

# Legged Robots Practical: Project 2

29.10.2024

# Plan

Single  
Leg

- **W1-3** (10.09, 17.09, 24.09)
  - Introduction + double pendulum kinematics and dynamics
  - Jacobian (Cartesian PD + Force Control)
  - Inverse Kinematics (compare with force control)
  - Single-leg hopping

Biped

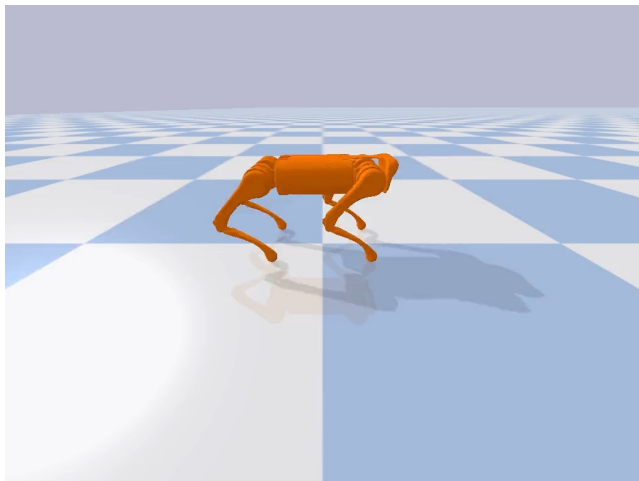
- **W4-6** (01.10, 08.10, 15.10)
  - Biped Walking with Divergent Component of Motion (BD Atlas)  
**[Biped Report – 15% of grade]**

Quadruped

- **W7-14** (29.10-17.12)
  - Quadruped CPG Trot
  - Quadruped Locomotion Project (CPGs, Deep RL)  
**[Quadruped Report – 35% of grade]**  
**[COMPETITION/DEMO 17.12.2024]**

# Part 1: Central Pattern Generators

Trot



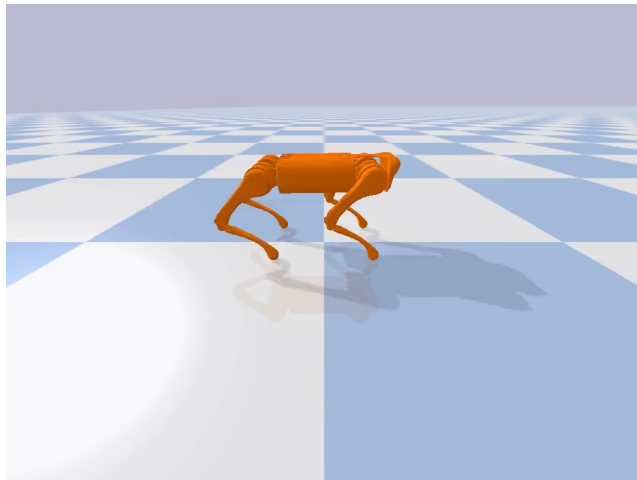
Bound



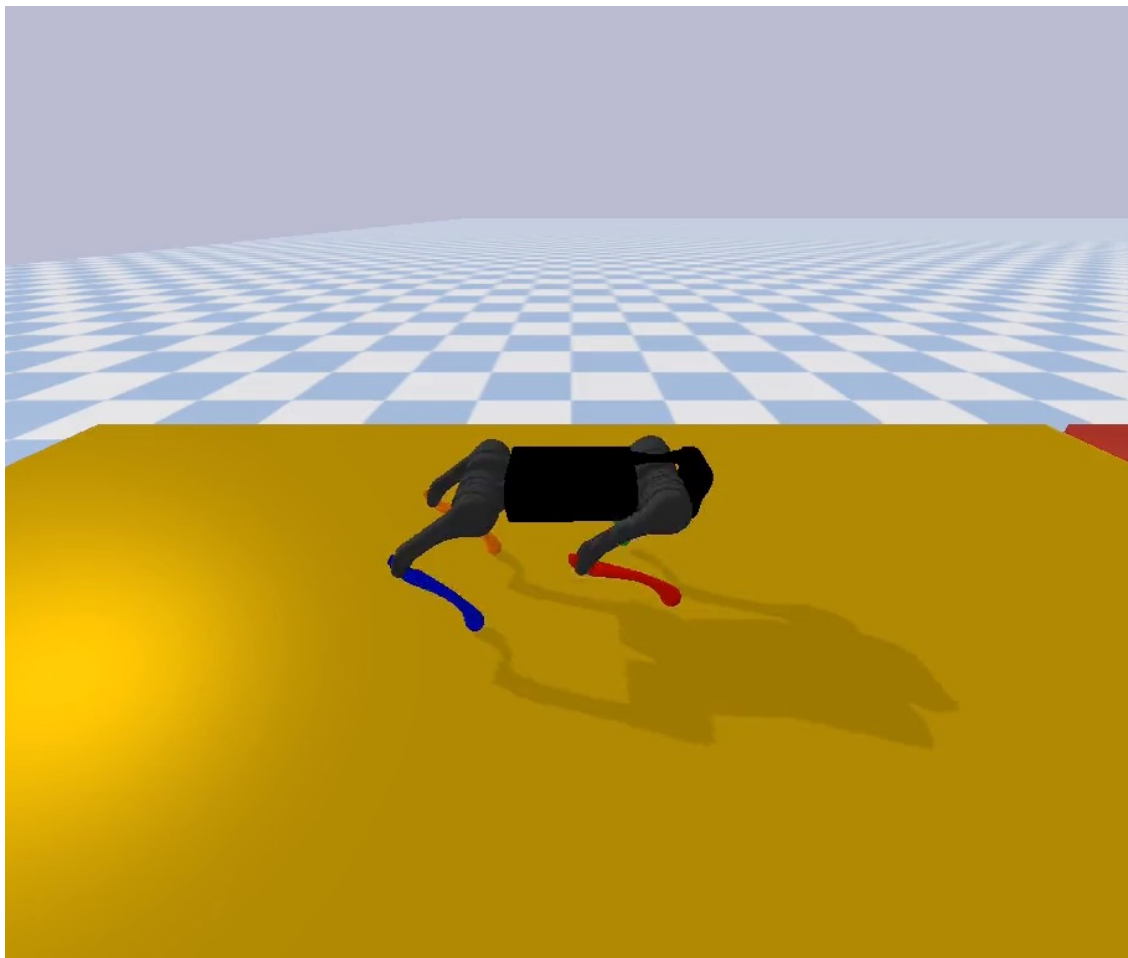
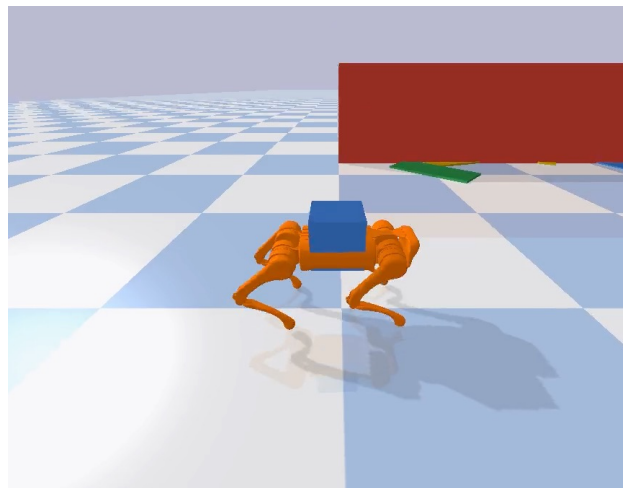
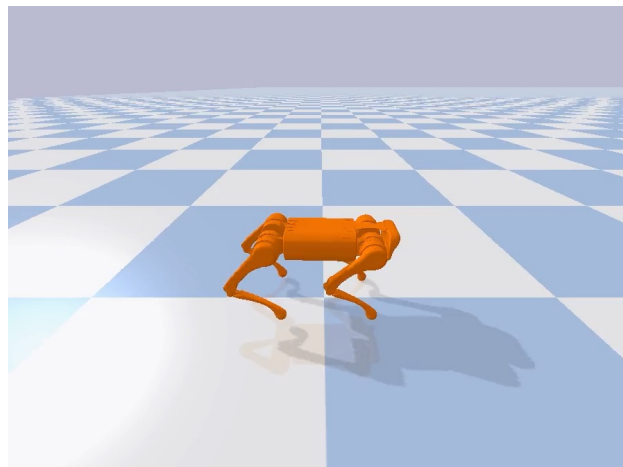
Pace



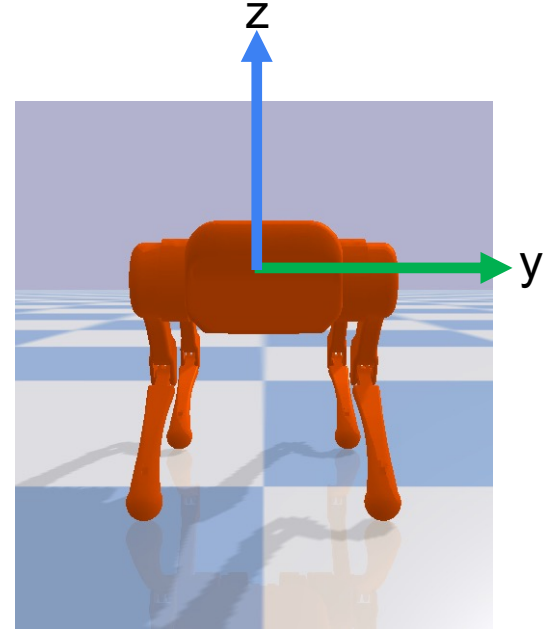
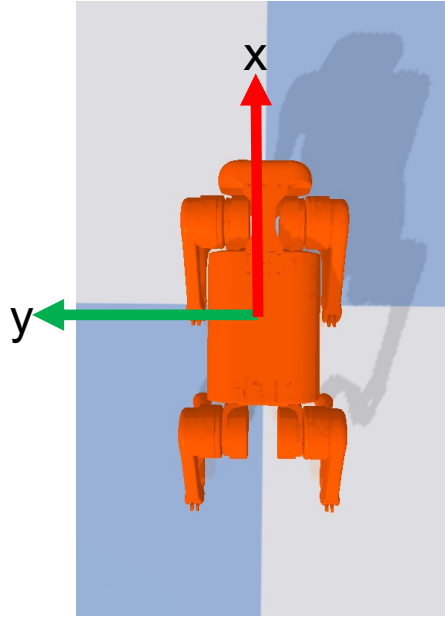
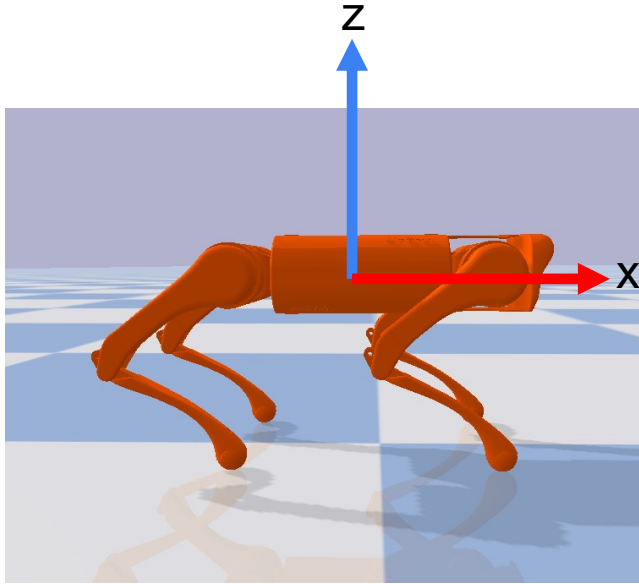
Walk



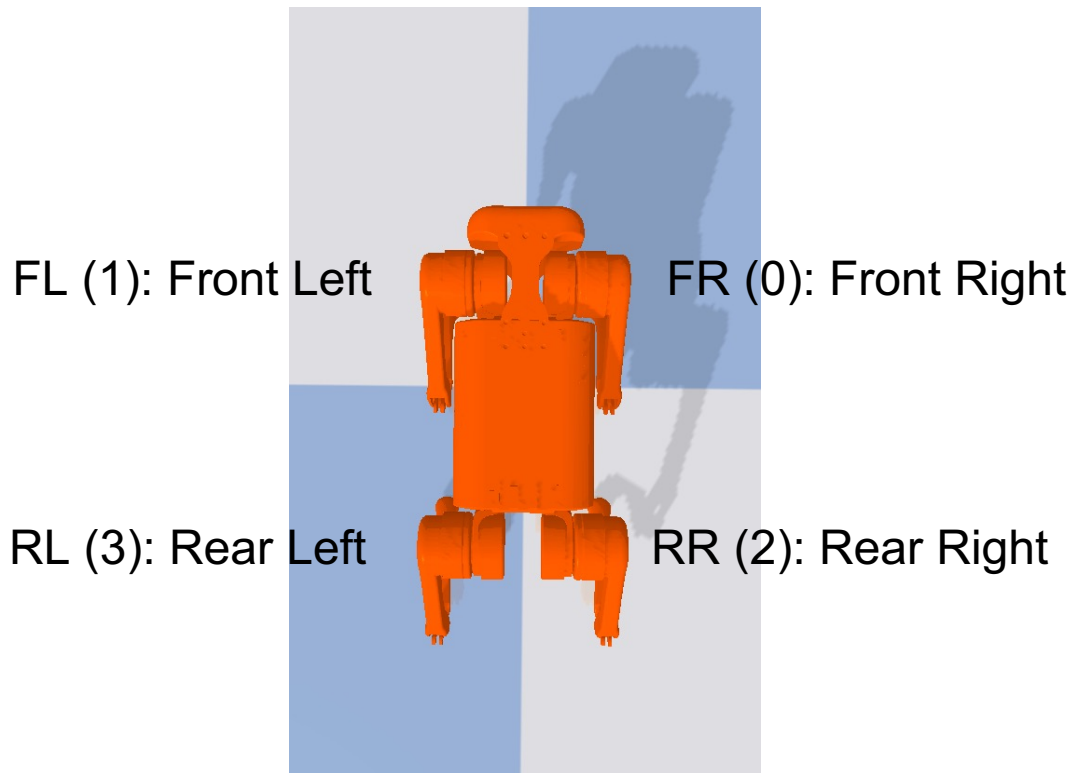
## Part 2: Deep Reinforcement Learning



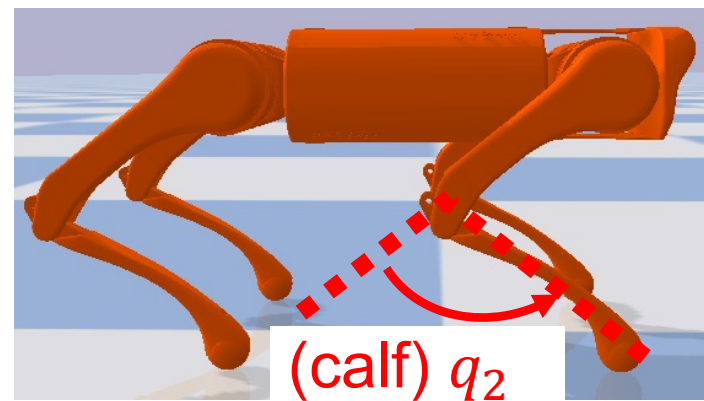
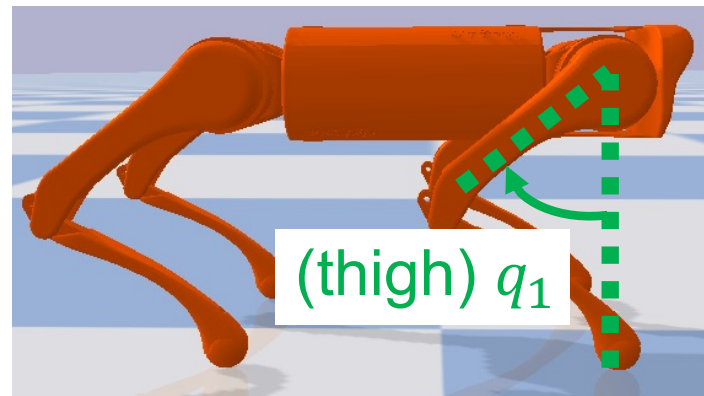
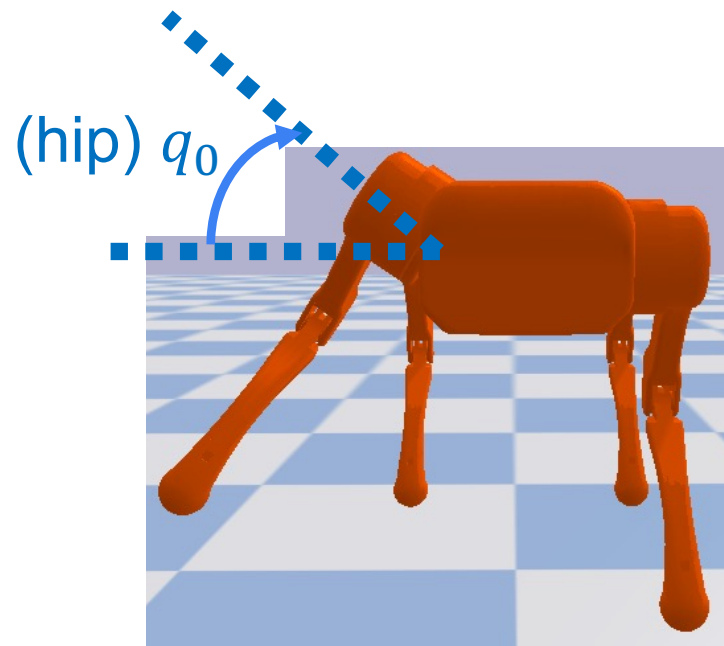
# Quadruped Model Reference Frame



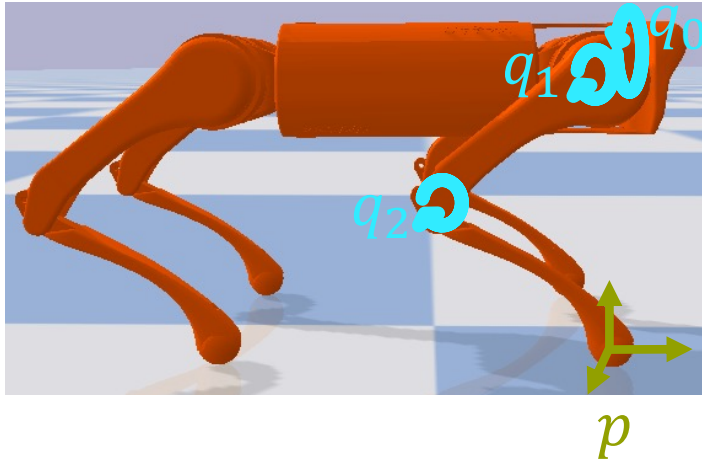
# Quadruped Model Leg References



# Quadruped Model Joint References



# Joint angles $\leftrightarrow$ Cartesian space (in leg frame)



$$p = f(q)$$

Forward kinematics

$$q = f^{-1}(p)$$

Inverse kinematics

$$\dot{p} = v = J(q)\dot{q}$$

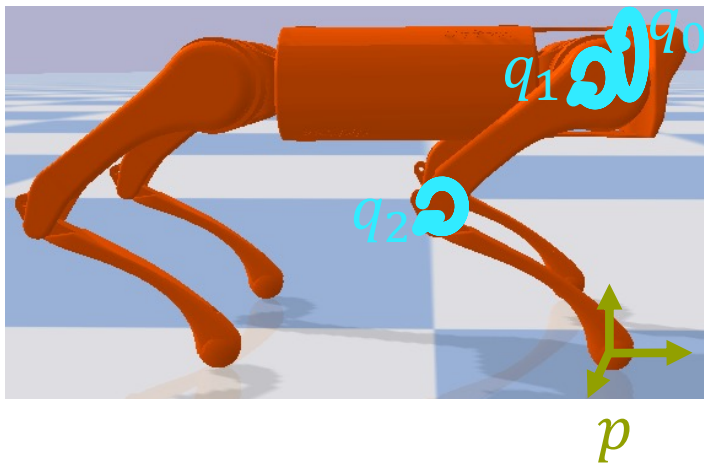
Foot linear velocity

$$\tau = J^T(q)F$$

Map desired end effector force to torques



# Joint angles $\leftrightarrow$ Cartesian space (leg frame control)



$$p = f(q)$$

Forward kinematics

$$q = f^{-1}(p)$$

Inverse kinematics

$$\dot{p} = v = J(q)\dot{q}$$

Foot linear velocity

$$\tau = J^T(q)F$$

Map desired end effector force to torques

$$\tau_{joint} = K_{p,joint}(q_d - q) + K_{d,joint}(\dot{q}_d - \dot{q})$$

Joint PD

$$\tau_{Cartesian} = J^T(q)[K_{p,Cartesian}(p_d - p) + K_{d,Cartesian}(v_d - v)]$$

Cartesian PD

$$\tau_{final} = \tau_{joint} + \tau_{Cartesian}$$

Contributions from both joint PD and Cartesian PD

# Central Pattern Generators: Review

100%

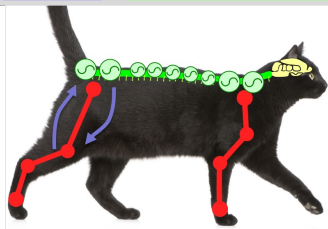
# From Lecture 4

Descending modulation

Spinal cord

Reflexes

Central pattern generators



Musculoskeletal system

Respective Role  
in motor control



lamprey



salamander



"Comp ima sp



cat



human

# Modeling the CPG with coupled oscillators (Quadruped)

Amplitude:

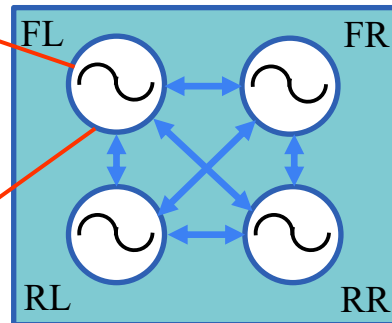
$$\dot{r}_i = \alpha(\mu - r_i^2)r_i$$

Phase:

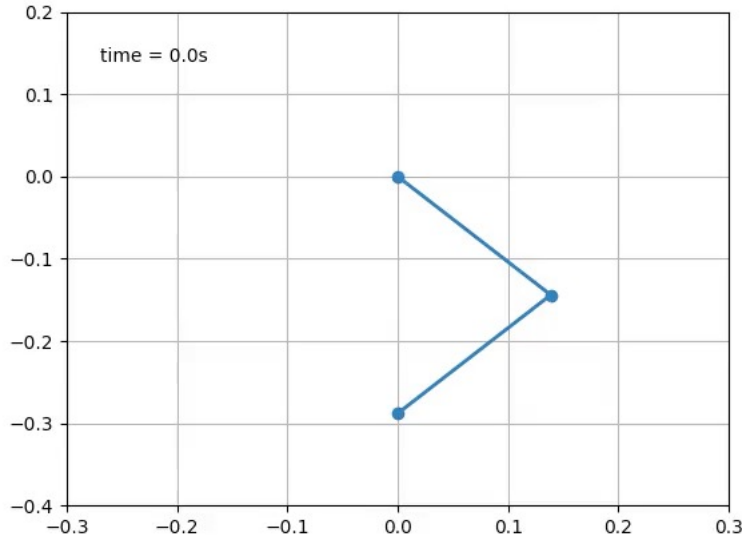
$$\dot{\theta}_i = \omega_i + \sum_{j=0}^3 r_j w_{ij} \sin(\theta_j - \theta_i - \phi_{ij})$$

Output:

$$x_{\text{foot}} = -d_{\text{step}} r_i \cos(\theta_i)$$
$$z_{\text{foot}} = \begin{cases} -h + g_c \sin(\theta_i) & \text{if } \sin(\theta_i) > 0 \\ -h + g_p \sin(\theta_i) & \text{otherwise} \end{cases}$$



# Mapping CPG States to Foot Positions with Inverse Kinematics



$$\dot{r}_i = \alpha(\mu - r_i^2)r_i$$

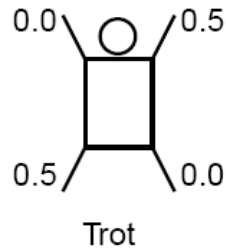
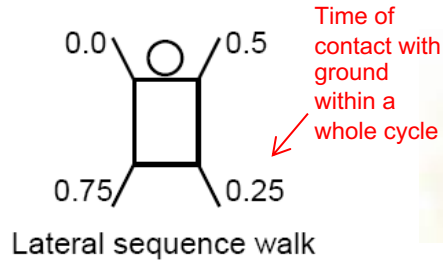
$$\dot{\theta}_i = \omega_i$$

$$x_{\text{foot}} = -d_{\text{step}}r_i \cos(\theta_i)$$

$$z_{\text{foot}} = \begin{cases} -h + g_c \sin(\theta_i) & \text{if } \sin(\theta_i) > 0 \\ -h + g_p \sin(\theta_i) & \text{otherwise} \end{cases}$$

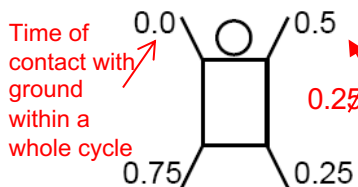
# Gait Terminology

- *Stride duration* = the duration of a complete cycle (the period)
- *Swing phase* of a limb (period during which the limb is off the ground)
- *Stance phase* (period during which the limb touches the ground)
- *Duty factor* = Stance duration / Stride duration

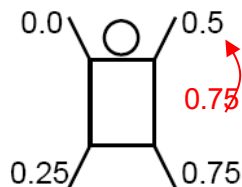


# Most common quadruped gaits

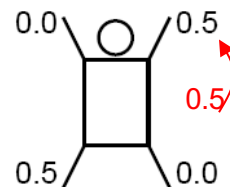
Classification in terms of the footfall sequences (mainly used in mathematical biology)



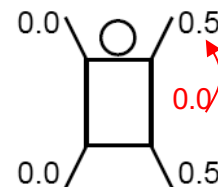
Lateral sequence walk



Diagonal sequence walk



Trot

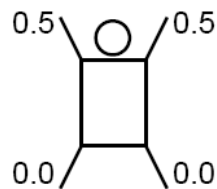


Pace

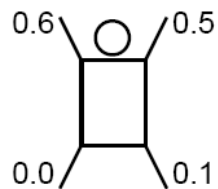
Symmetric

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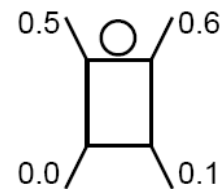
Asymmetric



Bound



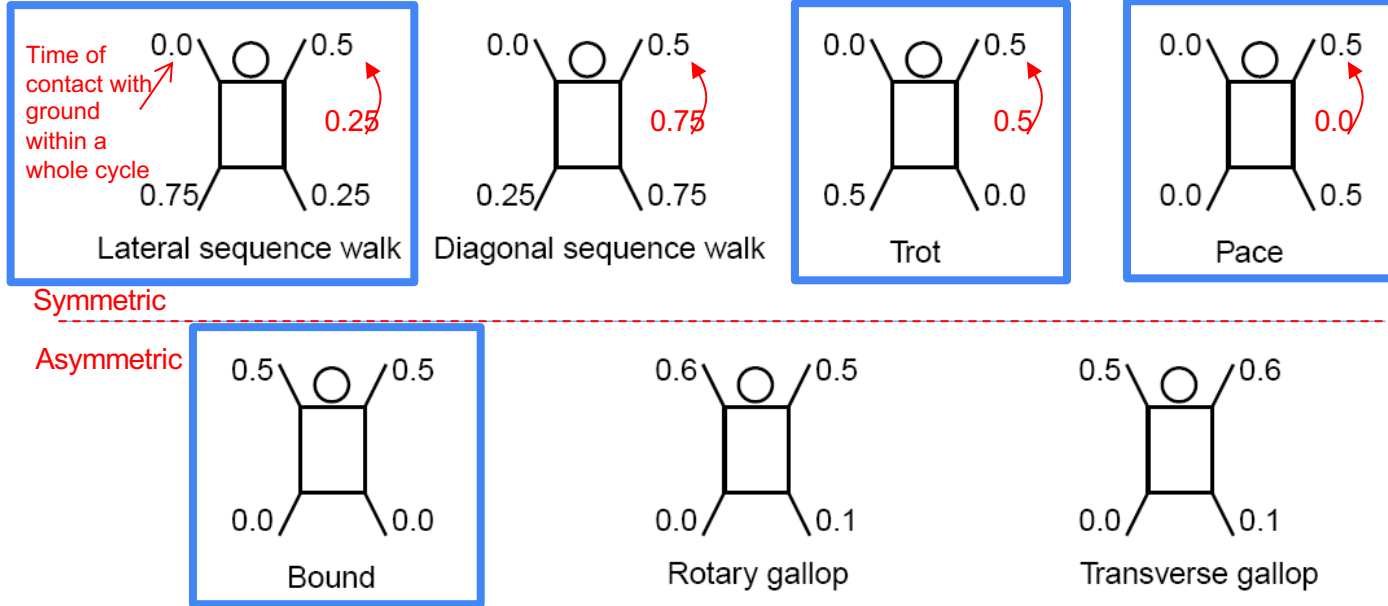
Rotary gallop



Transverse gallop

# Most common quadruped gaits

Classification in terms of the footfall sequences (mainly used in mathematical biology)



This project

$$\dot{r}_i = \alpha(\mu - r_i^2)r_i$$

$$\dot{\theta}_i = \omega_i + \sum_{j=0}^3 r_j w_{ij} \sin(\theta_j - \theta_i - \phi_{ij})$$

What should  $\phi$  be for each gait?



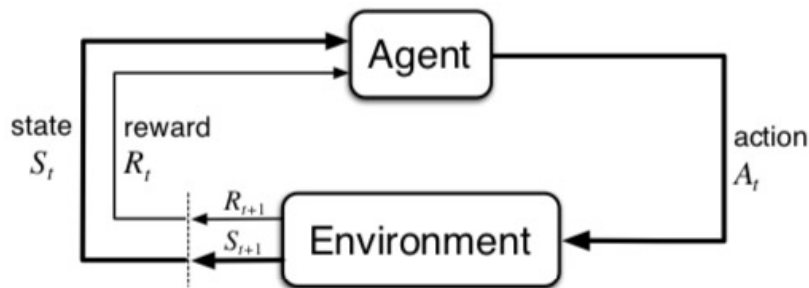
# Deep Reinforcement Learning: Review

## From Lecture 7

# Reinforcement Learning

An MDP is defined by:

- Set of states  $\mathcal{S}$
- Set of actions  $\mathcal{A}$
- Transition function  $P(s' | s, a)$
- Reward function  $R(s, a, s')$
- Start state  $s_0$
- Discount factor  $\gamma$
- Horizon  $H$



- Return over a trajectory  $\tau = (s_0, a_0, s_1, a_1, \dots)$

$$R(\tau) = \sum_{t=0}^{\infty} \gamma^t r_t$$

- Policy  $\pi(a_t | s_t)$  maps from states  $s_t$  to actions  $a_t$  (Goal: find policy maximizing above return)
- Value function:  $V^\pi(s) = \mathbb{E}_{\tau \sim \pi}[R(\tau) | s_0 = s]$
- Action-value function:  $Q^\pi(s, a) = \mathbb{E}_{\tau \sim \pi}[R(\tau) | s_0 = s, a_0 = a]$
- Advantage function:  $A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)$

## From Lecture 7

# Many Existing Tools for Reinforcement Learning

- RL algorithm implementations

- stable-baselines3 <https://github.com/DLR-RM/stable-baselines3>
- ray[rllib] <https://github.com/ray-project/ray>
- spinningup <https://github.com/openai/spinningup>
- tianshou <https://github.com/thu-ml/tianshou/>
- ... many others!

PPO, SAC

- Physics simulators

- pybullet <https://github.com/bulletphysics/bullet3>
- MuJoCo <https://mujoco.org>
- RaiSim <https://raisim.com>
- Isaac-Gym <https://developer.nvidia.com/isaac-gym>
- ... and others!

# RL Considerations

## Algorithm

- On/off policy
- Hyperparameters
- Network architecture
- Random seeds/trials

...implementation  
dependent!

## MDP Design Decisions

- Observation space
- Action space
- Reward function

## Environment Parameters

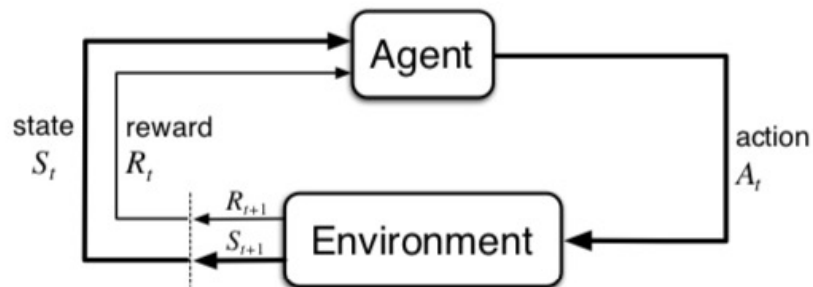
- Simulator dynamics
- Control gains –  
joint/Cartesian
- Control/environment  
time step
- Noise, latency

## From Lecture 7

# State/Action/Reward Space: A1

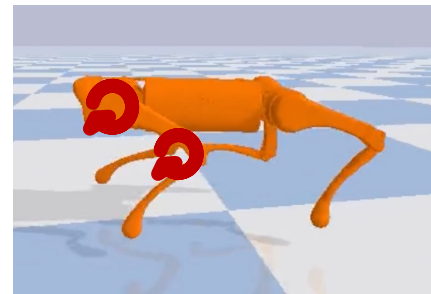
$s_t$  ? i.e.

- body (z, r, p, y)
- body velocities
- joint states



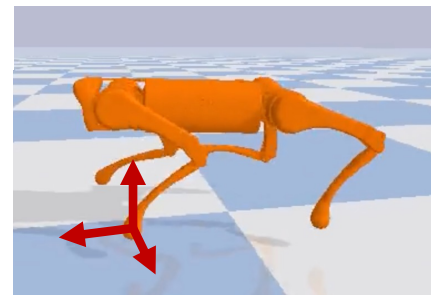
$r_t$  ? i.e.

- body linear velocity
- energy penalty



$a_t$  ?

- motor positions/torques
- Cartesian PD
- CPG state modulations

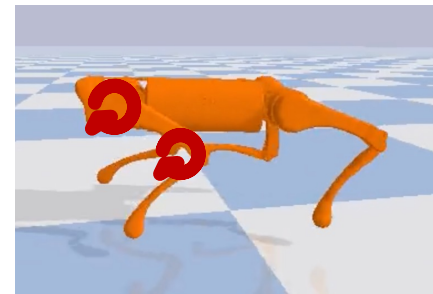
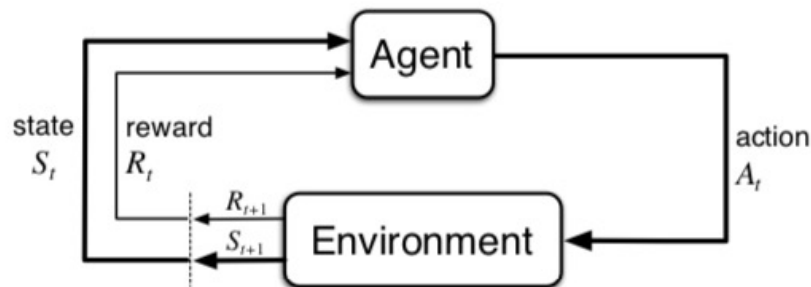


## From Lecture 7

# State/Action/Reward Space: A1

$s_t$  ? i.e.

- body (z, r, p, y)
- body velocities
- joint states

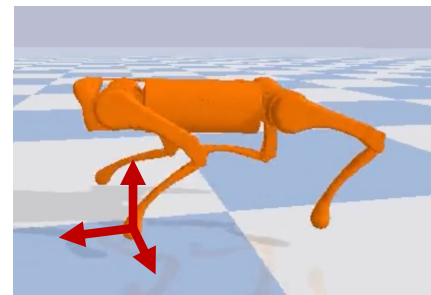


$a_t$  ?

- motor positions/torques
- Cartesian PD
- CPG state modulations

$r_t$  ? i.e.

- body linear velocity
- energy penalty

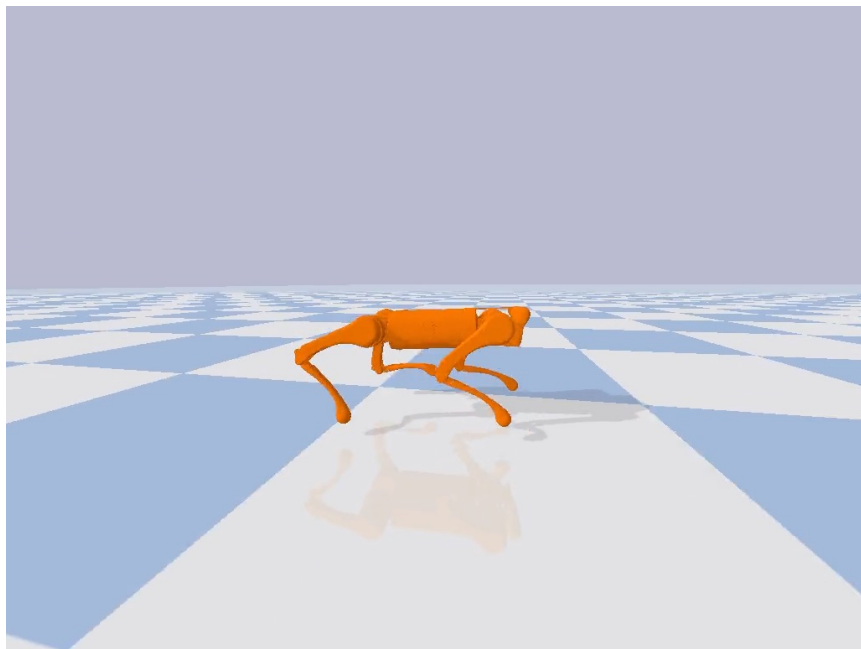


This project: construct the MDP

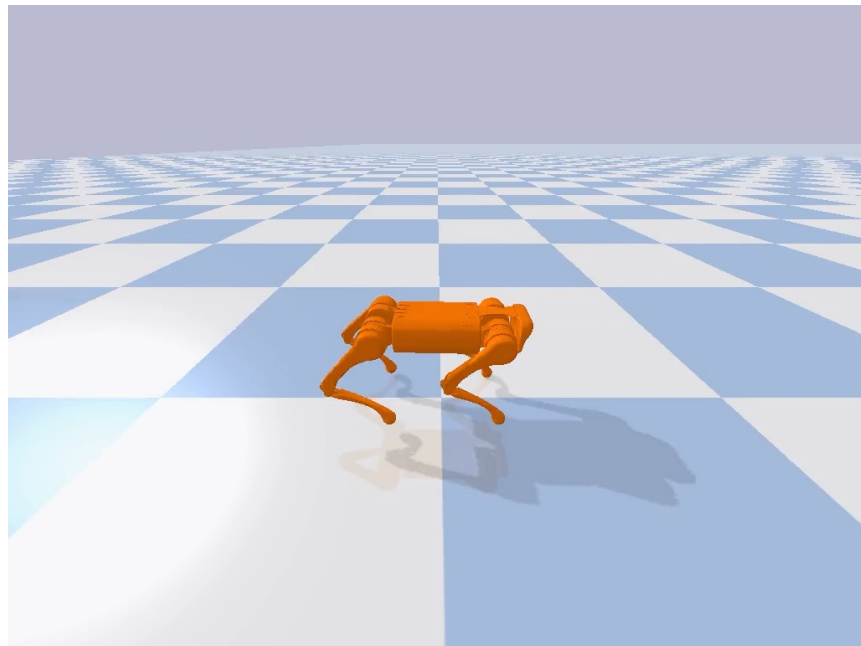
## From Lecture 7

# Joint Position Control vs. Cartesian PD Control (PPO/SAC)

Action Space:  $a_t = q_{1...N}$

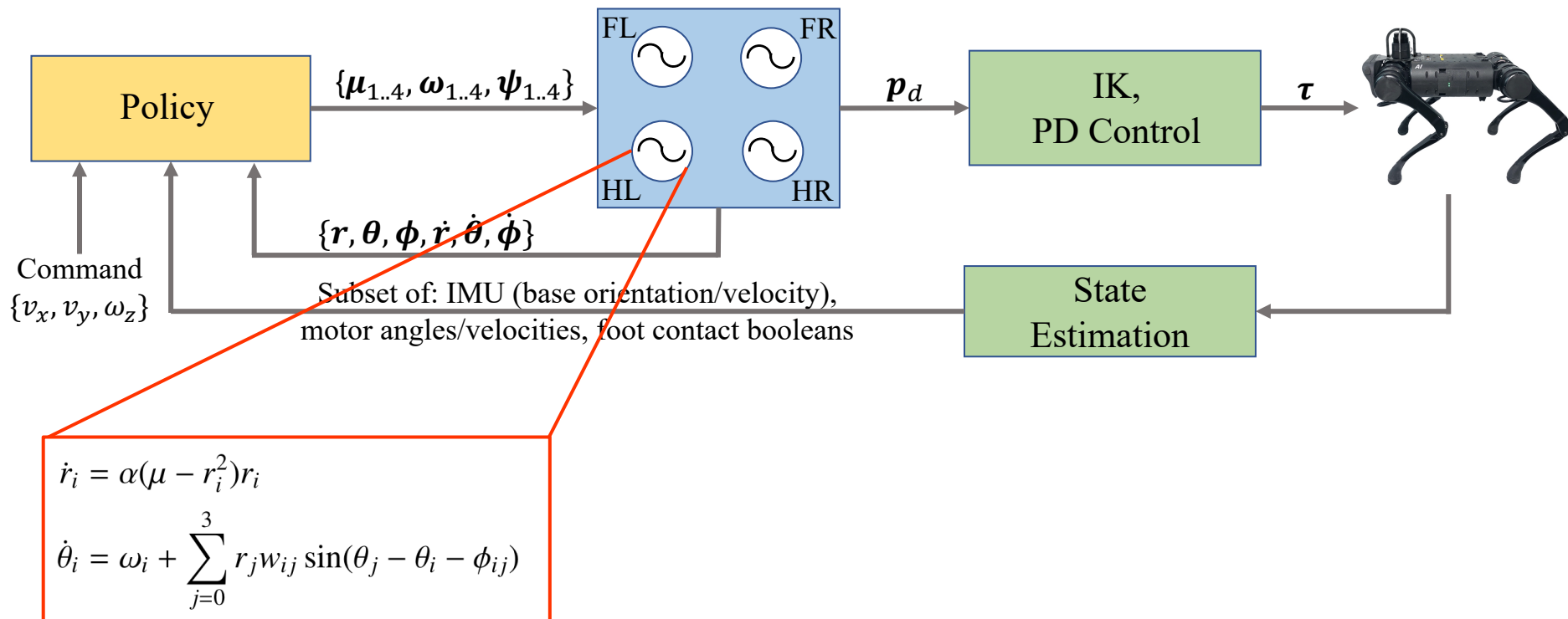


Action Space:  $a_t = [x_{ee_i}, y_{ee_i}, z_{ee_i}]$



# CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion

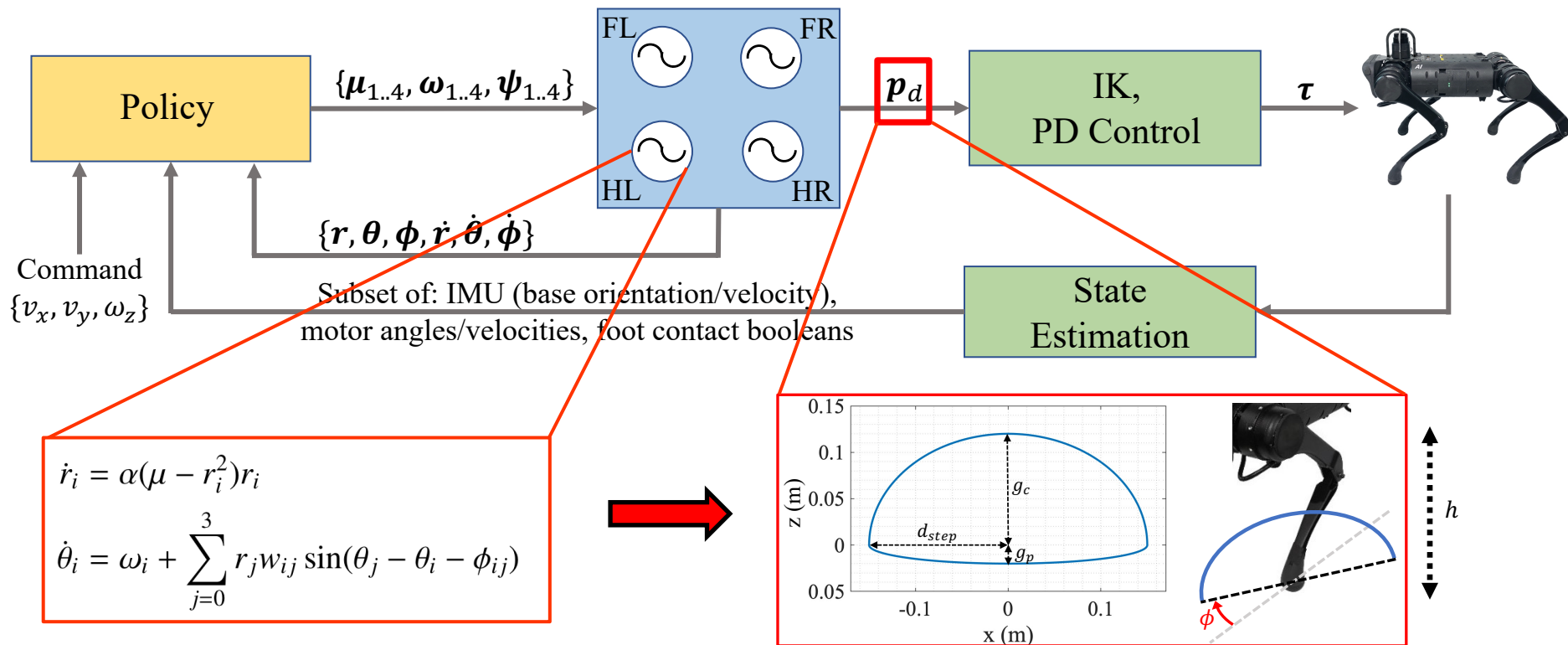
## From Lecture 7





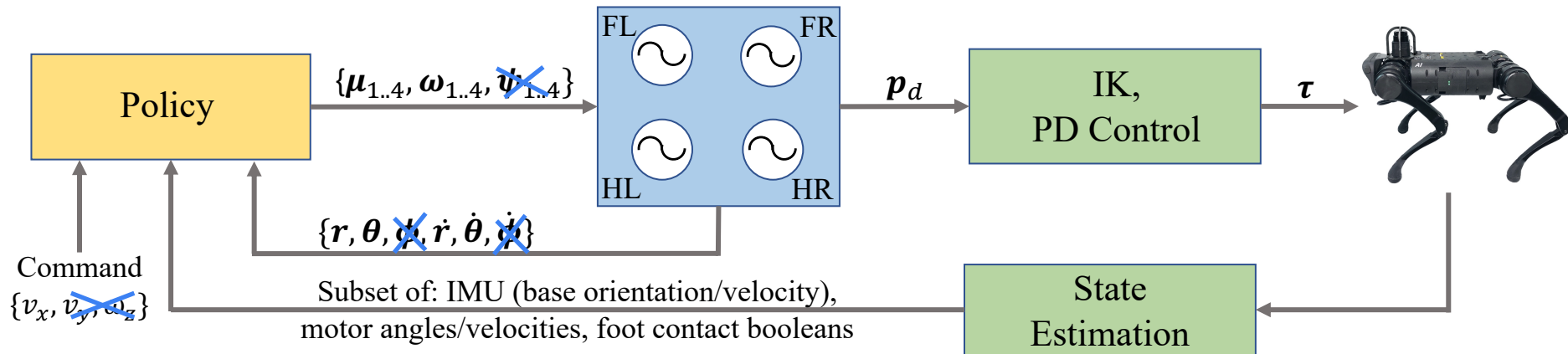
# CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion

## From Lecture 7



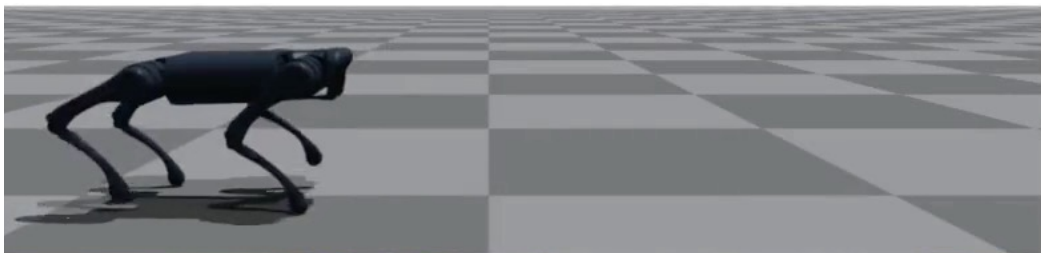
# CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion

From Lecture 7



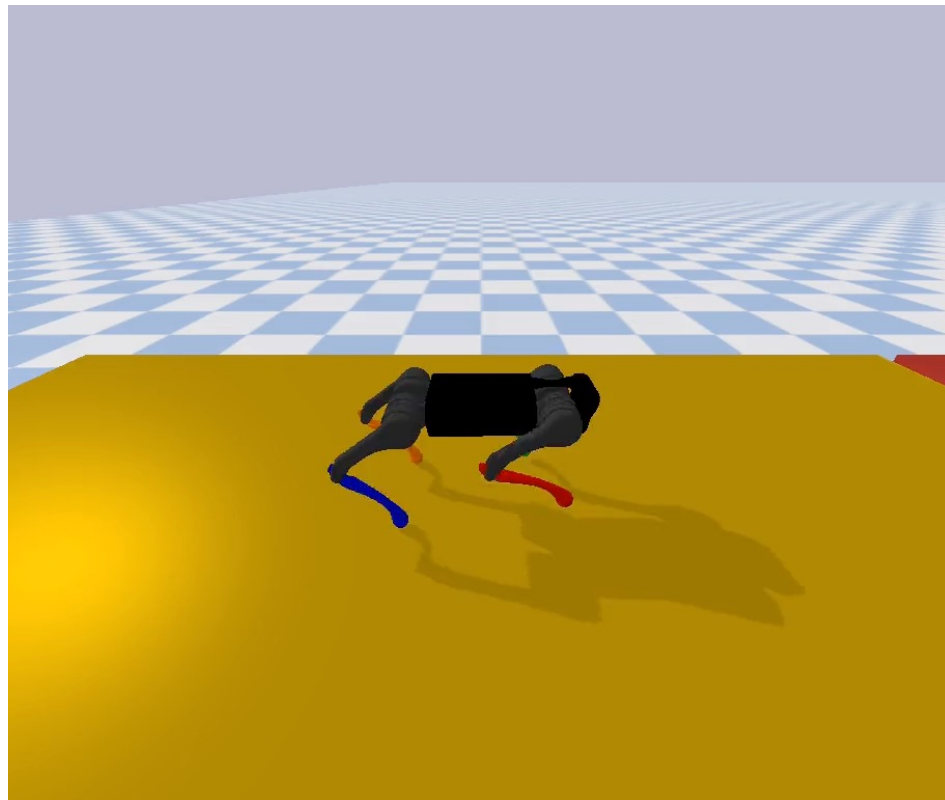
$$\dot{r}_i = \alpha(\mu - r_i^2)r_i$$

$$\dot{\theta}_i = \omega_i + \sum_{j=0}^3 r_j w_{ij} \sin(\theta_j - \theta_i - \phi_{ij})$$



# Choose a task and demonstrate your performance on 17.12.2024

- Gap crossing
  - Largest gap?
  - Smallest distance between gaps?
- Stairs
  - Width?
  - Height?
- Slopes
  - Largest angle?



# Tips

- Monitor episode length and reward mean during training
- Training should complete within a couple million timesteps for simple tasks with reasonable observation space, action space, and reward function choices (with no noise in the environment)
- Start training early!