Unit 26 — Mathematics for IT Practitioners

Assignment 1 — Matrix operations

Daniel Easteal

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Contents

1 Definition of matrices

In this initial section I have just recorded what matrices I will be using throughout the whole assignment for reference later on and so that I do not need to write them out for each question.

$$M = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} \qquad Q = \begin{pmatrix} 2 & 3 & 3 \\ 4 & 4 & -2 \\ 3 & -4 & 8 \end{pmatrix}$$

$$N = \begin{pmatrix} 4 & 3 \\ -3 & -1 \end{pmatrix} \qquad R = \begin{pmatrix} 9 & 2 & 6 \\ 12 & -4 & 7 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{pmatrix} \qquad S = \begin{pmatrix} -6 & 3 \\ -3 & -2 \\ -6 & -6 \end{pmatrix}$$

2 P2 - perform addition, subtraction and scalar multiplication

2.1 Addition

To add 2 matrices together you simply add each element in the first matrix to the corresponding number in the second matrix. The general case is shown below:

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} + \begin{pmatrix} A & D & G \\ B & E & H \\ C & F & I \end{pmatrix} = \begin{pmatrix} a+A & b+D & c+G \\ d+B & e+E & f+H \\ g+C & h+F & i+I \end{pmatrix}$$

From this general case, we just substitute in some numbers in to correct matrices that we need to add, and this is the flowing result:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix} = \begin{pmatrix} 1+1 & 4+2 & 7+3 \\ 2+4 & 5+5 & 8+6 \\ 3+7 & 6+8 & 9+9 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 6 & 10 \\ 6 & 10 & 14 \\ 10 & 14 & 18 \end{pmatrix}$$

2.2 Subtraction

In a similar way to addition, to subtract 2 matrices together you simply subtract each element in the first matrix to the corresponding number in the second matrix.

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} - \begin{pmatrix} A & D & G \\ B & E & H \\ C & F & I \end{pmatrix} = \begin{pmatrix} a-A & b-D & c-G \\ d-B & e-E & f-H \\ g-C & h-F & i-I \end{pmatrix}$$

With some random numbers inserted, an example of how this works would look like this: (again, you just substitute the numbers)

$$\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 1-1 & 4-2 & 7-3 \\ 2-4 & 5-5 & 8-6 \\ 3-7 & 6-8 & 9-9 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 2 & 4 \\ -2 & 0 & 2 \\ -4 & -2 & 0 \end{pmatrix}$$

2.3 Scalar Multiplication

Scalar multiplication is the simple to brother multiplication as it just consists of standard multiplication with no worrying about formatting. This is used for when you have a whole matrix multiplied by a single number. All you have to do is multiply each element in the matrix by the number, and that is it. Here is the general case for this:

$$n\begin{pmatrix} A & B & C \\ D & E & F \\ H & I & J \end{pmatrix} = \begin{pmatrix} An & Bn & Cn \\ Dn & En & Fn \\ Gn & Hn & In \end{pmatrix}$$

From here we can just add in some numbers so that you can see how it all woks out:

$$2.5\begin{pmatrix} 3 & 1 & 4 \\ 1.5 & 9 & 2.6 \\ 5 & 3.5 & 8 \end{pmatrix} = \begin{pmatrix} 3*2.5 & 1*2.5 & 4*2.5 \\ 1.5*2.5 & 9*2.5 & 2.6*2.5 \\ 5*2.5 & 3.5*2.5 & 8*2.5 \end{pmatrix} \Rightarrow \begin{pmatrix} 7.5 & 2.5 & 10 \\ 3.75 & 22.5 & 6.5 \\ 12.5 & 8.75 & 20 \end{pmatrix}$$

Now that we know how to do, addition subtraction and multiplication of matrices I can now carry on and do the questions that are at hand, and they are as follows:

2.4 Questions

For the assignment we are told to do a total of 5 addition, subtraction and scalar questions, and I will now go through them showing my calculations as they would be in the examples above.

2.4.1 M + N

$$M + N \Rightarrow \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ -3 & -1 \end{pmatrix} = \begin{pmatrix} 3+4 & -1+3 \\ 4+-3 & 2+-1 \end{pmatrix} \Rightarrow \begin{pmatrix} 7 & 2 \\ 1 & 1 \end{pmatrix}$$

$$\therefore M + N = \begin{pmatrix} 7 & 2 \\ 1 & 1 \end{pmatrix}$$

2.4.2 P + Q

$$P + Q \Rightarrow \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 3 \\ 4 & 4 & -2 \\ 3 & -4 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1+2 & 3+3 & 5+3 \\ -1+4 & 2+4 & 4+-2 \\ -3+3 & 4+-4 & 3+8 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & 6 & 8 \\ 3 & 6 & 2 \\ 0 & 0 & 11 \end{pmatrix}$$

$$\therefore P + Q = \begin{pmatrix} 3 & 6 & 8 \\ 3 & 6 & 2 \\ 0 & 0 & 11 \end{pmatrix}$$

2.4.3 M - N

$$M - N \Rightarrow \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 3 \\ -3 & -1 \end{pmatrix} = \begin{pmatrix} 3-4 & -1-3 \\ 4--3 & 2--1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -4 \\ 7 & 3 \end{pmatrix}$$

$$\therefore M + N = \begin{pmatrix} 1 & -4 \\ 1 & 3 \end{pmatrix}$$

2.4.4 3P

$$3P \Rightarrow 3\begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 3(1) & 3(3) & 3(5) \\ 3(-1) & 3(2) & 3(4) \\ 3(-3) & 3(4) & 3(3) \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & 9 & 15 \\ -3 & 6 & 12 \\ -9 & 12 & 9 \end{pmatrix}$$

$$\therefore 3P = \begin{pmatrix} 3 & 9 & 15 \\ -3 & 6 & 12 \\ -9 & 12 & 9 \end{pmatrix}$$

2.4.5 3P - 2Q

For this question we will first need to calculate what 2Q is, after this I can then use my value for 3P that I found in the previous section to calculate the actual answer.

$$2Q \Rightarrow 2\begin{pmatrix} 2 & 3 & 3 \\ 4 & 4 & -2 \\ 3 & -4 & 8 \end{pmatrix} = \begin{pmatrix} 2(2) & 2(3) & 2(3) \\ 2(4) & 2(4) & 2(-2) \\ 2(3) & 2(-4) & 2(8) \end{pmatrix} \Rightarrow \begin{pmatrix} 4 & 6 & 6 \\ 8 & 8 & -4 \\ 6 & -8 & 16 \end{pmatrix}$$

$$\therefore 2Q = \begin{pmatrix} 4 & 6 & 6 \\ 8 & 8 & -4 \\ 6 & -8 & 16 \end{pmatrix}$$

$$3P - 2Q \Rightarrow \begin{pmatrix} 3 & 9 & 15 \\ -3 & 6 & 12 \\ -9 & 12 & 9 \end{pmatrix} - \begin{pmatrix} 4 & 6 & 6 \\ 8 & 8 & -4 \\ 6 & -8 & 16 \end{pmatrix} = \begin{pmatrix} 3-4 & 9-6 & 15-6 \\ -3-8 & 6-8 & 12--4 \\ -9-6 & 12--8 & 9-16 \end{pmatrix}$$
$$\Rightarrow \begin{pmatrix} 1 & 3 & 9 \\ 11 & -2 & 16 \\ -15 & 20 & 7 \end{pmatrix}$$

$$\therefore 3P - 2Q = \begin{pmatrix} 1 & 3 & 9 \\ 11 & -2 & 16 \\ 15 & 20 & 7 \end{pmatrix}$$

3 P3 - Multiply two matricies

3.1 Multiplication

In order to multiply two matrices you must first ensure that the width of the first matrix is the same as the height of the second, if this is not the case then the multiplication cannot happen between them.

$$\rightarrow \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{pmatrix} \overset{\downarrow}{\begin{pmatrix} 4 & 7 \\ 5 & 8 \end{pmatrix}} = undefined$$

The number of items indicated by the arrows do not match.

If the multiplication can occur then you will have to add the values of each number in the first matrix row multiplied by its corresponding number in the second matrix column, this result would then go into the overlapping section for example if you just multiplied the first row by the first column then the result will go into the first row and first column of the answer matrix.

Please note that the answer matrix may be smaller than the initial matrices.

$$\begin{pmatrix} 3 & 5 & 1 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 9 & 2 \\ 6 & 1 \\ 10 & 3 \end{pmatrix} = \begin{pmatrix} 3*9+5*6+1*10 & 3*2+5*1+1*3 \\ 2*9+3*6+4*10 & 2*2+3*1+4*3 \end{pmatrix} \Rightarrow \begin{pmatrix} 67 & 14 \\ 76 & 19 \end{pmatrix}$$

3.2 Questions

3.2.1 MN

3.2.2 PQ

3.2.3 RS

3.2.4 SR

4 P4 - Inverse and transpose

4.1 Inverse

The process of generating an inverse matrix can be quite a difficult one to follow, however with the correct streps it can be done easily. To start off with, an inverse square is one where when the original and inverse square are multiplied together they will generate the identity matrix as an answer. The identity matrix is a matrix that contains all zeros apart from a single diagonal lines of ones from the top left down to the bottom right corner.

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig 1
The identity matrix

There are different ways to actually calculate the inverse of a matrix and these methods can differ based on the size of the matrices as well. I will now go through the general case that can be used for finding the inverse of a 2x2 matrix and then after that I will go through the general case to a higher order matrix.

4.1.1 2x2 Matrix Inverse

To find the inverse of a 2x2 Matrix you must first find the determinant of the matrix and then after that you can apply the general case rule that has to do with swapping and inverting numbers to get the inverse you need.

Determinant The determinant of a matrix is a special value that is used for calculations with the matrix like finding the inverse. To find the determinant you multiply the top left and bottom right values and then subtract the bottom left and top right values. The determinant is written as the letter of the matrix with pipes either side like it was an absolute value.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow |A| = ad - cb$$

To see what is going on, here is an example with random numbers filled in:

$$A = \begin{pmatrix} 5 & 6 \\ 3 & 2 \end{pmatrix} \Rightarrow |A| = 5(2) - 3(6) = 10 - 18 \Rightarrow -8$$

$$\therefore |A| = -8$$

Inverse formula Now that you have the determinant to find the inverse of the matrix you just need to swap the top left and bottom right values in the matrix, then inverse the other two values to their negatives, finally you multiply this new matrix by one over the determinant. Here is the general case formula:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{1}{ad-bc}d & -\frac{1}{ad-bc}b \\ -\frac{1}{ad-bc}c & \frac{1}{ad-bc}a \end{pmatrix}$$

Doing all the steps at once may seem quite confusing, so here is it again but done step by step so you can see what happens along the way:

$$\begin{aligned}
A &= \begin{pmatrix} a & b \\ c & d \end{pmatrix} && \text{starting equation} && (1) \\
&= \frac{1}{|A|}\begin{pmatrix} a & b \\ c & d \end{pmatrix} && \text{multiply by 1 over the determinant} && (2) \\
&= \frac{1}{ad-bc}\begin{pmatrix} a & b \\ c & d \end{pmatrix} && \text{calculate the determinant} && (3) \\
&= \frac{1}{ad-bc}\begin{pmatrix} d & b \\ c & a \end{pmatrix} && \text{swap } a \text{ and } b && (4) \\
&= \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} && \text{invert } c \text{ and } b && (5) \\
A^{-1} &= \begin{pmatrix} \frac{1}{ad-bc}d & -\frac{1}{ad-bc}b \\ -\frac{1}{ad-bc}c & \frac{1}{ad-bc}a \end{pmatrix} && \text{Multiply out} && (6)
\end{aligned}$$

Below you will see an example of how this would work when there are actually numbers in place to see how it all works:

$$\begin{aligned}
A &= \begin{pmatrix} 5 & 6 \\ 3 & 2 \end{pmatrix} \\
&= \frac{1}{|A|}\begin{pmatrix} 5 & 6 \\ 3 & 2 \end{pmatrix} \\
&= \frac{1}{5(2)-6(3)}\begin{pmatrix} 5 & 6 \\ 3 & 2 \end{pmatrix} \\
&= \frac{1}{-8}\begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix} \\
&= \frac{1}{-8}\begin{pmatrix} 2 & -6 \\ -3 & 5 \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{-8}2 & -\frac{1}{-8}6 \\ -\frac{1}{-8}3 & \frac{1}{-8}5 \end{pmatrix} \\
A^{-1} &= \begin{pmatrix} -\frac{1}{4} & \frac{3}{4} \\ \frac{3}{8} & -\frac{5}{8} \end{pmatrix}
\end{aligned}$$

4.1.2 Inverse larger matrices

In order to get the inverse of a matrix that is larger than 2x2 we need to work out both the determinant and the cofactor matrix of the main matrix we are working with. We need to work out these as we can follow the following equation to work out the inverse:

$$A^{-1} = \frac{1}{|A|}(cofactorMatrixofA)$$

As you can see, we need to work out the determinant and he cofactor matrix in order to solve the equation.

Large matrix determinant The process to get the determinant of a large matrix is not actually too difficult as there is just a simple pattern to follow that you need to apply. The way this pattern works is you add together all the different diagonals of the matrix going in one direction (assuming that the diagonals go through the 'walls' of the matrix and still count) and subtract the sum of the diagonals in the other direction.

When I say the diagonal pattern through the 'walls' of the matrix look to Fig 2 as all the same letters will be in the same FIRST pattern:

$$\begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}$$

Fig 2
The determinant pattern

Now I will go through the whole process in the general case so that you can see how it all comes together:

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \Rightarrow |A| = (aei + bfg + cdh) - (ceg + bdi + afh)$$

with numbers in place of the letters as an example it will play out like so:

$$\begin{aligned} A &= \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \\ |A| &= (1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8) - (3 \cdot 5 \cdot 7 + 2 \cdot 4 \cdot 9 + 1 \cdot 6 \cdot 8) \\ &= (45 + 84 + 96) - (105 + 72 + 48) \\ &= (225) - (225) \\ |A| &= 0 \end{aligned}$$

CoFactor Matrix The final part that I need for this assignment is the cofactor matrix. The process required to get the cofactor matrix is quite complicated and requires a lot of repetition and collating results as well as finding the determinant as well. To get the cofactor matrix for each item in the main matrix you need to temporally remove the row and column that that item is in, from here you then calculate the determinant for the remaining matrix. You will then put this determinant value in a new matrix the same size as the original in the place of the initial removed item. After you do this for all items in the main matrix you then make some of them negative based on the size of the matrix you are using, now you will have a filled new matrix that contains determinant values, this new matrix is the cofactor matrix.

On the next page I will go through the process in a general case so that you can see what I mean by this and how it works.

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$A_{11} = \begin{pmatrix} - & - & - \\ - & e & f \\ - & h & i \end{pmatrix} |A_{11}| = \textbf{ei-fh} \quad A_{12} = \begin{pmatrix} - & - & - \\ d & - & f \\ g & - & i \end{pmatrix} |A_{12}| = \textbf{di-fg} \quad A_{13} = \begin{pmatrix} - & - & - \\ d & e & - \\ g & h & - \end{pmatrix} |A_{13}| = \textbf{dh-eg}$$

$$A_{21} = \begin{pmatrix} - & b & c \\ - & - & - \\ - & h & i \end{pmatrix} |A_{21}| = \textbf{bi-ch} \quad A_{22} = \begin{pmatrix} a & - & c \\ - & - & - \\ g & - & i \end{pmatrix} |A_{22}| = \textbf{ai-cg} \quad A_{23} = \begin{pmatrix} a & b & - \\ - & - & - \\ g & h & - \end{pmatrix} |A_{23}| = \textbf{ah-bg}$$

$$A_{31} = \begin{pmatrix} - & b & c \\ - & e & f \\ - & - & - \end{pmatrix} |A_{31}| = \textbf{bf-ce} \quad A_{32} = \begin{pmatrix} a & - & c \\ d & - & f \\ - & - & - \end{pmatrix} |A_{32}| = \textbf{af-cd} \quad A_{33} = \begin{pmatrix} a & b & - \\ d & e & - \\ - & - & - \end{pmatrix} |A_{33}| = \textbf{ae-bd}$$

As you can see, the answer will now be the matrix that will consist of the answers of these answers that will look like this:

$$\begin{pmatrix} ei-fh & di-fg & dh-eg \\ bi-ch & ai-cg & ah-bg \\ bf-ce & af-cd & ae-bd \end{pmatrix}$$

From here I now just need to multiply the newest matrix by the determinant + and - matrix so that I know what the signs are. This matrix always consists of + and - that alternate starting with a + in the top left corner, in this case this matrix would look like so:

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

Fig 3
The + and - determinent pattern

Next I just need to apply the matrix by the determinant pattern to get just one step away from the answer as we have just calculate the co-factor matrix.

$$\begin{pmatrix} ei-fh & -(di-fg) & dh-eg \\ -(bi-ch) & ai-cg & -(ah-bg) \\ bf-ce & -(af-cd) & ae-bd \end{pmatrix}$$

Finally all that I need to do is multiply the new matrix by the determinant that I found earlier and the inverse will be found. This will follow the initial formula that I mentioned earlier that looks like the following:

$$A^{-1} = \frac{1}{|A|}(\text{cofactor Matrix of A})$$

And so now the equation with the numbers replaced with what they have been calculated to be will look like:

$$A^{-1} = \frac{1}{(aei+bfg+cdh)-(ceg+bdi+afh)} \begin{pmatrix} ei-fh & -(di-fg) & dh-eg \\ -(bi-ch) & ai-cg & -(ah-bg) \\ bf-ce & -(af-cd) & ae-bd \end{pmatrix}$$

4.2 Transpose

Another technique that can be used for the modification of matrices is the process of transposing. This is quite simple as all it does is transpose a matrix to its swapped dimension counterpart. For example a 3x2 matrix would become a 2x3 matrix.

The notation for this is the matrix with a 't' or a 'T' to the top right like a power.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^t Or \begin{pmatrix} a & b \\ c & d \end{pmatrix}^T$$

However it is still slightly complicated as you cannot just rotate the whole matrix as you have to rotate each line in an opposite way to its neighbours as shown in this following example:..

$$\begin{pmatrix} 3 & 5 & 1 \\ 2 & 3 & 4 \end{pmatrix}^T \Rightarrow \begin{pmatrix} 3 & 2 \\ 5 & 3 \\ 1 & 4 \end{pmatrix}$$

It may help if you think that the top left and bottom right corners are locked in place during this procedure, however this will only really work when you have a matrix that is 2xn as there is a more complicated procedure other wise.

The method that you actually use for a matrix of any size is actually very easy and you just need to remember the pattern although it may get a bit confusing. The pattern that you have to remember is that you keep the numbers that are in a diagonal line from the top left to the bottom right (a 45 degree lime) in the same place, and you rotate all the other numbers to the place that is on the other side of the line like a mirror. below you can see the general case for this:

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^T \Rightarrow \begin{pmatrix} \ddots & b & c \\ d & \ddots & f \\ g & h & \ddots \end{pmatrix} \text{flip along the line} \Rightarrow \begin{pmatrix} \ddots & d & g \\ b & \ddots & h \\ c & f & \ddots \end{pmatrix} \Rightarrow \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$$

$$\therefore \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^T = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$$

now for a worked example, the process is exactly the same for any size of matrix so is is very simple:

$$\begin{pmatrix} 40 & 9 & 6 \\ 23 & 42 & 7 \\ 18 & 98 & 19 \\ 37 & 21 & 1 \end{pmatrix}^T \Rightarrow \begin{pmatrix} \ddots & 9 & 6 \\ 23 & \ddots & 7 \\ 19 & 98 & \ddots \\ 37 & 21 & 1 \end{pmatrix} \text{flip along the line} \Rightarrow \begin{pmatrix} \ddots & 32 & 19 & 37 \\ 9 & \ddots & 98 & 21 \\ 6 & 7 & \ddots & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 40 & 32 & 19 & 37 \\ 9 & 42 & 98 & 21 \\ 6 & 7 & 19 & 1 \end{pmatrix}$$

4.3 Questions

4.3.1 M^{-1}

4.3.2 N^{-1}

4.3.3 P^{-1}

4.3.4 Q^{-1}

4.3.5 M^T

4.3.6 P^T

4.3.7 R^T

5 Simultaneous equations

for the final section that I will go through the maths of I will be going through the process of solving simultaneous equations using matrices. This process will require a few more steps and also an understanding of how matrices work and how this relates to the equations. For this I will not be going through a general case with letters but I well do an example to explain the steps. lets say that you have the following 2 equations:

$$x + 2y = 4$$
$$3x - 5y = 1$$

from the knowledge that hat been explained in the previous sections, you could work out that this could be re-written as the following:

$$\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

From here I can then reassign some temporary values to these sections so that I can explain them easier later on.

$$A = \begin{pmatrix} 1 & 3 \\ 3 & -5 \end{pmatrix} \qquad X = \begin{pmatrix} x \\ y \end{pmatrix} \qquad B = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

And this would therefore be able to be rearranged to this:

$$AX = B$$

From here I just need to work out what the X would be and then I have the answer to what both x and y are. To do that I need to rearrange the equation so that I have X by itself and everything on the other side as I can work those out to get X. To do this I need to multiply both sides of the equations by A^{-1}, and this would give:

$$A^{-1}AX = A^{-1}B$$

This can then be simplified as any matrix multiplied by its inverse creates the identity matrix as that is how the inverse is defined.
$A^{-1}A = I$
In addition to this, when any matrix is multiplied by the identity matrix nothing changes with the matrix as it is like multiplying a normal number by 1. From this we then get that

$Xi = X$
Knowing this we can then arrive to the conclusion that

$$A^{-1}AX = A^{-1}B \Rightarrow X = A^{-1}B$$

So now, all that we need to do is calculate the inverse of A (the initial matrix) and the multiply it by B (the answer matrix) and we will get the answer.
With this we can now just put in the equations that we have above and work out what x and y are equal to.

$$A^{-1} = \frac{1}{1(-5) - 2(3)} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} \Rightarrow -\frac{1}{11} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix}$$

Now that we have the inverse of A, we just need to multiply the inverted A by X and thee we are done

$$X = A^{-1}B = -\frac{1}{11} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$