CS7.302
Basics of Computer Graphics
Module: Ray Tracing

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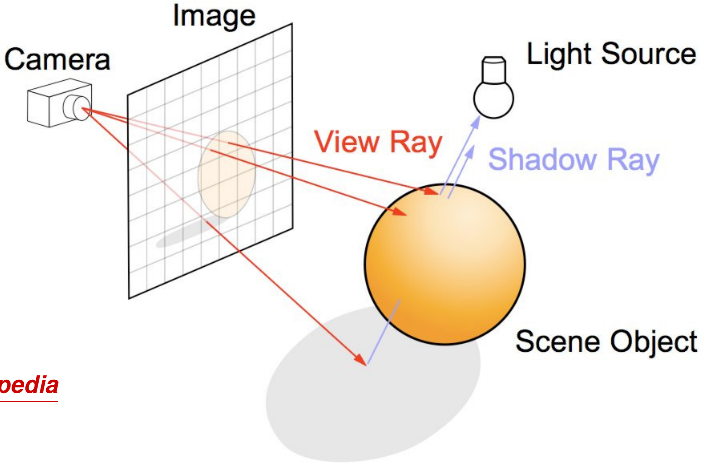
Image Generation

- Think of the image to be generated by graphics: Each pixel needs a colour. And there are $M \times N$ of them.
- Since we assume a pin-hole camera, the colour at each pixel is what the ray from the world point that falls on the pixel brings.
- Too many world points, but there are only MN pixels!
- Let us send a ray from the camera centre out through each pixel to the world. Let us see where that hits. Colour pixel accordingly!

Image Generation (cont.)



Wikipedia

Ray Tracing

- Send rays from CoP through each image pixel to the world and see what each finds

  for each pixel in the image

  Determine closest object in the direction of projector

  Draw the pixel with appropriate colours

- Called **ray tracing** or **ray casting**.

- Equation of the ray is known. World objects are known.
  Need to intersect the ray with objects!

Ray Equation

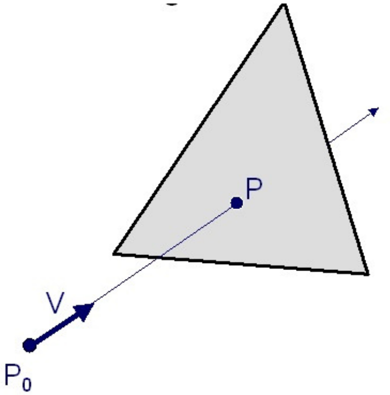
- If the CoP $\mathbf{P_0}$ is (x_0, y_0, z_0) and pixel point $\mathbf{P_1}$ is (x_1, y_1, z_1), the ray is given by $\mathbf{P} = \mathbf{P_0} + t(\mathbf{P_1} - \mathbf{P_0})$ or

$$(x_0 + t\Delta x,\ y_0 + t\Delta y,\ z_0 + t\Delta z), \quad t > 0$$

- Negative values of t: behind CoP.
  $t = 1$: the pixel plane.
  True region of interest: $t > 1$, in front of the camera

- Compute intersections with other objects. Closest object is the one with the smallest t value.

Intersection with a Polygon



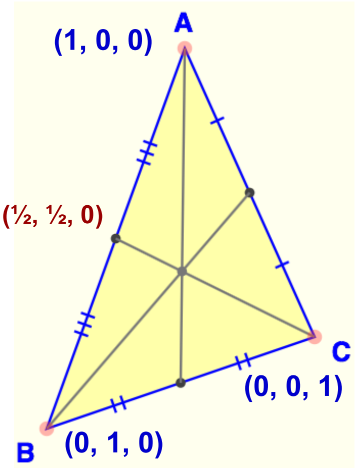
- Intersect the ray with the finite triangle: First intersect with the (infinite) plane.

Intersection with a Polygon

- Plane of the polygon is given by: $Ax + By + Cz + D = 0$
- Intersection point: $t = -\frac{Ax_0+By_0+Cz_0+D}{A\Delta x+B\Delta y+C\Delta z}$
- Does it lie within the polygon?
- Project to a coordinate plane and check for 2D polygon containment.
- There are more efficient methods using **barycentric** coordinates.

Barycentric Coords on Triangles

- A point $P = \alpha A + \beta B + \gamma C$
- **Barycentric coords:** α, β, γ
- Also, $\alpha + \beta + \gamma = 1$
- Coordinates of mid-points of sides? Centroid?
- Properties of inside of the triangle? Outside?
- α, β, γ given $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{P}$

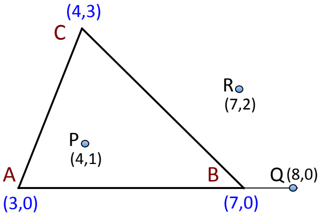


Computing Barycentric Coords

- $\mathbf{P} = \alpha\,\mathbf{A} + \beta\,\mathbf{B} + (\mathbf{1} - \alpha - \beta)\,\mathbf{C}$
- $\begin{bmatrix} x - x_3 \\ y - y_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \mathbf{T}\,\rho$
- Barycentric coordinates: $\rho = \mathbf{T}^{-1}\,(\mathbf{P} - \mathbf{C})$
- $\mathbf{T}$ is invertible as $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a triangle. (Determinant of T is related to area of the triangle ABC, which can not be zero, unless in case of degenerated scenarios).
  $\mathbf{T}$ is fixed for the triangle and is precomputed.
- Works for point **P** anywhere on the plane.
  Evaluate ρ for the intersection point.
  If any of $\alpha, \beta, \gamma \notin [0, 1]$, **P** is outside triangle

An Example

- Barycentric coords of P, Q, R?
- $\mathbf{T} : \begin{bmatrix} -1 & 3 \\ -3 & -3 \end{bmatrix}$, $\mathbf{T}^{-1} : \frac{1}{12}\begin{bmatrix} -3 & -3 \\ 3 & -1 \end{bmatrix}$
- $\mathbf{P} : \mathbf{T}^{-1}\begin{bmatrix} 0 \\ -2 \end{bmatrix} \equiv (?, ?, ?)$
- $\mathbf{Q} : \mathbf{T}^{-1}\begin{bmatrix} 4 \\ -3 \end{bmatrix} \equiv (?, ?, ?)$
- $\mathbf{R} : \mathbf{T}^{-1}\begin{bmatrix} 3 \\ -1 \end{bmatrix} \equiv (?, ?, ?)$



An Example

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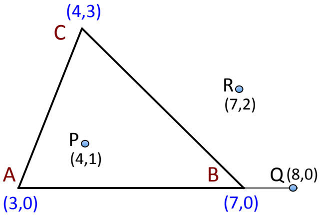
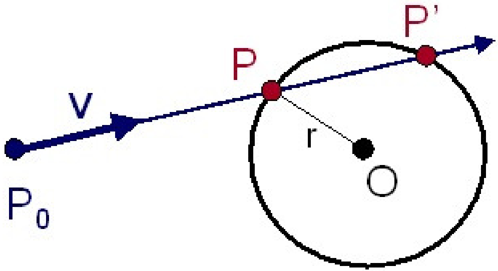
- $\mathbf{P} : \mathbf{T}^{-1} \begin{bmatrix} 0 \\ -2 \end{bmatrix} \equiv (\frac{1}{2}, \frac{1}{6}, \frac{1}{3})$
- $\mathbf{Q} : \mathbf{T}^{-1} \begin{bmatrix} 4 \\ -3 \end{bmatrix} \equiv (-\frac{1}{4}, \frac{5}{4}, \mathbf{0})$
- $\mathbf{R} : \mathbf{T}^{-1} \begin{bmatrix} 3 \\ -1 \end{bmatrix} \equiv (-\frac{1}{2}, \frac{5}{6}, \frac{2}{3})$



Intersection with a Sphere



- Ray is given by $(x_0 + t\Delta x,\ y_0 + t\Delta y,\ z_0 + t\Delta z),\ t > 0$
- Sphere is given by $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$.

Intersection with a Sphere

- Substituting ray into sphere: $(\Delta x^2 + \Delta y^2 + \Delta z^2)\ t^2 + 2[\Delta x(x_0 - a) + \Delta y(y_0 - b) + \Delta z(z_0 - c)]\ t + (x_0 - a)^2 + (y_0 - b)^2 + (z_0 - c)^2 - r^2 = 0$
- A quadratic equation. Solve for t. Two real solutions or two imaginary solutions.
- Real solution with smaller positive t is the one of interest. When are both equal??
- If both imaginary, no intersection.
- Can normalize such that the coefficient of t^2 is 1, since we are interested only in the relative values of t.

Intersection with Other Primitives

- Need an analytical method to intersect a primitive
- Possible to compute **exact intersections** with objects defined analytically (no approximations)
- Intersections are not known or easy! Higher polynomial surfaces? Sinusoids? Others? Parametric surfaces?
- Result: Quadratic, Cubic, and Quartic polynomials can be solved analytically. Beyond that ***only*** iterative solutions!
- Otherwise: First **tesselate** or subdivide to triangles and then use the triangle algorithm.

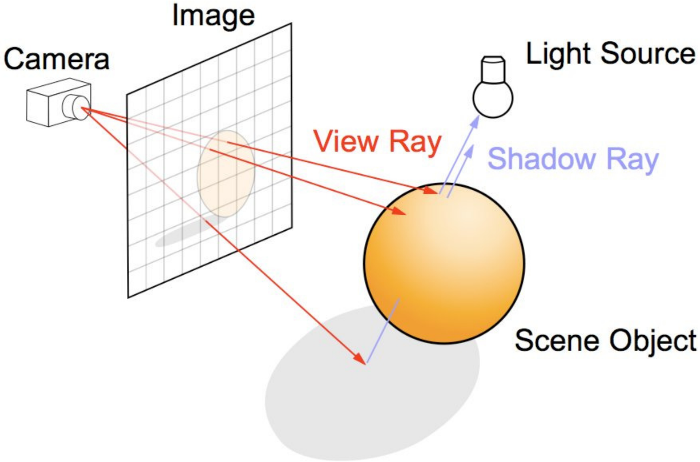
Scene with Multiple Primitives

- We now know how to intersect with **a single** primitive. Any typical scene has many primitives.
- Naive approach:
  - Check intersections with each primitive in a loop
  - Remember the primitive for which the t value is minimum positive.
- Very inefficient as each ray has to loop over each primitive!
- *We will be come back to this issue later!*

Can we do more?

- Let us look at the picture again ....

Can we do more? (cont.)



- We see: **light source, shadow, refections!**

3D Scene with Recursive Ray Tracing

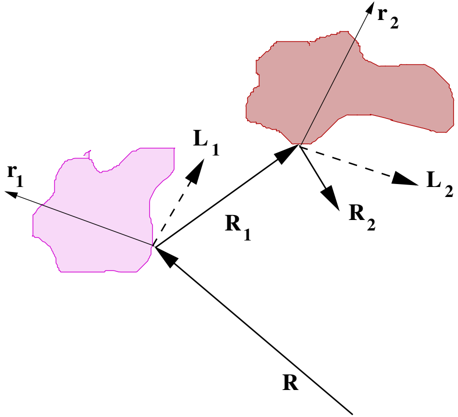
Wikipedia

Recursive Ray Tracing

- When a **primary ray** from CoP hits an object, it can
  - Reflect off the surface about the normal
  - Transmit into the object as per Snell's law of refraction
  - Collect light from all light sources by diffuse reflections. Or be shadowed from one more light sources.
- These **secondary rays** can bring in a colour/intensity by recursively applying the above principle.
  - As the rays from a pixel ricochet (rebound, bound or skip off) through the scene, each successively interested surface is added to a binary **ray-tracing tree**.

Recursive Ray Tracing (cont.)

- Net appearance is a combination of the individual colours.



- New rays may be spawned from every point, and handled the same way!

RRTracing: Main Algorithm

- Recursive ray tracing stops when:
  - The ray intersect no surface.
  - The ray intersect a light source that is not a reflective surface.
  - The tree has been generated to it's maximum allowable depth.
- Call the recursive ray tracing routine for every pixel to compute its colour.

```
for each scan-line do
        for each pixel in scan line do
        determine the ray for the pixel
        pixelColour ← RT_Trace(ray, 1)
```

Procedure `RayTrace`

- Intersect ray with closest object and compute colour using a shading routine

```
RT_Trace(ray, depth)
    Find the closest object for the ray
    if (object found)
        compute normal at intersection point
        return RT_Shade(obj, ray, intersect, normal, dpth)
    else
        return BackgroundColour
```

Procedure `RayShade`

- Combine all effects to compute colour

```
RT_Shade(obj, ray, pnt, n, d)
    clr = ambient term
    for each light L do
        lRay = ray to light from pnt
        if lRay is blocked, shadow
            Compute the light reaching pnt from L
        else
            clr += k_d * diffuse component due to L
        if (d >= maxDepth)
            return clr
    Onto recursive processing now
```

Procedure `RayShade` (cont.)

```
if (object is reflective)
    rRay = reflected ray from pnt
    rClr = RT_Trace(rRay, d + 1)
    clr += k_s * rClr
if (object is transparent)
    tRay = refracted ray from pnt
    if (no total internal reflection)
        tClr = RT_Trace(tRay, d + 1)
        clr += k_t * tClr
return clr
```

When do we Stop?

- Truth: *Recursion never stops in the real world!*
- However, the impact on original pixel due to a later *bounce* will diminish with the **depth** of the ray.
- Stop when further impact is negligible.
- In practice: Set a maximum limit on the depth.
- Add a step to `RT_Trace`:

  if (depth $>$ MAX_DEPTH) return 0;

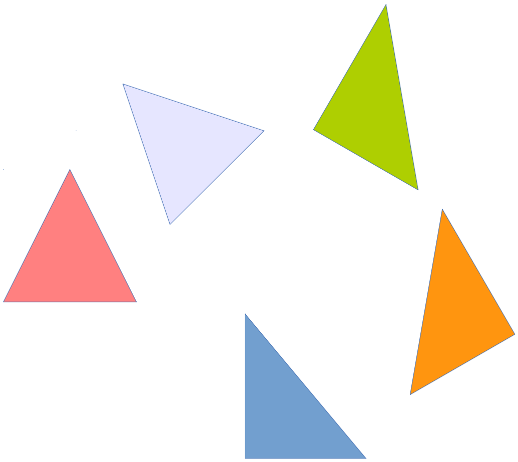
Acceleration Structure

- Use an **acceleration data structure** to quickly identify the subset of primitives the ray may intersect.
  Or, eliminate primitives that the ***ray will not intersect***
- Shouldn't miss intersections, but may have a few extra.
- Procedure for each ray:
  - Traverse structure, eliminate primitives with no intersection
  - Recurse, remember the one with the minimum t
- Efficiency depends on the acceleration structure used.
- Popular spatial structures: Grids, Octree, kd-Tree, Bounding Volume Hierarchy (BVH), ...

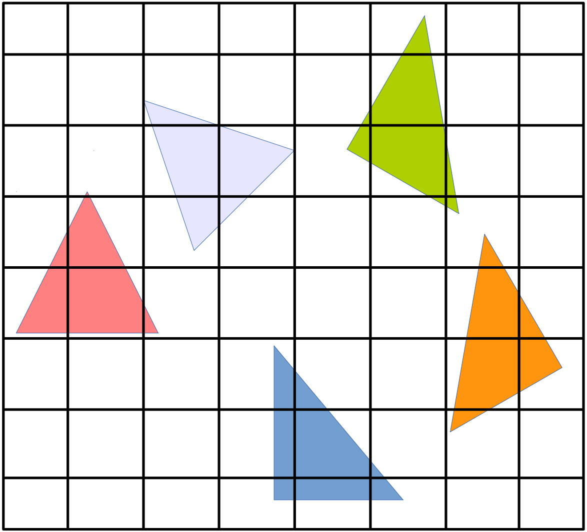
Identifying Primitives that Intersect

- Which primitive(s) does a ray intersect on its way?
- Organize primitives using a suitable data structure so that all primitives needn't be tested for each ray

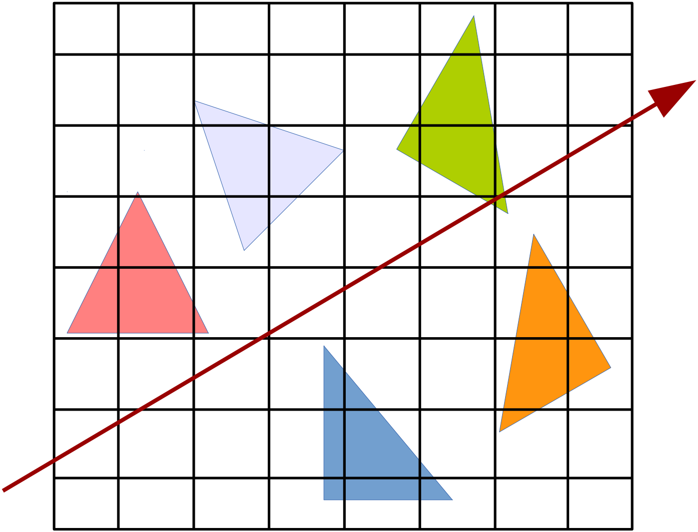
A Few Triangles



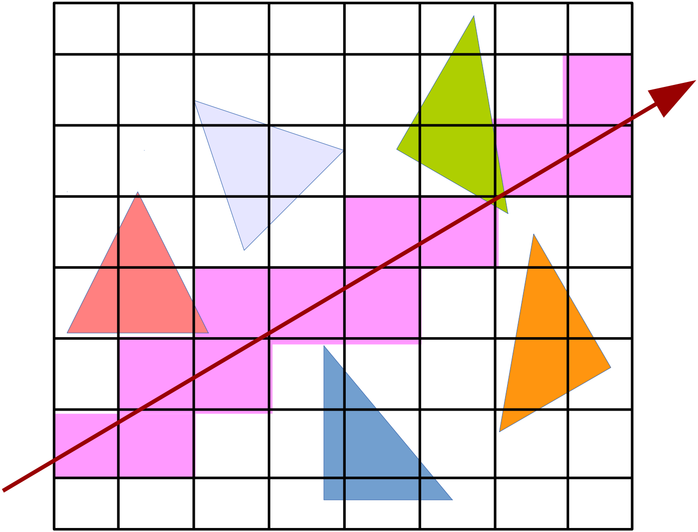
Triangles in a Grid



Triangles, Grid, and a Ray



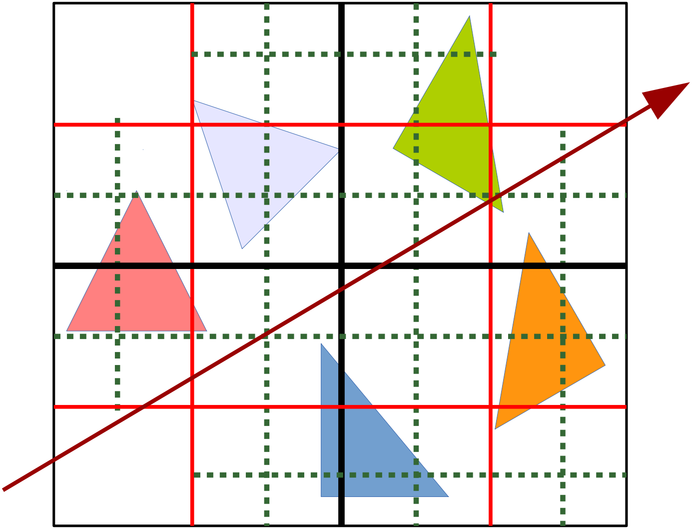
Evaluated Grid Cells



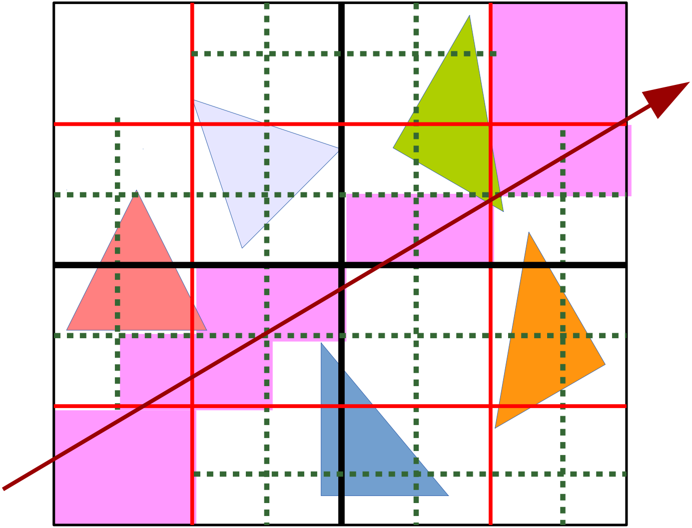
Teapot in a Stadium Situation

- Grids work well if distribution of primitives in the grid cells is nearly uniform
- What if there is a detailed **teapot** in a sparse **stadium**?
- Most parts of the stadium is empty, with triangles in a only a few cells
- The cell with the teapot has a large number of triangles and will take **a lot of time** to evaluate!
- Answer: non-uniform grid cells! Different ways to do this

Triangles and a Quad-Tree



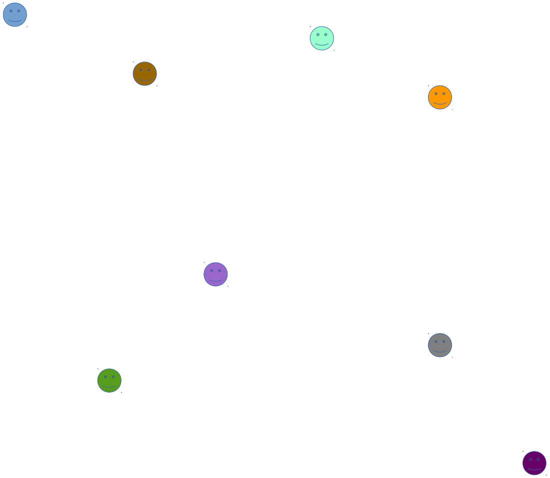
Evaluated Quad-Tree Nodes



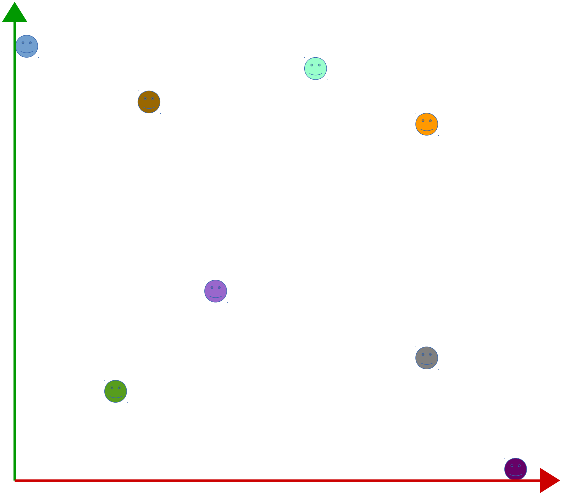
KD-Trees

- Grids, Quadtrees or Octrees, etc., divide the space using fixed planes, unmindful of the objects in it
- Objects may be split if the planes pass through them
- Can we minimize splits and be sensitive to the objects in the volume?
- Adaptive dividing plane selection: KD-Trees

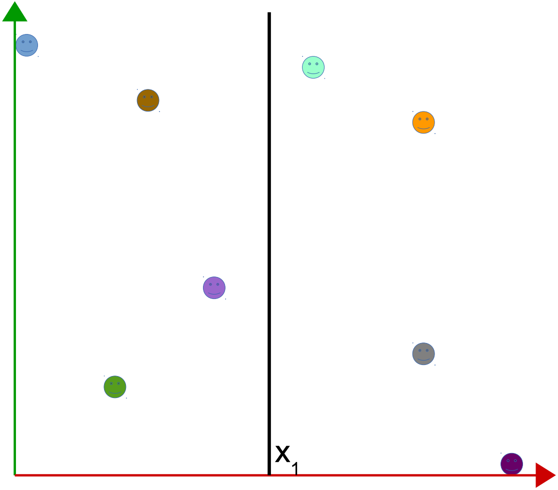
KD-Tree: Points



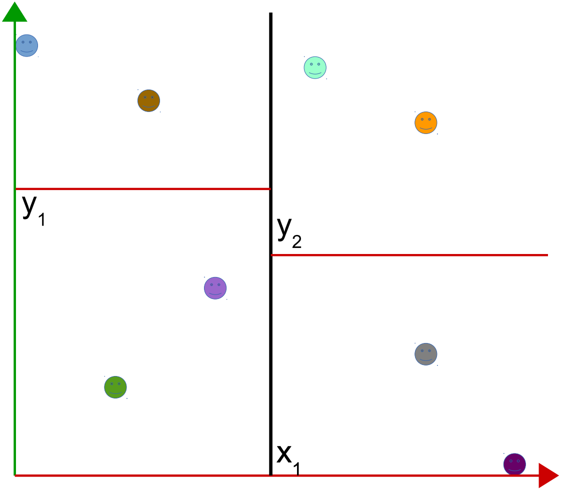
KD-Tree: Axes



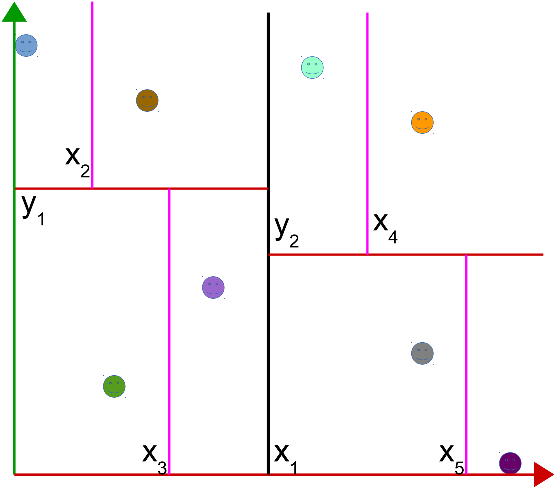
KD-Tree: First Division



KD-Tree: Second



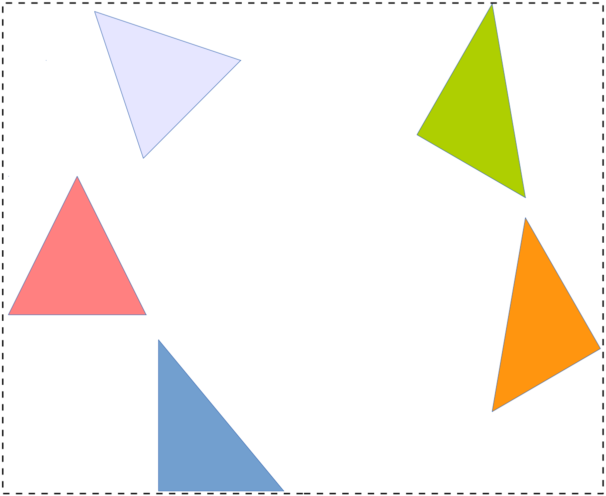
KD-Tree: Third



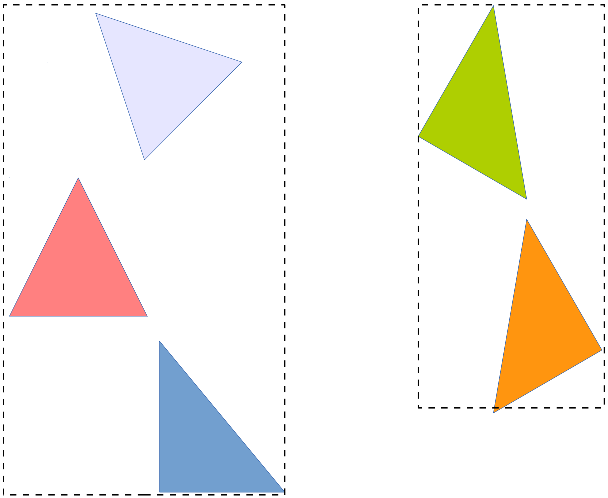
BVH Trees

- Spatial division can divide primitives.
- Can we keep primitives intact?
- Use bounding boxes to represent it and create a hierarchy
- Overlap may occur, but quite efficient
- BVH Tree: Bounding Volume Hierarchy Trees

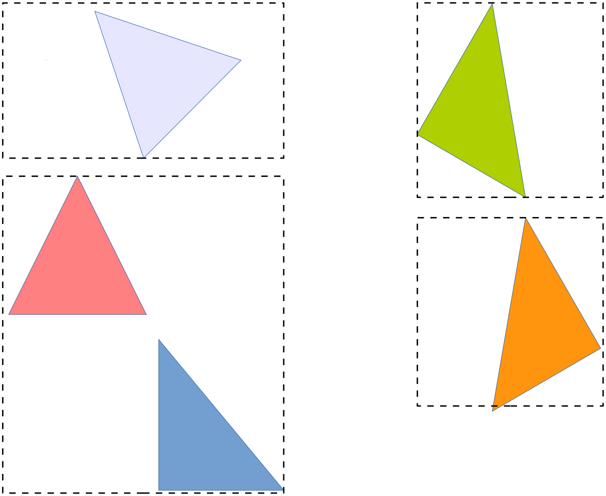
BVH Trees: Triangles



BVH Trees: Triangles



BVH Trees: Triangles



Ray Tracing: Discussion

- Ray tracing is for realistic rendering!
- Good acceleration structure critical to reduce the number of intersections computed.
- Coherence in image space enables us to trace a bunch of rays (beams) to be traced together.
- Very compute intensive as the ray tree can grow exponentially with spawning of new rays. Considered a **grand challenge** problem in parallel computing.
- Subject to numerical precision as small changes in secondary and tertiary rays can have large impact.
- Several simplifications: Trace a set of rays (beams, cones, pencils) to take advantage of coherence, stochastic sampling, etc.

Ray Tracing: Discussion (cont.)

- Used when really high quality rendered images are required at the expense of time.
- Powerful multicore CPUs and high-performance Graphics Processor Units (GPUs) have made it considerably fast today.

Thank you!