

Scratch work of Author

NOTES

HYPOTHESIS : If , employing the Einstein eqs (or Einstein - Lovelock eqs) , superluminal inflation / deflation exists,

then at time $x_4 = 0$ (before the particles of the standard model exist)
a pair of universes with masses $\pm M$ is created
(i.e., universes are created in pairs)
Dark matter / dark energy may be
related to wave function overlap of these two universes

THE FOLLOWING SCRATCH IS CONSISTENT WITH
UNIVERSES THAT ARE CREATED IN PAIRS, WITH MASSES $\pm M$.

Note that we are ONLY looking for
superluminal inflation or deflation type solutions,
and NOT solutions that are even / odd functions of t
(like $\cos[\nu[j][0] * t]$, $\sin[\nu[j][0] * t]$,
 $\text{Sech}[\nu[j][0] * t]$, $\tanh[\nu[j][0] * t]$, ...),
which also have $\pm M$ type eigenvalues,
or even solutions involving the
well-known special functions that might also have $\pm M$ type parameters.

Bigger Bang:

Question: Are Universe (s) of masses $\pm M$ created in pairs at time $x_4 = 0$
(before the particles of the standard model exist) ?

Scratch work

Patrick L. Nash, Ph.D.

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Professor, UTSA Physics and Astronomy, Retired (UTSA)

Patrick299Nash at gmail ...

Do not read if you are irascible; I apologize for typos and duplications and

git clone <https://github.com/43d168f3e/Eternal-DEFLATION-Inflation.git>

“git@github.com:43d168f3e/Eternal-DEFLATION-Inflation.git”

Ever wonder why there are 3 space dimensions and 1 time dimension?

Ever wonder if Nature loves the norm relation $\|XY\| = \|X\|\|Y\|$, for $X, Y \in \text{Nature}$?

Well, Hurwitz's theorem says that there are only 4 normed division algebras ($\|XY\| = \|X\|\|Y\|$) over the real numbers, which, up to isomorphism, are the 1-dim (over \mathbb{R}) real numbers \mathbb{R} , the 2-dim complex numbers, the 4-dim quaternions, and the 8-dim octonions.

Since this is the age of the Xtreme, let's go with the 8-dim octonions. Actually the split octonions, in this notebook, with 4 space dimensions and 4 time dimensions.

The origin of this calculation has its roots in "A Remarkable Representation of the 3+2 de Sitter Group" by P.A.M.Dirac, J.Math.Phys.4,901–909 (1963).

Comment: The term "reduced Brauer–Weyl generators", below, refers to a specific set of irreducible generators for certain complex Clifford algebras, particularly in the context of mathematical physics and the study of Dirac spinors. These generators were introduced by **E.A. Lord** (E. A. Lord. "The Dirac spinor in six dimensions". Mathematical Proceedings of the Cambridge Philosophical Society, vol. 64, pp. 765–778, 1968) and are described as "reduced" because they form an irreducible representation of the Clifford algebra C_{2n} from $C_{2(n-1)}$.

Here, we employ Lord's idea of reduced Brauer–Weyl generators to extend Dirac's 4×4 gamma matrices into this notebook's real tau8 8×8 and tau16 16×16 matrices, which are employed to write down the 'Dirac Equation for the Universe', [remember, this is before the particles of the standard model exist] formulated in terms of a $\mathcal{O}(4, 4)$ spinor Ψ_{16} (Ψ_{16} AKA '**WAVE FUNCTION OF the UNIVERSE**', which could possibly be named something more pompous). Then this equation is used to [WARNING: syncope, presyncope AHEAD] couple the **WAVE FUNCTION OF the UNIVERSE** to Gravity.

THIS WORK IS PARTIALLY BASED ON :

JOURNAL OF MATHEMATICAL PHYSICS

VOLUME 4, NUMBER 7

A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. DIRAC

*Cambridge University, Cambridge, England and
Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

Proc. Camb. Phil. Soc. (1968), 64, 765

765

PCPS 64-96

*Printed in Great Britain***The Dirac spinor in six dimensions**

BY E. A. LORD

*Department of Mathematics, King's College, University of London**(Received 17 November 1966)*

JOURNAL OF MATHEMATICAL PHYSICS 51, 042501 (2010)

Second gravityPatrick L. Nash^{a)}*Department of Physics and Astronomy, The University of Texas at San Antonio, San Antonio, Texas 78249-0697, USA**(Received 29 July 2009; accepted 8 February 2010; published online 5 April 2010)*

IL NUOVO CIMENTO

VOL. 105 B, N. 1

Gennaio 1990

On the Structure of the Split Octonion Algebra.

P. L. NASH

*Division of Earth and Physical Sciences
University of Texas at San Antonio, TX 78285-0663**(ricevuto il 24 Luglio 1989)***Identities satisfied by the generators of the Dirac algebra**

Patrick L. Nash

*M. S. 474, NASA Langley Research Center, Hampton, Virginia 23665**(Received 26 July 1983; accepted for publication 23 September 1983)***A new spin- $\frac{1}{2}$ wave equation**

Patrick L. Nash

*Division of Earth and Physical Sciences, University of Texas at San Antonio, San Antonio, Texas 78285**(Received 24 January 1984; accepted for publication 16 November 1984)*

A new spin- $\frac{1}{2}$ wave equation

Patrick L. Nash

Division of Earth and Physical Sciences, University of Texas at San Antonio, San Antonio, Texas 78285

4 | Pair_Creation_of_Universes_WaveFunctionOfUniverse-4+4-Einstein-Lovelock-Nash.nb

(Received 24 January 1984; accepted for publication 16 November 1984)

Here we use [spacetime coordinates](#) that are selected so that we are somewhat consistent with the [exact libraries](#),

(which we employ, in other included notebooks, in order to calculate the Einstein-Lovelock tensors):

[Cartesian coordinates:](#)

x_0 = hidden space (a small circle); x_1, x_2, x_3 are the usual 3-space coords;

x_4 = time coord,

and x_5, x_6, x_7 = superluminal [deflating](#) time coords}.

The split Octonion algebra carries basic 8-component representations of $\overline{\text{SO}(4, 4; \mathbb{R})}$ (left-spinor, right-spinor and vector), which are equivalent [Cartan's triality].

Here we find a solution of the Einstein-Lovelock vacuum field equations on a spacetime M^8 , whose tangent bundle has

$\overline{\text{SO}(4, 4; \mathbb{R})} \approx \text{Spin}(4, 4; \mathbb{R})$

as iso group, and in which

3 of the 4 space dimensions [superluminally INFLATE](#),

3 of the 4 time dimensions [superluminally DEFLATE](#) (no problems with causality in the surviving time dimension, although I am sure that the 3 deflated time dimensions are interesting),

and the 4th space dim, x_0 , curls up into a ring ([hence a particle whose wave function penetrates this ring acquires a mass contribution](#)).

In passing we remark that an analogous construction may be defined on an octonion space of [one time dimension and seven space dimensions](#), if the reader is allergic to multiple time dimensions (and employ the octonions rather than the split-octonions).

[Unsolved problem 1](#) (of many): All particles are initially massless in the standard model, due to gauge invariance under the symmetry group

$SU(3) \otimes SU(2)_L \otimes U(1)_Y$;

The photon should remain massless after the Higgs field acquires a vacuum expectation value (spontaneous symmetry breaking).

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Please cite this work and this web page if you use it.

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{(k)}^{\infty} \alpha_{(k)} g^{jl} \delta_{i_1 i_2 \dots i_{2k}}^{hh_1 \dots h_{2k}} R^{j_1 j_2}_{h_1 h_2} \dots R^{j_{2k-1} j_{2k}}_{h_{2k-1} h_{2k}} + \lambda \sqrt{g} g^{lh}, \quad (4.38)$$

The Einstein – Lovelock vacuum field equations are

$$A_a^b = 0, \text{ where}$$

Remark 2. The problem pertaining to (4.1)–(4.3) for arbitrary n has been completely settled (Lovelock [3, 5]) the result being

$$A^{lh} = \sqrt{g} \sum_{k=1}^{m-1} \alpha_{(k)} g^{jl} \delta_{jj_1 \cdots j_{2k}}^{hh_1 \cdots h_{2k}} R^{j_1 j_2}_{\quad h_1 h_2} \cdots R^{j_{2k-1} j_{2k}}_{\quad h_{2k-1} h_{2k}} + \lambda \sqrt{g} g^{lh}, \quad (4.38)$$

where $\alpha_{(k)}, \lambda$ are arbitrary constants and

$$m = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n+1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

and $m - 1 = \frac{8}{2} - 1 = 3$.

Citation : Tensors, Differential Forms,
and Variational Principles (Dover Books on Mathematics),
by David Lovelock and Hanno Rund

Let {w1, w2, w3, Δ} be pure numbers;

then by Eq. [4.38] the vacuum equations are

$$0 = -\Delta + H^{-2} w1 \text{ Lovelock1} + H^{-4} w2 \text{ Lovelock2} + H^{-6} w3 \text{ Lovelock3}$$

in a hopefully obvious notation.

Here H is a fundamental inverse length that exists in virtue of the fact that the Einstein – Lovelock tensors have different dimensions.

For simplicity, we do not attempt to associate H with the familiar inverse Planck length, or the Lemaitre – Hubble parameter. For now, let's pretend that they are independent.

NOTES:

Under the spacetime coordinate transformation

$$x^k \mapsto \bar{x}^k (x^j)$$

according to

$$\Psi16(x) \mapsto \overline{\Psi16(\bar{x})} = S(x(\bar{x})) \cdot \Psi16(x(\bar{x})),$$

$$\text{i.e., } \overline{\Psi16^{(a)}(\bar{x})} = \left(S_{(b)}^{(a)} \Psi16^{(b)}(x(\bar{x})) \right)$$

Therefore,

$$\frac{\partial}{\partial \bar{x}^k} \overline{\Psi16(\bar{x})} = \frac{\partial x^j}{\partial \bar{x}^k} \frac{\partial}{\partial x^j} S(x) \cdot \overline{\Psi16}(x) =$$

$$\frac{\partial x^j}{\partial \bar{x}^k} \left\{ S(x) \cdot \frac{\partial}{\partial x^j} \overline{\Psi16}(x) + \left[\frac{\partial}{\partial x^j} S(x) \right] \cdot \overline{\Psi16}(x) \right\}$$

Identify the Octad connection : $\frac{\partial}{\partial x^j} S(x) = -S \cdot \Gamma_j(x) + \frac{\partial \bar{x}^k}{\partial x^j} \bar{\Gamma}_k(\bar{x}) \cdot S$

hence

$$\frac{\partial}{\partial \bar{x}^k} \overline{\Psi16(\bar{x})} = \bar{\Gamma}_k \cdot \overline{\Psi16(\bar{x})} =$$

$$S(x(\bar{x})) \cdot \frac{\partial x^j}{\partial \bar{x}^k} \left\{ \frac{\partial}{\partial x^j} \overline{\Psi16}(x(\bar{x})) - \Gamma_j \cdot \overline{\Psi16}(x(\bar{x})) \right\}$$

or

$$\left(1_{16 \times 16} \frac{\partial}{\partial \bar{x}^k} - \bar{\Gamma}_k \right) \cdot \overline{\Psi16(\bar{x})} =$$

$$S(x(\bar{x})) \cdot \left[\frac{\partial x^j}{\partial \bar{x}^k} \left\{ 1_{16 \times 16} \frac{\partial}{\partial x^j} - \Gamma_j \right\} \cdot \overline{\Psi16}(x(\bar{x})) \right]$$

The connection transforms according to

$$\bar{\Gamma}_k(\bar{x}) = \frac{\partial \bar{x}^j}{\partial x^k} \left\{ S \cdot \Gamma_j(x) \cdot S^{-1} + S_{,j} \cdot S^{-1} \right\}$$

and is defined as

$$\Gamma_j = \frac{1}{2} [e_k^{(a)} \nabla_j e_{(b)}^k] SAB^{(b)}_{(a)}$$

Here, spin connection coefficients are $e_k^{(a)} (\nabla_j e_{(b)}^k)$

and

$$S = \exp \left(\frac{1}{2} \omega_{(A)(B)}(x) SAB^{(A)(B)} \right),$$

where $\omega_{(B)(A)} = -\omega_{(A)(B)}$;

In the spinor Lagrangian Lg[], below, M is the mass of the $\Psi16$ field [A.K.A., wave function characterizing the Universe(s) of masses $\pm M$];

TODO: prove **Universe(s) of masses $\pm M$ are created in pairs!**

K is used to track spin coefficients; K == 1; set K→1 to employ [total] covariant derivative of spinors; put K→0 to ignore

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:

- $\nabla_\mu e^a{}_\nu = 0$
- Here, ∇_μ is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_\nu$ is the tetrad component.

apologies, I lost the citation for this :

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^\omega_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_\mu e^I_\nu = \partial_\mu e^I_\nu - \Gamma^\rho_{\mu\nu} e^I_\rho + \omega^I_{\mu J} e^J_\nu = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^\omega_\mu \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigorous in the mathematical demonstrations, too rigorous at my taste :-))

? SOURCE terms $TU^{\mu\nu}$ for $g_{\alpha\beta}$ come from "Universes' Wave Function Ψ^{16} Lagrangian"

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]} \text{ Lagrangian}_{\Psi^{16}})$$

(hope that $TU^{\mu\nu} = \Lambda g^{\mu\nu}$, and ×

$H = \text{some function of } M, \text{ where } \times$
 Universe (s) of masses $\pm M$ created in pairs at time $x4 = 0$,
 before the particles of the standard model exist) ;

WARNING:

Universes $\Psi16$ source $g_{\alpha\beta}$;

The Euler–Lagrange equations for $\Psi16$ must have
 ‘solutions’ such that
 all off–diagonal terms of $TU^{\mu\nu}$ ARE ZERO .

The $\Psi16$ Lagrangian (please see below) =

$$\sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg[]} = \sqrt{\text{Det}[g_{\mu\nu}]} * \left(\text{Transpose}[\Psi16].\circ16.\text{Sum}\left[T16^\alpha[\alpha1 - 1].\left\{1_{16 \times 16} \frac{\partial}{\partial x^{\alpha1-1}} - \Gamma_{\alpha1-1}\right\}.\Psi16, \{\alpha1, 1, \text{Length}[X]\}\right] + \frac{mASs}{2} * \text{Transpose}[\Psi16].\circ16.\Psi16\right)$$

Next,

assume that $\Psi16soln$ solves the Euler – Lagrange equations .

SOURCE terms

$TU^{\mu\nu}$ [evaluate terms after performing
 differentiation] (using Lagrangian $\Psi16 \equiv$
 $(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{ Lg}[]) |_{\Psi16=\text{solution-toEL-eqs}}$) :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]} \text{ Lg}[])$$

$$= \left(\text{Lg}[] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{\text{Det}[g_{\alpha\beta}]} \text{ Lg}[]) \right] + \frac{\partial}{\partial g_{\mu\nu}} (\text{Lg}[]) \right) \quad \boxed{\Psi16=\Psi16soln}$$

$$\begin{aligned}
&= \left(\theta ? * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(-\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \frac{\sqrt{\text{Det}[g_{\alpha\beta}]}}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} (-Lg[]) \right) \quad \boxed{\Psi16=\Psi16soln} \\
&= \frac{\partial}{\partial g_{\mu\nu}} \left(\text{Transpose}[\Psi16].\circ16.\text{Sum}\left[D[\Psi16, X[\alpha1]] + \left(\left(\frac{1}{2}\right) \text{connectionMatrix} \right). \text{Sum}[\omega[\alpha1, a, b] * \text{SAB}[a, b], \{a, 1, 8\}, \{b, 1, 8\}].\Psi16, \{\alpha1, 1, \text{Length}[X]\}] + \right. \right. \\
&\quad \left. \left. mASs * \text{Transpose}[\Psi16].\text{symm16}[j, 1].\Psi16 \right] \right) \quad \boxed{\Psi16=\Psi16soln} \\
&= \text{Transpose}[\Psi16].\circ16.\text{Sum}\left[\frac{\partial}{\partial g_{\mu\nu}} (T16^\alpha[\alpha1-1]) . \Psi16_{\alpha1-1}, \{\alpha1, 1, \text{Length}[X]\} \right] \quad \boxed{\Psi16=\text{solution-toEL-eqs}} \\
&\stackrel{\sim}{=} \Psi16.\circ16.T16^A.\Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (g^{-1}{}^{\alpha\beta} e_\beta^B \eta_{BA}) = \stackrel{\sim}{=} \Psi16.\circ16.T16^A.\Psi16_{,\alpha} \frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha)
\end{aligned}$$

NOTES :

Let g be a square matrix; we could use : $\frac{\partial}{\partial q} (g^{-1}) = -g^{-1} \cdot \frac{\partial g}{\partial q} \cdot g^{-1}$,

where q is a parameter (result from $\frac{\partial}{\partial q} (g \cdot g^{-1}) = 0$),

In this notebook,

g is usually some 8×8 matrix [with unconstrained elements; call them $g_{\mu\nu}$] ,
 g^{-1} is its matrix inverse

Hence $\frac{\partial}{\partial g_{\mu\nu}} (g^{-1}) = -g^{-1} \cdot \frac{\partial g}{\partial g_{\mu\nu}} \cdot g^{-1}$;

here the $g_{\mu\nu}$ are independent parameters ; note that $g_{\nu\mu} \neq g_{\mu\nu}$ since the $g_{\mu\nu}$ are independent. In metric matrix g ,

we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$, and then differentiate, for example.

This should be understood before reading further.

Therefore $\frac{\partial}{\partial g_{\mu\nu}} (e_{(A)}^\alpha) = \frac{\partial}{\partial g_{\mu\nu}} (\text{Inverse}[e_\alpha^{(A)}]) =$
 $-\text{Inverse}[e_\alpha^{(A)}] \cdot \frac{\partial e_\alpha^{(A)}}{\partial g_{\mu\nu}} \cdot \text{Inverse}[e_\alpha^{(A)}] = -e_{(A)}^\alpha [\alpha1, A1] \frac{\partial e_\alpha^{(A)} [\alpha1, \alpha2]}{\partial g_{\mu\nu}} \cdot e_{(A)}^\alpha [\alpha2, A2]$

$$\text{Consider } \frac{\partial}{\partial g_{\mu\nu}} (g_{\alpha\beta}) = \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AB} e_\alpha^{(A)} e_\beta^{(B)}) = \eta_{AB} e_\beta^{(B)} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^{(A)} \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^{(B)});$$

$$e_c^\beta \frac{\partial g_{\alpha\beta}}{\partial g_{\mu\nu}} = \frac{\partial}{\partial g_{\mu\nu}} (\eta_{AC} e_\alpha^{(A)} e_\beta^{(B)}) = \eta_{AC} \frac{\partial}{\partial g_{\mu\nu}} (e_\alpha^{(A)}) + \eta_{AB} e_\alpha^{(B)} e_c^\beta \frac{\partial}{\partial g_{\mu\nu}} (e_\beta^{(B)})$$

In metric matrix g , we must replace element $g_{\mu\nu}$ with $\frac{g_{\nu\mu} + g_{\mu\nu}}{2}$,

and then differentiate.

Some of my calculations employ “http://www.xact.es/download/xAct_1.2.0.tgz” ; also, see Cyril Pitrou

Institut d’Astrophysique de Paris (France)

pitrou@iap.fr

<http://www2.iap.fr/users/pitrou/>

“FriedmannLemaitreMetric_CoordinatesApproach_xCoba.nb”

MyArrayComponents[expr_] := expr //ToBasis[BS] //ComponentArray //ToValues //ToValues //Simplify

I copied and pasted too often; however, I am going to leave this here.

Universe sources for $g_{\alpha\beta}$;
be sure to append these to the Einstein and/or Einstein–Lovelock field equations.

WARNING: all off-diagonal terms of all source terms [so that $g_{\mu\nu}$ is diagonal] MUST BE ZERO in order for the previous ‘hacked solutions’ for $\Psi16$ to be valid

Let $T^\alpha = \{T16^\alpha[0], T16^\alpha[4]\}$ and

$$\sqrt{\text{Det}[g_{\mu\nu}]} * \text{Lg}[\text{mASs}_-, j_-] = \\ \sqrt{\text{Det}[g_{\mu\nu}]} * (\text{Transpose}[\Psi16].\sigma16.\text{Sum}[T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \\ \{\alpha1, 1, \text{Length}[\text{vars}]\}] + \text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[134, 1].\Psi16)$$

Let $j = 134$, $\sigma16.(\text{mass Matrix}) = T16^\alpha[5].T16^\alpha[6].T16^\alpha[7]$

and

$$\text{usingLagrangianF16massive} = (\text{Transpose}[\Psi16].\sigma16. \\ \text{Sum}[T^\alpha[\alpha1 - 1].D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\}] + \\ \text{mASs} * \text{Transpose}[\Psi16].\text{symm16}[j = 134, 1].\Psi16);$$

where it is assumed that $\Psi16$ solves the Euler – Lagrange equations, above; then $\text{usingLagrangianF16massive} = 0$, as shown below.

SOURCE terms $TU^{\mu\nu}$ [evaluate terms after performing differentiation] :

$$\frac{1}{\kappa} TU^{\mu\nu} \equiv \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \text{Lg}[\] \right)$$

$$= \left(\text{Lg}[\] * \left[\frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) \right] + \right. \\ \left. \frac{\partial}{\partial g_{\mu\nu}} \left(\text{Lg}[\] \right) \right) \quad \Big| \text{ Lg}[\] = \text{usingLagrangianF16massive} \\ = \left(0 * \frac{1}{\sqrt{\text{Det}[g_{\alpha\beta}]}} \frac{\partial}{\partial g_{\mu\nu}} \left(\sqrt{\text{Det}[g_{\alpha\beta}]} \right) + \right.$$

remark: the term

$$\underset{\partial g_{\mu\nu}}{\overset{\sim}{\frac{\partial}{\partial}}} \Psi16 . \sigma16 . T16^A . \Psi16_{,\alpha} \underset{\partial}{\frac{\partial}{\partial}} (e_A^\alpha) = \underset{\partial g_{\mu\nu}}{\overset{\sim}{\frac{\partial}{\partial}}} \Psi16 . \sigma16 . T16^A . \Psi16_{,\alpha} e_A^\alpha =$$

$$\left(\underset{\partial g_{\mu\nu}}{\frac{\partial}{\partial}} \right) \left(\begin{array}{l} \text{Transpose}[\Psi16] . \sigma16 . \\ \\ \underbrace{T16^A[A1-1] . \Psi16_{,\alpha1} \eta_{A1B} e_B^\beta g^{-1} \alpha1 \beta}_{\text{Sum}\left[T^\alpha[\alpha1 - 1] . D[\Psi16, \text{vars}[\alpha1]], \{\alpha1, 1, \text{Length}[\text{vars}]\} \right]} \end{array} \right)$$

Frame fields [\[edit\]](#)

We use a set of `vierbein` or frame fields $\{e_\mu\} = \{e_0, e_1, e_2, e_3\}$, which are a set of vector fields (which are not necessarily defined globally on M). Their defining equation is

$$g_{ab} e_\mu^a e_\nu^b = \eta_{\mu\nu}.$$

The vierbein defines a local rest `frame`, allowing the constant `Gamma matrices` to act at each spacetime point.

In differential-geometric language, the vierbein is equivalent to a `section` of the `frame bundle`, and so defines a local trivialization of the frame bundle.

Incomplete theories [\[snow\]](#)

Scientists [\[show\]](#)

[V • T • E](#)

Spin connection [\[edit\]](#)

To write down the equation we also need the `spin connection`, also known as the connection (1-)form. The dual frame fields $\{e^\mu\}$ have defining relation

$$e_a^\mu e_\nu^a = \delta^\mu_\nu.$$

The connection 1-form is then

$$\omega^\mu_{\nu a} := e_b^\mu \nabla_a e_\nu^b$$

where ∇_a is a `covariant derivative`, or equivalently a choice of `connection` on the frame bundle, most often taken to be the `Levi-Civita connection`.

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space $T_p M$ to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

`Raising and lowering indices` is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract `connection on a principal bundle`, specifically on the `frame bundle`, which is defined on any smooth manifold, but which restricts to an orthonormal frame bundle on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [\[edit\]](#)

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\alpha\}$ is defined

Frame fields [edit]

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V • T • E

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where ∇_a is a **covariant derivative**, or equivalently a choice of **connection** on the frame bundle, most often taken to be the **Levi-Civita connection**.

One should be careful not to treat the abstract Latin indices and Greek indices as the same, and further to note that neither of these are coordinate indices: it can be verified that $\omega^\mu{}_{\nu a}$ doesn't transform as a tensor under a change of coordinates.

Mathematically, the frame fields $\{e_\mu\}$ define an isomorphism at each point p where they are defined from the tangent space $T_p M$ to $\mathbb{R}^{1,3}$. Then abstract indices label the tangent space, while greek indices label $\mathbb{R}^{1,3}$. If the frame fields are position dependent then greek indices do not necessarily transform tensorially under a change of coordinates.

Raising and lowering indices is done with g_{ab} for latin indices and $\eta_{\mu\nu}$ for greek indices.

The connection form can be viewed as a more abstract **connection on a principal bundle**, specifically on the **frame bundle**, which is defined on any smooth manifold, but which restricts to an **orthonormal frame bundle** on pseudo-Riemannian manifolds.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

$$\partial_\mu \psi = e_\mu^\alpha \partial_\alpha \psi,$$

and connection components with respect to a general orthonormal frame are

$$\omega^\mu{}_{\nu\rho} = e_\rho^\alpha \omega^\mu{}_{\nu\alpha}.$$

These components do not transform tensorially under a change of frame, but do when combined. Also, these are definitions rather than saying that these objects can arise as partial derivatives in some coordinate chart. In general there are non-coordinate orthonormal frames, for which the commutator of vector fields is non-vanishing.

It can be checked that under the transformation

$$\psi \mapsto \rho(\Lambda)\psi,$$

if we define the covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \frac{1}{2} (\omega_{\nu\rho})_\mu \sigma^{\nu\rho} \psi,$$

then $D_\mu \psi$ transforms as

$$D_\mu \psi \mapsto \rho(\Lambda) D_\mu \psi$$

This generalises to any representation R for the Lorentz group: if v is a vector field for the associated representation,

$$D_\mu v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu R(M^{\nu\rho}) v = \partial_\mu v + \frac{1}{2} (\omega_{\nu\rho})_\mu T^{\nu\rho} v.$$

When R is the fundamental representation for $SO(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

Covariant derivative for fields in a representation of the Lorentz group [edit]

Given a coordinate frame ∂_α arising from say coordinates $\{x^\alpha\}$, the partial derivative with respect to a general orthonormal frame $\{e_\mu\}$ is defined

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When R is the fundamental representation for $SO(1,3)$, this recovers the familiar covariant derivative for (tangent-)vector fields, of which the Levi-Civita connection is an example.

OTHER WORK THAT I USED:

JOURNAL OF MATHEMATICAL PHYSICS

VOLUME 4, NUMBER 7

A Remarkable Representation of the $3 + 2$ de Sitter Group

P. A. M. DIRAC

*Cambridge University, Cambridge, England and
Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

Proc. Camb. Phil. Soc. (1968), 64, 765

765

PCPS 64-96

*Printed in Great Britain***The Dirac spinor in six dimensions**

By E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

Identities satisfied by the generators of the Dirac algebra

Patrick L. Nash
M. S. 474, NASA Langley Research Center, Hampton, Virginia 23665

(Received 26 July 1983; accepted for publication 23 September 1983)

A new spin- $\frac{1}{2}$ wave equation

Patrick L. Nash
Division of Earth and Physical Sciences, University of Texas at San Antonio, San Antonio, Texas 78285
(Received 24 January 1984; accepted for publication 16 November 1984)

Begin

```
In[0]:= Print["CopyRight (C) 2022, Patrick L. Nash, under the General Public License."]
CopyRight (C) 2022, Patrick L. Nash, under the General Public License.

In[0]:= Print["Please cite this work, and this web page, if you use it"]
Please cite this work, and this web page, if you use it

In[0]:= Print["For simplicity, metric gαβ= gαβ(x0,x1,x2,x3,x4,x5,x6,x7)=
gαβ(x0,x4) = diag{ g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1,
g77(x0,x4),g77(x0,x4),g77(x0,x4) },"]
For simplicity, metric gαβ= gαβ(x0,x1,x2,x3,x4,x5,x6,x7)= gαβ(x0,x4) = diag{
g00(x0),g11(x0,x4),g11(x0,x4),g11(x0,x4),-1, g77(x0,x4),g77(x0,x4),g77(x0,x4) },

In[0]:= Print["where g77(x0,x4) = g11(x0,-x4) "]
where g77(x0,x4) = g11(x0,-x4)

ConvertMapleToMathematicaV2

In[0]:= FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]]
mapleToMathematicaV2PATH = FileNameJoin[{%, "ConvertMapleToMathematicaV2.m"}]

Out[0]=
C:\Users\nsh\Documents\8-dim\2026-01-04-Complex

Out[0]=
C:\Users\nsh\Documents\8-dim\2026-01-04-Complex\ConvertMapleToMathematicaV2.m

In[0]:= Get[mapleToMathematicaV2PATH] (*Loads new maple to mathematica parser*)
ConvertMapleToMathematicaV2 loaded successfully!
BUT, WARNING: DO NOT USE IF YOU WANT A CORRECT RESULT!

Load the Maple output strings.

In[0]:= (*SetDirectory[NotebookDirectory[]];*)

In[0]:= (*maplestringEQ1=Get["string-seq1.txt"]
maplestringEQ2=Get["thinkpad_stringEQ2.txt"]*)
```

```
In[®]:= NotebookFileName[]
Out[®]= C:\Users\nsh\Documents\8-dim\2026-01-04-Complex\2026-01-30-4sets-of-4-eqs-4+4-Einstein-
Lovelock-Nash-L9i.nb

In[®]:= Unprotect[dir];
dir = FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]];
Protect[dir];
If[$Failed == SetDirectory[dir], {.CreateDirectory[dir], SetDirectory[dir]}];

In[®]:= (*FileNameJoin[Drop[FileNameSplit[NotebookFileName[]], -1]]*)

In[®]:= names = FileNameSplit[NotebookFileName[]]
Out[®]= {C:, Users, nsh, Documents, 8-dim, 2026-01-04-Complex,
2026-01-30-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-L9i.nb}

In[®]:= name = StringReplace[names[[-1]], "nb" → "mx"]
Out[®]= 2026-01-30-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-L9i.mx

In[®]:= header = StringReplace[names[[-1]], ".nb" → "-"]
Out[®]= 2026-01-30-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-L9i-

In[®]:= SetOptions[Simplify, TimeConstraint → 1]
Out[®]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, 
TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[®]:= SetOptions[FullSimplify, TimeConstraint → 3]
Out[®]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, 
TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}

In[®]:= Options[Simplify]
Out[®]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, 
TimeConstraint → 1, TransformationFunctions → Automatic, Trig → True}

In[®]:= Options[FullSimplify]
Out[®]= {Assumptions → $Assumptions, ComplexityFunction → Automatic, ExcludedForms → {}, 
TimeConstraint → 3, TransformationFunctions → Automatic, Trig → True}

In[®]:= {Off[General::spell], Off[General::spell1]};
```

```
In[•]:= DIM8 = 8;
In[•]:= Protect[DIM8, M, K, H]
Out[•]= {DIM8, M, K, H}
```

Coordinates :

$\left\{ \underbrace{\text{hidden space}}, \underbrace{\text{3-space}}, \underbrace{\text{time}} \underbrace{\text{superluminal deflating time}} \right\}$
 $\{x_0, \overbrace{x_1, x_2, x_3}^{\text{3-space}}, x_4, \overbrace{x_5, x_6, x_7}^{\text{superluminal deflating time}}\} :$

```
In[•]:= X = {x0, x1, x2, x3, x4, x5, x6, x7};
Protect[X];
Protect[x0, x1, x2, x3, x4, x5, x6, x7];
```

```
In[•]:= sX0 = And @@ Thread[X > 0]
Out[•]=  $x_0 > 0 \&\& x_1 > 0 \&\& x_2 > 0 \&\& x_3 > 0 \&\& x_4 > 0 \&\& x_5 > 0 \&\& x_6 > 0 \&\& x_7 > 0$ 
```

```
In[•]:= ssX = H > 0 \&\& sX0 \&\& 6 H x0 > 0 \&\& 2 la[x4] > 0 \&\& Cot[6 H x0] > 0 \&\& Sin[6 H x0] > 0
Out[•]=  $H > 0 \&\& x_0 > 0 \&\& x_1 > 0 \&\& x_2 > 0 \&\& x_3 > 0 \&\& x_4 > 0 \&\& x_5 > 0 \&\& x_6 > 0 \&\&$   

 $x_7 > 0 \&\& 6 H x0[] > 0 \&\& 2 la[x4[]] > 0 \&\& Cot[6 H x0[]] > 0 \&\& Sin[6 H x0[]] > 0$ 
```

For Maple_2025 :

```
In[•]:= sreplaceZ = Z[#] → ToExpression["Z" <> ToString[#]] & /@ Range[0, 15]
Out[•]= {Z[0] → Z0, Z[1] → Z1, Z[2] → Z2, Z[3] → Z3, Z[4] → Z4,  

Z[5] → Z5, Z[6] → Z6, Z[7] → Z7, Z[8] → Z8, Z[9] → Z9, Z[10] → Z10,  

Z[11] → Z11, Z[12] → Z12, Z[13] → Z13, Z[14] → Z14, Z[15] → Z15}

In[•]:= sreplacenZ = nZ[#] → ToExpression["nZ" <> ToString[#]] & /@ Range[0, 15]
Out[•]= {nZ[0] → nZ0, nZ[1] → nZ1, nZ[2] → nZ2, nZ[3] → nZ3, nZ[4] → nZ4,  

nZ[5] → nZ5, nZ[6] → nZ6, nZ[7] → nZ7, nZ[8] → nZ8, nZ[9] → nZ9, nZ[10] → nZ10,  

nZ[11] → nZ11, nZ[12] → nZ12, nZ[13] → nZ13, nZ[14] → nZ14, nZ[15] → nZ15}
```

hacks (a.k.a., lies):

```
In[8]:= constraintTrig =
Sin[6 * H * x0] > 0 && Cos[6 * H * x0] > 0 && Csc[6 * H * x0] > 0 && Sec[6 * H * x0] > 0 &&
Tan[6 * H * x0] > 0 && Cot[6 * H * x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^3/2 > 0 &&
Sin[z]^1/2 > 0 && Sin[z]^-3/2 > 0 && Sin[z]^-1/2 > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]^3/2 > 0 && Cot[z]^1/2 > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

```
Out[8]= Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^3/2 > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]^(3/2) > 0 && 1/Sqrt[Sin[z]] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]^3/2 > 0 && Cot[z]^1/2 > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

more hacks (damn lies):

```
In[9]:= (*&&K^2-M^2>0&&Sqrt[K^2-M^2]>0&&e^(2 H Sqrt[K^2-M^2])^x4>0&&2 H Sqrt[K^2-M^2] x4>0&&
2 H Sqrt[1-M^2] x4>0&&Sqrt[1-M^2]>0&&e^(2 H Sqrt[1-M^2])^x4>0&&2 H Sqrt[1-M^2] x4>0*)
```

```
In[10]:= constraintX = x0 > 0 && x4 > 0 && H > 0 && 6 * H * x0 > 0 &&
3 * H * x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 && z > 0 && t > 0 && M > 0 &&
K > 0 && e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0 && e^2 a4[H x4] > 0 && e^a4[H x4] > 0
```

```
Out[10]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 && Q > 0 &&
z > 0 && t > 0 && M > 0 && K > 0 && e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0 && e^2 a4[H x4] > 0 && e^a4[H x4] > 0
```

statistics:

```
In[11]:= constraintVars = constraintX && constraintTrig
```

```
Out[11]= x0 > 0 && x4 > 0 && H > 0 && 6 H x0 > 0 && 3 H x4 > 0 && a4[x4] > 0 && A4[t] > 0 &&
Q > 0 && z > 0 && t > 0 && M > 0 && K > 0 && e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0 && e^2 a4[H x4] > 0 &&
e^a4[H x4] > 0 && Sin[6 H x0] > 0 && Cos[6 H x0] > 0 && Csc[6 H x0] > 0 && Sec[6 H x0] > 0 &&
Tan[6 H x0] > 0 && Cot[6 H x0] > 0 && Sin[z] > 0 && Sqrt[Sin[z]] > 0 && Sin[z]^3/2 > 0 &&
Sqrt[Sin[z]] > 0 && 1/Sin[z]^(3/2) > 0 && 1/Sqrt[Sin[z]] > 0 && Cot[z] > 0 && Sqrt[Cot[z]] > 0 &&
Cot[z]^3/2 > 0 && Cot[z]^1/2 > 0 && Tan[z] > 0 && Sec[z] > 0 && Sqrt[Csc[z]] > 0 && Csc[z] > 0
```

```
In[8]:= subsDefects = {  $\sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}$ ,  

 $\sqrt{e^{-2 a4[H x4]}} \rightarrow e^{-a4[H x4]}$ ,  $\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}} \rightarrow e^{a4[H x4]} \sin[6 H x4]^{1/6}$ ,  

 $\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}}} \rightarrow \frac{1}{e^{a4[H x4]} \sin[6 H x4]^{1/6}}$ ,  

 $\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{1}{e^{a4[H x4]} \sin[6 H x0]^{1/6}}$ ,  

 $\frac{1}{\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{1}{e^{-a4[H x4]} \sin[6 H x0]^{1/6}}$ ,  

 $\sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}} \rightarrow e^{a4[H x4]} \sin[6 H x0]^{1/6}$ ,  

 $\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/6}} \rightarrow e^{-a4[H x4]} \sin[6 H x0]^{1/6}$  }
```

Out[8]= { $\sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}$, $\sqrt{e^{-2 a4[H x4]}} \rightarrow e^{-a4[H x4]}$, $\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}} \rightarrow e^{a4[H x4]} \sin[6 H x4]^{1/6}$,
 $\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x4]^{1/3}}} \rightarrow \frac{e^{-a4[H x4]}}{\sin[6 H x4]^{1/6}}$, $\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{e^{-a4[H x4]}}{\sin[6 H x0]^{1/6}}$,
 $\frac{1}{\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/6}}} \rightarrow \frac{e^{a4[H x4]}}{\sin[6 H x0]^{1/6}}$, $\sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/6}} \rightarrow e^{a4[H x4]} \sin[6 H x0]^{1/6}$,
 $\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/6}} \rightarrow e^{-a4[H x4]} \sin[6 H x0]^{1/6}$ }

future coordinate transformation :

```
In[9]:= 6 H x0 == z && H x4 == t  

szt = Solve[%, {z, t}][[1]]  

sx0x4 = Solve[%%, {x0, x4}][[1]]  

Protect[sx0x4, szt];
```

Out[9]= $6 H x0 == z \& H x4 == t$

Out[10]= {z → 6 H x0, t → H x4}

Out[11]= $\left\{x0 \rightarrow \frac{z}{6 H}, x4 \rightarrow \frac{t}{H}\right\}$

In[12]:= sx0x4

Out[13]= $\left\{x0 \rightarrow \frac{z}{6 H}, x4 \rightarrow \frac{t}{H}\right\}$

In[14]:= (*sa4={a4→((A4[3 H #2])&)}*)

```
In[1]:= Protect[sx0x4, szt]
Out[1]= {}

SO(4, 4) Minkowski Lorentz metric  $\eta_{4488}$  : 4 + 4 spacetime; 8 × 8 dimensional :

In[2]:= ( $\eta_{4488} = \text{ArrayFlatten}[\{\{\text{IdentityMatrix}[4], 0\}, \{0, -\text{IdentityMatrix}[4]\}\}]$ ) // MatrixForm
Out[2]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


SO(4, 4) Spinor Lorentz metric  $\sigma$ ;
 $\tau^A$  are analogous to the Dirac gamma matrices :
all true :

```

$$\text{Table}\left[\left\{A, B, \text{FullSimplify}\left[\frac{1}{2}(\tau[A].\bar{\tau}[B] + \tau[B].\bar{\tau}[A]) = \eta_{4488}[[A+1, B+1]] * \text{ID8}\right]\right\}, \{A, 0, 7\}, \{B, 0, 7\}\right]$$

Type - 1 and type - 2 employ σ (the usual notational abuse),
since $\sigma = \sigma^{-1}$: 4 + 4 spacetime, 8 × 8 dimensional and 16 × 16 dimensional :

indices. We define the reduced Brauer-Weyl generators $\{\bar{\tau}^{A'}, \tau^{A'}\}$ of the generators of the two real 8 × 8 inequivalent irreducible representations of $SO(4, 4)$ (see eqs. (2.6) and (2.7)) by demanding that the tau matrices satisfy (the tilde denotes transpose)

$$(2.3) \quad \tilde{\tau}^{A'} \sigma = \sigma \bar{\tau}^{A'}$$

and

$$(2.4) \quad \tau^{A'} \bar{\tau}^{B'} + \tau^{B'} \bar{\tau}^{A'} = 2IG^{A'B'} = \bar{\tau}^{A'} \tau^{B'} + \bar{\tau}^{B'} \tau^{A'},$$

where I denotes the 8 × 8 unit matrix. Denoting the matrix elements of $\tau^{A'}$ by $\tau^{A'a}_b$, we may write eq. (3) as

$$(2.5) \quad \bar{\tau}^{A'}_{ab} = \tau^{A'}_{ba},$$

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix}$$

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4, 4)}$ and $L \in SO(4, 4)$. The canonical 2-1 homomorphism $\overline{SO(4, 4)} \rightarrow SO(4, 4)$ is given by

$$(2.8) \quad 8L^{A'}{}_{B'}(g) = \text{tr } \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4, 4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}{}_{B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

```
In[8]:= (σ = ArrayFlatten[{{ConstantArray[0, {4, 4}], IdentityMatrix[4]}, {IdentityMatrix[4], ConstantArray[0, {4, 4}]}}]) // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

below :

```
In[9]:= σ16 == ArrayFlatten[{{{-σ, 0}, {0, σ}}}]
```

below :

```
In[8]:= Table[T16^A[A1] = ArrayFlatten[{{(0, T[A1]), (T[A1], 0)}}, {(A1, 0, 7)}];
```

below:

```
In[9]:= c16.T16^A[U] = -Transpose[c16.T16^A[U]] & /@ Range[0, 7]
```

Out[9]=

{True, True, True, True, True, True, True, True}

below:

```
In[10]:= Table[{(A1, B1), FullSimplify[ExpandA11[
  1/2 (T16^A[A1].T16^A[B1] + T16^A[B1].T16^A[A1]) = n4488[A1 + 1, B1 + 1]*ID16]}, {(A1, 0, 7), (B1, 0, 7)}] // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} (\{0, 0\}) & (\{0, 1\}) & (\{0, 2\}) & (\{0, 3\}) & (\{0, 4\}) & (\{0, 5\}) & (\{0, 6\}) & (\{0, 7\}) \\ \text{True} & \text{True} \\ (\{1, 0\}) & (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) & (\{1, 5\}) & (\{1, 6\}) & (\{1, 7\}) \\ \text{True} & \text{True} \\ (\{2, 0\}) & (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) & (\{2, 5\}) & (\{2, 6\}) & (\{2, 7\}) \\ \text{True} & \text{True} \\ (\{3, 0\}) & (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) & (\{3, 5\}) & (\{3, 6\}) & (\{3, 7\}) \\ \text{True} & \text{True} \\ (\{4, 0\}) & (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) & (\{4, 5\}) & (\{4, 6\}) & (\{4, 7\}) \\ \text{True} & \text{True} \\ (\{5, 0\}) & (\{5, 1\}) & (\{5, 2\}) & (\{5, 3\}) & (\{5, 4\}) & (\{5, 5\}) & (\{5, 6\}) & (\{5, 7\}) \\ \text{True} & \text{True} \\ (\{6, 0\}) & (\{6, 1\}) & (\{6, 2\}) & (\{6, 3\}) & (\{6, 4\}) & (\{6, 5\}) & (\{6, 6\}) & (\{6, 7\}) \\ \text{True} & \text{True} \\ (\{7, 0\}) & (\{7, 1\}) & (\{7, 2\}) & (\{7, 3\}) & (\{7, 4\}) & (\{7, 5\}) & (\{7, 6\}) & (\{7, 7\}) \\ \text{True} & \text{True} \end{pmatrix}$$

defs; some Symbols; metric

$$\left\{ \left\{ a4[t] \rightarrow \frac{Mt(c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7])}{Q1} + \frac{1}{c_1 + \frac{1}{2} \text{ProductLog}\left[-e^{-\frac{2Mt(c[0]+c[1]+c[2]+c[3]+c[4]+c[5]+c[6]+c[7])}{Q1}-2c_1}\right]\}} \right\} \right\}$$

$$\begin{aligned} \text{DSolve}[\\ 0 & == -M(c[0] + c[1] + c[2] + c[3] + c[4] + c[5] + c[6] + c[7]) + (Q1 - e^{-2a4[t]} Q1) a4'[t], \\ a4[t], t] \end{aligned}$$

$$Mc[j] = (j+1)^{\text{st}} \text{ Energy Eigenvalue for Transpose[cayZ].}\Psi16, j = 0, \dots, 7$$

ProductLog[z]

gives the principal solution for w in $z = we^w$.

if z and w are any complex numbers, then

$$we^w = z$$

holds if and only if

$$w = W_k(z) \text{ for some integer } k.$$

When dealing with real numbers only, the two branches W_0 and W_{-1} suffice: for real numbers x and y the equation

$$ye^y = x$$

can be solved for y only if $x \geq \frac{-1}{e}$; yields $y = W_0(x)$ if $x \geq 0$ and the two values $y = W_0(x)$ and $y = W_{-1}(x)$ if $\frac{-1}{e} \leq x < 0$.

?todo: NEED DefScalarFunction[#]; & /@ (f16[#]&/@Range[0,15]), and allow xact to compute?

Introduce the wave function, Ψ_{16} , for a Universe:

```
In[•]:= Ψ16 = f16[#, x0, x4] & /@ Range[0, 15]
Out[•]= {f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}

In[•]:= processRawSets[rawSets_] := Module[{s1, s2, J, ret, L = Range[Length[rawSets]], r = {}},
  While[Length[L] > 1,
    s1 = Union[rawSets[[L[[1]]]]];
    L = Drop[L, 1];
    J = L[[1]];
    While[J ≤ 16,
      If[MemberQ[L, J],
        s2 = Union[rawSets[[J]]];
        If[Intersection[s1, s2] ≠ {}, s1 = Union[Flatten[{s1, s2}]]];
        L = Complement[L, {J}]; ##&[]];
      ##&[]];
    J++];
    AppendTo[r, s1];
  ];
  ret = Sort[r, #1[[1]] < #2[[1]] &];
  Return[ret];
]
```

```
In[1]:= rawSets¶16 = Cases [
  #,
  h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
  Infinity,
  Heads → True
] & /@ ¶16

Out[1]= {{}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}, {15}}
```

```
In[2]:= processRawSets[rawSets¶16]

Out[2]= {{}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}}
```

```
In[3]:= (*¶new16=F16[#][x0,x4]&/@Range[0,15]*)


```

```
In[4]:= Clear[sf16Aa];
sf¶16Aa = f16[#] → ToExpression["((Z[" <> ToString[#] <> "] [6*H*#1,H*#2])&)" ] & /@
Range[0, 15]
```

```
Out[4]= {f16[0] → (Z[0] [6 H #1, H #2] &), f16[1] → (Z[1] [6 H #1, H #2] &),
f16[2] → (Z[2] [6 H #1, H #2] &), f16[3] → (Z[3] [6 H #1, H #2] &),
f16[4] → (Z[4] [6 H #1, H #2] &), f16[5] → (Z[5] [6 H #1, H #2] &),
f16[6] → (Z[6] [6 H #1, H #2] &), f16[7] → (Z[7] [6 H #1, H #2] &),
f16[8] → (Z[8] [6 H #1, H #2] &), f16[9] → (Z[9] [6 H #1, H #2] &),
f16[10] → (Z[10] [6 H #1, H #2] &), f16[11] → (Z[11] [6 H #1, H #2] &),
f16[12] → (Z[12] [6 H #1, H #2] &), f16[13] → (Z[13] [6 H #1, H #2] &),
f16[14] → (Z[14] [6 H #1, H #2] &), f16[15] → (Z[15] [6 H #1, H #2] &) }
```

```
In[5]:= (*Clear[sf16Aa];
sf¶16Aa=f16[#]→ToExpression[
  "((Z["<>ToString[#]<>"] [6*H*#1,H*#2] 1/sin[6*H*#1]^(1/2))&)" ]&/@Range[0,15]*)
```

```
In[•]:= Clear[snewfψ16Aa];
snewfψ16Aa = f16[#] → ToExpression["((nZ[" <> ToString[#] <>
"])[6*H*#1,H*#2] 1
Sin[6*H*#1]^(1/2))&)" ] & /@ Range[0, 15]
```

```
Out[•]= {f16[0] → (nZ[0][6 H #1, H #2] &),
f16[1] → (nZ[1][6 H #1, H #2] &),
f16[2] → (nZ[2][6 H #1, H #2] &),
f16[3] → (nZ[3][6 H #1, H #2] &),
f16[4] → (nZ[4][6 H #1, H #2] &),
f16[5] → (nZ[5][6 H #1, H #2] &),
f16[6] → (nZ[6][6 H #1, H #2] &),
f16[7] → (nZ[7][6 H #1, H #2] &),
f16[8] → (nZ[8][6 H #1, H #2] &),
f16[9] → (nZ[9][6 H #1, H #2] &),
f16[10] → (nZ[10][6 H #1, H #2] &),
f16[11] → (nZ[11][6 H #1, H #2] &),
f16[12] → (nZ[12][6 H #1, H #2] &),
f16[13] → (nZ[13][6 H #1, H #2] &),
f16[14] → (nZ[14][6 H #1, H #2] &),
f16[15] → (nZ[15][6 H #1, H #2] &)}
```

0 (4, 4) related :

```
In[•]:= Needs["Notation`"]
```

```
In[•]:= Symbolize[σga1A]
```

```
In[•]:= Symbolize[σga2A]
```

```
In[•]:= Symbolize[τ̄]
```

```
In[•]:= Symbolize[T16A]
```

```
In[•]:= Symbolize[T16α]
```

```
In[•]:= Symbolize[ua]
```

```
In[•]:= Symbolize[Jy]
```

```
In[•]:= Symbolize[FAa]
```

In[1]:= **Symbolize**[F^a_A]

In[2]:= **Symbolize**[σ²²]

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1,1,1, - 1) \quad (5)$$

In[3]:= **Symbolize**[g_{αβ}]

note below: g_{AB} is really g_{(A)(B)}:: Minkowski – Lorentz metric

In[4]:= **Symbolize**[g_{AB}]

note below: G_{AB} is really G_{(A)(B)}:: Minkowski – Lorentz

In[5]:= (***Symbolize**[G_{AB}]*)

In[6]:= **Symbolize**[S^{44αβ}]

$$\Sigma^{\alpha\beta} = \xi S^{\alpha\beta} u,$$

In[7]:= **Symbolize**[Σ^{αβ}]

In[8]:= (***Symbolize**[Σ^{αβ}]*)

$$\frac{1}{2}\Sigma^{jk}\Sigma_{jk} = \frac{1}{4}\partial_a u^a u^b \partial_b - \frac{1}{4}\partial_a u^b u^b \partial_a,$$

In[9]:= **Symbolize**[Σ²]

NEED DefScalarFunction[#]; & /@ Flatten[Universe]

In[10]:= Universe = ToExpression["U" <> ToString[#]] & /@ Range[0, 7]

In[11]:= **Symbolize**[F^A_a]

In[12]:= **Symbolize**[F^a_A]

In[13]:= **Symbolize**[gtrye^(A)_α]

In[14]:= **Symbolize**[gtrye^α_(A)]

SPACETIME METRIC

In[15]:= (*einsteinsteinMetric=Array[g[#1-1, #2-1]&, {8,8}]*)

SPACETIME METRIC:

In[16]:= g4488 = Array[g[#1 - 1][#2 - 1][x0, x4] &, {8, 8}];

SPACETIME METRIC values:

MatrixMetric44

```

6 H x0 = z && H x4 = t
{{{a4''[t] == 0, a4'[t] == 2/3 (-1 + M)}, {{a4''[t] == 0, a4'[t] == 2 (1 + M)/3}}}}
In[=]:= (*β3=Exp[2 H x4 - √(K²-M²)]*)
(*β3=Exp[2 * a4[3*H*x4]]*)
β3 = Exp[2 * a4[H * x4]] (* /. {a4 → ((K1*2^(1+M)/3 + K2*2/3 (-1+M)) #) &} *)
β1 = Sin[6 * H * x0]^(1/3);
β2 = Cot[6 * H * x0]^2;
Out[=]= e^2 a4[H x4]

```

```

In[=]:= MatrixForm[MatrixMetric44 =
{{β2, 0, 0, 0, 0, 0, 0, 0}, {0, β1 β3, 0, 0, 0, 0, 0, 0}, {0, 0, β1 β3, 0, 0, 0, 0, 0},
{0, 0, 0, β1 β3, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -β1/β3, 0, 0},
{0, 0, 0, 0, 0, 0, -β1/β3, 0}, {0, 0, 0, 0, 0, 0, 0, -β1/β3}} // FullSimplify[#, constraintVars] &]

```

```

Out[=]//MatrixForm=

$$\begin{pmatrix} \cot[6Hx0]^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


```

```

In[=]:= (*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression[
"((Z["<>ToString[#]<>"] [6*H*#1,3*H*#2] 1/Sin[6*H*#1]^(1/2))&)" ]&/@Range[0,15]*)

```

```

In[=]:= (*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression["((Z["<>ToString[#]<>"] [6*H*#1,3*H*#2])&)" ]&/@
Range[0,15]*)

```

```

In[]:= (*Clear[sf16Aa];
sfψ16Aa=f16[#]→ToExpression[
"((Z["<>ToString[#]<>"] [6*H*#1,H*#2] 1/Sin[6*H*#1]^(1/2))&)"]&/@Range[0,15]*)

In[]:= (*Inactivate[((q)&)]Evaluate[Activate[Evaluate[q]]]*)

In[]:= Clear[makeSubstitution];
makeSubstitution[] := Module[{met, mat, t, f, p},
  met = Flatten[g4488][[All, 0]];
  mat = Flatten[MatrixMetric44];
  t = Thread[met → mat];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]

In[]:= makeSubstitution[a_, b_] := Module[{t, f, p},
  t = Thread[a → b];
  f = Table[FullForm[t[[j]] /. {x0 → #1, x4 → #2}], {j, 1, Length[t]}];
  p = Table[Block[{s, r, q, expr}, q = f[[j]][[1]][[2]];
    expr = Inactivate[((q) &)];
    s = f[[j]][[1]][[1]] → expr;
    r = Activate[s // Evaluate] // Evaluate;
    r], {j, 1, Length[f]}];
  Return[p];
]

```

In[•]:= **ssgm4488 = makeSubstitution[]**

Out[•]=

```
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
g[1][1] → (e^2 a4[H^#2] Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &),
g[1][4] → (0 &), g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &),
g[2][1] → (0 &), g[2][2] → (e^2 a4[H^#2] Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &),
g[2][5] → (0 &), g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &),
g[3][2] → (0 &), g[3][3] → (e^2 a4[H^#2] Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
g[5][5] → (-e^-2 a4[H^#2] Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
g[6][5] → (0 &), g[6][6] → (-e^-2 a4[H^#2] Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^-2 a4[H^#2] Sin[6 H #1]^(1/3) &) }
```

In[•]:= **Symbolize[E_α^(A)]**

In[•]:= **Symbolize[e_α^(A)]**

In[•]:= **e_α^(A) = Array[E_α^(A) [#1 - 1, #2 - 1] &, {8, 8}]**

Out[•]=

```
{ {Eα(A)[0, 0], Eα(A)[0, 1], Eα(A)[0, 2], Eα(A)[0, 3], Eα(A)[0, 4], Eα(A)[0, 5], Eα(A)[0, 6], Eα(A)[0, 7]}, {Eα(A)[1, 0], Eα(A)[1, 1], Eα(A)[1, 2], Eα(A)[1, 3], Eα(A)[1, 4], Eα(A)[1, 5], Eα(A)[1, 6], Eα(A)[1, 7]}, {Eα(A)[2, 0], Eα(A)[2, 1], Eα(A)[2, 2], Eα(A)[2, 3], Eα(A)[2, 4], Eα(A)[2, 5], Eα(A)[2, 6], Eα(A)[2, 7]}, {Eα(A)[3, 0], Eα(A)[3, 1], Eα(A)[3, 2], Eα(A)[3, 3], Eα(A)[3, 4], Eα(A)[3, 5], Eα(A)[3, 6], Eα(A)[3, 7]}, {Eα(A)[4, 0], Eα(A)[4, 1], Eα(A)[4, 2], Eα(A)[4, 3], Eα(A)[4, 4], Eα(A)[4, 5], Eα(A)[4, 6], Eα(A)[4, 7]}, {Eα(A)[5, 0], Eα(A)[5, 1], Eα(A)[5, 2], Eα(A)[5, 3], Eα(A)[5, 4], Eα(A)[5, 5], Eα(A)[5, 6], Eα(A)[5, 7]}, {Eα(A)[6, 0], Eα(A)[6, 1], Eα(A)[6, 2], Eα(A)[6, 3], Eα(A)[6, 4], Eα(A)[6, 5], Eα(A)[6, 6], Eα(A)[6, 7]}, {Eα(A)[7, 0], Eα(A)[7, 1], Eα(A)[7, 2], Eα(A)[7, 3], Eα(A)[7, 4], Eα(A)[7, 5], Eα(A)[7, 6], Eα(A)[7, 7]} }
```

In[•]:= **(*Symbolize[se_α^(A)]*)**

In[•]:= **Symbolize[sge_α^(A)]**

In[•]:= **Symbolize[sgtrye_α^(A)]**

In[•]:= **Symbolize[E_(A)^α]**

In[•]:= **Symbolize[sge_(A)^α]**

In[•]:= **Symbolize[sgtrye_(A)^α]**

In[•]:= **Symbolize[e_(A)^α]**

```

In[1]:= e(A)(A) = Array[E(A)(A) [##1 - 1, ##2 - 1] &, {8, 8}]

Out[1]= { {E(A)(A) [0, 0], E(A)(A) [0, 1], E(A)(A) [0, 2], E(A)(A) [0, 3], E(A)(A) [0, 4], E(A)(A) [0, 5], E(A)(A) [0, 6], E(A)(A) [0, 7]}, {E(A)(A) [1, 0], E(A)(A) [1, 1], E(A)(A) [1, 2], E(A)(A) [1, 3], E(A)(A) [1, 4], E(A)(A) [1, 5], E(A)(A) [1, 6], E(A)(A) [1, 7]}, {E(A)(A) [2, 0], E(A)(A) [2, 1], E(A)(A) [2, 2], E(A)(A) [2, 3], E(A)(A) [2, 4], E(A)(A) [2, 5], E(A)(A) [2, 6], E(A)(A) [2, 7]}, {E(A)(A) [3, 0], E(A)(A) [3, 1], E(A)(A) [3, 2], E(A)(A) [3, 3], E(A)(A) [3, 4], E(A)(A) [3, 5], E(A)(A) [3, 6], E(A)(A) [3, 7]}, {E(A)(A) [4, 0], E(A)(A) [4, 1], E(A)(A) [4, 2], E(A)(A) [4, 3], E(A)(A) [4, 4], E(A)(A) [4, 5], E(A)(A) [4, 6], E(A)(A) [4, 7]}, {E(A)(A) [5, 0], E(A)(A) [5, 1], E(A)(A) [5, 2], E(A)(A) [5, 3], E(A)(A) [5, 4], E(A)(A) [5, 5], E(A)(A) [5, 6], E(A)(A) [5, 7]}, {E(A)(A) [6, 0], E(A)(A) [6, 1], E(A)(A) [6, 2], E(A)(A) [6, 3], E(A)(A) [6, 4], E(A)(A) [6, 5], E(A)(A) [6, 6], E(A)(A) [6, 7]}, {E(A)(A) [7, 0], E(A)(A) [7, 1], E(A)(A) [7, 2], E(A)(A) [7, 3], E(A)(A) [7, 4], E(A)(A) [7, 5], E(A)(A) [7, 6], E(A)(A) [7, 7]}}

In[2]:= (*gtrye(A)(A) = (e(A)(A) /. sgtrye(A)(A))*)

In[3]:= (*gtrye(A)(A) = (e(A)(A) /. sgtrye(A)(A))*)

In[4]:= (*Symbolize[Γκα β]*)

In[5]:= (*preSpinConnection=Array[Γκβ [##]&, {8,8,8}] ;*)

In[6]:= (*Symbolize[ EAa ]*)

In[7]:= (*Symbolize[ EaA ]*)

In[8]:= (* FAa FaA *)

In[9]:= (*Do[ FAa=EAa[h]=Table[(u[[h]].σ.(τ[B])),{B,1,8}],{h,1,Length[u]}];
Do[ EaA[h]=FullSimplify[Inverse[EAa[h]]],{h,1,Length[u]}];
Table[ FaA=EaA[h]==(u[[h]].σ.u[[h]])Transpose[Table[
FullSimplify[ExpandAll[ η8[[B,B]]τ[B].u[[h]] ]],{B,1,8}]],{h,1,Length[u]}]*)

In[10]:= (*Table[
FullSimplify[ExpandAll[Transpose[EAa[h]].η8.EAa[h]]]- (u[[h]].σ.u[[h]])σ==Zero,
{h,1,Length[u}]]*)

In[11]:= (*inη88=FullSimplify[Inverse[η8]];*)

In[12]:= (*Clear[ω(a)(b)];*) Clear[ω]; Symbolize[ω(a)(b)]

In[13]:= ω = Array[ω(a)(b) [##]&, {8, 8, 8}];
Protect[ω]

Out[13]= {ω}

```

constants

```

← ERROR : 08similarityTransformation has 1 st index that transforms as  $\frac{\partial}{\partial \Psi}$  , not as  $\Psi$ 
In[=]:= (* 08similarityTransformation has 1st index that transforms as  $\frac{\partial}{\partial \Psi}$  , not as  $\Psi$  *)
          08similarityTransformation has 1 st index that transforms as  $\frac{\partial}{\partial \Psi}$  , not as  $\Psi$  :

In[=]:= (*ArrayFlatten[{{IdentityMatrix[4],0},{0,-IdentityMatrix[4]}}]*)

In[=]:= ID4 = IdentityMatrix[4];
          ID8 = IdentityMatrix[8];

In[=]:= η4488 // MatrixForm
Out[=]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$


In[=]:= η4488[[#, #]] & /@ Range[8]
Out[=]= {1, 1, 1, 1, -1, -1, -1, -1}

In[=]:= Zero4 = ConstantArray[0, {4, 4}]; Zero8 = ConstantArray[0, {8, 8}];
          mid = ExpandAll[-1 * ID4];
          ε3 = Array[Signature[{##}] &, {3, 3, 3}]
          ε4 = Array[Signature[{##}] &, {4, 4, 4, 4}];
          MId = ExpandAll[-1 * ID8];
          simp = {Zero4 → 0, ID4 → 1, mid → -1};
          Simp = {Zero8 → ZERO, ID8 → ONE, MId → MONE};

Out[=]= {{{{0, 0, 0}, {0, 0, 1}, {0, -1, 0}}, {{0, 0, -1}, {0, 0, 0}, {1, 0, 0}}}, {{{0, 1, 0}, {-1, 0, 0}, {0, 0, 0}}}}
```

In[=]:= Zero16 = ConstantArray[0, {16, 16}];
 G16 = ArrayFlatten[{{η8, 0}, {0, -η8}}];
 Id16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * Id16];
 Simp16 = {Zero16 → ZERO16, Id16 → ONE16, MId16 → MONE16};

In[=]:= ZERO16 = ConstantArray[0, {16, 16}];
 G16 = ArrayFlatten[{{η4488, 0}, {0, -η4488}}];
 ID16 = IdentityMatrix[16]; MId16 = ExpandAll[-1 * ID16];
 Simp16 = {ZERO16 → ZERO16, ID16 → ONE16, MId16 → MONE16};

```
In[®]:= ws = Flatten[Table[
  ToExpression["w" <> ToString[A1] <> ToString[B1]], {A1, 1, 7}, {B1, A1 + 1, 8}]]
Length[ws]

Out[®]= {w12, w13, w14, w15, w16, w17, w18, w23, w24, w25, w26, w27, w28,
w34, w35, w36, w37, w38, w45, w46, w47, w48, w56, w57, w58, w67, w68, w78}

Out[®]= 28

In[®]:= σ.σ - ID8 == Zero8
Tr[σ] == 0

Out[®]= True

Out[®]= True
```

some function definitions:

```
In[®]:= Clear[rawSets];
rawSets[l_, f_, o_] := Module[{t, h},
  t = Table[Cases[
    l[[j]],
    h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], f] & n + o,
    Infinity,
    Heads → True
  ], {j, 1, Length[l]}];
  Return[t];
]

In[®]:= (*rawSetsel16=rawSets[e16,"f16",0]*)

In[®]:= (*couplings=showCoupledEquations[rawSetsel16]
//Flatten//Sort
==Range[0,15]*)
```



```
In[®]:= Clear[dualRawSets];
dualRawSets[l_, f_, o_, h_, oh_] := Module[{r, r1, r2},
  r1 = rawSets[l, f, o];
  r2 = rawSets[l, h, oh];
  r = Union[Flatten[{r1, r2}, 1]] // Drop[#, 1] &;
  Return[r];
]
```

```

In[0]:= (* Helper to perform one pass of merging *)
MergeSetsStep[currentSets_List] :=
Module[{n = Length[currentSets], i, j, merged = currentSets},
(* We look for the first pair (i, j) that intersects *)
Catch[
For[i = 1, i <= Length[merged], i++,
For[j = i + 1, j <= Length[merged], j++,
If[Intersection[merged[[i]], merged[[j]]] != {}, {i, j}]];
(* Found intersection: Merge and Throw to restart/finish this step *)
merged = Delete[merged, {{i}, {j}}];
AppendTo[merged, Sort[Union[currentSets[[i]], currentSets[[j]]]]];
Throw[Sort[merged]]; (* Sort for canonical form *)
];
];
];
(* If no intersection found, return original sorted *)
Sort[merged]
];
];

In[0]:= showCoupledEquations[items_List] :=
FixedPoint[MergeSetsStep, items];

In[0]:= blockPartition[mat_] := Module[{{blocks, block11, block12, block21, block22, *} ret},
blocks = Partition[mat, {8, 8}];
(*block11=blocks[[1,1]] ;*) (*Top-left block*)
(*block12=blocks[[1,2]] ;*) (*Top-right block*)
(*block21=blocks[[2,1]] ;*) (*Bottom-left block*)
(*block22=blocks[[2,2]] ;*) (*Bottom-right block*)
ret = ArrayFlatten[
{{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}] === mat;
If[ret,
Return[{{blocks[[1, 1]], blocks[[1, 2]]}, {blocks[[2, 1]], blocks[[2, 2]]}}], ## &[]];
Return[Null];
]

In[0]:= extract[a_] := Extract[a, {#}] & /@ Range[0, Length[a]]

```

```
In[0]:= Clear[imes]
```

```
In[0]:= imes[a_] := {a}
```

```
In[0]:= imes[a_, b_] := {a, b}
```

```
In[0]:= imes[a_, b_, c_] := {a, b, c}
```

```

In[1]:= imes[a_, b_, c_, d_] := {a, b, c, d}

In[2]:= imes[a_, b_, c_, d_, f_] := {a, b, c, d, f}

In[3]:= imes[a_, b_, c_, d_, f_, h_] := {a, b, c, d, f, h}

In[4]:= imes[a_, b_, c_, d_, f_, h_, j_] := {a, b, c, d, f, h, j}

In[5]:= times[a__] := Flatten[{Flatten[##] & /@ a}]

In[6]:= (*times[a_,b_,c_,d_]:= {a,b}*)

In[7]:= (*imes[a_,b_,c_,d_][0]:= {a,d}*)

In[8]:= (*imes[a_,b_][0]^:= {a,b}*)

In[9]:= iid[a_, b_] := a

In[10]:= iid[a_, b_, c_] := {a, b}

In[11]:= iid[a_List] := a[[1]]

In[12]:= qid[a__] := a

In[13]:= Clear[der]; der[a__] [c_] [b__] := c

In[14]:= Clear[der3]; der3[a__] [c_] [b__] := {c, a, b}

In[15]:= (*der[a_, b_][c_]:= c*)

In[16]:= (*derz[a_, b_]:= a*)

In[17]:= (*dert[a_, b_]:= b*)

In[18]:= (*derivative[a__][c_]:= FullForm[c]/.{Derivative->der}*)

```

```
In[8]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}

FullForm[#[& /@ %

(*ToString[#[& /@%*)

% /. {Derivative -> der}

Out[8]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[9]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[10]= {Z[0][z, t], Z[0], Z[0]}
```

```

In[]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
% * (Prime[#] & /@ Range[Length[%]])
Plus @@ %
FullForm[%] (* #]&/@%*)
% /. {Plus -> qid, Times -> qid, Derivative -> der}
List[%]
Partition[%, 2]
eextract[#] & /@ %
(*eextract[#]&/@%*)
(*#__List[0]&/@%*)
#[[2]] & /@ %
#[[3]] & /@ %

Out[=] {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[=] {2 Z[0][z, t], 3 Z[0]^(1,0)[z, t], 5 Z[0]^(0,1)[z, t]}

Out[=] 2 Z[0][z, t] + 5 Z[0]^(0,1)[z, t] + 3 Z[0]^(1,0)[z, t]

Out//FullForm=
Plus[Times[2, Z[0][z, t]], Times[5, Derivative[0, 1][Z[0]][z, t]],
Times[3, Derivative[1, 0][Z[0]][z, t]]]

Out[=] Sequence[2, Z[0][z, t], 5, Z[0], 3, Z[0]]

Out[=] {2, Z[0][z, t], 5, Z[0], 3, Z[0]}

Out[=] {{2, Z[0][z, t]}, {5, Z[0]}, {3, Z[0]}}

Out[=] {{List, 2, Z[0][z, t]}, {List, 5, Z[0]}, {List, 3, Z[0]}}

Out[=] {2, 5, 3}

Out[=] {Z[0][z, t], Z[0], Z[0]}

Sequence[Subscript[expr, 1], Subscript[expr, 2], ...]
represents a sequence of arguments to be spliced automatically into any function .

```



```

In[]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@%
ToString[#] & /@%
StringSplit[#, "]"] & /@%
StringExtract[#, "[" → All] & /@%
(*StringExtract[#, {"", z, t"} → All]&/@%*)
InputForm[TextString[#]] & /@%

Out[]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[]= {Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]
{{Z[0], {z, t}}, {Derivative[1, 0, {Z[0], {z, t}}], {Derivative[0, 1, {Z[0], {z, t}}]}}

Out[=]
{{{Z, 0}, {, z, t}}, {{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}},
 {{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}}

Out[=]
"{{Z, 0}, {, z, t}}", "{{Derivative, 1, 0}, {, Z, 0}, {}, {, z, t}}",
 "{{Derivative, 0, 1}, {, Z, 0}, {}, {, z, t}}"

```

```

In[]:= {Z[0][z, t], D[Z[0][z, t], z], D[Z[0][z, t], t]}
FullForm[#] & /@%
ToString[#] & /@%
(*InputForm[TextString[#]]&/@%*)
StringExtract[#, "[" → All] & /@%
StringExtract[#, "]" → All] & /@%
StringExtract[#, "[" → 1] & /@%%
StringExtract[#, "]" → 1] & /@%%%%
StringExtract[#, "Z[" → 1] & /@%%%%%
Out[]= {Z[0][z, t], Z[0]^(1,0)[z, t], Z[0]^(0,1)[z, t]}

Out[=]
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]
{Z[0][z, t], Derivative[1, 0][Z[0]][z, t], Derivative[0, 1][Z[0]][z, t]}

Out[=]
{{Z, 0}, z, t}}, {Derivative, 1, 0], Z, 0]], z, t]}, {Derivative, 0, 1], Z, 0]], z, t}}]

Out[=]
{{Z[0], [z, t, ]}, {Derivative[1, 0], [Z[0], [z, t, ]}, {Derivative[0, 1], [Z[0], [z, t, ]}

Out[=]
{Z, Derivative, Derivative}

Out[=]
{Z[0, Derivative[1, 0], Derivative[0, 1]

Out[=]
{, Derivative[1, 0][, Derivative[0, 1][

In[]:= (*TemplateApply[StringTemplate["Z ` [` then ` b`"],
<|"a"→1234,"b"→5678|>]*)

In[]:= skelx[x_, matQ_ : True] := Module[{t = {666}},
If[matQ == True, t = Block[{r},
r = x;
Table[Block[{q}, q = SameQ[0, r[[j, i]]];
If[True == q, Style[0, Blue], Style[Length[r[[j, i]]], Red]]],
{j, 1, Length[r]}, {i, 1, Length[r[[1]]]}]];
If[matQ == False, t = Block[{r},
r = x;
Table[Block[{q}, q = SameQ[0, r[[j]]];
If[True == q, Style[0, {RGBColor → {0, 1/3, 0}, Bold}],
Style[Length[r[[j]]], Red]]], {j, 1, Length[r]}]];
MatrixForm[t, TableAlignments → Left]]

In[]:= makeSym[size_, fn_] := Module[{rtmp}, rtmp = Table[fn[i, j], {i, 1, size}, {j, 1, i}]];
MapThread[Join, {rtmp, Rest /@ Flatten[rtmp, {{2}, {1}}]}]]

```

```
In[8]:= makeAntiSym[size_, fn_] :=
Module[{rtmp}, rtmp = Table[If[i == j, 0, fn[i, j]], {i, 1, size}, {j, 1, i}]; 
MapThread[Join, {rtmp, Rest /@ Flatten[-rtmp, {{2}, {1}}]}]]]

In[9]:= Block[{MX}, MX = makeAntiSym[8, Subscript[\[omega], ##] &]]

Out[9]=
{ {0, -\[omega]2,1, -\[omega]3,1, -\[omega]4,1, -\[omega]5,1, -\[omega]6,1, -\[omega]7,1, -\[omega]8,1},
{\[omega]2,1, 0, -\[omega]3,2, -\[omega]4,2, -\[omega]5,2, -\[omega]6,2, -\[omega]7,2, -\[omega]8,2},
{\[omega]3,1, \[omega]3,2, 0, -\[omega]4,3, -\[omega]5,3, -\[omega]6,3, -\[omega]7,3, -\[omega]8,3},
{\[omega]4,1, \[omega]4,2, \[omega]4,3, 0, -\[omega]5,4, -\[omega]6,4, -\[omega]7,4, -\[omega]8,4},
{\[omega]5,1, \[omega]5,2, \[omega]5,3, \[omega]5,4, 0, -\[omega]6,5, -\[omega]7,5, -\[omega]8,5},
{\[omega]6,1, \[omega]6,2, \[omega]6,3, \[omega]6,4, \[omega]6,5, 0, -\[omega]7,6, -\[omega]8,6},
{\[omega]7,1, \[omega]7,2, \[omega]7,3, \[omega]7,4, \[omega]7,5, \[omega]7,6, 0, -\[omega]8,7},
{\[omega]8,1, \[omega]8,2, \[omega]8,3, \[omega]8,4, \[omega]8,5, \[omega]8,6, \[omega]8,7, 0} }
```

$$\Gamma_{\mu\alpha\beta} = \frac{1}{2} \left(\frac{\partial g_{\mu\alpha}}{\partial x^\beta} + \frac{\partial g_{\mu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\mu} \right),$$

$$\Gamma^\mu{}_{\alpha\beta} = g^{\mu\nu} \Gamma_{\nu\alpha\beta},$$

$$R^\mu{}_{\nu\alpha\beta} = \frac{\partial \Gamma^\mu{}_{\nu\beta}}{\partial x^\alpha} - \frac{\partial \Gamma^\mu{}_{\nu\alpha}}{\partial x^\beta} + \Gamma^\mu{}_{\rho\alpha} \Gamma^\rho{}_{\nu\beta} - \Gamma^\mu{}_{\rho\beta} \Gamma^\rho{}_{\nu\alpha}.$$

$$R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}, \quad R = g^{\mu\nu} R_{\mu\nu},$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

```
In[•]:= (*Christoffel, Einstein tensor,...*)
Clear[rt];
rt[g_, ass_ : {}] := Module[{t, ΓΓ, RicciΓ, RieΓ, RS, G, ginv},
  Print[Now];
  t = AbsoluteTiming[
    ginv = FullSimplify[Inverse[g], ass];
    ΓΓ = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] *
      (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}], {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
    RicciΓ =
      ParallelTable[FullSimplify[D[ΓΓ[[μ, ν, β]], X[[α]]] - D[ΓΓ[[μ, ν, α]], X[[β]]] + Sum[
        ΓΓ[[μ, s, α]]*ΓΓ[[s, ν, β]] - ΓΓ[[μ, s, β]]*ΓΓ[[s, ν, α]], {s, 1, DIM8}], ass],
      {μ, 1, DIM8}, {ν, 1, DIM8}, {α, 1, DIM8}, {β, 1, DIM8}];
    RieΓ = ParallelTable[ FullSimplify[
      Sum[ RicciΓ[[α, μ, α, ν]], {α, 1, DIM8}], ass], {μ, 1, DIM8}, {ν, 1, DIM8}];
    RS = FullSimplify[Tr[ ginv.RieΓ], ass];
    G = ParallelTable[
      FullSimplify[RieΓ[[α]] - 1/2 g[[α]] RS, ass], {α, 1, Length[RieΓ]}];
    ];
  Print[t];
  Print[Now];
  Return[{ginv, ΓΓ, RicciΓ, RieΓ, RS, G}]
]
```

If ‘paste’ Maple MathML to ToExpression[MathMLForm[]] fails, then try to edit this :

```
In[•]:= Clear[mapleztStringToMathematica];
mapleztStringToMathematica[input_String, toExpression_: False, debug_: False] :=
Module[{prepare, post, custom, use = input, save = input, linenumber = 0},
(*use=ToString[InputForm[TextString[input]]];*) (*Print[use];*)
prepare[in_String] := Module[{s = in}, linenumber++;
  s = in;
  (*If[s==ToString[InputForm[TextString[s]]]],
  (*##&[]*)Print[s],Print[ToUpperCase[s]]];*)
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,z)" >
    "D[Z[" <> ToString[j] <> "] [z,t],{z,2}]"], {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z,t)" >
    "D[D[Z[" <> ToString[j] <> "] [z,t],z],t]", {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,z)" >
    "D[D[Z[" <> ToString[j] <> "] [z,t],t],z]", {j, 0, 15}];
  Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t,t)" >
    "D[Z[" <> ToString[j] <> "] [z,t],{t,2}]"], {j, 0, 15}];
```

```

Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),t)" >
    "D[Z[" <> ToString[j] <> "] [z,t],t]"], {j, 0, 15}];
Do[s = StringReplace[s, "diff(Z" <> ToString[j] <> "(z,t),z)" >
    "D[Z[" <> ToString[j] <> "] [z,t],z]"], {j, 0, 15}];

s = StringReplace[s, "tan(z)" >> "Tan[z]"];
s = StringReplace[s, "cot(z)" >> "Cot[z]"];
s = StringReplace[s, "sin(z)" >> "Sin[z]"];
s = StringReplace[s, "cos(z)" >> "Cos[z]"];
s = StringReplace[s, "sec(z)" >> "Sec[z]"];
s = StringReplace[s, "csc(z)" >> "Csc[z]"];

s = StringReplace[s, "=" >> "->"];

(*Do[s=StringReplace[s,
    "Z"<>ToString[j]<>"(z,t)">>"Z["<>ToString[j]<>"[z,t]",{j,0,15}]];*)
(*s=StringReplace[s,"diff(a4(t),t)">>"D[a4[t],t]"];*)
(*s=StringReplace[s,"exp(-2*a4(t))">>"Exp[-2a4[t]]"];*)
(*s=StringReplace[s,"a4(t)">>"a4[t]"];*)

Return[s];
];

post[in_String] := Module[{s = in}, linenumber++;
s = in;
s = StringReplace[s, "a4(t)" >> "a4[t]"];
Do[s = StringReplace[s,
    "Z" <> ToString[j] <> "(z,t)" >> "Z[" <> ToString[j] <> "] [z,t]", {j, 0, 15}];
Return[s];
];
custom[in_String] := Module[{s = in}, linenumber++;
s = in;

s = StringReplace[s, "diff(a4(t),t)" >> "D[a4[t],t]"];
s = StringReplace[s, "exp(-2*a4(t))" >> "Exp[-2a4[t]]"];

Return[s];
];
use = prepare[use];
use = custom[use];
Return[post[use]];
(*If[toExpression,Return[ToExpression[s/.{z->#1,t->#2}],##&[]]];*)
]

```

Mathematica Lexer and Parser for Maple - like Syntax Supports : Basic arithmetic, function calls, lists {}, equations =, and specific Maple keywords .

```
In[1]:= (*---LEXER---*) (*Helper:Check if character is a digit*)
IsDigit[char_] := StringMatchQ[char, RegularExpression["\d"]];

(*Helper:Check if character is a letter*)
IsAlpha[char_] := StringMatchQ[char, RegularExpression"[a-zA-Z_"]];

(*Helper:Check if character is whitespace*)
IsSpace[char_] := StringMatchQ[char, RegularExpression["\s"]];

(*Main Lexer Function*)
GetTokens[str_String] := Module[
  {chars, len, i, char, tokens = {}, token, numStr, idStr}, chars = Characters[str];
  len = Length[chars];
  i = 1;
  While[i <= len, char = chars[[i]];
    Which[(*Skip Whitespace*)
      IsSpace[char], i++, (*Numbers*) IsDigit[char], numStr = char;
      i++;
      While[i <= len && IsDigit[chars[[i]]], numStr = numStr <> chars[[i]];
      i++];
    AppendTo[tokens, {"NUMBER", ToExpression[numStr]}],
    (*Identifiers and Keywords*) IsAlpha[char], idStr = char;
    i++;
    While[
      i <= len && (IsAlpha[chars[[i]]] || IsDigit[chars[[i]]]), idStr = idStr <> chars[[i]];
      i++];
    (*Check for specific keywords if needed,
    otherwise IDENTIFIER*) AppendTo[tokens, {"IDENTIFIER", idStr}],
    (*Operators and Punctuation*) char == "+", AppendTo[tokens, {"PLUS", "+"}];
    i++, char == "-", AppendTo[tokens, {"MINUS", "-"}];
    i++, char == "*", AppendTo[tokens, {"STAR", "*"}];
    i++, char == "/", AppendTo[tokens, {"SLASH", "/"}];
    i++, char == "^", AppendTo[tokens, {"CARET", "^"}];
    i++, char == "(", AppendTo[tokens, {"LPAREN", "("}];
    i++, char == ")", AppendTo[tokens, {"RPAREN", ")"}];
    i++, char == "{", AppendTo[tokens, {"LBRACE", "{"}];
    i++, char == "}", AppendTo[tokens, {"RBRACE", "}" }];
    i++, char == "=", AppendTo[tokens, {"EQUALS", "="}];
    i++, char == ",", AppendTo[tokens, {"COMMA", ","}];
    i++, char == "$", AppendTo[tokens, {"DOLLAR", "$"}]];
  ]
]
```

```

i++, (*Ellipsis... or Dot.*) char == ".", If[i + 2 ≤ len && chars[[i + 1]] == "." &&
    chars[[i + 2]] == ".", AppendTo[tokens, {"ELLIPSIS", "..."}];
i += 3, AppendTo[tokens, {"DOT", "."}];
i++], True, Print["Unknown character: ", char];
i++]];
AppendTo[tokens, {"EOF", "EOF"}];
tokens];

In[8]:= (*---PARSER---*) (*Grammar:Expression→Equation Equation→AddExp [= AddExp] AddExp→
MulExp {("+"|"-") MulExp} MulExp→PowerExp {("*"|" /") PowerExp} PowerExp→
Primary["^" PowerExp] (Right associative) Primary→
NUMBER|IDENTIFIER|(" Expression ")|IDENTIFIER "(" ArgList ")"|(
" ArgList ")" ArgList→Expression {," Expression}|Empty*)
ParseTokens[tokens_List] := Module[{pos = 1, currentToken, eat, peek,
parseExpression, parseEquation, parseAddExp, parseMulExp, parsePowerExp,
parseUnaryExp, parsePrimary, parseArgs, parseList}, currentToken := tokens[[pos]];
peek[] := tokens[[pos]];
eat[type_] := If[currentToken[[1]] == type, pos++;
True, Print["Error: Expected ", type, " but found ", currentToken[[1]]];
False];
(*Entry point*)parseExpression[] := parseEquation[];
(*Equation:a=b*)parseEquation[] := Module[{left, right}, left = parseAddExp[];
If[currentToken[[1]] == "EQUALS", eat["EQUALS"];
right = parseAddExp[];
{"Equation", left, right}, left]];
parseAddExp[] := Module[{node, right, op}, node = parseMulExp[];
While[MemberQ[{"PLUS", "MINUS"}, currentToken[[1]]], op = currentToken[[2]];
eat[currentToken[[1]]];
right = parseMulExp[];
node = {"BinaryOp", op, node, right};];
node];
parseMulExp[] := Module[{node, right, op}, node = parsePowerExp[];
While[True,
If[MemberQ[{"STAR", "SLASH"}, currentToken[[1]]], op = currentToken[[2]];
eat[currentToken[[1]]];
right = parsePowerExp[];
node = {"BinaryOp", op, node, right}, (*Implicit Multiplication*)
If[MemberQ[{"IDENTIFIER", "NUMBER", "LPAREN", "LBRACE", "ELLIPSIS"}, currentToken[[1]]], right = parsePowerExp[];
node = {"BinaryOp", "*", node, right},
Break[] (*Not an operator or start of expression*)]];
node];
parsePowerExp[] := Module[{node, right}, node = parseUnaryExp[];
If[currentToken[[1]] == "CARET", eat["CARET"];

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right = parsePowerExp[];
(*Right associative recursion*) {"BinaryOp", "^", node, right}, node]];
parseUnaryExp[] := Module[{op, node},
  If[MemberQ[{"PLUS", "MINUS"}, currentToken[[1]]], op = currentToken[[2]];
  eat[currentToken[[1]]];
  node = parseUnaryExp[];
  {"UnaryOp", op, node}, parsePrimary[]]];
parsePrimary[] := Module[{token, node, name, args}, token = currentToken;
  Switch[token[[1]], "NUMBER", eat["NUMBER"];
  {"Number", token[[2]]}, "IDENTIFIER", eat["IDENTIFIER"];
  name = token[[2]];
  (*Check if it's a function call:IDENTIFIER followed by LPAREN*)
  If[currentToken[[1]] == "LPAREN", eat["LPAREN"];
  args = parseArgs[];
  eat["Rparen"];
  {"Call", name, args}, {"Identifier", name}], "LPAREN", eat["LPAREN"];
  node = parseExpression[];
  eat["Rparen"];
  node, "LBRACE", parseList[], "ELLIPSIS", eat["ELLIPSIS"];
  {"Identifier", "..."}, "EOF", Print["Error: Unexpected EOF in Primary"];
  {"Error", "EOF"}, (*Do NOT eat EOF*)
  _, Print["Error: Unexpected token in Primary: ", token];
  eat[token[[1]]];
  (*Advance to avoid infinite loop*) {"Error", token}]];
(*Parse List:{a,b,c}*)
parseList[] := Module[{elements}, eat["LBRACE"];
  elements = parseArgs[];
  eat["RBRACE"];
  {"List", elements}];
(*Parse Argument List:expr,expr,...*)
parseArgs[] := Module[{args = {}, arg}, If[currentToken[[1]] != "Rparen" &&
  currentToken[[1]] != "RBRACE", arg = parseExpression[];
  AppendTo[args, arg];
  While[currentToken[[1]] == "COMMA", eat["COMMA"]];
  arg = parseExpression[];
  AppendTo[args, arg];];
  args];
parseExpression[]];

```

X

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In[•]:= (*---CONVERTER---*) (*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunction[funcName_String] :=
Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate", "DESol",
"DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan", "Tan", "sin",
"Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc", "Csc", "csch", "Csch",

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"tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh", "coth", "Coth", "sech", "Sech",
"det", "Det", "transpose", "Transpose", "sphericalbessel", "SphericalBesselJ",
"lambertw", "ProductLog", "gamma", "Gamma", "airy", "AiryAi", "besselJ",
"BesselJ", "besselY", "BesselY", "hypergeometricU", "HypergeometricU", "zeta",
"Zeta", "erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNode[nodeType_, nodeData_, childResults_] :=
Module[{funcName, cleanFuncName, mathFunc, argsStr}, Switch[nodeType, "Number",
ToString[nodeData], "Identifier", (*Handle special identifiers like _Y→Y*)
If[StringLength[nodeData] > 0 && StringTake[nodeData, 1] == "_",
StringDrop[nodeData, 1], nodeData], "BinaryOp",
"(" <> childResults[[1]] <> " " <> nodeData <> " " <> childResults[[2]] <> ")",
"UnaryOp", nodeData <> "(" <> childResults[[1]] <> ")", "Equation",
childResults[[1]] <> " == " <> childResults[[2]], "List",
"{" <> StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
(*Handle special identifiers like _Y→Y for function names too*)
cleanFuncName = If[StringLength[funcName] > 0 && StringTake[funcName, 1] == "_",
StringDrop[funcName, 1], funcName];
mathFunc = MapleToMathematicaFunction[cleanFuncName];
(*Special handling for DSolve-requires 3 arguments*)
If[cleanFuncName == "DESol" || mathFunc == "DSolve", (*DSolve[equations,
functions,variables]*) (*Ensure we have exactly 3 arguments,
pad with empty lists if needed*) argsStr = StringRiffle[Join[childResults,
Table["{}", {i, 1, Max[0, 3 - Length[childResults]]}]], ", "];
mathFunc <> "[" <> argsStr <> "]",
(*Default function call*) argsStr = StringRiffle[childResults, ", "];
mathFunc <> "[" <> argsStr <> "]"], _, "Error"]];

(*Convert AST to Mathematica String-Non-
recursive iterative version using post-order traversal*)
ToMathematicaString[ast_] := Module[{stack, outputStack, currentItem, node,
nodeType, children, childCount, childResults, i, result, nodeData},
(*Use two stacks:one for traversal,one for results*) stack = {{ast, False}};
(*{node,visited}*.)outputStack = {};
(*Post-order traversal:process children before parents*)
While[Length[stack] > 0, currentItem = Last[stack];
node = currentItem[[1]];
nodeType = node[[1]];
If[currentItem[[2]], (*Node already visited,process it*) stack = Most[stack];
(*Determine children and extract data*)
{children, nodeData} = Switch[nodeType, "Number", {{}, node[[2]]},
"Identifier", {{}, node[[2]]}, "BinaryOp", {{node[[3]], node[[4]]}, node[[2]]},

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"UnaryOp", {{node[3]}, node[2]}, "Equation", {{node[2]}, node[3]}, ""},
"List", {node[2], ""}, "Call", {node[3], node[2]}, _, {}, ""}];
(*Pop child results from output stack*)childCount = Length[children];
childResults = {};
If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
  outputStack = Most[outputStack];, {i, 1, childCount}]];
(*Process this node*)
result = ProcessASTNode[nodeType, nodeData, childResults];
AppendTo[outputStack, result];, (*Node not visited,
mark as visited and push children*)stack[[Length[stack]]] = {node, True};
(*Push children onto stack in reverse order (so they process left-to-right)*)
children = Switch[nodeType, "Number", {}, "Identifier", {},
  "BinaryOp", {node[3], node[4]}, "UnaryOp", {node[3]}, "Equation",
  {node[2], node[3]}, "List", node[2], "Call", node[3], _, {}];
Do[AppendTo[stack, {children[[i]], False}], {i, Length[children], 1, -1}]];
(*Return the final result*)
If[Length[outputStack] > 0, Last[outputStack], "Error"]];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematica[inputStr_String] :=
Module[{tokens, ast}, tokens = GetTokens[inputStr];
ast = ParseTokens[tokens];
ToMathematicaString[ast]];

In[1]:= (*---CONVERTER---*) (*Map Maple function names to Mathematica function names*)
MapleToMathematicaFunctionOLD[funcName_String] :=
Switch[funcName, "diff", "D", "int", "Integrate", "Int", "Integrate", "DESol",
"DSolve", "exp", "Exp", "log", "Log", "sqrt", "Sqrt", "tan", "Tan", "sin",
"Sin", "cos", "Cos", "cot", "Cot", "sec", "Sec", "csc", "Csc", "csch", "Csch",
"tanh", "Tanh", "sinh", "Sinh", "cosh", "Cosh", "coth", "Coth", "sech", "Sech",
"det", "Det", "transpose", "Transpose", "sphericalbessel", "SphericalBesselJ",
"lambertw", "ProductLog", "gamma", "Gamma", "airy", "AiryAi", "besselJ",
"BesselJ", "besselY", "BesselY", "hypergeometricU", "HypergeometricU", "zeta",
"Zeta", "erf", "Erf", "erfc", "Erfc", _, funcName (*Default:keep same name*)];

(*Helper:Process a single AST node-core conversion logic*)
ProcessASTNodeOLD[nodeType_, nodeData_, childResults_] :=
Module[{funcName, cleanFuncName, mathFunc, argsStr}, Switch[nodeType, "Number",
ToString[nodeData], "Identifier", (*Handle special identifiers like _Y→Y*)
If[StringLength[nodeData] > 0 && StringTake[nodeData, 1] == "_",
StringDrop[nodeData, 1], nodeData], "BinaryOp",
"(" <> childResults[[1]] <> " " <> nodeData <> " " <> childResults[[2]] <> ")",
"UnaryOp", nodeData <> "(" <> childResults[[1]] <> ")", "Equation",
childResults[[1]] <> " == " <> childResults[[2]], "List",

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"{" <> StringRiffle[childResults, ", "] <> "}", "Call", funcName = nodeData;
(*Handle special identifiers like _Y→Y for function names too*)
cleanFuncName = If[StringLength[funcName] > 0 && StringTake[funcName, 1] == "_",
  StringDrop[funcName, 1], funcName];
mathFunc = MapleToMathematicaFunction[cleanFuncName];
argsStr = StringRiffle[childResults, ", "];
mathFunc <> "[" <> argsStr <> "]", _, "Error"]];

(*Convert AST to Mathematica String-Non-
recursive iterative version using post-order traversal*)
ToMathematicaStringOLD[ast_] := Module[{stack, outputStack, currentItem, node,
 .nodeType, children, childCount, childResults, i, result, nodeData},
  (*Use two stacks:one for traversal,one for results*) stack = {{ast, False}};
  (*{node,visited}*-)outputStack = {};
  (*Post-order traversal:process children before parents*)
  While[Length[stack] > 0, currentItem = Last[stack];
  node = currentItem[[1]];
  nodeType = node[[1]];
  If[currentItem[[2]], (*Node already visited,process it*) stack = Most[stack];
   (*Determine children and extract data*)
   {children, nodeData} = Switch[nodeType, "Number", {}, node[[2]],
     "Identifier", {}, node[[2]], "BinaryOp", {{node[[3]], node[[4]]}, node[[2]]},
     "UnaryOp", {{node[[3]]}, node[[2]]}, "Equation", {{node[[2]], node[[3]]}}, "",
     "List", {node[[2]], ""}, "Call", {node[[3]], node[[2]]}, _, {}, ""}];
   (*Pop child results from output stack*) childCount = Length[children];
   childResults = {};
   If[childCount > 0, Do[PrependTo[childResults, Last[outputStack]];
    outputStack = Most[outputStack];, {i, 1, childCount}]];
   (*Process this node*)
   result = ProcessASTNode[nodeType, nodeData, childResults];
   AppendTo[outputStack, result];, (*Node not visited,
   mark as visited and push children*) stack[[Length[stack]]] = {node, True};
   (*Push children onto stack in reverse order (so they process left-to-right)*)
   children = Switch[nodeType, "Number", {}, "Identifier", {},
     "BinaryOp", {node[[3]], node[[4]]}, "UnaryOp", {node[[3]]}, "Equation",
     {node[[2]], node[[3]]}, "List", node[[2]], "Call", node[[3]], _, {}];
   Do[AppendTo[stack, {children[[i]], False}], {i, Length[children], 1, -1}]];
   (*Return the final result*)
   If[Length[outputStack] > 0, Last[outputStack], "Error"]];
  ];

(*---MAIN WRAPPER---*)
ConvertMapleToMathematicaOLD[inputStr_String] :=
Module[{tokens, ast}, tokens = GetTokens[inputStr];

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ast = ParseTokens[tokens];
ToMathematicaString[ast]];

In[]:= sta = "nZ10(z, t) = (c2C7*sin(C2Q1*t) +
c2C8*cos(C2Q1*t))*(c2C5*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) +
c2C6*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ11(z, t) = (c2C3*sin(C2Q1*t) +
c2C4*cos(C2Q1*t))*(c2C1*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) +
c2C2*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6)), nZ8(z, t) = ((((-C2Q1*c2C2*c2C3
+ M*c2C6*c2C8)*cos(C2Q1*t) + sin(C2Q1*t)*(C2Q1*c2C2*c2C4 +
M*c2C6*c2C7))*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) + ((C2Q1*c2C1*c2C3
- M*c2C5*c2C8)*cos(C2Q1*t) - sin(C2Q1*t)*(C2Q1*c2C1*c2C4 +
M*c2C5*c2C7))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6) + (-cos(M*t)*c2C10
+ sin(M*t)*c2C9)*sqrt(-C2Q1^2 + M^2))/sqrt(-C2Q1^2 + M^2),
nZ9(z, t) = (((C2Q1*c2C6*c2C7 + M*c2C2*c2C4)*cos(C2Q1*t) +
sin(C2Q1*t)*(-C2Q1*c2C6*c2C8 + M*c2C2*c2C3))*sin(z)^(-sqrt(-C2Q1^2
+ M^2)/3) + sqrt(-C2Q1^2 + M^2)*(cos(M*t)*c2C9 +
sin(M*t)*c2C10)*sin(z)^(-sqrt(-C2Q1^2 + M^2)/6) + (-C2Q1*c2C5*c2C7
- M*c2C1*c2C4)*cos(C2Q1*t) - sin(C2Q1*t)*(-C2Q1*c2C5*c2C8 +
M*c2C1*c2C3))*sin(z)^(sqrt(-C2Q1^2 + M^2)/6)/sqrt(-C2Q1^2 + M^2)}";

In[]:= ConvertMapleToMathematica[sta]

Out[]=
{nZ10[z, t] == (((c2C7 * Sin[(C2Q1 * t)]) + (c2C8 * Cos[(C2Q1 * t)])) * ((c2C5
* (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)) / 6])) + (c2C6 * (Sin[z] ^
(-Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)) / 6))))), nZ11[z, t] == (((c2C3 * Sin[(C2Q1
* t)] + (c2C4 * Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z] ^ (Sqrt[((-(C2Q1) ^ 2)
+ (M ^ 2)) / 6))) + (c2C2 * (Sin[z] ^ (-Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)) / 6))))),
nZ8[z, t] == ((((((((-(C2Q1) * c2C2) * c2C3) + ((M * c2C6) *
c2C8)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((C2Q1 * c2C2) * c2C4) +
((M * c2C6) * c2C7)))) * (Sin[z] ^ (-Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)) / 6))) +
((((((C2Q1 * c2C1) * c2C3) - ((M * c2C5) * c2C8)) * Cos[(C2Q1 * t)]) -
(Sin[(C2Q1 * t)] * (((C2Q1 * c2C1) * c2C4) + ((M * c2C5) * c2C7)))) * (Sin[z]
^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)) / 6))) + (((-(Cos[(M * t)]) * c2C10) +
(Sin[(M * t)] * c2C9)) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) / Sqrt[((-(C2Q1) ^
2) + (M ^ 2))]), nZ9[z, t] == (((((((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8)
+ ((M * c2C2) * c2C3)))) * (Sin[z] ^ (-Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)) / 3))) +
((Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * ((Cos[(M * t)] * c2C9) + (Sin[(M
* t)] * c2C10))) * (Sin[z] ^ (-Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)) / 6))) +
(((((-(C2Q1) * c2C5) * c2C7) - ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)])) -
(Sin[(C2Q1 * t)] * ((((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * (Sin[z]
^ (Sqrt[((-(C2Q1) ^ 2) + (M ^ 2)) / 6))) / Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]}

```

```

In[6]:= sti = 
  " {nZ10(z,t)=(c2C7*sin(C2Q1*t)+c2C8*cos(C2Q1*t))*(c2C5*sin(z)^(1/6*sqrt(-C2Q1^2+M^2))+c2C6*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))),nZ11(z,t)=(c2C3*sin(C2Q1*t)+c2C4*cos(C2Q1*t))*(c2C1*sin(z)^(1/6*sqrt(-C2Q1^2+M^2))+c2C2*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))),nZ8(z,t)=1/6*(6*sqrt(-C2Q1^2+M^2)*c2C6*(c2C7*sin(C2Q1*t)+c2C8*cos(C2Q1*t))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-6*sqrt(-C2Q1^2+M^2)*c2C5*(c2C7*sin(C2Q1*t)+c2C8*cos(C2Q1*t))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2))+(-6*M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3)))*cos(M*t)+(6*M*c2C9-Int(-sin(M*t)*((-C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+(C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*sin(M*t)-C2Q1*Int(-cos(z)*(((C2Q1*c2C6*c2C8+M*c2C2*c2C3)*cos(C2Q1*t)-sin(C2Q1*t)*(C2Q1*c2C6*c2C7+M*c2C2*c2C4))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+((-C2Q1*c2C5*c2C8+M*c2C1*c2C3)*cos(C2Q1*t)-sin(C2Q1*t)*(C2Q1*c2C5*c2C7+M*c2C1*c2C4))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1),z))/M,nZ9(z,t)=1/6*((6*M*c2C9-Int(-sin(M*t)*((-C2Q1^2+M^2)*Int(-cos(z)*((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+(C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z))/M,nZ10(z,t)=1/6*((6*M*c2C9-Int(-sin(M*t)*((-C2Q1^2+M^2)*Int(-cos(z)*((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+(C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*sin(M*t)+(6*M*c2C10+Int(-cos(M*t)*((-C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+(C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z)-6*sqrt(-C2Q1^2+M^2)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2))-sin(z)^(1/6*sqrt(-C2Q1^2+M^2))*((C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))),t))*sin(M*t)+M*Int(-cos(z)*(((C2Q1*c2C6*c2C7+M*c2C2*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C6*c2C8+M*c2C2*c2C3))*sin(z)^(-1/6*sqrt(-C2Q1^2+M^2)-1)+(C2Q1*c2C5*c2C7+M*c2C1*c2C4)*cos(C2Q1*t)+sin(C2Q1*t)*(-C2Q1*c2C5*c2C8+M*c2C1*c2C3))*sin(z)^(1/6*sqrt(-C2Q1^2+M^2)-1)),z))/M};"

```

In[•]:= **ConvertMapleToMathematica**[sti]

Out[•]=

$$\{nZ10[z, t] == (((c2C7 * \sin[(c2Q1 * t)]) + (c2C8 * \cos[(c2Q1 * t)])) * ((c2C5 * (\sin[z] ^ ((1 / 6) * \text{Sqrt}[(-(c2Q1) ^ 2) + (M ^ 2)])))) + (c2C6 * (\sin[z] ^$$

```

((-1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]]))), nZ11[z, t] == (((c2C3 *
Sin[(C2Q1 * t)])) + (c2C4 * Cos[(C2Q1 * t)])) * ((c2C1 * (Sin[z] ^ ((1 / 6) *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]])) + (c2C2 * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]])))), nZ8[z, t] == (((1 / 6) * (((((6 * Sqrt[((-(C2Q1)
^ 2) + (M ^ 2))]])) * c2C6) * ((c2C7 * Sin[(C2Q1 * t)])) + (c2C8 * Cos[(C2Q1 *
t)]))) * (Sin[z] ^ ((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]]))) - (((6 *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]])) * c2C5) * ((c2C7 * Sin[(C2Q1 * t)])) + (c2C8 * Cos[(C2Q1 *
t)]))) * (Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]]))) + ((((-6) * M) * c2C10) - Integrate[(-(Cos[(M * t)]) * (((-(C2Q1) ^ 2) +
(M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) *
c2C4)) * Cos[(C2Q1 * t)])) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]] - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))))], z]) - ((6 *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) *
c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]]))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) * (((((C2Q1
* c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] *
(((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * (((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) * (((-(C2Q1) ^ 2) +
(M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) *
c2C4)) * Cos[(C2Q1 * t)])) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]] - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))))], z]) - ((6 *
Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) *
c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]]))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) * (((((C2Q1
* c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] *
(((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * (((((C2Q1 * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]]))) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) * (((((C2Q1
* c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] *
(((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) * (((((C2Q1 * c2C6) * c2C7) +
((M * c2C2) * c2C4)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]]))) - 1))) + (((((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)) * Cos[(C2Q1 *
t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C6) * c2C7) + ((M * c2C2) *
c2C3)) * Cos[(C2Q1 * t)])) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C6) * c2C7) + ((M *
c2C2) * c2C4)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]]))) - 1))) + (((((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)) * Cos[(C2Q1 *
t)]) - (Sin[(C2Q1 * t)] * (((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)))) * (Sin[z]
^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))), z])))) / M), nZ9[z, t] ==
(((1 / 6) * (((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) * (((-(C2Q1)
^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) *
c2C4)) * Cos[(C2Q1 * t)])) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]] - 1)))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)])) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))))], z)))) / M), nZ9[z, t]
== (((1 / 6) * (((((6 * M) * c2C9) - Integrate[(-(Sin[(M * t)]) * (((-(C2Q1)
^ 2) + (M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2) *
c2C4)) * Cos[(C2Q1 * t)])) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
((M * c2C2) * c2C3)))) * (Sin[z] ^ (((-(1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]] - 1)))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)])) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) *
(Sin[z] ^ (((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]] - 1))))], z))))))

```

```

z]) - ((6 * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * (((((C2Q1 * c2C6) * c2C7) +
(M * c2C2) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6)
* c2C8) + ((M * c2C2) * c2C3)))) * (Sin[z] ^ ((-1) / 6) * Sqrt[((-(C2Q1) ^
2) + (M ^ 2))])) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) * (
(((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1
* t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))))), t]) * Cos[(M
* t)] + (((6 * M) * c2C10) + Integrate[(-(Cos[(M * t)]) * (((-(C2Q1) ^ 2)
+ (M ^ 2)) * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
(M * c2C2) * c2C3)))) * (Sin[z] ^ ((-1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]) - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) *
(Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1)))), z]) - ((6
* Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
(M * c2C2) * c2C3)))) * (Sin[z] ^ ((-1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]) - ((Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))])) * (((((C2Q1
* c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] *
(((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3))))), t]) * Sin[(M *
t)]) + (M * Integrate[(-(Cos[z]) * (((((C2Q1 * c2C6) * c2C7) + ((M * c2C2)
* c2C4)) * Cos[(C2Q1 * t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C6) * c2C8) +
(M * c2C2) * c2C3)))) * (Sin[z] ^ ((-1) / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^
2))]) - 1))) + (((((C2Q1 * c2C5) * c2C7) + ((M * c2C1) * c2C4)) * Cos[(C2Q1 *
t)]) + (Sin[(C2Q1 * t)] * (((-(C2Q1) * c2C5) * c2C8) + ((M * c2C1) * c2C3)))) *
(Sin[z] ^ ((1 / 6) * Sqrt[((-(C2Q1) ^ 2) + (M ^ 2))]) - 1)))), z])) / M)

```

gtry and Γ and ...

In[6]:= gtry = MatrixMetric44

Out[6]=

```

{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, E^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, E^(2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -E^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, -E^(-2 a4[H x4]) Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -E^(-2 a4[H x4]) Sin[6 H x0]^(1/3)}}

```

In[7]:= Protect[gtry]

Out[7]=

```
{gtry}
```

In[8]:= Unprotect[ginv, Γ , Ricci Γ , Rie Γ , RS, EinsteinG]

Out[8]=

```
{}
```

In[9]:= result = {ginv, Γ , Ricci Γ , Rie Γ , RS, EinsteinG} = rt[gtry];

```

Fri 30 Jan 2026 05:21:17 GMT-8
{5.31052, Null}

Fri 30 Jan 2026 05:21:23 GMT-8

In[•]:= Protect[ginv, \[Gamma], Ricci\[Gamma], Rie\[Gamma], RS, EinsteinG]
Out[•]= {ginv, \[Gamma], Ricci\[Gamma], Rie\[Gamma], RS, EinsteinG}

verify g\mu\nu|\alpha = 0 :

In[•]:= Simplify[Table[D[gtry[j, k], X[i]] - Sum[gtry[k, s] \[Cross] \[Gamma][s, i, j], {s, 1, DIM8}] -
Sum[gtry[j, s] \[Cross] \[Gamma][s, i, k], {s, 1, DIM8}],
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union
Out[•]= {0}

In[•]:= Table[g[j][k] \[Rule] If[j == k,
ToExpression["((g[" <> ToString[j] <> "][" <> ToString[k] <> "][#1,#2])&)"],
(0 &), {j, 0, 7}, {k, 0, 7}];
sg = % // Flatten
Out[•]= {g[0][0] \[Rule] (g[0][0][#1, #2] &), g[0][1] \[Rule] (0 &), g[0][2] \[Rule] (0 &),
g[0][3] \[Rule] (0 &), g[0][4] \[Rule] (0 &), g[0][5] \[Rule] (0 &), g[0][6] \[Rule] (0 &),
g[0][7] \[Rule] (0 &), g[1][0] \[Rule] (0 &), g[1][1] \[Rule] (g[1][1][#1, #2] &), g[1][2] \[Rule] (0 &),
g[1][3] \[Rule] (0 &), g[1][4] \[Rule] (0 &), g[1][5] \[Rule] (0 &), g[1][6] \[Rule] (0 &),
g[1][7] \[Rule] (0 &), g[2][0] \[Rule] (0 &), g[2][1] \[Rule] (0 &), g[2][2] \[Rule] (g[2][2][#1, #2] &),
g[2][3] \[Rule] (0 &), g[2][4] \[Rule] (0 &), g[2][5] \[Rule] (0 &), g[2][6] \[Rule] (0 &), g[2][7] \[Rule] (0 &),
g[3][0] \[Rule] (0 &), g[3][1] \[Rule] (0 &), g[3][2] \[Rule] (0 &), g[3][3] \[Rule] (g[3][3][#1, #2] &),
g[3][4] \[Rule] (0 &), g[3][5] \[Rule] (0 &), g[3][6] \[Rule] (0 &), g[3][7] \[Rule] (0 &), g[4][0] \[Rule] (0 &),
g[4][1] \[Rule] (0 &), g[4][2] \[Rule] (0 &), g[4][3] \[Rule] (0 &), g[4][4] \[Rule] (g[4][4][#1, #2] &),
g[4][5] \[Rule] (0 &), g[4][6] \[Rule] (0 &), g[4][7] \[Rule] (0 &), g[5][0] \[Rule] (0 &), g[5][1] \[Rule] (0 &),
g[5][2] \[Rule] (0 &), g[5][3] \[Rule] (0 &), g[5][4] \[Rule] (0 &), g[5][5] \[Rule] (g[5][5][#1, #2] &),
g[5][6] \[Rule] (0 &), g[5][7] \[Rule] (0 &), g[6][0] \[Rule] (0 &), g[6][1] \[Rule] (0 &), g[6][2] \[Rule] (0 &),
g[6][3] \[Rule] (0 &), g[6][4] \[Rule] (0 &), g[6][5] \[Rule] (0 &), g[6][6] \[Rule] (g[6][6][#1, #2] &),
g[6][7] \[Rule] (0 &), g[7][0] \[Rule] (0 &), g[7][1] \[Rule] (0 &), g[7][2] \[Rule] (0 &), g[7][3] \[Rule] (0 &),
g[7][4] \[Rule] (0 &), g[7][5] \[Rule] (0 &), g[7][6] \[Rule] (0 &), g[7][7] \[Rule] (g[7][7][#1, #2] &)}
In[•]:= g[7][7][x0, x4] /. ssgm4488
% /. sx0x4
Out[•]= -\[ExponentialE]^(-2 a4[H x4]) Sin[6 H x0]^(1/3)

Out[•]= -\[ExponentialE]^(-2 a4[t]) Sin[z]^(1/3)

```

```

In[•]:= MatrixMetric44
Out[•]=
{ {Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

In[•]:= ssgm4488[x0, x4] /. sx0x4
Out[•]=
{g[0][0] → (Cot[6 H #1]^2 &), g[0][1] → (0 &), g[0][2] → (0 &), g[0][3] → (0 &),
g[0][4] → (0 &), g[0][5] → (0 &), g[0][6] → (0 &), g[0][7] → (0 &), g[1][0] → (0 &),
g[1][1] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[1][2] → (0 &), g[1][3] → (0 &), g[1][4] → (0 &),
g[1][5] → (0 &), g[1][6] → (0 &), g[1][7] → (0 &), g[2][0] → (0 &), g[2][1] → (0 &),
g[2][2] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[2][3] → (0 &), g[2][4] → (0 &), g[2][5] → (0 &),
g[2][6] → (0 &), g[2][7] → (0 &), g[3][0] → (0 &), g[3][1] → (0 &), g[3][2] → (0 &),
g[3][3] → (e^2 a4[H #2] Sin[6 H #1]^(1/3) &), g[3][4] → (0 &), g[3][5] → (0 &),
g[3][6] → (0 &), g[3][7] → (0 &), g[4][0] → (0 &), g[4][1] → (0 &), g[4][2] → (0 &),
g[4][3] → (0 &), g[4][4] → (-1 &), g[4][5] → (0 &), g[4][6] → (0 &), g[4][7] → (0 &),
g[5][0] → (0 &), g[5][1] → (0 &), g[5][2] → (0 &), g[5][3] → (0 &), g[5][4] → (0 &),
g[5][5] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[5][6] → (0 &), g[5][7] → (0 &),
g[6][0] → (0 &), g[6][1] → (0 &), g[6][2] → (0 &), g[6][3] → (0 &), g[6][4] → (0 &),
g[6][5] → (0 &), g[6][6] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &), g[6][7] → (0 &),
g[7][0] → (0 &), g[7][1] → (0 &), g[7][2] → (0 &), g[7][3] → (0 &), g[7][4] → (0 &),
g[7][5] → (0 &), g[7][6] → (0 &), g[7][7] → (-e^-2 a4[H #2] Sin[6 H #1]^(1/3) &) } [z, t]
6 H

In[•]:= (*ssgGzt={G[0][0]→((Cot[#1]^2)&),G[0][1]→((0)&),G[0][2]→((0)&),G[0][3]→((0)&),
G[0][4]→((0)&),G[0][5]→((0)&),G[0][6]→((0)&),G[0][7]→((0)&),G[1][0]→((0)&),
G[1][1]→((e^2*a4[#2]*Sin[#1]^(1/3))&),G[1][2]→((0)&),G[1][3]→((0)&),
G[1][4]→((0)&),G[1][5]→((0)&),G[1][6]→((0)&),G[1][7]→((0)&),G[2][0]→((0)&),
G[2][1]→((0)&),G[2][2]→((e^2*a4[#2]*Sin[#1]^(1/3))&),G[2][3]→((0)&),
G[2][4]→((0)&),G[2][5]→((0)&),G[2][6]→((0)&),G[2][7]→((0)&),G[3][0]→((0)&),
G[3][1]→((0)&),G[3][2]→((0)&),G[3][3]→((e^2*a4[#2]*Sin[#1]^(1/3))&),
G[3][4]→((0)&),G[3][5]→((0)&),G[3][6]→((0)&),G[3][7]→((0)&),G[4][0]→((0)&),
G[4][1]→((0)&),G[4][2]→((0)&),G[4][3]→((0)&),G[4][4]→((-1)&),
G[4][5]→((0)&),G[4][6]→((0)&),G[4][7]→((0)&),G[5][0]→((0)&),
G[5][1]→((0)&),G[5][2]→((0)&),G[5][3]→((0)&),G[5][4]→((0)&),
G[5][5]→((-e^-2*a4[#2]*Sin[#1]^(1/3))&),G[5][6]→((0)&),G[5][7]→((0)&),
G[6][0]→((0)&),G[6][1]→((0)&),G[6][2]→((0)&),G[6][3]→((0)&),G[6][4]→((0)&),
G[6][5]→((0)&),G[6][6]→((-e^-2*a4[#2]*Sin[#1]^(1/3))&),G[6][7]→((0)&),
G[7][0]→((0)&),G[7][1]→((0)&),G[7][2]→((0)&),G[7][3]→((0)&),G[7][4]→((0)&),
G[7][5]→((0)&),G[7][6]→((0)&),G[7][7]→((-e^-2*a4[#2]*Sin[#1]^(1/3))&)}*)

```

```
In[•]:= (*G[#][z,t]/.ssgGzt&/@Range[0,7]*)
```

```
In[1]:= detgg = Det[g4488 /. sg] // FullSimplify[#, constraintVars] &
Out[1]=
g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]

In[2]:= g4488 /. sg // MatrixForm
Out[2]//MatrixForm=

$$\begin{pmatrix} g[0][0][x0, x4] & 0 & 0 & 0 & 0 & 0 & \epsilon \\ 0 & g[1][1][x0, x4] & 0 & 0 & 0 & 0 & \epsilon \\ 0 & 0 & g[2][2][x0, x4] & 0 & 0 & 0 & \epsilon \\ 0 & 0 & 0 & g[3][3][x0, x4] & 0 & 0 & \epsilon \\ 0 & 0 & 0 & 0 & g[4][4][x0, x4] & 0 & \epsilon \\ 0 & 0 & 0 & 0 & 0 & g[5][5] & \epsilon \\ 0 & 0 & 0 & 0 & 0 & 0 & \epsilon \\ 0 & 0 & 0 & 0 & 0 & 0 & \epsilon \end{pmatrix}$$

```

```
In[3]:= MatrixMetric44.η4488
(*eAa=*)FullSimplify[√%, constraintVars]
Out[3]=
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

Out[4]=
{{Cot[6 H x0], 0, 0, 0, 0, 0, 0}, {0, √e^2 a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, √e^2 a4[H x4] Sin[6 H x0]^(1/6), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, √e^-2 a4[H x4] Sin[6 H x0]^(1/6), 0, 0}, {0, 0, 0, 0, 0, 0, √e^-2 a4[H x4] Sin[6 H x0]^(1/6), 0}, {0, 0, 0, 0, 0, 0, 0, √e^-2 a4[H x4] Sin[6 H x0]^(1/6)}}

eAa
```

```
In[•]:= g4488.η4488 /. sg
```

$$\left(\text{eAa} = \text{FullSimplify}[\sqrt{\%}, \text{constraintVars}] \right) // \text{MatrixForm}$$

```
Out[•]=
```

```
{ {g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0},  

{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0},  

{0, 0, 0, 0, -g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, -g[5][5][x0, x4], 0, 0},  

{0, 0, 0, 0, 0, -g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, -g[7][7][x0, x4]} }
```

```
Out[•]//MatrixForm=
```

$$\begin{pmatrix} \sqrt{g[0][0][x0, x4]} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{g[1][1][x0, x4]} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{g[2][2][x0, x4]} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{g[3][3][x0, x4]} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{-g[4][4][x0, x4]} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[6]:= eAa

ssgm4488

(eAa04 = %% /. % // FullSimplify[#, constraintVars] &) // MatrixForm

Out[6]=

$$\left\{ \left\{ \sqrt{g[0][0][x0, x4]}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \sqrt{g[1][1][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, \sqrt{g[2][2][x0, x4]}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \sqrt{g[3][3][x0, x4]}, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, \sqrt{-g[4][4][x0, x4]}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \sqrt{-g[5][5][x0, x4]}, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, \sqrt{-g[6][6][x0, x4]}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \sqrt{-g[7][7][x0, x4]} \right\} \right\}$$

Out[6]=

$$\begin{aligned} g[0][0] &\rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ g[0][4] &\rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ g[1][1] &\rightarrow (\text{e}^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ g[1][4] &\rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ g[2][1] &\rightarrow (0 \&), g[2][2] \rightarrow (\text{e}^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ g[2][5] &\rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ g[3][2] &\rightarrow (0 \&), g[3][3] \rightarrow (\text{e}^{2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ g[3][6] &\rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ g[4][3] &\rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ g[5][0] &\rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ g[5][5] &\rightarrow (-\text{e}^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ g[6][0] &\rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ g[6][5] &\rightarrow (0 \&), g[6][6] \rightarrow (-\text{e}^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ g[7][0] &\rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ g[7][5] &\rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-\text{e}^{-2a4[H\#2]} \text{Sin}[6H\#1]^{1/3} \&) \end{aligned}$$

Out[6]//MatrixForm=

$\text{Cot}[6Hx0]$	0	0	0	0	0
0	$\sqrt{\text{e}^{2a4[Hx4]}} \text{Sin}[6Hx0]^{1/6}$	0	0	0	0
0	0	$\sqrt{\text{e}^{2a4[Hx4]}} \text{Sin}[6Hx0]^{1/6}$	0	0	0
0	0	0	$\sqrt{\text{e}^{2a4[Hx4]}} \text{Sin}[6Hx0]^{1/6}$	0	0
0	0	0	0	0	1
0	0	0	0	0	$\sqrt{\text{e}^{-2z}}$
0	0	0	0	0	0
0	0	0	0	0	0

OCTAD $e_{\alpha \leftarrow \text{spacetime}}^{(A) \leftarrow \text{Lorentz}}$:

$$e_{\alpha}^{(A)} = e_{\alpha}^{(A)} = eAa$$

```
In[•]:= (sgeα(A) = Thread[Flatten[eα(A)] → Flatten[eAa]]) (*//Column*)
```

Out[•]=

$$\begin{aligned} & \left\{ E_{\alpha}^{(A)} [0, 0] \rightarrow \sqrt{g[0][0][x0, x4]}, E_{\alpha}^{(A)} [0, 1] \rightarrow 0, E_{\alpha}^{(A)} [0, 2] \rightarrow 0, E_{\alpha}^{(A)} [0, 3] \rightarrow 0, E_{\alpha}^{(A)} [0, 4] \rightarrow 0, \right. \\ & E_{\alpha}^{(A)} [0, 5] \rightarrow 0, E_{\alpha}^{(A)} [0, 6] \rightarrow 0, E_{\alpha}^{(A)} [0, 7] \rightarrow 0, E_{\alpha}^{(A)} [1, 0] \rightarrow 0, E_{\alpha}^{(A)} [1, 1] \rightarrow \sqrt{g[1][1][x0, x4]}, \\ & E_{\alpha}^{(A)} [1, 2] \rightarrow 0, E_{\alpha}^{(A)} [1, 3] \rightarrow 0, E_{\alpha}^{(A)} [1, 4] \rightarrow 0, E_{\alpha}^{(A)} [1, 5] \rightarrow 0, E_{\alpha}^{(A)} [1, 6] \rightarrow 0, E_{\alpha}^{(A)} [1, 7] \rightarrow 0, \\ & E_{\alpha}^{(A)} [2, 0] \rightarrow 0, E_{\alpha}^{(A)} [2, 1] \rightarrow 0, E_{\alpha}^{(A)} [2, 2] \rightarrow \sqrt{g[2][2][x0, x4]}, E_{\alpha}^{(A)} [2, 3] \rightarrow 0, E_{\alpha}^{(A)} [2, 4] \rightarrow 0, \\ & E_{\alpha}^{(A)} [2, 5] \rightarrow 0, E_{\alpha}^{(A)} [2, 6] \rightarrow 0, E_{\alpha}^{(A)} [2, 7] \rightarrow 0, E_{\alpha}^{(A)} [3, 0] \rightarrow 0, E_{\alpha}^{(A)} [3, 1] \rightarrow 0, E_{\alpha}^{(A)} [3, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)} [3, 3] \rightarrow \sqrt{g[3][3][x0, x4]}, E_{\alpha}^{(A)} [3, 4] \rightarrow 0, E_{\alpha}^{(A)} [3, 5] \rightarrow 0, E_{\alpha}^{(A)} [3, 6] \rightarrow 0, E_{\alpha}^{(A)} [3, 7] \rightarrow 0, \\ & E_{\alpha}^{(A)} [4, 0] \rightarrow 0, E_{\alpha}^{(A)} [4, 1] \rightarrow 0, E_{\alpha}^{(A)} [4, 2] \rightarrow 0, E_{\alpha}^{(A)} [4, 3] \rightarrow 0, E_{\alpha}^{(A)} [4, 4] \rightarrow \sqrt{-g[4][4][x0, x4]}, \\ & E_{\alpha}^{(A)} [4, 5] \rightarrow 0, E_{\alpha}^{(A)} [4, 6] \rightarrow 0, E_{\alpha}^{(A)} [4, 7] \rightarrow 0, E_{\alpha}^{(A)} [5, 0] \rightarrow 0, E_{\alpha}^{(A)} [5, 1] \rightarrow 0, E_{\alpha}^{(A)} [5, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)} [5, 3] \rightarrow 0, E_{\alpha}^{(A)} [5, 4] \rightarrow 0, E_{\alpha}^{(A)} [5, 5] \rightarrow \sqrt{-g[5][5][x0, x4]}, E_{\alpha}^{(A)} [5, 6] \rightarrow 0, E_{\alpha}^{(A)} [5, 7] \rightarrow 0, \\ & E_{\alpha}^{(A)} [6, 0] \rightarrow 0, E_{\alpha}^{(A)} [6, 1] \rightarrow 0, E_{\alpha}^{(A)} [6, 2] \rightarrow 0, E_{\alpha}^{(A)} [6, 3] \rightarrow 0, E_{\alpha}^{(A)} [6, 4] \rightarrow 0, E_{\alpha}^{(A)} [6, 5] \rightarrow 0, \\ & E_{\alpha}^{(A)} [6, 6] \rightarrow \sqrt{-g[6][6][x0, x4]}, E_{\alpha}^{(A)} [6, 7] \rightarrow 0, E_{\alpha}^{(A)} [7, 0] \rightarrow 0, E_{\alpha}^{(A)} [7, 1] \rightarrow 0, E_{\alpha}^{(A)} [7, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)} [7, 3] \rightarrow 0, E_{\alpha}^{(A)} [7, 4] \rightarrow 0, E_{\alpha}^{(A)} [7, 5] \rightarrow 0, E_{\alpha}^{(A)} [7, 6] \rightarrow 0, E_{\alpha}^{(A)} [7, 7] \rightarrow \sqrt{-g[7][7][x0, x4]} \} \end{aligned}$$

```
In[•]:= (*Symbolize[sgeα(A)]*)
```

```
In[•]:= (*Symbolize[sgtryeα(A)]*)
```

```
In[•]:= (sgtryeα(A) = Thread[Flatten[eα(A)] → Flatten[eAa04]]) (*//Column*)
```

Out[•]=

$$\begin{aligned} & \left\{ E_{\alpha}^{(A)} [0, 0] \rightarrow \text{Cot}[6Hx0], E_{\alpha}^{(A)} [0, 1] \rightarrow 0, E_{\alpha}^{(A)} [0, 2] \rightarrow 0, E_{\alpha}^{(A)} [0, 3] \rightarrow 0, \right. \\ & E_{\alpha}^{(A)} [0, 4] \rightarrow 0, E_{\alpha}^{(A)} [0, 5] \rightarrow 0, E_{\alpha}^{(A)} [0, 6] \rightarrow 0, E_{\alpha}^{(A)} [0, 7] \rightarrow 0, E_{\alpha}^{(A)} [1, 0] \rightarrow 0, \\ & E_{\alpha}^{(A)} [1, 1] \rightarrow \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}, E_{\alpha}^{(A)} [1, 2] \rightarrow 0, E_{\alpha}^{(A)} [1, 3] \rightarrow 0, E_{\alpha}^{(A)} [1, 4] \rightarrow 0, \\ & E_{\alpha}^{(A)} [1, 5] \rightarrow 0, E_{\alpha}^{(A)} [1, 6] \rightarrow 0, E_{\alpha}^{(A)} [1, 7] \rightarrow 0, E_{\alpha}^{(A)} [2, 0] \rightarrow 0, E_{\alpha}^{(A)} [2, 1] \rightarrow 0, \\ & E_{\alpha}^{(A)} [2, 2] \rightarrow \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}, E_{\alpha}^{(A)} [2, 3] \rightarrow 0, E_{\alpha}^{(A)} [2, 4] \rightarrow 0, E_{\alpha}^{(A)} [2, 5] \rightarrow 0, \\ & E_{\alpha}^{(A)} [2, 6] \rightarrow 0, E_{\alpha}^{(A)} [2, 7] \rightarrow 0, E_{\alpha}^{(A)} [3, 0] \rightarrow 0, E_{\alpha}^{(A)} [3, 1] \rightarrow 0, E_{\alpha}^{(A)} [3, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)} [3, 3] \rightarrow \sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}, E_{\alpha}^{(A)} [3, 4] \rightarrow 0, E_{\alpha}^{(A)} [3, 5] \rightarrow 0, E_{\alpha}^{(A)} [3, 6] \rightarrow 0, \\ & E_{\alpha}^{(A)} [3, 7] \rightarrow 0, E_{\alpha}^{(A)} [4, 0] \rightarrow 0, E_{\alpha}^{(A)} [4, 1] \rightarrow 0, E_{\alpha}^{(A)} [4, 2] \rightarrow 0, E_{\alpha}^{(A)} [4, 3] \rightarrow 0, E_{\alpha}^{(A)} [4, 4] \rightarrow 1, \\ & E_{\alpha}^{(A)} [4, 5] \rightarrow 0, E_{\alpha}^{(A)} [4, 6] \rightarrow 0, E_{\alpha}^{(A)} [4, 7] \rightarrow 0, E_{\alpha}^{(A)} [5, 0] \rightarrow 0, E_{\alpha}^{(A)} [5, 1] \rightarrow 0, \\ & E_{\alpha}^{(A)} [5, 2] \rightarrow 0, E_{\alpha}^{(A)} [5, 3] \rightarrow 0, E_{\alpha}^{(A)} [5, 4] \rightarrow 0, E_{\alpha}^{(A)} [5, 5] \rightarrow \sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}, \\ & E_{\alpha}^{(A)} [5, 6] \rightarrow 0, E_{\alpha}^{(A)} [5, 7] \rightarrow 0, E_{\alpha}^{(A)} [6, 0] \rightarrow 0, E_{\alpha}^{(A)} [6, 1] \rightarrow 0, E_{\alpha}^{(A)} [6, 2] \rightarrow 0, \\ & E_{\alpha}^{(A)} [6, 3] \rightarrow 0, E_{\alpha}^{(A)} [6, 4] \rightarrow 0, E_{\alpha}^{(A)} [6, 5] \rightarrow 0, E_{\alpha}^{(A)} [6, 6] \rightarrow \sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}, \\ & E_{\alpha}^{(A)} [6, 7] \rightarrow 0, E_{\alpha}^{(A)} [7, 0] \rightarrow 0, E_{\alpha}^{(A)} [7, 1] \rightarrow 0, E_{\alpha}^{(A)} [7, 2] \rightarrow 0, E_{\alpha}^{(A)} [7, 3] \rightarrow 0, \\ & E_{\alpha}^{(A)} [7, 4] \rightarrow 0, E_{\alpha}^{(A)} [7, 5] \rightarrow 0, E_{\alpha}^{(A)} [7, 6] \rightarrow 0, E_{\alpha}^{(A)} [7, 7] \rightarrow \sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6} \} \end{aligned}$$

```
In[•]:= (sgtrye $_{(A)}^{\alpha}$  = Thread[Flatten[e $_{(A)}^{\alpha}$ ] → Flatten[Inverse[e $_{\alpha}^{(A)}$  /. sgtrye $_{\alpha}^{(A)}$ ]]]) (*//Column*)
```

Out[•]=

$$\begin{aligned} & \left\{ E_{(A)}^{\alpha}[0, 0] \rightarrow \tan[6Hx0], E_{(A)}^{\alpha}[0, 1] \rightarrow 0, E_{(A)}^{\alpha}[0, 2] \rightarrow 0, E_{(A)}^{\alpha}[0, 3] \rightarrow 0, \right. \\ & E_{(A)}^{\alpha}[0, 4] \rightarrow 0, E_{(A)}^{\alpha}[0, 5] \rightarrow 0, E_{(A)}^{\alpha}[0, 6] \rightarrow 0, E_{(A)}^{\alpha}[0, 7] \rightarrow 0, E_{(A)}^{\alpha}[1, 0] \rightarrow 0, \\ & E_{(A)}^{\alpha}[1, 1] \rightarrow \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}}, E_{(A)}^{\alpha}[1, 2] \rightarrow 0, E_{(A)}^{\alpha}[1, 3] \rightarrow 0, E_{(A)}^{\alpha}[1, 4] \rightarrow 0, \\ & E_{(A)}^{\alpha}[1, 5] \rightarrow 0, E_{(A)}^{\alpha}[1, 6] \rightarrow 0, E_{(A)}^{\alpha}[1, 7] \rightarrow 0, E_{(A)}^{\alpha}[2, 0] \rightarrow 0, E_{(A)}^{\alpha}[2, 1] \rightarrow 0, \\ & E_{(A)}^{\alpha}[2, 2] \rightarrow \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}}, E_{(A)}^{\alpha}[2, 3] \rightarrow 0, E_{(A)}^{\alpha}[2, 4] \rightarrow 0, E_{(A)}^{\alpha}[2, 5] \rightarrow 0, \\ & E_{(A)}^{\alpha}[2, 6] \rightarrow 0, E_{(A)}^{\alpha}[2, 7] \rightarrow 0, E_{(A)}^{\alpha}[3, 0] \rightarrow 0, E_{(A)}^{\alpha}[3, 1] \rightarrow 0, E_{(A)}^{\alpha}[3, 2] \rightarrow 0, \\ & E_{(A)}^{\alpha}[3, 3] \rightarrow \frac{1}{\sqrt{e^{2a4[Hx4]}} \sin[6Hx0]^{1/6}}, E_{(A)}^{\alpha}[3, 4] \rightarrow 0, E_{(A)}^{\alpha}[3, 5] \rightarrow 0, E_{(A)}^{\alpha}[3, 6] \rightarrow 0, \\ & E_{(A)}^{\alpha}[3, 7] \rightarrow 0, E_{(A)}^{\alpha}[4, 0] \rightarrow 0, E_{(A)}^{\alpha}[4, 1] \rightarrow 0, E_{(A)}^{\alpha}[4, 2] \rightarrow 0, E_{(A)}^{\alpha}[4, 3] \rightarrow 0, E_{(A)}^{\alpha}[4, 4] \rightarrow 1, \\ & E_{(A)}^{\alpha}[4, 5] \rightarrow 0, E_{(A)}^{\alpha}[4, 6] \rightarrow 0, E_{(A)}^{\alpha}[4, 7] \rightarrow 0, E_{(A)}^{\alpha}[5, 0] \rightarrow 0, E_{(A)}^{\alpha}[5, 1] \rightarrow 0, \\ & E_{(A)}^{\alpha}[5, 2] \rightarrow 0, E_{(A)}^{\alpha}[5, 3] \rightarrow 0, E_{(A)}^{\alpha}[5, 4] \rightarrow 0, E_{(A)}^{\alpha}[5, 5] \rightarrow \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}}, \\ & E_{(A)}^{\alpha}[5, 6] \rightarrow 0, E_{(A)}^{\alpha}[5, 7] \rightarrow 0, E_{(A)}^{\alpha}[6, 0] \rightarrow 0, E_{(A)}^{\alpha}[6, 1] \rightarrow 0, E_{(A)}^{\alpha}[6, 2] \rightarrow 0, \\ & E_{(A)}^{\alpha}[6, 3] \rightarrow 0, E_{(A)}^{\alpha}[6, 4] \rightarrow 0, E_{(A)}^{\alpha}[6, 5] \rightarrow 0, E_{(A)}^{\alpha}[6, 6] \rightarrow \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}}, \\ & E_{(A)}^{\alpha}[6, 7] \rightarrow 0, E_{(A)}^{\alpha}[7, 0] \rightarrow 0, E_{(A)}^{\alpha}[7, 1] \rightarrow 0, E_{(A)}^{\alpha}[7, 2] \rightarrow 0, E_{(A)}^{\alpha}[7, 3] \rightarrow 0, \\ & E_{(A)}^{\alpha}[7, 4] \rightarrow 0, E_{(A)}^{\alpha}[7, 5] \rightarrow 0, E_{(A)}^{\alpha}[7, 6] \rightarrow 0, E_{(A)}^{\alpha}[7, 7] \rightarrow \frac{1}{\sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6}} \} \end{aligned}$$

```
In[•]:= gtrye $_{\alpha}^{(A)}$  = (e $_{\alpha}^{(A)}$  /. sgtrye $_{\alpha}^{(A)}$ ) /. subsDefects
```

Out[•]=

$$\begin{aligned} & \left\{ \{\cot[6Hx0], 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0\}, \right. \\ & \{0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0\}, \\ & \left. \{0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}\} \right\} \end{aligned}$$

```
In[•]:= gtrye $_{(A)}^{\alpha}$  = (e $_{(A)}^{\alpha}$  /. sgtrye $_{(A)}^{\alpha}$ ) /. subsDefects
```

Out[•]=

$$\begin{aligned} & \left\{ \{\tan[6Hx0], 0, 0, 0, 0, 0, 0, 0\}, \left\{ 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0, 0, 0 \right\}, \\ & \{0, 0, 0, 0, 1, 0, 0, 0\}, \left\{ 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0, 0 \right\}, \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}} \right\} \right\} \end{aligned}$$

```

In[•]:= eα(A)
% /. sgtryeα(A) // MatrixForm
Out[•]= { {Eα(A)[0, 0], Eα(A)[0, 1], Eα(A)[0, 2], Eα(A)[0, 3], Eα(A)[0, 4], Eα(A)[0, 5], Eα(A)[0, 6], Eα(A)[0, 7]}, {Eα(A)[1, 0], Eα(A)[1, 1], Eα(A)[1, 2], Eα(A)[1, 3], Eα(A)[1, 4], Eα(A)[1, 5], Eα(A)[1, 6], Eα(A)[1, 7]}, {Eα(A)[2, 0], Eα(A)[2, 1], Eα(A)[2, 2], Eα(A)[2, 3], Eα(A)[2, 4], Eα(A)[2, 5], Eα(A)[2, 6], Eα(A)[2, 7]}, {Eα(A)[3, 0], Eα(A)[3, 1], Eα(A)[3, 2], Eα(A)[3, 3], Eα(A)[3, 4], Eα(A)[3, 5], Eα(A)[3, 6], Eα(A)[3, 7]}, {Eα(A)[4, 0], Eα(A)[4, 1], Eα(A)[4, 2], Eα(A)[4, 3], Eα(A)[4, 4], Eα(A)[4, 5], Eα(A)[4, 6], Eα(A)[4, 7]}, {Eα(A)[5, 0], Eα(A)[5, 1], Eα(A)[5, 2], Eα(A)[5, 3], Eα(A)[5, 4], Eα(A)[5, 5], Eα(A)[5, 6], Eα(A)[5, 7]}, {Eα(A)[6, 0], Eα(A)[6, 1], Eα(A)[6, 2], Eα(A)[6, 3], Eα(A)[6, 4], Eα(A)[6, 5], Eα(A)[6, 6], Eα(A)[6, 7]}, {Eα(A)[7, 0], Eα(A)[7, 1], Eα(A)[7, 2], Eα(A)[7, 3], Eα(A)[7, 4], Eα(A)[7, 5], Eα(A)[7, 6], Eα(A)[7, 7]}}

Out[•]//MatrixForm=
Cot[6 H x 0] 0 0 0 0 0
0 √e2 a4[H x 4] Sin[6 H x 0]1/6 0 0 0
0 0 √e2 a4[H x 4] Sin[6 H x 0]1/6 0 0
0 0 0 √e2 a4[H x 4] Sin[6 H x 0]1/6 0
0 0 0 0 0 1
0 0 0 0 0 0 √e-2 i
0 0 0 0 0 0
0 0 0 0 0 0

In[•]:= eηe = FullSimplify[Transpose[eα(A)] . η4488.eα(A), constraintVars];
% /. sgeα(A)
%% /. sgtryeα(A)
% == gtry // FullSimplify[#, constraintVars] &

Out[•]= {{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}

Out[•]= {{Cot[6 H x 0]2, 0, 0, 0, 0, 0, 0, 0}, {0, e2 a4[H x 4] Sin[6 H x 0]1/3, 0, 0, 0, 0, 0, 0}, {0, 0, e2 a4[H x 4] Sin[6 H x 0]1/3, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, e2 a4[H x 4] Sin[6 H x 0]1/3, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e-2 a4[H x 4] Sin[6 H x 0]1/3, 0, 0}, {0, 0, 0, 0, 0, 0, -e-2 a4[H x 4] Sin[6 H x 0]1/3, 0}, {0, 0, 0, 0, 0, 0, 0, -e-2 a4[H x 4] Sin[6 H x 0]1/3}}

Out[•]= True

```

In[1]:= **gtry**

Out[1]=

$$\left\{ \left\{ \text{Cot}[6Hx0]^2, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, e^{2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, -1, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, -e^{-2a4[Hx4]} \sin[6Hx0]^{1/3} \right\} \right\}$$

In[2]:= (*FullSimplify[Transpose[e_α^(A)] . η4488.e_α^(A), constraintVars]-MatrixMetric44*)

In[3]:= **ssgm4488**

Out[3]=

$$\begin{aligned} & \{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ & g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ & g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ & g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ & g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&) \} \end{aligned}$$

In[4]:= **MatrixMetric44 // MatrixForm**

Out[4]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e^{-2a4[Hx4]} \end{pmatrix}$$

In[•]:= $e_{\alpha}^{(A)}$

Out[•]= $\{ \{E_{\alpha}^{(A)}[0, 0], E_{\alpha}^{(A)}[0, 1], E_{\alpha}^{(A)}[0, 2], E_{\alpha}^{(A)}[0, 3], E_{\alpha}^{(A)}[0, 4], E_{\alpha}^{(A)}[0, 5], E_{\alpha}^{(A)}[0, 6], E_{\alpha}^{(A)}[0, 7]\},$
 $\{E_{\alpha}^{(A)}[1, 0], E_{\alpha}^{(A)}[1, 1], E_{\alpha}^{(A)}[1, 2], E_{\alpha}^{(A)}[1, 3], E_{\alpha}^{(A)}[1, 4], E_{\alpha}^{(A)}[1, 5], E_{\alpha}^{(A)}[1, 6], E_{\alpha}^{(A)}[1, 7]\},$
 $\{E_{\alpha}^{(A)}[2, 0], E_{\alpha}^{(A)}[2, 1], E_{\alpha}^{(A)}[2, 2], E_{\alpha}^{(A)}[2, 3], E_{\alpha}^{(A)}[2, 4], E_{\alpha}^{(A)}[2, 5], E_{\alpha}^{(A)}[2, 6], E_{\alpha}^{(A)}[2, 7]\},$
 $\{E_{\alpha}^{(A)}[3, 0], E_{\alpha}^{(A)}[3, 1], E_{\alpha}^{(A)}[3, 2], E_{\alpha}^{(A)}[3, 3], E_{\alpha}^{(A)}[3, 4], E_{\alpha}^{(A)}[3, 5], E_{\alpha}^{(A)}[3, 6], E_{\alpha}^{(A)}[3, 7]\},$
 $\{E_{\alpha}^{(A)}[4, 0], E_{\alpha}^{(A)}[4, 1], E_{\alpha}^{(A)}[4, 2], E_{\alpha}^{(A)}[4, 3], E_{\alpha}^{(A)}[4, 4], E_{\alpha}^{(A)}[4, 5], E_{\alpha}^{(A)}[4, 6], E_{\alpha}^{(A)}[4, 7]\},$
 $\{E_{\alpha}^{(A)}[5, 0], E_{\alpha}^{(A)}[5, 1], E_{\alpha}^{(A)}[5, 2], E_{\alpha}^{(A)}[5, 3], E_{\alpha}^{(A)}[5, 4], E_{\alpha}^{(A)}[5, 5], E_{\alpha}^{(A)}[5, 6], E_{\alpha}^{(A)}[5, 7]\},$
 $\{E_{\alpha}^{(A)}[6, 0], E_{\alpha}^{(A)}[6, 1], E_{\alpha}^{(A)}[6, 2], E_{\alpha}^{(A)}[6, 3], E_{\alpha}^{(A)}[6, 4], E_{\alpha}^{(A)}[6, 5], E_{\alpha}^{(A)}[6, 6], E_{\alpha}^{(A)}[6, 7]\},$
 $\{E_{\alpha}^{(A)}[7, 0], E_{\alpha}^{(A)}[7, 1], E_{\alpha}^{(A)}[7, 2], E_{\alpha}^{(A)}[7, 3], E_{\alpha}^{(A)}[7, 4], E_{\alpha}^{(A)}[7, 5], E_{\alpha}^{(A)}[7, 6], E_{\alpha}^{(A)}[7, 7]\} \}$

In[•]:= Block[{s, r}, s = FullSimplify[e_{\alpha}^{(A)} /. sgtrye_{\alpha}^{(A)}, constraintVars];
r = FullSimplify[Transpose[s].\eta4488.s, constraintVars] - MatrixMetric44;
FullSimplify[r, constraintVars]]

Out[•]= $\{ \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\},$
 $\{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\},$
 $\{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}$

In[•]:= (* $e_{\alpha}^{(A)} = eAa$ *) //MatrixForm)

In[•]:= (* $e_{\alpha}^{(A)}$
Inverse[%] *)

In[•]:= (* $e_{\alpha}^{(A)} = \text{Inverse}[$
 $e_{\alpha}^{(A)} /. \text{sgtryeSubsuperscript}_{\alpha}\text{andLeftParenthesis}_A\text{RightParenthesis}]$ *) //
MatrixForm*)

In[•]:= $e_{(A)}^{\alpha}$

Out[•]= $\{ \{E_{(A)}^{\alpha}[0, 0], E_{(A)}^{\alpha}[0, 1], E_{(A)}^{\alpha}[0, 2], E_{(A)}^{\alpha}[0, 3], E_{(A)}^{\alpha}[0, 4], E_{(A)}^{\alpha}[0, 5], E_{(A)}^{\alpha}[0, 6], E_{(A)}^{\alpha}[0, 7]\},$
 $\{E_{(A)}^{\alpha}[1, 0], E_{(A)}^{\alpha}[1, 1], E_{(A)}^{\alpha}[1, 2], E_{(A)}^{\alpha}[1, 3], E_{(A)}^{\alpha}[1, 4], E_{(A)}^{\alpha}[1, 5], E_{(A)}^{\alpha}[1, 6], E_{(A)}^{\alpha}[1, 7]\},$
 $\{E_{(A)}^{\alpha}[2, 0], E_{(A)}^{\alpha}[2, 1], E_{(A)}^{\alpha}[2, 2], E_{(A)}^{\alpha}[2, 3], E_{(A)}^{\alpha}[2, 4], E_{(A)}^{\alpha}[2, 5], E_{(A)}^{\alpha}[2, 6], E_{(A)}^{\alpha}[2, 7]\},$
 $\{E_{(A)}^{\alpha}[3, 0], E_{(A)}^{\alpha}[3, 1], E_{(A)}^{\alpha}[3, 2], E_{(A)}^{\alpha}[3, 3], E_{(A)}^{\alpha}[3, 4], E_{(A)}^{\alpha}[3, 5], E_{(A)}^{\alpha}[3, 6], E_{(A)}^{\alpha}[3, 7]\},$
 $\{E_{(A)}^{\alpha}[4, 0], E_{(A)}^{\alpha}[4, 1], E_{(A)}^{\alpha}[4, 2], E_{(A)}^{\alpha}[4, 3], E_{(A)}^{\alpha}[4, 4], E_{(A)}^{\alpha}[4, 5], E_{(A)}^{\alpha}[4, 6], E_{(A)}^{\alpha}[4, 7]\},$
 $\{E_{(A)}^{\alpha}[5, 0], E_{(A)}^{\alpha}[5, 1], E_{(A)}^{\alpha}[5, 2], E_{(A)}^{\alpha}[5, 3], E_{(A)}^{\alpha}[5, 4], E_{(A)}^{\alpha}[5, 5], E_{(A)}^{\alpha}[5, 6], E_{(A)}^{\alpha}[5, 7]\},$
 $\{E_{(A)}^{\alpha}[6, 0], E_{(A)}^{\alpha}[6, 1], E_{(A)}^{\alpha}[6, 2], E_{(A)}^{\alpha}[6, 3], E_{(A)}^{\alpha}[6, 4], E_{(A)}^{\alpha}[6, 5], E_{(A)}^{\alpha}[6, 6], E_{(A)}^{\alpha}[6, 7]\},$
 $\{E_{(A)}^{\alpha}[7, 0], E_{(A)}^{\alpha}[7, 1], E_{(A)}^{\alpha}[7, 2], E_{(A)}^{\alpha}[7, 3], E_{(A)}^{\alpha}[7, 4], E_{(A)}^{\alpha}[7, 5], E_{(A)}^{\alpha}[7, 6], E_{(A)}^{\alpha}[7, 7]\} \}$

```
In[1]:= (sgeα(A) = Thread[Flatten[eα(A)] → Flatten[Inverse[e(A)α /. sge(A)α]]]) (*//Column*)
```

Out[1]=

$$\left\{ \begin{array}{l} E_{(A)}^{\alpha}[0, 0] \rightarrow \frac{1}{\sqrt{g[0][0][x0, x4]}}, E_{(A)}^{\alpha}[0, 1] \rightarrow 0, E_{(A)}^{\alpha}[0, 2] \rightarrow 0, E_{(A)}^{\alpha}[0, 3] \rightarrow 0, E_{(A)}^{\alpha}[0, 4] \rightarrow 0, \\ E_{(A)}^{\alpha}[0, 5] \rightarrow 0, E_{(A)}^{\alpha}[0, 6] \rightarrow 0, E_{(A)}^{\alpha}[0, 7] \rightarrow 0, E_{(A)}^{\alpha}[1, 0] \rightarrow 0, E_{(A)}^{\alpha}[1, 1] \rightarrow \frac{1}{\sqrt{g[1][1][x0, x4]}}, \\ E_{(A)}^{\alpha}[1, 2] \rightarrow 0, E_{(A)}^{\alpha}[1, 3] \rightarrow 0, E_{(A)}^{\alpha}[1, 4] \rightarrow 0, E_{(A)}^{\alpha}[1, 5] \rightarrow 0, E_{(A)}^{\alpha}[1, 6] \rightarrow 0, E_{(A)}^{\alpha}[1, 7] \rightarrow 0, \\ E_{(A)}^{\alpha}[2, 0] \rightarrow 0, E_{(A)}^{\alpha}[2, 1] \rightarrow 0, E_{(A)}^{\alpha}[2, 2] \rightarrow \frac{1}{\sqrt{g[2][2][x0, x4]}}, E_{(A)}^{\alpha}[2, 3] \rightarrow 0, \\ E_{(A)}^{\alpha}[2, 4] \rightarrow 0, E_{(A)}^{\alpha}[2, 5] \rightarrow 0, E_{(A)}^{\alpha}[2, 6] \rightarrow 0, E_{(A)}^{\alpha}[2, 7] \rightarrow 0, E_{(A)}^{\alpha}[3, 0] \rightarrow 0, E_{(A)}^{\alpha}[3, 1] \rightarrow 0, \\ E_{(A)}^{\alpha}[3, 2] \rightarrow 0, E_{(A)}^{\alpha}[3, 3] \rightarrow \frac{1}{\sqrt{g[3][3][x0, x4]}}, E_{(A)}^{\alpha}[3, 4] \rightarrow 0, E_{(A)}^{\alpha}[3, 5] \rightarrow 0, \\ E_{(A)}^{\alpha}[3, 6] \rightarrow 0, E_{(A)}^{\alpha}[3, 7] \rightarrow 0, E_{(A)}^{\alpha}[4, 0] \rightarrow 0, E_{(A)}^{\alpha}[4, 1] \rightarrow 0, E_{(A)}^{\alpha}[4, 2] \rightarrow 0, E_{(A)}^{\alpha}[4, 3] \rightarrow 0, \\ E_{(A)}^{\alpha}[4, 4] \rightarrow \frac{1}{\sqrt{-g[4][4][x0, x4]}}, E_{(A)}^{\alpha}[4, 5] \rightarrow 0, E_{(A)}^{\alpha}[4, 6] \rightarrow 0, E_{(A)}^{\alpha}[4, 7] \rightarrow 0, E_{(A)}^{\alpha}[5, 0] \rightarrow 0, \\ E_{(A)}^{\alpha}[5, 1] \rightarrow 0, E_{(A)}^{\alpha}[5, 2] \rightarrow 0, E_{(A)}^{\alpha}[5, 3] \rightarrow 0, E_{(A)}^{\alpha}[5, 4] \rightarrow 0, E_{(A)}^{\alpha}[5, 5] \rightarrow \frac{1}{\sqrt{-g[5][5][x0, x4]}}, \\ E_{(A)}^{\alpha}[5, 6] \rightarrow 0, E_{(A)}^{\alpha}[5, 7] \rightarrow 0, E_{(A)}^{\alpha}[6, 0] \rightarrow 0, E_{(A)}^{\alpha}[6, 1] \rightarrow 0, E_{(A)}^{\alpha}[6, 2] \rightarrow 0, \\ E_{(A)}^{\alpha}[6, 3] \rightarrow 0, E_{(A)}^{\alpha}[6, 4] \rightarrow 0, E_{(A)}^{\alpha}[6, 5] \rightarrow 0, E_{(A)}^{\alpha}[6, 6] \rightarrow \frac{1}{\sqrt{-g[6][6][x0, x4]}}, \\ E_{(A)}^{\alpha}[6, 7] \rightarrow 0, E_{(A)}^{\alpha}[7, 0] \rightarrow 0, E_{(A)}^{\alpha}[7, 1] \rightarrow 0, E_{(A)}^{\alpha}[7, 2] \rightarrow 0, E_{(A)}^{\alpha}[7, 3] \rightarrow 0, \\ E_{(A)}^{\alpha}[7, 4] \rightarrow 0, E_{(A)}^{\alpha}[7, 5] \rightarrow 0, E_{(A)}^{\alpha}[7, 6] \rightarrow 0, E_{(A)}^{\alpha}[7, 7] \rightarrow \frac{1}{\sqrt{-g[7][7][x0, x4]}} \end{array} \right\}$$

In[2]:= (*Symbolize[sge^α_(A)]*)

In[3]:= (*Symbolize[sgtrye^α_(A)]*)

In[4]:= (*%/.sge^(A)
%/.sgtrye^(A)*)

In[5]:= (*FullSimplify[Transpose[e^(A)_α].η4488.e^(A)_α,constraintVars]-MatrixMetric44*)

For Spin (4, 4); τ tau; T16; OCTAD : Nash; Introduce the wave function, Ψ16, for this Universe::

O(4,4); evals, evecs of σ

In[6]:= {evals, evecs} = Eigensystem[σ]

Out[6]=

$$\left\{ \begin{array}{l} \{-1, -1, -1, -1, 1, 1, 1, 1\}, \{0, 0, 0, -1, 0, 0, 0, 1\}, \{0, 0, -1, 0, 0, 0, 1, 0\}, \\ \{0, -1, 0, 0, 0, 1, 0, 0\}, \{-1, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 1\}, \\ \{0, 0, 1, 0, 0, 0, 1, 0\}, \{0, 1, 0, 0, 0, 1, 0, 0\}, \{1, 0, 0, 0, 1, 0, 0, 0\} \end{array} \right\}$$

```

In[8]:= 
$$\left( \mathbf{u} = \text{ExpandAll} \left[ \frac{1}{\sqrt{2}} \text{evecs} \right] \right) // \text{MatrixForm}$$

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$


In[9]:= 
$$\text{Table}[\mathbf{u}[[h]].\sigma.\mathbf{u}[[h]], \{h, 1, \text{Length}[\mathbf{u}]\}]$$

Out[9]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[10]:= 
$$\text{Table}[(\text{Transpose}[\mathbf{u}][[h]]).\sigma.\mathbf{u}[[h]], \{h, 1, \text{Length}[\mathbf{u}]\}]$$

Out[10]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[11]:= 
$$\begin{aligned} \text{Table}[(\text{Transpose}[\mathbf{u}][[A]]) == -\sigma.\mathbf{u}[[A]], \{A, 1, 4\}] \\ \text{Table}[(\text{Transpose}[\mathbf{u}][[A]]) == \sigma.\mathbf{u}[[A]], \{A, 5, \text{Length}[\mathbf{u}]\}] \end{aligned}$$

Out[11]= {True, True, True, True}

Out[12]= {True, True, True, True}

Ax = 
$$\left( \frac{\text{KroneckerProduct}[dX, X] - \text{KroneckerProduct}[X, dX]}{2} \right). \sigma;$$

Au = 
$$\left( \frac{\text{KroneckerProduct}[dU, U] - \text{KroneckerProduct}[U, dU]}{2} \right). \sigma;$$

Lsquared =

$$\dot{x}^A \dot{x}_A = dX.\sigma.dX + X.\sigma.X \left( \frac{dU.\sigma.dU}{U.\sigma.U} - \left( \frac{U.\sigma.dU}{U.\sigma.U} \right)^2 \right) + \frac{2}{U.\sigma.U} \text{Tr}[\bar{\tau}^A.Ax.\tau_A.Au]$$

Timing[Simplify[ExpandAll[dY.η8.dY - Lsquared]]
{10.733, 0}]

```

```
In[•]:= FullSimplify[Sqrt[Flatten[MatrixMetric44.η4488]], constraintVars]
```

Out[•]=

$$\left\{ \text{Cot}[6 H x], 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{2 a4[H x^4]}} \sin[6 H x]^{\frac{1}{6}}, 0, 0, 0, 0, 0, 0, 0, 0, \right. \\ \left. \sqrt{e^{2 a4[H x^4]}} \sin[6 H x]^{\frac{1}{6}}, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{2 a4[H x^4]}} \sin[6 H x]^{\frac{1}{6}}, 0, 0, 0, \right. \\ \left. 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x^4]}} \sin[6 H x]^{\frac{1}{6}}, 0, 0, 0, 0, 0, 0, \right. \\ \left. 0, 0, \sqrt{e^{-2 a4[H x^4]}} \sin[6 H x]^{\frac{1}{6}}, 0, 0, 0, 0, 0, 0, 0, \sqrt{e^{-2 a4[H x^4]}} \sin[6 H x]^{\frac{1}{6}} \right\}$$

$$SO(4), \quad \gamma; M8, \{0, 1, 2, 3, 4, 5, 6, 7\}, \{+, +, +, -, -, -, -\}, \{x0, x1, x2, x3, x4, x5, x6, x7\}$$

Under the involutive automorphism $\gamma^{AB} \rightarrow -\tilde{\gamma}^{AB}$ of $so(3,3)$, the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra $so(4)$, which is the Lie algebra of $\overline{SO(4)}$, the maximal compact subgroup of $\overline{SL(4, \mathbb{R})}$. The subalgebra $so(4) \cong su(2) + su(2)$ may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of $so(4)$ whereby $\tau \in so(4)$ is mapped into its dual, ${}^*\tau$. The even subspace under $*$ of $so(4)$ corresponds to self-dual tensors, and, say, the first $su(2)$ in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second $su(2)$ in the direct sum. A basis for $so(4)$ may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by ($h = 1, 2, 3$),

$$2s^h = (\gamma^{23}, \gamma^{31}, \gamma^{21}), \quad (4)$$

and

$$2t^h = (\gamma^{45}, \gamma^{64}, \gamma^{65}). \quad (5)$$

Out[•]=

Under the involutive automorphism $\gamma^{AB} \rightarrow -\tilde{\gamma}^{AB}$ of $so(3,3)$, the Lie algebra decomposes into the eigenvalue (-1) and eigenvalue $(+1)$ subspaces corresponding to, respectively, the nine linearly independent real traceless symmetric 4×4 matrices, and the six linearly independent real skew-symmetric 4×4 matrices. The eigenvalue $(+1)$ subspace is the subalgebra $so(4)$, which is the Lie algebra of $\overline{SO(4)}$, the maximal compact subgroup of $\overline{SL(4,\mathbb{R})}$. The subalgebra $so(4) \cong su(2) + su(2)$ may be further decomposed into the even (eigenvalue $+1$) and odd (eigenvalue -1) subspaces of the linear transformation of $so(4)$ whereby $\tau \in so(4)$ is mapped into its dual, ${}^*\tau$. The even subspace under * of $so(4)$ corresponds to self-dual tensors, and, say, the first $su(2)$ in the direct sum; the odd subspace corresponds to anti-self-dual tensors, and the second $su(2)$ in the direct sum. A basis for $so(4)$ may be chosen as follows. Each of the six skew-symmetric γ matrices has the property that the square of the matrix is equal to $-\gamma_0$. By Eq. (1), these six matrices are given by ($h = 1, 2, 3$),

$$2s^h = (\gamma^{23}, \gamma^{31}, \gamma^{21}), \quad (4)$$

and

$$2t^h = (\gamma^{45}, \gamma^{64}, \gamma^{65}). \quad (5)$$

```
In[•]:= Qa[h_, p_, q_] := Signature[{h, p, q, 4}];
Qb[h_, p_, q_] := ID4[[p, 4]] * ID4[[q, h]] - ID4[[p, h]] * ID4[[q, 4]];
SelfDualAntiSymmetric[h_, p_, q_] := Qa[h, p, q] - Qb[h, p, q];
AntiSelfDualAntiSymmetric[h_, p_, q_] := (Qa[h, p, q] + Qb[h, p, q]);
```

```
In[•]:= Qa[1, 2, 3]
```

Out[•]=

1

```
In[•]:= e4[[1, 2, 3, 4]]
```

Out[•]=

1

```
In[•]:= Do[
  s4by4[h] = Table[Table[SelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}], {h, 1, 3}]
In[•]:= Do[t4by4[h] = Table[Table[AntiSelfDualAntiSymmetric[h, p, q], {q, 4}], {p, 4}], {h, 1, 3}]
In[•]:= (*Protect[{s4by4[1], s4by4[2], s4by4[3], t4by4[1], t4by4[2], t4by4[3]}]*)
```

```

In[1]:= Protect[s4by4, t4by4]
Out[1]= {s4by4, t4by4}

In[2]:= {s4by4[#] // MatrixForm, MatrixForm[t4by4[#]]} & /@ Range[3]
Out[2]= {{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}, {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}}}, {{{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}}, {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}, {{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}}, {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}}}}, {{{{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}, {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}}, {{1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}, {0, 0, 0, 1}}}, {{{{0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 0, 0, -1}}, {{0, 0, 0, -1}, {1, 0, 0, 0}, {0, 0, 1, 0}, {0, -1, 0, 0}}}, {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}}}, {{{{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}}, {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}}, {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}}}}}, {{{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, -1, 0}, {0, 0, -1, 0}}, {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}}, {{2, 2}, {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}}, {{2, 3}, {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}}}}, {{3, 1}, {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}, {{3, 2}, {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}}, {{3, 3}, {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}}}], Do[st[J, K] = s4by4[J].t4by4[K], {J, 1, 3}, {K, 1, 3}]
In[3]:= Table[{{J, K}, MatrixForm[st[J, K]]}, {J, 1, 3}, {K, 1, 3}]
Out[3]= {{{{1, 1}, {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}}, {{1, 2}, {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}}, {{1, 3}, {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}}}}, {{2, 1}, {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}}}, {{2, 2}, {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}}, {{2, 3}, {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}}}}, {{3, 1}, {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}, {{3, 2}, {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}}, {{3, 3}, {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}}}], Do[st[J, K] = st[J, K].st[K, J], {J, 1, 3}, {K, 1, 3}]
In[4]:= Protect[st]
Out[4]= {st}

```

```
In[6]:= Table[{{J, K}, MatrixForm[st[J, K] - Transpose[st[J, K]]]}, {J, 1, 3}, {K, 1, 3}]
Out[6]=
{{{{1, 1}, {0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0}}, {{1, 2}, {0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0}}, {{1, 3}, {0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0}}}, {{{{2, 1}, {0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0}}, {{2, 2}, {0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0}}, {{2, 3}, {0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0}}}, {{{{3, 1}, {0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0}}, {{3, 2}, {0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0}}, {{3, 3}, {0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0}}}}
```

In[7]:= Permutations[Range[3], 2]

```
Out[7]= {{}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 1}, {2, 3}, {3, 1}, {3, 2}}
```

In[8]:= KroneckerProduct[Range[3], Range[3]]

```
Out[8]= {{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}
```

In[9]:= Table[{J, K}, {J, 1, 3}, {K, 1, 3}]

```
st[##]& /@%
st[##]& @@ %
```

Out[9]=

```
{{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}}
```

Out[10]=

```
{st[{{1, 1}, {1, 2}, {1, 3}}], st[{{2, 1}, {2, 2}, {2, 3}}], st[{{3, 1}, {3, 2}, {3, 3}}]}
```

Out[11]=

```
st[{{1, 1}, {1, 2}, {1, 3}}, {{2, 1}, {2, 2}, {2, 3}}, {{3, 1}, {3, 2}, {3, 3}}]
```

```
In[•]:= Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]
Length[%]

Out[•]= {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
{{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
{{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}},
{{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

Out[•]= 16

In[•]:= Unprotect[γ];
γ = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
  Flatten[Table[st[J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}}, 1]

Out[•]= {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
{{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
{{0, 0, 1, 0}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}},
{{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}},
{{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}},
{{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}},
{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}},
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}},
{{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}},
{{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}},
{{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}},
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}

In[•]:= Length[γ]
Out[•]= 16
```

In[•]:= **Protect**[γ]

Out[•]=

{ γ }

In[•]:= $\gamma[[1]]$

Out[•]=

{ {0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0} }

In[•]:= $\gamma[[-1]]$

Out[•]=

{ {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1} }

Self Dual Anti-Symmetric

In[•]:= **Table** $\left[\left(\frac{1}{2} \sum_{j1=1}^4 \left(\sum_{j2=1}^4 \epsilon4[p, q, j1, j2] \times s4by4[h][j1, j2]\right)\right) - s4by4[h][p, q],$
 $\{h, 1, 3\}, \{q, 4\}, \{p, 4\}\right]$

Out[•]=

{ { {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} },
{ {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} },
{ {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} } }

Anti SelfDual Anti-Symmetric

In[•]:= **Table** $\left[\left(\frac{1}{2} \sum_{j1=1}^4 \left(\sum_{j2=1}^4 \epsilon4[p, q, j1, j2] \times t4by4[h][j1, j2]\right)\right) + t4by4[h][p, q],$
 $\{h, 1, 3\}, \{q, 4\}, \{p, 4\}\right]$

Out[•]=

{ { {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} },
{ {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} },
{ {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0} } }

SO(4,4), Spin(4,4) = $\overline{\text{SO}(4,4)}$, τ ; if using xact: M8, {0, 1, 2, 3, 4, 5, 6, 7}, {+, +, +, -, -, -}, {x0, x1, x2, x3, x4, x5, x6, x7}

In[•]:= $\overline{\tau} == \overline{\overline{\tau}}$

Out[•]=

False

In[•]:= $\overline{\tau}$

Head[%]

Out[•]=

$\overline{\tau}$

Out[•]=

Symbol

```
In[1]:=  $\overline{\tau}$ 
Head[%]
Out[1]=  $\overline{\tau}$ 
```

```
Out[1]= ParsedBoxWrapper
In[2]:=  $\overline{\tau}$ 
Head[%]
Out[2]=  $\overline{\tau}$ 
Out[2]= Symbol
```

```
In[3]:= Symbol
Out[3]= Symbol
```

```
In[4]:=  $\overline{\tau} == \overline{\tau}$ 
Out[4]= True
```

```
In[5]:=  $\overline{\tau} == \overline{\tau}$ 
Out[5]= False
```

```
In[6]:=  $\eta4488 // MatrixForm$ 
Out[6]//MatrixForm= 
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```

```
In[7]:=  $\eta4488 == \text{DiagonalMatrix}[\text{RotateRight}[\text{values}, 4]]$ 
```

```
Out[7]= True
```

```
In[6]:= DiagonalMatrix[RotateRight[values, 3]] // MatrixForm
```

```
Out[6]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[7]:= τ[0] = ID8;
Table[τ[7 - h] = ArrayFlatten[{{0, t4by4[h]}, {-t4by4[h], 0}}], {h, 1, 3}];
Table[τ[h] = ArrayFlatten[{{0, s4by4[h]}, {s4by4[h], 0}}], {h, 1, 3}];
(τ[7] = τ[1].τ[2].τ[3].τ[4].τ[5].τ[6]) // MatrixForm
```

```
Out[7]//MatrixForm=
```

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[8]:= σ == τ[1].τ[2].τ[3]
```

```
Out[8]=
```

```
True
```

```
In[9]:= Table[{A, B,
  FullSimplify[ExpandAll[ $\frac{1}{2} (\tau[A].\tau[B] + \tau[B].\tau[A]) = -\eta 4488 [A+1, B+1] * ID8]$ ]]},
{A, 1, 7}, {B, 1, 7}]
```

```
Out[9]=
```

```
{ {{1, 1, True}, {1, 2, True}, {1, 3, True}, {1, 4, True},
  {1, 5, True}, {1, 6, True}, {1, 7, True}}, {{2, 1, True}, {2, 2, True},
  {2, 3, True}, {2, 4, True}, {2, 5, True}, {2, 6, True}, {2, 7, True}},
  {{3, 1, True}, {3, 2, True}, {3, 3, True}, {3, 4, True}, {3, 5, True}, {3, 6, True},
  {3, 7, True}}, {{4, 1, True}, {4, 2, True}, {4, 3, True}, {4, 4, True},
  {4, 5, True}, {4, 6, True}, {4, 7, True}}, {{5, 1, True}, {5, 2, True},
  {5, 3, True}, {5, 4, True}, {5, 5, True}, {5, 6, True}, {5, 7, True}},
  {{6, 1, True}, {6, 2, True}, {6, 3, True}, {6, 4, True}, {6, 5, True},
  {6, 6, True}, {6, 7, True}}, {{7, 1, True}, {7, 2, True},
  {7, 3, True}, {7, 4, True}, {7, 5, True}, {7, 6, True}, {7, 7, True}}}
```

In[8]:= $\eta4488 // \text{MatrixForm}$

Out[8]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[9]:= Table[{A, - $\eta4488[A + 1, A + 1] * \tau[A]$ == Transpose[\tau[A]]}, {A, 1, 7}]

Out[9]=

{ {1, True}, {2, True}, {3, True}, {4, True}, {5, True}, {6, True}, {7, True} }

In[10]:= $(\sigma == \tau[1].\tau[2].\tau[3])$

Out[10]=

True

In[11]:= $\sigma.\tau[1].\tau[2].\tau[3] // \text{MatrixForm}$

Out[11]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[12]:= (* $\tau[0] = \text{ID8};$

$\text{Do}[\{\tau[A] = \text{FullSimplify}[\text{ExpandAll}[(\tau[A])]]\}, \{A, 1, 7\}]*$

(* $\text{Do}[\{\tau[A] = \text{FullSimplify}[\text{ExpandAll}[-(\sigma.\tau[1].\tau[2].\tau[3]).\tau[A]]]\}, \{A, 1, 7\}]*$

In[13]:= $(\sigma == \tau[1].\tau[2].\tau[3])$

Out[13]=

True

In[14]:= (* $\text{Symbolize}[\bar{\tau}]*)$

In[15]:= $\bar{\tau}[0] = \text{ID8};$

$\text{Do}[\{\bar{\tau}[A] = \text{FullSimplify}[\text{ExpandAll}[(\sigma.\text{Transpose}[\tau[A]).\sigma])]]\}, \{A, 1, 7\}]$

In[16]:= $\sigma.\bar{\tau}[\#] == \text{Transpose}[\sigma.\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[16]=

{True, True, True, True, True, True, True}

```
In[•]:= Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\tau[A] \cdot \bar{\tau}[B] + \tau[B] \cdot \bar{\tau}[A]) = \eta 4488 [A+1, B+1] * ID8$ ]]}, {A, 0, 7}, {B, 0, 7}] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 0 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 1 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 1 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 2 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 2 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 3 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 3 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 4 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 4 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 5 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 5 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 6 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 6 \\ 7 \\ \text{True} \end{pmatrix} \\ \begin{pmatrix} 7 \\ 0 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 1 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 2 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 3 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 4 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 5 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 6 \\ \text{True} \end{pmatrix} & \begin{pmatrix} 7 \\ 7 \\ \text{True} \end{pmatrix} \end{pmatrix}$$

In[•]:= $\tau[7]$ // MatrixForm

Out[•]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[•]:= $(\Omega = \sigma \cdot \tau[7]) // \text{MatrixForm}$

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:= $\Omega == \tau[4] \cdot \tau[5] \cdot \tau[6]$

Out[•]=

True

In[•]:= $\tau[5] \cdot \tau[6] \cdot \tau[7] // \text{MatrixForm}$

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

In[•]:= $(\sigma \Omega = \sigma \cdot \Omega) // \text{MatrixForm}$

Out[•]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[•]:= $\text{Inverse}[\sigma \Omega] // \text{MatrixForm}$

Out[•]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[•]:= $\tau[1] \cdot \tau[2] \cdot \tau[3] \cdot \tau[4] \cdot \tau[5] \cdot \tau[6] \cdot \tau[7] == \tau[0] == \text{ID8}$

Out[•]=

True

In[1]:= $(\sigma == \tau[1].\tau[2].\tau[3])$

Out[1]=

True

In[2]:= $(\sigma == \tau[4].\tau[5].\tau[6].\tau[7])$

Out[2]=

True

In[3]:= $\tau[1].\tau[2].\tau[3].\tau[\#] == -\text{Transpose}[\tau[1].\tau[2].\tau[3].\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[3]=

{False, True, True, True, True, True, True}

In[4]:= $\sigma.\bar{\tau}[\#] == \text{Transpose}[\sigma.\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[4]=

{True, True, True, True, True, True, True}

In[5]:= $\tau[5].\tau[6].\tau[7].\bar{\tau}[\#] == \text{Transpose}[\tau[5].\tau[6].\tau[7].\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[5]=

{False, True, True, True, False, True, True}

In[6]:= $\tau[4].\tau[\#] == \text{Transpose}[\tau[4].\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[6]=

{True, True, True, True, False, False, False}

In[7]:= $\tau[4].\bar{\tau}[\#] == \text{Transpose}[\tau[4].\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[7]=

{True, False, False, False, False, True, True}

In[8]:= $\tau[4].\tau[\#] == \text{Transpose}[\tau[4].\tau[\#]] \& /@ \text{Range}[0, 7]$

Out[8]=

{True, True, True, True, False, False, False}

$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}. \quad ; \text{ O } (4, 4) : \text{ SAB } ;$$

covariantDiffMatrix = T16^A[5].T16^A[6].T16^A[7]

$$S = \begin{pmatrix} a & b \\ c & d \end{pmatrix};$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix} \cdot \begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix} = \begin{pmatrix} b \cdot \tau^A & a \cdot \bar{\tau}^A \\ d \cdot \tau^A & c \cdot \bar{\tau}^A \end{pmatrix} \cdot \begin{pmatrix} \tilde{a} & \tilde{c} \\ \tilde{b} & \tilde{d} \end{pmatrix}$$

$$= \begin{pmatrix} b.\tau^A.\tilde{a} + a.\bar{\tau}^A.\tilde{b} & b.\tau^A.\tilde{c} + a.\bar{\tau}^A.\tilde{d} \\ d.\tau^A.\tilde{a} + c.\bar{\tau}^A.\tilde{b} & d.\tau^A.\tilde{c} + c.\bar{\tau}^A.\tilde{d} \end{pmatrix}$$

Need these in order to define the Lagrangian for the "universe," later :

```
In[•]:= (σ16 = T16A[0].T16A[1].T16A[2].T16A[3]) // MatrixForm
```

```
Out[•]//MatrixForm=
T16A[0].T16A[1].T16A[2].T16A[3]
```

```
In[•]:= Table[T16A[A1] = ArrayFlatten[{{0, τ[A1]}, {τ[A1], 0}}], {A1, 0, 7}];
```

```
In[•]:= (T16A[8] = T16A[0].T16A[1].T16A[2].T16A[3].T16A[4].T16A[5].T16A[6].T16A[7]) // MatrixForm
```

```
Out[•]//MatrixForm=
{{-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1}}
```

```
In[•]:= σ16 // MatrixForm
```

```
Out[•]//MatrixForm=
{{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0}}
```

```
In[®]:= T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#] ==
-Transpose[T16^A[0].T16^A[1].T16^A[2].T16^A[3].T16^A[#]] & /@ Range[0, 8]
```

```
Out[8]= {True, True, True, True, True, True, True, True, False}
```

```
In[®]:= T16A[4].T16A[5].T16A[6].T16A[7].T16A[#] ==
          Transpose[T16A[4].T16A[5].T16A[6].T16A[7].T16A[#]] & /@ Range[0, 8]
```

```
Out[6]= {True, True, True, True, True, True, True, True, True}
```

```
In[•]:= T16A[8] == σ16.T16A[4].T16A[5].T16A[6].T16A[7]
```

Out[•]=

In[•]:= **T16^A**[#] == Transpose[T16^A[#]] & /@ Range[0, 8]

```
Out[•]= {True, True, True, True, False, False, False, False, True}
```

```
In[•]:= T16A[#] == -Transpose[T16A[#]] & /@ Range[0, 8]
```

```
Out[1]= {False, False, False, False, True, True, True, True, True, False}
```

```
In[•]:= σ16.T16A[#] == -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

```
Out[•]= {True, True, True, True, True, True, True, True}
```

```
In[•]:= σ16.covariantDiffMatrix == Transpose[σ16.covariantDiffMatrix]
```

```
Out[•]=
```

```
True
```

```
In[•]:= ID16 // MatrixForm
```

```
Out[•]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[•]:= (σ16(*=T16^A[0].T16^A[1].T16^A[2].T16^A[3]*)) // MatrixForm
```

```
Out[•]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[•]:= σ16 == ArrayFlatten[{{{-σ, 0}, {0, σ}}}]
```

```
Out[•]=
```

```
True
```

later, after defining $e_{(A)}^\alpha$:

```
In[•]:= (*Table[T16^α[A1-1] = Sum[(e_{(A)}^\alpha [[α1,A1]] ) T16^A[A1-1], {A1,1,8}], {α1,1,8}];*)
```

```
In[8]:= (* (T16α[8]=T16α[0].T16α[1].T16α[2].T16α[3].T16α[4].T16α[5].T16α[6].T16α[7]) // MatrixForm*)

In[9]:= (*eLpairs*)

In[10]:= (*Ω16a=Array[0&, {16, 16}]; {1+#[[1]], 1+#[[2]]}&/@eLpairs
(Ω16a[[#[[1]], #[[2]]]=1)&/@%*)

In[11]:= (*Ω16a//MatrixForm*)

In[12]:= (* (Ω16=σ16. (Ω16a+Transpose[Ω16a])) //MatrixForm*)
```



```
In[•]:= σ16.Ω16 // MatrixForm
```

```

Out[•]//MatrixForm=
( 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 )

```

```
In[•]:= σ16.T16A[#] == -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

Out[•]:= {True, True, True, True, True, True, True, True, True}

```
In[•]:= σ16.T16A[#] == -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

Out[\oplus] = {True, True, True, True, True, True, True, True}

```
In[•]:= σ16.T16A[#] == -Transpose[σ16.T16A[#]] & /@ Range[0, 7]
```

```
Out[8]= {True, True, True, True, True, True, True, True}
```

```
In[•]:= σ16.newT16A[#] == -Transpose[σ16.newT16A[#]] & /@ Range[0, 7]
```



```

{0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0} } .newT16A[6] ==
- ({{0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, ,
{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0,
0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0},
{-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0,
-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0} } .newT16A[6])T

{{0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0,
0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0} } .newT16A[7] ==
- ({{0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, ,
{0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0,
0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0},
{-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0,
-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0} } .newT16A[7])T

```

```
In[8]:= newσ16.newT16A[#] == -Transpose[newσ16.newT16A[#]] & /@ Range[0, 7]
```

```
Out[8]=
{newo16.newT16A[0] == - (newo16.newT16A[0])T, newo16.newT16A[1] == - (newo16.newT16A[1])T,
 newo16.newT16A[2] == - (newo16.newT16A[2])T, newo16.newT16A[3] == - (newo16.newT16A[3])T,
 newo16.newT16A[4] == - (newo16.newT16A[4])T, newo16.newT16A[5] == - (newo16.newT16A[5])T,
 newo16.newT16A[6] == - (newo16.newT16A[6])T, newo16.newT16A[7] == - (newo16.newT16A[7])T}
```

```
In[9]:= Table[
 { {A1, B1}, FullSimplify[ExpandAll[1/2 ( T16A[A1].T16A[B1] + T16A[B1].T16A[A1]) ==
 η4488[[A1+1, B1+1]*ID16]]}], {A1, 0, 7}, {B1, 0, 7}] // MatrixForm
```

```
Out[9]//MatrixForm=
{{{{0, 0}, {0, 1}, {0, 2}, {0, 3}, {0, 4}, {0, 5}, {0, 6}, {0, 7}}, {{True, True, True, True, True, True, True, True}}},
 {{{1, 0}, {1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6}, {1, 7}}, {{True, True, True, True, True, True, True, True}}},
 {{{2, 0}, {2, 1}, {2, 2}, {2, 3}, {2, 4}, {2, 5}, {2, 6}, {2, 7}}, {{True, True, True, True, True, True, True, True}}},
 {{{3, 0}, {3, 1}, {3, 2}, {3, 3}, {3, 4}, {3, 5}, {3, 6}, {3, 7}}, {{True, True, True, True, True, True, True, True}}},
 {{{4, 0}, {4, 1}, {4, 2}, {4, 3}, {4, 4}, {4, 5}, {4, 6}, {4, 7}}, {{True, True, True, True, True, True, True, True}}},
 {{{5, 0}, {5, 1}, {5, 2}, {5, 3}, {5, 4}, {5, 5}, {5, 6}, {5, 7}}, {{True, True, True, True, True, True, True, True}}},
 {{{6, 0}, {6, 1}, {6, 2}, {6, 3}, {6, 4}, {6, 5}, {6, 6}, {6, 7}}, {{True, True, True, True, True, True, True, True}}},
 {{{7, 0}, {7, 1}, {7, 2}, {7, 3}, {7, 4}, {7, 5}, {7, 6}, {7, 7}}, {{True, True, True, True, True, True, True, True}}}}
```

```
In[8]:= Table[{{A1, B1}, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\text{newT16}^A[A1].\text{newT16}^A[B1] + \text{newT16}^A[B1].\text{newT16}^A[A1]) =$ 
   $\eta 4488 [[A1 + 1, B1 + 1]] * \text{ID16}]]}, {A1, 0, 7}, {B1, 0, 7}] // MatrixForm$ 
```

Out[8]//MatrixForm=

$$\begin{pmatrix} \text{newT16}^A[0]^2 = \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \frac{1}{2} (\text{newT16}^A[0].\text{newT16}^A[1] + \text{newT16}^A[1].\text{newT16}^A[0]) = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \frac{1}{2} (\text{newT16}^A[0].\text{newT16}^A[2] + \text{newT16}^A[2].\text{newT16}^A[0]) = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \frac{1}{2} (\text{newT16}^A[0].\text{newT16}^A[3] + \text{newT16}^A[3].\text{newT16}^A[0]) = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \frac{1}{2} (\text{newT16}^A[0].\text{newT16}^A[4] + \text{newT16}^A[4].\text{newT16}^A[0]) = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \frac{1}{2} (\text{newT16}^A[0].\text{newT16}^A[5] + \text{newT16}^A[5].\text{newT16}^A[0]) = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \frac{1}{2} (\text{newT16}^A[0].\text{newT16}^A[6] + \text{newT16}^A[6].\text{newT16}^A[0]) = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ \frac{1}{2} (\text{newT16}^A[0].\text{newT16}^A[7] + \text{newT16}^A[7].\text{newT16}^A[0]) = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \end{pmatrix}$$

In[9]:= **P16upper** = Take[**P16**, 8]
P16lower = Take[**P16**, -8]
P16 == Flatten[{**P16upper**, **P16lower**}]

Out[9]= {f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4]}

Out[10]= {f16[8][x0, x4], f16[9][x0, x4], f16[10][x0, x4], f16[11][x0, x4],
 f16[12][x0, x4], f16[13][x0, x4], f16[14][x0, x4], f16[15][x0, x4]}

Out[11]= True

$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$

```
Table[T16^A[A1] = ArrayFlatten[{{(0, \tau[A1]), (\tau[A1], 0)}}, {A1, 0, 7}],  

  \sigma16 = ArrayFlatten[{{(-\sigma, 0), (0, \sigma)}},  

  {{0, \tau[A1]}, {\tau[A1], 0}}] \cdot {{P16upper}}, {{\tau[A1], P16lower}} = {{\tau[A1], P16upper}}
```

```
In[•]:= (*σ16.T16A[#]&/@Range[0,7]*)
```

```
In[6]:= (*
(T16^A[8]=FullSimplify[T16^A[0].T16^A[1].T16^A[2].
T16^A[3].T16^A[4].T16^A[5].T16^A[6].T16^A[7]])//MatrixForm*)
```

In[●]:= **η4488**

```
Out[•]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, -1}}
```

```
In[•]:= η99 = DiagonalMatrix[{1, 1, 1, 1, -1, -1, -1, -1, -1, 1}]
```

```
Out[•]= {{1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

```
In[8]:= (*Table[{A1,B1,FullSimplify[
ExpandAll[1/2 (T16^A[A1].T16^A[B1]+
{A1,0,8},{B1,0,8}]]//MatrixForm*})
```

```
In[8]:= (*Table[(T16^A[A1].T16^A[B1]+T16^A[B1].T16^A[A1]),{A1,8},{B1,8}]*)
```

$$In[6]:= \left(P_L = \frac{1}{2} (ID16 - T16^A[8]) \right) // MatrixForm$$

```
In[6]:= P_R = 1/2 (ID16 + T16^A[8]) // MatrixForm
```

```
In[•]:= {ID16 == PL + PR, PL.PL == PL, PL.PR == PR.PL == ZERO16}
```

Out[\bullet] = {True, True, True}

$$SAB = \text{Table}\left[\frac{1}{4} \left(T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]\right), \{A1, 0, 7\}, \{B1, 0, 7\}\right];$$

$$SAB = \text{Table}\left[\frac{1}{4} \left(T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]\right), \{A1, 0, 7\}, \{B1, 0, 7\}\right];$$

```
In[8]:= ParallelTable[FullSimplify[\sigma16.SAB[[A1, B1]] == -Transpose[\sigma16.SAB[[A1, B1]]]], {A1, 1, 8}, {B1, 1, 8}] // Flatten // Union
```

Out[•]=

{ True }

```
In[8]:= ParallelTable[FullSimplify[SAB[[A1, B1]].SAB[[A2, B2]] - SAB[[A2, B2]].SAB[[A1, B1]] ==  
- (η4488[[A1, A2]] × SAB[[B1, B2]] - η4488[[A1, B2]] × SAB[[B1, A2]] -  
η4488[[B1, A2]] × SAB[[A1, B2]] + η4488[[B1, B2]] × SAB[[A1, A2]])],  
{A1, 1, 7}, {B1, A1 + 1, 8}, {A2, 1, 7}, {B2, A2 + 1, 8}] // Flatten // Union
```

Out[•]=

{ True }

```
In[•]:= ParallelTable[FullSimplify[SAB[[A1, B1]].T16^A[B2 - 1] - T16^A[B2 - 1].SAB[[A1, B1]] ==  
(-η4488[[B2, A1]] T16^A[B1 - 1] + η4488[[B2, B1]] T16^A[A1 - 1])],  
{A1, 1, 8}, {B1, 1, 8}, {B2, 1, 8}] // Flatten // Union
```

Out[•]=
{True}

```
In[•]:= (*ParallelTable[  
{ {A1,B1},FullSimplify[\sigma16.SAB[[A1,B1]]==Transpose[\sigma16.SAB[[A1,B1]]]] },  
{A1,1, 8},{B1,1,8}]*)
```

```
In[•]:= (*ParallelTable[  
{ {A1,B1,A2,B2},FullSimplify[SAB[[A1,B1]].SAB[[A2,B2]] -SAB[[A2,B2]].SAB[[A1,B1]] ==  
-(η4488[[A1,A2]]SAB[[B1,B2]]-η4488[[A1,B2]]SAB[[B1,A2]] -  
η4488[[B1,A2]]SAB[[A1,B2]]+η4488[[B1,B2]]SAB[[A1,A2]]) },  
{A1,1, 7},{B1,A1+1,8},{A2,1, 7},{B2,A2+1,8}]*)
```

```
In[•]:= (*ParallelTable[  
{ {A1,B1,B2},FullSimplify[SAB[[A1,B1]].T16^A[B2-1] -T16^A[B2-1].SAB[[A1,B1]] ==  
(-η4488[[B2,A1]]T16^A[B1-1]+η4488[[B2,B1]]T16^A[A1-1]) }],  
{A1,1, 8},{B1,1,8},{B2,1,8}]*)
```

```
In[•]:= Do[sAB[A1][B1] =  $\frac{1}{4}$  (T16^A[A1].T16^A[B1] - T16^A[B1].T16^A[A1]),  
{A1, 0, 7}, {B1, 0, 7}];
```

```
In[•]:= Dimensions[ArrayFlatten[{{0, ID8}, {ID8, 0}}]]  
Out[•]= {16, 16}
```

```
In[•]:= sAB[-1 + 1] [-1 + 2] == SAB[[1, 2]]  
Out[•]= True
```

```
In[•]:= σ16.SAB[[1, 2]] // MatrixForm
```

Out[•]//MatrixForm=

0	0	0	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	$-\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	0	0	0	0	0
0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	0	0	0	0	0	0	0
0	0	0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{2}$	0	0	0	0

```
In[•]:= σ16.sAB[1][2] // MatrixForm
```

Out[•]//MatrixForm=

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula: $\textcircled{8}$

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. $\textcircled{8}$

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The **$\mathfrak{so}(4,4)$** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature $(4, 4)$. $\textcircled{8}$

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) $\textcircled{8}$

Step 2: State the general commutation relation

The commutation relations for the generators of the **$\mathfrak{so}(p,q)$** Lie algebra are a generalization of the **$\mathfrak{so}(n)$** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. $\textcircled{8}$

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **$\mathfrak{so}(4,4)$** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a $(4,4)$ signature (four plus signs and four minus signs, or vice versa). $\textcircled{8}$

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). $\textcircled{8}$

what are the commutation relation for the generators of $so(4,4)$?

The commutation relations for the generators $J_{\mu\nu}$ of the $so(4,4)$ Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: ⓘ

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for $so(8)$ in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



↳ Why is the metric signature significant for $so(4,4)$ commutation relations?

Why is the metric signature significant for $so(4,4)$ commutation relations?

The metric signature is significant for the $so(p,q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). ⓘ

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p,q)$ is given by:
$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$
The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4,4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[•]:= **η4488**

Out[•] =

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},  

{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},  

{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1} }
```

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

```
In[6]:= SAB[[6, 2]].SAB[[6, 7]] - SAB[[6, 7]].SAB[[6, 2]] == \eta4488[[6, 6]] \times SAB[[7, 2]]
```

Out[•]=

True

In[•]:= { {6, 2, 6, 7}, False}

Out[•]=

```
{ {6, 2, 6, 7}, False }
```

```
In[6]:= SAB[[1, 2]].SAB[[1, 3]] - SAB[[1, 3]].SAB[[1, 2]] == \eta4488[[1, 1]] \times SAB[[2, 3]]
```

Out[•]=

False

In[•]:= **sAB**[θ][θ]

Out[•]=


```
In[1]:= Table[Transpose[T16^A[7].sAB[A1][B1]] == T16^A[7].sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[1]= {{True, True, True, True, False, False, True},
{True, True, True, True, False, False, True},
{True, True, True, True, False, False, True},
{True, True, True, True, False, False, True},
{False, False, False, False, True, True, False},
{False, False, False, False, True, True, False},
{False, False, False, False, True, True, False},
{True, True, True, True, False, False, True}]

In[2]:= Table[Transpose[s16.sAB[A1][B1]] == s16.sAB[A1][B1], {A1, 0, 7}, {B1, 0, 7}]
Out[2]= {{True, False, False, False, False, False, False},
{False, True, False, False, False, False, False},
{False, False, True, False, False, False, False},
{False, False, False, True, False, False, False},
{False, False, False, False, True, False, False},
{False, False, False, False, False, True, False},
{False, False, False, False, False, True, False},
{False, False, False, False, False, False, True}]

In[3]:= Table[Transpose[s16.SAB[[A1, B1]]] == s16.SAB[[A1, B1]], {A1, 1, 8}, {B1, 1, 8}]
Out[3]= {{True, False, False, False, False, False, False, False},
{False, True, False, False, False, False, False, False},
{False, False, True, False, False, False, False, False},
{False, False, False, True, False, False, False, False},
{False, False, False, False, True, False, False, False},
{False, False, False, False, False, True, False, False},
{False, False, False, False, False, False, True, False},
{False, False, False, False, False, False, False, True}]

In[4]:= SAB[[1, 2]][1]
Out[4]= {0, 0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0}

In[5]:= SAB[[1, 2]][2]
Out[5]= {0, 0, 0, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0}


$$t^A = \begin{pmatrix} 0 & \bar{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$


In[6]:= SAB1 = Table[Take[SAB[[A1, B1]][C1]], 8], {A1, 8}, {B1, 8}, {C1, 1, 8}];
```

```
In[8]:= SAB2 = Table[Take[SAB[[A1, B1]][[C1]], -8], {A1, 8}, {B1, 8}, {C1, 9, 16}];

In[9]:= SAB[[1, 2]] == ArrayFlatten[{{SAB1[[1, 2]], 0}, {0, SAB2[[1, 2]]}}]

Out[9]= True
```

```
(*Table[SAB[[A1,B1]]== ArrayFlatten[{{SAB1[[A1,B1]],0},{0,SAB2[[A1,B1]]}}],{A1,0,7},{B1,0,7}]*)
```

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4,4)}$ and $L \in SO(4,4)$. The canonical 2-1 homomorphism $\overline{SO(4,4)} \rightarrow SO(4,4)$ is given by

$$(2.8) \quad 8L^{A'}{}_{B'}(g) = \text{tr } \tilde{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4,4)}$

$$(2.9) \quad \tilde{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \tilde{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}{}_{B'} \tau^{B'} = \tilde{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \tilde{D}^{(2)-1}.$$

```
In[10]:= SAB2[[1, 2]]

Out[10]= {{0, 0, 0, 0, 0, 0, 0, -1/2}, {0, 0, 0, 0, 0, 0, 0, -1/2}, {0, 0, 0, 0, 0, 0, 1/2, 0}, {0, 0, 0, 0, 0, 1/2, 0, 0}, {0, 0, 0, -1/2, 0, 0, 0, 0}, {0, 0, -1/2, 0, 0, 0, 0, 0}, {0, 1/2, 0, 0, 0, 0, 0, 0}, {1/2, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[8]:= τ[θ]
Out[8]= {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}
```

```
(*Block[{b,A,F_a^(μ),F_a^a,ψ,Ψ},
b=Array[A,{8}];
ψ[1]=Array[Ψ[1],{8}];
ψ[2]=Array[Ψ[2],{8}];
F_a^(μ)=Table[(1/(Sqrt[ψ[1].σ.ψ[1]])ψ[1].σ.(τ[μL])),{μL,1,8}];
F_a^a=Transpose[Table[(1/(Sqrt[ψ[1].σ.ψ[1]])η4488[[μL,μL]]τ[μL].ψ[1]),{μL,1,8}]];
b==FullSimplify[F_a^(μ).ψ[2]/.Thread[ψ[2]→F_a^a.b]]]*)
```

```
(*Block[{b,A,B,q,F_a^(μ),ψ,φ,Ψ},
b=Array[A,{8}];
ψ[1]=Array[Ψ[1],{8}];
φ[1]=Array[Ψ[2],{8}];
q=ParallelTable[FullSimplify[
(1/(Sqrt[ψ[1].σ.ψ[1]])ψ[1].σ.1/2(τ[A1].τ[B1]-τ[B1].τ[A1]).φ[1])],
{A1,1,8},{B1,1,8}];
B=FullSimplify[
(1/(Sqrt[ψ[1].σ.ψ[1]])ParallelSum[η4488[[μL,μL]]τ[μL]b[[μL]],[μL,1,8]].ψ[1])];
FullSimplify[q/.Thread[φ[1]→B]]]*)
```

```
(*Block[{b,A,B,q,F_a^(μ),ψ,φ,Ψ},
b=Array[A,{8}];
ψ[1]=Array[Ψ[1],{8}];
φ[1]=Array[Ψ[2],{8}];
q=
ParallelTable[FullSimplify[(ψ[1].σ.(τ[A1].τ[B1]).φ[1])],{A1,1,8},{B1,1,8}];
B=FullSimplify[(ParallelSum[η4488[[μL,μL]]τ[μL]b[[μL]],[μL,1,8]].ψ[1])];
FullSimplify[(1/(ψ[1].σ.ψ[1]))q/.Thread[φ[1]→B]]]*)
```

T16 $^\alpha$ In[•]:= **ssgm4488**

Out[•]=

$$\begin{aligned} \{ & g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ & g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ & g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ & g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ & g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&) \} \end{aligned}$$

In[•]:= **e $^\alpha_{(A)}$ /. sge $^\alpha_{(A)}$**

Out[•]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{g[0][0][x0, x4]}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left\{ 0, 0, \frac{1}{\sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, 0, 0, \frac{1}{\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \\ & \left. \left\{ 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{-g[7][7][x0, x4]}} \right\} \right\} \end{aligned}$$

In[•]:= **Table[T16 $^\alpha[\alpha1 - 1] = \text{Sum}[(e $^\alpha_{(A)} / . \text{sge}_{(A)}^\alpha) [[\alpha1, A1]], T16^A[A1 - 1], {A1, 1, 8}], \{\alpha1, 1, 8\}]$; (*./ssgm4488*)$** In[•]:= **(T16 $^\alpha[8] = T16^\alpha[0].T16^\alpha[1].T16^\alpha[2].T16^\alpha[3].T16^\alpha[4].T16^\alpha[5].T16^\alpha[6].T16^\alpha[7])$;**In[•]:= **(*T16 $^\alpha[8] == T16^A[8]*\text{Sec}[6 H x0]*)$**

CHECK **Table[T16 $^\alpha[\alpha1 - 1] = \text{Sum}[(e $^\alpha_{(A)} [[\alpha1, A1]], T16^A[A1 - 1], {A1, 1, 8}], \{\alpha1, 1, 8\}]$, and ● ● ● :$**

In[•]:= **(*Table[T16 $^\alpha[\alpha1 - 1] = \text{Sum}[(e $^\alpha_{(A)} [[\alpha1, A1]], T16^A[A1 - 1], {A1, 1, 8}], \{\alpha1, 1, 8\}]*)$$**

```
In[•]:= (* (T16α[8]=T16α[0].T16α[1].T16α[2].T16α[3].T16α[4].T16α[5].T16α[6].T16α[7]) // MatrixForm*)
```

```
In[8]:= Table[
  { {A1, B1}, FullSimplify[ ExpandAll[  $\frac{1}{2} (T16^A[A1] \cdot T16^A[B1] + T16^A[B1] \cdot T16^A[A1]) ==$ 
     $\eta 4488 [A1 + 1, B1 + 1] * ID16]$  ] }, {A1, 0, 7}, {B1, 0, 7} ] // MatrixForm
```

```
Out[•]//MatrixForm=
( {0, 0} ) ( {0, 1} ) ( {0, 2} ) ( {0, 3} ) ( {0, 4} ) ( {0, 5} ) ( {0, 6} ) ( {0, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {1, 0} ) ( {1, 1} ) ( {1, 2} ) ( {1, 3} ) ( {1, 4} ) ( {1, 5} ) ( {1, 6} ) ( {1, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {2, 0} ) ( {2, 1} ) ( {2, 2} ) ( {2, 3} ) ( {2, 4} ) ( {2, 5} ) ( {2, 6} ) ( {2, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {3, 0} ) ( {3, 1} ) ( {3, 2} ) ( {3, 3} ) ( {3, 4} ) ( {3, 5} ) ( {3, 6} ) ( {3, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {4, 0} ) ( {4, 1} ) ( {4, 2} ) ( {4, 3} ) ( {4, 4} ) ( {4, 5} ) ( {4, 6} ) ( {4, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {5, 0} ) ( {5, 1} ) ( {5, 2} ) ( {5, 3} ) ( {5, 4} ) ( {5, 5} ) ( {5, 6} ) ( {5, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {6, 0} ) ( {6, 1} ) ( {6, 2} ) ( {6, 3} ) ( {6, 4} ) ( {6, 5} ) ( {6, 6} ) ( {6, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
( {7, 0} ) ( {7, 1} ) ( {7, 2} ) ( {7, 3} ) ( {7, 4} ) ( {7, 5} ) ( {7, 6} ) ( {7, 7} )
( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True ) ( True )
```

```
In[•]:= Table[{{α1, β1}, FullSimplify[
  1 - FullSimplify[((T16^α[α1].T16^α[β1] + T16^α[β1].T16^α[α1]) /. ssgm4488),
  constraintVars] == Inverse[MatrixMetric44][α1 + 1, β1 + 1] * ID16]}, {{α1, 0, 7}, {β1, 0, 7}}] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \{0, 0\} & \{0, 1\} & \{0, 2\} & \{0, 3\} & \{0, 4\} & \{0, 5\} & \{0, 6\} & \{0, 7\} \\ \text{True} & \text{True} \\ \{1, 0\} & \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} & \{1, 5\} & \{1, 6\} & \{1, 7\} \\ \text{True} & \text{True} \\ \{2, 0\} & \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} & \{2, 5\} & \{2, 6\} & \{2, 7\} \\ \text{True} & \text{True} \\ \{3, 0\} & \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} & \{3, 5\} & \{3, 6\} & \{3, 7\} \\ \text{True} & \text{True} \\ \{4, 0\} & \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} & \{4, 5\} & \{4, 6\} & \{4, 7\} \\ \text{True} & \text{True} \\ \{5, 0\} & \{5, 1\} & \{5, 2\} & \{5, 3\} & \{5, 4\} & \{5, 5\} & \{5, 6\} & \{5, 7\} \\ \text{True} & \text{True} \\ \{6, 0\} & \{6, 1\} & \{6, 2\} & \{6, 3\} & \{6, 4\} & \{6, 5\} & \{6, 6\} & \{6, 7\} \\ \text{True} & \text{True} \\ \{7, 0\} & \{7, 1\} & \{7, 2\} & \{7, 3\} & \{7, 4\} & \{7, 5\} & \{7, 6\} & \{7, 7\} \\ \text{True} & \text{True} \end{pmatrix}$$

In[•]:= MatrixMetric44 // MatrixForm

Out[•]//MatrixForm=

$$\begin{pmatrix} \cot[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} - e^{-2a4[Hx4]}$$

In[•]:= ((e^α_(A) /. sgtrye^α_(A)).MatrixMetric44.(e^α_(A) /. sgtrye^α_(A))) // MatrixForm

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[]:= Transpose[(e(A) /. sgtrye(A))].η4488.(e(A) /. sgtrye(A)) - MatrixMetric44 // FullSimplify[#, constraintVars] &

Out[]:= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[]:= Transpose[(e(A) /. sgtrye(A))].MatrixMetric44.(e(A) /. sgtrye(A)) - η4488 // (*Full*)Simplify[#, constraintVars] &

Out[]:= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[]:= Inverse[(e(A) /. sgtrye(A))] == (e(A) /. sgtrye(A))
```

```
Out[]:= True
```

```
In[]:= Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]
```

```
Out[]:= {{0, 0}, {0, 0}}
```

BELLOW : Lagrangian_{¶16} =

$\kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi16].\sigma16.T16^A[A1].\Psi16_{|\alpha} g^{-1}{}^{\alpha\beta} \eta_{A1B1} E_\beta^{B1} + \text{mass term}$

```
In[]:= e(A) /. sgtrye(A)
```

```
Out[]:= {{Cot[6 H x 0], 0, 0, 0, 0, 0, 0, 0}, {0, Sqrt[e2 a4[H x 4]] Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, Sqrt[e2 a4[H x 4]] Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, Sqrt[e-2 a4[H x 4]] Sin[6 H x 0]1/6, 0, 0}, {0, 0, 0, 0, 0, 0, Sqrt[e-2 a4[H x 4]] Sin[6 H x 0]1/6, 0}, {0, 0, 0, 0, 0, 0, 0, Sqrt[e-2 a4[H x 4]] Sin[6 H x 0]1/6}}
```

```

In[6]:= eα(A) /. sgtryeα(A)

Out[6]=
{ { Tan[6 H x0], 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1/(Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/6)]}, 0, 0, 0, 0, 0, 0}, {0, 0, 1/(Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/6)]}, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1/(Sqrt[e^(2 a4[H x4]) Sin[6 H x0]^(1/6)]}, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1/(Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)]}, 0, 0}, {0, 0, 0, 0, 0, 0, 1/(Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)]}, 0}, {0, 0, 0, 0, 0, 0, 0, 1/(Sqrt[e^(-2 a4[H x4]) Sin[6 H x0]^(1/6)]})} }

In[7]:= eα(A) /. sgeα(A)

Out[7]=
{ { 1/Sqrt[g[0][0][x0, x4]], 0, 0, 0, 0, 0, 0, 0}, {0, 1/Sqrt[g[1][1][x0, x4]], 0, 0, 0, 0, 0, 0}, {0, 0, 1/Sqrt[g[2][2][x0, x4]], 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1/Sqrt[g[3][3][x0, x4]], 0, 0, 0, 0}, {0, 0, 0, 0, 1/Sqrt[-g[4][4][x0, x4]], 0, 0, 0}, {0, 0, 0, 0, 0, 1/Sqrt[-g[5][5][x0, x4]], 0, 0}, {0, 0, 0, 0, 0, 0, 1/Sqrt[-g[6][6][x0, x4]], 0}, {0, 0, 0, 0, 0, 0, 0, 1/Sqrt[-g[7][7][x0, x4]]} }

```

```

In[8]:= Clear[con];
con[g_, ass_ : {}] := Module[{t, ΓΓ, ginv},
  ginv = FullSimplify[Inverse[g], ass];
  ΓΓ = FullSimplify[Table[(1/2) * Sum[(ginv[[i, s]] * (D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}], {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];
  Return[{ginv, ΓΓ}]];

```

In[•]:= **ssgm4488**

Out[•]=

$$\begin{aligned} & \{g[0][0] \rightarrow (\text{Cot}[6H\#1]^2 \&), g[0][1] \rightarrow (0 \&), g[0][2] \rightarrow (0 \&), g[0][3] \rightarrow (0 \&), \\ & g[0][4] \rightarrow (0 \&), g[0][5] \rightarrow (0 \&), g[0][6] \rightarrow (0 \&), g[0][7] \rightarrow (0 \&), g[1][0] \rightarrow (0 \&), \\ & g[1][1] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[1][2] \rightarrow (0 \&), g[1][3] \rightarrow (0 \&), \\ & g[1][4] \rightarrow (0 \&), g[1][5] \rightarrow (0 \&), g[1][6] \rightarrow (0 \&), g[1][7] \rightarrow (0 \&), g[2][0] \rightarrow (0 \&), \\ & g[2][1] \rightarrow (0 \&), g[2][2] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[2][3] \rightarrow (0 \&), g[2][4] \rightarrow (0 \&), \\ & g[2][5] \rightarrow (0 \&), g[2][6] \rightarrow (0 \&), g[2][7] \rightarrow (0 \&), g[3][0] \rightarrow (0 \&), g[3][1] \rightarrow (0 \&), \\ & g[3][2] \rightarrow (0 \&), g[3][3] \rightarrow (e^{2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[3][4] \rightarrow (0 \&), g[3][5] \rightarrow (0 \&), \\ & g[3][6] \rightarrow (0 \&), g[3][7] \rightarrow (0 \&), g[4][0] \rightarrow (0 \&), g[4][1] \rightarrow (0 \&), g[4][2] \rightarrow (0 \&), \\ & g[4][3] \rightarrow (0 \&), g[4][4] \rightarrow (-1 \&), g[4][5] \rightarrow (0 \&), g[4][6] \rightarrow (0 \&), g[4][7] \rightarrow (0 \&), \\ & g[5][0] \rightarrow (0 \&), g[5][1] \rightarrow (0 \&), g[5][2] \rightarrow (0 \&), g[5][3] \rightarrow (0 \&), g[5][4] \rightarrow (0 \&), \\ & g[5][5] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[5][6] \rightarrow (0 \&), g[5][7] \rightarrow (0 \&), \\ & g[6][0] \rightarrow (0 \&), g[6][1] \rightarrow (0 \&), g[6][2] \rightarrow (0 \&), g[6][3] \rightarrow (0 \&), g[6][4] \rightarrow (0 \&), \\ & g[6][5] \rightarrow (0 \&), g[6][6] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&), g[6][7] \rightarrow (0 \&), \\ & g[7][0] \rightarrow (0 \&), g[7][1] \rightarrow (0 \&), g[7][2] \rightarrow (0 \&), g[7][3] \rightarrow (0 \&), g[7][4] \rightarrow (0 \&), \\ & g[7][5] \rightarrow (0 \&), g[7][6] \rightarrow (0 \&), g[7][7] \rightarrow (-e^{-2a4[H\#2]} \sin[6H\#1]^{1/3} \&) \} \end{aligned}$$

In[•]:= **g4488**

g4488 / . sg

Out[•]=

$$\begin{aligned} & \{\{g[0][0][x0, x4], g[0][1][x0, x4], g[0][2][x0, x4], g[0][3][x0, x4], \\ & g[0][4][x0, x4], g[0][5][x0, x4], g[0][6][x0, x4], g[0][7][x0, x4]\}, \\ & \{g[1][0][x0, x4], g[1][1][x0, x4], g[1][2][x0, x4], g[1][3][x0, x4], \\ & g[1][4][x0, x4], g[1][5][x0, x4], g[1][6][x0, x4], g[1][7][x0, x4]\}, \\ & \{g[2][0][x0, x4], g[2][1][x0, x4], g[2][2][x0, x4], g[2][3][x0, x4], \\ & g[2][4][x0, x4], g[2][5][x0, x4], g[2][6][x0, x4], g[2][7][x0, x4]\}, \\ & \{g[3][0][x0, x4], g[3][1][x0, x4], g[3][2][x0, x4], g[3][3][x0, x4], \\ & g[3][4][x0, x4], g[3][5][x0, x4], g[3][6][x0, x4], g[3][7][x0, x4]\}, \\ & \{g[4][0][x0, x4], g[4][1][x0, x4], g[4][2][x0, x4], g[4][3][x0, x4], \\ & g[4][4][x0, x4], g[4][5][x0, x4], g[4][6][x0, x4], g[4][7][x0, x4]\}, \\ & \{g[5][0][x0, x4], g[5][1][x0, x4], g[5][2][x0, x4], g[5][3][x0, x4], \\ & g[5][4][x0, x4], g[5][5][x0, x4], g[5][6][x0, x4], g[5][7][x0, x4]\}, \\ & \{g[6][0][x0, x4], g[6][1][x0, x4], g[6][2][x0, x4], g[6][3][x0, x4], \\ & g[6][4][x0, x4], g[6][5][x0, x4], g[6][6][x0, x4], g[6][7][x0, x4]\}, \\ & \{g[7][0][x0, x4], g[7][1][x0, x4], g[7][2][x0, x4], g[7][3][x0, x4], \\ & g[7][4][x0, x4], g[7][5][x0, x4], g[7][6][x0, x4], g[7][7][x0, x4]\}\} \end{aligned}$$

Out[•]=

$$\begin{aligned} & \{\{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0, 0\}, \{0, g[1][1][x0, x4], 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, g[2][2][x0, x4], 0, 0, 0, 0, 0\}, \{0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0\}, \\ & \{0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0\}, \{0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0\}, \\ & \{0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0\}, \{0, 0, 0, 0, 0, 0, g[7][7][x0, x4]\}\} \end{aligned}$$

```
In[6]:= MatrixForm[#] & /@ Block[{eA $\alpha$ , e $\alpha$ A, (*g $_{inv}$ , $\Gamma$ ,gg,*) colVecs, rowVecs, ab, ba, r},
  (*gg=g4488/.sg;*)
  (*{g $_{inv}$ , $\Gamma$ }=con[gg];*)
  e $\alpha$ A = e $^\alpha_{(A)}$  /. sge $^\alpha_{(A)}$ ;
  eA $\alpha$  = e $^{\alpha(A)}$  /. sge $^{\alpha(A)}$ ;
  rowVecs = Table[eA $\alpha$ [[b, All]], {b, 1, Length[eA $\alpha$ ] }];
  colVecs = Table[e $\alpha$ A[[All, b]], {b, 1, Length[e $\alpha$ A] }];
  ab = rowVecs.colVecs // FullSimplify[#, constraintVars] &;
  ba = Transpose[colVecs].Transpose[rowVecs] // FullSimplify[#, constraintVars] &;
  (*r={{ab},{ba}};*)
  r = {ab, ba};
  r]
```

Out[6]=

$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

metric compatibility condition for the Octad

$$\nabla_\mu e_\nu^I = \partial_\mu e_\nu^I - \Gamma_{\mu\nu}^\rho e_\rho^I + \omega_{\mu J}^I e_\nu^J = 0$$

```
In[7]:= Clear[spinCoefficients];
spinCoefficients[eIv_] := Module[{evI, w, r, s},
  If[Dimensions[eIv] != {8, 8}, Return[{}]];
  If[Det[eIv] == 0, Return[{}]];
  evI = FullSimplify[Inverse[eIv], constraintVars] /. subsDefects;
  w =
    Table[-Table[FullSimplify[Sum[D[eIv][I1, v1], X][\mu1]] * evI[v1, Jprime]], {v1, 1, Length[evI]}] - Sum[ eIv[I1, \rho] * \Gamma[\rho, \mu1, v1] * evI[v1, Jprime]], {v1, 1, Length[evI]}, {\rho, 1, Length[X]}], constraintVars] /. subsDefects,
  {I1, 1, Length[eIv]}, {Jprime, 1, Length[evI]}], {\mu1, 1, Length[X]}];
  Assert[Dimensions[w] == {8, 8, 8}];
  Return[w];
]
```



```
In[8]:= Table[wμIJ[μ] =  
FullSimplify[-((D[gtryeα(A), X[[μ]]].gtryeα(A)) - (gtryeα(A).Γ[All, μ, All].gtryeα(A))),  
constraintVars] /. subsDefects, {μ, 1, Length[X]}]
```

In[•]:= (*gtrye $_{(A)}^{\alpha}$.gtrye $_{\alpha}^{(A)}$ *)

In[•]:= (*gtrye_α^(A) .gtrye_(A)^α *)

In[•]:= (*gtrye _{α} ^(A) *)

```

In[0]:= (*Γ
Dimensions[%]*)

In[0]:= (*Table[Γ[[All,μ,All]],{μ,1,Length[X]}]*)

In[0]:= (*Γ[[All,5,All]]*)

In[0]:= (*Γ[[All,1,All]]*)

In[0]:= (*spinCoeffs[[1]]*)

In[0]:= (*spinCoeffs[[5]]*)

In[0]:= (*{X[[1]],X[[5]]}*)

In[0]:= (*spinCoeffs[[1]][[1]][[1]]*)

In[0]:= (*ωμIJ[[1]][[1]][[1]]*)

In[0]:= (*Block[{μ=1},
FullSimplify[-((D[gtrye_α^(A),X[[μ]]].gtrye_α^(A))-(gtrye_α^(A).Γ[[All,μ,All]].gtrye_α^(A))) ,
constraintVars]/.subsDefects]
Dimensions[%]*)

In[0]:= (*Block[{μ=1},
FullSimplify[-((D[gtrye_α^(A),X[[μ]]].Transpose[gtrye_α^(A)])-
(gtrye_α^(A).Γ[[All,μ,All]].Transpose[gtrye_α^(A)])),constraintVars]/.subsDefects]*)

In[0]:= (*Block[{μ=1},
FullSimplify[-((Sum[D[gtrye_α^(A)][[All,v]],X[[μ]]]*gtrye_α^(A)[[v,All]] ,{v,1,8}]-(
gtrye_α^(A).Γ[[All,μ,All]].gtrye_α^(A))),constraintVars]/.subsDefects]
Dimensions[%]*)

In[0]:= (*Table[,{A1,1,8},{B1,1,8}]*)

In[0]:= (*Block[{μ=1},Table[
FullSimplify[-((Sum[D[gtrye_α^(A)][[A1,v]],X[[μ]]]*gtrye_α^(A)[[v,B1]] ,{v,1,8}]-(
Sum[gtrye_α^(A)][[A1,o]]*Γ[[o,μ,v]]*gtrye_α^(A)[[v,B1]] ,{o,1,8},{v,1,8})),constraintVars]/.subsDefects,{A1,1,8},{B1,1,8}]]
Dimensions[%]*)

In[0]:= Table[(ωμIJ[μ]-spinCoeffs[[μ]])/.subsDefects , {μ, 1, Length[X]}] // Flatten //
Union

```

Out[0]= {0}

- **Metric compatibility:** The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame. The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is:

$$\circ \quad \nabla_{\mu} e^a{}_{\nu} = 0$$

◦ Here, ∇_{μ} is the covariant derivative with respect to the connection $\Gamma^a{}_{b\nu}$, and $e^a{}_{\nu}$ is the tetrad component.

◦ Metric compatibility : The tetrad formalism uses a tetrad (or vierbein) to connect the curved spacetime to a local flat, Lorentzian frame . The connection coefficients are defined to satisfy the metric compatibility condition for the tetrad, which is :

he "

v=0

- Here, V_{μ} is the covariant derivative with respect to the connection $F^I{}_{b\nu}$, and $e^a{}_{\nu}$ is the tetrad component.

No it is not the torsionless condition which is:

$$T^I_{\mu\nu} = D^{\omega}_{[\mu} e^I_{\nu]} = \partial_{[\mu} e^I_{\nu]} + \omega^I_{[\mu J} e^J_{\nu]} = 0$$

This postulate says:

$$\nabla_{\mu} e^I_{\nu} = \partial_{\mu} e^I_{\nu} - \Gamma^{\rho}_{\mu\nu} e^I_{\rho} + \omega^I_{\mu J} e^J_{\nu} = 0$$

And is merely obtained by expressing a tensor in two different basis then putting the two components equals after returning in the natural basis, it is a sorte of "consistency" condition.

I see the anticommutativity of the spin-connection not implying the metricity but resulting from it! By imposing $D^{\omega}_{\mu} \eta_{IJ} = 0$ (nevertheless, after that it will IMPLY it)

So in the same spirit, I wonder if there is a way to obtain the "tetrad postulate" from a same requirement, say, the metricity of g for example (It will nevertheless IMPLY it once written down)

I think that the confusion in almost all papers I read is the problem, recently I read a paper which pointed out these confusions (An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields arXiv:math-ph/0411085v12 6 Jan 2008) The author said that this postulate comes from the metricity condition but didn't show how (even if he was very rigourous in the mathematical demonstrations, too rigourous at my tast :-))

```
In[6]:= Block[{eA $\alpha$ , e $\alpha$ A, t, ginv,  $\Gamma$ , gg},
  gg = g4488 /. sg;
  {ginv,  $\Gamma$ } = con[gg];
  e $\alpha$ A = e $_{(A)}^{\alpha}$  /. sge $_{(A)}^{\alpha}$ ; eA $\alpha$  = e $_{\alpha}^{(A)}$  /. sge $_{\alpha}^{(A)}$ ;
  t = Table[ Block[{v, V, contra, covari},
    V = eA $\alpha$ [[b, All]];
    v = e $\alpha$ A[[All, b]];
    contra =
      Table[D[v[[k]], X[[ $\mu$ ]]] + Sum[v[[i]]  $\times$   $\Gamma$ [[k, i,  $\mu$ ]], {i, 1, DIM8}], {k, 1, DIM8}];
      (*before  $\omega$  terms*)
    covari = Table[D[V[[k]], X[[ $\mu$ ]]] - Sum[V[[i]]  $\times$   $\Gamma$ [[i, k,  $\mu$ ]], {i, 1, DIM8}], {k, 1, DIM8}];
      (*before  $\omega$  terms*)
    {b - 1, X[[ $\mu$ ]], {{contra}, {covari}}}], {b, 1, Length[e $\alpha$ A]}, { $\mu$ , 1, DIM8}];
  t]
FullSimplify[#, # /. ssgm4488, constraintVars] & /@%
```

Out[6]=

$$\left\{ \left\{ 0, x_0, \left\{ \left\{ 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, -\frac{g[0][0]^{(0,1)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0 \right\} \right\},$$

$$\left\{ 0, x_1, \left\{ \left\{ 0, \frac{g[1][1]^{(1,0)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[1][1][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, \frac{g[1][1]^{(1,0)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[1][1][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\},$$

$$\left\{ 0, x_2, \left\{ \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[2][2][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[2][2][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\},$$

$$\left\{ 0, x_3, \left\{ \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[3][3][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, \frac{g[3][3]^{(1,0)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[3][3][x_0, x_4]}, 0, 0, 0, 0, 0, 0 \right\} \right\},$$

$$\left\{ 0, x_4, \left\{ \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, \frac{g[4][4]^{(1,0)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[4][4][x_0, x_4]}, 0, 0, 0, 0 \right\} \right\},$$

$$\left\{ 0, x_5, \left\{ \left\{ 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x_0, x_4]}{2\sqrt{g[0][0][x_0, x_4]} g[5][5][x_0, x_4]}, 0, 0 \right\} \right\},$$

$$\begin{aligned}
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0, 0 \right\} \right\}, \\
& \left\{ 4, x6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[6][6][x0, x4]}, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]}, 0 \right\} \right\}, \\
& \left\{ 4, x7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[7][7][x0, x4]} \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 g[4][4][x0, x4]} \right\} \right\}, \left\{ 5, x0, \right. \\
& \quad \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(1,0)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]}, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}} - \frac{g[5][5]^{(1,0)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]}, \right. \right. \\
& \quad \left. \left. 0, 0 \right\} \right\}, \left\{ 5, x1, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 5, x2, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 5, x3, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 5, x4, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[5][5]^{(0,1)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]}, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left. \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}} - \frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0 \right\} \right\}, \left\{ 5, x5, \left\{ \left\{ -\frac{g[5][5]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[5][5][x0, x4]}}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ -\frac{g[5][5]^{(1,0)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]}, 0, 0, 0, \right. \right. \\
& \quad \left. \left. \left. -\frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2 g[5][5][x0, x4]}, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 5, x6, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \right. \\
& \quad \left. \left\{ 5, x7, \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}, \left\{ 6, x0, \right. \right. \\
& \quad \left. \left. \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0 \right\} \right\}, \right. \\
& \quad \left. \left. \left\{ 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]}, \right. \right. \right. \\
& \quad \left. \left. \left. 0, 0 \right\} \right\} \right\}
\end{aligned}$$

$\{0\}\}\}\}, \{6, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
 $\{6, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
 $\{6, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{6, x4,$
 $\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0\right\}\right\},$
 $\left\{\left\{0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}} - \frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, 2\sqrt{-g[6][6][x0, x4]}\right\}\right\},$
 $\{0\}\}\}\}, \{6, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
 $\{6, x6, \left\{\left\{-\frac{g[6][6]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4]\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0,\right.\right.$
 $\left.-\frac{g[6][6]^{(0,1)}[x0, x4]}{2g[4][4][x0, x4]\sqrt{-g[6][6][x0, x4]}}, 0, 0, 0\right\}\},$
 $\left\{\left\{-\frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, 0, 0, 0,\right.\right.$
 $\left.-\frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2g[6][6][x0, x4]}, 0, 0, 0\right\}\}\},$
 $\{6, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \{7, x0,$
 $\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(1,0)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]}\right\}\right\},$
 $\{0, 0, 0, 0, 0, 0, 0,\}$
 $\left.-\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]}\right\}\}\},$
 $\{7, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
 $\{7, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
 $\{7, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \{7, x4,$
 $\left\{\left\{0, 0, 0, 0, 0, 0, 0, \frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]}\right\}\right\},$
 $\{0, 0, 0, 0, 0, 0, 0,\}$
 $\left.-\frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}} - \frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2g[7][7][x0, x4]}\right\}\}\},$
 $\{7, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
 $\{7, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},$
 $\{7, x7, \left\{\left\{-\frac{g[7][7]^{(1,0)}[x0, x4]}{2g[0][0][x0, x4]\sqrt{-g[7][7][x0, x4]}},$


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{3, x1, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {3, x2, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {3, x3, {{\left\{-\sqrt{e^{2 a4[H x4]}} H \text{Sec}[6 H x0] \text{Sin}[6 H x0]\right\}^{7/6}}, {0, 0, 0, \sqrt{e^{2 a4[H x4]}} H \text{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0}}}, {\left\{-\frac{\sqrt{e^{2 a4[H x4]}} H \text{Cos}[6 H x0]}{\text{Sin}[6 H x0]^{5/6}}, 0, 0, 0, -\sqrt{e^{2 a4[H x4]}} H \text{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\right\}}}}, {3, x4, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {3, x5, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {3, x6, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {3, x7, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {4, x0, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {4, x1, {{\{0, H a4'[H x4], 0, 0, 0, 0, 0, 0\}}, {\{0, -e^{2 a4[H x4]} H \text{Sin}[6 H x0]^{1/3} a4'[H x4], 0, 0, 0, 0, 0, 0\}}}}, {4, x2, {{\{0, 0, H a4'[H x4], 0, 0, 0, 0, 0\}}, {\{0, 0, -e^{2 a4[H x4]} H \text{Sin}[6 H x0]^{1/3} a4'[H x4], 0, 0, 0, 0, 0, 0\}}}}, {4, x3, {{\{0, 0, 0, H a4'[H x4], 0, 0, 0, 0\}}, {\{0, 0, 0, -e^{2 a4[H x4]} H \text{Sin}[6 H x0]^{1/3} a4'[H x4], 0, 0, 0, 0, 0, 0\}}}}, {4, x4, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {4, x5, {{\{0, 0, 0, 0, 0, -H a4'[H x4], 0, 0\}}, {\{0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \text{Sin}[6 H x0]^{1/3} a4'[H x4], 0, 0\}}}}, {4, x6, {{\{0, 0, 0, 0, 0, 0, -H a4'[H x4], 0\}}, {\{0, 0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \text{Sin}[6 H x0]^{1/3} a4'[H x4], 0\}}}}, {4, x7, {{\{0, 0, 0, 0, 0, 0, 0, -H a4'[H x4]\}}, {\{0, 0, 0, 0, 0, 0, -e^{-2 a4[H x4]} H \text{Sin}[6 H x0]^{1/3} a4'[H x4]\}}}}, {5, x0, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {5, x1, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {5, x2, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {5, x3, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {5, x4, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {5, x5, {{\left\{\sqrt{e^{-2 a4[H x4]}} H \text{Sec}[6 H x0] \text{Sin}[6 H x0]\right\}^{7/6}}, {0, 0, 0, \sqrt{e^{-2 a4[H x4]}} H \text{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0}}}, {\left\{-\frac{\sqrt{e^{-2 a4[H x4]}} H \text{Cos}[6 H x0]}{\text{Sin}[6 H x0]^{5/6}}, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} H \text{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\right\}}}}, {5, x6, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {5, x7, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {6, x0, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {6, x1, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {6, x2, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {6, x3, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}, {6, x4, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}},

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{6, x5, {{{{0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0}}}}}, 
{6, x6, {{\left\{\left\{\sqrt{e^{-2 a4[H x4]}} H \text{Sec}[6 H x0] \text{Sin}[6 H x0]^{7/6},\right.\right.},\\
0, 0, 0, \sqrt{e^{-2 a4[H x4]}} H \text{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\right\}\right\}},\\
\left\{\left\{-\frac{\sqrt{e^{-2 a4[H x4]}} H \text{Cos}[6 H x0]}{\text{Sin}[6 H x0]^{5/6}}, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} H \text{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\right\}\right\}\right\},\\
{6, x7, {{{{0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0}}}}},\\
\left\{\left\{7, x0, {{{{0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0}}}}},\right.\right.\\
{7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0}}}},\\
{7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0}}}},\\
{7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0}}}},\\
{7, x4, {{{0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0}}}},\\
{7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0}}}},\\
{7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0}}}},\\
{7, x7, {{\left\{\left\{\sqrt{e^{-2 a4[H x4]}} H \text{Sec}[6 H x0] \text{Sin}[6 H x0]^{7/6},\right.\right.},\\
0, 0, 0, \sqrt{e^{-2 a4[H x4]}} H \text{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\right\}\right\}},\\
\left\{\left\{-\frac{\sqrt{e^{-2 a4[H x4]}} H \text{Cos}[6 H x0]}{\text{Sin}[6 H x0]^{5/6}}, 0, 0, 0, \sqrt{e^{-2 a4[H x4]}} H \text{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0\right\}\right\}\right\}\right\}\\
In[8]:= (*Block[{eA\alpha,e\alpha A,t,ginv,\Gamma,gg},\\
gg=g4488/.sg;\\
{ginv,\Gamma}=con[gg];\\
e\alpha A=e_{(A)}^{\alpha}/.sge_{(A)}^{\alpha};\\
eA\alpha=e_{\alpha}^{(A)}/.sge_{\alpha}^{(A)};\\
t=Table[ Block[{v,V,d},v=e\alpha A[[All,b]];\\
d=Table[D[v[[k]],X[[\mu]] ]+Sum[ v[[i]] \Gamma[[k,i,\mu]],{i,1,DIM8}],{k,1,DIM8}];\\
{b-1,X[[\mu]],d}],{b,1,Length[e\alpha A]}, {\mu,1,DIM8}];t]\\
FullSimplify[#/ssgm4488,constraintVars]&/@%*)\\
\\
In[9]:= (*Block[{eA\alpha,e\alpha A,t,ginv,\Gamma,gg},\\
gg=g4488/.sg;\\
{ginv,\Gamma}=con[gg];\\
e\alpha A=e_{(A)}^{\alpha}/.sge_{(A)}^{\alpha};\\
eA\alpha=e_{\alpha}^{(A)}/.sge_{\alpha}^{(A)};\\
t=Table[ Block[{v,V,d},v=eA\alpha[[b,All]];\\
d=Table[D[v[[k]],X[[\mu]] ]-Sum[ v[[i]] \Gamma[[k,i,\mu]],{i,1,DIM8}],{k,1,DIM8}];\\
{b-1,X[[\mu]],d}],{b,1,Length[e\alpha A]}, {\mu,1,DIM8}];t]\\
FullSimplify[#/ssgm4488,constraintVars]&/@%*)\\
\\
In[10]:= Symbolize[\omega_\mu^{(a)}_{(b)}]\\
In[11]:= \omega = Array[\omega_\mu^{(a)}_{(b)} [\#\#] &, {8, 8, 8}];\\

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In[1]:= $\omega[[1]]$

Out[1]=

$$\left\{ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 1, 1], \omega_{\mu(b)}^{(a)} [1, 1, 2], \omega_{\mu(b)}^{(a)} [1, 1, 3], \omega_{\mu(b)}^{(a)} [1, 1, 4], \\ \omega_{\mu(b)}^{(a)} [1, 1, 5], \omega_{\mu(b)}^{(a)} [1, 1, 6], \omega_{\mu(b)}^{(a)} [1, 1, 7], \omega_{\mu(b)}^{(a)} [1, 1, 8] \end{array} \right\}, \right. \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 2, 1], \omega_{\mu(b)}^{(a)} [1, 2, 2], \omega_{\mu(b)}^{(a)} [1, 2, 3], \omega_{\mu(b)}^{(a)} [1, 2, 4], \\ \omega_{\mu(b)}^{(a)} [1, 2, 5], \omega_{\mu(b)}^{(a)} [1, 2, 6], \omega_{\mu(b)}^{(a)} [1, 2, 7], \omega_{\mu(b)}^{(a)} [1, 2, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 3, 1], \omega_{\mu(b)}^{(a)} [1, 3, 2], \omega_{\mu(b)}^{(a)} [1, 3, 3], \omega_{\mu(b)}^{(a)} [1, 3, 4], \\ \omega_{\mu(b)}^{(a)} [1, 3, 5], \omega_{\mu(b)}^{(a)} [1, 3, 6], \omega_{\mu(b)}^{(a)} [1, 3, 7], \omega_{\mu(b)}^{(a)} [1, 3, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 4, 1], \omega_{\mu(b)}^{(a)} [1, 4, 2], \omega_{\mu(b)}^{(a)} [1, 4, 3], \omega_{\mu(b)}^{(a)} [1, 4, 4], \\ \omega_{\mu(b)}^{(a)} [1, 4, 5], \omega_{\mu(b)}^{(a)} [1, 4, 6], \omega_{\mu(b)}^{(a)} [1, 4, 7], \omega_{\mu(b)}^{(a)} [1, 4, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 5, 1], \omega_{\mu(b)}^{(a)} [1, 5, 2], \omega_{\mu(b)}^{(a)} [1, 5, 3], \omega_{\mu(b)}^{(a)} [1, 5, 4], \\ \omega_{\mu(b)}^{(a)} [1, 5, 5], \omega_{\mu(b)}^{(a)} [1, 5, 6], \omega_{\mu(b)}^{(a)} [1, 5, 7], \omega_{\mu(b)}^{(a)} [1, 5, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 6, 1], \omega_{\mu(b)}^{(a)} [1, 6, 2], \omega_{\mu(b)}^{(a)} [1, 6, 3], \omega_{\mu(b)}^{(a)} [1, 6, 4], \\ \omega_{\mu(b)}^{(a)} [1, 6, 5], \omega_{\mu(b)}^{(a)} [1, 6, 6], \omega_{\mu(b)}^{(a)} [1, 6, 7], \omega_{\mu(b)}^{(a)} [1, 6, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 7, 1], \omega_{\mu(b)}^{(a)} [1, 7, 2], \omega_{\mu(b)}^{(a)} [1, 7, 3], \omega_{\mu(b)}^{(a)} [1, 7, 4], \\ \omega_{\mu(b)}^{(a)} [1, 7, 5], \omega_{\mu(b)}^{(a)} [1, 7, 6], \omega_{\mu(b)}^{(a)} [1, 7, 7], \omega_{\mu(b)}^{(a)} [1, 7, 8] \end{array} \right\}, \\ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 8, 1], \omega_{\mu(b)}^{(a)} [1, 8, 2], \omega_{\mu(b)}^{(a)} [1, 8, 3], \omega_{\mu(b)}^{(a)} [1, 8, 4], \\ \omega_{\mu(b)}^{(a)} [1, 8, 5], \omega_{\mu(b)}^{(a)} [1, 8, 6], \omega_{\mu(b)}^{(a)} [1, 8, 7], \omega_{\mu(b)}^{(a)} [1, 8, 8] \end{array} \right\} \right\}$$

In[2]:= $s\omega = \text{Block}[\{\text{eA}\alpha, \text{e}\alpha\text{A}, \text{t}, \text{V}, \text{ginv}, \Gamma, \text{gg}\},$

$\text{gg} = \text{g4488} /.\text{sg};$

$\{\text{ginv}, \Gamma\} = \text{con}[\text{gg}];$

$\text{e}\alpha\text{A} = \text{e}_{(\text{A})}^{\alpha} /.\text{sge}_{(\text{A})}^{\alpha};$

$\text{eA}\alpha = \text{e}_{\alpha}^{(\text{A})} /.\text{sge}_{\alpha}^{(\text{A})};$

$\text{Table}[\omega[[\mu, \alpha, b]] \rightarrow \text{Block}[\{\text{v}, \text{d}\}, \text{v} = \text{e}\alpha\text{A}[\text{All}, \text{b}];$

$\text{d} = \text{Sum}[(\text{D}[\text{v}[[\text{k}]], \text{X}[[\mu]]] + \text{Sum}[\text{v}[[\text{i}]] \times \Gamma[[\text{k}, \text{i}, \mu]], \{\text{i}, 1, \text{DIM8}\}]) * \text{eA}\alpha[[\text{a}, \text{k}],$
 $\{\text{k}, 1, \text{DIM8}\}];$

$\text{d}], \{\mu, 1, \text{DIM8}\}, \{\text{a}, 1, \text{Length}[\text{e}\alpha\text{A}]\}, \{\text{b}, 1, \text{Length}[\text{e}\alpha\text{A}]\}]$

Out[2]=

$$\left\{ \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [1, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 3] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [1, 1, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 5] \rightarrow \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \\ \omega_{\mu(b)}^{(a)} [1, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 1, 8] \rightarrow 0 \end{array} \right\} \right\}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [1, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 5, 1] \rightarrow -\frac{g[0][0]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[4][4][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 5, 2] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 5] \rightarrow \right. \\
& \quad \left. \left(\frac{g[4][4]^{(1,0)}[x0, x4]}{2(-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 3] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 6, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 6] \rightarrow \right. \\
& \quad \left. \left(\frac{g[5][5]^{(1,0)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]}g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 6, 8] \rightarrow 0 \right\}, \left\{ \omega_{\mu(b)}^{(a)} [1, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 2] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 7, 6] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 7, 7] \rightarrow \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]}g[6][6][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[6][6][x0, x4]}, \omega_{\mu(b)}^{(a)} [1, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [1, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [1, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [1, 8, 8] \rightarrow \right. \\
& \quad \left. \left(\frac{g[7][7]^{(1,0)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]}g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [2, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 2] \rightarrow -\frac{g[1][1]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{g[1][1][x0, x4]}}, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 5] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [2, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [2, 1, 8] \rightarrow 0 \right\}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [2, 2, 1] \rightarrow \frac{g[1][1]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[1][1][x0, x4]}}, \right. \\
& \quad \omega_{\mu(b)}^{(a)} [2, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 2, 4] \rightarrow 0, \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 2, 5] \rightarrow \frac{g[1][1]^{(0,1)}[x0, x4]}{2 \sqrt{g[1][1][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 2] \rightarrow -\frac{g[1][1]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 5] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [2, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 4] \rightarrow 0, \right. \\
& \quad \left. \omega_{\mu(b)}^{(a)} [2, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [2, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [3, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 1, 2] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 1, 3] \rightarrow -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [3, 1, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 1, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 4] \rightarrow 0, \right. \right. \\
& \quad \left. \left. \omega_{\mu(b)}^{(a)} [3, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 2, 8] \rightarrow 0 \right\}, \right. \\
& \quad \left. \left. \left\{ \omega_{\mu(b)}^{(a)} [3, 3, 1] \rightarrow \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, \right. \right. \right. \\
& \quad \left. \left. \left. \omega_{\mu(b)}^{(a)} [3, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 3, 4] \rightarrow 0, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \omega_{\mu(b)}^{(a)} [3, 3, 5] \rightarrow \frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 3, 8] \rightarrow 0 \\ \omega_{\mu(b)}^{(a)} [3, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 4, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 5, 2] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 5, 3] \rightarrow -\frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, \quad \omega_{\mu(b)}^{(a)} [3, 5, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 5, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 6, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 7, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [3, 8, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [3, 8, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [3, 8, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 1, 3] \rightarrow 0, \right. \\ \left. \omega_{\mu(b)}^{(a)} [4, 1, 4] \rightarrow -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \quad \omega_{\mu(b)}^{(a)} [4, 1, 5] \rightarrow 0, \right. \\ \left. \omega_{\mu(b)}^{(a)} [4, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 1, 8] \rightarrow 0 \right\}, \\ \left. \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 2, 8] \rightarrow 0 \end{array} \right\}, \right. \\ \left. \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 3, 8] \rightarrow 0 \end{array} \right\}, \right. \\ \left. \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [4, 4, 1] \rightarrow \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \\ \omega_{\mu(b)}^{(a)} [4, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [4, 4, 5] \rightarrow \frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \\ \omega_{\mu(b)}^{(a)} [4, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [4, 4, 8] \rightarrow 0 \end{array} \right\}, \right. \end{array} \right\}
\end{aligned}$$

$$\begin{aligned}
& \left\{ \omega_{\mu(b)}^{(a)} [4, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 5, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 5, 3] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 5, 4] \rightarrow -\frac{g[3][3]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, \omega_{\mu(b)}^{(a)} [4, 5, 5] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 6, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [4, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [4, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [4, 8, 8] \rightarrow 0 \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [5, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 1, 5] \rightarrow -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 1, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 1, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 2, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 3, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 4, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 5, 1] \rightarrow \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, \omega_{\mu(b)}^{(a)} [5, 5, 2] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 5, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 5, 5] \rightarrow \right. \right. \\
& \left. \left. \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 5, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 5, 8] \rightarrow 0 \right\}, \right. \\
& \left. \left\{ \omega_{\mu(b)}^{(a)} [5, 6, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 3] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [5, 6, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 6] \rightarrow \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2(-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2\sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \sqrt{-g[5][5][x0, x4]}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [5, 6, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 6, 8] \rightarrow 0 \right\}, \left\{ \omega_{\mu(b)}^{(a)} [5, 7, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 7, 2] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [5, 7, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 7, 4] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 7, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 7, 6] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [5, 7, 7] \rightarrow \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2(-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2\sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[6][6][x0, x4]}, \omega_{\mu(b)}^{(a)} [5, 7, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [5, 8, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [5, 8, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [5, 8, 8] \rightarrow \right. \\
& \left. \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \sqrt{-g[7][7][x0, x4]} \right\}, \\
& \left\{ \left\{ \omega_{\mu(b)}^{(a)} [6, 1, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 4] \rightarrow 0, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [6, 1, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 6] \rightarrow -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]} \sqrt{-g[5][5][x0, x4]}}, \right. \right. \\
& \left. \left. \omega_{\mu(b)}^{(a)} [6, 1, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 1, 8] \rightarrow 0 \right\}, \right. \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 2, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [6, 2, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 2, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 3, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [6, 3, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 3, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 4, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [6, 4, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 6] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 4, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 5, 1] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [6, 5, 5] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 6] \rightarrow -\frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4] \sqrt{-g[5][5][x0, x4]}}, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [6, 5, 7] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 5, 8] \rightarrow 0 \right\}, \\
& \left\{ \omega_{\mu(b)}^{(a)} [6, 6, 1] \rightarrow \frac{g[5][5]^{(1,0)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2\sqrt{g[0][0][x0, x4]} g[5][5][x0, x4]}, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [6, 6, 2] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 6, 3] \rightarrow 0, \omega_{\mu(b)}^{(a)} [6, 6, 4] \rightarrow 0, \right. \\
& \left. \omega_{\mu(b)}^{(a)} [6, 6, 5] \rightarrow \frac{g[5][5]^{(0,1)}[x0, x4] \sqrt{-g[5][5][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]} g[5][5][x0, x4]}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 1, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 7] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 1, 8] \rightarrow -\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[7][7][x0, x4]}} \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 7] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 5, 8] \rightarrow -\frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[7][7][x0, x4]}} \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 8, 1] \rightarrow \frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[7][7][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [8, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 4] \rightarrow 0, \\ \omega_{\mu(b)}^{(a)} [8, 8, 5] \rightarrow \frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}g[7][7][x0, x4]}, \\ \omega_{\mu(b)}^{(a)} [8, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 8] \rightarrow 0 \end{array} \right\} \}
\end{aligned}$$

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In[1]:= (*sw=Block[{eA\alpha,e\alpha A,t,V,ginv,\Gamma,gg},
gg=g4488/.sg;
{ginv,\Gamma}=con[gg];
e\alpha A=e\alpha^{(A)}/.sge^{(A)}_{(A)};
eA\alpha=e\alpha^{(A)}/.sge^{(A)}_{\alpha};
Table[ \omega_\mu^{(a)}{(b)} [\mu,a,b]\rightarrow Block[{v,d},v=e\alpha A[[All,b]];
d=Sum[(D[v[[k]],X[[\mu]]]+Sum[ v[[i]] \Gamma[[k,i,\mu]],{i,1,DIM8}])\*eA\alpha[[a,k]],{k,1,DIM8}];
d],{\mu,1,DIM8},{a,1,Length[e\alpha A]},{b,1,Length[e\alpha A]}]\]*)

In[2]:= swf=Flatten[sw];

In[3]:= (*\omega_\mu^{(a)}{(b)} *)
\omega;
wmat=% /. swf
Dimensions[%]

Out[3]=
\left\{\left\{0,0,0,0,\frac{g[0][0]^{(0,1)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{-g[4][4][x0,x4]}},0,0,0\right\},\right.
\{0,0,0,0,0,0,0,0,\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0\},\right.
\left.-\frac{g[0][0]^{(0,1)}[x0,x4]\sqrt{-g[4][4][x0,x4]}}{2\sqrt{g[0][0][x0,x4]}g[4][4][x0,x4]},0,0,0,\right.
\left.\left(\frac{g[4][4]^{(1,0)}[x0,x4]}{2(-g[4][4][x0,x4])^{3/2}}+\frac{g[4][4]^{(1,0)}[x0,x4]}{2\sqrt{-g[4][4][x0,x4]}g[4][4][x0,x4]}\right)\sqrt{-g[4][4][x0,x4]},\right.
0,0,0\},\left\{0,0,0,0,0,0,\right.
\left.\left(\frac{g[5][5]^{(1,0)}[x0,x4]}{2(-g[5][5][x0,x4])^{3/2}}+\frac{g[5][5]^{(1,0)}[x0,x4]}{2\sqrt{-g[5][5][x0,x4]}g[5][5][x0,x4]}\right)\sqrt{-g[5][5][x0,x4]},0,\right.
0\},\left\{0,0,0,0,0,0,0,\left(\frac{g[6][6]^{(1,0)}[x0,x4]}{2(-g[6][6][x0,x4])^{3/2}}+\frac{g[6][6]^{(1,0)}[x0,x4]}{2\sqrt{-g[6][6][x0,x4]}g[6][6][x0,x4]}\right)\right.
\sqrt{-g[6][6][x0,x4]},0\},\left\{0,0,0,0,0,0,0,\right.
\left.\left(\frac{g[7][7]^{(1,0)}[x0,x4]}{2(-g[7][7][x0,x4])^{3/2}}+\frac{g[7][7]^{(1,0)}[x0,x4]}{2\sqrt{-g[7][7][x0,x4]}g[7][7][x0,x4]}\right)\right.
\sqrt{-g[7][7][x0,x4]}\}\},\right.
\left\{0,-\frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[1][1][x0,x4]}},0,0,0,0,0,0\right\},\right.
\left\{\frac{g[1][1]^{(1,0)}[x0,x4]}{2\sqrt{g[0][0][x0,x4]}\sqrt{g[1][1][x0,x4]}},0,0,0,\right.
\left.\frac{g[1][1]^{(0,1)}[x0,x4]}{2\sqrt{g[1][1][x0,x4]}\sqrt{-g[4][4][x0,x4]}},0,0,0\right\},\{0,0,0,0,0,0,0,0\},

```

$$\begin{aligned}
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, -\frac{g[1][1]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[1][1][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, -\frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ \frac{g[2][2]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, \right. \\
& \left. \frac{g[2][2]^{(0,1)}[x0, x4]}{2 \sqrt{g[2][2][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, -\frac{g[2][2]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[2][2][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, -\frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ \frac{g[3][3]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{g[3][3][x0, x4]}}, \right. \\
& \left. 0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4]}{2 \sqrt{g[3][3][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, -\frac{g[3][3]^{(0,1)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[3][3][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ \frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]} \sqrt{-g[4][4][x0, x4]}}, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ \frac{g[4][4]^{(1,0)}[x0, x4] \sqrt{-g[4][4][x0, x4]}}{2 \sqrt{g[0][0][x0, x4]} g[4][4][x0, x4]}, 0, 0, 0, \right. \\
& \left. \left(\frac{g[4][4]^{(0,1)}[x0, x4]}{2 (-g[4][4][x0, x4])^{3/2}} + \frac{g[4][4]^{(0,1)}[x0, x4]}{2 \sqrt{-g[4][4][x0, x4]} g[4][4][x0, x4]} \right) \sqrt{-g[4][4][x0, x4]}, 0, \right. \\
& \left. 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, \left(\frac{g[5][5]^{(0,1)}[x0, x4]}{2 (-g[5][5][x0, x4])^{3/2}} + \frac{g[5][5]^{(0,1)}[x0, x4]}{2 \sqrt{-g[5][5][x0, x4]} g[5][5][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[5][5][x0, x4]}, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \\
& \left. \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) \sqrt{-g[6][6][x0, x4]}, 0 \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ 0, 0, 0, 0, 0, 0, 0, \left(\frac{g[7][7]^{(0,1)}[x0, x4]}{2(-g[7][7][x0, x4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x0, x4]}{2\sqrt{-g[7][7][x0, x4]} g[7][7][x0, x4]} \right) \right. \\
& \quad \left. \sqrt{-g[7][7][x0, x4]} \right\}, \left\{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[5][5][x0, x4]}}, 0, 0 \right\}, \\
& \left\{ \frac{g[5][5]^{(1,0)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[5][5][x0, x4]}, 0, 0, \right. \\
& \quad \left. 0, \frac{g[5][5]^{(0,1)}[x0, x4]\sqrt{-g[5][5][x0, x4]}}{2\sqrt{-g[4][4][x0, x4]}g[5][5][x0, x4]}, 0, 0, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[6][6][x0, x4]}}, 0 \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{g[6][6]^{(1,0)}[x0, x4]\sqrt{-g[6][6][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[6][6][x0, x4]}, 0, 0, 0, \right. \\
& \quad \left. 0, 0, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4]}{2\sqrt{g[0][0][x0, x4]}\sqrt{-g[7][7][x0, x4]}}, \right. \\
& \quad \left. 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4]\sqrt{-g[4][4][x0, x4]}}{2g[4][4][x0, x4]\sqrt{-g[7][7][x0, x4]}}, \right. \\
& \quad \left. 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4]\sqrt{-g[7][7][x0, x4]}}{2\sqrt{g[0][0][x0, x4]}g[7][7][x0, x4]}, \right. \\
& \quad \left. 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}
\end{aligned}$$

Out[•]=

{8, 8, 8}

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In[8]:= swgtry = Block[{eAα, eαA, t, V},
  eαA = e(A) /. sgtrye(A);
  eAα = e(A)α /. sgtrye(A)α;
  Table[(*V=eAα[[a,All]];*) ω(a)μ(b) [μ, a, b] → Block[{v, d}, v = eαA[[All, b]];
    d = Sum[(D[v[[k]], X[[μ]]] + Sum[v[[i]]×Γ[[k, i, μ]], {i, 1, DIM8}]) * eAα[[a, k]],
      {k, 1, DIM8}];
    FullSimplify[d, constraintVars] /. {Sqrt[e2 a4[Hx4]] → ea4[Hx4], Sqrt[e-2 a4[Hx4]] →
      e-a4[Hx4]}], {μ, 1, DIM8}, {a, 1, Length[eAα]}, {b, 1, Length[eαA]}]]
]

Out[8]=
{ { { { ω(a)μ(b) [1, 1, 1] → 0, ω(a)μ(b) [1, 1, 2] → 0, ω(a)μ(b) [1, 1, 3] → 0, ω(a)μ(b) [1, 1, 4] → 0,
    ω(a)μ(b) [1, 1, 5] → 0, ω(a)μ(b) [1, 1, 6] → 0, ω(a)μ(b) [1, 1, 7] → 0, ω(a)μ(b) [1, 1, 8] → 0 },
    { ω(a)μ(b) [1, 2, 1] → 0, ω(a)μ(b) [1, 2, 2] → 0, ω(a)μ(b) [1, 2, 3] → 0, ω(a)μ(b) [1, 2, 4] → 0,
      ω(a)μ(b) [1, 2, 5] → 0, ω(a)μ(b) [1, 2, 6] → 0, ω(a)μ(b) [1, 2, 7] → 0, ω(a)μ(b) [1, 2, 8] → 0 },
    { ω(a)μ(b) [1, 3, 1] → 0, ω(a)μ(b) [1, 3, 2] → 0, ω(a)μ(b) [1, 3, 3] → 0, ω(a)μ(b) [1, 3, 4] → 0,
      ω(a)μ(b) [1, 3, 5] → 0, ω(a)μ(b) [1, 3, 6] → 0, ω(a)μ(b) [1, 3, 7] → 0, ω(a)μ(b) [1, 3, 8] → 0 },
    { ω(a)μ(b) [1, 4, 1] → 0, ω(a)μ(b) [1, 4, 2] → 0, ω(a)μ(b) [1, 4, 3] → 0, ω(a)μ(b) [1, 4, 4] → 0,
      ω(a)μ(b) [1, 4, 5] → 0, ω(a)μ(b) [1, 4, 6] → 0, ω(a)μ(b) [1, 4, 7] → 0, ω(a)μ(b) [1, 4, 8] → 0 },
    { ω(a)μ(b) [1, 5, 1] → 0, ω(a)μ(b) [1, 5, 2] → 0, ω(a)μ(b) [1, 5, 3] → 0, ω(a)μ(b) [1, 5, 4] → 0,
      ω(a)μ(b) [1, 5, 5] → 0, ω(a)μ(b) [1, 5, 6] → 0, ω(a)μ(b) [1, 5, 7] → 0, ω(a)μ(b) [1, 5, 8] → 0 },
    { ω(a)μ(b) [1, 6, 1] → 0, ω(a)μ(b) [1, 6, 2] → 0, ω(a)μ(b) [1, 6, 3] → 0, ω(a)μ(b) [1, 6, 4] → 0,
      ω(a)μ(b) [1, 6, 5] → 0, ω(a)μ(b) [1, 6, 6] → 0, ω(a)μ(b) [1, 6, 7] → 0, ω(a)μ(b) [1, 6, 8] → 0 },
    { ω(a)μ(b) [1, 7, 1] → 0, ω(a)μ(b) [1, 7, 2] → 0, ω(a)μ(b) [1, 7, 3] → 0, ω(a)μ(b) [1, 7, 4] → 0,
      ω(a)μ(b) [1, 7, 5] → 0, ω(a)μ(b) [1, 7, 6] → 0, ω(a)μ(b) [1, 7, 7] → 0, ω(a)μ(b) [1, 7, 8] → 0 },
    { ω(a)μ(b) [1, 8, 1] → 0, ω(a)μ(b) [1, 8, 2] → 0, ω(a)μ(b) [1, 8, 3] → 0, ω(a)μ(b) [1, 8, 4] → 0,
      ω(a)μ(b) [1, 8, 5] → 0, ω(a)μ(b) [1, 8, 6] → 0, ω(a)μ(b) [1, 8, 7] → 0, ω(a)μ(b) [1, 8, 8] → 0 } },
  { { ω(a)μ(b) [2, 1, 1] → 0, ω(a)μ(b) [2, 1, 2] → -ea4[Hx4] H Sin[6 H x 0]1/6,
    ω(a)μ(b) [2, 1, 3] → 0, ω(a)μ(b) [2, 1, 4] → 0, ω(a)μ(b) [2, 1, 5] → 0,
    ω(a)μ(b) [2, 1, 6] → 0, ω(a)μ(b) [2, 1, 7] → 0, ω(a)μ(b) [2, 1, 8] → 0 },
    { ω(a)μ(b) [2, 2, 1] → ea4[Hx4] H Sin[6 H x 0]1/6, ω(a)μ(b) [2, 2, 2] → 0, ω(a)μ(b) [2, 2, 3] → 0,
      ω(a)μ(b) [2, 2, 4] → 0 } }
]

```


$$\begin{aligned}
& \omega_{\mu(b)}^{(a)} [8, 1, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 6] \rightarrow 0, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 1, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 1, 8] \rightarrow e^{-a4[Hx4]} H \sin[6Hx0]^{1/6} \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 2, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 2, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 2, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 3, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 3, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 3, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 4, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 4, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 4, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 5, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 3] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 5, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 6] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 5, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 5, 8] \rightarrow e^{-a4[Hx4]} H \sin[6Hx0]^{1/6} a4'[Hx4] \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 6, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 6, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 6, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 7, 1] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 3] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 4] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 7, 5] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 7, 8] \rightarrow 0 \end{array} \right\}, \\
& \left\{ \begin{array}{l} \omega_{\mu(b)}^{(a)} [8, 8, 1] \rightarrow e^{-a4[Hx4]} H \sin[6Hx0]^{1/6}, \quad \omega_{\mu(b)}^{(a)} [8, 8, 2] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 3] \rightarrow 0, \\
\omega_{\mu(b)}^{(a)} [8, 8, 4] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 5] \rightarrow -e^{-a4[Hx4]} H \sin[6Hx0]^{1/6} a4'[Hx4], \\
\omega_{\mu(b)}^{(a)} [8, 8, 6] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 7] \rightarrow 0, \quad \omega_{\mu(b)}^{(a)} [8, 8, 8] \rightarrow 0 \end{array} \right\} \}
\end{aligned}$$

```
In[6]:= swfgtry = Flatten[swgtry];
```



```

In[•]:= g4488 /. sg
% /. ssgm4488

Out[•]=
{{g[0][0][x0, x4], 0, 0, 0, 0, 0, 0}, {0, g[1][1][x0, x4], 0, 0, 0, 0, 0}, {0, 0, g[2][2][x0, x4], 0, 0, 0, 0}, {0, 0, 0, g[3][3][x0, x4], 0, 0, 0, 0}, {0, 0, 0, 0, g[4][4][x0, x4], 0, 0, 0}, {0, 0, 0, 0, 0, g[5][5][x0, x4], 0, 0}, {0, 0, 0, 0, 0, 0, g[6][6][x0, x4], 0}, {0, 0, 0, 0, 0, 0, 0, g[7][7][x0, x4]}}

Out[•]=
{{Cot[6 H x0]^2, 0, 0, 0, 0, 0, 0}, {0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0, 0}, {0, 0, 0, e^2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0, 0}, {0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3), 0}, {0, 0, 0, 0, 0, 0, 0, -e^-2 a4[H x4] Sin[6 H x0]^(1/3)}}

check metric compatibility condition for the Octad

In[•]:= Block[{eAα, eαA, t, ginv, Γ, gg(*, ωω*)},
(*ωω=ω(a)μ(b)/.Swf;*)
gg = g4488 /. sg /. ssgm4488;
{ginv, Γ} = con[gg];
eAα = eα(A) /. sgeα(A); eAα = e(A)α /. sge(A)α;
t = Table[Block[{v, V, contra, covari},
V = eAα[[b, All]];
v = eαA[[All, b]];
contra =
Table[D[v[[k]], X[[μ]]] + Sum[v[[i]]×Γ[[k, i, μ]], {i, 1, DIM8}], {k, 1, DIM8}];
(*no ω terms*)
covari = Table[D[V[[k]], X[[μ]]] - Sum[V[[i]]×Γ[[i, k, μ]], {i, 1, DIM8}] +
Sum[eAα[[j, All]][[k]]×ωmat[[μ, b, j]], {j, 1, DIM8}] //
FullSimplify[#, constraintVars && e^2 a4[H x4] > 0 && e^a4[H x4] > 0 &&
e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0] &, {k, 1, DIM8}]; (*ω terms*)
(*{b-1,X[[μ]],{{contra},{covari}}},{b,1,Length[eαA]},{μ,1,DIM8}];*)
{b-1, X[[μ]], covari}], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
t]
FullSimplify[#/ . ssgm4488,
constraintVars && e^2 a4[H x4] > 0 && e^a4[H x4] > 0 && e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0] & /@ %

Out[•]=
{{{{0, x0, {g[0][0]^(1,0)[x0, x4] + 24 H Csc[12 H x0] g[0][0][x0, x4],
2 √g[0][0][x0, x4]}}, {0, 0, 0, {g[0][0]^(0,1)[x0, x4],
2 √g[0][0][x0, x4]}, {0, 0, 0}}, {0, x1, {0, (-g[1][1]^(1,0)[x0, x4] + 2 e^2 a4[H x4] H Sec[6 H x0] Sin[6 H x0]^(4/3) g[0][0][x0, x4]) / (2 √g[0][0][x0, x4]), {0, 0, 0, 0, 0, 0}}}, {0, x2, {0, 0, 0}}}}

```

$$\begin{aligned}
& \left(-g[2][2]^{(1,0)}[x0, x4] + 2 e^{2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4] \right) / \\
& \quad (2 \sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0, 0 \}, \{0, x3, \{0, 0, 0, \\
& \left(-g[3][3]^{(1,0)}[x0, x4] + 2 e^{2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4] \right) / \\
& \quad (2 \sqrt{g[0][0][x0, x4]}), 0, 0, 0, 0 \}, \\
& \{0, x4, \left\{ \frac{g[0][0]^{(0,1)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2 \sqrt{g[0][0][x0, x4]}}, 0, 0, 0 \right\}, \\
& \{0, x5, \{0, 0, 0, 0, 0, 0, \\
& \left(-g[5][5]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4] \right) / \\
& \quad (2 \sqrt{g[0][0][x0, x4]}), 0, 0 \}, \{0, x6, \{0, 0, 0, 0, 0, 0, 0, \\
& \left(-g[6][6]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4] \right) / \\
& \quad (2 \sqrt{g[0][0][x0, x4]}), 0 \}, \{0, x7, \{0, 0, 0, 0, 0, 0, 0, 0, \\
& \left(-g[7][7]^{(1,0)}[x0, x4] - 2 e^{-2 a4[H x4]} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{4/3} g[0][0][x0, x4] \right) / \\
& \quad (2 \sqrt{g[0][0][x0, x4]}), 0 \} \}, \\
& \left\{ 1, x0, \left\{ 0, \frac{g[1][1]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \left\{ 1, x1, \left\{ \frac{g[1][1]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, \right. \\
& \left. 0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0 \right\}, \\
& \{1, x2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{1, x3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{ 1, x4, \left\{ 0, \frac{g[1][1]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[1][1][x0, x4]}{2 \sqrt{g[1][1][x0, x4]}}, 0, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \{1, x5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{1, x6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{1, x7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{ 2, x0, \left\{ 0, 0, \frac{g[2][2]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0 \right\} \right\}, \\
& \{2, x1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 2, x2, \left\{ \frac{g[2][2]^{(1,0)}[x0, x4] - 2 H \operatorname{Cot}[6 H x0] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, \right. \\
& \left. 0, \frac{g[2][2]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0 \right\}, \\
& \{2, x3, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 2, x4, \left\{ 0, 0, \frac{g[2][2]^{(0,1)}[x0, x4] - 6 H a4'[H x4] g[2][2][x0, x4]}{2 \sqrt{g[2][2][x0, x4]}}, 0, 0, 0, 0, 0 \right\} \right\},
\end{aligned}$$

$$\begin{aligned}
& \{2, x5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{2, x6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{2, x7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \left\{3, x0, \left\{0, 0, 0, \frac{g[3][3]^{(1,0)}[x0, x4] - 2H \operatorname{Cot}[6Hx0] g[3][3][x0, x4]}{2\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\right\}\right\}, \\
& \{3, x1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x2, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{3, x3, \left\{\frac{g[3][3]^{(1,0)}[x0, x4] - 2H \operatorname{Cot}[6Hx0] g[3][3][x0, x4]}{2\sqrt{g[3][3][x0, x4]}}, 0, 0,\right.\right. \\
& \left.\left.0, \frac{g[3][3]^{(0,1)}[x0, x4] - 6Ha4'[Hx4] g[3][3][x0, x4]}{2\sqrt{g[3][3][x0, x4]}}, 0, 0, 0\right\}\right\}, \\
& \left\{3, x4, \left\{0, 0, 0, \frac{g[3][3]^{(0,1)}[x0, x4] - 6Ha4'[Hx4] g[3][3][x0, x4]}{2\sqrt{g[3][3][x0, x4]}}, 0, 0, 0, 0\right\}\right\}, \\
& \{3, x5, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{3, x6, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{3, x7, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \left\{4, x0, \left\{\frac{g[0][0]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}\right\}, \{4, x1, \\
& \{0, (g[1][1]^{(0,1)}[x0, x4] + 6e^{2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4] g[4][4][x0, x4]) / \\
& (2\sqrt{-g[4][4][x0, x4]}), 0, 0, 0, 0, 0, 0\}\}, \{4, x2, \\
& \{0, 0, (g[2][2]^{(0,1)}[x0, x4] + 6e^{2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4] g[4][4][x0, x4]) / \\
& (2\sqrt{-g[4][4][x0, x4]}), 0, 0, 0, 0, 0, 0\}\}, \{4, x3, \{0, 0, 0, \\
& (g[3][3]^{(0,1)}[x0, x4] + 6e^{2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4] g[4][4][x0, x4]) / \\
& (2\sqrt{-g[4][4][x0, x4]}), 0, 0, 0, 0, 0\}\}, \\
& \left\{4, x4, \left\{-\frac{g[4][4]^{(1,0)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0, -\frac{g[4][4]^{(0,1)}[x0, x4]}{2\sqrt{-g[4][4][x0, x4]}}, 0, 0, 0\right\}\right\}, \{4, \\
& x5, \left\{0, 0, 0, 0, 0, 0, \frac{1}{2} \left(-6e^{-2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4] - \frac{g[5][5]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]}, 0, 0\right\}\}, \{4, x6, \\
& \{0, 0, 0, 0, 0, 0, \frac{1}{2} \left(-6e^{-2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4] - \frac{g[6][6]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]}, 0\right\}\}, \{4, x7, \\
& \{0, 0, 0, 0, 0, 0, \frac{1}{2} \left(-6e^{-2a4[Hx4]} H \operatorname{Sin}[6Hx0]^{1/3} a4'[Hx4] - \frac{g[7][7]^{(0,1)}[x0, x4]}{g[4][4][x0, x4]} \right) \right. \\
& \left. \sqrt{-g[4][4][x0, x4]}\right\}\}, \\
& \left\{5, x0, \left\{0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(1,0)}[x0, x4] - 2H \operatorname{Cot}[6Hx0] g[5][5][x0, x4]}{2\sqrt{-g[5][5][x0, x4]}}, 0, 0\right\}\right\}, \\
& \{5, x1, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}\},
\end{aligned}$$

$$\begin{aligned}
& \{5, x2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{5, x4, \left\{0, 0, 0, 0, 0, 0, -\frac{g[5][5]^{(0,1)}[x0, x4] + 6 H a4' [H x4] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0\right\}\right\}, \\
& \left\{5, x5, \left\{-\frac{g[5][5]^{(1,0)}[x0, x4] - 2 H \text{Cot}[6 H x0] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0,\right.\right. \\
& \left.\left.0, -\frac{g[5][5]^{(0,1)}[x0, x4] + 6 H a4' [H x4] g[5][5][x0, x4]}{2 \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0\right\}\right\}, \\
& \{5, x6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{5, x7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \left\{\{6, x0, \left\{0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4] - 2 H \text{Cot}[6 H x0] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0\right\}\right\}, \\
& \{6, x1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{6, x2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{6, x3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{6, x4, \left\{0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4] + 6 H a4' [H x4] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0\right\}\right\}, \\
& \{6, x5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{6, x6, \left\{-\frac{g[6][6]^{(1,0)}[x0, x4] - 2 H \text{Cot}[6 H x0] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0, 0,\right.\right. \\
& \left.\left.0, -\frac{g[6][6]^{(0,1)}[x0, x4] + 6 H a4' [H x4] g[6][6][x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0\right\}\right\}, \\
& \{6, x7, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}, \\
& \left\{\{7, x0, \left\{0, 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(1,0)}[x0, x4] - 2 H \text{Cot}[6 H x0] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}}\right\}\right\}, \\
& \{7, x1, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \{7, x2, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x3, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{7, x4, \left\{0, 0, 0, 0, 0, 0, 0, 0, -\frac{g[7][7]^{(0,1)}[x0, x4] + 6 H a4' [H x4] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}}\right\}\right\}, \\
& \{7, x5, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{7, x6, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \\
& \left\{7, x7, \left\{-\frac{g[7][7]^{(1,0)}[x0, x4] - 2 H \text{Cot}[6 H x0] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}}, 0, 0,\right.\right. \\
& \left.\left.0, -\frac{g[7][7]^{(0,1)}[x0, x4] + 6 H a4' [H x4] g[7][7][x0, x4]}{2 \sqrt{-g[7][7][x0, x4]}}, 0, 0, 0\right\}\right\}\}
\end{aligned}$$

Out[•]=

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{{0, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, \\
{0, x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {0, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}}, \\
{{1, x0, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x1, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x2, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x3, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, \\
x4, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x5, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x6, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {1, x7, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{2,

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x0, {0, 0, 0, 0, 0, 0, 0, 0}], {2, x1, {0, 0, 0, 0, 0, 0, 0, 0}], {2, x2, {0, 0, 0, 0, 0, 0, 0, 0}], {2, x3, {0, 0, 0, 0, 0, 0, 0, 0}], {2, x4,
{0, 0, 0, 0, 0, 0, 0, 0}], {2, x5, {0, 0, 0, 0, 0, 0, 0, 0}], {2, x6, {0, 0, 0, 0, 0, 0, 0, 0}], {2, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{3, x0,
{0, 0, 0, 0, 0, 0, 0, 0}], {3, x1, {0, 0, 0, 0, 0, 0, 0, 0}], {3, x2, {0, 0, 0, 0, 0, 0, 0, 0}], {3, x3, {0, 0, 0, 0, 0, 0, 0, 0}], {3, x4, {0,
0, 0, 0, 0, 0, 0, 0}], {3, x5, {0, 0, 0, 0, 0, 0, 0, 0}], {3, x6, {0, 0, 0, 0, 0, 0, 0, 0}], {3, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{4, x0, {0, 0,
0, 0, 0, 0, 0, 0}], {4, x1, {0, 0, 0, 0, 0, 0, 0, 0}], {4, x2, {0, 0, 0, 0, 0, 0, 0, 0}], {4, x3, {0, 0, 0, 0, 0, 0, 0, 0}], {4, x4, {0, 0, 0,
0, 0, 0, 0, 0}], {4, x5, {0, 0, 0, 0, 0, 0, 0, 0}], {4, x6, {0, 0, 0, 0, 0, 0, 0, 0}], {4, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{5, x0, {0, 0, 0, 0,
0, 0, 0, 0}], {5, x1, {0, 0, 0, 0, 0, 0, 0, 0}], {5, x2, {0, 0, 0, 0, 0, 0, 0, 0}], {5, x3, {0, 0, 0, 0, 0, 0, 0, 0}], {5, x4, {0, 0, 0, 0, 0,
0, 0, 0}], {5, x5, {0, 0, 0, 0, 0, 0, 0, 0}], {5, x6, {0, 0, 0, 0, 0, 0, 0, 0}], {5, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{6, x0, {0, 0, 0, 0, 0, 0, 0,
0}], {6, x1, {0, 0, 0, 0, 0, 0, 0, 0}], {6, x2, {0, 0, 0, 0, 0, 0, 0, 0}], {6, x3, {0, 0, 0, 0, 0, 0, 0, 0}], {6, x4, {0, 0, 0, 0, 0, 0, 0, 0}],
{6, x5, {0, 0, 0, 0, 0, 0, 0, 0}], {6, x6, {0, 0, 0, 0, 0, 0, 0, 0}], {6, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}, {{7, x0, {0, 0, 0, 0, 0, 0, 0, 0}],
{7, x1, {0, 0, 0, 0, 0, 0, 0, 0}], {7, x2, {0, 0, 0, 0, 0, 0, 0, 0}], {7, x3, {0, 0, 0, 0, 0, 0, 0, 0}], {7, x4, {0, 0, 0, 0, 0, 0, 0, 0}],
{7, x5, {0, 0, 0, 0, 0, 0, 0, 0}], {7, x6, {0, 0, 0, 0, 0, 0, 0, 0}], {7, x7, {0, 0, 0, 0, 0, 0, 0, 0}}}

In[=]:=
Block[{eAα, eαA, t, ginv, Γ, gg (*, ωω*)},
(*ωω = ωμ^(a)_(b) /. swf; *)
gg = g4488 /. sg;
{ginv, Γ} = con[gg];
eαA = eα^(A) /. sge^(A); eAα = eA^(A) /. sge^(A);
t = Table[ Block[{v, V, contra, covari},
V = eAα[b, All];
v = eαA[All, b];
contra =
Table[D[v[k], X[μ]] + Sum[v[i] × Γ[k, i, μ], {i, 1, DIM8}], {k, 1, DIM8}];
(*no ω terms*)
covari = Table[D[V[k], X[μ]] - Sum[V[i] × Γ[i, k, μ], {i, 1, DIM8}] + Sum[
eAα[J, All][k] × ωmat[μ, b, J], {J, 1, DIM8}], {k, 1, DIM8}]; (*ω terms*)
{b - 1, X[μ]}, {{contra}, {covari}}]], {b, 1, Length[eαA]}, {μ, 1, DIM8}];
t]
FullSimplify[#/ . ssqm4488,
constraintVars && e^2 a4[H x4] > 0 && e^a4[H x4] > 0 && e^-2 a4[H x4] > 0 && e^-a4[H x4] > 0] & /@ %

Out[=]=
{{{{0, x0, {{{{0, 0, 0, 0, 0, -g[0][0]^(0,1)[x0, x4]
2 √g[0][0][x0, x4] g[4][4][x0, x4], 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0}}}}}, {{{0, x1, {{{0, g[1][1]^(1,0)[x0, x4]
2 √g[0][0][x0, x4] g[1][1][x0, x4], 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0}}}}}, {{{0, x2, {{{0, 0, g[2][2]^(1,0)[x0, x4]
2 √g[0][0][x0, x4] g[2][2][x0, x4], 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0}}}}}, {{{0, x3, {{{0, 0, g[3][3]^(1,0)[x0, x4]
2 √g[0][0][x0, x4] g[3][3][x0, x4], 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0}}}}}, {{{0, x4, {{{0, 0, g[4][4]^(1,0)[x0, x4]
2 √g[0][0][x0, x4] g[4][4][x0, x4], 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0}}}}}, {{{0, x5, {{{0, 0, g[5][5]^(1,0)[x0, x4]
2 √g[0][0][x0, x4] g[5][5][x0, x4], 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0}}}}}, {{{0, x6, {{{0, 0, g[6][6]^(1,0)[x0, x4]
2 √g[0][0][x0, x4] g[6][6][x0, x4], 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0}}}}}, {{{0, x7, {{{0, 0, g[7][7]^(1,0)[x0, x4]
2 √g[0][0][x0, x4] g[7][7][x0, x4], 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0}}}}}
}
```



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{2, x5, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}},  

{2, x6, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}},  

{2, x7, {{{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}},  

{3, x0, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}},  

{3, x1, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}},  

{3, x2, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}},  

{3, x3, {{{-g[3][3]^(1,0)[x0,x4]}/{2 g[0][0][x0,x4] sqrt[g[3][3][x0,x4]]}, 0, 0, 0,  

-g[3][3]^(0,1)[x0,x4]/{2 sqrt[g[3][3][x0,x4]] g[4][4][x0,x4]}, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{3, x4, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}},  

{3, x5, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}},  

{3, x6, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}},  

{3, x7, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}},  

{4, x0, {{-g[0][0]^(0,1)[x0,x4]/{2 g[0][0][x0,x4] sqrt[-g[4][4][x0,x4]]}, 0, 0, 0,  

g[4][4]^(1,0)[x0,x4]/(2 (-g[4][4][x0,x4])^(3/2) + 2 sqrt[-g[4][4][x0,x4]] g[4][4][x0,x4]), 0, 0, 0}}},  

{{0, 0, 0, 0, -g[4][4]^(1,0)[x0,x4]/(2 sqrt[-g[4][4][x0,x4]]), -g[4][4]^(1,0)[x0,x4] sqrt[-g[4][4][x0,x4]]/(2 g[4][4][x0,x4]) -  

(g[4][4]^(1,0)[x0,x4]/(2 (-g[4][4][x0,x4])^(3/2) + 2 sqrt[-g[4][4][x0,x4]] g[4][4][x0,x4]))}, g[4][4][x0,x4], 0, 0, 0}}}},  

{4, x1, {{0, g[1][1]^(0,1)[x0,x4]/(2 g[1][1][x0,x4] sqrt[-g[4][4][x0,x4]]), 0, 0, 0, 0, 0, 0, 0}}},  

{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{4, x2, {{0, 0, g[2][2]^(0,1)[x0,x4]/(2 g[2][2][x0,x4] sqrt[-g[4][4][x0,x4]]), 0, 0, 0, 0, 0, 0}}},  

{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{4, x3, {{0, 0, 0, g[3][3]^(0,1)[x0,x4]/(2 g[3][3][x0,x4] sqrt[-g[4][4][x0,x4]]), 0, 0, 0, 0, 0}}},  

{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{4, x4, {{-g[4][4]^(1,0)[x0,x4]/{2 g[0][0][x0,x4] sqrt[-g[4][4][x0,x4]]}, 0, 0, 0,

```


$$\begin{aligned}
& g[5][5][x0, x4], 0, 0 \} \} \} \}, \\
& \left\{ 5, x5, \left\{ \left\{ -\frac{g[5][5]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{g[5][5]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[5][5][x0, x4]}}, 0, 0, 0 \right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \}, \\
& \{5, x6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{5, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \left\{ 6, x0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0 \right\} \right\} \right\} \right\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}} - \right. \right. \\
& \quad \left. \left. \frac{g[6][6]^{(1,0)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]} - \left(\frac{g[6][6]^{(1,0)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{g[6][6]^{(1,0)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) g[6][6][x0, x4], 0 \right\} \right\} \right\}, \\
& \{6, x1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{6, x3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \left\{ 6, x4, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]}, 0 \right\} \right\} \right\} \right\}, \\
& \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, -\frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]}} - \right. \right. \\
& \quad \left. \left. \frac{g[6][6]^{(0,1)}[x0, x4] \sqrt{-g[6][6][x0, x4]}}{2 g[6][6][x0, x4]} - \left(\frac{g[6][6]^{(0,1)}[x0, x4]}{2 (-g[6][6][x0, x4])^{3/2}} + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{g[6][6]^{(0,1)}[x0, x4]}{2 \sqrt{-g[6][6][x0, x4]} g[6][6][x0, x4]} \right) g[6][6][x0, x4], 0 \right\} \right\} \right\}, \\
& \{6, x5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \left\{ 6, x6, \left\{ \left\{ -\frac{g[6][6]^{(1,0)}[x0, x4]}{2 g[0][0][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0, \right. \right. \right. \\
& \quad \left. \left. \left. -\frac{g[6][6]^{(0,1)}[x0, x4]}{2 g[4][4][x0, x4] \sqrt{-g[6][6][x0, x4]}}, 0, 0, 0 \right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \right\}, \\
& \{6, x7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \left\{ 7, x_0, \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2(-g[7][7][x_0, x_4])^{3/2}} + \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \}, \\
& \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. - \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]}} - \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \frac{g[7][7]^{(1,0)}[x_0, x_4]\sqrt{-g[7][7][x_0, x_4]}}{2g[7][7][x_0, x_4]} - \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \left. \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\} \right\} \right\} \right\} \}, \\
& \{7, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_4, \left\{ \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2(-g[7][7][x_0, x_4])^{3/2}} + \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \}, \\
& \left\{ \left\{ \left\{ \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. - \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]}} - \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \frac{g[7][7]^{(0,1)}[x_0, x_4]\sqrt{-g[7][7][x_0, x_4]}}{2g[7][7][x_0, x_4]} - \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \left. \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2\sqrt{-g[7][7][x_0, x_4]} g[7][7][x_0, x_4]} \right\} \right\} \right\} \right\} \right\} \}, \\
& \{7, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\
& \{7, x_7, \left\{ \left\{ \left\{ \left\{ - \frac{g[7][7]^{(1,0)}[x_0, x_4]}{2g[0][0][x_0, x_4]\sqrt{-g[7][7][x_0, x_4]}}, 0, 0, 0, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. - \frac{g[7][7]^{(0,1)}[x_0, x_4]}{2g[4][4][x_0, x_4]\sqrt{-g[7][7][x_0, x_4]}}, 0, 0, 0 \right\} \right\} \right\} \right\} \right\} \right\} \right\} \\
& Out[8]= \\
& \left\{ \left\{ \left\{ \left\{ 0, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \{0, x_1, \{\{\{0, H, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \{0, x_2, \{\{\{0, 0, H, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \{0, x_3, \{\{\{0, 0, 0, H, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \{0, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \{0, x_5, \{\{\{0, 0, 0, 0, 0, H, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \{0, x_6, \{\{\{0, 0, 0, 0, 0, 0, H, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. \left. \left. \{0, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, H\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \right. \right. \right. \right. \right. \right. \right.
\end{aligned}$$

$$\left\{ \begin{array}{l} \{1, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x_1, \left\{ \begin{array}{l} \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \\ \left. \left. 3 \sqrt{e^{2 a4[H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \end{array} \right\}, \\ \{1, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{1, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x_2, \left\{ \begin{array}{l} \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \\ \left. \left. 3 \sqrt{e^{2 a4[H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \end{array} \right\}, \\ \{2, x_3, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{2, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_2, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_3, \left\{ \begin{array}{l} \left\{ \left\{ -\sqrt{e^{2 a4[H x4]}} H \operatorname{Sec}[6 H x0] \operatorname{Sin}[6 H x0]^{7/6}, 0, 0, 0, \right. \right. \\ \left. \left. 3 \sqrt{e^{2 a4[H x4]}} H \operatorname{Sin}[6 H x0]^{1/6} a4'[H x4], 0, 0, 0 \right\} \right\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\} \end{array} \right\}, \\ \{3, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_5, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{3, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{4, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{4, x_1, \{\{\{0, 3 H a4'[H x4], 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{4, x_2, \{\{\{0, 0, 3 H a4'[H x4], 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{4, x_3, \{\{\{0, 0, 0, 3 H a4'[H x4], 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{4, x_4, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{4, x_5, \{\{\{0, 0, 0, 0, 0, 0, -3 H a4'[H x4], 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{4, x_6, \{\{\{0, 0, 0, 0, 0, 0, 0, -3 H a4'[H x4], 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{4, x_7, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, -3 H a4'[H x4]\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{5, x_0, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \\ \{5, x_1, \{\{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}, \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\}\}\}\}, \end{array} \right.$$

```

{5, x2, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{5, x3, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{5, x4, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{5, x5, {{\sqrt{e^{-2 a4[Hx4]}} H Sec[6 Hx0] Sin[6 Hx0]^{7/6}, 0, 0, 0,  

  3 \sqrt{e^{-2 a4[Hx4]}} H Sin[6 Hx0]^{1/6} a4'[Hx4], 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{5, x6, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{5, x7, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{6, x0, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{6, x1, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{6, x2, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{6, x3, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{6, x4, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{6, x5, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{6, x6, {{\sqrt{e^{-2 a4[Hx4]}} H Sec[6 Hx0] Sin[6 Hx0]^{7/6}, 0, 0, 0,  

  3 \sqrt{e^{-2 a4[Hx4]}} H Sin[6 Hx0]^{1/6} a4'[Hx4], 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{6, x7, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{7, x0, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{7, x1, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{7, x2, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{7, x3, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{7, x4, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{7, x5, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{7, x6, {{{0, 0, 0, 0, 0, 0, 0, 0, 0}}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}},  

{7, x7, {{\sqrt{e^{-2 a4[Hx4]}} H Sec[6 Hx0] Sin[6 Hx0]^{7/6}, 0, 0, 0,  

  3 \sqrt{e^{-2 a4[Hx4]}} H Sin[6 Hx0]^{1/6} a4'[Hx4], 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0, 0}}}}}

```

```

In[1]:= (*\omega\Psi=Table[
  Sum[ExpandAll[(\omega_\mu^{(a)}_{(b)} [\mu,a,b] /. swf) Simplify[(Transpose[\Psi16].\sigma16.SAB[a,b].\Psi16)]],
  {a,1,8},{b,1,8}],{\mu,1,8}]*)

In[2]:= (*\omega\Psi=
  Table[Sum[ExpandAll[(wmat[\mu,a,b])] Simplify[(Transpose[\Psi16].\sigma16.SAB[a,b].\Psi16)]],{a,1,8},{b,1,8}],{\mu,1,8}]*)

In[3]:= (*j\omega\Psi=ParallelTable[{j,base16[[j,2]],
  Table[{X[\mu]},Simplify[(Transpose[\Psi16].\sigma16.(ZZZ0*T16^\alpha[0]+T16^\alpha[4]* ZZZ4 ) .
  base16[[j,1]].(Sum[wmat[\mu,a,b]SAB[a,b],{a,1,8},{b,1,8}])
  \Psi16)],{\mu,1,8}]},{j,1,Length[base16]]}]*)

```


In[•]:= SAB[[1, 2]]. Ψ 16

Out[•] =

$$\left\{ \begin{array}{l} \frac{1}{2} f16[7][x0, x4], \frac{1}{2} f16[6][x0, x4], -\frac{1}{2} f16[5][x0, x4], -\frac{1}{2} f16[4][x0, x4], \\ \frac{1}{2} f16[3][x0, x4], \frac{1}{2} f16[2][x0, x4], -\frac{1}{2} f16[1][x0, x4], -\frac{1}{2} f16[0][x0, x4], \\ -\frac{1}{2} f16[15][x0, x4], -\frac{1}{2} f16[14][x0, x4], \frac{1}{2} f16[13][x0, x4], \frac{1}{2} f16[12][x0, x4], \\ -\frac{1}{2} f16[11][x0, x4], -\frac{1}{2} f16[10][x0, x4], \frac{1}{2} f16[9][x0, x4], \frac{1}{2} f16[8][x0, x4] \end{array} \right\}$$

In[•]:= **T16^A[0]**

Out[•]=

In[•]:= Ψ16

Out[•] =

```
{f16[0][x0, x4], f16[1][x0, x4], f16[2][x0, x4], f16[3][x0, x4],
 f16[4][x0, x4], f16[5][x0, x4], f16[6][x0, x4], f16[7][x0, x4],
```

```
In[°]:= Sum[ExpandAll[( $\omega_{\mu(b)}^{(a)}$  [4, a, b] /. swfgtry) Simplify[(SAB[a, b])]], {a, 1, 8}, {b, 1, 8}]. $\Psi$ 16 // MatrixForm
```

```
Out[°]//MatrixForm=
```

$$\begin{pmatrix} -e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[5][x0, x4] \\ e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[4][x0, x4] \\ -e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[7][x0, x4] \\ e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[6][x0, x4] \\ -e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[1][x0, x4] \\ e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[0][x0, x4] \\ -e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[3][x0, x4] \\ e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[2][x0, x4] \\ e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[13][x0, x4] \\ -e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[12][x0, x4] \\ e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[15][x0, x4] \\ -e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[14][x0, x4] \\ e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[9][x0, x4] \\ -e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[8][x0, x4] \\ e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[11][x0, x4] \\ -e^{a4[Hx4]} H \sin[6Hx0]^{1/6} f16[10][x0, x4] \end{pmatrix}$$

```
In[°]:= (*Table[Block[{eA $\alpha$ , eA $A$ , t, V}, eA $A$ =e $^\alpha_{(A)}$  /. sgtrye $^\alpha_{(A)}$ ;  
eA $\alpha$ =e $^\alpha_{(A)}$  /. sgtrye $^\alpha_{(A)}$ ;  
V=eA $\alpha$ [[a,All]];  
t=Table[Block[{v,d},v=eA $A$ [[All,b]];  
d=D[v[[k]],X[[j]]]+Sum[v[[i]] $\Gamma$ [[k,i,j]],{i,1,DIM8]];  
d.V],{k,1,DIM8}];  
t],{j,1,DIM8},{a,1,Length[eA $A$ ]},{b,1,Length[eA $\alpha$ ]}]*)
```

```
In[°]:= Clear[rt];  
rt[g_, ass_ : {}] := Module[{t,  $\Gamma$ , Ricci $\Gamma$ , Rie $\Gamma$ , RS, G, ginv},  
Print[Now];  
t = AbsoluteTiming[  
ginv = FullSimplify[Inverse[g], ass];  
 $\Gamma$  = FullSimplify[Table[(1/2)*Sum[(ginv[[i, s]) *  
(D[g[[s, j]], X[[k]]] + D[g[[s, k]], X[[j]]] - D[g[[j, k]], X[[s]]]), {s, 1, DIM8}],  
{i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}], ass];  
Ricci $\Gamma$  =  
ParallelTable[FullSimplify[D[ $\Gamma$ [[ $\mu$ ,  $\nu$ ,  $\beta$ ]], X[[ $\alpha$ ]] - D[ $\Gamma$ [[ $\mu$ ,  $\nu$ ,  $\alpha$ ]], X[[ $\beta$ ]] +  
Sum[ $\Gamma$ [[ $\mu$ , s,  $\alpha$ ]  $\times$   $\Gamma$ [[s,  $\nu$ ,  $\beta$ ] -  $\Gamma$ [[ $\mu$ , s,  $\beta$ ]  $\times$   $\Gamma$ [[s,  $\nu$ ,  $\alpha$ ]], {s, 1, DIM8}], ass],  
{ $\mu$ , 1, DIM8}, { $\nu$ , 1, DIM8}, { $\alpha$ , 1, DIM8}, { $\beta$ , 1, DIM8}]];  
Rie $\Gamma$  = ParallelTable[ FullSimplify[  
Sum[ Ricci $\Gamma$ [[ $\alpha$ ,  $\mu$ ,  $\alpha$ ,  $\nu$ ], { $\alpha$ , 1, DIM8}], ass], { $\mu$ , 1, DIM8}, { $\nu$ , 1, DIM8}];  
RS = FullSimplify[Tr[ ginv.Rie $\Gamma$ ], ass];
```

```
G = ParallelTable[
  FullSimplify[Rie $\Gamma$ [\mathbf{\alpha}] -  $\frac{1}{2}$  g[\mathbf{\alpha}] RS, ass], {\mathbf{\alpha}, 1, Length[Rie $\Gamma$ ] }];
Print[t];
Print[Now];
Return[{ginv,  $\Gamma$ , Ricci $\Gamma$ , Rie $\Gamma$ , RS, G}]]
```

verify $g_{\mu\nu|\alpha} = 0$:

```
In[=]:= Simplify[Table[D[gtry[j, k], X[i]] - Sum[gtry[k, s]  $\times$   $\Gamma$ [s, i, j], {s, 1, DIM8}] -
  Sum[gtry[j, s]  $\times$   $\Gamma$ [s, i, k], {s, 1, DIM8}],
  {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union
```

Out[=]=
 $\{0\}$

CHECK

In[=]:= MatrixMetric44 // MatrixForm

Out[=]//MatrixForm=

$$\begin{pmatrix} \text{Cot}[6Hx0]^2 & 0 & 0 & 0 & 0 \\ 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 & 0 \\ 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 & 0 \\ 0 & 0 & 0 & e^{2a4[Hx4]} \sin[6Hx0]^{1/3} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[=]:= $(e_{(A)}^\alpha . \text{MatrixMetric44}.e_{(A)}^\alpha / . \text{sgtrye}_{(A)}^\alpha) // \text{MatrixForm}$

Out[=]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[=]:= Transpose[e_\alpha^{(A)}].\eta4488.e_\alpha^{(A)} - MatrixMetric44 /. sgtrye_\alpha^{(A)} //

FullSimplify[#, constraintVars] &

Out[=]=
 $\{ \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$
 $\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$
 $\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\} \}$

```

In[•]:= Transpose[e(A)].MatrixMetric44.e(A) - η4488 /. sgtrye(A) // FullSimplify[#, constraintVars] &
Out[•]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
In[•]:= Inverse[e(A) /. sgtrye(A)] == e(A) /. sgtrye(A) // FullSimplify[#, constraintVars] &
Out[•]= True

In[•]:= Block[{a, b, c, d, m}, m = {{a, b}, {c, d}}; Inverse[Transpose[m]] - Transpose[Inverse[m]]]
Out[•]= {{0, 0}, {0, 0}}

In[•]:= (*Inverse[Transpose[e(A)].MatrixMetric44.e(A)] ==
e(A).Inverse[MatrixMetric44].Transpose[e(A)] //FullSimplify[#,constraintVars]&*)
In[•]:= (*e(A).Inverse[MatrixMetric44].Transpose[e(A)] //FullSimplify[#,constraintVars]&
-Inverse[η4488] //Union[Flatten[#]]&*)
In[•]:= (*Transpose[e(A)] ==e(A) //FullSimplify[#,constraintVars]&*)

In[•]:= (* (FullSimplify[e(A).Inverse[MatrixMetric44].Transpose[e(A)] -Inverse[η4488] ,
constraintVars] //Union[Flatten[#]]&) //FullSimplify[#,constraintVars]&*)

In[•]:= (* (FullSimplify[Transpose[e(A)].Inverse[MatrixMetric44].e(A),constraintVars]-
Inverse[η4488] //Union[Flatten[#]]&) //FullSimplify[#,constraintVars]&*)

In[•]:= (* (FullSimplify[Transpose[e(A)].ginv.e(A),constraintVars]-Inverse[η4488] //
Union[Flatten[#]]&) //FullSimplify[#,constraintVars]&*)

In[•]:= (E(A) = Table[Sum[e(A)[[A1, α1] ginv[[α1, β1]] - η4488[A1, B1] /. sgtrye(A),
{α1, 1, 8}, {A1, 1, 8}], {β1, 1, 8}], {B1, 1, 8}] // FullSimplify[#, constraintVars] &) // Column
Out[•]= {Tan[6 H x0], 0, 0, 0, 0, 0, 0, 0, 0}
{0, e-a4[x4]/Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0, 0}
{0, 0, e-a4[x4]/Sin[6 H x0]1/6, 0, 0, 0, 0, 0, 0}
{0, 0, 0, e-a4[x4]/Sin[6 H x0]1/6, 0, 0, 0, 0, 0}
{0, 0, 0, 0, 1, 0, 0, 0, 0}
{0, 0, 0, 0, 0, ea4[x4]/Sin[6 H x0]1/6, 0, 0, 0}

```

$$\left\{ 0, 0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}}, 0 \right\}$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[x4]}}{\sin[6Hx0]^{1/6}} \right\}$$

Lagrangian $\Psi_{16} =$

$$\kappa \sqrt{\text{Det}[g_{\mu\nu}]} \text{Transpose}[\Psi_{16}] . \sigma_{16}.T_{16}^A[A1]. \Psi_{16,\alpha} g^{-1}{}^{\alpha\beta} \eta_{A1B1} E_B^{B1} + \text{mass term}$$

In[•]:= ass = constraintVars

$$\begin{aligned} Out[•]= & x0 > 0 \&& x4 > 0 \&& H > 0 \&& 6Hx0 > 0 \&& 3Hx4 > 0 \&& a4[x4] > 0 \&& A4[t] > 0 \&& \\ & Q > 0 \&& z > 0 \&& t > 0 \&& M > 0 \&& K > 0 \&& e^{-2a4[Hx4]} > 0 \&& e^{-a4[Hx4]} > 0 \&& e^{2a4[Hx4]} > 0 \&& \\ & e^{a4[Hx4]} > 0 \&& \sin[6Hx0] > 0 \&& \cos[6Hx0] > 0 \&& \csc[6Hx0] > 0 \&& \sec[6Hx0] > 0 \&& \\ & \tan[6Hx0] > 0 \&& \cot[6Hx0] > 0 \&& \sin[z] > 0 \&& \sqrt{\sin[z]} > 0 \&& \sin[z]^{3/2} > 0 \&& \\ & \sqrt{\sin[z]} > 0 \&& \frac{1}{\sin[z]^{3/2}} > 0 \&& \frac{1}{\sqrt{\sin[z]}} > 0 \&& \cot[z] > 0 \&& \sqrt{\cot[z]} > 0 \&& \\ & \cot[z]^{3/2} > 0 \&& \sqrt{\cot[z]} > 0 \&& \tan[z] > 0 \&& \sec[z] > 0 \&& \sqrt{\csc[z]} > 0 \&& \csc[z] > 0 \\ \\ In[•]:= & x0 > 0 \&& x4 > 0 \&& H > 0 \&& 6Hx0 > 0 \&& Hx4 > 0 \&& a4[x4] > 0 \&& A4[t] > 0 \&& Q > 0 \&& \\ & z > 0 \&& t > 0 \&& M > 0 \&& K > 0 \&& e^{-2a4[Hx4]} > 0 \&& e^{-a4[Hx4]} > 0 \&& e^{2a4[Hx4]} > 0 \&& \\ & e^{a4[Hx4]} > 0 \&& \sin[6Hx0] > 0 \&& \cos[6Hx0] > 0 \&& \csc[6Hx0] > 0 \&& \sec[6Hx0] > 0 \&& \\ & \tan[6Hx0] > 0 \&& \cot[6Hx0] > 0 \&& \sin[z] > 0 \&& \sqrt{\sin[z]} > 0 \&& \sin[z]^{3/2} > 0 \&& \\ & \sqrt{\sin[z]} > 0 \&& \frac{1}{\sin[z]^{3/2}} > 0 \&& \frac{1}{\sqrt{\sin[z]}} > 0 \&& \cot[z] > 0 \&& \sqrt{\cot[z]} > 0 \&& \\ & \cot[z]^{3/2} > 0 \&& \sqrt{\cot[z]} > 0 \&& \tan[z] > 0 \&& \sec[z] > 0 \&& \sqrt{\csc[z]} > 0 \&& \csc[z] > 0 \end{aligned}$$

$$\begin{aligned} Out[•]= & x0 > 0 \&& x4 > 0 \&& H > 0 \&& 6Hx0 > 0 \&& Hx4 > 0 \&& a4[x4] > 0 \&& A4[t] > 0 \&& \\ & Q > 0 \&& z > 0 \&& t > 0 \&& M > 0 \&& K > 0 \&& e^{-2a4[Hx4]} > 0 \&& e^{-a4[Hx4]} > 0 \&& e^{2a4[Hx4]} > 0 \&& \\ & e^{a4[Hx4]} > 0 \&& \sin[6Hx0] > 0 \&& \cos[6Hx0] > 0 \&& \csc[6Hx0] > 0 \&& \sec[6Hx0] > 0 \&& \\ & \tan[6Hx0] > 0 \&& \cot[6Hx0] > 0 \&& \sin[z] > 0 \&& \sqrt{\sin[z]} > 0 \&& \sin[z]^{3/2} > 0 \&& \\ & \sqrt{\sin[z]} > 0 \&& \frac{1}{\sin[z]^{3/2}} > 0 \&& \frac{1}{\sqrt{\sin[z]}} > 0 \&& \cot[z] > 0 \&& \sqrt{\cot[z]} > 0 \&& \\ & \cot[z]^{3/2} > 0 \&& \sqrt{\cot[z]} > 0 \&& \tan[z] > 0 \&& \sec[z] > 0 \&& \sqrt{\csc[z]} > 0 \&& \csc[z] > 0 \end{aligned}$$

In[•]:= ginv = FullSimplify[Inverse[gtry], ass]

$$Out[•]= \text{True}$$

In[•]:= ginv === Transpose[ginv]

$$Out[•]= \text{True}$$

verify metric covariant derivative again:

```
In[•]:= Simplify[Table[D[gtry[[j, k]], X[[i]]] - Sum[gtry[[k, s]] \[Cross] \[Gamma][[s, i, j]], {s, 1, DIM8}] - Sum[gtry[[j, s]] \[Cross] \[Gamma][[s, i, k]], {s, 1, DIM8}], {i, 1, DIM8}, {j, 1, DIM8}, {k, 1, DIM8}]] // Flatten // Union
```

```
Out[•]= {0}
```

```
In[•]:= RS
```

```
Out[•]= 6 H2 (-7 + a4' [H x4]2)
```

```
In[•]:= EinsteinG
skelx[%]
```

```
Out[•]= {{-3 H2 Cot[6 H x0]2 (-5 + a4' [H x4]2), 0, 0, 0, 0, 0, 0, 0, 0}, {0, -e-2 a4 [H x4] H2 Sin[6 H x0]1/3 (-15 + 3 a4' [H x4]2 - a4'' [H x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 0, -e2 a4 [H x4] H2 Sin[6 H x0]1/3 (-15 + 3 a4' [H x4]2 - a4'' [H x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -e-2 a4 [H x4] H2 Sin[6 H x0]1/3 (-15 + 3 a4' [H x4]2 - a4'' [H x4]), 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -3 H2 (7 + a4' [H x4]2), 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, e-2 a4 [H x4] H2 Sin[6 H x0]1/3 (-15 + 3 a4' [H x4]2 + a4'' [H x4]), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, e-2 a4 [H x4] H2 Sin[6 H x0]1/3 (-15 + 3 a4' [H x4]2 + a4'' [H x4]), 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, e-2 a4 [H x4] H2 Sin[6 H x0]1/3 (-15 + 3 a4' [H x4]2 + a4'' [H x4])}}}
```

```
Out[•]//MatrixForm=
```

$$\begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{pmatrix}$$

```
Union[
```

```
Table[Sum[\[Tau][A].bas64[[k, 1]].\sigma.\[Tau][A] \[Cross] n8[[A, A]], {A, 1, 8}] - ID8 Tr[bas64[[k, 1]].\sigma] + 4 eg[[k]] \frac{bas64[[k, 1]] - Transpose[bas64[[k, 1]]]}{2} . \sigma, {k, 1, 64}]]
```

```
{}{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

BASIS of 16 × 16 matrices :

```
In[•]:= 16 * 16
```

```
Out[•]=
```

```
256
```

```

In[1]:= 2^8
Out[1]= 256

In[2]:= Sum[Binomial[8, p], {p, 0, 8}]
Out[2]= 256

In[3]:= Id16 == ID16 == Dot @@ (T16^A[#] & /@ Range[0, 8])
Out[3]= True

In[4]:= Clear[t160]; t160 = {{Id16, {Range[0, 8]}}}; Length[t160]
Clear[t16A];
t16A = {};
Do[AppendTo[t16A, {T16^A[j], {j}}], {j, 0, 7}];
Length[t16A]
Clear[t16AB];
t16AB = {};
Do[AppendTo[t16AB, {T16^A[j].T16^A[k], {j, k}}], {j, 0, 6}, {k, j + 1, 7}];
Length[t16AB]
Clear[t16ABC];
t16ABC = {};
Do[AppendTo[t16ABC, {T16^A[j].T16^A[k].T16^A[h], {j, k, h}}], {j, 0, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[t16ABC]
Clear[t16ABCD];
t16ABCD = {};
Do[AppendTo[t16ABCD, {T16^A[j].T16^A[k].T16^A[h].T16^A[i], {j, k, h, i}}], {j, 0, 4}, {k, j + 1, 5}, {h, k + 1, 6}, {i, h + 1, 7}];
Length[t16ABCD]
Clear[t16ABCDE];
t16ABCDE = {};
Do[AppendTo[t16ABCDE, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l], {j, k, h, i, l}}], {j, 0, 3}, {k, j + 1, 4}, {h, k + 1, 5}, {i, h + 1, 6}, {l, i + 1, 7}];
Length[t16ABCDE]
Clear[t16ABCDEF];
t16ABCDEF = {};
Do[AppendTo[t16ABCDEF,
{T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q], {j, k, h, i, l, q}}], {j, 0, 2}, {k, j + 1, 3}, {h, k + 1, 4}, {i, h + 1, 5}, {l, i + 1, 6}, {q, l + 1, 7}];
Length[t16ABCDEF]
Clear[t16ABCDEFG];

```

```

t16ABCDEFG = {};
Do[AppendTo[t16ABCDEFG, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].T16^A[r],
{j, k, h, i, l, q, r}]], {j, 0, 1}, {k, j + 1, 2},
{h, k + 1, 3}, {i, h + 1, 4}, {l, i + 1, 5}, {q, l + 1, 6}, {r, q + 1, 7}];
Length[t16ABCDEFG]
Clear[t16ABCDEFGH];
t16ABCDEFGH = {};
Do[AppendTo[t16ABCDEFGH, {T16^A[j].T16^A[k].T16^A[h].T16^A[i].T16^A[l].T16^A[q].T16^A[r].
T16^A[s], {j, k, h, i, l, q, r, s}]], {j, 0, 1}, {k, j + 1, 1}, {h, k + 1, 2},
{i, h + 1, 3}, {l, i + 1, 4}, {q, l + 1, 5}, {r, q + 1, 6}, {s, r + 1, 7}];
Length[t16ABCDEFGH]
% + %% + %%% + %%%% + %%%%% + %%%%%% + %%%%%%% + %%%%%%%%

```

Out[•]=

1

Out[•]=

8

Out[•]=

28

Out[•]=

56

Out[•]=

70

Out[•]=

56

Out[•]=

28

Out[•]=

8

Out[•]=

1

Outro

2

```
In[•]:= (*base16=Flatten[  
    {t160,t16A,t16AB,t16ABC,t16ABCD,t16ABCDE,t16ABCDEF,t16ABCDEFG,t16ABCDEFGH},1];  
Length[base16]*)
```

```
In[6]:= base16 = Flatten[{t16A, t16AB, t16ABC, t16ABCD,  
    t16ABCDE, t16ABCDEF, t16ABCDEFG, t16ABCDEFGH, t160}, 1];  
Length[base16]
```

Out[•] =

256


```

In[®]:= (*BASE16[-1]*)
In[®]:= (*BASE16[-2])
%[[1]]==BASE16[-1][[1]]*)

In[®]:= 16 * (16 + 1) / 2
16 * (16 - 1) / 2
% + %%

Out[®]= 136

Out[®]= 120

Out[®]= 256

In[®]:= Clear[antisymm16];
antisymm16 = {};
Do[If[base16[[k, 1]] === -Transpose[base16[[k, 1]]], AppendTo[antisymm16,
{base16[[k, 1]], {{k}, base16[[k, 2]]}}], {k, 1, Length[base16]}];
Length[antisymm16]
antisymm16[%]

Out[®]= 120

Out[®]= {{ {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1}, {-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0}}, {{254}, {1, 2, 3, 4, 5, 6, 7}}}

```


In[8]:= **#[[2]] & /@ symm16**

Out[8]=

```

{{{{1}, {0}}, {{2}, {1}}, {{3}, {2}}, {{4}, {3}}, {{12}, {0, 4}}, {{13}, {0, 5}},
{{14}, {0, 6}}, {{15}, {0, 7}}, {{18}, {1, 4}}, {{19}, {1, 5}}, {{20}, {1, 6}},
{{21}, {1, 7}}, {{23}, {2, 4}}, {{24}, {2, 5}}, {{25}, {2, 6}}, {{26}, {2, 7}},
{{27}, {3, 4}}, {{28}, {3, 5}}, {{29}, {3, 6}}, {{30}, {3, 7}}, {{39}, {0, 1, 4}},
{{40}, {0, 1, 5}}, {{41}, {0, 1, 6}}, {{42}, {0, 1, 7}}, {{44}, {0, 2, 4}}, {{45}, {0, 2, 5}},
{{46}, {0, 2, 6}}, {{47}, {0, 2, 7}}, {{48}, {0, 3, 4}}, {{49}, {0, 3, 5}},
{{50}, {0, 3, 6}}, {{51}, {0, 3, 7}}, {{59}, {1, 2, 4}}, {{60}, {1, 2, 5}},
{{61}, {1, 2, 6}}, {{62}, {1, 2, 7}}, {{63}, {1, 3, 4}}, {{64}, {1, 3, 5}},
{{65}, {1, 3, 6}}, {{66}, {1, 3, 7}}, {{73}, {2, 3, 4}}, {{74}, {2, 3, 5}},
{{75}, {2, 3, 6}}, {{76}, {2, 3, 7}}, {{89}, {4, 5, 6}}, {{90}, {4, 5, 7}},
{{91}, {4, 6, 7}}, {{92}, {5, 6, 7}}, {{93}, {0, 1, 2, 3}}, {{102}, {0, 1, 4, 5}},
{{103}, {0, 1, 4, 6}}, {{104}, {0, 1, 4, 7}}, {{105}, {0, 1, 5, 6}}, {{106}, {0, 1, 5, 7}},
{{107}, {0, 1, 6, 7}}, {{112}, {0, 2, 4, 5}}, {{113}, {0, 2, 4, 6}}, {{114}, {0, 2, 4, 7}},
{{115}, {0, 2, 5, 6}}, {{116}, {0, 2, 5, 7}}, {{117}, {0, 2, 6, 7}}, {{118}, {0, 3, 4, 5}},
{{119}, {0, 3, 4, 6}}, {{120}, {0, 3, 4, 7}}, {{121}, {0, 3, 5, 6}}, {{122}, {0, 3, 5, 7}},
{{123}, {0, 3, 6, 7}}, {{132}, {1, 2, 4, 5}}, {{133}, {1, 2, 4, 6}}, {{134}, {1, 2, 4, 7}},
{{135}, {1, 2, 5, 6}}, {{136}, {1, 2, 5, 7}}, {{137}, {1, 2, 6, 7}}, {{138}, {1, 3, 4, 5}},
{{139}, {1, 3, 4, 6}}, {{140}, {1, 3, 4, 7}}, {{141}, {1, 3, 5, 6}}, {{142}, {1, 3, 5, 7}},
{{143}, {1, 3, 6, 7}}, {{148}, {2, 3, 4, 5}}, {{149}, {2, 3, 4, 6}}, {{150}, {2, 3, 4, 7}},
{{151}, {2, 3, 5, 6}}, {{152}, {2, 3, 5, 7}}, {{153}, {2, 3, 6, 7}}, {{162}, {4, 5, 6, 7}},
{{167}, {0, 1, 2, 4, 5}}, {{168}, {0, 1, 2, 4, 6}}, {{169}, {0, 1, 2, 4, 7}},
{{170}, {0, 1, 2, 5, 6}}, {{171}, {0, 1, 2, 5, 7}}, {{172}, {0, 1, 2, 6, 7}},
{{173}, {0, 1, 3, 4, 5}}, {{174}, {0, 1, 3, 4, 6}}, {{175}, {0, 1, 3, 4, 7}},
{{176}, {0, 1, 3, 5, 6}}, {{177}, {0, 1, 3, 5, 7}}, {{178}, {0, 1, 3, 6, 7}},
{{183}, {0, 2, 3, 4, 5}}, {{184}, {0, 2, 3, 4, 6}}, {{185}, {0, 2, 3, 4, 7}},
{{186}, {0, 2, 3, 5, 6}}, {{187}, {0, 2, 3, 5, 7}}, {{188}, {0, 2, 3, 6, 7}},
{{197}, {0, 4, 5, 6, 7}}, {{198}, {1, 2, 3, 4, 5}}, {{199}, {1, 2, 3, 4, 6}},
{{200}, {1, 2, 3, 4, 7}}, {{201}, {1, 2, 3, 5, 6}}, {{202}, {1, 2, 3, 5, 7}},
{{203}, {1, 2, 3, 6, 7}}, {{212}, {1, 4, 5, 6, 7}}, {{217}, {2, 4, 5, 6, 7}},
{{218}, {3, 4, 5, 6, 7}}, {{225}, {0, 1, 2, 4, 5, 6}}, {{226}, {0, 1, 2, 4, 5, 7}},
{{227}, {0, 1, 2, 4, 6, 7}}, {{228}, {0, 1, 2, 5, 6, 7}}, {{229}, {0, 1, 3, 4, 5, 6}},
{{230}, {0, 1, 3, 4, 5, 7}}, {{231}, {0, 1, 3, 4, 6, 7}}, {{232}, {0, 1, 3, 5, 6, 7}},
{{234}, {0, 2, 3, 4, 5, 6}}, {{235}, {0, 2, 3, 4, 5, 7}}, {{236}, {0, 2, 3, 4, 6, 7}},
{{237}, {0, 2, 3, 5, 6, 7}}, {{240}, {1, 2, 3, 4, 5, 6}}, {{241}, {1, 2, 3, 4, 5, 7}},
{{242}, {1, 2, 3, 4, 6, 7}}, {{243}, {1, 2, 3, 5, 6, 7}}, {{247}, {0, 1, 2, 3, 4, 5, 6}},
{{248}, {0, 1, 2, 3, 4, 5, 7}}, {{249}, {0, 1, 2, 3, 4, 6, 7}}, {{250}, {0, 1, 2, 3, 5, 6, 7}},
{{255}, {0, 1, 2, 3, 4, 5, 6, 7}}, {{256}, {{0, 1, 2, 3, 4, 5, 6, 7, 8}}}}

```

In[9]:= **$\sigma16 = T16^A[0].T16^A[1].T16^A[2].T16^A[3]$**

Out[9]=

True

In[10]:= **$\sigma16 == base16[[93]][[1]]$**

Out[10]=

True

```
In[1]:= Select[symmm16, #[[2]][1][1] == 250 & → "Index"]
Out[1]= {134}
```

```
In[1]:= T16A  
Out[1]= T16A
```

```
In[®]:= σ16.T16A[5].T16A[6].T16A[7] == symm16[[134]][1]
Out[®]= True
```

```
In[6]:= T16A[8] == base16[[255][1]]  
Out[6]= True
```

```
In[1]:= Select[symm16, #[[2]][[1]][[1]] == 255 &] // "Index"
Out[1]= {135}
```

similarly :

```
In[®]:= coupled1 = {4, 5, 0, 9, 9, 12, 4, 5, 1, 8, 8, 13} // Union
Out[®]= {0, 1, 4, 5, 8, 9, 12, 13}
```

```
In[1]:= coupled2 = Complement[Range[0, 15], coupled1]
Out[1]= {2, 3, 6, 7, 10, 11, 14, 15}
```


In[•]:= **linTrans // MatrixForm**

Out[•]//MatrixForm=

```
In[•]:= linTrans.Transpose[linTrans] // FullSimplify // MatrixForm
```

Out[•]//MatrixForm=

```

In[1]:= linTrans.¶16 // Column
Union[%[[1]]]

Out[1]=
f16[0] [x0, x4]
f16[1] [x0, x4]
f16[8] [x0, x4]
f16[9] [x0, x4]
f16[2] [x0, x4]
f16[3] [x0, x4]
f16[10] [x0, x4]
f16[11] [x0, x4]
f16[4] [x0, x4]
f16[5] [x0, x4]
f16[12] [x0, x4]
f16[13] [x0, x4]
f16[6] [x0, x4]
f16[7] [x0, x4]
f16[14] [x0, x4]
f16[15] [x0, x4]

Out[2]=
{f16[0] [x0, x4], f16[1] [x0, x4], f16[2] [x0, x4], f16[3] [x0, x4],
 f16[4] [x0, x4], f16[5] [x0, x4], f16[6] [x0, x4], f16[7] [x0, x4],
 f16[8] [x0, x4], f16[9] [x0, x4], f16[10] [x0, x4], f16[11] [x0, x4],
 f16[12] [x0, x4], f16[13] [x0, x4], f16[14] [x0, x4], f16[15] [x0, x4]}

In[3]:= positiveTrMM = Select[base16, Tr[#[[1]] . #[[1]]] > 0 & -> "Index"]

Out[3]=
{1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59,
 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116,
 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151,
 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198,
 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242,
 243, 247, 248, 249, 250, 255, 256}

In[4]:= negativeTrMM = Select[base16, Tr[#[[1]] . #[[1]]] < 0 & -> "Index"]

Out[4]=
{5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77,
 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128,
 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182,
 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221,
 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[5]:= {Length[positiveTrMM], Length[negativeTrMM]}

Out[5]=
{136, 120}

```

```
In[8]:= positiveTrMM = Select[base16, Tr[#[[1]].#[[1]]] > 0 & → "Index"]
Out[8]= {1, 2, 3, 4, 12, 13, 14, 15, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 89, 90, 91, 92, 93, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 197, 198, 199, 200, 201, 202, 203, 212, 217, 218, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 255, 256}

In[9]:= negativeTrMM = Select[base16, Tr[#[[1]].#[[1]]] < 0 & → "Index"]
Out[9]= {5, 6, 7, 8, 9, 10, 11, 16, 17, 22, 31, 32, 33, 34, 35, 36, 37, 38, 43, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 144, 145, 146, 147, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 179, 180, 181, 182, 189, 190, 191, 192, 193, 194, 195, 196, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 233, 238, 239, 244, 245, 246, 251, 252, 253, 254}

In[10]:= {Length[positiveTrMM], Length[negativeTrMM]}
Out[10]= {136, 120}
```

```

In[=]:= base16[[#]] & /@ positiveTrMM;
{#[[2]], Tr[Transpose[#[[1]]].linTrans]} & /@ %
Select[%, #[[2]] ≠ 0 &]
linTransSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];
Out[=]=
{{{0}, 0}, {{1}, 0}, {{2}, 0}, {{3}, 0}, {{0, 4}, 0}, {{0, 5}, 0}, {{0, 6}, 0}, {{0, 7}, -4},
{{1, 4}, 0}, {{1, 5}, 0}, {{1, 6}, 0}, {{1, 7}, 0}, {{2, 4}, 0}, {{2, 5}, 0}, {{2, 6}, 0},
{{2, 7}, 0}, {{3, 4}, -4}, {{3, 5}, 0}, {{3, 6}, 0}, {{3, 7}, 0}, {{0, 1, 4}, 0},
{{0, 1, 5}, 0}, {{0, 1, 6}, 0}, {{0, 1, 7}, 0}, {{0, 2, 4}, 0}, {{0, 2, 5}, 0}, {{0, 2, 6}, 0},
{{0, 2, 7}, 0}, {{0, 3, 4}, 0}, {{0, 3, 5}, 0}, {{0, 3, 6}, 0}, {{0, 3, 7}, 0},
{{1, 2, 4}, 0}, {{1, 2, 5}, 0}, {{1, 2, 6}, 0}, {{1, 2, 7}, 0}, {{1, 3, 4}, 0},
{{1, 3, 5}, 0}, {{1, 3, 6}, 0}, {{1, 3, 7}, 0}, {{2, 3, 4}, 0}, {{2, 3, 5}, 0},
{{2, 3, 6}, 0}, {{2, 3, 7}, 0}, {{4, 5, 6}, -4}, {{4, 5, 7}, 0}, {{4, 6, 7}, 0},
{{5, 6, 7}, 0}, {{0, 1, 2, 3}, 0}, {{0, 1, 4, 5}, 0}, {{0, 1, 4, 6}, 0}, {{0, 1, 4, 7}, 0},
{{0, 1, 5, 6}, 0}, {{0, 1, 5, 7}, 0}, {{0, 1, 6, 7}, 0}, {{0, 2, 4, 5}, 0}, {{0, 2, 4, 6}, 0},
{{0, 2, 4, 7}, 0}, {{0, 2, 5, 6}, 0}, {{0, 2, 5, 7}, 0}, {{0, 2, 6, 7}, 0}, {{0, 3, 4, 5}, 0},
{{0, 3, 4, 6}, 0}, {{0, 3, 4, 7}, 4}, {{0, 3, 5, 6}, 0}, {{0, 3, 5, 7}, 0}, {{0, 3, 6, 7}, 0},
{{1, 2, 4, 5}, 0}, {{1, 2, 4, 6}, 0}, {{1, 2, 4, 7}, 0}, {{1, 2, 5, 6}, 0}, {{1, 2, 5, 7}, 0},
{{1, 2, 6, 7}, -4}, {{1, 3, 4, 5}, 0}, {{1, 3, 4, 6}, 0}, {{1, 3, 4, 7}, 0}, {{1, 3, 5, 6}, 0},
{{1, 3, 5, 7}, 0}, {{1, 3, 6, 7}, 0}, {{2, 3, 4, 5}, 0}, {{2, 3, 4, 6}, 0}, {{2, 3, 4, 7}, 0},
{{2, 3, 5, 6}, 0}, {{2, 3, 5, 7}, 0}, {{2, 3, 6, 7}, 0}, {{4, 5, 6}, 0}, {{0, 1, 2, 4, 5}, 4},
{{0, 1, 2, 4, 6}, 0}, {{0, 1, 2, 4, 7}, 0}, {{0, 1, 2, 5, 6}, 0}, {{0, 1, 2, 5, 7}, 0},
{{0, 1, 2, 6, 7}, 0}, {{0, 1, 3, 4, 5}, 0}, {{0, 1, 3, 4, 6}, 0}, {{0, 1, 3, 4, 7}, 0},
{{0, 1, 3, 5, 6}, 0}, {{0, 1, 3, 5, 7}, 0}, {{0, 1, 3, 6, 7}, 0}, {{0, 2, 3, 4, 5}, 0},
{{0, 2, 3, 4, 6}, 0}, {{0, 2, 3, 4, 7}, 0}, {{0, 2, 3, 5, 6}, 0}, {{0, 2, 3, 5, 7}, 0},
{{0, 2, 3, 6, 7}, 0}, {{0, 4, 5, 6, 7}, 4}, {{1, 2, 3, 4, 5}, 0}, {{1, 2, 3, 4, 6}, 0},
{{1, 2, 3, 4, 7}, 0}, {{1, 2, 3, 5, 6}, 0}, {{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 6, 7}, 0},
{{1, 2, 4, 5, 6, 7}, 0}, {{2, 4, 5, 6, 7}, 0}, {{3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 4, 5, 6}, 0},
{{0, 1, 2, 4, 5, 7}, 0}, {{0, 1, 2, 4, 6, 7}, 0}, {{0, 1, 2, 5, 6, 7}, 0},
{{0, 1, 3, 4, 5, 6}, 0}, {{0, 1, 3, 4, 5, 7}, 0}, {{0, 1, 3, 4, 6, 7}, 0},
{{0, 1, 3, 5, 6, 7}, 0}, {{0, 2, 3, 4, 5, 6}, 0}, {{0, 2, 3, 4, 5, 7}, 0},
{{0, 2, 3, 4, 6, 7}, 0}, {{0, 2, 3, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6}, 0},
{{1, 2, 3, 4, 5, 7}, 0}, {{1, 2, 3, 4, 6, 7}, -4}, {{1, 2, 3, 5, 6, 7}, 0},
{{0, 1, 2, 3, 4, 5, 6}, 0}, {{0, 1, 2, 3, 4, 5, 7}, 0}, {{0, 1, 2, 3, 4, 6, 7}, 0},
{{0, 1, 2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6, 8}, 4}}
Out[=]=
{{{0, 7}, -4}, {{3, 4}, -4}, {{4, 5, 6}, -4}, {{0, 3, 4, 7}, 4},
{{1, 2, 6, 7}, -4}, {{0, 1, 2, 4, 5}, 4}, {{0, 4, 5, 6, 7}, 4},
{{1, 2, 3, 5, 7}, -4}, {{1, 2, 3, 4, 6, 7}, -4}, {{0, 1, 2, 3, 4, 5, 6, 7}, 4}}
}

```



```

In[]:= base16[[#]] & /@ negativeTrMM;
{#[[2]], Tr[Transpose[#[[1]]].linTrans]} & /@ %
Select[%, #[[2]] ≠ 0 &]
linTransAntiSymmetric = Table[Select[base16, #[[2]] == %[[j]][[1]] &], {j, 1, Length[%]}];
Out[]=
{{{4}, 0}, {{5}, 0}, {{6}, 0}, {{7}, 0}, {{0, 1}, 0}, {{0, 2}, 0}, {{0, 3}, 0},
{{1, 2}, 0}, {{1, 3}, 0}, {{2, 3}, 0}, {{4, 5}, 0}, {{4, 6}, 0}, {{4, 7}, 0},
{{5, 6}, 0}, {{5, 7}, 0}, {{6, 7}, 0}, {{0, 1, 2}, 0}, {{0, 1, 3}, 0}, {{0, 2, 3}, 0},
{{0, 4, 5}, 0}, {{0, 4, 6}, 0}, {{0, 4, 7}, 0}, {{0, 5, 6}, 0}, {{0, 5, 7}, 0},
{{0, 6, 7}, 0}, {{1, 2, 3}, 0}, {{1, 4, 5}, 0}, {{1, 4, 6}, 0}, {{1, 4, 7}, 0},
{{1, 5, 6}, 0}, {{1, 5, 7}, 0}, {{1, 6, 7}, 0}, {{2, 4, 5}, 0}, {{2, 4, 6}, 0},
{{2, 4, 7}, 0}, {{2, 5, 6}, 0}, {{2, 5, 7}, 0}, {{2, 6, 7}, 0}, {{3, 4, 5}, 0},
{{3, 4, 6}, 0}, {{3, 4, 7}, 0}, {{3, 5, 6}, -4}, {{3, 5, 7}, 0}, {{3, 6, 7}, 0},
{{0, 1, 2, 4}, 0}, {{0, 1, 2, 5}, 0}, {{0, 1, 2, 6}, -4}, {{0, 1, 2, 7}, 0},
{{0, 1, 3, 4}, 0}, {{0, 1, 3, 5}, 0}, {{0, 1, 3, 6}, 0}, {{0, 1, 3, 7}, 0}, {{0, 2, 3, 4}, 0},
{{0, 2, 3, 5}, 0}, {{0, 2, 3, 6}, 0}, {{0, 2, 3, 7}, 0}, {{0, 4, 5, 6}, 0}, {{0, 4, 5, 7}, 0},
{{0, 4, 6, 7}, 0}, {{0, 5, 6, 7}, 0}, {{1, 2, 3, 4}, 0}, {{1, 2, 3, 5}, 0}, {{1, 2, 3, 6}, 0},
{{1, 2, 3, 7}, 0}, {{1, 4, 5, 6}, 0}, {{1, 4, 5, 7}, 0}, {{1, 4, 6, 7}, 0}, {{1, 5, 6, 7}, 0},
{{2, 4, 5, 6}, 0}, {{2, 4, 5, 7}, 0}, {{2, 4, 6, 7}, 0}, {{2, 5, 6, 7}, 0}, {{3, 4, 5, 6}, 0},
{{3, 4, 5, 7}, 0}, {{3, 4, 6, 7}, 0}, {{3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4}, 0},
{{0, 1, 2, 3, 5}, -4}, {{0, 1, 2, 3, 6}, 0}, {{0, 1, 2, 3, 7}, 0}, {{0, 1, 4, 5, 6}, 0},
{{0, 1, 4, 5, 7}, 0}, {{0, 1, 4, 6, 7}, 0}, {{0, 1, 5, 6, 7}, 0}, {{0, 2, 4, 5, 6}, 0},
{{0, 2, 4, 5, 7}, 0}, {{0, 2, 4, 6, 7}, 0}, {{0, 2, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6}, 0},
{{0, 3, 4, 5, 7}, 0}, {{0, 3, 4, 6, 7}, 0}, {{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 6}, 0},
{{1, 2, 4, 5, 7}, 4}, {{1, 2, 4, 6, 7}, 0}, {{1, 2, 5, 6, 7}, 0}, {{1, 3, 4, 5, 6}, 0},
{{1, 3, 4, 5, 7}, 0}, {{1, 3, 4, 6, 7}, 0}, {{1, 3, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6}, 0},
{{2, 3, 4, 5, 7}, 0}, {{2, 3, 4, 6, 7}, 0}, {{2, 3, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5}, 0},
{{0, 1, 2, 3, 4, 6}, -4}, {{0, 1, 2, 3, 4, 7}, 0}, {{0, 1, 2, 3, 5, 6}, 0},
{{0, 1, 2, 3, 5, 7}, 0}, {{0, 1, 2, 3, 6, 7}, 0}, {{0, 1, 4, 5, 6, 7}, 0},
{{0, 2, 4, 5, 6, 7}, 0}, {{0, 3, 4, 5, 6, 7}, 0}, {{1, 2, 4, 5, 6, 7}, 0},
{{1, 3, 4, 5, 6, 7}, 0}, {{2, 3, 4, 5, 6, 7}, 0}, {{0, 1, 2, 3, 4, 5, 6}, 0},
{{0, 1, 2, 3, 4, 5, 7}, 0}, {{0, 2, 3, 4, 5, 6, 7}, 0}, {{1, 2, 3, 4, 5, 6, 7}, 0}}
Out[]=
{{{3, 5, 6}, -4}, {{0, 1, 2, 6}, -4}, {{0, 1, 2, 3, 5}, -4},
{{0, 3, 5, 6, 7}, 4}, {{1, 2, 4, 5, 7}, 4}, {{0, 1, 2, 3, 4, 6}, -4}}

```

```
In[]:= {Length[linTransSymmetric], Length[linTransAntiSymmetric]}
```

```
Out[]= {10, 6}
```

BASIS of 8×8 matrices :

```
In[0]:= tA = {}; Do[AppendTo[tA, {τ[j], {j}}], {j, 1, 7}]
Length[tA]
tAB = {};
Do[AppendTo[tAB, {τ[j].τ[k], {j, k}}], {j, 1, 6}, {k, j + 1, 7}]
Length[tAB]
tABC = {};
Do[AppendTo[tABC, {τ[j].τ[k].τ[h], {j, k, h}}], {j, 1, 5}, {k, j + 1, 6}, {h, k + 1, 7}];
Length[tABC]
```

Out[0]=

7

Out[0]=

21

Out[0]=

35

```
In[0]:= tA[[1]]
Out[0]= {{{{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0},
{0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}}, {1}}}

In[0]:= tA[[#]] [[1]] == -Transpose[tA[[#]] [[1]]] & /@ Range[Length[tA]]
Out[0]= {True, True, True, False, False, False}
```

```
In[0]:= bas64 = Flatten[{tA, tAB, tABC, {{Id, {64}}}}, 1]; Length[bas64]
```

Out[0]=

64

```
In[0]:= determineIF8AntiSymmetric[m_] := Module[{t, tt, sum, res, r},
t = m.τ[#] & /@ Range[0, 7];
tt = Transpose[#] & /@ t;
sum = FullSimplify[t[[#]] + tt[[#]]] & /@ Range[Length[t]];
res = Union[Flatten[#]] & /@ sum;
If[Length[#] > 1, Return[False]] & /@ res;
r = Union[Flatten[res]];
Return[r == {0}]];
]
```

```
In[0]:= determineIF8AntiSymmetric[Zero8]
```

Out[0]=

True


```
In[=]:= (*(*(*(*(*(*countTrace=Table[{A,B,FullSimplify[\frac{1}{8}Tr[bas64[A,1].bas64[B,1]]]}],{A,1,64},{B,A,64}])//MatrixForm*)*)*)*)*)
```

```
In[=]:= (*■■*)
```

```
In[=]:= 32 ≥ 65
```

```
Out[=]=
```

```
2080
```

```
In[=]:= (*Length[Flatten[countTrace,1]]*)
```

```
In[=]:= (*{Length[countTrace],Length[countTrace[[1]],Length[countTrace[[1,1]]]}*)
```

```
In[=]:= (*{Length[countTrace],Length[countTrace[[2]],Length[countTrace[[2,1]]]}*)
```

```
In[=]:= (*{Length[countTrace],Length[countTrace[[64]],Length[countTrace[[64,1]]]}*)
```

```
In[=]:= (*{Length[Flatten[countTrace,1][All,3]],Count[Flatten[countTrace,1][All,3],0],Count[Flatten[countTrace,1][All,3],1],Count[Flatten[countTrace,1][All,3],-1]}*)
```

```
In[=]:= 32 ≥ 63
```

```
Out[=]=
```

```
2016
```

```
In[=]:= anti = {};
```

```
Do[If[bas64[[k, 1]] == -Transpose[bas64[[k, 1]]],  
AppendTo[anti, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}], {k, 1, 64}];  
Length[anti]  
anti[[28]]
```

```
Out[=]=
```

```
28
```

```
Out[=]=
```

```
{{{0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0},  
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},  
{0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0}}, {{63}, {5, 6, 7}}}
```

```
In[=]:= anti[[28, 1]]
```

```
Out[=]=
```

```
{ {0, 1, 0, 0, 0, 0, 0, 0}, {-1, 0, 0, 0, 0, 0, 0, 0},  
{0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0},  
{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0} }
```

```
In[=]:= anti[[28, 2]]
```

```
Out[=]=
```

```
{ {63}, {5, 6, 7} }
```

```
In[•]:= symm = { };
Do [If [bas64[[k, 1]] == Transpose[bas64[[k, 1]]],  

  AppendTo[symm, {bas64[[k, 1]], {{k}, bas64[[k, 2]]}}]], {k, 1, 64}];
Length[symm]
symm[[28]]

Out[•]= 35

Out[•]= {{{-1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0},  

  {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0},  

  {0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 1}}, {{52}, {2, 5, 7}}}}
```

For the split orthogonal group Spin(4,4) and its associated Lie algebra, there exist three fundamentally equivalent 8-dimensional representations:

1. The vector representation V
2. The type-1 spinor representation S+ (or S1), positive chirality
3. The type-2 spinor representation S- (or S2), negative chirality

These representations are related by outer automorphisms of Spin(4,4). All three representations have the same dimension: $\dim(V) = \dim(S1) = \dim(S2) = 8$

This is unique to dimension 8 and forms the basis of triality: Cartan, E. (1925). La théorie des spineurs. Hermann, Paris.

metric-preserving transformations; find automorphism $\sigma g \leftrightarrow \sigma g_A^a$

$$\begin{aligned}\sigma.g &= \text{Transpose}[\sigma g^{-1}].g \\ \sigma &= \text{Transpose}[\sigma g^{-1}].g.\sigma g^{-1} \\ (\sigma g)_A^a &\equiv \sigma g_A^a \\ (\sigma g^{-1})_a^A &\equiv \sigma g_a^A ; (\text{Transpose}[\sigma g^{-1}])_a^A \equiv \sigma g_a^A \\ \sigma_{ab} &= \sigma g_a^A g_{AB} \sigma g_b^B\end{aligned}$$

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$N = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}$$

In[•]:= $\text{Clear}[\sigma g_{1A}^a]; \sigma g_{1A}^a = \frac{1}{\sqrt{2}} (\eta 4488 + \sigma)$

Out[•]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ \left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

In[•]:= $\sigma g_{2A}^a = \frac{1}{\sqrt{2}} \{ \{ 1, 0, 0, 0, 1, 0, 0, 0 \},$

$$\{ 0, 0, 1, 0, 0, 0, 1, 0 \}, \{ 0, 1, 0, 0, 0, 1, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, 1 \}, \\ \{ 1, 0, 0, 0, -1, 0, 0, 0 \}, \{ 0, 0, 1, 0, 0, 0, -1, 0 \}, \\ \{ 0, 1, 0, 0, 0, -1, 0, 0 \}, \{ 0, 0, 0, 1, 0, 0, 0, -1 \} \}$$

Out[•]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \right. \\ \left. \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

In[•]:= $(s1s2 = \sigma g_{1A}^a.\text{Transpose}[\sigma g_{2A}^a]) // \text{MatrixForm}$

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[•]:= $(*(s1s2=\text{Inverse}[\sigma g_{1A}^a.\text{Transpose}[\sigma g_{2A}^a]]) // \text{MatrixForm}*)$

```
In[]:= s1s2.s1s2 === ID8
Out[]= True

In[]:= (Thread[#1 ; 8] → s1s2.#1) // Column
          %[[1]] [[All, 1, 0]]
          %%[[1]] [[All, 2]] /. {x0 → #1, x4 → #2}
          ToExpression["((" <> ToString[#] <> ")")"] & /@ %
          forceDimensionReductionA = Thread[%%% → %]

Out[=]
f16[0][x0, x4] → f16[8][x0, x4]
f16[1][x0, x4] → f16[10][x0, x4]
f16[2][x0, x4] → f16[9][x0, x4]
f16[3][x0, x4] → f16[11][x0, x4]
f16[4][x0, x4] → f16[12][x0, x4]
f16[5][x0, x4] → f16[14][x0, x4]
f16[6][x0, x4] → f16[13][x0, x4]
f16[7][x0, x4] → f16[15][x0, x4]

Out[=]
{f16[0], f16[1], f16[2], f16[3], f16[4], f16[5], f16[6], f16[7]}

Out[=]
{f16[8][#1, #2], f16[10][#1, #2], f16[9][#1, #2], f16[11][#1, #2],
 f16[12][#1, #2], f16[14][#1, #2], f16[13][#1, #2], f16[15][#1, #2]}

Out[=]
{f16[8][#1, #2] &, f16[10][#1, #2] &, f16[9][#1, #2] &, f16[11][#1, #2] &,
 f16[12][#1, #2] &, f16[14][#1, #2] &, f16[13][#1, #2] &, f16[15][#1, #2] &}

Out[=]
{f16[0] → (f16[8][#1, #2] &), f16[1] → (f16[10][#1, #2] &),
 f16[2] → (f16[9][#1, #2] &), f16[3] → (f16[11][#1, #2] &), f16[4] → (f16[12][#1, #2] &),
 f16[5] → (f16[14][#1, #2] &), f16[6] → (f16[13][#1, #2] &), f16[7] → (f16[15][#1, #2] &)}
```

```
In[8]:= (Thread[¶16[[9 ;; 16]] → s1s2.¶16[[1 ;; 8]]]) // Column
%[[1]] [[All, 1, 0]]
%%[[1]] [[All, 2]] /. {x0 → #1, x4 → #2}
ToExpression["((\" <> ToString[#] <> \")&)" ] & /@ %
forceDimensionReduction = Thread[%%% → %]

Out[8]=
f16[8][x0, x4] → f16[0][x0, x4]
f16[9][x0, x4] → f16[2][x0, x4]
f16[10][x0, x4] → f16[1][x0, x4]
f16[11][x0, x4] → f16[3][x0, x4]
f16[12][x0, x4] → f16[4][x0, x4]
f16[13][x0, x4] → f16[6][x0, x4]
f16[14][x0, x4] → f16[5][x0, x4]
f16[15][x0, x4] → f16[7][x0, x4]

Out[9]=
{f16[8], f16[9], f16[10], f16[11], f16[12], f16[13], f16[14], f16[15]}

Out[10]=
{f16[0][#1, #2], f16[2][#1, #2], f16[1][#1, #2], f16[3][#1, #2],
 f16[4][#1, #2], f16[6][#1, #2], f16[5][#1, #2], f16[7][#1, #2]}

Out[11]=
{f16[0][#1, #2] &, f16[2][#1, #2] &, f16[1][#1, #2] &, f16[3][#1, #2] &,
 f16[4][#1, #2] &, f16[6][#1, #2] &, f16[5][#1, #2] &, f16[7][#1, #2] &}

Out[12]=
{f16[8] → (f16[0][#1, #2] &), f16[9] → (f16[2][#1, #2] &),
 f16[10] → (f16[1][#1, #2] &), f16[11] → (f16[3][#1, #2] &), f16[12] → (f16[4][#1, #2] &),
 f16[13] → (f16[6][#1, #2] &), f16[14] → (f16[5][#1, #2] &), f16[15] → (f16[7][#1, #2] &)}

In[13]:= (*σga2A====-#+σga1A&/@bas64[[All,1]]*)

Out[13]= σga2A.Transpose[σga2A] === ID8
```

Out[14]= True

In[15]:= σg^a_{1A}.Transpose[σg^a_{1A}] === ID8

Out[15]= True

metric - preserving transformation:

In[16]:= σg^a_{1A}.σ.Transpose[σg^a_{1A}] === η4488

Out[16]= True

metric - preserving transformation:

```

In[1]:= σga2A.σ.Transpose[σga2A] === η4488
Out[1]= True

In[2]:= Mc = 1/√2 (η4488 + σ)
Out[2]= { {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}, {0, 1/√2, 0, 0, 0, 1/√2, 0, 0}, {0, 0, 1/√2, 0, 0, 0, 1/√2, 0}, {0, 0, 0, 1/√2, 0, 0, 0, 1/√2}, {0, 0, 0, 0, 1/√2, 0, 0, 0}, {0, 0, 0, 0, 0, 1/√2, 0, 0}, {0, 0, 0, 0, 0, 0, 1/√2, 0}, {0, 0, 0, 0, 0, 0, 0, 1/√2} }

In[3]:= Nc = 1/√2 {{1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1}, {1, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 1, 0, 0, 0, -1, 0}, {0, 1, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 1, 0, 0, 0, -1}}
Out[3]= { {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}, {0, 0, 1/√2, 0, 0, 0, 1/√2, 0}, {0, 1/√2, 0, 0, 0, 1/√2, 0, 0}, {0, 0, 0, 1/√2, 0, 0, 0, 1/√2}, {0, 0, 0, 0, 1/√2, 0, 0, 0}, {0, 0, 0, 0, 0, 1/√2, 0, 0}, {0, 0, 0, 0, 0, 0, 1/√2, 0}, {0, 0, 0, 0, 0, 0, 0, 1/√2} }

In[4]:= (* (s1s2=Mc.Transpose[Nc]) //MatrixForm*)
Out[4]= Nc.Transpose[Nc] // MatrixForm
Out[4]/MatrixForm=

```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```

In[]:= Mc.Transpose[Mc] // MatrixForm
Out[//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$


In[]:= Mc.σ.Transpose[Mc] === η4488
Out[=]
True

In[]:= Nc.σ.Transpose[Nc] === η4488
Out[=]
True

In[]:= (Thread[Π16[[9;;16]] → s1s2.Π16[[1;;8]]]) // Column
Out[=
f16[8][x0, x4] → f16[0][x0, x4]
f16[9][x0, x4] → f16[2][x0, x4]
f16[10][x0, x4] → f16[1][x0, x4]
f16[11][x0, x4] → f16[3][x0, x4]
f16[12][x0, x4] → f16[4][x0, x4]
f16[13][x0, x4] → f16[6][x0, x4]
f16[14][x0, x4] → f16[5][x0, x4]
f16[15][x0, x4] → f16[7][x0, x4]

In[]:= f8[#][x0, x4] & /@ Range[0, 7] // Column
(s1s2.%[[1]]) // Column
Out[=
f8[0][x0, x4]
f8[1][x0, x4]
f8[2][x0, x4]
f8[3][x0, x4]
f8[4][x0, x4]
f8[5][x0, x4]
f8[6][x0, x4]
f8[7][x0, x4]

Out[=
f8[0][x0, x4]
f8[2][x0, x4]
f8[1][x0, x4]
f8[3][x0, x4]
f8[4][x0, x4]
f8[6][x0, x4]
f8[5][x0, x4]
f8[7][x0, x4]

```

Killing Vector Fields

```

), _DG([["vector", M8, []], [[7],
- 1/(sin(6 Hx0)^1/3)^(x7 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0)^1/3)], [[8],
1/(sin(6 Hx0)^1/3)^(x6 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0)^1/3)]]), _DG([["vector",
M8, []], [[7], - 1/(sin(6 Hx0)^1/3)^( (2 cos(Hx0)
- 1)^1/3 (cos(Hx0) + 1)^1/6 (-1 + 2 cos(2 Hx0))^1
/3 (cos(Hx0) - 1)^1/6 (2 cos(Hx0) + 1)^1/3 cos(Hx0)^1/3)]]
), _DG([["vector", M8, []], [[8],
- 1/(sin(6 Hx0)^1/3)^( (2 cos(Hx0) - 1)^1/3 (cos(Hx0)
+ 1)^1/6 (-1 + 2 cos(2 Hx0))^1/3 (cos(Hx0) - 1)^1
/6 (2 cos(Hx0) + 1)^1/3 cos(Hx0)^1/3)]]),
_DG([["vector", M8, []], [[2],
1/(sin(6 Hx0)^1/3)^(x2 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0)^1/3)], [[3],
- 1/(sin(6 Hx0)^1/3)^(x1 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0)^1/3)]]), _DG([["vector",
M8, []], [[2],
1/(sin(6 Hx0)^1/3)^(x3 cos(Hx0)^1/3 (cos(Hx0) - 1)^1
/6 (cos(Hx0) + 1)^1/6 (2 cos(Hx0) - 1)^1/3 (2 cos(Hx0)
+ 1)^1/3 (-1 + 2 cos(2 Hx0)^1/3)]]),
_DG([["vector", M8, []], [[2], 1/(sin(6 Hx0)^1/3)^( (2 cos(Hx0) - 1)^1
/3 (cos(Hx0) + 1)^1/6 (-1 + 2 cos(2 Hx0))^1
/3 (cos(Hx0) - 1)^1/6 (2 cos(Hx0) + 1)^1/3 cos(Hx0)^1/3]]])
]

```

Out[8]=

```

_DG([["vector", M8, []], [[6],

```

$$\begin{aligned}
& - \frac{1}{\sin(6Hx\theta)^{1/3}} (x_6 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [[7], \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x_5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]]), \text{DG}\left([["vector",\right. \\
& M8, []], [[3], \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x_3 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [[4], \\
& - \frac{1}{\sin(6Hx\theta)^{1/3}} (x_2 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]]), \text{DG}\left([["vector",\right. \\
& M8, []], [[3], \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} \\
& /^3 (\cos(Hx\theta) - 1)^{1/6} (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})]] \\
&], \text{DG}\left([["vector", M8, []], [[4], \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} (\cos(Hx\theta) - 1)^{1/3} \\
& (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})]]], \text{DG}\left([["vector",\right. \\
& M8, []], [[6], \\
& - \frac{1}{\sin(6Hx\theta)^{1/3}} (x_7 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})], [[8], \\
& \frac{1}{\sin(6Hx\theta)^{1/3}} (x_5 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]]), \text{DG}\left([["vector",\right. \\
& M8, []], [[6], - \frac{1}{\sin(6Hx\theta)^{1/3}} ((2 \cos(Hx\theta) - 1)^{1/3} (\cos(Hx\theta) + 1)^{1/6} (-1 + 2 \cos(2Hx\theta))^{1/3} \\
& /^3 (\cos(Hx\theta) - 1)^{1/6} (2 \cos(Hx\theta) + 1)^{1/3} \cos(Hx\theta)^{1/3})]] \\
&], \text{DG}\left([["vector", M8, []], [[7], \\
& - \frac{1}{\sin(6Hx\theta)^{1/3}} (x_7 \cos(Hx\theta)^{1/3} (\cos(Hx\theta) - 1)^{1/6} (\cos(Hx\theta) + 1)^{1/6} (2 \cos(Hx\theta) - 1)^{1/3} (2 \cos(Hx\theta) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx\theta))^{1/3})]]]
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})], [[8], \\
& \frac{1}{\sin(6Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})]]], _DG\left(\left[\begin{array}{l} "vector", \\ M8, [] \end{array}\right], \left[\begin{array}{l} [7], -\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3})] \end{array}\right]\right], \\
&), _DG\left(\left[\begin{array}{l} "vector", M8, [] \end{array}\right], \left[\begin{array}{l} [8], \\
-\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3})] \end{array}\right]\right], \\
& _DG\left(\left[\begin{array}{l} "vector", M8, [] \end{array}\right], \left[\begin{array}{l} [2], \\
\frac{1}{\sin(6Hx0)^{1/3}} (x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
+ 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})], [[3], \\
-\frac{1}{\sin(6Hx0)^{1/3}} (x1 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
+ 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})]]], _DG\left(\left[\begin{array}{l} "vector", \\ M8, [] \end{array}\right], \left[\begin{array}{l} [2], \\
\frac{1}{\sin(6Hx0)^{1/3}} (x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
+ 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})] \end{array}\right]\right], \\
& _DG\left(\left[\begin{array}{l} "vector", M8, [] \end{array}\right], \left[\begin{array}{l} [4], \\
-\frac{1}{\sin(6Hx0)^{1/3}} (x1 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
+ 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3})] \end{array}\right]\right], _DG\left(\left[\begin{array}{l} "vector", \\ M8, [] \end{array}\right], \left[\begin{array}{l} [2], \\
\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) + 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (\cos(Hx0) - 1)^{1/6} (2 \cos(Hx0) + 1)^{1/3} \cos(Hx0)^{1/3})] \end{array}\right]\right]
\end{aligned}$$

*In[1]:= (*ToExpression[MathMLForm[{"http://www.w3.org/TR/MathML", "math"}]]*)*

*In[2]:= (*ToExpression[MathMLForm[□]]*)*

*In[3]:= " [_DG([[["vector", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)*

```

+ 1)^^(1/3)*(-1 + 2*cos(2*H*x0))^^(1/3)/sin(6*H*x0)^(1/3)],  

[[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +  

1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"], M8,  

[]], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)  

+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0)  

- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)  

+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],  

_M8, []], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) +  

1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)  

+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],  

M8, []], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +  

1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],  

M8, []], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)  

+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x5*cos(H*x0)^(1/3)*(cos(H*x0)  

- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)  

+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],  

_M8, []], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) +  

1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0)  

+ 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],  

M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)  

+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8], x6*cos(H*x0)^(1/3)*(cos(H*x0)  

- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)  

+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],  

_M8, []], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +  

1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],  

M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1  

+ 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) +  

1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],  

M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)  

+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0)  

- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +  

1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[\"vector\"],  

_M8, []], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)  

+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +  

2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0)  

- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)

```

```
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]],  
_DG([["vector", M8, []], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0)  
+ 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) -  
1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])"
```

```

Out[8]=
[_DG([[{"vector", M8, []}], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[7], x5*cos(H*x0)^(1/3)*(cos(H*x0)
- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]], _DG([[{"vector",
M8, []}], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]], _DG([[{"vector", M8, []}], [[[3],
(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0)
- 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[{"vector", M8, []}], [[[4],
(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) - 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[{"vector", M8, []}], [[[6],
-x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8],
x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]], _DG([[{"vector", M8, []}], [[[6],
-(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[{"vector", M8, []}], [[[7],
-x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[8],
x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]], _DG([[{"vector", M8, []}], [[[7],
-(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[{"vector", M8, []}], [[[8],
-(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0) -
1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([[{"vector", M8, []}], [[[2],
x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[3],
-x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]], _DG([[{"vector", M8, []}], [[[2],
x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[4],
-x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]]], _DG([[{"vector", M8, []}], [[[2],
(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(cos(H*x0)
- 1)^(1/6)*(2*cos(H*x0) + 1)^(1/3)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]
]

```

In[8]:= **killingVectors =**

```
"[_DG([["vector"], M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) -
```

```

1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [[7], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector"], M8,
[]], [[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [
[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector"], M8,
[]], [[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]), _DG([["vector"], M8,
[]], [[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]), _DG([["vector"], M8,
[]], [[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [
[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector"], M8,
[]], [[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]), _DG([["vector"], M8,
[]], [[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [
[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector"], M8,
[]], [[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]), _DG([["vector"], M8,
[]], [[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]), _DG([["vector"], M8,
[]], [[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) -
1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)], [
[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]])

```

```
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector", M8,
[]], [[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)],
[[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) +
1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)]]], _DG([["vector", M8,
[]], [[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1
+ 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)]]]])]"
```

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Out[8]=
{_DG([["vector", M8, []], [[[6], -x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[7], x5*cos(H*x0)^(1/3)*(cos(H*x0)
- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]]]], _DG([["vector",
M8, []], [[[3], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[4], -x2*cos(H*x0)^(1/3)*(cos(H*x0)
- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
+ 1)^(1/3)*(-1 + 2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]]]], _DG([["vector",
M8, []], [[[3], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(-1 + 2*cos(2*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0)
+ 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[4], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[6], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[8], x5*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]]]], _DG([["vector",
M8, []], [[[6], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*x0)^(1/3)]]], _DG([["vector", M8, []], [[[7], -x7*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[8], x6*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) -
1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 + 2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]]]], _DG([["vector",
M8, []], [[[7], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) -
1)^(1/6)*(-1 + 2*cos(2*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[8], -(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(-1 +
2*cos(2*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(cos(H*x0) +
1)^(1/6)*cos(H*x0)^(1/3)/sin(6*x0)^(1/3)]]], _DG([["vector",
M8, []], [[[2], x2*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[3], -x1*cos(H*x0)^(1/3)*(cos(H*x0)
- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]]]], _DG([["vector",
M8, []], [[[2], x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)], [[4], -x1*cos(H*x0)^(1/3)*(cos(H*x0)
- 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0) +
1)^(1/3)*(-1 + 2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]]]], _DG([["vector",
M8, []], [[[2], (2*cos(H*x0) - 1)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0)
+ 1)^(1/6)*(-1 + 2*cos(2*x0))^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
2*cos(2*x0))^(1/3)/sin(6*x0)^(1/3)]]]]

```

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$$\begin{aligned}
& -DG\left(\left[["vector", M8, []], \left[\left[[6], \right.\right.\right. \right. \\
& \left. \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}}(x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \right. \\
& \left. \left. \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \\
& \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right], [[7], \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}(x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \right. \\
& \left. \left. \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \\
& \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) ]]]], \right. \right. \right. \right. \\
& \left. \left. \left. \left. -DG\left(\left[["vector", \right. \right. \right. \right. \\
& M8, []], \left[\left[[3], \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{1}{\sin(6Hx0)^{1/3}}(x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \right. \\
& \left. \left. \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \\
& \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right], [[4], \right. \right. \right. \right. \\
& \left. \left. \left. \left. -\frac{1}{\sin(6Hx0)^{1/3}}(x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \right. \right. \right. \right. \\
& \left. \left. \left. \left. /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \right. \right. \right. \right. \\
& \left. \left. \left. \left. + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) ]]]], \right. \right. \right. \right. \\
& \left. \left. \left. \left. -DG\left(\left[["vector", \right. \right. \right. \right. \\
& M8, []], \left[\left[[3], \frac{1}{\sin(6Hx0)^{1/3}}((2 \cos(Hx0) - 1)^1 \right. \right. \right. \right. \\
& \left. \left. \left. \left. /^3 (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^1 \right. \right. \right. \right. \\
& \left. \left. \left. \left. /^3 (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \right] \right] \right] \right] \right]
\end{aligned}$$


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$$\begin{aligned}
& - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1/3} (2 \cos(Hx0) + 1)^1 \\
& /^3 (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3})]]]), \_DG\left(\left[["vector",\right. \right. \\
& M8, [ ]], \left[ \left[ [6], \right. \right. \\
& - \frac{1}{\sin(6 Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \\
& /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2 Hx0))^{1/3})], \left[ [8], \right. \left. \right. \\
& \frac{1}{\sin(6 Hx0)^{1/3}} (x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \\
& /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2 Hx0))^{1/3})]]]), \_DG\left(\left[["vector",\right. \right. \\
& M8, [ ]], \left[ \left[ [6], - \frac{1}{\sin(6 Hx0)^{1/3}} ((2 \cos(Hx0) \\
& - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2 Hx0))^{1 \\
& /^3 (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3})]]] \\
& ), \_DG\left(\left[ ["vector", M8, [ ]], \left[ \left[ [7], \right. \right. \right. \\
& - \frac{1}{\sin(6 Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \\
& /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2 Hx0))^{1/3})], \left[ [8], \right. \left. \right. \\
& \frac{1}{\sin(6 Hx0)^{1/3}} (x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^1 \\
& /^6 (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) \\
& + 1)^{1/3} (-1 + 2 \cos(2 Hx0))^{1/3})]]]), \_DG\left(\left[ ["vector",\right. \right.
\end{aligned}$$

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M8, [ ]], \!\!\! \left[ \left[ 7\right], -\frac{1}{\sin(6 H x0)^{1/3}} \big( (2 \cos(H x0) - 1)^{1/3} (\cos(H x0) - 1)^{1/6} (-1 + 2 \cos(2 H x0))^1 /3 (2 \cos(H x0) + 1)^{1/3} (\cos(H x0) + 1)^{1/6} \cos(H x0)^{1/3}) \big) \right] ], \\

), _DG \Big( \left[ ["vector", M8, [ ]], \left[ \left[ 8\right], -\frac{1}{\sin(6 H x0)^{1/3}} \big( (2 \cos(H x0) - 1)^{1/3} (\cos(H x0) - 1)^{1/6} (-1 + 2 \cos(2 H x0))^1 /3 (\cos(H x0) + 1)^{1/6} \cos(H x0)^{1/3}) \big) \right] \right]), _DG \Big( \left[ ["vector", M8, [ ]], \left[ \left[ 2\right], \frac{1}{\sin(6 H x0)^{1/3}} \big( x2 \cos(H x0)^{1/3} (\cos(H x0) - 1)^1 /6 (\cos(H x0) + 1)^{1/6} (2 \cos(H x0) - 1)^{1/3} (2 \cos(H x0) + 1)^{1/3} (-1 + 2 \cos(2 H x0))^1 /3 \big) \right], \left[ \left[ 3\right], -\frac{1}{\sin(6 H x0)^{1/3}} \big( x1 \cos(H x0)^{1/3} (\cos(H x0) - 1)^1 /6 (\cos(H x0) + 1)^{1/6} (2 \cos(H x0) - 1)^{1/3} (2 \cos(H x0) + 1)^{1/3} (-1 + 2 \cos(2 H x0))^1 /3 \big) \big) \right] \right]), _DG \Big( \left[ ["vector", M8, [ ]], \left[ \left[ 2\right], \frac{1}{\sin(6 H x0)^{1/3}} \big( x3 \cos(H x0)^{1/3} (\cos(H x0) - 1)^1 /6 (\cos(H x0) + 1)^{1/6} (2 \cos(H x0) - 1)^{1/3} (2 \cos(H x0) + 1)^{1/3} (-1 + 2 \cos(2 H x0))^1 /3 \big) \right], \left[ \left[ 4\right], -\frac{1}{\sin(6 H x0)^{1/3}} \big( x1 \cos(H x0)^{1/3} (\cos(H x0) - 1)^1

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 $\sin(6Hx0)^{1/3} \cdot$ 
 $^{1/6}(\cos(Hx0) + 1)^{1/6}(2\cos(Hx0) - 1)^{1/3}(2\cos(Hx0)$ 
 $+ 1)^{1/3}(-1 + 2\cos(2Hx0))^{1/3})]]]), _DG\left( \left[ \text{"vector",}$ 
 $M8, [ ] \right], \left[ [2], \frac{1}{\sin(6Hx0)^{1/3}} \left( (2\cos(Hx0) - 1)^{1/3} \right.$ 
 $\left. (\cos(Hx0) - 1)^{1/6} (-1 + 2\cos(2Hx0))^{1/3} \right.$ 
 $\left. (2\cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3} \right)]]$ 
 $]\right]$ 

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Out[•]=

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$$-DG\left(\left[\left["vector", M8, [ ]\right], \left[\left[[6], -\frac{1}{\sin(6Hx0)^{1/3}}(x6 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}\right)\right], \left[[7], \frac{1}{\sin(6Hx0)^{1/3}}(x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}\right)\right]], _DG\left(\left[M8, [ ]\right], \left[\left[[3], \frac{1}{\sin(6Hx0)^{1/3}}(x3 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}\right)\right], \left[[4], -\frac{1}{\sin(6Hx0)^{1/3}}(x2 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}\right)\right]]\right), _DG\left(\left["vector",$$


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$\text{M8, []}, \left[\left[[3], \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right]$
 $]), \text{_DG}\left(\left[["vector", M8, []], \left[[4], \frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \right)], \text{_DG}\left(\left[["vector", M8, []], \left[[6], -\frac{1}{\sin(6Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right], [8], \frac{1}{\sin(6Hx0)^{1/3}} (x5 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (\cos(Hx0) + 1)^{1/6} (2 \cos(Hx0) - 1)^{1/3} (2 \cos(Hx0) + 1)^{1/3} (-1 + 2 \cos(2Hx0))^{1/3}) \right] \right] \right)], \text{_DG}\left(\left[["vector", M8, []], \left[[6], -\frac{1}{\sin(6Hx0)^{1/3}} ((2 \cos(Hx0) - 1)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \right)], \text{_DG}\left(\left[["vector", M8, []], \left[[7], -\frac{1}{\sin(6Hx0)^{1/3}} (x7 \cos(Hx0)^{1/3} (\cos(Hx0) - 1)^{1/6} (-1 + 2 \cos(2Hx0))^{1/3} (2 \cos(Hx0) + 1)^{1/3} (\cos(Hx0) + 1)^{1/6} \cos(Hx0)^{1/3}) \right] \right] \right)]$


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M8, [ ]], \!\!\! \left[ \!\!\! \left[ [2], \frac{1}{\sin(6 H x \theta )^{1/3}} \left(x 3 \cos(H x \theta )^{1/3} \left(\cos(H x \theta )-1\right)^{1/6} \left(\cos(H x \theta )+1\right)^{1/6} \left(2 \cos(H x \theta )-1\right)^{1/3} \left(2 \cos(H x \theta )+1\right)^{1/3} \left(-1+2 \cos(2 H x \theta )\right)^{1/3}\right), [4], -\frac{1}{\sin(6 H x \theta )^{1/3}} \left(x I \cos(H x \theta )^{1/3} \left(\cos(H x \theta )-1\right)^{1/6} \left(\cos(H x \theta )+1\right)^{1/6} \left(2 \cos(H x \theta )-1\right)^{1/3} \left(2 \cos(H x \theta )+1\right)^{1/3} \left(-1+2 \cos(2 H x \theta )\right)^{1/3}\right) ]]]], _DG \!\!\! \left( \!\!\! \left[ \!\!\! \left[ \text{"vector"}, M8, [ ]], \left[ [2], \frac{1}{\sin(6 H x \theta )^{1/3}} \left(\left(2 \cos(H x \theta )-1\right)^{1/3} \left(\cos(H x \theta )-1\right)^{1/6} \left(-1+2 \cos(2 H x \theta )\right)^{1/3} \left(2 \cos(H x \theta )+1\right)^{1/3} \left(\cos(H x \theta )+1\right)^{1/6} \cos(H x \theta )^{1/3}\right) \right] \right] \right]

```

In[1]:= **ConvertMapleToMathematicaV2[**

```

" -x6*Cos[H*x0]^(1/3)*(Cos[H*x0] - 1)^(1/6)*(Cos[H*x0] +
1)^(1/6)*(2*Cos[H*x0] - 1)^(1/3)*(2*Cos[H*x0] + 1)^(1/3)*(-1 +
2*Cos[2*H*x0])^(1/3)/Sin[6*H*x0]^(1/3), , x5*Cos[H*x0]^(1/3)*(Cos[H*x0] -
1)^(1/6)*(Cos[H*x0] + 1)^(1/6)*(2*Cos[H*x0] - 1)^(1/3)*(2*Cos[H*x0] +
1)^(1/3)*(-1 + 2*Cos[2*H*x0])^(1/3)/Sin[6*H*x0]^(1/3")]

```

Out[1]=

$$-\frac{1}{\text{Sin}[6 \text{H} \times 0]^{1/3}} x6 \left(-1+\text{Cos}[\text{H} \times 0]\right)^{1/6} \text{Cos}[\text{H} \times 0]^{1/3} \\ (1+\text{Cos}[\text{H} \times 0])^{1/6} \left(-1+2 \text{Cos}[\text{H} \times 0]\right)^{1/3} \left(1+2 \text{Cos}[\text{H} \times 0]\right)^{1/3} \left(-1+2 \text{Cos}[2 \text{H} \times 0]\right)^{1/3}$$

```
In[1]:= ConvertMapleToMathematica[
  "x3*cos(H*x0)^(1/3)*(cos(H*x0) - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0)
  - 1)^(1/3)*(2*cos(H*x0) + 1)^(1/3)*(-1 +
  2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3), -x2*cos(H*x0)^(1/3)*(cos(H*x0)
  - 1)^(1/6)*(cos(H*x0) + 1)^(1/6)*(2*cos(H*x0) - 1)^(1/3)*(2*cos(H*x0)
  + 1)^(1/3)*(-1 + 2*cos(2*H*x0))^(1/3)/sin(6*H*x0)^(1/3)"]
```

Out[1]=

$$\begin{aligned} & \left(\left(\left(\left(\left(x3 * (\text{Cos}[(H * x0)]^{\frac{1}{3}}) * ((\text{Cos}[(H * x0)] - 1)^{\frac{1}{6}}) * (\text{Cos}[(H * x0)] + 1)^{\frac{1}{6}} * (2 \text{Cos}[(H * x0)] - 1)^{\frac{1}{3}} * (2 \text{Cos}[(H * x0)] + 1)^{\frac{1}{3}} * (-1 + 2 \text{Cos}(2 H * x0))^{\frac{1}{3}} / \sin(6 H * x0)^{\frac{1}{3}} \right) \right) \right) \right) \\ & \quad * \left(\left(\left(\left((\text{Cos}[(H * x0)] + 1)^{\frac{1}{6}}) * ((2 * \text{Cos}[(H * x0)]) - 1)^{\frac{1}{3}} * ((2 * \text{Cos}[(H * x0)] + 1)^{\frac{1}{3}}) * ((-1) + (2 * \text{Cos}[(2 * H * x0)]))^{\frac{1}{3}} / (\sin((6 * H) * x0))^{\frac{1}{3}} \right) \right) \right) \end{aligned}$$

In[2]:= ConvertMapleToMathematica["(2*cos(H*x0) - 1)^(1/3)*(cos(H*x0)
 - 1)^(1/6)*(-1 + 2*cos(2*H*x0))^(1/3)*(2*cos(H*x0) +
 1)^(1/3)*(cos(H*x0) + 1)^(1/6)*cos(H*x0)^(1/3)/sin(6*H*x0)^(1/3)"]

Out[2]=

$$\begin{aligned} & \left(\left(\left(\left(\left((2 * \text{Cos}[(H * x0)] - 1)^{\frac{1}{3}}) * ((\text{Cos}[(H * x0)] - 1)^{\frac{1}{6}}) * ((-1) + (2 * \text{Cos}[(2 * H * x0)]))^{\frac{1}{3}} * ((2 * \text{Cos}[(H * x0)] + 1)^{\frac{1}{3}}) * ((\text{Cos}[(H * x0)] + 1)^{\frac{1}{6}}) * (\text{Cos}[(H * x0)]^{\frac{1}{3}}) / (\sin((6 * H) * x0))^{\frac{1}{3}} \right) \right) \right) \right) \end{aligned}$$

M8 > KV := KillingVectors(g);

KV :=

$$\begin{aligned} & \left[-\frac{x6 \cos(Hx0)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} (2 \cos(Hx0) - 1)^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} D_{x5}}{\sin(6 Hx0)^{\frac{1}{6}}}, \right. \\ & + \frac{x5 \cos(Hx0)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} (2 \cos(Hx0) - 1)^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} D_{x6}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & \frac{x3 \cos(Hx0)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} (2 \cos(Hx0) - 1)^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} D_{x2}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & - \frac{x2 \cos(Hx0)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} (2 \cos(Hx0) - 1)^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} D_{x3}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & \frac{(2 \cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} \cos(Hx0)^{\frac{1}{6}} D_{x2}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & \frac{(2 \cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} \cos(Hx0)^{\frac{1}{6}} D_{x3}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & - \frac{x7 \cos(Hx0)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} (2 \cos(Hx0) - 1)^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} D_{x5}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & + \frac{x5 \cos(Hx0)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} (2 \cos(Hx0) - 1)^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} D_{x7}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & - \frac{(2 \cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} \cos(Hx0)^{\frac{1}{6}} D_{x5}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & - \frac{x7 \cos(Hx0)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} (2 \cos(Hx0) - 1)^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} D_{x6}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & + \frac{x6 \cos(Hx0)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} (2 \cos(Hx0) - 1)^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} D_{x7}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & - \frac{(2 \cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} \cos(Hx0)^{\frac{1}{6}} D_{x6}}{\sin(6 Hx0)^{\frac{1}{6}}}, \\ & - \frac{(2 \cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} \cos(Hx0)^{\frac{1}{6}} D_{x7}}, \\ & \frac{x2 \cos(Hx0)^{\frac{1}{6}} (\cos(Hx0) - 1)^{\frac{1}{6}} (\cos(Hx0) + 1)^{\frac{1}{6}} (2 \cos(Hx0) - 1)^{\frac{1}{6}} (2 \cos(Hx0) + 1)^{\frac{1}{6}} (-1 + 2 \cos(2 Hx0))^{\frac{1}{6}} D_{x1}}{\sin(6 Hx0)^{\frac{1}{6}}} \end{aligned}$$

Out[•]=

In[•]:=

Out[•]=

$$\begin{aligned}
& - \frac{x^1 \cos(Hx0)^1 \sqrt{\sin(Hx0)}}{\sin(6Hx0)^1} (\cos(Hx0) - 1)^1 \sqrt{\sin(Hx0)} (\cos(Hx0) + 1)^1 \sqrt{\sin(Hx0)} (2 \cos(Hx0) - 1)^1 \sqrt{\sin(Hx0)} (2 \cos(Hx0) + 1)^1 \sqrt{\sin(Hx0)} (-1 + 2 \cos(2Hx0))^1 \sqrt{\sin(Hx0)} D_x x^2, \\
& x^3 \cos(Hx0)^1 \sqrt{\sin(Hx0)} (\cos(Hx0) - 1)^1 \sqrt{\sin(Hx0)} (\cos(Hx0) + 1)^1 \sqrt{\sin(Hx0)} (2 \cos(Hx0) - 1)^1 \sqrt{\sin(Hx0)} (2 \cos(Hx0) + 1)^1 \sqrt{\sin(Hx0)} (-1 + 2 \cos(2Hx0))^1 \sqrt{\sin(Hx0)} D_x x^1, \\
& - \frac{x^1 \cos(Hx0)^1 \sqrt{\sin(Hx0)}}{\sin(6Hx0)^1} (\cos(Hx0) - 1)^1 \sqrt{\sin(Hx0)} (\cos(Hx0) + 1)^1 \sqrt{\sin(Hx0)} (2 \cos(Hx0) - 1)^1 \sqrt{\sin(Hx0)} (2 \cos(Hx0) + 1)^1 \sqrt{\sin(Hx0)} (-1 + 2 \cos(2Hx0))^1 \sqrt{\sin(Hx0)} D_x x^3, \\
& \frac{(2 \cos(Hx0) - 1)^1 \sqrt{\sin(Hx0)} (\cos(Hx0) - 1)^1 \sqrt{\sin(Hx0)} (-1 + 2 \cos(2Hx0))^1 \sqrt{\sin(Hx0)} (2 \cos(Hx0) + 1)^1 \sqrt{\sin(Hx0)} (\cos(Hx0) + 1)^1 \sqrt{\sin(Hx0)} \cos(Hx0)^1 \sqrt{\sin(Hx0)} D_x x^1}{\sin(6Hx0)^1} \Big]
\end{aligned}$$

8 > 1

8 > 1

4×4 Dirac γ matrices:

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta], \quad (22)$$

$$S^{\alpha\beta} = -\frac{1}{2}\gamma^{\alpha\beta}. \quad (23)$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \quad (24)$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^\alpha \epsilon = -\epsilon \gamma^\alpha, \quad (25)$$

$$\tilde{S}^{\alpha\beta} \epsilon = -\epsilon S^{\alpha\beta}, \quad (26)$$

$$[S^{\alpha\beta}, \gamma_\mu] = \delta_\mu^\alpha \gamma^\beta - \delta_\mu^\beta \gamma^\alpha, \quad (27)$$

$$[S^{\alpha\beta}, S^{\mu\nu}] = g^{\alpha\mu} S^{\beta\nu} - g^{\alpha\nu} S^{\beta\mu} - g^{\beta\mu} S^{\alpha\nu} + g^{\beta\nu} S^{\alpha\mu}, \quad (28)$$

and

$$\gamma^5 S^{\alpha\beta} = \frac{1}{2} g^{\alpha\mu} g^{\beta\nu} \epsilon_{\mu\nu\lambda\sigma} S^{\lambda\sigma}. \quad (29)$$

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

```
In[1]:= (*Symbolize[ g<sub>αβ</sub> ]*)

In[2]:= η4488
Dimensions[%]
g<sub>αβ</sub> = η4488[[2;;5, 2;;5]]

Out[1]=
{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0},
 {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1} }

Out[2]=
{8, 8}

Out[3]=
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1} }

In[4]:= Protect[g<sub>αβ</sub>]

Out[4]=
{g<sub>Subscript</sub><sub>αβ</sub>}

In[5]:= g3 = DiagonalMatrix[{1, 1, -1}]

Out[5]=
{{1, 0, 0}, {0, 1, 0}, {0, 0, -1} }

In[6]:= Do[Do[Gr[h, k] = t4by4[h].s4by4[k]*(-g3[[h, h]]*g3[[k, k]]), {k, 1, 3}], {h, 1, 3}]
```

```
In[•]:= Gr[1, 1] // MatrixForm
Out[•]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

```

```
In[•]:= γ = {Gr[1, 1], Gr[1, 2], Gr[1, 3], t4by4[2]};
```

Set: Symbol γ is Protected. [i](#)

```
In[•]:= Table[FullSimplify[γ[h].γ[h] === gαβ[h, h] * ID4], {h, 1, 4}]
```

```
Out[•]= {False, False, False, True}
```

$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\gamma_0 g^{\alpha\beta}, \quad (4)$$

where

$$g^{\alpha\beta} = g_{\alpha\beta} = \text{diag}(1, 1, 1, -1) \quad (5)$$

and γ_0 denotes the 4×4 unit matrix. The $\Sigma^{\alpha\beta}$ of Eq. (1)

```
In[•]:= Table[{{h, k}, FullSimplify[(γ[h].γ[k] + γ[k].γ[h])/2] === gαβ[h, k] ID4}, {h, 1, 4}, {k, 1, 4}] // MatrixForm
```

```
Out[•]//MatrixForm=

$$\begin{pmatrix} (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) \\ (\text{False}) & (\text{True}) & (\text{True}) & (\text{False}) \\ (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) \\ (\text{True}) & (\text{False}) & (\text{True}) & (\text{False}) \\ (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) \\ (\text{True}) & (\text{True}) & (\text{False}) & (\text{False}) \\ (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) \\ (\text{False}) & (\text{False}) & (\text{False}) & (\text{True}) \end{pmatrix}$$

```

```
In[•]:= Table[{{h, k}, FullSimplify[(γ[h].γ[k] + γ[k].γ[h])/2] === η4488[h + k] ID4}, {h, 1, 4}, {k, 1, 4}] // MatrixForm
```

```
Out[•]//MatrixForm=

$$\begin{pmatrix} (\{1, 1\}) & (\{1, 2\}) & (\{1, 3\}) & (\{1, 4\}) \\ (\text{False}) & (\text{True}) & (\text{True}) & (\text{False}) \\ (\{2, 1\}) & (\{2, 2\}) & (\{2, 3\}) & (\{2, 4\}) \\ (\text{True}) & (\text{False}) & (\text{True}) & (\text{False}) \\ (\{3, 1\}) & (\{3, 2\}) & (\{3, 3\}) & (\{3, 4\}) \\ (\text{True}) & (\text{True}) & (\text{False}) & (\text{False}) \\ (\{4, 1\}) & (\{4, 2\}) & (\{4, 3\}) & (\{4, 4\}) \\ (\text{False}) & (\text{False}) & (\text{False}) & (\text{True}) \end{pmatrix}$$

```

$$\gamma^5 = -\gamma^1 \gamma^2 \gamma^3 \gamma^4$$

```
In[1]:= -γ[[1]] . γ[[2]] . γ[[3]] . γ[[4]]
Out[1]= {{0, 0, 0, 1}, {0, 0, -1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}
```

```
In[2]:= AppendTo[γ, (-γ[[1]] . γ[[2]] . γ[[3]] . γ[[4]])]
```

Set: Symbol γ is Protected. [i](#)

```
Out[2]= {{ {0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0} },
          { {0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0} },
          { {0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0} },
          { {0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0} },
          { {0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0} },
          { {0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0} },
          { {1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1} },
          { {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0} },
          { {0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0} },
          { {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0} },
          { {-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1} },
          { {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0} },
          { {0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0} },
          { {0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0} },
          { {-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1} },
          { {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1} },
          { {0, 0, 0, 1}, {0, 0, -1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0} }}
```

```
In[3]:= Protect[γ]
```

```
Out[3]= {}
```

```
In[4]:= γ[[5]]
```

```
Out[4]= {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}
```

```
In[5]:= γ[[5]] === -t4by4[3]
```

```
Out[5]= False
```

```
In[•]:= Table[{{h, k}, FullSimplify[\gamma[h].\gamma[k] + \gamma[k].\gamma[h]] === 2 * \eta 4488 [1 + h, 1 + k] ID4}, {h, 1, 5}, {k, 1, 5}] // MatrixForm
```

```
Out[•]//MatrixForm=

$$\begin{pmatrix} \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} & \{1, 5\} \\ \text{False} & \text{True} & \text{True} & \text{False} & \text{False} \\ \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} & \{2, 5\} \\ \text{True} & \text{False} & \text{True} & \text{False} & \text{False} \\ \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} & \{3, 5\} \\ \text{True} & \text{True} & \text{False} & \text{False} & \text{False} \\ \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} & \{4, 5\} \\ \text{False} & \text{False} & \text{False} & \text{True} & \text{True} \\ \{5, 1\} & \{5, 2\} & \{5, 3\} & \{5, 4\} & \{5, 5\} \\ \text{False} & \text{False} & \text{False} & \text{True} & \text{True} \end{pmatrix}$$

```

$$\gamma^\alpha = \gamma^{\alpha 6}, \quad (19)$$

$$\gamma^5 = \gamma^{56}, \quad (20)$$

$$\gamma^\alpha \gamma^5 = \gamma^{\alpha 5}, \quad (21)$$

$$\epsilon = \gamma^{64}.$$

```
In[•]:= -\gamma[4] // MatrixForm
```

```
Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

```

```
In[•]:= s\epsilon = {\epsilon \rightarrow t4by4[2]}
```

```
Out[•]= {\epsilon \rightarrow {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}
```

```
In[•]:= Transpose[\epsilon /. s\epsilon] === (-\epsilon /. s\epsilon)
```

```
Out[•]= True
```

```
In[•]:= ParallelTable[
  {{A1}}, FullSimplify[(\epsilon /. s\epsilon).\gamma[A1] === Transpose[(\epsilon /. s\epsilon).\gamma[A1]]]], {A1, 1, 5}]
```

```
Out[•]= {{1, True}, {2, True}, {3, True}, {4, False}, {5, True}}
```

```
In[•]:= \gamma[4] === t4by4[2]
```

```
Out[•]= False
```

```
In[1]:= t4by4[1]
Out[1]= {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}}
```

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta]. \quad (3)$$

```
In[2]:= S44\alpha\beta = -\frac{1}{4} Table[FullSimplify[\gamma[h].\gamma[k] - \gamma[k].\gamma[h]], {h, 1, 4}, {k, 1, 4}];
```

```
In[3]:= Protect[S44\alpha\beta]
```

```
Out[3]= {S44\alpha\beta}
```

```
In[4]:= \eta44 = DiagonalMatrix[{1, 1, 1, -1}]
```

```
Out[4]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -1}}
```

```
In[5]:= Protect[\eta44]
```

```
Out[5]= {\eta44}
```

S44 $\alpha\beta$ commutation relations; misc :

$$S^{\alpha\beta} = -\frac{1}{4}[\gamma^\alpha, \gamma^\beta], \quad (22)$$

$$S^{\alpha\beta} = -\frac{1}{2}\gamma^{\alpha\beta}. \quad (23)$$

The symplectic form ϵ on D_4 is defined to be

$$\epsilon = \gamma^{64}. \quad (24)$$

As a consequence of Eq. (1), and the definitions of Eqs. (16)–(24), are the identities

$$\tilde{\gamma}^\alpha \epsilon = -\epsilon \gamma^\alpha, \quad (25)$$

$$\tilde{S}^{\alpha\beta} \epsilon = -\epsilon S^{\alpha\beta}, \quad (26)$$

$$[S^{\alpha\beta}, \gamma_\mu] = \delta_\mu^\alpha \gamma^\beta - \delta_\mu^\beta \gamma^\alpha, \quad (27)$$

$$[S^{\alpha\beta}, S^{\mu\nu}] = g^{\alpha\mu} S^{\beta\nu} - g^{\alpha\nu} S^{\beta\mu} - g^{\beta\mu} S^{\alpha\nu} + g^{\beta\nu} S^{\alpha\mu}, \quad (28)$$

and

$$\gamma^5 S^{\alpha\beta} = \frac{1}{2} g^{\alpha\mu} g^{\beta\nu} \epsilon_{\mu\nu\lambda\sigma} S^{\lambda\sigma}. \quad (29)$$

```
S44\alpha\beta = -\frac{1}{4} Table[FullSimplify[\gamma[h].\gamma[k] - \gamma[k].\gamma[h]], {h, 1, 4}, {k, 1, 4}];
```

```
(*$SAB=Table[\frac{1}{4} (T16^A[A1].T16^A[B1]-T16^A[B1].T16^A[A1]), {A1,0, 7},{B1,0, 7}];*)
```

```
In[8]:= (*ParallelTable[FullSimplify[\[sigma]16.SAB[A1,B1]===Transpose[SAB[A1,B1]]],{A1,1,8},{B1,1,8}]/.Flatten/.Union*)


```

```
In[9]:= (*ParallelTable[{A1,B1},FullSimplify[SAB[A1,B1]===Transpose[SAB[A1,B1]]]],{A1,1,8},{B1,1,8}]*)


```

```
In[10]:= ParallelTable[
  FullSimplify[S44\[Alpha]\[Beta][A1,B1].S44\[Alpha]\[Beta][A2,B2]-S44\[Alpha]\[Beta][A2,B2].S44\[Alpha]\[Beta][A1,B1]==
  (\[Eta]44[A1,A2]\[Times]S44\[Alpha]\[Beta][B1,B2]-\[Eta]44[A1,B2]\[Times]S44\[Alpha]\[Beta][B1,A2]-
  \[Eta]44[B1,A2]\[Times]S44\[Alpha]\[Beta][A1,B2]+\[Eta]44[B1,B2]\[Times]S44\[Alpha]\[Beta][A1,A2])],
  {A1,1,3}, {B1,A1+1,4}, {A2,1,3}, {B2,A2+1,4}]/.Flatten/.Union


```

```
Out[10]= {False, True}
```

```
In[•]:= ParallelTable[{ {A1, B1, A2, B2},  
  FullSimplify[S44αβ[A1, B1].S44αβ[A2, B2] - S44αβ[A2, B2].S44αβ[A1, B1] ==  
   (η44[A1, A2] × S44αβ[B1, B2] - η44[A1, B2] × S44αβ[B1, A2] -  
    η44[B1, A2] × S44αβ[A1, B2] + η44[B1, B2] × S44αβ[A1, A2])],  
  {A1, 1, 4}, {B1, 1, 4}, {A2, 1, 4}, {B2, 1, 4}] // MatrixForm
```

Out[•]/MatrixForm=

$\begin{pmatrix} \{1, 1, 1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 1, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 1, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 1, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2, 1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2, 1, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{1, 1, 2, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 2, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 2, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 2, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2, 2, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2, 2, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{1, 1, 3, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 3, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 3, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 3, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2, 3, 1\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{1, 2, 3, 2\} \\ \text{False} \end{pmatrix}$
$\begin{pmatrix} \{1, 1, 4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 4, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 4, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 1, 4, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2, 4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{1, 2, 4, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{2, 1, 1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 1, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 1, 3\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 1, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2, 1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2, 1, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{2, 1, 2, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 2, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 2, 3\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 2, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2, 2, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2, 2, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{2, 1, 3, 1\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 3, 2\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 3, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 3, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2, 3, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2, 3, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{2, 1, 4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 4, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 4, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 1, 4, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2, 4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{2, 2, 4, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{3, 1, 1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 1, 2\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 1, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 1, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 2, 1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 2, 1, 2\} \\ \text{False} \end{pmatrix}$
$\begin{pmatrix} \{3, 1, 2, 1\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 2, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 2, 3\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 2, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 2, 2, 1\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{3, 2, 2, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{3, 1, 3, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 3, 2\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 3, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 3, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 2, 3, 1\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{3, 2, 3, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{3, 1, 4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 4, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 4, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 1, 4, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 2, 4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{3, 2, 4, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{4, 1, 1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 1, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 1, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 1, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 2, 1, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 2, 1, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{4, 1, 2, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 2, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 2, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 2, 4\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{4, 2, 2, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 2, 2, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{4, 1, 3, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 3, 2\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 3, 3\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 3, 4\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{4, 2, 3, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 2, 3, 2\} \\ \text{True} \end{pmatrix}$
$\begin{pmatrix} \{4, 1, 4, 1\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 4, 2\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 4, 3\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{4, 1, 4, 4\} \\ \text{True} \end{pmatrix}$	$\begin{pmatrix} \{4, 2, 4, 1\} \\ \text{False} \end{pmatrix}$	$\begin{pmatrix} \{4, 2, 4, 2\} \\ \text{True} \end{pmatrix}$

```
In[•]:= ParallelTable[FullSimplify[S44αβ[A1, B1].γ[B2] - γ[B2].S44αβ[A1, B1] ==  
  (η44[B2, A1] × γ[B1] - η44[B2, B1] × γ[A1])],  
  {A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // Flatten // Union
```

Out[•]= {False, True}

```
In[•]:= ParallelTable[
  {{A1, B1, B2}, FullSimplify[S44αβ[A1, B1].γ[B2] - γ[B2].S44αβ[A1, B1] ==
    (η44[B2, A1] × γ[B1] - η44[B2, B1] × γ[A1])]}, {A1, 1, 4}, {B1, 1, 4}, {B2, 1, 4}] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \{1, 1, 1\} & \text{True} & \{1, 2, 1\} & \text{False} & \{1, 3, 1\} & \text{False} & \{1, 4, 1\} & \text{False} \\ \{1, 1, 2\} & \text{True} & \{1, 2, 2\} & \text{False} & \{1, 3, 2\} & \text{True} & \{1, 4, 2\} & \text{True} \\ \{1, 1, 3\} & \text{True} & \{1, 2, 3\} & \text{True} & \{1, 3, 3\} & \text{False} & \{1, 4, 3\} & \text{True} \\ \{1, 1, 4\} & \text{True} & \{1, 2, 4\} & \text{True} & \{1, 3, 4\} & \text{True} & \{1, 4, 4\} & \text{False} \\ \{2, 1, 1\} & \text{False} & \{2, 2, 1\} & \text{True} & \{2, 3, 1\} & \text{True} & \{2, 4, 1\} & \text{True} \\ \{2, 1, 2\} & \text{False} & \{2, 2, 2\} & \text{True} & \{2, 3, 2\} & \text{False} & \{2, 4, 2\} & \text{False} \\ \{2, 1, 3\} & \text{True} & \{2, 2, 3\} & \text{True} & \{2, 3, 3\} & \text{False} & \{2, 4, 3\} & \text{True} \\ \{2, 1, 4\} & \text{True} & \{2, 2, 4\} & \text{True} & \{2, 3, 4\} & \text{True} & \{2, 4, 4\} & \text{False} \\ \{3, 1, 1\} & \text{False} & \{3, 2, 1\} & \text{True} & \{3, 3, 1\} & \text{True} & \{3, 4, 1\} & \text{True} \\ \{3, 1, 2\} & \text{True} & \{3, 2, 2\} & \text{False} & \{3, 3, 2\} & \text{True} & \{3, 4, 2\} & \text{True} \\ \{3, 1, 3\} & \text{False} & \{3, 2, 3\} & \text{False} & \{3, 3, 3\} & \text{True} & \{3, 4, 3\} & \text{False} \\ \{3, 1, 4\} & \text{True} & \{3, 2, 4\} & \text{True} & \{3, 3, 4\} & \text{True} & \{3, 4, 4\} & \text{False} \\ \{4, 1, 1\} & \text{False} & \{4, 2, 1\} & \text{True} & \{4, 3, 1\} & \text{True} & \{4, 4, 1\} & \text{True} \\ \{4, 1, 2\} & \text{True} & \{4, 2, 2\} & \text{False} & \{4, 3, 2\} & \text{True} & \{4, 4, 2\} & \text{True} \\ \{4, 1, 3\} & \text{True} & \{4, 2, 3\} & \text{True} & \{4, 3, 3\} & \text{False} & \{4, 4, 3\} & \text{True} \\ \{4, 1, 4\} & \text{False} & \{4, 2, 4\} & \text{False} & \{4, 3, 4\} & \text{False} & \{4, 4, 4\} & \text{True} \end{pmatrix}$$

```
In[•]:= ParallelTable[{ {A1, B1},
  FullSimplify[(ε /. se).S44αβ[A1, B1] === Transpose[(ε /. se).S44αβ[A1, B1]]}],
 {A1, 1, 3}, {B1, A1 + 1, 4}]
```

Out[•]=

$$\{\{\{1, 2\}, \text{True}\}, \{\{1, 3\}, \text{True}\}, \{\{1, 4\}, \text{True}\}\}, \{\{\{2, 3\}, \text{True}\}, \{\{2, 4\}, \text{True}\}\}, \{\{\{3, 4\}, \text{True}\}\}$$

The commutation relations for the generators J_{ab} of the $\mathfrak{so}(3,3)$ Lie algebra (using the physics convention where generators are Hermitian) are given by the formula: [🔗](#)

$$[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$$

where:

- The indices a, b, c, d range from 1 to 6.
- η_{ab} is the **metric tensor** with signature $(+, +, +, -, -, -)$ (or the reverse, depending on the convention). A common choice is a diagonal matrix $\eta = \text{diag}(1, 1, 1, -1, -1, -1)$.
- J_{ab} are the **generators** of the algebra, which are antisymmetric, $J_{ab} = -J_{ba}$.
- The terms on the right-hand side involve Kronecker deltas weighted by the metric, ensuring the resulting generator is also an element of the algebra and satisfies the required antisymmetry and Jacobi identity properties. [🔗](#)

The final answer is $[J_{ab}, J_{cd}] = i(\eta_{ac}J_{bd} - \eta_{ad}J_{bc} - \eta_{bc}J_{ad} + \eta_{bd}J_{ac})$.

Step 1: Define the generators and metric

The **so(4,4)** Lie algebra has 28 generators, which can be denoted as $J_{\mu\nu}$ where the indices μ, ν range from 0 to 7, and $J_{\mu\nu} = -J_{\nu\mu}$. The Lie algebra is defined in an 8-dimensional real vector space equipped with a metric $\eta_{\mu\nu}$ with signature (4, 4). ⓘ

- **Generators:** $J_{\mu\nu}$ with $\mu, \nu \in \{0, 1, \dots, 7\}$
- **Metric tensor:** $\eta_{\mu\nu} = \text{diag}(-1, -1, -1, -1, +1, +1, +1, +1)$ (or the opposite sign convention) ⓘ

Step 2: State the general commutation relation

The commutation relations for the generators of the **so(p,q)** Lie algebra are a generalization of the **so(n)** algebra relations, incorporating the metric tensor $\eta_{\mu\nu}$. ⓘ

Answer:

The commutation relations for the generators $J_{\mu\nu}$ of the **so(4,4)** Lie algebra are:

$$[J_{\mu\nu}, J_{\rho\sigma}] = \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho}$$

where $\eta_{\mu\nu}$ is the metric tensor with a (4,4) signature (four plus signs and four minus signs, or vice versa). ⓘ

(Note: In some physics conventions, an extra factor of i or $i\hbar$ is included depending on whether the generators are defined as Hermitian or not). ⓘ

what are the commutation relation for the generators of so(4,4)?

The commutation relations for the generators $J_{\mu\nu}$ of the *so*(4, 4) Lie algebra, where $\mu, \nu \in \{0, 1, \dots, 7\}$ and $J_{\mu\nu} = -J_{\nu\mu}$, are the **standard relations for orthogonal Lie algebras**, given by a generalization of the angular momentum commutation relations: ⓘ

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho} J_{\nu\sigma} - \eta_{\mu\sigma} J_{\nu\rho} - \eta_{\nu\rho} J_{\mu\sigma} + \eta_{\nu\sigma} J_{\mu\rho})$$

where $\eta_{\mu\nu}$ is the flat metric of 8-dimensional Minkowski space with signature (4,4) (or some other convention, the structure constants are the same as for *so*(8) in an appropriate basis), which is a diagonal matrix with four positive and four negative entries (e.g., $\eta = \text{diag}(+1, +1, +1, +1, -1, -1, -1, -1)$).



↳ Why is the metric signature significant for \$so(4,4)\$ commutation relations?

Why is the metric signature significant for \$so(4,4)\$ commutation relations?

The metric signature is significant for the $so(p, q)$ commutation relations because the metric tensor $\eta_{\mu\nu}$ acts as the **structure constants** of the Lie algebra, defining its specific algebraic structure and physical properties (if applicable). ⓘ

Here is a detailed explanation of its significance:

- **Definition of the Algebra:** The commutation relation for the generators $J_{\mu\nu}$ of any orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$
The values of the structure constants depend directly on the components of the metric $\eta_{\mu\nu}$. This means that the specific algebraic properties of $so(4, 4)$ are distinct from $so(8)$ (which has a (8,0) signature, i.e., a Euclidean metric), even though they operate in the same 8-dimensional vector space.
- **Physical Interpretation:** In physics, the metric signature determines the nature of spacetime. A mixed signature (like (4,4) or the more common (1,3) Minkowski signature) implies a causal structure, distinguishing between timelike and spacelike dimensions.
 - Generators associated with rotations in spacelike dimensions (positive $\eta_{\mu\nu}$ entries) typically lead to compact symmetry groups (like $SO(4)$ or $SO(8)$).

In[®]:= **η4488**

Out[®]=

$$\{\{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 0, 0, -1, 0\}, \{0, 0, 0, 0, 0, 0, 0, -1\}\}$$

orthogonal Lie algebra $so(p, q)$ is given by:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(\eta_{\mu\rho}J_{\nu\sigma} - \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\rho}J_{\mu\sigma} + \eta_{\nu\sigma}J_{\mu\rho})$$

$$t^A = \begin{pmatrix} 0 & \vec{\tau}^A \\ \tau^A & 0 \end{pmatrix}.$$

```
In[®]:= (*Table[SAB[[A1,B1]]== ArrayFlatten[{{SAB1[[A1,B1]],0},{0,SAB2[[A1,B1]]}}],{A1,0,7},{B1,0,7}]*)
```

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A$$

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A.$$

Let $g \in \overline{SO(4, 4)}$ and $L \in SO(4, 4)$. The canonical 2-1 homomorphism $\overline{SO(4, 4)} \rightarrow SO(4, 4)$ is given by

$$(2.8) \quad 8L^{A'}{}_{B'}(g) = \text{tr } \bar{D}^{(2)}(g) \tau^{A'} D^{(1)}(g) \bar{\tau}_{B'}.$$

As usual tr denotes the trace. Under the action of $\overline{SO(4, 4)}$

$$(2.9) \quad \bar{D}^{(1)} \sigma = \sigma D^{(1)-1},$$

$$(2.10) \quad \sigma^{-1} \bar{D}^{(2)} = D^{(2)-1} \sigma^{-1},$$

$$(2.11) \quad L^{A'}{}_{C'} G_{A'B'} L^{B'}{}_{D'} = G_{C'D'},$$

$$(2.12) \quad L^{A'}{}_{B'} \tau^{B'} = \bar{D}^{(2)} \tau^{A'} D^{(1)},$$

$$(2.13) \quad L^{A'}{}_{B'} \bar{\tau}^{B'} = D^{(1)-1} \bar{\tau}^{A'} \bar{D}^{(2)-1}.$$

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A Remarkable Representation of the 3 + 2 de Sitter Group

P. A. M. DIRAC

*Cambridge University, Cambridge, England and
Institute for Advanced Study, Princeton, New Jersey**
(Received 20 February 1963)

Identities satisfied by the generators of the Dirac algebra

Patrick L. Nash

M. S. 474, NASA Langley Research Center, Hampton, Virginia 23665

(Received 26 July 1983; accepted for publication 23 September 1983)

```
In[•]:= Unprotect[ $\gamma$ ABs];
 $\gamma$ ABs = Flatten[{s4by4[#] & /@ Range[3], t4by4[#] & /@ Range[3],
Flatten[Table[ $st$ [J, K], {J, 1, 3}, {K, 1, 3}], 1], {ID4}], 1]
```

```
Out[•]=
{{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}}, {{0, 0, -1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, -1, 0, 0}}}, {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, -1, 0}}}, {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {1, 0, 0, 0}}}, {{0, 0, -1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}, {{0, 1, 0, 0}, {-1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}}}, {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}}, {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}}, {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}}}, {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, -1}, {0, 0, -1, 0}}}, {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}}, {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}}}, {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}}}, {{0, 0, 0, -1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {-1, 0, 0, 0}}}, {{-1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}, {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}}}
```

```
In[•]:= Length[ $\gamma$ ABs]
```

```
Out[•]=
```

```
16
```

```
In[•]:= Protect[ $\gamma$ ABs]
```

```
Out[•]=
```

```
{ $\gamma$ ABs}
```

```
In[•]:=  $\gamma$ ABs[[1]]
```

```
Out[•]=
```

```
{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}}
```

```
In[•]:=  $\gamma$ ABs[[ -1 ]]
```

```
Out[•]=
```

```
{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Recall Self Dual Anti-Symmetric

```
In[•]:= Table[ $\left(\frac{1}{2} \sum_{j1=1}^4 \left(\sum_{j2=1}^4 \epsilon 4[p, q, j1, j2] \times s4by4[h][j1, j2]\right)\right) - s4by4[h][p, q],$ 
```

```
{h, 1, 3}, {q, 4}, {p, 4}]
```

```
Out[•]=
```

```
{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

Recall Anti SelfDual Anti-Symmetric

```
In[8]:= Table[(1/2) Sum[Sum[epsilon4[p, q, j1, j2] t4by4[h][j1, j2]], {j1, 1, 4}], {j2, 1, 4}] + t4by4[h][p, q], {h, 1, 3}, {q, 4}, {p, 4}]

Out[8]= {{{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}, {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}}}
```

BASIS of 4×4 matrices :

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The Dirac spinor in six dimensions

BY E. A. LORD

Department of Mathematics, King's College, University of London

(Received 17 November 1966)

3. Decomposition of the basic spinor representations. We have already stated that the basic spinor representations of $SO(n, c)$, for n even, are reducible. We shall consider here the reduction of the basic 8-component spinor representation of $SO(6, c)$ into constituent 4-component representations. The procedure is easily generalized to apply to any even-dimensional space. An irreducible representation of the Clifford algebra C_6 can be constructed from a given irreducible representation of C_5 . The $\gamma_\mu (\mu = 1, \dots, 5)$ of an irreducible representation of C_5 satisfy (2.5), so that the quantities $e_\alpha (\alpha = 1, \dots, 6)$ defined by

$$\left. \begin{aligned} e_\mu &= (-i) \begin{pmatrix} \gamma_\mu B \\ -B^{-1}\gamma_\mu \end{pmatrix} \quad (\mu = 1, \dots, 5), \\ e_6 &= \begin{pmatrix} B \\ B^{-1} \end{pmatrix}, \end{aligned} \right\} \quad (3.1)$$

satisfy

$$\frac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \dots, 6).$$

For the moment the matrix B is completely arbitrary. The e_α therefore give an irreducible representation of C_6 . They can be written more concisely in terms of the sets of matrices a_α and \bar{a}_α , defined as follows:

$$\left. \begin{aligned} a_\mu &= \gamma_\mu B, & \bar{a}_\mu &= -B^{-1}\gamma_\mu \quad (\mu = 1, \dots, 5), \\ a_6 &= iB, & \bar{a}_6 &= iB^{-1}. \end{aligned} \right\} \quad (3.2)$$

The generator e_α of C_6 can then be written

$$e_\alpha = (-i) \begin{pmatrix} a_\alpha \\ \bar{a}_\alpha \end{pmatrix} \quad (\alpha = 1, \dots, 6). \quad (3.3)$$

With $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1e_2e_3e_4e_5e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

and $e_{\alpha\beta} = \frac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2} \begin{pmatrix} a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha & \\ \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha & \end{pmatrix}.$

This latter expression shows that the infinitesimal generators of the 8-component spinor representation of $SO(6, c)$ is the direct sum of two 4-component representations. The basic spinor can be written

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix},$$

where ϕ and χ are four component spinors which transform according to representations with infinitesimal generators

$$\left. \begin{aligned} G_{\alpha\beta}^{(1)} &= -\frac{1}{4}(a_\alpha \bar{a} - a_\beta \bar{a}_\alpha) \\ \text{and} \quad G_{\alpha\beta}^{(2)} &= -\frac{1}{4}(\bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha), \end{aligned} \right\} \quad (3.4)$$

$$\underbrace{\text{sy}}_1, \underbrace{\text{sy}}_4, \underbrace{\text{asy}}_6, \underbrace{\text{sy}}_4, \underbrace{\text{sy}}_1$$

```

In[1]:= Plus @@ {1, 4, 6, 4, 1}
Out[1]=
16

{sy, sy, asy, sy, sy}
  1   4   6   4   1

In[2]:= Plus @@ {1, 4, 6, 4, 1}
Out[2]=
16

In[3]:= MatrixForm[If[#=2, ix, 1]*PauliMatrix[#]] & /@ Range[3]
Out[3]=
{{(0 1), (0 1), (1 0)}, {(1 0), (-1 0), (0 -1)}}

In[4]:= σ22 = Flatten[{ {IdentityMatrix[2]}, If[#=2, ix, 1]*PauliMatrix[#] & /@ Range[3]}, 1]
Out[4]=
{{{1, 0}, {0, 1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}}}

In[5]:= σ22 =
Flatten[{ { -IdentityMatrix[2]}, If[#=2, ix, 1]*PauliMatrix[#] & /@ Range[3]}, 1]
Out[5]=
{{{ -1, 0}, {0, -1}}, {{0, 1}, {1, 0}}, {{0, 1}, {-1, 0}}, {{1, 0}, {0, -1}}}

In[6]:= -IdentityMatrix[2] === Dot @@ (σ22[[#]] & /@ Range[Length[σ22]])
Out[6]=
True

In[7]:= Table[{{A, B}, FullSimplify[1/2 Tr[1/2 (σ22[[A]].σ22[[B]] + σ22[[B]].σ22[[A]])]]}], {{A, 1, 4}, {B, 1, 4}}] // MatrixForm
Out[7]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} \{1, 1\} \\ -1 \end{pmatrix} & \begin{pmatrix} \{1, 2\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{1, 3\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{1, 4\} \\ 0 \end{pmatrix} \\ \begin{pmatrix} \{2, 1\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{2, 2\} \\ 1 \end{pmatrix} & \begin{pmatrix} \{2, 3\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{2, 4\} \\ 0 \end{pmatrix} \\ \begin{pmatrix} \{3, 1\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{3, 2\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{3, 3\} \\ -1 \end{pmatrix} & \begin{pmatrix} \{3, 4\} \\ 0 \end{pmatrix} \\ \begin{pmatrix} \{4, 1\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{4, 2\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{4, 3\} \\ 0 \end{pmatrix} & \begin{pmatrix} \{4, 4\} \\ 1 \end{pmatrix} \end{pmatrix}$$


In[8]:= η2244 = DiagonalMatrix[{-1, 1, -1, 1}]
Out[8]=
{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, 1}}

```

```
In[•]:= Table[{{A, B}, FullSimplify[ExpandAll[ $\frac{1}{2} (\sigma_{22}[A] \cdot \bar{\sigma}_{22}[B] + \sigma_{22}[B] \cdot \bar{\sigma}_{22}[A]) ==$ 
 $\eta_{2244}[A, B] * IdentityMatrix[2]]]}], {A, 1, 4}, {B, 1, 4}] // MatrixForm$ 
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} \\ \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} \\ \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} \\ \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} \\ \text{True} & \text{True} & \text{True} & \text{True} \end{pmatrix}$$

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The Dirac spinor in six dimensions

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$$\left. \begin{aligned} e_\mu &= (-i) \begin{pmatrix} & \gamma_\mu B \\ -B^{-1}\gamma_\mu & \end{pmatrix} \quad (\mu = 1, \dots, 5), \\ e_6 &= \begin{pmatrix} & B \\ B^{-1} & \end{pmatrix}, \end{aligned} \right\}$$

$$\tfrac{1}{2}(e_\alpha e_\beta + e_\beta e_\alpha) = \delta_{\alpha\beta} \quad (\alpha, \beta = 1, \dots, 6).$$

$$\left. \begin{aligned} \alpha_\mu &= \gamma_\mu B, \quad \bar{a}_\mu = -B^{-1}\gamma_\mu \quad (\mu = 1, \dots, 5), \\ a_6 &= iB, \quad \bar{a}_6 = iB^{-1}. \end{aligned} \right\}$$

generator e_α of C_6 can then be written

$$e_\alpha = (-i) \begin{pmatrix} & a_\alpha \\ \bar{a}_\alpha & \end{pmatrix} \quad (\alpha = 1, \dots, 6).$$

Since $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we then find

$$e_7 = ie_1 e_2 e_3 e_4 e_5 e_6 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$e_{\alpha\beta} = \tfrac{1}{2}(e_\alpha e_\beta - e_\beta e_\alpha) = -\frac{1}{2} \begin{pmatrix} a_\alpha \bar{a}_\beta - a_\beta \bar{a}_\alpha & \\ & \bar{a}_\alpha a_\beta - \bar{a}_\beta a_\alpha \end{pmatrix}.$$

```

In[]:= Do[yyy[j] = ArrayFlatten[{{0, σ22[[j]]}, {-σ22[[j]], 0}}], {j, 4}];
yyy[5] = ArrayFlatten[{{0, IdentityMatrix[2]}, {IdentityMatrix[2], 0}}]

Out[=]
{{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}};

In[]:= yyy[5] = {{0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0}};

Out[=]
{{0, 0, -1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, 1, 0, 0}};

In[]:= (yyy[6] = Dot @@ (yyy[#] & /@ Range[5])) // MatrixForm
Out[=]/MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$


In[]:= MatrixForm[yyy[#]] & /@ Range[6]
Out[=]
{ $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$ }

In[]:= Do[xxx[j] = ArrayFlatten[{{0, σ22[[j]]}, {σ22[[j]], 0}}], {j, 4}]

In[]:= MatrixForm[xxx[#]] & /@ Range[4]
Out[=]
{ $\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$ }

In[]:= xxx[5] = Dot @@ (xxx[#] & /@ Range[4])
Out[=]
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};

In[]:= IdentityMatrix[4]
Out[=]
{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}

```

```
In[•]:= txA0 = {{IdentityMatrix[4], {0}}};  
Length[txA0]  
txA = {} ; Do[AppendTo[txA, {xxx[j], {j}}], {j, 1, 4}]  
Length[txA]  
txAB = {};  
Do[AppendTo[txAB, {xxx[j].xxx[k], {j, k}}], {j, 1, 3}, {k, j + 1, 4}]  
Length[txAB]  
txABC = {};  
Do[AppendTo[txABC, {xxx[j].xxx[k].xxx[h], {j, k, h}}],  
{j, 1, 2}, {k, j + 1, 3}, {h, k + 1, 4}];  
Length[txABC]  
txABCD = {{xxx[1].xxx[2].xxx[3].xxx[4], {1, 2, 3, 4}}};  
Length[txABCD]
```

```
Out[•]= 1  
  
Out[•]= 4  
  
Out[•]= 6  
  
Out[•]= 4  
  
Out[•]= 1
```

```
In[•]:= base4by4 = Flatten[{txA0, txA, txA, txA, txA}, 1]; Length[base4by4]
```

```
Out[•]= 16
```

```
In[8]:= Table[{ {A, B}, Block[{r}, r = FullSimplify[
  1/4 Tr[1/2 (base4by4[A, 1].base4by4[B, 1] + base4by4[B, 1].base4by4[A, 1])]];
  If[r == 0, Style[r, Red], Style[r, Darker[Green]]] ] },
  {A, 1, Length[base4by4]}, {B, 1, Length[base4by4]} ] // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} \{1, 1\} & \{1, 2\} & \{1, 3\} & \{1, 4\} & \{1, 5\} & \{1, 6\} & \{1, 7\} & \{1, 8\} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{2, 1\} & \{2, 2\} & \{2, 3\} & \{2, 4\} & \{2, 5\} & \{2, 6\} & \{2, 7\} & \{2, 8\} \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{3, 1\} & \{3, 2\} & \{3, 3\} & \{3, 4\} & \{3, 5\} & \{3, 6\} & \{3, 7\} & \{3, 8\} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{4, 1\} & \{4, 2\} & \{4, 3\} & \{4, 4\} & \{4, 5\} & \{4, 6\} & \{4, 7\} & \{4, 8\} \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{5, 1\} & \{5, 2\} & \{5, 3\} & \{5, 4\} & \{5, 5\} & \{5, 6\} & \{5, 7\} & \{5, 8\} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{6, 1\} & \{6, 2\} & \{6, 3\} & \{6, 4\} & \{6, 5\} & \{6, 6\} & \{6, 7\} & \{6, 8\} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{7, 1\} & \{7, 2\} & \{7, 3\} & \{7, 4\} & \{7, 5\} & \{7, 6\} & \{7, 7\} & \{7, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} \{8, 1\} & \{8, 2\} & \{8, 3\} & \{8, 4\} & \{8, 5\} & \{8, 6\} & \{8, 7\} & \{8, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \{9, 1\} & \{9, 2\} & \{9, 3\} & \{9, 4\} & \{9, 5\} & \{9, 6\} & \{9, 7\} & \{9, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{10, 1\} & \{10, 2\} & \{10, 3\} & \{10, 4\} & \{10, 5\} & \{10, 6\} & \{10, 7\} & \{10, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{11, 1\} & \{11, 2\} & \{11, 3\} & \{11, 4\} & \{11, 5\} & \{11, 6\} & \{11, 7\} & \{11, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{12, 1\} & \{12, 2\} & \{12, 3\} & \{12, 4\} & \{12, 5\} & \{12, 6\} & \{12, 7\} & \{12, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{13, 1\} & \{13, 2\} & \{13, 3\} & \{13, 4\} & \{13, 5\} & \{13, 6\} & \{13, 7\} & \{13, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{14, 1\} & \{14, 2\} & \{14, 3\} & \{14, 4\} & \{14, 5\} & \{14, 6\} & \{14, 7\} & \{14, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{15, 1\} & \{15, 2\} & \{15, 3\} & \{15, 4\} & \{15, 5\} & \{15, 6\} & \{15, 7\} & \{15, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \{16, 1\} & \{16, 2\} & \{16, 3\} & \{16, 4\} & \{16, 5\} & \{16, 6\} & \{16, 7\} & \{16, 8\} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[9]:= base4by4[[16]][1]
```

Out[9]=

```
{ {1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1} }
```

```
In[•]:= anti4by4 = {};
Do[If[base4by4[[k, 1]] == -Transpose[base4by4[[k, 1]]], AppendTo[anti4by4,
    {base4by4[[k, 1]], {{k}, base4by4[[k, 2]]}}]], {k, 1, Length[base4by4]}];
Length[anti4by4]
anti4by4[[%]]
```

Out[•]=

6

Out[•]=

{{{{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}}, {{14}, {1, 3, 4}}}}

```
In[•]:= symm4by4 = {};
Do[If[base4by4[[k, 1]] == Transpose[base4by4[[k, 1]]], AppendTo[symm4by4,
    {base4by4[[k, 1]], {{k}, base4by4[[k, 2]]}}]], {k, 1, Length[base4by4]}];
Length[symm4by4]
symm4by4[[%]]
```

Out[•]=

10

Out[•]=

{{{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}}, {{16}, {1, 2, 3, 4}}}}

```
In[•]:= η16 = DiagonalMatrix[Table[
  FullSimplify[ $\frac{1}{4} \text{Tr}[\text{symm4by4}[[A, 1]].\text{symm4by4}[[A, 1]]]$ ], {A, Length[symm4by4]}]];
Tr[η16]
```

Out[•]=

10

O(4,4); evals, evecs of σ

```
In[•]:= {evals, evecs} = Eigensystem[σ]
Out[•]= {{{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0},
{0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1},
{0, 0, 1, 0, 0, 0, 1, 0}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 1, 0, 0, 0, 0}}}}
```

```

In[8]:= 
$$\left( \text{u} = \text{ExpandAll} \left[ \frac{1}{\sqrt{2}} \text{ evecs} \right] \right) // \text{MatrixForm}$$

Out[8]//MatrixForm= 
$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}$$


In[9]:= 
$$\text{Table}[\text{u}[[\text{h}]] \cdot \sigma \cdot \text{u}[[\text{h}]], \{\text{h}, 1, \text{Length}[\text{u}]\}]$$

Out[9]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[10]:= 
$$\text{Table}[(\text{Transpose}[\text{u}][[\text{h}]]) \cdot \sigma \cdot \text{u}[[\text{h}]], \{\text{h}, 1, \text{Length}[\text{u}]\}]$$

Out[10]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[11]:= 
$$\begin{aligned} \text{Table}[(\text{Transpose}[\text{u}][[\text{A}]] == -\sigma \cdot \text{u}[[\text{A}]], \{\text{A}, 1, 4\})] \\ \text{Table}[(\text{Transpose}[\text{u}][[\text{A}]] == \sigma \cdot \text{u}[[\text{A}]], \{\text{A}, 5, \text{Length}[\text{u}]\})] \end{aligned}$$

Out[11]= {True, True, True, True}

Out[11]= {True, True, True, True}

In[12]:= hUSE = 8;
```

```
In[•]:= (Transpose[unit].σ.unit)
```

```
Out[•]=
```

```
1
```

```
In[•]:= (Transpose[unit].σ. (τ[θ]))
```

```
Out[•]=
```

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

```
In[•]:= myid[MX_] := Sum[τ[A].MX.τ[A] η4488[[A, A]], {A, 1, 8}]
```

```
Myid[MX_] := Sum[τ[A].MX.τ[A] × η4488[[A, A]], {A, 1, 8}]
```

```
Union[
```

$$\text{Table}\left[\text{Sum}[\tau[A].\text{bas64}[[k, 1]].\sigma.\bar{\tau}[A] \times \eta8[[A, A]], \{A, 1, 8\}] - \text{IdTr}[\text{bas64}[[k, 1]].\sigma] + 4 \text{ eg}[[k]] \frac{\text{bas64}[[k, 1]] - \text{Transpose}[\text{bas64}[[k, 1]]]}{2} .\sigma, \{k, 1, 64\}\right]$$

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}
```

```
In[•]:= τ[θ]
```

```
Out[•]=
```

$$\begin{aligned} & \{ \{1, 0, 0, 0, 0, 0, 0, 0\}, \{0, 1, 0, 0, 0, 0, 0, 0\}, \\ & \{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \\ & \{0, 0, 0, 0, 0, 1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 0\}, \{0, 0, 0, 0, 0, 0, 0, 1\} \} \end{aligned}$$

```
In[•]:= (Transpose[unit].σ. (τ[θ]))
```

```
Out[•]=
```

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

```
In[•]:= FA = Transpose[η4488[[#, 1], [#, 1]] * (τ[#].unit) & /@ Range[0, 7]]
```

```
Out[•]=
```

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ & \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \end{aligned}$$

```
In[1]:= FAa = Transpose[unit].σ. (τ[#]) & /@ Range[0, 7]

Out[1]=
{ { 1/√2, 0, 0, 0, 1/√2, 0, 0, 0 }, { 0, 0, 0, -1/√2, 0, 0, 0, -1/√2 },
{ 0, 0, 1/√2, 0, 0, 0, 1/√2, 0 }, { 0, -1/√2, 0, 0, 0, -1/√2, 0, 0 },
{ 0, 1/√2, 0, 0, 0, -1/√2, 0, 0 }, { 0, 0, -1/√2, 0, 0, 0, 1/√2, 0 },
{ 1/√2, 0, 0, 0, -1/√2, 0, 0, 0 } }

In[2]:= FaA.FAa === ID8

Out[2]=
True

pretend that the X are Minkowski Coordinates :

Block[{U, u = U[#] & /@ Range[8], fx = FaA.X, eq, sx, su, ret},
eq = And @@ Thread[u == fx];
sx = Solve[eq, X][[-1]];
su = Solve[eq, u][[-1]];
ret = {sx, su};
ret]

Out[3]=
{ {x0 → U[1]/√2 + U[5]/√2, x1 → -U[4]/√2 - U[8]/√2, x2 → U[3]/√2 + U[7]/√2,
x3 → -U[2]/√2 - U[6]/√2, x4 → 1/2 ( √2 U[2] - √2 U[6] ), x5 → 1/2 (- √2 U[3] + √2 U[7]),
x6 → 1/2 (- √2 U[4] + √2 U[8]), x7 → 1/2 ( √2 U[1] - √2 U[5] ) },
{U[1] → 1/2 ( √2 x0 + √2 x7 ), U[2] → 1/2 (- √2 x3 + √2 x4 ), U[3] → 1/2 ( √2 x2 - √2 x5 ),
U[4] → 1/2 (- √2 x1 - √2 x6 ), U[5] → 1/2 ( √2 x0 - √2 x7 ), U[6] → 1/2 (- √2 x3 - √2 x4 ),
U[7] → 1/2 ( √2 x2 + √2 x5 ), U[8] → 1/2 (- √2 x1 + √2 x6) } }

In[4]:= (sgtrye(A)α = Thread[Flatten[e(A)α] → Flatten[eAa04]]) (* //Column *)
```

In[•]:= **subsDefects**

Out[•]=

$$\left\{ \sqrt{e^{2a4[Hx4]}} \rightarrow e^{a4[Hx4]}, \sqrt{e^{-2a4[Hx4]}} \rightarrow e^{-a4[Hx4]}, \sqrt{e^{2a4[Hx4]} \sin[6Hx4]^{1/3}} \rightarrow e^{a4[Hx4]} \sin[6Hx4]^{1/6}, \right.$$

$$\frac{1}{\sqrt{e^{2a4[Hx4]} \sin[6Hx4]^{1/3}}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx4]^{1/6}}, \frac{1}{\sqrt{e^{2a4[Hx4]} \sin[6Hx0]^{1/6}}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}},$$

$$\frac{1}{\sqrt{e^{-2a4[Hx4]} \sin[6Hx0]^{1/6}}} \rightarrow \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, \sqrt{e^{2a4[Hx4]} \sin[6Hx0]^{1/6}} \rightarrow e^{a4[Hx4]} \sin[6Hx0]^{1/6},$$

$$\left. \sqrt{e^{-2a4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{-a4[Hx4]} \sin[6Hx0]^{1/6} \right\}$$

In[•]:= **Thread[X → 0]**

Out[•]=

$$\{x0 \rightarrow 0, x1 \rightarrow 0, x2 \rightarrow 0, x3 \rightarrow 0, x4 \rightarrow 0, x5 \rightarrow 0, x6 \rightarrow 0, x7 \rightarrow 0\}$$

Stop pretending:

```
In[•]:= FaA.e(A)α.X /. sgrtrye(A)α // FullSimplify[#, constraintVars] &;
```

```
frameOctadX = # /. { $\sqrt{e^{2a4[Hx4]}} \rightarrow e^{a4[Hx4]}$ ,  $\sqrt{e^{-2a4[Hx4]}} \rightarrow e^{-a4[Hx4]}$ } & /@ %
```

```
% /. {x1 → 0, x2 → 0, x3 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Out[•]=

$$\left\{ \frac{x0 \cot[6Hx0] + e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{x4 - e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right.$$

$$\frac{(e^{a4[Hx4]} x2 - e^{-a4[Hx4]} x5) \sin[6Hx0]^{1/6}}{\sqrt{2}}, -\frac{(e^{a4[Hx4]} x1 + e^{-a4[Hx4]} x6) \sin[6Hx0]^{1/6}}{\sqrt{2}},$$

$$\frac{x0 \cot[6Hx0] - e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, -\frac{x4 + e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}},$$

$$\left. \frac{(e^{a4[Hx4]} x2 + e^{-a4[Hx4]} x5) \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{(-e^{a4[Hx4]} x1 + e^{-a4[Hx4]} x6) \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\}$$

Out[•]=

$$\left\{ \frac{x0 \cot[6Hx0]}{\sqrt{2}}, \frac{x4}{\sqrt{2}}, 0, 0, \frac{x0 \cot[6Hx0]}{\sqrt{2}}, -\frac{x4}{\sqrt{2}}, 0, 0 \right\}$$

In[•]:= F^a_A.eAα.X - frameOctadX // FullSimplify[#, constraintVars] &

Out[•]=

$$\left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right.$$

$$\left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \right. \right.$$

$$\left. \left. \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right.$$

$$\left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}.eA\alpha. \right.$$

$$\begin{aligned}
& \left\{ x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7 \right\} - \frac{x_0 \operatorname{Cot}[6Hx_0] + e^{-a^4[Hx_4]} x_7 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \right. \right. \\
& \left. \left. -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \\
& eA\alpha. \left\{ x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7 \right\} + \frac{-x_4 + e^{a^4[Hx_4]} x_3 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \right. \right. \\
& \left. \left. -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \\
& eA\alpha. \left\{ x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7 \right\} + \frac{e^{-a^4[Hx_4]} (-e^{2a^4[Hx_4]} x_2 + x_5) \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \right. \right. \\
& \left. \left. -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \\
& eA\alpha. \left\{ x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7 \right\} + \frac{(e^{a^4[Hx_4]} x_1 + e^{-a^4[Hx_4]} x_6) \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \right. \right. \\
& \left. \left. -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \\
& eA\alpha. \left\{ x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7 \right\} + \frac{-x_0 \operatorname{Cot}[6Hx_0] + e^{-a^4[Hx_4]} x_7 \sin[6Hx_0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \right. \right. \\
& \left. \left. -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}.
\end{aligned}$$

$$\begin{aligned}
& \left\{ -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \}. \\
& eA\alpha. \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{x4 + e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \right. \right. \\
& \left. \left. -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \\
& eA\alpha. \{x0, x1, x2, x3, x4, x5, x6, x7\} - \frac{(e^{a4[Hx4]} x2 + e^{-a4[Hx4]} x5) \sin[6Hx0]^{1/6}}{\sqrt{2}}, \\
& \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\
& \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \right. \right. \\
& \left. \left. -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\}. \\
& eA\alpha. \{x0, x1, x2, x3, x4, x5, x6, x7\} + \frac{e^{-a4[Hx4]} (e^{2a4[Hx4]} x1 - x6) \sin[6Hx0]^{1/6}}{\sqrt{2}}
\end{aligned}$$

In[8]:= $\{\{F_A^a\}, \{eA\alpha\}, \{X\}\} // \text{Column}$

Out[8]=

$$\begin{aligned}
& \left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right. \\
& \left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \right. \\
& \left. \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \right\} \\
& \{eA\alpha\} \\
& \{x0, x1, x2, x3, x4, x5, x6, x7\}
\end{aligned}$$

```
In[6]:= {x2u, u2x} = Block[
  {U, u = U[#] & /@ Range[8], fx = frameOctadX, eq, sx, su, ret, altEQ, sCot, skot},
  altEQ = And @@ {Cot[6 H x0] == kot, Sin[6 H x0] == sin, a4[H x4] == A44};
  sCot = Solve[altEQ, {Cot[6 H x0], Sin[6 H x0], a4[H x4]}][[1]];
  skot = Solve[altEQ, {kot, sin, A44}][[1]];
  eq = And @@ Thread[u == (fx /. sCot)];
  sx = Solve[eq, X][[-1]];
  su = Solve[eq, u][[-1]];
  ret = {sx /. skot // FullSimplify, su /. skot // ExpandAll};
  ret]
```

```
Out[6]=
{ {x0 → Tan[6 H x0] (U[1] + U[5]) / Sqrt[2], x1 → -(E^-a4[H x4] (U[4] + U[8])) / (Sqrt[2] Sin[6 H x0]^(1/6)),
  x2 → (E^-a4[H x4] (U[3] + U[7])) / (Sqrt[2] Sin[6 H x0]^(1/6)), x3 → -(E^-a4[H x4] (U[2] + U[6])) / (Sqrt[2] Sin[6 H x0]^(1/6)),
  x4 → (U[2] - U[6]) / Sqrt[2],
  x5 → (E^a4[H x4] (-U[3] + U[7])) / (Sqrt[2] Sin[6 H x0]^(1/6)), x6 → (E^a4[H x4] (-U[4] + U[8])) / (Sqrt[2] Sin[6 H x0]^(1/6)),
  x7 → (E^a4[H x4] (U[1] - U[5])) / (Sqrt[2] Sin[6 H x0]^(1/6)) },
  {U[1] → (x0 Cot[6 H x0]) / Sqrt[2] + (E^-a4[H x4] x7 Sin[6 H x0]^(1/6)) / Sqrt[2], U[2] → (x4) / Sqrt[2] - (E^a4[H x4] x3 Sin[6 H x0]^(1/6)) / Sqrt[2],
  U[3] → (E^a4[H x4] x2 Sin[6 H x0]^(1/6)) / Sqrt[2] - (E^-a4[H x4] x5 Sin[6 H x0]^(1/6)) / Sqrt[2],
  U[4] → -(E^a4[H x4] x1 Sin[6 H x0]^(1/6)) / Sqrt[2] - (E^-a4[H x4] x6 Sin[6 H x0]^(1/6)) / Sqrt[2],
  U[5] → (x0 Cot[6 H x0]) / Sqrt[2] - (E^-a4[H x4] x7 Sin[6 H x0]^(1/6)) / Sqrt[2], U[6] → (x4) / Sqrt[2] - (E^a4[H x4] x3 Sin[6 H x0]^(1/6)) / Sqrt[2],
  U[7] → (E^a4[H x4] x2 Sin[6 H x0]^(1/6)) / Sqrt[2] + (E^-a4[H x4] x5 Sin[6 H x0]^(1/6)) / Sqrt[2],
  U[8] → -(E^a4[H x4] x1 Sin[6 H x0]^(1/6)) / Sqrt[2] + (E^-a4[H x4] x6 Sin[6 H x0]^(1/6)) / Sqrt[2] } }
```

In[•]:= **u2x**

Out[•]=

$$\begin{aligned} U[1] &\rightarrow \frac{x0 \operatorname{Cot}[6 H x0]}{\sqrt{2}} + \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \quad U[2] \rightarrow \frac{x4}{\sqrt{2}} - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[3] &\rightarrow \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[4] &\rightarrow -\frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[5] &\rightarrow \frac{x0 \operatorname{Cot}[6 H x0]}{\sqrt{2}} - \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \quad U[6] \rightarrow -\frac{x4}{\sqrt{2}} - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[7] &\rightarrow \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \\ U[8] &\rightarrow -\frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}} \} \end{aligned}$$

In[•]:=

```
{sumu2x, diffu2x} = Block[{U, u, a1, a2, a3 = {}, aa4 = {}},  
(*Clear[U,u,a1,a2,a3,a4]*);  
u = U[#] & /@ Range[8]; Print[u];  
a1 = u /. u2x; Print[a1];  
a2 = Transpose[{{Take[a1, 4]}, {Take[a1, -4]} }]; Print[a2];  
a3 = ExpandAll[FullSimplify[(#[[1]] + #[[2]] )/2 ] ] & /@ a2;  
aa4 = ExpandAll[FullSimplify[(#[[1]] - #[[2]] )/2 ] ] & /@ a2;  
{a3, aa4}]
```

```

{U[1], U[2], U[3], U[4], U[5], U[6], U[7], U[8]}

{ $\frac{x0 \operatorname{Cot}[6 H x0]}{\sqrt{2}} + \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}},$ 
  $\frac{x4 - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}},$ 
  $- \frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \frac{x0 \operatorname{Cot}[6 H x0]}{\sqrt{2}} - \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}},$ 
  $- \frac{x4 - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}},$ 
  $- \frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}} \}$ 
{{ $\left\{ \left\{ \frac{x0 \operatorname{Cot}[6 H x0]}{\sqrt{2}} + \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}},$ 
  $\frac{x4 - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}},$ 
  $- \frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} - \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}} \right\}, \left\{ \frac{x0 \operatorname{Cot}[6 H x0]}{\sqrt{2}} - \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}},$ 
  $- \frac{x4 - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}},$ 
  $- \frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} + \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}} \right\} \right\}}$ 
Out[•]=
{{ $\left\{ \left\{ \frac{x0 \operatorname{Cot}[6 H x0]}{\sqrt{2}}, - \frac{e^{a4[H x4]} x3 \sin[6 H x0]^{1/6}}{\sqrt{2}},$ 
  $\frac{e^{a4[H x4]} x2 \sin[6 H x0]^{1/6}}{\sqrt{2}}, - \frac{e^{a4[H x4]} x1 \sin[6 H x0]^{1/6}}{\sqrt{2}} \right\},$ 
  $\left\{ \left\{ \frac{e^{-a4[H x4]} x7 \sin[6 H x0]^{1/6}}{\sqrt{2}}, \frac{x4}{\sqrt{2}}, - \frac{e^{-a4[H x4]} x5 \sin[6 H x0]^{1/6}}{\sqrt{2}}, - \frac{e^{-a4[H x4]} x6 \sin[6 H x0]^{1/6}}{\sqrt{2}} \right\} \right\}$ 

```

split octonions; evals, evecs of σ

```

# Type - 1 spinor basis (constructed from Clifford algebra considerations)
# These create a maximal isotropic subspace with respect to the split form
self.type1_spinor_basis_matrix = sqrt2_inv*np.array([[1, 1, 0, 0, 0, 0, 0, 0], # s1_0[1, -1, 0, 0, 0, 0, 0, 0], # s1_
1[0, 0, 1, 1, 0, 0, 0, 0], # s1_2[0, 0, 1, -1, 0, 0, 0, 0], # s1_3[0, 0, 0, 1, 1, 0, 0], # s1_4[0, 0, 0, 1, -1, 0, 0], # s1_
5[0, 0, 0, 0, 0, 1, 1], # s1_6[0, 0, 0, 0, 0, 1, -1], # s1_7])

# Type - 2 spinor basis (the complementary spinor representation)
# Related to Type - 1 by another triality automorphism
self.type2_spinor_basis_matrix = sqrt2_inv*np.array([[1, 0, 1, 0, 0, 0, 0, 0], # s2_0[1, 0, -1, 0, 0, 0, 0, 0], # s2_
```

```
1[0, 1, 0, 1, 0, 0, 0, 0], # s2_2[0, 1, 0, -1, 0, 0, 0, 0], # s2_3[0, 0, 0, 0, 1, 0, 1, 0], # s2_4[0, 0, 0, 0, 1, 0, -1, 0], # s2_5[0, 0, 0, 0, 0, 1, 0, 1], # s2_6[0, 0, 0, 0, 0, 1, 0, -1], # s2_7])
```

$$\text{In}[0]:= M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

Out[0]=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

4 Transformation Matrices

We provide explicit 8×8 matrices that transform coordinates between the three bases.

4.1 Vector to Type-1 Spinor

The transformation matrix from vector basis to type-1 spinor basis is:

$$M_{V \rightarrow S_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (11)$$

In[1]:=

4.2 Vector to Type-2 Spinor

The transformation matrix from vector basis to type-2 spinor basis is:

$$M_{V \rightarrow S_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (12)$$

Out[1]:=

4 Transformation Matrices

We provide explicit 8×8 matrices that transform coordinates between the three bases.

4.1 Vector to Type-1 Spinor

The transformation matrix from vector basis to type-1 spinor basis is:

$$M_{V \rightarrow S_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (11)$$

4.2 Vector to Type-2 Spinor

The transformation matrix from vector basis to type-2 spinor basis is:

$$M_{V \rightarrow S_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (12)$$

```
In[•]:= (type1SpinorBasisMatrix = {{1, 1, 0, 0, 0, 0, 0, 0}, {1, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 1, 0, 0, 0, 0}, {0, 0, 1, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 1, 0, 0}, {0, 0, 0, 0, 0, 1, -1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 1}, {0, 0, 0, 0, 0, 0, 1, -1}}) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

```
In[•]:= (type2SpinorBasisMatrix = {{1, 0, 1, 0, 0, 0, 0, 0}, {1, 0, -1, 0, 0, 0, 0, 0}, {0, 1, 0, 1, 0, 0, 0, 0}, {0, 1, 0, -1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 1, 0}, {0, 0, 0, 0, 0, 1, 0, -1}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{pmatrix}$$

$$\text{In[1]:= } M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

Out[1]=

$$M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\text{In[2]:= } M_{S_1 \rightarrow S_2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

```

In[1]:= MS1S2 = type1SpinorBasisMatrix.Transpose[type2SpinorBasisMatrix] // MatrixForm
Out[1]= 
$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 \end{pmatrix}$$


In[2]:= gAB = η4488
Out[2]= {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}

In[3]:= {valuesAB, evecsAB} = Eigensystem[gAB]
Out[3]= {{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0}, {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}}

In[4]:= {values, evecs} = Eigensystem[σ]
Out[4]= {{-1, -1, -1, -1, 1, 1, 1, 1}, {{0, 0, 0, -1, 0, 0, 0, 1}, {0, 0, -1, 0, 0, 0, 1, 0}, {0, -1, 0, 0, 0, 1, 0, 0}, {-1, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1}, {0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0}}, {{0, 0, 0, 1/2, 0, 0, 0, -1/2}, {0, 0, -1/2, 0, 0, 0, 1/2, 0}, {0, -1/2, 0, 0, 0, 1/2, 0, 0}, {-1/2, 0, 0, 0, 1/2, 0, 0, 0}, {0, 0, 0, 1/2, 0, 0, 0, -1/2}, {0, 0, 1/2, 0, 0, 0, -1/2, 0}, {0, 1/2, 0, 0, 0, -1/2, 0, 0}, {1/2, 0, 0, 0, -1/2, 0, 0, 0}}}

In[5]:= Table[u[[h]].σ.u[[h]], {h, 1, Length[u]}]
Out[5]= {-1, -1, -1, -1, 1, 1, 1, 1}

```

```
In[1]:= Table[(Transpose[u][h]).σ.u[h], {h, 1, Length[u]}]
Out[1]= {-1, -1, -1, -1, 1, 1, 1, 1}

In[2]:= Table[(Transpose[u][A]) === -σ.u[A], {A, 1, 4}]
Table[(Transpose[u][A]) === σ.u[A], {A, 5, Length[u]}]

Out[2]= {True, True, True, True}

Out[3]= {True, True, True, True}
```

In[4]:= hUSE = 8;

Set: Symbol hUSE is Protected. [i](#)

```
In[5]:= Protect[hUSE]
Out[5]= {}
```

In[6]:= unit = u[hUSE]

Set: Symbol unit is Protected. [i](#)

```
Out[6]= {1/√2, 0, 0, 0, 1/√2, 0, 0, 0}
```

In[7]:= Protect[unit]

```
Out[7]= {}
```

In[8]:= (Transpose[unit].σ.unit)

Out[8]=

1

In[9]:= (Transpose[unit].σ.(τ[θ]))

Out[9]=

{1/√2, 0, 0, 0, 1/√2, 0, 0, 0}

```
In[8]:= Do[E^A_a[h] = Table[(u[[h]].σ.(τ[B])), {B, 0, 7}], {h, 1, Length[u]}];
```

$$\mathfrak{F}_a^A = \frac{1}{\sqrt{u}\sigma u} u^c \sigma_{cb} \bar{\tau}^{Ab}_a$$

```
Do[E^a_A[h] = FullSimplify[Inverse[E^A_a[h]]], {h, 1, Length[u]}];
```

$$\mathfrak{F}_A^a = \frac{1}{\sqrt{u}\sigma u} \tau_A^a{}_b u^b$$

```
Table[E^a_A[h] === (u[[h]].σ.u[[h]])
```

```
Transpose[Table[FullSimplify[ExpandAll[η4488[[B, B]] × τ[B-1].u[[h]]]], {B, 1, 8}]], {h, 1, Length[u]}]
```

Out[8]=

$$\mathfrak{F}_A^A = \frac{1}{\sqrt{u}\sigma u} u^c \sigma_{cb} \bar{\tau}^{Ab}_a$$

Out[8]=

$$\mathfrak{F}_A^a = \frac{1}{\sqrt{u}\sigma u} \tau_A^a{}_b u^b$$

Out[8]=

```
{True, True, True, True, True, True, True, True}
```

```
In[8]:= (*Do[E^A_a[h]=Table[(u[[h]].σ.(τ[B])),{B,0,7}],{h,1,Length[u]}];
```

```
Do[E^a_A[h]=FullSimplify[Inverse[E^A_a[h]]],{h,1,Length[u]}];
```

```
Table[E^a_A[h]===(u[[h]].σ.u[[h]])
```

```
Transpose[Table[FullSimplify[ExpandAll[η4488[[B,B]]τ[B-1].u[[h]]]],{B,1,8}]],{h,1,Length[u]}]*)
```

```
In[8]:= myid[MX_]:=Sum[τ[A].MX.τ[A] η4488[[A, A]], {A, 1, 8}]
```

```
Myid[MX_]:=Sum[τ[A].MX.τ[A] × η448[[A, A]], {A, 1, 8}]
```

```
Union[
```

```
Table[Sum[τ[A].bas64[[k, 1]].σ.τ[A] × η8[[A, A]], {A, 1, 8}] - Id Tr[bas64[[k, 1]].σ] +
4 eg[[k]]  $\frac{\text{bas64}[[k, 1]] - \text{Transpose}[\text{bas64}[[k, 1]]]}{2} . \sigma, \{k, 1, 64\}]$ ]
```

```
{{{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}, {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}}
```

In[8]:= $\bar{\tau}[θ]$

Out[8]=

```
{ {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0},
```

```
 {0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
```

```
 {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1} }
```

In[1]:= $(\text{Transpose}[\text{unit}] \cdot \sigma. (\bar{\tau}[0]))$

Out[1]=

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

In[2]:= $F_A^a = \text{Transpose}[\eta 4488 [\# + 1, \# + 1] * (\tau [\#]. \text{unit}) \& /@ \text{Range}[0, 7]]$

Out[2]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ & \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \end{aligned}$$

In[3]:= $F_a^A = \text{Transpose}[\text{unit}] \cdot \sigma. (\bar{\tau}[\#]) \& /@ \text{Range}[0, 7]$

Out[3]=

$$\begin{aligned} & \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\} \end{aligned}$$

In[4]:= $F_A^a \cdot F_a^A == ID8$

Out[4]=

True

$\text{gtrye}_{\alpha}^{(A)} = (e_{\alpha}^{(A)} /. \text{sgtrye}_{\alpha}^{(A)}) /. \text{subsDefects}$

$\text{gtrye}_{(A)}^{\alpha} = (e_{(A)}^{\alpha} /. \text{sgtrye}_{(A)}^{\alpha}) /. \text{subsDefects}$

$(\text{sgtrye}_{(A)}^{\alpha} = \text{Thread}[\text{Flatten}[e_{(A)}^{\alpha}] \rightarrow \text{Flatten}[\text{Inverse}[e_{\alpha}^{(A)} /. \text{sgtrye}_{\alpha}^{(A)}]]])$
 $(*//\text{Column}*)$

$\text{gtrye}_{(A)}^{\alpha} = (e_{(A)}^{\alpha} /. \text{sgtrye}_{(A)}^{\alpha}) /. \text{subsDefects}$

Out[5]=

$$\begin{aligned} & \left\{ \left\{ \text{Tan}[6 H x \theta], 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, \frac{e^{-a4[H x 4]}}{\text{Sin}[6 H x \theta]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, \frac{e^{-a4[H x 4]}}{\text{Sin}[6 H x \theta]^{1/6}}, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{e^{-a4[H x 4]}}{\text{Sin}[6 H x \theta]^{1/6}}, 0, 0, 0, 0, 0 \right\}, \right. \\ & \left. \left\{ 0, 0, 0, 0, 1, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, \frac{e^{a4[H x 4]}}{\text{Sin}[6 H x \theta]^{1/6}}, 0, 0 \right\}, \right. \end{aligned}$$

$$\left\{0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, 0, \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}\right\}$$

pretend that the X are Minkowski Coordinates :

```
In[•]:= Block[{U, u = U[#] & /@ Range[8], fx = Fa.X, eq, sx, su, ret},
  eq = And @@ Thread[u == fx];
  sx = Solve[eq, X][[-1]];
  su = Solve[eq, u][[-1]];
  ret = {sx, su};
  ret]
```

```
Out[•]=
{ {x0 →  $\frac{U[1]}{\sqrt{2}} + \frac{U[5]}{\sqrt{2}}$ , x1 →  $-\frac{U[4]}{\sqrt{2}} - \frac{U[8]}{\sqrt{2}}$ , x2 →  $\frac{U[3]}{\sqrt{2}} + \frac{U[7]}{\sqrt{2}}$ ,
  x3 →  $-\frac{U[2]}{\sqrt{2}} - \frac{U[6]}{\sqrt{2}}$ , x4 →  $\frac{1}{2} (\sqrt{2} U[2] - \sqrt{2} U[6])$ , x5 →  $\frac{1}{2} (-\sqrt{2} U[3] + \sqrt{2} U[7])$ ,
  x6 →  $\frac{1}{2} (-\sqrt{2} U[4] + \sqrt{2} U[8])$ , x7 →  $\frac{1}{2} (\sqrt{2} U[1] - \sqrt{2} U[5])$  },
  {U[1] →  $\frac{1}{2} (\sqrt{2} x0 + \sqrt{2} x7)$ , U[2] →  $\frac{1}{2} (-\sqrt{2} x3 + \sqrt{2} x4)$ , U[3] →  $\frac{1}{2} (\sqrt{2} x2 - \sqrt{2} x5)$ ,
   U[4] →  $\frac{1}{2} (-\sqrt{2} x1 - \sqrt{2} x6)$ , U[5] →  $\frac{1}{2} (\sqrt{2} x0 - \sqrt{2} x7)$ , U[6] →  $\frac{1}{2} (-\sqrt{2} x3 - \sqrt{2} x4)$ ,
   U[7] →  $\frac{1}{2} (\sqrt{2} x2 + \sqrt{2} x5)$ , U[8] →  $\frac{1}{2} (-\sqrt{2} x1 + \sqrt{2} x6)$  } }
```

```
In[•]:= (sgtrye(A)α = Thread[Flatten[e(A)α] → Flatten[eAa04]]) (*//Column*)
```

```
In[•]:= subsDefects
```

```
Out[•]=
{  $\sqrt{e^{2 a4[Hx4]}} \rightarrow e^{a4[Hx4]}$ ,  $\sqrt{e^{-2 a4[Hx4]}} \rightarrow e^{-a4[Hx4]}$ ,  $\sqrt{e^{2 a4[Hx4]}} \sin[6Hx4]^{1/3} \rightarrow e^{a4[Hx4]} \sin[6Hx4]^{1/6}$ ,
   $\frac{1}{\sqrt{e^{2 a4[Hx4]}} \sin[6Hx4]^{1/3}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx4]^{1/6}}$ ,  $\frac{1}{\sqrt{e^{2 a4[Hx4]}} \sin[6Hx0]^{1/6}} \rightarrow \frac{e^{-a4[Hx4]}}{\sin[6Hx0]^{1/6}}$ ,
   $\frac{1}{\sqrt{e^{-2 a4[Hx4]}} \sin[6Hx0]^{1/6}} \rightarrow \frac{e^{a4[Hx4]}}{\sin[6Hx0]^{1/6}}$ ,  $\sqrt{e^{2 a4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{a4[Hx4]} \sin[6Hx0]^{1/6}$ ,
   $\sqrt{e^{-2 a4[Hx4]}} \sin[6Hx0]^{1/6} \rightarrow e^{-a4[Hx4]} \sin[6Hx0]^{1/6}$  }
```

```
In[•]:= Thread[X → 0]
```

```
Out[•]=
{x0 → 0, x1 → 0, x2 → 0, x3 → 0, x4 → 0, x5 → 0, x6 → 0, x7 → 0}
```

Stop pretending :

```
(sgtrye(A)(A) = Thread[Flatten[e(A)(A)] → Flatten[Inverse[e(A)α /. sgtrye(A)α]]]) (*//Column*)
```



```
In[1]:= FaA.e(A)α.X /. sgrtrye(A)α // FullSimplify[#, constraintVars] &;
```

$$\text{frameOctadX} = \# /. \left\{ \sqrt{e^{2a4[Hx4]}} \rightarrow e^{a4[Hx4]}, \sqrt{e^{-2a4[Hx4]}} \rightarrow e^{-a4[Hx4]} \right\} \& /@ %$$

$$\% /. \{x1 \rightarrow 0, x2 \rightarrow 0, x3 \rightarrow 0, x5 \rightarrow 0, x6 \rightarrow 0, x7 \rightarrow 0\}$$

```
Out[1]=
```

$$\left\{ \frac{x0 \operatorname{Cot}[6Hx0] + e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{x4 - e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}}, \right.$$

$$\frac{e^{-a4[Hx4]} (e^{2a4[Hx4]} x2 - x5) \sin[6Hx0]^{1/6}}{\sqrt{2}}, -\frac{e^{-a4[Hx4]} (e^{2a4[Hx4]} x1 + x6) \sin[6Hx0]^{1/6}}{\sqrt{2}},$$

$$\frac{x0 \operatorname{Cot}[6Hx0] - e^{-a4[Hx4]} x7 \sin[6Hx0]^{1/6}}{\sqrt{2}}, -\frac{x4 + e^{a4[Hx4]} x3 \sin[6Hx0]^{1/6}}{\sqrt{2}},$$

$$\left. \frac{e^{-a4[Hx4]} (e^{2a4[Hx4]} x2 + x5) \sin[6Hx0]^{1/6}}{\sqrt{2}}, \frac{e^{-a4[Hx4]} (-e^{2a4[Hx4]} x1 + x6) \sin[6Hx0]^{1/6}}{\sqrt{2}} \right\}$$

```
Out[2]=
```

$$\left\{ \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}}, \frac{x4}{\sqrt{2}}, 0, 0, \frac{x0 \operatorname{Cot}[6Hx0]}{\sqrt{2}}, -\frac{x4}{\sqrt{2}}, 0, 0 \right\}$$

```
In[3]:= FaA.eAα.X - frameOctadX // FullSimplify[#, constraintVars] &
```

```
Out[3]= {0, 0, 0, 0, 0, 0, 0, 0}
```

```
In[4]:= eAα
```

```
Out[4]=
```

$$\left\{ \{\operatorname{Cot}[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0\}, \right.$$

$$\{0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0\},$$

$$\left. \{0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}\} \right\}$$

```
In[5]:= {{FaA}, {eAα}, {X}} // Column
```

```
Out[5]=
```

$$\left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \right. \right.$$

$$\left. \left. \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \right.$$

$$\left. \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \right\} \right\}$$

$$\left\{ \left\{ \operatorname{Cot}[6Hx0], 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \{0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0\}, \right.$$

$$\{0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, e^{a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0, 0, 0\},$$

$$\left. \{0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}, 0\}, \{0, 0, 0, 0, 0, 0, 0, e^{-a4[Hx4]} \sin[6Hx0]^{1/6}\} \right\}$$

$$\{x0, x1, x2, x3, x4, x5, x6, x7\}$$

split octonion multiplication constants:

$$m_{ab}^c = \mathbb{F}_a^A \tau_A \begin{matrix} c \\ b \end{matrix}$$

$$m_{AB}^C = \mathbb{F}_c^C \tau_A \begin{matrix} c \\ b \end{matrix} \mathbb{F}_B^b$$

$$\begin{aligned} m_{ab}^c &= \mathbb{F}_a^A \tau_A{}_b{}^c \\ m_{AB}^C &= \mathbb{F}_c^C \tau_A{}_b{}^c \mathbb{F}_B^b \end{aligned}$$

In[1]:= $(xw)w = xw^2$

Out[1]=

$$(xw)w = xw^2$$

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

Out[1]=

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

In[2]:=

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex structure J_u . If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on \mathbf{O} . However, because S^6 is connected, we see that the orientation of \mathbf{O} induced by the natural orientation of \mathbf{O}_u as a complex vector space does not depend on u . We refer to this orientation as the natural orientation of \mathbf{O} .

Out[2]=

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Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

In[1]:= so J_u is an isometry for each $u \in S$. Moreover, it follows that \mathbf{O}_u is endowed with a natural Hermitian structure with respect to the inner product \langle , \rangle . We denote the group of complex linear transformations of \mathbf{O}_u by $GL(\mathbf{O}_u)$ and the special unitary transformations of \mathbf{O}_u with its Hermitian metric by $SU(\mathbf{O}_u)$.

Out[1]=

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$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

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J_u

```
In[-]:= Table[FullSimplify[
Sum[ψ[A1] - J[B1] gAB[[A1, A1]] E^A_a[hUSE][[C]][c] τ[A1 - 1][[c]][b] E^a_A[hUSE][[b]][B1],
{c, 1, 8}, {b, 1, 8}, {A1, 1, 8}, {B1, 1, 8}], {C, 1, 8}]
```

Out[-]=

$$\begin{aligned}
& \{J[1] - \psi[1] - J[2] - \psi[2] - J[3] - \psi[3] - J[4] - \psi[4] + \\
& J[5] - \psi[5] + J[6] - \psi[6] + J[7] - \psi[7] + J[8] - \psi[8], J[2] - \psi[1] + J[1] - \psi[2] - \\
& J[4] - \psi[3] + J[3] - \psi[4] + J[6] - \psi[5] - J[5] - \psi[6] - J[8] - \psi[7] + J[7] - \psi[8], \\
& J[3] - \psi[1] + J[4] - \psi[2] + J[1] - \psi[3] - J[2] - \psi[4] + J[7] - \psi[5] + J[8] - \psi[6] - \\
& J[5] - \psi[7] - J[6] - \psi[8], J[4] - \psi[1] - J[3] - \psi[2] + J[2] - \psi[3] + \\
& J[1] - \psi[4] + J[8] - \psi[5] - J[7] - \psi[6] + J[6] - \psi[7] - J[5] - \psi[8], \\
& J[5] - \psi[1] + J[6] - \psi[2] + J[7] - \psi[3] + J[8] - \psi[4] + J[1] - \psi[5] - J[2] - \psi[6] - \\
& J[3] - \psi[7] - J[4] - \psi[8], J[6] - \psi[1] - J[5] - \psi[2] + J[8] - \psi[3] - \\
& J[7] - \psi[4] + J[2] - \psi[5] + J[1] - \psi[6] + J[4] - \psi[7] - J[3] - \psi[8], \\
& J[7] - \psi[1] - J[8] - \psi[2] - J[5] - \psi[3] + J[6] - \psi[4] + J[3] - \psi[5] - J[4] - \psi[6] + \\
& J[1] - \psi[7] + J[2] - \psi[8], J[8] - \psi[1] + J[7] - \psi[2] - J[6] - \psi[3] - \\
& J[5] - \psi[4] + J[4] - \psi[5] + J[3] - \psi[6] - J[2] - \psi[7] + J[1] - \psi[8] \}
\end{aligned}$$


```
In[8]:= Table[FullSimplify[ExpandAll[gAB[[B, B]] E^A_a[hUSE].\tau[B-1].E^a_A[hUSE]]], {B, 1, 8}]
```

Out[•]=

```

{{{1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0}, ,
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0},
{0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1}}, ,
{{0, -1, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0},
{0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, 1, 0}}}, ,
{{0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, 1},
{0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0}}, ,
{{0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0},
{1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, -1, 0},
{0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, -1, 0, 0, 0}}}, ,
{{0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 1, 0},
{0, 0, 0, 0, 0, 0, 1}, {1, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0}}}, ,
{{0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1},
{0, 0, 0, 0, 0, -1, 0}, {0, -1, 0, 0, 0, 0, 0, 0}, ,
{1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0, 0, 0}},
{{0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 0, -1}, {0, 0, 0, 0, 0, -1, 0, 0, 0},
{0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 0, 0},
{1, 0, 0, 0, 0, 0, 0, 0}, {0, -1, 0, 0, 0, 0, 0, 0, 0}}}, ,
{{0, 0, 0, 0, 0, 0, 0, 1}, {0, 0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0},
{0, 0, 0, 0, -1, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0}, ,
{0, 0, -1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0, 0, 0}}}}}

```

```
In[6]:= Table[Transpose[FullSimplify[ExpandAll[gAB[[B, B]] E^A_a[hUSE].\t[B - 1].E^a_A[hUSE]]]], {B, 1, 8}] // MatrixForm
```

Out[•]//MatrixForm=

In[•]:= EA = Array[eA, 8]

Out[•]=

```
{eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}
```

```

In[]:= e(A)α
Out[]= { {Cot[6 H x 0], 0, 0, 0, 0, 0, 0, 0}, {0, ea4[H x 4] Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, ea4[H x 4] Sin[6 H x 0]1/6, 0, 0, 0, 0, 0, 0}, {0, 0, 0, ea4[H x 4] Sin[6 H x 0]1/6, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, e-a4[H x 4] Sin[6 H x 0]1/6, 0, 0}, {0, 0, 0, 0, 0, 0, e-a4[H x 4] Sin[6 H x 0]1/6, 0}, {0, 0, 0, 0, 0, 0, 0, e-a4[H x 4] Sin[6 H x 0]1/6} }

In[]:= h = hUSE;
B = 1; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 2; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 3; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 4; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 5; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 6; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 7; FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]
B = 8;
FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]

Out[=] {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

Out[=] {eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}

Out[=] {eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}

Out[=] {eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]}

Out[=] {eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]}

Out[=] {eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]}

Out[=] {eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]}

Out[=] {eA[8], eA[7], -eA[6], -eA[5], -eA[4], -eA[3], eA[2], eA[1]}

In[]:= B = 1;
Grid[{EA, FullSimplify[ExpandAll[gAB[B, B] EA.EAa[h].τ[B - 1].EaA[h]]]}, Frame → All]

Out[=]


|       |       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|
| eA[1] | eA[2] | eA[3] | eA[4] | eA[5] | eA[6] | eA[7] | eA[8] |
| eA[1] | eA[2] | eA[3] | eA[4] | eA[5] | eA[6] | eA[7] | eA[8] |


```

```
In[•]:= gridx = Grid[Partition[Flatten[{ {x}, EA},  
Table[( {x}, FullSimplify[ExpandAll[gAB[B, B] EA.E^A_a[h].\tau[B - 1].E^a_A[h]]]),  
{x \[Rule] ToExpression["eA[" \& ToString[B] \& "]"]}],  
{B, 1, 8} ]}], 9], Frame \[Rule] All]
```

Out[•]=

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

In[•]:=

$\epsilon_B =$ $\epsilon_A =$	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8
ϵ_1	$-\epsilon_8$	ϵ_3	$-\epsilon_2$	$-\epsilon_5$	ϵ_4	$-\epsilon_7$	ϵ_6	ϵ_1
ϵ_2	$-\epsilon_3$	$-\epsilon_8$	ϵ_1	$-\epsilon_6$	ϵ_7	ϵ_4	$-\epsilon_5$	ϵ_2
ϵ_3	ϵ_2	$-\epsilon_1$	$-\epsilon_8$	$-\epsilon_7$	$-\epsilon_6$	ϵ_5	ϵ_4	ϵ_3
ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4
ϵ_5	$-\epsilon_4$	$-\epsilon_7$	ϵ_6	$-\epsilon_1$	ϵ_8	ϵ_3	$-\epsilon_2$	ϵ_5
ϵ_6	ϵ_7	$-\epsilon_4$	$-\epsilon_5$	$-\epsilon_2$	$-\epsilon_3$	ϵ_8	ϵ_1	ϵ_6
ϵ_7	$-\epsilon_6$	ϵ_5	$-\epsilon_4$	$-\epsilon_3$	ϵ_2	$-\epsilon_1$	ϵ_8	ϵ_7
ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8

Out[•]=

$\epsilon_B =$ $\epsilon_A =$	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8
ϵ_1	$-\epsilon_8$	ϵ_3	$-\epsilon_2$	$-\epsilon_5$	ϵ_4	$-\epsilon_7$	ϵ_6	ϵ_1
ϵ_2	$-\epsilon_3$	$-\epsilon_8$	ϵ_1	$-\epsilon_6$	ϵ_7	ϵ_4	$-\epsilon_5$	ϵ_2
ϵ_3	ϵ_2	$-\epsilon_1$	$-\epsilon_8$	$-\epsilon_7$	$-\epsilon_6$	ϵ_5	ϵ_4	ϵ_3
ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4
ϵ_5	$-\epsilon_4$	$-\epsilon_7$	ϵ_6	$-\epsilon_1$	ϵ_8	ϵ_3	$-\epsilon_2$	ϵ_5
ϵ_6	ϵ_7	$-\epsilon_4$	$-\epsilon_5$	$-\epsilon_2$	$-\epsilon_3$	ϵ_8	ϵ_1	ϵ_6
ϵ_7	$-\epsilon_6$	ϵ_5	$-\epsilon_4$	$-\epsilon_3$	ϵ_2	$-\epsilon_1$	ϵ_8	ϵ_7
ϵ_8	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8

In[•]:= **EA**

Out[•]= {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[•]:= **EA.τABC[[1]]**

Out[•]= {eA[1], eA[2], eA[3], eA[4], eA[5], eA[6], eA[7], eA[8]}

In[•]:= **EA.τABC[[2]]**

Out[•]= {eA[2], -eA[1], -eA[4], eA[3], -eA[6], eA[5], eA[8], -eA[7]}

In[•]:= **EA.τABC[[3]]**

Out[•]= {eA[3], eA[4], -eA[1], -eA[2], -eA[7], -eA[8], eA[5], eA[6]}

In[•]:= **EA.τABC[[4]]**

Out[•]= {eA[4], -eA[3], eA[2], -eA[1], -eA[8], eA[7], -eA[6], eA[5]}

In[•]:= **EA.τABC[[5]]**

Out[•]= {eA[5], eA[6], eA[7], eA[8], eA[1], eA[2], eA[3], eA[4]}

In[•]:= **EA.τABC[[6]]**

Out[•]= {eA[6], -eA[5], eA[8], -eA[7], -eA[2], eA[1], -eA[4], eA[3]}

In[•]:= **EA.τABC[[7]]**

Out[•]= {eA[7], -eA[8], -eA[5], eA[6], -eA[3], eA[4], eA[1], -eA[2]}

In[•]:= **TeXForm[gridx]**

Out[•]//TeXForm=

```
\begin{array}{cccccccc}
x & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \text{eA}(4) & \text{eA}(5) & \text{eA}(6) & \text{eA}(7) \\
& \text{eA}(1) & \text{eA}(1) & \text{eA}(2) & \text{eA}(3) & \text{eA}(4) & \text{eA}(5) & \text{eA}(6) \\
& \text{eA}(2) & \text{eA}(2) & \text{eA}(1) & \text{eA}(1) & \text{eA}(4) & \text{eA}(3) & \text{eA}(6) \\
& \text{eA}(3) & \text{eA}(3) & \text{eA}(4) & \text{eA}(1) & \text{eA}(1) & \text{eA}(2) & \text{eA}(7) \\
& \text{eA}(4) & \text{eA}(4) & \text{eA}(3) & \text{eA}(2) & \text{eA}(2) & \text{eA}(1) & \text{eA}(8) \\
& \text{eA}(5) & \text{eA}(5) & \text{eA}(6) & \text{eA}(7) & \text{eA}(8) & \text{eA}(1) & \text{eA}(9) \\
& \text{eA}(6) & \text{eA}(6) & \text{eA}(5) & \text{eA}(4) & \text{eA}(8) & \text{eA}(2) & \text{eA}(10) \\
& \text{eA}(7) & \text{eA}(7) & \text{eA}(6) & \text{eA}(5) & \text{eA}(8) & \text{eA}(3) & \text{eA}(11) \\
& \text{eA}(8) & \text{eA}(8) & \text{eA}(7) & \text{eA}(6) & \text{eA}(5) & \text{eA}(4) & \text{eA}(12)
\end{array}
```

In[•]:= (*StringReplace[ToString[TeXForm[gridx]], "\text{eA}" → "\epsilon"]*)

```
In[•]:= Grid[Partition[
  Flatten[{{{x}, EA}, Table[({{x}, Table[Sum[FullSimplify[ExpandAll[gAB[[B, B]] EA[[C1]] E^A_a[hUSE][[C1, c1]] \[Tau][B - 1][[c1, d1]] E^a_A[hUSE][[d1, B1]]]], {C1, 1, 8}, {c1, 1, 8}, {d1, 1, 8}], {B1, 1, 8}}] /. {x \[Rule] ToExpression["EA[" <> ToString[B] <> "]"]}, {B, 1, 8}]}, 9], Frame \[Rule] All]
Out[•]=
```

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

```
In[•]:= gridx
```

```
Out[•]=
```

x	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[1]	eA[1]	eA[2]	eA[3]	eA[4]	eA[5]	eA[6]	eA[7]	eA[8]
eA[2]	eA[2]	-eA[1]	-eA[4]	eA[3]	-eA[6]	eA[5]	eA[8]	-eA[7]
eA[3]	eA[3]	eA[4]	-eA[1]	-eA[2]	-eA[7]	-eA[8]	eA[5]	eA[6]
eA[4]	eA[4]	-eA[3]	eA[2]	-eA[1]	-eA[8]	eA[7]	-eA[6]	eA[5]
eA[5]	eA[5]	eA[6]	eA[7]	eA[8]	eA[1]	eA[2]	eA[3]	eA[4]
eA[6]	eA[6]	-eA[5]	eA[8]	-eA[7]	-eA[2]	eA[1]	-eA[4]	eA[3]
eA[7]	eA[7]	-eA[8]	-eA[5]	eA[6]	-eA[3]	eA[4]	eA[1]	-eA[2]
eA[8]	eA[8]	eA[7]	-eA[6]	-eA[5]	-eA[4]	-eA[3]	eA[2]	eA[1]

Complex Structure

J_u

```
In[•]:= J[#] \[Rule] 0 & /@ Range[0, 7]
```

```
Out[•]= {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 0, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}
```

```
In[•]:= sJu = {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 1, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}
```

```
Out[•]= {J[0] \[Rule] 0, J[1] \[Rule] 0, J[2] \[Rule] 0, J[3] \[Rule] 0, J[4] \[Rule] 1, J[5] \[Rule] 0, J[6] \[Rule] 0, J[7] \[Rule] 0}
```

```
for  $\frac{1}{i}$  :
```

```
In[•]:= Block[{ψ, J, t, r, sJu},
  sJu = {J[0] → 0, J[1] → 0, J[2] → 0, J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0};
  t = Table[
    FullSimplify[Sum[ψ[A1 - 1] × J[B1 - 1] gAB[[A1, A1]] E^A_a [hUSE][[C]][c] τ[A1 - 1][c][b]
      E^a_A [hUSE][[b]][B1], {c, 1, 8}, {b, 1, 8}, {A1, 1, 8}, {B1, 1, 8}], {C, 1, 8}]];
  r = t /. sJu
]
```

Out[•]=
 $\{\psi[4], -\psi[5], -\psi[6], -\psi[7], \psi[0], -\psi[1], -\psi[2], -\psi[3]\}$

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \tilde{\tau}^A \sigma$$

In[•]:= $\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2I_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$

8 unit matrix. Denoting the matrix elemen

$$\bar{\tau}_{ab}^A = \tau_{ba}^A$$

042501-7 Second gravity

J. Math. Phys. 51, 042501 (2010)

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix} \quad (10)$$

of the 16-component spinor representation of $\overline{\text{SO}(4,4;\mathbb{R})}$. We see that, as is, in fact, well known from the general theory, the 16-component spinor representation of $\overline{\text{SO}(4,4;\mathbb{R})}$ is the direct sum of two (inequivalent) real 8×8 irreducible spinor representations $D_{(1)}=D_{(1)}(g)$ and $D_{(2)}=D_{(2)}(g)$ of $\overline{\text{SO}(4,4;\mathbb{R})} \ni g$ that are generated by $D_{(1)}^{AB}$ and $D_{(2)}^{AB}$, respectively. This we record as

$$4D_{(1)}^{AB} = \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A \quad (11)$$

and

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A. \quad (12)$$

Out[8]=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \widetilde{\tau}^A \sigma$$

$$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2\mathbb{I}_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A,$$

8 unit matrix. Denoting the matrix elements

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

042501-7 Second gravity

J. Math. Phys. 51, 042501 (2010)

$$t^A t^B - t^B t^A = \begin{pmatrix} \bar{\tau}^A \tau^B - \bar{\tau}^B \tau^A & 0 \\ 0 & \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A \end{pmatrix} = 4 \begin{pmatrix} D_{(1)}^{AB} & 0 \\ 0 & D_{(2)}^{AB} \end{pmatrix} \quad (10)$$

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and

$$4D_{(2)}^{AB} = \tau^A \bar{\tau}^B - \tau^B \bar{\tau}^A. \quad (12)$$

042501-8 Patrick L. Nash

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$$\widetilde{D}_{(1)}^{AB}\sigma = -\sigma D_{(1)}^{AB} \Rightarrow \tilde{D}_{(1)}\sigma = \sigma D_{(1)}^{-1}, \quad (18)$$

$$\widetilde{D}_{(2)}^{AB}\sigma = -\sigma D_{(2)}^{AB} \Rightarrow \tilde{D}_{(2)}\sigma = \sigma D_{(2)}^{-1}, \quad (19)$$

$$L^A{}_C G_{AB} L^B{}_D = G_{CD} = \{\tilde{L}GL\}_{CD}, \quad (20)$$

$$L^A{}_B \bar{\tau}^B = D_{(1)}^{-1} \bar{\tau}^A D_{(2)}, \quad (21)$$

$$L^A{}_B \tau^B = D_{(2)}^{-1} \tau^A D_{(1)}, \quad (22)$$

The canonical 2-1 homomorphism $\overline{\text{SO}(4,4;\mathbb{R})} \rightarrow \text{SO}(4,4;\mathbb{R}): g \mapsto L(g)$ is given by

$$8L^A{}_B = \text{tr}(D_{(1)}^{-1} \bar{\tau}^A D_{(2)} \tau^B) G_{CB}, \quad (23)$$

where tr denotes the trace. Note that $D_{(1)}(g(\omega)) = D_{(2)}(g(\omega))$ when $\omega_{A8} = 0$, i.e., when one restricts $\overline{\text{SO}(4,4;\mathbb{R})}$ to

$$\overline{\text{SO}(3,4;\mathbb{R})} = \{g \in \overline{\text{SO}(4,4;\mathbb{R})} \mid ,$$

$$g = \begin{pmatrix} \exp\left(\frac{1}{4}\omega_{AB} D_{(1)}^{AB}\right) & 0 \\ 0 & \exp\left(\frac{1}{4}\omega_{AB} D_{(2)}^{AB}\right) \end{pmatrix} \quad \text{and} \quad \omega_{A8} = 0 \} \quad (24)$$

[one of the real forms of spin (7, C)].

Out[8]=

042501-8 Patrick L. Nash

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$$\widetilde{D}_{(1)}^{AB}\sigma = -\sigma D_{(1)}^{AB} \Rightarrow \tilde{D}_{(1)}\sigma = \sigma D_{(1)}^{-1}, \quad (18)$$

$$\widetilde{D}_{(2)}^{AB}\sigma = -\sigma D_{(2)}^{AB} \Rightarrow \tilde{D}_{(2)}\sigma = \sigma D_{(2)}^{-1}, \quad (19)$$

$$L^A{}_C G_{AB} L^B{}_D = G_{CD} = \{\tilde{L}GL\}_{CD}, \quad (20)$$

$$L^A{}_B \bar{\tau}^B = D_{(1)}^{-1} \bar{\tau}^A D_{(2)}, \quad (21)$$

$$L^A{}_B \tau^B = D_{(2)}^{-1} \tau^A D_{(1)}, \quad (22)$$

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[one of the real forms of spin (7, C)].

```
In[8]:= Block[{\psi, J, t, r, sJu},
  sJu =
  {J[0] → 0, J[1] → 0, J[2] → 0, J[3] → 0, J[4] → 1, J[5] → 0, J[6] → 0, J[7] → 0};
  r = {\psi[4], -ψ[5], -ψ[6], -ψ[7], ψ[0], -ψ[1], -ψ[2], -ψ[3]};
  ψ = ḡ[A].τ[B].Ψ16[[1;;8]];
]
```

Out[8]=

$$\begin{aligned} & \overline{\tau}[A].\{f16[7][x0, x4], f16[6][x0, x4], -f16[5][x0, x4], -f16[4][x0, x4], \\ & \quad f16[3][x0, x4], f16[2][x0, x4], -f16[1][x0, x4], -f16[0][x0, x4]\} \end{aligned}$$

$$\overline{\tau}[A1].\tau[B1] \rightarrow D_{(1)}^{-1}.\overline{\tau}[A1].\tau[B1].D_{(1)};$$

$$\tau[A1].\overline{\tau}[B1] \rightarrow D_{(2)}^{-1}.\tau[A1].\overline{\tau}[B1].D_{(2)};$$

$$\sigma.\overline{\tau}[A1].\tau[B1] = \text{Transpose}[\tau[A1]].\sigma.\tau[B1] =$$

$$\text{Transpose}[\tau[A1]].\text{Transpose}[\sigma.\overline{\tau}[B1]] = \text{Transpose}[\sigma.\overline{\tau}[B1].\tau[A1]];$$

$$\sigma.\overline{\tau}[A1].\tau[B1] = \text{Transpose}[\overline{\tau}[B1].\tau[A1]].\text{Transpose}[\sigma];$$

$$\sigma.\overline{\tau}[A1].\tau[B1].ψ_1 = \text{Transpose}[\overline{\tau}[B1].\tau[A1]].\text{Transpose}[\sigma].ψ_1;$$

$$\text{Transpose}[\psi_2].\sigma \leftrightarrow \text{Transpose}[\sigma].ψ_1 ? ;$$

$$\sigma.\overline{\tau}[A1].\tau[B1].ψ_1 \leftrightarrow \text{Transpose}[\overline{\tau}[B1].\tau[A1]].\text{Transpose}[\psi_2].\sigma;$$

$$\sigma.\overline{\tau}[A1].\tau[B1].ψ_1 \leftrightarrow \text{Transpose}[\psi_2.\overline{\tau}[B1].\tau[A1]].\sigma;$$

$$B1 = A1$$

```

 $\sigma.\psi_1 \leftrightarrow \text{Transpose}[\psi_2].\sigma^{-1} ?;$ 
 $\psi_1 \leftrightarrow \text{Transpose}[\psi_2].\sigma^{-1} ?;$ 
 $\text{Transpose}[\psi_2] \leftrightarrow \sigma.\psi_1 ?;$ 

In[-]:= {ψ1[4], -ψ1[5], -ψ1[6], -ψ1[7], ψ1[0], -ψ1[1], -ψ1[2], -ψ1[3]} /. {ψ1[#] → } ;

In[-]:= (*Block[{ψ1, ψ2, σψ, sσψ, r, sJu},  

    σψ = σ. (ψ2[#] & /@ Range[0, 7]);  

    sσψ = (ψ1[#] → σψ[[# + 1]] & /@ Range[0, 7]);  

    r = {ψ1[4], -ψ1[5], -ψ1[6], -ψ1[7], ψ1[0], -ψ1[1], -ψ1[2], -ψ1[3]};  

    (*ψ = τ[A].τ[B].Π16[[1;;8]]*)  

    r /. sσψ  

  ]*)

In[-]:= Block[{ψ1, ψ2, σψ, sσψ, r},  

    σψ = σ. (ψ2[#] & /@ Range[0, 7]);  

    sσψ = (ψ1[#] → σψ[[# + 1]] & /@ Range[0, 7]);  

    r = {ψ1[4], -ψ1[5], -ψ1[6], -ψ1[7], ψ1[0], -ψ1[1], -ψ1[2], -ψ1[3]};  

    (*ψ = τ[A].τ[B].Π16[[1;;8]]*)  

    r /. sσψ  

  ]

Out[-]= {ψ2[0], -ψ2[1], -ψ2[2], -ψ2[3], ψ2[4], -ψ2[5], -ψ2[6], -ψ2[7]}

For  $\frac{1}{ii}$  :

In[-]:= Π16cccc = Block[{ψ1, ψ2, σψ, sσψ, sψ1, sψ2, r},  

    σψ = σ. (Π16[[9;;16]]);  

    sσψ = (ψ1[#] → σψ[[# + 1]] & /@ Range[0, 7]);  

    sψ1 = Thread[(ψ1[#] & /@ Range[0, 7]) → Π16[[1;;8]]];  

    sψ2 = Thread[(ψ1[#] & /@ Range[0, 7]) → Π16[[9;;16]]];  

    r = {ψ1[4], -ψ1[5], -ψ1[6], -ψ1[7], ψ1[0], -ψ1[1], -ψ1[2], -ψ1[3]};  

    (*ψ = τ[A].τ[B].*)  

    Flatten[{r /. sψ1, r /. sψ2}]
  ]

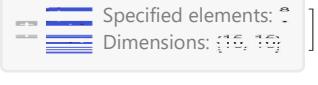
Out[-]= {f16[4][x0, x4], -f16[5][x0, x4], -f16[6][x0, x4], -f16[7][x0, x4],  

  f16[0][x0, x4], -f16[1][x0, x4], -f16[2][x0, x4], -f16[3][x0, x4],  

  f16[12][x0, x4], -f16[13][x0, x4], -f16[14][x0, x4], -f16[15][x0, x4],  

  f16[8][x0, x4], -f16[9][x0, x4], -f16[10][x0, x4], -f16[11][x0, x4]}

In[-]:= caΠccΠ = CoefficientArrays[Π16cccc - Π16, Π16cccc]

Out[-]= {SparseArray[ Specified elements: 12 Dimensions: {16, 16}], SparseArray[ Specified elements: 12 Dimensions: {16, 16}]}

```

```
In[•]:= (caΨccΨ2 = Normal[caΨccΨ[[2]]]) // MatrixForm
```

Out[•]//MatrixForm=

```
In[•]:= cawccw = CoefficientArrays [w16cccc - w16, w16]
```

Out[•]=

```
{SparseArray[ Specified elements: 0 Dimensions: {15}], SparseArray[ Specified elements: 32 Dimensions: {15, 15}]}
```

```
In[ -]:= (caΨccΨ2 = Normal[caΨccΨ[[2]]]) // MatrixForm
```

Out[-]//MatrixForm=

-1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	-1	0	0	0	0	0	0	0	0
0	0	-1	0	0	0	-1	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	-1	0	0	0	0	0	0
1	0	0	0	-1	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	-1	0	0	0	0	0	0	0	0
0	0	-1	0	0	0	-1	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	-1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	-1	0	0	-1	0
0	0	0	0	0	0	0	0	0	0	-1	0	0	-1
0	0	0	0	0	0	0	0	1	0	0	0	-1	0
0	0	0	0	0	0	0	0	0	-1	0	0	0	-1
0	0	0	0	0	0	0	0	0	0	-1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-1	0	0

In[=]:= **ca\Pcc\P2** === Transpose [**ca\Pcc\P2**]

Out[\cdots] =

True

```
In[•]:= caΨccΨ2.caΨccΨ2 // MatrixForm
Out[•]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

```
In[•]:= caΨccΨ2.Ψ16cccc - Ψ16 + Ψ16cccc
caΨccΨ2.Ψ16 - Ψ16cccc + Ψ16
```

```
Out[•]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
Out[•]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```
for  $\frac{1}{ii}$  :
```

```
In[•]:= jjcc = caΨccΨ2 + ID16;
```

```
In[•]:= jjcc.Ψ16 === Ψ16cccc
```

```
Out[•]= True
```

```
In[•]:=  $\bar{x} = 2\langle x, 1 \rangle - x.$ 
```

```
Out[•]=  $\bar{x} = 2\langle x, 1 \rangle - x.$ 
```

```
In[•]:=  $\bar{x} = 2\langle x, 1 \rangle - x.$ 
```

```
Out[•]=  $\bar{x} = 2\langle x, 1 \rangle - x.$ 
```

```
FAa = Transpose[n4488[[#, 1, # + 1]] * (τ[#].unit) & /@ Range[0, 7]]
```

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right.$$

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \\ \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\} \}$$

```
FaA = Transpose[unit].o. (v[#]) & /@ Range[0, 7]
```

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \\ \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \\ \left. \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\}$$

In[8]:= F_a^A

Out[8]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}} \right\}, \right. \\ \left\{ 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \left\{ 0, \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0 \right\}, \\ \left. \left\{ 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0 \right\}, \left\{ 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, -\frac{1}{\sqrt{2}}, 0, 0, 0 \right\} \right\}$$

In[9]:= unit

Out[9]=

$$\left\{ \frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 \right\}$$

In[10]:= (F_a^A.unit)

Out[10]=

$$\{1, 0, 0, 0, 0, 0, 0, 0\}$$

In[11]:= (F_a^A[8].unit)

Out[11]=

$$0$$

In[12]:= unit.F_A^a

Out[12]=

$$\{1, 0, 0, 0, 0, 0, 0, 0\}$$

In[13]:= unit.unit // FullSimplify

Out[13]=

$$1$$


```
In[•]:= ¶16cc = 2 realProjection16.¶16 - ¶16 // FullSimplify
Out[•]= {f16[4][x0, x4], -f16[1][x0, x4], -f16[2][x0, x4], -f16[3][x0, x4],
f16[0][x0, x4], -f16[5][x0, x4], -f16[6][x0, x4], -f16[7][x0, x4],
f16[12][x0, x4], -f16[9][x0, x4], -f16[10][x0, x4], -f16[11][x0, x4],
f16[8][x0, x4], -f16[13][x0, x4], -f16[14][x0, x4], -f16[15][x0, x4]}
```

```
In[•]:= ¶16ccA =
2 (Flatten[{unit.¶16[[1;;8]* (unit.FA), unit.¶16[[9;;16]* (unit.FA) }]) - ¶16 //
FullSimplify
```

```
Out[•]= {(-1 + √2) f16[0][x0, x4] + √2 f16[4][x0, x4], -f16[1][x0, x4], -f16[2][x0, x4],
-f16[3][x0, x4], -f16[4][x0, x4], -f16[5][x0, x4], -f16[6][x0, x4], -f16[7][x0, x4],
(-1 + √2) f16[8][x0, x4] + √2 f16[12][x0, x4], -f16[9][x0, x4], -f16[10][x0, x4],
-f16[11][x0, x4], -f16[12][x0, x4], -f16[13][x0, x4], -f16[14][x0, x4], -f16[15][x0, x4]}
```

```
In[•]:= ¶16cc - ¶16ccA // FullSimplify
```

```
Out[•]= {-((-1 + √2) (f16[0][x0, x4] + f16[4][x0, x4])), 0, 0, 0,
f16[0][x0, x4] + f16[4][x0, x4], 0, 0, 0, -((-1 + √2) (f16[8][x0, x4] + f16[12][x0, x4])),
0, 0, 0, f16[8][x0, x4] + f16[12][x0, x4], 0, 0, 0}
```

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \tilde{\tau}^A \sigma$$

$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2I_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A$,

8 unit matrix. Denoting the matrix elemen

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

```
Out[•]=

$$\sigma \bar{\tau}^A = \widetilde{\sigma \tau^A} = \tilde{\tau}^A \sigma$$

```

$\tau^A \bar{\tau}^B + \tau^B \bar{\tau}^A = 2I_{8 \times 8} G^{AB} = \bar{\tau}^A \tau^B + \bar{\tau}^B \tau^A$,

8 unit matrix. Denoting the matrix elemen

$$\bar{\tau}^A_{ab} = \tau^A_{ba}$$

```
Table[{A, B, FullSimplify[
ExpandAll[ $\frac{1}{2} (\bar{\tau}[B].\tau[A] + \bar{\tau}[A].\tau[B]) = \eta 4488 [A+1, B+1] * ID8$ ]}], {A, B}]
```

```
{A, 0, 7}, {B, 0, 7}] // MatrixForm
```

```
Table[{A, B, FullSimplify[
  ExpandAll[ $\frac{1}{2} (\tau[A] \cdot \bar{\tau}[B] + \tau[B] \cdot \bar{\tau}[A]) = \eta 4488 [A+1, B+1] * ID8]$ ]}],
{A, 0, 7}, {B, 0, 7}] // MatrixForm
```

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In[•]:=

SUBMANIFOLDS AND SPECIAL STRUCTURES ON THE OCTONIANS

ROBERT L. BRYANT

is described as follows: If we let $\text{Im } \mathbf{O} \subseteq \mathbf{O}$ be the hyperplane (through 0) orthogonal to $1 \in \mathbf{O}$, and we let $S^6 \subseteq \text{Im } \mathbf{O}$ be the space of unit vectors, then right multiplication by $u \in S^6$ induces a linear transformation $R_u: \mathbf{O} \rightarrow \mathbf{O}$ which is orthogonal and satisfies $(R_u)^2 = -1$. Thus, associated to each $u \in S^6$ is a complex structure on \mathbf{O} (induced by $J = R_u$) which is compatible with the natural inner product on \mathbf{O} . We denote by \mathbf{O}_u the Hermitian vector space whose underlying real vector space (with inner product) is \mathbf{O} and whose complex structure is given by R_u .

Classically, this observation was used to define an almost complex structure on S^6 as follows: If $u \in S^6$, then R_u preserves the 2-plane spanned by 1 and u and therefore preserves its orthogonal 6-plane, which may be identified with $T_u S^6 \subseteq \text{Im } \mathbf{O}$ after translation to the origin. Thus R_u induces a complex

In[•]:=

structure on $T_u S^6$ for each $u \in S^6$. This almost complex structure is not integrable (even locally) to a complex structure (see below). In [Ca], Calabi noticed that for any oriented $M^6 \subseteq \text{Im } \mathbf{O}$, $R_{N(x)}$ induces a complex structure on $T_x M^6$ (where $N(x)$ is the oriented unit normal). Thus every oriented $M^6 \subseteq \text{Im } \mathbf{O}$ inherits an almost complex structure. Moreover, M^6 inherits a metric from $\text{Im } \mathbf{O}$, so we actually have a $U(3)$ -structure on M^6 . (Calabi calls these structures “almost Hermitian.” He also proves that such M^6 possess a canonical $SU(3)$ -substructure but we will not need this.) Calabi shows that the second fundamental form Π of M decomposes with respect to the $U(3)$ -structure into a piece $\Pi^{1,1}$ of type (1,1) and a piece $\Pi^{0,2}$ of type (0,2). He then shows that the almost complex structure of M is integrable if and only if $\Pi^{1,1} = 0$ and that the canonical 2-form of the $U(3)$ -structure, say Ω , is closed if and only if $\Pi^{0,2} = 0$ and $\text{tr}_\Gamma \Pi^{1,1} = 0$. From this it follows that the $U(3)$ -structure on M^6 is Kähler if and only if $\Pi \equiv 0$, so that M^6 is a hyperplane (or a union of pieces of hyperplanes). Calabi then constructs nontrivial examples of $M^6 \subseteq \text{Im } \mathbf{O}$ for which the almost complex structure is integrable. His examples are of the form $S \times \mathbf{R}^4 \subseteq \text{Im } \mathbf{O}$, where $S \subseteq \mathbf{R}^3$ is a minimal surface, $\mathbf{R}^3 \subseteq \text{Im } \mathbf{O}$ is an associative 3-plane, and $\mathbf{R}^4 = (\mathbf{R}^3)^\perp$. Calabi leaves open the problem of determining whether or not there are nontrivial $M^6 \subseteq \text{Im } \mathbf{O}$ for which the canonical 2-form is closed.

In[•]:=

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex structure J_u . If $u \neq v$, then clearly $J_u \neq J_v$, so we actually have a six-sphere of *distinct* complex structures on \mathbf{O} . However, because S^6 is connected, we see that the orientation of \mathbf{O} induced by the natural orientation of \mathbf{O}_u as a complex vector space does not depend on u . We refer to this orientation as the natural orientation of \mathbf{O} .

Using (1.3), we see that if $u \in S$, then

$$\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$$

so J_u is an isometry for each $u \in S$. Moreover, it follows that \mathbf{O}_u is endowed with a natural Hermitian structure with respect to the inner product $\langle \cdot, \cdot \rangle$. We denote the group of complex linear transformations of \mathbf{O}_u by $GL(\mathbf{O}_u)$ and the special unitary transformations of \mathbf{O}_u with its Hermitian metric by $SU(\mathbf{O}_u)$.

We let $\text{Spin}(7) \subseteq SO(8)$ denote the subgroup generated by the set $\{J_u \mid u \in S^6\} \subseteq SO(8)$. It is known (see [12]) that $\text{Spin}(7)$ is a connected, simply connected, compact Lie group of real dimension 21. Its center is $\{\pm I_8\} \cong \mathbb{Z}/2$ and $\text{Spin}(7)/\{\pm I_8\}$ is isomorphic to $SO(7)$, a simple group. We want to make explicit the structure equations of $\text{Spin}(7)$ as a subgroup of $SO(8)$ in such a way that its relationship with the complex structures J_u is made clear.

Let $u \in S^6$ be an imaginary unit which is orthogonal to $\epsilon \in \mathbf{O}$. For each $\lambda \in \mathbb{R}$, $(\cos \lambda \epsilon + \sin \lambda u)$ is an imaginary unit. Hence $J_\epsilon \circ J_{(\cos \lambda \epsilon + \sin \lambda u)} = -\cos \lambda I + \sin \lambda J_\epsilon \circ J_u$ is an element of $\text{Spin}(7)$. We easily compute that $J_\epsilon \circ J_u + J_u \circ J_\epsilon = 0$ by using (1.10). Thus $(J_\epsilon \circ J_u)^2 = J_\epsilon \circ J_u \circ J_\epsilon \circ J_u = -J_\epsilon^2 \circ J_u^2 = -I$. It follows that

$$(1.16) \quad \exp(\lambda J_\epsilon \circ J_u) = \cos \lambda I + \sin \lambda J_\epsilon \circ J_u.$$

Thus, if $\text{spin}(7) \subseteq so(8)$ is the Lie algebra of $\text{Spin}(7)$, we see that $J_\epsilon \circ J_u \in \text{spin}(7)$ for all $u \in S^6$ with $\langle u, \epsilon \rangle = 0$. Since $\text{spin}(7)$ is a vector space, we see that $L \subseteq \text{spin}(7)$ where

$$(1.17) \quad L = \{J_\epsilon \circ J_w \mid w \in \text{Im } \mathbf{O}, \langle \epsilon, w \rangle = 0\}.$$

Note that $\dim_{\mathbb{R}} L = 6$.

```
Transpose[a].σ16.b == Transpose[J.a].σ16.J.b = a.Transpose[J].σ16.J.b
want ψcc.σ16.?Ψ16 = Ψ16.J.σ16.?Ψ16 ≠ 0
```

need 1 irrep for {i, u}

need 1 irrep for {i, u} _perp

1 == x0 or x4; u == x4 or x0

x0 has periodic fns; use x4

```
t16A[[All, 2]]
```

```
Out[1]=
```

```
{ {0}, {1}, {2}, {3}, {4}, {5}, {6}, {7} }
```

```
In[2]:= T16^A[4].T16^A[4] === -Id16
```

```
Out[2]=
```

True

```
In[3]:= (*JT4=T16^A[4]*)(* complex structure on S^6 *)
```

We let $S^6 = \{u \in \text{Im } \mathbf{O} \mid \langle u, u \rangle = 1\}$. The elements of S^6 are called the *imaginary units* of \mathbf{O} . For any $u \in S^6$, we have $u = -\bar{u}$, so $u^2 = -u\bar{u} = -\langle u, u \rangle = -1$. We may use u to define a map $J_u: \mathbf{O} \rightarrow \mathbf{O}$ given by

$$(1.15) \quad J_u(x) = xu.$$

The identity (1.9) shows that $J_u^2(x) = (xu)u = xu^2 = -x$, so J_u defines a complex structure on \mathbf{O} . We write \mathbf{O}_u to denote \mathbf{O} endowed with the complex

J_u == -Id16

```

matrix multiplication is associative
(split) octonion multiplication is NOT associative
```
 $\langle J_u(x), J_u(y) \rangle = \langle xu, yu \rangle = \langle x, y \rangle \langle u, u \rangle = \langle x, y \rangle$
Transpose[J].o16.J == o16

```

```

In[1]:= Clear[possibleComplexStructureIndex2];
(*Clear[possibleComplexStructureMatrix2];*)
possibleComplexStructureIndex2[j_] := Module[{m, r, jsj, jj},
 J = base16[[j]][1];
 jj = (FullSimplify[J.J] === -ID16);
 jsj = FullSimplify[Transpose[J].o16.J] === o16;
 Return[{jj, jsj}]];

```

```

In[2]:= possibleComplexStructureIndex2[#] & /@ {12, 97, 100, 109, 127, 225, 230, 236}
Out[2]= {{False, False}, {True, False}, {True, False}, {True, False},
{True, False}, {False, False}, {False, False}, {False, False}}

```

```

In[3]:= Clear[possibleComplexStructureIndex];
Clear[possibleComplexStructureMatrix];
possibleComplexStructureIndex = (*Parallel*)Table[
 Block[{J, r, m},
 Clear[J, r, m];
 m = r = {};
 J = base16[[j]][1];
 If[(J.J === -ID16) &&
 (FullSimplify[Transpose[J].o16.J] === o16), r = base16[[j]][2];
 m = o16.J;, ##&[]];
 (*If[! (m===-Transpose[m]),r=base16[[j]][2],r={}]*)]
 r]
 , {j, 1, Length[base16]}] // Union // Drop[#, 1] &

```

```

Out[3]= {{4}, {5}, {6}, {7}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {4, 5}, {4, 6},
{4, 7}, {5, 6}, {5, 7}, {6, 7}, {0, 1, 2, 3, 4}, {0, 1, 2, 3, 5}, {0, 1, 2, 3, 6},
{0, 1, 2, 3, 7}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7}, {0, 1, 5, 6, 7},
{0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 2, 5, 6, 7}, {0, 3, 4, 5, 6},
{0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {0, 3, 5, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7},
{1, 2, 4, 6, 7}, {1, 2, 5, 6, 7}, {1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7},
{1, 3, 5, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7}, {2, 3, 4, 6, 7}, {2, 3, 5, 6, 7},
{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 6}, {0, 1, 2, 3, 4, 7}, {0, 1, 2, 3, 5, 6},
{0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}, {0, 1, 4, 5, 6, 7}, {0, 2, 4, 5, 6, 7},
{0, 3, 4, 5, 6, 7}, {1, 2, 4, 5, 6, 7}, {1, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 7}, {3, 4, 5, 6, 7}}

```

```
In[6]:= Length[possibleComplexStructureIndex]
Out[6]= 56

In[7]:= sixersJ = If[Length[#] ≥ 6, #, ## &[]] & /@ possibleComplexStructureIndex
Out[7]= {{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 6}, {0, 1, 2, 3, 4, 7}, {0, 1, 2, 3, 5, 6},
{0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}, {0, 1, 4, 5, 6, 7}, {0, 2, 4, 5, 6, 7},
{0, 3, 4, 5, 6, 7}, {1, 2, 4, 5, 6, 7}, {1, 3, 4, 5, 6, 7}, {2, 3, 4, 5, 6, 7}}
```



```

Out[0]=
{{4}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {5, 6}, {5, 7},
{6, 7}, {0, 1, 2, 3, 4}, {0, 1, 4, 5, 6}, {0, 1, 4, 5, 7}, {0, 1, 4, 6, 7},
{0, 2, 4, 5, 6}, {0, 2, 4, 5, 7}, {0, 2, 4, 6, 7}, {0, 3, 4, 5, 6},
{0, 3, 4, 5, 7}, {0, 3, 4, 6, 7}, {1, 2, 4, 5, 6}, {1, 2, 4, 5, 7}, {1, 2, 4, 6, 7},
{1, 3, 4, 5, 6}, {1, 3, 4, 5, 7}, {1, 3, 4, 6, 7}, {2, 3, 4, 5, 6}, {2, 3, 4, 5, 7},
{2, 3, 4, 6, 7}, {0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7}}
```

In[0]:= JT4 = T16<sup>A</sup>[4] ; (\* complex structure on S^6 \*)
Jcomplement  $\leftrightarrow$  (  $\hat{q}[[7]] \cdot \text{Cos}[Q7] + \hat{q}[[6]] \cdot \text{Sin}[Q7] \text{Cos}[Q8] + \hat{q}[[5]] \cdot \text{Sin}[Q7] \text{Sin}[Q8]$  )

In[0]:= basisProjections16 =
Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@
Range[16];

In[0]:= basisProjections8 =
Block[{x = Table[KroneckerDelta[#, j], {j, 16}]}, KroneckerProduct[x, x]] & /@
Range[8];

In[0]:= JcomplementProjection8 =
(basisProjections8[[7]] + basisProjections8[[6]] + basisProjections8[[5]]);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8

```

Out[0]=
True
```

In[0]:= JcomplementProjection8 = (basisProjections8[[7]] +
basisProjections8[[6]] + basisProjections8[[5]] + basisProjections8[[4]]);
JcomplementProjection8.JcomplementProjection8 === JcomplementProjection8

```

Out[0]=
True
```

In[0]:= (\* JcomplementProjection8 = (basisProjections8[[7]] \* Cos[Q7] +
basisProjections8[[6]] \* Sin[Q7] Cos[Q8] + basisProjections8[[5]] \* Sin[Q7] Sin[Q8]); \*)





```
In[8]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 5, 6};
% === -T16^A[4].T16^A[7].T16^A[8]
```

Out[8]=

True

```
In[9]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 5, 7};
% === T16^A[4].T16^A[6].T16^A[8]
```

Out[9]=

True

```
In[10]:= T16^A[#1].T16^A[#2].T16^A[#3].T16^A[#4].T16^A[#5].T16^A[#6] &@@ {0, 1, 2, 3, 6, 7};
% === -T16^A[4].T16^A[5].T16^A[8]
```

Out[10]=

True



```
In[1]:= realProjection16 = ArrayFlatten[{{realProjection8, 0}, {0, realProjection8}}]

Out[1]= {{1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {1/2, 0, 0, 0, 1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[2]:= realProjection16.realProjection16 === realProjection16
```

```
Out[2]= True
```

In[3]:= We denote  $\langle x, 1 \rangle$  by  $\text{Re } x$  and  $(x - \text{Re } x)$  by  $\text{Im } x$ .

```
Out[3]= We denote $\langle x, 1 \rangle$ by $\text{Re } x$ and $(x - \text{Re } x)$ by $\text{Im } x$.
```

In[4]:=  $\bar{x} = 2\langle x, 1 \rangle - x.$

```
Out[4]= $\bar{x} = 2\langle x, 1 \rangle - x.$
```

In[5]:= **unit.unit**

```
Out[5]= 1
```

In[6]:= **unit. $\sigma$ .unit**

```
Out[6]= 1
```

In[7]:= **T16<sup>A</sup>[4].T16<sup>A</sup>[4] === -ID16**

```
Out[7]= True
```





```
In[•]:= Jcomplementcc = Jcomplement - (realProjection16.Jcomplement) // FullSimplify
Out[•]=
{ { 0, $\frac{\cos[Q7]}{2}$, $-\frac{1}{2} \cos[Q8] \sin[Q7]$, $-\frac{1}{2} \sin[Q7] \sin[Q8]$, 0,
 $-\frac{\cos[Q7]}{2}$, $\frac{1}{2} \cos[Q8] \sin[Q7]$, $\frac{1}{2} \sin[Q7] \sin[Q8]$, 0, 0, 0, 0, 0, 0, 0, 0, 0 },
{ 0, 0, $\sin[Q7] \sin[Q8]$, $-\cos[Q8] \sin[Q7]$, $\cos[Q7]$, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 },
{ $\cos[Q8] \sin[Q7]$, $-\sin[Q7] \sin[Q8]$, 0, 0, 0, 0, $\cos[Q7]$, 0, 0, 0, 0, 0, 0, 0, 0, 0 },
{ $\sin[Q7] \sin[Q8]$, $\cos[Q8] \sin[Q7]$, 0, 0, 0, 0, $-\cos[Q7]$, 0, 0, 0, 0, 0, 0, 0, 0, 0 },
{ 0, $-\frac{\cos[Q7]}{2}$, $\frac{1}{2} \cos[Q8] \sin[Q7]$, $\frac{1}{2} \sin[Q7] \sin[Q8]$, 0, $\frac{\cos[Q7]}{2}$,
 $-\frac{1}{2} \cos[Q8] \sin[Q7]$, $-\frac{1}{2} \sin[Q7] \sin[Q8]$, 0, 0, 0, 0, 0, 0, 0, 0 },
{ $\cos[Q7]$, 0, 0, 0, 0, 0, $\sin[Q7] \sin[Q8]$, $-\cos[Q8] \sin[Q7]$, 0, 0, 0, 0, 0, 0, 0, 0 },
{ 0, 0, 0, $\cos[Q7]$, $\cos[Q8] \sin[Q7]$, $-\sin[Q7] \sin[Q8]$, 0, 0, 0, 0, 0, 0, 0, 0, 0 },
{ 0, 0, $-\cos[Q7]$, 0, $\sin[Q7] \sin[Q8]$, $\cos[Q8] \sin[Q7]$, 0, 0, 0, 0, 0, 0, 0, 0 },
{ 0, 0, 0, 0, 0, 0, 0, 0, $-\frac{\cos[Q7]}{2}$, $\frac{1}{2} \cos[Q8] \sin[Q7]$,
 $\frac{1}{2} \sin[Q7] \sin[Q8]$, 0, $\frac{\cos[Q7]}{2}$, $-\frac{1}{2} \cos[Q8] \sin[Q7]$, $-\frac{1}{2} \sin[Q7] \sin[Q8]$ },
{ 0, 0, 0, 0, 0, 0, 0, 0, 0, $-\sin[Q7] \sin[Q8]$, $\cos[Q8] \sin[Q7]$, $-\cos[Q7]$, 0, 0, 0 },
{ 0, 0, 0, 0, 0, 0, 0, 0, 0, $-\cos[Q8] \sin[Q7]$, $\sin[Q7] \sin[Q8]$, 0, 0, 0, 0, $-\cos[Q7]$ },
{ 0, 0, 0, 0, 0, 0, 0, 0, 0, $-\sin[Q7] \sin[Q8]$, $-\cos[Q8] \sin[Q7]$, 0, 0, 0, 0, $\cos[Q7]$, 0 },
{ 0, 0, 0, 0, 0, 0, 0, 0, 0, $\frac{\cos[Q7]}{2}$, $-\frac{1}{2} \cos[Q8] \sin[Q7]$,
 $-\frac{1}{2} \sin[Q7] \sin[Q8]$, 0, $-\frac{\cos[Q7]}{2}$, $\frac{1}{2} \cos[Q8] \sin[Q7]$, $\frac{1}{2} \sin[Q7] \sin[Q8]$ },
{ 0, 0, 0, 0, 0, 0, 0, 0, 0, $-\cos[Q7]$, 0, 0, 0, 0, $-\sin[Q7] \sin[Q8]$, $\cos[Q8] \sin[Q7]$ },
{ 0, 0, 0, 0, 0, 0, 0, 0, 0, $-\cos[Q7]$, 0, 0, 0, 0, $-\cos[Q8] \sin[Q7]$, $\sin[Q7] \sin[Q8]$, 0, 0 },
{ 0, 0, 0, 0, 0, 0, 0, 0, 0, $\cos[Q7]$, 0, $-\sin[Q7] \sin[Q8]$, $-\cos[Q8] \sin[Q7]$, 0, 0 } }
```

```
In[•]:= FullSimplify[Jcomplementcc.Jcomplementcc]
```

```
Out[•]=
{ { $-\frac{1}{2}$, 0, 0, 0, $\frac{1}{2}$, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 },
{ 0, $\frac{1}{4} (-3 + \cos[2 Q7])$, $\frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7]$, $\frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8]$, 0,
 $\frac{\cos[Q7]^2}{2}$, $\frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7]$, $\frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8]$, 0, 0, 0, 0, 0, 0, 0, 0 },
{ 0, $\frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7]$, $-\cos[Q7]^2 + \frac{1}{4} (-3 + \cos[2 Q8]) \sin[Q7]^2$,
 $\frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8]$, 0, $\frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7]$,
 $\frac{1}{2} \cos[Q8]^2 \sin[Q7]^2$, $\frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8]$, 0, 0, 0, 0, 0, 0, 0 } }
```



$$\left\{ \begin{aligned} & 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \frac{1}{2} \cos[Q8]^2 \sin[Q7]^2, \\ & \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], 0, \frac{1}{2} \cos[Q7] \cos[Q8] \sin[Q7], \\ & -\cos[Q7]^2 + \frac{1}{4} (-3 + \cos[2Q8]) \sin[Q7]^2, \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8] \}, \\ & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], \right. \\ & \frac{1}{2} \sin[Q7]^2 \sin[Q8]^2, 0, \frac{1}{2} \cos[Q7] \sin[Q7] \sin[Q8], \\ & \left. \frac{1}{2} \cos[Q8] \sin[Q7]^2 \sin[Q8], -\cos[Q7]^2 - \frac{1}{4} (3 + \cos[2Q8]) \sin[Q7]^2 \right\} \end{aligned} \right.$$

In[8]:= T16<sup>A</sup>[4].Jcomplement // MatrixForm

Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \cos[Q7] & 0 & 0 & 0 & 0 \\ 0 & \cos[Q7] & 0 & 0 & 0 \\ 0 & 0 & \cos[Q7] & 0 & -\sin[Q7] \sin[Q8] -\cos \\ 0 & 0 & 0 & \cos[Q7] & \cos[Q8] \sin[Q7] -\sin \\ 0 & 0 & -\sin[Q7] \sin[Q8] & \cos[Q8] \sin[Q7] & -\cos[Q7] \\ 0 & 0 & -\cos[Q8] \sin[Q7] & -\sin[Q7] \sin[Q8] & 0 \\ \sin[Q7] \sin[Q8] & \cos[Q8] \sin[Q7] & 0 & 0 & 0 \\ -\cos[Q8] \sin[Q7] & \sin[Q7] \sin[Q8] & 0 & 0 & 0 \end{pmatrix}$$

In[8]:= T16<sup>A</sup>[4].Jcomplement === Jcomplement.T16<sup>A</sup>[4]

FullSimplify[Jcomplement.Jcomplement] === -ID16

T16<sup>A</sup>[4].T16<sup>A</sup>[4] === -ID16

Out[8]=

True

Out[8]=

True

Out[8]=

True

In[8]:= unit3 = {Cos[Q7], Sin[Q7] Cos[Q8], Sin[Q7] Sin[Q8]}

Out[8]=

{Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}

In[8]:= unit3.unit3 // FullSimplify

Out[8]=

1

```

In[=]:= unit3
sixers
Complement[{0, 1, 2, 3, 5, 6, 7}, #] & /@ %
eqsRQQ7Q8 = Thread[(Subscript[q, #\[1]] & /@ %) == RQ * unit3]
sRQQ7Q8 = Solve[And @@ %, {RQ, Q7, Q8}] // FullSimplify
seqsRQQ7Q8 = %% /. {Equal -> Rule}

Out[=]=
{Cos[Q7], Cos[Q8] Sin[Q7], Sin[Q7] Sin[Q8]}

Out[=]=
{{0, 1, 2, 3, 5, 6}, {0, 1, 2, 3, 5, 7}, {0, 1, 2, 3, 6, 7} }

Out[=]=
{{7}, {6}, {5} }

Out[=]=
{q7 == RQ Cos[Q7], q6 == RQ Cos[Q8] Sin[Q7], q5 == RQ Sin[Q7] Sin[Q8] }

Out[=]=
{RQ -> -Sqrt[q5^2 + q6^2 + q7^2], Q7 -> 2 \pi c1 - I Log[-(I Sqrt[q5^2 + q6^2 + q7^2] - q7)/Sqrt[q5^2 + q6^2 + q7^2]] if c1 \in \mathbb{Z} ,}

Q8 -> ArcTan[q6, q5] + 2 \pi c2 if c2 \in \mathbb{Z} , {RQ -> -Sqrt[q5^2 + q6^2 + q7^2] ,}

Q7 -> 2 \pi c1 - I Log[(I Sqrt[q5^2 + q6^2] + I q7)/Sqrt[q5^2 + q6^2 + q7^2]] if c1 \in \mathbb{Z} , Q8 -> 2 \pi c2 - I Log[-(I q5 - q6)/Sqrt[q5^2 + q6^2]] if c2 \in \mathbb{Z} ,

{RQ -> Sqrt[q5^2 + q6^2 + q7^2], Q7 -> 2 \pi c1 - I Log[-(I Sqrt[q5^2 + q6^2] + q7)/Sqrt[q5^2 + q6^2 + q7^2]] if c1 \in \mathbb{Z} ,}

Q8 -> 2 \pi c2 - I Log[-(I q5 - q6)/Sqrt[q5^2 + q6^2]] if c2 \in \mathbb{Z} , {RQ -> Sqrt[q5^2 + q6^2 + q7^2] ,}

Q7 -> 2 \pi c1 - I Log[(I Sqrt[q5^2 + q6^2] + q7)/Sqrt[q5^2 + q6^2 + q7^2]] if c1 \in \mathbb{Z} , Q8 -> ArcTan[q6, q5] + 2 \pi c2 if c2 \in \mathbb{Z} } }

Out[=]=
{q7 -> RQ Cos[Q7], q6 -> RQ Cos[Q8] Sin[Q7], q5 -> RQ Sin[Q7] Sin[Q8] }

```

In[1]:= FullSimplify[( $\sqrt{q_6^2 + q_5^2}$  /. seqsRQQ7Q8), Q7 > 0 && Sin[Q7] > 0 && RQ > 0]

% / q7 /. seqsRQQ7Q8

Solve[ $\frac{\sqrt{q_6^2 + q_5^2}}{q_7} = \text{FullSimplify}[\%, Q7 > 0 \&\& \text{Sin}[Q7] > 0 \&\& \text{RQ} > 0]$ , Q7]

sQ7 = {Q7  $\rightarrow$  ArcTan[q7,  $\sqrt{q_5^2 + q_6^2}$ ]}

Out[1]=

RQ Sin[Q7]

Out[2]=

Tan[Q7]

Out[3]=

{Q7  $\rightarrow$  ArcTan[ $\frac{\sqrt{q_5^2 + q_6^2}}{q_7}$ ] +  $\pi c_1$  if  $c_1 \in \mathbb{Z}$ }

Out[4]=

{Q7  $\rightarrow$  ArcTan[q7,  $\sqrt{q_5^2 + q_6^2}$ ]}

In[5]:= (\*sQ7=\*) Solve[

$\frac{\sqrt{q_6^2 + q_5^2}}{q_7} = \text{FullSimplify}\left[\left(\frac{\sqrt{q_6^2 + q_5^2}}{q_7}\right) /. \text{seqsRQQ7Q8}\right], Q7 > 0 \&\& \text{Sin}[Q7] > 0 \&\& \text{RQ} > 0\right],$

Q7] // FullSimplify

Out[5]=

{Q7  $\rightarrow$  ArcCot[ $\frac{q_7}{\sqrt{q_5^2 + q_6^2}}$ ] +  $\pi c_1$  if  $c_1 \in \mathbb{Z}$ }

```

In[=] := eqsRQQ7Q8[[2;;3]]
Transpose[{%, All, 1}, {%, All, 2}]]
%[[2]][[1]] / %[[1]][[1]] == %[[2]][[2]] / %[[1]][[2]] // FullSimplify
sQ8 = Solve[%, Q8][[1]] // FullSimplify
sQ8 = {Q8 → ArcTan[q6, q5]}
sRQQ7Q8 /. % // FullSimplify

Out[=]
{q6 == RQ Cos[Q8] Sin[Q7], q5 == RQ Sin[Q7] Sin[Q8]}

Out[=]
{{q6, RQ Cos[Q8] Sin[Q7]}, {q5, RQ Sin[Q7] Sin[Q8]}}

Out[=]

$$\frac{q_5}{q_6} == \tan[Q8]$$

Out[=]

$$\left\{ Q8 \rightarrow \text{ArcTan}\left[\frac{q_5}{q_6}\right] + \pi c_1 \text{ if } c_1 \in \mathbb{Z} \right\}$$

Out[=]
{Q8 → ArcTan[q6, q5]}

Out[=]

$$\left\{ \begin{array}{l} RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \log\left[\frac{-i\sqrt{q_5^2 + q_6^2} - q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}, \\ \text{ArcTan}[q_6, q_5] \rightarrow \text{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \end{array} \right\},$$

$$\left\{ \begin{array}{l} RQ \rightarrow -\sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \log\left[\frac{i\left(\sqrt{q_5^2 + q_6^2} + i q_7\right)}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}, \\ \text{ArcTan}[q_6, q_5] \rightarrow 2\pi c_2 - i \log\left[\frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z} \end{array} \right\},$$

$$\left\{ \begin{array}{l} RQ \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \log\left[\frac{-i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}, \\ \text{ArcTan}[q_6, q_5] \rightarrow 2\pi c_2 - i \log\left[\frac{-i q_5 - q_6}{\sqrt{q_5^2 + q_6^2}}\right] \text{ if } c_2 \in \mathbb{Z} \end{array} \right\},$$

$$\left\{ \begin{array}{l} RQ \rightarrow \sqrt{q_5^2 + q_6^2 + q_7^2}, Q7 \rightarrow 2\pi c_1 - i \log\left[\frac{i\sqrt{q_5^2 + q_6^2} + q_7}{\sqrt{q_5^2 + q_6^2 + q_7^2}}\right] \text{ if } c_1 \in \mathbb{Z}, \\ \text{ArcTan}[q_6, q_5] \rightarrow \text{ArcTan}[q_6, q_5] + 2\pi c_2 \text{ if } c_2 \in \mathbb{Z} \end{array} \right\}$$


```

```

In[1]:= sQ7Q8 = Flatten[{sQ7, sQ8}]
Out[1]= {Q7 → ArcTan[q7, √(q5^2 + q6^2)], Q8 → ArcTan[q6, q5]}

In[2]:= Block[{a, b, c},
 {a, b, c} * unit3]
Out[2]= {a Cos[Q7], b Cos[Q8] Sin[Q7], c Sin[Q7] Sin[Q8]}

In[3]:= base16[[#][1]].base16[[#][1]] === -ID16 & /@ {222, 223, 224}
Out[3]= {True, True, True}

In[4]:= (*sixersT4=%*{Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}
 {Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}.
 {Cos[Q7],Sin[Q7]Cos[Q8],Sin[Q7]Sin[Q8]}//FullSimplify*)
 Block[{J = T16^A[4], m = σ16.J},
 {J.J === -ID16, σ16 === Transpose[J].σ16.J, !(m === -Transpose[m])}]
Out[4]= {True, True, False}

In[5]:= (*Clear[possibleComplexStructureIndex];
 Clear[possibleComplexStructureMatrix];
 possibleComplexStructureIndex=Select[base16,Block[{J,a0,a1,a2,b1,b2,r},
 Clear[J,a0,a1,a2,b1,b2,r];
 a0=#[[1]];
 J=T16^A[4]*a0//FullSimplify;
 a1=J.J//FullSimplify;
 b1=(a1===-ID16);
 a2=Transpose[J].σ16.J//FullSimplify;
 b2=(a2===σ16);
 r=b1&&b2;
 If[r,Block[{c1,c2,c3},
 c1=Append[#,2,4]//Sort; (* for J = T16^A[4]*#[[1]]*)
 c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
 Print[#,";",Tr[a1]/16,";c1=",c1,
 ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
 Tr[base16[[c2]][[1]].base16[[c2]][[1]]/16],##&[]];
 r
]]&→"Index"]
 possibleComplexStructureMatrix=base16[[2]]&/@%*)
 (*Tr[(T16^A[3].T16^A[4]).(T16^A[3].T16^A[4])]/16*)

```

```

In[6]:= (*base16[[5]][[2]]*)

In[6]:= (*Select[base16,#[[2]]=={1,2,3,4,5,6}&→"Index"]*)

In[6]:= (*base16[[240]][[2]]
Tr[base16[[240]][[1]].base16[[240]][[1]]]/16*)

In[6]:= (*(*Parallel*)Do[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
Clear[J,a0,a1,a2,b1,b2,r];
a0=base16[[j]][[1]];
J=T16^A[4]*a0//FullSimplify;
a1=J.J//FullSimplify;
If[!(a1===-ID16),Continue[],##&[]];
(*b1=(a1===-ID16);*)
a2=Transpose[J].σ16.J//FullSimplify;
If[!(a2===σ16),Continue[],##&[]];
(*b2=(a2===σ16);*)
(*r=b1&&b2;*)

c1=Append[base16[[j]][[2]],4]//Sort; (* for J = T16^A[4]*#[[1]]*)
c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
Print[base16[[j]][[2]],";",Tr[a1]/16,";c1=",c1,
";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16];
{J,base16[[c2]][[1]]}];
,{j,1,Length[base16}}]*)

(*ParallelDo[
Block[{r,t,a1},
t=Tr[base16[[j]][[1]].base16[[k]][[1]]/16];
r=t^2==KroneckerDelta[j,k];
a1=If[r,##&[],Print[{{j,k},t,{base16[[j]][[2]],base16[[k]][[2]]}}]];
],{j,1,Length[base16]},{k,1,Length[base16}]]*)

```

```
In[8]:= (*(*Parallel*)Table[Catch[Block[{J,a0,a1,a2,b1,b2,c1,c2,c3,r},
 Clear[J,a0,a1,a2,b1,b2,r];
 a0=base16[[j]][[1]];
 J=T16^A[4]*a0//FullSimplify;
 a1=J.J//FullSimplify;
 If[!(a1===-ID16),Throw[Null],##&[]];
 (*b1=(a1===-ID16);*)
 a2=Transpose[J].σ16.J//FullSimplify;
 If[!(a2==σ16),Throw[Null],##&[]];
 (*b2=(a2==σ16);*)
 (*r=b1&b2;*)

 c1=Append[base16[[j]][[2]],4]//Sort; (* for J = T16^A[4]*#[[1]]*)
 c2=Select[base16,#[[2]]==c1&→"Index"][[1]];
 Print[base16[[j]][[2]],";",Tr[a1]/16,";c1=",c1,
 ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
 Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16];
 {{J},{base16[[c2]][[1]]},{J==base16[[c2]][[1]]||J===-base16[[c2]][[1]]}}]
 ,{j,1,Length[base16]/128}]*)
```

```
In[9]:= (*(*Parallel*)Table[
 Block[{J,a1,a2,b1,b2,c1,c2,c3,r},
 Clear[J,a1,a2,b1,b2,r];
 r={};
 J=T16^A[4].base16[[j]][[1]]//FullSimplify;
 a1=J.J//FullSimplify;
 If[(a1===-ID16),
 a2=Transpose[J].σ16.J//FullSimplify;
 If[(a2==σ16),
 c1=Append[base16[[j]][[2]],4]//Sort; (* for J = T16^A[4]*#[[1]]*)
 c3=Select[base16,#[[2]]==c1&→"Index"];
 If[!(c3=={}),
 c2=c3[[1]];
 Print[base16[[j]][[2]],";Tr[J.J]/16=",Tr[a1]/16,";c1=",c1,
 ";c2=",c2,";Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16=",
 Tr[base16[[c2]][[1]].base16[[c2]][[1]]]/16];
 r={{J},{base16[[c2]][[1]]},{J==base16[[c2]][[1]]||J===-base16[[c2]][[1]]}}];
 ,##&[],##&[],##&[]];
 r]
 ,{j,1,Length[base16}]]//Union*)
```

```
In[8]:= (*possibleComplexStructureIndex=
Select[base16,Block[{J=T16^A[4]*#[1]],a1,a2,b1,b2,r}
 a1=J.J;
 a2=Transpose[J].σ16.J;
 b1=a1==ID16;
 b2=a2==σ16;
 r=b1&&b2;
 (*If[r,Print[a1]];*)
 r
] &→ "Index"]
possibleComplexStructureMatrix=base16[[#][2]]&/@%*)
```

In[•]:= **base16**[[201]]<sub>2</sub>

*Out[•] =*

{1, 2, 3, 5, 6}

```
In[•]:= T16A[4].base16[[201]][1] // MatrixForm
```

*Out[•]//MatrixForm=*

```
In[•]:= T16^A[1].T16^A[2].T16^A[3].T16^A[4].T16^A[5].T16^A[6] // MatrixForm
```

```
Out[•]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[•]:= (*possibleComplexStructureIndex=
Select[base16,Block[{J=T16^A[4]*#[1],a1=J.J,a2=Transpose[J].σ16.J,
b1=a1===-ID16,b2=a2==σ16},b1&&b2]&→"Index"]
possibleComplexStructureMatrix=base16[[#]] [[2]]&/@%*)
```

```
In[•]:= possibleComplexStructureIndex = Select[base16,
Block[{J = T16^A[4]*#[1], m = σ16.J}, J.J == -ID16 && σ16 == Transpose[J].σ16.J
(*&&! (m===-Transpose[m]) *)] & → "Index"]
possibleComplexStructureMatrix = base16[[#]] [[2]] & /@ %
```

```
Out[•]= {4, 178, 187, 201}
```

```
Out[•]= {{3}, {0, 1, 3, 6, 7}, {0, 2, 3, 5, 7}, {1, 2, 3, 5, 6}}
```

```
In[•]:= Select[base16, #[2] == {1, 2, 3, 4, 5, 6} & → "Index"]
```

```
Out[•]= {240}
```

```
In[•]:= base16[[240]] [[2]]
```

```
Out[•]= {1, 2, 3, 4, 5, 6}
```

```
In[•]:= base16[[240]][[1]] // MatrixForm
Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```

```
In[•]:= (*possibleComplexStructureIndex=
Select[base16,Block[{J=#[[1]],m=σ16.#[[1]]},J.J===-ID16&&
σ16==Transpose[J].σ16.J&&! (m===-Transpose[m])]&→"Index"]
possibleComplexStructureMatrix=base16[[#]] [[2]]&/@%*)

```

```
findComplexStucture[] := Module[{list, filter(*,innerProduct*)},
(*innerProduct[a_,b_]:=FullSimplify[Transpose[a].σ16.b];*)
(*filter[J_][a_,b_]:=J.J===-ID16&&innerProduct[a,b]==innerProduct[J.a,J.b];*)
filter[J_]:=J.J===-ID16 && σ16==Transpose[J].σ16.J;
Return[list];
]
```

```
In[•]:= Ω16 === T16^A[1].T16^A[2].T16^A[3].T16^A[4]
```

```
Out[•]=
True
```

## Spinor Lagrangian

```
In[•]:= usegT16 = Table[FullSimplify[(T16^α[α1 - 1]), constraintVars], {α1, 1, Length[X]}];
In[•]:= useT16 = Table[FullSimplify[(T16^α[α1 - 1] /. ssgm4488), constraintVars] /.
{ $\sqrt{e^{-2 a4[H x4]}} \rightarrow e^{-a4[H x4]}$, $\sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/3}} \rightarrow e^{a4[H x4]} \sin[6 H x0]^{1/6}$,
 $\frac{1}{\sqrt{e^{2 a4[H x4]} \sin[6 H x0]^{1/3}}} \rightarrow \frac{1}{e^{a4[H x4]} \sin[6 H x0]^{1/6}}$ }, {α1, 1, Length[X]}] /.
{ $\frac{1}{\sqrt{e^{-2 a4[H x4]} \sin[6 H x0]^{1/3}}} \rightarrow \frac{1}{e^{a4[H x4]} \sin[6 H x0]^{1/6}}$, $\sqrt{e^{2 a4[H x4]}} \rightarrow e^{a4[H x4]}$ };
```

```

In[1]:= Dimensions [useT16]
Out[1]= {8, 16, 16}

In[2]:= Det [MatrixMetric44]
Out[2]= Cos [6 H x0]^2

In[3]:= useDSQRT = Cos [6 H x0]
Out[3]= Cos [6 H x0]

In[4]:= wmat /. sg /. ssqm4488 /. subsDefects;
 (% /. {Sqrt[e^2 a4[H x4]] -> e^a4[H x4], Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) -> e^a4[H x4] H Sin[6 H x0]^(1/6)}) -
 spinCoeffs // FullSimplify[#, constraintVars] & /. subsDefects /.
 {Sqrt[e^2 a4[H x4]] -> e^a4[H x4], Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) -> e^a4[H x4] H Sin[6 H x0]^(1/6)} //.
 FullSimplify[#, constraintVars] & /. subsDefects;
% // Flatten // Union
% // ExpandAll /. subsDefects /. {Sqrt[e^2 a4[H x4]] -> e^a4[H x4],
 Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) -> e^a4[H x4] H Sin[6 H x0]^(1/6)} /. subsDefects
% /. {a4[H x4] -> π^4, Sin[6 H x0] -> e^12}

Out[4]= {0, e^-a4[H x4] (-1 + e^a4[H x4] Sqrt[e^-2 a4[H x4]]) H Sin[6 H x0]^(1/6),
 -(((-e^a4[H x4] + Sqrt[e^2 a4[H x4]]) H Sin[6 H x0]^(1/6)), ((-e^a4[H x4] + Sqrt[e^2 a4[H x4]]) H Sin[6 H x0]^(1/6)),
 e^-a4[H x4] (1 - e^a4[H x4] Sqrt[e^-2 a4[H x4]]) H Sin[6 H x0]^(1/6) a4'[H x4],
 e^-a4[H x4] (-1 + e^a4[H x4] Sqrt[e^-2 a4[H x4]]) H Sin[6 H x0]^(1/6) a4'[H x4],
 ((-e^a4[H x4] + Sqrt[e^2 a4[H x4]]) H Sin[6 H x0]^(1/6) a4'[H x4])}

Out[5]= {0, -e^-a4[H x4] H Sin[6 H x0]^(1/6) + Sqrt[e^-2 a4[H x4]] H Sin[6 H x0]^(1/6),
 e^a4[H x4] H Sin[6 H x0]^(1/6) - Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6),
 -e^a4[H x4] H Sin[6 H x0]^(1/6) + Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6),
 e^-a4[H x4] H Sin[6 H x0]^(1/6) a4'[H x4] - Sqrt[e^-2 a4[H x4]] H Sin[6 H x0]^(1/6) a4'[H x4],
 -e^-a4[H x4] H Sin[6 H x0]^(1/6) a4'[H x4] + Sqrt[e^-2 a4[H x4]] H Sin[6 H x0]^(1/6) a4'[H x4],
 -e^a4[H x4] H Sin[6 H x0]^(1/6) a4'[H x4] + Sqrt[e^2 a4[H x4]] H Sin[6 H x0]^(1/6) a4'[H x4]}

Out[6]= {0, 0, 0, 0, 0, 0, 0}

```

In[7]:= Q2 = 0; Protect[Q1, Q2]

Out[7]= {Q1, Q2}

```
In[1]:= Clear[Lg];
Lg[] := Sqrt[detgg] *

$$\left(\text{Transpose}[\Psi_{16}] . \sigma_{16} . \text{Sum} \left[\text{FullSimplify}[(T_{16}^{\alpha} [\alpha_1 - 1] / . sg), \text{constraintVars}] . \right. \right.$$

$$\left(D[\Psi_{16}, X[\alpha_1]] + \left(\frac{Q_1}{2} \right) * \text{Sum}[\omega_{\text{mat}}[\alpha_1, a, b] * SAB[a, b] . \Psi_{16}, \right.$$

$$\{a, 1, 8\}, \{b, 1, 8\}] \left. \right), \{\alpha_1, 1, \text{Length}[X]\}] +$$

$$\left. (H * M) * \text{Transpose}[\Psi_{16}] . \sigma_{16} . \Psi_{16} \right] // \text{Simplify}[\#, \text{constraintVars}] &$$

```

In[2]:=  $\omega_{\mu IJ}[1]$

```
Out[2]= {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}}
```

```
In[3]:= Clear[La];
La[] := useDSQRT * $\left(\text{Transpose}[\Psi_{16}] . \sigma_{16} . \right.$

$$\text{Sum} \left[\text{useT16}[\alpha_1] . \left(D[\Psi_{16}, X[\alpha_1]] + \left(\frac{Q_1}{2} \right) * \text{Sum}[\omega_{\mu IJ}[\alpha_1][A_1, B_1] * SAB[A_1, B_1], \right. \right.$$

$$\{A_1, 1, 8\}, \{B_1, 1, 8\}] . \Psi_{16} \left. \right), \{\alpha_1, 1, \text{Length}[X]\}] +$$

$$\left. (H * M) * \text{Transpose}[\Psi_{16}] . \sigma_{16} . \Psi_{16} \right] // \text{Simplify}[\#, \text{constraintVars}] &$$

```

In[4]:= (\*testLa=La[]\*)

In[5]:= (\*testLg=Lg[]\*)

```
In[6]:= (*D[testLa,a4[H x4]]//Flatten//Union*)
(*D[testLa,a4'[H x4]]//(*Full*)Simplify[#,constraintVars]&//Flatten//Union*)
```

In[7]:= (\*testLgm=testL/.sg/.ssgm4488//(\*Full\*)Simplify[#,constraintVars]&/.subsDefects\*)

```
In[8]:= (*D[testLgm,a4[H x4]]
D[testLgm,a4'[H x4]]//(*Full*)Simplify[#,constraintVars]&*)
```

```
In[8]:= Clear[Lj];
Lj[j_]:= useDSQRT * ((((
Transpose[\!`16].\!`16.useT16[1].D[\!`16, X[1]] +
Transpose[\!`16].\!`16.useT16[5].D[\!`16, X[5]] +
K0
2
)*Sum[(usew[1,A1,B1]*\!`muIJ[1][A1, B1]*Transpose[\!`16].\!`16.
useT16[1].SAB[A1, B1].(base16[j, 1]).\!`16, {A1, 1, 8}, {B1, 1, 8}]) +
Transpose[\!`16].\!`16.useT16[5].D[\!`16, X[5]] + (K4
2
)*
Sum[(usew[5,A1,B1]*\!`muIJ[5][A1, B1]*Transpose[\!`16].\!`16.useT16[5].
SAB[A1, B1].(base16[j, 1]).\!`16, {A1, 1, 8}, {B1, 1, 8}]) +
(H*M)*Transpose[\!`16].\!`16.\!`16) // Simplify[#, constraintVars] &
```

In[9]:= Lj[92]

```
Out[9]= Cos[6 H x0] (-2 H M f16[2][x0, x4] \!`f16[6][x0, x4] -
2 H M f16[3][x0, x4] \!`f16[7][x0, x4] + 2 H M f16[8][x0, x4] \!`f16[12][x0, x4] +
2 H M f16[9][x0, x4] \!`f16[13][x0, x4] + 2 H M f16[10][x0, x4] \!`f16[14][x0, x4] +
2 H M f16[11][x0, x4] \!`f16[15][x0, x4] + 2 f16[9][x0, x4] f16[0]^(0,1)[x0, x4] -
2 f16[8][x0, x4] f16[1]^(0,1)[x0, x4] - 2 f16[11][x0, x4] f16[2]^(0,1)[x0, x4] +
2 f16[10][x0, x4] f16[3]^(0,1)[x0, x4] - 2 f16[13][x0, x4] f16[4]^(0,1)[x0, x4] +
2 f16[12][x0, x4] f16[5]^(0,1)[x0, x4] + 2 f16[15][x0, x4] f16[6]^(0,1)[x0, x4] -
2 f16[14][x0, x4] f16[7]^(0,1)[x0, x4] - 2 f16[3][x0, x4] f16[10]^(0,1)[x0, x4] +
2 f16[2][x0, x4] f16[11]^(0,1)[x0, x4] - 2 f16[5][x0, x4] f16[12]^(0,1)[x0, x4] +
2 f16[4][x0, x4] f16[13]^(0,1)[x0, x4] + 2 f16[7][x0, x4] f16[14]^(0,1)[x0, x4] -
2 f16[6][x0, x4] f16[15]^(0,1)[x0, x4] + Tan[6 H x0] f16[12][x0, x4] f16[0]^(1,0)[x0, x4] +
Tan[6 H x0] f16[13][x0, x4] f16[1]^(1,0)[x0, x4] +
Tan[6 H x0] f16[14][x0, x4] f16[2]^(1,0)[x0, x4] + Tan[6 H x0] f16[15][x0, x4]
f16[3]^(1,0)[x0, x4] + Tan[6 H x0] f16[8][x0, x4] f16[4]^(1,0)[x0, x4] +
Tan[6 H x0] f16[9][x0, x4] f16[5]^(1,0)[x0, x4] + Tan[6 H x0] f16[10][x0, x4]
f16[6]^(1,0)[x0, x4] + Tan[6 H x0] f16[11][x0, x4] f16[7]^(1,0)[x0, x4] -
Tan[6 H x0] f16[4][x0, x4] f16[8]^(1,0)[x0, x4] - Tan[6 H x0] f16[5][x0, x4]
f16[9]^(1,0)[x0, x4] - Tan[6 H x0] f16[6][x0, x4] f16[10]^(1,0)[x0, x4] -
Tan[6 H x0] f16[7][x0, x4] f16[11]^(1,0)[x0, x4] - f16[0][x0, x4]
(2 H M f16[4][x0, x4] + 2 f16[9]^(0,1)[x0, x4] + Tan[6 H x0] f16[12]^(1,0)[x0, x4]) - f16[1][x0,
x4] (2 H M f16[5][x0, x4] - 2 f16[8]^(0,1)[x0, x4] + Tan[6 H x0] f16[13]^(1,0)[x0, x4]) -
Tan[6 H x0] f16[2][x0, x4] f16[14]^(1,0)[x0, x4] -
Tan[6 H x0] f16[3][x0, x4] f16[15]^(1,0)[x0, x4])
```

```
In[1]:= detgg
Out[1]= g[0][0][x0, x4] × g[1][1][x0, x4] × g[2][2][x0, x4] × g[3][3][x0, x4] ×
g[4][4][x0, x4] × g[5][5][x0, x4] × g[6][6][x0, x4] × g[7][7][x0, x4]

In[2]:= Clear[eL];
eL[Lagrangian_Symbol, detsqrt_] := Module[{L, t},
 L = Lagrangian[];
 t = Table[
 FullSimplify[$\frac{1}{detsqrt} (D[L, f16[k][x0, x4]] - D[D[L, f16[k]^{(1,0)}[x0, x4]], x0] -$
 $D[D[L, f16[k]^{(0,1)}[x0, x4]], x4]), constraintVars], {k, 0, 15}]];
 Return[t /. subsDefects];
]$
```

---

**eLa**

```
In[3]:= (*Get[
 "2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLa.mx"];*)

In[4]:= eLa = eL[La, useDSQRT];
```

```
In[•]:= eLa
Out[•]=
{ -2 e^{-a4[Hx4]} H Q1 Sinh[a4[Hx4]] f16[9][x0, x4] a4'[Hx4] -
 2 (H M f16[4][x0, x4] + 3 H f16[12][x0, x4] +
 f16[9]^(0,1)[x0, x4] + Tan[6 H x0] f16[12]^(1,0)[x0, x4]),
 2 (-H M f16[5][x0, x4] - 3 H f16[13][x0, x4] + e^{-a4[Hx4]} H Q1 Sinh[a4[Hx4]] f16[8][x0, x4] a4'[Hx4] + f16[8]^(0,1)[x0, x4] - Tan[6 H x0] f16[13]^(1,0)[x0, x4]),
 -2 (H M f16[6][x0, x4] + 3 H f16[14][x0, x4] - f16[11]^(0,1)[x0, x4] +
 Tan[6 H x0] f16[14]^(1,0)[x0, x4]),
 -2 (H M f16[7][x0, x4] + 3 H f16[15][x0, x4] + f16[10]^(0,1)[x0, x4] +
 Tan[6 H x0] f16[15]^(1,0)[x0, x4]), -2 H M f16[0][x0, x4] -
 6 H f16[8][x0, x4] - 2 e^{-a4[Hx4]} H Q1 Sinh[a4[Hx4]] f16[13][x0, x4] a4'[Hx4] +
 2 f16[13]^(0,1)[x0, x4] - 2 Tan[6 H x0] f16[8]^(1,0)[x0, x4],
 -2 H M f16[1][x0, x4] - 6 H f16[9][x0, x4] +
 e^{-2 a4[Hx4]} (-1 + e^{2 a4[Hx4]}) H Q1 f16[12][x0, x4] a4'[Hx4] -
 2 (f16[12]^(0,1)[x0, x4] + Tan[6 H x0] f16[9]^(1,0)[x0, x4]),
 -2 (H M f16[2][x0, x4] + 3 H f16[10][x0, x4] + f16[15]^(0,1)[x0, x4] +
 Tan[6 H x0] f16[10]^(1,0)[x0, x4]), -2 (H M f16[3][x0, x4] +
 3 H f16[11][x0, x4] - f16[14]^(0,1)[x0, x4] + Tan[6 H x0] f16[11]^(1,0)[x0, x4]),
 2 (3 H f16[4][x0, x4] + H M f16[12][x0, x4] + e^{-a4[Hx4]} H Q1 Sinh[a4[Hx4]] f16[1] [x0, x4] a4'[Hx4] - f16[1]^(0,1)[x0, x4] + Tan[6 H x0] f16[4]^(1,0)[x0, x4]),
 2 (3 H f16[5][x0, x4] + H M f16[13][x0, x4] - e^{-a4[Hx4]} H Q1 Sinh[a4[Hx4]] f16[0] [x0, x4] a4'[Hx4] + f16[0]^(0,1)[x0, x4] + Tan[6 H x0] f16[5]^(1,0)[x0, x4]),
 2 (3 H f16[6][x0, x4] + H M f16[14][x0, x4] + f16[3]^(0,1)[x0, x4] +
 Tan[6 H x0] f16[6]^(1,0)[x0, x4]), 2 (3 H f16[7][x0, x4] +
 H M f16[15][x0, x4] - f16[2]^(0,1)[x0, x4] + Tan[6 H x0] f16[7]^(1,0)[x0, x4]),
 2 (3 H f16[0][x0, x4] + H M f16[8][x0, x4] + e^{-a4[Hx4]} H Q1 Sinh[a4[Hx4]] f16[5][x0, x4] a4'[Hx4] + f16[5]^(0,1)[x0, x4] + Tan[6 H x0] f16[0]^(1,0)[x0, x4]), 6 H f16[1][x0, x4] +
 2 H M f16[9][x0, x4] - 2 e^{-a4[Hx4]} H Q1 Sinh[a4[Hx4]] f16[4][x0, x4] a4'[Hx4] -
 2 f16[4]^(0,1)[x0, x4] + 2 Tan[6 H x0] f16[1]^(1,0)[x0, x4],
 2 (3 H f16[2][x0, x4] + H M f16[10][x0, x4] - f16[7]^(0,1)[x0, x4] +
 Tan[6 H x0] f16[2]^(1,0)[x0, x4]), 2 (3 H f16[3][x0, x4] +
 H M f16[11][x0, x4] + f16[6]^(0,1)[x0, x4] + Tan[6 H x0] f16[3]^(1,0)[x0, x4]) }
```

```
In[•]:= Length[eLa]
```

```
Out[•]=
```

16

```
In[•]:= DumpSave[ToString[header <> "eLa.mx"], eLa];
```

```
In[•]:= Print[ToString[header <> "eLa.mx"]]
```

2026-01-30-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-L9i-eLa.mx

```
In[8]:= (*Cases[
 eLa,
 h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
 Infinity,
 Heads → True
] *)
```

### WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE :

```
In[9]:= rawSets = Cases[
 #,
 h_Symbol[n_Integer] /; StringEndsQ[SymbolName[h], "f16"] :> n,
 Infinity,
 Heads → True
] & /@ eLa
Out[9]= {{9, 4, 12, 9, 12}, {5, 13, 8, 8, 13}, {6, 14, 11, 14}, {7, 15, 10, 15}, {0, 8, 13, 13, 8},
{1, 9, 12, 12, 9}, {2, 10, 15, 10}, {3, 11, 14, 11}, {4, 12, 1, 1, 4}, {5, 13, 0, 0, 5},
{6, 14, 3, 6}, {7, 15, 2, 7}, {0, 8, 5, 5, 0}, {1, 9, 4, 4, 1}, {2, 10, 7, 2}, {3, 11, 6, 3}}
```

```
In[10]:= Dimensions[rawSets]
```

```
Out[10]= {16}
```

```
rawSetsel16 = rawSets[eLa, "f16", 0]
% === rawSets

eLaCouplings = showCoupledEquations[rawSets]
% // Flatten // Sort
% === Range[0, 15]
```

```
In[11]:= (*eLaCouplings=processRawSets[rawSets]*)
```

```
Out[11]= {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
```

```
In[12]:= Union[Flatten[eLaztCouplings]]
```

**Flatten:** Nonatomic expression expected at position 1 in Flatten[eLaztCouplings]. [i](#)

```
Out[12]= Flatten[eLaztCouplings]
```

```
In[13]:= Union[Flatten[eLaztCouplings]] === Range[0, 15]
```

**Flatten:** Nonatomic expression expected at position 1 in Flatten[eLaztCouplings]. [i](#)

```
Out[13]= False
```

```
In[14]:= eLaztCouplings = eLaCouplings
```

```
Out[14]= {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
```

```
In[8]:= (*processRawSets[]:=Module[{s1,s2,J,L=Range[Length[rawSets]],r={}},
 While[Length[L]>1,
 s1=Union[rawSets[[L[[1]]]];
 L=Drop[L,1];
 J=L[[1]];
 While[J≤16,
 If[MemberQ[L,J],
 s2=Union[rawSets[[J]]];
 If[Intersection[s1,s2]≠{},s1=Union[Flatten[{s1,s2}]];
 L=Complement[L,{J}];,##&[]];,
 ##&[]];
 J++];
 AppendTo[r,s1];
];
 Return[r];
]*)
```

eLast

```
In[9]:= (*Get[
 "2025-12-14_Q2=0-WaveFunctionUniverse-4+4-Einstein-Lovelock-Nash-L9i-eLast.mx"];
 *)
```

```
In[•]:= time1 = Now
AbsoluteTiming[
 eLazt = $\frac{1}{2 \cdot H} \cdot eLa /. sf\psi16Aa /. sx0x4 // FullSimplify[\#, constraintVars] &$
Now - time1

Out[•]= Fri 30 Jan 2026 05:22:31 GMT-8

Out[•]= {7.58209, {-M Z[4][z, t] - 3 Z[12][z, t] - e-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] -
 Z[9](0,1)[z, t] - 6 Tan[z] Z[12](1,0)[z, t], -M Z[5][z, t] - 3 Z[13][z, t] +
 e-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + Z[8](0,1)[z, t] - 6 Tan[z] Z[13](1,0)[z, t],
 -M Z[6][z, t] - 3 Z[14][z, t] + Z[11](0,1)[z, t] - 6 Tan[z] Z[14](1,0)[z, t],
 -M Z[7][z, t] - 3 Z[15][z, t] - Z[10](0,1)[z, t] - 6 Tan[z] Z[15](1,0)[z, t],
 -M Z[0][z, t] - 3 Z[8][z, t] - e-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] +
 Z[13](0,1)[z, t] - 6 Tan[z] Z[8](1,0)[z, t], -M Z[1][z, t] - 3 Z[9][z, t] +
 e-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] - Z[12](0,1)[z, t] - 6 Tan[z] Z[9](1,0)[z, t],
 -M Z[2][z, t] - 3 Z[10][z, t] - Z[15](0,1)[z, t] - 6 Tan[z] Z[10](1,0)[z, t],
 -M Z[3][z, t] - 3 Z[11][z, t] + Z[14](0,1)[z, t] - 6 Tan[z] Z[11](1,0)[z, t],
 3 Z[4][z, t] + M Z[12][z, t] + e-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] -
 Z[1](0,1)[z, t] + 6 Tan[z] Z[4](1,0)[z, t], 3 Z[5][z, t] + M Z[13][z, t] -
 e-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] + Z[0](0,1)[z, t] + 6 Tan[z] Z[5](1,0)[z, t],
 3 Z[6][z, t] + M Z[14][z, t] + Z[3](0,1)[z, t] + 6 Tan[z] Z[6](1,0)[z, t],
 3 Z[7][z, t] + M Z[15][z, t] - Z[2](0,1)[z, t] + 6 Tan[z] Z[7](1,0)[z, t],
 3 Z[0][z, t] + M Z[8][z, t] + e-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] +
 Z[5](0,1)[z, t] + 6 Tan[z] Z[0](1,0)[z, t], 3 Z[1][z, t] + M Z[9][z, t] -
 e-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] - Z[4](0,1)[z, t] + 6 Tan[z] Z[1](1,0)[z, t],
 3 Z[2][z, t] + M Z[10][z, t] - Z[7](0,1)[z, t] + 6 Tan[z] Z[2](1,0)[z, t],
 3 Z[3][z, t] + M Z[11][z, t] + Z[6](0,1)[z, t] + 6 Tan[z] Z[3](1,0)[z, t]}}
```

```
Out[•]= 7.5977766 s

In[•]:= DumpSave[ToString[header <> "eLazt.mx"], eLazt];
Print[ToString[header <> "eLazt.mx"]]
2026-01-30-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-L9i-eLazt.mx
```

```
In[•]:= varZ = Z[#] & /@ Range[0, 15]
Out[•]= {Z[0], Z[1], Z[2], Z[3], Z[4], Z[5], Z[6],
Z[7], Z[8], Z[9], Z[10], Z[11], Z[12], Z[13], Z[14], Z[15]}
```

```
In[•]:= varZzt = #[z, t] & /@ varZ
Out[•]= {Z[0][z, t], Z[1][z, t], Z[2][z, t], Z[3][z, t], Z[4][z, t],
Z[5][z, t], Z[6][z, t], Z[7][z, t], Z[8][z, t], Z[9][z, t], Z[10][z, t],
Z[11][z, t], Z[12][z, t], Z[13][z, t], Z[14][z, t], Z[15][z, t]}
```

```

In[1]:= DzvarZ = D[varZzt, z]
Out[1]= {Z[0]^(1,0)[z, t], Z[1]^(1,0)[z, t], Z[2]^(1,0)[z, t], Z[3]^(1,0)[z, t],
Z[4]^(1,0)[z, t], Z[5]^(1,0)[z, t], Z[6]^(1,0)[z, t], Z[7]^(1,0)[z, t],
Z[8]^(1,0)[z, t], Z[9]^(1,0)[z, t], Z[10]^(1,0)[z, t], Z[11]^(1,0)[z, t],
Z[12]^(1,0)[z, t], Z[13]^(1,0)[z, t], Z[14]^(1,0)[z, t], Z[15]^(1,0)[z, t]}

In[2]:= DtvarZ = D[varZzt, t]
Out[2]= {Z[0]^(0,1)[z, t], Z[1]^(0,1)[z, t], Z[2]^(0,1)[z, t], Z[3]^(0,1)[z, t],
Z[4]^(0,1)[z, t], Z[5]^(0,1)[z, t], Z[6]^(0,1)[z, t], Z[7]^(0,1)[z, t],
Z[8]^(0,1)[z, t], Z[9]^(0,1)[z, t], Z[10]^(0,1)[z, t], Z[11]^(0,1)[z, t],
Z[12]^(0,1)[z, t], Z[13]^(0,1)[z, t], Z[14]^(0,1)[z, t], Z[15]^(0,1)[z, t]}

In[3]:= (DtvarZsubs = Solve[And @@ Thread[θ == eLazt], DtvarZ][[1]] //
FullSimplify[#, constraintVars] &) // Column
Out[3]=
Z[0]^(0,1)[z, t] →
-3 Z[5][z, t] - M Z[13][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] - 6 Tan[z] Z[5]^(1,0)[z, t]
Z[1]^(0,1)[z, t] →
3 Z[4][z, t] + M Z[12][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] + 6 Tan[z] Z[4]^(1,0)[z, t]
Z[2]^(0,1)[z, t] → 3 Z[7][z, t] + M Z[15][z, t] + 6 Tan[z] Z[7]^(1,0)[z, t]
Z[3]^(0,1)[z, t] → -3 Z[6][z, t] - M Z[14][z, t] - 6 Tan[z] Z[6]^(1,0)[z, t]
Z[4]^(0,1)[z, t] →
3 Z[1][z, t] + M Z[9][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] + 6 Tan[z] Z[1]^(1,0)[z, t]
Z[5]^(0,1)[z, t] →
-3 Z[0][z, t] - M Z[8][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] - 6 Tan[z] Z[0]^(1,0)[z, t]
Z[6]^(0,1)[z, t] → -3 Z[3][z, t] - M Z[11][z, t] - 6 Tan[z] Z[3]^(1,0)[z, t]
Z[7]^(0,1)[z, t] → 3 Z[2][z, t] + M Z[10][z, t] + 6 Tan[z] Z[2]^(1,0)[z, t]
Z[8]^(0,1)[z, t] →
M Z[5][z, t] + 3 Z[13][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + 6 Tan[z] Z[13]^(1,0)[z, t]
Z[9]^(0,1)[z, t] → -M Z[4][z, t] - 3 Z[12][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] - 6 Tan[z] Z[12]^(1,0)[z, t]
Z[10]^(0,1)[z, t] → -M Z[7][z, t] - 3 (Z[15][z, t] + 2 Tan[z] Z[15]^(1,0)[z, t])
Z[11]^(0,1)[z, t] → M Z[6][z, t] + 3 Z[14][z, t] + 6 Tan[z] Z[14]^(1,0)[z, t]
Z[12]^(0,1)[z, t] →
-M Z[1][z, t] - 3 Z[9][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] - 6 Tan[z] Z[9]^(1,0)[z, t]
Z[13]^(0,1)[z, t] →
M Z[0][z, t] + 3 Z[8][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] + 6 Tan[z] Z[8]^(1,0)[z, t]
Z[14]^(0,1)[z, t] → M Z[3][z, t] + 3 Z[11][z, t] + 6 Tan[z] Z[11]^(1,0)[z, t]
Z[15]^(0,1)[z, t] → -M Z[2][z, t] - 3 (Z[10][z, t] + 2 Tan[z] Z[10]^(1,0)[z, t])

```

```

In[•]:= ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Equal"] & /@ %;
(DtvarZEQS = ToExpression[#] & /@ %) // Column

Out[•]=
Z[0]^(0,1)[z, t] ==
-3 Z[5][z, t] - M Z[13][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] - 6 Tan[z] Z[5]^(1,0)[z, t]
Z[1]^(0,1)[z, t] ==
3 Z[4][z, t] + M Z[12][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] + 6 Tan[z] Z[4]^(1,0)[z, t]
Z[2]^(0,1)[z, t] == 3 Z[7][z, t] + M Z[15][z, t] + 6 Tan[z] Z[7]^(1,0)[z, t]
Z[3]^(0,1)[z, t] == -3 Z[6][z, t] - M Z[14][z, t] - 6 Tan[z] Z[6]^(1,0)[z, t]
Z[4]^(0,1)[z, t] ==
3 Z[1][z, t] + M Z[9][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] + 6 Tan[z] Z[1]^(1,0)[z, t]
Z[5]^(0,1)[z, t] ==
-3 Z[0][z, t] - M Z[8][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] - 6 Tan[z] Z[0]^(1,0)[z, t]
Z[6]^(0,1)[z, t] == -3 Z[3][z, t] - M Z[11][z, t] - 6 Tan[z] Z[3]^(1,0)[z, t]
Z[7]^(0,1)[z, t] == 3 Z[2][z, t] + M Z[10][z, t] + 6 Tan[z] Z[2]^(1,0)[z, t]
Z[8]^(0,1)[z, t] ==
M Z[5][z, t] + 3 Z[13][z, t] - e^-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + 6 Tan[z] Z[13]^(1,0)[z, t]
Z[9]^(0,1)[z, t] == -M Z[4][z, t] - 3 Z[12][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] - 6 Tan[z] Z[12]^(1,0)[z, t]
Z[10]^(0,1)[z, t] == -M Z[7][z, t] - 3 (Z[15][z, t] + 2 Tan[z] Z[15]^(1,0)[z, t])
Z[11]^(0,1)[z, t] == M Z[6][z, t] + 3 Z[14][z, t] + 6 Tan[z] Z[14]^(1,0)[z, t]
Z[12]^(0,1)[z, t] ==
-M Z[1][z, t] - 3 Z[9][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] - 6 Tan[z] Z[9]^(1,0)[z, t]
Z[13]^(0,1)[z, t] ==
M Z[0][z, t] + 3 Z[8][z, t] + e^-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] + 6 Tan[z] Z[8]^(1,0)[z, t]
Z[14]^(0,1)[z, t] == M Z[3][z, t] + 3 Z[11][z, t] + 6 Tan[z] Z[11]^(1,0)[z, t]
Z[15]^(0,1)[z, t] == -M Z[2][z, t] - 3 (Z[10][z, t] + 2 Tan[z] Z[10]^(1,0)[z, t])

```

```
In[•]:= ToString[FullForm[#]] & /@ DtvarZsubs;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(DtvarZrelations = ToExpression[#] & /@ %) // Column

Out[•]=
3 Z[5][z, t] + M Z[13][z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[0][z, t] a4'[t] + Z[0](0,1)[z, t] + 6 Tan[z] Z[5](1,0)[z, t]
-3 Z[4][z, t] - M Z[12][z, t] -
e-a4[t] Q1 Sinh[a4[t]] Z[1][z, t] a4'[t] + Z[1](0,1)[z, t] - 6 Tan[z] Z[4](1,0)[z, t]
-3 Z[7][z, t] - M Z[15][z, t] + Z[2](0,1)[z, t] - 6 Tan[z] Z[7](1,0)[z, t]
3 Z[6][z, t] + M Z[14][z, t] + Z[3](0,1)[z, t] + 6 Tan[z] Z[6](1,0)[z, t]
-3 Z[1][z, t] - M Z[9][z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[4][z, t] a4'[t] + Z[4](0,1)[z, t] - 6 Tan[z] Z[1](1,0)[z, t]
3 Z[0][z, t] + M Z[8][z, t] + e-a4[t] Q1 Sinh[a4[t]] Z[5][z, t] a4'[t] +
Z[5](0,1)[z, t] + 6 Tan[z] Z[0](1,0)[z, t]
3 Z[3][z, t] + M Z[11][z, t] + Z[6](0,1)[z, t] + 6 Tan[z] Z[3](1,0)[z, t]
-3 Z[2][z, t] - M Z[10][z, t] + Z[7](0,1)[z, t] - 6 Tan[z] Z[2](1,0)[z, t]
-M Z[5][z, t] - 3 Z[13][z, t] +
e-a4[t] Q1 Sinh[a4[t]] Z[8][z, t] a4'[t] + Z[8](0,1)[z, t] - 6 Tan[z] Z[13](1,0)[z, t]
M Z[4][z, t] + 3 Z[12][z, t] + e-a4[t] Q1 Sinh[a4[t]] Z[9][z, t] a4'[t] +
Z[9](0,1)[z, t] + 6 Tan[z] Z[12](1,0)[z, t]
M Z[7][z, t] + Z[10](0,1)[z, t] + 3 (Z[15][z, t] + 2 Tan[z] Z[15](1,0)[z, t])
-M Z[6][z, t] - 3 Z[14][z, t] + Z[11](0,1)[z, t] - 6 Tan[z] Z[14](1,0)[z, t]
M Z[1][z, t] + 3 Z[9][z, t] - e-a4[t] Q1 Sinh[a4[t]] Z[12][z, t] a4'[t] +
Z[12](0,1)[z, t] + 6 Tan[z] Z[9](1,0)[z, t]
-M Z[0][z, t] - 3 Z[8][z, t] - e-a4[t] Q1 Sinh[a4[t]] Z[13][z, t] a4'[t] +
Z[13](0,1)[z, t] - 6 Tan[z] Z[8](1,0)[z, t]
-M Z[3][z, t] - 3 Z[11][z, t] + Z[14](0,1)[z, t] - 6 Tan[z] Z[11](1,0)[z, t]
M Z[2][z, t] + Z[15](0,1)[z, t] + 3 (Z[10][z, t] + 2 Tan[z] Z[10](1,0)[z, t])
```

### WRITE OUT THE VARIABLES THAT ARE COUPLED TO ANOTHER VARIABLE :

```
In[•]:= eLaztCouplings = eLaCouplings
Out[•]= {{0, 5, 8, 13}, {1, 4, 9, 12}, {2, 7, 10, 15}, {3, 6, 11, 14}}
In[•]:= (*{{{5,13,0,8,5,5,0,5,13,8},{4,12,1,1,9,4},{7,15,2,2,10,7},{6,14,3,3,11,6}}*)
eLaztCouplings=Union[#]&/@%
(*eLaztCouplings=Union[Flatten[#]]&/@%*)
Union[Flatten[%]]==Range[0,15]*)
```

### SEPARATE VARIABLES THAT ARE COUPLED TO EACH OTHER :

```

In[]:= Thread[(Z[#] & /@ Flatten[eLastCouplings]) → (yZ[#] & /@ Range[0, 15])]
% // Sort
sZt0yZ
%- %%

Out[]= {Z[0] → yZ[0], Z[5] → yZ[1], Z[8] → yZ[2], Z[13] → yZ[3], Z[1] → yZ[4], Z[4] → yZ[5],
Z[9] → yZ[6], Z[12] → yZ[7], Z[2] → yZ[8], Z[7] → yZ[9], Z[10] → yZ[10],
Z[15] → yZ[11], Z[3] → yZ[12], Z[6] → yZ[13], Z[11] → yZ[14], Z[14] → yZ[15]}

Out[]= {Z[0] → yZ[0], Z[1] → yZ[4], Z[2] → yZ[8], Z[3] → yZ[12], Z[4] → yZ[5], Z[5] → yZ[1],
Z[6] → yZ[13], Z[7] → yZ[9], Z[8] → yZ[2], Z[9] → yZ[6], Z[10] → yZ[10],
Z[11] → yZ[14], Z[12] → yZ[7], Z[13] → yZ[3], Z[14] → yZ[15], Z[15] → yZ[11]}

Out[=]
sZt0yZ

Out[=]
{sZt0yZ - (Z[0] → yZ[0]), sZt0yZ - (Z[1] → yZ[4]),
sZt0yZ - (Z[2] → yZ[8]), sZt0yZ - (Z[3] → yZ[12]), sZt0yZ - (Z[4] → yZ[5]),
sZt0yZ - (Z[5] → yZ[1]), sZt0yZ - (Z[6] → yZ[13]), sZt0yZ - (Z[7] → yZ[9]),
sZt0yZ - (Z[8] → yZ[2]), sZt0yZ - (Z[9] → yZ[6]), sZt0yZ - (Z[10] → yZ[10]),
sZt0yZ - (Z[11] → yZ[14]), sZt0yZ - (Z[12] → yZ[7]), sZt0yZ - (Z[13] → yZ[3]),
sZt0yZ - (Z[14] → yZ[15]), sZt0yZ - (Z[15] → yZ[11])}

```

```
In[-]:= (*Thread[(Z[#]&/@eLaztCouplings[[1]])→(yZ[#]&/@Range[0,3])]
 Thread[(Z[#]&/@eLaztCouplings[[2]])→(yZ[#]&/@Range[4,7])]
 Thread[(Z[#]&/@eLaztCouplings[[3]])→(yZ[#]&/@Range[8,11])]
 Thread[(Z[#]&/@eLaztCouplings[[4]])→(yZ[#]&/@Range[12,15])] *)
sZt0yZ = Thread[(Z[#] & /@ Flatten[eLaztCouplings]) → (yZ[#] & /@ Range[0, 15])]
(*Union[Flatten[{%,%%,%%%,%%%%}]]*)
ZyZEQS = sZt0yZ /. {Rule → Equal}
ZyZforCaEQS = -sZt0yZ /. {Rule → Subtract}
(*ToString[FullForm[#]]&/@sZt0yZ;
StringReplace[#, "Rule" → "Equal"]&/@%;*)
(*(ZyZEQS=ToExpression[#]&/@%)*)
syZtoZ = Solve[And @@ ZyZEQS, (yZ[#] & /@ Range[0, 15])] [[1]]
Out[-]= {Z[0] → yZ[0], Z[5] → yZ[1], Z[8] → yZ[2], Z[13] → yZ[3], Z[1] → yZ[4], Z[4] → yZ[5],
Z[9] → yZ[6], Z[12] → yZ[7], Z[2] → yZ[8], Z[7] → yZ[9], Z[10] → yZ[10],
Z[15] → yZ[11], Z[3] → yZ[12], Z[6] → yZ[13], Z[11] → yZ[14], Z[14] → yZ[15]}

Out[-]= {Z[0] == yZ[0], Z[5] == yZ[1], Z[8] == yZ[2], Z[13] == yZ[3], Z[1] == yZ[4], Z[4] == yZ[5],
Z[9] == yZ[6], Z[12] == yZ[7], Z[2] == yZ[8], Z[7] == yZ[9], Z[10] == yZ[10],
Z[15] == yZ[11], Z[3] == yZ[12], Z[6] == yZ[13], Z[11] == yZ[14], Z[14] == yZ[15]}

Out[-]= {yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1],
yZ[5] - Z[4], yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}

Out[-]= {yZ[0] → Z[0], yZ[1] → Z[5], yZ[2] → Z[8], yZ[3] → Z[13], yZ[4] → Z[1], yZ[5] → Z[4],
yZ[6] → Z[9], yZ[7] → Z[12], yZ[8] → Z[2], yZ[9] → Z[7], yZ[10] → Z[10],
yZ[11] → Z[15], yZ[12] → Z[3], yZ[13] → Z[6], yZ[14] → Z[11], yZ[15] → Z[14]}

In[-]:= (*Thread[(Z[#]&/@Flatten[eLaztCouplings])→(yZ[#]&/@Range[0,15])]
%//Sort
sZt0yZ
%-%%*)
Out[-]= ToString[FullForm[#]] & /@ sZt0yZ;
StringReplace[#, "Rule" → "Subtract"] & /@ %;
(ZyZforCaEQS = -ToExpression[#] & /@ %)

Out[-]= {yZ[0] - Z[0], yZ[1] - Z[5], yZ[2] - Z[8], yZ[3] - Z[13], yZ[4] - Z[1],
yZ[5] - Z[4], yZ[6] - Z[9], yZ[7] - Z[12], yZ[8] - Z[2], yZ[9] - Z[7], yZ[10] - Z[10],
yZ[11] - Z[15], yZ[12] - Z[3], yZ[13] - Z[6], yZ[14] - Z[11], yZ[15] - Z[14]}

In[-]:= cayZ = CoefficientArrays[ZyZforCaEQS, (yZ[#] & /@ Range[0, 15])]
Out[-]= {SparseArray[Specified elements: 15 Dimensions: {15}], SparseArray[Specified elements: 15 Dimensions: {15, 15}]}}

```

In[–]:= **caZ = CoefficientArrays [ZyZforCaEQS, (Z[#] & /@ Range[0, 15]) ]**

Out[–]=

{SparseArray[ Specified elements: 16 Dimensions: {16}], SparseArray[ Specified elements: 16 Dimensions: {16, 16}]}

In[–]:= **(cayZ2 = cayZ[[2]] // Normal) // MatrixForm**

Out[–]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

**IDENTIFY ORTHOGONAL SIMILARITY TRANSFORMATION FROM SET OF VARIABLES Z[j] TO SET OF VARIABLES yZ[j] :**

**IDENTIFY ORTHOGONAL SIMILARITY TRANSFORMATION FROM SET OF VARIABLES Z[j] TO SET OF VARIABLES yZ[j] :**

In[–]:= **cayZ2. (yZ[#] & /@ Range[0, 15])**  
**(\* cayZ2.yZ == Z : \*)**  
**% /. syZtoZ**  
**(\* inverse: yZ === Transpose[cayZ2].Z \*)**  
**Transpose[cayZ2].% /. sZt0yZ**

Out[–]=

{yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],  
yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}

Out[–]=

{Z[0], Z[5], Z[8], Z[13], Z[1], Z[4], Z[9],  
Z[12], Z[2], Z[7], Z[10], Z[15], Z[3], Z[6], Z[11], Z[14]}

Out[–]=

{yZ[0], yZ[1], yZ[2], yZ[3], yZ[4], yZ[5], yZ[6], yZ[7],  
yZ[8], yZ[9], yZ[10], yZ[11], yZ[12], yZ[13], yZ[14], yZ[15]}

**ORTHOGONAL O(16) SIMILARITY TRANSFORMATION :**

```
In[•]:= cayZ2.Transpose[cayZ2] === ID16
```

Out[•]=

True

**BUT NOT ORTHOGONAL O(8,8) SIMILARITY TRANSFORMATION :**

```
In[•]:= cayZ2.σ16.Transpose[cayZ2] === σ16
```

Out[•]=

True

**Also, Not a Direct Sum :**

```
In[•]:= cayZ2 === ArrayFlatten[{{cayZ2[[1;;8, 1;;8]], cayZ2[[1;;8, 9;;16]]}, {cayZ2[[9;;16, 1;;8]], cayZ2[[9;;16, 9;;16]]}}]
```

Out[•]=

True

```
In[•]:= cayZ2 === ArrayFlatten[{{cayZ2[[1;;8, 1;;8]], 0}, {0, cayZ2[[9;;16, 9;;16]]}}]
```

Out[•]=

True

**TRANSFORM Euler Lagrange equations to yZ[j] :**

```
In[•]:= (DtYRelations = Transpose[cayZ2].DtvarZrelations /. szt0yz // FullSimplify) // Column
Out[•]=
```

$$\begin{aligned}
& 3yz[1][z, t] + MyZ[3][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[0][z, t] a4'[t] + \\
& yZ[0]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[1]^{(1,0)}[z, t] \\
& -3 yZ[5][z, t] - MyZ[7][z, t] - \\
& e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[4][z, t] a4'[t] + yZ[4]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[5]^{(1,0)}[z, t] \\
& -3 yZ[9][z, t] - MyZ[11][z, t] + yZ[8]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[9]^{(1,0)}[z, t] \\
& 3 yZ[13][z, t] + MyZ[15][z, t] + yZ[12]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[13]^{(1,0)}[z, t] \\
& -3 yZ[4][z, t] - MyZ[6][z, t] + \\
& e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[5][z, t] a4'[t] + yZ[5]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[4]^{(1,0)}[z, t] \\
& 3 yZ[0][z, t] + MyZ[2][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[1][z, t] a4'[t] + \\
& yZ[1]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[0]^{(1,0)}[z, t] \\
& 3 yZ[12][z, t] + MyZ[14][z, t] + yZ[13]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[12]^{(1,0)}[z, t] \\
& -3 yZ[8][z, t] - MyZ[10][z, t] + yZ[9]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[8]^{(1,0)}[z, t] \\
& -MyZ[1][z, t] - 3 yZ[3][z, t] + \\
& e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[2][z, t] a4'[t] + yZ[2]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[3]^{(1,0)}[z, t] \\
& MyZ[5][z, t] + 3 yZ[7][z, t] + e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[6][z, t] a4'[t] + \\
& yZ[6]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[7]^{(1,0)}[z, t] \\
& MyZ[9][z, t] + 3 yZ[11][z, t] + yZ[10]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[11]^{(1,0)}[z, t] \\
& -MyZ[13][z, t] - 3 yZ[15][z, t] + yZ[14]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[15]^{(1,0)}[z, t] \\
& MyZ[4][z, t] + 3 yZ[6][z, t] - e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[7][z, t] a4'[t] + \\
& yZ[7]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[6]^{(1,0)}[z, t] \\
& -MyZ[0][z, t] - 3 yZ[2][z, t] - \\
& e^{-a4[t]} Q1 \operatorname{Sinh}[a4[t]] yZ[3][z, t] a4'[t] + yZ[3]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[2]^{(1,0)}[z, t] \\
& -MyZ[12][z, t] - 3 yZ[14][z, t] + yZ[15]^{(0,1)}[z, t] - 6 \operatorname{Tan}[z] yZ[14]^{(1,0)}[z, t] \\
& MyZ[8][z, t] + 3 yZ[10][z, t] + yZ[11]^{(0,1)}[z, t] + 6 \operatorname{Tan}[z] yZ[10]^{(1,0)}[z, t]
\end{aligned}$$

**CHECK THAT  $yZ[j]$  ARE IN PROPER ORDER:**

```

In[1]:= (DtyZsubs =
 Solve[And @@ Thread[θ == DtyZrelations], (D[(yZ[#][z, t] & /@ Range[0, 15]), t])][
 1]] // FullSimplify[#, constraintVars] &) // Column

Out[1]=
yZ[0]^(0,1)[z, t] → -3 yZ[1][z, t] - MyZ[3][z, t] +
 e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - 6 Tan[z] yZ[1]^(1,0)[z, t]
yZ[1]^(0,1)[z, t] → -3 yZ[0][z, t] - MyZ[2][z, t] -
 e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t]
yZ[2]^(0,1)[z, t] → MyZ[1][z, t] + 3 yZ[3][z, t] -
 e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] + 6 Tan[z] yZ[3]^(1,0)[z, t]
yZ[3]^(0,1)[z, t] → MyZ[0][z, t] + 3 yZ[2][z, t] +
 e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]
yZ[4]^(0,1)[z, t] → 3 yZ[5][z, t] + MyZ[7][z, t] +
 e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] + 6 Tan[z] yZ[5]^(1,0)[z, t]
yZ[5]^(0,1)[z, t] → 3 yZ[4][z, t] + MyZ[6][z, t] -
 e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t]
yZ[6]^(0,1)[z, t] → -MyZ[5][z, t] - 3 yZ[7][z, t] -
 e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - 6 Tan[z] yZ[7]^(1,0)[z, t]
yZ[7]^(0,1)[z, t] → -MyZ[4][z, t] - 3 yZ[6][z, t] +
 e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]
yZ[8]^(0,1)[z, t] → 3 yZ[9][z, t] + MyZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t]
yZ[9]^(0,1)[z, t] → 3 yZ[8][z, t] + MyZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t]
yZ[10]^(0,1)[z, t] → -MyZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t])
yZ[11]^(0,1)[z, t] → -MyZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])
yZ[12]^(0,1)[z, t] → -3 yZ[13][z, t] - MyZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t]
yZ[13]^(0,1)[z, t] → -3 yZ[12][z, t] - MyZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t]
yZ[14]^(0,1)[z, t] → MyZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t]
yZ[15]^(0,1)[z, t] → MyZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]

In[2]:= (caZ2 = caZ[[2]] // Normal) // MatrixForm

```

```
In[•]:= (caZ2 = caZ[[2]] // Normal) // MatrixForm
```

```

In[1]:= sx0x4
Out[1]=
{ x0 → z
 6 H , x4 → t
 H }

In[2]:= (*DtvarZEQS*)
In[3]:= (*DtyZsubs*)
In[4]:= (* (DtvarZsubs=Solve[And@@Thread[θ==eLazt],DtvarZ][[1]]//FullSimplify[#,constraintVars]&)//Column*)
In[5]:= (* (DtyZsubs=
Solve[And@@Thread[θ==DtyZrelations],(D[(yZ[#][z,t]&/@Range[θ,15]),t])[[1]]//FullSimplify[#,constraintVars]&)//Column*)

In[6]:= ToString[FullForm[#]&/@DtyZsubs;
StringReplace[#, "Rule" → "Equal"]&/@%;
(DtvaryZEQS = ToExpression[#]&/@%) // Column
coupledyZeqs = Partition[%[[1]], 4]

Out[6]=
yZ[0]^(0,1)[z, t] == -3 yZ[1][z, t] - MyZ[3][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] - 6 Tan[z] yZ[1]^(1,0)[z, t]
yZ[1]^(0,1)[z, t] == -3 yZ[0][z, t] - MyZ[2][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t]
yZ[2]^(0,1)[z, t] == MyZ[1][z, t] + 3 yZ[3][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] + 6 Tan[z] yZ[3]^(1,0)[z, t]
yZ[3]^(0,1)[z, t] == MyZ[0][z, t] + 3 yZ[2][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]
yZ[4]^(0,1)[z, t] == 3 yZ[5][z, t] + MyZ[7][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] + 6 Tan[z] yZ[5]^(1,0)[z, t]
yZ[5]^(0,1)[z, t] == 3 yZ[4][z, t] + MyZ[6][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t]
yZ[6]^(0,1)[z, t] == -MyZ[5][z, t] - 3 yZ[7][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] - 6 Tan[z] yZ[7]^(1,0)[z, t]
yZ[7]^(0,1)[z, t] == -MyZ[4][z, t] - 3 yZ[6][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]
yZ[8]^(0,1)[z, t] == 3 yZ[9][z, t] + MyZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t]
yZ[9]^(0,1)[z, t] == 3 yZ[8][z, t] + MyZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t]
yZ[10]^(0,1)[z, t] == -MyZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t])
yZ[11]^(0,1)[z, t] == -MyZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])
yZ[12]^(0,1)[z, t] == -3 yZ[13][z, t] - MyZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t]
yZ[13]^(0,1)[z, t] == -3 yZ[12][z, t] - MyZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t]
yZ[14]^(0,1)[z, t] == MyZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t]
yZ[15]^(0,1)[z, t] == MyZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]

```

```

Out[8]=
{ {yZ[0]^(0,1)[z, t] == -3 yZ[1][z, t] - MyZ[3][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] -
 6 Tan[z] yZ[1]^(1,0)[z, t], yZ[1]^(0,1)[z, t] == -3 yZ[0][z, t] - MyZ[2][z, t] -
 e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t],
 yZ[2]^(0,1)[z, t] == MyZ[1][z, t] + 3 yZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] +
 6 Tan[z] yZ[3]^(1,0)[z, t], yZ[3]^(0,1)[z, t] == MyZ[0][z, t] + 3 yZ[2][z, t] +
 e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]}, ,
{yZ[4]^(0,1)[z, t] == 3 yZ[5][z, t] + MyZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
 6 Tan[z] yZ[5]^(1,0)[z, t], yZ[5]^(0,1)[z, t] == 3 yZ[4][z, t] + MyZ[6][z, t] -
 e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t],
 yZ[6]^(0,1)[z, t] == -MyZ[5][z, t] - 3 yZ[7][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] -
 6 Tan[z] yZ[7]^(1,0)[z, t], yZ[7]^(0,1)[z, t] == -MyZ[4][z, t] - 3 yZ[6][z, t] +
 e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]}, ,
{yZ[8]^(0,1)[z, t] == 3 yZ[9][z, t] + MyZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t],
 yZ[9]^(0,1)[z, t] == 3 yZ[8][z, t] + MyZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t],
 yZ[10]^(0,1)[z, t] == -MyZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),
 yZ[11]^(0,1)[z, t] == -MyZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])}, ,
{yZ[12]^(0,1)[z, t] == -3 yZ[13][z, t] - MyZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t],
 yZ[13]^(0,1)[z, t] == -3 yZ[12][z, t] - MyZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t],
 yZ[14]^(0,1)[z, t] == MyZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t],
 yZ[15]^(0,1)[z, t] == MyZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]}}

```

```

In[8]:= ToString[FullForm[#]] & /@ Take[coupledYzeqs, 2]
StringReplace[#, "Equal" → "Subtract"] & /@ %;
(ToExpression[#] & /@ %)
/. {Equal → Subtract} & /@ Take[coupledYzeqs, 2]
%-%%

```

```

Out[8]=
{List[Equal[Derivative[0, 1][yZ[0]][z, t], Plus[Times[-3, yZ[1][z, t]], Times[-1, M,
yZ[3][z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ[0][z, t],
Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][yZ[1]][z, t]]]], Equal[Derivative[0, 1][yZ[1]][z, t], Plus[Times[-3, yZ[0][z, t]], Times[-1, M,
yZ[2][z, t]], Times[-1, Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ[1][z,
t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][yZ[0]][z, t]]]], Equal[Derivative[0, 1][yZ[2]][z, t], Plus[Times[M, yZ[1][z, t]], Times[3,
yZ[3][z, t]], Times[-1, Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ[2][z,
t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][yZ[3]][z, t]]]], Equal[Derivative[0, 1][yZ[3]][z, t], Plus[Times[M, yZ[0][z, t]], Times[3,
yZ[2][z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ[3][z, t],
Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][yZ[2]][z, t]]]], List[Equal[Derivative[0, 1][yZ[4]][z, t], Plus[Times[3, yZ[5][z, t]], Times[M,
yZ[7][z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ[4][z, t],
Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][yZ[5]][z, t]]]], Equal[Derivative[0, 1][yZ[5]][z, t], Plus[Times[3, yZ[4][z, t]], Times[M,
yZ[6][z, t]], Times[-1, Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ[5][z,
t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][yZ[4]][z, t]]]], Equal[Derivative[0, 1][yZ[6]][z, t], Plus[Times[-1, M, yZ[5][z, t]], Times[-3,
yZ[7][z, t]], Times[-1, Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ[6][z,
t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][yZ[7]][z, t]]]], Equal[Derivative[0, 1][yZ[7]][z, t], Plus[Times[-1, M, yZ[4][z, t]], Times[-3,
yZ[6][z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ[7][z, t],
Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][yZ[6]][z, t]]]]} }

Out[9]=
{{3 yZ[1][z, t] + M yZ[3][z, t] - e^{-a4[t]} Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] +
yZ[0]^{(0,1)}[z, t] + 6 Tan[z] yZ[1]^{(1,0)}[z, t], 3 yZ[0][z, t] + M yZ[2][z, t] +
e^{-a4[t]} Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] + yZ[1]^{(0,1)}[z, t] + 6 Tan[z] yZ[0]^{(1,0)}[z, t],
-M yZ[1][z, t] - 3 yZ[3][z, t] + e^{-a4[t]} Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] +
yZ[2]^{(0,1)}[z, t] - 6 Tan[z] yZ[3]^{(1,0)}[z, t], -M yZ[0][z, t] - 3 yZ[2][z, t] -
e^{-a4[t]} Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + yZ[3]^{(0,1)}[z, t] - 6 Tan[z] yZ[2]^{(1,0)}[z, t]}, {-3 yZ[5][z, t] - M yZ[7][z, t] - e^{-a4[t]} Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
yZ[4]^{(0,1)}[z, t] - 6 Tan[z] yZ[5]^{(1,0)}[z, t], -3 yZ[4][z, t] - M yZ[6][z, t] +
e^{-a4[t]} Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + yZ[5]^{(0,1)}[z, t] - 6 Tan[z] yZ[4]^{(1,0)}[z, t],
M yZ[5][z, t] + 3 yZ[7][z, t] + e^{-a4[t]} Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] +
yZ[6]^{(0,1)}[z, t] + 6 Tan[z] yZ[7]^{(1,0)}[z, t], M yZ[4][z, t] + 3 yZ[6][z, t] -
e^{-a4[t]} Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] + yZ[7]^{(0,1)}[z, t] + 6 Tan[z] yZ[6]^{(1,0)}[z, t]}}}

```

```

Out[•]=
{ {3 yZ[1][z, t] + MyZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] +
yZ[0]^(0,1)[z, t] + 6 Tan[z] yZ[1]^(1,0)[z, t], 3 yZ[0][z, t] + MyZ[2][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] + yZ[1]^(0,1)[z, t] + 6 Tan[z] yZ[0]^(1,0)[z, t],
-MyZ[1][z, t] - 3 yZ[3][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] +
yZ[2]^(0,1)[z, t] - 6 Tan[z] yZ[3]^(1,0)[z, t], -MyZ[0][z, t] - 3 yZ[2][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + yZ[3]^(0,1)[z, t] - 6 Tan[z] yZ[2]^(1,0)[z, t]}, { -3 yZ[5][z, t] - MyZ[7][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
yZ[4]^(0,1)[z, t] - 6 Tan[z] yZ[5]^(1,0)[z, t], -3 yZ[4][z, t] - MyZ[6][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + yZ[5]^(0,1)[z, t] - 6 Tan[z] yZ[4]^(1,0)[z, t],
MyZ[5][z, t] + 3 yZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] +
yZ[6]^(0,1)[z, t] + 6 Tan[z] yZ[7]^(1,0)[z, t], MyZ[4][z, t] + 3 yZ[6][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] + yZ[7]^(0,1)[z, t] + 6 Tan[z] yZ[6]^(1,0)[z, t] } }

Out[•]=
{ {0, 0, 0, 0}, {0, 0, 0, 0} }

In[•]:= (resolve4CaEQS = (# /. {Equal → Subtract} & /@ coupledyzeqs)) // Column
Out[•]=
{3 yZ[1][z, t] + MyZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] +
yZ[0]^(0,1)[z, t] + 6 Tan[z] yZ[1]^(1,0)[z, t], 3 yZ[0][z, t] + MyZ[2][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] + yZ[1]^(0,1)[z, t] + 6 Tan[z] yZ[0]^(1,0)[z, t],
-MyZ[1][z, t] - 3 yZ[3][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] +
yZ[2]^(0,1)[z, t] - 6 Tan[z] yZ[3]^(1,0)[z, t], -MyZ[0][z, t] - 3 yZ[2][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + yZ[3]^(0,1)[z, t] - 6 Tan[z] yZ[2]^(1,0)[z, t] }

{ -3 yZ[5][z, t] - MyZ[7][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
yZ[4]^(0,1)[z, t] - 6 Tan[z] yZ[5]^(1,0)[z, t], -3 yZ[4][z, t] - MyZ[6][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + yZ[5]^(0,1)[z, t] - 6 Tan[z] yZ[4]^(1,0)[z, t],
MyZ[5][z, t] + 3 yZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] +
yZ[6]^(0,1)[z, t] + 6 Tan[z] yZ[7]^(1,0)[z, t], MyZ[4][z, t] + 3 yZ[6][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] + yZ[7]^(0,1)[z, t] + 6 Tan[z] yZ[6]^(1,0)[z, t] }

{ -3 yZ[9][z, t] - MyZ[11][z, t] + yZ[8]^(0,1)[z, t] - 6 Tan[z] yZ[9]^(1,0)[z, t],
-3 yZ[8][z, t] - MyZ[10][z, t] + yZ[9]^(0,1)[z, t] - 6 Tan[z] yZ[8]^(1,0)[z, t],
MyZ[9][z, t] + yZ[10]^(0,1)[z, t] + 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),
MyZ[8][z, t] + yZ[11]^(0,1)[z, t] + 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t]) }

{3 yZ[13][z, t] + MyZ[15][z, t] + yZ[12]^(0,1)[z, t] + 6 Tan[z] yZ[13]^(1,0)[z, t],
3 yZ[12][z, t] + MyZ[14][z, t] + yZ[13]^(0,1)[z, t] + 6 Tan[z] yZ[12]^(1,0)[z, t],
-MyZ[13][z, t] - 3 yZ[15][z, t] + yZ[14]^(0,1)[z, t] - 6 Tan[z] yZ[15]^(1,0)[z, t],
-MyZ[12][z, t] - 3 yZ[14][z, t] + yZ[15]^(0,1)[z, t] - 6 Tan[z] yZ[14]^(1,0)[z, t] }

In[•]:= Clear[yZ, yZvar]; yZvar = (yZ[#][z, t] & /@ Range[0, 15])
Out[•]=
{yZ[0][z, t], yZ[1][z, t], yZ[2][z, t], yZ[3][z, t], yZ[4][z, t],
yZ[5][z, t], yZ[6][z, t], yZ[7][z, t], yZ[8][z, t], yZ[9][z, t], yZ[10][z, t],
yZ[11][z, t], yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t] }

```

```

In[1]:= coupledYzeqs[[1]] // Column
Out[1]=
yZ[0]^(0,1)[z,t] == -3 yZ[1][z,t] - MyZ[3][z,t] +
 e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z,t] a4'[t] - 6 Tan[z] yZ[1]^(1,0)[z,t]
yZ[1]^(0,1)[z,t] == -3 yZ[0][z,t] - MyZ[2][z,t] -
 e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z,t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z,t]
yZ[2]^(0,1)[z,t] == MyZ[1][z,t] + 3 yZ[3][z,t] -
 e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z,t] a4'[t] + 6 Tan[z] yZ[3]^(1,0)[z,t]
yZ[3]^(0,1)[z,t] == MyZ[0][z,t] + 3 yZ[2][z,t] +
 e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z,t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z,t]

In[2]:= DSolve[coupledYzeqs[[1]], yZ[#][z,t] & /@ Range[0,3], {z,t}]
Out[2]=
DSolve[
{yZ[0]^(0,1)[z,t] == -3 yZ[1][z,t] - MyZ[3][z,t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z,t] a4'[t] -
 6 Tan[z] yZ[1]^(1,0)[z,t], yZ[1]^(0,1)[z,t] == -3 yZ[0][z,t] - MyZ[2][z,t] -
 e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z,t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z,t],
yZ[2]^(0,1)[z,t] == MyZ[1][z,t] + 3 yZ[3][z,t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z,t] a4'[t] +
 6 Tan[z] yZ[3]^(1,0)[z,t], yZ[3]^(0,1)[z,t] == MyZ[0][z,t] + 3 yZ[2][z,t] +
 e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z,t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z,t]},
{yZ[0][z,t], yZ[1][z,t], yZ[2][z,t], yZ[3][z,t]}, {z,t}]

In[3]:= yZ2xyZ = {yZ[0] \rightarrow (x yZ[0][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
Out[3]=
{yZ[1] \rightarrow (x yZ[1][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[2] \rightarrow (x yZ[2][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[3] \rightarrow (x yZ[3][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[4] \rightarrow (x yZ[4][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[5] \rightarrow (x yZ[5][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[6] \rightarrow (x yZ[6][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[7] \rightarrow (x yZ[7][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &)
}

In[4]:= Out[4]=
{yZ[0] \rightarrow (x yZ[0][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[1] \rightarrow (x yZ[1][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[2] \rightarrow (x yZ[2][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[3] \rightarrow (x yZ[3][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[4] \rightarrow (x yZ[4][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[5] \rightarrow (x yZ[5][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[6] \rightarrow (x yZ[6][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
yZ[7] \rightarrow (x yZ[7][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &) }

```

```
In[8]:= Flatten[{resolve4CaEQS[[1]], resolve4CaEQS[[2]]}];
% /. yZ2xyZ // FullSimplify
(DtxyZsubs = Solve[And @@ Thread[\theta == %], (D[(xyz[#][z, t] & /@ Range[\theta, 15]), t])][[1]] //
FullSimplify[#, constraintVars] &) (*//Column*)
coupledYZeqs1and2 = % /. {Rule -> Equal}

Out[8]=
{e-a4[t]-e-a4[t] Q1 Sinh[a4[t]] (-Q1 Cosh[a4[t]] xyz[0][z, t] a4'[t] +
ea4[t] (3 xyz[1][z, t] + M xyz[3][z, t] + xyz[0](0,1)[z, t] + 6 Tan[z] xyz[1](1,0)[z, t]),
e-a4[t]-e-a4[t] Q1 Sinh[a4[t]] (-Q1 (Cosh[a4[t]] - 2 Sinh[a4[t]]) xyz[1][z, t] a4'[t] +
ea4[t] (3 xyz[0][z, t] + M xyz[2][z, t] + xyz[1](0,1)[z, t] + 6 Tan[z] xyz[0](1,0)[z, t]),
e-a4[t]-e-a4[t] Q1 Sinh[a4[t]] (-Q1 (Cosh[a4[t]] - 2 Sinh[a4[t]]) xyz[2][z, t] a4'[t] +
ea4[t] (-M xyz[1][z, t] - 3 xyz[3][z, t] + xyz[2](0,1)[z, t] - 6 Tan[z] xyz[3](1,0)[z, t]),
e-a4[t]-e-a4[t] Q1 Sinh[a4[t]] (-Q1 Cosh[a4[t]] xyz[3][z, t] a4'[t] +
ea4[t] (-M xyz[0][z, t] - 3 xyz[2][z, t] + xyz[3](0,1)[z, t] - 6 Tan[z] xyz[2](1,0)[z, t]),
e-a4[t]-e-a4[t] Q1 Sinh[a4[t]] (-Q1 Cosh[a4[t]] xyz[4][z, t] a4'[t] +
ea4[t] (-3 xyz[5][z, t] - M xyz[7][z, t] + xyz[4](0,1)[z, t] - 6 Tan[z] xyz[5](1,0)[z, t]),
e-a4[t]-e-a4[t] Q1 Sinh[a4[t]] (-Q1 (Cosh[a4[t]] - 2 Sinh[a4[t]]) xyz[5][z, t] a4'[t] +
ea4[t] (-3 xyz[4][z, t] - M xyz[6][z, t] + xyz[5](0,1)[z, t] - 6 Tan[z] xyz[4](1,0)[z, t]),
e-a4[t]-e-a4[t] Q1 Sinh[a4[t]] (-Q1 (Cosh[a4[t]] - 2 Sinh[a4[t]]) xyz[6][z, t] a4'[t] +
ea4[t] (M xyz[5][z, t] + 3 xyz[7][z, t] + xyz[6](0,1)[z, t] + 6 Tan[z] xyz[7](1,0)[z, t]),
e-a4[t]-e-a4[t] Q1 Sinh[a4[t]] (-Q1 Cosh[a4[t]] xyz[7][z, t] a4'[t] +
ea4[t] (M xyz[4][z, t] + 3 xyz[6][z, t] + xyz[7](0,1)[z, t] + 6 Tan[z] xyz[6](1,0)[z, t])}
```

**Solve:** Equations may not give solutions for all "solve" variables. [?](#)

```

Out[8]=
{xyZ[0]^(0,1)[z, t] → -3 xyZ[1][z, t] - M xyZ[3][z, t] +
 e^-a4[t] Q1 Cosh[a4[t]] xyZ[0][z, t] a4'[t] - 6 Tan[z] xyZ[1]^(1,0)[z, t],
 xyZ[1]^(0,1)[z, t] → -3 xyZ[0][z, t] - M xyZ[2][z, t] -
 1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyZ[1][z, t] a4'[t] - 6 Tan[z] xyZ[0]^(1,0)[z, t],
 xyZ[2]^(0,1)[z, t] → M xyZ[1][z, t] + 3 xyZ[3][z, t] -
 1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyZ[2][z, t] a4'[t] + 6 Tan[z] xyZ[3]^(1,0)[z, t],
 xyZ[3]^(0,1)[z, t] → M xyZ[0][z, t] + 3 xyZ[2][z, t] +
 e^-a4[t] Q1 Cosh[a4[t]] xyZ[3][z, t] a4'[t] + 6 Tan[z] xyZ[2]^(1,0)[z, t],
 xyZ[4]^(0,1)[z, t] → 3 xyZ[5][z, t] + M xyZ[7][z, t] + e^-a4[t] Q1 Cosh[a4[t]] xyZ[4][z, t] a4'[t] +
 6 Tan[z] xyZ[5]^(1,0)[z, t], xyZ[5]^(0,1)[z, t] → 3 xyZ[4][z, t] + M xyZ[6][z, t] -
 1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyZ[5][z, t] a4'[t] + 6 Tan[z] xyZ[4]^(1,0)[z, t],
 xyZ[6]^(0,1)[z, t] → -M xyZ[5][z, t] - 3 xyZ[7][z, t] -
 1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyZ[6][z, t] a4'[t] - 6 Tan[z] xyZ[7]^(1,0)[z, t],
 xyZ[7]^(0,1)[z, t] → -M xyZ[4][z, t] - 3 xyZ[6][z, t] +
 e^-a4[t] Q1 Cosh[a4[t]] xyZ[7][z, t] a4'[t] - 6 Tan[z] xyZ[6]^(1,0)[z, t]}
}

Out[9]=
{xyZ[0]^(0,1)[z, t] == -3 xyZ[1][z, t] - M xyZ[3][z, t] +
 e^-a4[t] Q1 Cosh[a4[t]] xyZ[0][z, t] a4'[t] - 6 Tan[z] xyZ[1]^(1,0)[z, t],
 xyZ[1]^(0,1)[z, t] == -3 xyZ[0][z, t] - M xyZ[2][z, t] -
 1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyZ[1][z, t] a4'[t] - 6 Tan[z] xyZ[0]^(1,0)[z, t],
 xyZ[2]^(0,1)[z, t] == M xyZ[1][z, t] + 3 xyZ[3][z, t] -
 1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyZ[2][z, t] a4'[t] + 6 Tan[z] xyZ[3]^(1,0)[z, t],
 xyZ[3]^(0,1)[z, t] == M xyZ[0][z, t] + 3 xyZ[2][z, t] +
 e^-a4[t] Q1 Cosh[a4[t]] xyZ[3][z, t] a4'[t] + 6 Tan[z] xyZ[2]^(1,0)[z, t],
 xyZ[4]^(0,1)[z, t] == 3 xyZ[5][z, t] + M xyZ[7][z, t] + e^-a4[t] Q1 Cosh[a4[t]] xyZ[4][z, t] a4'[t] +
 6 Tan[z] xyZ[5]^(1,0)[z, t], xyZ[5]^(0,1)[z, t] == 3 xyZ[4][z, t] + M xyZ[6][z, t] -
 1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyZ[5][z, t] a4'[t] + 6 Tan[z] xyZ[4]^(1,0)[z, t],
 xyZ[6]^(0,1)[z, t] == -M xyZ[5][z, t] - 3 xyZ[7][z, t] -
 1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyZ[6][z, t] a4'[t] - 6 Tan[z] xyZ[7]^(1,0)[z, t],
 xyZ[7]^(0,1)[z, t] == -M xyZ[4][z, t] - 3 xyZ[6][z, t] +
 e^-a4[t] Q1 Cosh[a4[t]] xyZ[7][z, t] a4'[t] - 6 Tan[z] xyZ[6]^(1,0)[z, t]}
}

```

```

In[8]:= DSolve[Take[coupledYZeqs1and2, 4],
{xyz[0][z, t], xyz[1][z, t], xyz[2][z, t], xyz[3][z, t]}, {z, t}]

Out[8]= DSolve[{xyz[0]^(0,1)[z, t] == -3 xyz[1][z, t] - M xyz[3][z, t] +
e^-a4[t] Q1 Cosh[a4[t]] xyz[0][z, t] a4'[t] - 6 Tan[z] xyz[1]^(1,0)[z, t],
xyz[1]^(0,1)[z, t] == -3 xyz[0][z, t] - M xyz[2][z, t] -
1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyz[1][z, t] a4'[t] - 6 Tan[z] xyz[0]^(1,0)[z, t],
xyz[2]^(0,1)[z, t] == M xyz[1][z, t] + 3 xyz[3][z, t] -
1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyz[2][z, t] a4'[t] + 6 Tan[z] xyz[3]^(1,0)[z, t],
xyz[3]^(0,1)[z, t] == M xyz[0][z, t] + 3 xyz[2][z, t] +
e^-a4[t] Q1 Cosh[a4[t]] xyz[3][z, t] a4'[t] + 6 Tan[z] xyz[2]^(1,0)[z, t]},
{xyz[0][z, t], xyz[1][z, t], xyz[2][z, t], xyz[3][z, t]}, {z, t}]

In[9]:= DSolve[Take[coupledYZeqs1and2, -4],
{xyz[4][z, t], xyz[5][z, t], xyz[6][z, t], xyz[7][z, t]}, {z, t}]

Out[9]= DSolve[{xyz[4]^(0,1)[z, t] == 3 xyz[5][z, t] + M xyz[7][z, t] +
e^-a4[t] Q1 Cosh[a4[t]] xyz[4][z, t] a4'[t] + 6 Tan[z] xyz[5]^(1,0)[z, t],
xyz[5]^(0,1)[z, t] == 3 xyz[4][z, t] + M xyz[6][z, t] -
1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyz[5][z, t] a4'[t] + 6 Tan[z] xyz[4]^(1,0)[z, t],
xyz[6]^(0,1)[z, t] == -M xyz[5][z, t] - 3 xyz[7][z, t] -
1/2 e^-2 a4[t] (-3 + e^2 a4[t]) Q1 xyz[6][z, t] a4'[t] - 6 Tan[z] xyz[7]^(1,0)[z, t],
xyz[7]^(0,1)[z, t] == -M xyz[4][z, t] - 3 xyz[6][z, t] +
e^-a4[t] Q1 Cosh[a4[t]] xyz[7][z, t] a4'[t] - 6 Tan[z] xyz[6]^(1,0)[z, t}],
{xyz[4][z, t], xyz[5][z, t], xyz[6][z, t], xyz[7][z, t]}, {z, t}]

```

```

In[8]:= coupledyZeqs1 = coupledyZeqs[[1]] /. {yz[1] → (xyz[1][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
 yz[2] → (xyz[2][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
 yz[3] → (xyz[3][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &),
 yz[0] → (xyz[0][#1, #2] Exp[-e^-a4[#2] Q1 Sinh[a4[#2]]] &)} // FullSimplify

Out[8]=
{e^-a4[t]-e^-a4[t] Q1 Sinh[a4[t]] (-Q1 xyz[0][z, t] a4'[t] - e^a4[t] Q1 Sinh[a4[t]] xyz[0][z, t] a4'[t] +
e^2 a4[t] (3 xyz[1][z, t] + M xyz[3][z, t] + xyz[0]^(0,1)[z, t] + 6 Tan[z] xyz[1]^(1,0)[z, t])) == 0,
e^-a4[t]-e^-a4[t] Q1 Sinh[a4[t]] (-Q1 xyz[1][z, t] a4'[t] + e^a4[t] Q1 Sinh[a4[t]] xyz[1][z, t] a4'[t] +
e^2 a4[t] (3 xyz[0][z, t] + M xyz[2][z, t] + xyz[1]^(0,1)[z, t] + 6 Tan[z] xyz[0]^(1,0)[z, t])) == 0,
e^-a4[t]-e^-a4[t] Q1 Sinh[a4[t]] (Q1 xyz[2][z, t] a4'[t] - e^a4[t] Q1 Sinh[a4[t]] xyz[2][z, t] a4'[t] +
e^2 a4[t] (M xyz[1][z, t] + 3 xyz[3][z, t] - xyz[2]^(0,1)[z, t] + 6 Tan[z] xyz[3]^(1,0)[z, t])) == 0,
e^-a4[t]-e^-a4[t] Q1 Sinh[a4[t]] (Q1 xyz[3][z, t] a4'[t] +
e^a4[t] Q1 Sinh[a4[t]] xyz[3][z, t] a4'[t] + e^2 a4[t]
(M xyz[0][z, t] + 3 xyz[2][z, t] - xyz[3]^(0,1)[z, t] + 6 Tan[z] xyz[2]^(1,0)[z, t])) == 0}

In[9]:= DSolve[{(-Q1 xyz[0][z, t] a4'[t] - e^a4[t] Q1 Sinh[a4[t]] xyz[0][z, t] a4'[t] +
e^2 a4[t] (3 xyz[1][z, t] + M xyz[3][z, t] + xyz[0]^(0,1)[z, t] + 6 Tan[z] xyz[1]^(1,0)[z, t])) == 0,
(-Q1 xyz[1][z, t] a4'[t] + e^a4[t] Q1 Sinh[a4[t]] xyz[1][z, t] a4'[t] +
e^2 a4[t] (3 xyz[0][z, t] + M xyz[2][z, t] + xyz[1]^(0,1)[z, t] + 6 Tan[z] xyz[0]^(1,0)[z, t])) == 0,
(Q1 xyz[2][z, t] a4'[t] - e^a4[t] Q1 Sinh[a4[t]] xyz[2][z, t] a4'[t] +
e^2 a4[t] (M xyz[1][z, t] + 3 xyz[3][z, t] - xyz[2]^(0,1)[z, t] + 6 Tan[z] xyz[3]^(1,0)[z, t])) == 0,
(Q1 xyz[3][z, t] a4'[t] + e^a4[t] Q1 Sinh[a4[t]] xyz[3][z, t] a4'[t] + e^2 a4[t]
(M xyz[0][z, t] + 3 xyz[2][z, t] - xyz[3]^(0,1)[z, t] + 6 Tan[z] xyz[2]^(1,0)[z, t])) == 0},
{xyz[0][z, t], xyz[1][z, t], xyz[2][z, t], xyz[3][z, t}],
{z, t}]

Out[9]=
DSolve[{-Q1 xyz[0][z, t] a4'[t] - e^a4[t] Q1 Sinh[a4[t]] xyz[0][z, t] a4'[t] +
e^2 a4[t] (3 xyz[1][z, t] + M xyz[3][z, t] + xyz[0]^(0,1)[z, t] + 6 Tan[z] xyz[1]^(1,0)[z, t]) == 0,
-Q1 xyz[1][z, t] a4'[t] + e^a4[t] Q1 Sinh[a4[t]] xyz[1][z, t] a4'[t] +
e^2 a4[t] (3 xyz[0][z, t] + M xyz[2][z, t] + xyz[1]^(0,1)[z, t] + 6 Tan[z] xyz[0]^(1,0)[z, t]) == 0,
Q1 xyz[2][z, t] a4'[t] - e^a4[t] Q1 Sinh[a4[t]] xyz[2][z, t] a4'[t] +
e^2 a4[t] (M xyz[1][z, t] + 3 xyz[3][z, t] - xyz[2]^(0,1)[z, t] + 6 Tan[z] xyz[3]^(1,0)[z, t]) == 0,
Q1 xyz[3][z, t] a4'[t] + e^a4[t] Q1 Sinh[a4[t]] xyz[3][z, t] a4'[t] + e^2 a4[t]
(M xyz[0][z, t] + 3 xyz[2][z, t] - xyz[3]^(0,1)[z, t] + 6 Tan[z] xyz[2]^(1,0)[z, t]) == 0},
{xyz[0][z, t], xyz[1][z, t], xyz[2][z, t], xyz[3][z, t}], {z, t}]

```

```

In[6]:= DSolve[coupledYZeqs[[#]], yZ[#][z, t] & /@ Range[4*(# - 1), 4*(# - 1) + 3], {z, t}] & /@
Range[4] // Column

Out[6]= DSolve[
{yZ[0]^(0,1)[z, t] == -3 yZ[1][z, t] - MyZ[3][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[0][z, t] a4'[t] -
6 Tan[z] yZ[1]^(1,0)[z, t], yZ[1]^(0,1)[z, t] == -3 yZ[0][z, t] - MyZ[2][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[1][z, t] a4'[t] - 6 Tan[z] yZ[0]^(1,0)[z, t],
yZ[2]^(0,1)[z, t] == MyZ[1][z, t] + 3 yZ[3][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[2][z, t] a4'[t] +
6 Tan[z] yZ[3]^(1,0)[z, t], yZ[3]^(0,1)[z, t] == MyZ[0][z, t] + 3 yZ[2][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[3][z, t] a4'[t] + 6 Tan[z] yZ[2]^(1,0)[z, t]}, {z, t}]

DSolve[
{yZ[4]^(0,1)[z, t] == 3 yZ[5][z, t] + MyZ[7][z, t] + e^-a4[t] Q1 Sinh[a4[t]] yZ[4][z, t] a4'[t] +
6 Tan[z] yZ[5]^(1,0)[z, t], yZ[5]^(0,1)[z, t] == 3 yZ[4][z, t] + MyZ[6][z, t] -
e^-a4[t] Q1 Sinh[a4[t]] yZ[5][z, t] a4'[t] + 6 Tan[z] yZ[4]^(1,0)[z, t],
yZ[6]^(0,1)[z, t] == -MyZ[5][z, t] - 3 yZ[7][z, t] - e^-a4[t] Q1 Sinh[a4[t]] yZ[6][z, t] a4'[t] -
6 Tan[z] yZ[7]^(1,0)[z, t], yZ[7]^(0,1)[z, t] == -MyZ[4][z, t] - 3 yZ[6][z, t] +
e^-a4[t] Q1 Sinh[a4[t]] yZ[7][z, t] a4'[t] - 6 Tan[z] yZ[6]^(1,0)[z, t]}, {z, t}]

DSolve[{yZ[8]^(0,1)[z, t] == 3 yZ[9][z, t] + MyZ[11][z, t] + 6 Tan[z] yZ[9]^(1,0)[z, t],
yZ[9]^(0,1)[z, t] == 3 yZ[8][z, t] + MyZ[10][z, t] + 6 Tan[z] yZ[8]^(1,0)[z, t],
yZ[10]^(0,1)[z, t] == -MyZ[9][z, t] - 3 (yZ[11][z, t] + 2 Tan[z] yZ[11]^(1,0)[z, t]),
yZ[11]^(0,1)[z, t] == -MyZ[8][z, t] - 3 (yZ[10][z, t] + 2 Tan[z] yZ[10]^(1,0)[z, t])},
{yZ[8][z, t], yZ[9][z, t], yZ[10][z, t], yZ[11][z, t]}, {z, t}]

DSolve[{yZ[12]^(0,1)[z, t] == -3 yZ[13][z, t] - MyZ[15][z, t] - 6 Tan[z] yZ[13]^(1,0)[z, t],
yZ[13]^(0,1)[z, t] == -3 yZ[12][z, t] - MyZ[14][z, t] - 6 Tan[z] yZ[12]^(1,0)[z, t],
yZ[14]^(0,1)[z, t] == MyZ[13][z, t] + 3 yZ[15][z, t] + 6 Tan[z] yZ[15]^(1,0)[z, t],
yZ[15]^(0,1)[z, t] == MyZ[12][z, t] + 3 yZ[14][z, t] + 6 Tan[z] yZ[14]^(1,0)[z, t]},
{yZ[12][z, t], yZ[13][z, t], yZ[14][z, t], yZ[15][z, t]}, {z, t}]

```

---

**change of vars:**

```
In[8]:= yZ2xyz = {
 yZ[0] → (xyz[0][#1, #2] Exp[BY[#2]] &),
 yZ[1] → (xyz[1][#1, #2] Exp[BY[#2]] &),
 yZ[2] → (xyz[2][#1, #2] Exp[BY[#2]] &),
 yZ[3] → (xyz[3][#1, #2] Exp[BY[#2]] &),
 yZ[4] → (xyz[4][#1, #2] Exp[BY[#2]] &),
 yZ[5] → (xyz[5][#1, #2] Exp[BY[#2]] &),
 yZ[6] → (xyz[6][#1, #2] Exp[BY[#2]] &),
 yZ[7] → (xyz[7][#1, #2] Exp[BY[#2]] &)
}

Out[8]= {yZ[0] → (xyz[0][#1, #2] Exp[BY[#2]] &), yZ[1] → (xyz[1][#1, #2] Exp[BY[#2]] &),
yZ[2] → (xyz[2][#1, #2] Exp[BY[#2]] &), yZ[3] → (xyz[3][#1, #2] Exp[BY[#2]] &),
yZ[4] → (xyz[4][#1, #2] Exp[BY[#2]] &), yZ[5] → (xyz[5][#1, #2] Exp[BY[#2]] &),
yZ[6] → (xyz[6][#1, #2] Exp[BY[#2]] &), yZ[7] → (xyz[7][#1, #2] Exp[BY[#2]] &)}
```

```
In[8]:= Flatten[{resolve4CaEQS[[1]], resolve4CaEQS[[2]]}];

% /. yZ2xyZ // FullSimplify

(DtxyZsubs = Solve[And @@ Thread[\theta == %], (D[(xyz[#][z, t] & /@ Range[\theta, 15]), t])[[1]] // FullSimplify[#, constraintVars] &) // Column

Out[8]= {eBY[t] (3 xyz[1][z, t] + M xyz[3][z, t] + xyz[0][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] + BY'[t]) + xyz[0](0,1)[z, t] + 6 Tan[z] xyz[1](1,0)[z, t]), eBY[t] (3 xyz[0][z, t] + M xyz[2][z, t] + xyz[1][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] + BY'[t]) + xyz[1](0,1)[z, t] + 6 Tan[z] xyz[0](1,0)[z, t]), eBY[t] (-M xyz[1][z, t] - 3 xyz[3][z, t] + xyz[2][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] + BY'[t]) + xyz[2](0,1)[z, t] - 6 Tan[z] xyz[3](1,0)[z, t]), eBY[t] (-M xyz[0][z, t] - 3 xyz[2][z, t] + xyz[3][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] + BY'[t]) + xyz[3](0,1)[z, t] - 6 Tan[z] xyz[2](1,0)[z, t]), eBY[t] (-3 xyz[5][z, t] - M xyz[7][z, t] + xyz[4][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] + BY'[t]) + xyz[4](0,1)[z, t] - 6 Tan[z] xyz[5](1,0)[z, t]), eBY[t] (-3 xyz[4][z, t] - M xyz[6][z, t] + xyz[5][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] + BY'[t]) + xyz[5](0,1)[z, t] - 6 Tan[z] xyz[4](1,0)[z, t]), eBY[t] (M xyz[5][z, t] + 3 xyz[7][z, t] + xyz[6][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] + BY'[t]) + xyz[6](0,1)[z, t] + 6 Tan[z] xyz[7](1,0)[z, t]), eBY[t] (M xyz[4][z, t] + 3 xyz[6][z, t] + xyz[7][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] + BY'[t]) + xyz[7](0,1)[z, t] + 6 Tan[z] xyz[6](1,0)[z, t])}
```

**Solve:** Equations may not give solutions for all "solve" variables. [?](#)

```
Out[8]= xyz[0](0,1)[z, t] → -3 xyz[1][z, t] - M xyz[3][z, t] +
 xyz[0][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) - 6 Tan[z] xyz[1](1,0)[z, t]
xyz[1](0,1)[z, t] → -3 xyz[0][z, t] - M xyz[2][z, t] +
 xyz[1][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) - 6 Tan[z] xyz[0](1,0)[z, t]
xyz[2](0,1)[z, t] → M xyz[1][z, t] + 3 xyz[3][z, t] +
 xyz[2][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) + 6 Tan[z] xyz[3](1,0)[z, t]
xyz[3](0,1)[z, t] → M xyz[0][z, t] + 3 xyz[2][z, t] +
 xyz[3][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) + 6 Tan[z] xyz[2](1,0)[z, t]
xyz[4](0,1)[z, t] → 3 xyz[5][z, t] + M xyz[7][z, t] +
 xyz[4][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) + 6 Tan[z] xyz[5](1,0)[z, t]
xyz[5](0,1)[z, t] → 3 xyz[4][z, t] + M xyz[6][z, t] +
 xyz[5][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) + 6 Tan[z] xyz[4](1,0)[z, t]
xyz[6](0,1)[z, t] → -M xyz[5][z, t] - 3 xyz[7][z, t] +
 xyz[6][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) - 6 Tan[z] xyz[7](1,0)[z, t]
xyz[7](0,1)[z, t] → -M xyz[4][z, t] - 3 xyz[6][z, t] +
 xyz[7][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) - 6 Tan[z] xyz[6](1,0)[z, t]
```

```
In[~]:= (coupledYZRelations1and2 = DtxyzZsubs /. {Rule → Subtract}) // Column
Out[~]=
3 xyz[1][z, t] + M xyz[3][z, t] - xyz[0][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) +
xyz[0]^(0,1)[z, t] + 6 Tan[z] xyz[1]^(1,0)[z, t]
3 xyz[0][z, t] + M xyz[2][z, t] - xyz[1][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) +
xyz[1]^(0,1)[z, t] + 6 Tan[z] xyz[0]^(1,0)[z, t]
-M xyz[1][z, t] - 3 xyz[3][z, t] - xyz[2][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) +
xyz[2]^(0,1)[z, t] - 6 Tan[z] xyz[3]^(1,0)[z, t]
-M xyz[0][z, t] - 3 xyz[2][z, t] - xyz[3][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) +
xyz[3]^(0,1)[z, t] - 6 Tan[z] xyz[2]^(1,0)[z, t]
-3 xyz[5][z, t] - M xyz[7][z, t] - xyz[4][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) +
xyz[4]^(0,1)[z, t] - 6 Tan[z] xyz[5]^(1,0)[z, t]
-3 xyz[4][z, t] - M xyz[6][z, t] - xyz[5][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) +
xyz[5]^(0,1)[z, t] - 6 Tan[z] xyz[4]^(1,0)[z, t]
M xyz[5][z, t] + 3 xyz[7][z, t] - xyz[6][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) +
xyz[6]^(0,1)[z, t] + 6 Tan[z] xyz[7]^(1,0)[z, t]
M xyz[4][z, t] + 3 xyz[6][z, t] - xyz[7][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) +
xyz[7]^(0,1)[z, t] + 6 Tan[z] xyz[6]^(1,0)[z, t]
```

```
In[~]:= cacoupledYZRelations1and2 =
CoefficientArrays[coupledYZRelations1and2, D[(xyz[#][z, t] & /@ Range[0, 7]), t]]
```

```
Out[~]=
{SparseArray[Specified elements: 8 Dimensions: {8, 8}], SparseArray[Specified elements: 8 Dimensions: {8, 8}]}
```

```
In[~]:= cacoupledYZRelations1and2[[2]] // Normal // MatrixForm
```

```
Out[~]/MatrixForm=
(1 0 0 0 0 0 0 0
 0 1 0 0 0 0 0 0
 0 0 1 0 0 0 0 0
 0 0 0 1 0 0 0 0
 0 0 0 0 1 0 0 0
 0 0 0 0 0 1 0 0
 0 0 0 0 0 0 1 0
 0 0 0 0 0 0 0 1)
```

```
In[~]:= cacoupledYZRelations1and2[[1]] // Normal // MatrixForm
```

```
Out[~]/MatrixForm=
(3 xyz[1][z, t] + M xyz[3][z, t] - xyz[0][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) + 6 Tan[z]
 3 xyz[0][z, t] + M xyz[2][z, t] - xyz[1][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) + 6 Tan[z]
 -M xyz[1][z, t] - 3 xyz[3][z, t] - xyz[2][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) - 6 Tan[z]
 -M xyz[0][z, t] - 3 xyz[2][z, t] - xyz[3][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) - 6 Tan[z]
 -3 xyz[5][z, t] - M xyz[7][z, t] - xyz[4][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) - 6 Tan[z]
 -3 xyz[4][z, t] - M xyz[6][z, t] - xyz[5][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) - 6 Tan[z]
 M xyz[5][z, t] + 3 xyz[7][z, t] - xyz[6][z, t] (-e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) + 6 Tan[z]
 M xyz[4][z, t] + 3 xyz[6][z, t] - xyz[7][z, t] (e-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]) + 6 Tan[z])
```

```

In[8]:= sBY = DSolve[0 == (e^-a4[t] Q1 Sinh[a4[t]] a4'[t] - BY'[t]), BY[t], t][[1]]
% /. {t → #2}

Out[8]=
{BY[t] → 1/2 Q1 (1/2 e^-2 a4[t] + a4[t]) + c1}

Out[9]=
{BY[#2] → 1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2]) + c1}

In[10]:= yZ2xyZa = {
 yZ[0] → (xYZ[0][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[1] → (xYZ[1][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[2] → (xYZ[2][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[3] → (xYZ[3][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[4] → (xYZ[4][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[5] → (xYZ[5][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[6] → (xYZ[6][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[7] → (xYZ[7][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &)
}

Out[10]=
{yZ[0] → (xYZ[0][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[1] → (xYZ[1][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[2] → (xYZ[2][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[3] → (xYZ[3][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[4] → (xYZ[4][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[5] → (xYZ[5][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[6] → (xYZ[6][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[7] → (xYZ[7][#1, #2] Exp[1/2 Q1 (1/2 e^-2 a4[#2] + a4[#2])] &)}

```

```
In[8]:= Flatten[{resolve4CaEQS[[1]], resolve4CaEQS[[2]]}];
% /. yZ2xyZa // FullSimplify
(DtxyZsubsa = Solve[And @@ Thread[\theta == %], (D[(xyZ[#][z, t] & /@ Range[0, 15]), t])[[1]] //
FullSimplify[#, constraintVars] &) // Column

Out[8]=
$$\left\{ \begin{aligned} & \frac{1}{e^4} Q1 (e^{-2a4[t]} + 2a4[t]) \\ & (3xyZ[1][z, t] + MxyZ[3][z, t] + xyZ[0]^{(0,1)}[z, t] + 6\tan[z]xyZ[1]^{(1,0)}[z, t]), \\ & \frac{1}{e^4} (e^{-2a4[t]} Q1 + 2(-4+Q1)a4[t]) (-Q1xyZ[1][z, t]a4'[t] + e^{2a4[t]} (3xyZ[0][z, t] + MxyZ[2][z, t] + Q1xyZ[1][z, t]a4'[t] + xyZ[1]^{(0,1)}[z, t] + 6\tan[z]xyZ[0]^{(1,0)}[z, t])), \\ & -\frac{1}{e^4} (e^{-2a4[t]} Q1 + 2(-4+Q1)a4[t]) (Q1xyZ[2][z, t]a4'[t] + e^{2a4[t]} (MxyZ[1][z, t] + 3xyZ[3][z, t] - Q1xyZ[2][z, t]a4'[t] - xyZ[2]^{(0,1)}[z, t] + 6\tan[z]xyZ[3]^{(1,0)}[z, t])), \\ & \frac{1}{e^4} Q1 (e^{-2a4[t]} + 2a4[t]) (-MxyZ[0][z, t] - 3xyZ[2][z, t] + xyZ[3]^{(0,1)}[z, t] - 6\tan[z]xyZ[2]^{(1,0)}[z, t]), e^{\frac{1}{4} Q1 (e^{-2a4[t]} + 2a4[t])} \\ & (-3xyZ[5][z, t] - MxyZ[7][z, t] + xyZ[4]^{(0,1)}[z, t] - 6\tan[z]xyZ[5]^{(1,0)}[z, t]), \\ & -\frac{1}{e^4} (e^{-2a4[t]} Q1 + 2(-4+Q1)a4[t]) (Q1xyZ[5][z, t]a4'[t] + e^{2a4[t]} (3xyZ[4][z, t] + MxyZ[6][z, t] - Q1xyZ[5][z, t]a4'[t] - xyZ[5]^{(0,1)}[z, t] + 6\tan[z]xyZ[4]^{(1,0)}[z, t])), \\ & \frac{1}{e^4} (e^{-2a4[t]} Q1 + 2(-4+Q1)a4[t]) (-Q1xyZ[6][z, t]a4'[t] + e^{2a4[t]} (MxyZ[5][z, t] + 3xyZ[7][z, t] + Q1xyZ[6][z, t]a4'[t] + xyZ[6]^{(0,1)}[z, t] + 6\tan[z]xyZ[7]^{(1,0)}[z, t])), \\ & \frac{1}{e^4} Q1 (e^{-2a4[t]} + 2a4[t]) (MxyZ[4][z, t] + 3xyZ[6][z, t] + xyZ[7]^{(0,1)}[z, t] + 6\tan[z]xyZ[6]^{(1,0)}[z, t]) \end{aligned} \right\}$$

```

••• **Solve:** Equations may not give solutions for all "solve" variables. ⓘ

```
Out[8]=
$$\begin{aligned} xyZ[0]^{(0,1)}[z, t] &\rightarrow -3xyZ[1][z, t] - MxyZ[3][z, t] - 6\tan[z]xyZ[1]^{(1,0)}[z, t] \\ xyZ[1]^{(0,1)}[z, t] &\rightarrow -3xyZ[0][z, t] - MxyZ[2][z, t] + (-1 + e^{-2a4[t]}) Q1xyZ[1][z, t]a4'[t] - 6\tan[z]xyZ[0]^{(1,0)}[z, t] \\ xyZ[2]^{(0,1)}[z, t] &\rightarrow MxyZ[1][z, t] + 3xyZ[3][z, t] + (-1 + e^{-2a4[t]}) Q1xyZ[2][z, t]a4'[t] + 6\tan[z]xyZ[3]^{(1,0)}[z, t] \\ xyZ[3]^{(0,1)}[z, t] &\rightarrow MxyZ[0][z, t] + 3xyZ[2][z, t] + 6\tan[z]xyZ[2]^{(1,0)}[z, t] \\ xyZ[4]^{(0,1)}[z, t] &\rightarrow 3xyZ[5][z, t] + MxyZ[7][z, t] + 6\tan[z]xyZ[5]^{(1,0)}[z, t] \\ xyZ[5]^{(0,1)}[z, t] &\rightarrow 3xyZ[4][z, t] + MxyZ[6][z, t] + (-1 + e^{-2a4[t]}) Q1xyZ[5][z, t]a4'[t] + 6\tan[z]xyZ[4]^{(1,0)}[z, t] \\ xyZ[6]^{(0,1)}[z, t] &\rightarrow -MxyZ[5][z, t] - 3xyZ[7][z, t] + (-1 + e^{-2a4[t]}) Q1xyZ[6][z, t]a4'[t] - 6\tan[z]xyZ[7]^{(1,0)}[z, t] \\ xyZ[7]^{(0,1)}[z, t] &\rightarrow -MxyZ[4][z, t] - 3(xyZ[6][z, t] + 2\tan[z]xyZ[6]^{(1,0)}[z, t]) \end{aligned}$$

```

```

In[1]:= (coupledYZEquations1and2a = DtXYZsubsa /. {Rule -> Equal} // Flatten) // Column
Out[1]=
xyz[0]^(0,1)[z, t] == -3 xyz[1][z, t] - M xyz[3][z, t] - 6 Tan[z] xyz[1]^(1,0)[z, t]
xyz[1]^(0,1)[z, t] == -3 xyz[0][z, t] - M xyz[2][z, t] +
 (-1 + e^-2 a4[t]) Q1 xyz[1][z, t] a4'[t] - 6 Tan[z] xyz[0]^(1,0)[z, t]
xyz[2]^(0,1)[z, t] == M xyz[1][z, t] + 3 xyz[3][z, t] +
 (-1 + e^-2 a4[t]) Q1 xyz[2][z, t] a4'[t] + 6 Tan[z] xyz[3]^(1,0)[z, t]
xyz[3]^(0,1)[z, t] == M xyz[0][z, t] + 3 xyz[2][z, t] + 6 Tan[z] xyz[2]^(1,0)[z, t]
xyz[4]^(0,1)[z, t] == 3 xyz[5][z, t] + M xyz[7][z, t] + 6 Tan[z] xyz[5]^(1,0)[z, t]
xyz[5]^(0,1)[z, t] == 3 xyz[4][z, t] + M xyz[6][z, t] +
 (-1 + e^-2 a4[t]) Q1 xyz[5][z, t] a4'[t] + 6 Tan[z] xyz[4]^(1,0)[z, t]
xyz[6]^(0,1)[z, t] == -M xyz[5][z, t] - 3 xyz[7][z, t] +
 (-1 + e^-2 a4[t]) Q1 xyz[6][z, t] a4'[t] - 6 Tan[z] xyz[7]^(1,0)[z, t]
xyz[7]^(0,1)[z, t] == -M xyz[4][z, t] - 3 (xyz[6][z, t] + 2 Tan[z] xyz[6]^(1,0)[z, t])

In[2]:= DSolve[Take[coupledYZEquations1and2a, 4],
{xyz[0][z, t], xyz[1][z, t], xyz[2][z, t], xyz[3][z, t]}, {z, t}]
Out[2]=
DSolve[{xyz[0]^(0,1)[z, t] == -3 xyz[1][z, t] - M xyz[3][z, t] - 6 Tan[z] xyz[1]^(1,0)[z, t],
xyz[1]^(0,1)[z, t] == -3 xyz[0][z, t] - M xyz[2][z, t] + (-1 + e^-2 a4[t]) Q1 xyz[1][z, t] a4'[t] -
 6 Tan[z] xyz[0]^(1,0)[z, t], xyz[2]^(0,1)[z, t] == M xyz[1][z, t] + 3 xyz[3][z, t] +
 (-1 + e^-2 a4[t]) Q1 xyz[2][z, t] a4'[t] + 6 Tan[z] xyz[3]^(1,0)[z, t],
xyz[3]^(0,1)[z, t] == M xyz[0][z, t] + 3 xyz[2][z, t] + 6 Tan[z] xyz[2]^(1,0)[z, t}],
{xyz[0][z, t], xyz[1][z, t], xyz[2][z, t], xyz[3][z, t]}, {z, t}]

In[3]:= DSolve[Take[coupledYZEquations1and2a, -4],
{xyz[4][z, t], xyz[5][z, t], xyz[6][z, t], xyz[7][z, t]}, {z, t}]
Out[3]=
DSolve[{xyz[4]^(0,1)[z, t] == 3 xyz[5][z, t] + M xyz[7][z, t] + 6 Tan[z] xyz[5]^(1,0)[z, t],
xyz[5]^(0,1)[z, t] == 3 xyz[4][z, t] + M xyz[6][z, t] + (-1 + e^-2 a4[t]) Q1 xyz[5][z, t] a4'[t] +
 6 Tan[z] xyz[4]^(1,0)[z, t], xyz[6]^(0,1)[z, t] == -M xyz[5][z, t] - 3 xyz[7][z, t] +
 (-1 + e^-2 a4[t]) Q1 xyz[6][z, t] a4'[t] - 6 Tan[z] xyz[7]^(1,0)[z, t],
xyz[7]^(0,1)[z, t] == -M xyz[4][z, t] - 3 (xyz[6][z, t] + 2 Tan[z] xyz[6]^(1,0)[z, t])},
{xyz[4][z, t], xyz[5][z, t], xyz[6][z, t], xyz[7][z, t]}, {z, t}]

```

TRY MAPLE:

```
In[0]:= coupledyZEquions1and2a
Out[0]=
{xyz[0]^(0,1)[z,t] == -3 xyz[1][z,t] - M xyz[3][z,t] - 6 Tan[z] xyz[1]^(1,0)[z,t],
 xyz[1]^(0,1)[z,t] == -3 xyz[0][z,t] - M xyz[2][z,t] + (-1 + e^-2 a4[t]) Q1 xyz[1][z,t] a4'[t] -
 6 Tan[z] xyz[0]^(1,0)[z,t], xyz[2]^(0,1)[z,t] == M xyz[1][z,t] + 3 xyz[3][z,t] +
 (-1 + e^-2 a4[t]) Q1 xyz[2][z,t] a4'[t] + 6 Tan[z] xyz[3]^(1,0)[z,t],
 xyz[3]^(0,1)[z,t] == M xyz[0][z,t] + 3 xyz[2][z,t] + 6 Tan[z] xyz[2]^(1,0)[z,t],
 xyz[4]^(0,1)[z,t] == 3 xyz[5][z,t] + M xyz[7][z,t] + 6 Tan[z] xyz[5]^(1,0)[z,t],
 xyz[5]^(0,1)[z,t] == 3 xyz[4][z,t] + M xyz[6][z,t] + (-1 + e^-2 a4[t]) Q1 xyz[5][z,t] a4'[t] +
 6 Tan[z] xyz[4]^(1,0)[z,t], xyz[6]^(0,1)[z,t] == -M xyz[5][z,t] - 3 xyz[7][z,t] +
 (-1 + e^-2 a4[t]) Q1 xyz[6][z,t] a4'[t] - 6 Tan[z] xyz[7]^(1,0)[z,t],
 xyz[7]^(0,1)[z,t] == -M xyz[4][z,t] - 3 (xyz[6][z,t] + 2 Tan[z] xyz[6]^(1,0)[z,t])}

In[0]:= sxyzzt =
xyz[#] → ToExpression["((xyz" <> ToString[#] <> "[#1,#2])&]" & /@ Range[0, 15]
Out[0]=
{xyz[0] → (xyz0[#1, #2] &), xyz[1] → (xyz1[#1, #2] &),
 xyz[2] → (xyz2[#1, #2] &), xyz[3] → (xyz3[#1, #2] &), xyz[4] → (xyz4[#1, #2] &),
 xyz[5] → (xyz5[#1, #2] &), xyz[6] → (xyz6[#1, #2] &), xyz[7] → (xyz7[#1, #2] &),
 xyz[8] → (xyz8[#1, #2] &), xyz[9] → (xyz9[#1, #2] &), xyz[10] → (xyz10[#1, #2] &),
 xyz[11] → (xyz11[#1, #2] &), xyz[12] → (xyz12[#1, #2] &), xyz[13] → (xyz13[#1, #2] &),
 xyz[14] → (xyz14[#1, #2] &), xyz[15] → (xyz15[#1, #2] &)}
```

```

In[]:= maplexyZeqs = coupledyZEquions1and2a[[#]] /. sxyZzt & /@ Range[8];
FullForm[maplexyZeqs[[1 ;; 4]] /. sxyZzt]
FullForm[maplexyZeqs[[5 ;; 8]] /. sxyZzt]

Out[//FullForm=
List[Equal[Derivative[0, 1][xyZ0][z, t], Plus[Times[-3, xyZ1[z, t]],
Times[-1, M, xyZ3[z, t]], Times[-6, Tan[z], Derivative[1, 0][xyZ1][z, t]]]],
Equal[Derivative[0, 1][xyZ1][z, t], Plus[Times[-3, xyZ0[z, t]], Times[-1, M, xyZ2[z, t]],
Times[Plus[-1, Power[E, Times[-2, a4[t]]]], Q1, xyZ1[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][xyZ0][z, t]]]],
Equal[Derivative[0, 1][xyZ2][z, t], Plus[Times[M, xyZ1[z, t]], Times[3, xyZ3[z, t]],
Times[Plus[-1, Power[E, Times[-2, a4[t]]]], Q1, xyZ2[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][xyZ3][z, t]]]],
Equal[Derivative[0, 1][xyZ3][z, t], Plus[Times[M, xyZ0[z, t]],
Times[3, xyZ2[z, t]], Times[6, Tan[z], Derivative[1, 0][xyZ2][z, t]]]]]

Out[//FullForm=
List[Equal[Derivative[0, 1][xyZ4][z, t], Plus[Times[3, xyZ5[z, t]],
Times[M, xyZ7[z, t]], Times[6, Tan[z], Derivative[1, 0][xyZ5][z, t]]]],
Equal[Derivative[0, 1][xyZ5][z, t], Plus[Times[3, xyZ4[z, t]], Times[M, xyZ6[z, t]],
Times[Plus[-1, Power[E, Times[-2, a4[t]]]], Q1, xyZ5[z, t], Derivative[1][a4][t]],
Times[6, Tan[z], Derivative[1, 0][xyZ4][z, t]]]],
Equal[Derivative[0, 1][xyZ6][z, t], Plus[Times[-1, M, xyZ5[z, t]], Times[-3, xyZ7[z, t]],
Times[Plus[-1, Power[E, Times[-2, a4[t]]]], Q1, xyZ6[z, t], Derivative[1][a4][t]],
Times[-6, Tan[z], Derivative[1, 0][xyZ7][z, t]]]],
Equal[Derivative[0, 1][xyZ7][z, t], Plus[Times[-1, M, xyZ4[z, t]],
Times[-3, Plus[xyZ6[z, t], Times[2, Tan[z], Derivative[1, 0][xyZ6][z, t]]]]]]]

In[]:= maplexyZvars =
FullForm[xyZ[[#][z, t] & /@ Range[4*(# - 1), 4*(# - 1) + 3] /. sxyZzt] & /@ Range[2]
Out[=]
{List[xyZ0[z, t], xyZ1[z, t], xyZ2[z, t], xyZ3[z, t]],
List[xyZ4[z, t], xyZ5[z, t], xyZ6[z, t], xyZ7[z, t]]}

```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

In[8]:= **coupledyZeqs1and2**

Out[8]=

$$\left\{ \begin{aligned} xyZ[0]^{(0,1)}[z, t] &= -3 xyZ[1][z, t] - M xyZ[3][z, t] + \\ &\quad e^{-a4[t]} Q1 \cosh[a4[t]] xyZ[0][z, t] a4'[t] - 6 \tan[z] xyZ[1]^{(1,0)}[z, t], \\ xyZ[1]^{(0,1)}[z, t] &= -3 xyZ[0][z, t] - M xyZ[2][z, t] - \\ &\quad \frac{1}{2} e^{-2 a4[t]} (-3 + e^{2 a4[t]}) Q1 xyZ[1][z, t] a4'[t] - 6 \tan[z] xyZ[0]^{(1,0)}[z, t], \\ xyZ[2]^{(0,1)}[z, t] &= M xyZ[1][z, t] + 3 xyZ[3][z, t] - \\ &\quad \frac{1}{2} e^{-2 a4[t]} (-3 + e^{2 a4[t]}) Q1 xyZ[2][z, t] a4'[t] + 6 \tan[z] xyZ[3]^{(1,0)}[z, t], \\ xyZ[3]^{(0,1)}[z, t] &= M xyZ[0][z, t] + 3 xyZ[2][z, t] + \\ &\quad e^{-a4[t]} Q1 \cosh[a4[t]] xyZ[3][z, t] a4'[t] + 6 \tan[z] xyZ[2]^{(1,0)}[z, t], \\ xyZ[4]^{(0,1)}[z, t] &= 3 xyZ[5][z, t] + M xyZ[7][z, t] + e^{-a4[t]} Q1 \cosh[a4[t]] xyZ[4][z, t] a4'[t] + \\ &\quad 6 \tan[z] xyZ[5]^{(1,0)}[z, t], xyZ[5]^{(0,1)}[z, t] = 3 xyZ[4][z, t] + M xyZ[6][z, t] - \\ &\quad \frac{1}{2} e^{-2 a4[t]} (-3 + e^{2 a4[t]}) Q1 xyZ[5][z, t] a4'[t] + 6 \tan[z] xyZ[4]^{(1,0)}[z, t], \\ xyZ[6]^{(0,1)}[z, t] &= -M xyZ[5][z, t] - 3 xyZ[7][z, t] - \\ &\quad \frac{1}{2} e^{-2 a4[t]} (-3 + e^{2 a4[t]}) Q1 xyZ[6][z, t] a4'[t] - 6 \tan[z] xyZ[7]^{(1,0)}[z, t], \\ xyZ[7]^{(0,1)}[z, t] &= -M xyZ[4][z, t] - 3 xyZ[6][z, t] + \\ &\quad e^{-a4[t]} Q1 \cosh[a4[t]] xyZ[7][z, t] a4'[t] - 6 \tan[z] xyZ[6]^{(1,0)}[z, t] \end{aligned} \right\}$$

In[9]:= **sxyZzt** =

$xyZ[\#] \rightarrow \text{ToExpression}[ "((xyZ" \<> \text{ToString}[\#] \<> "[\#1,\#2])\&)" ] \& /@ \text{Range}[0, 15]$

Out[9]=

$$\{ \begin{aligned} xyZ[0] &\rightarrow (xyZ0[\#1, \#2] \&), xyZ[1] \rightarrow (xyZ1[\#1, \#2] \&), \\ xyZ[2] &\rightarrow (xyZ2[\#1, \#2] \&), xyZ[3] \rightarrow (xyZ3[\#1, \#2] \&), xyZ[4] \rightarrow (xyZ4[\#1, \#2] \&), \\ xyZ[5] &\rightarrow (xyZ5[\#1, \#2] \&), xyZ[6] \rightarrow (xyZ6[\#1, \#2] \&), xyZ[7] \rightarrow (xyZ7[\#1, \#2] \&), \\ xyZ[8] &\rightarrow (xyZ8[\#1, \#2] \&), xyZ[9] \rightarrow (xyZ9[\#1, \#2] \&), xyZ[10] \rightarrow (xyZ10[\#1, \#2] \&), \\ xyZ[11] &\rightarrow (xyZ11[\#1, \#2] \&), xyZ[12] \rightarrow (xyZ12[\#1, \#2] \&), xyZ[13] \rightarrow (xyZ13[\#1, \#2] \&), \\ xyZ[14] &\rightarrow (xyZ14[\#1, \#2] \&), xyZ[15] \rightarrow (xyZ15[\#1, \#2] \&) \end{aligned} \}$$

```

In[®]:= coupledyZeqs1and2;
maplexyZeq1s = FullForm[Take[%, 4] /. sxyZzt]
maplexyZeq2s = FullForm[Take[%%, -4] /. sxyZzt]

Out[®]//FullForm=
List[
 Equal[Derivative[0, 1][xyZ0][z, t], Plus[Times[-3, xyZ1[z, t]], Times[-1, M, xyZ3[z, t]]],
 Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ0[z, t], Derivative[1][a4][t]],
 Times[-6, Tan[z], Derivative[1, 0][xyZ1][z, t]]]],
 Equal[Derivative[0, 1][xyZ1][z, t], Plus[Times[-3, xyZ0[z, t]]],
 Times[-1, M, xyZ2[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]],
 Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ1[z, t], Derivative[1][a4][t]],
 Times[-6, Tan[z], Derivative[1, 0][xyZ0][z, t]]]],
 Equal[Derivative[0, 1][xyZ2][z, t], Plus[Times[M, xyZ1[z, t]],
 Times[3, xyZ3[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]],
 Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ2[z, t], Derivative[1][a4][t]],
 Times[6, Tan[z], Derivative[1, 0][xyZ3][z, t]]]],
 Equal[Derivative[0, 1][xyZ3][z, t], Plus[Times[M, xyZ0[z, t]], Times[3, xyZ2[z, t]]],
 Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ3[z, t], Derivative[1][a4][t]],
 Times[6, Tan[z], Derivative[1, 0][xyZ2][z, t]]]]

Out[®]//FullForm=
List[Equal[Derivative[0, 1][xyZ4][z, t], Plus[Times[3, xyZ5[z, t]], Times[M, xyZ7[z, t]]],
 Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ4[z, t], Derivative[1][a4][t]],
 Times[6, Tan[z], Derivative[1, 0][xyZ5][z, t]]]],
 Equal[Derivative[0, 1][xyZ5][z, t], Plus[Times[3, xyZ4[z, t]],
 Times[M, xyZ6[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]],
 Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ5[z, t], Derivative[1][a4][t]],
 Times[6, Tan[z], Derivative[1, 0][xyZ4][z, t]]]],
 Equal[Derivative[0, 1][xyZ6][z, t], Plus[Times[-1, M, xyZ5[z, t]],
 Times[-3, xyZ7[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]],
 Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ6[z, t], Derivative[1][a4][t]],
 Times[-6, Tan[z], Derivative[1, 0][xyZ7][z, t]]]],
 Equal[Derivative[0, 1][xyZ7][z, t], Plus[Times[-1, M, xyZ4[z, t]], Times[-3, xyZ6[z, t]]],
 Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ7[z, t], Derivative[1][a4][t]],
 Times[-6, Tan[z], Derivative[1, 0][xyZ6][z, t]]]]

```

```

In[8]:= maplexyZeqs = FullForm[coupledxyZeqs1and2[[#]] /. sxyZzt] & /@ Range[8]

Out[8]= {Equal[Derivative[0, 1][xyZ0][z, t], Plus[Times[-3, xyZ1[z, t]], Times[-1, M, xyZ3[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ0[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][xyZ1][z, t]]]], Equal[Derivative[0, 1][xyZ1][z, t], Plus[Times[-3, xyZ0[z, t]], Times[-1, M, xyZ2[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]], Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ1[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][xyZ0][z, t]]]], Equal[Derivative[0, 1][xyZ2][z, t], Plus[Times[M, xyZ1[z, t]], Times[3, xyZ3[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]], Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ2[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][xyZ3][z, t]]]], Equal[Derivative[0, 1][xyZ3][z, t], Plus[Times[M, xyZ0[z, t]], Times[3, xyZ2[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ3[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][xyZ2][z, t]]]], Equal[Derivative[0, 1][xyZ4][z, t], Plus[Times[3, xyZ5[z, t]], Times[M, xyZ7[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ4[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][xyZ5][z, t]]]], Equal[Derivative[0, 1][xyZ5][z, t], Plus[Times[3, xyZ4[z, t]], Times[M, xyZ6[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]], Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ5[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][xyZ4][z, t]]]], Equal[Derivative[0, 1][xyZ6][z, t], Plus[Times[-1, M, xyZ5[z, t]], Times[-3, xyZ7[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]], Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ6[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][xyZ7][z, t]]]], Equal[Derivative[0, 1][xyZ7][z, t], Plus[Times[-1, M, xyZ4[z, t]], Times[-3, xyZ6[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ7[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][xyZ6][z, t]]]]}

```

```
In[•]:= maplexyZeq1s =
 FullForm[Thread[0 == Take[coupledxyZeqs1and2, 4] [[#]] /. sxyZzt] & /@ Range[4]]
Out[•]//FullForm=
List[Equal[0,
 Equal[Derivative[0, 1][xyZ0][z, t], Plus[Times[-3, xyZ1[z, t]], Times[-1, M, xyZ3[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ0[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][xyZ1][z, t]]]],
 Equal[0, Equal[Derivative[0, 1][xyZ1][z, t], Plus[Times[-3, xyZ0[z, t]], Times[-1, M, xyZ2[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]], Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ1[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][xyZ0][z, t]]]],
 Equal[0, Equal[Derivative[0, 1][xyZ2][z, t], Plus[Times[M, xyZ1[z, t]], Times[3, xyZ3[z, t]], Times[Rational[-1, 2], Power[E, Times[-2, a4[t]]]], Plus[-3, Power[E, Times[2, a4[t]]]], Q1, xyZ2[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][xyZ3][z, t]]]], Equal[0,
 Equal[Derivative[0, 1][xyZ3][z, t], Plus[Times[M, xyZ0[z, t]], Times[3, xyZ2[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Cosh[a4[t]], xyZ3[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][xyZ2][z, t]]]]]
In[•]:= maplexyZvars =
 FullForm[xyZ[#][z, t] & /@ Range[4 * (# - 1), 4 * (# - 1) + 3] /. sxyZzt] & /@ Range[2]
Out[•]=
{List[xyZ0[z, t], xyZ1[z, t], xyZ2[z, t], xyZ3[z, t]],
 List[xyZ4[z, t], xyZ5[z, t], xyZ6[z, t], xyZ7[z, t]}]

“maple-output.py”:
import textwrap
filename = "maple-output.txt"
maple_long_string = "paste maple string here"
Wraps the text and joins with newlines
with open(filename, "w") as f:
 f.write("\n".join(textwrap.wrap(maple_long_string, width=80)))

In[•]:= syZzt = yZ[#] → ToExpression["((yZ" <> ToString[#] <> "[#1,#2]) &)"] & /@ Range[0, 15]
Out[•]=
{yZ[0] → (yZ0[#1, #2] &), yZ[1] → (yZ1[#1, #2] &),
 yZ[2] → (yZ2[#1, #2] &), yZ[3] → (yZ3[#1, #2] &), yZ[4] → (yZ4[#1, #2] &),
 yZ[5] → (yZ5[#1, #2] &), yZ[6] → (yZ6[#1, #2] &), yZ[7] → (yZ7[#1, #2] &),
 yZ[8] → (yZ8[#1, #2] &), yZ[9] → (yZ9[#1, #2] &), yZ[10] → (yZ10[#1, #2] &),
 yZ[11] → (yZ11[#1, #2] &), yZ[12] → (yZ12[#1, #2] &),
 yZ[13] → (yZ13[#1, #2] &), yZ[14] → (yZ14[#1, #2] &), yZ[15] → (yZ15[#1, #2] &)}
```

```

In[6]:= mapleyzeqs = FullForm[coupledyezeqs[[#]] /. syZzt] & /@ Range[4]

Out[6]= {List[
 Equal[Derivative[0, 1][yZ0][z, t], Plus[Times[-3, yZ1[z, t]], Times[-1, M, yZ3[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ0[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][yZ1][z, t]]]],
 Equal[Derivative[0, 1][yZ1][z, t], Plus[Times[-3, yZ0[z, t]], Times[-1, M, yZ2[z, t]], Times[-1, Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ1[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][yZ0][z, t]]]],
 Equal[Derivative[0, 1][yZ2][z, t], Plus[Times[M, yZ1[z, t]], Times[3, yZ3[z, t]], Times[-1, Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ2[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][yZ3][z, t]]]],
 Equal[Derivative[0, 1][yZ3][z, t], Plus[Times[M, yZ0[z, t]], Times[3, yZ2[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ3[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][yZ2][z, t]]]],
 List[Equal[Derivative[0, 1][yZ4][z, t], Plus[Times[3, yZ5[z, t]], Times[M, yZ7[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ4[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][yZ5][z, t]]]],
 Equal[Derivative[0, 1][yZ5][z, t], Plus[Times[3, yZ4[z, t]], Times[M, yZ6[z, t]], Times[-1, Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ5[z, t], Derivative[1][a4][t]], Times[6, Tan[z], Derivative[1, 0][yZ4][z, t]]]],
 Equal[Derivative[0, 1][yZ6][z, t], Plus[Times[-1, M, yZ5[z, t]], Times[-3, yZ7[z, t]], Times[-1, Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ6[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][yZ7][z, t]]]],
 Equal[Derivative[0, 1][yZ7][z, t], Plus[Times[-1, M, yZ4[z, t]], Times[-3, yZ6[z, t]], Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ7[z, t], Derivative[1][a4][t]], Times[-6, Tan[z], Derivative[1, 0][yZ6][z, t]]]],
 List[Equal[Derivative[0, 1][yZ8][z, t], Plus[Times[M, yZ11[z, t]], Times[3, yZ9[z, t]], Times[6, Tan[z], Derivative[1, 0][yZ9][z, t]]]],
 Equal[Derivative[0, 1][yZ9][z, t], Plus[Times[M, yZ10[z, t]], Times[3, yZ8[z, t]], Times[6, Tan[z], Derivative[1, 0][yZ8][z, t]]]],
 Equal[Derivative[0, 1][yZ10][z, t], Plus[Times[-1, M, yZ9[z, t]], Times[-3, Plus[yZ11[z, t], Times[2, Tan[z], Derivative[1, 0][yZ11][z, t]]]]]],
 Equal[Derivative[0, 1][yZ11][z, t], Plus[Times[-1, M, yZ8[z, t]], Times[-3, Plus[yZ10[z, t], Times[2, Tan[z], Derivative[1, 0][yZ10][z, t]]]]]],
 List[Equal[Derivative[0, 1][yZ12][z, t], Plus[Times[-3, yZ13[z, t]], Times[-1, M, yZ15[z, t]], Times[-6, Tan[z], Derivative[1, 0][yZ13][z, t]]]],
 Equal[Derivative[0, 1][yZ13][z, t], Plus[Times[-3, yZ12[z, t]], Times[-1, M, yZ14[z, t]], Times[-6, Tan[z], Derivative[1, 0][yZ12][z, t]]]],
 Equal[Derivative[0, 1][yZ14][z, t], Plus[Times[M, yZ13[z, t]], Times[3, yZ15[z, t]], Times[6, Tan[z], Derivative[1, 0][yZ15][z, t]]]],
 Equal[Derivative[0, 1][yZ15][z, t], Plus[Times[M, yZ12[z, t]], Times[3, yZ14[z, t]], Times[6, Tan[z], Derivative[1, 0][yZ14][z, t]]]]]
}

```

```
In[•]:= mapleyZvars =
 FullForm[yZ[#][z, t] & /@ Range[4 * (# - 1), 4 * (# - 1) + 3] /. syZzt] & /@ Range[4]
Out[•]= {List[yZ0[z, t], yZ1[z, t], yZ2[z, t], yZ3[z, t]],
 List[yZ4[z, t], yZ5[z, t], yZ6[z, t], yZ7[z, t]],
 List[yZ8[z, t], yZ9[z, t], yZ10[z, t], yZ11[z, t]],
 List[yZ12[z, t], yZ13[z, t], yZ14[z, t], yZ15[z, t]]}
```

FOR MAPLE : A set is an unordered sequence of distinct expressions enclosed in braces {}, representing a set in the mathematical sense.

A list is an ordered sequence of expressions enclosed in square brackets []. The ordering of the expressions is the ordering of es.

## "Solution" by Maple

Load the Maple output strings. Get is a lexer and parer and ... , so use ReadString

```
In[•]:= SetDirectory[NotebookDirectory[]];
In[•]:= (*maplestringEQ1=Hold[ToString[Get["thinkpad_stringEQ1.txt"]]];*)
In[•]:= (*maplestringEQ2=Inactivate[Get["thinkpad_stringEQ2.txt"]];*)
In[•]:= maplestringEQ1 = ReadString["thinkpad_stringEQ1.txt"];
In[•]:= maplestringEQ2 = ReadString["thinkpad_stringEQ2.txt"];
In[•]:= maplestringEQ2L9i = ReadString["stringSEQ2_2026_01_23_xyZ_2sets_L9i.txt"];
```

```
In[•]:= (*time1=Now
AbsoluteTiming[
 solvedEQ1=ConvertMapleToMathematicaV2[ReleaseHold[maplestringEQ1]]]
Now-time1
AbsoluteTiming[
 solvedEQ2=ConvertMapleToMathematicaV2[Activate[maplestringEQ2]]]
Now-time1*)
```

```
In[8]:= yZT0xyz = Flatten[{{
 yZ[0] → (xyz[0][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[1] → (xyz[1][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[2] → (xyz[2][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[3] → (xyz[3][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[4] → (xyz[4][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[5] → (xyz[5][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[6] → (xyz[6][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
 yZ[7] → (xyz[7][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &)
}, Table[yZ[j] → ToExpression["(xyz[" >> ToString[j] << "] [##1,##2]&)"], {j, 8, 15}]}]
```

```
Out[8]= {yZ[0] → (xyz[0][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
yZ[1] → (xyz[1][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
yZ[2] → (xyz[2][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
yZ[3] → (xyz[3][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
yZ[4] → (xyz[4][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
yZ[5] → (xyz[5][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
yZ[6] → (xyz[6][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &),
yZ[7] → (xyz[7][#1, #2] Exp[1/2 Q1(1/2 e^-2 a4[#2] + a4[#2])] &), yZ[8] → (xyz[8][#1, #2] &),
yZ[9] → (xyz[9][#1, #2] &), yZ[10] → (xyz[10][#1, #2] &), yZ[11] → (xyz[11][#1, #2] &),
yZ[12] → (xyz[12][#1, #2] &), yZ[13] → (xyz[13][#1, #2] &),
yZ[14] → (xyz[14][#1, #2] &), yZ[15] → (xyz[15][#1, #2] &)}
```

```

In[®]:= Ψ16;
% /. sfψ16Aa /. sx0x4;
% /. szt0yz
% /. yzt0xyz
(*Thread[Take[%,8]==Take[%%%,8]]*)
Thread[% == %%%]
Solve[And @@ %, xyz[#][z, t] & /@ Range[0, 15]] [[1]]
sxyztof16 = Thread[%[[All, 1]] → (%[[All, 2]] /. szt)]
equalxyzandf16 = sxyztof16 /. {Rule → Equal}

Out[®]=
{yZ[0][z, t], yZ[4][z, t], yZ[8][z, t], yZ[12][z, t], yZ[5][z, t],
yZ[1][z, t], yZ[13][z, t], yZ[9][z, t], yZ[2][z, t], yZ[6][z, t], yZ[10][z, t],
yZ[14][z, t], yZ[7][z, t], yZ[3][z, t], yZ[15][z, t], yZ[11][z, t]}

Out[®]=
{e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[0][z, t], e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[4][z, t], xyz[8][z, t], xyz[12][z, t],
e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[5][z, t], e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[1][z, t], xyz[13][z, t], xyz[9][z, t],
e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[2][z, t], e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[6][z, t], xyz[10][z, t], xyz[14][z, t],
e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[7][z, t], e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[3][z, t], xyz[15][z, t], xyz[11][z, t]}

Out[®]=
{e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[0][z, t] == f16[0][x0, x4],
e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[4][z, t] == f16[1][x0, x4], xyz[8][z, t] == f16[2][x0, x4],
xyz[12][z, t] == f16[3][x0, x4], e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[5][z, t] == f16[4][x0, x4],
e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[1][z, t] == f16[5][x0, x4], xyz[13][z, t] == f16[6][x0, x4],
xyz[9][z, t] == f16[7][x0, x4], e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[2][z, t] == f16[8][x0, x4],
e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[6][z, t] == f16[9][x0, x4], xyz[10][z, t] == f16[10][x0, x4],
xyz[14][z, t] == f16[11][x0, x4], e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[7][z, t] == f16[12][x0, x4],
e^(1/2) Q1((1/2 e^-2 a4[t]+a4[t])) xyz[3][z, t] == f16[13][x0, x4],
xyz[15][z, t] == f16[14][x0, x4], xyz[11][z, t] == f16[15][x0, x4]}

```

Out[•]=

$$\left\{ \begin{aligned} \text{xyz}[0][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[t]} + a4[t] \right)} f16[0][x0, x4], \\ \text{xyz}[1][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[t]} + a4[t] \right)} f16[5][x0, x4], \\ \text{xyz}[2][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[t]} + a4[t] \right)} f16[8][x0, x4], \\ \text{xyz}[3][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[t]} + a4[t] \right)} f16[13][x0, x4], \\ \text{xyz}[4][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[t]} + a4[t] \right)} f16[1][x0, x4], \\ \text{xyz}[5][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[t]} + a4[t] \right)} f16[4][x0, x4], \\ \text{xyz}[6][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[t]} + a4[t] \right)} f16[9][x0, x4], \\ \text{xyz}[7][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[t]} + a4[t] \right)} f16[12][x0, x4], \\ \text{xyz}[8][z, t] &\rightarrow f16[2][x0, x4], \text{xyz}[9][z, t] \rightarrow f16[7][x0, x4], \\ \text{xyz}[10][z, t] &\rightarrow f16[10][x0, x4], \text{xyz}[11][z, t] \rightarrow f16[15][x0, x4], \\ \text{xyz}[12][z, t] &\rightarrow f16[3][x0, x4], \text{xyz}[13][z, t] \rightarrow f16[6][x0, x4], \\ \text{xyz}[14][z, t] &\rightarrow f16[11][x0, x4], \text{xyz}[15][z, t] \rightarrow f16[14][x0, x4] \end{aligned} \right\}$$

Out[•]=

$$\left\{ \begin{aligned} \text{xyz}[0][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[H x4]} + a4[H x4] \right)} f16[0][x0, x4], \\ \text{xyz}[1][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[H x4]} + a4[H x4] \right)} f16[5][x0, x4], \\ \text{xyz}[2][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[H x4]} + a4[H x4] \right)} f16[8][x0, x4], \\ \text{xyz}[3][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[H x4]} + a4[H x4] \right)} f16[13][x0, x4], \\ \text{xyz}[4][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[H x4]} + a4[H x4] \right)} f16[1][x0, x4], \\ \text{xyz}[5][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[H x4]} + a4[H x4] \right)} f16[4][x0, x4], \\ \text{xyz}[6][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[H x4]} + a4[H x4] \right)} f16[9][x0, x4], \\ \text{xyz}[7][z, t] &\rightarrow e^{-\frac{1}{2} Q1 \left( \frac{1}{2} e^{-2 a4[H x4]} + a4[H x4] \right)} f16[12][x0, x4], \\ \text{xyz}[8][z, t] &\rightarrow f16[2][x0, x4], \text{xyz}[9][z, t] \rightarrow f16[7][x0, x4], \\ \text{xyz}[10][z, t] &\rightarrow f16[10][x0, x4], \text{xyz}[11][z, t] \rightarrow f16[15][x0, x4], \\ \text{xyz}[12][z, t] &\rightarrow f16[3][x0, x4], \text{xyz}[13][z, t] \rightarrow f16[6][x0, x4], \\ \text{xyz}[14][z, t] &\rightarrow f16[11][x0, x4], \text{xyz}[15][z, t] \rightarrow f16[14][x0, x4] \end{aligned} \right\}$$

```
Out[0]=
{xyz[0][z, t] == E-1/2 Q1((1/2 e-2 a4[H x4] + a4[H x4]) f16[0][x0, x4],

 xyz[1][z, t] == E-1/2 Q1((1/2 e-2 a4[H x4] + a4[H x4]) f16[5][x0, x4],

 xyz[2][z, t] == E-1/2 Q1((1/2 e-2 a4[H x4] + a4[H x4]) f16[8][x0, x4],

 xyz[3][z, t] == E-1/2 Q1((1/2 e-2 a4[H x4] + a4[H x4]) f16[13][x0, x4],

 xyz[4][z, t] == E-1/2 Q1((1/2 e-2 a4[H x4] + a4[H x4]) f16[1][x0, x4],

 xyz[5][z, t] == E-1/2 Q1((1/2 e-2 a4[H x4] + a4[H x4]) f16[4][x0, x4],

 xyz[6][z, t] == E-1/2 Q1((1/2 e-2 a4[H x4] + a4[H x4]) f16[9][x0, x4],

 xyz[7][z, t] == E-1/2 Q1((1/2 e-2 a4[H x4] + a4[H x4]) f16[12][x0, x4],

 xyz[8][z, t] == f16[2][x0, x4], xyz[9][z, t] == f16[7][x0, x4],

 xyz[10][z, t] == f16[10][x0, x4], xyz[11][z, t] == f16[15][x0, x4],

 xyz[12][z, t] == f16[3][x0, x4], xyz[13][z, t] == f16[6][x0, x4],

 xyz[14][z, t] == f16[11][x0, x4], xyz[15][z, t] == f16[14][x0, x4]}
```

### FindFile[name]

finds the file with the specified name that would be loaded by Get[name] and related functions.

```
In[0]:= Print[ToString[header <> "solvedEQ1.mx"]]
2026-01-30-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-L9i-solvedEQ1.mx

In[0]:= Print[ToString[header <> "solvedEQ2.mx"]]
2026-01-30-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-L9i-solvedEQ2.mx

In[0]:= Print[ToString[header <> "solvedEQ2L9i.mx"]]
2026-01-30-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-L9i-solvedEQ2L9i.mx
```

```
time1 = Now
If[FindFile[
 "2026-01-29-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-mmM4pro-solvedEQ1.mx"]
 === $Failed,
 AbsoluteTiming[solvedEQ1 = ConvertMapleToMathematicaV2[maplestringEQ1]];
 DumpSave[ToString[header <> "solvedEQ1.mx"], solvedEQ1];
 Print[ToString[header <> "solvedEQ1.mx"]];
 , Get[
 "2026-01-29-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-mmM4pro-solvedEQ1.mx"]
];
Now - time1
```

```

time1 = Now
If[FindFile[
 "2026-01-29-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-mmM4pro-solvedEQ2L9i.mx"
] === $Failed,
 AbsoluteTiming[solvedEQ2L9i = ConvertMapleToMathematicaV2[maplestringEQ2L9i]];
 DumpSave[ToString[header <> "solvedEQ2L9i.mx"], solvedEQ2L9i];
 Print[ToString[header <> "solvedEQ2L9i.mx"]];
 , Get[
 "2026-01-29-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-mmM4pro-solvedEQ2L9i.mx"
]
 Now - time1

```

27.45017720 min

```

In[②]:= time1 = Now
If[FindFile[
 "2026-01-29-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-mmM4pro-solvedEQ2.mx"
] === $Failed,
 AbsoluteTiming[solvedEQ2 = ConvertMapleToMathematicaV2[maplestringEQ2]];
 DumpSave[ToString[header <> "solvedEQ2.mx"], solvedEQ2];
 Print[ToString[header <> "solvedEQ2.mx"]];
 , Get[
 "2026-01-29-4sets-of-4-eqs-4+4-Einstein-Lovelock-Nash-mmM4pro-solvedEQ2.mx"]
 Now - time1

```

Out[②]=

Fri 30 Jan 2026 06:26:57 GMT-8

Out[③]=

0.0160550 s

27.47444487 min

too long to load:

solvedEQ2[[1]]

solvedEQ2L9i[[1]]

Out[④]=

\$Aborted

Out[⑤]=

\$Aborted

```
In[8]:= maplestringEQ3 =
" {yZ10(z, t) = (c37*sin(sqrt(M^2 - 36*C3 - 9)*t) + c38*cos(sqrt(M^2 - 36*C3 - 9)*t))*sin(z)^(-sqrt(1 + 4*C3)/2)*c36 + sin(z)^(sqrt(1 + 4*C3)/2)*sqrt(sin(z)), yZ11(z, t) = (c33*sin(sqrt(M^2 - 36*C3 - 9)*t))*sin(z)^(-sqrt(1 + 4*C3)/2)*c35 + c34*cos(sqrt(M^2 - 36*C3 - 9)*t))*sin(z)^(-sqrt(1 + 4*C3)/2)*c32 + sin(z)^(sqrt(1 + 4*C3)/2)*c31)/sqrt(sin(z)), yZ8(z, t) = -3*(sqrt(M^2 - 36*C3 - 9)*(sin(z)^(-sqrt(1 + 4*C3)/2)*c32 + sin(z)^(sqrt(1 + 4*C3)/2)*c31)*cos(sqrt(M^2 - 36*C3 - 9)*t)*c33 - sin(sqrt(M^2 - 36*C3 - 9)*t)*c34)/3 + sqrt(1 + 4*C3)*(c37*sin(sqrt(M^2 - 36*C3 - 9)*t) + c38*cos(sqrt(M^2 - 36*C3 - 9)*t))*(-sin(z)^(-sqrt(1 + 4*C3)/2)*c36 + sin(z)^(sqrt(1 + 4*C3)/2)*c35))/(M*sqrt(sin(z))), yZ9(z, t) = -(sqrt(M^2 - 36*C3 - 9)*(sin(z)^(-sqrt(1 + 4*C3)/2)*c36 + sin(z)^(sqrt(1 + 4*C3)/2)*c35)*cos(sqrt(M^2 - 36*C3 - 9)*t)*c37 - sin(sqrt(M^2 - 36*C3 - 9)*t)*c38) + 3*sqrt(1 + 4*C3)*(c33*sin(sqrt(M^2 - 36*C3 - 9)*t) + c34*cos(sqrt(M^2 - 36*C3 - 9)*t))*(-sin(z)^(-sqrt(1 + 4*C3)/2)*c32 + sin(z)^(sqrt(1 + 4*C3)/2)*c31))/(M*sqrt(sin(z))) }";
In[9]:= maplestringEQ4 =
" {yZ12(z, t) = -3*(-((sin(z)^(-sqrt(1 + 4*C4)/2)*c42 + sin(z)^(sqrt(1 + 4*C4)/2)*c41)*(-sin(sqrt(M^2 - 36*C4 - 9)*t)*c44 + cos(sqrt(M^2 - 36*C4 - 9)*t)*c43)*sqrt(M^2 - 36*C4 - 9))/3 + sqrt(1 + 4*C4)*(c47*sin(sqrt(M^2 - 36*C4 - 9)*t) + c48*cos(sqrt(M^2 - 36*C4 - 9)*t))*(-sin(z)^(-sqrt(1 + 4*C4)/2)*c46 + sin(z)^(sqrt(1 + 4*C4)/2)*c45))/(sqrt(sin(z))*M), yZ13(z, t) = ((sin(z)^(-sqrt(1 + 4*C4)/2)*c46 + sin(z)^(sqrt(1 + 4*C4)/2)*c45)*(-sin(sqrt(M^2 - 36*C4 - 9)*t)*c48 + cos(sqrt(M^2 - 36*C4 - 9)*t)*c47)*sqrt(M^2 - 36*C4 - 9) - 3*sqrt(1 + 4*C4)*(c43*sin(sqrt(M^2 - 36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4 - 9)*t))*(-sin(z)^(-sqrt(1 + 4*C4)/2)*c42 + sin(z)^(sqrt(1 + 4*C4)/2)*c41))/(sqrt(sin(z))*M), yZ14(z, t) = (sin(z)^(-sqrt(1 + 4*C4)/2)*c46 + sin(z)^(sqrt(1 + 4*C4)/2)*c45)*(c47*sin(sqrt(M^2 - 36*C4 - 9)*t) + c48*cos(sqrt(M^2 - 36*C4 - 9)*t))/sqrt(sin(z)), yZ15(z, t) = (sin(z)^(-sqrt(1 + 4*C4)/2)*c42 + sin(z)^(sqrt(1 + 4*C4)/2)*c41)*(c43*sin(sqrt(M^2 - 36*C4 - 9)*t) + c44*cos(sqrt(M^2 - 36*C4 - 9)*t))/sqrt(sin(z))) }";
```

In[6]:= solvedEQ3 = ConvertMapleToMathematicaV2[ReleaseHold[maplestringEQ3]]

Out[6]=

$$\begin{aligned} \text{YZ10}[z, t] &= \frac{1}{\sqrt{\sin[z]}} \left( c38 \cos[\sqrt{-9 - 36 C3 + M^2} t] + c37 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right) \\ &\quad \left( c36 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C3}} + c35 \sin[z]^{\frac{1}{2} \sqrt{1+4 C3}} \right), \\ \text{YZ11}[z, t] &= \frac{1}{\sqrt{\sin[z]}} \left( c34 \cos[\sqrt{-9 - 36 C3 + M^2} t] + c33 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right) \\ &\quad \left( c32 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C3}} + c31 \sin[z]^{\frac{1}{2} \sqrt{1+4 C3}} \right), \text{YZ8}[z, t] = \\ &- \frac{1}{M \sqrt{\sin[z]}} 3 \left( \frac{1}{3} \sqrt{-9 - 36 C3 + M^2} \left( c33 \cos[\sqrt{-9 - 36 C3 + M^2} t] - c34 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right) \right. \\ &\quad \left( c32 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C3}} + c31 \sin[z]^{\frac{1}{2} \sqrt{1+4 C3}} \right) + \sqrt{1 + 4 C3} \left( c38 \cos[\sqrt{-9 - 36 C3 + M^2} t] + \right. \\ &\quad \left. \left. c37 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right) \left( -c36 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C3}} + c35 \sin[z]^{\frac{1}{2} \sqrt{1+4 C3}} \right) \right), \\ \text{YZ9}[z, t] &= \frac{1}{M \sqrt{\sin[z]}} \left( -3 \sqrt{1 + 4 C3} \left( c34 \cos[\sqrt{-9 - 36 C3 + M^2} t] + c33 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right) \right. \\ &\quad \left( -c32 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C3}} + c31 \sin[z]^{\frac{1}{2} \sqrt{1+4 C3}} \right) - \\ &\quad \sqrt{-9 - 36 C3 + M^2} \left( c37 \cos[\sqrt{-9 - 36 C3 + M^2} t] - c38 \sin[\sqrt{-9 - 36 C3 + M^2} t] \right) \\ &\quad \left. \left( c36 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C3}} + c35 \sin[z]^{\frac{1}{2} \sqrt{1+4 C3}} \right) \right) \} \end{aligned}$$

$$\left\{ \begin{aligned} yZ10(z, t) &= \frac{1}{\sqrt{\sin(z)}} \left( \left( c37 \sin(\sqrt{M^2 - 36 C3 - 9} t) \right. \right. \\ &\quad \left. \left. + c38 \cos(\sqrt{M^2 - 36 C3 - 9} t) \right) \left( \sin(z)^{-\frac{\sqrt{1+4 C3}}{2}} c36 \right. \right. \\ &\quad \left. \left. + \sin(z)^{\frac{\sqrt{1+4 C3}}{2}} c35 \right) \right), yZ11(z, t) \\ &= \frac{1}{\sqrt{\sin(z)}} \left( \left( c33 \sin(\sqrt{M^2 - 36 C3 - 9} t) \right. \right. \\ &\quad \left. \left. + c34 \cos(\sqrt{M^2 - 36 C3 - 9} t) \right) \left( \sin(z)^{-\frac{\sqrt{1+4 C3}}{2}} c32 \right. \right. \\ &\quad \left. \left. + \sin(z)^{\frac{\sqrt{1+4 C3}}{2}} c31 \right) \right) \end{aligned} \right.$$

$$\begin{aligned}
& + \sin(z)^{\frac{-1}{2}} c31 \Big) \Big), yZ8(z, t) = \\
& - \frac{1}{M\sqrt{\sin(z)}} \left( 3 \left( \frac{1}{3} \left( \sqrt{M^2 - 36C3 - 9} \right. \right. \right. \\
& \left. \left. \left. \left( \sin(z)^{-\frac{\sqrt{1+4C3}}{2}} c32 \right. \right. \right. \right. \\
& \left. \left. \left. \left. + \sin(z)^{\frac{\sqrt{1+4C3}}{2}} c31 \right) \left( \cos(\sqrt{M^2 - 36C3 - 9} t) c33 \right. \right. \right. \\
& \left. \left. \left. \left. - \sin(\sqrt{M^2 - 36C3 - 9} t) c34 \right) \right. \right. \right. \\
& \left. \left. \left. + \sqrt{1+4C3} \left( c37 \sin(\sqrt{M^2 - 36C3 - 9} t) \right. \right. \right. \\
& \left. \left. \left. + c38 \cos(\sqrt{M^2 - 36C3 - 9} t) \right) \right) \left( -\sin(z)^{-\frac{\sqrt{1+4C3}}{2}} c36 \right. \right. \\
& \left. \left. \left. + \sin(z)^{\frac{\sqrt{1+4C3}}{2}} c35 \right) \right) \Big), yZ9(z, t) = \\
& - \frac{1}{M\sqrt{\sin(z)}} \left( \sqrt{M^2 - 36C3 - 9} \left( \sin(z)^{-\frac{\sqrt{1+4C3}}{2}} c36 \right. \right. \\
& \left. \left. + \sin(z)^{\frac{\sqrt{1+4C3}}{2}} c35 \right) \left( \cos(\sqrt{M^2 - 36C3 - 9} t) c37 \right. \right. \\
& \left. \left. - \sin(\sqrt{M^2 - 36C3 - 9} t) c38 \right) \right. \\
& \left. + 3\sqrt{1+4C3} \left( c33 \sin(\sqrt{M^2 - 36C3 - 9} t) \right. \right. \\
& \left. \left. + c34 \cos(\sqrt{M^2 - 36C3 - 9} t) \right) \right) \left( -\sin(z)^{-\frac{\sqrt{1+4C3}}{2}} c32 \right. \right. \\
& \left. \left. + \sin(z)^{\frac{\sqrt{1+4C3}}{2}} c31 \right) \right) \Big)
\end{aligned}$$

In[6]:= solvedEQ4 = ConvertMapleToMathematicaV2[ReleaseHold[maplestringEQ4]]

*Out[•] =*

$$\begin{aligned} \left\{ \begin{array}{l} YZ12[z, t] = \\ -\frac{1}{M \sqrt{\sin[z]}} 3 \left( -\frac{1}{3} \sqrt{-9 - 36 C4 + M^2} \left( c43 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] - c44 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \right. \\ \left. \left( c42 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c41 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) + \sqrt{1 + 4 C4} \left( c48 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + \right. \right. \\ \left. \left. c47 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \left( -c46 \sin[z]^{-\frac{1}{2} \sqrt{1+4 C4}} + c45 \sin[z]^{\frac{1}{2} \sqrt{1+4 C4}} \right) \right), \end{array} \right. \end{aligned}$$

$$\begin{aligned} YZ13[z, t] = & \frac{1}{M \sqrt{\sin[z]}} \left( -3 \sqrt{1+4C4} \left( c44 \cos \left[ \sqrt{-9-36C4+M^2} t \right] + c43 \sin \left[ \sqrt{-9-36C4+M^2} t \right] \right) \right. \\ & \left( -c42 \sin[z]^{-\frac{1}{2}} \sqrt{1+4C4} + c41 \sin[z]^{\frac{1}{2}} \sqrt{1+4C4} \right) + \\ & \sqrt{-9-36C4+M^2} \left( c47 \cos \left[ \sqrt{-9-36C4+M^2} t \right] - c48 \sin \left[ \sqrt{-9-36C4+M^2} t \right] \right) \\ & \left. \left( c46 \sin[z]^{-\frac{1}{2}} \sqrt{1+4C4} + c45 \sin[z]^{\frac{1}{2}} \sqrt{1+4C4} \right) \right), \end{aligned}$$

$$\begin{aligned} \text{YZ14}[z, t] &= \frac{1}{\sqrt{\sin[z]}} \left( c48 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c47 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \\ &\quad \left( c46 \sin[z]^{-\frac{1}{2}} \sqrt{1+4 C4} + c45 \sin[z]^{\frac{1}{2}} \sqrt{1+4 C4} \right), \end{aligned}$$

$$\begin{aligned} \text{YZ15}[z, t] &= \frac{1}{\sqrt{\sin[z]}} \left( c44 \cos \left[ \sqrt{-9 - 36 C4 + M^2} t \right] + c43 \sin \left[ \sqrt{-9 - 36 C4 + M^2} t \right] \right) \\ &\quad \left( c42 \sin[z]^{-\frac{1}{2}} e^{\frac{\sqrt{1+4 C4}}{2} t} + c41 \sin[z]^{\frac{1}{2}} e^{\frac{\sqrt{1+4 C4}}{2} t} \right) \end{aligned}$$

$$\begin{aligned} yZI2(z, t) = & -\frac{1}{\sqrt{\sin(z)} M} \left( 3 \left( -\frac{1}{3} \left( \left( \sin(z) \right)^{-\frac{\sqrt{1+4C4}}{2}} c42 \right. \right. \right. \right. \\ & \left. \left. \left. \left. + \sin(z)^{\frac{\sqrt{1+4C4}}{2}} c41 \right) \left( -\sin(\sqrt{M^2 - 36C4 - 9} t) c44 \right. \right. \right. \end{aligned}$$

$$+ \cos(\sqrt{M^2 - 36C4 - 9} t) c43) \sqrt{M^2 - 36C4 - 9} \Big)$$

$$+ \sqrt{1 + 4 C4} \left( c47 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right)$$

$$+ \sin(z)^{\frac{\sqrt{1+4C4}}{2} c45})\Big)\Big), yZI3(z, t)$$

$$1 - \frac{1}{\sqrt{1+4C4}}$$

$$\begin{aligned}
&= \frac{1}{\sqrt{\sin(z)} M} \left( \left( \sin(z) - \frac{\sqrt{1+4C4}}{2} c46 \right. \right. \\
&\quad \left. \left. + \sin(z) \frac{\sqrt{1+4C4}}{2} c45 \right) \left( -\sin(\sqrt{M^2 - 36C4 - 9} t) c48 \right. \right. \\
&\quad \left. \left. + \cos(\sqrt{M^2 - 36C4 - 9} t) c47 \right) \sqrt{M^2 - 36C4 - 9} \right. \\
&\quad \left. - 3\sqrt{1+4C4} \left( c43 \sin(\sqrt{M^2 - 36C4 - 9} t) \right. \right. \\
&\quad \left. \left. + c44 \cos(\sqrt{M^2 - 36C4 - 9} t) \right) \left( -\sin(z) - \frac{\sqrt{1+4C4}}{2} c42 \right. \right. \\
&\quad \left. \left. + \sin(z) \frac{\sqrt{1+4C4}}{2} c41 \right) \right), yZ14(z, t) \\
&= \frac{1}{\sqrt{\sin(z)}} \left( \left( \sin(z) - \frac{\sqrt{1+4C4}}{2} c46 \right. \right. \\
&\quad \left. \left. + \sin(z) \frac{\sqrt{1+4C4}}{2} c45 \right) \left( c47 \sin(\sqrt{M^2 - 36C4 - 9} t) \right. \right. \\
&\quad \left. \left. + c48 \cos(\sqrt{M^2 - 36C4 - 9} t) \right) \right), yZ15(z, t) \\
&= \frac{1}{\sqrt{\sin(z)}} \left( \left( \sin(z) - \frac{\sqrt{1+4C4}}{2} c42 \right. \right. \\
&\quad \left. \left. + \sin(z) \frac{\sqrt{1+4C4}}{2} c41 \right) \left( c43 \sin(\sqrt{M^2 - 36C4 - 9} t) \right. \right. \\
&\quad \left. \left. + c44 \cos(\sqrt{M^2 - 36C4 - 9} t) \right) \right)
\end{aligned}$$

In[1]:= solvedEQ1

Out[1]=

$$\begin{aligned}
&\left\{ \left\{ Xyz0[z, t] = \frac{1}{(-1 + e^{-2A4[t]}) M Q1 A4'[t]} \right. \right. \\
&\quad \left. \left. \left( (9 - M^2) Xyz3[z, t] - Q1 A4'[t] Xyz3^{(0,1)}[z, t] + e^{-2A4[t]} Q1 A4'[t] Xyz3^{(0,1)}[z, t] - \right. \right. \\
&\quad \left. \left. Xyz3^{(0,2)}[z, t] + 36 (2 \tan[z] + \tan[z]^3) Xyz3^{(1,0)}[z, t] + 36 \tan[z]^2 Xyz3^{(2,0)}[z, t] \right) , \right. \\
&\quad \left. Xyz1[z, t] = \frac{1}{(-1 + e^{-2A4[t]})^2 M Q1 \sqrt{\sin[z]} A4'[t]^2} \right\}
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{3} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( e^{-2A4[t]} A4'[t]^2 + \frac{1}{2} (1 - e^{-2A4[t]}) A4''[t] \right) XYZ3^{(0,2)}[z, t] + \right. \right. \\
& \quad \frac{1}{2} (-1 + e^{-2A4[t]}) \cos[z]^3 A4'[t] XYZ3^{(0,3)}[z, t] - \frac{1}{2} (-1 + e^{-2A4[t]}) A4''[t] \\
& \quad \left( (-3 + M) (3 + M) \cos[z]^3 XYZ3[z, t] - 36 (1 + \cos[z]^2) \sin[z] XYZ3^{(1,0)}[z, t] \right) + \\
& \quad A4'[t] \left( -\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 XYZ3^{(0,1)}[z, t] + \right. \\
& \quad \left. \left. e^{-2A4[t]} \left( (-3 + M) (3 + M) \cos[z]^3 \left( XYZ3[z, t] A4'[t] + \frac{1}{2} XYZ3^{(0,1)}[z, t] \right) - \right. \right. \\
& \quad \left. \left. 36 (1 + \cos[z]^2) \sin[z] A4'[t] XYZ3^{(1,0)}[z, t] \right) \right) - \\
& \quad 18 (-1 + e^{-2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] XYZ3^{(1,1)}[z, t] - \\
& \quad 36 \cos[z] \sin[z]^2 \left( e^{-2A4[t]} A4'[t]^2 + \frac{1}{2} (1 - e^{-2A4[t]}) A4''[t] \right) XYZ3^{(2,0)}[z, t] - \\
& \quad \left. 18 (-1 + e^{-2A4[t]}) \cos[z] \sin[z]^2 A4'[t] XYZ3^{(2,1)}[z, t] \right) dz + \\
& \quad \frac{1}{3} \left( -\frac{1}{2} - \frac{1}{2} e^{-4A4[t]} + e^{-2A4[t]} \right) Q1 \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) XYZ3[z, t] + XYZ3^{(0,2)}[z, t] + \right. \right. \\
& \quad \left. \left. (-72 \tan[z] - 36 \tan[z]^3) XYZ3^{(1,0)}[z, t] - 36 \tan[z]^2 XYZ3^{(2,0)}[z, t] \right) dz \right) A4'[t]^2 - \\
& \quad 3 \left( e^{-\frac{1}{2} Q1(e^{-2A4[t]} + 2A4[t])} - 2 e^{-\frac{1}{2} e^{-2A4[t]} Q1 - (2+Q1) A4[t]} + e^{-\frac{1}{2} e^{-2A4[t]} Q1 - (4+Q1) A4[t]} \right) \cos[M t] \\
& \quad \left( \int \frac{1}{(-1 + e^{2A4[t]})^3 A4'[t]^3} e^{\frac{1}{2} Q1(e^{-2A4[t]} + 2A4[t])} \cos[M t] \right. \\
& \quad \left( -\frac{4}{9} e^{3A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \frac{1}{8} \cos[z]^3 (A4'[t]^2 (18 e^{A4[t]} - 2 e^{A4[t]} M^2 + \right. \right. \\
& \quad \left. \left. e^{-A4[t]} (-9 + M^2) + e^{3A4[t]} (-9 + M^2) + 8 \cosh[A4[t]] A4'[t]^2) + \right. \right. \\
& \quad \left. \left. 4 \sinh[A4[t]] A4'[t]^2 A4''[t] + 2 (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 \right) \right. \\
& \quad \left. XYZ3^{(0,2)}[z, t] - \cos[z]^3 A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \right. \\
& \quad \left. \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) XYZ3^{(0,3)}[z, t] + \right. \\
& \quad \left. \frac{1}{8} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) \cos[z]^3 A4'[t]^2 XYZ3^{(0,4)}[z, t] + \frac{1}{4} \right. \\
& \quad \left. (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 XYZ3[z, t] - \right. \\
& \quad \left. 36 (1 + \cos[z]^2) \sin[z] XYZ3^{(1,0)}[z, t] + A4'[t]^3 ((9 - M^2) \cos[z]^3 \right. \\
& \quad \left. \sinh[A4[t]] XYZ3^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \right. \\
& \quad \left. \cos[z]^3 XYZ3[z, t] - 36 (1 + \cos[z]^2) \sin[z] XYZ3^{(1,0)}[z, t] \right) \right) + \\
& \quad \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 XYZ3^{(0,1)}[z, t] + \right. \\
& \quad \left. e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 XYZ3^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \right)
\end{aligned}$$

$$\begin{aligned}
& (3 + M) \cos[z]^3 \text{XYZ3}^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 \text{XYZ3}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t]) + \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \text{XYZ3}^{(1,1)}[z, t] - \\
& \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t]^2 \text{XYZ3}^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \right) \text{XYZ3}^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \\
& 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 \text{XYZ3}[z, t] + \\
& \cos[z]^3 \text{XYZ3}^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \text{XYZ3}^{(2,0)}[z, t]) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& \text{XYZ3}^{(2,1)}[z, t] - \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z] \sin[z]^2 A4'[t]^2 \\
& \text{XYZ3}^{(2,2)}[z, t] dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \left. \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \right. \\
& \left. \left. \text{XYZ3}^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] \text{XYZ3}^{(0,3)}[z, t] + \right. \\
& \left. \left. \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 \text{XYZ3}[z, t] - \right. \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t]) + A4'[t] \right. \right. \\
& \left. \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{XYZ3}^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right. \right. \right. \\
& \left. \left. \left. \cos[z]^3 \text{XYZ3}^{(0,1)}[z, t] + A4'[t] ((-3 + M) (3 + M) \cos[z]^3 \right. \right. \right. \\
& \left. \left. \left. \text{XYZ3}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t]) \right) + \right. \\
& 18 (-1 + e^{2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] \text{XYZ3}^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \text{XYZ3}^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] \text{XYZ3}^{(2,1)}[z, t] \right) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] ((-9 + M^2) \text{XYZ3}[z, t] + \text{XYZ3}^{(0,2)}[z, t] + (-72 \tan[z] - \right. \\
& \left. 36 \tan[z]^3) \text{XYZ3}^{(1,0)}[z, t] - 36 \tan[z]^2 \text{XYZ3}^{(2,0)}[z, t]) dz \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{1}{6} Q1 \left( (2 - 4 Q1 + e^{-2 A4[t]} Q1) A4'[t]^2 - A4''[t] \right) + e^{2 A4[t]} \right. \\
& \quad \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4 A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \\
& \quad \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6 A4[t]} (M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t]) \Big) - \\
& 396 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^3 \sin[z]^{7/2} Xyz3^{(1,0)}[z, t] - \\
& 216 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^5 \sin[z]^{11/2} Xyz3^{(1,0)}[z, t] + \\
& 2 \operatorname{Sec}[z] \sin[z]^{3/2} (e^{2 A4[t]} (-117 + M^2 - 6 Q1^2 A4'[t]^2) + \\
& e^{6 A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2 e^{4 A4[t]} (117 - M^2 + 2 Q1^2 A4'[t]^2)) \\
& Xyz3^{(1,0)}[z, t] + \sqrt{\sin[z]} (e^{2 A4[t]} Xyz3[z, t] (-9 + M^2 - 6 Q1^2 A4'[t]^2) - \\
& 2 e^{4 A4[t]} Xyz3[z, t] (-9 + M^2 - 2 Q1^2 A4'[t]^2) + e^{6 A4[t]} Xyz3[z, t] \\
& (-9 + M^2 - Q1^2 A4'[t]^2) + (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) Xyz3^{(0,2)}[z, t] - \\
& (-4 + e^{-2 A4[t]}) Q1^2 A4'[t]^2 (Xyz3[z, t] + 2 \tan[z] Xyz3^{(1,0)}[z, t])) + \\
& 2 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z] \sin[z]^{3/2} Xyz3^{(1,2)}[z, t] - \\
& 324 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^2 \sin[z]^{5/2} Xyz3^{(2,0)}[z, t] - \\
& 216 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^4 \sin[z]^{9/2} Xyz3^{(2,0)}[z, t] - \\
& 72 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^3 \sin[z]^{7/2} Xyz3^{(3,0)}[z, t] \Big) dt \\
& A4'[t]^2 - 3 \left( e^{-\frac{1}{2} Q1 (e^{-2 A4[t]} + 2 A4[t])} - 2 e^{-\frac{1}{2} e^{-2 A4[t]} Q1 - (2+Q1) A4[t]} + \right. \\
& \left. e^{-\frac{1}{2} e^{-2 A4[t]} Q1 - (4+Q1) A4[t]} \right) \\
& \left( \int \frac{1}{(-1 + e^{2 A4[t]})^3 A4'[t]^3} e^{\frac{1}{2} Q1 (e^{-2 A4[t]} + 2 A4[t])} \sin[Mt] \right. \\
& \left( -\frac{4}{9} e^{3 A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \operatorname{Sec}[z]^2 \left( \frac{1}{8} \cos[z]^3 (A4'[t]^2 (18 e^{A4[t]} - 2 e^{A4[t]} M^2 + \right. \right. \\
& e^{-A4[t]} (-9 + M^2) + e^{3 A4[t]} (-9 + M^2) + 8 \cosh[A4[t]] A4'[t]^2) + \\
& 4 \sinh[A4[t]] A4'[t]^2 A4''[t] + 2 (e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]}) A4''[t]^2) \\
& Xyz3^{(0,2)}[z, t] - \cos[z]^3 A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3 A4[t]} \right) A4''[t] \right) Xyz3^{(0,3)}[z, t] + \\
& \frac{1}{8} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]}) \cos[z]^3 A4'[t]^2 Xyz3^{(0,4)}[z, t] + \frac{1}{4} \\
& (e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 Xyz3[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t] + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \sinh[A4[t]] Xyz3^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz3[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t])) + \\
& \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz3^{(0,1)}[z, t] + \right.
\end{aligned}$$

$$\begin{aligned}
& e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz3^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \\
& (3 + M) \cos[z]^3 Xyz3^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz3[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t]) + \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] (\sinh[A4[t]] A4'[t]^2 + \\
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]) Xyz3^{(1,1)}[z, t] - \\
& \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t]^2 Xyz3^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \right) Xyz3^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \\
& 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz3[z, t] + \\
& \cos[z]^3 Xyz3^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 Xyz3^{(2,0)}[z, t]) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& Xyz3^{(2,1)}[z, t] - \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z] \sin[z]^2 A4'[t]^2 \\
& Xyz3^{(2,2)}[z, t] dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \left. \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \right. \\
& Xyz3^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] Xyz3^{(0,3)}[z, t] + \\
& \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz3[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t]) + A4'[t] \\
& \left. \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz3^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right. \right. \right. \\
& \cos[z]^3 Xyz3^{(0,1)}[z, t] + A4'[t] ((-3 + M) (3 + M) \cos[z]^3 \\
& Xyz3[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t]) + \\
& 18 (-1 + e^{2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] Xyz3^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) Xyz3^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] Xyz3^{(2,1)}[z, t] \right) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] ((-9 + M^2) Xyz3[z, t] + Xyz3^{(0,2)}[z, t] + (-72 \tan[z] - \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left( -\frac{1}{6} Q1 \left( (2 - 4 Q1 + e^{-2 A4[t]} Q1) A4'[t]^2 - A4''[t] \right) + e^{2 A4[t]} \right. \right. \\
& \left. \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4 A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \right. \\
& \left. \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6 A4[t]} (M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t]) \right) - \\
& 396 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^3 \sin[z]^{7/2} Xyz3^{(1,0)}[z, t] - \\
& 216 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^5 \sin[z]^{11/2} Xyz3^{(1,0)}[z, t] + \\
& 2 \operatorname{Sec}[z] \sin[z]^{3/2} (e^{2 A4[t]} (-117 + M^2 - 6 Q1^2 A4'[t]^2) + \\
& e^{6 A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2 e^{4 A4[t]} (117 - M^2 + 2 Q1^2 A4'[t]^2)) - \\
& Xyz3^{(1,0)}[z, t] + \sqrt{\sin[z]} (e^{2 A4[t]} Xyz3[z, t] (-9 + M^2 - 6 Q1^2 A4'[t]^2) - \\
& 2 e^{4 A4[t]} Xyz3[z, t] (-9 + M^2 - 2 Q1^2 A4'[t]^2) + e^{6 A4[t]} Xyz3[z, t] \\
& (-9 + M^2 - Q1^2 A4'[t]^2) + (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) Xyz3^{(0,2)}[z, t] - \\
& (-4 + e^{-2 A4[t]}) Q1^2 A4'[t]^2 (Xyz3[z, t] + 2 \tan[z] Xyz3^{(1,0)}[z, t])) + \\
& 2 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z] \sin[z]^{3/2} Xyz3^{(1,2)}[z, t] - \\
& 324 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^2 \sin[z]^{5/2} Xyz3^{(2,0)}[z, t] - \\
& 216 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^4 \sin[z]^{9/2} Xyz3^{(2,0)}[z, t] - \\
& 72 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^3 \sin[z]^{7/2} Xyz3^{(3,0)}[z, t] \Bigg) dt \Bigg) \\
& \sin[M t] A4'[t]^2 + e^{-\frac{1}{2} Q1 (e^{-2 A4[t]} + 2 A4[t])} M Q1 (c12 \cos[M t] - \\
& c11 \\
& \sin[M t]) \\
& A4'[t]^2 + e^{-\frac{1}{2} e^{-2 A4[t]} Q1 - (4 + Q1) A4[t]} M Q1 (c12 \cos[M t] - c11 \\
& \sin[M t]) \\
& A4'[t]^2 + 2 e^{-\frac{1}{2} e^{-2 A4[t]} Q1 - (2 + Q1) A4[t]} M Q1 \\
& (-c12 \\
& \cos[M t] + \\
& c11 \sin[M t]) A4'[t]^2 + \\
& 3 Q1 \sqrt{\sin[z]} A4'[t]^2 (-Xyz3[z, t] - \\
& 2 \\
& \tan[z] \\
& Xyz3^{(1,0)}[z, t]) + \\
& 3 e^{-4 A4[t]} Q1 \sqrt{\sin[z]} A4'[t]^2 (-Xyz3[z, t] - \\
& 2 \\
& \tan[z] \\
& Xyz3^{(1,0)}[z, t]) + \\
& 6 e^{-2 A4[t]} Q1 \sqrt{\sin[z]} A4'[t]^2 (Xyz3[
\end{aligned}$$

$$\begin{aligned}
& z, \\
& t] + 2 \\
& \text{Tan}[z] \\
& \left. \text{XYZ3}^{(1,0)}[z, t] \right), \\
& \text{XYZ2}[z, t] = \frac{1}{(-1 + e^{-2A4[t]}) M Q1 \sqrt{\sin[z]} A4'[t]} \\
& \left( 3 \right. \\
& e^{-\frac{1}{2} Q1 (e^{-2A4[t]} + 2 A4[t])} \\
& (-1 + \\
& e^{-2A4[t]}) \cos[M \\
& t] \\
& \left( \int \frac{1}{(-1 + e^{2A4[t]})^3 A4'[t]^3} e^{\frac{1}{2} Q1 (e^{-2A4[t]} + 2 A4[t])} \sin[M t] \right. \\
& \left( -\frac{4}{9} e^{3A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \frac{1}{8} \cos[z]^3 (A4'[t]^2 (18 e^{A4[t]} - 2 e^{A4[t]} M^2 + \right. \right. \\
& e^{-A4[t]} (-9 + M^2) + e^{3A4[t]} (-9 + M^2) + 8 \cosh[A4[t]] A4'[t]^2) + \\
& 4 \sinh[A4[t]] A4'[t]^2 A4''[t] + 2 (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2) \\
& \text{XYZ3}^{(0,2)}[z, t] - \cos[z]^3 A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \text{XYZ3}^{(0,3)}[z, t] + \\
& \frac{1}{8} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z]^3 A4'[t]^2 \text{XYZ3}^{(0,4)}[z, t] + \frac{1}{4} \\
& (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 \text{XYZ3}[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t]) + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \sinh[A4[t]] \text{XYZ3}^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 \text{XYZ3}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t])) + \\
& \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{XYZ3}^{(0,1)}[z, t] + \right. \\
& e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{XYZ3}^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \\
& (3 + M) \cos[z]^3 \text{XYZ3}^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 \text{XYZ3}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t]) \left. \right) + \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \text{XYZ3}^{(1,1)}[z, t] - 
\end{aligned}$$

$$\begin{aligned}
& \frac{9}{2} \left( e^{-A4[t]} - 2e^{A4[t]} + e^{3A4[t]} \right) \left( 1 + \cos[z]^2 \right) \sin[z] A4'[t]^2 Xyz3^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \right) Xyz3^{(2,0)}[z, t] - \frac{1}{8} \left( e^{-A4[t]} - \right. \\
& \left. 2e^{A4[t]} + e^{3A4[t]} \right) A4'[t] A4^{(3)}[t] \left( (-3 + M) (3 + M) \cos[z]^3 Xyz3[z, t] + \right. \\
& \left. \cos[z]^3 Xyz3^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t] - \right. \\
& \left. 36 \cos[z] \sin[z]^2 Xyz3^{(2,0)}[z, t] \right) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& Xyz3^{(2,1)}[z, t] - \frac{9}{2} \left( e^{-A4[t]} - 2e^{A4[t]} + e^{3A4[t]} \right) \cos[z] \sin[z]^2 A4'[t]^2 \\
& Xyz3^{(2,2)}[z, t] \right) dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \\
& Xyz3^{(0,2)}[z, t] - \frac{1}{2} \left( -1 + e^{2A4[t]} \right) \cos[z]^3 A4'[t] Xyz3^{(0,3)}[z, t] + \\
& \left. \frac{1}{2} \left( -1 + e^{2A4[t]} \right) A4''[t] \left( (-3 + M) (3 + M) \cos[z]^3 Xyz3[z, t] - \right. \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t] \right) + A4'[t] \right. \\
& \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz3^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right. \right. \\
& \left. \left. \cos[z]^3 Xyz3^{(0,1)}[z, t] + A4'[t] \left( (-3 + M) (3 + M) \cos[z]^3 \right. \right. \right. \\
& \left. \left. \left. Xyz3[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t] \right) \right) + \right. \\
& 18 \left( -1 + e^{2A4[t]} \right) \left( 1 + \cos[z]^2 \right) \sin[z] A4'[t] Xyz3^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) Xyz3^{(2,0)}[z, t] + \\
& 18 \left( -1 + e^{2A4[t]} \right) \cos[z] \sin[z]^2 A4'[t] Xyz3^{(2,1)}[z, t] \right) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) Xyz3[z, t] + Xyz3^{(0,2)}[z, t] + (-72 \tan[z] - \right. \right. \\
& \left. \left. 36 \tan[z]^3) Xyz3^{(1,0)}[z, t] - 36 \tan[z]^2 Xyz3^{(2,0)}[z, t] \right) dz \right) \\
& \left( -\frac{1}{6} Q1 \left( (2 - 4 Q1 + e^{-2A4[t]} Q1) A4'[t]^2 - A4''[t] \right) + e^{2A4[t]} \right. \\
& \left. \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \right. \\
& \left. \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6A4[t]} \left( M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t] \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& 396 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} \text{XYZ3}^{(1,0)}[z, t] - \\
& 216 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^5 \sin[z]^{11/2} \text{XYZ3}^{(1,0)}[z, t] + \\
& 2 \sec[z] \sin[z]^{3/2} \left( e^{2A4[t]} (-117 + M^2 - 6Q1^2 A4'[t]^2) + \right. \\
& \quad e^{6A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2e^{4A4[t]} (117 - M^2 + 2Q1^2 A4'[t]^2) \\
& \quad \text{XYZ3}^{(1,0)}[z, t] + \sqrt{\sin[z]} \left( e^{2A4[t]} \text{XYZ3}[z, t] (-9 + M^2 - 6Q1^2 A4'[t]^2) - \right. \\
& \quad 2e^{4A4[t]} \text{XYZ3}[z, t] (-9 + M^2 - 2Q1^2 A4'[t]^2) + e^{6A4[t]} \text{XYZ3}[z, t] \\
& \quad (-9 + M^2 - Q1^2 A4'[t]^2) + \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \text{XYZ3}^{(0,2)}[z, t] - \\
& \quad (-4 + e^{-2A4[t]}) Q1^2 A4'[t]^2 (\text{XYZ3}[z, t] + 2 \tan[z] \text{XYZ3}^{(1,0)}[z, t]) \left. \right) + \\
& 2 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z] \sin[z]^{3/2} \text{XYZ3}^{(1,2)}[z, t] - \\
& 324 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^2 \sin[z]^{5/2} \text{XYZ3}^{(2,0)}[z, t] - \\
& 216 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^4 \sin[z]^{9/2} \text{XYZ3}^{(2,0)}[z, t] - \\
& \left. 72 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} \text{XYZ3}^{(3,0)}[z, t] \right) dt \\
& A4'[t] - 3e^{-\frac{1}{2}Q1(e^{-2A4[t]}+2A4[t])} \left( -1 + e^{-2A4[t]} \right) \left( \int \frac{1}{(-1 + e^{2A4[t]})^3 A4'[t]^3} \right. \\
& e^{\frac{1}{2}Q1(e^{-2A4[t]}+2A4[t])} \cos[Mt] \\
& \left( -\frac{4}{9} e^{3A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \frac{1}{8} \cos[z]^3 (A4'[t]^2 (18e^{A4[t]} - 2e^{A4[t]} M^2 + \right. \right. \\
& \quad e^{-A4[t]} (-9 + M^2) + e^{3A4[t]} (-9 + M^2) + 8 \cosh[A4[t]] A4'[t]^2) + \\
& \quad 4 \sinh[A4[t]] A4'[t]^2 A4''[t] + 2 (e^{-A4[t]} - 2e^{A4[t]} + e^{3A4[t]}) A4''[t]^2) \\
& \quad \text{XYZ3}^{(0,2)}[z, t] - \cos[z]^3 A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \quad \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \text{XYZ3}^{(0,3)}[z, t] + \\
& \quad \frac{1}{8} \left( e^{-A4[t]} - 2e^{A4[t]} + e^{3A4[t]} \right) \cos[z]^3 A4'[t]^2 \text{XYZ3}^{(0,4)}[z, t] + \frac{1}{4} \\
& \quad (e^{-A4[t]} - 2e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 \text{XYZ3}[z, t] - \\
& \quad 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t] + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \quad \sinh[A4[t]] \text{XYZ3}^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \quad \cos[z]^3 \text{XYZ3}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t])) + \\
& \quad \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{XYZ3}^{(0,1)}[z, t] + \right. \\
& \quad e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{XYZ3}^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \\
& \quad (3 + M) \cos[z]^3 \text{XYZ3}^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \quad \cos[z]^3 \text{XYZ3}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ3}^{(1,0)}[z, t]) \left. \right) + \\
& \quad 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \quad \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \text{XYZ3}^{(1,1)}[z, t] -
\end{aligned}$$

$$\begin{aligned}
& \frac{9}{2} \left( e^{-A4[t]} - 2e^{A4[t]} + e^{3A4[t]} \right) \left( 1 + \cos[z]^2 \right) \sin[z] A4'[t]^2 Xyz3^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \right) Xyz3^{(2,0)}[z, t] - \frac{1}{8} \left( e^{-A4[t]} - \right. \\
& \left. 2e^{A4[t]} + e^{3A4[t]} \right) A4'[t] A4^{(3)}[t] \left( (-3 + M) (3 + M) \cos[z]^3 Xyz3[z, t] + \right. \\
& \left. \cos[z]^3 Xyz3^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t] - \right. \\
& \left. 36 \cos[z] \sin[z]^2 Xyz3^{(2,0)}[z, t] \right) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& Xyz3^{(2,1)}[z, t] - \frac{9}{2} \left( e^{-A4[t]} - 2e^{A4[t]} + e^{3A4[t]} \right) \cos[z] \sin[z]^2 A4'[t]^2 \\
& Xyz3^{(2,2)}[z, t] \right) dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \\
& Xyz3^{(0,2)}[z, t] - \frac{1}{2} \left( -1 + e^{2A4[t]} \right) \cos[z]^3 A4'[t] Xyz3^{(0,3)}[z, t] + \\
& \left. \frac{1}{2} \left( -1 + e^{2A4[t]} \right) A4''[t] \left( (-3 + M) (3 + M) \cos[z]^3 Xyz3[z, t] - \right. \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t] \right) + A4'[t] \right. \\
& \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz3^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right. \right. \\
& \left. \left. \cos[z]^3 Xyz3^{(0,1)}[z, t] + A4'[t] \left( (-3 + M) (3 + M) \cos[z]^3 \right. \right. \right. \\
& \left. \left. \left. Xyz3[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz3^{(1,0)}[z, t] \right) \right) + \right. \\
& 18 \left( -1 + e^{2A4[t]} \right) \left( 1 + \cos[z]^2 \right) \sin[z] A4'[t] Xyz3^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) Xyz3^{(2,0)}[z, t] + \\
& 18 \left( -1 + e^{2A4[t]} \right) \cos[z] \sin[z]^2 A4'[t] Xyz3^{(2,1)}[z, t] \right) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) Xyz3[z, t] + Xyz3^{(0,2)}[z, t] + (-72 \tan[z] - \right. \right. \\
& \left. \left. 36 \tan[z]^3) Xyz3^{(1,0)}[z, t] - 36 \tan[z]^2 Xyz3^{(2,0)}[z, t] \right) dz \right) \\
& \left( -\frac{1}{6} Q1 \left( (2 - 4 Q1 + e^{-2A4[t]} Q1) A4'[t]^2 - A4''[t] \right) + e^{2A4[t]} \right. \\
& \left. \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \right. \\
& \left. \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6A4[t]} \left( M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t] \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left. \left( 396 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} Xyz3^{(1,0)}[z, t] - \right. \right. \\
& 216 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^5 \sin[z]^{11/2} Xyz3^{(1,0)}[z, t] + \\
& 2 \sec[z] \sin[z]^{3/2} \left( e^{2A4[t]} (-117 + M^2 - 6Q1^2 A4'[t]^2) + \right. \\
& e^{6A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2e^{4A4[t]} (117 - M^2 + 2Q1^2 A4'[t]^2) \\
& Xyz3^{(1,0)}[z, t] + \sqrt{\sin[z]} \left( e^{2A4[t]} Xyz3[z, t] (-9 + M^2 - 6Q1^2 A4'[t]^2) - \right. \\
& 2e^{4A4[t]} Xyz3[z, t] (-9 + M^2 - 2Q1^2 A4'[t]^2) + e^{6A4[t]} Xyz3[z, t] \\
& (-9 + M^2 - Q1^2 A4'[t]^2) + \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) Xyz3^{(0,2)}[z, t] - \\
& (-4 + e^{-2A4[t]}) Q1^2 A4'[t]^2 (Xyz3[z, t] + 2 \tan[z] Xyz3^{(1,0)}[z, t]) \Big) + \\
& 2 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z] \sin[z]^{3/2} Xyz3^{(1,2)}[z, t] - \\
& 324 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^2 \sin[z]^{5/2} Xyz3^{(2,0)}[z, t] - \\
& 216 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^4 \sin[z]^{9/2} Xyz3^{(2,0)}[z, t] - \\
& \left. \left. 72 \left( e^{2A4[t]} - 2e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} Xyz3^{(3,0)}[z, t] \right) dt \right) \\
& \sin[Mt] A4'[t] + M \left( \frac{1}{6} \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) Xyz3[z, t] + Xyz3^{(0,2)}[z, t], \right. \right. \\
& t] + (-72 \tan[z] - 36 \tan[z]^3) Xyz3^{(1,0)}[z, t] - 36 \tan[z]^2 Xyz3^{(2,0)}[z, t] \Big) dz + \\
& e^{-\frac{1}{2} Q1 (e^{-2A4[t]} + 2A4[t])} (-1 + e^{-2A4[t]}) Q1 (c11 \cos[Mt] + c12 \sin[Mt]) \\
& A4' [ \\
& \left. \left. t \right] \right) \Big), \\
& Xyz3^{(0,4)}[z, t] = \frac{1}{(-1 + e^{2A4[t]})^2 A4'[t]^2} \\
& \left( \left( -2 \right. \right. \\
& A4'[t]^2 \\
& \left. \left. \left( -9 + \right. \right. \right. \\
& M^2 + \\
& \frac{1}{2} e^{-4A4[t]} Q1^2 A4'[t]^2 - \\
& 2e^{-2A4[t]} Q1 \left( \frac{1}{2} + Q1 \right) A4'[t]^2 + \\
& (2 + 2Q1 + 3Q1^2) A4'[t]^2 + \\
& e^{4A4[t]} \left( -9 + M^2 + \frac{1}{2} Q1^2 A4'[t]^2 \right) - \\
& \left. \left. \left( -2 - 3Q1 + e^{-2A4[t]} Q1 - e^{4A4[t]} Q1 \right) A4'[t]^2 \right) \\
& A4'' [ \\
& t] + \\
& (-2 - 2e^{4A4[t]}) A4''[t]^2 + e^{2A4[t]} \\
& \left( 4 A4'[t]^2 \left( -9 + M^2 + \left( -1 + \frac{Q1}{2} + Q1^2 \right) A4'[t]^2 \right) - \right.
\end{aligned}$$

$$\begin{aligned}
& 3 \left( \frac{2}{3} + Q1 \right) A4'[t]^2 A4''[t] + \\
& 4 A4''[t]^2 \Bigg) \Bigg) \\
& Xyz3^{(0,2)}[z, t] + 2 A4'[t] \left( (-2 - 3 Q1 + e^{-2 A4[t]} Q1 - e^{4 A4[t]} Q1) \right. \\
& A4'[t]^2 + e^{2 A4[t]} \\
& \left. ((2 + 3 Q1) A4'[t]^2 - \right. \\
& \left. 2 A4''[t]) + \right. \\
& \left. (1 + e^{4 A4[t]}) A4''[t] \right) Xyz3^{(0,3)}[z, t] - 2 \left( 1 + \right. \\
& e^{4 A4[t]} \left. \right) A4''[t]^2 \left( (-9 + M^2) \right. \\
& Xyz3[z, t] + \\
& \left. (-72 \tan[z] - 36 \tan[z]^3) Xyz3^{(1,0)}[z, t] \right) - A4'[t]^2 \left( e^{-4 A4[t]} M^2 Q1^2 \right. \\
& Xyz3[z, t] A4'[t]^2 + Xyz3[z, t] \\
& \left. A4'[t]^2 + \right. \\
& \left. ((-3 + M)^2 (3 + M)^2 + (-36 + 4 M^2 - 36 Q1 + 4 M^2 Q1 + 6 M^2 Q1^2) A4'[t]^2) + 6 \right. \\
& (-3 + M) \\
& (3 + M) \\
& \left( \frac{2}{3} + Q1 \right) \\
& A4'[t] Xyz3^{(0,1)}[z, t] - 72 \tan[z] \\
& \left. - 270 \tan[z]^6 + \right. \\
& 2 (1 + Q1) (2 + \tan[z]^2) A4'[t]^2 Xyz3^{(1,0)}[z, t] + \\
& e^{4 A4[t]} (Xyz3[z, t] ((-3 + M)^2 (3 + M)^2 + M^2 Q1^2 A4'[t]^2) + \\
& (-18 + 2 M^2) Q1 A4'[t] Xyz3^{(0,1)}[z, t] - \\
& 72 \tan[z] (-90 + 2 M^2 + (-405 + M^2) \tan[z]^2 - 594 \tan[z]^4 - 270 \tan[z]^6) \\
& Xyz3^{(1,0)}[z, t]) - \\
& 2 e^{-2 A4[t]} Q1 A4'[t] \left( 2 \left( -\frac{9}{2} + \frac{M^2}{2} + M^2 Q1 \right) Xyz3[z, t] A4'[t] + \right. \\
& \left. (-9 + M^2) Xyz3^{(0,1)}[z, t] - \right)
\end{aligned}$$



$$\begin{aligned}
& \left( -\frac{1}{2} A4'[t]^2 (-261 + M^2 - 504 \tan[z]^2 - 270 \tan[z]^4 + \right. \\
& \quad e^{4A4[t]} (-261 + M^2 - 504 \tan[z]^2 - 270 \tan[z]^4) - \\
& \quad \left. e^{-2A4[t]} Q1 A4'[t]^2 + (2 + 2 Q1) A4'[t]^2 \right) - \\
& \frac{1}{4} \left( -2 - 3 Q1 + e^{-2A4[t]} Q1 - e^{4A4[t]} Q1 \right) \\
& A4'[t]^2 \\
& A4''[ \\
& \quad t] + \left( -\frac{1}{2} - \frac{1}{2} e^{4A4[t]} \right) A4''[t]^2 + \\
& e^{2A4[t]} \left( A4'[t]^2 \left( -261 + M^2 - 504 \tan[z]^2 - 270 \tan[z]^4 + \left( -1 + \frac{Q1}{2} \right) A4'[t]^2 \right) - \right. \\
& \quad \left. \frac{3}{4} \left( \frac{2}{3} + Q1 \right) A4'[t]^2 A4''[t] + A4''[t]^2 \right) \\
& Xyz3^{(2,0)}[z, t] + (-1 + e^{2A4[t]})^2 A4'[ \\
& \quad t] \\
& A4^{(3)}[ \\
& \quad t] \\
& \left( (-9 + M^2) \right. \\
& \quad Xyz3[ \\
& \quad z, \\
& \quad t] + \\
& Xyz3^{(0,2)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3) \\
& Xyz3^{(1,0)}[ \\
& \quad z, \\
& \quad t] - 36 \tan[z]^2 Xyz3^{(2,0)}[ \\
& \quad z, \\
& \quad t] \left. \right) - \\
& 72 \tan[z]^2 A4'[t] \left( (-2 - 3 Q1 + e^{-2A4[t]} Q1 - e^{4A4[t]} Q1) \right. \\
& \quad A4'[t]^2 + \\
& \quad \left. e^{2A4[t]} \left( (2 + 3 Q1) A4'[t]^2 - 2 A4''[t] \right) + (1 + e^{4A4[t]}) \right. \\
& \quad A4''[ \\
& \quad t] \left. \right) \\
& Xyz3^{(2,1)}[z, t] + 72 (-1 + e^{2A4[t]})^2 \tan[ \\
& \quad z]^2 A4'[ \\
& \quad t]^2 \\
& Xyz3^{(2,2)}[z, t] - 7776 (-1 + e^{2A4[t]})^2 \\
& \tan[ \\
& \quad z]^3 \\
& \left( \frac{4}{3} + \tan[z]^2 \right) A4'[t]^2 Xyz3^{(3,0)}[z, t] - \\
& 1296 \\
& \left( -1 + \right. \\
& \quad \left. e^{2A4[t]} \right)^2 \tan[ 
\end{aligned}$$

$$z]^4 A4' [t]^2 Xyz3^{(4,0)} [z, t] \Big) \Big\}$$

In[•]:= solvedEQ2

Out[•]=

$$\begin{aligned} & \left\{ \begin{aligned} Xyz4 [z, t] &= \frac{1}{(-1 + e^{-2 A4[t]}) M Q1 A4'[t]} \\ & \left( (-9 + M^2) Xyz7 [z, t] + Q1 A4'[t] Xyz7^{(0,1)} [z, t] - e^{-2 A4[t]} Q1 A4'[t] Xyz7^{(0,1)} [z, t] + \right. \\ & Xyz7^{(0,2)} [z, t] + (-72 \tan[z] - 36 \tan[z]^3) Xyz7^{(1,0)} [z, t] - 36 \tan[z]^2 Xyz7^{(2,0)} [z, t] \left. \right), \\ Xyz5 [z, t] &= \frac{1}{(-1 + e^{-2 A4[t]})^2 M Q1 \sqrt{\sin[z]} A4'[t]^2} \\ & \left( \frac{1}{3} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( e^{-2 A4[t]} A4'[t]^2 + \frac{1}{2} (1 - e^{-2 A4[t]}) A4''[t] \right) Xyz7^{(0,2)} [z, t] + \right. \right. \\ & \frac{1}{2} (-1 + e^{-2 A4[t]}) \cos[z]^3 A4'[t] Xyz7^{(0,3)} [z, t] - \frac{1}{2} (-1 + e^{-2 A4[t]}) A4''[t] \\ & \left. \left. \left( (-3 + M) (3 + M) \cos[z]^3 Xyz7 [z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)} [z, t] \right) + \right. \right. \\ & A4'[t] \left( -\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)} [z, t] + \right. \\ & e^{-2 A4[t]} \left( (-3 + M) (3 + M) \cos[z]^3 \left( Xyz7 [z, t] A4'[t] + \frac{1}{2} Xyz7^{(0,1)} [z, t] \right) - \right. \\ & \left. \left. 36 (1 + \cos[z]^2) \sin[z] A4'[t] Xyz7^{(1,0)} [z, t] \right) \right) - \\ & 18 (-1 + e^{-2 A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] Xyz7^{(1,1)} [z, t] - \\ & 36 \cos[z] \sin[z]^2 \left( e^{-2 A4[t]} A4'[t]^2 + \frac{1}{2} (1 - e^{-2 A4[t]}) A4''[t] \right) Xyz7^{(2,0)} [z, t] - \\ & 18 (-1 + e^{-2 A4[t]}) \cos[z] \sin[z]^2 A4'[t] Xyz7^{(2,1)} [z, t] \Big) dz - \\ & \frac{1}{3} \left( \frac{1}{2} + \frac{1}{2} e^{-4 A4[t]} - e^{-2 A4[t]} \right) Q1 \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) Xyz7 [z, t] + Xyz7^{(0,2)} [z, t] + \right. \right. \\ & \left. \left. (-72 \tan[z] - 36 \tan[z]^3) Xyz7^{(1,0)} [z, t] - 36 \tan[z]^2 Xyz7^{(2,0)} [z, t] \right) dz \right) \\ & A4'[t]^2 - 3 e^{-\frac{1}{2} Q1 (\epsilon^{-2 A4[t]} + 2 A4[t])} (-1 + e^{-2 A4[t]})^2 \cos[M t] \\ & \left( \int \frac{1}{(-1 + e^{2 A4[t]})^3 A4'[t]^3} e^{\frac{1}{2} Q1 (\epsilon^{-2 A4[t]} + 2 A4[t])} \cos[M t] \right. \\ & \left( -\frac{4}{9} e^{3 A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \frac{1}{8} \cos[z]^3 (A4'[t]^2 (18 e^{A4[t]} - 2 e^{A4[t]} M^2 + \right. \right. \\ & e^{-A4[t]} (-9 + M^2) + e^{3 A4[t]} (-9 + M^2) + 8 \cosh[A4[t]] A4'[t]^2) + \\ & 4 \sinh[A4[t]] A4'[t]^2 A4''[t] + 2 (e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]}) A4''[t]^2) \\ & Xyz7^{(0,2)} [z, t] - \cos[z]^3 A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) Xyz7^{(0,3)}[z, t] + \\
& \frac{1}{8} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) \cos[z]^3 A4'[t]^2 Xyz7^{(0,4)}[z, t] + \frac{1}{4} \\
& (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \sinh[A4[t]] Xyz7^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t])) + \\
& \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \right. \\
& e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \\
& (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) \Big) + \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \Big) Xyz7^{(1,1)}[z, t] - \\
& \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t]^2 Xyz7^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \Big) Xyz7^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \\
& 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] + \\
& \cos[z]^3 Xyz7^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 Xyz7^{(2,0)}[z, t]) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& Xyz7^{(2,1)}[z, t] - \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z] \sin[z]^2 A4'[t]^2 \\
& Xyz7^{(2,2)}[z, t] \Big) dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \\
& Xyz7^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] Xyz7^{(0,3)}[z, t] + \\
& \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t] \\
& \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \cos[z]^3 \text{XYZ7}^{(0,1)}[z, t] + A4'[t] \left( (-3 + M) (3 + M) \cos[z]^3 \right. \\
& \quad \left. \text{XYZ7}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t] \right) + \\
& 18 (-1 + e^{2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] \text{XYZ7}^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \text{XYZ7}^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] \text{XYZ7}^{(2,1)}[z, t] \Big) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) \text{XYZ7}[z, t] + \text{XYZ7}^{(0,2)}[z, t] + (-72 \tan[z] - \right. \right. \\
& \quad \left. \left. 36 \tan[z]^3) \text{XYZ7}^{(1,0)}[z, t] - 36 \tan[z]^2 \text{XYZ7}^{(2,0)}[z, t] \right) dz \right) \\
& \left( -\frac{1}{6} Q1 \left( (2 - 4 Q1 + e^{-2A4[t]} Q1) A4'[t]^2 - A4''[t] \right) + e^{2A4[t]} \right. \\
& \quad \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \\
& \quad \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6A4[t]} (M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t]) \Big) - \\
& 396 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^3 \sin[z]^{7/2} \text{XYZ7}^{(1,0)}[z, t] - \\
& 216 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^5 \sin[z]^{11/2} \text{XYZ7}^{(1,0)}[z, t] + \\
& 2 \sec[z] \sin[z]^{3/2} (e^{2A4[t]} (-117 + M^2 - 6 Q1^2 A4'[t]^2) + \\
& \quad e^{6A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2 e^{4A4[t]} (117 - M^2 + 2 Q1^2 A4'[t]^2)) \\
& \quad \text{XYZ7}^{(1,0)}[z, t] + \sqrt{\sin[z]} (e^{2A4[t]} \text{XYZ7}[z, t] (-9 + M^2 - 6 Q1^2 A4'[t]^2) - \\
& \quad 2 e^{4A4[t]} \text{XYZ7}[z, t] (-9 + M^2 - 2 Q1^2 A4'[t]^2) + e^{6A4[t]} \text{XYZ7}[z, t] \\
& \quad (-9 + M^2 - Q1^2 A4'[t]^2) + (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \text{XYZ7}^{(0,2)}[z, t] - \\
& \quad (-4 + e^{-2A4[t]}) Q1^2 A4'[t]^2 (\text{XYZ7}[z, t] + 2 \tan[z] \text{XYZ7}^{(1,0)}[z, t])) + \\
& 2 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z] \sin[z]^{3/2} \text{XYZ7}^{(1,2)}[z, t] - \\
& 324 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^2 \sin[z]^{5/2} \text{XYZ7}^{(2,0)}[z, t] - \\
& 216 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^4 \sin[z]^{9/2} \text{XYZ7}^{(2,0)}[z, t] - \\
& 72 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^3 \sin[z]^{7/2} \text{XYZ7}^{(3,0)}[z, t] \Big) dt \Big)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) Xyz7^{(0,3)}[z, t] + \\
& \frac{1}{8} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) \cos[z]^3 A4'[t]^2 Xyz7^{(0,4)}[z, t] + \frac{1}{4} \\
& (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \sinh[A4[t]] Xyz7^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t])) + \\
& \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \right. \\
& e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \\
& (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) \Big) + \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \Big) Xyz7^{(1,1)}[z, t] - \\
& \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t]^2 Xyz7^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \Big) Xyz7^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \\
& 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] + \\
& \cos[z]^3 Xyz7^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 Xyz7^{(2,0)}[z, t]) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& Xyz7^{(2,1)}[z, t] - \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z] \sin[z]^2 A4'[t]^2 \\
& Xyz7^{(2,2)}[z, t] \Big) dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \\
& Xyz7^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] Xyz7^{(0,3)}[z, t] + \\
& \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t] \\
& \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \cos[z]^3 \text{XYZ7}^{(0,1)}[z, t] + A4'[t] \left( (-3 + M) (3 + M) \cos[z]^3 \right. \\
& \quad \left. \text{XYZ7}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t] \right) + \\
& 18 (-1 + e^{2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] \text{XYZ7}^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \text{XYZ7}^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] \text{XYZ7}^{(2,1)}[z, t] \Big) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) \text{XYZ7}[z, t] + \text{XYZ7}^{(0,2)}[z, t] + (-72 \tan[z] - \right. \right. \\
& \quad \left. \left. 36 \tan[z]^3) \text{XYZ7}^{(1,0)}[z, t] - 36 \tan[z]^2 \text{XYZ7}^{(2,0)}[z, t] \right) dz \right) \\
& \left( -\frac{1}{6} Q1 \left( (2 - 4 Q1 + e^{-2A4[t]} Q1) A4'[t]^2 - A4''[t] \right) + e^{2A4[t]} \right. \\
& \quad \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \\
& \quad \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6A4[t]} \left( M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t] \right) \Big) - \\
& 396 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} \text{XYZ7}^{(1,0)}[z, t] - \\
& 216 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^5 \sin[z]^{11/2} \text{XYZ7}^{(1,0)}[z, t] + \\
& 2 \sec[z] \sin[z]^{3/2} \left( e^{2A4[t]} (-117 + M^2 - 6 Q1^2 A4'[t]^2) + \right. \\
& \quad \left. e^{6A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2 e^{4A4[t]} (117 - M^2 + 2 Q1^2 A4'[t]^2) \right) \\
& \text{XYZ7}^{(1,0)}[z, t] + \sqrt{\sin[z]} \left( e^{2A4[t]} \text{XYZ7}[z, t] (-9 + M^2 - 6 Q1^2 A4'[t]^2) - \right. \\
& \quad \left. 2 e^{4A4[t]} \text{XYZ7}[z, t] (-9 + M^2 - 2 Q1^2 A4'[t]^2) + e^{6A4[t]} \text{XYZ7}[z, t] \right. \\
& \quad \left. (-9 + M^2 - Q1^2 A4'[t]^2) + (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \text{XYZ7}^{(0,2)}[z, t] - \right. \\
& \quad \left. (-4 + e^{-2A4[t]}) Q1^2 A4'[t]^2 (\text{XYZ7}[z, t] + 2 \tan[z] \text{XYZ7}^{(1,0)}[z, t]) \right) + \\
& 2 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z] \sin[z]^{3/2} \text{XYZ7}^{(1,2)}[z, t] - \\
& 324 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^2 \sin[z]^{5/2} \text{XYZ7}^{(2,0)}[z, t] - \\
& 216 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^4 \sin[z]^{9/2} \text{XYZ7}^{(2,0)}[z, t] - \\
& 72 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} \text{XYZ7}^{(3,0)}[z, t] \Big) dt \Big) \\
& \sin[Mt] A4'[t]^2 - e^{-\frac{1}{2} Q1 (e^{-2A4[t]} + 2 A4[t])} M Q1 (c22 \cos[Mt] - \\
& c21 \\
& \sin[ \\
& Mt] + \\
& e^{-4A4[t]} (c22 \cos[Mt] - c21 \sin[Mt]) + \\
& 2 \\
& e^{-2A4[t]} \\
& (-c22 \cos[Mt] + c21 \sin[Mt])) A4' [ \\
& t]^2 - \\
& 6 e^{-2A4[t]} Q1 \sqrt{\sin[z]} A4'[t]^2 (-\text{XYZ7}[z, t] - 
\end{aligned}$$

$$\begin{aligned}
& \frac{2}{\text{Tan}[z]} \\
& \frac{\text{Xyz7}^{(1,0)}[z, t] -}{z, t]}) - \\
& 3 Q1 \sqrt{\sin[z]} A4'[t]^2 (Xyz7[z, t] + \\
& 2 \\
& \frac{\text{Tan}[z]}{\text{Xyz7}^{(1,0)}[z, t]}) - \\
& 3 e^{-4 A4[t]} Q1 \sqrt{\sin[z]} A4'[t]^2 (Xyz7[z, t] + 2 \\
& \text{Tan}[z] \\
& \text{Xyz7}^{(1,0)}[z, t]) \Bigg), \\
& \frac{1}{\text{Xyz6}[z, t] = \frac{(-1 + e^{-2 A4[t]}) M Q1 \sqrt{\sin[z]} A4'[t]}{(-1 + e^{-2 A4[t]})^3 A4'[t]^3}} \\
& \left( -3 \right. \\
& e^{-\frac{1}{2} Q1 (e^{-2 A4[t]} + 2 A4[t])} \\
& (-1 + \\
& e^{-2 A4[t]}) \cos[M \\
& t] \\
& \left( \int \frac{1}{(-1 + e^{2 A4[t]})^3 A4'[t]^3} e^{\frac{1}{2} Q1 (e^{-2 A4[t]} + 2 A4[t])} \sin[M t] \right. \\
& \left( -\frac{4}{9} e^{3 A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \frac{1}{8} \cos[z]^3 (A4'[t]^2 (18 e^{A4[t]} - 2 e^{A4[t]} M^2 + \right. \right. \\
& e^{-A4[t]} (-9 + M^2) + e^{3 A4[t]} (-9 + M^2) + 8 \cosh[A4[t]] A4'[t]^2) + \\
& 4 \sinh[A4[t]] A4'[t]^2 A4''[t] + 2 (e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]}) A4''[t]^2) \\
& \text{Xyz7}^{(0,2)}[z, t] - \cos[z]^3 A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3 A4[t]} \right) A4''[t] \right) \text{Xyz7}^{(0,3)}[z, t] + \\
& \frac{1}{8} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]} \right) \cos[z]^3 A4'[t]^2 \text{Xyz7}^{(0,4)}[z, t] + \frac{1}{4} \\
& (e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 \text{Xyz7}[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] \text{Xyz7}^{(1,0)}[z, t]) + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \sinh[A4[t]] \text{Xyz7}^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 \text{Xyz7}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{Xyz7}^{(1,0)}[z, t])) + \\
& \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{Xyz7}^{(0,1)}[z, t] + \right. \\
& e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{Xyz7}^{(0,1)}[z, t] - \frac{1}{2} e^{3 A4[t]} (-3 + M)
\end{aligned}$$

$$\begin{aligned}
& (3 + M) \cos[z]^3 \text{XYZ7}^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 \text{XYZ7}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t]) + \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \text{XYZ7}^{(1,1)}[z, t] - \\
& \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t]^2 \text{XYZ7}^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \right) \text{XYZ7}^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \\
& 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 \text{XYZ7}[z, t] + \\
& \cos[z]^3 \text{XYZ7}^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \text{XYZ7}^{(2,0)}[z, t]) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& \text{XYZ7}^{(2,1)}[z, t] - \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z] \sin[z]^2 A4'[t]^2 \\
& \text{XYZ7}^{(2,2)}[z, t] \right) dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \\
& \left. \text{XYZ7}^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] \text{XYZ7}^{(0,3)}[z, t] + \right. \\
& \left. \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 \text{XYZ7}[z, t] - \right. \\
& \left. 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t]) + A4'[t] \right. \\
& \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{XYZ7}^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right. \right. \\
& \left. \left. \cos[z]^3 \text{XYZ7}^{(0,1)}[z, t] + A4'[t] ((-3 + M) (3 + M) \cos[z]^3 \right. \right. \\
& \left. \left. \text{XYZ7}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t]) \right) + \right. \\
& 18 (-1 + e^{2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] \text{XYZ7}^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \text{XYZ7}^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] \text{XYZ7}^{(2,1)}[z, t] \right) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] ((-9 + M^2) \text{XYZ7}[z, t] + \text{XYZ7}^{(0,2)}[z, t] + (-72 \tan[z] - \right. \\
& \left. 36 \tan[z]^3) \text{XYZ7}^{(1,0)}[z, t] - 36 \tan[z]^2 \text{XYZ7}^{(2,0)}[z, t]) dz \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{1}{6} Q1 \left( (2 - 4 Q1 + e^{-2 A4[t]} Q1) A4'[t]^2 - A4''[t] \right) + e^{2 A4[t]} \right. \\
& \quad \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4 A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \\
& \quad \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6 A4[t]} (M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t]) \Big) - \\
& 396 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^3 \sin[z]^{7/2} Xyz7^{(1,0)}[z, t] - \\
& 216 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^5 \sin[z]^{11/2} Xyz7^{(1,0)}[z, t] + \\
& 2 \operatorname{Sec}[z] \sin[z]^{3/2} (e^{2 A4[t]} (-117 + M^2 - 6 Q1^2 A4'[t]^2) + \\
& e^{6 A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2 e^{4 A4[t]} (117 - M^2 + 2 Q1^2 A4'[t]^2)) \\
& Xyz7^{(1,0)}[z, t] + \sqrt{\sin[z]} (e^{2 A4[t]} Xyz7[z, t] (-9 + M^2 - 6 Q1^2 A4'[t]^2) - \\
& 2 e^{4 A4[t]} Xyz7[z, t] (-9 + M^2 - 2 Q1^2 A4'[t]^2) + e^{6 A4[t]} Xyz7[z, t] \\
& (-9 + M^2 - Q1^2 A4'[t]^2) + (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) Xyz7^{(0,2)}[z, t] - \\
& (-4 + e^{-2 A4[t]}) Q1^2 A4'[t]^2 (Xyz7[z, t] + 2 \tan[z] Xyz7^{(1,0)}[z, t])) + \\
& 2 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z] \sin[z]^{3/2} Xyz7^{(1,2)}[z, t] - \\
& 324 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^2 \sin[z]^{5/2} Xyz7^{(2,0)}[z, t] - \\
& 216 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^4 \sin[z]^{9/2} Xyz7^{(2,0)}[z, t] - \\
& 72 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^3 \sin[z]^{7/2} Xyz7^{(3,0)}[z, t] \Big) dt \\
& A4'[t] + 3 e^{-\frac{1}{2} Q1 (e^{-2 A4[t]} + 2 A4[t])} (-1 + e^{-2 A4[t]}) \left( \int \frac{1}{(-1 + e^{2 A4[t]})^3 A4'[t]^3} \right. \\
& e^{\frac{1}{2} Q1 (e^{-2 A4[t]} + 2 A4[t])} \cos[M t] \\
& \left( -\frac{4}{9} e^{3 A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \operatorname{Sec}[z]^2 \left( \frac{1}{8} \cos[z]^3 (A4'[t]^2 (18 e^{A4[t]} - 2 e^{A4[t]} M^2 + \right. \right. \\
& e^{-A4[t]} (-9 + M^2) + e^{3 A4[t]} (-9 + M^2) + 8 \cosh[A4[t]] A4'[t]^2) + \\
& 4 \sinh[A4[t]] A4'[t]^2 A4''[t] + 2 (e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]}) A4''[t]^2) \\
& Xyz7^{(0,2)}[z, t] - \cos[z]^3 A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3 A4[t]} \right) A4''[t] \right) Xyz7^{(0,3)}[z, t] + \\
& \frac{1}{8} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]}) \cos[z]^3 A4'[t]^2 Xyz7^{(0,4)}[z, t] + \frac{1}{4} \\
& (e^{-A4[t]} - 2 e^{A4[t]} + e^{3 A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \sinh[A4[t]] Xyz7^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t])) + \\
& \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \right. \\
& \left. e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] - \frac{1}{2} e^{3 A4[t]} (-3 + M) \right)
\end{aligned}$$

$$\begin{aligned}
& (3 + M) \cos[z]^3 \text{XYZ7}^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 \text{XYZ7}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t]) + \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \text{XYZ7}^{(1,1)}[z, t] - \\
& \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t]^2 \text{XYZ7}^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \right) \text{XYZ7}^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \\
& 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 \text{XYZ7}[z, t] + \\
& \cos[z]^3 \text{XYZ7}^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \text{XYZ7}^{(2,0)}[z, t]) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& \text{XYZ7}^{(2,1)}[z, t] - \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z] \sin[z]^2 A4'[t]^2 \\
& \text{XYZ7}^{(2,2)}[z, t] dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \left. \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \right. \\
& \left. \left. \text{XYZ7}^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] \text{XYZ7}^{(0,3)}[z, t] + \right. \\
& \left. \left. \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 \text{XYZ7}[z, t] - \right. \right. \\
& \left. \left. 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t]) + A4'[t] \right. \right. \\
& \left. \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{XYZ7}^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right. \right. \right. \\
& \left. \left. \left. \cos[z]^3 \text{XYZ7}^{(0,1)}[z, t] + A4'[t] ((-3 + M) (3 + M) \cos[z]^3 \right. \right. \right. \\
& \left. \left. \left. \text{XYZ7}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{XYZ7}^{(1,0)}[z, t]) \right) + \right. \\
& 18 (-1 + e^{2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] \text{XYZ7}^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \text{XYZ7}^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] \text{XYZ7}^{(2,1)}[z, t] \right) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] ((-9 + M^2) \text{XYZ7}[z, t] + \text{XYZ7}^{(0,2)}[z, t] + (-72 \tan[z] - \right. \\
& \left. 36 \tan[z]^3) \text{XYZ7}^{(1,0)}[z, t] - 36 \tan[z]^2 \text{XYZ7}^{(2,0)}[z, t]) dz \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{1}{6} Q1 \left( (2 - 4 Q1 + e^{-2 A4[t]} Q1) A4'[t]^2 - A4''[t] \right) + e^{2 A4[t]} \right. \\
& \quad \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4 A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \\
& \quad \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6 A4[t]} (M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t]) \Big) - \\
& 396 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^3 \sin[z]^{7/2} Xyz7^{(1,0)}[z, t] - \\
& 216 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^5 \sin[z]^{11/2} Xyz7^{(1,0)}[z, t] + \\
& 2 \operatorname{Sec}[z] \sin[z]^{3/2} (e^{2 A4[t]} (-117 + M^2 - 6 Q1^2 A4'[t]^2) + \\
& e^{6 A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2 e^{4 A4[t]} (117 - M^2 + 2 Q1^2 A4'[t]^2)) \\
& Xyz7^{(1,0)}[z, t] + \sqrt{\sin[z]} (e^{2 A4[t]} Xyz7[z, t] (-9 + M^2 - 6 Q1^2 A4'[t]^2) - \\
& 2 e^{4 A4[t]} Xyz7[z, t] (-9 + M^2 - 2 Q1^2 A4'[t]^2) + e^{6 A4[t]} Xyz7[z, t] \\
& (-9 + M^2 - Q1^2 A4'[t]^2) + (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) Xyz7^{(0,2)}[z, t] - \\
& (-4 + e^{-2 A4[t]}) Q1^2 A4'[t]^2 (Xyz7[z, t] + 2 \tan[z] Xyz7^{(1,0)}[z, t])) + \\
& 2 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z] \sin[z]^{3/2} Xyz7^{(1,2)}[z, t] - \\
& 324 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^2 \sin[z]^{5/2} Xyz7^{(2,0)}[z, t] - \\
& 216 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^4 \sin[z]^{9/2} Xyz7^{(2,0)}[z, t] - \\
& 72 (e^{2 A4[t]} - 2 e^{4 A4[t]} + e^{6 A4[t]}) \operatorname{Sec}[z]^3 \sin[z]^{7/2} Xyz7^{(3,0)}[z, t] \Big) dt \\
& \sin[Mt] A4'[t] + M \left( -\frac{1}{6} \int \frac{1}{\sqrt{\sin[z]}} \cos[z] ((-9 + M^2) Xyz7[z, t] + Xyz7^{(0,2)}[z, t] \right. \\
& t] + (-72 \tan[z] - 36 \tan[z]^3) Xyz7^{(1,0)}[z, t] - 36 \tan[z]^2 Xyz7^{(2,0)}[z, t]) dz + \\
& e^{-\frac{1}{2} Q1 (e^{-2 A4[t]} + 2 A4[t])} (-1 + e^{-2 A4[t]}) Q1 (c21 \cos[Mt] + c22 \sin[Mt]) \\
& A4' [ \\
& t] \Big) \Big\}, \\
& Xyz7^{(0,4)}[z, t] = \frac{1}{(-1 + e^{2 A4[t]})^2 A4'[t]^2} \\
& \left( \left( -2 A4'[t]^2 \right. \right. \\
& \left. \left. \left( -9 + \right. \right. \right. \\
& M^2 + \\
& \left. \left. \left. \frac{1}{2} e^{-4 A4[t]} Q1^2 A4'[t]^2 - \right. \right. \\
& \left. \left. \left. 2 e^{-2 A4[t]} Q1 \left( \frac{1}{2} + Q1 \right) A4'[t]^2 + \right. \right. \\
& \left. \left. \left. (2 + 2 Q1 + 3 Q1^2) A4'[t]^2 + \right. \right. \right)
\end{aligned}$$

$$\begin{aligned}
& e^{4A4[t]} \left( -9 + M^2 + \frac{1}{2} Q1^2 A4'[t]^2 \right) - \\
& (-2 - 3 Q1 + e^{-2A4[t]} Q1 - e^{4A4[t]} Q1) A4'[t]^2 \\
& A4''[t] + \\
& (-2 - 2 e^{4A4[t]}) A4''[t]^2 + e^{2A4[t]} \\
& \left( 4 A4'[t]^2 \left( -9 + M^2 + \left( -1 + \frac{Q1}{2} + Q1^2 \right) A4'[t]^2 \right) - \right. \\
& 3 \left( \frac{2}{3} + Q1 \right) A4'[t]^2 A4''[t] + \\
& \left. 4 A4''[t]^2 \right) \\
& Xyz7^{(0,2)}[z, t] + 2 A4'[t] \left( (-2 - 3 Q1 + e^{-2A4[t]} Q1 - e^{4A4[t]} Q1) \right. \\
& A4'[t]^2 + \\
& e^{2A4[t]} \\
& \left( (2 + 3 Q1) A4'[t]^2 - \right. \\
& \left. 2 A4''[t] \right) + \\
& (1 + e^{4A4[t]}) A4''[t]) Xyz7^{(0,3)}[z, t] - 2 \left( 1 + \right. \\
& e^{4A4[t]} \left. \right) A4''[t]^2 \\
& \left( (-9 + M^2) Xyz7[z, t] + (-72 \tan[z] - 36 \tan[z]^3) \right. \\
& Xyz7^{(1,0)}[z, t] - A4'[t] \\
& t]^2 \left( e^{-4A4[t]} M^2 Q1^2 \right. \\
& Xyz7[z, t] A4'[t]^2 + Xyz7[z, t] \\
& \left. \left( (-3 + M)^2 (3 + M)^2 + (-36 + 4 M^2 - 36 Q1 + 4 M^2 Q1 + 6 M^2 Q1^2) A4'[t]^2 \right) + 6 \right. \\
& (-3 + M) \\
& (3 + M) \\
& \left( \frac{2}{3} + Q1 \right) \\
& A4'[t] Xyz7^{(0,1)}[z, t] - 72 \tan[z] \\
& \left. \left( -90 + 2 M^2 + (-405 + M^2) \tan[z]^2 - 594 \tan[z]^4 - \right. \right. \\
& 270 \tan[z]^6 + \\
& \left. \left. 2 (1 + Q1) (2 + \tan[z]^2) A4'[t]^2 \right) Xyz7^{(1,0)}[z, t] + \right)
\end{aligned}$$

$$\begin{aligned}
& e^{4A4[t]} \left( XYZ7[z, t] \left( (-3 + M)^2 (3 + M)^2 + M^2 Q1^2 A4'[t]^2 \right) + \right. \\
& \quad \left( -18 + 2M^2 \right) Q1 A4'[t] XYZ7^{(0,1)}[z, t] - \\
& \quad 72 \tan[z] \left( -90 + 2M^2 + (-405 + M^2) \tan[z]^2 - 594 \tan[z]^4 - 270 \tan[z]^6 \right) \\
& \quad \left. XYZ7^{(1,0)}[z, t] \right) - \\
& 2 e^{-2A4[t]} Q1 A4'[t] \left( 2 \left( -\frac{9}{2} + \frac{M^2}{2} + M^2 Q1 \right) XYZ7[z, t] A4'[t] + \right. \\
& \quad \left. \left( -9 + M^2 \right) XYZ7^{(0,1)}[z, t] - \right. \\
& \quad \left. 36 \tan[z] \left( 2 + \tan[z]^2 \right) A4'[t] XYZ7^{(1,0)}[z, t] \right) \Bigg) - \\
& A4'[t] A4''[t] \left( -2 (-3 + M) (3 + M) \left( \frac{3}{2} \left( \frac{2}{3} + Q1 \right) XYZ7[z, t] A4'[t] + \right. \right. \\
& \quad \left. \left. XYZ7^{(0,1)}[z, t] \right) + \right. \\
& \quad \left. 108 \left( \frac{2}{3} + Q1 \right) \tan[z] \left( 2 + \tan[z]^2 \right) A4'[t] XYZ7^{(1,0)}[z, t] \right) + \\
& z, \\
& t] + e^{-2A4[t]} Q1 A4'[ \\
& t] \\
& \left( (-9 + M^2) XYZ7[z, t] + (-72 \tan[z] - 36 \tan[z]^3) XYZ7^{(1,0)}[z, t] \right) + \\
& e^{4A4[t]} \left( -2 (-3 + M) (3 + M) \left( \frac{1}{2} Q1 XYZ7[z, t] A4'[t] + XYZ7^{(0,1)}[z, t] \right) + \right. \\
& \quad \left. 36 Q1 \tan[z] \left( 2 + \tan[z]^2 \right) A4'[t] XYZ7^{(1,0)}[z, t] \right) \Bigg) + \\
& e^{2A4[t]} \left( A4''[t]^2 \left( (-36 + 4M^2) XYZ7[z, t] + (-288 \tan[z] - 144 \tan[z]^3) \right. \right. \\
& \quad \left. \left. XYZ7^{(1,0)}[z, t] \right) - 4 A4'[t] A4''[t] \right. \\
& \quad \left. \left( (-3 + M) (3 + M) \left( \frac{3}{4} \left( \frac{2}{3} + Q1 \right) XYZ7[z, t] A4'[t] + XYZ7^{(0,1)}[z, t] \right) - \right. \right. \\
& \quad \left. \left. 27 \left( \frac{2}{3} + Q1 \right) \tan[z] \left( 2 + \tan[z]^2 \right) A4'[t] XYZ7^{(1,0)}[z, t] \right) + \right. \\
& \quad \left. 2 A4'[t]^2 \left( XYZ7[z, t] \left( (-3 + M)^2 (3 + M)^2 + (18 - 2M^2 - 9Q1 + M^2 Q1 + 2M^2 Q1^2) A4'[t]^2 \right) + \right. \right. \\
& \quad \left. \left. 3 (-3 + M) (3 + M) \left( \frac{2}{3} + Q1 \right) A4'[t] XYZ7^{(0,1)}[z, t] - \right. \right. \\
& \quad \left. \left. 72 \tan[z] \left( -90 + 2M^2 + (-405 + M^2) \tan[z]^2 - 594 \tan[z]^4 - \right. \right. \right. \\
& \quad \left. \left. \left. 270 \tan[z]^6 + \frac{1}{2} (-2 + Q1) \left( 2 + \tan[z]^2 \right) A4'[t]^2 \right) XYZ7^{(1,0)}[z, t] \right) \right) - \\
& 72 \tan[z] \left( 2 + \tan[z]^2 \right) A4'[t] \left( (-2 - 3Q1 + e^{-2A4[t]} Q1 - e^{4A4[t]} Q1) \right. \\
& \quad \left. A4'[t]^2 + e^{2A4[t]} \right. \\
& \quad \left. \left( (2 + 3Q1) A4'[t]^2 - 2 A4''[t] \right) + \right. \\
& \quad \left. \left( 1 + e^{4A4[t]} \right) \right. \\
& \quad \left. A4''[ \right. \\
& \quad \left. t] \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Xyz7}^{(1,1)}[z, t] + 72 (-1 + e^{2A4[t]})^2 \tan[z] \\
& (2 + \tan[z]^2) \\
& A4'[t]^2 \text{Xyz7}^{(1,2)}[z, t] - 144 \\
& \tan[z]^2 \\
& \left( -\frac{1}{2} A4'[t]^2 (-261 + M^2 - 504 \tan[z]^2 - 270 \tan[z]^4 + \right. \\
& e^{4A4[t]} (-261 + M^2 - 504 \tan[z]^2 - 270 \tan[z]^4) - \\
& e^{-2A4[t]} Q1 A4'[t]^2 + (2 + 2 Q1) A4'[t]^2 \Big) - \\
& \frac{1}{4} (-2 - 3 Q1 + e^{-2A4[t]} Q1 - e^{4A4[t]} Q1) \\
& A4'[t]^2 \\
& A4''[t] + \left( -\frac{1}{2} - \frac{1}{2} e^{4A4[t]} \right) A4''[t]^2 + \\
& e^{2A4[t]} \left( A4'[t]^2 \left( -261 + M^2 - 504 \tan[z]^2 - 270 \tan[z]^4 + \left( -1 + \frac{Q1}{2} \right) A4'[t]^2 \right) - \right. \\
& \left. \frac{3}{4} \left( \frac{2}{3} + Q1 \right) A4'[t]^2 A4''[t] + A4''[t]^2 \right) \\
& \text{Xyz7}^{(2,0)}[z, t] + (-1 + e^{2A4[t]})^2 A4'[t] \\
& A4^{(3)}[t] \\
& \left( (-9 + M^2) \right. \\
& \text{Xyz7}[z, t] + \\
& \left. \text{Xyz7}^{(0,2)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3) \right. \\
& \text{Xyz7}^{(1,0)}[z, t] - 36 \tan[z]^2 \text{Xyz7}^{(2,0)}[z, t] \\
& \left. \text{Xyz7}^{(2,1)}[z, t] + 72 (-1 + e^{2A4[t]})^2 \tan[z]^2 A4'[t]^2 \right. \\
& \left. e^{2A4[t]} ((2 + 3 Q1) A4'[t]^2 - 2 A4''[t]) + (1 + e^{4A4[t]}) A4''[t] \right) \\
& \text{Xyz7}^{(2,2)}[z, t] - 7776 (-1 + e^{2A4[t]})^2
\end{aligned}$$

$$\begin{aligned} & \frac{\tan[z]^3}{z} \\ & \left( \frac{4}{3} + \tan[z]^2 \right) A4'[t]^2 Xyz7^{(3,0)}[z, t] - \\ & 1296 \\ & (-1 + e^{2A4[t]})^2 \tan[z]^4 A4'[t]^2 Xyz7^{(4,0)}[z, t] \end{aligned}$$

In[•]:= solvedEQ2L9i

Out[•]=

$$\begin{aligned} & \left\{ Xyz4[z, t] = \frac{1}{(-1 + e^{-2A4[t]}) M Q1 A4'[t]} \right. \\ & \left( (-9 + M^2) Xyz7[z, t] + Q1 A4'[t] Xyz7^{(0,1)}[z, t] - e^{-2A4[t]} Q1 A4'[t] Xyz7^{(0,1)}[z, t] + \right. \\ & Xyz7^{(0,2)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3) Xyz7^{(1,0)}[z, t] - 36 \tan[z]^2 Xyz7^{(2,0)}[z, t] \left. \right), \\ & Xyz5[z, t] = \frac{1}{(-1 + e^{-2A4[t]})^2 M Q1 \sqrt{\sin[z]} A4'[t]^2} \\ & \left( \frac{1}{3} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( e^{-2A4[t]} A4'[t]^2 + \frac{1}{2} (1 - e^{-2A4[t]}) A4''[t] \right) Xyz7^{(0,2)}[z, t] + \right. \right. \\ & \frac{1}{2} (-1 + e^{-2A4[t]}) \cos[z]^3 A4'[t] Xyz7^{(0,3)}[z, t] - \frac{1}{2} (-1 + e^{-2A4[t]}) A4''[t] \\ & ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + \\ & A4'[t] \left( -\frac{1}{2} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \right. \\ & e^{-2A4[t]} \left( (-3 + M) (3 + M) \cos[z]^3 \left( Xyz7[z, t] A4'[t] + \frac{1}{2} Xyz7^{(0,1)}[z, t] \right) - \right. \\ & 36 (1 + \cos[z]^2) \sin[z] A4'[t] Xyz7^{(1,0)}[z, t] \left. \right) - \\ & 18 (-1 + e^{-2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] Xyz7^{(1,1)}[z, t] - \\ & 36 \cos[z] \sin[z]^2 \left( e^{-2A4[t]} A4'[t]^2 + \frac{1}{2} (1 - e^{-2A4[t]}) A4''[t] \right) Xyz7^{(2,0)}[z, t] - \\ & 18 (-1 + e^{-2A4[t]}) \cos[z] \sin[z]^2 A4'[t] Xyz7^{(2,1)}[z, t] \left. \right) dz - \\ & \frac{1}{3} \left( \frac{1}{2} + \frac{1}{2} e^{-4A4[t]} - e^{-2A4[t]} \right) Q1 \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) Xyz7[z, t] + Xyz7^{(0,2)}[z, t] + \right. \right. \\ & (-72 \tan[z] - 36 \tan[z]^3) Xyz7^{(1,0)}[z, t] - 36 \tan[z]^2 Xyz7^{(2,0)}[z, t] \left. \right) dz \right) A4'[t]^2 - \\ & 3 \left( e^{-\frac{1}{2} Q1 (e^{-2A4[t]} + 2A4[t])} - 2 e^{-\frac{1}{2} e^{-2A4[t]} Q1 - (2+Q1) A4[t]} + e^{-\frac{1}{2} e^{-2A4[t]} Q1 - (4+Q1) A4[t]} \right) \cos[M t] \\ & \left( \int \frac{1}{(-1 + e^{2A4[t]})^3 A4'[t]^3} e^{\frac{1}{2} Q1 (e^{-2A4[t]} + 2A4[t])} \cos[M t] \right. \end{aligned}$$

$$\begin{aligned}
& \left( -\frac{4}{9} e^{3A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec^2[z] \left( \frac{1}{8} \cos[z]^3 (A4'[t]^2 (18 e^{A4[t]} - 2 e^{A4[t]} M^2 + \right. \right. \\
& \quad \left. \left. e^{-A4[t]} (-9 + M^2) + e^{3A4[t]} (-9 + M^2) + 8 \cosh[A4[t]] A4'[t]^2 ) + \right. \right. \\
& \quad \left. \left. 4 \sinh[A4[t]] A4'[t]^2 A4''[t] + 2 (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 \right) \right. \\
& \quad \left. Xyz7^{(0,2)}[z, t] - \cos[z]^3 A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \right. \\
& \quad \left. \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) Xyz7^{(0,3)}[z, t] + \right. \\
& \quad \left. \frac{1}{8} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) \cos[z]^3 A4'[t]^2 Xyz7^{(0,4)}[z, t] + \frac{1}{4} \right. \\
& \quad \left. (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \right. \\
& \quad \left. 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t]^3 ((9 - M^2) \cos[z]^3 \right. \\
& \quad \left. \sinh[A4[t]] Xyz7^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \right. \\
& \quad \left. \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) \right) + \\
& \quad \left. \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \right. \right. \\
& \quad \left. \left. e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \right. \right. \\
& \quad \left. \left. (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \right. \right. \\
& \quad \left. \left. \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) \right) + \right. \\
& \quad \left. 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \right. \\
& \quad \left. \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) Xyz7^{(1,1)}[z, t] - \right. \\
& \quad \left. \frac{9}{2} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) (1 + \cos[z]^2) \sin[z] A4'[t]^2 Xyz7^{(1,2)}[z, t] - \right. \\
& \quad \left. 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \right. \\
& \quad \left. \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \right) Xyz7^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \right. \\
& \quad \left. 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] + \right. \\
& \quad \left. \cos[z]^3 Xyz7^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t] - \right. \\
& \quad \left. 36 \cos[z] \sin[z]^2 Xyz7^{(2,0)}[z, t] \right) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \quad \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& \quad Xyz7^{(2,1)}[z, t] - \frac{9}{2} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) \cos[z] \sin[z]^2 A4'[t]^2 \\
& \quad Xyz7^{(2,2)}[z, t] \right) dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \quad \left. \int \frac{1}{\sqrt{\sin[z]}} \sec^2[z] \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \text{Xyz7}^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] \text{Xyz7}^{(0,3)}[z, t] + \\
& \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 \text{Xyz7}[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] \text{Xyz7}^{(1,0)}[z, t]) + A4'[t] \\
& \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 \text{Xyz7}^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right. \\
& \cos[z]^3 \text{Xyz7}^{(0,1)}[z, t] + A4'[t] ((-3 + M) (3 + M) \cos[z]^3 \\
& \text{Xyz7}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{Xyz7}^{(1,0)}[z, t]) \Big) + \\
& 18 (-1 + e^{2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] \text{Xyz7}^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \text{Xyz7}^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] \text{Xyz7}^{(2,1)}[z, t] \Big) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] ((-9 + M^2) \text{Xyz7}[z, t] + \text{Xyz7}^{(0,2)}[z, t] + (-72 \tan[z] - \right. \\
& \left. 36 \tan[z]^3) \text{Xyz7}^{(1,0)}[z, t] - 36 \tan[z]^2 \text{Xyz7}^{(2,0)}[z, t]) dz \right) \\
& \left( -\frac{1}{6} Q1 ((2 - 4 Q1 + e^{-2A4[t]} Q1) A4'[t]^2 - A4''[t]) + e^{2A4[t]} \right. \\
& \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \\
& \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6A4[t]} (M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t]) \Big) - \\
& 396 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^3 \sin[z]^{7/2} \text{Xyz7}^{(1,0)}[z, t] - \\
& 216 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^5 \sin[z]^{11/2} \text{Xyz7}^{(1,0)}[z, t] + \\
& 2 \sec[z] \sin[z]^{3/2} (e^{2A4[t]} (-117 + M^2 - 6 Q1^2 A4'[t]^2) + \\
& e^{6A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2 e^{4A4[t]} (117 - M^2 + 2 Q1^2 A4'[t]^2)) \\
& \text{Xyz7}^{(1,0)}[z, t] + \sqrt{\sin[z]} (e^{2A4[t]} \text{Xyz7}[z, t] (-9 + M^2 - 6 Q1^2 A4'[t]^2) - \\
& 2 e^{4A4[t]} \text{Xyz7}[z, t] (-9 + M^2 - 2 Q1^2 A4'[t]^2) + e^{6A4[t]} \text{Xyz7}[z, t] \\
& (-9 + M^2 - Q1^2 A4'[t]^2) + (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \text{Xyz7}^{(0,2)}[z, t] - \\
& (-4 + e^{-2A4[t]}) Q1^2 A4'[t]^2 (\text{Xyz7}[z, t] + 2 \tan[z] \text{Xyz7}^{(1,0)}[z, t])) + \\
& 2 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z] \sin[z]^{3/2} \text{Xyz7}^{(1,2)}[z, t] - \\
& 324 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^2 \sin[z]^{5/2} \text{Xyz7}^{(2,0)}[z, t] - \\
& 216 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^4 \sin[z]^{9/2} \text{Xyz7}^{(2,0)}[z, t] - \\
& 72 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^3 \sin[z]^{7/2} \text{Xyz7}^{(3,0)}[z, t] \Big) dt \Big) \\
& A4'[t]^2 - 3 \left( e^{-\frac{1}{2} Q1 (e^{-2A4[t]} + 2 A4[t])} - 2 e^{-\frac{1}{2} e^{-2A4[t]} Q1 - (2 + Q1) A4[t]} + \right. \\
& \left. e^{-\frac{1}{2} e^{-2A4[t]} Q1 - (4 + Q1) A4[t]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \int \frac{1}{(-1 + e^{2A4[t]})^3 A4'[t]^3} e^{\frac{1}{2} Q1 (e^{-2A4[t]} + 2A4[t])} \sin[Mt] \right. \\
& \quad \left( -\frac{4}{9} e^{3A4[t]} \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \frac{1}{8} \cos[z]^3 (A4'[t]^2 (18 e^{A4[t]} - 2 e^{A4[t]} M^2 + \right. \right. \\
& \quad \left. \left. e^{-A4[t]} (-9 + M^2) + e^{3A4[t]} (-9 + M^2) + 8 \cosh[A4[t]] A4'[t]^2) + \right. \right. \\
& \quad \left. 4 \sinh[A4[t]] A4'[t]^2 A4''[t] + 2 (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 \right) \\
& \quad Xyz7^{(0,2)}[z, t] - \cos[z]^3 A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \quad \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) Xyz7^{(0,3)}[z, t] + \\
& \quad \frac{1}{8} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) \cos[z]^3 A4'[t]^2 Xyz7^{(0,4)}[z, t] + \frac{1}{4} \\
& \quad (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& \quad 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \quad \sinh[A4[t]] Xyz7^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \quad \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t])) + \\
& \quad \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \right. \\
& \quad \left. e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \right. \\
& \quad (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \quad \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + \\
& \quad 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \quad \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) Xyz7^{(1,1)}[z, t] - \\
& \quad \frac{9}{2} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) (1 + \cos[z]^2) \sin[z] A4'[t]^2 Xyz7^{(1,2)}[z, t] - \\
& \quad 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \quad \left. \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \right) Xyz7^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \\
& \quad 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] + \\
& \quad \cos[z]^3 Xyz7^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t] - \\
& \quad 36 \cos[z] \sin[z]^2 Xyz7^{(2,0)}[z, t]) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \quad \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& \quad Xyz7^{(2,1)}[z, t] - \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z] \sin[z]^2 A4'[t]^2 \\
& \quad Xyz7^{(2,2)}[z, t] \right) dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right)
\end{aligned}$$

$$\begin{aligned}
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \\
& \quad Xyz7^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] Xyz7^{(0,3)}[z, t] + \\
& \quad \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& \quad 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t] \\
& \quad \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right. \\
& \quad \cos[z]^3 Xyz7^{(0,1)}[z, t] + A4'[t] ((-3 + M) (3 + M) \cos[z]^3 \\
& \quad Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) \Big) + \\
& \quad 18 (-1 + e^{2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] Xyz7^{(1,1)}[z, t] - \\
& \quad 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) Xyz7^{(2,0)}[z, t] + \\
& \quad \left. 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] Xyz7^{(2,1)}[z, t] \right) dz + \\
& \quad \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] ((-9 + M^2) Xyz7[z, t] + Xyz7^{(0,2)}[z, t] + (-72 \tan[z] - \right. \\
& \quad \left. 36 \tan[z]^3) Xyz7^{(1,0)}[z, t] - 36 \tan[z]^2 Xyz7^{(2,0)}[z, t]) dz \right) \\
& \quad \left( -\frac{1}{6} Q1 ((2 - 4 Q1 + e^{-2A4[t]} Q1) A4'[t]^2 - A4''[t]) + e^{2A4[t]} \right. \\
& \quad \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \\
& \quad \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6A4[t]} (M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t]) \Big) - \\
& \quad 396 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^3 \sin[z]^{7/2} Xyz7^{(1,0)}[z, t] - \\
& \quad 216 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^5 \sin[z]^{11/2} Xyz7^{(1,0)}[z, t] + \\
& \quad 2 \sec[z] \sin[z]^{3/2} (e^{2A4[t]} (-117 + M^2 - 6 Q1^2 A4'[t]^2) + \\
& \quad e^{6A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2 e^{4A4[t]} (117 - M^2 + 2 Q1^2 A4'[t]^2)) \\
& \quad Xyz7^{(1,0)}[z, t] + \sqrt{\sin[z]} (e^{2A4[t]} Xyz7[z, t] (-9 + M^2 - 6 Q1^2 A4'[t]^2) - \\
& \quad 2 e^{4A4[t]} Xyz7[z, t] (-9 + M^2 - 2 Q1^2 A4'[t]^2) + e^{6A4[t]} Xyz7[z, t] \\
& \quad (-9 + M^2 - Q1^2 A4'[t]^2) + (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) Xyz7^{(0,2)}[z, t] - \\
& \quad (-4 + e^{-2A4[t]}) Q1^2 A4'[t]^2 (Xyz7[z, t] + 2 \tan[z] Xyz7^{(1,0)}[z, t])) + \\
& \quad 2 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z] \sin[z]^{3/2} Xyz7^{(1,2)}[z, t] - \\
& \quad 324 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^2 \sin[z]^{5/2} Xyz7^{(2,0)}[z, t] - \\
& \quad 216 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^4 \sin[z]^{9/2} Xyz7^{(2,0)}[z, t] - \\
& \quad \left. 72 (e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]}) \sec[z]^3 \sin[z]^{7/2} Xyz7^{(3,0)}[z, t] \right) dt \Big) \\
& \quad \sin[Mt] A4'[t]^2 - e^{-\frac{1}{2} Q1 (e^{-2A4[t]} + 2 A4[t])} M Q1 (c22 \cos[Mt] - 
\end{aligned}$$

$$\begin{aligned}
& c21 \\
& \left. \begin{aligned}
& \text{Sin}[ \\
& \quad \text{M}\text{t}]) \\
& A4'[t]^2 - e^{-\frac{1}{2} \text{e}^{-2 \text{A4}[t]} \text{Q1} - (4+\text{Q1}) \text{A4}[t]} \text{M Q1} (\text{c22} \text{Cos}[\text{M}\text{t}] - c21 \\
& \quad \text{Sin}[ \\
& \quad \quad \text{M}\text{t}]) \\
& A4'[t]^2 - 2 e^{-\frac{1}{2} \text{e}^{-2 \text{A4}[t]} \text{Q1} - (2+\text{Q1}) \text{A4}[t]} \text{M Q1} \\
& (-c22 \\
& \quad \text{Cos}[\text{M}\text{t}] + \\
& \quad c21 \text{Sin}[\text{M}\text{t}]) \text{A4}'[t]^2 - \\
& 6 \text{e}^{-2 \text{A4}[t]} \text{Q1} \sqrt{\text{Sin}[z]} \text{A4}'[t]^2 (-\text{XYZ7}[z, t] - \\
& 2 \\
& \quad \text{Tan}[z] \\
& \quad \text{XYZ7}^{(1,0)}[z, t]) - \\
& 3 \text{Q1} \sqrt{\text{Sin}[z]} \text{A4}'[t]^2 (\text{XYZ7}[z, t] + \\
& 2 \\
& \quad \text{Tan}[z] \\
& \quad \text{XYZ7}^{(1,0)}[z, t]) - \\
& 3 \text{e}^{-4 \text{A4}[t]} \text{Q1} \sqrt{\text{Sin}[z]} \text{A4}'[t]^2 (\text{XYZ7}[ \\
& \quad z, \\
& \quad t] + 2 \\
& \quad \text{Tan}[z] \\
& \quad \text{XYZ7}^{(1,0)}[z, t]) \Bigg), \\
& \text{XYZ6}[z, t] = \frac{1}{(-1 + \text{e}^{-2 \text{A4}[t]}) \text{M Q1} \sqrt{\text{Sin}[z]} \text{A4}'[t]} \\
& \left. \begin{aligned}
& (-3 \\
& \quad e^{-\frac{1}{2} \text{Q1} (\text{e}^{-2 \text{A4}[t]} + 2 \text{A4}[t])} \\
& \quad (-1 + \\
& \quad \text{e}^{-2 \text{A4}[t]}) \text{Cos}[\text{M} \\
& \quad \text{t}] \\
& \quad \left( \int \frac{1}{(-1 + \text{e}^{2 \text{A4}[t]})^3 \text{A4}'[t]^3} \text{e}^{\frac{1}{2} \text{Q1} (\text{e}^{-2 \text{A4}[t]} + 2 \text{A4}[t])} \text{Sin}[\text{M}\text{t}] \right. \\
& \quad \left. \left( -\frac{4}{9} \text{e}^{3 \text{A4}[t]} \int \frac{1}{\sqrt{\text{Sin}[z]}} \text{Sec}[z]^2 \left( \frac{1}{8} \text{Cos}[z]^3 (\text{A4}'[t]^2 (18 \text{e}^{\text{A4}[t]} - 2 \text{e}^{\text{A4}[t]} \text{M}^2 + \right. \right. \\
& \quad \left. \left. \text{e}^{-\text{A4}[t]} (-9 + \text{M}^2) + \text{e}^{3 \text{A4}[t]} (-9 + \text{M}^2) + 8 \text{Cosh}[\text{A4}[t]] \text{A4}'[t]^2 \right) + \right. \\
& \quad \left. 4 \text{Sinh}[\text{A4}[t]] \text{A4}'[t]^2 \text{A4}''[t] + 2 (\text{e}^{-\text{A4}[t]} - 2 \text{e}^{\text{A4}[t]} + \text{e}^{3 \text{A4}[t]}) \text{A4}''[t]^2 \right) \\
& \quad \text{XYZ7}^{(0,2)}[z, t] - \text{Cos}[z]^3 \text{A4}'[t] \left( \text{Sinh}[\text{A4}[t]] \text{A4}'[t]^2 + \right.
\end{aligned} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) Xyz7^{(0,3)}[z, t] + \\
& \frac{1}{8} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) \cos[z]^3 A4'[t]^2 Xyz7^{(0,4)}[z, t] + \frac{1}{4} \\
& (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \sinh[A4[t]] Xyz7^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t])) + \\
& \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \right. \\
& e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \\
& (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) \Big) + \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \Big) Xyz7^{(1,1)}[z, t] - \\
& \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t]^2 Xyz7^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \Big) Xyz7^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \\
& 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] + \\
& \cos[z]^3 Xyz7^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 Xyz7^{(2,0)}[z, t]) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& Xyz7^{(2,1)}[z, t] - \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z] \sin[z]^2 A4'[t]^2 \\
& Xyz7^{(2,2)}[z, t] \Big) dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \\
& Xyz7^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] Xyz7^{(0,3)}[z, t] + \\
& \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t] \\
& \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) Xyz7^{(0,3)}[z, t] + \\
& \frac{1}{8} \left( e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]} \right) \cos[z]^3 A4'[t]^2 Xyz7^{(0,4)}[z, t] + \frac{1}{4} \\
& (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) A4''[t]^2 ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t]^3 ((9 - M^2) \cos[z]^3 \\
& \sinh[A4[t]] Xyz7^{(0,1)}[z, t] + \cosh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t])) + \\
& \frac{1}{2} A4'[t] A4''[t] \left( -\frac{1}{2} e^{-A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \right. \\
& e^{A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] - \frac{1}{2} e^{3A4[t]} (-3 + M) \\
& (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \sinh[A4[t]] A4'[t] ((-3 + M) (3 + M) \\
& \cos[z]^3 Xyz7[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) \Big) + \\
& 36 (1 + \cos[z]^2) \sin[z] A4'[t] \left( \sinh[A4[t]] A4'[t]^2 + \right. \\
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \Big) Xyz7^{(1,1)}[z, t] - \\
& \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t]^2 Xyz7^{(1,2)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( \cosh[A4[t]] A4'[t]^4 + \frac{1}{2} \sinh[A4[t]] A4'[t]^2 A4''[t] + \right. \\
& \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t]^2 \Big) Xyz7^{(2,0)}[z, t] - \frac{1}{8} (e^{-A4[t]} - \\
& 2 e^{A4[t]} + e^{3A4[t]}) A4'[t] A4^{(3)}[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] + \\
& \cos[z]^3 Xyz7^{(0,2)}[z, t] - 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 Xyz7^{(2,0)}[z, t]) + 36 \cos[z] \sin[z]^2 A4'[t] \\
& \left( \sinh[A4[t]] A4'[t]^2 + \frac{1}{2} \left( \frac{1}{2} e^{-A4[t]} - e^{A4[t]} + \frac{1}{2} e^{3A4[t]} \right) A4''[t] \right) \\
& Xyz7^{(2,1)}[z, t] - \frac{9}{2} (e^{-A4[t]} - 2 e^{A4[t]} + e^{3A4[t]}) \cos[z] \sin[z]^2 A4'[t]^2 \\
& Xyz7^{(2,2)}[z, t] \Big) dz + A4'[t]^2 \left( -\frac{4}{9} \left( -\frac{1}{2} + e^{2A4[t]} - \frac{1}{2} e^{4A4[t]} \right) Q1 \right. \\
& \int \frac{1}{\sqrt{\sin[z]}} \sec[z]^2 \left( \cos[z]^3 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \right. \\
& Xyz7^{(0,2)}[z, t] - \frac{1}{2} (-1 + e^{2A4[t]}) \cos[z]^3 A4'[t] Xyz7^{(0,3)}[z, t] + \\
& \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] ((-3 + M) (3 + M) \cos[z]^3 Xyz7[z, t] - \\
& 36 (1 + \cos[z]^2) \sin[z] Xyz7^{(1,0)}[z, t]) + A4'[t] \\
& \left. \left( -\frac{1}{2} e^{2A4[t]} (-3 + M) (3 + M) \cos[z]^3 Xyz7^{(0,1)}[z, t] + \frac{1}{2} (-9 + M^2) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \cos[z]^3 \text{Xyz7}^{(0,1)}[z, t] + A4'[t] \left( (-3 + M) (3 + M) \cos[z]^3 \right. \\
& \quad \left. \text{Xyz7}[z, t] - 36 (1 + \cos[z]^2) \sin[z] \text{Xyz7}^{(1,0)}[z, t] \right) + \\
& 18 (-1 + e^{2A4[t]}) (1 + \cos[z]^2) \sin[z] A4'[t] \text{Xyz7}^{(1,1)}[z, t] - \\
& 36 \cos[z] \sin[z]^2 \left( A4'[t]^2 + \frac{1}{2} (-1 + e^{2A4[t]}) A4''[t] \right) \text{Xyz7}^{(2,0)}[z, t] + \\
& 18 (-1 + e^{2A4[t]}) \cos[z] \sin[z]^2 A4'[t] \text{Xyz7}^{(2,1)}[z, t] \Big) dz + \\
& \frac{1}{3} \left( \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) \text{Xyz7}[z, t] + \text{Xyz7}^{(0,2)}[z, t] + (-72 \tan[z] - \right. \right. \\
& \quad \left. \left. 36 \tan[z]^3) \text{Xyz7}^{(1,0)}[z, t] - 36 \tan[z]^2 \text{Xyz7}^{(2,0)}[z, t] \right) dz \right) \\
& \left( -\frac{1}{6} Q1 \left( (2 - 4 Q1 + e^{-2A4[t]} Q1) A4'[t]^2 - A4''[t] \right) + e^{2A4[t]} \right. \\
& \quad \left( -\frac{M^2}{6} + \left( \frac{2 Q1}{3} - Q1^2 \right) A4'[t]^2 - \frac{1}{2} Q1 A4''[t] \right) + e^{4A4[t]} \left( \frac{M^2}{3} + \frac{1}{3} (-Q1 + 2 Q1^2) \right. \\
& \quad \left. A4'[t]^2 + \frac{1}{2} Q1 A4''[t] \right) - \frac{1}{6} e^{6A4[t]} \left( M^2 + Q1^2 A4'[t]^2 + Q1 A4''[t] \right) \Big) - \\
& 396 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} \text{Xyz7}^{(1,0)}[z, t] - \\
& 216 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^5 \sin[z]^{11/2} \text{Xyz7}^{(1,0)}[z, t] + \\
& 2 \sec[z] \sin[z]^{3/2} \left( e^{2A4[t]} (-117 + M^2 - 6 Q1^2 A4'[t]^2) + \right. \\
& \quad \left. e^{6A4[t]} (-117 + M^2 - Q1^2 A4'[t]^2) + 2 e^{4A4[t]} (117 - M^2 + 2 Q1^2 A4'[t]^2) \right) \\
& \text{Xyz7}^{(1,0)}[z, t] + \sqrt{\sin[z]} \left( e^{2A4[t]} \text{Xyz7}[z, t] (-9 + M^2 - 6 Q1^2 A4'[t]^2) - \right. \\
& \quad 2 e^{4A4[t]} \text{Xyz7}[z, t] (-9 + M^2 - 2 Q1^2 A4'[t]^2) + e^{6A4[t]} \text{Xyz7}[z, t] \\
& \quad (-9 + M^2 - Q1^2 A4'[t]^2) + \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \text{Xyz7}^{(0,2)}[z, t] - \\
& \quad (-4 + e^{-2A4[t]}) Q1^2 A4'[t]^2 (\text{Xyz7}[z, t] + 2 \tan[z] \text{Xyz7}^{(1,0)}[z, t]) + \\
& 2 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z] \sin[z]^{3/2} \text{Xyz7}^{(1,2)}[z, t] - \\
& 324 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^2 \sin[z]^{5/2} \text{Xyz7}^{(2,0)}[z, t] - \\
& 216 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^4 \sin[z]^{9/2} \text{Xyz7}^{(2,0)}[z, t] - \\
& \left. 72 \left( e^{2A4[t]} - 2 e^{4A4[t]} + e^{6A4[t]} \right) \sec[z]^3 \sin[z]^{7/2} \text{Xyz7}^{(3,0)}[z, t] \right) dt \Big) \\
& \sin[M t] A4'[t] + M \left( -\frac{1}{6} \int \frac{1}{\sqrt{\sin[z]}} \cos[z] \left( (-9 + M^2) \text{Xyz7}[z, t] + \text{Xyz7}^{(0,2)}[z, t] \right. \right. \\
& \quad \left. \left. + (-72 \tan[z] - 36 \tan[z]^3) \text{Xyz7}^{(1,0)}[z, t] - 36 \tan[z]^2 \text{Xyz7}^{(2,0)}[z, t] \right) dz + \right. \\
& \quad \left. e^{-\frac{1}{2} Q1 (e^{-2A4[t]} + 2 A4[t])} (-1 + e^{-2A4[t]}) Q1 (c21 \cos[M t] + c22 \sin[M t]) \right. \\
& \quad \left. A4'[t] \right) \Bigg) \Bigg),
\end{aligned}$$

$$\begin{aligned}
& \text{XYZ7}^{(0,4)}[z, t] = \frac{1}{(-1 + e^{2A4[t]})^2 A4'[t]^2} \\
& \left( \left( -2 \right. \right. \\
& \quad \left. \left. A4'[t]^2 \right. \right. \\
& \quad \left. \left( -9 + \right. \right. \\
& \quad \quad \left. \left. M^2 + \right. \right. \\
& \quad \quad \left. \left. \frac{1}{2} e^{-4A4[t]} Q1^2 A4'[t]^2 - \right. \right. \\
& \quad \quad \left. \left. 2 e^{-2A4[t]} Q1 \left( \frac{1}{2} + Q1 \right) A4'[t]^2 + \right. \right. \\
& \quad \quad \left. \left. (2 + 2 Q1 + 3 Q1^2) A4'[t]^2 + \right. \right. \\
& \quad \quad \left. \left. e^{4A4[t]} \left( -9 + M^2 + \frac{1}{2} Q1^2 A4'[t]^2 \right) \right) - \right. \\
& \quad \left. (-2 - 3 Q1 + e^{-2A4[t]} Q1 - e^{4A4[t]} Q1) A4'[t]^2 \right. \\
& \quad A4''[ \\
& \quad t] + \\
& \quad \left( -2 - 2 e^{4A4[t]} \right) A4''[t]^2 + e^{2A4[t]} \\
& \quad \left( 4 A4'[t]^2 \left( -9 + M^2 + \left( -1 + \frac{Q1}{2} + Q1^2 \right) A4'[t]^2 \right) - \right. \\
& \quad \left. 3 \left( \frac{2}{3} + Q1 \right) A4'[t]^2 A4''[t] + \right. \\
& \quad \left. \left. 4 A4''[t]^2 \right) \right) \\
& \quad \text{XYZ7}^{(0,2)}[z, t] + 2 A4'[t] \left( (-2 - 3 Q1 + e^{-2A4[t]} Q1 - e^{4A4[t]} Q1) \right. \\
& \quad \left. A4'[t]^2 + e^{2A4[t]} \right. \\
& \quad \left( (2 + 3 Q1) A4'[t]^2 - \right. \\
& \quad \left. 2 A4''[t] \right) + \\
& \quad \left( 1 + e^{4A4[t]} \right) A4''[t] \left) \text{XYZ7}^{(0,3)}[z, t] - 2 \left( 1 + \right. \right. \\
& \quad \left. e^{4A4[t]} \right) A4''[ \\
& \quad t]^2 \left( (-9 + M^2) \right. \\
& \quad Xyz7[ \\
& \quad z, \\
& \quad t] + \\
& \quad \left( -72 \tan[z] - 36 \tan[z]^3 \right) \text{XYZ7}^{(1,0)}[ \\
& \quad z, \\
& \quad t] \left) - A4'[ \right. \\
& \quad t]^2 \left( e^{-4A4[t]} M^2 Q1^2 \right. \\
& \quad Xyz7[ \\
& \quad z, \\
& \quad t] A4'[t]^2 + Xyz7[ \\
& \quad z,
\end{aligned}$$

$$\begin{aligned}
& \left( (-3 + M)^2 (3 + M)^2 + (-36 + 4 M^2 - 36 Q1 + 4 M^2 Q1 + 6 M^2 Q1^2) A4' [t]^2 \right) + 6 \\
& (-3 + M) \\
& (3 + M) \\
& \left( \frac{2}{3} + Q1 \right) \\
& A4' [ \\
& t] Xyz7^{(0,1)} [ \\
& z, \\
& t] - 72 \tan [ \\
& z] \\
& (-90 + 2 M^2 + (-405 + M^2) \tan [z]^2 - 594 \tan [z]^4 - \\
& 270 \tan [z]^6 + \\
& 2 (1 + Q1) (2 + \tan [z]^2) A4' [t]^2) Xyz7^{(1,0)} [z, t] + \\
& e^{4 A4 [t]} (Xyz7 [z, t] ((-3 + M)^2 (3 + M)^2 + M^2 Q1^2 A4' [t]^2) + \\
& (-18 + 2 M^2) Q1 A4' [t] Xyz7^{(0,1)} [z, t] - \\
& 72 \tan [z] (-90 + 2 M^2 + (-405 + M^2) \tan [z]^2 - 594 \tan [z]^4 - 270 \tan [z]^6) \\
& Xyz7^{(1,0)} [z, t]) - \\
& 2 e^{-2 A4 [t]} Q1 A4' [t] \left( 2 \left( -\frac{9}{2} + \frac{M^2}{2} + M^2 Q1 \right) Xyz7 [z, t] A4' [t] + \right. \\
& \left. (-9 + M^2) Xyz7^{(0,1)} [z, t] - \right. \\
& \left. 36 \tan [z] (2 + \tan [z]^2) A4' [t] Xyz7^{(1,0)} [z, t] \right) \Bigg) - \\
& A4' [t] A4'' [t] \left( -2 (-3 + M) (3 + M) \left( \frac{3}{2} \left( \frac{2}{3} + Q1 \right) Xyz7 [z, t] A4' [t] + \right. \right. \\
& \left. \left. Xyz7^{(0,1)} [z, t] \right) + \right. \\
& \left. 108 \left( \frac{2}{3} + Q1 \right) \tan [z] (2 + \tan [z]^2) A4' [t] Xyz7^{(1,0)} [ \right. \\
& z, \\
& t] + e^{-2 A4 [t]} Q1 A4' [ \\
& t] \\
& \left( (-9 + M^2) Xyz7 [z, t] + (-72 \tan [z] - 36 \tan [z]^3) Xyz7^{(1,0)} [z, t] \right) + \\
& e^{4 A4 [t]} \left( -2 (-3 + M) (3 + M) \left( \frac{1}{2} Q1 Xyz7 [z, t] A4' [t] + Xyz7^{(0,1)} [z, t] \right) + \right. \\
& \left. 36 Q1 \tan [z] (2 + \tan [z]^2) A4' [t] Xyz7^{(1,0)} [z, t] \right) \Bigg) + \\
& e^{2 A4 [t]} \left( A4'' [t]^2 ((-36 + 4 M^2) Xyz7 [z, t] + (-288 \tan [z] - 144 \tan [z]^3) \right. \\
& Xyz7^{(1,0)} [z, t]) - 4 A4' [t] A4'' [t] \\
& \left( (-3 + M) (3 + M) \left( \frac{3}{4} \left( \frac{2}{3} + Q1 \right) Xyz7 [z, t] A4' [t] + Xyz7^{(0,1)} [z, t] \right) - \right. \\
& \left. 27 \left( \frac{2}{3} + Q1 \right) \tan [z] (2 + \tan [z]^2) A4' [t] Xyz7^{(1,0)} [z, t] \right) +
\end{aligned}$$

$$\begin{aligned}
& 2 A4'[t]^2 \left( Xyz7[z, t] \left( (-3 + M)^2 (3 + M)^2 + (18 - 2 M^2 - 9 Q1 + M^2 Q1 + 2 M^2 Q1^2) A4'[t]^2 \right) + \right. \\
& 3 (-3 + M) (3 + M) \left( \frac{2}{3} + Q1 \right) A4'[t] Xyz7^{(0,1)}[z, t] - \\
& 72 \tan[z] \left( -90 + 2 M^2 + (-405 + M^2) \tan[z]^2 - 594 \tan[z]^4 - \right. \\
& 270 \tan[z]^6 + \frac{1}{2} (-2 + Q1) (2 + \tan[z]^2) A4'[t]^2 \left. \right) Xyz7^{(1,0)}[z, t] \Big) - \\
& 72 \tan[z] (2 + \tan[z]^2) A4'[t] \left( (-2 - 3 Q1 + e^{-2 A4[t]} Q1 - e^{4 A4[t]} Q1) \right. \\
& A4'[t]^2 + e^{2 A4[t]} \\
& \left( (2 + 3 Q1) A4'[t]^2 - 2 A4''[t] \right) + \\
& \left. \left( 1 + e^{4 A4[t]} \right) \right. \\
& A4''[ \\
& t] \Big) \\
& Xyz7^{(1,1)}[z, t] + 72 (-1 + e^{2 A4[t]})^2 \tan[ \\
& z] \\
& (2 + \\
& \tan[z]^2) \\
& A4'[t]^2 Xyz7^{(1,2)}[z, t] - 144 \\
& \tan[ \\
& z]^2 \\
& \left( -\frac{1}{2} A4'[t]^2 (-261 + M^2 - 504 \tan[z]^2 - 270 \tan[z]^4 + \right. \\
& e^{4 A4[t]} (-261 + M^2 - 504 \tan[z]^2 - 270 \tan[z]^4) - \\
& e^{-2 A4[t]} Q1 A4'[t]^2 + (2 + 2 Q1) A4'[t]^2 \left. \right) - \\
& \frac{1}{4} (-2 - 3 Q1 + e^{-2 A4[t]} Q1 - e^{4 A4[t]} Q1) \\
& A4'[t]^2 \\
& A4''[ \\
& t] + \left( -\frac{1}{2} - \frac{1}{2} e^{4 A4[t]} \right) A4''[t]^2 + \\
& e^{2 A4[t]} \left( A4'[t]^2 \left( -261 + M^2 - 504 \tan[z]^2 - 270 \tan[z]^4 + \left( -1 + \frac{Q1}{2} \right) A4'[t]^2 \right) - \right. \\
& \frac{3}{4} \left( \frac{2}{3} + Q1 \right) A4'[t]^2 A4''[t] + A4''[t]^2 \left. \right) \\
& Xyz7^{(2,0)}[z, t] + (-1 + e^{2 A4[t]})^2 A4'[ \\
& t] \\
& A4^{(3)}[ \\
& t] \\
& \left( (-9 + M^2) \right. \\
& Xyz7[ \\
& z, \\
& t] + \\
& Xyz7^{(0,2)}[z, t] + (-72 \tan[z] - 36 \tan[z]^3)
\end{aligned}$$

$$\begin{aligned}
& \text{Xyz7}^{(1,0)} [ \\
& z, \\
& t] - 36 \tan[z]^2 \text{Xyz7}^{(2,0)} [ \\
& z, \\
& t] ) - \\
& 72 \tan[z]^2 A4'[t] \left( (-2 - 3 Q1 + e^{-2 A4[t]} Q1 - e^{4 A4[t]} Q1) \right. \\
& A4'[t]^2 + \\
& e^{2 A4[t]} \left( (2 + 3 Q1) A4'[t]^2 - 2 A4''[t] \right) + \left( 1 + e^{4 A4[t]} \right) \\
& A4''[ \\
& t] ) \\
& \text{Xyz7}^{(2,1)} [z, t] + 72 \left( -1 + e^{2 A4[t]} \right)^2 \tan[ \\
& z]^2 A4'[ \\
& t]^2 \\
& \text{Xyz7}^{(2,2)} [z, t] - 7776 \left( -1 + e^{2 A4[t]} \right)^2 \\
& \tan[ \\
& z]^3 \\
& \left( \frac{4}{3} + \tan[z]^2 \right) A4'[t]^2 \text{Xyz7}^{(3,0)} [z, t] - \\
& 1296 \\
& \left. \left( -1 + \right. \right. \\
& e^{2 A4[t]} \left. \right)^2 \tan[ \\
& z]^4 A4'[t]^2 \text{Xyz7}^{(4,0)} [z, t] \left. \right\}
\end{aligned}$$

## A new spin- $\frac{1}{2}$ wave equation

Patrick L. Nash

*Division of Earth and Physical Sciences, University of Texas at San Antonio, San Antonio, Texas 78285*

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the description of a (composite) massive spin- $\frac{1}{2}$  particle. In this model, the spin degrees of freedom of the particle are carried by the functional dependence of the wave function on a real spinor independent variable  $u$ . Here,  $u$  is an element of an internal space  $D_4$ , real four-dimensional Dirac space. The wave equation itself is cast on  $M_4 \times D_4$ , where  $M_4$  denotes Minkowski space-time. Solutions of the wave equation, when restricted to  $D_4$ , belong to the Hilbert space  $L^2(D_4)$  of square-integrable smooth functions on  $D_4$ .

$$\frac{1}{2}\Sigma^{\alpha\beta}\Sigma_{\alpha\beta} = j_0^2 - c^2 - 1 \quad (6)$$

and

$$\frac{1}{8}\epsilon_{\alpha\beta\mu\nu}\Sigma^{\alpha\beta}\Sigma^{\mu\nu} = j_0c, \quad (7)$$

where

$$j_0 = -\frac{1}{2}\xi\gamma^5 u, \quad (8)$$

with

$$\gamma^5 = -\gamma^1\gamma^2\gamma^3\gamma^4 \quad (9)$$

and

$$c = \frac{1}{2}(\xi_a u^a + u^a \xi_a). \quad (10)$$

Equations (6) and (7) are, of course, manifestations of the well-known fact that the unitary irreducible representations of  $\overline{\text{SO}(3,1)}$  can be labeled by two numbers  $(j_0, c)$  (see Refs. 2–

4). In Ref. 3, Dirac shows that the unitary irrep  $(j_0, c) = (\frac{1}{2}, 0)$  provides a representation of  $\text{SO}(3,2)$ . However, we emphasize that Dirac does not work on  $L^2(D_4)$ . In the Appendix we show that  $j_0$  takes on half-integer values on  $L^2(D_4)$  while  $c$  is not diagonal. Here,  $|j_0|$  is the so-called “minimum spin” of the representation. This nomenclature arises from the fact that  $\frac{1}{2}\Sigma^{jk}\Sigma_{jk}, j, k, \dots = 1, 2, 3$  has eigenvalues  $s(s+1)$ , where  $s$  is constrained to satisfy  $s \geq |j_0|$  (see Refs. 2 and 4).

Dirac spinor with components  $u^a, a = 1, 2, 3, 4$ ; irrep, and  $\text{DD}\xi, \dots$

- The Wigner D-function  $D_{m_1, m_2}^j$  gives the matrix element of a rotation operator parametrized by Euler angles in a  $2j+1$ -dimensional unitary representation of a rotation group when parameters  $j, m_1, m_2$  are physical, i.e. all integers or half-integers such that  $-j \leq m_1, m_2 \leq j$ .
- For unphysical parameters, WignerD is defined by an analytic continuation.
- The Wolfram Language uses phase conventions where  $D_{m_1, m_2}^j(\psi, \theta, \phi) = \exp(i m_1 \psi + i m_2 \phi) D_{m_1, m_2}^j(0, \theta, 0)$ .

`WignerD[{j, m1, m2}, \[Psi], \[Theta], \[Phi]]`  
gives the Wigner D-function  $D_{m_1, m_2}^j(\psi, \theta, \phi)$ .

`In[1]:= Head[U1by4]`

`In[2]:= Head[allMighty[##] & @@ U1by4]`

`In[3]:= Head[Sin[x]]`

```
In[1]:= Head[U1by4.U1by4]
In[2]:= processInput[input_] := Module[{f, vars},
 If[Head[input] === Function, f = input, vars = Variables[input]];
 If[Length[vars] === 1, f = Function[{vars[[1]]}, Evaluate[input]],
 Return["Error: Expression must have only one variable."]];
 (*Use 'f' here*) f[10]]
```

$$\xi = -i \frac{\partial}{\partial u}, \quad (2)$$


---

```
In[3]:= DD\xi[f_] := -i*D[f, #] & /@ U1by4
In[4]:= DD\xi[U1by4]
In[5]:= (*Symbolize[S44\alpha\beta]*)
In[6]:= (*S44\alpha\beta=Table[FullSimplify[\frac{\gamma[h].\gamma[k]-\gamma[k].\gamma[h]}{4}], {h,1,4}, {k,1,4}];*)
In[7]:= Dimensions[S44\alpha\beta]
In[8]:= S44\alpha\beta[[1, 1]]
\Sigma^{\alpha\beta} = \xi S^{\alpha\beta} u,
```

$$\xi = -i \frac{\partial}{\partial u}, \quad (2)$$


---

fixed:

```
In[1]:= Clear[\Sigma^{\alpha\beta}];
\Sigma^{\alpha\beta}[f_] (* /; Head[f] === Symbol *) := Table[Block[{Su, d},
 Su = f * S44\alpha\beta[[h, k]].U1by4;
 d = FullSimplify[Sum[-i*D[Su[[j]], U1by4[[j]]], {j, 1, Length[U1by4]}]];
 d], {h, 1, 4}, {k, 1, 4}]
In[2]:= (*Clear[\Sigma^{\alpha\beta}];
\Sigma^{\alpha\beta}[f_]:=Table[Block[{Su,d},
 Su=f*S44\alpha\beta[[h,k]].U1by4;
 d=FullSimplify[Sum[D[Su[[j]],U1by4[[j]]],{j,1,Length[U1by4]}]];
 d],{h,1,4},{k,1,4}]*)

```

$$\frac{1}{2}\sum^{jk}\Sigma_{jk} = \frac{1}{4}\partial_a u^a u^b \partial_b - \frac{1}{4}\partial_a u^b u^b \partial_a,$$

```
In[3]:= (*Table[Sum[S44\alpha\beta[[h,j]][[.S44\alpha\beta[[j,k]],{j,1,4}]],{h,1,3},{k,h+1,4}]]//MatrixForm*)
In[4]:= (sigma3 = {S44\alpha\beta[[1, 2]], S44\alpha\beta[[2, 3]], S44\alpha\beta[[3, 1]]}) // Column
MatrixForm[#] & /@ %[[1]]
```

```

In[1]:= commutator[a_?SquareMatrixQ, b_?SquareMatrixQ] /; Dimensions[a] === Dimensions[b] :=
FullSimplify[a.b - b.a]

In[2]:= commutator[sigma3[[1]], sigma3[[2]]] === sigma3[[3]]

In[3]:= (*Clear[sigmaSquared];
(sigmaSquared=Table[Block[{s2},
s2=Table[S44αβ[[h;;k1]].S44αβ[[k1;;k]],{k1,1,4}];
s2],{h,1,3},{k,h+1,4}])//MatrixForm*)

In[4]:= (*Symbolize[Σ2]*)

$$\frac{1}{2}\sum^{jk}\Sigma_{jk} = \frac{1}{4}\partial_a u^a u^b \partial_b - \frac{1}{4}\partial_a u^b u^b \partial_a,$$

In[5]:= Clear[Σ2];
Σ2[f_] (* /; Head[f]===Symbol*) := Module[{u, n, sa, sb, d},
u = U1by4; n = Length[u];
sa = Sum[D[u[[a1]] * Sum[u[[b]] × D[f, u[[b]]], {b, 1, n}], u[[a1]]], {a1, 1, n}];
sb = Sum[D[(Sum[u[[b]] × u[[b]], {b, 1, n}] × D[f, u[[a1]]]), u[[a1]]], {a1, 1, n}];
(*d=sa-sb;*)
d = FullSimplify[$\frac{1}{4}(sa - sb)$];
Return[d];]

In[6]:=
$$\frac{\Sigma^2[\#]}{\#} \& /@ U1by4$$

In[7]:= Σ2[U1by4.U1by4]

```

$$\frac{1}{2}\Sigma^{jk}\Sigma_{jk} = \frac{1}{4}\partial_a u^a u^b \partial_b - \frac{1}{4}\partial_a u^b u^b \partial_a, \quad (14)$$

where  $\partial_a = i\xi_a$ . Equation (14) is equivalent to Dirac's Eq. (24). If we put

$$u^a u^a = r^2, \quad u^a \partial_a = r \frac{\partial}{\partial r},$$

and

$$\partial_a \partial_a = \frac{1}{r^3} \frac{\partial}{\partial r} r^3 \frac{\partial}{\partial r} + \frac{1}{r^2} \Delta_0,$$

where  $\Delta_0$  is the restriction of the Laplacian on  $\mathbb{R}^4$  to the unit three-sphere, then  $\frac{1}{2}\Sigma^{jk}\Sigma_{jk}$  reduces to

$$\frac{1}{2}\Sigma^{jk}\Sigma_{jk} = -\frac{1}{4}\Delta_0. \quad (15)$$

Here,  $\Delta_0$  has eigenvalues  $-l(l+2)$ ,  $l = 0, 1, 2, \dots$  (see Ref. 5), so that  $\frac{1}{2}\Sigma^{jk}\Sigma_{jk}$  has eigenvalues  $s(s+1)$ ,  $s = 0, \frac{1}{2}, 1, \dots$ .

In[1]:= (\*S44αβ\*)

$$j_0 = -\frac{1}{2}\xi\gamma^5 u,$$

In[2]:= (\*Symbolize[j0]\*)

In[3]:= Clear[j0];

$$\begin{aligned} j_0[f_] (* /; Head[f] === Symbol *) := & \text{Block}[\{\text{Su}, \text{d}\}, \\ \text{Su} = & f * \gamma[5].\text{U1by4}; \\ \text{d} = & \text{FullSimplify}\left[-\frac{1}{2} \sum[-i * D[\text{Su}[j], \text{U1by4}[j]], \{j, 1, \text{Length}[\text{U1by4}]\}]\right]; \\ & d \end{aligned}$$

$$c = \frac{1}{2}(\xi_a u^a + u^a \xi_a).$$

In[4]:= j0[U1by4.U1by4]

In[5]:= (\*Symbolize[cop]\*)

In[6]:= Clear[cop];

$$\begin{aligned} \text{cop}[f_] (* /; Head[f] === Symbol *) := & \\ \text{FullSimplify}\left[\frac{1}{4} \left( \sum[-i * D[f * \text{U1by4}[j], \text{U1by4}[j]], \{j, 1, \text{Length}[\text{U1by4}]\} \right) + \right. \\ & \left. \text{Transpose}[\text{U1by4}].\text{Transpose}[DD\xi[f]] \right] \end{aligned}$$

In[7]:= r2

```
In[8]:= (*{alpha,beta,gamma}={-ArcTan[u^a[2],u^a[1]]+ArcTan[u^a[4],u^a[3]],2 ArcTan[Sqrt[u^a[3]^2+u^a[4]^2], Sqrt[u^a[1]^2+u^a[2]^2],ArcTan[u^a[2],u^a[1]]+ArcTan[u^a[4],u^a[3]]};*)
```

$\psi_{nj0ss3}$

### aiWignerSmallDRefactored:

```
(*Non-recursive,single-expression Wigner Small-d.Uses the explicit
summation formula (Wigner's form).Valid for arbitrary complex j,m,
mp,beta (though physically meaningful for spin).-No'If' statements
used.-No recursion used.-Returns a symbolic Sum expression that
can be differentiated.-Uses Gamma[n+1] instead of Factorial[n] for
analytic continuity.*)
aiWignerSmallDRefactored[j_, mp_, m_, beta_] :=
Module[{(*k*)}, (*Prefactor with Square Roots of Factorials*)
Sqrt[Gamma[j + mp + 1] * Gamma[j - mp + 1] * Gamma[j + m + 1] * Gamma[j - m + 1]] *
(*The Summation over k*)Sum[((*Alternating sign and phase (-1)^(k+mp-m)*)
(-1)^(k + mp - m) * (*Trigonometric terms*)
(Cos[beta/2])^(2*j + m - mp - 2*k) * (Sin[beta/2])^(mp - m + 2*k)) /
(*Denominators:Gamma functions handle the truncation of the series*)
(*1/(n!) becomes 1/Gamma[n+1] which is 0 for negative integers*)
Gamma[j - mp - k + 1] * Gamma[j + m - k + 1] * Gamma[k + mp - m + 1] * Gamma[k + 1]),(*Sum from 0 to Infinity:Terms naturally vanish when k exceeds bounds
due to 1/Gamma poles.GenerateConditions→False prevents Mathematica from
wrapping the result in conditional'If' blocks about convergence.*)
{k, 0, Infinity}, GenerateConditions → False]]
```

```
In[•]:= Clear[\psi_{nj0ss3}];

ψ_{nj0ss3}[\mu_, j0_, s_, s3_] := Module[{n, rSquared, wf, alpha, beta, gamma, ret},
 {alpha, beta, gamma} = { -ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]],
 2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]],
 ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]] };

 n = s + μ;
 rSquared = u^a[1]^2 + u^a[2]^2 + u^a[3]^2 + u^a[4]^2; (*AKA, 1, on boundary *)
 wf = 1/π * Sqrt[(2*s + 1) * (n - s - 1)! / (n + s)!] * Exp[-rSquared/2] *
 LaguerreL[n - s - 1, 2*s + 1, rSquared] * Exp[-I*j0*alpha] *
 aiWignerSmallDRefactored[s, -s3, j0, beta] * Exp[I*s3*gamma];
 Return[wf];
]

In[•]:= ParallelTable[Block[{a1, a2, a3, a4, a5, a6, a7, f}, f = ψ_{nj0ss3}[\mu, j0, s, s3];
 a1 = FullSimplify[FullSimplify[Sigma^2[f]/f] /. {u^a[1]^2 + u^a[2]^2 + u^a[3]^2 + u^a[4]^2 → r^2} /.
 {Sqrt[(u^a[1]^2 + u^a[2]^2)^2] → (u^a[1]^2 + u^a[2]^2),
 Sqrt[(u^a[3]^2 + u^a[4]^2)^2] → (u^a[3]^2 + u^a[4]^2)}];
 a2 = FullSimplify[FullSimplify[j0*f/f] /. {u^a[1]^2 + u^a[2]^2 + u^a[3]^2 + u^a[4]^2 → r^2} /.
 {Sqrt[(u^a[1]^2 + u^a[2]^2)^2] → (u^a[1]^2 + u^a[2]^2),
 Sqrt[(u^a[3]^2 + u^a[4]^2)^2] → (u^a[3]^2 + u^a[4]^2)}];
 a3 = s*(s + 1);
 a4 = j0;
 a5 = {a1, a2};
 a6 = {a3, a4};
 a7 = {a5, a6};
 {{s, s3, j0, μ}, If[a5 == a6, Style[a7, Red, Bold, 16], Style[a7, Blue, Italic, 12]]}],
 {s, 0, 2, 1/2}, {s3, -s, s}, {j0, -s, s}, {μ, 1, 3}]
```

howabout:

```

In[8]:= howabout[s_, s1_, s2_] := (-1)^(s1-s2) Sqrt[(s - s1) ! (s + s1) ! (s - s2) ! (s + s2) !] *
Exp[-I*s1*ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]]] *
Exp[-I*s2*ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]]] * If[s2 > s1,
(If[θ == 2 s - s2 + s1, 1, Cos[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^2]^(s-s2+s1)]*
((-1)^(s2-s1) HypergeometricPFQ[{-s + s2, -s - s1}, {1 + s2 - s1},
-Tan[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^2]
Sin[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^(s2-s1)])/
((s - s2) ! (s2 - s1) ! (s + s1) !), (If[θ == 2 s + s2 - s1, 1,
Cos[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^2]^(s+s2-s1)]*
(HypergeometricPFQ[{-s - s2, -s + s1}, {1 - s2 + s1},
-Tan[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^2]
Sin[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^(s2+s1)])/
((s + s2) ! (s - s1) ! (-s2 + s1) !)]

```

```

In[9]:= Block[{jθ = -1/2, s = 3/2, s3 = 1/2, n = s + 1, f, r, r1, ret},
f = Exp[-r2/2] LaguerreL[n - s - 1, 2*s + 1, r2] *
(*(WignerMatrix[jθ,aWigner])*) howabout[s, s3, jθ];
ret = {f, FullSimplify[Σ²[f]/f], FullSimplify[jθ[f]/f]};
ret]

```

```

In[10]:= Block[{jθ = 1/2, s = 3/2, s3 = -1/2, n = s + 1, f, r, r1, ret},
f = Exp[-r2/2] LaguerreL[n - s - 1, 2*s + 1, r2] *
(*(WignerMatrix[jθ,aWigner])*) howabout[s, s3, jθ];
ret = {f, FullSimplify[Σ²[f]/f], FullSimplify[jθ[f]/f]};
ret]

```

```

In[1]:= Block[{jθ = 3/2, s = jθ, s3 = -1/2, n = s + 1, f, r, r1, ret},
 f = Exp[-r2/2] LaguerreL[n - s - 1, 2*s + 1, r2] *
 (* (WignerMatrix[jθ,aWigner]) *) wignerDmatrixU[s, s3, jθ];
 ret = {f, FullSimplify[(Σ²[f])/f], FullSimplify[(jθ[f])/f]}];
 ret]

In[2]:= Block[{s = 3/2, s3 = -1/2, jθ = 1/2, n = s + 1, f, r, r1, ret, alpha, beta, gamma},
 {alpha, beta, gamma} = {-ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]],
 2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)],
 ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]]};
 f = Exp[-r2/2] * LaguerreL[n - s - 1, 2*s + 1, r2] * (wignerDmatrixU[s, s3, jθ]) *
 Exp[-I*s3*alpha] * Exp[-I*jθ*gamma] // FullSimplify;
 Print[f];
 ret = {f, FullSimplify[(Σ²[f])/f], FullSimplify[(jθ[f])/f]}];
 ret]

In[3]:= Block[{s = 3/2, s3 = -1/2, jθ = 3/2, n = s + 1, f, r, r1, ret, alpha, beta, gamma},
 {alpha, beta, gamma} = {-ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]],
 2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)],
 ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]]};
 (*r=WignerMatrix[1/2,aWigner];*)
 f = Exp[-r2/2] LaguerreL[n - s - 1, 2*s + 1, r2] * (wignerDmatrixU[s, s3, jθ]) *
 Exp[-I*jθ*alpha] * Exp[-I*s3*gamma] // FullSimplify;
 Print[f];
 (*r=Σ²[f] (*&@Flatten[f]*) // Simplify;
 r1=;*)
 (*r1=Transpose[{r,Flatten[f]}];*)
 ret = {f, FullSimplify[(Σ²[f])/f], FullSimplify[(jθ[f])/f]}];
 ret]

```

```

In[8]:= Block[{ss = 3/2, s3 = -1/2, jθ = 1/2,
n = ss + 1, f, h, s, s1, s2, r, r1, ret, alpha, beta, gamma},
{alpha, beta, gamma} = {-ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]],
2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)],
ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]]};

(*r=WignerMatrix[1/2,aWigner];*)
h[s_, s1_, s2_] := (-1)^s1-s2 √(s-s1) ! (s+s1) ! (s-s2) ! (s+s2) ! *
Exp[-I*s1*alpha]*Exp[-I*s2*gamma]*If[s2 > s1,
 If[θ == 2s - s2 + s1, 1, Cos[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(2s-s2+s1]
 ((-1)^s2-s1 HypergeometricPFQ[{-s+s2, -s-s1}, {1+s2-s1},
 -Tan[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^2]
 Sin[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(s2-s1)])/
((s-s2) ! (s2-s1) ! (s+s1) !), (If[θ == 2s + s2 - s1, 1,
Cos[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(2s+s2-s1]
HypergeometricPFQ[{-s-s2, -s+s1}, {1-s2+s1},
-Tan[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^2]
Sin[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(-s2+s1)])/
((s+s2) ! (s-s1) ! (-s2+s1) !)];
f = Exp[-r2/2] LaguerreL[n - ss - 1, 2*ss + 1, r2]*h[ss, s3, jθ]
(*(WignerMatrix[jθ,aWigner])*);
(*r=Σ^2[#]&/@Flatten[f]//Simplify;*)
(*r1=Transpose[{r,Flatten[f]}];*)
ret = {f, FullSimplify[Σ^2[f]/f], FullSimplify[jθ[f]/f]} (*&/@r1*);
ret]

```

```

In[8]:= Block[{ss = 3/2, s3 = -1/2, jθ = -1/2,
n = ss + 1, f, h, s, s1, s2, r, r1, ret, alpha, beta, gamma},
{alpha, beta, gamma} = {-ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]],
2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)],
ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]]};

(*r=WignerMatrix[1/2,aWigner];*)
h[s_, s1_, s2_] := (-1)^s1-s2 √(s-s1) ! (s+s1) ! (s-s2) ! (s+s2) ! *
Exp[-I*s1*alpha]*Exp[-I*s2*gamma]*If[s2 > s1,
 If[θ == 2s - s2 + s1, 1, Cos[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(2s-s2+s1]
 ((-1)^s2-s1 HypergeometricPFQ[{-s+s2, -s-s1}, {1+s2-s1},
 -Tan[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^2]
 Sin[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(s2-s1)])/
((s-s2) ! (s2-s1) ! (s+s1) !), If[θ == 2s + s2 - s1, 1,
Cos[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(2s+s2-s1]
 (HypergeometricPFQ[{-s-s2, -s+s1}, {1-s2+s1},
 -Tan[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^2]
 Sin[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(-s2+s1)])/
((s+s2) ! (s-s1) ! (-s2+s1) !)];
f = Exp[-r2/2] LaguerreL[n - ss - 1, 2*ss + 1, r2]*h[ss, s3, jθ]
(*(WignerMatrix[jθ,aWigner])*);
(*r=Σ^2[#]&/@Flatten[f]//Simplify;*)
(*r1=Transpose[{r,Flatten[f]}];*)
ret = {f, FullSimplify[Σ^2[f]/f], FullSimplify[jθ[f]/f]} (*&/@r1*);
ret]

```

```

In[8]:= Block[{ss = 3/2, s3 = 1/2, j0 = -1/2,
n = ss + 1, f, h, s, s1, s2, r, r1, ret, alpha, beta, gamma},
{alpha, beta, gamma} = {-ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]],
2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]],
ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]]};

(*r=WignerMatrix[1/2,aWigner];*)
h[s_, s1_, s2_] := (-1)^s1-s2 Sqrt[(s-s1)!(s+s1)!(s-s2)!(s+s2)!] *
Exp[-I*s1*alpha]*Exp[-I*s2*gamma]*If[s2 > s1,
 If[θ == 2s - s2 + s1, 1, Cos[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^2]^s-s2+s1]
 ((-1)^s2-s1 HypergeometricPFQ[{-s+s2, -s-s1}, {1+s2-s1},
 -Tan[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^2]
 Sin[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^s2-s1])/
((s-s2)!(s2-s1)!(s+s1)!), If[θ == 2s + s2 - s1, 1,
Cos[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^2]^s+s2-s1]
 ((HypergeometricPFQ[{-s-s2, -s+s1}, {1-s2+s1},
 -Tan[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^2]
 Sin[2 ArcTan[Sqrt[u^a[3]^2 + u^a[4]^2], Sqrt[u^a[1]^2 + u^a[2]^2]]]^-s2+s1])/
((s+s2)!(s-s1)!(-s2+s1)!]];
f = Exp[-r2/2] LaguerreL[n - ss - 1, 2*ss + 1, r2]*h[ss, s3, j0]
(*(WignerMatrix[j0,aWigner])*);
(*r=Σ^2[#]&/@Flatten[f]//Simplify;*)
(*r1=Transpose[{r,Flatten[f]}];*)
ret = {f, FullSimplify[Σ^2[f]/f], FullSimplify[j0[f]/f]}(*&/@r1*);
ret]

```

```

In[8]:= Block[{ss = 3/2, s3 = -3/2, jθ = -1/2,
n = ss + 1, f, h, s, s1, s2, r, r1, ret, alpha, beta, gamma},
{alpha, beta, gamma} = {-ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]],
2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)],
ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]]};

(*r=WignerMatrix[1/2,aWigner];*)
h[s_, s1_, s2_] := (-1)^s1-s2 √(s-s1) ! (s+s1) ! (s-s2) ! (s+s2) ! *
Exp[-I*s1*alpha]*Exp[-I*s2*gamma]*If[s2 > s1,
 If[θ == 2s - s2 + s1, 1, Cos[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(2s-s2+s1]
 ((-1)^s2-s1 HypergeometricPFQ[{-s+s2, -s-s1}, {1+s2-s1},
 -Tan[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^2]
 Sin[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(s2-s1)])/
((s-s2) ! (s2-s1) ! (s+s1) !), (If[θ == 2s + s2 - s1, 1,
Cos[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(2s+s2-s1]
HypergeometricPFQ[{-s-s2, -s+s1}, {1-s2+s1},
-Tan[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^2]
Sin[2 ArcTan[√(u^a[3]^2 + u^a[4]^2), √(u^a[1]^2 + u^a[2]^2)]]^(-s2+s1)])/
((s+s2) ! (s-s1) ! (-s2+s1) !)];
f = Exp[-r2/2] LaguerreL[n - ss - 1, 2*ss + 1, r2]*h[ss, s3, jθ]
(*(WignerMatrix[jθ,aWigner])*);
(*r=Σ^2[#]&/@Flatten[f]//Simplify;*)
(*r1=Transpose[{r,Flatten[f]}];*)
ret = {f, FullSimplify[Σ^2[f]/f], FullSimplify[jθ[f]/f]}(*&/@r1*);
ret]

```

```
In[1]:= Clear[wignerDmatrixU];
wignerDmatrixU[j_, m1_, m2_] :=
 (-1)^{-m1+m2} e^{-i m2} (-ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]]) - i m1 (ArcTan[u^a[2], u^a[1]] + ArcTan[u^a[4], u^a[3]])
 Sqrt[(j - m1) ! (j + m1) ! (j - m2) ! (j + m2) !] Sum[(-1)^s,
 {s, Max[0, m1 - m2]}, Min[j + m1, j - m2]] *
 If[2 j + m1 - m2 - 2 s == 0, 1, (Sqrt[u^a[3]^2 + u^a[4]^2]/Sqrt[u^a[1]^2 + u^a[2]^2 + u^a[3]^2 + u^a[4]^2])^(2 j + m1 - m2 - 2 s)] *
 If[-m1 + m2 + 2 s == 0, 1, (Sqrt[u^a[1]^2 + u^a[2]^2]/Sqrt[u^a[1]^2 + u^a[2]^2 + u^a[3]^2 + u^a[4]^2])^{-m1 + m2 + 2 s}] /
 ((j + m1 - s) ! (j - m2 - s) ! s ! (-m1 + m2 + s) !)
```

```
In[2]:= Block[{ss = 3/2, s3 = -3/2, j0 = -1/2, n = ss + 1, f, h, s, s1, s2, r, r1, ret},
```

```
f = Exp[-r2/2] LaguerreL[n - ss - 1, 2*ss + 1, r2] *
 wignerDmatrixU[ss, s3, j0] // FullSimplify;
ret = {f, FullSimplify[Sigma^2[f]/f], FullSimplify[j0*f/f]};
ret]
```

```
In[3]:= Block[{ss = 3/2, s3 = -3/2, j0 = -3/2, n = ss + 1, f, h, s, s1, s2, r, r1, ret},
```

```
f = Exp[-r2/2] LaguerreL[n - ss - 1, 2*ss + 1, r2] *
 wignerDmatrixU[ss, s3, j0] // FullSimplify;
ret = {f, FullSimplify[Sigma^2[f]/f], FullSimplify[j0*f/f]};
ret]
```

```
In[4]:= Block[{ss = 3/2, s3 = -3/2, j0 (*=-3/2*), n = ss + 1, f, h, s, s1, s2, r, r1, ret},
```

```
f = Exp[-r2/2] LaguerreL[n - ss - 1, 2*ss + 1, r2] *
 wignerDmatrixU[ss, s3, j0] // FullSimplify;
ret = {f, FullSimplify[Sigma^2[f]/f], FullSimplify[j0*f/f]};
ret]
```

```
Out[4]=
```

\$Aborted