j{}_k\, R[D]_{jl}{}^{mn}\, R[D]^{kl}{}_{mn} - \delta_a{}^b\, R[D]_{jk}{}^{np}\, R[D]^{jk}{}_{lm}\, R[D]^{lm}{}_{np}
\end{aligned}$$

In[◦]:=

```
DumpSave[ToString[header <> "Lovelock123.mx"], {Lovelock1, Lovelock2, Lovelock3}]
```

Out[◦]=

$$\begin{aligned}
&\Big\{ R[D]_a{}^b - \frac{1}{2}\,\delta_a{}^b\, R[D],\ -4\, R[D]_a{}^j\, R[D]^b{}_j + 2\left(R[D]_a{}^b\, R[D] - 2\, R[D]^j{}_k\, R[D]_{aj}{}^{bk} + R[D]_{aj}{}^{kl}\, R[D]^{bj}{}_{kl}\right) + \\
&\quad \frac{1}{2}\,\delta_a{}^b\left(4\, R[D]_j{}^k\, R[D]^j{}_k - R[D]^2 - R[D]_{jk}{}^{lm}\, R[D]^{jk}{}_{lm}\right), \\
&-8\,\delta_a{}^b\, R[D]_j{}^l\, R[D]^j{}_k\, R[D]^k{}_l + 6\,\delta_a{}^b\, R[D]_j{}^k\, R[D]^j{}_k\, R[D] - \frac{1}{2}\,\delta_a{}^b\, R[D]^3 - \\
&\quad 12\, R[D]^j{}_k\, R[D]\, R[D]_{aj}{}^{bk} + 24\, R[D]^b{}_j\, R[D]^k{}_l\, R[D]_{ak}{}^{jl} + 24\, R[D]_j{}^l\, R[D]^j{}_k\, R[D]_{al}{}^{bk} + \\
&\quad 6\, R[D]\, R[D]_{aj}{}^{kl}\, R[D]^{bj}{}_{kl} - 12\, R[D]^j{}_k\, R[D]_{aj}{}^{lm}\, R[D]^{bk}{}_{lm} - 24\, R[D]^j{}_k\, R[D]_{al}{}^{km}\, R[D]^{bl}{}_{jm} + \\
&\quad 12\, R[D]_a{}^j\left(2\, R[D]^b{}_k\, R[D]_j{}^k - R[D]^b{}_j\, R[D] + 2\, R[D]^k{}_l\, R[D]^{bl}{}_{jk} - R[D]^{bk}{}_{lm}\, R[D]_{jk}{}^{lm}\right) - \\
&\quad 12\,\delta_a{}^b\, R[D]^j{}_k\, R[D]^l{}_m\, R[D]_{jl}{}^{km} + 24\, R[D]^j{}_k\, R[D]_{al}{}^{bm}\, R[D]_{jm}{}^{kl} - 12\, R[D]^b{}_j\, R[D]_{ak}{}^{lm}\, R[D]^{jk}{}_{lm} - \\
&\quad \frac{3}{2}\,\delta_a{}^b\, R[D]\, R[D]_{jk}{}^{lm}\, R[D]^{jk}{}_{lm} + 3\, R[D]_a{}^b\left(-4\, R[D]_j{}^k\, R[D]^j{}_k + R[D]^2 + R[D]_{jk}{}^{lm}\, R[D]^{jk}{}_{lm}\right) - \\
&\quad 24\, R[D]_{aj}{}^{kl}\, R[D]^{bm}{}_{kn}\, R[D]^{jn}{}_{lm} + 6\, R[D]_{aj}{}^{kl}\, R[D]^{bj}{}_{mn}\, R[D]_{kl}{}^{mn} - \\
&\quad 12\, R[D]_{aj}{}^{bk}\, R[D]^{jl}{}_{mn}\, R[D]_{kl}{}^{mn} + 4\,\delta_a{}^b\, R[D]_{jn}{}^{lp}\, R[D]^{jk}{}_{lm}\, R[D]_{kp}{}^{mn} + \\
&\quad 12\,\delta_a{}^b\, R[D]^j{}_k\, R[D]_{jl}{}^{mn}\, R[D]^{kl}{}_{mn} - \delta_a{}^b\, R[D]_{jk}{}^{np}\, R[D]^{jk}{}_{lm}\, R[D]^{lm}{}_{np} \Big\}
\end{aligned}$$

Model metric g44

g44 → {{e^(2 a4[x4]) Sin[6 H x8]^(1/3), 0, 0, 0, 0, 0, 0, 0}, {0, e^(2 a4[x4]) Sin[6 H x8]^(1/3), 0, 0, 0, 0, 0, 0},
{0, 0, e^(2 a4[x4]) Sin[6 H x8]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
{0, 0, 0, 0, -e^(-2 a4[x4]) Sin[6 H x8]^(1/3), 0, 0, 0}, {0, 0, 0, 0, 0, -e^(-2 a4[x4]) Sin[6 H x8]^(1/3), 0, 0},
{0, 0, 0, 0, 0, 0, -e^(-2 a4[x4]) Sin[6 H x8]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, Cot[6 H x8]^2}}

We change the options of ToCanonical to prevent indices inside the derivations from moving up and down.

In[∘]:=
```
SetOptions[ToCanonical, UseMetricOnVBundle → None]
```

Out[∘]=
```
{Verbose → False, UseMetricOnVBundle → None,
 Method → {ChangeCovD, ExpandChristoffel → False}, MathLink :→ $xpermQ, TimeVerbose → False}
```

MatrixMetric44

In[∘]:=
```
DefConstantSymbol[Λ, PrintAs → "Λ"];
```

```
** DefConstantSymbol: Defining constant symbol Λ.
```

In[∘]:=
```
DefConstantSymbol[H, PrintAs → "H"]
```

```
** DefConstantSymbol: Defining constant symbol H.
```

In[∘]:=
```
DefScalarFunction[a4]
```

```
** DefScalarFunction: Defining scalar function a4.
```

In[∘]:=
```
β3 = Exp[2 a4[x4[]] ];
β1 = Sin[6 * H * x8[]]^(1/3);
β2 = Cot[6 * H * x8[]]^2;
DiagonalMatrix[Flatten[
   {β1 * {β3, β3, β3}, {-1}, -β1 * {1 / β3, 1 / β3, 1 / β3}, {β2}}]] // MatrixForm
```

We will consider this Nash metric (corrected typo, in red):

In[∘]:=
```
(*MatrixForm[MatrixMetric44=
  DiagonalMatrix[Flatten[{β1*{β3,β3,β3},{-1},-β1*{1/β3,1/β3,1/β3},{β2}}]]]*)
```

And we define a CTensor object which has these components with down indices.

In[∘]:=
```
β3 = Exp[2 a4[x4[]] ];
β1 = Sin[6 * H * x0[]]^(1/3);
β2 = Cot[6 * H * x0[]]^2;
DiagonalMatrix[
  Flatten[{{β2}, β1 * {β3, β3, β3}, {-1}, -β1 * {1 / β3, 1 / β3, 1 / β3}}]] // MatrixForm
```

Out[∘]//MatrixForm=

Cot[6 H x0]^2	0	0	0	0	0
0	e^(2 a4[x4]) Sin[6 H x0]^(1/3)	0	0	0	0
0	0	e^(2 a4[x4]) Sin[6 H x0]^(1/3)	0	0	0
0	0	0	e^(2 a4[x4]) Sin[6 H x0]^(1/3)	0	0
0	0	0	0	-1	0
0	0	0	0	0	-e^(-2 a4[x4]) Sin[6
0	0	0	0	0	0
0	0	0	0	0	0

In[ ]:=
```
MatrixForm[MatrixMetric44 =
  DiagonalMatrix[Flatten[{{β2}, β1 * {β3, β3, β3}, {-1}, -β1 * {1 / β3, 1 / β3, 1 / β3}}]]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0]^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -e^{-2\,a4[x4]}\,\mathrm{Sin}[6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
MatrixForm[MatrixMetric71 =
  DiagonalMatrix[Flatten[{{β2}, β1 * {β3, β3, β3}, {-1}, β1 * {1 / β3, 1 / β3, 1 / β3}}]]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cot}[6\,H\,x0]^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
g44 = CTensor[MatrixMetric44, {-chartM8, -chartM8}]
```

Out[ ]=

$$\mathrm{CTensor}\Big[\Big\{\{\mathrm{Cot}[6\,H\,x0]^2, 0, 0, 0, 0, 0, 0, 0\},$$
$$\{0, e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \{0, 0, e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0, 0\},$$
$$\{0, 0, 0, e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\},$$
$$\{0, 0, 0, 0, 0, -e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0, 0\}, \{0, 0, 0, 0, 0, 0, -e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}, 0\},$$
$$\{0, 0, 0, 0, 0, 0, 0, -e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\}\Big\}, \{-\mathrm{chartM8}, -\mathrm{chartM8}\}, 0\Big]$$

We decide that this tensor should be our metric

In[ ]:=
```
(*g71=CTensor[MatrixMetric71,{-chartM8,-chartM8}]*)
```

In[ ]:=
```
time1 = Now
AbsoluteTiming[MetricCompute[g44, chartM8, All, Parallelize → True, Verbose → True]]
Now - time1
```

Out[ ]=

Tue 26 Aug 2025 06:00:46 GMT−7

```
** ReportCompute: DetMetric[]
    Constructed in 0.000371 seconds
    Applied Simplify in 2.12666 seconds
    Stored in 0.000142 seconds and 288 bytes
** ReportCompute: Metric[1, 1]
    Constructed in 0.002343 seconds
    Applied Simplify in 0.036452 seconds
    Stored in 0.000124 seconds and 5112 bytes
```

```
** ReportCompute: DMetric[-1, -1, -1]
    Constructed in 0.000738 seconds
    Applied Simplify in 0.025861 seconds
** ReportCompute: DDMetric[-1, -1, -1, -1]
    Constructed in 0.003211 seconds
    Applied Simplify in 0.051229 seconds
** ReportCompute: Christoffel[-1, -1, -1]
    Constructed in 0.001476 seconds
    Applied Simplify in 0.036621 seconds
** ReportCompute: Christoffel[1, -1, -1]
    Constructed in 0.00171 seconds
    Applied Simplify in 0.025274 seconds
** ReportCompute: CovDOfMetric
** ReportCompute: Riemann[-1, -1, -1, -1]
    Constructed in 0.005766 seconds
    Applied Simplify in 0.03307 seconds
    Stored in 0.004264 seconds and 244 400 bytes
** ReportCompute: Riemann[-1, -1, -1, 1]
    Constructed in 0.015459 seconds
    Applied Simplify in 0.050324 seconds
    Stored in 0.004332 seconds and 224 624 bytes
** ReportCompute: Ricci[-1, -1]
    Constructed in 0.00032 seconds
    Applied Simplify in 0.017642 seconds
    Stored in 0.00015 seconds and 7968 bytes
** ReportCompute: RicciScalar[]
    Constructed in 0.000083 seconds
    Applied Simplify in 0.013311 seconds
    Stored in 0.000087 seconds and 592 bytes
** ReportCompute: Einstein[-1, -1]
    Constructed in 0.000146 seconds
    Applied Simplify in 0.018406 seconds
    Stored in 0.000154 seconds and 10 752 bytes
** ReportCompute: Weyl[-1, -1, -1, -1]
    Constructed in 0.003307 seconds
    Applied Simplify in 0.035625 seconds
```

```
    Stored in 0.004299 seconds and 269 664 bytes
** ReportCompute: Riemann[-1, -1, 1, 1]
    Constructed in 0.007304 seconds
    Applied Simplify in 0.014192 seconds
    Stored in 0.004186 seconds and 164 480 bytes
** ReportCompute: Kretschmann[]
    Constructed in 0.003157 seconds
    Applied Simplify in 0.013045 seconds
    Stored in 0.000089 seconds and 1448 bytes
** ReportCompute: CDRiemann[-1, -1, -1, -1, -1]
    Constructed in 0.084637 seconds
    Applied Simplify in 0.056469 seconds
    Stored in 0.03339 seconds and 1 237 712 bytes
3.00555
```

Out[ ]=

{3.00555, Null}

Out[ ]=

3.00797 s

In[ ]:= **Lovelock1**

Out[ ]=

$$R[D]_a{}^b - \frac{1}{2}\,\delta_a{}^b\,R[D]$$

xAct magic : CD = LC[g44]

In[ ]:= `CD = LC[g44]`

Out[ ]=

```
CCovD[PDchartM8,
 CTensor[{{{-12 H Csc[12 H x0], 0, 0, 0, 0, 0, 0, 0}, {0, -e^(2 a4[x4]) H Sec[6 H x0] Sin[6 H x0]^(4/3),
     0, 0, 0, 0, 0, 0}, {0, 0, -e^(2 a4[x4]) H Sec[6 H x0] Sin[6 H x0]^(4/3), 0, 0, 0, 0, 0},
    {0, 0, 0, -e^(2 a4[x4]) H Sec[6 H x0] Sin[6 H x0]^(4/3), 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, e^(-2 a4[x4]) H Sec[6 H x0] Sin[6 H x0]^(4/3), 0, 0},
    {0, 0, 0, 0, 0, 0, e^(-2 a4[x4]) H Sec[6 H x0] Sin[6 H x0]^(4/3), 0},
    {0, 0, 0, 0, 0, 0, 0, e^(-2 a4[x4]) H Sec[6 H x0] Sin[6 H x0]^(4/3)}},
   {{0, H Cot[6 H x0], 0, 0, 0, 0, 0, 0}, {H Cot[6 H x0], 0, 0, 0, a4'[x4], 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, a4'[x4], 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, H Cot[6 H x0], 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {H Cot[6 H x0], 0,
     0, 0, a4'[x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, a4'[x4], 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, H Cot[6 H x0], 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {H Cot[6 H x0], 0, 0, 0, a4'[x4], 0, 0, 0}, {0, 0, 0, a4'[x4], 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, 0}, {0, e^(2 a4[x4]) Sin[6 H x0]^(1/3) a4'[x4], 0, 0, 0, 0, 0, 0},
    {0, 0, e^(2 a4[x4]) Sin[6 H x0]^(1/3) a4'[x4], 0, 0, 0, 0, 0},
    {0, 0, 0, e^(2 a4[x4]) Sin[6 H x0]^(1/3) a4'[x4], 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, e^(-2 a4[x4]) Sin[6 H x0]^(1/3) a4'[x4], 0, 0}, {0, 0, 0, 0, 0, 0,
     e^(-2 a4[x4]) Sin[6 H x0]^(1/3) a4'[x4], 0}, {0, 0, 0, 0, 0, 0, 0, e^(-2 a4[x4]) Sin[6 H x0]^(1/3) a4'[x4]}},
   {{0, 0, 0, 0, 0, H Cot[6 H x0], 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -a4'[x4], 0, 0}, {H Cot[6 H x0], 0,
     0, 0, -a4'[x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, H Cot[6 H x0], 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -a4'[x4], 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {H Cot[6 H x0], 0, 0, 0, -a4'[x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}},
   {{0, 0, 0, 0, 0, 0, 0, H Cot[6 H x0]}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -a4'[x4]}, {0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0}, {H Cot[6 H x0], 0, 0, 0, -a4'[x4], 0, 0, 0}}},
  {chartM8, -chartM8, -chartM8}, 0], CTensor[{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0},
   {0, e^(2 a4[x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^(2 a4[x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0},
   {0, 0, 0, e^(2 a4[x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
   {0, 0, 0, 0, 0, -e^(-2 a4[x4]) Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, -e^(-2 a4[x4]) Sin[6 H x0]^(1/3), 0},
   {0, 0, 0, 0, 0, 0, 0, -e^(-2 a4[x4]) Sin[6 H x0]^(1/3)}}, {-chartM8, -chartM8}, 0]]
```

In[∘]:= `EinsteinCD = Einstein[CD]`

Out[∘]=

$$\mathrm{CTensor}\Big[\Big\{\big\{3\,\mathrm{Cot}[6\,H\,x0]^2\,\big(5\,H^2 - a4'[x4]^2\big),\,0,\,0,\,0,\,0,\,0,\,0,\,0\big\},\\
\big\{0,\,e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(15\,H^2 - 3\,a4'[x4]^2 + a4''[x4]\big),\,0,\,0,\,0,\,0,\,0,\,0\big\},\\
\big\{0,\,0,\,e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(15\,H^2 - 3\,a4'[x4]^2 + a4''[x4]\big),\,0,\,0,\,0,\,0,\,0\big\},\\
\big\{0,\,0,\,0,\,e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(15\,H^2 - 3\,a4'[x4]^2 + a4''[x4]\big),\,0,\,0,\,0,\,0\big\},\\
\big\{0,\,0,\,0,\,0,\,-3\,\big(7\,H^2 + a4'[x4]^2\big),\,0,\,0,\,0\big\},\\
\big\{0,\,0,\,0,\,0,\,0,\,e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(-15\,H^2 + 3\,a4'[x4]^2 + a4''[x4]\big),\,0,\,0\big\},\\
\big\{0,\,0,\,0,\,0,\,0,\,0,\,e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(-15\,H^2 + 3\,a4'[x4]^2 + a4''[x4]\big),\,0\big\},\\
\big\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(-15\,H^2 + 3\,a4'[x4]^2 + a4''[x4]\big)\big\}\Big\},\\
\{-\mathrm{chartM8},\,-\mathrm{chartM8}\},\,0\Big]$$

In[∘]:= `RiemannCD = Riemann[CD];`

In[∘]:= `RicciCD = Ricci[CD]`

Out[∘]=

$$\mathrm{CTensor}\Big[\Big\{\big\{-6\,H^2\,\mathrm{Cot}[6\,H\,x0]^2,\,0,\,0,\,0,\,0,\,0,\,0,\,0\big\},\\
\big\{0,\,e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(-6\,H^2 + a4''[x4]\big),\,0,\,0,\,0,\,0,\,0,\,0\big\},\\
\big\{0,\,0,\,e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(-6\,H^2 + a4''[x4]\big),\,0,\,0,\,0,\,0,\,0\big\},\\
\big\{0,\,0,\,0,\,e^{2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(-6\,H^2 + a4''[x4]\big),\,0,\,0,\,0,\,0\big\},\\
\big\{0,\,0,\,0,\,0,\,-6\,a4'[x4]^2,\,0,\,0,\,0\big\},\,\big\{0,\,0,\,0,\,0,\,0,\,e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(6\,H^2 + a4''[x4]\big),\,0,\,0\big\},\\
\big\{0,\,0,\,0,\,0,\,0,\,0,\,e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(6\,H^2 + a4''[x4]\big),\,0\big\},\\
\big\{0,\,0,\,0,\,0,\,0,\,0,\,0,\,e^{-2\,a4[x4]}\,\mathrm{Sin}[6\,H\,x0]^{1/3}\,\big(6\,H^2 + a4''[x4]\big)\big\}\Big\},\,\{-\mathrm{chartM8},\,-\mathrm{chartM8}\},\,0\Big]$$

In[∘]:= `RicciScalarCD = RicciScalar[CD]`

Out[∘]=

$$\mathrm{CTensor}\Big[6\,\big(-7\,H^2 + a4'[x4]^2\big),\,\{\},\,0\Big]$$

voila:

In[∘]:= `Lovelock1`

Out[∘]=

$$\boxed{\begin{matrix}
-6\,H^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -6\,H^2{+}a4''[x4] & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -6\,H^2{+}a4''[x4] & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -6\,H^2{+}a4''[x4] & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 6\,a4'[x4]^2 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -6\,H^2{-}a4''[x4] & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -6\,H^2{-}a4''[x4] & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -6\,H^2{-}a4''[x4]
\end{matrix}}{}_a{}^b\;-$$

$$3\,\delta_a{}^b\,\big(-7\,H^2 + a4'[x4]^2\big)$$

Check with MyArrayComponents

Let us wrap all these steps into one single function

In[∘]:= `(*Clear[MyArrayComponents]*)`

In[∘]:=
```
(*MyArrayComponents[expr_]:=
 expr//ToBasis[chartM8]//ComponentArray//ToValues//ToValues//Simplify*)
```

In[ ]:=
```
(*My4ArrayComponents[expr_]:=
 expr//ToBasis[chartM8]//ComponentArray//ToValues//ToValues//ToValues//ToValues//
  Simplify*)
```

In[ ]:=
```
MatrixForm[TableDelta = delta[Times[-1, a], b] // ToBasis[chartM8] // ComponentArray]
% // ToValues // ToValues // Simplify
```

```
** DefTensor: Defining JacobianchartM8[].
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[ ]=

```
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

In[ ]:=
```
MyArrayComponents@RicciScalarCD[]
```

Out[ ]=

$$6 \left(-7 H^2 + a4'[x4]^2\right)$$

In[ ]:=
```
ein = MyArrayComponents@EinsteinCD[-a, b]
```

Out[ ]=

$$\{\{15 H^2 - 3 a4'[x4]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, 15 H^2 - 3 a4'[x4]^2 + a4''[x4], 0, 0, 0, 0, 0, 0\},$$
$$\{0, 0, 15 H^2 - 3 a4'[x4]^2 + a4''[x4], 0, 0, 0, 0, 0\},$$
$$\{0, 0, 0, 15 H^2 - 3 a4'[x4]^2 + a4''[x4], 0, 0, 0, 0\},$$
$$\{0, 0, 0, 0, 3 \left(7 H^2 + a4'[x4]^2\right), 0, 0, 0\}, \{0, 0, 0, 0, 0, 15 H^2 - 3 a4'[x4]^2 - a4''[x4], 0, 0\},$$
$$\{0, 0, 0, 0, 0, 0, 15 H^2 - 3 a4'[x4]^2 - a4''[x4], 0\},$$
$$\{0, 0, 0, 0, 0, 0, 0, 15 H^2 - 3 a4'[x4]^2 - a4''[x4]\}\}$$

In[ ]:=
```
ein // MatrixForm
```

Out[ ]//MatrixForm=

$$\left(\begin{matrix} 15 H^2 - 3 a4'[x4]^2 & 0 & 0 & 0 \\ 0 & 15 H^2 - 3 a4'[x4]^2 + a4''[x4] & 0 & 0 \\ 0 & 0 & 15 H^2 - 3 a4'[x4]^2 + a4''[x4] & 0 \\ 0 & 0 & 0 & 15 H^2 - 3 a4'[x4]^2 + a4''[x4] \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}\right.$$

In[ ]:=
```
ein[[1]]
```

Out[ ]=

$$\{15 H^2 - 3 a4'[x4]^2, 0, 0, 0, 0, 0, 0, 0\}$$

In[∘]:= `MyArrayComponents@RicciCD[-a, -b]`

Out[∘]=

$$\{\{-6 H^2 \mathrm{Cot}[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0\},\ \{0, e^{2 a4[x4]} \mathrm{Sin}[6 H x0]^{1/3} (-6 H^2 + a4''[x4]), 0, 0, 0, 0, 0, 0\},\ \{0, 0, e^{2 a4[x4]} \mathrm{Sin}[6 H x0]^{1/3} (-6 H^2 + a4''[x4]), 0, 0, 0, 0, 0\},\ \{0, 0, 0, e^{2 a4[x4]} \mathrm{Sin}[6 H x0]^{1/3} (-6 H^2 + a4''[x4]), 0, 0, 0, 0\},\ \{0, 0, 0, 0, -6 a4'[x4]^2, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-2 a4[x4]} \mathrm{Sin}[6 H x0]^{1/3} (6 H^2 + a4''[x4]), 0, 0\},\ \{0, 0, 0, 0, 0, 0, e^{-2 a4[x4]} \mathrm{Sin}[6 H x0]^{1/3} (6 H^2 + a4''[x4]), 0\},\ \{0, 0, 0, 0, 0, 0, 0, e^{-2 a4[x4]} \mathrm{Sin}[6 H x0]^{1/3} (6 H^2 + a4''[x4])\}\}$$

In[∘]:= `MyArrayComponents@RicciCD[-a, b]`

Out[∘]=

$$\{\{-6 H^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, -6 H^2 + a4''[x4], 0, 0, 0, 0, 0, 0\},\ \{0, 0, -6 H^2 + a4''[x4], 0, 0, 0, 0, 0\}, \{0, 0, 0, -6 H^2 + a4''[x4], 0, 0, 0, 0\},\ \{0, 0, 0, 0, 6 a4'[x4]^2, 0, 0, 0\}, \{0, 0, 0, 0, 0, -6 H^2 - a4''[x4], 0, 0\},\ \{0, 0, 0, 0, 0, 0, -6 H^2 - a4''[x4], 0\}, \{0, 0, 0, 0, 0, 0, 0, -6 H^2 - a4''[x4]\}\}$$

In[∘]:= `MyArrayComponents@Lovelock1[-a, b]`

Out[∘]=

$$\left(\boxed{\begin{array}{cccccccc} -6 H^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6 H^2+a4''[x4] & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6 H^2+a4''[x4] & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 H^2+a4''[x4] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 a4'[x4]^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -6 H^2-a4''[x4] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -6 H^2-a4''[x4] & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -6 H^2-a4''[x4] \end{array}}{}_a{}^b - 3 \delta_a{}^b (-7 H^2 + a4'[x4]^2)\right)[-a, b]$$

Einstein - Lovelock Components

In[∘]:= `allLovelock1 = MyArrayComponents@Lovelock1`

Out[∘]=

$$\{\{15 H^2 - 3 a4'[x4]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, 15 H^2 - 3 a4'[x4]^2 + a4''[x4], 0, 0, 0, 0, 0, 0\},\ \{0, 0, 15 H^2 - 3 a4'[x4]^2 + a4''[x4], 0, 0, 0, 0, 0\},\ \{0, 0, 0, 15 H^2 - 3 a4'[x4]^2 + a4''[x4], 0, 0, 0, 0\},\ \{0, 0, 0, 0, 3 (7 H^2 + a4'[x4]^2), 0, 0, 0\}, \{0, 0, 0, 0, 0, 15 H^2 - 3 a4'[x4]^2 - a4''[x4], 0, 0\},\ \{0, 0, 0, 0, 0, 0, 15 H^2 - 3 a4'[x4]^2 - a4''[x4], 0\},\ \{0, 0, 0, 0, 0, 0, 0, 15 H^2 - 3 a4'[x4]^2 - a4''[x4]\}\}$$

In[ ]:= **allLovelock2 = MyArrayComponents@Lovelock2**

0.522352

Out[ ]=

```
{{12 (-15 H^4 + 14 H^2 a4'[x4]^2 + a4'[x4]^4), 0, 0, 0, 0, 0, 0, 0},
 {0, -4 (-3 a4'[x4]^4 + 6 a4'[x4]^2 (-7 H^2 + a4''[x4]) + 5 H^2 (9 H^2 + 2 a4''[x4])), 0, 0, 0, 0, 0, 0},
 {0, 0, -4 (-3 a4'[x4]^4 + 6 a4'[x4]^2 (-7 H^2 + a4''[x4]) + 5 H^2 (9 H^2 + 2 a4''[x4])), 0, 0, 0, 0, 0},
 {0, 0, 0, -4 (-3 a4'[x4]^4 + 6 a4'[x4]^2 (-7 H^2 + a4''[x4]) + 5 H^2 (9 H^2 + 2 a4''[x4])), 0, 0, 0, 0},
 {0, 0, 0, 0, -12 (35 H^4 + 10 H^2 a4'[x4]^2 + 3 a4'[x4]^4), 0, 0, 0},
 {0, 0, 0, 0, 0, 4 (-45 H^4 + 3 a4'[x4]^4 + 10 H^2 a4''[x4] + 6 a4'[x4]^2 (7 H^2 + a4''[x4])), 0, 0},
 {0, 0, 0, 0, 0, 0, 4 (-45 H^4 + 3 a4'[x4]^4 + 10 H^2 a4''[x4] + 6 a4'[x4]^2 (7 H^2 + a4''[x4])), 0},
 {0, 0, 0, 0, 0, 0, 0, 4 (-45 H^4 + 3 a4'[x4]^4 + 10 H^2 a4''[x4] + 6 a4'[x4]^2 (7 H^2 + a4''[x4]))}}
```

In[ ]:= **allLovelock3 = MyArrayComponents@Lovelock3**

2.78034

Out[ ]=

```
{{72 (5 H^6 - 27 H^4 a4'[x4]^2 - 9 H^2 a4'[x4]^4 - a4'[x4]^6), 0, 0, 0, 0, 0, 0, 0},
 {0, 72 (-a4'[x4]^6 + 5 H^4 (H^2 + a4''[x4]) + a4'[x4]^4 (-9 H^2 + 5 a4''[x4]) +
     a4'[x4]^2 (-27 H^4 + 6 H^2 a4''[x4])), 0, 0, 0, 0, 0, 0}, {0, 0, 72
   (-a4'[x4]^6 + 5 H^4 (H^2 + a4''[x4]) + a4'[x4]^4 (-9 H^2 + 5 a4''[x4]) + a4'[x4]^2 (-27 H^4 + 6 H^2 a4''[x4])),
  0, 0, 0, 0, 0}, {0, 0, 0, 72 (-a4'[x4]^6 + 5 H^4 (H^2 + a4''[x4]) +
     a4'[x4]^4 (-9 H^2 + 5 a4''[x4]) + a4'[x4]^2 (-27 H^4 + 6 H^2 a4''[x4])), 0, 0, 0, 0},
 {0, 0, 0, 0, 72 (35 H^6 + 15 H^4 a4'[x4]^2 + 9 H^2 a4'[x4]^4 + 5 a4'[x4]^6), 0, 0, 0}, {0, 0, 0, 0, 0,
  72 (-a4'[x4]^6 + 5 H^4 (H^2 - a4''[x4]) - a4'[x4]^4 (9 H^2 + 5 a4''[x4]) - 3 a4'[x4]^2 (9 H^4 + 2 H^2 a4''[x4])),
  0, 0}, {0, 0, 0, 0, 0, 0,
  72 (-a4'[x4]^6 + 5 H^4 (H^2 - a4''[x4]) - a4'[x4]^4 (9 H^2 + 5 a4''[x4]) - 3 a4'[x4]^2 (9 H^4 + 2 H^2 a4''[x4])),
  0}, {0, 0, 0, 0, 0, 0, 0,
  72 (-a4'[x4]^6 + 5 H^4 (H^2 - a4''[x4]) - a4'[x4]^4 (9 H^2 + 5 a4''[x4]) - 3 a4'[x4]^2 (9 H^4 + 2 H^2 a4''[x4]))}}
```

Einstein - Lovelock vacuum field equations

In[ ]:=
```
(EinsteinLovelockVacuumFieldEquations =
   (-Λ IdentityMatrix[8] + H^-2 w1 allLovelock1 + H^-4 w2 allLovelock2 + H^-6 w3 allLovelock3)) //
 MatrixForm[#, TableAlignments → Left] &
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -\Lambda + \frac{w1\,(15\,H^2-3\,a4'[x4]^2)}{H^2} + \frac{12\,w2\,(-15\,H^4+14\,H^2\,a4'[x4]^2+a4'[x4]^4)}{H^4} + \frac{72\,w3\,(5\,H^6-27\,H^4\,a4'[x4]^2-9\,H^2\,a4'[x4]^4-a4'[x4]^6)}{H^6} & 0 \\ 0 & -\Lambda + \frac{w1\,(15\,H^2-}{} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

0== EinsteinLovelockVacuumFieldEquations

In[ ]:=
```
(*(EinsteinLovelockVacuumFieldEquations=
   (allEL0+H^-2w1 allLovelock1+H^-4w2 allLovelock2+H^-6w3 allLovelock3))//
 MatrixForm[#,TableAlignments→Left]&*)
```

In[ ]:=
```
Union[Flatten[EinsteinLovelockVacuumFieldEquations]]
Drop[%, 1]
(eqsELei00 = %) // Column
```

Out[•]=

$$\Big\{0,\ -\Lambda + \frac{w1\,(15\,H^2 - 3\,a4'[x4]^2)}{H^2} + \frac{12\,w2\,(-15\,H^4 + 14\,H^2\,a4'[x4]^2 + a4'[x4]^4)}{H^4} + \frac{72\,w3\,(5\,H^6 - 27\,H^4\,a4'[x4]^2 - 9\,H^2\,a4'[x4]^4 - a4'[x4]^6)}{H^6},$$

$$-\Lambda + \frac{3\,w1\,(7\,H^2 + a4'[x4]^2)}{H^2} - \frac{12\,w2\,(35\,H^4 + 10\,H^2\,a4'[x4]^2 + 3\,a4'[x4]^4)}{H^4} + \frac{72\,w3\,(35\,H^6 + 15\,H^4\,a4'[x4]^2 + 9\,H^2\,a4'[x4]^4 + 5\,a4'[x4]^6)}{H^6},\ -\Lambda + \frac{w1\,(15\,H^2 - 3\,a4'[x4]^2 - a4''[x4])}{H^2} +$$

$$\frac{4\,w2\,(-45\,H^4 + 3\,a4'[x4]^4 + 10\,H^2\,a4''[x4] + 6\,a4'[x4]^2\,(7\,H^2 + a4''[x4]))}{H^4} + \frac{1}{H^6}\,72\,w3$$

$$(-a4'[x4]^6 + 5\,H^4\,(H^2 - a4''[x4]) - a4'[x4]^4\,(9\,H^2 + 5\,a4''[x4]) - 3\,a4'[x4]^2\,(9\,H^4 + 2\,H^2\,a4''[x4])),$$

$$-\Lambda + \frac{w1\,(15\,H^2 - 3\,a4'[x4]^2 + a4''[x4])}{H^2} - \frac{4\,w2\,(-3\,a4'[x4]^4 + 6\,a4'[x4]^2\,(-7\,H^2 + a4''[x4]) + 5\,H^2\,(9\,H^2 + 2\,a4''[x4]))}{H^4} + \frac{1}{H^6}\,72\,w3$$

$$(-a4'[x4]^6 + 5\,H^4\,(H^2 + a4''[x4]) + a4'[x4]^4\,(-9\,H^2 + 5\,a4''[x4]) + a4'[x4]^2\,(-27\,H^4 + 6\,H^2\,a4''[x4]))\Big\}$$

Out[•]=

$$\Big\{-\Lambda + \frac{w1\,(15\,H^2 - 3\,a4'[x4]^2)}{H^2} + \frac{12\,w2\,(-15\,H^4 + 14\,H^2\,a4'[x4]^2 + a4'[x4]^4)}{H^4} + \frac{72\,w3\,(5\,H^6 - 27\,H^4\,a4'[x4]^2 - 9\,H^2\,a4'[x4]^4 - a4'[x4]^6)}{H^6},$$

$$-\Lambda + \frac{3\,w1\,(7\,H^2 + a4'[x4]^2)}{H^2} - \frac{12\,w2\,(35\,H^4 + 10\,H^2\,a4'[x4]^2 + 3\,a4'[x4]^4)}{H^4} + \frac{72\,w3\,(35\,H^6 + 15\,H^4\,a4'[x4]^2 + 9\,H^2\,a4'[x4]^4 + 5\,a4'[x4]^6)}{H^6},\ -\Lambda + \frac{w1\,(15\,H^2 - 3\,a4'[x4]^2 - a4''[x4])}{H^2} +$$

$$\frac{4\,w2\,(-45\,H^4 + 3\,a4'[x4]^4 + 10\,H^2\,a4''[x4] + 6\,a4'[x4]^2\,(7\,H^2 + a4''[x4]))}{H^4} + \frac{1}{H^6}\,72\,w3$$

$$(-a4'[x4]^6 + 5\,H^4\,(H^2 - a4''[x4]) - a4'[x4]^4\,(9\,H^2 + 5\,a4''[x4]) - 3\,a4'[x4]^2\,(9\,H^4 + 2\,H^2\,a4''[x4])),$$

$$-\Lambda + \frac{w1\,(15\,H^2 - 3\,a4'[x4]^2 + a4''[x4])}{H^2} - \frac{4\,w2\,(-3\,a4'[x4]^4 + 6\,a4'[x4]^2\,(-7\,H^2 + a4''[x4]) + 5\,H^2\,(9\,H^2 + 2\,a4''[x4]))}{H^4} + \frac{1}{H^6}\,72\,w3$$

$$(-a4'[x4]^6 + 5\,H^4\,(H^2 + a4''[x4]) + a4'[x4]^4\,(-9\,H^2 + 5\,a4''[x4]) + a4'[x4]^2\,(-27\,H^4 + 6\,H^2\,a4''[x4]))\Big\}$$

Out[∘]=

$$-\Lambda+\frac{w1\left(15H^2-3\,a4'[x4]^2\right)}{H^2}+\frac{12\,w2\left(-15H^4+14H^2\,a4'[x4]^2+a4'[x4]^4\right)}{H^4}+\frac{72\,w3\left(5H^6-27H^4\,a4'[x4]^2-9H^2\,a4'[x4]^4-a4'[x4]^6\right)}{H^6}$$

$$-\Lambda+\frac{3\,w1\left(7H^2+a4'[x4]^2\right)}{H^2}-\frac{12\,w2\left(35H^4+10H^2\,a4'[x4]^2+3\,a4'[x4]^4\right)}{H^4}+\frac{72\,w3\left(35H^6+15H^4\,a4'[x4]^2+9H^2\,a4'[x4]^4+5\,a4'[x4]^6\right)}{H^6}$$

$$-\Lambda+\frac{w1\left(15H^2-3\,a4'[x4]^2-a4''[x4]\right)}{H^2}+\frac{4\,w2\left(-45H^4+3\,a4'[x4]^4+10H^2\,a4''[x4]+6\,a4'[x4]^2\left(7H^2+a4''[x4]\right)\right)}{H^4}+\frac{72\,w3\left(-a4'[x4]^6+5H^4\left(H^2-a4''[x4]\right)-a4'[x4]^4\left(9H^2+5\,a4''[x4]\right)-3\,a4'[x4]^2\left(9H^4+2H^2\,a4''[x4]\right)\right)}{H^6}$$

$$-\Lambda+\frac{w1\left(15H^2-3\,a4'[x4]^2+a4''[x4]\right)}{H^2}-\frac{4\,w2\left(-3\,a4'[x4]^4+6\,a4'[x4]^2\left(-7H^2+a4''[x4]\right)+5H^2\left(9H^2+2\,a4''[x4]\right)\right)}{H^4}+\frac{72\,w3\left(-a4'[x4]^6+5H^4\left(H^2+a4''[x4]\right)+a4'[x4]^4\left(-9H^2+5\,a4''[x4]\right)+a4'[x4]^2\left(-27H^4+6H^2\,a4''[x4]\right)\right)}{H^6}$$

Solution exhibiting

Eternal Inflation/Deflation :

$$\left\{w1\to\text{constant }(==1\text{ is OK}),\ \frac{w2}{w1}\to\frac{1}{2^5},\ \frac{w3}{w1}\to\frac{1}{2^5\times2^2\times3^2},\ \frac{\Lambda}{w1}\to10,\ a4'[x4]\to H\right\}$$

In[∘]:=

```
almightyWs =
 FullSimplify[Solve[And @@ Thread[eqsELei00 == Table[0, {Length[eqsELei00]}]], {w2, w3, Λ}],
  H > 0 && a4′[x4] > 0]
(*  CHECK  *) FullSimplify[(EinsteinLovelockVacuumFieldEquations) /. #, H > 0] & /@ %
```

Out[∘]=

$$\left\{\left\{w2\to\frac{H^2\,w1\left(H^2+a4'[x4]^2\right)}{4\left(5H^4+10H^2\,a4'[x4]^2+a4'[x4]^4\right)},\ w3\to\frac{H^4\,w1}{72\left(5H^4+10H^2\,a4'[x4]^2+a4'[x4]^4\right)},\ \Lambda\to w1\left(31-\frac{a4'[x4]^2}{H^2}-\frac{40H^2\left(3H^2+5\,a4'[x4]^2\right)}{5H^4+10H^2\,a4'[x4]^2+a4'[x4]^4}\right)\right\}\right\}$$

Out[∘]=

{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}

{w1, w2, w3, Λ} should be constants (⇔ **a4′[x4] → ±const**):

In[∘]:=
```
almightyWs /. {a4′[x4] → const} // FullSimplify[#, H > 0] &
% /. {const → H}
```

Out[∘]=

$$\left\{\left\{w2 \to \frac{H^2 \left(const^2 + H^2\right) w1}{4 \left(const^4 + 10\, const^2 H^2 + 5 H^4\right)},\ w3 \to \frac{H^4\, w1}{72 \left(const^4 + 10\, const^2 H^2 + 5 H^4\right)},\ \Lambda \to \left(7 - \frac{const^2}{H^2} + \frac{8 \left(3\, const^4 + 5\, const^2 H^2\right)}{const^4 + 10\, const^2 H^2 + 5 H^4}\right) w1\right\}\right\}$$

Out[∘]=

$$\left\{\left\{w2 \to \frac{w1}{32},\ w3 \to \frac{w1}{1152},\ \Lambda \to 10\, w1\right\}\right\}$$

In[∘]:=
```
almightyWs /. {a4 → ((const * # + const2) &)} // FullSimplify[#, H > 0] &
% /. {const → H}
```

Out[∘]=

$$\left\{\left\{w2 \to \frac{H^2 \left(const^2 + H^2\right) w1}{4 \left(const^4 + 10\, const^2 H^2 + 5 H^4\right)},\ w3 \to \frac{H^4\, w1}{72 \left(const^4 + 10\, const^2 H^2 + 5 H^4\right)},\ \Lambda \to \left(7 - \frac{const^2}{H^2} + \frac{8 \left(3\, const^4 + 5\, const^2 H^2\right)}{const^4 + 10\, const^2 H^2 + 5 H^4}\right) w1\right\}\right\}$$

Out[∘]=

$$\left\{\left\{w2 \to \frac{w1}{32},\ w3 \to \frac{w1}{1152},\ \Lambda \to 10\, w1\right\}\right\}$$