1. Statement 1: If A is a skew-Hermitian Matrix, then iA and -iA are Hermitian
   Statement 2: If A is any square matrix, the A - A* is a skew-Hermitian Matrix

   Which of the above statement(s) is/are correct? **(1 mark)**

   a. Statement 1
   b. Statement 2
   c. Both Statement 1 and 2
   d. None of the above

2. If A is symmetric and positive definite, then LU factorization can be arranged so that $A=LL^T$
   Where L is ? **(1 mark)**

   a. Lower Triangular with positive diagonal entries
   b. Upper Triangular with positive diagonal entries
   c. Lower Triangular with real diagonal entries
   d. upper Triangular with real diagonal entries

3. $$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ 9 \\ 9 \end{bmatrix}$$
   Find the values of x_1, x_2, and x_3 by using forward and back substitution. **(1 mark)**

   a. 2,3,3
   b. 3,2,3
   c. 3,3,2
   d. 3,3,3

4. A is a 3X3 invertible matrix. Tr(A)=11, $Tr(A^2)$=53, and $Tr(A^{-1})$ = 1/7. Find det(A). **(2 marks)**

   a. 245
   b. 236
   c. 238
   d. 224

5. A matrix P is an orthogonal projector if it is? **(1 mark)**

   a. Idempotent
   b. Symmetric
   c. Square
   d. Both A and B

6. Which of the following is true regarding invertible matrices A and B with eigenvalues λ_1 and λ_2 respectively? **(1 mark)**

    a. tr(A)=tr(B) when $B = A^{-1}$
    b. $\lambda_1\lambda_2$=1 when $B = A^{-1}$
    c. $\lambda_1\lambda_2$=1 when $B = A^T$
    d. det(A)det(B)=1 when $B = A^T$

7. Eigen values of a real symmetric matrix are always: **(1 mark)**

    a. Positive
    b. Negative
    c. Real
    d. Complex

8. How does the Householder transformation preserve orthogonality of a matrix in QR decomposition? **(1 mark)**

    a. By multiplying the matrix with the identity matrix
    b. By transforming the matrix into an upper triangular matrix
    c. By transforming the matrix into a lower triangular matrix
    d. By preserving the dot product of the columns of the matrix

9. Let x_0 be a least squares solution to Ax = b. Which of the following statement is true in general about the residual $r = Ax_0 - b$? **(1 mark)**

    a. r is the projection of b onto the null space of A^T
    b. r is the projection of b onto the column space of A
    c. r is perpendicular to b
    d. r lies in the null space of A

10. For the model below:

$$\dot{N} = rN\left(1 - \frac{N}{K}\right)$$

a) Find all the fixed points of this model. **(3 marks)**
b) Which of these fixed points are stable? **(2 marks)**
c) Solve it analytically and check whether long term evolution of the solution converges to the stable fixed point. **(5 marks)**

Sol) **Strogatz page 25**

EXAMPLE 2.4.2:

Classify the fixed points of the logistic equation, using linear stability analysis, and find the characteristic time scale in each case.

Solution: Here $f(N) = rN\left(1 - \frac{N}{K}\right)$, with fixed points $N^* = 0$ and $N^* = K$. Then $f'(N) = r - \frac{2rN}{K}$ and so $f'(0) = r$ and $f'(K) = -r$. Hence $N^* = 0$ is unstable and $N^* = K$ is stable, as found earlier by graphical arguments. In either case, the characteristic time scale is $1/|f'(N^*)| = 1/r$. ■

11. Find the eigenvalues and eigenvectors of A and A^2 and A^{-1} and $A + 4I$: **(8 marks)**

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}.$$

Check the trace $\lambda_1 + \lambda_2 = 4$ and the determinant $\lambda_1\lambda_2 = 3$. **(2 marks)**

Solution The eigenvalues of A come from $\det(A - \lambda I) = 0$:

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \qquad \det(A - \lambda I) = \begin{vmatrix} \mathbf{2-\lambda} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{2-\lambda} \end{vmatrix} = \lambda^2 - 4\lambda + 3 = 0.$$

This factors into $(\lambda - 1)(\lambda - 3) = 0$ so the eigenvalues of A are $\lambda_1 = \mathbf{1}$ and $\lambda_2 = \mathbf{3}$. For the trace, the sum $2+2$ agrees with $1+3$. The determinant 3 agrees with the product $\lambda_1\lambda_2$.

The eigenvectors come separately by solving $(A - \lambda I)\boldsymbol{x} = \mathbf{0}$ which is $A\boldsymbol{x} = \lambda\boldsymbol{x}$:

$$\boldsymbol{\lambda = 1}: \quad (A - I)\boldsymbol{x} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ gives the eigenvector } \boldsymbol{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\boldsymbol{\lambda = 3}: \quad (A - 3I)\boldsymbol{x} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ gives the eigenvector } \boldsymbol{x}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

A^2 and A^{-1} and $A + 4I$ keep the *same eigenvectors as* A. Their eigenvalues are λ^2 and λ^{-1} and $\lambda + 4$:

A^2 has eigenvalues $1^2 = 1$ and $3^2 = 9$ $\quad A^{-1}$ has $\dfrac{1}{1}$ and $\dfrac{1}{3}$ $\quad A + 4I$ has $\begin{matrix} 1 + 4 = 5 \\ 3 + 4 = 7 \end{matrix}$