

Unit 26 — Mathematics for IT Practitioners

Assignment 2 — Probability, Sequences, Number Systems and Statistics

Daniel Easteal

March 2017

Contents

1	Introduction	3
2	Series and Sequences and Probability	3
2.1	-3, 1, 5, 9, 13 ...	3
2.1.1	nth term	3
2.1.2	17th term	3
2.1.3	Sum of terms	4
2.2	81, -27, 9, -3 ...	4
2.2.1	nth term	4
2.2.2	10th term	5
2.2.3	Sum to the 5th element	5
2.2.4	Sum to infinity	5
2.3	$3r - 2r^2 + r^3$	5
2.4	5 balls in a bag	6
2.4.1	yellow ball	6
2.4.2	2 yellow balls in a row	6
2.4.3	Tree	6
2.5	In a year group	6
2.5.1	Venn diagram	6
2.5.2	Only Computer science	6
2.5.3	Engineering	6
2.6	Betting game	6
3	Number Systems	7
3.1	Binary, Octal, Decimal and Hexadecimal conversion	7
4	Number Systems calculations	7
4.1	$a + f$ in hexadecimal	7
4.2	$g * f$ in hexadecimal	7
4.3	$f - b$ in hexadecimal	7
4.4	$a + e$ in Octal	7
4.5	$e - b$ in Octal	7
4.6	$a + d$ in binary	7
4.7	$a * d$ in binary	8
4.8	$h + i$ in binary	8
4.9	$h * i$ in hexadecimal	8
5	Data task	8
6	Recursion	8
7	Use of Number Systems	9
7.1	binary - ASCII	10
8	Network Planning	10
9	Number Systems calculations	10

1 Introduction

test

2 Series and Sequences and Probability

2.1 -3, 1, 5, 9, 13 ...

2.1.1 nth term

Find a formula for the nth term of this sequence and find the 17th term using your nth term formula. Also calculate the sum of the first 17 terms of this sequence.

$$-3, 1, 5, 9, 13, \dots$$

To start off with I will need to find the difference between the numbers in the sequence that I have been given because I believe that this is a linear progression and this will get half the answer for me. Looking at the numbers, each number is 4 more than the previous one...

$$\begin{array}{cccc} \overset{+4}{\underbrace{-3, 1}} & \overset{+4}{\underbrace{1, 5}} & \overset{+4}{\underbrace{5, 9}} & \overset{+4}{\underbrace{9, 13}} \end{array}$$

Now that I know the distance from each number to the next, I can then formulate the initial equation to be

$$n = 4x + ?$$

so now I just need to work out what the "?" will be. Now, for the first term the x value will be one and so we would have 4 as the value, but the first value is actually -3, so we will need to adjust to this and have the equation subtract 7 from all the values so now the equation will look like this:

$$n = 4x - 7$$

We can then test this to ensure that it works by testing the 4th value as we know the answer to that:

$$4(4) - 7 \Rightarrow 16 - 7 \Rightarrow 9$$

And this answer lines up with the answer that we are given in the question so I know that the equation is correct.

$$\therefore n^{\text{th}} \text{ term} = 4x - 7$$

2.1.2 17th term

It is very simple to find the 17th term, or really any term of the sequence once you know the nth term formula, all that you have to do is replace the "x" with the term that you want and then you have to work the equation through. For this question it would go like this:

$$4x - 7 \Rightarrow 4(17) - 7 \Rightarrow 68 - 7 \Rightarrow 61$$

$$\therefore 17^{\text{th}} \text{ term} = 61$$

2.1.3 Sum of terms

To sum the sequence that we have been given we will have to use the sum of an arithmetic sequence progression equation and this looks like this:

$$\frac{n(a_1 + a_n)}{2}$$

this sequence is an arithmetic sequence because it advances in the series by adding numbers rather than multiplying by them. In this equation, all of the symbols stand for something and I will explain them below:

n = represents the number of terms that you want to add, in this case it is 17

a_1 = represents the initial number of the sequence, in this case it is -3

a_n = represents the last number of the sequence, in this case this is 61 as we calculated that before

Now I just need to place these numbers where their letter counterpart as in the equation and then work through the equation to get the answer:

$$\frac{17(-3 + 61)}{2} \Rightarrow \frac{17(58)}{2} \Rightarrow \frac{986}{2} \Rightarrow 493$$

\therefore the sum of the first 17 terms of the equation $4x-7$ is 493

2.2 81, -27, 9, -3 ...

2.2.1 nth term

As with before I will start off with working out the difference between the values in the list that I have been given.

$$\overbrace{81, -27}^{108} \quad \overbrace{-27, 9}^{36} \quad \overbrace{9, -3}^{12}$$

As you can see here, the numbers do not match up and they are not the same, this then presents a problem because instead of adding to get to the next number we are now multiplying, this can also be shown as the difference between the difference of all the numbers is 3. What I mean by this is that the above values that are 108, 36 and 12, you will find that to get to the next number you would need to divide by 3, this shows that this is a geometric progression and that I will have to use another equation.

Looking at the numbers you can see that to get to the next number in the series you would divide by -3. 3 because that is the way the differences work and then the - because the sign of the series changes every other number. Now that we know the multiple of the series we need to work out the offset.

Again like last time we write it out in a standard form and this would be:

$$a \times r^{n-1}$$

Now that we know "r" (the common ratio) to be -1/3 we just need to put in "a" as the first number and we know that that is 81, so then it would look like this:

$$a \times r^{n-1} \Rightarrow 81 \times \frac{1}{3}^{(n-1)}$$

2.2.2 10th term

This section is again very easy as all that I have to do is put 10 into the nth term formula and then I will get the answer that I want. This would then work through like so:

$$81 \times -\frac{1}{3}^{10-1} \Rightarrow 81 \times -\frac{1}{3}^9 \Rightarrow 81 \times -\frac{1}{19683} \Rightarrow -\frac{81}{19683} \Rightarrow -\frac{1}{243}$$

2.2.3 Sum to the 5th element

In order to sum a geometric series we need to know a new formula and this is the formula that we need:

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$

As the other ones I will just insert the numbers into the formula and then go about and solve the equation to get to the answer that we need to get to:

$$\frac{a_1(r^n - 1)}{r - 1} \Rightarrow \frac{81(-\frac{1}{3}^5 - 1)}{-\frac{1}{3} - 1} \Rightarrow \frac{81(-0.004115226... - 1)}{-1.\dot{3}} \Rightarrow \frac{81 \times -1.004115226...}{-1.\dot{3}} \Rightarrow \frac{-81.\dot{3}}{-1.\dot{3}} \Rightarrow 61$$

2.2.4 Sum to infinity

In order to sum a geometric series to infinity you need to ensure that the common ration is between -1 and 1 as otherwise the sum would be infinity and not calculated. The new equation that we need to sum to infinity is as follows:

$$S = \frac{a}{(1 - r)}$$

As in the previous examples, the "a" represents the first number in the series and the "r" represents the common ration. In this case that would be 81 and -1/3 so now the formula would look like this:

$$S = \frac{a}{(1 - r)} \Rightarrow \frac{81}{(1 - -\frac{1}{3})} \Rightarrow \frac{81}{(1 + \frac{1}{3})} \Rightarrow \frac{81}{(1.\dot{3})} \Rightarrow 60.75$$

2.3 3r - 2r² + r³

In this section I have to calculate the answer to the following:

$$\sum_{r=1}^6 (3r - 2r^2 + r^3)$$

The way that this works is that I have to calculate the part of the equation that is inside the brackets by replacing the "r" in this case each time from the value on the bottom to the value on the top. Then you add all these answers and you are done.

$$((3 \times 1) - (2 \times (1^2)) + (1^3)) + ((3 \times 2) - (2 \times (2^2)) + (2^3)) + ((3 \times 3) - (2 \times (3^2)) + (3^3)) + ((3 \times 4) - (2 \times (4^2)) + (4^3)) \\ + ((3 \times 5) - (2 \times (5^2)) + (5^3)) + ((3 \times 6) - (2 \times (6^2)) + (6^3))$$

$$3 - 2 + 1 + 6 - 8 + 16 + 9 - 18 + 27 + 12 - 32 + 64 + 15 - 50 + 125 + 18 - 72 + 216 \Rightarrow 322$$

$$\therefore \sum_{r=1}^6 (3r - 2r^2 + r^3) = 322$$

2.4 5 balls in a bag

There are 3 red and 2 yellow balls in a bag:

2.4.1 yellow ball

The probability that a yellow ball is selected is $\frac{2}{5}$ due to the fact that there are 2 yellow balls in the bag out of a total of 5. This can be written as 0.4

2.4.2 2 yellow balls in a row

When there are 2 chances that need to be added up in a row then the individual chances are multiplied. For this we would then multiply the 0.4 chance from the previous question by itself as it is the same question we just have to do it twice in a row.

$$0.4 * 0.4 = 0.16$$

\therefore the chance of getting two yellow balls in a row is 0.16 or $\frac{2}{25}$

2.4.3 Tree

This section will be on the insert provided with the document...

2.5 In a year group

2.5.1 Venn diagram

This section will be on the insert provided with the document...

2.5.2 Only Computer science

To start to answer this question I first need to add up how many different students there are in total and then I can work out the probability.

$$70 + 83 + 15 + 12 + 10 = 190$$

Now that I know there are 190 students in the year I can now look at the information and see that there are 70 students that fit the category of only studying computer science and to the probability is now:

$$70/190 \Rightarrow 7/19 \Rightarrow (100/19) + 7 = 12.26315789473684210526\%$$

$$\text{or a probability of } 7/19 = 0.36842105263157894736$$

2.5.3 Engineering

looking at the information that I have there are 83+12 students that study Engineering in one way or another and as such there is a probability as follows for one to be randomly selected:

$$95/190 \Rightarrow 1/2 \text{ That is } 50\% \text{ or a probability of } 0.5$$

2.6 Betting game

This section will be on the insert provided with the document...

3 Number Systems

3.1 Binary, Octal, Decimal and Hexadecimal conversion

	Denary	Binary	Octal	Hexadecimal
a	22	10110	26	16
b	11	1011	13	B
c	41	101001	51	29
d	20	10100	24	14
e	30	11110	36	1E
f	42	101010	52	2A
g	271	100001111	417	10F
h	44	101100	54	2C
i	62	111110	76	3E

4 Number Systems calculations

4.1 a + f in hexadecimal

$$22 + 42 = 64 \rightarrow 64_{10} = 40_{16}$$
$$\therefore A + F = 40$$

4.2 g * f in hexadecimal

$$271 * 42 = 11382 \rightarrow 11382_{10} = 2C76_{16}$$
$$\therefore g * f = 2C76$$

4.3 f - b in hexadecimal

$$42 - 11 = 31 \rightarrow 31_{10} = 1F_{16}$$
$$\therefore f - b = 1F$$

4.4 a + e in Octal

$$22 + 30 = 52 \rightarrow 52_{10} = 64_8$$
$$\therefore a + e = 64$$

4.5 e - b in Octal

$$30 - 11 = 19 \rightarrow 19_{10} = 23_8$$
$$\therefore e - b = 23$$

4.6 a + d in binary

$$22 + 20 = 42 \rightarrow 42_{10} = 101010_2$$
$$\therefore a + d = 101010$$

4.7 a * d in binary

$$22 * 20 = 440 \rightarrow 440_{10} = 110111000_2$$
$$\therefore a * d = 110111000$$

4.8 h + i in binary

$$44 + 62 = 106 \rightarrow 106_{10} = 1101010_2$$
$$\therefore h + i = 1101010$$

4.9 h * i in hexadecimal

$$44 * 62 = 27282728_{10} = AA8_{16}$$
$$\therefore h * i = AA8$$

5 Data task

6 Recursion

For this section I will go through and explaining how the binary search algorithm works and it fits in with the idea of recursion. Below you will see a basic format for the algorithm written in pseudo code:

```
for every item in the data set, read it into an array.
if the array is not sorted then sort it
start:
go to the middle of the array
is the current item what you are looking for?
....no: is the current item less then the item you are looking for?
.....yes: remove all items in the array that are lower and return to start.
.....no: remove all items in the array that are greater and return to start.
....yes: you are done as you have found the item you are looking for, end the program.
```

As you can see here there are not many stages that are needed for the search algorithm and it is very simple to do. Due to this, the whole part of the program after start: goto will be run over and over again and this will lead to the item that you want being found. I will now go through the algorithm step by step so that you know why each part is done like it is.

```
for every item in the data set, read it into an array.
```

This line will read all of the information that you need into the program so that you can actually modify it and run the search.

```
if the array is not sorted then sort it
```

This line will sort the data in the array so that it is in an order that can be compared easily, like a cost or number. This will mean that later when you test if the data is greater or lower, you will know that you are removing only data that will not be correct.

`start:`

This line just marks this section in the code with a name so that I can return to it later in the program.

`go to the middle of the array`

This line will set the location of the program to look at the middle of the array of numbers, if the length of the array is odd then the program should just go to either one of the middle values as it does not really matter.

`is the current item what you are looking for?`

This will check if the item in the array that you are currently looking at in the program is actually the one that you want. This will then lead off to two conditions depending on if you have found the value that you are looking for or not. If you have found the item that you are looking for then you are done, but otherwise the program carries on.

`....no: is the current item less than the item you are looking for?`

This line will run if the item that the program is on is not the value that you want, that is what the `no:` part means. Now this is where the clever part of the algorithm starts. If the value of the item that you want is lower than where you are at the moment then you can remove all of the items that are greater than the current item as it is less than this one so it is not higher. This is why the array was sorted to start off with as now you can just remove the half of the array that is higher, and just like that you have halved the values to search.

`.....yes: remove all items in the array that are lower and return to start.`

If the item that you are on is lower than what you want then you can remove all of the items that are lower and this is what this section does. After you have removed the values then you can return to the `start:` marker of the program and recur it again.

`.....no: remove all items in the array that are greater and return to start.`

If the item that you are on is greater than what you want then you can remove all of the items that are greater and this is what this section does. After you have removed the values then you can return to the `start:` marker of the program and recur it again.

`....yes: you are done as you have found the item you are looking for, end the program.`

If the value that you have at the middle of the selection is the number that you are looking for then you have found it and can see what information it has that you need. Now you just get that information and save it and end the program and you are done. This is the terminal condition.

7 Use of Number Systems

In this section I will go through how binary, octal and hexadecimal are used and are applied in areas of computing with how they are used and how they work. For this I will be showing how binary is used with ASCII, how hexadecimal is used with MIME and how Octal is used with Unix file permissions.

7.1 binary - ASCII

One way that binary is used in computer science is in the use of ASCII. ASCII stands for the American Standard Code for Information Interchange and this is the standard that is used for how the standard set of English characters and numbers are encoded in binary on the computer and how it deals with them. With ASCII the letter "a" for example

8 Network Planning

9 Number Systems calculations