

```

>
> restart; libname
    "/Library/Frameworks/Maple.framework/Versions/2022/lib"
(1)
> #with(inttrans);
> with(LargeExpressions) :
> listrep := r→map2(op, 1, march('list', r)) :
> #NumericEventHandler( invalid_operation = `Heaviside/EventHandler`( value_at_zero =  $\frac{1}{2}$  ) ) :
> [kernelopts(mapledir), kernelopts(homedir), currentdir( )]
    ["/Library/Frameworks/Maple.framework/Versions/2022", "/Users/nsh", "/Users/nsh"]
(2)
> #with(PDEtools, casesplit, declare) :
> # with(DEtools, gensys) :
> #with(MathML) :
> #with(StringTools) :
> #`with(FileTools) :
> #with(Worksheet) :
> #with(ListTools) :
> with(MmaTranslator);
    [FromMma, FromMmaNotebook, Mma, MmaToMaple]
(3)
>
> #FileTools:-JoinPath([ currentdir( ), "M4M_seq2.m" ] ) ;
>
>
> #cat(kernelopts(homedir), "\\", "Documents", "\\", "myFile");
Using kernelopts(dirsep)
> #cat(kernelopts(homedir), kernelopts(dirsep), "Documents",
    kernelopts(dirsep), "myFile");
Using FileTools:-JoinPath
> #FileTools:-JoinPath([ "Documents", "myFile"],base=homedir);
>
>
>
> zvars1 :=
    FromMma( ` List[yZ0[z,t],yZ1[z,t],yZ2[z,t],yZ3[z,t]] ` );
    zvars1 := [yZ0(z,t),yZ1(z,t),yZ2(z,t),yZ3(z,t)]
(4)
> zvars2 :=
    FromMma( ` List[yZ4[z,t],yZ5[z,t],yZ6[z,t],yZ7[z,t]] ` );
    zvars2 := [yZ4(z,t),yZ5(z,t),yZ6(z,t),yZ7(z,t)]
(5)
>

```

```
> zvars3 :=
  FromMma( ` List[yZ8[z,t],yZ9[z,t],yZ10[z,t],yZ11[z,t]] ` );
  zvars3 := [yZ8(z,t),yZ9(z,t),yZ10(z,t),yZ11(z,t)] (6)
```

```
> zvars4 :=
  FromMma( ` List[yZ12[z,t],yZ13[z,t],yZ14[z,t],yZ15[z,t]] ` );
  zvars4 := [yZ12(z,t),yZ13(z,t),yZ14(z,t),yZ15(z,t)] (7)
```

```
> sC1 := seq(cat(_C,i) = cat(c1,i), i = 1..32);
sC1 := _C1 = c11, _C2 = c12, _C3 = c13, _C4 = c14, _C5 = c15, _C6 = c16, _C7 = c17, _C8
= c18, _C9 = c19, _C10 = c110, _C11 = c111, _C12 = c112, _C13 = c113, _C14 = c114, _C15
= c115, _C16 = c116, _C17 = c117, _C18 = c118, _C19 = c119, _C20 = c120, _C21 = c121,
_C22 = c122, _C23 = c123, _C24 = c124, _C25 = c125, _C26 = c126, _C27 = c127, _C28
= c128, _C29 = c129, _C30 = c130, _C31 = c131, _C32 = c132 (8)
```

```
> sC2 := seq(cat(_C,i) = cat(c2,i), i = 1..32);
sC2 := _C1 = c21, _C2 = c22, _C3 = c23, _C4 = c24, _C5 = c25, _C6 = c26, _C7 = c27, _C8
= c28, _C9 = c29, _C10 = c210, _C11 = c211, _C12 = c212, _C13 = c213, _C14 = c214, _C15
= c215, _C16 = c216, _C17 = c217, _C18 = c218, _C19 = c219, _C20 = c220, _C21 = c221,
_C22 = c222, _C23 = c223, _C24 = c224, _C25 = c225, _C26 = c226, _C27 = c227, _C28
= c228, _C29 = c229, _C30 = c230, _C31 = c231, _C32 = c232 (9)
```

```
> sC3 := seq(cat(_C,i) = cat(c3,i), i = 1..32);
sC3 := _C1 = c31, _C2 = c32, _C3 = c33, _C4 = c34, _C5 = c35, _C6 = c36, _C7 = c37, _C8
= c38, _C9 = c39, _C10 = c310, _C11 = c311, _C12 = c312, _C13 = c313, _C14 = c314,
_C15 = c315, _C16 = c316, _C17 = c317, _C18 = c318, _C19 = c319, _C20 = c320, _C21
= c321, _C22 = c322, _C23 = c323, _C24 = c324, _C25 = c325, _C26 = c326, _C27 = c327,
_C28 = c328, _C29 = c329, _C30 = c330, _C31 = c331, _C32 = c332 (10)
```

```
> sC4 := seq(cat(_C,i) = cat(c4,i), i = 1..32);
sC4 := _C1 = c41, _C2 = c42, _C3 = c43, _C4 = c44, _C5 = c45, _C6 = c46, _C7 = c47, _C8
= c48, _C9 = c49, _C10 = c410, _C11 = c411, _C12 = c412, _C13 = c413, _C14 = c414,
_C15 = c415, _C16 = c416, _C17 = c417, _C18 = c418, _C19 = c419, _C20 = c420, _C21
= c421, _C22 = c422, _C23 = c423, _C24 = c424, _C25 = c425, _C26 = c426, _C27 = c427,
_C28 = c428, _C29 = c429, _C30 = c430, _C31 = c431, _C32 = c432 (11)
```

```
> FromMma( ` List[Equal[Plus[Times[3,yZ1[z,t]],Times[M,yZ3[z,t]],
Derivative[0,1][yZ0][z,t],Times[6,Tan[z],Derivative[1,0][yZ1][z,
t]]],Times[Power[E,Times[-1,a4[t]]],Q1,Sinh[a4[t]],yZ0[z,t],
Derivative[1][a4][t]]],Equal[Plus[Times[3,yZ0[z,t]],Times[M,yZ2
[z,t]],Times[Power[E,Times[-1,a4[t]]],Q1,Sinh[a4[t]],yZ1[z,t],
```

```

Derivative[1][a4][t]], Derivative[0, 1][yZ1][z, t], Times[6, Tan[z],
Derivative[1, 0][yZ0][z, t]]], 0], Equal[Plus[Times[M, yZ1[z, t]],
Times[3, yZ3[z, t]], Times[6, Tan[z], Derivative[1, 0][yZ3][z, t]]],
Plus[Times[Power[E, Times[-1, a4[t]]], Q1, Sinh[a4[t]], yZ2[z, t],
Derivative[1][a4][t]], Derivative[0, 1][yZ2][z, t]]], Equal[Plus
[Times[M, yZ0[z, t]], Times[3, yZ2[z, t]], Times[Power[E, Times[-1, a4
[t]]], Q1, Sinh[a4[t]], yZ3[z, t], Derivative[1][a4][t]], Times[6, Tan
[z], Derivative[1, 0][yZ2][z, t]], Derivative[0, 1][yZ3][z, t]]]
`
;

```

```
eq1 := %;
```

```
nops(%);
```

$$\begin{aligned}
& \left[\text{add} \left(i, i = \left[3 yZ1(z, t), M yZ3(z, t), D_{\substack{1, \dots, 1 \\ 0 \text{ times}}} \substack{2, \dots, 2 \\ 1 \text{ times}}} (yZ0)(z, t), \right. \right. \\
& \quad \left. \left. 6 \tan(z) D_{\substack{1, \dots, 1 \\ 1 \text{ times}}} \substack{2, \dots, 2 \\ 0 \text{ times}}} (yZ1)(z, t) \right] \right) = (e)^{(-1) a4(t)} Q1 \sinh(a4(t)) yZ0(z, \\
& \quad t) D_{\substack{1, \dots, 1 \\ 1 \text{ times}}} (a4)(t), \text{add} \left(i, i = \left[3 yZ0(z, t), M yZ2(z, t), (e)^{(-1) a4(t)} Q1 \sinh(a4(t)) yZ1(z, \right. \right. \\
& \quad \left. \left. t) D_{\substack{1, \dots, 1 \\ 1 \text{ times}}} (a4)(t), D_{\substack{1, \dots, 1 \\ 0 \text{ times}}} \substack{2, \dots, 2 \\ 1 \text{ times}}} (yZ1)(z, t), 6 \tan(z) D_{\substack{1, \dots, 1 \\ 1 \text{ times}}} \substack{2, \dots, 2 \\ 0 \text{ times}}} (yZ0)(z, t) \right] \right) = 0, \\
& \quad \text{add} \left(i, i = \left[M yZ1(z, t), 3 yZ3(z, t), 6 \tan(z) D_{\substack{1, \dots, 1 \\ 1 \text{ times}}} \substack{2, \dots, 2 \\ 0 \text{ times}}} (yZ3)(z, t) \right] \right) = \text{add} \left(i, i \right. \\
& \quad \left. = \left[(e)^{(-1) a4(t)} Q1 \sinh(a4(t)) yZ2(z, t) D_{\substack{1, \dots, 1 \\ 1 \text{ times}}} (a4)(t), D_{\substack{1, \dots, 1 \\ 0 \text{ times}}} \substack{2, \dots, 2 \\ 1 \text{ times}}} (yZ2)(z, t) \right] \right), \\
& \quad \text{add} \left(i, i = \left[M yZ0(z, t), 3 yZ2(z, t), (e)^{(-1) a4(t)} Q1 \sinh(a4(t)) yZ3(z, t) D_{\substack{1, \dots, 1 \\ 1 \text{ times}}} (a4)(t), \right. \right. \\
& \quad \left. \left. 6 \tan(z) D_{\substack{1, \dots, 1 \\ 1 \text{ times}}} \substack{2, \dots, 2 \\ 0 \text{ times}}} (yZ2)(z, t) \right] \right) = D_{\substack{1, \dots, 1 \\ 0 \text{ times}}} \substack{2, \dots, 2 \\ 1 \text{ times}}} (yZ3)(z, t) \left. \right]
\end{aligned}$$

$$\begin{aligned}
eq1 := & \left[3 yZ1(z, t) + M yZ3(z, t) + D_2(yZ0)(z, t) + 6 \tan(z) D_1(yZ1)(z, t) = (e) \right. \\
& \left. -a4(t) Q1 \sinh(a4(t)) yZ0(z, t) D(a4)(t), 3 yZ0(z, t) + M yZ2(z, t) + (e) \right. \\
& \left. -a4(t) Q1 \sinh(a4(t)) yZ1(z, t) D(a4)(t) + D_2(yZ1)(z, t) + 6 \tan(z) D_1(yZ0)(z, t) = 0, \right. \\
& \left. M yZ1(z, t) + 3 yZ3(z, t) + 6 \tan(z) D_1(yZ3)(z, t) = (e) -a4(t) Q1 \sinh(a4(t)) yZ2(z, \right. \\
& \left. t) D(a4)(t) + D_2(yZ2)(z, t), M yZ0(z, t) + 3 yZ2(z, t) + (e) \right. \\
& \left. -a4(t) Q1 \sinh(a4(t)) yZ3(z, t) D(a4)(t) + 6 \tan(z) D_1(yZ2)(z, t) = D_2(yZ3)(z, t) \right]
\end{aligned}$$

>

```
> FromMma( ` List[Equal[Plus[Times[3,yZ5[z,t]],Times[M,yZ7[z,t]],  
Times[Power[E,Times[-1,a4[t]]],Q1,Sinh[a4[t]],yZ4[z,t],  
Derivative[1][a4][t]],Times[6,Tan[z],Derivative[1,0][yZ5][z,t]]  
],Derivative[0,1][yZ4][z,t]],Equal[Plus[Times[3,yZ4[z,t]],Times  
[M,yZ6[z,t]],Times[6,Tan[z],Derivative[1,0][yZ4][z,t]]],Plus  
[Times[Power[E,Times[-1,a4[t]]],Q1,Sinh[a4[t]],yZ5[z,t],  
Derivative[1][a4][t]],Derivative[0,1][yZ5][z,t]],Equal[Plus  
[Times[M,yZ5[z,t]],Times[3,yZ7[z,t]],Times[Power[E,Times[-1,a4  
[t]]],Q1,Sinh[a4[t]],yZ6[z,t],Derivative[1][a4][t]],Derivative  
[0,1][yZ6][z,t],Times[6,Tan[z],Derivative[1,0][yZ7][z,t]]],0],  
Equal[Plus[Times[M,yZ4[z,t]],Times[3,yZ6[z,t]],Derivative[0,1]  
[yZ7][z,t],Times[6,Tan[z],Derivative[1,0][yZ6][z,t]]],Times  
[Power[E,Times[-1,a4[t]]],Q1,Sinh[a4[t]],yZ7[z,t],Derivative[1]  
[a4][t]]] ` );
```

eq2 := %;

nops(%);

$$\left[\text{add} \left(i, i = \left[3 yZ5(z, t), M yZ7(z, t), (e)^{(-1) a4(t)} Q1 \sinh(a4(t)) yZ4(z, t) D_{\underbrace{1, \dots, 1}_{1 \text{ times}}} (a4)(t), \right. \right. \right. \\ \left. \left. \left. 6 \tan(z) D_{\underbrace{1, \dots, 1, 2, \dots, 2}_{1 \text{ times } 0 \text{ times}}} (yZ5)(z, t) \right] \right) = D_{\underbrace{1, \dots, 1, 2, \dots, 2}_{0 \text{ times } 1 \text{ times}}} (yZ4)(z, t), \text{add} \left(i, i = \left[3 yZ4(z, t), \right. \right. \right. \\ \left. \left. \left. M yZ6(z, t), 6 \tan(z) D_{\underbrace{1, \dots, 1, 2, \dots, 2}_{1 \text{ times } 0 \text{ times}}} (yZ4)(z, t) \right] \right) = \text{add} \left(i, i = \left[(\right. \right. \right. \\ \left. \left. \left. e)^{(-1) a4(t)} Q1 \sinh(a4(t)) yZ5(z, t) D_{\underbrace{1, \dots, 1}_{1 \text{ times}}} (a4)(t), D_{\underbrace{1, \dots, 1, 2, \dots, 2}_{0 \text{ times } 1 \text{ times}}} (yZ5)(z, t) \right] \right), \text{add} \left(i, i \right. \\ \left. = \left[M yZ5(z, t), 3 yZ7(z, t), (e)^{(-1) a4(t)} Q1 \sinh(a4(t)) yZ6(z, t) D_{\underbrace{1, \dots, 1}_{1 \text{ times}}} (a4)(t), \right. \right. \right. \\ \left. \left. \left. D_{\underbrace{1, \dots, 1, 2, \dots, 2}_{0 \text{ times } 1 \text{ times}}} (yZ6)(z, t), 6 \tan(z) D_{\underbrace{1, \dots, 1, 2, \dots, 2}_{1 \text{ times } 0 \text{ times}}} (yZ7)(z, t) \right] \right) = 0, \text{add} \left(i, i = \left[M yZ4(z, \right. \right. \right. \\ \left. \left. \left. t), 3 yZ6(z, t), D_{\underbrace{1, \dots, 1, 2, \dots, 2}_{0 \text{ times } 1 \text{ times}}} (yZ7)(z, t), 6 \tan(z) D_{\underbrace{1, \dots, 1, 2, \dots, 2}_{1 \text{ times } 0 \text{ times}}} (yZ6)(z, t) \right] \right) = (\\ \left. e)^{(-1) a4(t)} Q1 \sinh(a4(t)) yZ7(z, t) D_{\underbrace{1, \dots, 1}_{1 \text{ times}}} (a4)(t) \right]$$

$$eq2 := \left[3 yZ5(z, t) + M yZ7(z, t) + (e)^{-a4(t)} Q1 \sinh(a4(t)) yZ4(z, t) D(a4)(t) \right]$$

$$+ 6 \tan(z) D_1(yZ5)(z, t) = D_2(yZ4)(z, t), 3 yZ4(z, t) + M yZ6(z, t)$$

$$+ 6 \tan(z) D_1(yZ4)(z, t) = (e)^{-a4(t)} Q1 \sinh(a4(t)) yZ5(z, t) D(a4)(t) + D_2(yZ5)(z,$$

$$\begin{aligned}
& t), MyZ5(z, t) + 3 yZ7(z, t) + (e)^{-a4(t)} QI \sinh(a4(t)) yZ6(z, t) D(a4)(t) \\
& + D_2(yZ6)(z, t) + 6 \tan(z) D_1(yZ7)(z, t) = 0, MyZ4(z, t) + 3 yZ6(z, t) + D_2(yZ7)(z, t) \\
& + 6 \tan(z) D_1(yZ6)(z, t) = (e)^{-a4(t)} QI \sinh(a4(t)) yZ7(z, t) D(a4)(t)]
\end{aligned}$$

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> FromMma( ` List[Equal[Plus[Times[M,yZ11[z,t]],Times[3,yZ9[z,t]],
Times[6,Tan[z],Derivative[1,0][yZ9][z,t]],Derivative[0,1][yZ8]
[z,t]],Equal[Plus[Times[M,yZ10[z,t]],Times[3,yZ8[z,t]],Times[6,
Tan[z],Derivative[1,0][yZ8][z,t]],Derivative[0,1][yZ9][z,t]],
Equal[Plus[Times[3,yZ11[z,t]],Times[M,yZ9[z,t]],Derivative[0,1]
[yZ10][z,t],Times[6,Tan[z],Derivative[1,0][yZ11][z,t]],0],Equal
[Plus[Times[3,yZ10[z,t]],Times[M,yZ8[z,t]],Derivative[0,1][yZ11]
[z,t],Times[6,Tan[z],Derivative[1,0][yZ10][z,t]],0]] ` );

```

eq3 := %;

nops(%);

$$\begin{aligned}
& \left[add \left(i, i = \left[MyZ11(z, t), 3 yZ9(z, t), 6 \tan(z) D_{\substack{1, \dots, 1 \\ 1 \text{ times } 0 \text{ times}}} (yZ9)(z, t) \right] \right) \right] \\
& = D_{\substack{1, \dots, 1, 2, \dots, 2 \\ 0 \text{ times } 1 \text{ times}}} (yZ8)(z, t), add \left(i, i = \left[MyZ10(z, t), 3 yZ8(z, t), \right. \right. \\
& \left. \left. 6 \tan(z) D_{\substack{1, \dots, 1, 2, \dots, 2 \\ 1 \text{ times } 0 \text{ times}}} (yZ8)(z, t) \right] \right) = D_{\substack{1, \dots, 1, 2, \dots, 2 \\ 0 \text{ times } 1 \text{ times}}} (yZ9)(z, t), add \left(i, i = \left[3 yZ11(z, t), \right. \right. \\
& \left. \left. MyZ9(z, t), D_{\substack{1, \dots, 1, 2, \dots, 2 \\ 0 \text{ times } 1 \text{ times}}} (yZ10)(z, t), 6 \tan(z) D_{\substack{1, \dots, 1, 2, \dots, 2 \\ 1 \text{ times } 0 \text{ times}}} (yZ11)(z, t) \right] \right) = 0, add \left(i, i \right. \\
& = \left[3 yZ10(z, t), MyZ8(z, t), D_{\substack{1, \dots, 1, 2, \dots, 2 \\ 0 \text{ times } 1 \text{ times}}} (yZ11)(z, t), 6 \tan(z) D_{\substack{1, \dots, 1, 2, \dots, 2 \\ 1 \text{ times } 0 \text{ times}}} (yZ10)(z, \right. \\
& \left. t) \right] = 0]
\end{aligned}$$

$$\begin{aligned}
eq3 := & [MyZ11(z, t) + 3 yZ9(z, t) + 6 \tan(z) D_1(yZ9)(z, t) = D_2(yZ8)(z, t), MyZ10(z, t) \\
& + 3 yZ8(z, t) + 6 \tan(z) D_1(yZ8)(z, t) = D_2(yZ9)(z, t), 3 yZ11(z, t) + MyZ9(z, t) \\
& + D_2(yZ10)(z, t) + 6 \tan(z) D_1(yZ11)(z, t) = 0, 3 yZ10(z, t) + MyZ8(z, t) \\
& + D_2(yZ11)(z, t) + 6 \tan(z) D_1(yZ10)(z, t) = 0]
\end{aligned}$$

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(14)

```

> FromMma( ` List[Equal[Plus[Times[3,yZ13[z,t]],Times[M,yZ15[z,t]],
Derivative[0,1][yZ12][z,t],Times[6,Tan[z],Derivative[1,0][yZ13]

```

```

[z,t]]],0],Equal[Plus[Times[3,yZ12[z,t]],Times[M,yZ14[z,t]],
Derivative[0,1][yZ13][z,t],Times[6,Tan[z],Derivative[1,0][yZ12]
[z,t]]],0],Equal[Plus[Times[M,yZ13[z,t]],Times[3,yZ15[z,t]],
Times[6,Tan[z],Derivative[1,0][yZ15][z,t]]],Derivative[0,1]
[yZ14][z,t]],Equal[Plus[Times[M,yZ12[z,t]],Times[3,yZ14[z,t]],
Times[6,Tan[z],Derivative[1,0][yZ14][z,t]]],Derivative[0,1]
[yZ15][z,t]]] `);

```

```
eq4 := %;
```

```
nops(%);
```

$$\begin{aligned}
 & \left[\text{add} \left(i, i = \left[3 yZ13(z, t), M yZ15(z, t), D_{\underbrace{1, \dots, 1}_{0 \text{ times}}, \underbrace{2, \dots, 2}_{1 \text{ times}}} (yZ12)(z, t), \right. \right. \right. \\
 & \left. \left. \left. 6 \tan(z) D_{\underbrace{1, \dots, 1}_{1 \text{ times}}, \underbrace{2, \dots, 2}_{0 \text{ times}}} (yZ13)(z, t) \right] \right) = 0, \text{add} \left(i, i = \left[3 yZ12(z, t), M yZ14(z, t), \right. \right. \right. \\
 & \left. \left. \left. D_{\underbrace{1, \dots, 1}_{0 \text{ times}}, \underbrace{2, \dots, 2}_{1 \text{ times}}} (yZ13)(z, t), 6 \tan(z) D_{\underbrace{1, \dots, 1}_{1 \text{ times}}, \underbrace{2, \dots, 2}_{0 \text{ times}}} (yZ12)(z, t) \right] \right) = 0, \text{add} \left(i, i = \right. \\
 & \left. \left[M yZ13(z, t), 3 yZ15(z, t), 6 \tan(z) D_{\underbrace{1, \dots, 1}_{1 \text{ times}}, \underbrace{2, \dots, 2}_{0 \text{ times}}} (yZ15)(z, t) \right] \right) \\
 & = D_{\underbrace{1, \dots, 1}_{0 \text{ times}}, \underbrace{2, \dots, 2}_{1 \text{ times}}} (yZ14)(z, t), \text{add} \left(i, i = \left[M yZ12(z, t), 3 yZ14(z, t), \right. \right. \\
 & \left. \left. 6 \tan(z) D_{\underbrace{1, \dots, 1}_{1 \text{ times}}, \underbrace{2, \dots, 2}_{0 \text{ times}}} (yZ14)(z, t) \right] \right) = D_{\underbrace{1, \dots, 1}_{0 \text{ times}}, \underbrace{2, \dots, 2}_{1 \text{ times}}} (yZ15)(z, t) \left. \right]
 \end{aligned}$$

$$\begin{aligned}
 eq4 := & \left[3 yZ13(z, t) + M yZ15(z, t) + D_2(yZ12)(z, t) + 6 \tan(z) D_1(yZ13)(z, t) = 0, \right. \\
 & 3 yZ12(z, t) + M yZ14(z, t) + D_2(yZ13)(z, t) + 6 \tan(z) D_1(yZ12)(z, t) = 0, M yZ13(z, t) \\
 & + 3 yZ15(z, t) + 6 \tan(z) D_1(yZ15)(z, t) = D_2(yZ14)(z, t), M yZ12(z, t) + 3 yZ14(z, t) \\
 & \left. + 6 \tan(z) D_1(yZ14)(z, t) = D_2(yZ15)(z, t) \right]
 \end{aligned}$$

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(15)

```

> seq3 := simplify( subs(sC3, _c1 = C3, pdsolve( eq3, zvars3 , build)) )
    assuming D(a4)(t) :: positive, M :: positive, Q1 :: positive, t
    :: positive, z :: positive , a4(t) :: positive;

```

$$seq3 := \begin{cases} yZ10(z, t) = \frac{1}{\sqrt{\sin(z)}} \left(c37 \sin(\sqrt{M^2 - 36 C3 - 9} t) \right. \end{cases} \quad (16)$$

$$+ c38 \cos(\sqrt{M^2 - 36 C3 - 9} t)) \left(c36 \sin(z) \frac{-\sqrt{1+4 C3}}{2} + c35 \sin(z) \frac{\sqrt{1+4 C3}}{2} \right) \Bigg),$$

$$yZ11(z, t) = \frac{1}{\sqrt{\sin(z)}} \left((c33 \sin(\sqrt{M^2 - 36 C3 - 9} t) \right.$$

$$+ c34 \cos(\sqrt{M^2 - 36 C3 - 9} t)) \left(c32 \sin(z) \frac{-\sqrt{1+4 C3}}{2} + c31 \sin(z) \frac{\sqrt{1+4 C3}}{2} \right) \Bigg),$$

$$yZ8(z, t) = -\frac{1}{\sqrt{\sin(z)} M} \left(3 \left(\frac{1}{3} \left((\cos(\sqrt{M^2 - 36 C3 - 9} t) c33 \right. \right. \right.$$

$$- \sin(\sqrt{M^2 - 36 C3 - 9} t) c34) \left(c32 \sin(z) \frac{-\sqrt{1+4 C3}}{2} \right.$$

$$+ c31 \sin(z) \frac{\sqrt{1+4 C3}}{2} \Bigg) \sqrt{M^2 - 36 C3 - 9} \Bigg) + (c37 \sin(\sqrt{M^2 - 36 C3 - 9} t) \Bigg)$$

$$+ c38 \cos(\sqrt{M^2 - 36 C3 - 9} t)) \sqrt{1+4 C3} \left(c35 \sin(z) \frac{\sqrt{1+4 C3}}{2} \right.$$

$$- c36 \sin(z) \frac{-\sqrt{1+4 C3}}{2} \Bigg) \Bigg), yZ9(z, t) = -\frac{1}{\sqrt{\sin(z)} M} \left(\left(c36 \sin(z) \frac{-\sqrt{1+4 C3}}{2} \right. \right.$$

$$+ c35 \sin(z) \frac{\sqrt{1+4 C3}}{2} \Bigg) (\cos(\sqrt{M^2 - 36 C3 - 9} t) c37$$

$$- \sin(\sqrt{M^2 - 36 C3 - 9} t) c38) \sqrt{M^2 - 36 C3 - 9}$$

$$+ 3 \sqrt{1+4 C3} \left(c31 \sin(z) \frac{\sqrt{1+4 C3}}{2} \right.$$

$$- c32 \sin(z) \frac{-\sqrt{1+4 C3}}{2} \Bigg) (c33 \sin(\sqrt{M^2 - 36 C3 - 9} t) \Bigg)$$

$$+ c34 \cos(\sqrt{M^2 - 36 C3 - 9} t)) \Bigg) \Bigg\}$$

```
> save seq3, cat("seq3_M4M_", StringTools:-FormatTime("%Y-%m-%d_%H-%M-%S") ,  
".m");  
> save seq3, "2026-02-01-M4M_seq3.m";
```

```
> seq4 := simplify( subs(sC4, _c1 = C4, pdsolve( eq4, zvars4) ))  
    assuming D(a4)(t) :: positive, M :: positive, Q1 :: positive, t  
    :: positive, z :: positive, a4(t) :: positive;
```

$$\begin{aligned} seq4 := & \left\{ yZ12(z, t) = -\frac{1}{\sqrt{\sin(z)} M} \left(3 \left(\frac{1}{3} \left(\sin(\sqrt{M^2 - 36 C4 - 9} t) c44 \right. \right. \right. \\ & - \cos(\sqrt{M^2 - 36 C4 - 9} t) c43) \left(c42 \sin(z) - \frac{\sqrt{1 + 4 C4}}{2} \right. \\ & + c41 \sin(z) \frac{\sqrt{1 + 4 C4}}{2} \left. \right) \sqrt{M^2 - 36 C4 - 9} \left. \right) \\ & + \sqrt{1 + 4 C4} \left(c47 \sin(\sqrt{M^2 - 36 C4 - 9} t) \right. \\ & + c48 \cos(\sqrt{M^2 - 36 C4 - 9} t) \left. \right) \left(c45 \sin(z) \frac{\sqrt{1 + 4 C4}}{2} - c46 \sin(z) - \frac{\sqrt{1 + 4 C4}}{2} \right) \left. \right) \right\}, \\ & yZ13(z, t) = -\frac{1}{\sqrt{\sin(z)} M} \left(\left(c46 \sin(z) - \frac{\sqrt{1 + 4 C4}}{2} \right. \right. \\ & + c45 \sin(z) \frac{\sqrt{1 + 4 C4}}{2} \left. \right) \left(\sin(\sqrt{M^2 - 36 C4 - 9} t) c48 \right. \\ & - \cos(\sqrt{M^2 - 36 C4 - 9} t) c47) \sqrt{M^2 - 36 C4 - 9} \\ & + 3 \sqrt{1 + 4 C4} \left(c41 \sin(z) \frac{\sqrt{1 + 4 C4}}{2} \right. \end{aligned} \quad (17)$$

$$- c42 \sin(z) - \frac{\sqrt{1+4C4}}{2} \Big) \Big(c43 \sin(\sqrt{M^2 - 36C4 - 9} t) \\$$

$$+ c44 \cos(\sqrt{M^2 - 36C4 - 9} t) \Big) \Big), yZ14(z, t) = \frac{1}{\sqrt{\sin(z)}} \Big(\Big(c46 \sin(z) - \frac{\sqrt{1+4C4}}{2} \\$$

$$+ c45 \sin(z) - \frac{\sqrt{1+4C4}}{2} \Big) \Big(c47 \sin(\sqrt{M^2 - 36C4 - 9} t) \\$$

$$+ c48 \cos(\sqrt{M^2 - 36C4 - 9} t) \Big) \Big), yZ15(z, t) = \frac{1}{\sqrt{\sin(z)}} \Big(\Big(c42 \sin(z) - \frac{\sqrt{1+4C4}}{2} \\$$

$$+ c41 \sin(z) - \frac{\sqrt{1+4C4}}{2} \Big) \Big(c43 \sin(\sqrt{M^2 - 36C4 - 9} t) \\$$

$$+ c44 \cos(\sqrt{M^2 - 36C4 - 9} t) \Big) \Big) \Big\}$$

```
> # SaveComplete(seq4);
> save seq4, cat("seq4_M4M_", StringTools:-FormatTime("%Y-%m-%d_%H-%M-%S") ,
    ".m")
> save seq4, "2026-02-01-M4M_seq4.m";
```

```
findme 1
```

```
> eq1;
nops(%);
> seq1 := simplify( subs(sC1, _c1 = C1,
    convert(
        pdsolve( eq1,
            zvars1)
        , int, method = value )
    assuming D(a4)(t) :: positive, M :: positive, Q1 :: positive, t
        :: positive, z :: positive , a4(t) :: positive;
> save seq1, cat("seq1_thinkpad_", StringTools:-
    FormatTime("%Y-%m-%d_%H-%M-%S") , ".m")
> save seq1, "2026-02-01-think_seq1.m";
```

```
findme 2
```

```
> eq2;
nops(%);
> seq2 := simplify( subs( sC2, _c1 = C2,
    convert(
        pdsolve( eq2,
```

```

    zvars2)          ,int, method = value ) )
assuming D(a4)(t) :: positive, M :: positive, Q1 :: positive, t
:: positive, z :: positive , a4(t) :: positive;
> save seq2, cat("seq2_thinkpad_", StringTools:-
  FormatTime("%Y-%m-%d_%H-%M-%S") , ".m")
>
> save seq2, "2026-02-10-think_seq2.m";
>
>
>

```