Question 8) Show that there exists a continuous function F : [0, 1] -> R whose derivative exists and equals zero almost everywhere but which is not constant.

1. **Correct Expected Solution:**
   a. A sample function is required in the answer to support your arguments about the existence of such a function.
   b. Correct Sample functions: Stepwise recursive fractal functions (Example: Cantor's function/ Devil Staircase Function).

2. **Common Incorrect Solutions**:
   a. **Answers using Rolle's Theorem/ Mean Value Theorem**: Rolle's theorem states that if a function is continuous on the closed interval [a, b] and differentiable on the open interval (a, b) such that $f(a) = f(b)$, then $f(x) = 0$ for some x with $a \leq x \leq b$.

      Problem with this solution: Proves the existence of only one such 'x' with $a \leq x \leq b$ where $f(x) = 0$. It does not prove the existence of a function whose $f(x) = 0$ almost everywhere.

   b. **Dirac Delta function:**

$$\delta(x) \simeq \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

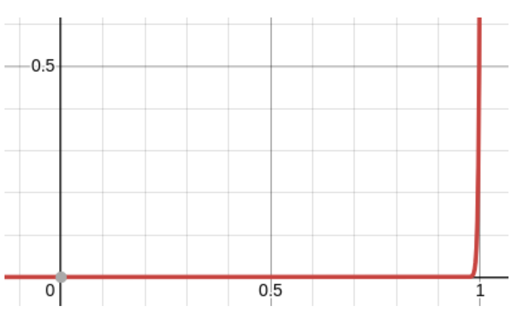
      and which is also constrained to satisfy the identity

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1.$$ [19]

      Problem with this Solution: Dirac delta is not a function which can be defined on real numbers. No function defined on real numbers can have such a property. It is only a heuristic used in various problems to get a calculated guess or an approximation.

   c. **Functions which peak once at the edges**

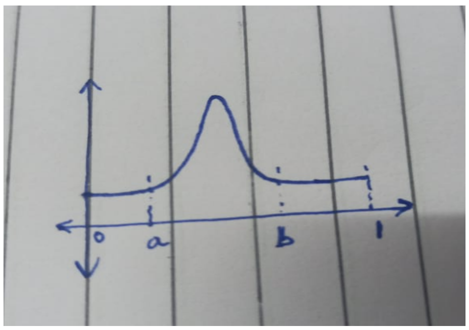
      Examples of such functions include: $f(x) = x^n$ where $n \to \inf$



Problem with this solution: Such functions do not have a value at the boundary points (here $f(1)$ is undefined) thereby making the function discontinuous at that point.

A similar function can be constructed by modifying the Dirac delta function peaking at the value x = 0. The function will become discontinuous at ‘0’ this time.

d. **Functions which peak somewhere in the middle**



It is impossible to make a function peak somewhere in the middle at a single point without making the function discontinuous. If the function retains continuity (as shown in the image), there are infinite points between the two real numbers ‘a’ and ‘b’ (irrespective of how close they are) where the function's derivative is not zero.

e. **Constant Functions are depicted as not constant.**

Functions should assume different values in the range. Sample functions such as F(x) = { 1 if x!=1; x if x = 1 } are considered constant.

Problem with the Solution: All values of x in [0,1] have F(x) = 1, thereby making the function constant. For a non-constant function, it should assume some other value(s) as well.

f. **Thomae’s function (Similar to Dirichlet function)**:

$$f(x) = \begin{cases} \frac{1}{q} & \text{if } x = \frac{p}{q} \quad (x \text{ is rational}), \text{ with } p \in \mathbb{Z} \text{ and } q \in \mathbb{N} \text{ coprime} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Problem with this Solution: This function is discontinuous at all rational numbers and is nowhere differentiable.

Even in incorrect solutions, you have been awarded marks based on the merit of the answer, the approach and the thinking behind it.