

1. Prove or disprove the negligibility of the following functions:

(a) $\frac{2^{-1000}}{n}$

(b) $\frac{1}{(\log n)!}$

(c) $\frac{1}{(\log \log n)!}$

(d) $2^{\frac{-n}{1000}}$

[10]

2. Using your experience in security definitions, provide a definition for perfect pseudorandom generators $G : \{0, 1\}^n \rightarrow \{0, 1\}^{n+1}$. Furthermore, prove that such perfect PRGs do not exist.

[10]

3. Assuming that DLP is hard in Z_{17}^* (of course, it isn't really), using 4-bits to represent each of its elements, design a corresponding PRG $G : \{0, 1\}^4 \rightarrow \{0, 1\}^*$, and output the first six bits if seed is set to be the last 4 bits of your choice (say, the last 4 bits of the last 2 digits of your roll number).

[10]

4. Prove that the shift cipher is perfectly secret as long as only one character in $[a, \dots, z]$ is encrypted.

[10]