

Scratch work of Author

NOTES

HYPOTHESIS : If , employing the Einstein eqs,
superluminal inflation / deflation exists,
then at time $x_4 = 0$ (before the particles of the standard model exist)
a pair of universes with MASSES $\pm M$ is created
(i.e., universes are created in pairs)

THE FOLLOWING SCRATCH IS CONSISTENT WITH
UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES $\pm M$.

Note that we are ONLY looking for
superluminal inflation or deflation type solutions,
and NOT solutions that are even / odd functions of t
(like $\cos[\nu[j]^\theta * t]$, $\sin[\nu[j]^\theta * t]$,
 $\text{Sech}[\nu[j]^\theta * t]$, $\text{Tanh}[\nu[j]^\theta * t]$, ...),
which also have $\pm M$ type eigenvalues,
or even solutions involving the well-known special functions that might also have $\pm M$ type eigenvalues.

Bigger Bang:

Question: Are Universe (s) of masses $\pm M$ created in pairs at time $x_4 = 0$

(before the particles of the standard model exist) ?

Scratch work

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Do not read if you are irascible; I apologize for typos.

git clone <https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git>

“git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git”

Ever wonder why there are 3 space dimensions and 1 time dimension?

Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in \text{Nature}$?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\| = \|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over \mathbb{R}) real numbers \mathbb{R} , the 2-dim complex numbers, the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in “A Remarkable Representation of the 3+2 de Sitter Group” by P.A.M.Dirac, J.Math.-Phys.4,901–909 (1963).

Comment: The term “reduced Brauer–Weyl generators”, below, refers to a specific set of irreducible generators for certain com-

plex Clifford algebras, particularly in the context of mathematical physics and the study of Dirac spinors. These generators were introduced by **E.A. Lord** (E. A. Lord. “The Dirac spinor in six dimensions”. Mathematical Proceedings of the Cambridge Philosophical Society, vol. 64, pp. 765–778, 1968) and are described as “reduced” because they form an irreducible representation of the Clifford algebra C_{2n} from $C_{2(n-1)}$.

Here, we employ Lord’s idea of reduced Brauer–Weyl generators to extend Dirac’s 4×4 gamma matrices into this notebook’s real tau8 8×8 and tau16 16×16 matrices, which are employed to write down the ‘Dirac Equation for the Universe’, [remember, this is before the particles of the standard model exist]

formulated in terms of a $\overline{O(4,4)}$ spinor Ψ_{16} (Ψ_{16} AKA ‘**WAVE FUNCTION OF the UNIVERSE**,’ which could possibly be named something more pompous).

Then this equation is used to

[WARNING: syncope, presyncope AHEAD]

couple the **WAVE FUNCTION OF the UNIVERSE** to Gravity.

THIS WORK IS PARTIALLY BASED ON :

JOURNAL OF MATHEMATICAL PHYSICS

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A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. DIRAC

*Cambridge University, Cambridge, England and
Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

*Proc. Camb. Phil. Soc. (1968), 64, 765
PCPS 64–96
Printed in Great Britain*

765

The Dirac spinor in six dimensions

By E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

Identities satisfied by the generators of the Dirac algebra

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(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin- $\frac{1}{2}$ wave equation

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(Received 24 January 1984; accepted for publication 16 November 1984)

Here we use spacetime coordinates that are selected so that we are somewhat consistent with the exact libraries,

(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):

Cartesian coordinates:

x_0 = hidden space (a small circle); x_1, x_2, x_3 are the usual 3-space coords;

x_4 = time coord,

and x_5, x_6, x_7 = superluminal deflating time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{\text{SO}(4, 4; \mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M8 , whose tangent bundle has

$\text{SO}(4, 4; \mathbb{R}) \approx \text{Spin}(4, 4; \mathbb{R})$

as iso group, and in which

3 of the 4 space dimensions **superluminally INFLATE**,

3 of the 4 time dimensions **superluminally DEFLATE** (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),

and the 4th space dim, x_0 , curls up into a ring (hence a particle whose wave function penetrates this ring acquires a mass contribution).

In passing we remark that an analogous construction may be defined on an octonion space of **one time dimension and seven space dimensions**, if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

Unsolved problem 1 (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group

$SU(3) \otimes \text{SU}(2)_L \otimes U(1)_Y$;

The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

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The Einstein - Lovelock vacuum field equations are

$$A_a^b = 0, \text{ where}$$

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta_{jj_1 \dots j_{2k}}^{hh_1 \dots h_{2k}} R^{j_1 j_2}_{ h_1 h_2} \dots R^{j_{2k-1} j_{2k}}_{\phantom{j_{2k-1} j_{2k}} h_{2k-1} h_{2k}} + \lambda \sqrt{g} g^{lh}, \quad (4.3)$$

where $\alpha_{(k)}, \lambda$ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

and $m-1 = \frac{8}{2} - 1 = 3$.

Citation: Tensors, Differential Forms,
and Variational Principles (Dover Books on Mathematics),
by David Lovelock and Hanno Rund

Let {w1, w2, w3, Δ} be pure numbers;

then by Eq. [4.38] the vacuum equations are

$$0 = -\Delta + H^{-2} w1 \text{ Lovelock1} + H^{-4} w2 \text{ Lovelock2} + H^{-6} w3 \text{ Lovelock3}$$

in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein - Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre - Hubble parameter. For now, let's pretend that they are independent.

NOTES :

Under the spacetime coordinate transformation

$$x^k \mapsto \bar{x}^k (x^j)$$

according to

$$\Psi_{16}(x) \mapsto \overline{\Psi_{16}(\bar{x})} = S(x(\bar{x})) \cdot \Psi_{16}(x(\bar{x})), \text{ i.e.,}$$

$$\overline{\Psi_{16}^{(a)}(\bar{x})} = (S_{(b)}^{(a)} \Psi_{16}^{(b)}(x(\bar{x})))$$

Therefore,

$$\frac{\partial}{\partial x^k} \overline{\Psi_{16}(\bar{x})} =$$

$$\frac{\partial x^j}{\partial x^k} \frac{\partial}{\partial x^j} S(x) \cdot \Psi_{16}(x) = \frac{\partial x^j}{\partial x^k} \left\{ s(x) \cdot \frac{\partial}{\partial x^j} \Psi_{16}(x) + \left[\frac{\partial}{\partial x^j} s(x) \right] \cdot \Psi_{16}(x) \right\}$$

Identify the Octad connection : $\frac{\partial}{\partial x^j} s(x) =$

$$-s \cdot \Gamma_j(x) + \frac{\partial x^k}{\partial x^j} \overline{\Gamma_k(\bar{x})} \cdot s$$

hence

$$\frac{\partial}{\partial x^k} \overline{\Psi_{16}(\bar{x})} - \overline{\Gamma_k} \cdot \overline{\Psi_{16}(\bar{x})} = S(x(\bar{x})) \cdot \frac{\partial x^j}{\partial x^k}$$

$$\left\{ \frac{\partial}{\partial x^j} \Psi_{16}(x(\bar{x})) - \Gamma_j \cdot \Psi_{16}(x(\bar{x})) \right\}$$

or

$$\left(1_{16 \times 16} \frac{\partial}{\partial x^k} - \overline{\Gamma_k} \right) \cdot \overline{\Psi_{16}(\bar{x})} =$$

$$S(x(\bar{x})) \cdot \left[\frac{\partial x^j}{\partial x^k} \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^j} - \Gamma_j \right\} \cdot_{\Psi 16} (x(\bar{x})) \right]$$

The connection transforms according to

$$\bar{\Gamma}_k(\bar{x}) = \frac{\partial \bar{x}^j}{\partial x^k} \left\{ S \cdot \Gamma_j(x) \cdot S^{-1} + S_{,j} \cdot S^{-1} \right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2} [e_k^{(a)} \nabla_j e_{(b)}^k] SAB^{(b)(a)}$$

Here, spin connection

$$\text{coefficients are } e_k^{(a)} (\nabla_j e_{(b)}^k)$$

and

$$S = \exp \left(\frac{1}{2} \omega_{(A)(B)}(x) SAB^{(A)(B)} \right),$$

where $\omega_{(B)(A)} = -\omega_{(A)(B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the $\Psi 16$ field [A.K.A., wave function characterizing the Universe(s) of masses $\pm M$];

TODO: prove Universe(s) of masses $\pm M$ are created in pairs!

K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:

- $\nabla_\mu e^a{}_\nu = 0$
- Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

apologies, I lost the citation for this :

No it is not the torsionless condition which is:

$$T_{\mu\nu}^I = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

? SOURCE terms

TU ^{$\mu\nu$} for $g_{\alpha\beta}$ come from
 "Universes' Wave Function
 Ψ^{16} Lagrangian"

$$\frac{1}{\mathcal{K}} \text{TU}^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lagrangian}_{\Psi16} \right)$$

(hope that $\text{TU}^{\mu\nu} = \Lambda g^{\mu\nu}$, and \times

$H = \text{some function of } M$, where \times

Universe(s) of masses \pm

M created in pairs at time $x_4 = 0$,

before the particles

of the standard model exist);

WARNING:

Universes $\Psi16$ source $g_{\alpha\beta}$;

The Euler–Lagrange equations for $\Psi16$ must have ‘solutions’ such that

all off-diagonal terms of $\text{TU}^{\mu\nu}$ ARE ZERO.

The $\Psi16$ Lagrangian (please see below) =

$$\sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg}[] = -$$

$$\sqrt{\text{Det}[g_{\mu\nu}]} * \left(\text{Transpose}[\Psi16].\sigma16. \right)$$

$$\text{Sum} \left[T16^\alpha [\alpha 1 - 1] . \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^{\alpha 1 - 1}} - \Gamma_{\alpha 1 - 1} \right\} . \right]$$

$$\Psi16, \{\alpha 1, 1, \text{Length}[X]\} \right] +$$

$$\frac{\text{mAss}}{2} * \text{Transpose}[\Psi16] . \sigma16 . \Psi16 \Big)$$

Next,

assume that $\Psi16soln$ solves the Euler – Lagrange equations.

SOURCE terms

$TU^{\mu\nu}$ [evaluate terms after performing differentiation] (using Lagrangian $\Psi16 \equiv$
 $(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{ Lg}[])$ | $_{\Psi16=\text{solution-toEL-eqs}}$) :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{ Lg}[] \right)$$

$$= \left(\text{Lg}[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]}) \right] + \frac{\partial}{\partial g_{\mu\nu}} (\text{Lg}[]) \right) \Big|_{\Psi16=\Psi16soln}$$

$$= \left(\Theta ? * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]}) + \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\text{Lg}[]) \right) \Big|_{\Psi16=\Psi16soln}$$

$$\begin{aligned}
&= \frac{\partial}{\partial \mathbf{g}_{\mu\nu}} \left(\text{Transpose}[\Psi16] . \right. \\
&\quad \sigma16.\text{Sum}\left[\left(D[\Psi16, X[\alpha1]] + \right. \right. \\
&\quad \left(\left(\frac{1}{2} \right) \text{connectionMatrix} \right) . \\
&\quad \text{Sum}[\omega[\alpha1, a, b] * \text{SAB}[a, b], \\
&\quad \{a, 1, 8\}, \{b, 1, 8\}] . \Psi16 \Big), \\
&\quad \left. \left. \{ \alpha1, 1, \text{Length}[X] \} \right] + \text{mASs} * \right. \\
&\quad \text{Transpose}[\Psi16].\text{symm16[j, 1]]. \\
&\quad \left. \Psi16 \right) \quad \Big|_{\Psi16=\Psi16soln} \\
&= \text{Transpose}[\Psi16].\sigma16. \\
&\quad \text{Sum}\left[\frac{\partial}{\partial \mathbf{g}_{\mu\nu}} (\text{T16}^\alpha[\alpha1 - 1]) . \Psi16_{\alpha1-1}, \right. \\
&\quad \left. \{ \alpha1, 1, \text{Length}[X] \} \right] \Big|_{\Psi16=solution_toEL_eqs} \\
&= \\
&\quad \tilde{\Psi16} . \sigma16. \text{T16}^A . \Psi16_{,\alpha} \frac{\partial}{\partial \mathbf{g}_{\mu\nu}} \left(g^{-1}{}^{\alpha\beta} \mathbf{e}_\beta^B \eta_{BA} \right) = \\
&\quad \tilde{\Psi16} . \sigma16. \text{T16}^A . \Psi16_{,\alpha} \frac{\partial}{\partial \mathbf{g}_{\mu\nu}} \left(\mathbf{e}_{(A)}^\alpha \right)
\end{aligned}$$

NOTES :

Let g be a square matrix;

we could use : $\frac{\partial}{\partial q} \left(g^{-1} \right) = - g^{-1} \cdot \frac{\partial g}{\partial q} \cdot g^{-1}$,

where q is a parameter

(result from $\frac{\partial}{\partial q} \left(g \cdot g^{-1} \right) = 0$),

In this notebook, g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] , g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}} \left(g^{-1} \right) = - g^{-1} \cdot \frac{\partial g}{\partial g_{\mu\nu}} \cdot g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ;
note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are
independent. In metric matrix g ,

we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate, for example.

This should be

understood before reading further.

Therefore $\frac{\partial}{\partial g_{\mu\nu}} \left(e_{(A)}^{\alpha} \right) =$
 $\frac{\partial}{\partial g_{\mu\nu}} \left(\text{Inverse}[e_{\alpha}^{(A)}] \right) =$

$$\begin{aligned}
 & - \text{Inverse} [e_{\alpha}^{(A)}] \cdot \frac{\partial e_{\alpha}^{(A)}}{\partial g_{\mu\nu}} \cdot \text{Inverse} [e_{\alpha}^{(A)}] = \\
 & - e_{(A)}^{\alpha} [\alpha 1, A1] \frac{\partial e_{\alpha}^{(A)} [\alpha 1, \alpha 2]}{\partial g_{\mu\nu}} \cdot e_{(A)}^{\alpha} [\alpha 2, A2]
 \end{aligned}$$

$$\begin{aligned}
 \text{Consider } \frac{\partial}{\partial g_{\mu\nu}} (g_{\alpha\beta}) &= \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_{\alpha}^{(A)} e_{\beta}^{(B)}) = \\
 \eta_{AB} e_{\beta}^{(B)} \frac{\partial}{\partial g_{\mu\nu}} (e_{\alpha}^{(A)}) + \eta_{AB} e_{\alpha}^{(B)} \frac{\partial}{\partial g_{\mu\nu}} (e_{\beta}^{(A)}) ; \\
 e_{C}^{\beta} \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} &= \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_{\alpha}^{(A)} e_{\beta}^{(B)}) = \\
 \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}} (e_{\alpha}^{(A)}) + \eta_{AB} e_{\alpha}^{(B)} e_{C}^{\beta} \frac{\partial}{\partial g_{\mu\nu}} (e_{\beta}^{(A)})
 \end{aligned}$$

In metric matrix g ,

we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,
and then differentiate.

Some of my calculations employ “http://www.xact.es/download-xAct_1.2.0.tgz” ; also, see

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<http://www2.iap.fr/users/pitrou/>
“FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb”
MyArrayComponents[expr_] := expr //ToBasis[BS] //Componen-
tArray //ToValues //ToValues //Simplify

I copied and pasted too often; however, I am going to leave this here.

Universe sources for $g_{\alpha\beta}$;

be sure to append these to the Einstein and/or Einstein–Love-lock field equations.

WARNING: all off-diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous ‘hacked solutions’ for $\Psi16$ to be valid

Let $T^\alpha = \{T16^\alpha[0], T16^\alpha[4]\}$ and

```
 $\sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg[]}[\text{mASs}_-, j_] =$ 
 $\sqrt{\text{Det}[g_{\mu\nu}]} * (\text{Transpose}[\Psi16].\sigma16.\text{Sum}[$ 
 $T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\}] +$ 
 $\text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[134, 1].\Psi16)$ 
```

Let $j = 134$, $\sigma16.\text{(mass Matrix)} = T16^\alpha[5].T16^\alpha[6].T16^\alpha[7]$

and

```
usingLagrangianF16massive = ( $\text{Transpose}[\Psi16].\sigma16.$ 
 $\text{Sum}[T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\}] +$ 
 $\text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[j = 134, 1].\Psi16];$ 
```

where it is assumed that $\Psi16$ solves the Euler – Lagrange equations, above; then **usingLagrangianF16massive** = 0, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lg}[] \right)$$

$$= \left(\text{Lg}[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \right. \\ \left. \frac{\partial}{\partial g_{\mu\nu}} (\text{Lg}[]) \right) \quad \mid \text{Lg}[] = \text{usingLagrangianF16massive}$$

$$= \left(\Theta * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \right. \\ \left. \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\text{Lg}[]) \right) \quad \mid \text{Lg}[] = \text{usingLagrangianF16massive}$$

remark: the term

$$\underset{\partial g_{\mu\nu}}{\overset{\sim}{\Psi}} \cdot \sigma \cdot T \cdot 16^A \cdot \underset{\alpha}{\Psi} \cdot 16, \alpha \frac{\partial}{\partial g_{\mu\nu}} (e_A^\alpha) = \underset{\partial g_{\mu\nu}}{\overset{\sim}{\Psi}} \cdot \sigma \cdot T \cdot 16^A \cdot \underset{\alpha}{\Psi} \cdot 16, \alpha e_A^\alpha =$$

$$\begin{aligned} & \frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\underset{\sim}{\Psi}] \cdot \sigma \cdot 16 \cdot \right. \\ & \left. \underbrace{T \cdot 16^A[A1-1] \cdot \underset{\alpha}{\Psi} \cdot 16, \alpha \eta_{A1B} e_B^\beta g^{-1} \alpha \beta}_{\text{Sum}} \right) \\ & \text{Sum} \left[T^\alpha [\alpha 1 - 1] \cdot D[\underset{\sim}{\Psi}, \text{vars}[\alpha 1]], \{\alpha 1, 1, \text{Length}[\text{vars}]\} \right] \end{aligned}$$

Frame fields [edit]

We use a set of [vierbein](#) or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab}e_\mu^a e_\nu^b = \eta_{\mu\nu}.$$

The vierbein defines a local rest [frame](#), allowing the constant [Gamma matrices](#) to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a [section](#) of the [frame bundle](#), and so defines a local trivialization of the frame bundle.

Incomplete theories

Scientists

[snow]

[show]

v • t • e

Spin connection [edit]

To write down the equation we also need the [spin connection](#), also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e_a^\mu e_b^\nu = \delta_a^\nu.$$

The connection 1-form is then

$$\omega^\mu{}_{\nu a} := e_b^\mu \nabla_a e_b^\nu$$

where ∇_a is a [covariant derivative](#), or equivalently a choice of [connection](#) on the frame bundle, most often taken to be the [Levi-Civita connection](#).

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space $T_p M$ to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

[Raising and lowering indices](#) is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract [connection on a principal bundle](#), specifically on the [frame bundle](#), which is defined on any smooth manifold, but which restricts to an [orthonormal frame bundle](#) on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu_{\nu\rho} = e_\rho^\alpha \omega^\mu_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2} (\omega_{\nu\rho})_\mu \sigma^{\nu\rho} \psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu R(M^{\nu\rho}) v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $SO(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

OTHER WORK THAT I USED :

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A Remarkable Representation of the 3 + 2 de Sitter Group

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Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

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765

The Dirac spinor in six dimensions

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Identities satisfied by the generators of the Dirac algebra

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A new spin- $\frac{1}{2}$ wave equation

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Begin

```
In[1]:= Print["CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]
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In[2]:= Print["Please cite this work, and this web page, if you use it"]
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In[3]:= Print["For simplicity, metric gαβ= gαβ(x0,x1,x2,x3,x4,x5,x6,x7)= gαβ(x0,x4) = diag{g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4)},"]
For simplicity, metric gαβ= gαβ(x0,x1,x2,x3,x4,x5,x6,x7)= gαβ(x0,x4) = diag{g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4)},

In[4]:= Print["where g77(x0,x4) = g11(x0,-x4) "]
where g77(x0,x4) = g11(x0,-x4)

In[5]:= Unprotect[dir];
In[6]:= dir = FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]];
Protect[dir];
If[$Failed == SetDirectory[dir], {.CreateDirectory[dir], SetDirectory[dir]}];

In[7]:= names = FileNameSplit[NotebookFileName[]]

Out[7]= {C:, Users, nsh, Documents, 8-dim, 2025-12-13-Partitioned-Determinant,
EDIT-2025-12-25-Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-
Nash-ALT.nb}
```

```
In[]:= name = StringReplace[names[-1], "nb" → "mx"]
Out[]= EDIT-2025-12-25-Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-
Nash-ALT.mx

In[]:= header = StringReplace[names[-1], ".nb" → "-"]
Out[=] EDIT-2025-12-25-Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-
Nash-ALT-

In[]:= DIM8 = 8;
In[]:= Protect[DIM8, M, K, H]
Out[=] {DIM8, M, K, H}
```

Coordinates :

$\left\{ \underbrace{\text{hidden space}}, \underbrace{\text{3-space}}, \right.$
 $\left. \underbrace{\text{time}}_{\text{superluminal deflating time}} \right\}$
 $\{x_0, \overbrace{x_1, x_2, x_3}, x_4, \overbrace{x_5, x_6, x_7}\} :$

```
In[]:= X = {x0, x1, x2, x3, x4, x5, x6, x7};
Protect[X];
Protect[x0, x1, x2, x3, x4, x5, x6, x7];
```

```
In[]:= sX0 = And @@ Thread[X > 0]
Out[=] x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 && x7 > 0

In[]:= ssX = H > 0 && sX0 && 6 H x0 > 0 && 2 la[x4] > 0 && Cot[6 H x0] > 0 && Sin[6 H x0] > 0
Out[=] H > 0 && x0 > 0 && x1 > 0 && x2 > 0 && x3 > 0 && x4 > 0 && x5 > 0 && x6 > 0 &&
x7 > 0 && 6 H x0[] > 0 && 2 la[x4[]] > 0 && Cot[6 H x0[]] > 0 && Sin[6 H x0[]] > 0
```

For Maple_2025 :

```
In[]:= sreplaceZ = Z[#] → ToExpression["Z" <> ToString[#]] & /@ Range[0, 15]
Out[=] {Z[0] → Z0, Z[1] → Z1, Z[2] → Z2, Z[3] → Z3, Z[4] → Z4, Z[5] → Z5, Z[6] → Z6, Z[7] → Z7, Z[8] → Z8,
Z[9] → Z9, Z[10] → Z10, Z[11] → Z11, Z[12] → Z12, Z[13] → Z13, Z[14] → Z14, Z[15] → Z15}

In[]:= sreplacenZ = nZ[#] → ToExpression["nZ" <> ToString[#]] & /@ Range[0, 15]
Out[=] {nZ[0] → nZ0, nZ[1] → nZ1, nZ[2] → nZ2, nZ[3] → nZ3, nZ[4] → nZ4,
nZ[5] → nZ5, nZ[6] → nZ6, nZ[7] → nZ7, nZ[8] → nZ8, nZ[9] → nZ9, nZ[10] → nZ10,
nZ[11] → nZ11, nZ[12] → nZ12, nZ[13] → nZ13, nZ[14] → nZ14, nZ[15] → nZ15}
```

hacks (a.k.a., lies):

```
In[8]:= constraintTrig = Sin[6 * H * x0] > 0 && Cos[6 * H * x0] > 0 && Csc[6 * H * x0] > 0 &&
Sec[6 * H * x0] > 0 && Tan[6 * H * x0] > 0 && Cot[6 * H * x0] > 0 && Sin[z] > 0 &&  $\sqrt{\sin[z]}$  > 0 &&
Sin[z]3/2 > 0 && Sin[z]1/2 > 0 && Sin[z]-3/2 > 0 && Sin[z]-1/2 > 0 && Cot[z] > 0 &&  $\sqrt{\cot[z]}$  > 0 &&
Cot[z]3/2 > 0 && Cot[z]1/2 > 0 && Tan[z] > 0 && Sec[z] > 0 &&  $\sqrt{\csc[z]}$  > 0 && Csc[z] > 0
```

```
Out[8]= Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 &&  $\sqrt{\sin[z]}$  > 0 && Sin[z]3/2 > 0 &&
 $\sqrt{\sin[z]}$  > 0 &&  $\frac{1}{\sin[z]^{3/2}}$  > 0 &&  $\frac{1}{\sqrt{\sin[z]}}$  > 0 && Cot[z] > 0 &&  $\sqrt{\cot[z]}$  > 0 &&
Cot[z]3/2 > 0 &&  $\sqrt{\cot[z]}$  > 0 && Tan[z] > 0 && Sec[z] > 0 &&  $\sqrt{\csc[z]}$  > 0 && Csc[z] > 0
```

more hacks (damn lies):

```
In[9]:= (*&&K2-M2>0&& $\sqrt{K^2-M^2}$ >0&&e2 H \sqrt{K^2-M^2} x4>0&&2 H \sqrt{K^2-M^2} x4>0&&
2 H \sqrt{1-M^2} x4>0&& $\sqrt{1-M^2}$ >0&&e2 H \sqrt{1-M^2} x4>0&&2 H \sqrt{1-M^2} x4>0*)
```

```
In[10]:= constraintX =
x0 > 0 && x4 > 0 && H > 0 && 6 * H * x0 > 0 && 3 * H * x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && ea4[H x4] > 0 && ea4[H x4] > 0
```

```
Out[10]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && ea4[H x4] > 0 && ea4[H x4] > 0
```

statistics :

```
In[11]:= constraintVars = constraintX && constraintTrig
```

```
Out[11]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && ea4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 &&  $\sqrt{\sin[z]}$  > 0 && Sin[z]3/2 > 0 &&
 $\sqrt{\sin[z]}$  > 0 &&  $\frac{1}{\sin[z]^{3/2}}$  > 0 &&  $\frac{1}{\sqrt{\sin[z]}}$  > 0 && Cot[z] > 0 &&  $\sqrt{\cot[z]}$  > 0 &&
Cot[z]3/2 > 0 &&  $\sqrt{\cot[z]}$  > 0 && Tan[z] > 0 && Sec[z] > 0 &&  $\sqrt{\csc[z]}$  > 0 && Csc[z] > 0
```

```
In[]:= subsDefects =
{  $\sqrt{e^{2a^4[Hx4]}} \rightarrow e^{a^4[Hx4]}$ ,  $\sqrt{e^{-2a^4[Hx4]}} \rightarrow e^{-a^4[Hx4]}$ ,  $\sqrt{e^{2a^4[Hx4]} \sin[6Hx4]^{1/3}} \rightarrow e^{a^4[Hx4]} \sin[6Hx4]^{1/6}$ ,
 $\frac{1}{\sqrt{e^{2a^4[Hx4]} \sin[6Hx4]^{1/3}}} \rightarrow \frac{1}{e^{a^4[Hx4]} \sin[6Hx4]^{1/6}}$ ,
 $\frac{1}{\sqrt{e^{2a^4[Hx4]} \sin[6Hx0]^{1/6}}} \rightarrow \frac{1}{e^{a^4[Hx4]} \sin[6Hx0]^{1/6}}$ ,
 $\frac{1}{\sqrt{e^{-2a^4[Hx4]} \sin[6Hx0]^{1/6}}} \rightarrow \frac{1}{e^{-a^4[Hx4]} \sin[6Hx0]^{1/6}}$ ,  $\sqrt{e^{2a^4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow$ 
 $e^{a^4[Hx4]} \sin[6Hx0]^{1/6}$ ,  $\sqrt{e^{-2a^4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{-a^4[Hx4]} \sin[6Hx0]^{1/6}$  }
```

```
Out[]:= {  $\sqrt{e^{2a^4[Hx4]}} \rightarrow e^{a^4[Hx4]}$ ,  $\sqrt{e^{-2a^4[Hx4]}} \rightarrow e^{-a^4[Hx4]}$ ,  $\sqrt{e^{2a^4[Hx4]} \sin[6Hx4]^{1/3}} \rightarrow e^{a^4[Hx4]} \sin[6Hx4]^{1/6}$ ,
 $\frac{1}{\sqrt{e^{2a^4[Hx4]} \sin[6Hx4]^{1/3}}} \rightarrow \frac{e^{-a^4[Hx4]}}{\sin[6Hx4]^{1/6}}$ ,  $\frac{1}{\sqrt{e^{2a^4[Hx4]} \sin[6Hx0]^{1/6}}} \rightarrow \frac{e^{-a^4[Hx4]}}{\sin[6Hx0]^{1/6}}$ ,
 $\frac{1}{\sqrt{e^{-2a^4[Hx4]} \sin[6Hx0]^{1/6}}} \rightarrow \frac{e^{a^4[Hx4]}}{\sin[6Hx0]^{1/6}}$ ,  $\sqrt{e^{2a^4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{a^4[Hx4]} \sin[6Hx0]^{1/6}$ ,
 $\sqrt{e^{-2a^4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{-a^4[Hx4]} \sin[6Hx0]^{1/6}$  }
```

future coordinate transformation :

```
In[]:= 6Hx0 = z && Hx4 = t
szt = Solve[%, {z, t}] [[1]]
sx0x4 = Solve[%, {x0, x4}] [[1]]
Protect[sx0x4, szt];
```

```
Out[]:= 6Hx0 == z && Hx4 == t
```

```
Out[]:= {z → 6Hx0, t → Hx4}
```

```
Out[]:= {x0 →  $\frac{z}{6H}$ , x4 →  $\frac{t}{H}$ }
```

```
In[]:= sx0x4
```

```
Out[]:= {x0 →  $\frac{z}{6H}$ , x4 →  $\frac{t}{H}$ }
```

```
In[]:= (*sa4={a4→((A4[3 H #2])&) }*)
```

```
In[]:= Protect[sx0x4, szt]
```

```
Out[]:= {}
```

SO (4, 4) Minkowski Lorentz metric

$\eta_{4488} : 4 + 4$ spacetime; 8×8 dimensional :

```
In[8]:= (η4488 = ArrayFlatten[{{{{IdentityMatrix[4]}, {0}, {0, -IdentityMatrix[4]}}}}]) // MatrixForm
Out[8]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```

SO (4, 4) Spinor Lorentz metric σ ;

τ^A are analogous to

the Dirac gamma matrices :

all true : Table[{A, B, FullSimplify[

$$\frac{1}{2} (\tau[A].\bar{\tau}[B] + \tau[B].\bar{\tau}[A]) ==$$

$$\eta4488[[A + 1, B + 1]] * ID8]$$
],
{A, 0, 7}, {B, 0, 7}]]

Type - 1 and type -

2 employ σ (the usual notational abuse),
since $\sigma = \sigma^{-1}$: 4 + 4 spacetime,
8 × 8 dimensional and 16 × 16 dimensional :

indices. We define the reduced Brauer-Weyl generators $\{\bar{\tau}^{A'}, \tau^{A'}\}$ of the generators of the two real 8×8 inequivalent irreducible representations of $\overline{SO(4, 4)}$ (see eqs. (2.6) and (2.7)) by demanding that the tau matrices satisfy (the tilde denotes transpose)

$$(2.3) \quad \tilde{\tau}^{A'} \sigma = \sigma \bar{\tau}^{A'}$$

and

$$(2.4) \quad \tau^{A'} \bar{\tau}^{B'} + \tau^{B'} \bar{\tau}^{A'} = 2IG^{A'B'} = \bar{\tau}^{A'} \tau^{B'} + \bar{\tau}^{B'} \tau^{A'},$$

where I denotes the 8×8 unit matrix. Denoting the matrix elements of $\tau^{A'}$ by $\tau^{A'a}_b$, we may write eq. (3) as

$$(2.5) \quad \bar{\tau}^{A'}_{ab} = \tau^{A'}_{ba},$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix}$$

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4, 4)}$ and $L \in SO(4, 4)$. The canonical 2-1 homomorphism $\overline{SO(4, 4)} \rightarrow SO(4, 4)$ is given by

$$(2.8) \quad 8L^{A'}_{\ B'}(g) = \text{tr} \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4, 4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}_{\ C'} G_{A'B'} L^{B'}_{\ D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}_{\ B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}_{\ B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

```
In[=]:= (σ = ArrayFlatten[{{ConstantArray[0, {4, 4}], IdentityMatrix[4]}, {IdentityMatrix[4], ConstantArray[0, {4, 4}]}}]) // MatrixForm
Out[=]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

below :

```
In[=]:= σ16 == ArrayFlatten[{{{-σ, 0}, {0, σ}}}]
```

below :

```
In[=]:= Table[T16^A[A1] = ArrayFlatten[ {{0, τ[A1]}, {τ[A1], 0}} ], {A1, 0, 7} ];
```

below :

```
In[=]:= σ16.T16^A[#[ ] == -Transpose[σ16.T16^A[#[ ] ] & /@ Range[0, 7]
```

Out[]=

{True, True, True, True, True, True, True, True}

below :

```
In[ ]:= Table[{{A1, B1}, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\text{T16}^A[\text{A1}] \cdot \text{T16}^A[\text{B1}] +$ 
 $\text{T16}^A[\text{B1}] \cdot \text{T16}^A[\text{A1}]) ==$ 
 $\eta 4488 [\text{A1} + 1, \text{B1} + 1] * \text{ID16}]}], {A1, 0, 7}, {B1, 0, 7}], // MatrixForm$ 
```

```
Out[=]//MatrixForm=

$$\begin{pmatrix} \{0, 0\} & \{0, 1\} & \{0, 2\} & \{0, 3\} & \{0, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} & \text{True} \\ \{1, 0\} & \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} & \text{True} \\ \{2, 0\} & \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} & \text{True} \\ \{3, 0\} & \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} & \text{True} \\ \{4, 0\} & \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} & \text{True} \\ \{5, 0\} & \{5, 1\} & \{5, 2\} & \{5, 3\} & \{5, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} & \text{True} \\ \{6, 0\} & \{6, 1\} & \{6, 2\} & \{6, 3\} & \{6, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} & \text{True} \\ \{7, 0\} & \{7, 1\} & \{7, 2\} & \{7, 3\} & \{7, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} & \text{True} \end{pmatrix}$$

```

defs; some Symbols; metric

$$\left\{ \left\{ a4[t] \rightarrow \frac{M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} + \right. \right.$$

$$\left. \left. c_1 + \frac{1}{2} \text{ProductLog}\left[-e^{-\frac{2 M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} - 2 c_1}\right]\right\} \right\}$$

DSolve[

$$\Theta = -M (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7]) + (Q1 - e^{-2 a4[t]} Q1) a4'[t], a4[t], t]$$

M c[j] = (j + 1)st Energy Eigenvalue for

Transpose[cayZ]. Ψ 16 , j = 0, ..., 7

ProductLog[z]

gives the principal solution for w in $z = we^w$.

if z and w are any complex numbers, then

$$we^w = z$$

holds if and only if

$$w = W_k(z) \text{ for some integer } k.$$

When dealing with real numbers only, the two branches W_0 and W_{-1} suffice: for real numbers x and y the equation

$$ye^y = x$$

can be solved for y only if $x \geq \frac{-1}{e}$; yields $y = W_0(x)$ if $x \geq 0$ and the two values $y = W_0(x)$ and $y = W_{-1}(x)$ if $\frac{-1}{e} \leq x < 0$.

$$\frac{2 M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} - 2 c_1 =$$

-1

$$\frac{2 M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} + 2 c_1 =$$

$$1, , t = 0 \Rightarrow c_1 = \frac{1}{2}$$

$$\frac{2 M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} + 1 = 1$$

$$\frac{2 M t (c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} =$$

0 \Rightarrow sum Energy Eigenvalues =

$$\sum_{j=0}^7 M c[j] = 0 \stackrel{?}{\Rightarrow} \text{if } +M \text{ then } -M$$

?todo: NEED DefScalarFunction[#]; & /@ (f16[#]&/@Range[0,15]), and allow xact to compute?

Introduce the wave function, $\Psi16$, for a Universe:

```
In[]:=  $\Psi_{16} = f16[\#][x_0, x_4] \& /@ Range[0, 15]$ 
```

Out[]:=

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

```
In[]:= (* $\Psi_{new16}=f16[\#][x_0, x_4] \& /@ Range[0, 15]$ *)
```

In[]:= Clear[sf16Aa];

```
sfp16Aa = f16[\#] → ToExpression[
  "((Z[" <> ToString[\#] <> "] [6 H\#1, H\#2]  $\frac{1}{\sqrt{\sin[6 H\#1]^{1/2}}}$ ) &)"] & /@ Range[0, 15]
```

Out[]:=

```
{f16[0] →  $\frac{Z[0][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &, f16[1] →  $\frac{Z[1][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &,
f16[2] →  $\frac{Z[2][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &, f16[3] →  $\frac{Z[3][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &,
f16[4] →  $\frac{Z[4][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &, f16[5] →  $\frac{Z[5][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &,
f16[6] →  $\frac{Z[6][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &, f16[7] →  $\frac{Z[7][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &,
f16[8] →  $\frac{Z[8][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &, f16[9] →  $\frac{Z[9][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &,
f16[10] →  $\frac{Z[10][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &, f16[11] →  $\frac{Z[11][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &,
f16[12] →  $\frac{Z[12][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &, f16[13] →  $\frac{Z[13][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &,
f16[14] →  $\frac{Z[14][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &, f16[15] →  $\frac{Z[15][6 H\#1, H\#2]}{\sqrt{\sin[6 H\#1]}}$  &}
```

```
In[1]:= Clear[snewf16Aa];
snewf16Aa = f16[#] & /@ Range[0, 15]
```

$$\frac{1}{\sin[6H\#1]^{1/2}})$$

```
Out[1]= {f16[0] &gt;> <math>\frac{nZ[0][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[1] &gt;> <math>\frac{nZ[1][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[2] &gt;> <math>\frac{nZ[2][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[3] &gt;> <math>\frac{nZ[3][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[4] &gt;> <math>\frac{nZ[4][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[5] &gt;> <math>\frac{nZ[5][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[6] &gt;> <math>\frac{nZ[6][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[7] &gt;> <math>\frac{nZ[7][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[8] &gt;> <math>\frac{nZ[8][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[9] &gt;> <math>\frac{nZ[9][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[10] &gt;> <math>\frac{nZ[10][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[11] &gt;> <math>\frac{nZ[11][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[12] &gt;> <math>\frac{nZ[12][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[13] &gt;> <math>\frac{nZ[13][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[14] &gt;> <math>\frac{nZ[14][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&, f16[15] &gt;> <math>\frac{nZ[15][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}}&&}

```

0 (4, 4) related :

```
In[2]:= Needs["Notation`"]
In[3]:= Symbolize[\tau]
In[4]:= Symbolize[newtau]
In[5]:= Symbolize[T16^A]
In[6]:= Symbolize[T16^a]
In[7]:= Symbolize[newT16^A]
In[8]:= Symbolize[newT16^a]
```

NEED DefScalarFunction[#]; & /@ Flatten[Universe]

Universe = ToExpression["U" \leftrightarrow ToString[#]] & /@ Range[0, 7]

```
In[9]:= Symbolize[F_a^A]
In[10]:= Symbolize[F_A^a]
```

```
In[=]:= Symbolize[gtrye $\alpha$ (A)]
In[=]:= Symbolize[gtrye(A) $\alpha$ ]
```

SPACETIME METRIC

```
In[=]:= (*einsteinsteinMetric=Array[g[#1-1,#2-1]&,{8,8}]*)
```

SPACETIME METRIC:

```
In[=]:= g4488 = Array[g[#1 - 1] [#2 - 1] [x0, x4] &, {8, 8}];
```

SPACETIME METRIC values:

MatrixMetric44

$$\begin{aligned} 6 H x0 &= z \quad \& \quad H x4 = t \\ \left\{ \left\{ a4''[t] \rightarrow 0, a4'[t] \rightarrow \frac{2}{3} (-1 + M) \right\}, \right. \\ \left. \left\{ a4''[t] \rightarrow 0, a4'[t] \rightarrow \frac{2 (1 + M)}{3} \right\} \right\} \end{aligned}$$

```
In[=]:= (* $\beta_3=\text{Exp}\left[2 H x4 \sqrt{K^2-M^2}\right];*$ )
(* $\beta_3=\text{Exp}[2 * a4[3 H x4]]$  ;*)
 $\beta_3 = \text{Exp}[2 * a4[H x4]] \text{(*}.\{a4 \rightarrow ((K1 \frac{2 (1+M)}{3} + K2 \frac{2}{3} (-1+M)) \#) \&)\}*)$ 
 $\beta_1 = \text{Sin}[6 * H x0]^{\frac{1}{3}};$ 
 $\beta_2 = \text{Cot}[6 * H x0]^2;$ 
```

```
Out[=]=
 $e^{2 a4[H x4]}$ 
```

```
In[=]:= MatrixForm[MatrixMetric44 =
\{ \{ \beta_2, 0, 0, 0, 0, 0, 0, 0 \}, \{ 0, \beta_1 \beta_3, 0, 0, 0, 0, 0, 0 \}, \{ 0, 0, \beta_1 \beta_3, 0, 0, 0, 0, 0 \},
\{ 0, 0, 0, \beta_1 \beta_3, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, -1, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0, -\frac{\beta_1}{\beta_3}, 0, 0 \}, \{ 0, 0,
0, 0, 0, -\frac{\beta_1}{\beta_3}, 0 \}, \{ 0, 0, 0, 0, 0, 0, -\frac{\beta_1}{\beta_3} \} \} // FullSimplify[\#, constraintVars] & ]
```

```
Out[=]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6 H x0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2 a4[H x4]} \text{Sin}[6 H x0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2 a4[H x4]} \text{Sin}[6 H x0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2 a4[H x4]} \text{Sin}[6 H x0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

```

In[1]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"] [6*H#1,3*H#2] 1/Sin[6*H#1]^(1/2))&)"]&/@Range[0,15]*)

In[2]:= (*Clear[sf16Aa];
sf16Aa=
f16[#]→ToExpression["((Z["<>ToString[#]<>"] [6*H#1,3*H#2])&)"]&/@Range[0,15]*)

In[3]:= (*Clear[sf16Aa];
sf16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"] [6*H#1,H#2] 1/Sin[6*H#1]^(1/2))&)"]&/@Range[0,15]*)

In[4]:= (*Inactivate[((q)&)]Evaluate[Activate[Evaluate[q]]]*)

In[5]:= Clear[makeSubstitution];
makeSubstitution[] := Module[{met, mat, t, f, p},
  met = Flatten[g4488][All, 0];
  mat = Flatten[MatrixMetric44];
  t = Thread[met → mat];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]

In[6]:= makeSubstitution[a_, b_] := Module[{t, f, p},
  t = Thread[a → b];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]

```

```
In[1]:= ssgm4488 = makeSubstitution[]
```

```
Out[1]= {g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
g[1][1] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
g[2][1] → (0 &), g[2][2] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
g[3][2] → (0 &), g[3][3] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
g[5][5] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
g[6][5] → (0 &), g[6][6] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &)}
```

```
In[2]:= Symbolize[Eα(A)]
```

```
In[3]:= Symbolize[eα(A)]
```

```
In[4]:= eα(A) = Array[Eα(A) [##1 - 1, ##2 - 1] &, {8, 8}]
```

```
Out[4]= { {Eα(A) [0, 0], Eα(A) [0, 1], Eα(A) [0, 2], Eα(A) [0, 3], Eα(A) [0, 4], Eα(A) [0, 5], Eα(A) [0, 6], Eα(A) [0, 7]}, {Eα(A) [1, 0], Eα(A) [1, 1], Eα(A) [1, 2], Eα(A) [1, 3], Eα(A) [1, 4], Eα(A) [1, 5], Eα(A) [1, 6], Eα(A) [1, 7]}, {Eα(A) [2, 0], Eα(A) [2, 1], Eα(A) [2, 2], Eα(A) [2, 3], Eα(A) [2, 4], Eα(A) [2, 5], Eα(A) [2, 6], Eα(A) [2, 7]}, {Eα(A) [3, 0], Eα(A) [3, 1], Eα(A) [3, 2], Eα(A) [3, 3], Eα(A) [3, 4], Eα(A) [3, 5], Eα(A) [3, 6], Eα(A) [3, 7]}, {Eα(A) [4, 0], Eα(A) [4, 1], Eα(A) [4, 2], Eα(A) [4, 3], Eα(A) [4, 4], Eα(A) [4, 5], Eα(A) [4, 6], Eα(A) [4, 7]}, {Eα(A) [5, 0], Eα(A) [5, 1], Eα(A) [5, 2], Eα(A) [5, 3], Eα(A) [5, 4], Eα(A) [5, 5], Eα(A) [5, 6], Eα(A) [5, 7]}, {Eα(A) [6, 0], Eα(A) [6, 1], Eα(A) [6, 2], Eα(A) [6, 3], Eα(A) [6, 4], Eα(A) [6, 5], Eα(A) [6, 6], Eα(A) [6, 7]}, {Eα(A) [7, 0], Eα(A) [7, 1], Eα(A) [7, 2], Eα(A) [7, 3], Eα(A) [7, 4], Eα(A) [7, 5], Eα(A) [7, 6], Eα(A) [7, 7]}}
```

```
In[5]:= (*Symbolize[seα(A)]*)
```

```
In[6]:= Symbolize[sgeα(A)]
```

```
In[7]:= Symbolize[sgtryeα(A)]
```

```
In[8]:= Symbolize[E(A)α]
```

```
In[9]:= Symbolize[sge(A)α]
```

```
In[10]:= Symbolize[sgtrye(A)α]
```

```
In[11]:= Symbolize[e(A)α]
```

```

In[]:= eα(A) = Array[Eα(A) [##1 - 1, ##2 - 1] &, {8, 8}]

Out[]=
{ {Eα(A) [0, 0], Eα(A) [0, 1], Eα(A) [0, 2], Eα(A) [0, 3], Eα(A) [0, 4], Eα(A) [0, 5], Eα(A) [0, 6], Eα(A) [0, 7]}, {Eα(A) [1, 0], Eα(A) [1, 1], Eα(A) [1, 2], Eα(A) [1, 3], Eα(A) [1, 4], Eα(A) [1, 5], Eα(A) [1, 6], Eα(A) [1, 7]}, {Eα(A) [2, 0], Eα(A) [2, 1], Eα(A) [2, 2], Eα(A) [2, 3], Eα(A) [2, 4], Eα(A) [2, 5], Eα(A) [2, 6], Eα(A) [2, 7]}, {Eα(A) [3, 0], Eα(A) [3, 1], Eα(A) [3, 2], Eα(A) [3, 3], Eα(A) [3, 4], Eα(A) [3, 5], Eα(A) [3, 6], Eα(A) [3, 7]}, {Eα(A) [4, 0], Eα(A) [4, 1], Eα(A) [4, 2], Eα(A) [4, 3], Eα(A) [4, 4], Eα(A) [4, 5], Eα(A) [4, 6], Eα(A) [4, 7]}, {Eα(A) [5, 0], Eα(A) [5, 1], Eα(A) [5, 2], Eα(A) [5, 3], Eα(A) [5, 4], Eα(A) [5, 5], Eα(A) [5, 6], Eα(A) [5, 7]}, {Eα(A) [6, 0], Eα(A) [6, 1], Eα(A) [6, 2], Eα(A) [6, 3], Eα(A) [6, 4], Eα(A) [6, 5], Eα(A) [6, 6], Eα(A) [6, 7]}, {Eα(A) [7, 0], Eα(A) [7, 1], Eα(A) [7, 2], Eα(A) [7, 3], Eα(A) [7, 4], Eα(A) [7, 5], Eα(A) [7, 6], Eα(A) [7, 7]}}

In[]:= (*gtryeα(A)=(eα(A)/.sgtryeα(A))*)

In[]:= (*gtryeα(A)=(eα(A)/.sgtryeα(A))*)

In[]:= (*Symbolize[Rκα β]*)

In[]:= (*preSpinConnection=Array[Rκα β[##]&,{8,8,8}];*)

In[]:= (*Symbolize[EAa]*)

In[]:= (*Symbolize[EaA]*)

In[]:= (* FAa FaA *)

In[]:= (*Do[FAa=EAa[h]=Table[(u[[h]].σ.(τ[B])),{B,1,8}],{h,1,Length[u]}];*
Do[EaA[h]=FullSimplify[Inverse[EAa[h]]],{h,1,Length[u]}];
Table[FaA=EaA[h]==(u[[h]].σ.u[[h]])
Transpose[Table[FullSimplify[ExpandAll[η8[[B,B]]τ[B].u[[h]]]],
{B,1,8}]],{h,1,Length[u]}]*)

In[]:= (*Table[FullSimplify[ExpandAll[Transpose[EAa[h]].η8.EAa[h]]]-
(u[[h]].σ.u[[h]])σ==Zero,{h,1,Length[u]}]*)

In[]:= (*iη88=FullSimplify[Inverse[η8]];*)

In[]:= (*Clear[w(a)(b)];*) Clear[w]; Symbolize[w(a)(b)]

In[]:= w = Array[w(a)(b) [##]&, {8, 8, 8}];
Protect[w]

Out[]=
{w}

```

constants

← ERROR : 08similarityTransformation has 1 st

index that transforms as $\frac{\partial}{\partial \Psi}$, not as Ψ s

```
In[8]:= (* 08similarityTransformation has 1st  
index that transforms as  $\frac{\partial}{\partial \Psi}$  , not as  $\Psi$  *)
```

08similarityTransformation has 1 st

index that transforms as $\frac{\partial}{\partial \Psi}$, not as Ψ :


```

In[]:= η4488 // MatrixForm
Out[//MatrixForm]=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}

In[]:= η4488[[#, #]] & /@ Range[8]
Out[=]
{1, 1, 1, 1, -1, -1, -1, -1}

In[=]
Zero4 = ConstantArray[0, {4, 4}]; Zero8 = ConstantArray[0, {8, 8}];
mid = ExpandAll[-1 * ID4];
ε3 = Array[Signature[{{##}}] &, {3, 3, 3}];
ε4 = Array[Signature[{{##}}] &, {4, 4, 4, 4}];
MId = ExpandAll[-1 * ID8];
simp = {Zero4 → 0, ID4 → 1, mid → -1};
Simp = {Zero8 → ZERO, ID8 → ONE, MId → MONE};

Out[=]
{{{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}}, {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}}, {{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}

In[=]
Zero16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η8, 0}, {0, -η8}}];
Id16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * Id16];
Simp16 = {Zero16 → ZERO16, Id16 → ONE16, MId16 → MONE16};

In[=]
ZERO16 = ConstantArray[0, {16, 16}];
G16 = ArrayFlatten[{{η4488, 0}, {0, -η4488}}];
ID16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * ID16];
Simp16 = {ZERO16 → ZERO16, ID16 → ONE16, MId16 → MONE16};

In[=]
ws = Flatten[
  Table[ToExpression["w" <> ToString[A1] <> ToString[B1]], {A1, 1, 7}, {B1, A1 + 1, 8}]];
Length[ws]

Out[=]
{w12, w13, w14, w15, w16, w17, w18, w23, w24, w25, w26, w27, w28,
 w34, w35, w36, w37, w38, w45, w46, w47, w48, w56, w57, w58, w67, w68, w78}$$

```

Out[=]

```
In[1]:= σ.σ - ID8 == Zero8
Tr[σ] == 0

Out[1]= True
Out[2]= True
```

some function definitions :

```
In[1]:= blockPartition[mat_] := Module[{(*blocks,block11,block12,block21,block22,*)ret},
  blocks = Partition[mat, {8, 8}];
  (*block11=blocks[[1,1]];*) (*Top-left block*)
  (*block12=blocks[[1,2]];*) (*Top-right block*)
  (*block21=blocks[[2,1]];*) (*Bottom-left block*)
  (*block22=blocks[[2,2]];*) (*Bottom-right block*)
  ret =
  ArrayFlatten[{{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}] === mat;
  If[ret, Return[{{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}], ## &[]];
  Return[Null];]

In[2]:= eextract[a_]:= Extract[a, {#}] & /@ Range[0, Length[a]]

In[3]:= Clear[iimes]

In[4]:= iimes[a_]:= {a}

In[5]:= iimes[a_, b_]:= {a, b}

In[6]:= iimes[a_, b_, c_]:= {a, b, c}

In[7]:= iimes[a_, b_, c_, d_]:= {a, b, c, d}

In[8]:= iimes[a_, b_, c_, d_, f_]:= {a, b, c, d, f}

In[9]:= iimes[a_, b_, c_, d_, f_, h_]:= {a, b, c, d, f, h}

In[10]:= iimes[a_, b_, c_, d_, f_, h_, j_]:= {a, b, c, d, f, h, j}

In[11]:= times[a__]:= Flatten[{Flatten[##] & /@ a}]

In[12]:= (*times[a_,b_,c_,d_]:= {a,b}*)

In[13]:= (*iimes[a_,b_,c_,d_][0]:= {a,d}*)

In[14]:= (*iimes[a_,b_][0]^:= {a,b}*)
```

```

In[1]:= iid[a_, b_] := a
In[2]:= iid[a_, b_, c_] := {a, b}
In[3]:= iid[a_List] := a[[1]]
In[4]:= qid[a__] := a
In[5]:= Clear[der]; der[a__][c_][b__] := c
In[6]:= Clear[der3]; der3[a__][c_][b__] := {c, a, b}
In[7]:= (*der[a_, b_][c_]:= c*)
In[8]:= (*derz[a_, b_]:= a*)
In[9]:= (*dert[a_, b_]:= b*)
In[10]:= (*derivative[a__][c_]:= FullForm[c]/.{Derivative->der}*)
In[11]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#]& /@ %
(*ToString[#]&/@%*)
% /. {Derivative -> der}
Out[1]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}
Out[2]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}
Out[3]= {Z[0][z, t], Z[0], Z[0]}

```

```
In[=]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
% * (Prime[#] & /@ Range[Length[%]])
Plus @@ %
FullForm[%] (* #]&/@%*)
% /. {Plus -> qid, Times -> qid, Derivative -> der}
List[%]
Partition[% , 2]
eextract[#] & /@ %
(*eextract[#]&/@%*)
(*#_List[[0]]&/@%*)
#[[2]] & /@ %
#[[3]] & /@ %

Out[=]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[=]= {2 Z[0][z, t], 3 Z[0]^(1,0)[z, t], 5 Z[0]^(0,1)[z, t]}

Out[=]= 2 Z[0][z, t] + 5 Z[0]^(0,1)[z, t] + 3 Z[0]^(1,0)[z, t]

Out[=]//FullForm=
Plus[Times[2, Z[0][z, t]], Times[5, Derivative[0, 1][Z[0]][z, t]],
Times[3, Derivative[1, 0][Z[0]][z, t]]]

Out[=]= Sequence[2, Z[0][z, t], 5, Z[0], 3, Z[0]]

Out[=]= {2, Z[0][z, t], 5, Z[0], 3, Z[0]}

Out[=]= {{2, Z[0][z, t]}, {5, Z[0]}, {3, Z[0]}}

Out[=]= {{List, 2, Z[0][z, t]}, {List, 5, Z[0]}, {List, 3, Z[0]}}

Out[=]= {2, 5, 3}

Out[=]= {Z[0][z, t], Z[0], Z[0]}

Sequence[Subscript[expr, 1], Subscript[expr, 2], ...]
represents a sequence of arguments to be spliced automatically
into any function .

In[=]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
(*ToString[#]&/@%*)
% /. {Derivative -> der3}

Out[=]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[=]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]= {Z[0][z, t], List[Z[0], 1, 0, z, t], List[Z[0], 0, 1, z, t]}
```

```

In[1]:= helpf[x__]:= Length[{x}]
In[2]:= {helpf[x, y, z], helpf[]}
Out[2]= {3, helpf[]}

In[3]:= f[##] & @@ X
Out[3]= f[x0, x1, x2, x3, x4, x5, x6, x7]

In[4]:= StringExtract["a--bbb--ccc--ddd", "--" → 3]
Out[4]= ccc

In[5]:= StringExtract["a bbb cccc aa d", -1]
Out[5]= d

In[6]:= StringRiffle[{{"a", "b", "c"}, {"d", "e", "f"}}, "\n", "\t"]
Out[6]=
      a      b      c
      d      e      f

In[7]:= {"", "z", "t"}
Out[7]= {, z, t}

{Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#]&/@%
ToString[#]&/@%
StringSplit[#, ""]&/@%
StringExtract[#, "[" → All]&/@%
StringExtract[#, "All]&/@%
InputForm[TextString[#]]&/@%

```

The code block shows a series of Mathematica commands. It starts by defining a function `helpf` that returns the length of its argument. Then it defines a function `f` that takes a variable number of arguments and applies them to a head `X`. Next, it uses `StringExtract` to extract the third part of a string containing multiple dashes. Following this, it uses `StringRiffle` to interleave two lists of characters with newlines and tabs. Finally, it performs a complex transformation on a list of expressions, including `FullForm`, `ToString`, `StringSplit`, `StringExtract`, and `InputForm[TextString]`.

```

In[]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
StringSplit[#, "]"] & /@ %
StringExtract[#, "[" → All] & /@ %
(*StringExtract[#, " {, z, t"} → All]&/@%*)
InputForm[TextString[#]] & /@ %

Out[=]
{Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[=]
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]
{{Z[0], [z, t], {Derivative[1, 0, [Z[0], , [z, t], {Derivative[0, 1, [Z[0], , [z, t]}}

Out[=]
{{{Z, 0}, {, z, t}}, {{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}},
{{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}}

Out[=]
{"{{Z, 0}, {, z, t}}", "{{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}",
 "{{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}"}

In[]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@ %
ToString[#] & /@ %
(*InputForm[TextString[#]]&/@%*)
StringExtract[#, "[" → All] & /@ %
StringExtract[#, "]" → All] & /@ %
StringExtract[#, "[" → 1] & /@ %%%
StringExtract[#, "]" → 1] & /@ %%%%
StringExtract[#, "Z[" → 1] & /@ %%%%%

Out[=]
{Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[=]
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]
{{Z, 0}, z, t}, {Derivative, 1, 0}, Z, 0]], z, t}, {Derivative, 0, 1}, Z, 0]], z, t}]

Out[=]
{{Z[0], [z, t, ], {Derivative[1, 0, [Z[0], , [z, t, ], {Derivative[0, 1, [Z[0], , [z, t, ]}

Out[=]
{Z, Derivative, Derivative}

Out[=]
{Z[0, Derivative[1, 0, Derivative[0, 1]

Out[=]
{, Derivative[1, 0][, Derivative[0, 1][}

In[]:= (*TemplateApply[StringTemplate["Z ` [` then `b`"],
<|"a"→1234,"b"→5678|>]*]

```

```

In[]:= skelx[x_, matQ_ : True] := Module[{t = {666}},
  If[matQ == True, t = Block[{r},
    r = x;
    Table[Block[{q}, q = SameQ[0, r[[j, i]]];
      If[True == q, Style[0, Blue], Style[Length[r[[j, i]]], Red]]],
      {j, 1, Length[r]}, {i, 1, Length[r[[1]]]}]];
  If[matQ == False, t = Block[{r},
    r = x;
    Table[Block[{q}, q = SameQ[0, r[[j]]];
      If[True == q, Style[0, {RGBColor → {0, 1/3, 0}, Bold}], Style[Length[r[[j]]], Red]],
      {j, 1, Length[r]}]];
  MatrixForm[t, TableAlignments → Left]]

In[]:= makeSym[size_, fn_] :=
  Module[{rtmp}, rtmp = Table[fn[i, j], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[rtmp, {{2}, {1}}]}]]

In[]:= makeAntiSym[size_, fn_] :=
  Module[{rtmp}, rtmp = Table[If[i == j, 0, fn[i, j]], {i, 1, size}, {j, 1, i}];
  MapThread[Join, {rtmp, Rest /@ Flatten[-rtmp, {{2}, {1}}]}]]

In[]:= Block[{MX}, MX = makeAntiSym[8, Subscript[ω, ##] &]

Out[]=
{{0, -ωω₂₁, -ωω₃₁, -ωω₄₁, -ωω₅₁, -ωω₆₁, -ωω₇₁, -ωω₈₁},
{ωω₂₁, 0, -ωω₃₂, -ωω₄₂, -ωω₅₂, -ωω₆₂, -ωω₇₂, -ωω₈₂},
{ωω₃₁, ωω₃₂, 0, -ωω₄₃, -ωω₅₃, -ωω₆₃, -ωω₇₃, -ωω₈₃},
{ωω₄₁, ωω₄₂, ωω₄₃, 0, -ωω₅₄, -ωω₆₄, -ωω₇₄, -ωω₈₄},
{ωω₅₁, ωω₅₂, ωω₅₃, ωω₅₄, 0, -ωω₆₅, -ωω₇₅, -ωω₈₅},
{ωω₆₁, ωω₆₂, ωω₆₃, ωω₆₄, ωω₆₅, 0, -ωω₇₆, -ωω₈₆},
{ωω₇₁, ωω₇₂, ωω₇₃, ωω₇₄, ωω₇₅, ωω₇₆, 0, -ωω₈₇},
{ωω₈₁, ωω₈₂, ωω₈₃, ωω₈₄, ωω₈₅, ωω₈₆, ωω₈₇, 0}}

```

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2} \left(\frac{\partial g_{\mu\alpha}}{\partial x^\beta} + \frac{\partial g_{\mu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\mu} \right),$$

$$\Gamma^\mu{}_{\alpha\beta} = g^{\mu\nu} \Gamma_\nu{}^{}_{\alpha\beta},$$

$$R^\mu{}_{\nu\alpha\beta} = \frac{\partial \Gamma^\mu{}_{\nu\beta}}{\partial x^\alpha} - \frac{\partial \Gamma^\mu{}_{\nu\alpha}}{\partial x^\beta} + \Gamma^\mu{}_{\rho\alpha} \Gamma^\rho{}_{\nu\beta} - \Gamma^\mu{}_{\rho\beta} \Gamma^\rho{}_{\nu\alpha}.$$

$$R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}, \quad R = g^{\mu\nu} R_{\mu\nu},$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

```
In[6]:= (*Christoffel, Einstein tensor, ...*)
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, rΓ, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    rΓ = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
      (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}], {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
    RicciΓ = ParallelTable[FullSimplify[D[rΓ[[μ, ν, β]], X[[α]]] - D[rΓ[[μ, ν, α]], X[[β]]] +
      Sum[rΓ[[μ, s, α]] * rΓ[[s, ν, β]] - rΓ[[μ, s, β]] * rΓ[[s, ν, α]], {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}];
    RieΓ = ParallelTable[FullSimplify[Sum[RicciΓ[[α, μ, α, ν]], {α, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}];
    RS = FullSimplify[Tr[ginv.RieΓ], ass];
    G = ParallelTable[FullSimplify[RieΓ[[α]] - 1/2 g[[α]] RS, ass], {α, 1, Length[RieΓ]}];
  ];
  Print[t];
  Print[Now];
  Return[{ginv, rΓ, RicciΓ, RieΓ, RS, G}]
]
```

If ‘paste’ Maple MathML to ToExpression[MathMLForm[]] fails, then try to edit this :

```
In[6]:= Clear[mapleztStringToMathematica];
mapleztStringToMathematica[input_String, toExpression_: False, debug_: False] :=
Module[{prepare, post, custom, use = input, save = input, linenumber = 0},
(*use=ToString[InputForm[TextString[input]]];*)(*Print[use];*)
prepare[in_String] := Module[{s = in}, linenumber++;
  s = in;
  (*If[s==ToString[InputForm[TextString[s]]],*
   (*##&[]*)Print[s],Print[ToUpperCase[s]]];*)
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,z)" >
    "D[Z[" <> ToString[j] <> "] [z,t],{z,2}]", {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,t)" >
    "D[D[Z[" <> ToString[j] <> "] [z,t],z],t]", {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,z)" >
    "D[D[Z[" <> ToString[j] <> "] [z,t],t],z]", {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,t)" >
    "D[Z[" <> ToString[j] <> "] [z,t],{t,2}]", {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,t)" >
    "D[Z[" <> ToString[j] <> "] [z,t],t]", {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t)" >
    "D[Z[" <> ToString[j] <> "] [z,t],t]", {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z)" >
    "D[Z[" <> ToString[j] <> "] [z,t],z]", {j, 0, 15}];
```

```

s = StringReplace[s, "tan(z)" → "Tan[z]"];
s = StringReplace[s, "cot(z)" → "Cot[z]"];
s = StringReplace[s, "sin(z)" → "Sin[z]"];
s = StringReplace[s, "cos(z)" → "Cos[z]"];
s = StringReplace[s, "sec(z)" → "Sec[z]"];
s = StringReplace[s, "csc(z)" → "Csc[z]"];

s = StringReplace[s, "=" → "->"];

(*Do[s=StringReplace[s,"Z"<>ToString[j]<>"(z,t)"→"Z["<>ToString[j]<>"][z,t]",{j,0,15}];*)
(*s=StringReplace[s,"diff(a4(t),t)"→"D[a4[t],t]"];*)
(*s=StringReplace[s,"exp(-2*a4(t))"→"Exp[-2a4[t]]"];*)
(*s=StringReplace[s,"a4(t)"→"a4[t]"];*)

Return[s];
];

post[in_String] := Module[{s = in}, linenumber++;
s = in;
s = StringReplace[s, "a4(t)" → "a4[t]"];
Do[s = StringReplace[s, "Z" <> ToString[j] <> "(z,t)" → "Z[" <> ToString[j] <> "] [z,t]", {j, 0, 15}];
Return[s];
];
custom[in_String] := Module[{s = in}, linenumber++;
s = in;

s = StringReplace[s, "diff(a4(t),t)" → "D[a4[t],t]"];
s = StringReplace[s, "exp(-2*a4(t))" → "Exp[-2a4[t]]"];

Return[s];
];
use = prepare[use];
use = custom[use];
Return[post[use]];
(*If[toExpression,Return[ToExpression[s/.{z→#1,t→#2}],##&[]]];*)
]

```

**Mathematica Lexer and Parser for Maple - like Syntax
Supports : Basic arithmetic, function calls, lists {},
equations =, and specific Maple keywords .**

```

In[]:= (*---LEXER---*) (*Helper:Check if character is a digit*)
IsDigit[char_] := StringMatchQ[char, RegularExpression["\d"]];

(*Helper:Check if character is a letter*)
IsAlpha[char_] := StringMatchQ[char, RegularExpression"[a-zA-Z_"]];

(*Helper:Check if character is whitespace*)
IsSpace[char_] := StringMatchQ[char, RegularExpression["\s"]];

(*Main Lexer Function*)
GetTokens[str_String] :=
Module[{chars, len, i, char, tokens = {}, token, numStr, idStr}, chars = Characters[str];
len = Length[chars];
i = 1;
While[i \leq len, char = chars[[i]];
Which[(*Skip Whitespace*) IsSpace[char], i++, (*Numbers*) IsDigit[char], numStr = char;
i++;
While[i \leq len && IsDigit[chars[[i]]], numStr = numStr \>>> chars[[i]];
i++];
AppendTo[tokens, {"NUMBER", ToExpression[numStr]}],
(*Identifiers and Keywords*) IsAlpha[char], idStr = char;
i++;
While[i \leq len && (IsAlpha[chars[[i]]] || IsDigit[chars[[i]]]), idStr = idStr \>>> chars[[i]];
i++];
(*Check for specific keywords if needed,
otherwise IDENTIFIER*) AppendTo[tokens, {"IDENTIFIER", idStr}],
(*Operators and Punctuation*) char == "+", AppendTo[tokens, {"PLUS", "+"}],
i++, char == "-", AppendTo[tokens, {"MINUS", "-"}];
i++, char == "*", AppendTo[tokens, {"STAR", "*"}];
i++, char == "/", AppendTo[tokens, {"SLASH", "/" }];
i++, char == "^", AppendTo[tokens, {"CARET", "^"}];
i++, char == "(", AppendTo[tokens, {"LPAREN", "("}];
i++, char == ")", AppendTo[tokens, {"RPAREN", ")" }];
i++, char == "{", AppendTo[tokens, {"LBRACE", "{" }];
i++, char == "}", AppendTo[tokens, {"RBRACE", "}" }];
i++, char == "=", AppendTo[tokens, {"EQUALS", "="}];
i++, char == ",", AppendTo[tokens, {"COMMA", ","}];
i++, char == "$", AppendTo[tokens, {"DOLLAR", "$"}];
i++, (*Ellipsis... or Dot.*) char == ".", If[i + 2 \leq len &&
chars[[i + 1]] == "." && chars[[i + 2]] == ".", AppendTo[tokens, {"ELLIPSIS", "..."}];
i += 3, AppendTo[tokens, {"DOT", "."}];
i++], True, Print["Unknown character: ", char];
i++];
AppendTo[tokens, {"EOF", "EOF"}];
tokens];

```

In[]:= (*---PARSER---*) (*Grammar:Expression→Equation Equation→AddExp["=" AddExp] AddExp→MulExp { ("+" | "-") MulExp} MulExp→

```

PowerExp { ("*" | "/") PowerExp} PowerExp→Primary["^" PowerExp] (Right associative)
Primary→NUMBER|IDENTIFIER|(" Expression ")"|IDENTIFIER "(" ArgList ")"|
"{" ArgList "}" ArgList→Expression {" Expression}|Empty*)
ParseTokens[tokens_List] := Module[{pos = 1, currentToken, eat, peek, parseExpression,
parseEquation, parseAddExp, parseMulExp, parsePowerExp, parseUnaryExp,
parsePrimary, parseArgs, parseList}, currentToken := tokens[[pos]];
peek[] := tokens[[pos]];
eat[type_] := If[currentToken[[1]] == type, pos++];
True, Print["Error: Expected ", type, " but found ", currentToken[[1]]];
False];
(*Entry point*)parseExpression[] := parseEquation[];
(*Equation:a=b*)parseEquation[] := Module[{left, right}, left = parseAddExp[];
If[currentToken[[1]] == "EQUALS", eat["EQUALS"];
right = parseAddExp[];
{"Equation", left, right}, left]];
parseAddExp[] := Module[{node, right, op}, node = parseMulExp[];
While[MemberQ[{"PLUS", "MINUS"}, currentToken[[1]]], op = currentToken[[2]];
eat[currentToken[[1]]];
right = parseMulExp[];
node = {"BinaryOp", op, node, right};];
node];
parseMulExp[] := Module[{node, right, op}, node = parsePowerExp[];
While[True, If[MemberQ[{"STAR", "SLASH"}, currentToken[[1]]], op = currentToken[[2]];
eat[currentToken[[1]]];
right = parsePowerExp[];
node = {"BinaryOp", op, node, right}, (*Implicit Multiplication*)
If[MemberQ[{"IDENTIFIER", "NUMBER", "LPAREN", "LBRACE", "ELLIPSIS"}, currentToken[[1]]], right = parsePowerExp[];
node = {"BinaryOp", "*", node, right},
Break[] (*Not an operator or start of expression*)]];
node];
parsePowerExp[] := Module[{node, right}, node = parseUnaryExp[];
If[currentToken[[1]] == "CARET", eat["CARET"];
right = parsePowerExp[];
(*Right associative recursion*) {"BinaryOp", "^", node, right}, node]];
parseUnaryExp[] := Module[{op, node},
If[MemberQ[{"PLUS", "MINUS"}, currentToken[[1]]], op = currentToken[[2]];
eat[currentToken[[1]]];
node = parseUnaryExp[];
 {"UnaryOp", op, node}, parsePrimary[]]];
parsePrimary[] := Module[{token, node, name, args}, token = currentToken;
Switch[token[[1]], "NUMBER", eat["NUMBER"]];
 {"Number", token[[2]]}, "IDENTIFIER", eat["IDENTIFIER"];
name = token[[2]];
(*Check if it's a function call:IDENTIFIER followed by LPAREN*)
If[currentToken[[1]] == "LPAREN", eat["LPAREN"];
args = parseArgs[]];

```

```

eat["RPAREN"];
 {"Call", name, args}, {"Identifier", name}], "LPAREN", eat["LPAREN"];
node = parseExpression[];
eat["RPAREN"];
node, "LBRACE", parseList[], "ELLIPSIS", eat["ELLIPSIS"];
 {"Identifier", "..."}, "EOF", Print["Error: Unexpected EOF in Primary"];
 {"Error", "EOF"}, (*Do NOT eat EOF*)
 _, Print["Error: Unexpected token in Primary: ", token];
eat[token[[1]]];
 (*Advance to avoid infinite loop*) {"Error", token}]];
(*Parse List:{a,b,c}* )parseList[] := Module[{elements}, eat["LBRACE"]];
elements = parseArgs[];
eat["RBRACE"];
 {"List", elements}];
(*Parse Argument List:expr,expr,...*)parseArgs[] := Module[{args = {}, arg},
 If[currentToken[[1]] != "RPAREN" && currentToken[[1]] != "RBRACE", arg = parseExpression[];
 AppendTo[args, arg];
 While[currentToken[[1]] == "COMMA", eat["COMMA"]];
 arg = parseExpression[];
 AppendTo[args, arg];];
 args];
parseExpression[]];

```

X

```

In[1]:= (*---CONVERTER---*) (*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunction[funcName_String] :=
Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate", "DESol",
 "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan", "Tan", "sin",
 "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc", "Csc", "csch", "Csch",
 "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh", "coth", "Coth", "sech", "Sech",
 "det", "Det", "transpose", "Transpose", "sphericalbessel", "SphericalBesselJ",
 "lambertw", "ProductLog", "gamma", "Gamma", "airy", "AiryAi", "besselJ",
 "BesselJ", "besselY", "BesselY", "hypergeometricU", "HypergeometricU", "zeta",
 "Zeta", "erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNode[nodeType_, nodeData_, childResults_] :=
Module[{funcName, cleanFuncName, mathFunc, argsStr},
Switch[nodeType, "Number", ToString[nodeData], "Identifier",
(*Handle special identifiers like _Y→Y*) If[StringLength[nodeData] > 0 &&
StringTake[nodeData, 1] == "_", StringDrop[nodeData, 1], nodeData], "BinaryOp",
 "(" <> childResults[[1]] <> " " <> nodeData <> " " <> childResults[[2]] <> ")",
 "UnaryOp", nodeData <> "(" <> childResults[[1]] <> ")", "Equation",
 childResults[[1]] <> " == " <> childResults[[2]], "List",
 "{" <> StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
(*Handle special identifiers like _Y→Y for function names too*)
cleanFuncName = If[StringLength[funcName] > 0 && StringTake[funcName, 1] == "_",

```

```

        StringDrop[funcName, 1], funcName];
mathFunc = MapleToMathematicaFunction[cleanFuncName];
(*Special handling for DSolve-requires 3 arguments*)
If[cleanFuncName == "DESol" || mathFunc == "DSolve",
  (*DSolve[equations,functions,variables]*) (*Ensure we have exactly 3 arguments,
  pad with empty lists if needed*) argsStr = StringRiffle[
    Join[childResults, Table["{}", {i, 1, Max[0, 3 - Length[childResults]]}]], ", ", ];
mathFunc <> "[" <> argsStr <> "]",
(*Default function call*) argsStr = StringRiffle[childResults, ", ", ];
mathFunc <> "[" <> argsStr <> "]"], _, "Error"]];

(*Convert AST to Mathematica String-Non-
recursive iterative version using post-order traversal*)
ToMathematicaString[ast_] := Module[{stack, outputStack, currentItem,
  node, nodeType, children, childCount, childResults, i, result, nodeData},
  (*Use two stacks:one for traversal,one for results*) stack = {{ast, False}};
  (*{node,visited}* )outputStack = {};
  (*Post-order traversal:process children before parents*)
  While[Length[stack] > 0, currentItem = Last[stack];
  node = currentItem[[1]];
  nodeType = node[[1]];
  If[currentItem[[2]], (*Node already visited,process it*) stack = Most[stack];
  (*Determine children and extract data*)
  {children, nodeData} = Switch[nodeType, "Number", {}, node[[2]],
    "Identifier", {}, node[[2]], "BinaryOp", {{node[[3]], node[[4]]}, node[[2]]},
    "UnaryOp", {{node[[3]]}, node[[2]]}, "Equation", {{node[[2]], node[[3]]}, ""},
    "List", {node[[2]], ""}, "Call", {node[[3]], node[[2]]}, _, {}, ""}];
  (*Pop child results from output stack*) childCount = Length[children];
  childResults = {};
  If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
  outputStack = Most[outputStack];, {i, 1, childCount}]];
  (*Process this node*) result = ProcessASTNode[nodeType, nodeData, childResults];
  AppendTo[outputStack, result];, (*Node not visited,
  mark as visited and push children*) stack[[Length[stack]]] = {node, True};
  (*Push children onto stack in reverse order (so they process left-to-right)*)
  children = Switch[nodeType, "Number", {}, "Identifier", {},
    "BinaryOp", {node[[3]], node[[4]]}, "UnaryOp", {node[[3]]}, "Equation",
    {node[[2]], node[[3]]}, "List", node[[2]], "Call", node[[3]], _, {}];
  Do[AppendTo[stack, {children[[i]], False}];, {i, Length[children], 1, -1}]];
  (*Return the final result*) If[Length[outputStack] > 0, Last[outputStack], "Error"]];
(*---MAIN WRAPPER---*)
ConvertMapleToMathematica[inputStr_String] :=
Module[{tokens, ast}, tokens = GetTokens[inputStr];
ast = ParseTokens[tokens];
ToMathematicaString[ast]];

In[]:= (*---CONVERTER---*) (*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunctionOLD[funcName_String] :=

```

```

Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate", "DESol",
  "DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan", "Tan", "sin",
  "Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc", "Csc", "csch", "Csch",
  "tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh", "coth", "Coth", "sech", "Sech",
  "det", "Det", "transpose", "Transpose", "sphericalbessel", "SphericalBesselJ",
  "lambertw", "ProductLog", "gamma", "Gamma", "airy", "AiryAi", "besselJ",
  "BesselJ", "bessely", "Bessely", "hypergeometricU", "HypergeometricU", "zeta",
  "Zeta", "erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNodeOLD[nodeType_, nodeData_, childResults_] :=
Module[{funcName, cleanFuncName, mathFunc, argsStr},
  Switch[nodeType, "Number", ToString[nodeData], "Identifier",
    (*Handle special identifiers like _Y→Y*) If[StringLength[nodeData] > 0 &&
      StringTake[nodeData, 1] == "_", StringDrop[nodeData, 1], nodeData], "BinaryOp",
    "(" <> childResults[[1]] <> " " <> nodeData <> " " <> childResults[[2]] <> ")",
    "UnaryOp", nodeData <> "(" <> childResults[[1]] <> ")", "Equation",
    childResults[[1]] <> " == " <> childResults[[2]], "List",
    "{" <> StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
    (*Handle special identifiers like _Y→Y for function names too*)
    cleanFuncName = If[StringLength[funcName] > 0 && StringTake[funcName, 1] == "_",
      StringDrop[funcName, 1], funcName];
    mathFunc = MapleToMathematicaFunction[cleanFuncName];
    argsStr = StringRiffle[childResults, ", "];
    mathFunc <> "[" <> argsStr <> "]", _, "Error"]];

(*Convert AST to Mathematica String-Non-
recursive iterative version using post-order traversal*)
ToMathematicaStringOLD[ast_] := Module[{stack, outputStack, currentItem,
  node, nodeType, children, childCount, childResults, i, result, nodeData},
  (*Use two stacks:one for traversal,one for results*) stack = {{ast, False}};
  (*{node,visited}*-) outputStack = {};
  (*Post-order traversal:process children before parents*)
  While[Length[stack] > 0, currentItem = Last[stack];
  node = currentItem[[1]];
  nodeType = node[[1]];
  If[currentItem[[2]], (*Node already visited,process it*) stack = Most[stack];
    (*Determine children and extract data*)
    {children, nodeData} = Switch[nodeType, "Number", {}, node[[2]],
      "Identifier", {}, node[[2]], "BinaryOp", {{node[[3]], node[[4]]}, node[[2]]},
      "UnaryOp", {{node[[3]]}, node[[2]]}, "Equation", {{node[[2]], node[[3]]}}, "",
      "List", {node[[2]], ""}, "Call", {node[[3]], node[[2]]}, _, {}, ""}];
    (*Pop child results from output stack*) childCount = Length[children];
    childResults = {};
    If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
      outputStack = Most[outputStack], {i, 1, childCount}]];
    (*Process this node*) result = ProcessASTNode[nodeType, nodeData, childResults];
    stack = Most[stack];
    outputStack = {result, True} && stack != {} ? stack : {{ast, False}}];
  ];
]

```

```

AppendTo[outputStack, result];, (*Node not visited,
mark as visited and push children*)stack[[Length[stack]]] = {node, True};
(*Push children onto stack in reverse order (so they process left-to-right)*)
children = Switch[nodeType, "Number", {}, "Identifier", {},
  "BinaryOp", {node[[3]], node[[4]]}, "UnaryOp", {node[[3]]}, "Equation",
  {node[[2]], node[[3]]}, "List", node[[2]], "Call", node[[3]], _, {}];
Do[AppendTo[stack, {children[[i]], False}];, {i, Length[children], 1, -1}];];
(*Return the final result*)If[Length[outputStack] > 0, Last[outputStack], "Error"]];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematicaOLD[inputStr_String] :=
Module[{tokens, ast}, tokens = GetTokens[inputStr];
ast = ParseTokens[tokens];
ToMathematicaString[ast]];

In[]:= sta = "{nZ10(z, t) = (c2C7*sin(C2Q1*t) + c2C8*cos(C2Q1*t))*(c2C5*sin(z)^(sqrt(-C2Q1^2
+ M^2)/6) + c2C6*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ11(z, t) =
(c2C3*sin(C2Q1*t) + c2C4*cos(C2Q1*t))*(c2C1*sin(z)^(sqrt(-C2Q1^2
+ M^2)/6) + c2C2*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ8(z, t) =
(((C2Q1*c2C2*c2C3 + M*c2C6*c2C8)*cos(C2Q1*t) + sin(C2Q1*t)*(C2Q1*c2C2*c2C4
+ M*c2C6*c2C7))*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) + ((C2Q1*c2C1*c2C3
- M*c2C5*c2C8)*cos(C2Q1*t) - sin(C2Q1*t)*(C2Q1*c2C1*c2C4 +
M*c2C5*c2C7))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) + (-cos(M*t)*c2C10 +
sin(M*t)*c2C9)*sqrt(-C2Q1^2 + M^2))/sqrt(-C2Q1^2 + M^2), nZ9(z, t) =
(((C2Q1*c2C6*c2C7 + M*c2C2*c2C4)*cos(C2Q1*t) + sin(C2Q1*t)*(-C2Q1*c2C6*c2C8
+ M*c2C2*c2C3))*sin(z)^(-sqrt(-C2Q1^2 + M^2)/3) + sqrt(-C2Q1^2 +
M^2)*(cos(M*t)*c2C9 + sin(M*t)*c2C10)*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) +
(-C2Q1*c2C5*c2C7 - M*c2C1*c2C4)*cos(C2Q1*t) - sin(C2Q1*t)*(-C2Q1*c2C5*c2C8
+ M*c2C1*c2C3))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6)/sqrt(-C2Q1^2 + M^2)}";

```

```
In[*]:= ConvertMapleToMathematica[sta]
Out[*]= {nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5 * (Sin[z]^
  ^ (Sqrt[((-(C2Q1)^2 + (M^2))/6)) + (c2C6 * (Sin[z]^ (-(Sqrt[((-(C2Q1)^2 + (M^2))/6)))) + (c2C2 * (Sin[z]^ (-(Sqrt[((-(C2Q1)^2 + (M^2))/6)))) + (c2C1 * (Sin[z]^ (Sqrt[((-(C2Q1)^2 + (M^2))/6)) * (c2C3 * Sin[(C2Q1 * t)]) + (c2C4 *
  Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z]^ (Sqrt[((-(C2Q1)^2 + (M^2))/6)) * (c2C2 * c2C3) + ((M * c2C6) * c2C8)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((C2Q1 * c2C2) * c2C4) + ((M * c2C6) * c2C7))) * (Sin[z]^ (-(Sqrt[((-(C2Q1)^2 + (M^2))/6)) + (((((C2Q1 * c2C1) * c2C3) - ((M * c2C5) * c2C8)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C1) * c2C4) + ((M * c2C5) * c2C7))) * (Sin[z]^ (Sqrt[((-(C2Q1)^2 + (M^2))/6)) + ((((-(Cos[(M * t]) * c2C10) + (Sin[(M * t)] * c2C9)) * Sqrt[((-(C2Q1)^2 + (M^2))/2)) / Sqrt[((-(C2Q1)^2 + (M^2))/6))), nZ9[z, t] == (((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1)^2 + (M^2))/3)) + (Sqrt[((-(C2Q1)^2 + (M^2))/6)) * ((Cos[(M * t)] * c2C9) + (Sin[(M * t)] * c2C10)) * (Sin[z]^ (-(Sqrt[((-(C2Q1)^2 + (M^2))/6)) / 6))) + (((-(C2Q1)^2 + (M^2))/6)) * ((Sqrt[((-(C2Q1)^2 + (M^2))/6)) * ((Cos[(C2Q1 * t)] * ((-(C2Q1)^2 + (M^2))/6)) + (Sin[(C2Q1 * t)] * (((-(C2Q1)^2 + (M^2))/6)) * (Sqrt[((-(C2Q1)^2 + (M^2))/6)) / Sqrt[((-(C2Q1)^2 + (M^2))/6))})}
```

```
In[]:= sti =
"
$$\begin{aligned}
nZ10(z, t) = & (c2C7 * \sin(C2Q1*t) + c2C8 * \cos(C2Q1*t)) * (c2C5 * \sin(z)^{(1/6 * \sqrt{-C2Q1^2 + M^2})} + \\
& c2C6 * \sin(z)^{(-1/6 * \sqrt{-C2Q1^2 + M^2})}), \\
nZ11(z, t) = & (c2C3 * \sin(C2Q1*t) + c2C4 * \cos(C2Q1*t)) * (c2C1 * \sin(z)^{(1/6 * \sqrt{-C2Q1^2 + M^2})} + \\
& c2C2 * \sin(z)^{(-1/6 * \sqrt{-C2Q1^2 + M^2})}), \\
nZ8(z, t) = & 1/6 * (6 * \sqrt{-C2Q1^2 + M^2}) * c2C6 * (c2C7 * \sin(C2Q1*t) + c2C8 * \cos(C2Q1*t)) * \sin(z)^{(-1/6 * \\
& \sqrt{-C2Q1^2 + M^2})} - 6 * \sqrt{-C2Q1^2 + M^2} * c2C5 * (c2C7 * \sin(C2Q1*t) + c2C8 * \cos(C2Q1*t)) * \\
& \sin(z)^{(1/6 * \sqrt{-C2Q1^2 + M^2})} + (-6 * M * c2C10 - \text{Int}(-\cos(M*t) * ((-C2Q1^2 + M^2) * \text{Int}(-\cos(z) * \\
& ((C2Q1 * c2C6 * c2C7 + M * c2C2 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * c2C6 * c2C8 + M * c2C2 * \\
& c2C3)) * \sin(z)^{(-1/6 * \sqrt{-C2Q1^2 + M^2}) - 1}) + ((C2Q1 * c2C5 * c2C7 + M * c2C1 * c2C4) * \cos(C2Q1*t) \\
& + \sin(C2Q1*t) * (-C2Q1 * c2C5 * c2C8 + M * c2C1 * c2C3)) * \sin(z)^{(1/6 * \sqrt{-C2Q1^2 + M^2}) - 1}), z) - 6 * \\
& \sqrt{-C2Q1^2 + M^2} * (((C2Q1 * c2C6 * c2C7 + M * c2C2 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * \\
& c2C6 * c2C8 + M * c2C2 * c2C3)) * \sin(z)^{(-1/6 * \sqrt{-C2Q1^2 + M^2}) - 1} - \sin(z)^{(1/6 * \sqrt{-C2Q1^2 + M^2})} * \\
& ((C2Q1 * c2C5 * c2C7 + M * c2C1 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * c2C5 * c2C8 + M * c2C1 * \\
& c2C3))), t) * \cos(M*t) + (6 * M * c2C9 - \text{Int}(-\sin(M*t) * ((-C2Q1^2 + M^2) * \text{Int}(-\cos(z) * ((C2Q1 * \\
& c2C6 * c2C7 + M * c2C2 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * c2C6 * c2C8 + M * c2C2 * c2C3)) * \sin(z) \\
& ^{(-1/6 * \sqrt{-C2Q1^2 + M^2}) - 1} + ((C2Q1 * c2C5 * c2C7 + M * c2C1 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * \\
& (-C2Q1 * c2C5 * c2C8 + M * c2C1 * c2C3)) * \sin(z)^{(1/6 * \sqrt{-C2Q1^2 + M^2}) - 1}), z) - 6 * \sqrt{-C2Q1^2 + M^2} * \\
& (((C2Q1 * c2C6 * c2C7 + M * c2C2 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * c2C6 * c2C8 + M * \\
& c2C2 * c2C3)) * \sin(z)^{(-1/6 * \sqrt{-C2Q1^2 + M^2}) - 1} - \sin(z)^{(1/6 * \sqrt{-C2Q1^2 + M^2})} * ((C2Q1 * \\
& c2C5 * c2C7 + M * c2C1 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * c2C5 * c2C8 + M * c2C1 * c2C3))), t) * \\
& \sin(M*t) - C2Q1 * \text{Int}(-\cos(z) * ((-C2Q1 * c2C6 * c2C8 + M * c2C2 * c2C3) * \cos(C2Q1*t) - \sin(C2Q1*t) * \\
& (C2Q1 * c2C6 * c2C7 + M * c2C2 * c2C4)) * \sin(z)^{(-1/6 * \sqrt{-C2Q1^2 + M^2}) - 1} + ((-C2Q1 * c2C5 * \\
& c2C8 + M * c2C1 * c2C3) * \cos(C2Q1*t) - \sin(C2Q1*t) * (C2Q1 * c2C5 * c2C7 + M * c2C1 * c2C4)) * \sin(z)^{(1/6 * \\
& \sqrt{-C2Q1^2 + M^2}) - 1}), z) / M, \\
nZ9(z, t) = & 1/6 * ((6 * M * c2C9 - \text{Int}(-\sin(M*t) * ((-C2Q1^2 + M^2) * \\
& \text{Int}(-\cos(z) * ((C2Q1 * c2C6 * c2C7 + M * c2C2 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * c2C6 * \\
& c2C8 + M * c2C2 * c2C3)) * \sin(z)^{(-1/6 * \sqrt{-C2Q1^2 + M^2}) - 1} + ((C2Q1 * c2C5 * c2C7 + M * c2C1 * c2C4) * \\
& \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * c2C5 * c2C8 + M * c2C1 * c2C3)) * \sin(z)^{(1/6 * \sqrt{-C2Q1^2 + M^2})} - \\
& 6 * \sqrt{-C2Q1^2 + M^2} * (((C2Q1 * c2C6 * c2C7 + M * c2C2 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * \\
& c2C5 * c2C8 + M * c2C1 * c2C3))), t) * \cos(M*t) + (6 * M * c2C10 + \text{Int}(-\cos(M*t) * ((-C2Q1^2 + M^2) * \\
& \text{Int}(-\cos(z) * ((C2Q1 * c2C6 * c2C7 + M * c2C2 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * \\
& c2C5 * c2C8 + M * c2C1 * c2C3))), t) * \cos(M*t) + M * \text{Int}(-\cos(z) * ((C2Q1 * c2C6 * c2C7 + M * c2C2 * c2C4) * \\
& \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * c2C6 * c2C8 + M * c2C2 * c2C3)) * \sin(z)^{(-1/6 * \sqrt{-C2Q1^2 + M^2}) - 1} + \\
& ((C2Q1 * c2C5 * c2C7 + M * c2C1 * c2C4) * \cos(C2Q1*t) + \sin(C2Q1*t) * (-C2Q1 * c2C5 * c2C8 + M * c2C1 * \\
& c2C3)) * \sin(z)^{(1/6 * \sqrt{-C2Q1^2 + M^2}) - 1}), z) / M);
\end{aligned}$$

```

In[]:= ConvertMapleToMathematica[sti]

```
Out[]:= {nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5 * (Sin[z]^((1/6 * \\
& Sqrt[((-C2Q1)^2 + (M^2))])) + (c2C6 * (Sin[z]^((-1/6 * \\
& Sqrt[((-C2Q1)^2 + (M^2)))))))), nZ11[z, t] == (((c2C3 * Sin[(C2Q1 * t)]) + \\
& (c2C4 * Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z]^((1/6 * \\
& Sqrt[((-C2Q1)^2 + (M^2)))) + (c2C2 * (Sin[z]^((-1/6 * \\
& Sqrt[((-C2Q1)^2 + (M^2)))))))), nZ8[z, t] == (((1/6 * (((((6 * \\
& Sqrt[((-C2Q1)^2 + (M^2)))])))
```

```

* c2C6) * ((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)]))) * (Sin[z] ^
((-1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) - (((6 * Sqrt[((-(C2Q1) ^ 2)
+ (M ^ 2))]) * c2C5) * ((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)]))) *
(Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) + ((((-6) * M) * c2C10) -
Integrate[(-(Cos[(M * t)]) * (((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) *
((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1
* t)] * (((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((((-1)
/ 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))) + (((((C2Q1 * c2C5) * c2C7) +
((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5)
* c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) +
(M ^ 2))]) - 1))), z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * ((((((C2Q1
* c2C6) * c2C7) + ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] *
(((-(C2Q1) * c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((((-1) / 6) *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) +
(M ^ 2))])) * (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)])
+ (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))), t]) *
Cos[(M * t)]) + (((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) *
((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7)
+ ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) *
c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((((-1) / 6) * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]) - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) *
Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) *
c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))), z]) - ((6 *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * ((((((C2Q1 * c2C6) * c2C7) +
((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) *
c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((((-1) / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))])) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]))) *
((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1
* t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))), t]) * Sin[(M
* t)]) - (C2Q1 * Integrate[(-(Cos[z]) * (((((-(C2Q1) * c2C6) * c2C8) + ((M *
c2C2) * c2C3)) * Cos[(C2Q1 * t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C6) * c2C7)
+ ((M * c2C2) * c2C4)))) * (Sin[z] ^ ((((-1) / 6) * Sqrt[((-(C2Q1) ^ 2) +
(M ^ 2))])) - 1))) + (((((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)) * Cos[(C2Q1
* t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))), z])) / M),
nZ9[z, t] == (((1 / 6) * (((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) *
((((-(C2Q1) ^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7)
+ ((M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) *
c2C6) * c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((((-1) / 6) * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]) - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) *
Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) *
c2C3)))) * (Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1)))), z])) / M,

```

```

* t) ] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))), t]) * Cos[(M
* t)]) + (((6 * M) * c2C10) + Integrate[(-(Cos[(M * t)]) * (((-(C2Q1) ^ 2)
+ (M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))) - 1])) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))) - 1]))), z]) - ((6 *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))) - ((Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * (((((C2Q1
* c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)]
* (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))))))), t]) * Sin[(M *
t)]) + (M * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))) - 1])) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1))), z])) / M)

```

gtry and Γ and ...

```

In[]:= gtry = MatrixMetric44
Out[]=
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}}
In[]:= Protect[gtry]
Out[]= {gtry}
In[]:= Unprotect[ginv, \[CapitalGamma], Ricci\[CapitalGamma], Rie\[CapitalGamma], RS, EinsteinG]
Out[]= {}
In[]:= result = {ginv, \[CapitalGamma], Ricci\[CapitalGamma], Rie\[CapitalGamma], RS, EinsteinG} = rt[gtry];

```

Thu 25 Dec 2025 04:09:28 GMT-8

```

{5.09382, Null}

```

Thu 25 Dec 2025 04:09:33 GMT-8

```

In[]:= Protect[ginv, \[CapitalGamma], Ricci\[CapitalGamma], Rie\[CapitalGamma], RS, EinsteinG]
Out[]= {ginv, \[CapitalGamma], Ricci\[CapitalGamma], Rie\[CapitalGamma], RS, EinsteinG}

```

verify $g_{\mu\nu}|\alpha = 0 :$

```

In[=]:= Simplify[Table[D[gtry[[j, k]], X[[i]]] -
  Sum[gtry[[k, s]] \[Cross] \[Gamma][[s, i, j]], {s, 1, DIM8}] - Sum[gtry[[j, s]] \[Cross] \[Gamma][[s, i, k]], {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union

Out[=]= {0}

In[=]:= Table[g[j][k] \[Rule]
  If[j == k, ToExpression["((g[" <> ToString[j] <> "] [" <> ToString[k] <> "] [#1, #2]) &)"],
  (0 &)], {j, 0, 7}, {k, 0, 7}];
sg = % // Flatten

Out[=]= {g[0][0] \[Rule] (g[0][0][#1, #2] &), g[0][1] \[Rule] (0 &), g[0][2] \[Rule] (0 &),
g[0][3] \[Rule] (0 &), g[0][4] \[Rule] (0 &), g[0][5] \[Rule] (0 &), g[0][6] \[Rule] (0 &),
g[0][7] \[Rule] (0 &), g[1][0] \[Rule] (0 &), g[1][1] \[Rule] (g[1][1][#1, #2] &), g[1][2] \[Rule] (0 &),
g[1][3] \[Rule] (0 &), g[1][4] \[Rule] (0 &), g[1][5] \[Rule] (0 &), g[1][6] \[Rule] (0 &),
g[1][7] \[Rule] (0 &), g[2][0] \[Rule] (0 &), g[2][1] \[Rule] (0 &), g[2][2] \[Rule] (g[2][2][#1, #2] &),
g[2][3] \[Rule] (0 &), g[2][4] \[Rule] (0 &), g[2][5] \[Rule] (0 &), g[2][6] \[Rule] (0 &), g[2][7] \[Rule] (0 &),
g[3][0] \[Rule] (0 &), g[3][1] \[Rule] (0 &), g[3][2] \[Rule] (0 &), g[3][3] \[Rule] (g[3][3][#1, #2] &),
g[3][4] \[Rule] (0 &), g[3][5] \[Rule] (0 &), g[3][6] \[Rule] (0 &), g[3][7] \[Rule] (0 &), g[4][0] \[Rule] (0 &),
g[4][1] \[Rule] (0 &), g[4][2] \[Rule] (0 &), g[4][3] \[Rule] (0 &), g[4][4] \[Rule] (g[4][4][#1, #2] &),
g[4][5] \[Rule] (0 &), g[4][6] \[Rule] (0 &), g[4][7] \[Rule] (0 &), g[5][0] \[Rule] (0 &), g[5][1] \[Rule] (0 &),
g[5][2] \[Rule] (0 &), g[5][3] \[Rule] (0 &), g[5][4] \[Rule] (0 &), g[5][5] \[Rule] (g[5][5][#1, #2] &),
g[5][6] \[Rule] (0 &), g[5][7] \[Rule] (0 &), g[6][0] \[Rule] (0 &), g[6][1] \[Rule] (0 &), g[6][2] \[Rule] (0 &),
g[6][3] \[Rule] (0 &), g[6][4] \[Rule] (0 &), g[6][5] \[Rule] (0 &), g[6][6] \[Rule] (g[6][6][#1, #2] &),
g[6][7] \[Rule] (0 &), g[7][0] \[Rule] (0 &), g[7][1] \[Rule] (0 &), g[7][2] \[Rule] (0 &), g[7][3] \[Rule] (0 &),
g[7][4] \[Rule] (0 &), g[7][5] \[Rule] (0 &), g[7][6] \[Rule] (0 &), g[7][7] \[Rule] (g[7][7][#1, #2] &)}

In[=]:= g[7][7][x0, x4] /. ssgm4488
% /. sx0x4

Out[=]= -e^-2 a4[H x4] Sin[6 H x0]^(1/3)

Out[=]= -e^-2 a4[t] Sin[z]^(1/3)

In[=]:= MatrixMetric44

Out[=]= {{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

```

```

In[]:= ssgm4488[x0, x4] /. sx0x4

Out[]= {g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
g[1][1] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
g[2][1] → (0 &), g[2][2] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
g[3][2] → (0 &), g[3][3] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
g[5][5] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
g[6][5] → (0 &), g[6][6] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &) } [z, t]
                                         6 H , H

In[]:= (*ssgGzt={G[0][0]→((Cot[#1]^2)&),G[0][1]→((0)&),G[0][2]→((0)&),
G[0][3]→((0)&),G[0][4]→((0)&),G[0][5]→((0)&),G[0][6]→((0)&),G[0][7]→((0)&),
G[1][0]→((0)&),G[1][1]→((e^2 a4[#2] Sin[#1]^(1/3))&),G[1][2]→((0)&),G[1][3]→((0)&),
G[1][4]→((0)&),G[1][5]→((0)&),G[1][6]→((0)&),G[1][7]→((0)&),G[2][0]→((0)&),
G[2][1]→((0)&),G[2][2]→((e^2 a4[#2] Sin[#1]^(1/3))&),G[2][3]→((0)&),G[2][4]→((0)&),
G[2][5]→((0)&),G[2][6]→((0)&),G[2][7]→((0)&),G[3][0]→((0)&),G[3][1]→((0)&),
G[3][2]→((0)&),G[3][3]→((e^2 a4[#2] Sin[#1]^(1/3))&),G[3][4]→((0)&),G[3][5]→((0)&),
G[3][6]→((0)&),G[3][7]→((0)&),G[4][0]→((0)&),G[4][1]→((0)&),G[4][2]→((0)&),
G[4][3]→((0)&),G[4][4]→((-1)&),G[4][5]→((0)&),G[4][6]→((0)&),G[4][7]→((0)&),
G[5][0]→((0)&),G[5][1]→((0)&),G[5][2]→((0)&),G[5][3]→((0)&),G[5][4]→((0)&),
G[5][5]→((-e^-2 a4[#2] Sin[#1]^(1/3))&),G[5][6]→((0)&),G[5][7]→((0)&),
G[6][0]→((0)&),G[6][1]→((0)&),G[6][2]→((0)&),G[6][3]→((0)&),G[6][4]→((0)&),
G[6][5]→((0)&),G[6][6]→((-e^-2 a4[#2] Sin[#1]^(1/3))&),G[6][7]→((0)&),
G[7][0]→((0)&),G[7][1]→((0)&),G[7][2]→((0)&),G[7][3]→((0)&),G[7][4]→((0)&),
G[7][5]→((0)&),G[7][6]→((0)&),G[7][7]→((-e^-2 a4[#2] Sin[#1]^(1/3))&)*)

In[]:= (*G[#[z,t]/.ssgGzt&/@Range[0,7]*)

In[]:= detgg = Det[g4488 /. sg] // FullSimplify[#, constraintVars] &

Out[]= g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]

```

In[•]:= g4488 /. sg // MatrixForm

Out[•]//MatrixForm=

$$\begin{pmatrix} g[0][0][x0, x4] & 0 & 0 & 0 & 0 & 0 \\ 0 & g[1][1][x0, x4] & 0 & 0 & 0 & 0 \\ 0 & 0 & g[2][2][x0, x4] & 0 & 0 & 0 \\ 0 & 0 & 0 & g[3][3][x0, x4] & 0 & 0 \\ 0 & 0 & 0 & 0 & g[4][4][x0, x4] & 0 \\ 0 & 0 & 0 & 0 & 0 & g[5][5][x0] \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:= MatrixMetric44.η4488

(*eAa=*) FullSimplify[√%, constraintVars]

Out[•]=

$$\left\{ \{\text{Cot}[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{2 a4[H x4]} \sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \{0, 0, e^{2 a4[H x4]} \sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{2 a4[H x4]} \sin[6 H x0]^{1/3}, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-2 a4[H x4]} \sin[6 H x0]^{1/3}, 0, 0\}, \{0, 0, 0, 0, 0, 0, e^{-2 a4[H x4]} \sin[6 H x0]^{1/3}, 0\}, \{0, 0, 0, 0, 0, 0, 0, e^{-2 a4[H x4]} \sin[6 H x0]^{1/3}\} \right\}$$

Out[•]=

$$\left\{ \{\text{Cot}[6 H x0], 0, 0, 0, 0, 0, 0, 0\}, \{0, \sqrt{e^{2 a4[H x4]}} \sin[6 H x0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \{0, 0, \sqrt{e^{2 a4[H x4]}} \sin[6 H x0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} \sin[6 H x0]^{1/6}, 0, 0\}, \{0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} \sin[6 H x0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} \sin[6 H x0]^{1/6}\} \right\}$$

eAa

In[•]:=

g4488.η4488 /. sg

(eAa = FullSimplify[√%, constraintVars]) // MatrixForm

Out[•]=

$$\left\{ \{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \{0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0\}, \{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0\}, \{0, 0, 0, 0, -g[4][4][x0, x4], 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, -g[5][5][x0, x4], 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, -g[6][6][x0, x4], 0\}, \{0, 0, 0, 0, 0, 0, 0, -g[7][7][x0, x4]\} \right\}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} \sqrt{g[0][0][x0, x4]} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{g[1][1][x0, x4]} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{g[2][2][x0, x4]} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{g[3][3][x0, x4]} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{-g[4][4][x0, x4]} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

In[*]:= eAa
ssgm4488
(eAa04 = %% /. % // FullSimplify[#, constraintVars] &) // MatrixForm

Out[*]=
{{\{\sqrt{g[0][0][x0, x4]}, 0, 0, 0, 0, 0, 0, 0\}, \{0, \sqrt{g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0\},
\{0, 0, \sqrt{g[2][2][x0, x4]}, 0, 0, 0, 0, 0\}, \{0, 0, 0, \sqrt{g[3][3][x0, x4]}, 0, 0, 0, 0\},
\{0, 0, 0, 0, \sqrt{-g[4][4][x0, x4]}, 0, 0, 0\}, \{0, 0, 0, 0, 0, \sqrt{-g[5][5][x0, x4]}, 0, 0\},
\{0, 0, 0, 0, 0, 0, \sqrt{-g[6][6][x0, x4]}, 0\}, \{0, 0, 0, 0, 0, 0, 0, \sqrt{-g[7][7][x0, x4]}\}}}

Out[*]=
{g[0][0] \rightarrow (\text{Cot}[6 H \#1]^2 &), g[0][1] \rightarrow (0 &), g[0][2] \rightarrow (0 &), g[0][3] \rightarrow (0 &),
g[0][4] \rightarrow (0 &), g[0][5] \rightarrow (0 &), g[0][6] \rightarrow (0 &), g[0][7] \rightarrow (0 &), g[1][0] \rightarrow (0 &),
g[1][1] \rightarrow (\text{e}^{2 a4[H \#2]} \text{Sin}[6 H \#1]^{1/3} &), g[1][2] \rightarrow (0 &), g[1][3] \rightarrow (0 &),
g[1][4] \rightarrow (0 &), g[1][5] \rightarrow (0 &), g[1][6] \rightarrow (0 &), g[1][7] \rightarrow (0 &), g[2][0] \rightarrow (0 &),
g[2][1] \rightarrow (0 &), g[2][2] \rightarrow (\text{e}^{2 a4[H \#2]} \text{Sin}[6 H \#1]^{1/3} &), g[2][3] \rightarrow (0 &), g[2][4] \rightarrow (0 &),
g[2][5] \rightarrow (0 &), g[2][6] \rightarrow (0 &), g[2][7] \rightarrow (0 &), g[3][0] \rightarrow (0 &), g[3][1] \rightarrow (0 &),
g[3][2] \rightarrow (0 &), g[3][3] \rightarrow (\text{e}^{2 a4[H \#2]} \text{Sin}[6 H \#1]^{1/3} &), g[3][4] \rightarrow (0 &), g[3][5] \rightarrow (0 &),
g[3][6] \rightarrow (0 &), g[3][7] \rightarrow (0 &), g[4][0] \rightarrow (0 &), g[4][1] \rightarrow (0 &), g[4][2] \rightarrow (0 &),
g[4][3] \rightarrow (0 &), g[4][4] \rightarrow (-1 &), g[4][5] \rightarrow (0 &), g[4][6] \rightarrow (0 &), g[4][7] \rightarrow (0 &),
g[5][0] \rightarrow (0 &), g[5][1] \rightarrow (0 &), g[5][2] \rightarrow (0 &), g[5][3] \rightarrow (0 &), g[5][4] \rightarrow (0 &),
g[5][5] \rightarrow (-\text{e}^{-2 a4[H \#2]} \text{Sin}[6 H \#1]^{1/3} &), g[5][6] \rightarrow (0 &), g[5][7] \rightarrow (0 &),
g[6][0] \rightarrow (0 &), g[6][1] \rightarrow (0 &), g[6][2] \rightarrow (0 &), g[6][3] \rightarrow (0 &), g[6][4] \rightarrow (0 &),
g[6][5] \rightarrow (0 &), g[6][6] \rightarrow (-\text{e}^{-2 a4[H \#2]} \text{Sin}[6 H \#1]^{1/3} &), g[6][7] \rightarrow (0 &),
g[7][0] \rightarrow (0 &), g[7][1] \rightarrow (0 &), g[7][2] \rightarrow (0 &), g[7][3] \rightarrow (0 &), g[7][4] \rightarrow (0 &),
g[7][5] \rightarrow (0 &), g[7][6] \rightarrow (0 &), g[7][7] \rightarrow (-\text{e}^{-2 a4[H \#2]} \text{Sin}[6 H \#1]^{1/3} &)}

Out[*]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6 H x0] & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\text{e}^{2 a4[H x4]}} \text{Sin}[6 H x0]^{1/6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\text{e}^{2 a4[H x4]}} \text{Sin}[6 H x0]^{1/6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\text{e}^{2 a4[H x4]}} \text{Sin}[6 H x0]^{1/6} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


```

OCTAD $e_\alpha^{(A) \leftarrow \text{Lorentz}}$: $e_\alpha^{(A) \leftarrow \text{spacetime}}$

■ $e_\alpha^{(A)} = e_\alpha^{(A)} = eAa$

```
In[1]:= (sg $e_\alpha^{(A)}$  = Thread[Flatten[e $\alpha^{(A)}$ ] → Flatten[eAa]]) (*//Column*)
```

Out[1]=

$$\left\{ E_\alpha^{(A)}[0, 0] \rightarrow \sqrt{g[0][0][x0, x4]}, E_\alpha^{(A)}[0, 1] \rightarrow 0, E_\alpha^{(A)}[0, 2] \rightarrow 0, E_\alpha^{(A)}[0, 3] \rightarrow 0, E_\alpha^{(A)}[0, 4] \rightarrow 0, \right. \\ E_\alpha^{(A)}[0, 5] \rightarrow 0, E_\alpha^{(A)}[0, 6] \rightarrow 0, E_\alpha^{(A)}[0, 7] \rightarrow 0, E_\alpha^{(A)}[1, 0] \rightarrow 0, E_\alpha^{(A)}[1, 1] \rightarrow \sqrt{g[1][1][x0, x4]}, \\ E_\alpha^{(A)}[1, 2] \rightarrow 0, E_\alpha^{(A)}[1, 3] \rightarrow 0, E_\alpha^{(A)}[1, 4] \rightarrow 0, E_\alpha^{(A)}[1, 5] \rightarrow 0, E_\alpha^{(A)}[1, 6] \rightarrow 0, E_\alpha^{(A)}[1, 7] \rightarrow 0, \\ E_\alpha^{(A)}[2, 0] \rightarrow 0, E_\alpha^{(A)}[2, 1] \rightarrow 0, E_\alpha^{(A)}[2, 2] \rightarrow \sqrt{g[2][2][x0, x4]}, E_\alpha^{(A)}[2, 3] \rightarrow 0, E_\alpha^{(A)}[2, 4] \rightarrow 0, \\ E_\alpha^{(A)}[2, 5] \rightarrow 0, E_\alpha^{(A)}[2, 6] \rightarrow 0, E_\alpha^{(A)}[2, 7] \rightarrow 0, E_\alpha^{(A)}[3, 0] \rightarrow 0, E_\alpha^{(A)}[3, 1] \rightarrow 0, E_\alpha^{(A)}[3, 2] \rightarrow 0, \\ E_\alpha^{(A)}[3, 3] \rightarrow \sqrt{g[3][3][x0, x4]}, E_\alpha^{(A)}[3, 4] \rightarrow 0, E_\alpha^{(A)}[3, 5] \rightarrow 0, E_\alpha^{(A)}[3, 6] \rightarrow 0, E_\alpha^{(A)}[3, 7] \rightarrow 0, \\ E_\alpha^{(A)}[4, 0] \rightarrow 0, E_\alpha^{(A)}[4, 1] \rightarrow 0, E_\alpha^{(A)}[4, 2] \rightarrow 0, E_\alpha^{(A)}[4, 3] \rightarrow 0, E_\alpha^{(A)}[4, 4] \rightarrow \sqrt{-g[4][4][x0, x4]}, \\ E_\alpha^{(A)}[4, 5] \rightarrow 0, E_\alpha^{(A)}[4, 6] \rightarrow 0, E_\alpha^{(A)}[4, 7] \rightarrow 0, E_\alpha^{(A)}[5, 0] \rightarrow 0, E_\alpha^{(A)}[5, 1] \rightarrow 0, E_\alpha^{(A)}[5, 2] \rightarrow 0, \\ E_\alpha^{(A)}[5, 3] \rightarrow 0, E_\alpha^{(A)}[5, 4] \rightarrow 0, E_\alpha^{(A)}[5, 5] \rightarrow \sqrt{-g[5][5][x0, x4]}, E_\alpha^{(A)}[5, 6] \rightarrow 0, E_\alpha^{(A)}[5, 7] \rightarrow 0, \\ E_\alpha^{(A)}[6, 0] \rightarrow 0, E_\alpha^{(A)}[6, 1] \rightarrow 0, E_\alpha^{(A)}[6, 2] \rightarrow 0, E_\alpha^{(A)}[6, 3] \rightarrow 0, E_\alpha^{(A)}[6, 4] \rightarrow 0, E_\alpha^{(A)}[6, 5] \rightarrow 0, \\ E_\alpha^{(A)}[6, 6] \rightarrow \sqrt{-g[6][6][x0, x4]}, E_\alpha^{(A)}[6, 7] \rightarrow 0, E_\alpha^{(A)}[7, 0] \rightarrow 0, E_\alpha^{(A)}[7, 1] \rightarrow 0, E_\alpha^{(A)}[7, 2] \rightarrow 0, \\ \left. E_\alpha^{(A)}[7, 3] \rightarrow 0, E_\alpha^{(A)}[7, 4] \rightarrow 0, E_\alpha^{(A)}[7, 5] \rightarrow 0, E_\alpha^{(A)}[7, 6] \rightarrow 0, E_\alpha^{(A)}[7, 7] \rightarrow \sqrt{-g[7][7][x0, x4]} \right\}$$

```
In[2]:= (*Symbolize[sg $e_\alpha^{(A)}$ ]*)
```

```
In[3]:= (*Symbolize[sgtrye $\alpha^{(A)}$ ]*)
```

```
In[4]:= (sgtrye $\alpha^{(A)}$  = Thread[Flatten[e $\alpha^{(A)}$ ] → Flatten[eAa04]]) (*//Column*)
```

Out[4]=

$$\left\{ E_\alpha^{(A)}[0, 0] \rightarrow \text{Cot}[6Hx0], E_\alpha^{(A)}[0, 1] \rightarrow 0, E_\alpha^{(A)}[0, 2] \rightarrow 0, E_\alpha^{(A)}[0, 3] \rightarrow 0, \right. \\ E_\alpha^{(A)}[0, 4] \rightarrow 0, E_\alpha^{(A)}[0, 5] \rightarrow 0, E_\alpha^{(A)}[0, 6] \rightarrow 0, E_\alpha^{(A)}[0, 7] \rightarrow 0, E_\alpha^{(A)}[1, 0] \rightarrow 0, \\ E_\alpha^{(A)}[1, 1] \rightarrow \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}, E_\alpha^{(A)}[1, 2] \rightarrow 0, E_\alpha^{(A)}[1, 3] \rightarrow 0, E_\alpha^{(A)}[1, 4] \rightarrow 0, \\ E_\alpha^{(A)}[1, 5] \rightarrow 0, E_\alpha^{(A)}[1, 6] \rightarrow 0, E_\alpha^{(A)}[1, 7] \rightarrow 0, E_\alpha^{(A)}[2, 0] \rightarrow 0, E_\alpha^{(A)}[2, 1] \rightarrow 0, \\ E_\alpha^{(A)}[2, 2] \rightarrow \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}, E_\alpha^{(A)}[2, 3] \rightarrow 0, E_\alpha^{(A)}[2, 4] \rightarrow 0, E_\alpha^{(A)}[2, 5] \rightarrow 0, \\ E_\alpha^{(A)}[2, 6] \rightarrow 0, E_\alpha^{(A)}[2, 7] \rightarrow 0, E_\alpha^{(A)}[3, 0] \rightarrow 0, E_\alpha^{(A)}[3, 1] \rightarrow 0, E_\alpha^{(A)}[3, 2] \rightarrow 0, \\ E_\alpha^{(A)}[3, 3] \rightarrow \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}, E_\alpha^{(A)}[3, 4] \rightarrow 0, E_\alpha^{(A)}[3, 5] \rightarrow 0, E_\alpha^{(A)}[3, 6] \rightarrow 0, \\ E_\alpha^{(A)}[3, 7] \rightarrow 0, E_\alpha^{(A)}[4, 0] \rightarrow 0, E_\alpha^{(A)}[4, 1] \rightarrow 0, E_\alpha^{(A)}[4, 2] \rightarrow 0, E_\alpha^{(A)}[4, 3] \rightarrow 0, \\ E_\alpha^{(A)}[4, 4] \rightarrow 1, E_\alpha^{(A)}[4, 5] \rightarrow 0, E_\alpha^{(A)}[4, 6] \rightarrow 0, E_\alpha^{(A)}[4, 7] \rightarrow 0, E_\alpha^{(A)}[5, 0] \rightarrow 0, E_\alpha^{(A)}[5, 1] \rightarrow 0, \\ E_\alpha^{(A)}[5, 2] \rightarrow 0, E_\alpha^{(A)}[5, 3] \rightarrow 0, E_\alpha^{(A)}[5, 4] \rightarrow 0, E_\alpha^{(A)}[5, 5] \rightarrow \sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}, \\ E_\alpha^{(A)}[5, 6] \rightarrow 0, E_\alpha^{(A)}[5, 7] \rightarrow 0, E_\alpha^{(A)}[6, 0] \rightarrow 0, E_\alpha^{(A)}[6, 1] \rightarrow 0, E_\alpha^{(A)}[6, 2] \rightarrow 0, \\ E_\alpha^{(A)}[6, 3] \rightarrow 0, E_\alpha^{(A)}[6, 4] \rightarrow 0, E_\alpha^{(A)}[6, 5] \rightarrow 0, E_\alpha^{(A)}[6, 6] \rightarrow \sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}, \\ E_\alpha^{(A)}[6, 7] \rightarrow 0, E_\alpha^{(A)}[7, 0] \rightarrow 0, E_\alpha^{(A)}[7, 1] \rightarrow 0, E_\alpha^{(A)}[7, 2] \rightarrow 0, E_\alpha^{(A)}[7, 3] \rightarrow 0, \\ \left. E_\alpha^{(A)}[7, 4] \rightarrow 0, E_\alpha^{(A)}[7, 5] \rightarrow 0, E_\alpha^{(A)}[7, 6] \rightarrow 0, E_\alpha^{(A)}[7, 7] \rightarrow \sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6} \right\}$$

```
In[1]:= (sgtrye $^{\alpha}_{(A)}$  = Thread[Flatten[e $^{\alpha}_{(A)}$ ]  $\rightarrow$  Flatten[Inverse[e $^{(\alpha)}_{\alpha}$  /. sgtrye $^{\alpha}_{\alpha}$ ]]]) (*//Column*)
```

$\text{Out}[\#]=$
 $E_{(A)}^{\alpha}[0, 0] \rightarrow \text{Tan}[6 H x 0], E_{(A)}^{\alpha}[0, 1] \rightarrow 0, E_{(A)}^{\alpha}[0, 2] \rightarrow 0, E_{(A)}^{\alpha}[0, 3] \rightarrow 0,$
 $E_{(A)}^{\alpha}[0, 4] \rightarrow 0, E_{(A)}^{\alpha}[0, 5] \rightarrow 0, E_{(A)}^{\alpha}[0, 6] \rightarrow 0, E_{(A)}^{\alpha}[0, 7] \rightarrow 0, E_{(A)}^{\alpha}[1, 0] \rightarrow 0,$
 $E_{(A)}^{\alpha}[1, 1] \rightarrow \frac{1}{\sqrt{e^{2 a4[H x 4]} \sin[6 H x 0]^{1/6}}}, E_{(A)}^{\alpha}[1, 2] \rightarrow 0, E_{(A)}^{\alpha}[1, 3] \rightarrow 0, E_{(A)}^{\alpha}[1, 4] \rightarrow 0,$
 $E_{(A)}^{\alpha}[1, 5] \rightarrow 0, E_{(A)}^{\alpha}[1, 6] \rightarrow 0, E_{(A)}^{\alpha}[1, 7] \rightarrow 0, E_{(A)}^{\alpha}[2, 0] \rightarrow 0, E_{(A)}^{\alpha}[2, 1] \rightarrow 0,$
 $E_{(A)}^{\alpha}[2, 2] \rightarrow \frac{1}{\sqrt{e^{2 a4[H x 4]} \sin[6 H x 0]^{1/6}}}, E_{(A)}^{\alpha}[2, 3] \rightarrow 0, E_{(A)}^{\alpha}[2, 4] \rightarrow 0, E_{(A)}^{\alpha}[2, 5] \rightarrow 0,$
 $E_{(A)}^{\alpha}[2, 6] \rightarrow 0, E_{(A)}^{\alpha}[2, 7] \rightarrow 0, E_{(A)}^{\alpha}[3, 0] \rightarrow 0, E_{(A)}^{\alpha}[3, 1] \rightarrow 0, E_{(A)}^{\alpha}[3, 2] \rightarrow 0,$
 $E_{(A)}^{\alpha}[3, 3] \rightarrow \frac{1}{\sqrt{e^{2 a4[H x 4]} \sin[6 H x 0]^{1/6}}}, E_{(A)}^{\alpha}[3, 4] \rightarrow 0, E_{(A)}^{\alpha}[3, 5] \rightarrow 0, E_{(A)}^{\alpha}[3, 6] \rightarrow 0,$
 $E_{(A)}^{\alpha}[3, 7] \rightarrow 0, E_{(A)}^{\alpha}[4, 0] \rightarrow 0, E_{(A)}^{\alpha}[4, 1] \rightarrow 0, E_{(A)}^{\alpha}[4, 2] \rightarrow 0, E_{(A)}^{\alpha}[4, 3] \rightarrow 0,$
 $E_{(A)}^{\alpha}[4, 4] \rightarrow 1, E_{(A)}^{\alpha}[4, 5] \rightarrow 0, E_{(A)}^{\alpha}[4, 6] \rightarrow 0, E_{(A)}^{\alpha}[4, 7] \rightarrow 0, E_{(A)}^{\alpha}[5, 0] \rightarrow 0, E_{(A)}^{\alpha}[5, 1] \rightarrow 0,$
 $E_{(A)}^{\alpha}[5, 2] \rightarrow 0, E_{(A)}^{\alpha}[5, 3] \rightarrow 0, E_{(A)}^{\alpha}[5, 4] \rightarrow 0, E_{(A)}^{\alpha}[5, 5] \rightarrow \frac{1}{\sqrt{e^{-2 a4[H x 4]} \sin[6 H x 0]^{1/6}}},$
 $E_{(A)}^{\alpha}[5, 6] \rightarrow 0, E_{(A)}^{\alpha}[5, 7] \rightarrow 0, E_{(A)}^{\alpha}[6, 0] \rightarrow 0, E_{(A)}^{\alpha}[6, 1] \rightarrow 0, E_{(A)}^{\alpha}[6, 2] \rightarrow 0,$
 $E_{(A)}^{\alpha}[6, 3] \rightarrow 0, E_{(A)}^{\alpha}[6, 4] \rightarrow 0, E_{(A)}^{\alpha}[6, 5] \rightarrow 0, E_{(A)}^{\alpha}[6, 6] \rightarrow \frac{1}{\sqrt{e^{-2 a4[H x 4]} \sin[6 H x 0]^{1/6}}},$
 $E_{(A)}^{\alpha}[6, 7] \rightarrow 0, E_{(A)}^{\alpha}[7, 0] \rightarrow 0, E_{(A)}^{\alpha}[7, 1] \rightarrow 0, E_{(A)}^{\alpha}[7, 2] \rightarrow 0, E_{(A)}^{\alpha}[7, 3] \rightarrow 0,$
 $E_{(A)}^{\alpha}[7, 4] \rightarrow 0, E_{(A)}^{\alpha}[7, 5] \rightarrow 0, E_{(A)}^{\alpha}[7, 6] \rightarrow 0, E_{(A)}^{\alpha}[7, 7] \rightarrow \frac{1}{\sqrt{e^{-2 a4[H x 4]} \sin[6 H x 0]^{1/6}}}$

```
In[1]:= gtrye $\alpha$ (A) = (e $\alpha$ (A) /. sgtrye $\alpha$ (A)) /. subsDefects
```

```

Out[7]=
{ {Cot[6 H x0], 0, 0, 0, 0, 0, 0, 0}, {0, ea4[H x4] Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, ea4[H x4] Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, 0, ea4[H x4] Sin[6 H x0]1/6, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e-a4[H x4] Sin[6 H x0]1/6, 0, 0}, {0, 0, 0, 0, 0, 0, e-a4[H x4] Sin[6 H x0]1/6, 0} }

```

```
In[1]:= gtryeα(A) = (eα(A) /. sgtryeα(A)) /. subsDefects
```

```

Out[8]=
{ { Tan[6 H x0], 0, 0, 0, 0, 0, 0, 0, 0}, {0, E^-a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0, 0}, {0, 0, E^-a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0, 0}, {0, 0, 0, E^-a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, E^-a4[H x4] Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, 0, E^-a4[H x4] Sin[6 H x0]^(1/6)} }

```

```

In[1]:= eα(A)
% /. sgtryeα(A) // MatrixForm

Out[1]= { {Eα(A)[0, 0], Eα(A)[0, 1], Eα(A)[0, 2], Eα(A)[0, 3], Eα(A)[0, 4], Eα(A)[0, 5], Eα(A)[0, 6], Eα(A)[0, 7]}, {Eα(A)[1, 0], Eα(A)[1, 1], Eα(A)[1, 2], Eα(A)[1, 3], Eα(A)[1, 4], Eα(A)[1, 5], Eα(A)[1, 6], Eα(A)[1, 7]}, {Eα(A)[2, 0], Eα(A)[2, 1], Eα(A)[2, 2], Eα(A)[2, 3], Eα(A)[2, 4], Eα(A)[2, 5], Eα(A)[2, 6], Eα(A)[2, 7]}, {Eα(A)[3, 0], Eα(A)[3, 1], Eα(A)[3, 2], Eα(A)[3, 3], Eα(A)[3, 4], Eα(A)[3, 5], Eα(A)[3, 6], Eα(A)[3, 7]}, {Eα(A)[4, 0], Eα(A)[4, 1], Eα(A)[4, 2], Eα(A)[4, 3], Eα(A)[4, 4], Eα(A)[4, 5], Eα(A)[4, 6], Eα(A)[4, 7]}, {Eα(A)[5, 0], Eα(A)[5, 1], Eα(A)[5, 2], Eα(A)[5, 3], Eα(A)[5, 4], Eα(A)[5, 5], Eα(A)[5, 6], Eα(A)[5, 7]}, {Eα(A)[6, 0], Eα(A)[6, 1], Eα(A)[6, 2], Eα(A)[6, 3], Eα(A)[6, 4], Eα(A)[6, 5], Eα(A)[6, 6], Eα(A)[6, 7]}, {Eα(A)[7, 0], Eα(A)[7, 1], Eα(A)[7, 2], Eα(A)[7, 3], Eα(A)[7, 4], Eα(A)[7, 5], Eα(A)[7, 6], Eα(A)[7, 7]}}

Out[1]//MatrixForm=
Cot[6 H x0] 0 0 0 0 0
0 √e2 a4[H x4] Sin[6 H x0]1/6 0 0 0
0 0 √e2 a4[H x4] Sin[6 H x0]1/6 0 0
0 0 0 √e2 a4[H x4] Sin[6 H x0]1/6 0
0 0 0 0 0 1
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0

In[2]:= eηe = FullSimplify[Transpose[eα(A)] . η4488.eα(A), constraintVars];
% /. sgeα(A)
% /. sgtryeα(A)
% == gtry // FullSimplify[#, constraintVars] &

Out[2]= {{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}

Out[3]= {{Cot[6 H x0]2, 0, 0, 0, 0, 0, 0, 0}, {0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0, 0}, {0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0}, {0, 0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0, 0}, {0, 0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0}, {0, 0, 0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3}}

Out[4]= True

In[5]:= gtry
Out[5]= {{Cot[6 H x0]2, 0, 0, 0, 0, 0, 0, 0}, {0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0, 0}, {0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0, 0}, {0, 0, 0, e2 a4[H x4] Sin[6 H x0]1/3, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0, 0}, {0, 0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3, 0}, {0, 0, 0, 0, 0, 0, 0, -e-2 a4[H x4] Sin[6 H x0]1/3}}

In[6]:= (*FullSimplify[Transpose[eα(A)] . η4488.eα(A), constraintVars]-MatrixMetric44*)

```

In[•]:= **ssgm4488**

Out[•]=

$$\begin{aligned} \{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&)\} \end{aligned}$$
In[•]:= **MatrixMetric44 // MatrixForm**

Out[•]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a^4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a^4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a^4[Hx4]} \sin[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:= **$e_\alpha^{(A)}$**

Out[•]=

$$\begin{aligned} &\{E_\alpha^{(A)}[0, 0], E_\alpha^{(A)}[0, 1], E_\alpha^{(A)}[0, 2], E_\alpha^{(A)}[0, 3], E_\alpha^{(A)}[0, 4], E_\alpha^{(A)}[0, 5], E_\alpha^{(A)}[0, 6], E_\alpha^{(A)}[0, 7]\}, \\ &\{E_\alpha^{(A)}[1, 0], E_\alpha^{(A)}[1, 1], E_\alpha^{(A)}[1, 2], E_\alpha^{(A)}[1, 3], E_\alpha^{(A)}[1, 4], E_\alpha^{(A)}[1, 5], E_\alpha^{(A)}[1, 6], E_\alpha^{(A)}[1, 7]\}, \\ &\{E_\alpha^{(A)}[2, 0], E_\alpha^{(A)}[2, 1], E_\alpha^{(A)}[2, 2], E_\alpha^{(A)}[2, 3], E_\alpha^{(A)}[2, 4], E_\alpha^{(A)}[2, 5], E_\alpha^{(A)}[2, 6], E_\alpha^{(A)}[2, 7]\}, \\ &\{E_\alpha^{(A)}[3, 0], E_\alpha^{(A)}[3, 1], E_\alpha^{(A)}[3, 2], E_\alpha^{(A)}[3, 3], E_\alpha^{(A)}[3, 4], E_\alpha^{(A)}[3, 5], E_\alpha^{(A)}[3, 6], E_\alpha^{(A)}[3, 7]\}, \\ &\{E_\alpha^{(A)}[4, 0], E_\alpha^{(A)}[4, 1], E_\alpha^{(A)}[4, 2], E_\alpha^{(A)}[4, 3], E_\alpha^{(A)}[4, 4], E_\alpha^{(A)}[4, 5], E_\alpha^{(A)}[4, 6], E_\alpha^{(A)}[4, 7]\}, \\ &\{E_\alpha^{(A)}[5, 0], E_\alpha^{(A)}[5, 1], E_\alpha^{(A)}[5, 2], E_\alpha^{(A)}[5, 3], E_\alpha^{(A)}[5, 4], E_\alpha^{(A)}[5, 5], E_\alpha^{(A)}[5, 6], E_\alpha^{(A)}[5, 7]\}, \\ &\{E_\alpha^{(A)}[6, 0], E_\alpha^{(A)}[6, 1], E_\alpha^{(A)}[6, 2], E_\alpha^{(A)}[6, 3], E_\alpha^{(A)}[6, 4], E_\alpha^{(A)}[6, 5], E_\alpha^{(A)}[6, 6], E_\alpha^{(A)}[6, 7]\}, \\ &\{E_\alpha^{(A)}[7, 0], E_\alpha^{(A)}[7, 1], E_\alpha^{(A)}[7, 2], E_\alpha^{(A)}[7, 3], E_\alpha^{(A)}[7, 4], E_\alpha^{(A)}[7, 5], E_\alpha^{(A)}[7, 6], E_\alpha^{(A)}[7, 7]\} \end{aligned}$$

In[•]:=

```
Block[{s, r}, s = FullSimplify[e_\alpha^{(A)} /. sgtrye_\alpha^{(A)}, constraintVars];
r = FullSimplify[Transpose[s].η4488.s, constraintVars] - MatrixMetric44;
FullSimplify[r, constraintVars]]
```

Out[•]=

$$\begin{aligned} &\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \end{aligned}$$

```

In[1]:= (*e(A)=eAa) //MatrixForm

In[2]:= (*e(A)
Inverse[%]*)

In[3]:= (*e(A)=Inverse[e(A) /. sgeSubscriptαandLeftParenthesisARightParenthesis])//MatrixForm

In[4]:= e(A)

Out[4]= {E(A)[0, 0], E(A)[0, 1], E(A)[0, 2], E(A)[0, 3], E(A)[0, 4], E(A)[0, 5], E(A)[0, 6], E(A)[0, 7]}, {E(A)[1, 0], E(A)[1, 1], E(A)[1, 2], E(A)[1, 3], E(A)[1, 4], E(A)[1, 5], E(A)[1, 6], E(A)[1, 7]}, {E(A)[2, 0], E(A)[2, 1], E(A)[2, 2], E(A)[2, 3], E(A)[2, 4], E(A)[2, 5], E(A)[2, 6], E(A)[2, 7]}, {E(A)[3, 0], E(A)[3, 1], E(A)[3, 2], E(A)[3, 3], E(A)[3, 4], E(A)[3, 5], E(A)[3, 6], E(A)[3, 7]}, {E(A)[4, 0], E(A)[4, 1], E(A)[4, 2], E(A)[4, 3], E(A)[4, 4], E(A)[4, 5], E(A)[4, 6], E(A)[4, 7]}, {E(A)[5, 0], E(A)[5, 1], E(A)[5, 2], E(A)[5, 3], E(A)[5, 4], E(A)[5, 5], E(A)[5, 6], E(A)[5, 7]}, {E(A)[6, 0], E(A)[6, 1], E(A)[6, 2], E(A)[6, 3], E(A)[6, 4], E(A)[6, 5], E(A)[6, 6], E(A)[6, 7]}, {E(A)[7, 0], E(A)[7, 1], E(A)[7, 2], E(A)[7, 3], E(A)[7, 4], E(A)[7, 5], E(A)[7, 6], E(A)[7, 7]}

In[5]:= (sge(A) = Thread[Flatten[e(A)] → Flatten[Inverse[e(A) /. sge(A)]]]) (*//Column*)

Out[5]= {E(A)[0, 0] → 1/((g[0][0][x0, x4])), E(A)[0, 1] → 0, E(A)[0, 2] → 0, E(A)[0, 3] → 0, E(A)[0, 4] → 0, E(A)[0, 5] → 0, E(A)[0, 6] → 0, E(A)[0, 7] → 0, E(A)[1, 0] → 0, E(A)[1, 1] → 1/((g[1][1][x0, x4])), E(A)[1, 2] → 0, E(A)[1, 3] → 0, E(A)[1, 4] → 0, E(A)[1, 5] → 0, E(A)[1, 6] → 0, E(A)[1, 7] → 0, E(A)[2, 0] → 0, E(A)[2, 1] → 0, E(A)[2, 2] → 1/((g[2][2][x0, x4])), E(A)[2, 3] → 0, E(A)[2, 4] → 0, E(A)[2, 5] → 0, E(A)[2, 6] → 0, E(A)[2, 7] → 0, E(A)[3, 0] → 0, E(A)[3, 1] → 0, E(A)[3, 2] → 0, E(A)[3, 3] → 1/((g[3][3][x0, x4])), E(A)[3, 4] → 0, E(A)[3, 5] → 0, E(A)[3, 6] → 0, E(A)[3, 7] → 0, E(A)[4, 0] → 0, E(A)[4, 1] → 0, E(A)[4, 2] → 0, E(A)[4, 3] → 0, E(A)[4, 4] → 1/((-g[4][4][x0, x4])), E(A)[4, 5] → 0, E(A)[4, 6] → 0, E(A)[4, 7] → 0, E(A)[5, 0] → 0, E(A)[5, 1] → 0, E(A)[5, 2] → 0, E(A)[5, 3] → 0, E(A)[5, 4] → 0, E(A)[5, 5] → 1/((-g[5][5][x0, x4])), E(A)[5, 6] → 0, E(A)[5, 7] → 0, E(A)[6, 0] → 0, E(A)[6, 1] → 0, E(A)[6, 2] → 0, E(A)[6, 3] → 0, E(A)[6, 4] → 0, E(A)[6, 5] → 0, E(A)[6, 6] → 1/((-g[6][6][x0, x4])), E(A)[6, 7] → 0, E(A)[7, 0] → 0, E(A)[7, 1] → 0, E(A)[7, 2] → 0, E(A)[7, 3] → 0, E(A)[7, 4] → 0, E(A)[7, 5] → 0, E(A)[7, 6] → 0, E(A)[7, 7] → 1/((-g[7][7][x0, x4])}

In[6]:= (*Symbolize[sge(A)]*)

```

```
In[=]:= (*Symbolize[sgtryeα(A)]*)
In[=]:= (*%/.sge(A)α
%/.sgtrye(A)α*)
In[=]:= (*FullSimplify[Transpose[e(A)α].η4488.e(A)α,constraintVars]-MatrixMetric44*)
```

For Spin (4, 4); τ tau; T16; OCTAD : Nash; Introduce the wave function, Ψ16, for this Universe::

O(4,4); evaluates, evecs of σ

```
In[=]:= {evals, evecs} = Eigensystem[σ]
Out[=]= {{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0}, {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1}, {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}

In[=]:= 
$$\mathbf{u} = \text{ExpandAll}\left[\frac{1}{\sqrt{2}} \text{evecs}\right] // \text{MatrixForm}$$

Out[=]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$


In[=]:= Table[u[[h]].σ.u[[h]], {h, 1, Length[u]}]
Out[=]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[=]:= Table[(Transpose[u][[h]]).σ.u[[h]], {h, 1, Length[u]}]
Out[=]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[=]:= Table[(Transpose[u][[A]]) == -σ.u[[A]], {A, 1, 4}]
Table[(Transpose[u][[A]]) == σ.u[[A]], {A, 5, Length[u]}]

Out[=]= {True, True, True, True}

Out[=]= {True, True, True, True}
```

$$\mathbf{Ax} = \left(\frac{1}{2} (\text{KroneckerProduct}[dX, X] - \right.$$

```

KroneckerProduct[X, dX])2.σ;
Au = 
$$\left( \frac{1}{2} (\text{KroneckerProduct}[dU, U] - \text{KroneckerProduct}[U, dU]) \right) . \sigma;$$

Lsquared = 
$$\dot{x}^A \dot{x}_A = dX.\sigma . dX + x.\sigma . x \left( \frac{dU.\sigma . dU}{U.\sigma . U} - \left( \frac{U.\sigma . dU}{U.\sigma . U} \right)^2 \right) + \frac{2}{U.\sigma . U} \text{Tr} [\tau^A . Ax . \tau_A . Au]$$

Timing[
Simplify[ExpandAll[dY.η8.dY - Lsquared]]]
{10.733, 0}

```

```

In[]:= Do[
  t4by4[h] = Table[Table[AntiSelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}],
  {h, 1, 3}]

In[]:= (*Protect[{s4by4[1],s4by4[2],s4by4[3],t4by4[1],t4by4[2],t4by4[3]}]*)

In[]:= Protect[s4by4, t4by4]

Out[]= {s4by4, t4by4}

In[]:= {s4by4[#] // MatrixForm, MatrixForm[t4by4[#]]} & /@ Range[3]

Out[=]

$$\left\{ \left\{ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right\},$$


$$\left\{ \left\{ \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\}, \left\{ \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right\} \right\}$$


In[]:= Table[{{J, K}, MatrixForm[s4by4[J].t4by4[K]]}, {J, 1, 3}, {K, 1, 3}]

Out[=]

$$\left\{ \left\{ \{1, 1\}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}, \{1, 2\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \{1, 3\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \right\},$$


$$\left\{ \left\{ \{2, 1\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \right\}, \{2, 2\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right\},$$


$$\left\{ \left\{ \{3, 1\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\}, \{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \{3, 3\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\} \right\}$$


In[]:= Do[st[J, K] = s4by4[J].t4by4[K], {J, 1, 3}, {K, 1, 3}]

In[]:= Table[{{J, K}, MatrixForm[st[J, K]]}, {J, 1, 3}, {K, 1, 3}]

Out[=]

$$\left\{ \left\{ \{1, 1\}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\}, \{1, 2\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \{1, 3\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \right\},$$


$$\left\{ \left\{ \{2, 1\}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \right\}, \{2, 2\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \{2, 3\}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right\},$$


$$\left\{ \left\{ \{3, 1\}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\}, \{3, 2\}, \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \{3, 3\}, \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\} \right\}$$


```

```

In[]:= Protect[st]
Out[]= {st}

In[]:= Table[{{J, K}, MatrixForm[st[J, K] - Transpose[st[J, K]]]}, {J, 1, 3}, {K, 1, 3}]
Out[=]

{{{{1, 1}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{1, 2}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{1, 3}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}, {{{2, 1}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{2, 2}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{2, 3}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}, {{{3, 1}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{3, 2}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}, {{3, 3}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}}

In[]:= Permutations[Range[3], 2]
Out[=] {{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}

In[]:= KroneckerProduct[Range[3], Range[3]]
Out[=] {{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}

In[]:= Table[{{J, K}, {J, 1, 3}, {K, 1, 3}]
st[##]& /@ %
st[##]& @@ %

Out[=] {{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}}

Out[=] {st[{{1, 1}, {1, 2}, {1, 3}}], st[{{2, 1}, {2, 2}, {2, 3}}], st[{{3, 1}, {3, 2}, {3, 3}}]}

Out[=] st[{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}]

```

```

In[]:= Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
Length[%]

Out[]=
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

Out[=]
16

In[]:= Unprotect[y];
y = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]

Out[=]
{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
 {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
 {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
 {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
 {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
 {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
 {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
 {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
 {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

In[]:= Length[y]

Out[=]
16

```

```
In[]:= Protect[\gamma]
Out[]= {\gamma}

In[]:= \gamma[[1]]
Out[= {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}]

In[]:= \gamma[[-1]]
Out[= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}]
```

Self Dual Anti-Symmetric

```
In[]:= Table[(1/2) Sum[Sum[\epsilon4[p, q, j1, j2] \times s4by4[h][j1, j2]], {j1, 1, 4}], {j2, 1, 4}] - s4by4[h][p, q],
{h, 1, 3}, {q, 4}, {p, 4}]

Out[= {{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

Anti SelfDual Anti-Symmetric

```
In[]:= Table[(1/2) Sum[Sum[\epsilon4[p, q, j1, j2] \times t4by4[h][j1, j2]], {j1, 1, 4}], {j2, 1, 4}] + t4by4[h][p, q],
{h, 1, 3}, {q, 4}, {p, 4}]

Out[= {{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}}
```

SO(4,4), Spin(4,4) = $\overline{\text{SO}(4, 4)}$, τ ; if using xact: M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+, +, +, -, -, -}, {x0, x1, x2, x3, x4, x5, x6, x7}

```
In[]:= \tau === \bar{\tau}
Out[= False

In[]:= \tau
Head[%]

Out[= \bar{\tau}

Out[= Symbol
```

```

In[1]:=  $\overline{\tau}$ 
Head[%]
Out[1]=  $\overline{\tau}$ 
Out[1]= ParsedBoxWrapper
In[2]:=  $\overline{\tau}$ 
Head[%]
Out[2]=
In[3]:=  $\overline{\tau}$ 
Head[%]
Out[3]= Symbol
In[4]:= Symbol
Out[4]= Symbol
In[5]:=  $\overline{\tau} === \overline{\tau}$ 
Out[5]= True
In[6]:=  $\overline{\tau} === \overline{\tau}$ 
Out[6]= False
In[7]:= η4488 // MatrixForm
Out[7]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[8]:= η4488 = DiagonalMatrix[RotateRight[evalues, 4]]
Out[8]=
In[9]:= DiagonalMatrix[RotateRight[evalues, 3]] // MatrixForm
Out[9]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


```

these also yield correct reps:

```

 $\zeta[0] = \text{ID8};$ 
Table[ $\zeta[7 - h] = \text{ArrayFlatten}[\{\{0, -t4by4[h]\}, \{t4by4[h], 0\}\}], \{h, 1, 3\}\];$ 
Table[ $\zeta[h] = \text{ArrayFlatten}[\{\{0, s4by4[h]\}, \{s4by4[h], 0\}\}], \{h, 1, 3\}\];$ 
 $(\zeta[7] = \zeta[1].\zeta[2].\zeta[3].\zeta[4].\zeta[5].\zeta[6]) // \text{MatrixForm}$ 

```

```

 $\zeta[0] = \text{ID8};$ 
Table[ $\zeta[h] = \text{ArrayFlatten}[\{\{0, t4by4[h]\}, \{t4by4[h], 0\}\}], \{h, 1, 3\}\];$ 
Table[ $\zeta[7 - h] = \text{ArrayFlatten}[\{\{0, -s4by4[h]\}, \{s4by4[h], 0\}\}], \{h, 1, 3\}\];$ 
 $(\zeta[7] = \zeta[1].\zeta[2].\zeta[3].\zeta[4].\zeta[5].\zeta[6]) // \text{MatrixForm}$ 

```

```

 $\zeta[0] = \text{ID8};$ 
Table[ $\zeta[h] = \text{ArrayFlatten}[\{\{0, t4by4[h]\}, \{t4by4[h], 0\}\}], \{h, 1, 3\}\];$ 
Table[ $\zeta[7 - h] = \text{ArrayFlatten}[\{\{0, s4by4[h]\}, \{-s4by4[h], 0\}\}], \{h, 1, 3\}\];$ 
 $(\zeta[7] = \zeta[1].\zeta[2].\zeta[3].\zeta[4].\zeta[5].\zeta[6]) // \text{MatrixForm}$ 

```

```

In[=]:=  $\zeta[0] = \text{ID8};$ 
Table[
   $\zeta[7 - h] = \text{ArrayFlatten}[\{\{0, t4by4[h]\}, \{-t4by4[h], 0\}\}], \{h, 1, 3\}\];$ 
Table[ $\zeta[h] = \text{ArrayFlatten}[\{\{0, s4by4[h]\}, \{s4by4[h], 0\}\}], \{h, 1, 3\}\];$ 
 $(\zeta[7] = \zeta[1].\zeta[2].\zeta[3].\zeta[4].\zeta[5].\zeta[6]) // \text{MatrixForm}$ 

```

```

Out[=]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


```

```
In[=]:=  $\sigma = \zeta[1].\zeta[2].\zeta[3]$ 
```

```
Out[=]=
True
```

```
In[1]:= Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\xi[A].\xi[B] + \xi[B].\xi[A]) == -\eta 4488 [A+1, B+1] * ID8]$ ]}],
  {A, 1, 7}, {B, 1, 7}]

Out[1]= {{1, 1, True}, {1, 2, True}, {1, 3, True}, {1, 4, True},
  {1, 5, True}, {1, 6, True}, {1, 7, True}}, {{2, 1, True}, {2, 2, True},
  {2, 3, True}, {2, 4, True}, {2, 5, True}, {2, 6, True}, {2, 7, True}},
  {{3, 1, True}, {3, 2, True}, {3, 3, True}, {3, 4, True}, {3, 5, True}, {3, 6, True},
  {3, 7, True}}, {{4, 1, True}, {4, 2, True}, {4, 3, True}, {4, 4, True},
  {4, 5, True}, {4, 6, True}, {4, 7, True}}, {{5, 1, True}, {5, 2, True},
  {5, 3, True}, {5, 4, True}, {5, 5, True}, {5, 6, True}, {5, 7, True}},
  {{6, 1, True}, {6, 2, True}, {6, 3, True}, {6, 4, True}, {6, 5, True},
  {6, 6, True}, {6, 7, True}}, {{7, 1, True}, {7, 2, True},
  {7, 3, True}, {7, 4, True}, {7, 5, True}, {7, 6, True}, {7, 7, True}}}

In[2]:=  $\eta 4488 // \text{MatrixForm}$ 

Out[2]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


In[3]:= Table[{A, - $\eta 4488 [A+1, A+1] * \xi[A] == \text{Transpose}[\xi[A]]$ }, {A, 1, 7}]

Out[3]= {{1, True}, {2, True}, {3, True}, {4, True}, {5, True}, {6, True}, {7, True}}
```

In[4]:= $(\sigma = \xi[1].\xi[2].\xi[3])$

```
Out[4]= True

In[5]:=  $\sigma.\xi[1].\xi[2].\xi[3] // \text{MatrixForm}$ 

Out[5]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```

```
In[8]:= τ[θ] = ID8;
Do[
 {τ[A] = FullSimplify[ExpandAll[(ξ[A])]]} ,
 {A, 1, 7}]
(*Do[
 {τ[A] = FullSimplify[ExpandAll[-(σ.ξ[1].
 ξ[2].ξ[3].ξ[A])]}} , {A, 1, 7}]*)
```

```
In[9]:= (σ == τ[1].τ[2].τ[3])
```

```
Out[9]= True
```

```
In[10]:= (*Symbolize[τ]*)
```

```
In[11]:= τ̄[θ] = ID8;
Do[{τ̄[A] = FullSimplify[ExpandAll[
 (σ.Transpose[τ[A]].σ)]]} , {A, 1, 7}]
```

```
In[12]:= σ.τ̄[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
```

```
Out[12]= {True, True, True, True, True, True, True}
```

```
In[8]:= Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\tau[A] \cdot \bar{\tau}[B] + \tau[B] \cdot \bar{\tau}[A]) = \eta 4488 [A+1, B+1] * ID8]$ ]}],
 {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \end{pmatrix} \begin{pmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \end{pmatrix} \begin{pmatrix} 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \end{pmatrix} \begin{pmatrix} 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \end{pmatrix} \begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \end{pmatrix} \begin{pmatrix} 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \end{pmatrix} \begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{True} & \text{True} \end{pmatrix}$$

In[9]:= $\tau[7] // \text{MatrixForm}$

Out[9]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[10]:= $(\Omega = \sigma \cdot \tau[7]) // \text{MatrixForm}$

Out[10]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

In[=]:= Ω == τ[4].τ[5].τ[6]
Out[=]= True

In[=]:= τ[5].τ[6].τ[7] // MatrixForm
Out[=]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$


In[=]:= (σΩ = σ.Ω) // MatrixForm
Out[=]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


In[=]:= Inverse[σΩ] // MatrixForm
Out[=]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


In[=]:= τ[1].τ[2].τ[3].τ[4].τ[5].τ[6].τ[7] == τ[0] == ID8
Out[=]= True

In[=]:= (σ == τ[1].τ[2].τ[3])
Out[=]= True

In[=]:= (σ == τ[4].τ[5].τ[6].τ[7])
Out[=]= True

In[=]:= τ[1].τ[2].τ[3].τ[#] == -Transpose[τ[1].τ[2].τ[3].τ[#]] & /@ Range[0, 7]
Out[=]= {False, True, True, True, True, True, True}

```

```
In[]:= σ.τ[#] == Transpose[σ.τ[#]] & /@ Range[0, 7]
Out[]= {True, True, True, True, True, True, True, True}

In[]:= τ[5].τ[6].τ[7].τ[#] == Transpose[τ[5].τ[6].τ[7].τ[#]] & /@
Range[0, 7]
Out[]= {False, True, True, True, False, True, True, True}

In[]:= τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
Out[= {True, True, True, True, True, False, False, False}

In[]:= τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
Out[= {True, False, False, False, False, True, True, True}

In[]:= τ[4].τ[#] == Transpose[τ[4].τ[#]] & /@ Range[0, 7]
Out[= {True, True, True, True, True, False, False, False}
```

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}. \quad ; \quad \mathbf{0} (4, 4) : \mathbf{SAB} ;$$

covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]

$$\mathbf{S} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix} ;$$

$$\begin{aligned} \left(\begin{matrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{matrix} \right) \cdot \left(\begin{matrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{matrix} \right) \cdot \left(\begin{matrix} \tilde{\mathbf{a}} & \tilde{\mathbf{c}} \\ \tilde{\mathbf{b}} & \tilde{\mathbf{d}} \end{matrix} \right) &= \left(\begin{matrix} \mathbf{b} \cdot \tau^A & \mathbf{a} \cdot \bar{\tau}^A \\ \mathbf{d} \cdot \tau^A & \mathbf{c} \cdot \bar{\tau}^A \end{matrix} \right) \cdot \left(\begin{matrix} \tilde{\mathbf{a}} & \tilde{\mathbf{c}} \\ \tilde{\mathbf{b}} & \tilde{\mathbf{d}} \end{matrix} \right) \\ &= \left(\begin{matrix} \mathbf{b} \cdot \tau^A \cdot \tilde{\mathbf{a}} + \mathbf{a} \cdot \bar{\tau}^A \cdot \tilde{\mathbf{b}} & \mathbf{b} \cdot \tau^A \cdot \tilde{\mathbf{c}} + \mathbf{a} \cdot \bar{\tau}^A \cdot \tilde{\mathbf{d}} \\ \mathbf{d} \cdot \tau^A \cdot \tilde{\mathbf{a}} + \mathbf{c} \cdot \bar{\tau}^A \cdot \tilde{\mathbf{b}} & \mathbf{d} \cdot \tau^A \cdot \tilde{\mathbf{c}} + \mathbf{c} \cdot \bar{\tau}^A \cdot \tilde{\mathbf{d}} \end{matrix} \right) \end{aligned}$$

In[8]:= 08similarityTransformation // MatrixForm

Out[8]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

In[9]:= blocks = Partition[08similarityTransformation, {8, 8}];

```

In[]:= (*To access the individual blocks*)
block11 = blocks[[1, 1]] (*Top-left block*)
block12 = blocks[[1, 2]] (*Top-right block*)
block21 = blocks[[2, 1]] (*Bottom-left block*)
block22 = blocks[[2, 2]] (*Bottom-right block*)

Out[]=
{{1, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}

Out[]=
{{0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 1, 0, 0}

Out[=]
{{0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}

Out[=]
{{0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 1}

In[]:= ArrayFlatten[{{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}] ===
08similarityTransformation

Out[=
True

```

$$\begin{pmatrix} \text{blocks}[1, 1] & \text{blocks}[1, 2] \\ \text{blocks}[2, 1] & \text{blocks}[2, 2] \end{pmatrix}.$$

$$\begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix} \cdot \begin{pmatrix} \tilde{\text{blocks}}[1, 1] & \tilde{\text{blocks}}[2, 1] \\ \tilde{\text{blocks}}[1, 2] & \tilde{\text{blocks}}[2, 2] \end{pmatrix} =$$

$$\begin{pmatrix} \text{blocks}[1, 2] \cdot \tau^A & \text{blocks}[1, 1] \cdot \bar{\tau}^A \\ \text{blocks}[2, 2] \cdot \tau^A & \text{blocks}[2, 1] \cdot \bar{\tau}^A \end{pmatrix}.$$

$$= \begin{pmatrix} \tilde{\text{blocks}}[1, 1] & \tilde{\text{blocks}}[2, 1] \\ \tilde{\text{blocks}}[1, 2] & \tilde{\text{blocks}}[2, 2] \end{pmatrix}$$

```
In[6]:= Table[
  newT16^A1 === ArrayFlatten[{{blocks[[1, 2]].τ[A1].Transpose[blocks[[1, 1]]] + blocks[[1, 1]].τ[A1].Transpose[blocks[[1, 2]]], blocks[[1, 2]].τ[A1].Transpose[blocks[[2, 1]]] + blocks[[1, 1]].τ[A1].Transpose[blocks[[2, 2]]]}, {blocks[[2, 2]].τ[A1].Transpose[blocks[[1, 1]]] + blocks[[2, 1]].τ[A1].Transpose[blocks[[1, 2]]], blocks[[2, 2]].τ[A1].Transpose[blocks[[2, 1]]] + blocks[[2, 1]].τ[A1].Transpose[blocks[[2, 2]]]}}], {A1, 0, 7}]
```

```
Out[=] = {False, False, False, False, False, False, False, False, False}
```

```
In[•]:= MatrixForm[newT16A[#]] & /@ Range[0, 8]
Out[•]= {newT16A[0], newT16A[1], newT16A[2], newT16A[3],
          newT16A[4], newT16A[5], newT16A[6], newT16A[7], newT16A[8]}
```

```
In[6]:= (*Example 16x16 matrix*) (*matrix=Table[RandomInteger[{1,100}],{16},{16}];*)
```

```
(*Partition the matrix into 8x8 blocks*)
(*blocks=Partition[matrix,{8,8}];*)
```

(*The result is a 2x2 list of 8x8 matrices*)
(*blocks*)

```
In[8]:= (*TakeList[08similarityTransformation,{8,8}]*)
```

```
blocks = TakeList[O8similarityTransformation, {8, 8}, {8, 8}]  
ArrayFlatten[%] === O8similarityTransformation
```

Out[•] =

```

0,0}, {0,0,0,1,0,0,0,0}}}, {{0,0,0,0,1,0,0,0}, {0,0,0,0,0,
0,1,0,0}, {0,0,0,0,0,0,0,0}, {0,0,0,0,0,0,0,0}, {0,0,0,0,0,
0,0,1,0}, {0,0,0,0,0,0,0,1}, {0,0,0,0,0,0,0,0}, {0,0,0,0,0,
0,0,0,0}}, {{0,0,0,0,0,0,0,0}, {0,0,0,0,0,0,0,0}, {0,0,0,0,0,
0,1,0,0}, {0,0,0,0,0,1,0,0}, {0,0,0,0,0,0,0,0}, {0,0,0,0,0,
0,0,0,0}, {0,0,0,0,0,0,0,1}, {0,0,0,0,0,0,0,1}}}}}

```

Out[•] =

True

In[•]:= Length [blocks]

Out[•]=

2

```
In[+]:= (*08similarityTransformation==ArrayFlatten[{{Take[],Take[]},{Take[],Take[]}}]*)
```

Take [*list*, *seq*₁, *seq*₂]

gives a nested list in which elements

specified by seq_i are taken at level i in $list$.

effectively extracts a submatrix from *list*.

In[•]:= **Take**[]

 **Take:** Take called with 0 arguments; 1 or more arguments are expected. [i](#)

Out[•]=

Take []

Need these in order to define the Lagrangian for the "universe," later :

$\ln[...]\equiv$

```
(σ16 = T16A[0].T16A[1].T16A[2].T16A[3]) // MatrixForm
```

Out[1]//MatrixForm=

$T16^A[0], T16^A[1], T16^A[2], T16^A[3]$

```
Table[T16A[A1] = ArrayFlatten[{{{\theta, \bar{\tau}[A1]}, {\tau[A1], 0}}}], {A1, 0, 7}];
```

```
In[1]:= 
$$\left( T16^A[8] = T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7] \right) // \text{MatrixForm}$$

```

Out[•]//MatrixForm=

```
In[•]:= Table[newT16A[A1] = 08similarityTransformation. T16A[A1].  
Transpose[08similarityTransformation], {A1, 0, 8} ];
```

```
In[8]:= Transpose[\!`16]. σ16 ===  
Transpose[Transpose[08similarityTransformation].08similarityTransformation.\!`16]. σ16 ===  
Transpose[08similarityTransformation.\!`16].08similarityTransformation. σ16
```

Out[•]=

True

```
In[1]:= Transpose[\u03a916]. \u03c316 . Transpose[08similarityTransformation] ===  
Transpose[Transpose[08similarityTransformation].08similarityTransformation.\u03a916]. \u03c316.  
Transpose[08similarityTransformation] === Transpose[08similarityTransformation.\u03a916].  
08similarityTransformation. \u03c316.Transpose[08similarityTransformation]
```

Out[•]=

True

```
In[=]:= (newσ16 = 08similarityTransformation. σ16.Transpose[08similarityTransformation]) // MatrixForm
```

Out[=]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[=]:= σ16 // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[=]:= T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#] ==
          -Transpose[T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#]] & /@ Range[0, 8]
```

Out[=]=

{True, True, True, True, True, True, True, False}

```
In[=]:= T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#] ==
          Transpose[T16^A[4].T16^A[5].T16^A[6].T16^A[7].T16^A[#]] & /@ Range[0, 8]
```

Out[=]=

{True, True, True, True, True, True, True, True}

```
In[=]:= T16^A[8] == σ16.T16^A[4].T16^A[5].T16^A[6].T16^A[7]
```

Out[=]=

True

```

In[=]:= T16^A[#=Transpose[T16^A[#]] & /@ Range[0, 8]

Out[=]= {True, True, True, True, False, False, False, False, True}

In[=]:= T16^A[#= -Transpose[T16^A[#]] & /@ Range[0, 8]

Out[=]= {False, False, False, False, True, True, True, True, False}

In[=]:= σ16.T16^A[#= -Transpose[σ16.T16^A[#]] & /@ Range[0, 7]

Out[=]= {True, True, True, True, True, True, True, True}

In[=]:= (covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]) // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


In[=]:= σ16.covariantDiffMatrix == Transpose[σ16.covariantDiffMatrix]

Out[=]= True

In[=]:= ID16 // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$


```

```
In[8]:=  $(\sigma16(*=T16^A[0].T16^A[1].T16^A[2].T16^A[3]*)) // MatrixForm$ 
```

```
Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

```
In[9]:=  $\sigma16 = \text{ArrayFlatten}[\{\{-\sigma, 0\}, \{0, \sigma\}\}]$ 
```

```
Out[9]=
True
```

later, after defining $e_{(A)}^\alpha$:

```
In[10]:= (*Table[T16^\alpha[\alpha1-1]=
Sum[(e_{(A)}^\alpha[[\alpha1,A1]] ) T16^A[A1-1],{A1,1,8}],{\alpha1,1,8}];*)
```

```
In[11]:= (*(T16^\alpha[8]=T16^\alpha[0].T16^\alpha[1].T16^\alpha[2].T16^\alpha[3].T16^\alpha[4].T16^\alpha[5].T16^\alpha[6].T16^\alpha[7])//MatrixForm*)
```

```
In[12]:= (*eLpairs*)
In[13]:= (*Ω16a=Array[0&,{16,16}];
{1+#[[1]],1+#[[2]]}&/@eLpairs
(Ω16a[[#[[1]],#[[2]]]]=1)&/@%*)
In[14]:= (*Ω16a//MatrixForm*)
In[15]:= (*(Ω16=σ16.(Ω16a+Transpose[Ω16a]))//MatrixForm*)
```


In[8]:= $\sigma16.\Omega16 // \text{MatrixForm}$

```
Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

In[9]:= $\sigma16.T16^A[\#] == -\text{Transpose}[\sigma16.T16^A[\#]] \& /@ \text{Range}[0, 7]$

Out[9]= {True, True, True, True, True, True, True}

In[10]:= $\sigma16.T16^A[\#] == -\text{Transpose}[\sigma16.T16^A[\#]] \& /@ \text{Range}[0, 7]$

Out[10]= {True, True, True, True, True, True, True}

In[11]:= $\sigma16.T16^A[\#] == -\text{Transpose}[\sigma16.T16^A[\#]] \& /@ \text{Range}[0, 7]$

Out[11]= {True, True, True, True, True, True, True}

In[12]:= $\sigma16.newT16^A[\#] == -\text{Transpose}[\sigma16.newT16^A[\#]] \& /@ \text{Range}[0, 7]$

Out[12]= {False, False, False, True, False, True, False}

In[13]:= $\text{new}\sigma16.newT16^A[\#] == -\text{Transpose}[\text{new}\sigma16.newT16^A[\#]] \& /@ \text{Range}[0, 7]$

Out[13]= {True, True, True, True, True, True, True}

```
In[8]:= Table[{ {A1, B1}, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\text{T16}^A[\text{A1}] \cdot \text{T16}^A[\text{B1}] + \text{T16}^A[\text{B1}] \cdot \text{T16}^A[\text{A1}]) -$ 
 $\eta 4488 [[\text{A1} + 1, \text{B1} + 1]] * \text{ID16}] ] }, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm$ 
```

```
Out[•]//MatrixForm=
( { { 0, 0} } { { 0, 1} } { { 0, 2} } { { 0, 3} } { { 0, 4} } { { 0, 5} } { { 0, 6} } { { 0, 7} } )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 1, 0} ) ( { 1, 1} ) ( { 1, 2} ) ( { 1, 3} ) ( { 1, 4} ) ( { 1, 5} ) ( { 1, 6} ) ( { 1, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 2, 0} ) ( { 2, 1} ) ( { 2, 2} ) ( { 2, 3} ) ( { 2, 4} ) ( { 2, 5} ) ( { 2, 6} ) ( { 2, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 3, 0} ) ( { 3, 1} ) ( { 3, 2} ) ( { 3, 3} ) ( { 3, 4} ) ( { 3, 5} ) ( { 3, 6} ) ( { 3, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 4, 0} ) ( { 4, 1} ) ( { 4, 2} ) ( { 4, 3} ) ( { 4, 4} ) ( { 4, 5} ) ( { 4, 6} ) ( { 4, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 5, 0} ) ( { 5, 1} ) ( { 5, 2} ) ( { 5, 3} ) ( { 5, 4} ) ( { 5, 5} ) ( { 5, 6} ) ( { 5, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 6, 0} ) ( { 6, 1} ) ( { 6, 2} ) ( { 6, 3} ) ( { 6, 4} ) ( { 6, 5} ) ( { 6, 6} ) ( { 6, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 7, 0} ) ( { 7, 1} ) ( { 7, 2} ) ( { 7, 3} ) ( { 7, 4} ) ( { 7, 5} ) ( { 7, 6} ) ( { 7, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
```

```
In[8]:= Table[{ {A1, B1}, FullSimplify[ExpandAll[
  1/2 (newT16^A[A1].newT16^A[B1] + newT16^A[B1].newT16^A[A1]) ==
  η4488[[A1 + 1, B1 + 1]] * ID16 ] ] }, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

```
Out[•]//MatrixForm=
( ( {0, 0} ) ( {0, 1} ) ( {0, 2} ) ( {0, 3} ) ( {0, 4} ) ( {0, 5} ) ( {0, 6} ) ( {0, 7} ) )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {1, 0} ) ( {1, 1} ) ( {1, 2} ) ( {1, 3} ) ( {1, 4} ) ( {1, 5} ) ( {1, 6} ) ( {1, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {2, 0} ) ( {2, 1} ) ( {2, 2} ) ( {2, 3} ) ( {2, 4} ) ( {2, 5} ) ( {2, 6} ) ( {2, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {3, 0} ) ( {3, 1} ) ( {3, 2} ) ( {3, 3} ) ( {3, 4} ) ( {3, 5} ) ( {3, 6} ) ( {3, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {4, 0} ) ( {4, 1} ) ( {4, 2} ) ( {4, 3} ) ( {4, 4} ) ( {4, 5} ) ( {4, 6} ) ( {4, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {5, 0} ) ( {5, 1} ) ( {5, 2} ) ( {5, 3} ) ( {5, 4} ) ( {5, 5} ) ( {5, 6} ) ( {5, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {6, 0} ) ( {6, 1} ) ( {6, 2} ) ( {6, 3} ) ( {6, 4} ) ( {6, 5} ) ( {6, 6} ) ( {6, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {7, 0} ) ( {7, 1} ) ( {7, 2} ) ( {7, 3} ) ( {7, 4} ) ( {7, 5} ) ( {7, 6} ) ( {7, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
```

```
In[]:= Ψ16upper = Take[Ψ16, 8]
Ψ16lower = Take[Ψ16, -8]
Ψ16 == Flatten[{Ψ16upper, Ψ16lower}]

Out[]= {f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4]}

Out[]= {f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}

Out[=]
True
```

$$t^A = \begin{pmatrix} 0 & \vec{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
Table[T16^A[A1] = ArrayFlatten[
  {{0, τ[A1]}, {τ[A1], 0}}], {A1, 0, 7}]

σ16 = ArrayFlatten[{{{-σ, 0}, {0, σ}}}]
( 0   τ[A1] ) ( Ψ16upper )
( τ[A1]   0   ) ( Ψ16lower ) =
( τ[A1].Ψ16lower )
( τ[A1].Ψ16upper )

σ16.( 0   τ[A1] ) ( Ψ16upper )
( τ[A1]   0   ) ( Ψ16lower ) =
( -σ τ[A1].Ψ16lower )
( σ τ[A1].Ψ16upper )

Ψ16.σ16.( 0   τ[A1] ).Ψ16 =
( -Ψ16upper.σ τ[A1].Ψ16lower )
( Ψ16lower.σ τ[A1].Ψ16upper )
```

```
In[]:= (*σ16.T16^A[#]&/@Range[0,7]*)
```

```

In[]:= (*
T16^A[8]=FullSimplify[
T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]])//MatrixForm*)

In[]:= η4488

Out[]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}

In[]:= η99 = DiagonalMatrix[{1, 1, 1, 1, -1, -1, -1, -1}]

Out[]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}]

In[]:= (*Table[{A1,B1,
FullSimplify[ExpandAll[1/2 ( T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1])= η99[[A1,B1]*ID16]]]}, {A1,0, 8},{B1,0, 8}]//MatrixForm*)

In[]:= (*Table[1/4 (T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]), {A1, 8}, {B1, 8}]*)

In[]:= 
$$P_L = \frac{1}{2} (ID16 - T16^A[8])$$
 // MatrixForm

Out[]= MatrixForm[
{{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}]

```

```
In[8]:= 
$$\mathbf{P}_R = \frac{1}{2} (\mathbf{ID16} + \mathbf{T16}^A[8]) // \text{MatrixForm}$$

```

```
In[•]:= {ID16 == PL + PR, PL.PL == PL, PL.PR == PR.PL == ZERO16}
```

Out[•] =

{True, True, True}

SAB = Table

$$\frac{1}{4} \left(T16^A[A1] \cdot T16^A[B1] - T16^A[B1] \cdot T16^A[A1] \right), \{ A1, 0, 7 \}, \{ B1, 0, 7 \} \right];$$

```
In[•]:= SAB = Table[ $\frac{1}{4} (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1])$ , {A1, 0, 7}, {B1, 0, 7}];
```

```
In[6]:= ParallelTable[FullSimplify[\sigma16.SAB[[A1, B1]] == -Transpose[\sigma16.SAB[[A1, B1]]]], {A1, 1, 8}, {B1, 1, 8}] // Flatten // Union
```

Out[•]=

{ True }

```
In[6]:= ParallelTable[{{A1, B1}, FullSimplify[SAB[A1, B1] === -Transpose[SAB[A1, B1]]]}, {A1, 1, 8}, {B1, 1, 8}]
```

```
Out[6]= {{{{1, 1}, True}, {{1, 2}, True}, {{1, 3}, True}, {{1, 4}, True}, {{1, 5}, False}, {{1, 6}, False}, {{1, 7}, False}, {{1, 8}, False}}, {{{2, 1}, True}, {{2, 2}, True}, {{2, 3}, True}, {{2, 4}, True}, {{2, 5}, False}, {{2, 6}, False}, {{2, 7}, False}, {{2, 8}, False}}, {{{3, 1}, True}, {{3, 2}, True}, {{3, 3}, True}, {{3, 4}, True}, {{3, 5}, False}, {{3, 6}, False}, {{3, 7}, False}, {{3, 8}, False}}, {{{4, 1}, True}, {{4, 2}, True}, {{4, 3}, True}, {{4, 4}, True}, {{4, 5}, False}, {{4, 6}, False}, {{4, 7}, False}, {{4, 8}, False}}, {{{5, 1}, False}, {{5, 2}, False}, {{5, 3}, False}, {{5, 4}, False}, {{5, 5}, True}, {{5, 6}, True}, {{5, 7}, True}, {{5, 8}, True}}, {{{6, 1}, False}, {{6, 2}, False}, {{6, 3}, False}, {{6, 4}, False}, {{6, 5}, True}, {{6, 6}, True}, {{6, 7}, True}, {{6, 8}, True}}, {{{7, 1}, False}, {{7, 2}, False}, {{7, 3}, False}, {{7, 4}, False}, {{7, 5}, True}, {{7, 6}, True}, {{7, 7}, True}, {{7, 8}, True}}, {{{8, 1}, False}, {{8, 2}, False}, {{8, 3}, False}, {{8, 4}, False}, {{8, 5}, True}, {{8, 6}, True}, {{8, 7}, True}, {{8, 8}, True}}}}
```

```
In[7]:= for044Similarity =
  ParallelTable[If[FullSimplify[SAB[A1, B1] === -Transpose[SAB[A1, B1]]],
    {A1, B1}, ## &[]], {A1, 1, 7}, {B1, A1 + 1, 8}] // Flatten[#, 1] &
Length[%]
```

```
Out[7]= {{1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {5, 6}, {5, 7}, {5, 8}, {6, 7}, {6, 8}, {7, 8}}
```

```
Out[8]= 12
```

```
In[9]:= SAB[##] &@@ for044Similarity[[1]]
MatrixExp[%]
```

```

Out[8]=
{ {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
{0, 0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
{0, 0, 0, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
{0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
{0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
{-1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2}, 
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2}, 
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2}, 
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/2}, 
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0} }

```

```
Out[6]=
{ {Cos[\frac{1}{2}], 0, 0, 0, 0, 0, 0, Sin[\frac{1}{2}], 0, 0, 0, 0, 0, 0, 0}, {0, Cos[\frac{1}{2}], 0, 0, 0, 0, Sin[\frac{1}{2}], 0, 0, 0, 0, 0, 0, 0}, {0, 0, Cos[\frac{1}{2}], 0, 0, -Sin[\frac{1}{2}], 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, Cos[\frac{1}{2}], -Sin[\frac{1}{2}], 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, Sin[\frac{1}{2}], Cos[\frac{1}{2}], 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, Sin[\frac{1}{2}], 0, 0, Cos[\frac{1}{2}], 0, 0, 0, 0, 0, 0, 0}, {0, -Sin[\frac{1}{2}], 0, 0, 0, 0, Cos[\frac{1}{2}], 0, 0, 0, 0, 0, 0, 0}, {-Sin[\frac{1}{2}], 0, 0, 0, 0, 0, 0, Cos[\frac{1}{2}], 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, Cos[\frac{1}{2}], 0, 0, 0, 0, 0, -Sin[\frac{1}{2}]}, {0, 0, 0, 0, 0, 0, 0, Cos[\frac{1}{2}], 0, 0, 0, 0, -Sin[\frac{1}{2}], 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[\frac{1}{2}], 0, 0, Sin[\frac{1}{2}], 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Cos[\frac{1}{2}], Sin[\frac{1}{2}], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -Sin[\frac{1}{2}], Cos[\frac{1}{2}], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -Sin[\frac{1}{2}], 0, 0, Cos[\frac{1}{2}], 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Sin[\frac{1}{2}], 0, 0, 0, 0, 0, Cos[\frac{1}{2}]}}}
```

```
In[7]:= ParallelTable[FullSimplify[SAB[A1, B1].SAB[A2, B2] - SAB[A2, B2].SAB[A1, B1]] ==
- (η4488[A1, A2] × SAB[B1, B2] - η4488[A1, B2] × SAB[B1, A2] -
η4488[B1, A2] × SAB[A1, B2] + η4488[B1, B2] × SAB[A1, A2])],
{A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}] // Flatten // Union
```

```
Out[7]=
{True}
```

```
In[8]:= ParallelTable[FullSimplify[SAB[A1, B1].T16^A[B2 - 1] - T16^A[B2 - 1].SAB[A1, B1]] ==
(-η4488[B2, A1] T16^A[B1 - 1] + η4488[B2, B1] T16^A[A1 - 1])],
{A1, 1, 8}, {B1, 1, 8}, {B2, 1, 8}] // Flatten // Union
```

```
Out[8]=
{True}
```



```

In[=]:= (044similarityTransformation = sqrtb.044similarityTransformation00.Inverse[sqrtb]) // MatrixForm
Out[=]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} \\ \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{i}{2} \\ -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$


In[=]:= (*044similarityTransformation=ArrayFlatten[{{0, ID8}, {ID8, 0}}]. Table[Block[{m=08similarityTransformation. (scale16*\#16), d}, d=D[m[[j]], #]&/@#16; d], {j, 1, 16}]*)

In[=]:= 044similarityTransformation.Transpose[044similarityTransformation] === Id16

Out[=]=
False

In[=]:= (044similarityTransformation = sqrtb.044similarityTransformation00.Transpose[sqrtb]);
044similarityTransformation.Transpose[044similarityTransformation] === Id16

Out[=]=
False

In[=]:= (044similarityTransformation = sqrtb.044similarityTransformation00.sqrtb);
044similarityTransformation.Transpose[044similarityTransformation] === Id16

Out[=]=
False

In[=]:= SAB[1, 2] // MatrixForm
08similarityTransformation. SAB[1, 2].
Transpose[08similarityTransformation] // MatrixForm
044similarityTransformation. SAB[1, 2].Transpose[044similarityTransformation] // MatrixForm

```


Out[•]//MatrixForm=

```
In[•]:= sAB[-1 + 1] [-1 + 2] == SAB[[1, 2]]
```

Out[•]=

True

```
In[•]:= σ16.SAB[[1, 2]] // MatrixForm
```

Out[•]//MatrixForm=

In[8]:= σ16.sAB[1][2] // MatrixForm

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula: [🔗](#)

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. [🔗](#)

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4). 🔗

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) 🔗

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. 🔗

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa). 🔗

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). 🔗

what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the *so*(4, 4) Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: 🔗

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for *so*(8) in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



↳ Why is the metric signature significant for \$so(4,4)\$ commutation relations?

Why is the metric signature significant for \$so(4,4)\$ commutation relations?

The metric signature is significant for the $so(p, q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). ⓘ

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$
The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4, 4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[1]:= **η4488**

Out[1]=

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1} }
```

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

In[2]:= SAB[6, 2].SAB[6, 7] - SAB[6, 7].SAB[6, 2] = η4488[6, 6] × SAB[7, 2]

Out[2]=

True

In[3]:= {{6, 2, 6, 7}, False}

Out[3]=

```
{ {6, 2, 6, 7}, False }
```

In[4]:= SAB[1, 2].SAB[1, 3] - SAB[1, 3].SAB[1, 2] = η4488[1, 1] × SAB[2, 3]

Out[4]=

False


```
In[1]:= SAB[[1, 2]] = ArrayFlatten[{{SAB1[[1, 2]], 0}, {0, SAB2[[1, 2]]}}]
Out[1]=
True

In[2]:= (*Table[
  SAB[[A1,B1]] = ArrayFlatten[{{SAB1[[A1,B1]], 0}, {0, SAB2[[A1,B1]]}}], 
  {A1,0, 7}, {B1,0,7}]*)

```

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \rightarrow SO(4,4)$ is given by

$$(2.8) \quad 8L^{A'}{}_{B'}(g) = \text{tr } \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}{}_{B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

```
In[2]:= SAB2[[1, 2]]
Out[2]=
{{0, 0, 0, 0, 0, 0, 0, -1/2}, {0, 0, 0, 0, 0, 0, 0, -1/2}, {0, 0, 0, 0, 0, 0, 1/2, 0}, {0, 0, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, -1/2, 0, 0, 0, 0}, {0, 0, -1/2, 0, 0, 0, 0, 0}, {0, 1/2, 0, 0, 0, 0, 0, 0}, {1/2, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[8]:= τ[θ]
```

```
Out[8]=
```

```
{ { 1, 0, 0, 0, 0, 0, 0, 0}, { 0, 1, 0, 0, 0, 0, 0, 0},  

{ 0, 0, 1, 0, 0, 0, 0, 0}, { 0, 0, 0, 1, 0, 0, 0, 0}, { 0, 0, 0, 0, 1, 0, 0, 0},  

{ 0, 0, 0, 0, 0, 1, 0, 0}, { 0, 0, 0, 0, 0, 0, 1, 0}, { 0, 0, 0, 0, 0, 0, 0, 1} }
```

```
In[9]:= (*Block[{b,A,F_a^(μ),F_a^a,ψ,Ψ},  

b=Array[A,{8}];  

ψ[1]=Array[Ψ[1],{8}];  

ψ[2]=Array[Ψ[2],{8}];  

F_a^(μ)=Table[(1/(Sqrt[ψ[1].σ.ψ[1]])ψ[1].σ.(τ[μL])),{μL,1,8}];  

F_a^a= Transpose[Table[(1/(Sqrt[ψ[1].σ.ψ[1]])η4488[[μL,μL]]τ[μL].ψ[1]),{μL,1,8}]];  

b==FullSimplify[F_a^(μ).ψ[2]/.Thread[ψ[2]→F_a^a.b]]]*)
```

```
In[10]:= (*Block[{b,A,B,q,F_a^(μ),ψ,φ,Ψ},  

b=Array[A,{8}];  

ψ[1]=Array[Ψ[1],{8}];  

φ[1]=Array[Ψ[2],{8}];  

q=ParallelTable[FullSimplify[  

(1/(Sqrt[ψ[1].σ.ψ[1]])ψ[1].σ.1/2(τ[A1].τ[B1]-τ[B1].τ[A1]).φ[1])],{A1,1,8},{B1,1,8}];  

B=FullSimplify[(1/(Sqrt[ψ[1].σ.ψ[1]])ParallelSum[η4488[[μL,μL]]τ[μL]b[[μL]],{μL,1,8}].ψ[1])];  

FullSimplify[q/.Thread[φ[1]→B]]*)
```

```
In[11]:= (*Block[{b,A,B,q,F_a^(μ),ψ,φ,Ψ},  

b=Array[A,{8}];  

ψ[1]=Array[Ψ[1],{8}];  

φ[1]=Array[Ψ[2],{8}];  

q=ParallelTable[FullSimplify[(ψ[1].σ.(τ[A1].τ[B1]).φ[1])],{A1,1,8},{B1,1,8}];  

B=FullSimplify[(ParallelSum[η4488[[μL,μL]]τ[μL]b[[μL]],{μL,1,8}].ψ[1])];  

FullSimplify[(1/(ψ[1].σ.ψ[1])q/.Thread[φ[1]→B])]*)
```

T16^α

In[•]:= ssgm4488

Out[•]=

$$\begin{aligned} \{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&)\} \end{aligned}$$

In[•]:= e^α_(A) /. sge^α_(A)

Out[•]=

$$\begin{aligned} &\left\{\left\{\frac{1}{\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0\right\}, \right. \\ &\left\{0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0\right\}, \left\{0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\right\}, \\ &\left\{0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}, \left\{0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0\right\}, \\ &\left.\left\{0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}}\right\}\right\} \end{aligned}$$

In[•]:= Table[T16^α[α1 - 1] =
Sum[((e^α_(A) /. sge^α_(A)) [[α1, A1]]) T16^A[A1 - 1], {A1, 1, 8}],
{α1, 1, 8}]; (*/.ssgm4488*)

In[•]:= (T16^α[8] = T16^α[0].T16^α[1].T16^α[2].T16^α[3].T16^α[4].T16^α[5].T16^α[6].T16^α[7]);

In[•]:= Table[newT16^α[A1] = 08similarityTransformation. T16^α[A1].
Transpose[08similarityTransformation], {A1, 0, 8}];

In[•]:= (*T16^α[8]==T16^A[8]*Sec[6 H x0]*)

CHECK

```
Table[T16a[α1 - 1] = Sum[(ea(A) [[α1, A1]] ) T16A[A1 - 1], {A1, 1, 8}], {α1, 1, 8}]; and ● ● ● :
```

```
In[=]:= (*Table[T16a[α1-1]=
Sum[(ea(A) [[α1,A1]] ) T16A[A1-1],{A1,1,8}],{α1,1,8}];*)
```

```
In[=]:= (*(T16a[8]=T16a[0].T16a[1].T16a[2].T16a[3].T16a[4].T16a[5].T16a[6].T16a[7])//MatrixForm*)
```

```
In[=]:= Table[{ {A1, B1}, FullSimplify[
ExpandAll[ 1/2 ( T16A[A1].T16A[B1] + T16A[B1].T16A[A1] ) ==
η4488[[A1 + 1, B1 + 1]] * ID16 ] ] }, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

```
Out[=]//MatrixForm=
{{{{0, 0}}, {{0, 1}}, {{0, 2}}, {{0, 3}}, {{0, 4}}, {{0, 5}}, {{0, 6}}, {{0, 7}}}, {{{1, 0}}, {{1, 1}}, {{1, 2}}, {{1, 3}}, {{1, 4}}, {{1, 5}}, {{1, 6}}, {{1, 7}}}, {{{2, 0}}, {{2, 1}}, {{2, 2}}, {{2, 3}}, {{2, 4}}, {{2, 5}}, {{2, 6}}, {{2, 7}}}, {{{3, 0}}, {{3, 1}}, {{3, 2}}, {{3, 3}}, {{3, 4}}, {{3, 5}}, {{3, 6}}, {{3, 7}}}, {{{4, 0}}, {{4, 1}}, {{4, 2}}, {{4, 3}}, {{4, 4}}, {{4, 5}}, {{4, 6}}, {{4, 7}}}, {{{5, 0}}, {{5, 1}}, {{5, 2}}, {{5, 3}}, {{5, 4}}, {{5, 5}}, {{5, 6}}, {{5, 7}}}, {{{6, 0}}, {{6, 1}}, {{6, 2}}, {{6, 3}}, {{6, 4}}, {{6, 5}}, {{6, 6}}, {{6, 7}}}, {{{7, 0}}, {{7, 1}}, {{7, 2}}, {{7, 3}}, {{7, 4}}, {{7, 5}}, {{7, 6}}, {{7, 7}}}}
```

```
In[8]:= Table[{ {α1, β1}, FullSimplify[
  1/2 FullSimplify[((T16^α[α1].T16^α[β1] + T16^α[β1].T16^α[α1]) /.
  2
    ssgm4488), constraintVars] ==
  Inverse[MatrixMetric44][[α1 + 1, β1 + 1]] * ID16 ] },
{α1, 0, 7}, {β1, 0, 7}] // MatrixForm
```

```
Out[•]//MatrixForm=
( { { 0, 0} } ) ( { 0, 1} ) ( { 0, 2} ) ( { 0, 3} ) ( { 0, 4} ) ( { 0, 5} ) ( { 0, 6} ) ( { 0, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 1, 0} ) ( { 1, 1} ) ( { 1, 2} ) ( { 1, 3} ) ( { 1, 4} ) ( { 1, 5} ) ( { 1, 6} ) ( { 1, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 2, 0} ) ( { 2, 1} ) ( { 2, 2} ) ( { 2, 3} ) ( { 2, 4} ) ( { 2, 5} ) ( { 2, 6} ) ( { 2, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 3, 0} ) ( { 3, 1} ) ( { 3, 2} ) ( { 3, 3} ) ( { 3, 4} ) ( { 3, 5} ) ( { 3, 6} ) ( { 3, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 4, 0} ) ( { 4, 1} ) ( { 4, 2} ) ( { 4, 3} ) ( { 4, 4} ) ( { 4, 5} ) ( { 4, 6} ) ( { 4, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 5, 0} ) ( { 5, 1} ) ( { 5, 2} ) ( { 5, 3} ) ( { 5, 4} ) ( { 5, 5} ) ( { 5, 6} ) ( { 5, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 6, 0} ) ( { 6, 1} ) ( { 6, 2} ) ( { 6, 3} ) ( { 6, 4} ) ( { 6, 5} ) ( { 6, 6} ) ( { 6, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( { 7, 0} ) ( { 7, 1} ) ( { 7, 2} ) ( { 7, 3} ) ( { 7, 4} ) ( { 7, 5} ) ( { 7, 6} ) ( { 7, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
```

```
In[=]:= Table[{α1, β1}, FullSimplify[
  1/2 FullSimplify[((newT16^α[α1].newT16^α[β1] + newT16^α[β1].
    newT16^α[α1]) /. ssgm4488), constraintVars] ==
  Inverse[MatrixMetric44][[α1 + 1, β1 + 1]] * ID16], {α1, 0, 7}, {β1, 0, 7}] // MatrixForm
```

Out[=]//MatrixForm=

$$\begin{pmatrix} (\{0, 0\}) & (\{0, 1\}) & (\{0, 2\}) & (\{0, 3\}) & (\{0, 4\}) & (\{0, 5\}) & (\{0, 6\}) & (\{0, 7\}) \\ \text{True} & \text{True} \\ (\{1, 0\}) & (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) & (\{1, 5\}) & (\{1, 6\}) & (\{1, 7\}) \\ \text{True} & \text{True} \\ (\{2, 0\}) & (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) & (\{2, 5\}) & (\{2, 6\}) & (\{2, 7\}) \\ \text{True} & \text{True} \\ (\{3, 0\}) & (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) & (\{3, 5\}) & (\{3, 6\}) & (\{3, 7\}) \\ \text{True} & \text{True} \\ (\{4, 0\}) & (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) & (\{4, 5\}) & (\{4, 6\}) & (\{4, 7\}) \\ \text{True} & \text{True} \\ (\{5, 0\}) & (\{5, 1\}) & (\{5, 2\}) & (\{5, 3\}) & (\{5, 4\}) & (\{5, 5\}) & (\{5, 6\}) & (\{5, 7\}) \\ \text{True} & \text{True} \\ (\{6, 0\}) & (\{6, 1\}) & (\{6, 2\}) & (\{6, 3\}) & (\{6, 4\}) & (\{6, 5\}) & (\{6, 6\}) & (\{6, 7\}) \\ \text{True} & \text{True} \\ (\{7, 0\}) & (\{7, 1\}) & (\{7, 2\}) & (\{7, 3\}) & (\{7, 4\}) & (\{7, 5\}) & (\{7, 6\}) & (\{7, 7\}) \\ \text{True} & \text{True} \end{pmatrix}$$

```
In[=]:= MatrixMetric44 // MatrixForm
```

Out[=]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6 H x \theta]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2 a4[H x 4]} \sin[6 H x \theta]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2 a4[H x 4]} \sin[6 H x \theta]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2 a4[H x 4]} \sin[6 H x \theta]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[=]:= ((e^α_(A) /. sgtrye^α_(A)).MatrixMetric44.(e^α_(A) /. sgtrye^α_(A))) // MatrixForm
```

Out[=]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[1]:= Transpose[(e(A)α /. sgtrye(A)α)].η4488.(e(A)α /. sgtrye(A)α) - MatrixMetric44 ///
FullSimplify[#, constraintVars] &
Out[1]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
In[2]:= Transpose[(e(A)(A) /. sgtrye(A)(A))].MatrixMetric44.(e(A)(A) /. sgtrye(A)(A)) - η4488 ///
(*Full*)Simplify[#, constraintVars] &
Out[2]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
In[3]:= Inverse[(e(A)(A) /. sgtrye(A)(A))] == (e(A)α /. sgtrye(A)α)
Out[3]= True
In[4]:= Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]
Out[4]= {{0, 0}, {0, 0}}
```

BELOW : Lagrangian_{Ψ16} =

$$\kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi16].\sigma16.T16^A[A1].$$

$$\Psi16_{|\alpha} g^{-1}{}^{\alpha\beta} \eta_{A1 B1} E_B^{B1} + \text{mass term}$$

```
In[5]:= e(A)α /. sgtrye(A)α
Out[5]= {{Cot[6 H x 0], 0, 0, 0, 0, 0, 0, 0}, {0, Sqrt[e^(2 a4[H x 4])] Sin[6 H x 0]^(1/6), 0, 0, 0, 0, 0, 0}, {0, 0, Sqrt[e^(2 a4[H x 4])] Sin[6 H x 0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, Sqrt[e^(2 a4[H x 4])] Sin[6 H x 0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, Sqrt[e^(2 a4[H x 4])] Sin[6 H x 0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, Sqrt[e^(2 a4[H x 4])] Sin[6 H x 0]^(1/6)}}}
```

```
In[1]:= eα(A) /. sgttryeα(A)
```

```
Out[1]=
```

$$\begin{aligned} & \left\{ \{\text{Tan}[6 H x_0], 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, \frac{1}{\sqrt{e^{2 a^4 [H x_4]}} \sin[6 H x_0]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{e^{2 a^4 [H x_4]}} \sin[6 H x_0]^{1/6}}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{e^{2 a^4 [H x_4]}} \sin[6 H x_0]^{1/6}}, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2 a^4 [H x_4]}} \sin[6 H x_0]^{1/6}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2 a^4 [H x_4]}} \sin[6 H x_0]^{1/6}}, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{e^{-2 a^4 [H x_4]}} \sin[6 H x_0]^{1/6}} \right\} \right\} \end{aligned}$$

```
In[2]:= eα(A) /. sgeα(A)
```

```
Out[2]=
```

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{g[0][0][x_0, x_4]}}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{g[1][1][x_0, x_4]}}, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, -\frac{1}{\sqrt{-g[5][5][x_0, x_4]}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x_0, x_4]}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x_0, x_4]}} \right\} \right\} \end{aligned}$$

```
In[3]:=
```

```
Clear[con];
con[g_, ass_ : {}] := Module[{t, rΓ, ginv},
  ginv = FullSimplify[Inverse[g], ass];
  rΓ = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
    (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]])), {s, 1, DIM8}], {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
  Return[{ginv, rΓ}]]
```

In[*n*]:= **ssgm4488**

Out[*n*]=

$$\begin{aligned} & \{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ & g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ & g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ & g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ & g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a^4[H\#2]} \sin[6H\#1]^{1/3} \&)\} \end{aligned}$$

In[*n*]:= **g4488**

g4488 /. **sg**

Out[*n*]=

$$\begin{aligned} & \{\{g[0][0][x0, x4], g[0][1][x0, x4], g[0][2][x0, x4], g[0][3][x0, x4], \\ & g[0][4][x0, x4], g[0][5][x0, x4], g[0][6][x0, x4], g[0][7][x0, x4]\}, \\ & \{g[1][0][x0, x4], g[1][1][x0, x4], g[1][2][x0, x4], g[1][3][x0, x4], \\ & g[1][4][x0, x4], g[1][5][x0, x4], g[1][6][x0, x4], g[1][7][x0, x4]\}, \\ & \{g[2][0][x0, x4], g[2][1][x0, x4], g[2][2][x0, x4], g[2][3][x0, x4], \\ & g[2][4][x0, x4], g[2][5][x0, x4], g[2][6][x0, x4], g[2][7][x0, x4]\}, \\ & \{g[3][0][x0, x4], g[3][1][x0, x4], g[3][2][x0, x4], g[3][3][x0, x4], \\ & g[3][4][x0, x4], g[3][5][x0, x4], g[3][6][x0, x4], g[3][7][x0, x4]\}, \\ & \{g[4][0][x0, x4], g[4][1][x0, x4], g[4][2][x0, x4], g[4][3][x0, x4], \\ & g[4][4][x0, x4], g[4][5][x0, x4], g[4][6][x0, x4], g[4][7][x0, x4]\}, \\ & \{g[5][0][x0, x4], g[5][1][x0, x4], g[5][2][x0, x4], g[5][3][x0, x4], \\ & g[5][4][x0, x4], g[5][5][x0, x4], g[5][6][x0, x4], g[5][7][x0, x4]\}, \\ & \{g[6][0][x0, x4], g[6][1][x0, x4], g[6][2][x0, x4], g[6][3][x0, x4], \\ & g[6][4][x0, x4], g[6][5][x0, x4], g[6][6][x0, x4], g[6][7][x0, x4]\}, \\ & \{g[7][0][x0, x4], g[7][1][x0, x4], g[7][2][x0, x4], g[7][3][x0, x4], \\ & g[7][4][x0, x4], g[7][5][x0, x4], g[7][6][x0, x4], g[7][7][x0, x4]\}\} \end{aligned}$$

Out[*n*]=

$$\begin{aligned} & \{\{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \{0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0\}, \{0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0\}, \{0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0\}, \{0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]\}\} \end{aligned}$$

```
In[8]:= MatrixForm[#] & /@ Block[{eA $\alpha$ , e $\alpha$ A, (*g $^{-1}$ , $\Gamma$ ,gg,*) colVecs, rowVecs, ab, ba, r},
(*gg=g4488/.sg;*)
(*{g $^{-1}$ , $\Gamma$ }={con[gg];*}
e $\alpha$ A = e $^{\alpha}_{(A)}$  /. sge $^{\alpha}_{(A)}$ ;
eA $\alpha$  = e $_{\alpha}^{(A)}$  /. sge $_{\alpha}^{(A)}$ ;
rowVecs = Table[eA $\alpha$ [[b, All]], {b, 1, Length[eA $\alpha$ ] }];
colVecs = Table[e $\alpha$ A[[All, b]], {b, 1, Length[e $\alpha$ A]}];
ab = rowVecs.colVecs // FullSimplify[#, constraintVars] &;
ba = Transpose[colVecs].Transpose[rowVecs] // FullSimplify[#, constraintVars] &;
(*r={{{ab},{ba}}};*)
r = {ab, ba};
r]
```

Out[8]=

$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

metric compatibility condition for the Octad

$$\nabla_{\mu} e_{\nu}^I = \partial_{\mu} e_{\nu}^I - \Gamma_{\mu\nu}^{\rho} e_{\rho}^I + \omega_{\mu J}^I e_{\nu}^J = 0$$

```
In[9]:= Clear[spinCoefficients];
spinCoefficients[eIv_] := Module[{evI, w, r, s},
  If[Dimensions[eIv] != {8, 8}, Return[{[]}]];
  If[Det[eIv] == 0, Return[{[]}]];
  evI = FullSimplify[Inverse[eIv], constraintVars] /. subsDefects;
  w = Table[-Table[
    FullSimplify[Sum[D[eIv[[I1, v1]], X[[\mu 1]]]*evI[[v1, Jprime]], {v1, 1, Length[evI]]] -
      Sum[ eIv[[I1, \rho]]*\Gamma[\rho, \mu 1, v1]*evI[[v1, Jprime]], {v1, 1, Length[evI]}],
      {\rho, 1, Length[X]}], constraintVars] /. subsDefects,
    {I1, 1, Length[eIv]}, {Jprime, 1, Length[evI]}], {\mu 1, 1, Length[X]}];
  Assert[Dimensions[w] == {8, 8, 8}];
  Return[w];
]
```



```

In[1]:= (*Γ[All,5,All]*)
In[2]:= (*Γ[All,1,All]*)
In[3]:= (*spinCoeffs[1]*)
In[4]:= (*spinCoeffs[5]*)
In[5]:= (*{X[1],X[5]}*)
In[6]:= (*spinCoeffs[1][1][1]*)
In[7]:= (*ωμIJ[1][1][1]*)
In[8]:= (*Block[{μ=1},FullSimplify[-((D[gtrye^(A),X[μ]].gtrye^(A))-({gtrye^(A).Γ[All,μ,All].gtrye^(A)})],constraintVars]/.subsDefects]
Dimensions[%]*)
In[9]:= (*Block[{μ=1},FullSimplify[
-((D[gtrye^(A),X[μ]].Transpose[gtrye^(A)])-({gtrye^(A).Γ[All,μ,All].Transpose[gtrye^(A)}]),constraintVars]/.subsDefects]*)
In[10]:= (*Block[{μ=1},
FullSimplify[-((Sum[D[gtrye^(A)[All,v],X[μ]]*gtrye^(A)[v,All],{v,1,8}]-({gtrye^(A).Γ[All,μ,All].gtrye^(A)}),constraintVars]/.subsDefects]
Dimensions[%]*)
In[11]:= (*Table[,{A1,1,8},{B1,1,8}]*)
In[12]:= (*Block[{μ=1},
Table[FullSimplify[-((Sum[D[gtrye^(A)[A1,v],X[μ]]*gtrye^(A)[v,B1],{v,1,8}]-({Sum[gtrye^(A)[A1,ρ]*Γ[ρ,μ,v]*gtrye^(A)[v,B1],{ρ,1,8},{v,1,8}})),constraintVars]/.subsDefects,{A1,1,8},{B1,1,8}]]
Dimensions[%]*)
In[13]:= Table[(ωμIJ[μ]-spinCoeffs[μ])/.subsDefects,{μ,1,Length[X]]]//Flatten//Union

```

Out[1]=

{0}

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:
 - $\nabla_\mu e^a{}_\nu = 0$
 - Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

Metric compatibility : The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame . The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is : ◦ ∇

he "

v=0

- Here, V_μ is the covariant derivative with respect to the connection F^μ_ν and e_μ^ν is the tetrad component.

No it is not the torsionless condition which is:

$$T_{\mu\nu}^I = D_{[\mu}^{\omega} e_{\nu]}^I = \partial_{[\mu} e_{\nu]}^I + \omega_{[\mu J}^I e_{\nu]}^J = 0$$

This postulate says:

$$\nabla_\mu e_\nu^I = \partial_\mu e_\nu^I - \Gamma_{\mu\nu}^\rho e_\rho^I + \omega_{\mu J}^I e_\nu^J = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D_\mu^\omega \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

```
In[1]:= Block[{eA\alpha, e\alpha A, t, ginv, \Gamma, gg},
gg = g4488 /. sg;
{ginv, \Gamma} = con[gg];
e\alpha A = e^\alpha_{(A)} /. sge^\alpha_{(A)}; eA\alpha = e_\alpha^{(A)} /. sge_\alpha^{(A)};
t = Table[ Block[{v, V, contra, covari},
V = eA\alpha[b, All];
v = e\alpha A[All, b];
contra = Table[D[v[k], X[\mu]] + Sum[v[i]*\Gamma[k, i, \mu], {i, 1, DIM8}], {k, 1, DIM8}];
(*before \omega terms*)
covari = Table[D[V[k], X[\mu]] - Sum[V[i]*\Gamma[i, k, \mu], {i, 1, DIM8}], {k, 1, DIM8}];
(*before \omega terms*)
{b - 1, X[\mu], {{contra}, {covari}}}], {b, 1, Length[e\alpha A]}, {\mu, 1, DIM8}];
t]
FullSimplify[#, ssym4488, constraintVars] & /@ %
Out[1]= { {{0, x0, {{0, 0, 0, 0, 0, -g[0][0]^(0,1)[x0, x4]/(2 Sqrt[g[0][0][x0, x4]] g[4][4][x0, x4])}}, 0, 0, 0}},
```

$$\begin{aligned}
& \left\{ \left\{ 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x1, \left\{ \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x2, \left\{ \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[2][2][x0, x4]}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x3, \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[3][3][x0, x4]}, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x4, \left\{ \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \right\}, \\
& \left\{ 0, x5, \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[5][5][x0, x4]}, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0, 0 \right\} \right\}, \\
& \left\{ 0, x6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[6][6][x0, x4]}, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}}, 0 \right\} \right\}, \\
& \left\{ 0, x7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]} \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}} \right\} \right\}, \\
& \left\{ 1, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{1, x1, \right. \\
& \quad \left. \left\{ \left\{ -\frac{g[1][1]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4]\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0 \right\} \right\}, \left\{ \left\{ -\frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, -\frac{g[1][1]^{(0,1)}[x0, x4]}{2\sqrt{g[1][1][x0, x4]}}, 0, 0, 0 \right\} \right\} \right\}, \\
& \{1, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\},
\end{aligned}$$

$$\begin{aligned} & \{1, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{1, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{1, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{1, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{2, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{2, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{2, x_2, \\ & \left\{ \left\{ -\frac{g[2][2]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]} g[4][4][x_0, x_4]} \right. \right. , \\ & \left. \left. 0, 0, 0 \right\} \right\}, \left\{ -\frac{g[2][2]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0, -\frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[2][2][x_0, x_4]}}, 0, 0, 0 \right\} \right\} \}, \\ & \{2, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{2, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{2, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{2, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{2, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{3, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{3, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{3, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{3, x_3, \\ & \left\{ \left\{ -\frac{g[3][3]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]} g[4][4][x_0, x_4]} \right. \right. , \\ & \left. \left. 0, 0, 0 \right\} \right\}, \left\{ -\frac{g[3][3]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[3][3][x_0, x_4]}}, 0, 0, 0 \right\} \right\} \}, \\ & \{3, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{3, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{3, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{3, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ & \{4, x_0, \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, \right. \right. , \\ & \left. \left. \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \}, \\ & \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, \right. \\ & \left. -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}} - \frac{g[4][4]^{(1,0)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \}, \\ & \{4, x_1, \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x_0, x_4]}{2 g[1][1][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\ & \left\{ 0, \frac{g[1][1]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \} \}, \end{aligned}$$

$$\begin{aligned}
& \left\{ 4, x_2, \left\{ \left\{ \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4]}{2 g[2][2][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_3, \left\{ \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4]}{2 g[3][3][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ 0, 0, 0, \frac{g[3][3]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_4, \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ -\frac{g[4][4]^{(1,0)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. -\frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}} - \frac{g[4][4]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_5, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_6, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[6][6][x_0, x_4]}, 0 \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]}, 0 \right\} \right\} \right\}, \\
& \left\{ 4, x_7, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]} \right\} \right\} \right\}, \left\{ 5, x_0, \right. \\
& \quad \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]}} - \frac{g[5][5]^{(1,0)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]}, \right. \right. \\
& \quad \left. \left. 0, 0 \right\} \right\} \right\}, \left\{ 5, x_1, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 5, x_2, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \right. \\
& \quad \left. \left\{ 5, x_3, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 5, x_4, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\}, \right. \right. \\
& \quad \left. \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]} \right\} \right\} \right\}
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}} - \frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2g[5][5][x0, x4]}, \right. \right. \\
& \left. \left. 0, 0 \right\} \right\}, \left\{ 5, x5, \left\{ \left\{ -\frac{g[5][5]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4]\sqrt{-g[5][5][x0, x4]}}, \right. \right. \right. \\
& \left. \left. \left. 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, 0, 0, 0 \right\}, \right. \right. \\
& \left. \left. \left\{ -\frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2g[5][5][x0, x4]}, 0, 0, 0, \right. \right. \right. \\
& \left. \left. \left. -\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2g[5][5][x0, x4]}, 0, 0, 0 \right\} \right\}, \right. \right. \\
& \left. \left. \left\{ 5, x6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \right. \right. \\
& \left. \left. \left. \left\{ 5, x7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 6, x0, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}g[6][6][x0, x4]}, 0 \right\}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 0 \right\} \right\}, \left\{ 6, x1, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 6, x2, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 6, x3, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 6, x4, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}g[6][6][x0, x4]}, 0 \right\}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 0 \right\} \right\}, \left\{ 6, x5, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 6, x6, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ -\frac{g[6][6]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4]\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2g[4][4][x0, x4]\sqrt{-g[6][6][x0, x4]}}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 0, 0, 0 \right\}, \left\{ -\frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, 0, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, 0, 0, 0 \right\} \right\}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 6, x7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 7, x0, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}g[7][7][x0, x4]} \right\}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. -\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]} \right\} \right\}, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left\{ 7, x1, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \right. \right. \right. \\
\end{aligned}$$

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{7, x2, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{7, x3, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {7, x4,
{{0, 0, 0, 0, 0, 0, 0, 0, 0}, g[7][7]^(0,1)[x0, x4] + g[7][7]^(0,1)[x0, x4]
2 (-g[7][7][x0, x4])^3/2 + 2 sqrt(-g[7][7][x0, x4]) g[7][7][x0, x4]}},
{{0, 0, 0, 0, 0, 0, 0, 0, 0}, -g[7][7]^(0,1)[x0, x4] - g[7][7]^(0,1)[x0, x4] sqrt(-g[7][7][x0, x4])
2 sqrt(-g[7][7][x0, x4])}, {{7, x5, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{7, x6, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {7, x7,
{{{-g[7][7]^(1,0)[x0, x4]
2 g[0][0][x0, x4] sqrt(-g[7][7][x0, x4]), 0, 0, 0, -g[7][7]^(0,1)[x0, x4]
2 g[4][4][x0, x4] sqrt(-g[7][7][x0, x4])}, {0, 0, 0}, {-g[7][7]^(1,0)[x0, x4]
2 g[7][7][x0, x4], 0, 0, 0, -g[7][7]^(0,1)[x0, x4]
2 g[7][7][x0, x4], 0, 0, 0}}}}}
Out[*]=
{{0, x0, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, x1, {{0, H, 0, 0, 0, 0, 0, 0, 0}}, {{0, e^(2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0, 0}}}}, {{0, x2, {{0, 0, H, 0, 0, 0, 0, 0, 0}}, {{0, 0, e^(2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}}}}, {{0, x3, {{0, 0, 0, H, 0, 0, 0, 0, 0}}, {{0, 0, 0, e^(2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}}}}, {{0, x4, {{0, 0, 0, 0, H, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, H, 0, 0, 0}}}, {{0, x5, {{0, 0, 0, 0, 0, H, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3), 0, 0}}}}, {{0, x6, {{0, 0, 0, 0, 0, 0, H, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3), 0}}}}, {{0, x7, {{0, 0, 0, 0, 0, 0, 0, H, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, -e^(-2 a4[H x4]) H Sin[6 H x0]^(1/3)}}}}, {{1, x0, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x1, {{-sqrt(e^(2 a4[H x4]) H Sec[6 H x0] Sin[6 H x0]^(7/6),
0, 0, 0, sqrt(e^(2 a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0)}}, {{-sqrt(e^(2 a4[H x4]) H Cos[6 H x0]
Sin[6 H x0]^(5/6)}, 0, 0, 0, -sqrt(e^(2 a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0)}}}}, {{1, x2, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x3, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x4, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x5, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x6, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{1, x7, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}, {{2, x0, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{2, x1, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{2, x2, {{-sqrt(e^(2 a4[H x4]) H Sec[6 H x0] Sin[6 H x0]^(7/6),
0, 0, 0, sqrt(e^(2 a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0)}}, {{-sqrt(e^(2 a4[H x4]) H Cos[6 H x0]
Sin[6 H x0]^(5/6)}, 0, 0, 0, -sqrt(e^(2 a4[H x4]) H Sin[6 H x0]^(1/6) a4'[H x4], 0, 0, 0)}}}}

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$$\begin{aligned}
& \left\{ \left\{ -\frac{\sqrt{e^{2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, 0, 0, 0, -\sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \{2, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{2, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x3, \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \sec[6 H x0] \sin[6 H x0]^{7/6}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, \sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \left\{ -\frac{\sqrt{e^{2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, 0, 0, 0, -\sqrt{e^{2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x1, \{\{\{0, H a4'[H x4], 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, -e^{2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x2, \{\{\{0, 0, H a4'[H x4], 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, -e^{2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x3, \{\{\{0, 0, 0, H a4'[H x4], 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, -e^{2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{4, x5, \{\{\{0, 0, 0, 0, 0, -H a4'[H x4], 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0, 0\}\}\}\}, \\
& \{4, x6, \{\{\{0, 0, 0, 0, 0, 0, -H a4'[H x4], 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4], 0\}\}\}\}, \\
& \{4, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, -H a4'[H x4]\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \sin[6 H x0]^{1/3} a4'[H x4]\}\}\}\}, \\
& \{5, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x5, \left\{ \left\{ \sqrt{e^{-2 a4[H x4]}} H \sec[6 H x0] \sin[6 H x0]^{7/6}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \left\{ -\frac{\sqrt{e^{-2 a4[H x4]}} H \cos[6 H x0]}{\sin[6 H x0]^{5/6}}, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} H \sin[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\},
\end{aligned}$$

In[•]:= Symbolize[$\omega_{\mu(b)}^{(a)}$]
In[•]:= $\omega = \text{Array}[\omega_{\mu(b)}^{(a)} [\#], \{8, 8, 8\}]$;

In[•]:= $\omega[[1]]$

Out[•]= $\left\{ \begin{array}{l} \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 1, 1], \omega_{\mu(b)}^{(a)} [1, 1, 2], \omega_{\mu(b)}^{(a)} [1, 1, 3], \omega_{\mu(b)}^{(a)} [1, 1, 4], \\ \omega_{\mu(b)}^{(a)} [1, 1, 5], \omega_{\mu(b)}^{(a)} [1, 1, 6], \omega_{\mu(b)}^{(a)} [1, 1, 7], \omega_{\mu(b)}^{(a)} [1, 1, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 2, 1], \omega_{\mu(b)}^{(a)} [1, 2, 2], \omega_{\mu(b)}^{(a)} [1, 2, 3], \omega_{\mu(b)}^{(a)} [1, 2, 4], \\ \omega_{\mu(b)}^{(a)} [1, 2, 5], \omega_{\mu(b)}^{(a)} [1, 2, 6], \omega_{\mu(b)}^{(a)} [1, 2, 7], \omega_{\mu(b)}^{(a)} [1, 2, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 3, 1], \omega_{\mu(b)}^{(a)} [1, 3, 2], \omega_{\mu(b)}^{(a)} [1, 3, 3], \omega_{\mu(b)}^{(a)} [1, 3, 4], \\ \omega_{\mu(b)}^{(a)} [1, 3, 5], \omega_{\mu(b)}^{(a)} [1, 3, 6], \omega_{\mu(b)}^{(a)} [1, 3, 7], \omega_{\mu(b)}^{(a)} [1, 3, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 4, 1], \omega_{\mu(b)}^{(a)} [1, 4, 2], \omega_{\mu(b)}^{(a)} [1, 4, 3], \omega_{\mu(b)}^{(a)} [1, 4, 4], \\ \omega_{\mu(b)}^{(a)} [1, 4, 5], \omega_{\mu(b)}^{(a)} [1, 4, 6], \omega_{\mu(b)}^{(a)} [1, 4, 7], \omega_{\mu(b)}^{(a)} [1, 4, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 5, 1], \omega_{\mu(b)}^{(a)} [1, 5, 2], \omega_{\mu(b)}^{(a)} [1, 5, 3], \omega_{\mu(b)}^{(a)} [1, 5, 4], \\ \omega_{\mu(b)}^{(a)} [1, 5, 5], \omega_{\mu(b)}^{(a)} [1, 5, 6], \omega_{\mu(b)}^{(a)} [1, 5, 7], \omega_{\mu(b)}^{(a)} [1, 5, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 6, 1], \omega_{\mu(b)}^{(a)} [1, 6, 2], \omega_{\mu(b)}^{(a)} [1, 6, 3], \omega_{\mu(b)}^{(a)} [1, 6, 4], \\ \omega_{\mu(b)}^{(a)} [1, 6, 5], \omega_{\mu(b)}^{(a)} [1, 6, 6], \omega_{\mu(b)}^{(a)} [1, 6, 7], \omega_{\mu(b)}^{(a)} [1, 6, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 7, 1], \omega_{\mu(b)}^{(a)} [1, 7, 2], \omega_{\mu(b)}^{(a)} [1, 7, 3], \omega_{\mu(b)}^{(a)} [1, 7, 4], \\ \omega_{\mu(b)}^{(a)} [1, 7, 5], \omega_{\mu(b)}^{(a)} [1, 7, 6], \omega_{\mu(b)}^{(a)} [1, 7, 7], \omega_{\mu(b)}^{(a)} [1, 7, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 8, 1], \omega_{\mu(b)}^{(a)} [1, 8, 2], \omega_{\mu(b)}^{(a)} [1, 8, 3], \omega_{\mu(b)}^{(a)} [1, 8, 4], \\ \omega_{\mu(b)}^{(a)} [1, 8, 5], \omega_{\mu(b)}^{(a)} [1, 8, 6], \omega_{\mu(b)}^{(a)} [1, 8, 7], \omega_{\mu(b)}^{(a)} [1, 8, 8] \end{array} \right\} \end{array} \right\}$

In[•]:= $\text{sw} = \text{Block}[\{\text{eA}\alpha, \text{eA}\alpha, \text{t}, \text{V}, \text{ginv}, \Gamma, \text{gg}\},$
 $\text{gg} = \text{g4488} /. \text{sg};$
 $\{\text{ginv}, \Gamma\} = \text{con}[\text{gg}];$
 $\text{eA}\alpha = \text{e}_{(A)}^\alpha /. \text{sge}_{(A)}^\alpha;$
 $\text{eA}\alpha = \text{e}_\alpha^{(A)} /. \text{sge}_\alpha^{(A)};$
 $\text{Table}[\omega[\mu, \alpha, b] \rightarrow \text{Block}[\{\text{v}, \text{d}\}, \text{v} = \text{eA}\alpha[\text{All}, b]],$
 $\text{d} = \text{Sum}[(\text{D}[\text{v}[k], X[\mu]] + \text{Sum}[\text{v}[i] \times \Gamma[k, i, \mu], \{i, 1, \text{DIM8}\}]) * \text{eA}\alpha[a, k], \{k, 1, \text{DIM8}\}];$
 $\text{d}], \{\mu, 1, \text{DIM8}\}, \{a, 1, \text{Length}[\text{eA}\alpha]\}, \{b, 1, \text{Length}[\text{eA}\alpha]\}]$

Out[•]=

$\left\{ \begin{array}{l} \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 1, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 5] \rightarrow -\frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}} \end{array} \right\} \end{array} \right\}$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [1, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 5, 1] \rightarrow -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 5, 2] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 5] \rightarrow \right. \\
& \quad \left. \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 6, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 6] \rightarrow \right. \\
& \quad \left. \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 8] \rightarrow 0 \right\}, \left\{ \omega_{\mu(b)}^{(a)} [1, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 2] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 7, 7] \rightarrow \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[6][6][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 8] \rightarrow \right. \\
& \quad \left. \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [2, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 2] \rightarrow -\frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 5] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 8] \rightarrow 0 \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [2, 2, 1] \rightarrow \frac{g[1][1]^{(1,0)} [x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, \right. \\
& \quad \omega_{\mu(b)}^{(a)} [2, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 2, 4] \rightarrow 0, \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 2, 5] \rightarrow \frac{g[1][1]^{(0,1)} [x0, x4]}{2 \sqrt{g[1][1][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 2] \rightarrow -\frac{g[1][1]^{(0,1)} [x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 5] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [3, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 1, 2] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 1, 3] \rightarrow -\frac{g[2][2]^{(1,0)} [x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [3, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 1, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [3, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu(b)}^{(a)} [3, 3, 1] \rightarrow \frac{g[2][2]^{(1,0)} [x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 3, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 3, 5] \rightarrow \frac{g[2][2]^{(0,1)} [x0, x4]}{2 \sqrt{g[2][2][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu}^{(a)} [3, 3, 6] \rightarrow 0, \omega_{\mu}^{(a)} [3, 3, 7] \rightarrow 0, \omega_{\mu}^{(a)} [3, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [3, 4, 1] \rightarrow 0, \omega_{\mu}^{(a)} [3, 4, 2] \rightarrow 0, \omega_{\mu}^{(a)} [3, 4, 3] \rightarrow 0, \omega_{\mu}^{(a)} [3, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [3, 4, 5] \rightarrow 0, \omega_{\mu}^{(a)} [3, 4, 6] \rightarrow 0, \omega_{\mu}^{(a)} [3, 4, 7] \rightarrow 0, \omega_{\mu}^{(a)} [3, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [3, 5, 1] \rightarrow 0, \omega_{\mu}^{(a)} [3, 5, 2] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [3, 5, 3] \rightarrow -\frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, \omega_{\mu}^{(a)} [3, 5, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [3, 5, 5] \rightarrow 0, \omega_{\mu}^{(a)} [3, 5, 6] \rightarrow 0, \omega_{\mu}^{(a)} [3, 5, 7] \rightarrow 0, \omega_{\mu}^{(a)} [3, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [3, 6, 1] \rightarrow 0, \omega_{\mu}^{(a)} [3, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)} [3, 6, 3] \rightarrow 0, \omega_{\mu}^{(a)} [3, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [3, 6, 5] \rightarrow 0, \omega_{\mu}^{(a)} [3, 6, 6] \rightarrow 0, \omega_{\mu}^{(a)} [3, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)} [3, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [3, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)} [3, 7, 2] \rightarrow 0, \omega_{\mu}^{(a)} [3, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)} [3, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [3, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)} [3, 7, 6] \rightarrow 0, \omega_{\mu}^{(a)} [3, 7, 7] \rightarrow 0, \omega_{\mu}^{(a)} [3, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [3, 8, 1] \rightarrow 0, \omega_{\mu}^{(a)} [3, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)} [3, 8, 3] \rightarrow 0, \omega_{\mu}^{(a)} [3, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [3, 8, 5] \rightarrow 0, \omega_{\mu}^{(a)} [3, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)} [3, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)} [3, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu}^{(a)} [4, 1, 1] \rightarrow 0, \omega_{\mu}^{(a)} [4, 1, 2] \rightarrow 0, \omega_{\mu}^{(a)} [4, 1, 3] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [4, 1, 4] \rightarrow -\frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \omega_{\mu}^{(a)} [4, 1, 5] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [4, 1, 6] \rightarrow 0, \omega_{\mu}^{(a)} [4, 1, 7] \rightarrow 0, \omega_{\mu}^{(a)} [4, 1, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu}^{(a)} [4, 2, 1] \rightarrow 0, \omega_{\mu}^{(a)} [4, 2, 2] \rightarrow 0, \omega_{\mu}^{(a)} [4, 2, 3] \rightarrow 0, \omega_{\mu}^{(a)} [4, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [4, 2, 5] \rightarrow 0, \omega_{\mu}^{(a)} [4, 2, 6] \rightarrow 0, \omega_{\mu}^{(a)} [4, 2, 7] \rightarrow 0, \omega_{\mu}^{(a)} [4, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu}^{(a)} [4, 3, 1] \rightarrow 0, \omega_{\mu}^{(a)} [4, 3, 2] \rightarrow 0, \omega_{\mu}^{(a)} [4, 3, 3] \rightarrow 0, \omega_{\mu}^{(a)} [4, 3, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [4, 3, 5] \rightarrow 0, \omega_{\mu}^{(a)} [4, 3, 6] \rightarrow 0, \omega_{\mu}^{(a)} [4, 3, 7] \rightarrow 0, \omega_{\mu}^{(a)} [4, 3, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu}^{(a)} [4, 4, 1] \rightarrow \frac{g[3][3]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [4, 4, 2] \rightarrow 0, \omega_{\mu}^{(a)} [4, 4, 3] \rightarrow 0, \omega_{\mu}^{(a)} [4, 4, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [4, 4, 5] \rightarrow \frac{g[3][3]^{(0,1)}[x0, x4]}{2\sqrt{g[3][3][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \omega_{\mu}^{(a)} [4, 4, 6] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [4, 4, 7] \rightarrow 0, \omega_{\mu}^{(a)} [4, 4, 8] \rightarrow 0 \right\}, \left\{ \omega_{\mu}^{(a)} [4, 5, 1] \rightarrow 0, \omega_{\mu}^{(a)} [4, 5, 2] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [4, 5, 3] \rightarrow 0, \omega_{\mu}^{(a)} [4, 5, 4] \rightarrow -\frac{g[3][3]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [4, 5, 5] \rightarrow 0, \omega_{\mu}^{(a)} [4, 5, 6] \rightarrow 0, \omega_{\mu}^{(a)} [4, 5, 7] \rightarrow 0, \omega_{\mu}^{(a)} [4, 5, 8] \rightarrow 0 \right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu}^{(a)} [4, 6, 1] \rightarrow 0, \omega_{\mu}^{(a)} [4, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)} [4, 6, 3] \rightarrow 0, \omega_{\mu}^{(a)} [4, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [4, 6, 5] \rightarrow 0, \omega_{\mu}^{(a)} [4, 6, 6] \rightarrow 0, \omega_{\mu}^{(a)} [4, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)} [4, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [4, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)} [4, 7, 2] \rightarrow 0, \omega_{\mu}^{(a)} [4, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)} [4, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [4, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)} [4, 7, 6] \rightarrow 0, \omega_{\mu}^{(a)} [4, 7, 7] \rightarrow 0, \omega_{\mu}^{(a)} [4, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [4, 8, 1] \rightarrow 0, \omega_{\mu}^{(a)} [4, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)} [4, 8, 3] \rightarrow 0, \omega_{\mu}^{(a)} [4, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [4, 8, 5] \rightarrow 0, \omega_{\mu}^{(a)} [4, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)} [4, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)} [4, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu}^{(a)} [5, 1, 1] \rightarrow 0, \omega_{\mu}^{(a)} [5, 1, 2] \rightarrow 0, \omega_{\mu}^{(a)} [5, 1, 3] \rightarrow 0, \omega_{\mu}^{(a)} [5, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 1, 5] \rightarrow -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[4][4][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 1, 6] \rightarrow 0, \omega_{\mu}^{(a)} [5, 1, 7] \rightarrow 0, \omega_{\mu}^{(a)} [5, 1, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu}^{(a)} [5, 2, 1] \rightarrow 0, \omega_{\mu}^{(a)} [5, 2, 2] \rightarrow 0, \omega_{\mu}^{(a)} [5, 2, 3] \rightarrow 0, \omega_{\mu}^{(a)} [5, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 2, 5] \rightarrow 0, \omega_{\mu}^{(a)} [5, 2, 6] \rightarrow 0, \omega_{\mu}^{(a)} [5, 2, 7] \rightarrow 0, \omega_{\mu}^{(a)} [5, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu}^{(a)} [5, 3, 1] \rightarrow 0, \omega_{\mu}^{(a)} [5, 3, 2] \rightarrow 0, \omega_{\mu}^{(a)} [5, 3, 3] \rightarrow 0, \omega_{\mu}^{(a)} [5, 3, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 3, 5] \rightarrow 0, \omega_{\mu}^{(a)} [5, 3, 6] \rightarrow 0, \omega_{\mu}^{(a)} [5, 3, 7] \rightarrow 0, \omega_{\mu}^{(a)} [5, 3, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu}^{(a)} [5, 4, 1] \rightarrow 0, \omega_{\mu}^{(a)} [5, 4, 2] \rightarrow 0, \omega_{\mu}^{(a)} [5, 4, 3] \rightarrow 0, \omega_{\mu}^{(a)} [5, 4, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 4, 5] \rightarrow 0, \omega_{\mu}^{(a)} [5, 4, 6] \rightarrow 0, \omega_{\mu}^{(a)} [5, 4, 7] \rightarrow 0, \omega_{\mu}^{(a)} [5, 4, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu}^{(a)} [5, 5, 1] \rightarrow \frac{g[4][4]^{(1,0)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[4][4][x0, x4]}, \omega_{\mu}^{(a)} [5, 5, 2] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 5, 3] \rightarrow 0, \omega_{\mu}^{(a)} [5, 5, 4] \rightarrow 0, \omega_{\mu}^{(a)} [5, 5, 5] \rightarrow \right. \right. \\
& \quad \left. \left. \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 5, 6] \rightarrow 0, \omega_{\mu}^{(a)} [5, 5, 7] \rightarrow 0, \omega_{\mu}^{(a)} [5, 5, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left\{ \omega_{\mu}^{(a)} [5, 6, 1] \rightarrow 0, \omega_{\mu}^{(a)} [5, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)} [5, 6, 3] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 6, 4] \rightarrow 0, \omega_{\mu}^{(a)} [5, 6, 5] \rightarrow 0, \omega_{\mu}^{(a)} [5, 6, 6] \rightarrow \right. \right. \\
& \quad \left. \left. \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)} [5, 6, 8] \rightarrow 0 \right\}, \left\{ \omega_{\mu}^{(a)} [5, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)} [5, 7, 2] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)} [5, 7, 4] \rightarrow 0, \omega_{\mu}^{(a)} [5, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)} [5, 7, 6] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu}^{(a)} [5, 7, 7] \rightarrow \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}g[6][6][x0, x4]} \right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \sqrt{-g[6][6][x0, x4]}, \omega_{\mu(b)}^{(a)} [5, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [5, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [5, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 8] \rightarrow \right. \\
& \quad \left. \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 1, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 6] \rightarrow -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[5][5][x0, x4]}}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 5, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 6] \rightarrow -\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 6, 1] \rightarrow \frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[5][5][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 6, 5] \rightarrow \frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}g[5][5][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 7, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 7, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [6, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [7, 1, 3] \rightarrow 0, \right.
\end{aligned}$$

$$\begin{aligned}
& \omega_{\mu(b)}^{(a)} [7, 1, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 1, 6] \rightarrow 0, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 1, 7] \rightarrow -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[6][6][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [7, 1, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 5, 3] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 5, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 5, 6] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 5, 7] \rightarrow -\frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[6][6][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [7, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 7, 1] \rightarrow \frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[6][6][x0, x4]}, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 7, 5] \rightarrow \frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}g[6][6][x0, x4]}, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [7, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [7, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [7, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [8, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [8, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 7] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [8, 1, 8] \rightarrow -\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[7][7][x0, x4]}} \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [8, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [8, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [8, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 4] \rightarrow 0, \right. \right.
\end{aligned}$$

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$$\left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 8] \rightarrow 0 \\ \omega_{\mu(b)}^{(a)} [8, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 8] \rightarrow 0 \end{array} \right\},$$


$$\left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 7] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 5, 8] \rightarrow -\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}} \end{array} \right\},$$


$$\left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 8] \rightarrow 0 \end{array} \right\},$$


$$\left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 8] \rightarrow 0 \end{array} \right\},$$


$$\left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 8, 1] \rightarrow \frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[0][0][x0, x4]} g[7][7][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [8, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 8, 5] \rightarrow \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [8, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 8] \rightarrow 0 \end{array} \right\} \right\}$$


$$In[1]:= (*sw=Block[{eA\alpha,e\alpha A,t,V,g\text{inv},\Gamma,gg},$$


$$gg=g4488/.sg;$$


$${g\text{inv},\Gamma}=con[gg];$$


$$e\alpha A=e_{(A)}^\alpha /.sge_{(A)}^\alpha ;$$


$$eA\alpha=e_\alpha^{(A)}/.sge_\alpha^{(A)};$$


$$Table[\omega_{\mu(b)}^{(a)} [\mu,a,b]\rightarrow Block[{v,d},v=e\alpha A\text{All},b];$$


$$d=Sum[(D[v[k],X[\mu]]+Sum[v[i]\Gamma[k,i,\mu],{i,1,DIM8}])\text{*}eA\alpha[a,k],{k,1,DIM8}];$$


$$d],{\mu,1,DIM8},{a,1,Length[e\alpha A]},{b,1,Length[e\alpha A}]\Big]*)$$


$$In[2]:= swf=Flatten[sw];$$


$$In[3]:= (*\omega_{\mu(b)}^*(*)$$


$$w;$$


$$wmat=%/.swf$$


$$Dimensions[%]$$


$$Out[3]=$$


$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} 0, 0, 0, 0, \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{-g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \end{array} \right\}, \\ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \end{array} \right.$$


```

$$\begin{aligned}
& \left\{ -\frac{g[0][0]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, \right. \\
& \left. \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \right. \\
& \left. 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \\
& \left. \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, 0, \right. \\
& \left. 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \\
& \left. \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[6][6][x0, x4]}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \right. \\
& \left. \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2 (-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2 \sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ 0, -\frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, \right. \\
& \left. \frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, -\frac{g[1][1]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, \right. \\
& \left. \frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \right. \\
& \left. 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\},
\end{aligned}$$


```

{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, - $\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4] \sqrt{-g[7][7][x0, x4]}}$ , {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},  $\left\{\frac{g[7][7]^{(1,0)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[7][7][x0, x4]}, 0, 0, 0, 0, 0, 0, 0\right\}$ ,
0, 0, 0,  $\left\{\frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[7][7][x0, x4]}}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]}, 0, 0, 0\right\}\right\}$ 

Out[•]= {8, 8, 8}

In[•]:= swgtry = Block[{eA $\alpha$ , e $\alpha$ A, t, V},
e $\alpha$ A = e $\alpha$ (A) /. sgtrye $\alpha$ (A);
eA $\alpha$  = e $\alpha$ (A) /. sgtrye $\alpha$ (A);
Table[(*V=eA $\alpha$ [[a,All]];*)  $\omega_{\mu(b)}^{(a)}$  [μ, a, b]  $\rightarrow$  Block[{v, d}, v = e $\alpha$ A[[All, b]];
d = Sum[(D[v[[k]], X[[μ]]] + Sum[v[[i]]  $\times$  r[[k, i, μ]], {i, 1, DIM8}]) * eA $\alpha$ [[a, k]], {k, 1, DIM8}];
FullSimplify[d, constraintVars] /. { $\sqrt{e^{2 a^4[H \times 4]}}$   $\rightarrow$  e $a^4[H \times 4]$ ,  $\sqrt{e^{-2 a^4[H \times 4]}}$   $\rightarrow$  e $-a^4[H \times 4]$ }],
{μ, 1, DIM8}, {a, 1, Length[eA $\alpha$ ]], {b, 1, Length[e $\alpha$ A]}]]

```

Out[•]=

```

{ { {  $\omega_{\mu(b)}^{(a)}$  [1, 1, 1]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 1, 2]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 1, 3]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 1, 4]  $\rightarrow$  0,
 $\omega_{\mu(b)}^{(a)}$  [1, 1, 5]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 1, 6]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 1, 7]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 1, 8]  $\rightarrow$  0 },
{  $\omega_{\mu(b)}^{(a)}$  [1, 2, 1]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 2, 2]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 2, 3]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 2, 4]  $\rightarrow$  0,
 $\omega_{\mu(b)}^{(a)}$  [1, 2, 5]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 2, 6]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 2, 7]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 2, 8]  $\rightarrow$  0 },
{  $\omega_{\mu(b)}^{(a)}$  [1, 3, 1]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 3, 2]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 3, 3]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 3, 4]  $\rightarrow$  0,
 $\omega_{\mu(b)}^{(a)}$  [1, 3, 5]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 3, 6]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 3, 7]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 3, 8]  $\rightarrow$  0 },
{  $\omega_{\mu(b)}^{(a)}$  [1, 4, 1]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 4, 2]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 4, 3]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 4, 4]  $\rightarrow$  0,
 $\omega_{\mu(b)}^{(a)}$  [1, 4, 5]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 4, 6]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 4, 7]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 4, 8]  $\rightarrow$  0 },
{  $\omega_{\mu(b)}^{(a)}$  [1, 5, 1]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 5, 2]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 5, 3]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 5, 4]  $\rightarrow$  0,
 $\omega_{\mu(b)}^{(a)}$  [1, 5, 5]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 5, 6]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 5, 7]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 5, 8]  $\rightarrow$  0 },
{  $\omega_{\mu(b)}^{(a)}$  [1, 6, 1]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 6, 2]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 6, 3]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 6, 4]  $\rightarrow$  0,
 $\omega_{\mu(b)}^{(a)}$  [1, 6, 5]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 6, 6]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 6, 7]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 6, 8]  $\rightarrow$  0 },
{  $\omega_{\mu(b)}^{(a)}$  [1, 7, 1]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 7, 2]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 7, 3]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 7, 4]  $\rightarrow$  0,
 $\omega_{\mu(b)}^{(a)}$  [1, 7, 5]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 7, 6]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 7, 7]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 7, 8]  $\rightarrow$  0 },
{  $\omega_{\mu(b)}^{(a)}$  [1, 8, 1]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 8, 2]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 8, 3]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 8, 4]  $\rightarrow$  0,
 $\omega_{\mu(b)}^{(a)}$  [1, 8, 5]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 8, 6]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 8, 7]  $\rightarrow$  0,  $\omega_{\mu(b)}^{(a)}$  [1, 8, 8]  $\rightarrow$  0 } },

```


$$\begin{aligned}
& \left\{ \omega_{\mu}^{(a)} [8, 2, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 2, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 2, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 3, 1] \rightarrow 0, \omega_{\mu}^{(a)} [8, 3, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 3, 3] \rightarrow 0, \omega_{\mu}^{(a)} [8, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 3, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 3, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 3, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 4, 1] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 3] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 4, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 5, 1] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 5, 4] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 5, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 5, 8] \rightarrow e^{-a4[H \times 4]} H \sin[6Hx0]^{1/6} a4'[H \times 4] \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 6, 1] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 3] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 6, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 7, 1] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 3] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 7, 5] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu}^{(a)} [8, 8, 1] \rightarrow e^{-a4[H \times 4]} H \sin[6Hx0]^{1/6}, \omega_{\mu}^{(a)} [8, 8, 2] \rightarrow 0, \omega_{\mu}^{(a)} [8, 8, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 8, 4] \rightarrow 0, \omega_{\mu}^{(a)} [8, 8, 5] \rightarrow -e^{-a4[H \times 4]} H \sin[6Hx0]^{1/6} a4'[H \times 4], \right. \\
& \quad \left. \omega_{\mu}^{(a)} [8, 8, 6] \rightarrow 0, \omega_{\mu}^{(a)} [8, 8, 7] \rightarrow 0, \omega_{\mu}^{(a)} [8, 8, 8] \rightarrow 0 \right\} \}
\end{aligned}$$

In[◦]:= **sWfgtry** = **Flatten**[**sWgtry**];


```

In[=]:= g4488 /. sg
% /. ssgm4488

Out[=]=
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}

Out[=]=
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

check metric compatibility condition for the Octad
Block[{eAα, eαA, t, ginv, Γ, gg(*, ωω*)},
(*ωω=ω^(a)_(b) /. swf;*)

gg = g4488 /. sg /. ssgm4488;
{ginv, Γ} = con[gg];
eαA = e^(α)_(A) /. sge^(α)_(A);
eAα = e^(A)_α /. sge^(A)_α;
t = Table[ Block[{v, V, contra, covari},
V = eAα[b, All];
v = eαA[All, b];
contra = Table[
D[v[k], X[μ]] + Sum[ v[i] × Γ[k, i, μ],
{i, 1, DIM8}], {k, 1, DIM8}];
(*no ω terms*)
covari = Table[
D[V[k], X[μ]] - Sum[ V[i] × Γ[i, k, μ],
{i, 1, DIM8}] + Sum[ eAα[j, All][k] ×
]
]
]
```

```

 $\omega \text{mat}[\mu, b, J], \{J, 1, \text{DIM8}\}] //$ 
 $\text{FullSimplify}[\#, \text{constraintVars} \&&$ 
 $e^{2 a4[H x4]} > 0 \&& e^{a4[H x4]} > 0 \&&$ 
 $e^{-2 a4[H x4]} > 0 \&& e^{-a4[H x4]} > 0] \&,$ 
 $\{k, 1, \text{DIM8}\}]; (*\omega terms*)$ 
 $(*\{b-1, X[\mu], \{\{\text{contra}\}, \{\text{covari}\}\}\}]*,$ 
 $\{b, 1, \text{Length}[e\alpha A]\}, \{\mu, 1, \text{DIM8}\}]; *)$ 
 $\{b - 1, X[\mu], \text{covari}\}],$ 
 $\{b, 1, \text{Length}[e\alpha A]\}, \{\mu, 1, \text{DIM8}\}];$ 
 $t]$ 
 $\text{FullSimplify}[\# /. \text{ssgm4488},$ 
 $\text{constraintVars} \&& e^{2 a4[H x4]} > 0 \&& e^{a4[H x4]} >$ 
 $0 \&& e^{-2 a4[H x4]} > 0 \&& e^{-a4[H x4]} > 0] \& /@ \%$ 

Out[8]=

 $\left\{ \left\{ \left\{ 0, x0, \left\{ \left( g[0][0]^{(1,0)} [x0, x4] + \right.$ 
 $24 H \text{Csc}[12 H x0] g[0][0][x0, x4] \right) / \right.$ 
 $\left( 2 \sqrt{g[0][0][x0, x4]} \right), 0, 0, 0,$ 
 $\frac{g[0][0]^{(0,1)} [x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \right\},$ 
 $\left\{ 0, x1, \left\{ 0, \left( -g[1][1]^{(1,0)} [x0, x4] + \right.$ 
 $2 e^{2 a4[H x4]} H \text{Sec}[6 H x0]$ 
 $\left. \text{Sin}[6 H x0]^{4/3} g[0][0][x0, x4] \right) / \right\}$ 

```

$$\begin{aligned}
& \left(2 \sqrt{g[0][0][x0, x4]} \right), 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ 0, x2, \{ 0, 0, (-g[2][2]^{(1,0)}[x0, x4] + \\
& 2 e^{2 a4[H x4]} H \operatorname{Sec}[6 H x0] \\
& \sin[6 H x0]^{4/3} g[0][0][x0, x4]) / \\
& (2 \sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0, 0 \} \}, \\
& \{ 0, x3, \{ 0, 0, 0, (-g[3][3]^{(1,0)}[x0, x4] + \\
& 2 e^{2 a4[H x4]} H \operatorname{Sec}[6 H x0] \\
& \sin[6 H x0]^{4/3} g[0][0][x0, x4]) / \\
& (2 \sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0 \} \}, \\
& \{ 0, x4, \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}} , 0, 0, 0, \right. \\
& \left. - \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\} \}, \\
& \{ 0, x5, \{ 0, 0, 0, 0, 0, 0, \\
& (-g[5][5]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H \\
& x0] \sin[6 H x0]^{4/3} g[0][0][x0, x4]) / \\
& (2 \sqrt{g[0][0][x0, x4]}), 0, 0 \} \}, \\
& \{ 0, x6, \{ 0, 0, 0, 0, 0, 0, 0, \\
& (-g[6][6]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H \\
& x0] \sin[6 H x0]^{4/3} g[0][0][x0, x4]) / \\
& (2 \sqrt{g[0][0][x0, x4]}), 0 \} \},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 0, x7, \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \\
& \quad \left(-g[7][7]^{(1,0)} [x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H \right. \\
& \quad \left. \left. x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4] \right) / \right. \\
& \quad \left. \left. (2 \sqrt{g[0][0][x0, x4]} \right) \right\} \}, \\
& \left\{ \left\{ 1, x0, \left\{ 0, \left(g[1][1]^{(1,0)} [x0, x4] - \right. \right. \right. \right. \\
& \quad \left. \left. 2 H \operatorname{Cot}[6 H x0] g[1][1][x0, x4] \right) / \right. \\
& \quad \left. \left. (2 \sqrt{g[1][1][x0, x4]} \right), 0, 0, 0, 0, 0, 0 \right\} \}, \\
& \left\{ 1, x1, \left\{ \left(g[1][1]^{(1,0)} [x0, x4] - \right. \right. \right. \\
& \quad \left. \left. 2 H \operatorname{Cot}[6 H x0] g[1][1][x0, x4] \right) / \right. \\
& \quad \left. \left. (2 \sqrt{g[1][1][x0, x4]} \right), 0, 0, \right. \\
& \quad \left. 0, \left(g[1][1]^{(0,1)} [x0, x4] - \right. \right. \\
& \quad \left. \left. 6 H a4'[H x4] g[1][1][x0, x4] \right) / \right. \\
& \quad \left. \left. (2 \sqrt{g[1][1][x0, x4]} \right), 0, 0, 0 \right\} \}, \\
& \left\{ 1, x2, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 1, x3, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 1, x4, \left\{ 0, \left(g[1][1]^{(0,1)} [x0, x4] - \right. \right. \right. \\
& \quad \left. \left. 6 H a4'[H x4] g[1][1][x0, x4] \right) / \right. \\
& \quad \left. \left. (2 \sqrt{g[1][1][x0, x4]} \right), 0, 0, 0, 0, 0, 0 \right\} \}, \\
& \left\{ 1, x5, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 1, x6, \right.
\end{aligned}$$

$$\begin{aligned}
& \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{1, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{\{2, x0, \{0, 0, (g[2][2]^{(1,0)}[x0, x4] - \\
& \quad 2 H \text{Cot}[6 H x0] g[2][2][x0, x4]) / \\
& \quad (2 \sqrt{g[2][2][x0, x4]}), 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x2, \{(g[2][2]^{(1,0)}[x0, x4] - \\
& \quad 2 H \text{Cot}[6 H x0] g[2][2][x0, x4]) / \\
& \quad (2 \sqrt{g[2][2][x0, x4]}), 0, 0, \\
& \quad 0, (g[2][2]^{(0,1)}[x0, x4] - \\
& \quad 6 H a4'[H x4] g[2][2][x0, x4]) / \\
& \quad (2 \sqrt{g[2][2][x0, x4]}), 0, 0, 0\}\}\}, \\
& \{2, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x4, \\
& \quad \{0, 0, (g[2][2]^{(0,1)}[x0, x4] - \\
& \quad 6 H a4'[H x4] g[2][2][x0, x4]) / \\
& \quad (2 \sqrt{g[2][2][x0, x4]}), 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{2, x5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{\{3, x0, \{0, 0, 0, (g[3][3]^{(1,0)}[x0, x4] -
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{2 H \operatorname{Cot}[6 H x_0] g[3][3][x_0, x_4]}{(2 \sqrt{g[3][3][x_0, x_4]})}, 0, 0, 0, 0 \right\} \}, \\
& \{3, x_1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x_2, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x_3, \{ (g[3][3]^{(1,0)}[x_0, x_4] - \\
& \quad \frac{2 H \operatorname{Cot}[6 H x_0] g[3][3][x_0, x_4]}{(2 \sqrt{g[3][3][x_0, x_4]})}, 0, 0, \\
& \quad 0, (g[3][3]^{(0,1)}[x_0, x_4] - \\
& \quad \frac{6 H a4'[H x_4] g[3][3][x_0, x_4]}{(2 \sqrt{g[3][3][x_0, x_4]})}, 0, 0, 0 \} \}, \\
& \{3, x_4, \{0, 0, 0, (g[3][3]^{(0,1)}[x_0, x_4] - \\
& \quad \frac{6 H a4'[H x_4] g[3][3][x_0, x_4]}{(2 \sqrt{g[3][3][x_0, x_4]})}, 0, 0, 0, 0 \} \}, \\
& \{3, x_5, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x_6, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x_7, \{0, 0, 0, 0, 0, 0, 0, 0\} \}, \\
& \left\{ \left\{ 4, x_0, \left\{ \frac{g[0][0]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. 0, -\frac{g[4][4]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]}}, 0, 0, 0 \right\} \right\}, \\
& \{4, x_1, \{0, (g[1][1]^{(0,1)}[x_0, x_4] + \\
& \quad 6 e^{2 a4[H x_4]} H \operatorname{Sin}[6 H x_0]^{1/3}
\end{aligned}$$

$$\begin{aligned}
& \left\{ 4, x_6, \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \\
& \frac{1}{2} \left(-6 e^{-2 a4[Hx4]} H \sin[6Hx0]^{1/3} a4'[Hx4] - \right. \\
& \left. \left. \frac{g[6][6]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]}, 0 \right\}, \\
& \left\{ 4, x_7, \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \\
& \frac{1}{2} \left(-6 e^{-2 a4[Hx4]} H \sin[6Hx0]^{1/3} a4'[Hx4] - \right. \\
& \left. \left. \frac{g[7][7]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]} \right\} \}, \left\{ \{5, x0, \right. \\
& \{0, 0, 0, 0, 0, -((g[5][5]^{(1,0)}[x0, x4] - \right. \\
& \left. 2 H \cot[6Hx0] g[5][5][x0, x4]) / \right. \\
& \left. (2 \sqrt{-g[5][5][x0, x4]})) , 0, 0 \right\} \}, \\
& \{5, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{5, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{5, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{5, x4, \\
& \{0, 0, 0, 0, 0, -((g[5][5]^{(0,1)}[x0, x4] + \right. \\
& \left. 6 H a4'[Hx4] g[5][5][x0, x4]) / \right. \\
& \left. (2 \sqrt{-g[5][5][x0, x4]})) , 0, 0 \right\} ,
\end{aligned}$$

$$\{5, x5, \{-((g[5][5]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[5][5][x0, x4]) / (2 \sqrt{-g[5][5][x0, x4]})), 0,$$

$$0, 0, -((g[5][5]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[5][5][x0, x4]) / (2 \sqrt{-g[5][5][x0, x4]})), 0, 0\}\},$$

$$\{5, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{5, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{\{6, x0, \{0, 0, 0, 0, 0, 0, 0, 0,$$

$$-((g[6][6]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[6][6][x0, x4]) / (2 \sqrt{-g[6][6][x0, x4]})), 0\}\},$$

$$\{6, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{6, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{6, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\{6, x4, \{0, 0, 0, 0, 0, 0, -((g[6][6]^{(0,1)}[x0, x4] + 6 H a4'[H x4] g[6][6][x0, x4]) / (2 \sqrt{-g[6][6][x0, x4]})), 0\}\},$$

$$\{6, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\},$$

$$\begin{aligned} & \{ 6, x6, \\ & \quad \{ - ((g[6][6]^{(1,0)} [x0, x4] - \\ & \quad \quad 2 H \operatorname{Cot}[6 H x0] g[6][6][x0, x4]) / \\ & \quad \quad (2 \sqrt{-g[6][6][x0, x4]})) , 0, \\ & \quad 0, 0, - ((g[6][6]^{(0,1)} [x0, x4] + \\ & \quad \quad 6 H a4' [H x4] g[6][6][x0, x4]) / \\ & \quad \quad (2 \sqrt{-g[6][6][x0, x4]})) , 0, 0, 0 \} \}, \\ & \{ 6, x7, \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ & \{ \{ 7, x0, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ & \quad - ((g[7][7]^{(1,0)} [x0, x4] - \\ & \quad \quad 2 H \operatorname{Cot}[6 H x0] g[7][7][x0, x4]) / \\ & \quad \quad (2 \sqrt{-g[7][7][x0, x4]})) \} \}, \\ & \{ 7, x1, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\ & \{ 7, x2, \\ & \quad \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\ & \{ 7, x3, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\ & \{ 7, x4, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \\ & \quad 0, - ((g[7][7]^{(0,1)} [x0, x4] + \\ & \quad \quad 6 H a4' [H x4] g[7][7][x0, x4]) / \\ & \quad \quad (2 \sqrt{-g[7][7][x0, x4]})) \} \}, \\ & \{ 7, x5, \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\ & \{ 7, x6, \end{aligned}$$

$$\begin{aligned}
& \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{7, x7, \{-((g[7][7]^{(1,0)}[x0, x4] - \\
& 2 H \operatorname{Cot}[6 H x0] g[7][7][x0, x4]) / \\
& (2 \sqrt{-g[7][7][x0, x4]}))\}, 0, \\
& 0, 0, -((g[7][7]^{(0,1)}[x0, x4] + \\
& 6 H a4'[H x4] g[7][7][x0, x4]) / \\
& (2 \sqrt{-g[7][7][x0, x4]}))\}, 0, 0, 0\}\}\}
\end{aligned}$$

Out[=]=

$$\begin{aligned}
& \{\{0, x0, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{0, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, \\
& x4, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x6, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{0, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{\{1, x0, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x2, \{0, \\
& 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x4, \{0, 0, 0, \\
& 0, 0, 0, 0, 0\}\}, \{1, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{1, x6, \{0, 0, 0, 0, 0, \\
& 0, 0, 0\}\}, \{1, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{\{2, x0, \{0, 0, 0, 0, 0, 0, \\
& 0, 0\}\}, \{2, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{2, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x4, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, \\
& x5, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, \\
& x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{\{3, x0, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, \\
& x1, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x3, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x4, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x5, \{0, \\
& 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x7, \{0, 0, 0, \\
& 0, 0, 0, 0, 0\}\}\}, \{\{4, x0, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x1, \{0, 0, 0, 0, \\
& 0, 0, 0, 0\}\}, \{4, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x3, \{0, 0, 0, 0, 0, 0, \\
& 0, 0\}\}, \{4, x4, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{4, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{4, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{\{5, \\
& x0, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x1, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{5, x2, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x3, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{5, x4, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x5, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{5, x6, \{0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x7, \{0, 0, 0, 0, 0, 0, 0, 0\}\}\}
\end{aligned}$$

```

{5, x4, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {5,
x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {5, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{6,
x0, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x2,
{0, 0, 0, 0, 0, 0, 0, 0}}, {6, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x4, {0,
0, 0, 0, 0, 0, 0, 0}}, {6, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {6, x6, {0, 0, 0,
0, 0, 0, 0, 0}}, {6, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{7, x0, {0, 0, 0, 0,
0, 0, 0, 0}}, {7, x1, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x2, {0, 0, 0, 0, 0, 0,
0, 0}}, {7, x3, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x4, {0, 0, 0, 0, 0, 0, 0, 0},
{7, x5, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x6, {0, 0, 0, 0, 0, 0, 0, 0}}, {7, x7,
{0, 0, 0, 0, 0, 0, 0, 0}}}}}

In[8]:= Block[{eA $\alpha$ , e $\alpha$ A, t, ginv,  $\Gamma$ , gg(*,  $\omega\omega*$ )},
(* $\omega\omega=\omega_{\mu}^{(a)}(b)/.s\omega f;$ *)]
gg = g4488 /. sg;
{ginv,  $\Gamma$ } = con[gg];
e $\alpha$ A = e $^{\alpha}_{(A)}$  /. sge $^{\alpha}_{(A)}$ ;
eA $\alpha$  = e $_{\alpha}^{(A)}$  /. sge $_{\alpha}^{(A)}$ ;
t = Table[ Block[{v, V, contra, covari},
V = eA $\alpha$ [b, All];
v = e $\alpha$ A[All, b];
contra = Table[
D[v[k], X $[\mu]$ ] + Sum[ v[i]  $\times$   $\Gamma$ [k, i,  $\mu$ ],
{i, 1, DIM8}], {k, 1, DIM8}];
(*no  $\omega$  terms*)
covari = Table[D[V[k], X $[\mu]$ ] -
Sum[ V[i]  $\times$   $\Gamma$ [i, k,  $\mu$ ], {i, 1, DIM8}] +
Sum[ eA $\alpha$ [J, All][k]  $\times$  omega $[\mu, b, J]$ ,

```

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{J, 1, DIM8}],  

{k, 1, DIM8}]; (* $\omega$  terms*)  

{b - 1, X[\mu], {{contra}, {covari}}}],  

{b, 1, Length[e\alpha A]}, {\mu, 1, DIM8}];  

t]  

FullSimplify[#, ssym4488,  

constraintVars && e^{2 a4[H x4]} > 0 && e^{a4[H x4]} >  

0 && e^{-2 a4[H x4]} > 0 && e^{-a4[H x4]} > 0] & /@ %  

Out[=]=
{{{{0, x0, {{{0, 0, 0, 0,
- g[0][0]^(0,1)[x0, x4]
2 sqrt[g[0][0][x0, x4]] g[4][4][x0, x4],
0, 0, 0}}},  

{{0, 0, 0, 0, 0, 0, 0, 0}}}}, {{0, x1,
{{{0, 1, 0, 0, 0, 0, 0, 0,
g[1][1]^(1,0)[x0, x4]
2 sqrt[g[0][0][x0, x4]] g[1][1][x0, x4],
0, 0, 0, 0, 0, 0}}},  

{{0, 0, 0, 0, 0, 0, 0, 0}}}},  

{{0, x2, {{{0, 0,
g[2][2]^(1,0)[x0, x4]
2 sqrt[g[0][0][x0, x4]] g[2][2][x0, x4],
0, 0, 0, 0, 0, 0}}},  

{{0, 0, 0, 0, 0, 0, 0, 0}}}}}

```

$\{ \{ \{ \{ \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \} \} \},$
 $\{ \{ \{ \{ \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \} \} \},$
 $\{ 0, x3, \{ \{ \{ \{ 0, 0, 0,$
 $\frac{g[3][3]^{(1,0)} [x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[3][3][x0, x4]}, 0,$
 $0, 0, 0 \} \}, \{ \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \},$
 $\{ 0, x4, \{ \{ \{ \{ 0, 0, 0, 0,$
 $\frac{g[4][4]^{(1,0)} [x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]},$
 $0, 0, 0 \} \}, \{ \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \},$
 $\{ 0, x5, \{ \{ \{ \{ 0, 0, 0, 0, 0,$
 $\frac{g[5][5]^{(1,0)} [x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[5][5][x0, x4]},$
 $0, 0 \} \}, \{ \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \},$
 $\{ 0, x6, \{ \{ \{ \{ 0, 0, 0, 0, 0, 0,$
 $\frac{g[6][6]^{(1,0)} [x0, x4]}{2 \sqrt{g[0][0][x0, x4]} g[6][6][x0, x4]},$
 $0 \} \}, \{ \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \},$
 $\{ 0, x7, \{ \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0,$

$$\begin{aligned}
& \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2 \sqrt{g[0][0][x_0, x_4]} g[7][7][x_0, x_4]} \Big\} \Big\}, \\
& \left\{ \{0, 0, 0, 0, 0, 0, 0, 0\} \right\} \Big\}, \\
& \left\{ \{1, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \left. \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{1, x_1, \right. \\
& \left. \left\{ \left\{ - \frac{g[1][1]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{g[1][1][x_0, x_4]}} \right. \right. \right. , \\
& \left. \left. \left. 0, 0, 0, \right. \right. \right. \\
& \left. \left. \left. - \frac{g[1][1]^{(0,1)}[x_0, x_4]}{2 \sqrt{g[1][1][x_0, x_4]} g[4][4][x_0, x_4]} \right. \right. \right. , \\
& \left. \left. \left. 0, 0, 0\right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\right\}, \\
& \{1, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{1, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \{3, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \right. \\
& \quad \left. \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \right. \\
& \{3, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \right. \\
& \quad \left. \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \{3, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \right. \\
& \quad \left. \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \quad \left. \left\{ 3, x3, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{g[3][3][x0, x4]}}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 0, 0, 0, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 0, 0, 0 \right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\}, \right. \\
& \{3, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \right. \\
& \quad \left. \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \{3, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \right. \\
& \quad \left. \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \{3, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \right. \\
& \quad \left. \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \{3, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \right. \\
& \quad \left. \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \right. \\
& \quad \left. \left\{ 4, x0, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[4][4][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, \frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \right. \right. \\
& \quad \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]}, \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left\{ \left\{ 0, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]}} - \right. \right. \\
& \quad \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]} \right. \right. \\
& \quad \left. \left. - \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{g[4][4]^{(1,0)}[x0, x4]}{(2 \sqrt{-g[4][4][x0, x4]})} \right. \right. \right. \\
& \quad \left. \left. \left. g[4][4][x0, x4] \right) \right) \right. \\
& \quad \left. \left. \left. g[4][4][x0, x4], 0, 0, 0 \right\} \right\} \right\}, \left\{ 4, x1, \right. \\
& \quad \left\{ \left\{ 0, \frac{g[1][1]^{(0,1)}[x0, x4]}{2 g[1][1][x0, x4] \sqrt{-g[4][4][x0, x4]}}, \right. \right. \\
& \quad \left. \left. 0, 0, 0, 0, 0, 0 \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \frac{g[4][4]^{(0,1)}[x_0, x_4] \sqrt{-g[4][4][x_0, x_4]}}{2 g[4][4][x_0, x_4]} \\
& - \left(\frac{g[4][4]^{(0,1)}[x_0, x_4]}{2 (-g[4][4][x_0, x_4])^{3/2}} + \right. \\
& \quad \frac{g[4][4]^{(0,1)}[x_0, x_4]}{(2 \sqrt{-g[4][4][x_0, x_4]}} \\
& \quad \left. g[4][4][x_0, x_4]) \right) \\
& g[4][4][x_0, x_4], 0, 0, 0 \} \} \} \}, \\
& \{ 4, x_5, \{ \{ \{ 0, 0, 0, 0, 0, \\
& \quad \frac{g[5][5]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[5][5][x_0, x_4]}, \\
& \quad 0, 0 \} \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 4, x_6, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, \\
& \quad \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[6][6][x_0, x_4]}, \\
& \quad 0 \} \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 4, x_7, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0,
\end{aligned}$$

$$\begin{aligned}
& \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 \sqrt{-g[4][4][x_0, x_4]} g[7][7][x_0, x_4]} \Big\} \Big\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \Big\}, \\
& \left\{ \left\{ 5, x_0, \left\{ \left\{ 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. \left. \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. \left. \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} g[5][5][x_0, x_4]} \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. 0, 0 \right\} \right\}, \left\{ \left\{ 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. \left. - \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 \sqrt{-g[5][5][x_0, x_4]} - \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. \left. \frac{g[5][5]^{(1,0)}[x_0, x_4] \sqrt{-g[5][5][x_0, x_4]}}{2 g[5][5][x_0, x_4]} \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. - \left(\frac{g[5][5]^{(1,0)}[x_0, x_4]}{2 (-g[5][5][x_0, x_4])^{3/2}} + \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. \left. \frac{g[5][5]^{(1,0)}[x_0, x_4]}{(2 \sqrt{-g[5][5][x_0, x_4]} \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. g[5][5][x_0, x_4]) \right) \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left. g[5][5][x_0, x_4], 0, 0 \right\} \right\} \right\} \right\} \right\}, \\
& \{5, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
\end{aligned}$$

$$\begin{aligned}
& \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{5, x4, \\
& \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}} g[5][5][x0, x4] \right. \right. \right. , \\
& \quad \left. \left. \left. \left. 0, 0 \right\} \right\}, \left\{ \left\{ 0, 0, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}} - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2g[5][5][x0, x4]} \right. \right. \right. , \\
& \quad \left. \left. \left. - \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \right. \right. \right. \\
& \quad \left. \left. \left. g[5][5]^{(0,1)}[x0, x4] / \right. \right. \right. \\
& \quad \left. \left. \left. (2\sqrt{-g[5][5][x0, x4]} \right. \right. \right. \\
& \quad \left. \left. \left. g[5][5][x0, x4]) \right) \right. \right. \right. \\
& \quad \left. \left. \left. g[5][5][x0, x4], 0, 0 \right\} \right\} \right\}, \{5, x5,
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ \left\{ - \frac{g[5][5]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[5][5][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{g[5][5]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[5][5][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0, 0 \right\} \right\}, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\} \right\}, \\
& \{5, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \left\{ \left\{ 6, x0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. 0 \right\} \right\}, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. - \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} - \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \frac{g[6][6]^{(1,0)}[x0, x4]}{2 g[6][6][x0, x4]} \sqrt{-g[6][6][x0, x4]} \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. 2 g[6][6][x0, x4] \right. \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \right. \\
& \quad \frac{g[6][6]^{(1,0)}[x0, x4]}{(2 \sqrt{-g[6][6][x0, x4]}} \\
& \quad \left. g[6][6][x0, x4]) \right) \\
& g[6][6][x0, x4], 0 \} \} \} \}, \\
& \{ 6, x1, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 6, x2, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 6, x3, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 6, x4, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, \\
& \quad \frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \\
& \quad \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, \\
& \quad 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, \\
& \quad - \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} -
\end{aligned}$$

$$\begin{aligned}
& \frac{g[6][6]^{(0,1)}[x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}{2 g[6][6][x_0, x_4]} \\
& - \left(\frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 (-g[6][6][x_0, x_4])^{3/2}} + \right. \\
& \quad \frac{g[6][6]^{(0,1)}[x_0, x_4]}{(2 \sqrt{-g[6][6][x_0, x_4]}} \\
& \quad \left. g[6][6][x_0, x_4]) \right) \\
& g[6][6][x_0, x_4], 0 \} \} \} \}, \\
\{ & 6, x_5, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
\{ & 6, x_6, \\
& \left\{ \left\{ \left\{ - \frac{g[6][6]^{(1,0)}[x_0, x_4]}{2 g[0][0][x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}}, \right. \right. \right. \\
& 0, 0, 0, \\
& \left. \left. \left. - \frac{g[6][6]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[6][6][x_0, x_4]}} \right. \right. \right. \\
& 0, 0, 0 \} \}, \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
\{ & 6, x_7, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
\{ & \{ 7, x_0, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0,
\end{aligned}$$

$$\begin{aligned}
& \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2(-g[7][7][x_0, x_4])^{3/2}} + \\
& \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \Big\} \Big\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \\
& - \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]}} - \\
& \frac{g[7][7]^{(0,1)}[x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}{2g[7][7][x_0, x_4]} \\
& - \left(\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2(-g[7][7][x_0, x_4])^{3/2}} + \right. \\
& \left. \left. \frac{g[7][7]^{(0,1)}[x_0, x_4]}{(2\sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4])} \right) g[7][7][x_0, x_4] \right\} \Big\} \Big\}, \\
& \{7, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{7, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{7, x_7, \\
& \left\{ \left\{ - \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2g[0][0][x_0, x_4]\sqrt{-g[7][7][x_0, x_4]}} \right. \right. , \\
& 0, 0, 0,
\end{aligned}$$

$$-\frac{g[7][7]^{(0,1)}[x_0, x_4]}{2 g[4][4][x_0, x_4] \sqrt{-g[7][7][x_0, x_4]}}, \\ \{0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}\}$$

Out[=]=

$$\left\{ \begin{array}{l} \{ \{ 0, x_0, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, x_1, \{ \{ \{ 0, H, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, x_2, \{ \{ \{ 0, 0, H, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, x_3, \{ \{ \{ 0, 0, 0, H, 0, 0, 0, 0 \} \} \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, x_4, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, x_5, \{ \{ \{ 0, 0, 0, 0, 0, 0, H, 0, 0 \} \} \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, x_6, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, H, 0 \} \} \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, x_7, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, H \} \} \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 1, x_0, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\ \{ 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \{ 1, x_1, \end{array} \right.$$

$$\begin{aligned}
& \left\{ \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, \right. \right. \right. \\
& \quad 0, 0, 0, 3 \sqrt{e^{2 a4[H x4]}} H \\
& \quad \left. \left. \left. \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \quad \left\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \right\} \Big\}, \\
& \{1, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}}, \\
& \{1, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}}, \\
& \{1, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}}, \\
& \{1, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}}, \\
& \{1, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}}, \\
& \{1, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}}, \\
& \Big\{ \{2, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}}, \\
& \{2, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}}, \\
& \{2, x2, \left\{ \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\} \Big\}
\end{aligned}$$

$$\begin{aligned}
& \text{Sin}[6 H x_0]^{7/6}, 0, 0, 0, 0, 3 \sqrt{e^{2 a4[H x4]}} \\
& H \text{Sin}[6 H x_0]^{1/6} a4'[H x4], 0, 0, 0 \Big\} \Big\}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \Big\}, \\
& \{ 2, x3, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 2, x4, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 2, x5, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 2, x6, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 2, x7, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \Big\{ \{ 3, x0, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 3, x1, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 3, x2, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \{ 3, x3, \{ \{ \{ -\sqrt{e^{2 a4[H x4]}} H \text{Sec}[6 H x0] \\
& \text{Sin}[6 H x_0]^{7/6}, 0, 0, 0, 0, 3 \sqrt{e^{2 a4[H x4]}}
\end{aligned}$$

$$\begin{aligned}
& H \sin[6 H x_0]^{1/6} a4' [H x_4], 0, 0, 0 \Big\} \Big\}, \\
& \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \Big\}, \\
& \{ 3, x_4, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 3, x_5, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 3, x_6, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 3, x_7, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ \{ 4, x_0, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 4, x_1, \{ \{ \{ 0, 3 H a4' [H x_4], 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 4, x_2, \{ \{ \{ 0, 0, 3 H a4' [H x_4], 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 4, x_3, \{ \{ \{ 0, 0, 0, 3 H a4' [H x_4], 0, 0, 0, 0, 0 \} \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \\
& \{ 4, x_4, \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \}, \\
& \quad \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}, \{ 4, x_5, \\
& \quad \{ \{ \{ 0, 0, 0, 0, 0, -3 H a4' [H x_4], 0, 0 \} \} \}, \{ 4, x_6, \\
& \quad \{ \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \} \} \}
\end{aligned}$$

$$\begin{aligned}
& \{\{\{0, 0, 0, 0, 0, 0, -3 H a4'[H x4], 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{4, x7, \\
& \quad \{\{\{0, 0, 0, 0, 0, 0, 0, -3 H a4'[H x4]\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \left\{ \begin{array}{l} \{5, x0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{5, x5, \\
& \quad \left\{ \begin{array}{l} \left\{ \left\{ \sqrt{e^{-2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, \right. \right. \\
& \quad 0, 0, 0, 3 \sqrt{e^{-2 a4[H x4]}} H \\
& \quad \left. \left. \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \quad \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\},
\end{array} \right. \right. \\
\end{aligned}$$

In[•]:= T16^A[θ]

Out[•] =

In[•]:=

```
SAB = Table[ $\frac{1}{4}$ 
  (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]), 
  {A1, 0, 7}, {B1, 0, 7}];
```

In[•]:= Ψ16

Out[•] =

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
 f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}
```

```
In[6]:= Sum[ExpandAll[(wμ(a)(b) [4, a, b] /. swfgtry) Simplify[(SAB[a, b])]], {a, 1, 8}, {b, 1, 8}].
```

¶16 // MatrixForm

Out[•]//MatrixForm=

```

- ea4[H x4] H Sin[6 H x0]1/6 f16[5][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[4][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[7][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[6][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[1][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[0][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[3][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[2][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[13][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[12][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[15][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[14][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[9][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[8][x0, x4]
ea4[H x4] H Sin[6 H x0]1/6 f16[11][x0, x4]
- ea4[H x4] H Sin[6 H x0]1/6 f16[10][x0, x4]

```

```
In[8]:= (*Table[Block[{eA $\alpha$ , eA $A$ , t, V}, eA $A$ =e $^{\alpha}_{(A)}$  /.sgtrye $^{\alpha}_{(A)}$ ;
eA $\alpha$ =e $^{\alpha}_{\alpha}$  /.sgtrye $^{\alpha}_{\alpha}$ ;
V=eA $\alpha$ [[a,All]];
t=Table[Block[{v,d},v=eA $A$ [[All,b]];
d=D[v[[k]],X[[j]] ]+Sum[ v[[i]]  $\Gamma$ [[k,i,j]],{i,1,DIM8}];
d.V],{k,1,DIM8}];
t],[{j,1,DIM8},{a,1,Length[eA $A$ ]},{b,1,Length[eA $A$ ]})]*)
```

```
In[9]:= Clear[rt];
rt[g_, ass_ : {}] :=
Module[{t,  $\Gamma$ , Ricci $\Gamma$ , Rie $\Gamma$ , RS, G, ginv},
Print[Now];
t = AbsoluteTiming[
ginv = FullSimplify[Inverse[g], ass];
 $\Gamma$  = FullSimplify[
Table[(1/2) * Sum[(ginv[[i, s]) * (D[g[[s, j]], X[[k]] +
D[g[[s, k]], X[[j]] ] - D[g[[j, k]],
X[[s]]]), {s, 1, DIM8}],
{i, 1, DIM8},
{j, 1, DIM8}, {k, 1, DIM8}], ass];
Ricci $\Gamma$  = ParallelTable[
FullSimplify[D[ $\Gamma$ [[ $\mu$ ,  $\nu$ ,  $\beta$ ]], X[[ $\alpha$ ]] -
D[ $\Gamma$ [[ $\mu$ ,  $\nu$ ,  $\alpha$ ]], X[[ $\beta$ ]] + Sum[
 $\Gamma$ [[ $\mu$ , s,  $\alpha$ ]  $\times$   $\Gamma$ [[s,  $\nu$ ,  $\beta$ ] -  $\Gamma$ [[ $\mu$ , s,  $\beta$ ]  $\times$ 
 $\Gamma$ [[s,  $\nu$ ,  $\alpha$ ]], {s, 1, DIM8}], ass],
{ $\mu$ , 1, DIM8}, { $\nu$ , 1, DIM8},
```

```

{α, 1, DIM8}, {β, 1, DIM8}] ;
RieΓ = ParallelTable[ FullSimplify[
Sum[ RicciΓ[α, μ, α, ν], {α, 1, DIM8}] ,
ass], {μ, 1, DIM8}, {ν, 1, DIM8}] ;
RS = FullSimplify[Tr[ ginv.RieΓ] , ass];
G = ParallelTable[
FullSimplify[RieΓ[α] -  $\frac{1}{2}$  g[α] RS, ass],
{α, 1, Length[RieΓ]}];];
Print[t];
Print[Now];
Return[{ginv, Γ, RicciΓ, RieΓ, RS, G}] ]

```

verify $g_{\mu\nu|\alpha} = 0$:

```

In[6]:= Simplify[Table[D[gtry[[j, k]], X[[i]]] -
Sum[gtry[[k, s]] X[[s, i, j]], {s, 1, DIM8}] - Sum[gtry[[j, s]] X[[s, i, k]], {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}] ] // Flatten // Union

```

Out[6]=

{0}

▪ CHECK

```
In[7]:= MatrixMetric44 // MatrixForm
```

```

Out[7]//MatrixForm=

$$\begin{pmatrix} \cot[6Hx\theta]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a4[Hx4]} \sin[6Hx\theta]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a4[Hx4]} \sin[6Hx\theta]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a4[Hx4]} \sin[6Hx\theta]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e^{-2a4[Hx4]} S \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


```

```

In[1]:=  $(e_{(A)}^{\alpha} \cdot \text{MatrixMetric44} \cdot e_{(A)}^{\alpha}) / . \text{sgtrye}_{(A)}^{\alpha}) // \text{MatrixForm}$ 

Out[1]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


In[2]:=  $\text{Transpose}[e_{\alpha}^{(A)}] \cdot \eta4488 \cdot e_{\alpha}^{(A)} - \text{MatrixMetric44} / . \text{sgtrye}_{\alpha}^{(A)} // \text{FullSimplify}[\#, \text{constraintVars}] &$ 

Out[2]=
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
In[3]:=  $\text{Transpose}[e_{(A)}^{\alpha}] \cdot \text{MatrixMetric44} \cdot e_{(A)}^{\alpha} - \eta4488 / . \text{sgtrye}_{(A)}^{\alpha} // \text{FullSimplify}[\#, \text{constraintVars}] &$ 

Out[3]=
{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
In[4]:=  $\text{Inverse}[e_{(A)}^{\alpha} / . \text{sgtrye}_{(A)}^{\alpha}] = e_{\alpha}^{(A)} / . \text{sgtrye}_{\alpha}^{(A)} // \text{FullSimplify}[\#, \text{constraintVars}] &$ 

Out[4]=
True

In[5]:=  $\text{Block}[\{a, b, c, d, m\}, m = \{ \{a, b\}, \{c, d\} \}; \text{Inverse}[\text{Transpose}[m]] - \text{Transpose}[\text{Inverse}[m]]]$ 

Out[5]=
{{0, 0}, {0, 0}}
In[6]:= (*Inverse[Transpose[e_{(A)}^{\alpha}]] * MatrixMetric44 * e_{(A)}^{\alpha} == e_{\alpha}^{(A)} * Inverse[MatrixMetric44] * Transpose[e_{\alpha}^{(A)}] // FullSimplify[#, constraintVars] &*)

In[7]:= (*e_{\alpha}^{(A)} * Inverse[MatrixMetric44] * Transpose[e_{\alpha}^{(A)}] // FullSimplify[#, constraintVars] &
        %-Inverse[\eta4488] // Union[Flatten[\#]] &*)

In[8]:= (*Transpose[e_{\alpha}^{(A)}] == e_{\alpha}^{(A)} // FullSimplify[\#, constraintVars] &*)

In[9]:= (*FullSimplify[e_{\alpha}^{(A)} * Inverse[MatrixMetric44] * Transpose[e_{\alpha}^{(A)}] - Inverse[\eta4488], constraintVars] // Union[Flatten[\#]] &) // FullSimplify[\#, constraintVars] &*)

In[10]:= (*FullSimplify[Transpose[e_{\alpha}^{(A)}] * Inverse[MatrixMetric44] * e_{\alpha}^{(A)}, constraintVars] - Inverse[\eta4488] // Union[Flatten[\#]] &) // FullSimplify[\#, constraintVars] &*)

In[11]:= (*FullSimplify[Transpose[e_{\alpha}^{(A)}] * ginv[e_{\alpha}^{(A)}, constraintVars] - Inverse[\eta4488] // Union[Flatten[\#]] &) // FullSimplify[\#, constraintVars] &*)

In[12]:=  $(E_{(A)}^{\alpha} = \text{Table}[\text{Sum}[e_{\alpha}^{(A)} \cdot A1, \alpha1] \cdot \text{ginv}[\alpha1, \beta1] \times$ 
 $\eta4488 \cdot A1, B1] / . \text{sgtrye}_{\alpha}^{(A)},$ 

```

```
{\alpha1, 1, 8}, {A1, 1, 8}], {\beta1, 1, 8},
{B1, 1, 8}] // FullSimplify[#, constraintVars] &) // Column
```

Out[=]

$$\begin{aligned} & \{\tan[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0\} \\ & \left\{0, \frac{e^{-a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0\right\} \\ & \left\{0, 0, \frac{e^{-a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0\right\} \\ & \left\{0, 0, 0, \frac{e^{-a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0\right\} \\ & \{0, 0, 0, 0, 1, 0, 0, 0\} \\ & \left\{0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}, 0, 0\right\} \\ & \left\{0, 0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}, 0\right\} \\ & \left\{0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}\right\} \end{aligned}$$
Lagrangian Ψ_{16} =
$$\begin{aligned} & \kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi_{16}] . \sigma_{16} . T_{16}^A[A1]. \\ & \Psi_{16,\alpha} g^{-1}{}^{\alpha\beta} \eta_{A1 B1} E_B^B + \text{mass term} \end{aligned}$$

In[=]:= ass = constraintVars

Out[=]=

$$\begin{aligned} & x0 > 0 \&& x4 > 0 \&& H > 0 \&& 6Hx0 > 0 \&& 3Hx4 > 0 \&& a4[x4] > 0 \&& A4[t] > 0 \&& \\ & Q > 0 \&& z > 0 \&& t > 0 \&& M > 0 \&& K > 0 \&& e^{-2a4[Hx4]} > 0 \&& e^{-a4[Hx4]} > 0 \&& e^{2a4[Hx4]} > 0 \&& \\ & e^{a4[Hx4]} > 0 \&& \sin[6Hx0] > 0 \&& \cos[6Hx0] > 0 \&& \csc[6Hx0] > 0 \&& \sec[6Hx0] > 0 \&& \\ & \tan[6Hx0] > 0 \&& \cot[6Hx0] > 0 \&& \sin[z] > 0 \&& \sqrt{\sin[z]} > 0 \&& \sin[z]^{3/2} > 0 \&& \\ & \sqrt{\sin[z]} > 0 \&& \frac{1}{\sin[z]^{3/2}} > 0 \&& \frac{1}{\sqrt{\sin[z]}} > 0 \&& \cot[z] > 0 \&& \sqrt{\cot[z]} > 0 \&& \\ & \cot[z]^{3/2} > 0 \&& \sqrt{\cot[z]} > 0 \&& \tan[z] > 0 \&& \sec[z] > 0 \&& \sqrt{\csc[z]} > 0 \&& \csc[z] > 0 \end{aligned}$$

```

In[]:= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]3/2 > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]3/2 > 0 && 1/Sqrt[Sin[z]] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]3/2 > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0

Out[]=
x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e-2 a4[H x4] > 0 && e-a4[H x4] > 0 && e2 a4[H x4] > 0 &&
ea4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]3/2 > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]3/2 > 0 && 1/Sqrt[Sin[z]] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]3/2 > 0 && Sqrt[Cot[z]] > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0

In[]:= ginv = FullSimplify[Inverse[gtry], ass]

Out[]=
True

In[]:= ginv == Transpose[ginv]

Out[]=
True

```

verify metric covariant derivative again:

```

In[]:= Simplify[Table[D[gtry[[j, k]], X[[i]]] -
    Sum[gtry[[k, s]] \[Cross] \[Gamma][s, i, j], {s, 1, DIM8}] - Sum[gtry[[j, s]] \[Cross] \[Gamma][s, i, k], {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union

```

```

Out[]=
{0}

In[]:= RS

Out[]=
6 H2 (-7 + a4'[H x4]2)

```

```
In[•]:= EinsteinG
skelx[%]

Out[•]=
{ {-3 H^2 Cot[6 H x0]^2 (-5 + a4'[H x4]^2), 0, 0, 0, 0, 0, 0, 0, 0}, {0, -e^2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 0, -e^2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -e^2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 - a4''[H x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -3 H^2 (7 + a4'[H x4]^2), 0, 0, 0, 0}, {0, 0, 0, 0, 0, e^-2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4]), 0, 0, 0}, {0, 0, 0, 0, 0, 0, e^-2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4]), 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, e^-2 a4[H x4] H^2 Sin[6 H x0]^{1/3} (-15 + 3 a4'[H x4]^2 + a4''[H x4])} }

Out[•]//MatrixForm=

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{pmatrix}$$

```

```
Union[Table[
Sum[\tau[A].bas64[k, 1].\sigma.\bar{\tau}[A] \times \eta8[A, A], {A,
1, 8}] - ID8 Tr[bas64[k, 1].\sigma] + 4 eg[k]
bas64[k, 1] - Transpose[bas64[k, 1]]], {\sigma,
2
{k, 1, 64}}]]
```

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}
```

BASIS of 16 × 16 matrices :

```
In[•]:= 16 * 16
```

```
Out[•]=
256
```

```
In[]:= 28
Out[]:= 256
In[]:= Sum[Binomial[8, p], {p, 0, 8}]
Out[]:= 256
In[]:= Id16 == ID16 == Dot @@ (T16A[#] & /@ Range[0, 8])
Out[]:= True
```

```

In[6]:= Clear[t160]; t160 = {{Id16, {Range[0, 8]}}}; Length[t160]
Clear[t16A];
t16A = {};
Do[AppendTo[t16A, {T16^A[j], {j}}], {j, 0, 7}];
Length[t16A]
Clear[t16AB];
t16AB = {};
Do[AppendTo[t16AB, {T16^A[j].T16^A[k], {j, k}}], {j, 0, 6}, {k, j + 1, 7}];
Length[t16AB]
Clear[t16ABC];
t16ABC = {};
Do[AppendTo[t16ABC, {T16^A[j].T16^A[k].T16^A[h], {j, k, h}}], {j, 0, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[t16ABC]
Clear[t16ABCD];
t16ABCD = {};
Do[AppendTo[t16ABCD, {T16^A[j].T16^A[k].T16^A[h].T16^A[i], {j, k, h, i}}], {j, 0, 4}, {k, j + 1, 5}, {h, k + 1, 6}, {i, h + 1, 7}];
Length[t16ABCD]
Clear[t16ABCDE];
t16ABCDE = {};
Do[AppendTo[t16ABCDE, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l], {j, k, h, i, l}}], {j, 0, 3}, {k, j + 1, 4}, {h, k + 1, 5}, {i, h + 1, 6}, {l, i + 1, 7}];
Length[t16ABCDE]
Clear[t16ABCDEF];
t16ABCDEF = {};
Do[AppendTo[t16ABCDEF, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q], {j, k, h, i, l, q}}], {j, 0, 2}, {k, j + 1, 3}, {h, k + 1, 4}, {i, h + 1, 5}, {l, i + 1, 6}, {q, l + 1, 7}];
Length[t16ABCDEF]
Clear[t16ABCDEFG];
t16ABCDEFG = {};
Do[AppendTo[t16ABCDEFG, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].T16^A[r], {j, k, h, i, l, q, r}}], {j, 0, 1}, {k, j + 1, 2}, {h, k + 1, 3}, {i, h + 1, 4}, {l, i + 1, 5}, {q, l + 1, 6}, {r, q + 1, 7}];
Length[t16ABCDEFG]
Clear[t16ABCDEFGH];
t16ABCDEFGH = {};
Do[AppendTo[t16ABCDEFGH, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].T16^A[r].T16^A[s], {j, k, h, i, l, q, r, s}}], {j, 0, 1}, {k, j + 1, 1}, {h, k + 1, 2}, {i, h + 1, 3}, {l, i + 1, 4}, {q, l + 1, 5}, {r, q + 1, 6}, {s, r + 1, 7}];
Length[t16ABCDEFGH]
% + % + %% + %%% + %%%% + %%%%% + %%%%%% + %%%%%%% + %%%%%%%%

```

Out[•]=

1

```

Out[=]=
8

Out[=]=
28

Out[=]=
56

Out[=]=
70

Out[=]=
56

Out[=]=
28

Out[=]=
8

Out[=]=
1

Out[=]=
256

In[=]:= (*base16=Flatten[
  {t160,t16A,t16AB,t16ABC,t16ABCD,t16ABCDE,t16ABCDEF,t16ABCDEFG,t16ABCDEFGH},1];
Length[base16]*)

In[=]:= base16 = Flatten[
  {t16A, t16AB, t16ABC, t16ABCD, t16ABCDE, t16ABCDEF, t16ABCDEFG, t16ABCDEFGH, t160}, 1];
Length[base16]

Out[=]=
256

In[=]:= Length[Union[base16[[All, 1]]]]
Out[=]=
256

In[=]:= Tr[#.#/16] & /@ base16[[All, 1]] // Flatten // Union
Out[=]=
{-1, 1}

In[=]:= positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]
Out[=]=
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44,
45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91,
92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121,
122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151,
152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185,
186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229,
230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

```


0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-1	0
0	0	0	0	0	0	0	0	0	0	0	0	-1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	-1
0	0	0	0	0	0	0	0	0	0	0	0	0	-1
-1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0

In[•]:= (*BASE16[-1]*)

In[•]:= (*BASE16[]-2]

%[1]==BASE16[-1][1]*)


```
In[1]:= Clear[symmm16];
symmm16 = {};
Do[If[base16[[k, 1]] === Transpose[base16[[k, 1]]],
AppendTo[symmm16, {base16[[k, 1]], {{k}, base16[[k, 2]]}}]], {k, 1, Length[base16]}];
Length[symmm16]
symmm16[[%]]
```

Out[1]= 136

```
Out[2]= {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}
```

```
In[3]:= dupssymmm16 = Select[symmm16, MemberQ[symmm16[[All, 1]], -#[[1]]] &]; Length[dupssymmm16]
```

Out[3]= 0

```
In[]:= #[[2]] & /@ symmm16
Out[]=
{{{{1}, {0}}, {{2}, {1}}, {{3}, {2}}, {{4}, {3}}, {{12}, {0, 4}}, {{13}, {0, 5}}, {{14}, {0, 6}}, {{15}, {0, 7}}, {{18}, {1, 4}}, {{19}, {1, 5}}, {{20}, {1, 6}}, {{21}, {1, 7}}, {{23}, {2, 4}}, {{24}, {2, 5}}, {{25}, {2, 6}}, {{26}, {2, 7}}, {{27}, {3, 4}}, {{28}, {3, 5}}, {{29}, {3, 6}}, {{30}, {3, 7}}, {{39}, {0, 1, 4}}, {{40}, {0, 1, 5}}, {{41}, {0, 1, 6}}, {{42}, {0, 1, 7}}, {{44}, {0, 2, 4}}, {{45}, {0, 2, 5}}, {{46}, {0, 2, 6}}, {{47}, {0, 2, 7}}, {{48}, {0, 3, 4}}, {{49}, {0, 3, 5}}, {{50}, {0, 3, 6}}, {{51}, {0, 3, 7}}, {{59}, {1, 2, 4}}, {{60}, {1, 2, 5}}, {{61}, {1, 2, 6}}, {{62}, {1, 2, 7}}, {{63}, {1, 3, 4}}, {{64}, {1, 3, 5}}, {{65}, {1, 3, 6}}, {{66}, {1, 3, 7}}, {{73}, {2, 3, 4}}, {{74}, {2, 3, 5}}, {{75}, {2, 3, 6}}, {{76}, {2, 3, 7}}, {{89}, {4, 5, 6}}, {{90}, {4, 5, 7}}, {{91}, {4, 6, 7}}, {{92}, {5, 6, 7}}, {{93}, {0, 1, 2, 3}}, {{102}, {0, 1, 4, 5}}, {{103}, {0, 1, 4, 6}}, {{104}, {0, 1, 4, 7}}, {{105}, {0, 1, 5, 6}}, {{106}, {0, 1, 5, 7}}, {{107}, {0, 1, 6, 7}}, {{112}, {0, 2, 4, 5}}, {{113}, {0, 2, 4, 6}}, {{114}, {0, 2, 4, 7}}, {{115}, {0, 2, 5, 6}}, {{116}, {0, 2, 5, 7}}, {{117}, {0, 2, 6, 7}}, {{118}, {0, 3, 4, 5}}, {{119}, {0, 3, 4, 6}}, {{120}, {0, 3, 4, 7}}, {{121}, {0, 3, 5, 6}}, {{122}, {0, 3, 5, 7}}, {{123}, {0, 3, 6, 7}}, {{132}, {1, 2, 4, 5}}, {{133}, {1, 2, 4, 6}}, {{134}, {1, 2, 4, 7}}, {{135}, {1, 2, 5, 6}}, {{136}, {1, 2, 5, 7}}, {{137}, {1, 2, 6, 7}}, {{138}, {1, 3, 4, 5}}, {{139}, {1, 3, 4, 6}}, {{140}, {1, 3, 4, 7}}, {{141}, {1, 3, 5, 6}}, {{142}, {1, 3, 5, 7}}, {{143}, {1, 3, 6, 7}}, {{148}, {2, 3, 4, 5}}, {{149}, {2, 3, 4, 6}}, {{150}, {2, 3, 4, 7}}, {{151}, {2, 3, 5, 6}}, {{152}, {2, 3, 5, 7}}, {{153}, {2, 3, 6, 7}}, {{162}, {4, 5, 6, 7}}, {{167}, {0, 1, 2, 4, 5}}, {{168}, {0, 1, 2, 4, 6}}, {{169}, {0, 1, 2, 4, 7}}, {{170}, {0, 1, 2, 5, 6}}, {{171}, {0, 1, 2, 5, 7}}, {{172}, {0, 1, 2, 6, 7}}, {{173}, {0, 1, 3, 4, 5}}, {{174}, {0, 1, 3, 4, 6}}, {{175}, {0, 1, 3, 4, 7}}, {{176}, {0, 1, 3, 5, 6}}, {{177}, {0, 1, 3, 5, 7}}, {{178}, {0, 1, 3, 6, 7}}, {{183}, {0, 2, 3, 4, 5}}, {{184}, {0, 2, 3, 4, 6}}, {{185}, {0, 2, 3, 4, 7}}, {{186}, {0, 2, 3, 5, 6}}, {{187}, {0, 2, 3, 5, 7}}, {{188}, {0, 2, 3, 6, 7}}, {{197}, {0, 4, 5, 6, 7}}, {{198}, {1, 2, 3, 4, 5}}, {{199}, {1, 2, 3, 4, 6}}, {{200}, {1, 2, 3, 4, 7}}, {{201}, {1, 2, 3, 5, 6}}, {{202}, {1, 2, 3, 5, 7}}, {{203}, {1, 2, 3, 6, 7}}, {{212}, {1, 4, 5, 6, 7}}, {{217}, {2, 4, 5, 6, 7}}, {{218}, {3, 4, 5, 6, 7}}, {{225}, {0, 1, 2, 4, 5, 6}}, {{226}, {0, 1, 2, 4, 5, 7}}, {{227}, {0, 1, 2, 4, 6, 7}}, {{228}, {0, 1, 2, 5, 6, 7}}, {{229}, {0, 1, 3, 4, 5, 6}}, {{230}, {0, 1, 3, 4, 5, 7}}, {{231}, {0, 1, 3, 4, 6, 7}}, {{232}, {0, 1, 3, 5, 6, 7}}, {{234}, {0, 2, 3, 4, 5, 6}}, {{235}, {0, 2, 3, 4, 5, 7}}, {{236}, {0, 2, 3, 4, 6, 7}}, {{237}, {0, 2, 3, 5, 6, 7}}, {{240}, {1, 2, 3, 4, 5, 6}}, {{241}, {1, 2, 3, 4, 5, 7}}, {{242}, {1, 2, 3, 4, 6, 7}}, {{243}, {1, 2, 3, 5, 6, 7}}, {{247}, {0, 1, 2, 3, 4, 5, 6}}, {{248}, {0, 1, 2, 3, 4, 5, 7}}, {{249}, {0, 1, 2, 3, 4, 6, 7}}, {{250}, {0, 1, 2, 3, 5, 6, 7}}, {{255}, {0, 1, 2, 3, 4, 5, 6, 7}}, {{256}, {0, 1, 2, 3, 4, 5, 6, 7, 8}}}

In[]:= σ16 = T16^A[0].T16^A[1].T16^A[2].T16^A[3]
Out[]=
True

In[]:= σ16 == base16[[93]][1]
Out[]=
True
```



```
In[•]:= linTrans.Transpose[linTrans] // FullSimplify // MatrixForm
```

```

In[1]:= linTrans.¶16 // Column
Union[%[[1]]]

Out[1]=
f16[0] [x0, x4]
f16[1] [x0, x4]
f16[8] [x0, x4]
f16[9] [x0, x4]
f16[2] [x0, x4]
f16[3] [x0, x4]
f16[10] [x0, x4]
f16[11] [x0, x4]
f16[4] [x0, x4]
f16[5] [x0, x4]
f16[12] [x0, x4]
f16[13] [x0, x4]
f16[6] [x0, x4]
f16[7] [x0, x4]
f16[14] [x0, x4]
f16[15] [x0, x4]

Out[2]=
{f16[0] [x0, x4], f16[1] [x0, x4], f16[2] [x0, x4], f16[3] [x0, x4],
 f16[4] [x0, x4], f16[5] [x0, x4], f16[6] [x0, x4], f16[7] [x0, x4],
 f16[8] [x0, x4], f16[9] [x0, x4], f16[10] [x0, x4], f16[11] [x0, x4],
 f16[12] [x0, x4], f16[13] [x0, x4], f16[14] [x0, x4], f16[15] [x0, x4]}

In[3]:= positiveTrMM = Select[base16, Tr[#[[1]] . #[[1]]] > 0 & ->
 "Index"]

Out[3]=
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29,
 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63,
 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105,
 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122,
 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143,
 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172,
 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197,
 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228,
 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247,
 248, 249, 250, 255, 256}

In[4]:= negativeTrMM = Select[base16, Tr[#[[1]] . #[[1]]] < 0 & ->
 "Index"]

Out[4]=
{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43,
 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81}

```

82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[]:= {Length[positiveTrMM], Length[negativeTrMM]}

Out[]=

{136, 120}

In[]:= positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]

Out[]=

{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[]:= negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]

Out[]=

{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[]:= {Length[positiveTrMM], Length[negativeTrMM]}

Out[]=

{136, 120}

```

In[=]:= base16[[#]] & /@ positiveTrMM;
{#[[2]], Tr[Transpose[#[[1]].linTrans]]} & /@ %
Select[% , #[[2]] ≠ 0 &]
linTransSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];

Out[=]=
{{{{0}, 0}, {{1}, 0}, {{2}, 0}, {{3}, 0}, {{0, 4}, 0}, {{0, 5}, 0}, {{0, 6}, 0}, {{0, 7}, -4},
{{1, 4}, 0}, {{1, 5}, 0}, {{1, 6}, 0}, {{1, 7}, 0}, {{2, 4}, 0}, {{2, 5}, 0}, {{2, 6}, 0},
{{2, 7}, 0}, {{3, 4}, -4}, {{3, 5}, 0}, {{3, 6}, 0}, {{3, 7}, 0}, {{0, 1, 4}, 0},
{{0, 1, 5}, 0}, {{0, 1, 6}, 0}, {{0, 1, 7}, 0}, {{0, 2, 4}, 0}, {{0, 2, 5}, 0}, {{0, 2, 6}, 0},
{{0, 2, 7}, 0}, {{0, 3, 4}, 0}, {{0, 3, 5}, 0}, {{0, 3, 6}, 0}, {{0, 3, 7}, 0},
{{1, 2, 4}, 0}, {{1, 2, 5}, 0}, {{1, 2, 6}, 0}, {{1, 2, 7}, 0}, {{1, 3, 4}, 0},
{{1, 3, 5}, 0}, {{1, 3, 6}, 0}, {{1, 3, 7}, 0}, {{2, 3, 4}, 0}, {{2, 3, 5}, 0},
{{2, 3, 6}, 0}, {{2, 3, 7}, 0}, {{4, 5, 6}, -4}, {{4, 5, 7}, 0}, {{4, 6, 7}, 0},
{{5, 6, 7}, 0}, {{0, 1, 2, 3}, 0}, {{0, 1, 4, 5}, 0}, {{0, 1, 4, 6}, 0}, {{0, 1, 4, 7}, 0},
{{0, 1, 5, 6}, 0}, {{0, 1, 5, 7}, 0}, {{0, 1, 6, 7}, 0}, {{0, 2, 4, 5}, 0}, {{0, 2, 4, 6}, 0},
{{0, 2, 4, 7}, 0}, {{0, 2, 5, 6}, 0}, {{0, 2, 5, 7}, 0}, {{0, 2, 6, 7}, 0}, {{0, 3, 4, 5}, 0},
{{0, 3, 4, 6}, 0}, {{0, 3, 4, 7}, 4}, {{0, 3, 5, 6}, 0}, {{0, 3, 5, 7}, 0}, {{0, 3, 6, 7}, 0},
{{1, 2, 4, 5}, 0}, {{1, 2, 4, 6}, 0}, {{1, 2, 4, 7}, 0}, {{1, 2, 5, 6}, 0}, {{1, 2, 5, 7}, 0},
{{1, 2, 6, 7}, -4}, {{1, 3, 4, 5}, 0}, {{1, 3, 4, 6}, 0}, {{1, 3, 4, 7}, 0}, {{1, 3, 5, 6}, 0},
{{1, 3, 5, 7}, 0}, {{1, 3, 6, 7}, 0}, {{2, 3, 4, 5}, 0}, {{2, 3, 4, 6}, 0}, {{2, 3, 4, 7}, 0},
{{2, 3, 5, 6}, 0}, {{2, 3, 5, 7}, 0}, {{2, 3, 6, 7}, 0}, {{4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5}, 4},
{{0, 1, 2, 4, 6}, 0}, {{0, 1, 2, 4, 7}, 0}, {{0, 1, 2, 5, 6}, 0}, {{0, 1, 2, 5, 7}, 0},
{{0, 1, 2, 6, 7}, 0}, {{0, 1, 3, 4, 5}, 0}, {{0, 1, 3, 4, 6}, 0}, {{0, 1, 3, 4, 7}, 0},
{{0, 1, 3, 5, 6}, 0}, {{0, 1, 3, 5, 7}, 0}, {{0, 1, 3, 6, 7}, 0}, {{0, 2, 3, 4, 5}, 0},
{{0, 2, 3, 4, 6}, 0}, {{0, 2, 3, 4, 7}, 0}, {{0, 2, 3, 5, 6}, 0}, {{0, 2, 3, 5, 7}, 0},
{{0, 2, 3, 6, 7}, 0}, {{0, 4, 5, 6, 7}, 4}, {{1, 2, 3, 4, 5}, 0}, {{1, 2, 3, 4, 6}, 0},
{{1, 2, 3, 4, 7}, 0}, {{1, 2, 3, 5, 6}, 0}, {{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 6, 7}, 0},
{{1, 4, 5, 6, 7}, 0}, {{2, 4, 5, 6, 7}, 0}, {{3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6}, 0},
{{0, 1, 2, 4, 5, 7}, 0}, {{0, 1, 2, 4, 6, 7}, 0}, {{0, 1, 3, 4, 5, 6}, 0}, {{0, 1, 3, 4, 5, 7}, 0},
{{0, 1, 3, 4, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6}, 0}, {{0, 2, 3, 4, 5, 7}, 0}, {{0, 2, 3, 4, 6, 7}, 0},
{{0, 1, 2, 3, 4, 5, 6}, 0}, {{0, 1, 2, 3, 4, 5, 7}, 0}, {{0, 1, 2, 3, 4, 6, 7}, 0},
{{0, 1, 2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 7, 8}, 4}}

```



```

In[]:= base16[[#]] & /@ negativeTrMM;
{#[[2]], Tr[Transpose[#[[1]].linTrans]]} & /@ %
Select[% , #[[2]] ≠ 0 &]
linTransAntiSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];

Out[]=
{{{4}, 0}, {{5}, 0}, {{6}, 0}, {{7}, 0}, {{0, 1}, 0}, {{0, 2}, 0}, {{0, 3}, 0}, {{1, 2}, 0},
{{1, 3}, 0}, {{2, 3}, 0}, {{4, 5}, 0}, {{4, 6}, 0}, {{4, 7}, 0}, {{5, 6}, 0}, {{5, 7}, 0},
{{6, 7}, 0}, {{0, 1, 2}, 0}, {{0, 1, 3}, 0}, {{0, 2, 3}, 0}, {{0, 4, 5}, 0}, {{0, 4, 6}, 0},
{{0, 4, 7}, 0}, {{0, 5, 6}, 0}, {{0, 5, 7}, 0}, {{0, 6, 7}, 0}, {{1, 2, 3}, 0}, {{1, 4, 5}, 0},
{{1, 4, 6}, 0}, {{1, 4, 7}, 0}, {{1, 5, 6}, 0}, {{1, 5, 7}, 0}, {{1, 6, 7}, 0},
{{2, 4, 5}, 0}, {{2, 4, 6}, 0}, {{2, 4, 7}, 0}, {{2, 5, 6}, 0}, {{2, 5, 7}, 0},
{{2, 6, 7}, 0}, {{3, 4, 5}, 0}, {{3, 4, 6}, 0}, {{3, 4, 7}, 0}, {{3, 5, 6}, -4},
{{3, 5, 7}, 0}, {{3, 6, 7}, 0}, {{0, 1, 2, 4}, 0}, {{0, 1, 2, 5}, 0}, {{0, 1, 2, 6}, -4},
{{0, 1, 2, 7}, 0}, {{0, 1, 3, 4}, 0}, {{0, 1, 3, 5}, 0}, {{0, 1, 3, 6}, 0}, {{0, 1, 3, 7}, 0},
{{0, 2, 3, 4}, 0}, {{0, 2, 3, 5}, 0}, {{0, 2, 3, 6}, 0}, {{0, 2, 3, 7}, 0}, {{0, 4, 5, 6}, 0},
{{0, 4, 5, 7}, 0}, {{0, 4, 6, 7}, 0}, {{0, 5, 6, 7}, 0}, {{1, 2, 3, 4}, 0}, {{1, 2, 3, 5}, 0},
{{1, 2, 3, 6}, 0}, {{1, 2, 3, 7}, 0}, {{1, 4, 5, 6}, 0}, {{1, 4, 5, 7}, 0}, {{1, 4, 6, 7}, 0},
{{1, 5, 6, 7}, 0}, {{2, 4, 5, 6}, 0}, {{2, 4, 5, 7}, 0}, {{2, 4, 6, 7}, 0}, {{2, 5, 6, 7}, 0},
{{3, 4, 5, 6}, 0}, {{3, 4, 5, 7}, 0}, {{3, 4, 6, 7}, 0}, {{3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4}, 0},
{{0, 1, 2, 3, 5}, -4}, {{0, 1, 2, 3, 6}, 0}, {{0, 1, 2, 3, 7}, 0}, {{0, 1, 4, 5, 6}, 0},
{{0, 1, 4, 5, 7}, 0}, {{0, 1, 4, 6, 7}, 0}, {{0, 1, 5, 6, 7}, 0}, {{0, 2, 4, 5, 6}, 0},
{{0, 2, 4, 5, 7}, 0}, {{0, 2, 4, 6, 7}, 0}, {{0, 2, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6}, 0},
{{0, 3, 4, 5, 7}, 0}, {{0, 3, 4, 6, 7}, 0}, {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 6}, 0},
{{1, 2, 4, 5, 7}, 4}, {{1, 2, 4, 6, 7}, 0}, {{1, 2, 5, 6, 7}, 0}, {{1, 3, 4, 5, 6}, 0},
{{1, 3, 4, 5, 7}, 0}, {{1, 3, 4, 6, 7}, 0}, {{1, 3, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6}, 0},
{{2, 3, 4, 5, 7}, 0}, {{2, 3, 4, 6, 7}, 0}, {{2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5}, 0},
{{0, 1, 2, 3, 4, 6}, -4}, {{0, 1, 2, 3, 4, 7}, 0}, {{0, 1, 2, 3, 5, 6}, 0},
{{0, 1, 2, 3, 5, 7}, 0}, {{0, 1, 2, 3, 6, 7}, 0}, {{0, 1, 4, 5, 6, 7}, 0},
{{0, 2, 4, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6, 7}, 0}, {{1, 2, 4, 5, 6, 7}, 0},
{{1, 3, 4, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6, 7}, 0},
{{0, 1, 3, 4, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6, 7}, 0}
}

Out[]=
{{{3, 5, 6}, -4}, {{0, 1, 2, 6}, -4}, {{0, 1, 2, 3, 5}, -4},
{{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 7}, 4}, {{0, 1, 2, 3, 4, 6}, -4}}
}

In[]:= {Length[linTransSymmetric], Length[linTransAntiSymmetric]}

Out[=
{10, 6}
]

```

BASIS of 8×8 matrices :

```
In[1]:= tA = {}; Do[AppendTo[tA, {τ[j], {j}}], {j, 1, 7}]
Length[tA]
tAB = {} ; Do[AppendTo[tAB, {τ[j].τ[k], {j, k}}], {j, 1, 6}, {k, j + 1, 7}]
Length[tAB]
tABC = {};
Do[AppendTo[tABC, {τ[j].τ[k].τ[h], {j, k, h}}], {j, 1, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[tABC]

Out[1]= 7

Out[2]= 21

Out[3]= 35

In[4]:= tA[[1]]
Out[4]= {{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {1} }

In[5]:= tA[[#]] [[1]] == -Transpose[tA[[#]] [[1]]] & /@ Range[Length[tA]]
Out[5]= {True, True, True, False, False, False}

In[6]:= bas64 = Flatten[{tA, tAB, tABC, {{Id, {64}}}}, 1]; Length[bas64]

Out[6]= 64

In[7]:= determineIF8AntiSymmetric[m_] := Module[{t, tt, sum, res, r},
t = m.τ[#] & /@ Range[0, 7];
tt = Transpose[#] & /@ t;
sum = FullSimplify[t[[#]] + tt[[#]]] & /@ Range[Length[t]];
res = Union[Flatten[#]] & /@ sum;
If[Length[#] > 1, Return[False]] & /@ res;
r = Union[Flatten[res]];
Return[r == {0}];
]

In[8]:= determineIF8AntiSymmetric[Zero8]
Out[8]= True

In[9]:= determineIF8AntiSymmetric[ID8]
Out[9]= False
```

```
In[6]:= anti = {};
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],
    AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[anti]
anti[[28]]
```

```

Out[6]=
28

Out[7]=
{{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}

```

```
In[6]:= symm = {};
Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
    AppendTo[symm, {bas64[[k, 1]], {{k}}, bas64[[k, 2]]}]], {k, 1, 64}];
Length[symm]
symm[[%]]
```

```
Out[•]= 35
Out[•]= {{ {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}}, {{59}, {3, 6, 7}}}}
```

```
In[6]:= η64 = DiagonalMatrix[
  Table[FullSimplify[ $\frac{1}{8} \text{Tr}[\text{bas64}[A, 1].\text{bas64}[A, 1]]$ ], {A, 64}]];
Tr[η64]
```

Out[\circ] =

$$7 + \frac{1}{8} \operatorname{Tr}[\operatorname{Id}.\operatorname{Id}]$$

```
In[6]:= (* (* (* (* (* countTrace=
Table[{A,B,FullSimplify[ $\frac{1}{8}$ Tr[bas64[A,1].bas64[B,1]]]}, {A,1, 64},{B,A, 64}])//MatrixForm*)*)*)*)*)*)
```

$|n[\bullet]| := (\star\blacksquare\star)$

```

In[]:= 32 × 65
Out[]= 2080
In[]:= (*Length[Flatten[countTrace,1]]*)
In[]:= (*{Length[countTrace],Length[countTrace[[1]],Length[countTrace[[1,1]]]}*)
In[]:= (*{Length[countTrace],Length[countTrace[[2]],Length[countTrace[[2,1]]]}*)
In[]:= (*{Length[countTrace],Length[countTrace[[64]],Length[countTrace[[64,1]]]}*)
In[]:= (*{Length[Flatten[countTrace,1][All,3]],Count[Flatten[countTrace,1][All,3],0],
Count[Flatten[countTrace,1][All,3],1],Count[Flatten[countTrace,1][All,3],-1]}*)
In[]:= 32 × 63
Out[]= 2016

In[]:= anti = {};
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],
AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[anti]
anti[[28]]

Out[]= 28
Out[=]

{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}

In[]:= anti[[28, 1]]
Out[=]

{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0},
{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}}

In[]:= anti[[28, 2]]
Out[=]

{{63}, {5, 6, 7}}


In[]:= symm = {};
Do[If[bas64[[k, 1]] == Transpose[bas64[[k, 1]]],
AppendTo[symm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[symm]
symm[[28]]

Out[=]
35
Out[=]

{{{ -1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},
{0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, {{52}, {2, 5, 7}}}

```

Spinor Lagrangian

```
In[1]:= uset16 = Table[FullSimplify[(T16α[α1 - 1]), constraintVars], {α1, 1, Length[X]}];

In[2]:= useT16 = Table[FullSimplify[(T16α[α1 - 1] /. ssgm4488), constraintVars] /.
  {Sqrt[e-2 a4[H x4]] → e-a4[H x4], Sqrt[e2 a4[H x4] Sin[6 H x0]1/3] → ea4[H x4] Sin[6 H x0]1/6,
   1/Sqrt[e2 a4[H x4] Sin[6 H x0]1/3] → 1/(ea4[H x4] Sin[6 H x0]1/6)}, {α1, 1, Length[X]}] /.

{1/Sqrt[e2 a4[H x4] Sin[6 H x0]1/3] → 1/(ea4[H x4] Sin[6 H x0]1/6), Sqrt[e2 a4[H x4]] → ea4[H x4]}];

In[3]:= newuseT16 = Table[FullSimplify[(newT16α[α1 - 1] /. ssgm4488), constraintVars] /.
  {Sqrt[e-2 a4[H x4]] → e-a4[H x4], Sqrt[e2 a4[H x4] Sin[6 H x0]1/3] → ea4[H x4] Sin[6 H x0]1/6,
   1/Sqrt[e2 a4[H x4] Sin[6 H x0]1/3] → 1/(ea4[H x4] Sin[6 H x0]1/6)}, {α1, 1, Length[X]}] /.

{1/Sqrt[e2 a4[H x4] Sin[6 H x0]1/3] → 1/(ea4[H x4] Sin[6 H x0]1/6), Sqrt[e2 a4[H x4]] → ea4[H x4]}];

In[4]:= Dimensions[useT16]

Out[4]= {8, 16, 16}

In[5]:= Det[MatrixMetric44]

Out[5]= Cos[6 H x0]2

In[6]:= useDSQRT = Cos[6 H x0]

Out[6]= Cos[6 H x0]
```



```
In[=]:= (*Protect[Q1,Q2]*)
In[=]:= (*(matTest=(Q1*ID16 +Q2* covariantDiffMatrix))//MatrixForm*)
In[=]:= (newmatTest = 08similarityTransformation.(Q1 * ID16 + Q2 * covariantDiffMatrix).
Transpose[08similarityTransformation]) // MatrixForm
Out[=]//MatrixForm=

$$\begin{pmatrix} Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q1 \end{pmatrix}$$

```

```
In[=]:= Clear[Lg];
Lg[] :=

$$\sqrt{\det gg} * \left( \text{Transpose}[\Psi_{16}].\sigma_{16}. \text{Sum} \left[ \text{FullSimplify}[(T_{16}^{\alpha}[\alpha_1 - 1] /. sg), \text{constraintVars}], \left( D[\Psi_{16}, X[\alpha_1]] + \left( \frac{K}{2} /. \{K \rightarrow 1\} \right) * \text{Sum}[\omega_{\text{mat}}[\alpha_1, a, b] * SAB[a, b].\text{matTest}.\Psi_{16}, \{a, 1, 8\}, \{b, 1, 8\}] \right), \{\alpha_1, 1, \text{Length}[X]\}] + (H * M) * \text{Transpose}[\Psi_{16}].\sigma_{16}.\Psi_{16} \right) // \text{Simplify}[\#, \text{constraintVars}] &$$

```

```
In[=]:= Clear[newLg];
newLg[] := 
$$\sqrt{\det gg} * \left( \text{Transpose}[\Psi_{16}].\text{new}\sigma_{16}. \text{Sum} \left[ \text{FullSimplify}[(\text{new}T_{16}^{\alpha}[\alpha_1 - 1] /. sg), \text{constraintVars}], \left( D[\Psi_{16}, X[\alpha_1]] + \left( \frac{K}{2} /. \{K \rightarrow 1\} \right) * \text{Sum}[\omega_{\text{mat}}[\alpha_1, a, b] * 08\text{similarityTransformation}.SAB[a, b].\text{Transpose}[08\text{similarityTransformation}].\text{newmatTest}.\Psi_{16}, \{a, 1, 8\}, \{b, 1, 8\}] \right), \{\alpha_1, 1, \text{Length}[X]\}] + (H * M) * \text{Transpose}[\Psi_{16}].\text{new}\sigma_{16}.\Psi_{16} \right) // \text{Simplify}[\#, \text{constraintVars}] &$$

```

```
In[6]:= Block[{A1 = 1, B1 = 2}, 08similarityTransformation.  
          SAB[[A1, B1]].Transpose[08similarityTransformation] // MatrixForm]
```

Out[•]//MatrixForm=

```
In[8]:= Block[{A1 = 1, B1 = 2, m1, ms, m, j1, s1},
  m1 = ArrayFlatten[{{0, ID8}, {ID8, 0}}]; Print[m1 // MatrixForm];
  {s1, j1} = JordanDecomposition[m1];
  Print[m1 == s1.j1.Inverse[s1]];
  ms =  $\sqrt{j_1}$ ;
  m = s1.ms.Inverse[s1];
  Print[m // MatrixForm];
  Print[ $\sqrt{m_1}$  // MatrixForm];
  m.08similarityTransformation.SAB[A1, B1].
  Transpose[m.08similarityTransformation] // MatrixForm]
```

0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0

True

```
In[•]:= wμIJ[1]
Out[•]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
          {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```

In[8]:= Clear[La];
La[] :=

useDSQRT * 
$$\left( \text{Transpose}[\Psi_{16}] . \sigma_{16} . \text{Sum}[\text{useT16}[\alpha_1] . \left( D[\Psi_{16}, X[\alpha_1]] + \left( \frac{K}{2} / . \{K \rightarrow 1\} \right) * \text{Sum}[ \right. \right.$$


$$\left. \left. \omega_{\mu IJ}[\alpha_1][A_1, B_1] * SAB[A_1, B_1], \{A_1, 1, 8\}, \{B_1, 1, 8\} \right] . \text{matTest}[\Psi_{16}], \{\alpha_1, 1, \right.$$


$$\left. \text{Length}[X]\} \right] + (H * M) * \text{Transpose}[\Psi_{16}] . \sigma_{16} . \Psi_{16} \right) // \text{Simplify}[\#, constraintVars] &$$

```

NOT O8similarityTransformation . Ψ16 :

```

In[8]:= Clear[newLa];
newLa[] :=
useDSQRT * 
$$\left( \text{Transpose}[\Psi_{16}] . \text{new}\sigma_{16} . \text{Sum} \left[ \text{newuseT16}[\alpha_1] . \left( D[\Psi_{16}, X[\alpha_1]] + \left( \frac{K}{2} / . \{K \rightarrow 1\} \right) * \right. \right. \right.$$


$$\text{Sum}[\omega_{\mu IJ}[\alpha_1][A_1, B_1] * (O8similarityTransformation.$$


$$SAB[A_1, B_1].\text{Transpose}[O8similarityTransformation]),$$


$$\left. \left. \left. \{A_1, 1, 8\}, \{B_1, 1, 8\} \right] . \text{newmatTest}.\Psi_{16} \right), \{\alpha_1, 1, \text{Length}[X]\} \right] +$$


$$(H * M) * \text{Transpose}[\Psi_{16}] . \text{new}\sigma_{16}.\Psi_{16} \right) // \text{Simplify}[\#, constraintVars] &$$

```

```

In[1]:= (*testLa=La[]*)

In[2]:= (*testLg=Lg[];*)

In[3]:= (*D[testLa,a4[H x4]]//Flatten//Union*)
          (*D[testLa,a4'[H x4]]//(*Full*)Simplify[#,constraintVars]&//Flatten//Union*)

In[4]:= (*testLgm=testL/.sg/.ssgm4488//(*Full*)Simplify[#,constraintVars]&/.subsDefects*)

In[5]:= (*D[testLgm,a4[H x4]]
          D[testLgm,a4'[H x4]]//(*Full*)Simplify[#,constraintVars]&*)

In[6]:= Clear[Lj];
Lj[j_]:= useDSQRT * 
$$\left( \left( \left( \text{Transpose}[\Psi_{16}] . \sigma_{16} . \text{useT16}[1] . D[\Psi_{16}, X[1]] + \right. \right. \right.$$


$$\left. \left. \left. \text{Transpose}[\Psi_{16}] . \sigma_{16} . \text{useT16}[5] . D[\Psi_{16}, X[5]] + \right. \right. \right.$$


$$\left( \frac{K_0}{2} \right) * \text{Sum}[(*\text{usew}[1,A1,B1]*), \omega_{\mu IJ}[1][A1, B1] \times \text{Transpose}[\Psi_{16}] . \sigma_{16} .$$


$$\text{useT16}[1] . SAB[A1, B1] . (\text{base16}[j, 1]) . \Psi_{16}, \{A1, 1, 8\}, \{B1, 1, 8\}] \Big) +$$


$$\text{Transpose}[\Psi_{16}] . \sigma_{16} . \text{useT16}[5] . D[\Psi_{16}, X[5]] + \left( \frac{K_4}{2} \right) * \text{Sum}[(*\text{usew}[5,A1,B1]*),$$


$$\omega_{\mu IJ}[5][A1, B1] \times \text{Transpose}[\Psi_{16}] . \sigma_{16} . \text{useT16}[5] . SAB[A1, B1] . (\text{base16}[j, 1]) . \Psi_{16},$$


$$\{A1, 1, 8\}, \{B1, 1, 8\}] \Big) +$$


$$(H * M) * \text{Transpose}[\Psi_{16}] . \sigma_{16} . \Psi_{16} \Big) // \text{Simplify}[#, constraintVars] &$$


```

```

In[]:= Lj[92]
Out[]=

Cos[6 H x0] (-2 H M f16[2][x0, x4] × f16[6][x0, x4] -
2 H M f16[3][x0, x4] × f16[7][x0, x4] + 2 H M f16[8][x0, x4] × f16[12][x0, x4] +
2 H M f16[9][x0, x4] × f16[13][x0, x4] + 2 H M f16[10][x0, x4] × f16[14][x0, x4] +
2 H M f16[11][x0, x4] × f16[15][x0, x4] + 2 f16[9][x0, x4] f16[0]^(0,1)[x0, x4] -
2 f16[8][x0, x4] f16[1]^(0,1)[x0, x4] - 2 f16[11][x0, x4] f16[2]^(0,1)[x0, x4] +
2 f16[10][x0, x4] f16[3]^(0,1)[x0, x4] - 2 f16[13][x0, x4] f16[4]^(0,1)[x0, x4] +
2 f16[12][x0, x4] f16[5]^(0,1)[x0, x4] + 2 f16[15][x0, x4] f16[6]^(0,1)[x0, x4] -
2 f16[14][x0, x4] f16[7]^(0,1)[x0, x4] - 2 f16[3][x0, x4] f16[10]^(0,1)[x0, x4] +
2 f16[2][x0, x4] f16[11]^(0,1)[x0, x4] - 2 f16[5][x0, x4] f16[12]^(0,1)[x0, x4] +
2 f16[4][x0, x4] f16[13]^(0,1)[x0, x4] + 2 f16[7][x0, x4] f16[14]^(0,1)[x0, x4] -
2 f16[6][x0, x4] f16[15]^(0,1)[x0, x4] + Tan[6 H x0] f16[12][x0, x4] f16[0]^(1,0)[x0, x4] +
Tan[6 H x0] f16[13][x0, x4] f16[1]^(1,0)[x0, x4] +
Tan[6 H x0] f16[14][x0, x4] f16[2]^(1,0)[x0, x4] + Tan[6 H x0] f16[15][x0, x4]
f16[3]^(1,0)[x0, x4] + Tan[6 H x0] f16[8][x0, x4] f16[4]^(1,0)[x0, x4] +
Tan[6 H x0] f16[9][x0, x4] f16[5]^(1,0)[x0, x4] + Tan[6 H x0] f16[10][x0, x4]
f16[6]^(1,0)[x0, x4] + Tan[6 H x0] f16[11][x0, x4] f16[7]^(1,0)[x0, x4] -
Tan[6 H x0] f16[4][x0, x4] f16[8]^(1,0)[x0, x4] - Tan[6 H x0] f16[5][x0, x4]
f16[9]^(1,0)[x0, x4] - Tan[6 H x0] f16[6][x0, x4] f16[10]^(1,0)[x0, x4] -
Tan[6 H x0] f16[7][x0, x4] f16[11]^(1,0)[x0, x4] - f16[0][x0, x4]
(2 H M f16[4][x0, x4] + 2 f16[9]^(0,1)[x0, x4] + Tan[6 H x0] f16[12]^(1,0)[x0, x4]) - f16[1][x0,
x4] (2 H M f16[5][x0, x4] - 2 f16[8]^(0,1)[x0, x4] + Tan[6 H x0] f16[13]^(1,0)[x0, x4]) -
Tan[6 H x0] f16[2][x0, x4] f16[14]^(1,0)[x0, x4] -
Tan[6 H x0] f16[3][x0, x4] f16[15]^(1,0)[x0, x4])

In[]:= detgg
Out[]=

g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]

In[]:= Clear[eL];
eL[Lagrangian_Symbol, detsqrt_] := Module[{L, t},
L = Lagrangian[];
t = Table[FullSimplify[ $\frac{1}{\text{detsqrt}}$  (D[L, f16[k][x0, x4]] - D[D[L, f16[k]^(1,0)[x0, x4]], x0] -
D[D[L, f16[k]^(0,1)[x0, x4]], x4]), constraintVars], {k, 0, 15}];
Return[t /. subsDefects];
]

```

eLa

```

In[]:= (*Get["2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLa.mx"];*)
In[]:= eLa = eL[La, useDSQRT];

```

```

In[]:= eLa

Out[]=
{-2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[9][x0, x4] a4'[H x4] -
2 (H M f16[4][x0, x4] + 3 H f16[12][x0, x4] +
f16[9](0,1)[x0, x4] + Tan[6 H x0] f16[12](1,0)[x0, x4]),
2 (-H M f16[5][x0, x4] - 3 H f16[13][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[8][x0, x4] a4'[H x4] + f16[8](0,1)[x0, x4] - Tan[6 H x0] f16[13](1,0)[x0, x4]),
-2 (H M f16[6][x0, x4] + 3 H f16[14][x0, x4] - f16[11](0,1)[x0, x4] +
Tan[6 H x0] f16[14](1,0)[x0, x4]),
-2 (H M f16[7][x0, x4] + 3 H f16[15][x0, x4] + f16[10](0,1)[x0, x4] +
Tan[6 H x0] f16[15](1,0)[x0, x4]), -2 H M f16[0][x0, x4] -
6 H f16[8][x0, x4] - 2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[13][x0, x4] a4'[H x4] +
2 f16[13](0,1)[x0, x4] - 2 Tan[6 H x0] f16[8](1,0)[x0, x4],
-2 H M f16[1][x0, x4] - 6 H f16[9][x0, x4] +
e-2 a4[H x4] (-1 + e2 a4[H x4]) H Q1 f16[12][x0, x4] a4'[H x4] -
2 (f16[12](0,1)[x0, x4] + Tan[6 H x0] f16[9](1,0)[x0, x4]),
-2 (H M f16[2][x0, x4] + 3 H f16[10][x0, x4] + f16[15](0,1)[x0, x4] +
Tan[6 H x0] f16[10](1,0)[x0, x4]), -2 (H M f16[3][x0, x4] +
3 H f16[11][x0, x4] - f16[14](0,1)[x0, x4] + Tan[6 H x0] f16[11](1,0)[x0, x4]),
2 (3 H f16[4][x0, x4] + H M f16[12][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[1][x0, x4] a4'[H x4] - f16[1](0,1)[x0, x4] + Tan[6 H x0] f16[4](1,0)[x0, x4]),
2 (3 H f16[5][x0, x4] + H M f16[13][x0, x4] - e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[0][x0, x4] a4'[H x4] + f16[0](0,1)[x0, x4] + Tan[6 H x0] f16[5](1,0)[x0, x4]),
2 (3 H f16[6][x0, x4] + H M f16[14][x0, x4] + f16[3](0,1)[x0, x4] +
Tan[6 H x0] f16[6](1,0)[x0, x4]), 2 (3 H f16[7][x0, x4] +
H M f16[15][x0, x4] - f16[2](0,1)[x0, x4] + Tan[6 H x0] f16[7](1,0)[x0, x4]),
2 (3 H f16[0][x0, x4] + H M f16[8][x0, x4] + e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[5][x0, x4] a4'[H x4] +
f16[5](0,1)[x0, x4] + Tan[6 H x0] f16[0](1,0)[x0, x4]),
6 H f16[1][x0, x4] + 2 H M f16[9][x0, x4] - 2 e-a4[H x4] H Q1 Sinh[a4[H x4]] f16[4][x0, x4] a4'[H x4] -
2 f16[4](0,1)[x0, x4] + 2 Tan[6 H x0] f16[1](1,0)[x0, x4],
2 (3 H f16[2][x0, x4] + H M f16[10][x0, x4] - f16[7](0,1)[x0, x4] +
Tan[6 H x0] f16[2](1,0)[x0, x4]), 2 (3 H f16[3][x0, x4] +
H M f16[11][x0, x4] + f16[6](0,1)[x0, x4] + Tan[6 H x0] f16[3](1,0)[x0, x4])}

In[]:= Length[eLa]

Out[]=
16

In[]:= DumpSave[ToString[header <> "eLa.mx"], eLa];

In[]:= Print[ToString[header <> "eLa.mx"]]

EDIT-2025-12-25-Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-
Nash-ALT-eLa.mx

In[]:= (*Get["2025-11-29-Q1-Q28-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-mmM4p-fix
3-eLa.mx"]*)

In[]:= (*newLa[]*)

```

```
In[1]:= (*Get["2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eneWLa.mx"]*)

In[2]:= (*eneWLa=eL[newLa,useDSQRT]*)

In[3]:= (*DumpSave[ToString[header<>"eneWLa.mx"],eneWLa];*)

In[4]:= (*Print[ToString[header <> "eneWLa.mx"]]*)
```

eLazt

```
In[1]:= (*Get["2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLazt.mx"];*)
```

```
In[2]:= time1 = Now
AbsoluteTiming[
eLazt = 
$$\frac{2 \sqrt{\sin[z]}}{6 H} * eLa /. sfy16Aa /. sx0x4 // FullSimplify[\#, constraintVars] &$$

Now - time1
```

Out[2]=

Thu 25 Dec 2025 04:10:24 GMT-8

Out[6]=

$$\left\{ 6.01921, \left\{ -\frac{2}{3} e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] - \frac{2}{3} (\mathbf{M} Z[4][z, t] + Z[9]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t]), \right. \right.$$

$$\frac{2}{3} (-\mathbf{M} Z[5][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + Z[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t]), -\frac{2}{3} (\mathbf{M} Z[6][z, t] - Z[11]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t]),$$

$$-\frac{2}{3} (\mathbf{M} Z[7][z, t] + Z[10]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t]), \frac{2}{3} (-\mathbf{M} Z[0][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + Z[13]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t]),$$

$$-\frac{2}{3} (\mathbf{M} Z[1][z, t] + Q1 \operatorname{Sinh}[a4[t]] (-\operatorname{Cosh}[a4[t]] + \operatorname{Sinh}[a4[t]])) Z[12][z, t] a4'[t] + Z[12]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t]),$$

$$-\frac{2}{3} (\mathbf{M} Z[2][z, t] + Z[15]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t]),$$

$$-\frac{2}{3} (\mathbf{M} Z[3][z, t] - Z[14]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t]),$$

$$\frac{2}{3} (\mathbf{M} Z[12][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] - Z[1]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t]), \frac{2}{3} (\mathbf{M} Z[13][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] + Z[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t]),$$

$$\frac{2}{3} (\mathbf{M} Z[14][z, t] + Z[3]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t]),$$

$$\frac{2}{3} (\mathbf{M} Z[15][z, t] - Z[2]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t]),$$

$$\frac{2}{3} (\mathbf{M} Z[8][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] + Z[5]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t]), \frac{2}{3} (\mathbf{M} Z[9][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] - Z[4]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t]),$$

$$\frac{2}{3} (\mathbf{M} Z[10][z, t] - Z[7]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t]),$$

$$\left. \frac{2}{3} (\mathbf{M} Z[11][z, t] + Z[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t]) \right\}$$
Out[7]=

6.0257371 s

$$\left\{ 6.06786, \left\{ -\frac{2}{3} e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] - \frac{2}{3} (\mathbf{M} Z[4][z, t] + Z[9]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t]), \right. \right.$$

$$\frac{2}{3} (-\mathbf{M} Z[5][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + Z[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t]),$$

$$\begin{aligned}
& -\frac{2}{3} (\text{M Z}[6][z, t] - \text{Z}[11]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[14]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[7][z, t] + \text{Z}[10]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[15]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (-\text{M Z}[0][z, t] - e^{-a4[t]} \text{Q1 Sinh}[a4[t]] \text{Z}[13][z, t] a4'[t] + \\
& \quad \text{Z}[13]^{(0,1)}[z, t] - 6 \tan[z] \text{Z}[8]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[1][z, t] + \text{Q1 Sinh}[a4[t]] (-\text{Cosh}[a4[t]] + \text{Sinh}[a4[t]]) \text{Z}[12][z, t] a4'[t] + \\
& \quad \text{Z}[12]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[9]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[2][z, t] + \text{Z}[15]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[10]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[3][z, t] - \text{Z}[14]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[11]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[12][z, t] + e^{-a4[t]} \text{Q1 Sinh}[a4[t]] \text{Z}[1][z, t] a4'[t] - \\
& \quad \text{Z}[1]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[4]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[13][z, t] - e^{-a4[t]} \text{Q1 Sinh}[a4[t]] \text{Z}[0][z, t] a4'[t] + \\
& \quad \text{Z}[0]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[5]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[14][z, t] + \text{Z}[3]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[6]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[15][z, t] - \text{Z}[2]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[7]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[8][z, t] + e^{-a4[t]} \text{Q1 Sinh}[a4[t]] \text{Z}[5][z, t] a4'[t] + \\
& \quad \text{Z}[5]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[0]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[9][z, t] - e^{-a4[t]} \text{Q1 Sinh}[a4[t]] \text{Z}[4][z, t] a4'[t] - \\
& \quad \text{Z}[4]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[1]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[10][z, t] - \text{Z}[7]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[2]^{(1,0)}[z, t]), \\
& -\frac{2}{3} (\text{M Z}[11][z, t] + \text{Z}[6]^{(0,1)}[z, t] + 6 \tan[z] \text{Z}[3]^{(1,0)}[z, t]) \Big\} \Big\}
\end{aligned}$$

Out[6]=

6.0791309 s

In[7]:= DumpSave[ToString[header <> "eLazt.mx"], eLazt];

In[8]:= Print[ToString[header <> "eLazt.mx"]]

EDIT-2025-12-25-Pair_Creation_of_Universes_WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-ALT-eLazt.mx

```

In[]:= varZ = Z[#] & /@ Range[0, 15]
Out[]= {Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}

In[]:= varZzt = #[z, t] & /@ varZ
Out[=] {Z[0][z, t], Z[1][z, t], Z[2][z, t], Z[3][z, t], Z[4][z, t],
Z[5][z, t], Z[6][z, t], Z[7][z, t], Z[8][z, t], Z[9][z, t], Z[10][z, t],
Z[11][z, t], Z[12][z, t], Z[13][z, t], Z[14][z, t], Z[15][z, t]}

In[]:= DzvarZ = D[varZzt, z]
Out[=] {Z[0]^(1,0)[z, t], Z[1]^(1,0)[z, t], Z[2]^(1,0)[z, t], Z[3]^(1,0)[z, t],
Z[4]^(1,0)[z, t], Z[5]^(1,0)[z, t], Z[6]^(1,0)[z, t], Z[7]^(1,0)[z, t],
Z[8]^(1,0)[z, t], Z[9]^(1,0)[z, t], Z[10]^(1,0)[z, t], Z[11]^(1,0)[z, t],
Z[12]^(1,0)[z, t], Z[13]^(1,0)[z, t], Z[14]^(1,0)[z, t], Z[15]^(1,0)[z, t]}

In[]:= DtvarZ = D[varZzt, t]
Out[=] {Z[0]^(0,1)[z, t], Z[1]^(0,1)[z, t], Z[2]^(0,1)[z, t], Z[3]^(0,1)[z, t],
Z[4]^(0,1)[z, t], Z[5]^(0,1)[z, t], Z[6]^(0,1)[z, t], Z[7]^(0,1)[z, t],
Z[8]^(0,1)[z, t], Z[9]^(0,1)[z, t], Z[10]^(0,1)[z, t], Z[11]^(0,1)[z, t],
Z[12]^(0,1)[z, t], Z[13]^(0,1)[z, t], Z[14]^(0,1)[z, t], Z[15]^(0,1)[z, t]}

In[]:= DtvarZsubs = Solve[And @@ Thread[0 == eLazt], DtvarZ][[1]] // FullSimplify[#, constraintVars] &
Out[=] {Z[0]^(0,1)[z, t] \rightarrow -M Z[13][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] - 6 Tan[z] Z[5]^(1,0)[z, t], Z[1]^(0,1)[z, t] \rightarrow
M Z[12][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] + 6 Tan[z] Z[4]^(1,0)[z, t],
Z[2]^(0,1)[z, t] \rightarrow M Z[15][z, t] + 6 Tan[z] Z[7]^(1,0)[z, t],
Z[3]^(0,1)[z, t] \rightarrow -M Z[14][z, t] - 6 Tan[z] Z[6]^(1,0)[z, t],
Z[4]^(0,1)[z, t] \rightarrow M Z[9][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] + 6 Tan[z] Z[1]^(1,0)[z, t],
Z[5]^(0,1)[z, t] \rightarrow
-M Z[8][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] - 6 Tan[z] Z[0]^(1,0)[z, t],
Z[6]^(0,1)[z, t] \rightarrow -M Z[11][z, t] - 6 Tan[z] Z[3]^(1,0)[z, t],
Z[7]^(0,1)[z, t] \rightarrow M Z[10][z, t] + 6 Tan[z] Z[2]^(1,0)[z, t], Z[8]^(0,1)[z, t] \rightarrow
M Z[5][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + 6 Tan[z] Z[13]^(1,0)[z, t],
Z[9]^(0,1)[z, t] \rightarrow -M Z[4][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] -
6 Tan[z] Z[12]^(1,0)[z, t], Z[10]^(0,1)[z, t] \rightarrow -M Z[7][z, t] - 6 Tan[z] Z[15]^(1,0)[z, t],
Z[11]^(0,1)[z, t] \rightarrow M Z[6][z, t] + 6 Tan[z] Z[14]^(1,0)[z, t], Z[12]^(0,1)[z, t] \rightarrow
-M Z[1][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] - 6 Tan[z] Z[9]^(1,0)[z, t],
Z[13]^(0,1)[z, t] \rightarrow M Z[0][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] +
6 Tan[z] Z[8]^(1,0)[z, t], Z[14]^(0,1)[z, t] \rightarrow M Z[3][z, t] + 6 Tan[z] Z[11]^(1,0)[z, t],
Z[15]^(0,1)[z, t] \rightarrow -M Z[2][z, t] - 6 Tan[z] Z[10]^(1,0)[z, t]}

```

```
In[•]:= ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(DtvarZEQS = ToExpression[#] & /@ %) // Column
```

Out[•]=

$$\begin{aligned} Z[0]^{(0,1)}[z, t] &= -M Z[13][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t] \\ Z[1]^{(0,1)}[z, t] &= M Z[12][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t] \\ Z[2]^{(0,1)}[z, t] &= M Z[15][z, t] + 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t] \\ Z[3]^{(0,1)}[z, t] &= -M Z[14][z, t] - 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t] \\ Z[4]^{(0,1)}[z, t] &= M Z[9][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t] \\ Z[5]^{(0,1)}[z, t] &= -M Z[8][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t] \\ Z[6]^{(0,1)}[z, t] &= -M Z[11][z, t] - 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t] \\ Z[7]^{(0,1)}[z, t] &= M Z[10][z, t] + 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t] \\ Z[8]^{(0,1)}[z, t] &= M Z[5][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t] \\ Z[9]^{(0,1)}[z, t] &= -M Z[4][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t] \\ Z[10]^{(0,1)}[z, t] &= -M Z[7][z, t] - 6 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t] \\ Z[11]^{(0,1)}[z, t] &= M Z[6][z, t] + 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t] \\ Z[12]^{(0,1)}[z, t] &= \\ &\quad -M Z[1][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] - 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t] \\ Z[13]^{(0,1)}[z, t] &= M Z[0][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t] \\ Z[14]^{(0,1)}[z, t] &= M Z[3][z, t] + 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t] \\ Z[15]^{(0,1)}[z, t] &= -M Z[2][z, t] - 6 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t] \end{aligned}$$

```
In[•]:= ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(DtvarZrelations = ToExpression[#] & /@ %) // Column
```

Out[•]=

$$\begin{aligned} M Z[13][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[0][z, t] a4'[t] + Z[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[5]^{(1,0)}[z, t] \\ -M Z[12][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[1][z, t] a4'[t] + Z[1]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[4]^{(1,0)}[z, t] \\ -M Z[15][z, t] + Z[2]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[7]^{(1,0)}[z, t] \\ M Z[14][z, t] + Z[3]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[6]^{(1,0)}[z, t] \\ -M Z[9][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[4][z, t] a4'[t] + Z[4]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[1]^{(1,0)}[z, t] \\ M Z[8][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[5][z, t] a4'[t] + Z[5]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[0]^{(1,0)}[z, t] \\ M Z[11][z, t] + Z[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[3]^{(1,0)}[z, t] \\ -M Z[10][z, t] + Z[7]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[2]^{(1,0)}[z, t] \\ -M Z[5][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[8][z, t] a4'[t] + Z[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[13]^{(1,0)}[z, t] \\ M Z[4][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[9][z, t] a4'[t] + Z[9]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[12]^{(1,0)}[z, t] \\ M Z[7][z, t] + Z[10]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[15]^{(1,0)}[z, t] \\ -M Z[6][z, t] + Z[11]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[14]^{(1,0)}[z, t] \\ M Z[1][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[12][z, t] a4'[t] + Z[12]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[9]^{(1,0)}[z, t] \\ -M Z[0][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] Z[13][z, t] a4'[t] + Z[13]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[8]^{(1,0)}[z, t] \\ -M Z[3][z, t] + Z[14]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] Z[11]^{(1,0)}[z, t] \\ M Z[2][z, t] + Z[15]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] Z[10]^{(1,0)}[z, t] \end{aligned}$$

WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE :

```
In[]:= {{4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13, 0, 1, 4, 13, 13, 8, 0, 5, 12, 1, 12, 9},
{6, 7, 2, 11, 14, 6, 3, 7, 15, 2, 3, 6, 15, 10, 2, 3, 7, 14, 11}}
eLastCouplings = Union[Flatten[#]] & /@ %
Union[Flatten[%]] === Range[0, 15]

Out[]= {{4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13, 0, 1, 4, 13, 13, 8, 0, 5, 12, 1, 12, 9},
{6, 7, 2, 11, 14, 6, 3, 7, 15, 2, 3, 6, 15, 10, 2, 3, 7, 14, 11}}

Out[=]
{{0, 1, 4, 5, 8, 9, 12, 13}, {2, 3, 6, 7, 10, 11, 14, 15}}

Out[=]
True
```

SEPARATE VARIABLES THAT ARE COUPLED TO EACH OTHER :

```
In[]:= Thread[(Z[#] & /@ eLastCouplings[[1]]) → (yZ[#] & /@ Range[0, 7])]
Thread[(Z[#] & /@ eLastCouplings[[2]]) → (yZ[#] & /@ Range[8, 15])]
sZt0yZ = Union[Flatten[{%, %}]]
ToString[FullForm[#]] & /@ sZt0yZ;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(ZyZEQS = ToExpression[#] & /@ %)
syZtoZ = Solve[And @@ %, (yZ[#] & /@ Range[0, 15])] [[1]]

Out[=]
{Z[0] → yZ[0], Z[1] → yZ[1], Z[4] → yZ[2], Z[5] → yZ[3],
Z[8] → yZ[4], Z[9] → yZ[5], Z[12] → yZ[6], Z[13] → yZ[7]}

Out[=]
{Z[2] → yZ[8], Z[3] → yZ[9], Z[6] → yZ[10], Z[7] → yZ[11],
Z[10] → yZ[12], Z[11] → yZ[13], Z[14] → yZ[14], Z[15] → yZ[15]}

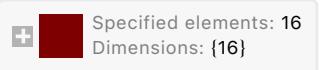
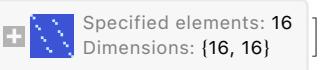
Out[=]
{Z[0] → yZ[0], Z[1] → yZ[1], Z[2] → yZ[8], Z[3] → yZ[9], Z[4] → yZ[2], Z[5] → yZ[3],
Z[6] → yZ[10], Z[7] → yZ[11], Z[8] → yZ[4], Z[9] → yZ[5], Z[10] → yZ[12],
Z[11] → yZ[13], Z[12] → yZ[6], Z[13] → yZ[7], Z[14] → yZ[14], Z[15] → yZ[15]}

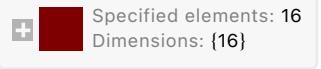
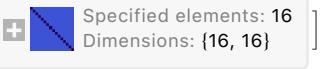
Out[=]
{Z[0] == yZ[0], Z[1] == yZ[1], Z[2] == yZ[8], Z[3] == yZ[9], Z[4] == yZ[2], Z[5] == yZ[3],
Z[6] == yZ[10], Z[7] == yZ[11], Z[8] == yZ[4], Z[9] == yZ[5], Z[10] == yZ[12],
Z[11] == yZ[13], Z[12] == yZ[6], Z[13] == yZ[7], Z[14] == yZ[14], Z[15] == yZ[15]}

Out[=]
{yZ[0] → Z[0], yZ[1] → Z[1], yZ[2] → Z[4], yZ[3] → Z[5], yZ[4] → Z[8], yZ[5] → Z[9],
yZ[6] → Z[12], yZ[7] → Z[13], yZ[8] → Z[2], yZ[9] → Z[3], yZ[10] → Z[6],
yZ[11] → Z[7], yZ[12] → Z[10], yZ[13] → Z[11], yZ[14] → Z[14], yZ[15] → Z[15]}

In[]:= ToString[FullForm[#]] & /@ sZt0yZ;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(ZyZforCaEQS = -ToExpression[#] & /@ %)

Out[=
{yZ[0] - Z[0], yZ[1] - Z[1], yZ[8] - Z[2], yZ[9] - Z[3], yZ[2] - Z[4], yZ[3] - Z[5],
yZ[10] - Z[6], yZ[11] - Z[7], yZ[4] - Z[8], yZ[5] - Z[9], yZ[12] - Z[10],
yZ[13] - Z[11], yZ[6] - Z[12], yZ[7] - Z[13], yZ[14] - Z[14], yZ[15] - Z[15]}
```

```
In[=]:= cayZ = CoefficientArrays[ZyZforCaEQS, {yZ[#] & /@ Range[0, 15]}]
Out[=]= {SparseArray[, SparseArray[]}
```

```
In[=]:= caZ = CoefficientArrays[ZyZforCaEQS, {Z[#] & /@ Range[0, 15]}]
Out[=]= {SparseArray[, SparseArray[]}
```

```
In[=]:= (cayZ2 = cayZ[[2]] // Normal) // MatrixForm
Out[=]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

IDENTIFY ORTHOGONAL SIMILARITY TRANSFORMATION FROM SET OF VARIABLES Z[j] TO SET OF VARIABLES yZ[j] :

```
In[=]:= cayZ2.(yZ[#] & /@ Range[0, 15]) /. syZtoZ
Transpose[cayZ2].% /. sZt0yZ
Out[=]= {Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}
Out[=]= {yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],
yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}
In[=]:= cayZ2.Transpose[cayZ2] === ID16
Out[=]= True
```

TRANSFORM Euler Lagrange equations to yZ[j] :

```
In[]:= (DtYRelations = Transpose[cayZ2].DtvarZrelations /. sZt0yZ // FullSimplify) // Column
Out[]=
MyZ[7][z, t] - e-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] +
yZ[0](0,1)[z, t] + 6 Tan[z] yZ[3](1,0)[z, t]
-M yZ[6][z, t] - e-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] +
yZ[1](0,1)[z, t] - 6 Tan[z] yZ[2](1,0)[z, t]
-M yZ[5][z, t] + e-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] +
yZ[2](0,1)[z, t] - 6 Tan[z] yZ[1](1,0)[z, t]
MyZ[4][z, t] + e-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] +
yZ[3](0,1)[z, t] + 6 Tan[z] yZ[0](1,0)[z, t]
-M yZ[3][z, t] + e-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
yZ[4](0,1)[z, t] - 6 Tan[z] yZ[7](1,0)[z, t]
MyZ[2][z, t] + e-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] +
yZ[5](0,1)[z, t] + 6 Tan[z] yZ[6](1,0)[z, t]
MyZ[1][z, t] - e-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] +
yZ[6](0,1)[z, t] + 6 Tan[z] yZ[5](1,0)[z, t]
-M yZ[0][z, t] - e-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] +
yZ[7](0,1)[z, t] - 6 Tan[z] yZ[4](1,0)[z, t]
-M yZ[15][z, t] + yZ[8](0,1)[z, t] - 6 Tan[z] yZ[11](1,0)[z, t]
MyZ[14][z, t] + yZ[9](0,1)[z, t] + 6 Tan[z] yZ[10](1,0)[z, t]
MyZ[13][z, t] + yZ[10](0,1)[z, t] + 6 Tan[z] yZ[9](1,0)[z, t]
-M yZ[12][z, t] + yZ[11](0,1)[z, t] - 6 Tan[z] yZ[8](1,0)[z, t]
MyZ[11][z, t] + yZ[12](0,1)[z, t] + 6 Tan[z] yZ[15](1,0)[z, t]
-M yZ[10][z, t] + yZ[13](0,1)[z, t] - 6 Tan[z] yZ[14](1,0)[z, t]
-M yZ[9][z, t] + yZ[14](0,1)[z, t] - 6 Tan[z] yZ[13](1,0)[z, t]
MyZ[8][z, t] + yZ[15](0,1)[z, t] + 6 Tan[z] yZ[12](1,0)[z, t]
```

CHECK THAT yZ[j] ARE IN PROPER ORDER:

```

In[]:= (DtyZsubs =
  Solve[And @@ Thread[θ == DtyZrelations], (D[(yZ[#][z, t] & /@ Range[0, 15]), t])[[1]] // 
  FullSimplify[#, constraintVars] &) // Column

Out[]=
yZ[0]^(0,1) [z, t] →
-M yZ[7] [z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[0] [z, t] a4'[t] - 6 Tan[z] yZ[3]^(1,0) [z, t]
yZ[1]^(0,1) [z, t] →
M yZ[6] [z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[1] [z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0) [z, t]
yZ[2]^(0,1) [z, t] →
M yZ[5] [z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[2] [z, t] a4'[t] + 6 Tan[z] yZ[1]^(1,0) [z, t]
yZ[3]^(0,1) [z, t] →
-M yZ[4] [z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[3] [z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0) [z, t]
yZ[4]^(0,1) [z, t] →
M yZ[3] [z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[4] [z, t] a4'[t] + 6 Tan[z] yZ[7]^(1,0) [z, t]
yZ[5]^(0,1) [z, t] →
-M yZ[2] [z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[5] [z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0) [z, t]
yZ[6]^(0,1) [z, t] →
-M yZ[1] [z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[6] [z, t] a4'[t] - 6 Tan[z] yZ[5]^(1,0) [z, t]
yZ[7]^(0,1) [z, t] →
M yZ[0] [z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[7] [z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0) [z, t]
yZ[8]^(0,1) [z, t] → M yZ[15] [z, t] + 6 Tan[z] yZ[11]^(1,0) [z, t]
yZ[9]^(0,1) [z, t] → -M yZ[14] [z, t] - 6 Tan[z] yZ[10]^(1,0) [z, t]
yZ[10]^(0,1) [z, t] → -M yZ[13] [z, t] - 6 Tan[z] yZ[9]^(1,0) [z, t]
yZ[11]^(0,1) [z, t] → M yZ[12] [z, t] + 6 Tan[z] yZ[8]^(1,0) [z, t]
yZ[12]^(0,1) [z, t] → -M yZ[11] [z, t] - 6 Tan[z] yZ[15]^(1,0) [z, t]
yZ[13]^(0,1) [z, t] → M yZ[10] [z, t] + 6 Tan[z] yZ[14]^(1,0) [z, t]
yZ[14]^(0,1) [z, t] → M yZ[9] [z, t] + 6 Tan[z] yZ[13]^(1,0) [z, t]
yZ[15]^(0,1) [z, t] → -M yZ[8] [z, t] - 6 Tan[z] yZ[12]^(1,0) [z, t]

In[]:= (caZ2 = caZ[[2]] // Normal) // MatrixForm

Out[//MatrixForm=
{{-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}]

In[]:= sx0x4

Out[]=
{x0 → z/6 H, x4 → t/H}

```

ENERGY EIGENVALUES OF THE Euler Lagrange equations:

TRY TO FIND ENERGY EIGENFUNCTIONS AND ENERGY EIGENVALUES OF THE Euler Lagrange equations of the form

$$yZ[j] = \frac{1}{\sin[6*H*x\theta]^{1/2}} \text{Exp}[E_j[x4]*H] =$$

$$yZ[j] = \frac{1}{\sin[z]^{1/2}} \text{Exp}[E_j[t]]$$

(recall

that

sfψ16Aa =

```
f16[#] → ToExpression["((Z[" <> ToString[#] <>
    "] [6*H*#1,H*#2]  $\frac{1}{\sin[6*H*\#1]^{1/2}}$ ) &) "] & /@
Range[0, 15]
```

):

```
In[1]:= syZtOnly = yZ[#] → ToExpression["((Exp[v[" <> ToString[#] <> "] [#2]]) &)"] & /@ Range[0, 15]
```

Out[1]=

```
{yZ[0] → (Exp[v[0][#2]] &), yZ[1] → (Exp[v[1][#2]] &),
yZ[2] → (Exp[v[2][#2]] &), yZ[3] → (Exp[v[3][#2]] &), yZ[4] → (Exp[v[4][#2]] &),
yZ[5] → (Exp[v[5][#2]] &), yZ[6] → (Exp[v[6][#2]] &), yZ[7] → (Exp[v[7][#2]] &),
yZ[8] → (Exp[v[8][#2]] &), yZ[9] → (Exp[v[9][#2]] &), yZ[10] → (Exp[v[10][#2]] &),
yZ[11] → (Exp[v[11][#2]] &), yZ[12] → (Exp[v[12][#2]] &),
yZ[13] → (Exp[v[13][#2]] &), yZ[14] → (Exp[v[14][#2]] &), yZ[15] → (Exp[v[15][#2]] &)}
```

```
In[2]:= syZtc = v[#][0] → Log[T[#]] & /@ Range[0, 15]
```

Out[2]=

```
{v[0][0] → Log[T[0]], v[1][0] → Log[T[1]], v[2][0] → Log[T[2]], v[3][0] → Log[T[3]],
v[4][0] → Log[T[4]], v[5][0] → Log[T[5]], v[6][0] → Log[T[6]], v[7][0] → Log[T[7]],
v[8][0] → Log[T[8]], v[9][0] → Log[T[9]], v[10][0] → Log[T[10]], v[11][0] → Log[T[11]],
v[12][0] → Log[T[12]], v[13][0] → Log[T[13]], v[14][0] → Log[T[14]], v[15][0] → Log[T[15]]}
```

```
In[3]:= DtyZrelations /. syZtOnly
```

```
FullSimplify[Normal[Series[#, {t, 0, 0}]]] & /@ %
```

```
% /. syZtc
```

```
DtyZrelationsA = %
```

```

Out[=] = {e^(7)[t] M - e^(-a4[t]+v[0])[t] Q1 Sinh[a4[t]] a4'[t] + e^(v[0])[t] v[0]'[t], -e^(6)[t] M - e^(-a4[t]+v[1])[t] Q1 Sinh[a4[t]] a4'[t] + e^(v[1])[t] v[1]'[t], -e^(5)[t] M + e^(-a4[t]+v[2])[t] Q1 Sinh[a4[t]] a4'[t] + e^(v[2])[t] v[2]'[t], e^(4)[t] M + e^(-a4[t]+v[3])[t] Q1 Sinh[a4[t]] a4'[t] + e^(v[3])[t] v[3]'[t], -e^(3)[t] M + e^(-a4[t]+v[4])[t] Q1 Sinh[a4[t]] a4'[t] + e^(v[4])[t] v[4]'[t], e^(2)[t] M + e^(-a4[t]+v[5])[t] Q1 Sinh[a4[t]] a4'[t] + e^(v[5])[t] v[5]'[t], e^(1)[t] M - e^(-a4[t]+v[6])[t] Q1 Sinh[a4[t]] a4'[t] + e^(v[6])[t] v[6]'[t], -e^(0)[t] M - e^(-a4[t]+v[7])[t] Q1 Sinh[a4[t]] a4'[t] + e^(v[7])[t] v[7]'[t], -e^(15)[t] M + e^(v[8])[t] v[8]'[t], e^(v[14])[t] M + e^(v[9])[t] v[9]'[t], e^(v[13])[t] M + e^(v[10])[t] v[10]'[t], -e^(12)[t] M + e^(v[11])[t] v[11]'[t], e^(v[11])[t] M + e^(v[12])[t] v[12]'[t], -e^(10)[t] M + e^(v[13])[t] v[13]'[t], -e^(v[9])[t] M + e^(v[14])[t] v[14]'[t], e^(v[8])[t] M + e^(v[15])[t] v[15]'[t]}

Out[=] = {e^(7)[0] M + e^(v[0])[0] (-e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[0]'[0]), -e^(6)[0] M + e^(v[1])[0] (-e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[1]'[0]), -e^(5)[0] M + e^(v[2])[0] (e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[2]'[0]), e^(4)[0] M + e^(v[3])[0] (e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[3]'[0]), -e^(3)[0] M + e^(v[4])[0] (e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[4]'[0]), e^(2)[0] M + e^(v[5])[0] (e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[5]'[0]), e^(1)[0] M + e^(v[6])[0] (-e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[6]'[0]), -e^(0)[0] M + e^(v[7])[0] (-e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[7]'[0]), -e^(15)[0] M + e^(v[8])[0] v[8]'[0], e^(v[14])[0] M + e^(v[9])[0] v[9]'[0], e^(v[13])[0] M + e^(v[10])[0] v[10]'[0], -e^(12)[0] M + e^(v[11])[0] v[11]'[0], e^(v[11])[0] M + e^(v[12])[0] v[12]'[0], -e^(10)[0] M + e^(v[13])[0] v[13]'[0], -e^(v[9])[0] M + e^(v[14])[0] v[14]'[0], e^(v[8])[0] M + e^(v[15])[0] v[15]'[0]}

Out[=] = {M T[7] + T[0] (-e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[0]'[0]), -M T[6] + T[1] (-e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[1]'[0]), -M T[5] + T[2] (e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[2]'[0]), M T[4] + T[3] (e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[3]'[0]), -M T[3] + T[4] (e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[4]'[0]), M T[2] + T[5] (e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[5]'[0]), M T[1] + T[6] (-e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[6]'[0]), -M T[0] + T[7] (-e^(-a4[0]) Q1 Sinh[a4[0]] a4'[0] + v[7]'[0]), -M T[15] + T[8] v[8]'[0], M T[14] + T[9] v[9]'[0], M T[13] + T[10] v[10]'[0], -M T[12] + T[11] v[11]'[0], M T[11] + T[12] v[12]'[0], -M T[10] + T[13] v[13]'[0], -M T[9] + T[14] v[14]'[0], M T[8] + T[15] v[15]'[0]}

```

```

Out[=]=
{M T[7] + T[0] (-e^-a4[0] Q1 Sinh[a4[0]] a4'[0] + v[0]'[0]), 
 -M T[6] + T[1] (-e^-a4[0] Q1 Sinh[a4[0]] a4'[0] + v[1]'[0]), 
 -M T[5] + T[2] (e^-a4[0] Q1 Sinh[a4[0]] a4'[0] + v[2]'[0]), 
 M T[4] + T[3] (e^-a4[0] Q1 Sinh[a4[0]] a4'[0] + v[3]'[0]), 
 -M T[3] + T[4] (e^-a4[0] Q1 Sinh[a4[0]] a4'[0] + v[4]'[0]), 
 M T[2] + T[5] (e^-a4[0] Q1 Sinh[a4[0]] a4'[0] + v[5]'[0]), 
 M T[1] + T[6] (-e^-a4[0] Q1 Sinh[a4[0]] a4'[0] + v[6]'[0]), 
 -M T[0] + T[7] (-e^-a4[0] Q1 Sinh[a4[0]] a4'[0] + v[7]'[0]), 
 -M T[15] + T[8] v[8]'[0], M T[14] + T[9] v[9]'[0], M T[13] + T[10] v[10]'[0], 
 -M T[12] + T[11] v[11]'[0], M T[11] + T[12] v[12]'[0], 
 -M T[10] + T[13] v[13]'[0], -M T[9] + T[14] v[14]'[0], M T[8] + T[15] v[15]'[0]}

In[=]:= (*DtyZrelations/.syZtOnly;
%/.syZtc
DtyZrelationsA=%*)

In[=]:= Ts = (T[#] & /@ Range[0, 15])

Out[=]=
{T[0], T[1], T[2], T[3], T[4], T[5], T[6],
 T[7], T[8], T[9], T[10], T[11], T[12], T[13], T[14], T[15]}

In[=]:= caDtyZrelationsA = CoefficientArrays[DtyZrelationsA, Ts]

Out[=]=
{SparseArray[ Specified elements: 0 Dimensions: {16}], SparseArray[ Specified elements: 32 Dimensions: {16, 16}]}

In[=]:= (caDtyZrelationsA2 = caDtyZrelationsA[[2]] // Normal) // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} -e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]] a4'[0] + v[0]'[0] & 0 \\ 0 & -e^{-a4[0]} Q1 \operatorname{Sinh}[a4[0]] a4'[0] + v[1]'[0] \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & M \\ -M & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$


In[=]:= caDtyZrelationsA2 === ArrayFlatten[
{{caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]], {0, caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]]}}]

Out[=]=
True

```

CHECK THAT THERE ARE NO $\nu[]$ TERMS :

```
In[8]:= (caDtyZrelationsA1 = caDtyZrelationsA[[1]] // Normal) // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
In[9]:= time1 = Now
AbsoluteTiming[caDtyZrelationsA21Y0 = Det[caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]] - Y ID8]]
Now
caDtyZrelationsA21Y = (*Full*) Simplify[caDtyZrelationsA21Y0, constraintVars]
Now - time1
```

Out[9]=

Thu 25 Dec 2025 04:10:43 GMT-8

Out[•]=

$$\begin{aligned} & \{0.0859364, e^{-8 a4[0]} \\ & \quad (e^{2 a4[0]} M^2 (e^{2 a4[0]} M^2 (e^{2 a4[0]} M^2 (e^{2 a4[0]} M^2 (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[3]'[0]) \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[4]'[0])) + \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[2]'[0]) (e^{2 a4[0]} M^2 + \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[3]'[0]) (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] \\ & \quad a4'[0] + e^{a4[0]} v[4]'[0])) (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[5]'[0])) + \\ & \quad (-e^{a4[0]} Y - Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[1]'[0]) \\ & \quad (e^{2 a4[0]} M^2 (e^{2 a4[0]} M^2 + (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[3]'[0]) \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[4]'[0])) + \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[2]'[0]) (e^{2 a4[0]} M^2 + \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[3]'[0]) (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] \\ & \quad a4'[0] + e^{a4[0]} v[4]'[0])) (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[5]'[0])) + \\ & \quad (-e^{a4[0]} Y - Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[6]'[0])) + \\ & \quad (-e^{a4[0]} Y - Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[0]'[0]) \\ & \quad (e^{2 a4[0]} M^2 (e^{2 a4[0]} M^2 (e^{2 a4[0]} M^2 + (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[3]'[0]) \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[4]'[0])) + \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[2]'[0]) \\ & \quad (e^{2 a4[0]} M^2 + (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[3]'[0]) \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[4]'[0])) + \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[5]'[0])) + (-e^{a4[0]} Y - \\ & \quad Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[1]'[0]) \\ & \quad (e^{2 a4[0]} M^2 (e^{2 a4[0]} M^2 + (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[3]'[0]) \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[4]'[0])) + \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[2]'[0]) (e^{2 a4[0]} M^2 + \\ & \quad (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[3]'[0]) (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] \\ & \quad a4'[0] + e^{a4[0]} v[4]'[0])) (-e^{a4[0]} Y + Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[5]'[0])) + \\ & \quad (-e^{a4[0]} Y - Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[6]'[0])) (-e^{a4[0]} Y - \\ & \quad Q1 \operatorname{Sinh}[a4[0]] a4'[0] + e^{a4[0]} v[7]'[0])) \} \end{aligned}$$
Out[•]=

Thu 25 Dec 2025 04:10:43 GMT-8

Out[•]=

$$\begin{aligned} & \frac{1}{256} e^{-16 a4[0]} (Q1^2 a4'[0]^2 + e^{4 a4[0]} (4 M^2 + (2 Y - Q1 a4'[0] - 2 v[3]'[0]) (2 Y - Q1 a4'[0] - 2 v[4]'[0])) - \\ & \quad 2 e^{2 a4[0]} Q1 a4'[0] (-2 Y + Q1 a4'[0] + v[3]'[0] + v[4]'[0])) \\ & \quad (Q1^2 a4'[0]^2 + e^{4 a4[0]} (4 M^2 + (2 Y - Q1 a4'[0] - 2 v[2]'[0]) (2 Y - Q1 a4'[0] - 2 v[5]'[0])) - \\ & \quad 2 e^{2 a4[0]} Q1 a4'[0] (-2 Y + Q1 a4'[0] + v[2]'[0] + v[5]'[0])) \\ & \quad (Q1^2 a4'[0]^2 + e^{4 a4[0]} (4 M^2 + (2 Y + Q1 a4'[0] - 2 v[1]'[0]) (2 Y + Q1 a4'[0] - 2 v[6]'[0])) - \\ & \quad 2 e^{2 a4[0]} Q1 a4'[0] (2 Y + Q1 a4'[0] - v[1]'[0] - v[6]'[0])) \\ & \quad (Q1^2 a4'[0]^2 + e^{4 a4[0]} (4 M^2 + (2 Y + Q1 a4'[0] - 2 v[0]'[0]) (2 Y + Q1 a4'[0] - 2 v[7]'[0])) - \\ & \quad 2 e^{2 a4[0]} Q1 a4'[0] (2 Y + Q1 a4'[0] - v[0]'[0] - v[7]'[0])) \end{aligned}$$
Out[•]=

1.4155398 s

```
In[1]:= sumEvalscaDtyZrelationsA21Y =  $\frac{1}{7!} D[caDtyZrelationsA21Y, \{Y, 7\}] /. \{Y \rightarrow 0\} // Simplify$ 
```

```
Out[1]= -v[0]'[0] - v[1]'[0] - v[2]'[0] - v[3]'[0] - v[4]'[0] - v[5]'[0] - v[6]'[0] - v[7]'[0]
```

```
In[2]:= time1 = Now
AbsoluteTiming[caDtyZrelationsA22Y0 = Det[caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]] - Y ID8]]
Now
caDtyZrelationsA22Y = (*Full*)Simplify[caDtyZrelationsA22Y0, constraintVars]
Now - time1
```

```
Out[2]=
```

Thu 25 Dec 2025 04:10:44 GMT-8

```
Out[2]=
```

```
{0.0025971,
M^2 (M (M^3 + M Y^2 - M Y v[11]'[0] - M Y v[12]'[0] + M v[11]'[0] v[12]'[0]) + (-Y + v[10]'[0])
(M^2 + Y^2 - Y v[11]'[0] - Y v[12]'[0] + v[11]'[0] v[12]'[0]) (-Y + v[13]'[0])) +
(-Y + v[9]'[0]) (M (M^3 + M Y^2 - M Y v[11]'[0] - M Y v[12]'[0] + M v[11]'[0] v[12]'[0]) +
(-Y + v[10]'[0]) (M^2 + Y^2 - Y v[11]'[0] - Y v[12]'[0] + v[11]'[0] v[12]'[0])
(-Y + v[13]'[0])) (-Y + v[14]'[0])) + (-Y + v[8]'[0])
(M^2 (M (M^3 + M Y^2 - M Y v[11]'[0] - M Y v[12]'[0] + M v[11]'[0] v[12]'[0]) + (-Y + v[10]'[0])
(M^2 + Y^2 - Y v[11]'[0] - Y v[12]'[0] + v[11]'[0] v[12]'[0]) (-Y + v[13]'[0])) +
(-Y + v[9]'[0]) (M (M^3 + M Y^2 - M Y v[11]'[0] - M Y v[12]'[0] + M v[11]'[0] v[12]'[0]) +
(-Y + v[10]'[0]) (M^2 + Y^2 - Y v[11]'[0] - Y v[12]'[0] + v[11]'[0] v[12]'[0])
(-Y + v[13]'[0])) (-Y + v[14]'[0])) (-Y + v[15]'[0]))}
```

```
Out[2]=
```

Thu 25 Dec 2025 04:10:44 GMT-8

```
Out[2]=
```

```
(M^2 + (Y - v[11]'[0]) (Y - v[12]'[0])) (M^2 + (Y - v[10]'[0]) (Y - v[13]'[0]))
(M^2 + (Y - v[9]'[0]) (Y - v[14]'[0])) (M^2 + (Y - v[8]'[0]) (Y - v[15]'[0]))
```

```
Out[2]=
```

0.0492286 s

```
In[3]:= sumEvalscaDtyZrelationsA22Y =  $\frac{1}{7!} D[caDtyZrelationsA22Y, \{Y, 7\}] /. \{Y \rightarrow 0\} // Simplify$ 
```

```
Out[3]=
```

```
-v[8]'[0] - v[9]'[0] - v[10]'[0] - v[11]'[0] - v[12]'[0] - v[13]'[0] - v[14]'[0] - v[15]'[0]
```

```
In[4]:= sumEvalscaDtyZrelationsA21Y =
 $\frac{1}{7!} D[caDtyZrelationsA21Y, \{Y, 7\}] /. \{Y \rightarrow 0\} // Simplify$ 
```

```
-v[0]'[t] - v[1]'[t] - v[2]'[t] - v[3]'[t] -
v[4]'[t] - v[5]'[t] - v[6]'[t] - v[7]'[t]
```

```
In[5]:= sumEvalscaDtyZrelationsA22Y =
```

```

$$\frac{1}{7!} D[caDtyZrelationsA22Y, \{Y, 7\}] /. \{Y \rightarrow 0\} // Simplify$$

```

```

$$-\nu[8]'[t] - \nu[9]'[t] - \nu[10]'[t] - \nu[11]'[t] - \\ \nu[12]'[t] - \nu[13]'[t] - \nu[14]'[t] - \nu[15]'[t]$$

```

{Eigenvals, Eigenvecs} = Eigensystem[m];

Jordan decomposition of a square matrix m .

The result is a list {s, j} where s is a similarity matrix and j is the Jordan canonical form of m.

The j matrix is called the Jordan normal form or Jordan canonical form.

j is a special block-diagonal, upper-triangular matrix representation for any square matrix, featuring eigenvalues on the diagonal and ones on the superdiagonal (just above the main diagonal) for non-diagonalizable cases

{s, j} = JordanDecomposition[m]

m == s.j.Inverse[s] **s.j.Inverse[s] - m**

```
In[8]:= {sca2, jca2} = JordanDecomposition[caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]]]
```

$Out[=] =$

$$\left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right.$$

$$- \frac{-\nu[8]'[0] + \nu[15]'[0] + \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2}}{2 M},$$

$$- \frac{-\nu[8]'[0] + \nu[15]'[0] - \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2}}{2 M} \},$$

$$\left\{ 0, 0, 0, 0, - \frac{\nu[9]'[0] - \nu[14]'[0] - \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2}}{2 M}, \right.$$

$$- \frac{\nu[9]'[0] - \nu[14]'[0] + \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2}}{2 M}, 0, 0 \},$$

$$\left\{ 0, 0, - \frac{\nu[10]'[0] - \nu[13]'[0] - \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2}}{2 M}, \right.$$

$$- \frac{\nu[10]'[0] - \nu[13]'[0] + \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2}}{2 M}, 0, 0, 0, 0 \},$$

$$\left\{ - \frac{-\nu[11]'[0] + \nu[12]'[0] + \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2}}{2 M}, \right.$$

$$- \frac{-\nu[11]'[0] + \nu[12]'[0] - \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2}}{2 M},$$

$$0, 0, 0, 0, 0, 0 \}, \{ 1, 1, 0, 0, 0, 0, 0, 0 \},$$

$$\{ 0, 0, 1, 1, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 1, 1, 0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 1, 1 \} \},$$

$$\left\{ \frac{1}{2} \left(\nu[11]'[0] + \nu[12]'[0] - \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2} \right), \right.$$

$$0, 0, 0, 0, 0, 0, 0 \},$$

$$\left\{ 0, \frac{1}{2} \left(\nu[11]'[0] + \nu[12]'[0] + \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2} \right), \right.$$

$$0, 0, 0, 0, 0, 0, 0 \},$$

$$\left\{ 0, 0, \frac{1}{2} \left(\nu[10]'[0] + \nu[13]'[0] - \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2} \right), \right.$$

$$0, 0, 0, 0, 0, 0 \},$$

$$\left\{ 0, 0, 0, \frac{1}{2} \left(\nu[10]'[0] + \nu[13]'[0] + \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2} \right), \right.$$

$$0, 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0,$$

$$\frac{1}{2} \left(\nu[9]'[0] + \nu[14]'[0] - \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2} \right), 0, 0, 0 \},$$

$$\left\{ 0, 0, 0, 0, 0, \frac{1}{2} \left(\nu[9]'[0] + \nu[14]'[0] + \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2} \right), \right.$$

$$0, 0 \}, \{ 0, 0, 0, 0, 0, 0, 0,$$

$$\frac{1}{2} \left(\nu[8]'[0] + \nu[15]'[0] - \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2} \right), 0 \}, \{ 0, 0, 0, 0,$$

$$0, 0, 0, 0, \frac{1}{2} \left(\nu[8]'[0] + \nu[15]'[0] + \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2} \right) \} \}$$

```

In[]:= {valscaDtyZrelationsA22, vecscaDtyZrelationsA22} =
Eigensystem[caDtyZrelationsA2[[9 ;; 16, 9 ;; 16]]]

Out[]=
{ $\left\{\frac{1}{2} \left(\nu[11]'[0] + \nu[12]'[0] - \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2}\right), \right.$ 
 $\frac{1}{2} \left(\nu[11]'[0] + \nu[12]'[0] + \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2}\right),$ 
 $\frac{1}{2} \left(\nu[10]'[0] + \nu[13]'[0] - \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2}\right),$ 
 $\frac{1}{2} \left(\nu[10]'[0] + \nu[13]'[0] + \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2}\right),$ 
 $\frac{1}{2} \left(\nu[9]'[0] + \nu[14]'[0] - \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2}\right),$ 
 $\frac{1}{2} \left(\nu[9]'[0] + \nu[14]'[0] + \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2}\right),$ 
 $\frac{1}{2} \left(\nu[8]'[0] + \nu[15]'[0] - \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2}\right),$ 
 $\frac{1}{2} \left(\nu[8]'[0] + \nu[15]'[0] + \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2}\right)\},$ 
 $\left\{0, 0, 0, -\frac{-\nu[11]'[0] + \nu[12]'[0] + \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2}}{2 M}, \right.$ 
 $1, 0, 0, 0\}, \{0, 0, 0,$ 
 $- \frac{-\nu[11]'[0] + \nu[12]'[0] - \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2}}{2 M}, 1, 0, 0, 0\},$ 
 $\left\{0, 0, -\frac{\nu[10]'[0] - \nu[13]'[0] - \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2}}{2 M}, 0, 0, 1, \right.$ 
 $0, 0\}, \{0, 0, -\frac{\nu[10]'[0] - \nu[13]'[0] + \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2}}{2 M},$ 
 $0, 0, 1, 0, 0\},$ 
 $\left\{0, -\frac{\nu[9]'[0] - \nu[14]'[0] - \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2}}{2 M}, 0, 0, 0, 0, 1, \right.$ 
 $0\}, \{0, -\frac{\nu[9]'[0] - \nu[14]'[0] + \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2}}{2 M}, 0, 0, 0,$ 
 $0, 1, 0\}, \left\{-\frac{-\nu[8]'[0] + \nu[15]'[0] + \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2}}{2 M}, 0, 0, \right.$ 
 $0, 0, 0, 0, 1\}, \left\{-\frac{-\nu[8]'[0] + \nu[15]'[0] - \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2}}{2 M}, \right.$ 
 $0, 0, 0, 0, 0, 1\}\}$ 

In[]:= {valscaDtyZrelationsA21, vecscaDtyZrelationsA21} =
Eigensystem[caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]] //.
FullSimplify[#, e6 a4[t] > 0 && e3 a4[t] > 0 && constraintVars] & //
FullSimplify[#, e6 a4[t] > 0 && e3 a4[t] > 0 && a4[t] > 0] &

Out[]=
{ $\left\{\frac{1}{2} e^{-3 a4[0]} \left(-e^{a4[0]} Q1 a4'[0] - \right.\right.$ 

```

$$\begin{aligned}
& \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[3]'[0] - \nu[4]'[0])^2)} + \epsilon^{3 a4[0]} (Q1 a4'[0] + \nu[3]'[0] + \nu[4]'[0]) \Big), \\
& \frac{1}{2} \epsilon^{-3 a4[0]} \left(-\epsilon^{a4[0]} Q1 a4'[0] + \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[3]'[0] - \nu[4]'[0])^2)} + \right. \\
& \quad \left. \epsilon^{3 a4[0]} (Q1 a4'[0] + \nu[3]'[0] + \nu[4]'[0]) \right), \\
& \frac{1}{2} \epsilon^{-3 a4[0]} \left(-\epsilon^{a4[0]} Q1 a4'[0] - \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[2]'[0] - \nu[5]'[0])^2)} + \right. \\
& \quad \left. \epsilon^{3 a4[0]} (Q1 a4'[0] + \nu[2]'[0] + \nu[5]'[0]) \right), \\
& \frac{1}{2} \epsilon^{-3 a4[0]} \left(-\epsilon^{a4[0]} Q1 a4'[0] + \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[2]'[0] - \nu[5]'[0])^2)} + \right. \\
& \quad \left. \epsilon^{3 a4[0]} (Q1 a4'[0] + \nu[2]'[0] + \nu[5]'[0]) \right), \\
& \frac{1}{2} \epsilon^{-3 a4[0]} \left(\epsilon^{a4[0]} Q1 a4'[0] - \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[1]'[0] - \nu[6]'[0])^2)} + \right. \\
& \quad \left. \epsilon^{3 a4[0]} (-Q1 a4'[0] + \nu[1]'[0] + \nu[6]'[0]) \right), \\
& \frac{1}{2} \epsilon^{-3 a4[0]} \left(\epsilon^{a4[0]} Q1 a4'[0] + \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[1]'[0] - \nu[6]'[0])^2)} + \right. \\
& \quad \left. \epsilon^{3 a4[0]} (-Q1 a4'[0] + \nu[1]'[0] + \nu[6]'[0]) \right), \\
& \frac{1}{2} \epsilon^{-3 a4[0]} \left(\epsilon^{a4[0]} Q1 a4'[0] - \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[0]'[0] - \nu[7]'[0])^2)} + \right. \\
& \quad \left. \epsilon^{3 a4[0]} (-Q1 a4'[0] + \nu[0]'[0] + \nu[7]'[0]) \right), \\
& \frac{1}{2} \epsilon^{-3 a4[0]} \left(\epsilon^{a4[0]} Q1 a4'[0] + \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[0]'[0] - \nu[7]'[0])^2)} + \right. \\
& \quad \left. \epsilon^{3 a4[0]} (-Q1 a4'[0] + \nu[0]'[0] + \nu[7]'[0]) \right) \Big\}, \\
& \left\{ \{0, 0, 0, \frac{\epsilon^{-3 a4[0]} \left(\sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[3]'[0] - \nu[4]'[0])^2)} + \epsilon^{3 a4[0]} (-\nu[3]'[0] + \nu[4]'[0]) \right)}{2 M}, \right. \\
& \quad \left. 1, 0, 0, 0\right\}, \\
& \left\{ \{0, 0, 0, -\frac{\nu[3]'[0] + \epsilon^{-3 a4[0]} \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[3]'[0] - \nu[4]'[0])^2)} - \nu[4]'[0]}{2 M}, 1, 0, 0, 0\right\}, \\
& \left\{ \{0, 0, -\frac{-\nu[2]'[0] + \epsilon^{-3 a4[0]} \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[2]'[0] - \nu[5]'[0])^2)} + \nu[5]'[0]}{2 M}, 0, 0, 1, 0, 0\right\}, \\
& \left\{ \{0, 0, \frac{\epsilon^{-3 a4[0]} \left(\sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[2]'[0] - \nu[5]'[0])^2)} + \epsilon^{3 a4[0]} (\nu[2]'[0] - \nu[5]'[0]) \right)}{2 M}, \right. \\
& \quad \left. 0, 0, 1, 0, 0\right\}, \\
& \left\{ \{0, -\frac{-\nu[1]'[0] + \epsilon^{-3 a4[0]} \sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[1]'[0] - \nu[6]'[0])^2)} + \nu[6]'[0]}{2 M}, 0, 0, 0, 0, 1, 0\right\}, \\
& \left\{ \{0, \frac{\epsilon^{-3 a4[0]} \left(\sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[1]'[0] - \nu[6]'[0])^2)} + \epsilon^{3 a4[0]} (\nu[1]'[0] - \nu[6]'[0]) \right)}{2 M}, \right. \\
& \quad \left. 0, 0, 0, 0, 1, 0\right\}, \\
& \left\{ \{0, 1, 0\}, \left\{ \frac{\epsilon^{-3 a4[0]} \left(\sqrt{\epsilon^{6 a4[0]} (-4 M^2 + (\nu[0]'[0] - \nu[7]'[0])^2)} + \epsilon^{3 a4[0]} (-\nu[0]'[0] + \nu[7]'[0]) \right)}{2 M}, \right. \right. \\
& \quad \left. \left. 0, 0, 0, 0, 0, 1, 0\right\}, \right.
\end{aligned}$$

```

In[=] := {scal, jca1} = JordanDecomposition[caDtyZrelationsA2[[1 ;; 8, 1 ;; 8]] // 
  FullSimplify[#, e^6 a4[t] > 0 && e^3 a4[t] > 0 && constraintVars] & //
  FullSimplify[#, e^6 a4[t] > 0 && e^3 a4[t] > 0 && a4[t] > 0] &

Out[=]=
{ { { 0, 0, 0, 0, 0, 0, 0, 1 } } }

{ { { 0, 0, 0, 0, 0, 0, 0, 0 } , 
  e^-3 a4[0] ( sqrt[e^6 a4[0] (-4 M^2 + (v[0]'[0] - v[7]'[0])^2)] - v[7]'[0] ) 
  / ( 2 M ) , 
  e^-3 a4[0] ( sqrt[e^6 a4[0] (-4 M^2 + (v[0]'[0] - v[7]'[0])^2)] + e^3 a4[0] (-v[0]'[0] + v[7]'[0]) ) 
  / ( 2 M ) , 
  e^-3 a4[0] ( sqrt[e^6 a4[0] (-4 M^2 + (v[0]'[0] - v[7]'[0])^2)] - v[7]'[0] ) 
  / ( 2 M ) } , 

{ 0, 0, 0, 0, 0, 0, 0, 0 } , 
- { -v[1]'[0] + e^-3 a4[0] ( sqrt[e^6 a4[0] (-4 M^2 + (v[1]'[0] - v[6]'[0])^2)] + v[6]'[0] ) 
  / ( 2 M ) , 
  e^-3 a4[0] ( sqrt[e^6 a4[0] (-4 M^2 + (v[1]'[0] - v[6]'[0])^2)] + e^3 a4[0] (v[1]'[0] - v[6]'[0]) ) 
  / ( 2 M ) , 
  e^-3 a4[0] ( -v[2]'[0] + e^-3 a4[0] ( sqrt[e^6 a4[0] (-4 M^2 + (v[2]'[0] - v[5]'[0])^2)] + v[5]'[0] ) 
  / ( 2 M ) , 
  e^-3 a4[0] ( sqrt[e^6 a4[0] (-4 M^2 + (v[2]'[0] - v[5]'[0])^2)] + e^3 a4[0] (v[2]'[0] - v[5]'[0]) ) 
  / ( 2 M ) , 
  e^-3 a4[0] ( sqrt[e^6 a4[0] (-4 M^2 + (v[3]'[0] - v[4]'[0])^2)] + e^3 a4[0] (-v[3]'[0] + v[4]'[0]) ) 
  / ( 2 M ) , 
  e^-3 a4[0] ( -v[3]'[0] + e^-3 a4[0] ( sqrt[e^6 a4[0] (-4 M^2 + (v[3]'[0] - v[4]'[0])^2)] - v[4]'[0] ) 
  / ( 2 M ) , 
  0, 0, 0, 0, 0, 0, 0 } , 

{ 1, 1, 0, 0, 0, 0, 0, 0 } , { 0, 0, 1, 1, 0, 0, 0, 0 } , 
{ 0, 0, 0, 0, 1, 1, 0, 0 } , { 0, 0, 0, 0, 0, 0, 1, 1 } } , 

{ { { 1/2 e^-3 a4[0] ( -e^a4[0] Q1 a4'[0] - sqrt[e^6 a4[0] (-4 M^2 + (v[3]'[0] - v[4]'[0])^2)] + 
  e^3 a4[0] (Q1 a4'[0] + v[3]'[0] + v[4]'[0]) ) , 0, 0, 0, 0, 0, 0, 0 } , 
  { 0, 1/2 e^-3 a4[0] ( -e^a4[0] Q1 a4'[0] + sqrt[e^6 a4[0] (-4 M^2 + (v[3]'[0] - v[4]'[0])^2)] + 
  e^3 a4[0] (Q1 a4'[0] + v[3]'[0] + v[4]'[0]) ) , 0, 0, 0, 0, 0, 0 } , 
  { 0, 0, 1/2 e^-3 a4[0] ( -e^a4[0] Q1 a4'[0] - sqrt[e^6 a4[0] (-4 M^2 + (v[2]'[0] - v[5]'[0])^2)] + 
  e^3 a4[0] (Q1 a4'[0] + v[2]'[0] + v[5]'[0]) ) , 0, 0, 0, 0, 0, 0 } , 
  { 0, 0, 0, 1/2 e^-3 a4[0] ( -e^a4[0] Q1 a4'[0] + sqrt[e^6 a4[0] (-4 M^2 + (v[2]'[0] - v[5]'[0])^2)] + 
  e^3 a4[0] (Q1 a4'[0] + v[2]'[0] + v[5]'[0]) ) , 0, 0, 0, 0, 0, 0 } , 
  { 0, 0, 0, 0, 1/2 e^-3 a4[0] ( e^a4[0] Q1 a4'[0] - sqrt[e^6 a4[0] (-4 M^2 + (v[1]'[0] - v[6]'[0])^2)] + 
  e^3 a4[0] (Q1 a4'[0] + v[1]'[0] + v[6]'[0]) ) , 0, 0, 0, 0, 0, 0 } }

```

$$\begin{aligned}
& \left. e^{3 a4[0]} (-Q1 a4'[0] + \nu[1]'[0] + \nu[6]'[0]) \right), 0, 0, 0 \Big\}, \\
& \left. \left\{ 0, 0, 0, 0, 0, \frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[1]'[0] - \nu[6]'[0])^2)} \right) + \right. \right. \\
& \left. \left. e^{3 a4[0]} (-Q1 a4'[0] + \nu[1]'[0] + \nu[6]'[0]) \right), 0, 0 \Big\}, \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] - \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[0]'[0] - \nu[7]'[0])^2)} \right) + \right. \right. \\
& \left. \left. e^{3 a4[0]} (-Q1 a4'[0] + \nu[0]'[0] + \nu[7]'[0]) \right), 0 \Big\}, \\
& \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[0]'[0] - \nu[7]'[0])^2)} \right) + \right. \right. \\
& \left. \left. e^{3 a4[0]} (-Q1 a4'[0] + \nu[0]'[0] + \nu[7]'[0]) \right) \right\} \Big\}
\end{aligned}$$

```
In[8]:= eigenValues = Union[Flatten[{valscaDtyZrelationsA21, valscaDtyZrelationsA22}]]
eigenValues00 = FullSimplify[Normal[Series[#, {t, 0, 0}]]] & /@ eigenValues
```

Out[1]=

$$\left\{ \frac{1}{2} e^{-3 a4[0]} \left(-e^{a4[0]} Q1 a4'[0] - \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[3]'[0] - \nu[4]'[0])^2)} + e^{3 a4[0]} (Q1 a4'[0] + \nu[3]'[0] + \nu[4]'[0]) \right), \right.$$

$$\frac{1}{2} e^{-3 a4[0]} \left(-e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[3]'[0] - \nu[4]'[0])^2)} + e^{3 a4[0]} (Q1 a4'[0] + \nu[3]'[0] + \nu[4]'[0]) \right),$$

$$\frac{1}{2} e^{-3 a4[0]} \left(-e^{a4[0]} Q1 a4'[0] - \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[2]'[0] - \nu[5]'[0])^2)} + e^{3 a4[0]} (Q1 a4'[0] + \nu[2]'[0] + \nu[5]'[0]) \right),$$

$$\frac{1}{2} e^{-3 a4[0]} \left(-e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[2]'[0] - \nu[5]'[0])^2)} + e^{3 a4[0]} (Q1 a4'[0] + \nu[2]'[0] + \nu[5]'[0]) \right),$$

$$\frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] - \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[1]'[0] - \nu[6]'[0])^2)} + e^{3 a4[0]} (-Q1 a4'[0] + \nu[1]'[0] + \nu[6]'[0]) \right),$$

$$\frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[1]'[0] - \nu[6]'[0])^2)} + e^{3 a4[0]} (-Q1 a4'[0] + \nu[1]'[0] + \nu[6]'[0]) \right),$$

$$\frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] - \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[0]'[0] - \nu[7]'[0])^2)} + e^{3 a4[0]} (-Q1 a4'[0] + \nu[0]'[0] + \nu[7]'[0]) \right),$$

$$\frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[0]'[0] - \nu[7]'[0])^2)} + e^{3 a4[0]} (-Q1 a4'[0] + \nu[0]'[0] + \nu[7]'[0]) \right),$$

$$\frac{1}{2} \left(\nu[11]'[0] + \nu[12]'[0] - \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2} \right),$$

$$\frac{1}{2} \left(\nu[11]'[0] + \nu[12]'[0] + \sqrt{-4 M^2 + \nu[11]'[0]^2 - 2 \nu[11]'[0] \nu[12]'[0] + \nu[12]'[0]^2} \right),$$

$$\frac{1}{2} \left(\nu[10]'[0] + \nu[13]'[0] - \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2} \right),$$

$$\frac{1}{2} \left(\nu[10]'[0] + \nu[13]'[0] + \sqrt{-4 M^2 + \nu[10]'[0]^2 - 2 \nu[10]'[0] \nu[13]'[0] + \nu[13]'[0]^2} \right),$$

$$\frac{1}{2} \left(\nu[9]'[0] + \nu[14]'[0] - \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2} \right),$$

$$\frac{1}{2} \left(\nu[9]'[0] + \nu[14]'[0] + \sqrt{-4 M^2 + \nu[9]'[0]^2 - 2 \nu[9]'[0] \nu[14]'[0] + \nu[14]'[0]^2} \right),$$

$$\frac{1}{2} \left(\nu[8]'[0] + \nu[15]'[0] - \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2} \right),$$

$$\left. \frac{1}{2} \left(\nu[8]'[0] + \nu[15]'[0] + \sqrt{-4 M^2 + \nu[8]'[0]^2 - 2 \nu[8]'[0] \nu[15]'[0] + \nu[15]'[0]^2} \right) \right\}$$

$$\begin{aligned}
Out[\#] = & \left\{ \frac{1}{2} e^{-3 a4[0]} \left(-e^{a4[0]} Q1 a4'[0] - \right. \right. \\
& \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[3]'[0] - \nu[4]'[0])^2)} + e^{3 a4[0]} (Q1 a4'[0] + \nu[3]'[0] + \nu[4]'[0]) \Big), \\
& \frac{1}{2} e^{-3 a4[0]} \left(-e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[3]'[0] - \nu[4]'[0])^2)} + \right. \\
& \left. e^{3 a4[0]} (Q1 a4'[0] + \nu[3]'[0] + \nu[4]'[0]) \right), \\
& \frac{1}{2} e^{-3 a4[0]} \left(-e^{a4[0]} Q1 a4'[0] - \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[2]'[0] - \nu[5]'[0])^2)} + \right. \\
& \left. e^{3 a4[0]} (Q1 a4'[0] + \nu[2]'[0] + \nu[5]'[0]) \right), \\
& \frac{1}{2} e^{-3 a4[0]} \left(-e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[2]'[0] - \nu[5]'[0])^2)} + \right. \\
& \left. e^{3 a4[0]} (Q1 a4'[0] + \nu[2]'[0] + \nu[5]'[0]) \right), \\
& \frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] - \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[1]'[0] - \nu[6]'[0])^2)} + \right. \\
& \left. e^{3 a4[0]} (-Q1 a4'[0] + \nu[1]'[0] + \nu[6]'[0]) \right), \\
& \frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[1]'[0] - \nu[6]'[0])^2)} + \right. \\
& \left. e^{3 a4[0]} (-Q1 a4'[0] + \nu[1]'[0] + \nu[6]'[0]) \right), \\
& \frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] - \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[0]'[0] - \nu[7]'[0])^2)} + \right. \\
& \left. e^{3 a4[0]} (-Q1 a4'[0] + \nu[0]'[0] + \nu[7]'[0]) \right), \\
& \frac{1}{2} e^{-3 a4[0]} \left(e^{a4[0]} Q1 a4'[0] + \sqrt{e^{6 a4[0]} (-4 M^2 + (\nu[0]'[0] - \nu[7]'[0])^2)} + \right. \\
& \left. e^{3 a4[0]} (-Q1 a4'[0] + \nu[0]'[0] + \nu[7]'[0]) \right), \\
& \frac{1}{2} \left(\nu[11]'[0] - \sqrt{-4 M^2 + (\nu[11]'[0] - \nu[12]'[0])^2} + \nu[12]'[0] \right), \\
& \frac{1}{2} \left(\nu[11]'[0] + \sqrt{-4 M^2 + (\nu[11]'[0] - \nu[12]'[0])^2} + \nu[12]'[0] \right), \\
& \frac{1}{2} \left(\nu[10]'[0] - \sqrt{-4 M^2 + (\nu[10]'[0] - \nu[13]'[0])^2} + \nu[13]'[0] \right), \\
& \frac{1}{2} \left(\nu[10]'[0] + \sqrt{-4 M^2 + (\nu[10]'[0] - \nu[13]'[0])^2} + \nu[13]'[0] \right), \\
& \frac{1}{2} \left(\nu[9]'[0] - \sqrt{-4 M^2 + (\nu[9]'[0] - \nu[14]'[0])^2} + \nu[14]'[0] \right), \\
& \frac{1}{2} \left(\nu[9]'[0] + \sqrt{-4 M^2 + (\nu[9]'[0] - \nu[14]'[0])^2} + \nu[14]'[0] \right), \\
& \frac{1}{2} \left(\nu[8]'[0] - \sqrt{-4 M^2 + (\nu[8]'[0] - \nu[15]'[0])^2} + \nu[15]'[0] \right), \\
& \left. \frac{1}{2} \left(\nu[8]'[0] + \sqrt{-4 M^2 + (\nu[8]'[0] - \nu[15]'[0])^2} + \nu[15]'[0] \right) \right\}
\end{aligned}$$

If superluminal inflation/deflation then pair of universes

with MASSES $\pm M$?

THE FOLLOWING IS CONSISTENT WITH UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES $\pm M$;

Note that we are ONLY looking for superluminal inflation or deflation type solutions,

and NOT solutions that are even/odd functions of t

(like $\text{Cos}[v[j][0] * t]$, $\text{Sin}[v[j][0] * t]$, $\text{Sech}[v[j][0] * t]$, $\text{Tanh}[v[j][0] * t]$, ...),

which might also have $\pm M$ type eigenvalues, or even solutions involving the well-known special functions that might also have $\pm M$ type eigenvalues.

```
In[1]:= sDt v3Dt v4 = Solve[And @@ Thread[0 == eigenValues00[[1 ;; 2]], {v[3]'[0], v[4]'[0]}] // TrigToExp(*//FullSimplify*)]
```

```
Out[1]= {{v[3]'[0] → M - 1/2 Q1 a4'[0] + 1/2 e^-2 a4[0] Q1 a4'[0], v[4]'[0] → -M - 1/2 Q1 a4'[0] + 1/2 e^-2 a4[0] Q1 a4'[0]}, {v[3]'[0] → -M - 1/2 Q1 a4'[0] + 1/2 e^-2 a4[0] Q1 a4'[0], v[4]'[0] → M - 1/2 Q1 a4'[0] + 1/2 e^-2 a4[0] Q1 a4'[0]}}
```

```
In[2]:= sDt v2Dt v5 = Solve[And @@ Thread[0 == eigenValues00[[3 ;; 4]], {v[2]'[0], v[5]'[0]}] // TrigToExp(*//FullSimplify*)]
```

```
Out[2]= {{v[2]'[0] → M - 1/2 Q1 a4'[0] + 1/2 e^-2 a4[0] Q1 a4'[0], v[5]'[0] → -M - 1/2 Q1 a4'[0] + 1/2 e^-2 a4[0] Q1 a4'[0]}, {v[2]'[0] → -M - 1/2 Q1 a4'[0] + 1/2 e^-2 a4[0] Q1 a4'[0], v[5]'[0] → M - 1/2 Q1 a4'[0] + 1/2 e^-2 a4[0] Q1 a4'[0]}}
```

```
In[3]:= sDt v1Dt v6 = Solve[And @@ Thread[0 == eigenValues00[[5 ;; 6]], {v[1]'[0], v[6]'[0]}] // TrigToExp(*//FullSimplify*)]
```

```
Out[3]= {{v[1]'[0] → M + 1/2 Q1 a4'[0] - 1/2 e^-2 a4[0] Q1 a4'[0], v[6]'[0] → -M + 1/2 Q1 a4'[0] - 1/2 e^-2 a4[0] Q1 a4'[0]}, {v[1]'[0] → -M + 1/2 Q1 a4'[0] - 1/2 e^-2 a4[0] Q1 a4'[0], v[6]'[0] → M + 1/2 Q1 a4'[0] - 1/2 e^-2 a4[0] Q1 a4'[0]}}
```

```
In[1]:= sDtv0Dtv7 = Solve[And @@ Thread[0 == eigenValues00[[7 ;; 8]], {v[0]'[0], v[7]'[0]}] // TrigToExp(*//FullSimplify*)]

Out[1]=
{ {v[0]'[0] → M + 1/2 Q1 a4'[0] - 1/2 e^-2 a4[0] Q1 a4'[0], v[7]'[0] → -M + 1/2 Q1 a4'[0] - 1/2 e^-2 a4[0] Q1 a4'[0]}, {v[0]'[0] → -M + 1/2 Q1 a4'[0] - 1/2 e^-2 a4[0] Q1 a4'[0], v[7]'[0] → M + 1/2 Q1 a4'[0] - 1/2 e^-2 a4[0] Q1 a4'[0]} }

In[2]:= sDtv11Dtv12 =
Solve[And @@ Thread[0 == eigenValues00[[9 ;; 10]], {v[11]'[0], v[12]'[0]}] // FullSimplify]

Out[2]=
{ {v[11]'[0] → M, v[12]'[0] → -M}, {v[11]'[0] → -M, v[12]'[0] → M} }

In[3]:= sDtv10Dtv13 =
Solve[And @@ Thread[0 == eigenValues00[[11 ;; 12]], {v[10]'[0], v[13]'[0]}] // FullSimplify]

Out[3]=
{ {v[10]'[0] → M, v[13]'[0] → -M}, {v[10]'[0] → -M, v[13]'[0] → M} }

In[4]:= sDtv9Dtv14 =
Solve[And @@ Thread[0 == eigenValues00[[13 ;; 14]], {v[9]'[0], v[14]'[0]}] // FullSimplify]

Out[4]=
{ {v[9]'[0] → M, v[14]'[0] → -M}, {v[9]'[0] → -M, v[14]'[0] → M} }

In[5]:= sDtv8Dtv15 =
Solve[And @@ Thread[0 == eigenValues00[[15 ;; 16]], {v[8]'[0], v[15]'[0]}] // FullSimplify]

Out[5]=
{ {v[8]'[0] → M, v[15]'[0] → -M}, {v[8]'[0] → -M, v[15]'[0] → M} }
```

TRY TO FIND ANY SOLUTION TO the Euler-Lagrange EQS :

Task: WRITE OUT THE VARIABLES in a ROW, THAT ARE COUPLED TO ANOTHER VARIABLE in that row :

```
In[1]:= Clear[resolveWithRemainder];
resolveWithRemainder[xy_, wrt_List] := Module[{b},
  b = D[xy, #] & /@ wrt;
  Return[Flatten[{b, {xy - b.wrt}}]]];
```

Reminder: In Wolfram Mathematica, `FullForm` is a display wrapper .

When you call `FullForm[list]`, the resulting expression has the Head `FullForm`, not `List` .

Because `Take` expects its first argument to have a head like `List`, `Association`, or another expression it can act upon, it fails when it sees the `FullForm` wrapper .

Order of Evaluation: `Take[FullForm[any_list], -1]` fails because `FullForm` is essentially a "tag" telling Mathematica how to print the list, but `Take` does not know how to handle that tag.

Even when an output omits `FullForm` from the top level, it is not stripped from subexpressions:

```

In[=]:= Clear[resolve18];
resolve18[j_] :=
Module[{yZs1, a0, a1, a2, a3, a4, a5, a6, a7, a8, a9, b0, b1, b2, b3, b4, b5},
a0 = resolved18[[j]];
a1 = FullSimplify[a0 /. {Times → imes, Derivative → der3}];
a2 = Flatten[a1];
a3 = Union[a2];
yZs1 = Take[a3, -4];
a4 = eextract[#] & /@ yZs1;
a5 = Flatten[a4];
a6 = Union[a5];
AppendTo[yZs1, Take[a6, 2]];
a7 = Take[a6, -4];
a8 = eextract[#] & /@ a7;
a9 = Flatten[a8];
b0 = Union[a9];
AppendTo[yZs1, Take[b0, 2]];
AppendTo[yZs1, Union[Flatten[Take[yZs1, -2]]]];
Return[{j, yZs1[[-1]]}];

In[=]:= Clear[resolve28];
resolve28[j_] :=
Module[{yZs1, a0, a1, a2, a3, a4, a5, a6, a7, a8, a9, b0, b1, b2, b3, b4, b5},
a0 = resolved28[[j]];
a1 = FullSimplify[a0 /. {Times → imes, Derivative → der3}];
a2 = Flatten[a1];
a3 = Union[a2];
yZs1 = Take[a3, -3];
a4 = eextract[#] & /@ yZs1;
a5 = Flatten[a4];
a6 = Union[a5];
AppendTo[yZs1, Take[a6, 2]];
a7 = Take[a6, -1];
a8 = eextract[#] & /@ a7;
a9 = Flatten[a8];
b0 = Union[a9];
AppendTo[yZs1, Take[b0, 1]];
AppendTo[yZs1, Union[Flatten[Take[yZs1, -2]]]];
Return[{8 + j, yZs1[[-1]]}];
```

```
In[8]:= ToString[FullForm[#]] & /@ DtyZsubs;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(DtyZsubsrelations = ToExpression[##] & /@ %) // Column
resolveWithRemainder[#, {M, a4'[t], Tan[z]}] & /@ %[[1]] // 
Column (* INSPECT: DEDUCE 0-7, 8-15 COUPLINGS *)
```

```
Out[8]= M yZ[7][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] +
yZ[0]^(0,1)[z, t] + 6 Tan[z] yZ[3]^(1,0)[z, t]
-M yZ[6][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] +
yZ[1]^(0,1)[z, t] - 6 Tan[z] yZ[2]^(1,0)[z, t]
-M yZ[5][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] +
yZ[2]^(0,1)[z, t] - 6 Tan[z] yZ[1]^(1,0)[z, t]
M yZ[4][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] +
yZ[3]^(0,1)[z, t] + 6 Tan[z] yZ[0]^(1,0)[z, t]
-M yZ[3][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
yZ[4]^(0,1)[z, t] - 6 Tan[z] yZ[7]^(1,0)[z, t]
M yZ[2][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] +
yZ[5]^(0,1)[z, t] + 6 Tan[z] yZ[6]^(1,0)[z, t]
M yZ[1][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] +
yZ[6]^(0,1)[z, t] + 6 Tan[z] yZ[5]^(1,0)[z, t]
-M yZ[0][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] +
yZ[7]^(0,1)[z, t] - 6 Tan[z] yZ[4]^(1,0)[z, t]
-M yZ[15][z, t] + yZ[8]^(0,1)[z, t] - 6 Tan[z] yZ[11]^(1,0)[z, t]
M yZ[14][z, t] + yZ[9]^(0,1)[z, t] + 6 Tan[z] yZ[10]^(1,0)[z, t]
M yZ[13][z, t] + yZ[10]^(0,1)[z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t]
-M yZ[12][z, t] + yZ[11]^(0,1)[z, t] - 6 Tan[z] yZ[8]^(1,0)[z, t]
M yZ[11][z, t] + yZ[12]^(0,1)[z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t]
-M yZ[10][z, t] + yZ[13]^(0,1)[z, t] - 6 Tan[z] yZ[14]^(1,0)[z, t]
-M yZ[9][z, t] + yZ[14]^(0,1)[z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t]
M yZ[8][z, t] + yZ[15]^(0,1)[z, t] + 6 Tan[z] yZ[12]^(1,0)[z, t]
```

```

Out[ ]=
{yZ[7][z, t], -e-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t],
 6 yZ[3](1,0)[z, t], yZ[0](0,1)[z, t]}

{-yZ[6][z, t], -e-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t],
 -6 yZ[2](1,0)[z, t], yZ[1](0,1)[z, t]}

{-yZ[5][z, t], e-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t],
 -6 yZ[1](1,0)[z, t], yZ[2](0,1)[z, t]}

{yZ[4][z, t], e-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t],
 6 yZ[0](1,0)[z, t], yZ[3](0,1)[z, t]}

{-yZ[3][z, t], e-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t],
 -6 yZ[7](1,0)[z, t], yZ[4](0,1)[z, t]}

{yZ[2][z, t], e-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t],
 6 yZ[6](1,0)[z, t], yZ[5](0,1)[z, t]}

{yZ[1][z, t], -e-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t],
 6 yZ[5](1,0)[z, t], yZ[6](0,1)[z, t]}

{-yZ[0][z, t], -e-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t],
 -6 yZ[4](1,0)[z, t], yZ[7](0,1)[z, t]}

{-yZ[15][z, t], 0, -6 yZ[11](1,0)[z, t], yZ[8](0,1)[z, t]}

{yZ[14][z, t], 0, 6 yZ[10](1,0)[z, t], yZ[9](0,1)[z, t]}

{yZ[13][z, t], 0, 6 yZ[9](1,0)[z, t], yZ[10](0,1)[z, t]}

{-yZ[12][z, t], 0, -6 yZ[8](1,0)[z, t], yZ[11](0,1)[z, t]}

{yZ[11][z, t], 0, 6 yZ[15](1,0)[z, t], yZ[12](0,1)[z, t]}

{-yZ[10][z, t], 0, -6 yZ[14](1,0)[z, t], yZ[13](0,1)[z, t]}

{-yZ[9][z, t], 0, -6 yZ[13](1,0)[z, t], yZ[14](0,1)[z, t]}

{yZ[8][z, t], 0, 6 yZ[12](1,0)[z, t], yZ[15](0,1)[z, t]}

```

BEGIN: Create 2 SETS OF COUPLED VARIABLES :

```
In[1]:= (Clear[resolved18];
resolved18 = resolveWithRemainder[#, {M, a4'[t], Tan[z]}] & /@
Take[DtyZsubsrelations, 8]) // Column
```

```
Out[1]= {yZ[7][z, t], -e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t],
6 yZ[3]^(1,0)[z, t], yZ[0]^(0,1)[z, t]}
{-yZ[6][z, t], -e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t],
-6 yZ[2]^(1,0)[z, t], yZ[1]^(0,1)[z, t]}
{-yZ[5][z, t], e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t],
-6 yZ[1]^(1,0)[z, t], yZ[2]^(0,1)[z, t]}
{yZ[4][z, t], e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t],
6 yZ[0]^(1,0)[z, t], yZ[3]^(0,1)[z, t]}
{-yZ[3][z, t], e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t],
-6 yZ[7]^(1,0)[z, t], yZ[4]^(0,1)[z, t]}
{yZ[2][z, t], e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t],
6 yZ[6]^(1,0)[z, t], yZ[5]^(0,1)[z, t]}
{yZ[1][z, t], -e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t],
6 yZ[5]^(1,0)[z, t], yZ[6]^(0,1)[z, t]}
{-yZ[0][z, t], -e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t],
-6 yZ[4]^(1,0)[z, t], yZ[7]^(0,1)[z, t]}
```

```
In[2]:= (Clear[resolved28];
resolved28 = resolveWithRemainder[#, {M, a4'[t], Tan[z]}] & /@
Take[DtyZsubsrelations, -8]) // Column
```

```
Out[2]= {-yZ[15][z, t], 0, -6 yZ[11]^(1,0)[z, t], yZ[8]^(0,1)[z, t]}
{yZ[14][z, t], 0, 6 yZ[10]^(1,0)[z, t], yZ[9]^(0,1)[z, t]}
{yZ[13][z, t], 0, 6 yZ[9]^(1,0)[z, t], yZ[10]^(0,1)[z, t]}
{-yZ[12][z, t], 0, -6 yZ[8]^(1,0)[z, t], yZ[11]^(0,1)[z, t]}
{yZ[11][z, t], 0, 6 yZ[15]^(1,0)[z, t], yZ[12]^(0,1)[z, t]}
{-yZ[10][z, t], 0, -6 yZ[14]^(1,0)[z, t], yZ[13]^(0,1)[z, t]}
{-yZ[9][z, t], 0, -6 yZ[13]^(1,0)[z, t], yZ[14]^(0,1)[z, t]}
{yZ[8][z, t], 0, 6 yZ[12]^(1,0)[z, t], yZ[15]^(0,1)[z, t]}
```

BEGIN: WRITE OUT THE VARIABLES in a ROW, THAT ARE COUPLED TO ANOTHER VARIABLE in that row :

```
In[=]:= Clear[rowsAndVarsFor18]; rowsAndVarsFor18 = {resolve18[1]};
AppendTo[rowsAndVarsFor18, resolve18[#]] & /@ Range[2, 8];
rowsAndVarsFor18
Dimensions[rowsAndVarsFor18]
```

```
Out[=]= {{1, {0, 3, 7}}, {2, {1, 2, 6}}, {3, {1, 2, 5}}, {4, {0, 3, 4}}, {5, {3, 4, 7}}, {6, {2, 5, 6}}, {7, {1, 5, 6}}, {8, {0, 4, 7}}}
Out[=]= {8, 2}
```

```
In[=]:= Clear[rowsAndVarsFor28]; rowsAndVarsFor28 = {resolve28[1]};
AppendTo[rowsAndVarsFor28, resolve28[#]] & /@ Range[2, 8];
rowsAndVarsFor28
Dimensions[rowsAndVarsFor28]
```

```
Out[=]= {{9, {8, 11, 15}}, {10, {9, 10, 14}}, {11, {9, 10, 13}}, {12, {8, 11, 12}}, {13, {11, 12, 15}}, {14, {10, 13, 14}}, {15, {9, 13, 14}}, {16, {8, 12, 15}}}
Out[=]= {8, 2}
```

WRITE OUT 1st SET of VARIABLES in a ROW, THAT ARE COUPLED TO ANOTHER VARIABLE in that row :

```
In[=]:= Select[rowsAndVarsFor18, Intersection[#[[2]], {0}] == {0} &];
(* STARTED, ALMOST THERE; KEEP GOING *)
Complement[%[[1]][[2]], {0}][[1]];
Select[rowsAndVarsFor18,
Intersection[#[[2]], {0}] == {} || Intersection[#[[2]], {0}] == {0} &];
rows1For18 = #[[1]] & /@ %
vars1For18 = Union[Flatten[#[[2]] & /@ %]]
rows2For18 = Complement[Range[8], %]
vars2For18 = Complement[Range[0, 7], %]
```

```
Out[=]= {1, 4, 5, 8}
```

```
Out[=]= {0, 3, 4, 7}
```

```
Out[=]= {2, 3, 6, 7}
```

```
Out[=]= {1, 2, 5, 6}
```

WRITE OUT 2nd SET of VARIABLES in a ROW, THAT ARE COUPLED TO ANOTHER VARIABLE in that row :

```

In[]:= Select[rowsAndVarsFor28, Intersection[#[[2]], {8}] == {8} &];
(* STARTED, ALMOST THERE; KEEP GOING *)
Complement[%[[1]][[2]], {8}][[1]];
Select[rowsAndVarsFor28,
  Intersection[#[[2]], {8}] == {8} || Intersection[#[[2]], {8}] == {8} &];
rows1For28 = #[[1]] & /@ %
vars1For28 = Union[Flatten[#[[2]] & /@ %]];
rows2For28 = Complement[Range[9, 16], %];
vars2For28 = Complement[Range[8, 15], %];

Out[]= {9, 12, 13, 16}

Out[]= {8, 11, 12, 15}

Out[]= {10, 11, 14, 15}

Out[]= {9, 10, 13, 14}

In[]:= (eq118 = ToExpression[#] & /@ (StringReplace[#, "Rule" → "Equal"] & /@
  ToString[FullForm[#]] & /@ (DtyZsubs[[#]] & /@ rows1For18))) // Column

Out[=]
yZ[0]^(0,1)[z, t] ==
-M yZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - 6 Tan[z] yZ[3]^(1,0)[z, t]
yZ[3]^(0,1)[z, t] ==
-M yZ[4][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t]
yZ[4]^(0,1)[z, t] ==
M yZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] + 6 Tan[z] yZ[7]^(1,0)[z, t]
yZ[7]^(0,1)[z, t] ==
M yZ[0][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t]

In[]:= (eq218 = ToExpression[#] & /@ (StringReplace[#, "Rule" → "Equal"] & /@
  ToString[FullForm[#]] & /@ (DtyZsubs[[#]] & /@ rows2For18))) // Column

Out[=]
yZ[1]^(0,1)[z, t] ==
M yZ[6][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]
yZ[2]^(0,1)[z, t] ==
M yZ[5][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] + 6 Tan[z] yZ[1]^(1,0)[z, t]
yZ[5]^(0,1)[z, t] ==
-M yZ[2][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]
yZ[6]^(0,1)[z, t] ==
-M yZ[1][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - 6 Tan[z] yZ[5]^(1,0)[z, t]

```

```
In[=]:= (eq128 = ToExpression[##] & /@ (StringReplace[#, "Rule" → "Equal"] & /@
(ToString[FullForm[##]] & /@ (DtyZsubs[##] & /@ rows1For28)))) // Column
Out[=]=
yZ[8]^(0,1)[z, t] == M yZ[15][z, t] + 6 Tan[z] yZ[11]^(1,0)[z, t]
yZ[11]^(0,1)[z, t] == M yZ[12][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t]
yZ[12]^(0,1)[z, t] == -M yZ[11][z, t] - 6 Tan[z] yZ[15]^(1,0)[z, t]
yZ[15]^(0,1)[z, t] == -M yZ[8][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t]

In[=]:= (eq228 = ToExpression[##] & /@ (StringReplace[#, "Rule" → "Equal"] & /@
(ToString[FullForm[##]] & /@ (DtyZsubs[##] & /@ rows2For28)))) // Column
Out[=]=
yZ[9]^(0,1)[z, t] == -M yZ[14][z, t] - 6 Tan[z] yZ[10]^(1,0)[z, t]
yZ[10]^(0,1)[z, t] == -M yZ[13][z, t] - 6 Tan[z] yZ[9]^(1,0)[z, t]
yZ[13]^(0,1)[z, t] == M yZ[10][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]
yZ[14]^(0,1)[z, t] == M yZ[9][z, t] + 6 Tan[z] yZ[13]^(1,0)[z, t]

UNSURPRISING:

In[=]:= DSolve[eq228, yZ[##][z, t] & /@ vars2For28, {z, t}]
Out[=]=
DSolve[{yZ[9]^(0,1)[z, t] == -M yZ[14][z, t] - 6 Tan[z] yZ[10]^(1,0)[z, t],
yZ[10]^(0,1)[z, t] == -M yZ[13][z, t] - 6 Tan[z] yZ[9]^(1,0)[z, t],
yZ[13]^(0,1)[z, t] == M yZ[10][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t],
yZ[14]^(0,1)[z, t] == M yZ[9][z, t] + 6 Tan[z] yZ[13]^(1,0)[z, t]},
{yZ[9][z, t], yZ[10][z, t], yZ[13][z, t], yZ[14][z, t]}, {z, t}]
```

TRY MAPLE :

FOR MAPLE :

```
In[=]:= syZzt =
yZ[##] → ToExpression["((yZ" <> ToString[##] <> "[#1,#2])&]" & /@ Range[0, 15]
Out[=]=
{yZ[0] → (yZ0[#1, #2] &), yZ[1] → (yZ1[#1, #2] &), yZ[2] → (yZ2[#1, #2] &),
yZ[3] → (yZ3[#1, #2] &), yZ[4] → (yZ4[#1, #2] &), yZ[5] → (yZ5[#1, #2] &),
yZ[6] → (yZ6[#1, #2] &), yZ[7] → (yZ7[#1, #2] &), yZ[8] → (yZ8[#1, #2] &),
yZ[9] → (yZ9[#1, #2] &), yZ[10] → (yZ10[#1, #2] &), yZ[11] → (yZ11[#1, #2] &),
yZ[12] → (yZ12[#1, #2] &), yZ[13] → (yZ13[#1, #2] &),
yZ[14] → (yZ14[#1, #2] &), yZ[15] → (yZ15[#1, #2] &)}
```

```

In[=]:= eq228 /. syZzt
FullForm[%]

Out[=]= {yZ9^(0,1)[z, t] == -M yZ14[z, t] - 6 Tan[z] yZ10^(1,0)[z, t],
yZ10^(0,1)[z, t] == -M yZ13[z, t] - 6 Tan[z] yZ9^(1,0)[z, t],
yZ13^(0,1)[z, t] == M yZ10[z, t] + 6 Tan[z] yZ14^(1,0)[z, t],
yZ14^(0,1)[z, t] == M yZ9[z, t] + 6 Tan[z] yZ13^(1,0)[z, t]}

Out[=]//FullForm=
List[Equal[Derivative[0, 1][yZ9][z, t], Plus[Times[-1, M, yZ14[z, t]],
Times[-6, Tan[z], Derivative[1, 0][yZ10][z, t]]]],
Equal[Derivative[0, 1][yZ10][z, t], Plus[Times[-1, M, yZ13[z, t]],
Times[-6, Tan[z], Derivative[1, 0][yZ9][z, t]]]],
Equal[Derivative[0, 1][yZ13][z, t],
Plus[Times[M, yZ10[z, t]], Times[6, Tan[z], Derivative[1, 0][yZ14][z, t]]]],
Equal[Derivative[0, 1][yZ14][z, t],
Plus[Times[M, yZ9[z, t]], Times[6, Tan[z], Derivative[1, 0][yZ13][z, t]]]]]

In[=]:= yZ[#][z, t] & /@ vars2For28
% /. syZzt
FullForm[%]

Out[=]= {yZ[9][z, t], yZ[10][z, t], yZ[13][z, t], yZ[14][z, t]}

Out[=]= {yZ9[z, t], yZ10[z, t], yZ13[z, t], yZ14[z, t]}

Out[=]//FullForm=
List[yZ9[z, t], yZ10[z, t], yZ13[z, t], yZ14[z, t]]

```

$$\begin{aligned}
& \left\{ yZ10(z, t) = \frac{1}{M} \left(\sin(z)^{-\frac{\sqrt{M^2 - c228^2}}{6}} \left(-(\cos(c228t)c4 \right. \right. \right. \\
& + \sin(c228t)c3) \left(\sin(z)^{\frac{\sqrt{M^2 - c228^2}}{3}} c1 \right. \\
& \left. \left. \left. - c2 \right) \sqrt{M^2 - c228^2} + \left(\sin(z)^{\frac{\sqrt{M^2 - c228^2}}{3}} c5 \right. \right. \\
& \left. \left. \left. + c6 \right) (\cos(c228t)c7 - \sin(c228t)c8)c228 \right) \right), yZ13(z, t) \right. \\
& = (c7 \sin(c228t) + c8 \cos(c228t)) \left(c5 \sin(z)^{\frac{\sqrt{M^2 - c228^2}}{6}} \right. \\
& \left. \left. + c6 \sin(z)^{-\frac{\sqrt{M^2 - c228^2}}{6}} \right), yZ14(z, t) = (\cos(c228t)c4 \right. \\
& + \sin(c228t)c3) \left(c1 \sin(z)^{\frac{\sqrt{M^2 - c228^2}}{6}} \right. \\
& \left. \left. + c2 \sin(z)^{-\frac{\sqrt{M^2 - c228^2}}{6}} \right), yZ9(z, t) \right. \\
& = \frac{1}{M} \left(\sin(z)^{-\frac{\sqrt{M^2 - c228^2}}{6}} \left(- \left(\sin(z)^{\frac{\sqrt{M^2 - c228^2}}{3}} c5 \right. \right. \right. \\
& \left. \left. \left. - c6 \right) (c7 \sin(c228t) + c8 \cos(c228t)) \sqrt{M^2 - c228^2} \right. \right. \\
& \left. \left. \left. + \left(\sin(z)^{\frac{\sqrt{M^2 - c228^2}}{3}} c1 + c2 \right) c228 (c3 \cos(c228t) \right. \right. \right. \\
& \left. \left. \left. - \sin(c228t)c4) \right) \right) \right\}
\end{aligned}$$

Out[]=

$$\begin{aligned}
& \left\{ yZI0(z, t) = \frac{1}{M} \left(\sin(z) - \frac{\sqrt{M^2 - c228^2}}{6} \left(-(\cos(c228t)c4 \right. \right. \right. \\
& + \sin(c228t)c3) \left(\sin(z) \frac{\sqrt{M^2 - c228^2}}{3} cI \right. \\
& \left. \left. \left. - c2 \right) \sqrt{M^2 - c228^2} + \left(\sin(z) \frac{\sqrt{M^2 - c228^2}}{3} c5 \right. \right. \\
& \left. \left. \left. + c6 \right) (\cos(c228t)c7 - \sin(c228t)c8)c228 \right) \right), yZI3(z, t) \right. \\
& = (c7 \sin(c228t) + c8 \cos(c228t)) \left(c5 \sin(z) \frac{\sqrt{M^2 - c228^2}}{6} \right. \\
& \left. \left. + c6 \sin(z) - \frac{\sqrt{M^2 - c228^2}}{6} \right), yZI4(z, t) = (\cos(c228t)c4 \right. \\
& \left. + \sin(c228t)c3) \left(cI \sin(z) \frac{\sqrt{M^2 - c228^2}}{6} \right. \right. \\
& \left. \left. + c2 \sin(z) - \frac{\sqrt{M^2 - c228^2}}{6} \right), yZ9(z, t) \right. \\
& = \frac{1}{M} \left(\sin(z) - \frac{\sqrt{M^2 - c228^2}}{6} \left(- \left(\sin(z) \frac{\sqrt{M^2 - c228^2}}{3} c5 \right. \right. \right. \\
& \left. \left. \left. - c6 \right) (c7 \sin(c228t) + c8 \cos(c228t)) \sqrt{M^2 - c228^2} \right. \right. \\
& \left. \left. \left. + \left(\sin(z) \frac{\sqrt{M^2 - c228^2}}{3} cI + c2 \right) c228 (c3 \cos(c228t) \right. \right. \right. \\
& \left. \left. \left. - \sin(c228t)c4) \right) \right) \right\}
\end{aligned}$$

```
In[=]:= ConvertMapleToMathematica[
"yZ10(z,t)=((sin(z))^(-(sqrt(M^2-c228^2))/6) (-cos(c228 t)
c4+sin(c228 t) c3) ((sin(z))^((sqrt(M^2-c228^2))/3) c1-c2)
sqrt(M^2-c228^2)+((sin(z))^((sqrt(M^2-c228^2))/3) c5+c6)
(cos(c228 t) c7-sin(c228 t) c8) c228)/M,yZ13(z,t)=(c7 sin(c228
t)+c8 cos(c228 t)) (c5 (sin(z))^((sqrt(M^2-c228^2))/6)+c6
(sin(z))^(-(sqrt(M^2-c228^2))/6)),yZ14(z,t)=(cos(c228 t)
c4+sin(c228 t) c3) (c1 (sin(z))^((sqrt(M^2-c228^2))/6)+c2
(sin(z))^(-(sqrt(M^2-c228^2))/6)),yZ9(z,t)=((sin(z))^(-(sqrt(M^2-c228^2)/6)
(-((sin(z))^((sqrt(M^2-c228^2))/3) c5-c6) (c7 sin(c228
t)+c8 cos(c228 t)) sqrt(M^2-c228^2)+((sin(z))^((sqrt(M^2-c228^2))/3)
c1+c2) c228 (c3 cos(c228 t)-sin(c228 t) c4))/M)]"
Out[=]=
{yZ10[z, t] == (((Sin[z] ^ (-(Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6)) *
(((-(Cos[(c228 * t)] * c4) + (Sin[(c228 * t)] * c3))) * (((Sin[z]
^ (Sqrt[((M ^ 2) - (c228 ^ 2))]) / 3)) * c1) - c2)) * Sqrt[((M ^
2) - (c228 ^ 2))]) + (((((Sin[z] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))])
/ 3)) * c5) + c6) * ((Cos[(c228 * t)] * c7) - (Sin[(c228 * t)] *
c8))) * c228)) / M, yZ13[z, t] == (((c7 * Sin[(c228 * t)]) + (c8
* Cos[(c228 * t)])) * ((c5[Sin[z]] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))])
/ 6)) + (c6[Sin[z]] ^ (-(Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6))), yZ14[z, t] == (((Cos[(c228 * t)] * c4) + (Sin[(c228 * t)] * c3)) *
((c1[Sin[z]] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6)) + (c2[Sin[z]] ^
(-Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6))), yZ9[z, t] == (((Sin[z] ^
(-Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6)) * ((((-(((Sin[z] ^ (Sqrt[((M
^ 2) - (c228 ^ 2))]) / 3)) * c5) - c6)) * ((c7 * Sin[(c228 * t)])
+ (c8 * Cos[(c228 * t)])) * Sqrt[((M ^ 2) - (c228 ^ 2))]) +
(((Sin[z] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))]) / 3)) * c1) + c2) *
c228[((c3 * Cos[(c228 * t)]) - (Sin[(c228 * t)] * c4)))) / M)}
```

```
In[8]:= syZ9yZ10yZ13yZ14 =
{yZ[9][z, t] → (((Sin[z] ^ (-(Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6)) *
(((-((((Sin[z] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))]) / 3)) * c5) - c6)) *
((c7 * Sin[(c228 * t)]) + (c8 * Cos[(c228 * t)]))) * *
Sqrt[((M ^ 2) - (c228 ^ 2))]) +
(((Sin[z] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))]) / 3)) * c1) + c2) *
c228[((c3 * Cos[(c228 * t)]) - (Sin[(c228 * t)] * c4))) / M)
, yZ[10][z, t] → (((Sin[z] ^ (-(Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6)) *
(((-(Cos[(c228 * t)] * c4) + (Sin[(c228 * t)] * c3))) * *
((Sin[z] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))]) / 3)) * c1) - c2)) * *
Sqrt[((M ^ 2) - (c228 ^ 2))]) +
((((Sin[z] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))]) / 3)) * c5) + c6) *
((Cos[(c228 * t)] * c7) -
(Sin[(c228 * t)] * c8))) * c228)) / M),
yZ[13][z, t] → (((c7 * Sin[(c228 * t)]) + (c8 * Cos[(c228 * t)])) * *
((c5[Sin[z]] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6)) +
(c6[Sin[z]] ^ (-(Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6))), ,
yZ[14][z, t] → (((Cos[(c228 * t)] * c4) + (Sin[(c228 * t)] * c3)) * *
((c1[Sin[z]] ^ (Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6)) +
(c2[Sin[z]] ^ (-(Sqrt[((M ^ 2) - (c228 ^ 2))]) / 6))))}
```

Out[8]=

$$\left\{ \begin{aligned} yZ[9][z, t] &\rightarrow \frac{1}{M} \sin[z]^{-\frac{1}{6}} \sqrt{-c228^2 + M^2} \\ &\quad \left(c228[c3 \cos[c228 t] - c4 \sin[c228 t]] \left(c2 + c1 \sin[z]^{\frac{1}{3}} \sqrt{-c228^2 + M^2} \right) + \right. \\ &\quad \left. \sqrt{-c228^2 + M^2} (c8 \cos[c228 t] + c7 \sin[c228 t]) \left(c6 - c5 \sin[z]^{\frac{1}{3}} \sqrt{-c228^2 + M^2} \right) \right), \\ yZ[10][z, t] &\rightarrow \frac{1}{M} \sin[z]^{-\frac{1}{6}} \sqrt{-c228^2 + M^2} \\ &\quad \left(\sqrt{-c228^2 + M^2} (-c4 \cos[c228 t] - c3 \sin[c228 t]) \left(-c2 + c1 \sin[z]^{\frac{1}{3}} \sqrt{-c228^2 + M^2} \right) + \right. \\ &\quad \left. c228 (c7 \cos[c228 t] - c8 \sin[c228 t]) \left(c6 + c5 \sin[z]^{\frac{1}{3}} \sqrt{-c228^2 + M^2} \right) \right), \\ yZ[13][z, t] &\rightarrow \left(c5[\sin[z]]^{\frac{1}{6}} \sqrt{-c228^2 + M^2} + c6[\sin[z]]^{-\frac{1}{6}} \sqrt{-c228^2 + M^2} \right) \\ &\quad (c8 \cos[c228 t] + c7 \sin[c228 t]), \\ yZ[14][z, t] &\rightarrow \left(c1[\sin[z]]^{\frac{1}{6}} \sqrt{-c228^2 + M^2} + c2[\sin[z]]^{-\frac{1}{6}} \sqrt{-c228^2 + M^2} \right) \\ &\quad (c4 \cos[c228 t] + c3 \sin[c228 t]) \end{aligned} \right\}$$

```
In[9]:= syZ9yZ10yZ13yZ14[[All, 1, 0]]
```

Out[9]= {yZ[9], yZ[10], yZ[13], yZ[14]}

```

In[=]:= syZ9yZ10yZ13yZ14[[All, 2]] /. {z → #1, t → #2}
ToExpression["((" <> ToString[FullForm[#]] <> ")")"] & /@ %
ssyZ9yZ10yZ13yZ14 = Thread[syZ9yZ10yZ13yZ14[[All, 1, 0]] → %]

Out[=]=

$$\left\{ \frac{1}{M} \sin[\#1]^{-\frac{1}{6}} \sqrt{-c228^2 + M^2} \right. \\ \left( c228 [c3 \cos[c228 \#2] - c4 \sin[c228 \#2]] \left( c2 + c1 \sin[\#1]^{\frac{1}{3}} \sqrt{-c228^2 + M^2} \right) + \right. \\ \left. \sqrt{-c228^2 + M^2} \left( c6 - c5 \sin[\#1]^{\frac{1}{3}} \sqrt{-c228^2 + M^2} \right) (c8 \cos[c228 \#2] + c7 \sin[c228 \#2]) \right), \\ \frac{1}{M} \sin[\#1]^{-\frac{1}{6}} \sqrt{-c228^2 + M^2} \left( \sqrt{-c228^2 + M^2} \left( -c2 + c1 \sin[\#1]^{\frac{1}{3}} \sqrt{-c228^2 + M^2} \right) \right. \\ \left( -c4 \cos[c228 \#2] - c3 \sin[c228 \#2] \right) + \\ c228 \left( c6 + c5 \sin[\#1]^{\frac{1}{3}} \sqrt{-c228^2 + M^2} \right) (c7 \cos[c228 \#2] - c8 \sin[c228 \#2]), \\ \left( c5 [\sin[\#1]]^{\frac{1}{6}} \sqrt{-c228^2 + M^2} + c6 [\sin[\#1]]^{-\frac{1}{6}} \sqrt{-c228^2 + M^2} \right) \\ (c8 \cos[c228 \#2] + c7 \sin[c228 \#2]), \\ \left( c1 [\sin[\#1]]^{\frac{1}{6}} \sqrt{-c228^2 + M^2} + c2 [\sin[\#1]]^{-\frac{1}{6}} \sqrt{-c228^2 + M^2} \right) \\ \left. \left( c4 \cos[c228 \#2] + c3 \sin[c228 \#2] \right) \right\}

Out[=]=

$$\left\{ \frac{1}{M} \sin[\#1]^{\text{Rational}[-1, 6]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} \left( c228 [c3 \cos[c228 \#2] - c4 \sin[c228 \#2]] \right. \right. \\ \left( c2 + c1 \sin[\#1]^{\text{Rational}[1, 3]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} \right) + \\ \left. \left( -c228^2 + M^2 \right)^{\text{Rational}[1, 2]} \left( c6 - c5 \sin[\#1]^{\text{Rational}[1, 3]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} \right) \right. \\ \left. (c8 \cos[c228 \#2] + c7 \sin[c228 \#2]) \right) \&, \\ \frac{1}{M} \sin[\#1]^{\text{Rational}[-1, 6]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} \\ \left( (-c228^2 + M^2)^{\text{Rational}[1, 2]} \left( -c2 + c1 \sin[\#1]^{\text{Rational}[1, 3]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} \right) \right. \\ \left( -c4 \cos[c228 \#2] - c3 \sin[c228 \#2] \right) + \\ c228 \left( c6 + c5 \sin[\#1]^{\text{Rational}[1, 3]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} \right) \\ \left. (c7 \cos[c228 \#2] - c8 \sin[c228 \#2]) \right) \&, \\ \left( c5 [\sin[\#1]]^{\text{Rational}[1, 6]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} + c6 [\sin[\#1]]^{\text{Rational}[-1, 6]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} \right) \\ (c8 \cos[c228 \#2] + c7 \sin[c228 \#2]) \&, \\ \left( c1 [\sin[\#1]]^{\text{Rational}[1, 6]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} + c2 [\sin[\#1]]^{\text{Rational}[-1, 6]} (-c228^2 + M^2)^{\text{Rational}[1, 2]} \right) \\ \left. (c4 \cos[c228 \#2] + c3 \sin[c228 \#2]) \& \right\}$$$$

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```

Out[6]=
{yZ[9] →

$$\left( \frac{1}{M} \sin[\#1]^{\text{Rational}[-1,6]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \left( c228 [c3 \cos[c228 \#2] - c4 \sin[c228 \#2]] \right. \right.$$


$$\left. \left( c2 + c1 \sin[\#1]^{\text{Rational}[1,3]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \right) + \right.$$


$$\left. \left( -c228^2 + M^2 \right)^{\text{Rational}[1,2]} \left( c6 - c5 \sin[\#1]^{\text{Rational}[1,3]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \right) \right. \\ \left. \left( c8 \cos[c228 \#2] + c7 \sin[c228 \#2] \right) \right) \&,$$

yZ[10] → 
$$\left( \frac{1}{M} \sin[\#1]^{\text{Rational}[-1,6]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \right. \\ \left. \left( \left( -c228^2 + M^2 \right)^{\text{Rational}[1,2]} \left( -c2 + c1 \sin[\#1]^{\text{Rational}[1,3]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \right) \right. \right. \\ \left. \left. \left( -c4 \cos[c228 \#2] - c3 \sin[c228 \#2] \right) + \right. \right. \\ \left. \left. c228 \left( c6 + c5 \sin[\#1]^{\text{Rational}[1,3]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \right) \right. \right. \\ \left. \left. \left( c7 \cos[c228 \#2] - c8 \sin[c228 \#2] \right) \right) \&, \right.$$

yZ[13] → 
$$\left( \left( c5 [\sin[\#1]]^{\text{Rational}[1,6]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \right. \right. \\ \left. \left. + c6 [\sin[\#1]]^{\text{Rational}[-1,6]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \right) \right. \\ \left. \left( c8 \cos[c228 \#2] + c7 \sin[c228 \#2] \right) \&, \right.$$

yZ[14] → 
$$\left( \left( c1 [\sin[\#1]]^{\text{Rational}[1,6]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \right. \right. \\ \left. \left. + c2 [\sin[\#1]]^{\text{Rational}[-1,6]} (-c228^2 + M^2)^{\text{Rational}[1,2]} \right) \right. \\ \left. \left( c4 \cos[c228 \#2] + c3 \sin[c228 \#2] \right) \& \right\}$$


```

```

In[=]:= chkeq228 = eq228 /. ssyZ9yZ10yZ13yZ14 // FullSimplify[#, constraintVars] &
Out[=]=
{c2 [Sin[z]]-1/6  $\sqrt{-c228^2 + M^2}$  (c4 Cos[c228 t] + c3 Sin[c228 t])
 (-M2 Sin[z]1/6  $\sqrt{-c228^2 + M^2}$  + c2 [Sin[z]]1/6  $\sqrt{-c228^2 + M^2}$ 
 (-M2 (c1 [Sin[z]] Sin[z])1/6  $\sqrt{-c228^2 + M^2}$  + (c2 + c1 Sin[z]1/3  $\sqrt{-c228^2 + M^2}$ )
 (-c2282 + M2 + c228 c228'[c3 Cos[c228 t] - c4 Sin[c228 t]]))) == 0,
c2 c228 c3  $\sqrt{-c228^2 + M^2}$  Cos[c228 t] + M2 (-c6 + (c6 [Sin[z]] Csc[z])-1/6  $\sqrt{-c228^2 + M^2}$ )
 (c8 Cos[c228 t] + c7 Sin[c228 t]) +
M2 (c8 Cos[c228 t] + c7 Sin[c228 t]) (c5 [Sin[z]] Sin[z])1/6  $\sqrt{-c228^2 + M^2}$  ==
c2  $\sqrt{-c228^2 + M^2}$  c228 [c3 Cos[c228 t] - c4 Sin[c228 t]] +
c2 c228 c4  $\sqrt{-c228^2 + M^2}$  Sin[c228 t] +
(c1  $\sqrt{-c228^2 + M^2}$  (-c228 [c3 Cos[c228 t] - c4 Sin[c228 t]] + c228 c3 Cos[c228 t] -
c228 c4 Sin[c228 t]) + c5 M2 (c8 Cos[c228 t] + c7 Sin[c228 t])) +
Sin[z]1/3  $\sqrt{-c228^2 + M^2}$ , c228 (c5 [Sin[z]]1/6  $\sqrt{-c228^2 + M^2}$  + c6 [Sin[z]]-1/6  $\sqrt{-c228^2 + M^2}$ )
(c7 Cos[c228 t] - c8 Sin[c228 t]) == Sin[z]-1/6  $\sqrt{-c228^2 + M^2}$ 
( $\sqrt{-c228^2 + M^2}$  (c4 Cos[c228 t] + c3 Sin[c228 t]) (c2 - c1 Sin[z]1/3  $\sqrt{-c228^2 + M^2}$ ) +
c228 (c7 Cos[c228 t] - c8 Sin[c228 t]) (c6 + c5 Sin[z]1/3  $\sqrt{-c228^2 + M^2}$ ) ) +
 $\sqrt{-c228^2 + M^2}$  (c4 Cos[c228 t] + c3 Sin[c228 t]) Sin[z]
(c1 [Sin[z]]-1+1/6  $\sqrt{-c228^2 + M^2}$  c1'[Sin[z]] - c2 [Sin[z]]-1-1/6  $\sqrt{-c228^2 + M^2}$  c2'[Sin[z]]),
c228 (c1 [Sin[z]]1/6  $\sqrt{-c228^2 + M^2}$  + c2 [Sin[z]]-1/6  $\sqrt{-c228^2 + M^2}$ )
(c3 Cos[c228 t] - c4 Sin[c228 t]) == Sin[z]-1/6  $\sqrt{-c228^2 + M^2}$ 
(c228 [c3 Cos[c228 t] - c4 Sin[c228 t]] (c2 + c1 Sin[z]1/3  $\sqrt{-c228^2 + M^2}$ ) +
 $\sqrt{-c228^2 + M^2}$  (c8 Cos[c228 t] + c7 Sin[c228 t]) (c6 - c5 Sin[z]1/3  $\sqrt{-c228^2 + M^2}$ ) ) +
 $\sqrt{-c228^2 + M^2}$  (c8 Cos[c228 t] + c7 Sin[c228 t]) Sin[z]
(c5 [Sin[z]]-1+1/6  $\sqrt{-c228^2 + M^2}$  c5'[Sin[z]] - c6 [Sin[z]]-1-1/6  $\sqrt{-c228^2 + M^2}$  c6'[Sin[z]]) }

```

```
In[1]:= "yZ10(z,t)=((sin(z))^(-(sqrt(M^2-c228^2))/6) (-cos(c228
t) c4+sin(c228 t) c3) ((sin(z))^((sqrt(M^2-c228^2))/3) c1-c2)
sqrt(M^2-c228^2)+((sin(z))^((sqrt(M^2-c228^2))/3) c5+c6)
(cos(c228 t) c7-sin(c228 t) c8) /M,yZ13(z,t)=(c7 sin(c228
t)+c8 cos(c228 t)) (c5 (sin(z))^((sqrt(M^2-c228^2))/6)+c6
(sin(z))^(-(sqrt(M^2-c228^2))/6)),yZ14(z,t)=(cos(c228 t)
c4+sin(c228 t) c3) (c1 (sin(z))^((sqrt(M^2-c228^2))/6)+c2
(sin(z))^(-(sqrt(M^2-c228^2))/6)),yZ9(z,t)=((sin(z))^(-(sqrt(M^2-c228^2))/
6) (-((sin(z))^((sqrt(M^2-c228^2))/3) c5-c6) (c7 sin(c228 t)+c8
cos(c228 t)) sqrt(M^2-c228^2)+((sin(z))^((sqrt(M^2-c228^2))/3)
c1+c2) c228 (c3 cos(c228 t)-sin(c228 t) c4))/M}"
```

Out[1]= {yZ10(z,t)=((sin(z))^(-(sqrt(M^2-c228^2))/6) (-cos(c228
t) c4+sin(c228 t) c3) ((sin(z))^((sqrt(M^2-c228^2))/3) c1-c2)
sqrt(M^2-c228^2)+((sin(z))^((sqrt(M^2-c228^2))/3) c5+c6)
(cos(c228 t) c7-sin(c228 t) c8) /M,yZ13(z,t)=(c7 sin(c228
t)+c8 cos(c228 t)) (c5 (sin(z))^((sqrt(M^2-c228^2))/6)+c6
(sin(z))^(-(sqrt(M^2-c228^2))/6)),yZ14(z,t)=(cos(c228 t)
c4+sin(c228 t) c3) (c1 (sin(z))^((sqrt(M^2-c228^2))/6)+c2
(sin(z))^(-(sqrt(M^2-c228^2))/6)),yZ9(z,t)=((sin(z))^(-(sqrt(M^2-c228^2))/
6) (-((sin(z))^((sqrt(M^2-c228^2))/3) c5-c6) (c7 sin(c228 t)+c8
cos(c228 t)) sqrt(M^2-c228^2)+((sin(z))^((sqrt(M^2-c228^2))/3)
c1+c2) c228 (c3 cos(c228 t)-sin(c228 t) c4))/M}

```
In[2]:= ToExpression[MathMLForm[{yZ10 (z, t) =
sin (z)^(-sqrt (M^2 - c228^2) / 6) * (-cos (c228*t) * c4 + sin (c228*t) * c3) *
(sin (z)^sqrt (M^2 - c228^2) / 3) * c1 - c2) * sqrt (M^2 - c228^2) +
(sin (z)^sqrt (M^2 - c228^2) / 3) * c5 + c6) *
(cos (c228*t) * c7 - sin (c228*t) * c8) * c228) / M,
yZ13 (z, t) = (c7 * sin (c228*t) + c8 * cos (c228*t)) *
(c5 * sin (z)^sqrt (M^2 - c228^2) / 6) +
c6 * sin (z)^(-sqrt (M^2 - c228^2) / 6)),
yZ14 (z, t) = (cos (c228*t) * c4 + sin (c228*t) * c3) *
(c1 * sin (z)^sqrt (M^2 - c228^2) / 6) +
c2 * sin (z)^(-sqrt (M^2 - c228^2) / 6)),
yZ9 (z, t) = sin (z)^(-sqrt (M^2 - c228^2) / 6) *
(-sin (z)^sqrt (M^2 - c228^2) / 3) * c5 - c6) *
(c7 * sin (c228*t) + c8 * cos (c228*t)) * sqrt (M^2 - c228^2) +
(sin (z)^sqrt (M^2 - c228^2) / 3) * c1 + c2) * c228 *
(c3 * cos (c228*t) - sin (c228*t) * c4)) / M}]]
```

In[1]:= $yZ10[z, t]$

Out[1]=

$yZ10[z, t]$

FYI: wave functions?

In[2]:= $\{\# + 0, 7 - \#\} \& /@ Range[0, 3]$

ToExpression["sDtv" <> ToString[#[1]] <> "Dtv" <> ToString[#[2]]] & /@ %;

plusM03 = Flatten[#[1]] & /@ %

minusM03 = Flatten[#[2]] & /@ %%

Out[2]=

$\{\{0, 7\}, \{1, 6\}, \{2, 5\}, \{3, 4\}\}$

Out[3]=

$$\left\{ \begin{aligned} \nu[0]'[0] &\rightarrow M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \nu[7]'[0] &\rightarrow -M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \\ \nu[1]'[0] &\rightarrow M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \nu[6]'[0] &\rightarrow -M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \\ \nu[2]'[0] &\rightarrow M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \nu[5]'[0] &\rightarrow -M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \\ \nu[3]'[0] &\rightarrow M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \nu[4]'[0] &\rightarrow -M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \end{aligned} \right\}$$

Out[4]=

$$\left\{ \begin{aligned} \nu[0]'[0] &\rightarrow -M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \nu[7]'[0] &\rightarrow M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \\ \nu[1]'[0] &\rightarrow -M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \nu[6]'[0] &\rightarrow M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \\ \nu[2]'[0] &\rightarrow -M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \nu[5]'[0] &\rightarrow M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \\ \nu[3]'[0] &\rightarrow -M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0], \nu[4]'[0] &\rightarrow M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \end{aligned} \right\}$$

In[5]:= $\{\# + 8, 15 - \#\} \& /@ Range[0, 3]$

ToExpression["sDtv" <> ToString[#[1]] <> "Dtv" <> ToString[#[2]]] & /@ %

plusM811 = Flatten[#[1]] & /@ %

minusM811 = Flatten[#[2]] & /@ %%

Out[5]=

$\{\{8, 15\}, \{9, 14\}, \{10, 13\}, \{11, 12\}\}$

Out[6]=

$$\left\{ \begin{aligned} \{\{\nu[8]'[0] \rightarrow M, \nu[15]'[0] \rightarrow -M\}, \{\nu[8]'[0] \rightarrow -M, \nu[15]'[0] \rightarrow M\}\}, \\ \{\{\nu[9]'[0] \rightarrow M, \nu[14]'[0] \rightarrow -M\}, \{\nu[9]'[0] \rightarrow -M, \nu[14]'[0] \rightarrow M\}\}, \\ \{\{\nu[10]'[0] \rightarrow M, \nu[13]'[0] \rightarrow -M\}, \{\nu[10]'[0] \rightarrow -M, \nu[13]'[0] \rightarrow M\}\}, \\ \{\{\nu[11]'[0] \rightarrow M, \nu[12]'[0] \rightarrow -M\}, \{\nu[11]'[0] \rightarrow -M, \nu[12]'[0] \rightarrow M\}\} \end{aligned} \right\}$$

Out[7]=

$$\{\nu[8]'[0] \rightarrow M, \nu[15]'[0] \rightarrow -M, \nu[9]'[0] \rightarrow M, \nu[14]'[0] \rightarrow -M, \\ \nu[10]'[0] \rightarrow M, \nu[13]'[0] \rightarrow -M, \nu[11]'[0] \rightarrow M, \nu[12]'[0] \rightarrow -M\}$$

Out[8]=

$$\{\nu[8]'[0] \rightarrow -M, \nu[15]'[0] \rightarrow M, \nu[9]'[0] \rightarrow -M, \nu[14]'[0] \rightarrow M, \\ \nu[10]'[0] \rightarrow -M, \nu[13]'[0] \rightarrow M, \nu[11]'[0] \rightarrow -M, \nu[12]'[0] \rightarrow M\}$$

```

In[]:= plusM03plusM811 = Flatten[{plusM03, plusM811}]

Out[]=
{v[0]'[0] → M +  $\frac{1}{2}$  Q1 a4'[0] -  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0],  

 v[7]'[0] → -M +  $\frac{1}{2}$  Q1 a4'[0] -  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0], v[1]'[0] → M +  $\frac{1}{2}$  Q1 a4'[0] -  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0],  

 v[6]'[0] → -M +  $\frac{1}{2}$  Q1 a4'[0] -  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0], v[2]'[0] → M -  $\frac{1}{2}$  Q1 a4'[0] +  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0],  

 v[5]'[0] → -M -  $\frac{1}{2}$  Q1 a4'[0] +  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0], v[3]'[0] → M -  $\frac{1}{2}$  Q1 a4'[0] +  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0],  

 v[4]'[0] → -M -  $\frac{1}{2}$  Q1 a4'[0] +  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0], v[8]'[0] → M, v[15]'[0] → -M,  

 v[9]'[0] → M, v[14]'[0] → -M, v[10]'[0] → M, v[13]'[0] → -M, v[11]'[0] → M, v[12]'[0] → -M}

In[]:= minusM03minusM811 = Flatten[{minusM03, minusM811}]

Out[]=
{v[0]'[0] → -M +  $\frac{1}{2}$  Q1 a4'[0] -  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0],  

 v[7]'[0] → M +  $\frac{1}{2}$  Q1 a4'[0] -  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0], v[1]'[0] → -M +  $\frac{1}{2}$  Q1 a4'[0] -  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0],  

 v[6]'[0] → M +  $\frac{1}{2}$  Q1 a4'[0] -  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0], v[2]'[0] → -M -  $\frac{1}{2}$  Q1 a4'[0] +  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0],  

 v[5]'[0] → M -  $\frac{1}{2}$  Q1 a4'[0] +  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0], v[3]'[0] → -M -  $\frac{1}{2}$  Q1 a4'[0] +  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0],  

 v[4]'[0] → M -  $\frac{1}{2}$  Q1 a4'[0] +  $\frac{1}{2}$  e-2 a4[0] Q1 a4'[0], v[8]'[0] → -M, v[15]'[0] → M,  

 v[9]'[0] → -M, v[14]'[0] → M, v[10]'[0] → -M, v[13]'[0] → M, v[11]'[0] → -M, v[12]'[0] → M}

```

```

In[*]:= sfy16t =
Table[f16[j] → ToExpression["((Y[" <> ToString[j] <> "] [6*H##1,H##2]" <> "Exp[" <> ToString[
  "(v[" <> ToString[j] <> "]')'[0]"] <> "*#2]" <> "*1/Sqrt[Sin[6*H##1]]))&)", {j, 0, 15}]

Out[*]=
{f16[0] → 
$$\frac{Y[0][6H\#1, H\#2]\ Exp[v[0]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[1] → 
$$\frac{Y[1][6H\#1, H\#2]\ Exp[v[1]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[2] → 
$$\frac{Y[2][6H\#1, H\#2]\ Exp[v[2]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[3] → 
$$\frac{Y[3][6H\#1, H\#2]\ Exp[v[3]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[4] → 
$$\frac{Y[4][6H\#1, H\#2]\ Exp[v[4]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[5] → 
$$\frac{Y[5][6H\#1, H\#2]\ Exp[v[5]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[6] → 
$$\frac{Y[6][6H\#1, H\#2]\ Exp[v[6]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[7] → 
$$\frac{Y[7][6H\#1, H\#2]\ Exp[v[7]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[8] → 
$$\frac{Y[8][6H\#1, H\#2]\ Exp[v[8]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[9] → 
$$\frac{Y[9][6H\#1, H\#2]\ Exp[v[9]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[10] → 
$$\frac{Y[10][6H\#1, H\#2]\ Exp[v[10]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[11] → 
$$\frac{Y[11][6H\#1, H\#2]\ Exp[v[11]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[12] → 
$$\frac{Y[12][6H\#1, H\#2]\ Exp[v[12]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[13] → 
$$\frac{Y[13][6H\#1, H\#2]\ Exp[v[13]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[14] → 
$$\frac{Y[14][6H\#1, H\#2]\ Exp[v[14]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&,$$
,
 f16[15] → 
$$\frac{Y[15][6H\#1, H\#2]\ Exp[v[15]'[0]\ \#2]}{\sqrt{\sin[6H\#1]}} \&}$$

}

```

In[]:= **sfy16Aa**

Out[]:=

$$\begin{aligned} \{f16[0] \rightarrow \left(\frac{Z[0][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), f16[1] \rightarrow \left(\frac{Z[1][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), \\ f16[2] \rightarrow \left(\frac{Z[2][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), f16[3] \rightarrow \left(\frac{Z[3][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), \\ f16[4] \rightarrow \left(\frac{Z[4][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), f16[5] \rightarrow \left(\frac{Z[5][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), \\ f16[6] \rightarrow \left(\frac{Z[6][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), f16[7] \rightarrow \left(\frac{Z[7][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), \\ f16[8] \rightarrow \left(\frac{Z[8][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), f16[9] \rightarrow \left(\frac{Z[9][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), \\ f16[10] \rightarrow \left(\frac{Z[10][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), f16[11] \rightarrow \left(\frac{Z[11][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), \\ f16[12] \rightarrow \left(\frac{Z[12][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), f16[13] \rightarrow \left(\frac{Z[13][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), \\ f16[14] \rightarrow \left(\frac{Z[14][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right), f16[15] \rightarrow \left(\frac{Z[15][6H\#1, H\#2]}{\sqrt{\sin[6H\#1]}} \& \right) \} \end{aligned}$$

In[]:= **sfy16t1t = Table[**

```

    f16[j] \rightarrow (((ToExpression["((Y[" <> ToString[j] <> "])[1][6*H\#1,H\#2]" <> "Exp[" <> ToString[
        "(\v[" <> ToString[j] <> "])'[0]" <> "*\#2]" <> "*1/Sqrt[Sin[6*H\#1]]))"] /.
    plusM03plusM811) + (ToExpression["((Y[" <> ToString[j] <> "])[2][6*H\#1,H\#2]" <>
        "Exp[" <> ToString["(\v[" <> ToString[j] <> "])'[0]" <> "*\#2]" <>
        "*1/Sqrt[Sin[6*H\#1]]))"] /. minusM03minusM811)), {j, 0, 15}]

```

sfy16TRY = Table[Block[{s}, s = %\[j]];**s[[1]] \rightarrow ToExpression["((" <> ToString[FullForm[s[[2]]]] <> ")")], {j, 1, Length[%]}]**

Out[]:=

$$\begin{aligned} \{f16[0] \rightarrow \frac{e^{\frac{\pi i}{2} \left(M+\frac{1}{2}\right) Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0]}}{\sqrt{\sin[6H\#1]}} Y[0][1][6H\#1, H\#2] + \\ \frac{e^{\frac{\pi i}{2} \left(-M+\frac{1}{2}\right) Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0]}}{\sqrt{\sin[6H\#1]}} Y[0][2][6H\#1, H\#2], \\ f16[1] \rightarrow \frac{e^{\frac{\pi i}{2} \left(M+\frac{1}{2}\right) Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0]}}{\sqrt{\sin[6H\#1]}} Y[1][1][6H\#1, H\#2] + \\ \frac{e^{\frac{\pi i}{2} \left(-M+\frac{1}{2}\right) Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0]}}{\sqrt{\sin[6H\#1]}} Y[1][2][6H\#1, H\#2], \\ f16[2] \rightarrow \frac{e^{\frac{\pi i}{2} \left(M-\frac{1}{2}\right) Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0]}}{\sqrt{\sin[6H\#1]}} Y[2][1][6H\#1, H\#2] + \end{aligned}$$

$$\begin{aligned}
& \frac{e^{\frac{H^2}{2} \left(-M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[2][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[3] & \rightarrow \frac{e^{\frac{H^2}{2} \left(M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[3][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
& \frac{e^{\frac{H^2}{2} \left(-M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[3][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[4] & \rightarrow \frac{e^{\frac{H^2}{2} \left(-M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[4][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
& \frac{e^{\frac{H^2}{2} \left(M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[4][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[5] & \rightarrow \frac{e^{\frac{H^2}{2} \left(-M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[5][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
& \frac{e^{\frac{H^2}{2} \left(M - \frac{1}{2} Q1 a4'[0] + \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[5][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[6] & \rightarrow \frac{e^{\frac{H^2}{2} \left(-M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[6][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
& \frac{e^{\frac{H^2}{2} \left(M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[6][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[7] & \rightarrow \frac{e^{\frac{H^2}{2} \left(-M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[7][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
& \frac{e^{\frac{H^2}{2} \left(M + \frac{1}{2} Q1 a4'[0] - \frac{1}{2} e^{-2 a4[0]} Q1 a4'[0] \right)} Y[7][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[8] & \rightarrow \frac{e^{M \frac{H^2}{2}} Y[8][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}} + \frac{e^{-M \frac{H^2}{2}} Y[8][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[9] & \rightarrow \frac{e^{M \frac{H^2}{2}} Y[9][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}} + \frac{e^{-M \frac{H^2}{2}} Y[9][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[10] & \rightarrow \frac{e^{M \frac{H^2}{2}} Y[10][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}} + \frac{e^{-M \frac{H^2}{2}} Y[10][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[11] & \rightarrow \frac{e^{M \frac{H^2}{2}} Y[11][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}} + \frac{e^{-M \frac{H^2}{2}} Y[11][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}}, \\
f16[12] & \rightarrow \frac{e^{-M \frac{H^2}{2}} Y[12][1][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}} + \frac{e^{M \frac{H^2}{2}} Y[12][2][6 H \#1, H \#2]}{\sqrt{\sin[6 H \#1]}},
\end{aligned}$$

$f16[13] \rightarrow \frac{e^{-M \# 2} Y[13][1][6 H \# 1, H \# 2]}{\sqrt{\sin[6 H \# 1]}} + \frac{e^{M \# 2} Y[13][2][6 H \# 1, H \# 2]}{\sqrt{\sin[6 H \# 1]}},$
 $f16[14] \rightarrow \frac{e^{-M \# 2} Y[14][1][6 H \# 1, H \# 2]}{\sqrt{\sin[6 H \# 1]}} + \frac{e^{M \# 2} Y[14][2][6 H \# 1, H \# 2]}{\sqrt{\sin[6 H \# 1]}},$
 $f16[15] \rightarrow \frac{e^{-M \# 2} Y[15][1][6 H \# 1, H \# 2]}{\sqrt{\sin[6 H \# 1]}} + \frac{e^{M \# 2} Y[15][2][6 H \# 1, H \# 2]}{\sqrt{\sin[6 H \# 1]}} \}$

$Out[\#]=$
 $\{ f16[0] \rightarrow$
 $(e^{\# 2 (M + Rational[1, 2] Q1 a4'[0] + Rational[-1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2] Y[0][1][6 H \# 1, H \# 2] +$
 $e^{\# 2 (-M + Rational[1, 2] Q1 a4'[0] + Rational[-1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2] Y[1][1][6 H \# 1, H \# 2] +$
 $e^{\# 2 (M + Rational[1, 2] Q1 a4'[0] + Rational[-1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2] Y[1][2][6 H \# 1, H \# 2] \&), f16[1] \rightarrow$
 $(e^{\# 2 (M + Rational[1, 2] Q1 a4'[0] + Rational[-1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2] Y[1][1][6 H \# 1, H \# 2] +$
 $e^{\# 2 (-M + Rational[1, 2] Q1 a4'[0] + Rational[-1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2] Y[1][2][6 H \# 1, H \# 2] \&), f16[2] \rightarrow$
 $(e^{\# 2 (M + Rational[-1, 2] Q1 a4'[0] + Rational[1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2] Y[2][1][6 H \# 1, H \# 2] +$
 $e^{\# 2 (-M + Rational[-1, 2] Q1 a4'[0] + Rational[1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2] Y[2][2][6 H \# 1, H \# 2] \&), f16[3] \rightarrow$
 $(e^{\# 2 (M + Rational[-1, 2] Q1 a4'[0] + Rational[1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2] Y[3][1][6 H \# 1, H \# 2] +$
 $e^{\# 2 (-M + Rational[-1, 2] Q1 a4'[0] + Rational[1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2] Y[3][2][6 H \# 1, H \# 2] \&),$
 $f16[4] \rightarrow (e^{\# 2 (-M + Rational[-1, 2] Q1 a4'[0] + Rational[1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2]$
 $Y[4][1][6 H \# 1, H \# 2] + e^{\# 2 (M + Rational[-1, 2] Q1 a4'[0] + Rational[1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2]$
 $Y[4][2][6 H \# 1, H \# 2] \&),$
 $f16[5] \rightarrow (e^{\# 2 (-M + Rational[-1, 2] Q1 a4'[0] + Rational[1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2]$
 $Y[5][1][6 H \# 1, H \# 2] + e^{\# 2 (M + Rational[-1, 2] Q1 a4'[0] + Rational[1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2]$
 $Y[5][2][6 H \# 1, H \# 2] \&),$
 $f16[6] \rightarrow (e^{\# 2 (-M + Rational[1, 2] Q1 a4'[0] + Rational[-1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2]$
 $Y[6][1][6 H \# 1, H \# 2] + e^{\# 2 (M + Rational[1, 2] Q1 a4'[0] + Rational[-1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2]$
 $Y[6][2][6 H \# 1, H \# 2] \&),$
 $f16[7] \rightarrow (e^{\# 2 (-M + Rational[1, 2] Q1 a4'[0] + Rational[-1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2]$
 $Y[7][1][6 H \# 1, H \# 2] + e^{\# 2 (M + Rational[1, 2] Q1 a4'[0] + Rational[-1, 2] e^{-2 a4[0]} Q1 a4'[0])} \sin[6 H \# 1] Rational[-1, 2]$
 $Y[7][2][6 H \# 1, H \# 2] \&),$
 $f16[8] \rightarrow (e^{M \# 2} \sin[6 H \# 1] Rational[-1, 2] Y[8][1][6 H \# 1, H \# 2] +$
 $e^{-M \# 2} \sin[6 H \# 1] Rational[-1, 2] Y[8][2][6 H \# 1, H \# 2] \&),$
 $f16[9] \rightarrow (e^{M \# 2} \sin[6 H \# 1] Rational[-1, 2] Y[9][1][6 H \# 1, H \# 2] +$
 $e^{-M \# 2} \sin[6 H \# 1] Rational[-1, 2] Y[9][2][6 H \# 1, H \# 2] \&),$
 $f16[10] \rightarrow (e^{M \# 2} \sin[6 H \# 1] Rational[-1, 2] Y[10][1][6 H \# 1, H \# 2] +$
 $e^{-M \# 2} \sin[6 H \# 1] Rational[-1, 2] Y[10][2][6 H \# 1, H \# 2] \&),$
 $f16[11] \rightarrow (e^{M \# 2} \sin[6 H \# 1] Rational[-1, 2] Y[11][1][6 H \# 1, H \# 2] +$

```
e-M#2 Sin[6 H#1]Rational[-1,2] Y[11][2][6 H#1, H#2] & ,  
f16[12] → (e-M#2 Sin[6 H#1]Rational[-1,2] Y[12][1][6 H#1, H#2] +  
eM#2 Sin[6 H#1]Rational[-1,2] Y[12][2][6 H#1, H#2] & ) ,  
f16[13] → (e-M#2 Sin[6 H#1]Rational[-1,2] Y[13][1][6 H#1, H#2] +  
eM#2 Sin[6 H#1]Rational[-1,2] Y[13][2][6 H#1, H#2] & ) ,  
f16[14] → (e-M#2 Sin[6 H#1]Rational[-1,2] Y[14][1][6 H#1, H#2] +  
eM#2 Sin[6 H#1]Rational[-1,2] Y[14][2][6 H#1, H#2] & ) ,  
f16[15] → (e-M#2 Sin[6 H#1]Rational[-1,2] Y[15][1][6 H#1, H#2] +  
eM#2 Sin[6 H#1]Rational[-1,2] Y[15][2][6 H#1, H#2] & ) }
```