Q1.

It is given in the question that $aabb$ is a perfect square and a decimal number.

$$aabb = 10^3a + 10^2a + 10b + b$$

$$= 10^2a(10+1) + 11b$$

$$= 11(10^2a + b)$$

For the above quantity to be a perfect square, $(10^2a + b)$ should be divisible by 11 **and** the quotient should be a perfect square, i.e.

$$(10^2a + b) = 11n^2$$

[3 Marks till here] - So if guessing starts here and concludes in right answer, 4 Marks (No extra mark if final answer is wrong).

Since $a, b \in Z^+$ and $a, b \in [0, 9]$,

$$10a^2 + b = (a0b)_{10}$$

The divisibility rule of 11 dictates that the quantity $(a + b - 0)$ should be zero or divisible by 11. Since the former is not possible, and $a, b \in [0, 18]$,

$$\boldsymbol{a + b = 11}$$

[6 Marks till here] – If guessing starts here and concludes in right answer, 7 Marks (No extra mark if final answer is wrong).

Performing the division,

$$\begin{array}{l} 11\,]\ a\ \ 0\ \ b\,[\,x \\ \underline{\quad\ \ x\ \ x\qquad\quad} \end{array}$$

The remainder of the first step above contains two digits $(a - 1 - x)$, and $(10 - x)$. The remainder should also $\in [0, 10]$.

$$\Rightarrow a - 1 - x = 1 \; \boldsymbol{AND} \; 10 - x = 0$$

$$(OR)$$

$$a - 1 - x = 0 \; (the\ second\ digit \in [0, 9])$$

The first condition above leads to $a = 13$, which is a contradiction.

$$\Rightarrow a = x + 1$$

Now, continuing the division

$$\begin{array}{l} 11\,]\ a\ \ 0\ \ b\,[\,xy \\ \underline{\quad\ \ x\ \ x\qquad\qquad\ } \\ \qquad\quad (10 - x)\ \ \ b \\ \qquad\qquad y \qquad\ \ y \end{array}$$

$$\Rightarrow 10 - x = y \; \boldsymbol{AND} \; b = y$$

$$(OR)$$

$$\Rightarrow 9 - x = y \; \boldsymbol{AND} \; b + 10 = y$$

Expanding the first condition,

$$\Rightarrow b = 10 - x$$

$$\Rightarrow a + b = 10 - x + x + 1 = 11$$

which is true. Therefore,

$$a = x + 1 \Rightarrow x = a - 1$$

$$b = y = 10 - x$$

$$\Rightarrow y = 10 - a + 1 = 11 - a$$

The final condition is that $(xy)_{10}$ should be a perfect square.

$$(xy)_{10} = 10(a - 1) + 11 - a$$

$$= 9a + 1$$

$$\Rightarrow \boldsymbol{9a + 1 = n^2}$$

[9 Marks till here] – If guessing starts here and concludes in right answer, 10 Marks (No extra mark if final answer is wrong).

Since $a \in [0, 9]$ and $a \in Z^+$

$$\Rightarrow n^2 \in [1, 82]$$

$$\Longrightarrow n \in [1, 9] \; and \; n \in Z^+$$

Now

$$a = \frac{n^2 - 1}{9}$$

$$= \left(\frac{n-1}{3}\right)\left(\frac{n+1}{3}\right)$$

For $a \in Z^+$, both $n - 1$ and $n + 1$ should be divisible by 3. However, $n - 1$, n, $n + 1$ are consecutive integers. Hence, if $n - 1$ is divisible by 3, $n + 1$ is not — and vice versa. The only possibility for $a \in Z^+$ is if either $n - 1$ or $n + 1$ is equal to 9 (Multiple of 9 is also enough, but $n \in [1, 9]$).

$$\Longrightarrow n = 8$$

$$\Longrightarrow \boldsymbol{a = 7}$$

$$\Longrightarrow \boldsymbol{b = 4}$$

Therefore, the original decimal number is **7744**.